

EXOGENOUS FACTORS THAT AFFECT THE VOTER'S CALCULUS IN THE RATIONAL VOTER MODEL

by

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(Under the direction of Arthur Snow)

ABSTRACT

One of the chief assumptions of economic theory is that agents are rational. This is usually regarded as the best predictor of any given economic agents behavior. However, despite this fact, public choice economists have long been puzzled by the fact that every election cycle, millions of voters turn out to participate in a seemingly irrational act: voting. This paper introduces the classic rational voter model, which hinges on an agent only deciding to vote when the agent thinks his or her vote has a chance of affecting the outcome of the election, and proposes an extension of it. When taking these other factors are taken into account, it may actually become rational to vote, thus saving the rational voter model from needing to be discarded. Three past cases are discussed, and an extended rational voter model is proposed that takes these exogenous factors into account.

INDEX WORDS: Public choice, Rational Voter Hypothesis, Exogenous Factors, Voting, Recounts

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DEDICATION

I dedicate this paper to my Double Dragon brother, who has made the whole thing possible, the man with the intestinal fortitude of a dragon; Wesley.

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CHAPTER 1

INTRODUCTION

Economists have used the concept of the rational, utility maximizing economic agent “homo economicus” since long before Pareto coined the term in 1906 (Pareto, 206). The idea dates back to Adam Smith, who first suggested it in *The Wealth of Nations* in 1776 (Smith, 742). Although rational choice theory has dominated the paradigm of microeconomics throughout the 20th century, it has yet to be successfully applied to one seemingly irrational act that economic agents participate in at least every two years: voting. Despite their best efforts, public choice theorists have been unable to explain how seemingly self-interested agents could consistently, from year to year cast a vote when doing so has real costs, and no real benefits. This is traditionally called the “paradox of voting”, and has been the Achilles Heel of public choice theory since its inception. This paper proposes a theoretical adjustment to the classical rational voter that will help to resolve that conundrum, and explain why a single vote can actually upset an election. Although there is surely a myriad of factors involved in the voter’s calculus, the factor that will be focused on in this paper is the impact of recounts, and other follow up processes on the voter’s calculus

The rational voting hypothesis was first developed by Downs in 1957, only to be later further elaborated on by Gordon Tullock in 1967, then by William Riker and Peter Ordeshook in both 1968 and 1973. It was in their 1968 work in which

the term “calculus of voting” was coined. The rationale for voting could be motivated by the following argument in the mind of the rational, utility maximizing individual, given the simple assumptions of a two candidate race and a “majority rule” decision process:

- 1) The individual, i , must believe that candidate x is preferred to candidate y by a given margin of B
- 2) The individual must also believe that candidate x will lose unless i casts a vote in favor of x
- 3) Therefore, individual i votes for candidate x

The above syllogism explains the given voter's rationale for voting, but step 2 requires such extreme assumptions as to be virtually implausible in the mind of public choice theorists.

Once more fully developed, the rational voter hypothesis developed by Downs is explained as follows: Given the fact that rational agents do what is in their best interest, they will only pursue a given course of action if the perceived benefits of the action outweigh the perceived costs. The associated benefits and costs of voting are explained more fully below

THE BENEFITS OF VOTING

Voters will benefit from an election by the additional utility they will receive when the candidate that they most prefer is elected and that candidate's policies are implemented, relative to the opposition candidate. These benefits may take the form of a simple transfer payment that benefits the voter, a policy change or

implementation of a new policy that provides additional utility, or as some political scientists have suggested, voters gain utility from simply being represented by someone with a political philosophy closer to their own (Tullock, 311).

It must be noted that this utility is representative of the *difference* in expected utilities between the two candidates. Thus, the expected utility of voting is the additional utility from one's candidate winning (B) times the probability that the voter casts the crucial vote. It should also be noted that these benefits from voting do *not* include any of the "psychic gains" from voting, which would be representative of the pleasure of participating in a civic process, etc. These gains are strictly in terms of the gains that the voter will receive simply from having the candidate of choice seated in office.

We'll call the probability P (it is the sum of the two probabilities of either casting the deciding vote or the tie-making vote). The probability, P , is a function of two variables; the size of the electorate, as well as the voter's subjective assessment of how close the election will be.

THE COSTS OF VOTING

The costs of voting are simply the values of time and other resources (driving to the polls, physical effort etc.) that a voter expends in order to cast a vote, in addition to all other relevant opportunity costs. Assuming that the voter does research on the candidates and spends time deciding which candidate to vote for, these costs will be included in the analysis. Let C represent all of these costs. While the costs of voting appear to be small, they can be relatively large

when compared to the expected benefits of voting in an election where the probability of casting the crucial vote is exceedingly minute.

THE DECISION TO VOTE

As long as the difference between the benefits and costs are greater than zero, voters should rationally vote. This can be expressed by the decision formula below:

$$PB - C > 0$$

In effect, the voter maximizes his expected utility by voting when the expected benefits of voting (PB) outweigh the expected costs (C).

CHAPTER 2

ANALYZING THE PROBABILITY

The probability of actually deciding the outcome is the crucial variable in the calculus. With sufficiently large voting populations, the probability becomes all but negligible. In fact, Skinner (1948) noted that the probability of getting run over by a car on the way to or from the polling location is often greater than the probability of casting the deciding vote.ⁱ The probability also rests on the voter's assessment of how close the election outcome will be (closer elections will increase the likelihood of casting the deciding vote).

The probability is expressed as follows:

$$P' = \left(\frac{3e^{-2(N-1)(p-.5)^2}}{2\sqrt{2\pi(N-1)}} \right)$$

Where the variables in the model are defined as follows:

N: The size of the electorate in the election

p: The voter's best guess as to the preferred candidate's final share of the vote

This probability was derived by Owen and Groford in 1984, and is the result of using an applied mathematical technique. In that case, a circle is used to represent the electorate, and they are divided into two categories: people voting for candidate one, and people voting for candidate two. The circle is

divided into two parts by probability p . Now, the voter decides which candidate to vote for, and increases his or her candidate's area of the circle. The winner of the election is then determined by randomly throwing a dart at the circle, and declaring that candidate the winner. The probability that the candidate's vote matters is simply the increased area of the circle as a result of the voter's vote, which is given above.

Thus, the probability of casting the crucial vote is a negative function of the voting population size (N) and the voter's estimation of their candidate's share of the final vote (p) as it differs from .5 or 50% of the votes cast. Note that even if the voter expected the outcome to be an even split between the two candidates ($p = .5$), the probability of casting the deciding vote is exceedingly small even for a small voting population. For example, if $p = .5$ and there are 1,000 voters (a very small population), the expected probability of casting the deciding vote is a mere .026; or 1 chance in 39. With a population of 1,000,000, the probability is .0006 or one chance in approximately 1,700.

It seems that in any large election, the probability of one vote actually making the difference is so low that voting becomes an irrational activity.ⁱⁱ To get an idea of how small the probability is when a voter expects an extremely close election (but not an even split), take the case of a voting population of 6,000,000 and $p = .49$ (a 1% split between candidates). The chances of deciding the election with one vote becomes one out of $10^{-1,046}$, or essentially zero. In fact, economist Steven Landsburg noted that this is just as likely to happen as winning the lottery's Powerball jackpot...128 times in a row (Landsburg, 47)! This is

definitely something that would yield much more positive benefits to the voter than simply having the voter's candidate of choice win an election. This is why voting is often viewed as irrational; P is miniscule, and B is not large enough to make the net benefits of voting positive.

OTHER BENEFITS OF VOTING

Another benefit of voting can be described as the additional utility a voter gets from participating in the voting process. This idea has been posited by Mueller as having a "taste" for voting, meaning that voting is somehow a utility generating action, like eating or sex. Rationale for this explanation has been attributed to people being endowed with some sort of itch to do their "civic duty", and somehow voting scratches that itch (Mueller, 492). Another idea is that people enjoy following politics in much the same way that some people follow sports, and voting is the sports equivalent to "rooting" for your "team".

If we assign this "psychic gain" the variable D in our decision rule, then the adjusted model would be as follows:

If $PB + D - C > 0$, then vote

Otherwise, abstain

If we slightly adjust this model by assuming (as has been asserted previously) that P is essentially equal to zero, then the modified model is simple given by the decision rule:

$D - C > 0$

This is extremely problematic, because although it saves the rational voter model in the sense that voting now becomes a rational act, it now has no

predictive power because every variable in the model is now exogenous! What we have now is a totally useless model that doesn't predict voter behavior at all. For the purposes of this paper, we will set any other benefits of voting aside and employ the model with only non-civic benefits included.

CHAPTER 3

THE ELECTION PROCESS IN THE UNITED STATES

Voter turnout in the United States has been perplexingly low in the opinion of many analysts. For example, presidential general elections since 1960 have had turnouts that range from 50.1% to 63.1% of the total voting age population. Compare that to voter turnouts in at least 35 countries for their leaders that are above 80% of the voting population (IDEA, 2000). It's hard to know whether this can be attributed to the fact that US voters as a whole are more "rational" than other nations' or if other factors are involved. Regardless, the United States has a few peculiarities when it comes to the election process that is worth discussing here

The assumptions of the rational voter model are that we have a two-candidate race, and the winner is simply the candidate who receives a majority of the votes. Reality though, is not that simple. One count of the number of legitimate and legal candidates for President in the historic 2000 race put the number at 15. For all intents and purposes, however, it was a two man race. We are going to make the simplifying assumption that the only affect that the other candidates have is to change the margin of victory of the winner, and not determine the actual outcome of the race.

Another simplifying assumption that conflicts with the reality of the election of the President in the U.S. is the winner simply being the candidate who

receives the most votes. Due to the nature of the electoral college system, however, it is possible for not only an individual vote not to matter in the selection of the President, it is possible for the votes of an entire *state* not to matter depending on how close the electoral college vote count is in the election. This isn't a problem for the theoretical proposition of this paper, however, simply because relaxing the assumption of a "majority rule" voting structure serves only to strengthen the argument.

Another election reality, the one this paper focuses on, is that the majority of votes may not decide the winner after all. Whenever the results of an election are sufficiently close, the election is determined by what we'll call a "follow up process" which is quite often a function of almost anything but simply the number of votes for each candidate. The next section of the paper will give more concrete examples, but it is assumed that the outcome of this follow up process is determined by variables that are totally unrelated to the number of votes cast for each candidate, such as the strength of each candidates legal team, the political party and philosophy of election officials, judges, and other relevant elected representatives that decide how to apply recounts, and other follow up procedures. This proposition, if true, can save the rational voter model from the throes of destruction for the following reason: Although the probability of casting the deciding vote is now negligible and for all intents and purposes equal to zero, the probability of casting a vote that makes the election "too close to call", and voting becomes rational. Because each voter has no idea how close the election

has to be for it to be “too close to call”, and for a winner to be seated, the perceived probability of casting an influential (not deciding) vote increases.

This type of result has happened more than once in American politics; the “winner” of the election did not **clearly win** the most votes. Three such examples are given in the next section, followed by an evaluation of how such cases might affect the rational voter model, and therefore future turnout.

THREE EXAMPLES OF RANDOM OUTCOMES

Here are some cases where the outcome of an election appears to be less than a cold calculation of the number of votes cast and more of a random process that determined who would be seated in office. They are presented in chronological order and are examples of a House election, Presidential election, and Senate election, respectively.

“THE BLOODY EIGHTH”ⁱⁱⁱ

According to Indiana election records, the 1984 election race for the 8th district U.S. Congressional seat has Richard McIntyre (R) with 114,278 votes, and Frank McCloskey (D) with 113,860 votes. McIntyre, the winner of the election (from Indiana’s perspective), however, was never seated as a U.S. representative. Instead, after 4 months of counting, recounting, debate, and legal maneuvering, McCloskey was seated on May 1, 1985 with a House vote of 236 to 190. The democrat-controlled congress was able to strong-arm McCloskey into office after a considerable ballot dispute. The *CQ Almanac* stated it this way:

The McCloskey-McIntyre race was the closest House contest of the 20th century, according to the three-member House Administration Committee task force that investigated it.

McCloskey was the apparent winner by 72 votes – out of more than 234,000 cast – right after the November 6th election. But in two precincts in one of the district's 15 counties, ballots had been counted twice. Correction of that arithmetical error gave McIntyre an apparent 34-vote victory. On that basis, Indiana Secretary of State, Edwin J. Simcox, a Republican, on Dec. 14 certified McIntyre as the winner.

But when Congress convened Jan. 3, House Democrats refused to seat McIntyre, voting instead to declare the seat vacant.

A full recount was completed Jan. 22. It showed McIntyre's lead had increased to 418 votes after more than 4,800 ballots were thrown out for technical reasons.

...After four months of partisan wrangling, the House May 1 finally settled an issue that had plagued it since the first day of the session, voting 239-190 along party lines to seat Democrat Frank McCloskey as the representative of the 8th District of Indiana.

“Partisan wrangling” and subjective determination of what was actually a valid ballot had decided the election—far from a simple, objective tallying of votes. The taskforce mentioned above went through the process of examining absentee ballots to see if they should or should not be counted. Enough were included and counted to satisfy House Democrats that McCloskey had won the race by 4 votes. In the end, both sides of the contest and many voters still believe the outcome was anything but objective.

Although this might appear at first to be damaging to the rational voter hypothesis, it is important to consider what allowed the election to enter the random process in the first place: the fact that the election was extremely close. So although no individual vote mattered in the sense that the election wasn't

decided by a tie breaking vote, the sheer closeness of the election the election forced it to enter the follow up process.

GORE-BUSH 2000

The election held on November 7th, 2000 officially resulted in a vote count that had George W. Bush winning the key electoral votes in Florida by 1,785 votes. The difference of 1,785 votes was within the mandated threshold of 0.5% for an automatic recount of the votes. The subsequent recount revealed that Bush had won, by a margin of 327 votes. In this case, the mandated follow-up process for determining the winner resulted in a new, even closer margin of victory for the same candidate. However, that was not the end of the follow-up. An election controversy ensued and was fought in the courts and public arena for another 34 days. The final winner of the race was, not necessarily who got more votes, but arguably who was most successful in those arenas (one legal decision the other way around could have reversed the outcome). The counting of the votes is still seen by many as anything but accurate. And this is the most important issue concerning the rationality of voting: the **voter's perception** is that the end result comes out of a process that involves a random component of determining the winner (i.e. who wins the legal battle, who actually counts the votes, the degree to which partisanship affects the outcome, etc.).

FRANKEN-COLEMAN 2008

The 2008 Minnesota Senate Election went down in history as both one of the closest and longest Senate elections to date with the legal battle lasting until 238 days after the election. Democratic-Farmer-Labor candidate Al Franken

faced off against the incumbent Republican, Norm Coleman. When the initial count of the votes was completed on November 18, Franken was trailing Coleman by a mere 215 votes, out of the more than 286,000 cast. The closeness of the initial results triggered a mandated recount, which was fraught with legal problems for both sides to debate over. After reviewing ballots that had been challenged during the recount and counting 953 wrongly rejected absentee ballots, the State Canvassing Board officially certified the recount results with Franken holding a 225-vote lead over Coleman.

This election is a classic example of how both arbitrary and exogenous factors can swing the results of elections one way or another after the casting of ballots has been completed. With 3,600 votes disputed in the recount (more than ten times Franken's final margin of victory) it is easy to see how the subjective nature of what counts as a legitimate and legal ballot could easily have swung the election one way or the other. In addition to other factors, such as the infamous lost envelope containing 133 votes that went missing during the recount, and the decision of the Minnesota Supreme Court to dismiss Coleman's appeal, it is easy to see how the nature of the aforementioned follow up processes could positively affect the voter's calculus in the context of the rational voter model.

CONCLUSIONS FROM THE EXAMPLES

The Indiana 8th congressional district election in 1984, the presidential election of 2000, as well as the 2008 senate race are only three cases of how election outcomes may or may not have been determined by a mere counting of votes, but rather by a random follow-up procedure. To clarify, we refer to the follow-up procedure as “random” only because the outcome involves some uncertainty and is not necessarily strictly a function of how many votes were cast.

For example, manual recounts involve randomness in how the questionable ballots are assessed (i.e. lenient versus strict interpretations of dimpled chads). Although not truly “random”, the factors that determine the winner of an election once the initial tally has been made quite often has nothing to do with the number of votes cast for each candidate.

Other examples of random components are when a candidate possesses a stronger legal team, or if there is partisanship in the follow-up process. In addition to the three cases discussed here, many other cases, no doubt, exist. The question is how the knowledge of randomness in the follow-up procedure might affect voter behavior. That will be analyzed in the next chapter.

One final note about the welfare effects of post election “wrangling” is that if the public really enjoys the entertainment value of it, then this alone could be one welfare enhancing effect of voting. It is theoretically possible that people vote just to make the election close enough to have an entertaining legal battle

afterwards! Although that is one interesting possibility, such considerations will be put aside for future research.

CHAPTER 4

THE MODIFIED VOTER'S CALCULUS FUNCTION

If the aforementioned cases of “random” follow up processes really do affect the voter’s estimated costs and benefits of voting in the rational voter model, then it is important for this adjustment of the model to be included mathematically. How then will the voters go about making these estimates? There are a few things to consider.

First, we must make a few assumptions about how the variables in the model affect the voter’s calculus. We will assume as the size of the electorate (N) increases, the probability for accuracy in the follow up process goes down. This assumption makes intuitive sense, because as the scale of the election goes up, there are more people tallying more votes in more precincts, and it becomes much more difficult to both tally and collect accurate scores from each precinct.

In order to mathematically represent this, we will now introduce a “randomness” function into the model, which is a function of the electorate size (N). Because this function affects the probability of a voter casting a “deciding vote”, it must be represented by a likelihood function that takes on a value between zero and one. We will mathematically define this function to be as follows:

Randomness function: $r(N)$, where $0 < r(N) < 1$

$$r'(N) > 0$$

THE PROBABILITY OF CASTING A DECIDING VOTE

To include the new information in the voter's calculus, we must reconsider what constitutes a "deciding vote." One way to adjust the voter's calculus is to look at the aforementioned probability, P , of casting the crucial vote. If the follow-up procedure involves a random component, then the number of votes cast does not necessarily determine the winner. In other words, the probability of casting the crucial vote is diminished, or quite possibly zero, when the randomness is due to partisanship or legal advantages and not due to merely re-tallying the votes.

Thus, the traditional, straightforward calculus of considering the size of the voting population and estimating the closeness of the election is obscured by the "noise" of close elections being decided by factors other than a simple tallying of votes. P is lower. But, how do we show that P is lower mathematically?

Arguably the degree of the uncertainty is a positive function of the scale of the election. Larger elections leave more room for disagreement and the voter subjectively determines how random the process is based on past election outcomes. Therefore, the rational voter model should include an additional term, r , to capture the subjective estimate of the randomness to get the adjusted probability, P' :

$$P' = \left(\frac{3e^{-2(N-1)(p-.5)^2}}{2\sqrt{2\pi(N-1)}} \right) (r(N))$$

where $1 > r(N) > 0$

and $r'(N) > 0$

So it should become immediately apparent that the addition of a random follow up process has a two prong affect, one of which is illustrated here. Because of the “randomness” of the partisan follow up process, the probability of casting the deciding vote goes down. As will be shown later though, when the concept of a “threshold vote” is introduced, the probability of a single vote mattering actually goes up!

CASTING A “THRESHOLD VOTE”

In addition to a lower probability of one’s vote being counted as the deciding vote, the initial count of the votes must be close enough to warrant the follow-up process in the first place. Thus, given the fact that the method of tallying votes involves some error (i.e. punch-card machines); a close election is not guaranteed to be sent to the follow-up process if the initial count incorrectly shows margin that is too disparate. Conversely, a race that is not close enough to warrant a follow-up process may be inappropriately assigned to the process if the initial tally makes the race appear closer than it actually was.

With randomness in the follow-up process, such a race could be awarded to the candidate actually receiving fewer votes. Thus, just as important as the follow-up process is to determining the winner, so is the *decision* to apply the follow-up process. Granted, the cases we are hypothesizing are on the fringes of the possibilities, but the mere possibility of seating a false-winner will affect the voter’s calculus albeit a very small amount. Thus, another crucial vote could be

the vote that either sends the election to the follow-up process or prevents it from going there. We will call this crucial vote the “threshold vote.”

Let Z be the probability of casting the threshold vote. In a similar way to looking at a vote being either the tie-breaking or tie-making vote, we can model the threshold vote. One’s vote can be significant if it is the one that makes the margin so close as to send the election to the follow-up process where their candidate is declared the winner. Another way for the threshold vote to be significant is for the vote to make the margin of the election too large to warrant a follow-up process (where their candidate could have been defeated).

At first glance, determining the value of Z should be analogous to the value of P in the original voter model. Implying that the likelihood of casting a particular vote is a function of the voting population (N) and the voter’s assessment of how close the election will be to the threshold of the random process. If viewed that way, the value of Z should be exactly equal to the value of P for an election that the voter thinks will be very close to the follow-up threshold. For example, if the voter estimates the threshold to be T (i.e. 1%), where T is the percent difference from an even 50-50 split of the overall vote count, then Z is calculated as follows:

$$Z = \left(\frac{3e^{-2(N-1)(t-T)^2}}{2\sqrt{2\pi(N-1)}} \right)$$

Here, t is the voter’s subjective assessment of how the initial outcome of the election will differ from a threshold of T difference between the two candidates (in percentage terms). Note the somewhat perverse result that comes from a voter expecting a dead-heat; the probability of casting the

threshold vote (Z) is ***smaller*** than if the voter expected the difference in votes to be at the threshold.

At this point our rational voter model has taken a form that is different from the conventional form above. Including the randomness involved in a voter considering his probability in casting the crucial vote, and the probability of casting a threshold vote the model becomes:

If $ZB + P'B - C > 0$, vote

Otherwise, abstain

The function can be rewritten more simply as:

$(Z+P')B - C > 0$

A voter will rationally vote when the expected benefits $(Z+P')B$ exceed the costs of voting, C .

CHAPTER 5

THE THRESHOLD VOTE AND PSYCHOPHYSICS

At this point, it appears that the model has decreased the rationality of voting by the reduced probability P' , but also increased the rationality of voting by the addition of a new probability of a vote affecting the election outcome, Z . However, the final model will, in fact, result in a higher overall probability of one's vote deciding the outcome of the election.

To reach this conclusion, we must examine the theoretical underpinnings of the rational voter model. Let's take the mathematical model upon which the probability for casting the deciding vote in an election was originally based, which is just a circle whose area sums to one. Let's pretend that this is the face of a clock, and in the classical rational voter model, the probability of casting a deciding vote is equivalent to spinning a dial on the clock, and having it land exactly on 12:00. This is a zero probability event, because the area of this line is equal to zero. There isn't a range of values that this "deciding vote" can take, because the winner of an election is determined by who receives the majority of the votes.

It's no wonder then, that Public Choice economists consider voting to be an irrational action. With the introduction of the "threshold vote" concept however, all of a sudden, voters have a new way to affect the outcome of an election. In addition to casting the deciding vote (which still has a negligible

probability), the voter can now affect the election in favor of their candidate in two ways.

First, by casting a vote in favor of the candidate of choice, the voter can make an election in which his or her candidate received fewer votes close enough to warrant a follow up process, one in which the candidate of choice is declared the winner. Because we are working with perceived probability on the part of the voter, voters are actually more likely to vote if the threshold with which election officials make their decision whether or not to apply a follow up process is unknown. Once we have a discrete, known threshold, such as in Florida in the 2000 Presidential Election (less than a 0.5% margin between the candidates was required for a recount), then the probability of casting a threshold vote becomes analogous to casting a deciding vote, and we once again have a zero probability event.

Secondly, an additional way in which a single vote can matter is when it makes the margin between the candidate of choice and the opposing candidate large enough, that the election is kept out of a follow up process, and the candidate of choice is declared the winner. The same theoretical arguments hold for this way of affecting an election, though obviously, some exogenous factors that *would* matter in the recount process wouldn't have an affect now, such as the strength of the candidate's legal team, the likelihood that a judge will side with the candidate, etc. The most important exogenous factor to the voter's calculus in this case is whether or not the election officials are going to decide to

apply a follow up process to the election. Now, we must mathematically include these conclusions in the model.

To incorporate this into the model, the voter will only realize the probability Z of casting a threshold vote, if in fact the election officials decide that the election outcome did in fact break the threshold. In other words, the threshold vote also rests on the probability that the officials will react to the one-vote difference as a finite threshold. Thus, another probability term must be included in the calculation. We'll call the probability that a single-vote is seen as threshold breaking, W ; and also view it as negatively related to the size of the election. For example, in a small election of 1,000 votes, one might easily see an 11-vote margin as more significant than a 10-vote margin. But, in a large election of 1 million votes, it is less reasonable to see a 10,001-vote margin as more significant than 10,000-vote margin.

These theoretical conclusions are supported by research in the field of psychophysics (how people respond to signals in various environments). The general ideas of psychophysics are that an individual's ability to respond to a stimulus depends on the strength of the stimulus and the amount of "noise" that exists in the individual's environment. Noise is "any random event that can influence a person's decision-making process" (Pittenger, 2001). People will not react to a stimulus if either the stimulus is too weak or if there are sufficient distractions affecting the individual as to make the signal indiscernible.

The primary issue of psychophysics research as it is applied to our version of the rational voter model is the strength of the signal as it affects election

officials' decisions of whether to assign a follow-up procedure. That is, is the difference of one more vote (the signal) sufficient to make the margin close enough to warrant a follow-up process (the individual responds to the stimulus)? For example, in the hypothetical case of a large election of 1 million voters, does a final election margin of 1,513 votes make a follow-up process any more likely than if the margin were 1,514? Is the follow-up process more likely with a margin of 2,888 than if the margin were 2,889? That decision is sometimes mandated (as it was for the Florida elections), but in many cases it is not.^{iv} In these other cases, the decision is left to some election officials or left to the courts. In either situation, it is individuals or small groups that decide whether to respond to the stimulus.

Research by noted psychophysicist Gustav Fechner suggests that a change in the signal (or stimulus) will elicit a response only if the *difference threshold* is large enough for the individual to notice a difference (Haberlandt, 1994). This “difference threshold” is how significant the signal is compared to the standard environment with no signal.

The basic idea behind psychophysics (Haberlandt 1994) explains that, based on Fechner’s work, “psychophysicists [find] that the difference threshold [is] relative; a change is more easily noticed if the standard stimulus is of smaller magnitude.”

Using this logic then, it is easy to see why a single vote in a smaller election can have a stronger impact on an election than a single vote in a larger election: Election officials perceive the vote as a stronger “signal”. This has

particular relevance to U.S. Presidential Elections, which are really multiple small statewide elections, as opposed to a single large election, as it is traditionally thought of. In Presidential Elections, the candidate receiving the majority of the votes receives all of the state's electoral votes, which makes the probability of one vote making a difference even greater in that state, because that one vote is seen as a stronger signal.

One last issue in the mix of casting threshold votes is that even if one does cast the threshold vote (and it's recognized as such); that only creates the *possibility* that the voter had an impact on the election. For the vote to matter, the final outcome would have to be different than if the voter had not voted. In other words, the difference of votes between the candidates would have to be overcome by the random process that followed. We will include a probability, S , to represent this likelihood. This probability will be a negative function of the threshold, T , which is chosen by the officials. A larger threshold would thus imply that the probability of a random outcome switching from what it would have been without the follow up process decreases.

A REVISED RATIONAL VOTER MODEL

Finally, with all the relevant factors included, the rational voter model takes the following form:

If $Z \bullet W(N) \bullet S(T)[B] + P'B - C > 0$, then vote

Otherwise, abstain

Or with simplified notation:

If $(ZWS + P')B - C > 0$, then vote

Otherwise, abstain

The theoretical evidence presented above suggests that $ZWS + P' > P$, which would imply, that although still improbable, it is now theoretically rational for a vote to make a difference in an election. With the inclusion of r (randomness in the follow-up process) in determining P' , we know that P' is in fact less than P , but the voters' have been given a new avenue with which to determine the election in the follow up process.

CHAPTER 6

CONCLUSIONS

While the conventional rational voter framework suggests that close elections increase the probability of one casting the deciding vote, the reality of post-election resolution processes suggest that the point may be moot. By including in the model the reality that close elections are resolved by processes that involve some degree of uncertainty, it is possible for a rational voter to influence the outcome of an election by either pushing the election into such a process, or keeping it out of one.

If the theory presented in this paper is true, it not only saves the rational voter model, but it has some interesting policy implications as well. Given randomness in the follow-up, however, another interesting public choice issue is the notion of not setting a mandated threshold for election follow-up processes to take place. More specifically, however, it is not essential that a mandated threshold not exist, just that it remain secret from the electorate as a whole. Given the average voter's knowledge of voting rules and regulations, however, it is not a far stretch of the imagination to act under the assumption that even if there was a mandated threshold did exist, and it was made available to the public, that the majority of people wouldn't know about it, and would behave as if it was an uncertain outcome.

From a policy perspective, when deciding whether or not an election deserves a recount then becomes an interesting choice. Choosing a discrete

threshold too high means that many elections will be unnecessarily sent to follow-up procedures where the probability of changing the outcome is exceedingly small (S near zero). Choosing a threshold too low, means that elections that would have been overturned on a re-examination of the votes, would remain as the original count stood. Choosing the threshold then becomes an optimization decision for minimizing the costs of getting accurate election outcomes. A low threshold that is strictly followed is the best policy for increasing voter rationality. Of course, the cost of a low threshold is that some elections may result in the wrong candidate being seated when the initial vote count involves a tabulation error greater than the threshold.

A more complete analysis would allow for utility to be derived from watching the post election battle unfold, as people clearly enjoy that form of entertainment, and that would make choosing a higher T become a more efficient outcome than we have under a model with such effects ignored.

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Appendix

ⁱ The noted example is cited in Mueller (1997).

ⁱⁱ There is the possibility, however, that the small P can be offset by a sufficiently large set of benefits. But again, except for only the extreme cases, voting is virtually always irrational. For example, if we assume costs of voting are a mere \$1, in the 1 million voter election with a 1% expected margin, the amount of benefit that a voter would need to receive to make voting rational is an astounding $\$1.2 \times 10^{90}$. In a 1 million-voter election, with the \$1 cost and an expected dead-heat, a voter would have to expect \$1,668 in additional benefits to make voting rational.

ⁱⁱⁱ Ben Ginsburg, an attorney in the case, refers to the contest as the “bloody eighth.”

^{iv} Even in the case of a mandate, a follow-up process can be undertaken when the margin is outside the range, if officials wish to do so