PROBLEM POSING IN MIDDLE-GRADES MATHEMATICS CLASSES

by

CLAYTON NEAL KITCHINGS

(Under the Direction of James W. Wilson)

ABSTRACT

Problem posing refers to the generation of new problems or the reformulation of previously given problems. Problem posing has received increased attention recently, but studies indicate problem posing is an emerging topic in mathematics education research. In this observational, interpretive study, I observed 88 mathematics lessons from six teachers in Grades 5 through 7. I analyzed transcripts from the 88 filmed lessons over 1 school year of instruction. I identified instances of problem posing across those 88 lessons in order to better understand when and how problem posing occurs. The teachers most commonly asked students to generate problems like an example problem, or they asked students to create contexts for routine mathematical exercises. I also focused on one teacher who engaged students in problem posing more than others in the sample. I interviewed that teacher in order to try to understand more about motivation to use problem posing and her past experience with problem posing. I asked stimulated recall questions and showed her video clips from her class in order to prompt her to reflect on various instances of problem posing from the school year. For Ms. Green, motivation to use problem posing included differentiation, connections to real-life contexts, and student engagement. Based on these findings, I recommend additional research to learn about the prevalence of problem posing as well as ways to encourage more teachers to use problem posing in mathematics classes.

INDEX WORDS: Problem posing, problem solving, mathematics education, mathematics teacher education

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DEDICATION

For the love and pursuit of Jesus Christ, who "is the image of the invisible God, the firstborn of all creation. For by him all things were created, in heaven and on earth, visible and invisible, whether thrones or dominions or rulers or authorities—all things were created through him and for him. And he is before all things, and in him all things hold together. And he is the head of the body, the church. He is the beginning, the firstborn from the dead, that in everything he might be preeminent. For in him all the fullness of God was pleased to dwell, and through him to reconcile to himself all things, whether on earth or in heaven, making peace by the blood of his cross" Col. 1:15–20 (English Standard Version).

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CHAPTER 1

INTRODUCTION

I have had an interest in problem *solving* since a very young age. As an elementary and middle school student, I encountered what teachers called “problem solving” in most of my mathematics classes. I remember asking on more than one occasion questions such as, “When will you teach us *problem solving*?” or “How do you *do word problems*?” In my undergraduate studies in mathematics education, I began to understand problem solving from a different perspective. I began to view it as a means for teaching mathematics, and I also viewed mathematics as a means for helping individuals learn to solve problems.

One initial experience with problem posing had a large impact on me. Only in hindsight did I realize that the experience was a problem-posing (PP) experience. In my post-student-teaching seminar, our instructor asked us about some of our concerns (a very open-ended question). Many in our class mentioned a concern with our lack of ability to explain fractions to our students who had trouble with fractions. At the time, many of us indicated that we thought we understood fractions well, but that we lacked the ability to explain the topic well.

The instructor responded the next day by bringing in manipulatives to us to increase our conceptual understanding of fractions. When she asked us about the procedure for dividing two fractions, most of us responded with the typical “flip and multiply” response (or some similar algorithmic device sufficient for dividing two fractions procedurally). Most of us indicated that we thought we knew very well how to do it. The instructor asked us to provide a “real” (or at least feasible) problem that would model the division of two fractions. When she requested this example from our class, the proverbial crickets chirped loudly, as we had no good responses. It occurred to me immediately that my conceptual understanding of fractions was sorely lacking. Anecdotally (and upon reflecting on this event later), I learned from a problem-posing scenario about my lack of a robust, conceptual understanding of fractions. I was enthusiastic to recently read such a scenario described in *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001, pp. 386–388).

On occasions, as a practicing secondary mathematics teacher, I informally experimented with problem posing on my own without a formal understanding of the notion of problem posing. I warmed to the informal idea of problem posing while teaching students in an honors Algebra 2 class. I occasionally remember that an honors student might ask, “What happens if (a constraint is modified or manipulated)?” within some mathematical context. It seemed at the time that the inquisitive nature of the honors or accelerated students stimulated more discussion and deeper understanding. Over time, I ended up asking students to test questions “in reverse” by providing some sort of answer and asking them to find the original problem, question, or equation. For example, when estimating student’s understanding of polynomials and their roots, I told them that a certain polynomial function had roots of $x = 2$, $x = 3$, and $x = –4$. I told them to use those roots to find the problem (or a problem or function) consistent with the above information. I posed the question “in reverse” because they typically saw a polynomial generated by an external authority (such as their teacher or their textbook), and they were prompted to identify the roots of the polynomial. I determined that I learned quite a bit about what my students thought about polynomials and their roots by asking such a question in reverse. I also found that such questions provided students with at least some suggestion of autonomy in their learning. It seemed that such question reversals created a different kind of thinking environment for my students. In another example I sometimes provided a rough drawing of a graph of a function and asked them to create reasonable equations that could possibly match with the presented graph as opposed to providing them with an equation and asking them to produce a graph. In this way students demonstrated their levels of understanding about what happens with transformations of various functions. I did not think formally about the notion of problem posing during that time, but I recognized that there seemed to be some value in giving students the answer they typically gave and asking them to describe a problem that fit well with the given answer.

Problem posing may be regarded as a means by which students may take progressive steps in anticipation of becoming productive citizens in a largely democratic society. For example, the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000) claimed that students should experience the opportunity to "formulate interesting problems based on a wide variety of situations, both *within and outside mathematics* [emphasis added]" (p. 258). One feasible aim of problem posing in a mathematics class is to provide students with experiences where they may question constraints provided in a given scenario, identify problematic features of the scenario, carefully formulate (or reformulate) the problem, and begin searching for potential solution methods or solutions. Mathematical problem-posing experiences may provide students with at least some entry-level acquaintance with larger scaled problematic scenarios that they may encounter outside formal school environment. With this in mind, teachers should give all students opportunities to experience problem posing throughout all grade levels.

The *Professional Standards for Teaching Mathematics* (NCTM, 1991) state, "Students should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95). It therefore seems reasonable to assume that problem posing is well suited for all mathematics courses and grade levels. It is important to increase the awareness of how the tool of problem posing may be used by more mathematics teachers. Why, then, do students rarely, if ever, experience opportunities to publically pose mathematics problems (Silver, 1994, p. 19)? Perhaps problem posing will increase as more teachers consider problem posing as a viable pedagogical tool.

Background and Rationale

Einstein and Infeld (1938) claimed that

> the formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, requires creative imagination and marks real advance in science. (p. 92)

Silver, Kilpatrick, and Schlesinger (1990) observed that problem posing is frequently overlooked in discussions of problem solving (p. 15). They also called for problem posing to receive an emphasis of equal importance to problem solving because problem posing and problem solving are related activities (p. 16).

The present study is important because it helps to reveal environments or situations that may help engender problem posing. It also helps to paint a plausible picture of the extent to which problem posing may occur in certain contexts. Brown and Walter (2005) provided examples of several benefits from the use of problem posing (pp. 1–6). They also argued that "the activity of problem posing ought to assume a greater degree of centrality in education" (p. 6). In order to nudge problem posing closer to the forefront of the discussion of what is important in mathematics education, I thought it reasonable to ask questions about what may count as problem posing in addition to when or how it might already occur in current mathematics classes.

Anecdotally, it seems that problem posing is not present (formally) in many mathematics education courses or mathematics education programs. Problem posing and problem solving are related activities. An analysis of some of Polya's writings (Polya, 1981a; 1981b; 2004) suggests that the two constructs are indeed connected, even though he did not explicitly use the term *problem posing*. The term *problem posing* recently occurred in two National Council of Teachers of Mathematics (NCTM) conference session titles (including session abstracts) out of 614 total session titles at the 2014 NCTM Annual Meeting and Exposition in New Orleans. By contrast, a document search of the *Program Book* (NCTM, 2014a) for references to problem *solving* occurred on 69 *pages* (not mere sessions) with multiple references on most of those pages. References to *problem posing* were much less frequent than references to *problem solving*. The lack of mention of problem posing is noteworthy even if problem-posing activities existed within the sessions on problem solving. The main point is that problem posing received almost no direct attention relative to problem solving in the list of all of the sessions.

Kilpatrick, Swafford, and Findell (2001) related problem posing to strategic competence (p. 124). They described strategic competence as a strand "similar to what has been called problem solving and problem formulation in the literature of mathematics education and cognitive science, and mathematical problem-solving, in particular, has been studied extensively" (p. 124). They implied that problem formulation was not widely discussed in the literature. They argued that students should have experiences formulating problems in order that they might use mathematics to solve them (p. 124). They also noted that many situations that students encounter outside of the classroom require students to determine what the problem actually is, and this is typically a difficult task (p. 124).

It seems that *problem posing* is not discussed as frequently among mathematics educators as *problem solving* is. In fact, Cai, Hwang, Jiang, and Silber (in press) observed, “Problem-posing research is a relatively new endeavor” (p. 4). They also observed that “the importance of problem posing in school mathematics has required slightly more explanation” (p. 6). They identified 14 unanswered questions concerning problem-posing research along with a brief explanation for each such question. Some of the unanswered questions include:

- What kinds of problem-posing tasks might help reveal mathematical understandings or misunderstandings (p. 26)?
- To what extent are problem-posing tasks represented in mathematics curricula (p. 26)?
- What features of the classroom environment are necessary for problem-posing instruction in classrooms (p. 35)?

Cai et al. also specifically identified the need to carefully analyze classroom practices where problem posing occurs (p. 30). They further observed, “few researchers have tried to carefully describe the dynamics of classroom instruction where students are engaged in problem-posing activities” (p. 34). They continued, “…researchers will need to identify those features that are most relevant for problem posing and which may be most influenced by the introduction of problem-posing activities” (pp. 35–36). It seems that they wish to identify features *of instruction* that may engender or encourage students to pose problems. They did not ask about what counts as problem posing, but rather their goal was to list characteristics of instruction that support problem posing.

Wilson, Fernandez, and Hadaway (1993) observed that problem posing and problem solving are very much related. They also commented on the amount of research on problem formulation versus problem solving. They claimed, "Although there has been *little research in this area* [emphasis added], this activity [problem posing] has been gaining considerable attention in U. S. mathematics education in recent years" (p. 61). In fact, attention to problem posing increased in 2013 with the publication of a special issue of the journal *Educational Studies in Mathematics* (Singer, Ellerton, & Cai, 2013a) that was devoted solely to mathematical problem posing. In that issue, Singer, Ellerton, and Cai called for more studies on problem posing "[when] the solver is *required* [emphasis added] to reformulate the problem statement in order to develop solutions" (p. 3). They did not address situations where students pose a problem when they are *not* required to do so or when they were not reformulating a problem. They also observed "a need to study problem-posing techniques that are already practiced in some classrooms in order to analyze and extend those strategies" (p. 3). They also claimed, "The field of problem posing is still very diverse and lacks definition and structure" (p. 4). In the same issue, Ellerton (2013) found a disproportionate amount of attention that problem solving received compared with problem posing (p. 87). She also wrote of the silence of the literature concerning whether or not students need "supportive scaffolding" (p. 89) in order to venture into "the unexplored world of problem posing" (p. 89). Leung (2013) called for additional research to investigate the implementation of problem posing as well as environmental conditions that may favor mathematical problem posing (p. 114).

At first I was uncertain as to how or where I might locate teachers who engaged their students in problem posing. Nevertheless, I observed problem posing while filming mathematics lessons as a part the Discourse in Mathematics Classrooms (DIMaC) (Bishop, 2012; Stockero, Bishop, & Conner, 2014) research project. The project focused on mathematics classroom discourse in which I participated. My initial observation of problem posing caused me to question the frequency with which problem posing might occur within participant's classes in the research project in addition to the environmental conditions surrounding such problem-posing episodes. That curiosity initialized this study of problem posing. If fact, some evidence exists to suggest a link between discourse and problem posing. Singer, Ellerton, and Cai (2013b) proposed a possible link between discourse and problem posing (p. 4) that convinced me to pursue a study in search of instances of problem posing situated within another broader study of discourse in middle grades mathematics classes. In other words, given a sample of teachers with a perceived reputation for higher levels of mathematical discourse, what does problem posing look like, if it occurs at all? The literature is silent on this issue perhaps because researchers do not currently have a useful way to detect it.

Singer et al. (2013b) claimed that although problem posing is not new, there is a new "awareness that problem posing needs to pervade the education systems around the world, both as a means of instruction … and as an object of instruction" (p. 5). Kilpatrick (1987) provided an identical emphasis in his call for more problem formulation in mathematics (p. 123). He also claimed, "How to design instruction that will help students learn to formulate mathematical problems is itself a problem in need of a more complete formulation" (p. 139). Cai et al. (2013b) observed that "little research has been done to identify instructional strategies that can effectively promote productive problem posing or even to determine whether engaging students in problem-posing activities is an effective pedagogical strategy" (p. 58). They called for future research that might "focus on ways to integrate problem posing into regular classroom activities" (p. 67).

An Introduction to the Problem-Posing Framework

This study did not situate problem posing as an antecedent in a conditional statement. The following conditional statement summarizes much of the research on problem posing: "If problem posing occurs, then [something] happens." In other words, with one possible exception, all problem-posing research I analyzed contained reports where problem posing was a goal of the lesson. Problem posing was regularly described as free, semi-structured, or structured (Stoyanova & Ellerton, 1996). The only possible exception is the study from Da Ponte and Henriques (2013), but their study occurred in a university-level numerical analysis mathematics course. They observed students who engaged in mathematical activities and investigated how those activities led the students to pose problems.

This study occurred with a different assumption that did not assume problem posing as a given. In the study, I observed six middle grades classrooms in which the teachers had a reputation for numerous student-to-student and student-to-teacher discussions about mathematics. The main assumption was that *considerable* discourse already occurs in these classrooms. Given the assumption that discourse occurs, to what extent might problem posing occur, and how might it be detected? The over-arching question is as follows: "If these teachers engage students in discourse, does problem posing occur as well, and if so, what does it look like?"

I formulated the following research questions to help focus my study to address some of the above calls for additional research in problem posing in mathematics:

1. How often does problem posing occur in six Grades 5–7 mathematics classrooms? What types of problem posing occur in those classrooms? What mathematical processes do students use when engaged in problem posing?

2. What kind of classroom environment exists when students engage in problem posing activities? (Singer, Ellerton, Cai, & Leung, 2011, p. 149).
3. What understanding and perspectives on mathematical problem posing do teachers possess? For a teacher who uses mathematical problem posing in her instruction:
    a. To what extent are problem-posing episodes planned in advance?
    b. What prompts the problem-posing episodes that are planned?
    c. What is the participant's overall assessment of the potential learning impact of the problem-posing episodes?

I continued to reflect on these questions as I observed lessons and analyzed data. The second research question contains the citation because Singer, Ellerton, Cai and Leung (2011) identified it as an unanswered question in the literature.

CHAPTER 2

REVIEW OF RELATED LITERATURE

"Everybody knows what a problem is."

-Student S, Ms. Gold's 5th-grade class, August 30, 2013

The following is a review of literature related to problem posing. My own experience as well as anecdotal evidence suggests that the term *problem* is not always defined clearly. As a result, it seemed appropriate to trace the construct of problem prior to the construct of problem posing. How have previous authors treated the construct of problem in general terms and specifically in mathematics? How might their definitions (or lack of definitions) inform a study about problem posing? I begin with definitions for *problems* and *problem posing*, followed by a review of the literature. This review provides a trace of the treatment of the construct of problem in general and in mathematical terms. The review of the literature on problem posing follows the trace of the construct of problem in the literature.

For the purposes of this study, a *mathematics problem* is any command or invitation to engage (or by any internal or external stimuli) in mathematical action for the purposes of producing answers to mathematical questions requiring at lease *some* observable (even if minimal) form of cognitive effort; or, for producing additional mathematical questions. As a secondary definition, a task or scenario that has been traditionally considered a *problem* by the mathematics community shall be considered a *problem*, even if such a problem has been proven to have no solution (such as trisecting an angle by compass and straightedge). Any scenario that a teacher identifies as a problem was also considered a problem for the purposes of this study. In such cases I sometimes classifies such problems as routine exercises. I define *problem posing* as the generation of new problems or the reformulation of previously given problems (Duncker, 1945; Silver, 1994).

The Construct of Problem

Introduction

I follow the admonition of other researchers to keep a deliberately broad definition for the purposes of discussing problems and problem posing in efforts to somewhat mitigate the "agony" (Wilson, Fernandez, & Hadaway, 1993, p. 58). The construct of problem is often vague and ambiguous. A problem for one individual may not be a problem for another individual. Brown and Walter (2005) observed that the definition of the term *problem* is quite complicated. My first experience with this idea occurred in a reading of a report from Progressive Education Association (1940) entitled *Mathematics in General Education: A Report*. This report challenged my notions of the construct of a *mathematical problem* as contrasted with a *mathematical exercise*. At the same time, it is plausible to consider that an exercise for one student may be a problem for another student, depending on his or her experience with mathematics. Nevertheless, the report encouraged me to rethink what I meant when I used the term *problem*.

The definitions of *problem* in the literature range from broad to narrow. Agre (1982) provided an analysis of the concept of problem (without specificity to any content area). He claimed that all problems had some type of undesirable trait, at least in the eyes of the solver. He wrote, "A problem is an undesirable situation which may be solvable by some agent although probably with some difficulty" (p. 122). Charles and Lester (as cited in Bush, Leinwand, & Beck, 2000, p. 21) provided a definition in a similar light:

A problem is a task for which–

- *The person confronting the task wants or needs to find a solution;*
- *The person has no readily available procedure for finding a solution;*
- *The person makes an attempt to find a solution;*
- *A variety of solution routes may be appropriate for solving the problem*.

Bush, Leinwand, and Beck (2000) claimed a preference for this definition even though most mathematicians are not fond of using the defined word as a part of the definition. The point is that they preferred a broad definition because it "allows us to include many mathematics tasks as problems—from simple word problems to extended investigations" (p. 21). Reitman defined a problem as something that exists "when you have been given the description of something but do not yet have anything that satisfies that description" (as cited in Wilson et al., 1993). Others situated the definition in terms of the problem solver, and described him or her as one with a goal, a hindrance to obtaining the goal, and an acceptance of the goal (Henderson & Pingry, 1953).

Brownell (1942) described many possibilities for interpretations of the term *problem*: "Problems may be thought of as occupying intermediate territory in a continuum which stretches from the 'puzzle' at one extreme to the completely familiar and understandable situation at the other" (p. 416). Brownell's observation represents more of a continuum of ideas, whereas Schoenfeld's (1992) classifications are more discrete. Perhaps Brownell's (1942) most important observation is the idea that distinguishing problems from other learning tasks is "wholly or largely subjective" (p. 416). He further wrote, "The criteria could scarcely be other than subjective, for the crux of the distinction between problems and other situations lies in the peculiar relationship which exists between the learner and his task" (p. 416). Perhaps it is this peculiar relationship that makes the identification of a problem difficult for an outside observer.

Table 1 presents a synthesis of the construct of problem by looking at multiple scholars' research on problems and problem solving.

Table 1

Synthesizing Research on Problems

Researcher	Problem defined	Characteristics of a problem	Categories of problems (if applicable)	Related issues/ideas
Brownell, 1942	May be thought of as occupying intermediate territory in a continuum which stretches from 'puzzle' at one extreme to the completely familiar and understandable situation at the other; deliberately left vague	N/A	N/A	Problems exist on a continuum; classifications are highly subjective
Henderson & Pingry, 1953		The existence of some goal, some hindrance to achieving the goal, and an acceptance of the goal		
Polya, 1957/2004	N/A		1. Problems to find 2. Problems to prove	1. Practical problems 2. Mathematical problems
Reitman, (1965)	Something for which you have been given a description but do not yet have anything to satisfy that description.			
Kantowski, 1980	A situation for which the individual who confronts it has no algorithm that will guarantee a solution	Distinguished problems from exercises;	Routine exercises vs. nonroutine problems	N/A
Polya, 1962/1981a, Vol. 1	Partial definition: To have a problem means: to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim	N/A	1. Problems to find 2. Problems to prove	N/A
Polya, 1965/1981b, Vol. 2	(Same as Vol. 1)	N/A	1. One rule under your nose 2. Application with	N/A

			some choice 3. Choice of combination 4. Approaching research level	
Agre, 1982	An undesirable situation which may be solvable by some agent although probably with some difficulty	1. Problems must be solvable (have a viable solution), or at least thought to have a solution at one time 2. Outcome in doubt 3. Some aspect of situation is undesirable (affective domain); "undesirable" has broad meaning	N/A	Subjectivity of problem identification (for whom is this a problem); Problem implies consciousness of physical, social, psychological, or intellectual situation or object which is said to be the problem
Kilpatrick, 1985	1. Psychological problem: a situation in which a goal is to be attained and a direct route to the goal is blocked 2. Social-anthropological: a task; given and received in a transaction 3. Mathematical problem: a problem as constructed 4. Pedagogical problem: a problem as a vehicle	N/A	1. Psychological 2. Social-anthropological 3. Mathematical 4. Pedagogical	Psychological vs anthropological stance; Important to situate oneself in one of the four perspectives prior to providing definition
Kilpatrick, 1987	No definition given	N/A	N/A	N/A
Schoenfeld, 1992	N/A	Based on categories	1. Routine exercises 2. Traditional problems 3. Problems that are problematic	Discrete, mutually exclusive categories
Kilpatrick, Swafford, & Findell, 2001)	Routine: problems that the learner knows how to solve based on past experience Nonroutine: problems for which the learner does not immediately know a usable solution (p. 126)	N/A	N/A	N/A

Rutledge & Norton, 2008	N/A	Must cause solver to exhibit some threshold of cognitive struggle; subjective	N/A	N/A
Brown & Walter, 2005	Refused definition; referred to Agre, 1982	N/A	N/A	N/A
Bonotto, 2013	No definition; only examples given	N/A	Distinguished between problems and exercises	N/A
Koichu & Kontorovich, 2013	A task involving mathematical concepts and principles, for which the solution method is unknown in advance by solver	Should be mathematically meaningful for the solver	N/A	Meaningfulness of problem is highly subjective to the individual

Polya: On Mathematical Problems and Problem Solving

Polya (2004) wrote much concerning problem solving in mathematics. He did not provide a definition of *problem* despite all of his writings on problem solving. He instead provided contrasts for various problem types. He wrote, “Practical problems are different in various respects from purely mathematical problems, yet the principal motives and procedures of the solution are essentially the same” (p. 149). Why did he distinguish between *practical problems* and *mathematical problems*? He wrote, for a mathematics problem, solvers typically start with well-ordered concepts as compared with practical problems that begin with “hazy concepts” (p. 151)—concepts that become important parts of the problem once they are clarified. He continued:

> In a perfectly stated mathematical problem all data and all clauses of the condition are essential and must be taken into account. In practical problems we have a multitude of data and conditions; we take into account as many as we can but we are obliged to neglect some. (p. 152)

In context, it seems that he simply meant to distinguish the mathematical from the non-mathematical problems rather than to imply that mathematical problems are impractical.

Indirectly, he also introduced an additional classification for a mathematical problem: a mathematical problem that is *perfectly stated*. Kilpatrick (1987) somewhat merged the two distinctions by writing of the existence of "practical problem(s) involving mathematics" (p. 125).

Problems to Find and Problems to Prove

Polya also provided another distinction of types of problems: problems to find and problems to prove (p. 33). These two distinctions exist both directly and indirectly throughout his book, *How to Solve It*. He observed that the purpose of a *problem to find* is to find the unknown (p. 154). He identified numerous potential unknowns: "We may try to find, to obtain, to acquire, to produce, or to construct all imaginable kinds of objects" (p. 154). The principal parts of these kinds of problems are the unknown, the data, and the condition (p. 155). He believed that *problems to find* were more important in elementary mathematics and *problems to prove* were more important in advanced mathematics (p. 156). His notions of *problems to find* were not the same as finding or generating problems. Indeed, he also wrote, "To *find a new problem* (emphasis added) which is both interesting and accessible, is not so easy; we need experience, taste, and good luck" (p. 65). Of *problems to prove*, he wrote "the aim of a 'problem to prove' is to show conclusively that a certain clearly stated assertion is true, or else to show that is false" (p. 154). Of these kinds of problems, he emphasized the necessity of the problem solver to know "exactly, its principal parts, the hypothesis, and the conclusion" (p. 156).

Polya distinguished other types of problems as well. For example, he acknowledged the existence of *routine problems* (p. 171). He wrote,

> **Routine problem** may be called the problem to solve the equation $x^2 - 3x + 2 = 0$ if the solution of the general quadratic was explained and illustrated before so that the student has nothing to do but to substitute numbers –3 and 2 for certain letters which appear in

> the general solution. Even if the quadratic equation was not solved generally in "letters" but half a dozen similar quadratic equations with the numerical coefficients were just solved before, the problem should be called a "routine problem." (p. 171)

He failed to mention how to classify such a problem if students possessed no instruction or experience concerning methods of solving quadratic equations. How would the degree of a student's experience change the classification, if at all? The answer is not clear, but perhaps it would no longer be a routine problem depending on the solver. Regarding the routine problems, he cautioned that although routine problems may be necessary to teach mathematics, it is inexcusable to "make the students do no other kind" (p. 172). His classifications are helpful, but it would have been more helpful if he either (1) provided a definition for a *mathematics problem* or (2) explained why he decided not to do so (as some others decided, as described below). His views are repeated below as a means to contrast others' notions of *problems*. Incidentally, Kilpatrick, Swafford, and Findell (2001) similarly classified problems as either routine or nonroutine (p. 124). They also did not provide a definition for a problem in the more general sense. Because I use the *routine exercise* as a classification of a type of problem, I now discuss additional examples of *routine problems* in the literature.

Polya's (2004) use of the term *routine problem* as a classification of a certain type of problem is a strength of his discussion of problems. Schoenfeld (1992) tweaked this distinction and used the term *routine exercise* instead. But why did Schoenfeld consider such *routine exercises* as one of three classifications of *problems*? Are these items *problems* or *exercises*? Is a *routine exercise* also a *problem*? The answer is in the affirmative for Schoenfeld, possibly because some teachers use the term *problem* even when referring to routine exercises, and the prevalence of the use of the term *problem* led him to include it as one type of problem.

In Polya's (1981a) later work, *Mathematical Discovery, Volume 1*, he began a discussion for the definition of *problem* in mathematics by providing a nonexample of a *problem*. He observed, "If the desire brings to my mind immediately, without any difficulty, some obvious action that is likely to attain the *desired* [emphasis added] object, there is no problem" (p. 117). Note that for Polya, in this case there is reference to the affective domain with the mention of the term *desired*. Polya wrote, in comparison to the nonexample of a problem, "to have a problem means: *to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim*" (p. 117). Observe that this is not a definition but rather an implication of some underlying definition. One may generalize his statement as: "If problem, then action occurs" as opposed to "if certain criteria, then problem exists." He once again mentioned the same two classifications of problems: (1) problems to *find* and (2) problems to *prove* (p. 119). These classifications remained as descriptions (as was the case in his 2004, *How To Solve It*) and did not rise to the level of a definition.

In the second volume of Polya's *Mathematical Discovery* (Polya, 1981b), he provided four additional problem classifications (p. 139). These types differ from the two classifications he provided earlier. Polya identified the following new problem classifications:

(1) *One rule under your nose*.

(2) *Application with some choice*.

(3) *Choice of combination*.

(4) *Approaching research level* (p. 139).

The first classification implies little cognitive stress, whereas the fourth classification is a more robust level of problem, perhaps somewhat analogous to the high cognitive demand level that Stein, Smith, Henningsen, and Silver (2009) branded as *Doing Mathematics* (pp. 3–4). Polya's four categorizations represent lower to higher levels of critical thinking, with each subsequent level requiring a higher level of cognition than the preceding level.

Kilpatrick (1985) parsed the definition for problem by situating it first inside of a particular perspective. For example, he determined that Polya's four classifications emerged from a "pedagogical perspective" (p. 4) rather than a "*psychological*" (p. 2) perspective. Note that, for Kilpatrick, it seemed important to situate oneself within a particular perspective (presumably consciously or perhaps even subconsciously) prior to providing a definition of *problem*. He wrote, "A problem is defined generally as a situation in which a goal is to be attained and a direct route to the goal is blocked" (p. 2). He further noted that the term *problem* is subjective in nature and is "often overlooked in discussions of problem solving" (p. 2). Elsewhere, Kilpatrick (1987) observed that psychologists "are fond of reminding us that a problem is not a problem for you until you accept it and interpret it as your own" (p. 124). In Schoenfeld's (1987) discussion of Kilpatrick's chapter, Schoenfeld wrote, "There are various classes of problems but … students rarely work on *real problems* [emphasis added]. Most of their work is on exercises that are called problems but that aren't problematic" (p. 144). Perhaps it is indeed useful to consider the notion of *exercises* as an entry-level type of *problem*.

Kilpatrick (1985) contrasted a *psychological perspective* with a *social-anthropological perspective* (pp. 2–3). He described the socio-anthropological problem situation as *transactional*: the problem "is given and received in a transaction" (p. 3). In such a view the problem is a task given by one entity to another entity for interaction and for solving. Perhaps this perspective is what Brown and Walter (2005) demonstrated when they wrote, "People tend to view a situation or even a problem as something that is *given* and that must be responded to in a small number of ways" (p. 5). It thus seems the implication is that people do not typically view themselves as creators of problems.

By way of a brief summary so far, it seems the notion of *problem* is quite complex, and the term *problem* is difficult to define. Below I explain how I chose three classifications of problems to help describe instances of problem posing.

Problem Posing: Definitions, Influences, and Research

What is Problem Posing?

Scholars have reported the difficulty inherent in attempts to define *problem posing*. For example, Singer et al. (2013b) acknowledged, "The field of problem posing is still very diverse and lacks definition and structure" (p. 4). My study affirms Singer et al.'s claim that "challenges remain … in defining the characteristics of problem posing, identifying possible relationships between the various subcategories of problem posing, and investigating possible interrelationships and interdependence between problem posing and problem solving in theory and in practice" (p. 3). One might also question what Singer et al. meant by "defining the characteristics of problem posing."

Stoyanova (1998) found it useful to use a definition for *problem posing* that was "deliberately broad" (p. 165). She claimed that a broad definition "provides the freedom in the design of a wider range of problem-posing situations and interrelationships between problem posing and problem solving" (p. 165). She also explained that a broad definition is useful in light of the idea that problem posing may have potential to "fit within the goals of mathematical instruction in the larger context of school mathematics" (p. 165).

Singer et al. (2013b) claimed that although problem posing is not new, there is a new "awareness that problem posing needs to pervade the education systems around the world, both as a means of instruction… and as an object of instruction…" (p. 5). They did not cite Kilpatrick (1987), who also hypothesized, "Problem formulating should be viewed not only as a *goal* of instruction but also as a *means* of instruction" (p. 123).

I trace some of the history of the topic of problem posing in the literature, beginning with two publications that began my journey into problem-posing research. I then provide a framework for identifying and describing instances of problem posing followed by a critique of a few research studies on problem posing.

Initial Exposure to Problem Posing

I begin with Brown and Walter (1983) because I encountered their work, *The Art of Problem Posing*, as a starting point in my research on problem posing. They provided an updated edition in 2005 (Brown & Walter, 2005). This work is cited regularly in problem-posing research. For example, a recent special issue of *Educational Studies in Mathematics* (Singer, Ellerton, & Cai, 2013c) contained 10 empirical research articles concerning problem posing. Eight of the 10 articles referenced Brown and Walter's (1983 or 2005) work on problem posing. Brown and Walter did not provide an empirical study in their book. As the title of their book states, they presented problem posing as something of an art. They provided numerous strategies that may help individuals (teachers or students) increase their ability to pose mathematics problems. They feared that individuals were "robbed of the opportunity of asking questions" (p. 3) when they were not allowed or not encouraged to reformulate problems. They also saw a correspondence between asking questions and problem posing (p. 3). They questioned, "When given the opportunity to pose problems on our own, what *sorts of questions* [emphasis added] do we ask? (p. 11). They also identified *what-if-not* questions as forms of problem posing. They also pressed for more centrality of problem posing in school mathematics (p. 6).

Brown and Walter initiated their discussion of problem posing with an example of a basic Diophantine equation: the specific case of Pythagorean Triples (though not initially identified as such). They began with the equation $x^2 + y^2 = z^2$ (p. 12). They then observed the equation is not at all a question or a command to act, but "if anything, it begs for you to *ask a question* or to *pose a problem* rather than to answer a question" (p. 13). One example of questioning in this instance is to inquire about the domain restrictions for x, y, and z. The distinction between allowing integers only versus also allowing rational numbers is a form of problem posing if one questions "what happens if …" for both sets of domains. They observed that any questions one may ask about the equation above may be classified as problem posing. Either any questions or curiosities that arise from some context (or even some equation) are problem posing, or such curiosities may quickly suggest a problem to pose. They also provided a "Handy List of Questions" (pp. 30–31) as potential questions to use to begin a problem-posing episode, even though some questions may not be germane for all contexts. The only research paper I located with a small critique of Brown and Walter was from Wilson et al. (1993). They noted a lack of any empirical research (p. 65) in Brown and Walter's book.

In addition to Brown and Walter, Silver (1994) influenced my thoughts about mathematical problem posing. Silver's article was theoretical and not empirical. It provided a synthesis of research and highlighted the calls for increased problem posing in respected mathematics reform documents, including NCTM (1989). He theorized, "In classrooms where children are encouraged to be autonomous learners, problem posing would be a natural and frequent occurrence" (p. 21). Silver summarized three conclusions from the research at the time of his article: (1) problem posing can help researchers view students' mathematical thinking; (2) problem posing can provide insight into the possible connection between cognitive and affective dimensions of students' mathematical learning; and (3) much more research is necessary into problem posing (p. 25).

Influences on Problem Posing in the Literature

Polya (2004) provided much discussion about mathematical problem solving. He also contributed to the discussion of problem posing. However, as Wilson et al. (1993) observed, Polya "did not talk specifically about problem posing, but much of the spirit and format of problem posing is included in his illustrations of looking back" (p. 61). It seems clear that Polya valued the generation of mathematics problems. For example, he wrote, "The mathematical experience of the student is incomplete if he never had an opportunity to solve a *problem invented by himself*" (p. 68). It seems Polya had a clear expectation that students should have opportunities to create problems. He also had much advice concerning the notion of "looking back" (p. 14) as a problem-solving strategy. As mentioned above, it seems he expected the looking back phase to frequently generate new questions or problems. He claimed:

> We should not fail to look around for more good problems *when we have succeeded* [emphasis added] in solving one. Good problems and mushrooms of certain kinds have something in common; they grow in clusters. Having found one, you should look around; there is a good chance there are some more quite near. (p. 65)

Kilpatrick (1987) observed looking back on *incomplete* or *incorrect* problems may lead the solver to reformulate the problem as well (p. 131).

Polya's (2004) admonition to "look around" may prompt the reader to recall his contrast of *problems to find* versus *problems to prove* (p. 33; pp. 78–85). Regarding problems to find, he observed the following means by which one may create a new problem:

(1) Keep the unknown and change the rest (the data and the condition)

(2) Keep the data and change the rest (the unknown and the condition)

(3) Change both the unknown and the data. (p. 78)

Regarding problems to prove, he observed the following possible ways to generate a new problem:

(1) Keep the conclusion and change the hypothesis

(2) Keep the hypothesis and change the conclusion

(3) Change both the hypothesis and conclusion. (p. 85)

Polya did not define *problem posing* or *problem formulation* in either of his works presented here, nor did Kilpatrick (1987) in his chapter on problem formulation. In contrast, Silver (1994) echoed Duncker (1945) and defined problem posing as "both the generation of new problems and the re-formulation, of given problems" (p. 19).

Kilpatrick's (1987) question concerning the genesis of good problems also provided much fodder for researchers in problem solving and problem posing. His chapter provides advice for the pedagogical practice of problem posing in ways that expand on Brown and Walter (2005). In the special issue of *Educational Studies in Mathematics* (Singer, Ellerton, & Cai, 2013) mentioned above, six of the ten articles referenced Kilpatrick's chapter, despite the fact that his chapter did not contain new, empirical research. He identified a *lack of research* on student problem posing (p. 141) and referenced a few findings from the research that existed prior to 1987. It appears that his chapter helped motivate others to study and write about problem posing.

In summary, problem posing is difficult to define. Neither Kilpatrick nor Polya provided a definition for it. Problem posing may be found in Polya's work on problem solving, although he did not specifically mention it. He provided ways to modify problems, which essentially translate into problem posing. There seems to be a growing tide of opinion that problem posing should increase in mathematics instruction, and there is a need for additional research in this area.

A Framework for Research on Problem Posing

In this study I primarily used a pedagogical perspective (Kilpatrick, 1987) on problem posing. I also found that Schoenfeld's three categorizations seem reasonable to use in order to classify the types of problems posed. I describe Stoyanova and Ellerton's (1996) framework and explain its modification for this study.

Stoyanova and Ellerton (1996) provided a potentially useful framework to describe problem posing episodes. They proposed their framework based on "the notion that every problem-posing situation can be classified as free, semi-structured or structured" (p. 519). At the same time, in none of the situations described in their article did problem posing occur outside of a teacher's directive to pose problems. Each explanation of the three codes referenced some form of the phrase "students were asked to pose problems" (p. 521). It seems that students can engage in the act of posing problems without being specifically asked to do so, but Stoyanova and Ellerton's framework does not account for such instances.

In each classification, Stoyanova and Ellerton (indirectly) reported the *teacher as the catalyst* for problem posing rather than the student. Their classifications seem to assume the following question: "If free, semi-structured, or structured problem posing occurs, then what happens?" These three classifications assume problem posing will occur prior to a particular mathematics lesson. The classifications do not consider the question in reverse: "If students engage in mathematics tasks, when and how do they pose problems?"

Because Stoyanova and Ellerton's framework did not consider occasions when students initiated problem posing, I revised their framework by adding the *student-as-catalyst* category. I planned to look for and identify instances of problem posing in each observed lesson. If a student initiated the problem-posing episode, I classified it as student-initiated (not as free, semi-structured, or structured). Because I was interested in the environment surrounding student-initiated instances of problem posing, I wanted to distinguish these cases from the other three types. In addition, I added Schoenfeld's (1992) categories of problems (routine exercises, traditional problems, or problems that are problematic) to create a means by which I could try to detect instances of problem posing and compare them with the classifications of problem types. *Figure 1* illustrates this framework and provides a means to classify observed instances of problem-posing episodes from the data.

<table>
<tr><td colspan="2" rowspan="2"></td><td rowspan="2">Student as catalyst</td><td colspan="3">Teacher as catalyst: Type of Problem-posing episode</td></tr>
<tr><td>Structured</td><td>Semi-structured</td><td>Free</td></tr>
<tr><td rowspan="3">Classification of Problem</td><td>Routine exercises</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Traditional problems</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Problems that are problematic</td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 1. A framework for identifying instances of problem posing.

I analyzed transcripts and use my framework to identify and describe the frequency and types of problem-posing situations that occurred, and considered the intersection of the *type of problem-posing* situation and the *type of problem* posed. I also identified mathematical topics and prominent strands of mathematical proficiency that accompanied each episode. I applied this framework with six middle grades mathematics teachers across 88 total days of instruction focused on fractions, integers, or algebra standards. I used this grid to search for discernable instances of problem posing. I report some basic summary statistics of the instances for which I observed problem posing. After identifying these episodes, I analyzed them and attempted to describe characteristics of the environment that were present during the episodes. I also report the nature of the mathematics in these episodes by identifying primary strands of mathematical proficiency that emerged from the observation of each episode.

Critiques From a Recent Special Issue Journal on Problem Posing

The following is a discussion and critique of a recent journal publication of 10 research studies on problem posing. Singer et al. (2013c) edited a special issue of *Educational Studies in Mathematics* devoted to problem posing. The special journal issue is particularly relevant because it represents the most recent, focused publication of research devoted to problem posing in mathematics. The contributors expanded on the limited research that existed prior to 2013. Although some research exists prior to 2013, this recent issue provided a good opportunity for review and critique.

In the introduction article, Singer et al. (2013b) identified “a need to study problem-posing techniques that are already practiced in some classrooms, in order to analyze and extend those strategies that proved to be effective” (p. 3). Their article consisted of a synthesis of problem-posing research. They further noted “the time has come for more systematic analyses that can *organize* [emphasis added] the research and theory of problem posing as well as its applications to the practice of teaching” (p. 4). It appears that they consider the theory of problem posing as *unorganized* at present. They encouraged “a large spectrum of opinions and visions [to] contribute to both the development and the structure of the field” (p. 4). They also observed, “the field of problem posing is still very diverse and lacks definition and structure” (p. 4). In other words, it appears that there exists a wide call for much more inquiry concerning problem posing.

In the first reported study in the journal, Singer and Voica (2013) identified a potential conceptual framework to connect problem solving and problem posing. One concern with their study is that neither of their research questions directly concerned problem posing, but rather problem solving. As mentioned before, the two constructs are very much related and often exist simultaneously, but problem solving typically receives more attention than problem posing in the literature. Their main research question was as follows: “What cognitive characteristics of the problem-solving approach might help teachers to better design learning tasks for students in the mathematics school contexts?” (pp. 10–11). They collected data from 150 experienced teachers and 120 students in Grades 3 to 6. The students voluntarily attended a mathematics summer camp. They interviewed 40 students who answered their calls to pose problems in the camp. They were interested in students’ ways of thinking more than any statistical analyses. They concluded that their proposed conceptual framework was particularly useful for creating multiple-choice problems, which addresses only the kinds of problems that *teachers* pose to their students. Their conclusion did not address how students pose problems. Their conclusion suggests teachers can pose problems for students who make mistakes at various phases in solving a problem. They did not describe student-initiated instances of problem posing.

Olson and Knott (2013) also examined problem posing from the teacher’s perspective, which means they did not consider episodes where students posed problems. They conjectured that the problems posed by a teacher might increase (or decrease) the cognitive demand of the lesson (p. 29). Their review of the literature consisted of only three paragraphs, which seems insufficient to ground a solid study. Their study could be more appropriately placed in a special journal on using questioning to maintain cognitive demand more so than a journal on problem posing. They concluded their study with the claim that a teacher’s mindset can influence the ways in which the teacher engages the students (p. 35). This finding does not seem like much of a new insight.

Bonotto (2013) observed problem posing from the students' perspective rather than the teacher's perspective. She reported results from two studies on problem posing and provided hypotheses for each study but did not provide a clear research question for either study. She classified the first study as a teaching experiment (p. 42) and the second study as an exploratory study (p. 45). She aimed to connect problem solving and problem posing in the first study. She reported results that were consistent with the data she presented. She provided artifacts in the form of coupons (such as those found in weekly circulars for supermarkets or other stores) to students and asked them to create mathematics problems based on the coupons. She found the problem-posing activity helped create interest and motivation (p. 43) and that the children "had no difficulty translating typical, everyday data … into problems suitable for mathematical treatment, and all pairs [of students] succeeded in solving the problems created" (p. 43). She also found that the "less able children" (p. 45) participated actively because the activities were meaningful to them. One problem with this study is that it provides only an existence proof because she reported these results from one classroom with 18 students. It is unclear whether or not one may observe a similar response given another sample of students.

In the second study, Bonotto (2013) aimed to explore possible relationships between creativity and problem posing. In this study, the teacher provided students with an artifact (a popular amusement park menu) followed by a problem-posing activity and a problem-solving activity. She studied the problems posed by the students and analyzed them in terms of their solvability. She concluded, "By solving problems created by their peers, the students become able to analyze them in a more detailed and critical way" (p. 51). She claimed that the second study confirmed the potential of students (p. 52). She did not attempt to classify the episode of problem posing. It seems her goal was to provide the problem-posing task for the students and then analyze students' creativity and problem-solving ability. She should have clarified that the study only confirmed the potential of the 63 students in her study rather than generalize to all students. She also concluded "an artifact provides a useful context for the creation of problems and the mathematization of reality as a result of its accessibility to all students" (p. 52). Her conclusions about the sample seemed reasonable given the data she reported, and her task of using artifacts to engender problem posing seems like a potentially useful pedagogical tool.

Cai et al. (2013) used problem posing to try to measure the effect of the Connected Mathematics Program (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002) curriculum for middle school students. They hypothesized that students who experienced the Connected Mathematics Program (CMP) curriculum in middle school might perform better with conceptual understanding and problem-solving tasks in high school than those who did not experience the same curriculum. They used problem posing as a means to test their hypothesis (p. 62). They provided each high school student with two tasks. Both tasks contained problem-posing and problem-solving components. They then provided 10 additional open-ended problem-solving tasks. They found that 84% of the students who found a correct equation for a graphing task were also able to pose a problem in the context beyond the given numbers (p. 65). They found supporting evidence in favor of their hypothesis but admitted that the evidence was not as strong as they had hoped (p. 67). Although their evidence was not strong, the question seems relevant and important. It seems that their study did more to provide rationale for additional research than it did to address their hypothesis.

Koichu and Kontorovich (2013) inquired about the traits of the problem-posing processes that led two prospective mathematics teachers to pose problems (p. 72). They presented a careful analysis of their observations of two prospective teachers who engaged in a task that involved angles of reflection on a billiard table." Koichu and Kontorovich sought to identify "some of the traits of problem-posing processes that lead the posers … to formulate interesting problems" (p. 72). Certain common traits were identified despite different levels of success with the two cases (p. 81). They analyzed the participants' work in terms of the following problem-posing stages:

(a) warming-up

(b) searching for an interesting mathematical phenomenon

(c) hiding the problem-posing process in the problem formulation

(d) reviewing (p. 81)

They also noted that these stages are interlaced as opposed to linear stages (p. 81)

Koichu and Kontorovich (2013) found it important that the two participants established "personal relationships with the task" (p. 83) and extended their work beyond that which was necessary to formally fulfill it (p. 83). They also observed that the phenomenon of "posing problems that would be interesting to solve also to the poser … was rarely observed in the past studies" (p. 83). They also noted that this task was not isolated from the proposed context of the Billiard Task.

The Koichu and Kontorovich (2013) study was observational, and they reported the following finding as a pedagogical implication (p. 83). They determined posing interesting problems "involved searching for an interesting mathematical phenomenon and hiding the problem-posing process in the problem formulation stages" (p. 84). They also determined that "problem-posing tasks should not be separated from mathematical explorations" (p. 84). It seems that a student may initiate problem posing in order to establish a personal relationship with the task.

Ellerton (2013) studied 154 prospective middle grades teachers who were in their teacher preparation programs. She followed two classes for three semesters. She aimed to "identify and document the effect of incorporating problem posing as part of the mathematics content curriculum for students in an undergraduate middle-school mathematics teacher-education course" (p. 90). She proposed a "conceptual framework for positioning problem posing within the context of mathematical learning" (p. 90). She proposed the development of an active learning framework for interpreting the role of problem posing (p. 87). She claimed the proposed active learning framework illustrates "how the students' classroom experiences are … cut short" (p. 99) if problem posing is left out of the curriculum. She asked the following research questions:

1. To what extent did pre-service middle-school students see mathematics problem posing as more of a challenge than solving similar problems?
2. To what extent did pre-service middle-school students find the problem-posing process helpful in understanding the mathematical structures of problems?
3. To what extent did pre-service middle-school students enjoy creating mathematics problems?
4. In what ways did the mathematics problems created by students as part of their mathematics teacher education course support students' learning of mathematical structures? (p. 90)

Ellerton (2013) found the active learning framework supported "the incorporation of both routine and project problem-posing activities in mathematics courses for pre-service teacher-education students" (p. 100). The proposed link between the active learning framework and the incorporation of problem-posing activities seems intuitively plausible, but I did not think the findings of this study overwhelmingly supported such an intuition. In addition, at times it was unclear as to whether the framework was about more about active learning or problem posing. Perhaps it seems reasonable to conclude better problem posing (or any problem posing at all) is more likely to occur in an active learning environment. This study did not provide convincing empirical evidence of such a connection. Indeed, Ellerton admitted that additional research is necessary before such a framework could be applied in mathematics classrooms in general (p. 100).

Leung (2013) reported an teaching intervention in a study with first-year schoolteachers in Taiwan. Leung asked, "Why and how do teachers enact problem-posing task materials in an elementary mathematics classroom?" (p. 104). Leung provided a strong link between problem posing and problem solving. Like Wilson et al. (1993), Leung proposed the idea that Polya's (2004) phases of problem solving may be related to problem posing (pp. 104–105). In particular, Leung hypothesized the act of posing a (new) problem is analogous to Polya's phase of understanding the problem. Leung's report could have been strengthened by adding a reference to Wilson et al., as they essentially provided the same idea 20 years earlier. Leung also explained that problem posing "can occur at many points … before or after solving [a problem]" (p. 105). Leung taught the teachers about problem posing and asked the teacher-participants to prompt their students to pose problems. The teachers collected the problems posed by their students and provided them for Leung to analyze. He coded the "problems" posed by students as follows: 1) Not a Problem, 2) Non Math, 3) Impossible, 4) Insufficient, and 5) Sufficient (p. 108).

Leung (2013) explained that elementary teachers in Taiwan do not have class on Wednesday afternoons, and they use this time for professional development (p. 107). This regular access to professional development implies that a replication of this study could be problematic in locations where teachers do not have such opportunities for regular, sustained professional learning. The presented data were insufficient to answer the research question of "Why … do teachers enact problem-posing task materials in an elementary mathematics curriculum?" (p. 104). Leung did not answer this question. Presumably, the teachers in the study implemented problem-posing tasks to their students simply because they were asked to do so. There is no other explanation as to why they used problem posing. The coding framework used to code the problems posed by the students seemed useful in determining the level of sophistication of the problems posed.

Van Harpen and Presmeg (2013) performed a comparative study in problem posing from the student's perspective with high school students in the United States and China. The students in the study were all 18 years old, and they took a course in advanced topics in mathematics (p. 120). Van Harpen and Presmeg asked the following research questions:

1) Are students who are stronger in mathematics content knowledge also stronger in their problem-posing abilities?
2) Are the relationships between students' mathematics content knowledge and their mathematical problem-posing abilities in different cultures the same? If not, what are the differences? (p. 118)

Van Harpen and Presmeg (2013) administered a mathematics content test and a mathematics problem-posing test. They found that students from the three groups (the United States, Shanghai, and Jiaozhou) all posed nonviable problems. Nonviable problems were problems posed with an insufficient amount of necessary information. For example, one such nonviable problem was, "if students were told to stand in two rows, how many girls would be in the first row?" (p. 124). They claimed that a limitation of the study was that the students were not asked to pose "viable" (p. 129) problems. At the same time, one might question, "How many *advanced* students would intentionally pose nonviable problems when asked to pose mathematical problems?" Van Harpen and Presmeg admitted inconsistent findings with regards to their first research question. In spite of the lack of solid, significant evidence, they claimed that the overall study suggested that "content knowledge in mathematics does have a significant influence on students' performances in posing new mathematical problems" (p. 130). Similarly, for the second research question, they found no statistically significant relationship but nevertheless suggested that "US students depend less on their mathematics content knowledge in posing problems than Chinese students" (p. 130). This is a mere suggestion, not a finding or implication. The reported implications were weak, and consisted only of one paragraph (p. 130). In this study, Van Harpen and Presmeg proposed a hypothesis and conducted a study that provided inconclusive evidence. From the inconclusive evidence, they concluded by suggesting that their hypothesis was valid, but additional study (with a comparison group) was needed for verification.

Tichá and Hošpesová (2013) attempted to use problem posing to identify shortcomings in preservice primary school teacher content knowledge. In particular, they were interested in preservice teachers' conceptual understanding of fractions (p. 133). They classified their study as an "educational experiment" (p. 136). They asked 56 preservice teachers to post three to five problems using the fractions one half and three fourths and then to solve the problems and engage in joint reflection with one another about the problems. They demonstrated the potential to use problem posing as a means to help assess the content knowledge of the preservice teachers in their sample. Tichá and Hošpesová found that “problem posing is first of all an appropriate way to introduce pre-service primary school teachers to the teaching of mathematics” (p. 141). Further, students with little teaching experience and some problem-solving experience “were able to transcend gradually to the position of teachers who pose mathematics problems, modify the problems, offer help in their solution, and evlauate solution procedures” (p. 141). They claimed that their study provided “strong evidence that problem posing can be a significant motivational force resulting in deeper exploration of the mathematical content” (p. 142). Their study provided strong evidence that problem posing can be motivational, at least for their sample of preservice teachers. They hypothesized that “problem posing, supplemented by joint reflection, should be one of the central themes in mathematics teacher education” (p. 142). Such claims need additional inquiry for substantiation.

Da Ponte and Henriques (2013) conducted an observational, interpretive study of problem posing in a university numerical analysis course. They did not explicitly provide a research question, but they stated that their goal was to identify “the mathematical processes used by university students exploring investigations in the classroom … from the interpretation of the situations to the justification of the results” (p. 146). They looked for problem posing that might occur during mathematical processes. They sought to determine how the students’ mathematical processes might engender problem posing. Their study is the only study I found that sought to *identify* instances of problem posing during the context of mathematics instruction. This report was in contrast with Leung’s (2013) paper because Leung specifically taught the teachers in the study about problem posing. It seems that Da Ponte and Henriques chose the sample of students because they believed the students would engage in “mathematical investigations" (p. 146). They believed such investigations were fertile ground for problem posing.

The Da Ponte and Henriques (2013) study caused me to wonder what their approach might look like in classrooms with younger students, such as classrooms with middle grades students. The study supported my claim that the framework presented by Stoyanova and Ellerton is useful only in instances when the instructor intentionally asks students to pose problems of a certain variety. As a result, it seems these episodes of problem posing were student-initiated episodes. The students in the Da Ponte and Henriques (2013) study were presented with mathematical tasks in numerical analysis that contained ambiguity. They did not have a clear course of action to complete these tasks. As a result they began to pose "subproblems" (p. 149) in order to solve the original problem.

In summary, the special issue of *Educational Studies in Mathematics* (Singer, Ellerton & Cai, 2013) contained several different themes in research on problem posing. The following research ideas appeared in the journal: students' cognition in problem posing activities, links to problem solving, characteristis of problem-posing tasks, and links to cognitive demand. The articles reported research on problem posing from the perspective of practicing teachers, prospective teachers, and students at various levels. From this review I learned that little research has been done to explain how often problem posing occurs in mathematics classes. In addition, I did not find evidence of instances where teachers used problem posing as a regular instructional tool. I did not find any literature that explained circumstances surrounding student-initiated instances of problem posing. None of the articles referenced the strands of mathematical proficiency (Kilpatrick et al., 2001). Thus, I wanted to understand more about how and why teachers use problem posing in their classrooms.

CHAPTER 3

METHODOLOGY

My initial intention was to identify one teacher who used problem posing in his or her classroom as a case study. I identified one teacher within the context of the DIMaC (Bishop, 2012; Stockero, Bishop, & Conner, 2014) research project. In addition, I found additional participants in this larger research study that seemed suitable for an observation study on problem posing within the interpretivist, qualitative tradition. The research questions guiding this study were the following:

1. How often does problem posing occur in six Grades 5–7 mathematics classrooms? What types of problem posing occur in those classrooms? What mathematical processes do students use when engaged in problem posing?
2. What kind of classroom environment exists when students engage in problem posing activities? (Singer, Ellerton, Cai, & Leung, 2011, p. 149).
3. What understanding and perspectives on mathematical problem posing do teachers possess? For a teacher who uses mathematical problem posing in her instruction:
    a. To what extent are problem-posing episodes planned in advance?
    b. What prompts the problem-posing episodes that were planned?
    c. What is the participant's overall assessment of the potential learning impact of the problem-posing episodes?

Setting, Participants, and Data Collection: Description of Larger Study

This study occurred within the context of a larger DIMaC research project that studied mathematics discourse in middle grades classrooms. I selected a subset of the participants in that study for my research. In the following sections, I share details about the larger study and the data collection process for the DIMaC Project. I then discuss how I selected the participants for the present study.

Participants

The DIMaC Project examined discourse (student to teacher or student to student discussions) that occurred in middle grades mathematics classrooms during lessons on the topics of fractions, integers, or algebra. The participants in this study were teachers recruited on the basis of their reputations for having mathematics discussions in their classrooms. They were selected and recruited for the larger discourse study based on recommendations of school or district administrators, a demonstrated reputation for excellence in teaching mathematics, or recommendations from other mathematics education leaders. All participants had at least 5 years of teaching experience.

During the 2013–2014 school year, nine middle grades teachers participated in the DIMaC Project. I was a member of the research team. Three of the teachers were located in the western United States, and the remaining six teachers practiced in the southeastern United States. Each of the teachers taught in a wide variety of socio-economic and diverse environments. Four of the teachers taught fifth grade, four of the teachers taught sixth grade, and one teacher taught seventh grade. Members of the research team filmed 132 mathematics lessons during the school year on one of these topics: fractions, integers, or algebra. Each teacher notified the research team prior to teaching lessons on fractions, integers, or algebra in which he or she expected student discussions as a part of the lesson. The filmed lessons occurred over the course of the school year during both semesters, depending on the individual teacher's sequencing of the lessons.

The research team met weekly during the fall semester to discuss any initial observations from the first few rounds of filming. During the research meetings, it became apparent that some of these teachers used problem posing in their classrooms. For example, we discussed how Ms. Green explicitly asked her students to create their own problems during a lesson on fraction division. These meetings influenced my decision to pursue at least a subset of the participants for my study on problem posing in addition to a case study of Ms. Green.

Data Collection

The research team used two digital video cameras to record each mathematics lesson. One camera was designated as the teacher camera. The videographer (a member of the research team) used the teacher camera to follow the teacher during each lesson. The teacher camera contained a highly directional, condenser, "shotgun" microphone as well as a wireless microphone system that was attached to the teacher for better audio acquisition. The shotgun microphone and the wireless microphone enabled us to acquire the audio from which we developed written transcripts of each lesson. The other camera was a stationary camera focused on one small group of students for the entirety of each lesson based on the recommendation of the teacher. This camera was connected to an omnidirectional, condenser microphone placed near the center of the small group table to acquire sound from the student discussions in the small group. The small group camera was useful in instances where additional audio was necessary, or when that set of students' interactions were desired. The videographer remained only with the teacher camera and not with the small group camera. In general the data from the small-group camera were not analyzed for the DIMaC Project. Each lesson ranged from 45 minutes to 75 minutes of instruction depending on the teacher's schedule.

The videographer transferred all data from both cameras onto a project computer after filming each mathematics lesson. The data from the teacher camera served as the primary data source for analysis. Immediately after the data transfer, audio levels were tested and adjusted as necessary using video-editing software. The videographer then exported the file from the software program to a movie file as well as an audio file suitable for digital playback on typical computer platforms.

Members of the research team created a lesson summary sheet as an initial form of analysis for a particular lesson after completing the video export. Lesson summary sheets contained highlights from the lesson or other pertinent interactions. The lesson summary served as an outline of a transcript for each lesson. The summaries provided information about activities during the lesson. Additionally, each lesson was transcribed using the video recording and corresponding audio files. Members of the project team transcribed most of the lessons, but some lessons were transcribed externally in order to expedite data analysis with such a large number of lessons. Members of the research team checked and corrected transcriptions received from external transcription sources by comparing them with the video file from each episode. The video files allowed members of the research team to provide a better transcription including names of students (with pseudonyms) in the classroom as they shared ideas with each other and the teacher. The transcripts contained speech as spoken in the classroom including restarts, fillers, and overtalk.

In March 2014, each of the nine teachers also participated in least one formal interview (about integers, fractions, and algebraic reasoning), an assessment of beliefs about mathematics, and an assessment for mathematical knowledge for teaching. Each formal interview lasted between 60 and 90 minutes. The teachers also spent approximately 90 minutes on each written assessment.

I selected six teachers from the original nine teachers in the project. I selected only those teachers who had a complete set of transcripts available for all of the lessons filmed in his or her classroom by the summer of 2014. Because the filmed lessons were spread across three mathematical topics, I wanted to include teachers with all transcripts completed in order to determine if variation in problem posing might occur across mathematical topics. Of the six teachers who had a complete set of transcripts, each had at least one instance of problem posing. Table 2 illustrates some basic information concerning the teachers in the present study. The asterisk indicates classrooms for which I served as the videographer at least once during the year. I did not enter the classrooms of Mr. Blue or Ms. White.

Table 2

Summary of the 88 Lessons Filmed

Grade	Teacher	Location	Algebra	Integers	Fractions	Total
5	Mr. Blue	West	5	0	8	13
5	*Ms. Gold	South	7	0	7	14
5	*Ms. Violet	South	4	0	10	14
6	*Ms. Green	South	6	7	5	18
6	Ms. White	West	5	3	4	12
7	*Ms. Lavender	South	6	5	6	17

Selection of a Subset of Participants from the Larger Study

Initially, I was skeptical that I would find other teachers (besides Ms. Green) who also engaged their students in problem posing. I soon visited Ms. Gold, a fifth grade teacher who participated in the DIMaC project, and while filming, I observed episodes that I thought might qualify as problem posing. (The classifications of observed instances of problem posing in Ms.

Gold’s classroom appear in Chapter 4.) These kinds of observations suggested to me that more teachers use problem posing than I had initially thought, or at least the teachers in this sample did so. During the fall semester, members of the research team and I identified instances of problem posing in other teachers’ classes as well. After identifying more instances, I decided that my study might be more meaningful if I reported overall instances of problem posing I observed from the DIMaC Project in addition to a case study of Ms. Green. By the summer of 2014 our research team complied completed transcripts for 88 lessons from six of the nine participants in the larger study. I selected these six teachers for my naturalistic observation study because of the availability of their data in addition to the fact that I had identified instances of problem posing in at least one of their lessons.

Selection of Ms. Green for a Case Study

I visited Ms. Green’s classroom as my first filming assignment as a member of the DIMaC Study in August of 2013. I had no prior experience or communication with Ms. Green before I entered her classroom to film her lesson. I became interested in learning more about Ms. Green’s teaching practices when she asked her students to pose problems during my first visit to film her lesson. I became interested in performing a case study with Ms. Green and her students after reflecting on my first two filming visits in her classroom in August and early September of 2013. I initially assumed (incorrectly) that she was the only teacher of the nine teachers who used problem posing. At a minimum, I wanted to learn more about why she asked her students to pose problems and to try to understand her pedagogical decisions surrounding her requests that students should pose their own problems. During the fall of 2013, I continued to visit Ms. Green’s classroom, and I filmed several of her lessons on fractions and algebraic reasoning. Each member of the research team was responsible for at least one of the nine teachers in the project. I became the primary videographer for Ms. Green's classes for the rest of the year. I did not film all of her remaining lessons during the school year, but I was responsible for creating lesson summaries for each of her lessons as well as the completed transcripts for each of her lessons. I was also able to observe the classroom norms in her classroom in person as the primary videographer. Ms. Green did not learn of my desire to see problem posing until the spring semester as she completed an updated consent form in order to participate in an additional interview about problem posing.

Over time, I observed that Ms. Green used problem posing with more frequency than the other teachers. She was a good candidate for a case study because of her relatively frequent use of problem posing in her classroom in addition to her eagerness to participate in the DIMaC Study. She was also a good candidate because she was within a reasonable driving distance for visits and interviews.

I transcribed each of Ms. Green's lessons by viewing the exported video files from her lessons except for three of her lessons that were transcribed externally. I received the copies of the external transcriptions and checked them by watching the lessons again on the computer. I made any necessary corrections, and I added the pseudonyms of her students in order to provide a better description of the interactions of all of the students.

I interviewed Ms. Green for approximately 90 minutes after the end of the school year to ask her additional questions about problem posing. Because I wanted to understand more about her past experiences with problem posing, her motivations for using problem posing and her reflections about the use of problem posing, I structured interview questions to target these topics. I recorded the interview and transcribed it within one week. Prior to the interview, I selected a few video clip samples from her class during the school year in which problem posing occurred. I played these video clips for her individually in order to stimulate her recall of these lessons to try to understand more about her perspective, motivation, and reflection of the instances of problem posing.

Overall, the research team acquired more filmed lessons from Ms. Green than any other teacher in the larger discourse study. In addition to filming more lessons, Ms. Green's class typically lasted 70 to 75 minutes—the longest of all of the teachers. As a videographer in her class, I observed several instances where she explicitly required her students to create their own mathematics problems.

Ms. Green's Classroom and Students

We filmed Ms. Green's *accelerated* sixth-grade mathematics class. The term *accelerated* does not necessarily mean *gifted*. The accelerated students were expected to move quickly through the sixth- grade mathematics curriculum and begin the official seventh-grade mathematics curriculum during their sixth-grade school year. Ms. Green had a reputation for infusing technology into her classes. She issued iPads to each of the 30 students in her accelerated mathematics class. Her students typically performed all of their written work on the iPad and submitted their work to her via email. This class began before 8:00am each morning during the first block of the day. Ms. Green taught science to the same group of students immediately after their mathematics class each day. She also taught "on track" sixth-grade mathematics classes each day. I observed only Ms. Green's first period accelerated mathematics class.

Analysis of Data

The analysis of data began by identifying instances of problem posing throughout the data set. I focused much of my early identification and analyses on possible problem posing episodes in Ms. Green and Ms. Gold's classrooms. I was most familiar with these classrooms and potential problem-posing episodes because I was one of the primary videographers for those teachers. Additionally, members of the research team indicated potential episodes of problem posing on lesson summary sheets, and I flagged such transcripts for additional analysis. The development of the problem-posing framework was iterative and went through multiple revisions as I coded more of the data. For example, I modified Stoyanova and Ellerton's (1996) classification scheme to include instances of problem posing that were generated by students. I also added codes related to the mathematics content addressed in the problem posting situations to more fully characterize the episodes.

For the purposes of this study, I classified problems in the same manner as Schoenfeld (1992). He partitioned problem scenarios into three separate groups: "routine exercises" (pp. 337–338), "traditional problems" (p. 338), and "problems that are problematic" (pp. 338– 340). Schoenfeld associated *routine* exercises with actions performed by students in order to practice some specific mathematical skill or technique. He considered such exercises as quite different from the other two categories. His second category of *traditional problems* referred to tasks students perform as a mean to a focused end (p. 338). He distinguished the third category of *problems that are problematic* as "problems of the perplexing kind" (p. 338). In this study, I used his three categories of *problems* as a means to classify the kinds of mathematics problems that students pose in mathematics classes.

The final problem-posing framework is in Figure 1 (p. 27). I used the framework to identify instances of problem posing for each of the six teachers across all of their video-recorded lessons. I coded each episode in terms of whether or not the episode was a teacher- or student-initiated episode of problem posing. I coded episodes initiated solely by a student question or comment in the "Student as catalyst" column, followed by a classification of the type of problem posed. I coded episodes initiated by the teacher as Structured, Semi-Structured, or Free (Stoyanova & Ellerton, 1996), also followed by the type of problem posed. Final coding using this framework was done at the end of summer using all of the completed transcripts from the six participating teachers.

An Example of Problem Posing With the Teacher as Catalyst

Ms. Green taught a lesson on division of fractions on August 30, 2013. She began class that day by asking students to create their own problem based on division. Her directions to the class were as follows:

> [You have] ten minutes to *design the problem* [emphasis added], and come up with your solution, and then, … ten minutes to solve your neighbors' [posed problem] and then we're gonna play a little game. Okay? So, let's get started.

In this instance, Ms. Green engaged her students in groups for the majority of the class period. I coded this instance using the framework from Figure 1. I observed that Ms. Green asked her students to create a traditional problem in a semi-structured, problem-posing environment. The column labeled "Student as catalyst" was not applicable since Ms. Green, rather than a student, initiated the action to pose a problem.

An Example of Problem Posing With the Student as Catalyst

The following is an example of a PP situation where the student was the catalyst rather than the teacher. This episode is also from Ms. Green's sixth-grade mathematics class. Ms. Green initially provided the students with an equal sharing division problem to solve. Ms. Green's essential question for the day was as follows: "When I divide one number by another number, is the quotient always smaller than my original number?" The problem involved taking one-fourth of a cake and sharing it equally among four teachers. The students worked in groups, and Ms. Green circulated around the room and visited the groups to monitor their progress. When she arrived at Student L's group, Student L questioned the initial constraints of the problem. He acknowledged the directions to partition the cake into four pieces, but he wanted to question whether or not Ms. Green was included in the four teachers. Ms. Green quickly regained the attention of the class and redirected them. She said,

> What if I wanted to be added to these 4 teachers? What if it was me and 4 teachers? What would—. I want you to think about that. That's a good, oooh, thank you for bringing that up. Okay, so now instead of me splitting it among 4 teachers, I wanna be included. So now I want you to split it among how many teachers?

Ms. Green also followed up with the question, "What changed, and why did it change?" before asking the students to continue working in their groups. In other words, Ms. Green used Student L's question as an occasion to modify the constraints of the problem based on the student's questioning of the constraints of the problem. In this episode, the student asked a clarifying question. As a result, Ms. Green asked a *what-if-not* (Brown & Walter, 2005) question that changed a constraint from the initial problem. As such, the student initiated the action to reformulate the initial problem by changing the constraint of the number of shared groups from four groups to five groups. Even though Ms. Green asked the class to officially modify the problem, the student was the catalyst and motivation for the reformulation of the problem. Thus, I considered Student L's question to Ms. Green as an instance of problem posing. Note that the coding of "Student as catalyst" was appropriate because the teacher did not initially intend to modify the problem in this manner, nor did she set an expectation that students modify the initial problem. Even if Ms. Green did plan the situation, the evidence from the video shows that her reaction and response was engendered by Student L's question. The student either initiated the question out of his own curiosity or as a result of the discussions that occurred in his group with three other students, apart from a prompt from Ms. Green.

Video and Transcript Analysis for Study 1

In order to address the first two research questions, I created counts of the various types of problem posing episodes (student, structured, semi-structured, or free) along with the categorization of problems (routine exercises, traditional problems, or problems that are problematic) posed from all of the lessons from the six teachers. I analyzed the transcripts from 88 lessons. I chose not to focus on whether the problems posed were sensible, nonsensical, or mathematically viable. I was only interested in the overall trajectory of each episode. For example, if the teacher directed students to create a situation to model a routine exercise, I used the framework to identify the problem-posing episode teacher-directed, routine exercise, followed by whether or not it was free, semi-structured, or structured.

I also described the mathematics present in the lessons. In particular, I identified the predominant strand of mathematical proficiency (Kilpatrick et al., 2001) present in each episode. I used conceptual reasoning, procedural fluency, and adaptive reasoning as three different codes of mathematical proficiency. Because some episodes appeared to support both procedural fluency and conceptual reasoning I added a fourth code, building procedural fluency from conceptual understanding. I also looked to see if patterns emerged across the topics of fractions, integers, and algebraic reasoning.

Video and Transcript Analysis for Study 2

In order to answer the third research question, I used a case-study methodology with Ms. Green as the focal teacher. According to Yin (2003), a case study should be used when the aim of the study is to answer "why" questions. In the case of Ms. Green, the goal was to understand why she used problem posing and to learn more about her experience with problem posing. Stake (2000) identified three classifications of case studies: intrinsic, instrumental, and collective. Because I wanted to better understand how problem posing occurred in the case of Ms. Green, I classified this case study as an intrinsic case study. Stake also observed, "Case study is not a methodological choice but a choice of what is to be studied" (p. 435).

As mentioned above, I obtained data in the form of filmed instructional lessons from 18 lessons in Ms. Green's class during the 2013–2014 school year. Members of the DIMaC research team assisted with lesson summaries across all teachers to provide additional input and support for identifying instances of problem posing. For example, each lesson summary template contained a prompt to identify potential instances of problem posing. Team members flagged potential instances of problem posing. If a lesson contained a possible episode of problem posing, I viewed the portion of the video lesson in question to look for evidence of problem posing. I applied the definitions and the framework to determine the classification of problem posing, or I determined that problem posing did not occur. I also shared the coding of each episode with the DIMaC principal investigator to create consensus about the episode. There were no contested episodes from Ms. Green's class.

I did not film each lesson in Ms. Green's class, but I began an informal analysis of each lesson immediately after I filmed an episode from her class. If another member of the DIMaC research team filmed one of her lessons, I watched the filmed episode and completed a lesson summary sheet. An example of the lesson summary sheet is included in Appendix A. The summary sheets helped me to begin to formulate questions to ask Ms. Green in the follow-up interview at the end of the school year. As I continued to observe Ms. Green's lessons on video and in person I began to question her past experience with problem posing. I wondered where she first learned about problem posing, or where she first observed problem posing in action. I also wondered if she had formal training in problem posing. Because she seemed to use problem posing relatively frequently, I also wondered how Ms. Green used it to meet her instructional goals. I also wanted to understand her motivations for asking her students to pose problems. Based on my observations of Ms. Green, my reflections of my observations of Ms. Green, and discussions with the principal investigator and others on the DIMaC research team and dissertation committee, I formulated an interview protocol to guide the follow-up interview. The Follow-Up Interview Protocol is given in Appendix B.

In March 2014, Ms. Green completed the IMAP Beliefs Survey (Philipp & Sowder, 2003) and an initial interview about teaching algebraic reasoning, integers, and fractions as a part of her participation in the DIMaC Study. I performed that initial interview. The purpose of the interview was to prompt Ms. Green to reflect on her own understanding of integers, fractions, and algebraic reasoning. I also asked Ms. Green about her mathematical goals for her students. This portion of the initial DIMaC Interview Protocol is included in Appendix C.

I used thematic analysis to search for themes and patterns in her lessons, in her IMAP Beliefs Survey, and in her follow-up interview. Between March and June, I completed written transcripts of each of Ms. Green's lessons and continued to revise my follow-up interview protocol. As I completed each transcript, I continued to look for potentially missed episodes of problem posing. I began to input the codes of problem-posing episodes into a spreadsheet to better organize the data. I parsed the data to identify instances of problem posing across the topics of fractions, integers, and algebraic reasoning to see if there were any patterns.

I identified 11 instances of problem posing in Ms. Green's class. I returned to the video of the filmed lessons when each of these instances occurred. I watched each of the episodes again and continued to revise the interview protocol. I sent the interview protocol to members of my committee for review and critique. I conducted the follow-up interview in June 2014 (after the end of the school year). I chose clips that included student-as-catalyst codes as well as teacher-as-catalyst. I also chose instances when problem posing occurred quickly and instances when problem posing occurred as an extended episode (e.g., most of the class period). I provided the students' posed problems for the last problem-posing clip in order to remind Ms. Green of the final products of the problems posed by the groups of students.

I followed the interview protocol during the follow-up interview. I tried to remain flexible during that interview and listened for possible key themes in her statements. Because I did not know the extent of Ms. Green's experiences with problem posing, I tried to ask broad questions without biasing her responses. The follow-up interview was audio-recorded. The audio captured our voices as well as the audio from the selected video clips of problem posing. I transcribed the interview within 2 weeks of the interview.

After transcribing the follow-up interview, I searched for moments in the interview when Ms. Green identified her perspectives about problem posing. I tried to connect instances in the interview to either one or two other sources of data: the first DIMaC interview or specific lessons in her class. For example, if she expressed a possible belief in the follow-up interview, I reviewed her scores from the IMAP Survey to support or refute the possible belief. I learned about Ms. Green's ideas about using problem posing as I compared the filmed lessons with her DIMaC and follow-up interviews.

Subjectivity: Identification of Potential Bias

To what extent do my own beliefs and experiences jeopardize my findings? I believe that problem posing is important in that it is a potential tool that teachers can use when they feel it is appropriate, yet I do not believe many possess formal knowledge about problem posing. At issue is whether or not teachers understand what problem posing is or how to wield it as a potentially useful educational tool, not to mention whether or not problem posing is actually useful. My desire is that teachers should have exposure to some basic notions for what constitutes problem posing. Brown and Walter (2005) provided a helpful initiation to problem posing. I suspect that some teachers may like the idea much like I did before studying it more formally. Still other teachers may find the notion foreign or perhaps even superfluous to mathematics instruction.

Before the study began I held to the belief that few (if any) local teachers engaged in problem-posing activities. This belief existed because of my lack of experience with problem posing and because of a lack of experience discussing problem posing with others. This belief is also problematic if one assumes that problem posing and problem solving are indeed companions. Initially I felt this would influence my study in that it would necessitate some form of "intervention" for any of my research participants to try to engage them in problem posing and to teach them about problem posing. Fortunately, my initial experiences in the larger research group troubled my bias in this regard as I observed instances of problem posing in my field research assignment. This bias lingered as I initially planned to limit my study to a case study of Ms. Green because I was convinced at the time that no other teachers in the project used problem posing. I addressed this bias by asking other members of the research team to notify me if, at any point, they observed anything that seemed remotely like occasions when students posed or created problems. Over time, with additional observed instances of problem posing, it became obvious that a case study of Ms. Green in addition to an observation of some of the other teachers would strengthen my study. I then included five additional teachers as participants.

At present, my bias is that I sense perhaps many more teachers have, at a minimum, a favorable disposition towards mathematical problem posing, but that they may not possess a formal awareness of the topic. This bias is also based on the perceived absence of a discussion of problem posing in my formal education experiences and in my time as a mathematics teacher. This bias could lead me to want to introduce problem posing to participants as a formal topic of a pedagogical tool for mathematics instruction.

I tried to minimize bias by not introducing the topic of problem posing to the participants in the study. This helped to prevent them from artificially using problem posing in their lessons in order to provide the types of lessons that I wanted to see. I avoided a formal discussion for what I meant by problem posing until after the school year so as to try not to force a participant to engage in problem posing. The only formal discussion for problem posing occurred with Ms. Green during the spring semester consent process and during the follow-up interview in the summer after the school year ended. I also presented my determinations about each instance of problem posing to the principal investigator on the larger project for criticism with the expectation that I could decrease bias and increase the trustworthiness of my findings.

My present bias also leads me to conclude that teachers who do not use problem posing (either formally or informally) might neglect to provide powerful learning experiences for their students. This bias could have influenced my thoughts about those I selected as research participants. Fortunately, the research participants were selected for reasons outside of my requirements. They were selected because other educators and administrators recommended them based on their reputations, and they entered the research group with scrutiny from the principal investigator, thus reducing some potential participant selection bias on my part. As a result I looked to identify any episodes that qualified as problem-posing episodes by any of these teachers while they taught about fractions, integers, or algebraic topics.

I find that I have a desire to see episodes of problem posing. As such, I used caution to not over-identify episodes of problem posing in my eagerness to identify such episodes. My classification framework had to be such that I could justify whether or not a certain episode did or did not qualify as a problem-posing episode. I used Brown and Walter's (2005) influence on problem posing in addition to broad definitions of problem posing as proposed by researchers who contributed to the problem-posing literature (Duncker, 1945; Silver, 1994).

Possible Limitations

There are potential limitations of this study. The first limitation is that this is an observational study that occurred in only six classrooms. As result, it is not appropriate to generalize to other classrooms beyond those in the sample set. Another limitation is the fact that three additional teachers participated in the DIMaC Project, and I did not analyze the data from their classrooms because their data were not available soon enough to meet key deadlines. It is unclear how the addition of the data from the other three participants would have changed the findings. It is plausible that I may over-represent the frequency of problem posing because the teachers were identified as exemplary in relation to their ability to facilitate mathematical discussions. A random sample of participants would likely result in different findings.

It is also appropriate to question the reliability of my coding of the episodes. Although I consulted with the principal investigator of the DIMaC Project about my codes, it is possible that other outside observers may disagree with my coding scheme after observing an episode. In addition, at times a single problem-posing episode constituted an entire class period. Does it make sense to code one entire class period as one instance of problem posing when there are several group of students, each posing different problems? I argue in the affirmative. Perhaps it is plausible that one could count each problem posed as a separate instance of problem posing. These judgments required that I make a decision that I felt I could apply in a consistent fashion. Another observer might pose legitimate critiques of my coding.

Also in reference to the coding of the episodes, some may view my detection of instances of problem posing as flawed. How can one determine whether problem posing occurred, particularly in episodes initiated by the student rather than the teacher? Did I appropriately and consistently use the same criteria for detecting problem posing (such as suggested by Brown and Walter, 2005)? I argue that I used criteria consistently, but others may disagree. In addition, I did not distinguish between the reformulation of a given problem versus the creation of a new problem. In the present study I only reported whether or not an instance of problem posing occurred.

The classifications of free, semi-structured, and structured may not be the most useful ways to classify instances of problem posing. In particular, the distinction between semi-structured and structured problem posing is not always easy to see. Indeed, Stoyanova and Ellerton were not in search of problem posing situations when they used these classifications. Rather, they specifically created problem-posing tasks to suit their descriptions of each classification.

Another limitation of the study is the fact that I did not provide a detailed analysis all of the problems produced by the students in the problem-posing episodes. As result, I did not attempt to correlate problem-solving ability with problem-posing ability in the students I observed. For example, I did not identify episodes where students failed to pose viable mathematics problems, or problems that were appropriately cognitively demanding in a particular mathematical context. I report only those instances where either the teacher requested that the students pose problems or instances where students engaged in problem posing of their own volition.

CHAPTER 4

DATA ANALYSIS AND RESULTS

I report findings from this study in two parts. In Study 1, I address the first two research questions by providing an analysis of problem posing of the six teachers participating in the study. The analysis of the six participants provides descriptive data about the frequency and types of problem posing occurring across the six teachers. This broader analysis helps to situate the case study of the problem posing that occurred in Ms. Green's classroom, which comprises Study 2, which addresses the last research question. Ms. Green's data are included in both studies. The research questions guiding this study on problem posing were as follows:

1. How often does problem posing occur in six Grades 5–7 mathematics classrooms? What types of problem posing occur in those classrooms? What mathematical processes do students use when engaged in problem posing?
2. What kind of classroom environment exists when students engage in problem-posing activities? (Singer, Ellerton, Cai, & Leung, 2011, p. 149).
3. What understanding and perspectives on mathematical problem posing do teachers possess? For a teacher who uses mathematical problem posing in her instruction:
    a. To what extent are problem-posing episodes planned in advance?
    b. What prompts the problem-posing episodes that were planned?
    c. What is the participant's overall assessment of the potential learning impact of the problem-posing episodes?

Study 1: Overall Findings from the Six Participants

Description of PP Environments

During the 88 lessons, I observed 24 distinct instances of problem posing. Table 3 contains counts that summarize the 24 instances of problem posing across all teachers and categorizes these instances by the following three categories: (a) catalyst for each episode (student or teacher), (b) the problem type (routine exercise, traditional problem, or problem that is problematic) and (c) problem-posing type (student, structured, semi-structured, or free). On average, problem posing occurred once out of every four lessons.

Table 3

Aggregate Counts of Problem Posing by Teacher

		Catalyst		Problem Type			PP Type			
Grade	Participant	Tchr.	Std.	Rout.	Trad.	Prob	Std.	Struct	Semi-St.	Free
5	Mr. Blue	2	1	3	0	0	1	2	0	0
5	Ms. Gold	0	3	2	1	0	3	0	0	0
5	Ms. Violet	1	0	0	1	0	0	0	1	0
6	Ms. Green	8	3	6	5	0	3	2	6	0
6	Ms. White	3	1	2	2	0	1	1	2	0
7	Ms. Lavender	2	0	2	0	0	0	0	2	0
	Total	16	8	15	9	0	8	5	11	0

The Catalyst of Problem-Posing Episodes

First, note that Table 3 indicates the teacher initiated two-thirds of the problem-posing episodes (see the teacher-as-catalyst column). In other words, these were instances in which the teacher specifically asked the students to engage in problem posing. The eight student-as-catalyst episodes often appeared spontaneously within the flow of classroom instruction, as if the teacher did not plan for them. These instances of problem posing occurred when students posed questions or questioned the constraints of the task in ways that the teacher did not clearly anticipate. Sometimes the teacher took up these unexpected ideas and questions, and other times the teacher did not. The identification of the catalyst is useful data for two reasons. First, as reported above, the most widely used framework to study problem posing (Stoyanova & Ellerton, 1996) does not account for such instances. Second, student-initiated problem posing episodes may help us understand how teachers respond to students' questions and intuitions about the mathematics. This type of catalyst-analysis can help us gain insight into how willing a teacher is to capitalize on students' mathematical ideas (see *Student-as-Catalyst Example* section below).

Teacher-as-Catalyst Example. The teachers often initiated a problem-solving episode by asking the students to create problems similar to those solved earlier in the class. On some occasions, the teacher asked the students to pose problems in groups for classwork after a lecture. In some instances, the students created problems and traded them with classmates to solve. That was the case in Ms. Green's class on two occasions. In other instances, the teacher asked the students to create problems for a homework assignment. Ms. White provided an assignment for homework during one of the episodes from her class. In that episode, Ms. White asked students to create contexts for routine exercises. She provided the exercises and asked the students to assign a "real world scenario" in each case and to compute the final answers in terms of the scenario they generated. In summary, the teacher-initiated episodes always contained a specific directive from the teacher to create problems. By way of an example, consider the following episode from Ms. Green's class.

Ms. Green: All right, so what I want you to do on your paper, okay? You are gonna come up with a real-life problem that we might see that has to, that deals with integers. We could, it could deal with integers; it could deal with

> absolute values; um, it could deal with opposites; anything like that. I want you to think about, and you think about that in your group and come up with an idea to, to test your fellow classmates. So I don't want you to put an answer on this paper. I don't want you to [write] "What is the, the absolute value of 12 is 12"—that's not what I want you to do. I want you to come up with a real-life example. So for example, I might, my group might say, "Okay, … we are,… it is tax season, okay? And we've paid all these taxes into the government, and now we have filled out our tax return, and we owe the government a hundred dollars. Represent that using integers or something like that." Does that make sense? Or, "I am on the high dive at sea level, and I'm gonna jump into the ocean, and I go 7 feet below the surface of the ocean. Represent that, um, with integers, okay?" Anything like that, anything that you can think of that has to do with anything we've talked about in the past week. But I want you to come up with an example for the class to somehow test their knowledge. Can ya'll do that? Do you think you can do that? All right, we'll set the timer for 10 minutes. Okay? So brainstorm.

Ms. Green then walked around and visited each group of students as they created problems relating to integers or signed numbers. The students discussed and created problems in groups for the rest of the period. The students wrote their problems on a large sheet of paper. Ms. Green posted each group's problems on the wall at the end of the class. She read each problem for the class and asked students to solve each group's problem on their iPads. The students then submitted their responses to Ms. Green electronically on their iPads.

Student-as-Catalyst Example. As mentioned above, a student-initiated episode may help provide insight into the willingness of a teacher to capitalize on a student's mathematical ideas. A student-initiated episode occurred in Ms. Gold's class when a student engaged in problem posing and questioned the constraints of a given problem. Ms. Gold provided a scenario prompt for her students involving the measurement interpretation of the division expression 5 ÷ 1/3 (in other words, the number of groups of one-third that are in 5). Ms. Gold initially posted the following question on the SmartBoard: "Mac has 5 cups of dog food left. If he feeds his dog, Nick, 1/3 cup a day, how many days will it be when Nick runs out of food?" During the discussion, Student J asked a "what if not" question (Brown & Walter, 2005). The discussion centered on counting the number of groups of size 1/3 that are in 5 whole groups. Student J's question caused Ms. Gold to change the discussion, and she asked the class to modify the divisor from 1/3 to 2/3.

Student J: But what if it was like two-thirds. What would you do? Like, if… If the one third were— were like, two thirds, what would you do? Like, how would you do that?

Ms. Gold: All right. So I have a question. What if the serving was two thirds?

Student J: Would you like—?

Ms. Gold: Wait a minute. Let's – let's switch it up. So now we don't have—. We have five cups. But, Student J wants to change this to two-thirds. All right. How many two-thirds? [overtalk]

Student J: [overtalk] Would it be thirty?

Ms. Gold: [overtalk] are there in five?

Student J: Would it be thirty? Would that be thirty?

Ms. Gold: Look, I don't know. You see. Can you prove it? What would that be if two-thirds was the serving?

Ms. Gold took Student J's question and asked the class to pursue the new scenario with 2/3 as the divisor. It is possible that Ms. Gold intended to eventually modify the divisor later, but, in this instance, she leveraged Student J's question by asking the entire class to consider the problem Student J posed. This example illustrates a student-initiated episode of problem posing based on changing a constraint in the original problem. It also illustrates how the teacher took the student's mathematical idea and used it for the rest of the lesson.

The Types of Problems Posed

Second, note that the only types of problems the students posed were routine exercises or traditional problems, with over 60% of the problems classified as routine exercises. For the routine exercises, sometimes the students created the exercises and other times they created contexts for a given, routine exercise. Nine of the routine exercises episodes occurred as a result of occasions where a student or group of students created a context for a routine exercise. Consider an example where students provided the contextualization of a given, routine exercise from Ms. Lavender's class. The routine exercise was to compute the sum of positive five and negative eight. Ms. Lavender asked her students to create a story or context to make sense of or model the addition of the two numbers. Student B created a story involving borrowing and paying off debt. In other cases, the students created an exercise.

In contrast to the creation of routine exercise, consider the following from Ms. Green's class as an example of the creation of a traditional problem. Ms. Green asked the students to create a problem to represent integers. One group created a problem involving the subtraction of a negative integer from a positive integer. They created the following problem: "As we were driving home, Student M drove us off a cliff. Beneath the cliff was a pond that was 25 feet deep. How far did we fall?" Aside from the tragic ramifications of the context they created, this problem has the characteristics of a typical, traditional word problem in a textbook, and I coded it as a traditional problem. I found it noteworthy that the students did not pose any problems that were problematic. It is unclear why the students did not pose such problems.

Problem-Posing Episode Classifications

As Table 3 shows, I observed both structured and semi-structured episodes of problem posing in the teacher-initiated episodes. The majority (two-thirds) of the problem-posing episodes were either structured or semi-structured. Five of the episodes were structured episodes, and 11 were semi-structured. Seven out of the 11 semi-structured problem-posing episodes comprised situations in which the teacher asked the students to contextualize (create a story or context for) a routine exercise. I did not observe any instances of free problem posing.

Strands of Mathematical Proficiency

I chose to identify the most prominent strand of mathematical proficiency (Kilpatrick et al., 2001, p. 5) for each problem-posing episode to highlight the mathematical processes in which students were engaged when problem posing occurred. Kilpatrick et al. (2001) described problem posing as a form of strategic competence. It is reasonable, then, to conclude that episodes of problem posing support strategic competence at some level for all instances of problem posing. As a result, I did not code for strategic competence in these episodes, as I assumed every problem-posing episode included strategic competence by definition. It also seems reasonable that problem posing might support productive disposition because it can help students to "see sense in mathematics, to perceive it as both useful and worthwhile … and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001, p. 131).

Therefore, I did not analyze the episodes for productive disposition or strategic competence. I chose, instead, to focus instead on the presence of procedural fluency, conceptual understanding, and adaptive reasoning. Because some episodes supported the development of both procedural fluency and conceptual understanding, it was difficult to choose only one strand as being most prevalent. Thus, I used the following four codes to identify the strands of mathematical proficiency in each episode: building procedural from conceptual, procedural fluency, conceptual understanding, and adaptive reasoning.

Table 4 illustrates the predominant strand of mathematical proficiency addressed across the episodes. The counts in the table need not imply an absence of the other strands of mathematical proficiency. For example, episodes with the adaptive reasoning code likely also had influences from some of the other strands. In fact, Kilpatrick et al. (2001) claimed that the proficiency strands are interwoven and support one another (pp. 133–134).

Table 4

Problem Posing and Strands of Mathematical Proficiency

Primary strand of mathematical proficiency addressed in PP episode	Total
Building procedural from conceptual	11
Procedural fluency	6
Conceptual understanding	2
Adaptive reasoning	5

For many of the problem-posing episodes, conceptual understanding helped to engender procedural fluency. Six of the 11 instances of *building procedural from conceptual* occurred within a context concerning the interpretation of the division of fractions or within a context of a discussion about the interpretation of division.

Consider the following example from Ms. Green's class. Ms. Green engaged students in a discussion about dividing with fractions. Ms. Green asked her students to create a new problem after they worked in groups for the first part of the class. She said, "I want you to come up with your own example similar to the [equal sharing] cake question, but with fractions." She continued to ask students to use multiplicative reasoning when they constructed pictures of the division problems created. For example, in one group she questioned, "We are already dividing here, but when we say the word *of* what operation are we doing?" Some students began using the typical, procedural algorithm by switching to multiplication and taking the reciprocal of the second fraction. Ms. Green praised them by saying, "I am super impressed by the fact that a lot of us are just taking regular math problems where we might use our long division algorithm, and instead we're talking about how to turn those into fractions and multiplying and dividing with fractions." This example illustrates how Ms. Green used the students' conceptual understanding of multiplicative reasoning to relate the algorithm for dividing fractions to multiplication.

In these cases, the problem-posing task served as a potential bridge between the procedural and the conceptual. The precursor to such episodes included grounding in conceptual understanding of fractions or division. The prompt to create a new, similar problem or exercise helped to reinforce conceptual understanding while motivating a possible procedure for the exercise.

Another finding was that students initiated *every* instance of problem-posing episodes that addressed adaptive reasoning. According to Kilpatrick et al. (2001), adaptive reasoning is "capacity for logical thought, reflection, explanation, and justification" (p. 116). In these cases either the student created a new problem that illustrated such logical thought, reflection, or justification, or the teacher asked for the justification or proof as a result of the problem posed by the student. Not once did I observe any problem-posing episodes with adaptive reasoning that were initiated by the teacher. This observation might support the idea that students possess strong intuitions and curiosities about mathematical ideas, including reasoning and proof even in late elementary and early middle grades.

The Interaction Between Problem Posing and Mathematical Content

I was interested to see if problem posing occurred more frequently in certain mathematical topics and whether patterns, if they existed, were consistent across teachers. I observed lessons involving fractions, integers, and algebraic reasoning and categorized episodes of problem posing within each of these mathematics topics. In all, there were 16 lessons dealing with integers, and in these 16 lessons there were 5 episodes of problem posing. There were 37 lessons on fractions, and 10 instances of problem posing during those lessons. There were 35 lessons on algebraic reasoning, and 9 instances of problem posing. Table 5 provides the counts and percentages for problem-posing episodes within each type of mathematics content in each lesson across all six teachers.

Table 5

Counts Problem-Posing Episodes by Topic and Teacher

	Total Lessons Observed			Problem-Posing Total		
Participant	Int	Frac	Alg	Int	Frac	Alg
Mr. Blue	0	8	5	0	1	2
Ms. Gold	0	7	7	0	3	0
Ms. Violet	0	7	7	0	1	0
Ms. Green	7	5	6	2	3	6
Ms. White	3	4	5	1	2	1
Ms. Lavender	6	6	5	2	0	0
Totals	16	37	35	5	10	9

Problem Posing at the Teacher Level

Problem Posing in Mr. Blue's Classroom. Mr. Blue taught fifth-grade in the Western United States. I observed 3 instances of problem posing from 13 lessons from his class. One episode was student-initiated, and the remaining two were teacher-initiated.

The student-initiated problem-posing episode in Mr. Blue's class involved adaptive reasoning in the form of a logical thought, reflection, and explanation (Kilpatrick et al. 2001, p. 116). Three or four students comprised each table in the room. Mr. Blue provided the following instructions: "See if your table can come up with one or two reasons that you think this is true, three divided,… True or false: three divided by four equals three-fourths. Is it true or false? Why? Prove it." Mr. Blue wrote "$3 \div 4 = \frac{3}{4}$" on the board in the front of the room as a visual reference for the students. It seems that Mr. Blue did not necessarily expect a rigorous mathematical proof. For example, he told them to come up with "one or two reasons why you think it is true or false." In fact, he used the term "proof" synonymously with the term "mathematical model" when giving the directions for this task.

The students collaborated in groups of three or four during this task. Several students initially conjectured that the two expressions were not equivalent. Some students referred back to a previous problem from a demonstration earlier in the period (five candy bars shared equally by eight students). Student L seemed to want a context to make sense of the division. Mr. Blue interrupted Student L's idea:

Student L: If three candy bars are shared by…[overtalk]

Mr. Blue: [overtalk] three candy bars shared by four … so when you don't have that context, it's harder to think about it. Okay, interesting, interesting.

As the groups continued to discuss possibilities, Student R eventually made an appropriate connection once she had provided a context, or story, to accompany the proposed equivalency. The following excerpt from the transcript helps demonstrate Student R's reasoning about this task.

Mr. Blue: All right. I'm going to have [Student R] come up, and this is kind of just the next step, this is where we're going to be going, uh, more this year, this is not for… I don't expect you guys to know this right now, but if you do that's great.

Student R: Um, I, well, we put it into cents, and we made it quarters because three-fourths equals, um, seventy-five; zero point seventy-five. So, um, we put three quarters into, like, four spots pretty much right here.

Mr. Blue: How did you … Can I interrupt you for one second? How did you know to change it into three quarters, three quarters?

Student R: Because if you, if there was, if it was four-fourths, that would be a whole, like a dollar and [overtalk].

Mr. Blue: Okay, so four-fourths [overtalk] would be a whole dollar, but we didn't have four-fourths, we had …

Student R: Three-fourths.

Mr. Blue: Three-fourths.

Student R: So we put the three quarters into four slots, and there was a leftover slot. So we got seventy-five cents, which—. Seventy-five cents, when you divide it into four slots, um, one is empty, and you, pretty much, your change is seventy-five cents, just like three fourths. So it's true because

> three-fourths, uh, is equal to, um, three divided by four is, that's what we did up here.

This was another example of the contextualization of a routine exercise. In this case, the contextualization of the exercise helped Student R to conclude that the two quantities were equivalent. I coded this episode as adaptive reasoning because the task demonstrated students' capacity for logical thought, reflection, explanation, and justification. Note once again that the notion of "proof" here did not indicate a rigorous mathematical proof, but rather a reasoning process that was appropriate for a typical fifth-grade mathematics student.

Problem Posing in Ms. Gold's Classroom. Ms. Gold taught a fifth grade, on level, mathematics class. Her students were classified as neither gifted nor accelerated. I observed 14 of Ms. Gold's fifth-grade mathematics lessons and identified 3 episodes of problem posing. Problem-posing episodes occurred at a rate of approximately one episode every 5 lessons in Ms. Gold's class.

The instances of problem posing in Ms. Gold's classes were unique because her students initiated all three. In addition, all three involved adaptive reasoning. None of the transcripts contained evidence to suggest that she ever asked her students to directly pose or create a problem. In contrast, the only other teacher in this study with three student-initiated instances of problem posing was Ms. Green. All the other teachers in the study explicitly provided at least one opportunity for students to create their own problems, yet Ms. Gold's students accounted for three out of the eight student-initiated problem-posing episodes.

One problem-posing episode also occurred in Ms. Gold's class when Student J proposed a conjecture during a discussion concerning the addition of fractions. During the course of the class discussion one student suggested that it is appropriate to add denominators when adding two fractions. Student J disagreed. He claimed, “The denominator has to be bigger than the numerator to reach a whole number.” Ms. Gold identified Student J’s statement as a “conjecture.” She said, “… I like that … I’m going to write that *conjecture* down.” Moments later, Student M revoiced the conjecture as follows: “If you add the denominators, then you’re never going to get to a whole number.” Ms. Gold used this conjecture and supported a class discussion for approximately 15 minutes. Ms. Gold used this situation to help students see that adding denominators is not appropriate when adding two fractions. Ms. Gold concluded, “if you add your numerators (corrects herself), um, denominators, then you’re always getting a fraction that is smaller… and you’re changing the whole so that you would never reach a whole. Okay… I’ve never thought of it that way.” Student J initiated problem posing because he vocalized a conjecture and Ms. Gold used it to guide students to make sense of what it means to add fractions.

Ms. Gold regularly questioned her students about how they thought about mathematics, and she often required precise language from them. For example, she did not typically allow the use of unclear pronoun referents when students spoke of mathematical ideas. If the student said, “You divide it,” Ms. Gold followed up with the question, “What do you mean by *it*?” She regularly pressed her students for precision in mathematical language. It was also not unusual to observe Ms. Gold say to a student, “I want you to prove it to me or to your class,” when a student presented his or her work. In the earlier example of teacher-student interaction in her class, Ms. Gold asked the student, “Can you prove it?” after the student provided a response to the 15 ÷ 2/3 scenario. For that reason, adaptive reasoning occurred regularly in Ms. Gold’s classroom.

Additionally, Ms. Gold also used journal writing as a means for her students to engage in written discourse about mathematics. In at least one example, she used the journal as a means to assess students' understanding about a particular problem. In one lesson she posed an equal sharing division problem on the SmartBoard screen. She gave students 1 minute to read the problem without writing anything in their journals. Then she removed the problem from the screen and asked the students to write in their journals, in their own words, the meaning of the problem. Several of the students were asked to share what they had written. She then placed the problem on the SmartBoard once again for students to read and requested that they compare what they wrote with the original problem statement in order to reflect on their understanding of what the problem was asking.

In another equal-sharing division problem, Ms. Gold again pressed her students to make sense by posing the following problem: "If eight children shared five hamburgers equally, how many would each child get?" She then posed the following series of questions to the class about the problem:

- Who or what is the problem about?
- What is the situation in the problem?
- Does it say *dividing* in the problem?
- Is there anything unusual about this problem … anything you might not have seen before in this situation (referring to the fact that eight is not divisible by five)?
- Are we dividing eight by five?
- Are we dividing up children?
- What are we sharing?
- Who is sharing the burgers?
- What does it say about how those children are going to share the burgers?
- What does it mean that they are going to share them equally?

Ms. Gold demonstrated a pattern of asking these kinds of questions about problems she posed for the students. To put it another way, she regularly questioned the constraints of problems that she presented. It is possible that her consistent questioning of the constraints of problems encouraged her students to question the constraints as well. Perhaps that is why each instance of problem posing in her classroom was a student-initiated episode.

Problem Posing in Ms. Violet's Classroom. I observed 14 lessons in Ms. Violet's fifth-grade mathematics class. Only one of her lessons contained problem posing. In addition, that instance occurred for only one group of students. During that lesson, each group (station) was assigned a different task as a means to investigate the division of fractions. One group was asked to create such a problem relating to the division of fractions. There is no evidence that students outside of that group engaged in problem posing during this episode. Ms. Violet directed the students in the group to include wording in their problem so as to ensure that division was unambiguous. Ms. Violet was the only teacher with one observed instance of problem posing.

Problem Posing in Ms. Green's Classroom. I observed 18 lessons in Ms. Green's classroom. Problem posing episodes occurred in Ms. Green's class, on average, at approximately once in every two lessons. Table 5 illustrates the 11 problem posing episodes I identified in her class. I provide more details about Ms. Green's classroom, including some of her reflections on problem posing in the case study presented in the second half of Chapter 4.

Problem Posing in Ms. White's Classroom. Ms. White taught sixth-grade mathematics and also practiced in the same district as Mr. Blue. In the 12 lessons we observed with Ms. White, there were 4 instances of problem posing. I will describe a few details from the first episode. This episode began on September 30 as a homework assignment concerning one-step, algebraic equations. The lesson on September 30 concerned identifying the unknown in a problem and then representing the details of the problem in the form of an algebraic equation. Ms. White gave the following directions at the end of class:

> I want you to write me a word problem that involves an … adding an algebraic expression. Try not to make it too wild and crazy and out of the scope of plausibility. I mean it could be … creative, but within being appropriate.

The next day, Ms. White asked the students to conceal the answers to the problems they created for the homework task. The students then exchanged and attempted to solve each others' problems. The students exchanged problems more than once over the course of the first 20 minutes of class. After the exchanges, Ms. White led the class in a discussion about the various types of expressions they created. She used their examples as an opportunity to discuss the placement of the unknown in the algebraic equations.

Problem Posing in Ms. Lavender's Class. The only teacher with two observed instances of problem posing was Ms. Lavender. Both of the episodes in Ms. Lavender's seventh-grade mathematics class involved the addition of integers. One occurred near the beginning of the fall semester, and one near the end of the fall semester. The first episode occurred during a discussion about adding negative integers. Ms. Lavender asked students to create story problems to match the integer expression. She asked, "Who can come up with a story? … Tell me a story that could represent this problem." She used the students' responses to help frame her discussion of integers during the next day's class. This is an example of teacher-initiated, structured problem posing of routine exercises. It is also an example of the contextualization of a routine exercise (creating or assigning a story problem to fit some given numerical or algebraic exercise, expression, or equation).

Summary of Study 1

The teachers in each of the six classrooms provided an expectation for students to engage in mathematical discussions during instruction. This expectation was most often evidenced by the fact that students were always in pairs or larger groups during each lesson. It was also common for the students to present their work to the class and for other students to comment and question one another. In addition, all problem-posing episodes contained some type of group-work dynamic. It is plausible to suggest that classrooms where mathematical discussions occur regularly may provide an environment to promote mathematical problem posing. Perhaps it is less likely to expect students to spontaneously pose their own problems during a mathematics lesson if they do not discuss their ideas with each other.

Most of the problem-posing episodes in this study involved teacher-initiated, semi-structured, routine problems. Most of the semi-structured problem-posing episodes concerned the contextualization of routine exercises. The students initiated all of the instances for which adaptive reasoning was the predominant mathematical strand of proficiency.

The students in this study initiated problem-posing episodes even when their teacher did not ask them to pose problems. It seems that each instance involved a student's desire to better understand the content or to satisfy his or her own mathematical curiosity. In particular, none of Ms. Gold's 14 lessons contained any teacher prompts for students to create their own problems. Each of the three posed problems from Ms. Gold's class occurred, at least in part, because of Ms. Gold's classroom structure and expectations. She regularly modeled the questioning of constraints and she regularly asked students to think deeply about mathematical ideas.

Study 2: A Case Study of Problem Posing in Ms. Green's Classroom

Ms. Green taught mathematics and science courses at a local middle school. I observed only Ms. Green's first block, accelerated sixth-grade mathematics course. Ms. Green had recently participated in a technology partnership with another large technical university in the Southeastern United States. As a result of her partnership, she had received financing to provide iPads for her students and other forms of technology including a Promethean board, an Apple TV, a large screen television, and a video camera. I did not observe the use of the video camera, but Ms. Green occasionally used it for video conferencing with those outside of her school, including an instance when her students presented their projects to a research scientist at a rocket propulsion laboratory. Ms. Green had a reputation for using technology in her classes, and she served as the Webmaster for her school.

Ms. Green completed a Bachelor of Science degree in Middle Grades Language Arts and Social Studies in 2006. She began teaching in the Piedmont School District (pseudonym) in 2006, and she continued to teach in the same school district. We began observing her class during the start of her eighth year of teaching. She completed a Master of Education in Middle Grades Mathematics and Science in 2008, and an Education Specialist degree in Media and Instructional Technology in 2011. Ms. Green's colleagues selected her as their school's teacher of the year for the 2012–2013 school year. Ms. Green regularly used small group work. Her students were in pairs or groups in each visit to her classroom. Her qualifications, experience, reputation, and disposition for problem posing with her students made her a good candidate for this case study.

Lessons Observed With Problem Posing in Ms. Green's Classroom

Members of our research team filmed 18 lessons from Ms. Green's classroom during the 2013–2014 school year. The lessons were filmed between late September and late February. The topics of those lessons included fractions, integers, and algebraic reasoning. We narrowed these broad content areas to more specific standards and topics (e.g., fraction division) to help with lesson selection. We planned to film approximately 5 lessons per topic focused on specific curricular standards, but also relied on Ms. Green to identify lessons on those topics that might lend themselves to problem solving and discussions.

Problem-Posing and Problem Characterizations

Table 6 reveals the counts and percentages of the 11 instances of problem posing I observed during the 18 lessons that we filmed. The counts and percentages are categorized by catalyst (teacher or student), problem type (Schoenfeld, 1992), and type of problem posing (Stoyanova & Ellerton, 1996) with the additional student category.

Table 6

Frequencies of Problem-Posing Episodes in Ms. Green's Classroom

	Catalyst?		Problem Type			PP Type			
	Tchr	Stud	Rout.	Trad.	Prob.	Stud.	Struct.	Semi-St.	Free
Totals	8	3	6	5	0	3	2	6	0

Although Ms. Green was the catalyst for the majority of the problem-posing episodes (73%), there were instances in her classroom where students initiated problem posing. In fact, over one-fourth of the problem-posing episodes were student-initiated. The predominant type of problem posed was that of a routine exercise, though traditional problems were also commonly used. Half of the routine exercise episodes (three out of six) occurred when students provided a contextualization of a routine exercise.

Consider the following example of a semi-structured, routine exercise. During the episode on October 24, Ms. Green wrote "3 + *a* = 12" on her iPad and displayed it on the large screen television for the students to see. She asked, "Can someone come up with a word problem or an explanation for this that we could use?" She provided a routine exercise—solving a one step linear equation with one unknown—and she prompted the class to think about a situation in which this equation could describe a "real" scenario. In response, Student S gave the following context: "Jack owned three mountain peaks in the Appalachian Mountains, and Susie owned *a* [the unknown] mountain peaks in the Himalayan Mountains. How many mountain peaks do they own together?" That was how Student S viewed this routine exercise in a particular context, although the context seems quite contrived and not particularly "real." Also note that the question posed by Student S was not exactly what Ms. Green intended. It seems that in this context, the total number of mountain peaks that was owned between Susie and Jack was already clear. Ms. Green addressed that issue by asking the student to clarify, once again, what was known and what was unknown in the context that she provided. The student then clarified that the desired solution (the unknown, *a*) answers the question of the number of mountain peaks that Susie owned as indicated in the transcript excerpt below. I begin with Student S's initial context:

Student S: Jack owned three mountain peaks in the Appalachian Mountains, and Susie owned *a* mountain peaks in the Himalayan Mountains. How many mountain peaks do they own together?

Ms. Green: Okay, so they owned how many together, because we could tell you that information…

Student S: 12.

Ms. Green: 12, so we want to know how many, who?

Student S: Susie.

Ms. Green: Susie. How many did Susie own?

Several: 9.

Ms. Green: 9, and how do you know that?

Several: (murmuring, inaudible).

Ms. Green: Because we know, what?

Student N: That 3 plus 9 equals 12.

Ms. Green: We know that 3 + 9 = 12. It's always going to equal … 3 + 9 is always going to equal—.

Several 12.

Ms. Green: 12, right?

Student S corrected her mistake quickly after Ms. Green asked clarifying questions. Ms. Green continued the discussion to help the students see the relationship between the equation $3 + a = 12$ and the equation $a = 12 – 3$. She then demonstrated how to perform the subtraction on both sides of an equation as a procedure to solve it.

And, finally, all teacher-initiated episodes were structured or semi-structured, which is likely related to the nature of the problems Ms. Green requested. Semi-structured and structured problem-posing contexts may not provide the conditions for students to engage in problems that are problematic. Alternatively, the choice of problem or activity (in this case, routine exercises and traditional problems) may have constrained the nature of problem posing so that the activities provided more structure and less flexibility to students. Because structured problems are based on a specific problem or solution (Stoyanova, 1996; 1998), it seems that structured problem-posing tasks are unlikely to yield problems that are problematic.

The Mathematics in Ms. Green's Problem-Posing Episodes

Table 7 contains information about the topics of each lesson taught by Ms. Green that we also observed. We did not observe Ms. Green's lessons on geometry, data analysis, or measurement standards.

Table 7

Lessons With and Without Problem Posing in Ms. Green's Classroom

	Fractions	Integers	Algebra	Total
All lessons observed	5	7	6	18
PP observed	2	2	4	8
PP not observed	3	5	2	10
% of episodes with PP	40	29	67	44

Table 7 provides a contrast by illustrating the counts of the lessons observed by topic in which problem posing occurred versus those with no observed instances of problem posing. This table does not report instances of problem posing, but rather the overall lessons for which either problem posing did or did not occur. For example, Ms. Green's entire class on August 30 consisted of students posing problems related to fractions and fraction division. In that lesson, students posed problems in groups throughout most of the instructional period. That counted as one problem-posing episode. By contrast, a lesson in her class from October 9 contained an episode in which one student posed a numerical expression for the purpose of illustrating the order of operations. That episode ended within 10 minutes. Later in the same lesson, Ms. Green then also initiated a problem-posing scenario by asking the students to come up with a real world problem to model a one-step, linear, algebraic equations with one unknown. For that reason it was possible to observe two problem-posing episodes in the same lesson.

Strands of Mathematical Proficiency

I analyzed each of Ms. Green's lessons containing one or more episodes of problem posing in search of examples of the strands of mathematical proficiency (Kilpatrick et al., 2001). Table 8 lists the counts of the types of mathematical proficiency observed during episodes of problem posing in Ms. Green's classroom.

Table 8

Mathematical Proficiency During Problem Posing in Ms. Green's Lessons

Procedural	Conceptual	Adaptive	Building Procedural from Conceptual
3	2	0	6

The table shows that there were no examples of problem posing where adaptive reasoning was the predominant strand, although Ms. Green did ask students to justify their answers and engage in appropriate mathematical reasoning, but not in a problem-posing context. In her problem-posing activities, Ms. Green emphasized conceptual understanding along with procedural fluency—below I share additional evidence from an interview that supports this interpretation. Table 9 contains a mapping of the mathematical topics from each problem-posing episode onto the strands of mathematical proficiency I used in this study.

Table 9

Mathematical Topics and Mathematical Proficiency in Ms. Green's Classroom

Episode	Strands of Mathematical Proficiency			
	Procedural	Conceptual	Adaptive	Building Procedural from Conceptual
1				Interpretation of division of fractions

2			Interpretation of division of fractions
3			Fraction division; measurement division
4	Algebraic expressions; associating words with procedures		
5		Creating algebraic expressions; comparing unknowns	
6		Creating algebraic expressions; adding like terms	
7	Order of operations		
8	Order of operations; evaluating numerical expressions		
9			Solving one-step linear equations; maintaining equality
10			Integers and their opposites; absolute value
11			Integer addition; addition of signed numbers

An Example of an Episode With an Emphasis on Procedural Fluency

The following example of a procedural fluency episode occurred on October 9. A discussion occurred about the order of operations during the last half of the class. This discussion was not an introduction to the order of operations for these students because Student B said, “Last year (implying, in the fifth grade), when we learned about the order of operations, there was this song that we learned.” A few moments later, Student C brought up the acronym, PEMDAS (parentheses, exponents, multiplication, division, addition, subtraction). Ms. Green asked, “Why do we have this? …What is the purpose?” Student W responded by creating a numerical expression with several operations as a justification for the necessity of PEMDAS. Ms. Green transcribed his expression on the iPad and Apple TV for all to see. Figure 2 is a still video capture of the expression Student W created.

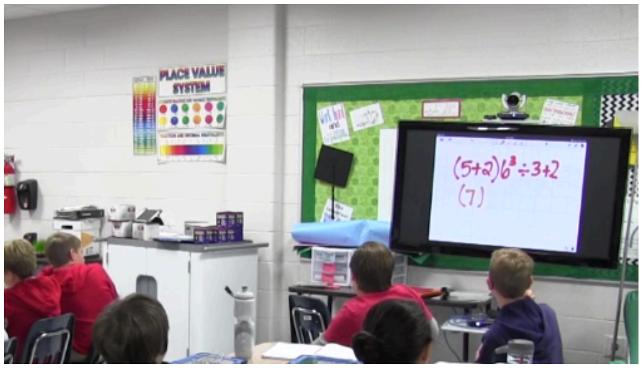

Figure 2. Student W's numerical expression on order of operations

The screen displays "$(5 + 2)6^3 \div 3 + 2$." The beginning step of the simplification of the 5 + 2 is also displayed as "7." Student W then related each operation back to the PEMDAS acronym in order to explain the procedure for using order of operations to simplify numerical expressions. Student W created a numerical expression, or posed a mathematical problem, that was a routine exercise for these students in order to illustrate the order of operations. This problem-posing episode promoted procedural fluency. Ms. Green then prompted the class to talk through the simplification of the expression using the appropriate order of operations.

An Example of an Episode With an Emphasis on Conceptual Understanding

In an episode on October 8, Ms. Green's students were working on a task provided by the state department of education. Students were presented with a scenario in which a certain mathematics class planned a field trip to a theater. They were told that the price was \$10 for the school bus and the price of the ticket was \$13 for each student. They were instructed to create an expression to determine the amount of money required for the class to make the trip. Ms. Green indicated in a follow up interview that the intent of the task was that only one bus of students would travel to the theater. She explained that the intent was that the students would create an algebraic expression to represent the cost for taking one bus to the theater when tickets cost \$13 per student. She expected some kind of equation analogous to $C = 13x + 10$, where C represented the cost of attendance for the class, $13x + 10$ represented the corresponding algebraic expression for the cost, and x represented the number of students in attendance. Instead, in this episode students asked a what-if-not (Brown & Walter, 2005) question as they worked in groups on this task.

Students in several groups expressed concern that one bus would be insufficient to take all of the students to the theater. For example, Student S claimed, “If you have more than 30 students, there has to be an extra bus, because the bus can only hold, like, 30 students.” Students in multiple groups struggled with this issue. It is unclear as to whether one group overheard another group’s discussion of this issue and adopted it as their own or whether individuals in each of these groups shared the same concern for this issue of the number of buses. Regardless, it became a part of the discussion that Ms. Green did not anticipate, according to her follow-up interview. Ms. Green tried to assure the students that one bus would be sufficient, but they continued to insist that more buses were necessary. Later in the discussion, Student K said, “I put S times 13, and then 10 times B … for the number of busses.”

This problem-posing episode emphasizes conceptual understanding of algebraic expressions and equations because students demonstrated a deeper “comprehension of mathematical concepts, operations, and relations” (Kilpatrick et al., 2001, p. 116) than was expected. In particular, the students were comfortable extending the problem from one to two variables. Their extension of the task seems unlikely without conceptual understanding of the role of the variable in this problem. The students seemed satisfied after they considered an unknown number of buses as well as an unknown number of students. In addition, in a follow up interview Ms. Green mentioned that their extension of this scenario to include two variables helped her to see that they understood the role of the coefficients.

I used this episode as one of the stimulated-recall questions, and I showed a portion of this video to Ms. Green during the follow-up interview. In the interview, I asked Ms. Green, “What, if anything, were you able to tell about [these students] as mathematics learners [as result of this episode]?” Ms. Green responded, “They *understood* that there were multiple factors to the price. It was good algebraic reasoning.” She also acknowledged that perhaps their approach to this scenario signaled that her students were ready for experiences with algebraic expressions and equations with more than one variable. She did not indicate the extent of those experiences—only that they could be ready for more mathematically robust situations.

An Example of Building Procedural Fluency from Conceptual Understanding

The episode from February 3 contains an example of problem posing that helped to build procedural fluency from conceptual understanding. I provided an excerpt of the video clip from this lesson for Ms. Green to view during the interview, and I asked her to reflect on it. The episode was from a lesson concerning integers and other signed numbers. Ms. Green intended for this lesson to provide students with an opportunity to identify integers and other signed numbers in real-world contexts, and to connect those contexts to mathematics. The primary discussion concerned integers, but she allowed other signed rational numbers as well because some students provided examples that were not actually integer values. Some of the students discussed stock prices, most of which contained (positive and negative) decimals. Each group posed one problem and displayed it on a sheet of poster paper. Ms. Green gave the following instructions:

> You are going to come up with a real-life problem that we might see … that deals with integers.… It could deal with integers, it could deal with absolute values, it could deal with opposites—anything like that. I want you to think about—, That, in your group and come up with an idea to … test your fellow classmates.

Ms. Green acknowledged in the follow-up interview that she did not expect the students to begin to subtract integers or signed numbers during this lesson. In particular, she did not expect the students to create a scenario that involved subtracting a negative integer from a positive integer. She claimed that she simply wanted them to represent real-world scenarios that involved integers or signed numbers. Nevertheless, one of the groups did create a problem that involved the subtraction of two integers, which was beyond the scope and intent of the lesson. Student M's group (four students in all) created the following scenario: "As we're driving home, [Student M] drove us off a cliff that was 72 feet tall. Beneath the cliff was a pond that was 25 feet deep. How far did we fall?" The problem did not state that the car came to rest on the bottom of the pond. When Ms. Green read this problem to the class she added, "They landed on the bottom of the pond—before you start getting technical." The students in this group seemed to intuitively understand the important role of absolute value in this scenario as evidenced by their discussion with Ms. Green. For example, Student M explained, "We need to know both of the absolute values." Ms. Green then asked the group, "What kind of mistake do you think many students would make when trying to solve this problem?" Student M answered, "They would subtract, they might subtract." Of course, one way to find the overall difference between the height of the car and the depth of the fall is to use subtraction (as in 72 – -25) but the students in this group were taking the absolute values of the quantities and adding them. It appears that Student M thought that the subtraction mistake was to subtract 25 from 72 without the consideration of the signs of both numbers.

In this situation, Ms. Green's problem-posing activity provided an opportunity for students to connect the mathematical concepts of integers, signed numbers, and absolute value to real-life contexts (regardless of whether or not those contexts were exaggerated or even feasible).

Student M's group used a conceptual understanding of integers and created a scenario that involved the subtraction of a negative integer from a positive integer. I coded this as "building procedural from conceptual" because Ms. Green indicated in the follow-up interview, "We referenced [this] poster later on… when we talked about adding and subtracting integers" (interview). As a result, this problem scenario was used later to support procedural fluency for adding and subtracting integers.

A Culture of Problem Posing in Ms. Green's Classroom

Planning for Problem Posing

In the follow-up interview, I asked Ms. Green about how she planned for problem-posing episodes. Ms. Green always had a lesson plan for her classes, but she admitted that a lesson "never goes [according] to plan." She said that her administrators wanted to see more lesson plans in response to a new teacher evaluation system in her state. Her flexibility mantra appears in her writing of formal lesson plans. For example, she summarized her response to her administrators' requests for more detailed lesson plans as follows: "I can give you written lesson plans but that is not what you're going to see when you come into the room." She reported that the practice of questioning her students sometimes extended "into something that I had not anticipated, but I go with it." She also identified this characteristic as one of her strengths as a teacher. She planned activities in which her students would create their own problems because she wanted "to get that [mathematical] connection more concrete." She believed that problem posing was critical for her accelerated mathematics students because "making them think about what they're doing and applying it to *their own question* or problem solving.… It's going to make it more interesting to them… [and] keep them engaged." In summary, Ms. Green planned most of her problem-posing episodes, but she remained flexible and attentive to students' questions. She made in-the-moment decisions as to whether or not to follow a student's questioning of the constraints of a problem based on various factors including time constraints, the scope of the state mathematics standards, and the type of task provided to the student. Some of the motivations to engage in problem posing also affected her planning, as evidenced in the discussion below.

Classroom Expectations in Ms. Green's Classroom

During the follow-up interview Ms. Green explained that she thought her flexibility played an important role in her overall instructional decisions. She also attributed her personality as part of her motivation for flexibility and group work. She said, "My personality leads to … talking, I like to talk. I like to talk to the kids. I like … getting to know what they think.… It's more of my personality to have this *fluctuating*, go-with-the-flow kind of room, I guess."

One common classroom expectation in Ms. Green's classroom was student collaboration in groups, as evidenced by observations and the follow-up interview. Ms. Green's students were arranged in pairs or groups during each of our observations. Ms. Green also mentioned, "I did a lot of … group work and … stations." She contrasted her flair for group work with than her own. In other words, her colleague's students worked individually in class more than they worked in groups. By contrast, Ms. Green moved around from group to group to interact with the students and to discuss mathematics with them in every lesson observed. The observations and the interview suggest that group work and classroom discussions were common occurrences in her classroom. Ms. Green had expectations for how students should interact in group settings and spent time early in the year establishing those expectations.

At the beginning of the school year, it was common to observe Ms. Green redirect students who were off task or disengaged in group work activities. It was also common, initially, for students to assume a *divide-and-conquer approach* to group work. For example, Student A might complete one subset of a given task, and Student B might complete another. Once the two students had completed their respective subtasks, they swapped and copied one another's work. Ms. Green redirected this kind of approach near the beginning of the school year, but she found it less necessary as the year progressed as students adjusted to her expectations. She emphasized that the students should collaborate (talk and discuss) on the mathematics tasks she provided. I observed that the students slowly moved away from the divide-and-conquer approach as the year progressed by involving one another in all areas of the problem.

Ms. Green expected students in groups to move through a task as a cohesive unit, not as individuals who merely compared answers with each group member once the group finished a task. On more than one occasion, Ms. Green assisted one person in a group and then directed that student to explain a problem or concept to the rest of the group members. If one group member got ahead of the other members, it was not unusual to observe Ms. Green admonish that student to go back and collaborate with other group members. She encouraged students to hold one another accountable for understanding the mathematics together and for progressing through tasks.

Not only did Ms. Green's students often work in groups, but she also regularly changed the seating arrangement of the groups in her room. In fact, the seating arrangement was different each time we observed one of her lessons. In addition, in the follow up interview she said, "Probably every time you came into my room my desks were different. They changed almost every day – every single day the kids were like, 'Oh! Where do I sit now?'" She also indicated that she did not "do seating charts." She said,

> I might group them the way I need them to group. I might say, "Okay, you guys come and sit here." I think having that fluidity… helps me make this questioning, this discussion, this [creating problems] easier to happen versus if I have them in their rows and columns.

She compared her classroom dynamics with the classroom dynamics of a colleague who also taught another section of sixth-grade accelerated mathematics. As mentioned above, Ms. Green's colleague's students rarely engaged in group work and group discussions. She reported that although her colleague used the same state-provided task, her colleagues' students did not extend the task as her students extended it. She said, "I don't think [her colleague's students] thought about it that way either." It is not clear whether the individualized classroom dynamics prevented her colleagues' students from extending the task, but Ms. Green thought it noteworthy to mention the distinction when prompted to conjecture as to why the other teacher's students did not try to extend the task. She indicated that she thought it would be difficult to create a problem-posing environment outside of a collaborative or group atmosphere. Consider the following exchange from the interview:

> Q: Do you think that [problem posing] is common to other teachers that you know, or is it something that you do probably more than they do? Any ideas as to why that might be?
>
> Ms. Green: I think … my personality leads to … talking, I like to talk, I like to talk to the kids. I like to… getting to know what they think, that kind of thing. I have– my best friend is a math teacher in seventh grade, and she is very… "no, I'm talking, I'm the teacher, you sit and listen, you sit and practice, you sit and do this." Whereas I am, like, "Okay we are going to do

groups, we're…." It's just, it's more of my personality to have this fluctuating … go-with- the-flow kind of room, I guess.

Q: So do you think the groups help you to do this better?

Ms. Green: Yes.

Q: Would it be hard to do if it, if you didn't have a group environment all the time, do you think?

Ms. Green: Yes. … I think having that fluidity is, to me, it helps make this questioning, this discussion, this type thing, easier to happen versus if I have them in their rows and their columns, and they are going to "sit and get" (referring to students working independently as passive learners).

Ms. Green indicated that she used problem posing "very often" with her accelerated mathematics students. She contrasted the frequency of problem posing in the accelerated class with her other classes by stating, "With my other classes, … maybe once a week we did something where they were having to create the problem." It is unclear whether problem posing was a common occurrence in her classes, but it is safe to say that she used problem posing regularly in all of her mathematics classes.

It was also common to observe Ms. Green ask her students to interpret the meaning of quantities and calculations in the context of the given problem or to explain why they had performed certain mathematical operations when solving a problem. While interacting with students, Ms. Green often asked, "What does this mean?" while pointing to a student's work on his or her desk. For example, during a lesson on fraction division, she presented the following task for her students to do in groups: "In preparing to make hair bows for friends, Samantha realizes she needs two-thirds yards of ribbon for each bow. She has two yards of ribbon. Does she have enough ribbon for four bows?" Ms. Green observed students in several groups create the division expression 2 ÷ 2/3, and simplify the expression to get 3 as a result. Without fail, each student who explained his or her thinking to Ms. Green gave an answer of 3 without considering what it meant in the context of the problem. During the lesson, Ms. Green asked the students to connect the meaning of the 3 to the given scenario. She obtained a piece of ribbon and asked the students to use the ribbon to demonstrate the answer in the context of whether or not there was enough ribbon to create four bows. She continued to discuss this task with several groups, and she questioned them until they realized the connection between the answer of 3 and the question of whether or not there was enough ribbon for four bows.

This type of emphasis on the meaning of answers was not unusual, occurring throughout the majority of video-recorded lessons in Ms. Green's classroom, regardless of whether or not problem posing occurred. In fact, during every observed lesson, Ms. Green asked the question, "What does it [or this] mean?" at least once. She apparently wanted her students on a regular basis to make sense of the mathematics they used.

The use of technology was another feature of Ms. Green's classroom. As mentioned above, she issued iPads to all of her students. The students used iPads during every lesson observed. On some occasions, the students displayed the work that they did on their iPads by taking control of the Apple TV through the iPad. On one occasion, the students created their own instructional videos in groups to display their work on the problems they did during class. Ms. Green frequently asked her students to submit their work to her by sending an email through their iPads. She also believed that "having the technology in my room also helps a lot … with student engagement."

Differences in Problem Posing Across Ms. Green's Other Class Periods

The duration of the episodes of problem posing varied by lesson. I observed two episodes of problem posing that spanned the majority of the class period. Five of the problem-posing episodes in her class lasted between 15 and 30 minutes. The remaining four episodes were short—completed within 15 minutes or less. Occasionally, these episodes involved ticket-out-the-door types of tasks. Ms. Green contrasted the way she structured problem posing in her accelerated class with the way she structured it in her nonaccelerated classes. She indicated that with her collaborative classes, the problem-posing episodes occurred over a whole class period. She said, "It would take longer with my collaborative classes, … and I wouldn't want them to do it at home because a lot of times they would not do it at home." She also noted that problems posed in the accelerated class often had more steps or more humor in them than problems posed in the collaborative class.

I asked Ms. Green to reflect on some of the other occasions of problem posing in classes I did not observe (such as in her collaborative classes—classes with students with diverse learning needs). She reported a memorable instance of problem posing from her collaborative class. It involved a scenario in which she distributed sale papers (advertisements typically found in the newspaper) to her students. This scenario is somewhat similar to the coupon problem-posing study reported by Bonotto (2013). The difference is that Ms. Green intended for the students only to use the sale prices of various items in the paper to compute unit rates for the sale items. Instead, the students in the collaborative class decided that they wanted to expand the scenario after they found a unit rate for a type of dog food. They then considered how they might expand the unit rate in order to find the price of a ton of dog food, based on the unit rate. Ms. Green indicated that she was impressed that they expanded the scenario. She said that her students "looked in their agenda [a notebook for keeping track of assignments that contains lists of common weights and measures preprinted in it] to see how many ounces and [other conversions they could identify]." She said she was pleased with the results of this episode even though she did not initially plan it as a problem-posing episode.

Ms. Green's Motivations to Use Problem Posing

Ms. Green traced her experience with problem posing to her eighth-grade physics teacher. She said that her physics teacher motivated her to become a middle school teacher. She recalled a time in her physics class when her teacher asked the students to create something using simple machines. She said, "He let us decide what the problem was and how we can fix it." She also referenced a professor of curriculum in college who encouraged teacher candidates to "get the kids to think, not just drill and kill, or regurgitate the information." Both of these teachers influenced Ms. Green's beliefs about teaching, and they also engendered in her a desire to use problem posing.

Ms. Green took the IMAP Beliefs Assessment (Philipp & Sowder, 2003) in the spring of 2014. The IMAP instrument assesses the extent to which there is evidence an individual holds the following seven beliefs:

1. Mathematics is a web of interrelated concepts and procedures (school mathematics should be, too).
2. One's knowledge of how to apply mathematical procedures does not necessarily go with understanding of the underlying concepts. Students or adults may know a procedure they do not understand.
3. Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures.

4. If students learn mathematical concepts before they learn procedures, they are more likely to understand the procedures when they learn them. If they learn the procedures first, they are less likely ever to learn the concepts.
5. Children can solve problems in novel ways before being taught how to solve such problems. Children in primary grades generally understand more mathematics and have more flexible solution strategies than their teachers, or even their parents, expect.
6. The ways children think about mathematics are generally different from the ways adults would expect them to think about mathematics. For example, real-world contexts support children's initial thinking, whereas symbols do not.
7. During interactions related to the learning of mathematics, the teacher should allow the children to do as much of the thinking as possible.

The assessment estimates a person's beliefs by providing scores from zero to four. Each belief was scored on a five-point scale (0–4) to reflect the level of intensity with which Ms. Green held the above beliefs. A score of 0 or 1 indicated no evidence or weak evidence of the belief, respectively. A score of 2 reflected evidence of the belief, and scores of 3 or 4 indicated strong or very strong evidence for the belief, respectively. Table 10 contains Ms. Green's scores for Beliefs 1–7.

Table 10

IMAP Scores of Ms. Green's Beliefs

Belief	Belief 1	Belief 2	Belief 3	Belief 4	Belief 5	Belief 6	Belief 7
Ms. Green's Score	3	3.2	4	3	2.4	3.2	1

The IMAP Beliefs Assessment data for Ms. Green indicated a score of 4 (very strong evidence) on the belief that students should learn mathematical concepts before they learn procedures (IMAP, Belief 4). The data from this instrument also indicate a strong belief that "Understanding mathematical concepts is more powerful and more generative than remembering mathematical procedures" (IMAP, Belief 3). I observed additional evidence to support Beliefs 3 and 4 in the initial DIMaC teacher beliefs interview. Consider Ms. Green's statement from that interview:

> Ms. Green: I don't want to teach [my students] just, well, here's the process: Keep, change, flip, when you're dividing fractions. Its– because it doesn't make any sense, and … so what I want them to do is be able to understand what the fraction means, like, what is it, in real life? … I want them to understand what it means to divide a fraction.

Later, in the same interview, Ms. Green referenced a similar belief concerning the use of algorithms without understanding in the context of adding integers. She said, "Like the algorithm [for adding integers], like that's… you're just going through the motions. I don't want the kid to go through the motions." (Later she said, "So [elementary students] don't start out learning the algorithm, because if they start out learning the algorithm, they're not really learning what they're doing. They're just going through the process." Regarding algorithms, she saw "the value in learning it my own way, working… struggling with it my own way before I do that." Ms. Green regularly emphasized conceptual understanding to motivate or explain algorithms.

The IMAP instrument also indicated Ms. Green had a strong belief about the ways children think about mathematics and how they learn differently than adults. She believed that real-world contexts help support student understanding. Consistent with Belief Six, Ms. Green stated in the interview, “My personality leads to talking – I like to talk. I like to talk to the kids. I like… getting to know what they think– that kind of thing.” She also stated in the interview, “I also wanted real-life examples. I don’t want… some arbitrary… textbook problem.” Ms. Green claimed these factors helped motivate her to use problem posing.

Differentiation also motivated Ms. Green to use problem posing. She recalled an earlier experience when she began teaching sixth grade. Not only did she change grade levels (having previously taught eighth grade), but she also began teaching in a collaborative classroom (i.e., an inclusion classroom with students diagnosed with learning disabilities and requiring special education accommodations). She thought that the change from the eighth to sixth grade curriculum was challenging because it was a new curriculum and because it was in a collaborative classroom environment. She thought that problem posing was beneficial for her as a collaborative teacher in an inclusion classroom, stating that problem posing helped her to meet the needs of the “higher kids that were in the same classroom with [the] inclusion kids.” Problem posing was one tool she used to help her successfully teach a new curriculum that year. She stated that her use of problem posing began during that year and then she “just kind of ran with it from there.”

Ms. Green also used problem posing because she believed in the importance of mathematics standards, and she believed that they could help to make mathematics “relevant for the students.” In her first DIMaC interview, she said: “We have to focus on what the Common Core (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) wants us to do. … That is my ultimate goal.” She later claimed that the Common Core helped “guide me into, okay, we’re looking at more application versus just drill and kill, because that’s not what they’re, … that’s not what they need.” She thought that the habit of drill, kill, and recall discouraged the use of critical thinking skills. She also viewed problem posing as a means to bring critical thinking "back into the classroom."

In summary, it seems that Ms. Green was motivated to engage her students in problem posing because of (1) her past experiences as a student along with influential teachers, (2) her beliefs about the teaching and learning of mathematics, (3) the need to differentiate instruction for diverse learners in a collaborative classroom, (4) her perception that problem posing helps improve critical thinking, and (5) her perception that problem posing helps students to engage in mathematics and apply it in ways that are meaningful to them. There were also some factors that Ms. Green considered as hindrances to problem posing.

Ms. Green was reluctant to pursue the problem-posing scenario about the students taking buses to the theater mentioned above. Her students wanted to expand the task beyond the original intent of the standard addressed by the task. When I asked her to reflect on her reluctance to allow a discussion about including more than one bus in the task, she referenced the fact that the original task was a state frameworks task. She referenced the wording of the standard in her reflection on this episode. She said, "I think that the way the standard, or the problem was worded, it wanted you to take one bus." She continued by stating, "I think, that just shows you how much we're drilled into standards, standards, standards." Consider the following exchange from the follow-up interview, when I asked her to consider what triggered her to decide whether or not to proceed with a student's suggestion to modify a given problem.

Ms. Green: I think it would be dependent upon the situation. … I'll tell you one thing that could have been [a] factor is the fact that this [field trip bus problem] was a frameworks task.

Q: Okay.

Ms. Green: Versus me coming up with the question myself. You know what I'm saying?

Q: Okay, so you felt a little more compelled to try to keep the framework as it w– [overtalk]

Ms. Green: [overtalk] mm-hmm [indicates in the affirmative].

I asked Ms. Green if she could identify other factors that could discourage problem posing from her perspective. She reported some reluctance to venture into problem-posing episodes in situations that required excessive amounts of time. She also reported a reluctance to pursue problem posing as the calendar approached the standardized testing season.

Q: So are there, other than the frameworks tasks, other than the fact that it might be a frameworks task, are there other reasons that might make you not want to pursue the student idea? What kinds of things might make you say, "Ehh, I'm not going to do this right now?"

Ms. Green: Um, It could be time.

Q: Ok

Ms. Green: Um, It could be, um, what in my mind we have left to do that day, um.… Tthere could, there could be different factors—I think, I feel like a lot of times I do, um, I try to do that, I try to say, "Okay, well, let's talk about that, let's see, you know, what do you think about that?" And I think that's … the flexibility of a, of a teacher comes into play there, um.… Whereas … there are classes where that would never happen, you know?

Q: Would it, and it wouldn't happen because ... of time, maybe, or…?

Ms. Green: Um, it could,… um, … I feel like I would be a lot more … able to do that earlier in the year

Q: Okay.

Ms. Green: Um, versus closer to CRCT (Criterion-Reference Competency Test).

According to Ms. Green, another possible hindrance to problem posing is classroom management and student personalities. She conjectured that problem posing was less likely to occur in one of her classes because "I [had] to be a lot more strict in that classroom; otherwise, it was a little bit too chaotic." She balanced this conjecture by stating that a teacher "can still maintain her classroom management expectation of how she wants her classroom [to be] run and still have that flexibility [to engage students in problem posing]." It seems that for Ms. Green, classroom management and behavioral issues could also hinder problem posing.

Ms. Green's Reflections About the Perceived Usefulness of Problem-Posing Experiences

Ms. Green was also motivated to use problem posing because she viewed it as useful for both her and her students. She identified its usefulness for teachers in that it can be a helpful formative assessment strategy, particularly when addressing various mathematics standards. She thought it was useful for the students because it helped prepare them for life as well as for the next grade.

Ms. Green used problem-posing episodes as a formative assessment tool, regardless of whether or not she recorded a grade for the task. She said that she "definitely" saw the potential for assessment because it could "help tell you whether or not [the students] understand." She thought problem posing was very useful for teachers and that it "should be … another tool in the [teacher's pedagogical] toolbox." I asked her to explain how she informally assessed her students from some of the episodes. She said she learned that her students "are ready for two-step equations," referring to the episode where students took buses to the theater. When she reflected on the episode with the integers, she said, "I think that they understand the concept of a negative number or positive number… and that [the negative and positive numbers] are useful in what we're doing even if we are being silly about driving off of a cliff."

Ms. Green observed that over the course of the year, "the problems they were creating became more independent of what I was saying." She reported that her students basically reworded her problems in the beginning of the year, but as the year progressed, their problem-posing ability improved. She thought the students improved in their ability to pose problems as they continued to practice and gain experience.

Ms. Green claimed problem posing is a good fit for standards-driven instruction. She said, "We are a standards-driven educational system now." I asked Ms. Green to reflect on whether problem posing helped her achieve her instructional goals. Consider her response:

Ms. Green: Well I, do you mean like, how do, how do these— Having kids create the questions fit into that, that the ultimate goal for the kids?

Q: Yeah

Ms. Green: Yeah okay. I think that, you know, we are a standards drive educational system now. We have to, we have to make sure that we are, the kids are getting their common core, um, so that we can make sure that they are prepared, Because I, I definitely want them to be (A) prepared for life and (B) prepared for the next grade level, Because, The way it is structured in America, they have— it's a, it's a process, I guess.

She also said problem posing helps to promote good "habits of mind and things like that."

Ms. Green reported the use of problem posing in lessons with other mathematics, including geometry. She said that such instances in geometry often provided opportunities to take “an engineering turn” and connect geometry to engineering. She claimed that her teacher-initiated problem-posing tasks did accomplish her instructional goals.

Ms. Green did not report a distinction in the types of problems posed by her students. In other words, she did not distinguish between the types of problems described by Schoenfeld (1992). In particular, she did not identify or speak of routine exercises versus problems. She distinguished only between a problem and an example. She said, “I think examples are going to have more information written out in their notebook … versus, I give them $2x = 3$, what is x? And they solve it.” She reported she was not aware of occasions when she sometimes treated the words *example* and *problem* as synonymous when asking students to pose problems.

Ms. Green concluded her thoughts about problem posing by calling for more teachers to provide opportunities for their students to pose mathematics problems. She encouraged teachers to “just start asking the questions, [have] the students ask the questions” and to not be too concerned if such scenarios extend beyond the scope of the published standards. She continued, “It goes with the enrichment of the class; it makes it better.”

In this chapter I provided some additional insight into how problem posing occurs in Ms. Green’s classroom. I began by reviewing Ms. Green’s educational background and teaching experience. My first research question concerned the frequency with which problem posing occurred in the classrooms I observed. I provided the counts and frequencies of various types of problem posing that occurred in Ms. Green’s class and the catalyst for those episodes. My second research question considered the environmental conditions that exist surrounding problem-posing episodes. To better describe that environment I explained Ms. Green’s flexible attitude and her frequent use of group work in her classroom. I mapped each problem-posing episode from Ms. Green's classroom onto one predominant strand of mathematical proficiency for each episode. I provided some examples of problem posing that featured a predominant strand of mathematical proficiency, whether conceptual understanding, procedural fluency, adaptive reasoning, or building procedural fluency from conceptual understanding.

In response to the first two research questions, problem posing occurred in roughly one out of every four lessons across the set of six teachers' observed lessons. When considering the topics of the lessons (integers, fractions, or algebra) individually problem posing occurred 31.3%, 27%, and 25.7% of the time, respectively. Most of the problem-posing episodes in this study involved teacher-initiated, semi-structured, routine problems. Most of the routine exercises occurred when students were asked to create a context to fit a given, routine exercise. Students initiated all of the instances for which adaptive reasoning was the predominant mathematical strand of proficiency. Five of the six teachers in the data set used problem posing at least once as part of instruction on fractions or division. Ms. Violet was the only teacher with one observed instance of problem posing, and it involved division of fractions.

It was difficult to describe a predominant strand of mathematical proficiency when procedural fluency and conceptual understanding were both evident in the episode. I created a blend of the two with the code, *building procedural from conceptual*. I used this code for nearly half (46%) of the observed instances of problem posing. The next most common, predominant strand of mathematical proficiency was procedural fluency (25%).

There were no observed instances of problems that are problematic, and there were no observed instances of free problem posing initiated by the teacher. Teachers' did not present students with problems that were problematic in this study. All posed problems, whether student-or teacher-initiated, occurred as a result of the specific mathematics content from the teacher's lesson plans or the mathematics standards. In all of the problem-posing episodes, the problems posed served as a tool for understanding mathematics.

It also seems students can and do engage in problem posing even without a prompt to do so from their teacher (as was the case in Ms. Gold's class). Nearly half of the student-initiated problem-posing episodes occurred in a classroom (Ms. Gold's classroom) without any formal prompts to pose problems. In addition, an environment in which the constraints of problems are questioned regularly may encourage students to engage in problem posing on their own.

In response to my third research question, I investigated Ms. Green's understanding and perspective concerning mathematical problem posing. I also looked for motivating factors that engendered episodes of problem posing. She sometimes planned problem posing and sometimes she allowed a student's question to change the trajectory of the initial intent of the lesson. She attributed the student-initiated episodes to the curiosity of her students and to her flexibility in being willing to allow them to share their curiosity. She was initially motivated to use problem posing, in part, because she moved from an eighth grade class to teach in a collaborative (a classroom with students with diverse learning needs) classroom. She felt problem posing helped her differentiate instruction to better meet the needs of all learners. Her motivations to plan for problem posing also included her past experiences as a learner, influences from previous teachers, her perceived notion that it helped differentiate instruction to accommodate the needs of diverse learners, a helpful means to meet mathematics standards (such as the Common Core), a perceived increase in student motivation and engagement, and an opportunity to help students connect mathematics to real-world phenomena.

Ms. Green sometimes felt hindered by time constraints, but she allowed student-initiated instances of problem posing if she determined, in the moment, that the venture seemed worthwhile for her students. She also felt that the approach of testing season may cause some teachers to feel less freedom to use problem posing in their classes. Ms. Green indicated that she felt that problem posing was useful for both her because it has implications for formative assessment by helping her see what her students do or do not understand about a particular mathematics topic. She felt it was useful for her students in that she thought it helped them to connect mathematics to real-world contexts.

CHAPTER 5

SUMMARY AND CONCLUSIONS

The following is a summary of my study, beginning with the research questions that helped focus my study. After the summary I provide some conclusions based on the data in the study. I also discuss some potential implications based on those conclusions. Then, in conclusion, I suggest some possible future directions for research.

The following research questions framed the study:

1. How often does problem posing occur in six Grades 5–7 mathematics classrooms? What types of problem posing occur in those classrooms? What mathematical processes do students use when engaged in problem posing?
2. What kind of classroom environment exists when students engage in problem-posing activities? (Singer, Ellerton, Cai, & Leung, 2011, p. 149).
3. What understanding and perspectives on mathematical problem posing do teachers possess? For a teacher who used mathematical problem posing in her instruction:
    a. To what extent are problem-posing episodes planned in advance?
    b. What prompts the problem-posing episodes that were planned?
    c. What is the participant's overall assessment of the potential learning impact of the problem-posing episodes?

The second research question contains the reference because Singer, Ellerton, Cai and Leung (2011) identified it as an unanswered question in the literature.

Summary

I observed and reviewed 88 filmed lessons and transcripts from six mathematics teachers in Grades 5 through 7. The six teachers were initially recruited to participate in the Discourse in Mathematics Classrooms (DIMaC) study because they had a reputation for an increased amount of student-to-student and teacher-to-student discussions in mathematics classes. I observed only lessons on fractions, integers, or algebraic reasoning because those topics were used exclusively in the DIMaC Study. In the first filmed lesson, I observed a sixth-grade teacher engage her students in problem posing. After reflecting on this lesson, I wondered whether problem posing occurred in other teachers' classrooms in the DIMaC Study. I decided to search for a way to identify and describe instances of problem posing. I did not find references in the literature to the prevalence of problem posing in mathematics instruction. If anything, I found evidence from the literature to suggest problem posing is not common in mathematics classes. None of the studies in the special issue of *Educational Studies in Mathematics* (Singer, Ellerton, & Cai, 2013) reported the prevalence of problem posing from a given sample of teachers. I decided to report the frequency of observed instances of problem posing across teachers in the DIMaC study. Because I observed the most instances of problem posing in one sixth-grade teacher's classroom, I decided to also pursue a case study of that teacher to understand more about why problem posing occurred in her classroom.

As I reviewed the literature, I found several references to Stoyanova and Ellerton's (1996) proposed framework for describing different types of problem posing. They classified episodes of problem posing as free, semi-structured, or structured. As I observed lessons, I found some episodes of problem posing initiated by students rather than teachers. Because their framework did not account for student-initiated episodes of problem posing I added the student-as-catalyst category as an amendment to their framework. I also merged Schoenfeld's (1992) classifications of problems and created a framework to describe and classify observed instances of problem posing. After applying the framework, I identified the mathematical topic as well as the predominant strand of mathematical proficiency observed in each episode (Kilpatrick, Swafford, & Findell, 2001).

Results

In response to the first research question, the teachers regularly engaged their students in discussions during mathematical instruction during my observations. There were 24 observed instances of problem posing in 88 lessons from the 6 teachers. Plenty of studies suggest problem posing needs more attention, but they do not describe how much more attention is required. In contrast, none of the other studies I analyzed reported the prevalence of problem posing from a given set of teachers. It seems researchers need to know how often it occurs within a given environment. An accumulation of similar reports of the instances of problem posing over time may help advance the discussion of problem posing in mathematics education.

On average, problem posing occurred in approximately one out of every four lessons I observed. Most of the episodes involved teacher-initiated, semi-structured, routine problems. Most of the semi-structured problem-posing episodes were instances in which students created a story or context to accompany a given mathematical exercise. Students did not have opportunities to pose problems that were problematic, nor did they have the opportunity to engage in free problem posing (Stoyanova & Ellerton, 1996). The observed instances of problem posing seemed to be at an emerging level. For example, the most common form of problem posing was the contextualization of routine exercises.

Students used procedural fluency, conceptual understanding and adaptive reasoning in the observed instances of problem posing. Nearly half of the episodes of problem posing involved building procedural fluency from conceptual understanding. Five instances of problem posing occurred in which adaptive reasoning was the predominant strand of mathematical proficiency, and students initiated each episode.

The second research question concerned the classroom environment conditions when problem posing occurred. All teachers in the study had a reputation for asking students to speak with each other about mathematical topics. All observed lessons contained examples of students working in groups or pairs. As a result, all observed instances of problem posing occurred in environments in which the students interacted with each other. It is possible that the group work and collaborative environment provided fertile ground for problem posing. As mentioned in Chapter 4, the case study teacher expressed her belief that problem posing was less likely in classes taught by one of her colleagues—a classroom in which group work did not occur. She also used problem posing because she thought it was important to help students exercise their curiosity and share their ideas with one another.

I also found that students sometimes initiated problem posing on their own during their progression through a given mathematics task. One fifth-grade teacher did not explicitly require her students to pose problems during our observations of her class, but when they did, she pursued their newly posed problems. All problem-posing episodes in her class were student-initiated, and she either allowed students to pursue the new problems or she used their problems as examples later in the lesson. The three student-initiated episodes accounted for nearly half of the student-initiated episodes I observed. Perhaps that teacher's regular modeling of the questioning of the constraints of problems engendered a culture of questioning in her students, which in turn encouraged problem posing.

The third research question concerned the case-study teacher. As mentioned in Chapter 4, the case study teacher indicated that differentiation was an initial motivating factor to use problem posing. She explained that her move from teaching eighth-grade mathematics to sixth-grade mathematics with students in an inclusion classroom (i.e., many students with diverse needs) required some adjustments to her teaching methods. She identified problem posing as one tool she used to help differentiate instruction to help meet the needs of all learners in her class. She explained that she had used problem posing each year since her move to sixth grade. She also claimed problem posing increased access to mathematics for students with diverse learning needs.

The case study teacher was motivated to plan for instances of problem posing based on her goal to promote critical thinking. She also expressed her desire to use problem posing as a means to demonstrate the relevance of mathematics for her students. She planned for instances of problem posing on a regular basis. She reported using problem posing a minimum of once each week.

Linking Results to Research

The *Principles to Actions* (National Council of Teachers of Mathematics, 2014b, p. 93) document identifies the use of contextualization of routine exercises as a useful tool to promote conceptual understanding and reasoning. This conclusion also relates to the first research question. As mentioned above, the contextualization of routine exercises was the most common form of problem posing in this study. The contextualization episodes seemed to help students make sense of important mathematical concepts. For example, in one class, students initially (and erroneously) thought 3 divided by 4 was not equivalent to three-fourths. One student corrected her error after she assigned a brief context to the routine division exercise. She then verbalized the proper conclusion. This example is also consistent with Stoyanova and Ellerton's (1996) observation that such instances "help students learn how to generalise, as well as [to make] mathematics more meaningful to them" (p. 520). The example in this class only differs in that it was student-initiated whereas those in Stoyanova and Ellerton's study were teacher-initiated. Regardless of the catalyst, it seems activities that prompt students to create contexts for abstract mathematical equations or expressions help students understand the mathematics on a deeper level.

Olson and Knott (2013) contrasted a growth versus a fixed mindset within problem-posing episodes. They found that "a teacher's mindset influences how the teacher engages students during the class" (p. 35). They claimed that the existence of problem posing in a teacher's class suggests the teacher has a growth versus a fixed teaching mindset. One of the fifth-grade teachers demonstrated what Olson and Knott referred to as a teacher's growth mindset. In contrast, teachers with a fixed mindset "tend to focus on the products of mathematical activity and emphasize the answers" (p. 29). The fifth-grade teacher demonstrated a willingness to pursue student-initiated problem posing during instruction. In addition, students in her class created conjectures and increased the demands of the given task or problem. This may also be an example of what Olson and Knott (2013) referred to as "evidence of students' belief in a growth mindset" (p. 34) as well.

I did not find any problem-posing literature with references to the impacts of group work on problem-posing activity. No such reports occurred in the special journal issue (Singer, Ellerton, & Cai, 2013) on problem posing. Brown and Walter (2005), however, suggested cooperative activity as a means to promote problem posing (pp. 167–168), and the examples of problem posing from the present study support their suggestion.

The case study teacher's report is consistent with Contreras's (2009) analysis on differentiation with tasks using dynamic geometry software. Although Contreras focused on problem posing with conjectures and theorems in geometry, he identified differentiation as an important result of problem posing. He claimed, "Differentiating instruction by focusing not only on proofs, but also on *generating problems and formulating and testing conjectures* [emphasis added], we can *provide access to mathematics* [emphasis added] and engage more students in doing mathematics than with the traditional approach" (p. 83).

Implications

Perhaps an initial step in promoting problem posing is that it can become more prevalent in the mathematics educator conversation as a whole. How can researchers study the potential effectiveness of problem posing as a pedagogical strategy if there is little awareness of it as implied by the literature (Ellerton, 2013; Silver, 1996)? Mathematics teacher educators can model various forms of problem posing in mathematics education methods courses. In addition, instructors for methods courses could explicitly identify problem posing as a topic for discussion with prospective teachers. Practicing teachers can also participate in professional learning opportunities on the topic of problem posing. As the discourse about problem posing increases problem posing could become more prevalent in influential mathematics education documents along with other topics such as mathematical reasoning and problem solving.

Another implication of this study concerns the existence of problem posing in the school mathematics curriculum. Problem solving is widespread in the mathematics curriculum, but if problem posing is a close companion, why does it receive little mention or attention (Ellerton, 2013; Silver, 1996)? It appears that there is a place for problem posing in mathematics curricula, but might it already exist in some? For example, is it necessary to create an additional Standard for Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) to specifically address problem posing? Perhaps, but perhaps not if one uses problem posing as a means to promote the other mathematical practices. Consider that the *Principles to Actions* (National Council of Teachers of Mathematics, 2014b) document suggests the following writing prompt for students: “Create a situation that could be modeled with 6 ÷ ¾” (p. 93) followed by “Write three equations, one with no solution, one with exactly one solution, and one with infinitely many solutions” (p. 93). The authors of this document did not identify these as problem posing, but I classify the prompts as problem posing because students have to create a “situation” (or problem or model) that corresponds to the arithmetic or algebraic procedure. The further implication is that if the above prompts are examples of problem posing, then problem posing may provide important data about students’ background knowledge if used when introducing new mathematical concepts (NCTM, 2014, pp. 93–94). NCTM claimed that one possible means by which to gather evidence about student’s learning is to “reverse givens and unknowns in a problem situation” (p. 92). In other words, problem posing can help provide a useful form of evidence to inform teachers about what students do and do not understand.

Mathematics teachers can use problem posing as a pedagogical tool. Teachers can ask students to create new problems or reformulate previously given problems as a means to promote additional conceptual understanding and procedural fluency. Teachers can also use problem posing to link abstract mathematics to real world phenomena and increase the relevance of mathematics to students’ experiences. Teachers can also use problem posing to gain insight into how their students think about mathematics. Teachers can improve the relevance of routine exercises in mathematics by asking students to create contexts for given routine exercises or they can ask students to create similar routine exercises on their own.

Mathematics teachers can also create a classroom environment that encourages students to question constraints of given problems. Teachers who model the questioning of constraints may encourage students to initiate problem posing on their own. In such cases, problem posing may become more of a mathematical activity than a pedagogical tool.

Future Research

This study leaves some unanswered questions and provides an opportunity to pose more questions (and problems) for research. Researchers in mathematics education can find teachers who use problem posing and study more about how they use it. As a result of this study, I propose additional research that (1) identifies teachers who purposefully use problem posing, and (2) examines their instructional practices surrounding problem posing to determine potential impacts on student learning.

One lingering unanswered question is the following: “What are the demands on teachers who use problem posing in mathematics classes?” It seems there may be at least two avenues for inquiry here. One research trajectory might investigate the teacher knowledge (Ball, Thames, & Phelps, 2008) necessary for teachers to *pose problems for students* during the course of interactions with students. In other words, how does teacher knowledge influence or inform the kinds of problems teachers pose for their students? Another research trajectory might ask, “What teacher knowledge is necessary for teachers to create problem-posing tasks for students to pose problems?”

Researchers in mathematics education can investigate instances of free problem posing. In particular, since I did not observe free problem posing of problems that were problematic, researchers can create opportunities for students to engage in free problem posing. What kinds of reasoning do students use when they are engaged in free problem-posing episodes? How might those episodes support students' learning of mathematics content? How can students have experiences where they pose problems that are problematic? For example, is it reasonable to expect a student to pose a problem that is problematic in a structured problem-posing environment? It seems problems that are problematic may occur more easily in free problem-posing environments, but I have no empirical evidence to support that supposition. A future study could examine the types of student-initiated instances of problem posing during the course of regular instruction in mathematics classes.

Another study might focus solely on identifying and describing student-initiated instances of problem posing. Other than direct requests from teachers, why do students to engage in problem posing on their own? Do student-initiated instances of problem posing occur more in classrooms in which the teachers regularly model the questioning of constraints of mathematics problems? I propose an additional 1-year observational study of a teacher who regularly models the questioning of the constraints of given problems in his or her classroom discussions. In the observational study, I propose the recording and analysis of all student-initiated instances of problem posing in order to understand how and why students question constraints and initiate problem posing on their own.

I propose one final suggestion for additional research. In the present study the case-study teacher used problem posing as an informal assessment tool. And, as mentioned above, problem posing can provide teachers with evidence about students' understanding. I echo Silver's (2013) call to “explore ways in which problem-posing tasks might be used as assessments of desired mathematics learning outcomes” (p. 161). This question might somewhat diverge along two paths. The first idea concerns how researchers might assess mathematical problem posing itself. For example, how can an assessment attempt to measure or estimate a student’s problem-posing ability, and what would such an assessment look like? In addition, how can a problem-posing assessment be used to estimate students’ mathematical understanding? Are these two ideas indeed distinct? Future studies concerning problem posing and assessment must distinguish clearly the goals to either assess mathematical problem posing or to assess mathematical understanding using problem posing. It seems to me that the two may not be identical. In summary, what are the implications for assessment with mathematical problem posing? In an age where assessment discussions abound, perhaps it will be useful to create a study of problem posing and assessment to contribute to the discussion of assessment.

Final Remarks

It is perhaps somewhat ironic that my initial task was to pose a problem to study for my dissertation. When I began this process, I had no idea I would pose a problem to study problem posing. The process of identifying and posing a research problem was difficult. After identifying the problem, generating questions to address the problem also consumed large amounts of effort and research. In mathematics, problem posing serves as a means to pique students’ curiosity, and it encourages them to ask questions. As mentioned in Chapter 1, Einstein and Infeld (1938) observed that the asking of good questions is as important as finding answers to questions.

I began the present study on a quest to learn more about problem posing in mathematics classrooms. I wanted to understand how often it happened in a sample of teachers. I also wanted to gain a better idea of why it happened from one teacher’s perspective. I wanted to observe it in action and present my observations in a form that might encourage other researchers and practitioners to increase the exposure of students to problem posing. For teachers who have not engaged with problem posing, I hope to encourage them to give it serious consideration. It seems teachers can take incremental steps towards using problem posing more and more. As additional research continues in this area and discussions of problem posing increase, I believe problem posing will help students engage with mathematics more deeply and that they will also develop a deeper understanding of mathematical concepts.

REFERENCES

Agre, G. P. (1982). The concept of problem. *Educational Studies*, *13*(2), 121–142.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special? *Journal of Teacher Education*, *59*(5), 389–407. doi:10.1177/0022487108324554

Bishop, J. P. (2012). *CAREER: Characterizing Critical Aspects of Mathematics Classroom Discourse*. External Grant received: National Science Foundation (NSF) Faculty Early Career Development [CAREER] Program, 2012–2017.

Bonotto, C. (2013). Artifacts as sources for problem-posing activities. *Educational Studies in Mathematics*, *83*(1), 37–55.

Brownell, W. A. (1942). Problem solving. In N. B. Henry (Ed.), *The psychology of learning (41st Yearbook of the National Society for the Study of Education, Part 2)* (pp. 415–443). Chicago, IL: University of Chicago Press.

Brown, S. I., & Walter, M. I. (1983). *The art of problem posing*. Philadelphia, PA: Franklin Institute.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing* (3rd ed.). Mahwah, NJ: Erlbaum.

Bush, W. S., Leinwand, S., & Beck, P. (2000). *Mathematics assessment: A practical handbook for grades 6-8*. National Council of Teachers of Mathematics.

Cai, J., Hwang, S., Jiang, C., & Silber, S. (in press). Problem posing research in mathematics education: Some answered and unanswered questions. In F. M. Singer & J. Cai (Eds.), *Problem posing: From research to effective practice*. New York, NY: Springer.

Cai, J., Moyer, J. C., Wang, N., Hwang, S., Nie, B., & Garber, T. (2013). Mathematical problem posing as a measure of curricular effect on students' learning. *Educational Studies in Mathematics*, *83*(1), 57–69.

Contreras, J. (2009). Generating problems, conjectures, and theorems with interactive geometry: An environment for fostering mathematical thinking for all students. In A. Flores & C. E. Malloy (Eds.), *Mathematics for every student: Responding to diversity, grades 9–12* (pp. 71–84). Reston, VA: National Council of Teachers of Mathematics.

Da Ponte, J. P., & Henriques, A. (2013). Problem posing based on investigation activities by university students. *Educational Studies in Mathematics*, *83*(1), 145–156. doi:10.1007/s10649-012-9443-5

Duncker, K. (1945). On problem solving. *Psychological Monographs*, *58*(5, Whole No. 270).

Einstein, A., & Infeld, L. (1938). *The evolution of physics from early concepts to relativity and quanta*. New York, NY: Simon & Schuster.

Ellerton, N. F. (2013). Engaging pre-service middle-school teacher-education students in mathematical problem posing: Development of an active learning framework. *Educational Studies in Mathematics*, *83*(1), 87–101.

Henderson, K. B., & Pingry, R. E. (1953). Problem solving in mathematics. In *The learning of mathematics: Its theory and practice (21st yearbook of the National Council of Teachers of Mathematics)* (pp. 228–270). Washington, DC: National Council of Teachers of Mathematics.

Kilpatrick, J. (1985). A retrospective account of the past twenty-five years of research on teaching mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning* *mathematical problem solving: Multiple research perspectives* (pp. 1–16). Hillsdale, NJ: Lawrence Erlbaum Associates.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123–147). Hillsdale, NJ: Erlbaum.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Koichu, B., & Kontorovich, I. (2013). Dissecting success stories on mathematical problem posing: A case of the billiard task. *Educational Studies in Mathematics*, *83*(1), 71–86.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2002). *Connected mathematics*. Needham, MA: Prentice Hall.

Leung, S. S. (2013). Teachers implementing mathematical problem posing in the classroom: Challenges and strategies. *Educational Studies in Mathematics*, *83*(1), 103–116.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2014a). *2014 NCTM annual meeting & exposition program book*. Reston, VA: National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/uploadedFiles/Conferences/2014_Annual_Program_Book.pdf

National Council of Teachers of Mathematics. (2014b). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author.

Olson, J. C., & Knott, L. (2013). When a problem is more than a teacher's question. *Educational Studies in Mathematics*, *83*(1), 27–36.

Philipp, R. A., & Sowder, J. (2003). *IMAP web-based beliefs survey manual*. San Diego, CA: San Diego State University. Retrieved from http://www.sci.sdsu.edu/CRMSE/IMAP/Beliefs-Survey_Manual.pdf

Polya, G. (1981a). *Mathematical discovery: On understanding, learning, and teaching problem solving* (2nd ed., Vol. 2). New York, NY: John Wiley & Sons.

Polya, G. (1981b). *Mathematical discovery: On understanding, learning, and teaching problem solving* (2nd ed., Vol. 1). New York, NY: John Wiley & Sons.

Polya, G. (2004). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.

Progressive Education Association. (1940). *Mathematics in general education: A report* (Commission on the Secondary School Curriculum. Committee on the Function of Mathematics in General Education.). New York, NY: Appleton-Century.

Reitman, W. R. (1965). *Cognition and thought*. New York, NY: Wiley.

Schoenfeld, A. H. (1987). Discussion. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 144–145). Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York, NY: Macmillan.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, *14*(1), 19–28.

Silver, E. A. (2013). Problem-posing research in mathematics education: Looking back, looking around, and looking ahead. *Educational Studies in Mathematics*, *83*(1), 157–162.

Silver, E. A., Kilpatrick, J., & Schlesinger, B. (1990). *Thinking through mathematics: Fostering inquiry and communication in mathematics classrooms*. New York, NY: The College Entrance Examination Board.

Singer, F. M., Ellerton, N., & Cai, J. (Eds.). (2013a). Problem posing in mathematics teaching and learning [Special issue]. *Educational Studies in Mathematics*, *83*(1).

Singer, F. M., Ellerton, N., & Cai, J. (2013b). Problem-posing research in mathematics education: New questions and directions. *Educational Studies in Mathematics*, *83*(1), 1–7.

Singer, F. M., Ellerton, N., Cai, J., & Leung, E. (2011). Problem posing in mathematics learning and teaching: A research agenda. In B. Ubuz (Ed.), *Developing mathematical thinking. Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 137–166). Ankara, Turkey: Psychology of Mathematics Education. Retrieved from http://www.academia.edu/2117717/Singer_F._M._Ellerton_N._Cai_J._Leung_E._2011_._Problem_Posing_in_Mathematics_Learning_and_Teaching_a_Research_Agenda._In_U buz_B._Ed._Developing_mathematical_thinking._Proceedings_of_the_35th_PME_vol_1_pp.137-166_Research_forum_._Ankara_Turkey_PME._ISBN_978-975-429-294-7

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2013c). Problem posing in mathematics teaching and learning: Establishing a framework for research (Special issue). *Educational Studies in Mathematics*, *83*(1).

Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics*, *83*(1), 9–26.

Stake, R. (2000). Case studies. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed., pp. 435–454). Thousand Oaks, CA: Sage.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development*. Reston, VA: National Council of Teachers of Mathematics.

Stockero, S., Bishop, J. P., & Conner, A. (2014, August). *Perspectives on mathematics classroom discourse.* Presentation at the Discovery Research K-12 PI Meeting, CADRE [Community for Advancing Discovery Research in Education], Washington, D.C.

Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh & N. Ellerton (Eds.), *Research in mathematics education: A contemporary perspective* (pp. 164–185). Perth, Australia: Edith Cowan University, Mathematics, Science and Technology Education Centre.

Stoyanova, E., & Ellerton, N. F. (1996). A framework for research into students’ problem posing in school mathematics. In P. Clarkson (Ed.), *Technology in mathematics education* (pp. 518–525). Melbourne, Australia: Mathematics Education Research Group of Australia.

Tichá, M., & Hošpesová, A. (2013). Developing teachers' subject didactic competence through problem posing. *Educational Studies in Mathematics*, *83*(1), 133–143.

Van Harpen, X. Y., & Presmeg, N. C. (2013). An investigation of relationships between students' mathematical problem-posing abilities and their mathematical content knowledge. *Educational Studies in Mathematics*, *83*(1), 117–132.

Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). Mathematical problem solving. In P. S. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 57–78). New York, NY: Macmillan.

Yin, R. K. (2003). *Case study research* (3rd ed.). Thousand Oaks, CA: Sage.

APPENDIX A

LESSON SUMMARY SHEET TEMPLATE

Lesson Summary Sheet

Teacher:
Topic: Algebra, Fractions, Integers (circle one)
Your name:

Date filmed:
Videographer:
Date (summ sheet completed):

Brief description of math topic/objectives in the observed lesson:

Lesson Map:
Think of this as a play-by-play of the lesson. Try to keep the length of the lesson maps to 2 pages. Reconstruct the flow of the lesson. What problem(s) were students considering? Were strategies discussed and presented? If the lesson was lecture-based, what problems were shown or what concepts were addressed during the lecture and in what order? For problem solving/discussion lessons, note when new problems are posed or different strategies are discussed. This raises the issue of what 'counts' as a problem which we haven't come to consensus on.

Use the following class activities as possible headings to organize the Lesson Map you create. Check homework, Whole-class discussion of problem, small-group/partner work, seat work, student problem solving, math journal writing, student presentations of solutions, whole-class lecture, etc. (Note that lecture is not a typical college lecture, but teacher-directed whole-class conversation with little student input.)

Possible video clips
Are there 3-4 minute clips (or shorter) of whole-class discussion, teacher-student or small group interactions that are possible video clips we could use as exemplary or interesting or problematic in some way? Note approximate time and circumstances and what you found noteworthy.

Teacher similarities or questions about practice
For this teacher, are you noticing similarities across lessons? Is there a question you would like to ask him/her about some aspect of her teaching? If so, describe any similarities or questions.

Noteworthy instances/events (this may be redundant with the video clip section above)
Describe any noteworthy instances and be sure to include why they were noteworthy to you.

Did you notice any problem posing? (If so make a note and try to remember to email CK.) Did you notice any use of metaphor? (If so make a note and try to remember to email ES.)

APPENDIX B

MS. GREEN'S FOLLOW-UP INTERVIEW PROTOCOL

STIMULATED RECALL INTERVIEW

Say, **Please remember to talk out loud as you are talking about the interview questions. Interview with Ms. Green. Today is ____________________________. Hello, Ms. Green!**

General PP Experiences

1. **Thank you once again for allowing us to learn about teaching mathematics in your classroom this year with your sixth grade students. We noticed that, on some of our visits to observe, you sometimes asked students to "create a problem" or to "come up with a problem," and then you provided some constraints for them with which to work.**
    a. **Why do you ask your students to engage in this kind of activity? (If needed follow up with, What do you hope they gain or learn from creating a problem?)**
    b. **Do you recall learning about this technique in any of your formal learning experiences (undergraduate, graduate, RESA, or other professional learning)? Please explain.**
        i. **(*Alternate question: When and how did you first hear about having students generate their own mathematics problems*?)**
    c. **Please think back on your experiences as a mathematics teacher. When, to the best of your recollection, did you first give or first decide to provide this type of learning activity?**
    d. **Do you think it is more likely that you would plan such an activity in advance, or is it more likely that such an activity might occur to you more spontaneously during instruction?**
    e. **Do you believe this kind of learning experience is more appropriate for certain types of mathematics classes (or certain types of students, etc.)?**
        i. **Example: accelerated vs. "on level" classes**
    f. **How did these kinds of activities, in general, impact your instructional goals as a teacher of mathematics, specifically?**

2. **Take a moment and think back over the previous school year's mathematics lessons where you asked students to create their own mathematics problems. Think about some of the submissions you received from them.**
    a. **Do any of their submitted problems stand out in your memory? (What about them stands out to you?)**
    b. **What kinds of responses do you want to see from your students when you ask them to create their own problems?**

c. **Other than what we observed (integers, fractions, and algebra lessons), did you ask students to create problems in other content areas?**

Stimulated Recall 1

It has been quite some time since last August. I'd like to show you a brief clip from August 28, 2013, see what you recall about it, and ask a few questions about it.

- **Start video: 0:25:00.0**
- **End Video: 0:26:30.0**

[Transcript portion – for reference, if necessary]

Ms. Green 1/20th of the cake. Alright so we have discussed this, now I'm gonna set the timer for 5 minutes; maybe a little bit more than that. I want you to come up with your own example similar to the cake question, like the cake question, but with fractions, okay? Okay, a real-world, like something we will legitimately need to divide up. … So, we're gonna write it up on one page of Notability and then somebody else is gonna write up the solution on one page of Notability. I need to get, I need to be able to see it on most, on most on mostly one screen and then we're gonna present, so maybe try to find someone who has internet access, on their ipad. …

[0:27:00.0]

Ms. Green This is as a group, guys – as a group come up with your own <u>problem</u>.

1. **To what extent do you remember this lesson?**
2. **What do you remember about this activity?**
3. **What helped to motivate this particular activity?**
4. **What were your goals or objectives for this particular activity?**
5. **How were these goals situated within your larger instructional goals for your students?**
6. **What did this activity tell you, if anything, about the level of mathematical understanding of your students?**
7. **How often do you ask your students to do activities like this one? Can you tell me more?**

Stimulated Recall 2: IMAX Bus(ses)

In this episode the students were working in groups on a State Frameworks task in which students paid $13 each for a ticket and the cost of the bus was $10. It seemed that several students were insistent that multiple busses should be available for the field trip.

October 8, 2013

- **Start Video: 0:45:40.0**
- **End Video: 0:47:00.0**

Again:

- **Start Video: 0:49:00.0**
- **End Video: 0:50:00.0**

Again: (Student K's Group)

- **Start Video: 0:57:40.0**
- **End Video: 0:58:17.0**

Again

- **Video Start: 1:06:15.0**
- **Video End: 1:06:58.0**

1. **To what extent do you remember this lesson?**
2. **What do you remember about this activity?**
3. **To what do you attribute your students' desire to include more busses in the constraints (or "givens") of the problem?**
4. **Earlier in the year a student proposed somewhat similar question when he asked whether or not to count you in the distribution of cake in a division problem (available on transcript if necessary for reminder). In that situation you asked the class to change the problem, changing the divisor from 4 to 5 people.**
    a. **In the case of the busses, under what circumstances might you elect to ask the class to go ahead and change the bus problem and treat the busses as a varying quantity? (Perhaps later, such as 7th grade?)**
    b. **Given your time and experience with this group of students, please take a moment and try to predict what would have happened if you then asked the class to also answer the question about multiple busses.**
5. **Sometimes you change problems based on students' questions, input, or ideas, like in the cake problem. And other times you do not change problems in response to student ideas or questions, like in the bus problem. Can you share with me the factors that you consider when deciding to make changes or not to make changes based on students' questions or ideas? What might cause you to go with the student idea and change the problem/task or to NOT pursue the student idea and keep the problem/task the same?**

6. **Did you use this task with any of your other classes this year?**
    a. **If so, did other classes consider the number of busses as a varying quantity or as a constant of only one bus?**
    b. **Any other comparisons here?**
7. **How were these goals situated within your larger instructional goals for your students?**
8. **What did this activity tell you, if anything, about the level of mathematical understanding of your students?**
9. **How often do you ask your students to do activities like this one? Can you tell me more?**

Stimulated Recall 3

Let's return to the ice age of 2014: February 3, 2014. In this episode you supplied your students with a piece of poster-paper and asked them to create a problem concerning integers.

- **Video Start: 0:14:00.0**
- **Video End: 0:16:10.0**

Partial Transcript:

"*You are gonna come up with a real-life problem that we might see that has to, that deals with integers. We could, it could deal with integers, it could deal with absolute values, um, it could deal with opposites; anything like that, I want you to think about and you think about that in your group and come up with an idea to, to test your fellow classmates… come up with an example for the class to somehow test their knowledge*"

1. **To what extent do you remember this lesson?**
2. **What do you remember about this activity?**
3. **What helped to motivate this particular activity?**
4. **What were your goals or objectives for this particular activity?**
5. **How were these goals situated within your larger instructional goals for your students?**
6. **What did this activity tell you, if anything, about the level of mathematical understanding of your students?**
7. **You mentioned the idea that the students could "test" each other's knowledge.**
    a. **To what extent do you see assessment potential (from a teacher's perspective) in asking students to create problems?**
    b. **Did you ever use these kinds of tasks to assess students' mathematics understanding?**
8. **How often do you ask your students to do activities like this one? Can you tell me more?**

In this episode problems were posed by each of the 8 groups. Eight separate problems were posed by the class. (Another option: show her each of the 8 problems posed and ask her to comment on them; perhaps state which ones stood out to her (and why)…)

End: Follow-Up Questions

1. **Let's think for a moment about creating mathematics problems and assessments.**
    a. **Please tell me the extent to which (if any) you used these kinds of activities for student assessment on quizzes or tests or for informal, formative feedback.**
    b. **Please tell me about your perspective on how students' engagement in these kinds of activities impacts their problem-solving skills – based on your judgment. Please tell me more about this.**

2. **Imagine that you are mentoring a new middle school mathematics teacher who wants to get your advice about how to structure her new classroom. In particular, she wants to know what you think is important if she wants to also incorporate some problem posing in her classroom on somewhat regular occasions. How would you advise her?**

APPENDIX C

DIMaC Interview Protocol

Teacher Interview General Questions

Goals with respect to student learning within each topic area (may uncover procedural/conceptual orientations)

1a. This question is an *imagine* question. I would like you to imagine that your students learned everything about **fractions** that you wanted them to learn. Everything! I told you that this was an imagine question. Talk about what they would have learned about **fractions**. Just to be clear, I'm not referring here to *how* they learned or *how* you taught. That's important, but that is not what this question is about. It is just about what they would end up *knowing*.

1b. How do you find teaching fractions—enjoyable, challenging, difficult, rewarding, some other adverb? *(Do not spend a significant amount of time on this question)*

2a. This question is another *imagine* question. I would like you to imagine that your students learned everything about **algebraic reasoning** that you wanted them to learn. Everything! Talk about what they would have learned about **algebraic reasoning**. Again, I'm not referring here to *how* they learned or *how* you taught. This question is just about what they would end up *knowing*.

2b. How do you find teaching algebraic reasoning—enjoyable, challenging, difficult, rewarding, some other adverb?

3a. This question is the final *imagine* question. I would like you to imagine that your students learned everything about **integers** that you wanted them to learn. Everything! Talk about what they would have learned about integers. Again, I'm not referring here to *how* they learned or *how* you taught, but what your students would end up *knowing*.

3b. How do you find teaching integers—enjoyable, challenging, difficult, rewarding, some other adjective?

4. What other mathematical goals do you have for your students? These goals may not be specific to a single mathematical topic, and do not need to be content specific. (Are there other frames besides content that teachers use to think about what they want their students to learn? For example, participating in discussions? Critiquing each others' ideas? Mathematical Practices from CCSS)

Instruction

5. You've discussed some of your goals for your students with respect to algebra, fractions and integers. Now think about how you *teach* each of these topics. Do you teach any of these topics differently? For example, do you use discussion more or use discussion less, do you use more or less group work, do you emphasize practice problems more with one topic than others, do you teach procedures before concepts or practical applications for one topic, etc.?

Professional development experiences

6. Describe your professional development experiences with respect to math. Have you had any PD that you felt has particularly influenced your teaching of mathematics? [This PD could be focused on mathematics in some way or more general PD, even in a different subject.]

Discussion/Discourse Specific Questions

7. Sometimes in your mathematics classes you have discussions. As we have talked about before, there are many different ways to have a discussion. If you think back to discussions you facilitate in your math class, talk about what makes a discussion good. What are you trying to accomplish? What would it sound like or look like?

8. Do you think the lessons we've been out to film this past year generally meet this image of a good discussion? [Follow-up asking if the lessons we filmed were normal or out-of-the-ordinary.] Were these lessons pretty typical for your math instruction? Do you have a harder time facilitating discussions for some topics? Which topics? Why?