A NEW NONPARAMETRIC BIVARIATE SURVIVAL FUNCTION ESTIMATOR UNDER RANDOM RIGHT CENSORING

by

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(Under the direction of Somnath Datta)

ABSTRACT

This thesis examines the efficiency and accuracy of a new nonparametric bivariate survival estimator under right censoring. The estimator and its doubly modified version were simulated in a number of design settings to investigate their finite sample behaviors. Both S-Plus and Fortran 90 codes are employed in the simulation to speed up the running of program. The bias, standard deviation and mean squared error are investigated to study the efficiency and accuracy of estimators with contour and wireframe plots. The results show that, overall, the new bivariate survival estimator and its modified version work fairly well. These new estimators overcome a number of drawbacks in several earlier nonparametric estimators in terms of their finite sample properties, such as monotonicity, simple computation and reduction to the empirical distribution in the uncensored case.

INDEX WORDS: Bivariate survival estimator, Nonparametric estimator, Right censoring, Simulation, Paired failure times

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1.1.1 Introduction

Survival analysis examines and models the time it takes for events to occur. The prototypical event is death, from which the name 'survival analysis' and much of its terminology derives. Survival techniques were primarily developed in the medical and biological sciences, but they are also widely employed in a variety of disciplines, for example, "event-history analysis" in sociology and 'reliability and failure time analysis' in engineering.

Imagine that a researcher in a hospital is observing the survival situation of some severe disease patients. The major interest here is the number of days that the respective patients survive. Some patients have large probability to survival the entire study period, especially those coming into trial very late or being relatively healthy. But, others, who left during the study due to certain reasons, fail to keep the contact with the researcher, these patients are called 'dead' or 'censored' in statistical study. This kind of method is applied usually to compare the effectiveness of a new treatment with the conventional method. Those survivors reflect the success of new treatment method if they are majority in study.

The survival function gives the probability that a subject will survive beyond time, it ranges from 1 to 0, being non-increasing and non-negative. The probability at time 0 is 1, but as time goes to infinity, the survival curve falls monotonically to 0. The survival function is smooth, theoretically, but in practice, we observe events on a discrete time scale in days, weeks, years or other units. The survival function is the complement of the cumulative distribution function, we will never know the exact true function**,** it can only be estimated.

Some other relevant functions are the density function, the cumulative distribution function, the hazard function and the cumulative hazard function. They have broad utilizations in survival analysis.

1.1.2 Discussion of Survival Analysis Assumptions

1). Random sample. The sample is assumed to be randomly collected from the population and represent the same population. All subjects in sample should be identical and independent to each other, the correlation between any two subjects is as little as possible.

2). $(C_1, C_2) \perp (T_1, T_2)$. The pair of censoring time (C_1, C_2) is statistically independent of the pair of failure time (T_1, T_2).

1.1.3 Data Censoring

A nearly universal feature of survival data is censoring, the most common form is right-censoring. Data where the event beyond a particular temporal point is unobserved, they supposed to be missing values. In all, there are three common types of censorings (Figure 1).

1) *Left censoring*: The patient experiences the event in question before the beginning of the study observation period, such as ‘dies or leaves’, but the patient is still in the study;

2) *Interval censoring*: The survival time is between two certain times, which is only roughly known;

3) *Right censoring*: The patient ‘dies or leaves’ after the study period. The patient does not experience event during the study observation period or the patient is lost to follow-up within the study period. The patient may experience event multiple times after study observation ends, referred as multiple censorings.

1.1.4 Univariate Survival Analysis

In addition to the intuitive life table, the classical methods for univariate survival analysis are the Kaplan-Meier estimator and the Cox proportional regression model (Cox, 1972). Both are based on the hazard function.

The Kaplan-Meier Estimator of survival function (Figure 2), also named the Product Limit Estimator in some literature, can be shown in the curve in Appendix section. If the time that the curve covers is broken up into intervals, then the percentage surviving at the start of any interval is equal to the probability of surviving each of the preceding intervals all multiplied together. Differently, the proportional hazard model keeps in consideration a baseline hazard function. Its dependence on covariates in paired failure times exhibits the effect on hazard function. This model would be explained later in bivariate analysis section of this review.

1.2 Bivariate Survival Function

1.2.1 Bivariate Failure Times

In recent years, a large quantity of research effort has been contributed to develop the methodology for multivariate failure time data; the particular emphasis is put on bivariate failure time analysis. The application of bivariate failure time data arises in various areas where the subject units in study are paired; the paired failure times are correlated due to their natural relationship.

In biomedical research, study subjects from the same cluster (e.g. family) share common genetic and/or environmental factors. Moreover, in these studies, either or both failure times might not be observed due to censoring (Figure 3). In twin studies, for example, the correlation exists due to similar gene structure, and the same educational and living environments. Even fathers and sons have correlation to some degree, which constructs the paired failure times in study.

The paired units such as the eyes, ears, lungs, legs, breasts and kidneys from the same person are also good sources for bivariate studies. Bivariate data are also recorded in hospitals when two related diseases happened in one patient or two recurrence times of a certain disease are encountered. For example, an HIV-infected person will develop AIDS and eventually expire, and the times to both events are important response variables in AIDS research. Some other examples are the onset of blindness of diabetic retinopathy, the durability of dental crowns within a set of teeth, the onset of disease within a family, or the onset of smoking within pupils of different classes.

In addition to the biomedical field, bivariate failure times and survival analysis also occur in many other fields, such as demography, economics, engineering, ecology and food science. In economic studies, the time of unemployment is a kind of censoring example. Reduced healthy condition accompanied by lower outcomes can be treated as bivariate problems. In engineering, the analysis of failure time on machine in a production line is a similar example. An obvious illustration in food preservation is that fish deterioration can be a severe consequence of protein oxidation and bacterial infection together. Genetic scientists may put out a lot of examples, like coexisted genes. The bivariate survival analysis provides the most reasonable resolution to the relationship between red face and alcoholic reaction in body.

An example for Bivariate Survival Data with Right Censoring is the Ventilation Tube Study for Otitis Media Patients. As an illustration, the ventilation tubes duration data set from Le and Lindgren (1992) is a frequently cited example involving bivariate failure times. The inflammation of middle ear or otitis media (OM) is a very common childhood disease; the usual surgical intervention is to install ventilating tubes inside two ears of the patient. 40 of 78 patients were randomly chosen to receive the surgery, and then followed by prednisone and sulphamethoprim treatment for two weeks. No surgery was performed in control group. Whether the surgery can prolong the life of tubes was investigated. Thus, the survival time is the lifetime of a functioning tube. Since each patient had one tube in each of his or her two ears, the survival times of the two tubes in one subject are correlated. 12 out of 156 survival times were censored before the trial was terminated due to several reasons; all others were uncensored.

1.2.2 Four Cases of Bivariate Survival Estimators

In general, there are four bivariate survival estimators proposed in the statistical literatures. They are:

i. Path-dependent estimator, adopting empirical functions of observed and censored data, such as Campbell estimator (1982), Lin-Ying estimator (1993) , Wang-Wells estimator (1997) and Tsai and Crowley estimator (1998);

ii. Using conditional survival function such as in Langberg and Shaked (1982) and Akritas (1986);

iii. Using several identifiable sub-survival functions, as in papers of Tsai, Leurgans & Crowley (1986) and Dabrowska (1988);

iv. Using copula censoring model, such as models in Oakes (1982).

1.2.3 Bivariate Right Censored Data Structure

1 *Survival times*: $T = (T_1, T_2) \in \Re_+^2$, positive bivariate lifetime vector with bivariate distribution F_0 and survival function S_0; $F_0(t) \equiv \Pr(T \le t)$ and $S_0(t) \equiv \Pr(T > t)$;

2 *Censoring times*: : $C = (C_1, C_2) \in \Re_+^2$, positive bivariate censoring vector with bivariate distribution G_0 and survival function H_0 ; $G_0 \equiv \Pr(C \le t)$ and $H_0 \equiv \Pr(C > t)$;

3 *Censoring indicator*: $\delta = (\delta_1, \delta_2)$ bivariate censoring indicator vector 0 or 1; δ_i =1 when $C_i > T_i$ or 0 when $C_i < T_i$, i =1,…,n;

4 T and C are independent: $(T, C) \in \Re^4$, such that $T \perp C$ and (T_i, C_i), i =1,…,n are n independent copies of (T, C);

5 *The observed data*: iid copies of $Y_i = ((Y_{1i}, \delta_{1i}), (Y_{2i}, \delta_{2i}))$, i =1,…,n, where $Y_{1i} \equiv T_{1i} \wedge C_{1i}$, $Y_{2i} \equiv T_{2i} \wedge C_{2i}$ and $\delta_{1i} \equiv I(T_{1i} \le C_{1i})$, $\delta_{2i} \equiv I(T_{2i} \le C_{2i})$;

6 *Parameter of interest*: the typical of interested parameter is survival function $S(t_1, t_2)$ at a given point (t_1, t_2).

Based on this assumption, a classification of region for observations $Y_i = (T_{1i}, T_{2i}, C_{1i}, C_{2i})$ is generated in four groups (Figure 3).

1.2.4 Analysis of Censored Paired Data

Traditionally, there are two main approaches to analyze the censored paired data, the non-parametric estimation of the bivariate survival function and the extension of the Cox proportional hazard model. With the development of smoothing theory and applications, much effort has been put into spline survival analysis in recent years.

1.2.4.1 Non-parametric Estimation of Bivariate Survival Function

When no exact distribution of data can be exploited, non-parametric estimation plays the major role in estimation of the survival estimator for paired failure times.

Non-parametric estimation of the bivariate survival function in the presence of censoring is of great importance in applications. This estimator is not only useful in prediction of joint survival experience, but also plays key roles in estimating the degree of dependence, model building and testing, and strengthening marginal analyses.

Non-parametric estimators have been proposed by Campbell and Foldes (1982), Tsai and coworkers (1986), Dabrowska (1988), Pruitt (1991b), Prentice and Cai (1992a), van der Laan (1996b) and Wang and Wells (1997) among others. Dabrowska and Prentice and Cai gave the main contributions to this typical non-parametric estimation of bivariate survival function. Their estimators have been proved to have good practical performance (Bakker, 1990, Prentice and Cai 1992a, Pruitt 1993, van der laan 1997).

By defining a bivariate hazard and applying it to the estimation of joint survival function, Dabrowska extended the estimation from the normal univariate Kaplan-Meier estimator. The consistency of the estimates was confirmed but the covariance of estimates was ignored in her study.

In contrast, Prentice and Cai (1992a) worked out the bivariate survival function directly from marginal survival functions by applying the Volterra structure suggested by Bickel (Dabrowska, 1988). However, this estimator is still related to Dabrowska estimator. The covariance is

$$\operatorname{cov}\left(\int_0^{T_1} h_1(t)dt, \int_0^{T_2} h_2(t)dt\right)$$

with two paired failure times T_1 and T_2 and the hazard h_i, $i=1,2$. In their estimation, consistency was shown, but estimates of standard errors were not.

Based on smooth functions of the data, these estimators hold consistency, asymptotic normality, and correctness of the bootstrap and consistent estimation of the variance of the influence curve.

One year later, Lin and Ying (1993) proposed a simple estimator of the bivariate survival function with covariance estimation based on univariate censoring. They assumed that there is only one censoring time influencing both failure times.

Dabrowska, Prentice-Cai and Pruitt estimators are not, non-parametrically efficient in general. On the contrary, van der Laan's (1996b) SOR-NPMLE (Sequence of Reductions - Non-Parametric Maximum Likelihood Estimator) is non-parametrically efficient. However, it is computationally challenging. Furthermore, the SOR-NPMLE needs a choice of bandwidth. van der Lann removed the inconsistency of NPMLE due to the singly censored observations by replacing them with interval censoring, a small predetermined interval around uncensored T_i. The further reduction by van der Laan (1997) based on the discretization of the C_i, was made to facilitate factorization of the join likelihood into a F part and a G part so that one could avoid estimating G.

Recently, Quale and coworkers (2001) proposed a new estimator of the bivariate survival function based on the locally efficient (LE) estimation theory. Their estimator is:

$$\hat{S}_{LE}(t_1,t_2) = \hat{S}_0(t_1,t_2) + \frac{1}{n}\sum_{i=1}^{n} IC(Z_i \mid F_n, G_n, \hat{S}_0(t_1,t_2))$$

where $\hat{S}_0(t_1,t_2)$ is a consistent initial estimator of $\hat{S}_{LE}(t_1,t_2)$ and F_n and G_n are estimators of F and G respectively, and $IC(Z_i \mid F_n, G_n, \hat{S}_0(t_1,t_2))$ is an estimate of the efficient influence curve of the parameter $\hat{S}(t_1,t_2)$.

This LE estimator is guaranteed to be asymptotically normal; being efficient if the user proves the estimator for F is consistent and may overcome the 'curse of dimensionality' by guessing a lower dimensional (semi-parametric or parametric) model for F (with non-parametric estimator, such as Dabrowska for G). Thus, it could work well even for small parametric estimator. The proposed estimator is consistent if either one of the models of F and G is correctly specified and locally efficient, or if both are correctly specified. The practical founding is that it is better to parameterize F than G .

Wang and Wells (1997) estimated the nonparametric estimators of bivariate survival function under simplifications of three different censoring schemes: univariate censoring, independent censoring and the copula censoring model. The joint survival functions of observables were in the following forms respectively:

$$\hat{S}(t_1,t_2)=\frac{\hat{H}(t_1,t_2)}{\hat{G}(t_1,t_2)}=\begin{cases}\dfrac{\hat{H}(t_1,t_2)}{\hat{G}_1(t_1)\wedge\hat{G}_2(t_2)} & \text{(Unvariate Censoring)}\\[2ex] \dfrac{\hat{H}(t_1,t_2)}{\hat{G}_1(t_1)\hat{G}_2(t_2)} & \text{(Independent Censoring)}\\[2ex] \dfrac{\hat{H}(t_1,t_2)}{C_{\hat{\alpha}}\{\hat{G}_1(t_1),\hat{G}_2(t_2)\}} & \text{(The Copula Censoring)}\end{cases}$$

where, $\hat{S}(t_1,t_2)$ is a common joint survival function of independent and identically distributed pairs of bivariate failure times, $\hat{H}(t_1,t_2)$ is the joint survival function of observables, and $\hat{G}(t_1,t_2)$ is a common joint survival function of independent and identically distributed pairs of bivariate censoring times. $C_{\hat{\alpha}}$: [0, 1] x [0, 1], a copula function, describes the local dependence structure. The strong consistency and weak convergence of three $\hat{S}(t_1,t_2)$ were established.

1.2.4.2 Extension of the Cox Proportional Hazards Model to Paired Data

The hazard function of the Cox proportional model (Cox, 1972) is

$$h(t \mid z) = h_0(t)\exp(\sum \beta z)$$

where h_0 is the baseline hazard, z is a covariate vector and β is unknown. In this model, all observations are assumed to be independent. It is further assumed that hazards are proportional according to a covariate within a pair, the baseline hazard h_0 depends on the pair and the effect of the covariate β is constant across pairs.

Clayton (1978) and Oakes (1982) presented one more parameter to describe the association within a pair in their fully parametric model for paired survival data. A few years later, Huster, Brookmeyer and Self (1989) extended the Clayton-Oakes model with censoring, the correlation within a pair was also considered.

In a model proposed by Wei, Lin and Weissfeld (1989), the multivariate failure times were assumed to follow the distribution of Cox proportional hazards model with respect to some covariates. The different baselines hazards exist for different strata, the regression parameters vary by stratum. The parameter estimators for covariate effects are asymptotically jointly normal distributed, and the covariance matrix allows the consistent estimation.

In addition to the consistency and asymptotic normality of the estimated coefficient in the Cox proportional model, Lee, Wei and Amato (1992) proposed a correct variance-covariance estimate taking the correlation within a pair into account. The dependence of members in pairs was ignored, however. In their model, a common baseline hazard function is assumed for different failure times.

A nested structure was considered by Spiekerman and Lin (1998), allowing both of the above two assumptions to be included. It is safe to say that the models presented by Wei, Lin and Weissfeld (1989) and Lee, Wei and Amato (1992) can be regarded as special cases of this model.

For the afore-mentioned three works, consistent regression parameter estimates with robust variance estimates were derived based on the quasi-partial likelihood method under the independence working model assumption. The common theme is that no baseline hazard function is estimated, it is regarded as nuisance.

Alternatively, Liang, Self and Chang (1993) proposed a pseudo-likelihood approach to estimate the regression parameters under unstratified proportional hazard model. Cai and Prentice (1995, 1997) applied a weighted procedure to the estimation under stratified and unstratified marginal proportional hazard models respectively. Lipsitz and Parzzen model (1996) adopted a 'one-step' jackknife estimator of variance, which equals asymptotically to the variance presented by Wei, Lin and Weissfeld (1989) and Lee, Wei and Amato (1992). In 1994, Parzen, Wei and Ying proposed a resampling procedure by taking the data as fixed constant and generating random deviates to mimic the distributional behavior of the estimating functions.

1.2.4.3. Spline Applications

The development of smoothing theory and methods provided many people opportunity for new trials of splines in survival analysis. Spline methods approximate density functions, survival functions, hazard functions or baseline hazard functions in the presence of censoring.

Abrahamowicz, Ciampi and Ramsay (1992) applied B-splines to density function estimation; O'Sullivan (1988) used smoothing splines in the estimation of log hazard functions. Linear B-splines and their tensor products were tried by Kooperberg, Stone and Troung in 1995 to estimate the conditional log-hazard function, which is a function of survival time t and a covariate vector z. The Cox proportional model was utilized as a submodel. Considering that cubic spline approximate models much better than linear splines, Kooperberg and co-workers put forward a model using cubic B-splines to estimate the log hazard function. The only drawback is that the estimation and model selection procedure are rather complicated, and estimating covariates for such models was out of consideration. Nonetheless, this method holds hope for the future.

1.2.5 Disadvantages of Survival Estimators

Many estimators are not proper bivariate distributions, having no nice formula relating the hazard and survival functions, since there is no canonical way to define past, present and future at 'time' t. They either don't perform completely satisfactorily or depend on the choice of smoothing parameters. Although certain efficiency results were obtained, most were obtained under very restrictive conditions. None of these bivariate estimators have been proved to be the best in all regards. Moreover, estimation of the marginal distribution has not been discussed explicitly in most literatures.

In comparison of bivariate survival models with discrete and continuous frailties, Begun, Yashin and Iachine (1999) indicated that the available sample size of bivariate survival data alone does not allow one to make reliable conclusions about the nature of the genetic influence on life span of twins. Additional information about the tails of the bivariate distribution or about risk factors may help to understand it more.

CHAPTER 2

CONSTRUCTION OF THE NEW BIVARIATE SURVIVAL ESTIMATORS

In this section we introduce a new estimator of the bivariate survival function, called 'Modified Bivariate Survival Estimator (MBV)'. It is based on the construction of a preliminary estimator, named 'Weighted Bivariate Survival Estimator (BV)'. For added estimation efficiency, the basic modification is applied twice leading to a Doubly-Modified Bivariate Survival Estimator (MMBV).

In terms of notation, suppose we are interested in the survival function of (T_1, T_2), assuming T_j is censored by C_j, $(j = 1, 2)$ and $\delta_j (j = 1, 2)$ is the censoring indicator. An underlying assumption is that the censoring variables are independent of the failure time variables.

Akritas and Van Keilegom (2003) put forward the idea of a weighted bivariate survival function estimator. The bivariate estimator is computed by a weighted average of conditional density estimators obtained by kernel smoothing and the Kaplan Meier technique. This method adopts the approach of considering only those pairs with the conditional variable uncensored, which is accomplished by using an "inverse probability of censoring" weighting. In the following description of the construction of conditional bivariate survival estimator, $\hat{g}_1$ and $\hat{g}_2$ are two conditional bivariate survival estimators for paired failure times T_1 and T_2 separately. They are the basis for the construction of the other bivariate survival estimators, including BV, MBV and MMBV. Of this computation, bandwidth h may vary to choose the appropriate conditional bivariate survival estimators.

For s, $t_1 \geq 0$, the conditional estimator

$$\hat{g}_1(t_1, T_{i2}) = P(T_1 > t_1 \mid T_2 = s),$$

$$\approx \prod_{t \leq t_1} (1 - \frac{P\{T_1 \in [t, t+dt), \delta_1 = 1, T_2 \in [s-h, s+h], \delta_2 = 1\}}{P\{T_1 \geq t, C_1 \geq t, T_2 \in [s-h, s+h], \delta = 1\}})$$

where h is a bandwidth tending to zero with the sample size, therefore

$$:= \prod_{t \leq t_1} (1 - \frac{dN(t; s, h)}{Y(t; s, h)})$$

where, $N(t; s, h) = \sum_{i=1}^{n} I(T_{i1} \leq t, \delta_{i1} = 1, T_{i2} \in [s-h, s+h], \delta_{i2} = 1)$

and $Y(t;, s, h) = \sum_{i=1}^{n} I(T_{i1} \wedge C_{i1} \geq t, \delta_{i1} = 1, T_{i2} \in [s-h, s+h], \delta_{i2} = 1)$

then, the joint survival functin is

$$\hat{S}(t_1, t_2) := \hat{P}(T_1 > t_1, T_2 > t_2) = \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{g}_1(t_1; T_{i2}) \delta_{i2}}{\hat{K}_2(T_{i2})} I_{I[T_{i2} > t_2]}$$

where, $\hat{K}_2$ is a Kaplan Meier of C_2 censored by T_2.

In the same way, $\hat{g}_2(T_{i1}, t_2)$ is obtained.

BV treats the two failure times symmetrically by interchanging their roles on evaluation of conditional distribution and averaging the two conditional distributions again. Therefore, the proposed estimator is in practice a weighted average of two bivariate estimators:

$$\hat{S}_W(t_1, t_2) = \frac{1}{2n} \{ \sum_{i=1}^{n} (\frac{\hat{g}_1(t_1, T_{i2}) \delta_{i2}}{\hat{K}_2(T_{i2})} I_{[T_{i2} > t_2]} + \frac{\hat{g}_2(t_2, T_{i1}) \delta_{i1}}{\hat{K}_1(T_{i1})} I_{[T_{i1} > t_1]}) \} \qquad \text{(BV)}$$

where, $\hat{K}_i$ is a Kaplan-Meier of C_i censored by T_i; $\hat{g}_i$ are the conditional survival estimators obtained using Kaplan-Meier; $I_{[T_i > t]}$ is the indicator of whether T_i is greater than t.

As far as MBV is concerned, the details behind its theoretical statistical justification are beyond the scope of this thesis. We discuss the construction in an intuitive fashion here including its explicit mathematical formula. The MBV consists of four cases overall corresponding to (i) both failure times are uncensored, (ii) the first failure time is uncensored but the second component is censored, (iii) the first component is censored but the second component is not and finally (iv) both coordinates of the pair are right censored. In case (i), it is easy to determine whether the term contributes to the empirical proportions of tail values since both coordinates are known. For terms like (ii) and (iii) we use conditional univariate estimators for T_1 and T_2 separately, based on the Kaplan-Meier technique where censored evaluation is conditioned by uncensored variable. The final part of the formulae corresponds to the situation where both the failure times within a pair are censored. In essence, for the last case, we depend on an existing (or preliminary) bivariate survival function estimator, namely BV mentioned previously, to impute this needed proportion. These lead to the following precise definition of the modified estimator:

$$\begin{aligned}\hat{S}_M(t_1,t_2) &= n^{-1}\sum_{i=1}^{n}\delta_{i1}\delta_{i2}I(T_{i1}>t_1,T_{i2}>t_2)+n^{-1}\sum_{i=1}^{n}\delta_{i1}\delta_{i2}^{-}I(T_{i1}>t_1)\frac{\hat{g}_2(t_2\vee C_{i2};T_{i1})}{\hat{g}_2(C_{i2},T_{i1})}\\ &+n^{-1}\sum_{i=1}^{n}\delta_{i1}^{-}\delta_{i2}I(T_{i2}>t_2)\frac{\hat{g}_1(t_1\vee C_{i1};T_{i2})}{\hat{g}_1(C_{i1},T_{i2})}+n^{-1}\sum_{i=1}^{n}\delta_{i1}^{-}\delta_{i2}^{-}\frac{\hat{S}_W(t_2\vee C_{i2};T_{i1}\vee C_{i2})}{\hat{S}_W(C_{i1},C_{i2})}\end{aligned} \qquad \text{(MBV)}$$

where $\hat{s}_w$ is the weighted estimator (MBV) and $\hat{g}_1(t_1,t_2)$ is the Kaplan-Meier estimator at time t_1 based on the subset of samples whose T_2 values lie in a small neighborhood of t_2; $\hat{g}_2$ is defined similarly. Moreover, $\delta_i^{-}=1-\delta_i$, where $i=1$ or 2.

Estimation of a bivariate survival function under right censoring following this approach is praiseworthy in its efficiency. It is to be noted that unlike most other estimators, this estimator puts mass even on doubly censored data points. Moreover, it is not too difficult to code this estimator due to its closed form expression. To speed up our simulation, we have combined Fortran 90 with S-Plus.

As a further step, the modified bivariate survival estimator serves as the basis for a more precise estimator, namely, the doubly modified bivariate survival function estimator. The procedure replaces the preliminary estimator (BV) needed in the fourth term of the MBV with MBV itself again; the resulting estimator is knows as the MMBV estimator.

$$\hat{S}_{MM}(t_1,t_2) = n^{-1}\sum_{i=1}^{n}\delta_{i1}\delta_{i2}I(T_{i1} > t_1, T_{i2} > t_2) + n^{-1}\sum_{i=1}^{n}\delta_{i1}\bar{\delta}_{i2}I(T_{i1} > t_1)\frac{\hat{g}_2(t_2 \vee C_{i2};T_{i1})}{\hat{g}_2(C_{i2},T_{i1})}$$
$$+n^{-1}\sum_{i=1}^{n}\bar{\delta}_{i1}\delta_{i2}I(T_{i2} > t_2)\frac{\hat{g}_1(t_1 \vee C_{i1};T_{i2})}{\hat{g}_1(C_{i1},T_{i2})} + n^{-1}\sum_{i=1}^{n}\bar{\delta}_{i1}\bar{\delta}_{i2}\frac{\hat{S}_M(t_2 \vee C_{i2};T_{i1} \vee C_{i2})}{\hat{S}_M(C_{i1},C_{i2})} \quad \text{(MMBV)}$$

The simulation in the next chapter obtains finite sample performances and explicit comparisons on bivariate survival functions: the weighted estimator, the modified estimator and the doubly-modified estimator. Overall, based on simulations with various sample sizes and correlation associations, we conclude that these newly developed estimators deserve to hold a serious place in the list of nonparametric bivariate survival estimates.

The new estimators overcome several drawbacks found in existing bivariate survival estimators in the statistical literature. Unlike some others, this estimator can be calculated in a closed form. This estimator has several desirable small sample properties such as non-negativity and monotonicity; in addition, it reduces to the empirical survival function in the case of uncensored data.

CHAPTER 3

SIMULATIONS

The simulation study was conducted to examine the properties of the bivariate survival function $\hat{S}(t_1, t_2)$ introduced in Chapter 2. This includes comparing performances across five choices of correlation within the bivariate data from weak to moderate to strong. I considered scenarios with the sample sizes covering small, medium and large values. However, to relieve some of the computational burden and to present the results explicitly, the bandwidth choice fixed at $h = 0.5$ throughout the study. We estimated the bivariate survival function on a time grid of five percentiles (0.1 to 0.9) of each marginal distribution. For each simulation setting we compute bias, standard deviation and mean squared error of the estimators. In addition, we looked at the contour plot and the wireframe plot for visual evaluation of functions. The weighted estimator seems to work well and the corresponding modified estimator works even better. The doubly-modified estimator may be worthy of the extra effort and will be investigated further in the future.

3.1 Parameters Applied in Simulations

The relevant parameters employed in the simulations are listed in the Table 1 of Appendices.

3.1.1 Simulation Times

Fortran 9.0 and S-Plus were integrated to run the simulations. I set the simulation times at 200, which is a good compromise between computational burden and acceptability of results. The computational speed of the doubly modified survival estimator was still unacceptably slow with 200 simulations.

3.1.2 Bandwidth

Bandwidth, *h,* a smoothing parameter, is needed in the construction of our bivariate survival estimate. Optimal bandwidth selection is a hard problem even in the univariate context (e.g., density estimation or regression). Some acceptable criteria, like cross-validation (CV) and asymptotic consideration, is not suitable for such a complicated complex. The necessary theoretical development behind this simulation is beyond the scope of this thesis. Moreover, it was confirmed by Akritas and Van Keilegom (2003) that the conditional bivariate survival estimate is not sensitive to choices of bandwidth. As a result, we used a somewhat arbitrarily chosen bandwidth, h=0.5, for the entire simulation.

3.1.3 Correlation Coefficients

The paired failure times with a variety of correlations are inspected. They are all positive, since negative correlations are rare in practice. The correlation relationship was taken on a range from the independence to strong correlation, 0, 0.1, 0.2, 0.5 and 0.8. It is widely known that the association between two failure times is capable of affecting the estimation of bivariate survival. Thus, the association coefficient is an essential design parameter in this simulation study.

3.2 Simulations

3.2.1 Sample Generation

The failure times were randomly produced from a bivariate log normal distribution with correlation coefficient from 0 to 0.8. Both components of normal distribution are 1.5 and 0.2. The correlation coefficient varies from 0 to 0.8 at five stages. The bivariate censored times were independently generated from a Gamma distribution. No correlation exists between paired censoring times as well as between the failure time and censoring time. The degree of censoring was maintained at the high level (Table 2). This depends on the selection of parameters in random number generation mechanism. When the two groups of random numbers (T_i, C_i) are close to each other on average, the censoring proportion is about to be relatively high, even up to 90% or above sometimes. By adjusting the mean values and standard deviations of two groups, the censoring rate is controlled accordingly.

3.2.2 Simulation Diagram

After obtaining the random numbers for paired failure times T_i and censoring times C_i, the samples are entered into a baseline computation of Kaplan-Meier estimator. This is the basis for BV. As a sequence of averageness, BV can be generated from the conditional distributions of T_1 and T_2. Next, there are four parts: univariate part, conditional bivariate parts for both paired failure times and bivariate in case of censoring in both sides, together bring MBV into existence based on BV. If we put MBV back into the calculation tunnel in same way we calculated MBV with BV, finally, MMBV is drugged out to be a more accurate and efficient estimator than MBV. All these are shown in the following diagram:

Samples-->Kaplan-Meier Estimator-->BV-->MBV-->MMBV-->Diagnostics

3.2.3 Accuracy Diagnostics

In each simulation process, the performance of the proposed bivariate survival function was evaluated on its accuracy and efficiency. The measurements consist of bias, standard deviation (SD) and mean squared error (MSE). Smaller values of these quantities give an indication of more accurate estimation. In addition, the contour plots are also presented to verify that the estimator in a suitable visual angle. Same as contour plots, the wireframe plots provide a much nice visual confirmation on the estimation accuracy of the new estimators.

CHAPTER 4

RESULTS

The simulation results from the combination of five correlation coefficients (0, 0.1, 0.2, 0.5 and 0.8) and three sample sizes (30, 50, 100 and 150) are shown in form of Tables and Figures in Appendices. For each of these combinations, three performance measurements, bias, standard deviation and mean squared error, are calculated separately. All of these statistics are listed for each of three estimators, BV, MBV and MMBV, at five quantile points (0.1, 0.3, 0.5, 0.7 and 0.9). In the visual evaluation, the contour plots and wireframe plots were employed for single samples.

4.1 BV and MBV Estimations

4.1.1 Bias

A 5 by 5 table gives all the biases generated from five quantile cutoffs of the failure times. All values are less than 0.05; some are even lower than 0.005. It is not surprising that most biases in BV are smaller than those in MBV especially in small sample of size 50 or lower. Obviously, the small samples can not produce enough statistical evidences. Given the sample size up to 100 or 150, the biases of MBV became smaller than those in BV in most cases.

It is not hard to see that, in bias tables of BV and MBV, the values closer to small quantiles of two failure times, like $\tau = 0.1$, are substantially lower than those near large quantiles of failure times, such as $\tau = 0.9$. The values increase gradually in table from upper left to lower right and values in lower right corner are the largest.

Stronger association between two failure times may weaken the biases of estimators under most circumstances, but not always. Where the association coefficient decreased from 0.8

to 0.1 gradually, the generated biases for estimators climb up, as a matter of interest, from 0.1 to 0.9.

The effect of sample size on estimation is considered. As the sample size increased, the bias of MBV becomes closer and closer to the bias of BV; eventually they are even smaller than the bias of BV. This can be seen clearly, for example, in simulation with 150 replications at association coefficient 0.8.

4.1.2 Standard Deviation (SD)

Concerning the SD, MBV has the smaller values than BV.

There is also a pattern appearing in SD tables. We find that the values in lower left and upper right corners of table tend to be larger, while values on the other two corners tend to be smaller. In particular, it is interesting to note that the larger quantile points in both time axes are with larger bias but smaller standard deviation. Perhaps it is a reflection of the edge effect in kernel smoothing.

The values are asymmetric in tables with nonzero association coefficients. The magnitude of the SD decreases with the sample size at around $n^{1/2}$ rate provided the bandwidth is fixed.

4.1.3 Mean Squared Error (MSE)

The mean squared error (MSE) is a comprehensive measurement of bias and variance. It is considered to be the ultimate marker of estimation accuracy. All MSE values in BV and MBV are less than 0.01 and MSE of MBV is noticeably smaller than that of BV. This indicates that although both the methods produce good bivariate survival function estimates, MBV is more accurate.

As was the case with SD, the variation of association strength has a certain effect on the overall behavior of MSE. For example, when T_1 is fixed at a small quantile, say (0.1, 0.1), the MSE value rises up with the increased quantile of T_2, but for large quantile points of T_1 (0.9, 0.9), the opposite variation occurs. This characteristic is more transparent in MBV. The on-diagonal entries of the MSE tend to be smaller than the off-diagonal ones.

It is safe to say that MSE's of both BV and MBV drop off with the sample size increasing.

4.2 MMBV Estimation

Since MBV has smaller MSE compared to BV, we hope that the same modification to MBV will lead to further improvement. The simulation based on 200 simulation times, 50 individuals and association coefficient 0.5 is performed (Table 4). The variation pattern in biases of MBV and BV is followed by MMBV as well; but most values for MMBV are much less than those for MBV, which indicates a definite achievement. Both SD and MSE continue to be lower for MMBV as compared to MBV and BV. On the other side, the time for computing MMBV is unusually long by the present method, which is why complete simulations for MMBV were not attempted.

4.3 Contour Plots

The contour plots of true bivariate survival function, estimated BV, MBV and MMBV are listed in Figure 4. The estimated bivariate survival functions were drawn on the conditions of n=50 samples, 200 simulation times and correlation coefficient ρ=0.5. These plots give quite illustrative visual evidence for the advantages of the developed estimators.

The true bivariate survival function contains the really nice and smooth curves centered at quantile point (0.1, 0.1). The pronounced curves extend from the center in a parallel manner; the outer line is more linear.

Taking the true survival function as the reference, the common characteristics of BV, MBV and MMBV are that the estimators near the origin or small quantile points, say (0.1, 0.1), are closer to the true survival function. Of the three bivariate survival estimators, the MMBV demonstrates the smallest departure from the true contours.

4.4 Wireframe Plots

The wireframe plots (Figure 5) draw 3D surfaces. The similar specifications are set up for wireframe plots, just replacing 5 points with 25 points to make the plots nicer. Comparisons among true bivariate survival estimator, true function, BV and MBV were made for coefficient at 0.5. Then, MBV plots at 0.2 and 0.8 were drawn to compare with MBV at 0.5 respectively. Due to huge calculation in MMBV, its efficiency confirmation is left for future study.

From these surface plots, we reach the same conclusion as that from contour plot. By naked eyes, it is really hard to tell a big difference between BV and MBV. The MBV plot is overall closer to the true surfaces and the BV plot seems to be the worst. Given the large sample and simulation times, the result should be much clearer.

CHAPTER 5

DISCUSSIONS

5.1 Censoring Rate

The specification of censoring rate in simulation plays a key role in the results. It affects not only the simulation results, but also the running time of program in computer. I control it at a high level throughout although it is likely that other censoring rates will be considered for future publications.

Suppose the mean of first censoring value (C_1) is close to the mean of the second censoring value (C_2) and they are very close to the survival times (T_1, T_2), the ratio of censored times is very high, as a result, MMBV employs MBV and MBV employs BV more often, the running time of simulation program increases remarkably.

In the same situation, if censoring times are far from the survival times, thus a knowledge of survival times are censored, BV and MBV are called by MBV and MMBV rarely, the running time does not go up sharply compared to the program without MMBV. In a compromise with a large quantity of calculations on MMBV, the variance, bias and mean square error of the three estimators would not change a lot.

Otherwise, provided that the mean values of two censoring times (C_1, C_2) does not approach to each other too much, the issue changes definitely and walks into a more complex text.

It is easy to verify that the expected value of the proposed estimating function is centered at the place a little higher or lower than the censoring point. The censoring rate in this study is controlled at high level by keeping the failure and censoring times in an appropriate distance.

In this simulation project, it is easy to see the variations of correlation effect on censoring rate (Table 2). When correlation coefficient becomes stronger, the single censoring rate will drops down, but the double censoring rate goes up, total censoring rate decreases gradually. Therefore, in case of bivariate survival data, the more mass of doubly-censored points are counted in simulation, the evaluation of new estimator becomes better and better.

5.2 Simulation Times

Specifying the simulation times at 200, the simulations of BV and MBV for sample sizes at 30, 50 and 100 take 10min, 1h 40min and 3 hours respectively. With the addition of MMBV into the simulation, the running time rises exponentially due to the quantity of calculations in program. For example, the running time increased from 3 hours to 4 days for the same simulation times (200) and sample size (50). But, the improvement is obvious without any doubtness.

Together with simulation times, sample size and addition of MMBV, the sequence points of failure times also contribute to the running speed. Increasing the points from 5 to 25 increases the running time from nine hours to more than two days or more. Furthermore, S-Plus is well known to be slower in performance than Fortran language. Therefore, the proposed program is suggestive of total Fortran form. It is assumed that all the above parameters are balanced at the desirable cutoff in simulation.

5.3 Estimator Performances and Comparisons

The new estimators considered here have the excellent properties. MBV approaches the true survival function more than BV, and the MMBV works even better in spite of some limitations. The small bias, SD and MSE of MBV indicates its advantage over BV, thus for MMBV over MBV too. The contour plot is another evidence to illustrate the better simulation performance of MMBV against MBV, and MBV against BV. In 3D angle, the wireframe plot disclosed the visual evidence more actively.

The best estimation of bivariate survival function should belong to the small quartile point of paired failure times, (T_1, T_2) = (0.1, 0.1). As the contour curve moves outward, the estimation of BV is furthest from the true value compared to the other two. The assessment at the large quartile of paired failure times does not give very good suggestion, as in other places on plot, such as somewhere close to axes of T_1 and T_2. For these reasons, the small quartile point is the best place to show the improvement in estimation.

Correlation between paired failure times has an apparent influence on the behavior of the bivariate survival function. Strong association between paired times drags the estimate of bivariate survival function close to the true value. In contrast, the lower association leads to impaired estimation. Unfortunately, provided the complete independence of two failure times happens, the estimation of bivariate survival function was turned to be less accurate. These are shown in the investigation of biases, standard deviation and MSE of three estimators. One phenomenon worthy of attention is that the effect of association between paired failure times on simulated estimators has a little uncertainty; probably the larger sample is a requirement for future clarification.

The bias, SD and MSE of simulation alone are short of determination of better estimation of bivariate survival function. Only combined with contour plot or other criteria, might the overall evaluation lead to the more sensible conclusion.

One aspect of simulation without any doubt is its sample size. The larger sample always beats the smaller one in the efficiency and accuracy in simulation. Samples less than 50 individuals will not benefit much from these new proposed estimators, in my opinion.

CHAPTER 6

CONCLUSIONS

Based on the investigations of bias, standard deviation, mean square error, contour plot and wireframe plot, we can conclude that both the modified bivariate survival estimator (MBV) and the doubly-modified bivariate survival estimator (MMBV) work fairly well in simulation. This proves them to be prospective estimation method and helps to fill in the blank in this field of research. These two estimators have the ability of predicting the true bivariate survival function with great efficiency and accessibility. It is the case with most survival estimators, the proposed new estimators work best with large sample sizes and at small quantiles.

Besides the excellent performance, these estimators overcome some disadvantages shown in some previous estimators with respect to consistency, monotonicity and easy calculation.

REFERENCES

Abrahamowicz, M., Ciampi, A. and Ramsay, J.O. 1992. Nonparametric density estimation for censored survival data: Regression-spline approach. The Canadian Journal of Statistics 20, 171-185.

Abrahamowicz, M., MacKenzie, T., and Esdaile, J. M. 1996. Time-dependent hazard ratio: Modeling and hypothesis testing with application in lupus nephritis. Journal of the American Statistical Association 91, 1432-1439.

Akritas, M.G. 1986. Bootstrapping the Kaplan-Meier Estimator. Journal of the American Statistical Association, 81, 1032-1038.

Akritas, M.G. and Ingrid Van Keilegom 2003. Estimation of bivariate and marginal distributions with censored data. Journal of Royal Statistical Society (B), 65, part 2, 457-441.

Bakker D.M. 1990. 'Two nonparametric estimators of the survival function of bivariate right censored observations. Technical Report BSR9035, centre for mathematics and computer science, Amsterdam.

Begun A.Z., Yashin A.I., Iachine I.A. 1999. 'Comparison of bivariate survival models with discrete and continuous frailties. 20th Annual Conference of the International Society for Clinical Biostatistics.

Cox, D.R. 1972. Regression models with life tables. Journal of the Royal Statistical Society, 34, 187-220.

Cai, J. and Prentice, R.L. 1995. Estimating equations for hazard ratio parameters based on correlated failure time data. Biometrika, 82,151-164.

Cai, J. and Prentice, R.L. 1997. Regression estimation using multivariate failure time data and a common baseline hazard function model. Lifetime Data Analysis, 3, 197-213.

Campbell, G. and Foldes, A. 1982. Large sample properties of nonparametric bivariate estimators with censored data. Biometrika, 68, 417-42.

Clayton, D. G. 1978. A Model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. Biometrika, 65, 141-151.

Dabrowska D.M. 1988. Kaplan Meier estimate on the plan. The Annals of Statistics, 18, 308-325.

Diabetic Retinopathy Study Research Group. 1981. 'Diabetic retinopathy study', Investigative Opthalmology and Visual Science, 21, 149-226.

Huster, W. J., Brookmeyer, R. and Self, S.G. 1989. Modeling Paired Survival Data with Covariates. Biometrics, 45, 145-156.

Kooperberg, C., Stone, C., and Troung, Y. 1995. Hazard regression. Journal of the American Statistical Association. 90, 78-94.

Koziol, J.A. and Green, S.B. 1976. A Cramer-von Mises statistic for randomly censored data. IEEE Trans. Reliability 48, 68-72.

Langberg, N.A. and Shaked, M. 1982. On the identifiability of multivariate life distribution functions. Annual of Probability, 10, 773-779.

Le, C.T. and Lindgren, B.R. 1992. Duration of ventilating tubes: a test for comparing two clustered samples of censored data, Biometrics, 52, 328-334.

Lee, E.W., Wei, L.J. and Amato, D.A. 1992. Cox-type regression for large numbers of small groups of correlated failure time observations. In Survival Analysis: State of the Art, Ed.

J.P. Klein and P.K. Goel, Dordrecht the Netherlands: Kluwer Academic Publisher, 237-247.

Liang, K.Y., Self, S.G. and Chang, Y.C. 1993. Modeling marginal hazards in multivariate failure time data. Journal of the Royal Statistical society B, 55(2), 441-453.

Lin, D.Y. and Ying, Z. 1993. A simple nonparametric of the bivariate survival function under univariate censoring. Biometrika 80, 573-81.

Lipsitz, S.R. and Parzzen, M. 1996. A jackknife estimator of variance for Cox regression for correlated survival data. Biometrics, 52, 291-298.

Oakes, D. 1982. A model for Association in Bivariate Survival Data. Journal of the Royal Statistical Society, Series B, 44, 414-428.

O'Sullivan, F. 1988. Nonparametric estimation of relative risk using splines and cross validation. SIAM journal on scientific and statistical computing, 9, 531-542.

Parzen, MI, Wei, LJ, and Ying, Z. 1994 A resampling method based on pivotal estimating functions. Biometrika, 81: 341-350

Prentice, R.L. and Cai, J. 1992a. Covariance and survivor function estimation using censored multivariate failure time data. Biometrika, 79, 495-512.

Pruitt, R.C. 1991b. Strong consistency of self-consistent estimators: General theory and an application to bivariate survival analysis. Technical Report 543, University of Minnesota.

Pruitt, R.C. 1993. Small sample comparisons of six bivariate survival curve estimators. Journal of Statistical Computing and Simulation, 45, 147-167.

Quale C.M., Mark J. van der Laan and Robins J.M. 2002. Locally efficient estimation with bivariate right censored data, http://www.bepress.com/ucbbiostat/paer119.

Spiekerman CF, Lin DY (1998). Marginal regression models for multivariate failure time data. Journal of the American Statistical Association, 93:1164–1175.

Tsai, W.Y., Leurgans, S., and Crowley, J. 1986. Nonparametric estimation of a bivariate survival functions in the presence of censoring. The Annals of Statistics, 14, 1351-1365.

Tsai W.Y. and Crowley J. 1998. A note on nonparametric estimators of the bivariate survival function under univariate censoring, Biometrika, 85(3), 573-580.

Wang W.J. and Wells M.T. 1997. Nonparametric estimators of the bivariate survival function under simplified censoring conditions, Biometrika, 84(4), 863-880.

Wei Pan and Kooperberg, Charles. 1999. Linear regression for bivariate censored data via multiple imputation, Statistics in Medicine, 18, 3111-3121.

Wei, L.J., Lin, D.Y. and Weissfeld, L, 1989. Regression Analysis of Multivariate Incomplete Failure Time Data by Modeling Marginal Distributions. Journal of the American Statistical Association, 84, 1065-1073.

van der Laan, M.J. 1996b. Efficient estimation of the bivariate censoring model and repairing NPMLE. The Annals of Statistics, 24, 596-627.

van der Laan, M.J. 1997. Nonparametric estimators of the bivariate survival function under random censoring. Statistica Neerlandica, 51, 178-200.

APPENDICES

Table 1. Parameters Applied in Simulation

Parameters	Level	Values
Association Coefficients (ρ)	5	0, 0.1, 0.2, 0.5, 0.8
Failure Time Quantiles (τ)	5	0.1, 0.3, 0.5, 0.7, 0.9
Sample Size (n)	4	30, 50, 100, 150
Simulation Times	1	200
Diagnostics	4	Bias, SD, MSE and plots
Bandwidth (h)	1	0.5
Estimators	3	BV, MBV and MMBV

*SD is the standard deviation of estimator from the real value; MSE is the mean square error; BV is the weighted bivariate survival estimator from Akrita and Keilegom; MBV is the modified bivariate survival estimator; MMBV is the doubly-modified bivariate survival estimator.

Table 2. Effect of Correlation Coefficient on Censoring Rate

	Censoring Rate (%)		
Correlation Coefficient	Singly Censoring	Doubly Censoring	Total
0	49.9	28.7	78.6
0.1	47.1	30.1	77.2
0.2	44.4	31.5	75.9
0.5	36.6	35.3	71.9
0.8	27.7	39.6	67.3

* When correlation becomes stronger, the double censoring rate increases, more mass are counted in estimation, therefore, the evaluation of new estimator becomes better.

Table 3. Biases, SD and MSE Result of BV and MBV

Biases, standard deviations and mean squared errors of two bivariate survival estimators: The weighted estimator (BV) and the modified estimator (MBV), are listed in the following tables, where, ‘sample’ is the sample size, only 50, 100 and 150 are shown here; ‘1’ is BV, ‘2’ is MBV; ‘Std’ is Standard deviation; ‘Mse’ is Mean squared error.

Sample=50 and ρ= 0:

Bias1:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.002323696	0.0004305224	-0.000586147	0.005503741	0.01135896
	0.3	0.006321591	0.0054021805	0.005249287	0.010706079	0.01530922
	0.5	0.015109291	0.0167508048	0.016803381	0.022896836	0.02743952
	0.7	0.018741144	0.0193278418	0.021322765	0.028042171	0.03086665
	0.9	0.019973726	0.0208712223	0.022693144	0.026982080	0.03136652

Bias2:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.005949585	0.003992355	0.003839928	0.01200165	0.02166955
	0.3	0.013272258	0.011650576	0.011814756	0.01891643	0.02580398
	0.5	0.021040399	0.022643373	0.022471201	0.03006264	0.03567874
	0.7	0.025767594	0.025493283	0.027139788	0.03448411	0.03721262
	0.9	0.030160767	0.029540029	0.029887895	0.03414461	0.03741106

Std1:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.07949553	0.1052714	0.12510120	0.12325521	0.12155069
	0.3	0.09707816	0.1059513	0.11747617	0.11025040	0.11227414
	0.5	0.10601326	0.1064119	0.11114530	0.10273086	0.10565069
	0.7	0.11711330	0.1100179	0.10845294	0.09888831	0.09618934
	0.9	0.11700077	0.1069765	0.09846593	0.08635888	0.08327217

Std2:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.06880322	0.09397204	0.11419477	0.11304575	0.11295375
	0.3	0.08457491	0.09557847	0.10865956	0.10229015	0.10698931
	0.5	0.09464491	0.09561201	0.10302520	0.09627411	0.10092740
	0.7	0.10688027	0.09954024	0.09857958	0.09134211	0.09090708
	0.9	0.11165936	0.10035036	0.09136899	0.08136666	0.07961076

Mse1:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.006324939	0.01108224	0.01565065	0.015222138	0.014903597
	0.3	0.009464132	0.01125486	0.01382821	0.012269770	0.012839855
	0.5	0.011467103	0.01160408	0.01263563	0.011077894	0.011914995
	0.7	0.014066754	0.01247750	0.01221670	0.010565262	0.010205138
	0.9	0.014088131	0.01187958	0.01021052	0.008185889	0.007918112

Mse2:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.004769281	0.008846683	0.013055190	0.012923381	0.013228120
	0.3	0.007329068	0.009270980	0.011946489	0.010821105	0.012112557
	0.5	0.009400358	0.009654378	0.011119146	0.010172466	0.011459313
	0.7	0.012087361	0.010558167	0.010454502	0.009532535	0.009648877
	0.9	0.013377484	0.010942808	0.009241579	0.007786388	0.007737460

Sample=50 and ρ= 0.1:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	-0.002790686	-0.0007723648	-0.0001630791	0.004229096	0.02337557
	0.3	-0.001879741	0.0009922674	0.0021830245	0.007655802	0.02630582
	0.5	-0.004027839	-0.0016729827	-0.0011101856	0.005158703	0.02304319
	0.7	0.000740686	0.0025060045	0.0059691104	0.012534445	0.03048679
	0.9	0.025200613	0.0226574366	0.0206478955	0.023203944	0.03661025

Bias2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t2 \ t1	0.1	0.3	0.5	0.7	0.9
	0.1	-0.00027402568	0.001989064	0.004038089	0.01010463	0.02917281
	0.3	0.00002572992	0.002814522	0.004206348	0.01103185	0.02967810
	0.5	0.00202673015	0.003109216	0.002903322	0.01039979	0.02751145
	0.7	0.00852574899	0.008834515	0.011828072	0.01859868	0.03511519
	0.9	0.03430804735	0.029338037	0.026517926	0.02858054	0.04033573

Std1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.08496312	0.1015004	0.1228970	0.13058240	0.11911939
	0.3	0.10081201	0.1037081	0.1221683	0.12463172	0.11300831
	0.5	0.11379843	0.1106339	0.1169682	0.11292205	0.10240543
	0.7	0.11903978	0.1113360	0.1095045	0.10162198	0.09210269
	0.9	0.11387012	0.1072126	0.1034760	0.09285301	0.08314164

Std2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.07243777	0.08812836	0.10965256	0.11529918	0.10585912
	0.3	0.08854525	0.09208994	0.10954381	0.11006644	0.10000059
	0.5	0.10120610	0.09849613	0.10346896	0.09883719	0.09203071
	0.7	0.10828251	0.10176876	0.09862839	0.08941522	0.08304389
	0.9	0.10576027	0.09998476	0.09622242	0.08394409	0.07503147

Mse1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.00722652	0.01030292	0.01510369	0.017069650	0.014735846
	0.3	0.01016660	0.01075635	0.01492985	0.015591677	0.013462874
	0.5	0.01296631	0.01224265	0.01368280	0.012778002	0.011017860
	0.7	0.01417102	0.01240199	0.01202687	0.010484139	0.009412350
	0.9	0.01360148	0.01200789	0.01113361	0.009160105	0.008252843

Mse2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t2 \ t1	0.1	0.3	0.5	0.7	0.9
	0.1	0.005247305	0.007770564	0.012039991	0.013396005	0.012057206
	0.3	0.007840262	0.008488479	0.012017540	0.012236324	0.010880908
	0.5	0.010246782	0.009711154	0.010714255	0.009876946	0.009226532
	0.7	0.011797790	0.010434928	0.009867463	0.008340993	0.008129364
	0.9	0.012362277	0.010857672	0.009961955	0.007863458	0.007256693

Sample=50 and ρ= 0.2:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.001388009	0.0006391774	0.001515629	0.001045055	0.01434941
	0.3	0.001391645	0.0009630596	0.001937269	0.002090579	0.01604797
	0.5	0.015916378	0.0150235537	0.014752706	0.013656421	0.02438800
	0.7	0.017170574	0.0154159187	0.015346367	0.016781070	0.02756782
	0.9	0.020171149	0.0188793579	0.020020963	0.024092317	0.03301460

Bias2:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.0008562972	0.003250068	0.005973096	0.007193825	0.02216410
	0.3	0.0040896863	0.005020895	0.006602981	0.007803728	0.02248218
	0.5	0.0170316343	0.017997781	0.018252984	0.017345391	0.02831626
	0.7	0.0191284649	0.018349689	0.018077317	0.019877599	0.03079739
	0.9	0.0243066883	0.022769173	0.023471674	0.027428870	0.03552979

Std1:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.07498642	0.1021200	0.1157370	0.11850925	0.11774159
	0.3	0.08827634	0.1028076	0.1105904	0.11375421	0.11253809
	0.5	0.10334945	0.1093671	0.1083747	0.10696006	0.10681223
	0.7	0.10357395	0.1050491	0.1045114	0.09733884	0.09645730
	0.9	0.10481828	0.1043503	0.1009247	0.09162004	0.08784696

Std2:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.06611873	0.08986529	0.09980070	0.10642802	0.10785422
	0.3	0.07827332	0.08913467	0.09475996	0.10174131	0.10185748
	0.5	0.09274473	0.09740636	0.09429461	0.09573219	0.09710110
	0.7	0.09352203	0.09334857	0.09279167	0.08795381	0.08737236
	0.9	0.09705631	0.09605097	0.09185988	0.08405690	0.08084156

Mse1:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.005624890	0.01042891	0.01339735	0.014045535	0.014068989
	0.3	0.007794649	0.01057032	0.01223399	0.012944390	0.012922360
	0.5	0.010934440	0.01218687	0.01196271	0.011626953	0.012003626
	0.7	0.011022391	0.01127297	0.01115814	0.009756455	0.010063996
	0.9	0.011393747	0.01124541	0.01058664	0.008974671	0.008807053

Mse2:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.004372419	0.008086333	0.009995858	0.011378675	0.012123780
	0.3	0.006143437	0.007970199	0.009023050	0.010412193	0.010880395
	0.5	0.008891661	0.009811918	0.009224644	0.009465514	0.010230433
	0.7	0.009112268	0.009050667	0.008937083	0.008130992	0.008582409
	0.9	0.010010743	0.009744225	0.008989157	0.007817905	0.007797723

Sample=50 and ρ= 0.5:

Bias1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	-0.007121340	-0.010666511	-0.010335047	-0.004957156	0.008708517
0.3	-0.010404065	-0.014177085	-0.014338946	-0.007439232	0.006055411
0.5	-0.007800112	-0.011819202	-0.013598818	-0.006014854	0.006383238
0.7	0.001228078	-0.006645094	-0.009651901	-0.003268787	0.011665121
0.9	0.014983912	0.007780344	0.003845007	0.005513949	0.016754156

Bias2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	-0.003404466	-0.0059495635	-0.002387513	0.0040312671	0.02001414
0.3	-0.006097900	-0.0090929938	-0.007459085	0.0007196102	0.01502303
0.5	-0.001826010	-0.0060232589	-0.006444343	0.0024195666	0.01463409
0.7	0.009314873	0.0007143042	-0.001607782	0.0058032538	0.01976063
0.9	0.025762882	0.0175751732	0.013773177	0.0158256460	0.02540635

Std1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.07379979	0.1037394	0.12121059	0.1261015	0.11888222
0.3	0.09740614	0.1107298	0.11977827	0.1249966	0.11658001
0.5	0.10732461	0.1131457	0.11749187	0.1186741	0.11112759
0.7	0.11311359	0.1141225	0.11007565	0.1092210	0.10371832
0.9	0.10432495	0.1026976	0.09902493	0.0956639	0.09004421

Std2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.06492549	0.08973755	0.1075804	0.11261016	0.1068421
0.3	0.08873032	0.09662998	0.1067933	0.11288714	0.1049916
0.5	0.09956220	0.10132737	0.1056276	0.10754720	0.1011695
0.7	0.10583231	0.10514578	0.1005904	0.09936999	0.0945625
0.9	0.09882723	0.09753673	0.0943859	0.09041268	0.0855738

Mse1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.005497123	0.01087564	0.01479882	0.015926174	0.014208820
0.3	0.009596201	0.01246209	0.01455244	0.015679501	0.013627567
0.5	0.011579414	0.01294164	0.01398927	0.014119723	0.012390087
0.7	0.012796192	0.01306810	0.01220981	0.011939908	0.010893566
0.9	0.011108212	0.01060732	0.00982072	0.009181985	0.008388661

Mse2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.004226909	0.008088226	0.011579239	0.012697300	0.011815790
0.3	0.007910254	0.009420036	0.011460447	0.012744024	0.011248937
0.5	0.009915966	0.010303515	0.011198711	0.011572254	0.010449422
0.7	0.011287245	0.011056145	0.010121021	0.009908073	0.009332549
0.9	0.010430548	0.009822301	0.009098399	0.008424905	0.007968358

Sample=50 and ρ= 0.8:

Bias1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.0015820973	0.009452882	0.003644968	0.006576117	0.0032877909
0.3	0.0004254655	0.011316062	0.006772921	0.010799555	0.0004682468
0.5	0.0001953908	0.01067258	0.01222933	0.01672921	0.006285127
0.7	0.0018529713	0.01082449	0.01575675	0.01918273	0.010813090
0.9	0.008633902	0.01353292	0.01686297	0.01717069	0.01273141

Bias2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.001191652	0.002288697	0.0028134286	0.002173214	0.013468941
0.3	0.003034050	0.003950600	0.0009799658	0.002366590	0.008947642
0.5	0.005040324	0.001276129	0.002597636	0.007202223	0.002878603
0.7	0.005280081	0.001723513	0.006481018	0.009217620	0.001228997
0.9	0.001419023	0.003774967	0.007078932	0.007972787	0.004202174

Std1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.06503163	0.09262955	0.1024439	0.1154602	0.1088610
0.3	0.08218018	0.09080006	0.1030584	0.1142250	0.1070059
0.5	0.09760712	0.09627305	0.1045381	0.1085429	0.10396809
0.7	0.11179267	0.10784726	0.1095950	0.1050558	0.09432648
0.9	0.10243600	0.10264150	0.1017297	0.0962958	0.08958365

Std2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.05875711	0.08246749	0.08968209	0.1034712	0.1003728
0.3	0.07387712	0.08102445	0.09072741	0.1025569	0.0980978
0.5	0.08854637	0.08578609	0.09090581	0.0960245	0.09435898
0.7	0.10277590	0.09812260	0.09820809	0.0937624	0.08557904
0.9	0.09459576	0.09473213	0.09353654	0.0877416	0.08183446

Mse1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.004231616	0.008669590	0.01050804	0.01337431	0.01186153
0.3	0.006753762	0.008372705	0.01066690	0.01316398	0.01145048
0.5	0.009527188	0.009382405	0.01107776	0.01206143	0.010848866
0.7	0.012501034	0.011748200	0.01225934	0.01140470	0.009014408
0.9	0.010567670	0.010718410	0.01063330	0.009567714	0.00818732

Mse2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.003453817	0.006806125	0.008050793	0.01071102	0.010256111
0.3	0.005467034	0.006580570	0.008232423	0.01052351	0.009703239
0.5	0.007865864	0.007360882	0.008270613	0.009272576	0.008911903
0.7	0.010590765	0.009631014	0.009686833	0.008876353	0.007325282
0.9	0.008950372	0.008988427	0.008799196	0.007762154	0.006714537

Sample=100 and ρ= 0:

Bias1:

	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.012293687	0.012073539	0.01173584	0.001618101	0.001889551
0.3	0.008147136	0.009607541	0.01103261	0.003466071	0.004814457
0.5	0.009937159	0.010914723	0.01493422	0.008109547	0.009064429
0.7	0.005248513	0.006505502	0.01312012	0.007762182	0.010127575
0.9	0.008476013	0.008790411	0.01493659	0.011815342	0.014382937

($T_2=F_2^{-1}(t_2)$)

Bias2:

	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.012432113	0.012140462	0.01267847	0.004304526	0.005466081
0.3	0.007489556	0.008974139	0.01090751	0.005290009	0.007499173
0.5	0.010759465	0.011498678	0.01596381	0.010736336	0.011634734
0.7	0.007634892	0.008495496	0.01508290	0.010912583	0.012604632
0.9	0.011964421	0.011266727	0.01716988	0.014616316	0.016564249

($T_2=F_2^{-1}(t_2)$)

Std1:

	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.05309466	0.06573453	0.07479908	0.08032510	0.07820168
0.3	0.07572074	0.07849434	0.07882633	0.07626125	0.07266232
0.5	0.08153267	0.07942589	0.07648790	0.06968642	0.06833939
0.7	0.08027485	0.07586524	0.07032647	0.06120959	0.06042874
0.9	0.08053242	0.07431296	0.06717259	0.05674977	0.05203698

($T_2=F_2^{-1}(t_2)$)

Std2:

	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.04663763	0.05871172	0.06626536	0.07274558	0.06965712
0.3	0.06782042	0.07123168	0.07043305	0.06946448	0.06528057
0.5	0.07361639	0.07250052	0.06801744	0.06420742	0.06200595
0.7	0.07249041	0.06959540	0.06420700	0.05894354	0.05681284
0.9	0.07366529	0.06905273	0.06332788	0.05502790	0.04951856

($T_2=F_2^{-1}(t_2)$)

Mse1:

	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.002970177	0.004466799	0.005732632	0.006454741	0.006119073
0.3	0.005800006	0.006253666	0.006335309	0.005827792	0.005302992
0.5	0.006746323	0.006427603	0.006073430	0.004921962	0.004752436
0.7	0.006471599	0.005797856	0.005117950	0.003806865	0.003754200
0.9	0.006557314	0.005599687	0.004735258	0.003360138	0.002914716

($T_2=F_2^{-1}(t_2)$)

Mse2:

	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.002329626	0.003594457	0.004551841	0.005310448	0.004881993
0.3	0.004655703	0.005154487	0.005079788	0.004853299	0.004317790
0.5	0.005535139	0.005388545	0.004881215	0.004237861	0.003980105
0.7	0.005313151	0.004915693	0.004350033	0.003593425	0.003386576
0.9	0.005569722	0.004895219	0.004305225	0.003241707	0.002726462

($T_2=F_2^{-1}(t_2)$)

Sample=100 and ρ= 0.1:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.0024425982	-0.0003837144	0.006423143	0.01010465	0.01486983
	0.3	0.0002488866	-0.0033226224	0.004560090	0.01093237	0.01640339
	0.5	0.0085424518	0.0041983197	0.009413581	0.01445635	0.01768912
	0.7	0.0123011690	0.0078424961	0.011716439	0.01683369	0.01929397
	0.9	0.0136204123	0.0100243716	0.012330490	0.01619709	0.01943781

Bias2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.001203034	0.001413222	0.008484471	0.01205736	0.01795976
	0.3	0.001504620	-0.002351754	0.005566314	0.01173974	0.01797206
	0.5	0.008570902	0.005558961	0.010595009	0.01514879	0.01898975
	0.7	0.012060900	0.008612620	0.012255683	0.01672342	0.02000290
	0.9	0.015473928	0.012414072	0.013925749	0.01691531	0.02043755

Std1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.05313274	0.06515281	0.07470666	0.07859102	0.08299983
	0.3	0.06989588	0.07035257	0.07558026	0.07592576	0.08028584
	0.5	0.07764652	0.07628565	0.07790283	0.07423123	0.07712402
	0.7	0.08452519	0.08145330	0.07962115	0.07503253	0.07182082
	0.9	0.08343392	0.07827643	0.07412229	0.06685636	0.06136770

Std2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.04563361	0.05614916	0.06542057	0.06982165	0.07721005
	0.3	0.06043983	0.06156837	0.06604340	0.06760472	0.07438412
	0.5	0.06959155	0.06850728	0.06985517	0.06746880	0.07183802
	0.7	0.07821794	0.07502196	0.07291702	0.06961788	0.06703512
	0.9	0.07850095	0.07417970	0.06923493	0.06227292	0.05717364

Mse1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.002829055	0.004245036	0.005622341	0.006278652	0.007110084
	0.3	0.004885496	0.004960524	0.005733171	0.005884238	0.006714887
	0.5	0.006101955	0.005837126	0.006157466	0.005719262	0.006261020
	0.7	0.007295827	0.006696145	0.006476803	0.005913254	0.005530487
	0.9	0.007146735	0.006227687	0.005646155	0.004732118	0.004143823

Mse2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.002083873	0.003154726	0.004351838	0.005020442	0.006283945
	0.3	0.003655237	0.003796195	0.004392715	0.004708220	0.005855992
	0.5	0.004916444	0.004724150	0.004991999	0.004781524	0.005521311
	0.7	0.006263512	0.005702472	0.005467094	0.005126321	0.004893823
	0.9	0.006401842	0.005656738	0.004987402	0.004164044	0.003686518

Sample=100 and ρ= 0.2:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.0013932502	-0.0073346085	-0.0084136289	-0.0031188802	0.004524034
	0.3	-0.0004974811	-0.0062576788	-0.0065369611	-0.0021690337	0.003519364
	0.5	0.0009731995	-0.0052152789	-0.0051399418	-0.0005197865	0.004586102
	0.7	0.0050285108	-0.0006974311	-0.0007765757	0.0024905593	0.006402165
	0.9	0.0066074496	0.0042626086	0.0042159717	0.0072097150	0.012007905

Bias2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.0004242299	-0.0036552749	-0.00321219846	0.002269600	0.010072531
	0.3	0.0028127411	-0.0008805324	-0.00049515758	0.003200520	0.008932683
	0.5	0.0048209215	0.0001034276	0.00008181865	0.004574467	0.009436314
	0.7	0.0091720574	0.0048271511	0.00450417142	0.007184090	0.010756281
	0.9	0.0110971229	0.0098475826	0.00944367392	0.011853899	0.016006623

Std1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.05424097	0.06788459	0.07731287	0.08519168	0.08355206
	0.3	0.07157778	0.07139772	0.07795843	0.08137464	0.07878996
	0.5	0.08191312	0.07867197	0.07978140	0.07848704	0.07232995
	0.7	0.08864505	0.08530885	0.07854003	0.07337498	0.06688112
	0.9	0.08681310	0.08159674	0.07390028	0.06778881	0.06155829

Std2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04691925	0.06099553	0.06794244	0.07709996	0.07755200
	0.3	0.06195050	0.06240538	0.06850655	0.07317461	0.07272760
	0.5	0.07137658	0.06941892	0.07025098	0.06987139	0.06586011
	0.7	0.07904282	0.07684178	0.06955069	0.06649004	0.06166327
	0.9	0.08037461	0.07487403	0.06723513	0.06311642	0.05755044

Mse1:

		$T_1=F_1^{-1}(t_1)$				
	t2 \ t1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.002944024	0.004662114	0.006048069	0.007267349	0.007001413
	0.3	0.005123626	0.005136792	0.006120248	0.006626537	0.006220244
	0.5	0.006710706	0.006216478	0.006391491	0.006160486	0.005252655
	0.7	0.007883230	0.007278086	0.006169140	0.005390091	0.004514072
	0.9	0.007580173	0.006676197	0.005479026	0.004647303	0.003933613

Mse2:

		$T_1=F_1^{-1}(t_1)$				
	t2 \ t1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.002201596	0.003733816	0.004626493	0.005949555	0.006115769
	0.3	0.003845775	0.003895207	0.004693392	0.005364768	0.005369097
	0.5	0.005117858	0.004818997	0.004935207	0.004902937	0.004426598
	0.7	0.006331894	0.005927961	0.004857586	0.004472536	0.003918056
	0.9	0.006583223	0.005703095	0.004609745	0.004124197	0.003568265

Sample=100 and ρ= 0.5:

Bias1:

	$T_1=F_1^{-1}(t_1)$					
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.004688332	0.006625752	0.003410993	0.000203666	0.006594565
	0.3	0.009713582	0.013118398	0.007709607	0.003396369	0.005490257
	0.5	0.002070990	0.007289794	0.005602662	0.002469806	0.004624709
	0.7	0.001251035	0.004614033	0.004202489	0.002471422	0.003718430
	0.9	0.0002124346	0.003832204	0.004470948	0.003942739	0.001703898

Bias2:

	$T_1=F_1^{-1}(t_1)$					
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.002272723	-0.001378681	0.001650087	0.006054118	0.01252041
	0.3	-0.006107232	-0.007034097	-0.001420970	0.003322888	0.01128109
	0.5	0.002043502	-0.001395426	-0.000050322	0.003106992	0.00982029
	0.7	0.006599399	0.002055974	0.002416279	0.003561689	0.00967582
	0.9	0.006953801	0.002969724	0.001723424	0.001925555	0.00718503

Std1:

	$T_1=F_1^{-1}(t_1)$					
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04835805	0.05931924	0.07083907	0.07480994	0.08517736
	0.3	0.06379299	0.06714615	0.07096440	0.07242641	0.08226200
	0.5	0.07470627	0.07589361	0.07606413	0.07550614	0.08189037
	0.7	0.07875263	0.07651583	0.07371920	0.06992820	0.07461600
	0.9	0.08385547	0.08271177	0.07562211	0.06988589	0.06790553

Std2:

	$T_1=F_1^{-1}(t_1)$					
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04315413	0.05307875	0.06380972	0.06851466	0.07961370
	0.3	0.05807101	0.06074012	0.06393212	0.06608589	0.07700378
	0.5	0.06805587	0.06893759	0.06864928	0.06905273	0.07577277
	0.7	0.07176271	0.07049891	0.06820548	0.06550922	0.07010727
	0.9	0.07627413	0.07639775	0.06963582	0.0635362	0.06275353

Mse1:

	$T_1=F_1^{-1}(t_1)$					
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.002360481	0.003562672	0.005029809	0.005596569	0.007298672
	0.3	0.004163899	0.004680698	0.005095384	0.005257120	0.006797180
	0.5	0.005585316	0.005812982	0.005817141	0.005707278	0.006727420
	0.7	0.006203542	0.005875962	0.005452182	0.004896061	0.005581373
	0.9	0.007031786	0.006855923	0.005738693	0.004899583	0.004614065

Mse2:

	$T_1=F_1^{-1}(t_1)$					
	t2 \ t1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.001867444	0.002819254	0.004074404	0.004730911	0.006495102
	0.3	0.003409540	0.003738840	0.004089335	0.004378386	0.006056846
	0.5	0.004635777	0.004754339	0.004712726	0.004777934	0.005837951
	0.7	0.005193438	0.004974323	0.004657825	0.004304144	0.005008651
	0.9	0.005866099	0.005845435	0.004852118	0.004040557	0.00398963

Sample=100 and ρ= 0.8:

Bias1:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.007753966	0.006280581	0.006212641	0.007227082	-0.008518748
	0.3	0.012107688	0.016509320	0.017262180	0.014927658	-0.012327307
	0.5	0.011233150	0.018571272	0.023992933	0.021917367	-0.017043912
	0.7	0.008685073	0.015158838	0.021357461	0.022479795	-0.019348898
	0.9	0.010883656	0.014303189	0.017793407	0.020554428	-0.019050467

Bias2:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.0041186798	0.0002897428	0.0002205864	0.0002375267	0.0002313664
	0.3	0.0058075532	0.0067661376	0.0077743476	0.0053108286	0.0034663498
	0.5	0.0033907287	0.0083163616	0.0130888592	0.0111303996	0.0074555154
	0.7	0.0003963155	0.0055567522	0.0108093029	0.0122392583	0.0099191976
	0.9	0.0019765947	0.0049686791	0.0079896732	0.0100563886	0.0092403415

Std1:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.04254481	0.05887262	0.07310033	0.08226180	0.07841149
	0.3	0.06345705	0.06558058	0.07268956	0.07830770	0.07615196
	0.5	0.06739461	0.06812791	0.07125040	0.07706659	0.07471990
	0.7	0.07967523	0.07776053	0.07535288	0.07684791	0.07171101
	0.9	0.07751614	0.07636738	0.07532856	0.07095190	0.06641456

Std2:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.03847181	0.05329613	0.06767373	0.07598353	0.07296261
	0.3	0.05745918	0.05851147	0.06672405	0.07204270	0.07053542
	0.5	0.06092322	0.06147413	0.06423912	0.07049139	0.06880044
	0.7	0.07330163	0.07178276	0.07001526	0.07075947	0.06628482
	0.9	0.07294205	0.07198205	0.07138158	0.06669615	0.06292220

Mse1:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.001870185	0.003505431	0.005382255	0.006819234	0.006220930
	0.3	0.004173394	0.004573369	0.005581755	0.006354930	0.005951084
	0.5	0.004668217	0.004986304	0.005652280	0.006419630	0.005873559
	0.7	0.006423573	0.006276491	0.006134198	0.006410943	0.005516849
	0.9	0.006127206	0.006036559	0.005990997	0.005456656	0.004773814

Mse2:

	$T_1=F_1^{-1}(t_1)$					
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.001497044	0.002840562	0.004579783	0.005773553	0.005323596
	0.3	0.003335285	0.003469372	0.004512539	0.005218356	0.004987261
	0.5	0.003723136	0.003848231	0.004297982	0.005092922	0.004789085
	0.7	0.005373286	0.005183642	0.005018977	0.005156702	0.004492068
	0.9	0.005324449	0.005206103	0.005159166	0.004549507	0.004044587

Sample=150 and ρ= 0:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 / t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.003787223	-0.0081711339	-0.004896328	0.0005632452	0.00164880
	0.3	0.005178242	0.0006935874	0.003933314	0.0077344701	0.00766800
	0.5	0.011521404	0.0083983794	0.011823868	0.0151271557	0.01391570
	0.7	0.013464641	0.0111871045	0.016570562	0.0202838189	0.01829404
	0.9	0.008592854	0.0083435537	0.012966848	0.0166663785	0.01645879

Bias2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 / t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.002751012	-0.0076280789	-0.003432349	0.001347589	0.003046517
	0.3	0.005851379	0.0006989963	0.004850572	0.007771111	0.008264456
	0.5	0.012435557	0.0086608165	0.012864733	0.015181566	0.014293435
	0.7	0.013534616	0.0108461885	0.016527201	0.019817507	0.017787620
	0.9	0.009799572	0.0086068075	0.013124652	0.015874019	0.015197992

Std1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 / t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04637803	0.05975952	0.06978309	0.06864939	0.06572805
	0.3	0.05912079	0.06790377	0.07047333	0.06786855	0.06177916
	0.5	0.06437752	0.06896565	0.06878186	0.06609027	0.06030329
	0.7	0.06848152	0.06914881	0.06560862	0.06199210	0.05580537
	0.9	0.07415159	0.06958161	0.06304854	0.05839387	0.04824878

Std2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 / t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04186128	0.05363106	0.06247339	0.06201766	0.05991576
	0.3	0.05220572	0.06017470	0.06267641	0.06173072	0.05591791
	0.5	0.05879672	0.06245600	0.06163854	0.06000303	0.05450885
	0.7	0.06220294	0.06215366	0.05854583	0.05577086	0.05037046
	0.9	0.06929241	0.06289222	0.05716776	0.05258029	0.04320434

Mse1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 / t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.002165265	0.003637967	0.004893654	0.004713056	0.004322894
	0.3	0.003522082	0.004611404	0.004981962	0.004665962	0.003875463
	0.5	0.004277208	0.004826793	0.004870748	0.004596755	0.003830134
	0.7	0.004871015	0.004906709	0.004579075	0.004254454	0.003448912
	0.9	0.005572295	0.004911215	0.004143257	0.003687613	0.002598836

Mse2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 / t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.001759934	0.002934478	0.003914706	0.003848006	0.003599179
	0.3	0.002759676	0.003621483	0.003951860	0.003871072	0.003195113
	0.5	0.003611697	0.003975762	0.003964811	0.003830844	0.003175517
	0.7	0.004052391	0.003980717	0.003700763	0.003503122	0.002853583
	0.9	0.004897470	0.004029508	0.003440409	0.003016671	0.002097594

Sample=150 and ρ= 0.1:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.00092476267	-0.00002981852	0.0003769303	0.002488250	0.003783715
	0.3	0.00003225365	0.00166004383	0.0008788428	0.004276379	0.006322184
	0.5	0.00421221298	0.00506763096	0.0017577628	0.004392892	0.007441042
	0.7	0.00691988460	0.00686587352	0.0029461558	0.005409697	0.009202119
	0.9	0.01668194303	0.01590362637	0.0104796436	0.010848800	0.013262098

Bias2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.00004761863	0.001326116	0.002981586	0.005065077	0.005909830
	0.3	0.00069132381	0.002507982	0.002547088	0.005353681	0.007460413
	0.5	0.00528401133	0.006830907	0.003613242	0.005605075	0.008622290
	0.7	0.00893197796	0.009086274	0.005049684	0.007120631	0.010541732
	0.9	0.01850654119	0.017812468	0.012227089	0.011827368	0.013890630

Std1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04786369	0.06156914	0.06322413	0.06754350	0.06143685
	0.3	0.06157318	0.06823315	0.06762860	0.06664223	0.06090704
	0.5	0.07157234	0.07134853	0.06855667	0.06526674	0.05923380
	0.7	0.06771587	0.06618035	0.06494668	0.06084760	0.05284336
	0.9	0.06910771	0.06533994	0.06171136	0.05732031	0.04937504

Std2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04227202	0.05509767	0.05645992	0.06264926	0.05693472
	0.3	0.05456953	0.06071184	0.05953780	0.05945459	0.05522068
	0.5	0.06439230	0.06371981	0.06071004	0.05839293	0.05328357
	0.7	0.06184333	0.05940718	0.05822963	0.05524710	0.04786331
	0.9	0.06337361	0.05899816	0.05521621	0.05165709	0.04463303

Mse1:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.002291788	0.003790759	0.003997432	0.004568316	0.003788803
	0.3	0.003791258	0.004658519	0.004574400	0.004459475	0.003749638
	0.5	0.005140343	0.005116294	0.004703107	0.004279045	0.003564012
	0.7	0.004633324	0.004426978	0.004226751	0.003731695	0.002877099
	0.9	0.005054163	0.004522233	0.003918115	0.003403315	0.002613778

Mse2:

		$T_1=F_1^{-1}(t_1)$				
	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.001786926	0.003037512	0.003196613	0.003950585	0.003276488
	0.3	0.002978312	0.003692218	0.003551238	0.003563511	0.003104981
	0.5	0.004174289	0.004106875	0.003698765	0.003441152	0.002913483
	0.7	0.003904378	0.003611773	0.003416190	0.003102945	0.002402025
	0.9	0.004358707	0.003798067	0.003198331	0.002808342	0.002185057

Sample=150 and ρ= 0.2:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.001784637	-0.0010931300	-0.001248650	0.0012335787	0.0006175876
	0.3	-0.001051670	-0.0001168056	-0.001510264	0.0009377303	0.0010461928
	0.5	0.004681819	0.0049781942	0.001225026	0.0028640656	0.0030875967
	0.7	0.006687321	0.0060126623	0.001252744	0.0023797481	0.0041980279
	0.9	0.015404818	0.0140088342	0.008571129	0.0080900774	0.0090089481

Bias2:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	-0.0006796835	0.000706170	0.0017013205	0.003947363	0.003396841
	0.3	0.0001534984	0.001365519	0.0007968125	0.002674327	0.002930902
	0.5	0.0062503778	0.007159896	0.0036134996	0.004663600	0.005023976
	0.7	0.0091899207	0.008692325	0.0039629819	0.004696284	0.006152859
	0.9	0.0175104202	0.016542217	0.0109549151	0.009660972	0.010332505

Std1:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.04565769	0.05818696	0.06227992	0.06680797	0.06164962
	0.3	0.05928861	0.06408329	0.06759527	0.06656532	0.06039861
	0.5	0.07069046	0.06919501	0.06847053	0.06665546	0.06063491
	0.7	0.06719373	0.06419001	0.06610162	0.06351857	0.05511210
	0.9	0.06821477	0.06397275	0.06313487	0.05920805	0.05070890

Std2:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(\alpha_2)$	0.1	0.04120253	0.05260180	0.05559147	0.06120267	0.05696890
	0.3	0.05404100	0.05773912	0.06003795	0.05914501	0.05485576
	0.5	0.06379247	0.06130527	0.06047078	0.05896036	0.05448200
	0.7	0.06133774	0.05730474	0.05911974	0.05703743	0.04998357
	0.9	0.06318031	0.05772169	0.05704649	0.05321620	0.04603389

Mse1:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.002087810	0.003386917	0.003880347	0.004464827	0.003801057
	0.3	0.003516245	0.004106682	0.004571402	0.004431821	0.003649087
	0.5	0.005019060	0.004812732	0.004689714	0.004451154	0.003686126
	0.7	0.004559717	0.004156510	0.004370994	0.004040273	0.003054967
	0.9	0.004890563	0.004288760	0.004059476	0.003571042	0.002652554

Mse2:

		$T_1=F_1^{-1}(t_1)$				
	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
$T_2=F_2^{-1}(t_2)$	0.1	0.001698111	0.002767448	0.003093306	0.003761349	0.003256994
	0.3	0.002920454	0.003335670	0.003605191	0.003505284	0.003017745
	0.5	0.004108547	0.003809601	0.003669773	0.003498073	0.002993529
	0.7	0.003846773	0.003359389	0.003510849	0.003275323	0.002536214
	0.9	0.004298366	0.003605438	0.003374313	0.002925299	0.002225880

Sample=150 and ρ= 0.5:

Bias1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	-0.005914055	4.120453e-003	-0.004067851	-0.004140102	-0.003577162
	0.3	-0.007545709	7.708675e-003	-0.007512085	-0.008661815	-0.005766116
	0.5	-0.004405233	5.795839e-003	-0.007913478	-0.009110999	-0.005027474
	0.7	0.001957807	1.004440e-003	-0.004236414	0.007203274	-0.002980001
	0.9	0.001883846	7.214301e-006	-0.002909357	0.006497441	0.002030010

Bias2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.00280714344	0.0006678108	0.0004033940	0.0007670206	0.0010177872
	0.3	0.00309506024	0.0027893259	0.0019054715	0.0029796154	-0.0007265865
	0.5	0.00002617093	0.0011128204	0.0028807978	0.0038409164	-0.0003962057
	0.7	0.00572689564	0.0031179781	0.0002731021	0.0022992436	0.0012870483
	0.9	0.00663649337	0.0048501977	0.0021492252	0.0012940766	0.0022699213

Std1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.04128808	0.05356375	0.06448368	0.06782885	0.06755896
	0.3	0.05689719	0.05916545	0.06465725	0.06681923	0.06594642
	0.5	0.06465328	0.06206113	0.06485207	0.06752753	0.06553878
	0.7	0.06921029	0.06499215	0.06530963	0.06548186	0.06118326
	0.9	0.06614734	0.06357523	0.06233412	0.06176469	0.05654036

Std2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.03773132	0.04939878	0.05878589	0.06197367	0.06261644
	0.3	0.05232329	0.05388792	0.05873477	0.06074648	0.06030772
	0.5	0.05838846	0.05478276	0.05753637	0.06076258	0.05929507
	0.7	0.06225503	0.05800087	0.05795335	0.05869033	0.05555235
	0.9	0.06122307	0.05816419	0.05648357	0.05635574	0.05231465

Mse1:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.001739682	0.002886054	0.004174692	0.004617893	0.004577010
	0.3	0.003294228	0.003559974	0.004236991	0.004539837	0.004382178
	0.5	0.004199453	0.003885176	0.004268414	0.004642978	0.004320607
	0.7	0.004793898	0.004224988	0.004283294	0.004339761	0.003752272
	0.9	0.004379020	0.004041810	0.003894007	0.003857093	0.003200933

Mse2:

		$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
	0.1	0.001431533	0.002440686	0.003455943	0.003841324	0.003921854
	0.3	0.002747306	0.002911688	0.003453404	0.003699013	0.003637549
	0.5	0.003409213	0.003002389	0.003318732	0.003706844	0.003516062
	0.7	0.003908486	0.003373823	0.003358666	0.003449841	0.003087720
	0.9	0.003792307	0.003406597	0.003195013	0.003177644	0.002741975

Sample=150 and ρ= 0.8:

Bias1:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$: t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.0093070380	0.007511904	0.0004572592	0.005157625	-0.01292620
0.3	0.0092720338	0.014566014	0.0101287988	0.011163536	-0.01524811
0.5	0.0042337037	0.012861582	0.0145185538	0.015423357	-0.01820249
0.7	0.0029920113	0.008885915	0.0141210369	0.017455976	-0.01979071
0.9	0.0005397264	0.002720543	0.0077844215	0.013070121	0.01674401

Bias2:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$: t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	-0.0052833230	0.003629851	-0.003412051	-0.0002083728	-0.006924794
0.3	-0.0037198155	-0.007772070	-0.003525416	-0.0045037046	-0.008973138
0.5	0.0001515472	-0.006804993	-0.008055003	-0.0080950018	-0.011771709
0.7	0.0013077530	-0.003442370	-0.007712718	-0.0107449515	-0.013566048
0.9	0.0055804187	0.002379110	-0.002200967	-0.0065169276	-0.010337838

Std1:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$: t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.03727129	0.04780309	0.05932667	0.06616124	0.06617874
0.3	0.04986998	0.05309995	0.05906237	0.06418565	0.06509240
0.5	0.05795501	0.05598238	0.05619404	0.06061767	0.06307864
0.7	0.06557098	0.06265181	0.06083535	0.06171263	0.06177395
0.9	0.05926685	0.05871967	0.05632078	0.05760167	0.05610240

Std2:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$: t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.03438352	0.04475383	0.05457532	0.06086380	0.06184174
0.3	0.04522639	0.04815396	0.05338389	0.05886866	0.06096056
0.5	0.05227622	0.04965907	0.04988151	0.05488698	0.05871826
0.7	0.05976157	0.05646659	0.05476581	0.05587550	0.05679331
0.9	0.05470113	0.05411596	0.05163044	0.05296118	0.05210525

Mse1:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$: t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.001475770	0.002341564	0.003519862	0.004403910	0.004546713
0.3	0.002572985	0.003031774	0.003590957	0.004244422	0.004469526
0.5	0.003376707	0.003299447	0.003368558	0.003912382	0.004310245
0.7	0.004308506	0.004004208	0.003900343	0.004113160	0.004207693
0.9	0.003512851	0.003455401	0.003232628	0.003488781	0.003427841

Mse2:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$: t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.001210140	0.002014547	0.002991641	0.003704446	0.003872353
0.3	0.002059263	0.002379209	0.002862269	0.003485803	0.003796708
0.5	0.002732826	0.002512331	0.002553048	0.003078109	0.003586407
0.7	0.003573156	0.003200326	0.003058780	0.003237526	0.003409517
0.9	0.003023355	0.002934197	0.002670547	0.002847357	0.002821828

Table 4. Biases, SD and MSE Result of BV, MBV and MMBV

The comparisons among biases, standard deviations and mean squared errors of three bivariate survival estimators: The weighted estimator (BV), the modified estimator (MBV) and doubly-modified estimator (MMBV) are listed in the following tables. The sample size is specified as 50, the number of simulation is 200, and correlation coefficient is fixed at 0.5. The order number 1, 2 and 3 in the following table represent BV, MBV and MMBV. And, 'Std' is Standard deviation; 'Mse' is Mean squared error.

Biases:

Bias1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	-0.007121340	-0.010666511	-0.010335047	-0.004957156	0.008708517
0.3	-0.010404065	-0.014177085	-0.014338946	-0.007439232	0.006055411
0.5	0.007800112	0.011819202	0.013598818	0.006014854	0.006383238
0.7	0.001228078	0.006645094	0.009651901	0.003268787	0.011665121
0.9	0.014983912	0.007780344	0.003845007	0.005513949	0.016754156

Bias2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	-0.003404466	-0.0059495635	-0.002387513	0.0040312671	0.02001414
0.3	-0.006097900	-0.0090929938	-0.007459085	0.0007196102	0.01502303
0.5	-0.001826010	-0.0060232589	-0.006444343	0.0024195666	0.01463409
0.7	0.009314873	0.0007143042	-0.001607782	0.0058032538	0.01976063
0.9	0.025762882	0.0175751732	0.013773177	0.0158256460	0.02540635

Bias3:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	-0.0027497042	-0.004865380	-0.0004693435	0.006688259	0.02363094
0.3	-0.0051565928	-0.007844552	-0.0057208002	0.003208956	0.01805091
0.5	-0.0005421396	-0.004583624	-0.0046048071	0.004955065	0.01736974
0.7	0.0111866611	0.002603233	0.0005212827	0.008535874	0.02237977
0.9	0.0288534239	0.020550232	0.0168770402	0.019271996	0.02832405

Standard deviations:

Std1:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.07379979	0.1037394	0.12121059	0.1261015	0.11888222
0.3	0.09740614	0.1107298	0.11977827	0.1249966	0.11658001
0.5	0.10732461	0.1131457	0.11749187	0.1186741	0.11112759
0.7	0.11311359	0.1141225	0.11007565	0.1092210	0.10371832
0.9	0.10432495	0.1026976	0.09902493	0.0956639	0.09004421

Std2:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.06492549	0.08973755	0.1075804	0.11261016	0.1068421
0.3	0.08873032	0.09662998	0.1067933	0.11288714	0.1049916
0.5	0.09956220	0.10132737	0.1056276	0.10754720	0.1011695
0.7	0.10583231	0.10514578	0.1005904	0.09936999	0.0945625
0.9	0.09882723	0.09753673	0.0943859	0.09041268	0.0855738

Std3:

$T_2=F_2^{-1}(t_2)$	$T_1=F_1^{-1}(t_1)$				
t_2 \ t_1	0.1	0.3	0.5	0.7	0.9
0.1	0.06405201	0.08721445	0.10466285	0.10952838	0.10352378
0.3	0.08741806	0.09413822	0.10398160	0.11015148	0.10186748
0.5	0.09847788	0.09945135	0.10334386	0.10533204	0.09872202
0.7	0.10455301	0.10364033	0.09884642	0.09764151	0.09263330
0.9	0.09791754	0.09694572	0.09426318	0.09019150	0.08550119

Mean squared errors:

Mse1:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.005497123	0.01087564	0.01479882	0.015926174	0.014208820
	0.3	0.009596201	0.01246209	0.01455244	0.015679501	0.013627567
	0.5	0.011579414	0.01294164	0.01398927	0.014119723	0.012390087
	0.7	0.012796192	0.01306810	0.01220981	0.011939908	0.010893566
	0.9	0.011108212	0.01060732	0.00982072	0.009181985	0.008388661

Mse2:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.004226909	0.008088226	0.011579239	0.012697300	0.011815790
	0.3	0.007910254	0.009420036	0.011460447	0.012744024	0.011248937
	0.5	0.009915966	0.010303515	0.011198711	0.011572254	0.010449422
	0.7	0.011287245	0.011056145	0.010121021	0.009908073	0.009332549
	0.9	0.010430548	0.009822301	0.009098399	0.008424905	0.007968358

Mse3:

	$T_1=F_1^{-1}(t_1)$				
$T_2=F_2^{-1}(t_2)$	t_1 \ t_2	0.1	0.3	0.5	0.7	0.9
	0.1	0.004110221	0.007630033	0.010954532	0.012041199	0.011275595
	0.3	0.007668508	0.008923542	0.010844902	0.012143646	0.010702819
	0.5	0.009698187	0.009911581	0.010701158	0.011119391	0.010047745
	0.7	0.011056474	0.010748094	0.009770886	0.009606725	0.009081783
	0.9	0.010420366	0.009820784	0.009170382	0.008505916	0.008112705

Figure 1. Basic Types of Censoring

There are three types of censoring existing in biomedical bivariate survival data, of them, right censoring is most common.

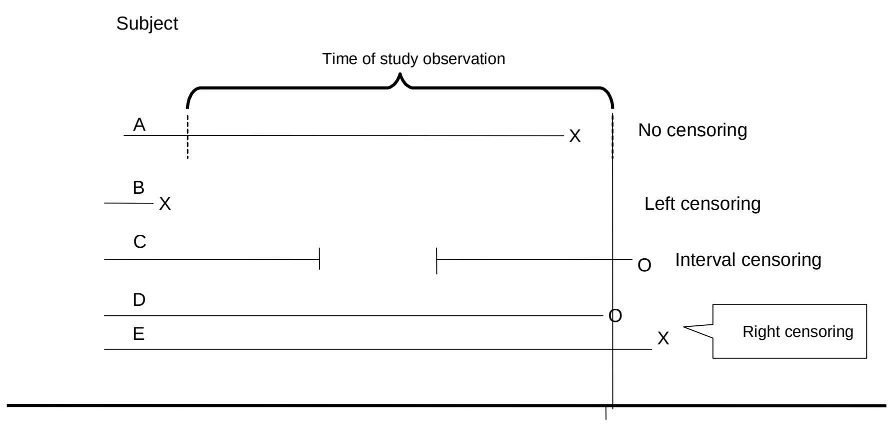


Figure 2. Kaplan Meier Estimation Curve

If the time that the curve covers is broken up into intervals, then the percentage surviving at the start of any interval is equal to the probability of surviving each of the preceding intervals all multiplied together.

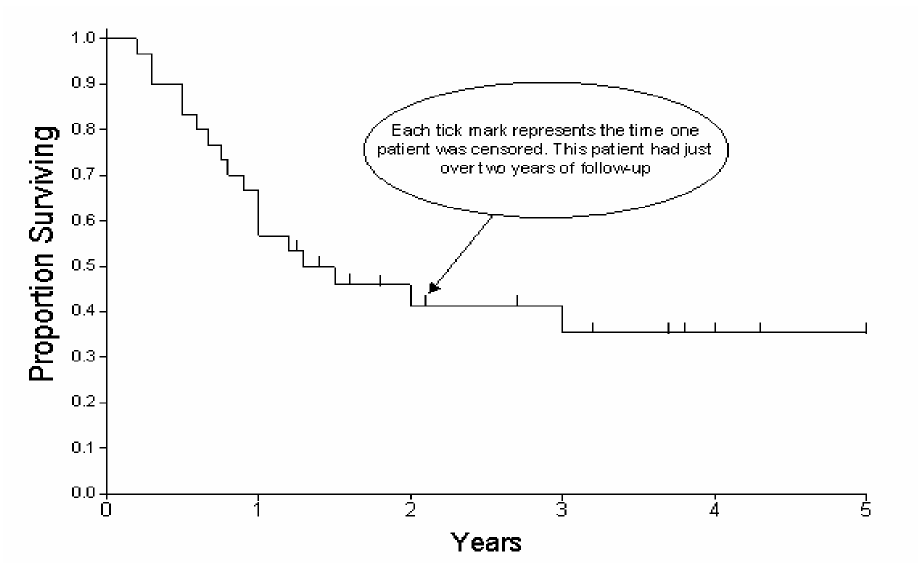


Figure 3. Bivariate Right Censored Data

In this graph, T stands for survival time point, C is censoring time point. The four types of data points are shown, including uncensored points, two kinds of singly censored points and doubly censored points.

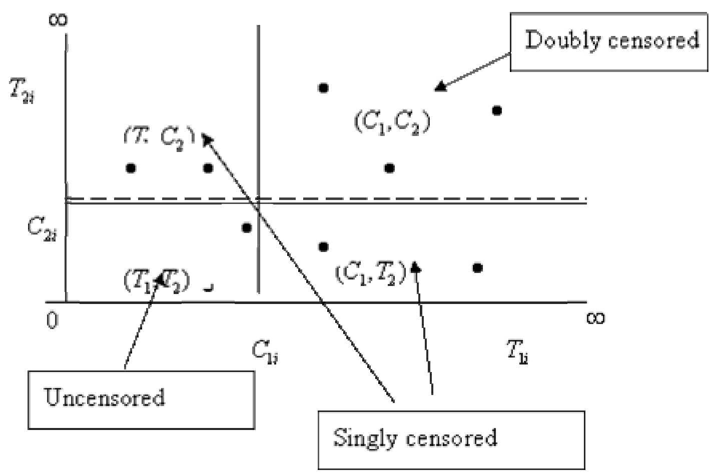
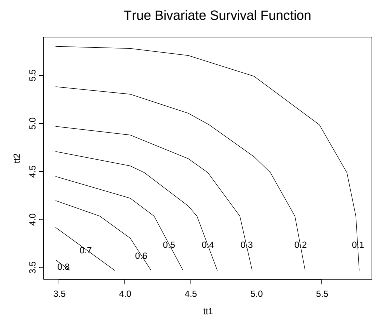
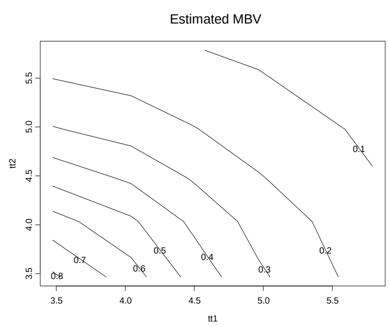


Figure 4. Contour Plots

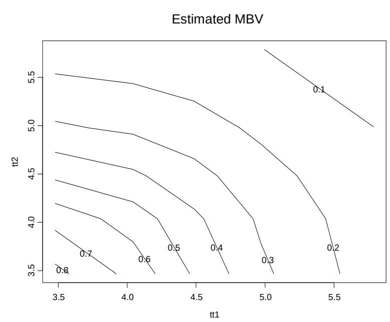
In part of contour plots, only ρ=0.5 is employed to show the comparison among the real bivariate survival function, estimated MBV and MMBV. For the influence of association coefficient on estimation, ρ =0.2, 0.5 and 0.8 are adopted.



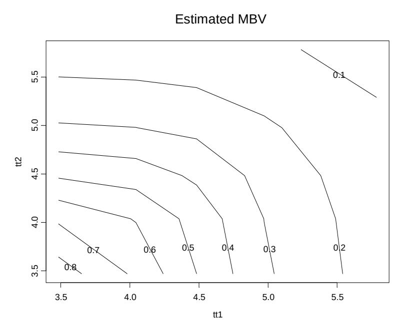
ρ=0.5



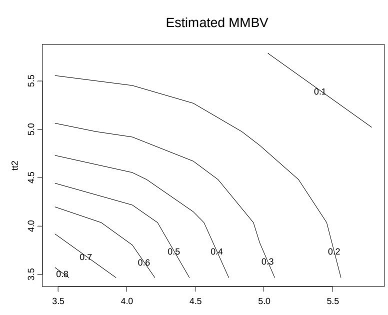
ρ=0.2



ρ=0.5



ρ=0.8

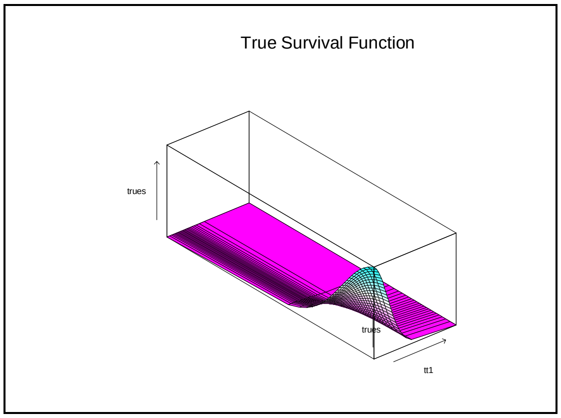


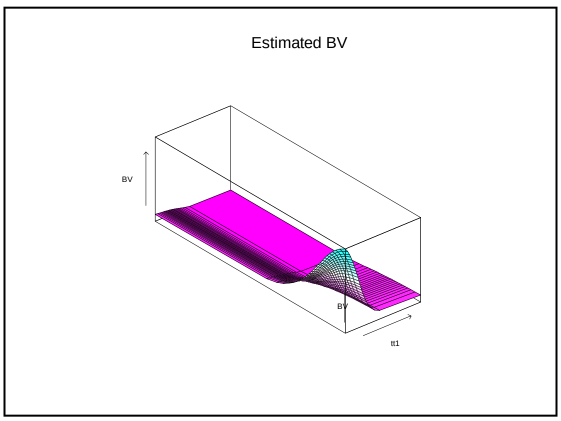
ρ=0.5

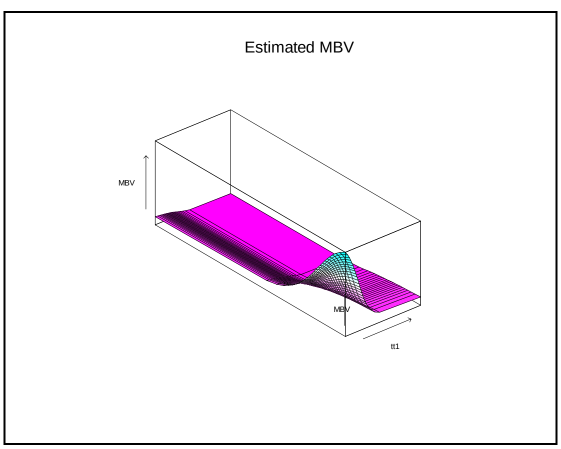
Figure 5. Wireframe Plots

The same specifications as those in contour plots are good for wireframe plots too. For wireframe plots, at $\rho=0.5$, true function, BV and MBV are exhibited to tell the efficiency. The reason for omitted MMBV is that the huge calculation leads to absolutely slow running and large quantity of time in simulation. MBV plots at $\rho=0.2$ and 0.8 were also rendered to compare the variation due to dependence levels.

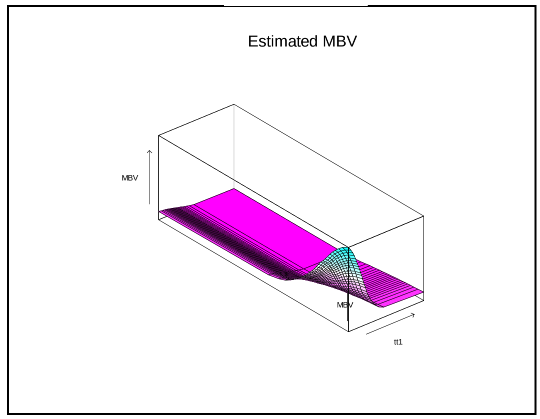
ρ= 0.5


True Survival Function
trues
trues
tt1

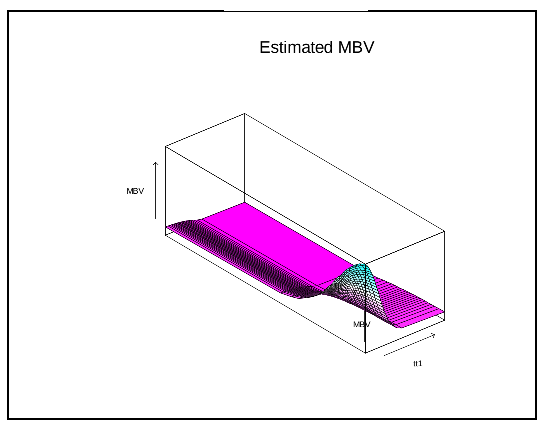

Estimated BV
BV
BV
tt1


Estimated MBV
MBV
MBV
tt1

ρ= 0.2


Estimated MBV
MBV
MBV
tt1

ρ= 0.8


Estimated MBV
MBV
MBV
tt1

PROGRAMMING CODES

FORTRAN part 1: Sorting
###

```
! quick sort alorithm

RECURSIVE SUBROUTINE quick_sort(list, order,asize)

IMPLICIT NONE
integer:: asize;
REAL, DIMENSION (asize), INTENT(IN OUT)  :: list
INTEGER, DIMENSION (asize), INTENT(OUT)  :: order

INTEGER :: i

DO i = 1, SIZE(list)
  order(i) = i
END DO

CALL quick_sort_1(1, SIZE(list))

CONTAINS

RECURSIVE SUBROUTINE quick_sort_1(left_end, right_end)

INTEGER, INTENT(IN) :: left_end, right_end

INTEGER             :: i, j, itemp
REAL                :: reference, temp
INTEGER, PARAMETER  :: max_simple_sort_size = 6

IF (right_end < left_end + max_simple_sort_size) THEN
  CALL interchange_sort(left_end, right_end)

ELSE
  reference = list((left_end + right_end)/2)
  i = left_end - 1; j = right_end + 1

  DO
    DO
      i = i + 1
      IF (list(i) >= reference) EXIT
    END DO
    ! Scan list from right end until element <= reference is found
```

```
    DO
      j = j - 1
      IF (list(j) <= reference) EXIT
    END DO

    IF (i < j) THEN
      ! Swap two out-of-order elements
      temp = list(i); list(i) = list(j); list(j) = temp
      itemp = order(i); order(i) = order(j); order(j) = itemp
    ELSE IF (i == j) THEN
      i = i + 1
      EXIT
    ELSE
      EXIT
    END IF
  END DO

  IF (left_end < j) CALL quick_sort_1(left_end, j)
  IF (i < right_end) CALL quick_sort_1(i, right_end)
END IF

END SUBROUTINE quick_sort_1


SUBROUTINE interchange_sort(left_end, right_end)

INTEGER, INTENT(IN) :: left_end, right_end

INTEGER             :: i, j, itemp
REAL                :: temp

DO i = left_end, right_end - 1
  DO j = i+1, right_end
    IF (list(i) > list(j)) THEN
      temp = list(i); list(i) = list(j); list(j) = temp
      itemp = order(i); order(i) = order(j); order(j) = itemp
    END IF
  END DO
END DO

END SUBROUTINE interchange_sort

END SUBROUTINE quick_sort
```

FORTRAN part 2: Simulation

```
######################################################################################
!kaplan-Meier estimator
SUBROUTINE mkm(t,TT,del,n1,n2, ans)

!DEC$ ATTRIBUTES DLLEXPORT::mkm

implicit NONE

integer:: n1, n2,i,j,l,kt
REAL(4):: t(n1)
real(4):: TT(n2), del(n2)
real(4):: ans(n1)
Real(4):: tau(n2)
integer, ALLOCATABLE ::order(:)
Real, ALLOCATABLE ::y(:)
integer:: k=0

k=0

do i=1, n2
if (del(i)==1) then
k=k+1
tau(k)=TT(i)
end if
end do

if (k==0) then
  do i=1, n1
    ans(i)=1
  end do
  return
end if
allocate(order(k))
allocate(y(k))
call quick_sort(tau, order,k)

do i=1, k
y(i)=0
do l=1, n2
if (TT(l)>=tau(i)) y(i)=y(i)+1
end do
end do

do i=1, n1
```

```
kt=0

do j=1, k

if (tau(j)<=t(i)) kt=kt+1

end do

ans(i)=1

if (k==0 .OR. kt==0) then
  ans(i)=1
     else
do l=1, kt;
ans(i)=ans(i)*(1-1/y(l))
end do

end if

end do

end SUBROUTINE mkm

!Marginal survival function with T1 not-censored
SUBROUTINE g1(t,s,Ts1,Ts2,ds1, ds2, h,n1,n2,n3,ans)

integer:: n1,n2,n3

REAL(4):: t(n1), s, Ts1(n2),ds1(n2), Ts2(n3), ds2(n3), ans(n1),Ts1m(n2),ds1m(n2)

k=0

do i=1, n2

if (Ts2(i)<=(s+h) .AND. Ts2(i) >=(s-h) .AND. ds2 (i)==1) then

k=k+1

Ts1m(k)=Ts1(i)

ds1m(k)=ds1(i)

end if

end do
```

```
do i=1, n1
ans(i)=1
end do

if (k>0) call mkm(t,Ts1m,ds1m,n1,k,ans)

end SUBROUTINE g1

!Marginal survival function with T2 not-censored
SUBROUTINE g2(t2,s,Ts1,Ts2,ds1, ds2, h,n1,n2,n3,ans)

integer:: n1,n2,n3

REAL(4):: t(n1), s, Ts1(n2),ds1(n2), Ts2(n3), ds2(n3), ans(n1),Ts2m(n3),ds2m(n3)

k=0

do i=1, n3

if (Ts1(i)<=(s+h) .AND. Ts1(i) >=(s-h) .AND. ds1 (i)==1) then

k=k+1

Ts2m(k)=Ts2(i)

ds2m(k)=ds2(i)

end if

end do

do i=1, n1
ans(i)=1
end do

if (k>0) call mkm(t2,Ts2m,ds2m,n1,k,ans)

end SUBROUTINE g2

!Weighted bivariate survival funciton with both T1 and T2 censored
SUBROUTINE bv(t1, t2,Ts1,Ts2,ds1,ds2, h,n1,n2,ans)

!DEC$ ATTRIBUTES DLLEXPORT::bv

integer:: n1,n2
```

```
REAL(4):: t1, t2, Ts1(n1), Ts2(n2), ds1(n1), ds2(n2), h, ans

REAL(4):: f1(n1), f2(n1), f3(n1), temp2, anst(1), ds2i(n2),ds1i(n2)

do j=1, n2
ds2i(j)=1-ds2(j)
ds1i(j)=1-ds1(j)
end do

do i=1, n1

if (Ts2(i)>t2) then
f2(i)=1
else
f2(i)=0
end if

call g1(t1,Ts2(i),Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)

f1(i)=anst(1)

call mkm(Ts2(i),Ts2,ds2i,1,n2, anst)

temp2=anst(1)

if (temp2==0) then
f3(i)=1
else
f3(i)=temp2
end if

end do

bv1=sum(f1*ds2*f2/f3)/n1

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

do i=1, n1

if (Ts1(i)>t1) then
f2(i)=1
else
f2(i)=0
end if
```

```
temp=Ts1(i)

call g2(t2,Ts1(i),Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)

f1(i)=anst(1)

call mkm(Ts1(i),Ts1,ds1i,1,n2, anst)

temp2=anst(1)

if (temp2==0) then
f3(i)=1
else
f3(i)=temp2
end if

end do

bv2=sum(f1*ds1*f2/f3)/n1

ans=0.5*(bv1+bv2)

end SUBROUTINE bv



!Modified survival function
SUBROUTINE mbv(t1, t2,Ts1,Ts2,ds1,ds2, h,n1,n2,ans)

!DEC$ ATTRIBUTES DLLEXPORT::mbv

integer:: n1,n2

REAL(4):: t1, t2, Ts1(n1), Ts2(n2), ds1(n1), ds2(n2), h, ans

REAL(4):: f1(n1), f2(n1), f3(n1),f4(n1), f5(n1), temp1,temp2,temp3,temp4, anst(1), ansbv

do i=1, n1

if (Ts1(i)>t1) then
f1(i)=1
else
f1(i)=0
end if

if (Ts2(i)>t2) then
```

```
f2(i)=1
else
f2(i)=0
end if

temp1=Ts1(i)

if (t2>=Ts2(i)) then
temp2=t2
else
temp2=Ts2(i)
end if

temp3=Ts2(i)

if (t1>=Ts1(i)) then
temp4=t1
else
temp4=Ts1(i)
end if

anst(1)=0

if (ds2(i)==1) then
f3(i)=0
else
call g2(temp3,temp1,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
end if

if (anst(1)==0) then
f3(i)=0
else
call g2(temp2,temp1,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f3(i)=anst(1)
call g2(temp3,temp1,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f3(i)=f3(i)/anst(1)
end if

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
anst(1)=0
if (ds1(i)==1) then
f4(i)=0
else
call g1(temp1,temp3,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
end if
```

```
if (anst(1)==0) then
f4(i)=0
else
call g1(temp4,temp3,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f4(i)=anst(1)
call g1(temp1,temp3,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f4(i)=f4(i)/anst(1)
end if

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

ansbv=0
if (ds1(i)==1 .OR. ds2(i)==1) then
f5(i)=0
else
call bv(temp1, temp3,Ts1,Ts2,ds1,ds2, h,n1,n2,ansbv)
end if

if (ansbv ==0 ) then
f5(i)=0
else
call bv(temp4, temp2,Ts1,Ts2,ds1,ds2, h,n1,n2,ansbv)
f5(i)=ansbv
call bv(temp1, temp3,Ts1,Ts2,ds1,ds2, h,n1,n2,ansbv)
f5(i)=f5(i)/ansbv
end if

end do

ans=0

ans=sum(ds1*ds2*f1*f2 +ds1*(1-ds2)*f1*f3+ (1-ds1)*ds2*f2*f4 + (1-ds1)*(1-ds2)*f5)/n1

end SUBROUTINE mbv


!Double-modified survival function
SUBROUTINE mmbv(t1, t2,Ts1,Ts2,ds1,ds2, h,n1,n2,ans)

!DEC$ ATTRIBUTES DLLEXPORT::mmbv

integer:: n1,n2

REAL(4):: t1, t2, Ts1(n1), Ts2(n2), ds1(n1), ds2(n2), h, ans

REAL(4):: f1(n1), f2(n1), f3(n1),f4(n1), f5(n1), temp1,temp2,temp3,temp4, anst(1), ansbv
```

```
do i=1, n1

if (Ts1(i)>t1) then
f1(i)=1
else
f1(i)=0
end if

if (Ts2(i)>t2) then
f2(i)=1
else
f2(i)=0
end if

temp1=Ts1(i)

if (t2>=Ts2(i)) then
temp2=t2
else
temp2=Ts2(i)
end if

temp3=Ts2(i)

if (t1>=Ts1(i)) then
temp4=t1
else
temp4=Ts1(i)
end if

anst(1)=0

if (ds2(i)==1) then
f3(i)=0
else
call g2(temp3,temp1,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
end if

if (anst(1)==0) then
f3(i)=0
else
call g2(temp2,temp1,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f3(i)=anst(1)
call g2(temp3,temp1,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
```

```
f3(i)=f3(i)/anst(1)
end if

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
anst(1)=0
if (ds1(i)==1) then
f4(i)=0
else
call g1(temp1,temp3,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
end if

if (anst(1)==0) then
f4(i)=0
else
call g1(temp4,temp3,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f4(i)=anst(1)
call g1(temp1,temp3,Ts1,Ts2,ds1,ds2,h,1,n1,n2,anst)
f4(i)=f4(i)/anst(1)
end if

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

ansbv=0
if (ds1(i)==1 .OR. ds2(i)==1) then
f5(i)=0
else
call mbv(temp1, temp3,Ts1,Ts2,ds1,ds2, h,n1,n2,ansbv)
end if

if (ansbv ==0 ) then
f5(i)=0
else
call mbv(temp4, temp2,Ts1,Ts2,ds1,ds2, h,n1,n2,ansbv)
f5(i)=ansbv
call mbv(temp1, temp3,Ts1,Ts2,ds1,ds2, h,n1,n2,ansbv)
f5(i)=f5(i)/ansbv
end if

end do

ans=0

ans=sum(ds1*ds2*f1*f2 +ds1*(1-ds2)*f1*f3+ (1-ds1)*ds2*f2*f4 + (1-ds1)*(1-ds2)*f5)/n1

end SUBROUTINE mmbv
```

S-Plus part: Simulation

```
#################################################################################
dll.load("N:/DFORRT.DLL")
dll.load("N:/dll.dll",c("MKM", "BV", "MBV","MMBV"))

mkm <- function(t,TT,del)
{
ans <- rep(1,length(t))
if (length(TT)>0)
{
t3<-
   .Fortran("MKM",as.single(t),as.single(TT),as.single(del),as.integer(length(t)),as.i
   nteger(length(TT)),ans=as.single(ans))
ans <-t3$ans
}
ans
}

#######################################

bv <- function(t1,t2,Ts1,Ts2,ds1,ds2)
{
t3<-
   .Fortran("BV",as.single(t1),as.single(t2),as.single(Ts1),as.single(Ts2),as.single(d
   s1),as.single(ds2),as.single(h), as.integer(length(Ts1)),as.integer(length(Ts2)),
   ans=as.single(ans))
t3$ans
}

mbv <- function(t1,t2,Ts1,Ts2,ds1,ds2) {
t3<-
   .Fortran("MBV",as.single(t1),as.single(t2),as.single(Ts1),as.single(Ts2),as.single(
   ds1),as.single(ds2),
as.single(h), as.integer(length(Ts1)),as.integer(length(Ts2)), ans=as.single(ans))
t3$ans
}

mmbv <- function(t1,t2,Ts1,Ts2,ds1,ds2) {
t3<-
   .Fortran("MMBV",as.single(t1),as.single(t2),as.single(Ts1),as.single(Ts2),as.single
   (ds1),as.single(ds2),
as.single(h), as.integer(length(Ts1)),as.integer(length(Ts2)), ans=as.single(ans))
t3$ans
}
###########################################
h <- 0.5

simu<-function(iteration,sample, real)
{
###########################################
# Theoretical survival function;
options(object.size=500000000)

# ρ varies at 0, 0.1,0.2, 0.5, 0.8
t<-rmvnorm(real, mean = c(1.5, 1.5), cov=matrix(c(1,ρ,ρ,1),ncol=2)
t<-exp(t)
Ys1 <- t[,1]
Ys2 <- t[,2]
Xs1 <- Ys1
Xs2 <- Ys2
```

```
tt1 <- quantile(Xs1,prob=seq(0.1,0.9,0.2))
tt2 <- quantile(Xs2,prob=seq(0.1,0.9,0.2))
trues <- matrix(rep(0,25),5,5)
for (i in 1:5)
 for (j in 1:5)
trues[i,j] <- length(Xs1[Xs1>tt1[i] & Xs2>tt2[j]])/real

###########################################
#Simulated survival functions
est1 <- est2 <-est3 <- rep(1,25*iteration)
dim(est1) <- dim(est2) <- dim(est3)<- c(iteration,5,5)
for (iter in 1:iteration) {
cat(iter,"\n")

# ρ varies at 0, 0.1,0.2, 0.5, 0.8
t<-rmvnorm(real, mean = c(1.5, 1.5), cov=matrix(c(1,ρ,ρ,1),ncol=2)
t<-exp(t)
Ys1 <- t[,1]
Ys2 <- t[,2]
Xs1 <- Ys1
Xs2 <- Ys2

Cs1 <- rgamma(sample,4.5)
Cs2 <- rgamma(sample,4.5)
ds1 <- ds2 <- rep(0,sample)
ds1[Xs1<=Cs1] <- 1
ds2[Xs2<=Cs2] <- 1
Ts1 <- pmin(Xs1,Cs1)
Ts2 <- pmin(Xs2,Cs2)
for (i in 1:5)
 for (j in 1:5)
  {
   est1[iter,i,j] <- bv(tt1[i],tt1[j],Ts1,Ts2,ds1,ds2)
   est2[iter,i,j] <- mbv(tt1[i],tt1[j],Ts1,Ts2,ds1,ds2)
   est3[iter,i,j] <- mmbv(tt1[i],tt1[j],Ts1,Ts2,ds1,ds2)
  }
est1[iter,,] <- est1[iter,,]/bv(0,0,Ts1,Ts2,ds1,ds2)
est2[iter,,] <- est2[iter,,]/mbv(0,0,Ts1,Ts2,ds1,ds2)
est3[iter,,] <- est3[iter,,]/mmbv(0,0,Ts1,Ts2,ds1,ds2)
                 }
bias1 <- bias2 <-bias3<-  std1<- std2<-std3<- ms1 <- ms2 <- ms3 <-BV<-MBV<-MMBV<-
   matrix(rep(0,25),5,5)
for (i in 1:5)
 for (j in 1:5)
 {
   std1[i,j] <- stdev(est1[,i,j])
   std2[i,j] <- stdev(est2[,i,j])
   std3[i,j] <- stdev(est3[,i,j])

   bias1[i,j] <- mean(est1[,i,j])-trues[i,j]
   bias2[i,j] <- mean(est2[,i,j])-trues[i,j]
   bias3[i,j] <- mean(est3[,i,j])-trues[i,j]

   ms1[i,j] <- (bias1[i,j])^2 + var(est1[,i,j])
   ms2[i,j] <- (bias2[i,j])^2 + var(est2[,i,j])
    ms3[i,j] <- (bias3[i,j])^2 + var(est3[,i,j])

    BV[i,j]<- mean(est1[,i,j])
    MBV[i,j]<- mean(est2[,i,j])
    MMBV[i,j]<- mean(est3[,i,j])

 }
```

```
list(bias1=bias1,bias2=bias2,std1=std1,std2=std2,ms1=ms1,ms2=ms2,BV=BV,MBV=MBV,MMBV=MM
   BV)

   ##########################################

   #Contour plot;
   contour(tt1,tt2,trues,nlevel=8,plotit=TRUE)
   title(main="True Bivariate Survival Function")

   contour(tt1,tt2,mean1,nlevel=8,plotit=TRUE)
   title(main="Estimated BV")

   contour(tt1,tt2,mean2,nlevel=8,plotit=TRUE)
   title(main="Estimated MBV")

   contour(tt1,tt2,mean3,nlevel=8,plotit=TRUE,triangles=F)
   title(main="Estimated MMBV")

   ###########################################

   #Surface plot;
   g <- expand.grid(tt1 = tt1, tt2 = tt2)

   g$trues<-trues
   wireframe(trues ~ tt1 * tt2, g, drape = TRUE, perspective = FALSE, aspect = c(3,1),
   colorkey = FALSE)
   title(main="True Survival Function")

   g$BV<-BV
   wireframe(BV ~ tt1 * tt2, g, drape = TRUE, perspective = FALSE, aspect = c(3,1),
   colorkey = FALSE)
   title(main="Estimated BV at rho=0.5")

   g$MBV<-MBV
   wireframe(MBV ~ tt1 * tt2, g, drape = TRUE, perspective = FALSE, aspect = c(3,1),
   colorkey = FALSE)
   title(main="Estimated MBV at rho=0.5")

   g$MMBV<-MMBV
   wireframe(MMBV ~ tt1 * tt2, g, drape = TRUE, perspective = FALSE, aspect = c(3,1),
   colorkey = FALSE)
   title(main="Estimated MMBV at rho=0.5")

}


ans<-0

set.seed(153)
#Sample=50 is just an example, by changing it, the simulation can be varied
t1<-simu(iteration=200,sample=50,real=50000)
t1
```