DIAMETER DISTRIBUTION PREDICTION MODELS FOR THINNED SLASH AND LOBLOLLY PINE PLANTATIONS IN THE SOUTHEAST

by

SAMMY KIBET YATICH

(Under the Direction of BRUCE E. BORDERS)

ABSTRACT

Slash pine (*Pinus elliotii* Englem.) and loblolly pine (*Pinus taeda* L.) are important commercial species in the Southeastern United States. With intensive management of these pine plantations gaining popularity, growth and yield models that incorporate cultural treatments such as thinning, fertilization and weed control are required. This study developed such models to predict stand structure of thinned slash and loblolly pine stands. Preliminary results showed that after first thinning selective thinning from below applied at 33%, 40% and 50% thinning intensities shifted the diameter distributions towards the right increasing significantly the $0^{th}$ and $25^{th}$ percentiles. Row thinning irrespective of thinning intensity did not change the percentiles. $0^{th}$, $25^{th}$, $50^{th}$ and $95^{th}$ percentile prediction equations were developed to account both for these effects and information that is possibly available after thinning. Weibull distributions recovered from predicted percentiles were not significantly different from observed diameter distributions. This indicates that Weibull that has been well documented for unthinned stands can be used to describe thinned pine diameter distributions. Height-dbh prediction equations to be used with diameter distribution prediction models to predict stand volume were developed. Effects of thinning on average total tree heights were such that when compared to row thinning and unthinned stands, selective thinning from below, depending on intensity, increased the average height of trees towards the left of the diameter distribution.

An empirical evaluation of generalized stand table projection method (GSTP) of Pienaar and Harrison (1988) against relative size relationship model (CRS) showed that both models performed reasonably well in projecting current stand tables. Results also indicated absence of consistency in the estimate for the rate parameter in GSTP model and when thinning was accounted for in this model, it implied conflicting trends in relative size for row and select thinned stands. On the other hand plots of observed data showed no significant change in relative size over time. This indicates it is reasonable to assume that relative tree size for these data remains constant over time and with respect to type and timing of thinning.

INDEX WORDS: Slash pine, Loblolly pine, Diameter distribution, Growth and yield models, Thinning, Percentile prediction equation, Height-dbh equation, Weibull, Generalized stand table Projection, Constant relative size

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# CHAPTER 1

## INTRODUCTION

Slash pine (*Pinus elliotii* Engelm.) and loblolly pine (*Pinus taeda* L.) are important commercial pine species in the Southeastern United States. They provide raw materials for the forest industry in the southeast and offer an attractive alternative land investment in these regions because they grow well in the soils that are low in nutrients required for hardwood growth or agricultural crops.  In plantations, loblolly pine in particular has been shown to respond well to intensive cultural treatments such as fertilization, weed control, thinning etc**.** With such intensive management gaining popularity, growth and yield models that incorporate these treatments are required. Few of these models exist. However there is a continuing interest to develop them.

The increase in demand for wood products such as lumber and veneer and the diminishing supply of these products from natural stands have made thinning an integral part of intensive management of slash and loblolly pine plantations (Bailey et al. 1981). With this comes the need to predict stand structure. While many diameter distribution model systems are available for unthinned pine stands few exist for thinned pine stands. This is likely because of the difficulty in specifying how various types and intensity of thinning affect the distribution initially and through time (Farrar, Jr., 1979). In spite of this, efforts have been made to develop such models. This includes in slash pine stands (Clutter and Jones 1980, Bailey et al. 1981, Borders et al 1987, and others) and in loblolly pine stands (Daniels and Burkhart 1975, Strub et al. 1981, Matney and Sullivan 1982, Cao and Burhart 1984, McTague and Bailey 1987 and others). This study expanded on this list.

A height by diameter equation is an essential component of a diameter distribution yield model. It predicts individual tree heights, which are used with predicted diameter distributions in a volume equation to calculate volume by dbh class which, when summed up over all classes, provides total stand volume. Most thinned diameter distribution systems in the literature do not have a companion height-dbh equation. This suggests these systems have assumed that individual tree height does not change after thinning despite evidence indicating otherwise (Dell and Collicott 1968, Parker 1979 among others). Towards filling this gap this study developed height-dbh equations that reflected both the type and intensity of thinning.

Previous modeling studies of thinned stands have assumed that complete information about a thinning is available. Rarely is this the case. There are cases where partial information is known about a thinning to where nothing at all is known except that the stand was previously thinned. Accounting for some of the most practical of these cases this study developed models for when complete, partial and no thinning information is available.

Relative tree size defined by Pienaar and Harrison (1988) as the ratio of tree basal area to the mean tree basal area has been reported in previous studies (Clutter 1993, McTague and Stansfield 1994, Knowe et al. 1997, Borders and Patterson 1990 among others) to change over time. Such changes indicate that trees smaller than average tree size become even smaller than average tree size and trees that are larger than average tree size become even larger than average tree size over time. This study evaluated these changes against the assumption of constant relative size using thinned slash and loblolly pine data.

Data for this study come from McIntyre-Stennis project number 33 (MS33) thinning study. This thinning study was installed from 1981 to 1984 on industry owned plantations in piedmont, upper and lower coastal plain provinces of Alabama, Florida, Georgia and South

Carolina to investigate the effect of type, intensity and timing of thinning in slash and loblolly pine plantations. The study is comprised of a total of twenty installations, fourteen of which were loblolly pine and seven were slash pine installations. These installations were constituted of young (13-15 years) and old (16-19 years) age classes in each physiographic region. Since initial thinning, research plots have been remeasured up to four times at three year intervals. For plots designated for second thinning the second thin was implemented 9 years following first thinning.

To summarize, the objectives of this study included:

- Develop diameter distribution prediction equations for thinned slash and loblolly pine plantations. Towards this goal effects of thinning on the diameter distributions were evaluated. These effects were incorporated into percentile prediction equations. A non-iterative parameter recovery procedure (Bailey et al 1989) was used to recover the Weibull parameters hence the diameter distribution.
- Develop height-dbh equations to predict average total tree height in thinned stands of slash and loblolly pine. Effects of type and intensity of thinning were accounted for in these equations.
- Evaluate empirically generalized stand table projection system of Pienaar and Harrison (1988) against constant relative size relationship.

# CHAPTER 2

# DIAMETER DISTRIBUTION PREDICTION MODELS FOR THINNED SLASH AND LOBLOLLY PINE PLANTATIONS IN THE SOUTHEAST

## Introduction

Slash and loblolly pine plantations are important sources of lumber and wood fiber in the Southeastern United States. For a sustained supply of these products forest resource managers and owners have realized the need to manage these pine plantations intensively. This kind of management includes cultural practices and tree improvement programs such as thinning, fertilization, weed control etc. Where these practices are now routinely applied new growth and yield models are needed. In particular there is a growing need for models to predict the stand structure of thinned pine plantations.

For over forty years diameter distribution model systems have been developed mainly for unthinned stands - slash pine (Bennett and Clutter 1968, Clutter ad Belcher 1977, Dell et al. 1979, Schreuder et al. 1979, Lenhart 1988, Borders et al 1987, Bailey et al. 1989, Pienaar and Harrison 1988 and others) and loblolly pine (Lenhart and Clutter 1971, Matney and Sullivan 1982, Clutter et al. 1984, Amateis et al. 1984, Baldwin and Feduccia 1987, Murphy and Farrar Jr. 1988, Burk and Burkhart 1984, Cao 2004, Bullock and Burkhart 2005 and others). The few diameter distribution models that have been developed for thinned stands - slash pine (Clutter and Jones 1980, Bailey et al. 1981, Borders et al 1987, and others) and loblolly pine (Daniels and Burkhart (1975), Strub et al. 1981, Matney and Sullivan 1982, Cao and Burhart 1984, McTague and Bailey 1987 and others) were expanded in this study. This expansion was achieved by accounting for the effects of type and intensity of thinning on diameter distributions.

Equally important is that this expansion addressed various pieces of information that are practically available after thinning. The assumption in previous modeling studies has been that complete information about thinning is available. Rarely is this the standard. There are cases where partial information is known about a thinning to those where nothing at all is known except that the stand was previously thinned. This study thus developed diameter distribution prediction models for the following practical scenarios:

1. complete pre-thin and post-thin stand information is available
2. thinning type and after thin stand information is available
3. thinning intensity and after thin stand information is available
4. age of thinning and type of thinning are known
5. only age of thinning is known
6. no thinning information is available.

## Literature Review

### Growth and Yield Modeling

Growth and yield estimates in early forestry practices were obtained using normal yield tables. A well-known example of these tables is the normal yield tables prepared for natural stands of southern pine species (USDA 1929). These tables were based on data collected in fully stocked stands. Unfortunately there were no objective criteria for deciding whether these stands were fully stocked or not. This problem led to the development of variable density yield tables that explicitly incorporated some measure of stand density into the yield prediction equation.

MacKinney et al. (1937) and Schumacher (1939) originally suggested variable yield density models. These models were refined and improved by Buckman (1962) and Clutter (1963) who recognized independently that a mathematical relationship exists between growth and yield. That is the integral of a growth function should result in a yield function. Sullivan and Clutter (1972) developed further improvements on these models. They developed a method to simultaneously estimate the parameters of the model while retaining desirable model properties of consistency and path invariance. These explicit stand yield models usually provide a total per acre yield estimate of the stand. For management purposes, forest managers need this information as well as information about stand structure. This more detailed information can be obtained from diameter distribution models such as (in thinned slash pine plantations: Bailey et al., 1981; Borders et al., 1987; and others and in thinned loblolly pine plantations: Daniels and Burkhart 1975; Strub et al., 1981; Cao et al., 1982; Matney and Sullivan, 1982; Cao and Burhart, 1984; McTague and Bailey, 1987; and others). This study improved further on some of these models by accounting for both the information that is practically available after thinning as well as the effect of thinning type and intensity on diameter distributions.

**Diameter Distribution Functions**

Diameter distribution modeling dates back to1898 when De Liocourt published the first numerical studies of growing stock distribution in uneven-aged forests (Meyer, 1952). Since that time many functions have been used to describe diameter distributions. For example Meyer (1930) used a Gram-Charlier series to characterize distributions in even-aged stands of shortleaf pine. Osborne and Schumacher (1935) applied Pearl-Reed population growth curve to construct red gum stand tables and Meyer and Stevenson (1943) applied the exponential probability density function to predict the diameter distribution of uneven-aged stands. This function was later determined to closely represent the probability distribution of tree diameter as measured in uneven aged stands by among others Meyer (1952) and, Schmelz and Lindsey (1965). Nelson (1964) determined that the Pearl-Reed function provides excellent characterization of diameter distributions of even-aged managed loblolly pine stands and Bliss and Reinker (1964) used a three-parameter lognormal curve to describe the diameter distributions of even aged stands of Douglas-fir.

Clutter and Bennett (1965) introduced the beta distribution function to describe diameter distribution of old-field slash pine plantations:

$$\begin{aligned} f(x) &= \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\Gamma(\beta+1)} x^{\alpha}(1-x)^{\beta} \qquad 0 \le x \le 1 \\ &= 0, \quad \text{otherwise} \end{aligned} \tag{2.1}$$

where,

f(x) = probability density function associated with random variable x

$\Gamma$ = gamma function

x = random variate (dbh)

$\alpha$ and $\beta$ are shape parameters of the beta distribution to be estimated

This distribution (2.1) assumes a wide range of shapes such as normal, bimodal, uniform etc depending on the values of both $\alpha$ and$\beta$ but does not have a closed form cumulative distribution function (cdf). As such, numerical techniques are used to determine the proportion of trees in each diameter class. In spite of this disadvantage, Bennett and Clutter (1968) used it to develop a yield model to predict multiple-product yields for unthinned slash pine plantations in the coastal plain of Georgia and north Florida. McGee and Della-Bianca (1967), Bennett (1970), Lenhart and Clutter (1971), Bukhart (1971), and Lenhart (1972) among others have also used it in their diameter distribution modeling studies.

Bailey and Dell (1973) introduced the Weibull distribution function presented below to describe diameter distributions of even-aged stands.

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} \exp\left\{-\left(\frac{x-a}{b}\right)^{c}\right\} \quad (a \le x < \infty), b > 0, c > 0 \tag{2.2}$$
$$= 0, \text{otherwise}$$

where,

f(x) = cumulative distribution function of x (cdf)

x = random variate

a = location parameter

b = scale parameter

c = shape parameter

This distribution is flexible in shape allowing it to describe various unimodal shapes or curves. For example if c<1 the curve is reverse J-shape, for c = 1 the exponential distribution is obtained, for c < 3.6 the curve is mound shaped and positively skewed, when c = 2 the Rayleigh results, when c = 3.6 Weibull approximates the normal distribution and when c > 3.6 the distribution is negatively skewed.

The *a* parameter in function (2.2) is essentially the lower limit of the diameter distribution. If it is zero or assumed to be zero the three-parameter Weibull pdf is transformed to a two-parameter Weibull pdf using the relationship x = y + a:

$$f(y) = \frac{c}{b}\left(\frac{y}{b}\right)^{c-1} \exp\left\{-\left(\frac{y}{b}\right)^{c}\right\} \quad y \ge 0, b > 0, c > 0 \tag{2.3}$$
$$= 0, \text{otherwise}$$

By integrating the Weibull pdf (2.2) using the change of variable technique, the following cumulative distribution function (cdf) is obtained (Bailey and Dell, 1973):

$$F(x) = 1 - \exp\left\{ -\left( \frac{x-a}{b} \right)^c \right\} \qquad (a \le x < \infty), b > 0, c > 0$$
$$= 0, \text{ otherwise} \tag{2.4}$$

Shape flexibility and existence of closed form cumulative distribution function have made the Weibull probability density function the most popular for describing diameter distributions of even-aged stands. Some of the studies that have used it include models for slash pine plantations (Smalley and Bailey, 1974; Lohrey and Bailey, 1977; Clutter and Belcher, 1978; Dell et al., 1979; Souter 1979 among others) and loblolly pine plantations (Smalley and Bailey, 1974a; Alvarez-Gonzalez, 2002; Cao, 2004; Borders et al., 2004; Bullock and Burkhart, 2005; Nord-Larsen and Cao 2006 and others).

Despite the popularity of Weibull, some researchers have argued for the suitability of other probability density functions to describe diameter distributions. Hafley and Schreuder (1977) comparing beta, Johnson's $S_B$, Weibull, lognormal, gamma and normal distributions in terms of their flexibility using skewness and kurtosis found Johnson's $S_B$ to be relatively stable, with considerable flexibility and in addition relatively simple to apply.

Most diameter distribution model systems in the literature are based on a probability density function such as Weibull or beta function. A departure from this approach is pdf free methods of Borders et al. (1987) who introduced percentile-based distribution methods to characterize diameter distributions of unthinned and thinned slash pine plantations. Their method assumes trees are uniformly distributed between adjacent percentiles yielding the following empirical probability density function:

$$f(D) = \begin{cases} N_i/N(P_i - P_{i-1}) & P_{i-1} < D < P_i \\ 0 & \text{elsewhere} \end{cases} \tag{2.5}$$

where,

$D$ = dbh

$N_i$ = number of trees between percentiles Pi and Pi-1,

$N$ = number of trees per acre

$P_i$-1, $P_i$ are adjacent percentiles in the system

Based on (2.5) they derived the following closed form cumulative distribution function to be used to calculate the number of trees per acre in each diameter class:

$$N_k = \left\{ \frac{P_i - {}_L D_K}{P_i - P_{i-1}} (t_i - t_{i-1}) + (t_j - t_i) + (t_{j+1} - t_j) \frac{{}_U D_K - P_j}{P_{j+1} - P_j} \right\} \tag{2.6}$$

where,

$N_k$ = number of trees per acre in the Kth diameter class

${}_L D_K$ = lower limit of the Kth diameter class

${}_U D_K$ = upper limit of the Kth diameter class

$i$ = -j, -j+1, …, -1, 1, 2, …, k

$P_i$ = adjacent percentiles

$t_i$ = proportion of trees per acre with D less than $P_i$;

$t_{i-1}$ = proportion of trees per acre with D less than $P_{i-1}$;

Borders et al (1987) used 12 percentiles ($0^{th}$, $5^{th}$, $15^{th}$, $25^{th}$, $35^{th}$, $45^{th}$, $55^{th}$ , $65^{th}$, $75^{th}$, $85^{th}$, $95^{th}$ and the $100^{th}$ ) to characterize diameter distributions of unthinned and thinned slash pine plantations in South Georgia, north Florida and Gulf Coast of Alabama and Mississipi. They reported that their system characterized well both unimodal and multimodal stand tables. In latter applications Maltamo et al. (2000) reported it to be better than Weibull in describing diameter distributions of heterogeneous Scots pine.

**Parameter Estimation**

Diameter distribution models based on beta and Weibull functions have used maximum likelihood, method of moments or parameter recovery methods to estimate the parameters of the function. For both the maximum likelihood and method of moments approaches, functions were then developed to predict parameters from stand characteristics such as age, trees/acre, site index, etc. Such prediction systems have come to be known as parameter prediction systems. Early diameter distribution studies (Smalley and Bailey, 1974; Schreuder at al., 1974) used the method of maximum likelihood. Generally this approach involved first generating maximum likelihood estimates of Weibull parameters then predicting these parameters directly from stand characteristics. An example of this is the following Smalley and Bailey (1974) diameter distribution prediction system, which they developed for short-leaf pine plantations in Tennessee, Alabama and Georgia:

$$a = \beta_0 + \beta_1 H \tag{2.7a}$$

$$a + b = \alpha_0 + \alpha_1 T_s + \alpha_2 \log(H) + \alpha_3 / T_s \tag{2.7b}$$

$$c = \delta_0 + \delta_1 H + \delta_2 T_s \tag{2.7c}$$

where,

H = average height of domininants and codominants

$T_s$ = number of surviving trees per acre

a, b and c are the Weibull parameters

$\beta_0$, $\beta_1$, $\alpha_0$, $\alpha_1$, $\alpha_1$, $\delta_0$, $\delta_1$ are the regression coefficients to be estimated.

For dominant height less than 26 feet the *a* parameter was set to zero. The drawback of this approach was poor fits with low R-squares for (2.7a) and (2.7b). Despite this Smalley and Bailey (1974) reported only 20% of the predicted stand tables were not the same as observed stand tables.

Poor fits with maximum likelihood approach has discouraged many from using this approach of parameter prediction. In more recent diameter distribution modeling studies it has been replaced by parameter recovery methods (Cao et al. 1982; Matney and Sullivan, 1982; Bailey et al., 1981; Bailey et al. 1985; Baldwin and Feduccia, 1987; Burk and Newberry, 1984; Borders and Patterson, 1990; McTague and Bailey, 1987; Bailey et al. 1989, Brooks et al. 1992; Harrison and Borders, 1996; among others). This method typically involves selecting a suitable pdf such as beta and Weibull and then using regression analysis to predict stand variables such as diameter percentiles, basal area and quadratic mean diameter. Estimated stand variables are used to solve for pdf parameters using either iterative or non-iterative methods. An example of this method is the non-iterative parameter recovery procedure developed by Bailey et al (1989) to recover Weibull parameters from predicted 0$^{th}$, 25$^{th}$, 50$^{th}$ and 95$^{th}$ percentiles. Assuming initially that the shape, c, parameter is 3.0, they predicted the location, a, parameter using predicted 0$^{th}$ and 50$^{th}$ percentile as follows:

$$\hat{a} = \frac{\left(n^{1/3} P_0 - P_{50}\right)}{\left(n^{1/3} - 1\right)} \tag{2.8a}$$

They then set up simultaneous equations by substituting equation (2.8a) for *a* in cumulative distribution functions for 25$^{th}$ and 95$^{th}$ percentile and solved for *c* to obtain:

$$\hat{c} = \frac{2.343088}{\left(\ln(P_{95} - \hat{a}) - \ln(P_{25} - \hat{a})\right)} \tag{2.8b}$$

They solved for *b* (2.8c) by using the second moment of a three parameter and asuming a and c are known.

$$\hat{b} = -\hat{a}\left(\Gamma_1 / \Gamma_2\right) + \left[\left(\hat{a}^2 / \Gamma_2^2\right)\left(\Gamma_1^2 - \Gamma_2\right) + Dq^2 / \Gamma_2\right]^{1/2} \tag{2.8c}$$

where,

n = sample size/number of trees in the plot

$P_i$ = ith percentile (i = 0, 25, 50, 95)

Dq = quadratic mean diameter

$\Gamma$ = gamma function

$\Gamma_1 = \Gamma\left(1 + \frac{1}{c}\right)$

$\Gamma_2 = \Gamma\left(1 + \frac{2}{c}\right)$

All else are as defined in equation (2.7).

Some of the advantages for using a parameter recovery procedure include the fact that stand variables such as diameter percentiles can be estimated with a better accuracy and confidence than pdf parameters (Borders et al 1987) and according to Burk and Newberry (1984) it is sensitive to small changes in stand attributes.

Apart from $0^{th}$, $25^{th}$, $50^{th}$, and $95^{th}$ used by Bailey et al. (1989) several other combinations of percentiles have been found to be efficient in estimating the Weibull parameters. For example Dubey (1967) determined that the $24^{th}$ and $93^{rd}$ percentiles were best for estimating the scale and shape parameter in a two-parameter Weibull. Bailey et al. (1981) showed the $24^{th}$, $63^{rd}$ and $93^{rd}$ percentiles to be efficient in estimating the parameters for a three-parameter Weibull. McTague and Bailey (1987) chose the $10^{th}$ and $63^{rd}$ percentiles for their two-parameter Weibull function. They found the two percentiles to be far apart to provide a good indication of the spread of the diameter distribution. The equations they developed to predict these percentiles were as follows:

$$\begin{aligned} D10 = {} & \beta_0 + \beta_1 \ln(A/N) + \beta_2 SA \\ & + \beta_4 XA \ln(N) + \beta_5 XS \ln(N) \end{aligned} \tag{2.9a}$$

$$D63 = \beta_0 + \beta_1 \ln(A/N) + \beta_2 SA + \beta_4 SX + \beta_5 X\ln(N) \quad (2.9b)$$

where,

D10 = 10th percentile

D63 = 63rd percentile

A = plantation age

S = base-age-15 site index

N = trees per acre

$$X = \begin{cases} 1 \text{ if stand in unthinned} \\ 0 \text{ if stand has been thinned at least once} \end{cases}$$

Cao and Burkhart (1984) used the 0th, 25th, 50th, 75th and 100th percentile in their segmented cumulative distribution function (cdf) to describe diameter distributions of unthinned and thinned stands. They reported this function to be superior to the Weibull because they found it had greater flexibility allowing it to fit irregularities found in diameter distributions of thinned stands. Most recently Bullock and Burkart (2005) used the 25th and 97th to characterize diameter distributions of juvenile loblolly stands with two-parameter Weibull function. However all these combinations of percentiles were chosen arbitrarily. That is, there was no biological or theoretical basis for choosing them but rather common sense and empirical deductions dictated their selection (Borders et al., 1987).

**Effects of Thinning**

Thinning removes some of the trees growing in a stand in order to concentrate growth on remaining trees. This affects various aspects of stand development. In particular it has been shown to affect survival, diameter growth, stand mean height, stand density, and stand mean diameter, among others. Effects on these stand characteristics vary depending on type, intensity, timing and frequency of thinning, and also on species, age and quality of the site. A brief review of these effects follows.

Slash and loblolly pine thinning studies have reported that thinning reduces mortality. This should be expected because thinning, particularly thinning from below, removes small, suppressed and diseased trees that would otherwise have died naturally (Harrington, 2002). Among thinning types greater reduction in mortality was observed in thinning from below than in row thinning (Baldwin et al., 1989; Brooks, 1992; Harrison et al. 1998) and with respect to thinning intensity mortality was found to decrease with increase in intensity (Dell and Collicott, 1968; Mann and Enghardt, 1972; Brooks ,1992; Harrison et al., 1998). However as thinning was delayed mortality tended to increase (Mann and Enghardt 1972).

In all thinning studies the major stand response to thinning has been increased diameter growth. This diameter response varies with intensity and timing of thinning, and also on quality of the site. For example diameter response has been reported to increase inversely with stand density (Dell and Collicott, 1968; Mann and Enghardt, 1972; Parker, 1979; Brooks, 1992; Harrison et al 1998) but this increase diminishes as the stand takes up the extra growing space. Parker (1979), Brooks (1992) and Harrison et al (1998) showed that thinning irrespective of thinning type and intensity increased diameter growth significantly. To realize these diameter responses, the stand must be thinned early because as trees age they may fail to respond to any form of thinning (Mann and Enghardt, 1972).

Several researchers have reported thinning to have little impact on height growth of remaining trees. This is because height growth is not sensitive to changes in stand density (Clutter et al., 1983). Depending on the thinning method, average stand height might be changed a little by thinning but the height growth of dominant and codominant trees has been found not to change with thinning (Dell and Collicott, 1968; Parker, 1979; Brooks, 1992; Harrison et al., 1998). In cases where very heavy thinning was implemented a slight short-term reduction in height growth has been reported in loblolly pine stands (Ginn et al., 1991; Haywood, 1994). This response, which has been mislabeled as "thinning shock", is believed to occur when photosynthate is redirected to maintain increases in crown diameter growth (Harrington, 2002).

Total basal area has generally been known to be greater in unthinned than in thinned stands. However thinning studies have shown net basal area growth may be greater in thinned than in unthinnned stands. This growth varies with thinning type and intensity. Stands thinned from below tend to have faster growth rates than row thinned stands and light thinned stands tend to have greater basal area growth rates than heavily thinned stands (Dell and Collicott, 1968; Brooks, 1992; Harrison et al., 1998). With increased basal area growth rates the total basal area of a thinned stand has been suggested to approach that of an unthinned counterpart over time (Pienaar and Shiver, 1984).

Thinning does not generally increase gross cubic volume. In fact total cubic foot volume per acre in thinned stands has been found in many studies to be less than that of unthinned stands (Clutter and Jones, 1980; Mann and Dell, 1971; Feduccia, 1977; Baldwin et al., 1989; Brooks, 1992; Harrison et al., 1998). However exceptions to this exist, for example, where severe overcrowding has restricted root and crown development in unthinned stands (Clutter et al., 1983). Considering only thinned stands, volume growth and yield depend on type, intensity and timing of thinning. Selectively thinned stands have been shown to have greater volume growth and yield than row thinned stands (Baldwin et al., 1989; Parker, 1979; Brooks, 1992, Harrison et al., 1998). Light thinned stands tend to have more volume growth than heavily thinned stands (Dell and Collicott, 1968; Mann and Enghardt, 1972; Brooks, 1992; Harrison et al., 1998).

Thinning studies also indicate that volume growth decreases as thinning is delayed (Mann and Enghardt, 1972).

The effect of thinning on diameter distribution of even-aged pine stands also depends on the type and intensity of thinning. Selective methods, depending on the form of selection, generally shift the distribution either to the right or left. Thinning from below which removes more smaller trees than larger trees shifts the distribution towards the right. This increases the average size of the remaining trees. By removing large trees, crown thinning has been shown to have the opposite effect and shifts the distribution towards smaller trees. Row thinning removes rows of trees irrespective of size and as such has generally been assumed not to change the distribution (Bailey et al., 1981; Bailey and Ware, 1983; Matney and Sullivan, 1982).

**Diameter Distribution Models for Thinned Stands**

Few diameter distribution models have been developed to predict stand structure for thinned southern pine stands. This is because of the difficulty to quantify the effects of thinning on diameter distributions both immediately after thinning and through time (Farrar 1979). Despite this some efforts have been made. Daniels and Burkhart (1975) developed a computer simulation model (PTAEDA) for managed loblolly pine plantations that includes the option of thinning. Depending on the values specified for this option the model uses some function of crown size to estimate the decrease in competition of remaining trees due to thinning. Clutter and Jones (1980) presented an algorithm to predict the future stand table structure for thinned old-field slash stands. They adjusted the current stand table for thinning by removing reasonably sufficient number of trees from the diameter distribution at the age of thinning. Other efforts include Strub et al. (1981) who accounted for the shift to the right in thinned loblolly pine distributions. They modified the location Weibull parameter estimate obtained at the start of the growth period. Their modifier, which is comprised of age at first thinning and remaining basal area after first thinning adjusted the origin parameter as follows:

$$\alpha_1 = \beta_0 + \beta_1 \log(A_0) + \beta_2 \log(H_0) + \beta_3 \log(T_0) + \Delta P + CF \tag{2.10a}$$

$$\Delta P = \beta_1[\log(A_1) - \log(A_0)] + \beta_2[\log(H_1) - \log(H_0)] + \beta_2[\log(T_1) - \log(T_0)] \tag{2.10b}$$

$$CF = [\alpha_0 + \alpha_1 TA \bullet TB][A_1 - A_0] \tag{2.10c}$$

where,

$\alpha_i$ = projected Weibull origin parameter for thinned stand at time i;

$A_i$= age of thinned stand at time i

$H_i$ = average height of dominant and codominant trees in thinned stand at time i

$T_i$ = number of surviving trees per acre in thinned stand at time i

$\Delta P$ = change in the respective Weibull parameter (here $\alpha$) between time 0 and 1

CF = correction factor

TA = age of the stand at first thinning

TB = residual basal area per acre for the first thinning

Other efforts have focused on introducing a thinning term into the percentile prediction equations. For example Bailey et al. (1981) adjusted before thinning percentiles for stands to obtain after thin percentiles in slash pine plantations. They achieved this by using the proportion of basal removed as the modifier term which entered their percentile prediction equations as follows:

$$D_{24a} = \beta_0 + \beta_1 D_{24b} + \beta_2 D_{qb} + \beta_3 B_r \tag{2.11a}$$

$$D_{63a} = \alpha_0 D_{63b} + \alpha_1 A + \alpha_2 B_r \tag{2.11b}$$

$$D_{93a} = \delta_0 + \delta_1 D_{93b} + \delta_2 D_{qb} + \delta_3 B_r \qquad (2.11c)$$

where,

$D_{pa}$ = p'th percentile after thinning

$D_{pb}$ = p'th percentile before thinning

$D_{qb}$ = Dq before thinning, all in inches

A = age in years

$B_r$ = basal area removed as a proportion of basal area before thinning

To recover diameter distributions Bailey et al. (1981) developed a percentile based parameter recovery system:

$$\frac{D_{63} - D_{24}}{D_{93} - D_{63}} = \frac{1 - (0.2744368)^{1/c}}{(2.6592600)^{1/c} - 1} \qquad (2.12a)$$

$$b = \frac{D_{93} - D_{63}}{1 - (0.2744368)^{1/c}} \qquad (2.12b)$$

$$a = D_{63} - b \qquad (2.12c)$$

where,

$D_p$ = p'th percentile (p = 24, 63, 93)

For the same repeatedly thinned slash pine stand Bailey et al. (1981) developed percentile projection equations (2.13a), (2.13b) and (2.13c):

$$D_{24e} = \beta_0 + \beta_1 D_{24s} + \beta_2 A + \beta_3 B_s + \beta_4 S + \beta_5 t \qquad (2.13a)$$

$$D_{63e} = \alpha_0 + \alpha_1 D_{63s} + \alpha_2 A + \alpha_3 B_s + \alpha_4 S + \alpha_5 t \qquad (2.13b)$$

$$D_{93e} = \delta_0 + \delta_1 D_{93s} + \delta_2 A + \delta_3 B_s + \delta_4 S + \delta_5 t \qquad (2.13c)$$

where,

$D_{pe}$ = p'th percentile at the end of the growth period (p = 24, 63, 93)

$D_{ps}$ = p'th percentile at the start of the growth period (after thinning if thinned), all in inches

A = age in years

$B_s$ = basal area in square feet per acre at the start of the growth period

S = base-age-25 site index in feet

T = length of the growth period in years

As mentioned earlier segmented cumulative distribution function of Cao and Burkhart (1984), percentile-based methods of Borders et al. (1987) and parameter recovery methods of Cao et al. (1982) and Matney and Sullivan (1982) models have also been developed to describe diameter distributions in thinned pine stands.

**Explicit Growth and Yield Models for Thinned Stands**

It is considered relevant to review some of the efforts made to account for thinning in explicit growth and yield models. Bailey and Ware (1983) accounted for thinning by incorporating a thinning term into the basal area projection model they developed for slash pine plantations in the plains of Georgia, Florida, Alabama and Mississipi:

$$B_2 = B_1^{(A_1/A_2)} \exp\{\beta_1(1 - A_1/A_2) + \beta_2 X_t (1/A_2 - 1/A_1)/A_t A_2 + \beta_3 S(1 - A_1/A_2)\} \quad (2.14)$$

where,

$B_i$ = basal area at age i

S = site index base age 25 years

$A_i$ = age i

$A_t$ = age at most recent thinning

$$X_t = \begin{cases} 1 - R_t, \text{if } R_t \neq 0 \\ 0, \quad \text{if } R_t = 0 \end{cases}$$

$R_t$ = quadratic mean diameter removed in thinning / quadratic mean diameter before thinning

$\beta_1$, $\beta_2$ and $\beta_3$ are parameters to be estimated.

The thinning term, $X_t$, represents the type of thinning. For thinning from below, which removes smaller trees, it is positive and for thinning from above which removes larger trees it is negative. It is equal to zero for thinning with indifference to diameter such as row thinning or for no thinning (Bailey and Ware, 1983). This term assumes that no matter the thinning intensity, row thinning removes an equal proportion of small and large trees. A drawback of this approach is that one must have past thinning information, which might be hard to find if no records about the thinned stand were kept.

Bailey et al (1985) used the thinning term above in their survival model which they developed for thinned and unthinned slash pine plantations in north Florida and Southeast Georgia:

$$\begin{aligned} N_2 = {} & N_1(A_2/A_1)^{\beta_1} e\{(\beta_0 + \beta_2 S)(A_2 - A_1) \\ & + \beta_3 X_t Z[(1/A_2) - (1/A_1)]/A_t\} \end{aligned} \tag{2.15}$$

where,

$N_i$ = number of trees at time i

$A_i$ = age at time i

$$Z = \begin{cases} 1 \text{ if } A_2 < 22.5 \\ 0 \text{ if } A_2 \geq 22.5 \end{cases}$$

$X_t$ is as defined in model (2.14)

$\beta_1$, $\beta_2$ and $\beta_3$ are parameters to be estimated.

Model (2.15) predicts less survival for stands thinned from below than unthinned counterpart when projection age is less than 22.5 years the model. For crown thinned stands the model predicts higher survival than unthinned counterpart. The model assumes that there is no difference in survival between row thinned and unthinned stands.

Pienaar and Shiver (1986) developed a basal area prediction equation that included a term which accounted for intensity of thinning. This term was a ratio of the number of trees per acre remaining after thinning to the number of trees per acre before thinning. It entered their basal area prediction equation as a multiplier:

$$\begin{aligned} \ln B = \beta_0 + \beta_1\left(\frac{1}{A}\right) + \beta_2(\ln N) + \beta_3(\ln H) \\ + \beta_4\left(\frac{\ln N}{A}\right) + \beta_5\left(\frac{\ln H}{A}\right) + \beta_6\left(\frac{N_t A_t}{N_a A}\right) \end{aligned} \qquad (2.16)$$

where,

B = basal area per acre

A = plantation age

$A_t$ = plantation age at last thinning

N = present number of trees per acre

$N_t$ = number of trees remaining removed in last thinning

$N_a$ = number of trees remaining after last thinning

H = average dominant height

$\left(\beta_6 \frac{N_t A_t}{N_a A}\right)$ = thinning term

The thinning term adjusts the basal area upwards for light thinned stands and younger thinned stands. For no thinning the thinning term drops from the model.

Several studies modeling diameter distributions have assumed that row thinning has no effect on diameter distribution and little effect on growth following a thinning. This is tenable because in row thinning the proportion of trees removed is equal to the proportion of basal area removed. The growth assumption following a row thinning is that row thinning releases trees adjacent to the rows but the trees between the rows retains the same properties as if the stand was not thinned. On average future stand diameter will be close or the same as that for unthinned counterpart (Hafley and Buford, 1985).

**Data**

Data used to investigate diameter distribution relationships and develop percentile prediction models come from a thinning study known as the MS33 thinning study. This thinning study was installed from 1981 to 1984 in piedmont, upper and lower coastal plain provinces of Alabama, Florida, Georgia and South Carolina (Figure 2.1) to investigate the effect of type, intensity and timing of thinning in slash and loblolly pine plantations. The study is comprised of a total of twenty installations, fourteen of which were loblolly pine and seven were slash pine installations. These installations were constituted of young (13-15 years) and old (16-19 years) age classes in each physiographic region. In each installation ten treatments were allocated randomly to each of two blocks with each block representing a complete replication. Each of the main factors namely thinning type and intensity comprised of three levels. These were represented in this study by the codes defined in Table 2.1. The letter in the code represented thinning type and the two digit numeral represented thinning intensity for example R50 is a row thinning which removed 50% of trees per acre.

Brooks (1992) and Harrison et al (1998) provide a detailed description of MS33 study. In row thinning 2nd, (2nd & 5th) and 3rd rows were removed under 50%, 40% and 33% thinning intensity levels respectively whereas in row-selective combination thinning, rows removed included 3rd row under 50%, 4th row under 40% and 5th row under 33% thinning intensity level. Of the total trees removed in row-selective thinning, the proportion of trees selectively removed was 34%, 37.5% and 39% under 50%, 40% and 33% thinning intensity levels, respectively. On each ½-acre treatment plot there was an interior ¼-acre measurement plot. The minimum side length of the treatment plot was 120 feet while that of the measurement plot was 80 feet. The measurement plot was surrounded by the greater of 20 feet or two planting spaces. Trees within each measurement plot were measured before and immediately following initial thinning. Remaining trees were tagged and remeasured at three year intervals. For plots designated for second thinning the second thin was implemented 9 years following first thinning. Tree measurements of dbh to the nearest tenth inch on all trees, total height to the nearest foot on a subsample across the diameter distribution and crown class were taken. In addition, stem quality and absence or presence of fusiform rust (stem canker) was recorded. A tree was identified as a potential sawtimber-quality tree given that its stem was free of cankers, crook, sweep, fork and breakage so that at least one 16-foot sawlog could be obtained from the tree at some time in the future. Stand summaries such as average height of dominants and codominants, trees per acre, basal area per and volume per acre were calculated from field measurements. Site index was

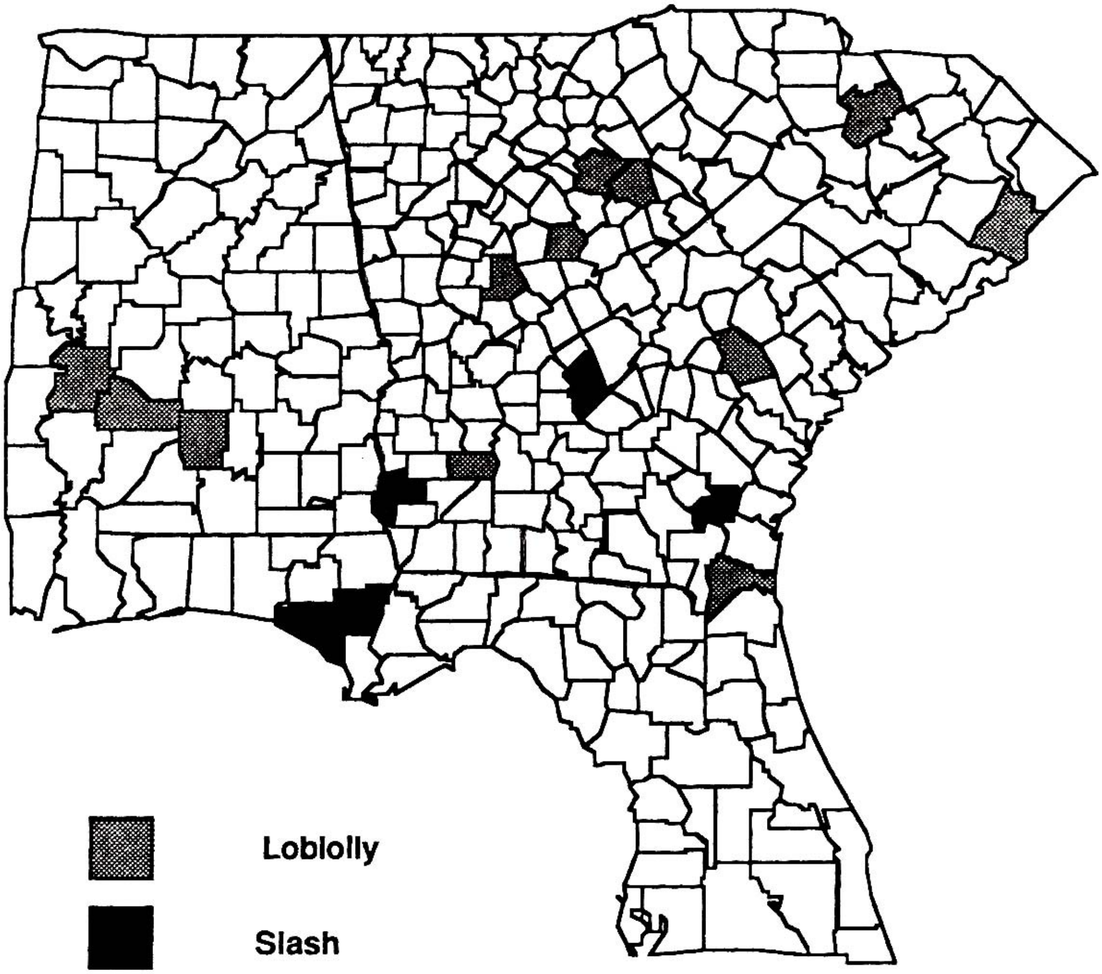



Figure 2.1. Distribution of MS33 installations by species, state and county.

Table 2.1. Thinning treatment codes for MS33 thinning study.

| Thinning Treatment Code | Thinning Type | Thinning Intensity (% of trees per acre removed) |
|---|---|---|
| R33 | Row | 33 |
| R40 | Row | 40 |
| R50 | Row | 50 |
| C33 | Row-Selective Combination | 33 |
| C40 | Row-Selective Combination | 40 |
| C50 | Row-Selective Combination | 50 |
| S33 | Selective | 33 |
| S40 | Selective | 40 |
| S50 | Selective | 50 |
| U0 | Unthinned Check | 0 |

calculated for slash and loblolly pine plantations using Pienaar et al (1988) and Harrison and Borders (1996) site index equations respectively. Brooks (1992) provided 6-year and Harrison et al (1998) presented 9-year results and analysis of the data. Unlike previous studies this study analyzed both first and second thinning data.

The slash pine data that was used in this study are comprised of 14 unthinned check and 126 thinned plots. The distribution of these plots by age, trees per acre, basal area per acre and site index is presented in Tables 2.2 and 2.3. Of the unthinned check plots, 10 were in the lower coastal plain province and 4 were in the upper coastal plain province. The 10 plots in the lower coastal plain are comprised of 6 in the young age class and 4 in the old age class, and trees per acre that ranged from 324 to 635, basal area per acre from 42 to 113 square per acre and site class (base age 25 years) from 50 to 70. All unthinned check plots in the upper coastal plain were in the old age class. These plots contained 55 to 68 square feet per acre of basal area, stand density that ranged from 220 to 475 trees per acre and a site class of 60.

Of the 126 thinned slash pine plots 90 were in the lower coastal plain and 36 were in the upper coastal plain province. Immediately after thinning summary statistics of lower coastal plain plots indicated age ranged from 14 to 17 years, site class from 50 to 70, and basal area per acre from 26 to 77 square feet per acre and trees per acre from 160 to 446 with 86% of the plots containing between 200 and 400 trees per acre. The 36 plots in the upper coastal plain contained a minimum of 25 to a maximum of 59 square feet per acre of basal area, stand density that ranged from 101 to 371 trees per acre, plots divided equally between young and old age classes and two site classes of 50 and 60.

Of the 280 loblolly pine plots that were analyzed 28 were unthinned check plots and 252 were thinned. The distribution of these plots by age, basal area per acre and site index is presented in Tables 2.4 and 2.5. Unthinned check plots included 8, 12, and 8 plots in piedmont, upper coastal plain and lower coastal plain, respectively. Unthinned check plots in the piedmont region had stand density that ranged from 316 to 516 trees per acre, site class that ranged from 40 to 60 and basal area that ranged from 69 to 117 square feet per acre. Unthinned check plots in the upper coastal plain province contained 102 to 177 square feet per acre of basal area, stand density of 353 to 827 trees per acre, plots divided equally between young and old age classes and site class that ranged from 60 to 80. The 8 unthinned check plots in the lower coastal province had stand density ranging from 172 to 804 trees per acre, basal area from 70 to 160 square feet per acre, age from 14.92 to 19.08 years and site class from 50 to 70.

Thinned loblolly pine plots were distributed as follows: 72 piedmont province, 108 upper coastal plain province and 72 lower coastal plain province. Summary statistics of piedmont plots indicated a minimum age of 14.58 years to a maximum age of 17.67 years, site class that ranged from 40 to 60, trees per acre from 140 to 435 with 83% of the plots containing between 200 and 400 trees per acre, and basal area per that ranged from 40 to 104 square feet per acre. Thinned plots in the upper coastal plain province contained 50 to 137 square feet per acre of basal area, stand density that ranged from 187 to 579 trees per acre, plots divided equally between young and old age classes and site class that ranged from 60 to 80. Thinned plots in the lower coastal plain province had basal area per acre ranging from 45 to 119 square feet, age from 14.92 to 19 years, site class from 60 to 80 and trees per acre from 116 to 526 with 73% of o the plots containing between 200 and 400 trees per acre.

**Methods**

Preliminary analyses indicated thinned slash and loblolly pine diameter distributions were predominantly unimodal in nature. As such the Weibull distribution function (Bailey and Dell, 1973) that has been well developed for unthinned stands was chosen to characterize the distributions. Weibull parameter estimates were obtained using Bailey et al. (1989) non-iterative parameter recovery procedure. This procedure takes advantage of the fact that for a Weibull distribution:

$$E[P_0] = a + \left(\frac{b}{n^{1/c}}\right)\Gamma\left(1 + 1/c\right) \tag{2.17}$$

$$P_{50} = a + b\left[-\ln(0.5)\right]^{1/c} \tag{2.18}$$

where,

$P_0$ = minimum percentile (a first order statistic)

$P_{50}$ = 50th percentile

n = sample size (if not known TPA is multiplied by 0.1)

$\Gamma$ = gamma function

*a*, *b* and *c* are Weibull parameters.

Letting $c = 3$ and solving for $a$ using models (2.17) and (2.18) yield,

$$\hat{a} = \frac{\left(n^{1/3}P_0 - P_{50}\right)}{\left(n^{1/3} - 1\right)} \tag{2.19}$$

if $\hat{a} < 0$ then set $\hat{a} = 0$.

25th and 95th percentiles are defined as:

$$P_{25} = a + b\left[-\ln(0.75)\right]^{1/c} \tag{2.20}$$

Table 2.2. Distribution of unthinned slash pine plots by age and trees per acre, basal area per acre and site index in lower and upper coastal plain provinces.

| | | **Lower Coastal** Age (years) | | | | **Upper Coastal** Age (years) | | |
|---|---|---|---|---|---|---|---|---|
| Stand parameter | | 14 | 15 | 17 | | 13 | 18 | |
| | | number of plots | | | Total | number of plots | | Total |
| Trees per acre | 200-299 | - | - | - | - | 0 | 2 | 4 |
| | 300-399 | 1 | 0 | 1 | 2 | 0 | 0 | 0 |
| | 400-499 | 3 | 0 | 3 | 6 | 2 | 0 | 0 |
| | 500-599 | 0 | 0 | 0 | 0 | - | - | - |
| | 600-699 | 0 | 2 | 0 | 2 | - | - | - |
| | Total | 4 | 2 | 4 | 10 | 2 | 2 | 4 |
| Basal area per acre ($ft^2$/ac) | 40-59 | 2 | 0 | 0 | 2 | 1 | 1 | 2 |
| | 60-79 | 2 | 0 | 0 | 2 | 1 | 1 | 2 |
| | 80-99 | 0 | 1 | 3 | 4 | - | - | - |
| | 100-119 | 0 | 1 | 1 | 2 | - | - | - |
| | Total | 4 | 2 | 4 | 10 | 2 | 2 | 4 |
| Site index | 50 | 3 | 1 | 0 | 4 | - | - | - |
| | 60 | 1 | 1 | 2 | 4 | 2 | 2 | 2 |
| | 70 | 0 | 0 | 2 | 2 | - | - | - |
| | Total | 4 | 2 | 4 | 10 | 2 | 2 | 4 |

Table 2.3 Distribution of slash pine plots after thinning by age, trees per acre, basal area per acre and site index in lower and upper coastal plain provinces.

| Stand parameter | | **Lower Coastal** Age (years) 14 | 15 | 17 | | **Upper Coastal** Age (years) 13 | 18 | |
|---|---|---|---|---|---|---|---|---|
| | | number of plots | | | Total | number of plots | | Total |
| Trees per acre | 100-199 | 5 | 0 | 1 | 6 | 0 | 18 | 18 |
| | 200-299 | 30 | 2 | 25 | 57 | 12 | 0 | 12 |
| | 300-399 | 1 | 9 | 10 | 20 | 6 | 0 | 6 |
| | 400-499 | 0 | 7 | 0 | 7 | - | - | - |
| | Total | 36 | 18 | 36 | 90 | 18 | 18 | 36 |
| Basal area per acre ($ft^2$/ac) | 20-39 | 23 | 0 | 0 | 23 | 5 | 7 | 12 |
| | 40-59 | 13 | 9 | 24 | 46 | 13 | 11 | 24 |
| | 60-79 | 0 | 9 | 12 | 21 | - | - | - |
| | Total | 36 | 18 | 36 | 90 | 18 | 18 | 36 |
| Site index | 50 | 20 | 11 | 0 | 31 | 0 | 10 | 10 |
| | 60 | 16 | 7 | 29 | 52 | 18 | 8 | 26 |
| | 70 | 0 | 0 | 7 | 7 | - | - | - |
| | Total | 36 | 18 | 36 | 90 | 18 | 18 | 36 |

Table 2.4 Distribution of unthinned loblolly pine plots by age and trees per acre, basal area per acre and site index in piedmont, upper and lower coastal plain provinces.

| | | **Piedmont** | | | | | **Upper Coastal Plain** | | | | | **Lower Coastal Plain** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Age (years) | | | | | Age (years) | | | | | Age (years) | | | |
| Stand parameter | | 15 | 16 | 17 | 18 | | 12 | 14 | 16 | 17 | | 15 | 18 | 19 | |
| | | number of plots | | | | Total | number of plots | | | | Total | | | | Total |
| Trees per acre | 100-199 | - | - | - | - | - | - | - | - | - | - | 0 | 0 | 1 | 1 |
| | 200-299 | - | - | - | - | - | - | - | - | - | - | 0 | 0 | 0 | 0 |
| | 300-399 | 2 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 2 |
| | 400-499 | 1 | 0 | 1 | 2 | 4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| | 500-599 | 0 | 1 | 1 | 0 | 2 | 0 | 2 | 0 | 3 | 5 | 0 | 2 | 0 | 2 |
| | 600-699 | - | - | - | - | - | 1 | 0 | 1 | 0 | 2 | 0 | 0 | 0 | 0 |
| | 700-799 | - | - | - | - | - | 3 | 0 | 0 | 0 | 3 | 0 | 1 | 0 | 1 |
| | 800-899 | - | - | - | - | - | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| | Total | 3 | 1 | 2 | 2 | 8 | 4 | 2 | 2 | 4 | 12 | 2 | 4 | 2 | 8 |
| Basal area per acre ($ft^2/ac$) | 60-79 | 2 | 0 | 0 | 0 | 2 | - | - | - | - | - | 1 | 0 | 0 | 1 |
| | 80-99 | 1 | 1 | 1 | 2 | 5 | - | - | - | - | - | 1 | 0 | 2 | 3 |
| | 100-119 | 0 | 0 | 1 | 0 | 1 | 3 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 |
| | 120-139 | - | - | - | - | - | 1 | 1 | 1 | 2 | 5 | 0 | 1 | 0 | 1 |
| | 140-159 | - | - | - | - | - | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 0 | 2 |
| | 160-179 | - | - | - | - | - | 0 | 1 | 1 | 1 | 3 | 0 | 1 | 0 | 1 |
| | Total | 3 | 1 | 2 | 2 | 8 | 4 | 2 | 2 | 4 | 12 | 2 | 4 | 2 | 8 |
| Site index | 40 | 0 | 0 | 0 | 1 | 1 | - | - | - | - | - | - | - | - | - |
| | 50 | 2 | 1 | 1 | 1 | 5 | - | - | - | - | - | 0 | 2 | 0 | 2 |
| | 60 | 1 | 0 | 1 | 0 | 2 | 2 | 0 | 1 | 3 | 6 | 1 | 2 | 1 | 4 |
| | 70 | - | - | - | - | - | 1 | 1 | 1 | 1 | 4 | 1 | 0 | 1 | 1 |
| | 80 | - | - | - | - | - | 1 | 1 | 0 | 0 | 2 | - | - | - | - |
| | Total | 3 | 1 | 2 | 2 | 8 | 4 | 2 | 2 | 4 | 12 | 2 | 4 | 2 | 8 |

Table 2.5. Distribution of loblolly pine plots after thinning by age, trees per acre, basal area per acre and site index in piedmont, upper and lower coastal plain provinces.

| | | **Piedmont** | | | | | **Upper Coastal Plain** | | | | | **Lower Coastal Plain** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Age (years) | | | | | Age (years) | | | | | Age (years) | | | |
| Stand parameter | | 15 | 16 | 17 | 18 | | 12 | 14 | 16 | 17 | | 15 | 18 | 19 | |
| | | number of plots | | | | Total | number of plots | | | | Total | | | | Total |
| Trees per acre | 100-199 | 3 | 1 | 1 | 1 | 6 | 0 | 0 | 0 | 4 | 4 | 0 | 0 | 11 | 11 |
| | 200-299 | 11 | 6 | 11 | 8 | 36 | 2 | 8 | 5 | 18 | 33 | 10 | 2 | 7 | 19 |
| | 300-399 | 9 | 5 | 3 | 7 | 24 | 13 | 9 | 10 | 14 | 46 | 8 | 11 | 0 | 19 |
| | 400-499 | 1 | 2 | 1 | 2 | 6 | 14 | 1 | 3 | 0 | 18 | 0 | 18 | 0 | 18 |
| | 500-599 | - | - | - | - | - | 7 | 0 | 0 | 0 | 7 | 0 | 5 | 0 | 5 |
| | Total | 24 | 14 | 16 | 18 | 72 | 36 | 18 | 18 | 36 | 108 | 18 | 36 | 18 | 72 |
| Basal area per acre ($ft^2$/ac) | 20-39 | 3 | 0 | 0 | 0 | 3 | - | - | - | - | - | - | - | - | - |
| | 40-59 | 7 | 7 | 9 | 6 | 29 | 8 | 1 | 2 | 0 | 11 | 6 | 0 | 6 | 12 |
| | 60-79 | 9 | 6 | 6 | 10 | 31 | 18 | 1 | 6 | 3 | 28 | 11 | 9 | 5 | 25 |
| | 80-99 | 5 | 1 | 1 | 1 | 8 | 10 | 6 | 8 | 19 | 43 | 1 | 19 | 5 | 25 |
| | 100-119 | 0 | 0 | 0 | 1 | 1 | 0 | 6 | 2 | 10 | 18 | 0 | 8 | 2 | 10 |
| | 120-139 | - | - | - | - | - | 0 | 4 | 0 | 4 | 8 | - | - | - | - |
| | Total | 24 | 14 | 16 | 18 | 72 | 36 | 18 | 18 | 36 | 108 | 18 | 36 | 18 | 72 |
| Site index | 40 | 0 | 0 | 0 | 3 | 3 | - | - | - | - | - | - | - | - | - |
| | 50 | 7 | 11 | 5 | 13 | 36 | - | - | - | - | - | 0 | 11 | 0 | 11 |
| | 60 | 17 | 3 | 11 | 2 | 33 | 8 | 1 | 11 | 3 | 23 | 7 | 25 | 5 | 37 |
| | 70 | - | - | - | - | - | 23 | 10 | 7 | 26 | 66 | 10 | 0 | 11 | 21 |
| | 80 | - | - | - | - | - | 5 | 7 | 0 | 7 | 19 | 1 | 0 | 2 | 3 |
| | Total | 24 | 14 | 16 | 18 | 72 | 36 | 18 | 18 | 36 | 108 | 18 | 36 | 18 | 72 |

$$P_{95} = a + b\left[-\ln(0.05)\right]^{1/c} \tag{2.21}$$

Solving for *c* using equations (2.20) and (2.21) and assuming *a* is known yield:

$$\hat{c} = \frac{2.343088}{\left(\ln(P_{95} - \hat{a}) - \ln(P_{25} - \hat{a})\right)} \tag{2.22}$$

Weibull *b* parameter estimate is obtained using the second moment of a three parameter Weibull distribution function and assuming *a* and *c* are known, that is,

$$E[X^2] = b^2\Gamma\left(1 + 2/c\right) + 2ab\Gamma\left(1 + 1/c\right) + a^2 \tag{2.23}$$

Letting

$$\Gamma_1 = \Gamma\left(1 + 1/c\right) \tag{2.24a}$$

$$\Gamma_2 = \Gamma\left(1 + 2/c\right) \tag{2.24b}$$

and re-arranging equation (2.23) to obtain a quadratic in *b* gives:

$$b^2\Gamma_2 + 2ab\Gamma_1 + a^2 - E[X^2] = 0 \tag{2.25}$$

Solving for *b* using quadratic formula and using the positive root the estimate for *b* is:

$$\hat{b} = -\hat{a}\left(\Gamma_1/\Gamma_2\right) + \left[\left(\hat{a}^2/\Gamma_2^2\right)\left(\Gamma_1^2 - \Gamma_2\right) + E[X^2]/\Gamma_2\right]^{1/2} \tag{2.26}$$

Quadratic mean diameter squared ($Dq^2$) is substituted for $E[X^2]$ in equation (2.26).

This procedure constrains the predicted stand table to have the same quadratic diameter as that obtained from trees per acre and basal area per acre. To apply this procedure estimates of percentiles are required. These are predicted from stand information that include age, trees per acre, basal area per acre, site index and/or dominant and codominant height using linear regression analysis.

This study developed operational percentile prediction equations for the following most common cases:  thinned stand case 1 (TSC1) where complete information about before and after thin stand is available; thinned stand case 2 (TSC2) where information about thinning type, age of thin and after thin dominant height, basal area per acre and trees per acre are available; thinned stand case 3 (TSC3) where information about thinning intensity, age of thin and after thin  dominant height, basal area per acre and trees per acre are available; thinned stand case 4 (TSC4) where information about age and type of thinning is available; thinned stand case 5 (TSC5) where age of thinning is known and thinned stand case 6 (TSC6) where no thinning information is available. For ease of discussion abbreviation for each case will be used. A brief description of how percentile prediction equations were fitted follows.

For TSC1 where complete thinning information is available, immediately after thin stand percentiles were predicted by adjusting before thin stand information such as percentiles, basal area per acre and trees per acre. This adjustment was achieved using a thinning term. Candidate terms that were initially considered included ratios of before and after thin basal area per acre, trees per acre and quadratic mean diameter. These ratios were well correlated with after thin percentiles. The best ratio was obtained by running all possible regression equations between selected base percentile prediction equations and the thinning ratios. Fit statistics of these runs indicated that the ratio of quadratic mean diameter of trees after thinning to the quadratic mean diameter of trees before thinning age performed well in both slash and loblolly pine plantations. This ratio had the lowest root mean square error (RMSE) and largest percent variation explained (PVE). In the next step thinned stand percentiles for TSC1 were predicted from before thin, immediately after thin and thinned stand information.

For TSC2 and TSC3 cases percentile prediction equations were also developed for both immediately after thin and thinned stands. In both cases the development was carried out over two phases. In the first phase percentiles were predicted directly from known stand information. In the second phase indicator variables were introduced into the best percentile prediction equations to represent type (TSC2) and intensity (TSC3) of thinning. For TSC4, TSC5 and TSC6 percentiles were predicted directly form thinned stand information such as age, age of thin (for TSC4 and TSC5), dominant and co-dominant height, site index, trees per acre, basal area per acre and their transformations. Type of thin was introduced into TSC4 equations using an indicator variable.

Preliminary analyses indicated that physiographic province was significant. As such separate fits for each region were generated. Goodness of fit measures namely percent variation explained and root mean square error were calculated for each equation as follows:

$$PVE = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{2.27}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} \tag{2.28}$$

where,

PVE = percent variation explained

RMSE = root mean square error

$y_i$ = observed $i^{th}$ percentile

$\bar{y}$ = mean $i^{th}$ percentile

$\hat{y}$ = predicted $i^{th}$ percentile n = number of observations

Predicted stand tables were compared with observed stand tables using Kolmogrov-Smirnoff (KS) two-sample test at significance levels of 0.01, 0.05 and 0.1. Predicted stand tables were further assessed using the error index suggested by Reynolds et al. (1988). This index is defined as the weighted sum of the absolute differences between predicted and observed numbers of diameters in each diameter class:

$$e = \sum_{j=1}^{k} \left| w(x)(N_p - N_O) \right| \quad (2.29)$$

where,

e = error index

w(x) = weighting factor

$N_p$ = predicted number of trees in dbh class

$N_o$ = observed number of trees per acre in dbh class

j = 1,…, k dbh class

Total volume was used as the weight in this study. Since *e* in equation (2.29) is the sum of the absolute differences this means that an over prediction in one dbh class will not offset an under prediction in another dbh class. In this case error is only small when the model performs well in all dbh classes (Reynolds et al. 1988). For simulation purposes slash and loblolly pine stand survival and basal area per acre were predicted using Logan (2005) and Borders et al (2004) whole stand projection equations (Appendices A and B).

## Results and Analysis

### Slash pine

Preliminary analysis of diameter distributions in slash pine plantations indicated that first thinning changed the distributions significantly. These changes depended on the type and intensity of thinning (Figure 2.2). Relative distributions after row thinning (R33, R40 and R50) remained essentially the same as pre-thin relative distributions. After selective thinning (S33, S40, and S50) there was a shift in the distributions towards larger trees. The result of this was narrower distributions and significant increases in the $0^{th}$ and $25^{th}$ diameter percentiles. Corresponding relative diameter class frequencies of these distributions (Table 2.6) show that in row thinned stands relative frequencies remained the same or fairly close to those of the pre-thinned stand. In select thinned stands the relative frequencies of small dbh classes (2 – 5 inch) were smaller than before thin and for larger dbh classes they were larger or equal to pre-thin frequencies. These differences were reflected in a KS two-sample test that compared pre-thin and after thin distributions. 6 of 126 plots were rejected. Of these plots 5 were from select thinned, 1 from row-select thinned and none from row thinned plots. Other initial analysis indicated that over time thinned and unthinned diameter distributions remained generally unimodal. However at the end of the growth period average increase in $0^{th}$, $25^{th}$, and $50^{th}$ percentiles were significantly larger in selective and row-selective thinned stands than in unthinned stands. Percentile prediction equation regression results follow for unthinned, immediately after thin and thinned slash pine stands.

### Unthinned stands

Several existing percentile prediction equations were evaluated for this study. Equations developed by Borders et al. (1990) were found to fit the data well. Fitted equations are presented in Table 2.8 and their fit statistics are given in Tables 2.11 and 2.12. All parameter estimates in the equations were significant at $\alpha = 0.05$. Large PVE and small RMSE values were obtained for all equations except the $0^{th}$ percentile prediction equation (2.31a) in lower coastal plain. Residual plots did not indicate serious violation of regression assumptions and plots of percentile development curves seemed reasonable.

KS two-sample test detected no significant difference at commonly used alpha levels of 0.01, 0.05 and 0.1 between predicted and observed stand tables in lower and upper coastal plain provinces. The mean difference between observed and predicted quadratic mean diameters was 0.05 inches in both physiographic regions. In lower coastal plain 80% and in upper coastal plain 89% of the predicted quadratic mean diameters were within ±0.1 inches of the observed quadratic mean diameters. Error indices were 93.69 in lower coastal plain and 84.81 in upper coastal plain. These results compare well with previous work using the Weibull to model diameter distributions of unthinned slash pine plantations. Bailey et al. (1981) reported a rejection rate of 3.1% based on KS test and 78 percent of predicted quadratic mean diameters differed from observed quadratic mean diameters by ±0.5 inches. Clutter and Belcher (1978) reported 99% of quadratic mean diameters were within ±0.5 inches of the observed quadratic mean diameters.

For an example of how fitted percentile prediction equations were used to recover the stand tables consider a 15 year old unthinned slash pine stand that was established in the lower coastal plain and has 450 trees per acre, 90 $ft^2$ of basal area and site index of 60 feet (base age 25 years). $P_0$, $P_{25}$, $P_{50}$ and $P_{95}$ were predicted using Equations (2.31a), (2.31b), (2.31c) and (2.31d). These predictions were substituted for in Equations (2.19), (2.22) and (2.26) to obtain Weibull parameter estimates. The proportion of trees per acre in each dbh class for example in the 5 inch dbh class was obtained by subtracting the value of the lower diameter class limit Weibull cumulative distribution function (cdf) from the value of the upper diameter class limit Weibull cdf as shown below:

$$\begin{aligned} \text{Prob}_5 &= \left\{1-\exp\left[-\left(\frac{5.5-0.65738}{5.74949}\right)\right]^{4.84907}\right\} \\ &\quad -\left\{1-\exp\left[-\left(\frac{4.5-0.65738}{5.74949}\right)\right]^{4.84907}\right\} \\ &= 0.22062 \end{aligned} \tag{2.30}$$

The predicted number of trees per acre in each diameter class was obtained by multiplying the current stand trees per acre by the proportion obtained e.g. for 5 inch class above: 0.22062*450 = 99.28 (Table 2.7).

**Immediately after thin stands**

Fitted percentile prediction equations for immediately after thin slash pine TSC1, TSC2 and TSC3 are presented in Table 2.9. All coefficients in the equations were significantly different from zero ($\alpha = 0.05$) except the intercept for equation (2.33c). Reasonably good fit statistics (Tables 2.11 and 2.12) were obtained for all equations except for the $0^{th}$ percentile prediction equation in lower coastal plain. A plot of the residuals against predicted values for all equations did not indicate serious departures from regression assumptions. Percentile development curves implied by all fitted equations seemed logical with none crossing each other within the 45-year growth period. The KS two-sample test ($\alpha = 0.01$, 0.05 and 0.1) did not detect a significant difference between observed and predicted stand tables in both lower and upper coastal plains. The smallest error index was obtained with after thin TSC1 equations and the largest with after thin TSC3 equations in both provinces (Tables 2.11 and 2.12).

**Thinned stands**

Fitted operational percentile prediction equations for thinned slash plantations in lower and upper coastal plain are presented in Table 2.10. All parameter estimates in the equations were statistically significant at $\propto = 0.05$. Large PVE and small RMSE values (Tables 2.11 and 2.12) were obtained for the 25$^{th}$, 50$^{th}$ and 95$^{th}$ percentile prediction equations in both regions. Relatively poor fit statistics were obtained for the 0$^{th}$ percentile in lower coastal plain particularly for TSC4, TSC5 and TSC6. However residual plots of all equations did not did not indicate any serious problems and plots of predicted percentile development curves appeared reasonable (Figures 2.5-2.6, 2.9-2.10).

KS two-sample tests indicated no significant difference (at ($\alpha$ = 0.01, 0.05 and 0.1) between observed and predicted stand tables for all cases. For comparison with previous work modeling thinned slash pine distributions Bailey et al (1981) reported none of the predicted distributions of repeatedly slash pine plantations to be significantly different. Borders et al. (1987) using percentile based method to characterize slash pine distributions reported 96% of predicted stand tables not to be significantly different.

In lower coastal plain TSC2 had the smallest error index and TSC5 and TSC6 had the largest error index (Table 2.11). In upper coastal plain TSC1 had the smallest and TSC6 had the largest error index (Table 2.12). However error indices for TSC1 and TSC2 were fairly similar in both regions.

When second thinned diameter distributions were predicted using developed equations none were rejected by KS two-sample test. Overall error indices obtained with this prediction were as follows: 142.86 for TSC1, 167.36 for TSC2, 165.41 for TSC3, 154.63 for TSC4, 153.33 for TSC5 and 152.43 for TSC 6.

**Simulation**

Application of fitted percentile prediction to estimate future stand structure of thinned stands requires site index and estimates of future surviving trees per acre and basal area per acre. This was obtained using whole stand functions for slash pine plantations developed by Logan (2005). For an illustration of how simulation proceeded consider a 15 year old unthinned slash pine plantation in lower coastal plain that has 450 trees per acre each, 40 $ft^2$ per acre of basal area and site index of 40 feet. Suppose this stand was thinned from below at age 15 to 300 trees per acre, and to 29 $ft^2$ per acre basal area. Given estimates of future survival and basal area per acre for this stand, prediction of future percentiles and diameter distributions proceeded as follows for each of the cases considered. For TSC1, equations (2.31a)-(2.31d), (2.33a)-(2.33d) and (2.39a)-(2.39d) were used to predict before thin, immediately after thin and future thinned stand percentiles and diameter distributions respectively. Immediately after thin and future thinned stand percentiles and diameter distributions were determined using equations (2.35a)-(2.35d) and (2.41a)-(2.41d) for TSC2, and equations (2.37a)-(2.37d) and (2.43a)-(2.43d) for TSC3. Future thinned stand percentiles and diameter distributions were predicted using equations (2.45a)-(2.45d) for TSC4, and (2.47a)-(2.47d) for both TSC5 and TSC6. TSC1 and TSC2 simulations for site indices 40 and 70 and, 300 and 600 starting trees per acre are presented for illustration.

Predicted before and immediately after thin diameter distributions, (Figures 2.3-2.4), indicated a shift of the distribution towards the right and, narrower after thin distributions than before thin. This reflected selective thinning from below that was assumed for the stand under consideration. The number of trees in larger diameter classes in after thin stand was close to that of pre-thin stand and generally, after thin distributions of TSC1 were relatively similar to those of TSC2. Simulated percentile development curves and diameter distributions (Figures 2.5–2.12) indicated trends that were quite reasonable. There was no percentile cross over. In addition, these plots indicated trends that are commonly expected in even-aged stands such as for a given site index starting trees per acre was inversely related with tree size, and for a given starting trees per acre site index was directly related with tree size.

**Loblolly pine**

Preliminary analysis of before and after thin diameter distributions of loblolly pine plantations data indicated changes that were dictated by thinning type and intensity (Figure 2.13 and Table 2.13). In row thinned stands (R33, R40, R50) there was essentially no change in the shape of the distributions compared to pre-thin distributions. With thinning from below there was a shift of the distributions towards larger dbh classes and as selective thinning increased in intensity (S40 and S50) after thin distributions became more narrow than pre-thin distributions. These differences were reflected in a KS two-sample test that compared pre-thin and after thin distributions. In 23 of 252 plots that were rejected 20 of these plots were from select thinned, 3 from row-select thinned and none from row thinned plots. Other initial results indicated that over time thinned and unthinned distributions remained generally unimodal. However at the end of the growth period average increase in $0^{th}$, $25^{th}$, and $50^{th}$ percentiles were significantly larger in selective and row-selective thinned stands than in unthinned stands. Regression results for unthinned, immediately after thin and thinned stand percentile prediction equations follow.

**Unthinned Stands**

As in unthinned slash pine plantations Border's et al (1990) percentile prediction equations were found to fit well unthinned loblolly pine data. All coefficients in the equations (Table 2.14) were significantly different from zero at $\alpha = 0.05$. Reasonably large PVE values (Tables 2.17-2.19) were obtained in all regions except for the $0^{th}$ percentile prediction equation

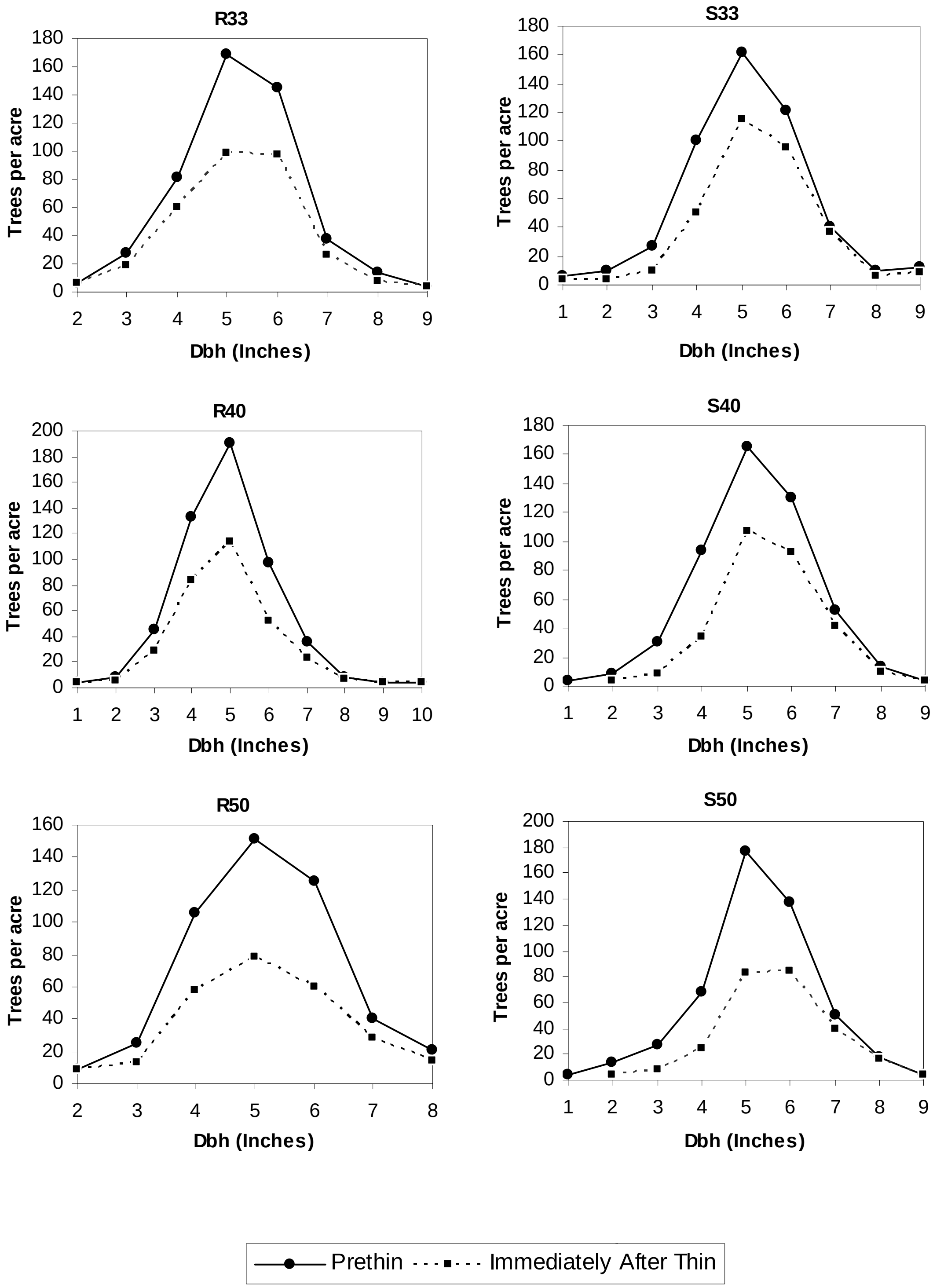



Figure 2.2. Average pre-thin and immediately after thin diameter distributions by thinning treatment for young age class slash pine plantation in lower coastal plain.

Table 2.6. Relative diameter class frequencies for diameter distributions presented in Figure 2.2.

| Dbh Class | Pre-R33 | After R33 | Pre-R40 | After R40 | Pre-R50 | After R50 | Pre-S33 | After S33 | Pre-S40 | After S40 | Pre-S50 | After S50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | 0.01 | 0.01 | | | 0.01 | 0.01 | 0.01 | | 0.01 | |
| 2 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 | 0.02 | 0.01 | 0.02 | 0.01 | 0.03 | 0.01 |
| 3 | 0.06 | 0.06 | 0.09 | 0.09 | 0.05 | 0.05 | 0.06 | 0.03 | 0.06 | 0.03 | 0.05 | 0.03 |
| 4 | 0.17 | 0.19 | 0.25 | 0.26 | 0.22 | 0.22 | 0.21 | 0.15 | 0.19 | 0.11 | 0.14 | 0.09 |
| 5 | 0.35 | 0.31 | 0.37 | 0.36 | 0.32 | 0.30 | 0.33 | 0.35 | 0.33 | 0.35 | 0.35 | 0.32 |
| 6 | 0.30 | 0.31 | 0.19 | 0.17 | 0.26 | 0.23 | 0.25 | 0.29 | 0.26 | 0.31 | 0.28 | 0.32 |
| 7 | 0.08 | 0.08 | 0.07 | 0.07 | 0.09 | 0.11 | 0.08 | 0.11 | 0.11 | 0.14 | 0.10 | 0.15 |
| 8 | 0.03 | 0.02 | 0.01 | 0.02 | 0.04 | 0.05 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.06 |
| 9 | 0.01 | 0.01 | 0.01 | 0.01 | | | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 |
| 10 | | | 0.01 | 0.01 | | | | | | | | |

Table 2.7. Predicted stand table for 15 year old unthinned slash pine plantation in the lower coastal plain.

| Dbh Class | Probability | Trees/ac |
|---|---|---|
| 1 | 0.00009 | 0.04 |
| 2 | 0.00392 | 1.76 |
| 3 | 0.02832 | 12.74 |
| 4 | 0.09981 | 44.91 |
| 5 | 0.22062 | 99.28 |
| 6 | 0.30801 | 138.60 |
| 7 | 0.24153 | 108.69 |
| 8 | 0.08667 | 39.00 |
| 9 | 0.01104 | 4.97 |
| Total | 1 | 450 |

Table 2.8. Fitted percentile prediction equations for unthinned slash pine plantations in lower and upper coastal plain.

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| Unthinned | Lower coastal plain | $\text{In}(P_{0u}) = 1.3195 + 0.3394\text{In}(B_u/N_u)$ | (2.31a) |
| | | $\text{In}(P_{25u}) = 2.4021 + 0.4816\text{In}(B_u/N_u)$ | (2.31b) |
| | | $\text{In}(P_{50u}) = 2.6274 + 0.5166\text{In}(B_u/N_u)$ | (2.31c) |
| | | $\text{In}(P_{95u}) = 2.8496 + 0.4909\text{In}(B_u/N_u)$ | (2.31d) |
| | Upper coastal plain | $\text{In}(P_{0u}) = 6.57423 - 1.006\text{In}(H) + 1.1095\text{In}(B_u/N_u)$ | (2.32a) |
| | | $\text{In}(P_{25u}) = 2.378 + 0.4738\text{In}(B_u/N_u)$ | (2.32b) |
| | | $\text{In}(P_{50u}) = 2.5876 + 0.495\text{In}(B_u/N_u)$ | (2.32c) |
| | | $\text{In}(P_{95u}) = 2.9056 + 0.5199\text{In}(B_u/N_u)$ | (2.32d) |

where,

$P_{iu}$ = $i^{th}$ percentile of unthinned stand table (i = 0, 25, 50, 95)

$B_u$ = unthinned stand basal area per acre

$N_u$ = unthinned stand number of trees per acre

ln = natural logarithm

Table 2.9. Fitted percentile prediction equations for immediately after thin slash pine plantations in lower and upper coastal plain.

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| After thin TSC1 | Lower coastal plain | $\ln(P_{0at}) = -1.5596 + 0.4659\ln(B_{bt}/N_{bt}) + 3.3422R_d$ | (2.33a) |
| | | $\ln(P_{25at}) = -1.4247 + 0.00262S + 0.8616\ln(P_{25bt}) + 1.4891R_d$ | (2.33b) |
| | | $\ln(P_{50at}) = 0.1499 + 0.2318(B_{at}/N_{at}) + 0.5562\ln(P_{50bt}) + 1.0174R_d$ | (2.33c) |
| | | $\ln(P_{95at}) = -0.302 + 0.9479\ln(P_{95bt}) + 0.4009R_d$ | (2.33d) |
| | Upper coastal plain | $\ln(P_{0at}) = -3.0324 + 0.8565\ln(D_{qat}) + 0.4428\ln(P_{0bt}) + 2.1584R_d$ | (2.34a) |
| | | $\ln(P_{25at}) = -0.9891 + 0.4387\ln(D_{qat}) + 0.5919\ln(P_{25bt}) + 0.8783R_d$ | (2.34b) |
| | | $\ln(P_{50at}) = -0.872 + 0.9847\ln(P_{50bt}) + 0.8972R_d$ | (2.34c) |
| | | $\ln(P_{95at}) = -0.6274 + 1.014\ln(P_{95bt}) + 0.599R_d$ | (2.34d) |
| After thin TSC2 | Lower coastal plain | $\ln(P_{0at}) = 1.994 + 0.5085\ln(B_{at}/N_{at}) - 2.4477Z_l/A_t$ | (2.35a) |
| | | $\ln(P_{25at}) = 2.496 + 0.5156\ln(B_{at}/N_{at}) - 0.0061Z_l\ln(B_{at})$ | (2.35b) |
| | | $\ln(P_{50at}) = 2.6367 + 0.5233(B_{at}/N_{at})$ | (2.35c) |
| | | $\ln(P_{95at}) = 2.7668 + 0.4607\ln(B_{at}/N_{at}) + 0.0887Z_l\ln(B_{at})/A_t$ | (2.35d) |
| | Upper coastal plain | $\ln(P_{0at}) = 2.4787 + 0.8135\ln(B_{at}/N_{at})$ | (2.36a) |
| | | $\ln(P_{25at}) = 2.4457 + 2.0577/A_t + 0.5792\ln(B_{at}/N_{at})$ | (2.36b) |
| | | $\ln(P_{50at}) = 2.54 + 0.4717(B_{bt}/N_{bt})$ | (2.36c) |
| | | $\ln(P_{95at}) = 2.9095 + 0.5358\ln(B_{at}/N_{at}) + 0.0006Z_lS$ | (2.36d) |

Table 2.9. Continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| After thin TSC3 | Lower coastal plain | $\ln(P_{0at}) = 2.1206 + 0.613\ln(B_{at}/N_{at})$ | (2.37a) |
| | | $\ln(P_{25at}) = 2.5083 + 0.5272\ln(B_{at}/N_{at})$ | (2.37b) |
| | | $\ln(P_{50at}) = 2.6367 + 0.5233(B_{at}/N_{at})$ | (2.37c) |
| | | $\ln(P_{95at}) = 2.7517 + 0.4477\ln(B_{at}/N_{at})$ | (2.37d) |
| | Upper coastal plain | $\ln(P_{0at}) = 2.4787 + 0.8135\ln(B_{at}/N_{at})$ | (2.38a) |
| | | $\ln(P_{25at}) = 2.4457 + 2.0577/A_t + 0.5792\ln(B_{at}/N_{at})$ | (2.38b) |
| | | $\ln(P_{50at}) = 2.54 + 0.4717(B_{at}/N_{at})$ | (2.38c) |
| | | $\ln(P_{95at}) = 2.9092 + 0.5291\ln(B_{at}/N_{at})$ | (2.38d) |

where,

$P_{iat}$ = $i^{th}$ percentile of after thin stand table (i = 0, 25, 50, 95)
$P_{ibt}$ = $i^{th}$ percentile of before thin stand table
$B_{at}$ = basal area per acre of after thin stand
$N_{at}$ = number of trees per acre of after thin stand
$B_{bt}$ = basal area per acre of before thin stand
$N_{bt}$ = number of trees per acre of before thin stand
$D_{qat}$ = quadratic mean diameter of the stand after thinning
S = site index in feet - base age 25
$R_d$ = ratio of quadratic mean diameter of the stand after thinning to the quadratic mean diameter of the stand before thinning
$A_t$ = age of the stand at time of first thinning

$$Z_1 = \begin{cases} 1 & \text{if stand is row thinned} \\ 0 & \text{if otherwise} \end{cases}$$

Table 2.10. Fitted operational percentile prediction equations for thinned slash pine plantations in lower and upper coastal plain.

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC1 | Lower coastal plain | $\ln(P_0) = 0.8254 + 11.7072/A + 0.2973\ln(B/N) + 0.8479\ln(P_{0at}) - 0.9056(A_t/A)R_d$ | (2.39a) |
| | | $\ln(P_{25}) = 1.7865 + 4.6687/A + 0.3959\ln(B/N) + 0.3769\ln(P_{25at}) - 0.4026(A_t/A)R_d$ | (2.39b) |
| | | $\ln(P_{50}) = 2.4025 + 1.6831/A + 0.4837\ln(B/N) + 0.1148\ln(P_{50at}) - 0.1422(A_t/A)R_d$ | (2.39c) |
| | | $\ln(P_{95}) = 2.0036 + 1.914/A + 0.0016S + 0.2738\ln(B/N) + 0.3796\ln(P_{95at}) - 0.5093(A_t/A)R_d$ | (2.39d) |
| | Upper coastal plain | $\ln(P_0) = 1.6931 - 6.7156/A_t + 0.7093\ln(P_{0at}) - 0.983(A_t/A)R_d$ | (2.40a) |
| | | $\ln(P_{25}) = 2.1228 + 4.4865/A - 0.0033S + 0.4926\ln(B/N) + 0.2691\ln(P_{25at}) - 0.2196(A_t/A)R_d$ | (2.40b) |
| | | $\ln(P_{50}) = 2.3836 + 1.583/A + 0.4706\ln(B/N) + 0.1122\ln(P_{50at}) - 0.1459(A_t/A)R_d$ | (2.40c) |
| | | $\ln(P_{50}) = 2.4249 - 2.4654/A + 0.3039\ln(B/N) + 0.2736\ln(P_{50at}) - 0.3037(A_t/A)R_d$ | (2.40d) |
| TSC2 | Lower coastal plain | $\ln(P_0) = 0.1605 + 9.6466/A_t + 0.3582\ln(B_{at}/N_{at}) + 0.8055\ln(P_{0at}) + 1.466Z_1/A + 0.8203Z_2\ln(P_{0at})/A$ | (2.41a) |
| | | $\ln(P_{25}) = 1.6156 + 3.9122/A_t + 0.4459\ln(B_{at}/N_{at}) + 0.3147\ln(P_{25at}) - 3.0375Z_2/A + 0.1163Z_2\ln(B_{at}/N_{at})$ | (2.41b) |
| | | $\ln(P_{50}) = 2.4339 + 0.8304/A_t + 0.4997\ln(B_{at}/N_{at}) + 0.0633\ln(P_{50at}) + 0.0035Z_1\ln(B_{at}/N_{at})$ | (2.41c) |
| | | $\ln(P_{95}) = 2.6951 - 2.1817/A + 0.4123\ln(B_{at}/N_{at}) + 0.0785\ln(P_{95at}) \; 0.0164Z_2\ln(B_{at}/N_{at})$ | (2.41d) |

Table 2.10 Continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC2 | Upper coastal plain | $\ln(P_0) = 1.4058 + 0.4876\ln(B_{at}/N_{at}) + 0.4473\ln(P_{0at})$ | (2.42a) |
| | | $\ln(P_{25}) = 2.2144 + 2.2806/A + 0.517\ln(B_{at}/N_{at}) + 0.0948\ln(P_{25at}) - 0.0258Z_1\ln(P_{25at}) + 0.01656Z_2\ln(B_{at}/N_{at})$ | (2.42b) |
| | | $\ln(P_{50}) = 2.3622 + 1.1547/A_t + 0.4866\ln(B_{at}/N_{at}) + 0.0735\ln(P_{50at})$ | (2.42c) |
| | | $\ln(P_{95}) = 2.7983 - 5.3857/A + 0.3509\ln(B_{at}/N_{at}) + 0.0752\ln(P_{95at}) - 0.0168Z_2\ln(B_{at}/N_{at})$ | (2.42d) |
| TSC3 | Lower coastal plain | $\ln(P_0) = 0.1799 + 9.5014/A_t + 0.3452\ln(B_{at}/N_{at}) + 0.8007\ln(P_{0at}) + 0.008653X\ln(N_{at})$ | (2.43a) |
| | | $\ln(P_{25}) = 1.6633 + 3.7394/A_t + 0.4423\ln(B_{at}/N_{at}) + 0.2896\ln(P_{25at})$ | (2.43b) |
| | | $\ln(P_{50}) = 2.4244 + 0.8921/A_t + 0.5012\ln(B_{at}/N_{at}) + 0.0676\ln(P_{50at}) - 0.0012X\ln(B_{at})$ | (2.43c) |
| | | $\ln(P_{95}) = 2.6749 - 2.6857/A + 0.3966\ln(B_{at}/N_{at}) + 0.087\ln(P_{95at})$ | (2.43d) |
| | Upper coastal plain | $\ln(P_0) = 1.5296 + 0.4582\ln(B_{at}/N_{at}) + 0.3212\ln(P_{0at}) - 0.41453X + 0.3157X\ln(P_{0at})$ | (2.44a) |
| | | $\ln(P_{25}) = 2.18143 + 2.26570/A + 0.5185\ln(B_{at}/N_{at}) + 0.1071\ln(P_{25at}) - 0.0082X\ln(P_{25at})$ | (2.44b) |
| | | $\ln(P_{50}) = 2.3905 + 1.0396/A_t + 0.4869\ln(B_{at}/N_{at}) + 0.0644\ln(P_{50at}) - 0.003X\ln(B_{at})$ | (2.44c) |
| | | $\ln(P_{95}) = 2.7896 - 4.9448/A + 0.3594\ln(B_{at}/N_{at}) + 0.0748\ln(P_{95at}) + 0.21803X/A_t$ | (2.44d) |

Table 2.10 Continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC4 | Lower coastal plain | $\ln(P_0) = 1.8345 + 0.5075\ln(B/N) + 0.7331Z_2\ln(B)/A$ | (2.45a) |
| | | $\ln(P_{25}) = 2.4136 + 1.7945/A + 0.5421\ln(B/N) + 0.2679Z_2/A_t$ | (2.45b) |
| | | $\ln(P_{50}) = 2.6088 + 0.5829/A + 0.5299\ln(B/N)$ | (2.45c) |
| | | $\ln(P_{95}) = 2.8723 - 1.6003/A + 0.4502\ln(B/N) - 0.3115Z_2/A_t$ | (2.45d) |
| | Upper coastal plain | $\ln(P_0) = 2.8374 - 11.2905/A_t + 0.6044\ln(B/N) + 8.0277Z_1A_t - 0.2977Z_1\ln(D_q)$ | (2.46a) |
| | | $\ln(P_{25}) = 2.3887 + 3.7645/A + 0.5911\ln(B/N) - 0.0201Z_1\ln(D_q) - 0.3239Z_2/A_t$ | (2.46b) |
| | | $\ln(P_{50}) = 2.5729 + 0.4864\ln(B/N) + 0.0071\ln(B/N)$ | (2.46c) |
| | | $\ln(P_{95}) = 2.9495 - 4.2768/A + 0.4083\ln(B/N) + 0.1866Z_1\ln(B) + 0.5225Z_2/A_t$ | (2.46d) |
| TSC5 | Lower coastal plain | $\ln(P_0) = 1.9188 + 0.5367\ln(B/N)$ | (2.47a) |
| | | $\ln(P_{25}) = 2.4201 + 2.1445/A + 0.5552\ln(B/N)$ | (2.47b) |
| | | $\ln(P_{50}) = 2.6088 + 0.5829/A + 0.5299\ln(B/N)$ | (2.47c) |
| | | $\ln(P_{95}) = 2.8648 - 2.0073/A + 0.4351\ln(B/N)$ | (2.47d) |
| | Upper coastal plain | $\ln(P_0) = 2.5798 - 8.1305/A_t + 0.5828\ln(B/N)$ | (2.48a) |
| | | $\ln(P_{25}) = 2.3601 + 4.1382/A + 0.6005\ln(B/N)$ | (2.48b) |
| | | $\ln(P_{50}) = 2.5737 + 0.4896\ln(B/N)$ | (2.48c) |
| | | $\ln(P_{95}) = 2.9559 - 3.8443/A + 0.4111\ln(B/N)$ | (2.48d) |

Table 2.10 Continued

| | | |
|---|---|---|
| TSC6 | Lower coastal plain | Equation (2.47a)<br>Equation (2.47b)<br>Equation (2.47c)<br>Equation (2.47d) |
| | Upper coastal plain | $\ln(P_0) = 1.9188 + 0.5367\ln(B/N)$ (2.49)<br>Equation (2.48b)<br>Equation (2.48c)<br>Equation (2.48d) |

where,

$$Z_1 = \begin{cases} 1 \text{ if stand is row thinned} \\ 0 \text{ if otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 \text{ if stand is select thinned} \\ 0 \text{ if otherwise} \end{cases}$$

$$X = \begin{cases} 1 \text{ if thinning intensity} > 33\% \\ 0 \text{ if otherwise} \end{cases}$$

All else are as defined as previously defined.

Table 2.11. Fit statistics and error indices for percentile prediction equations fitted to slash pine data in lower coastal plain.

| Stand | Equation | PVE | | | | RMSE | | | | Error Index |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | |
| Unthinned | 2.31a-2.31d | 10.58 | 94.34 | 98.68 | 96.89 | 0.63 | 0.19 | 0.11 | 0.22 | 93.69 |
| After Thin TSC1 | 2.33a-2.33d | 41.63 | 95.09 | 97.77 | 94.76 | 0.59 | 0.13 | 0.10 | 0.17 | 48.25 |
| After Thin TSC2 | 2.35a-2.35d | 37.33 | 91.33 | 97.40 | 94.76 | 0.61 | 0.18 | 0.10 | 0.21 | 51.31 |
| After Thin TSC3 | 2.37a-2.37d | 29.07 | 90.39 | 97.40 | 92.15 | 0.65 | 0.18 | 0.10 | 0.22 | 53.66 |
| TSC1 | 2.39a-2.39d | 70.74 | 95.16 | 98.23 | 93.37 | 0.52 | 0.18 | 0.13 | 0.27 | 101.63 |
| TSC2 | 2.41a-2.41d | 72.25 | 95.85 | 98.18 | 91.28 | 0.50 | 0.17 | 0.13 | 0.31 | 101.00 |
| TSC3 | 2.43a-2.43d | 71.50 | 95.43 | 98.19 | 92.91 | 0.51 | 0.18 | 0.13 | 0.32 | 102.58 |
| TSC4 | 2.45a-2.45d | 36.77 | 93.54 | 98.07 | 93.11 | 0.76 | 0.21 | 0.13 | 0.31 | 104.24 |
| TSC5 | 2.47a-2.47d | 28.23 | 93.30 | 98.07 | 92.70 | 0.81 | 0.21 | 0.13 | 0.32 | 106.54 |
| TSC6 | 2.47a-2.47d | 28.23 | 93.30 | 98.07 | 92.70 | 0.81 | 0.21 | 0.13 | 0.32 | 106.54 |

Table 2.12. Fit statistics and error indices for percentile prediction equations fitted to slash pine data in upper coastal plain.

| Stand | Equation | PVE | | | | RMSE | | | | Error Index |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | |
| Unthinned | 2.32a-2.32d | 72.42 | 95.46 | 99 | 97.74 | 0.47 | 0.20 | 0.12 | 0.24 | 84.81 |
| After Thin TSC1 | 2.34a-2.34d | 83.61 | 98.89 | 98.59 | 96.27 | 0.45 | 0.10 | 0.11 | 0.27 | 52.91 |
| After Thin TSC2 | 2.36a-2.36d | 76.60 | 98.15 | 98.07 | 95.22 | 0.54 | 0.12 | 0.13 | 0.31 | 57.16 |
| After Thin TSC3 | 2.38a-2.38d | 76.60 | 98.15 | 98.07 | 94.43 | 0.54 | 0.12 | 0.13 | 0.33 | 58.03 |
| TSC1 | 2.40a-2.40d | 71.61 | 95.54 | 98.21 | 94.86 | 0.70 | 0.24 | 0.17 | 0.39 | 112.64 |
| TSC2 | 2.42a-2.42d | 75.79 | 96.23 | 98.13 | 94.65 | 0.65 | 0.22 | 0.17 | 0.40 | 113.04 |
| TSC3 | 2.44a-2.44d | 78.01 | 95.36 | 98.29 | 94.59 | 0.61 | 0.24 | 0.16 | 0.39 | 115.81 |
| TSC4 | 2.46a-2.46d | 71.25 | 96.11 | 98.06 | 95.34 | 0.69 | 0.22 | 0.17 | 0.36 | 115.08 |
| TSC5 | 2.48a-2.48d | 65.82 | 94.95 | 98.02 | 94.16 | 0.76 | 0.25 | 0.17 | 0.41 | 118.87 |
| TSC6 | 2.49 2.48b-2.48d | 62.09 | 94.95 | 98.02 | 94.16 | 0.80 | 0.25 | 0.17 | 0.41 | 118.92 |

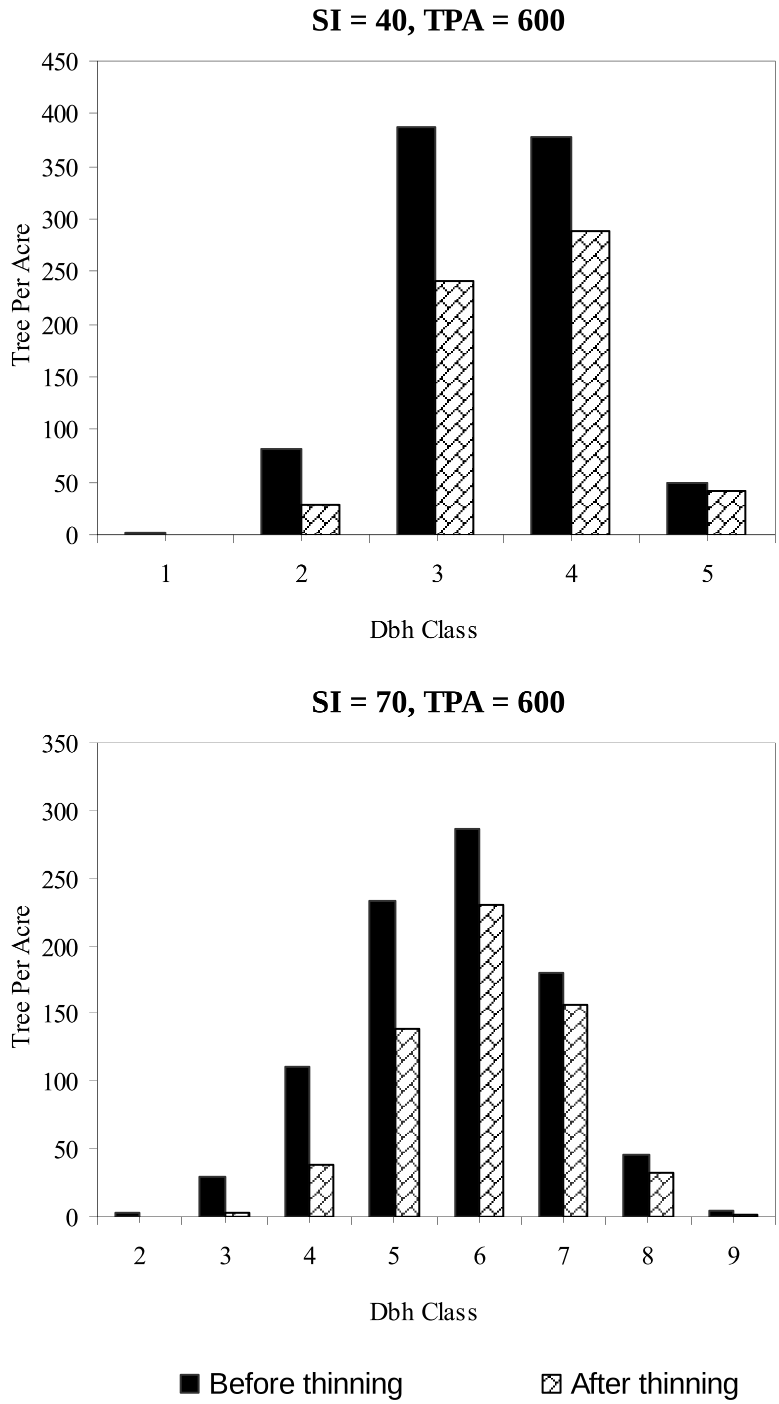



Figure 2.3. Predicted before (equations 2.31a–2.31d → unthinned TSC1) and immediately after thin (equations 2.33a – 2.33d → after thin TSC1) diameter distributions for a 15 year old slash pine stand thinned from below.

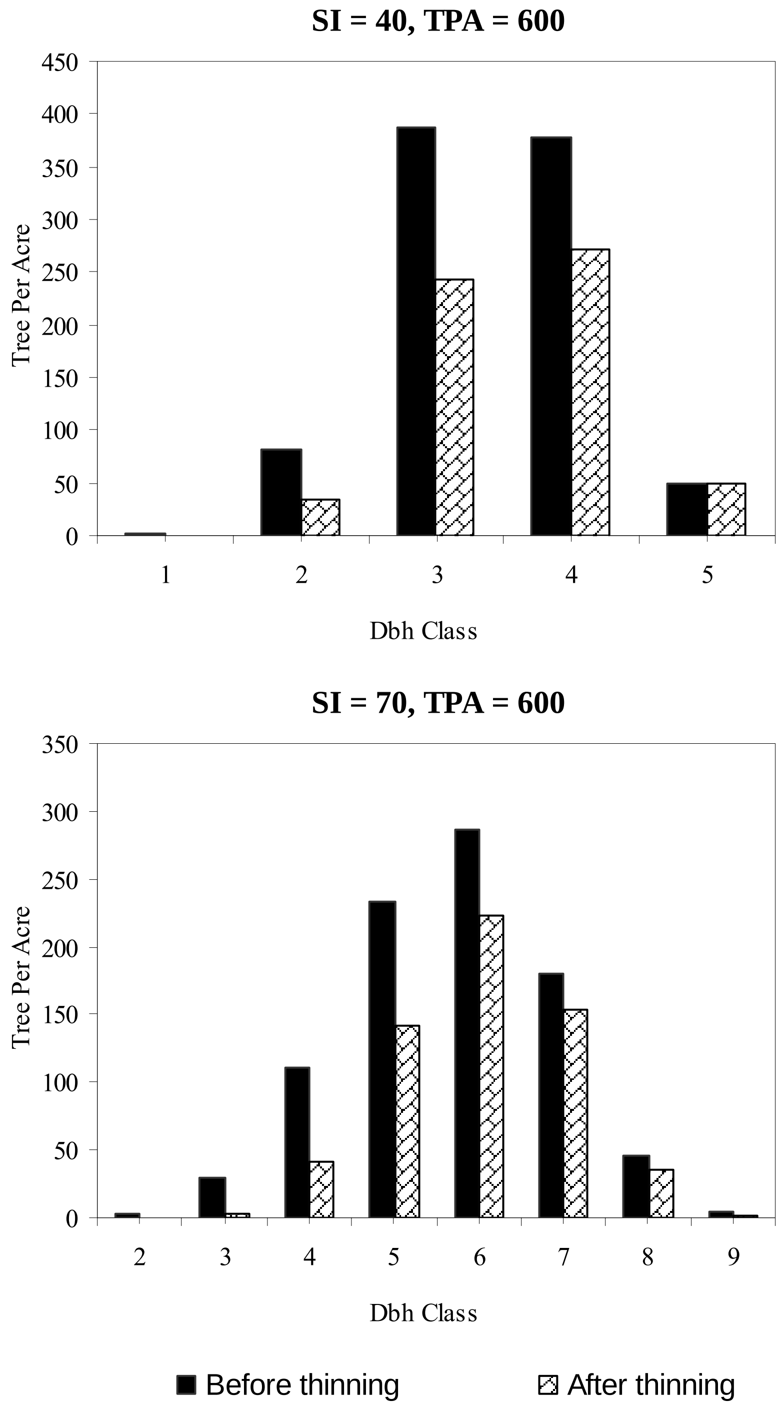



Figure 2.4. Predicted before (equations 2.31a–2.31d → unthinned TSC2) and immediately after thin (equations 2.35a – 2.35d → after thin TSC2) diameter distributions for a 15 year old slash pine stand thinned from below.

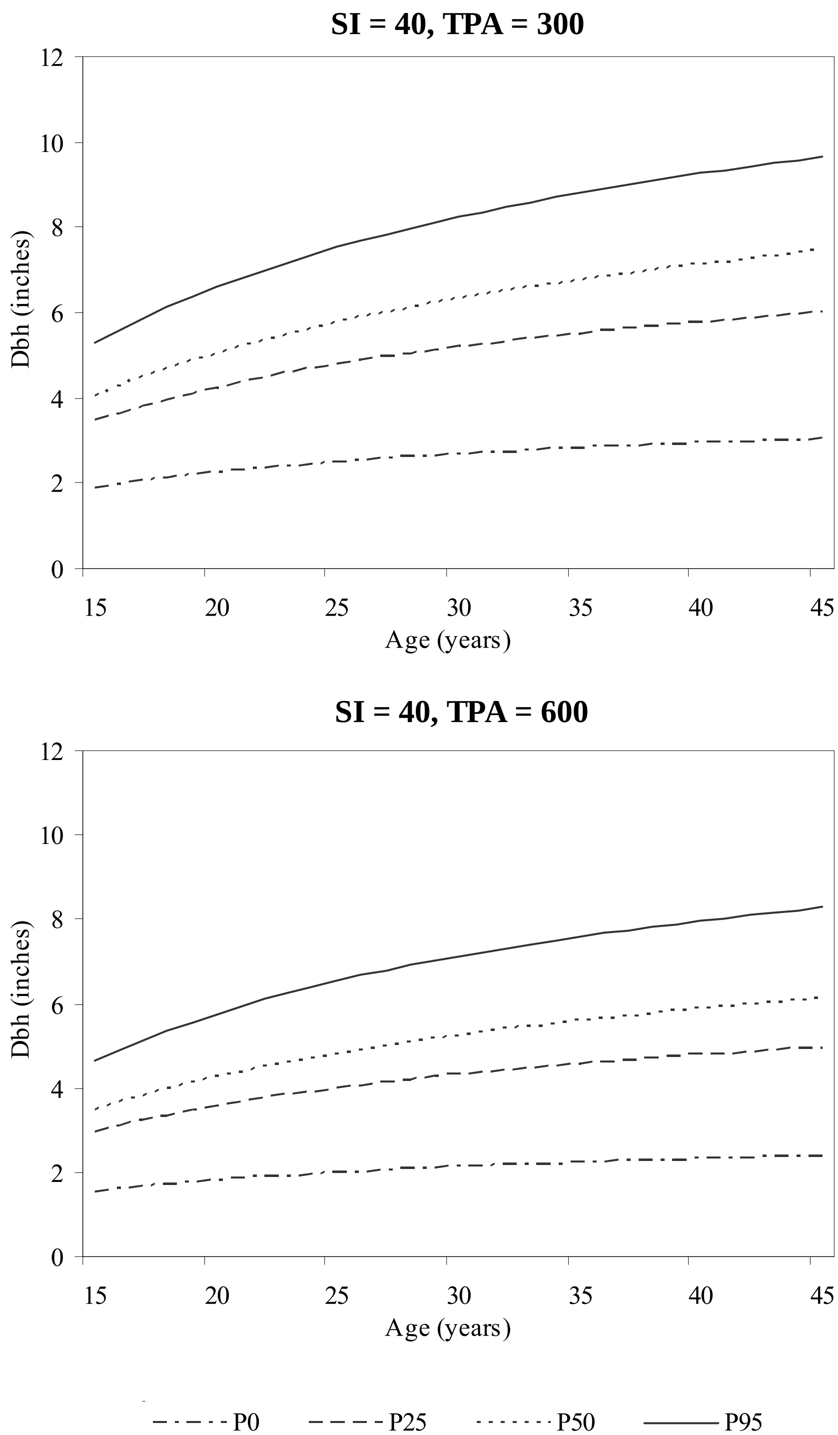



Figure 2.5. Projected thinned slash pine stand table percentiles (equations 2.39a–2.39d → TSC1) for site index 40 feet, 300 and 600 trees per acre.

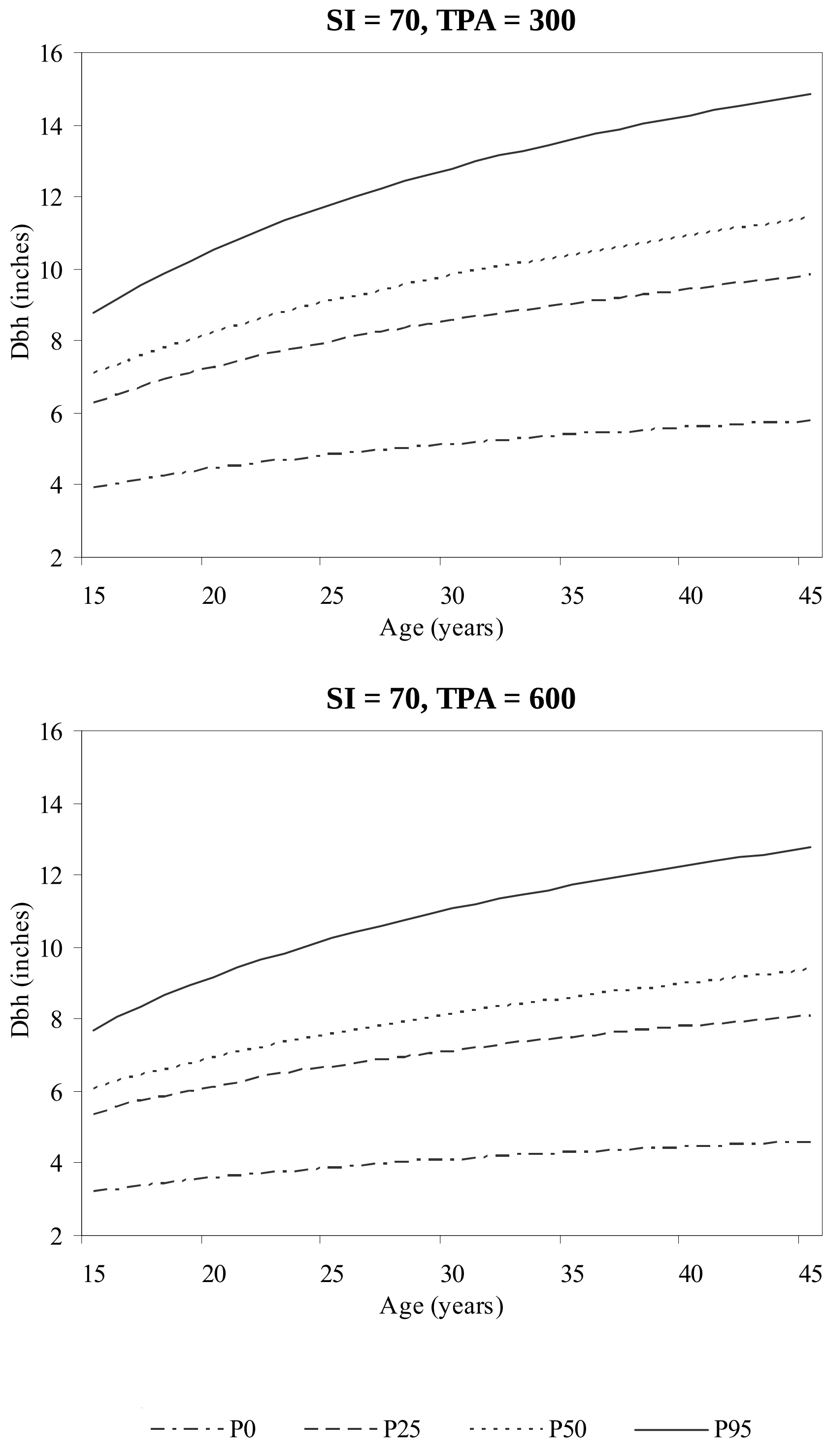



Figure 2.6. Projected thinned slash pine stand table percentiles (equations 2.39a–2.39d → TSC1) for site index 70 feet, 300 and 600 trees per acre.

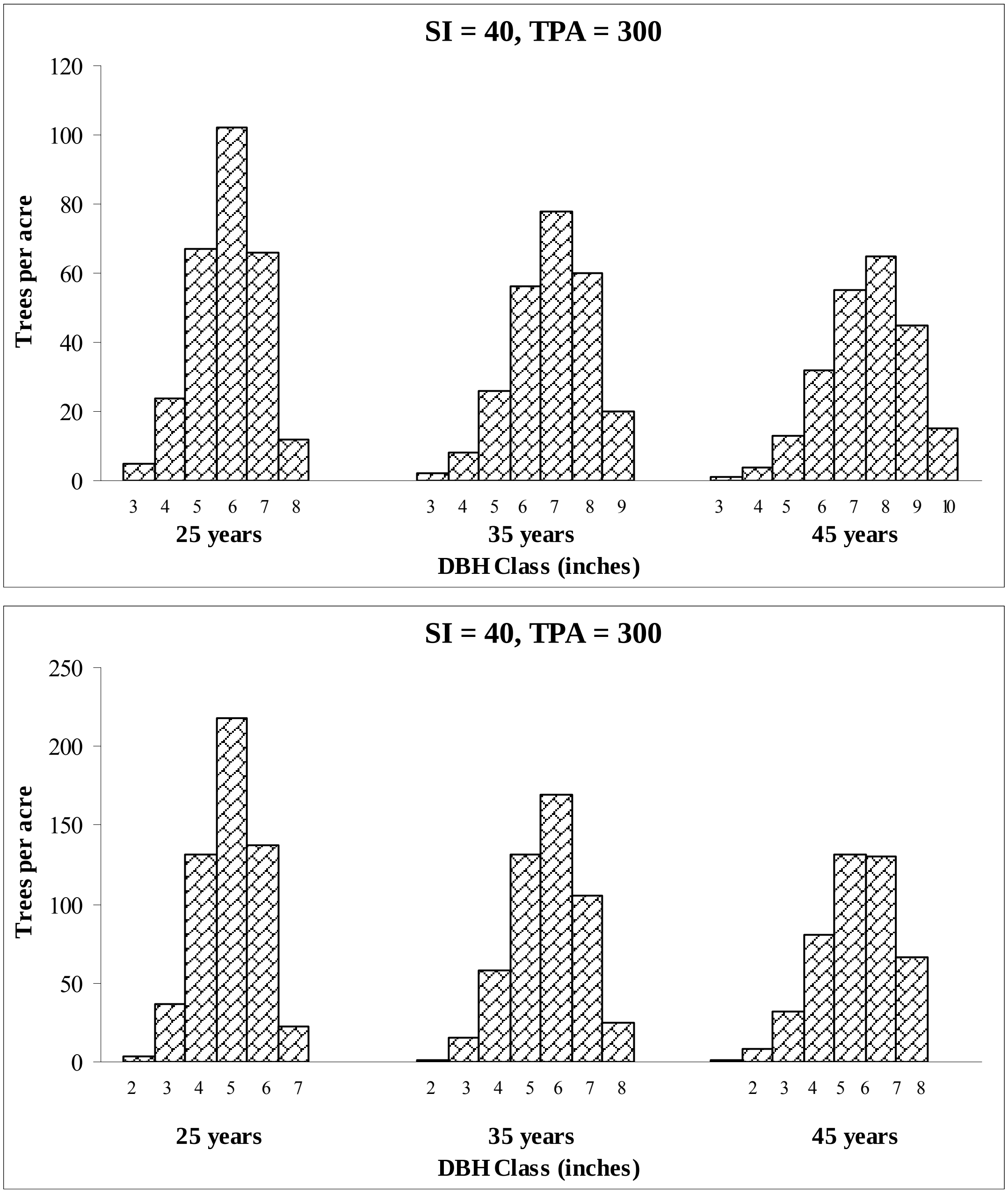
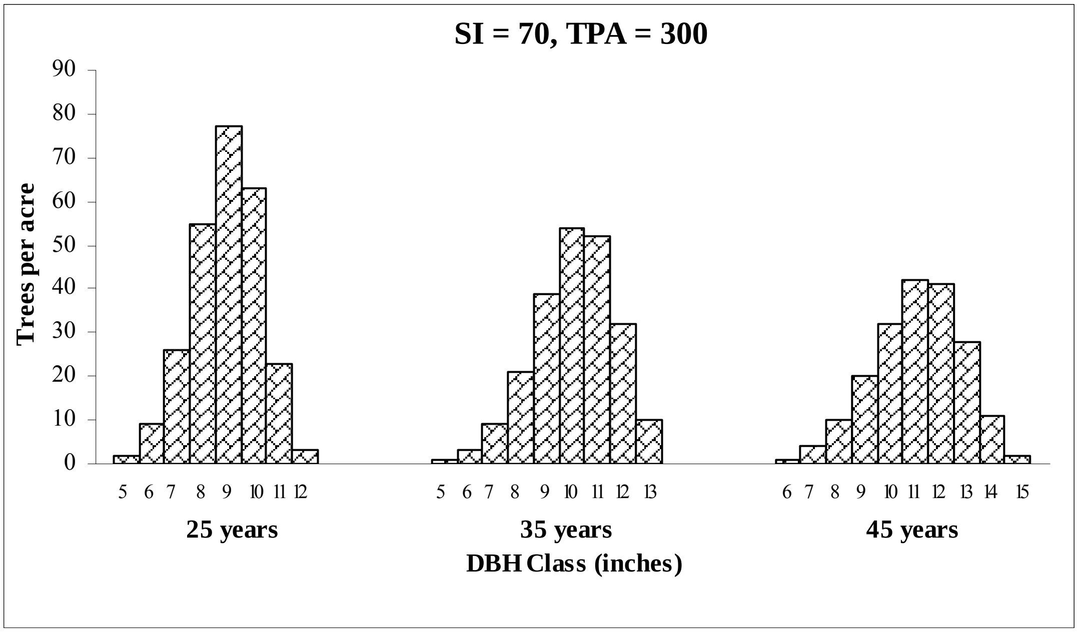



Figure 2.7. Estimated stand tables (equations 2.39a–2.39d $\rightarrow$ TSC1) at 25, 35 and 45 years of age for site index 40 feet, 300 and 600 trees per acre slash pine plantations thinned lightly from below at age 15.



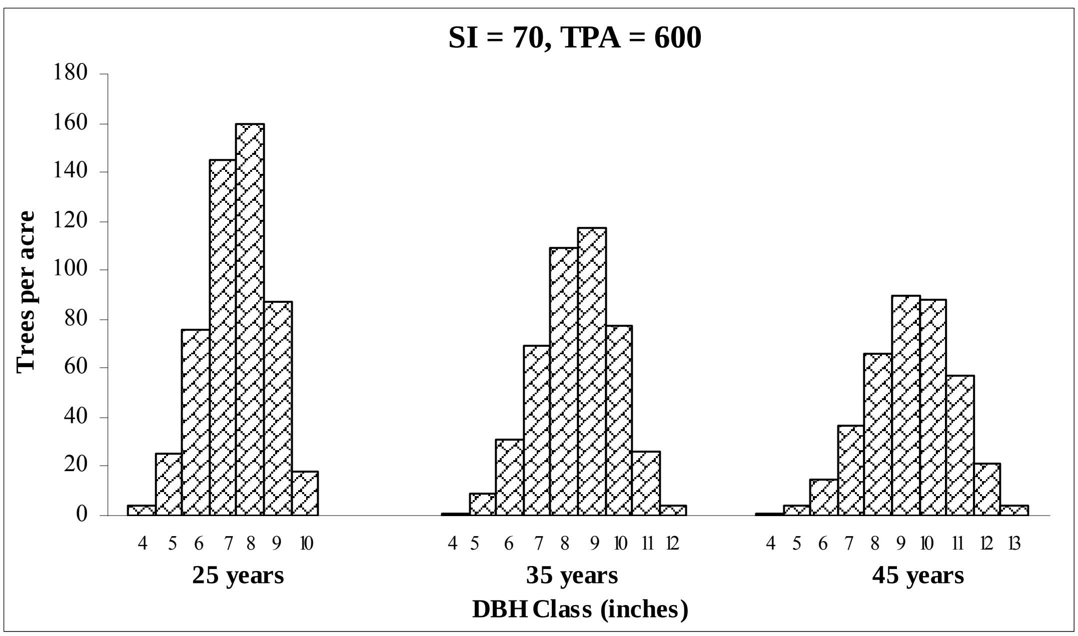




Figure 2.8. Estimated stand tables (equations 2.39a–2.39d → TSC1) at 25, 35 and 45 years of age for site index 70 feet, 300 and 600 trees per acre slash pine plantations thinned lightly from below at age 15.

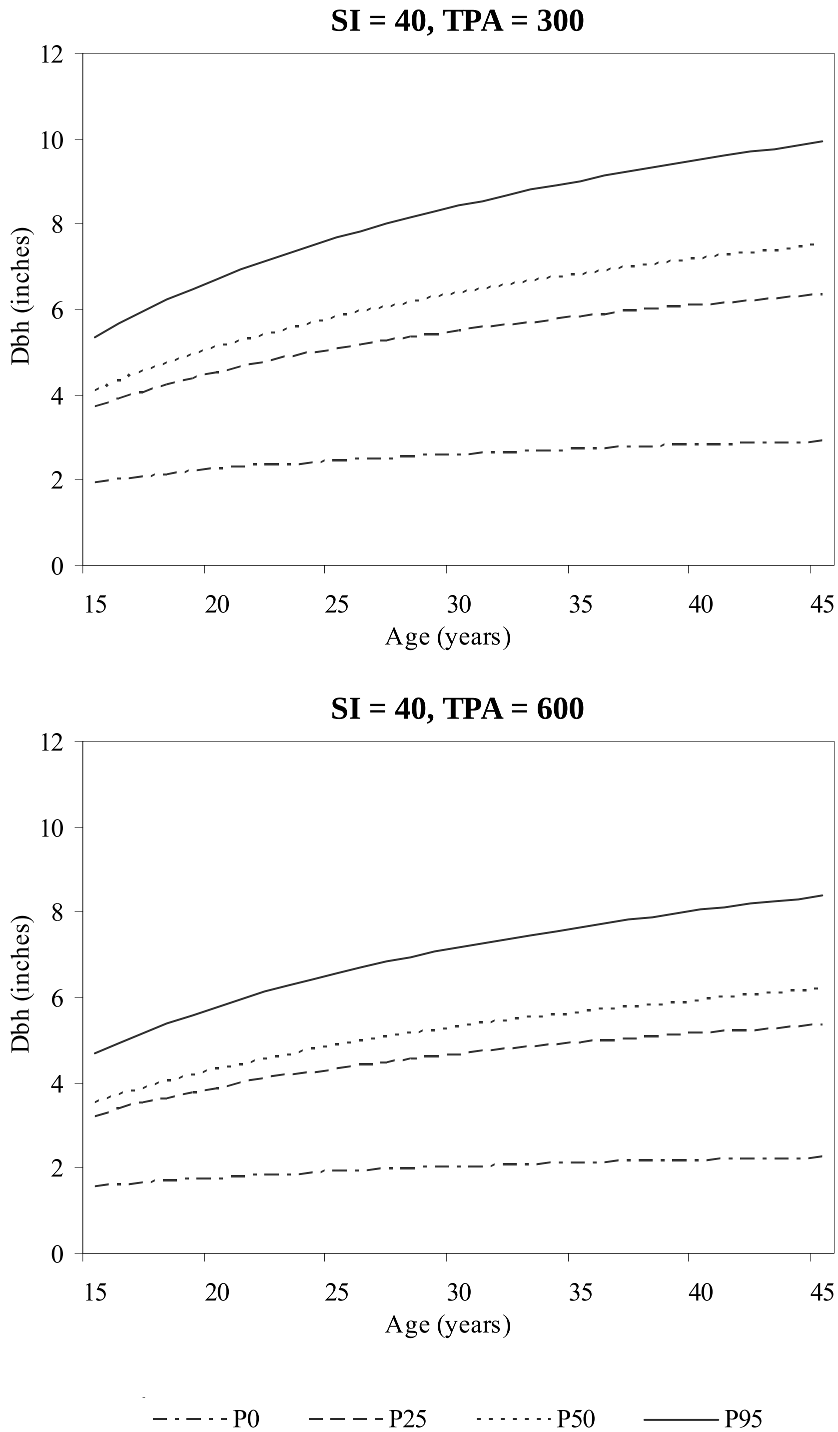
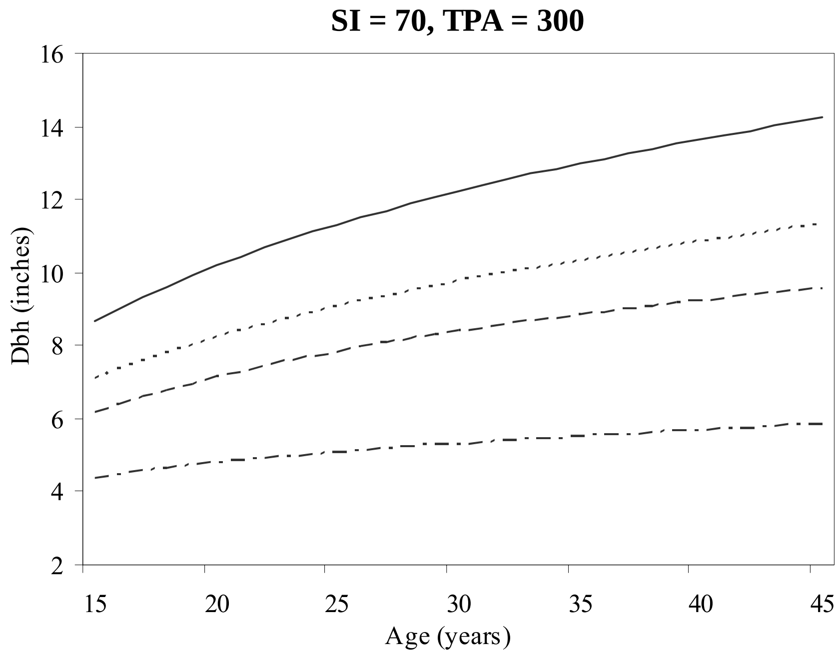



Figure 2.9. Projected thinned slash pine stand table percentiles (equations 2.41a–2.41d → TSC2) for site index 40 feet, 300 and 600 trees per acre.



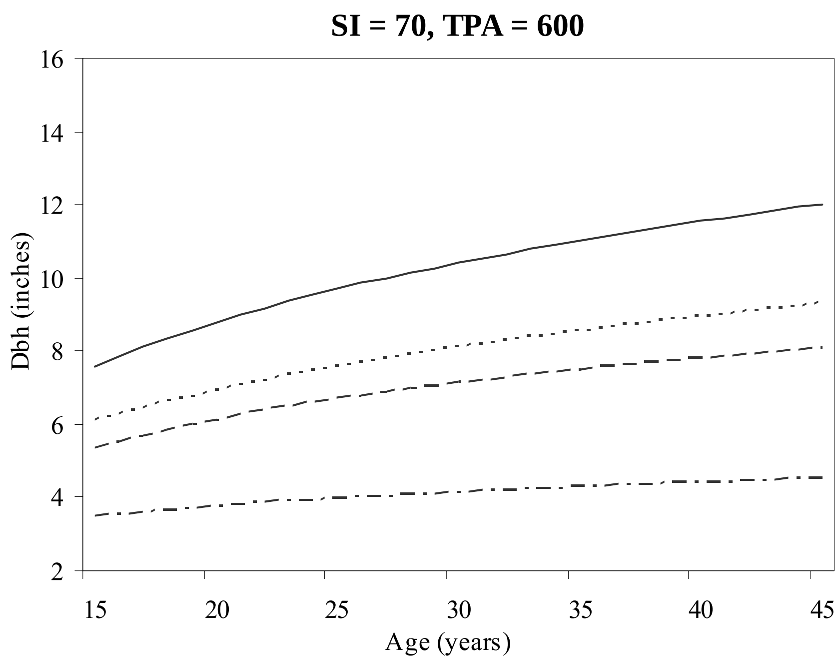
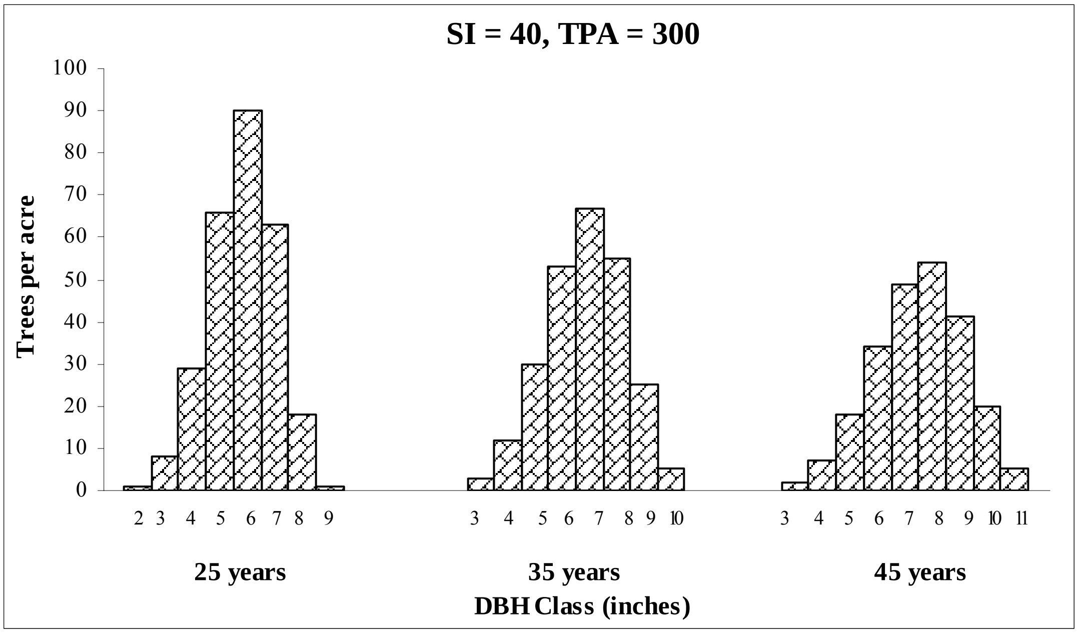




Figure 2.10. Projected thinned slash pine stand table percentiles (equations 2.41a–2.41d → TSC2) for site index 70 feet, 300 and 600 trees per acre.



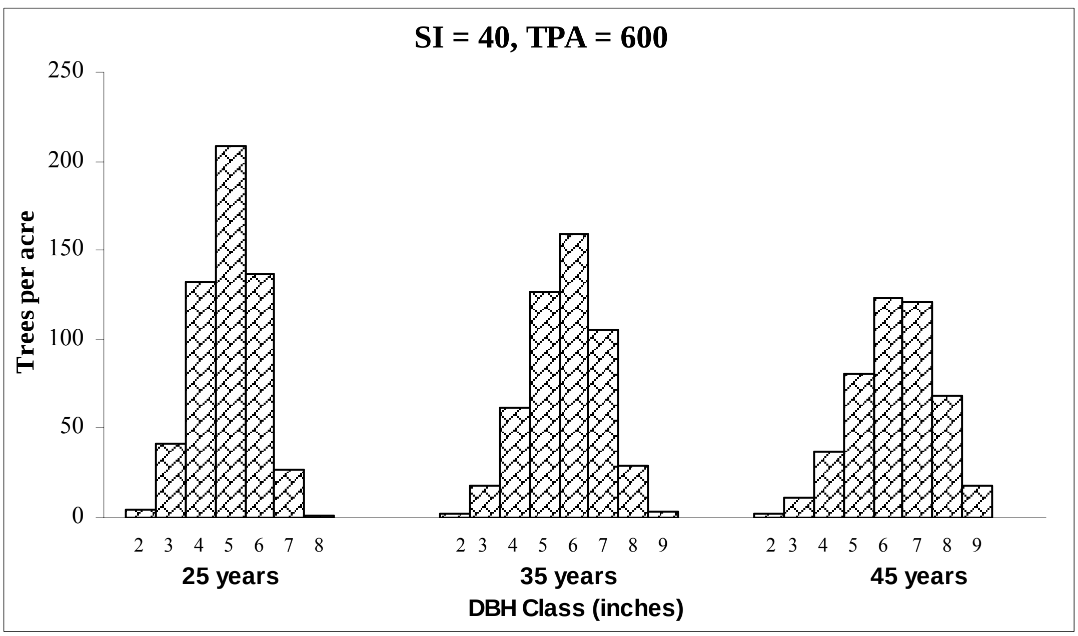
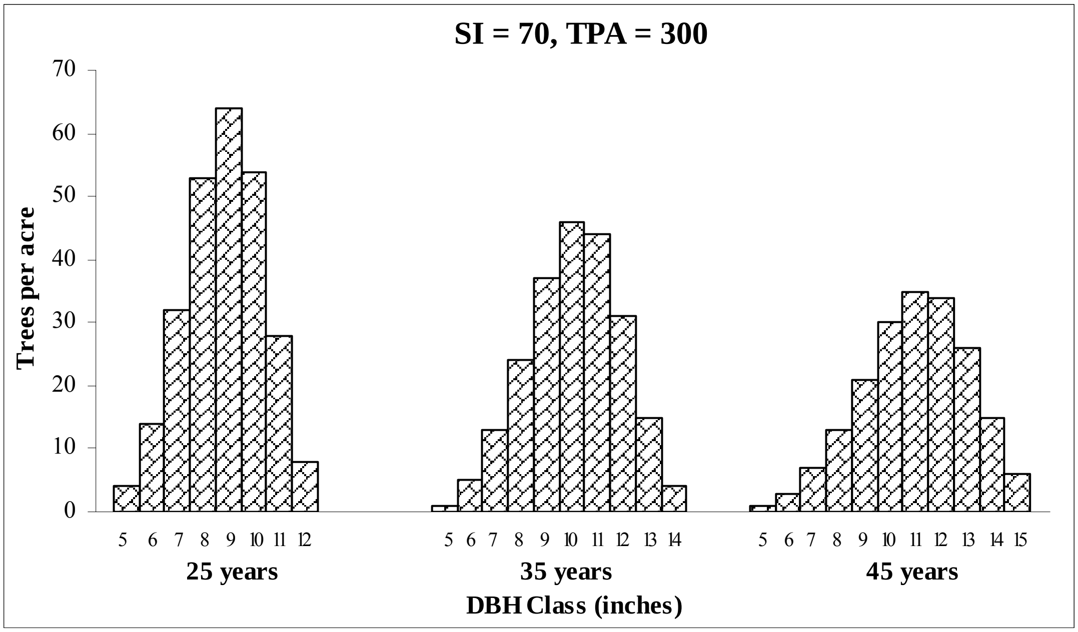




Figure 2.11. Estimated stand tables (equations 2.41a–2.41d → TSC2) at 25, 35 and 45 years of age for site index 40 feet, 300 and 600 trees per acre slash pine plantations thinned lightly from below at age 15.



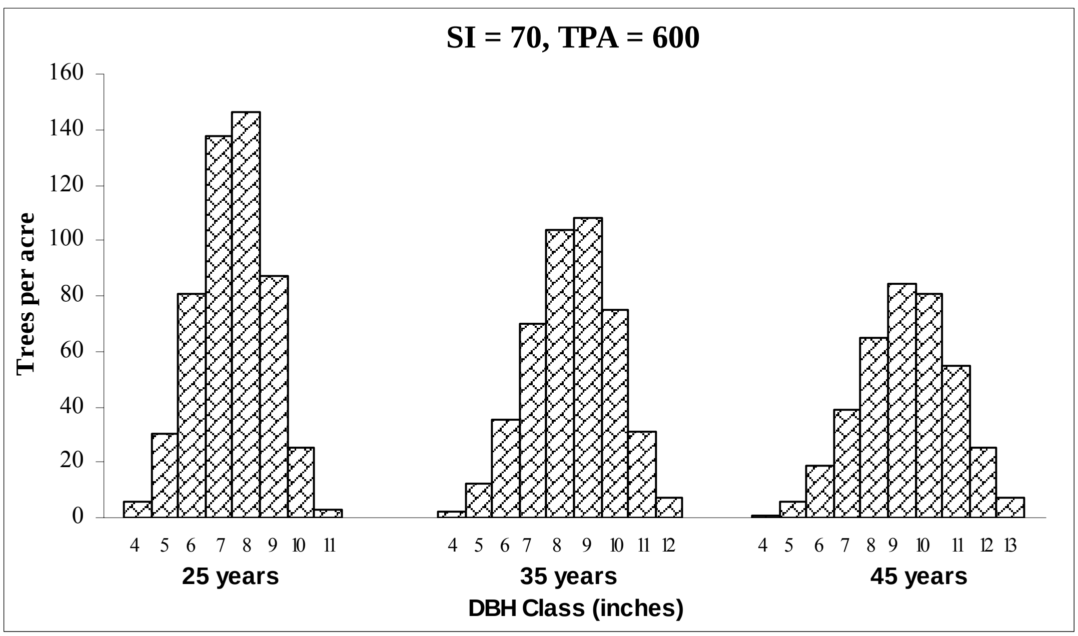




Figure 2.12. Estimated stand tables (equations 2.41a–2.41d → TSC2) at 25, 35 and 45 years of age for site index 70 feet, 300 and 600 trees per acre slash pine plantations thinned lightly from below at age 15.

in piedmont region. Residual plots did not show any serious departures from regression assumptions and plots of predicted percentile development curves seemed reasonable. KS two-sample test at $\alpha = 0.01$, 0.05 and 0.1 detected no significant difference between predicted and observed stand tables for any of the models in all physiographic regions. For a comparison with previous work using Weibull to model diameter distributions of unthinned loblolly pine plantations, Smalley and Bailey (1974) reported 20% of the predicted stand tables were significantly different.

**Immediately after thin stands**

Fitted percentile prediction equations for immediately after thin stands in piedmont, upper and lower coastal plain are presented in Table 2.15. All parameter estimates in the models were highly significant and in addition large PVE were obtained with all equations except the $0^{th}$ percentile prediction equation in piedmont region (Tables 2.17-2.19). Comparatively, the $0^{th}$ percentile had larger RMSE than other percentiles for all cases. However residual plots of all models did not suggest any serious problems and plots of percentile growth curves seemed logical. KS two-sample test (at $\alpha = 0.01$, 0.05 and 0.1) did not detect a significant difference between observed and predicted stand tables for all three cases. The smallest error index was obtained with TSC1 equations and the largest was obtained with TSC3 equations.

**Thinned stands**

Operational percentile prediction equations that were developed for thinned loblolly pine plantations in piedmont, upper and lower coastal plain regions are presented in Table 2.16. All parameter estimates in the equations were significantly different from zero at $\propto = 0.05$. The fit statistics shown in Tables 2.17-2.19 indicated that over all cases the best fit statistics were obtained with the $50^{th}$ percentile prediction equation and the worst fit statistics were obtained with the $0^{th}$ percentile prediction equation. However visual inspection of the residual plots did not indicate serious departures from regression model assumptions and plots of predicted percentile development curves appeared reasonable (Figure 2.16–2.17, 2.20-2.21). KS two-sample test found no significant difference (at $\alpha = 0.01$, 0.05 and 0.1) between observed and predicted stand tables. The smallest error index was obtained with TSC1. The largest error index was obtained with TSC6 equations in piedmont and upper coastal plain and with TSC4 equations in lower coastal plain province. Second thinned diameter distributions were predicted well by developed percentile prediction equations with no significance between predicted and observed stand tables. Overall error indices obtained for second thin stand table predictions were as follows: 146.25 for TSC1, 144.56 for TSC2, 143.85 for TSC3, 151.04 for TSC4, 149.75 for TSC5 and 150.43 for TSC6.

Generation of percentile development curves and the corresponding diameter distributions followed the same procedure and stand characteristics as outlined for slash pine plantations. Equations developed for piedmont were used for illustration. Predicted before and immediately after thin distributions (Figures 2.14 – 2.15) indicated a shift of the distribution towards the right and a narrower after thin distribution than before thin. This reflected selective thinning from below that was assumed for the example stand. Percentile development curves and projected diameter distributions indicated trends that were logical. There was no percentile cross-over and, in addition, commonly observed trends were evident. For example for a given site index starting trees per acre was inversely related with tree size and for a given initial trees per acre site index was directly related with tree size (Figures 2.16 – 2.23).

**Discussion**

Operational percentile prediction equations were developed for thinned slash and loblolly pine plantations in the piedmont, upper coastal plain and lower coastal plains of Alabama, Georgia, South Carolina and Florida. All equations fitted the data well except for the $0^{th}$ percentile prediction equation for slash pine plantations in lower coastal plain and loblolly pine plantations in piedmont region. Despite this, predicted percentiles estimated well Weibull parameters as demonstrated by KS test results which indicated no significant difference ($\alpha$ = 0.01, 0.05 and 0.1) between predicted and observed diameter distributions. Developed equations also predicted quite well diameter distributions of second thinned plantations. These results imply that Weibull can be used to describe diameter distributions of thinned slash and loblolly pine plantations.

Percentile prediction equations developed by Borders et al. (1990) fitted well unthinned slash and loblolly pine stands. The appealing feature of this equation is that it eliminates a great deal of the problem associated with percentiles crossing one another. The single explanatory variable in this equation which is the natural logarithm of the ratio of basal area per acre and trees per acre is well correlated with the percentiles. This variable which is a function of quadratic mean diameter was also determined to explain most of the variation in both immediately after thin and thinned percentiles and was the key in providing logical percentile development curves.

Selective thinning from below shifted diameter distributions towards the right and as a result diameters corresponding to the $0^{th}$ and $25^{th}$ percentiles increased significantly under all thinning intensity levels considered. On the other hand row thinning under all thinning intensity levels had no significant effect on the percentiles. Given these effects, immediately after thin percentiles in TSC1, were predicted by adjusting before thin stand information such as percentiles, basal area per acre and trees per acre. The adjustment was achieved using a thinning term in the form of the ratio of quadratic mean diameter of trees after thinning to the quadratic mean diameter of trees before thinning. This term was highly correlated with the percentiles and was determined to be a significant predictor of thinned percentiles several years after thinning. In TSC2, TSC3 and TSC4 equations, type and intensity of thinning were represented using indicator variables. An interesting result obtained with TSC3 equations was that immediately after thinning, intensity of thinning was not significant in all plantations except for loblolly pine plantations in upper coastal plain (Tables 2.9 and 2.15). This was possibly because thinning intensities applied were relatively high and similar in their effects.

Over all cases considered, the general trend from best to worst fit statistics was as follows: TSC1, TSC2, TSC3, TSC4, TSC5 and TSC6 (Tables 2.11-2.12, 2.17-2.19). This came as no surprise because with more information about a thinning better predictions are expected. However KS test results indicated there was no significant difference among these cases since no predicted stand table was rejected. In addition there were no large differences in error indices between the best fit (TSC1) and the least fit (TSC6). This implies that reasonably reliable predictions can be obtained even when no information about a thinning is available. Information about age of thinning alone was not important in most of the cases and as such TSC5 and TSC6 equations were similar.

Simulated percentile development curves and diameter distributions for TSC1 –TSC6 were quite reasonable. For all cases there was no percentile cross over. Predicted before and immediately after thin diameter distributions, (Figures 2.3-2.4, 2.14-.2.15), indicated a shift of the distribution towards the right and, narrower after thin distributions than before thin. This reflected selective thinning from below that was assumed for the example stand. Projected diameter distributions indicated trends that are expected in even-aged stands. That is for a given site index, starting trees per acre was inversely related with tree size and for a given initial trees per acre, site index was directly related with tree size. For example in site 40, the stand with 300 trees per acre had larger tree sizes than the stand with 600 trees per acre and with starting trees per acre of 300 trees per acre, site 70 had larger trees than site 40. In addition to these trends there was a greater difference between the smallest and largest trees in site 70 than in site 40, and with 300 trees per acre than with 600 trees per acre (Figures 2.5–2.12, 2.16-2.23).

Above simulations illustrate the versatility of developed percentile prediction equations. That is they can be used to recover diameter distribution for any given stand values. Thus if current stand values such as age, trees per acre, basal area per acre are used in the calculation then current diameter distribution is recovered. However, if future stand values are used an estimate of future diameter distribution at the projection age will be obtained. Current and future stand values can be obtained from growth and yield systems that have been developed for slash and loblolly pine plantations such as Pienaar et al. (1996), Harrison and Borders (1996), Logan (2005), Borders et al (2004) among others.

Developed percentile equations were used to predict the stand tables for second thinned slash and loblolly plantations. KS two-sample test was used to compare observed and predicted stand tables. Of the 120 slash pine plots and 74 loblolly pine plots that were available for analysis, none were rejected at $\alpha = 0.01$, 0.05 and 0.1. This implies that second thinning did not significantly change the distribution and that developed equations can be used to predict diameter distributions of second thinned plantations. This result is in agreement with other studies that found that once a thinning regime has started, second and subsequent thinnings do not shift much the diameter distributions (Bailey et al. 1981).

**Conclusion**

Percentile prediction equations were developed to estimate the 0th, 25th, 50th and 95th percentiles for thinned slash and loblolly pine stand TSC1, TSC2, TSC3, TSC4, TSC5 and TSC6 cases in the piedmont, upper coastal plain and lower coastal plain of Alabama, Georgia, South Carolina and Florida. Weibull distributions that were recovered using predicted percentiles described well observed diameter distributions. KS two-sample test found no significant difference ($\alpha$ = 0.01, 0.05 and 0.1) between predicted and observed diameter distributions. It can be concluded therefore that Weibull that has been well documented for unthinned stands can be used to describe diameter distributions of thinned slash and loblolly pine plantations.

Availability of after and before thin information provided the most precise percentile prediction equations. Model precision was in the order of TSC1, TSC2, TSC3, TSC4, TSC5 and TSC6. However all predicted stand tables were not rejected for any of these six cases. Absence of large differences in error indices between the best fit (TSC1) and the least fit (TSC6) implied that reasonably reliable predictions can be obtained even when no information about a thinning is available.

Developed percentile equations predicted quite well diameter distributions of second thinned plantations. KS two sample test at $\alpha$ = 0.01, 0.05 and 0.1 detected no significant difference between predicted and observed distributions. This suggests that second thinning did not significantly change the distribution and thus developed equations can be used to predict diameter distributions of second thinned plantations.

The percentile prediction equations that were developed can be used to recover both current future diameter distributions. This is defined by the stand values that are used in the computations. If future stand values are used as illustrated in the simulations presented an estimate of future diameter distribution at the projection age will be obtained. Current and future stand values can be obtained from regional growth and yield systems.

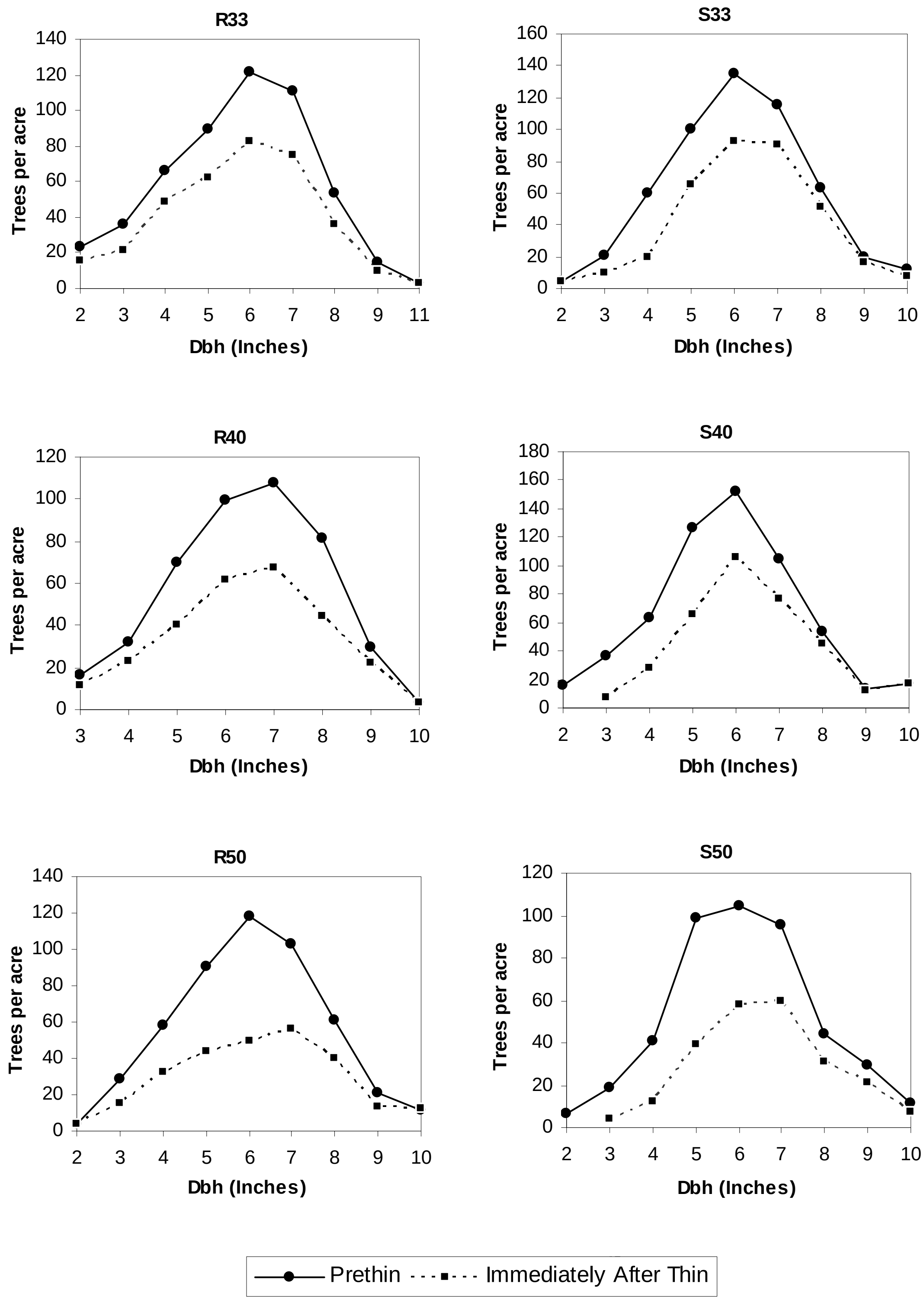



Figure 2.13. Average pre-thin and immediately after thin diameter distributions by thinning treatment for old age class loblolly pine plantations in piedmont region.

Table 2.13. Relative diameter class frequencies for distributions presented in Figure 2.13.

| Dbh Class | Pre-R33 | After R33 | Pre-R40 | After R40 | Pre-R50 | After R50 | Pre-S33 | After S33 | Pre-S40 | After S40 | Pre-S50 | After S50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.04 | 0.04 | | | 0.01 | 0.02 | 0.01 | 0.01 | 0.03 | | 0.01 | |
| 3 | 0.07 | 0.06 | 0.04 | 0.04 | 0.06 | 0.06 | 0.04 | 0.03 | 0.06 | 0.02 | 0.04 | 0.02 |
| 4 | 0.13 | 0.14 | 0.07 | 0.08 | 0.12 | 0.12 | 0.11 | 0.05 | 0.11 | 0.08 | 0.09 | 0.05 |
| 5 | 0.17 | 0.18 | 0.16 | 0.15 | 0.18 | 0.16 | 0.19 | 0.18 | 0.22 | 0.18 | 0.22 | 0.17 |
| 6 | 0.23 | 0.23 | 0.23 | 0.23 | 0.24 | 0.19 | 0.26 | 0.26 | 0.26 | 0.29 | 0.23 | 0.25 |
| 7 | 0.21 | 0.21 | 0.25 | 0.25 | 0.21 | 0.21 | 0.22 | 0.25 | 0.18 | 0.21 | 0.21 | 0.26 |
| 8 | 0.10 | 0.10 | 0.19 | 0.16 | 0.12 | 0.15 | 0.12 | 0.14 | 0.09 | 0.13 | 0.10 | 0.13 |
| 9 | 0.03 | 0.03 | 0.07 | 0.08 | 0.04 | 0.05 | 0.04 | 0.05 | 0.02 | 0.04 | 0.07 | 0.09 |
| 10 | | | 0.01 | 0.01 | 0.02 | 0.05 | 0.02 | 0.02 | 0.03 | 0.05 | 0.03 | 0.03 |
| 11 | 0.01 | 0.01 | | | | | | | | | | |

Table 2.14. Fitted percentile prediction equations for unthinned loblolly pine plantations in piedmont, upper and lower coastal plain provinces.

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| Unthinned | Piedmont | $\text{In}(P_{0u}) = 1.7463 + 0.4759\text{In}(B_u/N_u)$ | (2.50a) |
| | | $\text{In}(P_{25u}) = 2.4229 + 3.0291/A + 3.0291\text{In}(B_u/N_u)$ | (2.50b) |
| | | $\text{In}(P_{50u}) = 2.6169 + 0.5174\text{In}(B_u/N_u)$ | (2.50c) |
| | | $\text{In}(P_{95u}) = 2.9095 - 4.2647/A + 0.3513\text{In}(B_u/N_u)$ | (2.50d) |
| | Upper coastal plain | $\text{In}(P_{0u}) = 2.2423 + 0.8643\text{In}(B_u/N_u)$ | (2.51a) |
| | | $\text{In}(P_{25u}) = 2.3816 + 1.7329/A + 0.563\text{In}(B_u/N_u)$ | (2.51b) |
| | | $\text{In}(P_{50u}) = 2.5827 + 0.4941\text{In}(B_u/N_u)$ | (2.51c) |
| | | $\text{In}(P_{95u}) = 2.8587 + 0.4819\text{In}(B_u/N_u)$ | (2.51d) |
| | Lower coastal plain | $\text{In}(P_{0u}) = 2.1631 + 0.8439\text{In}(B_u/N_u)$ | (2.52a) |
| | | $\text{In}(P_{25u}) = 2.3599 + 2.0177/A + 0.5528\text{In}(B_u/N_u)$ | (2.52b) |
| | | $\text{In}(P_{50u}) = 2.6099 + 0.5139\text{In}(B_u/N_u)$ | (2.52c) |
| | | $\text{In}(P_{95u}) = 2.8442 + 0.4625\text{In}(B_u/N_u)$ | (2.52d) |

Table 2.15. Fitted percentile prediction equations for immediately after thin loblolly pine plantations in piedmont, upper and lower coastal plain provinces.

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| After thin TSC1 | Piedmont | $\ln(P_{0at}) = -3.0715 + 0.6374\ln(P_{0bt}) + 3.4978R_d$ | (2.53a) |
| | | $\ln(P_{25at}) = -1.0123 + 3.9506/A_t + 0.1759\ln(B_{bt}/N_{bt}) + 0.7221\ln(P_{25bt}) + 1.5001R_d$ | (2.53b) |
| | | $\ln(P_{50at}) = -0.2109 + 0.16745\ln(B_{bt}/N_{bt}) + 0.7115\ln(P_{50bt}) + 0.9936R_d$ | (2.53c) |
| | | $\ln(P_{95at}) = -0.0652 - 2.3316/A_t + 0.0349\ln(B_{bt}) + 0.9098\ln(P_{95bt}) + 0.2477R_d$ | (2.53d) |
| | Upper coastal plain | $\ln(P_{0at}) = -3.203 + 0.253\ln(B_{bt}/N_{bt}) + 0.6678\ln(P_{0bt}) + 3.9589R_d$ | (2.54a) |
| | | $\ln(P_{25at}) = -1.189 + 0.0993\ln(B_{bt}/N_{bt}) + 0.8352\ln(P_{25bt}) + 1.6128R_d$ | (2.54b) |
| | | $\ln(P_{50at}) = -0.9119 + 0.9935\ln(P_{50bt}) + 0.9249R_d$ | (2.54c) |
| | | $\ln(P_{95at}) = -0.3852 + 1.0045\ln(P_{95bt}) + 0.3712R_d$ | (2.54d) |
| | Lower coastal plain | $\ln(P_{0at}) = -1.6383 + 0.2787\ln(B_{bt}/N_{bt}) + 0.707\ln(P_{0bt}) + 2.4063R_d$ | (2.55a) |
| | | $\ln(P_{25at}) = -0.5551 + 0.2312\ln(B_{bt}/N_{bt}) + 0.5693\ln(P_{25bt}) + 1.6134R_d$ | (2.55b) |
| | | $\ln(P_{50at}) = -1.0712 + 1.0196\ln(P_{50bt}) + 1.0365R_d$ | (2.55c) |
| | | $\ln(P_{95at}) = -0.8454 + 0.1672\ln(H) + 0.8771\ln(P_{95bt}) + 0.4613R_d$ | (2.55d) |

Table 2.15 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| After thin TSC2 | Piedmont | $\ln(P_{0at}) = 1.6884 + 0.3266\ln(B_{at}/N_{at}) - 1.4841Z_1/A_t$ | (2.56a) |
| | | $\ln(P_{25at}) = 2.1378 + 5.5643/A_t + 0.5259\ln(B_{at}/N_{at}) + 3.2283Z_1/A_t + 0.1517Z_2\ln(B_{at}/N_{at})$ | (2.56b) |
| | | $\ln(P_{50at}) = 2.5083 + 2.2276/A_t + 0.5375\ln(B_{at}/N_{at})$ | (2.56c) |
| | | $\ln(P_{95at}) = 2.847 - 10.6241/A_t + 0.00624S + 0.2745\ln(B_{at}/N_{at}) + 0.0068Z_1\ln(B_{at})$ | (2.56d) |
| | Upper coastal plain | $\ln(P_{0at}) = 2.3023 + 0.7344\ln(B_{at}/N_{at}) - 2.8759Z_1/A_t$ | (2.57a) |
| | | $\ln(P_{25at}) = 2.4153 + 1.6335/A_t + 0.5562\ln(B_{at}/N_{at}) - 0.0062Z_1\ln(N_{at})$ | (2.57b) |
| | | $\ln(P_{50at}) = 2.6074 + 0.5136\ln(B_{at}/N_{at}) - 0.044Z_1 - 0.034Z_1\ln(B_{at}/N_{at})$ | (2.57c) |
| | | $\ln(P_{95at}) = 2.8804 - 1.382/A_t + 0.4467\ln(B_{at}/N_{at})$ | (2.57d) |
| | Lower coastal plain | $\ln(P_{0at}) = 3.1824 + 0.7956\ln(B_{at}/N_{at}) - 0.2262\ln(B_{at})$ | (2.58a) |
| | | $\ln(P_{25at}) = 2.3674 + 2.6125/A_t + 0.574\ln(B_{at}/N_{at})$ | (2.58b) |
| | | $\ln(P_{50at}) = 2.6169 + 0.5183\ln(B_{at}/N_{at}) - 0.0072Z_1\ln(B_{at}/N_{at})$ | (2.58c) |
| | | $\ln(P_{95at}) = 2.8145 + 0.4509\ln(B_{at}/N_{at})$ | (2.58d) |

Table 2.15 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| After thin TSC3 | Piedmont | $\ln(P_{0at}) = 1.7745 + 0.4049\ln(B_{at}/N_{at})$ | (2.59a) |
| | | $\ln(P_{25at}) = 2.1682 + 6.7732/A_t + 0.604\ln(B_{at}/N_{at})$ | (2.59b) |
| | | $\ln(P_{50at}) = 2.5083 + 2.2276/A_t + 0.5375\ln(B_{at}/N_{at})$ | (2.59c) |
| | | $\ln(P_{95at}) = 2.8077 - 10.6708/A_t + 0.0064S + 0.2476\ln(B_{at}/N_{at})$ | (2.59d) |
| | Upper coastal plain | $\ln(P_{0at}) = 2.3524 + 0.822\ln(B_{at}/N_{at})$ | (2.60a) |
| | | $\ln(P_{25at}) = 2.3877 + 2.2802/A_t + 0.5835\ln(B_{at}/N_{at}) + 0.0041X_1\ln(B_{at})$ | (2.60b) |
| | | $\ln(P_{50at}) = 2.5933 + 0.5017\ln(B_{at}/N_{at})$ | (2.60c) |
| | | $\ln(P_{95at}) = 2.8804 - 1.382/A_t + 0.4467\ln(B_{at}/N_{at})$ | (2.60d) |
| | Lower coastal plain | $\ln(P_{0at}) = 3.1824 + 0.7956\ln(B_{at}/N_{at}) - 0.2262\ln(B_{at})$ | (2.61a) |
| | | $\ln(P_{25at}) = 2.3674 + 2.6125/A_t + 0.574\ln(B_{at}/N_{at})$ | (2.61b) |
| | | $\ln(P_{50at}) = 2.6174 + 0.5164\ln(B_{at}/N_{at})$ | (2.61c) |
| | | $\ln(P_{95at}) = 2.8145 + 0.4509\ln(B_{at}/N_{at})$ | (2.61d) |

Table 2.16. Fitted operational percentile prediction equations for thinned loblolly pine plantations in piedmont, upper and lower coastal plain provinces.

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC1 | Piedmont | $\ln(P_0) = 1.0487 - 11.0191/A + 0.43817\ln(B/N) + 0.7843\ln(P_{0at}) + 0.4757(A_t/A)R_d$ | (2.62a) |
| | | $\ln(P_{25}) = 2.0181 + 3.5612/A + 0.4929\ln(B/N) + 0.2424\ln(P_{25at}) - 0.2252(A_t/A)R_d$ | (2.62b) |
| | | $\ln(P_{50}) = 2.0613 + 1.9134/A + 0.3927\ln(B/N) + 0.3132\ln(P_{50at}) - 0.3361(A_t/A)R_d$ | (2.62c) |
| | | $\ln(P_{95}) = 1.9236 - 4.3369/A + 0.218\ln(B/N) + 0.4805\ln(P_{95at}) - 0.2432(A_t/A)R_d$ | (2.62d) |
| | Upper coastal plain | $\ln(P_0) = 0.7469 - 5.8159/A + 0.3202\ln(B/N) + 0.4203\ln(P_{0at}) + 0.7621R_d$ | (2.63a) |
| | | $\ln(P_{25}) = 1.619 + 1.0956/A + 0.3285\ln(B/N) + 0.4467\ln(P_{25at}) - 0.2764(A_t/A)R_d$ | (2.63b) |
| | | $\ln(P_{50}) = 2.1041 + 0.3891\ln(B/N) + 0.237\ln(P_{50at}) - 0.1115(A_t/A)R_d$ | (2.63c) |
| | | $\ln(P_{95}) = 1.6891 - 1.2623/A + 0.205\ln(B/N) + 0.5676\ln(P_{95at}) - 0.3852(A_t/A)R_d$ | (2.63d) |
| | Lower coastal plain | $\ln(P_0) = -0.2122 + 0.2416\ln(B/N) + 0.6064\ln(P_{0at}) + 1.0214R_d$ | (2.64c) |
| | | $\ln(P_{25}) = 1.5974 + 0.3025\ln(B/N) + 0.3914\ln(P_{25at}) - 0.1044(A_t/A)R_d$ | (2.64b) |
| | | $\ln(P_{50}) = 2.2632 + 0.7872/A_t + 0.4613\ln(B/N) + 0.1257\ln(P_{50at})$ | (2.64c) |
| | | $\ln(P_{95}) = 2.192 + 0.2929\ln(B/N) + 0.2683\ln(P_{95at}) - 0.1605(A_t/A)R_d$ | (2.64d) |

Table 2.16 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC2 | Piedmont | $\ln(P_0) = 1.0797 - 5.9936/A_t + 0.3696\ln(B/N) + 0.8017\ln(P_{0at})$ | (2.65a) |
| | | $\ln(P_{25}) = 1.8631 + 5.2529/A_t + 0.4936\ln(B/N) + 0.1414\ln(P_{25at}) + 0.0242Z_1\ln(B/N)$ | (2.65b) |
| | | $\ln(P_{50}) = 2.2116 + 2.2889/A_t + 0.4969\ln(B/N) + 0.1285\ln(P_{50at}) - 0.0027Z_2\ln(B)$ | (2.65c) |
| | | $\ln(P_{95}) = 2.1129 - 6.4324/A + 0.2712\ln(B/N) + 0.379\ln(P_{95at}) - 0.0022Z_2\ln(B)$ | (2.65d) |
| | Upper coastal plain | $\ln(P_0) = 1.4467 - 5.8483/A_t + 0.295\ln(B/N) + 0.4789\ln(P_{0at})$ | (2.66a) |
| | | $\ln(P_{25}) = 1.8499 + 1.9199/A_t + 0.45\ln(B/N) + 0.2264\ln(P_{25at}) + 0.0158Z_1\ln(B/N)$ | (2.66b) |
| | | $\ln(P_{50}) = 2.3104 + 0.448\ln(B/N) + 0.116\ln(P_{50at})$ | (2.66c) |
| | | $\ln(P_{95}) = 2.1972 - 2.8734/A + 0.3197\ln(B/N) + 0.2931\ln(P_{95at}) - 0.0051Z_2\ln(B)$ | (2.66d) |
| | Lower coastal plain | $\ln(P_0) = 0.7856 + 0.2431\ln(B/N) + 0.6224\ln(P_{0at}) + 0.0011Z_2S$ | (2.67a) |
| | | $\ln(P_{25}) = 1.631 + 2.0147/A_t + 0.3843\ln(B/N) + 0.3065\ln(P_{25at}) + 0.0041Z_1\ln(B)$ | (2.67b) |
| | | $\ln(P_{50}) = 2.2632 + 0.7871/A_t + 0.4613\ln(B/N) + 0.1257\ln(P_{50at})$ | (2.67c) |
| | | $\ln(P_{95}) = 2.0819 - 4.8806/A + 0.2098\ln(B/N) + 0.3175\ln(P_{95at})$ | (2.67d) |

Table 2.16 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC3 | Piedmont | $\ln(P_0)=1.0783-6.5929/A+0.3503\ln(B/N)+0.7959\ln(P_{0at})+0.7131X/A_t$ | (2.68a) |
| | | $\ln(P_{25})=1.7833+4.7914/A_t+0.4956\ln(B/N)+0.1993\ln(P_{25at})+0.0022X\ln(N)$ | (2.68b) |
| | | $\ln(P_{50})=2.2185+2.2564/A_t+0.4957\ln(B/N)+0.1231\ln(P_{50at})$ | (2.68c) |
| | | $\ln(P_{95})=2.1091-6.5225/A+0.2678\ln(B/N)+0.3794\ln(P_{95at})$ | (2.68d) |
| | Upper coastal plain | $\ln(P_0)=1.4411-7.0877/A_t+0.2219\ln(B/N)+0.477\ln(P_{0at})+4.278X/A+0.2258X\ln(B/N)$ | (2.69a) |
| | | $\ln(P_{25})=1.7856+2.0722/A_t+0.4509\ln(B/N)+0.2516\ln(P_{25at})+0.0024X\ln(B)$ | (2.69b) |
| | | $\ln(P_{50})=2.3104+0.448\ln(B/N)+0.116\ln(P_{50at})$ | (2.69c) |
| | | $\ln(P_{95})=2.1327-3.3461/A+0.3002\ln(B/N)+0.3214\ln(P_{95at})+0.2614X-0.0562X\ln(B)$ | (2.69d) |
| | Lower coastal plain | $\ln(P_0)=0.8403+0.2644\ln(B/N)+0.6143\ln(P_{0at})$ | (2.70a) |
| | | $\ln(P_{25})=1.616+2.062/A_t+0.3824\ln(B/N)+0.316\ln(P_{25at})$ | (2.70b) |
| | | $\ln(P_{50})=2.267+0.7721/A_t+0.4632\ln(B/N)+0.1242\ln(P_{50at})-0.0056X\ln(B/N)$ | (2.70c) |
| | | $\ln(P_{95})=2.0719-4.941/A+0.2072\ln(B/N)+0.3199\ln(P_{95at})+0.003X\ln(B)$ | (2.70d) |

Table 2.16 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC4 | Piedmont | $\ln(P_0) = 1.9342 + 0.5096\ln(B/N) - 0.251Z_1\ln(N)/A + 19.9413Z_1/A_t$ | (2.71a) |
| | | $\ln(P_{25}) = 2.0888 + 1.8898/A + 5.2608/A_t + 0.5604\ln(B/N) - 0.1485Z_1\ln(B)$ | (2.71b) |
| | | $\ln(P_{50}) = 2.5746 + 1.9123/A + 0.5682\ln(B/N) + 0.0332Z_1\ln(N) - 2.6439Z_1/A_t$ | (2.71c) |
| | | $\ln(P_{95}) = 3.1137 - 4.087/A - 3.2857/A_t + 0.3705\ln(B/N) + 0.6572Z_1\ln(B)/A - 1.8878Z_2/A_t$ | (2.71d) |
| | Upper coastal plain | $\ln(P_0) = 2.199 + 0.684\ln(B/N) - 0.494Z_1\ln(N)/A$ | (2.72a) |
| | | $\ln(P_{25}) = 2.3432 + 1.1039/A + 1.441/A_t + 0.5565\ln(B/N) - 0.6604Z_1/A$ | (2.72b) |
| | | $\ln(P_{50}) = 2.5778 + 0.4933\ln(B/N) + 0.1019Z_2/A_t$ | (2.72c) |
| | | $\ln(P_{95}) = 2.8974 - 1.1339/A + 0.4701\ln(B/N) + 0.0649Z_1\ln(B)/A - 0.3336Z_2/A_t$ | (2.72d) |
| | Lower coastal plain | $\ln(P_0) = 2.1671 + 0.832\ln(B/N) - 7.2965Z_1/A_t + 1.5587Z_1\ln(N)/A$ | (2.73a) |
| | | $\ln(P_{25}) = 2.2939 + 2.3368/A + 2.0423/A_t + 0.6006\ln(B/N) + 0.0047Z_2\ln(B)$ | (2.73b) |
| | | $\ln(P_{50}) = 2.6168 + 0.5234\ln(B/N) + 2.8754Z_2/A - 0.6351Z_2\ln(B)/A$ | (2.73c) |
| | | $\ln(P_{95}) = 2.8729 - 2.338/A + 0.3974\ln(B/N) + 0.0887Z_1 - 1.8844Z_1/A$ | (2.73d) |

Table 2.16 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC5 | Piedmont | $\ln(P_0) = 1.9839 + 0.5869\ln(B/N)$ | (2.74a) |
| | | $\ln(P_{25}) = 2.1159 + 2.281/A + 4.6832/A_t + 0.5774\ln(B/N)$ | (2.74b) |
| | | $\ln(P_{50}) = 2.5804 + 1.4292/A + 0.5491\ln(B/N)$ | (2.74c) |
| | | $\ln(P_{95}) = 3.0828 - 3.4911/A - 3.2587/A_t + 0.3658\ln(B/N)$ | (2.74d) |
| | Upper coastal plain | $\ln(P_0) = 2.2114 + 0.7448\ln(B/N)$ | (2.75a) |
| | | $\ln(P_{25}) = 2.3318 + 1.2111/A + 1.6381/A_t + 0.5755\ln(B/N)$ | (2.75b) |
| | | $\ln(P_{50}) = 2.5812 + 0.4944\ln(B/N)$ | (2.75c) |
| | | $\ln(P_{95}) = 2.9002 - 1.6021/A + 0.4513\ln(B/N)$ | (2.75d) |
| | Lower coastal plain | $\ln(P_0) = 2.111 + 0.7886\ln(B/N)$ | (2.76a) |
| | | $\ln(P_{25}) = 2.2997 + 2.5817/A + 2.0178/A_t + 0.6081\ln(B/N)$ | (2.76b) |
| | | $\ln(P_{50}) = 2.6159 + 0.5223\ln(B/N)$ | (2.76c) |
| | | $\ln(P_{95}) = 2.904 - 3.0679/A + 0.3945\ln(B/N)$ | (2.76d) |

Table 2.16 continued

| Stand | Province | Fitted Equation | |
|---|---|---|---|
| TSC6 | Piedmont | Equation (2.74a) | |
| | | $\ln(P_{25}) = 2.3741 + 4.2382/A + 0.6263\ln(B/N)$ | (2.77a) |
| | | Equation (2.74c) | |
| | | $\ln(P_{95}) = 2.9031 - 4.853/A + 0.3318\ln(B/N)$ | (2.77b) |
| | Upper coastal plain | Equation (2.75a) | |
| | | $\ln(P_{25}) = 2.3835 + 2.236/A + 0.5674\ln(B/N)$ | (2.78) |
| | | Equation (2.75c) | |
| | | Equation (2.75d) | |
| | Lower coastal plain | Equation (2.76a) | |
| | | $\ln(P_{25}) = 2.3645 + 3.612/A + 0.6046\ln(B/N)$ | (2.79) |
| | | Equation (2.76c) | |
| | | Equation (2.76d) | |

Table 2.17. Fit statistics and error indices for percentile prediction equations fitted to loblolly pine data in piedmont region.

| Stand | Equation | PVE | | | | RMSE | | | | Error Index |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | |
| Unthinned | 2.50a-2.50d | 39.91 | 93.05 | 96.61 | 91.49 | 0.35 | 0.16 | 0.13 | 0.26 | 113.33 |
| After Thin TSC1 | 2.53a-2.53d | 39.12 | 91.80 | 95.55 | 86.68 | 0.46 | 0.15 | 0.10 | 0.21 | 67.93 |
| After Thin TSC2 | 2.56a-2.56d | 13.69 | 86.39 | 92.51 | 71.22 | 0.54 | 0.19 | 0.13 | 0.31 | 76.45 |
| After Thin TSC3 | 2.59a-2.59d | 7.80 | 83.58 | 92.51 | 67.57 | 0.56 | 21 | 0.13 | 0.33 | 76.86 |
| TSC1 | 2.62a-2.62d | 61.03 | 92.96 | 98.20 | 93.52 | 0.67 | 0.28 | 0.15 | 0.32 | 119.35 |
| TSC2 | 2.65a-2.65d | 60.40 | 93.91 | 97.84 | 93.20 | 0.68 | 0.26 | 0.16 | 0.33 | 120.17 |
| TSC3 | 2.68a-2.68d | 60.94 | 93.56 | 97.70 | 93.05 | 0.67 | 0.26 | 0.17 | 0.33 | 119.87 |
| TSC4 | 2.71a-2.71d | 44.22 | 93.78 | 97.58 | 90.42 | 0.80 | 0.26 | 0.17 | 0.39 | 123.95 |
| TSC5 | 2.74a-2.74d | 39.89 | 93.29 | 97.30 | 89.90 | 0.83 | 0.27 | 0.18 | 0.40 | 124.52 |
| TSC6 | 2.74a, 2.77a, 2.74c, 2.77b | 39.89 | 92.68 | 97.30 | 89.32 | 0.83 | 0.28 | 0.18 | 0.41 | 126.01 |

Table 2.18. Fit statistics and error indices for percentile prediction equations fitted to loblolly pine data in upper coastal plain province.

| Stand | Equation | PVE | | | | RMSE | | | | Error Index |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | |
| Unthinned | 2.51a-2.51d | 80.09 | 96.57 | 98.99 | 97.18 | 0.53 | 0.21 | 0.13 | 0.28 | 171.25 |
| After Thin TSC1 | 2.54a-2.54d | 75.83 | 98.08 | 98.77 | 97.06 | 0.61 | 0.16 | 0.13 | 0.26 | 112.57 |
| After Thin TSC2 | 2.57a-2.57d | 70.90 | 95.67 | 98.83 | 93.93 | 0.67 | 0.24 | 0.13 | 0.38 | 124.82 |
| After Thin TSC3 | 2.60a-2.60d | 65.67 | 95.32 | 98.70 | 93.93 | 0.73 | 0.24 | 0.14 | 0.38 | 126.49 |
| TSC1 | 2.63a-2.63d | 74.62 | 95.37 | 98.20 | 96.09 | 0.69 | 0.28 | 0.19 | 0.37 | 199.99 |
| TSC2 | 2.66a-2.66d | 73.46 | 95.55 | 98.13 | 95.04 | 0.71 | 0.28 | 0.19 | 0.42 | 204.49 |
| TSC3 | 2.69a-2.69d | 73.90 | 95.50 | 98.13 | 94.66 | 0.70 | 0.25 | 0.19 | 0.44 | 206.08 |
| TSC4 | 2.72a-2.72d | 65.07 | 94.70 | 97.95 | 94.01 | 0.81 | 0.30 | 0.20 | 0.49 | 212.49 |
| TSC5 | 2.75a-2.75d | 61.78 | 94.03 | 97.91 | 93.01 | 0.85 | 0.32 | 0.20 | 0.50 | 216.95 |
| TSC6 | 2.75a, 2.78, 2.75c-2.75d | 61.78 | 93.81 | 97.91 | 93.01 | 0.85 | 0.33 | 0.20 | 0.50 | 218.46 |

Table 2.19. Fit statistics and error indices for percentile prediction equations fitted to loblolly pine data in lower coastal plain province.

| Stand | Equation | PVE | | | | RMSE | | | | Error Index |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | $P_0$ | $P_{25}$ | $P_{50}$ | $P_{95}$ | |
| Unthinned | 2.52a-2.52d | 78 | 95.29 | 98.86 | 95.89 | 0.55 | 0.24 | 0.14 | 0.33 | 166.44 |
| After Thin TSC1 | 2.55a-2.55d | 80.70 | 97.33 | 99.17 | 97.01 | 0.50 | 0.18 | 0.11 | 0.24 | 108.26 |
| After Thin TSC2 | 2.58a-2.58d | 71.63 | 95.95 | 98.51 | 93.28 | 0.60 | 0.22 | 0.14 | 0.36 | 114.69 |
| After Thin TSC3 | 2.61a-2.61d | 71.63 | 95.95 | 98.48 | 93.28 | 0.60 | 0.22 | 0.15 | 0.36 | 114.65 |
| TSC1 | 2.64a-2.64d | 81.27 | 95.66 | 98.66 | 93.12 | 0.55 | 0.25 | 0.15 | 0.39 | 165.01 |
| TSC2 | 2.67a-2.67d | 80.85 | 95.98 | 98.66 | 94.14 | 0.56 | 0.24 | 0.15 | 0.36 | 166.23 |
| TSC3 | 2.70a-2.70d | 80.02 | 98.74 | 98.68 | 94.32 | 0.57 | 0.24 | 0.15 | 0.36 | 164.48 |
| TSC4 | 2.73a-2.73d | 62.48 | 94.19 | 98.41 | 92.79 | 0.79 | 0.29 | 0.16 | 0.40 | 172.96 |
| TSC5 | 2.76a-2.76d | 60.64 | 93.85 | 98.31 | 92.48 | 0.80 | 0.29 | 0.17 | 0.41 | 171.47 |
| TSC6 | 2.76a, 2.79, 2.76c-2.76d | 60.64 | 93.68 | 98.31 | 92.48 | 0.80 | 0.29 | 0.17 | 0.41 | 169.96 |

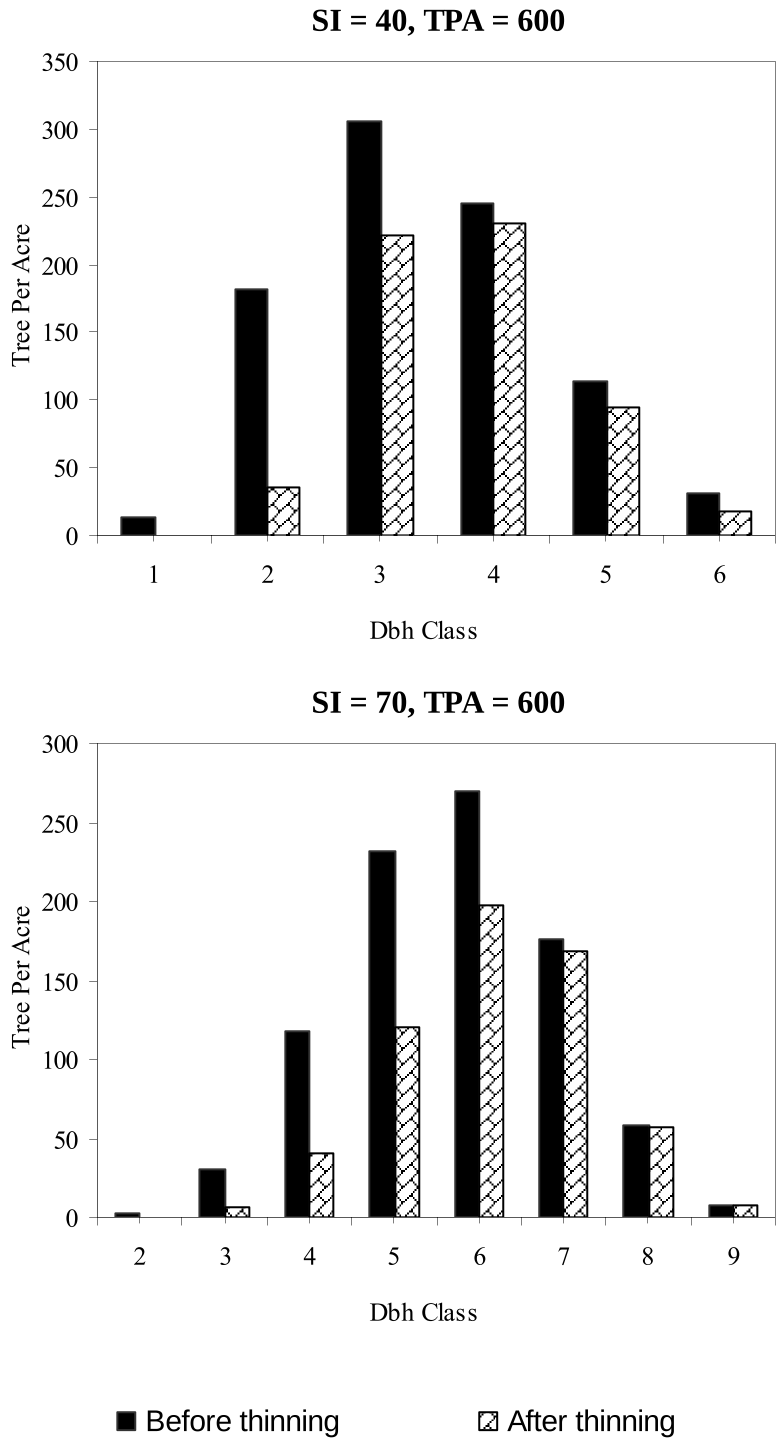



Figure 2.14. Predicted before (equation 2.50a-2.50d → unthinned TSC1) and immediately after thin (equation 2.53a-2.53d → after thin TSC1) diameter distributions for a 15 year old loblolly pine stand thinned from below.

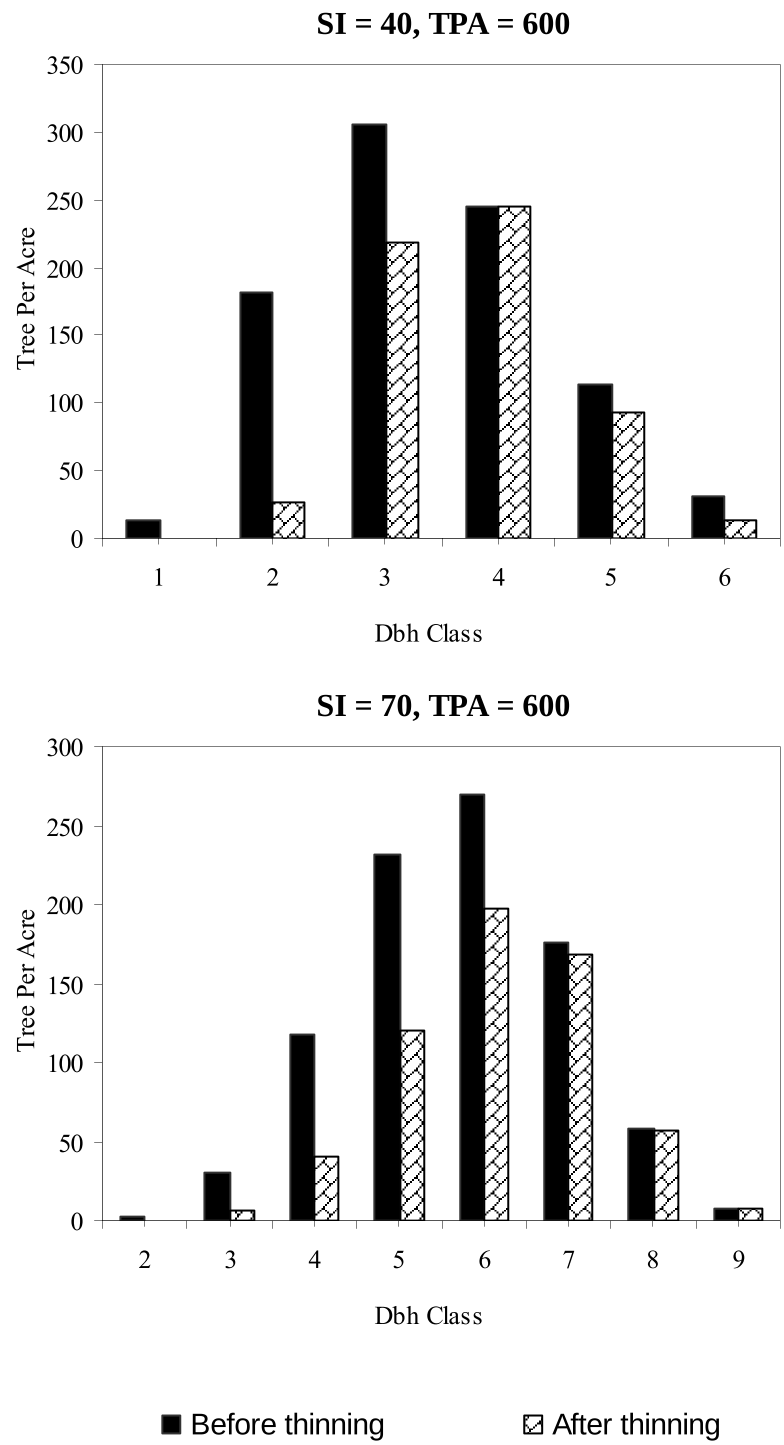



Figure 2.15. Predicted before (equation 2.50a-2.50d → unthinned TSC2) and immediately after thin (equation 2.56a-2.56d → after thin TSC2) diameter distributions for a 15 year old loblolly pine stand thinned from below.

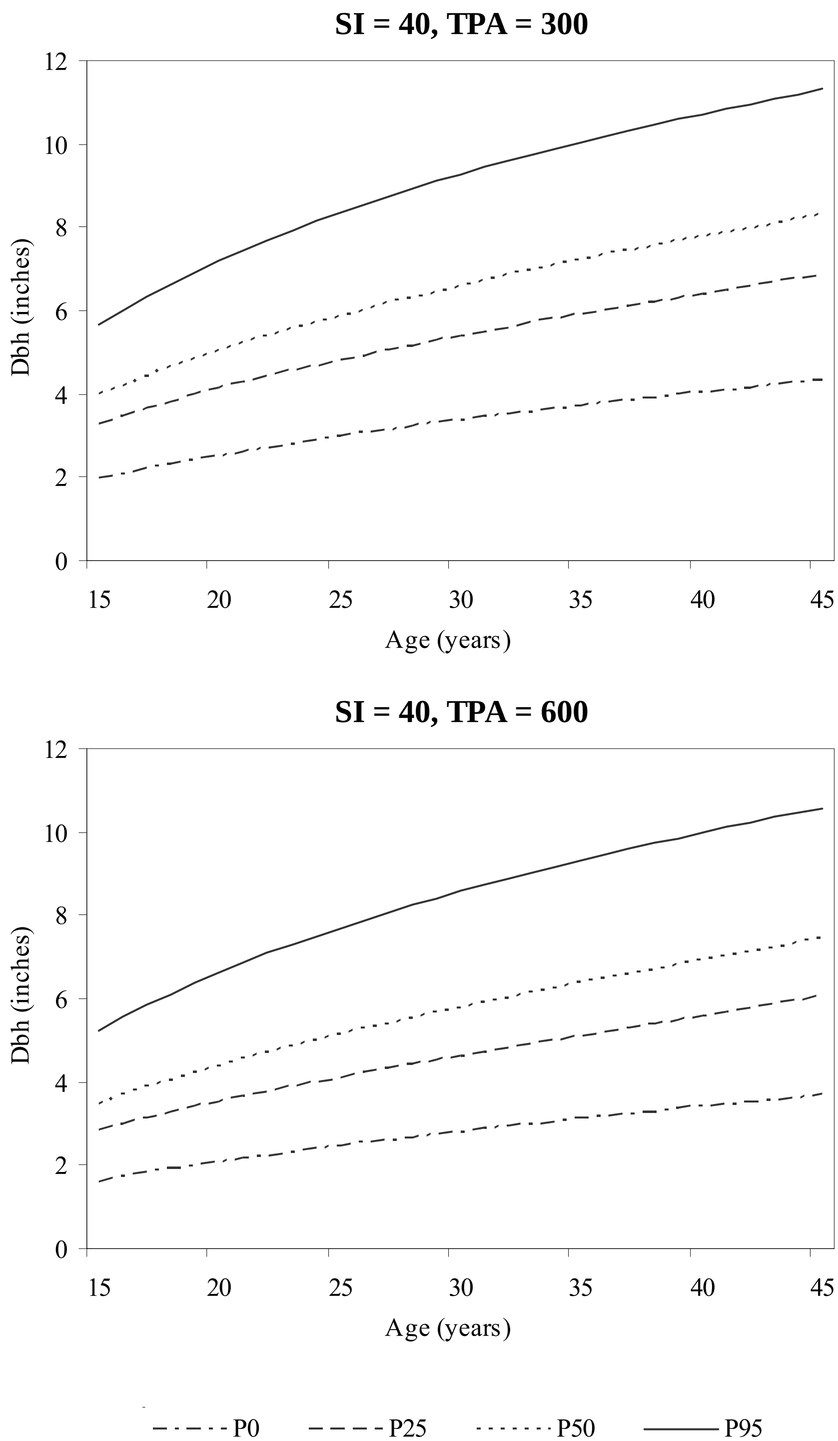



Figure 2.16. Projected thinned loblolly pine diameter distribution percentiles (equations 2.62a–2.62d → TSC1) for site index 40 feet, 300 and 600 trees per acre.

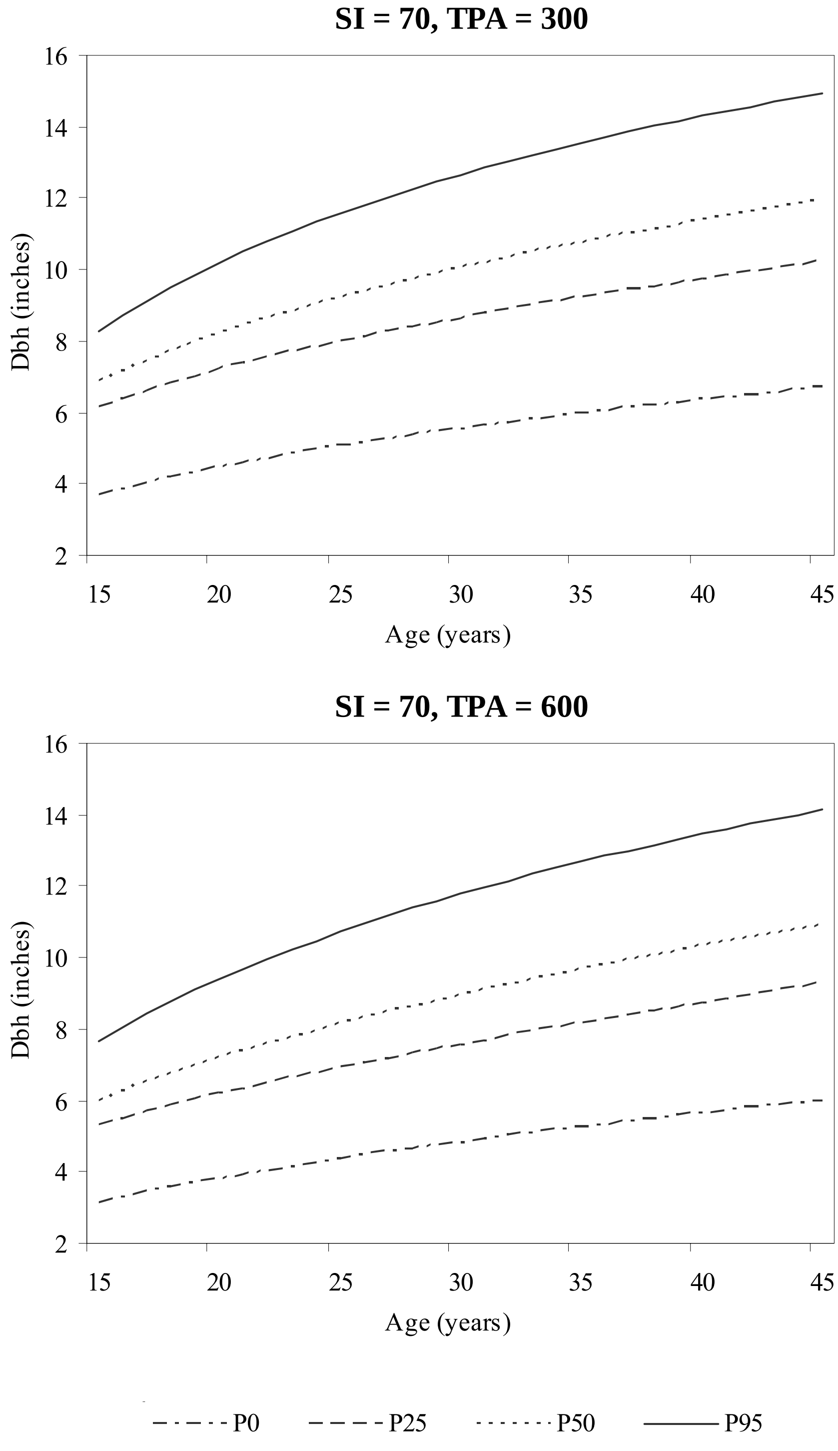
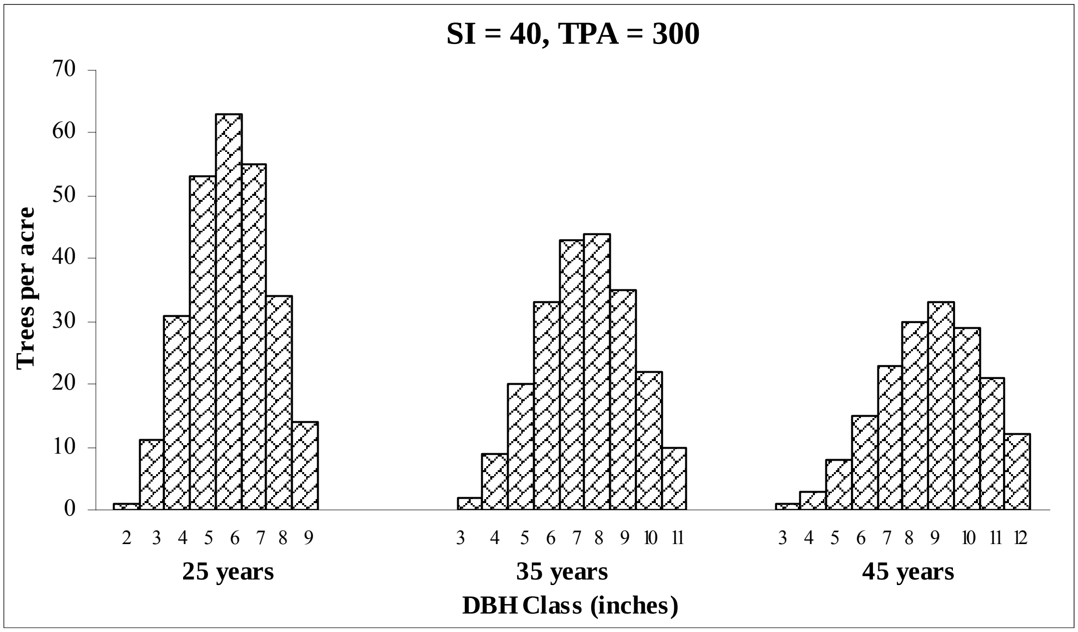



Figure 2.17. Projected thinned loblolly pine diameter distribution percentiles (equations 2.62a–2.62d → TSC1) for site index 70 feet, 300 and 600 trees per acre.



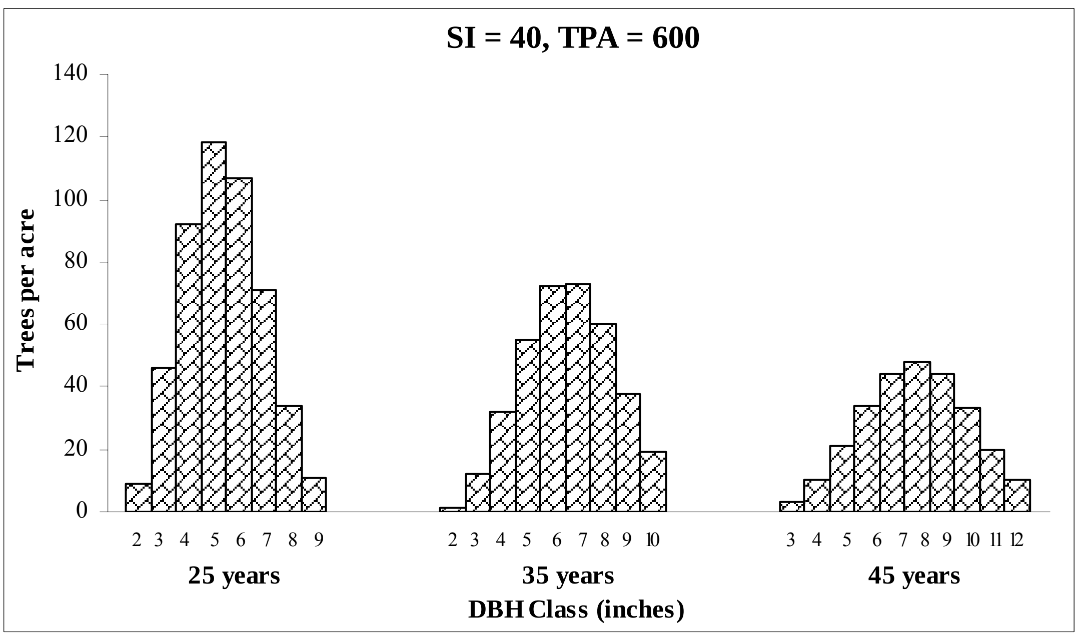




Figure 2.18. Estimated stand tables (equations 2.62a–2.62d → TSC1) at 25, 35 and 45 years of age for site index 40 feet, 300 and 600 trees per acre loblolly pine plantations thinned lightly from below at age 15.

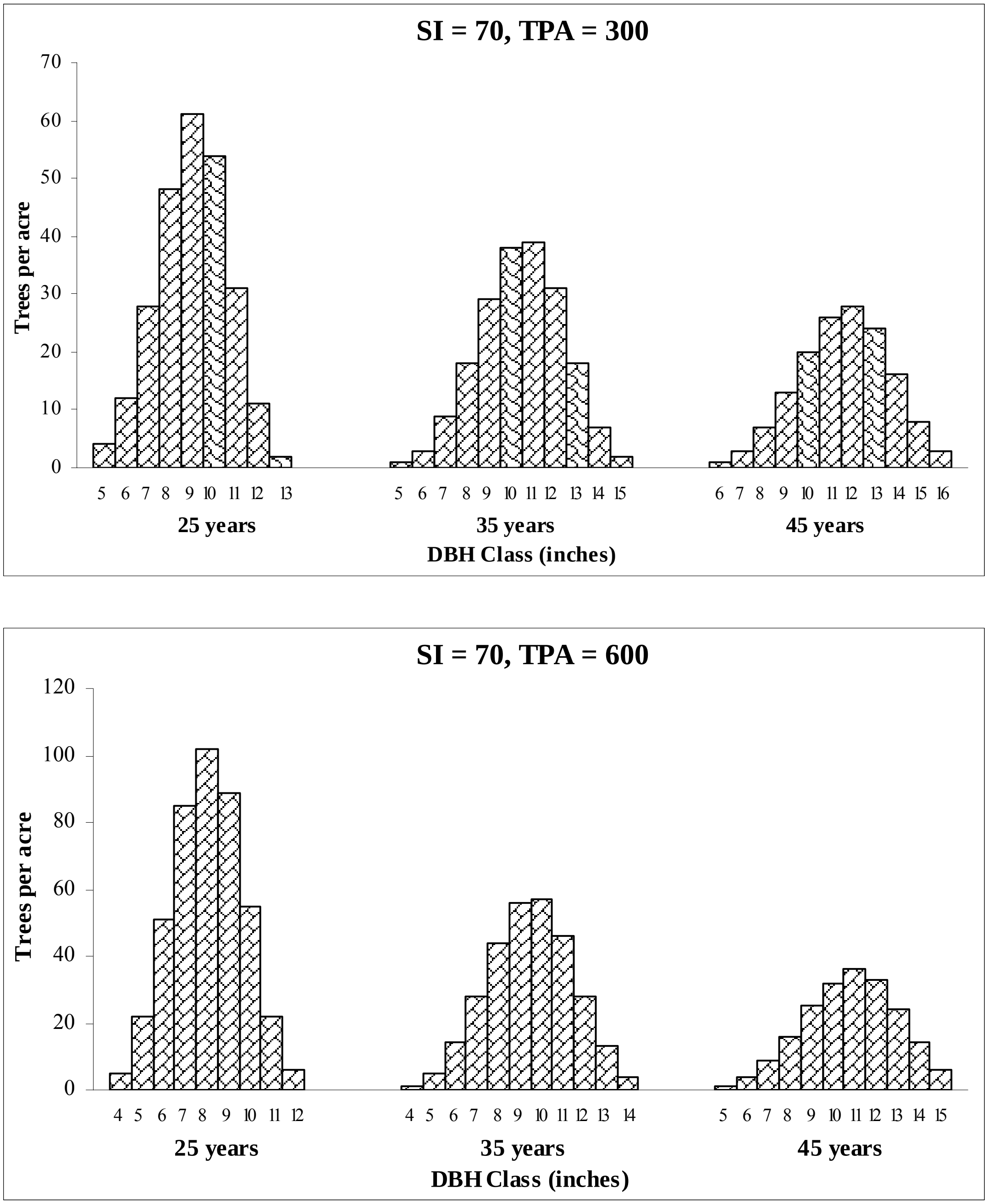



Figure 2.19. Estimated stand tables (equations 2.62a–2.62d → TSC1) at 25, 35 and 45 years of age for site index 70 feet, 300 and 600 trees per acre loblolly pine plantations thinned lightly from below at age 15.

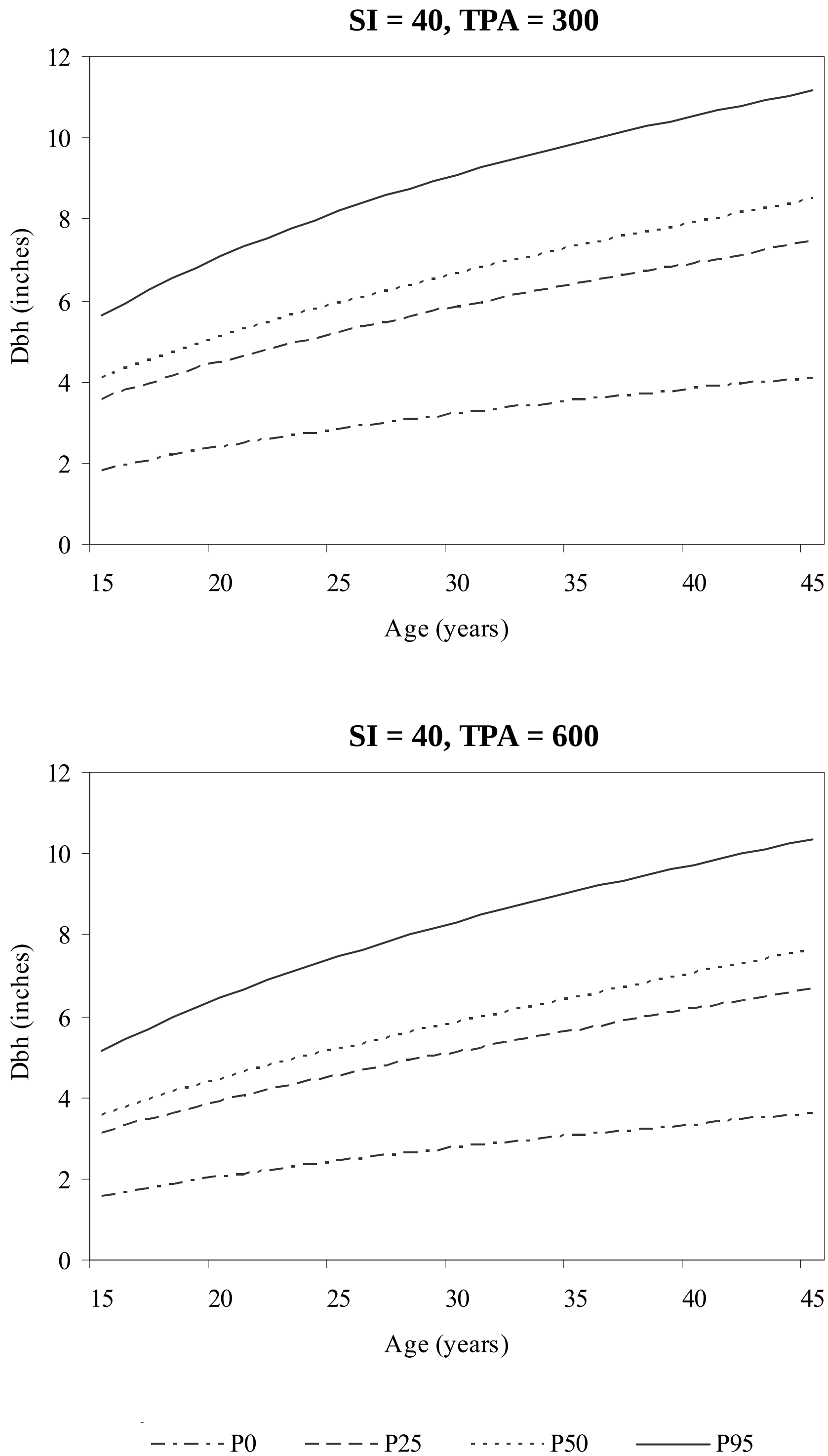



Figure 2.20. Projected thinned loblolly pine diameter distribution percentiles (equations 2.65a–2.65d → TSC2) for site index 40 feet, 300 and 600 trees per acre.

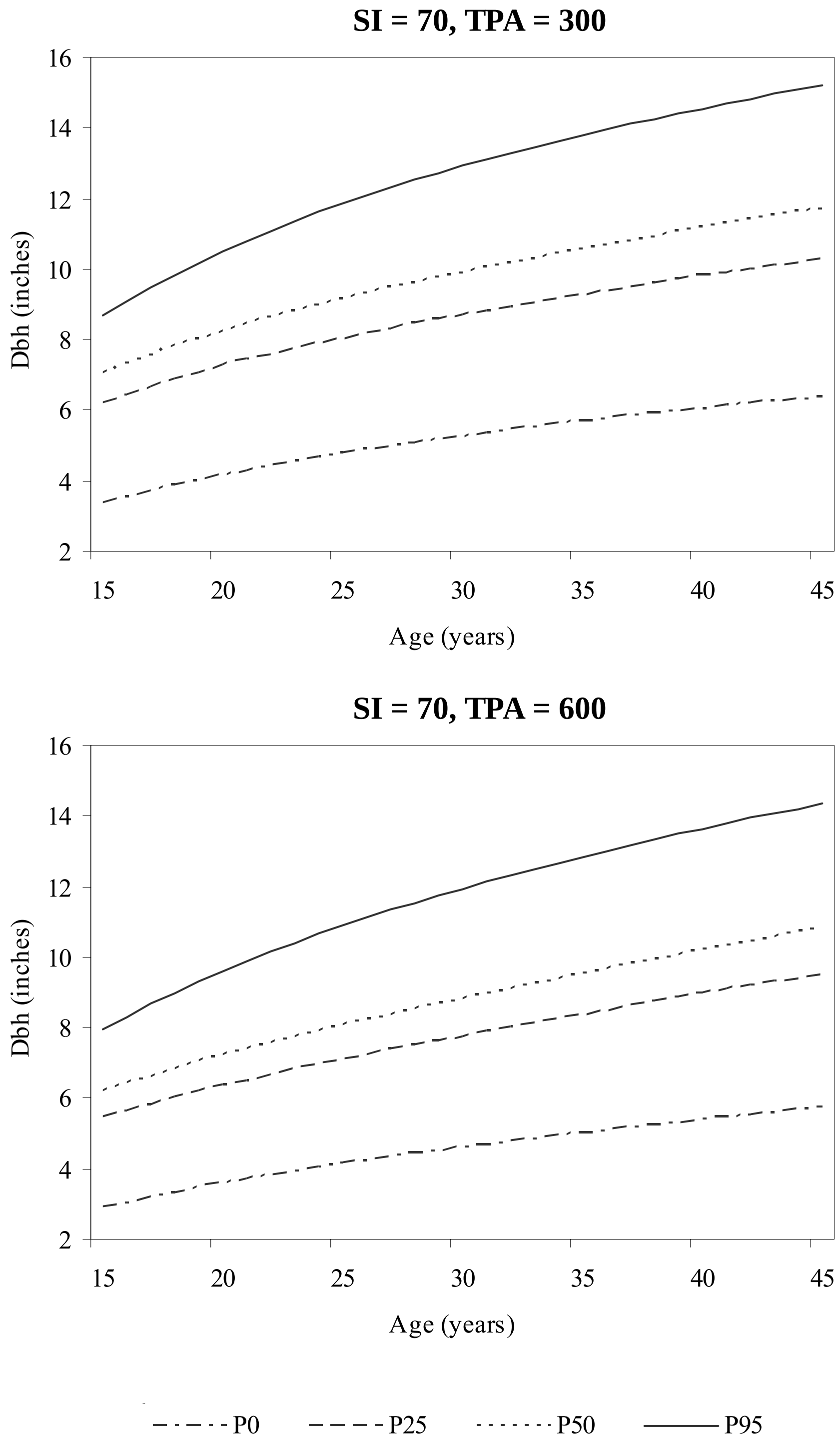
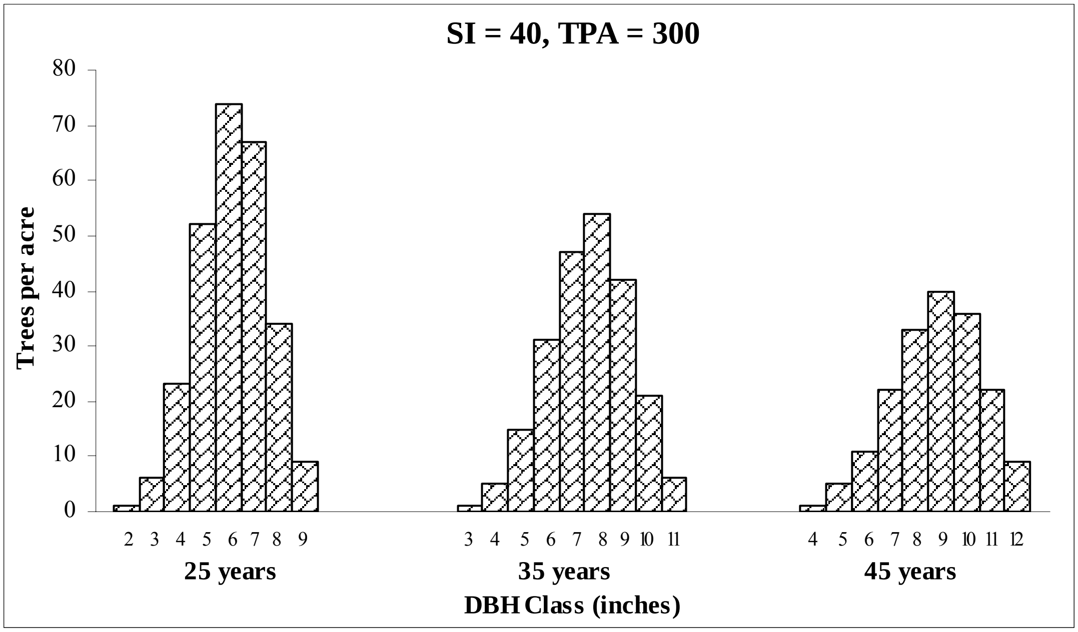



Figure 2.21. Projected thinned loblolly pine diameter distribution percentiles (equations 2.65a–2.65d $\rightarrow$ TSC2) for site index 70 feet, 300 and 600 trees per acre.



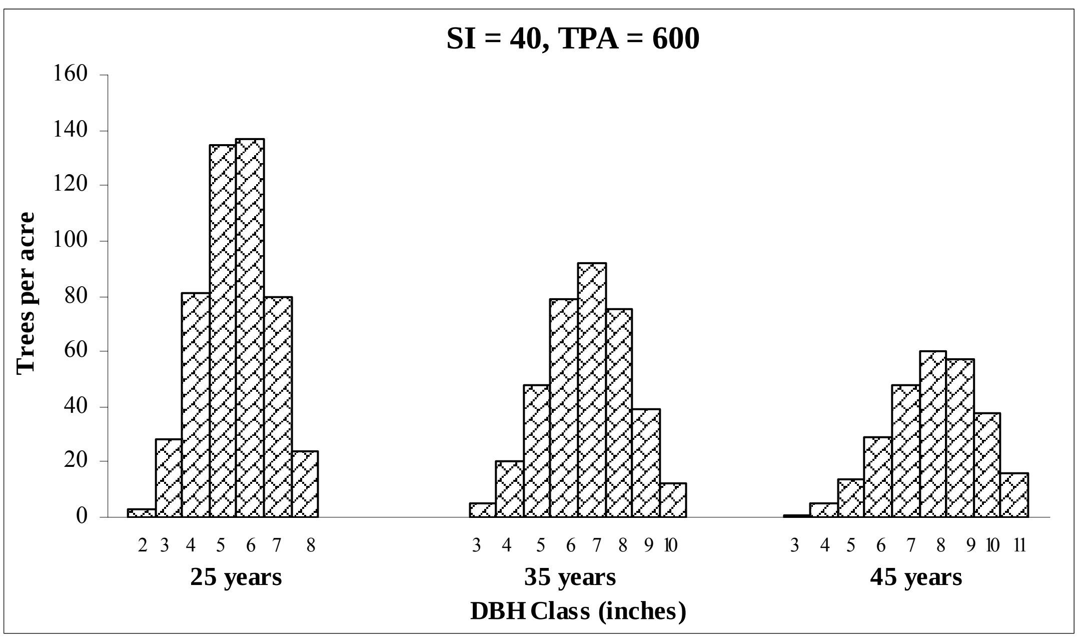




Figure 2.22. Estimated stand tables (equations 2.65a–2.657d → TSC2) at 25, 35 and 45 years of age for site index 40 feet, 300 and 600 trees per acre loblolly pine plantations thinned lightly from below at age 15.

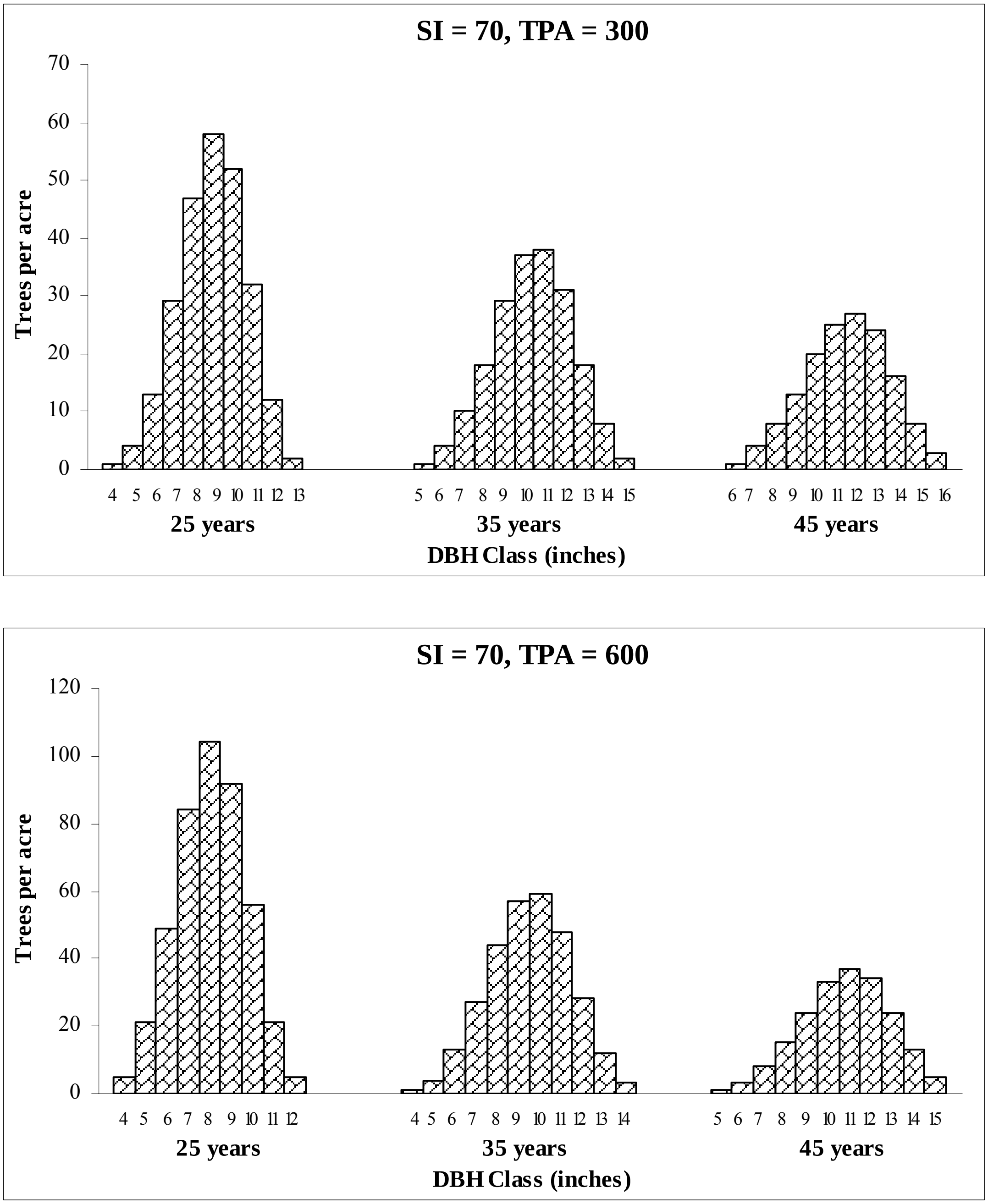



Figure 2.23. Estimated stand tables (equations 2.65a–2.65d → TSC2) at 25, 35 and 45 years of age for site index 70 feet, 300 and 600 trees per acre loblolly pine plantations thinned lightly from below at age 15.

# CHAPTER 3

# HEIGHT BY DIAMETER PREDICTION EQUATIONS FOR THINNED SLASH AND LOBLOLLY PINE PLANTATIONS

## Introduction

A height by diameter equation is an essential component of a diameter distribution yield model. It predicts individual tree heights, which are used with predicted diameter distributions in a volume equation to calculate volume by dbh class which, when summed up over all classes, provides total stand volume. It has generally been developed from stand age, dominant height (or site index), trees per acre and dbh. Like diameter distribution models, most if not all, of the existing height/dbh prediction equations (Bennett and Clutter, 1968; Lenhart, 1968; Smith, 1978; Pienaar et al., 1988 among others) have been developed mainly for unthinned pine stands. Except for Matney and Sullivan's (1982) work, thinned diameter distribution model systems in the literature use height/dbh equations developed for unthinned stands. This suggests that these studies have assumed that there is no change in individual tree height after thinning. However Dell and Collicott (1968), Parker (1979), Brooks (1992) and Harrison et al (1998) reported a change in average stand height of pine plantations after thinning and Clutter[1] in unpublished work with MS33 data reported that thinning from below increased average height of trees in the lower end of the diameter distribution of slash and loblolly pine plantations. Despite these changes, height growth, especially for dominant and codominant trees, in pine plantations has been observed to be affected very little by thinning (Bennett 1960, Dell and Collicott 1968, Parker 1979, Brooks 1992, Harrison et al 1998).

[1] Clutter, M. Personal Communication

The history of height/dbh equations for even-aged stands goes back to the pioneering diameter distribution work of Bennett and Clutter (1968). They developed a companion individual tree height equation for their beta distribution model system (3.1).

$$\log(h_i) = \beta_0 + \beta_1 S + \beta_2 (N/100) + \beta_3/A + \beta_5/D_i \qquad (3.1)$$

where,

$h_i$ = height of dbh class i

S = site index

A = age of the stand

N = number of surviving trees per acre in the stand

$D_i$ = dbh class i

log = logarithm base 10

Equation (3.1) estimates maximum heights to be close to site index. It simplifies to equation (3.2) for given stand values of S, N and A (Clutter et al.1983).

$$\ln(h_i) = \alpha + \delta/D_i \qquad (3.2)$$

A versatile height equation that spawned most of the height/dbh equations used in diameter distribution modeling studies in the 1970's and 1980's is Lenhart's (1968) individual tree total height equation (3.3):

$$\log\left(\frac{h_i}{HD}\right) = \beta_0 + \left(\frac{1}{D_i} - \frac{1}{Dmax}\right)\left\{\beta_1 + \frac{\beta_2}{A} + \beta_3 \log(N)\right\} \qquad (3.3)$$

where,

Dmax = midpoint of the largest diameter class

HD = average height of dominants and codominants in the stand

$h_i$, A, N and $D_i$ are as defined in equation (3.1)

Equation (3.3) assumes that a linear relationship exists between individual tree height and reciprocal of diameter such that the tree with the smallest diameter will be the shortest and the tree with the largest diameter will be the tallest. These patterns are easily evident when the equation takes on different $D_i$ values. As $D_i$ become smaller the second term in the right hand side of the model assumes larger negative values leading to smaller $H_i$. On the other hand when $D_i$ is equal to Dmax the second term is equal to zero resulting to larger $H_i$ (that is $\log(H_i) = \log(HD) + \beta_0$).

Variations of model (3.3) used to predict height include the Lenhart and Clutter (1971) height equation:

$$\begin{aligned}\log(HD/h_i) = \beta_0 + (1/D_i - 1/DMAX)\{\beta_1 + \beta_2\log(T_s) \\ + \beta_3/A + \beta_4\log(HD)\end{aligned} \tag{3.4}$$

and the Smalley and Bailey (1974) height prediction model:

$$\begin{aligned}\log\left(\frac{HD}{H_i}\right) = \beta_0 + \left(\frac{1}{D_i} - \frac{1}{Dmax}\right)\left\{\beta_1 + \beta_2 AN + \beta_3\frac{N}{A}\right. \\ \left. + \beta_4\log\left(\frac{N}{A}\right) + \log\left(\frac{H}{A}\right)\right\}\end{aligned} \tag{3.5}$$

Using a different approach, Smith (1978) predicted individual tree height from age, dominant height and sample dbh class percentile:

$$\begin{aligned}h_i/H = \beta_0 + \beta_1(1/A)H + \beta_2\ln[F(D)+1] \\ + \beta_3\ln[F(D)+1](1/A)H\end{aligned} \tag{3.6}$$

where,

F[D) = sample dbh class percentile

$h_i$, HD and A are as defined in model (3.3).

A desirable feature of equation (3.6) is that once Weibull parameters are estimated F[D] is readily obtained.

Matney and Sullivan (1982) suggested a total tree height prediction equation for both thinned and unthinned old-field loblolly pine plantations in Arkansas, Mississippi and Tennessee:

$$\ln(H) = \ln(c_0) + c_1/DBH \tag{3.7}$$

where,

$$c_0 = \overline{H}\{1 + a_1\exp[a_2(Q/\overline{H})^{a_3}]\}$$

$$c_1 = [\ln(\overline{\overline{H}}) - \ln(c_0)]Q$$

$$\overline{\overline{H}} = \overline{H}\exp\{b_1[\overline{H}^{b_2}/(b_3 + Q)^{b_4}]\}$$

H = total tree height in feet

$\overline{H}$ = average height of dominant and codominant trees in feet

Q = quadratic mean diameter of stand in inches

$a_i$'s and bi's are parameters to be estimated

They defined $\overline{H}$ as the predicted height of a tree having a diameter equal to the quadratic mean diameter (Q). Their height equation conditions asymptotic height to be greater than the height of dominants and codominants and height of the tree of mean basal area to be smaller than the height of dominants and codominants.

Working with natural longleaf pine stands in south Alabama, Farrar (1985) developed the following height/dbh prediction equation to predict mean height:

$$\begin{aligned} h = HD[\exp\{\beta_0 + \beta_1/HD + (1/D_i)(\beta_2 \\ + \beta_3 HD + \beta_4 T_s + \beta_5 HD^2 \end{aligned} \tag{3.8}$$

where,

$D_i$ = midpoint of the ith inch dbh class, inches

h, and HD and A are as defined in model (3.3).

Equation (3.8) basically predicts the proportion for each diameter class then uses it to adjust average dominant and codominant height.

Pienaar et al. (1988) suggested an individual tree height prediction that accounts for the relative position of the dbh class in the diameter distribution:

$$h_i = \beta_0 HD[1 - \beta_1(\exp\{-\beta_2(D_i/D_q)\})] \tag{3.9}$$

where,

$h_i$ = average total height of dbh class i

HD = average height of dominant and codominant trees

$D_i$ = midpoint of dbh class i

$D_q$ = quadratic mean diameter

Equation (3.9) modifies average dominant and codominant height based upon the size of the dbh class relative to the quadratic mean dbh. That is as $D_i$ becomes larger the second term in the model becomes larger resulting in larger average total heights. This equation is relatively simple in structure and has proven to be very robust for use in both loblolly and slash pine plantations (Harrison and Borders 1996, Pienaar et al. 1996). A modified version of this equation that was originally suggested by Clutter[1] was used in the development of height equation in this study.

**Data**

Data used to develop height/dbh prediction equation come from MS33 thinning study. This study was installed from 1981 to 1984 in piedmont, upper and lower coastal plain provinces of Alabama, Florida, Georgia and South Carolina (Figure 2.1 in Chapter 2) to investigate the effect of type, intensity and timing of thinning in slash and loblolly pine plantations. Tree measurements of dbh to the nearest tenth inch on all trees, total height to the nearest foot on a subsample across the diameter distribution and crown class were taken. These trees were tagged and remeasured at three year intervals. A total of 140 thinned and unthinned slash pine plots were used in this study. These plots ranged in density from 101 to 475 trees per acre, age ranged from 13 to 18 years, site class from 50 to 70, and basal area per acre from a minimum of 25 to a maximum of 113. Of the 280 loblolly pine plots that were analyzed 28 were unthinned and 252 were thinned plots. These plots had initial minimum age of 12 years to a maximum of 19 years, site class that ranged from 40 to 80, trees per acre from 116 to 827 trees per acre, and basal area per from 40 to 177 square feet per acre. A detailed description of MS33 thinning study is presented in Chapter 2.

**Methods**

Trees that were measured for total height in MS33 thinning study were identified. In each experimental plot dbh class, maximum dbh class, average height by dbh class, average height of dominants and codominants, quadratic mean diameter, number of trees before thinning and number of trees thinned from below were determined. Preliminary analyses indicated physiographic province, and type and intensity of thinning impacted the height dbh relationship. As such, separate height prediction equations were fitted for each region. Thinning type and intensity were represented using indicator variables.

Clutter[1], in previous work with MS33 data, observed that when thinning from below short and small trees had a higher probability of removal. This changed the average height by dbh class in the left tail of the diameter distributions. Row thinning had no influence on the average height because it removed on average small and large trees in equal proportions. That is it had a neutral effect on the size of remaining trees. He suggested adding a thinning term to the Pienaar et al. (1988) height/diameter function (model 3.9) to reflect these patterns:

$$\begin{aligned} h_i = \beta_0 HD[1-\beta_1 \exp[-\beta_2(D_i/Dq) \\ -\beta_3(N_s/N_b)(1-D_i/Dmax)]] \end{aligned} \quad (3.10)$$

where

$h_i$ = average total tree height in feet of the $i^{th}$ dbh class

HD = average height of dominants and codominants in feet

D = midpoint of the $i^{th}$ dbh class

Dq = quadratic mean diameter in inches

$N_s$ = number of trees removed selectively from below

$N_b$ = number of trees before thinning

Dmax = maximum occupied dbh class

The thinning term in equation (3.10) drops out of the model for row thinning and unthinned stand. This term when significant accounts for the intensity of selective thinning and the position of the dbh class in the diameter distribution. As intensity of the thinning increases the size of this term becomes larger hence greater differences between thinned and unthinned height predictions. However the magnitude of these differences is dependent on the position of dbh class in the distribution with the largest differences obtained with the minimum dbh class and the smallest with the maximum dbh class.

Height/dbh prediction equations were developed for each of the following cases that were also considered in diameter distribution modeling. Specifically, thinned stand case 1 (TSC1) where complete information about before and after thin stand is available; thinned stand case 2 (TSC2) where information about thinning type, age of thin and after thin dominant height, basal area per acre and trees per acre are available; thinned stand case 3 (TSC3) where information about thinning intensity, age of thin and after thin dominant height, basal area per acre and trees per acre are available; thinned stand case 4 (TSC4) where information about age and type of thinning is available; thinned stand case 5 (TSC5) where age of thinning is known and thinned stand case 6 (TSC6) where no thinning information is available except it has been thinned (Table 3.1). Note that for the available data thinning intensity is restricted to 33%, 40% and 50% of trees per acre removed. Height prediction equations fitted for these cases included the following. The height prediction equation (3.10) suggested by Clutter[1] was developed for TSC1. Pienaar et al's height prediction equation (3.9) was developed for TSC2, TSC3 and TSC4 by accounting for the effects of type and intensity of thinning using indicator variables. Equation (3.9) was fitted for TSC5 and TSC6. PROC NLIN in SAS was used to fit these equations. Convergence was easily achieved. Goodness of fit measures namely percent variation explained and root mean square error were calculated for each equation as follows:

$$PVE = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{3.11}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} \tag{3.12}$$

where,

PVE = percent variation explained

RMSE = root mean square error

$y_i$ = observed average total tree height by dbh class

$\bar{y}$ = mean $y_i$

$\hat{y}$ = predicted average total tree height by dbh class

n = number of observations

Table 3.1. Information available (√) about thinning for TSC1-TSC6.

| TSC# | Thinning Type | Thinning Intensity | $A_t$ | $N_{bt}$ | $N_{at}$ | $B_{bt}$ | $B_{at}$ | $HD_{bt}$ | $HD_{at}$ |
|---|---|---|---|---|---|---|---|---|---|
| TSC1 | √ | √ | √ | √ | √ | √ | √ | √ | √ |
| TSC2 | √ | | √ | | √ | | √ | | √ |
| TSC3 | | √ | √ | | √ | | √ | | √ |
| TSC4 | √ | | √ | | | | | | |
| TSC5 | | | √ | | | | | | |
| TSC6 | | | | | | | | | |

where,

$A_t$ = age of the stand at time of thinning

$N_{bt}$ = number of trees per acre before thinning

$N_{at}$ = number of trees per acre after thinning

$B_{bt}$ = basal area per acre before thinning

$B_{at}$ = basal area per acre after thinning

$HD_{bt}$ = average height of dominants and codominants before thinning

$HD_{at}$ = average height of dominants and codominants after thinning

## Results and Analysis

### Slash Pine

When thinning type was represented using indicator variables in equation (3.9), the following equations were obtained for slash pine TSC2 and TSC4,

Lower Coastal Plain:

$$h_i = (\beta_0 + \beta_{01}Z_1)HD\{1 - (\beta_1 + \beta_{12}Z_2)\exp[-(\beta_2 + \beta_{21}Z_1)(D_i/Dq)]\} \tag{3.13}$$

Upper Coastal Plain:

$$h_i = \delta_0 HD\{1 - (\delta_1 + \delta_{11}Z_1 + \delta_{12}Z_2)\exp[-(\delta_2 + \delta_{21}Z_1)(D_i/Dq)]\} \tag{3.14}$$

where,

$$Z_1 = \begin{cases} 1 & \text{if row thinned} \\ 0 & \text{if otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 & \text{if select thinned} \\ 0 & \text{if otherwise} \end{cases}$$

All else are as defined in model (3.9).

For slash pine TSC3 thinning intensity was accounted for in equation (3.9) to derive the following equations,

Lower Coastal Plain:

$$h_i = (\beta_0 + \beta_{01}Z_1 + \beta_{02}Z_2)HD\{1 - \beta_1\exp[-(\beta_2 + \beta_{21}Z_1)(D_i/Dq)]\} \tag{3.15}$$

Upper Coastal Plain:

$$h_i = \delta_0 HD\{1 - \delta_1\exp[-(\delta_2 + \delta_{21}Z_1)(D_i/Dq)]\} \tag{3.16}$$

where,

$$Z_1 = \begin{cases} 1 \text{ if thinning intensity is } 50\% \\ 0 \text{ if otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 \text{ if thinning intensity is } 40\% \\ 0 \text{ if otherwise} \end{cases}$$

All else are as defined in model 3.9.

The regression analysis results of fitting equations (3.9-3.10), and (3.13)-(3.16) to slash pine height-dbh data are presented in Tables 3.2 and 3.3. All parameter estimates in the fitted equations were significantly different from zero at $\alpha = 0.05$. Percent variation explained by these equations ranged from 93.69 for TSC5 and TSC6 height equation (3.9) in upper coastal plain province, to 96.06 for TSC1 height equation (3.10) in lower coastal plain province. Standard errors ranged from 2.52 feet for TSC1 height equation (3.10) in lower coastal plain province, to 2.91 feet for fitted height equations in upper coastal plain province. A plot of standardized residuals against projected height did not indicate trends that violated regression assumptions.

Figure 3.1 shows plots of tree height predictions obtained with fitted equation (3.10) for a stand thinned from below and for an unthinned stand. For dbh classes 2-9 tree heights were larger in the thinned stand than in the unthinned stand. These differences were greater for small dbh classes and increased with thinning intensity. However as dbh class increased in size tree heights in the thinned stand approached that of the unthinned stand. These results are in agreement with previous studies that reported a change in average stand height of pine plantations after thinning (Dell and Collicott 1968, Parker 1979).

When thinning types were accounted for using indicator variables (equations 3.13 and 3.14), similar trends were observed as in equation (3.10). That is, average total tree heights for dbh classes 2-8 were smaller in a row thinned stand than in a row-select thinned stand, and were smaller in row-select thinned stands than in select thinned stands. These differences increased with decrease in tree size and decreased with increase in tree sizes (Figure 3.2). Like thinning type, intensity of thinning was determined to be significant (equations 3.15 and 3.16). There were no differences in tree heights between light and moderate thinned stands in lower coastal plain. However, in both physiographic regions average total tree heights were smaller in heavily thinned stands than in lightly and moderately thinned stands (Figure 3.3). This result was rather surprising in that it did not seem logical. It suggested opposing trends of what previous models had indicated. This is probably a weakness of the height prediction models developed for TSC3.

**Loblolly Pine**

For TSC2 and TSC4 thinning types were represented by indicator variables in equation (3.9). Non-significant variables were dropped from the model to yield the following height prediction equations:

Piedmont:

$$\begin{aligned} h_i = (\beta_0 + \beta_{02} Z_2) HD\{1 - (\beta_1 + \beta_{11} Z_1) \exp[ \\ -(\beta_2 + \beta_{22} Z_2)(D_i/Dq)]\} \end{aligned} \tag{3.17}$$

Upper Coastal Plain:

$$\begin{aligned} h_i = (\alpha_0 + \alpha_{01} Z_1) HD\{1 - (\alpha_1 + \alpha_{12} Z_2) \exp[ \\ -\alpha_2 (D_i/Dq)]\} \end{aligned} \tag{3.18}$$

where,

$$Z_1 = \begin{cases} 1 \text{ if row thinned} \\ 0 \text{ if otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 \text{ if select thinned} \\ 0 \text{ if otherwise} \end{cases}$$

In lower Coastal Plain an equation similar to height prediction equation (3.14) was obtained. For loblolly pine TSC3 intensity of thinning was accounted for in equation (3.9). The final height prediction equations obtained for piedmont and upper coastal plain were as follows:

Piedmont:

$$h_i = \beta_0 HD\{1-(\beta_1+\beta_{11}Z_1)\exp[-\beta_2(D_i/Dq)]\} \tag{3.19}$$

Upper Coastal Plain:

$$h_i = (\alpha_0+\alpha_{01}Z_1)HD\{1-(\alpha_1+\alpha_{11}Z_1)\exp[-\alpha_2(D_i/Dq)]\} \tag{3.20}$$

$$Z_1 = \begin{cases} 1 \text{ if thinning intensity} > 33\% \\ 0 \text{ if otherwise} \end{cases}$$

In lower Coastal Plain thinning intensity was not significant.

When equations (3.9-3.10, 3.14, and 3.17-3.20) were fitted to loblolly pine height-dbh data the regression results obtained (Tables 3.4 and 3.5) indicated that all the parameter estimates in the fitted equations were statistically different from zero at $\alpha = 0.05$. Reasonably good statistics of fit were obtained for all equations. This was demonstrated by PVE values that ranged from 92.48% to 95.21% and RSME values that ranged from 2.88 feet to 3.11 feet. Visual analyses of the residual plots did not indicate any evidence of serious departures from regression assumptions.

Simulations of fitted equations are presented in Figures 3.4-3.6. Predictions obtained with equation (3.10) indicated larger average total tree heights for dbh classes 2-9 in a stand thinned from below than in an unthinned stand. These differences were greater for small dbh classes. In addition, differences increased with thinning intensity and decreased with tree size, approaching zero for the maximum dbh class (Figure 3.4). When thinning type was represented using indicator variables (equations 3.14, 3.17 and 3.18) predicted average total tree heights were larger in selectively thinned stands than in row-select, and larger in row-select than in row thinned stands. These differences increased with decrease in tree size but decreased towards the right end of the diameter distribution (Figure 3.5). When thinning intensity was considered (equations 3.19 and 3.20) generally average total height of trees in heavily thinned stand were smaller than in lightly and moderately thinned stands. In the upper coastal plain tree heights for dbh classes 2-8 were larger in light thinned than in moderate thinned stands (Figure 3.6). This result did not seem logical and raised questions about the reliability of equations (3.19) and (3.20).

**Discussion**

This study developed average total tree height-dbh equations for TSC1-TSC6 using MS33 slash and loblolly pine data. Results show that both equations (3.9) and (3.10) performed reasonably well in predicting average total tree height. This was shown by percent variation explained that were greater than 92% and root mean square errors that were less than 3.2 feet. All parameter estimates in fitted equations were significantly different from zero at $\alpha = 0.05$ (Tables 3.2-3.5). Thus the thinning term in equation (3.10) was significant hence the observed differences in average total tree height between select thinned (from below) and unthinned stands (Figures 3.1 and 3.4). These differences increased with thinning intensity. For small dbh classes average total tree heights were larger in thinned stands than in unthinned stands. However these differences decreased with dbh class size approaching zero for the maximum dbh. These results imply that the thinning term in equation (3.10) accounted for both the effects of type and intensity of thinning, and the relative position of the dbh class in the diameter distribution (Clutter[1]).

For TSC2, TSC3 and TSC4 where pre-thin information is not available, type and intensity of thinning were accounted for in height prediction equation (3.9) using indicator variables. These variables were determined to be significant. This implied the changes shown in Figures 3.2-3.3 and Figures 3.5-3.6. A comparison of thinning types show that average total tree heights for small dbh classes were larger in select thinned than in row-select thinned stand, and larger in row-select thinned than in row thinned stands. These differences appeared to decrease with tree size. This implies that selective thinning by removing smaller trees alters the height distribution of the trees in such a manner that it increases the average total height of smaller trees (Clutter[1]). However, when thinning intensity was considered (TSC3) generally average total heights of trees in heavily thinned stands were smaller than in lightly and moderately thinned stands. This result did not seem reasonable. It seems to suggest that by removing a larger proportion of trees heavy thinning increased quadratic mean diameters making the term $D_i/D_q$ in the model smaller hence smaller average total tree heights than in the light thinned stands. The data showed quadratic mean diameters to be generally larger in heavily thinned stands than in lightly thinned stands. However observed differences were not large to explain the trend suggested by the model. This suggests that thinning intensity significance in the model might be an over-fitting problem especially given the size of the data used to fit these models. It is clear that intensity alone did not appear to reflect the height distribution in the data. Its effects on height-dbh relationship can best be understood when the type of thinning is known.

**Conclusion**

The objective of this study was to develop individual total tree height prediction equation for MS33 slash and loblolly pine plantations. This equation is to be used with predicted diameter distributions to predict stand volume. The height equation (3.10) originally suggested by Clutter[1] was determined to be suitable for TSC1. This equation had the best fit statistics and its thinning term was significant. However a limitation of this equation is that it requires complete information about pre and post-thin stand conditions.

Equation (3.9) with effects of type and intensity thinning accounted for was suitable for TSC2, TSC3 and TSC4. No changes were required when it was applied to TSC5 and TSC6 since very limited information is assumed to be available from the pre and post-thin stand conditions. This equation had reasonably good fit statistics and represented well the height distribution of the trees. In addition it demonstrated like equation (3.10) that type and intensity of thinning significantly affected average total tree heights. These effects were such that when compared to row thinning and unthinned stands, selective thinning, depending on thinning intensity, increased the average height of trees towards the left end of the diameter distribution. However thinning intensity when considered alone showed average total tree heights were smaller in heavily thinned stands than in lightly thinned stands. This result seems somewhat counterintuitive and is likely a limitation of this model form. Consequently, when limited information is available about a thinning a reasonable attempt should be made to ascertain both the type of thinning (row, select, row-select) as well as the intensity of thinning.

Table 3.2. Parameter estimates and fit statistics for height prediction equations (3.10), (3.13) and (3.14) fitted to slash pine data in lower and upper coastal plain provinces.

| **Lower Coastal Plain** | | | | **Upper Coastal Plain** | | | |
|---|---|---|---|---|---|---|---|
| **TSC1 Height Equation (3.10)** | | | | **TSC1 Equation Height (3.10)** | | | |
| Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| |
| $\beta_0$ | 1.2586 | 0.0069 | <.0001 | $\beta_0$ | 1.1823 | 0.0095 | <.0001 |
| $\beta_1$ | 1.0725 | 0.0114 | <.0001 | $\beta_1$ | 1.1129 | 0.0334 | <.0001 |
| $\beta_2$ | 1.5688 | 0.0301 | <.0001 | $\beta_2$ | 1.8918 | 0.0711 | <.0001 |
| $\beta_3$ | 0.2783 | 0.0277 | <.0001 | $\beta_3$ | 0.1837 | 0.0609 | 0.0026 |
| PVE | | 96.06 | | PVE | | 93.73 | |
| RMSE | | 2.52 | | RMSE | | 2.91 | |
| **TSC2 & TSC4 Height Equation (3.13)** | | | | **TSC2 & TSC4 Height Equation (3.14)** | | | |
| Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| |
| $\beta_0$ | 1.2429 | 0.0069 | <.0001 | $\alpha_0$ | 1.1801 | 0.0093 | <.0001 |
| $\beta_{01}$ | 0.0235 | 0.0087 | 0.0068 | $\alpha_1$ | 1.1025 | 0.0326 | <.0001 |
| $\beta_1$ | 1.0649 | 0.0113 | <.0001 | $\alpha_{12}$ | -0.0287 | 0.0131 | 0.0286 |
| $\beta_{12}$ | -0.0346 | 0.0062 | <.0001 | $\alpha_2$ | 1.8930 | 0.0709 | <.0001 |
| $\beta_2$ | 1.6180 | 0.0312 | <.0001 | | | | |
| $\beta_{21}$ | -0.0884 | 0.0237 | 0.0002 | | | | |
| PVE | | 96.03 | | PVE | | 93.71 | |
| RMSE | | 2.52 | | RMSE | | 2.91 | |

Table 3.3. Parameter estimates and fit statistics for height prediction equations (3.9), (3.15) and (3.16) fitted to slash pine data in lower and upper coastal plain provinces.

| **Lower Coastal Plain** | | | | **Upper Coastal Plain** | | | |
|---|---|---|---|---|---|---|---|
| **TSC3 Height Equation (3.15)** | | | | **TSC3 Height Equation (3.16)** | | | |
| Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| |
| $\beta_0$ | 1.2536 | 0.0073 | <.0001 | $\alpha_0$ | 1.1796 | 0.0093 | <.0001 |
| $\beta_{01}$ | -0.0242 | 0.0085 | 0.0046 | $\alpha_1$ | 1.0987 | 0.0323 | <.0001 |
| $\beta_{02}$ | 0.00764 | 0.0028 | 0.0060 | $\alpha_2$ | 1.8875 | 0.0707 | <.0001 |
| $\beta_1$ | 1.0593 | 0.0112 | <.0001 | $\alpha_{21}$ | 0.0440 | 0.0166 | 0.0081 |
| $\beta_2$ | 1.5749 | 0.0309 | <.0001 | | | | |
| $\beta_{21}$ | 0.0841 | 0.0252 | 0.0009 | | | | |
| PVE | | 95.96 | | PVE | | 93.72 | |
| RMSE | | 2.55 | | RMSE | | 2.91 | |
| **TSC5 & TSC6 Height Equation (3.9)** | | | | **TSC5 & TSC6 Height Equation (3.9)** | | | |
| Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| |
| $\beta_0$ | 1.2482 | 0.0066 | <.0001 | $\alpha_0$ | 1.1799 | 0.0093 | <.0001 |
| $\beta_1$ | 1.0607 | 0.0112 | <.0001 | $\alpha_1$ | 1.0973 | 0.0323 | <.0001 |
| $\beta_2$ | 1.6019 | 0.0302 | <.0001 | $\alpha_2$ | 1.8975 | 0.0711 | <.0001 |
| PVE | | 95.94 | | PVE | | 93.69 | |
| RMSE | | 2.55 | | RMSE | | 2.91 | |

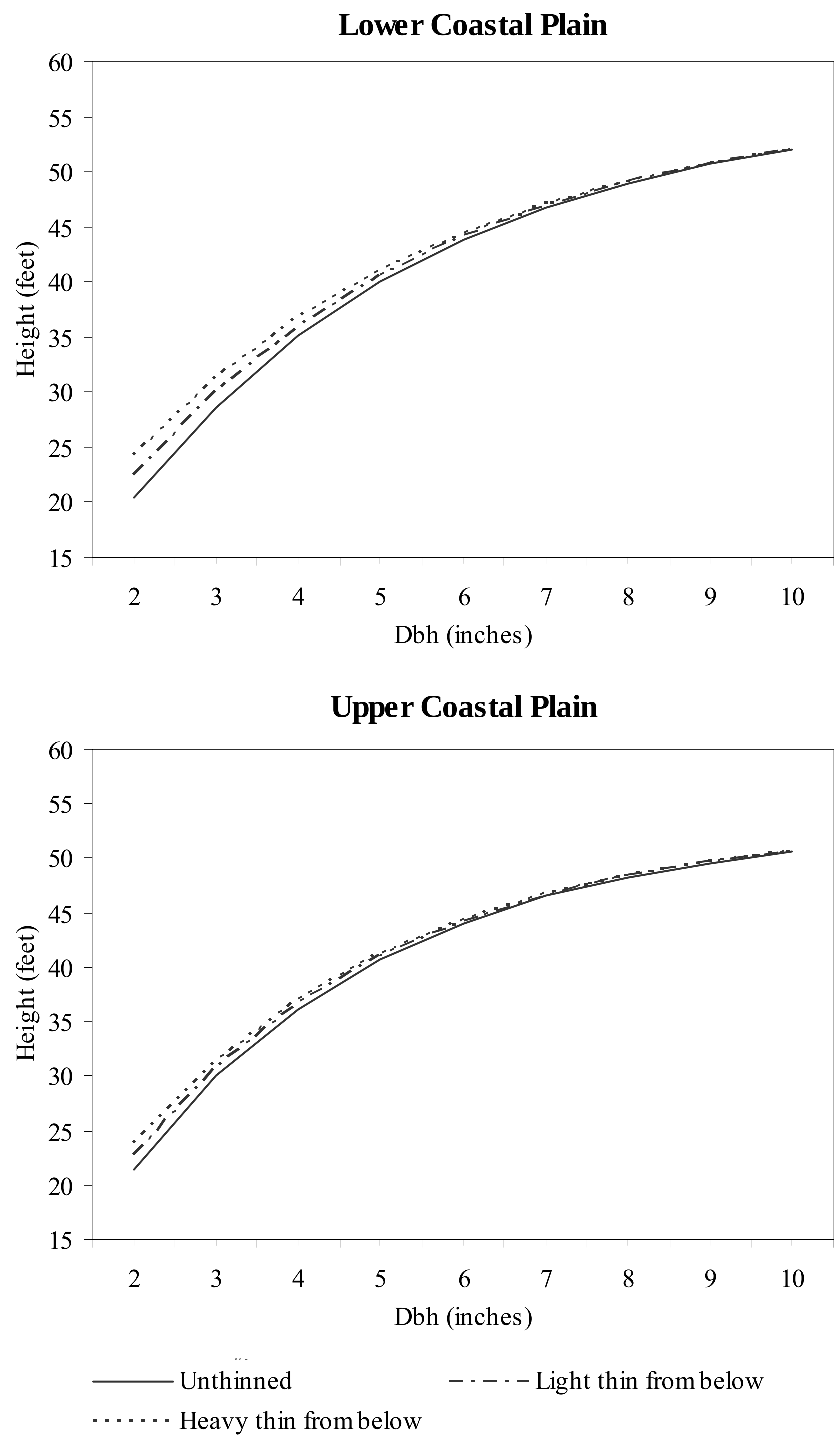



Figure 3.1. Predicted average height by dbh class (equation 3.10 – TSC1) for unthinned and selectively (light and heavy) thinned slash pine stand in lower and upper coastal plains.

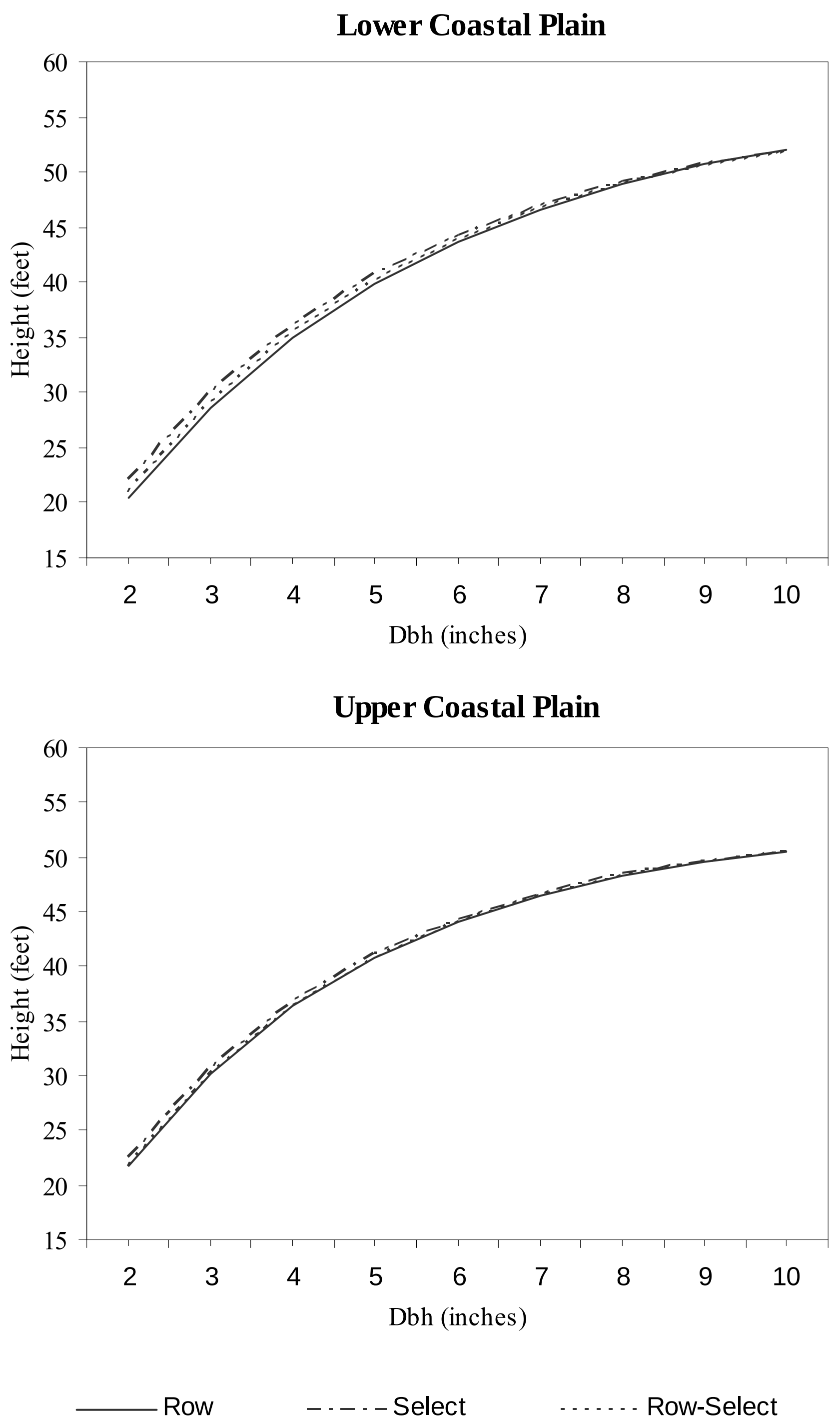



Figure 3.2. Predicted average height by dbh class (equations 3.13 and 3.14 – TSC2 and TSC4) for row, select and row-select thinned slash pine stands in lower and upper coastal plain.

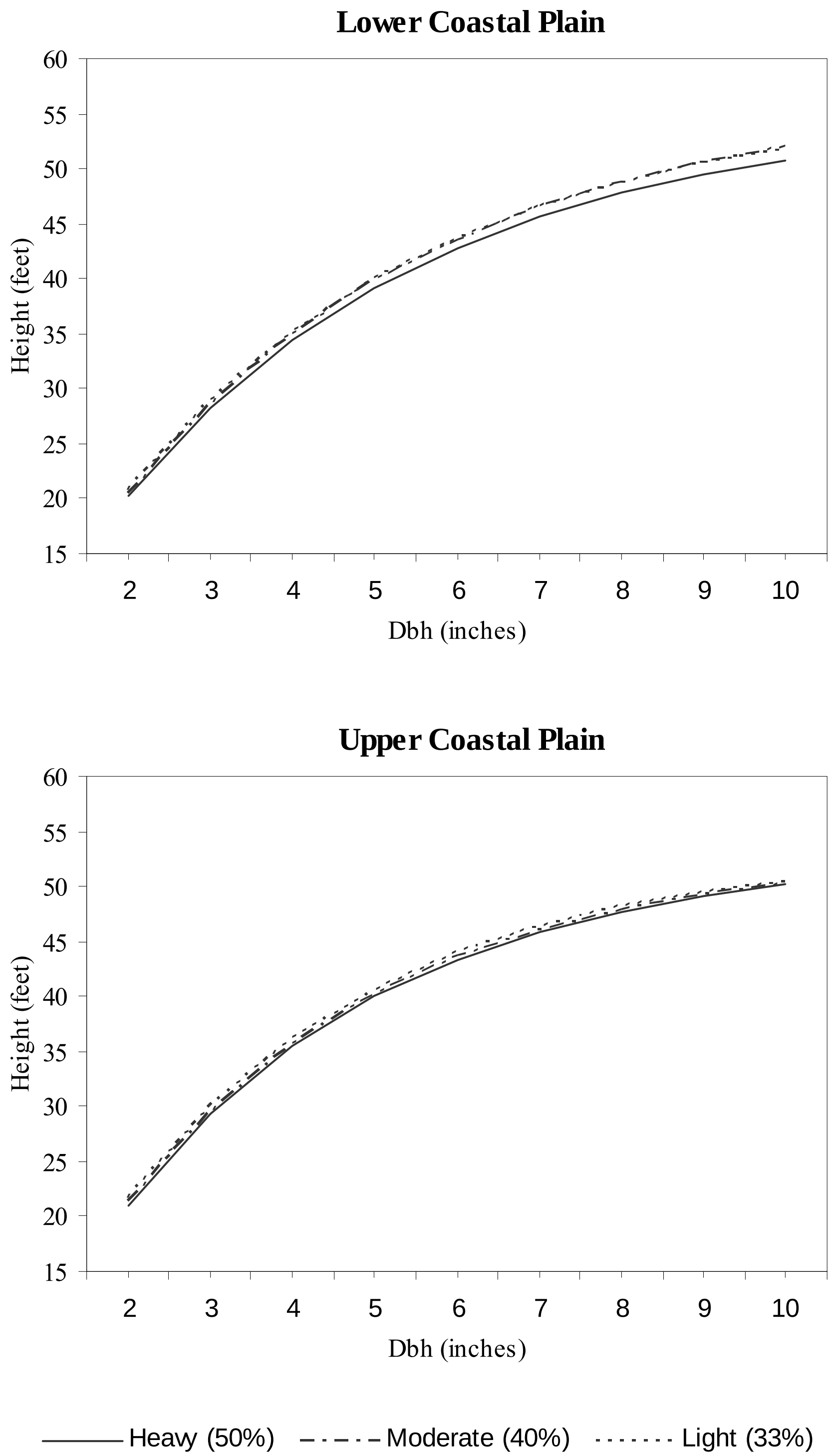



Figure 3.3. Predicted average height by dbh class (equations 3.15 and 3.16 – TSC3) for heavy, moderate and light thinned slash pine stands in lower and upper coastal plain provinces.

Table 3.4. Parameter estimates and fit statistics for height prediction equations (3.10), (3.17), (3.18) and (3.14) fitted to loblolly pine data in piedmont, upper and lower plain provinces.

| **Piedmont** | | | | **Upper Coastal Plain** | | | | **Lower Coastal Plain** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **TSC1 Height Equation (3.10)** | | | | **TSC1 Height Equation (3.10)** | | | | **TSC1 Height Equation (3.10)** | | | |
| Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| |
| $\beta_0$ | 1.2641 | 0.0128 | <.0001 | $\beta_0$ | 1.1787 | 0.0060 | <.0001 | $\beta_0$ | 1.2217 | 0.0079 | <.0001 |
| $\beta_1$ | 0.9321 | 0.0177 | <.0001 | $\beta_1$ | 0.9853 | 0.0183 | <.0001 | $\beta_1$ | 0.9942 | 0.0157 | <.0001 |
| $\beta_2$ | 1.4156 | 0.0544 | <.0001 | $\beta_2$ | 1.8084 | 0.0464 | <.0001 | $\beta_2$ | 1.5932 | 0.0425 | <.0001 |
| $\beta_3$ | 0.3426 | 0.0409 | <.0001 | $\beta_3$ | 0.3615 | 0.0453 | <.0001 | $\beta_3$ | 0.1293 | 0.0191 | <.0001 |
| PVE | 93.17 | | | PVE | 95.21 | | | PVE | 92.60 | | |
| RMSE | 2.88 | | | RMSE | 2.94 | | | RMSE | 3.08 | | |
| **TSC2 & TSC4 Height Equation (3.17)** | | | | **TSC2 & TSC4 Height Equation (3.18)** | | | | **TSC2 & TSC4 Height Equation (3.14)** | | | |
| Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| | Parameter | Estimate | Std.Error | Pr > \|t\| |
| $\beta_0$ | 1.2600 | 0.0132 | <.0001 | $\alpha_0$ | 1.1742 | 0.0057 | <.0001 | $\delta_0$ | 1.2204 | 0.0079 | <.0001 |
| $\beta_{02}$ | -0.0281 | 0.0141 | 0.0464 | $\alpha_{01}$ | -0.0089 | 0.0025 | 0.0003 | $\delta_1$ | 0.9789 | 0.0167 | <.0001 |
| $\beta_1$ | 0.9014 | 0.0167 | <.0001 | $\alpha_1$ | 0.9844 | 0.0187 | <.0001 | $\delta_{11}$ | 0.0414 | 0.0200 | 0.0385 |
| $\beta_{11}$ | 0.0294 | 0.0074 | <.0001 | $\alpha_{12}$ | -0.0206 | 0.0083 | 0.0129 | $\delta_{12}$ | -0.0443 | 0.0079 | <.0001 |
| $\beta_2$ | 1.4097 | 0.0558 | <.0001 | $\alpha_2$ | 1.8541 | 0.0462 | <.0001 | $\delta_2$ | 1.5821 | 0.0432 | <.0001 |
| $\beta_{22}$ | 0.1228 | 0.0447 | 0.0060 | | | | | $\delta_{21}$ | 0.0544 | 0.0268 | 0.0425 |
| PVE | 93.14 | | | PVE | 95.16 | | | PVE | 92.60 | | |
| RMSE | 2.88 | | | RMSE | 2.96 | | | RMSE | 3.08 | | |

Table 3.5. Parameter estimates and fit statistics for height prediction equations (3.9), (3.19) and (3.20) fitted to loblolly pine data in piedmont, upper and lower plain provinces.

**Piedmont**

**TSC3 Height Equation (3.19)**

| Parameter | Estimate | Std.Error | Pr > \|t\| |
|---|---|---|---|
| $\beta_0$ | 1.2526 | 0.0121 | <.0001 |
| $\beta_1$ | 0.9170 | 0.0177 | <.0001 |
| $\beta_{11}$ | -0.0135 | 0.0063 | 0.0330 |
| $\beta_2$ | 1.4394 | 0.0550 | <.0001 |
| PVE | | 93.01 | |
| RMSE | | 2.91 | |

**TSC5 & TSC6 Height Equation (3.9)**

| Parameter | Estimate | Std.Error | Pr > \|t\| |
|---|---|---|---|
| $\beta_0$ | 1.2519 | 0.0121 | <.0001 |
| $\beta_1$ | 0.9101 | 0.0172 | <.0001 |
| $\beta_2$ | 1.4433 | 0.0551 | <.0001 |
| PVE | | 92.99 | |
| RMSE | | 2.91 | |

**Upper Coastal Plain**

**TSC3 Height Equation (3.20)**

| Parameter | Estimate | Std.Error | Pr > \|t\| |
|---|---|---|---|
| $\alpha_0$ | 1.1650 | 0.0058 | <.0001 |
| $\alpha_{01}$ | 0.0079 | 0.0036 | 0.0284 |
| $\alpha_1$ | 0.9676 | 0.0196 | <.0001 |
| $\alpha_{11}$ | 0.0295 | 0.0123 | 0.0165 |
| $\alpha_2$ | 1.8714 | 0.0463 | <.0001 |
| PVE | | 95.13 | |
| RMSE | | 2.97 | |

**TSC5 & TSC6 Height Equation (3.9)**

| Parameter | Estimate | Std.Error | Pr > \|t\| |
|---|---|---|---|
| $\beta_0$ | 1.1695 | 0.0055 | <.0001 |
| $\beta_1$ | 0.9852 | 0.0186 | <.0001 |
| $\beta_2$ | 1.8724 | 0.0463 | <.0001 |
| PVE | | 95.13 | |
| RMSE | | 2.97 | |

**Lower Coastal Plain**

**TSC3,TSC5 & TSC6 Height Equation (3.9)**

| Parameter | Estimate | Std.Error | Pr > \|t\| |
|---|---|---|---|
| $\beta_0$ | 1.2190 | 0.0079 | <.0001 |
| $\beta_1$ | 0.9808 | 0.0153 | <.0001 |
| $\beta_2$ | 1.6076 | 0.0429 | <.0001 |
| PVE | | 92.48 | |
| RMSE | | 3.11 | |

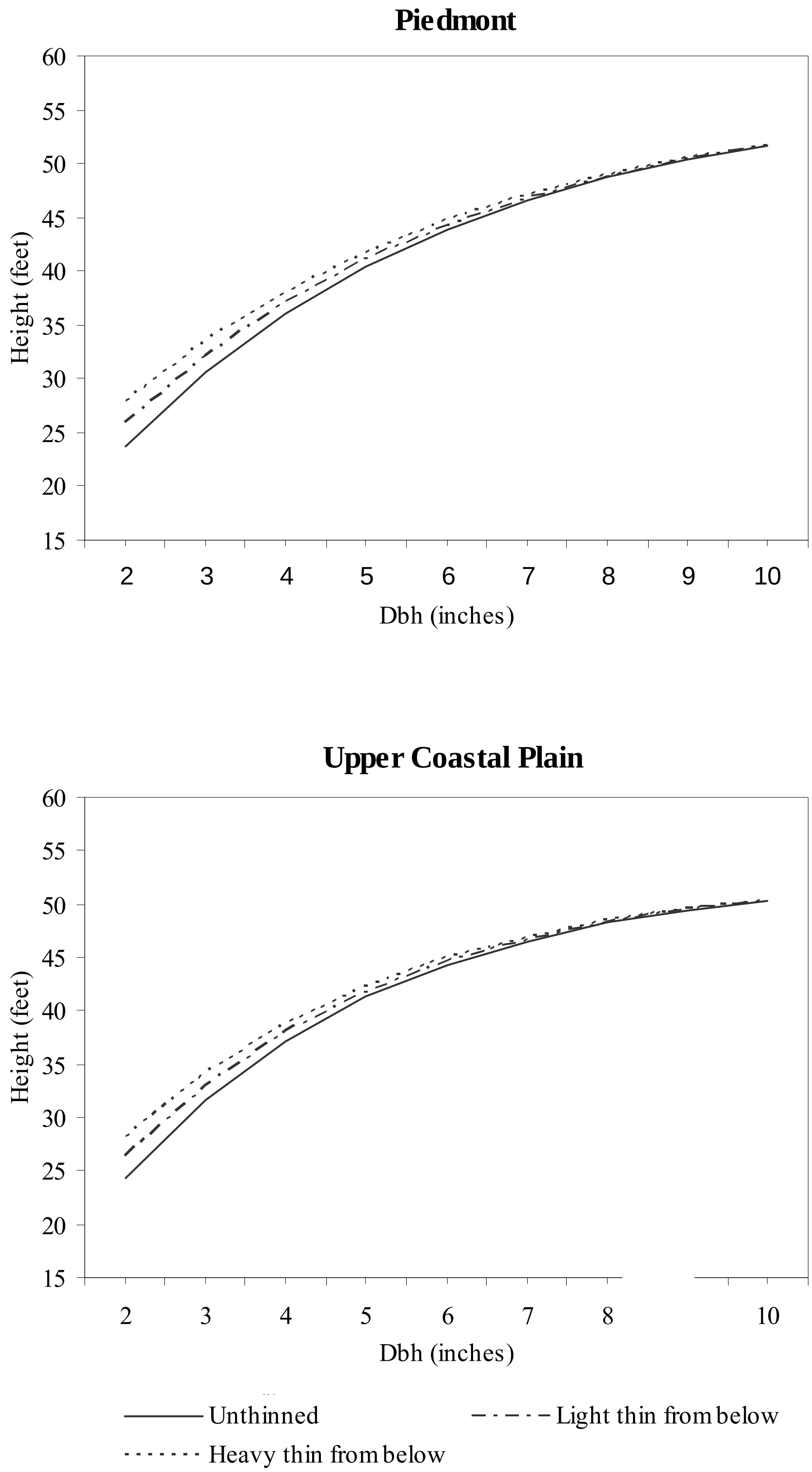



Figure 3.4. Predicted average height by dbh class (equation 3.10 – TSC1) for unthinned, lightly and heavily thinned loblolly pine stand in piedmont and upper coastal plain.

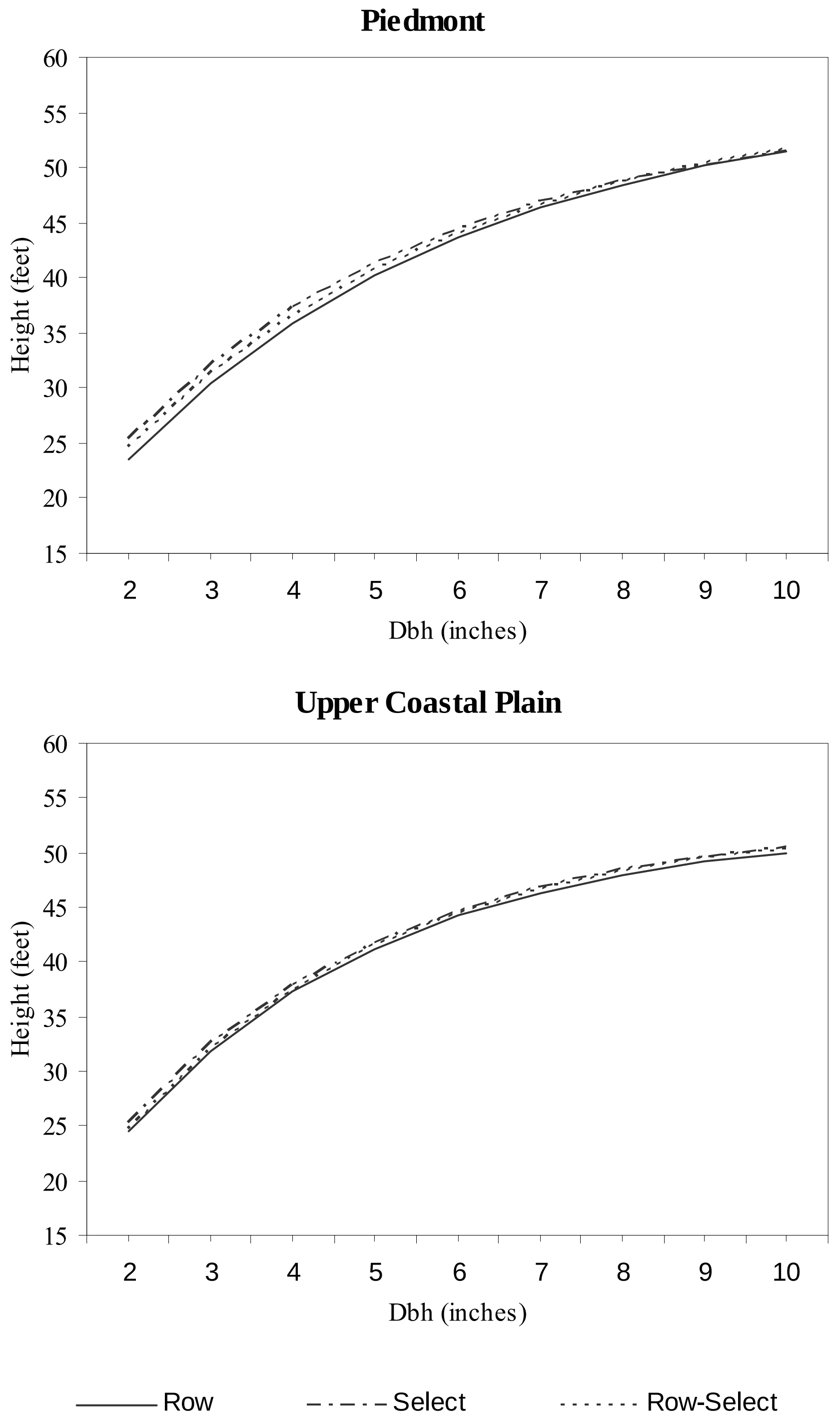



Figure 3.5. Predicted average height by dbh class (equations 3.17 and 3.18 –TSC2 and TSC4 cases) for row, select and row-select thinned loblolly pine stands in piedmont and upper coastal plain province.

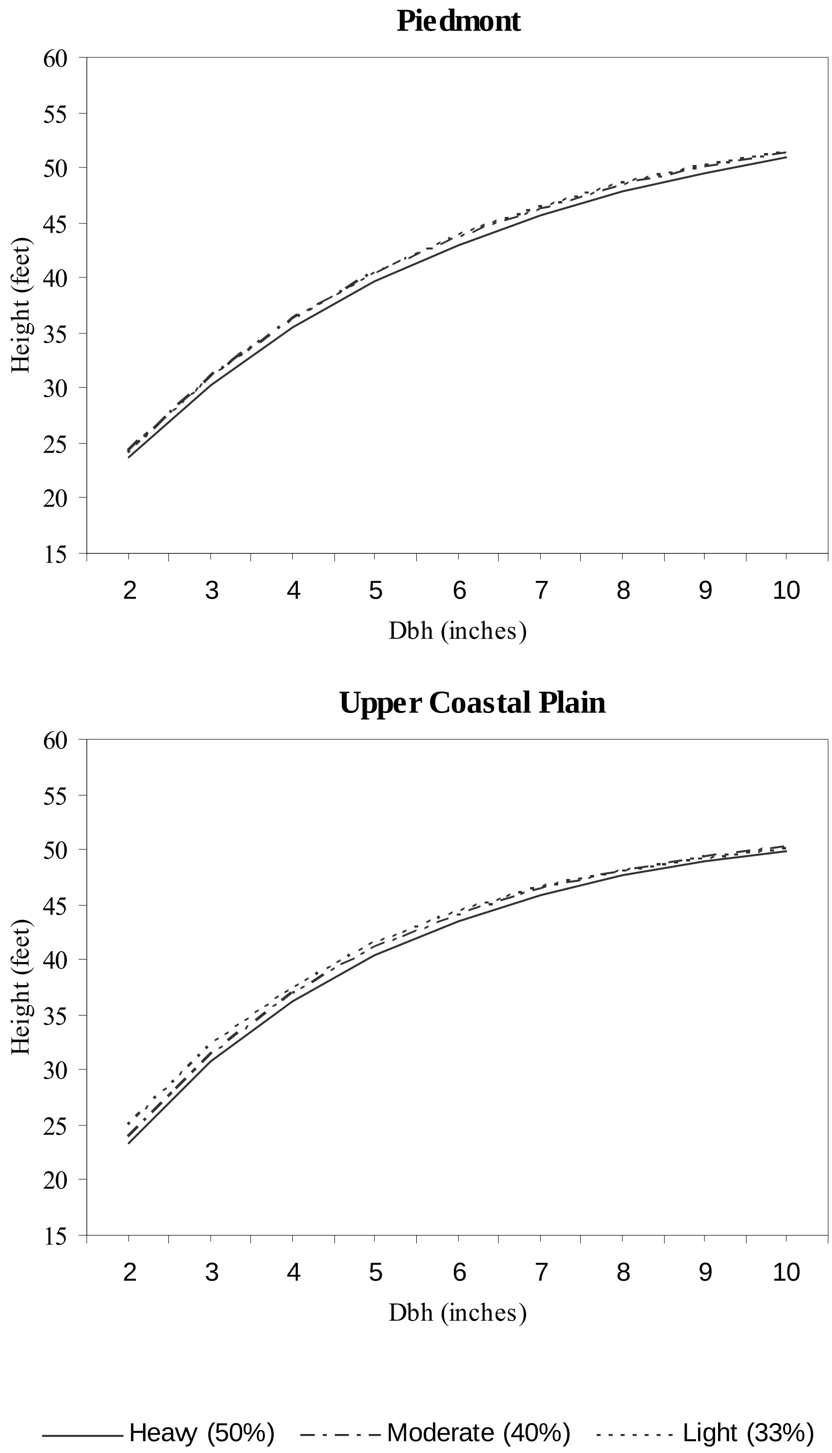



Figure 3.6. Predicted average height by dbh class (equations 3.19 and 3.20 – TSC3) for heavy, moderate and light thinned loblolly pine stands in piedmont and upper coastal plain province.

# CHAPTER 4

# EMPIRICAL EVALUATION OF RELATIVE SIZE RELATIONSHIPS USING THINNED SLASH AND LOBLOLLY PINE PLANTATION DATA

## Introduction

The importance of accurate and reliable forest stand projections for southern pines cannot be overstated. Such projections are important for updating inventories, scheduling stand management practices, and making economic evaluations of future stands (Nepal and Somers, 1992). For some cases it is essential and desirable to have future number of trees per acre by diameter classes. This kind of information can be obtained from stand table projection methods. The method suitable for a particular situation is more often defined by whether an initial stand table is available from inventory. If it is not available either a diameter distribution method or the traditional stand table projection method may be used. The latter method uses estimates of past diameter growth to predict a future stand table. On the other hand, if a stand table is available the diameter distribution method can be used. However, methods that take advantage of detailed individual dbh class information often prove superior to these other methods for predicting future stand tables. A relatively well known method that takes advantage of such information was originally suggested by Clutter and Allison (1974). This procedure was modified and refined by, among others, Clutter and Jones (1980) and Pienaar and Harrison (1988) who renamed it generalized stand table projection. Compared with Weibull diameter distribution and percentile-based recovery methods, this method was shown by Borders and Patterson (1990) to provide better estimates and to be able to reproduce multimodal diameter distributions. These properties in addition to its considerable flexibility, have been demonstrated by Clutter (1993), Pienaar and Rheney (1993), McTague and Stansfield (1994), Knowe et al. (1997), Borders et al. (2004) among others.

Given tree diameter measurements from a stand, Pienaar and Harrison (1988) defined tree relative size as the ratio of individual tree basal area to the average basal area of the trees in the stand:

$$\frac{b_i}{\bar{b}} \tag{4.1}$$

where,

$b_i$ = basal area of the ith tree

$\bar{b}$ = average basal area of the trees in the stand

Based on this ratio they predicted future tree relative size from current tree relative size as:

$$\frac{b_{2i}}{\bar{b}_2} = \left(\frac{b_{1i}}{\bar{b}_1}\right)^{(A_2/A_1)^{\beta}} \tag{4.2}$$

where,

$b_{2i}$ = basal area of the ith tree at time 2

$b_{1i}$ = basal area of the ith tree at time 1

$\bar{b}_2$ = average basal area of trees that survived at time 2

$\bar{b}_1$ = average basal area of trees that survived at time 1

$A_2$ = age of the stand at time 2

$A_1$ = age of the stand at time 1

The value of β indicates the change in future size of individual trees relative to the future average tree size. For example, if β is positive trees that are smaller than average tree size become even smaller than average tree size and trees that are larger than average tree size become even larger than average tree size over time. The opposite is true if $\beta$ is negative. That is, smaller trees become larger and larger trees become smaller relative to the average tree size over time. In addition to these trends a smaller change in relative size will be realized for a given projection length if starting projection age is delayed.

Application of the generalized stand table projection method follows two main steps namely mortality allocation and stand table projection in such a manner that it is consistent with predicted whole stand survival and total stand basal area. Total mortality in the stand determined using a survival function is allocated to individual dbh classes based on probability of mortality estimated for each dbh class using any of the methods suggested by Clutter and Allison (1974), Clutter and Jones (1980) and Pienaar and Harrison (1988).

Clutter and Allison (1974) assumed that the probability of a tree dying is inversely proportional to its relative size and that it is also directly proportional to the diameter class frequency. Given these relationships they suggested that probability of mortality in each individual dbh class be predicted using the equation:

$$p_i = \frac{n_{1i} e^{\delta R_{1i}}}{\sum n_{1i} e^{\delta R_{1i}}} \tag{4.3}$$

so that,

$$n_{2i} = n_{1i} - p_i M_n \tag{4.4}$$

where,

$p_i$ = the proportion of mortality in dbh class i

$R_{1i}$ = $(b_{1i}/\overline{b}_1)$

$n_{1i}$ = number of trees in dbh class i at age $A_1$

$n_{2i}$ = predicted number if trees in dbh class i at age $A_2$

$M_n$ = predicted total mortality between $A_1$ and $A_2$

$\delta$ is parameter to be estimated

Later on Clutter and Jones (1980) used a different approach. They predicted probability of survival in each dbh class as:

$$\text{probit}(p_i) = \alpha_1 + \alpha_2 \ln(b_{1i}/\overline{b}_{1i}) \tag{4.5}$$

so that:

$$n_{2i} = \left( \frac{n_{1i} p_i}{\sum_i n_{1i} p_i} \right) N_2 \tag{4.6}$$

where,

$\alpha_1$, $\alpha_2$ are parameters to be estimated

All else are as defined above.

Pienaar and Harrison (1988) assumed that the probability a tree will die is inversely proportional to its relative size. As such, if a tree died they could calculate the probability that it was of a given size. Based on this they predicted the probability of mortality in each dbh class as follows:

$$p_i = \frac{\left(\overline{b}_1/b_{1i}/\sum_i \overline{b}_1/b_{1i}\right)\left(n_{1i}/\sum_i n_{1i}\right)}{\sum_i \left(\overline{b}_1/b_{1i}/\sum_i \overline{b}_1/b_{1i}\right)\left(n_{1i}/\sum_i n_{1i}\right)} \tag{4.7}$$

All are as previously defined.

It is evident that equation (4.7) is based on empirical observation. The value derived for $p_i$ is substituted in equation (4.4) to determine the number of survivors in each dbh class. This number is used with an estimate of $\beta$ in equation (4.2) to project the current stand table to the future in such a manner that it is consistent with projected total basal area. This is achieved using the following constraint:

$$b_{2i} = B_2 \frac{n_{2i}(b_{1i}/\bar{b}_1)^{(A_2/A_1)^{\beta}}}{\sum_{i=1}^{k} n_{2i}(b_{1i}/\bar{b}_1)^{(A_2/A_1)^{\beta}}} \qquad (4.8)$$

All are as defined above.

Clutter (1993) evaluated the effect of cultural treatments such as thinning, competition control and mid-rotation nitrogen fertilization on the change in relative size in thinned loblolly pine plantations. He accounted for these effects using indicator variables in equation (4.2) to obtain the following equation:

$$\frac{b_{2i}}{\bar{b}_2} = \left(\frac{b_{1i}}{\bar{b}_1}\right)^{(A_2/A_1)^{(\beta_1+\beta_2 X_1+\beta_3 X_2+\beta_4 X_3)}} \qquad (4.9)$$

where,

$X_1$ = 1 if competition control, 0 otherwise

$X_2$ = 1 if fertilization had occurred, 0 otherwise

$X_3$ = 1 if thinning had occurred, 0 otherwise

$\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$ are parameters to be estimated.

He observed that even though the inclusion of joint effects in (4.9) was not significant the differences in actual projected size would be apparent among thinning, competition control and fertilization because these treatments affect projected mean basal area per tree. That is as projected mean basal area per tree change so does projected relative size.

A modification for equation (4.2) was suggested by Knowe (1994). He defined relative size as the ratio of individual dbh to the quadratic mean diameter (equation 4.9):

$$\frac{d_{2i}}{D_{q2}} = \left(\frac{d_{1i}}{D_{q1}}\right)^{(A_2/A_1)^{\alpha}} \tag{4.11}$$

where,

$d_{2i}$ = dbh of the tree at the end of the growth period

$d_{1i}$ = dbh of the tree at the start of the growth period

$D_{q2}$ = Dq of the tree at the end of the growth period

$D_{q1}$ = Dq of the tree at the start the growth period

$A_2$ = age of the stand at the end of the growth period

$A_1$ = age of the stand at the start of the growth period

$\alpha$ = rate parameter to be estimated

He used this ratio to project stand tables of Douglas fir plantations in Oregon and Washington state. Like generalized stand table projection this procedure combines features of stand table projection and whole stand models. This model was successfully used by Knowe and Stain (1995), Knowe and Hibbs (1996), and Knowe et al. (1997).

The objective of this study was to empirically evaluate the generalized stand table projection system of Pienaar and Harrison (1988) against a constant relative size relationship using MS33 thinned slash and loblolly pine data.

**Data**

Data used to compare tree relative size relationships come from a thinning study known as the MS33 thinning study. This study was installed from 1981 to 1984 in piedmont, upper and lower coastal plain provinces of Alabama, Florida, Georgia and South Carolina (Figure 2.1 in Chapter 2) to investigate the effect of type, intensity and timing of thinning in slash and loblolly pine plantations. Tree measurements of dbh to the nearest tenth inch on all trees, total height to the nearest foot on a subsample across the diameter distribution and crown class were taken. These trees were tagged and remeasured at three year intervals. The 126 slash pine plots ranged from 101 to 446 trees per acre, age ranged from 13 to 18 years, site class from 50 to 70, and basal area per acre from 25 to 77 square feet per acre. The 280 loblolly pine plots that were studied had an initial minimum age of 12 years and a maximum of 19 years, site class that ranged from 40 to 80, trees per acre ranged from 116 to 579 trees per acre, and basal area per ranged from 40 to 137 square feet per acre. A further description of this study is presented in Chapter 2.

**Methods**

Trees that were tagged in the MS33 thinning study were identified. Tree basal area, plot mean basal area and relative sizes of these trees both at the start and end of growth period were calculated. These were merged with corresponding stand information such as site index, trees per acre and basal area per acre. Using this data the changes in relative size over time were investigated for each pine species. Preliminary analyses indicated thinning type and physiographic province explained a significant amount of variation. Thus separate regression analyses were performed for each region. The effect of thinning type was represented using indicator variables (equation 4.11). Equally important is that initial analyses suggested that individual tree relative size remained constant over time (equation 4.12).

$$\frac{b_{2i}}{\bar{b}_2} = \left(\frac{b_{1i}}{\bar{b}_1}\right)^{(A_2/A_1)^{(\alpha_0+\alpha_1 Z_1+\alpha_2 Z_2)}} \tag{4.11}$$

$$\frac{b_{2i}}{\bar{b}_2} = \left(\frac{b_{1i}}{\bar{b}_1}\right) \tag{4.12}$$

where,

$$Z_1 = \begin{cases} 1 \text{ if stand is row thinned} \\ 0 \text{ if otherwise} \end{cases}$$

$$Z_2 = \begin{cases} 1 \text{ if stand is select thinned} \\ 0 \text{ if otherwise} \end{cases}$$

PROC NLIN in SAS was used to fit equations (4.2) and (4.11). Convergence(s) was easily achieved. Goodness of fit measures namely percent variation explained and root mean square error were calculated for each equation as follows:

$$PVE = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{4.13}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y})^2}{n}} \tag{4.14}$$

where,

PVE = percent variation explained

RMSE = root mean square error

$y_i$ = observed relative size

$\bar{y}$ = mean relative size

$\hat{y}$ = projected relative size

n = number of observations

Projected stand tables were compared with observed stand tables using Kolmogrov-Smirnoff (KS) two-sample test at significance levels of 0.01, 0.05 and 0.1. Predicted stand tables were further assessed using the error index suggested by Reynolds et al. (1988). This index is defined as the weighted sum of the absolute differences between predicted and observed numbers of diameters in each diameter class:

$$e = \sum_{j=1}^{k} \left| w(x)(N_p - N_O) \right| \tag{4.14}$$

where,

e = error index

w(x) = weighting factor

$N_p$ = predicted number of trees in dbh class

$N_o$ = observed number of trees per acre in dbh class

j = 1,…, k dbh class

Total volume was used as the weight in this study. Since *e* in equation (4.14) is the sum of the absolute differences this means that an over prediction in one dbh class will not offset an under prediction in another dbh class. In this case error is only small when the model performs well in all dbh classes (Reynolds et al. 1988). For simulation purposes slash and loblolly pine stand survival and basal area per acre were predicted using Logan (2005) and Borders et al. (2004) whole stand projection equations (Appendices A and B) for slash and loblolly plots, respectively. Predicted stand mortality was allocated to each diameter class using Pienaar and Harrison (1988) method.

## Results

### Slash Pine

Preliminary analysis of remeasured slash pine data indicated that tree relative sizes did not change significantly over remeasurement intervals considered which varied from a minimum of 3 to a maximum of 15 years for some stands (Figure 4.1). Despite this, tree basal area and average basal area increased significantly over time. This is shown in Figure 4.2 where tree basal area increased as average basal area size increased. Given these results a plausible relationship for thinned slash pine plantation is that relative size remains constant over time. This relationship was compared against generalized stand table projection of Pienaar and Harrison (1988).

Table 4.1 presents regression analysis results obtained when equations (4.2) and (4.11) were fitted to remeasured thinned slash pine data. All parameter estimates in the models were statistically significant at $\propto = 0.05$ except for $\beta$ in the upper coastal plain region. In fitted equation (4.11) the estimate for the rate parameter was as follows: 0.0668 in row thinned and 0.0216 in select and row-select thinned stands in lower coastal plain, and 0.0403 in row thinned and -0.0237 in select and row-select thinned stands in upper coastal plain. Large PVE and small RMSE values were obtained with fitted equations (4.2) and (4.11). Comparatively similar fit statistics were obtained with the constant relative size equation (4.12). Residual plots of fitted equations did not indicate serious departures from regression assumptions.

KS two-sample tests indicated 1 of 294 stand tables predicted with equation (4.2), (4.11) or (4.12), was significantly different from observed stand tables in the lower coastal plain at commonly used alpha levels of 0.01, 0.05 and 0.1. There were no significant differences between any given pair of predicted stand tables. In the upper coastal plain province no predicted stand table was rejected. A pairwise comparison of predicted stand stables found no significant differences. Fairly close error indices (Reynolds et al. 1988) were obtained with all projection equations. In the lower coastal plain, the error index was 84.17 for equation (4.2), 83.79 for equation (4.11) and 83.66 for equation (4.12). In the upper coastal plain the error index was 99.82 for equation (4.2), 99.46 for equation (4.11) and 93.37 for equation (4.12).

Predicted changes in relative size over time are presented in Figure 4.3 for a 15 year old thinned slash pine stand in the lower coastal plain with 300 trees per acre and 43 $ft^2$ per acre of basal area and a site index of 50. In this stand, trees with relative size less than 1 were smaller than average size and trees with relative size greater than 1 were larger than average size. As such, constant relative size equation (4.12) predicted smaller future relative size for larger trees and larger future relative size for smaller trees than equations (4.2) and (4.11). Future relative size predicted with equations (4.2) and (4.11) decreased for smaller trees, and increased for larger trees. The size of these changes depended on initial tree size. Trees that were initially smallest had the largest decrease in future relative size and similarly trees that were initially largest trees had the largest increase in relative size over time. Changes due to thinning type were apparent with row thinning showing the largest increase in relative sizes for larger trees and the largest decrease in relative sizes for smaller trees than selective thinning methods. Effects of initial projection ages of 15, 20, 25 and 30 on future relative size for small and large trees are shown in Table 4.2. As initial projection age was delayed small changes in relative size for any given projection length were predicted for both small and large trees using equations (4.2) and (4.11). These trends contrast with constant relative size relationship that does not change with time. Despite this relatively similar future diameter distributions were implied by all equations (Figure 4.4). This was in agreement with KS test results that found no significant differences at levels of 0.01, 0.05 and 0.1 among projection methods.

**Loblolly Pine**

Plots of remeasured loblolly pine tree relative size indicated tree relative sizes neither increased nor decreased significantly over time (Figure 4.5). However tree basal area and average basal area increased significantly over time. This is summarized in Figure 4.6 which shows tree basal area increased as average basal area size increased. It is reasonable thus that relative size remains constant over time in MS33 loblolly pine plantations. This relationship was compared against generalized stand table projection of Pienaar and Harrison (1988).

When equations (4.2) and (4.11) were fitted to thinned loblolly pine data all parameter estimates in the models were significantly different from zero at $\propto = 0.05$ (Table 4.3). In fitted equation (4.11) the estimate for the rate parameter was as follows: in lower coastal plain it was -0.0004 in row thinned and -0.0382 in select and row-select thinned stands; in upper coastal plain it was -0.1594 in row thinned, -0.002 in select thinned and 0.0801 in row-select thinned stands and in piedmont region it was -0.1489 in row thinned, -0.1642 in select thinned and -0.1062 in row-select thinned stands. Reasonably good fit statistics were obtained with both equations (4.2) and (4.11). In addition residual plots did not show any serious departures from regression assumptions. For constant relative size at time 1 and time 2 (equation 4.12) relatively good fit statistics were also obtained.

KS two-sample test at $\alpha = 0.01$, 0.05 and 0.1 detected no significant difference between predicted and observed stand tables for any of the models in all physiographic regions. No significant differences were found between any given pair of predicted stand tables in all regions. That is stand tables predicted with constant relative size equation were not significantly different from those predicted using equations (4.2) and (4.11) and similarly the predictions of these two equations were not significantly different. This was reflected by the following relatively similar error indices (Reynolds et al., 1988). In piedmont region an error index of 102.06 was obtained for equation (4.2), 101.87 for equation (4.11) and 99.74 for equation for equation (4.12). In lower coastal plain the error index was 117.81 for equation (4.2), 117.83 for equation (4.11) and 118.28 for equation (4.12). In upper coastal plain an error index of 160.68 was obtained for equation (4.2), 161.47 for equation (4.4) and 159.54 for equation (4.5).

Projected relative sizes for a 15 year old loblolly pine stand in piedmont region with 400 trees per acre and 70 $ft^2$ per acre of basal area and a site index of 60 are shown in Figure 4.7. In this stand trees smaller than average size had relative size less than 1 and trees larger than average size had relative size greater than 1. As such constant relative size equation (4.12) predicted smaller future relative size for smaller trees and larger future relative size for larger trees than equations (4.2) and (4.11). Future relative size predicted with (4.2) and (4.11) increased for smaller trees, and decreased for larger trees. The size of these changes depended on initial tree size. Trees that were initially smallest had the largest increase in future relative size and similarly trees that were initially largest had the largest decrease in relative size over time. Irrespective of the projection equation, trees with an initial relative size of 1 had no change in relative size over time. Figure 4.7 also indicated that thinning type affected future relative size. This was not surprising given that thinning term in the model was found to be significant. The effects of thinning were such that select thinned trees had largest increase in relative size and row/select thinned trees had the smallest change in relative size over time. These effects became more pronounced with increase or decrease in initial tree size. For projection lengths of 5 and 10 years small changes were observed in future relative size as initial projection age was increased for both small and large trees (Table 4.4). Projected diameter distributions (Figure 4.8) compare well for all projections methods at stand ages 20 and 30. At stand ages 40 and 50 constant relative size equation seems to predict less trees per acre for modal classes than equations (4.2) and (4.11). These differences were not sufficiently large as they were not detected by KS test results that found no significant difference among relative size projection methods.

**Discussion**

This study investigated the performance of the generalized stand table projection method (GSTP model – equations (4.2) and (4.11)) of Pienaar and Harrison (1988) against a constant relative size relationship (CRS model – equation (4.12)) using MS33 slash and loblolly pine data. Results show that both GSTP and CRS models performed reasonably well in projecting current stand tables. Based on goodness of fit measures, KS two-sample test, error index of Reynolds et al. (1988) and implied diameter distributions neither of the models was found to be superior even when thinning was accounted for in GSTP model. Reasonably similar PVE and RMSE values were obtained with both projection models. According to KS two-sample tests each model rejected 1 of 294 stand stables in slash pine plantations in lower coastal plain province and rejected no stand tables in the rest of slash and loblolly pine plantations. In addition no significant difference was found between the stand tables predicted with GSTP and CRS model. The error indices (Reynolds et al. 1988) obtained for the two models in any given physiographic region were fairly close and reasonably similar future diameter distributions were implied by both models.

All parameter estimates in GSTP model were significant except for the estimate of $\beta$ in slash pine plantations in upper coastal plain region. This suggested that relative size significantly changed over time and that it was justified to account for thinning type in the model. The value of the rate parameter in equation (4.11) varied with species, thinning type and physiographic region. For example a positive value for this parameter was estimated for row-select thinned loblolly pine plantations in upper coastal plain and for all thinned slash pine plantations except for select and row-select thinned slash pine plantations in upper coastal plain. This indicated that the relative size of trees smaller than average size decreased whereas that for trees larger than average size increased over time and also that as initial projection age was delayed the model predicted small changes in relative size over time for any given projection length. Similar results were reported by Pienaar and Harrison (1988) for unthinned slash pine plantations and by Clutter (1993) for thinned loblolly pine plantations. The opposite of these trends were implied for select and row-select thinned slash pine plantations in upper coastal plain and in all thinned loblolly pine plantations except row-select thinned loblolly pine plantations in upper coastal plain. In these plantations the estimate of $\beta$ was negative. This implied that the relative size of trees smaller than average size increased whereas that for trees larger than average size decreased over time. These results indicated inconsistencies in the estimate for the rate parameter. This was hardly surprising given the poor correlation seen in plots of relative size over time. A further inconsistency was the different thinning effects implied by the model. For example in slash pine plantations in lower coastal plain the model implied larger changes (decrease or increase) in relative size in row thinned than in select thinned stands. Fitted model for loblolly pine plantations in piedmont region implied the opposite of this. However select thinning which removes smaller trees than larger trees shifts the distribution towards the right. This increases the average size of the remaining trees. Row thinning on the other hand removes rows of trees irrespective of size and as such has generally been assumed not to change the distribution (Bailey et al., 1981; Bailey and Ware, 1983; Matney and Sullivan, 1982). In addition previous MS33 studies (Brooks, 1992; Harrison et al. 1998) indicated that select thinned stands had larger basal area growth than row thinned stands.

CRS model on the other hand implied no change in relative size over time. Relative size patterns observed in MS33 data favor this relationship. That is, relative size did not increase or decrease significantly over time (Figures 4.1 and 4.5). This implied that future tree contribution to future total stand basal area remained constant and that tree basal area increased as average basal area size increased (Figures 4.2 and 4.6) thus the diameter distribution will show more spread over time. In summary small trees will remain small and big trees will remain big. This is reasonable for loblolly and slash pine stands in this study whose minimum ages were 12 and 13 years respectively. At this age these stands are known to have differentiated well into crown classes (Harrington 2002).

An important question arises when choosing between the GSTP and CRS models. Why is it that a model that does not require fitting to the data (CRS) performs as well as another model with several parameters estimated from the data (GSTP)? From plots of the data (Figures 4.1-4.2 and 4.5-4.6) it appears that the constant relative size assumption is reasonable for these data. How then, is it that the GSTP model has significant parameter estimates that imply a rate of change over time and with respect to thinning type and timing? Further, why is it that others have reported similar changes in relative size over time (Pienaar and Harrison 1988, Clutter 1993, McTague and Stansfield 1994, Knowe et al. 1997, Borders and Patterson 1990). It appears that the issue is likely related to sample size issues in the fitting datasets used to estimate parameters in the relative size projection function. Since the function is usually fitted with a very large number of individual trees, it is not surprising that parameter estimates are statistically significantly different from zero. When sample size becomes large the chance of rejecting a null hypothesis of a parameter being equal to zero is a near certainty. Of course, this may lead to over-fitting of the available empirical data for functions such as the relative size projection function discussed above. This simple idea may explain the conflicting results concerning the nature of the individual tree relative size relationship over time that have been reported by others (Pienaar and Harrison 1988, Clutter 1993, McTague and Stansfield 1994, Knowe et al. 1997, Borders and Patterson 1990 among others) and also the conflicting results with respect to thinning type and timing demonstrated by the GSTP model in this study. This study demonstrates that it is reasonable to assume that relative tree size remains constant over time and with respect to type and timing of thinning treatment.

**Conclusion**

The objective of this study was to empirically evaluate GSTP model against CRS model using MS33 thinned slash and loblolly pine data. Results show that both GSTP and CRS models performed reasonably well in projecting current stand tables. Based on goodness of fit measures, KS two-sample test, error index and implied diameter distributions neither of the models was found to be superior. Results also show that there was absence of consistency in the estimate for $\beta$ in GSTP and when thinning was accounted for in this model, it implied conflicting trends for row and select thinned slash and loblolly pine plantations. A conjecture is that the data may likely be too complex for this model hence a problem of over-fitting or simply this model may not be appropriate for this data.

Plots of raw data showed there was no significant change in relative size over time. Thus a clear choice for MS33 thinned data is the CRS model. Such a relationship is plausible for the pine stands under study. Given the minimum ages for these stands which were 13 for slash and

12 for loblolly pine these stands are known to have well defined diameter distributions. As such future tree contribution to future total stand basal area will remain constant and tree basal area will increases with average basal area. Consequently, the diameter distribution will show more spread over time.

Since CRS model compared well with GSTP model in terms of PVE, RSME, KS tests, error indices and implied diameter distributions, it seems to be a reasonable alternative to GSTP model for MS33 thinned data. It is a parsimonious model yet nothing was lost. Application of this model requires that thinning effects should be accounted for in the stand level projection equations. However the suitability of this model will need to be validated with long term thinned remeasurement data with projection intervals exceeding the 15 years that was considered.

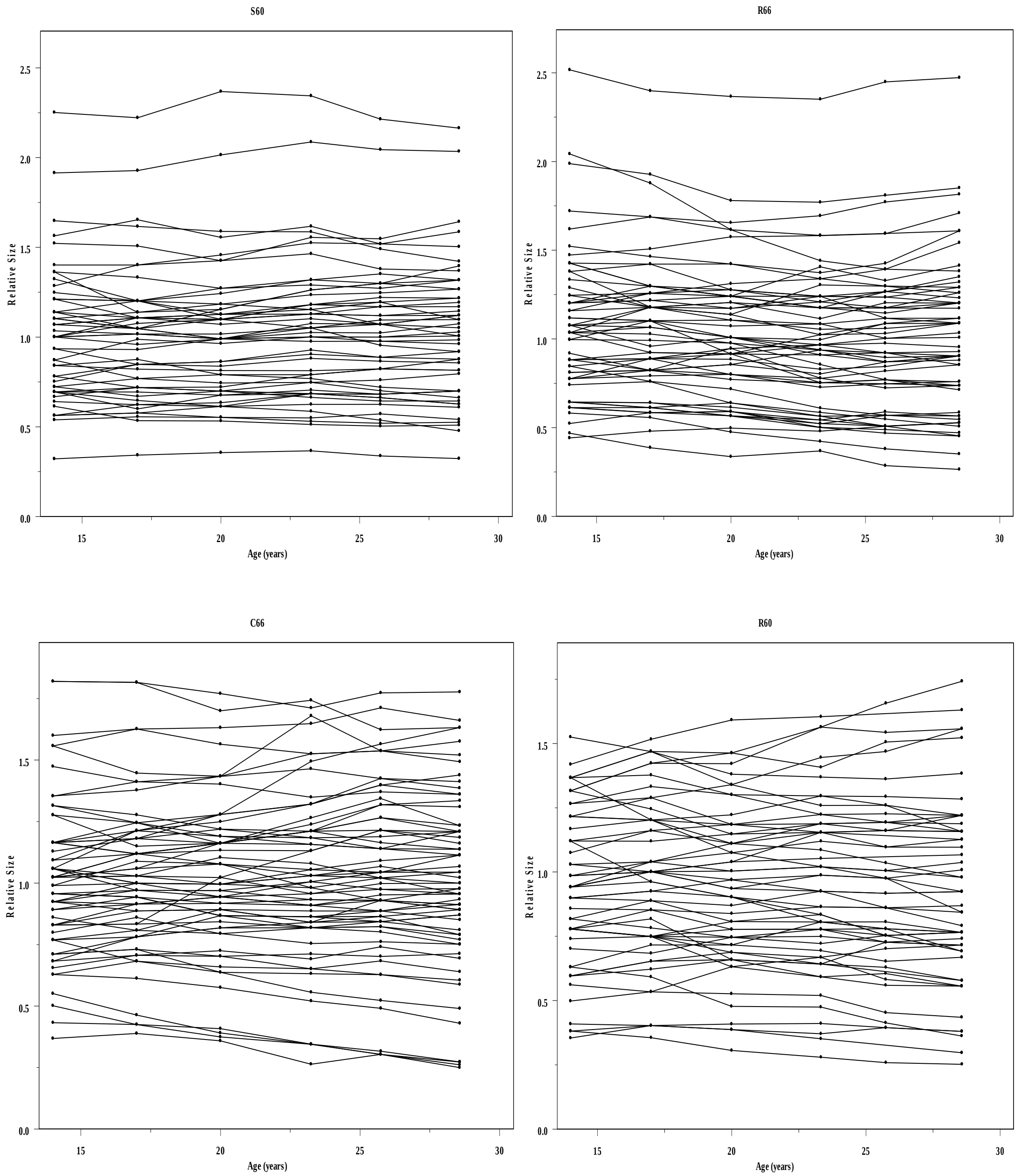



Figure 4.1. Observed changes in tree relative size over time in sample slash pine stands in lower coastal plain (S = Select thinned; C = Row-Select thinned; R = Row thinned; 60 and 66 are percent of trees remaining after thinning).

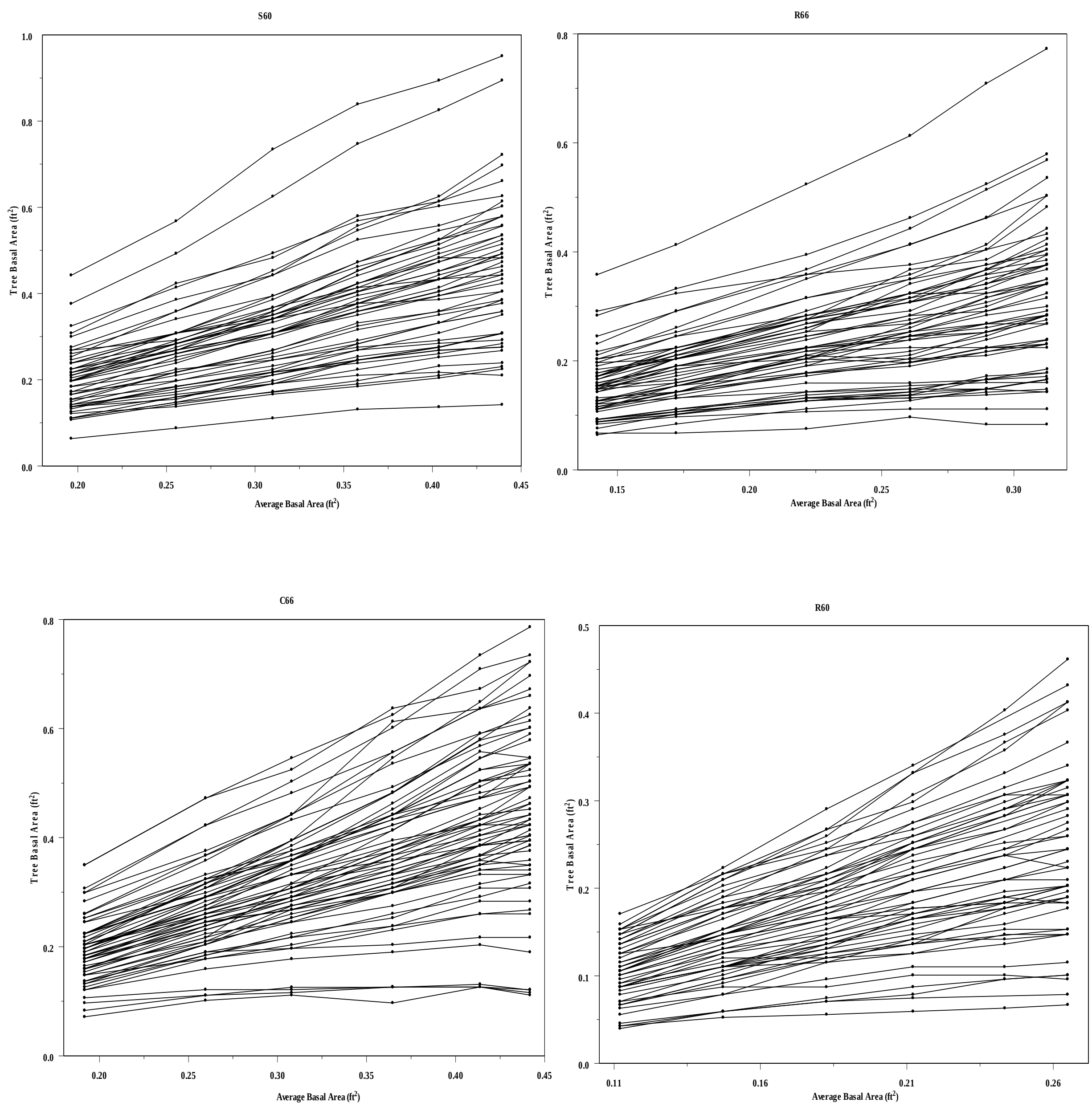



Figure 4.2. Observed relationship between tree basal area and average stand basal in sample slash pine stands in lower coastal plain.

Table 4.1. Parameter estimates and fit statistics for equations (4.2), (4.11) and (4.12) fitted to thinned slash pine data in lower and upper coastal plain provinces.

| | | Lower Coastal Plain | | | Upper Coastal Plain | | |
|---|---|---|---|---|---|---|---|
| **Equation** | Parameter | Estimate | Std.Error | Pr > \|t\| | Estimate | Std.Error | Pr > \|t\| |
| 4.2 | $\beta$ | 0.0391 | 0.00376 | <.0001 | -0.00058 | 0.00712 | 0.9351 |
| | PVE | | 93.31 | | | 90.09 | |
| | RMSE | | 0.10 | | | 0.13 | |
| 4.11 | Parameter | Estimate | Std.Error | Pr > \|t\| | Estimate | Std.Error | Pr > \|t\| |
| | $\alpha_0$ | 0.0216 | 0.00484 | <.0001 | -0.0237 | 0.00900 | 0.0085 |
| | $\alpha_1$ | 0.0452 | 0.00767 | <.0001 | 0.0640 | 0.0147 | <.0001 |
| | PVE | | 93.31 | | | 90.11 | |
| | RMSE | | 0.10 | | | 0.13 | |
| 4.12 | PVE | | 93.29 | | | 90.09 | |
| | RMSE | | 0.10 | | | 0.13 | |

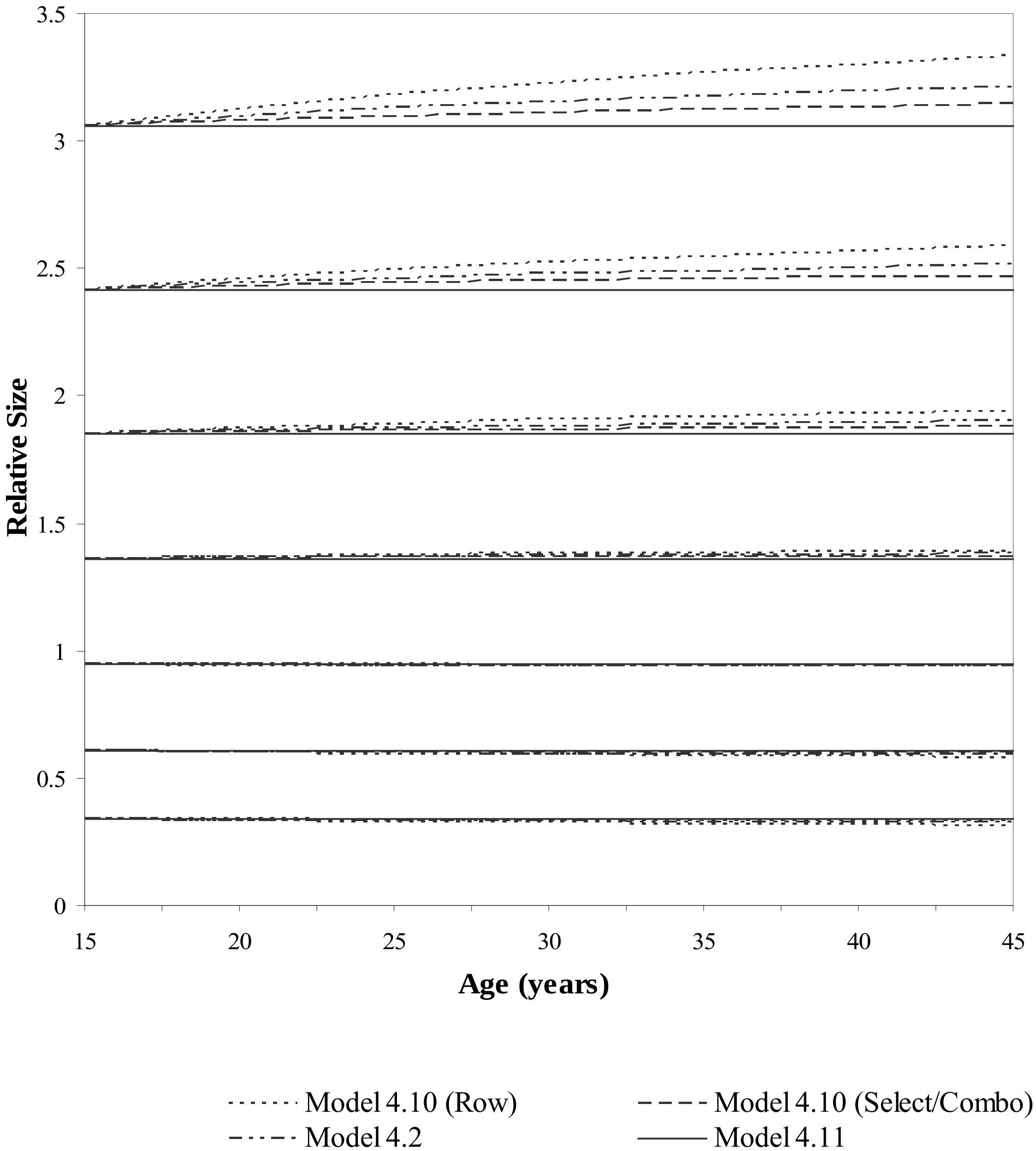



Figure 4.3. Projected changes in tree relative size in a 15 year old slash pine stand in lower coastal plain province.

Table 4.2. Predicted changes in relative size for small and large trees given different initial projection ages and two projection lengths of 5 and 10 years in slash pine stand.

| | | Predicted Change in Relative Size | | |
|---|---|---|---|---|
| $A_1$ | $A_2$ | Eq. 4.2 | Eq. 4.11 (Row) | Eq. 4.11 (Select) |
| | | Initial Relative Size = 0.34 (Small Tree) | | |
| 15 | 20 | -0.0041 | -0.0070 | -0.0023 |
| 20 | 25 | -0.0032 | -0.0055 | -0.0018 |
| 25 | 30 | -0.0026 | -0.0045 | -0.0014 |
| 30 | 35 | 0.0022 | -0.0038 | -0.0012 |
| 15 | 25 | -0.0073 | -0.0125 | -0.0040 |
| 20 | 30 | -0.0058 | -0.0099 | -0.0032 |
| 25 | 35 | -0.0048 | -0.0082 | -0.0027 |
| 30 | 40 | -0.0041 | -0.0070 | -0.0023 |
| | | Initial Relative Size = 1.36 (Large Tree) | | |
| 15 | 20 | 0.0047 | 0.0081 | 0.0026 |
| 20 | 25 | 0.0037 | 0.0063 | 0.0020 |
| 25 | 30 | 0.0030 | 0.0051 | 0.0017 |
| 30 | 35 | 0.0025 | 0.0043 | 0.0014 |
| 15 | 25 | 0.0085 | 0.0146 | 0.0046 |
| 20 | 30 | 0.0067 | 0.0115 | 0.0037 |
| 25 | 35 | 0.0055 | 0.0095 | 0.0031 |
| 30 | 40 | 0.0047 | 0.0081 | 0.0026 |

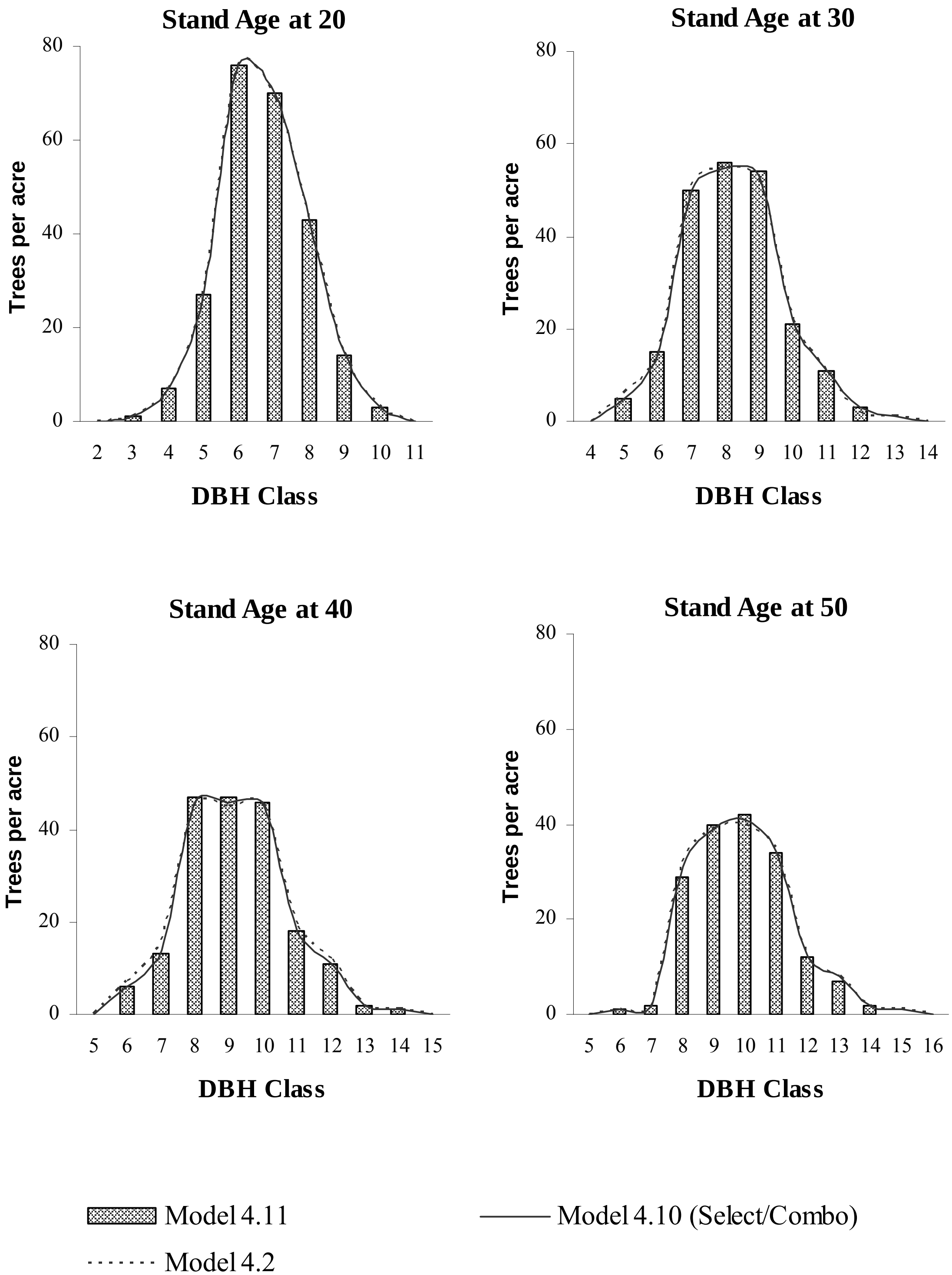



Figure 4.4. Projected diameter distributions for a 15 year old slash pine plantation in lower coastal plain province.

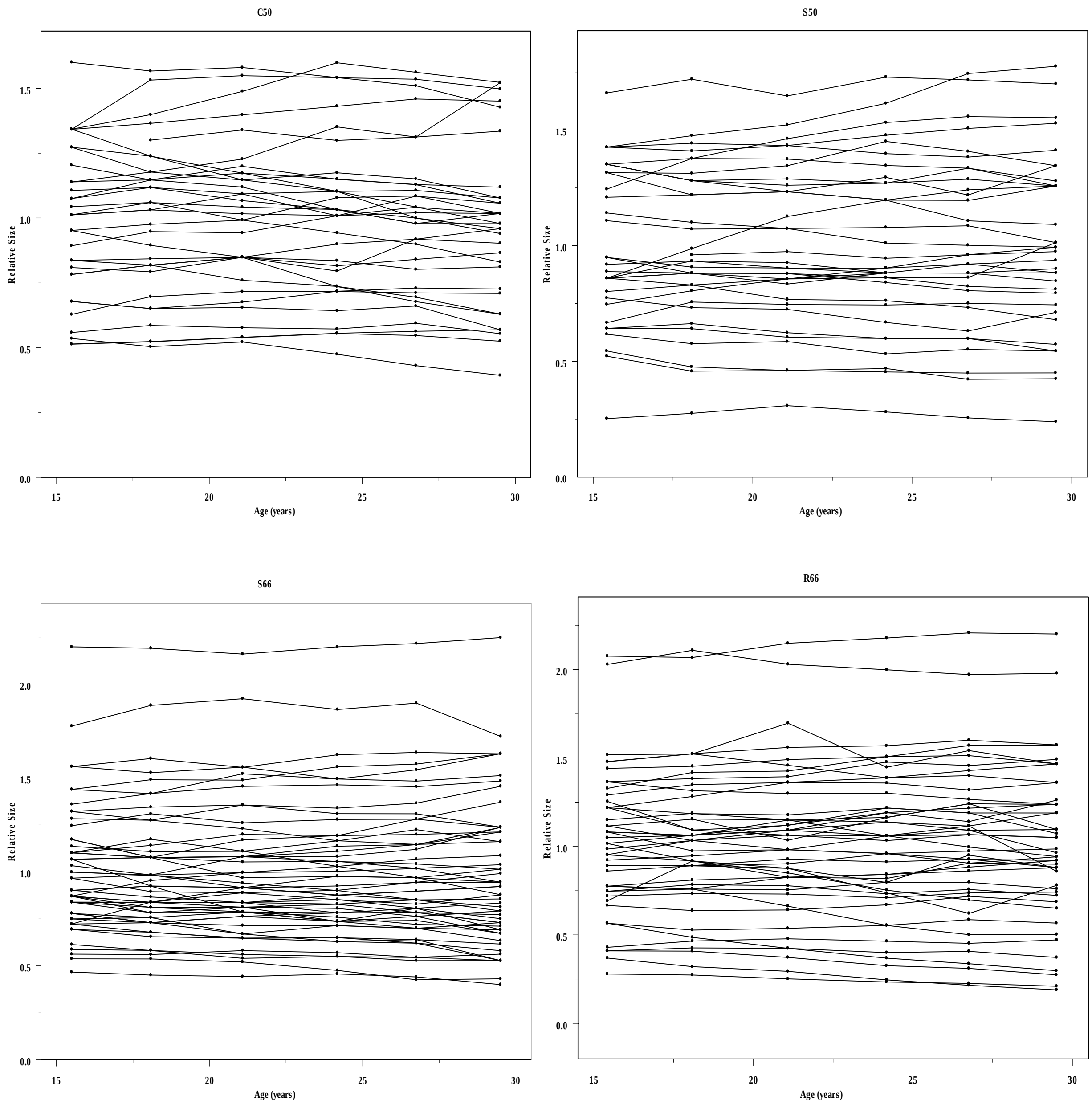



Figure 4.5. Observed changes in tree relative size over time in sample loblolly pine stands in piedmont region.

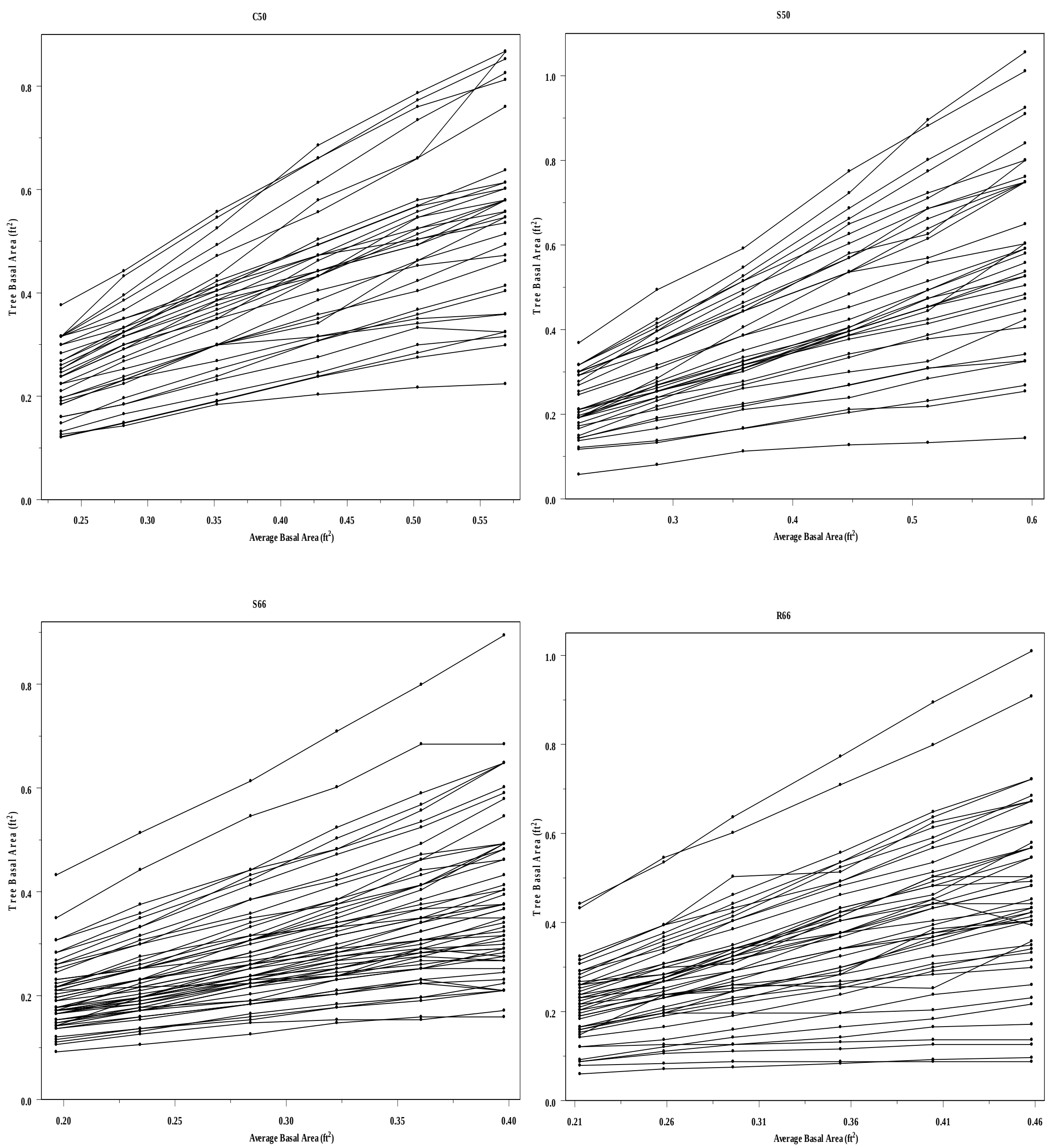



Figure 4.6. Observed relationship between tree basal area and average stand basal in sample loblolly pine stands in piedmont region.

Table 4.3. Parameter estimates and fit statistics for equations (4.2), (4.11) and (4.12) fitted to thinned loblolly pine data in piedmont, upper and lower coastal plain regions.

| | | **Piedmont** | | | **Upper Coastal Plain** | | | **Lower Coastal Plain** | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Equation | Parameter | Estimate | Std.Error | Pr > \|t\| | Estimate | Std.Error | Pr > \|t\| | Estimate | Std.Error | Pr > \|t\| |
| 4.2 | $\beta$ | -0.1395 | 0.00663 | <.0001 | -0.0454 | 0.00601 | <.0001 | -0.0246 | 0.00592 | <.0001 |
| | PVE | | 90.83 | | | 91.83 | | | 95.3 | |
| | RMSE | | 0.12 | | | 0.11 | | | 0.10 | |
| 4.11 | Parameter | Estimate | Std.Error | Pr > \|t\| | Estimate | Std.Error | Pr > \|t\| | Estimate | Std.Error | Pr > \|t\| |
| | $\alpha_0$ | -0.1062 | 0.0114 | <.0001 | 0.0801 | 0.0104 | <.0001 | -0.0382 | 0.00746 | <.0001 |
| | $\alpha_1$ | -0.0427 | 0.0159 | 0.0072 | -0.2395 | 0.0141 | <.0001 | 0.0377 | 0.0123 | 0.0022 |
| | $\alpha_2$ | -0.0580 | 0.0166 | 0.0005 | -0.0781 | 0.0155 | <.0001 | | | |
| | PVE | | 90.83 | | | 91.93 | | | 95.31 | |
| | RMSE | | 0.12 | | | 0.11 | | | 0.10 | |
| 4.12 | PVE | | 90.64 | | | 91.82 | | | 95.3 | |
| | RMSE | | 0.12 | | | 0.11 | | | 0.10 | |

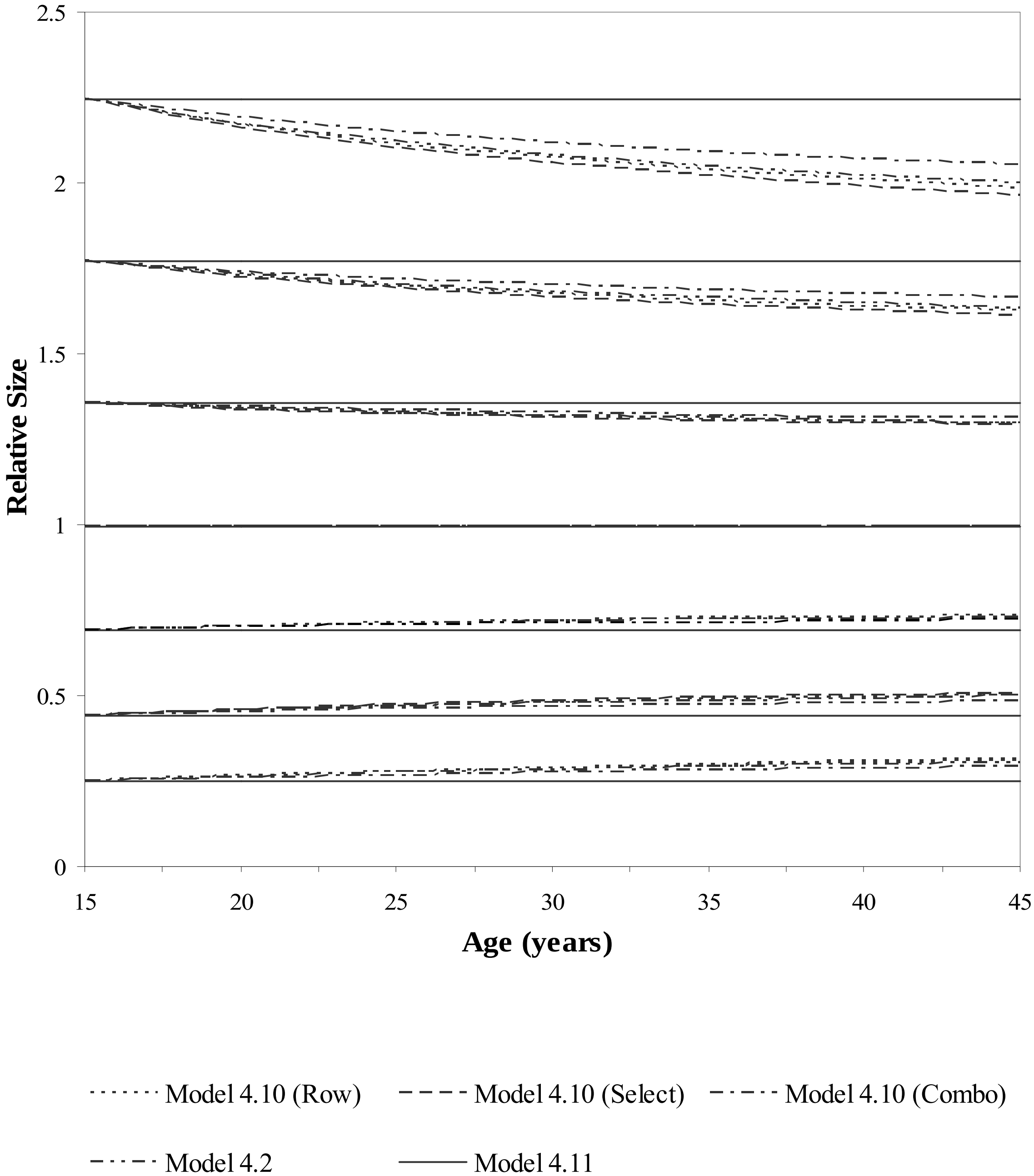



Figure 4.7. Projected changes in relative size over time years for a 15 year old loblolly pine stand in piedmont province (Combo is a combination of row and select thinning).

Table 4.4. Predicted changes in relative size for small and large trees given different initial projection ages and two projection lengths of 5 and 10 years in a loblolly pine stand.

| | | Predicted Change in Relative Size | | |
|---|---|---|---|---|
| $A_1$ | $A_2$ | Eq. 4.2 | Eq. 4.11 (Row) | Eq. 4.11 (Select) |
| | | Initial Relative Size = 0.44 (Small Tree) | | |
| 15 | 20 | 0.0144 | 0.0154 | 0.0170 |
| 20 | 25 | 0.0112 | 0.0119 | 0.0132 |
| 25 | 30 | 0.0092 | 0.0098 | 0.0108 |
| 30 | 35 | 0.0077 | 0.0083 | 0.0091 |
| 15 | 25 | 0.0255 | 0.0272 | 0.0300 |
| 20 | 30 | 0.0203 | 0.0216 | 0.0239 |
| 25 | 35 | 0.0169 | 0.0180 | 0.0198 |
| 30 | 40 | 0.0144 | 0.0154 | 0.0170 |
| | | Initial Relative Size = 1.77 (Large Tree) | | |
| 15 | 20 | -0.0394 | -0.0420 | -0.0462 |
| 20 | 25 | -0.0308 | -0.0328 | -0.0361 |
| 25 | 30 | -0.0253 | -0.0269 | -0.0297 |
| 30 | 35 | -0.0214 | -0.0229 | -0.0252 |
| 15 | 25 | -0.0684 | -0.0727 | -0.0797 |
| 20 | 30 | -0.0549 | -0.0584 | -0.0641 |
| 25 | 35 | -0.0459 | -0.0489 | -0.0537 |
| 30 | 40 | -0.0394 | -0.0420 | -0.0462 |

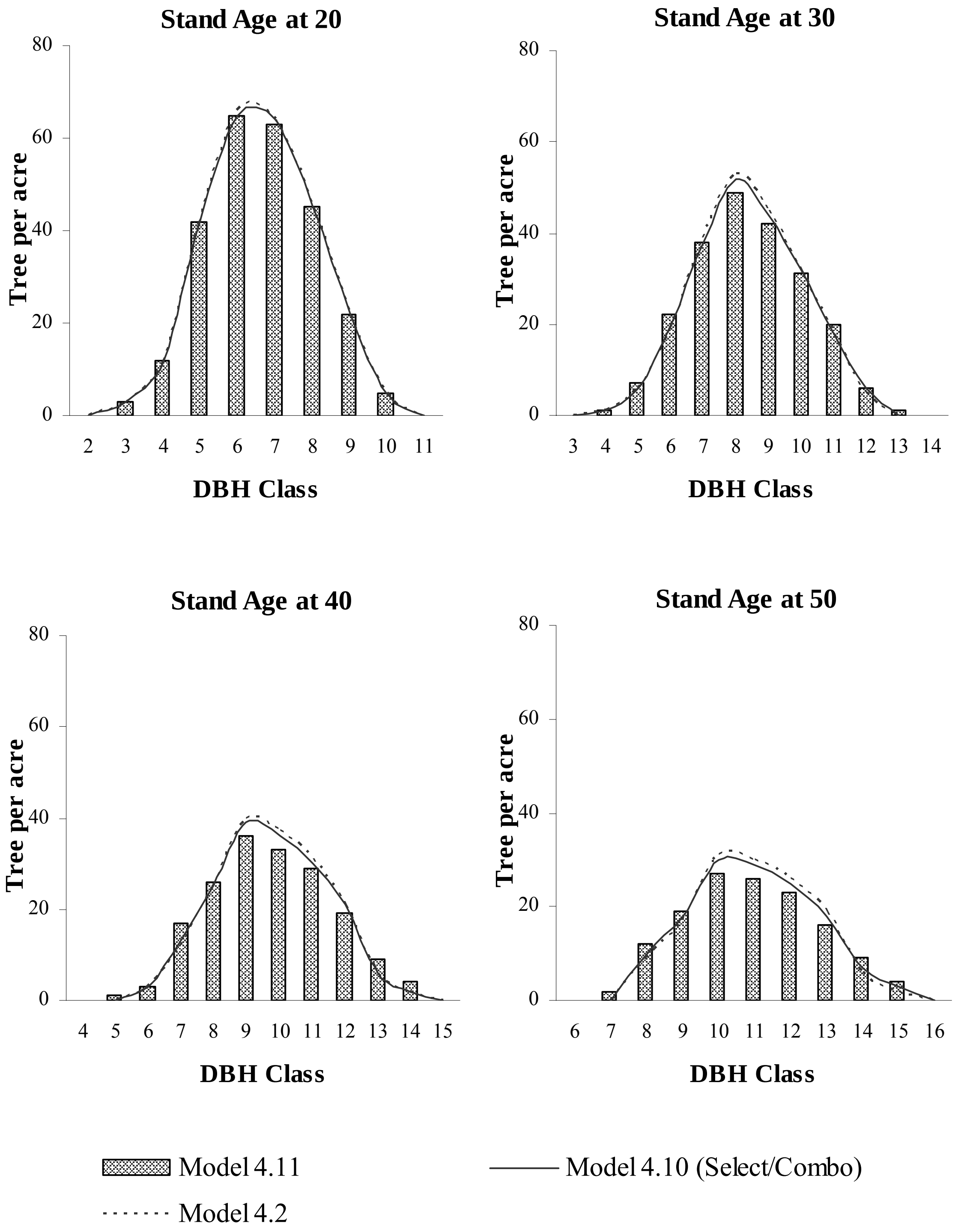



Figure 4.8. Projected diameter distributions for a 15-year old loblolly pine stand in piedmont region.

# CHAPTER 5

# CONCLUSION

This study developed operational diameter distribution prediction models for thinned slash and loblolly pine stand TSC1, TSC2, TSC3, TSC4, TSC5 and TSC6 cases in the piedmont, upper coastal plain and lower coastal plain of Alabama, Georgia, South Carolina and Florida. Percentile prediction equations were developed to estimate the $0^{th}$, $25^{th}$, $50^{th}$ and $95^{th}$ percentiles. All prediction equations fitted the data well except for the $0^{th}$ percentile prediction equation for slash pine plantations in lower coastal plain and loblolly pine plantations in piedmont region. A non-iterative parameter recovery method (Bailey et al. 1980) was used to recover Weibull distributions from predicted percentiles. KS two-sample test found no significant difference ($\alpha$ = 0.01, 0.05 and 0.1) between recovered distributions and observed diameter distributions. These results implied that Weibull that has been well documented for unthinned stands can be used to describe diameter distributions of thinned slash and loblolly pine plantations.

Model precision was in the order of TSC1, TSC2, TSC3, TSC4, TSC5 and TSC6. However all predicted stand tables were not rejected for any of these six cases. Absence of large differences in error indices between the best fit (TSC1) and the least fit (TSC6) implied that reasonably reliable predictions can be obtained even when no information about a thinning is available.

Developed percentile equations predicted quite well diameter distributions of second thinned plantations. KS two sample test at $\alpha$ = 0.01, 0.05 and 0.1 detected no significant difference between predicted and observed distributions. This suggests that second thinning did not significantly change the distribution.

Developed operational percentile prediction equations can be used to recover both current future diameter distributions. This is defined by the stand values that are used in the computations. For example if future stand values are used an estimate of future diameter distribution at the projection age will be obtained. Current and future stand values can be obtained from regional growth and yield systems.

This study also developed average total tree height-dbh equations for the diameter distribution prediction system for TSC1-TSC6. Results showed that Pienaar et al's equation (3.9) and Clutter[1]'s equation (3.10) performed reasonably well in predicting average total tree height. This was shown by percent variation explained that were greater than 92% and root mean square errors that were less than 3.2 feet. Equation (3.10) was determined to be suitable for TSC1 and equation (3.9) with effects of type and intensity thinning accounted for was suitable for TSC2, TSC3 and TSC4. No changes were required when it was applied to TSC5 and TSC6 since very limited information is assumed to be available from the pre and post-thin stand conditions. Developed equations indicated that type and intensity of thinning significantly affected average total tree heights. These effects were such that when compared to row thinning and unthinned stands, selective thinning, depending on thinning intensity, increased the average height of trees towards the left end of the diameter distribution. However thinning intensity when considered alone did not represent well the height distribution. This indicates the effects of thinning intensity on height-dbh relationship can best be understood when the type of thinning is known.

Empirical evaluation of the generalized stand table projection method (GSTP model) of Pienaar and Harrison (1988) against constant relative size relationship (CRS model) indicated that both models performed reasonably well in projecting current stand tables. Based on goodness of fit measures, KS two-sample test, error index of Reynolds et al. (1988) and implied diameter distributions neither of the models was found to be superior even when thinning was accounted for in GSTP model. However results also show that there was absence of consistency in the estimate for $\beta$ in GSTP and when thinning was accounted for in this model, it implied conflicting trends for row and select thinned slash and loblolly pine plantations. On the other hand plots of raw data showed there was no significant change in relative size over time. These results indicate that assumption of constant relative is reasonable for these data.

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**APPENDICES**

A. PMRC slash pine stand projection equations (Logan, 2005).

| Stand Attribute | Equation | | Parameter Estimates |
|---|---|---|---|
| Dominant height | $\ln(H_2) = \exp[\alpha_1((1/A_1) - (1/A_2))][\ln(H_1) - \alpha_2(1/A_1) + \alpha_3] + \alpha_2(1/A_2) - \alpha_3$ | (A.1) | $\alpha_1$ = -9.07052<br>$\alpha_2$ = -15.9819<br>$\alpha_3$ = -4.46547 |
| Trees per acre | $N_2 = 0.202724N_1\exp\{-(\delta_1 S/100)(A_2 - A_1)\} + \delta_0(0.79N_1)\exp\{-(\delta_2 S/100)(A_2 - A_1)\} + (0.79N_1)\exp\{-(\delta_2 S/100)(A_2 - A_1)\}$ | (A.2) | $\delta_0$ = 0.00921<br>$\delta_1$ = 0.036139<br>$\delta_2$ = 0.018215 |
| Basal area per acre | $\ln(B_2) = \beta_0 + (A_1/A)[\ln(B_1) - \beta_0 - \beta_2\ln(N_1) + \beta_3\ln(H_1) - \beta_4\ln(N_1)/A_1 - \beta_5\ln(H_1)/A_1] + \beta_2\ln(N_2) + \beta_3\ln(H_2) - \beta_4\ln(N_2)/A_2 - \beta_5\ln(H_2)/A_2$ | (A.3) | $\beta_0$ = -3.21012<br>$\beta_1$ = -22.1226<br>$\beta_2$ = 0.406383<br>$\beta_3$ = 1.330237<br>$\beta_4$ = 2.697179<br>$\beta_5$ = 1.888084 |

where,

B = basal area per acre

A = plantation age ($A_i$ = age at time i)

N = number of trees per acre ($N_i$ = number of trees at time i)

H = average dominant height

S = site index (base age 25 years)

PMRC = Plantation Mangement Research Co-operative

$\alpha_i$, $\delta_i$, and $\beta_i$ are parameters to be estimated.

B. PMRC loblolly pine stand projection equations (Borders et al. 2004).

| Stand Attribute | Equation | | Parameter Estimates | |
|---|---|---|---|---|
| | | | Piedmont/UC | LCP |
| Dominant height | $$H_2 = \rho_0 \left\{ 1 - \left[ 1 - \left( \frac{H_1}{\rho_0} \right)^{1/\rho_2} \right]^{A_2/A_1} \right\}^{\rho_2}$$ | (B.1) | $\rho_0 = 117.6$<br>$\rho_2 = 1.336527$ | $\rho_0 = 136.6$<br>$\rho_2 = 1.202941$ |
| Trees per acre | $$N_2 = S^{\varphi} + \left[ (N_1 - S^{\varphi})^{\lambda} + \beta S(A_2^{\theta} - A_1^{\theta}) \right]^{1/\lambda}$$ | (B.2) | $\varphi = 0.85$<br>$\lambda = -0.7$<br>$\theta = 2.3$<br>$\beta = 0.4*10^{(-7)}$ | |
| | | | Piedmont/UCP | LCP |
| Basal area per acre | Equation (A.3) | | $\beta_0 = -2.63847$<br>$\beta_1 = -34.5563$<br>$\beta_2 = 0.525931$<br>$\beta_3 = 0.99077$<br>$\beta_4 = 2.041639$<br>$\beta_5 = 6.849555$ | $\beta_0 = -1.96561$<br>$\beta_1 = -31.6234$<br>$\beta_2 = 0.408805$<br>$\beta_3 = 1.038358$<br>$\beta_4 = 2.59909$<br>$\beta_5 = 4.304848$ |

where,

B = basal area per acre

A = plantation age ($A_i$ = age at time i)

N = number of trees per acre ($N_i$ = number of trees at time i)

H = average dominant height

S = site index (base age 25 years)

PMRC = Plantation Management Research Co-operative

$\rho_i$, $\varphi$, $\theta$ and $\lambda$ are parameters to be estimated.