Model Comparison with Squared Sharpe Ratios of Mimicking Portfolios

by

Jihyeon Kim

(Under the direction of Cesare Robotti)

Abstract

There are various asset pricing models proposed in the field of finance by using different traded and non-traded factors. In this paper, a variety of statistical methodologies are introduced for comparisons based on differences between squared Sharpe ratios of such models. Especially, in order to compare mimicking portfolios with non-traded factors, different computations are used for squared Sharpe ratios defined by whether two or more models are nested or non-nested. For empirical analysis, five asset pricing models with traded factors are used; Fama-French 3 factor model by Fama and French (1992), Fama-French 5 factor model by Fama and French (2017), Fama-French models with a momentum factor by Jegadeesh and Titman (1993), and Carhart (1997), and the betting against beta model by Frazzini and Pedersen (2014). For mimicking portfolios, four different non-traded factors, which are proxies for consumption, are compared with those five models.

Index words: Asset pricing, Sharpe ratio, Portfolio theory, Mimicking portfolio, Non-traded factor

Model Comparison with Squared Sharpe Ratios of Mimicking Portfolios

by

Jihyeon Kim

B.A., Ulsan National Institute of Science and Technology, 2016

A Thesis Submitted to the Graduate Faculty
of The University of Georgia in Partial Fulfillment
of the
Requirements for the Degree

Master of Science

Athens, Georgia

2018

Model Comparison with Squared Sharpe Ratios of Mimicking Portfolios

by

Jihyeon Kim

Approved:

Major Professor: Cesare Robotti

Committee: Cheolwoo Park
Jeongyoun Ahn

Electronic Version Approved:

Suzanne Barbouri
Dean of the Graduate School
The University of Georgia
May 2018

Dedication

To my best heroes; my dad, mom, and sister who always believe in me with unfathomable love. Last but not least, I dedicate this thesis to Dr. Cesare Robotti who has provided valuable guidance to me so that I can devote myself to academic pursuits.

Table of Contents

List of Tables

Chapter 1

Introduction

For several decades, researchers in the field of finance have been looking for asset pricing model in order to predict expected returns on assets. Following the capital asset pricing model (CAPM) by Sharpe (1964) and Lintner (1965), a variety of models have been presented based on market excess returns and market risks, including intertemporal CAPM by Merton (1973).

Fama and French (1993) introduced a new asset pricing model with three factors comprised of two additional factors in addition to the market risk factor used in the previous CAPM, so the demand for comparisons with each model has emerged as a research interest.

In a response to the need for comparisons with different asset pricing models, researchers have attempted to quantify the different models. Barillas, Kan, Robotti, and Shanken (2017) proposed that comparison methodologies based on squared Sharpe ratios can be obtained from the GRS test of Gibbons, Ross, and Shanken (1989).

Throughout this paper, various methodologies of computation for squared Sharpe ratios, depending on whether two or more models are nested or non-nested, are proposed and tested. And also, after various statistical likelihood tests and pairwise comparisons, non-traded factors are added to Fama and French three factor model (FF3) constructing mimicking portfolios. All of the non-traded factors in this paper denote values of consumption based on different measures, such as using different time points on consumption data or the amount of garbage as proxies for consumption.

Those mimicking portfolios are compared with traded factor models in terms of squared Sharpe ratios, and for our empirical analysis, five models are selected as traded factor models consisting of both nested and non-nested models. Especially, under the general assumptions of distributions, squared Sharpe ratios are computed asymptotically. Since original factors have only 43 yearly observations due to the limitation of the available number of observations for some factors, simple linear interpolation methodology is used to convert yearly data to quarterly data so that asymptotic statistical tests can be conducted. Thus, a total of 172 quarterly observations are used for comparison of squared Sharpe ratios. Following tests of each single model, pairwise tests, and other asymptotic statistical comparisons, the difference between mimicking portfolios and traded factor models are reported in the last chapter of this paper.

Chapter 2

Literature Review

2.1 Asset pricing model

In order to predict excess returns from stock portfolio as accurately as possible, a variety of asset pricing models have been introduced based on various factors in the field of finance. Among those models, CAPM model and Fama-French models are widely used and explored for asset pricing in a various way.

2.1.1 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM), introduced by Sharpe (1964), and Linter (1965), is a model shows the relationship between the expected return (R_i) on an asset including the stock portfolio and the market risk where i denotes an individual asset or portfolio.

The expected return of the asset (E(R_i)) on the left-hand side is computed by beta (β_i) which is the sensitivity of the return on asset to the market return (R_m) and the risk-free rate of interest (R_f) such as government bond and T-bill rates, and CAPM can be explained as

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \quad where \quad \beta_i = \frac{cov(R_i, R_m)}{\sigma_m}$$

In terms of the excess returns, the formula can be described as

$$R_i - R_f = \alpha_i^* + \beta_i(R_m - R_f) + \varepsilon_i \quad where \quad \alpha_i^* = \alpha_i - (1 - \beta_i)R_f$$

Then, taking expectations on both sides gives

$$E(R_i) = R_f + \alpha_i^* + \beta_i(E(R_m) - R_f)$$

and CAPM implies $\alpha_i^* = 0,\ or\ \alpha_i = (1 - \beta_i)R_f$. So, testing for CAPM is usually testing $H_0 : \alpha_i^* = 0$.

According to the model above, the expected return on assets increases as the value of beta increases and it has been used as a major asset pricing model.

2.1.2 Fama-French 3 factor model

While the traditional CAPM explains the expected return of asset based on a market risk factor, Fama and French (1992) argued that the market risk (β_i) does not fully explain expected returns on stock or portfolio, and they introduced a multi-factor asset pricing model by adding size and value factors.

By using market capitalization and book-to-market values which are traded factors, they categorized those values from small(low) to large(high) indicated as SMB (Small market capitalization minus Big) and HML (High book-to-market minus Low) factors in the model. Also, they proposed that value and small stocks which have higher book-to-market values tend to have higher values of expected returns compared to big and growth stocks which have lower book-to-market values. Fama-French 3 factor (FF3) model can be expressed as,

$$E(R_i) = R_f + \beta_{MKT}^i E(MKT) + \beta_{SMB}^i E(SMB) + \beta_{HML}^i E(HML)$$

where MKT denotes the difference between returns on market portfolio and risk-free asset.

In the equation, three different betas based on each factor can be obtained by conducting the cross-sectional regression and actual return of asset is calculated by,

$$R_{it} = R_{ft} + \alpha + \beta_m^i (R_{mt} - R_{ft}) + \beta_{SMB}^i R_{SMBt} + \beta_{HML}^i R_{HMLt} + \varepsilon_{it}$$

where ε is a white noise term by time t.

2.2 Mimicking portfolio

In addition to those two asset pricing models prevalent in the finance field, there have been a myriad of different pricing models introduced since then. Those models including

CAPM and Fama-French three factor model which consider value-weighted portfolio as an equilibrium factor. These implications have prompted up researchers to study mimicking portfolios with non-traded factors which also could be called macroeconomic factors such as GDP growth or consumption growth.

Chapter 3

Methodology

3.1 Sharpe ratio

Along with newly explored asset pricing models, there have been several measurements to evaluate the fit of such models. Barillas and Shanken (2017a) introduced a method using Sharpe ratio based on the mean-variance efficient. The Sharpe ratio is simply obtained by dividing the difference between the risk free rate (R_f) and the expected rate of return of the portfolio ($E(R_p)$) by the standard deviation of portfolio returns (σ_f), which is considered as the total risk. Generally, the model with the highest squared Sharpe ratio is considered a preferred model in asset pricing.

$$Sharpe\ ratio = \frac{E(R_p) - R_f}{\sigma_p}$$

In order to find a model without any redundant factor that satisfies the tangency portfolio, Barillas, Kan, Robotti, and Shanken (2017) concentrated on the comparison of the squared Sharpe ratios of models with an asymptotic analysis under the general assumptions for distributions. There are different statistical methodologies to compute mean-variance depending on whether the models are nested or non-nested. For nested models, the squared Sharpe ratio of the larger model is always higher than that of the nested model. However, to account for non-nested modes which are consist of some factors not included in the other models, Barillas, Kan, Robotti, and Shanken (2017) developed a test for comparing non-nested models which accommodates the difference between two tangency portfolios constructed by different sets of factors. This test will be explained in the empirical analysis chapter.

Balduzzi and Robotti (2008), Lewellen, Nagel, and Shanken (2010), and Barillas and Shanken (2017a) indicated that it is possible that the differences between squared Sharpe ratios of models can be explored by a quadratic form from the generalized least squared pricing errors. This asymptotic methodology will also be shown in the next chapter.

If each factor portfolio has the minimum variance, it implies $\alpha(intercept) = 0$. The Gibbsons, Ross, and Shanken (GRS) test is a statistical hypothesis test that whether all αs are zero in pricing models by using Wald statistic. Since alpha is an estimated intercept, the test statistic shows that

$$\alpha_R' \Sigma^{-1} \alpha_R = \left(\frac{\mu_q}{\sigma_q}\right)^2 - \left(\frac{\mu_p}{\sigma_p}\right)^2$$

where portfolio q is the tangency portfolio constructed with assets and test factors, while portfolio p is the market portfolio with only test factors, which is the difference between squared Sharpe ratios of portfolios. Thus, the GRS test, which is a hypothesis test of $\alpha = 0$, can derive a hypothesis test if the market portfolio shows the maximum squared Sharpe ratio. Suppose R_t is a vector of returns comprised of factors f_t. Then, a vector equation can be expressed as

$$R_t = \alpha_R + \beta f_t + \epsilon_t, \qquad t = 1, \ldots, T$$

And from the GRS test,

$$\alpha_R' \Sigma^{-1} \alpha_R = Sh^2\left(f, R\right) - Sh^2\left(f\right)$$

denotes that when the extra test asset R is added, the equation shows the improvement of the new model in terms of the squared Sharpe ratio. In Appendix, various methodologies regarding the asymptotic distribution of the difference in the squared Sharpe ratios of non-nested models are cited by Barillas, Kan, Robotti, and Shanken (2017).

3.2 Mimicking Portfolio

A mimicking portfolio is used in asset pricing by projection of non-traded (macroeconomic) factors on asset returns with time series regression, and it represents unobservable information by using non-trade factors. Models with one or more factors, such as consumption CAPM or intertemporal CAPM, do not represent asset returns themselves. Barillas, Kan, Robotti, and Shanken (2017) discussed the asymptotic distribution of the sample squared Sharpe ratio of mimicking portfolio, and the difference between the sample squared Sharpe ratio of mimicking portfolio and asset pricing models. For a traded factor, the mimicking portfolio is equivalent to the factor itself; thus, model comparison now depends on non-traded factors. Barillas, Kan, Robotti, and Shanken (2017) noted that suppose f_t are comprised of traded and non-traded factors and R_t is a vector of returns of both of those traded factors and non-traded factors, which are also called basis assets where $Y_t = [f'_t,\ R'_t]'$. And then, assuming that variance matrices of f and R are invertible and that V_{fR} is full rank, the mean matrix and the variance matrix can be constructed as

$$\mu = E\left[Y_t\right] \equiv \begin{bmatrix} \mu_f \\ \mu_R \end{bmatrix}$$

$$V = Var\left[Y_t\right] \equiv \begin{bmatrix} V_f\ V_{fR} \\ V_{Rf}\ V_R \end{bmatrix}$$

In terms of projecting f_t on to R_t, mimicking portfolio returns can be presented as

$$f_t^* = V_{fR}{V_R}^{-1}R_t$$

where f_t^* denotes the return of mimicking portfolio

$$\mu^* = E\left[f_t^*\right] = V_{fR}{V_R}^{-1}\mu_R$$

$$V^* = Var\left[f_t^*\right] = V_{fR}{V_R}^{-1}V_R\left(V_{fR}{V_R}^{-1}\right)'$$

Let θ^2 denote the population squared Sharpe ratio of the mimicking portfolio, and $\hat{\theta}^2$ denote the sample squared Sharpe ratio of the mimicking portfolio. Then,

$$\theta^2 = \mu^{*'} V^{*-1} \mu^* \equiv \mu_R' V_R{}^{-1} V_{Rf} \left(V_{fR} V_R{}^{-1} V_{Rf} \right)^{-1} V_{fR} V_R{}^{-1} \mu_R$$
$$\hat{\theta}^2 = \hat{\mu}^{*'} \hat{V}^{*-1} \hat{\mu}^* \equiv \hat{\mu}_R' \hat{V}_R^{-1} \hat{V}_{Rf} \left(\hat{V}_{fR} \hat{V}_R^{-1} \hat{V}_{Rf} \right)^{-1} \hat{V}_{fR} \hat{V}_R^{-1} \hat{\mu}_R$$

In Appendix chapter, various methodologies regarding the asymptotic distribution of the sample squared Sharpe ratio of the mimicking portfolio and the pairwise model comparison with mimicking portfolios are introduced.

Chapter 4

Data

Sample period for all non-traded factors are from 1964 to 2006 which have only 43 observations each in total due to limited data range on some factors. For further analysis, since methodologies require data to be asymptotic, simple linear interpolation method is used in order to convert yearly data to quarterly data for statistical tests.

4.1 Non-traded factors

For model comparisons by using squared Sharpe ratio, four non-traded factors are selected for empirical analysis. All of factors are proxies for the consumption with different measures and those will be compared each other in the later section.

Each factor is used as a non-traded factor attached to the Fama-French three factor model (1993) which have the value-weighted market excess return (MKT), the size factor (SMB), and the book-to-market factor (HML) constructing mimicking portfolios.

4.1.1 Three-year consumption growth – 3Y

Parker and Julliard (2005) measured the risk of the return of an asset by the covariance of the assets return and cumulated consumption over three years (twelve quarters) and showed this measurement for the risk may be better than the true risk of an asset. The real personal consumption expenditures on nondurable goods per capita from the National Income and Product Accounts (NIPA) are used for analysis and data range is from 1964 to 2006.

4.1.2 Consumption growth by four quarter – 4Q

When betas known as risks are obtained from consumption growth by each of the fourth quarter points, they explain better in cross section of stock returns than consecutive consumption growth traditionally used (Jagannathan and Wang (2007)). Consumption data is the annual and quarterly seasonally adjusted aggregate nominal consumption expenditures on nondurables and services for the period 1964 to 2006 from National Income and Product Accounts (NIPA).

4.1.3 Garbage measurement for prediction of consumption – GAR

Savov (2011) introduced interesting measure of consumption by using the amount of garbage. In cross-section of asset pricing, garbage measures better than NIPA expenditure on nondurables and services as a proxy for consumption. The amount of garbage is annually collected from U.S. Environmental Protection Agency which is from 1964 to 2006. The key variable for this measure is called Municipal Solid Waste – commonly known as trash or garbage.

4.1.4 Unfiltered consumption factor – UNFIL

A lot of researchers have been trying to estimate and look for the most precise factors as proxies for consumption and Kroencke (2017) proposed that garbage factor by Savov (2011) may not properly explain consumption and offered another alternative measurement which is called unfiltered NIPA consumption in order to measure consumption factors.

4.2 Asset pricing models

For comparisons of squared Sharpe ratios, several asset pricing models are used. The first model is the Fama and French (1993) three factor model (FF3) which adds size (SMB) and book-to-market (HML) factors on the traditional CAPM model (MKT). The second model is Fama and French (2017) five factor model (FF5) comprised of investment (CMA) and cash profitability (RMW) factors in addition to the three factors from the Fama and French three factor model; market return, size, and book-to-market.

The third model is the Carhart (1997) four factor model (FF3+UMD) which introduces a momentum factor (UMD) along with the Fama and French three factors. The momentum in the stock market shows that the price tends to continue to go up if it is increasing while the price tends to keep going down if it is decreasing. Similarly, the fourth model (FF5 + UMD) is combined with Fama and French five factor model with a momentum factor proposed by Jegadeesh and Titman (1993).

The last model is called the Frazzini and Pedersen (2014) model (MKT + BAB) contained the market excess return (MKT) and the betting against beta factor (BAB) which denotes long leveraged low-beta assets and short high-beta assets.

Table 1 shows summary statistics for yearly returns of the traded factors and Table 2 shows the same statistics for quarterly adjusted traded factors. Panel A of both tables contains means, standard deviations, and t-statistics of factor returns and all of the factors have fairly large values of factor returns. BAB and UMD factors show higher expected returns among both yearly and quarterly data, while MKT and BAB factors seem volatile in both yearly and quarterly datasets. Panel B of Table 1 and Table 2 present correlations of each factor. Book-to-market (HML) and investment (CMA) factors are highly positively correlated among both yearly and quarterly data, and correlations between investment (CMA) and BAB factors are also high.

Chapter 5

Empirical Results

5.1 Tests of the equality of squared Sharpe ratios for competing traded-factor models

Before the analysis of non-traded factor models, pairwise tests for traded-factor models, which are comprised of either nested or non-nested models, are conducted. Panel A in Table 3 (yearly) and Table 4 (quarterly) present the differences in sample squared Sharpe ratios for all of the possible pairs which are under the tests of $H_0 : \theta_i^2 = \theta_j^2$ in column i and row j, and Panel B shows p-values corresponding to those tests. In Panel A, * indicates significance at the 5% level, and ** indicates significance at the 1% level, and those p-values are obtained by different calculations depending on whether the model is nested and non-nested based on methodologies in Appendix.

For the nested models, a single alternative model that contains all the factors and the hypothesis that whether the nested model has the same squared Sharpe ratio are tested using the methodologies introduced by Barillas, Kan, Robotti, and Shanken (2017). That is, the values of α from projecting all factors, which are not included in the nested model onto the nested factors, are tested whether these are equal to zero or not under the null hypothesis. From this test, given the null hypothesis test is rejected, it indicates that the nested model is dominated by larger models while the nested model is as good as larger models if null hypothesis is not rejected.

For the non-nested models, a different methodology is used to test the equality as we can see the Proposition 1 in Appendix chapter. The difference between squared Sharpe ratios of the model with nested factors and the models that contains nested and non-nested factors are tested whether it is different from zero or not. Then, it can be concluded that the model with nested factors performs as good as the one with non-nested factors if the null hypothesis is rejected.

In summary of Table 3 and Table 4, with yearly returns data, most of factor models are outperformed at 1% of significance level except MKT+BAB model with FF3 model and FF5, and those with a momentum factor. When a momentum factor is added to FF3 and FF5 model, those models become statistically significant at a 1% of significance level indicating FF3 and FF5 models with the momentum factor outperforms MKT+BAB model. With quarterly adjusted returns data, the FF3 model with a momentum factor outperforms all other models at the 1% significance level.

The second test conducted in this paper is to examine whether a model has the highest squared Sharpe ratio among all other models based on the multivariate inequality analysis by Wolak (1989). The null hypothesis of the test is described as none of the other model is superior to the testing model so called benchmark, while the alternative hypothesis is that at least one model represents larger squared Sharpe ratio than that of the testing model. According to Barillas, Kan, Robotti, and Shanken (2017), –

Suppose there are p models and let $\delta = (\delta_2, \ldots, \delta_p)$ and $\hat{\delta} = (\hat{\delta}_2, \ldots, \hat{\delta}_p)$ which follows asymptotic normal distribution with mean δ and covariance $\sum_\delta$, where $\delta_i = \theta_1^2 - \theta_i^2$ and $\hat{\delta}_i = \hat{\theta}_1^2 - \hat{\theta}_i^2$ for $i = 2, \ldots, p, r = p - 1$, and t indicates the number of periods (observations).

Then, the hypothesis whether the testing model performs better than other models and, the hypothesis test can be constructed as

$$H_0 : \delta \geq 0_r$$

$$H_a : \delta \in \Re^r$$

Kan, Robotti, and Shanken (2013) discussed test statistics corresponding to the hypothesis test, that is,

$$\tilde{\delta} = \min_{\delta}(\hat{\delta} - \delta)\hat{\sum}_{\hat{\delta}}^{-1}(\hat{\delta} - \delta) \quad s.t. \delta \geq 0_r$$

where $\hat{\sum}_{\hat{\delta}}^{-1}$ is a consistent estimator of $\sum_{\delta}$. Thus, the likelihood ratio is obtained by,

$$LR = T(\hat{\delta} - \tilde{\delta})' \hat{\sum}_{\hat{\delta}}^{-1} (\hat{\delta} - \tilde{\delta})$$

And it indicates that given the LR has a higher value, it would represent that the null hypothesis can be rejected. Kan, Robotti, and Shanken (2013) explained the LR value follows $\bar{\chi}^2$ distribution with asymptotic distribution. Thus,

$$LR = T(\hat{\delta} - \tilde{\delta})' \hat{\sum}_{\hat{\delta}}^{-1} (\hat{\delta} - \tilde{\delta}) \overset{A}{\sim} \sum_{i=0}^{r} w_{r-i}(\textstyle\sum_{\hat{\delta}}) X_i$$

is computed in order to obtain p-values where X_i is independent χ^2 random variables with degrees of freedom of i, and w indicates the weight of X_i.

Based on the above methodology, in this paper, each of non-nested models is tested with 100,000 times of bootstrapping as we see in Table 5. For the analysis, only quarterly adjusted data is investigated. The first column shows sample squared Sharpe ratio, the r denotes the number of alternative models in each comparison, and LR indicates the likelihood ratio followed by p-value on the fifth column. In summary of the table 5, FF3+UMD and FF5+UMD have higher sample squared Sharpe ratio showing that p-values corresponding to the squared Sharpe ratios and likelihood ratios are higher than p-values of other models. The result indicates those two models asymptotically perform better than or as good as other models. However, considering Table 4 regarding the pairwise tests of traded factor models, although the FF3+UMD and FF5+UMD models are superior over other models in Table 5, each model fails to reject the null hypothesis that either FF3+UMD or FF5+UMD has a squared Sharpe ratio that has larger or equal to other alternative models in Table 4.

5.2 Model comparison with non-traded factors

In the previous chapter, various methodologies are introduced in order to compare asset pricing models with mimicking portfolios. In this section, four non-traded consumption factors obtained by different measures are used for our empirical analysis. These four factors

– 3YR, 4Q, GAR, and UNFIL – are augmented with the FF3 model thus, four mimicking portfolios are constructed.

At first, those four mimicking portfolios with five asset pricing models, are compared using the linear regression method. All of the traded factors in the five models are consist of the returns of MKT, SMB, HML, UMD, RMW, CMA, and BAB. As we can see in Table 1 and Table 2, some of factor combinations are highly correlated, however, in this analysis, those individual weights do not matter since only overall mimicking returns are considered as a result. In the cross-sectional regressions, Kan and Zhang (1999) proposed that – the correlation should be significantly different from zero so as to avoid complications akin to the "useless factor" problems. In Table 6, when regressing each of four non-traded factors on all of the traded factors, adjusted R^2 are acceptable especially with quarterly adjusted data. Quarterly data is only considered for our empirical analysis instead of yearly data due to the asymptotic distribution. For the F-test with these regressions, p-values show all factors have their p-values less than 0.05, and quarterly adjusted 4Q and GAR data have p-values less than the 1% level. It indicates that some of asset returns would be able to mimic some of non-traded factors.

To examine how well the models augmented with non-traded factors perform compared to mimicking portfolios, the differences of squared Sharpe ratios are obtained as seen in Table 7 and Table 8. In Panel A in both tables, the values of difference are computed by subtraction the squared Sharpe ratio of non-traded factor mimicking portfolio from that of each of traded factor models. Statistical values are also obtained based on the proposition 4 in the Appendix chapter which explains the asymptotic statistics regarding the difference in squared Sharpe ratios of models. As our statistical methodologies for this analysis are based on asymptotic distribution, only Table 8 with quarterly data is considered due to the number of observations. With the FF3+3Y model, most of traded factor models are dominated by the FF3+3Y model except for the FF3 model considering differences and p-values corresponding to those differences. In the same way, with 4Q data, the FF3+4Q model dominates over only the FF3+UMD and the FF5+UMD models and it's dominated by the rest of asset pricing models. Also, the garbage consumption factor models (FF3+GAR) presents all of the traded factor models except for the FF3 model are dominated by the FF3+GAR model while only the MKT+BAB and the FF5 models are dominated by the FF3+UNFIL model.

Chapter 6

Conclusion

Throughout various asymptotic methodologies and testing, multiple asset pricing models were compared based on their squared Sharpe ratios. From the GRS test, the values of differences between squared Sharpe ratios are obtained and those are used for the most of analyses in this paper.

The first analysis conducted is the comparison of asset pricing models with traded factors. Five models are selected; FF3, FF5, FF3+UMD, FF5+UMD, and MKT+BAB, and each model contains some of seven traded factors; MKT, SMB, HML, UMD, RMW, CMA, and BAB. Summary statistics show those seven factors have fairly large value of returns and t-statistics.

With five asset pricing models with traded factors, pairwise tests are conducted between each of models. Under the null hypothesis that squared Shape ratios of two models are the same, p-values are computed and compared differently depending on whether models are nested and non-nested at 1% and 5% significance levels. With both yearly and quarterly data, most of models are dominated by the FF3 model except for the MKT+BAB model. Next, individual squared Sharpe ratios of each model are analyzed with the hypothesis test whether there is any difference between the test (benchmark) asset and other asset models. From the test, although the FF3+UMD and the FF5+UMD show that they have large values of squared Sharpe ratio, since p-values are higher than 0.5, it can be concluded that those models fail to reject the null hypothesis.

Finally, mimicking portfolios with four different non-traded factors are compared in the last chapter. Before the comparison of the difference between squared Sharpe ratios, F-statistic were obtained from the regression of each non-traded factor onto all the seven traded factors. With both yearly and quarterly adjusted data, the values of adjusted R^2 are fair, and p-values of each F-statistics are mostly at 5% significance level except for quarterly adjusted 3YR and 4Q factors. As the last comparison of this paper, non-traded factor mimicking portfolios are compared with asset pricing models with traded-factor based on their squared Sharpe ratios. With quarterly adjusted mimicking portfolios of 3Y factor augmented with the FF3 model (FF3+3Y) and GAR factor augmented with the FF3 model (FF3+GAR), the results indicate that they perform better than other traded factor models except for only one traded factor model. However, FF3 model with unfiltered factor (FF3+UNFIL) shows only MKT+BAB model is dominated by the FF3+UNFIL portfolio.

Chapter 7

Appendix

Barillas, Kan, Robotti, and Shanken (2017) proposed different methodologies for each comparison and those methodologies are cited below.

7.1 Asymptotic distribution of the difference in squared Sharpe ratio of non-nested models

Suppose there are two non-nested models, A and B with returns from factors, f_{At} and f_{Bt} with time period T. Then, the squared Sharpe ratios of factors as $\theta_A^2 = \mu'_A V_A^{-1} \mu_A$ and $\theta_B^2 = \mu'_B V_B^{-1} \mu_B$, and the sample squared Sharpe ratios are $\hat{\theta}_A^2 = \hat{\mu}'_A \hat{V}_A^{-1} \hat{\mu}_A$ and $\hat{\theta}_B^2 = \hat{\mu}'_B \hat{V}_B^{-1} \hat{\mu}_B$, respectively.

PROPOSITION 1: The asymptotic distribution of the difference in sample squared Sharpe ratios is given by

$$\sqrt{T}[\hat{\theta}_A^2 - \hat{\theta}_B^2] - [{\theta_A}^2 - {\theta_B}^2] \overset{A}{\sim} N(0, E[d_t^2])$$

provided that $E[d_t^2] > 0$, where $d_t = 2(u_{At} - u_{Bt}) - (u_{At}^2 - u_{Bt}^2) + (\theta_A^2 - \theta_B^2)$ with $u_{At} = \mu'_A V_A^{-1}(f_{At} - \mu_A)$ and $u_{Bt} = \mu'_B V_B^{-1}(f_{Bt} - \mu_B)$

LEMMA 1: When the traded-factor returns are i.i.d. multivariate elliptically distributed with kurtosis parameter κ, the asymptotic variance of the difference in sample squared Sharpe ratios is given by

$$E[d_t^2] = \theta_A^2[4 + (2+3\kappa)\theta_A^2] + \theta_B^2[4 + (2+3\kappa)\theta_B^2 - 2\{2\rho\theta_A\theta_B[2 + (1+\kappa)\rho\theta_A\theta_B] + \kappa\theta_A^2\theta_B^2\}$$

where $\rho = corr[u_{At}, u_{Bt}] = \frac{E[u_{At}u_{Bt}]}{\theta_A\theta_B}$ is the correlation between the returns on the tangency portfolios of f_{At} and f_{Bt}.

7.2 Asymptotic distribution of the sample squared Sharpe ratio of mimicking portfolios

Suppose $Y_t = [f_t', R_t']'$, then $f_t = a + AR_t + \eta_t$ where $A = V_{fR}V_R^{-1}$.

PROPOSITION 2: The asymptotic distribution of $\hat{\theta}^2$ is given by

$$\sqrt{T}(\hat{\theta}^2 - \theta^2) \overset{A}{\sim} N(0, E[d_t^2])$$

provided that $E[d_t^2] < 0$, where

$$h_t = 2u_t(1 - y_t) - u_t^2 + 2y_t v_t + \theta^2$$

$$v_t = \mu_R' V_R^{-1}(R_t - \mu_R), \quad u_t = \mu^{*\prime} V^{*-1}(f_t^* - \mu^*), \textit{ and } y_t = \mu^{*\prime} V^{*-1}\eta_t$$

LEMMA 2: When the factors and returns are *i.i.d.* multivariate elliptically distributed with kurtosis parameterκ, the asymptotic variance of $\hat{\theta}^2$ is given by

$$E[h_t^2] = \theta^2[4 + (2 + 3\kappa)\theta^2] + 4(1 + \kappa)E[y_t^2](\theta_R^2 - \theta^2)$$

where $\theta_R^2 = \mu_R' V_R^{-1}\mu_R$ represents the squared Sharpe ratio of the tangency portfolio of R,

$$E[y_t^2] = \mu^{*'} V^{*-1} V_{f\cdot R} V^{*-1}\mu^*,$$

$$\textit{and } \; V_{f\cdot R} = V_f - V_{fR}V_R^{-1}V_{Rf}$$

is the covariance matrix of the residuals from projecting the factors on the returns.

7.3 Pairwise model comparison with mimicking portfolios

Nested models: Suppose model A has $f_{At}^* = [f_{1t}^{*\prime}, f_{2t}^{*\prime}]'$, and model B has $f_{Bt}^* = f_{1t}^*$. Then, $\mu_1^* = E[f_{1t}^*]$, $\mu_2^* = E[f_{2t}^*]$, $V_{11}^* = Var(f_{1t}^*)$, $V_{12}^* = Cov(f_{1t}^*, f_{2t}^*)$, $V_{22}^* = Var(f_{2t}^*)$, $and\, V_{21}^* = V_{12}^*$. Let f_{1t}^* is a K1-vector and f_{2t}^* is a K2-vector, with K=K1+K2.

PROPOSITION 3: Under the null hypothesis $H_0 : \alpha_{21}^* = 0_{K_2}$,

$$T\hat{\alpha}_{21}^{*\prime}\hat{V}(\hat{\alpha}_{21}^*)^{-1}\hat{\alpha}_{21}^* \overset{A}{\sim} \chi_{K_2}^2,$$

where $\hat{V}(\hat{\alpha}^*_{21}) = E[q_t q_t']$, with $q_t = \xi_t(1 - y_{1t}) + w_t(v_t - u_{1t})$,

$$\xi_t = (f^*_{2t} - \mu^*_2) - V^*_{21}V^{*-1}_{11}(f^*_{1t} - \mu^*_1),\ y_{1t} = \mu^{*\prime}_1 V^{*-1}_{11}(f_{1t} - \mu_1),$$

$$\eta_{1t} = (f_{1t} - \mu_1) - (f^*_{1t} - \mu^*_1),\ \eta_{2t} = (f_{2t} - \mu_2) - (f^*_{2t} - \mu^*_2),$$

$$u_{1t} = \mu^{*\prime}_1 V^{*-1}_{11}(f^*_{1t} - \mu^*_1),\ and\, w_t = \eta_{2t} - V^*_{21}V^{*-1}_{11}\eta_{1t}.$$

Non-nested models: Suppose two non-nested models, A and B, with mimicking portfolios f^*_{At} and f^*_{Bt}, respectively. Then, $\mu^*_A = E[f^*_{At}],\ \mu^*_B = E[f^*_{Bt}],\ V^*_A = Var(f^*_{At}),\ and\, V^*_B = Var(f^*_{Bt})$. Let θ^2_A and θ^2_B denote population squared Sharpe ratios, with $\hat{\theta}^2_A$ and $\hat{\theta}^2_B$ as sample squared Sharper ratios.

PROPOSITION 4: The asymptotic distribution of the difference in sample squared Sharpe ratios is given by

$$\sqrt{T}([\hat{\theta}^2_A - \hat{\theta}^2_B] - [\theta^2_A - \theta^2_B]) \overset{A}{\sim} N(0, E[d^2_t])$$

provided that $E[d^2_t] > 0$, where $d_t = h_{At} - h_{Bt}$, with $u_{At} = \mu^{*\prime}_A V^{*-1}_A(f^*_{At} - \mu^*_A)$ and $y_{At} = \mu^{*\prime}_A V^{*-1}_A \eta_{At},\ h_{At} = 2u_{At}(1 - y_{At}) - u^2_{At} + 2y_{At}v_t + \theta^2_A$, and similarly for model B. As defined earlier, $\eta_{jt} = (f_{jt} - \mu_j) - (f^*_{jt} - \mu^*_j)$ for $j = A, B$.

Bibliography

[1] Balduzzi, P.; Robotti, C. (2008) *Mimicking portfolios, economic risk premia, and tests of multi-beta models.* Journal of Business and Economic Statistics, Vol 26, No. 3, 354-368.

[2] Barillas, F.; Kan, R.; Robotti, C.; Shanken, J. (2017) *Model comparison with sharpe ratios.* Working Paper.

[3] Carhart, M. M. (1997) *On persistence in mutual fund performance.* Journal of Finance, Vol 52, No. 1, 57-82.

[4] Cochrane, J. H. (1996) *A cross-sectional test of an investment-based asset pricing model.* Journal of Political Economy, Vol 104, No. 3, 572-621.

[5] Fama, E. F.; French, K. R. (1992) *The crosssection of expected stock returns.* Journal of Finance, Vol 57, No. 2, 427-465.

[6] Fama, E. F.; French, K. R. (2015) *A five-factor asset pricing model.* Journal of Financial Economics, Vol 116, 1-22.

[7] Fama, E. F.; French, K. R. (2012) *Size, value, and momentum in international stock returns.* Journal of Financial Economics, Vol 105, 457-472.

[8] Frazzini, A. ; Pedersen, L. H. (2014) *Betting against beta.* Journal of Financial Economics, Vol 111, No. 1, 1-25.

[9] Gibbons, M. R.; Ross, S. A.; Shanken, J (1989) *A test of the efficiency of a given portfolio.* Econometrica, Vol 57, No. 5, 1121-1152.

[10] Gospodinova, N.; Kan, R.; Robotti, C. (2013) *Chi-squared tests for evaluation and comparison of asset pricing models*. Journal of Econometrics, Vol 173, No. 1, 108-125.

[11] How, K.; Xue, C.; Zhang, L. (2015) *Digesting anomalies: an investment approach*. Review of Financial Studies, Vol 28, Issue 3, 650-705.

[12] Jagannathan, R.; Wang, Z. (1996) *The conditional CAPM and the cross-section of expected returns*. Journal of Finance, Vol. 51, No. 1, 3-53.

[13] Jagannathan, R.; Wang, Y. (2007) *Lazy investors, discretionary consumption, and the cross-section of stock returns*. Journal of Finance, Vol 62, No. 4, 1623-1661.

[14] Jegadeesh, N.; Titman, S. (1993) *Returns to buying winners and selling losers: implications for stock market efficiency*. Journal of Finance, Vol 48, No. 1, 65-91.

[15] Kleibergen, F.; Zhan, Z. (2017) *Asset pricing with consumption and robust inference*. Working Paper.

[16] Kroencke, T. A. (2017) *Asset pricing without garbage*. Journal of Finance, Vol 72, No. 1, 47-98.

[17] Lettau, M.; Ludvigson, S. (2001) *Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying*. Journal of Political Economy, Vol 109, No. 61, 1238-1287.

[18] Liew, J.; Vassalou M. (2000) *Can book-to-market, size and momentum be risk factors that predict economic growth?*. Journal of Financial Economics, Vol 57, 221-245.

[19] Merton, R. C. (1973) *An intertemporal capital asset pricing model*. Econometrica, Vol 41, No. 5, 867-887.

[20] Parker, J. A.; Julliard, C. (2005) *Consumption risk and the cross section of expected returns*. Journal of Political Economy, Vol 113, No. 1, 185-222.

[21] Savov, A. (2011) *Asset pricing with garbage*. Journal of Finance, Vol 66, No. 1, 177-201.

[22] Sharpe. W. F. (1964) *Capital asset prices: A theory of market equilibrium under conditions of risk*. Journal of Finance, Vol 19, No. 3, 425-442.

[23] Sharpe. W. F. (1966) *Mutual fund performance*. Journal of Business, Vol 39, No. 1, 119-138.

[24] Vassalou, M (2003) *News related to future GDP growth as a risk factor in equity returns.* Journal of Financial Economics, Vol 68, 4773.

[25] Wolar, F. A. (1989) *Testing inequality constraints in linear econometric models.* Journal of Econometrics, Vol 41, No. 2, 205-235.

Table 1: Summary Statistics for Traded Factors (Yearly)

Two tables below give the summary statistics for all yearly observations of traded factors. Market excess returns (MKT), size effect factor (SMB), and book-to-market ratios factor (HML) are used for both Fama-French 3 factor and 5 factor models. The profitability factor (RMW) and investment factor (CMA) are used for Fama-French 5 factor model and Fama-French models with a momentum factor are consist of original FF model factors and the momentum factor (UMD). At last, Frazzini and Pedersen (2014) introduced a model with the betting against beta factor (BAB). Panel A shows mean, standard deviation and t-statistic values of all factors, and Panel B shows correlations between each factor.

Panel A : Means, Standard Deviations, and t-statistics (percentage)			
	Mean	**Standard Deviation**	**t-statistic**
MKT	6.18	17.01	2.38
SMB	3.87	14.67	1.73
HML	6.52	13.92	3.07
UMD	10.65	13.80	5.06
RMW	3.07	10.11	1.99
CMA	4.67	10.20	3.00
BAB	13.55	16.88	5.26

Panel B : Correlations						
	SMB	**HML**	**UMD**	**RMW**	**CMA**	**BAB**
MKT	0.285	-0.296	-0.040	-0.215	-0.411	0.139
SMB		-0.039	-0.111	-0.215	-0.135	0.173
HML			-0.397	0.127	0.738	0.573
UMD				0.045	-0.305	-0.235
RMW					-0.057	0.319
CMA						0.339

Table 2: Summary Statistics for Traded Factors (Quarterly)

Two tables below give the summary statistics for all quarterly observations of traded factors. Market excess returns (MKT), size effect factor (SMB), and book-to-market ratios factor (HML) are used for both Fama-French 3 factor and 5 factor models. The profitability factor (RMW) and investment factor (CMA) are used for Fama-French 5 factor model and Fama-French models with a momentum factor are consist of original FF model factors and the momentum factor (UMD). At last, Frazzini and Pedersen (2014) introduced a model with the betting against beta factor (BAB). Panel A shows mean, standard deviation and t-statistic values of all factors, and Panel B shows correlations between each factor.

Panel A : Means, Standard Deviations, and t-statistics (percentage)			
	Mean	**Standard Deviation**	**t-statistic**
MKT	0.30	3.78	1.05
SMB	0.49	2.79	2.31
HML	0.27	2.51	1.43
UMD	1.70	3.53	6.31
RMW	0.43	2.02	2.81
CMA	0.23	1.94	1.57
BAB	1.23	3.20	5.05

Panel B : Correlations						
	SMB	**HML**	**UMD**	**RMW**	**CMA**	**BAB**
MKT	0.217	-0.363	0.017	-0.182	-0.463	-0.137
SMB		-0.269	0.144	-0.387	-0.187	-0.006
HML			-0.230	0.144	0.745	0.510
UMD				-0.084	-0.077	-0.029
RMW					-0.079	0.324
CMA						0.418

Table 3: Tests of equality of squared Sharpe Ratios (Yearly)

Tables below present the differences of sample squared Sharpe ratios between traded factor asset pricing models with yearly observations and p-values corresponding to those differences. Each value in Panel A shows the difference of all possible pairs which is the test of $H_0 : \theta_i^2 = \theta_j^2$ in column i and row j. * denotes 1% of significance level and ** denotes 5% of significance level.

Panel A : Differeces in sample squared Sharpe ratios				
Model	**FF5+UMD**	**FF3**	**FF5**	**MKT+BAB**
FF3+UMD	0.107**	0.234**	0.356**	0.253*
FF5+UMD		0.126**	0.248**	0.146
FF3			0.122**	0.020
FF5				-0.103

Panel B : p-values				
Model	**FF5+UMD**	**FF3**	**FF5**	**MKT+BAB**
FF3+UMD	0.001	0.000	0.000	0.039
FF5+UMD		0.231	0.000	0.167
FF3			0.000	0.759
FF5				0.118

Table 4: Tests of equality of squared Sharpe Ratios (Quarterly)

Tables below present the differences of sample squared Sharpe ratios between traded factor asset pricing models with quarterly observations and p-values corresponding to those differences. Each value in Panel A shows the difference of all possible pairs which is the test of $H_0 : \theta_i^2 = \theta_j^2$ in column i and row j. * denotes 1% of significance level and ** denotes 5% of significance level.

Panel A : Differeces in sample squared Sharpe ratios				
Model	**FF5+UMD**	**FF3**	**FF5**	**MKT+BAB**
FF3+UMD	0.218**	1.111**	1.368**	1.156**
FF5+UMD		0.893	1.150**	0.938*
FF3			0.257*	0.045
FF5				-0.212

Panel B : p-values				
Model	**FF5+UMD**	**FF3**	**FF5**	**MKT+BAB**
FF3+UMD	0.008	0.000	0.000	0.003
FF5+UMD		0.061	0.000	0.027
FF3			0.017	0.815
FF5				0.237

Table 5: Multiple Model Comparison Tests

The table below shows multiple model comparison tests including sample squared Sharpe ratios and likelihood ratios. Since statistical analysis is based on the asymptotic methodology, only yearly data (172 observations) is used for these tests. $\hat{\theta}^2$ denotes the values of sample squared Sharpe ratios and r denotes the number of alternative models in each multiple non-nested model comparison. And LR indicates the values of likelihood ratio, and p-values corresponding to those LR values are shown in the fifth column.

Testing Model	$\hat{\theta}^2$	**r**	**LR**	p**-value**
FF3	0.048	6	2.445	0.059
FF5	0.170	6	1.432	0.214
FF3+UMD	0.297	6	0.000	0.598
FF5+UMD	0.404	6	0.000	0.500
MKT+BAB	0.151	6	4.254	0.020

Table 6: Regression Statistics

Tables below present regression statistics of four non-traded factors when traded factors are regressed onto each of non-traded factors. Panel A shows the statistics with yearly observations and Panel B shows the statistics with quarterly observations. The second columns of both Panels indicate adjusted R^2 values and the third and fifth columns give us F-statistics and p-values corresponding those statistics.

Panel A : Yearly Data				
	Adj. R^2	**F-statistic**	**Deg. of Freedom**	**p-value**
3YR	-0.0561	40.6799	7	<0.001
4Q	-0.0021	30.4821	7	<0.001
GAR	0.2395	143.7213	7	<0.001
UNFIL	0.0265	62.9522	7	<0.001

Panel B : Quarterly Data				
	Adj. R^2	**F-statistic**	**Deg. of Freedom**	**p-value**
3YR	-0.0140	16.9526	7	0.0177
4Q	0.0349	21.5911	7	0.0003
GAR	0.0462	18.6388	7	0.0094
UNFIL	0.0216	17.6995	7	0.0134

Table 7: Model Comparisons with Non-Traded Factor Models (Yearly)

Tables show the results of pairwise tests with FF3 model augmented with each non-traded factor and other asset pricing models; FF3, FF5, FF3+UMD, FF5+UMD, and MKT+BAB. Tables are sorted by p-values and the difference of squared Sharpe ratios are presented in each of the first Panel of all tables.

Differences in Sample Squared Sharpe Ratios **(3Y)**					
	MKT+BAB	**FF3**	**FF5**	**FF3+UMD**	**FF5+UMD**
FF3+3Y	0.107	-0.088	0.304	1.290	1.722
Differences in Sample Squared Sharpe Ratios **(3Y)**					
	MKT+BAB	**FF3**	**FF5**	**FF3+UMD**	**FF5+UMD**
FF3+3Y	0.888	0.791	0.699	0.113	0.045

Differences in Sample Squared Sharpe Ratios **(4Q)**					
	FF3+UMD	**FF5+UMD**	**FF5**	**FF3**	**MKT+BAB**
FF3+4Q	-0.079	0.354	-1.065	-1.456	-1.262
Differences in Sample Squared Sharpe Ratios **(4Q)**					
	FF3+UMD	**FF5+UMD**	**FF5**	**FF3**	**MKT+BAB**
FF3+4Q	0.914	0.596	0.240	0.176	0.172

Differences in Sample Squared Sharpe Ratios **(GAR)**					
	FF3+UMD	**FF5**	**MKT+BAB**	**FF5+UMD**	**FF3**
FF3+GAR	0.019	-0.105	-0.146	0.148	-0.245
Differences in Sample Squared Sharpe Ratios **(GAR)**					
	FF3+UMD	**FF5**	**MKT+BAB**	**FF5+UMD**	**FF3**
FF3+GAR	0.944	0.719	0.614	0.570	0.288

Differences in Sample Squared Sharpe Ratios **(UNFIL)**					
	FF5	**MKT+BAB**	**FF3**	**FF3+UMD**	**FF5+UMD**
FF3+UNFIL	0.028	-0.168	-0.363	1.014	1.447
Differences in Sample Squared Sharpe Ratios **(UNFIL)**					
	FF5	**MKT+BAB**	**FF3**	**FF3+UMD**	**FF5+UMD**
FF3+UNFIL	0.959	0.760	0.287	0.217	0.061

Table 8: Model Comparisons with Non-Traded Factor Models (Quarterly)

Tables show the results of pairwise tests with FF3 model augmented with each non-traded factor and other asset pricing models; FF3, FF5, FF3+UMD, FF5+UMD, and MKT+BAB. Tables are sorted by p-values and the difference of squared Sharpe ratios are presented in each of the first Panel of all tables.

Differences in Sample Squared Sharpe Ratios **(3Y)**					
	FF3	**MKT+BAB**	**FF5**	**FF3+UMD**	**FF5+UMD**
FF3+3Y	-0.016	0.082	0.124	0.248	0.376
Differences in Sample Squared Sharpe Ratios **(3Y)**					
	FF3	**MKT+BAB**	**FF5**	**FF3+UMD**	**FF5+UMD**
FF3+3Y	0.888	0.716	0.596	0.354	0.192

Differences in Sample Squared Sharpe Ratios **(4Q)**					
	FF3+UMD	**FF5**	**MKT+BAB**	**FF5+UMD**	**FF3**
FF3+4Q	0.019	-0.105	-0.146	0.148	-0.245
Differences in Sample Squared Sharpe Ratios **(4Q)**					
	FF3+UMD	**FF5**	**MKT+BAB**	**FF5+UMD**	**FF3**
FF3+4Q	0.944	0.719	0.614	0.570	0.288

Differences in Sample Squared Sharpe Ratios **(GAR)**					
	MKT+BAB	**FF5**	**FF3**	**FF3+UMD**	**FF5+UMD**
FF3+GAR	0.014	0.056	-0.085	0.179	0.308
Differences in Sample Squared Sharpe Ratios **(GAR)**					
	MKT+BAB	**FF5**	**FF3**	**FF3+UMD**	**FF5+UMD**
FF3+GAR	0.944	0.784	0.374	0.311	0.114

Differences in Sample Squared Sharpe Ratios **(UNFIL)**					
	FF3	**MKT+BAB**	**FF5**	**FF3+UMD**	**FF5+UMD**
FF3+UNFIL	0.000	0.098	0.140	0.263	0.392
Differences in Sample Squared Sharpe Ratios **(UNFIL)**					
	FF3	**MKT+BAB**	**FF5**	**FF3+UMD**	**FF5+UMD**
FF3+UNFIL	0.933	0.174	0.051	0.010	0.004