TAPER, VOLUME, AND GREEN WEIGHT EQUATIONS FOR PLANTED LONGLEAF PINE IN GEORGIA, UNITED STATES

by

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(Under the Direction of Bronson P. Bullock)

ABSTRACT

Once common in the Southeastern United States (SE US), longleaf pine (*Pinus palustris* Mill.) ecosystems, have been reduced to 3% of its native range. The goal of this research project is to improve quantitative estimates of wood volume and green weight for unthinned planted longleaf pine growing in Georgia (GA). We surveyed 20 unthinned longleaf pine stands across GA from old-field and cut-over sites, and destructively sampled 400 trees to obtain outside bark diameter and green weight measurements. The sample trees had ages 12–25 years old, diameter at breast height 3.9–12.2 in, and total height 28.5–73.9 ft. Stem taper for inside and outside bark was best modeled with the Max and Burkhart (1976) form. Stem taper, implied volume, and green weight did not vary with stand origin. On average 36% of longleaf pine trees surveyed had stem defects. The volume and green weight in forked trees was about 11% higher than non-defect trees and about 15% less in crooked trees compared to non-defect trees with the same DBH and total height. These models will be useful for determining the value of the existing longleaf pine stands and newly established stands across GA and the SE US on old-field and cut-over sites.

INDEX WORDS: *Pinus palustris*, longleaf pine, taper, volume, green weight

Taper, volume, and green weight equations for planted longleaf pine in Georgia, United States

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Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction and Literature Review

To understand the important changes that have affected longleaf pine (*Pinus palustris* Mill.) ecosystems across the Southeastern United States (SE US) we begin this thesis with a chapter to review the history of the species as well as quantitative models used to describe trees and forests. The objective is to form the rationale and significance behind this research as well as to introduce the methods and concepts that will be used later to test the hypotheses about the growth of unthinned longleaf pine plantations in Georgia and the SE US.

1.1 Introduction

As a commercial timber species, longleaf pine (*Pinus palustris* Mill.) has a commonly understood history of exploitation and extirpation from its native range. Historical pressure from turpentining exhausted millions of acres of longleaf pine. The destructive logging that followed many of the turpentine operations further reduced the quantity and distribution of longleaf pine all across the South (Walker et al., 2006). In the early 20$^{\text{th}}$ century, petroleum distillates, such as kerosene, were quickly displacing naval stores (Outland, 2001). Around the same time in the early 20$^{\text{th}}$ century, chemical kraft pulping and the revolution of paper production in the US south allowed mills to extract turpentine and tall oil from the pulping processes. There was no longer a need to tap the longleaf pines, when the same products could be supplied from the byproducts of the chemical pulping process in softwood paper manufacturing (Ragauskas et al., 2006). This essentially eliminated the need for turpentining in the South at that time. Other longleaf pine stands were clearcut to support the expansion of European settlement further south and west. Another significant contribution to the conversion of longleaf to other pine cover types occurred in the mid-20th century when many lumber mills transferred timberlands to pulp and paper mills that were springing up across the SE US (Farrar, 1978). Those timberlands owned by pulp and paper mills were converted to more productive stands of loblolly pine (*Pinus taeda* L.) and slash pine (*Pinus elliottii* Engelm.) to supply wood fiber for the mills. Widespread agriculture was another major force in eliminating longleaf pine from many landscapes.

The main reason for the loss of longleaf pine following disturbance from harvesting was the inability for those stands to regenerate naturally. It is longleaf pine, of all the southern hard pines that is most difficult to cultivate from the silvicultural perspective. For effective afforestation to occur, there must be a heavy seed crop, ample exposed mineral soil near the seed trees, and usually a growing season fire to expose the soil. These conditions will prepare the site for the particular needs of longleaf pine seeds, which germinate immediately upon reaching mineral soil in the fall, unlike other southern hard pine species which germinate in the spring following seed maturation. Therefore, the timing of these past tree harvests rarely coincided with the discrete series of events necessary to regenerate longleaf pine from seed. Thus, those areas recruited different overstory woody species in place of longleaf pine. The exclusion of longleaf pine from the overstory eliminated any potential seed source for those affected areas across the native range of longleaf pine.

Today, planting is necessary to re-introduce the longleaf pine germplasm to natural systems and provide sources of future seeds and natural reproduction. In the past few decades, cut-over and old-field sites have been planted with longleaf pine across Georgia and the Southeastern United States (SE US), with over 700,000 acres between the ages of 10–25 in 2020 (Oswalt et al., 2012). Today, longleaf pine seedlings are being produced at the same rate as slash pine, accounting for roughly 10% of all conifer seedlings in the SE US (Enebak, 2018); this shows the continued interest in the future of longleaf pine.

Foresters managing these young unthinned longleaf pine plantations must prepare for future harvesting and management activities. In order to do that, they need accurate and precise estimates of wood volume for their stands. A big proportion of the published work on planted longleaf originates from east Texas and Louisiana; with a few recent papers based on the work by Brooks and Jack (2006); Brooks et al. (2007); Brooks and Jack (2016) and their data from two counties in southwest Georgia. Further work and data collection in the Atlantic coastal plain region is needed to better understand the stem form and quality of longleaf pine across the entire native range.

Longleaf pine wood is used by many forest product industries including for utility poles, dimensional lumber, and pulp and paper. Another important contemporary use for old longleaf pine stumps and roots is the production of fragrances and other chemicals from old longleaf pine stumps and roots.

The overall goal of this research project is to improve quantitative estimates of wood volume for planted longleaf pine growing in the Atlantic coastal plain. These models will be useful for the existing longleaf pine stands and newly established stands across GA and the SE US on old-field and cut-over sites. More accurate predictions of wood volume will allow landowners to better manage their forestland, for more accurate estimates of carbon storage and sequestration in those forests, for projections of wildlife habitat structure, and more consistent methods to project the growth and yield of those forest resources into the future (Earley 2004).

1.2 Literature Review

Interest in estimating the wood volume and quantity of longleaf pine occurred early on in the history of American forest science. In 1899 Carl Schenck collected data in Walker County, Alabama on growth ring width to estimate productivity in virgin longleaf pine stands being harvested (Schenck, 1955). Ultimately data was transferred to Gifford Pinchot and no publication of the results was produced. Schenck did point out the results may have underwhelmed a prospective audience, as he advised the rotation length for mature longleaf pine could take as long as 150 years between harvests. A key work by Chapman

(1909) provided a quantitative response to the need for understanding the structure and volume of wood supply in those natural longleaf stands. His work offered a snapshot for the volume tables and age distribution of natural stands in a single county in Texas. Although these figures would fail to be useful for a wide range of natural longleaf stands, a major contribution was his understanding of the silvics of longleaf pine, especially in terms of its needs for regeneration following harvest.

Although extensive work and effort have been applied to research in natural longleaf pine stands, this research will focus on planted longleaf pine systems. As summarized by Nyland (2016), forest plantations have distinct differences from natural stands, namely more uniform height and diameter among crop trees in the stand as compared to natural stands. This difference in stand development results in distinct changes in individual tree growth within planted stands. Based on this information, we have a more discrete realm of published work to evaluate the history of individual tree equations for modeling growth of individual planted longleaf pine stems. Furthermore, our work with unthinned stands leads us to focus on those papers addressing unthinned stand conditions.

1.2.1 Growth and Yield Systems

The ability to model and predict standing tree volume is a principal driver in forest biometrics research. Estimating wood volume is useful for landowners focused on managing forests for profit and maximizing the net present value (NPV). Likewise, estimating stem biomass or carbon storage is necessary to evaluate progress toward achieving other management objectives or conservation goals. The way this is done in forestry is the use of growth and yield models. There are countless models for many different species and stand conditions. Land managers must be judicious in selecting the correct model suited to the forest stands they are intending to model.

The initial effort in producing growth and yield models for planted longleaf pine brought together the long history of work in longleaf pine. The first growth and yield model for longleaf plantations came from Lohrey and Bailey (1977) using data from Texas and Louisiana unthinned longleaf pine stands. Goelz and Leduc (2002) crafted a system specially adapted for longleaf pine for both thinned and unthinned stands and included the flexibility to use separate individual tree equations for predicting volume. Their data included those plots from Lohrey and Bailey (1977) with the addition of 250 plots in the Western Gulf region from Texas to Alabama. The next iteration in the growth and yield modeling came from Brooks and Jack (2006, 2016) who provided models using data from two longleaf pine stands, in southwest Georgia. More recent work from Gonzalez-Benecke et al. (2012) provides updated models for many different variables including new site index equations for longleaf plantations. These models are all limited based on the data available at the time of modeling. However, they have one thing in common, the incorporation of individual tree models to scale up prediction for volume, green weight and biomass. Those individual tree models have a similar development history, which will be addressed next.

1.2.2 Individual Tree Models

The basis for which some growth and yield models are built on are the individual tree equations utilized in the final step of prediction. It is important to note the first individual tree volume equation for planted longleaf pine published by Schmitt and Bower (1970). They destructively sampled 200 trees in southern Mississippi and fit a simple combined variable equation to predict total stem volume:

$$V = \beta_0 + \beta_1 D^2 H \tag{1.1}$$

where

V = volume (ft^3)

D= diameter at breast height (DBH) 4.5 feet above ground

H = total tree height

β_0, β_1, ... ,β_i = parameters to be estimated

This early work set the baseline for taper and integrated volume modeling for planted longleaf. The first taper models for longleaf pine plantations were produced by Baldwin and Polmer (1981) with data from 113 trees. Data was fit to three sets of three different model forms based on the taper model applied to slash pine by Dell (1979). Baldwin and Polmer (1981) also compared their model with the segmented polynomial taper model from Max and Burkhart (1976). The authors only pursued different models based on the three groups of crown ratios. The reliance on the crown ratio for model selection might improve the fit, but is a limiting approach for future application because crown ratio is rarely measured in operational forest inventory in the SE US. Therefore, it would be difficult for forest managers to select the single correct set of parameters because they vary widely between the crown ratio classes (Baldwin and Polmer, 1981). The next significant work on individual stem models was by Baldwin (1983) using some of the data from Baldwin and Polmer (1981) to publish new equations for volume, both inside bark and outside bark diameter, green weight, and biomass in unthinned stands. These functions were fit simultaneously with the linearized allometric volume equation:

$$\log V = \beta_0 + \beta_1 \log(D^2 H) \qquad (1.2)$$

Thomas et al. (1995) used data from 147 sites in Louisiana and Texas in both thinned and unthinned longleaf plantations, and fit a form of a trigonometric taper equation, as well as implied volume functions, for inside bark diameters only:

$$\frac{d_i^2}{D^2} = (\beta_1(x_i - 1) + \beta_2 \sin(c\pi x_i) + \beta_3 \cot(\frac{\pi}{2} x_i)) + \epsilon_i \qquad (1.3)$$

where

d_i = diameter at height h_i along the stem

$x_i = h_i/H$

D= diameter at breast height (DBH) 4.5 feet above ground

h_i = height at point along the stem

H = total tree height

β_0, β_1, ... ,β_i = parameters to be estimated

More recently, Brooks et al. (2007) fit a Max and Burkhart (1976) segmented polynomial taper model form:

$$\frac{d_i^2}{D^2} = \beta_1 - 1(Z_i - 1) + \beta_2(Z_i^2 - 1) + \beta_3(a_1 - Z_i)^2 I_1 + \beta_4(a_2 - Z_i)^2 I_2, \tag{1.4}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} \quad j = 1, 2; Z_i = h_i/H \tag{1.5}$$

to data from 42 trees growing in southwest Georgia. Recently, Gonzalez-Benecke et al. (2014, 2018) published individual tree equations for longleaf pine from data across the native range of longleaf pine.

The usefulness of taper and volume equations has a major limiting factor; roundwood in the SE US is bought and sold based on weight, not volume. Therefore, having accurate estimates of individual tree green weight is critical to planning forest management activities. Equations for green weight along with merchantable stem volume were provided by Baldwin (1983). These green weight equations were revisited by Brooks et al. (2007) and fitted simultaneously with stem taper and volume. These green weight estimates relied on measuring the wood density of disks and applying that to estimated volume of bolts in the 42 sample trees. Gonzalez-Benecke et al. (2014) recently published volume and taper equations from data in the Western Gulf. To date, no models based on the measured total green weight of the stem have been published from planted longleaf pine growing in the Atlantic Coastal Plain.

1.3 Rationale and Significance

For landowners to make management decisions about their forests, they must know the current volume and green weight of wood on the stump; furthermore, they must have a way to project the growth and yield of their forest stands into the future. This work of determining the volume and green weight of wood begins with individual tree equations and then stand-level equations to predict the density and size distribution of those individual trees across the stand. Errors in individual tree equations are quickly expanded when you have several hundred trees per acre in a planted stand.

We modeled a stand consistent with the higher growth rates we observed so we used an initial age of 12 with dominant height = 41 ft, basal area = 85 ft^2/ac, and initial trees per acre (TPA) = 550. In Figure 1.1, the projected growth of a hypothetical stand is displayed using five currently available growth and yield systems built for longleaf pine plantations; large differences in projections were observed. We see two main trends, the Brooks and Jack (2006, 2016); Gonzalez-Benecke et al. (2012) models with a major over projection of growth, approaching 250–325 tons/ac at age 50, which is not realistic given growth rates in these systems. The second trend is the much more conservative projection from the "SIMS" model from ForesTech Inc., which shows green weight approaching 180 tons/ac which is still high but a more reasonable estimate. Although it is unlikely these large differences in tons/ac projections arise from bias in estimating individual tree volume or green weight, improvements for every part of the system must be considered to bring about more uniformity in the model projections. Therefore, focusing on developing robust and accurate equations to predict individual tree volume, taper, and green weight are essential. Also, this work will bolster the information available for longleaf pine growing in the eastern portion of the native range, because much of the data collected in past studies is from the Western Gulf region (Brooks et al., 2007).

1.4 Goals, Objectives, and Hypotheses

It is hypothesized that existing taper and tree volume models for longleaf pine may not differentiate between the variation in site characteristics for stands currently growing in GA and the SE US. Therefore, collecting new data from a wide range of ages and locations across GA allows us to test this hypothesis. The objectives of this research are to:

1. Develop new individual tree equations to predict volume, taper, and green weight
2. Compare the effect of stand origin on taper, volume, and green weight estimation
3. Evaluate the rate of stem defects and rust incidence across measured stands

1.5 Methods

The data for this research was collected from across southern GA, in the native range of longleaf pine. A total of 400 trees were destructively sampled, and the diameters along the stem up to a 3-inch top diameter outside bark (DOB) and the green weight of the stem was recorded. Sample trees were selected in two groups, the first was straight stems, either in, dominant or co-dominant canopy positions, roughly 80% of sample trees. These sample trees were sampled proportionally across the distribution of diameters measured in each stand. The second group was trees with major stem defects such as forking or excessive sweep, roughly 20% of sample trees.

Study Site Selection

We selected study sites extending across the current planted range of longleaf pine in GA. The result was 24 sites that were identified as suitable and inventoried; with 20 final sites used in the full data collection. Longleaf pine growing in pure successful plantations were the ideal stands to sample from. The four stand level requirements were:

1. Plantations of longleaf pine

2. Age after planting from 10–25 years old

3. Clear documentation for prior land use as either old-agricultural field or cut-over

4. Unthinned at the time of our study

The motivation for selecting stands in ages between 10–25 years after planting was driven to select trees that had reached merchantable size, but not too physiologically old or advanced in stand development. These small stems would be suitable for pine pulpwood, the general specification for treelength pulpwood in the US south would include any stem that has a 1 in top diameter at thirty feet above ground. This size requirement allows for tree-length stems to be harvested and easily transported on log trucks in the standard trailer lengths, while also conforming to the mills delivered wood size standards, see Figure 1.2. An additional consideration is lower DBH limits for pulpwood of 6 in as described in TimberMart-South (Harris, 2019). The reason for using these sizes as a guide was that small unmerchantable stems represent very little volume; secondly, small trees have not reached the size of interest in commercial timber production as it is deemed pre-merchantable because it is too small to load on a truck and deliver to the mill profitably. The use of these volume and taper equations will be to estimate individual stem volume from cruise inventory data, and most landowners do not conduct a detailed inventory until the trees have reached a merchantable size. Therefore, we focused our attention on longleaf pine stands with high proportions of merchantable stems. Likewise, choosing the upper age limit of 25 years coincided with the point in stand development in which stem exclusion would typically occur on poor sites with slower growth. Stands that are older, especially if not commercially thinned, would begin to self thin and therefore begin to alter the individual tree characteristics such as live-crown-ratio, taper, and wood specific gravity. Selecting stands with consistent stand attributes across a wide geographic range in Georgia was the goal of the study. Those stands with fertilization or other non-routine management activities as compared with other longleaf pine stands were noted. Any stands with poor survival that affected the stand dynamics and individual tree canopy structure and competition with neighboring trees were excluded.

Many longleaf pine stands have intense management activities compared with what might be considered in other approaches such as the Stoddard-Neel approach for managing uneven aged systems (Neel et al., 2011). Many of the stands visited had management activities such as intensive site preparation in connection with restoration from old-agricultural fields. Later in the rotation, some stands had periodic herbicide application in connection with pine straw raking. These stands with intensive silvicultural inputs comprise a significant portion of the longleaf pine plantation area and are regarded as representative of the total longleaf pine growing stock and were included for sampling. Additionally, many stands had intermittent burning, and that was to be expected with the management of longleaf pine with goals of maintaining diverse understory plant communities and improving wildlife grazing and forage opportunities (Walker et al., 2006). The longleaf pine plantations growing on old-field sites express the recent past trends in agriculture, especially fragmentation and parcelization. So, even small stands (<10 acres) of longleaf pine were considered for this sample because they represent the current growing conditions of other longleaf pine stands that can be modeled successfully from the products of this research.

To identify potential landowners who would allow us to sample their trees, I contacted a variety of landowners and organizations. Because no single landowner has longleaf pine plantations across a wide geographic range and at all different ages and site characteristics we require to test our hypothesis, many stakeholders were required to achieve the desired sampling intensity. See Table 1.1 for a complete list of sites sampled. The stand and stock table is displayed in Table 1.2, note the site index base age 50 is calculated from the equation by Gonzalez-Benecke et al. (2012). The map showing the complete distribution of sites is shown in Figure 1.3.

Stand Level Inventory Data Collection

For each of the 24 sample sites that was located, we visited each one to take initial stand-level measurements using three 1/10 ac fixed radius plots. That data allowed us to estimate stand basal area per acre, dominant height, site index, and additional forest inventory values that can be calculated as needed. In a rolling process new sites were sought out to balance the distribution of tree sizes, age, and geographic distribution while maintaining the balance between old-field and cut-over sites, resulting in a final selection of 20 sites for destructive sampling.

To achieve the objective of comparing growth differences in longleaf pine plantations based on stand origin, the sites will be divided roughly equally between old-field (11 sites) and cut-over (9 sites) (1 site that was described as cut-over was later found out to be old-field in origin). At each site, 20 trees were destructively sampled, yielding a total sample of 400 trees. Within each 1/10 ac fixed radius plot we recorded every living tree with a DBH>0 at 4.5 feet. For each sample tree we recorded:

1. Tree species
2. DBH to the nearest 0.1 in with a logger's tape
3. Total height to the nearest 1/2 foot with a laser hypsometer
4. Defects including: forking, sweep, ramicorn, and broken top
5. Incidence of fusiform rust

Across all 24 sites we selected for initial inventory, we measured 2214 trees. This data will allow a regional height diameter comparison, regional fusiform rust incidence, as well as supporting stand level characteristics for use in the individual tree taper and volume models which require such inputs including basal area (BA), trees per acre (TPA), and dominant height.

Individual tree sample selection

The stand level inventory data and visual inspection from site visits allowed us to select the most appropriate 20 sites for our sample. We then selected 20 trees within each stand to be included for the intensive destructive sampling. One drawback to the longleaf pine plantations that exist today are the high rate of stem defects from forking, sweep, and to a lesser extent fusiform rust (*Cronartium quercuum* (Berk.) Miyabe ex. Shirai f. sp. *fusiforme*). So, for our sample we were interested to collect data on straight trees as is traditionally done as well as those defect trees. Trees free from defect such as forking, sweep, broken top, and fusiform rust galls were selected for about 16 trees (80%) in each stand, and the remaining 4 trees (20%) in each stand were intentionally chosen with major defect such as forking, sweep, and high stem sinuosity. The data was split into two groups and used for comparison to understand the relationship between defects in trees and the expected green-weight or volume in longleaf pine. All trees were selected from the interior of the stand to avoid the effects of increased sunlight and decreased crop tree competition on the stand edge. The trees from each stand were selected across the diameter distribution measured in prior stand level inventories to ensure a representative sample of trees from different sizes in each stand.

Once chosen for destructive sampling, prior to felling, the total height was measured using a Nikon Forestry Pro laser hypsometer. This data was compared with the total length of the felled tree measured with a tape measure. Additionally, the sample tree was marked at 0.5, 2, and 4.5 ft above ground level prior to felling. After felling, the limbs were removed and the height-to-live-crown (HTLC) and total height were measured with a tape measure to the nearest 0.1 ft. Each sample tree was measured for DOB at 0.5, 2, 4.5 and 8 ft, and every 4 ft above 8 ft until a 3 in DOB top was reached. All diameter measurements were collected with a diameter tape and recorded to the nearest 0.1 in. Where a measurement point coincided with a branch or some other defect, the measurement position was moved slightly up or down the stem to avoid the anomaly in the diameter and the new measurement position was recorded. These measurement procedures follow established protocols in the literature including those in Sherrill et al. (2011).

The stem was bucked into bolts and the green weight measured. Every tree was sampled and the green weight recorded within a few hours of felling. The green weight of each bolt was measured with a digital balance scale (resolution 0.2 lbs). The balance was tared between each measurement. The tip of the tree, that portion of the stem above the 3 in DOB top to the terminal bud, was also weighed. Longleaf pine grows dense needles directly from the tip of the stem, without additional branching structure to support the foliage. Those 'tufts' of needles along the leader were pulled out by hand. In some cases where this growth pattern was exaggerated, the needles alone could weigh several pounds, corresponding to a significant proportion of the total tip weight, and therefore it was essential to remove them before obtaining accurate measurements of the stem green-weight.

In the case of crooked trees with high stem sinuosity, the same process was carried out. However, in the case of forked trees, the process was modified to account for the unique shape and growth form. The main objective was to collect data for three stem sections from each forked tree: the main stem below the fork, the crotch base, and both stems above the fork all the way to the apical bud. The same process of measurements for taper and green weight were carried out along the stem up to the point identified as the crotch base. This crotch base was the point along the stem which shifted from decreasing diameter with increasing relative height, to an inflection point where the stem began to increase in diameter as a result of the added radial growth from two competing leaders in the fork. The crotch was calculated as the stem segment from the crotch base up to the point where the fork separated enough to allow for the diameter measurement to be taken for each of the component stems in the fork. At that point, the fork with a larger base diameter after the crotch (fork a) and the smaller fork (fork b) were identified. The fork pieces were measured for DOB at the base of each fork and the point at which each fork had a 3 in DOB top. The green weight for the forked trees followed the same process as noted earlier for the portion of the stem below the crotch base, and then the crotch was weighed, and then the three pieces of each fork were weighed, see Figure 1.4.

Sample Disks from Bolts

The process of sampling the 400 trees to collect DOB and green weight measurement resulted in 6–17 bolts per tree. From the base of each bolt and the tip a disk was collected to take additional measurement of wood and bark quality back in the wood quality laboratory. Those disks were cut with a chainsaw labeled and stored in plastic zip seal bags. Then the disks were transported back to the laboratory and stored in a freezer until they were processed. Each disk was processed to collect data for modeling wood and bark specific gravity, relative bark content and several other variables. This stage of sampling allowed us to recover the diameter inside bark (DIB) measurement from the sample trees previously destructively sampled in the field. However, due to the COVID-19 pandemic the complete collection of disks could not be processed. Instead, a subset of 16 stands (320 trees) were completed. This provides a data set to address the taper, volume, and weight measurement for both outside bark and inside bark.

An obvious point of interest was comparing the DOB we measured in the field with that of the DOB that was measured in the lab. The analysis showed a consistent trend in the field measurements being larger than those observed in the lab, see Figure 1.5. These consistent differences in DOB measurements made it so that we would model inside bark and outside bark separately.

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1.7 Tables and Figures

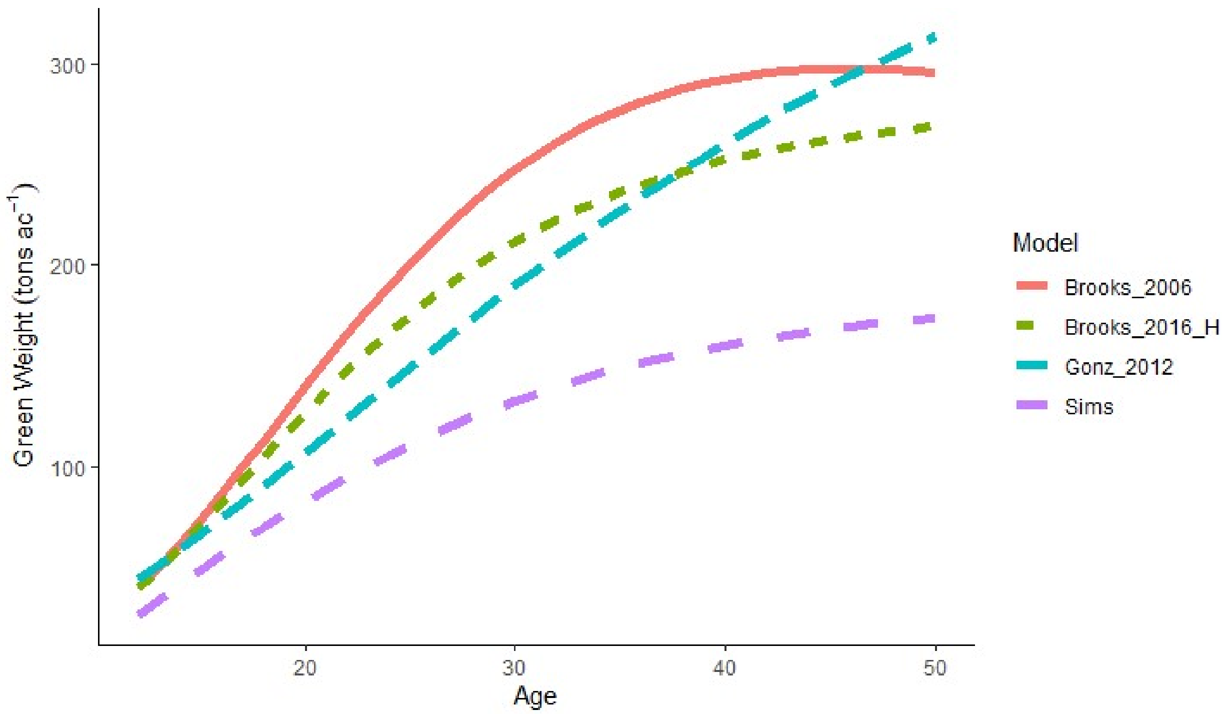

Figure 1.1: Green weight projections for planted longleaf stand: initial age =12; initial BA=85 ft^2/ac; initial TPA=550; dominant height@age 12 = 41 ft. "Brooks 2006", see Brooks and Jack (2006); "Brooks 2016 H", see Brooks and Jack (2016); "Gonz 2012", see Gonzalez-Benecke et al. (2012); and "Sims", see ForesTech International, LLC (2011).

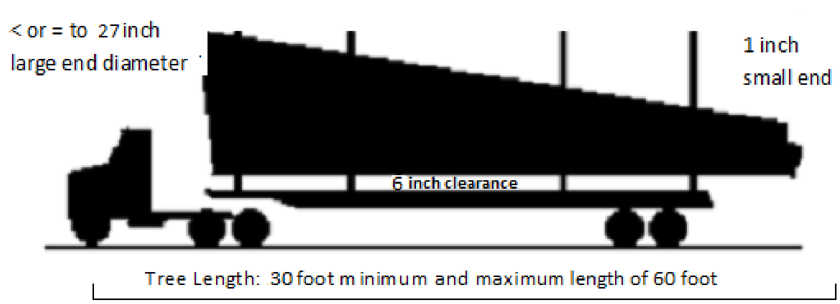

Figure 1.2: Treelength pulpwood size specifications, adapted from International Paper (2015).

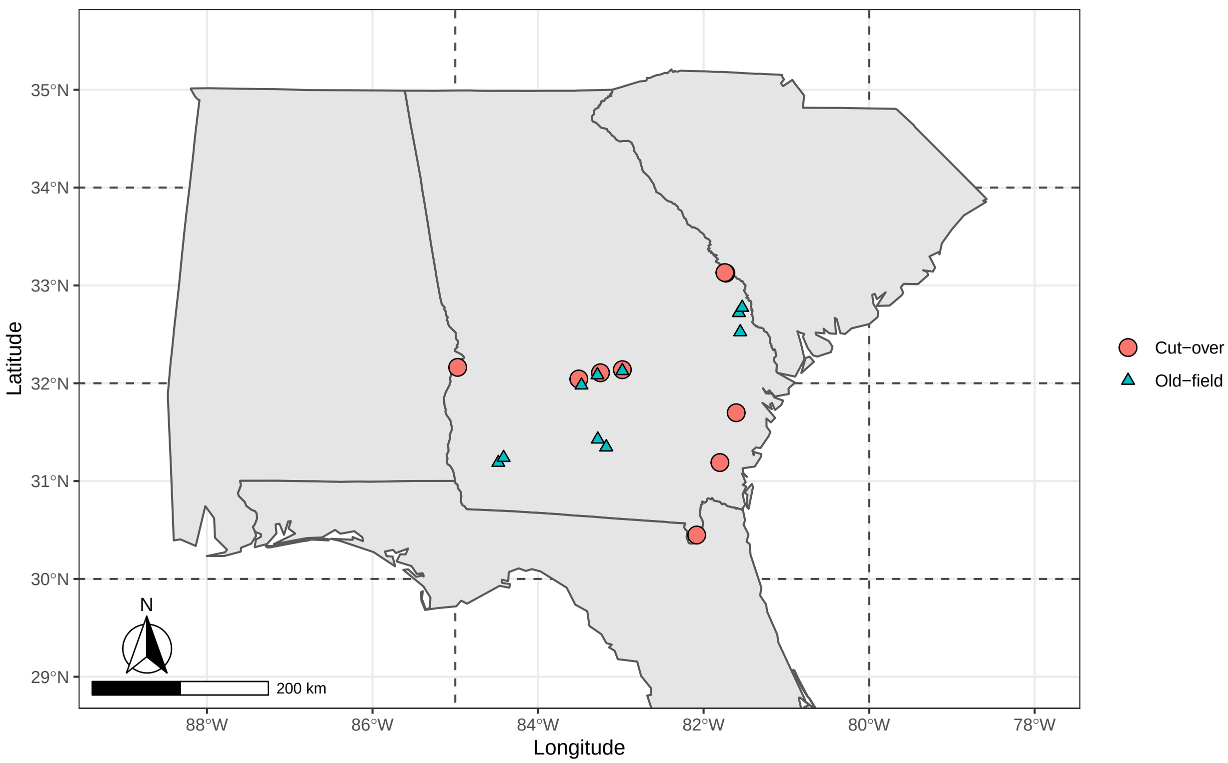

Figure 1.3: Study site locations of longleaf pine stands sampled in GA in this research.

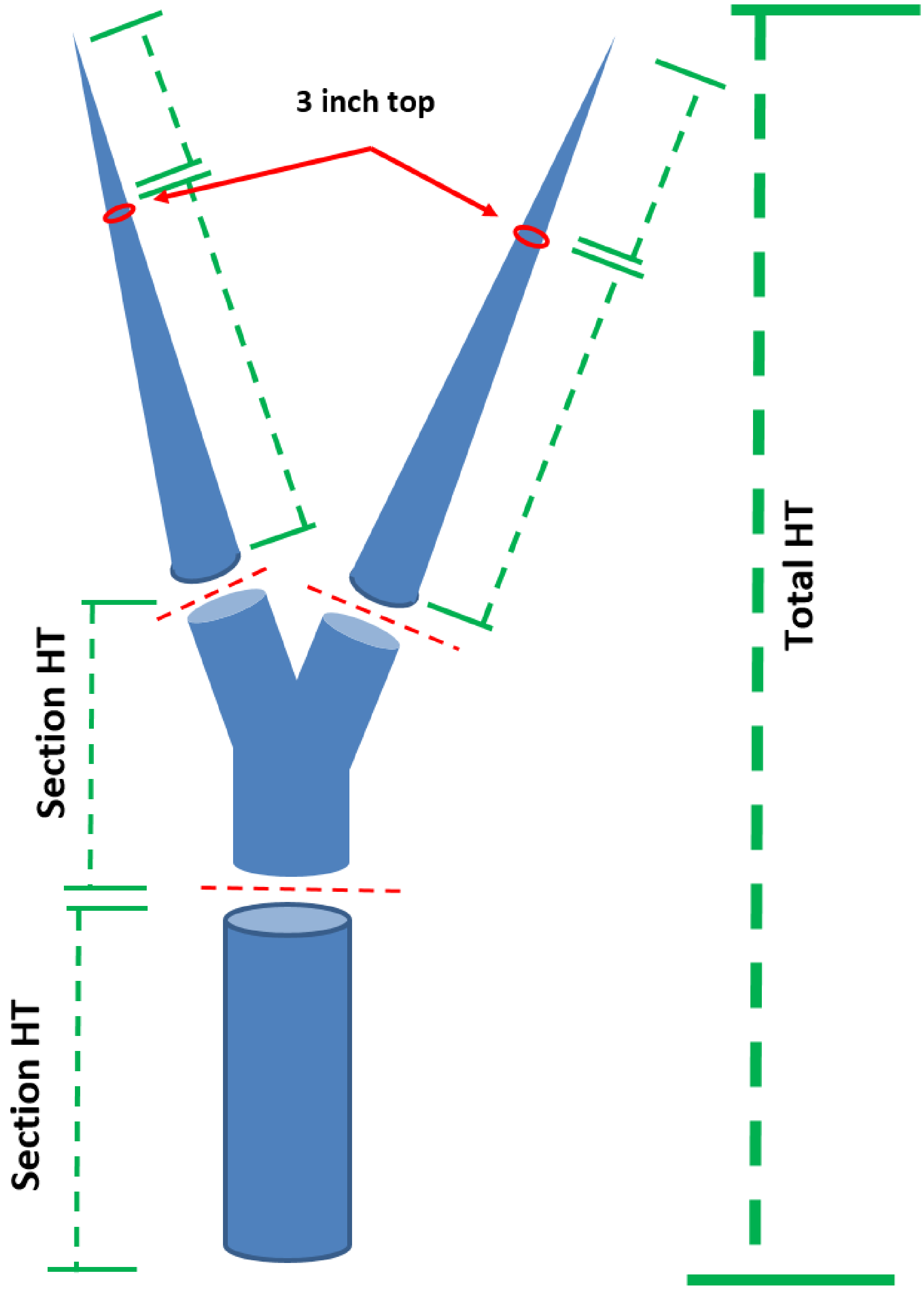

Figure 1.4: Fork tree sample protocol diagram. Note: HT = Height

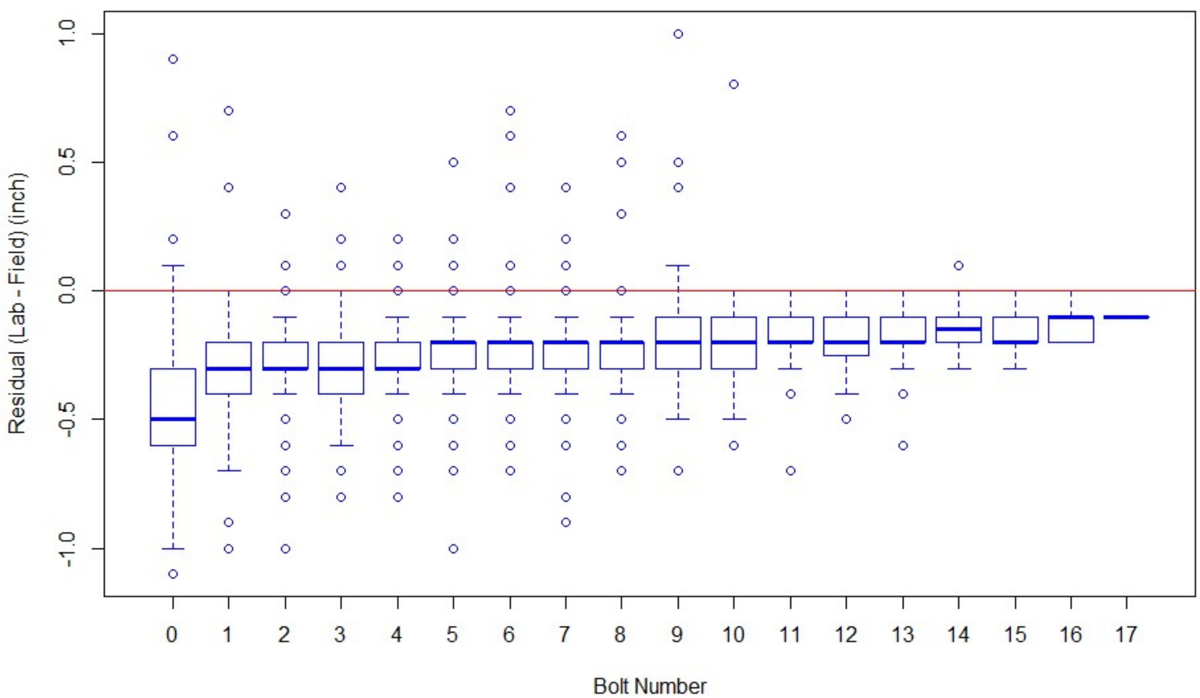

Figure 1.5: Differences in DOB for each bolt.

Table 1.1: Detailed stand description for all sites used in longleaf pine study. Planting spacing was unknown for some sites, shown with "-".

Site ID	Landowner	County	Lat.	Long.	Origin	n	Est. Year	Age	Spacing	Raking	Acres	Burned
1	1	Dodge	32.1388	−82.9816	cut-over	20	2003	16	8x14	No	35	Yes
2	1	Dodge	32.1271	−82.9815	old-field	20	2001	18	7x12	Yes	73	Yes
4	2	Dodge	32.1072	−83.2458	cut-over	20	2007	12	5x10	No	11	No
5	3	Dodge	32.0841	−83.2828	old-field	20	2007	12	6x13	Yes	29	Yes
6	4	Wilcox	32.0436	−83.5076	cut-over	20	2007	12	8x13	No	149	Yes
7	4	Wilcox	31.9802	−83.4751	old-field	20	2001	18	7x12	Yes	33	Yes
8	5	Berrien	31.3465	−83.1758	old-field	20	1999	20	-	Yes	4	No
9	5	Berrien	31.3474	−83.1756	old-field	20	1999	20	-	Yes	18	No
10	6	Berrien	31.4276	−83.2781	old-field	20	2004	15	6x12	Yes	29	No
11	GA Power	Burke	33.1262	−81.7308	cut-over	20	2003	16	8x10	Yes	52	Yes
12	GA Power	Burke	33.1310	−81.7428	cut-over	20	2000	19	8x10	Yes	16	Yes
13	GA Power	Stewart	32.1625	−84.9713	cut-over	20	2004	15	9x10	No	64	Yes
14	Rayonier	Charlton	30.4489	−82.0832	cut-over	20	1994	25	-	No	7	No
15	Rayonier	Brantley	31.1896	−81.8029	cut-over	20	2000	19	-	No	13	No
18	Rayonier	Long	31.6986	−81.6057	cut-over	20	1997	22	7x12	Yes	69	No
20	7	Screven	32.7201	−81.5731	old-field	20	2000	19	7x12	Yes	43	Yes
21	8	Screven	32.7741	−81.5332	old-field	20	2000	19	9x9	Yes	35	Yes
22	9	Screven	32.5241	−81.5569	old-field	20	2005	14	7x11	No	10	Yes
23	Jones Center	Baker	31.1884	−84.4798	old-field	20	2002	17	-	No	51	No
24	Jones Center	Baker	31.2392	−84.4162	old-field	20	2001	18	-	No	91	No

Table 1.2: Stand and Stock Table for all 20 sample sites used in taper data. Site index “SI” is base age 50.

Site ID	Age	Avg. DBH (in)	Avg. H (ft)	BA (ft^2/ac)	Dom. H (ft)	SI (ft)	TPA	Tons/ac	Vol/ac (ft^3/ac)
1	16	6.1	38.3	62	47.7	102.0	290	37.7	1218
2	18	7.1	50.9	96	50.3	107.6	333	75.9	2448
4	12	5.3	34.9	49	46.8	104.1	313	27.2	882
5	12	6.9	38.2	87	41.8	93.9	323	52.5	1697
6	12	5.9	29.9	38	40.0	89.3	190	18.2	589
7	18	7.6	46.2	103	53.3	105.4	307	75.4	2432
8	20	8.4	46.3	93	48.0	104.1	233	65.6	2114
9	20	9.0	51.4	115	47.8	104.1	257	88.5	2852
10	15	7.2	43.5	120	39.2	93.8	420	79.1	2554
11	16	5.9	38.1	69	48.7	109.8	357	41.7	1349
12	19	7.0	45.6	89	47.2	106.3	320	62.4	2014
13	15	6.0	36.5	68	49.3	108.8	335	39.0	1262
14	25	5.7	45.2	85	39.7	96.3	453	62.7	2027
15	19	7.1	58.3	104	39.9	95.2	370	93.6	3019
18	22	8.2	61.9	109	52.4	96.6	280	104.3	3362
20	19	7.6	48.8	118	48.6	91.3	357	88.3	2848
21	19	7.6	48.4	121	53.8	107.7	367	90.0	2905
22	14	6.0	34.5	85	49.8	107.7	423	46.4	1501
23	17	9.0	51.4	68	50.0	106.5	150	52.9	1704
24	18	7.7	42.7	58	48.1	104.6	170	38.6	1245

Chapter 2

Stem taper, volume, and Green Weight Equations for Longleaf Pine Plantations: Outside Bark[1]

[1]Harris, T. B., B. P. Bullock, C. R. Montes, and J. Dahlen. To be submitted to *Forest Ecology and Management*.

Abstract

The ability to predict volume or green weight from the inventory data collected in the forest is an important tool for managing forestland. Taper equations that predict diameters at any point on the stem are essential in forest inventory calculations as well. This chapter provides an in-depth review of the available taper, total volume, and green weight models published for longleaf pine plantations and their performance compared with our new set of measurements from 400 trees destructively sampled from 20 old-field and cut-over stands distributed across Georgia. More importantly, a suite of models were fit to the data and their performance was compared. The best model for taper used the Max and Burkhart (1976) form (RMSD = 0.277 in), the best model for total volume used the generalized logarithmic equation (Abs. Bias = 0.044 %), and the best green-weight equation used the Bullock and Burkhart (2003) form (RMSD = 0.02 lbs). Stem taper and volume did not vary with stand origin. The model selected for each variable of interest balanced the needs of precise predictions while also being adaptable to the geographic range of longleaf pine across the Atlantic Coastal Plain.

2.1 Introduction

Tree growth can be measured and modeled to predict numerous characteristics from diameter at a given height to the weight of the foliage for a specific tree. The emphasis of this chapter pertains to the modeling of the bole (stem) of the tree. The bole is the result of many years of growth of secondary xylem as well as the phloem, bark, and bark cambium. This portion of the tree has significant commercial importance, because this is the piece that is sold by landowners and utilized in the forest products industry to produce countless products our society depends on. Accurate models are needed to be used with measurements obtained during forest inventory and provide estimates of tree-level volume and green weight. It is generally true that the tree stem could be regarded as a cone, where the diameter is widest at the base, diminishing as you increase in height up to the terminal bud of the tree where the diameter becomes zero. However attempting to model stem volume using a cone will overlook the biological growth pattern in conifer stems. It has been widely shown that stem form is better regarded in three geometric shapes. The profile at the base of the tree is regarded as neilod, the middle section as a paraboloid, and the top as a cone (Burkhart and Tomé, 2012). A successful taper and volume equation will accurately predict changes in diameter and volume along the entire length of the stem.

The early work in taper models was the parabolic taper function developed by Kozak et al. (1969):

$$\frac{d_i^2}{D^2} = \beta_0 + \beta_1(h_i/H) + \beta_2(h_i^2/H^2) \tag{2.1}$$

where

d_i = diameter (inches) at any point on the stem corresponding to height h_i

D = diameter (inches) at breast height (DBH) 4.5 feet above ground

h_i = height at point along the stem

H = total tree height

Along the same lines of early taper models is the simple non-linear form proposed by Ormerod (1973) which includes only one parameter to be estimated.

$$d_i = D\left(\frac{(H - h_i)}{(H - 4.5)}\right)^{\beta_1} \tag{2.2}$$

One of the most impactful of the early taper models is that developed by Max and Burkhart (1976) which has been adapted to numerous species and geographic areas:

$$\frac{d_i^2}{D^2} = \beta_1 - 1(Z_i - 1) + \beta_2(Z_i^2 - 1) + \beta_3(a_1 - Z_i)^2 I_1 + \beta_4(a_2 - Z_i)^2 I_2, \tag{2.3}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} \quad j = 1, 2; Z_i = h_i/H \tag{2.4}$$

In pursuit of a more parsimonious model Thomas and Parresol (1991) formulated a trigonometric taper function:

$$\frac{d_i^2}{D^2} = \beta_1(\frac{h_i}{H} - 1) + \beta_2 \sin(\beta_3 \pi \frac{h_i}{H}) + \beta_4 \cot(\pi \frac{h_i}{H}/2) \tag{2.5}$$

This model still requires the estimation of 4 parameters and does not always predict $d_i = 0$ at the tip of the tree.

In several cases, additional predictor variables such as crown ratio (CR) and age are incorporated into the taper function. As noted in the previous chapter Baldwin and Polmer (1981) fit outside bark taper and volume models based on three separate classes of CR. Another example from longleaf pine is the paper by Thomas et al. (1995) which uses age in the final form of the proposed taper function. Although they can be shown to improve the model fit, they will simultaneously impose restrictions on the usability of the function when those additional variables are not known. In general, a more parsimonious model relying on DBH and total height is the most universally applicable form in forest mensuration (Burkhart and Tomé, 2012).

For every taper model, the developer needs a data set of many trees with many diameters recorded for each stem at different heights. One key point in the data structure that is used to fit these models is that the measurements from each stem are correlated. Such that, the diameter at a point along the stem is closely correlated in space with any nearby point on the same stem. To account for this auto-correlation Gregoire and Schabenberger (1996) incorporated a mixed effect model to account for the intra-tree relatedness in observations, and the approach improved the model fit to their data.

The models evaluated in this chapter include many of the previously mentioned. An attempt is made to evaluate the performance of the published models compared with the taper data set we collected from our destructively sampled 400 longleaf pine stems. As noted by Amateis and Burkhart (1987) taper and volume can vary by stand origin, where using an F-test they showed evidence that different models would be used in the different loblolly pine stand types. The stand origin categories included loblolly pine plantations and natural loblolly pine. At that point in time, the sites with planted loblolly pine would have been established with improved genetics and could have led to the significant difference in stem taper and volume between the natural and planted sites. So we will test to see if significant differences exist between the old-agricultural field and cut-over sites in our sample. Additionally, we will fit several different taper models and select the best model with parameters fit to predict values for planted longleaf pine growing in Georgia and the Atlantic Coastal Plain.

2.2 Methods

2.2.1 Study Site

There were a total of 20 different longleaf pine stands sampled across Georgia, with 11 from old-field stands with previous cover types including row crops and hay pasture, the other 9 stands were from cut-over stands with previous cover types of longleaf, slash, or loblolly pine stands.

2.2.2 Data

As previously mentioned, several different taper equations were fit to the longleaf pine taper data set to ascertain the model form with the best fit. The data represent the taper stem measurements from 400 destructively sampled trees. The data will be used to fit the taper equations and the estimated parameters will be recorded and reported. Although 400 total stems were sampled, 324 of those had straight stems and were free from defects of forking, rust, and excessive sweep. The distribution of the DBH and total height for the 324 sample trees without defects by stand origin is shown in Figure 2.1, and the DBH vs. total volume is shown in Figure 2.2. The predictor variable for most of the models is D^2H, the, roughly linear, relationship between that variable and the total volume is shown in Figure 2.3.

Another useful visualisation is the relative DOB plotted against relative height (Figure 2.5). Where relative DOB $= d_i/D$, and relative height $= h_i$/H. This shows the total variation that the taper models will be fit to using different methods of least squares and non-linear least squares regression models. Another view of the data is the overlapping stem profiles in the real measurement scales of diameter outside bark (in) and height (ft) (Figure 2.6).

2.2.3 Comparing Taper Model Forms

The analysis of the results was conducted using R (R Core Team, 2018). To compare the various taper model forms, we evaluated the predicted values from each fit taper model with the data measured in the field. The goodness of fit for the models was compared on three metrics: average bias, relative absolute bias in percent, and the root mean square distance (RMSD).

$$\text{Bias} = (y_i - \hat{y}_i) \tag{2.6}$$

$$\text{Relative Absolute Bias (\%)} = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{\sum_{i=1}^{n} y_i} 100 \tag{2.7}$$

$$\text{RMSD} = \sqrt{\left(\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}\right)} \tag{2.8}$$

Where: y_i = observation value, $\hat{y}_i$ = the predicted value for the ith observation, n = sample size. These measures of goodness of fit provide information about both spread and magnitude of the residuals and performance of different model forms.

2.2.4 Comparing Volume Model Forms

From every taper equation, one can derive the implied volume equation. However, it is also useful to evaluate the separate volume equations common in forestry. Equations can be formulated to estimate the volume of any portion of the stem. However, we are concerned with estimating total stem volume because the size of all the trees in our data collection were pulpwood size, which implies the majority of their volume can be harvested and utilized by a mill.

We evaluated four common equations for total stem volume:

$$\text{Combined variable} \quad V = \beta_0 + \beta_1 D^2 H \tag{2.9}$$

$$\text{Constant form factor} \quad V = \beta_1 D^2 H \tag{2.10}$$

$$\text{Logarithmic} \quad V = \beta_1 D^{\beta_2} H^{\beta_3} \tag{2.11}$$

$$\text{Generalized logarithmic} \quad V = \beta_0 + \beta_1 D^{\beta_2} H^{\beta_3} \tag{2.12}$$

Where: V = stem volume; D = DBH; H = total height; and β_0, β_1, β_2, β_3 are parameters to be estimated (Burkhart and Tomé, 2012).

2.2.5 Comparing Green weight equations

The green weight of the complete stem was estimated using two different equation forms. First we followed the form by Baldwin (1987) which used a modified Schumacher and Hall (1933) logarithmic form:

$$W = \beta_1 D^{\beta_2} H^{\beta_3} \tag{2.13}$$

Where W is the green weight in pounds, β_1, β_2, and β_3 are parameters to be estimated, D = DBH in inches, and H = total height in feet. This equation was proven to work well for loblolly pine (Baldwin, 1987). The second equation we tested was the green weight equation from Bullock and Burkhart (2003) that was also proven to work well with loblolly pine:

$$W = \beta_0 + \beta_1 (D^2 H) \tag{2.14}$$

Where W is the green weight in pounds, β_0 and β_1 are parameters to be estimated, D = DBH in inches, and H = total height in feet. We also compared the effect of stand origin using the form in Equation 3.14, and adding dummy variables β_0' and β_1' to the β_0 and β_1 parameters:

$$W = (\beta_0 + \beta_0' p) + (\beta_1 + \beta_1' p)(D^2 H) \tag{2.15}$$

$$p = \begin{cases} 1 & \text{if Old-field} \\ 0 & \text{if Cut-over} \end{cases} \tag{2.16}$$

where W is the green weight in pounds, β_0, β_0', β_1, and β_1' are parameters to be estimated, D = DBH in inches, and H = total height in feet.

2.2.6 Merchantable volume and green weight

Thus far, we have presented the different models for estimating total stem values for taper, volume and green weight. Although the total stem can be utilized in some cases it is also common that the stem would only be merchantable up to some top diameter. In the process of collecting our destructively sampled data, we imposed a 3 in top DOB to demarcate between the last bolt and the top. But, many mills have different top size requirements, which range from "buying to the bud" where the whole stem is delivered to the mill to the more common minimum top diameter of 2-3 in for pulp and paper mills up to 4 inches for super pulpwood or oriented strand board (OSB) mills (Harris, 2019). The top diameter restriction is imposed to reduce the losses from breakage in the debarking process and low pulp yield from the younger top portion of the stem. Mills do not want to pay pulpwood prices for portions of the stem that can not be utilized in the paper making process. So, the impact is that the logger will trim the stems that have been loaded onto the trailer before leaving the woods, leaving the tops behind to be scattered across the harvest site. So, for the landowner to have an accurate estimate of merchantable volume or weight of their trees that will go across the scales, they need a way to calculate the portion of a whole stem that will be used at a specified top DOB.

We have followed the process from Bullock and Burkhart (2003) to fit ratio equations for volume and green weight. The ratio equations modify the estimated volume to predict the merchantable volume at a specified top DOB.

$$R_{vol} = 1 + \beta_1 \left(\frac{d^{\beta_2}}{D^{\beta_3}} \right) \tag{2.17}$$

where

d = upper diameter limit, outside bark (in)

β_i = coefficients to be estimated, i = 1,2,3

The ratio equation to predict the merchantable green weight at a specified top DOB.

$$R_{wt} = 1 + \beta_1 \left(\frac{d^{\beta_2}}{D^{\beta_3}} \right) \tag{2.18}$$

where

d = upper diameter limit, outside bark (in)

β_i = parameters to be estimated, i = 1,2,3

2.3 Results

Following the process outlined in the methods for each subsection, we present the results for each component. We review the taper equations for outside bark, total volume, total green weight, ratio equations for merchantable volume and green weight, and the comparison of published volume and green weight models.

2.3.1 Taper Models: Outside Bark

Past work with loblolly pine has demonstrated differences based on stand origin in cut-over and old-field sites by Amateis and Burkhart (1987), so we wanted to determine if this relationship existed in our sample data. We evaluated the estimated parameters from the Kozak et al. (1969) confidence intervals for the full model with all 324 trees, the reduced model with 175 trees in the old-field group, and 149 trees in the cut-over site group. The variance in estimated parameters based on stand origin was significantly different based on stand origin, but only slightly. In fact the parameter estimates were very similar, and only the parameter corresponding to the relative height variable was significantly different in all three groupings, see Table 2.1. A closer look at the confidence intervals revealed that the one parameter corresponding to the relative height variable had a significant difference in the confidence intervals based on alpha-value = 0.05. If the alpha level was raised to = 0.01, then all three estimated parameters had overlap in the confidence intervals and showed no significant difference based on stand origin. This process of evaluation was straightforward for the linear model from Kozak et al. (1969). We were interested to see if the same relationship existed in other taper models. So, we selected the non-linear segmented polynomial taper equation from Max and Burkhart (1976) to evaluate as well. Rather than recovering the estimated parameter confidence intervals, we choose a more robust way to evaluate the effect of stand origin on taper by simultaneously estimating the model form and using an indicator variable according to stand origin. In our study, every tree belonged to either of two groups, cut-over sites or old-field sites. So, if there was a significant effect on the model of taper it would be shown in those indicator variables. Initially, we added dummy variables β_1', β_2', β_3', β_4' to each of the original β_1, β_2, β_3, β_4 parameters

$$\frac{d_i^2}{D^2} = (\beta_1 + \beta_1' p)(Z_i - 1) + (\beta_2 + \beta_2' p)(Z_i^2 - 1) + (\beta_3 + \beta_3' p)(a_1 - Z_i)^2 I_1 + (\beta_4 + \beta_4' p)(a_2 - Z_i)^2 I_2, \tag{2.19}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} \quad j = 1, 2; Z_i = h/H \tag{2.20}$$

$$p = \begin{cases} 1 & \text{if Old-field} \\ 0 & \text{if Cut-over} \end{cases} \tag{2.21}$$

However, the additional dummy variables β_1', β_2', β_3' proved to be insignificant predictors in stem DOB taper, and only β_4' was significant, so the model was revised and we dropped the insignificant dummy variables and re-fitted the model with the addition of the β_4' dummy variable

$$\frac{d^{i2}}{D^2} = \beta_1(Z_i - 1) + \beta_2(Z_i^2 - 1) + \beta_3(a_1 - Z_i)^2 I_1 + (\beta_4 + \beta_4' p)(a_2 - Z_i)^2 I_2, \tag{2.22}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} \quad j = 1, 2; Z_i = h/H \tag{2.23}$$

$$p = \begin{cases} 1 & \text{if Old-field} \\ 0 & \text{if Cut-over} \end{cases} \tag{2.24}$$

We then set out to see if the numerically significant dummy variable had any practical effect on the predictions from the model. So, we simulated the taper predictions for 3 trees at the extreme ends of our data set and the average tree, one tree with DBH = 3.9 inches and total height of 28 feet, a second tree with DBH = 7 inches and total height = 50 feet, and the third tree with DBH = 12 inches and total height = 74 feet. These hypothetical trees will apply the difference in the proposed taper modeling including a dummy variable for stand origin. In Figure 2.7 we notice that the predicted stem profile for all three trees is almost exactly the same between the cut-over, old-field, and combined models. In fact, the only noticeable difference was at the very base of the tree below the usual stump height of 0.5 feet which would not typically be utilized in a traditional harvest. This figure shows essentially no difference in stem taper based on stand origin. Thus, we fail to reject the hypothesis that there is no difference in modeling stem taper based on stand origin, and therefore the implied volume, in longleaf pine plantations. Although there may be a small numerical significance based on stand origin, when applied to new observations, the effect is so minimal that it does not justify imposing a more complex model with an additional parameter. The result is the proposition of using the more parsimonious model in Equation

2.3. Based on our analysis of the different model forms for taper, the best model is the Max and Burkhart (1976) fit to the entire data set of 324 trees.

We did not have sufficient evidence that there is significant difference in the taper models estimated for longleaf pine plantations based on stand origin. Therefore, as we move forward with this analysis we will present the full model using all 324 non-defect trees for outside bark model fitting and errors comparison. The differences in wood quality from stand origin may exaggerate the effects of insignificant difference in taper and volume to produce significant difference based on stand origins for the variable of green weight. So, the effect of stand origin on the estimation of green weight will be evaluated in that analysis. However, in Figure 2.4 we show the break out of the relationship in total height and DBH. There seems to be different trends based on stand origin here, and it might be helpful to model height-by-diameter equations separately.

Based on the comparison of criteria for taper models, the models from Kozak et al. (1969) in Equation 2.1 is the worst with the highest RMSD = 0.473 in. The best performance is attained similarly for Max and Burkhart (1976) in Equation 2.3 (RMSD = 0.277 in) and then fit with mixed effects for β_1 and β_2 following to the process from Trincado and Burkhart (2006) (RMSD = 0.276 in). Given that the model performance is not greatly improved from the inclusion of the mixed effects, either fitting method would be satisfactory to model taper and predict the DOB for this data set on longleaf pine. See Table 2.2 for complete goodness of fit criteria for all 5 model forms.

We sought to test the outside bark taper model fit to our data with an external data set. We used the taper data set from Brooks et al. (2007) to validate our outside bark taper model. The validation data set had 42 trees, with DBH ranging from 1.9–7.2 inches and total height from 17.6–43.9 feet. The distribution of height by diameter is shown in Figure 2.8 and the stem profiles for all 42 trees are shown in Figure 2.9. These data were collected from longleaf pine plantations growing in southwest Georgia. One key point to note is that many of the trees were smaller and outside the size range of the data we used to fit our taper models. However, the fit appeared to be adequate especially in the lower portion of the stems. The upper stem diameters had more bias from the prediction of our model. In the upper 10%, or relative heights from 0.9–1.0, had an average relative absolute bias of 53% , which may seem quite large but in actual measurements that comes out to be 0.09 inches, which is within the measurement error for DOB using a loggers tape. Over all the heights, the model relative absolute bias was 7.5% compared with 3.5% within the data used to fit the model. So we are satisfied with the performance and propose the Max and Burkhart (1976) model form as our final model (Table 2.3).

In addition to the models that we have fit in this work there are three papers that we evaluated which had taper models for longleaf pine plantations. The papers from Baldwin and Polmer (1981); Brooks et al. (2007) published taper equations for both inside and outside bark; while the paper from Thomas et al. (1995) provides a taper model for inside bark only. So, we wanted to compare the values predicted using these models and the parameters estimated and included in the publication with the observations we had. This comparison was interesting because the model from Baldwin and Polmer (1981) performed the best. However, this model incorporates a total of 15 parameters, and requires the input of the crown ratio to operate. The model by Brooks et al. (2007) performed well, and supports our selection of the same base model based on the performance of the segmented taper equation from Max and Burkhart (1976).

2.3.2 Total Volume Equations

We compared four model forms to evaluate total volume for our data set. The selection of the best volume model form depends on your criteria. In this case, the generalized logarithmic is the best choice on all three criteria of bias, relative bias, and absolute relative bias, see Table 2.4. The combined variable equation over predicted volume in small and large trees and slightly over predicted volume in medium size tree, diameter classes (D-class) 7–9 in. The constant form factor equation underestimates volume in the smaller D-classes 4–8 in, and over predicted volume in the larger d-classes up to 12 in. The logarithmic equation is similar to the combined variable in over estimating volume in the smaller and larger d-classes with underestimation in the middle. The generalized logarithmic has mixed bias in either direction based on d-class and seems to avoid systematic bias based on the D-class and is centered at 0.000 in bias overall. However, the model forms perform very well when fitted with least squares regression, all four forms would work suitably well in modeling total volume. The estimated parameters fit to our data set are shown in Table 2.5.

2.3.3 Total green weight equations

We tested two model forms for green weight equations. We compared the mean relative absolute bias in each model and found that Equation 2.14 performed better than Equation 2.13 and had one less parameter to be estimated, see Table 2.6. We then used Equation 2.15 to test the effect of stand origin. However, the dummy variables β_0' and β_1' were insignificant predictors of green weight and did not provide evidence to support the more complex model accounting for stand origin. So that indicates the basic form in Equation 2.14 is acceptable for modeling green weight in either old-field or cut-over stands. Therefore, we would select the more parsimonious model for total green weight comparison. The complete prediction model is:

$$W = 3.470008 + 0.152546(D^2H) \tag{2.25}$$

Where W is the green weight in pounds, D = DBH in inches, and H = total height in feet.

2.3.4 Merchantable volume and green weight

We fit the models for merchantable volume and green weight described by Bullock and Burkhart (2003) to our data from 324 straight non-defect trees. These equations are used in conjunction with the total volume or green weight equations identified above as the best fit to our data. The full equation to predict volume at a specified top diameter combines the generalized logarithmic equation for total volume with the volume ratio equation

$$V_{merch} = \left(0.1830191 + 0.0034072(0.0041133(D^{1.8644660})(H^{0.9695934}))\right) \times \left(1 - 0.5969959\left(\frac{d^{3.2529816}}{D^{3.3692120}}\right)\right) \quad (2.26)$$

where

V_{merch} = merchantable volume outside bark (ft^3)

d = upper diameter limit, outside bark (in.)

D = diameter at breast height (DBH), 4.5 feet above ground (in.)

H = total height (feet)

The full equation to predict green weight at a specified top DOB combines the green weight equation using the Bullock and Burkhart (2003) form and the green weight ratio equation

$$GWT_{merch} = \left(3.470008 + 0.152546(D^2H)\right)\left(1 - 0.6692696\left(\frac{d^{3.1217456}}{D^{3.1909342}}\right)\right) \quad (2.27)$$

where

GWT_{merch} = merchantable green weight outside bark (lbs)

d = upper diameter limit, outside bark (in.)

D = diameter at breast height (DBH), 4.5 feet above ground (in.)

H = total height (feet)

2.3.5 Comparing Published Volume and Green Weight Models

Looking back to the history of published individual tree volume models is a key point to see how the models perform compared with our data set. To evaluate the Gonzalez-Benecke et al. (2014) models, we started with the first volume outside-bark prediction equation which uses only diameter as a predictor variable for total volume. Here we show the model bias was, on average, overestimating total volume by 2.36 ft^3, 95% CI(2.19, 2.53), in terms of relative bias 38.3% overestimation of total volume, 95% CI(35.8%, 40.2%), see Figure 2.10. We also tested the second version of the outside bark total volume equation from Gonzalez-Benecke et al. (2014) which added the tree height as a predictor variable. Combining height and diameter as predictor variables greatly increased the model accuracy and reduced the bias in the model predictions. Comparing the model from the publication to our data we showed a mean bias of -0.39 ft^3. So, the model overestimated the volume with a mean bias of 0.39 ft^3, 95% CI(0.34, 0.44) ft^3. On a relative basis, it comes out to be a 6.2% overestimation on average, 95% CI (5.5%, 7.0%), see Figure 2.11. In summary of the models from Gonzalez-Benecke et al. (2014), the models have a statistically significant difference in both volume and height predictions compared with the new observation we collected.

The second publication of interest is Baldwin (1983) to compare equations for volume and green weight. First, let us compare the proposed model of total volume outside bark. Here we see the Baldwin (1983) model overestimates the total volume, with mean bias of 0.161 ft^3, 95% CI (0.113, 0.210) ft^3, and to be overestimated by 1.8%, 95% CI(1.2%, 2.5%), see Figure 2.12. Then, we compared the green weight prediction equation which uses height and DBH to estimate the total green weight in pounds. The results of our analysis show that the equation overestimated green weight by 25.4 lbs, 95% CI(21.9, 28.9). On a percent basis, we find that weight is overestimated by 7.7% on average, 95% CI(6.8%, 8.6%), see Figure 2.13.

Next we will take a look at the first volume equation for planted longleaf pine published by Schmitt and Bower (1970). This study used destructive sampling for 200 trees in southern Mississippi. All trees were age 7 and from three large plantations, and the DBH ranged from 1–8 inch. They used a simple combined variable equation. The model performed poorly compared with our data. Although, the trees used for comparison were larger than those used to construct the model by Schmitt and Bower (1970). The results of our analysis show that the equation underestimated outside bark volume on average by 1.7 ft^3, 95% CI(1.63, 1.84) ft^3. On a percent basis, we find that it is underestimated by 24.2% on average, 95% CI(23.7%, 24.7%), see Figure 2.14.

A comparison of total volume was used for a recent publication by Brooks et al. (2007). These models were constructed using data from n=42 sample longleaf pine trees in southwest Georgia. The results of our analysis show that the equation underestimated outside bark volume on average by 1.12 ft^3, 95% CI(1.01, 1.22) ft^3. On a percent basis, we find that it is underestimated by 14.6% on average, 95% CI(13.8%, 15.4%), see Figure 2.15.

2.4 Discussion and Conclusions

A key finding in this analysis is that the estimated taper, implied volume, and green weight models did not vary with the stand origin. We tested two different taper models and forms of evaluation, and reached the same conclusion. The stand origin has the slightest effect on the model, but not enough to be significant in application, and can be disregarded and a simple model for taper and volume can be proposed for all sites.

One interesting finding was the inclusion of mixed effects did not greatly improve the basic segmented taper model from Max and Burkhart (1976). Perhaps, the use of this large data set, with 324 trees, provides a more comprehensive observation of the many relationships between relative height along the stem and the predicted DOB. Perhaps, if fewer trees were used to fit the model, the use of the mixed effects modeling could improve the fit. Also, we have a roughly equal number of observations of DOB per tree and stand which could contribute to similar parameter estimates using non-linear least squares and mixed effect modeling. This finding does not invalidate the usefulness of mixed effects modeling in taper, but rather is one observation on its effects with the data we collected for longleaf pine stands.

For those looking for a complex model and have information about crown ratio in their data, they could use the Baldwin and Polmer (1981) model. If you did not know the crown ratio or wanted a good regional model for Georgia, we suggest using our version of the Max and Burkhart (1976) fit with all 324 non-defect sample trees, see Table 2.7 for estimated parameters.

The main point to take away from comparing all of these volume and green weight equations is defining the bias they would introduce compared with our observed data for longleaf pine growing in Georgia. Therefore, this supports our effort to evaluate models that will perform better for the stand conditions we included in our sample and provide the new estimated parameters.

2.5 References

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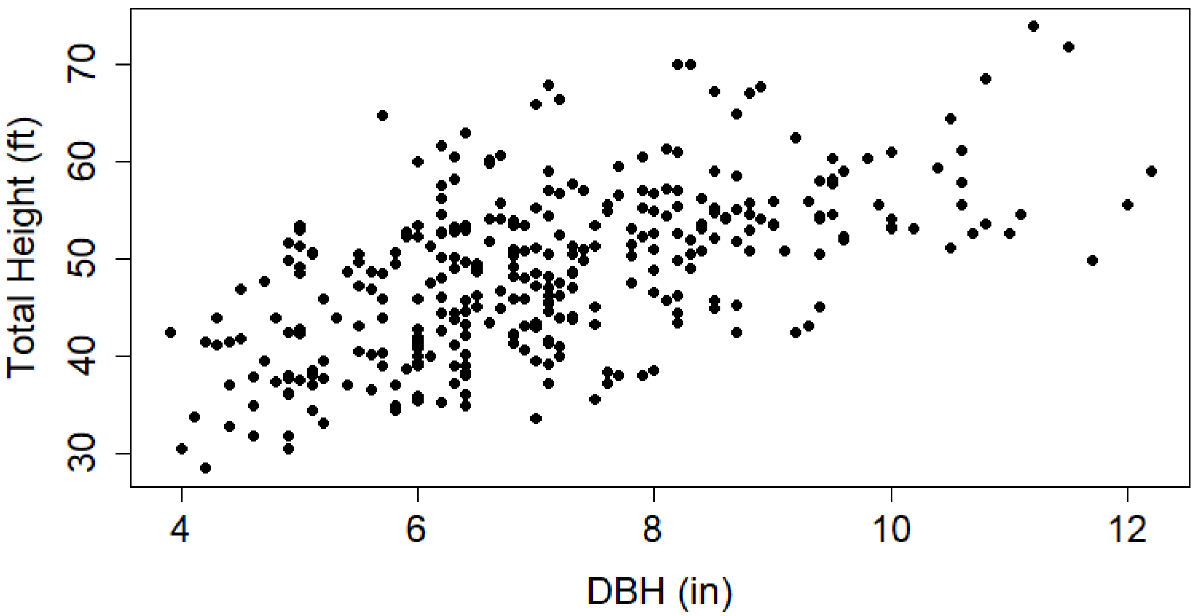

Figure 2.1: Total height versus diameter at breast height (DBH), for 324 non-defect sample trees of longleaf pine sampled at 20 different locations in GA.

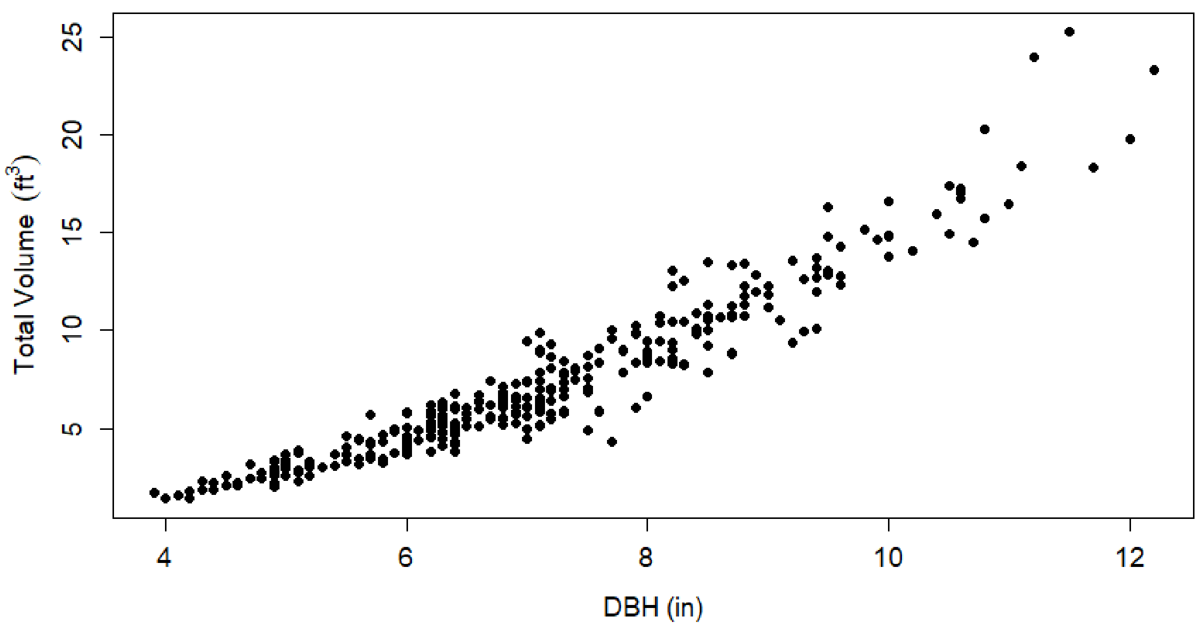

Figure 2.2: Total volume versus DBH, for 324 non-defect sample trees.

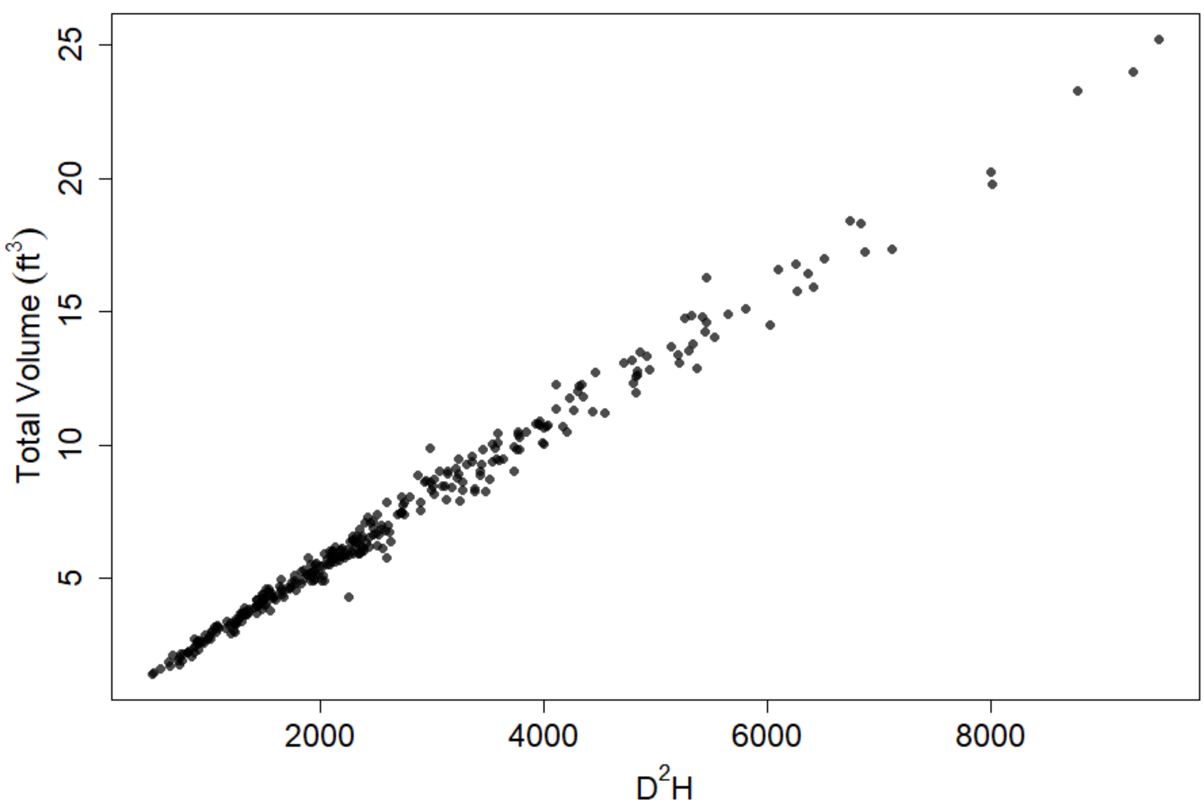

Figure 2.3: Total volume versus D^2H, for 324 non-defect sample trees. Where D = diameter at breast height (in) and H = total height (ft)

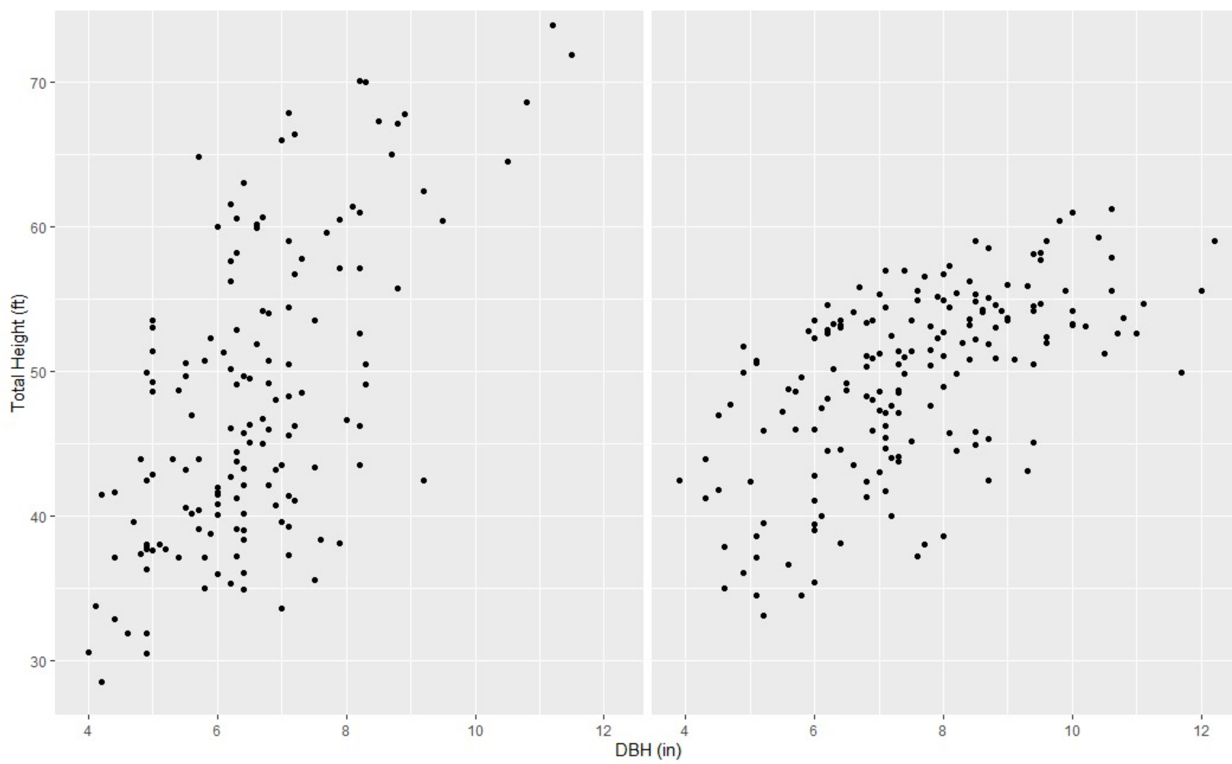

Figure 2.4: Total height and DBH for 324 non-defect sample trees. Left figure for cut-over sites, right figure for old-field sites.

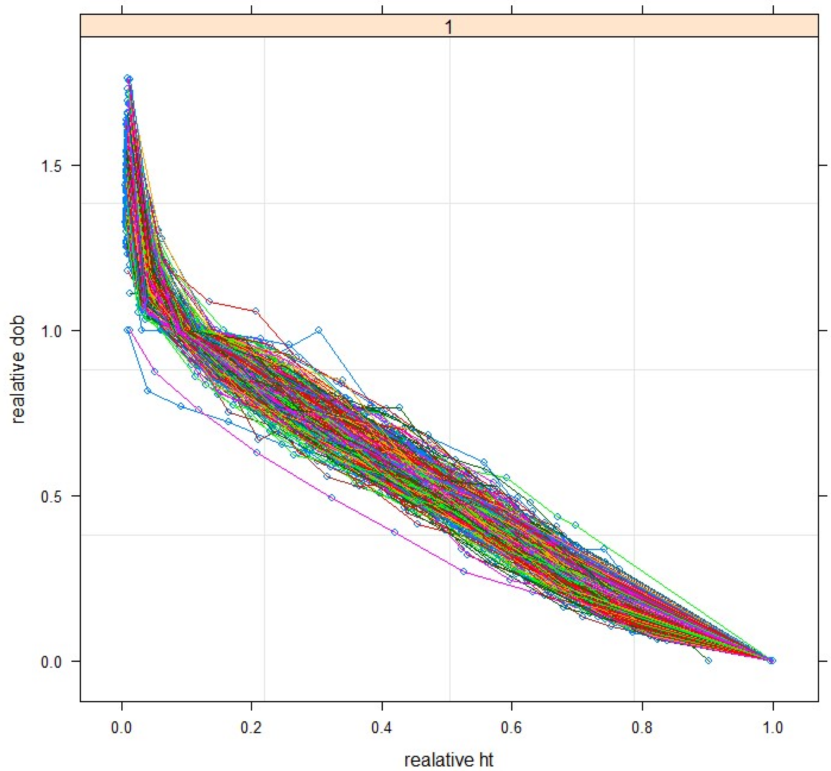

Figure 2.5: Stem profiles in relative DOB (d_i/D) and relative height (h_i/H) for 324 non-defect sample trees. Note: D = DBH and H = total height.

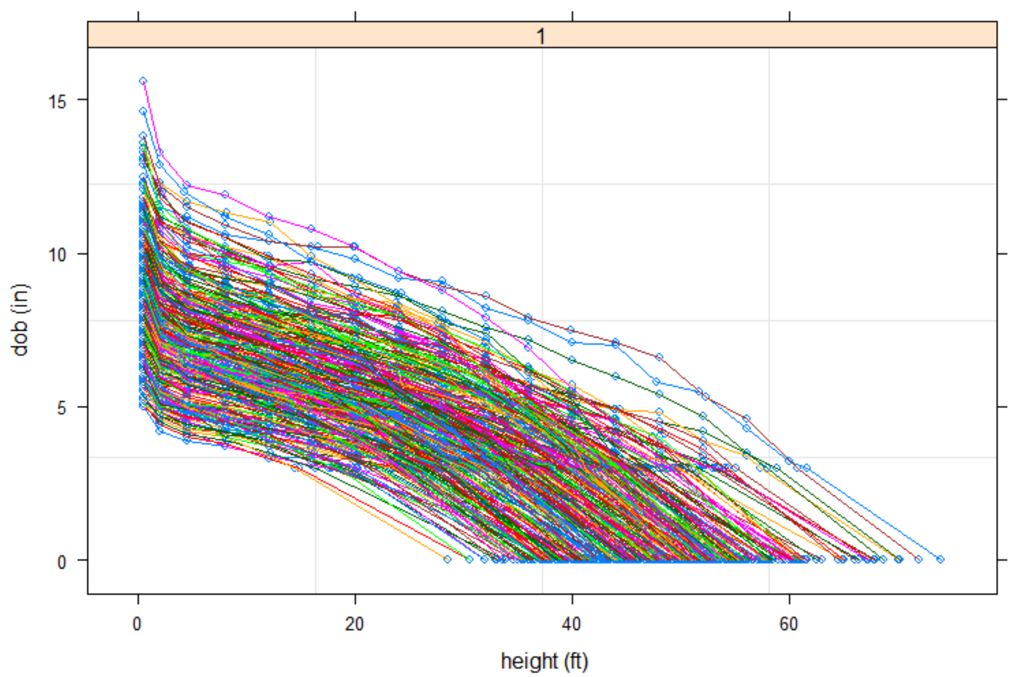

Figure 2.6: Stem profiles in DOB (in) and height (ft) for 324 non-defect sample trees.

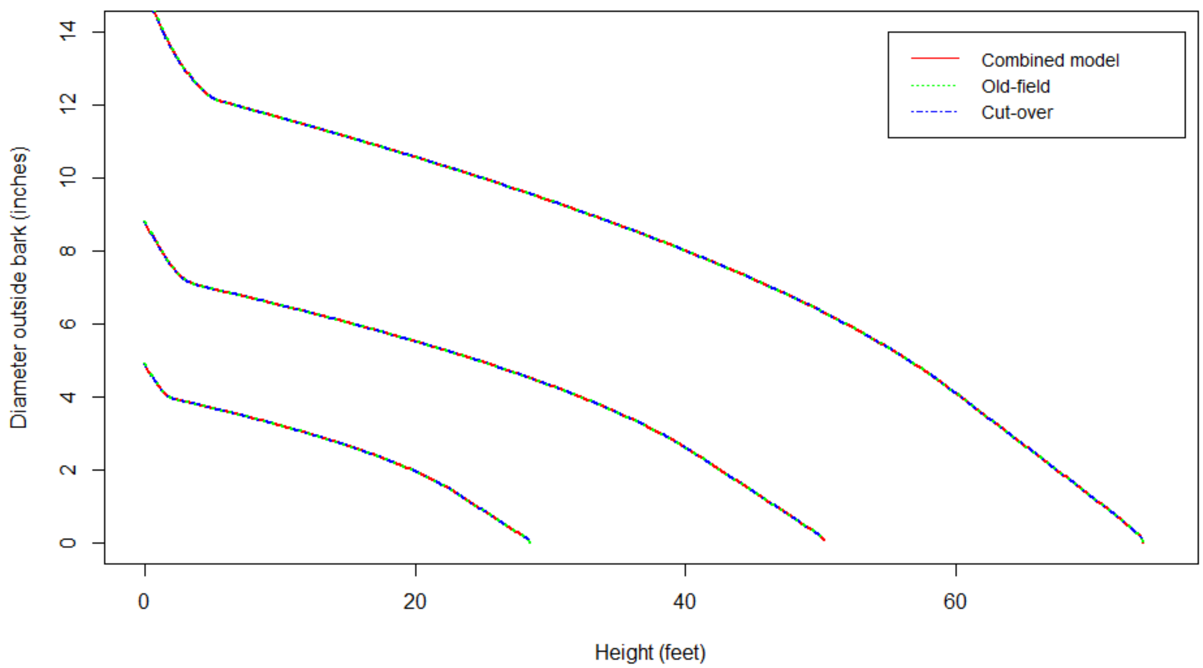

Figure 2.7: The predicted taper for three example trees: Small DBH = 3.9 inch, total height=28 feet; Average DBH = 7 inch, total height = 50 feet; and Large DBH = 12 in, total height = 74 ft of either old-field or cut-over origin, and the combined model. Note: all three lines are plotted on top of each other and may be indistinguishable.

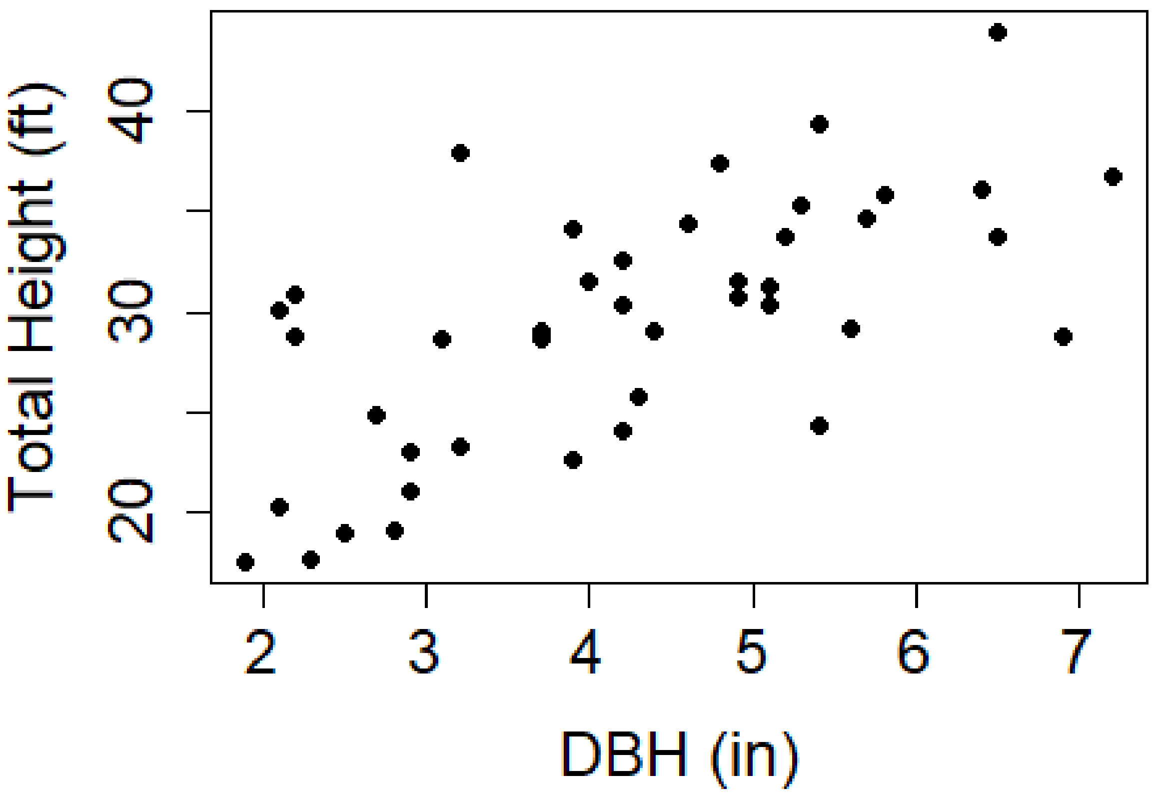

Figure 2.8: Total height and DBH for 42 trees in the validation data set. Data comes from the study by Brooks et al. (2007).

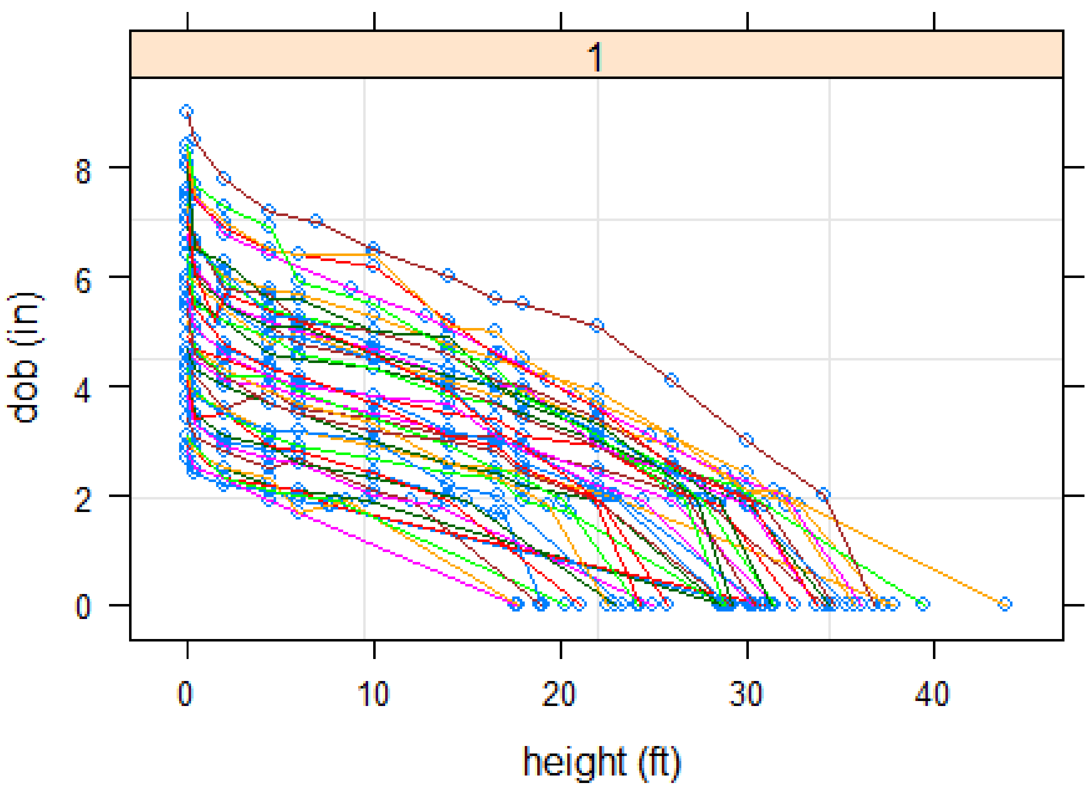

Figure 2.9: Stem profiles for 42 trees in the validation data set. Data comes from the study by Brooks et al. (2007).

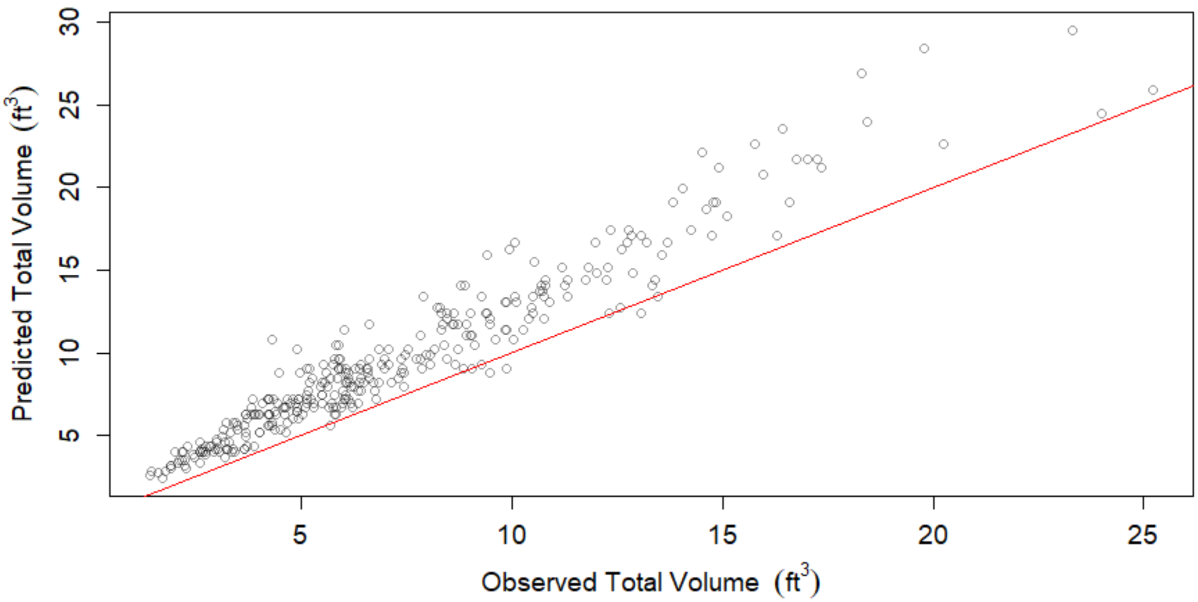

Figure 2.10: Total volume o.b. and predicted volume o.b. EQ. 1 from Gonzalez-Benecke et al. (2014). The 1:1 line is plotted as a solid red line.

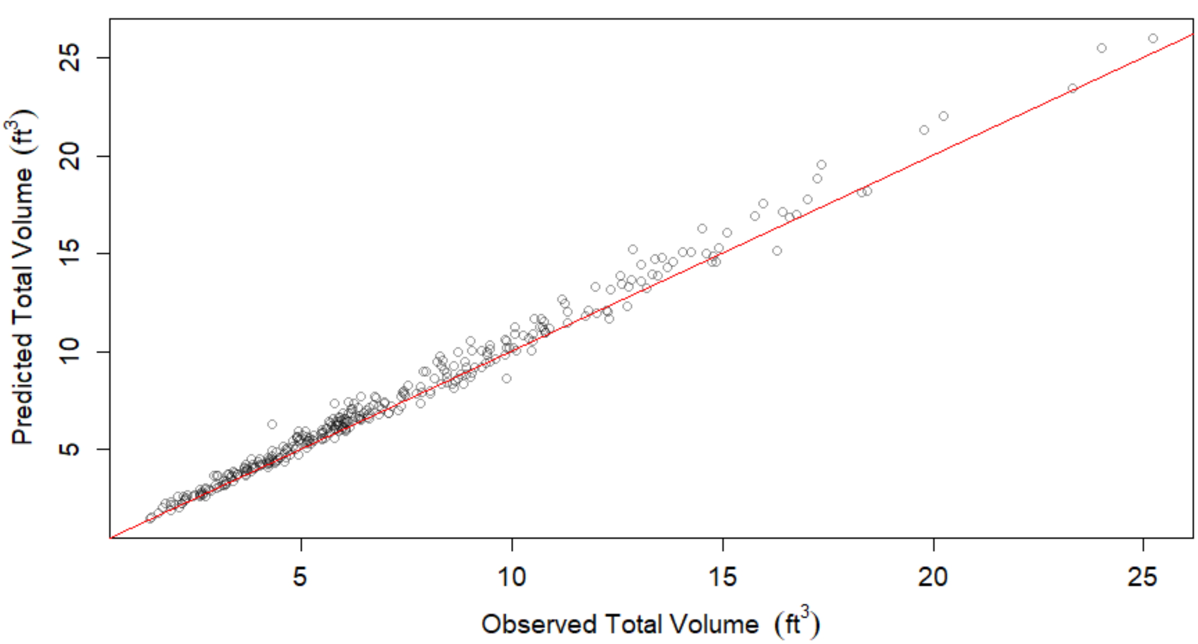

Figure 2.11: Total volume o.b. and predicted volume o.b. EQ. 2 from Gonzalez-Benecke et al. (2014). The 1:1 line is plotted as a solid red line.

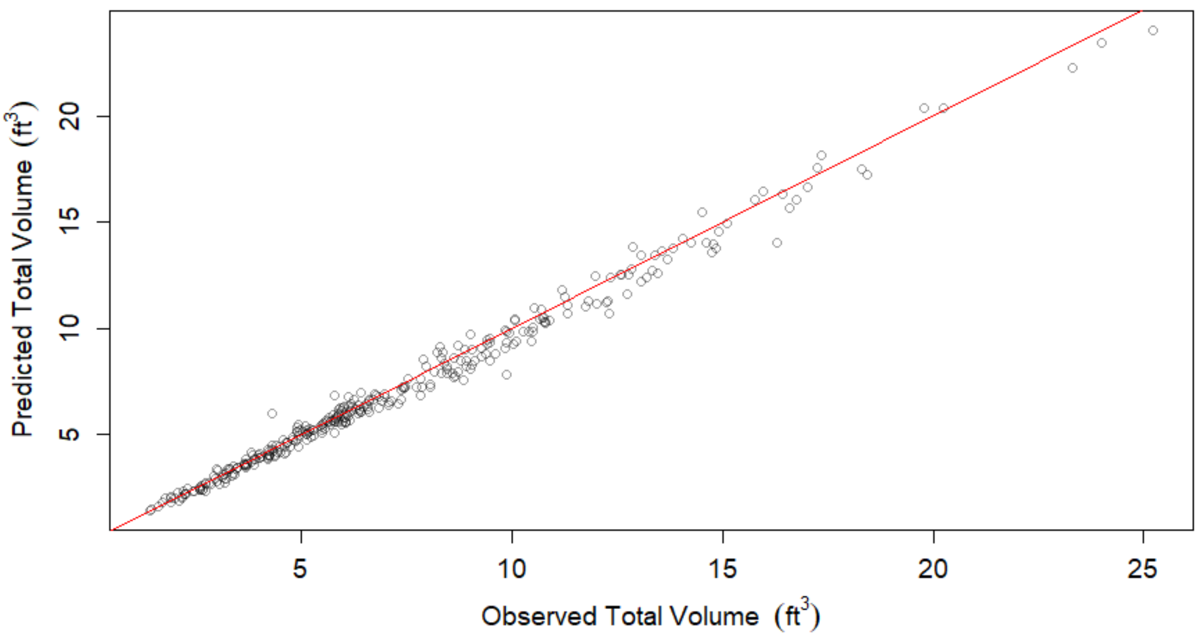

Figure 2.12: Total volume o.b. and predicted volume o.b. from Baldwin (1983). The 1:1 line is plotted as a solid red line.

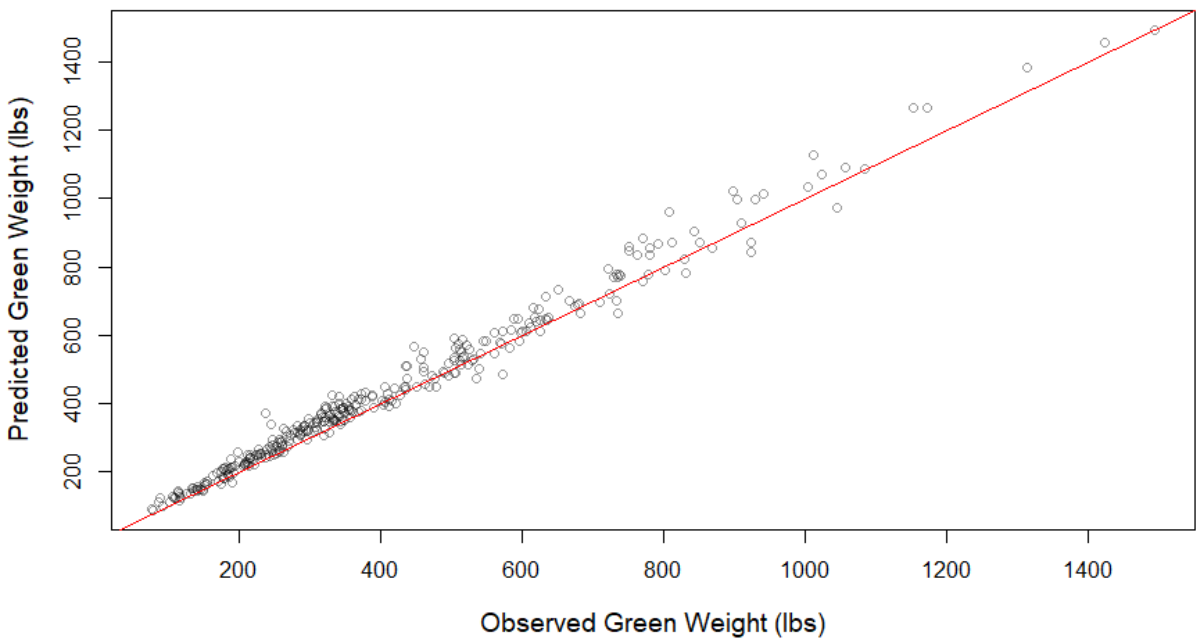

Figure 2.13: Total green weight and predicted green weight from Baldwin (1983). The 1:1 line is plotted as a solid red line.

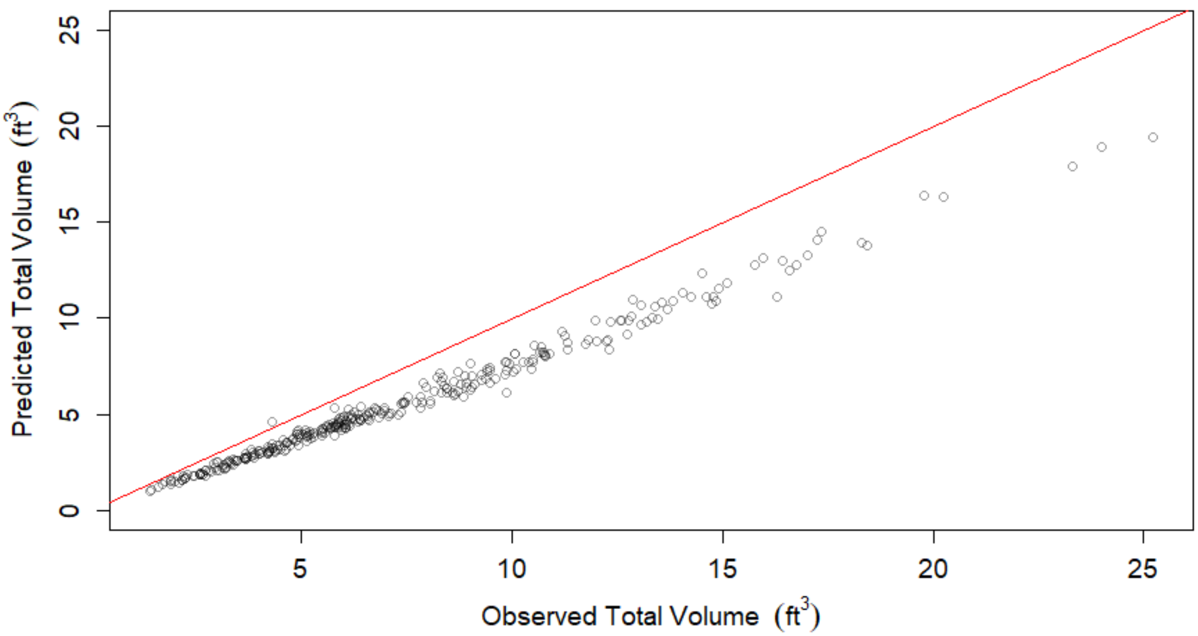

Figure 2.14: Total volume o.b. and predicted volume o.b. from Schmitt and Bower (1970). The 1:1 line is plotted as a solid red line.

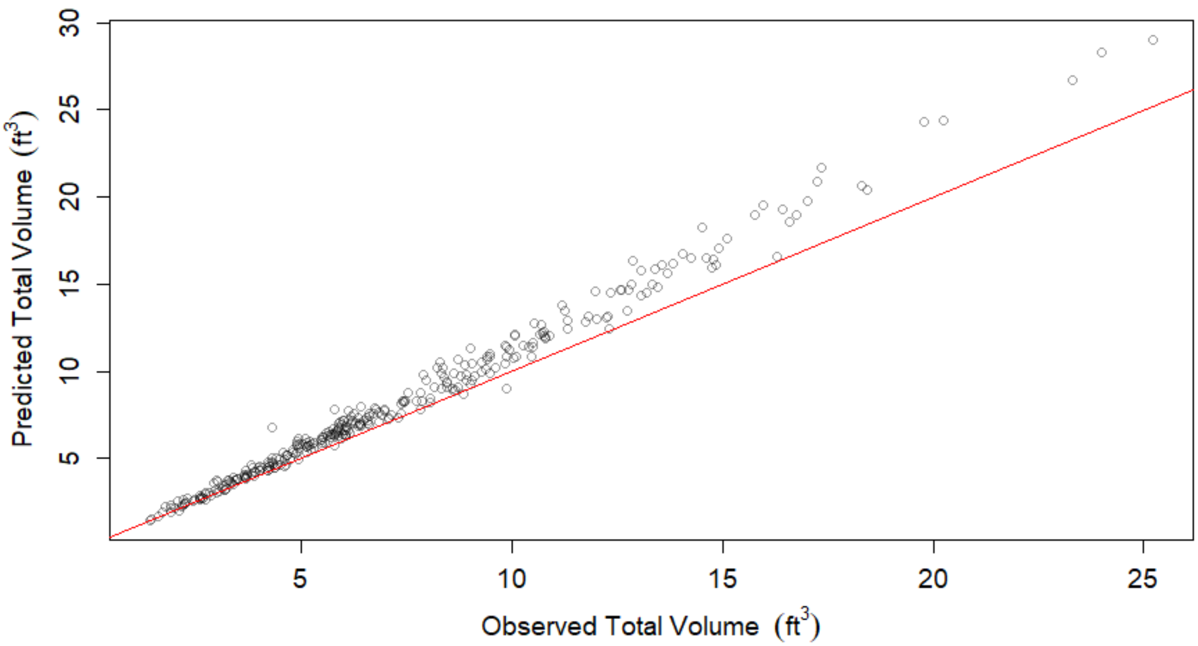

Figure 2.15: Total volume o.b. and predicted volume o.b. from Brooks et al. (2007). The 1:1 line is plotted as a solid red line.

Table 2.1: Estimated parameters of the Kozak et al. (1969) taper model with the full data set and reduced data with either old-field or cut-over. Confidence intervals estimated at $\alpha = 0.05$.

		$\widehat{\beta_0}$		$\widehat{\beta_1}$		$\widehat{\beta_2}$		
Kozak (1969)	df	Estimate	Conf. Int.	Estimate	Conf. Int.	Estimate	Conf. Int.	Adj. R^2
Full	4009	1.288037	1.28167, 1.29439	-2.033571	-2.06563, -2.00150	0.766205	0.73331, 0.79909	0.9489
Old-Field	2219	1.290302	1.28169, 1.29890	-2.060229	-2.10363, -2.01682	0.789066	0.74446, 0.83366	0.9495
Cut-over	1787	1.285201	1.27576, 1.29463	-2.000716	-2.04825, -1.95317	0.738596	0.68993, 0.78725	0.9482

Table 2.2: Goodness of fit criteria for five taper model forms by relative height groups.

Relative Height	n	Kozak 1969			Kozak 1977			Ormerod			Max and Burkhart			Trincado and Burkhart		
		Bias	RMSD	Abs. Bias (%)	Bias	RMSD	Abs. Bias (%)	Bias	RMSD	Abs. Bias (%)	Bias	RMSD	Abs. Bias (%)	Bias	RMSD	Abs. Bias (%)
0.0-0.1	862	0.027	0.506	5.279	0.030	0.508	5.295	0.482	0.717	6.263	−0.020	0.293	2.550	−0.014	0.292	2.551
0.1-0.2	398	−0.294	0.372	4.459	−0.299	0.376	4.520	−0.005	0.159	1.520	−0.013	0.191	2.131	−0.025	0.193	2.127
0.2-0.3	406	−0.161	0.303	3.628	−0.172	0.309	3.708	0.034	0.229	2.709	−0.046	0.247	2.957	−0.062	0.252	3.001
0.3-0.4	398	0.005	0.266	3.463	−0.008	0.266	3.460	0.095	0.270	3.543	−0.026	0.263	3.392	−0.042	0.266	3.425
0.4-0.5	389	0.134	0.293	4.320	0.124	0.289	4.247	0.153	0.302	4.421	0.004	0.262	3.777	−0.009	0.263	3.796
0.5-0.6	401	0.179	0.337	5.588	0.177	0.336	5.571	0.159	0.331	5.436	0.000	0.292	4.657	−0.006	0.293	4.661
0.6-0.7	405	0.096	0.334	6.732	0.114	0.337	6.834	0.072	0.335	6.736	−0.046	0.333	6.527	−0.037	0.331	6.495
0.7-0.8	358	−0.103	0.386	8.803	−0.046	0.364	8.489	−0.065	0.371	8.593	−0.075	0.363	8.312	−0.038	0.354	8.144
0.8-0.9	71	−0.419	0.601	15.571	−0.294	0.508	13.102	−0.256	0.482	12.528	−0.053	0.385	10.628	−0.037	0.380	10.564
All	4012	−0.098	0.473	6.646	−0.010	0.367	5.110	0.143	0.417	4.925	−0.025	0.277	3.652	−0.026	0.276	3.652

Table 2.3: Validation with external data set performance.

Relative Height	n	Max and Burkhart (1976) Bias (in)	Abs. Bias (%)	RMSD (in)
0.0-0.1	123	0.164	5.176	0.321
0.1-0.2	60	0.159	3.819	0.192
0.2-0.3	37	0.171	5.512	0.250
0.3-0.4	28	0.144	5.653	0.252
0.4-0.5	43	0.191	6.883	0.310
0.5-0.6	41	0.166	9.727	0.380
0.6-0.7	29	0.182	12.160	0.411
0.7-0.8	30	0.272	16.147	0.455
0.8-0.9	26	0.728	36.327	0.752
0.9-1.0	46	0.090	53.929	0.308
All	463	0.198	7.582	0.358

Table 2.4: Error comparison for 4 volume models: Combined Variable, Constant Form Factor, Logarithmic, and Generalized Logarithmic by diameter class (D-class) inches.

		Combined Variable			Constant Form Factor			Logarithmic			Generalized Logarithmic		
D-class (in)	n	Bias (ft^3)	Bias (%)	Abs. Bias (%)	Bias (ft^3)	Bias (%)	Abs. Bias (%)	Bias (ft^3)	Bias (%)	Abs. Bias (%)	Bias (ft^3)	Bias (%)	Abs. Bias (%)
4	12	-0.223	-0.121	0.121	0.010	0.007	0.057	-0.129	-0.067	0.072	-0.057	-0.027	0.063
5	45	-0.078	-0.03	0.051	0.125	0.042	0.061	-0.036	-0.013	0.048	0.008	0.003	0.049
6	79	-0.013	-0.004	0.036	0.139	0.030	0.045	-0.027	-0.006	0.037	-0.017	-0.003	0.037
7	79	0.084	0.007	0.046	0.180	0.022	0.049	0.040	0.001	0.045	0.029	-0.001	0.045
8	51	0.087	0.001	0.054	0.104	0.004	0.053	0.038	-0.004	0.053	0.016	-0.006	0.052
9	32	0.046	0.001	0.041	-0.018	-0.003	0.041	0.026	0.000	0.041	0.007	-0.002	0.041
10	13	-0.132	-0.008	0.029	-0.289	-0.019	0.033	-0.083	-0.005	0.029	-0.085	-0.005	0.029
11	10	-0.305	-0.019	0.033	-0.584	-0.034	0.040	-0.152	-0.011	0.031	-0.108	-0.009	0.031
12	3	-0.185	-0.010	0.031	-0.519	-0.026	0.030	0.120	0.005	0.036	0.180	0.008	0.036
All	324	0.000	-0.009	0.047	0.076	0.017	0.049	-0.005	-0.007	0.044	0.000	-0.003	0.044

Table 2.5: Estimated parameters for 4 volume models: Combined Variable, Constant Form Factor, Logarithmic, and Generalized Logarithmic.

model	Estimated Parameter $\widehat{\beta_0}$	$\widehat{\beta_1}$	$\widehat{\beta_2}$	$\widehat{\beta_3}$
Combined variable	0.288 911 5	0.002 586 9		
Constant form factor		0.002 666 1		
Logarithmic		0.003 407 2	1.903 227 7	0.991 062 5
Generalized logarithmic	−0.183 019 1	0.004 113 3	1.864 466 0	0.969 593 4

Table 2.6: Green Weight equation goodness of fit criteria by D-class.

D-class	n	Baldwin (1987)			Bullock (2003)		
(in)		Bias (lbs)	Abs. Bias (%)	RMSD (lbs)	Bias (lbs)	Abs. Bias (%)	RMSD (lbs)
4	12	-9.32	9.74	32.28	-9.18	9.46	31.81
5	45	-0.60	5.59	4.01	0.56	5.43	3.76
6	79	-4.99	5.32	44.37	-3.08	4.94	27.35
7	79	-1.26	6.12	11.23	0.55	6.22	4.90
8	51	9.51	6.05	67.94	10.83	6.38	77.32
9	32	6.16	4.55	34.83	4.95	4.50	28.01
10	13	-4.11	4.99	14.80	-9.54	5.26	34.40
11	10	-17.86	3.11	56.49	-23.61	3.66	74.67
12	3	3.09	3.69	5.36	-21.85	4.07	37.85
All	324	-0.54	5.66	9.63	0.00	5.64	0.02

Table 2.7: Estimated parameters of the Max and Burkhart (1976) outside bark taper model with the full data set of 324 trees.

Parameter	Estimate
$\widehat{\beta_1}$	−6.022 10
$\widehat{\beta_2}$	2.980 04
$\widehat{\beta_3}$	−2.959 86
$\widehat{\beta_4}$	79.865 94
$\widehat{a_1}$	0.808 38
$\widehat{a_2}$	0.077 03

Chapter 3

Stem taper: inside bark and volume and green weight equations for defective stems in longleaf pine plantations[1]

[1]Harris, T. B., B. P. Bullock, C. R. Montes, and J. Dahlen. To be submitted to *Journal of Forestry*.

Abstract

We fit equations to model stem taper for diameter inside bark (DIB) and, volume and green weight outside bark for defective stems for longleaf pine plantations in Georgia. We used DIB measurements from 3005 disks cut from 320 trees, across 16 sample sites established for this research, to test these models. Stem taper inside bark, and the implied volume inside bark, had subtle variation due to stand origin. On average 36% of longleaf pine trees had stem defects. Stands from old-field origin had double the rate of stem defects compared with cut-over sites. The volume and green weight in forked trees was about 11% higher than non-defect trees and about 15% less in crooked trees compared to non-defect trees with the same DBH and total height. These models will be useful for the existing longleaf pine stands and newly established stands across GA and the SE US on old-field and cut-over sites.

3.1 Introduction

The forest products manufacturing industry in the southern US predominantly purchase tree length stems with bark on, but it is useful to understand the components of the stems from longleaf pine plantations and what proportions of volume and green weight can be attributed to the wood and bark of the stem. Pulp, paper, and saw mills in the southern US purchase the entire stem, but use the components of the stem wood and bark differently. The wood is used to produce pulp, paper, or sawn lumber depending on the size of the stem and the type of mill.The bark is typically burned on site for energy production and provides a significant proportion of the energy for the mill. Understanding the relationship between the inside and outside bark volume and weight will provide useful information about the expected yield of bark and wood from longleaf pine stems in Georgia.

Longleaf pine is continuing to be a popular choice for landowners in the southeastern US. Today 10% of all southern pine seedlings produced are longleaf pine, the same proportions as slash pine (Enebak, 2018). These young stands have been established for a variety of reasons, but a key objective for many landowners is timber production. However, to be acceptable quality for sawtimber longleaf pine trees must be free from major defects such as rust galls, excessive sweep, and forking.

In forest inventory practices, data is collected to inform landowners about the condition of their forest. The principal interest for many landowners is determining the volume and quality of the trees growing in their forests. To make predictions of volume and green weight, DBH itself can be used, but more accurate estimates are made using the DBH and total height. So, forest cruisers must have a way to measure the height of trees in their samples. The common instruments include Biltmore stick, clinometer, laser hypsometer, and vertex hypsometer. We used the Nikon Forestry Pro laser hypsometer in our inventory and individual tree measurements. We compared the rate of stem defects to address the frequency of the problem and several model forms to improve predictions for trees with defective stems.

Our objective is to model volume in both non-defect and defect trees as well as green weight in non-defect and defect trees because trees in the southeastern US, where longleaf pine is native, are bought and sold based on weight. We hypothesized that these trees with excessive sweep and high stem sinuosity will have lower volume and green weight as compared with a straight tree with the same total height and DBH. Our reasoning is that those crooked trees were in poor health and coincided with fusiform rust infection and overall less vigorous growth.

3.2 Methods

3.2.1 Data

In 2019, we destructively sampled 400 trees across 20 sites in the native range of longleaf pine in Georgia. For each tree, the stem was bucked into bolts at 0.5 ft, 2 ft, 4.5 ft, 8 ft, and every 4 ft up to a 3 inch o.b. top diameter. From each bolt, we cut a cookie and brought them back to Athens, GA to be processed in the laboratory. However, due to the COVID-19 pandemic the complete collection of disks could not be processed. Instead, a subset of 16 stands (320 trees) were completed. This provides a data set to address the taper, volume, and weight measurement for both outside and inside bark diameter. Among the 320 trees with disks that were processed, 259 were from non-defect trees and were used to fit the taper models. Although the disks and the data come from the same sample sites, we will model the inside bark models for taper separately from the outside bark models because the sample size and distribution of tree sizes is different.

3.2.2 Taper model evaluation

Following on the findings for outside bark taper and volume models, we selected the Max and Burkhart (1976) model form for the basis for modeling inside bark taper.

$$\frac{d_i^2}{D^2} = \beta_1(Z_i - 1) + \beta_2(Z_i^2 - 1) + \beta_3(a_1 - Z_i)^2 I_1 + \beta_4(a_2 - Z_i)^2 I_2, \tag{3.1}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} \quad j = 1, 2; Z_i = h_i/H \tag{3.2}$$

Although we had shown earlier that there was not evidence to support separate models based on stand origin for taper and volume equations for outside bark, we wanted to see if the relationship was true for the inside bark observations as well. So, we used the Max and Burkhart (1976) model form with indicator variables based on stand origin to modify each of the estimated parameters in the model. Initially, we added dummy variables β_1', β_2', β_3', β_4' to each of the original β_1, β_2, β_3, β_4 parameters

$$\frac{d_{ib}^2}{D^2} = (\beta_1 + \beta_1' p)(Z_i - 1) + (\beta_2 + \beta_2' p)(Z_i^2 - 1) + (\beta_3 + \beta_3' p)(a_1 - Z_i)^2 I_1 + (\beta_4 + \beta_4' p)(a_2 - I)^2 I_2, \tag{3.3}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} \quad j = 1, 2; Z_i = h_i/H; d_{ib} = \text{diameter inside bark} \tag{3.4}$$

$$p = \begin{cases} 1 & \text{if Old-field} \\ 0 & \text{if Cut-over} \end{cases} \tag{3.5}$$

However, the dummy variables β_2' and β_3' proved to be insignificant predictors in stem d.i.b. taper, and only β_1' and β_4' were significant. So, the model was revised and we dropped the insignificant dummy variables and we fit the model again with the addition of the β_1' and β_4' dummy variables

$$\frac{d_{ib}^2}{D^2} = (\beta_1 + \beta_1' p)(Z_i - 1) + \beta_2(Z_i^2 - 1) + \beta_3(a_1 - Z_i)^2 I_1 + (\beta_4 + \beta_4' p)(a_2 - i)^2 I_2, \tag{3.6}$$

where

$$I_j = \begin{cases} 1 & \text{if } Z \leq a_j \\ 0 & \text{if } Z > a_j \end{cases} j = 1, 2; Z_i = h_i/H; d_{ib} = \text{diameter inside bark} \tag{3.7}$$

$$p = \begin{cases} 1 & \text{if Old-field} \\ 0 & \text{if Cut-over} \end{cases} \tag{3.8}$$

3.2.3 Volume and green weight in Forked and Crooked Trees

We surveyed 24 stands across Georgia installing three 1/10 ac plots in each stand. Within each 1/10 ac fixed radius plot we recorded every living tree with a DBH>0 at 4.5 feet. For each sample tree we recorded:

1. Tree species
2. DBH to the nearest 0.1 inch with a loggers tape
3. Total height to the nearest 1/2 foot with a laser hypsometer
4. Defects including: forking, sweep, ramicorn, and broken top
5. Incidence of fusiform rust

Across all 24 sites we selected for initial inventory, we measured 2,214 trees. This data allows us to compare stem defect rates from different defect categories. We can compare how the rates of stem defects vary according to stand origin as well.

We were interested in comparing the predicted values of volume and green weight with the binary condition defect or non-defect. Although forking occurs at different points up the stem on different trees, we wanted to test the overall relationship when the DBH and total height were modeled using the equations fit for the normal, straight non-defective stem condition.

A difficult question in determining the volume of a forked tree is ascribing a volume to the crotch section of the tree. Because the shape of each crotch section is unique, the traditional method such as Smalian's volume equation may not estimate the true geometric solid volume, given that there are three distinct stems united into one unit as they come together in a union at the center of the crotch. Therefore, we tested five methods of estimating the crotch volume to see their relative predictions for each tree.

$$\#1 \qquad V_{crotch} = a \times l \tag{3.9}$$

$$\#2 \qquad V_{crotch} = \left[(a \times \frac{a}{a+b+c}) + (b \times \frac{b}{a+b+c}) + (c \times \frac{c}{a+b+c}) \right] \times l \tag{3.10}$$

$$\#3 \qquad V_{crotch} = \frac{a + (\frac{b+c}{2})}{2} \times l \tag{3.11}$$

$$\#4 \qquad V_{crotch} = \frac{a+b+c}{3} \times l \tag{3.12}$$

$$\#5 \qquad V_{crotch} = \left(a \times \frac{1}{2} l \right) + \left(b \times \frac{1}{2} l \right) + \left(c \times \frac{1}{2} l \right) \tag{3.13}$$

Where V_{crotch} = the volume in cubic feet of the crotch section, a = large base of the section surface area in ft^2, b and c represent the surface area of the smaller upper fork base surfaces in ft^2, and the l = length of the fork crotch in feet.

In Equation 3.9, the volume is molded after a cylinder with base a, the larger base below the fork. For Equation 3.10 a proportional weighting method is proposed for the volume of the distinct shape of the crotch base that has three separate surfaces a, b, and c that project a column of wood into the amorphic geometric solid. The method in Equation 3.11 forces the weighting of the surfaces such that the base is 50% of the total and the upper fork bases represent 25% each, the average of these is multiplied by the length to estimate the total volume. In Equation 3.12, all three surfaces are given equal weighting, and the average is multiplied by length, similar to Smalian's formula, but with three faces instead of two. In the final equation, Equation 3.13 we propose a system that projects a cylinder from each surface in half the length of the crotch. To compare the estimation from each of the four methods, we calculated the crotch volume as a proportion of the total tree volume under each method.

We added all the values for the stem below the fork, the fork crotch, and the fork tops to calculate the total stem volume. We then compared the observed stem volume with the predicted stem volume using the generalized logarithmic volume equation fit from the data set of 324 straight trees. We used a simple ratio equation which has been proven to work well in forked stems of ponderosa pine (*Pinus ponderosa* P. Lawson & C. Lawson), to adjust the volume estimates for forked trees (Hann and Bare, 1978) .

$$V_f = V * R_f \tag{3.14}$$

Where the V_f is the predicted total cubic foot volume for a forked tree, V is the volume predicted for a normal un-forked tree, and R_f is the ratio of the observed cubic foot volume in the forked tree to the predicted volume from V.

Following the process for estimating volume of forked trees, we then modeled the green weight of forked trees. Understanding the green weight of the trees will provide landowners the best estimation of the standing timber weight and, therefore value. We found that overall the model using the Bullock and Burkhart (2003) form for green weight produced an average bias of 10.0%. This aligns with the previous finding that forked trees have larger volumes than un-forked trees with the same DBH and total height. It makes sense, intuitively that if these trees had more volume, then they would also have a higher green weight. So, we approached the issue the same way by proposing a ratio equation to modify the green weight prediction

$$W_f = W * R_{f,w} \tag{3.15}$$

Where the W_f is the predicted total green weight for a forked tree, W is the green weight predicted for a normal un-forked tree, and $R_{f,w}$ is the ratio of the observed green weight in the forked tree to the predicted green weight from W. The equation modifies the predicted green weight for a normal tree and adjusts it to the average ratio of green weight for forked trees.

We followed a similar process for the trees with excessive sweep. From the original sample of 400 trees destructively sampled and measured for green weight and taper as described previously, 21 trees were classified as crooked or having excessive sweep. We will use these measurements as a baseline for comparing volume and green weight with the non-defect trees.

When comparing the volume outside bark for these crooked trees, we found the model was overestimating the volume with an average bias of -21%. This is a major deviation in the expected volume based on total height and DBH compared with the normal non-defect trees. So, given the relative performance of the volume equation to predict these stem volumes we suggest using a ratio equation to adjust these estimates for total outside bark volume.

$$V_c = V * R_c \tag{3.16}$$

Where the V_c is the predicted total cubic foot volume for a crooked tree, V is the volume predicted for a normal non-defect tree, and R_c is the ratio of the observed cubic foot volume in the crooked tree to the predicted volume from V.

We then evaluated the comparison of expected green weight and actual green weight for the excessive sweep, or crooked, trees we observed. We found that overall the model for green weight produced an average bias of -21.6%. This aligns with the previous finding that crooked trees have lower volume than non-defect trees with the same DBH and total height. So, we approached the issue the same way by proposing a ratio equation to modify the green weight prediction:

$$W_c = W * R_{c,w} \tag{3.17}$$

Where the W_c is the predicted total green weight for a crooked tree, W is the green weight predicted for a normal non-defect tree, and $R_{c,w}$ is the ratio of the observed green weight in the crooked tree to the predicted green weight from W.

3.2.4 Paired Tree Height Measurements: Hypsometer vs. Tape Measure

We collected data to test the height measurements for a laser hypsometer with the height measurement from the length of the felled tree using a tape measure. For each of the 400 trees we felled and destructively sampled for taper and green weight measurements, we also measured the height with the laser hypsometer before felling. So, we have 400 paired observations of total height using the two methods. One important detail about the data structure is that the laser hypsometer only measures height to the nearest 0.5 feet, while the tape measure used in our work was accurate to the nearest 0.1 feet.

To test the effect of stem defect on the difference in expected height from hypsometer and tape measurements, we constructed a linear model with the hypsometer height as the predictor variable and each class of stem defect as additive indicator variables

$$H = H_{hyp} + (\beta_{fork} * I_{fork}) + (\beta_{sweep} * I_{sweep}) + (\beta_{ramicorn} * I_{ramicorn}) + (\beta_{rust} * I_{rust}) \tag{3.18}$$

Where H is the total height measured on the ground with tape measure, H_{hyp} is the total height measured with the hypsometer, and β_i is the i^{th} parameter for each stem defect class. Those β_is are added to the total height from the hypsometer measurement if the defect is present and the indicator I_i is $= 1$. This resulted in insignificant parameter estimates for fork, seep, rust, and ramicorn defects.

3.3 Results

The results are broken out into each subsection of taper models, volume and green weight for forked and crooked trees, and comparing height measurement methods.

3.3.1 Taper model evaluation

The inside bark taper equation using the Max and Burkhart (1976) form was well fit to the data and the summary of the estimated parameters is shown in Table 3.1. The visualization of the model is plotted over the scatter plot of observations of d.i.b. and relative height in Figure 3.1. These parameters vary widely from those estimated by Brooks et al. (2007) using the same model form for inside bark taper equation in planted longleaf pine.

Although the parameter estimates in the model accounting for stand origin were significantly different, they did not change the predicted stem taper for inside bark to a great degree. In Figure 3.2 we show predicted stem profiles based on the DBH and total height of the smallest, average, and largest tree in the inside bark taper data set. Some differences can be seen as the lower part of the stem, especially in the model for a tree on cut-over sites where the predicted diameter inside bark is less than the full model and the old-field variant. This suggests that the trees from cut-over sites may have slightly less decrease in d.i.b as the relative height increases. The estimated parameters for the two equations forms are shown in Table 3.2. However, these differences may not be significant in application, as most trees are not harvested exactly at the ground level, but instead are cut above that point, around $0.5 - 1$ (ft) depending on ground conditions and the harvester being used. Because of the subtle differences, we are presenting a combined model, and models for cut-over or old-field sites.

3.3.2 Volume and green weight in Forked and Crooked Trees

The knowledge that stem defects occur at such high rates in longleaf pine provides the motivation to understand the relationship between volume and green weight in trees with forking and excessive sweep compared with those trees free from defects. Landowners who have established, or intend to establish, longleaf pine plantations on old-field sites will be even more interested in this, as nearly half of their trees can be expected to have stem defects.

Across all 24 stands, the average stem defect rate was 36%. However, there was a disparity in stem quality and stand origin. The old-field sites we inventoried had significantly higher defect rates then the cut-over sites, 46% and 26%, respectively. In the extreme cases, we observed stem defect rates as high as 68% and 78% in two old-field sites and as low as 7% in one cut-over stand. The most common defect was forking, accounting for 38.3% of all defects, note that some trees have multiple stem defects. Similar to the overall trend in defects, forking occurred more frequently in the old-field sites than the cut over sites, 22% and 11%, respectively (Table 3.4). In fact, the relationship of more frequent stem defects in old-field sites held steady at about double the rate compared to cut-over sites for forking, sweep, broken top, and ramicorn. The incidence of rust was not associated with stand origin, and is likely affected by the virulence of rust fungi in each locality. Based on US Forest Service FIA data from loblolly and slash pine, Randolph et al. (2015) show the degrees by which loblolly and slash pine rust incidence varies by stand origin. They postulate that the difference in rust incidence based on stand origin is an effect of the genetic variation in both loblolly and slash pine seedlings, and showed rust incidence had decreased more for loblolly pine over time. Overall rust infection was 6.8% in our sites, which is higher than the 4.9% rate of infection found by Barnard and Van Loan (2003) for longleaf plantations in Florida. Although other stem defects may be frequent problems for longleaf pine, the species remains more resistant to fusiform rust compared to either loblolly or slash pine where it can be devastating (Randolph et al., 2015).

The data we collected in forked trees had forking heights of at least 4.6 feet above ground. It is still preferred to count trees with low forks below DBH as two separate stems, (Burkhart et al., 2019). The range in our data of 34 forked trees had forking heights from 4.6 feet to 39 feet, with a mean fork height of 17.4 feet.

When comparing the results of these five estimation methods for the crotch section volume, we evaluated the crotch volume as a proportion of the total tree volume in cubic feet. The method in Equation 3.13 had the largest estimate for volume, resulting in an average of 5.48% of the total tree volume coming from the crotch section. The method in Equation 3.9 had the second largest estimate for volume, resulting in an average of 5.21% of the total tree volume. The method in Equation 3.10 estimated a lower volume of the crotch by incorporating the upper bases of each fork into the estimate, resulting in the crotch section accounting for 4.03% of total volume on average. Equation 3.11 performed similarly to Equation 3.10, where the assigned weighting for all three surfaces slightly reduced the volume estimated for the crotch section, accounting for 4.02% of the total volume. The method in Equation 3.12 resulted in the lowest volume estimates based on the even weight given to the smaller upper portions as the single lower surface, and resulted in an estimated volume of the crotch section as 3.61% of the total tree volume. The true volume cannot be known from our measurements unless by water submersion or another suitable method, but choosing the method in Equation 3.10 provides an intermediate estimate of the crotch section volume while also providing flexibility to the specific relationship between the surface area for all three surfaces a, b, and c.

Overall the predicted volume for forked trees was underestimated by an average of 8.6%. This showed the need to formulate a new equation for the forked trees. The volume ratio equation for forked trees modifies the predicted volume for a normal tree and adjusts it to the average ratio of volume for forked trees. In our case, the average volume ratio for forked trees, $R_f = 1.097605$. So, the formula will expand the volume estimate from the generalized logarithmic equation to more closely predict total volume in the forked trees.

The result of applying the correction from the ratio equation reduced the bias in predictions and centered them around zero, see Figure 3.3. The average of the relative bias in the basic model had a mean of 8.6%, but in the corrected ratio equation from the relative bias was reduced to 0.2% on average. Although equations to predict the ratio of merchantable volume or green weight might be useful, because the size of the trees we have in our data set, they are only suitable for pulpwood, and therefore the entire stem can be utilized in a harvest. So, the basic functions to predict total volume will satisfy the needs for predicting individual tree volume and green weight.

From the previous work in the non-defect trees we found the best equation to predict green weight was the Bullock and Burkhart (2003) model form, and equation with the estimated parameters is:

$$W = 3.470008 + 0.152546(D^2H) \tag{3.19}$$

Where W is the green weight in pounds, D =DBH in inches, and H = total height in feet.

The green weight ratio for forked trees, $R_{f,w}$ did not vary consistently with respect to stand origin, so we anticipate that a single average value will work well for all trees on all sites. The equation modifies the predicted green weight for a normal tree and adjusts it to the average ratio of green weight for forked trees. In our case, the average ratio, $R_f, w =$ 1.116774, which means that the average predicted green weight for a forked tree is about 11% higher than the green weight of an un-forked tree with the same DBH and total height.

By applying the correction from the ratio equation for green weight, the bias in predictions was reduced and centered around zero, see Figure 3.4. The average of the relative bias in the basic model had a mean of 10.0%, but in the corrected ratio equation from the relative bias was reduced to -0.04% on average.

The second most common defect in longleaf pine was excessive sweep or crook. For these 21 crooked trees we found the average volume ratio in crooked trees, $R_c = 0.8515328$. Using this equation (3.16) will correct for the overestimation in the volume of crooked trees. The volume ratio for crooked trees, R_c did not vary consistently in relation with stand origin, so we propose a single average value will work well for all forked trees on all sites. To compare the performance of the regular volume equation for non-defect trees and the ratio equation for crooked trees, we plotted the residuals and saw the ratio equation reduced the bias and centered it around zero, see Figure 3.5.

The green weight ratio for crooked trees, $R_{c,w}$ did not vary consistently with stand origin, so we anticipate that a single average value will work well for all trees on all sites. The equation modifies the predicted green weight for a normal tree and adjusts it to the average ratio of green weight for non-defect trees. For the crooked trees green weight the average ratio, $R_c, w = 0.8445943$, which means that the average predicted green weight for a crooked tree is overestimated and will be reduced using the ratio equation to more accurately adjust for the stem defect. The performance of the ratio equation can be seen in the residuals that are reduced and centered around zero in Figure 3.6.

3.3.3 Paired Tree total Height Measurements: Hypsometer vs. Tape

We found a consistent trend for the observation of total height using the laser hypsometer to be lower than the observed height when measuring the felled tree on the ground with a tape measure. Overall, the height measured with the laser hypsometer was 1–2 feet lower than the height of the felled tree, middle 50% of residuals was 0.1 – 2.0 feet, with median 1.1 feet. That difference in observations works out to be an average underestimation of 2.4%, and absolute bias of 3.3%. Overall we found a high correlation (0.977) between measured height with hypsometer and the height measured with tape measure. The full distribution of total height measured with tape measure and laser hypsometer is shown in Figure 3.7.

We fit the model in Equation 3.18 to our data for stem defects, see Table 3.3 for model parameter estimates and error. The addition of the indicator variables proved to be insignificant parameters in the model. So, there is not sufficient evidence to support the hypothesis that stem defects influence total height measured on the ground. So, it seems that the bias in measurements remains somewhat steady across all trees, and tends to underestimate height compared to trees felled and measured with a tape measure.

3.4 Discussion

For the forked trees, we saw that the volume was higher than predicted using the total height and DBH as well as the green weight for forked trees with the same DBH and total height as un-forked trees. So, we believe the relationship of increased volume arose from the development of two separate stems in the place of one larger stem. Conceptually, the resources of the forked tree are being directed into developing two competing stems which require higher initial inputs for primary growth than would have been required to develop a single stem with the same height and DBH. But, as the crown develops with two upper portions, there is likely increased capability to intercept light and increase overall growth of the tree in volume and green weight. As the tree grows in time and space, the two forked stems begin to add radial growth, and perhaps this also increases the rate of cumulative volume growth than could be achieved with a single stem. Therefore, at the time of observation and measurement, the forked trees have larger volume and green weight than their non-defect counterparts. Another position to consider would be that the assumption about the relationship between total height and DBH could be influenced itself by the defect in the tree. So, although the DBH may have been similar to a non-defect tree at the same age and the same site, the total height could be different due to the growth response to the stem defect. The opposite situation could also be true, where perhaps the trajectory of total height growth was not so greatly impacted by the stem defect but the allocation of photosynthate and corresponding growth in the different parts of the vascular cambium could be influenced. In a forked tree, it could be the case where the development of a forked top intercepts more of the products of photosynthesis to develop the competing simultaneous height growth as well as the radial growth of each fork, perhaps decreasing the radial growth rate of the stem below the fork. More detailed investigations of the relationships between radial growth and the height and timing of the development of forked tops in a tree could help to answer those questions. The data we collected could allow future research into the spatial-temporal development of growth within forked and crooked trees by using measurements of radial growth and imputed heights based on the growth rings from the disks we collected in each sample tree.

In our study design, we were interested to see the general relationship between forked trees volume and green weight compared to non-defect trees. However, there are still many questions as to the drivers that cause forking and the physiological changes it invokes in longleaf pine and how that regulates the growth of the tree. These questions are outside the scope of our research but would be excellent points for continued work. So, for those interested in modeling the total volume of forked longleaf pine trees we would recommend using the ratio equation presented. If the user desires to predict the green weight of the stem the ratio equation could improve the estimates of green weight as well.

Although equations to predict the ratio of green weight might be useful, because the size of the trees we have in our data set, they are only suitable for pulpwood, and therefore the majority of the stem can be utilized in a harvest. If a large fork is present on a tree, the loader operator can possibly break off half of the fork section with the knuckle boom loader, leaving one tree length stem and the broken fork piece to be sorted as top-wood.

In the case of crooked trees, the volume and green weight were overestimated based on the models fit for the 324 straight trees. The average ratio is close to 85% of total volume and green weight of crooked trees compared to straight trees would necessitate the prediction for those crooked trees using a different model. It is preferred to use the ratio equations presented here to adjust those predictions to more accurately estimate volume and green weight.

The analysis of total height measurements brings up the question of what is the true height of a tree. For instance, is the true height that is measured for a standing tree, or one that has been felled and lies on the ground. However, Bragg (2014) highlights that the vertical height of the tree need not be equal to the bole length of a felled tree, as they are derived from separate concepts and measurements of tree height.

When we measure tree heights with a laser hypsometer, we can be confident that the height will be within a few feet of the total tree height measured on the ground. This is consistent with the results from Larjavaara and Muller-Landau (2013) that showed height estimation using the Nikon Forestry 550 could achieve accurate height reading with less than 0.65 ft error. This is re-assuring for the use of efficient height measurement tools in forest inventory. This is still contingent upon our position that the true height of the tree is that which can be observed when the tree is one the ground. This seems to be the most reasonable because after the tree has been felled, it resembles the way the tree stem will continue its horizontal position through the steps in processing it into forest products.

3.5 Conclusions

There is a significant difference in the volume and green weight for longleaf pine stems depending on the defects present. To summarize the best equation to use in each case, when someone sets out to estimate the volume or green weight for planted longleaf pine, they should use the standard equations for straight non-defect trees. When the intent is to predict volume or green weight of forked trees, we suggest using the volume ratio equation and the green weight ratio equation. When the volume or green weight of crooked trees is needed, the ratio equations for green weight and volume in crooked trees should be used. These equations did not vary consistently with stand origin and therefore using the appropriate equation for each stem form would be applicable to all sites.

3.6 References

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3.7 Tables and Figures

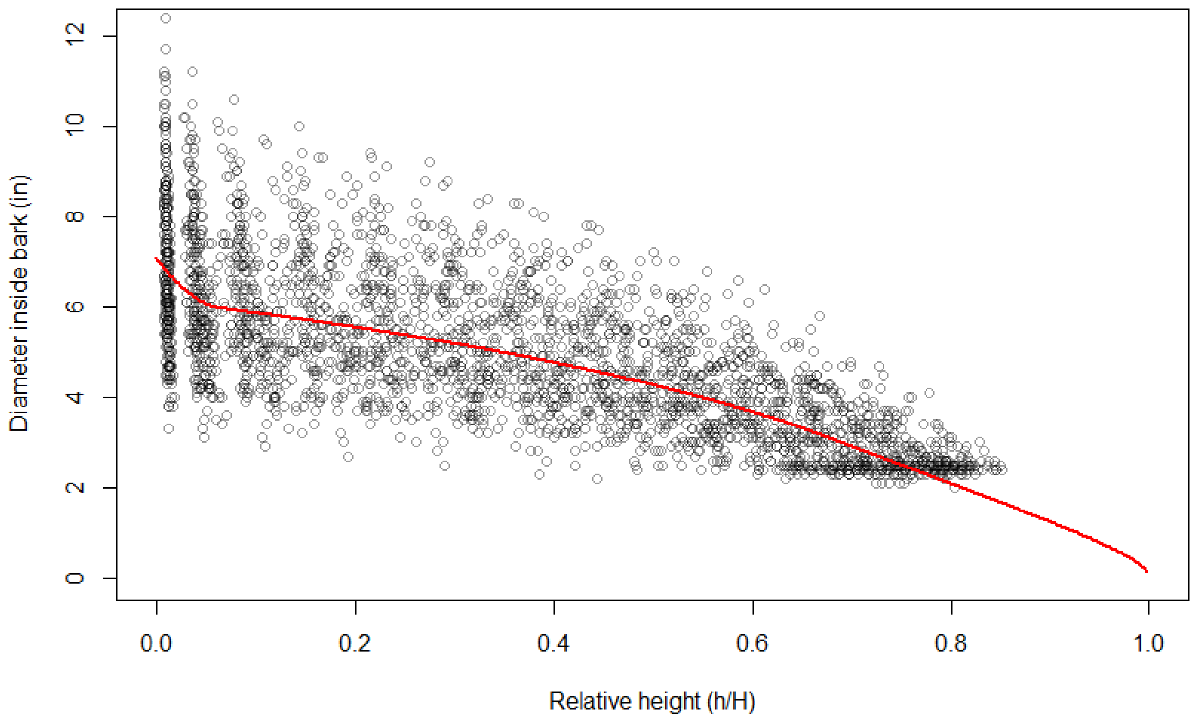

Figure 3.1: Fitted line for the average tree using the Max and Burkhart (1976) inside bark taper equation plotted over the data points corresponding to the diameter inside bark vs. relative height.

Table 3.1: Estimated parameters of the Max and Burkhart (1976) inside bark taper model with the full data set of 259 trees.

Parameter	Estimate
$\widehat{\beta_1}$	−2.846 21
$\widehat{\beta_2}$	1.333 56
$\widehat{\beta_3}$	−1.622 59
$\widehat{\beta_4}$	59.317 64
$\widehat{a_1}$	0.676 83
$\widehat{a_2}$	0.065 10

Table 3.2: Estimated parameters of the Max and Burkhart (1976) inside bark taper model for the full model and the stand origin variant.

	Model Form	
Parameter	Full	Stand Origin Variant
$\widehat{\beta_1}$	−2.846 213	−2.870 07
$\widehat{\beta_2}$	1.333 556	1.360 31
$\widehat{\beta_3}$	−1.622 585	−1.661 74
$\widehat{\beta_4}$	59.317 635	45.259 16
$\widehat{a_1}$	0.676 831	0.677 38
$\widehat{a_2}$	0.065 099	0.067 28
$\widehat{\beta_1'}$		−0.041 28
$\widehat{\beta_2'}$		20.676 79

Table 3.3: Stem defect average rates across all 24 stands inventoried across Georgia. Note: the "Defect Tree/Stand (%)" does not equal the summation of the individual categories because some trees have multiple stem defects. Note: "n" is the total number of trees measured in each stand across all 1/10 ac plots.

Stand	n	Old-field	Forked (%)	Sweep (%)	Ramicorn (%)	Broken Top (%)	Rust (%)	Defect Tree/Stand (%)
1	87	No	7	2	6	0	15	26
4	94	No	15	4	10	0	10	37
6	57	No	2	0	5	0	0	7
11	107	No	15	0	2	0	3	20
12	96	No	10	6	8	6	2	26
13	134	No	14	4	2	4	7	28
14	136	No	6	8	0	1	1	14
15	111	No	22	14	2	2	8	41
16	87	No	8	6	1	0	0	15
17	113	No	5	13	1	2	5	23
18	84	No	19	20	2	7	4	45
19	106	No	10	19	3	2	7	33
2	100	Yes	23	5	3	1	14	43
5	97	Yes	24	4	5	4	12	44
7	92	Yes	17	16	14	0	10	49
8	70	Yes	23	3	9	1	4	36
9	77	Yes	19	1	5	3	8	34
10	126	Yes	25	2	5	1	4	33
20	107	Yes	29	28	13	5	10	68
21	110	Yes	28	42	7	10	9	78
22	127	Yes	12	11	6	8	6	35
23	45	Yes	24	22	7	9	4	49
24	51	Yes	18	29	0	2	6	47
All	2214		16	11	5	3	7	36

Table 3.4: Estimated parameters form Equation 3.17 to model total height as a function of observed height and additive effects of stem defects

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	2.0438	0.5219	3.92	0.0001
hyp_ht	0.9811	0.0108	90.88	0.0000
fork	-0.5417	0.3180	-1.70	0.0893
sweep	0.3260	0.3102	1.05	0.2938
ramicorn	0.4396	0.7511	0.59	0.5587
rust	0.6601	0.7991	0.83	0.4093

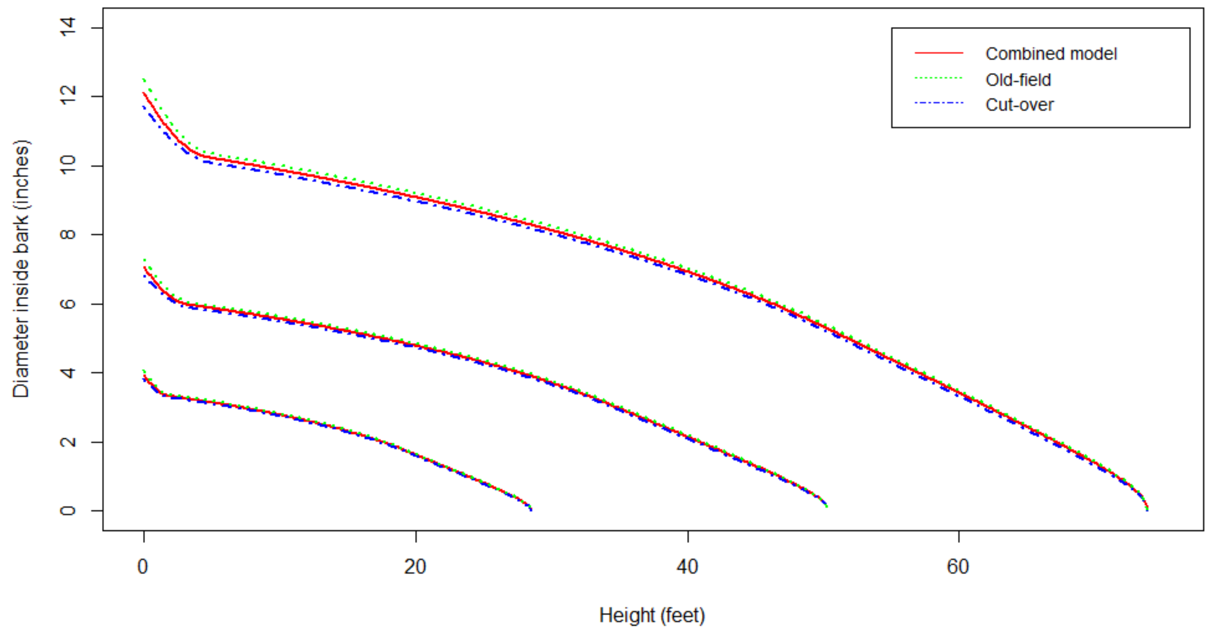

Figure 3.2: Different taper predictions from Max and Burkhart (1976) inside bark taper equation according to the full model and the stand origin variant.

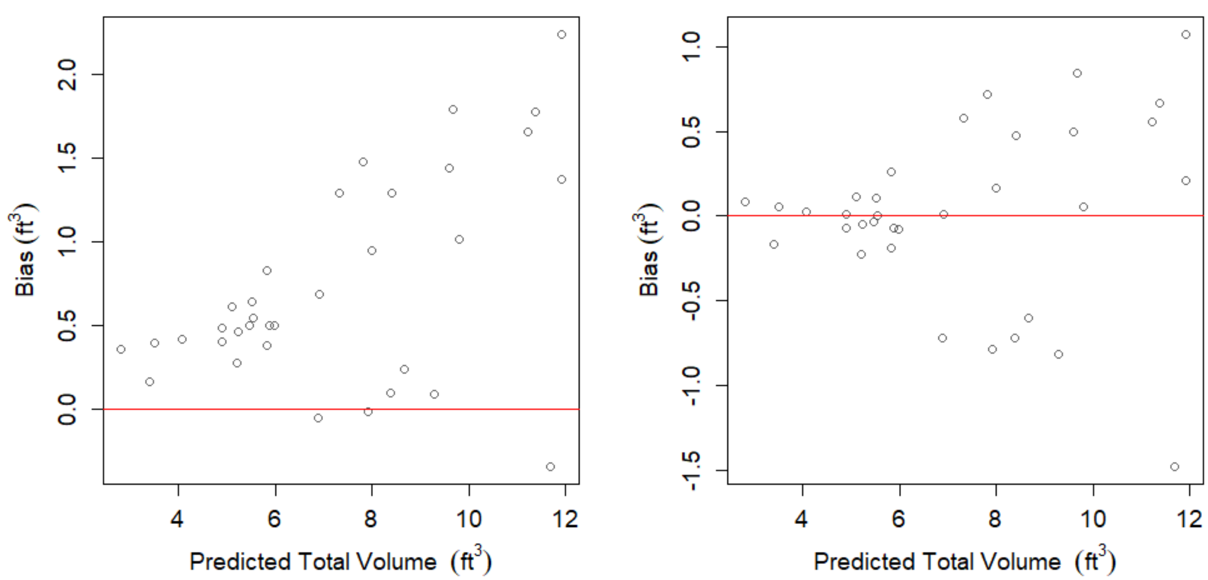

Figure 3.3: Forked tree residuals in volume prediction with generalized logarithmic equation on the left and the corrected ratio equation for forked trees on the right.

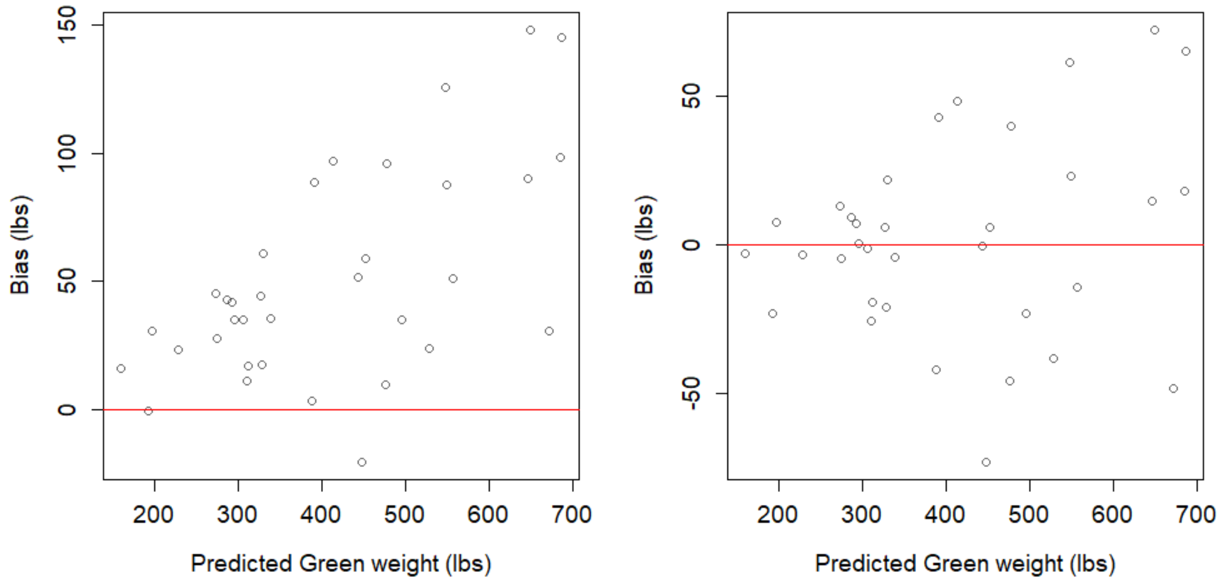

Figure 3.4: Forked tree residuals in green weight prediction with Bullock and Burkhart (2003) equation on the left and the corrected ratio equation for forked trees on the right.

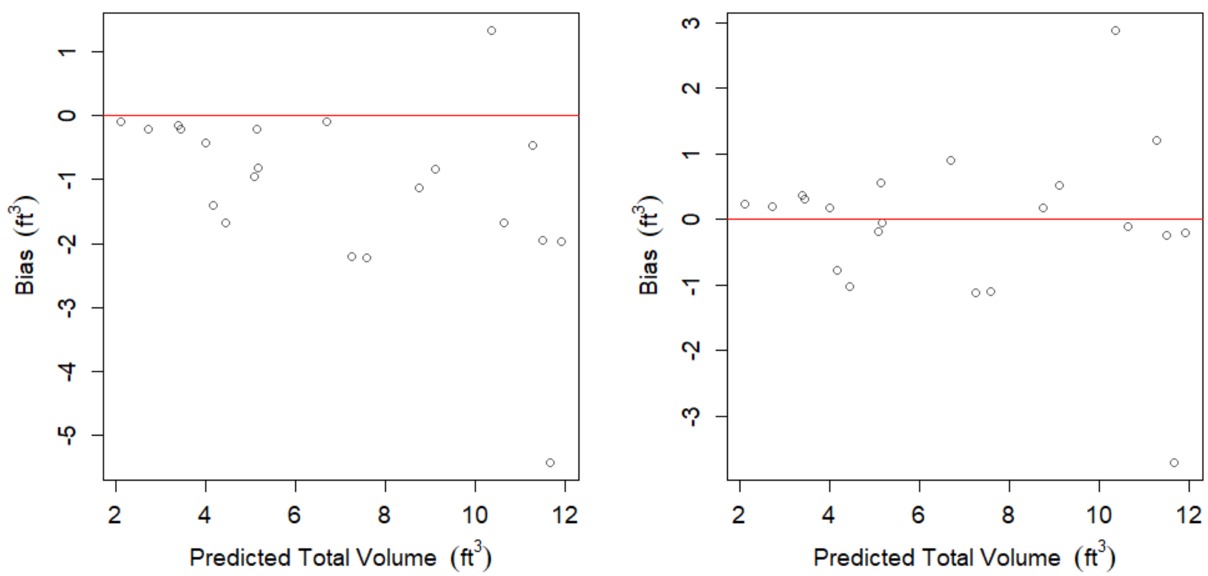

Figure 3.5: Crooked or excessive sweep tree residuals in volume prediction with basic logarithmic equation on the left and the corrected ratio equation for crooked trees on the right.

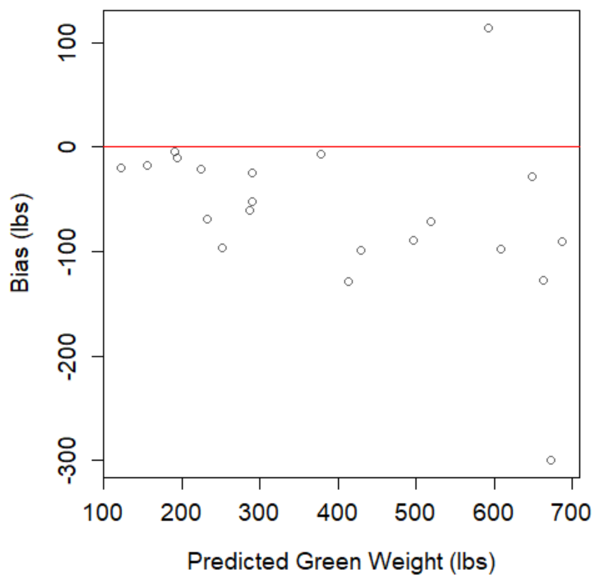

Figure 3.6: Crooked or excessive sweep trees residuals in green weight prediction with Bullock and Burkhart (2003) equation on the left and the corrected ratio equation for crooked trees on the right.

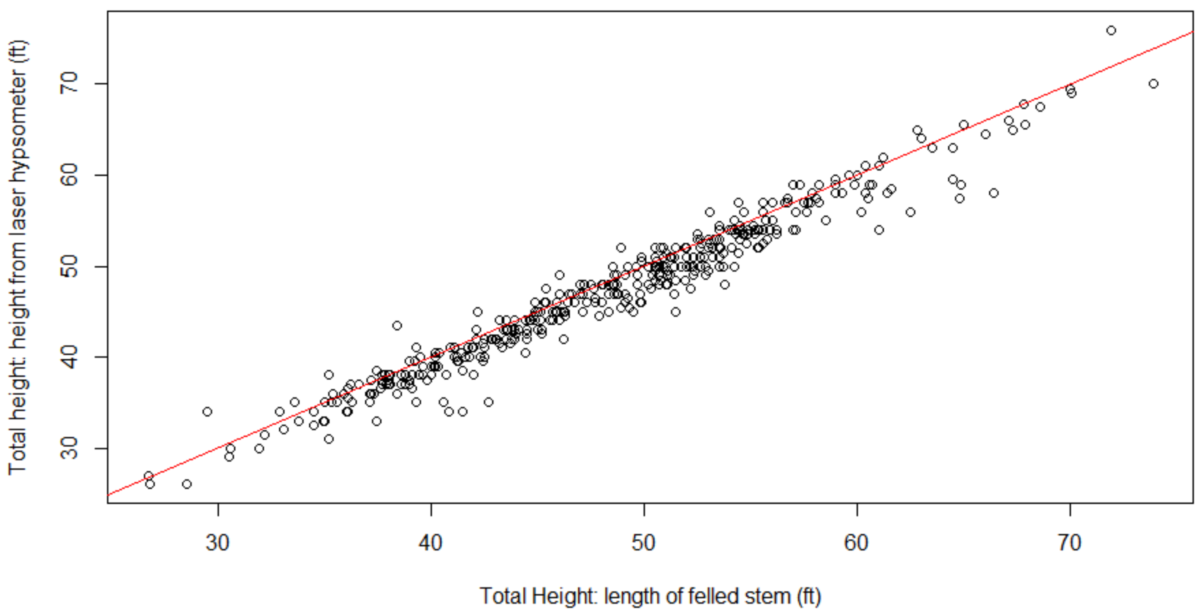

Figure 3.7: The total height from the felled tree length measured with tape measure vs. total height measured with laser hypsometer.

Appendix A

Metric Figures and Tables

Table A.1: Detailed stand description for all sites used in longleaf pine study. Note: planting spacing was unknown for some sites, shown with "-"; "n" is the number of trees destructively sampled at each site.

Site ID	Landowner	County	Lat.	Long.	Origin	n	Est. Year	Age	Spacing (m)	Raking	Area (ha)	Burned
1	1	Dodge	32.1388	−82.9816	cut-over	20	2003	16	2.4x4.3	No	86	Yes
2	1	Dodge	32.1271	−82.9815	old-field	20	2001	18	2.1x3.7	Yes	180	Yes
4	2	Dodge	32.1072	−83.2458	cut-over	20	2007	12	1.5x3	No	27	No
5	3	Dodge	32.0841	−83.2828	old-field	20	2007	12	1.8x4	Yes	72	Yes
6	4	Wilcox	32.0436	−83.5076	cut-over	20	2007	12	2.4x4	No	368	Yes
7	4	Wilcox	31.9802	−83.4751	old-field	20	2001	18	2.1x3.7	Yes	82	Yes
8	5	Berrien	31.3465	−83.1758	old-field	20	1999	20	-	Yes	10	No
9	5	Berrien	31.3474	−83.1756	old-field	20	1999	20	-	Yes	44	No
10	6	Berrien	31.4276	−83.2781	old-field	20	2004	15	1.8x3.7	Yes	72	No
11	GA Power	Burke	33.1262	−81.7308	cut-over	20	2003	16	2.4x3	Yes	128	Yes
12	GA Power	Burke	33.131	−81.7428	cut-over	20	2000	19	2.4x3	Yes	40	Yes
13	GA Power	Stewart	32.1625	−84.9713	cut-over	20	2004	15	2.7x3	No	158	Yes
14	Rayonier	Charlton	30.4489	−82.0832	cut-over	20	1994	25	-	No	17	No
15	Rayonier	Brantley	31.1896	−81.8029	cut-over	20	2000	19	-	No	32	No
18	Rayonier	Long	31.6986	−81.6057	cut-over	20	1997	22	2.1x3.7	Yes	170	No
20	7	Screven	32.7201	−81.5731	old-field	20	2000	19	2.1x3.7	Yes	106	Yes
21	8	Screven	32.7741	−81.5332	old-field	20	2000	19	2.7x2.7	Yes	86	Yes
22	9	Screven	32.5241	−81.5569	old-field	20	2005	14	2.1x3.4	No	25	Yes
23	Jones Center	Baker	31.1884	−84.4798	old-field	20	2002	17	-	No	126	No
24	Jones Center	Baker	31.2392	−84.4162	old-field	20	2001	18	-	No	225	No

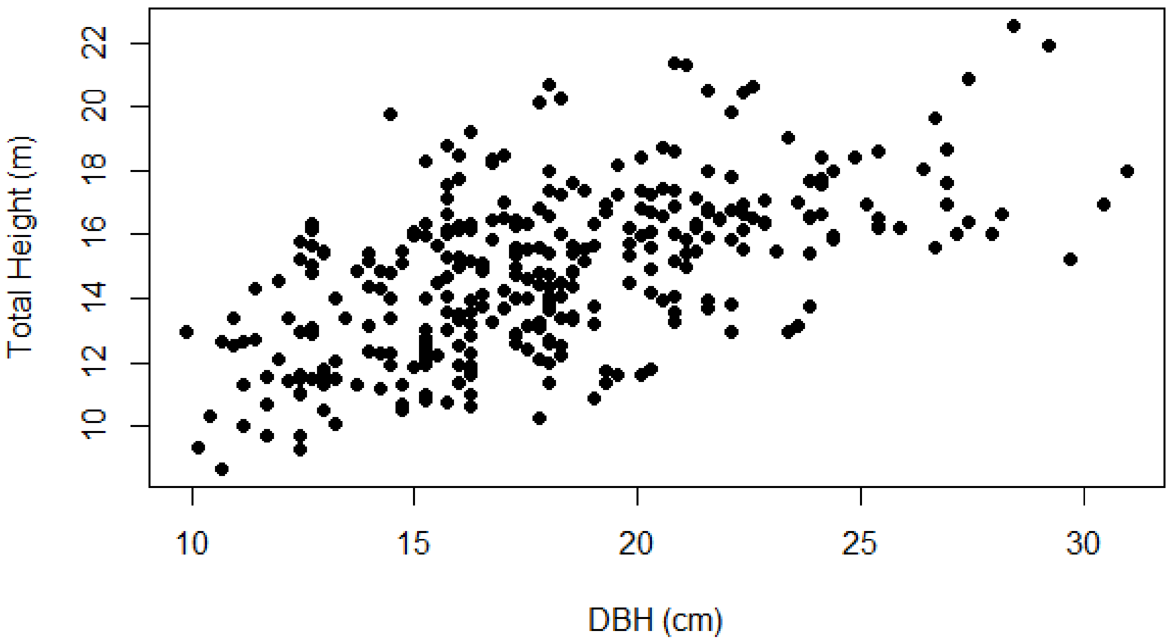

Figure A.1: Total height vs. DBH, for 324 non-defect sample trees.

Table A.2: Stand and Stock Table for all 20 sample sites used in taper data. Site index "SI" is base age 50.

Site ID	Age	Avg. DBH (cm)	Avg. Ht. (m)	BA (m^2/ha)	Dom. Ht. (m)	SI (m)	TPHa	Tons/ha	Vol/ac (m^3/ha)
1	16	15.5	11.7	14.2	14.5	31.1	717	84.5	85.2
2	18	18.0	15.5	22.0	15.3	32.8	823	170.1	171.3
4	12	13.5	10.6	11.2	14.3	31.7	773	61.0	61.7
5	12	17.5	11.6	20.0	12.7	28.6	798	117.7	118.7
6	12	15.0	9.1	8.7	12.2	27.2	470	40.8	41.2
7	18	19.3	14.1	23.6	16.2	32.1	759	169.0	170.2
8	20	21.3	14.1	21.3	14.6	31.7	576	147.1	147.9
9	20	22.9	15.7	26.4	14.6	31.7	635	198.4	199.6
10	15	18.3	13.3	27.5	11.9	28.6	1038	177.3	178.7
11	16	15.0	11.6	15.8	14.8	33.5	882	93.5	94.4
12	19	17.8	13.9	20.4	14.4	32.4	791	139.9	140.9
13	15	15.2	11.1	15.6	15.0	33.2	828	87.4	88.3
14	25	14.5	13.8	19.5	12.1	29.4	1119	140.6	141.8
15	19	18.0	17.8	23.9	12.2	29.0	914	209.8	211.3
18	22	20.8	18.9	25.0	16.0	29.4	692	233.8	235.3
20	19	19.3	14.9	27.1	14.8	27.8	882	197.9	199.3
21	19	19.3	14.8	27.8	16.4	32.8	907	201.8	203.3
22	14	15.2	10.5	19.5	15.2	32.8	1045	104.0	105.0
23	17	22.9	15.7	15.6	15.2	32.5	371	118.6	119.2
24	18	19.6	13.0	13.3	14.7	31.9	420	86.5	87.1

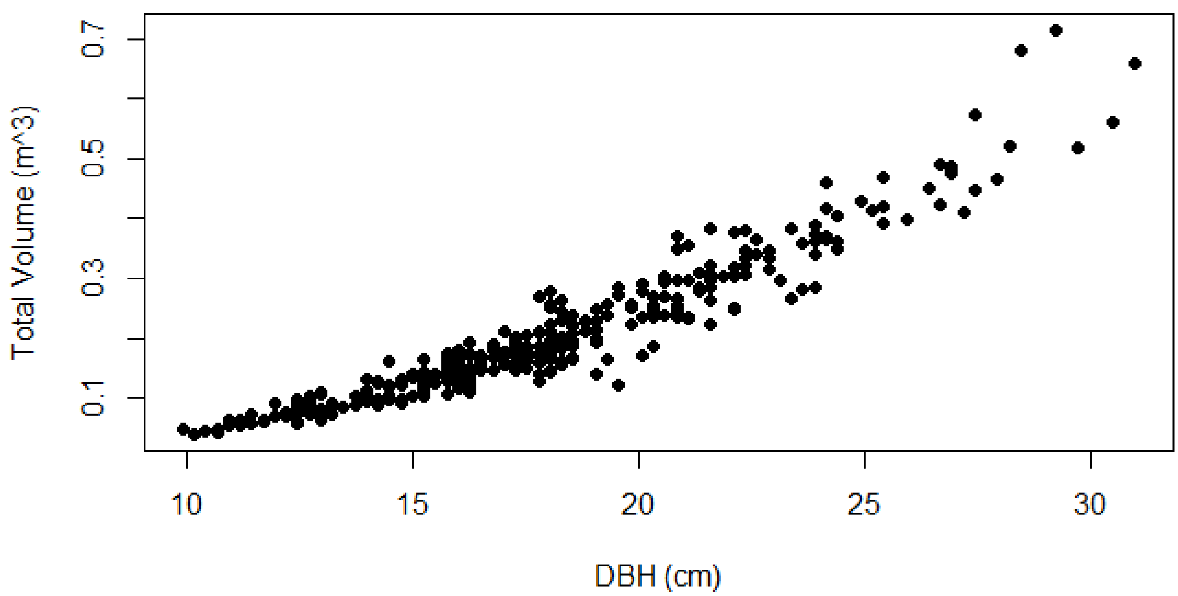

Figure A.2: Total volume o.b. vs. DBH, for 324 non-defect sample trees.

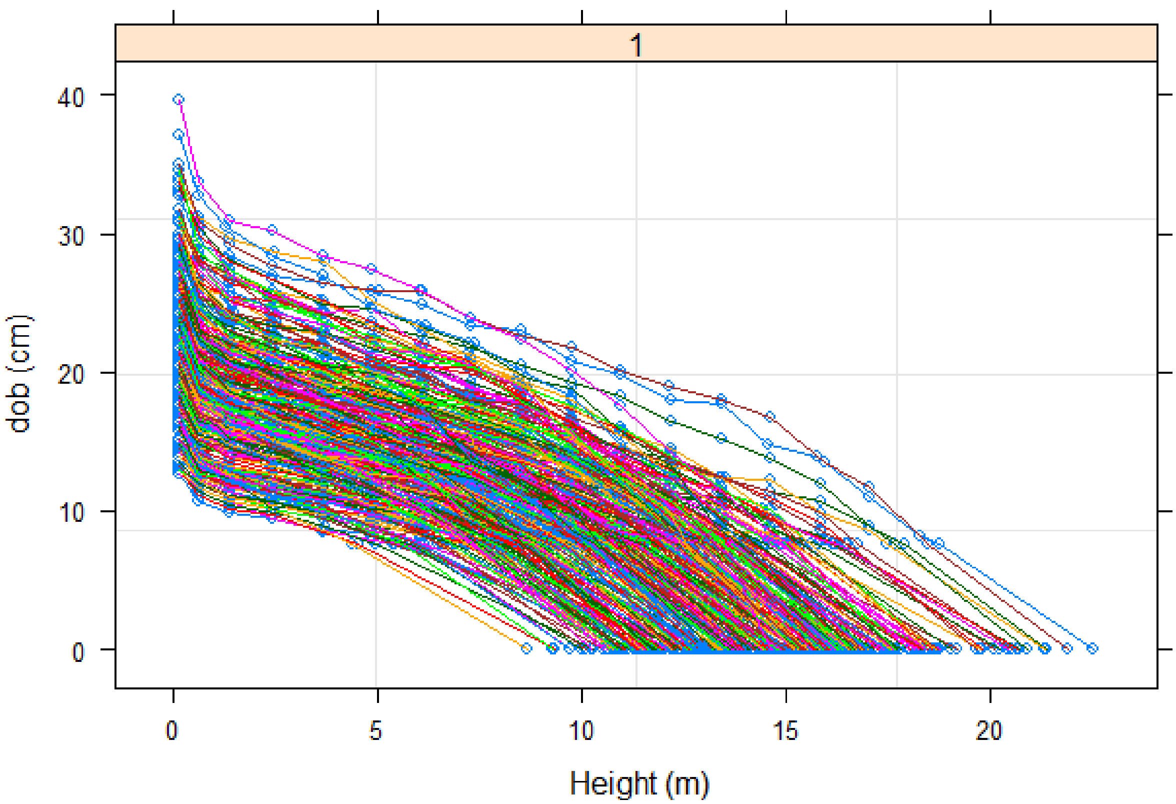

Figure A.3: Stem profiles in d.o.b. (cm) and height (m) for 324 non-defect sample trees.

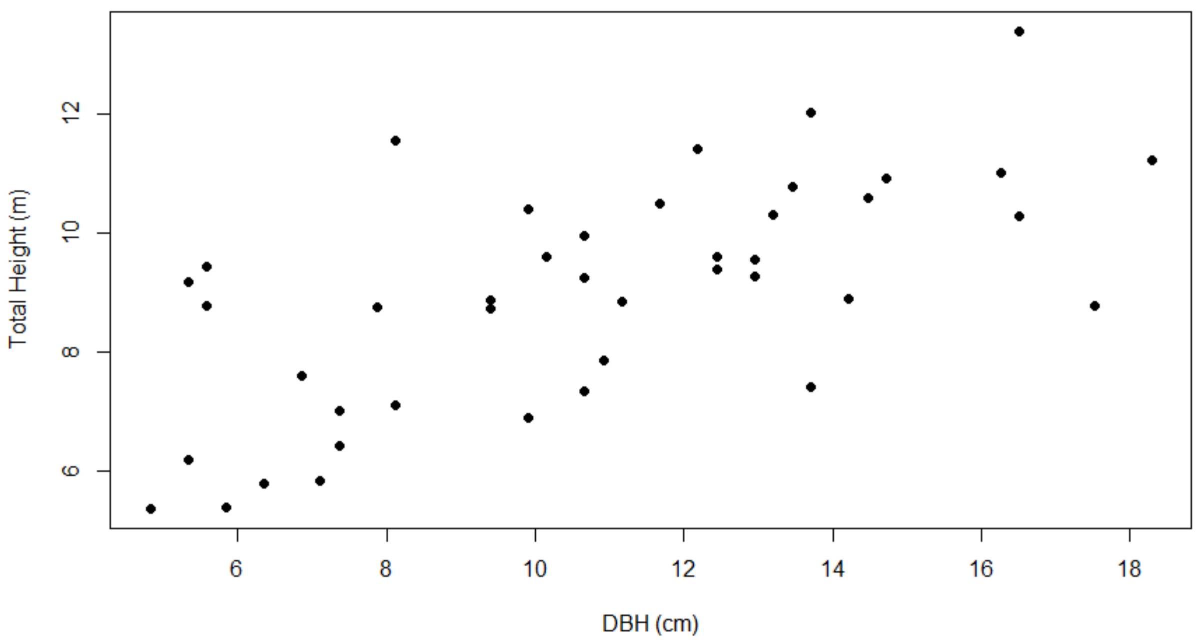

Figure A.4: Total height and DBH for 42 trees in the validation data set. Data comes from the study by Brooks et al. (2007).

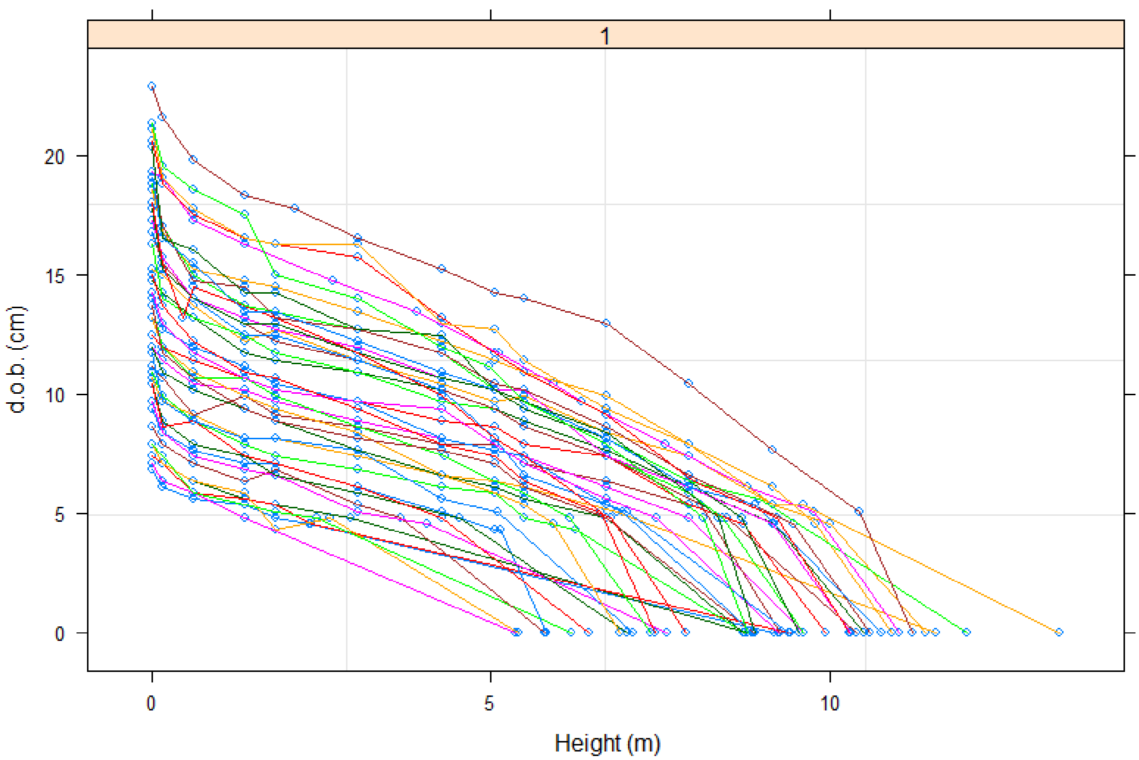

Figure A.5: Stem profiles for 42 trees in the validation data set. Data comes from the study by Brooks et al. (2007).

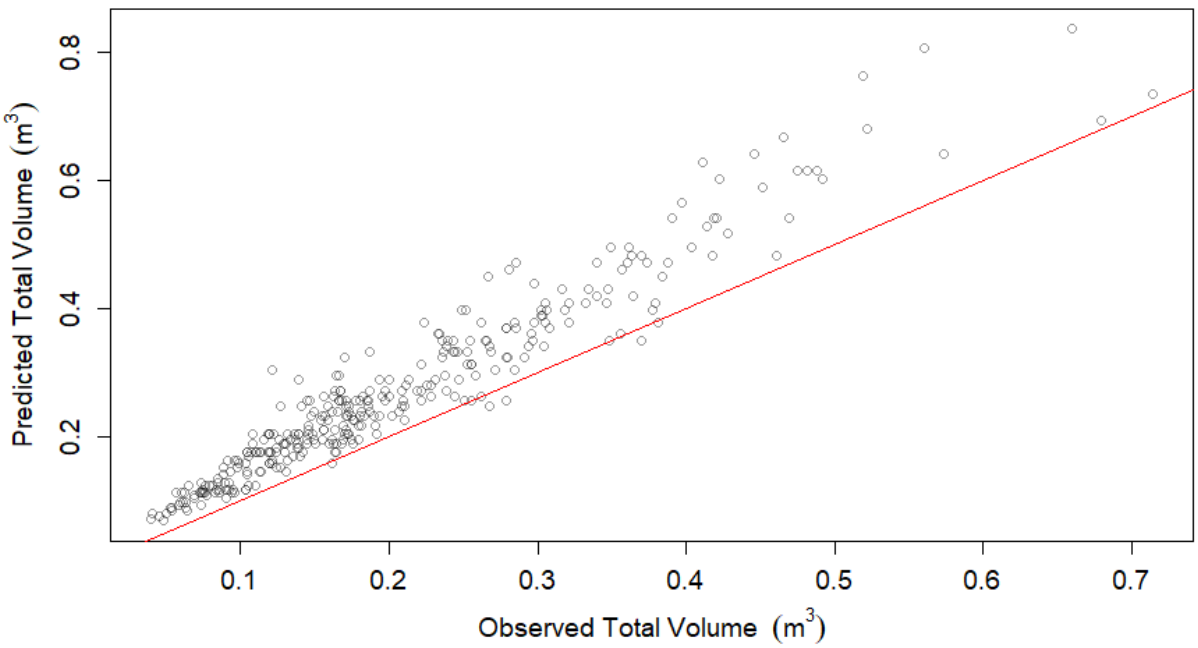

Figure A.6: Total volume o.b. and predicted volume o.b. EQ. 1 from Gonzalez-Benecke et al. (2014).

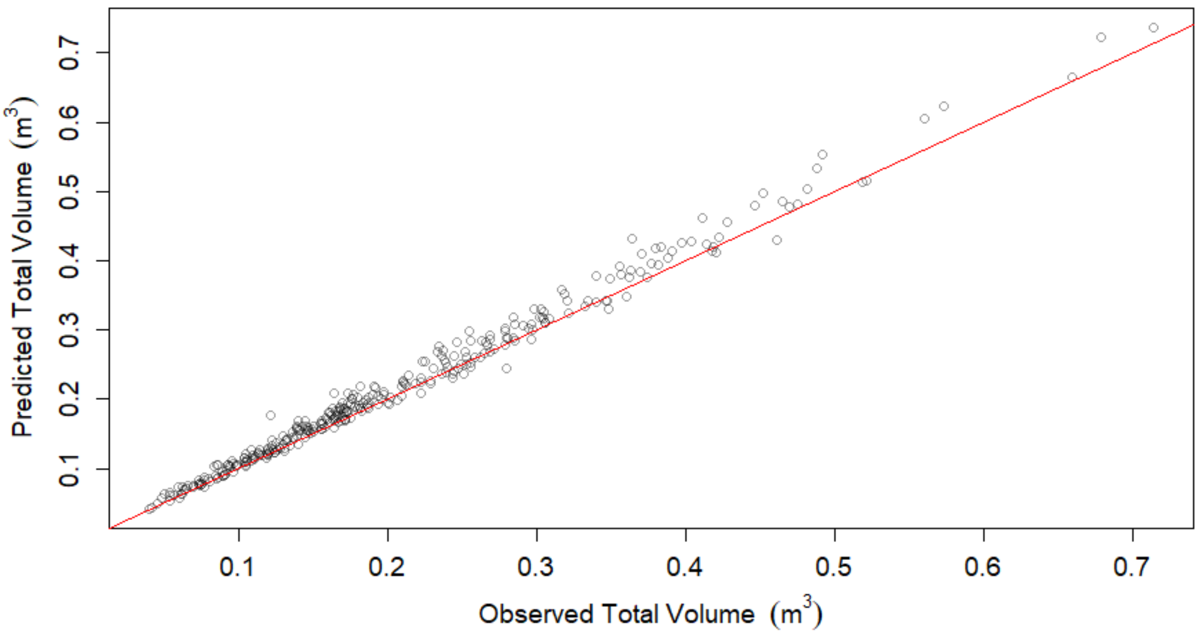

Figure A.7: Total volume o.b. and predicted volume o.b. EQ. 2 from Gonzalez-Benecke et al. (2014).

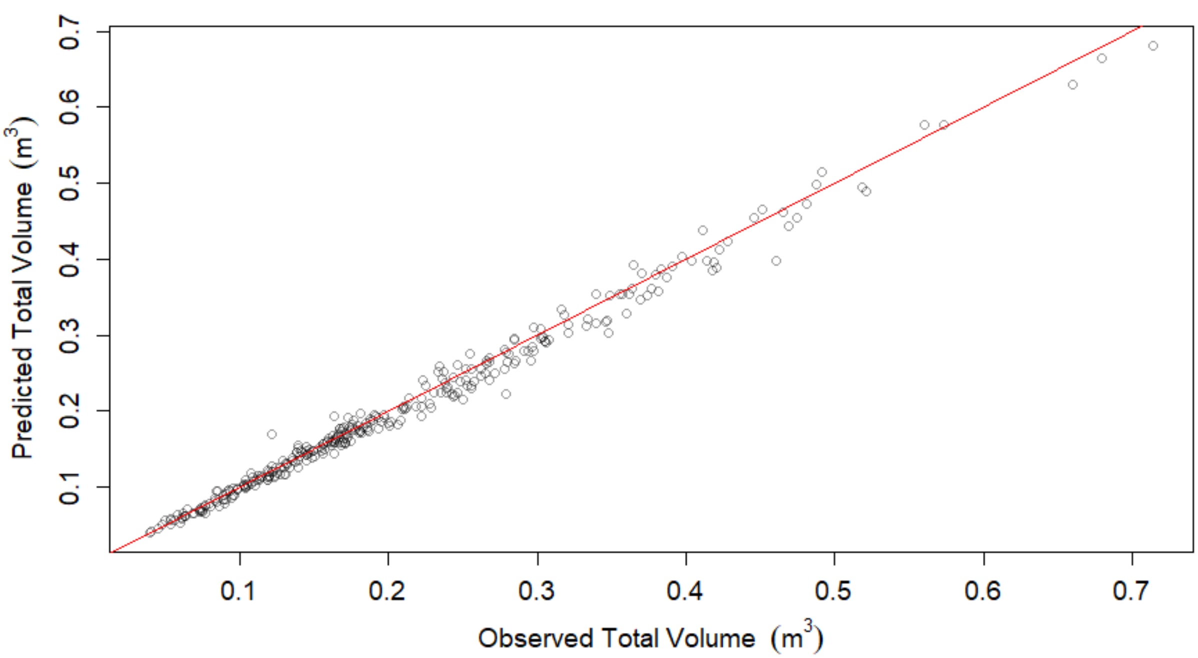

Figure A.8: Total volume o.b. and predicted volume o.b. from Baldwin (1983).

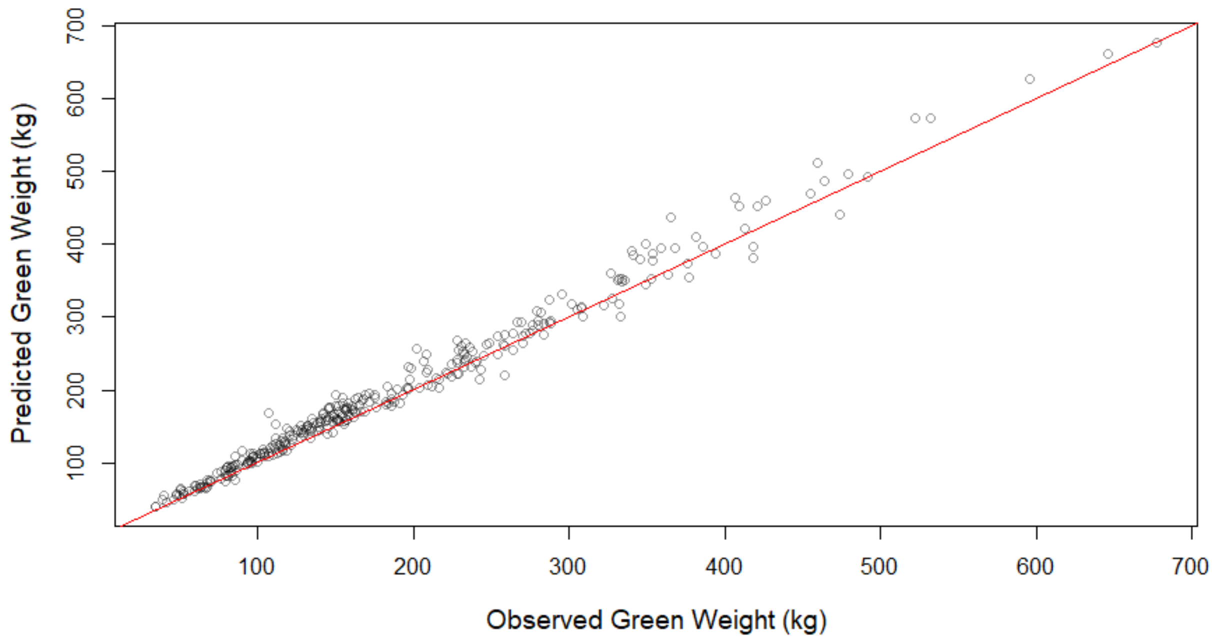

Figure A.9: Total green weight and predicted green weight from Baldwin (1983).

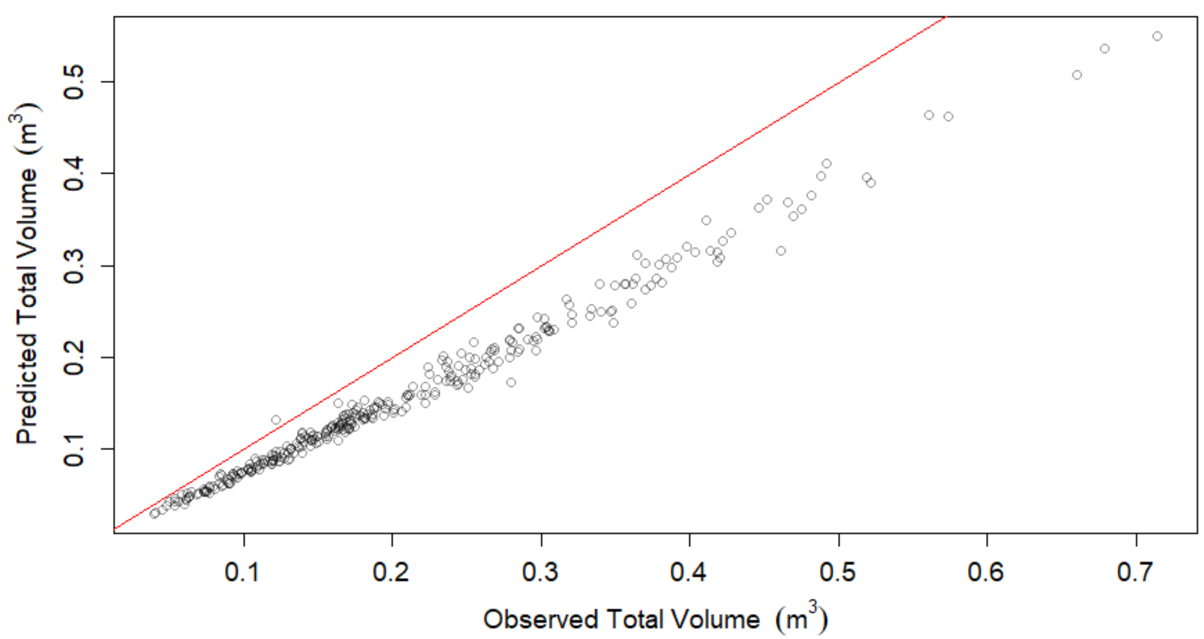

Figure A.10: Total volume o.b. and predicted volume o.b. from Schmitt and Bower (1970).

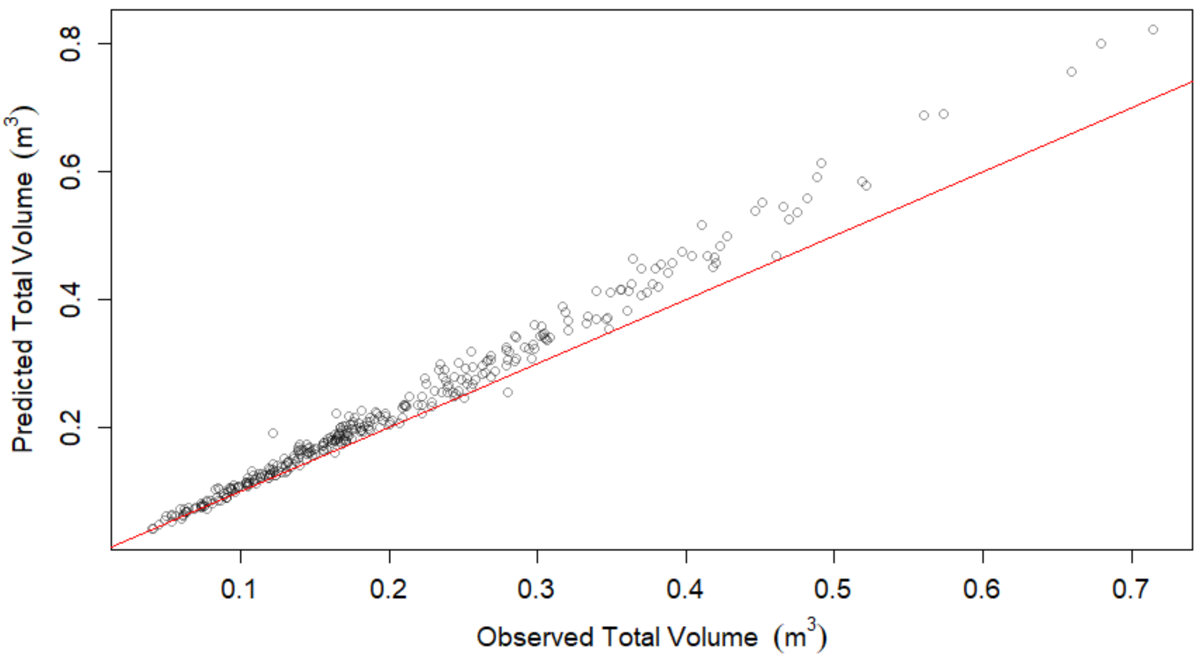

Figure A.11: Total volume o.b. and predicted volume o.b. from Brooks et al. (2007).

Table A.3: Error comparison for 4 volume models: Combined Variable, Constant Form Factor, Logarithmic, and Generalized Logarithmic by diameter class (D-class) inches.

D-class	n	Combined Variable			Constant Form Factor			Logarithmic			Generalized Logarithmic		
(in)		Bias (m^3)	Bias (%)	Abs. Bias (%)	Bias (m^3)	Bias (%)	Abs. Bias (%)	Bias (m^3)	Bias (%)	Abs. Bias (%)	Bias (m^3)	Bias (%)	Abs. Bias (%)
4	12	-0.006	-0.121	0.121	0.000	0.007	0.057	-0.004	-0.067	0.072	0.104	-0.027	0.063
5	45	-0.002	-0.03	0.051	0.004	0.042	0.061	-0.001	-0.013	0.048	0.112	0.003	0.049
6	79	0.000	-0.004	0.036	0.004	0.03	0.045	-0.001	-0.006	0.037	0.121	-0.003	0.037
7	79	0.002	0.007	0.046	0.005	0.022	0.049	0.001	0.001	0.045	0.000	-0.001	0.045
8	51	0.002	0.001	0.054	0.003	0.004	0.053	0.001	-0.004	0.053	0.000	-0.006	0.052
9	32	0.001	0.001	0.041	-0.001	-0.003	0.041	0.001	0.000	0.041	0.000	-0.002	0.041
10	13	-0.004	-0.008	0.029	-0.008	-0.019	0.033	-0.002	-0.005	0.029	0.000	-0.005	0.029
11	10	-0.009	-0.019	0.033	-0.017	-0.034	0.04	-0.004	-0.011	0.031	0.000	-0.009	0.031
12	3	-0.005	-0.01	0.031	-0.015	-0.026	0.03	0.003	0.005	0.036	0.000	0.008	0.036
All	324	0.000	-0.009	0.047	0.002	0.017	0.049	0.000	-0.007	0.044	0.000	-0.003	0.044

Table A.4: Goodness of fit criteria for five taper model forms by relative height groups.

Relative Height	n	Kozak 1969 Bias (cm)	Kozak 1969 RMSD	Kozak 1969 Abs. Bias (%)	Kozak 1977 Bias (cm)	Kozak 1977 RMSD	Kozak 1977 Abs. Bias (%)	Ormerod Bias (cm)	Ormerod RMSD	Ormerod Abs. Bias (%)	Max and Burkhart Bias (cm)	Max and Burkhart RMSD	Max and Burkhart Abs. Bias (%)	Trincado and Burkhart Bias (cm)	Trincado and Burkhart RMSD	Trincado and Burkhart Abs. Bias (%)
0.0-0.1	862	0.069	0.506	5.279	0.076	0.508	5.295	1.224	0.717	6.263	−0.051	0.293	2.550	−0.036	0.292	2.551
0.1-0.2	398	−0.747	0.372	4.459	−0.759	0.376	4.520	−0.013	0.159	1.520	−0.033	0.191	2.131	−0.064	0.193	2.127
0.2-0.3	406	−0.409	0.303	3.628	−0.437	0.309	3.708	0.086	0.229	2.709	−0.117	0.247	2.957	−0.157	0.252	3.001
0.3-0.4	398	0.013	0.266	3.463	−0.020	0.266	3.460	0.241	0.270	3.543	−0.066	0.263	3.392	−0.107	0.266	3.425
0.4-0.5	389	0.340	0.293	4.320	0.315	0.289	4.247	0.389	0.302	4.421	0.010	0.262	3.777	−0.023	0.263	3.796
0.5-0.6	401	0.455	0.337	5.588	0.450	0.336	5.571	0.404	0.331	5.436	0.000	0.292	4.657	−0.015	0.293	4.661
0.6-0.7	405	0.244	0.334	6.732	0.290	0.337	6.834	0.183	0.335	6.736	−0.117	0.333	6.527	−0.094	0.331	6.495
0.7-0.8	358	−0.262	0.386	8.803	−0.117	0.364	8.489	−0.165	0.371	8.593	−0.191	0.363	8.312	−0.097	0.354	8.144
0.8-0.9	71	−1.064	0.601	15.571	−0.747	0.508	13.102	−0.650	0.482	12.528	−0.135	0.385	10.628	−0.094	0.380	10.564
All	4012	−0.249	0.473	6.646	−0.025	0.367	5.110	0.363	0.417	4.925	−0.064	0.277	3.652	−0.066	0.276	3.652

Table A.5: Green Weight equation goodness of fit criteria by D-class.

D-class (in)	n	Baldwin (1987) Bias (kg)	Baldwin (1987) Abs. Bias (%)	Baldwin (1987) RMSD (kg)	Bullock (2003) Bias (kg)	Bullock (2003) Abs. Bias (%)	Bullock (2003) RMSD (kg)
4	12	-4.23	9.74	32.28	-4.16	9.46	31.81
5	45	-0.27	5.59	4.01	0.25	5.43	3.76
6	79	-2.26	5.32	44.37	-1.40	4.94	27.35
7	79	-0.57	6.12	11.23	0.25	6.22	4.9
8	51	4.31	6.05	67.94	4.91	6.38	77.32
9	32	2.79	4.55	34.83	2.25	4.5	28.01
10	13	-1.86	4.99	14.8	-4.33	5.26	34.4
11	10	-8.10	3.11	56.49	-10.71	3.66	74.67
12	3	1.40	3.69	5.36	-9.91	4.07	37.85
All	324	-0.24	5.66	9.63	0.00	5.64	0.02

Table A.6: Validation with external data set performance.

Relative Height	n	Max and Burkhart (1976) Bias (cm)	Max and Burkhart (1976) Abs. Bias (%)	Max and Burkhart (1976) RMSD (cm)
0.0-0.1	123	0.417	5.176	0.815
0.1-0.2	60	0.404	3.819	0.488
0.2-0.3	37	0.434	5.512	0.635
0.3-0.4	28	0.366	5.653	0.640
0.4-0.5	43	0.485	6.883	0.787
0.5-0.6	41	0.422	9.727	0.965
0.6-0.7	29	0.462	12.160	1.044
0.7-0.8	30	0.691	16.147	1.156
0.8-0.9	26	1.849	36.327	1.910
0.9-1.0	46	0.229	53.929	0.782
All	463	0.503	7.582	0.909

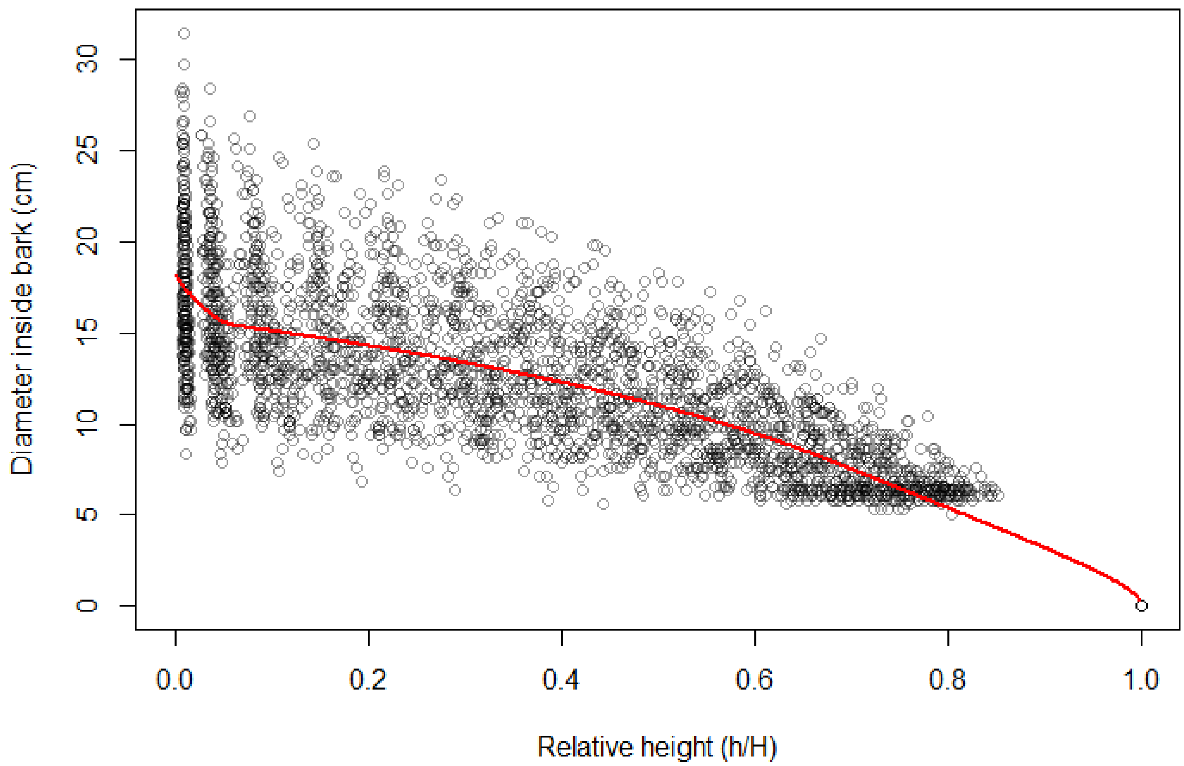

Figure A.12: Fitted line for the average tree using the Max and Burkhart (1976) inside bark taper equation plotted over the data points corresponding to the diameter inside bark vs. relative height.

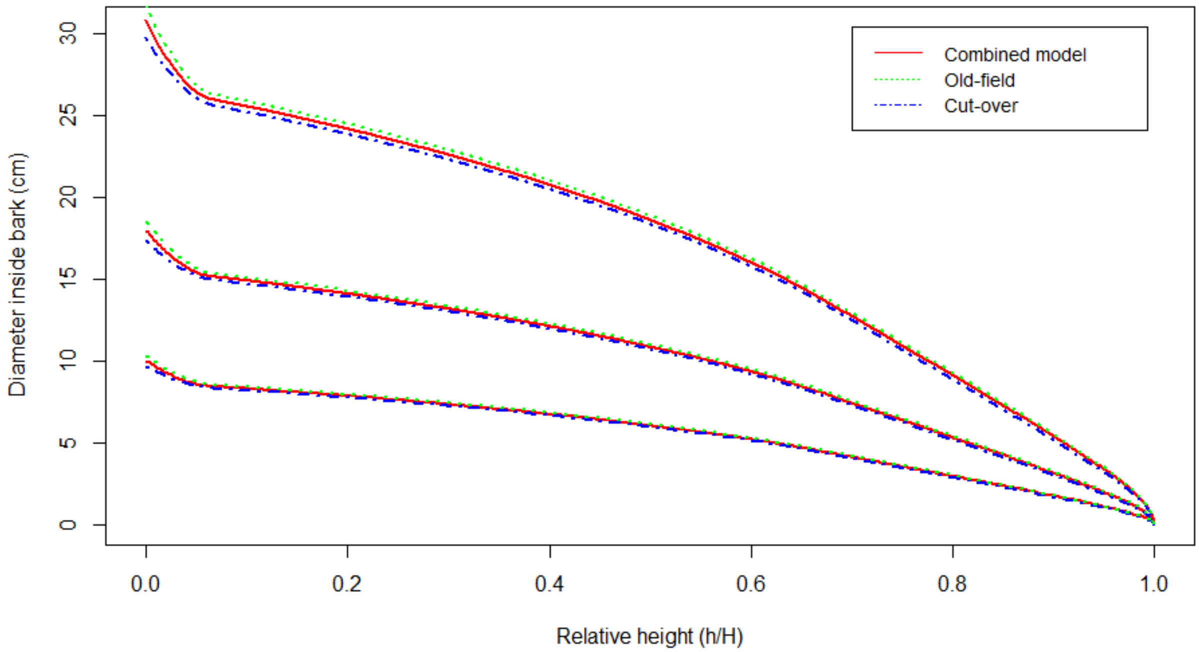

Figure A.13: Different taper predictions from Max and Burkhart (1976) inside bark taper equation according to the full model and the stand origin variant.

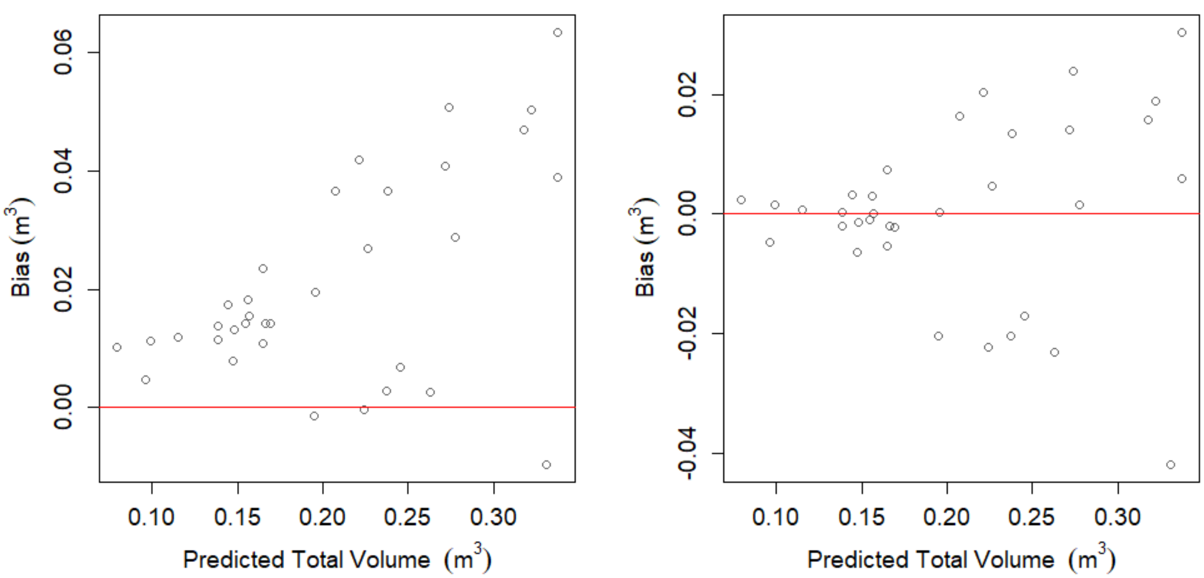

Figure A.14: Forked tree residuals in volume prediction with basic logarithmic equation on the left and the corrected ratio equation for forked trees on the right.

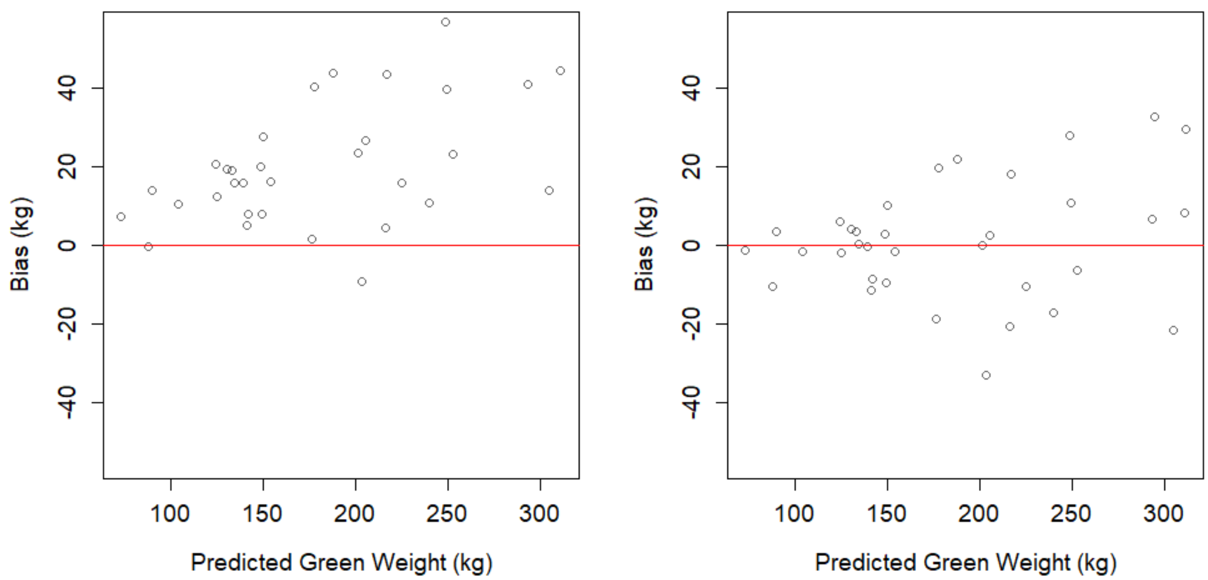

Figure A.15: Forked tree residuals in green weight prediction with Bullock and Burkhart (2003) equation on the left and the corrected ratio equation for forked trees on the right.

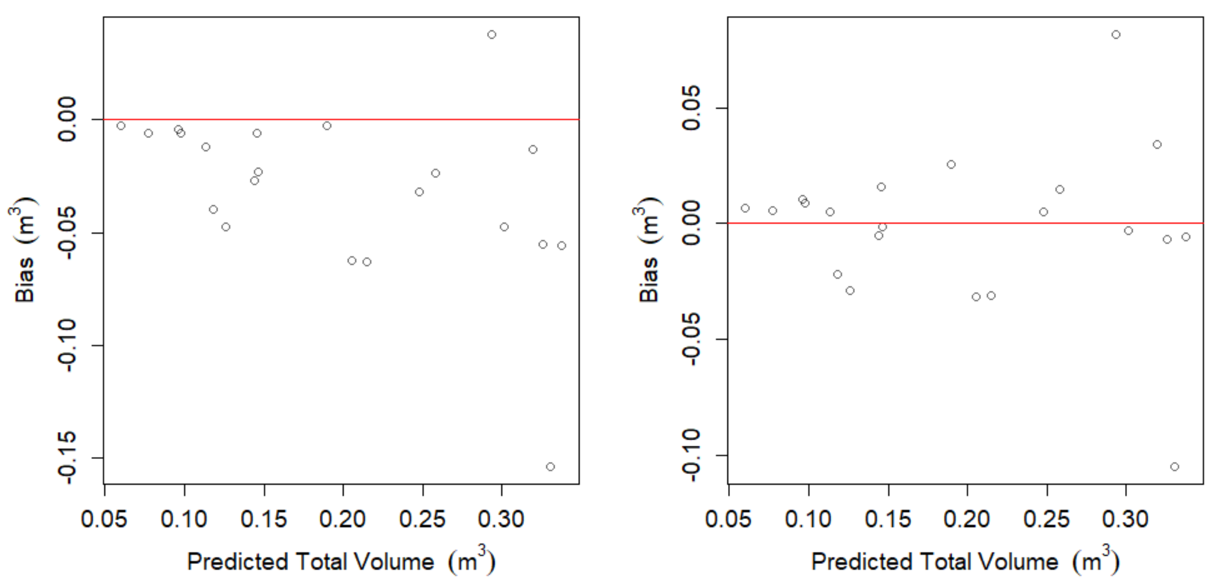

Figure A.16: Crooked or excessive sweep tree residuals in volume prediction with basic logarithmic equation on the left and the corrected ratio equation for crooked trees on the right.

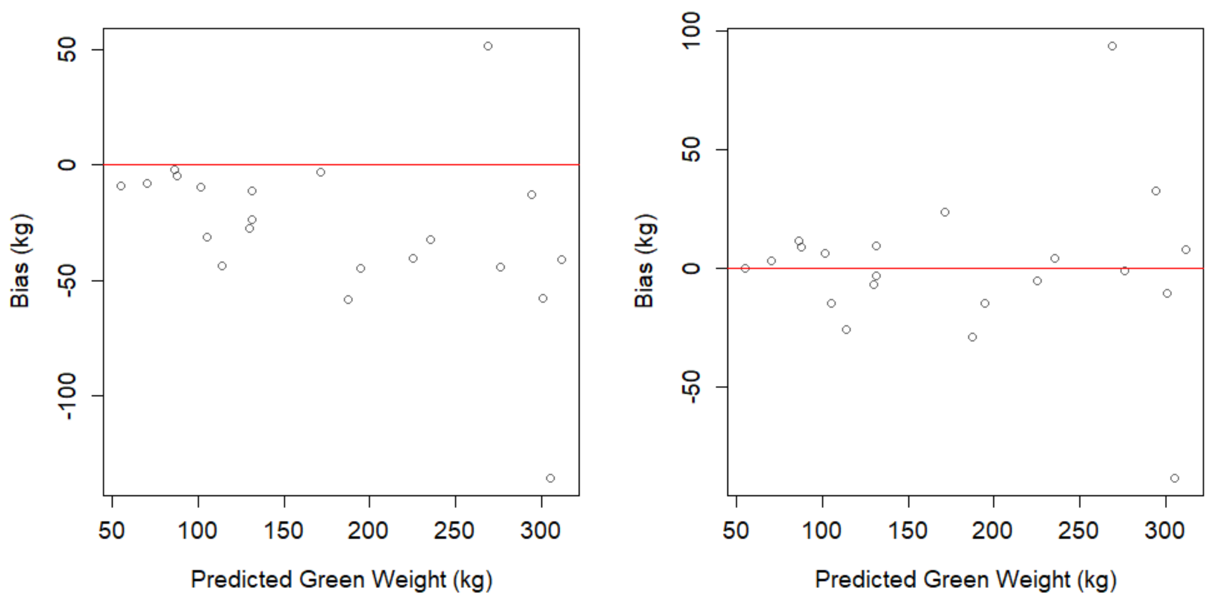

Figure A.17: Crooked or excessive sweep trees residuals in green weight prediction with Bullock and Burkhart (2003) equation on the left and the corrected ratio equation for crooked trees on the right.

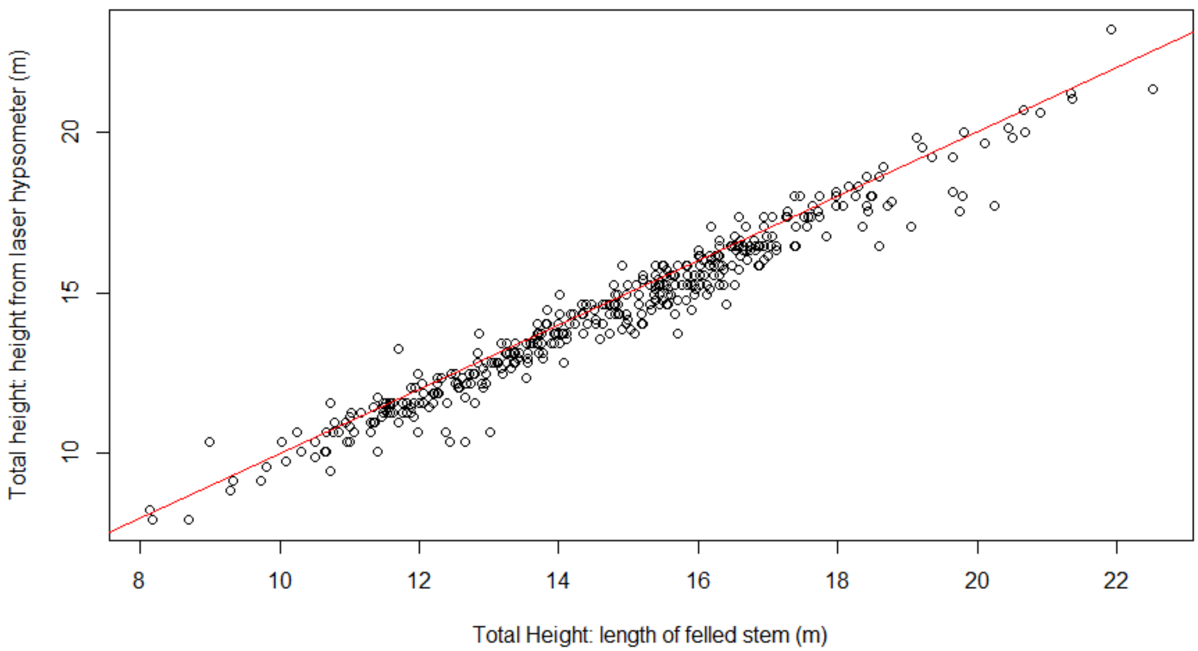

Figure A.18: The total height from the felled tree length measured with tape measure vs. total height measured with laser hypsometer.