COMPARISON OF THE MALMQUIST MULTIFACTOR PRODUCTIVITY INDEX AND THE MALMQUIST PRODUCTIVITY INDEX AND THEIR DECOMPOSITION

by

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(Under the Direction of Knox Lovell)

ABSTRACT

This study provides the empirical test of the comparison of the partially oriented Malmquist productivity index(Caves et al, 1982) and Malmquist Multifactor productivity index (Bjurek, 1994, 1996) considering both output index and input index simultaneously and compare the Bjurek(1994,1996) decomposition and the Fare et al(1994b), with a panel data of agricultural output(crop and livestock) and inputs(intermediate, capital, land, and labor) of time period (1960-1996) for the 48 contiguous states.

This present study shows a characteristics and closeness of the Malmquist productivity index and the Malmquist multifactor productivity index under the constant return to scale and the variable return to scale and its economically meaningful decomposition among Bjurek(1994,1996) and the Fare et al(1994b), with a result that Bjurek(1994, 1996) has a economically meaningful interpretation because in Bjurek(1994, 1996), a productivity is decomposed by variable return to scale technology

INDEX WORDS: Malmquist mulfifactor productivity index, Malmquist productivity index, Decomposition, Agricultural productivity

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CHAPTER 1

INTRODUCTION[1]

Since Shephard (1953) introduced the input distance function in the context of production analysis. At the same time Malmquist (1953) introduced the input distance function in the context of consumption analysis. However Malmquist went a step further than Shephard, by developing a standard of living (or consumption quantity) index as the ratio of a pair of input distance functions. In the context of production analysis, Malmquist's standard of living index becomes an input quantity index. There is an analogous output quantity index based on the output distance function introduced by Shephard (1970).

An obvious extension is to define a productivity index based on distance functions. Two approaches have been developed. The first approach (Malmquist multifactor productivity index) is simultaneously oriented, being based on a ratio of output distance functions contained in an output quantity index and a ratio of input distance functions contained in an input quantity index. This approach was noted and neglected by Fisher & Shell (1972), Caves et al. (1982) and Diewert (1992), and finally proposed by Bjurek (1994,1996). The second approach (Malmquist productivity index) also uses distance functions, but because it uses only output distance functions or input distance functions it is partially oriented, and so it does not define a productivity index as the ratio of an

1.This content is excerpt from the Lovell(2001) output quantity index to an input quantity index. In its basic output-oriented form it defines a productivity index as the ratio of a pair of output distance functions, and in its basic input-oriented form it defines a productivity index as the ratio of a pair of input distance functions. This is the Malmquist productivity index introduced by Caves et al. (1982). Except under severe restrictions[2] on the underlying technology, the two indexes generate different measures of productivity change.

Malmquist productivity index has been related to the Törnqvist productivity index by Caves et al. (1982), and to the Fisher productivity index by Färe & Grosskopf (1992) and Balk (1993). It decomposes into various sources of productivity change, as Färe et al. (1994a) first demonstrated.

Here, I show the Malmquist multifactor productivity index and Malmquist productivity index and its decomposition by Bjurek and Fare et al respectively with a panel made by USDA (1960-1996)

[2] Färe et al. (1996) proved that the two productivity indexes are equal if, and only if, technology exhibits inverse homotheticity and constant returns to scale. Although these restrictions are unlikely to hold empirically, the available evidence suggests that the two indexes generate similar results. Bjurek (1994) found little difference between the two indexes, using a sample of Swedish day care centers. Bjurek et al. (1998) found little difference using a sample of Swedish electricity retail distributors. Balk (1998) found little difference using a sample of firms in the Dutch rubber processing industry.

3 Methodology is excerpt from Lovell(2001)

CHAPTER 2

METHODOLOGY

2.1 Malmquist multifactor productivity

2.1.1 "True" Quantity Indexes

Following Moorsteen (1961), we define a "true" output quantity index as

$Y_o^s(x^s,y^{t+1},y^t) = D_o^s(x^s,y^{t+1}) / D_o^s(x^s,y^t)$, (2.1.1)

and following Malmquist (1953), we define a "true" input quantity index as

$X_i^s(y^s,x^{t+1},x^t) = D_i^s(y^s,x^{t+1}) / D_i^s(y^s,x^t)$, (2.1.2)

where s refers to either period t or period t+1. In the former case, s=t yields base period Laspeyres-Malmquist quantity indexes, and in the latter case s=t+1 yields comparison period Paasche-Malmquist quantity indexes.

The output quantity index is illustrated in Figure 2.1, where $Y_o^t(x^t,y^{t+1},y^t)$ compares y^{t+1} and y^t by comparing their radial distances to period t technology as represented by $IsoqP^t(x^t)$, and $Y_o^{t+1}(x^{t+1},y^{t+1},y^t)$ compares y^{t+1} and y^t by comparing their radial distances to period t+1 technology as represented by $IsoqP^{t+1}(x^{t+1})$. If $Y_o^s(x^s,y^{t+1},y^t)$ > (<) 1 we say that more (less) output is produced in period t+1 than in period t, relative to period s technology and period s input usage.

The input quantity index is illustrated in Figure 2.2, where $X_i^t(y^t,x^{t+1},x^t)$ compares x^{t+1} and x^t by comparing their radial distances to period t technology as represented by $IsoqL^t(y^t)$, and $X_i^{t+1}(y^{t+1},x^{t+1},x^t)$ compares x^{t+1} and x^t by comparing their radial distances to period t+1 technology as represented by $IsoqL^{t+1}(y^{t+1})$. If $X_i^s(y^s,x^{t+1},x^t) < (>)\ 1$, less (more) input is used in period t+1 than in period t, relative to period s technology and period s output production.

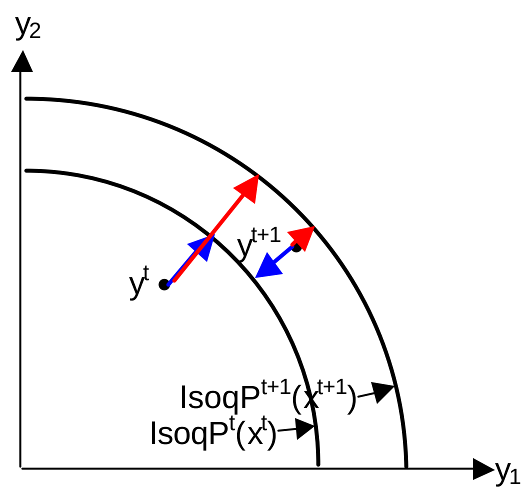

Figure 1 Input quantity index

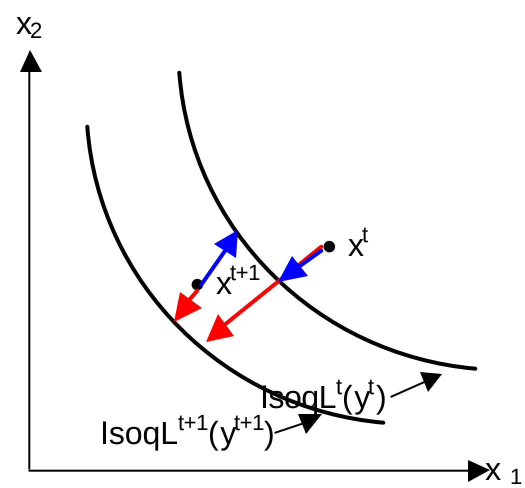

Figure 2 Output quantity index

2.1.2 The Malmquist Multifactor Productivity Index

This productivity index was introduced by Bjurek (1994,1996), who called it the Malmquist "total factor productivity" index. The period t Laspeyres-Malmquist multifactor productivity index is

$$MM^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{Y_o^t\left(x^t, y^{t+1}, y^t\right)}{X_i^t\left(y^t, x^{t+1}, x^t\right)}, (2.2.1)$$

where the period t Laspeyres-Malmquist output quantity index

$$Y_o^{\,t}\left(x^t, y^{t+1}, y^t\right) = \frac{D_o^{\,t}\left(x^t, y^{t+1}\right)}{D_o^{\,t}\left(x^t, y^t\right)} \quad (2.2.2)$$

compares y^{t+1} to y^t by comparing their radial distances from $IsoqP^t(x^t)$, and the period t Laspeyres-Malmquist input quantity index

$$X_i^{\,t}\left(y^t, x^{t+1}, x^t\right) = \frac{D_i^{\,t}\left(y^t, x^{t+1}\right)}{D_i^{\,t}\left(y^t, x^t\right)} \quad (2.2.3)$$

compares x^{t+1} to x^t by comparing their radial distances from $IsoqL^t(y^t)$. $MM^t(x^t,y^t,x^{t+1},y^{t+1}) \begin{smallmatrix} > \\ = \\ < \end{smallmatrix} 1$ according as productivity growth, stagnation or decline has occurred between periods t and t+1, from the forward-looking perspective of period t.

There is an analogous period t+1 Paasche-Malmquist multifactor productivity index

$$MM^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{Y_o^{\,t+1}\left(x^{t+1}, y^{t+1}, y^t\right)}{X_i^{\,t+1}\left(y^{t+1}, x^{t+1}, x^t\right)}, \quad (2.2.4)$$

where the period t+1 Paasche-Malmquist output quantity index

$$Y_o^{\,t+1}\left(x^{t+1}, y^{t+1}, y^t\right) = \frac{D_o^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^{\,t+1}\left(x^{t+1}, y^t\right)} \quad (2.2.5)$$

compares y^{t+1} to y^t by comparing their radial distances from $IsoqP^{t+1}(x^{t+1})$, and the period t+1 Paasche-Malmquist input quantity index

$$X_i^{\,t+1}\left(y^{t+1}, x^{t+1}, x^t\right) = \frac{D_i^{\,t+1}\left(y^{t+1}, x^{t+1}\right)}{D_i^{\,t+1}\left(y^{t+1}, x^t\right)} \quad (2.2.6)$$

compares x^{t+1} to x^t by comparing their radial distances from $\text{Isoq}L^{t+1}(x^{t+1})$. $MM^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) \gtreqless 1$ according as productivity growth, stagnation or decline has occurred between periods t and t+1, from the backward-looking perspective of period t+1.

A comparison of the right sides of (2.2.2) and (2.2.5) shows that $Y_o^t(x^t,y^{t+1},y^t)$ and $Y_o^{t+1}(x^{t+1},y^{t+1},y^t)$ are not necessarily equal, and a comparison of the right sides of (2.2.3) and (2.2.6) shows that $X_i^t(y^t,x^{t+1},x^t)$ and $X_i^{t+1}(y^{t+1},x^{t+1},x^t)$ are not necessarily equal. It follows that $MM^t(x^t,y^t,x^{t+1},y^{t+1})$ in (2.2.1) and $MM^{t+1}(x^t,y^t,x^{t+1},y^{t+1})$ in (2.2.4) are not necessarily equal. This makes it desirable to merge the forward-looking and backward-looking perspectives to define a geometric mean Fisher-Malmquist output quantity index

$$Y_o(x^t,y^t,x^{t+1},y^{t+1}) = [Y_o^t(x^t,y^{t+1},y^t) \times Y_o^{t+1}(x^{t+1},y^{t+1},y^t)]^{1/2} \quad (2.2.7)$$

and a geometric mean Fisher-Malmquist input quantity index

$$X_i(x^t,y^t,x^{t+1},y^{t+1}) = [X_i^t(y^t,x^{t+1},x^t) \times X_i^{t+1}(y^{t+1},x^{t+1},x^t)]^{1/2}. \quad (2.2.8)$$

The ratio of the two provides a geometric mean Fisher-Malmquist multifactor productivity index

$$MM(x^t,y^t,x^{t+1},y^{t+1}) = [MM^t(x^t,y^t,x^{t+1},y^{t+1}) \times MM^{t+1}(x^t,y^t,x^{t+1},y^{t+1})]^{1/2}$$

$$= [Y_o^t(x^t,y^{t+1},y^t) \times Y_o^{t+1}(x^{t+1},y^{t+1},y^t)]^{1/2} / [X_i^t(y^t,x^{t+1},x^t) \times X_i^{t+1}(y^{t+1},x^{t+1},x^t)]^{1/2}$$

$$= Y_o(x^t,y^t,x^{t+1},y^{t+1}) / X_i(x^t,y^t,x^{t+1},y^{t+1}), \quad (2.2.9)$$

which measures productivity change as the ratio of a Fisher-Malmquist output quantity index to a Fisher-Malmquist input quantity index.

The Malmquist multifactor productivity index is illustrated in Figure 3. $MM^t(x^t,y^t,x^{t+1},y^{t+1}) > 1$ because output has increased proportionately more than input relative to period t technology T^t. $MM^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) > 1$ because output has increased proportionately more than input relative to period t+1 technology T^{t+1}.

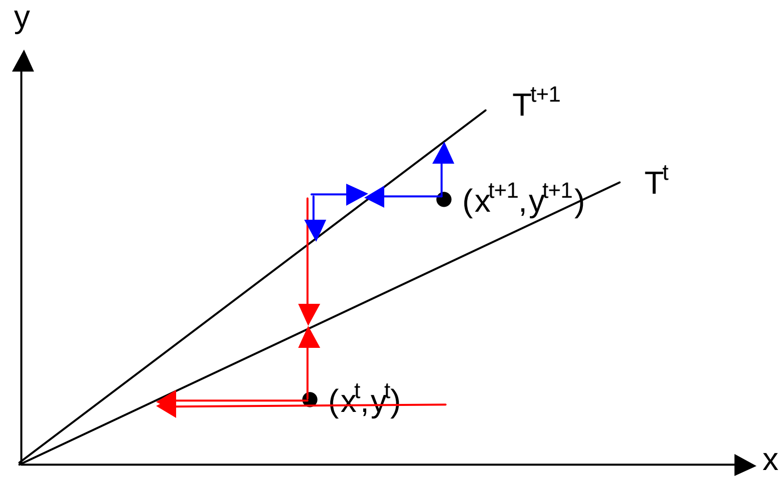

Figure 3 **The Malmquist Multifactor Productivity Index**

All three versions of the index given in (2.2.1), (2.2.4) and (2.2.9) are simultaneously oriented, involving both output distance functions and input distance functions.

All three versions take the form of the ratio of an output quantity index to an input quantity index. Consequently all three are faithful to the notion of productivity as the ratio of output produced to input consumed. All three versions of the index involve distance functions that use mixed-period data. This is occasionally mentioned as a drawback of the index, since no producer uses mixed-period data. However it is solidly based in the theory of "true" quantity indexes, which themselves involve distance functions using mixed-period data

2.1.3 Decomposition of the Malmquist Multifactor Productivity Index

Though Bjurek suggested MM(x^t,y^t,x^{t+1},y^{t+1}), he did not decompose that index. Førsund (1996) claimed that the index can be decomposed in a similar way as the Malmquist productivity index but he did not provide such a decomposition. According to Lovell(2003) the Fisher Malmquist multifactor productivity index is possible to obtain an economically meaningful decomposition of MM(x^t,y^t,x^{t+1},y^{t+1}).

Recall from Section 2.1.2 that the period t Malmquist multifactor productivity index is

$$MM^t\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{Y_o^{\,t}\left(x^t, y^{t+1}, y^t\right)}{X_i^{\,t}\left(y^t, x^{t+1}, x^t\right)}, \quad (2.3.1)$$

where the period t output quantity index is defined in (2.2.2) and the period t input quantity index is defined in (2.2.3). For notational convenience I decompose the period t index rather than the geometric mean version given in (2.2.7). I also use the variable returns to scale best practice technologies, rather than the constant returns to scale benchmark technologies, in the decomposition.

The output quantity index (2.2.2) decomposes as

$$Y_o^{\,t}\left(x^t, y^{t+1}, y^t\right) = \left[\frac{D_o^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^{\,t}\left(x^t, y^t\right)}\right] \times \left[\frac{D_o^{\,t}\left(x^{t+1}, y^{t+1}\right)}{D_o^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}\right] \times \left[\frac{D_o^{\,t}\left(x^t, y^{t+1}\right)}{D_o^{\,t}\left(x^{t+1}, y^{t+1}\right)}\right]$$

$$= TE\Delta_o\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \times T\Delta_o\left(x^{t+1}, y^{t+1}\right) \times \left[\frac{D_o^{\,t}\left(x^t, y^{t+1}\right)}{D_o^{\,t}\left(x^{t+1}, y^{t+1}\right)}\right], \quad (2.3.2)$$

Where $TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1})$ is an output-oriented measure of technical efficiency change, $T\Delta_o(x^{t+1},y^{t+1})$ is an output-oriented measure of technical change, and the third component is as yet undefined.

The input quantity index (2.2.3) decomposes as

$$X_i^t(y^t, x^{t+1}, x^t) = \left[\frac{D_i^{t+1}(y^{t+1}, x^{t+1})}{D_i^t(y^t, x^t)}\right] \times \left[\frac{D_i^t(y^{t+1}, x^{t+1})}{D_i^{t+1}(y^{t+1}, x^{t+1})}\right] \times \left[\frac{D_i^t(y^t, x^{t+1})}{D_i^t(y^{t+1}, x^{t+1})}\right]$$

$$= TE\Delta_i(x^t, y^t, x^{t+1}, y^{t+1}) \times T\Delta_i(x^{t+1}, y^{t+1}) \times \left[\frac{D_i^t(y^t, x^{t+1})}{D_i^t(y^{t+1}, x^{t+1})}\right], \quad (2.3.3)$$

Where $TE\Delta_i(x^t,y^t,x^{t+1},y^{t+1})$ is an input-oriented measure of technical efficiency change, $T\Delta_i(x^{t+1},y^{t+1})$ is an input-oriented measure of technical change, and the third component is as yet undefined.

Adopting a partially oriented approach to decomposing the Fisher Malmquist multifactor productivity index, and isolating $TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1})$ and $T\Delta_o(x^{t+1},y^{t+1})$, the Malmquist multifactor productivity index decomposes as

$$MM^t(x^t,y^t,x^{t+1},y^{t+1}) = TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^{t+1},y^{t+1})$$

$$\times \{Y_o^t(x^t,y^t,y^{t+1}) / [TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^{t+1},y^{t+1})]\} \div X_i^t(y^t,x^t,x^{t+1})$$

$$= TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^{t+1},y^{t+1})$$

$$\times Y_o^t[x^t,y^t/D_o^t(x^t,y^t),y^{t+1}/D_o^t(x^{t+1},y^{t+1})] \div X_i^t(y^t,x^t,x^{t+1})$$

$$= TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^{t+1},y^{t+1}) \times S\Delta^t(x^t,y^t,x^{t+1},y^{t+1}). \quad (2.3.4)$$

The first equality in (2.3.4) shows that the third term is the ratio of an output quantity index (adjusted for technical efficiency change and technical change) to an input quantity index. The second equality clarifies the adjustment, showing that the output quantity index uses outputs shifted to the surface of T^t. The third equality expresses the third term in terms of distance functions. The final equality claims that the third term identifies the contribution of scale economies on period t technology to productivity change.

If a decomposition is adopted in the period of t+1, the decomposed outcome is expressed as

$$\begin{aligned} MM^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) &= TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^t,y^t) \\ &\times \{Y_o^{t+1}(x^{t+1},y^t,y^{t+1}) / [TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^t,y^t)]\} \div X_i^{t+1}(y^{t+1},x^t,x^{t+1}) \\ &= TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^t,y^t) \\ &\times Y_o^{t+1}[x^t,y^{t+1}/D_o^{t+1}(x^{t+1},y^{t+1}),y^{t+1}/D_o^t(x^{t+1},y^{t+1})] \div X_i^{t+1}(y^t,x^t,x^{t+1}) \\ &= TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_o(x^{t+1},y^{t+1}) \times S\Delta^{t+1}(x^t,y^t,x^{t+1},y^{t+1}). \qquad (2.3.5) \end{aligned}$$

Considering a simultaneous combination of time periods t and t+1, Geometric mean of 2.3.4 and 2.3.5 is decomposed as

$$\begin{aligned} MM(x^t,y^t,x^{t+1},y^{t+1}) &= MM^t(x^t,y^t,x^{t+1},y^{t+1}) \times MM^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) \\ &= TE\Delta_o(x^t,y^t,x^{t+1},y^{t+1}) \times [T\Delta_o(x^{t+1},y^{t+1}) \times T\Delta_o(x^t,y^t)]^{1/2} \\ &\times [S\Delta^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) \times S\Delta^{t+1}(x^t,y^t,x^{t+1},y^{t+1})]^{½} \qquad (2.3.6) \end{aligned}$$

By same reasoning of 2.3.4 decomposition, the first term is efficiency change effect and the second term is the geometric mean of technological effect of t and t+1 period , Finally, the third term identifies the geometric mean of the contribution of scale economies on period t and t+1 to productivity change.

2.2 The Malmquist Productivity Index

2.2.1 The Malmquist Productivity index

The Malmquist productivity index was introduced as a theoretical index by Caves et al. (1982) and popularized as an empirical index by Färe et al. (1994a). The Malmquist productivity index is defined on a benchmark technology satisfying constant returns to scale, which is to be distinguished from a best practice technology allowing for variable returns to scale. This convention enables it to incorporate the influence of scale economies, as a departure of the best practice technology from the benchmark technology.

Using the period t benchmark technology, the output-oriented Malmquist productivity index is written as

$$M_{oc}^{\ t}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_{oc}^{\ t}\left(x^{t+1}, y^{t+1}\right)}{D_{oc}^{\ t}\left(x^t, y^t\right)}, \quad (2.4.1)$$

Where the notation "$_{oc}{}^{t}$" indicates that the output distance functions comprising the Malmquist productivity index are defined on the period t benchmark technology.

$M_{oc}^{t}(x^t,y^t,x^{t+1},y^{t+1}) \gtreqless 1$ according as productivity growth, stagnation or decline has occurred between periods t and t+1, from the forward-looking perspective of period t technology.

Using the period t+1 benchmark technology, the output-oriented Malmquist productivity index is written as

$$M_{oc}^{t+1}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \frac{D_{oc}^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_{oc}^{t+1}\left(x^t, y^t\right)}, \quad (2.4.2)$$

Where the notation "$_{oc}^{t+1}$" indicates that the output distance functions comprising the Malmquist productivity index are defined on the period t+1 benchmark technology. $M_{oc}^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) \gtreqless 1$ according as productivity growth, stagnation or decline has occurred between periods t and t+1, from the backward-looking perspective of period t+1 technology.

The two Malmquist productivity indexes are illustrated in Figure 4, with both $M_{oc}^{t}(x^t,y^t,x^{t+1},y^{t+1}) > 1$ and $M_{oc}^{t+1}(x^t,y^t,x^{t+1},y^{t+1}) > 1$ indicating productivity growth.

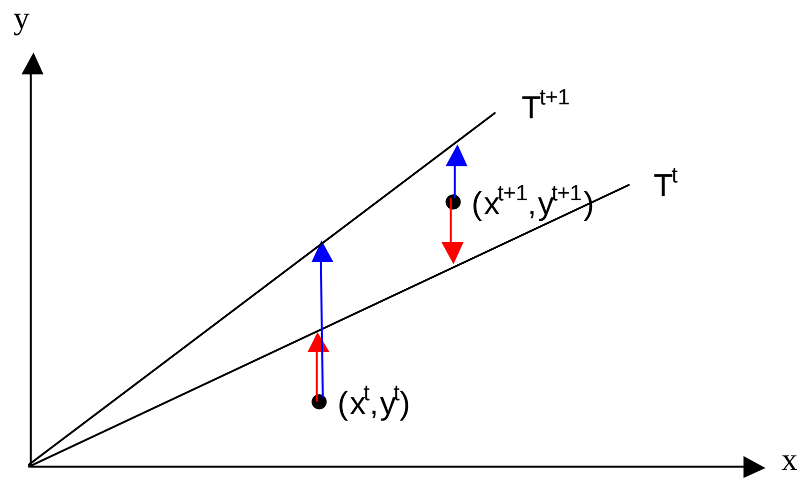

Figure 4. The Malmquist Productivity Index

Both indexes compare (x^{t+1},y^{t+1}) to (x^t,y^t), but they use different benchmark technologies. Since the choice of benchmark technology is arbitrary, and since the two indexes are not necessarily equal, it is conventional to define the Malmquist productivity index as the geometric mean of the two, and so

$$M_{oc}(x^t,y^t,x^{t+1},y^{t+1}) = \{[M_{oc}{}^t(x^t,y^t,x^{t+1},y^{t+1}) \times M_{oc}{}^{t+1}(x^t,y^t,x^{t+1},y^{t+1})]\}^{1/2}$$

$$= \left[\frac{D_{oc}{}^t\left(x^{t+1}, y^{t+1}\right)}{D_{oc}{}^t\left(x^t, y^t\right)} \times \frac{D_{oc}{}^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_{oc}{}^{t+1}\left(x^t, y^t\right)}\right]^{1/2} \quad .(2.4.3)$$

$M_{oc}(x^t,y^t,x^{t+1},y^{t+1}) \; \substack{> \\ = \\ <} \; 1$ according as productivity growth, stagnation or decline has occurred between periods t and t+1.

2.2.2 Decomposition of the Malmquist Productivity Index

Färe et al. (1994a) provided an initial decomposition of the index as

$$M_{oc}\left(x^t, y^t, x^{t+1}, y^{t+1}\right) = \left[\frac{D_{oc}^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}{D_{oc}^{\,t}\left(x^t, y^t\right)}\right] \times \left\{\left[\frac{D_{oc}^{\,t}\left(x^{t+1}, y^{t+1}\right)}{D_{oc}^{\,t+1}\left(x^{t+1}, y^{t+1}\right)} \times \frac{D_{oc}^{\,t}\left(x^t, y^t\right)}{D_{oc}^{\,t+1}\left(x^t, y^t\right)}\right]\right\}^{1/2} \quad (2.5.1)$$
$$= TE\Delta_c\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \times T\Delta_c\left(x^t, y^t, x^{t+1}, y^{t+1}\right),$$

Where $TE\Delta_c(x^t,y^t,x^{t+1},y^{t+1})$ measures technical efficiency change and $T\Delta_c(x^t,y^t,x^{t+1},y^{t+1})$ measures the geometric mean of the magnitudes of technical change along rays through (x^{t+1},y^{t+1}) and (x^t,y^t). Both components are measured on the benchmark technologies. Since the best practice technologies may exhibit variable returns to scale, it is desirable to redefine both components on best practice technologies, to see what is left over, and to see if what is left over can be given a meaningful economic interpretation.

Färe et al. (1994b) redefined one component. They decomposed the technical efficiency change component to obtain

$$TE\Delta_c(x^t, y^t, x^{t+1}, y^{t+1}) = \left[\frac{D_o^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^{\,t}\left(x^t, y^t\right)}\right] \times \left\{\left[\frac{D_{oc}^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}{D_o^{\,t+1}\left(x^{t+1}, y^{t+1}\right)}\right] \div \left[\frac{D_{oc}^{\,t}\left(x^t, y^t\right)}{D_o^{\,t}\left(x^t, y^t\right)}\right]\right\}$$
$$= TE\Delta\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \times \left[\frac{SE^{t+1}\left(x^{t+1}, y^{t+1}\right)}{SE^t\left(x^t, y^t\right)}\right]$$
$$= TE\Delta\left(x^t, y^t, x^{t+1}, y^{t+1}\right) \times SE\Delta\left(x^t, y^t, x^{t+1}, y^{t+1}\right),$$

(2.5.2)

Where $TE\Delta(x^t,y^t,x^{t+1},y^{t+1})$ measures technical efficiency change on the best practice technologies and $SE\Delta(x^t,y^t,x^{t+1},y^{t+1})$ measures the change in scale efficiency from period t to period t+1. Inserting (2.5.2) into (2.5.1) gives the Färe et al. (1994b) decomposition of the Malmquist productivity index

$M_{oc}(x^t,y^t,x^{t+1},y^{t+1}) = TE\Delta(x^t,y^t,x^{t+1},y^{t+1}) \times SE\Delta(x^t,y^t,x^{t+1},y^{t+1}) \times T\Delta_c(x^t,y^t,x^{t+1},y^{t+1})$.(2.5.3)

In this decomposition technical efficiency change $TE\Delta(x^t,y^t,x^{t+1},y^{t+1})$ is measured relative to the best practice technologies. However the relationship between scale efficiency change $SE\Delta(x^t,y^t,x^{t+1},y^{t+1})$ and the contribution of scale economies is unclear, despite the efforts of Førsund (1996). Finally, technical change $T\Delta_c(x^t,y^t,x^{t+1},y^{t+1})$ remains measured as a shift in the benchmark technology, and so does not provide a meaningful measure of change in the best practice technology.

CHAPTER 3

DATA

For the comparison of the Malmquist productivity index and multifactor productivity index and its decomposition, data is used from USDA (ERS) website .According to USDA report(Technical bulletin no. 1895, May 2001), The definition of output and inputs(intermediate, capital, land, labor) is followed.

3.1 Output

The output quantity for each crop and livestock category consists of quantities of commodities sold off the farm, additions to inventory, and quantities consumed as part of final demand in farm households during the calendar year. As discussed above, off-farm sales in the aggregate accounts are defined only in terms of output leaving the sector. Off-farm sales in the State accounts include sales to the farm sector in other States as well. The price corresponding to each disaggregated output reflects the value of that output to the sector. That is, subsidies are added and indirect taxes are subtracted from market values.

3.2 Intermediate Input

Intermediate input consists of goods used in production during the calendar year, whether withdrawn from beginning inventories or purchased from outside the farm sector or (in the case of the State production accounts) from farms in other States. The inclusion and treatment of open-market purchases of feed, seed, and livestock inputs require little discussion. These inputs should enter both State and aggregate farm sector intermediate goods accounts. However, the treatment of withdrawals from producers' inventories requires elaboration.

3.3 Capital Input

This study requires measures of capital input and capital service prices for each State. Construction of these series begins with estimating the capital stock and rental price for each asset type for each State. The perpetual inventory method is used to develop capital stocks from data on investment. Implicit rental prices for each asset are based on the correspondence between the purchase price of the asset and the discounted value of future service flows derived from that asset.

Indexes of capital input in each State are constructed by aggregating over the different capital assets using as weights the asset-specific rental prices. Service prices for capital input are formed implicitly as the ratio of the total current dollar value of capital service flows to the quantity index.

3.4 Land Input

To obtain a constant-quality land stock, we compile data on land area and average value (excluding buildings) per acre in each Agricultural Statistics District in each State. We further disaggregate land input into irrigated and dry cropland, grazing land, and other land in 11 Western States. The land area in each district and use category is reported in the quinquennial Census of Agriculture (U.S. Department of Agriculture). USDA's National Agricultural Statistics Service annually updates State estimates of total land in farms. For the years intermediate to the censuses, percentages in each district and use category are interpolated. Land values per acre are used to aggregate across the different land categories in each State

3.5 Labor Input

The USDA labor accounts for the aggregate farm sector incorporate the demographic cross-classification of the agricultural labor force developed by Jorgenson, Gollop, and Fraumeni (1987). Matrices of hours worked and compensation per hour have been developed for laborers cross-classified by sex, age, education, and employment class—employee versus self-employed and unpaid family workers.

Indexes of labor input are constructed for each State and the aggregate farm sector over the 1960-96 period using the demographically cross-classified hours and compensation data. Labor hours having higher marginal productivity (wages) are given higher weights in forming the index of labor input than are hours having lower marginal productivities. Doing so explicitly adjusts State and aggregate farm sector indexes of labor input for quality change in hours as originally defined by Jorgenson and Griliches (1967)

CHAPTER 4

EMPIRICAL TEST

In this section, we will show the difference of both the indices. From the below, the Malmquist multifactor productivity index expressed as Bc,t, Bc,t+1, Bv,t and Bv,t+1, and the Malmquist productivity index expressed as a Moc,t, Moc,t+1, Mov,t and Mov,t+1 by a technology perspective of Laspeyres and Paasche and a return to scale(CRS and VRS) respectively. Also, a geometric mean of the Malmquist productivity indexes combined in time perspectives is showed as the Moc and Mov, and a geometric mean of the Malmquist multifactor productivity index is the Bc and Bv.

4.1. The Malmquist indexes

According to the calculated outcome in the Table 1 and 2 of the Malmquist multifactor productivity index and the Malmquist productivity index, A index value has a different value on the point of technology perspective. Laspeyres index have a forward looking perspective of period t, Meanwhile, Paasche index has a backward looking perspective of period t+1. Geometric mean is a combination of a forward looking and a backward looking perspective.

In this section, we will show the difference of both the indices. For the whole period, we have a panel data of 49 states and 37 years(1960~1996).

In the Graph 1 and 2, we show the Malmquist indexes for an agricultural productivity in a constant return to scale is nearly similar. The Malmquist indexes under VRS are also similar.

The geometric mean value of the Moc in a whole period is 1.0174 and that of the Bc is 1.0191. On the other hand, the value of the Mov is 1.0193 and that of the Bv is 1.0203. Moc is much closer to Bv than any other indexes. The Malmquist indexes on the variable return to scale are all higher than the Malmquist indexes on the constant return to scale

In the difference of t and t+1 technologies of the geometric mean of theMalmquist indexes, a gap of the Malmquist indexes on the CRS is smaller than that of the Malmquist indexes on the VRS. The gap of the Bc,t and the Bc,t+1. Bc,t+1 is the smallest(0.0089).

In the deviation of productivity, the Malmquist multifactor productivity index is a little smaller than the Malmquist productivity index in both CRS and VRS condition.

4.1.1 The Malmquist indexes(CRS)

The Malmquist productivity indexes on the constant return to scale shows a lower standard deviation than the Malmquist multifactor productivity indexes. The gap of the Moc,t and the Moc,t+1 is smaller than the gap of the Bc,t and the Bc,t+1.

The Malmquist indexes have the highest values(1.102,1.103) in 1980 and, the lowest values(0.941, 0.942) in the lowest value, due to a poor crop made by a nationwide drought.

4.1.2 The Malmquist indexes(VRS)

In the Table 2, the Malmquist productivity index on the variable return to scale shows a higher value than the Malmquist multifactor productivity index. In the point of a deviation, the Mov's deviation(0.0403) is nearly similar to the Bv's(0.0400) .

In the highest value of productivity, the malmquist indexes(VRS) was highes in 1980, but in the lowest value of productivity, the malmquist indexes(VRS) was lowest in 1982, different from the malmquist indexes(CRS).

4.1.3 The ratios of the Malmquist indexes

In the table 3, ratios of the geometric means of the Malmquist indexes all are under 5% deviation level, but there are a lot of variation in individual states. Although most ratios are much close to one in most years, but in some years, there are some observations which deviate to some extent. All deviation from one are in the range of 5%. The standard deviation of the ratio between Malmquist indexes is very small,. At most, the biggest one is 1.8%. those facts mean that all indexes are nearly same, indicating a possibility of an inverse homotheticity.

In the level of each year, there are 48 states standard deviations which are above 5% range. Thus, in the state level, an inverse homotheticity does not work.

4.2 Comparison of the Malmquist indexes' decompositions

Adopting a equation 2.6.4's decomposition method , then getting a geometric mean of the decomposed outcomes (Efficiency change, Technical change, and Scale factor) of the Bv,t and the Bv,t+1, we can have a decomposed outcome of productivity of the Malmquist multifactor productivity index. Here, Scale factor identifies the contribution of scale economies on a production function.

In the table 5 and table 6, In the decomposed outcome of the Malmquist multifactor productivity index , geometric mean of each component is 0.999(Efficiency change), 1.027(technical change), and 0.994(Scale factor). In case of the Malmquist productivity index, geometric mean of each component is 0.998(Efficiency change), 1.022(Technical change), and 0.999(scale efficiency change). Technical change in both indexes is only above 1 and the other components are below 1. Here, we conclude that, only in technical change, there is a development, and that there is a decline in efficiency change and scale factor.

As discussed in Lovell (2001), In the decomposition of Mo of Fare et al(1994b) in the table 6, technical change is measured at a benchmarking technology instead of a best practice technology. Because a best practice technology means a variable return to scale, the relationship between scale efficiency change and the contribution of scale economies is unclear, and further, Technical change does not provide a meaningful measure of change in the best practice technology.

Further, if we check the outcome of the Malmquist multifactor productivity index in the table 6, this reasoning looks more trustworthy. Thus, We can conclude that the decomposed outcome done by Bjurek(1994,1996) show the true characteristics of US agricultural productivity in 1960-1996.

Reviewing the decomposed outcomes, we can say that an agricultural productivity is developed by a pure production technology rather than an efficient allocation and a scale factor because the technical change is above 1 and the other components are below 1.

In the view of a volatility, the Bv's standard deviation is 0.040, while those of efficiency change, technical change, and scale factor is 0.024, 0.052, and 0.027 respectively. Those standard deviations of the Bc's are 0.031, 0.033, and 0.007. Only technical change is higher fluctuated than the other componets. Meanwhile, Moc's standard deviation is 0.407 compared with each component's volatility (0.024, 0.033, and 0.014). Regarding a volatility of each component, all component standard deviation of the Moc are less than those of the Bv. The volatility of technical change in the Bv is nearly two times higher than the other components'. Meanwhile the other component's standard deviations are not different much.

In the point of a correlation between the decomposed components of the Bv and the Moc, we can support Bjurek's(1994, 1996) proposition, In the Table 6, scale efficiency change is below 1, thus meaning that scale efficiency was deteriorated, but the value of a correlation between scale efficiency change and the Moc is positive(0.368), this shows an inconsistency of scale efficiency change's contribution to the Moc because scale efficiency change is declined and the Moc is developed. On the other hand, the correlation value between the Bv's scale factor and the Bv itself is negative(-0.793), and thus shows a consistency between scale factor and the Bv.

In the table 10, Both a contribution of scale economy and a scale efficiency change shows a negative correlation value (-0.201), assuring that a scale efficiency change of the Moc don't explain a positive agricultural productivity trend.

CHAPTER 5

CONCLUSION

Through this paper, I tried to show a characteristics of the Malmquist multifactor productivity index, the Malmquist productivity index and their decomposition method. Here, we have some facts that Moc,Mov , Bv, and Bc is nearly close in a time trend, except some years.

The other is that in the decomposition of productivity, Bjurek(1994,1997) decomposition is more preferred to Fare et (1994b) in the economically meaningful interpretation of a productivity. Explained in a methodology part, the Fare et al(1994b) decomposed a productivity in the constant return to scale frontier, inducing a mixed effect of technical change and scale efficiency change.

Further, the next assignment will be to make a more sophisticated production frontier considering an exogenous effect like weather.

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Table 1 The Malmquist indexes (CRS)

Malmquist multifactor productivity index					Malmquist productivity index		
			CRS				CRS
Year	Bc,t	Bc,t+1			Moc,t	Moc,t+1	Moc
1960	1.0403	1.0352	1.0378	1	1.0407	1.0338	1.0372
1961	1.0633	0.9988	1.0305	2	1.0338	1.0290	1.0314
1962	1.0403	1.0266	1.0335	3	1.0367	1.0268	1.0317
1963	1.0157	1.0105	1.0131	4	1.0159	1.0093	1.0126
1964	1.0334	1.0290	1.0312	5	1.0338	1.0276	1.0307
1965	0.9869	0.9767	0.9818	6	0.9869	0.9764	0.9816
1966	1.0278	1.0328	1.0303	7	1.0388	1.0324	1.0356
1967	1.0218	1.0076	1.0147	8	1.0212	1.0069	1.0140
1968	1.0180	1.0143	1.0162	9	1.0188	1.0147	1.0167
1969	1.0124	1.0047	1.0086	10	1.0130	1.0029	1.0080
1970	1.0654	1.0546	1.0600	11	1.0654	1.0554	1.0604
1971	0.9994	0.9917	0.9955	12	0.9998	0.9907	0.9952
1972	1.0137	1.0023	1.0080	13	1.0126	1.0020	1.0073
1973	0.9615	0.9381	0.9497	14	0.9610	0.9382	0.9495
1974	1.0138	1.0811	1.0469	15	1.0959	1.0821	1.0889
1975	0.9858	0.9779	0.9818	16	0.9850	0.9772	0.9811
1976	1.0476	1.0372	1.0423	17	1.0476	1.0371	1.0423
1977	1.0022	0.9761	0.9890	18	1.0028	0.9760	0.9893
1978	1.0203	1.0062	1.0132	19	1.0202	1.0060	1.0131
1979	0.9597	0.9495	0.9546	20	0.9608	0.9519	0.9563
1980	1.1066	1.0977	1.1021	21	1.1063	1.0994	1.1028
1981	1.0527	1.0423	1.0475	22	1.0520	1.0431	1.0476
1982	0.9519	0.9403	0.9461	23	0.9525	0.9388	0.9456
1983	1.0673	1.0520	1.0596	24	1.0676	1.0526	1.0601
1984	1.0666	1.0523	1.0594	25	1.0657	1.0531	1.0594
1985	1.0343	1.0346	1.0345	26	1.0354	1.0332	1.0343
1986	1.0372	1.0210	1.0291	27	1.0373	1.0206	1.0289
1987	0.9654	0.9545	0.9600	28	0.9646	0.9559	0.9602
1988	1.0447	1.0374	1.0410	29	1.0458	1.0386	1.0422
1989	1.0039	1.0158	1.0098	30	1.0318	1.0167	1.0242
1990	1.0159	1.0049	1.0104	31	1.0157	1.0044	1.0100
1991	1.0805	1.0716	1.0760	32	1.0795	1.0728	1.0762
1992	0.9768	0.9658	0.9713	33	0.9775	0.9666	0.9720
1993	1.0596	1.0526	1.0561	34	1.0594	1.0527	1.0560
1994	0.9459	0.9371	0.9414	35	0.9475	0.9369	0.9422
1995	1.0763	1.0667	1.0715	36	1.0768	1.0664	1.0716
Geometric Mean	1.0219	1.0130	1.0174	0.0089	1.0244	1.0138	1.0191
STD	0.0391	0.0413	0.0392		0.0401	0.0415	0.0407
Max	1.1066	1.0977	1.1021		1.1063	1.0994	1.1028
Min	0.9459	0.9371	0.9414		0.9475	0.9369	0.9422
GAP	0.1607	0.1606	0.1607		0.1588	0.1625	0.1607

Table 2 The Malmquist indexes (VRS)

Malmquist multifactor productivity index					Malmquist productivity index		
			VRS				VRS
Year	Bv,t	Bv,t+1	Bv		Mov,t	Mov,t+1	Mov
1960	1.0459	1.0319	1.0389	1	1.0591	1.0334	1.0462
1961	1.1731	0.9064	1.0312	2	1.0369	1.0245	1.0307
1962	1.0437	1.0245	1.0341	3	1.0412	1.0263	1.0337
1963	1.0459	1.0071	1.0263	4	1.0674	1.0061	1.0363
1964	1.0459	1.0212	1.0335	5	1.0379	1.0232	1.0305
1965	1.0459	0.9727	1.0086	6	1.0439	0.9278	0.9842
1966	1.0298	1.0707	1.0500	7	1.0487	1.0268	1.0377
1967	1.0213	1.0031	1.0122	8	1.0233	1.0043	1.0138
1968	1.0459	1.0086	1.0271	9	1.0251	1.0081	1.0166
1969	1.0150	1.0016	1.0083	10	1.0129	1.0000	1.0064
1970	1.0681	1.0497	1.0588	11	1.0668	1.0505	1.0586
1971	1.0017	0.9880	0.9948	12	1.0055	0.9919	0.9987
1972	1.0154	1.0014	1.0084	13	1.0143	1.0018	1.0080
1973	0.9618	0.9343	0.9479	14	0.9598	0.9336	0.9466
1974	1.0162	1.0782	1.0467	15	1.0932	1.0753	1.0842
1975	0.9884	0.9717	0.9800	16	0.9897	0.9791	0.9844
1976	1.0486	1.0353	1.0419	17	1.0508	1.0370	1.0439
1977	1.0110	0.9714	0.9910	18	1.0075	0.9701	0.9886
1978	1.0208	1.0019	1.0113	19	1.0236	1.0046	1.0140
1979	0.9628	0.9481	0.9554	20	0.9625	0.9500	0.9562
1980	1.1072	1.0950	1.1011	21	1.1115	1.0933	1.1024
1981	1.0603	1.0308	1.0454	22	1.0524	1.0319	1.0421
1982	0.9571	0.9260	0.9414	23	0.9541	0.9303	0.9422
1983	1.0720	1.0480	1.0599	24	1.0700	1.0468	1.0584
1984	1.0728	1.0518	1.0622	25	1.0703	1.0519	1.0611
1985	1.0463	1.0254	1.0358	26	1.0442	1.0244	1.0343
1986	1.0449	1.0181	1.0314	27	1.0418	1.0176	1.0296
1987	0.9763	0.9155	0.9454	28	0.9703	0.9523	0.9613
1988	1.0487	1.0372	1.0429	29	1.0524	1.0387	1.0455
1989	1.0087	1.0162	1.0125	30	1.0369	1.0174	1.0271
1990	1.0243	0.9953	1.0097	31	1.0217	1.0017	1.0117
1991	1.0915	1.0656	1.0785	32	1.0851	1.0658	1.0754
1992	0.9856	0.9648	0.9751	33	0.9883	0.9670	0.9776
1993	1.0666	1.0469	1.0567	34	1.0636	1.0480	1.0557
1994	0.9592	0.9330	0.9460	35	0.9541	0.9357	0.9449
1995	1.0862	1.0586	1.0723	36	1.0804	1.0574	1.0689
Geometric Mean	1.0328	1.0060	1.0193	0.0268	1.0317	1.0090	1.0203
STD	0.0451	0.0476	0.0400		0.0401	0.0424	0.0403
Max	1.1731	1.0950	1.1011		1.1115	1.0933	1.1024
Min	0.9571	0.9064	0.9414		0.9541	0.9278	0.9422
GAP	0.2160	0.1887	0.1597		0.1574	0.1655	0.1602

Table 3 : Comparison of the ratio of the Malmquist indexes

Year	Bc/Bv	Moc/Mov	Mov/Bv	Moc/Bv	Mov/Bc	Moc/Bc
1960	0.999	0.991	1.007	0.998	1.008	1.000
1961	0.999	1.001	1.000	1.000	1.000	1.001
1962	0.999	0.998	1.000	0.998	1.000	0.998
1963	0.987	0.977	1.010	0.987	1.023	0.999
1964	0.998	1.000	0.997	0.997	0.999	0.999
1965	0.973	0.997	0.976	0.973	1.002	1.000
1966	0.981	0.998	0.988	0.986	1.007	1.005
1967	1.002	1.000	1.002	1.002	0.999	0.999
1968	0.989	1.000	0.990	0.990	1.000	1.001
1969	1.000	1.002	0.998	1.000	0.998	0.999
1970	1.001	1.002	1.000	1.001	0.999	1.000
1971	1.001	0.997	1.004	1.000	1.003	1.000
1972	1.000	0.999	1.000	0.999	1.000	0.999
1973	1.002	1.003	0.999	1.002	0.997	1.000
1974	1.000	1.004	1.036	1.040	1.036	1.040
1975	1.002	0.997	1.004	1.001	1.003	0.999
1976	1.000	0.999	1.002	1.000	1.001	1.000
1977	0.998	1.001	0.998	0.998	1.000	1.000
1978	1.002	0.999	1.003	1.002	1.001	1.000
1979	0.999	1.000	1.001	1.001	1.002	1.002
1980	1.001	1.000	1.001	1.002	1.000	1.001
1981	1.002	1.005	0.997	1.002	0.995	1.000
1982	1.005	1.004	1.001	1.004	0.996	0.999
1983	1.000	1.002	0.999	1.000	0.999	1.000
1984	0.997	0.998	0.999	0.997	1.002	1.000
1985	0.999	1.000	0.999	0.999	1.000	1.000
1986	0.998	0.999	0.998	0.998	1.001	1.000
1987	1.015	0.999	1.017	1.016	1.001	1.000
1988	0.998	0.997	1.002	0.999	1.004	1.001
1989	0.997	0.997	1.014	1.012	1.017	1.014
1990	1.001	0.998	1.002	1.000	1.001	1.000
1991	0.998	1.001	0.997	0.998	0.999	1.000
1992	0.996	0.994	1.003	0.997	1.006	1.001
1993	0.999	1.000	0.999	0.999	1.000	1.000
1994	0.995	0.997	0.999	0.996	1.004	1.001
1995	0.999	1.003	0.997	0.999	0.998	1.000
STD	0.007	0.005	0.009	0.010	0.008	0.007

Table 4: Decomposed outcome of the Malmquist multifactor productivity index(CRS)

Year	Efficiency change	Technical change	Scale factor
1960	0.970	1.069	1.000
1961	0.986	1.046	0.999
1962	1.058	0.976	0.999
1963	0.999	1.014	1.001
1964	1.002	1.028	1.001
1965	0.975	1.007	1.000
1966	0.998	1.038	0.978
1967	1.033	0.982	1.001
1968	0.981	1.036	0.999
1969	1.015	0.993	1.001
1970	1.005	1.055	1.000
1971	0.964	1.032	1.000
1972	0.986	1.021	1.001
1973	0.976	0.973	1.000
1974	1.009	1.080	0.966
1975	0.977	1.004	1.001
1976	1.038	1.005	1.000
1977	1.004	0.986	1.000
1978	1.031	0.982	1.000
1979	0.930	1.028	0.998
1980	1.038	1.063	0.999
1981	1.035	1.012	1.000
1982	0.974	0.971	1.001
1983	1.022	1.037	1.000
1984	0.986	1.074	1.000
1985	1.031	1.003	1.000
1986	0.988	1.041	1.000
1987	0.916	1.048	1.000
1988	1.050	0.993	0.999
1989	0.975	1.050	0.974
1990	0.984	1.027	1.000
1991	1.035	1.040	1.000
1992	0.964	1.008	0.999
1993	1.014	1.042	1.000
1994	0.985	0.957	0.999
1995	0.992	1.080	1.000
Geometric mean	0.998	1.022	0.998
STD	0.032	0.034	0.008
Max	1.058	1.087	1.001
Min	0.916	0.957	0.966
GAP	0.141	0.130	0.035

Table 5: Decomposed outcome of the Malmquist multifactor productivity index(VRS)

Year	Efficiency change	Technical Change	Scale factor
1960	0.987	1.095	0.961
1961	0.989	1.055	0.988
1962	1.040	1.007	0.987
1963	0.997	1.056	0.974
1964	1.003	1.039	0.991
1965	0.972	0.962	1.078
1966	1.006	1.045	0.999
1967	1.025	0.991	0.996
1968	0.987	1.035	1.006
1969	1.010	0.996	1.002
1970	1.002	1.083	0.976
1971	0.988	1.007	1.000
1972	0.986	1.024	0.999
1973	0.970	0.943	1.036
1974	1.014	1.108	0.931
1975	0.988	0.986	1.006
1976	1.019	1.043	0.980
1977	0.998	0.975	1.018
1978	1.030	0.986	0.995
1979	0.935	0.997	1.025
1980	1.040	1.108	0.955
1981	1.024	1.034	0.988
1982	0.992	0.916	1.036
1983	1.004	1.079	0.979
1984	0.987	1.103	0.976
1985	1.026	1.021	0.989
1986	0.995	1.044	0.993
1987	0.934	1.004	1.008
1988	1.029	1.036	0.979
1989	1.009	1.027	0.977
1990	0.990	1.022	0.997
1991	1.011	1.098	0.971
1992	0.989	0.972	1.014
1993	0.987	1.095	0.978
1994	0.994	0.920	1.035
1995	1.001	1.098	0.976
Geometric mean	0.999	1.027	0.994
STD	0.024	0.052	0.027
Max	1.040	1.108	1.078
Min	0.934	0.916	0.931
GAP	0.106	0.192	0.147

Table 6 Decomposed outcome of the Malmquist Productivity index(CRS)

year	Efficiency Change	Technical Change	scale efficiency change
1960	0.975	1.070	0.995
1961	0.989	1.046	0.997
1962	1.040	0.975	1.017
1963	0.997	1.014	1.002
1964	1.003	1.029	0.999
1965	0.972	1.008	1.002
1966	1.006	1.038	0.992
1967	1.026	0.982	1.007
1968	0.986	1.036	0.994
1969	1.010	0.994	1.005
1970	1.002	1.054	1.003
1971	0.988	1.032	0.976
1972	0.986	1.021	1.001
1973	0.970	0.973	1.006
1974	1.015	1.080	0.994
1975	0.988	1.004	0.989
1976	1.019	1.005	1.018
1977	0.998	0.986	1.005
1978	1.030	0.982	1.001
1979	0.935	1.028	0.995
1980	1.040	1.063	0.998
1981	1.024	1.012	1.011
1982	0.992	0.971	0.982
1983	1.004	1.037	1.019
1984	0.987	1.074	0.999
1985	1.026	1.004	1.005
1986	0.995	1.041	0.993
1987	0.934	1.048	0.981
1988	1.029	0.993	1.020
1989	1.009	1.050	0.966
1990	0.990	1.027	0.994
1991	1.011	1.040	1.023
1992	0.989	1.008	0.975
1993	0.987	1.042	1.027
1994	0.994	0.957	0.991
1995	1.001	1.080	0.991
Geometric mean	0.9982	1.0218	0.9992
STD	0.024	0.033	0.014
Max	1.040	1.080	1.027
Min	0.934	0.957	0.966
GAP	0.106	0.123	0.061

Table 7: Correlation of the Malmquist multifactor productivity(CRS) and its components

	Efficiency change(TE)	**Technical change(T)**	**Scale factor(S)**
Correlation with the Bc	0.645	0.585	-0.095
	TE, T	T, S	TE, S
Correlation of the components	-0.217	-0.328	0.032

Table 8: Correlation of the Malmquist multifactor productivity(VRS) and its components

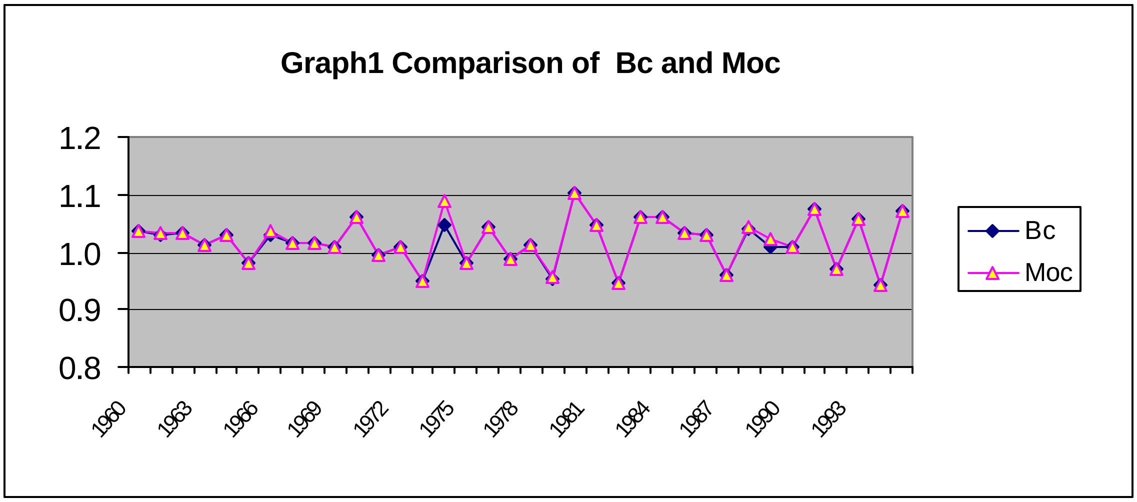
	Efficiency change(TE)	**Technical change(T)**	**Scale factor(S)**
Correlation with the Bv	0.595	0.891	-0.793
	TE, T	T, S	TE, S
Correlation of the components	0.242	-0.848	-0.493

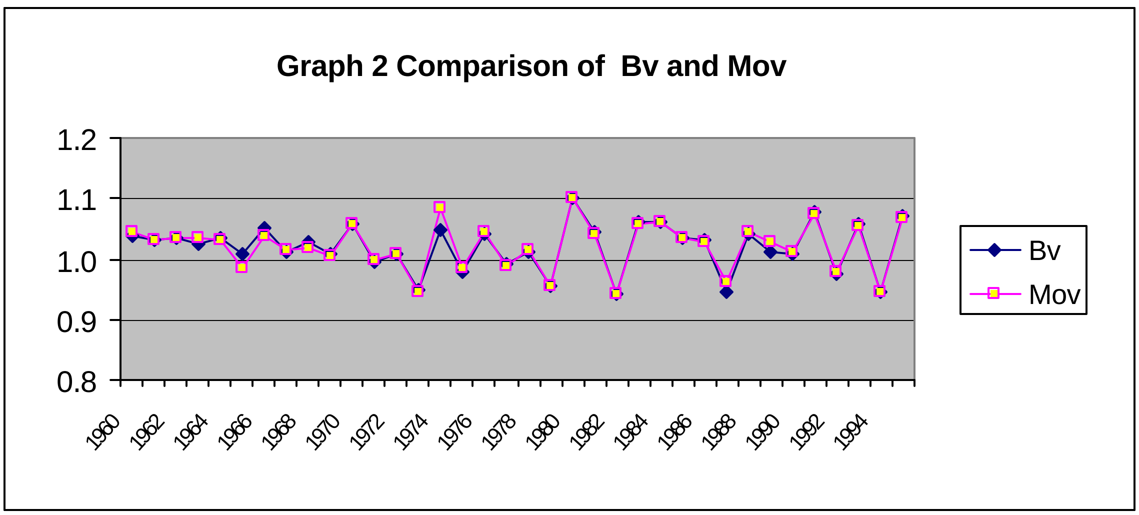
Table 9: Correlation of the Malmquist productivity(CRS) and its components

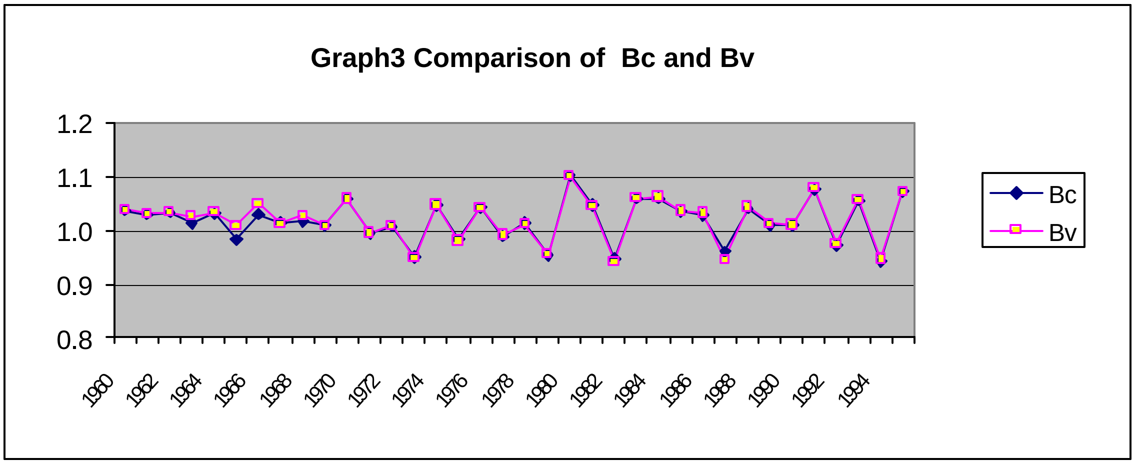
	Efficiency change (TE)	**Technical change (T)**	**Scale efficiency change(SE)**
Correlation with the Moc	0.582	0.636	0.412
	TE, T	T, SE	TE, SE
Correlation of the components	-0.175	-0.184	-0.368

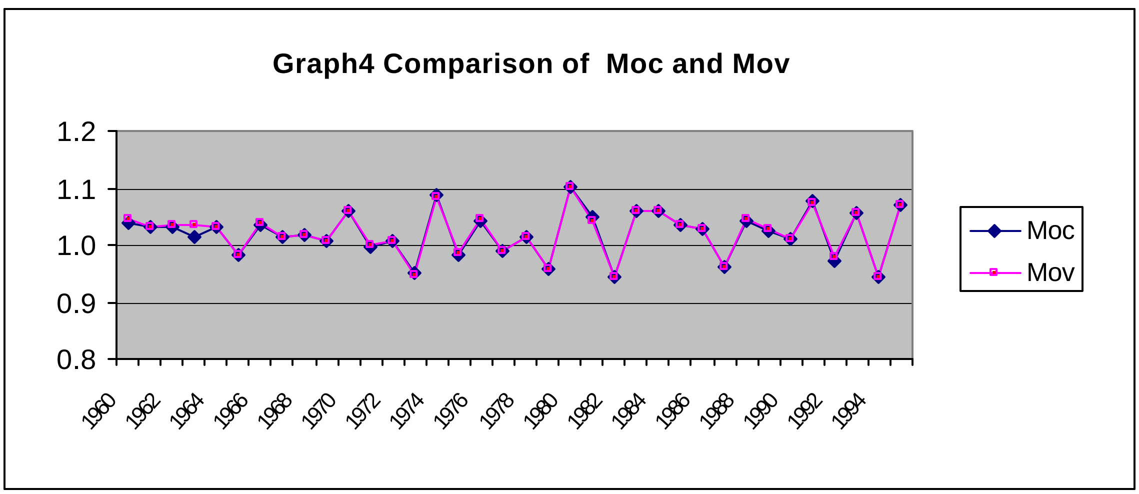
Table 10: Correlation of the same components of the Malmquist indexes

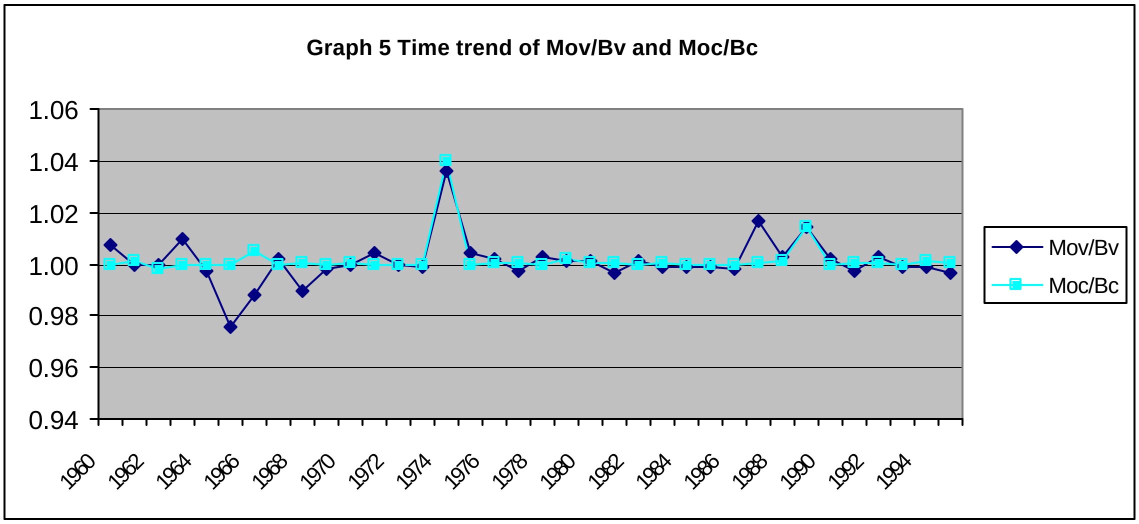
	TEs	Ts	SE, S
Correlation	0.997	0.839	-0.201

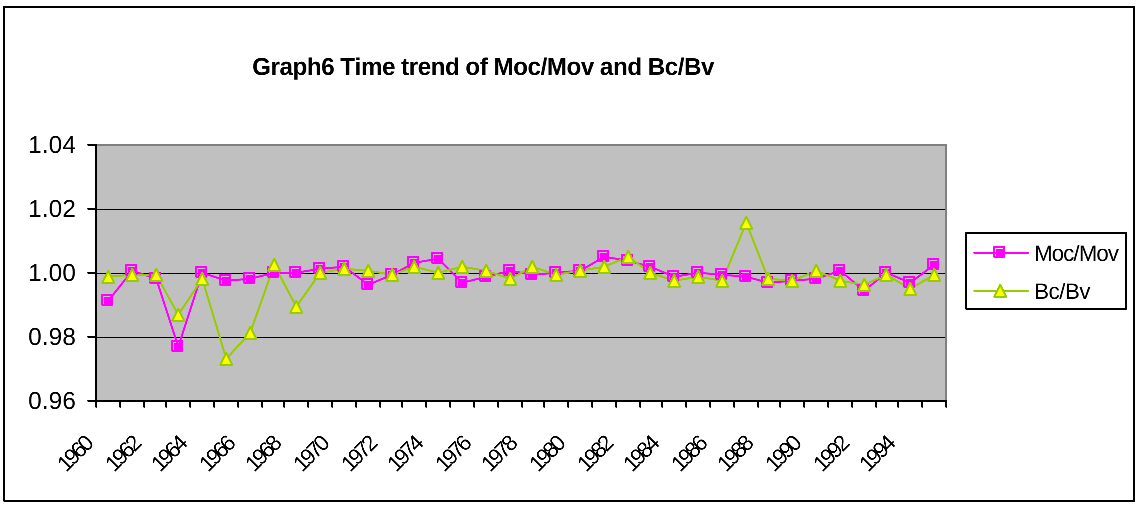

Graph1 Comparison of Bc and Moc
1.2
1.1
1.0
0.9
0.8
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
Bc
Moc

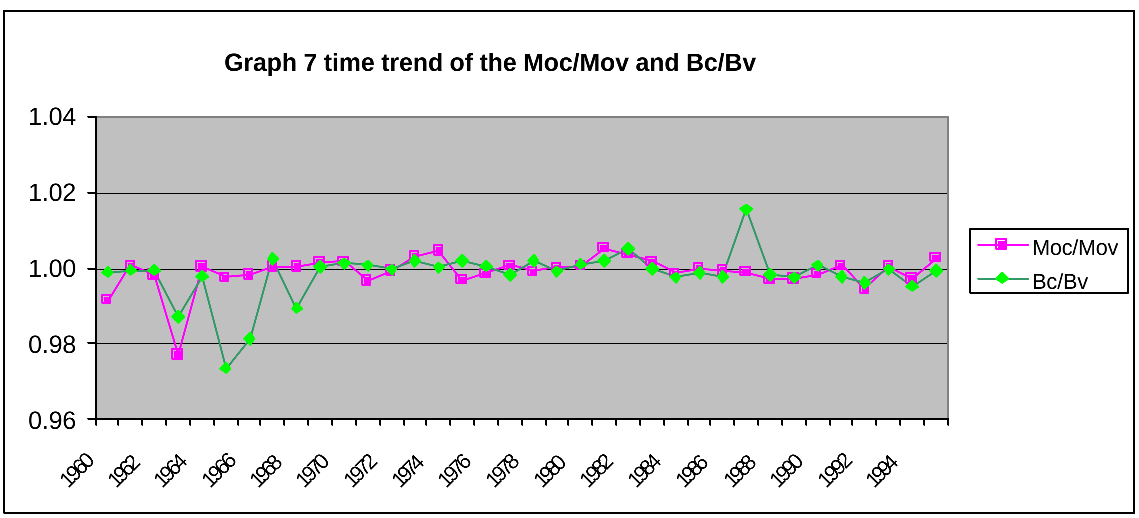

Graph 2 Comparison of Bv and Mov
1.2
1.1
1.0
0.9
0.8
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
Bv
Mov

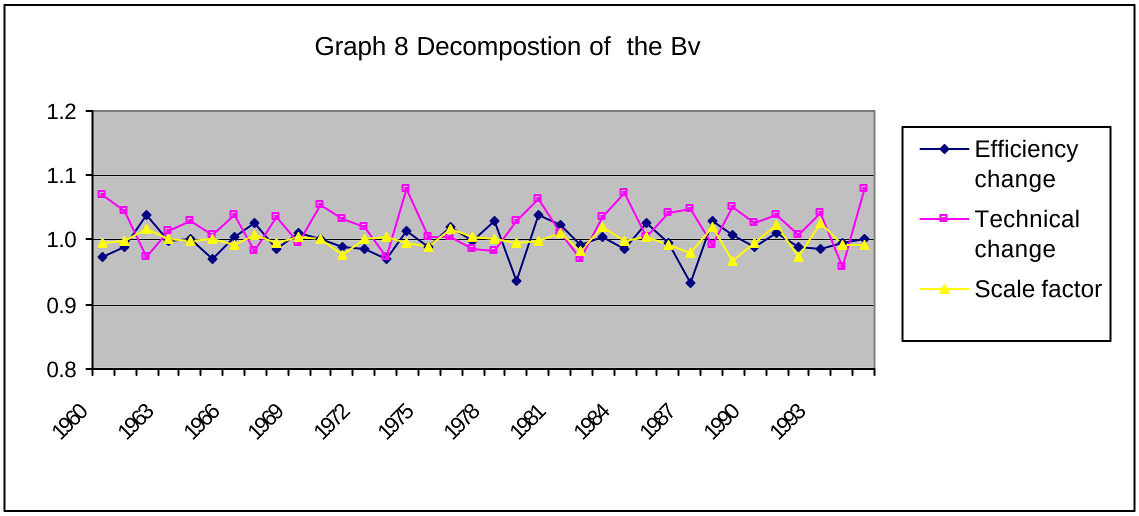

Graph3 Comparison of Bc and Bv
1.2
1.1
1.0
0.9
0.8
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
Bc
Bv

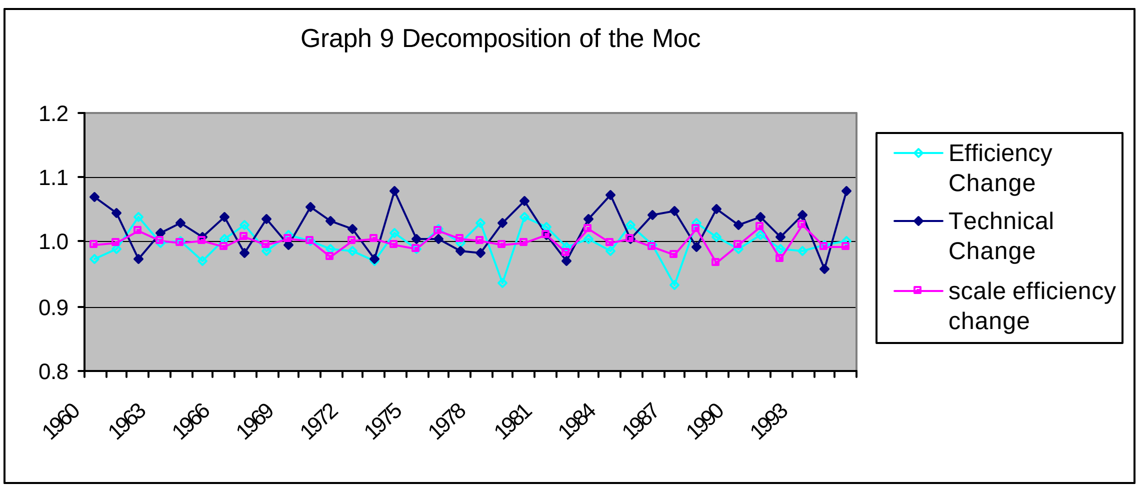

Graph4 Comparison of Moc and Mov
1.2
1.1
1.0
0.9
0.8
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
Moc
Mov

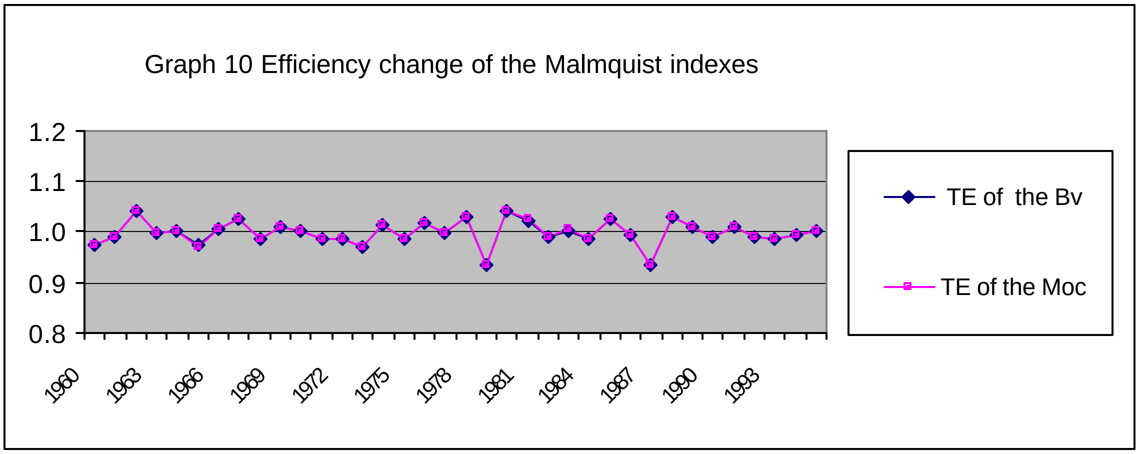

Graph 5 Time trend of Mov/Bv and Moc/Bc
1.06
1.04
1.02
1.00
0.98
0.96
0.94
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
Mov/Bv
Moc/Bc

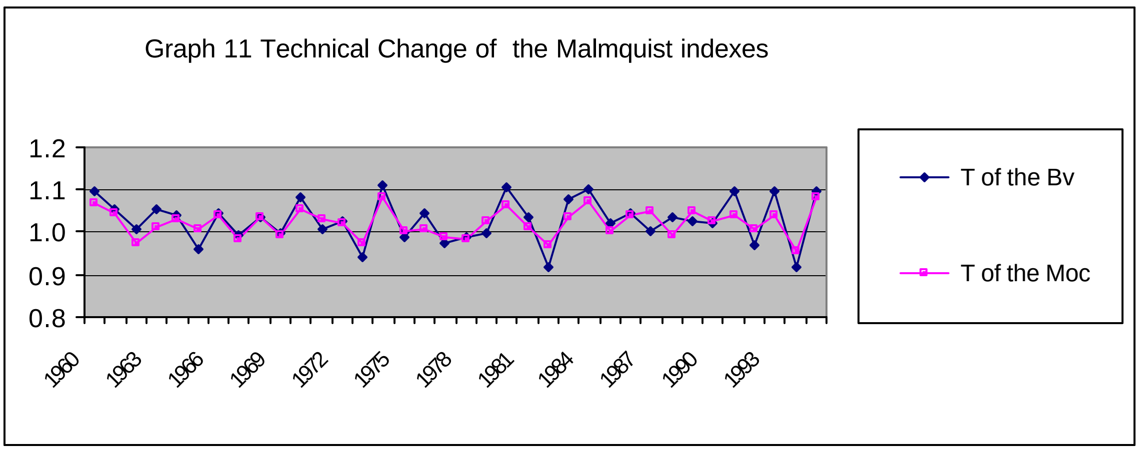

Graph6 Time trend of Moc/Mov and Bc/Bv
1.04
1.02
1.00
0.98
0.96
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
Moc/Mov
Bc/Bv

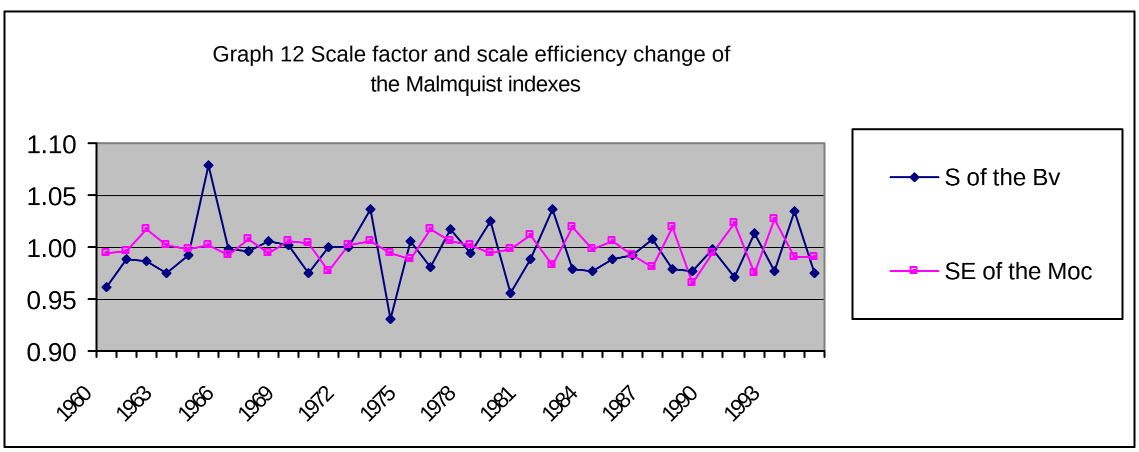

Graph 7 time trend of the Moc/Mov and Bc/Bv
1.04
1.02
1.00
0.98
0.96
1960
1962
1964
1966
1968
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
Moc/Mov
Bc/Bv


Graph 8 Decompostion of the Bv
1.2
1.1
1.0
0.9
0.8
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
Efficiency change
Technical change
Scale factor


Graph 9 Decomposition of the Moc
1.2
1.1
1.0
0.9
0.8
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
Efficiency Change
Technical Change
scale efficiency change


Graph 10 Efficiency change of the Malmquist indexes
1.2
1.1
1.0
0.9
0.8
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
TE of the Bv
TE of the Moc


Graph 11 Technical Change of the Malmquist indexes
1.2
1.1
1.0
0.9
0.8
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
T of the Bv
T of the Moc


Graph 12 Scale factor and scale efficiency change of the Malmquist indexes
1.10
1.05
1.00
0.95
0.90
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
S of the Bv
SE of the Moc