A Comparison of a Bayesian and Maximum Likelihood Algorithms for Estimation of a Multilevel IRT Model

by

Insuk Kim

(Under the direction of Deborah Bandalos)

Abstract

Multilevel Item Response Theory (IRT) models provide an analytic approach that formally incorporates the hierarchical structure characteristic of much educational and psychological data. In this study, maximum likelihood (ML) estimation, which is the method most widely used in current applied multilevel IRT analyses and Bayesian estimation, which has become a viable alternative to ML-based estimation techniques were examined. Item and ability parameter estimates from Bayesian and ML methods were compared using both empirical data and simulated data. It was found that Bayesian estimation using WinBUGS performed better than ML estimations in all conditions with regard to the item parameter estimates. For the individual (Level 2) variance estimates, PQL estimation using HLM showed less bias than the others. However, Bayesian and ML estimations performed similarly to each other for the group (Level 3) variance parameter estimates.

Index words: Multilevel Item Response Theory, Maximum Likelihood Estimation, Bayesian Estimation

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by

Insuk Kim

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M.A., The University of Georgia, 2001

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by

Insuk Kim

Approved:

Major Professor: Deborah Bandalos

Committee: Allan Cohen
Steve Olejnik
Ted Baumgartner

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
May 2007

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Chapter 1

Introduction

1.1 Preview of the Study

Hierarchical linear models (HLM) allow the natural multilevel structure characteristics of much educational and psychological data to be represented formally in the analysis of the data (Bryk & Raudenbush, 1992; Goldstein, 1987; Longford, 1993). Multilevel item response theory (IRT) models have been proposed as extensions of hierarchical generalized linear models (HGLM: Stiratelli, Laird, & Ware, 1984; Wong & Mason, 1985). The multilevel IRT models provide an analytic approach that formally incorporates the hierarchical structure characteristics of much educational and psychological data. The combination of HLM with IRT incorporates measurement error in estimates of the latent trait, θ, into the estimation of model parameters (Adams, Wilson, & Wu, 1997; Maier, 2001). Adams et al. (1997) and Patz and Junker (1999b) note that even the simplest of IRT models can be viewed as a multilevel model in which item responses are nested within persons. In this view, multilevel modeling provides a framework covering most IRT models and applications, for example, equating and differential item functioning (Rijmen, Tuerlinckx, De Boeck, & Kuppens, 2003).

In this study, some estimation issues in fitting multilevel IRT models were examined. The combination of HLM and IRT has led to the development of psychometric models for item response data that contain a hierarchical structure thus enabling a researcher to study the impact of covariates (e.g., schools, curriculum) on the lower level units such as students (e.g., Adams et al., 1997; Kamata, 2001; Maier, 2001). Adams et al. (1997), for example, describe a two-level model, in which person-characteristics are added as fixed parameters.

Kamata (2001) extended this model to a three-level formulation in which person-level variables are incorporated as random effects.

Kamata's multilevel model is a Rasch model (Rasch, 1960) and was estimated using the computer program HLM (Raudenbush, Bryk, Cheong, & Congdon, 2000) under a maximum likelihood (ML) algorithm. The HLM software uses the penalized or predictive quasi-likelihood (PQL) method to approximate a maximum likelihood. One problem with the PQL method is that it is known to underestimate the variances for dichotomous responses with small samples at level-3 (Rodriguez & Goldman, 1995; Goldstein & Rashbash, 1996). This may be a potential source of problems when estimating multilevel IRT models for tests scored dichotomously. An alternative estimation method, such as a Bayesian estimation (Maier, 2001), however, may be able to improve the variance estimates of the level-3 parameters. In this study a Bayesian alternative to the HLM approach with a focus on improvement in the variance estimates for the level-3 parameters was considered.

Multilevel models can become complex, making ML estimation difficult. Bayesian estimation can be particularly useful when models become complex, as is the case when additional linear constraints are added or when more complicated item formats are used on a test. In reality, there could be situations in which the assumption of normality is not met or for which sample sizes are too small for typical estimation of IRT model parameters. For situations in which these assumptions cannot be met, Bayesian estimation procedures may provide a useful alternative (Maier, 2001).

Efforts to estimate model parameters using a Bayesian algorithm are not new (e.g., Swaminathan & Gifford, 1982) although comparisons of Bayesian and ML algorithms in a multilevel context are. Swaminathan and Gifford (1982) found Bayesian estimation to be more accurate than ML estimation procedures for a Rasch model when the numbers of items and examinees were small. More recently, Patz and Junker (1999a, 1999b) described a fully Bayesian approach in the context of a multilevel IRT model and noted that Bayesian estimation is particularly useful when models become very complex, as is easily

possible in the context of multilevel IRT models. Given the increasing complexity of IRT models, it is likely that Bayesian estimation is going to be used more frequently.

In this study, therefore, I compared a Bayesian and a ML solution, using Kamata's three-level IRT model as a framework for the comparison. ML estimates of model parameters were obtained using the computer programs HLM 6.02 (Raudenbush et al., 2005) and Mplus 4.1 (Muthén & Muthén, 2006). Bayesian estimation was done using a Markov chain Monte Carlo (MCMC) estimation algorithm, implementing Gibbs sampling, and written using the computer software WinBUGS 1.4 (Spiegelhalter, Thomas, & Best, 2003). Item and ability parameter estimates from Bayesian and ML methods were compared using both empirical data and simulated data.

1.2 Significance of the Study

The natural hierarchical nature of much educational and psychological data has provided a context within which multilevel models have been developed. Inclusion of Item response theory (IRT) models in this multilevel framework is a natural extension of IRT and of multilevel models. One important benefit of multilevel models is that sparseness in some parts of the data can sometimes be compensated for by information in the remainder of the data. Estimation of parameters at different levels of these models is a potential concern, particularly given the extensive use of dichotomous scoring in most educational tests used today. The Bayesian algorithms to be studied in this paper should be useful in estimating model parameters when this type of scoring is used. It is also important to understand the solutions provided by Bayesian algorithms in comparison to solutions provided by more well-known algorithms such as maximum likelihood (ML).

This study presents a summary of the implementation of the computer programs HLM (Raudenbush et al., 2005), Mplus (Muthén & Muthén, 2006), and WinBUGS (Spiegelhalter et al., 2003) for the analyses of multilevel IRT model under ML and Bayesian estimation. In addition, a simulation study comparing model parameter estimates under practical

testing conditions should provide useful information for interpreting model results. Studies such as this one will provide useful information for researchers seeking to use ML as well as Bayesian estimation for estimation of multilevel IRT model parameters.

1.3 Overview of Later Chapters

In the remaining chapters of this dissertation, Chapter 2 provides some background on Hierarchical Linear Model (HLM), Item Response Theory (IRT), and Multilevel IRT including reviews of the statistical methods used for these models. Chapter 3 describes the multilevel IRT model; especially Kamata's three-level IRT model used as a framework for the comparison and the corresponding methods of parameter estimation including Bayesian estimation based on Gibbs sampling as well as Maximum Likelihood (ML) using both simulated data and real data. Chapter 4 shows results of the analyses. Chapter 5 summarizes the results and discusses limitations and possible future work.

Chapter 2

Literature Review

2.1 Hierarchical Linear Model

In educational and any other research, data with hierarchical structures are fairly common. These types of data exist when individuals are grouped in some way. For example, in educational research students are nested in classrooms or schools.

Traditionally, data collected within groups have been analyzed using different types of ordinary-least-squares (OLS) techniques. There are some known problems when hierarchical data are analyzed using these traditional methods. Hierarchical data often violate statistical assumptions such as linearity, normality, homoscedasticity, and independence. For example, hierarchical data generally violate the statistical assumption that observations or individuals are independent of each other, because individuals in the same group are likely to be more similar than individuals in different groups. Ignoring violations of the assumption of independence can result in mis-estimating the errors, which can lead to incorrect inferences.

Osborne (2000) cites three problems with this traditional approach. (1) Under aggregation, the properties of a higher-level (e.g., group) are described in terms of the sums of the properties of a lower-level (e.g., individual) nested in that group, resulting in loss of statistical power because in this process all individual information is lost. (2) Under disaggregation, the assignment of a group characteristic to an individual, does not satisfy the assumption of independence of observations, which can lead to over-optimistic estimates of significance. (3) Under either aggregation or disaggregation, there is the danger of the ecological fallacy: there is no necessary correspondence between

individual-level and group-level variable relationships (e.g., race and literacy correlate little at the individual-level but correlate well at the group-level).

Hierarchical Linear Model (HLM) was formulated to account for the interdependence of individuals within the same group and model both group-level and individual-level variance in the outcome (Bryk & Raudenbush, 1992). That is, HLM explicitly considers that individuals within a group unit may be more similar to one another than those in other groups and, thus, may not generate independent cases. Also, HLM examines both individual-level and group-level variance in outcome measures, while maintaining the proper level of analysis for independent variables. For instance, a researcher can model both individual- and group-level variance in individual outcomes while utilizing individual predictors at the individual-level and group predictors at the group-level. Therefore, HLM overcomes the weaknesses of the two data analysis methods in that people can model explicitly both within- and between-group variance, as well as examine the impact of groups on individual outcomes while maintaining the appropriate level of analysis (Lee, 2003).

Pollack (1998) discussed some advantages of HLM over OLS regression. First, improved estimation of regression path coefficients for multilevel predictors occurs because of the simultaneous estimation of the relationships between group predictors and individual predictors, if there is some between-group variability in the outcome. Second, HLM separates the explained variance in the outcome into the variance explained at each level and estimates variance explained at the group level that the OLS procedure does not produce. Third, HLM can model slopes of individual-level relationships within groups to enable an understanding of and why some group properties might affect the strength of these associations. Fourth, HLM simplifies the sampling procedure. Only random sampling at the highest level of analysis must be conducted because levels nested within the highest level are assumed to be intercorrelated. Finally, the researcher can choose whether he or she is interested in comparing individuals to all others in the population of interest or relative to those within the same groups (Pollack, 1998).

As mentioned previously with regard to hierarchical data, individuals in the same group are likely to be more similar than individuals in different groups. Due to this, the variations in outcome may be due to differences between groups, and to individual differences within a group. The IntraClass Correlation (ICC) is the proportion of the variance in the outcome that is between groups, which tells the extent to which observations are not independent of a grouping variable (ex., schools). This value is computed using the following equation:

$$\rho = \frac{\tau_{00}}{\sigma^2 + \tau_{00}}$$

where σ^2 is the Level-one variance component and τ_{00} is the Level-two variance component.

Kreft (1996) concluded that multilevel modeling can be more useful for revealing differences in variance among units in different groups which comprise the levels. Also, multilevel modeling may be a preferred method when data are sparse, including studies (e.g., twin studies) where groups are sparse. Kreft (1996) compared estimates of regression parameters from multilevel analysis with those obtained from more traditional regression techniques. In both cases, the fixed effects estimates were unbiased. The main difference, however, was that the standard errors of these parameters were underestimated if significant intraclass correlation was present and traditional regression analyses were used. The presence of a significant intraclass correlation, in other words, is an indicator of the need to employ multilevel modeling rather than conventional regression.

2.2 Item Response Theory

Item response theory (IRT) provides a family of mathematical models that specify the relation of item characteristics to a person's item responses (Embretson & Reise, 2000). That is, an IRT model provides a prediction that a given person will provide a given response to a given item. IRT requires stronger assumptions than Classical Test Theory (CTT) to provide these item-level predictions. It does so using a modeling approach that has a number of advantages over CTT. In IRT, the true score is defined on the latent trait of interest rather than on the test, as is the case in CTT.

IRT assumes a single common factor accounts for all item covariances. This is stated as *unidimensionality*, meaning the test measures only one latent trait. This trait is commonly referred to as 'ability'. If unidimensionality holds, then *local independence* holds as well and ability, as considered in the model, fully determines the response to a given item. Also, IRT assumes item and sample invariance for parameter estimates. Item invariance means that all items that are calibrated to a given scale measure that scale. Any subset of those same items measures the same scale. Sample invariance means that any simple random sample of examinees from the population will yield calibrations that are invariant from the first sample, up to a normalizing constant.

The simplest IRT model is a one-parameter logistic (1PL) model, which is also known as the Rasch model. The Rasch model has only one parameter to describe an item, difficulty, commonly denoted as β. In addition, each person has an ability parameter, denoted with θ.

This model is given as

$$P_i(\theta) = \frac{exp(\theta - \beta_i)}{1 + exp(\theta - \beta_i)} \ , \tag{2.1}$$

where $P_i(\theta)$ is the probability that a person with ability θ answers item i correctly. The β_i is the difficulty parameter of item i. The difficulty is the value of ability at which a person has a 50% probability of responding correctly to the item. Usually, the difficulty is standardized and typically ranges from -3 to $+3$ with higher values indicating more difficult items. The Rasch model assumes that all items are equally discriminating.

The Rasch model is known to be appropriate for modeling dichotomous responses and models the probability of an individual's correct response on a dichotomous item. The logistic item characteristic curve (ICC) forms the boundary between the probability areas of answering an item incorrectly and answering the item correctly. This one-parameter logistic model assumes that the discriminations of all items are equal to one.

2.3 Multilevel Item Response Theory

Both hierarchical linear models (HLM) and item response theory (IRT) are used in a variety of social science research applications. The use of HLM allows the natural multilevel structure present in so much social science data to be represented formally in data analysis (Bryk & Raudenbush, 1992; Goldstein, 1987; Longford, 1993). IRT allows connections to be made between observed categorical responses provided by students and an underlying unobservable trait, such as ability or attitude (Hambleton & Swaminathan, 1985; Lord & Novick, 1968).

The borrowing of strength advantage of multilevel modeling can be used to obtain accurate estimates of the relationships within groups, regardless of sample size (Raudenbush & Bryk, 2002). Person-level characteristics have been included in IRT models to help improve estimation of item difficulty parameters, or to model the effects of person characteristics upon the estimated latent trait measures (Mislevy, 1987; Patz & Junker, 1999a; Patz & Junker, 1999b). That is, the combination of HLM with IRT, called multilevel IRT, incorporates the hierarchical structure directly in the estimation of model parameters including persons' ability for person-level and group-level item characteristics and effects of covariates (Kamata, 2001).

An IRT model can be formulated as a multilevel model where item responses (level-1) are nested within persons (level-2) and random effect variance terms (persons' ability) vary across the items at level-1 (e.g. Hedeker, 2004; Kamata, 2001; Rijmen et al., 2003). There have been some attempts to reformulate IRT models as multilevel models (Stiratelli et al., 1984; Wong & Mason, 1985; Adams et al., 1997; Kamata, 1998 & 2001).

Multilevel item response theory (IRT) models have been proposed as extensions of hierarchical generalized linear models (HGLM: Stiratelli et al, 1984; Wong & Mason, 1985). The intent of such extensions is to incorporate the estimation of IRT item and ability parameters into the multilevel framework provided by HGLM.

Adams et al. (1997) described the reformulation of a regular IRT model as a multi-level model, in which person-characteristic variables are added as fixed parameters that are related to a latent trait. This was a two-level model in which person-level variables were included as linear constraints in a multilevel framework. Within a multilevel framework, the item response function can be viewed as a within person model and the person population distribution model can be viewed as a between persons model. The person population parameters are assumed to be random variables and used mainly for the purpose of parameter estimation. They are decomposed into a linear combination of multiple parameters. However, it is limited to a two-level model.

Kamata (1998, 2001) formulated a two-level item analysis model by the use of a hierarchical generalized linear model (HGLM) and showed that it is algebraically equivalent to the Rasch model. The level-1 model is an item level where the logit link function is utilized to relate the probability of answering correctly to linear predictors of item dummy codes. The level-2 model is the person level where the intercept coefficient of level-1 is assumed to be a random effect across persons, but the item coefficients or slope parameters are constrained to be constant across persons. When level-1 and level-2 are combined, the model is algebraically equivalent to the Rasch model.

Kamata (1998) also showed the advantages of casting IRT models as multilevel models by including person variables at the level-2 and extending to a three-level model to model group variations and group characteristic variables. Pastor (2003) also discussed this expansion of multilevel IRT models to three levels, allowing not only the dependency typically found in hierarchical data to be accommodated, but also the estimation of (a) latent traits at different levels and (b) the relationships between predictor variables and latent traits at different levels.

2.4 Maximum Likelihood Estimation

A variety of methods have been used to estimate the parameters of these expanded Item response theory (IRT) models. Maximum likelihood (ML) estimation is the method most widely used in current applied multilevel IRT analyses. In ML estimation, population parameters (e.g., item difficulty, ability) are treated as unknown but fixed quantities. That is, the ability estimates are considered to be a random sample from an underlying ability distribution and are integrated out to allow for maximization of likelihood for item parameters.

In the IRT model, an individual's ability is based on the probability of a given response as a function of characteristics of items presented to an individual. For instance, a person taking a test with i items can obtain one of $i + 1$ observed scores $(0, 1, \ldots, i)$. However, the number of the possible responses to the test (the response patterns) is 2^i. Each response pattern has a certain probability. Also, IRT assumes local independence, which means that the responses given to the separate items in a test are mutually independent given ability. The probability that a person of ability θ will respond to the test with a certain pattern, which is the likelihood function is written as:

$$L(\theta) = \prod P_{ij}(\theta_j, \beta_i)^{u_i} Q_{ij}(\theta_j, \beta_i)^{1-u_i} \tag{2.2}$$

where $u_i \in (0, 1)$ is the score on item i, $P_{ij}(\theta_j, \beta_i)$ is the probability of the correct response from the interaction between the individual ability θ_j and the item parameter β_i, and $Q_{ij}(\theta_j, \beta_i)$ is the probability of the wrong response, equal to $1 - P_{ij}(\theta_j, \beta_i)$. In ML, the ability estimate will be the ability which has the highest likelihood given the observed pattern and the item parameters.

In the context of multilevel IRT model, to maximize a likelihood two steps are required: first, evaluating an integral and, second, maximizing that integral (Raudenbush & Bryk, 2002, p. 455).

> Let Y denote a vector of all level-1 outcomes, let u denote the vector of all random effects at level 2 and higher, and let ω denote a vector containing all parameters to be estimated. Then we can denote as $f(Y|u,\omega)$ the probability distribution of the outcome at level 1, given the random effects and parameters. The higher-level models specify as $p(u|\omega)$ the distribution of the random effects given the parameters. The likelihood of the data given only the parameters is then
>
> $$L(Y|\omega) = \int f(Y|u,\omega)p(u|\omega)du.$$

The aim of ML is to maximize the integral with respect to ω in order to make inferences about ω.

Among the procedures commonly used are full maximum likelihood (FML, Goldstein, 1986; Longford, 1987) and restricted maximum likelihood estimation (RML, Mason, Wong, & Entwistle, 1983; Bryk & Raudenbush, 1986). Under FML, variance-covariance parameters and second-level fixed coefficients are estimated by maximizing their joint likelihood. Under RML, variance-covariance components are estimated via maximum likelihood, averaging over all possible values of the fixed effects, and fixed effects are estimated via Generalized Least Squares (GLS) given these variance-covariance estimates (Raudenbush, Bryk, Cheong, & Congdon, 2001, p. 7). GLS is then used to obtain estimates and standard errors for the fixed effects (Goldstein, 1995; Raudenbush & Bryk, 2002).

According to Jones and Steenbergen (1997), the variance components for FML will tend to be underestimated with small sample sizes since FML does not adjust for the number of fixed effects that are estimated. Because RML estimates variance components after removing the fixed effects from the model, it can lead theoretically to less bias than FML (Raudenbush & Bryk, 2002). However, many authors (e.g., Rodriguez & Goldman, 1995; Goldstein & Rashbash, 1996; Breslow & Clayton, 1993) have reported that these approximation methods exhibit downward biases for both the fixed effects and the variance components for dichotomous responses with small cluster sizes.

A number of approaches to compute and maximize the likelihood have been developed. The techniques to obtain full information ML can be classified along two dimensions (Rijmen et al., 2003): the method of numerical integration of the intractable integrals used to approximate the marginal likelihood and the type of algorithm used to maximize the approximate marginal likelihood.

The performance of estimation methods has been the subject of several studies. Rodriguez and Goldman (2001) reviewed several types of approximate procedures: the first order MQL, the second order MQL, the first order PQL (PQL-1), and the second order PQL (PQL-2, Goldstein & Rasbash, 1996) and a bootstrapped version (PQL-B) of PQL-1 and compared them, in estimating a three-level model, against the exact maximum likelihood and the Gibbs sampling. The results showed that even PQL-2 sometimes produces biased estimates, particulary when the clusters are small.

Snijders and Bosker (2000) are critical of such an estimation technique, arguing that the algorithms for PQL are not very stable; whether or not algorithms converge may be dependent upon the data set, the complexity of the model, and the starting values. To overcome these problems with PQL, Raudenbush, Yang, and Yosef (2000) proposed a sixth order Laplace approximation, known as LaPlace6, to approximate the maximum likelihood. HLM provides this for two-level models with dichotomous responses.

Recently, Callens and Croux (2004) compared the performance of three different likelihood-based estimation procedures: PQL, non-adaptive Gaussian quadrature, and adaptive Gaussian quadrature (AGQ) in estimating parameters for multilevel logistic regression models. In their study, comparing PQL with AGQ showed that the bias, as measured by mean squared error (MSE), was larger for PQL.

I focus on two likelihood-based estimation procedures implemented in two of the most commonly used statistical programs currently available for multilevel analysis, HLM and Mplus.

As mentioned, HLM uses the PQL approach, which approximates the joint posterior density of a parameter with a multivariate normal density having the same mode and curvature at the mode as the true posterior. When Taylor series expansions around the approximate posterior mode are used, this approach is called PQL.

Mplus performs full information ML estimation via an EM algorithm, solving the integrals with adaptive Gaussian quadrature. In a Gaussian approach, the integral is approximated by numerical integration and then the likelihood with approximate values for the integrals is maximized. Using the adaptive approach, the variable of integration is centered around its approximate posterior mode.

2.5 Bayesian Estimation

Bayesian methods have become a viable alternative to traditional maximum likelihood-based estimation techniques and may be the only solution currently available for more complex psychometric data structures (Rupp, Dey, & Zumbo, 2004). Given the increasing complexity of Item Response Theory (IRT) models, it is likely that Bayesian estimation is going to become used more frequently.

Even though the use of multilevel IRT model is advantageous when the assumptions of independence of observations as well as independence and identical distribution of errors are not met, it also requires satisfying the assumptions of both Hierarchical Linear Model (HLM) and IRT such as (1) a single dimension underlying the item responses and (2) the latent trait is a random parameter, is normally distributed within and between groups, and the variance of which is the same across groups (Pastor, 2003). In reality, there could be situations in which these assumptions cannot be met. Maier (2001) recommended the use of fully Bayesian estimation procedures for situations in which the assumption of normality is not met or when sample sizes are too small for typical estimation of IRT model parameters.

In contrast to Maximum Likelihood (ML), Bayesian inference does not rely on asymptotic approximations to sampling distributions. In the Bayesian paradigm, the model

parameters are treated as random variables that follow a certain distribution. The distributions of these parameters are conditional on the observed data, which are assumed to be fixed.

Bayesian analysis proceeds by assuming a model $f(Y|\theta)$ for the data Y conditional upon the unknown parameters θ. When $f(Y|\theta)$ is considered as a function of θ for fixed Y, it is referred to as the likelihood $L(\theta)$. A prior probability distribution $f(\theta)$ describes our knowledge of the model parameters before the data is actually observed. Then, the likelihood and prior distributions are combined according to Bayes' theorem[1] in order to form the conditional distribution of θ given the observed data, Y, that is,

$$f(\theta|Y) \propto f(\theta)L(\theta)$$

This conditional distribution is the posterior distribution for the model parameters and describes our knowledge of the data after they have been observed.

Bayesian methods were first introduced by Edwards, Lindman, & Savage (1963), but it was not long ago that Bayesian methods became the preferred approach for many researchers, especially for relatively complex models or when data were sparse and asymptotic theory was unlikely to hold.

Efforts to estimate model parameters using a Bayesian algorithm are not new (e.g., Swaminathan & Gifford, 1982). Swaminathan and Gifford (1982) compared Bayesian and ML estimation procedures for a Rasch model. In their study, Bayesian estimation was found to be more accurate than ML when the numbers of items and examinees were small. In the context of a multilevel IRT model, Patz and Junker (1999a, 1999b) described a fully Bayesian approach utilizing MCMC estimation. As Patz and Junker note, Bayesian

[1] A rule for computing the conditional probability distribution of a random variable A given B in terms of the conditional probability distribution of variable B given A and the marginal probability distribution of A alone

$$P(A|B) = \frac{P(A \textit{ and } B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

estimation is particularly useful when models become very complex. This is easily possible in the context of multilevel IRT models.

Also, researchers have expanded traditional IRT models in a number of ways that are appropriate in a variety of applications. Maier (2001) presented the integration of a hierarchical linear model and a one-parameter logistic item response model using an estimation method that does not rely on large-sample theory and normal approximations. According to her study, simultaneous estimation allows for better estimation of the true relationship by incorporating the standard errors of the latent traits into the model. More recently, Browne and Draper (2006) compared Bayesian and likelihood-based methods for fitting variance-components (VC) and random-effects logistic regression (RELR) models. They found that quasi-likelihood methods for estimating random effects variances perform poorly with respect to bias and coverage in the simulated example and Bayesian methods were well-calibrated in estimation for all parameters of the model.

2.6 Markov Chain Monte Carlo

A major limitation to more widespread use of Bayesian estimation is that obtaining the posterior distribution often requires integration which can be computationally very difficult, but several approaches have been proposed. The main emphasis is placed on one Markov Chain Monte Carlo (MCMC) method known as the Gibbs sampling, which was used in this study. MCMC is a particular Bayesian data analysis method used to estimate model parameters. A specific MCMC technique, Gibbs sampling, is a method for generating random variables from a distribution by sampling from the collection of full conditional distributions of the complete posterior distribution (Gelfand, Hills, Racine-Poon, & Smith, 1990).

As the name suggests, MCMC methods produce chains in which each of the simulated values is dependent on the preceding values. A Markov chain is a stochastic process with the property that any specified state in the series, $\theta^{[t]}$, is dependent only on the previous

value of the chain, $\theta^{[t-1]}$, and is therefore conditionally independent of all other previous values: $\theta^{[0]}, \theta^{[1]}, \ldots, \theta^{[t-1]}$. This can be stated:

$$P(\theta^{[t]} \in A|\theta^{[0]}, \theta^{[1]}, \ldots, \theta^{[t-2]}, \theta^{[t-1]}) = P(\theta^{[t]} \in A|\theta^{[t-1]}) \tag{2.3}$$

where t is time and A is any event.

Gibbs sampling is a widely used MCMC technique. It requires specific knowledge about the conditional nature of the relationship between the variables of interest. The basic idea is that if it is possible to express each of the coefficients to be estimated as conditional on all of the others, then by cycling through these conditional statements we can eventually reach the true joint distribution of interest. Gibbs sampling uses the following steps (Casella & George, 1992).

1. The first step involves the selection of a starting value for ϕ, say ϕ^0.

2. Then, one needs to generate a random value of θ^1 from the conditional distribution $p(\theta|y, \phi = \phi^0)$.

3. Next, ϕ^1 is generated from the conditional distribution $p(\phi|y, \theta = \theta^1)$.

4. This procedure continues for a large number of iterations, generating θ^i from $p(\theta|y, \phi = \phi^{i-1}$ and ϕ^i from $p(\phi|y, \theta = \theta^i)$ for $i = 1, 2, 3, \ldots$.

5. After a large enough number of iterations, the samples θ^i drawn from this process converge to the target posterior distribution $p(\theta|y)$.

where ϕ is a vector of unknown parameters.

Geman and Geman (1984) described the Gibbs sampler as a method for obtaining difficult posterior quantities. Gelfand and Smith (1990) illustrated the power of the Gibbs sampler to address a wide variety of statistical issues. Gibbs sampling was first applied by Albert (1992) for estimating the posterior distribution of the item and person parameters of the two-parameter normal ogive model. Patz and Junker (1999a) demonstrated MCMC

techniques that are particularly well-suited to complex models with IRT assumptions and showed MCMC methods which treat item and subject parameters at the same time by incorporating standard errors of item estimates into trait inferences, and vice versa. More recently, Browne and Draper (2006) developed a hybrid Metropolis-Gibbs approach in which Gibbs sampling is used for variance components and univariate-update random-walk Metropolis sampling with Gaussian proposal distributions is used for fixed effects and residuals. MCMC methods have significantly simplified Bayesian estimation, yet bring along with them new issues such as convergence and specification of proposal densities (Rupp, Dey, & Zumbo, 2004), which are discussed in a later chapter.

2.7 Research Questions

As previously stated, the penalized or predictive quasi-likelihood (PQL) estimation method is known to underestimate the variance estimates for dichotomous responses with small level-3 sizes (Rodriguez & Goldman, 1995; Goldstein & Rashbash, 1996). Some researchers (i.e., Kamata & Binici, 2003) found that the variance estimates produced by HLM software, which uses the PQL method, are substantially negatively biased. This study extends their work by considering a Bayesian alternative to the HLM approach with a focus on improvement in the variance estimates for the level-3 parameters.

This study is designed to compare the performance of parameter estimates of Bayesian and Maximum likelihood (ML) estimation in the context of Kamata's three-level IRT model to assess how varying sample sizes at each level affects parameter estimates of interest. This is done using simulated multilevel data, which have values similar to those obtained from HLM analysis conducted by the researcher on actual data. Bayesian estimates of model parameters are obtained from a Gibbs sampler run of 11,000 iterations after eliminating the first 4,000 iterations as burn-in, using the computer software WinBUGS 1. 4 (Spiegelhalter et al., 2003). ML estimates of model parameters are obtained using the computer programs HLM 6.02 (Raudenbush et al., 2005) and Mplus 4.1 (Muthén

& Muthén, 2006). Results of a recovery study of generating parameters for the models are presented. The magnitude of the bias of estimation from the true value is estimated by the root mean squared error (RMSE), which is the square root of the averaged squared deviation between the estimated parameter values and the generating parameter.

$$RMSE = \sqrt{\frac{\sum_{r=1}^{R}(\hat{a_r} - a_r)^2}{R}} \tag{2.4}$$

where R is the total number of replications (i.e., R = 5), $\hat{a_r}$ is the estimate for the generated value a_r in the r^{th} simulated sample. A non-zero bias means the estimate is, on the average, overestimating the parameters (positive bias) or underestimating the parameter (negative bias).

I hypothesize: First, the larger the sample size at each level, the smaller the bias of the estimators. Second, the RMSEs of the estimators from Bayesian estimation are smaller than those from ML methods under the conditions which have the smallest sample sizes at each level (i.e., I = 10, J = 450, and G = 15). Third, the RMSEs of the estimators from Bayesian and ML methods are similar under the conditions which have the largest sample sizes at each level (i.e., I = 20, J = 1500, and G = 60).

Chapter 3

Methodology

3.1 Multilevel Item Response Theory

For present purposes, groups are defined as collections of individuals. Below, a description of Kamata's (2002) three-level IRT model is presented to provide a context for the comparisons. The models used in this study are (1) the unconditional model, (2) the model with Level-2 predictor variable(s), and (3) the model with Level-2 and Level-3 predictor variables. In each model, Level 1 is the item-level model, Level 2 is the individual-level model, and Level 3 is the group-level model.

3.1.1 The Unconditional Model

The unconditional model is one which includes no Level-2 or Level-3 predictor variables and is estimated before adding any predictors to the model. In this model, ability estimates are allowed to vary randomly across individuals within a group and randomly across groups. This enables determination as to whether or not to include individual-level or group-level predictors.

The first level of the unconditional model is used to show variation of item responses within individuals, given below as the log-odds of the probability that individual j $(j = 1, \ldots, J)$ in group g $(g = 1, \ldots, G)$ answers item i $(i = 1, \ldots, k-1)$ correctly:

$$\begin{aligned} \log(\tfrac{p_{ijg}}{1-p_{ijg}}) &= \beta_{0jg} + \beta_{1jg}X_{1ijg} + \beta_{2jg}X_{2ijg} + \ldots + \beta_{(k-1)jg}X_{(k-1)ijg} \\ &= \beta_{0jg} + \textstyle\sum_{q=1}^{k-1} \beta_{qjg}X_{qijg} \end{aligned} \tag{3.1}$$

where P_{ijg} is the probability of answering item i correctly by individual j in group g; X_{qijg} is the qth dummy variable $(q = 1, \ldots, k-1)$ for item i for individual j in group g with

values 1 when $q = i$ and 0 otherwise; β_{0jg} is the effect of the reference item and β_{qjg} represents the effect of the qth item compared to the reference item. The classical item difficulty is used to determine the easiest item. The usual procedure in multilevel IRT modeling, which is also used in this study, is to take the easiest item as the reference item.

The second level considers only variation of individual ability within groups, so the item effects $(\beta_{1jg}, \ldots, \beta_{(k-1)jg})$ are fixed across students.

$$\begin{cases} \beta_{0jg} &= \gamma_{00g} + u_{0jg} \\ \beta_{1jg} &= \gamma_{10g} \\ \quad\vdots & \\ \beta_{(k-1)jg} &= \gamma_{(k-1)0g} \end{cases} \tag{3.2}$$

where $u_{0jg} \sim N(0, \tau_\beta)$. τ_β represents the variation among individuals within groups and is assumed to be homogeneous across groups (Pastor, 2003). Here γ_{00g} is an effect of the reference item in group g, and γ_{q0g} is the effect of the i^{th} item (for $i = q$) in group g. u_{0jg} indicates the deviation of individual j from the average in group g.

The third level models variation among groups, so item effects are constant across groups.

$$\begin{cases} \gamma_{00g} &= \pi_{000} + r_{00g} \\ \gamma_{10g} &= \pi_{100} \\ \gamma_{20g} &= \pi_{200} \\ \quad\vdots & \\ \gamma_{(k-1)0g} &= \pi_{(k-1)00} \end{cases} \tag{3.3}$$

where $r_{00g} \sim N(0, \tau_\gamma)$. τ_γ is the variation among groups. Here π_{000} is the ability estimate of the overall sample, or the effect of the reference item in the overall sample. γ_{00g} is the average ability of individuals in group g. Again, item effects are fixed across groups (as specified in Level 3) and across individuals (as specified in Level 2), such that all groups and all individuals receive the same item effect estimates.

Kamata (2001) suggested the following combined model:

$$p_{ijg} = \frac{1}{1 + exp(-[(r_{00g} + u_{0jg}) - (-\pi_{q00} - \pi_{000})])} \tag{3.4}$$

where $-\pi_{q00} - \pi_{000}$ is an item difficulty for item i for $i = q$ $(i = 1, \ldots, k-1)$, and $-\pi_{000}$ is the item difficulty for item i. In addition, $r_{00g} + u_{0jg}$ can be considered to be the ability for individual j in group g.

3.1.2 The Model with Level-2 Predictor Variables

If the unconditional model indicates significant variation within and between groups, within group variation is modeled followed by between group variation. A predictor variable(s), (for example, Age and Gender of individuals), can be included at the second level of the model in order to determine if variation is associated with the predictor variable(s). An interaction between predictor variables would indicate that the relationship of ability with one predictor differed for another predictor variable. In this case, the Level 1 model would remain the same as in equation 1 and the Level 2 model would be specified as follows:

$$\begin{cases} \beta_{0jg} &= \gamma_{00g} + \gamma_{01g}(first)_{jg} + \gamma_{02g}(second)_{jg} \\ & \quad +\gamma_{03g}(interact)_{jg} + u_{0jg} \\ \beta_{1jg} &= \gamma_{10g} \\ \quad \vdots & \\ \beta_{(k-1)jg} &= \gamma_{(k-1)0g} \end{cases} \tag{3.5}$$

Here, γ_{01g} is the first predictor variable (first) effect in group g controlling for all other variables in the model. γ_{02g} is the second predictor variable (second) effect for group g controlling for all other variables in the model. γ_{03g} is the interaction effect of the first predictor variable and the second predictor variable (interact) in group g controlling for all other variables in the model. The residual, u_{0jg}, or latent ability estimate for individual j in group g is individual j's level of ability controlling for the effects of the first predictor, the second predictor, and the first predictor by the second predictor variable interaction.

The Level 3 model contains no group-level predictor variables and is specified as follows:

$$\begin{cases} \gamma_{00g} = \pi_{000} + r_{00g} \\ \gamma_{01g} = \pi_{010} + r_{01g} \\ \gamma_{02g} = \pi_{020} + r_{02g} \\ \gamma_{03g} = \pi_{030} + r_{03g} \\ \gamma_{10g} = \pi_{100} \\ \gamma_{20g} = \pi_{200} \\ \vdots \\ \gamma_{(k-1)0g} = \pi_{(k-1)00} \end{cases} \tag{3.6}$$

where the effects of the individual-level predictor variables of Level 2 are allowed to vary randomly over groups. The parameters π_{000}, π_{010}, π_{020}, and π_{030}, have similar interpretation as γ_{00g}, γ_{01g}, γ_{02g}, and γ_{03g} except that the interpretation now applies to the overall sample, not just to the group. If the fixed effect for the variable is not significant and if the variation of the random effect is not significant, the variable is deleted from the model. This is the model building process recommended by Raudenbush and Bryk (2002).

3.1.3 The Model with Level-2 and Level-3 Predictor Variables

If the results indicate that there is significant variation in ability across groups even after the Level-2 predictor variable(s) is in the model, it is necessary to build a model with a Level-3 predictor variable (i.e., teachers' experience in students' school). In this model, the first and second level would remain the same as in equations 3.1 and 3.5.

The third level of the model can be specified as follows:

$$\begin{cases} \gamma_{00g} &= \pi_{000} + \pi_{001}(third)_g + r_{00g} \\ \gamma_{01g} &= \pi_{010} + \pi_{011}(third)_g + r_{01g} \\ \gamma_{10g} &= \pi_{100} \\ \gamma_{20g} &= \pi_{200} \\ \quad\vdots & \\ \gamma_{(k-1)0g} &= \pi_{(k-1)00} \end{cases} \tag{3.7}$$

Here, π_{001} is the level-3 predictor (third) effect, π_{011} indicates the magnitude of the interaction effect between the first Level-2 predictor variable and the Level-3 predictor variable. The associated p-value indicates whether the effect of the first Level-2 predictor variable is significantly different across groups depending on the Level-3 predictor variable.

3.2 Computer Programs

There are a variety of statistical programs currently available that can be directly used for multilevel analysis. In this study, Maximum Likelihood estimates of model parameters were obtained using the computer programs HLM 6.02 (Raudenbush et al., 2005) and Mplus 4.1 (Muthén & Muthén, 2006). Bayesian estimation was done using a Markov chain Monte Carlo (MCMC) estimation algorithm, implementing Gibbs sampling, and written using the computer software WinBUGS 1.4 (Spiegelhalter et al., 2003). Item and ability parameter estimates from Bayesian and ML methods were compared using both empirical data and simulated data.

3.2.1 HLM

The HLM program can fit models to outcome variables that generate a linear model with explanatory variables that account for variation at each level, utilizing variables specified at each level. HLM not only estimates model coefficients at each level, but it also predicts the random effects associated with each sampling unit at every level.

The HLM3 component of the software program HLM 6.02 (Raudenbush et al., 2005) was used to perform the analyses. The estimation procedure used in the HLM3 component of HLM 6 is PQL estimation (Breslow & Clayton, 1993), one of the more frequently used estimation procedures for hierarchical generalized linear models (Guo & Zhao, 2000). Also, HLM produces two separate residual files for a three-level model, one for Level-2 and another for Level-3. The variable labelled as *eb00* is an empirical Bayes estimate of group ability, while the variable labelled as *ol00* is an ordinary least square estimate of group ability. According to Kamata (2000), empirical Bayes estimates behave in a way that is similar to Bayesian estimates, and least squares estimates behave similarly to maximum likelihood estimates from the usual Item Response Theory (IRT) estimation. For further information on how to use the HLM software to estimate multilevel IRT models, see Kamata (2002).

3.2.2 Mplus

Mplus estimates a 2-parameter normal ogive model assuming a single factor to obtain Item Response Theory (IRT) model estimates. The conditional probability formulation is used:

$$P(y_{ij} = 1|\theta_j) = Logistic[\alpha_i(\theta_j - \beta_i)] \tag{3.8}$$

where y_{ij} is the response of item i by individual j, θ_j is latent ability for individual j, which has a normality assumption, α_i is item discrimination or factor loading, and β_i is item difficulty or threshold.

A transformation to the usual IRT parameters such as item discrimination, α and item difficulty, β is straightforward (e.g., Muthén, Kao, & Burstein, 1991). A Rasch model can be estimated by holding factor loadings equal across items. The thresholds in this model translate to the Rasch difficulties in terms of the logistic IRT model as $logit = \alpha(factor - \beta)$ and $\beta = threshold \div factor\ loading$, where $factor$ is ability (θ), $threshold$ is item difficulty (β), and $factor\ loading$ is item discrimination (α).

For the multilevel IRT model, Level 1 and Level 2 are combined into the "within" part of the model, i.e., the part describing variation across individuals. The "between" part describes across-group variation and corresponds to Level 3 of HLM. The conditional probability formulation for the Unconditional model for multilevel IRT can be specified as follows:

$$P(y_{ijg} = 1|\theta_{jg}) = Logistic[\alpha_i(\theta_{jg} - \beta_i)] \tag{3.9}$$

where y_{ijg} is the response of item i by individual j in group g, θ_{jg} is latent ability for individual j in group g, α_i is item discrimination, and β_i is item difficulty.

Following is the Mplus input file for the Unconditional model of TIMSS data.

```
TITLE: TIMMS DATA

DATA: FILE IS K_TIMSS.DAT;
      FORMAT IS F3.0 17F1.0;

VARIABLE: NAMES ARE SCH Q1-Q17;
          CLUSTER = SCH;
          USEVARIABLES ARE Q1-Q17;
          CATEGORICAL ARE Q1-Q17;

ANALYSIS: TYPE = TWOLEVEL GENERAL;
          ESTIMATOR = ML;

MODEL:
%WITHIN%
THETA BY Q1-Q16*(1);
THETA BY Q17@1;
%BETWEEN%
THETAB BY Q1-Q16*(1);
THETAB BY Q17@1;

OUTPUT: STANDARDIZED TECH1 TECH8;
```

The TITLE command is used to provide a title for the analysis. The title specified will be printed in the output.

The DATA command is used to provide information about the data set to be analyzed.

The FILE option is used to specify the name of the file that contains the data to be analyzed.

The FORMAT option is used to describe the format of the data to be analyzed.

The VARIABLE command is used to provide information about the variables in the data set to be analyzed.

The NAMES option is used to assign names to the variables in the data set. The data set in this example contains 18 variables: school code (SCH) and 17 items (Q1-Q17).

The CLUSTER option is used to identify the variable that contains clustering information.

The USEVARIABLES option identifies the variables that will be used in an analysis. Variables with special functions such as grouping variables are not included in the USEVARIABLES statement.

The CATEGORICAL option is used to specify which variables are treated as binary in the model and its estimation. In the example above, Q1-Q17 are binary variables.

The ANALYSIS command is used to describe the technical details of the analysis.

The TYPE option is used to describe the type of analysis that is to be carried out. Here, TWOLEVEL indicates to Mplus that a multilevel model is to be estimated.

The ESTIMATOR option can be used to select a different estimator. Here, ML, which is maximum likelihood parameter estimates with conventional standard errors and chi-square test statistic is used.

The MODEL command is used to describe the model to be estimated. In multilevel models, a model is specified for both within and between parts of the model.

The WITHIN option is used to identify variation across individuals.

The BETWEEN option is used to identify variation across groups.

In these models, the continuous latent variables, which are ‘THETA’ and ‘THETAB’ represent factors and random effects. That is, a single continuous factor, which is ‘THETA’

or 'THETAB' is measured by 17 binary factor indicators. To get factor variances, one factor loading is fixed to one.

The OUTPUT command is used to request additional output not included as the default.

The STANDARDIZED option is used to request two types of standardized coefficients.

The TECH1 option is used to request the arrays containing parameter specifications and starting values for all free parameters in the model.

The TECH8 option is used to request the optimization history in estimating the model. When TECH8 is requested, it is printed to the screen during the computations. The TECH8 screen printing is useful for determining how long the analysis takes.

The Mplus codes used for the other models are provided in the Appendices A.

3.2.3 WinBUGS

Bayesian estimation was done using a Markov Chain Monte Carlo (MCMC) estimation algorithm with Gibbs sampling as implemented in the computer software WinBUGS (Spiegelhalter et al., 2003). MCMC estimation with Gibbs sampling simulates a Markov chain in which the values representing parameters of the model are sampled repeatedly from their full conditional posterior distributions typically over a large number of iterations.

The multilevel IRT formulation in WinBUGS is set as follows:

$$P(y_{ijg} = 1|\theta_{jg}) = \sum_{g=1}^{G} \frac{exp(u_{jg} - \beta_i)}{1 + exp(u_{jg} - \beta_i)} \tag{3.10}$$

where $i = 1, \ldots, I$ items, $j = 1, \ldots, J$ individuals, $g = 1, \ldots, G$ groups, u_{jg} is the latent ability of an individual j within group g, and β_i is the difficulty parameter of item i.

The unconditional model for multilevel IRT can be specified as follows:

$$r_{ijg} \sim Bernoulli(p_{ijg})$$

$$logit(p_{ijg}) = u2_{jg} + u3_g - \beta_i$$

where

- $i = 1, 2, \ldots, I$ items, $j = 1, 2, \ldots, J$ individuals, and $g = 1, 2, \ldots, G$ groups
- $u2_{jg}$ is the ability of individual j. It is a random effect and is assumed to be normally distributed (i.e., $u2_{jg} \sim N(\mu 1, \tau_\beta)$). $\mu 1$ is the mean of individual ability and τ_β is the variance of the individual ability.
- $u3_g$ is the random effect for group g. It is assumed to be normally distributed (i.e., $u3_g \sim N(\mu 2, \tau_\gamma)$). $\mu 2$ is the mean of group ability and τ_γ is the variance of the group ability.
- β_i is a fixed effect representing the difficulty of item i.

As a first step in Bayesian estimation, the prior distributions for all model parameters must be specified. Then posterior distributions are calculated from prior distributions and the likelihood function. It is clear that different choices of the prior distribution may make the integral more or less difficult to calculate. For certain choices of the prior, the posterior has the same algebraic form as the prior. Such a choice is a conjugate prior.

In our prior distributions, the fixed effects (item difficulty, β_i and mean of individual ability, $\mu 1$) and the random effect parameters (individual ability, $u2_{jg}$ and group ability, $u3_g$) were assumed to be normally distributed. According to Lord and Novick (1968), it is reasonable to assume that the latent trait is drawn from a normal distribution. For the standard deviation parameters (τ_β and τ_γ), the gamma distribution is the conjugate prior for precision in the normal distribution. It is the most commonly used prior for a variance component (Gelman, Carlin, Stern, & Rubin, 1995, p. 71) and was used in this study as well.

For the initial values of the fixed effect parameters and standard deviation parameters in this model, the values obtained from the maximum likelihood estimation were used. In MCMC sampling with multilevel models it is natural to use as starting values the

likelihood and quasi-likelihood results (Browne & Draper, 2006). The initial values for the remaining model parameters were generated by WinBUGS 1.4 (Spiegelhalter et al., 2003).

The final estimates are taken as the means of the posteriors for each parameter estimated following the discarding of the burn-in iterations, which are some information from the initial iterations. Results from the convergence analyses (which will be discussed in a later chapter) indicated that a conservative burn-in of 4,000 iterations would be appropriate. After eliminating the first 4,000 iterations, the results were obtained from a Gibbs sampler run of 11,000 iterations. However, determining how long is "sufficiently long" in particular settings is an ongoing topic of research (Rosenthal, 1995; Polson, 1996; Roberts, 1996) and is discussed in the next section.

The WinBUGS code used for the models are given in the Appendices B.

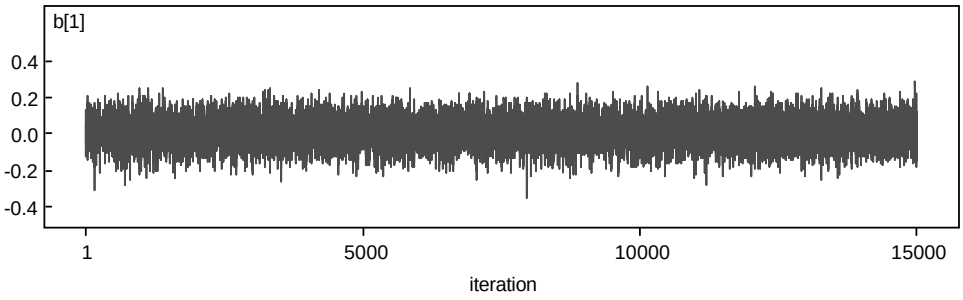
3.2.4 Convergence Diagnostics

MCMC methods have significantly simplified Bayesian estimation, yet bring along with them new issues such as convergence (Rupp et al., 2004). Convergence refers to the idea that the Gibbs Sampling or other MCMC technique will eventually reach a stationary distribution. There are several strategies for monitoring convergence, but there is no systematic, universal, guaranteed way. The basic idea is to solve for the number of iterations required to estimate some quantile of interest within an acceptable range of accuracy, at a specified probability level.

In this study, I considered the following: First, one intuitive and easily implemented diagnostic tool is a trace plot (or history plot) which plots the parameter value at time t against the iteration number, which is provided by WinBUGS. If the model has converged, the plot will move around the mode of the distribution (See Figure 3.1).

Second, Geweke's test (Geweke, 1992) is based on a time-series analysis approach. It splits the sample into two parts: say the first 10% and last 50%. If the chain is at stationarity, the means of the two samples should be equal. A modified z-test, referred to

Figure 3.1: A Sample of History Plot from WinBUGS

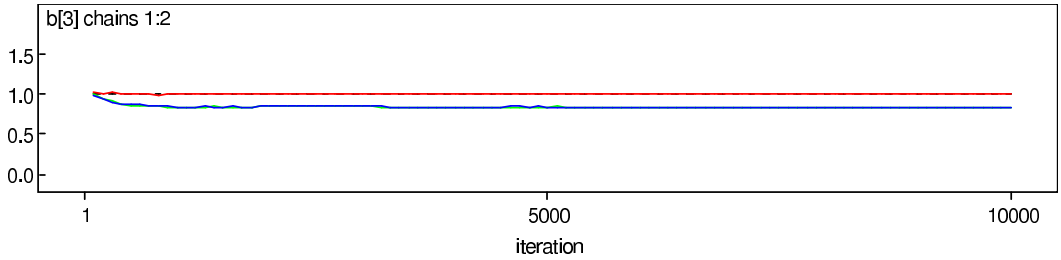


as a Geweke z-score, larger than 2 indicates that the mean of the series is still drifting, and a longer burn-in is required before monitoring the chain can begin.

Third, Gelman and Rubin (1992) proposed a test based on output from two or more runs of the MCMC simulation. The Gelman-Rubin approach was improved by Brooks and Gelman (1998). This diagnostic requires multiple Markov chains. Two separate sets of Markov chains were run for each of the simulated data sets, assuming two different starting values for the parameters of interest (i.e., β_i, τ_β, and τ_γ for the Unconditional Model). These starting values were provided in the WinBUGS code (See Appendices B). This method compares the within and between chain variances for each variable. On the computer monitor, one (green) line presents the width of the central 80% interval of the pooled runs. Another (blue) line represents the average width of the 80% interval within the individual runs. The third (red) line represents the ratio of pooled over within (= R). When the chains have converged, the variance within each sequence and the variance between sequences for each variable will be roughly equal, so R should approximately equal one. The Gelman-Rubin statistic based on this intuition is reported by WinBUGS (See Figure 3.2).

Fourth, the method of Raftery and Lewis (1992) is based on how many iterations are necessary to estimate the posterior for a given quantity. Here, a particular quantile (q) of

Figure 3.2: A Sample of Gelman and Rubin Statistic from WinBUGS



the distribution of interest (typically 2.5% and 97.5%, to give a 95% confidence interval), an accuracy of the quantile, and a power for achieving this accuracy on the specified quantile need to be specified. With these three parameters set, the Raftery-Lewis test breaks the chain into a $(1, 0)$ sequence (i.e., 1 if $\theta_t \leq q$, zero otherwise). This generates a two-state Markov chain, and the Raftery-Lewis test uses the sequence to estimate the transition probabilities. With these probabilities in hand, the number of addition burn-ins required to approach stationarity and the total chain length required to achieve the preset level of accuracy can be estimated.

According to the comprehensive review of 13 diagnostics by Cowles and Carlin (1996), the convergence diagnostics of Gelman and Rubin (1992) and of Raftery and Lewis (1992) currently are the most popularly used. These are used in this study.

WinBUGS does not estimate Geweke's and Raftery and Lewis's statistics. In order to get these, we export the CODA (Best, Cowles, & Vines, 2005) chain for each parameter of interest to S-PLUS (Insightful Corp., 2006) software. The program BOA (Bayesian Output Analysis) reports the Geweke statistic and Raftery and Lewis statistic. Examples of convergence diagnostics for item parameters for the TIMSS data are included in the Appendices C.

After the model has converged, samples from the conditional distributions are used to summarize the posterior distribution of parameters of interest, in this case, item difficulty, β and variance, τ.

3.3 Empirical Study

I provide two different types of comparisons, real data examples using data sets from (1) the Florida Comprehensive Assessment Test (FCAT: Florida Department of Education, 2003), (2) the Third International Mathematics and Science Study (TIMSS), and (3) the third edition of the Culture Free Self-Esteem Inventories (CFSEI-3; Battle, 2002) and a simulation study designed to explore possible differences in estimation for practical measurement situations.

3.3.1 FCAT

The Florida Comprehensive Assessment Test (FCAT: Florida Department of Education, 2003) is a standards-based statewide measure of student achievement in reading and mathematics in Grades 3 through 10 and in science in Grades 5, 8, and 10. The sample consists of 33 item responses from 5^{th} graders born between 1990 and 1992 to the 2003 FCAT Mathematics Test. The sample of students was randomly drawn from all students in Grade 5 by sampling proportionally from districts with at least 400 students in Grade 5. The final sample consisted of 1,855 students from 15 Florida school districts (see Table 3.1). Students' gender and age, and the interaction between gender and age created by multiplying the values of age by the values of gender are used as Level-2 predictor variables. Two Level-3 predictor variables, the percentage of free lunch in a district and sample size for each district are considered.

Table 3.1: FCAT Data Description

District	**N**	**Male**	**Female**	**1990**	**1991**	**1992**	**Lunch (%)**
5	83	43	40	3	36	44	28.9
6	320	178	142	13	133	174	45.6
13	428	237	191	15	156	257	65.9
17	59	36	23	3	27	29	66.1
27	21	12	9	0	6	15	47.6
29	213	112	101	12	86	115	53.1
31	18	12	6	0	8	10	50
36	84	36	48	1	33	50	56
37	40	23	17	4	17	19	47.5
48	177	94	83	3	68	106	42.4
49	52	28	24	2	19	31	50
50	189	87	102	8	76	105	48.7
53	99	47	52	6	53	40	61.6
59	63	37	26	3	26	34	28.6
66	9	4	5	0	3	6	66.7

3.3.2 TIMSS

The Third International Mathematics and Science Study (TIMSS) data set was selected for purposes of illustrating the comparison of two different estimation approaches, because it is relatively accessible and because a part of it has previously been analyzed in Kamata (2002). The data consisted of 1,130 high school seniors from 68 schools. A multiple-choice test consisting of 17 science literacy items was used for the example. A dichotomous student variable was included indicating whether the student studied at home or not, and a school characteristic variable was included indicating what the percentage was of teachers with five or more years of experience in the school.

3.3.3 CFSEI

The third edition of the Culture Free Self-Esteem Inventories (CFSEI-3; Battle, 2002) is a norm-referenced, self-report instrument used to measure both the global and specific dimensions of child and adolescent self-esteem. Eight Academic Self Esteem (ASE) items that were used by Pastor (2003) using the hierarchical generalized linear models (HGLM) component of the software program HLM 5 (Raudenbush et al., 1999) were analyzed. Data were collected between the fall of 1998 and the fall of 2000 for the norming of the CFSEI-3 from adolescents between the ages of 12 and 18 years. The final sample consisted of 905 respondents from 13 data collection sites located throughout the United States.

Three Level-2 predictor variables were used. They are students' gender, where 0 = female and 1 = male, age, and the interaction between gender and age created by multiplying the values of age by the values of gender. The education index, which is the number of years of education for a typical person in the site, and sample size for each site are used for the Level-3 predictor variables.

3.3.4 Summary

The following Table (see Table 3.2) is the summary for the empirical data sets used in this study.

Table 3.2: The Empirical Data Sets Used

Data	Item Size	Sample Size	Group Size
TIMSS	17	1130	68
CFSEI	8	905	13
FCAT	33	1855	15

3.4 Simulation Study

I used a simulation study, whose design is realistic for educational research, to compare Bayesian and Maximum Likelihood (ML) estimation for fitting multilevel IRT models. Estimated parameter values are compared across the conditions using root mean squared errors (RMSEs) between estimated and true parameter values.

Statistical power in multilevel models depends on the total sample sizes for each level (Snijders, 2005). That is, power for individual-level estimates depends on the number of individuals observed, and power for group-level estimates depends on the number of groups. Also, the efficiency and power of multilevel tests rests on pooled data across the units comprising two or more levels, which implies large data sets. For instance, simulation studies by Kreft (1996) found there was adequate statistical power for parameter estimates of interest with 30 groups of 30 observations each; 60 groups with 25 observations each; and 150 groups with 5 observations each. The number of groups has a greater effect on statistical power than the number of observations, though both are important. In a series of sampling simulations, Mok (1995) found that for smaller samples ($N < 800$) there is less bias in designs involving relatively more level-2 units and fewer subjects per unit, than in sample designs involving fewer units and more subjects per unit. Therefore, in this study, combinations of three conditions for each level were used: two item sizes ($I = 10$ and 20 items), three individual sizes ($J = 450$, 750, and 1500 individuals), and three group sizes ($G = 15$, 30, and 60 groups).

In addition, multilevel IRT modeling is advantageous in that it can obtain more accurate estimates of the relationship between predictor variables and the latent traits by simultaneous estimation of not only the IRT item and latent trait parameters, but also the parameters that describe the effects of the predictor variables (Pastor, 2003). In order to examine the effect of predictor variables at each level on the latent trait parameters, binary (i.e., 0 or 1) variables with equal sample sizes are considered for Level-2 and Level-3 predictor variables.

In this study, informative priors were used on the parameters of interest. That is, item difficulty, individual ability, and group ability are assumed to be normally distributed. According to Lord and Novick (1968), it is reasonable to assume that the latent trait is drawn from a normal distribution. For the variance parameters, the gamma distribution, which is the conjugate distribution of the normal distribution are used. The use of priors helps Bayesian estimation with model identification, which leads to the accuracy of the estimates. If convergence has not been reached, different priors would be considered. Gelman (2006) suggests to use a uniform prior on the hierarchical standard deviation.

3.4.1 Conditions

Data were simulated under the following conditions: two test lengths (I = 10 and 20 items), three individual-level sample sizes (J = 450, 750, and 1500 individuals), and three group-level sample sizes (G = 15, 30, and 60 groups). The values used in the simulations are given in Table 3.3, where the item difficulty $b[i]$ for item i = 1, . . . , I, the variance of the individual ability τ_β, the variance of the group ability τ_γ, the coefficient for Level-2 Predictor Variable γ_{01}, and the Coefficient for Level-3 Predictor Variable π_{001}.

Ability was simulated as $\sim$ Normal (0, 1). Item difficulty parameter values were determined so that values were roughly uniformly spaced, when items were ordered by difficulty, with a range between -2 and $+2$. The 10-item difficulty values that were used in this study are (-2.0, -1.5, -1.0, -0.5, 0.0, 0.0, 0.5, 1.0, 1.5, and 2.0). For the 20-item conditions, the pattern of item difficulties for the 10-item test was repeated twice. Five replications were simulated for each of the two test lengths $\times$ three individual-level sample sizes $\times$ three group-level sample sizes. All data were simulated by running the computer program WinBUGS 1.4 (Spiegelhalter et al, 2003) for one iteration after fixing the true parameters and saving the current state of the sampler only for item responses.

Table 3.3: Generating Parameters for the Models

Parameter	Value
$b[1]$	-2.0
$b[2]$	-1.5
$b[3]$	-1.0
$b[4]$	-0.5
$b[5]$	0.0
$b[6]$	0.0
$b[7]$	0.5
$b[8]$	1.0
$b[9]$	1.5
$b[10]$	2.0
$b[11]$	-2.0
$b[12]$	-1.5
$b[13]$	-1.0
$b[14]$	-0.5
$b[15]$	0.0
$b[16]$	0.0
$b[17]$	0.5
$b[18]$	1.0
$b[19]$	1.5
$b[20]$	2.0
τ_β	1.0
τ_γ	0.2
γ_{01}	0.5
π_{001}	0.1

Chapter 4

Results

4.1 Empirical Data

4.1.1 Analysis of the FCAT Data

Maximum Likelihood Estimation

Maximum likelihood estimates of model parameters were obtained using the computer programs HLM 6.02 (Raudenbush et al., 2005) and Mplus 4.1 (Muthén & Muthén, 2006).

The Unconditional Model. Significant within-district, $\tau_{\beta 11}$, and between-district variation, $\tau_{\gamma 11}$, were observed (see Table 4.1). These indicated that mathematics

Table 4.1: Results for the Unconditional Model of the FCAT Data from HLM Program

Estimates of Fixed Effects		Coefficient	SE	t	df	p	
π_{000}	Intercept	-1.096	0.073	-14.956	14	$< .000$	
Estimates of Random Effects		Reliability	Variance	SE	df	χ^2	p
Level 2 $\tau_{\beta 11}$	Intercept	0.849	0.981	0.038	1840	10273.461	$< .000$
Level 3 $\tau_{\gamma 11}$	Intercept	0.556	0.020	0.012	14	39.415	.001

performance varied within and between districts before any predictor variables were included in the model. Although the between-district variation was statistically different from zero, it accounted for only 2% of the between- and within-district variance combined. Again, the effect for π_{000} represents the estimate in the overall sample, or the effect of the

reference item (item 14). How items relate to one another was estimated by subtracting the effect for each item, π_{q00} from the fixed effect, π_{000}, of the reference item.

The Model with Level-2 Predictor Variables. The final model (see Table 4.2) yielded a significant effect for gender ($\pi_{010} = 0.149$), indicating a gender difference when age was controlled for. The positive coefficient for gender, where 0 = female and 1 = male,

Table 4.2: Results for the Model with Level-2 Predictor Variables of the FCAT Data from HLM Program

Estimates of Fixed Effects		Coefficient	SE	t	df	p	
π_{000}	Intercept	-1.584	0.103	-15.398	14	$< .000$	
π_{010}	Gender	0.149	0.050	3.015	1852		
π_{020}	Age	0.271	0.043	6.273	1852		
Estimates of Random Effects		Reliability	Variance	SE	df	χ^2	p
Level 2							
$\tau_{\beta 11}$	Intercept	0.845	0.951	0.037	1838	9998.910	$< .000$
Level 3							
$\tau_{\gamma 11}$	Intercept	0.572	0.021	0.012	14	41.027	.001

indicated that mathematics performance for males was higher than for females. Controlling for gender, there was also a significant effect for age ($\pi_{020} = 0.271$), with performance increasing with age.

Including gender and age in the model reduced the student-to-student variation by only 3% relative to the unconditional model. After the student level predictor variables were added into the model, the district random effect was no longer significant indicating that the gender and age effects applied to all districts. As the results indicated that there was no significant variation in mathematics performance across districts, it was not necessary to build a model with Level-3 predictors.

Bayesian Estimation

Bayesian estimates of model parameters were obtained from a Gibbs sampler run of 11,000 iterations after eliminating the first 4,000 iterations using the computer software WinBUGS 1. 4 (Spiegelhalter et al., 2003).

The Unconditional Model. The parameters we focus on in this study are the item difficulty parameter (b_i) and ability variance parameters (the variance of the individual ability, τ_β and the variance of the group ability, τ_γ). Examining the results (see Table 4.3) for the item difficulty parameter estimates of the FACT data sets first reveals that the agreement between the mean of the posterior distribution of the estimate and the parameter estimates by maximum likelihood estimation is quite good, which is not surprising because the number of items used are large. Also, a repeated measures one-way ANOVA was conducted to determine whether item parameter estimates differed across the three estimation methods. Mauchly's test indicated that the assumption of sphericity had been violated ($\chi^2(2) = 171.835$, $p < .000$), therefore degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity ($\epsilon = .501$). The results revealed that there were no significant differences in the item parameter estimates between the three estimation methods, F(1.002,32.063) = .000, $p = .998$, $\eta_p^2 = .000$.

The Model with Level-2 Predictor Variables. This model, in WinBUGS, can be written as:

$$r_{ijg} \sim Bernoulli(p_{ijg})$$

$$logit(p_{ijg}) = u2_{jg} + u3_g - b_i$$

$$u2_{jg} \sim N(\mu_{jg}, \tau_\beta)$$

$$\mu_{jg} = \alpha \times gender_{jg} + \beta \times age_{jg}$$

where α is the gender effect in district g controlling for all other variables in the model and β is the age effect for district g controlling for all other variables in the model. The other terms are defined as in the unconditional model.

Table 4.3: Parameter Estimates of the FCAT Data
for the Unconditional Model

	WinBUGS		HLM		Mplus	
Parameter	Mean	SD	Estimate	SE	Estimate	SE
$b[1]$	-2.096	0.082	-2.059	0.102	-1.681	0.099
$b[2]$	-1.261	0.064	-1.231	0.086	-1.010	0.083
$b[3]$	-0.529	0.056	-0.514	0.079	-0.426	0.076
$b[4]$	-0.172	0.053	-0.165	0.077	-0.141	0.074
$b[5]$	0.531	0.051	0.518	0.075	0.418	0.072
$b[6]$	-0.157	0.052	-0.151	0.077	-0.130	0.074
$b[7]$	-0.137	0.053	-0.132	0.077	-0.115	0.073
$b[8]$	0.065	0.052	0.066	0.076	0.047	0.073
$b[9]$	0.377	0.050	0.369	0.076	0.296	0.072
$b[10]$	0.131	0.052	0.129	0.076	0.099	0.073
$b[11]$	0.665	0.051	0.649	0.075	0.524	0.072
$b[12]$	1.614	0.055	1.570	0.078	1.281	0.075
$b[13]$	0.814	0.051	0.793	0.076	0.643	0.072
$b[14]$	1.737	0.055	1.689	0.073	1.548	0.070
$b[15]$	-0.169	0.053	-0.162	0.077	-0.139	0.074
$b[16]$	-0.148	0.052	-0.143	0.077	-0.123	0.074
$b[17]$	-0.093	0.052	-0.088	0.077	-0.078	0.073
$b[18]$	1.435	0.054	1.395	0.077	1.138	0.074
$b[19]$	-1.624	0.071	-1.589	0.091	-1.300	0.089
$b[20]$	-0.518	0.055	-0.502	0.079	-0.416	0.076
$b[21]$	-0.758	0.058	-0.738	0.081	-0.609	0.077
$b[22]$	-0.681	0.057	-0.662	0.080	-0.547	0.077
$b[23]$	-0.998	0.060	-0.972	0.083	-0.799	0.080
$b[24]$	0.080	0.051	0.079	0.076	0.058	0.073
$b[25]$	0.700	0.051	0.682	0.075	0.552	0.072
$b[26]$	-0.824	0.058	-0.802	0.081	-0.661	0.078
$b[27]$	0.002	0.052	0.004	0.076	-0.003	0.073
$b[28]$	1.304	0.053	1.269	0.077	1.034	0.074
$b[29]$	0.316	0.050	0.310	0.076	0.247	0.072
$b[30]$	0.567	0.051	0.554	0.075	0.447	0.072
$b[31]$	0.950	0.051	0.925	0.076	0.752	0.072
$b[32]$	-0.358	0.054	-0.347	0.078	-0.290	0.075
$b[33]$	-0.766	0.059	-0.744	0.081	-0.614	0.077
τ_β	1.068	0.045	0.981	0.038	0.681	0.111
τ_γ	0.028	0.021	0.020	0.012	0.014	0.010

Table 4.4 reports the model parameter estimates from maximum likelihood estimation and Bayesian estimation. This also showed agreement between the mean of the posterior distribution of the Bayesian estimate and the parameter estimates by maximum likelihood estimation is quite good. Also, a repeated measures one-way ANOVA was conducted to determine whether the item parameter estimates differed across the three estimation methods. Mauchly's test indicated that the assumption of sphericity was violated ($\chi^2(2) = 171.192$, $p < .000$). As a result, degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity ($\epsilon = .501$). The results revealed that there were no significant differences in the item parameter estimates between the three estimation methods, F(1.002,32.064) = .000, $p = .999$, $\eta_p^2 = .000$.

4.1.2 Example using the TIMSS Data

The Unconditional Model. The results from HLM, Mplus, and WinBUGS indicated that all programs yielded comparable parameter estimates of Kamata's 3-level IRT models. After mean-centering the parameters by subtracting them from their mean, item parameter estimates from the HLM, Mplus, and WinBUGS runs of the TIMSS data under the unconditional model are presented in Table 4.5. For the item parameter estimates, a repeated measures one-way ANOVA was conducted to determine whether there was a difference across the three estimation methods. Mauchly's test indicated that the assumption of sphericity had been violated ($\chi^2(2) = 79.865$, $p < .000$). As a result, the degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity ($\epsilon = .501$). The results revealed that there were no significant differences in the item parameter estimates between the three estimation methods, F(1.002,16.039) = .000, $p = .996$, $\eta_p^2 = .000$.

The Model with Level-2 Predictor Variables. The Level-2 predictor is an indicator variable (i.e., scistud) of whether the student studies science regularly at home or not, where 0 = no and 1 = yes. In this model, the primary interest is the magnitude of the

Table 4.4: Parameter Estimates of the FCAT Data
for the Model with Level-2 Predictor Variables

	WinBUGS		HLM		Mplus	
Parameter	Mean	SD	Estimate	SE	Estimate	SE
$b[1]$	-2.098	0.083	-2.060	0.102	-1.681	0.122
$b[2]$	-1.262	0.063	-1.231	0.086	-1.010	0.109
$b[3]$	-0.530	0.055	-0.514	0.079	-0.427	0.103
$b[4]$	-0.171	0.053	-0.165	0.077	-0.142	0.102
$b[5]$	0.531	0.051	0.518	0.075	0.417	0.101
$b[6]$	-0.156	0.053	-0.151	0.077	-0.130	0.102
$b[7]$	-0.138	0.053	-0.132	0.077	-0.114	0.102
$b[8]$	0.065	0.051	0.066	0.076	0.048	0.101
$b[9]$	0.378	0.051	0.369	0.076	0.295	0.101
$b[10]$	0.131	0.051	0.130	0.076	0.099	0.101
$b[11]$	0.665	0.051	0.649	0.075	0.525	0.101
$b[12]$	1.615	0.055	1.570	0.078	1.282	0.103
$b[13]$	0.813	0.051	0.793	0.075	0.643	0.101
$b[14]$	1.738	0.055	1.689	0.103	1.547	0.092
$b[15]$	-0.169	0.053	-0.162	0.077	-0.139	0.102
$b[16]$	-0.148	0.052	-0.143	0.077	-0.123	0.102
$b[17]$	-0.092	0.052	-0.088	0.077	-0.078	0.101
$b[18]$	1.435	0.054	1.395	0.077	1.138	0.102
$b[19]$	-1.624	0.071	-1.589	0.091	-1.300	0.113
$b[20]$	-0.517	0.055	-0.502	0.079	-0.416	0.103
$b[21]$	-0.758	0.057	-0.738	0.081	-0.609	0.104
$b[22]$	-0.680	0.057	-0.662	0.080	-0.547	0.104
$b[23]$	-0.998	0.061	-0.972	0.083	-0.800	0.106
$b[24]$	0.079	0.051	0.079	0.076	0.058	0.101
$b[25]$	0.699	0.051	0.682	0.075	0.552	0.101
$b[26]$	-0.823	0.058	-0.802	0.081	-0.661	0.105
$b[27]$	0.002	0.052	0.004	0.076	-0.003	0.101
$b[28]$	1.304	0.053	1.269	0.077	1.034	0.102
$b[29]$	0.315	0.051	0.310	0.076	0.247	0.101
$b[30]$	0.568	0.051	0.554	0.075	0.447	0.101
$b[31]$	0.951	0.052	0.926	0.076	0.751	0.101
$b[32]$	-0.359	0.054	-0.347	0.078	-0.290	0.102
$b[33]$	-0.766	0.057	-0.744	0.081	-0.614	0.104
α	0.157	0.051	0.149	0.050	0.067	0.037
β	0.281	0.045	0.271	0.043	0.130	0.047
τ_β	1.038	0.043	0.981	0.038	0.678	0.110
τ_γ	0.028	0.022	0.020	0.012	0.015	0.015

Table 4.5: Item Difficulty Estimates of the TIMSS Data for the Unconditional Model

	WinBUGS		HLM		Mplus	
Item	Mean	SD	Estimate	SE	Estimate	SE
1	-0.179	0.066	-0.140	0.092	-0.187	0.096
2	-0.288	0.065	-0.226	0.090	-0.301	0.096
3	0.637	0.068	0.504	0.079	0.676	0.097
4	-0.571	0.067	-0.452	0.090	-0.600	0.097
5	-0.731	0.067	-0.580	0.089	-0.769	0.097
6	-0.284	0.066	-0.222	0.087	-0.296	0.096
7	1.265	0.074	1.004	0.082	1.339	0.102
8	-0.153	0.066	-0.119	0.080	-0.159	0.096
9	-1.325	0.072	-1.063	0.100	-1.398	0.102
10	0.284	0.065	0.225	0.094	0.303	0.096
11	1.770	0.082	1.416	0.106	1.874	0.108
12	2.486	0.097	2.020	0.102	2.635	0.123
13	-0.876	0.069	-0.697	0.095	-0.924	0.098
14	-1.979	0.083	-1.614	0.107	-2.091	0.111
15	-1.140	0.071	-0.912	0.100	-1.203	0.100
16	-0.175	0.066	-0.137	0.077	-0.182	0.096
17	1.259	0.074	0.999	0.086	1.288	0.111

coefficient for the Level-2 predictor (γ_{01}). γ_{01} was estimated as 0.360 with a p-value less than 0.001 from HLM program. This implies that, on average, students who study science regularly at home had higher ability (0.360 logits) than those who do not, and the difference was statistically significant at $\alpha = 0.05$.

The Model with Level-2 and Level-3 Predictor Variables. The parameter of interest for this model is the coefficient estimate for the Level-3 predictor variable (π_{001}), which is teachers' experience in their school. As you can see, teachers' experience does not have a large effect on students' performance.

The following Table 4.6 shows the results from three programs for the individual variance (τ_β), the group variance (τ_γ), the coefficient for the Level-2 predictor (γ_{01}), and the coefficient estimate for the Level-3 predictor variable (π_{001})

Table 4.6: Parameter Estimates of the TIMSS Data
for the Model with Level-2 and Level-3 Predictor Variables

	WinBUGS		HLM		Mplus	
Parameter	Mean	SD	Estimate	SE	Estimate	SE
τ_β	1.082	0.070	0.931	0.055	1.260	0.232
τ_γ	0.244	0.060	0.201	0.048	0.252	0.075
γ_{01}	0.436	0.075	0.359	0.069	0.385	0.077
π_{001}	-0.003	0.003	-0.002	0.002	-0.004	0.003

Examining the result for the group variance estimates (τ_γ) reveals that there is agreement between Bayesian estimation and Maximum Likelihood estimation, which is not surprising since there are large group sizes (i.e., 68 schools).

4.1.3 Example with the CFSEI Data

The Unconditional Model. For this model, the comparisons of the parameters of interest, item effect (b), the individual variance (τ_β), and the group variance (τ_γ), are presented in Table 4.7. For the item parameter estimates, a repeated measures one-way ANOVA was conducted to determine whether there was a difference across the three estimation methods. Mauchly's test indicated that the assumption of sphericity had been violated ($\chi^2(2) = 10.975$, $p = .004$), therefore degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity ($\epsilon = .544$). The results revealed that there were no significant differences in the item parameter estimates between the three estimation methods, F(1.087,7.611) = .000, $p = .994$, $\eta_p^2 = .000$.

There was statistically significant within-site, τ_β, and between-site, τ_γ, variation in academic self-esteem (ASE). To examine if ASE variation between students was associated with their characteristics, the model with Level-2 predictor variables is considered.

Table 4.7: Parameter Estimates of the CFSEI Data
for the Unconditional Model

	WinBUGS		HLM		Mplus	
Parameter	Mean	SD	Estimate	SE	Estimate	SE
$b[1]$	0.010	0.078	0.008	0.054	0.020	0.222
$b[2]$	-0.512	0.082	-0.344	0.195	-0.384	0.225
$b[3]$	-0.445	0.080	-0.297	0.147	-0.332	0.224
$b[4]$	1.225	0.081	0.810	0.156	0.965	0.222
$b[5]$	0.453	0.078	0.303	0.157	0.365	0.221
$b[6]$	-0.117	0.078	-0.077	0.094	-0.078	0.223
$b[7]$	-0.423	0.081	-0.282	0.178	-0.315	0.224
$b[8]$	-0.191	0.079	-0.124	0.155	-0.237	0.177
τ_β	2.957	0.241	1.938	0.128	1.789	0.352
τ_γ	0.539	0.363	0.279	0.140	0.236	0.144

The Model with Level-2 Predictor Variables. For this model, the comparisons of the parameters of interest, item effect (b), the individual variance (τ_β), the group variance (τ_γ), and the coefficients for student-level predictors (gender effect, γ_{01}, and age effect, γ_{02}) are presented in Table 4.8. The negative coefficient for gender effects indicates that female academic self-esteem (ASE) is higher than male ASE. Also, the coefficient for age means that ASE decreased as the age of the students increased. For the item parameter estimates, a repeated measures one-way ANOVA was conducted to see that there was a difference across the three estimation methods. Mauchly's test indicated that the assumption of sphericity had been violated ($\chi^2(2) = 10.915$, $p < .004$), therefore, degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity ($\epsilon = .544$). The results revealed that there were no significant differences in the item parameter estimates between the three estimation methods, F(1.088,7.618) = .000, $p = .998$, $\eta_p^2 = .000$.

The Model with Level-2 and Level-3 Predictor Variables. After the student level predictor variables were added into the model, the site random effect was no longer

Table 4.8: Parameter Estimates of the CFSEI Data
for the Model with Level-2 Predictor Variables

	WinBUGS		HLM		Mplus	
Parameter	Mean	SD	Estimate	SE	Estimate	SE
$b[1]$	0.006	0.078	0.009	0.059	0.019	0.230
$b[2]$	-0.510	0.082	-0.350	0.210	-0.391	0.233
$b[3]$	-0.443	0.081	-0.303	0.162	-0.338	0.233
$b[4]$	1.224	0.082	0.828	0.162	0.979	0.228
$b[5]$	0.452	0.078	0.309	0.165	0.369	0.229
$b[6]$	-0.119	0.079	-0.078	0.106	-0.081	0.231
$b[7]$	-0.421	0.081	-0.287	0.193	-0.321	0.233
$b[8]$	-0.189	0.079	-0.126	0.167	-0.235	0.200
τ_β	2.768	0.224	1.821	0.123	1.791	0.352
τ_γ	0.331	0.265	0.287	0.142	0.149	0.109
γ_{01}	-0.633	0.128	-0.432	0.101	-0.275	0.064
γ_{02}	-0.204	0.038	-0.145	0.074	-0.165	0.035

significant indicating that the gender and age effects applied to all sites. As the results indicated that there was no significant variation in academic self-esteem across sites, it was not necessary to build a model with Level-3 predictors.

4.1.4 Summary of Real Data Analyses

It should be noted that the item fixed effects remained fairly constant across models (i.e., the unconditional model, the model with Level-2 predictor variable(s), and the model with Level-2 and Level-3 predictor variables), as would be expected. Kamata (1998) noted that including predictor variables in the multilevel IRT model should not alter the item effect estimates. This study lends support to such a conclusion.

As can be seen in Table 4.9, there was little difference between the fixed effect estimates for the items obtained using the unconditional model and those obtained using the model with level-2 predictor variables for the FCAT data.

Table 4.9: Item Fixed Effects for FCAT Data for the Unconditional Model and the Model with Level-2 Predictor Variables from HLM program

Item	The Unconditional Model	SE	The Model with Level-2 Variables	SE
1	3.748	0.102	3.749	0.102
2	2.920	0.086	2.920	0.086
3	2.204	0.079	2.203	0.079
4	1.854	0.077	1.854	0.077
5	1.171	0.075	1.171	0.075
6	1.840	0.077	1.840	0.077
7	1.821	0.077	1.821	0.077
8	1.623	0.076	1.623	0.076
9	1.320	0.076	1.320	0.076
10	1.560	0.076	1.559	0.076
11	1.041	0.075	1.040	0.075
12	0.119	0.078	0.119	0.078
13	0.896	0.076	0.896	0.075
14	-1.096	0.073	-1.584	0.103
15	1.852	0.077	1.851	0.077
16	1.832	0.077	1.832	0.077
17	1.777	0.077	1.777	0.077
18	0.294	0.077	0.294	0.077
19	3.278	0.091	3.278	0.091
20	2.191	0.079	2.191	0.079
21	2.427	0.081	2.427	0.081
22	2.351	0.080	2.351	0.080
23	2.661	0.083	2.661	0.083
24	1.610	0.076	1.610	0.076
25	1.007	0.075	1.007	0.075
26	2.491	0.081	2.491	0.081
27	1.685	0.076	1.685	0.076
28	0.420	0.077	0.420	0.077
29	1.380	0.076	1.379	0.076
30	1.135	0.075	1.135	0.075
31	0.764	0.076	0.763	0.076
32	2.036	0.078	2.036	0.078
33	2.433	0.081	2.433	0.081

From the empirical study, it was clear that maximum likelihood and Bayesian analyses give similar results for TIMSS data, which had the largest group sizes (68 schools) compared to the other data sets. However, the group variances for the CFSEI data, which had the smallest group sizes (13 sites) were not as similar (See Table 4.10).

Table 4.10: Group Variance Estimates for the Unconditional Model from Maximum Likelihood and Bayesian Estimation

Data	Item	Sample (Individual)	Group	ML HLM(SE)	Estimate Mplus(SE)	Bayesian Estimate WinBUGS(SD)
CFSEI	8	905	13	0.279(0.140)	0.236(0.144)	0.582(0.427)
FCAT	33	1180	15	0.020(0.012)	0.014(0.010)	0.028(0.021)
TIMSS	17	1130	68	0.218(0.052)	0.267(0.079)	0.258(0.063)

Finally, one advantage of using the ML method is that it was much faster to run than the Bayesian method and therefore made the study of power and other statistical properties of the system computationally more feasible. The advantage of the Bayesian method is that it is more flexible than the ML method and can easily be modified to accommodate more complicated models.

4.2 Simulated Data

Each data set for the unconditional model, the model with one level-2 predictor variable, and the model with one level-2 and one level-3 predictor variables was analyzed using HLM, Mplus, and WinBUGS programs. The values used in the simulations are given in Table 3.3, where the item difficulty $b[i]$ for item i = 1, . . . , I, the variance of the individual ability τ_β, the variance of the group ability τ_γ, the coefficient for level-2 predictor variable γ_{01}, and the coefficient for level-3 predictor variable π_{001}. The parameter estimates were obtained from five replications for each of the study conditions. After standardizing the parameters by subtracting them from their mean, the means of parameter estimates from

five replications and their line graphs for one condition from each model are presented in Appendices D and E.

In this study, recovery of item and variance parameters was assessed using root mean squared errors (RMSEs) between the generating parameter and the parameter estimate, which is a common measure of the robustness of parameter estimates. RMSEs for each parameter under ML and Bayesian were collected for all conditions. These RMSEs are provided in Tables 4.11, 4.12, 4.19, 4.20, and 4.23 to 4.26, for each parameter of interest from each model.

4.2.1 The Unconditional Model

The parameters I focus on in this model are the item difficulty ($b[i]$ for item i = 1, ..., I) and ability variance parameters (the variance of the individual ability, τ_β and the variance of the group ability, τ_γ). The values used in the simulations are given in Table 3.3.

First, I illustrate some typical Gibbs sample output using one data set from the model (I = 10, J = 750, and G = 30). Sample history (trace) plots and density plots are given for selected parameters for 11,000 iterations after eliminating the first 4,000 iterations and the values used in the simulations are circled. The trace plots for the parameters of interest are shown in Figure 4.1, where $b[1]$ is the item difficulty for item 1, $b[10]$ is the item difficulty for item 10, $sigma2$ is the individual variance estimate, and $sigma3$ is the group variance estimate. Each parameter of interest becomes stationary by 4,000 iterations, indicating that convergence has been reached by 4,000 iterations.

The density plots (Figures 4.2) show unimodal distributions which are nearly symmetric, and look close to normal except for the plot of $sigma3$, which is the group variance estimate. It has a long right tail and a high peak close to zero. This result is likely due to the values of $sigma3$ being close to zero, the lower boundary of the parameter space.

For HLM, the classical item difficulty was used to determine the easiest item. The usual procedure in multilevel IRT modeling, which was also used in this study, is to take the

Figure 4.1: History Plots of Parameter Estimates
for the Unconditional Model

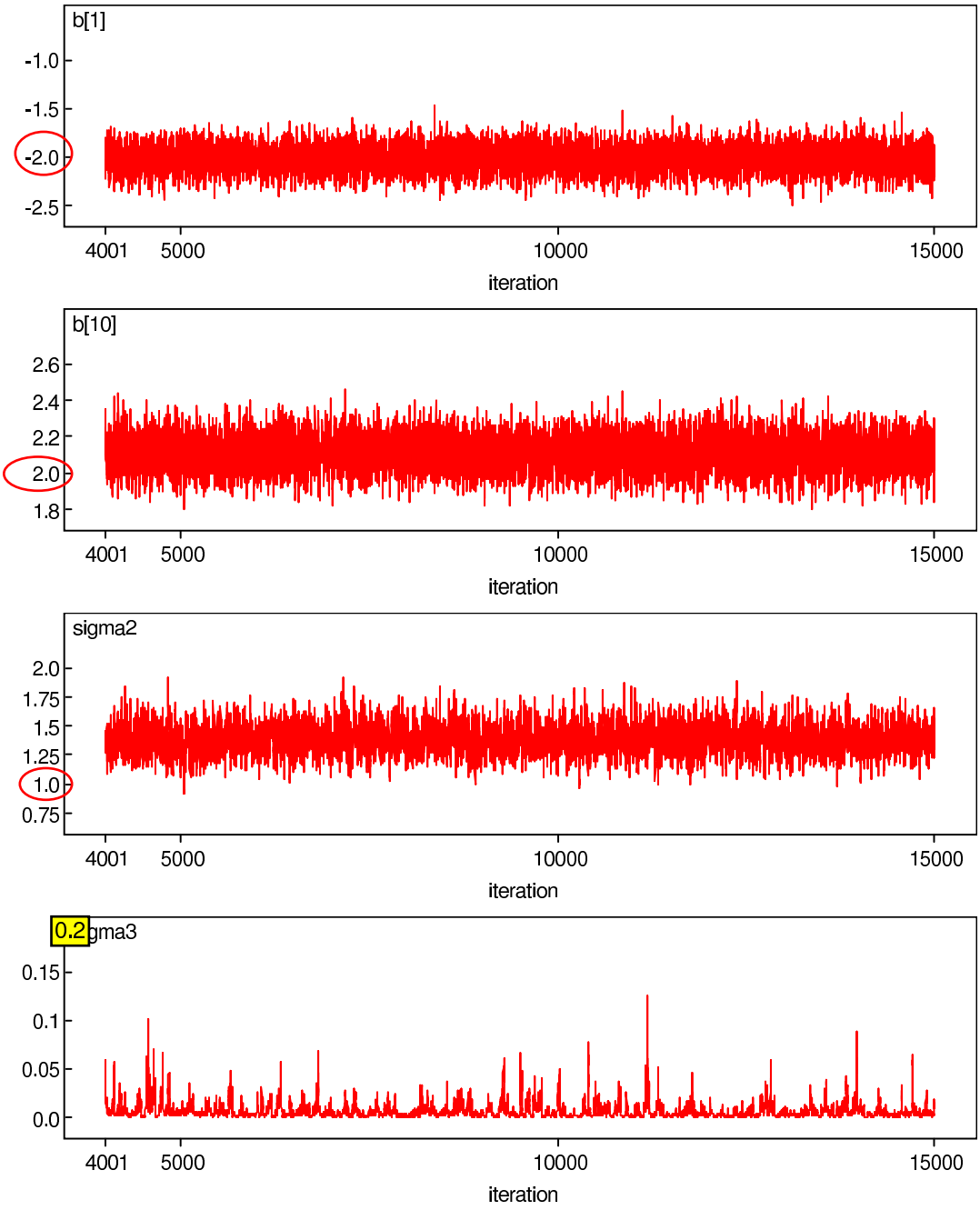
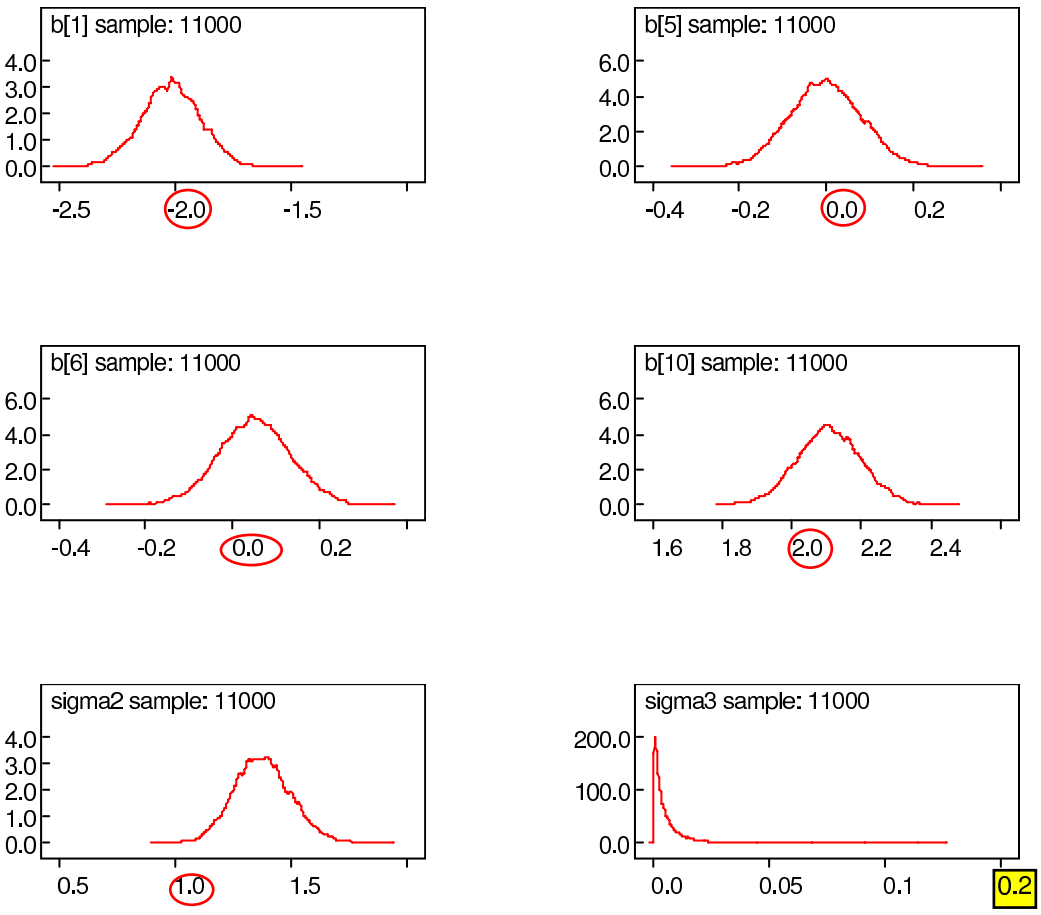


Figure 4.2: Density Plots of Parameter Estimates
for the Unconditional Model



easiest item as the reference item. Again, the effect for π_{000} represents the effect of the reference item (item 1), or the difficulty of the reference item, $b[1]$. The item difficulties for the others ($b[i]$, for i = 2, . . . , I) were estimated by subtracting the effect for each item, π_{q00} from the effect of the reference item, π_{000}.

Item parameters were mean-centered by subtracting them from their mean. The means of the centered parameter estimates from five replications and their line graphs for one condition (I = 10, J = 750, and G = 30) for the model are presented in Appendices D and E. From this table, we see that the means of each parameter estimates from HLM are smaller than those from the other programs, WinBUGS and Mplus. This makes sense since the HLM software uses the penalized or predictive quasi-likelihood (PQL) method to estimate the parameter estimates for dichotomous responses. Many authors (e.g., Rodriguez & Goldman, 1995; Goldstein & Rashbash, 1996; Breslow & Clayton, 1993) have reported that these approximation methods exhibit downward (i.e., estimated parameters are smaller than generated parameters) biases for both the fixed effects and the variance components for dichotomous responses with small cluster sizes. This conclusion was supported by the present study.

In order to study the effect of varying the number of items (I), individuals (J), and groups (G) on the bias, the RMSEs under each condition for the unconditional model are presented in Table 4.11 for 20 items and Table 4.12 for 10 items, where β = Item Difficulty (averaged over all the items), τ_β = Individual Variance, and τ_γ = Group Variance.

For each parameter in the unconditional model, a repeated measures ANOVA with the three estimation method (M) as the repeated factor and item (I), individual (J), and group (G) size as the independent variables was performed using the RMSEs across all the conditions used in this study as the dependent variable. For the item parameter estimates (β), Mauchly's criterion showed that there was not a problem with the sphericity assumption ($\chi^2(2) = 5.047$, $p = .080$). Univariate tests of within-subjects effects showed that the significant effects were method, method × test length interaction, method ×

Table 4.11: RMSEs for the Unconditional Model
from Maximum Likelihood and Bayesian Estimation

			Item = 20		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
1500	60	β	0.077	0.251	0.143
		τ_β	0.595	0.360	0.835
		τ_γ	0.174	0.172	0.163
	30	β	0.067	0.266	0.100
		τ_β	0.633	0.398	0.762
		τ_γ	0.187	0.189	0.185
	15	β	0.067	0.263	0.178
		τ_β	0.562	0.341	0.546
		τ_γ	0.190	0.195	0.194
750	60	β	0.098	0.254	0.199
		τ_β	0.531	0.306	0.703
		τ_γ	0.163	0.160	0.150
	30	β	0.096	0.267	0.153
		τ_β	0.566	0.338	0.658
		τ_γ	0.173	0.171	0.164
	15	β	0.099	0.260	0.165
		τ_β	0.493	0.284	0.727
		τ_γ	0.193	0.199	0.199
450	60	β	0.142	0.285	0.211
		τ_β	0.629	0.371	0.718
		τ_γ	0.041	0.042	0.050
	30	β	0.119	0.262	0.179
		τ_β	0.680	0.434	0.400
		τ_γ	0.178	0.183	0.184
	15	β	0.136	0.287	0.222
		τ_β	0.607	0.374	0.841
		τ_γ	0.160	0.168	0.162

Table 4.12: RMSEs for the Unconditional Model
from Maximum Likelihood and Bayesian Estimation

			Item = 10		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
1500	60	β	0.053	0.237	0.100
		τ_β	0.335	0.074	0.427
		τ_γ	0.175	0.177	0.171
	30	β	0.066	0.227	0.119
		τ_β	0.320	0.055	0.474
		τ_γ	0.184	0.187	0.184
	15	β	0.063	0.234	0.138
		τ_β	0.271	0.025	0.480
		τ_γ	0.189	0.193	0.190
750	60	β	0.078	0.228	0.167
		τ_β	0.374	0.059	0.326
		τ_γ	0.157	0.154	0.148
	30	β	0.096	0.234	0.130
		τ_β	0.291	0.092	0.250
		τ_γ	0.190	0.195	0.196
	15	β	0.093	0.240	0.174
		τ_β	0.260	0.073	0.364
		τ_γ	0.184	0.190	0.187
450	60	β	0.117	0.278	0.316
		τ_β	0.346	0.151	0.934
		τ_γ	0.138	0.139	0.246
	30	β	0.113	0.237	0.262
		τ_β	0.419	0.156	0.861
		τ_γ	0.150	0.161	0.146
	15	β	0.150	0.273	0.264
		τ_β	0.417	0.134	0.648
		τ_γ	0.174	0.182	0.175

individual sample size interaction, and method × *test length × individual sample size interaction (See Table 4.13). Significant method effect means that mean RMSEs of β

Table 4.13: Tests of Within-Subjects Effects
for the Item Parameters
from the Unconditional Model

Source	SS	df	MS	F	p	η_p^2
Method (M)	.226	2	.113	491.817	.000	.992
M * I	.003	2	.002	6.522	.021	.620
M * J	.012	4	.003	13.506	.001	.871
M * G	.002	4	.000	2.024	.184	.503
M * I * J	.005	4	.001	5.596	.019	.737
M * I * G	.001	4	.000	1.129	.408	.361
M * J * G	.002	8	.000	1.190	.406	.543
Error	.002	8	.000			

changes across the three estimation methods. The mean RMSEs of β were .096 for WinBUGS, .255 for HLM, and .179 for Mplus, respectively. Post hoc tests revealed that the three means were significantly different from each other at the $\alpha = .05$ level. A significant two-way interaction effect means that the influence of test length or individual sizes on RMSEs of β depends on the estimation method. ($M \times I$ and $M \times J$). A significant three-way interaction effect means that the influence of test length on RMSEs of β depends on individual sample size and the estimation method ($M \times I \times J$). This three-way interaction is broken down into a series of two-way interactions, one two-way within each level of test length (See Figure 4.3).

Among the between-subjects effects, the individual and group main effects and the interaction effect of test length × individual sample size were significant (See Table 4.14). There was a significant interaction effect between test length and individual sample size, F(2,4) = 12.462, $p = .019$, $\eta_p^2 = .862$. Although the RMSEs for β decreas ed as total sample increased, the results indicate that there was a strong tendency for the RMSEs of β under each individual sample size condition to increase as number of items increased, except for the condition, J = 450 (See Figure 4.4, where b indicates item parameter estimates). Also,

Figure 4.3: The RMSEs of Item Parameters
for the Unconditional Model
under Each Item Size

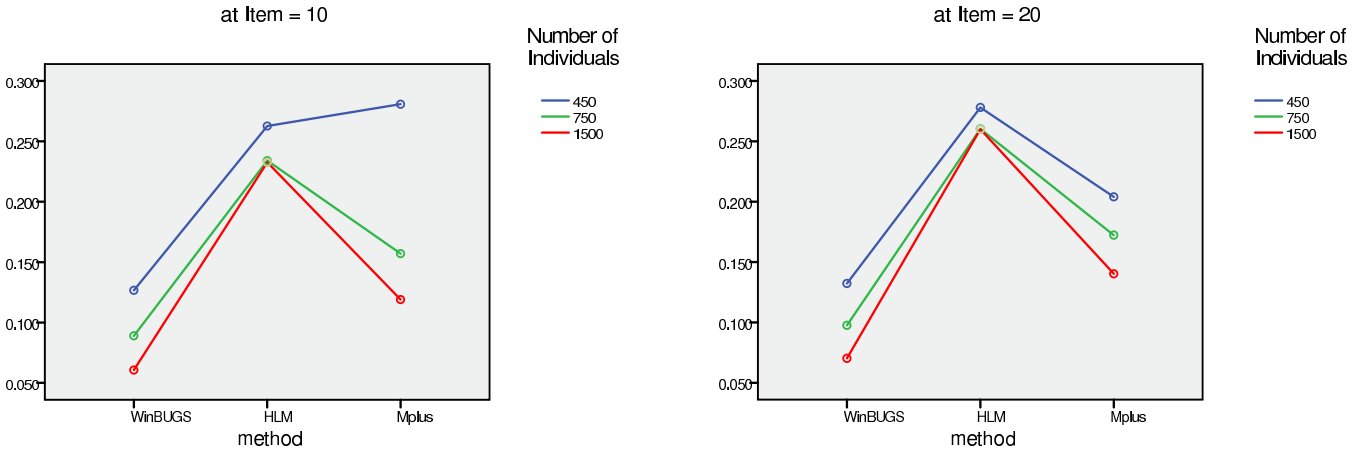
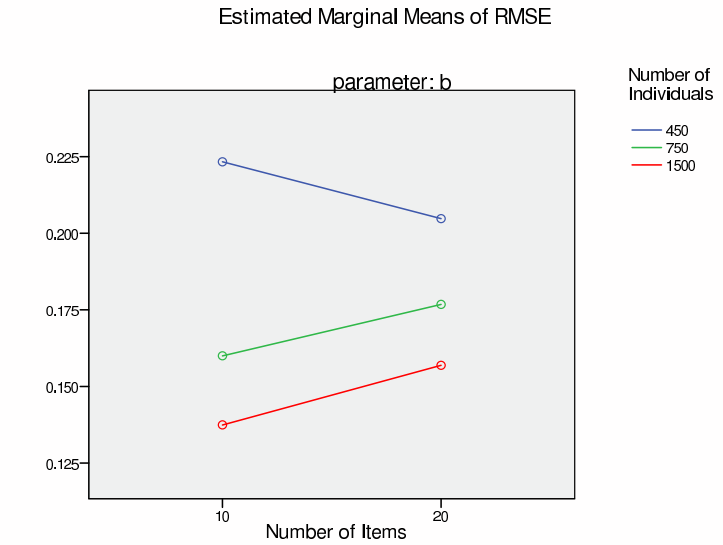


Table 4.14: Tests of Between-Subjects Effects
for the Item Parameters
from the Unconditional Model

Source	SS	df	MS	F	p	η_p^2
Item(I)	.000	1	.000	2.882	.165	.419
Individual (J)	.042	2	.021	129.437	.000	.985
Group (G)	.003	2	.001	9.189	.032	.821
I * J	.004	2	.002	12.462	.019	.862
I * G	.000	2	.000	.324	.740	.140
J * G	.001	4	.000	2.174	.235	.685
Error	.001	4	.000			

Figure 4.4: The RMSEs of Item Parameters
for the Unconditional Model



there was a significant difference between the RMSEs of β under each group size, F(2,4) = 9.189, $p = .032$, $\eta_p^2 = .821$. The mean RMSEs of β across all the conditions were .184 for G = 15, .166 for G = 30, and .180 for G = 60. Post hoc tests revealed that there was a difference only between the group size of 15 and the group size of 30 ($p = .045$).

For the individual variance estimates (i.e., τ_β), Mauchly's criterion showed that there was not a problem with the sphericity assumption ($\chi^2(2) = 5.783$, $p = .055$). Univariate tests of within-subjects effects only found estimation method to differ (See Table 4.15). Significant method effect means that mean RMSEs of τ_β changes across the three estimation methods. The mean RMSEs of τ_β were .463 for WinBUGS, .224 for HLM, and .609 for Mplus, respectively. Post hoc tests revealed that there was a difference only between WinBUGS and HLM ($p < .000$), and between HLM and Mplus ($p = .002$).

Among the between-subjects effects, the main effects of item and individual were significant (See Table 4.16). There was a significant difference between the RMSEs for τ_β

Table 4.15: Tests of Within-Subjects Effects
for the Individual Variance Parameters
from the Unconditional Model

Source	SS	df	MS	F	p	η_p^2
Method (M)	1.360	2	.680	57.115	.000	.935
M * I	.030	2	.015	1.275	.331	.242
M * J	.041	4	.010	.863	.525	.302
M * G	.026	4	.007	.555	.702	.217
M * I * J	.162	4	.040	3.399	.066	.630
M * I * G	.018	4	.004	.369	.824	.156
M * J * G	.034	8	.004	.358	.916	.263
Error	.095	8	.012			

Table 4.16: Tests of Between-Subjects Effects
for the Individual Variance Parameters
from the Unconditional Model

Source	SS	df	MS	F	p	η_p^2
Item(I)	.684	1	.684	77.886	.001	.951
Individual (J)	.170	2	.085	9.667	.029	.829
Group (G)	.011	2	.005	.624	.581	.238
I * J	.090	2	.045	5.113	.079	.719
I * G	.001	2	.000	.037	.964	.018
J * G	.012	4	.003	.347	.835	.258
Error	.035	4	.009			

under each test length, F(1,4) = 77.886, $p = .001$, $\eta_p^2 = .951$. The mean RMSEs for τ_β were .319 for I = 10 and .544 for I = 20. Also, there was a significant difference between the RMSEs of β under each individual size, F(2,4) = 9.667, $p = .029$, $\eta_p^2 = .829$. The mean RMSEs of τ_β across all models were .507 for J = 450, .372 for J = 750, and .416 for J = 1500. Post hoc tests revealed that there was a difference only between the individual sample sizes of 450 and 1500 ($p = .038$).

For the group variance (τ_γ) estimates, Mauchly's test indicated that the assumption of sphericity had been violated ($\chi^2(2) = 12.371$, $p = .002$), therefore, degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity ($\epsilon = .504$). However, there was no significant within-subjects effect (See Table 4.17).

Table 4.17: Tests of Within-Subjects Effects
for the Group Variance Parameters
from the Unconditional Model

Source	SS	df	MS	F	p	η_p^2
Method (M)	.000	1.008	.000	.603	.482	.131
M * I	.000	1.008	.000	.942	.387	.191
M * J	.001	2.016	.001	1.430	.340	.417
M * G	.001	2.016	.000	.830	.500	.293
M * I * J	.000	2.016	.000	.533	.624	.210
M * I * G	.001	2.016	.000	1.123	.410	.360
M * J * G	.003	4.033	.001	1.695	.310	.629
Error	.002	4.033	.000			

For this parameter, there was no significant between-subjects effect, either (See Table 4.18).

Next, in order to examine the effect of predictor variables at each level on the latent trait parameters, dichotomous (i.e., coded 0 or 1) variables with equal sample sizes were considered for level-2 and level-3 predictor variables.

Table 4.18: Tests of Between-Subjects Effects
for the Group Variance Parameters
from the Unconditional Model

Source	SS	df	MS	F	p	η_p^2
Item(I)	.003	1	.003	.662	.461	.142
Individual (J)	.012	2	.006	1.534	.320	.434
Group (G)	.016	2	.008	2.033	.246	.504
I * J	.004	2	.002	.515	.632	.205
I * G	.006	2	.003	.758	.526	.275
J * G	.004	4	.001	.243	.900	.196
Error	.016	4	.004			

4.2.2 The Model with Level-2 Predictor Variables

If the unconditional model indicates significant variation within and between groups, within group variation is modeled followed by between group variation. A predictor variable was included at the second level of the model in order to determine if variation was associated with the predictor variable. The values used in the simulations are in Table 3.3. γ_{01} was generated as 0.5, which implies that, on average, individuals who are coded 1 have higher ability (0.5 logits) than those who are coded 0.

For this model, sample history (trace) plots and density plots are given for selected parameters for 11,000 iterations after eliminating the first 4,000 iterations (I = 20, J = 1500, and G = 60) and the values used in the simulations are circled. The trace plots are shown in Figure 4.5, where $b[1]$ is the item difficulty for item 1, $sigma2$ is the individual variance estimate, $sigma3$ is the group variance estimate, and $alpha$ is the coefficient for Level-2 predictor variable. All parameter converged by 4,000 iterations.

The density plots (Figures 4.6) show unimodal distributions which are nearly symmetric, and look close to normal except for the plot of $sigma3$, which is the group variance estimate. It has a long right tail and a high peak close to zero.

Figure 4.5: History Plots of Parameter Estimates
for the Model with Level-2 Predictor Variable

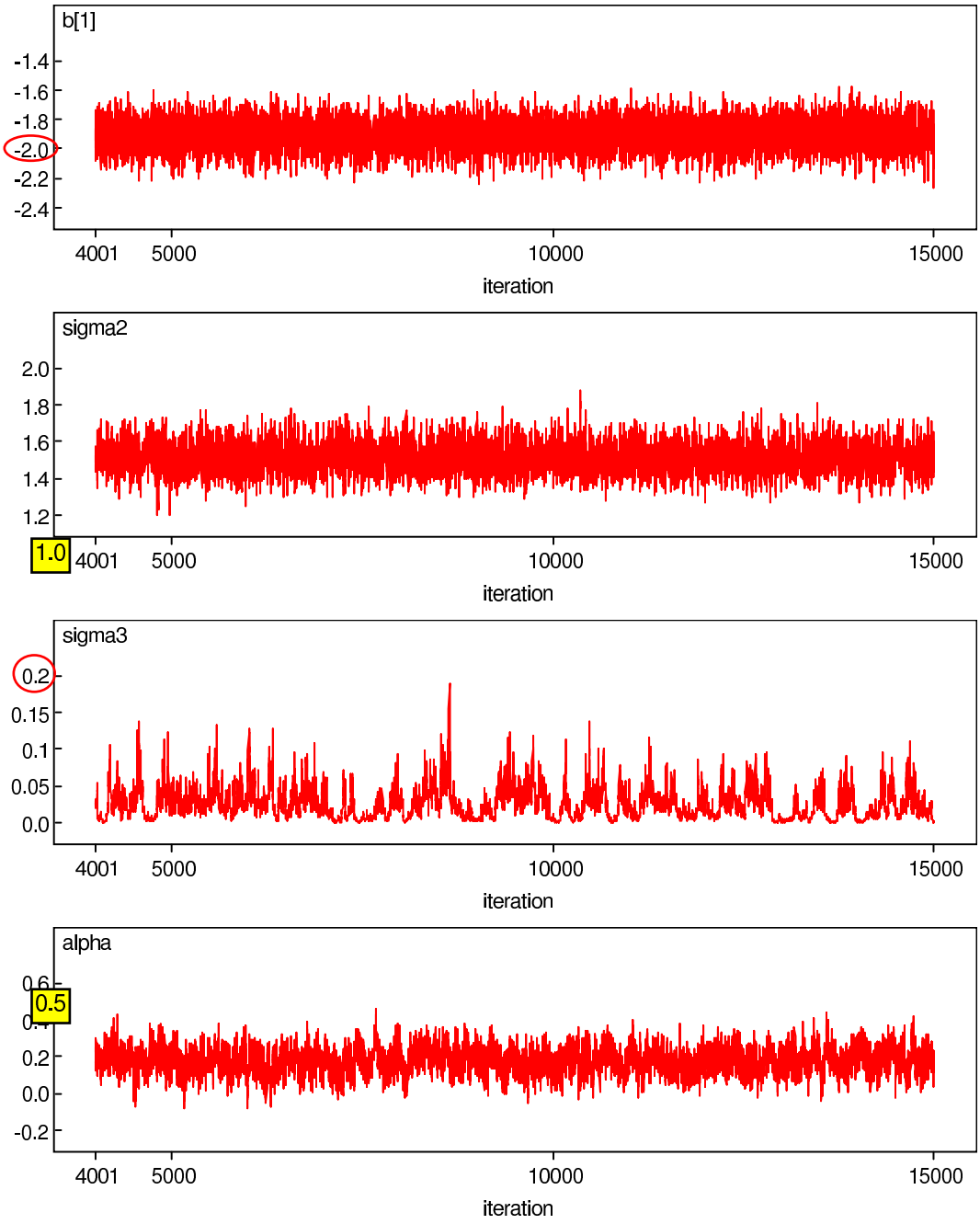
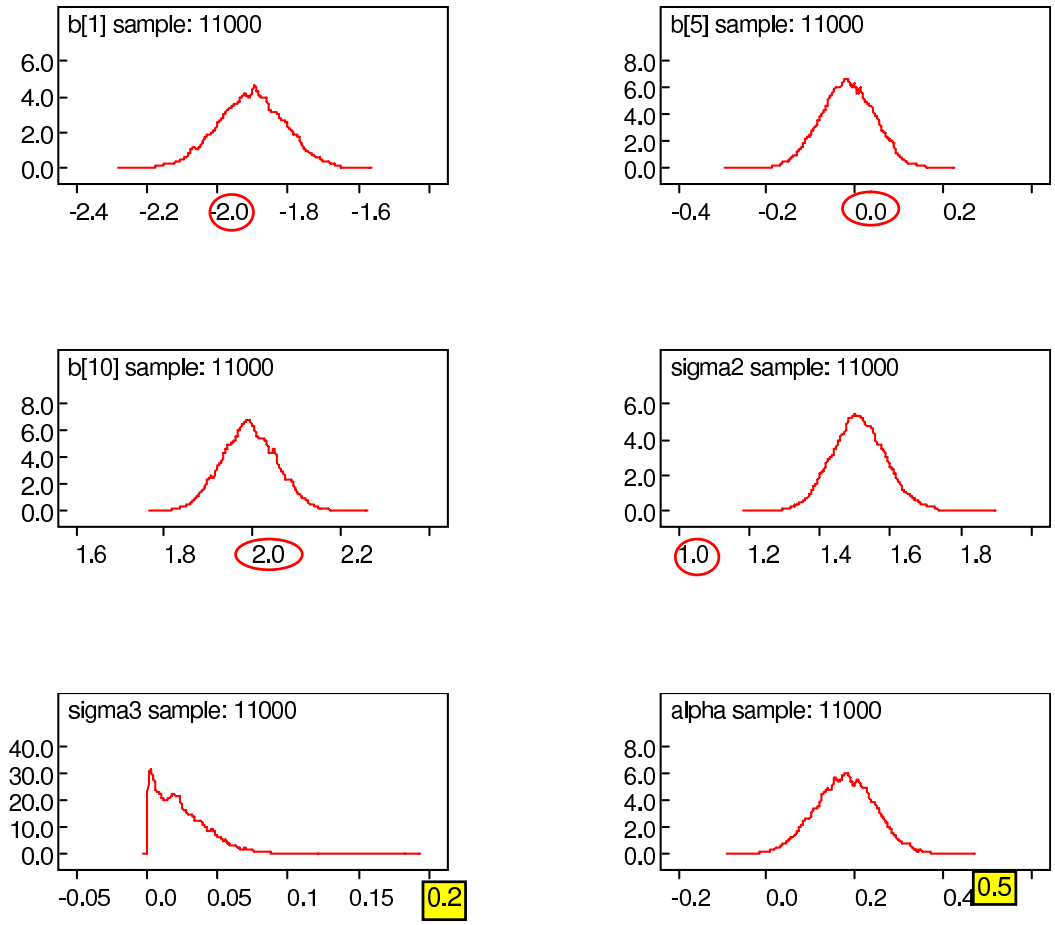


Figure 4.6: Density Plots of Parameter Estimates
for the Model with Level-2 Predictor Variable



The RMSEs under each condition for the model with a level-2 predictor variable are presented in Tables 4.19 and 4.20, where β = item difficulty (averaged over all the items), τ_β = individual variance, τ_γ = group variance, and γ_{01} = the coefficient for the Level-2 predictor variable. Individual-level characteristics have been included in the models to see the differences in the estimation of item difficulty parameters and to model the effects of individual characteristics upon the estimated latent trait measures. The use of an individual predictor variable did not change the item or group variance parameter estimates, but appeared to provide an improvement in the recovery of individual variance parameter estimates. The RMSEs of individual variance estimates (τ_β) with the individual predictor variable was slightly lower than those without a predictor variable. The mean RMSEs of τ_β were .385 for this model and .432 for the unconditional model, respectively.

In this model, the primary interest was in the coefficient estimates for the Level-2 predictor variable (γ_{01}). For this parameter estimate, Mauchly's criterion showed that sphericity was not violated ($\chi^2(2) = 5.687$, $p = .058$). Univariate tests of within-subjects effects showed that the significant effect was only method (See Table 4.21). Significant method effect means that mean RMSEs of γ_{01} changes across the three estimation methods. The mean RMSEs of γ_{01} were .316 for WinBUGS, .355 for HLM, and .371 for Mplus, respectively. Post hoc tests revealed that there was a difference only between WinBUGS and HLM ($p = .005$).

There was no significant between-subjects effect for γ_{01} (See Table 4.22).

For the other parameters (i.e., item difficulty, individual variance, and group variance estimates), the results were consistent with those from the unconditional model.

4.2.3 The Model with Level-2 and Level-3 Predictor Variables

If after the individual level predictor variables are added in the model, the group random effect is still significant, it is necessary to build a model with Level-3 predictors. The values

Table 4.19: RMSEs for the Model with the Level-2 Predictor Variable from Maximum Likelihood and Bayesian Estimation

			Item = 20		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
1500	60	β	0.069	0.262	0.100
		τ_β	0.638	0.400	0.805
		τ_γ	0.185	0.187	0.181
		γ_{01}	0.352	0.386	0.408
	30	β	0.065	0.255	0.118
		τ_β	0.553	0.333	0.675
		τ_γ	0.193	0.198	0.196
		γ_{01}	0.287	0.331	0.425
	15	β	0.074	0.258	0.123
		τ_β	0.520	0.306	0.338
		τ_γ	0.193	0.197	0.197
		γ_{01}	0.411	0.434	0.436
750	60	β	0.102	0.256	0.143
		τ_β	0.551	0.313	0.656
		τ_γ	0.157	0.151	0.135
		γ_{01}	0.402	0.436	0.440
	30	β	0.112	0.245	0.124
		τ_β	0.478	0.288	0.456
		τ_γ	0.178	0.179	0.176
		γ_{01}	0.510	0.517	0.511
	15	β	0.104	0.220	0.135
		τ_β	0.493	0.286	0.537
		τ_γ	0.194	0.199	0.200
		γ_{01}	0.227	0.280	0.313
450	60	β	0.148	0.279	0.303
		τ_β	0.567	0.310	1.129
		τ_γ	0.083	0.070	0.099
		γ_{01}	0.066	0.126	0.214
	30	β	0.140	0.284	0.165
		τ_β	0.575	0.342	0.482
		τ_γ	0.169	0.174	0.168
		γ_{01}	0.170	0.249	0.267
	15	β	0.140	0.282	0.197
		τ_β	0.466	0.253	0.216
		τ_γ	0.173	0.180	0.178
		γ_{01}	0.534	0.532	0.534

Table 4.20: RMSEs for the Model with the Level-2 Predictor Variable from Maximum Likelihood and Bayesian Estimation

			Item = 10		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
1500	60	β	0.060	0.220	0.142
		τ_β	0.354	0.068	0.668
		τ_γ	0.169	0.173	0.165
		γ_{01}	0.237	0.290	0.317
	30	β	0.055	0.228	0.159
		τ_β	0.208	0.079	0.097
		τ_γ	0.176	0.179	0.179
		γ_{01}	0.334	0.369	0.345
	15	β	0.070	0.233	0.126
		τ_β	0.274	0.062	0.084
		τ_γ	0.182	0.188	0.187
		γ_{01}	0.266	0.310	0.297
750	60	β	0.072	0.244	0.164
		τ_β	0.334	0.137	0.212
		τ_γ	0.160	0.154	0.157
		γ_{01}	0.530	0.559	0.538
	30	β	0.104	0.241	0.164
		τ_β	0.357	0.057	0.488
		τ_γ	0.187	0.193	0.191
		γ_{01}	0.363	0.390	0.398
	15	β	0.087	0.231	0.133
		τ_β	0.305	0.070	0.493
		τ_γ	0.180	0.187	0.181
		γ_{01}	0.327	0.359	0.372
450	60	β	0.132	0.263	0.339
		τ_β	0.347	0.142	0.649
		τ_γ	0.034	0.028	0.134
		γ_{01}	0.338	0.370	0.350
	30	β	0.122	0.250	0.335
		τ_β	0.378	0.057	1.149
		τ_γ	0.164	0.168	0.147
		γ_{01}	0.102	0.179	0.217
	15	β	0.107	0.236	0.189
		τ_β	0.321	0.054	0.398
		τ_γ	0.157	0.165	0.159
		γ_{01}	0.240	0.286	0.288

Table 4.21: Tests of Within-Subjects Effects
for the Coefficient of the Level-2 Predictor
from the Model with the Level-2 Predictor Variable

Source	SS	df	MS	F	p	η_p^2
Method (M)	.028	2	.014	12.568	.003	.759
M * I	.002	2	.001	.864	.457	.178
M * J	.002	4	.000	.411	.796	.170
M * G	.001	4	.000	.254	.900	.113
M * I * J	.000	4	.000	.090	.983	.043
M * I * G	.001	4	.000	.228	.915	.102
M * J * G	.006	8	.001	.624	.740	.384
Error	.009	8	.001			

Table 4.22: Tests of Between-Subjects Effects
for the Coefficient of the Level-2 Predictor
from the Model with the Level-2 Predictor Variable

Source	SS	df	MS	F	p	η_p^2
Item(I)	.013	1	.013	.320	.602	.074
Individual (J)	.161	2	.081	2.037	.245	.505
Group (G)	.007	2	.004	.093	.914	.044
I * J	.023	2	.011	.289	.763	.126
I * G	.083	2	.042	1.051	.430	.345
J * G	.231	4	.058	1.459	.362	.593
Error	.158	4	.040			

used in the simulations are in Table 3.3. π_{001} was generated as 0.1, which implies that, on average, groups that are coded 1 have higher ability (0.1 logits) than those that are coded 0.

For this model, sample history (trace) plots and density plots are given for selected parameters for 11,000 iterations after eliminating the first 4,000 iterations (I = 10, J = 450, and G = 15) and the values used in the simulations are circled. The trace plots are shown in Figure 4.7, where *sigma*2 is the individual variance estimate, *sigma*3 is the group variance estimate, *alpha* is the coefficient for the Level-2 predictor variable, and *beta* is the coefficient for the Level-3 predictor variable. Each parameter of interest becomes stationary by 4,000 iterations, indicating that convergence has been reached by 4,000 iterations. The density plots (Figures 4.8) show unimodal distributions which are nearly symmetric, and look close to normal except for the plot of *sigma*3, which is the group variance estimate. It has a long right tail and a high peak close to zero.

The RMSEs under each condition for the model with level-2 and level-3 predictor variables are presented in Tables 4.23 to 4.26, where β = Item Difficulty (averaged over all the items), τ_β = Individual Variance, τ_γ = Group Variance, γ_{01} = the coefficient for the level-2 predictor variable, and π_{001} = the coefficient for the level-3 predictor variable.

The RMSEs of the individual variance parameter estimates (τ_β) across all the conditions from this model were larger in comparison to those from the unconditional model or the model with a Level-2 predictor variable. The mean RMSEs of τ_β across all conditions were .431 for the unconditional model, and .385 for the model with a Level-2 predictor variable, and .441 for the model with a Level-2 and a Level-3 predictor variables. It suggests that the use of a group predictor variable does not appear to provide an improvement in the recovery of individual variance parameters.

Also, the inclusion of a group-level (Level 3) predictor variable in the model leads to a loss of efficiency for the individual level (Level 2) predictor variable for small individual sample sizes (J = 450). This means that choosing a group-level predictor variable is costly in terms of the RMSEs. The mean RMSEs of the coefficient of individual level predictor

Figure 4.7: History Plots of Parameter Estimates
for the Model with Level-2 and Level-3 Predictor Variables

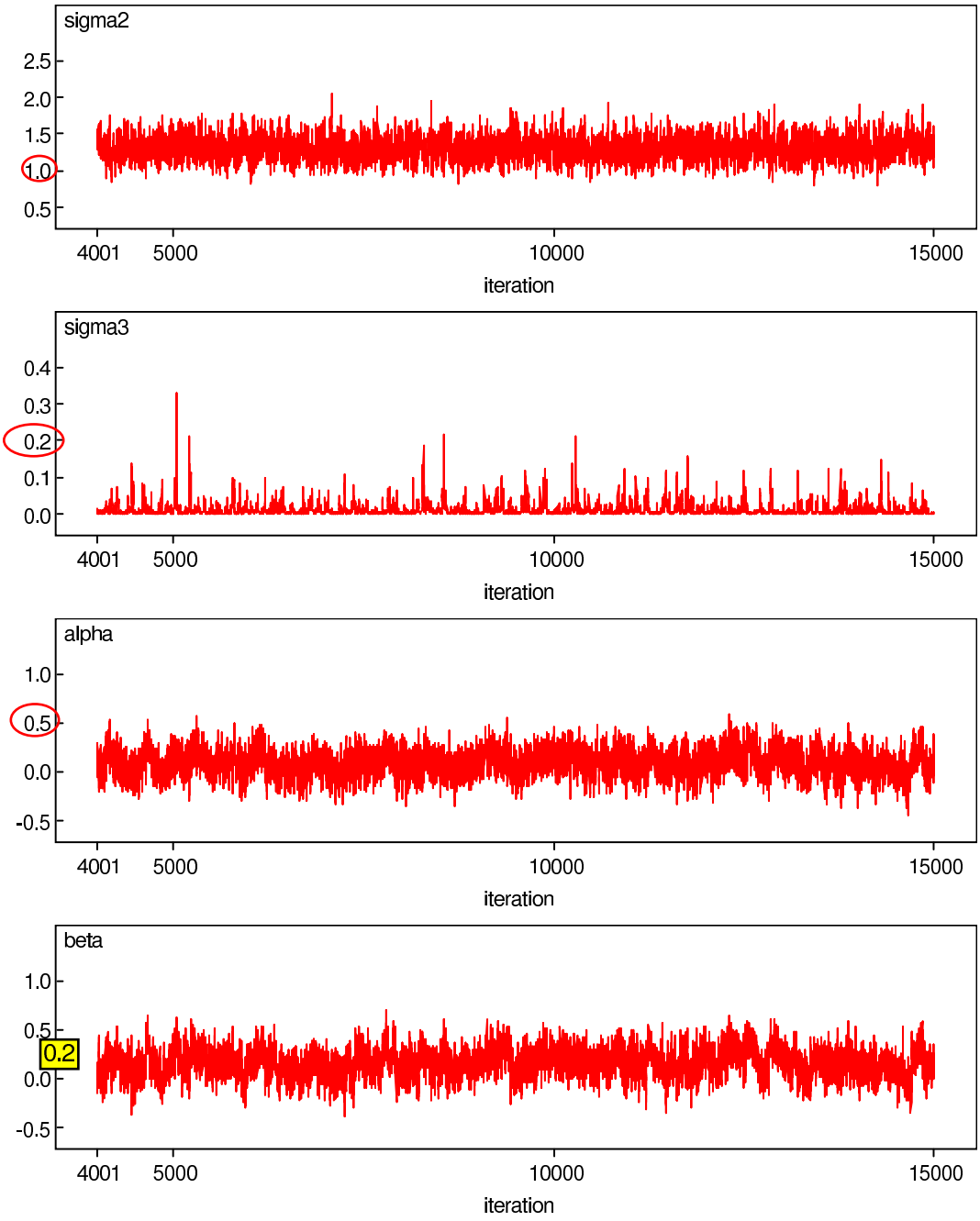


Figure 4.8: Density Plots of Parameter Estimates
for the Model with Level-2 and Level-3 Predictor Variables

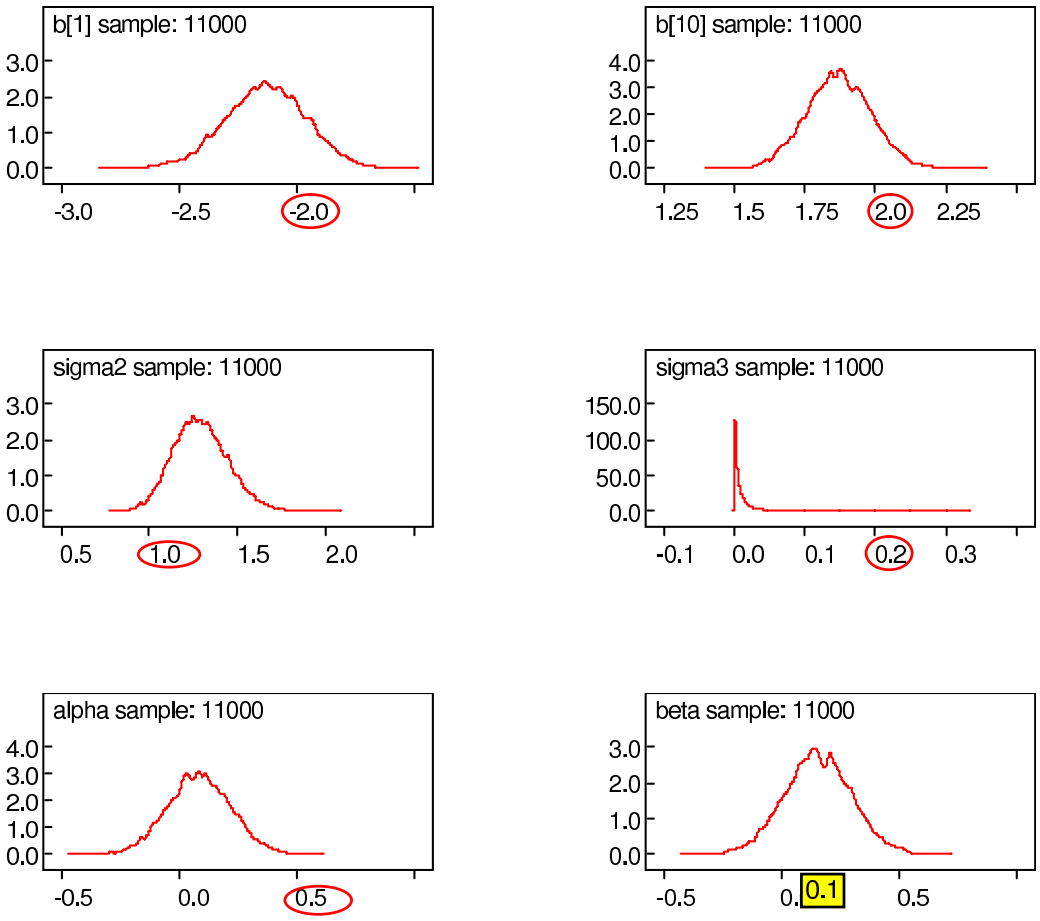


Table 4.23: RMSEs for the Model with the Level-2 Predictor Variable and the Level-3 Predictor Variable from Maximum Likelihood and Bayesian Estimation

			Item = 20		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
1500	60	β	0.077	0.269	0.098
		τ_β	0.646	0.402	0.726
		τ_γ	0.184	0.184	0.179
		γ_{01}	0.353	0.395	0.414
		π_{001}	0.156	0.148	0.135
	30	β	0.059	0.249	0.138
		τ_β	0.485	0.275	0.665
		τ_γ	0.191	0.195	0.194
		γ_{01}	0.296	0.336	0.364
		π_{001}	0.135	0.132	0.125
	15	β	0.067	0.252	0.143
		τ_β	0.625	0.389	0.894
		τ_γ	0.191	0.197	0.196
		γ_{01}	0.366	0.400	0.422
		π_{001}	0.053	0.049	0.058
750	60	β	0.116	0.245	0.202
		τ_β	0.581	0.338	0.565
		τ_γ	0.162	0.160	0.154
		γ_{01}	0.447	0.462	0.463
		π_{001}	0.096	0.056	0.057
	30	β	0.096	0.267	0.129
		τ_β	0.539	0.311	0.576
		τ_γ	0.167	0.171	0.164
		γ_{01}	0.492	0.501	0.493
		π_{001}	0.217	0.151	0.123
	15	β	0.100	0.248	0.154
		τ_β	0.509	0.296	0.938
		τ_γ	0.193	0.199	0.199
		γ_{01}	0.218	0.279	0.317
		π_{001}	0.057	0.029	0.023

Table 4.24: RMSEs for the Model with the Level-2 Predictor Variable and the Level-3 Predictor Variable from Maximum Likelihood and Bayesian Estimation - Continued

			Item = 20		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
450	60	β	0.130	0.279	0.244
		τ_β	0.472	0.238	0.936
		τ_γ	0.089	0.084	0.081
		γ_{01}	0.073	0.132	0.191
		π_{001}	0.251	0.217	0.199
	30	β	0.108	0.268	0.203
		τ_β	0.499	0.285	0.854
		τ_γ	0.180	0.190	0.181
		γ_{01}	0.153	0.229	0.278
		π_{001}	0.312	0.281	0.237
	15	β	0.114	0.290	0.386
		τ_β	0.662	0.410	1.762
		τ_γ	0.170	0.184	0.178
		γ_{01}	0.524	0.519	0.514
		π_{001}	0.193	0.121	0.045

Table 4.25: RMSEs for the Model with the Level-2 Predictor Variable and the Level-3 Predictor Variable from Maximum Likelihood and Bayesian Estimation

			Item = 10		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
1500	60	β	0.062	0.224	0.104
		τ_β	0.284	0.093	0.378
		τ_γ	0.188	0.192	0.187
		γ_{01}	0.204	0.260	0.263
		π_{001}	0.036	0.045	0.044
	30	β	0.055	0.233	0.111
		τ_β	0.281	0.034	0.355
		τ_γ	0.184	0.189	0.185
		γ_{01}	0.335	0.368	0.372
		π_{001}	0.069	0.070	0.070
	15	β	0.082	0.221	0.107
		τ_β	0.263	0.035	0.162
		τ_γ	0.192	0.198	0.198
		γ_{01}	0.251	0.298	0.294
		π_{001}	0.156	0.144	0.145
750	60	β	0.110	0.225	0.119
		τ_β	0.382	0.096	0.374
		τ_γ	0.175	0.178	0.172
		γ_{01}	0.473	0.485	0.490
		π_{001}	0.104	0.103	0.102
	30	β	0.087	0.234	0.140
		τ_β	0.421	0.156	0.427
		τ_γ	0.190	0.197	0.197
		γ_{01}	0.316	0.359	0.368
		π_{001}	0.118	0.117	0.115
	15	β	0.099	0.233	0.209
		τ_β	0.276	0.127	0.305
		τ_γ	0.183	0.192	0.190
		γ_{01}	0.320	0.358	0.339
		π_{001}	0.104	0.109	0.115

Table 4.26: RMSEs for the Model with the Level-2 Predictor Variable and the Level-3 Predictor Variable from Maximum Likelihood and Bayesian Estimation - Continued

			Item = 10		
Sample Size	Group Size	Parameter Estimate	Bayesian WinBUGS	Maximum HLM	Likelihood Mplus
450	60	β	0.141	0.275	0.201
		τ_β	0.471	0.116	0.769
		τ_γ	0.123	0.109	0.138
		γ_{01}	0.287	0.338	0.353
		π_{001}	0.064	0.076	0.076
	30	β	0.148	0.265	0.283
		τ_β	0.409	0.093	0.905
		τ_γ	0.117	0.130	0.091
		γ_{01}	0.148	0.220	0.277
		π_{001}	0.097	0.094	0.090
	15	β	0.127	0.255	0.224
		τ_β	0.333	0.069	0.332
		τ_γ	0.167	0.199	0.199
		γ_{01}	0.355	0.375	0.369
		π_{001}	0.086	0.079	0.083

variable (γ_{01}) was .281 for the model with a Level-2 predictor variable. The mean RMSEs of γ_{01} was .296 for the model with a Level-2 and a Level-3 predictor variables.

The parameter of most interest for this model was in the coefficient estimate for the Level-3 predictor variable, denoted as π_{001}. For this parameter estimate, Mauchly's criterion showed sphericity was violated ($\chi^2(2) = 6.621$, $p = .036$). Univariate tests of within-subjects effects (using Greenhouse-Geisser correction) showed that the significant effects were method and method × test length interaction (See Table 4.27). Significant

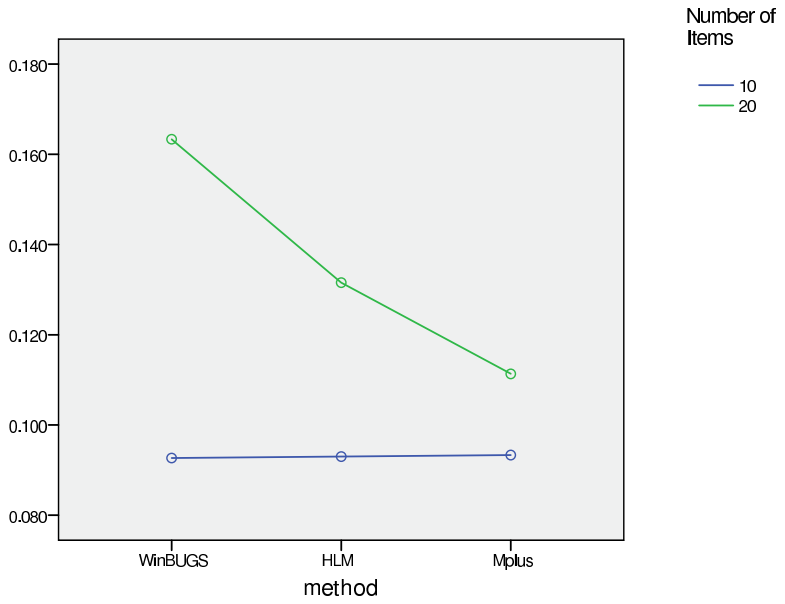
Table 4.27: Tests of Within-Subjects Effects
for the Coefficient of the Level-3 Predictor
from the Model with the Level-2 and Level-3 Predictor Variables

Source	SS	df	MS	F	p	η_p^2
Method (M)	.006	1.058	.006	14.401	.017	.783
M * I	.006	1.058	.006	15.146	.015	.791
M * J	.003	2.116	.001	3.214	.141	.616
M * G	.000	2.116	.000	0.566	.614	.220
M * I * J	.003	2.116	.001	3.438	.129	.632
M * I * G	.000	2.116	.000	0.200	.836	.091
M * J * G	.002	4.233	.000	1.171	.439	.539
Error	.002	4.233	.000			

method effect means that mean RMSEs of π_{001} changes across the three estimation methods. The mean RMSEs of π_{001} were .128 for WinBUGS, .112 for HLM, and .102 for Mplus, respectively. Post hoc tests revealed that there was a difference only between WinBUGS and HLM ($p = .026$). Significant interaction effect means that the influence of number of items on RMSEs of π_{001} depends on the estimation method. As you can see from the Figure 4.9, there was no difference among the three estimation methods for a test length of 10. The mean RMSEs of π_{001} were .093 for all three programs. However, the means were different for a test length of 20. The mean RMSEs of π_{001} were .163 for WinBUGS, .132 for HLM, and .111 for Mplus, respectively.

Among the between-subjects effects, the main effect of individual sample size and the interaction effects of test length × individual sample size and test length × group sample

Figure 4.9: The RMSEs of the Coefficient of Level-3 Predictor for the Model with Level-2 and Level-3 Predictor Variables



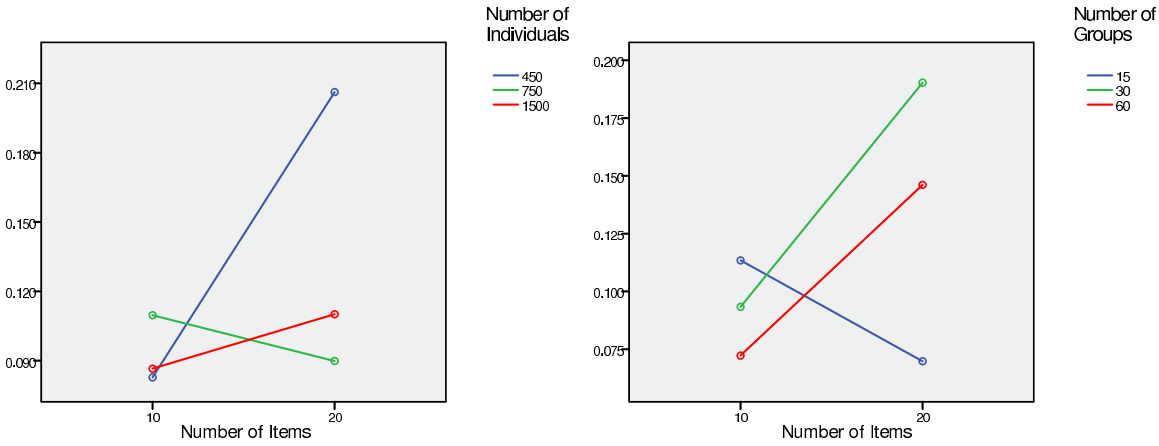
size were significant (See Table 4.28). There was a significant interaction effect between test length and individual sample size, F(2,4) = 9.100, $p = .032$, $\eta_p^2 = .820$. As can be seen from Figure 4.10, there was a positive relationship between the RMSEs of π_{001} and test length for the small (J = 450) and large (J = 1500) individual sample sizes, but not for the medium (J = 750) individual sample size. There was a significant interaction effect between test length and group sample size, F(2,4) = 9.598, $p = .030$, $\eta_p^2 = .828$. Although the RMSEs for π_{001} diminished as test length decreased for the conditions of group sample sizes are 30 and 60, the results indicated that there was a tendency for the RMSEs of π_{001} to decrease as number of items increased for group sample size of 15 (See Figure 4.10).

For the other parameter estimates (i.e., item difficulty, individual variance, group variance, and the coefficient of level-2 predictor), the results were consistent with those from the previous models.

Table 4.28: Tests of Between-Subjects Effects
for the Coefficient of the Level-3 Predictor
from the Model with the Level-2 and Level-3 Predictor Variables

Source	SS	df	MS	F	p	η_p^2
Item(I)	.024	1	.024	9.100	.039	.695
Individual (J)	.025	2	.012	4.648	.090	.699
Group (G)	.023	2	.012	4.383	.098	.687
I * J	.049	2	.024	9.100	.032	.820
I * G	.051	2	.026	9.598	.030	.828
J * G	.013	4	.003	1.241	.420	.554
Error	.011	4	.003			

Figure 4.10: The RMSEs of the Coefficient of Level-3 Predictor
for the Model with Level-2 and Level-3 Predictor Variables



4.2.4 All Models

This study compared item parameters (β), variances of latent ability at different levels (τ_β and τ_γ), and the relationship between predictor variables at different levels and latent ability (γ_{01} and π_{001}, respectively) from WinBUGS, HLM, and Mplus programs. As Kamata (1998) has suggested, inclusion of predictor variables in the multilevel IRT model did not change the item effect estimates from those in the unconditional model. As can be seen in Table 4.29, there was little difference between the item parameter estimates obtained from the unconditional model and those obtained from the model with a level-2 predictor variables or the model with a level-2 and a level-3 predictor variables using Mplus. This holds for the other two programs.

Table 4.29: Item Parameter Estimates for the Unconditional Model, the Model with Level-2 Predictor Variable, and the Model with Level-2 and Level-3 Predictor Variables from Mplus program (I = 10, J = 450, G = 15)

Item	The Unconditional Model	SE	The Model with Level-2 Variable	SE	The Model with Level-2 and Level-3 Variables	SE
1	-2.162	0.223	-2.162	0.223	-2.162	0.223
2	-1.389	0.173	-1.389	0.173	-1.388	0.173
3	-0.913	0.152	-0.913	0.152	-0.912	0.152
4	-0.653	0.144	-0.653	0.144	-0.653	0.144
5	0.163	0.126	0.163	0.126	0.163	0.126
6	0.105	0.127	0.106	0.127	0.106	0.127
7	0.518	0.122	0.518	0.122	0.518	0.122
8	1.132	0.121	1.132	0.121	1.132	0.121
9	1.368	0.123	1.368	0.123	1.368	0.123
10	1.832	0.134	1.833	0.134	1.836	0.135

For the common parameters of interest across all models used in this study (i.e., β, τ_β, and τ_γ), a repeated measures ANOVA with the three estimation method as the repeated factor and item, individual, and group size as the independent variables was performed using the RMSEs across all models used in this study as the dependent variable. Mauchly's

Table 4.30: Tests of Within-Subjects Effects
for the Item Parameters from All Models

Source	SS	df	MS	F	p	η_p^2
Method (M)	.643	1.147	.561	493.829	.000	.925
M * I	.007	1.147	.006	5.359	.021	.118
M * J	.053	2.294	.023	20.166	.000	.502
M * G	.001	2.294	.001	.468	.655	.023
M * I * J	.002	2.294	.001	.889	.431	.043
M * I * G	.006	2.294	.003	2.218	.114	.100
M * J * G	.003	4.588	.001	.540	.730	.051
Error	.052	45.878	.001			

criterion showed that sphericity was violated for item parameter estimates (β) from all conditions in all models. Univariate tests of within-subjects effects (using Greenhouse-Geisser correction) showed that the significant effects were method, method $\times$ test length interaction, and method $\times$ individual sample size interaction (See Table 4.30). Significant method effect means that mean RMSEs of β changes across the three estimation methods. Significant interaction effect means that the influence of test length or individual sample sizes on RMSEs of β depends on the estimation method (See Figure 4.11). As you can see from the figure, WinBUGS has the lowest RMSEs under all the conditions for this parameter.

Among the between-subjects effects, only the main effect of individual sample size was significant (See Table 4.31). That is, decreasing sample size, particularly at the individual level (Level 2), increased the RMSEs of β, $F(2,40) = 110.302$, $p < .000$, $\eta_p^2 = .847$. The mean RMSEs of β across all models were .217 for J = 450, .165 for J = 750, and .145 for J =1500. Post hoc tests revealed that all three means were significantly different at the $\alpha = .05$ level.

Figure 4.11: The RMSEs for the Item Parameters from All Models

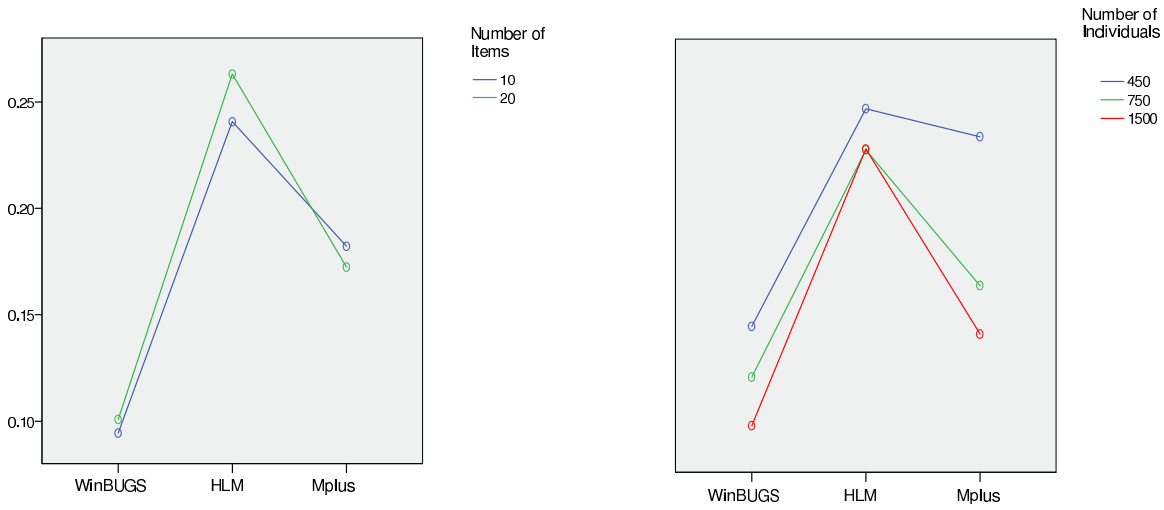


Table 4.31: Tests of Between-Subjects Effects for the Item Parameters from All Models

Source	SS	df	MS	F	p	η_p^2
Item(I)	.002	1	.002	2.412	.128	.057
Individual (J)	.149	2	.074	110.302	.000	.847
Group (G)	.002	2	.001	1.349	.271	.063
I * J	.002	2	.001	1.580	.218	.073
I * G	.003	2	.002	2.335	.110	.105
J * G	.002	4	.001	.919	.462	.084
Error	.027	40	.001			

Table 4.32: Tests of Within-Subjects Effects
for the Individual Variance Parameters from All Models

Source	SS	df	MS	F	p	η_p^2
Method (M)	4.174	1.035	4.031	107.648	.000	.729
M * I	.002	1.035	.002	.058	.819	.001
M * J	.443	2.071	.214	5.719	.006	.222
M * G	.032	2.071	.015	.409	.674	.020
M * I * J	.082	2.071	.040	1.063	.357	.050
M * I * G	.154	2.071	.074	1.984	.149	.090
M * J * G	.124	4.142	.030	.800	.536	.074
Error	1.551	41.418	.037			

Mauchly's criterion showed that sphericity assumption was violated for bias for the individual variance parameter estimates (τ_β) from all conditions in all models. Univariate tests of within-subjects effects (using Greenhouse-Geisser correction) showed that the only significant effects wer method and method $\times$ individual sample size (See Table 4.32). Significant method effect means that mean RMSEs of τ_β changes across the three estimation methods. Significant interaction effect means that the influence of individual sample sizes on RMSEs of τ_β depends on the estimation method (See Figure 4.12). As you can see from the figure, HLM has the lowest RMSEs under all the conditions for this parameter.

Among the between-subjects effects, the main effects of item and individual were significant (See Table 4.33). That is, the RMSEs of τ_β for test length of 10 were smaller than those for test length of 20, $F(1,40) = 81.530$, $p < .000$, $\eta_p^2 = .671$. The mean RMSEs of τ_β across all models were .301 for I = 10 and .537 for I = 20. Also, there was a significant difference among the RMSEs at each individual sample size, $F(2,40) = 7.760$, $p = .001$, $\eta_p^2 = .280$. The mean RMSEs of τ_β across all models were .492 for J = 450, .378 for J = 750, and .388 for J = 1500. Post hoc tests revealed that there were significant

Figure 4.12: The RMSEs for the Individual Variance Parameters from All Models

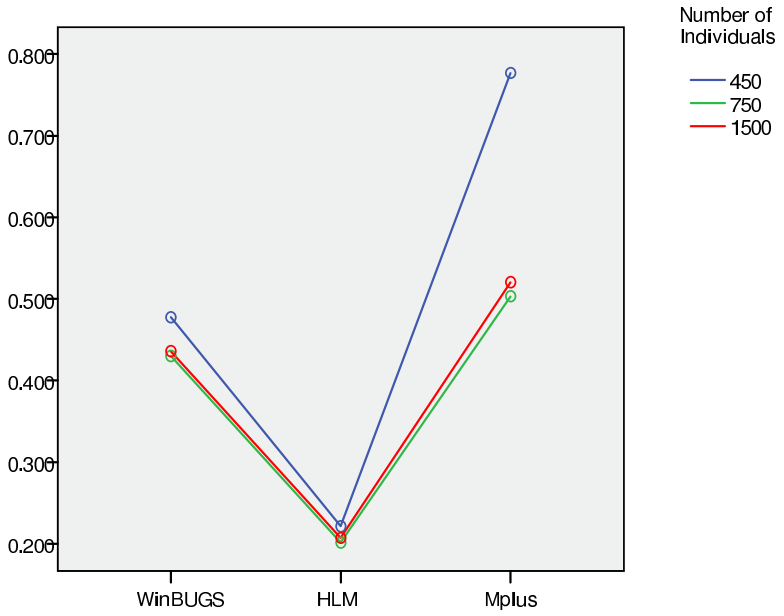


Table 4.33: Tests of Between-Subjects Effects of the Individual Variance Parameters from All Models

Source	SS	df	MS	F	p	η_p^2
Item (I)	2.259	1	2.259	81.530	.000	.671
Individual (J)	.430	2	.215	7.760	.001	.280
Group (G)	.080	2	.040	1.444	.248	.067
I * J	.150	2	.075	2.704	.079	.119
I * G	.105	2	.053	1.897	.163	.087
J * G	.058	4	.015	.524	.718	.050
Error	1.109	40	.028			

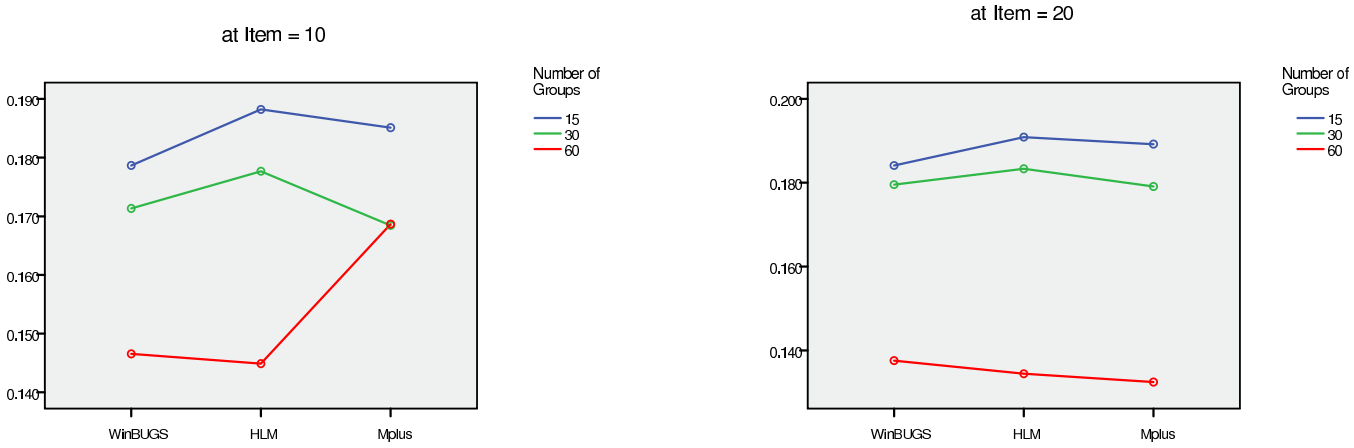
Table 4.34: Tests of Within-Subjects Effects
for the Group Variance Parameters from All Models

Source	SS	df	MS	F	p	η_p^2
Method (M)	.001	1.122	.000	3.249	.074	.075
M * I	.001	1.122	.000	3.201	.076	.074
M * J	.001	2.244	.001	4.309	.016	.177
M * G	.002	2.244	.001	4.928	.009	.198
M * I * J	.000	2.244	.000	1.453	.244	.068
M * I * G	.002	2.244	.001	4.749	.011	.192
M * J * G	.006	4.488	.001	8.253	.000	.452
Error	.007	44.878	.000			

differences between the individual sample sizes of 450 and 750 ($p = .003$) as well as between the individual sample sizes of 450 and 1500 ($p = .007$).

Mauchly's criterion showed that sphericity for bias for the group variance parameter estimates (τ_γ) for all conditions by all models. Univariate tests of within-subjects effects (using Greenhouse-Geisser correction) showed that the significant effects were method $\times$ individual sample size interaction, method $\times$ group sample size interaction, method $\times$ test length $\times$ group sample size interaction, and method $\times$ individual sample size $\times$ group sample size interaction (See Table 4.34). Significant two-way interaction effects mean that the influence of individual or group sample sizes on τ_γ depended on the estimation method (i.e., $M \times J$ and $M \times G$). A significant three-way interaction effect means that the influence of test length or individual sample sizes on τ_γ depended on group sample size and the estimation method ($M \times I \times G$ and $M \times J \times G$). As can be seen from Figure 4.13, the differences among three estimation methods are negligible under each group sample size when number of items are 20. However, Mplus performed poorly compared to the other programs under number of items are 10 and group sample sizes are 60. Figure 4.14 presents the RMSEs of τ_γ from each estimation method under each individual and group sample

Figure 4.13: The RMSEs for the Group Variance Parameters
under Each Test Length from All Models



size. The differences among the three estimation methods were found between Mplus and the other two programs under the group sample sizes are 60 under each sample size.

Among the between-subjects effects, significant main effects were found for individual and group sample size as well as for individual × group sample size interaction (See Table 4.35). A significant interaction effect means that the influence of number of individuals on RMSEs of τ_γ depends on number of groups (See Figure 4.15). The biases of the group variance estimates were inversely related to amount of information in the data that they are based on. As you can see from the figure, this bias became smaller as the number of groups increased under each individual sample size.

This implies that the robustness of the group-level variance estimates (τ_γ) seemed to depend largely on the group sample size, particularly when sample size was small at the individual level (J = 450). The mean RMSEs of τ_γ were .096 for G = 60, .160 for G = 30, and .174 for G = 15, respectively under J = 450. These results are consistent with the previous findings indicated by Kasim and Raudenbush (1998), which showed that group-level variance components are more biased when sample size was small. These

Figure 4.14: The RMSEs for the Group Variance Parameters
under Each Individual Sample Size
from All Models

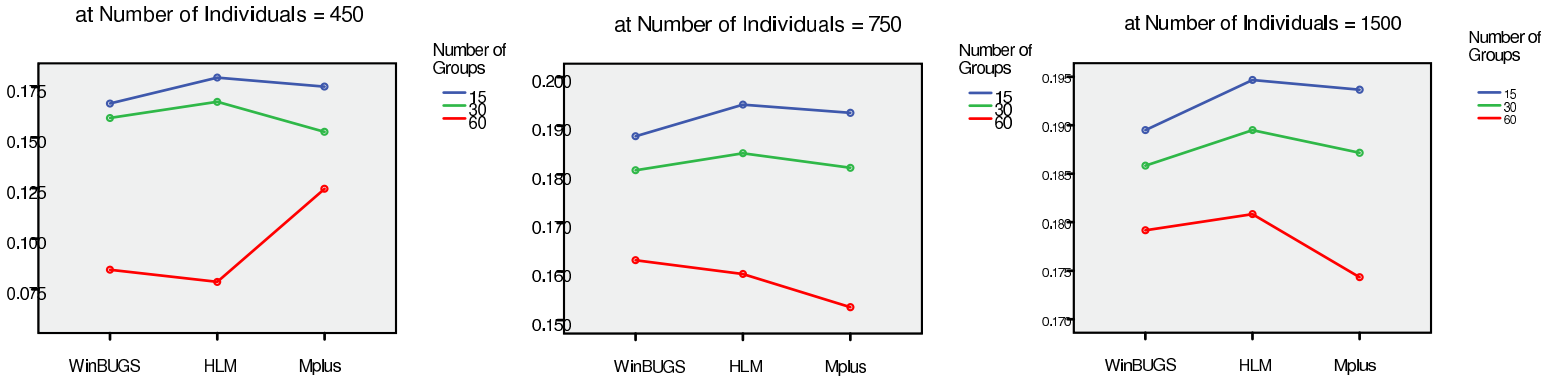
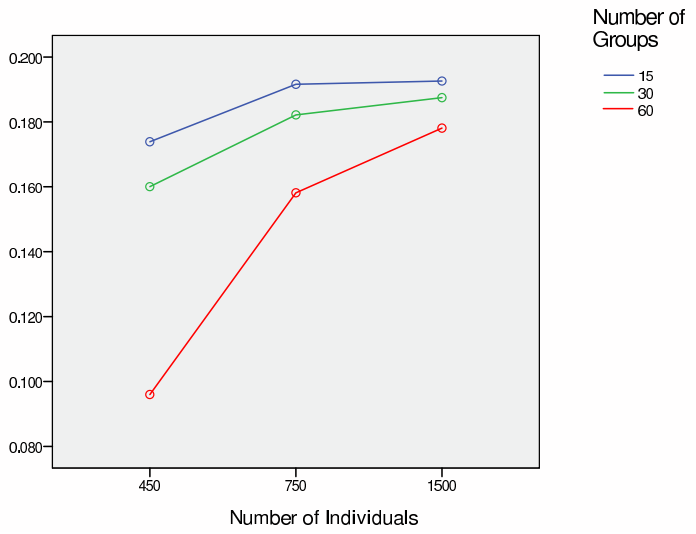


Table 4.35: Tests of Between-Subjects Effects
for the Group Variance Parameters from All Models

Source	SS	df	MS	F	p	η_p^2
Item (I)	.000	1	.000	.164	.688	.004
Individual (J)	.055	2	.028	25.314	.000	.559
Group (G)	.052	2	.026	24.015	.000	.546
I * J	.001	2	.000	.427	.656	.021
I * G	.006	2	.003	2.572	.089	.114
J * G	.023	4	.006	5.179	.002	.341
Error	.044	40	.001			

Figure 4.15: The RMSEs for the Group Variance Parameters from All Models



findings to some extent agreed with what Kim (1990) and Mok (1995) reported, which is that the random-effect estimates are affected more by the number of groups (sample size at the third level) than by the number of observations per group (sample size at the second level).

However, the findings of this study did not agree with Shieh's (1999) conclusion that the bias for the estimates of the variance at the first level was affected by total sample size, and at the second level is affected by second-level sample size. Even though there were more individuals than groups, the individual variance estimates (τ_β) were more biased, which means that they had larger RMSEs. The mean RMSEs of τ_β was .419 and .169 for τ_γ across all models. This held for all three estimation approaches. The mean RMSEs were .448 (τ_β) and .166 (τ_γ) from WinBUGS; .210 (τ_β) and .170 (τ_γ) from HLM; .660 (τ_β) and .171 (τ_β) from Mplus across all models.

Chapter 5

Discussion

This chapter presents the general summary and interpretation of the results, the limitations of the study, and suggestions for future research.

5.1 Summary

This study compared the performance of parameter estimates of maximum likelihood (ML) estimation, which is the method most widely used in current applied multilevel IRT analyses and Bayesian estimation, which has become a viable alternative to ML-based estimation techniques in the context of Kamata's three-level IRT model. The unconditional model, the model with a level-2 predictor variable, and the model with a level-2 and a level-3 predictor variable were used to assess how varying sample sizes at each level affects on parameter estimates of interest using simulated data as well as actual data.

Bayesian estimation using WinBUGS was found to perform better than ML estimations in all conditions for item parameter estimates. For the individual (Level 2) variance estimates, PQL estimation using HLM showed less bias than the other two methods. However, Bayesian and ML estimations performed similarly for the other parameters of interest, the group variance (Level 3) estimates and the coefficient estimates of level-2 and level-3 predictor variables.

Overall, the findings of the present study were consistent with those of previous studies with regard to the effect of sample size on bias of parameter estimates. As expected, there was an inverse relationship between magnitude of RMSEs of parameter estimates and the sample size. However, RMSEs increased as test length increased. The use of a predictor

variable did not change the item parameter estimates, but appeared to provide an improvement in the recovery of variance parameters. However, the inclusion of a group-level (Level 3) predictor variable in the model leads to a loss of efficiency for the individual level (Level 2) predictor variable.

This study also presented a summary of the implementation of the computer programs HLM (Raudenbush et al., 2005), Mplus (Muthén & Muthén, 2006), and WinBUGS (Spiegelhalter et al., 2003) for the analyses of multilevel IRT model. Studies such as this one will provide useful information for researchers seeking to use ML as well as Bayesian estimation for estimation of multilevel IRT model parameters.

Even though HLM has some disadvantages such as inflexibility of creating data files and convergence problems, it is a fast estimation method. In applying these models, it was also found that the choice of reference item in the multilevel IRT model does not have an impact on the results. The estimates of the fixed effects for individual-level and group-level predictor variables, the estimates of the random effects, and the rank order and distance between item parameters are not affected by which item is chosen as the reference item.

Mplus showed good performance on item parameter and group variance parameter estimates even with small sample sizes. To get factor variances, one factor loading was fixed to one. It was not examined whether the choice of item had an impact on the results. Currently, Mplus cannot handle cross-level interactions.

WinBUGS appears to be a time-consuming method, but has the advantage of simple code changes needed for each model used in this study and the results are straightforward.

One important aspect of this study should be noted here. Uninformative priors (i.e., $\gamma_{01} \sim N(0, 1.0E-6)$) were used to obtain parameter estimates in Bayesian estimation. When the information used by the estimating algorithm comes solely from the data, or when priors are uninformative, then a Bayesian estimate would be comparable to that obtained from a non-Bayesian algorithm such as a maximum likelihood algorithm.

Another important ML-Bayesian comparison should be noted is computational speed, where ML approaches have a distinct advantage. For example, for a test length of 20 items, individual sample size od 1500, and group sample size of 15, HLM or Mplus took just a couple of minutes, but WinBUGS took about 12 hours using a PC with 3GB RAM and a clock speed of 3.6GHz. However, steady improvements in recent years in both hardware speed and efficiency of Monte Carlo algorithms make MCMC-based Bayesian fitting of multilevel models an attractive approach, even with rather large data sets.

5.2 Limitations and Suggestions

There are several limitations in this study. As with any simulation study, this one includes only a small subset of all possible interesting conditions. Issues of sample sizes at each level, which plays an important role in item parameter and ability estimation, were mainly addressed. In addition, the range of conditions was limited due to computational time with Bayesian estimation using WinBUGS.

The present study was limited to balanced (that is, equal sample sizes) for group-level data. An extension to unbalanced data would be useful. Raudenbush and Bryk (2002) reported that ML bias with unbalanced data can be smaller than that with balanced data.

The models used were restricted to those modeling binary data. Moreover, individual- and group-level predictor variables were limited in the simulated data set. Dichotomous equal size Level-2 predictor variable and Level-3 predictor variables were used in order to look at their effect on estimates of ability. There are other factors which also might be examined for their influence on these estimates such as unequal numbers of individuals in a group.

Also, the generalization of the results was limited by the design of the study. This study employed only the unconditional model, the model with one level-2 predictor variable, and the model with one level-2 and one level-3 predictor variable. Neither the interaction between level-2 predictor variables nor the interaction between level-3 predictor variables

was considered. The cross-level interaction (i.e., the effect of the group-level predictors on the relationship between the individual-level predictor variable and individual ability) was not considered in the study, either. Future research studying interaction effects of predictor variables would be useful.

The true values of item and ability parameters used were selected from previous studies and the data used. Clearly, these values could have affected the results. Other values for the parameters are possible and could be adequate for other educational or non-educational settings and might lead to different conclusions. Therefore, more comprehensive simulation studies that resemble real data should be conducted to predict the pattern of effects associated with data structure, or assumption violations. Multilevel IRT models require satisfying the assumptions such as (1) a single dimension underlying the item responses and (2) the latent trait is a random parameter, is normally distributed within and between groups, and the variance of which is the same across groups (Pastor, 2003). It was suggested that this assumption violations should be taken into consideration, particularly when using samples with small sizes (Darandari, 2004).

Finally, the results of this study provide important information for the application of multilevel IRT models. In particular, the strengths and weaknesses of three different algorithms were compared, providing useful information as to their respective utility for this type of situation.

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Appendix A

Mplus Code

A.1 The Unconditional Model

```
TITLE: TIMMS DATA

DATA: FILE IS K_TIMSS.DAT;
      FORMAT IS F3.0 17F1.0;

VARIABLE: NAMES ARE SCH Q1-Q17;
          CLUSTER = SCH;
          USEVARIABLES ARE Q1-Q17;
          CATEGORICAL ARE Q1-Q17;

ANALYSIS: TYPE = TWOLEVEL GENERAL;
          ESTIMATOR = ML;

MODEL:
%WITHIN%
THETA BY Q1-Q16*(1);
THETA BY Q17@1;
%BETWEEN%
THETAB BY Q1-Q16*(1);
THETAB BY Q17@1;

OUTPUT: STANDARDIZED TECH1 TECH8;
```

A.2 The Model with Level-2 Predictor Variable

```
TITLE: TIMMS DATA

DATA: FILE IS K_TIMSS_P2.DAT;
      FORMAT IS F3.0 18F1.0;

VARIABLE: NAMES ARE SCH Q1-Q17 SCISTUD;
          WITHIN = SCISTUD;
          CLUSTER = SCH;
          USEVARIABLES ARE Q1-Q17 SCISTUD;
          CATEGORICAL ARE Q1-Q17 SCISTUD;

ANALYSIS: TYPE = TWOLEVEL GENERAL;
          ESTIMATOR = ML;

MODEL:
%WITHIN%
THETA BY Q1-Q16*(1);
THETA BY Q17@1;
THETA BY SCISTUD;
%BETWEEN%
THETAB BY Q1-Q16*(1);
THETAB BY Q17@1;

OUTPUT: STANDARDIZED TECH1 TECH8;
```

A.3 The Model with Level-2 and Level-3 Predictor Variables

```
TITLE: TIMMS DATA

DATA: FILE IS K_TIMSS_P3.DAT;
      FORMAT IS F3.0 18F1.0 F2.0;

VARIABLE: NAMES ARE SCH Q1-Q17 SCISTUD EXP;
          WITHIN = SCISTUD EXP;
          CLUSTER = SCH;
          USEVARIABLES ARE Q1-Q17 SCISTUD EXP;
          CATEGORICAL ARE Q1-Q17 SCISTUD;

ANALYSIS: TYPE = TWOLEVEL GENERAL;
          ESTIMATOR = ML;

MODEL:
%WITHIN%
THETA BY Q1-Q16*(1);
THETA BY Q17@1;
THETA BY SCISTUD;
THETA ON EXP;
%BETWEEN%
THETAB BY Q1-Q16*(1);
THETAB BY Q17@1;

OUTPUT: STANDARDIZED TECH1 TECH8;
```

Appendix B

WinBUGS Code

B.1 The Unconditional Model

```
# The Unconditional Model for the TIMSS Data
# CONDITION: I=17, J=1130, G=68
# I : # of items
# J : # of examinees
# G : # of groups


model
{
for (j in 1:J) {
    for (i in 1:I) {
    r[j,i]<-resp[j,i]
}}


#  1PL model
for (j in 1:J) {
    for (i in 1:I) {
     logit(p[j,i]) <-  u2[j] + u3[group[j]] - b[i]
     r[j,i] ~ dbern(p[j,i])
}}


# Higher level definition
for (j in 1:J) {
    u2[j] ~ dnorm(mu, tau.u2)
}

for (g in 1:G){
    u3[g] ~ dnorm(0, tau.u3)
}
```

```
#  Priors
mu ~ dnorm(0,1)
tau.u2 ~ dgamma(0.1, 0.001)
sigma2 <- 1/tau.u2  # Variance of ability for level 2


tau.u3 ~ dgamma(0.1, 0.001)
sigma3 <- 1/tau.u3  # Variance of ability for level 3


# added for identification
for (i in 1:I) {
    bb[i]~dnorm(mub,sigb)
    b[i] <- bb[i] - mean(bb[1:I])
 }

mub ~ dnorm(0,1)
sigb ~ dchisqr(.5)
}


# First Initial Values
list(b=c(-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2,-2) ,
tau.u2=1, tau.u3=5)


# Second Initial Values
list(b=c(2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2 ) , tau.u2=2, tau.u3=10)


list(I=17, J=1130, G=68, group=c( 1,
1,
...
68,
68 ) ,
resp=structure(.Data=c(0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,
0,1,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,1,0,
...
1,1,1,0,0,1,1,1,1,1,1,0,0,1,1,1,1,0,1,
0,1,1,1,1,0,0,0,0,1,0,0,1,0,0,0,0 ), .Dim=c(1130,17 )))
```

B.2 The Model with Level-2 Predictor Variable

```
# The Model with Level-2 Predictor Variable for the TIMSS Data
# CONDITION: I=17, J=1130, G=68
# I : # of items
# J : # of examinees
# G : # of groups


model
{
for (j in 1:J) {
    for (i in 1:I) {
    r[j,i]<-resp[j,i]
}}


#  1PL model
for (j in 1:J) {
    for (i in 1:I) {
     logit(p[j,i]) <-  u2[j] + u3[group[j]] - b[i]
     r[j,i]~dbern(p[j,i])
}}


# Higher level definition
for (j in 1:J) {
    u2[j] ~ dnorm(mu1[j], tau.u2)
}

for (j in 1:J) {
    mu1[j] <- alpha*scistud[j]
}

for (g in 1:G){
    u3[g] ~ dnorm(mu2, tau.u3)
}


#  Priors
alpha ~ dnorm(0,1.0E-6)

mu2 ~ dnorm(0,1)
```

```
tau.u2 ~ dgamma(0.1, 0.001)
sigma2 <- 1/tau.u2  # Variance of ability for level 2

tau.u3 ~ dgamma(0.1, 0.001)
sigma3 <- 1/tau.u3  # Variance of ability for level 3


# added for identification
for (i in 1:I) {
    bb[i]~dnorm(mub,sigb)
    b[i] <- bb[i] - mean(bb[1:I])
}

mub ~ dnorm(0,1)
sigb ~ dchisqr(.5)
}


list(tau.u2=1, tau.u3=5, alpha=0.393)


list(I=17, J=1130, G=68, group=c( 1,
1,
...
68,
68 ) ,
scistud=c(1,
1,
...
1,
1 ) ,
resp=structure(.Data=c(0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,
0,1,0,0,0,0,0,1,1,0,1,0,0,0,0,1,0,
...
1,1,1,0,0,1,1,1,1,1,0,0,1,1,1,0,1,
0,1,1,1,1,0,0,0,0,1,0,0,1,0,0,0,0 ), .Dim=c(1130,17 )))
```

B.3 The Model with Level-2 and Level-3 Predictor Variables

```
# The Model with Level-2 and Level-3 Predictor Variables
# CONDITION: I=17, J=1130, G=68
# I : # of items
# J : # of examinees
# G : # of groups


model
{
for (j in 1:J) {
    for (i in 1:I) {
    r[j,i]<-resp[j,i]
}}


#  1PL model
for (j in 1:J) {
    for (i in 1:I) {
     logit(p[j,i]) <-  u2[j] + u3[group[j]] - b[i]
     r[j,i]~dbern(p[j,i])
}}


# Higher level definition
for (j in 1:J) {
    u2[j] ~ dnorm(mu1[j], tau.u2)
}

for (j in 1:J) {
    mu1[j] <- alpha*scistud[j] + beta*exp[j]
}

for (g in 1:G){
    u3[g] ~ dnorm(mu2, tau.u3)
}


#  Priors
alpha ~ dnorm(0,1.0E-6)
beta ~ dnorm(0,1.0E-6)

tau.u2 ~ dgamma(0.1, 0.001)
```

```
sigma2 <- 1/tau.u2  # Variance of ability for level 2

tau.u3 ~ dgamma(0.1, 0.001)
sigma3 <- 1/tau.u3  # Variance of ability for level 3


# added for identification
for (i in 1:I) {
    bb[i]~dnorm(mub,sigb)
    b[i] <- bb[i] - mean(bb[1:I])
}

mub ~ dnorm(0,1)
sigb ~ dchisqr(.5)
}


list(tau.u2=1, tau.u3=5, alpha=0.393, beta=0)


list(I=17, J=1130, G=68, group=c( 1,
1,
...
68,
68 ) ,
scistud=c(1,
1,
0,
...
1,
1 ) ,
exp=c(85,
85,
85,
...
31,
31 ),
resp=structure(.Data=c(0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,
0,1,0,0,0,0,0,1,1,0,1,0,0,0,0,1,0,
...
1,1,1,0,0,1,1,1,1,1,0,0,1,1,1,0,1,
0,1,1,1,1,0,0,0,0,1,0,0,1,0,0,0,0 ), .Dim=c(1130,17 )))
```

Appendix C

Convergence Diagnostics

C.1 Geweke

```
GEWEKE CONVERGENCE DIAGNOSTIC:
==============================
Fraction in first window = 0.1
Fraction in last window = 0.5

Chain: b
--------

numeric matrix: 2 rows, 17 columns.
             b[10]       b[11]         b[12]      b[13]      b[14]      b[15]
Z-Score -0.8858132 3.163559312 7.416149e+000 -1.5441484 -0.7823772 -0.9010875
p-value  0.3757182 0.001558526 1.205702e-013  0.1225524  0.4339929  0.3675418

             b[16]      b[17]       b[1]       b[2]       b[3]        b[4]
Z-Score 0.08318674 -0.2368495 -1.2664404 -1.7952147 -1.4092248 -1.78364199
p-value 0.93370305  0.8127735  0.2053554  0.0726195  0.1587687  0.07448186

               b[5]       b[6]      b[7]        b[8]       b[9]
Z-Score -2.13921383 -1.4652862 -1.106207 -1.82312174 0.09608914
p-value  0.03241835  0.1428429  0.268637  0.06828496 0.92344978
```

C.2 Raftery and Lewis

```
RAFTERY AND LEWIS CONVERGENCE DIAGNOSTIC:
=========================================
Quantile = 0.025
Accuracy = +/- 0.005
Probability = 0.95

Chain: b
--------

numeric matrix: 17 rows, 5 columns.
      Thin Burn-in Total Lower Bound Dependence Factor
b[10]    1       2  3757        3746          1.002936
b[11]    1       2  4008        3746          1.069941
b[12]    1       2  3847        3746          1.026962
b[13]    1       2  3792        3746          1.012280
b[14]    1       2  3745        3746          0.999733
      Thin Burn-in Total Lower Bound Dependence Factor
b[15]    1       2  3805        3746          1.015750
b[16]    1       2  3914        3746          1.044848
b[17]    1       3  4052        3746          1.081687
 b[1]    1       2  3929        3746          1.048852
 b[2]    1       2  3895        3746          1.039776
     Thin Burn-in Total Lower Bound Dependence Factor
b[3]    1       1  3726        3746          0.994661
b[4]    1       2  3788        3746          1.011212
b[5]    1       2  3882        3746          1.036305
b[6]    1       2  3851        3746          1.028030
b[7]    1       2  3923        3746          1.047250
     Thin Burn-in Total Lower Bound Dependence Factor
b[8]    1       2  3819        3746          1.019487
b[9]    1       2  3890        3746          1.038441
```

Appendix D

Estimated Parameter Means From Five Replications

D.1 The Unconditional Model

For the Condition I = 10, J = 750, G = 30

Parameter	True Value	WinBUGS Mean	SD	HLM Estimate	SE	Mplus Estimate	SE
$b[1]$	-2.0	-1.986	0.130	-1.677	0.092	-1.897	0.151
$b[2]$	-1.5	-1.426	0.110	-1.180	0.114	-1.357	0.128
$b[3]$	-1.0	-1.120	0.101	-0.915	0.099	-1.062	0.119
$b[4]$	-0.5	-0.488	0.089	-0.381	0.109	-0.469	0.105
$b[5]$	0.0	0.020	0.083	0.034	0.097	0.013	0.098
$b[6]$	0.0	-0.012	0.083	0.010	0.104	-0.011	0.098
$b[7]$	0.5	0.552	0.080	0.464	0.102	0.518	0.094
$b[8]$	1.0	1.033	0.081	0.848	0.099	0.975	0.093
$b[9]$	1.5	1.415	0.083	1.154	0.105	1.334	0.095
$b[10]$	2.0	2.019	0.089	1.644	0.104	1.956	0.111
τ_β	1.0	1.263	0.118	0.986	0.067	1.141	0.324
τ_γ	0.2	0.010	0.012	0.006	0.015	0.004	0.005

where item parameter estimate $b[i]$ for item $i = 1, \ldots, I$,

the individual variance estimate τ_β, and

the group variance estimate τ_γ

D.2 The Model with Level-2 Predictor Variable

For the Condition I = 20, J = 1500, G = 60

	True	WinBUGS		HLM		Mplus	
Parameter	Value	Mean	SD	Estimate	SE	Estimate	SE
$b[1]$	-2.0	-1.924	0.097	-1.570	0.098	-2.031	0.109
$b[2]$	-1.5	-1.527	0.086	-1.223	0.111	-1.608	0.097
$b[3]$	-1.0	-1.014	0.075	-0.789	0.107	-1.066	0.086
$b[4]$	-0.5	-0.510	0.068	-0.378	0.101	-0.534	0.079
$b[5]$	0.0	-0.010	0.062	0.018	0.100	-0.008	0.074
$b[6]$	0.0	0.036	0.062	0.053	0.102	0.041	0.074
$b[7]$	0.5	0.506	0.060	0.416	0.096	0.536	0.071
$b[8]$	1.0	0.986	0.059	0.781	0.098	1.040	0.070
$b[9]$	1.5	1.484	0.060	1.161	0.097	1.564	0.071
$b[10]$	2.0	2.018	0.063	1.569	0.098	2.125	0.073
$b[11]$	-2.0	-2.054	0.102	-1.684	0.119	-2.167	0.113
$b[12]$	-1.5	-1.518	0.086	-1.215	0.115	-1.599	0.097
$b[13]$	-1.0	-0.994	0.074	-0.773	0.098	-1.043	0.086
$b[14]$	-0.5	-0.431	0.066	-0.314	0.102	-0.451	0.078
$b[15]$	0.0	0.004	0.062	0.027	0.101	0.006	0.074
$b[16]$	0.0	-0.021	0.063	0.009	0.094	-0.019	0.074
$b[17]$	0.5	0.514	0.060	0.423	0.095	0.545	0.071
$b[18]$	1.0	0.968	0.059	0.769	0.097	1.020	0.070
$b[19]$	1.5	1.541	0.060	1.203	0.102	1.625	0.071
$b[20]$	2.0	1.948	0.062	1.516	0.099	2.026	0.079
τ_β	1.0	1.610	0.082	1.371	0.064	1.769	0.266
τ_γ	0.2	0.015	0.014	0.014	0.015	0.020	0.020
γ_{01}	0.5	0.149	0.095	0.115	0.052	0.092	0.044

where item parameter estimate $b[i]$ for item $i = 1, \ldots, I$,

the individual variance estimate τ_β,

the group variance estimate τ_γ, and

the coefficient estimate for level-2 predictor variable γ_{01}

D.3 The Model with Level-2 and Level-3 Predictor Variables

For the Condition I = 10, J = 450, G = 15

Parameter	True Value	WinBUGS Mean	SD	HLM Estimate	SE	Mplus Estimate	SE
$b[1]$	-2.0	-1.980	0.168	-1.676	0.149	-1.845	0.199
$b[2]$	-1.5	-1.480	0.145	-1.295	0.185	-1.434	0.175
$b[3]$	-1.0	-0.894	0.125	-0.728	0.162	-0.818	0.148
$b[4]$	-0.5	-0.457	0.115	-0.333	0.146	-0.402	0.135
$b[5]$	0.0	-0.045	0.109	0.029	0.150	-0.001	0.127
$b[6]$	0.0	-0.048	0.108	-0.023	0.153	-0.066	0.128
$b[7]$	0.5	0.438	0.105	0.416	0.143	0.443	0.122
$b[8]$	1.0	0.978	0.104	0.805	0.161	0.898	0.120
$b[9]$	1.5	1.553	0.109	1.241	0.163	1.400	0.123
$b[10]$	2.0	1.934	0.114	1.563	0.154	1.826	0.136
τ_β	1.0	1.319	0.157	1.017	0.110	1.123	0.411
τ_γ	0.2	0.047	0.039	0.001	0.020	0.001	0.009
γ_{01}	0.5	0.149	0.133	0.129	0.106	0.140	0.118
π_{001}	0.1	0.093	0.167	0.041	0.099	0.041	0.116

where item parameter estimate $b[i]$ for item $i = 1, \ldots, I$,

the individual variance estimate τ_β,

the group variance estimate τ_γ,

the coefficient estimate for level-2 predictor variable γ_{01}, and

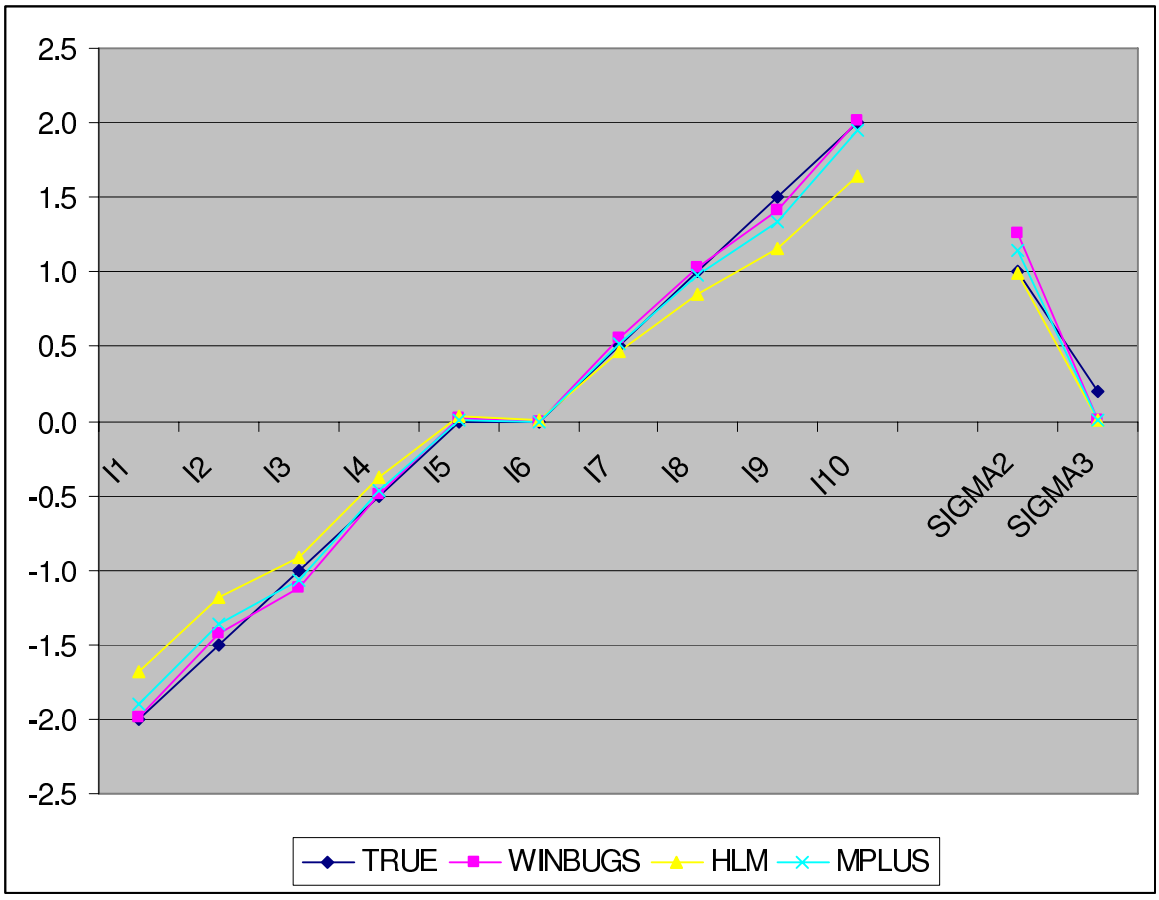
the coefficient estimate for level-3 predictor variable π_{001}

Appendix E

Line Graphs Of Estimated Parameter Means From Five Replications

E.1 The Unconditional Model

For the Condition I = 10, J = 750, G = 30



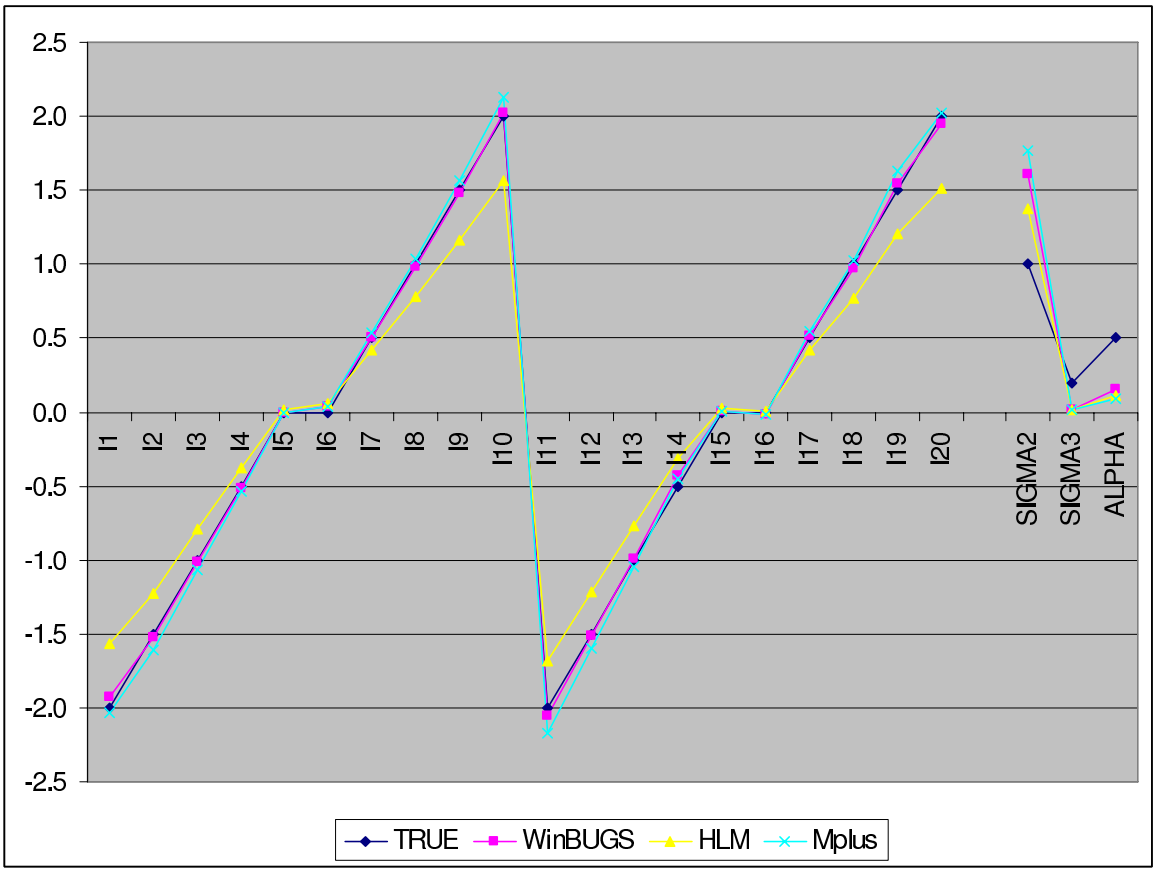
where $I[i]$ is the item parameter estimate for item $i = 1, \ldots, I$,

$SIGMA2$ is the individual variance estimate, and

$SIGMA3$ is the group variance estimate

E.2 The Model with Level-2 Predictor Variable

For the Condition I = 20, J = 1500, G = 60



where $I[i]$ is the item parameter estimate for item $i = 1, \ldots, I$,

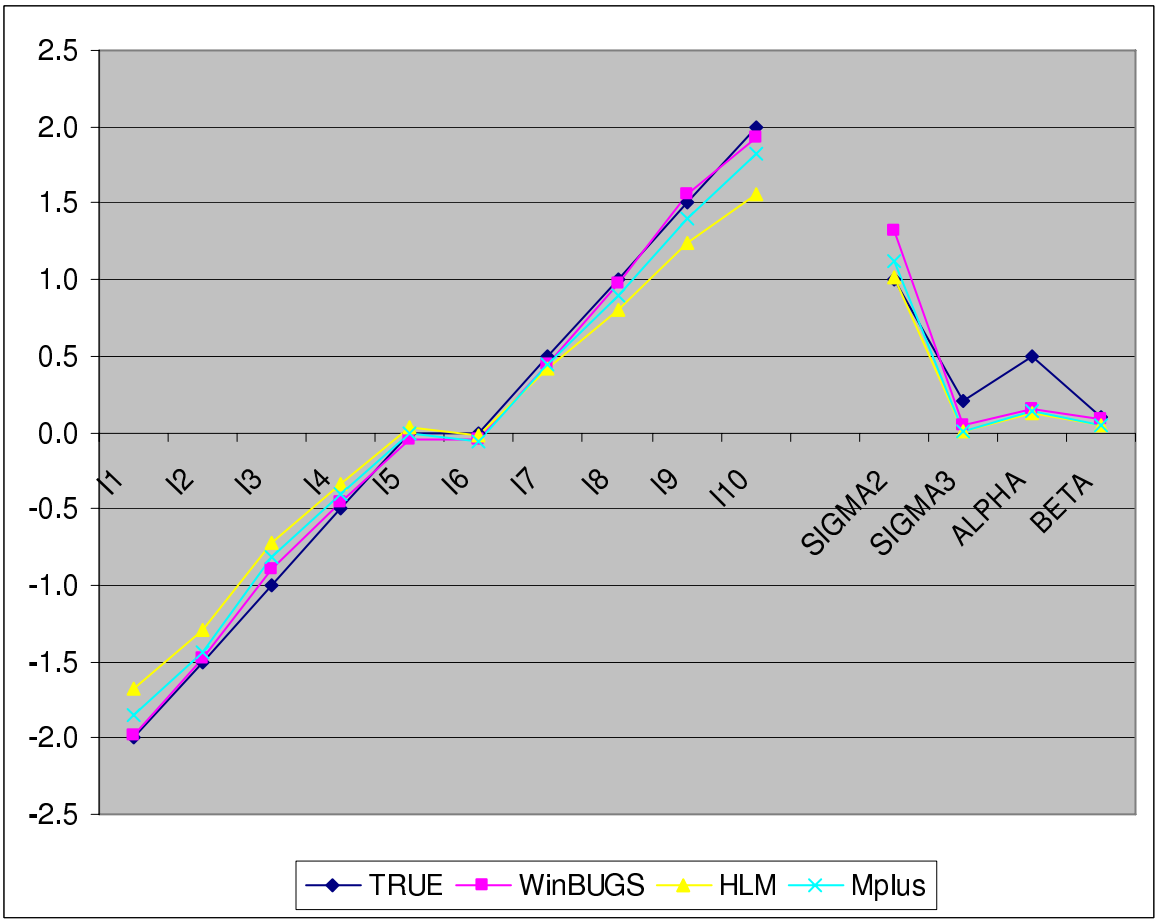
$SIGMA2$ is the individual variance estimate,

$SIGMA3$ is the group variance estimate, and

$ALPHA$ is the coefficient estimate for the Level-2 predictor variable

E.3 The Model with Level-2 and Level-3 Predictor Variables

For the Condition I = 10, J = 450, G = 15



where $I[i]$ is the item parameter estimate for item $i = 1, \ldots, I$,

$SIGMA2$ is the individual variance estimate,

$SIGMA3$ is the group variance estimate,

$ALPHA$ is the coefficient estimate for the Level-2 predictor variable, and

$BETA$ is the coefficient estimate for the Level-3 predictor variable