GUIDANCE AND STRUCTURE IN MATHEMATICS INSTRUCTION: HOW MUCH GUIDANCE DO STUDENTS NEED? AN INTERVENTION STUDY ON KINDERGARTEN MATHEMATICS WITH MANIPULATIVES

by

ERIN MARIA HORAN

(Under the Direction of Martha M. Carr)

ABSTRACT

The use of manipulatives, such as fingers, blocks, or coins, has been shown to positively impact students' learning of mathematics. Recent research has pointed out that the efficacy of learning with manipulatives is affected by multiple variables, including the amount of guidance teachers provide during learning. Guidance and structure have both been linked to higher achievement but the two terms are not clearly defined and, thus, are used interchangeably. This dissertation aimed to solve two issues. First, to clearly define guidance and distinguish it from structure. Second, an experimental intervention examined the optimal level of guidance during kindergarten mathematics instruction with manipulatives. Results showed there was no difference in learning across the conditions, even after controlling for pretest performance. These results provide valuable information to teachers on the areas of mathematics that do not require high guidance. Practical implications and areas of future research are discussed.

INDEX WORDS: guidance, mathematics, manipulatives, elementary school

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DEDICATION

I dedicate my dissertation work to my family; especially Janet, Jimmy, Colleen, Alanna, Shannon, and Kiera; and friends of the past, present, and future. I am fortunate to be surrounded by people who fuel my passion to continuously grow and learn. Thank you.

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Purpose of the Study

Teachers have a tremendous amount of work to do and little time to do it. Teachers can spend significant amounts of time providing guidance to students with the goal of improving learning. Knowing how much guidance to employ when teaching is essential information for teachers. When teachers have information about when it is best to use guidance during instruction, their lesson planning and instruction can be more efficient. This is because guided instruction requires a substantial amount of preparation and a higher level of expertise than less guided activities, such as independent seat work or computer instruction (Gerard, Matuk, McElhaney, & Linn, 2015). Although teachers are often encouraged to provide plenty of guidance the problem is we do not really know when guidance is best employed during instruction.

One reason for this problem is that guidance as a construct is not consistently defined and, as a result, research on guidance has not provided a clear picture of its effects. Research that uses the term guidance can include guidance as student-teacher interaction (e.g., Terwel, van Oers, van Dijk, & van den Eeden, 2009; Mayer, 2004) or the structured use of materials and information (e.g., Baroody, Purpura, Eiland, & Reid, 2015; Chen, Kalyuga, & Sweller, 2015), which are quite different. To understand the effects of guidance on learning it is necessary to better define guidance. Specifically, we must separate guidance as student-teacher interaction from guidance as structure.

In this dissertation, I discuss the research on guidance and differentiate the construct of guidance into two separate categories: guidance and structure. In manuscript one, chapter two, we will first describe the diversity of studies on guidance and the implications of a lack of a clear definition of guidance. Then, we will propose a clear definition of guidance that distinguishes it from structure, as well as a clear definition of structure. Finally, we will review the research in elementary school mathematics that uses the term “guidance” and meets our revised definition of guidance. We will then review the research that uses the term “guidance” that meets our definition of structure.

After clearly distinguishing guidance from structure, I implemented an experimental intervention with kindergarten students to investigate how much guidance students need when learning with manipulatives. The utility of manipulatives to support learning has been widely accepted and recommended (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, investigations by Carbonneau and her colleagues (Carbonneau & Marley, 2015; Carbonneau, Marley, & Selig, 2013) have shown the efficacy of learning with manipulatives is not consistent and depends on many variables of the instruction, including the level of guidance (e.g., high guidance or low guidance) during instruction and the student’s prior knowledge. There is evidence that at least some guidance during mathematics instruction is necessary for optimal learning but the literature is unclear as to when teachers should provide guidance and when they should allow students to practice alone without teacher help. In the study described in manuscript two, chapter three, I examined students’ learning with manipulatives (pennies and nickel strips) with varying levels of guidance. I implemented an experiment in which the amount and timing of guidance with manipulatives was tested using four conditions.

Literature Review

The lack of a clear definition of guidance was illuminated in a 2006 article by Kirschner, Sweller, and Clark as well as the subsequent commentary articles. In their review, the authors argued that instruction that involved low guidance is less effective than instruction that involved high guidance for improving mathematics learning. Kirschner and his colleagues (2006) classified low guidance instruction broadly, including studies that implemented constructivist, discovery, problem-based, experiential, and inquiry-based learning. These studies were classified as low guidance because they required the learner to construct and discover information independent of the teacher. It was concluded that low guidance instruction does not provide novice learners with sufficient information to set up and solve problems alone. This article prompted multiple responses that pointed out the considerable variability in the amount of guidance in the studies categorized as low guidance and subsequent problems with the conclusions.

Other articles have defined guidance broadly often including a range of instructional characteristics. Alfieri, Brooks, Aldrich, and Tenenbaum (2010) conducted two meta-analyses investigating the effectiveness of discovery learning. The first meta-analysis compared unassisted discovery learning, defined as no guidance or feedback during learning, with explicit instruction, described in the meta-analysis as explicit training or instruction. In their second meta-analysis, Alfieri and his colleagues (2010) compared enhanced discovery learning, defined as discovery learning with guidance eliciting self-explanation, with other types of discovery learning. As with the work of Kirschner and his colleagues, both of these meta-analyses involved unclear, vague definitions of guidance that collapsed very different forms of instruction into single categories (e.g., treating teacher scaffolded instruction the same as elicited student explanation). Baroody and his colleagues (2015) created categories of guided discovery learning, which included highly guided, moderately guided, minimally guided, and unguided. The levels differed in the amount of guidance and structure including scaffolding, organization, and feedback. This made it difficult to determine the impact of student-teacher interaction in comparison to the impact of organization and materials on student outcomes. Thus, as defined in the current literature, guidance is poorly defined and confounded with structure.

Because of the absence of a clear, common definition of guidance, it is difficult to determine the efficacy of guidance. The first purpose of this dissertation is to clearly define and differentiate guidance and structure, which have been confounded. The second purpose is to review the research literature on guidance and structure as defined here. After clearly defining guidance I implemented an intervention to study the effect of different levels of guidance for students learning with mathematics manipulatives.

Manipulatives refer to any concrete materials, objects, or drawings used during instruction to support students' learning of number and operations. Manipulatives can be simple, such as counting on fingers or unit blocks, or complex, such as using base ten sticks and blocks. In elementary school mathematics classrooms, students learn to count using individual manipulatives to determine "how many" (National Research Council, 2009). Later, students move on to complex manipulatives that represent values of the base-ten system. In elementary school, manipulatives are incorporated into mathematics curricula to aid students' mathematics reasoning and problem solving skills (e.g., Expressions, Investigations, Saxon).

Research studies have shown that guidance can affect the usefulness of manipulatives. For example, Terwel, van Oers, van Dijk, & van den Eeden (2009) found students learned how to represent percentages and graphs better with teacher guidance compared to learning alone.

Conversely, Fennema (1972) found that second graders taught with manipulatives actually performed worse on transfer tasks compared to typical instruction with a textbook. Given the inconsistent findings on the use of manipulatives during guided instruction we do not know much about when and with whom guided instruction is most effective.

In the study presented in chapter three I compared student performance on measures of mathematics achievement after one of four different five-day treatments that differed in the amount and/or timing of guidance. In the high guidance condition, students were taught with consistent high guidance for all five days. In the low guidance condition, students were taught with low guidance for all five days. In the high to low guidance condition, students were taught with high guidance for the first two days, low guidance for the last two days, with the third day utilized as a transition day where the researcher limited the guidance but did not eliminate it until day 4. In the low to high guidance condition, students were taught with low guidance for the first two days, high guidance for the last two days, with the third day utilized as a transition day where the researcher added some high guidance questions and comments. This study specifically investigated four questions:

1. Which of the four approaches to teaching with manipulatives is best for improving elementary students' performance on the intervention content specific task?

Based on the meta-analysis by Carbonneau et al. (2013), high guidance is optimal for improving student performance on an intervention content specific task. Research has also shown support for low guidance at some point during instruction, but it is not clear if high guidance should be faded out or if it should come after low guidance instruction. Therefore, we predicted that one of the transitioning conditions (high to low or low to high) would be best.

2. Which of the four approaches to teaching with manipulatives is best for elementary students' performance on a transfer task?

Carbonneau et al. (2013) found that studies that implemented low guidance interventions with manipulatives had higher effect sizes for transfer than the studies that implemented high guidance interventions with manipulatives. On the other hand, students in low guidance instruction may not learn at all, and may need guidance from the teacher in order to learn not just the material, but enough to be able to transfer to another task. Prior studies have found lower achieving students need more guidance to understand the content in order to transfer knowledge (Tournaki, 2003). First, we predicted the consistently low guidance condition would not be the optimal condition for transfer because not all students would be able to learn completely on their own. We predicted that the low to high and high to low guidance conditions would lead to better transfer because students will be given the opportunity to make meaningful connections on their own.

3. Which approach to teaching with manipulatives is best for improving number sense, as measured by the Test of Early Numeracy (TEN)?

The TEN can be considered far transfer and the same issues and predictions held for the impact of the different conditions on the TEN.

4. How does elementary students' pre-test knowledge affect which condition is best for learning?

It was hypothesized there would be an interaction effect. Students with low prior knowledge would perform best with consistent high guidance or high to low guidance in order to learn with manipulatives. If students are not given enough guidance to start with they may learn information incorrectly or may not know where to begin when exploring with manipulatives alone. Students with high prior knowledge may need consistent low guidance or low to high guidance in order to learn with manipulatives. These students need time to explore alone and already have enough prior knowledge to do this effectively. Starting with high guidance may confuse students with high prior knowledge.

CHAPTER 2

A REVIEW OF GUIDANCE AND STRUCTURE IN MATHEMATICS INSTRUCTION[1]

[1] Horan, E.M. and Carr, M.M. To be submitted to *Instructional Science*.

Abstract

Guidance and structure have both been linked to higher achievement but the two terms are not clearly defined and, thus, are used interchangeably. While one researcher might describe a lesson as highly structured, another could describe that same lesson as highly guided. This makes it difficult to determine the effectiveness of guidance and structure for instruction. This also makes it difficult to determine the practical implications for interventions and how teachers should apply guidance and structure in their own classrooms. This paper defined guidance and structure and distinguished the differences between the two constructs in elementary school mathematics research. Guidance describes interactive and responsive student-teacher interactions during teaching while structure refers to the explicitness of the lesson plan, curriculum or materials. First, the problems that have stemmed from a lack of clear and consistent definitions for guidance and structure are described. Next, the two terms will be defined followed by a review of the research literature using these definitions. Limitations to the current research and implications for future research will also be discussed.

Keywords: instructional guidance, elementary school, mathematics, guidance, structure

Introduction

Teachers have a tremendous amount of work to do and little time to do it. Teachers can spend significant amounts of time providing guidance to students with the goal of improving learning. Knowing how much guidance to employ when teaching is essential information for teachers. When teachers have information about when it is best to use guidance during instruction, their lesson planning and instruction can be more efficient. This is because guided instruction requires a substantial amount of preparation and a higher level of expertise than less guided activities, such as independent seat work or computer instruction (Gerard, Matuk, McElhaney, & Linn, 2015). Although teachers are often encouraged to provide plenty of guidance the problem is we do not really know when guidance is best employed during instruction.

One reason for this problem is that guidance as a construct is not consistently defined and, as a result, research on guidance has not provided a clear picture of its effects. Research that uses the term guidance can include guidance as student-teacher interaction (e.g., Terwel, van Oers, van Dijk, & van den Eeden, 2009; Mayer, 2004) or the structured use of materials and information (e.g., Baroody, Purpura, Eiland, & Reid, 2015; Chen, Kalyuga, & Sweller, 2015), which are quite different. To understand the effects of guidance on learning it is necessary to better define guidance. Specifically, we must separate guidance as student-teacher interaction from guidance as structure.

In this review, we discuss the research on guidance and differentiate the studies into separate categories: guidance and structure. We will first describe the diversity of studies on guidance and the implications of a lack of a clear definition of guidance. Then, we will propose a clear definition of guidance that distinguishes it from structure, as well as a clear definition of structure. Finally, we will review the research in elementary school mathematics that uses the term “guidance” and meets our revised definition of guidance. We will then review the research on structure that meets our definition of structure.

A Variety of Definitions of Guidance

The lack of a clear definition of guidance was illuminated in a 2006 article by Kirschner, Sweller, and Clark as well as the subsequent commentary articles. In their review, the authors argued that instruction that involved low guidance is less effective than instruction that involved high guidance for improving mathematics learning. Kirschner and his colleagues (2006) classified low guidance instruction broadly, including studies that implemented constructivist, discovery, problem-based, experiential, and inquiry-based learning. These studies were classified as low guidance because they required the learner to construct and discover information independent of the teacher. It was concluded that low guidance instruction does not provide novice learners with sufficient information to set up and solve problems alone. This article prompted multiple responses that pointed out the considerable variability in the amount of guidance in the studies categorized as low guidance and subsequent problems with the conclusions.

Other articles have defined guidance broadly often including a range of instructional characteristics. Alfieri, Brooks, Aldrich, and Tenenbaum (2010) conducted two meta-analyses investigating the effectiveness of discovery learning. The first meta-analysis compared unassisted discovery learning, defined as no guidance or feedback during learning, with explicit instruction, described in the meta-analysis as explicit training or instruction. In their second meta-analysis, Alfieri and his colleagues (2010) compared enhanced discovery learning, defined as discovery learning with guidance eliciting self-explanation, with other types of discovery learning. As with the work of Kirschner and his colleagues, both of these meta-analyses involved unclear, vague definitions of guidance that collapsed very different forms of instruction into single categories (e.g., treating teacher scaffolded instruction the same as elicited student explanation). Baroody and his colleagues (2015) created categories of guided discovery learning, which included highly guided, moderately guided, minimally guided, and unguided. The levels differed in the amount of guidance and structure including scaffolding, organization, and feedback. This made it difficult to determine the impact of student-teacher interaction in comparison to the impact of organization and materials on student outcomes. Thus, as defined in the current literature, guidance is poorly defined and confounded with structure.

Because of the absence of a clear, common definition of guidance, it is difficult to determine the efficacy of guidance. This paper serves two purposes. The first is to clearly define and differentiate guidance and structure, which we argue have been confounded. The second is to review the research literature on guidance and structure as defined here. We limited this review to elementary school mathematics because it is likely that as students mature their need for guidance will change and the effectiveness of guidance is likely affected by subject domain. Limiting the scoop of the review avoided potential age confounds.

Guidance and Structure Defined

Guidance Defined

For this paper we base our definition of guidance in the social constructivist theory of Vygotsky (1962, 1978), which focuses on the co-construction of knowledge by teacher and student. Guidance is defined as the purposeful interaction between teacher and students, specifically, the amount of feedback teachers provide in response to students' questions and learning difficulties, the quantity and quality of teachers' responsiveness to students' questions and concerns, scaffolding provided by the teacher (i.e., not provided worksheets or materials), and how often teachers ask students questions that are designed to cause students to think more deeply with the goal of guidance being to enhance and support student learning. For the purposes of this article, guidance refers only to student-teacher interaction and not peer interaction or parent-student interaction, as the nature of these interactions is different from teacher-student interaction and beyond the scope of the current review. In addition, while nonverbal cues and gestures from the teacher could be considered aspects of responsive teacher guidance what research has been done does not exemplify the interactive nature of guidance as we have defined it.

Our definition of guidance aligns with Mayer's (2004) definition of guided instruction, as he distinguished between "pure discovery" learning and "guided discovery" learning, where guided discovery learning includes teacher provided guidance focused on the learning objective through hints, direction, coaching, feedback, or modeling. For the purposes of our review, we created four levels of guidance including high guidance, moderate-high guidance, moderate-low guidance, and low guidance. Table 2.1 shows the categories of the levels of guidance as we describe them here and the criteria for inclusion in that category. We categorized guidance as high when there is substantial student-teacher interaction where the teacher is responsive to students' learning needs during a lesson or during problem solving and in which teachers support the deep learning of concepts. Examples of high guidance instruction include a teacher monitoring student responses during problem solving and providing assistance as needed, teachers providing feedback and responding to questions from students, students responding verbally to questions from teachers, and teachers creating opportunities for reflection based on students' performance and needs. High guidance is effortful on the teacher's part. It requires full attention as teachers monitor student progress and respond to student needs during instruction.

We included two moderate levels of guidance to encompass studies that were neither high nor low guidance. Moderate-high-guidance included instances of corrective feedback in which the teacher could provide more information on why an answer is correct or incorrect, provided information on how to find the correct answer, or provided a correct answer. This differed from high guidance as high guidance included more elaborate feedback and included more in-depth conversation and student-teacher interaction focused on deep learning of concepts.

Table 2.1
Categories of Guidance with Examples

	Description	Example
High Guidance	Collaborative construction of knowledge, teacher and student involved; lots of dialogue; substantial interaction; supports deep learning	Combination of several: co-construction; monitoring and providing assistance; feedback and responding to questions; opportunities for reflection
Moderate-High Guidance	Some interaction but not as responsive to students' needs as high guidance; more feedback than just Yes/No, Correct/Incorrect; prompts for student to talk	Scripted co-construction; feedback + this is the correct answer or what do we do next
Moderate-Low Guidance	No more than simple feedback; no elaboration; nor prompts to student; minimal interaction	Accuracy feedback from the teacher
Low Guidance	No interaction; no intervention/dialogue other than to direct students to task	Solo work on worksheets, non-interactive lecture

Moderate-low guidance included instances of scripted feedback that provided nothing more than accuracy information to the student. Low guidance was indicated when the teacher and students were not actively engaged in discussion and problem solving, such as independent work on worksheets or non-interactive lecture. Examples of low guidance would be students working alone or teachers lecturing students with no opportunities for student questions.

Structure Defined

Whereas the definition of guidance focuses on the quantity and quality of student-teacher interactions during instruction, structure is defined here as the purposeful explicitness and organization of the lesson plan, curriculum, or materials. This is consistent with Miller's (1980) definition of structure as "the purposeful ordering or placement of people, materials, and resources in time" (p.163). This definition of structure separates instructional assistance through teacher provided verbal help and instruction (i.e., guidance) from scaffolds and information provided through materials. Research on structure typically compares conditions with the presence or absence of an instructional component, such as worked examples or ordered problems. As we will discuss in our literature review of structure, some forms of structure can be helpful during instruction while in other cases, structure is not necessary for learning.

Proponents of highly structured instruction (e.g., Bruner, 1960; Cobb 1995) highlight the need to support less mature working memory or limited prior knowledge through the materials being used. Highly structured lessons can include instruction in which problems are ordered from easy to difficult or worksheets that scaffold learning with steps or ordered tasks. Worked examples or formula sheets are also good examples of structure. Static, instructional components not influenced by students, such as directing them to certain aspects of the lesson are also examples of structure, as these instructions do not provide the same guidance as questions to students (Sidney & Alibali 2015).

High structure can also be computer-based instruction in which items are organized by difficulty, that provides hints, or that provides accuracy feedback. The research on computer instruction has grown tremendously with the development of more advanced, responsive technologies. Although computer programs that provide hints or feedback might seem to be guidance we included them as structure because computer instruction is based on preplanned responses to events and these responses are not as responsive as those of a teacher. As such these studies better fit our definition of structure.

Proponents of low structure (e.g., Piaget, 1977) argue that students benefit from less structure because it forces them to construct their own knowledge. Low structure, in this paper, refers to instruction without plans as described above. Items are not presented in any specific order and students must organize any information that is presented. Students are not provided scaffolding; such as hints or partially completed problems. Low structure could take the form of students working on worksheets that are not organized in any way by the teacher. For example, having students decide how to set up problems as opposed to being given the steps to setting up the problem would be an instance of low structure. Computer-based instruction that does not organize content based on difficulty is another example of low structure.

Distinguishing between high and low structure can be difficult. Most lesson plans include some level of structure, otherwise they would not be lesson plans at all. Typically, studies on structure compare conditions with two forms structure. For example, they could include worked examples versus no worked examples, or ordered problems versus randomly displayed problems. Therefore, in our review in structure, we describe conditions and distinguish between higher or lower structure when the distinction is clear within the study (e.g., giving worked examples versus not giving worked examples).

Method

Literature Search

The literature we identified came from searches within two major education databases: Educational Resources Information Center (ERIC) and PsycInfo. Several criteria were followed. First, we included studies in elementary school (i.e., up to grade 5) mathematics education. We limited the age range and topic area because the level of teacher guidance and lesson structure can have different effects depending on the subject, topic, and age group, making it difficult to draw conclusions. Three studies were kept that included sixth graders as they also included fifth grade students. Second, we only included studies with an experimental design that manipulated different levels of guidance or different levels structure, so case studies of new program implementations were not included. Articles on parent guidance or peer guidance were not included as these are different from teacher guidance, which is the focus of this paper. Finally, we only used studies that had some measure of mathematics learning, so studies focused solely on motivation or other variables were not of interest.

Our search was limited to English-speaking, peer-reviewed journals published after 1996. We implemented searches for guidance then searches for structure. In our first search for guidance the following key terms were used: "'mathematics' and 'guidance'" with the limiters "Elementary Education" for ERIC and "Childhood" for PsycInfo. Of the 111 articles this search brought up, 37 were irrelevant as they discussed topics such as test anxiety, parent guidance, motivation, achievement gaps, and vocational guidance; 24 investigated new curricula and programs; 14 investigated teacher education; 13 were not elementary school mathematics topics; eight were not experimental comparisons of guidance or structure; seven investigated methodology and test development; and one was a duplicate. Therefore, only seven were relevant to our research criteria. Next, we searched for "'mathematics' and 'guided and instruction' not 'guidance'" with the same limiters. Of the 76 articles this search brought up, 23 investigated new curricula and programs; 16 investigated teacher education, professional development, and views; 12 were irrelevant as they discussed topics such as achievement gaps, reviews of literature, and culturally relevant instruction; six were not elementary school mathematics topics; six were not experimental studies; three investigated methodology and test development; one was a duplicate; and one was not in English. Therefore, eight studies from the second search met our criteria and were relevant to our search criteria. Combined, the two searches of the guidance literature resulted in 13 studies that met our criteria. Of all 15 studies found in our search for guidance, seven studies were reclassified as structure based on our criteria for structure; they were reviewed with other research on structure.

In our search for structure we used the same databases and limiters with the terms "'structure' and 'mathematics' and 'achievement.'" This brought up 184 articles many of which were irrelevant because they focused on family or goal structure, so we added "NOT 'family structure' NOT 'goal structure'" to our search. Of the 167 articles this search brought up, 61 were related to topics other than mathematics achievement including school/class structure, student-teacher relations, motivation; 43 were related to psychometric research including factor structure of mathematics skills and test development/validation; 14 did not investigate elementary school mathematics (e.g., science, reading, older group); 13 were not in English; 10 compared conditions that manipulated different content or problem features not related to lesson structure; six examined family variables such as parent involvement; five were review articles or reports; four explored teacher variables such as pay and education; two were comparisons between the United States and other countries; and two investigated a program or curriculum without a comparison group. Therefore, seven fit our search criteria. Our literature review on structure includes the seven studies that initially used the term guidance but that we reclassified as structure, the seven studies that emerged from the search described above, and one study from our guidance search that also manipulated different levels of structure.

To organize our literature review, we first review studies that compared different levels of guidance. Next we review studies that compared different types of structure. In our review of structure articles, we denote articles found in our guidance search with an asterisk. One study by Terwel and his colleagues (2009) manipulated both guidance and structure. We include this article in both the guidance and structure sections.

Literature Review

Guidance Studies

The eight studies we identified as guidance used terms such as cooperative learning, guided inquiry, guided play, questioning, feedback, prompts, and co-construction techniques in their description of the teacher-student interactions. In each of these studies, the authors compared at least two conditions that differed in their levels of guidance. Below we discuss these studies, focusing on how the authors described their conditions and classifying the conditions as high, moderate-high, moderate-low, or low guidance based on our own criteria-based categories of guidance.

Terwel, van Oers, van Dijk, and van den Eeden (2009) compared the impact of two problem solving lessons on student learning of percentages and graphs. In the condition they labeled as high guidance, fifth grade students were taught through the process of guided co-construction; students and teachers created representations of the percentages through teacher-initiated, guided discussions. In the low guidance condition students were provided with ready-made, completed representations, but were not engaged in discussion with the teacher. This mapped onto our low guidance category. Controlling for pretests scores, children in the high guidance condition performed better on a posttest and transfer test. This provided support for guided, interactive teaching when students are learning problem solving strategies for percentages and graphs.

Sengupta-Irving and Enyedy (2014) also compared a high guidance, interactive condition to a low guidance condition. Specifically, the study compared instruction labeled as “guided” to instruction labeled as “unguided” with fifth grade students learning data analysis and statistics. In their guided condition the teacher led students through the problem solving process with the teacher defining the problem and then leading students through the problem via interactive discussion. In their unguided, open approach, students completed the problem without any assistance from the teacher. They found no difference in learning outcomes between conditions, students in both conditions showed statistically significant gains in knowledge from a pretest to posttest on data analysis and probability.

Similar to the first two studies, Fisher, Hirsh-Pasek, Newcombe, and Golinkoff (2013) described guided instruction as a collaborative construction by students and teachers. In their study, Fisher et al. (2013) taught preschool students properties of shapes in three conditions: free play, where student activity was self-directed with no goals for learning, guided play, which was described as discovery learning with the presence of an active teacher participant, and didactic instruction, where the student observed the instructor talking through the material. For this study the free play was categorized as low guidance instruction as the teacher played no role. The guided play and didactic instruction conditions included the exact same script, however, they were different in that the guided play included the prompts and questions directed at the student during the lesson (moderate, high guidance) while in the didactic instruction the teacher read the script and answer questions while the student watched rather than asking the student for input (low guidance). The authors found that students in the guided play showed improved understanding of shapes over the other two conditions, and those improvements were still observed one week later. These findings showed that for understanding properties of shapes, moderately high guidance, even when scripted, was better than instruction that involved the student passively listening to the teacher or playing alone without any guidance.

Carbonneau and Marley (2015) also worked with preschool students to compare the impact of different levels of guidance. The study investigated the impact of guidance on students' conceptual and procedural knowledge on a quantity discrimination task (which side has more) with manipulatives. In the high guidance condition, the researcher would make two piles of objects and the child would have a crocodile mouth with instructions that the crocodile should eat the bigger number. After making the piles the researcher would ask, "Which way should this symbol go?" followed by prompts to the student to read the number sentence. Then, the researcher would read the number sentence after the child and correct them if any errors were made. The low guidance condition was the same but the teacher did not read the number sentence after the child or correct them for errors. Based on these descriptions, this study compared two conditions of moderate, high guidance. The only difference was the authors' high guidance condition included a slightly higher level of guidance. Carbonneau and Marley (2015) found that students in the higher guidance condition improved their conceptual and procedural knowledge more than students in the condition without the extra feedback as guidance.

Fyfe, Rittle-Johnson, and DeCaro (2012) examined the impact of prior knowledge on feedback during problem solving. Low performing second and third grade students worked on problem solving on mathematics equivalency problems with one of three types of feedback: no feedback, outcome feedback (i.e., correct or incorrect answer), or strategy feedback (i.e., correct or incorrect strategy used). All conditions were followed with brief, conceptual instruction. Based on our definitions, the no feedback condition was categorized as low guidance because there was no feedback or interaction between the teacher and student, while the outcome and strategy feedback conditions were both categorized as moderate-low guidance because they received nonelaborative feedback on performance. The authors found that performance interacted with prior knowledge. Students with low prior knowledge in the two feedback conditions (moderate-low guidance) performed significantly better on a procedural knowledge posttest compared to the no feedback (low guidance) condition. Students with moderate prior knowledge in the no feedback (low guidance) condition performed significantly better on a procedural knowledge posttest compared to the feedback (moderate-low guidance) conditions. These results suggest that students with no prior knowledge benefit from some guidance, but this is not the case for students with moderate prior knowledge.

Building on this work, Fyfe, DeCaro, and Rittle-Johnson (2015) examined the impact of working memory on feedback on equivalency problems. In their study, they gave second and third grade students accuracy only feedback (moderate-low guidance) or strategy feedback (moderate-low guidance). Fyfe and her colleagues (2015) found students with lower working memory benefitted more from accuracy feedback than strategy feedback, while students with higher working memory benefitted from both types of feedback. These results show that, like prior knowledge, working memory affects students' ability to utilize different types of feedback. In this case, although both forms of feedback were categorized as moderate-low, the low working memory group did not benefit as much from the strategy feedback.

Kroesbergen and Van Luit (2002) compared guided instruction to structured instruction of multiplication for low-performing students in both special education and regular education classes. The authors defined guided instruction as teacher supported learning through the use of teacher-generated questions and problems selected based on the students' performance during the lesson. Structured instruction, in contrast, involved teachers following a clear lesson plan in a prescribed order for each lesson, but without opportunities for the teacher to prompt learning through questions or adjusting the instruction based on performance. Based on these descriptions, we categorized the guided instruction as high guidance and structured instruction as low guidance teaching. The study also implemented a control condition, which involved no extra instruction to their regular mathematics curriculum. Students in both treatment conditions improved on their problem solving skills, however, the high guidance instruction condition resulted in more improvement than the low guidance condition for low performing students, especially students in regular education classes. Students in special education classes improved more in the low guidance condition in comparison to the high guidance condition, which may be the result of the cognitive load and is consistent with the work of Fyfe and her colleagues (Fyfe et al., 2015).

In another study, Kroesbergen and Van Luit (2005) taught elementary school students (age not specified) with mild intellectual disabilities strategies for multiplication over four months. They compared a high guidance condition (labeled guided instruction) with a moderate-low guidance condition (labeled direct instruction). The high guidance condition involved discussions between the instructor and students on multiplication solution procedures. Specifically, the teacher focused on learning difficulties students were having and initiated discussions with the students by asking and answering questions. The moderate-low guidance instruction involved the teacher instructing students on multiplication solution strategies, but students did not have the same opportunities to ask and answer questions. Teachers could give feedback about the correctness of the strategy being used but no other feedback was given. The authors found that students in both conditions improved on measures of multiplication automaticity and ability, but students in the low guidance condition showed greater improvement, supporting earlier findings that direct instruction involving less guidance was advantageous when teaching students with mild intellectual disabilities.

Summary. Of the eight studies discussed three studies (Terwel et al., 2009; Fisher et al., 2013; Carbonneau & Marley, 2015) indicated more guidance was better than less guidance, while one study Sengupta-Irving and Enyedy (2014) found no differences in learning as a function of guidance. The remaining four studies pointed to the need to consider a number of variables as moderators of the impact of guidance. Kroesbergen and Van Luit (2005) found low guidance was better than high guidance instruction for students with mild intellectual disabilities. Kroesbergen and Van Luit (2002) likewise found that students in special education classes benefited more from low guidance than high guidance, however they found that low performing students not identified as having a learning disability benefited more from high guidance. These findings indicated that students with learning disabilities may have characteristics that differentiate the impact of guidance on learning. This needs to be further explored in the research literature. In regard to prior knowledge, students who lack background knowledge seem to benefit from guidance whereas students with some background knowledge actually were hurt by guidance (Fyfe et al. 2012). Further, Fyfe and colleagues (2015) found that working memory moderated the effectiveness of the form of feedback on learning. While some studies indicated the usefulness of guidance for learning, teachers should consider the ability level of the child, their prior knowledge, and working memory capacity. These and likely other variables appear to moderate the effectiveness of guidance.

Structure Studies

The studies we discuss below implemented interventions of various types of structure. Seven were found in our search for structure, while seven were found in our search for guidance and were recategorized based on our definition. Research by Terwel et al. (2009) manipulated both guidance and structure, so we have included this study with our structure studies, in addition to the guidance section above. We identify studies found in our search for guidance with an asterisk. A number of interventions were used in these articles including worked examples, presentation of formulae or information, computer feedback, or sequential arrangement of problems (e.g., grouping similar problems together). Below we review the research on these different forms of structure including structure of solutions, structure of sequencing, structure of materials and structure of learning environments. We distinguish between high and low structure whenever possible, but we review articles that compare different types of structure (e.g., group versus individual work) that do not neatly fit into high and low structure categories.

Structure of information for problem solving. Five studies investigated structure as the information or instructions provided to students for their problem solving. Chen, Kalyuga, and Sweller (2015)* compared two forms of structure for learning geometry, worked examples (high structure) and generating formulas for problem solving (low structure). Worked examples included all of the steps needed to solve a problem so students could solve similar problems. This fits our definition of high structure because it provided students with materials to aid with problem solving. The comparison group for this variable was considered an instance of low structure because students were not given worked examples. The authors were specifically interested in determining if providing worked examples leads to better learning than having students generate formulas during problem solving (low structure) rather than being provided formulas. Therefore, they also presented half of the students with geometry formulas, compared to the other half of students who were not presented the formulas. These two variables created four conditions (worked examples + formulas; no worked examples + formulas; worked examples + no formulas; no worked examples + no formulas). The authors found that the optimal form of structure was dependent on element interactivity (i.e., cognitive load as a combination of the complexity of problems and learner expertise). Worked examples were optimal for problems high in element interactivity, while creating formulas (versus being presented with formulas) was optimal for materials low in element interactivity. The authors also found that as expertise increased, having students generate formulas was found to be optimal over and above providing worked examples. From this study, we can conclude that high structure in the form of worked examples is useful for non-experts solving complex geometry problems, while low structure in the form of not being presented with formulas is useful for experts solving less complex geometry problems.

Timmermans, Van Lieshout, and Verhoeven (2007)* worked with low-performing fourth grade students on their subtraction problem solving. In their “guided” instruction condition, which we categorized as low structure, students were left to their own devices to develop multiple strategies while solving subtraction problems, i.e., no structure was provided in the form of order or selection of strategies. In the direct instruction condition, which we categorized as high structure, students were trained on one strategy for solving subtraction problems. Overall, they found no difference in gains from pretest to posttest between conditions. The authors concluded that their guided instruction condition was not a satisfactory alternative to typical direct instruction for teaching low performing students.

As discussed above in our guidance section, Terwel et al. (2009)* compared the impact of two problem solving lessons on student learning of percentages and graphs. They effectively compared high and low guidance but also compared high and low structure. In the first condition, fifth grade students were taught through the process of guided co-construction, where students and teachers created representations of the percentages through teacher-initiated, guided discussions (low structure). In the second condition students were provided with ready-made, completed representations, but were not engaged in discussion with the teacher (high structure). Controlling for pretests scores, children in the co-construction condition performed better on a posttest and transfer test than students in the ready-made condition. While earlier we stated that this provided support for guided, interactive teaching when students are learning problem solving strategies for percentages and graphs, it also provides support for the construction of graphic representations over being provided ready-made representations.

Sidney and Alibali (2015) compared two forms of structure for the division of fractions: one condition explicitly asked students to link practice problems to previously practiced problems; another condition did not explicitly ask students to link the problems to their practice problems. We categorized the linking as high structure and the non-linking as low structure. All students learned fraction procedures equally well, but students in the non-linking, low structure group performed better than students in the linking, high structure group on items that assessed conceptual knowledge. The authors proposed that prompting students to link information without providing any sort of parameters may cause more problems for learning than not asking students to link at all.

Fyfe and Rittle-Johnson (2016)* investigated the impact of computer feedback on learning equivalency problems for second grade students. There were three conditions within computer based problem solving: no-feedback; immediate, correct answer feedback after each problem; and summative, correct answer feedback after all 12 problems were solved. We categorized these conditions as low structure, high structure, and high structure, respectively. In their analyses, students were grouped as having high or low prior knowledge. The impact of feedback structure differed as a function of prior knowledge. Students with lower prior knowledge, performed better in the feedback conditions than no feedback conditions on solving equivalency problems. For students with higher prior knowledge, all conditions resulted in improvement on solving equivalency problems. These findings support the use of computer feedback during problem solving, especially for students with lower prior knowledge.

To briefly summarize the types of structure in this section, the five articles that compared different forms of structure in the form of information for problem solving used the following high structure conditions: provided worked examples, provided formulas for solving problems, provided strategies for solving problems, provided ready-made graphs, asked students to link similar problems together during problem solving, and provided computer feedback to students. Given the variety of the types of conditions utilized for structure, we can already notice the lack of overarching conclusions about structure we will be able to draw. The value of these studies is that within each study and each type of structure we notice there are differences the different structure conditions can have on learning. Looking to the articles in this section that were found in our search for guidance, we further highlight the problems that arise when we use the term guidance to describe materials during problem solving.

Structure as sequencing. Three studies implemented worksheets and problem solving activities in a prescribed order to encourage learning of mathematics rules or shortcuts. In one instance of this, Baroody, Purpura, Eiland, and Reid (2014)*compared the effectiveness of three conditions that differed in the order of problem presentation on students' fluency with two rules: subtract to add and add-with-10. The first condition, "guided" subtraction, was labeled as high structure because subtraction problems were ordered so that families of addition and subtraction problems were grouped together (e.g., 3+9 and 12-9); the second condition, "guided" use-a-10, was labeled as high structure because it presented problems in a prescribed order designed to show students how to use the add-with-10 strategy. A final third condition was a control group that did not place the problems in any specified order and was categorized as low structure for that reason. The authors found the subtract to add group outperformed both the use-a-10 and control groups on student fluency on unpracticed subtraction problems.

In a subsequent, similar study, Baroody et al., (2015)* focused on teaching K-2 grade students the add-1 rule (any number plus one is the next number) and the doubles rule (using doubles to compute answers to close numbers). The add-1 rule condition was categorized as high structure because problems were ordered so that students saw primarily x+1 problems. The doubles rule condition was categorized as high structure because the problems were ordered so that students saw primarily doubles problems (e.g., 8+8). The control group was categorized as low structure because there was no order to problem presentation. They found students learned the add-1 rule in both the ordered and random conditions but only learned the doubles rule in the ordered conditions.

Similarly, Purpura, Baroody, Eiland, and Reid (2016)* compared conditions for at-risk first grade students learning the add-1 rule and the doubles rule with first graders. In their high structure conditions, rather than ordering addition problems to induce understanding, the computer program included problems to highlight relations (e.g., "What number comes after 5) followed immediately by an addition problem (e.g., "5+1=?"). In their low structure, practice-only condition, addition problems were ordered randomly with no problems that highlighted relations. They found for learning the doubles rule, only the high structure condition was effective. Both conditions were effective for learning the add-1 rule, providing additional support that learning less salient number rules requires more than unstructured practice.

Regarding structure as sequencing, Baroody and colleagues (2014; 2015) and Purpura et al. (2016) found situations where the ordering of problems can affect learning, but may not be as necessary for learning salient rules for problem solving. As all three of these studies came up in our search for guidance, we again highlight the importance of accurately defining guidance and separating teacher guidance from computer programs and other materials for instruction.

Structure of materials. Tournaki, Bae, and Kerekes (2008) investigated learning with and without the use of a rekrenrek, an instrument similar to an abacus but with a base-five structure instead of a base-ten structure. Forty-five first grade students with mathematics disabilities were randomly assigned to one of two instruction groups or a third control group that received no instruction. Both instruction groups included counting songs, counting activities with manipulatives, counting comparison activities, and fact family activities (e.g., 5 has the fact families 4 and 1 or 2 and 3). The only difference between the instruction groups was the use of the rekrenrek in a higher structure group. Students in the rekenrek instruction group performed significantly better than the no rekrenrek condition and the control condition on posttest addition and subtraction problems, but the lack of pretest makes it difficult to determine whether the use of rekenrek was the cause. There was no difference between the fingers-only group and the control group. This provides support for the use of an additional manipulative in the form of a rekrenrek.

Tsang, Blair, Bofferding, and Schwartz (2015) compared structure as different uses of a number line. The authors compared three conditions for fourth grade students counting positive and negative integers on a number line. One condition had students jump a figure along a number line, one condition had students stack blocks along a number line, and one condition had students fold the positive and negative sides of a number line together to emphasize cancelling out when problem solving. The authors found that students in the folding condition showed evidence of incorporating symmetry into their mental representations of integers and performed higher on transfer tasks compared to students in the other two conditions.

Moreno and Duran (2004)* compared whether adding verbal instructions to graphic representations of problem solving on a number line improved the performance of fifth and sixth graders' problem solving. All students in the study had low prior knowledge on solving addition and subtraction problems. The authors also assessed computer experience, as this was thought to potentially impact the effectiveness of any computer based instructional program. In the study, students were shown a -9 to +9 number line with a bunny that moved along the line. For each problem, the problem solution was shown by moving the bunny along the number line. One of the two conditions included the addition of a verbal explanation (higher structure). Students with high computer experience who received verbal explanations performed higher than all other students on the posttest.

Kaminski and Sloutsky (2013) also investigated different types of information during instruction but were interested in the effect of extra, irrelevant information in problems. They compared kindergarten, first, and second grade students' performance reading bar graphs after instruction that included extraneous information (e.g., extra designs within the bars) to no extraneous information. Regarding structure, we categorized the extra information condition as lower structure, as it detracted from the structure of the study by adding information. Thus, this extra information is not purposeful organization towards enhancing student learning. The authors found students who learned without the extra information learned more from pretest to posttest compared to students learning with the extra information.

The research on structure as materials investigated structure as the inclusion of a manipulative, uses of a number line during problem solving instruction, the addition of verbal instructions during computer instruction, and a lack of extra information within problems during learning. To summarize, Tournaki et al. (2008) found the use of a manipulative was better than using fingers alone, Tsang et al. (2015) found folding a positive/negative number line improved learning over other uses of a number line, Moreno and Duran (2004) found the addition of verbal instructions while watching problems be solved on a number line increased performance for students who were familiar with computer learning environments, and Kaminski and Sloutsky (2013) found extra information on bar graph reading can hinder performance. As with our first structure section on structure as information for problem solving, we again highlight the broad range of structure, and the attached problem of using guidance for these varied types of instruction.

Structure of learning environment. Our final section on structure encompasses the structure of the learning environment, which we use to describe studies that investigate computer versus teacher mediated instruction and group versus individual instruction. Leh and Jitendra (2013) compared supplemental computer mediated problem solving with supplemental teacher-mediated problem solving on word problems for third grade students struggling in mathematics. The computer mediated lesson included hints and corrected errors for the students and there was no student-teacher interaction, while the teacher-mediated lesson had teachers read a problem and model the problem-solving process using think-alouds. Both were highly structured because the computer instruction personalized problems for students to motivate learning (e.g., baseball problems if they are interested in baseball) and the teacher instruction included planned steps and worked examples. The study found no significant change from pretest to posttest and no group differences between the conditions.

Roschelle and colleagues (2010) also investigated computer feedback on fourth grade students' learning of fractions. They compared two computer programs, one that provided feedback to individual students during problem solving, and one that provided feedback to groups of students problem solving together. The mathematical topics in both programs were aligned, so the only difference was the group or individual feedback. Comparing these two conditions in terms of high and low guidance is difficult, so we only differentiate them as different types of structure, rather than different levels. Students who problem solved with the group based computer program learned significantly more than the individual problem solving group as measured by students' gain scores from a pretest to posttest on fractions. The authors attributed the increased learning to the cooperative activities students could engage in during structured group work, which provided support for social processing of content.

Fuchs and colleagues (2000) investigated structure in two forms by comparing (a) the effectiveness of students working in groups versus working in pairs and (b) the effectiveness of different background structures (individual, collaborative, or collaborative with structure) taught to students on how to work in their groups or pairs. There were three conditions. The individual condition received no instructions for groupwork, the collaborative condition gave instructions on how to participate in group work (e.g., stay on task, participate), and the collaborative with structure condition gave additional instructions on the roles for group work (e.g., reader, monitor, checker, and writer). For our categorizations of high and low structure, we classified collaborative with structure as the highest structure, collaborative as high structure, and individual as low structure. Third and fourth grade classrooms were assigned to conditions in a 2x3 design (pairs/groups and background structure condition). The authors found that regardless of prior mathematics achievement, pairs earned higher scores than small groups on participation, helpfulness, cooperation, quality of talk, and performance assessment. The background structure training (individual, collaborative, and collaborative with structure) had no impact on student performance.

Regarding the structure of the learning environment, the three studies discussed investigated computer versus teacher mediated instruction, group versus individual computer feedback, and group versus individual problem solving with different instructions for collaborating in the group conditions. Leh and Jitendra (2013) found no difference between computer-mediated problem solving and equally structured teacher-mediated problem solving. Roschelle and colleagues (2010) found computer provided group feedback was better for learning than computer provided individual feedback. Fuchs et al. (2000) found that training students for group learning activities did not affect learning compared to group learning without training. As we have already discussed up to this point, these are valuable findings for each of these types of structure but, again, these studies are not close enough in content to allow us to draw overall conclusions about high or low structure.

Summary

Interestingly, many studies found support for using less structure over more structure or at least found no difference when comparing high versus low structure. Like guidance, we found prior knowledge and cognitive load impacted whether certain levels of structure were optimal over other forms of structure. These findings point to the importance of studies on structure to determine when extra planning may help students learn and when it is not necessary. As we have discussed in each section of structure, drawing overall conclusions about high versus low structure is not possible and, further, may not even be practical given the wide range of variables "structure" can encompass. One conclusion we can draw comes from our section on structure as sequencing. Baroody and colleagues (2014; 2015) and Purpura et al. (2016) found situations where the ordering of problems positively impacted learning, but this was not found for learning salient rules for problem solving. A second conclusion we can draw relates back to the purpose of our paper, which is to differentiate guidance from structure. We see from this section the research on structure encompass many types of variables. Separating this group of research from research on teacher guidance is important for allowing us to effectively research teacher guidance and move the research forward to draw conclusions and expand our knowledge on guidance.

Conclusion

The goal of this review was to understand how guidance impacts learning. Kirschner, Sweller, and Clark's (2006) review of "low guidance" in the literature, and the commentary articles that followed it, highlighted the lack of a clear, consistent definition of guidance, with discrepancies on what constitutes guidance (e.g., teacher help, materials) and, relatedly, what types of teaching constitute high versus low guidance. Therefore, to accomplish this goal, we first needed clear definitions of guidance and structure.

We then searched for articles that manipulated different forms of guidance and found we had to separate articles that investigated teacher guidance from other constructs present during instruction, which we labeled as structure. Based on our literature review of articles on elementary school mathematics education, we found there are multiple variables at play when determining how much guidance to provide to students. For example, when teaching mathematics to students with mild intellectual disabilities, direct instruction was found to be more effective than a lesson with more guidance and discussion based instruction (Kroesbergen & Van Luit, 2005), but high guidance was more effective than low guidance for low performing students (Kroesbergen & Van Luit, 2002). Related to student variables, Fyfe and her colleagues (2012, 2015) found that prior knowledge and working memory also moderated the impact of high and low guidance on learning. Previous research has shown working memory and prior knowledge influence student learning during mathematics instruction and problem solving, and may require extra attention in the classroom (Alloway and Gathercole, 2008). As we found in our review, this support can be accomplished by attending to guidance or structure. Based on our review, it is important to continue research on guidance so we can work to provide specific recommendations about guidance in different subject domains and with different students.

The second component of our review focused on the structure of mathematics lessons. The structure of lessons can encompass a wide variety of features including worked examples, ordering problems from easy to difficult, or providing formula sheets during problem solving. As such, the articles we reviewed covered a variety of studies, so we did not attempt to draw generalizable conclusions about structure. While this may be a limitation of our literature review, we feel this was important for accomplishing our review of teacher guidance and to highlight the variety of uses of the term guidance. Additionally, reviews of different components of structure can be easily accomplished (e.g., by specifically searching for worked examples or ordered problems), but a full review of each component of structure is beyond the scope of this review.

Limitations

To review guidance and structure with a clear, concise methodology, we greatly limited the studies reviewed to elementary school, experimental studies on mathematics. Additionally, we only reviewed articles that explicitly described the instruction in all conditions to determine if they effectively compared two different levels of guidance. Consequently, this may have limited our ability to find articles that investigated guidance per our definition and categorizations. There may be more studies that effectively compared different levels of guidance, but due to the lack of consistent terminology in the literature, finding these articles, even with a clear methodology, would be impossible.

Another limitation of our review relates to the classification of conditions as high, moderate-high, moderate-low, and low guidance. As is the case with defining discovery learning methods (c.f., Alfieri et al., 2010), classifying instruction as high or low guidance or high or low structure may be based more on the comparison group in a study than by the teaching methods. Our four categories allowed us to accomplish this as best as we could based on the studies we reviewed. As more studies investigate guidance, there may be a need to add categories that effectively describe small differences between levels. This will allow us to continue to make comparisons of conditions within studies and comparisons of conditions between studies.

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CHAPTER 3

HOW MUCH GUIDANCE DO STUDENTS NEED? AN INTERVENTION STUDY ON KINDERGARTEN MATHEMATICS WITH MANIPULATIVES[2]

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Abstract

The use of manipulatives, such as fingers, blocks, or coins, has been shown to positively impact students' learning of mathematics. Recent research has pointed out that the efficacy of learning with manipulatives is affected by multiple variables, including the amount of guidance teachers provide during learning. However, there is no consensus on how much guidance is necessary when learning with manipulatives. The goal of this study was to examine the optimal level of guidance during instruction with manipulatives. The focus was on the timing and the level of guidance needed for mathematics understanding in kindergarten. The researcher implemented different levels of guidance in a lesson on counting from one to 10 with pennies and nickel strips. Kindergarten students were taught over five consecutive days in one of four conditions: high guidance, low guidance, high guidance that transitioned to low guidance, and low guidance that transitioned to high guidance. Analysis of variance and hierarchical linear regression were used to compare the students' performance between the four groups on a content related task comprised of counting to ten with pennies and nickel strips, a transfer task comprised of counting on from five, and a number sense battery (Test of Early Numeracy). Results showed that there was no difference in learning across the conditions, after controlling for pretest performance. These results provide valuable information to teachers on the areas of mathematics that do not require high guidance. Practical implications and areas of future research are discussed.

Introduction

The utility of manipulatives to support learning has been widely accepted and recommended (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, investigations by Carbonneau and her colleagues

(Carbonneau & Marley, 2015; Carbonneau, Marley, & Selig, 2013) have shown the efficacy of learning with manipulatives is not consistent and depends on many variables of the instruction, including the level of guidance (e.g., high guidance or low guidance) during instruction and the student's prior knowledge. There is evidence that at least some guidance during mathematics instruction is necessary for optimal learning but the literature is unclear as to when teachers should provide guidance and when they should allow students to practice alone without teacher help. In the study described below, we examined students' learning with manipulatives (pennies and nickel strips) with varying levels of guidance. We implemented an experiment in which the amount and timing of guidance with manipulatives was tested using four conditions.

Manipulatives refer to any concrete materials, objects, or drawings used during instruction to support students' learning of number and operations. Manipulatives can be simple, such as counting on fingers or unit blocks, or complex, such as using base ten sticks and blocks. In elementary school mathematics classrooms, students learn to count using individual manipulatives to determine "how many" (National Research Council, 2009). Later, students move on to complex manipulatives that represent values of the base-ten system. In elementary school, manipulatives are incorporated into mathematics curricula to aid students' mathematics reasoning and problem solving skills (e.g., Expressions, Investigations, Saxon).

Research studies have shown that guidance during learning can affect the usefulness of manipulatives. For example, Terwel, van Oers, van Dijk, & van den Eeden (2009) found students learned how to represent percentages and graphs better with teacher guidance compared to learning alone. Conversely, Fennema (1972) found that second graders taught with manipulatives actually performed worse on transfer tasks compared to typical instruction with a textbook. Given the inconsistent findings on the use of manipulatives during guided instruction we do not know much about when and with whom guided instruction is most effective. In the study presented here, we examined students' learning with manipulatives with varying levels of guidance.

For the purposes of this study, guidance is defined as the interaction between a teacher and students, specifically, the quantity and quality of teachers' responsiveness to students' questions and concerns, and teachers' tendency to promote reflection and critical thought with questions and comments. Examples of high quality interaction include a teacher monitoring student responses during problem solving and providing assistance as needed, teachers providing feedback and responding to questions from students, students responding verbally to questions from teachers, and teachers creating opportunities for reflection based on students' performance and needs. In contrast, simply providing performance feedback (i.e., correct or incorrect) that is not responsive to students' needs would be considered low guidance.

Research on Manipulatives, Guidance, and Prior Knowledge

In this section we review research on the effectiveness of manipulatives, guidance, and prior knowledge for learning mathematics. While research in general provides support for implementing instruction with manipulatives in the classroom, the research on guidance, and especially guidance with manipulatives, is less clear. Further, understanding how prior knowledge impacts guidance and manipulatives is difficult. Therefore, we first describe the overall findings from research on manipulatives, followed by research on guidance for mathematics learning, and finally the research on prior knowledge for mathematics learning.

The Research on Manipulatives

Sowell (1989) conducted a meta-analysis on the effectiveness of using manipulatives during mathematics instruction and found that using manipulatives was better than not using manipulatives. Younger students, especially, benefit from using manipulatives as they provide concrete objects to students who may not yet be able to think abstractly (DeLoache, 2000; Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009). Carbonneau, Marley, and Selig (2013) followed up on this research and conducted a meta-analysis of 55 studies that explored the efficacy of teaching with complex manipulatives and found that teaching with manipulatives compared to teaching with abstract symbols showed small to medium sized effects on student learning. The research has shown that manipulatives can aid learning, but there are certain variables (i.e., guidance and prior knowledge) that can mitigate their helpfulness.

In regard to the effectiveness of guidance when using manipulatives, Laski, Jordan, Daoust, and Murray (2015) summarized general findings on young children's learning with mathematics manipulatives and recommended explicit guidance that relates the concrete manipulatives to the abstract numbers they represent. Providing consistent guidance was found to allow students to devote their limited working memory to understanding the content of the mathematics lesson rather than other, extraneous content. In their meta-analysis, Carbonneau, Marley, and Selig (2013) found that high guidance instruction was associated with higher retention and problem solving performance, while low guidance instruction was associated with higher transfer performance when using manipulatives. Carbonneau et al. (2013) also investigated the impact of age on learning and found that students age 3-6 (preoperational age) struggled more when learning with manipulatives compared with students in the concrete operational age group (7-11) or formal operational age group (12 and older). The authors attributed this finding to young students' tendency to struggle with understanding that objects can represent larger mathematical concepts.

Overall, the research shows support for using manipulatives for mathematics learning. However, the research on implementing guidance with manipulatives is less clear, as some research supports high guidance but other research supports low guidance (Carbonneau et al., 2013). For example, while the utility of guidance for helping students is clear, research also shows low guidance can allow students to form their own knowledge, which is important for transfer. Furthermore, manipulatives were less effective for very young students, which indicates the potential for guidance and prior knowledge to impact learning with manipulatives

Research on Guidance and Prior Knowledge

In order to further investigate the impact of guidance and prior knowledge for learning mathematics, we will briefly explore the research on guidance for mathematics learning both with and without manipulatives. As discussed, while high guidance has been shown to be useful for learning, we are finding that there is a benefit to implementing lessons with low guidance, so we also explore the possibility of transitioning the level of guidance during learning. Finally, we look to the research on prior knowledge for learning mathematics, to further understand how prior knowledge may determine the usefulness of high or low guidance on learning.

Support for high guidance. Support for high guidance instruction comes from researchers and practitioners who argue that without teacher guidance, students left to their own devices will not learn concepts or, worse, learn the wrong concepts (Rogoff, 1990; Cobb, 1995). Social constructivist theorists posit high guidance during learning with manipulatives is essential because the manipulatives are culturally-specific, external representations that allow children to count before having an internal representation of the number. In order to support the eventual development of an internal representation of numbers, students need guidance to be able to recognize what the concrete manipulatives represent (Bruner, 1966; Vygotsky, 1978).

Empirical research supports implementing high guidance during learning, explaining that exploration without the guidance of an instructor can result in students never interacting with the content to be learned (Mayer, 2004). For example, Terwel, van Oers, van Dijk, and van den Eeden (2009) found students learned how to represent percentages and graphs better when they co-constructed representations with teacher guidance compared to learning on their own with ready-made representations. Hunt (2014) found third grade students significantly outperformed a control group on equivalency concepts when they were provided teacher guided instruction that included modeling, practice with prompts, and error correction. Further, Ramani, Siegler and Hitti (2012) found pre-kindergarten students who played a number board game guided by feedback from paraprofessionals significantly improved on number sense measures, while students in a control group without the number game did not improve. While these studies provide support for high guidance, other research provides support for low guidance during instruction.

Support for low guidance. While the importance of high teacher guidance is shared amongst researchers and teachers, others acknowledge the equal importance of providing learners with time for their own exploration (e.g., Bruner, 1961; Schwartz, 1992). Low guidance instruction gives learners the opportunity to formulate and understand mathematical concepts on their own, which is important for deeper learning of mathematics knowledge (Piaget, 1977; Fuson, 2009). Low guidance can also avoid the effects of overwhelming students' working memory with too many questions or comments from a teacher (Kroesbergen and Van Luit, 2005). Pure discovery learning, where students are left with no guidance or instruction and only materials, has not been found to help students learn; instead, researchers advocate for learning that incorporates some guidance from the teacher (Alfieri et al., 2011). This can come in the form of feedback on steps the students is taking or outcome feedback on their answers.

Looking to empirical support for low guidance instruction, Kroesbergen and Van Luit (2005) found low guidance was better than high guidance instruction for students with mild intellectual disabilities who were learning multiplication solution procedures. Kroesbergen and Van Luit (2002) likewise found that students in special education classes benefitted more from low guidance than high guidance, however they found that low performing students not identified as having a learning disability benefitted more from high guidance. These findings indicated that students with learning disabilities may have characteristics that differentiate the impact of guidance on learning.

Support for transitioning guidance. Another approach to implementing guidance involves starting with high guidance and then transitioning to low guidance as students gain skill and fluency. This use of guidance was studied by Fuchs et al (2003) who investigated whether initial high guidance instruction followed by exploratory problem solving is superior to exploration followed by guided instruction. Fuchs et al. (2003) found that problem solving improved for students who had high guidance instruction followed by low guidance problem solving with fully worked examples compared to a high guidance, instruction-only condition. However, high guidance instruction followed by low guidance problem solving with partially worked examples, rather than fully worked examples, was not better than high guidance, instruction-only. These findings showed that the optimal level and timing of guidance may depend on multiple variables, such as the age of students, mathematical topic, and structure and content of the instruction or problems. This points to the need for research on guidance under multiple conditions.

The role of prior knowledge. Cognitive load theory assumes that students with less knowledge need more guidance so as not to exceed their cognitive load. Students with more domain specific knowledge will not need as much guidance because the information is stored in long term memory (Sweller, Ayres, & Kalyuga, 2011). Guidance should be given to support the acquisition of new knowledge, and not to focus on information that has already been learned because this could confuse the students if conflicting information is given (Kalyuga, 2007). This means teachers need to monitor the amount of guidance to give based on students' prior knowledge and experience with a topic.

Fyfe and Rittle-Johnson (2016) investigated the impact of computer feedback on learning equivalency problems for second grade students with high or low prior knowledge. They found students with lower prior knowledge performed better with feedback versus no feedback on solving equivalency problems. For students with higher prior knowledge, all conditions resulted in improvement on solving equivalency problems. Jitendra et al. (2013) found a different effect of prior knowledge on learning with high and low guidance. They compared a high guidance condition that utilized schema-based instruction to a low guidance, business-as-usual group. The high guidance condition involved a curriculum in which the teacher prompted students to use think-alouds to encourage monitoring and reflection during problem solving. The low guidance condition involved a school-provided, inquiry-based curriculum, in which students worked alone to develop multiple solutions for an ordered set of problems presented on worksheets. Surprisingly, students with higher pretest scores (high prior knowledge) were found to perform significantly better with the high guidance, schema-based curriculum whereas students with lower pretest scores performed better with the low guidance curriculum. Tournaki (2003) compared performance on mathematics addition tasks for second grade students, half of which were general education students and half of which were students with learning disabilities. For students with learning disabilities, significant improvements from pretest to posttest were only found for students in the high guidance instruction group. General education students improved in both the low and high guidance groups. For both the general education students and the students with learning disabilities significant improvements on the transfer task were only found for students in the high guidance condition.

The results of the studies discussed do not paint a clear picture of the role of prior knowledge. Carbonneau, Marley, and Selig (2013) found that high guidance interventions with manipulatives produced better retention than low guidance interventions, but low guidance interventions produced better transfer than high guidance interventions. Another alternative is to include both high and low guidance in the instruction and determine the optimal sequence of guidance (e.g., Darch, Carnine, & Gersten, 1984; Fyfe, Rittle-Johnson, & DeCaro, 2012). Even further, the optimal level or sequence of guidance may also be influenced by students' prior knowledge (e.g., Jitendra et al., 2013; Tournaki, 2003).

Current Study

In the current study I compared student performance on measures of mathematics achievement after one of four different five-day treatments that differed in the amount and/or timing of guidance. In the high guidance condition, students were taught with consistent high guidance for all five days. In the low guidance condition, students were taught with low guidance for all five days. In the high to low guidance condition, students were taught with high guidance for the first two days, low guidance for the last two days, with the third day utilized as a transition day where the researcher limited the guidance but did not eliminate it until day 4. In the low to high guidance condition, students were taught with low guidance for the first two days, high guidance for the last two days, with the third day utilized as a transition day where the researcher added some high guidance questions and comments. Our study specifically investigated four questions:

1. Which of the four approaches to teaching with manipulatives is best for improving elementary students' performance on the intervention content specific task?

Based on the meta-analysis by Carbonneau et al. (2013), high guidance is optimal for improving student performance on an intervention content specific task. Research has also shown support for low guidance at some point during instruction, but it is not clear if high guidance should be faded out or if it should come after low guidance instruction. Therefore, we predicted that one of the transitioning conditions (high to low or low to high) would be best.

2. Which of the four approaches to teaching with manipulatives is best for elementary students' performance on a transfer task?

Carbonneau et al. (2013) found that studies that implemented low guidance interventions with manipulatives had higher effect sizes for transfer than the studies that implemented high guidance interventions with manipulatives. On the other hand, students in low guidance instruction may not learn at all, and may need guidance from the teacher in order to learn not just the material, but enough to be able to transfer to another task. Prior studies have found lower achieving students need more guidance to understand the content in order to transfer knowledge (Tournaki, 2003). First, we predicted the consistently low guidance condition would not be the optimal condition for transfer because not all students would be able to learn completely on their own. We predicted that the low to high and high to low guidance conditions would lead to better transfer because students will be given the opportunity to make meaningful connections on their own.

3. Which approach to teaching with manipulatives is best for improving number sense, as measured by the Test of Early Numeracy (TEN)?

The TEN can be considered far transfer and the same issues and predictions held for the impact of the different conditions on the TEN.

4. How does elementary students' pre-test knowledge affect which condition is best for learning?

Based on the review by Kalyuga (2007) on the expertise reversal effect, it was hypothesized there would be an interaction effect. Students with low prior knowledge would perform best with consistent high guidance or high to low guidance in order to learn with manipulatives. If students are not given enough guidance to start with they may learn information incorrectly or may not know where to begin when exploring with manipulatives alone. Students with high prior knowledge may need consistent low guidance or low to high guidance in order to learn with manipulatives. These students need time to explore alone and already have enough prior knowledge to do this effectively. Starting with high guidance may confuse students with high prior knowledge.

Method

Participants

Permission slips were distributed to kindergarten students at four elementary schools from a southeastern school district. Students at this school district are comprised of 61% white, 17% Hispanic, 13% black, 5% multi-racial, and 4% Asian. One hundred sixty-seven students returned permission slips. Of those, one student was absent during the week of the intervention and one student with special needs could not complete the measures for testing so the final sample was 165 (99 males, 66 females). At the start of the study in fall 2015, the average age of students was 5.56 years, *SD*=0.36.

Materials and procedure

All students were assessed at pretests, posttest, and delayed posttest on a counting with manipulatives task, the Test of Early Numeracy (TEN), and a transfer task. All three tests were given at all three time points. The pretest was administered the week before the intervention took place. The posttest was administered the week after the intervention and a delayed posttest was administered two weeks after the intervention. All pretest, posttest, and delayed posttest measures were individually administered in a quiet area. Administration took approximately 10-15 minutes per student, per test.

Counting task. The counting tasks were designed to assess student ability to count manipulatives. The counting tasks utilized ten boards which are used to teach kindergarten students the order of numbers as part of the *Math Expressions* curriculum. The counting task designed for the pretest was different from the counting task designed for the posttests because students had not yet been introduced to the nickel strips at the time of the pretest and the researcher did not want to provide instruction on the nickel strips until the time of the intervention. For the pretest, students were given a board with the numbers one through ten at the top of the board. Below each number was a column for the student to place pennies to show the value of the number (see Figure 3.1). The experimenter asked students, “Place the number of pennies that are written at the top of the column”. The students used pennies to show the numbers given. For the pretest the experimenter asked students to place the correct number of pennies under the columns five, eight, three, one, and six. The nickel strips were not used for the pretest. As this task had students fill in five total columns the scores for the pretest counting task could range from 0-5.

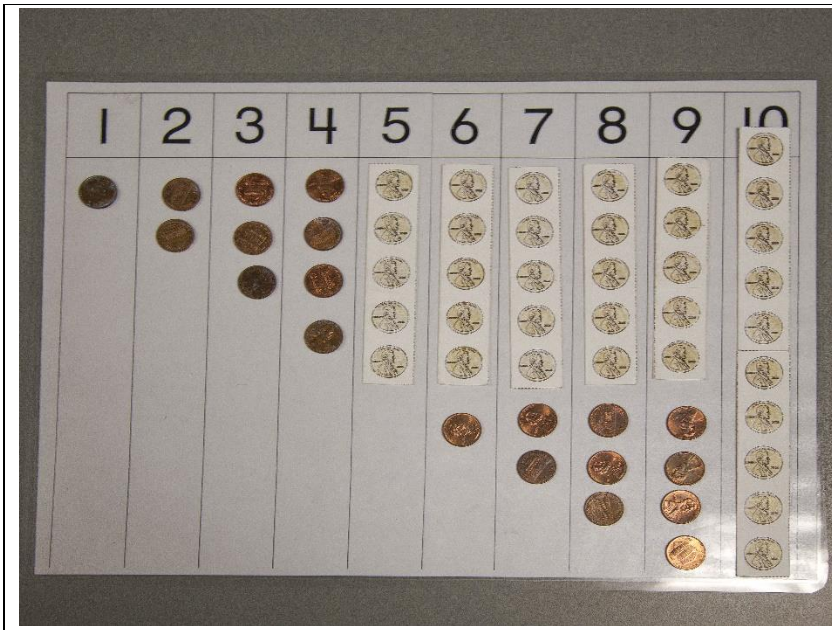

Figure 3.1. Intervention and the intervention specific task. One through ten board from Math Expressions. The instructions were: "You have a board, pennies and nickel strips. You are going to make the numbers one through ten on your boards using pennies and nickel strips. You can use pennies to make all of the numbers. You can also use nickel strips. Each nickel strip stands for five pennies (show nickel strip which has pictures of five pennies on it). You can use one nickel strip to take the place of five pennies. The number in the first column is one, can you make the number one with pennies?"

The posttest and delayed posttest had students place all pennies and nickel strips under all columns from one to ten, as shown in Figure 3.1. This measure was scored as either correct or incorrect which resulted in two categories; 0=not correct; 1=correct. To be scored correctly students needed to place the pennies *and* nickel strips correctly. The possible range of scores for this task was 0-1. Cronbach's alpha for the pretest, posttest, and delayed posttest for the consistent low guidance condition found counting tasks to be reliable (3 tests; α = .529).

Number sense measure. Number sense was measured with the Test of Early Numeracy (TEN). The TEN is individually administered and includes four measures; each measure lasts for one minute for a total of about five minutes per student. The four measures on the TEN are oral counting (possible scores 0-100), number identification (possible scores 0-56), quantity discrimination (possible scores 0-28), and missing number (possible scores 0-21) (Clarke & Shinn, 2002). The oral counting measure has students count as high as they can for one minute. The number identification measure has students identify numbers between 1 and 10 for kindergarteners. The quantity discrimination measure has students identify the larger of two numbers between 1 and 10 for kindergarteners. The missing number measure has students identify the missing number for a set of three numbers with two numbers given. Rather than one, summative score, the TEN yields four separate scores for number sense, which were analyzed individually.

The TEN has been shown to be a valid measure of number sense for kindergarten and first grade students. Clarke and Shinn (2004) found the TEN was correlated with the Woodcock-Johnson Applied Problems (Woodcock & Johnson, 1989) subtest for first grade students, which measures mathematics achievement based on mathematics operations problems and applied mathematics problem. Martinez, Missall, Graney, Aricak, and Clarke (2009) found that the TEN was correlated with Stanford 10 Achievement Test (SAT-10) (Harcourt Assessment Inc., 2002), which measures if students are meeting standards for reading, mathematics, and language. Alternate form reliability was measured by testing students with an alternate form of all subtests except for the oral counting measure because there is no alternate form for counting as high as you can (Clarke & Shinn, 2004) . Reliabilities were measured as .93 for oral counting, .93 for number identification, .92 for quantity discrimination, and .78 for missing number (Clarke & Shinn, 2004). Salvia and Ysseldyke (2001) assigned a reliability of .90 or greater for making educational decisions about individual students, .80 or greater for making screening decisions about individual students, and .60 or greater for making educational decisions about groups of students. According to these guidelines all measures of the TEN can be used to make educational decisions about individuals except the missing number measure, but .78 is still a moderately high reliability.

Transfer task. Transfer was assessed with a task that required students to count on from five. Students were shown a number between six and ten and five circles. Sample transfer problems are shown in Figure 3.2. Students were given the following instructions: “Do you see that we have 1,2,3,4,5 circles? Can you draw more circles so we have X circles in the box?” This task was scored as zero correct, one correct, or both correct. Cronbach’s alpha for the pretest, posttest, and delayed posttest for the consistent low guidance condition found the transfer task to be reliable (3 tests; $\alpha = .667$).

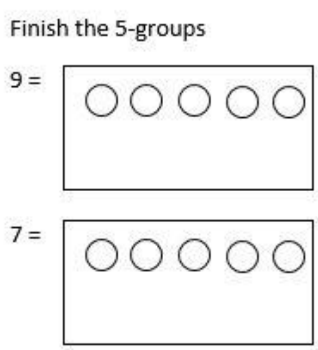

Figure 3.2. Transfer worksheet for pre-test post-test, and delayed post-test.

Intervention

Students were pulled from class in groups of five to seven. Each group was assigned to one condition (i.e., high guidance, low guidance, high to low guidance, or low to high guidance). Teaching took place for six to nine minutes per day for five days. As described below, the consistent high guidance group implemented only the high guidance lesson throughout the entire week and the consistent low guidance group implemented only the low guidance lesson throughout the entire week of the intervention. The high to low guidance group began the week with high guidance lessons then shifted to low guidance lessons. The low to high guidance group began the week with low guidance lessons and then shifted to high guidance lessons. To assure fidelity of the high and low guidance modifications, all lessons were recorded and coded as described below.

The four conditions utilized ten boards which are used to teach kindergarten students the order of numbers and are a part of the Math Expressions curriculum. For this task, students make the numbers one through ten by using pennies and nickel strips. Nickel strips are white pieces of paper that fit perfectly under five pennies. The students first counted out the number of pennies requested then added a nickel strip under sets of five pennies. For example, if the number eight was counted the student would count out eight pennies, then replace five of those pennies with a nickel strip.

High guidance modification. Three of the four conditions include lessons that have high guidance. A high guidance lesson is defined as the teacher asking many questions during learning. A list of possible questions is included in Table 3.1. The teacher was not required to use every question on this list nor is was the list an exhaustive list of questions asked. The high guidance instruction used these questions to increase student learning and understanding. The teacher also provided elaborate feedback about performance during the lesson (not just right or wrong but why), helped students if they needed help, and answered students' questions.

Table 3.1

Sample High guidance questions

"Why did you (not) use a nickel strip in this column?"
"How is this column different from the last column?"
"How is the 8 column the same as the 3 column? How is it different?"
"Can we use a nickel strip in this column? Why (not)?"
"Can you count the pennies to check your answer?"
"How many more pennies would we need for a nickel strip?"
"How many more pennies would we need for another nickel strip?"
"How many more pennies would we need for 5?"
"How many more pennies would we need for 10?"

Low guidance modification. Three of the four conditions include lessons that have low guidance. Per the definition of low guidance for this paper the teacher could provide feedback to students in the form of "yes" or "no" but provided no further information. In the case of the activity to learn the numbers one to ten, low guidance included instructions to make the numbers one to ten and corrective feedback, but did not include any back and forth questioning. In addition to the instructions given in Figure 3.1, students were provided the following instruction once they reached the five column: "When you reach the number five on the number board you take away the five pennies and use a nickel strip instead." For the numbers six through 10 these instructions were repeated. Questions to keep the students on task could be asked, but questions about the content (e.g., “which number is bigger?”) were not.

Transitioning conditions. There were two transitioning conditions; high to low guidance and low to high guidance. For the first two days students were taught with either high or low guidance. Day three was a transition day where the level of guidance started to taper off so that high guidance was tapered to low guidance or increased so that low guidance increased to high guidance. On days four and five students were taught with the second type of guidance so that students who were given low guidance on days one and two were given high guidance and students who were given high guidance on days one and two were given low guidance.

Fidelity. To assure fidelity of the high and low guidance modification all lessons were audio recorded and coded. Each lesson was rated as high guidance or low guidance based on the number of questions asked by the researcher to students; less than five indicated low guidance, more than five indicated high guidance. Five was chosen as the cutoff to allow room for the low guidance conditions to include minimal questioning such as to keep students on task, as completely cutting out questions is not realistic or practical in everyday teaching. The author and a trained independent rater (a graduate student) coded 20% (31) of the sessions. Interrater reliability between Erin Horan and the independent coder was established as 96.8%.

Results

Performance on Counting with Manipulatives

Means and standard deviations of performance on the pretest, posttest, and delayed posttest counting tasks for students each condition are shown in Table 3.2. Analysis of variance showed that pretest scores on the content task did not significantly differ across conditions, $F(3,164) = 1.506$ $p = .225$. Posttest scores on the content task controlling for pretest scores were not significantly different across conditions, $F(3,160) = 0.735$, $p = .532$. Delayed posttest scores on the content task controlling for pretest scores were not significantly different across conditions, $F(3,160) = 1.128$, $p = .339$.

Table 3.2

Mean Performance on Content Tasks and Standard Deviation

	H-H	H-L	L-H	L-L	Full sample
Pretest	4.73 (.554)	4.49 (1.00)	4.54 (.745)	4.72 (.701)	4.62 (,769)
Posttest	0.68 (.474)	0.63 (.488)	0.51 (.506)	0.63 (.489)	0.61 (.489)
Delayed Posttest	0.70 (.464)	0.63 (.488)	0.51 (.506)	0.70 (.465)	0.64 (.483)
N	40	41	41	43	165

Note: Pretests were scored as 0-5. Posttests and delayed posttests were scored as 0 (incorrect) or 1 (correct).

Subsequent analyses were performed after removing students who scored with the highest score on the pretest (five out of five) to account for ceiling effects. Means and standard deviations of performance on the pretest, posttest, and delayed posttest counting tasks for students each condition are shown in Table 3.3. Again, posttest scores on the content task controlling for pretest scores were not significantly different across conditions, $F(3,38) = 0.040$, $p = .989$. Delayed posttest scores on the content task controlling for pretest scores were not significantly different across conditions, $F(3,38) = 0.126$, $p = .944$.

Table 3.3

Performance on Content Tasks with Highest Performers Removed

	H-H	H-L	L-H	L-L	Full sample
Pretest	3.78 (.441)	3.38 (1.19)	3.64 (.633)	3.29 (.756)	3.53 (.827)
Posttest	0.56 (.527)	0.54 (.519)	0.57 (.514)	0.57 (.535)	0.56 (.502)
Delayed Posttest	0.56 (.527)	0.54 (.519)	0.50 (.519)	0.57 (.535)	0.53 (.505)
N	9	13	14	7	43

Note: Pretests were scored as 0-5. Posttests and delayed posttests were scored as 0 (incorrect) or 1 (correct).

Table 3.4

Mean Performance on Transfer Tasks and Standard Deviation Controlling for Pretest

	H-H	H-L	L-H	L-L	Full sample
Pretest	1.53 (.784)	1.46 (.745)	1.37 (.767)	1.47 (.855)	1.45 (.784)
Posttest	1.63 (.667)	1.61 (.628)	1.44 (.776)	1.63 (.618)	1.58 (.673)
Delayed Posttest	1.68 (.616)	1.59 (.741)	1.71 (.602)	1.60 (.660)	1.64 (.653)
N	40	41	41	43	165

Note: Pretest, posttest, and delayed posttest were scored from 0-2.

Performance on Transfer

The transfer pretest, posttest, and delayed posttest were two item tasks scored from 0-2. Means and standard deviations of performance on the pretest, posttest, and delayed posttest transfer tasks for students in each condition are shown in Table 3.4. Analysis of variance showed that pretest scores on the transfer task did not significantly differ across conditions, $F(3,164) = 0.283$ $p = .838$. Posttest scores on the transfer task controlling for pretest scores were not significantly different across conditions, $F(3,160) = 0.544$ $p = .653$. Delayed posttest scores on the transfer task controlling for pretest scores were not significantly different across conditions, $F(3,160) = 0.618$ $p = .604$. Subsequent analyses were performed after removing students who scored with the highest score on the pretest (five out of five) to account for ceiling effects but these findings were not significant.

Performance on TEN

A three stage hierarchical linear regression was conducted with each posttest TEN score as the dependent variable. Pretest TEN score was entered at stage one of the regression to control for prior knowledge. The four conditions were dummy coded into three variables and were included at stage two. Interactions between the conditions and pretest scores were included at stage three. Performance on each component of the TEN (i.e., counting, missing number, number identification, and quantity discrimination) was analyzed separately.

The hierarchical multiple regression revealed that at stage one, pretest scores on each component of the TEN contributed significantly to the regression model. Beyond stage one, only the model for the counting component of the TEN showed significant contributions by other variables (see Table 3.5). For the counting component, pretest scores contributed significantly to the regression model, $F(1,163) = 326.0$, $p < .001$ and accounted for 66.7% of the variation in

Table 3.5

Summary of Hierarchical Regression Analysis for Variables predicting Posttest Counting

Variable	β	*t*	sr^2	*R*	R^2	ΔR^2
Step 1				0.817	0.667	0.667
Pretest TEN Counting	0.765	18.056*	0.667			
Step 2				0.818	0.670	0.003
Pretest TEN Counting	0.761	17.557*	0.637			
Low-High Condition	0.606	-0.280	0.001			
High-Low Condition	-2.552	-1.178	0.003			
High-High Condition	-1.078	-0.495	0.001			
Step 3				0.826	0.682	0.012
Pretest TEN Counting	0.854	11.597*	0.272			
Low-High Condition	14.541	1.927	0.008			
High-Low Condition	-0.542	-0.075	0.001			
High-High Condition	11.868	1.276	0.003			
Low-High*Pretest	-0.240	-2.096*	0.008			
High-Low*Pretest	-0.026	-0.236	0.001			
High-High*Pretest	-0.192	-1.447	0.004			

Note. N=165; * $p < .05$

posttest scores. Introducing the experimental conditions explained an additional .3% of the variation in posttest scores and this change in R^2 was significant, F (4,160) = 81.1, $p < .001$. Adding the interaction terms to the regression model explained an additional 1.2% of the variation in posttest scores and this change in R^2 was significant, F (7,157) = 48.0, $p < .001$. When all seven variables were included in stage three of the regression model, only two variables were significant predictors of posttest score: pretest score and the interaction between pretest score and the low-high guidance condition. The pretest score uniquely explained 27% of the variation in posttest score and the interaction between pretest and the low-high guidance condition uniquely explained .8% of the variation in posttest score. The same hierarchical linear regression model was performed with delayed TEN counting posttest as the dependent variable but these findings did not remain, only pretest was a significant predictor of delayed posttest score. Subsequent analyses were performed after removing students who scored with the highest score on the counting with manipulatives pretest (five out of five). Subsequent analyses were performed after removing students who scored with the highest score on the counting with manipulatives pretest (five out of five) to account for ceiling effects but these models showed no significant predictors of TEN posttest scores other than TEN pretest scores.

Discussion

Based on the limited prior research on guidance and prior knowledge, we made several hypotheses. We predicted the transitioning high to low and low to high guidance conditions would be the best for students' learning on the intervention content specific task, transfer task, and number sense task. Regarding prior knowledge, we predicted an interaction effect; students with low prior knowledge would perform best on counting with manipulatives with consistent high guidance or high to low guidance on counting with manipulatives. Students with high prior knowledge would perform best with consistent low guidance or low to high guidance in order to learn with manipulatives. Overall, none of these findings were supported by the results of this study, with most comparisons between conditions showing no difference in learning, even after controlling for prior knowledge.

Overall, student performance on counting with pennies and nickel strips did not differ between conditions even after controlling for pretest scores and possible ceiling effects. These current findings contradict prior research, which has typically shown high guidance groups outperform control groups when highly guided instruction is implemented (e.g., Carbonneau et al., 2013; Hunt, 2014; Terwel, van Oers, van Dijk, and van den Eeden, 2009). There are several possible explanations for this finding. First, guidance may not be a moderator of learning for counting to ten with manipulatives. This skill may not require explicit explanation or questioning from an instructor. Simply allowing students to practice and count on their own may be all that is needed. Another explanation could be the ineffectiveness of this task for assessing deeper learning. The questions included in the high guidance modifications targeted deeper learning, as they focused on comparing columns and noticing similarities and differences in the quantities. This task assessing the intervention content only had students count pennies and nickel strips, which did not relate to the questions used in the high guidance modifications. Perhaps asking questions to target this deeper learning would have shown differences in learning by condition.

Overall, student performance on the transfer task also did not differ across conditions even after controlling for pretest scores and possible ceiling effects. As with the intervention content specific task, it could be guidance is not a moderator of learning for transfer to counting on from five. However, based on the research on guidance, it is surprising this study did not show differences between conditions. Specifically, the meta-analysis by Carbonneau et al. (2013) suggested that conditions that implemented low guidance (i.e., high to low and low to high) would show greater performance on transfer tasks. Typically allowing students time to practice while also incorporating guidance (i.e., the high to low or low to high guidance conditions) fosters deeper learning. The contradictory findings of this study could indicate an issue with this measure of transfer as we only included two items to yield a score of 0-2. Perhaps a longer or more in depth test of transfer would have provided better insight into students' learning for transfer.

Student performance on the four tasks for the Test of Early Numeracy showed a difference between conditions for the counting task only, where students counted as high as they could, up to 100, for one minute. After controlling for pretest scores, students in the low to high guidance condition scored significantly lower than students in the consistently low guidance condition. The consistently high and high to low guidance groups did not perform significantly different from the consistently low guidance group. This indicates that providing students with time to practice alone followed by providing guidance can hinder counting fluency; it was better to allow students to practice counting with manipulative on their own with no additional guidance or questions. These results deviate from prior research that included assessments of number sense, where student performance was significantly higher after high guidance instruction compared to low guidance instruction on counting tasks (Ramani, Siegler, and Hitti, 2012). Perhaps providing guidance in the form of additional questions can distract students from the main task of counting. But, it is interesting students performed lower in the low to high guidance condition but not the high to low condition.

We hypothesized that after controlling for pretest scores we would find interaction effects; students with higher prior knowledge would excel with less guidance while students with lower prior knowledge would excel with more guidance. After controlling for pretest scores we found no differences in the results for any of the measures. As with our other findings, this could be related to the task (i.e., counting to ten with manipulatives) being too simple, or our measures not being powerful enough to detect differences between groups.

Based on the overall findings for this study and the tasks used, the only difference in learning was found for the low-high guidance condition, where students performed significantly lower than the consistent low guidance condition on the counting portion of the Test of Early Numeracy. On the TEN counting posttest, students in the low to high guidance condition scored significantly lower when compared to the consistently low guidance condition. However, with this being the only measure to show a difference, we cannot draw conclusions about the optimal level and timing of guidance for learning with mathematics manipulatives.

The current research indicates that level and timing of teacher guidance is not important for teaching kindergarten students to count to ten with manipulatives. Controlling for prior knowledge (i.e., pretest) did not impact these results. It should be made clear that while this is true for this task it may not be true for more complex or challenging tasks. For this less complex and age appropriate task, the amount and timing of guidance was not important.

Future research should focus on variations of the timing and level of guidance with other tasks and age groups. The current research design could also be implemented with preschool students. Preschool students do not have the same base level of knowledge for counting in general and counting with manipulatives specifically. Perhaps implementing this research with preschool students would show some differences in learning based on the timing and level of guidance.

Future research should also implement stronger measures that focus on the deeper learning guidance intends to foster. The measures used in this study were not long or complex enough to determine if deeper learning occurred. Questions that targeted comparisons between number columns (e.g., “how many more pennies are in the seven column than the five column?”) would provide more insight into whether or not deeper, more meaningful learning took place beyond simply counting pennies and nickel strips on a board.

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CHAPTER 4

CONCLUSION

The purpose of this dissertation was to define and differentiate guidance and structure in order to effectively study guidance in elementary school mathematics education. Kirschner, Sweller, and Clark's (2006) review of "low guidance" in the literature, and the commentary articles that followed it, highlighted the lack of a clear, consistent definition of guidance, with discrepancies on what constitutes guidance (e.g., teacher help, materials) and, relatedly, what types of teaching constitute high versus low guidance. Therefore, to accomplish this goal, we first needed clear definitions of guidance and structure.

Guidance versus Structure

In our search for articles that manipulated different forms of guidance, we found we had to separate articles that investigated teacher guidance from other constructs present during instruction, which we labeled as structure. Based on our literature review of articles on elementary school mathematics education, we found there are multiple variables at play when determining how much guidance to provide to students. For example, when teaching mathematics to students with mild intellectual disabilities, direct instruction was found to be more effective than a lesson with more guidance and discussion based instruction (Kroesbergen & Van Luit, 2005), but high guidance was more effective than low guidance for low performing students (Kroesbergen & Van Luit, 2002). Related to student variables, Fyfe and her colleagues (2012, 2015) found that prior knowledge and working memory also moderated the impact of high and low guidance on learning. Previous research has shown working memory and prior knowledge influence student learning during mathematics instruction and problem solving, and may require extra attention in the classroom (Alloway and Gathercole, 2008). As we found in our review, this support can be accomplished by attending to guidance or structure. Based on our review, it is important to continue research on guidance so we can work to provide specific recommendations about guidance in different subject domains and with different students.

The second component of our review focused on the structure of mathematics lessons. The structure of lessons can encompass a wide variety of features including worked examples, ordering problems from easy to difficult, or providing formula sheets during problem solving. As such, the articles we reviewed covered a variety of studies, so we did not attempt to draw generalizable conclusions about structure. While this may be a limitation of our literature review, we feel this was important for accomplishing our review of teacher guidance and to highlight the variety of uses of the term guidance. Additionally, reviews of different components of structure can be easily accomplished (e.g., by specifically searching for worked examples or ordered problems), but a full review of each component of structure is beyond the scope of this review.

Limitations

To review guidance and structure with a clear, concise methodology, we greatly limited the studies reviewed to elementary school, experimental studies on mathematics. Additionally, we only reviewed articles that explicitly described the instruction in all conditions to determine if they effectively compared two different levels of guidance. Consequently, this may have limited our ability to find articles that investigated guidance per our definition and categorizations. There may be more studies that effectively compared different levels of guidance, but due to the lack of consistent terminology in the literature, finding these articles, even with a clear methodology, would be impossible.

Another limitation of our review relates to the classification of conditions as high, moderate-high, moderate-low, and low guidance. As is the case with defining discovery learning methods (c.f., Alfieri et al., 2010), classifying instruction as high or low guidance or high or low structure may be based more on the comparison group in a study than by the teaching methods. Our four categories allowed us to accomplish this as best as we could based on the studies we reviewed. As more studies investigate guidance, there may be a need to add categories that effectively describe small differences between levels. This will allow us to continue to make comparisons of conditions within studies and comparisons of conditions between studies.

Guidance when Teaching Kindergarteners with Manipulatives

The second manuscript in this dissertation detailed an experimental intervention to examine the optimal level of guidance during kindergarten mathematics instruction with manipulatives. Based on the limited prior research on guidance and prior knowledge when teaching kindergarteners with manipulatives, we made several hypotheses. We predicted the transitioning high to low and low to high guidance conditions would be the best for students' learning on the intervention content specific task, transfer task, and number sense task. Regarding prior knowledge, we predicted an interaction effect; students with low prior knowledge would perform best on counting with manipulatives with consistent high guidance or high to low guidance on counting with manipulatives. Students with high prior knowledge would perform best with consistent low guidance or low to high guidance in order to learn with manipulatives. Overall, none of these findings were supported by the results of this study, with most comparisons between conditions showing no difference in learning, even after controlling for prior knowledge.

Overall, student performance on counting with pennies and nickel strips did not differ between conditions even after controlling for pretest scores and possible ceiling effects. These current findings contradict prior research, which has typically shown high guidance groups outperform control groups when highly guided instruction is implemented (e.g., Carbonneau et al., 2013; Hunt, 2014; Terwel, van Oers, van Dijk, and van den Eeden, 2009). There are several possible explanations for this finding. First, guidance may not be a moderator of learning for counting to ten with manipulatives. This skill may not require explicit explanation or questioning from an instructor. Simply allowing students to practice and count on their own may be all that is needed. Another explanation could be the ineffectiveness of this task for assessing deeper learning. The questions included in the high guidance modifications targeted deeper learning, as they focused on comparing columns and noticing similarities and differences in the quantities. This task assessing the intervention content only had students count pennies and nickel strips, which did not relate to the questions used in the high guidance modifications. Perhaps asking questions to target this deeper learning would have shown differences in learning by condition.

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