LISTENING TO STRING SOUND:

A PEDAGOGICAL APPROACH TO EXPLORING THE COMPLEXITIES OF VIOLA TONE PRODUCTION

by

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(Under the Direction of Maggie Snyder)

ABSTRACT

String tone acoustics is a topic that has been largely overlooked in pedagogical settings. This document aims to illuminate the benefits of a general knowledge of practical acoustic science to inform teaching and performance practice. With an emphasis on viola tone production, the document introduces aspects of current physical science and psychoacoustics, combined with established pedagogy to help students and teachers gain a richer and more comprehensive view into aspects of tone production. The document serves as a guide to demonstrate areas where knowledge of the practical science can improve on playing technique and listening skills.

The document is divided into three main sections and is framed in a way that is useful for beginning, intermediate, and advanced string students. The first section introduces basic principles of sound, further delving into complex string tone and the mechanism of the violin and viola. The second section focuses on psychoacoustics and how it relates to the interpretation of string sound. The third section covers some of the pedagogical applications of the practical science in performance practice. A sampling of spectral analysis throughout the document demonstrates visually some of the relevant topics. Exercises for informing intonation practices utilizing combination tones are also included.

INDEX WORDS: string tone acoustics, psychoacoustics, string pedagogy, viola, tone production, combination tones, popular science, spectral analysis

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CHAPTER 1

INTRODUCTION

Need for Study

There are many string students who can benefit from an empirical approach to tone production. By combining aspects of physics, psychoacoustics, and string pedagogy, the challenging task of teaching tone production becomes more manageable, meeting the needs of students with differing learning styles: e.g., the visual learner, the kinesthetic learner, the aural learner, etc. Theorist and pianist Clarence G. Hamilton introduces his 1912 book, *Sound and its Relation to Music* by saying, “Every intelligent musician should be familiar with the physical laws which underline his art.”[1] By connecting practical acoustics and string pedagogy, students gain a better understanding of the limits and potential of their instrument, providing a basis for learning how to manipulate their technique to access a wider variety of sounds.

In numerous pedagogical string methods, practical acoustic science is overlooked due to the complexity of relevant topics, and, as a result, intuitive methods of instruction have emerged over time. Intuitive methods include language and semantics that do not fully align with the practical science. Music is also a highly subjective art, much pedagogical language is geared toward emotional, psychological, and physical sensations that can become highly personalized.

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[1] Clarence G. Hamilton, *Sound and Its Relation to Music* (Boston: Oliver Ditson Co., [1912]), [v], https://archive.org/details/sounditsrelation00hami/page/n9. Accessed July 2, 2019.

String players respond instinctively to the physical laws that govern the natural responses of their instruments. And although intuitive teaching methods have proven to be successful for achieving high quality string sounds, we live in an age of rapid advances in science and technology. We have access to a precise and efficient conceptual foundation for sound production based on more exact tools for measuring sound and vibration and a better understanding of the mechanism of the inner ear. By enhancing established pedagogy with recent scientific advances, both teachers and students can gain a broader comprehension of how different tones can be achieved. This document acquaints the string player with some essential acoustical science for approaching the complexities of string tone and, on this basis, offers alternative methods for improving intonation and listening practices.

Goals of the Study

My primary goal is to use insights from a review of recent studies in acoustics and psychoacoustics to develop a comprehensive approach to tone production on bowed string instruments, with focus on the viola. Although much of the science is unnecessary to teaching tone production, I draw attention to the most useful material for the string player. By applying aspects of practical science, students begin to listen with intent and utilize the natural responses of their instruments to inform their choices in fingerings and bowings. A general understanding of the basilar membrane and organ of Corti (the main components of the inner ear responsible for registering frequency) can provide deeper insights into how our ear and brain process sound.

Currently, considerable scientific literature exists regarding string tone acoustics, though much of it is specific to the violin. This document explores some of the differences in the acoustical properties between the violin and viola and demonstrates these differences and similarities with visual representations using spectral analysis.

I also summarize recent research on psychoacoustics, focusing on relevant aspects of perception and inner-ear processing that are useful for informing tone production and intonation. The new conceptual understanding of the mechanism of the inner ear justifies a reconsideration of the age-old technique of utilizing combination tones (specifically difference tones or Tartini tones) as a means of producing the intervals of the just intonation tuning system. After I review the psychoacoustical concepts underlying the phenomenon of combination tones, I then advocate for exercises involving them as a means for improving the identification of intervals and better intonation overall.

A better understanding of string tone acoustics and the psychoacoustics of hearing string tones justifies a different approach to the pedagogy of tone production on string instruments, at all levels of instruction. For beginning students, a basic conceptual understanding of the simpler aspects of string tone acoustics sharpens listening strategies and develops a larger palette of timbres. Intermediate students benefit from the more advanced combination tones exercises, which can lead to easier execution of double stops. Listening practices devoted to hearing overtones can lead to a greater overall awareness of the resonant response of their instruments. Advanced students can utilize the information to inform mature fingerings and bowings as well as improving listening skills. A careful consideration of the acoustics and psychoacoustics of string tone provides insight into how and why changes in bow placement, weight, and speed produce strikingly different results.

While recent explorations of spectral and timbre-centered composition will not be discussed, I hope that aspects of this document can provide insight into the acoustical properties of the viola that can benefit composers and performers.

Literature Review

Understandably, pedagogues of the past did not have access to modern physical measurements of sound and a comprehensive understanding of the workings of the inner ear until after the time of physicist Hermann von Helmholtz. Helmholtz's 1863 book, *The Sensations of Tone*, on acoustics and the perception of sound led the way for musicians and scientists alike to delve into the physics and psychoacoustics of "musical" tone.

John R. Pierce's book *The Science of Musical Sound* regarding acoustics (1992) is slightly more general and less in depth than Helmholtz. However, visual examples provided are very clear and the material presented is palatable for the non-physicist. *Measured Tones* by Ian Johnston, published in 2002, is arranged in a historical sequence and written by a physicist with the intent of informing both musicians and physicists. Dale Purves' recent book, *Music as Biology* (2017), is a concise introduction to basic aspects of audio cognition and focuses on a biological approach to demonstrate how aural perception in humans has evolved to focus on sounds likened to human vocalization. *Physics and the Sound of Music*, by John S. Rigden, though relatively less modern (1977), provides clear mathematical examples for combination tones and the main material is presented in a sort of textbook format, with exercises and problems at the end of each chapter.

The Physics and Psychophysics of Music by Juan G. Roederer, published in 1995, provides information regarding psychoacoustics and the function of the inner ear regarding musical perception. Like many systems found in nature, the ear functions as a nonlinear system, where the change of the output of the system is not proportional to the change of the input. Due to the nonlinearity of the systems employed in interpreting tone, much is yet to be understood and research in the area of psychoacoustics and neurological processing needs further consideration. Roederer provides a clear scientific synthesis of current research on this topic.

James Beament's 1997 book, *The Violin Explained: Components, Mechanism, and Sound* is another resource for the document. Though an entomologist by profession, Beament had a fascination with the acoustics of bowed string instruments. His book includes a fifty-page chapter specifically devoted to aural perception of violin sound and the other members of the violin family. His combined knowledge of science and music lead to a palatable description of string tone accessible to musicians and relevant to the topic as a reminder of what aspects of science are beneficial to the string player.[2]

Paul Hindemith's book, *The Craft of Musical Composition,* provides a comprehensive guide to some of his compositional idiosyncrasies. His focus on the overtone series, a part of what he calls Series 1, and combination tones, Series 2, is relevant to this approach of teaching tone production since an awareness of what informs his compositional style is especially useful for violists learning his music.

Regarding pedagogy, Simon Fischer's series of instructional videos, *The Secrets of Tone Production on All Bowed String Instruments,* is one of the most specifically tone-oriented pedagogical resources available. His DVD set is comprised of his own demonstrations of exercises as well as students ages 11-20 who are new to these techniques. The four-hour, two-disk set is broken into three main parts, "Knowing the instrument," "Five fundamental tone

[2] There are some concerns with claims that Beament presents in his book, but much of these questionable claims will not be referenced in this document. The material from his research that I include in my study are supported by the other references previously mentioned.

exercises," and "Combinations." The first section explains some of the physical limitations of string playing (bow pressure, five sound points, resonance, and upper partials). The second section is devoted to what Fischer calls "the five fundamental tone exercises," which includes demonstrations of exercises that he has compiled from great teachers, spanning 250 years of violin pedagogy. The last part of his DVD set is a combination of these techniques.

The Suzuki method is also a source for the applications portion of this study. I am a registered Suzuki instructor and have taught violin and viola using the method for over ten years. Through my research, I have come to find that facets of the Suzuki method are elucidated by aspects of science. Although, as with any comprehensive pedagogical method, the method may have some undeniable weaknesses, awareness of the practical acoustical science provides an explanation for why the method has been so successful over the years.

Methodology

My research for the topic began with authoritative recent research on the acoustics of string tone and psychoacoustic research and studies of the inner ear. My applied research included experimentation with combination tones on the viola to create numerous exercises and the necessary bowing and left-hand techniques for achieving them. Combination tones are achievable when playing bowed double stops in higher registers on a stringed instrument. The basic linear math for combination tones (specifically, difference tones) is demonstrated in the appendix of the document. The math for difference tones is presented by subtracting the lower frequency of a double-stopped interval from the higher frequency. It demonstrates how small variances in left hand finger placement affect the resulting pitch, and also demonstrates how combination tones function best with just intonation (I provide an explanation along with these exercises for the bowing technique necessary to achieve these tones).[3] My exercises include sample recordings that inform the student of how a bass line, created from the difference tones, can act as an aural guide to practice training the ear to listen for them.

I also use spectral analysis to underscore my study. I recorded specific sounds (i.e. open strings, natural harmonics, etc.) on two violas and a violin in the studio and observed them through the digital audio production application, Reaper, and the spectral analysis program Melda Production Audio Technologies, MAnalyzer. The resulting data from the analysis demonstrates what overtones are present in specific sounds created on two of these instruments.

How the following material can be utilized in teaching practices is a matter of individual preference, and this study serves as an introduction to some concepts associated with practical acoustics and string pedagogy. The document focuses on musicians that play viola but serves as a reference for all members of the string family. Whenever appropriate, terminology may refer to the broader string family as opposed to viola specifically.

This document attempts to keep the language in terms musicians can understand. It does not delve deeply into the psychology and neurology of interpreting sound as music as presented in psychoacoustics but will be limited to a preliminary explanation of the functions of the inner ear and only basic aspects of neural cognition useful for string players. In order to make the topic approachable for the music student and music instructor, the spectral analysis focuses primarily on sustained tone. Visual representations will be presented in the form of a Fourier analyses, based on the Fourier transform model, demonstrating decomposed waveforms into a sum of sine waves at different frequencies with different amplitudes and phase relationships. Only the basic linear math used for determining first order difference tones will be displayed in the document.

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[3] Differences in just intonation and the modern standard system of equal temperament are further discussed at the end of Chapter 3.

Chapter Organization

Chapter One introduces the topic and relevant literature. Chapter Two (String Sound Acoustics), provides a brief comparison of simple and complex tone, delving further into specific characteristics of string tone with a primary focus on the viola. The chapter also includes a surface-scratching look into the mechanism of the physical body of stringed instruments and will cover certain bowing techniques as described in acoustical terms.

Chapter Three (Psychoacoustics and the Inner Ear) is an introduction to the mechanism of the inner ear and the function of the basilar membrane within the cochlea. Chapter three is about psychoacoustics, our human perception of sound, and how string tone is registered in the inner ear. This chapter also covers the "missing fundamental" effect and why combination tones are perceived and briefly touches on the differences between just intonation and equal temperament.

Chapter Four is a practical application of the material for pedagogical use where I present more of my findings from recorded viola sounds and analysis. To explore our approach to listening to string sound, I demonstrate what overtones are present in specific viola sounds. Images derived from spectral analysis technology compare a viola and a violin, focusing primarily on my 1998 Bronek Cison viola. I also offer some suggestions for the learning musician to listen to overtones and examine combination tones as a means of improving intonation in their playing (I also include exercises for practicing combination tones). Overall, I hope to demonstrate how incorporating elements of physics and psychoacoustics into string pedagogy methods leads to improvements in technique, attentiveness in listening practices, and access of a greater palette of sounds for the string player.

CHAPTER 2

STRING SOUND ACOUSTICS

The process of examining string sound is a complicated one, intriguing scientists and musicians alike since before the time of Pythagoras. To this day, a precise, empirically grounded explanation of all aspects of string tone does not exist. Though visual representations in the form of Fourier analyses provide us with a much clearer interpretation, many aspects of string tone cannot be entirely explained in these terms. The topic begins to quickly diverge into the field of psychology. Therefore, an "all-purpose" explanation of the topic may be impossible on account of all the differences in playing technique, the response of instruments, and our individual perception of sound. However, there are many things that strings players would gain from a better comprehension of current research in string tone acoustics. Knowledge of practical acoustics can provide the string player with insights into bowing technique and listening awareness.

In analyzing string tone, we first need a generic understanding of how sound works. A sound wave acts as an auditory disturbance which travels through some transmission medium, be it a gas, solid, or liquid. According to music producer, sound educator, and psychoacoustician, Joshua Leeds, "Wherever there is motion, there is frequency. Though inaudible at times, all frequencies make a sound."[4] *Frequency* is the rate at which a vibration occurs that constitutes a

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[4] Joshua Leeds, *The Power of Sound: How to Be Healthy and Productive Using Music and Sound*. (Rochester, Vermont: Healing Arts Press, 2001, rev. 2010), 7. Modern book on psychoacoustics, covering aspects of sound and aural awareness as well as

wave, usually measured in *Hertz*. Hertz, named after Heinrich Rudolf Hertz, is the SI unit (international system of units) of frequency, equal to one cycle per second.

For string tone, the vibration of the string displaces molecules in the air causing them to vibrate at the same rate (frequency) as the string, and these air molecules cause our eardrums to vibrate in response to this frequency. The information is encoded in the inner ear and sent to the brain, signaling our *perception* of pitch and timbre.[5] Leeds goes on to explain that "sound is vibrating energy… It is a frequency of vibration that we audibly hear between 20 and 20,000 Hz."[6] Although this is typically the range associated with the human ear, this range can vary quite substantially from person to person, due to several factors.[7] Oleg Marchuk, from his dissertation on violin intonation, tells us that "One has to understand that the musical sound is not just the domain of physiology or acoustics but also in psychology and aesthetics."[8] This is important to remember. Physics and anatomy can explain many aspects of musical tone, but much further research is necessary to be able to fully explain why certain sounds are considered "musical."

Noise vs. Musical Sound

One way to begin to sort sound as a meaningful musical concept is to differentiate noise from musical sound. Helmholtz explains that, "The sensation of a musical tone is due to a rapid

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contributions to modern medicine and health. Also approaches the work of Alfred Tomatis and his studies of the ear and listening.

[5] Pitch determined as frequency is attributed to Felix Savart in 1833. His work was preceded by Robert Hooke in 1681, with a demonstration of what is now known as the Savart Wheel.

[6] Leeds, *The Power of Sound*, 14.

[7] Hearing loss is one of these factors, which is experienced to varying degrees throughout our lives, typically getting more pronounced with age. Here is one of many free online hearing tests: https://www.resound.com/en-us/online-hearing-test/take-test

[8] Oleg Marchuk, *Violin Intonation* (Music Performance thesis, Lahti University of Applied Sciences, 2012), 7. *How* we *perceive* sound is vitally important to our understanding of it and varies substantially across cultures and individuals.

periodic motion of the sonorous body: the sensation of a noise to non-periodic motions."[9] When we can perceive a periodicity of vibration, we can differentiate musical sounds from everyday noises. If we concentrate on a noise, say a machine doing road construction, we may be able to discern periodic elements from the sound. Our ears, however, are drawn to periodic sound signals within certain ranges, explaining why noise can be so distracting and computed as nonessential. Below, Figure 2.1, is an example of the sine function, showing a visual representation of a periodic sound signal in two complete periods. A sine wave demonstrates a single frequency and the clear discernible pattern one can expect from a musical sound signal.

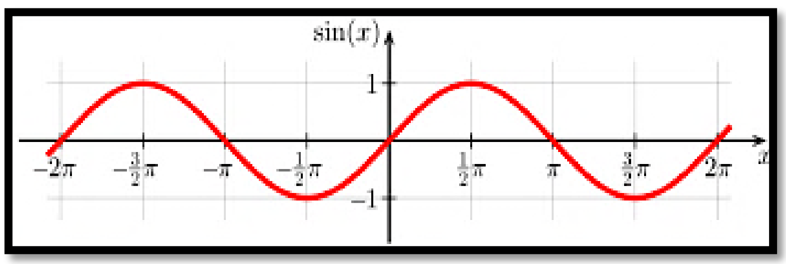

Figure 2.1 Graph of the sine function demonstrating two complete periods[10]

String players experience noise (non-periodic sounds) as well as musical sound in simply playing sustained tones. String players may recognize this as a raspy sound, audible when we play: however, the noise is typically not experienced by the audience and heard by the player on account of their proximity to the instrument. The noise experienced in string sound is due to friction during the slip phase of bowing caused by the rosin on the bow. Another type of noise is generated in the lower-sounding members of the string family (cello and bass) that is not

[9] Hermann L.F. Helmholtz, trans. from German, *The Sensations of Tone as a Physiological Basis for the Theory of Music,* 6th ed. (New York, NY: Longmans, Green and Co., 1948), 8.

[10] "Periodic function," *Wikipedia*, https://en.wikipedia.org/wiki/Periodic_function attributed https://commons.wikimedia.org/wiki/User:Geek3, https://commons.wikimedia.org/wiki/File:Sine.svg#/media/File:Sine.svg. Accessed July 2, 2019.

physically present in the sound. This noise is a psychoacoustic phenomenon called "roughness," which is caused by overlapping higher harmonics and distortions in the inner ear.[11] This explains, in part, why *clarity* of tone is more difficult to achieve on lower instruments of the string family. The ear also *prefers* a certain range of frequencies and amplitudes which will be further discussed in Chapter 3.

Pure Tones and Complex String Sound

The term "pure tone" is referenced in both science and music but holds quite different implications. In music, we tend to regard pure tones as relative to the pitch of the fundamental and the absence of noise. Though similar in acoustical science, a pure tone is described as an even Hertz frequency that forms an s-shaped sine wave or, in simpler terms, a single frequency (figure 2.1 above). "Complex tone" is a compilation of two or of these single frequencies, typically including a fundamental and series of overtones. Figure 2.2 below is a simplified visual representation of waveforms. The category on the left shows the "pure" tones or single frequency sine waves presented as a fundamental followed by some respective overtones. The next category shows complex waveforms and their composite representation which is what is being processed in our ear when we listen to string sound.

[11] The topic of "*roughness*" goes deep into the field of psychoacoustics and will only be briefly covered in this paper.

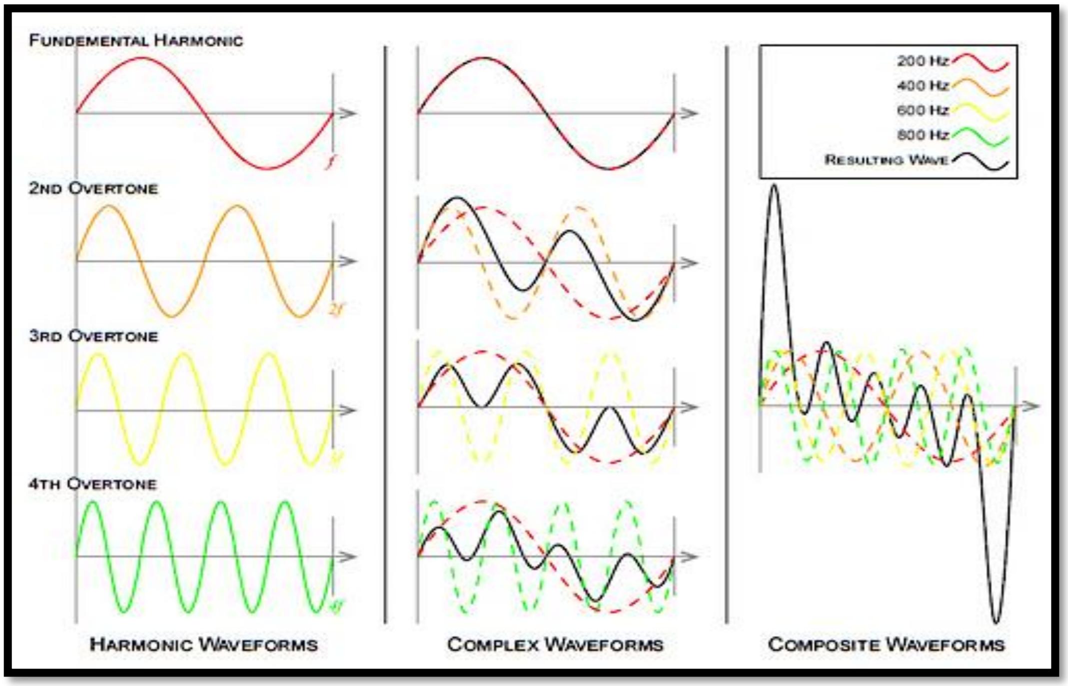

Figure 2.2 Diagram illustrating waveforms[12]

Pure tones can only be produced by means of technology, i.e. certain oscillators and sine wave frequency generators. All tones produced by bowed string instruments are considered *complex* tones because they include a fundamental plus numerous constituent overtones occurring at differing levels. These overtones and their varying strengths are responsible for the timbre of sounds: they enable us to differentiate between viola and violin sounds as well as all other sounds. The varying strengths of the present overtones can help the string player notice even more subtle changes, like playing an open A as opposed to a fingered A on another string.

A Fourier transform demonstrates the presence and varying levels of these overtones visually. Theorist and composer Reginald Bain explains that "In 1822, the French mathematician Jean Baptist Fourier (1768-1830) came forth with a mathematical proof demonstrating that any

[12] Merate A. Barakat, "Musicology Concepts and Spatial Awareness," February 17, 2016, http://www.meratebarakat.com/research-blog/2016/2/16/musicology. Accessed July 2, 2019.

waveform or signal, no matter how complex, could be reduced to an infinite set of sine wave components."[13] In a Fourier transform, these sine wave components are called Fourier coefficients. When you use software to take the Fourier transform of a composite waveform, you get back a bumpy function (the spectrum) which tells you the strength of each of its constituent overtones. The composite waveform can be described as the weighted sum over all its overtones. This sum is called a Fourier series and it has applications in many diverse areas of science, engineering, and math. The overtone series is just one example of a Fourier series.

In order to make sense of complex tones, it is necessary to understand how the overtone series works. Here, again, terminology can be a pedagogical bump in the road for students trying to make sense of these concepts. Frequently, the harmonic series and the overtone series are used synonymously, but they have notable differences. Harmonics refer to sounds we can achieve on stringed instruments by lightly dampening the string on specific proportionally related points along the string. Paul Hindemith wrote that "The string is free to vibrate on both sides of this dividing point- which is not true in the case of the normally produced tone, in which the part of the string which lies below the stopping finger is excluded from the production of tone."[14] Harmonics are not to be confused with overtones because they are a whole number integer multiple of the fundamental frequency. An overtone is any frequency that is greater than the fundamental frequency of a sound and does not necessarily have to be harmonic. The fundamental plus its numerical integer multiples are called partials. Although these terms are

[13] Reginald Bain, "The Harmonic Series: A path to understanding musical intervals, scales, tuning and timbre," 2003, http://in.music.sc.edu/fs/bain/atmi02/hs/hs.pdf. Accessed July 2, 2019.
[14] Paul Hindemith, *The Craft of Musical Composition, Book 1,* 4th ed. (New York: Associated Music Publishers, 1945), 19-20.

frequently used synonymously when discussing music acoustics, the differences are worth noting.[15] The example table (2.1) below demonstrates how these terms correspond and overlap.

Frequency	Order	Name 1	Name 2	Name 3
1*f* = 440 Hz	n=1	Fundamental tone	Fundamental tone	First partial
2*f* = 880 Hz	n=2	First overtone	First harmonic	Second partial
3*f* = 1320 Hz	n=3	Second overtone	Second harmonic	Third partial
4*f* = 1760 Hz	n=4	Third overtone	Third harmonic	Fourth partial

Table 2.1 An example of harmonic overtones[16]

The closest that a stringed instrument can come to producing pure tones is through natural and fingered harmonics. Some of the highest harmonics achievable can come close to acting as pure tones, demonstrating only a single sine wave of the fundamental with minimal overtones. The presence of other sound signals in the analysis of these harmonics are the reason why they are not considered "pure" tones.

Figure 2.3 (below) shows a spectral image of a natural harmonic on the G string of a viola.[17] The numbers to the left represent the amplitude in decibels (dB), the numbers below the image represent the Hertz frequencies. Red demonstrates the most prominent levels present in the sound, followed by orange, green, blue, and indigo in respective order. As can be seen, there

[15] Many systems of tuning, dating back to the time of Pythagoras, have been devised based on the monochord and simple integer ratios. The systems of just and equal temperament most currently relevant to string players will be further discussed at the end of Chapter 3.
[16] Edited table derived from "Harmonics, Overtones and the Fundamental" *Tontechnik-Rechner – sengpielaudio* http://www.sengpielaudio.com/calculator-harmonics.htm More details can be found in John R. Pierce, *The Science of Musical Sound*, revised ed. (New York, NY: W.H. Freeman and Company, 1992): Chapter 3: Sine Waves and Resonance. A similar table is presented on page 43.
[17] These images are derived my own recordings in the studio with sound engineer Suny Lyons. The spectral analysis is from a program found here: "MAnalyzer," *Meldaproduction* https://www.meldaproduction.com/MAnalyzer
***Please take note that in this program the octave identification system is incorrect. All octave designations are one octave too low. (Ex. C4 is presented as C3). This is consistent throughout. In the document, I have edited them to reflect the correct octave registers. Open strings on the viola are represented (low to high) as C3, G3, D4, A4

are still many overtones present in the sound, though the fundamental (G4), an octave above the open G string, is most prominent.

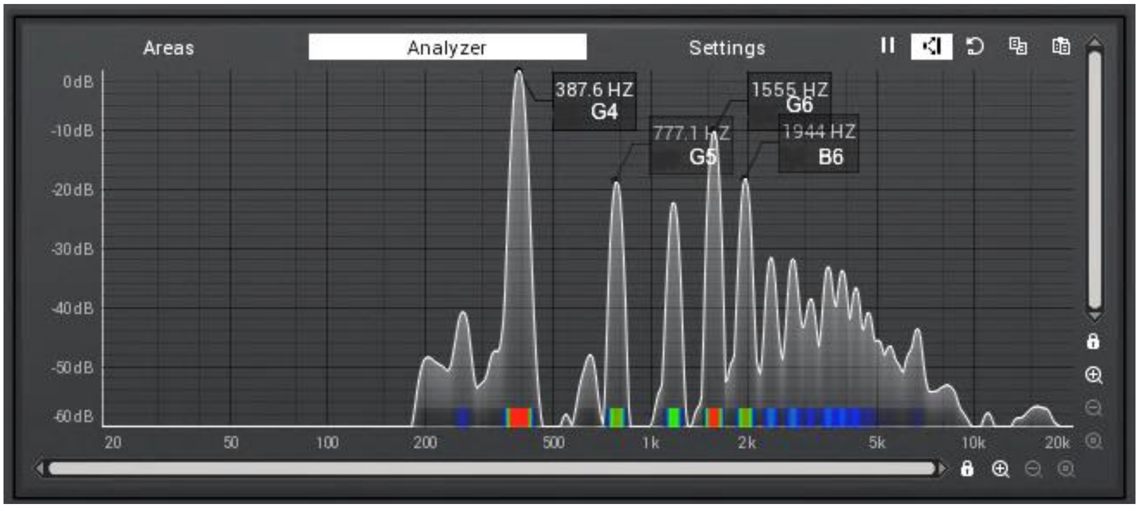

Figure 2.3 Open G string natural harmonic (2:1) on a viola

We now understand that true "pure" tones are unachievable on a stringed instrument. However, there are some harmonics that are closer approximations of sine waves. These vary from instrument to instrument and are dependent on a range of factors. Figure 2.4 shows the natural harmonic D7, the 8th partial in the overtone series on the open D string. Notice that this harmonic's fundamental is most prominent but does not represent a single sine wave.

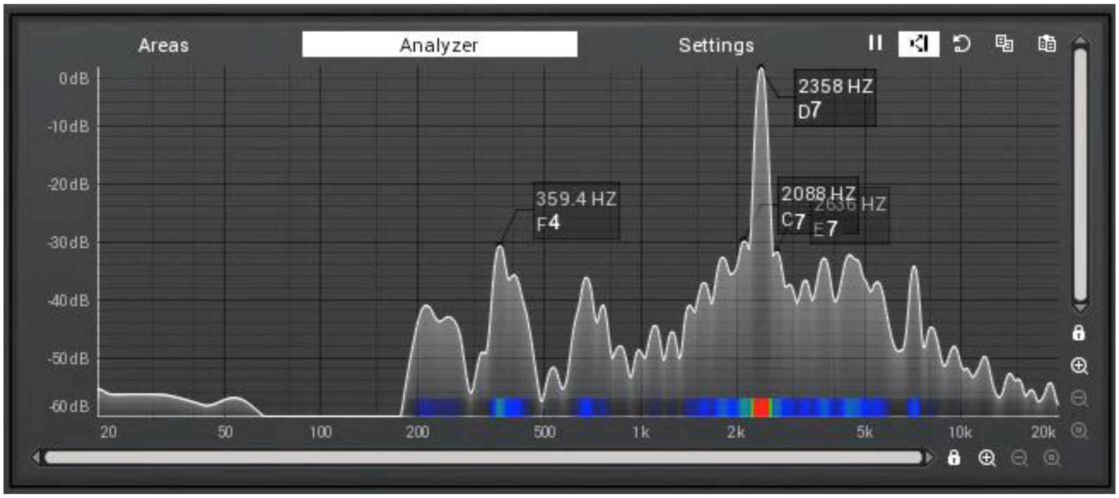

Figure 2.4 Natural harmonic on the D string (viola)

The different overtones and their varying levels present in a sound are responsible for the quality of the sound, otherwise known as timbre, and the *Grove Dictionary*'s third definition for tone. Paul Hindemith explains that, "The ear hardly hears [overtones] separately; it only perceives the disappearance of some or the addition of others as changes in tone-color."[18] Subtle changes in tone color can be manipulated by bowing technique and finger placement to be further discussed in Chapter 4.

The Missing Fundamental

Stanford professor John Pierce says that "For periodic musical sounds, the pitch is tied firmly to their periodicity, the frequency of the first harmonic partial."[19] Pierce is describing the fundamental—what we *perceive* as the "pitch" of a sound. Sometimes, the fundamental frequency will not be physically present in a tone, but our ears will "hear" it because of the

[18] Hindemith, *The Craft of Musical Composition,* 17.
[19] Pierce, *The Science of Musical Sound,* 36-37.

varying strength of the overtones present in the sound. This is known as the "missing fundamental effect," which is explained further in Chapter 3.[20]

Figures 2.5 and 2.6, demonstrate the "missing fundamental" effect on the open G and C strings of the viola in a spectral analysis.[21] The analysis shows how the 2nd partial in both the G and C open strings are more present than the fundamental. The open C string displays that the 3rd partial is even more pronounced than the second. The same is true of a violin G string, where again the fundamental is less present than its subsequent overtones.

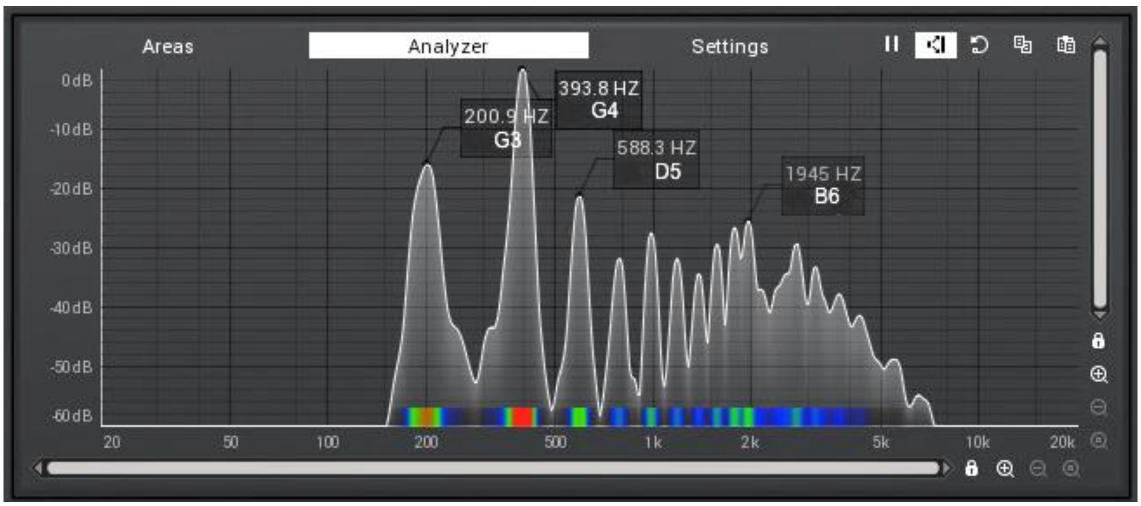

Figure 2.5 Open viola G string sustained

[20] Further reading: "Topic: How Do You Hear Tones?" *Hydrogen Audio* https://hydrogenaud.io/index.php/topic,40690.0.html
Sound clip: http://www.khoomei.com/forum/uploads/1180135123.mp3
[21] This is an analysis of a 1999 Bronek Cison viola and a violin.

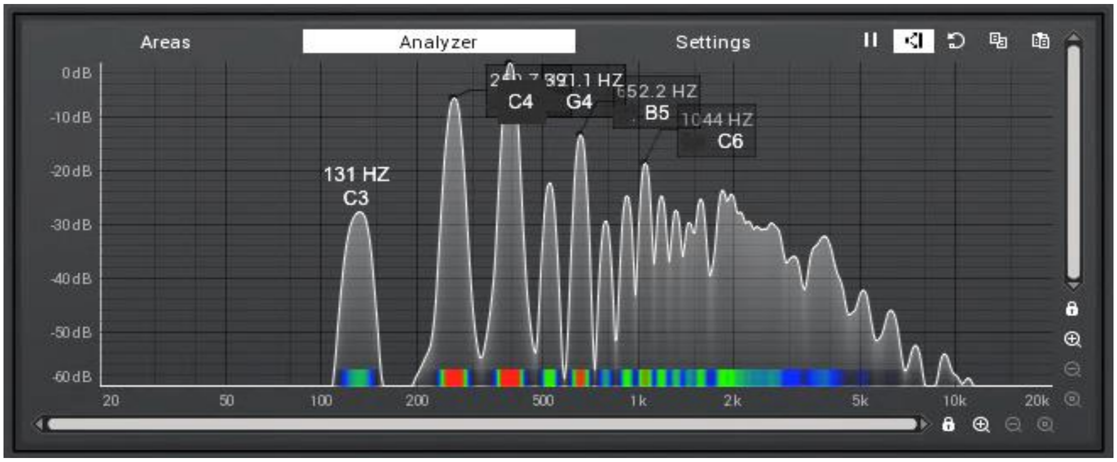

Figure 2.6 Open viola C string sustained

Regardless of the player's ability level, the complex tone of a stringed instrument is in a relatively constant state of fluctuation. The images presented are only snapshots of the sound. For a true representation of sustained tone, one would need to experience the analysis in real time, ideally observing it as a sequence of events.

Terminological problems in defining musical tone are often overlooked in the realm of string pedagogy. Although tone is a frequently used term in pedagogy and performance, references to tone production are often nebulous. A fundamental understanding of string tone is derived from the laws of physical acoustics. As musicians, reference to tone can mean any number of things, i.e. quality of sound, timbre, texture, and color, to name a few, and the term is frequently referenced in highly subjective ways that can be difficult to codify. For example, a mentor might describe the need for a "clean" tone or a "warmer" tone, which might leave students without a clear understanding of *how* to achieve these certain qualities in their sound. For the purpose of my research, I would like to create some delineations of the terminology in order to clarify how the science of tone corresponds to our understanding of it as musicians, beginning with tone.

There are five main definitions in the *Grove Dictionary of Music (Oxford Music Online)* for tone. The second and third definitions are most relevant here. The second definition describes tone as "any steady sound, especially one used in making measurements," i.e. pure tones, test tones, standard tones, combination tones, etc.[22] The third definition of tone refers to the "quality" of musical sounds and sends the reader to further reference the term *timbre*.[23] This definition can be explained further, thanks to Hermann Helmholtz's research and discovery of related harmonics dating back to 1870. Helmholtz is known for identifying the various frequencies of the sine wave components of complex tones. His research has led to our current understanding of how the overtones present in a sound inform "quality" of tone and timbre.

James Beament, entomologist and string sound enthusiast, in his 1997 book *The Violin Explained*, derives a more comprehensive definition of tone from Helmholtz's discoveries. Beament says, "Tone is the *sensation* produced by which harmonics are present and how big each is in this sound."[24] Beament's definition shows us how psychology factors in to our understanding of tone. Our perception of tone is not just about the harmonics and their relative strengths but our individualized *sensation* of them. Therefore, there is no easy way of establishing a concise definition for the term. However, in pedagogical settings, a student should be primarily aware of the crucial differences between "pure" and "complex" tones in order to avoid confusion.

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[22] William Drabkin, 2001 "Tone (ii)." *Grove Music Online,* https://www.oxfordmusiconline.com/grovemusic/view/10.1093/gmo/9781561592630.001.0001/omo-9781561592630-e-0000053933. Accessed June 13, 2019.

[23] William Drabkin. 2001 "Tone (iii)." *Grove Music Online,* https://www.oxfordmusiconline.com/grovemusic/view/10.1093/gmo/9781561592630.001.0001/omo-9781561592630-e-0000053934, Accessed June 13, 2019.

[24] James Beament, *The Violin Explained: Components, Mechanism, and Sound.* (Oxford: Clarendon Press, 1997), 235.

Some Aspects of the Mechanism of Stringed Instruments

The viola is known to have a darker, muted sound quality when compared to a violin. We understand this as resulting from the prominence of the second and third partials and the lack of a prominent fundamental on the lower strings as previously discussed. The brilliance of the violin tone, with the exception of the G string, is due to a prominent fundamental and a greater prominence of higher partials. Part of the reason for this difference is the size of the viola. Unlike the violin, there is no standard size, i.e. 4/4, but a wide range of sizes. "A full-size viola's body is between 25 mm (1 in) and 100 mm (4 in) longer than the body of a full-size violin (i.e., between 38 and 46 cm [15–18 in]), with an average length of 41 cm (16 in)."[25] The viola's unique sound is derived from its vibrational capacity, which is manipulated through the "tuning" of the vibrational modes of the instrument.[26]

According to luthier Carleen Hutchins, there are five main vibrational modes on a stringed instrument: A0, B0, A1, B1, and B-1.[27] Among these, the main design challenge for constructing a desirable viola is the matching of the vibration modes of the wood of the box to those of the air modes. This is difficult to achieve due to the dimensions of the viola.[28] The frequency of the A0 mode, otherwise known as the Helmholtz resonance or "breathing mode," in the words of Curtin and Rossing, "refers to the "f-hole" resonance associated with strong sound radiation from the f-holes, as elegantly described by [Lothar] Cremer (1984)." Curtin and Rossing expand that thought: "[In a violin] system, A1 is a mode around 470 Hz in which air "sloshes" from the upper to lower bout in response to plate motion. Bn (B = beam) labels are attached to modes with

[25] "Viola," *Wikipedia*, https://en.wikipedia.org/wiki/Viola. Accessed July 2, 2019.
[26] A useful website for further understanding of modes and plate tuning: "Acoustics for Violin and Guitar Makers," *Speech, Music and Hearing* http://www.speech.kth.se/music/acviguit4/index.html. For those looking to buy a new instrument, I would recommend checking out this luthier's blog: "Keith Hill – Instrument Maker" http://keithhillharpsichords.com/new-page-3
[27] For those wanting to see how the modes are tuned on your own instrument, I would recommend referring to this chart: Carleen M. Hutchins and Duane Voskuil, "Mode Tuning for the Violin Maker," *CAS Journal* 2, no. 4 (Series II, November 1993): 5-9 http://www.catgutacoustical.org/research/articles/modetune/modechrt.html.
[28] Carleen M. Hutchins, "The Acoustics of the Viola," *Journal of the American Viola Society* 4, no. 2 (Summer 1988): 5, http://www.americanviolasociety.org/PDFs/Journal/JAVS-4_2.pdf. Accessed July 2, 2019.

beam-like bending motions of the violin body."[29] Luthiers "tune" these different resonance modes by scraping bits of wood from different parts of the top and bottom plates as they match their frequencies using tap tones (knocking on the wood to hear the resonance) or Chladni patterns. Awareness of the natural resonance responses of our instruments can further our understanding of the capabilities and limitations of the instruments themselves.

The research of Carleen Hutchins elucidates the acoustical properties of the viola. She is known for improving on plate tuning technology, utilizing cymatics (visual representation of sound) and specifically the Chladni plate. The original Chladni plate consists of a metal plate covered with sand that is bowed at different frequencies to create patterns in the sand. Many modern luthiers now tune the plates (top and bottom pieces of wood) using this method, as it is more specific and accurate than tuning the plates with tap tones by ear. (This does not account for the works of Italian luthiers of the past who tuned entirely by ear with much success).[30]

Utilizing Chladni plate technology, Hutchins was able to demonstrate specifically the peculiarities of the viola's dimensions. As previously stated, there is no standard size for the viola as with the violin because, in the attempt to make the viola playable on the shoulder like the violin, matching the quality of viola sound to violin sound is not practical. Hutchins says, "In the violin, the two lower cavity modes, the AO mode and the next higher in frequency, the Al mode, lie within a semitone of the two open middle strings, the D and A. In the viola, they are higher in relation to string tuning, namely, up around B- Bb− A on the G string and F-F# on the

[29] J. Curtin and T.D. Rossing, "Violin," *The Science of String Instruments,* Chapter 13 (New York: Springer, 2010): 222, https://logosfoundation.org/kursus/The%20Science%20of%20String%20Instruments.pdf. Accessed July 2, 2019.
[30] Here is a short video that demonstrates using Chladni patterns on the plates of the violin: Jonathan McKinley, "Violin Back Plate Tuning," *YouTube* (2009) https://www.youtube.com/watch?v=3uMZzVvnSiU.

D string. The placement of these cavity modes in relation to string tone has been found to be the most critical factor in tone quality between the violin and other members of the violin family."[31]

According to James S. Rigdon, "Typically, the MWR (main wood resonance) and MAR (main air resonance) of the viola and cello fall at frequencies three or four semitones higher with respect to the unstopped frequencies of the two middle strings than they do in the violin. These resonances are therefore too high to provide adequate support for frequencies in the lower ranges of these instruments."[32] Rigdon says that, in accordance to how the viola is tuned, a fifth below the violin, the longest wavelength the viola could create is 1.5 times that of the violin. This indicates that the viola would need to be 1.5 times larger to produce the same resonances as a violin, but typically, violas are only built to be about 1.17 times larger. [33] Since Rigdon does not specify measurements, it makes sense that he is referring to a viola that is 16-16 1/2 inches long.

James Beament suggests that another main difference in the tone of the viola and violin is due to the size and weight of the bridge and tailpiece. The bridge of the viola is substantially heavier than that of a violin and is important to consider because the bridge is responsible for transmitting and filtering the vibrations that enter the instrument. Woodhouse explains that, "The bridge serves to translate the mainly lateral force of the strings into mainly vertical forces on the top via the bridge feet."[34] Also, the flexibility of the bridge causes the string to function somewhat differently than a string suspended between two immobile points. String players should be aware of these subtleties as they directly affect the sounds created as a result of bowing technique.

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[31] Hutchins, "The Acoustics of the Viola": 6, http://www.americanviolasociety.org/PDFs/Journal/JAVS-4_2.pdf. Accessed July 2, 2109.
[32] John S. Rigden, *Physics and the Sound of Music.* (New York, NY: John Wiley & Sons, Inc., 1977), 143.
[33] Ibid. 142-143
[34] J. Curtin and T.D. Rossing, "Violin," *The Science of String Instruments,* Chapter 13 (New York: Springer, 2010): 233, https://logosfoundation.org/kursus/The%20Science%20of%20String%20Instruments.pdf. Accessed July 2, 2019.

	Violin	Viola	Cello	Double Bass
Body length in inches (mm)	14 (355)	16 (407)	29 (735)	43 (1090)
Length needed to make wood resonances correspond with a violin	14 (355)	21 (538)	42 (1065)	68 (1725)

Table 2.2 Comparative lengths of the four instruments[35]

Carleen Hutchins is also known for her inventing a set of paired string instruments designed to match the parameters of the violin known as a "Violin Octet,"[36] in effect a set of eight violins of varying sizes. These instruments were created to homogenize the string family's timbre. The alto violin here is 20" long, the ideal size to replicate the quality of the violin sound (featuring a prominent fundamental in the lower registers). Hutchins' 20" alto violin is probably the closest that anyone has come to creating a viola that replicates the acoustical properties of the violin, and due to its size, many people hold it like one would a cello. Below (Figure 2.7) is an image of the Violin Octet that shows where the instrument registers lie and their related sizes to that of the traditional sizes of the string family.

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[35] Beament, *The Violin Explained*, 86.
[36] Carleen Hutchins, "The Violin Octet," *The New Violin Family Association, Inc.*, https://nvfa.org/8tet.html. Accessed July 2, 2019.

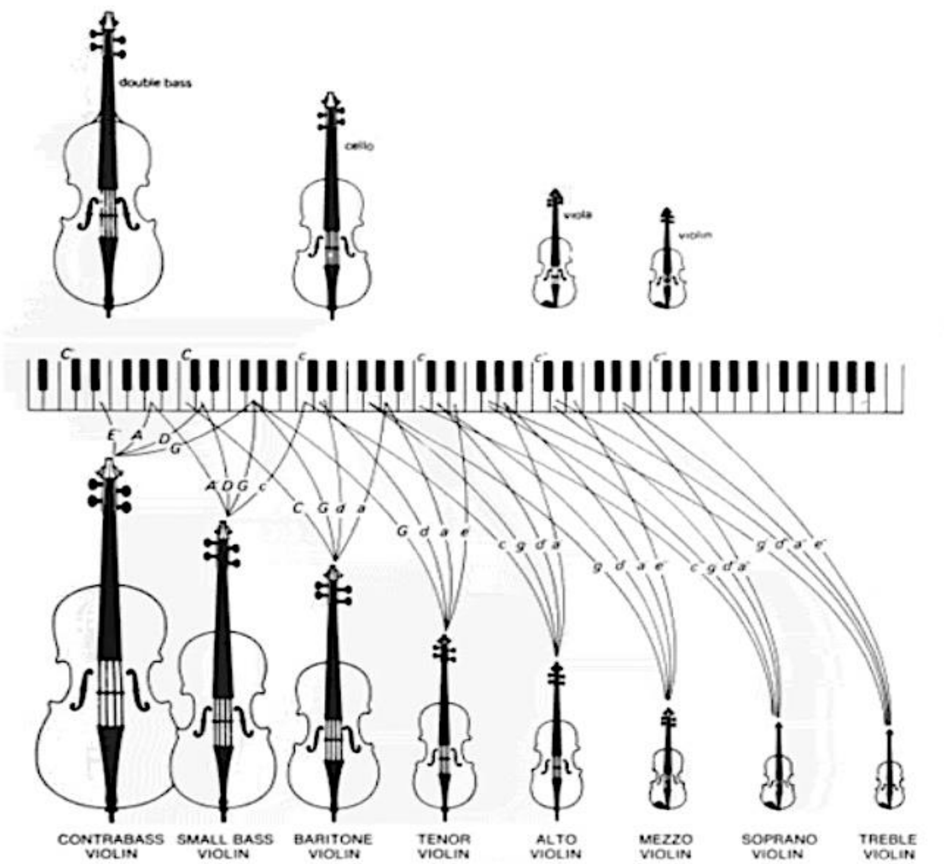

Figure 2.7 Carleen Hutchins's violin octet[37]

In the violist community, there is general agreement that the peculiarities of the viola tone are not undesirable—quite the opposite! Many musicians are drawn to the viola for these reasons. The highly diverse acoustic qualities of the string family add to the wonderful textures and timbres that we associate with chamber and orchestral music. Wolfgang Amadeus Mozart is a famous example of one of many proponents for the curiosities of the viola sound. One example

[37] Paul R. Laird, "Carleen Maley Hutchins: The Violin Octet" *The Catgut Acoustical Society,* https://www.catgutacoustical.org/people/cmh/laird6.htm. Accessed on July 2, 2019.

of Mozart's awareness of the unusual characteristics of the viola sound can be found in his Symphonie Concertante for Violin, Viola and Orchestra in E-flat major, K. 364. Here, the solo viola part is written in D major, so the violist incorporates *scordatura* and tunes all the strings up by one half step. This way, the player can still use fingerings that they are accustomed to, yet the raising of pitch was intended to add to the brilliance of the viola tone in order to highlight the viola as a solo instrument in the work and possibly counteract issues of projection over an orchestra.

Ian Johnston suggests that the most desired timbres in Mozart's quintets for two violas are with air resonances that lie between B and Bb on the G string.[38] He claims that the quintets were intended for a viola with these resonance qualities and would therefore not sound appropriate on an instrument such as one with timbral qualities like that of the alto violin from the Violin Octet.[39] The viola also potentially emerged as a result of the *lira da braccio* (lyre of the arm), an accompaniment instrument dating back to the Renaissance (early 16th century).[40] The *lira da braccio* was used to accompany recitative in the courts and functioned mainly as harmonic support, incorporating drones and chordal configurations. Although the *lira da braccio* was later replaced during the rise of the madrigal with the more soloistic vocal violin, one could imagine that viola was always considered, by luthiers and composers alike, to act as a harmony instrument, therefore its timbral quality was not viewed as a negative trait, but an intentional one.[41]

Resonant responses differ greatly from instrument to instrument. Luthiers have a comprehensive understanding of how the instruments resonate most effectively and how to

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[38] Ian Johnston, *Measured Tones: The Interplay of Physics and Music, 2nd ed.* (Philadelphia, PA: Institute of Physics Publishing, 2002), 130.
[39] Ibid. 129-130
[40] Maurice W. Riley, *The History of the Viola* (Ann Arbor, MI: Braun-Brumfield, 1980), 7-11.
[41] This is only personal speculation. More research would be necessary to prove this was the case.

achieve these resonances. The information is useful to guide string players' understanding of our own instruments' limitations and potential. We can also use the information to better guide our choices when purchasing new instruments or having work done to them. It is important to remember that the spectral comparisons in this document only apply to the specific instruments being compared, and while some commonalities can be observed between specific members of the violin family, there will always be some amount of difference when comparing them.

Slip-Stick Motion

Another common misconception that many string players have is how the vibration of the string is generated by bow motion. William Atwood writes, "The string, [Helmholtz] determined, does not just wag back and forth upon being agitated by the bow. Rather a sharp kink makes round-trips between the bridge and the nut, flipping over at each reflection. As the kink passes by the contact point of the bow on its way to the bridge, it first releases the grip of the bow hairs on the string and then an instant later grabs them again. The result is a continuous plucking action often referred to as "slip stick," or "Helmholtz" motion."[42]

Further studies following Helmholtz, outlined by J. Woodhouse and based on the research of Lothar Cremer show that the angular "corners" (Figures 2.8 and 2.9) may not always be so defined as described by the Helmholtz model. Woodhouse says, "the "corner" is significantly rounded when the normal force exerted by the bow on the string is small but becomes sharper when the force is increased."[43]

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[42] William Atwood, "A Physicist in the World of Violins," *Beam Line* (Summer 1998): 21, http://www.slac.stanford.edu/pubs/beamline/28/2/28-2-atwood.pdf. Accessed July 2, 2019.
[43] J. Woodhouse and P. M. Galluzzo, "The Bowed String As We Know It Today," *Acta Acustica United with Acustica* 90 (2004): 580, http://www2.eng.cam.ac.uk/~jw12/JW%20PDFs/BowedStringReview.pdf. Accessed July 2, 2019.

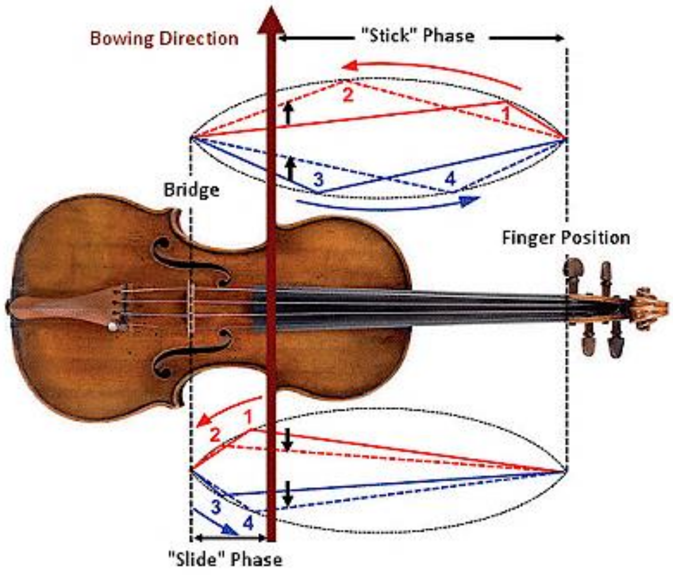

Figure 2.8 Visual representation of slip-stick (Helmholtz) motion[44]

Slip-stick motion could be likened in physics to moving a piano across a floor: getting the instrument in motion is harder than keeping it in motion, where static friction (sticking), is greater than kinetic friction (sliding). In string playing, friction is provided by the rosin on the bow and pressure increases both static and kinetic friction. Therefore, as Beament explains, "If too-high speed is used with low pressure, the bow skates instead of gripping the string and produces a variety of high-pitched noises which are a combination of frictional whistles and the string vibrating in different modes. If too-high pressure is used with low speed, a different kind of nasty sound is produced."[45] Again, it is important for the string player to understand that the string does not simply move back and forth like we interpret the motion with the naked eye. Instead, the bow twists the string as well. Beament explains that "the larger the diameter of the

[44] Klaus Roth, "Chemical Secrets of the Violin Virtuosi – Part 3" *Chemistry Views,* August 2012, https://www.chemistryviews.org/details/ezine/2085627/Chemical_Secrets_of_the_Violin_Virtuosi__Part_3.html. Accessed July 2, 2019.
[45] Beament, *The Violin Explained*, 22.

string, the greater the twisting."[46] He describes the motion of the cylindrical string as a "wind-up" motion that takes the energy from the bow, thus reducing the energy that would move the string sideways in a normal force on the bridge.

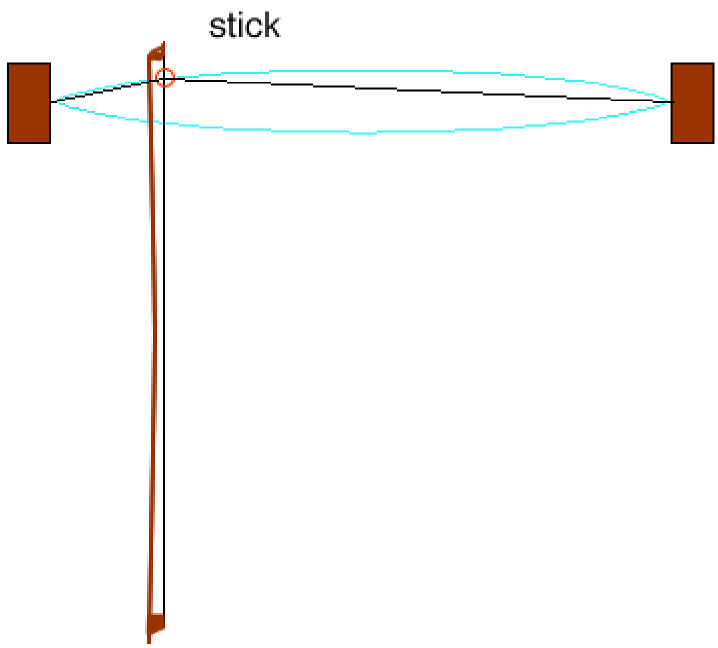

Figure 2.9 Image demonstrating slip-stick motion [47]

This describes what we experience in playing and the reason why it requires so much effort to bow, especially on the lower strings. Woodhouse explains that this twisting motion or "the conversion of transverse waves to torsional waves ... may account for a significant part of the energy dissipation during bowing."[48] In my own teaching, I describe the experience as

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[46] Beament, *The Violin Explained*, 29.
[47] Joe Wolfe, "Bows and Strings," *Music Acoustics,* The University New South Wales To view the animation visit: http://newt.phys.unsw.edu.au/jw/Bows.html
[48] Woodhouse and Galluzzo, "The Bowed String As We Know It Today": 582 http://www2.eng.cam.ac.uk/~jw12/JW%20PDFs/BowedStringReview.pdf

"feeling the resistance" of the string. This resistance then, is not just a simple back and forth motion, but also a twisting of the string.

The "Flattening Effect"

Another curious acoustic phenomenon specific to the bowed string family is what is referred to by acousticians as "the flattening effect." Woodhouse and Galluzzo write, "An explanation was given for the "flattening effect," whereby at high values of the normal force from the bow, the period of the Helmholtz motion (slip-stick) can be systematically lengthened so that in musical terms the note "plays flat."[49] In string playing, manipulations in bow speed and pressure can affect not only the quality of the sound, but also the pitch. The flattening effect and its resulting pitches, typically referred to by musicians as "subharmonics," have been utilized in the compositions of violinist Mari Kimura. A further explanation of this phenomenon can be found in chapter four.

Transients

An important challenge for the string player, and possibly the largest factor in creating pleasurable sounds on our instruments has to do with the sound phenomena known as transients. Transients are described as the "changing sound," and are largely responsible for how we are able to differentiate timbres.[50] Ian Johnston discusses three distinct phases of changing sound as, "the *attack* (or *starting transient*) when the sound rises from nothing to its highest value; the *steady state*, when it remains more or less at the same level; and the *decay*, when it

[49] Woodhouse and Galluzzo, "The Bowed String As We Know It Today": 581, http://www2.eng.cam.ac.uk/~jw12/JW%20PDFs/BowedStringReview.pdf. Accessed July 2, 2019.
[50] Beament, *The Violin Explained*, 127.

falls away again to silence."[51] Transients commonly contain a large amount of non-periodic sound and higher frequencies and are most easily identified at the beginning of a sound. One study on transients' results, conducted by researchers Guettler and Askenfelt, is described in the words of J. Woodhouse below:

> Both parts of the study showed that length of "pre-Helmholtz" transient is indeed a matter of critical importance for good violin sound. Listeners have a narrow and well-defined "acceptance band" for transient length, around 50 ms. Under a range of different bowing conditions, the professional players almost invariably produced transients within this acceptance range, including a significant proportion of "perfect transients" in which Helmholtz motion was established essentially from the first cycle. Learning bowing gestures which produce short transients is obviously a significant goal of the long hours of practice needed to master the violin.[52]

Transients are a large part of how we qualify tone production as string players. Mastery of the beginnings of strokes, according to acoustical science, is one of the most important aspects of good string playing. Chapter 4 expands on applications for further discussion of these concepts.

[51] Ian Johnston, *Measured Tones: The Interplay of Physics and Music, 2nd ed.* (Philadelphia, PA: Institute of Physics Publishing, 2002), 296.
[52] Woodhouse and Galluzzo, "The Bowed String As We Know It Today": 584 http://www2.eng.cam.ac.uk/~jw12/JW%20PDFs/BowedStringReview.pdf

CHAPTER 3

PSYCHOACOUSTICS AND THE INNER EAR

Psychoacoustics has been a rapidly emerging field since the time of Georg von Bekesy (around the 1940s–60s). Lately, much has been revealed about the inner ear due to advances in technology and access to the cochlear region and the organ of Corti. From the time of Helmholtz up until Bekesy, there was little to no access to this part of the ear due to its small size and hard bone covering. Bekesy worked on cadavers and was limited in his research. Now we can look into the cochlea of a living person and leave it intact thanks to optical coherence tomography, a visualization technology used to examine the eye.[53]

There are three main parts of the ear: the outer ear, the middle ear, and the inner ear.[54] The outer ear is responsible for funneling sounds into the middle ear. The middle ear begins with the eardrum, which vibrates according to the frequency it receives and then vibrates three small chains of bones, know together as the auditory ossicles (which increase the amplitude of the vibration by around 30 dBs before sending it to the inner ear and cochlear region. The inner ear consists of the cochlea (Greek for "snail"), the vestibular system (responsible for balance) and the auditory nerve, which sends the information to the brain. Due to how the ear processes sound, frequency, the rate at which a vibration occurs that constitutes a sound wave, measured in

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[53] Linköping Universitet, "New Discovery on How the Inner Ear Works," *Science Daily*, July 11, 2016, https://www.sciencedaily.com/releases/2016/07/160711092510.htm There is also much work being conducted using otoacoustic emissions, a standard way of testing hearing in infants and adults. Accessed July 2, 2019.

[54] For a quick visual introduction to how the ear works, I would recommend watching these two short, informative videos: "How Hearing Works," *Med-El*, https://www.medel.com/us/how-hearing-works/ and "2-Minute Neuroscience: The Cochlea," *YouTube*, June 19, 2015, https://www.youtube.com/watch?v=WeQluId1hnQ.

Hertz (cycles per second), and *amplitude*, the maximum extent of a vibration or oscillation, measured from the position of equilibrium, are the most important considerations for our aural perception of musical sounds. Figure 3.1 below is a visual demonstration of sound waves as processed in the ear.

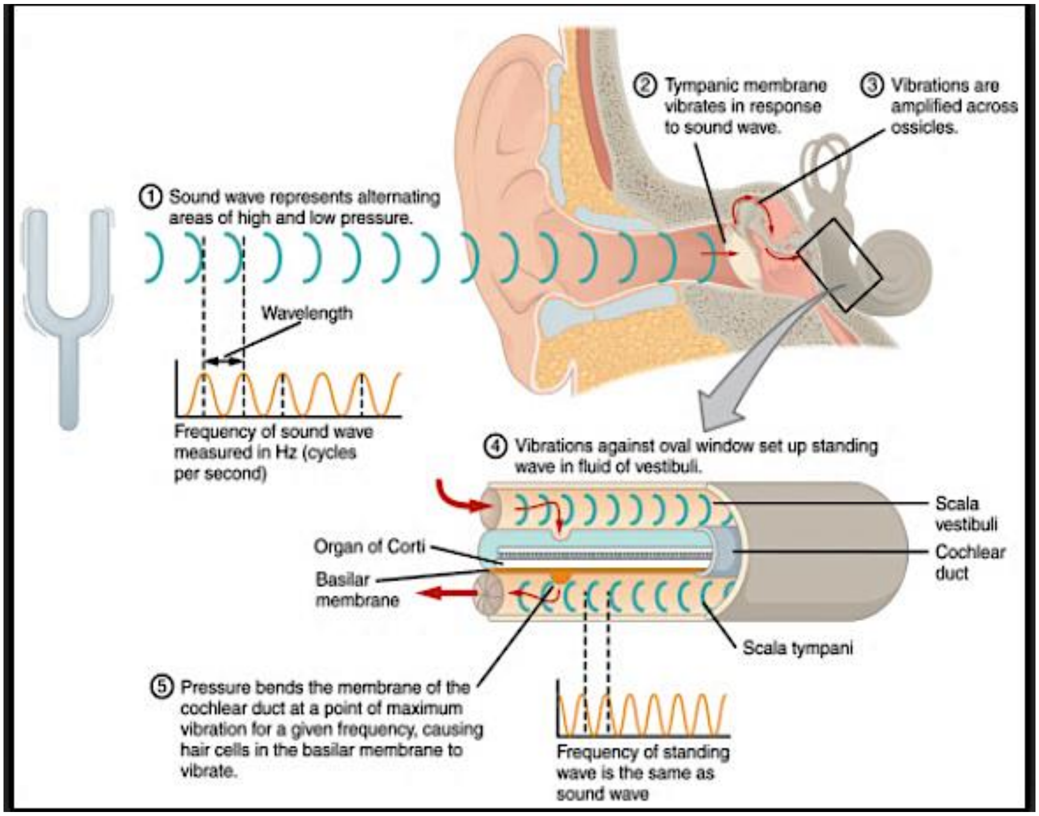

Figure 3.1 Sound waves and the ear[55]

Inside of the cochlea is the basilar membrane, an elastic membrane along which frequencies are registered. The organ of Corti rests on top of the basilar membrane (which is about 35 mm. in length) and runs from the base of the cochlea all the way to the apex. The Corti

[55] OpenStax, *OpenStax Anatomy and Physiology*, May 18, 2016, https://cnx.org/contents/FPtK1zmh@8.25:s3XqfSLV@7/Sensory-Perception. Accessed July 2, 2019.

contains the tiny hair cells that move according to the vibration in the fluid surrounding them encased in chambers within the cochlea (Figure 3.2). These sensitive hairs cells are responsible for sending the information along to the brain via the auditory nerve.

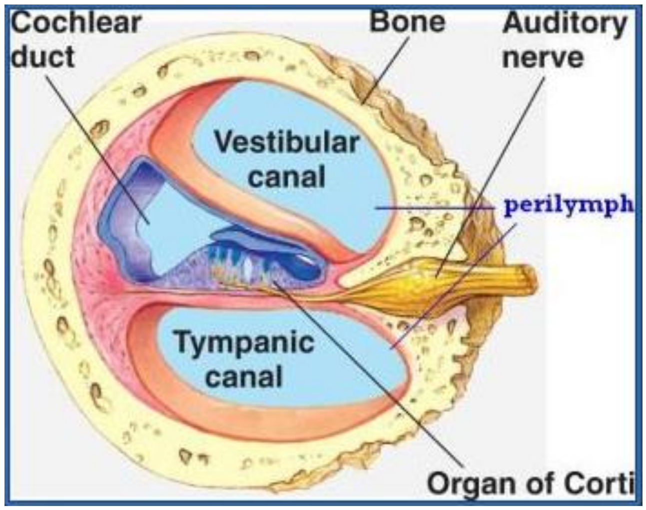

Figure 3.2 Cross section of the cochlea[56]

At the base of the cochlea, along the basilar membrane, high frequencies are registered, and at the apex, the low frequencies are registered, as you can see in Figure 3.3, a visual representation of an "uncoiled" cochlea.

[56] "Cochlear Anatomy," *StudyBlue – A Clegg Service*, https://www.studyblue.com/notes/note/n/3-cochlear-anatomy/deck/15276678. Accessed July 2, 2019.

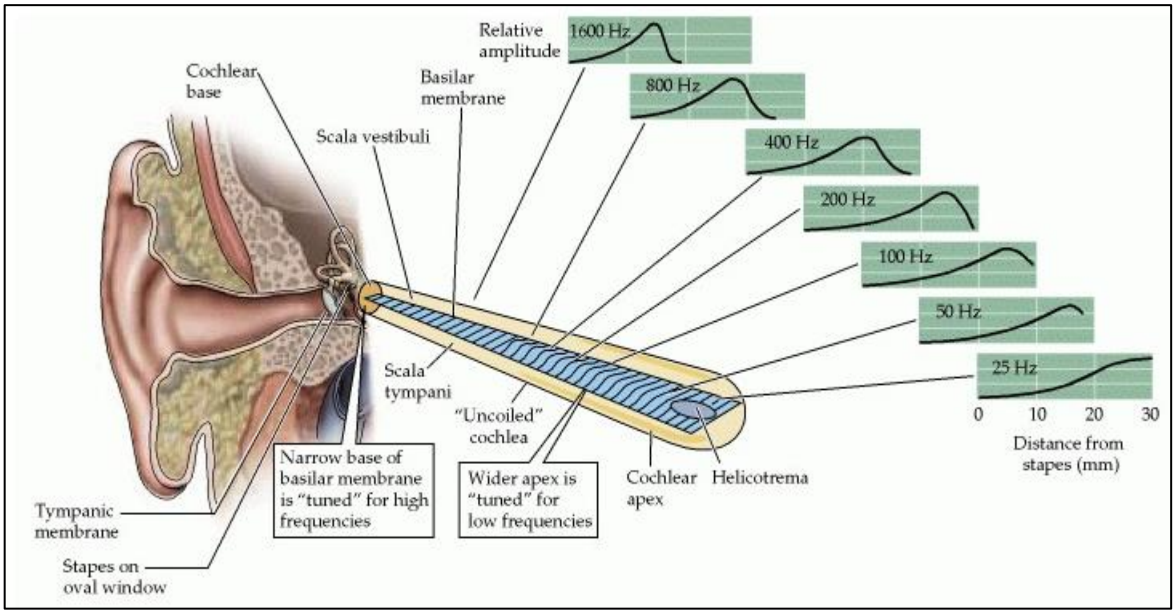

Figure 3.3 Image of an uncoiled cochlea[57]

Our hearing is sensitive to higher pitched sounds and prefers a range of frequencies that could suggest that hearing is drawn to human vocalization, however, more research would be necessary to prove this to be true. The ear also amplifies the preferred range of frequencies our ears are drawn to. Beament explains, "The range that had the most value for mankind was 1-5 kHz; otherwise we would be most sensitive in a different range."[58] His diagram below (Figure 3.4) demonstrates the particular range of hearing the ear "prefers." Notice how the range of the violin is within the "high sensitivity" range, whereas the lower instruments of the string family are not. High sensitivity occurs at around C above the treble staff.

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[57] Purves D, Augustine GJ, Fitzpatrick D, et al., editors. Neuroscience. 2nd edition. Sunderland (MA): Sinauer Associates; 2001, https://www.ncbi.nlm.nih.gov/books/NBK10946/, Accessed July 2, 2019.

[58] Beament, *The Violin Explained*, 106.

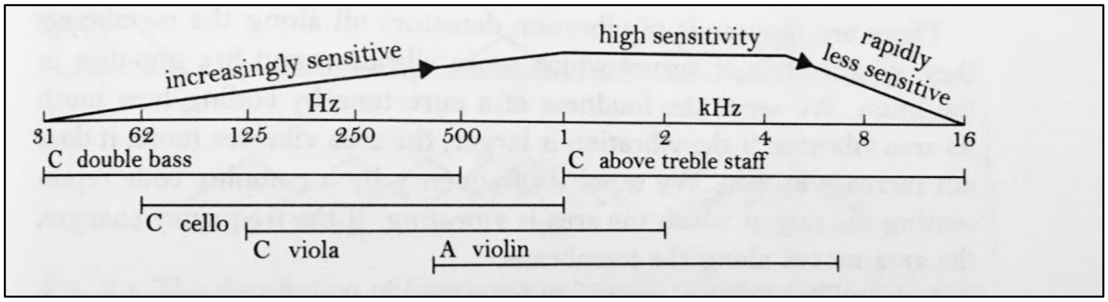

Figure 3.4 Sensitivity of hearing at different frequencies[59]

Psychoacoustics and the Missing Fundamental Effect

String players can use this biological information to inform our understanding of preferences of the ear and also how we interpret sounds that are produced by our instruments. An aforementioned topic, the missing fundamental effect, is a neurological phenomenon that occurs when the fundamental in a complex periodic sound is absent or less present than its corresponding overtones. The higher harmonics present in a sound, their varying strengths, and their interaction can imply a fundamental pitch below. The pitch is interpreted as being present because of neurological processes and cannot be explained by the mechanism of the inner ear as previously thought. In the words of Juan Roederer, "the corresponding pitch sensation is called *periodicity pitch, subjective pitch, residue tone, or virtual pitch."*[60] A periodicity pitch can also be achieved when two "component" tones are fed, one in each ear.[61]

On the C and G string of a viola, the brain is filling in missing information in order to perceive the fundamental pitch. The missing fundamental effect is yet another attribute to viola tone especially on the lower two strings of the instrument.

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59 Ibid. 99.

60 Juan G. Roederer, *The Physics and Psychophysics of Music: An Introduction,* 3rd ed. (New York, NY: Springer-Verlag, 1995), 45.

61 Ibid., 45. For further study see also Joshua Leeds, *The Power of Sound: How to Be Healthy and Productive Using Music and Sound*. (Rochester, Vermont: Healing Arts Press, 2001, rev. 2010): 182-188. Chapter on binaural beats, another neuroacoustical phenomenon discovered by German researcher H.W. Dove in 1839.

Psychoacoustics and Combination Tones

The basilar membrane is elastic, therefore physically responding similarly to a string, as strings are also elastic, but to varying degrees based on their materials. Gut strings, traditionally used on period instruments, are more elastic than a synthetic or steel wound string. Like the vibrating string, Roederer explains that "it is frequency ratios, not their differences, that determine the displacement of the resonance region along the basilar membrane. A relationship of this kind is called "logarithmic."[62]

When more than one tone enters the ear simultaneously, the tones are registered accordingly along this membrane. Dependent on their proximity to one another, the tones can interact with differing results. This is known as "superposition of sound," which is further broken down into linear superposition (where the tones do not interfere with one another), and nonlinear superposition (where the tones influence one another). If the processing is "mechanical," and based solely in the ear, it is known as a "first order superposition effect." A "second order" effect would include neural processing and is much more difficult to diagnose.[63]

If the difference between the two tones is large enough, we can distinguish two separate pitches. However, Roederer explains that "if the frequency difference is smaller than a certain amount, the resonance regions overlap, and we hear only one tone of intermediate pitch with modulated or "beating loudness."[64] Here, he is specifically referring to two pure tones played at the same amplitude, yet most musicians and even non-musicians are aware of this occurrence even with complex tones (beats). Roederer further explains that, "As long as [the difference in frequency] is less than about 10 Hz, these beats are perceived very clearly. When the frequency difference (**Δ**f) exceeds, say, 15 Hz, the beat sensation disappears, giving way to a quite

[62] Ibid., 26.
[63] Roederer, *The Physics and Psychophysics of Music*, 28-29.
[64] Ibid., 30-31.

characteristic *roughness* or unpleasantness of the resulting tone sensation."[65] The sensation of roughness persists until an area referred to as the "critical bandwidth" has been surpassed, and it is then that we can clearly distinguish two clear tones. Beyond critical bandwidth, when one of these tones remains constant and the other increases in frequency, especially in higher ranges with higher amplitudes, combination tones can be perceived. A combination tone is a pitch sensation that is not presented in the original sound stimulus but is a "result of a so-called nonlinear distortion of the acoustical signal in the ear."[66]

There are two main types of combination tones. Summation (sum) tones, which are achieved by adding the two primary tone frequencies together. Sum tones are difficult to hear due to masking of the primary tones. Difference tones, achievable when playing double stops in higher registers on stringed instruments are computed by subtracting the lower frequency primary tone from the higher frequency primary tone. Difference tones are not masked because they fall in a lower range than the primary tones.

Although there are presentable math equations (See Appendix) that can demonstrate approximations to achieving the combination tones in the following exercises, the Hertz values are based on an equal tempered system, which is why there is more than one possible tone presented in these terms. Due to the logarithmic processing of the inner ear, Ian Johnston says, "the natural way to express the 'difference' between the pitches of two notes involves forming the *ratio* of two frequencies, rather than *subtracting* those frequencies."[67] Ideally, we would account for the math in terms of ratios, as seen previously in the section on psychoacoustics and how the basilar membrane processes frequency. The linear equations used for these exercises

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[65] Ibid., 31-32. This is also variant dependent on frequency ranges, for example some lower ranges fall outside of our range of perception.
[66] Ibid., 37.
[67] Ian Johnston, *Measured Tones: The Interplay of Physics and Music, 2nd ed.* (Philadelphia, PA: Institute of Physics Publishing, 2002), 234.

include the most easily perceived, first order combination tones, $f_x = f_2 - f_1$, if the frequency of f_2 is greater than f_1. There are still more perceivable combination tones, second order and beyond, not expressed in the exercises. For those desiring to explore more of the math involved for discovering these difference tones, John S. Rigden provides a clear and straightforward account in Table 3.1 below.

Interval	f_1	f_2	f_{C1}	f_{C2}	f_{C3}
Octave	f_1	$2f_1$	f_1	—	—
Fifth	f_1	$\frac{3}{2}f_1$	$\frac{1}{2}f_1$	$\frac{1}{2}f_1$	—
Fourth	f_1	$\frac{4}{3}f_1$	$\frac{1}{3}f_1$	$\frac{2}{3}f_1$	$\frac{1}{3}f_1$
Major third	f_1	$\frac{5}{4}f_1$	$\frac{1}{4}f_1$	$\frac{3}{4}f_1$	$\frac{1}{2}f_1$
Major sixth	f_1	$\frac{5}{3}f_1$	$\frac{2}{3}f_1$	$\frac{1}{3}f_1$	—
Minor third	f_1	$\frac{6}{5}f_1$	$\frac{1}{5}f_1$	$\frac{4}{5}f_1$	$\frac{3}{5}f_1$

Table 3.1 Ratio relationships for achieving combination tones[68]

Here, Rigden is demonstrating the order of combination tones. The tones played are written as ratios in the first two columns. The last three columns are the perceived combination tones in the order of their perceptibility. The diagram demonstrates how the combination tones relate proportionally to the tones physically present in the sound.

Below is a spectral representation of a combination tone played on the viola (Figure 3.5). Notice that the expected difference tone (Ab2) based on the linear math, is not present in the

[68] John S. Rigden, *Physics and the Sound of Music* (New York, NY: John Wiley & Sons, Inc., 1977), 71.

spectral analysis. However, we can see that both octaves above Ab2 are present, though much less pronounced than the combinations of overtones and fundamentals of C5 and Eb5.

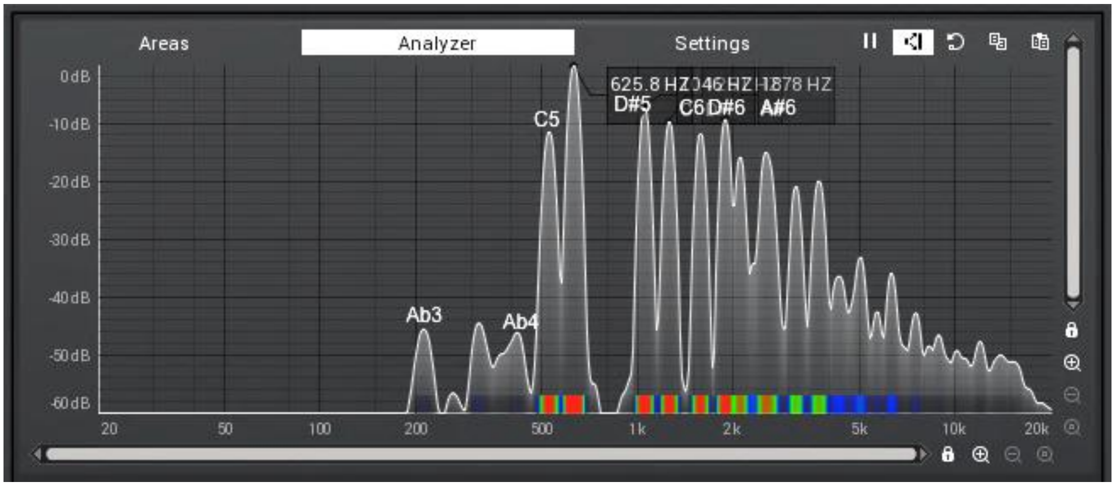

Figure 3.5 Difference tone Ab2

Listen here: https://soundcloud.com/maria-kindt-299629075/eb5-c5-ab2/s-3v0aB

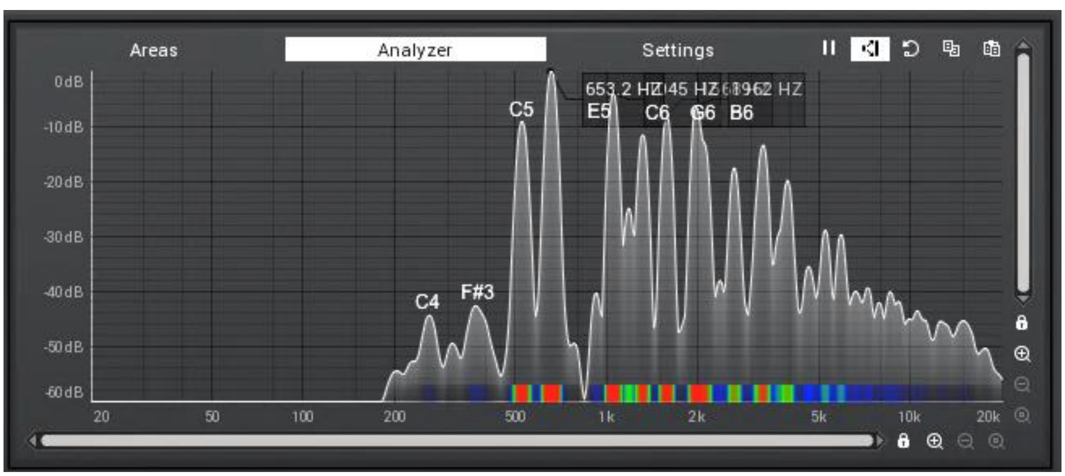

Figure 3.6 Difference tone C3

Listen here: https://soundcloud.com/maria-kindt-299629075/e5-c5-c3/s-mSMT1

Again, the higher octave is present in Figure 3.6 above. C3 is what we "hear," yet what is present in the spectral analysis is one octave above, C4. And again, in Figure 3.7, A3, the octave higher than the combination tone that we perceive, is present in the spectral analysis, yet the expected pitch is not.

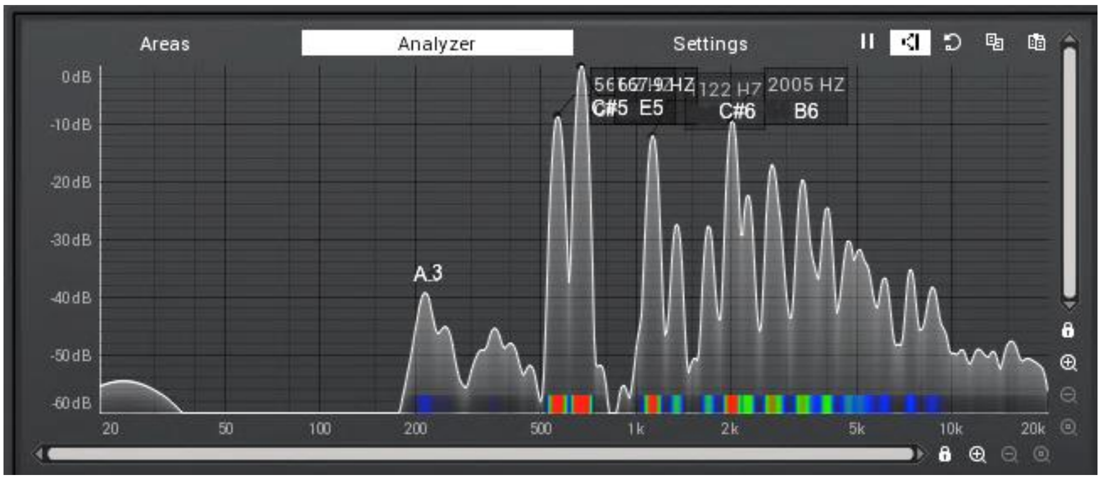

Figure 3.7 Difference tone A2

Listen here: https://soundcloud.com/maria-kindt-299629075/e5-c5-a2-flat/s-Rvo6A

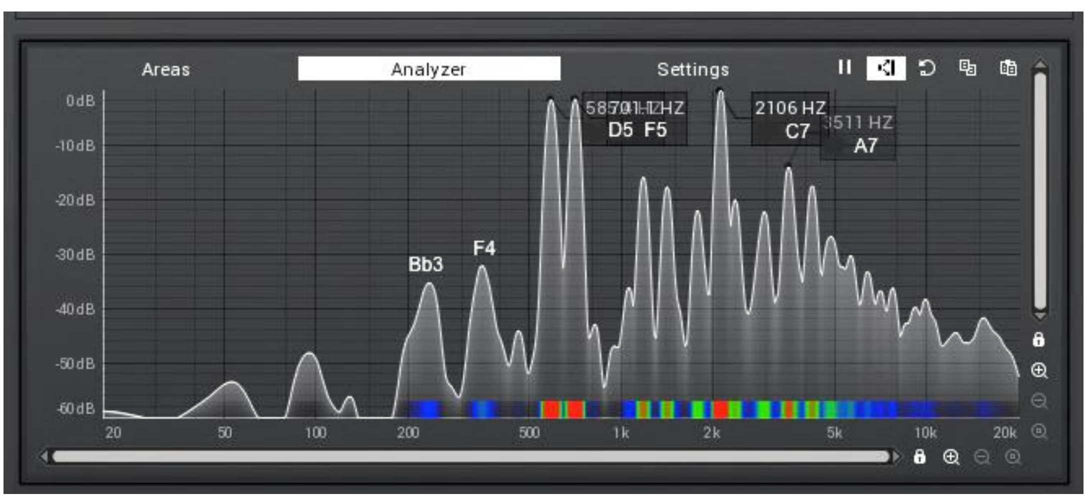

Figure 3.8 Difference tone Bb2

Listen here: https://soundcloud.com/maria-kindt-299629075/f5-d5-bb2/s-DcExu

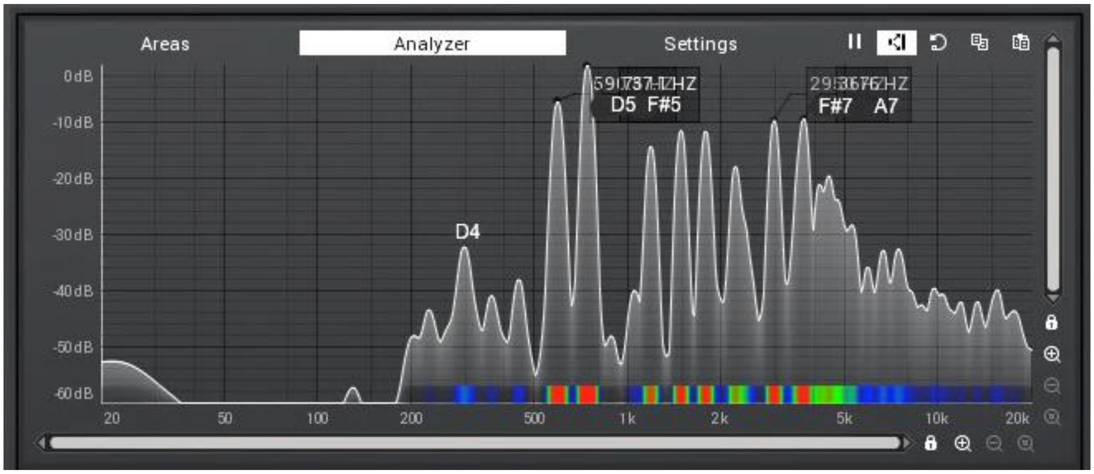

Figure 3.9 Difference tone D3

Listen here: https://soundcloud.com/maria-kindt-299629075/f5-d5-d3/s-R7Zjo

Pictured above, Figures 3.8 and 3.9, are two more demonstrations of the octave register difference. Combination tones are most easily perceived when the double stop intervals are tuned in just intonation. Why the ear "prefers" the just tuning system is due to the elasticity of the basilar membrane. [69]

Just Intonation and Equal Temperament

Just intonation, also known as pure intonation, is credited to the astronomer Ptolemy, dating back to 166 A.D.[70] This system of tuning relies on whole number ratios of intervals and the natural division of the string according to the overtone series. Therefore, it is considered

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[69] Though there is reason to believe that our ear prefers just tuning, there are limited studies that have been able to show that this is the case, and further research is necessary. Much information needs to be considered, such as cultural influences and preferences, musical training, etc. This concept goes deep into the field of psychoacoustics and cognition psychology. Here is one such study: Katarzyna J. Blinowska, Konrad Kwaskiewicz, W. Wiktor Jedrzejczak, and Henryk Skarzynski, "Musical Ratios in Sounds from the Human Cochlea," *Plos One* (May 24, 2012) https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0037988

[70] Alexander J. Ellis and Arthur Mendel, *Studies in the History of Musical Pitch.* (Amsterdam: Frits Knuf, 1968), 12. This is an historical approach to understanding Western classical pitch and its development and includes a nice compact explanation of temperament.

"natural" tuning, as it functions according to physical laws. Equal temperament is the current standardized Western system of tuning. In this tuning system, all semitones are equidistant. The system became standardized because it is the closest approximation for intervals to sound relatively acceptable to the ear when moving from different tonal centers. In equal temperament there are 100 cents between each semitone. Cents refer to a standardized logarithmic unit of measure for intervals dating back to the 1830s. The system is a necessary compromise for a chordal instrument like the piano functioning within musical systems that incorporate frequent modulation. Paul Hindemith describes the compromise well. "Purity must be neglected, or the possibility of unhindered polyphony sacrificed."[71] Table 3.2 below demonstrates the difference in cents between equal temperament and just (natural) tuning.

Interval	Equally-tempered intervals in cents	Natural intervals in cents	Difference in cents
Tonic	0	0	0
Minor second	100	111.73	-11.73
Major second	200	203.91	-3.91
Minor third	300	315.64	-15.64
Major third	400	386.31	13.69
Fourth	500	498.04	1.96
Tritone	600	590.22	9.78
Fifth	700	701.96	-1.96
Minor sixth	800	813.69	-13.69
Major sixth	900	884.36	15.64
Minor seventh	1000	996.09	3.91
Major seventh	1100	1088.27	11.73
Octave	1200	1200	0

Table 3.2 Difference (in cents) of a natural (just) scale to that of an equal tempered one[72]

[71] Paul Hindemith, *The Craft of Musical Composition,* trans. By Arthur Mendel (New York: Associated Music Publishers, 1945), 28.
[72] Chart from Oleg Marchuk, *Violin Intonation.* (Music Performance thesis, Lahti University of Applied Sciences, 2012), 16.

As we can see from the chart, the octave and unison in equal temperament are the only intervals that exactly correspond to the just system. Perfect fifths and fourths are very close. The difference for thirds is quite substantial.

Piano tuners frequently employ "beats" (as previously mentioned) when tuning a piano. On an equal tempered piano, octaves are perfect (beatless), fifths are narrow, beating 1-2 beats per second, and fourths are wide, also beating 1-2 beats per second. The rest of the intervals beat around 6-7 beats per second.[73] Important to note for the string player is that these differences affect our tuning when playing with an equal tempered instrument. Even though musicians will typically adjust by ear, for best results it is useful to know what specific intervals are affected.[74]

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[73] To hear the audible difference, check out the audio file on the Wikipedia page on cents: "Cent (music)," *Wikipedia* https://en.wikipedia.org/wiki/Cent_(music). The one cent difference is almost imperceptible, but more noticeable at a 6-cent difference, and very obvious at a 10-cent difference. The amount that this is perceptible varies from person to person.
[74] To understand more about tuning and temperaments, take a look at this website: Kenneth P. Scholtz, "Algorithms for Mapping Diatonic Keyboard Tunings and Temperaments," *Music Theory Online* 4.4, http://www.mtosmt.org/issues/mto.98.4.4/mto.98.4.4.scholtz_frames.html

Interval	Notes in Interval	Ratio of Frequencies in Equal Temperament	Ratio of Frequencies in Just Temperament
perfect unison	C	261.63/261.63 = 1.000	1/1 = 1.000
minor second	C C#	277.18/261.63 ≈ 1.059	16/15 ≈ 1.067
major second	C D	293.66/261.63 ≈ 1.122	9/8 = 1.125
minor third	C D E_	311.13/261.63 ≈ 1.189	6/5 = 1.200
major third	C D E	329.63/261.63 ≈ 1.260	5/4 ≈ 1.250
perfect fourth	C D E F	349.23/261.63 ≈ 1.335	4/3 ≈ 1.333
augmented fourth	C D E F#	369.99/261.63 ≈ 1.414	7/5 = 1.400
perfect fifth	C D E F G	392.00/261.63 ≈ 1.498	3/2 = 1.500
minor sixth	C D E F G A♭	415.30/261.63 ≈ 1.587	8/5 = 1.600
major sixth	C D E F G A	440.00/261.63 ≈ 1.682	5/3 ≈ 1.667
minor seventh	C D E F G A B♭	466.16/261.63 ≈ 1.782	7/4 = 1.750
major seventh	C D E F G A B	493.88/261.63 ≈ 1.888	15/8 = 1.875
perfect octave	C C	523.26/261.63 = 2.000	2/1 = 2.000

Table 3.3 Differences between equal tempered and just intervals according to ratios[75]

Table 3.3 above, demonstrates the discrepancies between the ratios of frequencies between just intonation and equal temperament. These discrepancies are due to the fact that dividing the octave (2:1 ratios) into twelve half-steps as in equal temperament makes the half-step a $\sqrt[12]{2}$:1 ratio which causes a necessary mismatch in all the intervals between the two systems other than in the case of octaves and unisons.

Differences in tuning, in accordance with just and equal temperament, are important considerations for the string player. As evidence from psychoacoustics suggests, the basilar membrane (responsible for registering frequency) naturally functions according the physical properties of a vibrating elastic membrane, demonstrating similar properties to the physical

[75] "Equal Tempered vs. Just Tempered Intervals," *Digital Sound & Music* http://digitalsoundandmusic.com/3-3-2-equal-tempered-vs-just-tempered-intervals/. Accessed July 2, 2019.

response of a vibrating string. More research is necessary to prove that the ear musically "prefers" a just intonation system. However, we do understand that the natural responses of our instruments demonstrate this preference. Although members of the string family function most naturally within a just tuning system, equal temperament is the standard method of tuning for most Western classical music. Therefore, it is important to be aware of these differences when playing with equal tempered instruments. Even though most string players can make quick adjustments by ear, clearer understanding of the discrepancies between tuning systems is useful for achieving appropriate intonation in both settings.

CHAPTER 4

PEDAGOGICAL APPLICATIONS

Currently there are a wide array of successful teaching methods available to the string player based upon centuries of pedagogy. However, these methods could be greatly complemented with further attention to string tone acoustics. Technological advances and spectral analysis have provided musicians access to descriptive and visual evaluations of string tone. This access allows for more predictable and definitive results in pedagogical settings.

Also, much of the current literature for violists involves more timbral concerns. The viola spent many of its early years as a harmony instrument in the orchestra due to its understated, subdued timbre. And although many earlier composers took an interest in the peculiar timbre of the instrument, solo literature written for the viola was relatively sparse until the early 20th century, when composers were already taking an interest in new approaches to sound and harmony. Hence, much of the post-20th-century repertoire for violists uses extended techniques and alternatives to the major and minor modes and the well-established equal temperament system. Awareness of acoustics for string players, violists in particular, can only lead to better execution of modern techniques.

Furthermore, an approach to string tone in accordance with the physical nature of the vibrating string sheds light on the glaring differences between just temperament and the equal temperament system. The mechanism of the inner ear also functions according to frequency preferences in specific ranges and amplitudes that seem to imply partiality towards whole number ratios and proportionality. Due to concerns with the equal temperament system, many modern composers such as Harry Partch, Ben Johnston have revisited alternative tunings and temperaments. And although there is a strong rationale in classical music and beyond for the use of an equal temperament system, as performers, we have become so comfortable in the system that we often overlook the way in which it does not fully align with the resonant responses of the vibrating string and body of the instrument. In examining the differences in these systems, we can achieve a more informed approach to intonation, be it in solo performance practice, string chamber ensembles, or playing with equal tempered instruments.

In this chapter I will be giving examples of how precise data pertaining to acoustics and psychoacoustics can be relevant to instructing viola students on developing various tone qualities. I will begin with some established opinions on basic principles of tone production from violin pedagogue Simon Fischer and physicists William Atwood and James Beament. I will give examples of how changes in the physical parameters can cause marked changes in tone quality and provide tactics for more efficient maneuvering within these parameters. Reproductions of spectral representations will demonstrate the results. I will also advocate the use of listening exercises devoted to overtones and combination tones as an effective means for helping students to improve upon just intonation and listening practices. The end of the chapter includes written exercises for practicing combination tones.

Fundamentals of Bowing Technique

There are four main components of bowing technique that most string players know well. Physicist William Atwood describes these variables as “the speed of the bow, the downward pressure on the string, and the point of contact on the string with the bow hair. (To a lesser extent the amount of hair which is in contact with the string also plays a role.)"[76] In pedagogy, the terms weight and pressure are interchangeable. In experimenting within these four parameters, pressure, speed, point of contact (placement of the bow corresponding to the bridge), and to a lesser degree, the angle of the wood of the bow (affecting the amount of hair distributed on the string), we find many possibilities for different sounds achievable on a stringed instrument.

Pedagogue Simon Fischer's DVD set, entitled "The Secrets of Tone Production," defines these four parameters as the "language of proportions." The DVD set covers the language of proportions (the four parameters) as well as five sounding points.[77] The five sounding points that Fischer describes are specifically related to the bow's proximity to the bridge and act as a subset of what is commonly referred to in pedagogical settings as the "point of contact," one of the four parameters of bowing technique.

Placement of the bow in relationship to the bridge can lead to many changes in tone production. Within Simon Fischer's "five sounding points," (assuming bow direction parallel to the bridge) sound point five is farthest from the bridge, just over the fingerboard (as for *sul tasto* bowing) and sound point one is closest to the bridge (as for *sul ponticello* bowing). Sound point three, *normale,* is in the middle between the fingerboard and bridge, and so on. The closer the bow to the bridge, the more weight and the less speed are desired for clarity of the fundamental, the lowest frequency. As the bow gets farther from the bridge, less weight and more speed are desired for clarity of the fundamental frequency. If we practice according to these principles, the

[76] William Atwood, "A Physicist in the World of Violins," *Beam Line* (Summer 1998): 27, http://www.slac.stanford.edu/pubs/beamline/28/2/28-2-atwood.pdf . Accessed July 2, 2019. I tend to avoid using the word "pressure" in my teaching, as it could subtlety implies tension, preferring the term "weight" instead. Pressure would be the correct term in physics, acting as the force on an object that is spread over a certain area.

[77] Simon Fischer, *The Secrets of Tone Production on All Bowed String Instruments.* [London]: Edition Peters, 2010, DVD. His tutorial is derived from many famous pedagogues of the past and includes masterclass type settings utilizing his techniques with all members of the string family. Although I personally find his work with the viola a bit limited, there are some really useful tips throughout that incorporate the science and complexity of string tone in a manner in which the student can understand.

fullest sound we can achieve is nearest to the bridge with heavy weight. In moving further away from the bridge, the sound becomes thinner, requiring less weight and faster bow speed.

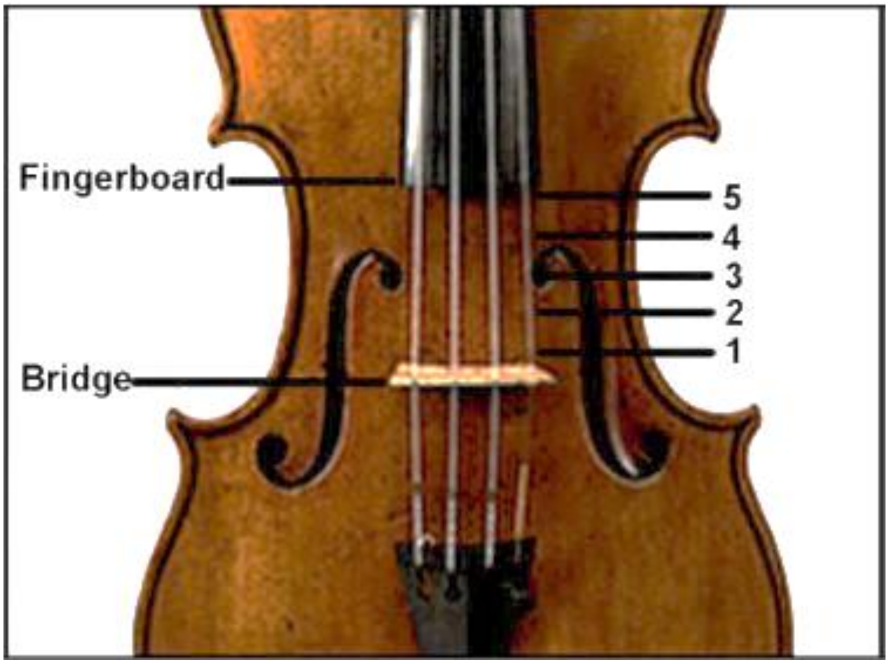

Figure 4.1: Image of the 5 sounding points on a violin[78]

When trying to achieve a full resonant tone, Simon Fischer suggests that the string player should also visually observe the amplitude of the vibration of the string, looking for the widest point of the string moving back and forth. To the naked eye, we see the string increase in its width as the tone reaches maximum amplitude. However, as we learned before with slip-stick motion, the string is not simply wagging back and forth, but also twisting. For best results, the instrument should ideally be as flat as possible (parallel with the floor) in order to allow the strings to better support the bow and gravity to aid with weight distribution. The varying

[78] Cheri Collins, "Contact Point Techniques," *Ovation Press: String Visions,* http://stringvisions.ovationpress.com/2011/07/contact-point-techniques/. Accessed July 2, 2019.

thicknesses of the strings are something to consider as well. This is especially important in viola tone, as the C string is very thick and requires much more weight to achieve a clear and rich tone. Fischer describes playing on the C string as pulling the bow "through thick mud."[79] Also, even the slightest change in bow speed can increase or decrease the amplitude of a vibration and the weight applied can affect the pitch of the sound. Much attention must be placed on these fluctuations of bow speed in sustained tones in order not to create what is commonly referred to in pedagogy as "banana bows," where an audible crescendo occurs within a sustained stroke. This fluctuation is also more sensitive in lower registers.

Understanding how sounding points affect the timbre of our tone can lead to optimization of basic techniques. Even the tuning of open strings can be informed with greater attention to sounding points. For tuning open strings, James Beament recommends bowing the string "lightly at the fingerboard because the timbre is less pronounced than that of a string bowed strongly at the bridge."[80] The desired effect is to come closer to that of a pure tone, so that timbral concerns do not distract us from tuning the fundamental pitch. This can be challenging when trying to tune with an orchestra, especially for the timbre of the viola, so tuning in quieter settings when possible is ideal.

On Transients

The beginning of a bow stroke requires careful attention for all levels of playing ability. In string tone acoustics, the beginning of a bow stroke is called the transient. String players often refer to the transient as the "attack," which precedes the sustained tone. Transients describe any momentary variation in frequency. The time required to set up a steady-state oscillation is called

[79] Ibid.
[80] Ibid., 125.

the transient time of the instrument. More information is encoded in the transient than in any aspect of sustained tone. Transients are largely responsible for our discernment of timbre and inform how the brain interprets a sound. Lower frequencies have longer transients, which is why the viola takes longer to speak. This requires the response time in relation to higher pitches to be faster, meaning violists need to play almost ahead of the beat to stay in time with instruments with a faster transient time.[81] Woodhouse explains that on "low frequency strings (such as the C string of a cello or any string of a double bass), almost any interruption to the regular stick-slip pattern of the 'perfect transient' is likely to be audible."[82] Therefore, when playing in lower registers, we need to be extra careful in order not to create unwanted disturbances in the force. For initiating a low dynamic sound, James Beament recommends starting at the tip or the frog where the hair is slightly less flexible, so that the initial impulse can still be achieved.[83]

Simon Fischer explains one way of practicing the initial attack called the "catch/pull" sequence. I have heard also heard it referred to as listening for *consonant* versus *vowel* sounds at the transient. To experiment with this, you can try to achieve "clicks" with the bow, grabbing the string with the bow due to downward pressure (weight) and the friction of the rosin. This exercise is useful for improving right hand and arm dexterity. In experimenting with these clicks, I found that resonant pitches result and change according to the distance of the bow from the bridge. The closer to the bridge, the lower the resonant click, almost as if you are playing a fingered note. The result is the same as you get further from the bridge: the frequency is increased.

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[81] Knut Guettler, "Onset Transient Times" unpublished commentarial article on transients from his website, http://knutsacoustics.com/files/Onset-transient-times-Rev.2.pdf. Accessed July 2, 2019.
[82] Woodhouse and Galluzzo, "The Bowed String As We Know It Today": 586, http://www2.eng.cam.ac.uk/~jw12/JW%20PDFs/BowedStringReview.pdf. Accessed July 2, 2019.
[83] James Beament, *The Violin Explained*, 24.

Another useful method of practicing transients is employed in the Suzuki method.[84] Suzuki felt that tone was of utmost importance in the study of the violin and incorporated the method of practicing "bow circles" in his teaching. Bow circles are incredibly helpful in practicing transients because the circle requires motion and momentum, clarifying, through practice, how much weight and speed are necessary to initiate the sound.

On Listening for Overtones

Simon Fischer suggests that projection is not necessarily determined by loudness, but by the perceived "quality" of the sound. Here Fischer is referring to the overtones and their relative amplitudes present in the sound. Therefore "quality" of tone is derived from the varying levels of the various overtones present in a sound. Paul Hindemith explains that "the ear hardly hears [overtones] separately; it only perceives the disappearance of some or the addition of others as changes in tone-color."[85] Listening for overtones can be particularly challenging but is not impossible for a student to achieve. On hearing upper partials, Helmholtz says, "Success depends rather on a peculiar power of mental abstraction or *a peculiar mastery over attention,* than on musical training. But a musically trained observer has an essential advantage over one not so trained in his power of figuring to himself how the simple tones sought for ought to sound."[86] Knowing what to listen for is the most efficient strategy for hearing overtones in our playing.

The success of the Suzuki method for violin is due largely to this way of learning music. Shinichi Suzuki revolutionized violin pedagogy with his idea of vocalization and the "mother

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[84] Suzuki Violin School. Suzuki is said to have students practice 10,000 bow cycles (circles) in a week.
See also: Violist Kim Kashkashian gives a nice demonstration on bow circles in a masterclass here: "Kim Kashkashian Teaches Circular Bow Motion," *YouTube*. https://www.youtube.com/watch?v=dKjhCv5Iew0
For further reading, see William Starr, *The Suzuki Violinist: A Guide for Teachers and Parents* (USA: Alfred Publishing Co. and Summy-Birchard Inc., 1976, 2000), 95, 142.
[85] Hindemith, *The Craft of Musical Composition*, 17.
[86] Helmholtz, *Sensations of Tone as a Physiological Basis for the Theory of Music*, 49.

tongue," likening musical practice to that of learning a language with repetitive imitation.[87] The Suzuki method comes with a CD of the songs included in the book and students are expected to make listening to the CD a large part of their practice routine. That way, students are already familiar with how the songs sound before they learn them. The method has been criticized over the years in part because of this practice, causing some students to be unable to proficiently read the notes and instead rely heavily on the ear. However, the vital success of the method proves that there is much to be gained from trying to replicate sounds that are already aurally available to us. If we know what tones we are trying to achieve, our chances of achieving them are much greater. This is a large part of why learning new music is so challenging. There may not be an existing recording to refer to in order to hasten the learning process, and simultaneously, the harmonic and melodic content may be less familiar.

In listening for overtones, I would first recommend listening to the intervals of the harmonic series, so you know what to expect when trying to recreate and listen for them on your instrument. On account of the flexibility of the bridge and the type of strings in use, harmonics will not behave as an idealized string would behave. For example, an octave difference is not exactly halfway along the string, but slightly closer to the bridge.[88]

Below are some listening examples of the overtone series on all open strings of a viola. The examples were recorded using a 1999 Bronek Cison viola with Evah Pirazzi strings. The open C and A strings were the most difficult to produce on this instrument: therefore, there are only 11 subsequent overtones for those recordings, the D and G include 13.

[87] Starr, *The Suzuki Violinist,* 1-6.
[88] Typically, the harmonic series can "sound out of tune" due to these variances and our familiarity with equal temperament.

1. Overtone Series Open C: https://www.youtube.com/watch?v=7HwfTNbkiIM
2. Overtone Series Open G: https://www.youtube.com/watch?v=j1tNyRnjvUo
3. Overtone Series Open D: https://www.youtube.com/watch?v=U9OmnMZYHJI
4. Overtone Series Open A: https://www.youtube.com/watch?v=F9dGKP8EI_g

Getting more aurally accustomed to hearing the series will begin to train the ear on what to listen for. Next, we must open our minds to hearing these higher tones on our instruments. Beament explains, "One mystery about our hearing is why we ascribe the pitch of a musical sound to the slowest rate of vibration present, regardless of the many higher rates of vibration which are also there, and… it has led people to attach far too much importance to the fundamental in the sensations we hear."[89] Listening to the overtone series pattern can draw our awareness to the just ratios present even when we are playing sustained tones on our instruments. And as we begin to draw our aural awareness away from the fundamental frequency of a note, we can notice how many more sounds are occurring.

Below in Figure 4.2 is a notated version of the harmonic series on the open C string, as performed by violist Anne Lanzilotti.[90] Notice the pitches that strike us as "out of tune." From the diagram below, we can see the deviation (in cents) from the equal tempered system.

[89] James Beament, *The Violin Explained*, 9.
[90] Anne Leilehua Lanzilotti, "Harmonics, Waveforms, and the Overtone Series," February 12, 2016, http://annelanzilotti.com/blog/2016/2/11/on-harmonics-waveforms-the-overtone-series. Accessed July 2, 2019.

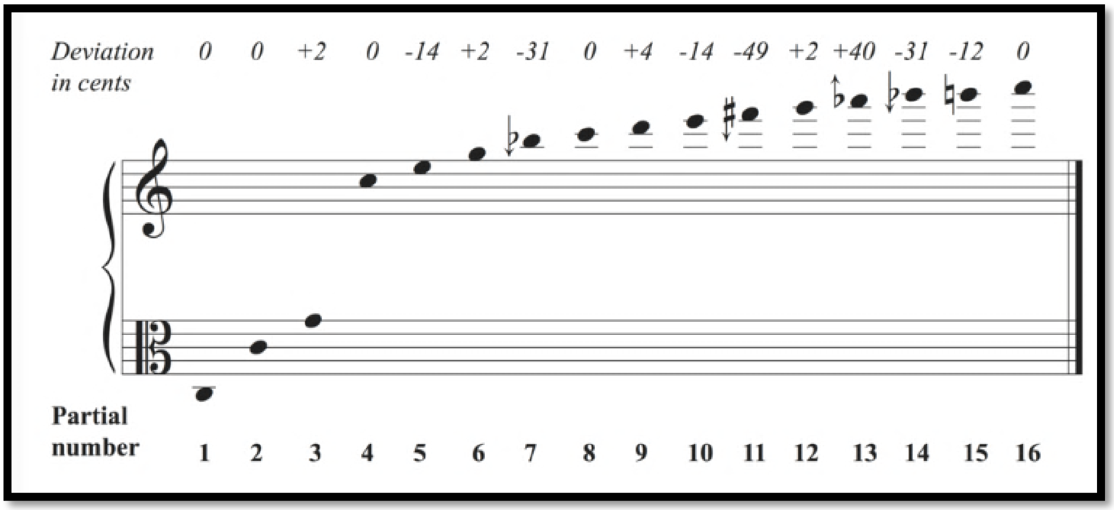

Figure 4.2 Overtone Series on open C. Partials 1-16 (arrows indicate quarter tone alterations which occur naturally in the harmonic series)

Once aural awareness of the overtone series is achieved, it becomes much easier to listen for them on our instruments. A good exercise to start experimenting with hearing them is to use stopped staccato bow strokes on open strings. Utilizing the catch/pull sequence discussed earlier, it is quite easy to hear the ringing overtones after stopping the bow.

Again, issues of semantics and descriptive words for defining tone can lead to confusion. The adjectives used to describe tone vary greatly from teacher to teacher and can be challenging to dissect. James Beament describes some general attributes to string tone, as “thinness from few harmonics, clarity from weak higher harmonics, and richness from the balance between higher and lower harmonics.”[91] What Beament is suggesting here is that “thinness” is attributed to the presence of just a few overtones in the sound, presumably creating a weaker, less present sound. “Clarity” seems to imply a strong presence of the fundamental frequency. And “richness” includes a good balance of both high and low overtones present in the sound. Descriptive

[91] Beament, *The Violin Explained*, 133.

language in teaching practices could be greatly enhanced with a sense of what specific overtones we are trying to emphasize in our playing to achieve these different "qualities" in tone production.

Three Useful Bowing Techniques

There are many different bowing techniques that can be further described and explained in these terms. Much modern music requires use of different timbral elements that are manipulated by tweaking traditional bowing techniques. Some of the techniques are not easily described by way of a musical score, but if we understand specific guidelines for achieving different timbres, we can communicate with composers more effectively and have access to a greater range of sounds.

To begin discussion of this topic, I will start with two fairly basic bowing techniques that were developed centuries ago but continue to be incorporated in new music: *sul tasto* and *sul ponticello*. Simon Fischer explains that upper partials are less audible away from the bridge and dissipate more quickly than the lower ones. Therefore, a *sul tasto* (over the fingerboard) bowing technique has a reduced number of upper partials and more lower ones, hence the softer and slightly muted quality of the sound. *Sul ponticello* (over the bridge), on the other hand, emphasizes a larger spectrum of higher harmonics. James Beament confirms these claims by saying that "the distribution and number of enhanced and strong harmonics are such that stronger bowing at 1/10 string length or nearer the bridge makes the sound richer but also increases the roughness. Bowed towards the fingerboard, the string produces weaker higher harmonics, and the balance is tilted towards the lower ones."[92] The quality of sound we experience with *sul*

[92] Beament, *The Violin Explained*, 116-117.

ponticello bowing can be partially explained by "roughness." Roughness, generally speaking, is a subjective impression of any sound fluctuation that cannot be tracked by the ear and further interpreted by the brain due to the rapid rate of the change. Roughness in this case could be considered as the overlap of the higher harmonics combined with the friction of the bow, causing the cognitive interpretation of numerous inharmonic elements to present themselves as noise. Also, bowing closer to the bridge is difficult to control due to the proximity of the fixed point and requires much right arm and hand dexterity to maintain a consistent tone. Experiments with bow pressure when playing close to the bridge can lead to some very interesting changes in timbre. Lightening up the weight of the bow can draw out specific higher harmonics, whereas heavier bow weight causes more sensation of roughness. Using this information to guide execution of these bowing techniques can allow for composers to be more specific with their requests and allow performers to feel more comfortable with the execution.

Below is a spectral representation of *sul tasto* bowing and *sul ponticello* bowing, in Figures 4.3 and 4.4. Here we see more lower harmonics present in the *sul tasto* bowing, whereas *sul ponticello* includes more "roughness" and an overall greater number of upper partials.

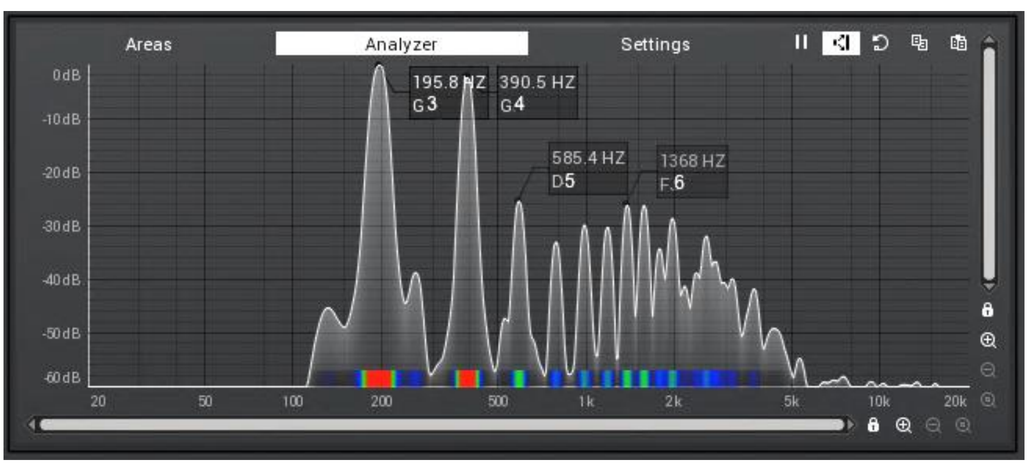

Figure 4.3: *Sul tasto* bowing on the G string (viola)

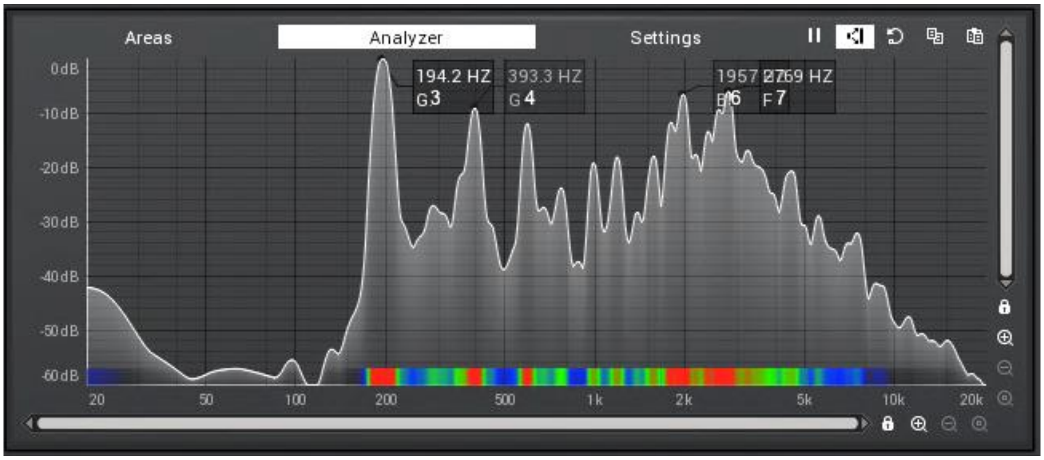

Figure 4.4 *Sul ponticello* bowing on the G string (viola)

Another interesting bowing technique that is emerging in modern music is the “flattening effect.” String players have the ability to subtly, and not so subtly, manipulate pitch with bow weight (pressure). When applying too much weight, the frequency is lowered, and with too little bow weight, the frequency is raised. If bow pressure exceeds maximum capacity, less subtle sounds emerge. When bowing with extreme weight and consistent speed, we experience different pitches that are flatter than the open string. These sounds are referred to as *anomalous low frequencies* (AFL) in physics, but as musicians we frequently hear them described as “subharmonics.” Modern violinist and composer, Mari Kimura, is known for utilizing AFLs in her compositions and has standardized the term “subharmonic.”[93] The term subharmonic is misleading because, as you can see from the spectral analysis below, in Figures 4.5- 4.8, although some actual subharmonics are present (left of the fundamental frequency), the fundamental frequency is most prominent followed by a series of partials.

[93] Mari Kimura playing her piece entitled “Gemini Subharmonics.” “Mari Kimura - Gemini Subharmonics,” *YouTube*. https://youtu.be/oPTt5u681so
See also: Larry Greenemeier, “String Theory: Violinist Taps Artificial Intelligence to Interact with Her Unique Sound [Video],” *Scientific American* (May 31, 2011) https://www.scientificamerican.com/article/kimura-augmented-violin-subharmonics/

Knut Guettler explains the science behind ALFs, regarding slip-stick motion, as when "the rotating corner fails to trigger a string release when hitting the bow on its way toward the bridge. This gives rise to two reflected waves, one torsional and one transversal, both capable of triggering a delayed release when passing the bow after one or more additional nut reflections."[94] Below are spectral representations of some achievable anomalous low frequencies on the open strings of the viola including links to recordings of them.

Even highly skilled performers demonstrate that sustained string tone always includes some amount of fluctuation in its constituent overtones. This fluctuation is controlled and manipulated by the parameters previously discussed.

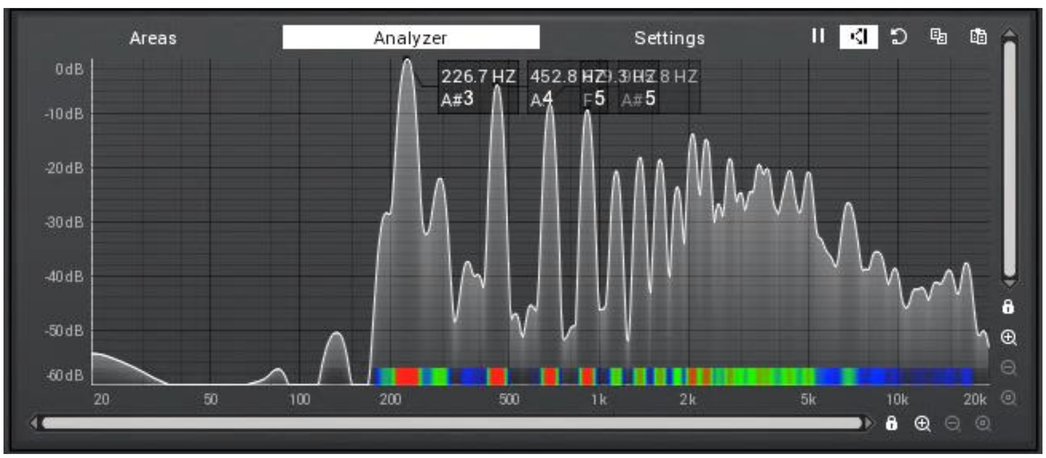

Figure 4.5 Anomalous low frequency ("subharmonic") on open A (viola)

Listen: https://www.youtube.com/watch?v=K3qap4wgSZE

[94] Knut Guettler, "Bows, String, and Bowing," in *The Science of String Instruments*, ed. Thomas D. Rossing (New York: Springer, 2010), 290. https://logosfoundation.org/kursus/The%20Science%20of%20String%20Instruments.pdf

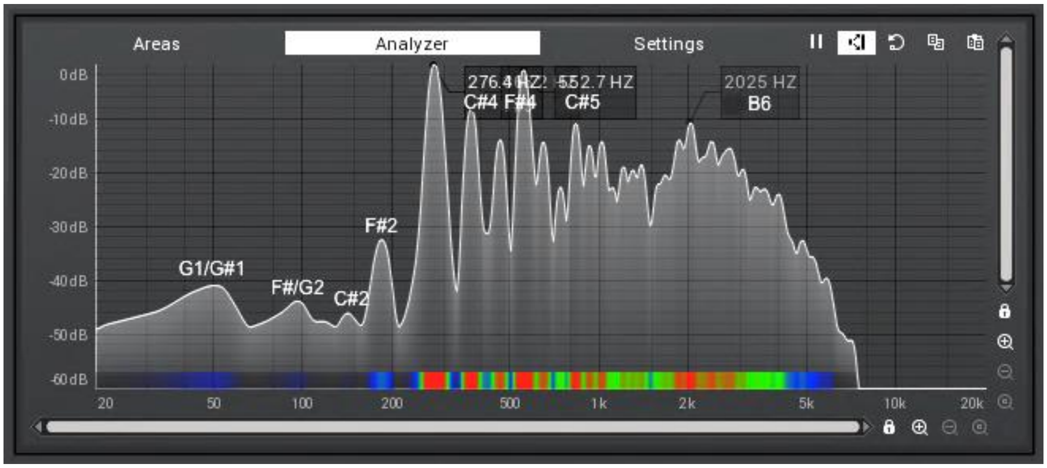

Figure 4.6 Anomalous low frequency (“subharmonic”) on open D (viola)

Listen: https://www.youtube.com/watch?v=rLYBFZP5FF4

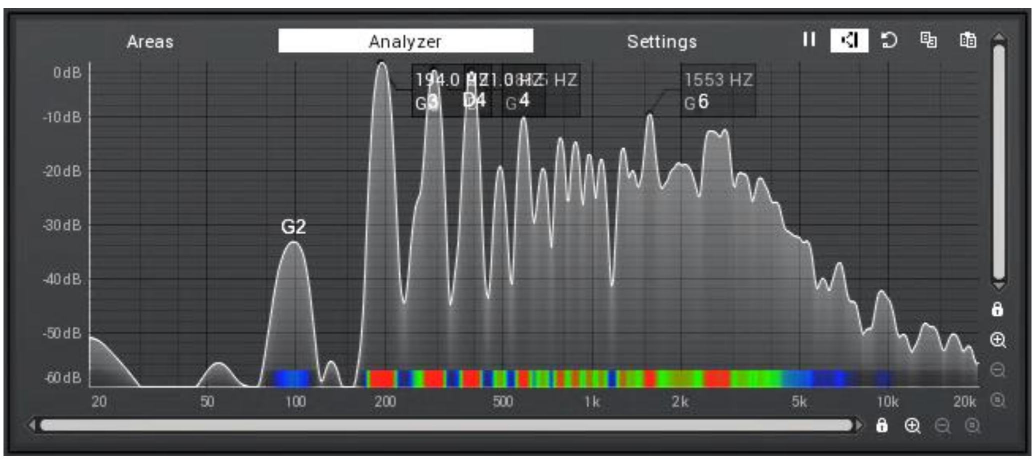

Figure 4.7 Anomalous low frequency (“subharmonic”) on open G (viola)

Listen: https://www.youtube.com/watch?v=AJGXqgL8mo4

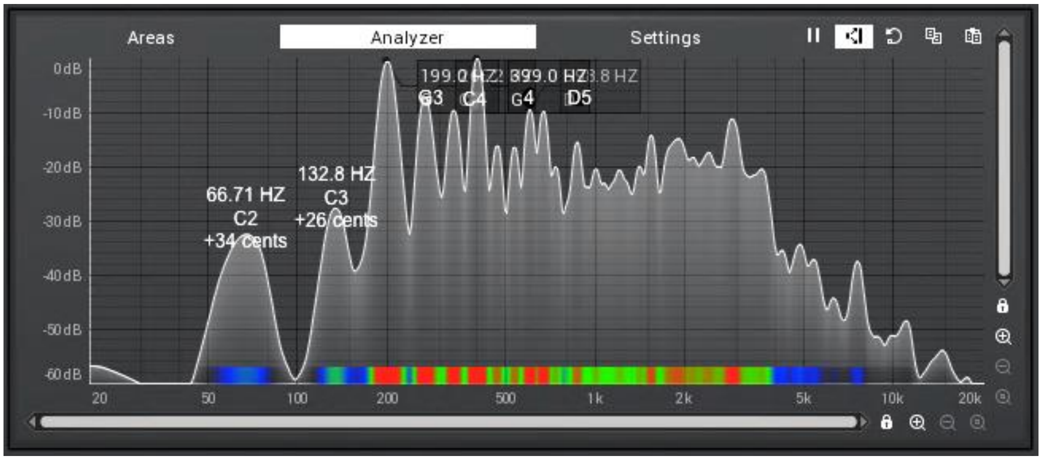

Figure 4.8 Anomalous low frequency (“subharmonic”) on open C (viola)

Listen: https://www.youtube.com/watch?v=_G24VXq4zjI

Fingered Notes

Once a student has begun to understand some basic principles of tone production on open strings, the information can be further applied to fingered notes. Fingered notes on stringed instruments shorten the length of the string, changing the properties of the string’s vibrations. Thus, it is imperative to adjust the placement of the bow closer to the bridge, in relation to the amount the string is being shortened, to maintain the tone quality. As the string length is shortened, the tension of the string is increased making it necessary to also adjust the speed and weight of the bow to maintain tone. As we move into different positions on our instruments, again shortening the length of the string, responses vary from instrument to instrument. Below are some spectral images demonstrating the change in the prominence of the fundamental as we get into the higher positions. See Figures 4.9- 4.14.

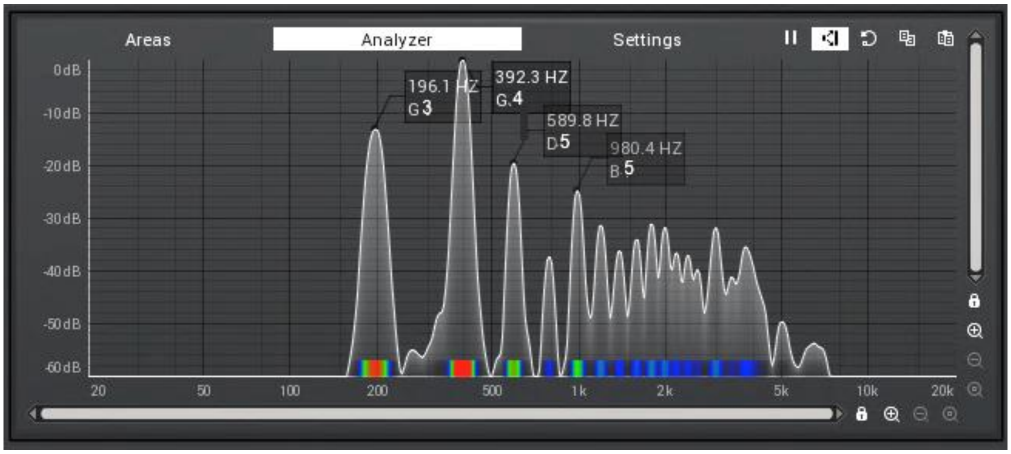

Figure 4.9 Fingered G3 on the C string (viola)

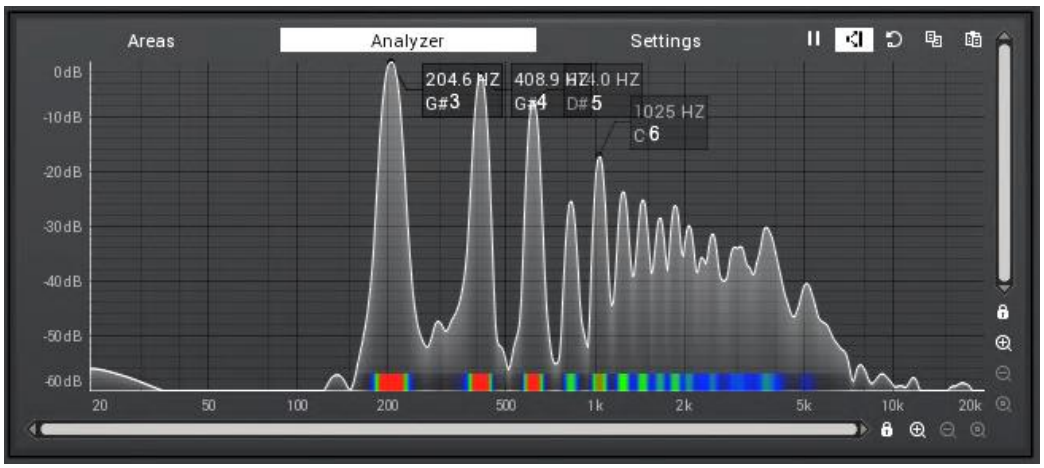

Figure 4.10 Fingered G#3 on the C string (viola)

Due to the resonances of these particular instruments, changes in the prominence of the fundamental occur on different notes at different distances between the nut and the bridge. For the C string viola examples above (Figures 4.9 and 4.10), notice how the fundamental peaks once we arrive at G#3, one half step above a normalized fourth position.

In the violin G string examples below, the change in the prominence of the fundamental occurs between fingered notes B3 and C4, or the normalized third position. See Figures 4.11 and 4.12 below.

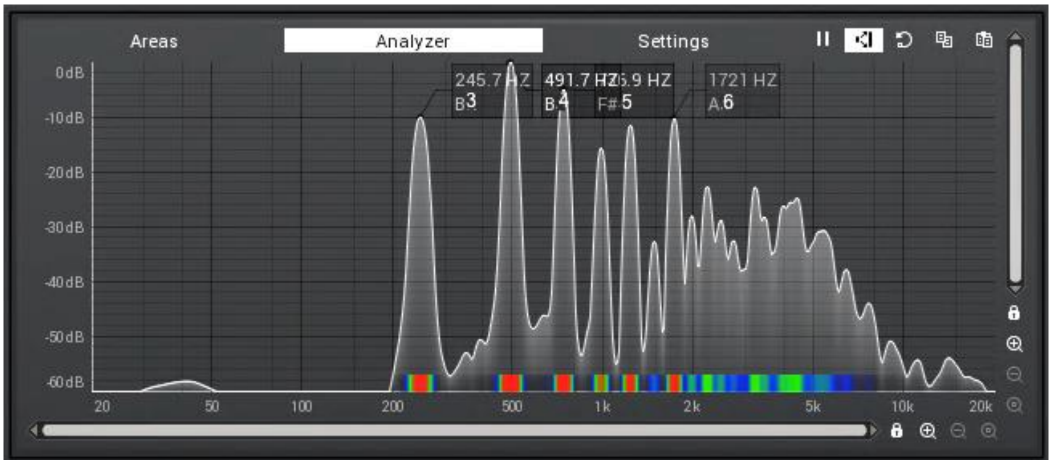

Figure 4.11 Fingered B3 on the G string (violin)

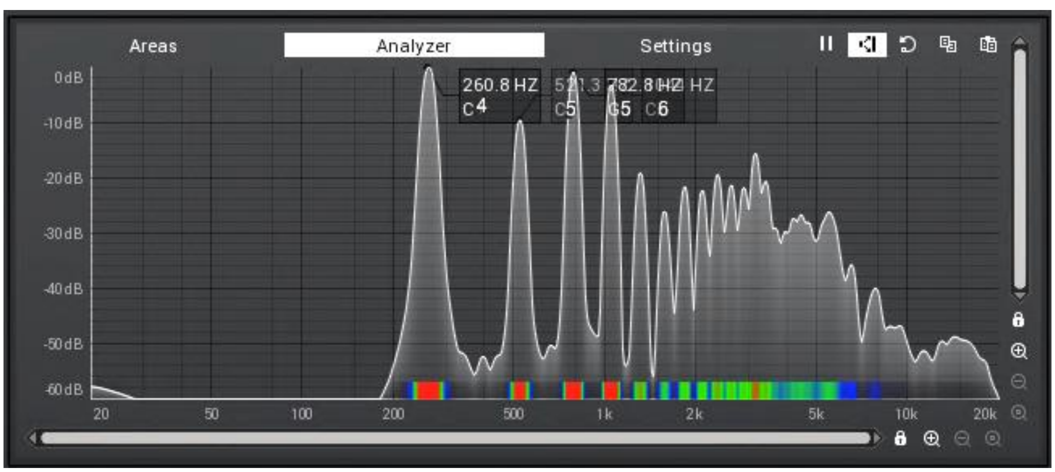

Figure 4.12 Fingered C4 on the G string (violin)

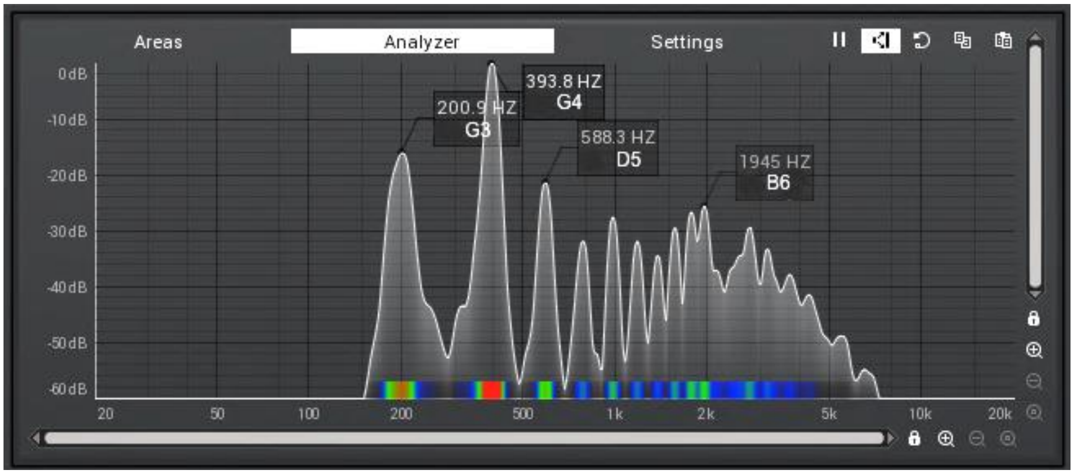

Figure 4.13 Open G3 (viola)

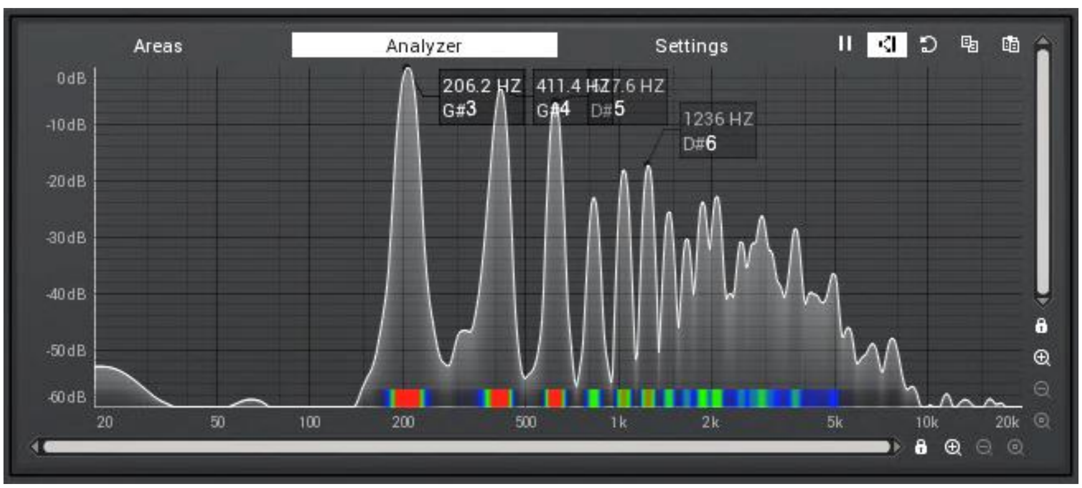

Figure 4.14 Fingered G#3 on the G string (viola)

For the viola G string, a strong fundamental occurs already at fingered note G#3, in normalized half position. The quality of the tone changes greatly due to where pitches are played on our instruments. This is an important detail to consider when trying to create different tone colors utilizing the same pitches played on different strings. Part of the reason for tone quality changes with fingered notes is due to sympathetic resonances.

Sympathetic Resonances

Sympathetic resonances are frequently referred to as "ringing tones" in pedagogy and can be achieved by watching and even touching open strings that are harmonically related to the fingered note. When the note is justly in tune with the open strings, the related open string will vibrate in response to the frequency of the note being played. This phenomenon is due to resonance and resonant frequencies. A resonant frequency is the natural vibrating frequency of an object. All objects have resonant frequencies. When you match a resonant frequency of an object and increase the amplitude, it forces the object into vibrational motion.[95] Sympathetic resonances can be used to inform fingering choices and are also an excellent way to improve just intonation practices and utilize the natural resonance responses of our instruments.

In order to be able to listen for the sympathetic resonances of the open strings, Simon Fischer recommends practicing his bowing technique exercises with a placed third finger in first position.[96] Third finger in first position is an octave above the lower open string. More specifically, the lower adjacent string resonates sympathetically with the third finger because the fingered note is harmonically related to the second harmonic partial (first overtone) of the open string. That is why there are many achievable ringing tones on a string instrument. They are related harmonically within the overtone series. Another example of this is the fingered 'E' on the A string. When the pitch is justly in tune, it will vibrate the open C string, due to its harmonic relationship to the fifth partial of the open C string within the overtone series. Listening for sympathetic resonances on stringed instruments can greatly help with intonation and tone production.

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[95] One way of considering resonant frequencies is the classic wine glass experiment (which, by the way, actually works). You can watch a YouTube video of a young boy demonstrating here- "Boy Breaks Wine Glass with Voice," *YouTube.* https://www.youtube.com/watch?v=sH7XSX10QkM).

[96] Fischer, *The Secrets of Tone Production on All Bowed String Instruments.*

To practice sympathetic resonances on your instrument, first look closely at the strings to see the vibrations. Once you get the sympathetic string to vibrate, you can further observe the vibration by lightly tapping with another finger on the sympathetically vibrating string, alternately stopping the vibration with your finger and then allowing it to vibrate again. This is useful for younger students that respond well to kinesthetic learning practices. The practice also functions well with aural learners, as the dampening of the sympathetic vibration is audible.

Utilizing Combination Tones to Inform Intonation

Combination tones are another way of informing just intonation listening practices for string players. Hindemith describes them as “double stops in pure intonation.”[97] As previously discussed in Chapter Two, combination tones are a rather complex phenomena with their basis in the realm of psychoacoustics and due to nonlinear distortions within the inner ear. Giuseppe Tartini (1692-1770), famous violinist and composer, is well known for using combination tones ("terzo suono" or third sound) to inform intonation practices. In 1754, he published his discoveries in a treatise entitled "Trattato di musica secondo la vera scienza dell'armonia," (*Treatise of Music According to the True Science of Harmony*). Although he was not the first to be aware of them, Tartini is credited with the discovery of them. Tartini used specifically the difference tone for informing violin intonation.

Following Tartini, the topic was largely overlooked until the time of Paul Hindemith, a renowned violist as well as a composer. Hindemith was aware of many aspects of string tone acoustics. He created an entire style of composition and scales based around some of these physical laws. He even writes extensively in his book, *The Craft of Musical Composition* (first

[97] Hindemith, *The Craft of Musical Composition*, 59.

published in 1937), about combination tones (categorized in his *Series 2*) and utilizes them in his compositions.[98] Hindemith was also inspired by the overtone series (*Series 1*) and discusses ratios and proportions in his writing. As violists, awareness of his intentions can help us approach some of the difficulties of his music with more ease.

Hindemith used combination tones to inform aspects of the harmonies he utilized in his compositions. He explains that "[combination tones] are usually so weak that the superficial ear does not perceive them, but this makes them all the more important for the subconscious ear. They are the third point of a triangle whose other two points are in the sounding interval, making possible for the ear a sort of trigonometry by which it is enabled to form a judgement of the purity of an interval."[99] He explains them further by dividing them into two main categories, first and second order combination tones. First order combination tones (he is referencing specifically difference tones) are created by subtracting the lower pitch from the higher pitch. They are significantly louder than second order difference tones. Hindemith says, "If a combination tone consists of the difference between the proportion numbers (or frequency numbers) of two tones, then by the same process we have already used we can easily find the combination tones of the second order."[100] By this process we could then create a series that is "theoretically infinite," though not necessarily audible.[101] He likens Series 2 to his first Series 1 (based on overtones), as both are potentially infinite, but only perceptible to a certain point.

Combination tones are extremely useful in guiding loud, higher pitched double stop intonation. Instead of referencing both tones simultaneously, the player can direct their focus to the tuning of one pitch, the difference tone. Once pure intonation is achieved in higher registers,

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[98] Ibid. Series 1 is his compositional exploration of tonal relations derived from the overtone series. Series 2 is his compositional exploration of intervallic relations derived from combination tones.
[99] Ibid. 58.
[100] Ibid. 61-62.
[101] Ibid. 63.

where the combination tones are most audible, the intonation of related intervals in different registers can be informed by their relative correspondence to the higher octaves.

To optimize achievement of these tones, fast bow speed, more bow pressure, and a closer proximity to the bridge must be employed. Once the tones are clearly established, the string player can experiment within the bowing parameters to discover the threshold of audibility within different dynamic levels on their individual instruments.[102]

Exercises and Listening Examples for Combination Tones

Again, to eliminate the difficulty of knowing what to listen for, I am providing some exercises that include the first order combination tones achievable with major and minor thirds, major and minor sixths, and perfect fourths, fifths, and tritones. Also included are some listening examples to accompany the exercises for combination tones.[103] The recordings work best over headphones or with good speakers. The intervals of seconds and sevenths are not included in the exercises below because, due to pitch placement along the basilar membrane of the inner ear, interference and distortion lead to greater variances in pitch discrimination. Each of the following listening exercises are organized according to intervals.

[102] Please keep in mind that the ability to perceive these tones differs greatly from person to person.

[103] The provided listening examples do not include the entire exercise but provide a clearly audible sample for listening to the bass line created by the difference tones themselves. Due to variance within the just system, transience, the components of the instrument in use, the physiology and psychology of the player and listener, equal temperament training, audio equipment, and the acoustical space, etc., some of the resulting pitches do not quite match what sounds “in tune.” They also fluctuate rapidly and require steady bow speed and pressure to achieve the best results.

1. **Major and Minor Thirds**. Example starting from the second system, chords **7-14** https://soundcloud.com/maria-kindt-299629075/combination-tones-major-and-minor-thirds

2. **Major and Minor Sixths**. Example starting from the first system, chords **5-11** https://soundcloud.com/maria-kindt-299629075/combination-tones-major-and-minor-sixths

3. **Perfect Fourths, Fifths, and Tritones**. Example starting from the second system, chords **8-16** https://soundcloud.com/maria-kindt-299629075/combination-tones-4ths-tt-5ths

The printable PDF versions of the exercises (for viola, violin, and cello) are available below.

Viola- Exercise 1: Major and Minor Thirds

Exercise 2: Major and Minor Sixths

Exercise 3: Perfect Fourths, Fifths, and Tritones

Violin- Exercise 1: Major and Minor Thirds

Exercise 2: Major and Minor Sixths

Exercise 3: Perfect Fourths, Fifths, and Tritones

Cello- Exercise 1: Major and Minor Thirds

Exercise 2: Major and Minor Sixths

Exercise 3: Perfect Fourths, Fifths, and Tritones

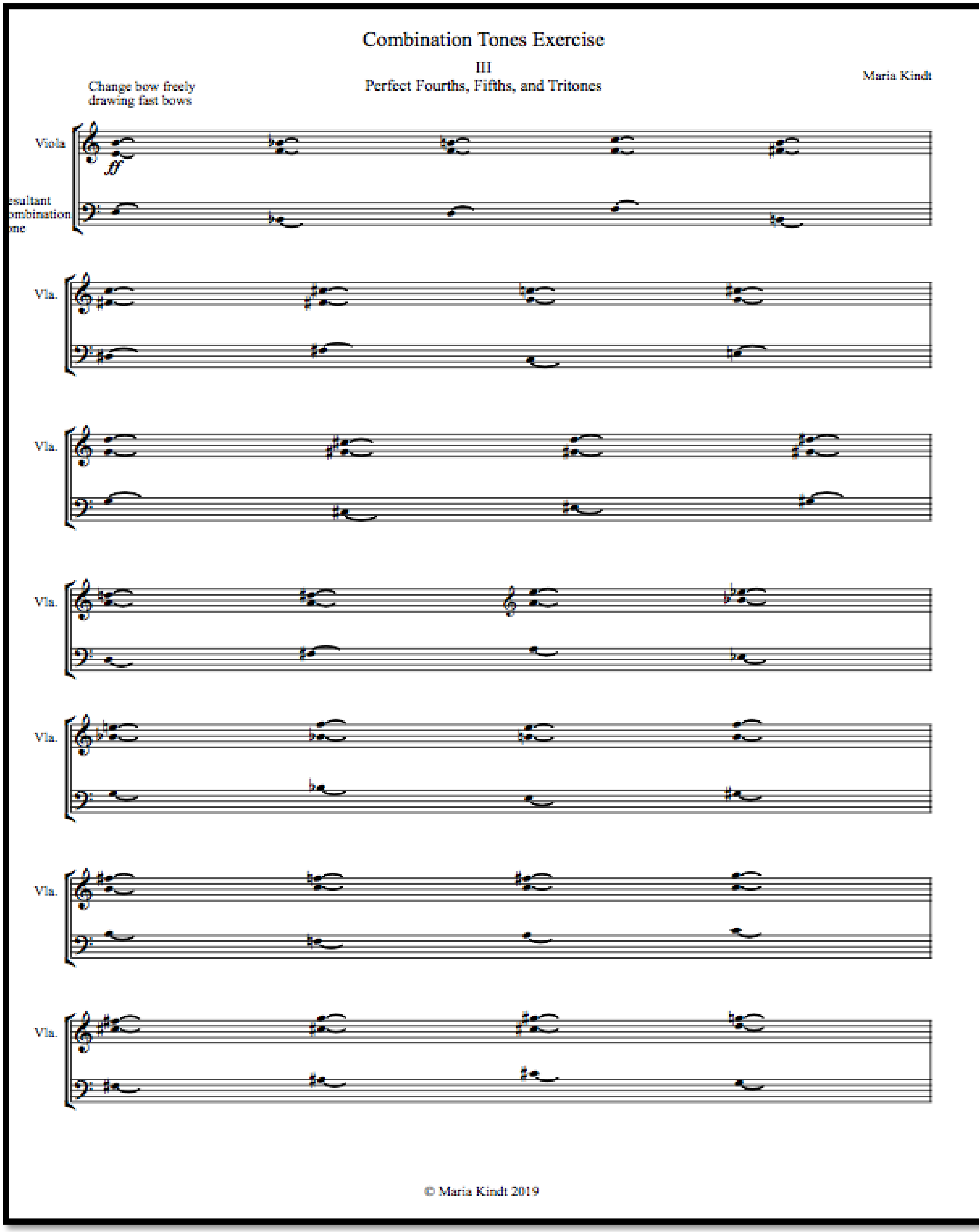
In experimenting with these exercises, I recommend pulling fast, heavy bows closer to the bridge. The tones are most audible in higher positions at louder volumes. When playing the chords in succession, listen for the bass line that is created by the double stop. This is the first order difference tone notated in the bass line of the exercise. Bows can be drawn freely without a specific rhythm or tempo as the main goal of the exercise is to be able to hear the combination tones. Do not be alarmed if you hear more than one combination tone at the same time. Those are second and third order combinations tones and are very apparent with the intervals of the minor sixth and the tritone. Also, please note that the bass line in Exercise II is written in alto clef to avoid ledger lines.

Combination Tones Exercise

I
Major and Minor Thirds

Maria Kindt


Combination Tones Exercise
II
Major and Minor Sixths
Maria Kindt
Change bow freely
drawing fast bows
Viola
ff
Resultant
Combination
Tone
Vla.
Vla.
Vla.
Vla.
Vla.
© Maria Kindt 2019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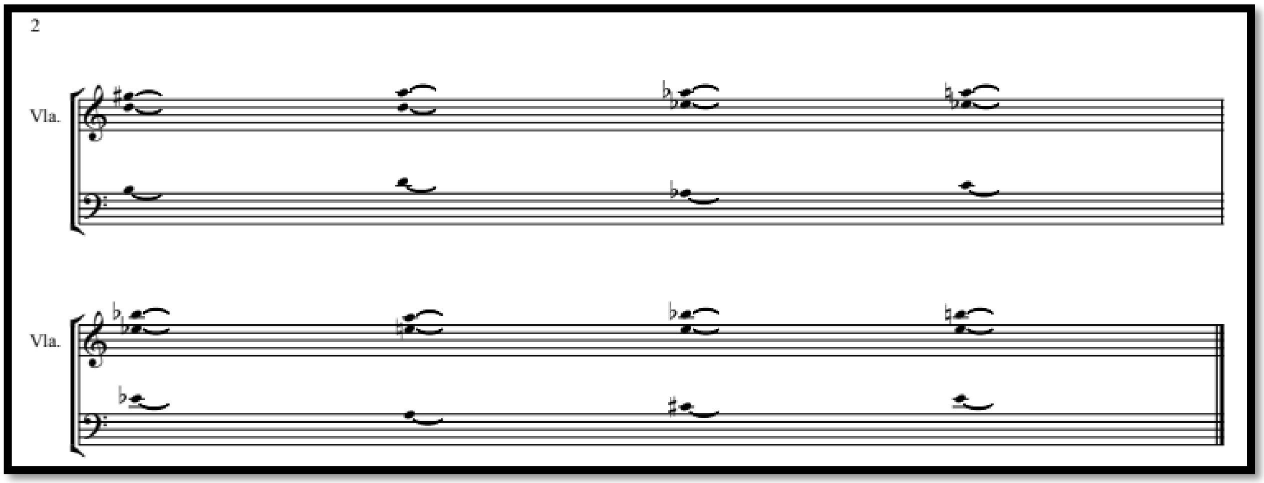

Physicist William Atwood describes, in a quote directed towards luthiers, that, "physics can be a tool for understanding what parameters of the instrument control various aspects of its sound, thereby providing a guide for shaping it."[104] This is also true for the players trying to shape the sounds by aspects of bowing technique. If we understand the physical constraints and resonance capacity of our instruments, we can have greater access to ways of manipulating the sounds we create.

This chapter was intended to act as an introductory guide for beginning to experiment within some of these physical parameters. Some of the practices are easier than others, therefore varying levels of success can be expected based on a range of differing factors, i.e. level of instruction, ear training experience, and hearing loss, to name a few. For beginners, I would suggest starting with some of the easier things to listen for such as sympathetic resonances and changes in timbre based on sounding points and the language of proportions. Listening for overtones can only come with time and practice, and combination tones can be frustrating because of the wide margin of error, various perceivable pitches, and their inaudibility in lower

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[104] William Atwood, "A Physicist in the World of Violins," *Beam Line* (Summer 1998): 27, http://www.slac.stanford.edu/pubs/beamline/28/2/28-2-atwood.pdf. Accessed on July 2, 2019.

registers. However, with practice, the great challenge of listening to the natural resonances of our instruments can greatly improve our listening awareness and ability to make conscientious decisions regarding bowings and fingerings in our music practice.

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APPENDICES

Equal tempered frequencies are derived from this website:
http://pages.mtu.edu/~suits/notefreqs.html

Linear Math Accompanying Exercises using equal tempered intervals

Combination Tones Exercise 1: Major and Minor Thirds

(Open Strings C3, G3, D4, A4)

*numbered according to measure numbers, (e)=equal tempered, based on A=440 hertz

1. **Eb4-C4≅ Ab1, *G1***
   (e) 311.13- 261.63= **49.50**
   (G1=49.0, G#/Ab1=51.91)
   *Ab (+2.41 Hz)
2. **E4-C4≅ C2, *C#/Db2***
   (e) 329.63- 261.63= **68**
   (C2=65.41, C#/Db2=69.30)
   *C2 (-2.59 Hz)
3. **E4-C#4≅ A1, *G#1***
   (e) 329.63- 277.18= **52.45**
   (Ab1/G#1= 51.91, A1 55.00)
   *A1 (-0.54)
4. **F4-Db4≅ Db2, *D2***
   (e) 349.23- 277.18= **72.05**
   (C#/Db2= 69.30, D2 73.42)
5. **F4-D4≅ Bb1, *A1***
   (e) 349.23- 293.66= **55.57**
   (A1= 55.00, A#/Bb1=58.27)
6. **F#4-D4≅ D2, *Eb2***
   (e) 369.99- 293.66= **76.33**
   (D2=73.42, D#3/Eb2=77.78)
7. **F#4-D#4≅ B1, *A#1***
   (e) 369.99- 311.13= **58.86**
   (A#/Bb1=58.27, B1=61.74)
8. **G4-Eb4≅ Eb2, *E2***
   (e) 392.00- 311.13= **80.87**
   (D#/Eb2=77.78, E2=82.41)
9. **G4-E4≅ C2, *B1***
   (e) 392.00- 329.63= **62.37**
   (B1=61.74, C2=65.41)
10. **G#4-E4≅ E2, *F2***
    (e) 415.30- 329.63= **85.67**
    (E2=82.41, F2=87.31)
11. **Ab4-F4≅ Db2, *C2***
    (e) 415.30- 349.23= **66.07**
    (C2=65.41, C#/Db2=69.30)
12. **A4-F4≅ F2, *Gb2***
    (e) 440.00- 349.23= **90.77**
    (F2=87.31, F#/Gb2=92.50)
13. **A4-F#4≅ D2, *C#2***
    (e) 440.00- 369.99= **70.01**
    (C#/Db2=69.30, D2=73.42)

14. **A#4-F#4≅ F#2, *G2***
    (e) 466.16- 369.99= **96.17**
    (F#/Gb2=92.50, G2=98.00)
15. **Bb4-G4≅ *Eb2, D2***
    (e) 466.16- 392.00= **74.16**
    (D2=73.42, D#/Eb2=77.78)
16. **B4-G4≅ G2, *Ab2***
    (e) 493.88- 392.00= **101.88**
    (G2=98.00, G#/Ab2=103.83)
17. **B4-G#4≅ E2, *D#2***
    (e) 493.88- 415.30= **78.58**
    (D#/Eb2=77.78, E2=82.41)
18. **C5-Ab4≅ Ab2, *A2***
    (e) 523.25- 415.30= **107.95**
    (G#/Ab2=103.83, A2=110.00)
19. **C5-A4≅ F2, *E2***
    (e)523.25- 440.00= **83.25**
    (E2=82.41, F2=87.31)
20. **C#5-A4≅ A2, *Bb2***
    (e) 554.37- 440.00= **114.37**
    (A2=110.00, A#/Bb2=116.54)
21. **C#5-A#4≅ F#2, *F2***
    (e) 554.37- 466.16= **88.21**
    (F2=87.31, F#/Gb2=92.50)
22. **D5-Bb4≅ Bb2, *B2***
    (e) 587.33- 466.16= **121.17**
    (A#/Bb2=116.54, B2=123.47)
23. **D5-B4≅ G2, *F#2***
    (e) 587.33- 493.88= **93.45**
    (F#/Gb2=92.50, G2=98.00)
24. **D#5-B4≅ B2, *C2***
    (e) 622.25- 493.88= **128.37**
    (B2=123.47, C2=130.81)
25. **Eb5-C5≅ Ab2, *G2***
    (e) 622.25- 523.25= **99**
    (G2=98.00, G#/Ab2=103.83)
26. **E5-C5≅ C3, *Db3***
    (e) 659.25- 523.25= **136**
    (C3=130.81, C#/Db3=138.59)
27. **E5-C#5≅ A2, *G#2***
    (e) 659.25- 554.37= **104.88**
    (G#/Ab2=103.83, A2=110.00)
28. **F5-Db5≅ Db3, *D3***
    (e) 698.46- 554.37= **144.09**
    (C#3/Db3=138.59, D3=146.83)
29. **F5-D5≅ Bb2, *A2***
    (e) 698.46- 587.33= **111.13**
    (A2=110.00, A#/Bb2=116.54)
30. **F#5-D5≅ D3, *Eb3***
    (e) 739.99- 587.33= **152.66**
    (D3=146.83, D#/Eb3=155.56)
31. **F#5-D#5≅ B2, *A#2***
    (e) 739.99- 622.25= **117.74**
    (A#/Bb2=116.54, B2=123.47)
32. **G5-Eb5≅ Eb3, *E3***
    (e) 783.99- 622.25= **161.74**
    (D#/Eb3=155.56, E3=164.81)
33. **G5-E5≅ C3, *B2***
    (e) 783.99- 659.25= **124.74**
    (B2=123.47, C3=130.81)
34. **G#5-E5≅ E3, *F3***
    (e) 830.61- 659.25= **171.36**

(E3=164.81, F3=174.61)

35. **Ab5-F5≅ Db3, *C3***
(e) 830.61- 698.46= **132.15**
(C3=130.81, C#/Db3=138.59)

36. **A5-F5≅ F3, *Gb3***
(e) 880.00- 698.46= **181.54**
(F3=174.61, F#/Gb3=185.00)

37. **A5-F#5≅ D3, *C#3***
(e) 880.00- 739.99= **140.01**
(C#/Db3=138.59, D3=146.83)

38. **Bb5-Gb5≅ Gb3, *G3***
(e) 932.33- 739.99= **192.34**
(F#/Gb3=185.00, G3=196.00)

39. **Bb5-G5≅ Eb3, *D3***
(e) 932.33- 783.99= **148.34**
(D3=146.83, D#/Eb3=155.56)

40. **B5-G5≅ G3, *Ab3***
(e) 987.77- 783.99= **203.78**
(G3=196.00, G#/Ab3=207.65)

41. **Cb6-Ab5≅ E3, *Eb3***
(e) 987.77- 830.61= **157.16**
(D#/Eb3=155.56, E3=164.81)

42. **C6-Ab5≅ Ab3, *A3***
(e) 1046.50- 830.61= **215.89**
(G#/Ab3=207.65, A3=220.00)

43. **C6-A5≅ F3, *E3***
(e) 1046.50- 880.00= **166.5**
(E3=164.81, F3=174.61)

44. **C#6-A5≅ A3, *Bb3***
(e) 1108.73- 880.00= **228.73**
(A3=220.00, A#/Bb3=233.08)

45. **Db6-Bb5≅ Gb3, *F3***
(e) 1108.73- 932.33= **176.4**
(F3=174.61, F#/Gb3=185.00)

46. **D6-Bb5≅ Bb3, *B3***
(e) 1174.66- 932.33= **242.33**
(A#/Bb3=233.08, B3=246.94)

47. **D6-B5≅ G3, *F#3***
(e) 1174.66- 987.77= **186.89**
(F#/Gb3=185.00, G3=196.00)

48. **D#6-B5≅ B3, *C4***
(e) 1244.51- 987.77= **256.74**
(B3=246.94, C4=261.63)

49. **Eb6-C6≅ Ab3, *G3***
(e) 1244.51- 1046.50= **198.01**
(G3=196.00, G#/Ab3=207.65)

50. **E6-C6≅ C4, *Db4***
(e) 1318.51- 1046.50= **272.01**
(C4=261.63, C#/Db4=277.1)

Combination Tones Exercise 2: Major and Minor 6ths

(Open Strings C3, G3, D4, A4)

*No math presented for perceived pitches in italics

1. **Bb4-D4≅ F3**
   (e) 466.16- 293.66= **172.50**
   (E3=164.81, F3= 174.61)
2. **B4-D4≅ G3, *G2***
   (e) 493.88- 293.66= **200.22**
   (G3=196.00, G#/Ab3=207.65)
3. **B4-D#4≅ F#3, *B2***
   (e) 493.88- 311.13= **182.75**
   (F3=174.61, F#/Gb3=185.00)
4. **C5-Eb4≅ Ab3, *Ab2***
   (e) 523.25- 311.13= **212.12**
   (G#/Ab3=207.65, A3=220.00)
5. **C5-E4≅ G3, *C3, C2***
   (e) 523.25- 329.63= **193.62**
   (F#/Gb3=185.00, G3=196.00)
6. **C#5-E4≅ A3, *A2***
   (e) 554.37- 329.63= **224.74**
   (A3=220.00, A#/Bb3=233.08)
7. **C#5-F4≅ G#3, *C#3***
   (e) 554.37- 349.23= **205.14**
   (G3=196.00, G#/Ab3=207.65)
8. **D5-F4≅ Bb3, *Bb2***
   (e) 587.33- 349.23= **238.10**
   (A#/Bb3=233.08, B3=246.94)
9. **D5-F#4≅ A3, *D2***
   (e) 587.33- 369.99= **217.34**
   (G#/Ab3=207.65, A3=220.00)
10. **D#5-F#4≅ B3, *B2***
    (e) 622.25- 369.99= **252.26**
    (B3=246.94, C4=261.63)
11. **Eb5-G4≅ Bb3, *Eb3***
    (e) 622.25- 392.00= **230.25**
    (A3=220.00, A#/Bb3=233.08)
12. **E5-G4≅ C4, *C3***
    (e) 659.25- 392.00= **267.25**
    (C4=261.63, C#/Db4= 277.18)
13. **E5-G#4≅ B3, *E3***
    (e) 659.25- 415.30= **243.95**
    (A#/Bb3=233.08, B3=246.94)
14. **F5-Ab4≅ Db4, *Db3***
    (e) 698.46- 415.30= **283.16**
    (C#/Db4= 277.18, D4=293.66)
15. **F5-A4≅ C4, *F3***
    (e) 698.46- 440.00= **258.60**
    (B3=246.94, C4=261.63)
16. **F#5-A4≅ D4, *D3***
    (e) 739.99- 440.00= **299.99**
    (D4=293.66, D#/Eb4=311.13)
17. **Gb5-Bb4≅ Db4, *Gb3***
    (e) 739.99- 466.16= **273.83**
    (C4=261.63, C#/Db4= 277.18)
18. **G5-Bb4≅ Eb4, *Eb3***
    (e) 783.99- 466.16= **317.83**
    (D#/Eb4=311.13, E4=329.63)

19. G5-B4≅ D4, *G3*

(e) 783.99- 493.88= **290.11**

(C#/Db4= 277.18, D4=293.66)

20. G#5-B4≅ E4, *E3*

(e) 830.61- 493.88= **336.73**

(E4=329.63, F4=349.23)

21. Ab5-C5≅ Eb4, *Ab2*

(e) 830.61- 523.25= **307.36**

(D4=293.66, D#/Eb4=311.13, Ab2=103.83)

22. A5-C5≅ F4, *F3*

(e) 880.00- 523.25= **356.75**

(F4=349.23, F#4/Gb4=369.99)

23. A5-Db5≅ E4, *A2*

(e) 880.00- 554.37= **325.63**

(D#/Eb4=311.13, E4=329.63)

24. Bb5-Db5≅ Gb4, *Gb3*

(e) 932.33- 554.37= 377.96

(F#4/Gb4=369.99, G4=392.00)

25. Bb5-D5≅ F4, *B2*

(e) 932.33- 587.33= **345.00**

(E4=329.63, F4=349.23)

26. B5-D5≅ G4, *G3*

(e) 987.77- 587.33= 400.44

(G4=392.00, G#/Ab4=415.30)

27. B5-D#5≅ F#4, *B2*

(e) 987.77- 622.25= **365.52**

(F4=349.23, F#4/Gb4=369.99)

28. C6-D#5≅ G#4, *G#2*

(e) 1046.50- 622.25= **424.25**

(G#/Ab4=415.30, A=440.00)

29. C6-E5≅ G4, *C3*

(e) 1046.50- 659.25= **387.25**

(F#4/Gb4=369.99, G4=392.00)

30. C#6-E5≅ A4, *A2*

(e) 1108.73- 659.25= **449.48**

(A=440.00, A#/Bb=466.16)

31. C#6-F5≅ G#4, *C#3*

(e) 1108.73- 698.46= **410.27**

(G4=392.00, G#/Ab4=415.30)

32. D6-F5≅ Bb4, *Bb2*

(e) 1174.66- 698.46= **476.20**

(A#/Bb4=466.16, B4=493.88)

33. D6-F#5≅ A4, *D3*

(e) 1174.66- 739.99= **434.67**

(G#/Ab4=415.30, A=440.00)

34. D#6-F#5≅ B4, *B2*

(e) 1244.51- 739.99= **504.52**

(B4=493.88, C5=523.25)

Combination Tones Exercise 3: (Perfect) Fourths, Tritones, and (Perfect) Fifths

(Open Strings C3, G3, D4, A4)

1. **A4-E4≅ A2**
   (e) 440.00- 329.63= **110.37**
   (A2=110.00)
2. **Bb4-E4≅ Db3**
   (e) 466.16- 329.63= **136.53**
   (C3=130.81, C#/Db3= 138.59)
3. **B4-E4≅ E3**
   (e) 493.88- 329.63= **164.25**
   (E3=164.81)
4. **Bb4-F4≅ Bb2**
   (e) 466.16- 349.23= **116.93**
   (Bb2=116.54)
5. **B4-F4≅ D3**
   (e) 493.88- 349.23= **144.65**
   (C#/Db3=138.59, D3=146.83)
6. **C5-F4≅ F3**
   (e) 523.25- 349.23= **174.02**
   (F3=174.61)
7. **B4-F#4≅ B2**
   (e) 493.88- 369.99= **123.89**
   (B2=123.47)
8. **C5-Gb4≅ Eb3**
   (e) 523.25- 369.99=**153.26**
   (D3=146.83, D#/Eb3=155.56)
9. **Db5-Gb4≅ Gb3**
   (e) 554.37- 369.99= **184.38**
   (F#/Gb3=185.00)
10. **C5-G4≅ C3**
    (e) 523.25- 392.00= **131.25**
    (C3=130.81)
11. **C#5-G4≅ E3**
    (e) 554.37- 392.00= **162.37**
    (D#/Eb3=155.56, E3=164.81)
12. **D5-G4≅ G3**
    (e) 587.33- 392.00=**195.33**
    (G3=196.00)
13. **C#5-G#4≅ C#4**
    (e) 554.37- 415.30= **139.07**
    (C#/Db4=138.59)
14. **D5-G#4≅ F3**
    (e) 587.33- 415.30= **172.03**
    (E3=164.81, F3=174.61)
15. **D#5-G#4≅ G#3**
    (e) 622.25- 415.30= **206.95**
    (G#3=207.65)
16. **D5-A4≅ D3**
    (e) 587.33- 440.00= **147.33**
    (D3=146.83)
17. **D#5-A4≅ F#3**
    (e) 622.25- 440.00= **182.25**
    (F3=174.61, F#/Gb3=185.00)
18. **E5-A4≅ A3**
    (e) 659.25- 440.00= **219.25**
    (A3=220.00)
19. **Eb5-Bb4≅ Eb3**
    (e) 622.25- 466.16= **156.09**

(D#/Eb3=155.56)

20. E5-Bb4≅ G3

(e) 659.25- 466.16= **193.09**

(F#/Gb3=185.00, G3=196.00)

21. F5-Bb4≅ Bb3

(e) 698.46- 466.16= **232.30**

(A#/Bb3=233.08)

22. E5-B4≅ E3

(e) 659.25- 493.88= **165.37**

(E3=164.81)

23. F5-B4≅ G#3

(e) 698.46- 493.88= **204.58**

(G3=196.00, G#/Ab3=207.65)

24. F#5-B4≅ B3

(e) 739.99- 493.88= **246.11**

(B3=246.94)

25. F5-C5≅ F3

(e) 698.46- 523.25= **175.21**

(F3=174.61)

26. F#5-C5≅ A3

(e) 739.99- 523.25= **216.74**

(G#/Ab3=207.65, A3=220.00)

27. G5-C5≅ C4

(e) 783.99- 523.25= **260.74**

(C4=261.63)

28. F#5-C#5≅ F#3

(e) 739.99- 554.37= **185.62**

(F#/Gb3=185.00)

29. G5-C#5≅ A#3

(e) 783.99- 554.37= **229.62**

(A3=220.00, A#/Bb3=233.08)

30. G#5-C#5≅ C#4

(e) 830.61- 554.37= **276.24**

(C#/Db4=277.18)

31. G5-D5≅ G3

(e) 783.99- 587.33= **196.66**

(G3=196.00)

32. G#5-D5≅ B3

(e) 830.61- 587.33= **243.28**

(A#/Bb3=233.08, B3=246.94)

33. A5-D5≅ D4

(e) 880.00- 587.33= **292.67**

(D4=293.66)

34. Ab5-Eb5≅ Ab3

(e) 830.61- 622.25= **208.36**

(G#/Ab3=207.65)

35. A5-Eb5≅ C4

(e) 880.00- 622.25= **257.75**

(B3=246.94, C4=261.63)

36. Bb5-Eb5≅ Eb4

(e) 932.33- 622.25= **310.08**

(D#/Eb4=311.13)

37. A5-E5≅ A3

(e) 880.00- 659.25= **220.75**

(A3= 220.00)

38. Bb5-E5≅ Db4

(e) 932.33- 659.25= **273.08**

(C4=261.63, C#/Db4= 277.18)

39. B5-E5≅ E4

987.77- 659.25= **328.52**

(E4=329.6)