BUFFER SIZING FOR CONTSTRAINED NETWORKS

by

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(Under the Direction of John H. Blackstone, Jr.)

ABSTRACT

A methodology is developed and tested for sizing convergence, resource, and completion buffers for networks using Critical Chain Project Management (CCPM). The testing was done using computer simulation. Activities along the critical chain were started upon completion of the preceding activity, often called the “roadrunner” method. The normal distribution was used to model all activity durations in the models. Activities along non-critical paths were started upon completion of both the previous activity and the scheduled start date. This second method of starting activities was found to modify the normally distributed activity times on the non-critical path to where they behaved like truncated normal activity times. As a result of the behavior of the non-critical paths, the path durations did not behave as expected in the literature.

INDEX WORDS: Theory of Constraints, Convergence Buffers, Completion Buffers, normal Distribution, Truncated Normal Distribution, Arena, Computer Simulation, Project Scheduling

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A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2004

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CHAPTER I

INTRODUCTION AND OVERVIEW

This dissertation deals with project scheduling using a basic methodology developed by Eliyahu Goldratt under the Theory of Constraints. Goldratt (1997), Leach (2000), and Newbold (1998) indicate that there is a chronic problem in project management that the majority of projects managed using traditional project management are late. Critical Chain Project Management (CCPM) was developed by Goldratt (1997) to overcome this problem. The purpose of this dissertation is to extend the research in the field of Critical Chain Project Management by developing methods for sizing the three buffers, completion, convergence, and resource, essential to the success of CCPM as identified by Pittman (1994) and Goldratt (1997).

Traditional methods of estimating project duration have not been accurate. The results of this dissertation will improve the accuracy of buffer sizing for projects using the three types of buffers. It will develop an understanding of the effect of various network characteristics, such as variability, number of converging paths, and other things found to have an impact on project duration. This understanding will then be used to develop methods for setting the size of network buffers so that these findings can be used in practice. This study will be restricted to single project networks using normally distributed activity durations.

Project management is a fairly well researched area. Davis (1973, p. 298) says that there are “15 books and over 300 articles” in print about standard CPM procedures. Much project management research has occurred since 1973 but has not addressed all of the issues. In an unpublished dissertation, Pittman (1994), eight fundamental problems in project management were identified. Some of these problems had been identified in the literature but not all had been addressed and resolved. The problems are discussed in the literature review section of this proposal.

Pittman (1994) used Goldratt's (1990) concept of a critical chain versus the traditional critical path. This new concept incorporates the impact of resource contention on the technical precedence of the network. Resource contention between two activities using a common resource on different paths can cause the critical path to include an activity on a parallel path that must be performed after an activity in the critical path sequence and cause the critical path to shift to this second path in the network because of contention for a common resource in the same time span. This disjoint path, which crosses the network from the critical path to another path caused by resource contention, is called a critical chain.

Pittman (1994) identified and discussed eight problems with traditional project management. The problems are presented in Table 1.1 and discussed in chapter two in detail.

Table 1.1: Pittman's (1994) problems and whether they are included in this study.

Problem Description	Included	Excluded
1. Capacity planning and resource contention	X	
2. Priority planning		X
3. Variability and convergence points	X	
4. Dependent activities and resource contention in a stochastic environment	X	
5. Planning to time rather than completion	X	
6. Early consumption of project slack		X
7. Protecting project completion by increasing planned activity times		X

8. Rescheduling/replanning for deviations above planned activity times		X

In addition to those problems mentioned by Pittman that are excluded from this dissertation, two other factors aren't considered. One is buffer management, i.e., the acceleration of some task if the project falls behind schedule. The second is rescheduling of a network that is behind schedule. The network is simulated as it is originally laid out.

One recommendation made by Pittman was to use an approach proposed by Goldratt (1990) and locate time buffers at three different types of locations in the network (see Figure 1). The first location common to all networks is at the end of the critical chain to protect the project completion date against variability in activity durations. This is the project completion buffer. The second type of buffer is to protect for resource contention. This occurs when two activities, one on the critical chain and the other on a non-critical path require the same resource at the same time. This is the resource contention buffer. The time buffer would cause the contending activity on the non-critical path to begin early enough to avoid this scheduling contention (including an allowance for randomly long durations). The third type of buffer involves where a non-critical path of activities intersects with the critical chain. A time buffer located at the point of convergence would protect the critical chain from being delayed by a late finish of the non-critical path. This is the convergence buffer. These time buffers are located so that they protect the accuracy of the planned completion date from disruption of non-critical activities. The three types of buffer placement are illustrated in Figure 1.1.

Pittman (1994) included these buffers in a simulation using his findings and found the buffers to be effective in reducing the impact of statistical fluctuations on project completion when compared to traditional project management methods. But the determination of the buffer sizes was beyond the scope of Pittman's study. The buffers were sized “based upon their location in the project schedule, the relative length of project activities, and the variability of activity times” (Pittman, 1994, p. 177). Development of a methodology for setting the sizes of the three buffers was left to future research.

Research Questions

Three people have made significant contributions toward Critical Chain Project Management (CCPM) of which buffers are a part. Goldratt (1997) has played a central role in applying the Theory of Constraints to project management. Leach (2003) is another person who has developed knowledge and techniques in field. Newbold has also contributed to the knowledge and use of CCPM. These people and Tony Rizzo will be part of this study because their methods of buffer sizing will be compared to the methods developed in this research.

The following seven research questions formed the basis of this study.

1. Is the completion buffer sized using the approach described in Aquilano et al. (1995) capable of protecting the expected project duration at a predetermined success probability of 95%. No converging paths are involved. (Success is defined as a project length being less than the buffered value).

2. Do the techniques for completion buffer sizing presented by Goldratt/Newbold and Leach/Rizzo create completion buffers that protect the project duration at the 90% probability? The value of 90% comes from Goldratt discussion about activity durations. (success is defined as a project length being less than the buffered value). There are no converging paths involved.

3. Can a technique be developed for sizing convergence buffers, that provides approximately 100% protection to the critical chain, and does it without wasted buffer?

4. Do Goldratt/Newbold and Leach/Rizzo methods protect project completion times, by properly sizing convergence and completion buffers for 100%, work as well as the Wray method.

5. Does the Wray method for sizing convergence buffers work for sizing resource buffers?

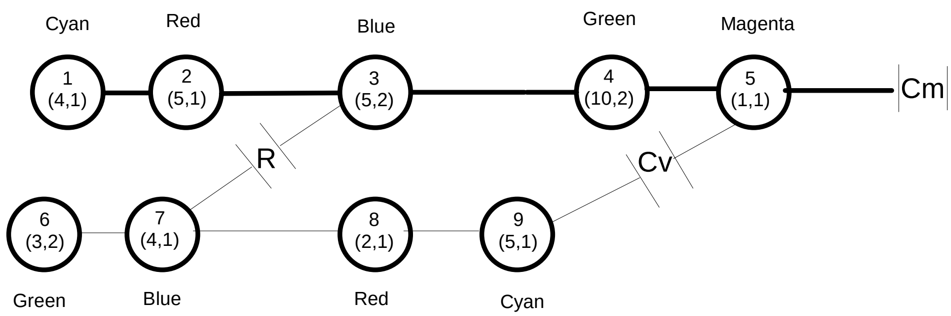
6. Does the Wray, method for sizing convergence buffers protect the critical chain for two path convergences at the same point on the critical chain?

7. In the case of insufficient space for convergence buffers calculated using the Wray method, does the Wray method of adding the buffer shortfall to the completion buffer, compensating for the shortfall, protect the completion time of the project?

The result of this research will be methods for setting the optimal size of each of the three buffers so that the resulting estimate of project duration will have a specific probability of being attained. This dissertation will use a four phase approach to develop methodologies for setting the buffers sizes and operationalizing them for practitioner use.

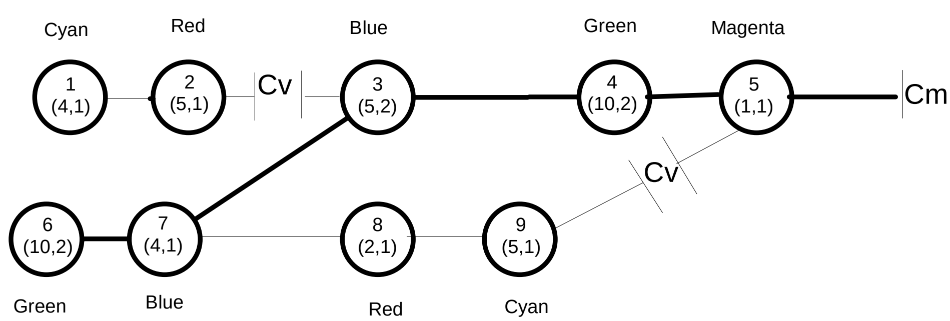
Case 1:

Resource Contention between two paths for the same resource: Slack exists on the parallel path that allows earlier scheduling of the activity 7 contending for the same blue resource, including a resource buffer to ensure that activity 3 on the critical chain is not delayed by a late finish on the other path (6, 7). The completion buffer (Cm) protects the chain of activities 1, 2, 3, 4, and 5.



Case 2:

Resource Contention between two paths for the same blue resource: Slack does not exist on the parallel path that allows earlier scheduling of the activity contending for the same resource. Therefore, the critical chain begins on the path(6, 7) and moves to path(3, 4, 5). This creates a path convergence where none had existed before. The completion buffer (Cm) protects the chain of activities 6, 7, 3, 4, and 5.



Case 3:

Path convergence without resource contention: The critical chain must be protected against the variation in the parallel path (6, 7, 8,9) that converges with the critical chain (1, 2, 3, 4, 5) by inserting a convergence buffer. The completion buffer (Cm) protects the chain of activities 1, 2, 3, 4, and 5. The convergence buffer (CV) protects the critical chain from activities 6, 7, 8, 9.

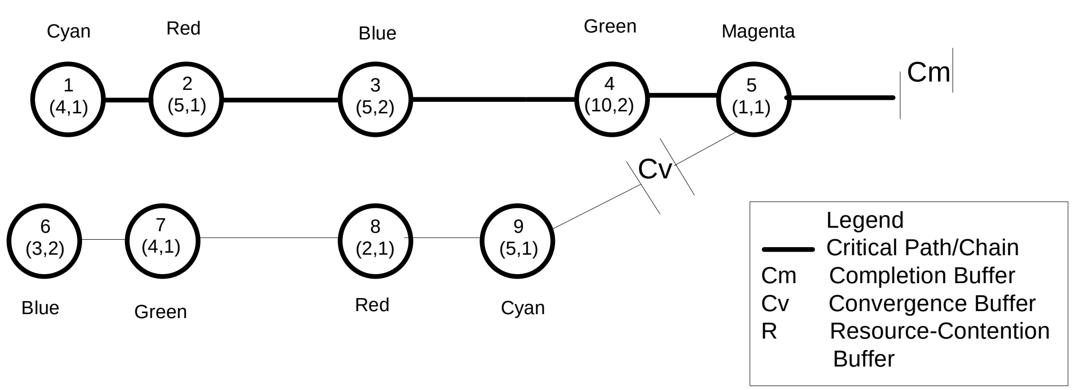


Figure 1.1: Location of the time buffers in resource-constrained networks.

Deliverables and Contribution to Knowledge

The following deliverables will be provided by this dissertation when completed.

1. A method will be provided for setting the convergence buffer size for a project network based variability on both the critical chain and the converging non-critical path of the network.
2. A method will be provided for setting the resource-contention buffer size for a project network based upon the resource contention and the availability of moving activities on the non-critical path of the network.
3. A method will be provided for augmenting the completion buffer to compensate for lack of space for the convergence buffers.
4. These methods will be tested using simulation. The Goldratt/Newbold method, and the Leach/Rizzo methods will also be evaluated for effectiveness against a baseline system and the Wray method. The estimated durations of the networks with buffers will be compared to the observed, simulated duration.

Organization of the Dissertation

Chapter I provides an introduction to the dissertation. Chapter II describes how the literature search was conducted, what was found regarding the sizing of buffers for protecting project duration. Chapter III presents the research methodology. This includes a theoretical development of the model, the research question, and hypotheses, and the research design/methodology. Research methods are presented in chapter IV. This includes the description of the simulation models used in developing the findings and their results. Chapter IV also presents the results of the research. Chapter V presents the conclusions and indicates areas for future research.

CHAPTER II

LITERATURE REVIEW

Introduction

The ABI/Inform database has over 5000 articles on project management. The literature in the field of project management was searched for articles relevant to critical chain and buffer sizing. The search was restricted to those articles dealing with the single project, and single resource environment. The methodology search was restricted to simulation because of stochastic activity durations and the complex interactions of resources and parallel paths within the networks. All other articles were excluded. The articles are classified in this section under the following headings: 1. Pittman's findings, 2. simulation as a methodology, 3. statistical distributions of activities, 4. network characteristics, 5. networks, and 6. analysis and measurements, 7. Critical Chain Project Management and buffer sizing.

Pittman's Eight Problems

Pittman (1994) investigated the problems with CPM and PERT. While numerous other researchers describe narrow problems with project management techniques, Pittman found eight problems in the use of the two techniques. These problems are important to this study in that several impact on the area of study. All eight problems are discussed in the following paragraphs and each discussion refers to Figure 2.1. Pittman's classification is used as he provides the most comprehensive listing of problems, detailed explanations of the causes and illustrations of each problem, and suggestions for solutions.

Capacity Planning and Resource Contention:

Both CPM and PERT do not consider the availability of resources when used to develop schedules. Such an oversight can result in project schedules that are infeasible in that they require the same limited resource to be used in two or more activities at the same time. Resource contention has been recognized by researchers. The problem has been addressed by many researchers (Cooper, 1976; Davis, 1973; Davis, 1975; Davis and Heidorn, 1971; Davis and Patterson, 1975, and Holloway, Nelson and Suraphongschai, 1979). Some of the same researchers have proposed solutions to this problem (Davis, 1973; Davis and Patterson, 1975).

Pittman (1994) found that while this resource contention did not exist over the entire project but rather at one or more points in the project network where the same resource was required by two or more activities over the same time span, this contention could increase project duration. Using a Gedanken experiment with the first network, panel 1 in Figure 2.1, Pittman (1994) found the traditional critical path for the network would be 30. When resource contention, between activities D1 and D2, is considered, the completion time is increased to 38. This is a rather simple example and the effect could be considerably greater for networks with more activities.

Priority Planning

Panel 2 of Figure 2 shows a network that exhibits the problem of priority planning. Activities C1 and C2 both require the same resource. If a strict first-come first-serve assignment rule were used, activity C1 would begin before activity C2. The result would be a completion time of 34 time units. If activity C2 were started first, the completion time would be 30 time units. This is a small example. As the project networks grow in size, the number of alternative paths increase and therefore the chance of more priority problems.

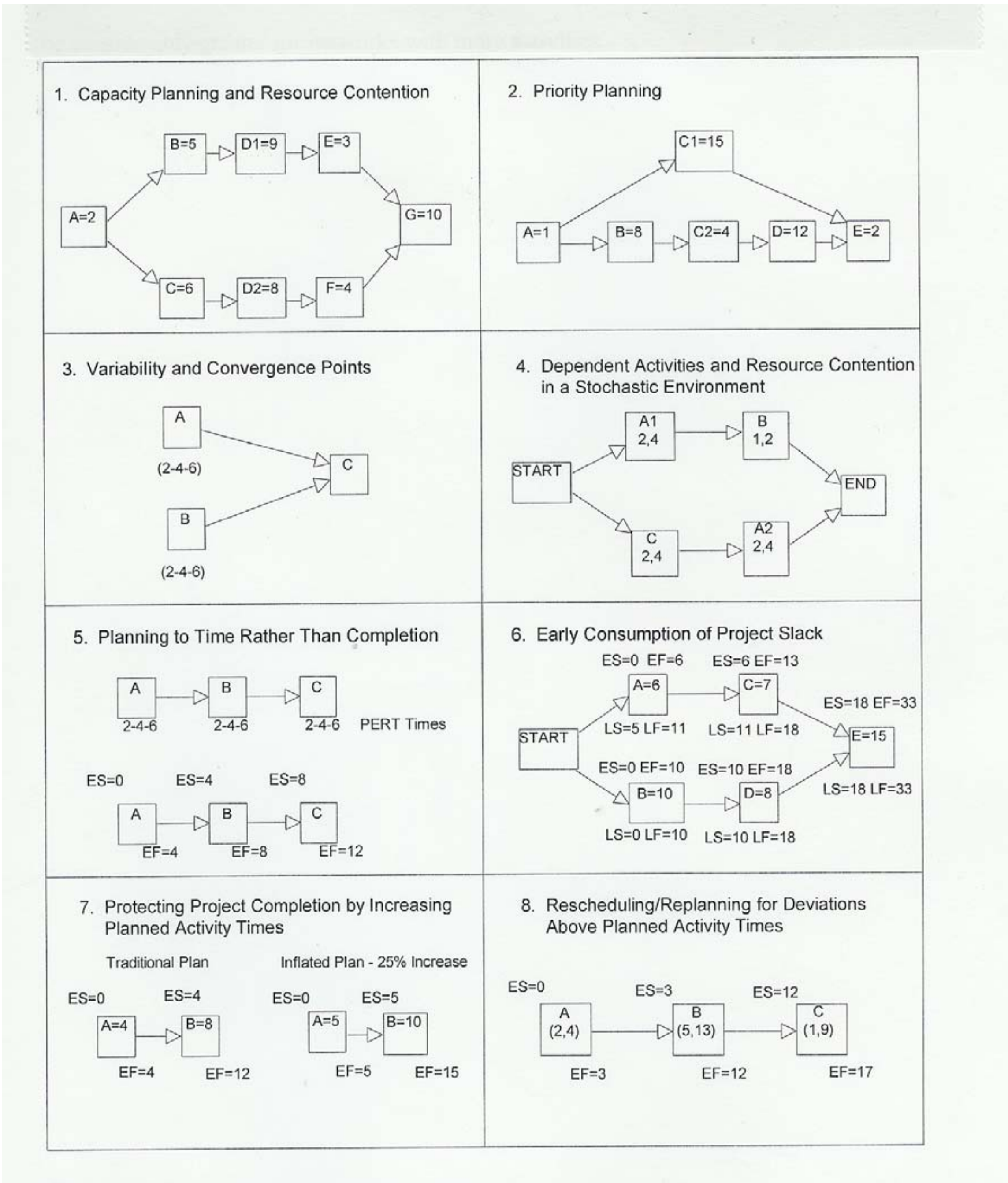


Figure 2.1: Eight problems with CPM and PERT found by Pittman (1994)

Researchers have studied this issue and developed heuristics to overcome the problem of priority planning. These studies dealt with the potentially large number of "feasible" paths in the network. A feasible path or schedule is defined as one in which the resource requirement of any category for all the activities scheduled during any particular period does not exceed the level available during the period (Pittman, 1994, p. 59).

Pittman (1994, p. 59) pointed out that "improper prioritization of activities is one of the drawbacks of heuristic solution procedures which lends strong support for developing more efficient optimal solution procedures." More discussion of heuristics used in research is contained in a later section dealing with heuristics in the literature.

In the preceding example of priority planning, the result was a "chain" of activities that needed to be performed in order to complete the network in the shortest time. This "critical chain" actually moves from one parallel path to the other because of the need to sequence activities using the same resource during the same relative time period. Pittman (1994, p.140) defines a critical chain "as the longest sequence of critical activities in a network which considers both the technological sequencing of activities and the simultaneous demand for common resources."

Variability and Convergence Points

In panel 3 of Figure 2.1, Pittman (1994) demonstrates the problem of variability in networks and how it impacts the network at points where parallel paths converge with the critical chain. Where a parallel path converges with the critical chain, its lateness can result in a delay of that critical chain and thus delay the entire network completion. Because all feeding activities must be completed before the merging activity can begin, the latest completion time for any of the converging paths transfers that lateness to the critical chain and thus the project completion.

Pittman (1994) compensated for this phenomenon by inserting a time buffer into the parallel path at the point of convergence. This time buffer resulted in starting activities on the parallel path prior to their late start date and thus allowed slack time at the right place (the convergent point) to compensate for the variation. Compensating for variation on non-critical paths means that the non-critical path would rarely impact the critical chain and the network completion time. Pittman (1994) estimated the buffer size but did not determine it based upon the characteristics of the network such as the total variation along the path. Goldratt (1997) inserted the same type of buffer at the point of convergence and calculated a size. The size of the convergence and completion buffers is calculated by cutting each activity duration by half and using one half of the reduction (summed along the activities on the affected path) to set the size of the buffer. This approach is based upon the assumption that each estimated activity duration has been inflated to compensate for numerous management practices (multitasking student syndrome, failure to provide 50% activity time estimates). This situation will be further discussed in the explanation of the network in panel 7 of Figure 2.1.

Dependent Activities and Resource Contention in a Stochastic Environment

A network in panel 4 of Figure 2.1, demonstrates this situation. Each of the activity durations follows a uniform distribution. There are four activities with three resources. Two of the activities use a common resource, A. Traditional methods of calculating the network duration would use average values of 3, 3, 3, and 1.5 for the activities A1, A2, C, and B respectively with an expected completion in 6 time units. Enumeration of all possible combinations of these activities results in an expected duration of 6.5 time units. Pittman (1994, p.70) concludes "that resource contention in a stochastic environment makes traditionally calculated estimates incorrect. When a mild degree of convergence is included in the model, the average completion time is further delayed." This demonstrates the need for the convergence buffer.

Planning to Time Rather than Completion

Project managers use the project networks to plan their projects and then to manage them until completion. These plans usually indicate what date each activity on the network is to begin and end. In practice the resources are prepared to start their activities on the scheduled date and not before. This results in not taking advantage of any early completions of activities. Late completion of a preceding activity still impacts the next activity by delaying the start of the activity past its scheduled date.

The Google web search engine was used to search the internet for project management software packages. Five web sites were found that provided project management software. All five web sites were scanned to determine what method was used to begin activities. Prochain, Scitor, and Concerto are three companies that provide the software but do not provide enough information on their web sites to determine whether they use activity-begins-on-date (AOD) or activity-begins-on-completion (AOC). Web sites for Sciforma and Focused Performs provide software and reported that it runs all paths using activity-begins-on-completion. it was not possible to determine whether AOC has become the norm.

Academic research in project networks and project management has used simulation (Van Slyke, 1963, Schonberger, 1981) to model stochastic activity durations. These simulations did not declare any special efforts to begin their activities by schedule date, so it would be reasonable to assume that the start of each activity took place based upon the completion of the preceding activity. This major difference between practice and research can make it difficult to transfer results from one to the other. This is illustrated in panel 5 of Figure 2.1.

Early Consumption of Project Slack

By definition, a path with slack is considered non-critical. This slack can be used up at any point on the non-critical path by delaying the start of an activity or by running over the expected time to complete an activity. In many cases the sequence of activities on the path is done by different people so there is a "local" view of things. Given a chance to decide when to start the first activity, the person responsible for performing that work sometimes tends to delay beginning the work until the calculated late-start-date. Once this consumption of slack at any activity on that path has occurred, any delay in completing the work of subsequent activities that goes past the expected time of the activity will result in the non-critical path using all of its slack, running late, and thus delaying the critical chain at the point where the non-critical path converges with the critical chain. Panel 6, Figure 2.1, illustrates this phenomenon.

Protecting Project Completion by Increasing Planned Activity Times

One way of protecting yourself when working on a project is to increase the time estimate given for completing your activities Goldratt (1997). High estimates may protect the individual responsible for completing the activity but it does not protect the projected duration of the project. The project duration is calculated by summing the expected value of the estimated duration for each activity on the critical chain. If each activity has slack in its estimate, the overall duration will be longer. Perhaps it is too long to get a job against the competition. But since the activities are started on their scheduled date, early completion of an activity is not taken advantage of by the project and the local protection afforded to each activity is not carried forward to protected the remainder of the project. Removing protection from each activity by reducing its estimated activity duration and applying the protection to the path or entire project reduces overall duration and affords greater overall protection.

Goldratt (1997) identified a phenomenon called "the student syndrome." This is a tendency by people responsible for work having additional time (a form of slack) in its estimate, delay starting that activity until the slack in the estimate has been consumed. If the activity then takes more time than expected, it is late. In addition there is a tendency for work to expand to fill the time allotted. The end result is that adding time to each activity duration estimate only extends the estimated time to do the project, it does not protect the project manager's ability to achieve completion by that time.

Rescheduling/Replanning for Deviations Above Planned Activity Times

Pittman (1994) identified replanning as a problem with existing methods for managing projects. Once a project has been delayed, there is a tendency to replan the project network to show the new dates for activities to begin. So long as the original plan was active, activities would begin on their scheduled date or later if the preceding activity completed late. Even if the following activity started past its schedule date, there is always a possibility that it might complete earlier than the expected completion date, thus making up for some or all of the lost time. Setting schedule dates later than shown in the previous plan causes the project to not be able to make up the lost time. The result is a late completion of the project. This is illustrated in panel 8, Figure 2.1. It shows a network that is rescheduled because of late completion of an activity.

Simulation as a Methodology

Watson and Blackstone (1989, p. 2) describe a system as a set of interrelated elements that function in a purposeful manner. A system consists of more than a single element; it is a set of elements that are interrelated and interact with one another. Furthermore, a system's behavior is purposeful; that is, it is goal seeking. The resources and project networks studied in this dissertation meet that definition and are therefore systems.

Simulation is one methodology that can be used to describe the behavior of a system, especially one that is complicated by statistical processes. Law and Kelton (1991, p.5) describe a mathematical model that represents the system "in terms of logical and quantitative relationships that are then manipulated and changed to see how the model reacts, and thus how the system would react - if the mathematical model is a valid one."

Some simple models lend themselves to analytical solution which produces an exact solution. More complex models are difficult or impossible to solve exactly. These models can often be solved by simulation which Law and Kelton (1991, p. 6) describe as "numerically exercising the model for the inputs in question to see how they affect the output measures of performance."

Simulation has been chosen as the method of analysis for this dissertation because of the stochastic nature of the activity duration times. Further, there are multiple activities processing on more than one path at any time. These complexities are best modeled using simulation. The simulation package chosen for the pilot study is GEMS. It was chosen for its availability, and the researcher's experience with its capability to handle a variety of statistical distributions without resorting to programming. The version of GEMS was limited to smaller networks. A second package called Arena is used for the larger networks.

Distributions

The two approaches to setting activity durations are the use of deterministic and stochastic times. Using deterministic activity times is a simplification that results in optimistic completion estimates and thus reduces the applicability of a heuristic. The most popular statistical distribution used in PERT has been the beta distribution. It has been used not because it has been proven the best or most accurate but because it is easy to estimate mean and variance parameters from the optimistic, most likely, and pessimistic estimates for each activity, Clark (1962**).** Many other distributions have been used in the literature for modeling activity durations.

Sculli (1983) used the normal distribution to model activity durations. This required the assumption of independence among the activity paths and did not involve resources. Variances for the activity durations were controlled by setting an upper limit based upon a percentage of the mean value. It would be difficult to directly use this approach with resource-constrained projects because the use of common resources violates his assumption of independence of activities.

Moder et. al. (1983) discussed in their overview of the PERT techniques the conventional method for "setting earliest and latest start and finish times is made in the same way (as CPM) using expected activity performance times only." (p 271) The activity duration variability is used in setting the PERT critical path completion probabilities. This causes the PERT activity durations to be slightly biased on the low side and "the PERT probabilities considerably (biased) on the high side." (p. 271). Since in practice, both CPM and PERT methodologies use the average duration of each activity in the network to calculate expected project duration (sum of activity times along the critical path) and early and late start times for activities on non-critical paths, this study will also use the same method for these calculations.

The effects of three distributions on simulation of a JIT environment were studied by Muralidhar, Scott, and Wilson (1992). They used the truncated normal, gamma and log-normal distributions to model processing times. A two part criteria was used to select these three distributions (p. 2):

1. They exist only for non-negative values.
2. As processing time variability decreases, the form of the distribution changes from:

   a. Monotonic decreasing to

   b. unimodal distribution heavily skewed to the right, to

   c. normal type distributions, truncated at zero

Only the log-normal distribution failed to meet the monatomic decreasing condition but it was included because it is "heavy skewed to the right when the variation is high." Exponential, Erlang, and the normal distributions were excluded from the study because they failed to meet the above conditions as set forth by Muralidhar, Scott, and Wilson (1992). The research did not find any significant differences between the distributions for modeling activity processing times so long as each distribution closely fit the activity processing times. Each distribution had the same coefficient of variation (CV). The authors found one operational problem with using the truncated normal because the computer processing time became excessive (terminated after 24 hours) for high CV values (CV=1). The gamma was recommended because it "specifically meets the requirements for describing processing times in JIT environment and is computationally efficient." (p. 1)

Ranasinghe (1994) also used the log-normal distribution for activities in a study to quantify the uncertainty of project durations. The distribution was used because it is strictly positive and positively skewed. These were described as important characteristics for describing activity durations.

Based upon the review of research using different statistical distributions to model activity durations, it will be necessary to have more than one distribution to model activity durations. The distributions chosen are the normal, log-normal, beta, and gamma. Analysis of the simulation runs may make it necessary to drop some of these distributions because of some unforeseen difficulty.

Measurements

Measurement in network studies has been used to compare the project duration obtained using the target method against some baseline measurement. Normally this is the difference between the observed value and the baseline measure presented as a percentage of the baseline measurement. Research studies in this area use two different base line measurements of project duration. The most prevalent for comparison is the duration of the critical path. This is calculated by summing the expected durations for each activity on the critical path. A second measurement that has been less utilized is optimum duration developed using optimization techniques. This second approach has been used primarily with smaller networks because optimization techniques have not been uniformly successful for larger networks.

Davis and Patterson (1975) used bounded enumeration to develop optimum project durations for each of the networks studied. They then computed the difference in time between the heuristic-based duration and the optimum as a percentage of the optimum. Davis (1975) in a study dealing with network measures for resource-constrained networks, used the non-resource-constrained critical path duration as the base line duration. The performance measure was the ratio of difference between the observed duration and this base line as compared to the base line.

Bowers (1995) in a study dealing with criticality in resource-constrained networks developed resource-constrained (RC) float as a performance measure. RC float is defined as: “the time by which a single activity can be extended or delayed without affecting the project duration, assuming that the resource allocation is unchanged.” (p. 82) This measure would be more useful in determining activities that may require closer management because of less RC float and thus more chance for causing late projects. Badiru (1988) indicates that this RC float is conditional because it depends on the particular resource allocation.

Truncated Normal Distributions

Cohen (1991) presented formulas for calculating the truncated normal parameters for the mean and variance from the symmetric normal mean and distribution. Barr and Sherrill (1999) developed another method of calculating the mean and variance of truncated normal random variables from the mean and variance of the symmetric normal distribution. The Barr and Sherrill (1999) method was compared to Cohen's (1991) and both produced the same results.

Robert (1995) presents a method of simulating truncated normal random variables from the parameters of the normal distribution. The technique cannot be used for generating random numbers using the truncated normal distribution parameters.

Nelson (1964) studied issues about adding two distributions, one normal and one truncated normal and found that the effort proves to be difficult and complicated. Too involved to be used for a network with many activities.

Critical Chain Project Management

Leach (1999) gives a very good recap of the CCPM process contained in *Critical Chain*. Additional information added by Leach (1999) includes insights into where non-critical paths tend to merge with the critical chain. This point tends to be near the end of the project because the merger is at the point of assembly or test. An additional insight provided by Leach (1999) is:

> Activity path merging creates a filter that eliminates positive fluctuations, and passes on the longest delay. The reason is that merging activity paths means that all of the feeding paths are required to start the successor activity. Therefore, the successor activity cannot start until the latest of the merging activities completes. (page 43)

Leach (1999) restates Goldratt's approach to buffer sizing "(use one half of the sum of the activity durations in the chain of activities that precedes the buffer"). He also describes an approach that he attributes to Lucent Technologies. "Sum the "D" (from above, the difference between the low-risk and average estimates). Following the "law of aggregation" use the square root of the sum of the squares (SSQ) to sum the Ds."(p. 46)

In reference to the PERT calculations for mean and variance based on the beta distribution, Leach (2003) states that "…the uncertainty in actual task data usually far outweighs the variation introduced by the choice of mathematical model." (p. 35)
A table is presented by Leach (2003) to demonstrate the differences in scheduled duration for CPM, PERT, and Critical Chain Project Management (CCPM).

Table 2.1 Example of PERT Calculation (Leach, 2003,p. 35).

Task	O	M	P	Xbar	s	s^2
1	2	5	10	5.33	1.33	1.78
2	3	5	12	5.83	1.50	2.25
3	4	6	8	6.00	.67	0.44
4	7	10	15	10.33	1.33	1.78
		CPM =	45	27.50	2.50	6.25
			3 x s =	7.50		
			PERT =	35.00		

Where O = optimistic time for the task, M = most likely, P = pessimistic time for the task, Xbar is calculated from the PERT formula for the task mean, s = standard deviation as calculated using the PERT formula for the standard deviation, and s^2 is the square of the standard deviation. The PERT project duration calculation would use the sum of the PERT activity means on the critical path and would be 27.50, without any buffer. Thus there would only be a 50% probability of obtaining that duration. Leach recommends adding 2 or 3 standard deviations to this duration resulting in 35.00 days.

Leach (2003) references a method he proposed in his book Leach (2000) for setting buffer sizes. The table from the article is presented here:

Table 2.3: Example CCPM calculations (Leach, 2003,P. 36).

Task	M	P	u	u^2
1	5	10	5	25
2	5	12	7	49
3	6	8	2	4
4	10	15	5	25
	26			103
SSQ	10			
CCPM	36			

Where M= most likely time for the task, P = pessimistic time for the task, u = P – M which is a measure of variability, and u^2 = sum of squares. SSQ = 10 is derived from the square root of 103. The value for CCPM is the sum of the sum of the M values and SSQ.

Leach (2003) points out that the closeness of the critical chain project duration and the PERT duration is only by chance. Often the critical chain is not the same as the critical path in PERT. The author states that "the sum of the mean estimate plus the buffer is always less than the sum of the low-risk (pessimistic) estimates for the individual tasks." (p. 36)

Leach (2003) and Hutchin (2001) tie Deming's concepts of variation to project management. Deming defined two types of variation, special and common. The common cause is defined as variation that is inherent to the process and cannot be changed without modification of the basic process. Special causes of variation are things that can be identified and reduced by modification short of changing the process.

Leach (2000) describes the issues that should be considered when setting resource buffer size:

> Size resource buffers to the needs of the resource provider. The size should depend on the quantity of the resource, the length of the resource's usual task, and special considerations such as required training, travel, or other lead time. (P. 170)

These buffers are related to making it worthwhile for resource providers to be ready to start their activity on the critical chain prior to the estimated date for starting. This is the method by which the project manager can induce resources to be ready to take advantage of early completions of activities preceding their own.

Leach (2000) recommends the following for setting the project (completion) buffer size:

> Size the project buffer using the square root of the sum of the squares method. Determine the D value for each task as the difference between the initial task duration estimate and the reduced estimate. The following guidelines will help ensure an effective buffer:
>
> - Seek to have at least 10 activities on the critical chain. Reason: The more activities in the critical chain, the more effective the sum of the squares and central limit theorem.
>
> - Do not allow any one activity to be more than 20% of the critical chain. Reason: The uncertainty of one large activity will dominate the chain, leaving little

possibility for the other tasks in the chain to make up overruns on the dominant task.

- Do not allow the project buffer to be less than 25% of the critical chain. Reason: Chains with many tasks of uniform length may calculate a relatively small buffer, providing inadequate protection. (p. 167)

Leach (2000) recommends the following for setting the feeding (converging) buffer size:

Size the feeding buffers using the square root of the sum of the squares method. Determine the D value for each task as the difference between the initial task duration estimate and the reduced estimate.
If there are fewer than four tasks in the feeding chain, make sure the feeding buffer is at least equal to the longest activity in the feeding chain. (p. 169)

Leach (2000) explains the sum of the squares method as follows: First, estimate the most likely and the most pessimistic times for each task. The difference between these two values for each task is "D". Square the D values and sum them. The buffer size for that path is then the square root of that sum of squares.

Herroelen and Leus (2001) evaluate the strengths and weaknesses of Critical Chain project scheduling. They express concerns that using Goldratt's approach to buffer sizing at 50% of the path could create unnecessarily large protection. They also evaluate the use of the sum of squares method for buffer sizing. They point out that the approach assumes project activities are independent. Finally, they have concerns with Newbold 's (1998) use of the lognormal distribution to model the activities in the network. They contend that Newbold's assertion that "average expected duration and the worst-case duration will be about 2 standard deviation" for each task. Herroelen and Leus (2001) assert that the range is more likely between 0.05 and 1.5 standard deviations rather than 22 standard deviations.

Newbold (1998) appears to have included the lognormal not as a preferred but a more scientific approach to setting buffer size.

Hoel and Taylor (1999) studied methods for sizing the completion and convergence (feeding) buffers. They used simulation of the network with normally distributed activity times to develop a graph of the simulated project completion probabilities (cumulative probability distribution) against the project time. The buffer was sized by locating the desired probability of completion and the corresponding number of weeks duration. The difference between the selected number of weeks duration for the percent chosen and the expected project completion time determined by the simulation represent the completion buffer. Convergence buffers were set by the Hoel and Taylor (1999) by allocating all slack in the non-critical path to the buffer. This researcher believes that allocating all of the available slack is overkill and will result in greater cost (time value of money) without increased benefits.

Goldratt's (1997) approach depends upon the group for whom the network is prepared. If the project team is new to CCPM, then their estimates should be cut by 50% to arrive at the mean time for each activity. If the project team is more experienced with CCPM, then their estimates should be cut by a lesser amount. The buffers should be sized by taking the amount of the activity time estimates that was cut and using half of it for the buffer at the end of that path (convergence or completion).

Rizzo (2004) critiques Goldratt's (1997) approach to sizing buffers. He states that one good feature is that the mathematics are easy. Several negatives in Goldratt's approach are also identified. One is that his method "provides a linear model of variation." Rizzo points out that variation does not add linearly, only variance. Rizzo states that the approach to variation is "inconsistent with sound mathematics."

Rizzo (2004) identifies a second problem with Goldratt's approach. By cutting the estimates of the project team, Goldratt is destroying the trust between the project manager and his project team.

Rizzo (2004) presents and discusses another approach to setting buffer sizes. He describes it as using a control chart. His description is:

> By this approach, we simply graph normalized values of project duration on a control chart. The planned (baseline) duration estimates of the projects are used as the normalizing values. For example, a project that had a planned duration of 100 business days and an actual duration of 140 business days would be represented in the control chart with a normalized duration of 1.4. The difference between the control limit and the mean of the normalized duration values serves as the basis for calculating subsequent project tolerances.

Rizzo (2004) points out that the method captures all the variation in the project. It would also be a very difficult technique to use to size the buffers. Finally, it would require having accurate information about how the project would perform before the project had been conducted. This might be done with a simulation. This would be beyond what might be expected from the normal project manager.

Rizzo (2004) describes the "sum of squares" or RSE method for setting buffers. The RSE method is illustrated in the next figure. In support of the RSE method, each developer provides two estimates of duration per task. First the developer provides an estimate that corresponds to a high level of confidence. We call this the "safe" estimate. Then the developer provides an estimate of the mean process time. We call this the "average" estimate. The difference, D, between safe and average estimates for each task gives us a measure of the expected variation for the task. The component tolerance is calculated as the square root of the sum of the squares of the differences, for the tasks in each component sequence. The same calculation also provides an estimate of the project tolerance, with the difference values being those that correspond to the tasks of the primary sequence in the project.

Rizzo (2004) describes the resulting buffers developed using the RSE method as the absolute minimum values. He describes them as “inappropriately small.” He leaves it to the individual what method of buffer sizing they should use but he indicates that he prefers RSE.

The control chart approach would not be useful for this study. It requires more knowledge about how the project would complete than would be available without other like projects or perhaps simulation. Goldratt’s method is simple, easy to understand, and has been widely used by people employing CCPM. The RSE approach described by Rizzo (2004) (also referred to by Leach as Lucent) and the approach described by Leach (2000) appear to be the same. They have enough grounding in statistical theory to make them plausible although the source of their values for the task means and standard deviations could be questioned. Goldratt and Rizzo/Leach are reasonable candidates for use in this study.

Aquilano et al. (1995) describe a technique for setting the completion buffer for a project. The technique utilizes the additive characteristic of the normal distribution. If the activities along the critical chain have a normally distributed activity time, the means and variances of these activities can be added to obtain the mean and variance of the duration for the critical chain. Then, using these characteristics of the standard normal distribution and the duration value that the researcher does not want to exceed, the probability of exceeding that value can be determined. Prior studies of buffer sizing, and the type(s) of buffer studied, are shown in Table 2.3.

Table: 2.3: Classification of the Literature Discussing Sizing Project Buffers

Source	Completion	Convergence	Resource	Other
Aquilano et al. (1995)	X			
Goldratt (1997)	X	X	X	
Leach (2000)	X	X	X	
Rizzo (2004)	X	X	X	
Herroelen and Leus (2001)				X
Hoel and Taylor (1999)	X	X		
Newbold (1998)	X	X	X	

Aquilano et al. (1995) presents a method for determining the probability of being able to meet an established duration for the project. This method uses standard statistical methods, is understandable, and should be usable for anyone with a college level understanding of statistics. It does not take into account the interaction of the variability on the two paths for converging and resource buffering. It will not be used for setting the convergence and resource buffer sizes. It will be used for setting the completion buffer size.

CHAPTER III

RESEARCH DESIGN

Introduction

This chapter describes the design of the experiment conducted in chapter four. A pilot was conducted in order to learn more about how the networks behave using activity-begins-on-date and activity-begins-on-completion. Based on the knowledge gained in the pilot experiment, a decision was made about what distribution would be used to model network activity times. The two alternative methods proposed by Goldratt/Newbold and Leach/Rizzo for sizing buffers were described. The Wray method for sizing the buffers was developed and described. The experiment to be conducted in chapter four is further described including a description of the networks used in the experiment. The questions to be resolved are presented with their hypotheses.

Pilot

A pilot study was conducted to learn more about the behavior of project networks using different activity starting rules, statistical distributions, and degrees of variability. The two starting rules studied here were activity-begins-on-date (AOD) and activity-begins-on-completion (AOC). There are two starting rules because the AOC or "road runner" concept proposed by Goldratt (1997) is used for the critical chain to get the shortest duration for the project. This method requires more management attention and cost than AOD to make it work, because succeeding workers must wait around in case of an early completion. A second method is proposed for managing the non-critical paths because it requires less management attention and thus cost. This method sets a date for the activity to begin; no resources are available for a succeeding activity until that date arrives. This is called activity-begins-on-date.

Using activity-begins-on-date, activities are scheduled using a project management technique such as CPM to begin on a specific date. The assumption is that the preceding activity will finish at that time. This assumption is not necessarily correct. The preceding activity may complete early but the advantage of the early completion is not transferred to the following activity. The following activity will wait until its scheduled date to begin. The result is that the network does not take advantage of early finishes.

The rule using activity-begins-on-completion assumes that the following activity is ready to begin immediately after the completion of the activities preceding it. This requires resources to be waiting for the completion of the preceding activity. The result of using this starting rule is that early finishes are taken advantage of in completing the network. This method has been used in research involving simulation by Schonberger, (1981), and Van Slyke, (1963).

The 84 pilot experiments were completed using GEMS simulation software with the two types of networks shown in Figure 6. The top network was used to simulate a simple network where activities start on completion. The bottom network represents a simple network where activities begin on date. The path of activities 1 through 15 represents the actual activities and the path of activities 20 through 31 represent "timing" boxes. All activities, #1 - #15, have the same expected value and standard deviation. The purpose of boxes, 20 – 31, is to restrict the boxes 1 through 15 to start on date or completion, whichever is latest. Box 2 does not need a timing box because it starts at time now.

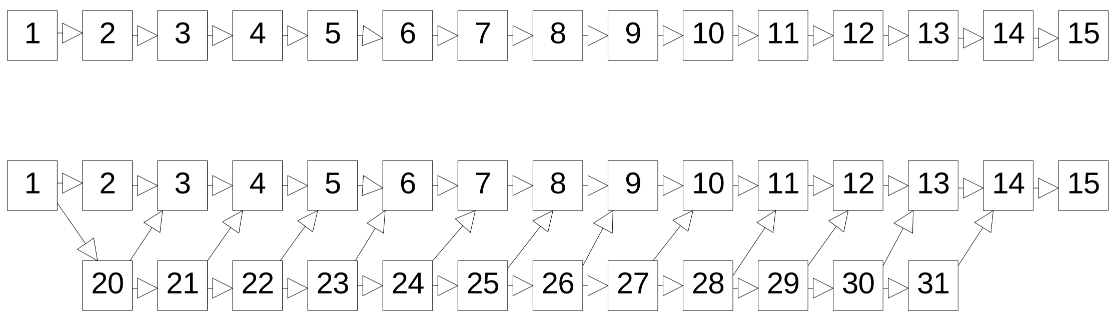


Figure 3.1: Sample networks used in pilot study.

Overview of the results of both starting rules:

Figures 3.2, 3.3, 3.4, and 3.5 compare the mean, minimum, maximum durations and the standard deviations of the two types of networks using the different rules.

The simulations in Figure 3.2 were done using the same inputs: activity distributions, means, and standard deviations. Figure 3.2 shows that the mean value for path durations, where activity-begins-on-completion, does not change with a change in variability (standard deviation) of individual activity times. The opposite is true of activity-begins-on-date. For this second rule, the mean network duration is larger and increases as the variability of the activity durations increase. This because of the effect of activity-begins-on-date has on the activity times of the non-critical path. Truncated normal distributions have a larger mean value and smaller variance. Therefore, two activities requiring the same resource can not be accurately located by summing the expected activity durations from a common point on both paths if each path uses a different rule. This can make it more difficult to fully evaluate possible resource contention on critical chain and a parallel non-critical path.

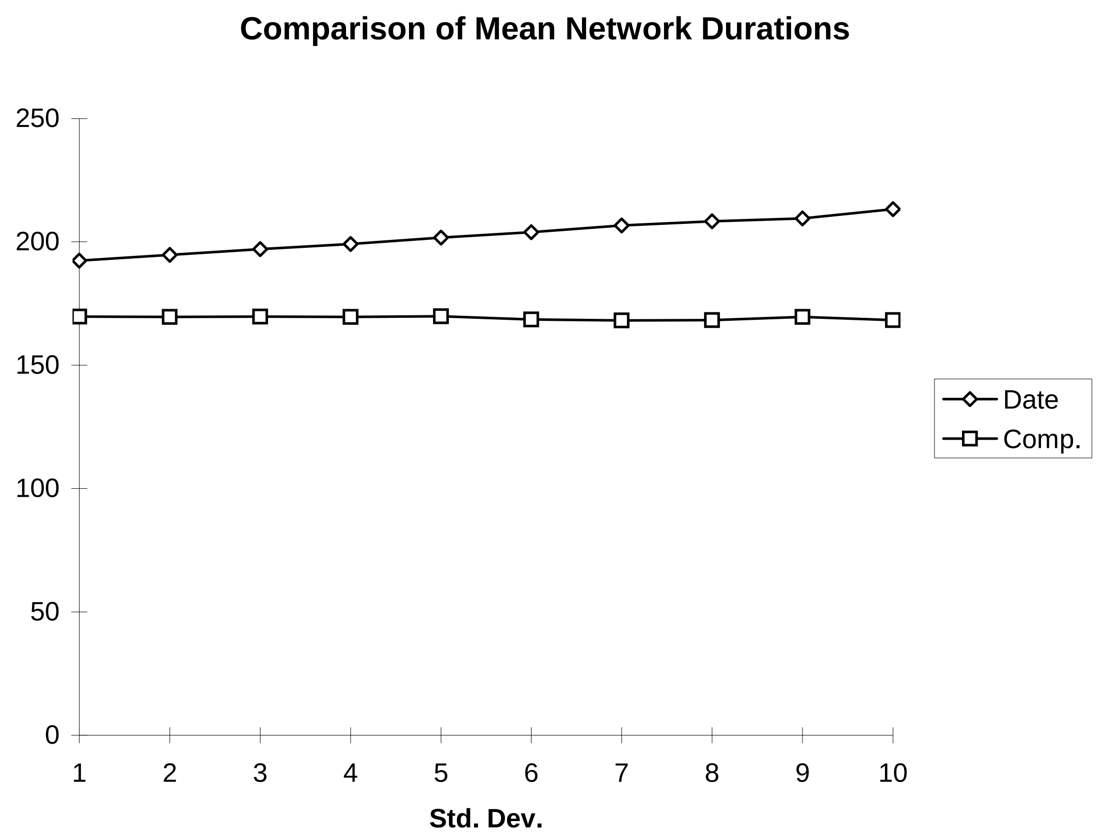


Figure 3.2: Comparison of activity-begins-on-completion (Comp.) and activity-begins-on-date (Date) for standard deviations from 1 to 10. The vertical axis (Y) represents network durations.

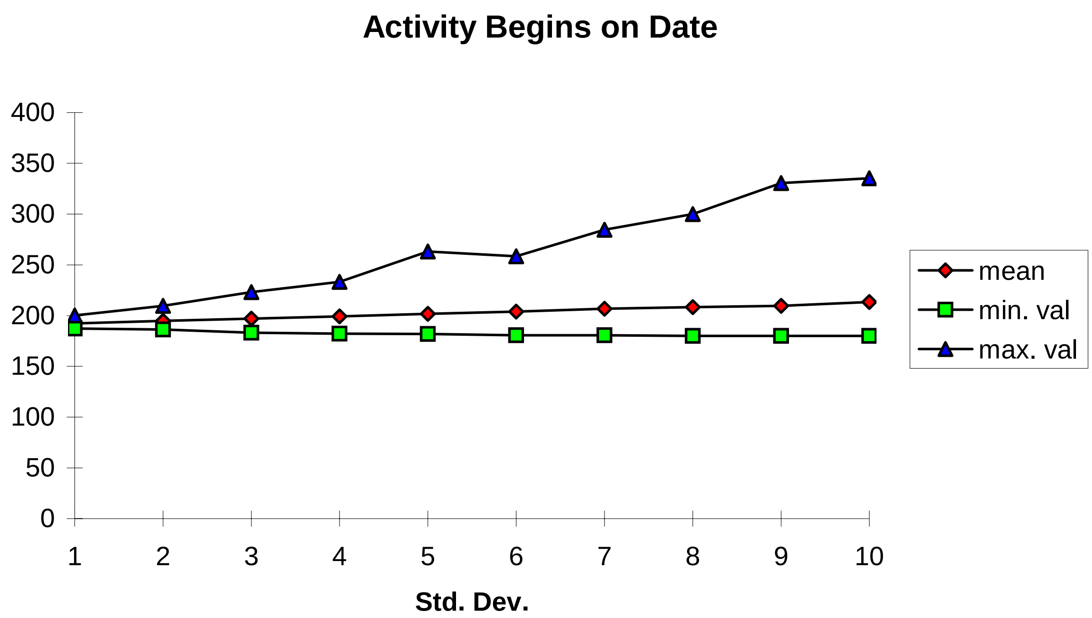


Figure 3.3: Mean, minimum and maximum durations for a network of 500 iterations using activity-begins-on-date. The vertical axis (Y) represents the network duration.

In Figure 3.3, the minimum network durations of AOD runs do not vary much over the different standard deviations used in the experiment. This is substantially different from the behavior of the same network using activity-begins-on-completion in Figure 3.4. It indicates that the distribution of durations resulting from AOD is bounded on the low end, because of the scheduled start dates..

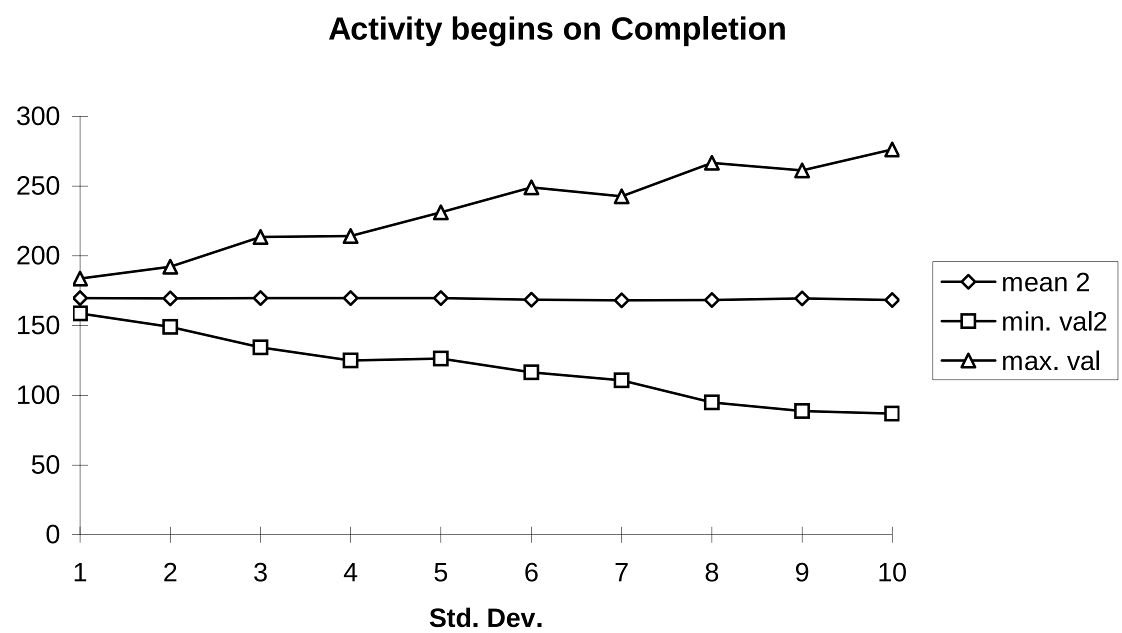


Figure 3.4: Mean, minimum, and maximum network durations of 500 iterations using activity-begins-on-completion. The vertical axis (Y) represents the network duration.

Figure 3.4 shows the variability of the maximum and minimum network durations for AOC networks. This variability is caused by changes in the standard deviation of the activity durations. As the standard deviation increases from 1 to 10, variability steadily increases. This pattern is consistent with the sum of activity distributions that are normally distributed. This pattern would be expected for durations for activities lying on the critical chain.

According to Figure 3.5, the maximum duration for the network using activity-begins-on-date is larger than networks using the other rule. This amount is not appreciable but is consistent over the range of standard deviations used in the experiment (around 350 versus around 300). This is a result of not being able to take advantage of early finish for activities on the non-critical path. This can be important for the project manager who uses the traditional approach of treating the activity times of activities on the path as constants when estimating the path duration. It creates estimates of path durations for non-critical activities that are consistently too low.

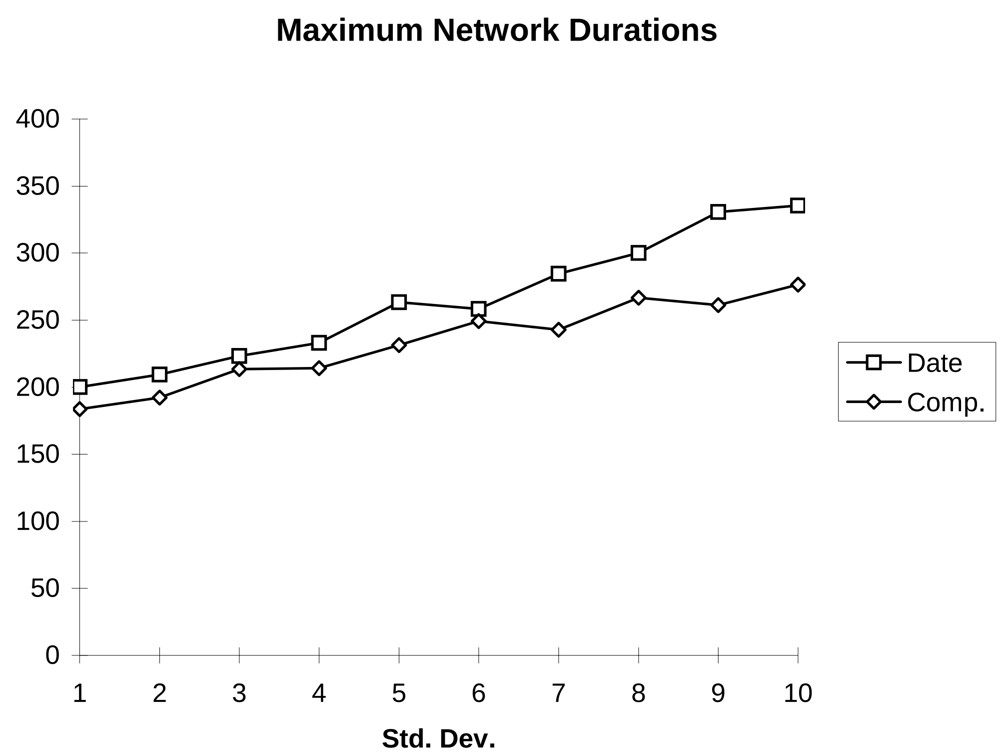


Figure 3.5: The maximum network durations for activity-begins-on-completion (Comp.) and activity-begins-on-date (Date). The vertical axis (Y) represents the network duration.

Figure 3.6 shows that the minimum durations of the non-critical path networks are less affected by variability of the activity times than the critical chain with its starting rule of activity-begins-on-completion. This difference will have to be taken into account when identifying activities on parallel paths that may be in contention for resources.

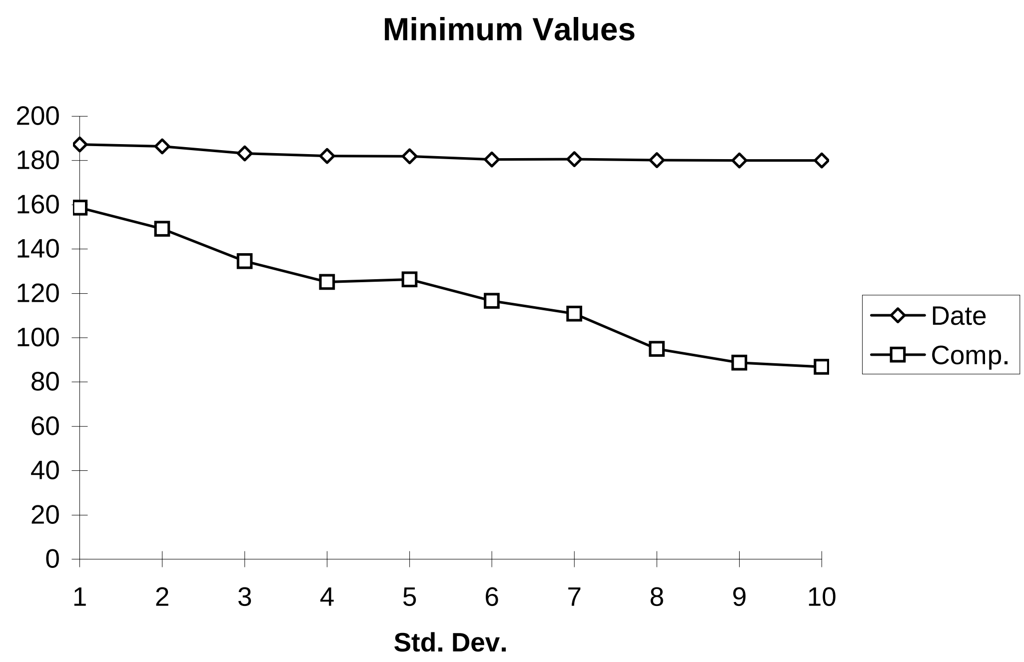


Figure 3.6: Comparison of minimum network durations for networks using activity-begins-on-completion (Comp.) and activity-begins-on-date (Date). The vertical axis (Y) represents the network duration.

According to Figure 3.7, the variability of networks using activity-begins-on-completion is greater than the same network using activity-begins-on-date. Networks started using activity-begins-on-completion have duration times that tend to be normally distributed and thus symmetric. This symmetry results in variability both above and below the mean value. Running networks using activity-begins-on-date results in duration times that are bounded on the lower end and slowly decrease on the upper end. The result of having the duration values fall in a narrower range of values is a smaller variance of the duration times.

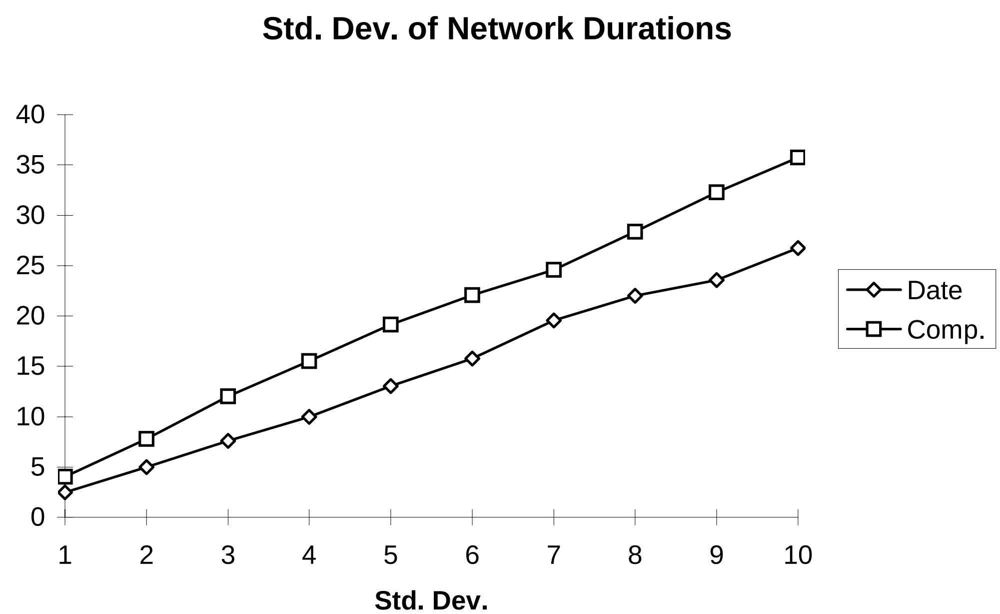


Figure 3.7: Comparison of standard deviations for networks using activity-begins-on-completion (Comp.) and activity-begins-on-date (Date). The vertical axis (Y) represents standard deviation of the network duration.

Findings from Figures 3.2, 3.3, 3.4, 3.5, 3.6, and 3.7, pertaining to the different starting rules

There is a definite effect on the performance of the networks depending on the starting rule used.

- The activity-begins-on-date starting rule results in a larger minimum duration for the network because of the effects of truncation preventing lower values.
- For the same network and inputs, the activity-begins-on-date results in a larger mean duration the activity-begins-on-completion because truncation causes the mean to be larger than in the case of the normal distribution.

- For the same networks and inputs, the activity-begins-on-date results in a larger maximum duration than the activity-begins-on-completion because of the truncation causing larger mean values.
- The activity-begins-on-date results in a smaller variance for the path duration than the activity-begins-on-completion because the distribution has lost smaller values due to the effects of truncation

There is enough difference in the results obtained from activity-begins-on-date and activity-begins-on-completion, that they must be dealt with using different methods. Treating the non-critical paths the same as the critical chain will result in inaccuracy that would significantly reduce the usefulness of this study.

Critical Chain and Activity-Begins-on-Completion

Table 3.1 indicates that the simulations of the critical chains using the starting rule of activity-begins-on-completion results in fairly consistent results. With some exceptions, all of the statistical distributions (gamma, normal, beta, log-normal) fit the duration time data. One point needs to be raised about these results. Not all of these distributions have liberal additive properties. When the gamma distributions are summed, they result in a gamma distribution with predictable parameters but only under restricted conditions. Each of the gamma variables summed must have the same parameters in order for the summation to work. Information was found to indicate that the log-normal has additive properties but the mathematics are problematical. Nothing was found dealing with any additive property of the beta distribution. The normal distribution has the most predictable additive property in that the sum of the means and the sum of the variances result in the mean and variance of the total.

Lack of ability to sum the activities along a path, when they are modeled by the gamma, beta, and log-normal distributions and thus estimate the parameters of that sum, makes it difficult to use the results shown in Table 3.1. The normal distribution is the exception. Being able to model activity times using the normal distribution would simplify the process and make it available to more potential users.

Findings about modeling the critical chain using activity-begins-on-date

- All statistical distributions would work for modeling the duration of the critical chain but all but the normal distribution present difficulties. The normal distribution is preferred.

Table 3.1: These are all activity-begin-on-completion. This table presents the results of pilot simulations and what distributions fit the duration data for each type of path. "X" indicates fit. Read GA-3 as "gamma" distributed activity times, with Std. Dev. = 3

Number of Boxes - Path	Activity Distribution Used - Std. Dev.	Gamma - GA	DATA Normal - NO	Beta - BE	Log-Normal LN
	GA-3	X			X
	NO-3	X	X*	X	X*
	BE-3	X	X	X	X
	LN-3	X	X	X	X
	GA-5	X		X	X
3	NO-5		X	X	
	BE-5	X	X	X	
	LN-5	X		X	X
	GA-8	X			
	NO-8	No C	X	No C	No C
	BE-8	X		X	
_____	LN-8	X			X
	GA-3	X	X	X	X
	NO-3	X	X	X	X
	BE-3	X	X	X	X
	LN-3	X	X	X	X
	GA-5	X	X	X	X
	NO-5	X	X*	X	X
8	BE-5	X	X	X	X
	LN-5	X		X*	X

	GA-8	X*			
	NO-8	X	X	X	X
	BE-8	X	X*	X	X*
	LN-8	X*		X	X
	GA-3	X	X	X	X
	NO-3	X	X	X	X*
	BE-3	X	X	X*	X
	LN-3	X	X	X	X
15	GA-5	X	X	X	X
	NO-5	X	X	X	X*
	BE-5	X	X		X
	LN-5	X	X	X*	X
	GA-8	X	X	X	X
	NO-8	X*	X	X	X*
	BE-8	X	X	X	X*
	LN-8	X	X	X	X

Chi-square goodness-of-fit tests with N=500, alpha = .05, and 24 degrees of freedom.
"X*" indicates that fit was achieved by combining adjacent cells.
"No C" indicates that the beta parameters would not calculate.

Non-Critical Paths and Activity-Begins-on-Date

Frequency histograms are shown in Figures 3.8, 3.9, 3.10, and 3.11. Each of these histograms is for a network of the same size (8 boxes), with the same rule (activity-on-date), and the same mean (10 time units) and same standard deviation (3 time units). The only difference is the statistical distribution for the activity times for each network. Looking at the tails can give the reader some idea of why not all of these duration times were fit by all of the distributions. Each of the histograms shows a chopped left tail and an elongated right tail. This is probably caused by the activity-begins-on-date starting rule. All histograms show a shift of the most likely values toward the left of the histogram.

The failure of the normal distribution to fit all of these distributions of summed values causes some concern. The standard deviation of the input data is not extreme and the number and the network size is increasing. Larger networks would be expected to be more normal like. The results for smaller simulations were more encouraging for the normal distribution than for the larger runs. After some consideration, it was determined that the lack of normality was caused by the starting rule and the probability of some correlations between the activities in the non-critical paths. The correlations violate one of the assumptions of the central limit theorem and may be a major cause of the data not showing a fit to the normal distribution.

No statistical evidence is available for this belief in correlations but there is some logic to support this belief. Each activity on a path must wait for two events before it can begin; the completion of the previous activity and the scheduled date for its start. If the previous activity finishes but the date has not arrived, the next activity must wait for the date. If the date has arrived but the previous activity has not completed, the next activity must wait for its completion. Under normal circumstances, each activity must take at least the expected value to complete, no less. If there are a succession of late finishes for

Table 3.2: These are all activity-on-date. This table presents the results of pilot simulations and what distributions fit the duration data for each type of path. "X" indicates fit. Read GA-3 as "gamma" distributed activity times, with Std. Dev. = 3

Number of Boxes - Path	Activity Distribution Used - Std. Dev.	Gamma - GA	DATA Normal - NO	Beta - BE	Log-Normal LN
	GA-3	X		X	X
	NO-3	X	X	X	X
	BE-3	X	X*	X	X
	LN-3	X*		X	X
	GA-5	X		X	X
2	NO-5	X	X	X	X
	BE-5	X	X	X	X
	LN-5	X*		X	X*
	GA-8			X	X*
	NO-8	X	X*	No C	
	BE-8	X*		X	X
	LN-8			X*	X*
	GA-3	X	X*	X	X
	NO-3	X	X	X	X
Number of Boxes - Path	Activity Distribution Used - Std. Dev.	Gamma -GA	Normal - NO	Beta - BE	Log-Normal LN

	BE-3	X*	X*	X*	X*
	LN-3	X*	X*	X	X
	GA-5	X*	X*	X	X
	NO-5	X	X	X	X
4	BE-5	X*		X	X*
	LN-5	X*		X	X*
	GA-8	X*		X	X*
	NO-8	X	X*	No C	X
	BE-8	X*			X*
	LN-8			X	
	GA-3	X*	X*	X	X*
	NO-3	X	X*	X	X
	BE-3			X	X*
	LN-3			X*	X*
6	GA-5	X*		X	X*
	NO-5	X	X*	No C	X
	BE-5	X*	X*	X	X
	LN-5			X	X*
	GA-8			X	
	NO-8	X*	X	No C	X
	BE-8	X*		X	X*
	LN-8			X	
	GA-3			X	
	NO-3	X		X	X
	BE-3	X*	X*	X	X*
	LN-3	X*		X*	X*
	GA-5			X	
	NO-5	X		X	X
8	BE-5			X	X*
	LN-5			X	
	GA-8			X	
	NO-8	X	X*	No C	X
	BE-8	X*		X	X*
	LN-8			X	

Chi-square goodness-of-fit tests with N=500, alpha = .05, and 24 degrees of freedom.
“X*” indicates that fit was achieved by combining adjacent cells.
“No C” indicates that the distribution parameters would not calculate.

activities on the path such that the activities are starting after the date has arrived and gone, there exists an opportunity for an activity to finish early and the accompanying date has already happened. So the succeeding activity can take advantage of the early finish and begin. This is the source of the expected correlation between activity times. The “effective” activity time is dependent on the adjacent activity and the amount of lateness.

Table 3.2 shows the usefulness of the four distributions to model non-critical paths with the different statistical distributions for activity times. The three distributions gamma, beta, and log-normal appear to model the path durations better than the normal

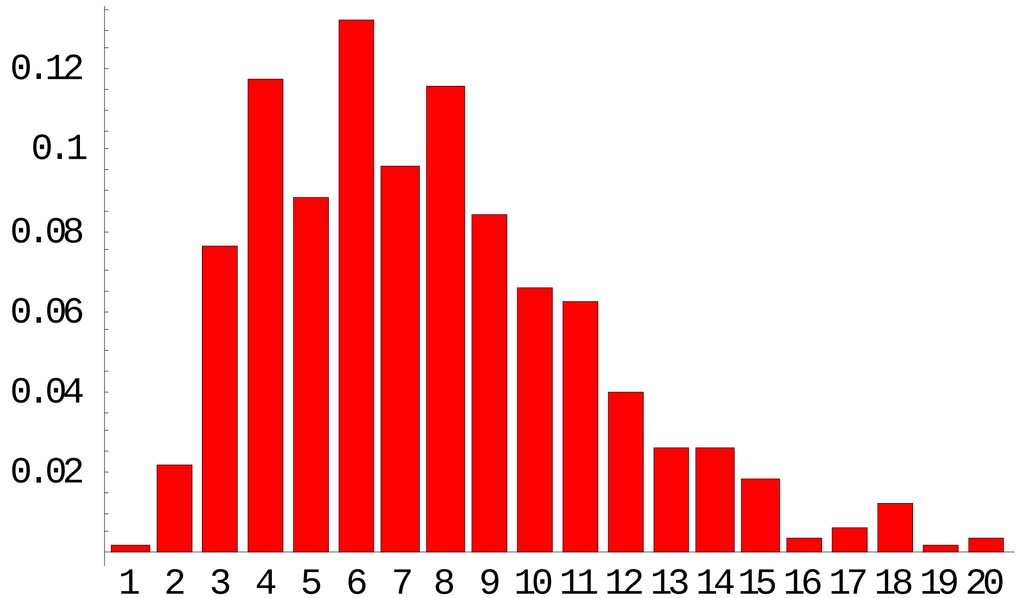


Figure 3.8: Frequency histogram of an 8 box, beta generated activity times, mean = 10 and standard deviation = 3, for all boxes. This is an activity-on-date simulation (see Table 3.2, 8 boxes, BE-3)

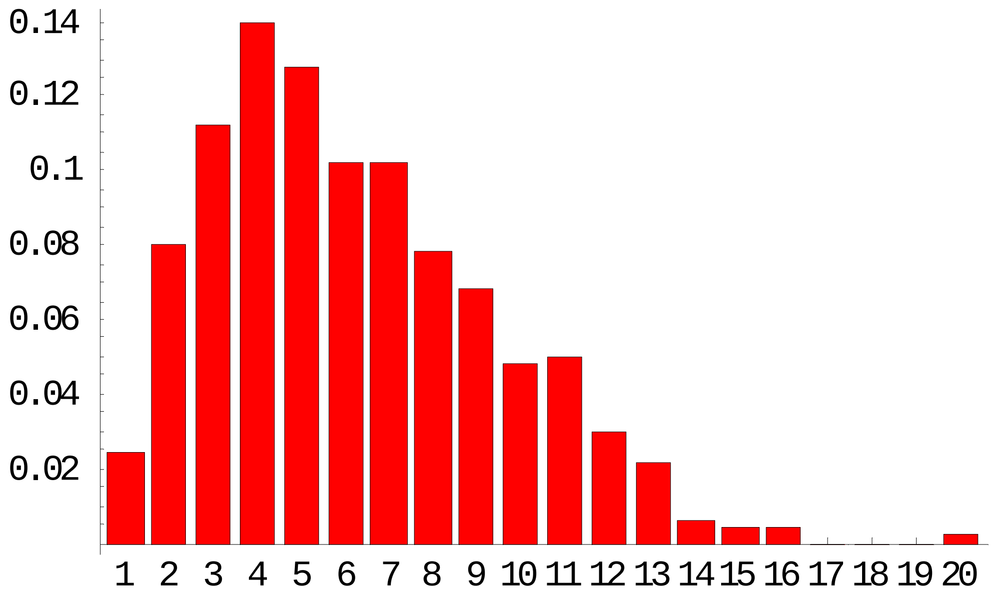


Figure 3.9: Frequency histogram of an 8 box, gamma generated activity times, mean = 10 and standard deviation = 3, for all boxes. This is an activity-on-date simulation (see Table 3.2, 8 boxes, GA-3)

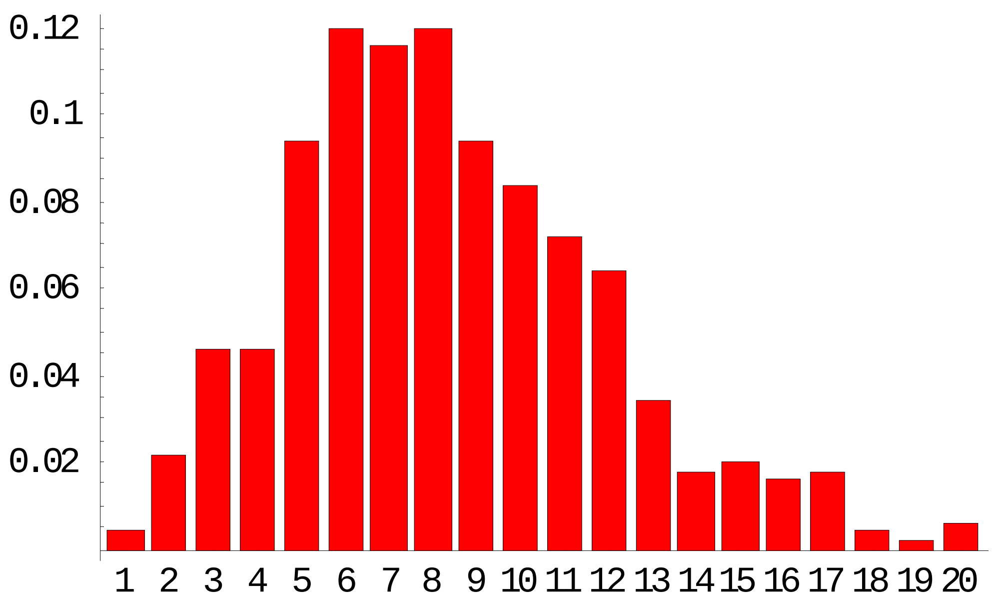


Figure 3.10: Frequency histogram of an 8 box, normal generated activity times, mean = 10 and standard deviation = 3, for all boxes. This is an activity-on-date simulation (see Table 3.2, 8 boxes, NO-3)

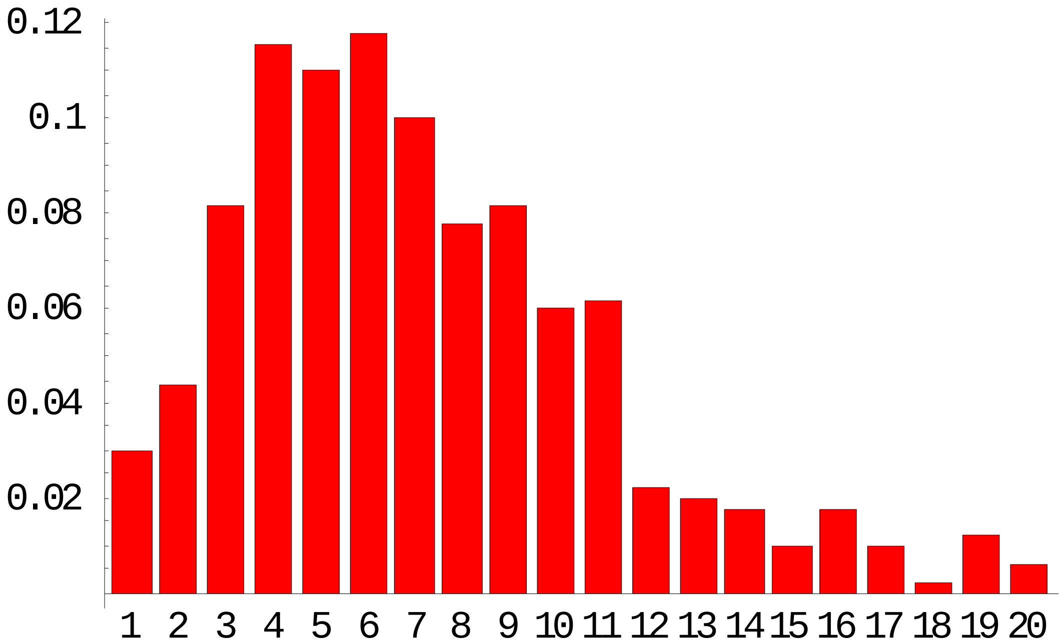


Figure 3.11: Frequency histogram of an 8 box, log-normal generated activity times, mean = 10 and standard deviation = 3, for all boxes. This is an activity-on-date simulation (see Table 3.2, 8 boxes, LN-3)

distribution. This is probably due to the creation of truncated activity times and the correlations between activities. None of these three distributions are very easy to work with in their truncated form. The truncated normal distribution has the most information about it in the literature.

Findings for non-critical paths, activity-begins-on-date

- These activity duration times for the non-critical paths will have to be modeled using truncated distributions.
- The truncated normal distribution is the most studied of all four truncated distributions and therefore should be used for modeling the activity times in this study.

Resource contention and convergence buffers

In Figure 3.12, we have two activities (A and B) on parallel paths. These activities must be done with A preceding B. This precedence could be caused by use of the same resource. A second cause of precedence could be activity A is on a parallel path and converges with the second path at activity B. The dotted line of activity A represents the range of times when the activity will finish. A's location on the common time scale is determined by its own duration and the durations of the activities preceding it. The solid line for B represents its range of expected starting times relative to the time scale. B is solely determined by the durations of the activities preceding activity B. The bold line C represents the potential for activity A to delay activity B from starting as soon as possible. Starting as soon as possible is a requirement if calculations using expected values are to be accurate, which is the case for activity begins-on-completion. If activities on the critical chain cannot start as early as possible, then the expected mean duration for the network cannot be accurately calculated by summing the expected durations of activities on the critical chain.

This problem is as follows: given a probability distribution of the completion time for a non-critical path, when will it impact on the critical chain at the converging activity and thus delay the project? Part of the solution is to take into account the variability along both paths when sizing the convergence buffers.

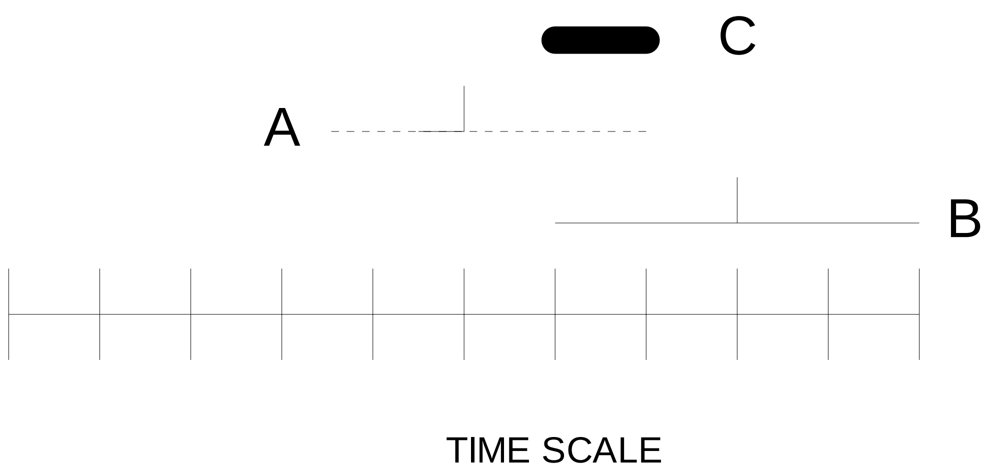


Figure 3.12: Impact of one activity on another on a parallel path.

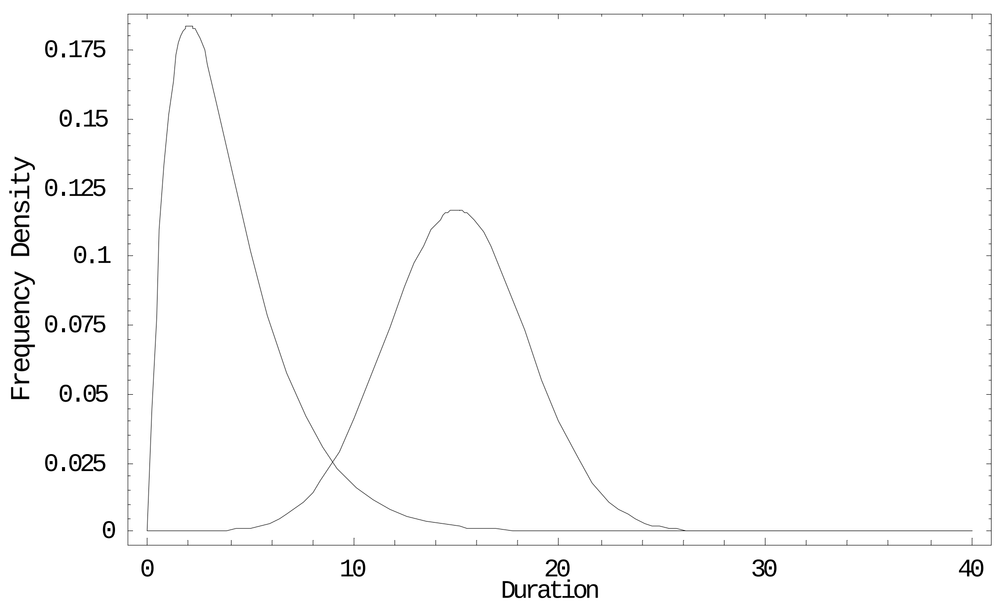


Figure 3.13: Relationship between the completion times of two parallel paths, one starting-on-date and the other starting-on-completion.

The probability distributions for the duration of single path networks using activity-begins-on-date were found to be substantially different from activity-begins-on-completion. The differences in these distributions are important because the actual delay or interference between two activities on parallel paths is related to the amount of overlap in their two distributions as measured on the same time scale. (see Figure 3.13)

Findings for the pilot study

- There is no one best distribution to model activities for activity-begins-on-date. All distributions work to some degree.
- There is no one best distribution to model path durations for activity-begins-on-date.
- Aquilano et al. (1995) proposed a very workable method of setting the completion buffer size using the normal distribution.
- The use of the starting rule, activity-begins-on-date, creates truncated activity times on the non-critical path.
    - A good deal of research has been done with the truncated normal distribution and is available in the literature.
    - Nothing was found in the literature about the truncated beta distribution and parameter estimation for the gamma and log-normal was described as "presents considerable difficulty" p. 241, Johnson et al.(1994), "…technical problems become formidable." 380, Johnson et al. (1994)
- All distributions appear to be workable for activity-begins-on-completion.
- The normal distribution is the best known of the four distributions. There are tables readily available and the parameters are easy to estimate. The normal distribution will be used to model the activity times for all networks used in this study.

- The gamma, beta, and log-normal distributions are less well known. Estimating the parameters is more of a challenge. There are few if any tables of probabilities available so the probabilities would have to be calculated which would make them difficult for practitioners to use.
- When sizing buffers for resource and convergence, it will be necessary to take into account the variability of both networks (critical chain and non-critical path).

Research Methodology

Goldratt/Newbold approach to sizing buffers

Goldratt (1997) and Newbold (1998) use the same approach to buffer sizing. They both take the initial estimates of the activity time (assumed to be pessimistic and 90% probable of completing on time) and cut them in half to achieve what would be considered a "most likely" or average time estimate for the activities. These estimates would be expected to occur 50% of the time. The half of the original estimates is then summed and half of that sum is used for the buffer on that set of activities, whether critical chain, converging path, or buffer between resources on different paths. Leach (2000) uses that difference between 90% and 50% as one standard deviation. Newbold (1998) says that it is more like 2 standard deviations. If you use the standard normal tables, the difference between 50% and 90% is 1.28 standard deviations.

Rizzo/Leach approach to sizing buffers

Based on the calculations, Leach (2000) and Rizzo (2004) treated the difference between the most likely (50%) and the pessimistic (90%) as one standard deviation. In order to calculate the buffer, the standard deviation for each activity on the path would be squared. These values would be summed and the square root taken to develop the standard deviation of the entire path. Once the standard deviation of the path has been calculated, both Leach (2000) and Rizzo (2004) are unclear about how many standard deviations are needed for the buffer. In order to be able to compare the Goldratt/Newbold approach and the Rizzo/Leach approach for setting buffer sizes to the approach developed in this research effort, the same number of standard deviations (2.5) will be used for all three to set buffers.

Wray method of sizing buffers

Introduction

Practitioners need to be able to set buffer sizes based on the characteristics of the project networks. It is not practical to always run simulations. The variability of both paths will play a part in setting the buffer sizes. This means that we must be able to develop reasonably accurate estimates of the variability at the end of non-critical paths and at convergence points on the critical chain. These estimates must be compared against the observed results from the simulations in order to determine the accuracy of any technique.

Because we are modeling the activity times with the normal distribution, the first effort was to compare the observed values to the sum of the means (Figure 3.14) and the sum of the variances (Figure 3.15) of each activity. In Figure 3.14, we can see that the sum of the activity means is a good estimator of the observed mean for the duration of the path. The same approach to estimating the observed variance for the path (Figure 3.15) proved to be an inaccurate estimator of the variance of the path duration.

Summing the means of each activity on the non-critical path, is a good approximation of the observed mean for the path duration. This can be seen in Figure 3.14. Comparison of the sum of the activity variances to the observed path variance, shown in Figure 3.15, indicates that this approach does not give good results.

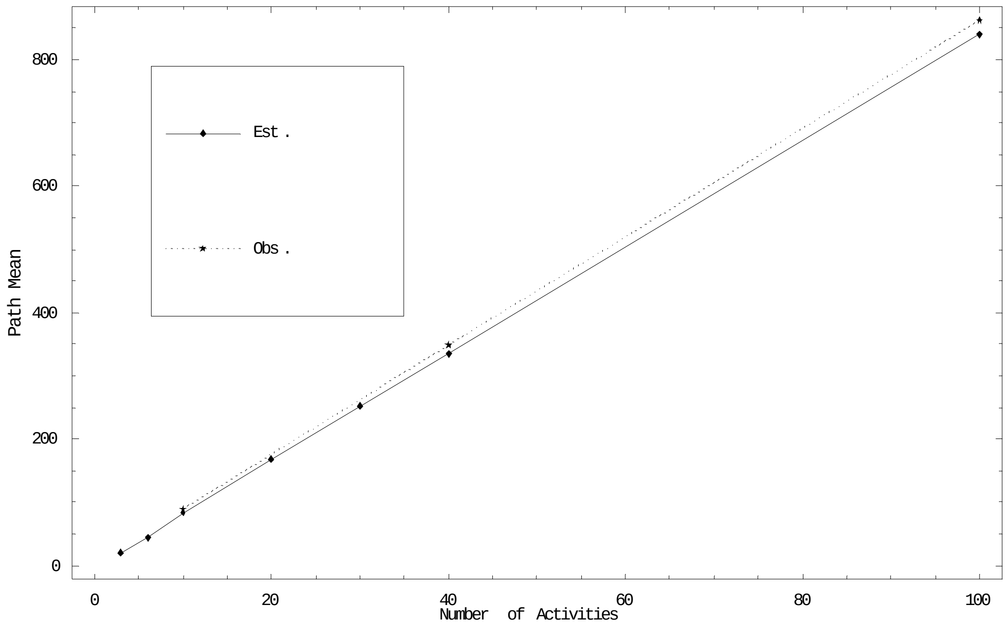


Figure 3.14: Comparison of observed path mean on non-critical path with sum of the mean activity times for each activity on the path.

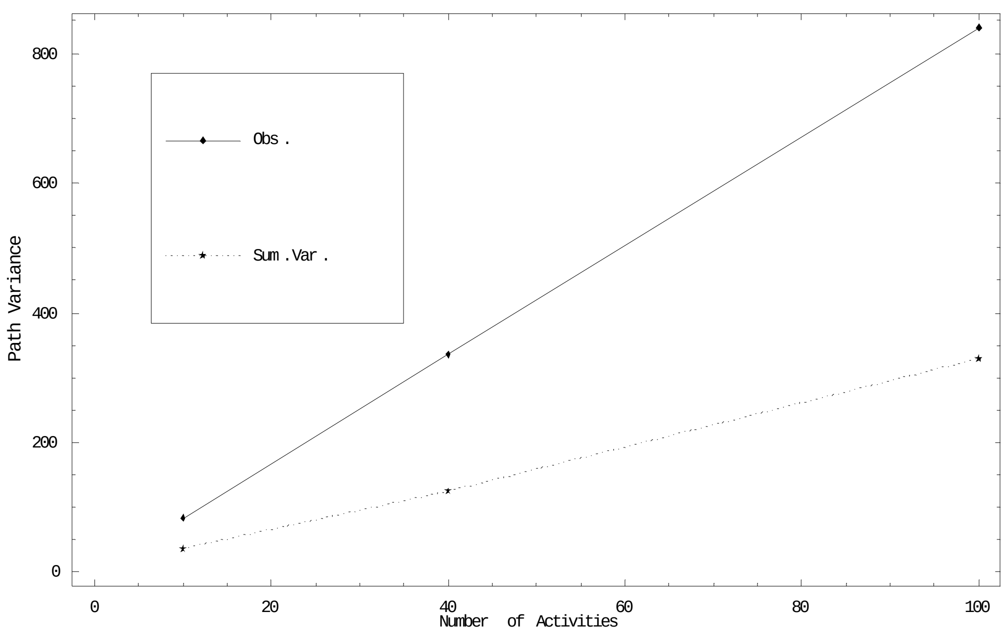


Figure 3.15: Comparison of observed path variance on non-critical path with sum of the variances of the activity times for each activity on the path.

Nothing was found in the literature about the truncated normal distribution having additive properties which means that adding multiple truncated random variables would result in a variable that was distributed in some specific manner and had parameters that were the result of summing the constituent parameters. Lacking clear direction and hoping that the theoretical relationship between the normal distribution and the truncated normal distribution, this researcher looked at the results of summing truncated normal random variables. The Cohen values are normal variable parameters transformed into truncated normal random variables using the formulas developed by Cohen (1991) and presented below. Estimation of Cohen values can be found in appendix B.

The probability density function for the truncated normal distribution is as follows: From Cohen (1991), the pdf for a single truncated distribution, left side only, is given by the following formula:

$$f_T(x : \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}\ (1 - F(T))} \operatorname{Exp}\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right), \quad T \le x < \infty,$$

$$= 0 \text{ elsewhere}$$

Where: x is the random variable,
μ is the mean or expected value of the normal distribution,
σ is the standard deviation of the normal distribution.
T is the point of truncation on the left side of the equation.

and

$$V(X_T) = \mu_2' - (\mu_1')^2 = \sigma^2 (1 - Q(Q - \xi))$$

and

$$E(X_T - T) = \mu_1' = \sigma(Q - \xi)$$

Where X_T designates the truncated random variable

Table 3.3: Comparison of observed mean and variance to means and variances calculated using values derived using formula from Cohen (1991). Does using Cohen values work?

A	B	C	D	E	F
Box & std	variable	Observed	Cohen Calculated	Difference (D-C)	% diff. (E/C)
2 box	xbar	20.39544	20.7979	0.40246	1.973284224
sigma = 1	var.	1.2522	1.3634	0.1112	8.880370548
2 box	xbar	21.1862	22.3927	1.2065	5.694744692
sigma = 3	var.	11.2693	12.2704	1.0011	8.883426655
2 box	xbar	21.9906	23.9894	1.9988	9.089338172
sigma = 5	var.	30.9871	34.0845	3.0974	9.995772434
2 box	xbar	23.4442	26.3831	2.9389	12.53572312
sigma = 8	var.	72.7554	87.2563	14.5009	19.93102917
4 box	xbar	40.8922	42.3937	1.5015	3.671849399
sigma = 1	var.	2.0127	2.0902	0.0775	3.850549014
4 box	xbar	42.6785	47.1781	4.4996	10.54301346
sigma = 3	var.	18.075	18.8112	0.7362	4.073029046
4 box	xbar	44.5132	51.9382	7.425	16.68044535
sigma = 5	var.	49.3525	52.2535	2.901	5.878121676
4 box	xbar	47.7239	59.1493	11.4254	23.94062514
sigma = 8	var.	116.465	133.7689	17.3039	14.8575967
6 box	xbar	61.4313	63.9895	2.5582	4.164326654
sigma = 1	var.	2.8812	2.817	-0.0642	2.228238234
6 box	xbar	64.2939	71.9635	7.6696	11.92896993
sigma = 3	var.	25.9307	25.352	-0.5787	2.231717617
6 box	xbar	67.201	79.947	12.746	18.96697966
sigma = 5	var.	71.0313	70.4225	-0.6088	0.857086946
6 box	xbar	72.008	91.9155	19.9075	27.64623375
sigma = 8	var.	175.034	180.2815	5.2475	2.997988962

Table 3.3 (continued): Comparison of observed mean and variance to means and variances calculated using values derived using formula from Cohen (1991).

A	B	C	D	E	F
Box & std	variable	Observed	Cohen Calculated	Difference (D-C)	% diff. (E/C)
8 box	xbar	81.7212	85.5853	3.8641	4.728393611
sigma = 1	var.	3.6594	3.6538	-0.0056	0.153030551
8 box	xbar	85.1637	96.7489	11.5852	13.60344842
sigma = 3	var.	32.9344	31.8928	-1.0416	3.162650602
8 box	xbar	88.6475	107.9258	19.2783	21.74714459
sigma = 5	var.	90.9117	88.5915	-2.3202	2.552146753
8 box	xbar	94.5658	124.6815	30.1157	31.84629115
sigma = 8	var.	226.243	226.7941	0.5511	0.243587647

Although there was nothing found in the literature to indicate that the variances of truncated normal variables could be summed to obtain the variance of the path duration, it was tried to see if some relationship existed. The results of this effort are contained in Table 3.3 where the sums of truncated normal means and variances are compared to the observed means and variances of non-critical paths. The last column (F) in Table 3.3 indicates the degree of fit for both variables as compared to the observed values. The mean value (xbar) is not as good as summing the means of the normal activity times. The variance of the path developed using the sum of the Cohen values provides a good estimate of the path variance. A graphic comparison of the means (Figure 3.16) and the variances (Figure 3.17) support these findings.

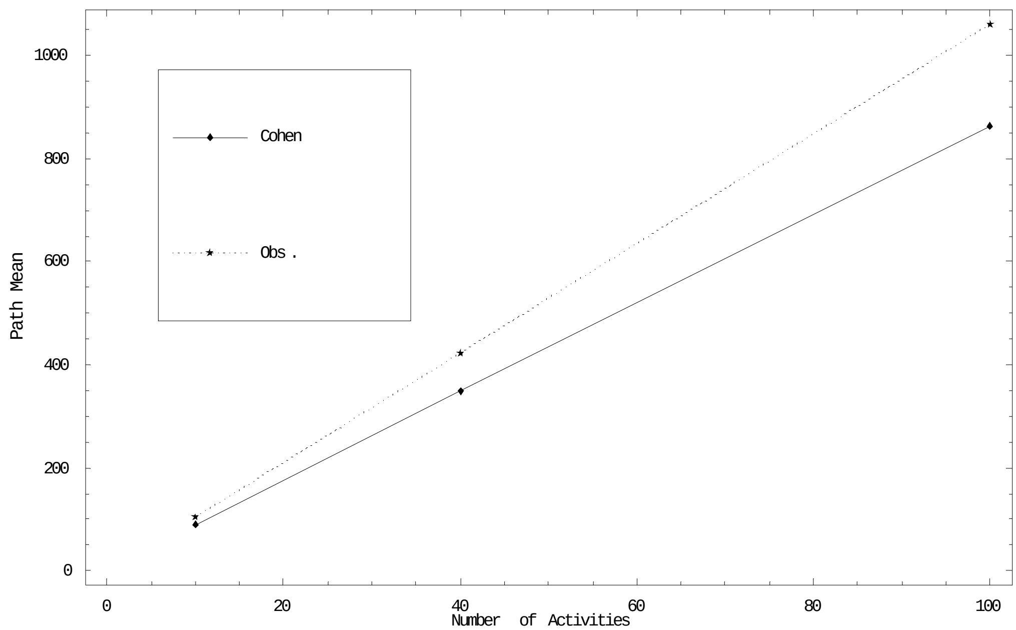


Figure 3.16: Comparison of observed path means to sums of activity mean times converted to truncated normal means using Cohen's formulas.

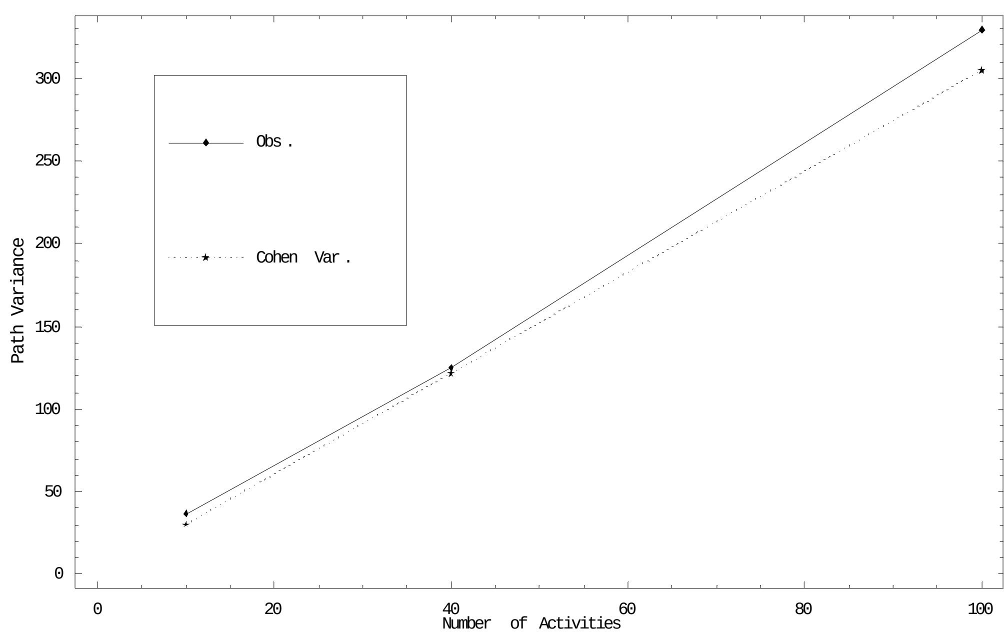


Figure 3.17: Comparison of observed path variances to sums of activity variances converted to truncated normal variances using Cohen's formulas.

Wray method of calculating the sizes of completion, convergence, and resource buffers:

- Completion buffers will be sized using the approach described in Aquilano et al. (1995).
- Convergence and resource buffers will be sized using the following approach:
    - Estimate the mean duration at the convergence point on the critical chain by summing the means of the activities prior to that point.
    - Estimate the variance at the convergence point on the critical chain by summing the variance of each activity prior to that point.
    - Sum the mean of each activity on the non-critical path that is converging.
    - Calculate Cohen variances for each activity on the converging non-critical path and sum these to arrive at a variance for that path.
    - Take the square root of both variances to get the standard deviations. Add these standard deviations to arrive at one value for the variability at the convergence point.
    - Multiply this value for the variability by the factor that you have chosen from Table 3.4 (choice of factor depends on how much risk you will accept) to arrive at the size of the buffer that will protect the critical chain from this path.
    - Use the mean value from the convergence point on the critical chain, the duration of the converging non-critical path, and the buffer calculated in the previous step to determine the start time for the non-critical path.

The information contained in Table 3.4 was developed using a Monte Carlo simulation of two paths with a buffer of different sizes. The buffer was sized using the sum of the standard deviations for both paths times the factor. The converging path was modeled using the actual duration times for a simulation run. Each critical chain was modeled using a random number generator for specific mean and variance. An ending duration was generated for each path and then compared to each other. If the converging path duration was greater than the value for the critical chain, the comparison was scored as a delay for the project and a failure of the buffer to protect the critical chain. If the converging path value was less than the critical chain, then the buffer had protected the critical chain and it was scored as a success. The probability values in the body of Table 3.4 represent the ratio of failures to total comparisons for each buffer. See appendix D for the Mathematica programs used to create the table.

Table 3.4 contains the values for the factors that are used to siz the convergence and resource contention buffers. Once the estimates of the standard deviation of the critical chain up to the point of convergence and the standard deviation of the converging non-critical path have been added to obtained the combined standard deviation, it is multiplied by the chosen factor to get the buffer size. The selection of the factor depends on the size of the combined standard deviations and the protection desired for the critical chain. The factor of 2.5 was used for all of the experiments in this study so that the protection of the critical chain was very close to 100%. Thus the critical chain should not be impacted by the convergence. This made it possible to test the effects of the converging path against the critical chain, without converging paths.

Table 3.4: The results of calculations of the probability of a non-critical path converging with the critical chain late and delaying the critical chain.

	Probability of Two Paths Contending		
Factor	Combined Std. Dev. 9.23	Combined Std. Dev 14.23	Combined Std. Dev 22.59
0.0	.484	.498	.536
0.1	.422	.442	.482
0.2	.322	.372	.420
0.3	.280	.350	.372
0.4	.322	.310	.284
0.5	.286	.262	.280
0.6	.218	.220	.252
0.7	.162	.196	.192
0.8	.142	.130	.158
0.9	.088	.150	.138
1.0	.080	.102	.104
1.1	.088	.094	.090
1.2	.044	.062	.082
1.3	.022	.058	.058
1.4	.018	.034	.044
1.5	.028	.032	.030
1.6	.010	.006	.024
1.7	.016	.020	.024
1.8	.008	.024	.024
1.9	.008	.004	.008
2.0	.004	.002	.018
2.1	.002	0	.010
2.2	0	.004	.002

Table 3.4 (continued): The results of calculations of the probability of a non-critical path converging with the critical chain late and delaying the critical chain.

	Probability of Two Paths Contending		
Factor	Combined Std. Dev. 9.23	Combined Std. Dev. 14.23	Combined Std. Dev. 22.59
2.3	0	.004	.006
2.4	0	0	.006
2.5	0	0	.004
2.6	0	.002	.006
2.7	0	0	.002
2.8	0	0	0
2.9	0	0	0
3.0	0	0	0

Research Questions

This section presents the research questions to be answered in this study. A model is presented to describe the hypothesized network characteristics that have the greatest impact on the variability of the traditional method of estimating expected project duration. Definitions of variables and important terms are presented.

The conventional method of calculating the expected duration of a project network uses the sum of the expected duration of each activity on the critical path. In using this method, a project manager ignores statistical fluctuation associated with each activity, in order to simplify calculations. In addition, many project managers calculate project duration with the assumption that activities begin on completion of prior activities and then manage the project using activity-begins-on-date. The result is an expected project duration that is often wrong. The model in Figure 3.18 indicates the sources of this variability in project durations. Additional explanation is provided in the following paragraphs.

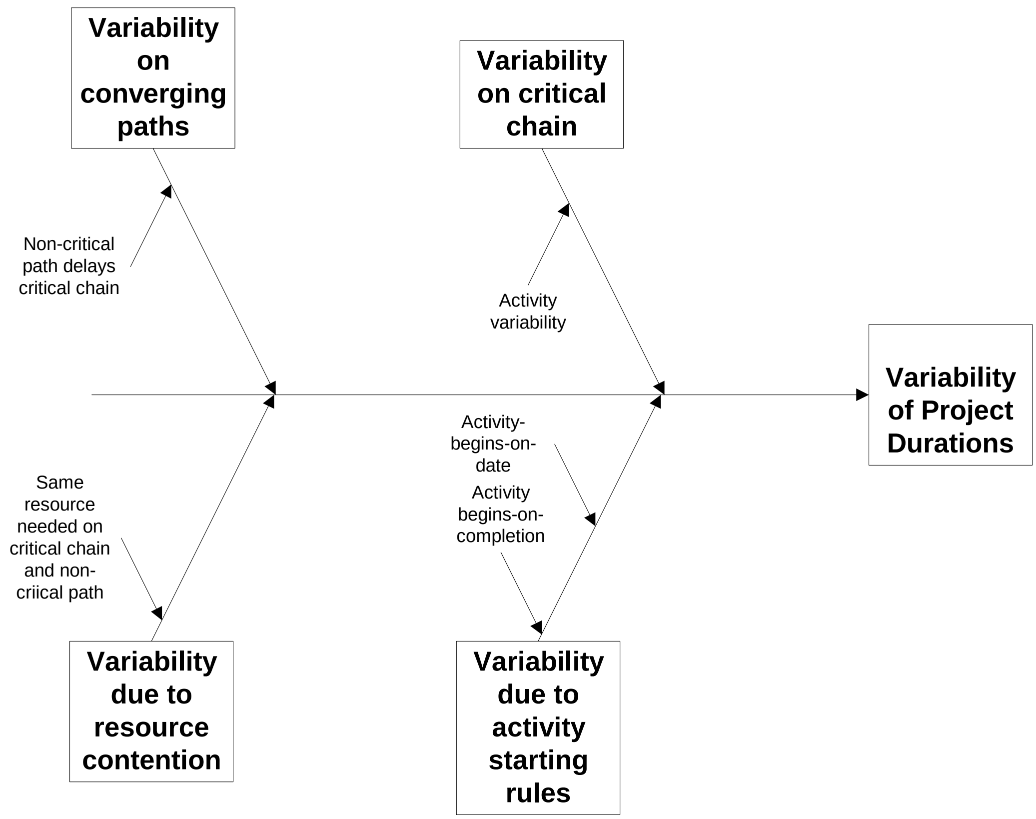


Figure 3.18: Model of the causes of variability of project durations

Model

1. Variability on the critical chain is due to the natural variability of the activities on that critical chain. The conventional solution is to insert a completion buffer that is large enough to protect the promised date of the project.

2. The variability of activities on converging paths can cause the path to finish late and delay the critical chain. The conventional solution is to add protection to each activity.

3. Resource contention between the critical chain and another non-critical parallel path can cause the critical chain to be delayed. The conventional solution has been to ignore it.

4. Not understanding the effects of the starting rules activity-begins-on-date and activity-begins-on-completion can create unreasonable schedules. The conventional solution has been to ignore it because the phenomena was not understood.

The following restrictions will be used in this study to include the recommendations made by Pittman to improve the methodology for scheduling and running projects:

a. Activities on the critical chain are started upon completion of the preceding activities and not by schedule date.

b. Activities on the non-critical chains are started on their specified start date unless they have run over their expected completion time. At that point the activity will begin upon completion of the preceding activity. Thus the completion time will normally not be less than the expected completion and could be more. This method consumes convergence buffer but is deemed to be a easier way to exert control over the project.

c. Activities on the network will not be rescheduled when the critical chain becomes behind schedule.

d. Activity durations for each activity will be estimated without bias and will not include any additional time to compensate for variability.

Operationalization of the Variables

1. Critical chain: "In the theory of constraints, the longest route through a project network considering both technological precedence and resource contention constraints

in completing the project. Where no resource contention exists the critical chain would be the same as the critical path." (APICS Dictionary 11th ed., p. 24)

2. Non-critical path: any path in a project network that is not a critical chain or critical path.

3. Buffers: "In the heory of constraints, buffers can be ime or material and support thoughput and/or due date performance" (APICS Dictionary, 11th ed., p. 12)

Description of the Simulation Networks Used in the Tests

Introduction

This section describes the type of structure that the networks used in the tests would have. There are two basic types of paths described and used in the testing. First there is the critical chain for each network. The 120 box network shown in Figure 3.19 is representative of that critical chain. The basic structure of the non-critical path is shown in Figure 3.20 where there is an additional path added to the simulation to account for the schedule date when the activities are to begin. These two figures give the basics from which all of the networks are constructed.

The activity times for all activities on these simulations were developed the same way. The values for the activity means and standard deviations were generated randomly. The mean values were generated from a uniform distribution with acceptable values being between 3 and 20. The standard deviations were generated the same way with the accepted values being no more than one third of the mean for each activity. The actual values for the different networks are contained in the Appendices and are referenced in the following text.

The remaining figures give the reader an understanding of how the networks are constructed from these building blocks described in the preceding paragraph. Some of the tests are run with just the critical chain to test hypothesis about the completion buffer and those networks would be like Figure 3.21. Tests involving single convergence of a non-critical path would look like Figure 3.22. A test involving multiple converging paths would look like Figure 3.23. Figure 3.24 shows what resource contention would look like prior to insertion of a resource buffer. The insertion of the resource buffer and the network used to test its effectiveness is shown in Figure 3.25. Additional information about the actual networks used in the tests is provided in the description attached to each figure.

Examples of the Types of Networks Used in the Tests

The 120 box network shown in Figure 3.19 exemplifies the model of the critical chain where ever one is used in the tests. The network runs activity-begins-on-completion of the preceding activity. The expected mean of the network calculated by summing the activity times of the 120 activities is 1426. The expected variance is 926 which is calculated by summing the variance of each activity in the network. A complete listing of these values for each activity is contained in Appendix .

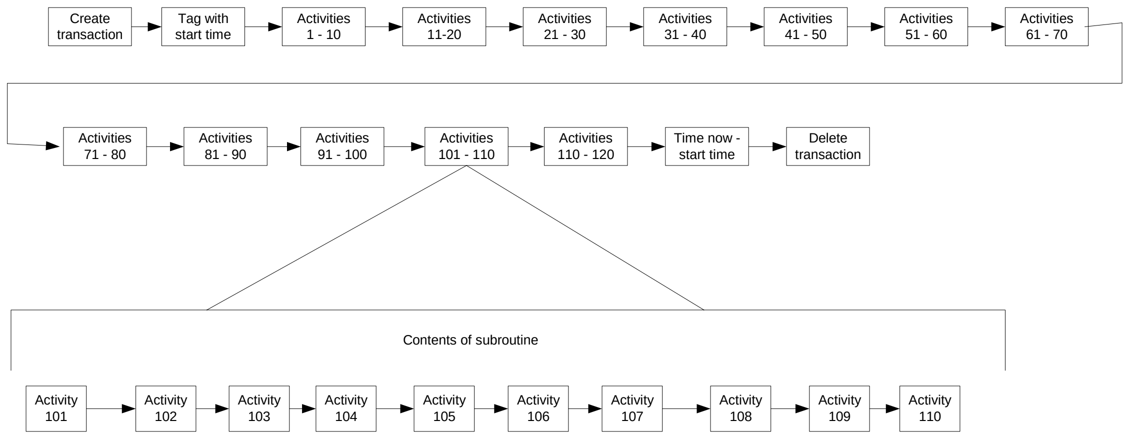


Figure 3.19: 120 box network used to model the critical chain and runs activity-begins-on-completion.

The 120 box network shown in Figure 3.19 was constructed in Arena using subroutines. Each subroutine consists of 10 activities. The contents of the subroutine for activities 101 to 110 is shown in the figure. It is at the bottom of the figure and is labeled "contents of subroutine". All of the networks basically run the same way. The first box creates the transaction. The second box time stamps each transaction with its creation time. The activities run using their normal distributed activity times and standard deviations. The next to last box calculates the elapsed time for each transaction by taking the "time now" and subtracting the creation time to get the elapsed time. The elapsed time is written to a file. The last box destroys the transaction.

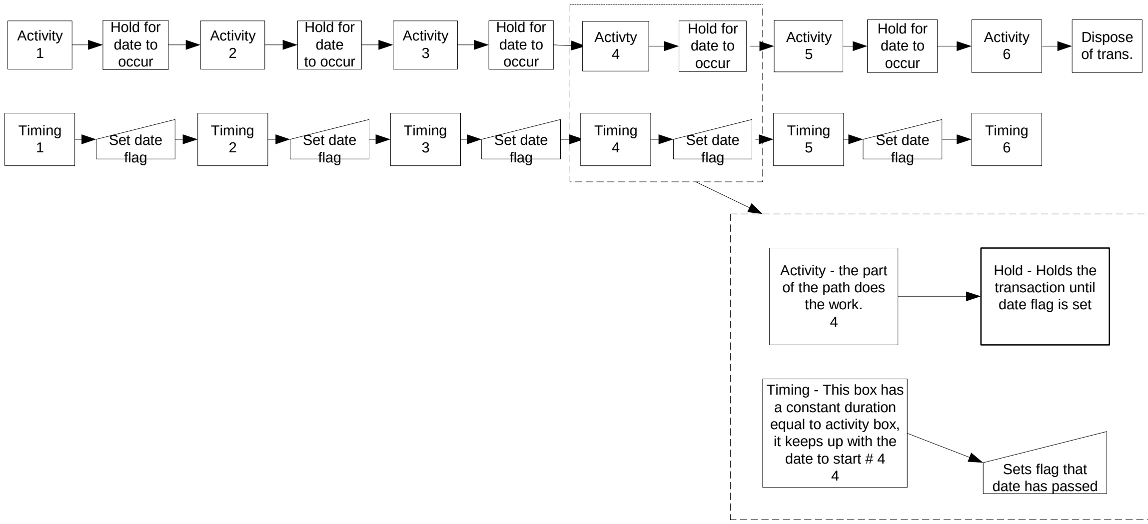


Figure 3.20: General structure of simulation models for all non-critical paths run activity-begins-on-date.

The general network shown in Figure 3.20 is the pattern used to simulate all non-critical paths. There are actually two parts to each of these non-critical "paths". The top line of activities in the figure represent the boxes that perform "work" or delay the project by some set amount of time. These boxes are labeled "activity" and "hold". Activity 4 represents work time needed to do something to the project. The second row of boxes labeled "timing" and "assign" keep track of the scheduled times for each of the "activities" on the upper path to begin. Each of the timing boxes have a constant value for processing equal to the mean of the associated activity box. Since this mean value of the activity box is what was used to create the schedule, the timing box 4 delays for that amount, the "assign" box sets a flag that the "date" of the next activity is right for it to begin and when activity 4 completes, activity 5 can begin. This arrangement is the reason why the normally distributed activity times result in truncated normal activity times and require the Cohen values to calculate the path variances.

There are three non-critical paths used in the testing. The activity times, standard deviations, variances, Cohen values, and the associated measures of variation for

Goldratt/Newbold and Leach/Rizzo are contained in the Appendices. All of the networks (120 box, 30 box, 20, box, 6 box, and 3 box) paths are fully described in Appendix A,

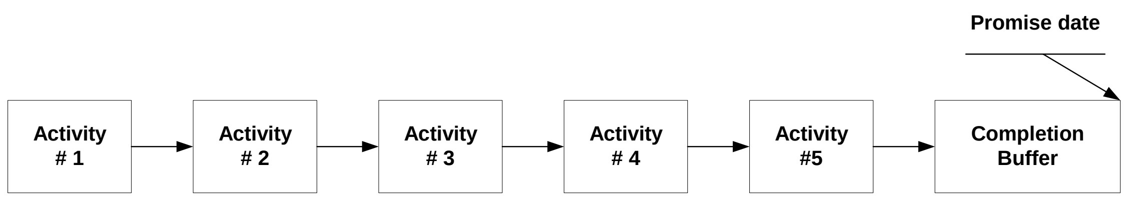


Figure 3.21: Location of the completion buffer at the end of the critical chain.

In Figure 3.21, the completion buffer is located at the end of the critical chain. It compensates for variability on the critical path and protects the project completion from that variability. If the convergence buffers are properly sized, then the completion buffer will only have to contend with that variability on the critical chain and it is possible to size the completion buffer to provide a set level of safety.

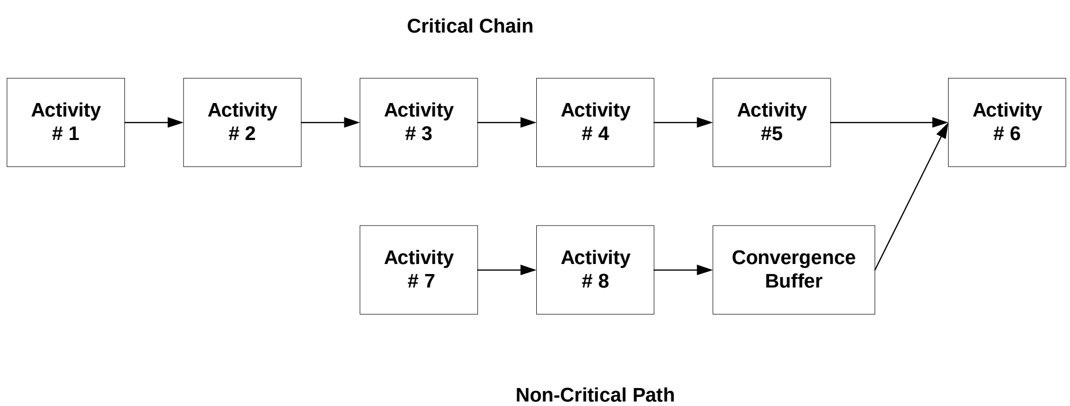


Figure 3.22: Example of a convergence of a single non-critical path with the critical chain.

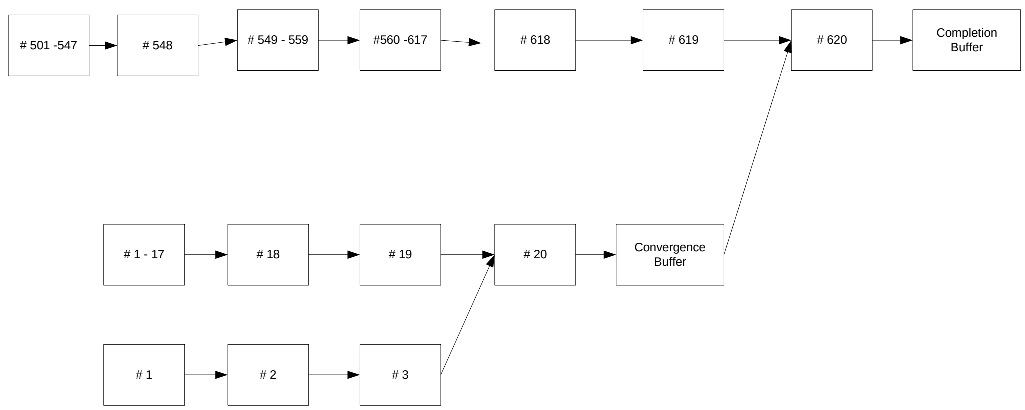


Figure 3.23: An example of a network with multiple converging non-critical paths.

Figures 3.22 and 3.23 present examples of convergence of non-critical paths with the critical chain. Figure 3.22 represents the case where one non-critical path intersects with the critical chain. This arrangement is the most prevalent in the testing. The multiple convergence example in Figure 3.23 is only used once in the testing.

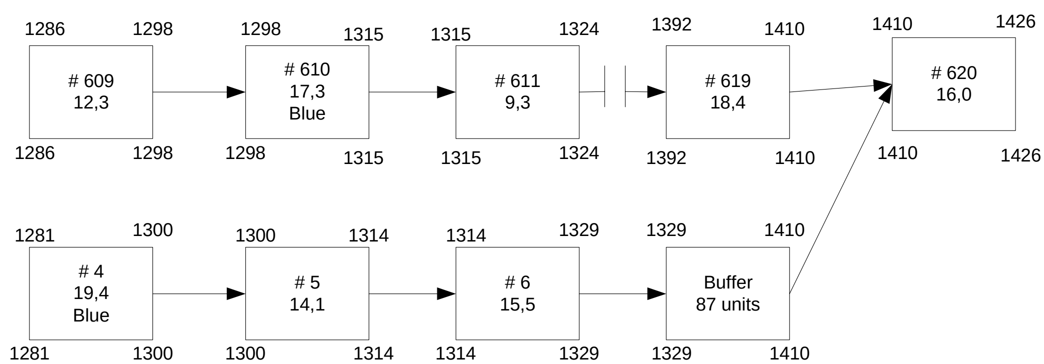


Figure 3.24: An example of resource contention between activities #610 and #4 for the blue resource.

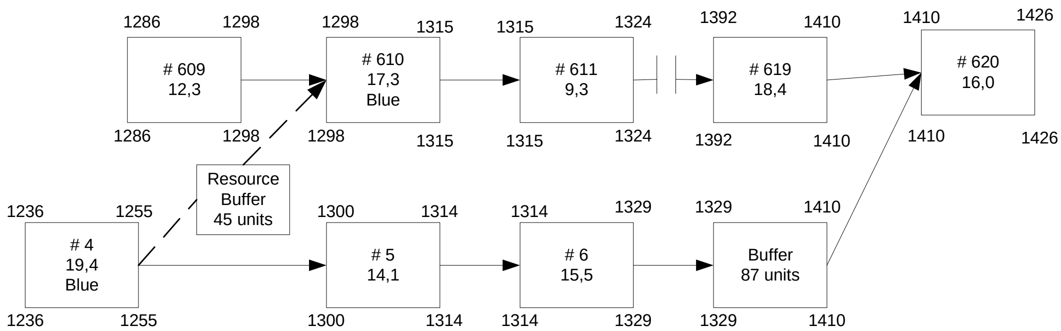


Figure 3.25: An example of use of a resource buffer to solve the resource contention problem found in Figure 3.24.

Figures 3.24 and 3.25 are both dealing with resource contention and resource buffers. Figure 3.24 shows the situation where resource contention has been identified. Activity #610 on the critical chain needs the "blue" resource at the same time as activity #4 on the non-critical path. In order to resolve this resource contention, activity #4 is moved into an earlier time period with a resource buffer inserted between the two scheduled uses of the "blue" resource. There is only one test case that deals with this type of situation.

Research Questions and Hypotheses

Research Q1: Is the completion buffer sized using the approach described in Aquilano et al. (1995) capable of protecting the expected project duration at a predetermined success probability of 95%. No converging paths are involved. (Success is defined as a project length being less than the buffered value).

Comparison: Run the 120 box critical chain without any converging paths or resource contention. Calculate the completion buffer using Aquilano et al. (1995) approach. Using the computer listing of the sorted durations for the 1000 runs, visually identify how many runs exceeded the completion buffer. Run a p-test for significance.

H(1): H_0: There is no statistically significant difference between the proportion of failures observed and the predetermined proportion of 0.05 using the approach described in Aquilano et al. (1995).

H_1: The observed proportion of failures is not equal to 0.05.

Research Q2: Do the techniques for completion buffer sizing presented by Goldratt/Newbold and Leach/Rizzo create completion buffers that protect the project duration at the 90% probability? The value of 90% comes from Goldratt discussion about activity durations. (success is defined as a project length being less than the buffered value). There are no converging paths involved.

Comparison: Run the 120 box critical chain without any converging paths or resource contention. Calculate the completion buffer using the Goldratt/Newbold approach and the Leach/Rizzo approach. Using the computer listing of the sorted durations for the 1000 runs, visually identify how many runs exceeded the completion buffer. Run a p-test for significance.

H(2): H_0: There is no statistically significant difference between the proportion of failures observed using the completion buffers sized by the Goldratt/Newbold method and the predetermined proportion of 0.10.

H_1: The proportion of failures observed using the completion buffers sized using the Goldratt/Newbold method is not statistically equal to 0.10.

H(3): H_0: There is no statistically significant difference between the proportion of failures observed using the completion buffers sized by the Leach/Rizzo method and the predetermined proportion of 0.10.

H_1: The proportion of failures observed, using the completion buffers sized using the Leach/Rizzo method, is not statistically equal to 0.10.

Research Q3: Can a technique be developed for sizing convergence buffers, that provides approximately 100% protection to the critical chain, and does it without wasted buffer?

Comparison: The convergence buffer is to protect the critical chain from encroachment by the non-critical path. Compare the results of the network with a convergence buffer to the critical chain without any convergence. If the buffer protected the critical chain, there should not be any difference between the critical chain alone and the critical chain with convergence. If the test run fails, then the convergence buffer is too small.

H(4): H_0: When sizing the convergence buffer, with the Wray method, for approximately 100% protection of the critical chain, there is no statistically significant impact on the critical chain completion times.

H_1: The convergence buffer sized, with the Wray method, for pproximately 100% protection of the critical chain fails to prevent statistically significant impact on the critical chain completion times.

H(5): H_0: When using the Wray method to size the convergence buffer, for approximately 90% protection of the critical chain, there is no statistically significant impact on the critical chain completion times.

H_1: The convergence buffer sized, with the Wray method, for approximately 90% protection of the critical chain fails to prevent statistically significant impact on the critical chain completion times.

Research Q4: Do Goldratt/Newbold and Leach/Rizzo methods protect project completion times, by properly sizing convergence and completion buffers for 100%, work as well as the Wray method.

Comparison; The test runs are compared to a test run with the same structure. The comparison test run has a convergence buffer sized using the Wray method. The mean are compared to the means of test runs with buffers sized using Goldratt/Newbold and Leach/Rizzo methods.

H(6): H_0: The use of convergence and completion buffers sized by the Goldratt/Newbold method and the Wray method protect (at 100%) the project completion times statistically the same.

H_1: The Wray method and Goldratt/Newbold methods for sizing the convergence and completion buffers do not provide statistically equal protection to the project duration.

H(7): H_0: The use of convergence and completion buffers sized by the Leach/Rizzo method and the Wray method protect (at 100%) the project completion times statistically the same.

H_1: The Wray method and Leach/Rizzo methods for sizing the convergence and completion buffers do not provide statistically equal protection to the project duration.

Research Q5: Does the Wray, method for sizing convergence buffers work for sizing resource buffers?

Comparison: Run a simulation with the resource buffer and compare the results to a run of the 120 box critical chain without convergence or resource contention.

H(8): H_0: The Wray method properly sizes the resource buffer such that the critical chain and project completion are not statistically impacted by the contention of resources.

H_1: The Wray method does not properly size the resource buffer such that The critical chain and project completion are not statistically impacted by the contention of resources

Research Q6. Does the Wray, method for sizing convergence buffers protect the critical chain for two path convergences at the same point on the critical chain?

Comparison: The convergence buffer is to protect the critical chain from encroachment by the non-critical path. Compare the results of the network with a convergence buffer to the critical chain without any convergence. If the buffer protected the critical chain, there should not be any difference between the critical chain alone and the critical chain with convergence. If the test run fails, then the convergence buffer is too small.

H(9): H_0: The Wray method for sizing the convergence buffer (100%), for two path convergence at the same point, protects the project completion time as compared to the completion time of the critical chain without convergence.

H_1: The Wray method for sizing the convergence buffer (100), for two path convergences at the same point, does not protect the completion time as compared to the completion time of the critical chain without convergence.

Research Q7: In the case of insufficient space for convergence buffers calculated using the Wray method, does the Wray method of adding the buffer shortfall to the completion buffer, compensating for the shortfall, protect the completion time of the project?

Comparison: This is determined by statistically comparing the project completion time, with convergence, against the critical chain completion time, without any convergence.

H(10): H_0: The Wray method provides effective protection to the project completion time in the case of an insufficient space for a convergence buffer by adding the shortfall in the convergence buffer to the completion buffer.

H_1: The Wray method for compensating for undersized buffers does not protect the project completion time.

CHAPTER IV

RESEARCH METHODS

Introduction

In this chapter, the experiments described in chapter III are performed and the information reported. After this introduction to the chapter, there are two more parts to the chapter. The first part briefly discusses the statistical methods that are used for analysis of the experiments. The second part details the questions, hypothesis, and the results of the experiments. The conclusions drawn from these experiments are discussed in chapter V.

Statistics Used in Experiments

Two statistical tests were selected for analysis of the test results. Wilcoxon Rank Sum Test was used because it is non-parametric and thus does not require the sample distributions to both be normally distributed. This test was implemented using the JMP In statistical analysis package from SAS Institute. The P-test as described in Neter, Wasserman, and Whitmore (1988) was selected for analysis of whether a proportion was the same as a target proportion. This test was programmed into an Excel spreadsheet. The confidence and sample size was the same for all tests and were set at, alpha = 0.05, N = 1000.

Experiments

Research Q1: Is the completion buffer sized using the approach described in Aquilano et al. (1995) capable of protecting the expected project duration at a predetermined success probability of 95%. No converging paths are involved. (success is defined as a project length being less than the buffered value).

Comparison: Run the 120 box critical chain without any converging paths or resource contention. Calculate the completion buffer using Aquilano et al. (1995) approach. Using the computer listing of the sorted durations for the 1000 runs, visually identify how many runs exceeded the completion buffer. Run a p-test for significance.

H(1): H_0: There is no statistically significant difference between the proportion of failures observed and the predetermined proportion of 0.05 using the approach described in Aquilano et al. (1995).

H_1: The observed proportion of failures is not equal to 0.05.

Results: The completion buffer was calculated to be 1426 + 49 =1475.38 using Aquilano et al.'s approach and using this value and the sorted completion times, 64 values were found to be above the buffered completion times. The mean of the 120 box critical chain was 1427.36 and the variance was 955.563. The expected mean was calculated to be 1426 and the expected variance was calculated to be 926. The decision rule for accepting H_0 was $-1.96 \leq$ test statistic ≤ 1.96 and the test statistic was 1.81. Conclusion, do not reject H_0.

Research Q2: Do the techniques for completion buffer sizing presented by Goldratt/Newbold and Leach/Rizzo create completion buffers that protect the project duration at the 90% probability? The value of 90% comes from Goldratt's discussion about activity durations. (success is defined as a project length being less than the buffered value). There are no converging paths involved.

Comparison: Run the 120 box critical chain without any converging paths or resource contention. Calculate the completion buffer using the Goldratt/Newbold approach and the Leach/Rizzo approach. Using the computer listing of the sorted durations for the

1000 runs, visually identify how many runs exceeded the completion buffer. Run a p-test for significance.

H(2): H_0: There is no statistically significant difference between the proportion of failures observed using the completion buffers sized by the Goldratt/Newbold method and the predetermined proportion of 0.10.

H_1: The proportion of failures observed using the completion buffers sized by the Goldratt/Newbold method is not statistically equal to 0.10.

Results: The Goldratt/Newbold completion buffer was calculated to be 713 + 1426 = 2139. The 120 box critical chain was run without any convergence. When this was added to the expected mean duration for the critical chain, the buffered duration was 2,139. This number was compared to the 1000 sorted durations created from the test run. The largest test duration was 1530.33. There were no failures using the completion buffer calculated using the Goldratt/Newbold method of sizing the buffer. There was for this test run an excess of 608.67 of time units of protection in the completion buffer. The test statistic did not compute because of division by zero. The p-test failed, reject H_0.

H(3): H_0: There is no statistically significant difference between the proportion of failures observed using the completion buffers sized by the Leach/Rizzo method and the predetermined proportion of 0.10.

H_1: The proportion of failures observed, using the completion buffers sized using the Leach/Rizzo method, is not statistically equal to 0.10.

Results: The Leach/Rizzo completion buffer was calculated to be 1426 + 180 = 1506. The 120 box critical chain was run without any convergence. When this was added to the expected mean duration for the critical chain, the buffered duration was 1,606. This number was compared to the 1000 sorted durations created from the test run. The largest test duration was 1530.33. There were no failures using the completion buffer calculated using Leach/Rizzo method of sizing the buffer. There was for this test run an excess of 75.67 time units of protection in the completion buffer. The test statistic did not compute because of division by zero. The p-test failed, reject H_0.

Research Q3: Can a technique be developed for sizing convergence buffers, that provides approximately 100% protection to the critical chain, and does it without wasted buffer?

Comparison: The convergence buffer is to protect the critical chain from encroachment by the non-critical path. Compare the results of the network with a convergence buffer to the critical chain without any convergence. If the buffer protected the critical chain, there should not be any difference between the critical chain alone and the critical chain with convergence. If the test run fails, then the convergence buffer is too small.

H(4): H_0: When sizing the convergence buffer, with the Wray method, for approximately 100% protection of the critical chain, there is no statistically significant impact on the critical chain completion times.

H_1: The convergence buffer sized, with the Wray method, for approximately 100% protection of the critical chain fails to prevent statistically significant impact on the critical chain completion times.

Results: The convergence buffer was 96 for the 30 box path converging at box 619. There was not any completion buffer in this test. The test simulation was run with a 30 box non-critical path converging with the 120 box critical chain at the last box of the critical chain. The convergence buffer was sized using a factor = 2.5 (see development of

Wray method in chapter 3) which was expected to provide close to 100% protection of the critical chain. The results were compared to the test run for the 120 box critical chain, without convergence. The means were compared using the Wilcoxon Rank Sum Test to determine if they are statistically equal. The mean value for the critical chain alone was 1427.36 compared to the expected mean of 1426. The variance for the critical chain alone was 955.563 as compared to the expected variance of 926 and the observed variance of . The mean and variance of the test run with convergence was 1427.5 and 891.051 respectively. The decision rule is p-value $\leq$ .025, reject H_0. The observed p-value was 0.9632. The decision is not to reject H_0.

H(5): H_0: When using the Wray method to size the convergence buffer, for approximately 90% protection of the critical chain, there is no statistically significant impact on the critical chain completion times.

H_1: The convergence buffer sized, with the Wray method, for approximately 90% protection of the critical chain fails to prevent statistically significant impact on the critical chain completion times.

Results: The convergence buffer was sized at 39 (factor = 1.0) which should give 90% protection. The 100% buffer would be 96 (factor = 2.5). The test simulation was run with a 30 box non-critical path converging with the 120 box critical chain at the last box of the critical chain. The convergence buffer was sized using a factor = 1.0, which is expected to give 90% protection of the critical chain from the convergence. The results were compared to the test run for the 120 box critical chain, without convergence. The means were compared using the Wilcoxon Rank Sum Test to determine if they are statistically equal. The mean value for the critical chain alone was 1427.36 compared to the expected mean of 1426. The observed variance for the critical chain alone was 955.563 as compared to the expected variance of 926. The mean and variance of the test run with convergence was 1432.16 and 483.412 respectively. The decision rule is p-value ≤ .025, reject H_0. The observed p-value was 0.0014. The decision is to reject H_0.

Research Q4: Do Goldratt/Newbold and Leach/Rizzo methods protect project completion times, by properly sizing convergence and completion buffers for 100%, work as well as the Wray method.

Comparison; The test runs are compared to a test run with the same structure. The comparison test run has a convergence buffer sized using the Wray method. The mean are compared to the means of test runs with buffers sized using Goldratt/Newbold and Leach/Rizzo methods.

H(6): H_0: The use of convergence and completion buffers sized by the Goldratt/Newbold method and the Wray method protect (at 100%) the project completion times statistically the same.

H_1: The Wray method and Goldratt/Newbold methods for sizing the convergence and completion buffers do not provide statistically equal protection to the project duration.

Results: The Goldratt/Newbold convergence buffer was sized at 183. No completion buffer was used in the test. The test run with convergence buffers set using the Goldratt/Newbold method were run with a 30 box non-critical path converging with the critical chain at the last activity. This was the same approach as the test for the Wray method. The statistics for the Goldratt/Newbold test run were mean = 1427.46 and variance = 883.037. The mean and variance of the test run with convergence using the

Wray method was 1427.5 and 891.051 respectively. The means of the two runs were compared using the Wilcoxon rank sums test. The decision rule is p-value ≤ .025, reject H_0. The observed p-value was 0.9720. The decision is to not reject H_0.

H(7): H_0: The use of convergence and completion buffers sized by the Leach/Rizzo method and the Wray method protect (at 100%) the project completion times statistically the same.

H_1: The Wray method and Leach/Rizzo methods for sizing the convergence and completion buffers do not provide statistically equal protection to the project duration.

Results: The Leach/Rizzo convergence buffer was sized at 180 using a factor of 2.5. No completion buffer was used in this test. The test run with convergence buffers set using the Leach/Rizzo method were run with a 30 box non-critical path converging with the critical chain at the last activity. This was the same approach as the test for the Wray method. The statistics for the Leach/Rizzo test run were mean = 1427.08 and variance = 874.13. The mean and variance of the test run with convergence using the Wray method was 1427.5 and 891.051 respectively. The means of the two runs were compared using the Wilcoxon rank sums test. The decision rule is p-value ≤ .025, reject H_0. The observed p-value was 0.8143. The decision is to not reject H_0.

Research Q5: Does the Wray, method for sizing convergence buffers work for sizing resource buffers?

Comparison: Run a simulation with the resource buffer and compare the results to a run of the 120 box critical chain without convergence or resource contention.

H(8): H_0: The Wray method properly sizes the resource buffer such that the critical chain and project completion are not statistically impacted by the contention of resources.

H_1: The Wray method does not properly size the resource buffer such that the critical chain and project completion are not statistically impacted by the contention of resources

Results: The resource buffer was sized at 45 after a 87 convergence buffer. This was a six box non-critical path converging with the 120 box critical chain at box 619. No completion buffer was used in this test. A test was run with a 120 box critical chain and a 6 box non-critical path converging with the critical chain at the last activity. After inserting the properly sized convergence buffer, two activities in the network were found to be contending for the same resource. The activity on the non-critical path was moved to an earlier time period and a resource buffer was calculated to protect the use of the same resource on the critical chain. The run was then made to determine if the contention still existed and statistically impacted the project duration. The critical chain without contention has a mean = 1426 and variance = 955.563. The results of the test for buffering resources had a mean = 1427.79 and variance = 908.434. The means of the two runs were compared using the Wilcoxon rank sums test. The decision rule is p-value ≤ .025, reject H_0. The observed p-value was 0.7725. The decision is to not reject H_0.

Research Q6: Does the Wray, method for sizing convergence buffers protect the critical chain for two path convergences at the same point on the critical chain?

Comparison: The convergence buffer is to protect the critical chain from encroachment by the non-critical path. Compare the results of the network with a convergence buffer to the critical chain without any convergence. If the buffer protected the critical chain, there should not be any difference between the critical chain alone and the critical chain with convergence. If the test run fails, then the convergence buffer is too small.

H(9): H_0: The Wray method for sizing the convergence buffer (100%), for two path convergence at the same point, protects the project completion time as compared to the completion time of the critical chain without convergence.

H_1: The Wray method for sizing the convergence buffer (100), for two path convergences at the same point, does not protect the completion time as compared to the completion time of the critical chain without convergence.

Results: A convergence buffer (96) was sized using the Wray method and inserted into the network. A second non-critical path of 20 boxes converged with the 30 box path at the last activity but no buffer was included. No completion buffer was in this test. This test involved a 120 box critical chain with mean = 1426 and variance = 955.563. A 30 box non-critical path converged with the critical chain at the last box. The results of the test were a mean = 1427.22 and a variance = 837.146. The means of the two runs were compared using the Wilcoxon rank sums test. The decision rule is p-value ≤ .025, reject H_0. The observed p-value was 0.9468. The decision is to not reject H_0.

Research Q7: In the case of insufficient space for convergence buffers calculated using the Wray method, does the Wray method of adding the buffer shortfall to the completion buffer, compensating for the shortfall, protect the completion time of the project?

Comparison: This is determined by statistically comparing the project completion time, with convergence, against the critical chain completion time, without any convergence.

H(10): H_0: The Wray method provides effective protection to the project completion time in the case of an insufficient space for a convergence buffer by adding the shortfall in the convergence buffer to the completion buffer.

H_1: The Wray method for compensating for undersized buffers does not protect the project completion time.

Results: The recommended convergence buffer for 100% protection would be 96 time units. This buffer was not used in the test but rather a buffer of 23 time units (80%) was used in its place. The test had two parts, the first part consisted of a 120 box critical chain, expected mean = 1426 and variance = 926, which are the values needed to calculate the completion buffer for 95% protection. One non-critical path of 30 boxes converged at the last activity on the critical chain. The results was a test run with mean = 1440.42 and a variance = 317.63. The means of the two runs were compared using the Wilcoxon rank sums test. The decision rule is p-value ≤ .025, reject H_0. The observed p-value was 0.0000 which indicates that the means are not equal. In the second part of the test, the completion buffer was augmented by the shortfall in the convergence buffer which was 82 time units. The old completion buffer was for 1475.38 and it would be 1557.38. Using the 1475.38 against the data, 55 runs (5.5%) had greater durations than the buffer allowed. This was the same result without the buffer shortfall because the completion buffer was sized for 5% failures. Using the 1557.38, there were no failures (max value was 1524.12). Since the method provided a solid 100% protection, H_0 can not be rejected. The same result can be had, at least in this case, without augmenting the completion buffer.

Table 4.1: Summary of test results

Quest.	Hyp.	Null Hypothesis	Action	Remarks
1	H(1)	Aquilano et al. (1995) method works for sizing the completion buffer for a predicted safety.	Accept Z=1.81<1.96	
2	H(2)	Goldratt/Newbold completion buffers protect at 90% safety.	Reject Z=∞>1.28	Protection was in excess of 100%.
	H(3)	Leach/Rizzo completion buffers protect at 90% safety	Reject Z=∞>1.28	Protection was in excess of 100%.
3	H(4)	The Wray method sized convergence buffers for 100% protection of critical chain.	Accept P=.9632>.025	The buffer was not too small to protect the critical chain.
	H(5)	The Wray method sized convergence buffers for 90% protection of critical chain.	Reject P=.0014<.025	The buffer was too small to protect the critical chain.
4	H(6)	The Wray method and Goldratt/Newbold work equally well for sizing convergence & completion buffers.	Accept Converg Reject complet.	Goldratt/Newbold provided larger buffers that provided greater safety.
	H(7)	The Wray method and Goldratt/Newbold work equally well for sizing convergence & completion buffers.	Accept Converg Accept complet.	

5	H(8)	Wray method properly sizes the resource buffer	Accept P=.7725>.025	
6	H(9)	Wray method properly sizes the convergence buffer for multiple path convergence.	Accept P=.9468>.025	
7	H(10)	Wray method for compensating for not enough space for a proper convergence buffer works.	Accept	Completion buffer protected at 95% without augmenting.

CHAPTER V

CONCLUSIONS

Introduction

This chapter presents the conclusions about the research, the contribution to knowledge, and the opportunity for future research. The results of the study show that the new method, called the Wray method, is capable of setting effective convergence, completion, and resource buffers. The addition of this new method of buffer sizing should provide the practitioner with additional options for scheduling their projects.

1. Is the completion buffer sized using the approach described in Aquilano et al. (1995) capable of protecting the expected project duration at a predetermined success probability of 95%. No converging paths are involved. (success is defined as a project length being less than the buffered value).

H(1): H_0: There is no statistically significant difference between the proportion of failures observed and the predetermined proportion of 0.05 using the approach described in Aquilano et al. (1995).

H_1: The observed proportion of failures is not equal to 0.05.

Conclusion: H_0 passed. The Aquilano et al. (1995) approach to sizing the completion buffer works. It effectively identifies the variability of the critical chain and provides a method for setting a buffer that has proved in the test that it can accurately deliver the level of safety desired.

2. Do the techniques for completion buffer sizing presented by Goldratt/Newbold and Leach/Rizzo create completion buffers that protect the project duration at the 90% probability? The value of 90% comes from Goldratt discussion about activity durations. (success is defined as a project length being less than the buffered value). There are no converging paths involved.

H(2): H_0: There is no statistically significant difference between the proportion of failures observed using the completion buffers sized by the Goldratt/Newbold method and the predetermined proportion of 0.10.

H_1: The proportion of failures observed using the completion buffers sized using the Goldratt/Newbold method is not statistically equal to 0.10.

H(3): H_0: There is no statistically significant difference between the proportion of failures observed using the completion buffers sized by the Leach/Rizzo method and the predetermined proportion of 0.10.

H_1: The proportion of failures observed, using the completion buffers sized using the Leach/Rizzo method, is not statistically equal to 0.10.

Conclusion: H_0 failed, the Goldratt/Newbold method protects the project completion time in excess of 90% and more likely 100%. The completion buffers sized using the Goldratt/Newbold approach do protect the completion time of the project but do it to an excess. The amount of buffer was more than required to deliver the desired protection. This additional buffer may in the long term affect the competitiveness of the company in competitive bidding for projects.

H_0 failed, the Leach/Rizzo method protects the project completion time in excess of 90% and more likely 100%. The Leach/Rizzo approach was also effective in protecting the project completion time. The method did not produce as large a completion buffer as the Goldratt/Newbold method. If the project manager wanted to use Leach/Rizzo to set the completion buffer for a predetermined level of protection, it would necessary to determine different factors than are used in the Wray method.

3. Can a technique be developed for sizing convergence buffers, that provides approximately 100% protection to the critical chain, and does it without wasted buffer?

H(4): H_0: When sizing the convergence buffer, with the Wray method, for approximately 100% protection of the critical chain, there is no statistically significant impact on the critical chain completion times.

H_1: The convergence buffer sized, with the Wray method, for pproximately 100% protection of the critical chain fails to prevent statistically significant impact on the critical chain completion times.

H(5): H_0: When using the Wray method to size the convergence buffer, for approximately 90% protection of the critical chain, there is no statistically significant impact on the critical chain completion times.

H_1: The convergence buffer sized, with the Wray method, for approximately 90% protection of the critical chain fails to prevent statistically significant impact on the critical chain completion times.

Conclusion: H_0 passed. The Wray method protected the convergence at 100% but not at 90% which means that the protection is sufficient and probably not in excess. The Wray method of sizing convergence buffers provides buffers that cover the variability of the converging non-critical path without much excess. The test for hypothesis H(4) indicated that the convergence buffer effectively handled the variability in the convergence such that the critical chain was not affected. The test for hypothesis H(5) indicates that the factor for 90% coverage is indeed too low to effectively cover the variability from the converging path. From these two tests, it is reasonable to conclude that the sizing factor of 2.5 produced a buffer that got the job done without wasting time.

4. Do Goldratt/Newbold and Leach/Rizzo methods protect project completion times, by properly sizing convergence and completion buffers for 100%, work as well as the Wray method.

H(6): H_0: The use of convergence and completion buffers sized by the Goldratt/Newbold method and the Wray method protect (at 100%) the project completion times statistically the same.

H_1: The Wray method and Goldratt/Newbold methods for sizing the convergence and completion buffers do not provide statistically equal protection to the project duration.

H(7): H_0: The use of convergence and completion buffers sized by the Leach/Rizzo method and the Wray method protect (at 100%) the project completion times statistically the same.

H_1: The Wray method and Leach/Rizzo methods for sizing the convergence and completion buffers do not provide statistically equal protection to the project duration.

Conclusion: H_0 passed. All three methods, Wray, Goldratt/Newbold, and Leach/Rizzo provided buffers that protected the project completion times.

5. Does the Wray, method for sizing convergence buffers work for sizing resource buffers?

H(8): H_0: The Wray method properly sizes the resource buffer such that the critical chain and project completion are not statistically impacted by the contention of resources.

H_1: The Wray method does not properly size the resource buffer such that The critical chain and project completion are not statistically impacted by the contention of resources

Conclusion: H_0 passed. The Wray method for sizing convergence buffers works to size resource buffers that are properly sized to protect the critical chain. Since the resource contention can only be determined after the convergence buffer has been sized and inserted into the network, the additional protection afforded to the non-critical path provides more protection than is warranted by the variability on the non-critical path. It is better to have too much protection for the critical chain than not enough.

6. Does the Wray, method for sizing convergence buffers protect the critical chain for two path convergences at the same point on the critical chain?

H(9): H_0: The Wray method for sizing the convergence buffer (100%), for two path convergence at the same point, protects the project completion time as compared to the completion time of the critical chain without convergence.

H_1: The Wray method for sizing the convergence buffer (100), for two path convergences at the same point, does not protect the completion time as compared to the completion time of the critical chain without convergence.

Conclusion: H_0 passed. The use of one convergence buffer, sized using the Wray method, to protect two converging paths worked. Further testing is required to conclude the suitability of the method for more than two merging paths.

7. In the case of insufficient space for convergence buffers calculated using the Wray method, does the Wray method of adding the buffer shortfall to the completion buffer, compensating for the shortfall, protect the completion time of the project?

H(10): H_0: The Wray method provides effective protection to the project completion time in the case of an insufficient space for a convergence buffer by adding the shortfall in the convergence buffer to the completion buffer.

H_1: The Wray method for compensating for undersized buffers does not protect the project completion time.

Conclusion: H_0 but may not be necessary. The results of this test indicate that augmenting the completion buffer may not be necessary to compensate for the lack of space for the converging buffer. Further testing needs to be done because this situation may change depending on how much shortfall in convergence buffer occurs. This test dealt with only a drop from 100% to 80% safety. Larger shortfalls would probably have a different effect.

There are difficulties in comparing the three methods of buffer sizing. Each uses different time units. Goldratt/Newbold use time units directly from the activity time estimator. These times have the additional time added to cover special causes of variability such as student syndrome, multi-tasking, and other conditions that people have become used to dealing with on traditional projects. Goldratt/Newbold's method takes the time for these special causes and creates the expected value for the project duration and buffers.

Leach/Rizzo deal with some of the same issues but they do it with a finer measure. Rather than taking all of the time for special causes in one lump amount, they break it into finer amounts that roughly equate to standard deviations. Leach/Rizzo are now faced with the decision of how many standard deviations should be in each convergence and completion buffers.

The Wray method deals with expected durations and an associated standard deviation for each of the activities. Thus the special causes of variation have been removed and the researcher is left with only common causes. This allows for a more controlled analysis.

Goldratt/Newbold and Leach/Rizzo are both attempting with their methods to protect the overall project completion time. To do this, both the convergence and completion buffers work together to absorb variability. The Wray method attempted to almost totally take care of the variability due to the convergence, leaving the variability along the critical chain to be protected by the completion buffer. This approach provides the degree of control over the sources of variability that is necessary for setting the completion buffer for a specified level of protection.

Limitations

There were limitations placed on the data and networks used in this dissertation. The symmetric normal distribution was used to model the activities in the simulations. The normal distribution was selected after it was determined that the truncated normal distribution of activity times would result from the starting rule of activity-begins-on-date. Goldratt (1997) indicated that the right tail of the activity time distribution would tend to extend farther than the left tail because of the tendency of the work to take longer rather than less time to perform. These tails may make a difference in accuracy. This right tail can also be looked at as a skew that can affect the ability to fit distributions.

The results may not extend to the use of other distributions such as the log-normal, gamma, and beta distributions to model activity durations.

A second possible issue is the selection criteria of the activity times used in the simulation models. In order to make them behave more normal, the standard deviations of each activity were randomly selected to be no more than one third of the associated mean. This would seriously reduce the incidence of truncation in the generation of individual activity times during the simulation runs. A standard deviation that is too large compared to the mean, will tend to generate more values that are less than or equal to zero. The model would drop these times and generate another value. This would be done until a positive time was available. The model would not allow a negative process time for an activity. Larger variances might change he results.

Contributions

The results of this research provides a third approach to setting buffer sizes with new opportunities for project scheduling. The other two approaches associated with the Theory of Constraints are Goldratt/Newbold and Leach/Rizzo. Each of the three methods have their strengths and weaknesses. Having the opportunity to size the completion buffer to fit a specific level of safety, gives the project manager new options.

The findings that the starting rule of activity-begins-on-date causes the non-critical paths to be chains of truncated activity times should be of interest to both researchers and practitioners alike. The use of this knowledge gives the researcher and practitioner an opportunity for greater accuracy.

Future Research

There is an opportunity for further research using different statistical distributions to model the activity times for both critical chains and non-critical paths. Although nothing was found in the literature about how to easily work with truncated gamma, log-normal, and beta distributions, the findings in the pilot experiment that the sums of these distributions are fit by the same distributions may provide signs of how to approach the problem of how to estimate the parameters of the path durations.

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APPENDIX A

Activity Times for each box in each network used in testing

Simulation run 120 box critical chain

Box Name	Action	Delay Type	Units	Mean	Std. Dev.	Variance		goldratt buffer	Leach std. dev^2
Process 501	Delay	Normal	Hours	8	2	4		4	64
Process 502	Delay	Normal	Hours	14	2	4		7	196
Process 503	Delay	Normal	Hours	17	1	1		8.5	289
Process 504	Delay	Normal	Hours	15	1	1		7.5	225
Process 505	Delay	Normal	Hours	16	3	9		8	256
Process 506	Delay	Normal	Hours	17	1	1		8.5	289
Process 507	Delay	Normal	Hours	20	5	25		10	400
Process 508	Delay	Normal	Hours	12	3	9		6	144
Process 509	Delay	Normal	Hours	20	1	1		10	400
Process 510	Delay	Normal	Hours	16	4	16		8	256
Process 511	Delay	Normal	Hours	9	2	4		4.5	81
Process 512	Delay	Normal	Hours	12	1	1		6	144
Process 513	Delay	Normal	Hours	10	2	4		5	100
Process 514	Delay	Normal	Hours	14	2	4		7	196
Process 515	Delay	Normal	Hours	16	2	4		8	256
Process 516	Delay	Normal	Hours	20	4	16		10	400
Process 517	Delay	Normal	Hours	16	4	16		8	256
Process 518	Delay	Normal	Hours	16	3	9		8	256
Process 519	Delay	Normal	Hours	19	1	1	C	9.5	361
Process 520	Delay	Normal	Hours	13	2	4		6.5	169
Process 521	Delay	Normal	Hours	18	2	4		9	324
Process 522	Delay	Normal	Hours	13	2	4		6.5	169
Process 523	Delay	Normal	Hours	12	1	1		6	144
Process 524	Delay	Normal	Hours	13	3	9		6.5	169
Process 525	Delay	Normal	Hours	13	3	9		6.5	169
Process 526	Delay	Normal	Hours	4	1	1		2	16
Process 527	Delay	Normal	Hours	7	2	4		3.5	49
Process 528	Delay	Normal	Hours	10	1	1		5	100
Process 529	Delay	Normal	Hours	19	6	36		9.5	361
Process 530	Delay	Normal	Hours	10	2	4		5	100
Process 531	Delay	Normal	Hours	15	1	1		7.5	225
Process 532	Delay	Normal	Hours	16	5	25		8	256
Process 533	Delay	Normal	Hours	13	3	9		6.5	169
Process 534	Delay	Normal	Hours	12	4	16		6	144
Process 535	Delay	Normal	Hours	7	1	1		3.5	49
Process 536	Delay	Normal	Hours	8	2	4		4	64
Process 537	Delay	Normal	Hours	6	2	4		3	36
Process 538	Delay	Normal	Hours	12	1	1		6	144
Process 539	Delay	Normal	Hours	12	3	9		6	144
Process 540	Delay	Normal	Hours	13	1	1		6.5	169
Process 541	Delay	Normal	Hours	17	4	16		8.5	289
Process 542	Delay	Normal	Hours	20	5	25		10	400
Process 543	Delay	Normal	Hours	9	2	4		4.5	81
Process 544	Delay	Normal	Hours	15	3	9		7.5	225
Process 545	Delay	Normal	Hours	9	2	4		4.5	81
Process 546	Delay	Normal	Hours	13	4	16		6.5	169

Process 547	Delay	Normal	Hours	5	1	1		2.5	25
Process 548	Delay	Normal	Hours	6	2	4		3	36
Process 549	Delay	Normal	Hours	15	2	4	B	7.5	225
Process 550	Delay	Normal	Hours	9	2	4		4.5	81
Process 551	Delay	Normal	Hours	6	1	1		3	36
Process 552	Delay	Normal	Hours	14	3	9		7	196
Process 553	Delay	Normal	Hours	13	4	16		6.5	169
Process 554	Delay	Normal	Hours	7	2	4		3.5	49
Process 555	Delay	Normal	Hours	18	4	16		9	324
Process 556	Delay	Normal	Hours	19	3	9		9.5	361
Process 557	Delay	Normal	Hours	12	3	9		6	144
Process 558	Delay	Normal	Hours	5	1	1		2.5	25
Process 559	Delay	Normal	Hours	3	1	1		1.5	9
Process 560	Delay	Normal	Hours	12	1	1		6	144
Process 561	Delay	Normal	Hours	5	1	1		2.5	25
Process 562	Delay	Normal	Hours	15	2	4		7.5	225
Process 563	Delay	Normal	Hours	14	4	16		7	196
Process 564	Delay	Normal	Hours	4	1	1		2	16
Process 565	Delay	Normal	Hours	18	6	36		9	324
Process 566	Delay	Normal	Hours	7	1	1		3.5	49
Process 567	Delay	Normal	Hours	14	4	16		7	196
Process 568	Delay	Normal	Hours	4	1	1		2	16
Process 569	Delay	Normal	Hours	6	2	4		3	36
Process 570	Delay	Normal	Hours	17	4	16		8.5	289
Process 571	Delay	Normal	Hours	8	1	1		4	64
Process 572	Delay	Normal	Hours	4	1	1		2	16
Process 573	Delay	Normal	Hours	10	1	1		5	100
Process 574	Delay	Normal	Hours	8	2	4		4	64
Process 575	Delay	Normal	Hours	9	2	4		4.5	81
Process 576	Delay	Normal	Hours	12	4	16		6	144
Process 577	Delay	Normal	Hours	18	4	16		9	324
Process 578	Delay	Normal	Hours	3	1	1		1.5	9
Process 579	Delay	Normal	Hours	10	1	1		5	100
Process 580	Delay	Normal	Hours	3	1	1		1.5	9
Process 581	Delay	Normal	Hours	14	4	16		7	196
Process 582	Delay	Normal	Hours	13	4	16		6.5	169
Process 583	Delay	Normal	Hours	16	1	1		8	256
Process 584	Delay	Normal	Hours	15	4	16		7.5	225
Process 585	Delay	Normal	Hours	17	2	4		8.5	289
Process 586	Delay	Normal	Hours	18	6	36		9	324
Process 587	Delay	Normal	Hours	11	2	4		5.5	121
Process 588	Delay	Normal	Hours	8	2	4		4	64
Process 589	Delay	Normal	Hours	15	2	4		7.5	225
Process 590	Delay	Normal	Hours	11	2	4		5.5	121
Process 591	Delay	Normal	Hours	12	1	1		6	144
Process 592	Delay	Normal	Hours	11	1	1		5.5	121
Process 593	Delay	Normal	Hours	19	4	16		9.5	361
Process 594	Delay	Normal	Hours	13	3	9		6.5	169
Process 595	Delay	Normal	Hours	17	3	9		8.5	289
Process 596	Delay	Normal	Hours	9	3	9		4.5	81

Process 597	Delay	Normal	Hours	12	2	4		6	144
Process 598	Delay	Normal	Hours	6	1	1		3	36
Process 599	Delay	Normal	Hours	12	4	16		6	144
Process 600	Delay	Normal	Hours	15	4	16		7.5	225
Process 601	Delay	Normal	Hours	3	1	1		1.5	9
Process 602	Delay	Normal	Hours	6	1	1		3	36
Process 603	Delay	Normal	Hours	14	2	4		7	196
Process 604	Delay	Normal	Hours	6	1	1		3	36
Process 605	Delay	Normal	Hours	18	6	36		9	324
Process 606	Delay	Normal	Hours	9	2	4		4.5	81
Process 607	Delay	Normal	Hours	6	2	4		3	36
Process 608	Delay	Normal	Hours	11	3	9		5.5	121
Process 609	Delay	Normal	Hours	12	3	9		6	144
Process 610	Delay	Normal	Hours	17	3	9		8.5	289
Process 611	Delay	Normal	Hours	9	3	9		4.5	81
Process 612	Delay	Normal	Hours	5	1	1		2.5	25
Process 613	Delay	Normal	Hours	4	1	1		2	16
Process 614	Delay	Normal	Hours	19	5	25		9.5	361
Process 615	Delay	Normal	Hours	3	1	1		1.5	9
Process 616	Delay	Normal	Hours	3	1	1		1.5	9
Process 617	Delay	Normal	Hours	18	2	4		9	324
Process 618	Delay	Normal	Hours	16	2	4		8	256
Process 619	Delay	Normal	Hours	18	4	16	A	9	324
Process 620	Delay	Normal	Hours	16	5	25		8	256
				1426	290	926		713	19728
						30.43			140.46

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Point A: mean = 1410, variance = 901, box 619
Point B: mean = 642, variance =361
Poing C: mean = 287, variance = 130, box 519

Goldratt act. Time = same as my activity time
Godratt buffer = .5 x act. Time
Leach act. Time = same as my activity time
Leach variance = same as my activity time^2

Ranomized activity times for test runs.. These were developed using an HP 21S calculator and its random number function kesys. Seed = 120100. Means weere choosen from 3 to 20, and standard deviations were randomlly choosen to be less than 1/3 of the mean.

30 box, non-critical path

	Mean	Std. Dev	Coh mean	Coh var.		Goldratt buffers calc.		Leach std. dev.
1	6	2	7.59577	1.45352		3		36
2	11	3	13.39337	3.27042		5.5		121
3	6	1	6.79788	0.36338		3		36
4	17	1	17.7979	0.36338		8.5		289
5	16	4	19.1915	5.81408		8		256
6	19	5	22.9894	9.08451		9.5		361
7	9	3	11.3937	3.27042		4.5		81
8	5	1	5.79788	0.36338		2.5		25
9	13	2	14.5958	1.45352		6.5		169
10	16	2	17.5958	1.45352		8		256
11	15	3	17.3937	3.27042		7.5		225
12	13	1	13.7979	0.36338		6.5		169
13	7	1	7.79788	0.36338		3.5		49
14	15	2	16.5958	1.45352		7.5		225
15	4	1	4.79788	0.36338		2		16
16	3	1	3.79788	0.36338		1.5		9
17	10	3	12.3937	3.27042		5		100
18	9	1	9.79788	0.36338		4.5		81
19	8	1	8.79788	0.36338		4		64
20	16	2	17.5958	1.45352		8		256
21	19	6	23.7873	13.0817		9.5		361
22	11	3	13.3937	3.27042		5.5		121
23	19	2	20.5958	1.45352		9.5		361
24	20	4	23.1915	5.81408		10		400
25	19	2	20.5958	1.45352		9.5		361
26	18	2	19.5958	1.45352		9		324
27	11	1	11.7979	0.36338		5.5		121
28	10	2	11.8958	1.45352		5		100
29	12	1	12.7979	0.36338		6		144
30	8	1	8.79788	0.36338		4		64
	365	64	416.3647	67.58871	buffer =	182.5		5181
							std. dev. =	71.98

6 Box, non-critical path

1	13	4	16.1915	5.81408
2	15	2	16.5958	1.45352
3	5	1	5.79788	0.36338
4	19	4	22.1915	5.81408
5	14	1	14.7979	0.36338
6	15	5	18.9894	9.08451
	81	17	94.56398	22.89295

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Goldratt starts with the pessimistic number which is expected to
be able to make 90% of the times. He cuts this in half to arrive
at a "most likely" time which should make 50% of the time.
This
would the expected value of the mean that I use in my simulations.
One half of the amount cut would go into the buffer. The
amount
cut is equal to the half left, so take half of the mean and that is
the buffer.

Leach takes the difference between the pessimistic and the
most
likely as being one standard deviation. He squares the amount,
adds them for the path, and then takes the square root of the
sum to equal one standard deviation of the path duration.

20 box non-critical path

Box #	mean	Std dev.	Variance	Cohen mean	Cohen var
1	3	1	1		0.36338
2	13	1	1		0.36338
3	15	2	4		1.45352
4	17	1	1		0.36338
5	16	2	4		1.45352
6	20	1	1		0.36338
7	11	2	4		1.45352
8	8	2	4		1.45352
9	20	2	4		1.45352
10	20	2	4		1.45352
11	16	3	9		3.27042
12	9	3	9		3.27042
13	8	1	1		0.36338
14	17	4	16		5.81408
15	16	5	25		9.08451
16	20	6	36		13.0817
17	16	2	4		1.45352
18	4	1	1		0.36338
19	13	1	1		0.36338
20	15	4	16		16
	277	46	146	0	63.23943

Random number seed =
501234

Using the same process as for 30 box
activity times.

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path.xls

calculating the delay for the multiple convergence, for 20 box H(9)
For 30 box, delay = 957
There are 365 time units for all 30 boxes but only 357 for boxes
1 through 29.

time to box 1 or 30	957
time to box 29 of 30 (+)	357
time to box 29 from start (sum)	1314
Time for 20 box (-)	277
Delay for 20 box (no buffer)	1037

3 box non-critical path

Box #	mean	Std dev.	Variance	Cohen mean	Cohen var
1	8	2	4		1.45352
2	4	1	1		0.36338
3	11	2	4		1.45352
	23	5	9	0	3.27042

Random number seed =
501234

Using the same process as for 30 box
activity times.

Buffer size calculatios for H(2) and H(3)

30 box path		Convergence buffer	
Goldratt Buffer			Leach Buffer
	183		180
		Completion buffer (Leach)	* std. dev. = 140.46
	713		180

Leach completion buffer calculations:
Std. dev. = 1240.46 for critical chain.

$$Z = \frac{X - mu}{\text{std. dev.}}$$

$$Z(.10) = 1.2817 = \frac{X - 1426}{140.46}$$

X = 1606 (This is the duration for the critical path plus the completion buffer)

interpolation to get Z(.10)
Z(.10) = 1.2817

For Wray method at 90%

$$Z(.10) = 1.2817 = \frac{X - 1426}{30.02}$$

X = 1464.4766 = 1464.48

Calculation of delays for Goldratt and Leach for 120 box critical chain and 30 box convergence.

Goldratt		Calc.	Leach
	1410	120 box	1410
	365	30 box	365
	183	buffer	180
	862	delay	865

Critical path plus completion buffer			
	2139		1606

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APPENDIX B

Example calculations for Cohen values

$\mu = 5$
$\sigma = 1$
$T = 5$

$$ft = \int_{-\infty}^{5} \frac{1}{\sigma\sqrt{2\pi}}\, Exp[-1/2\left(\frac{x-\mu}{\sigma}\right)^2]\, dx\, //\, N$$

$$mu1 = \frac{1}{\sigma\sqrt{2\pi}\,(1-ft)} \int_{5}^{\infty} (x-T) * Exp[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2]\, dx\, //\, N$$

```
5
1
5
0.5
0.797885
```

$$mu2 = \frac{1}{\sigma\sqrt{2\pi}\,(1-ft)} \int_{5}^{\infty} (x-T)^2 * Exp[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2]\, dx\, //\, N$$

```
1.
```

$$Var = mu2 - (mu1)^2\, //\, N$$

```
0.36338
xbar=mu1+T//N
5.79788
(*c:\quest3\cohen1.nb*)
```

μ = **10**
σ = **4**
T = **10**

$$ft = \int_{-\infty}^{10} \frac{1}{\sigma\sqrt{2\pi}}\, Exp\left[-1/2\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \,//\, N$$

$$mu1 = \frac{1}{\sigma\sqrt{2\pi}\,(1-ft)} \int_{10}^{\infty} (x-T) * Exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \,//\, N$$

```
10
4
10
0.5
3.19154
```

$$mu2 = \frac{1}{\sigma\sqrt{2\pi}\,(1-ft)} \int_{10}^{\infty} (x-T)^2 * Exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] dx \,//\, N$$

```
16.
```

$$Var = mu2 - (mu1)^2 \,//\, N$$

```
5.81408
xbar=mu1+T//N
13.1915
(*c:\quest3\cohen104.nb*)
```

APPENDIX C

Example Mathematica calculations for test analysis

```
Date[]
 {2004,12,2,20,23,16.2500000}
 (*Run for file name: Aoc120.ASC  *)
```

INTRODUCTION

This run is for a critical chain of 120 boxes with one late convergeing path of 30 boxes. The delay is set at 862. This same network is used for the non optimal 30 box converging buffer (delay at 1039). This network, as is, is the base system for hypothesis H(2) and H(3). This is for H(2) testing Goldratt's convergence and completion buffers for 90% protection.

A. Read in the data

The data will be read into *Mathematica* using the "get file" option in the " input menu". Once it has been read into the system, it may be referred to by data1 or "%" for manipulation. The data is sorted into ascending order for calucluation of the Beta distribution parameters.

```
(*The following section loads the statistical packages used
later in the program.*)
 <<Statistics`DataManipulation`
<<Statistics`ContinuousDistributions`
<<Statistics`DescriptiveStatistics`
<<Graphics`Graphics`

  Null^2
  Null^2
 (*The following section reads in data directly from the
ASCII file created by piping from the GEMSII run.*)

 s=ReadList["C:\\documents and settings\\edwin wray\\my
documents\\from desk
top\\DISSERT\\arena\\h2conv300.asc",Number];
(*pickdata[r_]:=Module[{rec=StringToStream[r], x},
           Skip[rec, Word, 2];
               x=Read[rec, Number];
               Close[rec];
               x]*)
(*datain = Map[pickdata, s];*)
datain = s;
avgduration = Mean[datain]
maximumval = Max[datain]
minimumval = Min[datain]
vardur = Variance[datain]
skew=Skewness[datain]
data1 = Sort[datain]
nn =Length[data1]


 1427.46
 1541.75
 1328.08
 883.037
 0.120921
```

{1328.08,1333.27,1340.38,1352.8,1353.02,1354.98,1354.99,1357
.34,1357.6,1357.9,1359.13,1360.74,1361.29,1361.97,1362.41,13
63.01,1364.47,1365.51,1368.07,1368.3,1368.54,1370.36,1370.43
,1371.23,1372.17,1372.44,1372.55,1372.67,1372.71,1373.28,137
3.71,1373.94,1374.36,1374.4,1375.03,1375.08,1375.66,1375.98,
1377.64,1377.86,1378.24,1378.53,1379.,1379.31,1379.44,1379.4
7,1379.77,1379.91,1380.04,1380.11,1380.64,1380.87,1381.22,13
81.28,1381.31,1381.54,1382.11,1382.3,1382.39,1383.2,1383.26,
1383.37,1383.76,1384.11,1384.17,1384.35,1384.37,1384.7,1384.
91,1385.02,1385.08,1385.22,1385.39,1385.42,1385.53,1385.69,1
386.36,1386.4,1386.5,1386.6,1386.63,1386.65,1386.66,1386.9,1
387.15,1387.19,1387.35,1387.45,1387.5,1387.54,1387.75,1388.0
2,1388.1,1388.32,1388.46,1388.74,1389.29,1389.74,1389.85,138
9.88,1390.02,1390.1,1390.19,1390.51,1390.62,1390.65,1390.78,
1390.95,1391.3,1391.57,1391.74,1391.77,1392.06,1392.42,1392.
58,1392.68,1392.77,1392.8,1392.9,1392.95,1393.32,1393.55,139
3.55,1393.71,1393.78,1393.84,1393.87,1393.98,1394.2,1394.28,
1394.36,1394.47,1394.49,1395.,1395.08,1395.32,1395.39,1395.5
3,1395.59,1395.63,1395.75,1395.8,1395.89,1395.89,1396.03,139
6.4,1396.55,1396.8,1396.81,1396.94,1397.1,1397.19,1397.29,13
97.46,1397.66,1397.67,1397.67,1397.88,1397.9,1397.97,1398.23
,1398.43,1398.54,1398.56,1398.74,1399.09,1399.09,1399.13,139
9.23,1399.31,1399.32,1399.66,1399.69,1399.79,1399.84,1399.87
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.8,1404.84,1404.87,1404.9,1404.95,1405.,1405.14,1405.15,1405
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36,1406.39,1406.43,1406.73,1406.77,1406.77,1406.82,1406.9,14
06.92,1406.96,1407.07,1407.17,1407.32,1407.51,1407.66,1407.6
6,1407.69,1407.77,1407.8,1407.83,1407.93,1407.93,1408.,1408.
07,1408.11,1408.14,1408.14,1408.23,1408.34,1408.34,1408.5,14
08.56,1408.59,1408.66,1408.74,1408.99,1409.08,1409.11,1409.1
7,1409.26,1409.35,1409.36,1409.42,1409.45,1409.47,1409.52,14
09.57,1409.63,1409.63,1409.7,1409.71,1409.71,1409.83,1409.97
,1410.27,1410.32,1410.4,1410.43,1410.49,1410.55,1410.67,1410
.71,1410.71,1410.76,1411.,1411.08,1411.1,1411.21,1411.31,141
1.36,1411.41,1411.45,1411.56,1411.72,1411.73,1411.86,1411.92
,1412.03,1412.24,1412.34,1412.45,1412.49,1412.55,1412.74,141
2.75,1412.88,1413.08,1413.12,1413.3,1413.37,1413.4,1413.49,1
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27,1414.43,1414.44,1414.6,1414.73,1414.76,1414.93,1415.,1415
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.04,1417.06,1417.07,1417.14,1417.17,1417.24,1417.3,1417.39,1
417.45,1417.56,1417.61,1417.66,1417.7,1417.71,1417.72,1417.7
5,1417.8,1417.81,1417.85,1418.12,1418.17,1418.28,1418.29,141
8.35,1418.38,1418.43,1418.45,1418.46,1418.54,1418.71,1418.94
,1418.96,1419.1,1419.34,1419.42,1419.55,1419.59,1419.68,1420

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9,1429.79,1429.95,1430.05,1430.1,1430.11,1430.12,1430.14,143
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.04,1431.11,1431.21,1431.23,1431.27,1431.35,1431.43,1431.53,
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.1,1433.11,1433.13,1433.29,1433.41,1433.52,1433.6,1433.61,14
33.79,1433.98,1434.1,1434.13,1434.18,1434.26,1434.35,1434.58
,1434.66,1434.68,1434.68,1434.83,1434.86,1434.96,1435.07,143
5.1,1435.24,1435.24,1435.25,1435.33,1435.5,1435.5,1435.62,14
35.74,1435.86,1436.22,1436.26,1436.28,1436.34,1436.38,1436.3
9,1436.5,1436.55,1436.56,1436.58,1436.59,1436.6,1436.61,1436
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1436.94,1436.98,1437.07,1437.11,1437.14,1437.18,1437.28,1437
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,1439.68,1439.69,1439.98,1440.03,1440.1,1440.1,1440.12,1440.
24,1440.29,1440.33,1440.38,1440.39,1440.47,1440.54,1440.59,1
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41.71,1441.75,1441.92,1441.94,1442.12,1442.19,1442.25,1442.3
1,1442.41,1442.46,1442.64,1442.65,1442.75,1442.9,1443.08,144
3.12,1443.46,1443.48,1443.59,1443.83,1443.96,1444.02,1444.71
,1444.72,1444.79,1444.85,1445.08,1445.13,1445.28,1445.36,144
5.47,1445.71,1445.73,1445.76,1445.78,1445.87,1446.04,1446.18
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.93,1446.93,1447.08,1447.1,1447.11,1447.3,1447.5,1447.7,1447
.88,1448.03,1448.2,1448.24,1448.25,1448.46,1448.48,1448.51,1
448.51,1448.53,1448.58,1448.67,1448.77,1449.02,1449.06,1449.
29,1449.31,1449.57,1449.66,1449.86,1449.86,1449.87,1450.01,1
450.13,1450.23,1450.5,1450.74,1450.84,1450.94,1451.03,1451.0
8,1451.11,1451.15,1451.32,1451.56,1451.65,1451.87,1452.11,14
52.12,1452.12,1452.29,1452.35,1452.36,1452.37,1452.52,1452.6
4,1452.8,1452.89,1453.12,1453.26,1453.27,1453.32,1453.39,145
3.51,1453.6,1453.67,1453.9,1453.95,1453.97,1454.04,1454.06,1

```
454.07,1454.36,1454.46,1454.5,1454.62,1454.74,1454.94,1455.0
1,1455.08,1455.08,1455.15,1455.22,1455.22,1455.32,1455.32,14
55.37,1455.44,1455.75,1455.78,1455.81,1455.84,1455.95,1456.0
2,1456.03,1456.36,1456.44,1456.58,1456.65,1456.66,1456.7,145
6.88,1456.94,1457.14,1457.44,1457.54,1457.76,1457.85,1458.03
,1458.04,1458.09,1458.26,1458.35,1458.81,1459.27,1459.43,145
9.47,1459.59,1459.78,1459.88,1460.,1460.43,1460.56,1460.62,1
460.75,1461.12,1461.17,1461.19,1461.31,1461.32,1461.35,1461.
41,1461.62,1461.76,1462.16,1462.56,1462.66,1462.71,1462.88,1
463.09,1463.32,1463.55,1463.58,1463.81,1464.06,1464.24,1464.
39,1464.46,1464.47,1464.55,1464.67,1464.79,1464.9,1465.32,14
65.34,1465.38,1465.39,1465.73,1465.74,1465.76,1465.94,1465.9
7,1466.12,1466.28,1466.31,1466.88,1467.24,1467.4,1467.45,146
8.,1468.21,1468.54,1468.64,1468.99,1469.34,1469.41,1469.69,1
469.75,1470.32,1470.54,1470.86,1470.96,1471.24,1471.41,1471.
82,1471.84,1472.35,1472.47,1472.5,1472.73,1473.02,1473.07,14
73.38,1474.1,1474.65,1474.74,1475.23,1475.23,1475.44,1475.73
,1476.33,1477.7,1477.89,1478.08,1478.47,1478.66,1478.74,1478
.77,1479.01,1479.02,1479.14,1479.21,1479.85,1480.58,1480.8,1
481.21,1481.33,1481.74,1482.48,1482.78,1483.16,1483.17,1483.
38,1484.76,1484.83,1485.6,1486.,1486.07,1486.21,1486.65,1486
.65,1487.17,1487.27,1487.34,1489.64,1490.43,1492.11,1492.63,
1492.81,1494.82,1494.9,1495.89,1498.08,1498.18,1501.45,1501.
48,1502.02,1503.05,1506.22,1506.25,1508.86,1509.47,1511.17,1
528.6,1541.75}
 1000
```

B. Calculate the parameters for four distributions using the data in data1

(2) Normal Distribution

(a.) Parameter Estimation

```
avgx = Mean[data1]
varx = Variance[data1]
```

```
 1427.46
 883.037
```

(b.). Chisquare Goodness-of-fit Tests and Their Results

```
stddev = varx^.5
 29.7159
```

```
ndist = NormalDistribution[avgx,stddev]
```

```
 NormalDistribution[1427.46,29.7159]
```

```
Clear[freqincr]
```

```
quart = {0.00001,.05, .1, .15, .2, .25, .30, .35, .4, .45,
.5, .55, .6, .65, .7, .75, .8, .85, .9, .95,.9999999};
Map[ Quantile[ndist, #]&, quart]//TableForm;
newlist = Table[Quantile[ndist, quart[[i]]], {i, 1,
Length[quart]}];

freqincr = Quantile[ndist, quart];
obsfrn = RangeCounts[data1,freqincr];
obsfrn = Delete[obsfrn, 1]
```

```
{42,55,50,45,57,63,43,49,50,54,59,35,55,59,38,45,57,46,44,54
,0}
```

```
Map[CDF[ndist, #]&, freqincr] //TableForm;
ptab=Table[CDF[ndist, freqincr[[i]]]-CDF[ndist, freqincr[[i-
1]]], {i, 2, Length[freqincr]}];
ptab //TableForm;
TableForm[ptab];
expfreq = ptab * nn
```

```
{49.99,50.,50.,50.,50.,50.,50.,50.,50.,50.,50.,50.,50.,50.,5
0.,50.,50.,50.,50.,49.9999}
```

```
teststat = Table[(obsfrn[[i]] -
expfreq[[i]])^2/expfreq[[i]],   {i, 1, Length[expfreq]}]
```

{1.27706, 0.5, 2.83637×10^{-27}, 0.5, 0.98, 3.38, 0.98, 0.02, 1.40484×10^{-25},
0.32, 1.62, 4.5, 0.5, 1.62, 2.88, 0.5, 0.98, 0.32, 0.72, 0.320017}

```
tabl=Table[{freqincr[[i]]," ",freqincr[[i+1]], "
",obsfrn[[i]]," ",expfreq[[i]],  teststat[[i]], "\n"},{i,
Length[freqincr]-1}];
Print["Test for normal distribution fit"]
TableForm[tabl]
Print["Chisquare test statistic"]
testtotal = Apply[Plus, teststat]
```

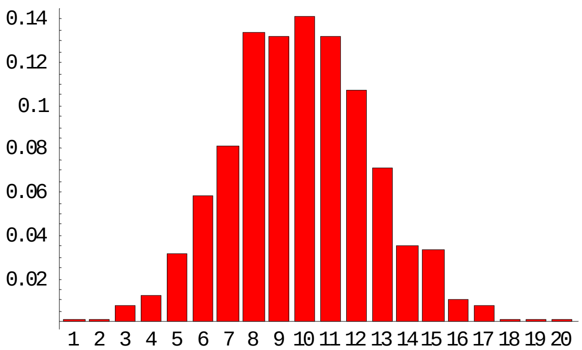
Test for normal distribution fit

1300.73	1378.58	42	49.99	1.27706
1378.58	1389.38	55	50.	0.5
1389.38	1396.66	50	50.	2.83637×10^{-27}
1396.66	1402.45	45	50.	0.5
1402.45	1407.42	57	50.	0.98
1407.42	1411.88	63	50.	3.38
1411.88	1416.01	43	50.	0.98
1416.01	1419.93	49	50.	0.02
1419.93	1423.73	50	50.	1.40484×10^{-25}
1423.73	1427.46	54	50.	0.32
1427.46	1431.2	59	50.	1.62
1431.2	1434.99	35	50.	4.5
1434.99	1438.91	55	50.	0.5
1438.91	1443.05	59	50.	1.62
1443.05	1447.51	38	50.	2.88
1447.51	1452.47	45	50.	0.5
1452.47	1458.26	57	50.	0.98
1458.26	1465.55	46	50.	0.32
1465.55	1476.34	44	50.	0.72
1476.34	1581.97	54	49.9999	0.320017

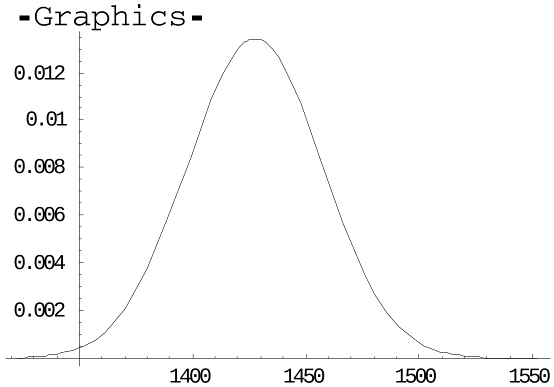
Chisquare test statistic
21.9171

```
minvalp = Min[data1]
maxvalp = Max[data1]
incrementp = (maxvalp - minvalp)/20

obsfreqp = BinCounts[data1, {minvalp, maxvalp, incrementp}]
obsfreqp[[1]] = obsfreqp[[1]] + Count[data3, minvalp]
histdata = obsfreqp/nn;
BarChart[histdata]
Plot[PDF[ndist, x], {x,minvalp -5, maxvalp + 10}];
```

```
1328.08
1541.75
10.6833
{1,1,8,12,32,58,81,134,132,141,132,107,71,35,33,10,8,1,1,1}
1
```



-Graphics-



APPENDIX D

Mathematica programs for calculating factors for convergence buffers.

```
(*This program is designed to calculate the size of the
convergnce buffer.  The method is as follows:
      1.  Use the actual duration data set generated by the
activity on date networks to simulate the convergence path.
          2. Use a normal distribution to simulate the
critical path (chain) at the point of convergence.
          3.  Move the relative location of the convergence
point by changing the value of the mean of the critical path
(chain) value in relation to the mean of the converging
path.
          4. Simulate the convergence by simulating the
ending values of the  converging path and the critical path.
          5. Compare the two values.  If the ending value of
the converging path is larger than the ending value of the
critical path, then the critical path is delayed at the
point of convergence and a 1 is added to the counter. *)

(*The following section loads the statistical packages used
later in the program.*)
<<Statistics`DataManipulation`
<<Statistics`ContinuousDistributions`
<<Statistics`DescriptiveStatistics`
<<Graphics`Graphics`
<<Graphics`MultipleListPlot`
s=ReadList["C:\\DISSERT\\arena\\aod10.asc",Number];
(*pickdata[r_]:=Module[{rec=StringToStream[r], x},
           Skip[rec, Word, 2];
               x=Read[rec, Number];
               Close[rec];
               x]*)
(*datain = Map[pickdata, s];*)
aoddata = s;
nn =Length[aoddata]


 500
 (*Statistics for aod40.asc *)
(*avgx = Mean[data1] =  344.679*)
(* varx = Variance[data1] =  57.6081 *)


(*stddev = varx^.5 = 7.59   *)
 ndist005 = NormalDistribution[87.79, 5];
aocdata005 = RandomArray[ndist005, 500];
ndist5005 =NormalDistribution[92.41, 5]
aocdata5005 = RandomArray[ndist5005, 500];
ndist105 = NormalDistribution[97.02, 5]
aocdata105 =RandomArray[ndist105, 500];
ndist11505 = NormalDistribution[101.64, 5]
aocdata11505 = RandomArray[ndist11505, 500];
ndist2005 = NormalDistribution[106.25, 5]
aocdata2005 = RandomArray[ndist2005, 500];
ndist25005 = NormalDistribution[110.87, 5]
aocdata25005 = RandomArray[ndist25005, 500];
```

```
ndist3005 = NormalDistribution[105.48,5]
aocdata3005 = RandomArray[ndist3005, 500];
ndist02505 = NormalDistribution[90.10, 5]
aocdata02505 = RandomArray[ndist02505, 500];
ndist7505 = NormalDistribution[94.71, 5]
aocdata7505 = RandomArray[ndist7505, 500];
ndist12505 = NormalDistribution[99.33, 5]
aocdata12505= RandomArray[ndist12505, 500];
counter25005 = 0
counter7505 = 0
counter12505 = 0
counter005 = 0
counter5005 = 0
counter105 = 0
counter11505 = 0
counter2005 = 0
counter02505 = 0
counter3005 = 0
Do[If[aoddata[[i]]≥aocdata005[[i]], counter005 = counter005
+1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata5005[[i]], counter5005 =
counter5005 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata105[[i]], counter105 = counter105
+ 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata11505[[i]], counter11505 =
counter11505 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata2005[[i]], counter2005 =
counter2005 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata25005[[i]], counter25005 =
counter25005 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata3005[[i]], counter3005 =
counter3005 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata02505[[i]], counter02505 =
counter02505 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata7505[[i]], counter7505 =
counter7505 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata12505[[i]], counter12505 =
counter12505 + 1,], {i,1,500}];
 General::spell1: Possible spelling error: new symbol name
"ndist5005" is similar to existing symbol "ndist005".
 NormalDistribution[92.41,5]
 General::spell1: Possible spelling error: new symbol name
"aocdata5005" is similar to existing symbol "aocdata005".
 NormalDistribution[97.02,5]
 NormalDistribution[101.64,5]
 General::spell1: Possible spelling error: new symbol name
"ndist2005" is similar to existing symbol "ndist005".
 NormalDistribution[106.25,5]
 General::spell1: Possible spelling error: new symbol name
"aocdata2005" is similar to existing symbol "aocdata005".
 General::spell: Possible spelling error: new symbol name
"ndist25005" is similar to existing symbols
{ndist2005,ndist5005}.
```

```
NormalDistribution[110.87,5]
General::spell: Possible spelling error: new symbol name
"aocdata25005" is similar to existing symbols
{aocdata2005,aocdata5005}.
General::spell1: Possible spelling error: new symbol name
"ndist3005" is similar to existing symbol "ndist005".
NormalDistribution[105.48,5]
General::spell1: Possible spelling error: new symbol name
"aocdata3005" is similar to existing symbol "aocdata005".
NormalDistribution[90.1,5]
NormalDistribution[94.71,5]
NormalDistribution[99.33,5]
0
0
0
0
General::spell: Possible spelling error: new symbol name
"counter5005" is similar to existing symbols
{counter005,counter25005}.
0
0
0
General::spell: Possible spelling error: new symbol name
"counter2005" is similar to existing symbols
{counter005,counter25005}.
0
0
General::spell1: Possible spelling error: new symbol name
"counter3005" is similar to existing symbol "counter005".
0
```

probability05 =

0	□
.25	□
.5	□
.75	□
1.0	□
1.25	□
1.5	□
2.0	□
2.5	□
3.0	□

```
{{0,□},{0.25,□},{0.5,□},{0.75,□},{1.,□},{1.25,□},{1.5,□},{2
.,□},{2.5,□},{3.,□}}
```

probability05[[1,2]] = counter005/500 //N

0.486

probability05[[3,2]] = counter5005/500 //N

0.21

probability05[[5,2]] = counter105/500 //N

0.094

probability05[[7,2]] = counter11505/500 //N

```
0.016
probability05[[8,2]] = counter2005/500 //N
0.002
probability05[[9,2]] = counter25005/500 //N
0.
probability05[[10,2]] = counter3005/500 //N
0.004
probability05[[2,2]] = counter02505/500 //N
0.342
probability05[[4,2]] = counter7505/500 //N
0.122
probability05[[6,2]] = counter12505/500 //N
0.044
```

probabilities15 =

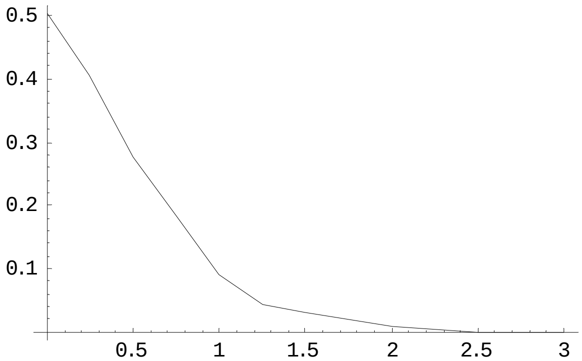
0	.504
.25	.404
.5	.276
.75	.182
1.0	.092
1.25	.044
1.5	.03
2.0	.008
2.5	.0
3.0	0

probabilities20 =

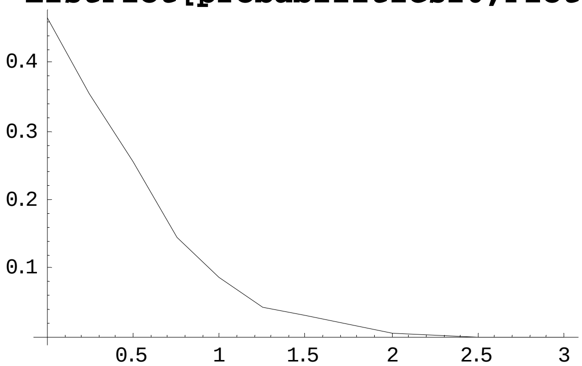
0	.466
.25	.354
.5	.256
.75	.146
1.0	.086
1.25	.044
1.5	.032
2.0	.004
2.5	0
3.0	0

```
ListPlot[probabilities15, PlotJoined → True]
```

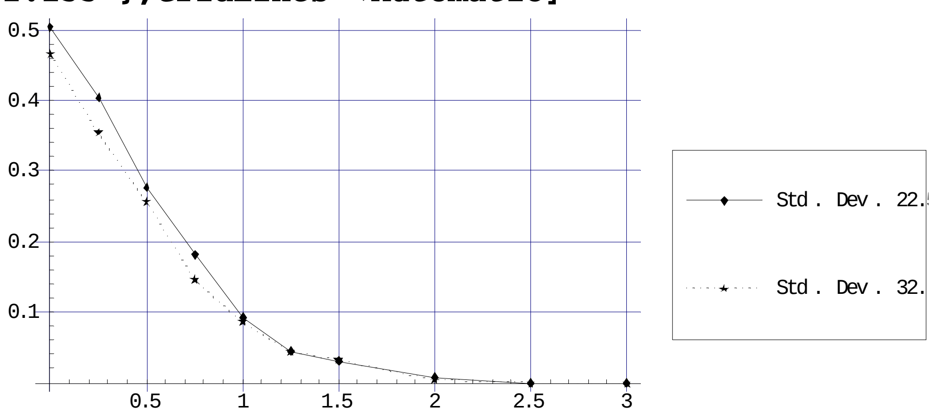
```
{{0,0.504},{0.25,0.404},{0.5,0.276},{0.75,0.182},{1.,0.092},
{1.25,0.044},{1.5,0.03},{2.,0.008},{2.5,0.},{3.,0}}

{{0,0.466},{0.25,0.354},{0.5,0.256},{0.75,0.146},{1.,0.086},
{1.25,0.044},{1.5,0.032},{2.,0.004},{2.5,0},{3.,0}}
```



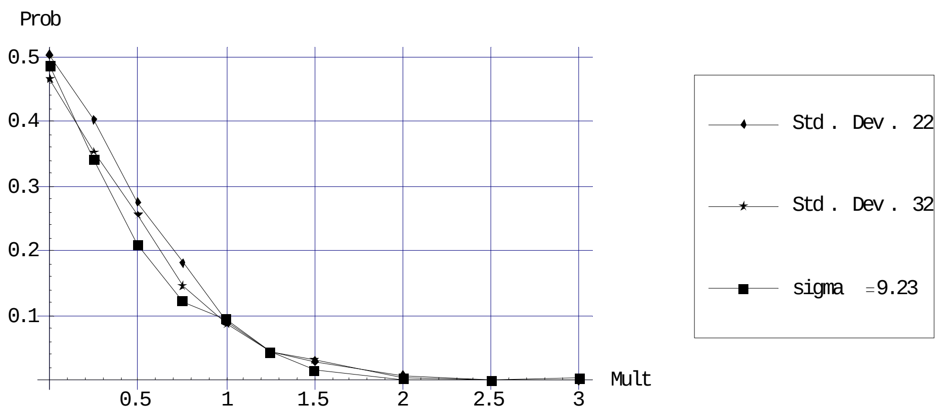

```
-Graphics-
ListPlot[probabilities20,PlotJoined→True]
```




```
-Graphics-

MultipleListPlot[probabilities15,probabilities20,PlotJoined
→True, PlotLegend→{"Std. Dev. 22.59", "Std. Dev.
32.135"},GridLines →Automatic]
```




```
-Graphics-

MultipleListPlot[probabilities15,probabilities20,probability
05,PlotJoined→True, PlotLegend→{"Std. Dev. 22.59", "Std.
Dev. 32.135", "sigma =9.23"}, GridLines →Automatic,
AxesLabel→{"Mult", "Prob "}, PlotStyle→{Thickness[.001]}]
```



Prob
0.5
0.4
0.3
0.2
0.1
0.5
1
1.5
2
2.5
3
Mult
Std. Dev. 22
Std. Dev. 32
sigma =9.23

-Graphics-

```
(*This program is designed to calculate the size of the
convergnce buffer.  The method is as follows:
      1.  Use the actual duration data set generated by the
activity on date networks to simulate the convergence path.
          2. Use a normal distribution to simulate the
critical path (chain) at the point of convergence.
          3.  Move the relative location of the convergence
point by changing the value of the mean of the critical path
(chain) value in relation to the mean of the converging
path.
          4. Simulate the convergence by simulating the
ending values of the  converging path and the critical path.
          5. Compare the two values.  If the ending value of
the converging path is larger than the ending value of the
critical path, then the critical path is delayed at the
point of convergence and a 1 is added to the counter. *)

(*The following section loads the statistical packages used
later in the program.*)
<<Statistics`DataManipulation`
<<Statistics`ContinuousDistributions`
<<Statistics`DescriptiveStatistics`
<<Graphics`Graphics`
<<Graphics`MultipleListPlot`
s=ReadList["C:\\DISSERT\\arena\\aod10.asc",Number];
(*pickdata[r_]:=Module[{rec=StringToStream[r], x},
           Skip[rec, Word, 2];
               x=Read[rec, Number];
               Close[rec];
               x]*)
(*datain = Map[pickdata, s];*)
aoddata = s;
nn =Length[aoddata]


 500
 (*Statistics for aod40.asc *)
(*avgx = Mean[data1] =  344.679*)
(* varx = Variance[data1] =  57.6081 *)


(*stddev = varx^.5 = 7.59   *)
 ndist010= NormalDistribution[87.79, 10];
aocdata010 = RandomArray[ndist010, 500];
ndist5010 =NormalDistribution[94.91, 10]
aocdata5010 = RandomArray[ndist5010, 500];
ndist110 = NormalDistribution[102.02, 10]
aocdata110 =RandomArray[ndist110, 500];
ndist1510 = NormalDistribution[109.14, 10]
aocdata1510 = RandomArray[ndist1510, 500];
ndist2010 = NormalDistribution[116.25, 10]
aocdata2010 = RandomArray[ndist2010, 500];
ndist25010 = NormalDistribution[123.37, 10]
aocdata25010 = RandomArray[ndist25010, 500];
```

```
ndist3010 = NormalDistribution[130.48, 10]
aocdata3010 = RandomArray[ndist3010, 500];
ndist02510 = NormalDistribution[91.35, 10]
aocdata02510 = RandomArray[ndist02510, 500];
ndist7510 = NormalDistribution[98.46, 10]
aocdata7510 = RandomArray[ndist7510, 500];
ndist12510 = NormalDistribution[105.58, 10]
aocdata12510= RandomArray[ndist12510, 500];
counter25010 = 0
counter7510 = 0
counter12510 = 0
counter010 = 0
counter5010 = 0
counter110 = 0
counter1510 = 0
counter2010 = 0
counter02510 = 0
counter3010 = 0
Do[If[aoddata[[i]]≥aocdata010[[i]], counter010 = counter010
+1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata5010[[i]], counter5010 =
counter5010 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata110[[i]], counter110= counter110 +
1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata1510[[i]], counter1510 =
counter1510+ 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata2010[[i]], counter2010 =
counter2010 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata25010[[i]], counter25010=
counter25010 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata3010[[i]], counter3010 =
counter3010 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata02510[[i]], counter02510 =
counter02510 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata7510[[i]], counter7510 =
counter7510 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata12510[[i]], counter12510 =
counter12510 + 1,], {i,1,500}];
 General::spell1: Possible spelling error: new symbol name
"ndist5015" is similar to existing symbol "ndist015".
 NormalDistribution[355.97,20]
 General::spell1: Possible spelling error: new symbol name
"aocdata5015" is similar to existing symbol "aocdata015".
 NormalDistribution[367.27,20]
 General::spell1: Possible spelling error: new symbol name
"ndist1515" is similar to existing symbol "ndist115".
 NormalDistribution[378.56,20]
 General::spell1: Possible spelling error: new symbol name
"aocdata1515" is similar to existing symbol "aocdata115".
 General::spell1: Possible spelling error: new symbol name
"ndist2015" is similar to existing symbol "ndist015".
 NormalDistribution[389.86,20]
```

```
 General::spell1: Possible spelling error: new symbol name
"aocdata2015" is similar to existing symbol "aocdata015".
 General::spell: Possible spelling error: new symbol name
"ndist25015" is similar to existing symbols
{ndist2015,ndist5015}.
 NormalDistribution[401.15,20]
 General::spell: Possible spelling error: new symbol name
"aocdata25015" is similar to existing symbols
{aocdata2015,aocdata5015}.
 General::spell1: Possible spelling error: new symbol name
"ndist3015" is similar to existing symbol "ndist015".
 NormalDistribution[412.45,20]
 General::spell1: Possible spelling error: new symbol name
"aocdata3015" is similar to existing symbol "aocdata015".
 NormalDistribution[350.33,20]
 NormalDistribution[361.62,20]
 General::spell1: Possible spelling error: new symbol name
"ndist12515" is similar to existing symbol "ndist1515".
 NormalDistribution[372.92,20]
 General::spell1: Possible spelling error: new symbol name
"aocdata12515" is similar to existing symbol "aocdata1515".
 0
 0
 0
 0
 General::spell: Possible spelling error: new symbol name
"counter5015" is similar to existing symbols
{counter015,counter25015}.
 0
 0
 General::spell: Possible spelling error: new symbol name
"counter1515" is similar to existing symbols
{counter115,counter12515}.
 0
 General::spell: Possible spelling error: new symbol name
"counter2015" is similar to existing symbols
{counter015,counter25015}.
 0
 0
 General::spell1: Possible spelling error: new symbol name
"counter3015" is similar to existing symbol "counter015".
 0
```

probofcont010 = counter010/500 //N

0.516

probofcont5010 = counter5010/500 //N

0.308

probofcont110 = counter110/500 //N

0.158

probofcont1510 = counter1510/500 //N

0.086

probofcont2010 = counter2010/500 //N

0.018

probofcont25010 = counter25010/500 //N

```
0.
probofcont3010 = counter3010/500 //N
0.002
probofcont02510 = counter02510/500 //N
0.412
probofcont7510 = counter7510/500 //N
0.228
probofcont12510 = counter12510/500 //N
0.09
```

probabilities15 =

0	.504
.25	.404
.5	.276
.75	.182
1.0	.092
1.25	.044
1.5	.03
2.0	.008
2.5	.0
3.0	0

probabilities20 =

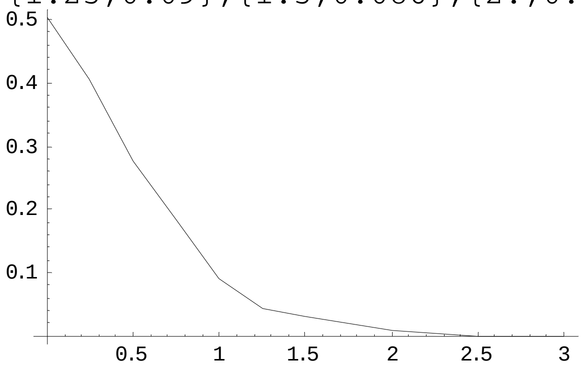
0	.466
.25	.354
.5	.256
.75	.146
1.0	.086
1.25	.044
1.5	.032
2.0	.004
2.5	0
3.0	0

probabilities10 =

0	.516
.25	.412
.5	.308
.75	.228
1.0	.158
1.25	.09
1.5	.086
2.0	.018
2.5	0
3.0	.002

```
ListPlot[probabilities15, PlotJoined → True]

{{0,0.504},{0.25,0.404},{0.5,0.276},{0.75,0.182},{1.,0.092},
{1.25,0.044},{1.5,0.03},{2.,0.008},{2.5,0.},{3.,0}}
```

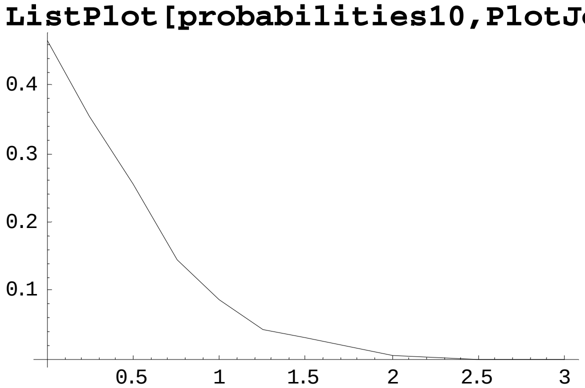
```
{{0,0.466},{0.25,0.354},{0.5,0.256},{0.75,0.146},{1.,0.086},
{1.25,0.044},{1.5,0.032},{2.,0.004},{2.5,0},{3.,0}}

{{0,0.516},{0.25,0.412},{0.5,0.308},{0.75,0.228},{1.,0.158},
{1.25,0.09},{1.5,0.086},{2.,0.018},{2.5,0},{3.,0.002}}
```



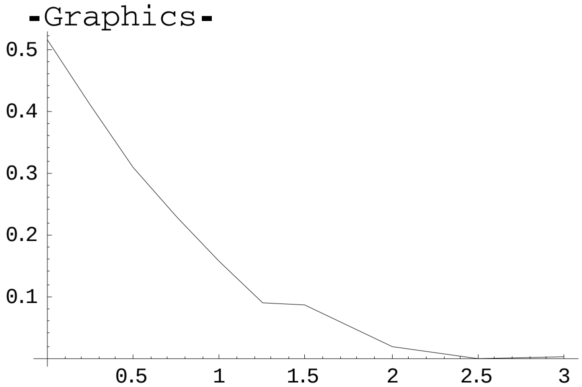
-Graphics-

ListPlot[probabilities20,PlotJoined→True]

ListPlot[probabilities10,PlotJoined→True]

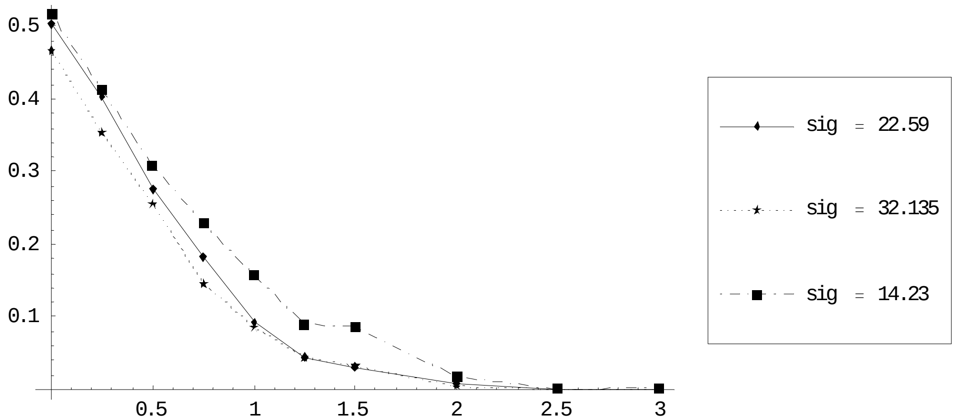


-Graphics-



-Graphics-

MultipleListPlot[probabilities15,probabilities20,probabilities10, PlotJoined→True, PlotLegend→{"sig = 22.59", "sig = 32.135", "sig = 14.23"}]



-Graphics-

```
(*This program is designed to calculate the size of the
convergnce buffer.  The method is as follows:
      1.  Use the actual duration data set generated by the
activity on date networks to simulate the convergence path.
          2. Use a normal distribution to simulate the
critical path (chain) at the point of convergence.
          3.  Move the relative location of the convergence
point by changing the value of the mean of the critical path
(chain) value in relation to the mean of the converging
path.
          4. Simulate the convergence by simulating the
ending values of the  converging path and the critical path.
          5. Compare the two values.  If the ending value of
the converging path is larger than the ending value of the
critical path, then the critical path is delayed at the
point of convergence and a 1 is added to the counter. *)

(*The following section loads the statistical packages used
later in the program.*)
<<Statistics`DataManipulation`
<<Statistics`ContinuousDistributions`
<<Statistics`DescriptiveStatistics`
<<Graphics`Graphics`
<<Graphics`MultipleListPlot`
s=ReadList["C:\\DISSERT\\arena\\aod40.asc",Number];
(*pickdata[r_]:=Module[{rec=StringToStream[r], x},
           Skip[rec, Word, 2];
               x=Read[rec, Number];
               Close[rec];
               x]*)
(*datain = Map[pickdata, s];*)
aoddata = s;
nn =Length[aoddata]


 500
 (*Statistics for aod40.asc *)
(*avgx = Mean[data1] =  344.679*)
(* varx = Variance[data1] =  57.6081 *)


(*stddev = varx^.5 = 7.59   *)
 ndist015 = NormalDistribution[344.679, 15];
aocdata015 = RandomArray[ndist015, 500];
ndist5015 =NormalDistribution[355.97, 15]
aocdata5015 = RandomArray[ndist5015, 500];
ndist115 = NormalDistribution[367.27, 15]
aocdata115 =RandomArray[ndist115, 500];
ndist1515 = NormalDistribution[378.56, 15]
aocdata1515 = RandomArray[ndist1515, 500];
ndist2015 = NormalDistribution[389.86, 15]
aocdata2015 = RandomArray[ndist2015, 500];
ndist25015 = NormalDistribution[401.15, 15]
aocdata25015 = RandomArray[ndist25015, 500];
```

```
ndist3015 = NormalDistribution[412.45, 15]
aocdata3015 = RandomArray[ndist3015, 500];
ndist02515 = NormalDistribution[350.33, 15]
aocdata02515 = RandomArray[ndist02515, 500];
ndist7515 = NormalDistribution[361.62, 15]
aocdata7515 = RandomArray[ndist7515, 500];
ndist12515 = NormalDistribution[372.92, 15]
aocdata12515= RandomArray[ndist12515, 500];
counter25015 = 0
counter7515 = 0
counter12515 = 0
counter015 = 0
counter5015 = 0
counter115 = 0
counter1515 = 0
counter2015 = 0
counter02515 = 0
counter3015 = 0
Do[If[aoddata[[i]]≥aocdata015[[i]], counter015 = counter015
+1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata5015[[i]], counter5015 =
counter5015 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata115[[i]], counter115 = counter115
+ 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata1515[[i]], counter1515 =
counter1515 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata2015[[i]], counter2015 =
counter2015 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata25015[[i]], counter25015 =
counter25015 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata3015[[i]], counter3015 =
counter3015 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata02515[[i]], counter02515 =
counter02515 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata7515[[i]], counter7515 =
counter7515 + 1,], {i,1,500}];
Do[If[aoddata[[i]]≥aocdata12515[[i]], counter12515 =
counter12515 + 1,], {i,1,500}];
 NormalDistribution[355.97,15]
 NormalDistribution[367.27,15]
 NormalDistribution[378.56,15]
 NormalDistribution[389.86,15]
 NormalDistribution[401.15,15]
 NormalDistribution[412.45,15]
 NormalDistribution[350.33,15]
 NormalDistribution[361.62,15]
 NormalDistribution[372.92,15]
 0
 0
 0
 0
 0
 0
```

```
0
0
0
0
probofcont015 = counter015/500 //N

0.504
probofcont5015 = counter5015/500 //N
0.276
probofcont115 = counter115/500 //N
0.092
probofcont1515 = counter1515/500 //N
0.03
probofcont2015 = counter2015/500 //N
0.008
probofcont25015 = counter25015/500 //N
0.
probofcont3015 = counter3015/500 //N
0.
probofcont02515 = counter02515/500 //N
0.372
probofcont7515 = counter7515/500 //N
0.182
probofcont12515 = counter12515/500 //N
0.044
```

probabilities15 =

0	.504
.25	.404
.5	.276
.75	.182
1.0	.092
1.25	.044
1.5	.03
2.0	.008
2.5	.0
3.0	0

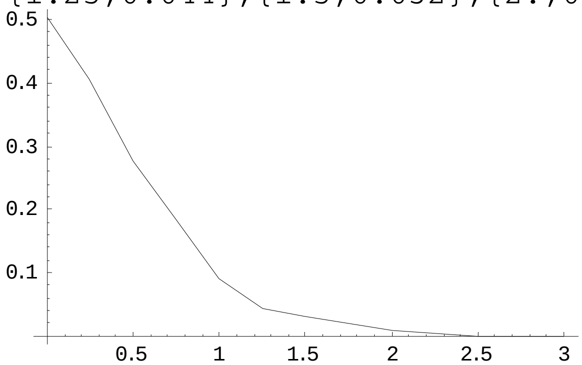
probabilities20 =

0	.466
.25	.354
.5	.256
.75	.146
1.0	.086
1.25	.044
1.5	.032
2.0	.004
2.5	0
3.0	0

```
ListPlot[probabilities15, PlotJoined → True]
```

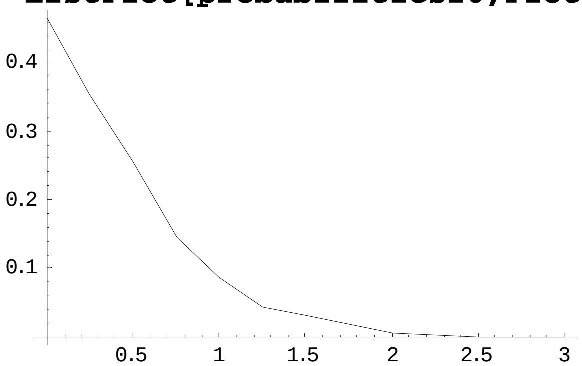
{{0,0.504},{0.25,0.404},{0.5,0.276},{0.75,0.182},{1.,0.092},
{1.25,0.044},{1.5,0.03},{2.,0.008},{2.5,0.},{3.,0}}

{{0,0.466},{0.25,0.354},{0.5,0.256},{0.75,0.146},{1.,0.086},
{1.25,0.044},{1.5,0.032},{2.,0.004},{2.5,0},{3.,0}}



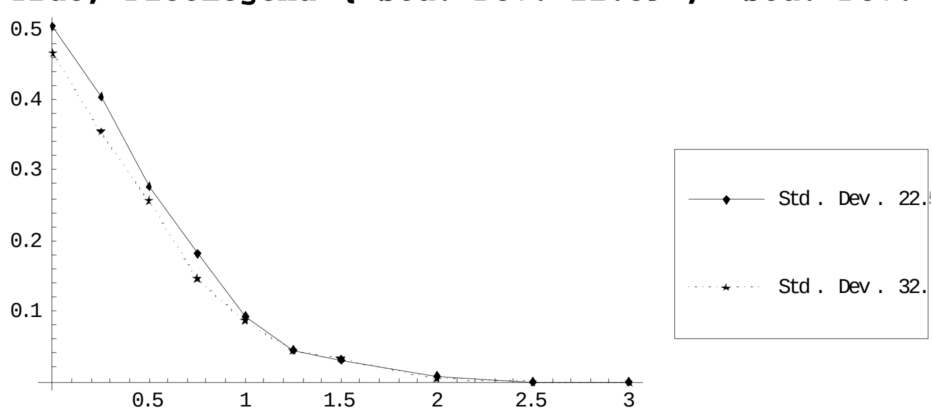
-Graphics-

ListPlot[probabilities20,PlotJoined→True]



-Graphics-

**MultipleListPlot[probabilities15,probabilities20,PlotJoined
→True, PlotLegend→{"Std. Dev. 22.59", "Std. Dev. 32.135"}]**



-Graphics-