EXHUMING THE RINDSKOPF REPARAMETERIZATION:

A COMPARISON OF THREE ALTERNATIVES TO THE ANALYSIS OF MTMM DATA

by

YI FAN

(Under the Direction of Nathan Carter)

ABSTRACT

This study evaluated a neglected parameterization approach by Rindskof (1984) and its application to analyzing Multitrait-Multimethod (MTMM) data. Through taking analyses on a Monte Carlo simulation study and a large review of MTMM studies, I examined the Rindskopf raparameterization model (CTCM-R model) and compared its performance with the other two widely applied CFA-MTMM models, namely, the correlated trait-correlated method model (CTCM) and the correlated trait-correlated uniqueness model (CTCU), in regards to their convergence, admissibility, model fit, and parameter estimation biases. Results from analyzing both simulated MTMM data and previous published MTMM data showed that the CTCM-R model serves as a favorable alternative approach to MTMM studies.

INDEX WORDS: Multitrait-Multimethod, Monte Carlo Simulation, Rindskopf Reparameterization

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CHAPTER 1

INTRODUCTION

Since Campbell and Fiske (1959) first proposed the multitrait-multimethod (MTMM) matrix, the use of MTMM data has become essential for establishing the construct validity of psychological measures by demonstrating evidence for or against discriminant and convergent validity (Kenny & Kashy, 1992). Campbell and Fiske's article is one of the most highly cited and influential papers in the history of *Psychological Bulletin* (Sternberg, 1992). As of May 1st, 2015, it had been cited 3,149 times in the PsychInfo database and 13,735 times in Google Scholar.

Campbell and Fiske (1959) introduced the MTMM matrix, which involves all of the intercorrelations between measures of a set of traits, and each trait is assessed by a set of various measurement procedures (i.e., methods). Using the MTMM matrix allows one to make inferences of the variances due to traits distinguishing from the variances due to method effect. As such, the primary use of this technique is establishing construct validity of psychology measurement, including convergent and discriminant validity. In addition to validating psychological tests, the MTMM framework is also applied with a wide variety of purposes, such as assessing consistency of multisource performance ratings, comparing differences across time or occasions, and evaluating performance across different exercises in assessment centers (Lance, Woehr, &Meade, 2007). Researchers have applied the MTMM matrices in wide and diverse disciplines besides psychology, such as social science (Watson & Clark, 1992), physical science (Marsh, Martin, & Jackson, 2010), education (Wong, Day, Maxwell, & Meara, 1995), management (Arthur, Woehr, & Maldegen, 2000), and communication studies (Kotowski, Levine, Baker, & Bolt, 2009).

Along with their proposal, Campbell and Fiske introduced an informative approach for analyzing MTMM data. This approach received criticism regarding its subjective nature, and several quantitative approaches for analyzing MTMM data have later been developed, including the analysis of variance(ANOVA, Guilford, 1954), confirmatory factor analysis (CFA, Joreskog, 1969), the composite direct product model (CDP, Browne, 1983), and generalizability theory (Woehr, Putka & Bowler, 2012). Of these various approaches, CFA models for the analysis of MTMM data has received the most attention (Kenny & Kashy, 1992; Schmitt & Stults, 1986).

The CTCM model and the CTCU model are the two most popular CFA parameterization approaches to MTMM data, but each has their empirical and/ or theoretical limitations. The CTCM model, though faithful to Campbell and Fiske's (1959) primary MTMM proposal, suffers from nonconvergence and/or inadmissibility issues. Whereas, the CTCU model, as the most commonly used alternate, generates convergent and proper solutions more frequently than the complete CTCM model (Marsh & Bailey, 1991). However, criticisms have been raised by several researchers that challenge the conceptual soundness and estimation accuracy of the CTCU model (Lance, Nobel, & Scullen, 2002; Conway, Lievens, Scullen, & Lance, 2004).

Another recommended alternative model derived from a reparameterized form of the general CTCM model. Rindskopf (1984) proposed a parameterization model of a general CFA model, which imposes non-negative restrictions on uniqueness estimates. This reparameterized form of a CFA-MTMM model, namely the CTCM-R model, is a mathematically equivalent specification of the CTCM. The CTCM-R model overcomes several deficiencies with the CTCM model yet not changing the conceptualization of method variance.

Many studies compared the CTCM model and the CTCU model with simulated data (Conway et al., 2004., Marsh & Bailey, 1991; Zhang, Jin, Leite, & Algina, 2014), or previous published data (Lance et al., 2002). In contrast, much fewer studies have focused on the CTCM-R model (Dillon, Kumar, & Mulani, 1987; Lance & Fan, 2014). Dillon and colleagues compared the CTCM-R model with another two alternative models to CTCM model in their simulation study, and suggested the CTCM-R model may not provide solutions to negative error variance estimates due to theoretical lack-of-fit or model misspecifications. In contrast, Lance & Fan (2014) found the CTCM-R model largely solves convergence and admissibility problems of the CTCM model in their re-analysis study on 318 MTMM matrices surveyed 258 studies. The current study is designed to test the CTCM-R model not only with simulated data, but also with published MTMM data. The purpose of the current study is to exhume the long proposed but ignored CTCM-R model, and to systematically compare its performance with the other well-known models (the CTCM and CTCU models). Finally, this study provides suggestions for choosing CFA-MTMM models for the analysis of MTMM data.

The following chapter reviews literatures on CFA-MTMM models. It first begins with Campbell and Fiske's (1959) proposal and briefly introduces their original approach to the analysis of the MTMM data. Next, it offers an overview on CTCM model and the CTCU, and explains several theoretical and/or empirical problems associated with these two models. Rindskopf's reparameterization of the CTCM model (i.e. CTCM-R model), as follows, is introduced as an alternative approach to MTMM data. In the end of the chapter, the purposes of the current study are presented.

The present research effort consists of two studies, a simulation study and a study using previously published data. Chapters 3 and 4 describe these two studies, respectively. Each chapter describes the study design, method, results, and a brief discussion. Chapter 5 summarizes the present study's results, limitations, and proposes recommendations for future MTMM studies.

CHAPTER 2

LITERATURE REVIEW

Campbell and Fiske's (1959) Proposal

MTMM data refers to measures of a set of traits, and each trait is assessed by as a set of various measurement procedures (i.e., methods). Each measure, that reflects a particular trait measured by a particular measurement procedure, is called a Trait-Method Unit (TMU; Campbell & Fiske, 1959). A typical MTMM matrix is a correlation matrix that contains all possible inter-correlations across the TMUs. Campbell and Fiske (1959) provided an informative approach to establish construct validity based on the MTMM matrix. As shown in *Table 1,* an MTMM matrix can be divided into three components: the heterotrait-monomethod correlations (HTMM), the monotrait-heteromethod correlations (MTHM), and the heterotrait-heteromethod correlations (HTHM). Campbell and Fiske's proposal for establishing inferences of construct validity is based on the patterns among these trait-method correlations: (a) significantly non-zero and sufficiently large MTHM correlations indicate convergent validity and encourage further validity tests, (b) MTHM correlations are larger than HTHM and HTMM correlations, and (c) the pattern of trait interrelationship is consistent across all of the HTMM blocks. The last two criteria support the discriminant validity.

Although Campbell and Fiske's approach has served as an informative guideline for establishing construct validity of a psychological measure (Messick, 1995), their guidelines receive criticism for their qualitative nature (Widaman, 1985). Based on arbitrary judgments upon subjective comparisons, it does not provide a quantitative computation or a statistical significance test for convergent validity, discriminant validity, or the presence of method effects (Widaman, 1985; Marsh & Grayson, 1995). Therefore, a number of more quantitative approaches have been further proposed for the analysis of MTMM data, including the analysis of variance (ANOVA, Guilford, 1954), confirmatory factor analysis (CFA, Joreskog, 1969), the composite direct product model (CDP, Browne, 1983), and generalizability theory (Woehr, Putka & Bowler, 2012). While controversies and debates over these MTMM analytical methods continue, the most popular approach for the analysis of MTMM matrices is some application of CFA modeling (Schmitt & Stults, 1986; Kenny & Kashy, 1992).

Previous researchers (Schmitt & Stults, 1986; Marsh & Hocevar, 1985) explained several advantages of the CFA-MTMM approach. First, the CFA approach allows estimation of the effect of individual factors, such as the size of trait and method factors. Researchers can test hypotheses on specific questions on these factors, such as the statistical significance of trait or method factor loadings, trait inter-correlations, and method inter-correlations. Given estimated individual factors, it also allows an omnibus model-fit test on the overall hypothesized model, as well as tests on a set of specified nested models. Widaman (1985) developed a general procedure of testing a CFA-MTMM model, which specified a comprehensive taxonomy of models with nested relationships and provided a procedure for comparing these models. This comparative process allows one to make quantitative inferences of the degree of convergent and discriminant validity, as well as the presence of method effect. Because of the estimation and theoretical advantages, the CFA-MTMM models became the top choices to MTMM data.

The Complete CFA Model (CTCM Model)

A CFA model specifies the relations of observed variables to underlying latent constructs. For MTMM data, a CFA approach specifies the relations between MTMM measures with underlying Traits and Methods factors. For example, a Trait-Method Unit (TMU) (e.g., Depression score assessed the Beck Depression Inventory) can be decomposed and effected by three additive components: a Trait component (Depression), a Method component (BDI scale), and a Uniqueness component. The model specification equation can be expressed as:

$$TMU_{ij} = \lambda_{T_{ij}} T_i + \lambda_{T_{ij}} M_j + \delta_{ij} \qquad (1)$$

where TMU_{ij} is the trait-method unit that corresponded to the *i*th Trait as measured by the *j*th Method, $\lambda_{T_{ij}}$ and $\lambda_{M_{ij}}$ are the factor loadings of the TMU_{ij} on the corresponding *i*th Trait(T_i) and *j*th Method(M_j) respectively, and δ_{ij} refers to the residual, or error in predicting the TMU_{ij}.

Given that the standard assumptions of the CFA model (Lance & Vandenberg, 2001), it can be assumed that the variance-covariance matrix (Σ) for the observed TMU_{ij}'s is of the form:

$$\sum = [\Lambda_T|\Lambda_M] \frac{[\Phi_{TT'}|0]}{[0|\Phi_{MM'}]} [\Lambda_T|\Lambda_M]' + \Theta_\delta \qquad (2)$$

where Λ refers to the factor pattern matrix, which can be partitioned as$[\Lambda_T|\Lambda_M]$, where Λ_T and Λ_M refer to the factor loadings linking each TMU_{ij} to the corresponding *i*th Trait(T_i) and *j*th Method(M_j); $\Phi_{TT'}$,and $\Phi_{MM'}$ denote the covariance matrix of Traits and Methods factors, respectively; and finally, Θ_δ is a diagonal variance-covariance matrix of residuals δ_{ij}. Because this model parameterizes inter-correlated Traits factors and inter-correlated Methods factors (*Figure 1*), it is called the CTCM model and is most faithful to Campbell Fiske's conceptualization of MTMM design (Lance et al., 2002; Kenny & Kashy, 1992). Notably, the CTCM model is the same as the Model 3C in Widaman's (1985) taxonomy. Widaman developed the general taxonomy of CFA-MTMM models, which includes other sub-models of the CTCM model for empirically testing the convergent and discriminant validity and the presence of method effects.

Limitations to the CTCM Model

Although the CTCM model has theoretical strength in explaining underlying factor structures of MTMM data, it suffers from statistical estimation problems of nonconvergence, underidentification, and inadmissibility (Brannick & Spector, 1990; Kenny & Kashy, 1991). Nonconvergence refers to the problems when estimation algorithms (e.g., maximum likelihood) cannot achieve minimal improvement in parameter estimates during the iterative process. Studies have shown that the CTCM model frequently encounters difficulties of convergence in statistics programs, especially when sample size is small (e.g., Brannick & Spector, 1990; Kenney & Kashy, 1992; Marsh & Bailey, 1991). Model identification problems refer to the situations in which the model has more parameters to estimate than pieces of information that the data provided. As summarized by Lance et al. (2002), the CTCM model has negative degrees of freedom for 2T2M, 2T3M, and 3T2M design. Furthermore, a number of studies have suggested that the CTCM model experiences empirical underidentification problems due to small sample sizes (Marsh & Bailey, 1991; Marsh & Grayson, 1995) and equal Method and Trait loadings (Brannick & Spector, 1990; Kenny & Kashy, 1991). As such, the CTCM model is known to return inadmissible solutions with ill-defined parameter estimates, such as negative unique variances, and standard factor loadings and correlations have absolute values are greater than one. One simulation study suggested the CTCM model often performs poorly and returns proper solutions for only 22%-24% of MTMM matrices (Marsh & Bailey, 1991).

The CTCU Model

Due to the estimation problems inherent in the CTCM, more flexible alternative parameterizations have been sought. Perhaps the most frequently used substitute for the CTCM is the correlated trait-correlated uniqueness (CTCU) model. Developed by Marsh (1989), the CTCU model specifies the relations between TMUs with the underlying Traits and Uniqueness variances, and there are inter-correlations between the Uniqueness variances if they share a common Method (*Figure 2*). CTCU model does not explicitly specify Method factors, but instead it incorporates method effects with uniqueness covariance between indicators arising from common method sources. The model expression for the CTCU model can be written as:

$$TMU_{ij} = \lambda_{T_{ij}} T_i + \delta_{ij} \tag{3}$$

where $\lambda_{T_{ik}}$represents the factor loadings of TMU_{ik} on corresponding Trait factor (T_i); and δ_{ij} refers to a composite effect of a systematic uniqueness (s_{ij}), a random error (e_{ij}), and a Method effect corresponding to this measure (M_{ij}) ($\delta_{ij} = s_{ij} + e_{ij} + M_{ij}$). In the CTCU model, all Trait variables T_i are inter-correlated; and residuals of variables that measured by the same Method are allowed to intercorrelate. Accordingly, the CTCU model expresses a covariance MTMM matrix as:

$$\Sigma = \Lambda_T \phi_{TT'} \Lambda_T^{'} + \Theta_\delta \tag{4}$$

where Σ represents the MTMM covariance matrix; Λ_T refers to the factor loadings pattern matrix linking each TMU_{ij} to the corresponding Trait(T_i); $\phi_{TT'}$ denotes the covariance matrix of among Trait factors; and Θ_δ is a symmetric covariance matrix with *m* sub-covariance matrices along the diagonal (*m* = number of Methods in MTMM design). These sub-covariance matrices are composed of residual variances corresponded to TMUs on the diagonal and covariance among residuals that share the same Method off the diagonal. Residual covariances between methods and traits are set to zero.

The CTCU model more frequently produces empirically proper solutions and results with better model fit than the CTCM model (Marsh, 1989,1992). However, the CTCU model has received substantial criticism. Lance et al. (2002) argued that the CTCU model has severe model design and analytic problems. For example, since the CTCU model doesn't estimate the method effect explicitly, it creates an "unmeasured variables problem" which leads to biased estimates of model parameters. In addition, the CTCU model was built on the assumption that Method factors are independent from each other, which is unrealistic in practice and in contradiction to the average method factor correlation of .31 that was found in a review study on published MTMM matrices (Conway, Lievens, Scullen & Lance, 2004). Moreover, due to the model misspecification inherent in the CTCU, estimates of trait variance tend to be biased upward (Conway et al.).

In sum, the CTCU model outperforms the CTCM model in terms of empirical convergence and admissibility (e.g. Marsh and Bailey, 1991). However, the CTCU model has severe theoretical and statistical problems (Lance et al., 2002). Given the empirical problems associated with CTCM model, and the theoretical and empirical issues with CTCU model, the present study evaluated a reparameterization form of the CTCM model. This reparameterization form of CFA models was developed by Rindskopf thirty years ago, yet has not received nearly as much attention as the above two models.

Rindskopf's Reparameterization of the CTCM Model (CTCM-R)

Rindskopf (1983, 1984) presented a case to reparameterize a CFA model in order to implement non-negative restrictions on uniqueness estimates. This modification was subsequently applied to MTMM studies. Applying Rindskopf's reparameterization to CFA-MTMM model creates a mathematically equivalent transformation of the CTCM model (i.e., CTCM-R model). Specifically, for a model with *m* Method and *t* Trait factors in a fully-crossed MTMM matrix (i.e. a p*p MTMM matrix, where p= m*t), the CTCM-R model introduces *m*t* unique variables to represent the effect of uniqueness. Each Trait-Method Unit (TMU) is linked to one Method factor, one Trait factor, as well as one Unique factor. Variances of the unique variables are fixed to be 1.0, as same with the variances of the Trait and Method factors. Estimatied unique variance is calculated as the squares of the factor loadings linking the unique variables to TMUs. As such, no matter whether positive or negative the estimated factor loadings are, estimated uniqueness variances are constrained to be non-negative.

Rindskopf's reparameterization of the CTCM model Σ can be written as:

$$\Sigma = \left[\Lambda_T \, \Lambda_M \, \sqrt{\theta_U}\right] \begin{bmatrix} \Phi_{TT'} & & \\ 0 & \Phi_{MM'} & \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \Lambda_T' \\ \Lambda_M' \\ \sqrt{\theta_U'} \end{bmatrix} \tag{5}$$

where Λ_T and Λ_M refer to the Trait and Method factor pattern matrix, respectively, and $\sqrt{\theta_U}$ is a diagonal matrix linking unique variables to TMUs; $\Phi_{TT'}$ and $\Phi_{MM'}$ denote the covariance matrix of Traits and Methods factors, respectively; variance of unique variables are fixed to 1.0, as the identity matrix in the Phi matrix.

In addition to ensuring non-negativity of unique variances, the CTCM-R model enjoys its implementation flexibility using computer programs, such as LISREL (Rindskopf, 1983) and Mplus (Lance & Fan, 2014). As a result, the CTCM-R model has been applied to several MTMM studies (e.g., Widaman 1985; La Du & Tanaka, 1989; Lance, Dawson, Birkelbach & Hoffman, 2010). However, very few studies have examined the performance of the CTCM-R model and compare its effectiveness with other models. Dillon, Kumar, & Mulani (1987) compared the CTCM-R model, an alternative parameterization model developed by Bentler (1976), and a model with uniqueness variance set to zero when Heywood cases occur. Dillon et al. didn't recommend the first two modified parameterizations of CFA model, and commented that those reparameterizations may produce zero error variance with large standard error estimates. However, the CTCM-R model guarantees to prevent Heywood Cases and shows its advantages over the CTCM model in re-analyzing reviewed MTMM matrices (Lance, Fan, Siminovsky, Morgan, & Shaikh, 2014). The CTCM-R model is the focus of the study, and its performance for analyzing MTMM data was evaluated in a simulation study for the first time.

Study Purposes

The goal of this study is to compare the Rindskopf reparameterization (CTCM-R) model to the other two frequently-used CFA models in the analysis of MTMM data. The performance of the CTCM and CTCU models has been systematically studied by a number of researchers (Conway et al., 2004; Lance et al., 2002; Marsh & Bailey, 1991). However, there is a lack of studies focus on the performance of the CTCM-R model, which ensures to solve the Heywood problem associated with the CTCM model.

In the following chapters, three CFA-MTMM models are empirically evaluated based on three criteria: model convergence and admissibility rates, model fit, and accuracy of the parameter estimates. These model performance criteria are assessed in two separate studies. Study 1 is a simulation study that employs the Monte Carlo technique. In addition to model convergence, model admissibility, and model fit, since population parameters are pre-defined in the simulation study, Study 1 compares of the parameter estimates to known population parameter values for examining accuracy of parameter estimates. To increase the ecological validity of the current study, a review study of published MTMM matrices is introduced. In essence, Study 2 re-analyzes published matrices using the CTCM, CTCM-R and CTCU models. By doing so, Study 2 provides an overview on the performance of the CTCM-R model with the real data comparing with the other two competing models.

CHAPTER 3

STUDY 1: EVALUATION OF THE MTMM MODELS ON THE BASIS OF SIMULATED MTMM MATRICES

METHOD

Study 1 is a simulation study that aims to examine the CTCM-R model performance under different simulated conditions. CTCM and CTCU models serve as benchmarks for comparison purposes. This study has four major steps: (a) generating model-implied population covariance matrices based on pre-defined population values, (b) analyzing sample data with three CFA-MTMM models (i.e., CTCM, CTCM-R, and CTCU models), (c) obtaining model convergence and admissibility rates, goodness-of-fit indices, and parameter estimates, and finally, (d) examining and comparing the CTCM-R model performance with the other models, in these aspects: model convergence and admissibility, model-data fit, and accuracy of parameter estimates (i.e., estimation bias of model estimates from true population values).

Population Values

In simulation studies, population covariance matrices are simulated to represent variations in data characteristics that might occur in real data. Accordingly, in the current study, population parameters and their values were defined to represent a wide range of realistic MTMM studies toward maximizing the ecological validity of the simulation and the applicability of its findings. As such, population values for the current simulation study came from the latest large-scale review of MTMM studies (Lance et al., 2014), with references from several previous MTMM reviews and simulation studies (Conway et al. 2004, Lance et al, 2007, Marsh & Bailey, 1991). Below I describe the generation of population matrices to reflect various conditions likely to be faced by data analysts in application.

Number of Traits/ Number of Methods. The 20th, 50th, and 80th percentiles for number of traits and methods from Lance et al.'s literature review are presented in *Table 2*. To make the simulation study feasible and manageable, two levels were chosen for number of Traits/Methods: three and five. As such, the population matrix had four matrix sizes: (1) three traits and three methods (3T3M), (2) three traits and five methods (3T5M), (3) five traits and three methods (5T3M), and (4) five traits and five methods (5T5M). These matrix sizes cover a wide range of matrix sizes based on the literature review, and can be found in other simulation studies (Conway et al., 2004; Lance, Woehr, & Mead, 2007).

Sample size. Performance of model estimation was evaluated at four sample sizes: (1) 100; (2) 250; (3) 500; and (4) 1000. These values were chosen with reference to the 20th, 50th, and 80th percentiles of sample sizes (N=71, 183, and 421) associated with MTMM matrices reviewed by Lance et al. (2014) and the sample size values from other simulation studies (Conway et al., 2004; Marsh & Bailey, 1991; Tomas et al., 2000). The lowest chosen population value for the current study was between the 20th percentile from Lance et al.'s review (N=71) and the lowest values chosen in other two other simulation studies (N=125; Conway et al., 2004; Marsh & Bailey, 1991). The largest sample size value was seen in one simulation study by Marsh & Bailey (1991). The other two values were chosen because they were not only frequently replicated by previous simulation studies (Conway, et al., 2004; Marsh & Bailey, 1991; Tomas et al., 2000), but also better represented the wide range of sample sizes in reviewed MTMM studies (Lance et al., 2014; Conway et al., 2004).

Trait and Method Factor Loadings. Population values for factor loadings were chosen based on the general rule of setting population values, that is, the 20th 50th, and 80th percentiles of the summary statistics from the literature review by Lance et al. (2014). As such, the population values for Trait loadings were .31, .50, and .69. The population values for Method loadings were .17, .28, and .48. Factor loadings within factors were varied by $\pm$.10 so that the averages of factor loadings equal the mean population values. An example of factor loading matrix for a 5T3M design is presented as the Lambda Matrix in *Table 3*.

Trait and Method Factor Correlations. Population Traits and Method correlations also comply with the 20th 50th, and 80th percentiles from Lance et al.'s (2014) literature review. As such, the population Trait correlations were .07, .36, and .61; and the population Method correlations were .02, .29 and .58. Similar to factor loadings, factor correlations were varied by $\pm$.10 so that the average factor correlations equal to the population values. An example of Trait correlations matrix (Phi Matrix) for a 5T3M design is presented in *Table 3*.

To summarize, the current study can be expressed in terms of the factorial design: 4 (Sample Size: 100, 250, 500, 1000) $\times$ 3 (Trait Factor Loading Size: .31, .50, .69) $\times$ 3 (Trait Correlation Size: .07, .36, .61) $\times$ 3 (Method Factor Loading Size: .17, .28, .48) $\times$ 3 (Method Correlation Size: .02, .29,.58) $\times$ 2 (Number of Traits Factors: 3, 5) $\times$ 2 (Number of Method Factors: 3, 5). There were a total of $4\times3\times3\times3\times3\times2\times2$=1296 unique conditions considering all above combinations. With reference to a number of previous simulation and review studies on MTMM data, the current study have the most comprehensive simulation design that includes population parameters covering the greatest range of values.

Procedures

The population covariance matrices were constructed through the general formula:

$$\Sum = [\Lambda_T|\Lambda_M]\frac{[\Phi_{TT'}|0]}{[0|\Phi_{MM'}]}\frac{\Lambda_T'}{\Lambda_M'} + \Theta_\delta \tag{6}$$

A number of statistics programs can be used to generate covariance matrices (e.g., R, SAS, Mplus) for Monte Carlo study. In the current study, I wrote syntax in R programs to generate population covariance matrix and sample data. Specifically, population MTMM matrices were created with pre-defined population values. Once all model-implied population matrices were generated, the synthetic data of 100 replications for each population condition were generated from the multivariate norm random function (rmvnorm()) in R. In the end, a total of 129,600 sample covariance matrices were created (1296 population conditions times 100 replications).

Next, LISREL 8 (Jöreskog & Sörbom, 2004) was used to analyze sample data with the three CFA models (i.e., CTCM, CTCM-R, and CTCU). LISREL provided outputs containing model convergence and admissibility indicators, a set of model-fit indices (e.g. CFI, TLI, χ^2, RMSEA), and parameter estimates. Finally, these outputs were saved externally, gathered, and further summarized for model performance comparisons.

Result

Convergence and Admissibility

Model performance was summarized and analyzed using SAS. There was a total of 388,791 solutions, of which 331,937 of the solutions (85.4%) were convergent and 213,131 of the solutions (54.8%) were convergent and admissible. A solution was considered convergent if it met default convergence criteria within 500 iterations. The convergent solution is also admissible if this solution does not have out-of-boundary values for the parameter estimates, such as negative unique variances, or standard factor loadings and correlations with absolute values greater than one.

Figure 3 presents the convergence and admissibility rates under different models (i.e., CTCM, CTCM-R, and CTCU). All three models had fair convergence rates between 70% and 95%. The CTCM model, which was known for its tendency to suffer nonconvergence problems (Marsh & Bailey, 1990), showed a lower rate (70%) than the other two models (95% for the CTCM-R model and 91% for the CTCU model). Although all the three models resulted in convergent solutions fairly frequently, the solutions were less likely to be admissible. Only around 28% of the outputs from the CTCM model were both convergent and admissible, which means parameter estimates from the CTCM model were prone to have ill-defined values. This result was consistent with previous simulation studies that the CTCM model often resulted in poor admissibility rates (i.e., 36% in Conway et al., 2004; 22% ~24% in Marsh & Bailey, 1991; 39% in Lance et al., 2007), while the CTCM-R model and the CTCU model, as expected, showed superiority over the CTCM model in terms of admissibility with success rates at 63% and 69%, respectively.

Table 4 presents the influences of MTMM study designs on three models' convergence and admissibility. A small sample size adversely affected the convergence and admissibility of all three models, yet the influences were not equal. The CTCM model, in particular, was greatly impacted by the small sample size and showed an extremely low admissibility rate at 7% when the sample size was 100, while the CTCM-R model and CTCU model maintained intermediate admissibility rates at 47% and 53%, respectively. In addition to the small sample size, fewer Methods reduced the successful admissibility rates of the three models. Admissibility rates for CTCM, CTCM-R and CTCU models were higher for MTMM matrices with five Methods than those matrices with three Methods. Surprisingly, the effect of the number of Traits was not significant.

Model Fit

When evaluating model fit, I analyzed only the datasets for which convergent and admissible solutions were found. As noted by Brannick and Spector (1990), researchers should not directly report parameter estimates or model-fit indices from ill-defined solutions. Thus, this study tested the model fit indices from the convergent and admissible solutions only (N=213,131).

Five model-fit indices were reported: χ^2, SRMR, RMSEA, TLI, and CFI. These indices were chosen based on their performance in detecting model-fitting discrepancies, sensitivity to sample size, and model misspecifications (Hu & Bentler, 1999), and their general popularity in the literature. *Table 5* shows the averaged indices scores across different analytic models, matrix sizes, and sample sizes. These mean scores fell into the suggested ranges for good model-data fit (Hu & Bentler, 1999; TLI$\geq$.95, CFI$\geq$.95, SRMR$\leq$.08, and RMSEA$\leq$0.06), which implied the convergent and admissible solutions from all three models fit simulated data quite well. When looking at each model closely, the CTCM model complied with the population matrix generating formula (6), so it is not surprising to find that the CTCM model has lower SRMR and RMSEA means and higher TLI and CFI means than the other two models. Same for the CTCM-R model, which offers a mathematically equivalent form of the CTCM model, resulted with good model fit to synthetic data. However, the CTCU model, which did not comply with simulated model structure, still appeared to fit the synthetic MTMM data quit well. Lance, Woehr, & Mead (2007) also found in their simulation study that, the CTCU model could achieve good model-data fit, even though it was the wrong model. Moreover, for smaller matrix sizes (<5T5M), the mean χ^2

for the CTCM and CTCM-R models were less than the degrees of freedom. In contrast, the mean χ^2 from the admissible and convergent solutions for the CTCU models were usually slightly greater than degrees of freedom. One possible explanation is that convergence and admissibility criteria had filtering out the poor fitting solutions from the CTCM and CTCM-R models so that fit indices from these two models suggested well model fit. However, the CTCU model displayed a good convergence, admissibility, and even model-data fit, even though it was not the correct model.

Based on suggested cutoffs from Hu and Bentler (1999), the good model-fit rates of three models are presented in *Table 6*. Data for *Table 6* were obtained from convergent and admissible solutions (N=213131). (a) CTCM model appeared to have highest good-fit rate because the best fitting models were not censored out as being improper – so there are far fewer number of convergent and admissible solutions for CTCM model than the other models. (b) CTCM-R model also fit data well with high good-fit rate, but there are a good amount of convergent and admissible solutions generated. And again (c) CTCU model has a decent good-fit rate, even though it is the wrong model. Therefore, CTCM-R model appears best for generating convergent and admissible solutions that are good-fit to synthetic data. Additionally, it should be noted that the CTCM-R kept a decent model-fit rate at 65% when the sample size was 100, compared to the 54% and 36% success rates for the CTCM and CTCU models. Finally, the number of Traits/Methods did not have significant effect on models fit.

Accuracy of Parameter Estimation

One primary advantage of simulation design is that simulation studies allow researchers to examine the accuracy of parameter estimates. Therefore, in the current simulation study, comparing the trait parameter estimates (i.e., trait loadings and trait correlations) with pre-defined trait parameter values could cause inferences about the accuracy of trait parameter estimates. Estimation biases were the dependent variables, which were obtained by subtracting the averaged trait parameter estimates from the corresponded pre-defined population values. An estimation bias score can be positive or negative, which indicates upward or downward estimation bias. The variances and standard deviations of estimation biases lead inferences about the reliability of estimation. As such, for trait loadings and trait correlation estimates, there were two dependent variables for accuracy of parameter estimation, including the bias of trait loadings and the bias of trait correlation estimates.

Figure 3 and *Figure 4* presented the distributions of estimation biases for trait loadings and trait correlations, respectively. A distribution with the mean at zero and small standard deviation indicates unbiased and accurate parameter estimation. As shown in *Figure 3*, trait loadings biases distributions for three models closely centered at zero ($\mu_{CTCM=.00}$, $\mu_{CTCM\text{-}R=.03}$, $\mu_{CTCU=.00}$). The standard deviation for the estimation bias of trait loadings for the CTCU model was 0.14, smaller than those for the CTCM and CTCM-R models, with both standard deviations at 0.23. It was noted that there were a considerable proportion of solutions from the CTCM and CTCM-R models that had a positive estimation biases of trait loadings between .15 and .25. This indicated the factor loading estimates from these two models tended to be greater than the corresponded population values. In other words, the CTCM and CTCM-R models tended to overestimate trait loadings.

Figure 4 presented the distributions of the estimation biases of trait correlations. The bias scores from the CTCM and CTCM-R models both formed bell-shaped normal distributions with centers at zeros (μ_{CTCM}= -.02, $\mu_{CTCM\text{-}R}$= -.00) and standard deviations at .27. However, the distribution of the trait correlations biases from the CTCU was skewed with the mean of .17. For three models, the standard deviations of the estimation biases of Trait Correlation were similar to one another ($\sigma_{CTCM=.27,}$ $\sigma_{CTCM\text{-}R=.27,}$ $\sigma_{CTCU=.23)}$.

Since the population factor loadings and trait loadings greatly determine the corresponding estimates (Conway et al., 2004), *Table 7* presents the means of trait loading deviation by population Trait and Method values.

Estimation biases in Trait loadings were influenced by population Trait and Method Loading values, but the biases were not significantly different among the three models. A homogenizing effect for Trait Loading estimate biases was found. That is, all three models appeared to overestimate the Trait loadings at a low population of Trait Loading values/ at a high population of Method loading values; they appeared to underestimate Trait loadings at high population Trait Loading values / at low population Method Loading values. For example, the CTCM-R model presented the greatest underestimation of Trait Loadings for the highest population Trait Loading value of .69 (bias = -.16), and its greatest overestimation for its lowest population Trait Loading value of .31 (bias = .13). The estimation biases also varied by population Method Loading values, where a substantial underestimation was seen for the lowest population Method Loading at .17 (bias= -.15). The other two models suffered from the same homogenizing tendency with different levels, but the differences were not significant across models (F (2, 15) = 0.133, p = 0.876).

However, CTCM-R and CTCM models showed significantly smaller estimation biases in Trait correlations than the CTCU model ($F_{2,33}$=20.1, p<.01). The CTCM-R model showed homogeneity estimation effect, which is an overestimation of Trait correlations for the low population trait value (bias=.15), an underestimation of Trait correlations for the high average population (median bias= -.15), and a relatively accurate estimation of Trait correlations for the median average population trait value (bias= -.04). Trait estimates returned from the CTCM-R model were also determined by population method correlation values, but in the other direction. It showed the CTCM-R model overestimated the Trait correlations for the high population method correlation (bias= .12), underestimated the Trait correlations for the low population method correlation (bias= -.12), and accurately estimated the Trait correlations for the median population method correlation (bias= .01). Though the CTCM-R model has the above mentioned estimation biases, from a t-test, the model showed as good of an estimation ability as the optimum CTCM model (t=2.1, p=.06) and outperformed the CTCU model significantly (t=16.7, p<.01). The CTCU model tended to overestimate trait correlations with mean estimation biases ranging from .05 to .30 across different population values.

Discussion

Study 1 presented a comprehensive e of the CTCM-R model and compared it with two other widely applied CFA-MTMM models. The three models were thoroughly evaluated across different simulation conditions, in three areas: (a) convergence and admissibility, (b) model fit, and (c) accuracy of parameter estimates.

First, as expected, the CTCM-R model largely overcame the convergence and admissibility difficulties associated with the CTCM model and kept an aggregate admissible rate at 63%. It is noted that the CTCM-R retained an impressive admissible rate when the sample size was small (p=55%).

Second, solutions from the CTCM-R model presented a good model fit. In particular, the CTCM-R kept a model fit rate at 65% when the sample size was 100, higher than the rates of 54% and 36% for the CTCM and CTCU models, respectively.

Third, the CTCM-R model returned accurate estimates of Trait factors. When estimating Trait correlations, the CTCM-R model's showing was as good as the optimal CTCM model and significantly outperformed the CTCU model. Consistent with previous studies, population parameter values affected estimations of the CTCM-R model. Median values of method correlations and/or trait correlations resulted in more precise estimates for the CTCM-R. Similar to Trait correlation estimates, Trait loading estimates from the CTCM-R model were influenced by large and small population Trait and Method loading values. Except for high or low population values, generally, analyzing the sample data generated from median levels of population Trait and/or Factor loading values often resulted in accurate estimates from the CTCM-R model. Lastly, when estimating Trait loadings, the CTCM-R model perform as well as the other two models.

In summary, the CTCM-R model showed a satisfactory performance in the three examined criteria: (a) owning to reparameterization, it generated convergent and admissible solutions more frequently than the mathematically equivalent CTCM model; (b) The model fit rate of the CTCM-R model was as high as the theoretically optimal CTCM model and was higher than the success rate of the CTCU model; and (c) The CTCM-R returned acceptable trait estimates. Trait estimates from the CTCM-R model were accurate for the population values at median levels, but were affected by extreme population values. There was a tendency of estimation bias, but more research is needed, to study how extreme values affect trait estimates.

All of these advantages of the CTCM-R model make it a great candidate for MTMM studies. Generally, the CTCM-R model outperformed the other two widely applied competitive models. To display the strengths of the CTCM-R model, to validate its usefulness with real examples, and to improve the ecological validity of the current study, Study 2 compared convergence, admissibility, and model fit of the CTCM, CTCM-R and CTCU models using the largest sample of previously published MTMM datasets ever assembled.

CHAPTER 4

STUDY 2: EVALUATION OF THE MTMM MODELS ON THE BASIS OF REVIEWED MTMM MATRICES

Method

The reviewed MTMM matrices in Study 2 come from the latest literature review of MTMM studies. Study 2 first updated the large-scale database of MTMM studies that had previously constructed by Lance et al. (2014). A total of 570 MTMM matrices from 489 studies were identified. Next, in order to perform model comparisons, matrices included in Study 2 had to satisfy the following requirements: (a) reported matrices should have sizes at least 3T3M, 2T4M or 4T2M; (b) matrices have trait and method factors that can be classified into the trait and method codes that Lance et al. (2014) developed; (c) the matrices should return at least one convergent and admissible solution from the three targeted CFA models (i.e. CTCM model, CTCU model, CTCM-R model). Finally, a total of 266 matrices were identified, and LISREL8.8 (Jöreskog & Sörbom, 1993) was used to fit CTCM model, the CTCU model, and the CTCM-R model to these identified matrices.

Through LISREL analyses, the model performance indicators were collected, including (a) model convergence, which suggested whether a model converged within limited iterations (within 500 iterations in the study), (b) admissibility, indicating whether estimates of a solution has proper values, such as no negative unique variances, no standardized factor loadings or factor correlations greater than 1.00 in absolute value, and (c) model goodness-of-fit, which was evaluated based on the selected goodness of fit indices, including the χ^2 statistic, the standardized root mean squared residual(SRMSR), the root mean squared error of approximation(RMSEA), the comparative fit index (CFI; Bentler, 1990), and the Tucker-Lewis Index (TLI). Hu and Bentler (1998) have suggested a set of model-fit cut-offs for these indices (a statistically nonsignificant χ^2, SRMSR≤.08, RMSEA≤.06, and CFI/TLI ≥.95). In addition to these model performance indicators, sample size and numbers of traits and methods were also collected as descriptive information about the matrices.

Results

Convergence and Admissibility

The CTCM-R model generated convergent and admissible solutions more frequently (84%) than the CTCM model and the CTCU model, whose convergence and admissibility rates were 23% and 79%, respectively (Table 6). When the proper solution rates were evaluated by sample sizes, the CTCM-R model showed results consistent with those of Study 1: All three models were more likely to converge and return an admissible solution as the sample size increased (Table 7). But the sample size had a particularly great effect on the CTCM model. Only 9% of the CTCM model solutions were convergent and admissible at sample sizes less than 100. In contrast, 86% of the CTCM-R solutions and 79% of the CTCU solutions were convergent and admissible at sample sizes less than 100.

Model Fit.

Table 8 presented the overall model fit. Since CTCM rarely returned admissible solutions when the sample size was small, the median df and χ^2 from the CTCM models were significantly higher than those of the other two models. It should also be noted that χ^2s are sample size dependent and easily become significant when the sample size is large. Thus, solutions from the CTCM models tend to have more solutions with significant χ^2 (57%) than the other two models (43% for CTCM-R model and 38% for CTCU model). Because of this deficiency of using χ^2, a number of goodness-of-fit indices were examined to justify model-fit performance. As shown in Table 8, all mean model-fit indicator scores fell into the suggested good ranges (Hu & Bentler, 1998). In addition, based on suggested cut-offs of indices for good model fit, the percentages of good-fit solutions among proper solutions were calculated and appeared in the last column of Table 7. The results suggested that 96% of the CTCM-R solutions showed a good-fit to the data; 97% of the CTCM solutions showed a good fit. And last, the CTCU showed a slightly suboptimal model-fit at a rate of 94%.

Discussion

Study 2 examined model performance with real MTMM data and served as complementary research to the previous study. The results of Study 2 showed findings consistent with those of Study 1. The CTCM-R model demonstrated a superior performance over the other two models: (a) It generated convergent and admissible solutions more frequently than the other two models; and (b) The model fit rates of the CTCM-R model were as high as those of the theoretical optimal CTCM model and showed model fit superior to that of the CTCU model.

CHAPTER 6

DISCUSSION

The primary goal of the current study is to evaluate the neglected CTCM-R model for MTMM data. This reparameterization approach imposes nonnegative constraints on uniqueness estimates, and therefore theoretically reduces improper solutions. What makes it even more effective is that this model structure complied with the Traits and Methods relationships in the optimal CTCM model. Additionally, it is able to estimate trait and method factor effects as well as the inter-correlations among the Trait factors and the Method factors.

To demonstrate the theoretical strength of the CTCM-R, in the current study the CTCM-R model has been applied with both synthetic and real MTMM data. Two widely applied models, the CTCM and the CTCU models, served as benchmarks in this study. Model performance was examined with a focus on three aspects: convergence and admissibility, model fit, and accuracy of parameter estimates. All these performance criteria were investigated and the results were compared to those of the other two models in order to answer the question, "Is the CTCM-R model a good option for MTMM data analysis?" And the answer is "Yes."

The results from both simulation and reviewed studies have suggested that the CTCM-R model achieved the desired convergence and admissibility rates. Compared with the CTCM model, the CTCM-R model was able to return a satisfactory number of proper solutions, even when the sample size was small. Moreover, these solutions showed good model-data fit. Both simulation and reviewed studies showed the CTCM-R model has favorable model-fit indices, which suggests it complies with data structures.

In terms of parameter estimates, the simulation study suggested that the CTCM-R model gives acceptable parameter estimates. Even though it tended to overestimate Trait Factor loadings, the model provided accurate unbiased trait correlations. Additionally, compared to the competitive CTCU model, the CTCM-R model is significantly more accurate in estimating Trait correlations.

The findings of these studies imply that, the CTCM-R model successfully overcomes the empirical difficulties associated with the CTCM model, and the estimation problems with the CTCU model. It achieves high convergence and admissible rates, shows good model-data fit, and estimates Trait factors accurately. However, some limitations to the CTCM-R model have also been found in the current study. As mentioned earlier, the CTCM-R model tends to overestimate the trait loadings, though the mechanism is unknown.

It is interesting to find that the convergence and admissibility rates in Study 2 were higher than those from Study 1. For example, the aggregate admissible rate for the CTCM-R model for reviewed MTMM data was 84%, higher than its 63% rate for simulation data. The CTCM-R model's admissibility rate for real MTMM data (84%) was almost double that for simulation data (p=44%). These discrepancies revealed that the MTMM data has more complex structure than that of a CFA model. George Box said, "Essentially, all models are wrong, but some are useful" (p. 424). Similarly, the proposed model CTCM-R may be wrong still, but it approaches to the unknown correct model. For the CFA-MTMM model, connections between Traits and Methods factors might exist.

Limited by available statistical techniques, these connections are not able to be programed or estimated. More work is always needed. However, one promising approach to increasing chance of getting convergent and admissible MTMM-CFA solutions is to increase the indicator–factor ratio (Monahan, Hoffman, Lance, Jackson, & Foster, 2013). Increasing the number of manifest indicators for each Assessment Centers dimension factor resulted in convergent and admissible solutions for the

CTCM model. Monahan et al. (2013) suggested to use multiple items per factor more broadly in other area of organizational research. Therefore, future research is needed applying multiple –indicator approach to the CTCM-R model.

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Table 1

Hypothetical MTMM Matrix of 3T3M

		Method 1			Method 2			Method 3		
		T1	T2	T3	T1	T2	T3	T1	T2	T3
Method1										
	T1	1.0								
	T2	HTMM	1.0							
	T3	HTMM	HTMM	1.0						
Method2										
	T1	MTHM	HTHM	HTHM	1.0					
	T2	HTHM	MTHM	HTHM	HTMM	1.0				
	T3	HTHM	HTHM	MTHM	HTMM	HTMM	1.0			
Method3										
	T1	MTHM	HTHM	HTHM	MTHM	HTHM	HTHM	1.0		
	T2	HTHM	MTHM	HTHM	HTHM	MTHM	HTHM	HTMM	1.0	
	T3	HTHM	HTHM	MTHM	HTHM	HTHM	MTHM	HTMM	HTMM	1.0

Table 2

Population Values of Simulation Study

	Reviewed MTMM Studies			Current Study
	Median	20th percentile	80th percentile	Population Values
Number of Method	3	2	3	3,5
Number of Trait	4	3	6	3,5
Sample Size	183	90	421	100, 250, 500, 1000
Method Loading	.28	.17	.48	.17, .28, .48
Trait Loading	.50	.31	.69	.31, .50, .69
Method Correlation	.29	.02	.58	.02, .29, .58
Trait Correlation	.36	.07	.61	.07, .36, .61

Note. N=258. Reviewed studies come from "Convergence, Admissibility and Fit of Alternative Confirmatory Factor Analysis Models for Multitrait-Multimethod (MTMM) Data," by C. Lance & Y. Fan (2015)

Table 3

Example of Lambda and Phi Matrices for Generating a 5T3M Population Matrix

Population Matrix		Population Design Matrix							
		M1	M2	M3	T1	T2	T3	T4	T5
Lambda Matrix	T1M1	0.18			0.4				
	T1M2	0.28				0.5			
	T1M3	0.38					0.6		
	T1M4	0.18						0.4	
	T1M5	0.28							0.6
	T2M1		0.28		0.5				
	T2M2		0.38			0.4			
	T2M3		0.18				0.6		
	T2M4		0.28					0.5	
	T2M5		0.38						0.4
	T3M1			0.38	0.6				
	T3M2			0.18		0.5			
	T3M3			0.28			0.4		
	T3M4			0.38				0.6	
	T3M5			0.18					0.5
Phi Matrix									
	T1	1							
	T2	0.29	1						
	T3	0.19	0.39	1					
	M1				1				
	M2				0.36	1			
	M3				0.26	0.46	1		
	M4				0.46	0.26	0.36	1	
	M5				0.36	0.36	0.46	0.26	1

Note. The pattern matrix for 5T3M population MTMM matrix with averaged Method Loading at .28, Trait loading at .5, Method Correlation at .29, Trait correlation at .36.

Table 4

Convergence and Admissible Rates for Three Models

Population Values			CTCM	CTCM-R	CTCU
Sample size	100	Convergence	57%	90%	84%
		Admissibility	7%	44%	53%
	1000	Convergence	81%	97%	96%
		Admissibility	49%	79%	88%
Number of Traits	3	Convergence	80%	96%	93%
		Admissibility	20%	65%	74%
	5	Convergence	60%	93%	90%
		Admissibility	35%	61%	74%
Number of Methods	3	Convergence	57%	93%	84%
		Admissibility	16%	58%	61%
	5	Convergence	83%	96%	99%
		Admissibility	39%	67%	87%

Table 5

Mean Model Goodness-of-Fit Indices Scores across Analytic Models, Matrix Sizes and Sample Sizes

		CTCM						CTCM-R						CTCU					
Matrix size	**Sample size**	**DF**	**CHIQ**	**C&A Rates**	**SRMR**	**TLI**	**CFI**	**DF**	**CHIQ**	**C&A Rates**	**SRMR**	**TLI**	**CFI**	**DF**	**CHIQ**	**C&A Rates**	**SRMR**	**TLI**	**CFI**
3T3M	**100**	12	6.8	2%	.03	1.11	1	12	9.1	49%	.04	1.11	.99	15	15.0	43%	.05	1.04	.99
	250		7.7	5%	.02	1.07	1		9.0	59%	.02	1.07	1		15.6	63%	.03	1.01	.99
	500		8.5	9%	.02	1.02	1		9.1	64%	.02	1.03	1		16.4	74%	.02	1	1
	1000		9.2	14%	.01	1.01	1		9.4	69%	.01	1.01	1		17.8	83%	.02	1	1
3T5M	**100**	62	56.1	6%	.05	1.03	1	62	58.7	48%	.06	1.05	.99	72	80.4	64%	.07	.98	.98
	250		56.6	22%	.03	1.01	1		56.9	65%	.03	1.03	1		80.8	81%	.04	1	.99
	500		58.0	41%	.02	1.01	1		57.5	76%	.02	1.02	1		87.0	91%	.03	.99	.99
	1000		58.8	61%	.02	1	1		58.5	87%	.02	1.01	1		100.2	96%	.03	.99	.99
5T3M	**100**	62	56.0	4%	.05	1.03	1	62	57.9	37%	.06	1.04	.99	50	53.1	31%	.06	1	.99
	250		56.0	17%	.03	1.01	1		56.7	51%	.03	1.02	1		54.7	52%	.04	1	.99
	500		57.6	32%	.02	1.01	1		57.1	64%	.02	1.01	1		57.8	65%	.03	1	1
	1000		58.9	47%	.02	1	1		58.2	73%	.02	1	1		64.9	75%	.02	.99	1
5T5M	**100**	230	244.8	14%	.07	.99	.99	230	245.4	42%	.07	.99	.98	215	243.8	75%	.07	.96	.97
	250		232.8	37%	.04	1	1		231.1	61%	.04	1	1		230.5	93%	.05	.99	.99
	500		229.3	57%	.03	1	1		228.4	74%	.03	1	1		231.5	99%	.03	.99	.99
	1000		228.7	73%	.02	1	1		227.8	85%	.02	1	1		243.8	99%	.02	1	1

Table 6

Percentages of Model Good Fit

		Model					
		CTCM		**CTCM-R**		**CTCU**	
		C&A Percentage[a]	**Good-fit Percentage**[b]	**C&A Percentage**	**Good-fit Percentage**	**C&A Percentage**	**Good-fit Percentage**
Sample Size	100	7%	95%	44%	92%	53%	74%
	250	20%	99%	59%	98%	73%	92%
	500	35%	100%	70%	99%	82%	96%
	1000	49%	100%	79%	100%	88%	98%
nT	3	20%	100%	65%	98%	74%	91%
	5	35%	99%	61%	98%	74%	93%
nM	3	16%	100%	58%	98%	61%	94%
	5	39%	99%	67%	98%	87%	90%

Note. [a]C&A Percentage: the proportion of convergent and admissible solutions [b]Good-fit Percentage: the proportion of the good-fit solutions among convergent and admissible solutions. Good-fit solutions are determined by model-fit indicators based on Hu and Benter suggested cut-offs (1989).

Table 7

Mean Estimation Biases of Trait Loading and Trait Correlations

Population Trait Factors		Estimation Bias of Trait Loadings			Estimation Bias of Trait Correlations		
		CTCM	CTCM-R	CTCU	CTCM	CTCM-R	CTCU
TL	Low(0.31)	0.14	0.13	0.10	-0.03	-0.04	0.10
	Median(0.5)	0.03	-0.01	0.00	-0.03	-0.01	0.17
	High(0.69)	-0.11	-0.16	-0.08	0.00	0.03	0.23
ML	Low(0.17)	-0.15	-0.15	-0.08	0.06	0.05	0.20
	Median(0.28)	-0.06	-0.07	-0.05	-0.01	0.00	0.19
	High(0.48)	0.06	0.05	0.08	-0.04	-0.04	0.13
TR	Low(0.07)	0.00	-0.04	-0.05	0.11	0.15	0.24
	Median(0.36)	0.00	-0.03	-0.01	-0.03	-0.01	0.20
	High(0.61)	-0.01	-0.03	0.04	-0.14	-0.15	0.08
MR	Low(0.02)	0.00	-0.03	-0.02	-0.14	-0.12	0.05
	Median(0.29)	0.00	-0.03	0.00	-0.01	0.00	0.17
	High(0.58)	-0.02	-0.05	0.01	0.12	0.12	0.30

Table 8

Convergence and Admissibility for Three Models for Analyzing Published Data

	CTCM		CTCM-R		CTCU	
	Number	Percentage	Number	Percentage	Number	Percentage
Convergence and Admissibility	62	23%	224	84%	210	79%
Non-Convergence	114	43%	11	4%	12	5%
Inadmissibility	90	34%	31	12%	44	17%

Note. N=266

Table 9

Model Convergence and Admissibility across Different Sample Sizes for Published Data

Sample Size	Number of Studies	CTCM		CTCM-R		CTCU	
		Number	Percentage	Number	Percentage	Number	Percentage
Less than 100	66	6	9%	57	86%	47	71%
100-200	78	12	15%	62	79%	56	72%
200-500	74	27	36%	63	85%	64	86%
More than 500	48	17	35%	42	88%	43	90%

Table 10

Model Goodness Fit Indices for Convergent and Admissible Solutions.

	k	df	χ^2	%p<.01	RMSEA	NFI	CFI	RMR	TLI	% of good-fit solutions
CTCM	62	33	87.9	56.5%	0.04	0.98	0.99	0.03	0.99	97%
CTCM-R	224	14	39.1	42.9%	0.04	0.98	0.99	0.04	0.98	96%
CTCU	210	15	31.8	37.6%	0.05	0.98	0.99	0.04	0.98	94%

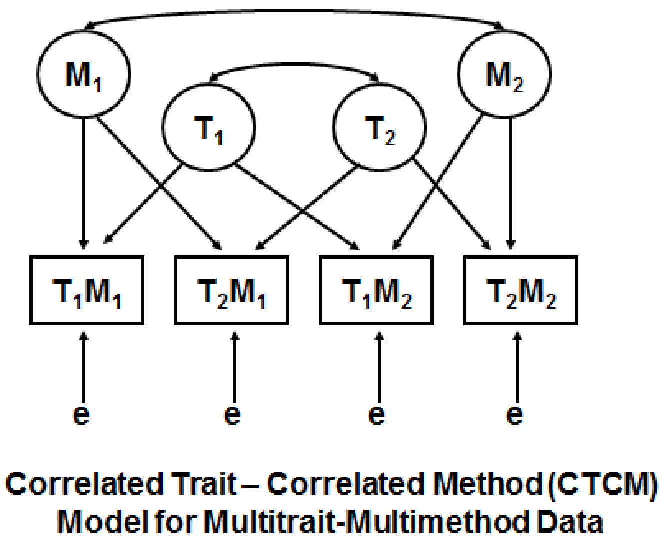

Figure 1. Model Specification of the CTCM model for MTMM Data

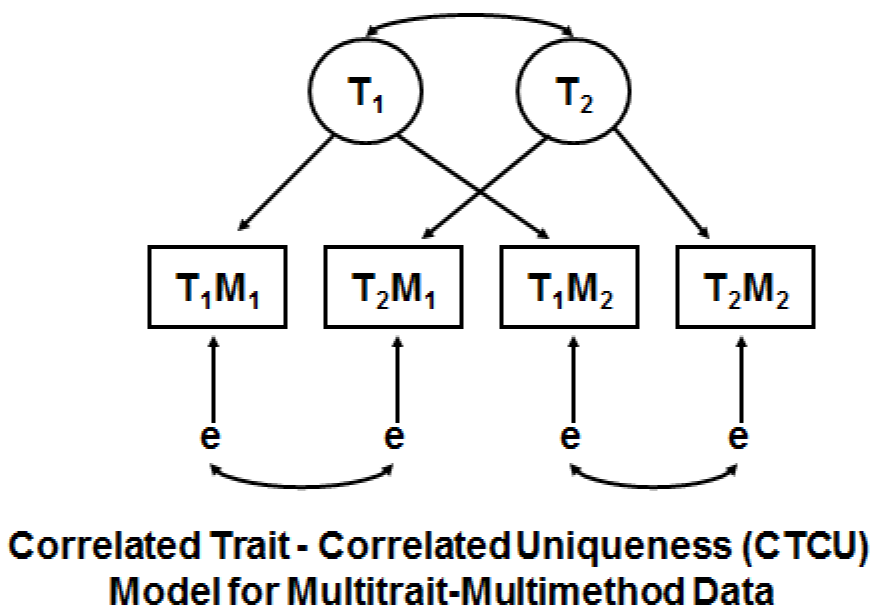

Figure 2. Model Specification of the CTCM model for MTMM Data

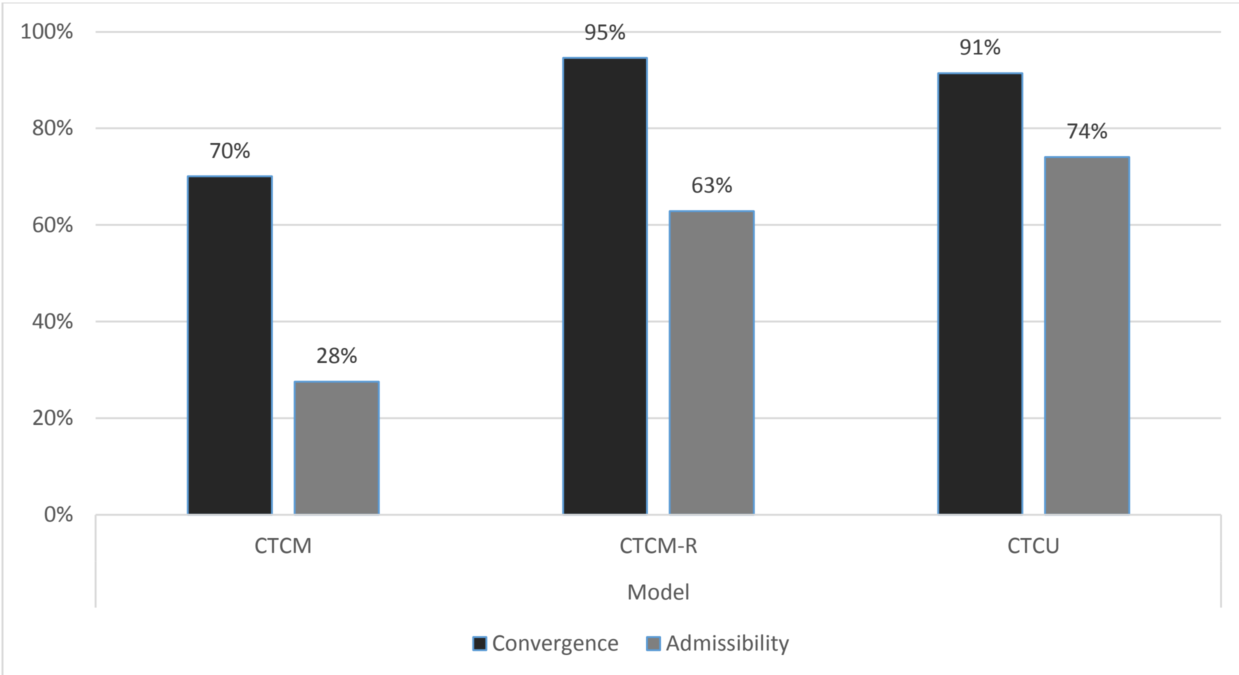

Figure 3. Convergence and admissibility rates of three models.

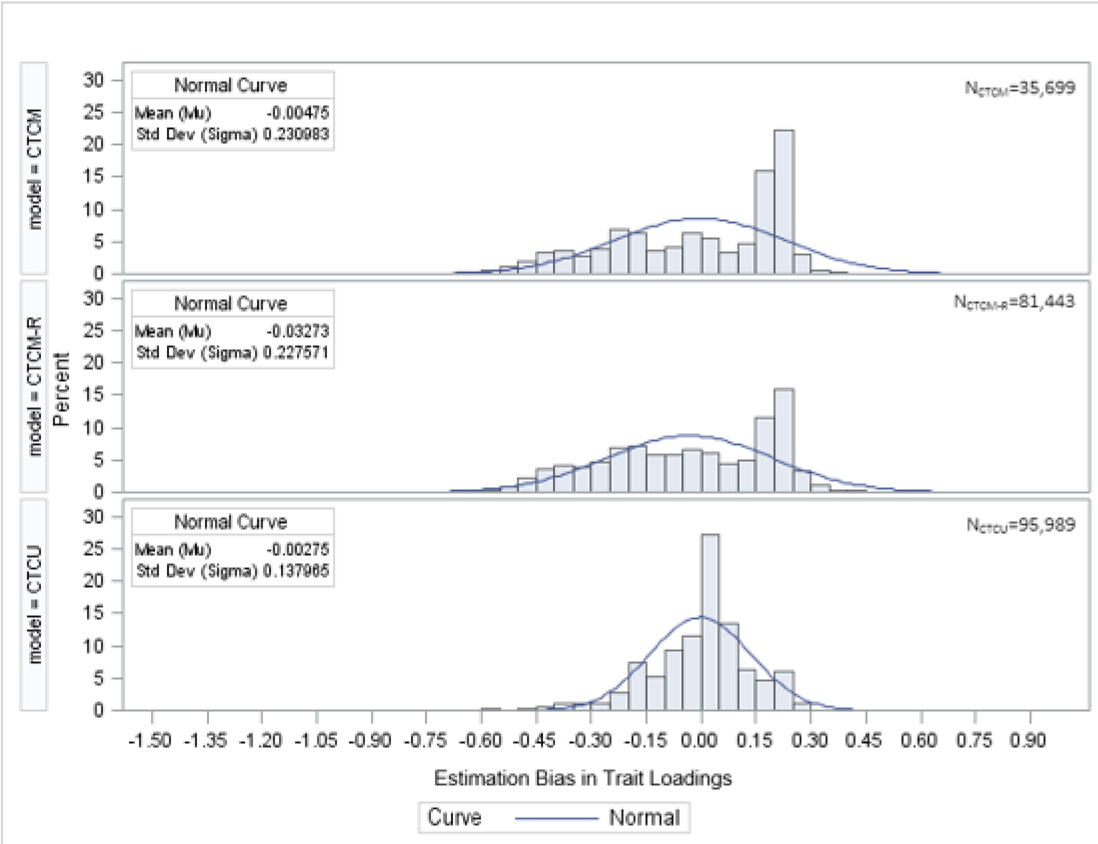

Figure 4. Distribution of Estimation Bias in Trait Loadings

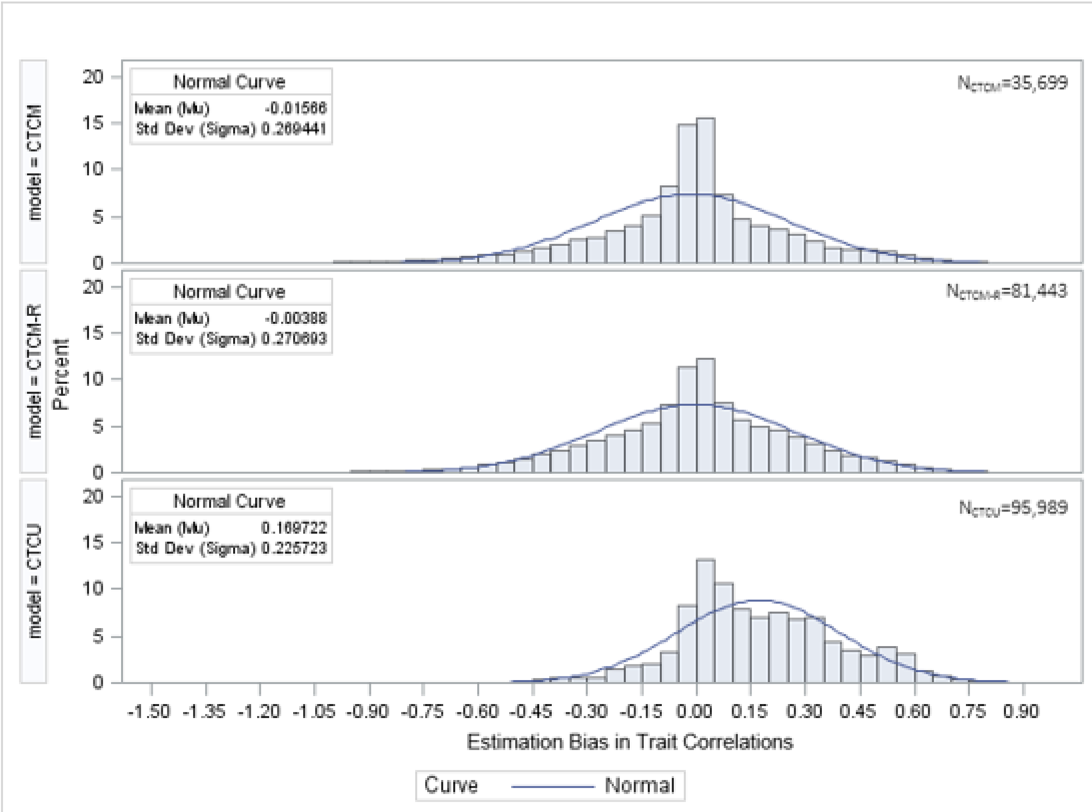

Figure 5. Distribution of estimation bias in Trait Correlations