

THE COLLATERAL CONSTRAINT  
AS THE RESALE VALUE OF CAPITAL

by

Andrew Kane

(Under the Direction of Julio Garín)

**Abstract**

Financial shocks are responsible for large fluctuations in real as well as financial variables. To explore a potential link between real shocks and financial conditions, I study a real business cycle model with collateral constraints. I modify the RBC model of Jermann and Quadrini (2012) by interpreting the borrowing limit for firms as the resale value of the capital they offer as collateral. For reasonable specifications of investment adjustment costs, the dynamic responses of my model mirror those of the original. The similar performance of my model suggests that the resale value of capital may be a channel by which real shocks affect the collateral constraint.

Borrowing Limits, Collateral Constraints, Credit Constraints, Financial Shocks, Investment Adjustment Costs

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ANDREW KANE

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Approved:

Major Professor: Julio Garín

Committee: William Lastrapes  
Svetlana Pashchenko

Electronic Version Approved:

Suzanne Barbour  
Dean of the Graduate School  
The University of Georgia  
May 2016

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# 1 Introduction

The 2008 crisis and subsequent recession have underlined the importance of the financial sector to the macroeconomy. The bank failures and sharp decline in credit concurrent with the recession suggest that deteriorating financial conditions contributed to the decline in output. Financial shocks have been found to explain up to one third of the variation in GDP for advanced countries (D'Agostino et al. 2013), demonstrating their importance to the real as well as financial sector. Financial shocks may also correlate to output more closely than productivity (Nolan and Thoenissen 2008). These results suggest that financial shocks have a meaningful and immediate effect on the real economy.

One potentially important propagator of financial shocks is borrowing constraints on firms. If firms are suddenly unable to borrow and cannot finance themselves from elsewhere due to financial frictions, they will be unable to invest or hire workers at the same level as before the shock. Consequently, output and real variables will be affected by a tightening constraint. Jermann and Quadrini (2012), henceforth JQ, find that debt repurchases by firms are countercyclical in the United States from 1954 to 2010, suggesting that tightening borrowing constraints (higher debt repurchases) correspond to lower output.

To shed light on the causes of tightening borrowing conditions, I endogenize the financial conditions variable in the JQ model, and compare the moments of the dynamic responses both to the original model and to the data. The JQ model is a real business cycle (RBC) model where firms use capital as collateral for loans. The variability in financial conditions is captured by the proportion of the firm's capital net of debt that lenders accept as collateral. When financial conditions worsen, this proportion falls. Due to financial frictions,

firms are unable to finance themselves by other means and therefore reduce expenses and output. In the original model, this constraint is exogenous.

I construct an RBC model where lenders' perception of the resale price of capital determines the collateral value of capital. I modify the firm's problem to give it the option of purchasing capital from a lender instead of investing in future capital itself. The firm does not pay investment adjustment costs to use this capital. Instead, the lender pays costs proportional to the amount of capital used as collateral. I then derive the first order condition with respect to purchased capital and solve for its price. The fact that in equilibrium firms do not default, and therefore that there is no capital to purchase from lenders, gives the resale price of capital.

The resale price is a function of investment adjustment costs: the more expensive it is for firms to produce their own capital, the more they are willing to pay for capital that bypasses these costs. I show that for moderately convex investment adjustment costs, the model with the endogenized borrowing limit closely tracks the performance of JQ's model for real variables. When the convexity of adjustment costs is set appropriately, the dynamic responses of my model mirror those of the JQ model. The similarities of the dynamics of both models suggest that the resale value of capital may be a channel by which real shocks affect financial conditions.

## 2 Literature Review

Financial shocks have been studied extensively both empirically and theoretically. The empirical work has found that these shocks drive large fluctuations in both real and financial variables. These results hold for economies with differing levels of development and varying financial structures. The theoretical work has focused on borrowing costs and credit constraints. The former make financing more expensive for firms under financial shocks, while the latter impose a hard limit on how much firms or households may borrow.

Empirical studies have found that fluctuations in financial conditions have significant effects on real economic variables in a wide variety of settings. Fornari and Stracca (2013) conclude that financial fluctuations in the OECD have significant effects on the non-financial economy regardless of a country's financial structure, level of financial development, or point in the business cycle. Nolan and Thoenissen (2008) conclude that financial shocks in the United States are more highly correlated with recessions than total factor productivity (TFP) shocks and that they explain a significant proportion of GDP fluctuations. D'Agostino et al. (2013) find that financial shocks account for up to one third of the variation in GDP for the European Union from 1980 to 2010, providing further evidence that shocks in this sector strongly influence the rest of the economy.

One of the more popular frameworks for modeling these financial fluctuations are models with credit constraints, where either households or firms face borrowing limits. These borrowing limits are frequently specified as functions of the collateral a firm or agent can offer to secure a loan. The collateral value of an asset can lead to overleveraging and large fluctuations in its price. Fostel and Geanakoplos (2013) develop a framework where

credit-constrained agents value assets for their collateral value as well as the consumption goods they produce. Agents' demand for collateral causes asset price bubbles that pop when financial conditions deteriorate.

Bernanke and Gertler (1989) introduce financial accelerators, borrowing frictions that vary as a borrower's net worth rises or falls. Because the cost of financing falls as net worth rises, financial costs are counter-cyclical. After incorporating the accelerator into a New Keynesian model, Bernanke et al. find that these accelerators amplify the effects of real shocks on macroeconomic variables beyond what would be expected from the original shocks themselves (Bernanke, Gertler, and Gilchrist 1999).

Financial accelerators are fundamentally different from credit constraints, however: the former merely increase the cost of financing, whereas the latter set a hard limit on borrowing. Kiyotaki and Moore (1997) analyze credit constraints by constructing a model where borrowers cannot commit to paying the lender more than the value of their collateral, which depends on firm value. Negative TFP shocks reduce the productivity of firms and therefore the value of their capital and collateral, which reduces their borrowing ability. This further reduces their output and value, resulting in a vicious cycle that lowers production far more than what the initial productivity shock would have implied on its own.

Most relevant to this paper is the work by JQ, who incorporate collateral constraints into an RBC framework. Firms are able to issue equity and take on loans to finance themselves by using a fraction of their capital net of debt as collateral. However, this fraction depends on exogenous financial conditions that limit the collateral value of firms' capital. A tax benefit on debt ensures that firms take on loans to the point where the constraint binds, and a cost function for equity payout deviations punishes firms for reducing dividend payments to finance themselves when loans are unavailable due to a tighter constraint. Negative shocks to financial conditions force firms to reduce output, employment, and debt.

### 3 The Model Economy

I build on JQ by endogenizing the collateral value of firms' capital net of debt. In the RBC model, there is a continuum of households and firms. Households maximize utility derived from consumption and leisure. They purchase shares from and provide labor and loans to firms. Firms maximize their market value, based on equity payout. They own capital, provide dividends to shareholders, and pay wages to households.

Firms use capital net of debt as collateral for loans. Their ability to borrow, and therefore their ability to hire workers, invest, and pay dividends, is subject to an exogenous financial conditions variable. A debt subsidy, financed by lump-sum taxes on households, encourages firms to take on loans until the collateral constraint binds. Furthermore, firms cannot switch financing from debt to equity costlessly: they face equity payout costs when dividend payments deviate from their long-run target. Thus, firms incur costs when financial conditions worsen.

#### 3.1 Household's Problem

Households choose consumption  $c$ , working hours  $n$ , loans  $b'$ , and shares  $s$  in each period to maximize lifetime utility subject to a budget constraint. Their time discount factor is  $\beta$ . They take interest rate  $r$ , wages  $w$ , share price  $p$ , loans due  $b$ , and dividends  $d$  as given. They pay a lump-sum tax  $T$  to finance the debt subsidy for firms:

$$\max_{c,n,s,b'} \mathbf{E} \sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \tag{1}$$

such that

$$wn + b + s(d + p) = \frac{b'}{1 + r} + s'p + c + T \quad (2)$$

The first order condition with respect to shares is

$$U_c(c, n)p - \beta \mathbf{E}(d' + p')U_c(c', n') = 0 \quad (3)$$

The price of shares can be solved for by forward substitution:

$$p_t = \sum_{i=1}^{\infty} \left[ \frac{\beta^i U_c(c_{t+i}, n_{t+i})}{U_c(c_t, n_t)} \right] d_{t+i} \quad (4)$$

The first order conditions for debt, consumption, and labor yield the Euler equation and the labor supply equation:

$$wU_c(c, n) + U_n(c, n) = 0 \quad (5)$$

$$U_c(c, n) = \beta \frac{1}{1 + r} \mathbf{E}U_c(c', n') \quad (6)$$

### 3.2 Firm's Problem

The firm's problem is to maximize its dividend-based market value subject to its budget and collateral constraints. Given current capital  $k$ , gross interest rate  $R$ , wage  $w$ , debt due  $b$ , productivity  $z$ , and financial conditions  $\xi$ , firms choose future capital  $k'$ , labor  $n$ , debt issued  $b'$ , and dividends  $d$  to maximize their present value. Firms pay wages, invest, issue dividends, pay back loans, and issue debt before production occurs. To finance these costs, firms take on an intraperiod loan  $l$ :

$$l = wn + b + d + k' - (1 - \delta)k - \frac{b'}{R} \quad (7)$$

$R$  is the gross interest rate paid by firms on debt. The government offers a debt subsidy  $\tau$ . Instead of paying a rate of  $1 + r$  per unit of debt from the previous period, the interest paid to the lender, firms pay only  $R = 1 + r(1 - \tau)$ .

Letting  $F(z, k, n)$  denote the firm's production function, its budget constraint is

$$b + wn + d + k' = (1 - \delta)k + \frac{b'}{R} + F(z, k, n) \quad (8)$$

We can combine the intraperiod loan and the budget constraint to see that  $l$  is equal to production:

$$l = F(z, k, n) \quad (9)$$

Firms can choose to default on the intraperiod loan after production has occurred. Output is assumed to be sufficiently liquid that lenders cannot recover it in the event of default, and thus firms cannot use it as collateral. The only asset lenders can recover is capital at the end of the period, net of debt. However, households do not offer loans at the collateral's face value. Instead, they limit firms' borrowing to a fraction of their capital net of debt. This fraction is the financial conditions variable  $\xi$ .

$$\xi \left( k' - \frac{b'}{1 + r} \right) \geq l \quad (10)$$

This is the collateral constraint. JQ show that for a sufficiently high debt subsidy, this constraint binds in and around the steady state.

At the beginning of the period, firms can adjust only labor and equity payout. If there are no financial frictions, the Modigliani-Miller Indifference Theorem implies that firms can simply issue equity to offset their inability to borrow, making the collateral constraint irrelevant to the real economy (Modigliani and Miller 1958). However, firms incur costs for deviating from the steady state target for equity payouts  $\bar{d}$ . The cost function for equity payouts is

$$\phi(d) = d + \kappa(d - \bar{d})^2 \quad (11)$$

These costs have several interpretations, such as managers' desire for smoothing dividends (Lintner 1956), or increasing marginal equity payout costs associated with large share issuances or repurchases. If  $\kappa > 0$ , the assumptions of the Modigliani-Miller Indifference Theorem do not hold and firms cannot costlessly substitute equity for debt. To lower equity payout costs, firms reduce labor as well as dividends in response to a deterioration in financial conditions. Through the labor market, financial shocks propagate to the real economy.

Given the debt subsidy and equity payout costs, the firm's problem is to maximize its value function subject to budget and collateral constraints. Letting  $m'$  denote the stochastic discount factor,

$$V(s, k, n) = \max_{d, n, k', b'} \{d + \mathbf{E}m'V(s', k', n')\} \quad (12)$$

such that

$$(1 - \delta)k + F(z, k, n) - wn + \frac{b'}{R} = b + \phi(d) + k' \quad (13)$$

$$\xi \left( k' - \frac{b'}{1+r} \right) \geq F(z, k, n) \quad (14)$$

The collateral constraint and equity adjustment costs drive significant fluctuations in real as well as financial variables. Letting  $\mu$  denote the multiplier on the collateral constraint, the FOC for labor is given by

$$F_n(z, k, n) = \frac{w}{1 - \mu\phi_d(d)} \quad (15)$$

The tightness of the collateral constraint in the presence of financial frictions drives a wedge between the wage paid to workers and the marginal product of labor, lowering employment. In the presence of debt subsidies and equity payout costs, fluctuations in financial conditions propagate through labor demand to the real economy. As shown in Section 4.2, these shocks can have a stronger effect in the short run than productivity shocks. JQ find that fluctuations in this financial conditions variable account for 30% of the variance of output over the period 1984-2010.

### 3.3 Value of Capital

While introducing an extra financial conditions variable significantly improves the performance of the RBC model, precisely what drives financial conditions remains to be answered. Understanding why and when the borrowing limit fluctuates would shed light on the relationship between real and financial variables, and potentially on ways to moderate the transmission of real shocks to the financial sector and vice versa. I modify the JQ model by incorporating an endogenized financial conditions variable and comparing the moments of dynamic responses of the new model to the original.

An interpretation of financial conditions are that they represent the value the lender places on the capital offered as collateral by the firm. This value of capital, in turn, depends on how the lender can use the capital he receives in the event of default. Assuming capital cannot be consumed directly and that only firms can use it for production, households (who provide the loans) will value it based solely on the price it fetches when being sold to firms. I examine this interpretation in a context where firms face non-constant investment adjustment costs. I denote the endogenized financial conditions variable based on the resale price of capital by  $\xi_k$ .

The timeline for firms' decisions is similar to that in JQ. Firms begin the period by making expenditures financed with an intraperiod loan. After the expenses are paid, production occurs and firms either default or repay the loan. If a firm defaults, the lender

takes the capital net of debt used as collateral for the intraperiod loan,  $k^d = k' - \frac{b'}{1+r}$ . The next period begins with the lender offering to sell  $k^d$  units of capital at price  $p^k$ .

Capital obtained from defaulted firms can be installed and used by other firms as if there were no investment adjustment costs, while capital obtained by investment is affected by investment adjustment costs. The intuition is that the legal, transportation, storage, and other costs of moving capital from defaulted firms to purchasers are proportional to the amount of capital being purchased, whereas the costs firms incur by switching equipment from the production of consumption to capital goods or from ‘breaking in’ new equipment vary with the amount of capital they own. Regardless, in the presence of high investment adjustment costs, firms offer a higher price for seized capital. On the other hand, firms facing low adjustment costs are able to produce their capital for less, and are less willing to purchase capital from the lender.

To find the resale price of capital, I derive the maximum price a firm would be willing to pay a lender for seized capital, denoted by  $p^f$ . Once I have appropriately specified the investment adjustment cost function  $\Phi(\cdot)$  and fixed costs of installing previously used capital  $C$ , I show that an RBC model with endogenized financial conditions  $\xi_k$  mirrors the dynamics of the JQ model with exogenous borrowing limits.

### 3.4 Endogenizing the Borrowing Limit

I assume that lenders have acquired capital from a defaulted firm in the previous period, and that the number of lenders with capital is sufficiently small that the lenders capture all surplus from the sale of capital. The firms’ lack of bargaining power implies that they are indifferent between obtaining capital by purchasing it or by building it themselves, and that  $p^f$  is equal to the cost of gaining an additional unit of capital by investment.

Now that firms may purchase capital, the new demand for an intraperiod loan is

$$l = wn + b + d + i - \frac{b'}{R} + p^d k^d \tag{16}$$

where  $i$  denotes investment. The firm's new problem is to maximize

$$V(s, k, n) = \max_{d, n, k', b'} \{d + \mathbf{E}m'V(s', k', n')\} \quad (17)$$

such that

$$F(z, k, n) + \frac{b'}{R} = b + \phi(d) + i + p^f k^d \quad (18)$$

$$\xi\left(k' - \frac{b'}{1+r}\right) \geq F(z, k, n) \quad (19)$$

The law of motion for capital follows

$$k' = (1 - \delta)(k + k^d) + \Phi(i)k \quad (20)$$

Solving the law of motion for investment and plugging it into the budget constraint,

$$F(z, k, n) + \frac{b'}{R} = b + \phi(d) + \Phi^{-1}\left[\frac{k' - (1 - \delta)(k + k^d)}{k}\right] + p^f k^d \quad (21)$$

I now pinpoint how much a firm would be willing to pay for a unit of capital unaffected by investment adjustment costs. Taking the first order condition with respect to capital purchased from a lender,

$$p^f = \frac{(1 - \delta)}{k} (\Phi^{-1})' \left[ \frac{k' - (1 - \delta)(k + k^d)}{k} \right] \quad (22)$$

If the financial conditions variable is between zero and one, the firms' assets exceed their liabilities, and thus they do not default.<sup>1</sup> Because this is always the case, there is no seized capital to purchase from lenders, and  $k^d = 0$ .<sup>2</sup>

The resale value of capital is equal to its resale price minus acquisition and transportation costs proportional to the amount of capital used as collateral:

$$\xi_k = p^f - C \tag{23}$$

I assume that these costs account for the difference between the steady state values of the resale price of capital and the financial conditions variable  $\xi$ , whose steady state value is denoted by  $\bar{\xi}$ . In the steady state, the resale price is  $1 - \delta$ . Therefore,

$$C = 1 - \delta - \bar{\xi} \tag{24}$$

My endogenized financial conditions variable becomes

$$\xi_k = p^f + \bar{\xi} - (1 - \delta) \tag{25}$$

### 3.5 Equilibrium

A competitive equilibrium consists of (1) household policies for consumption, labor, and loans (2) firm policies for dividend payments, labor, capital, and debt, and (3) a wage and interest rate such that (i) the labor and bond markets clear and (ii) households and firms

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<sup>1</sup>A firm's assets exceed its liabilities if  $k' \geq \frac{b'}{1+r} + l$ , which is equivalent to  $(k' - \frac{b'}{1+r}) \geq l$ . If the financial conditions variable is less than one, the collateral constraint implies that  $(k' - \frac{b'}{1+r}) > \xi(k' - \frac{b'}{1+r}) \geq l$ , and therefore that assets exceed liabilities.

<sup>2</sup>Of course, in reality firms default and their assets are seized. The change in the resale price of capital with respect to seized capital is  $\frac{\partial p^f}{\partial k^d} = -(\frac{1-\delta}{k})^2 (\Phi^{-1})''(\cdot)$ , which is negative for reasonable specifications of investment adjustment costs (see Section 4.1). Incorporating default into the model would likely exacerbate the effects of worsening borrowing conditions: if the collateral constraint tightens, more firms would default and the stock of seized capital would rise. More seized capital would lower its resale value, further tightening the collateral constraint.

act optimally given the wage, interest rate, productivity, the previous period's debt, the previous period's capital, and their budget and collateral constraints.

## 4 Quantitative Exercises

### 4.1 Parameters and Functional Forms

#### Investment Adjustment Costs

The volatility of the resale price of capital is highly sensitive to the specification and convexity of investment adjustment costs. If these costs increase more dramatically in the presence of greater investment, the firm is willing to pay more for capital from a lender to avoid the cost of producing capital itself, raising the resale (and collateral) value of capital. Similarly, if investment falls, the firm can build its own capital for relatively little cost, lowering the resale value of capital.

I employ the investment adjustment costs supplied by JQ,

$$\Phi(i) = \left[ \frac{\delta v \left(\frac{i}{k}\right)^{1-\nu} - \delta v}{1 - \nu} \right] \quad (26)$$

where  $v$  determines the convexity of adjustment costs.  $\Phi_i(\cdot)k = 1$  in the steady state, implying that Tobin's  $q$  is equal to 1 in the steady state (JQ).

The tightness of the collateral constraint is countercyclical in Kiyotaki and Moore (1997) and JQ. Worsening credit conditions cause loans to fall, which restrict firms' ability to pay expenses and produce goods. Therefore, investment adjustment costs should be specified so that the resale value of capital is procyclical. The cyclicity of the resale value can be determined by examining how it responds to a change in the percent accumulation of capital, a procyclical variable. Letting  $\Delta k = k' - (1 - \delta)k$ , (22) and (25) imply that

$$\frac{\partial \xi_k}{\partial \Delta k/k} = \frac{\beta(1-\delta)}{k} \left( \Phi^{-1} \right)'' \left( \frac{\Delta k}{k} \right) \quad (27)$$

For  $\xi_k$  to be procyclical,  $\left( \Phi^{-1} \right)'' \left( \frac{\Delta k}{k} \right)$  must be positive. Therefore,  $\Phi^{-1}(\cdot)$  must be convex: marginal investment adjustment costs must increase as output increases. While there is evidence that individual firms face concave adjustment costs, models with convex adjustment costs are more widely used and produce better empirical results on the macroeconomic level (Cooper 2005).

### Other Parameters and Functional Forms

Households derive utility from consumption  $c$  and disutility from labor  $n$ :

$$U(c, n) = \ln(c) + \alpha \ln(1 - n) \quad (28)$$

$\alpha$  is chosen so that the steady state value of labor is .3: households spend one third of their time working. Firms use Cobb-Douglas production with elasticity of output with respect to capital  $\theta$ . Firms use capital  $k$  and labor  $n$  to produce output  $y$ :

$$y = F(z, k, n) = zk^\theta n^{1-\theta} \quad (29)$$

Parameter values are based on the values from JQ. The time discount factor is chosen to imply average annual returns from shares of 7.3%. The elasticity of production to capital and depreciation rate are standard in the RBC literature. The tax benefit on debt is sufficiently large so that the collateral constraint binds for 1984-2010. The steady state value of financial conditions is chosen so that the steady state ratio of debt to GDP is equal to 3.36, the average for the sample period. The equity payout cost parameter yields the same standard deviation in dividends over output as the data. The persistence and standard deviation of productivity shocks are computed by applying the Solow Residuals method to deviations in output, labor, and capital in the sample period. Letting hats

denote the percent deviation of each variable from its trend, log linearizing (29) implies that

$$\hat{z} = \hat{y} - \theta \hat{k} - (1 - \theta) \hat{n} \tag{30}$$

Productivity is normalized to 1 in the steady state. Lastly, the investment adjustment cost convexity parameter is selected so that the response of the resale price of capital to a productivity shock in the first period matches the standard deviation of a financial shock in the original model.

Parameter	Description	Value
$\beta$	Time Discount Factor	.9825
$\alpha$	Work Disutility Coefficient	1.883
$\theta$	Output Elasticity of Capital	.36
$\delta$	Depreciation Rate	.025
$\kappa$	Equity Cost Function Coefficient	.146
$\tau$	Tax Benefit on Debt	.35
$\bar{d}$	Long-Run Equity Payout Target	.115
$\bar{\xi}$	Steady State Financial Conditions	.1634
$v$	Investment Adjustment Cost Convexity	.095
$\rho_z$	Productivity Shock Persistence	.946

Table 4.1: Parameters

## 4.2 Dynamic Responses

The performance of the model depends heavily on the convexity of investment adjustment costs. To demonstrate this, I estimate models with differing levels of  $v$  and record the correlations between each variable and output.  $\xi$  denotes the correlations of JQ’s model with exogenous financial conditions, and  $\xi_k$  refers to the correlations from my model with the endogenous collateral constraint:

Variable	Correlation with Output					
	Data	$\xi$	$\xi_k$			
		$v = .5$	$v = .10$	$v = .15$	$v = .25$	$v = .5$
$y$	1	1	1	1	1	1
$i$	.94	.99	.99	.99	.99	.99
$c$	.77	.95	.98	.99	.99	1
$n$	.84	.79	.93	.95	.98	.95
$k$	.79	.32	.35	.35	.34	.33
$b$	.38	.07	.76	.70	.63	.58
$w$	.39	.95	.99	.99	.99	.99
$d$	.26	.75	.58	.76	.89	.96
$R$	.46	-.52	-.89	-.95	-.98	-.99
$\mu$		-.82	-.85	-.93	-.98	-.99

Table 4.2: Comparison of Model Correlations

The higher  $v$  is, the more extreme the correlations between variables and output. This is due to the greater sensitivity of the collateral constraint when  $\Phi$  is more concave. The impulse responses of the multiplier on the collateral constraint in response to a standard negative shock for different values of  $v$  demonstrate that more convex costs lead to a more volatile collateral constraint:

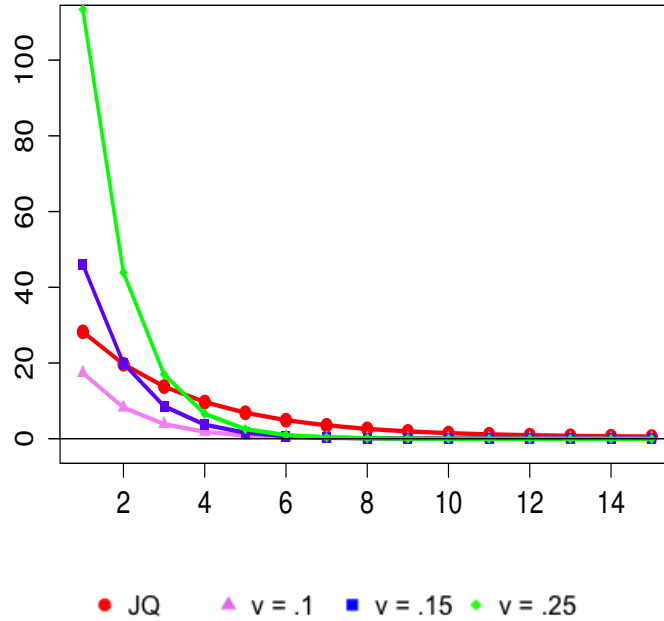


Figure 1: *Response of  $\mu$  to Exogenous Shocks*

The JQ response is denoted by circles, and is the maximum of the dynamic responses to a standard negative financial shock and standard negative productivity shock at each time period. The triangular, square, and diamond lines are the dynamic responses of the multiplier to a standard productivity shock when  $v = .10$ ,  $v = .15$ , and  $v = .25$  in the new model, respectively. These responses confirm that more convex adjustment costs result in more volatile resale values of capital and thus more extreme variations in the tightness of the collateral constraint.

While the tightness of the collateral constraint in my model is highly dependent on the convexity of investment adjustment costs, its persistence is not. Credit shocks in the original model have an autocorrelation coefficient of .971 (set as an exogenous parameter), while the autocorrelation of the resale value ranges from .276 to .593, depending on adjustment cost convexity. Because less convex adjustment costs yield more persistent fluctuations in the resale value of capital, the tightness of the collateral constraint falls to slightly above 0 within six or seven periods, regardless of the peak value of the multiplier.

This faster loosening is due to the specification of the resale value. When an initial shock to resale value occurs, the firm reduces investment both because capital is less productive and because it cannot finance itself as before. Once the capital stock falls, however, the denominator of (22) decreases, raising the resale price. This rapid raising of the resale value of capital, together with the fall in productivity, works to reduce the tightness of the collateral constraint more quickly than in the original model. This behavior implies that the collateral constraint tightens dramatically, but its importance to firms' production diminishes quickly. This short-term tightening is consistent with the responses of real variables in the original model to credit shocks, which are large but short-lived.

I set  $v$  so that the first period response of the resale value to a productivity shock is equal to the first period response of JQ's financial conditions variable to a financial shock in the original model (-.0016). The correlations between variables are as follows:

Variable	Model	$y$	$i$	$c$	$n$	$k$	$b$	$w$	$d$	$R$	$\mu$
$y$	$\xi$	1	.99	.95	.79	.32	.07	.95	.75	-.52	-.66
	$\xi_k$	1	.99	.98	.92	.36	.75	.99	.57	-.82	-.82
	Data	1	.94	.77	.84	.79	.38	.39	.26	.46	
$i$	$\xi$	.99	1	.90	.84	.24	.04	.96	.81	-.59	-.71
	$\xi_k$	.99	1	.95	.96	.24	.68	.99	.65	-.87	-.87
	Data	.94	1	.62	.79	.70	.24	.26	.28	.46	
$c$	$\xi$	.95	.90	1	.61	.47	.13	.85	.56	-.93	-.46
	$\xi_k$	.98	.95	1	.83	.53	.86	.97	.43	-.72	-.71
	Data	.77	.62	1	.82	.48	.82	.12	-.15	.23	
$n$	$\xi$	.79	.84	.61	1	.32	.26	.93	.99	-.93	-.98
	$\xi_k$	.92	.96	.83	1	.02	.44	.94	.84	-.98	-.98
	Data	.84	.79	.82	1	.57	.58	-.06	.14	.48	
$k$	$\xi$	.32	.24	.47	.32	1	.72	.42	.99	-.38	-.34
	$\xi_k$	.36	.24	.53	.02	1	.80	.35	.84	.09	.12
	Data	.79	.70	.48	.57	1	.18	.60	.14	.23	
$b$	$\xi$	.07	.04	.13	.26	.72	1	.23	.13	-.38	-.32
	$\xi_k$	.75	.68	.86	.44	.80	1	.72	-.09	-.26	-.25
	Data	.38	.24	.82	.58	.18	1	-.11	-.58	-.16	
$w$	$\xi$	.95	.96	.85	.93	.42	.23	1	.90	-.38	-.32
	$\xi_k$	.99	.99	.97	.94	.35	.72	1	.62	-.85	-.85
	Data	.39	.26	.12	-.06	.60	-.11	1	.08	-.09	
$d$	$\xi$	.75	.81	.56	.99	.24	.13	.90	1	-.93	-.97
	$\xi_k$	.57	.65	.43	.84	-.33	-.09	.62	1	-.94	-.94
	Data	.26	.28	-.15	.14	.18	-.58	.08	1	.63	
$R$	$\xi$	-.52	-.59	-.93	-.93	-.38	-.38	-.38	-.93	1	.98
	$\xi_k$	-.82	-.87	-.72	-.98	.09	-.26	-.85	-.94	1	.99
	Data	.46	.46	.23	.48	.23	-.16	-.09	.63	1	
$\mu$	$\xi$	-.66	-.71	-.46	-.98	-.34	-.32	-.32	-.97	.98	1
	$\xi_k$	-.82	-.87	-.71	-.98	.12	-.25	-.85	-.94	.99	1

Table 4.3: Variable Correlations

The variable cross correlations in both models are broadly similar to each other. Equity payouts in the JQ model are more procyclical than in mine, and the opposite is true for debt. The correlations for labor are closer to the data in JQ, but my correlations are closer for dividends. Both models yield similar moments between the other real variables.

I compare the impulse response functions of the variables in my model to productivity shocks to those of the original model to both productivity and financial shocks. Based on

the multiplier's behavior, in the first few periods the variables should respond to shocks in the new model similarly to their counterparts in the original model given a financial conditions shock. As the collateral constraint becomes less tight, they should behave as if a TFP shock had occurred in the original model.

I use JQ's estimates of the persistence of shocks and correlation between productivity and financial conditions. The persistence of productivity fluctuations is .9457 and the persistence of financial fluctuations is .9703. The variance of shocks to productivity is .0045, and the variance of shocks to financial conditions is .0098.

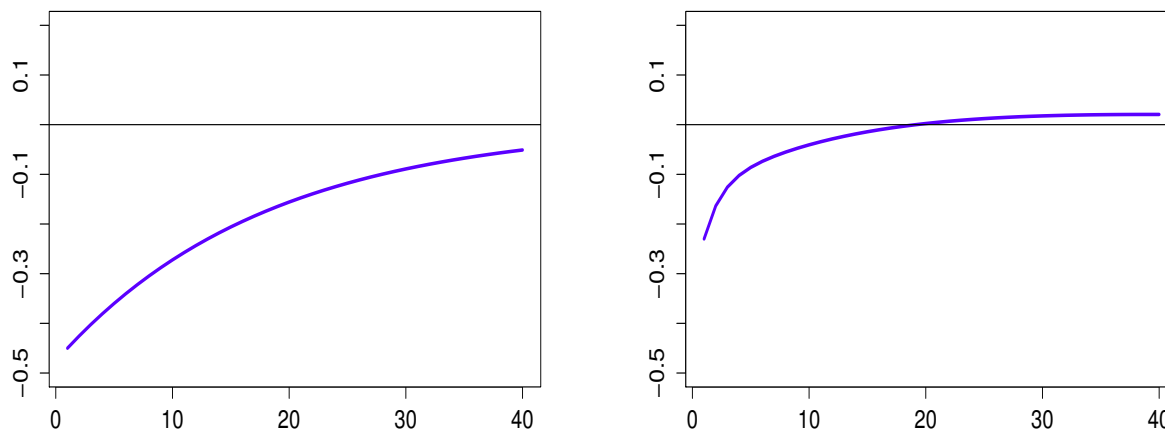


Figure 2: *Response of Productivity and Resale Price to Productivity Shock*

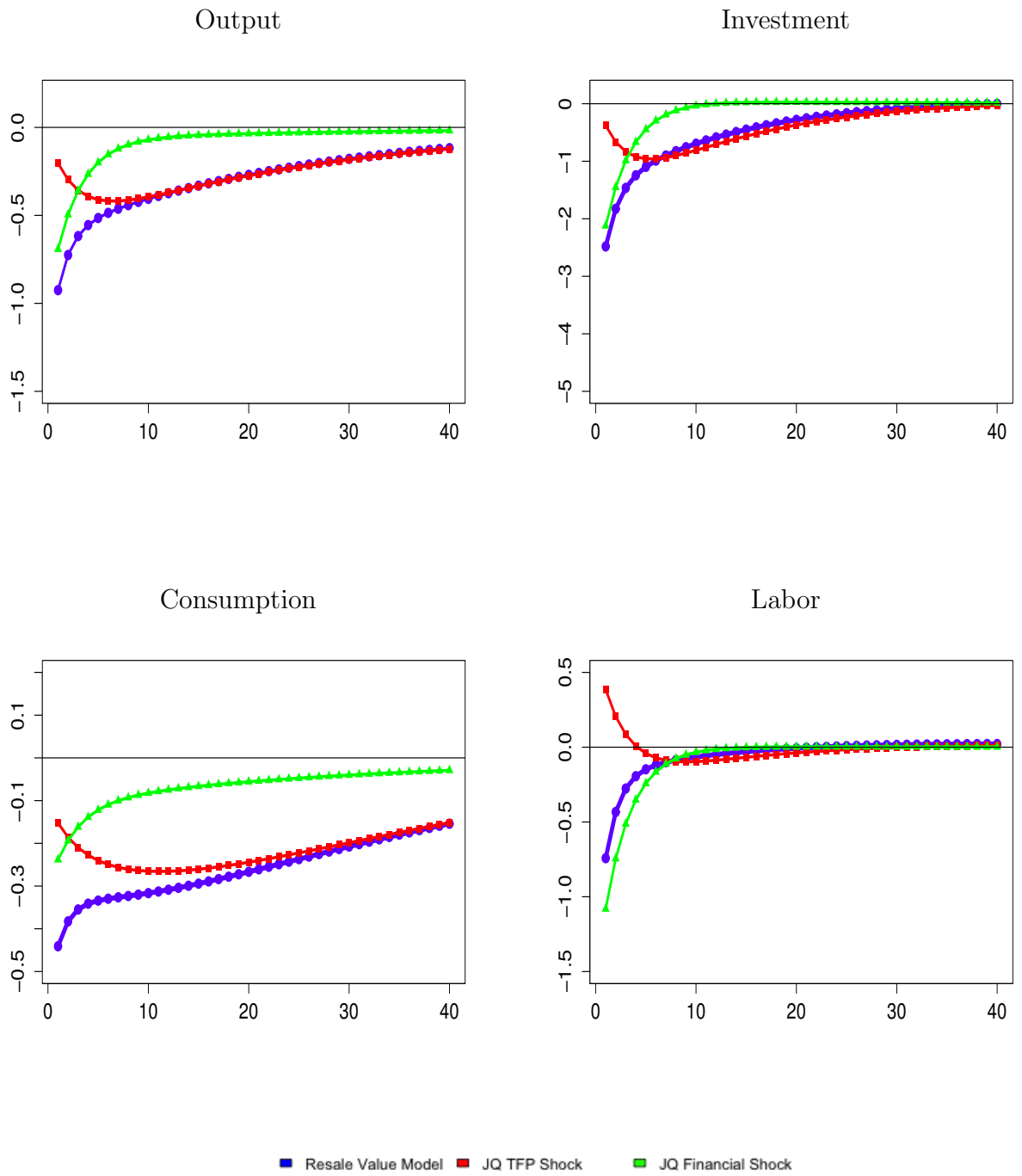


Figure 3: *Impulse Response Functions to Exogenous Shocks*

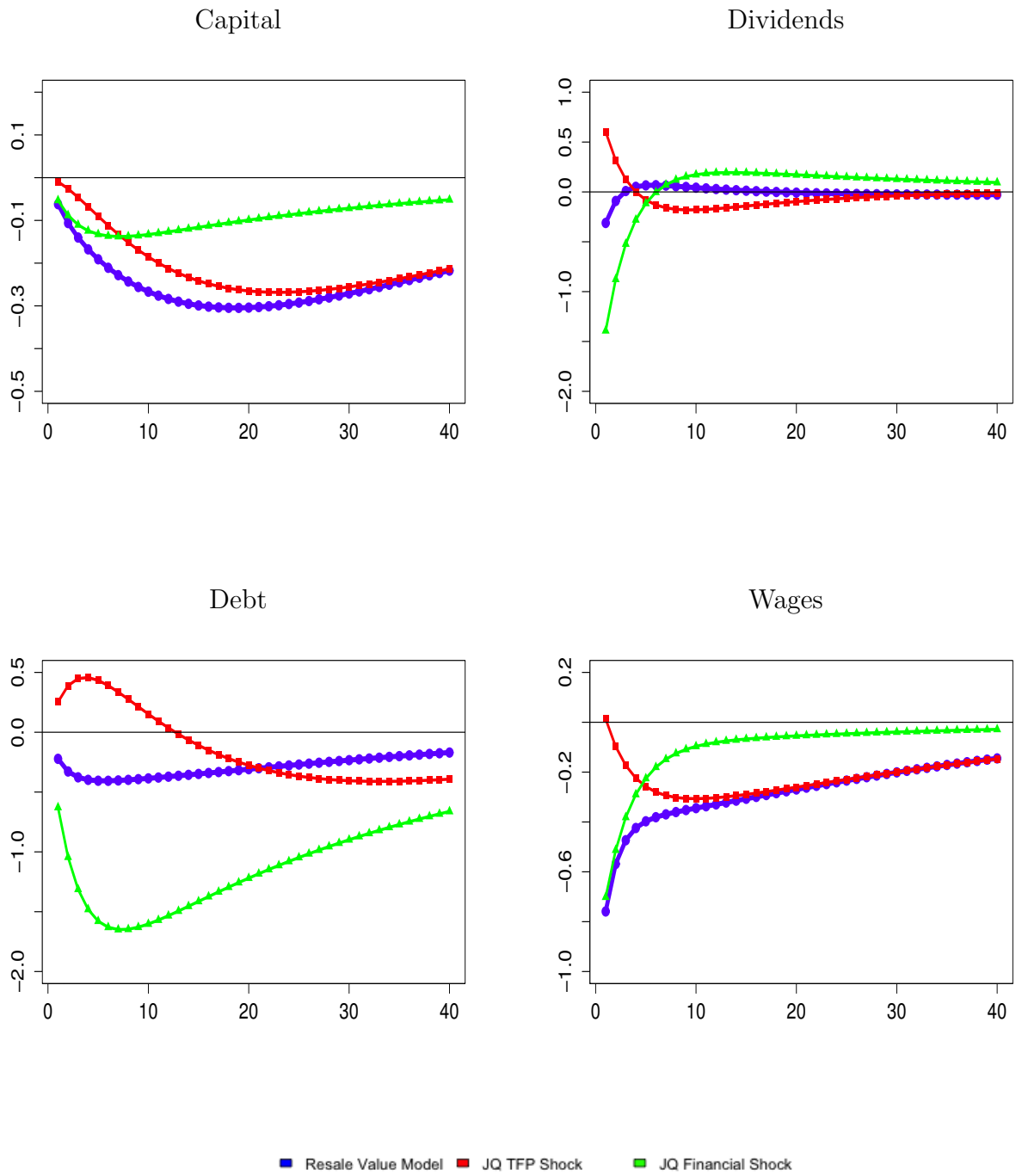


Figure 4: *Impulse Response Functions to Exogenous Shocks (Continued)*

The impulse responses for output, investment, labor, capital, and wages exhibit behavior consistent with an initial shock to financial conditions, followed by the effect of a

productivity shock dominating: they initially trace the impulse response of the JQ model to a deterioration in financial conditions, and then to a fall in productivity. While consumption reacts more dramatically in my model than in the original, its shape mirrors that of consumption's response to a borrowing limit shock, and then a TFP shock. These responses show that the real variables in my model behave similarly to those of the original.

## 5 Conclusion

Financial shocks are an important source of variation in real macroeconomic variables. Empirical studies have found that these shocks explain a high proportion of fluctuations in the real economy for countries at different levels of development and with varying financial systems. In the United States, financial shocks account for up to one third of variance in GDP.

Several specifications have been used to model these shocks, including financial accelerators and credit constraints on households and firms. I examine the JQ model of collateral constraints on firms and replace their exogenous financial conditions variable with an endogenous version. I define financial shocks as changes in the expected resale value of capital. Under the assumption that firms can use capital purchased from a lender without incurring investment adjustment costs, the resale price of capital is equal to the amount a firm must invest to attain one additional unit of capital in the future. When investment adjustment costs are convex, the resale value of capital is procyclical.

Using the new model with endogenous financial conditions, I simulate a negative shock to productivity. Under a reasonable specification of adjustment costs, the resulting correlations between variables and dynamic responses to exogenous shocks from my model are similar to those of JQ. These similarities suggest that investment adjustment costs may be a channel between real shocks and financial conditions.

## 6 Appendix A: First Order Conditions and Constraints

The first order conditions for my model are similar to those of JQ's base model with the addition of the endogenous financial conditions variable  $\xi_k$  and investment adjustment costs.

### 6.1 Firms

$n$ :

$$F_n(z, k, n) = \frac{w}{1 - \mu\phi_d(d)} \quad (31)$$

$k$ :

$$M' \left[ \frac{i'}{k'} \left( 1 + \frac{k'}{k\delta^v} \right) + F'_k(z, k, n)(1 - \mu)\phi'_d(d) \right] + \mu\phi_d(d)\xi_k = \frac{i}{k} \quad (32)$$

$b$ :

$$REM' + \mu\phi_d(d) \frac{R(R - \tau)}{1 - \tau} = 1 \quad (33)$$

Budget Constraint:

$$F(z, k, n) + \frac{b'}{R} = b + \phi(d) + i \quad (34)$$

Collateral Constraint:

$$\xi_k \left( k' - b' \frac{1 - \tau}{R - \tau} \right) = F(z, k, n) \quad (35)$$

Investment Adjustment Costs:

$$i = \Phi^{-1} \left( \frac{\Delta k}{k} \right) = \left[ \frac{1 - v}{\delta^v} \left( \frac{\Delta k}{k} + \frac{\delta v}{1 - v} \right) \right]^{\frac{1}{1-v}} k \quad (36)$$

Endogenized Financial Conditions:

$$\xi_k = \frac{(1-\delta)}{k} \left( \Phi^{-1} \right)' \left( \frac{\Delta k}{k} \right) + \frac{\bar{\xi}}{\beta} - (1-\delta) \quad (37)$$

## 6.2 Households

$n$ :

$$wU_c(c, n) + U_n(c, n) = 0 \quad (38)$$

Euler Equation:

$$U_c(c, n) = \beta \frac{R - \tau}{1 - \tau} \mathbf{E}U_c(c', n') \quad (39)$$

Budget Constraint:

$$wn + b - \frac{b'}{R} + d - c = 0 \quad (40)$$

The effective stochastic discount factor is  $M' = \beta \frac{U_c(c', n')}{U_c(c, n)} \frac{\phi_d(d)}{\phi_d(d')}$ . The first order conditions and constraints give ten equations. Substituting (36) into (32) and (34) to eliminate investment, there are nine equations with nine endogenous variables,  $c$ ,  $n$ ,  $w$ ,  $d$ ,  $b'$ ,  $R$ ,  $k'$ ,  $\mu$ , and  $\xi_k$ , that are affected by three exogenous variables,  $z$ ,  $b$ , and  $k$ .

## 7 Appendix B: Data Sources

### 7.1 Construction of Capital and Debt

The series for these variables are constructed by accumulating quarterly flows from the first quarter of 1952 to the last quarter of 2010. FOF denotes Flow of Funds accounts, NIPA denotes the National Income and Product Accounts, and FRED denotes the Federal Reserve Bank of St. Louis.

Capital follows the law of motion  $k' = (1 - \delta)k + \Delta k$ , where  $\Delta k$  is the net increase in the capital stock.  $\Delta k$  consists of total capital expenditures for nonfinancial businesses (FOF FA105050005.Q) minus consumption of fixed capital by nonfinancial business (FOF FA116300001.Q + FOF FA106300083.Q), all deflated by the price index for business value added (NIPA 1.3.5). The initial value for capital is chosen so that the log of the output to capital ratio displays no trend from 1952-2010.

Similarly, debt follows  $b^{e'} = b^e + \Delta b^e$ , where  $\Delta b^e$  is net new borrowing.  $b^e = \frac{b'}{1+r}$  is used instead of  $b$  because the data series is for the end of the period. After the initial value of debt is set at 94.12, the series for  $b^e$  is constructed by deflating credit market instruments for nonfinancial business (FRED BCNSDODNS) by the price index for business value added.

## 7.2 Other variables

Variable	Data Source
Output	NIPA 1.3.5: Business Value Added
Labor	BLS: Index of Aggregate Weekly Hours: Total Private Industries
Wage	FRED: Nonfarm Business Sector: Real Compensation Per Hour
Interest Rate	FOF: Effective Federal Funds Rate
Consumption	FRED: Real Personal Consumption Expenditures
Dividends	FOF: Nonfinancial corporate business; net dividends paid - Nonfinancial corporate business; corporate equities; liability - Nonfinancial noncorporate business; proprietors' equity in noncorporate business
Investment	NIPA Table 1.1.6: Gross Private Domestic Investment

Table 7.1: Data Sources

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