USING ONLINE LEARNING TO ENHANCE TEACHERS' MATHEMATICAL CONTENT KNOWLEDGE IN FRACTIONS, DECIMALS, AND PERCENTAGES

By

RACHEAL LANDERS

(Under the Direction of Robert Maribe Branch)

ABSTRACT

Improving teachers' content knowledge in mathematics is one way to address the ongoing national concern of poor performance in math. The purpose of this study was to investigate the impact of the online Math and Parent Partnerships (MAPPS) program as a professional development for teachers in fractions, decimals, and percentages, as well as factors prompting that impact. Quantitative analysis consisted of pre-test and posttests which showed significant improvements in teachers' content knowledge in fractions, decimals, and percentages. Moreover, teachers' writing tasks displayed evidence of reflective learning skills both individually and within learning communities. The use of online manipulatives, videos, and collaborative discussion boards enhanced the teachers experience and promoted deeper understanding of fractions, decimals, and percentages. The results of this study indicate that online professional development in fractions, decimals, and percentages increases teachers' content knowledge.

INDEX WORDS: fractions, decimals, and percentages content knowledge, online teacher professional development, reflection learning

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DEDICATION

"I am only one, but I am one. I cannot do everything, but I can do something. And I will not let what I cannot do interfere with what I can do."**– Edward Everett Hale**

To my heavenly Father who has been my "I can" – Thank you.

I dedicate my completed work wholeheartedly to my family, who supported me throughout this journey.

If in sharing information, we build community and contribute to effective change, then we need to keep sharing. I dedicate this dissertation to the multiple scholars and colleagues whose work has inspired my inquiry. I dedicate my research to my learning community and to all who believe that the gaps in mathematics knowledge can be eliminated. I hope that my work will add value for the educators who seek out resources to improve mathematics performance for their students.

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CHAPTER 1:

INTRODUCTION

Student performance in mathematics in the United States is a concern to policy makers. Statistics from the National Center of Education Statistics (NCES) reveal that the United States (U.S.) has lagged behind other developed countries (2012). In math, U.S. ranks 36th out of 79 countries in the Program for International Student Assessment (PISA) managed by the Organization for International Cooperation and Development (OECD) (Barshay, 2019). This ranking is below the international average. According to Barshay, math performance has not changed much since 2000. Poor performance in mathematics is not limited to grade schools, but it also creates concerns about the shortage of science, technology, engineering, and mathematics (STEM) college graduates. This has led to a shortage of STEM workers for the U.S. workforce. Wodka (2012) explained, that the high attrition rate of STEM majors among college students is attributed to failing mathematics grades. Wodka further explained that the failing math grades can be attributed to inadequate high school preparation. Slavin and Karweit (1985), agreed that, "One of the most troublesome and enduring problems of mathematics is the accommodating heterogeneity in student preparation and learning rate," (1985, p.351). Daun-Barnett and St. John (2012) reported that the varying levels of math course outcomes are a concern for educators, researchers, and policy makers for education. The National Governors Association for Best Practices (NGA) and the Council of Chief State Officers (CCSSO) introduced the national common core standards as an effort to enforce more consistent standards across the curriculum (2010). Due to a lack of standardization in curriculum and pedagogy, knowledge gaps exist that are detrimental to students' achievement. College administrators and policy makers recognize that this knowledge gap exists and have implemented developmental math courses, also known as remedial math, as prerequisites to college mathematics.

As a result of poor student performance, teacher quality has also come under scrutiny and has become a national concern. The Federal Department of Education introduced initiatives such as, Race to the Top to address this concern. Race to the Top was a component of the American Recovery Act signed into law in 2009. A 4.35 billion dollar United States Department of Education competitive grant was implemented to address the four core education reforms: 1) Establishing high, challenging learning standards aligned with readiness for college and careers, and transforming instructional practices to enable students to meet the more challenging expectations, 2) Developing and supporting effective teachers and leaders, 3) Creating data systems and using technology to inform and enhance instruction, 4) Turning around the lowest-performing schools, (Pierce, 2016). One of those reforms was to address teacher and principal effectiveness. Students testing scores were one of the benchmarks for evaluating teacher effectiveness. There is increasing evidence from literature that teacher quality is linked to student achievement (Cohen & Hill, 1998; Darling- Hammond, 2000; Hanuscheck, 2007). Thus, the need to focus on teacher quality has come to attention at a national level.

For several years, the Department of Education has focused on education standards for formal school years. Ball, Lubienski, and Mewborn (2001) also expressed concern that formal schooling years in America are not producing the desired level of mathematical proficiency required by adults. The lack of proficiency in mathematics includes adults who work as professional educators. The National Mathematics Advisory Panel (2008) stressed that K-8 proficiency with fractions needs to be addressed since knowledge of fractions is fundamental to proficiency in algebra. Fractions instruction in schools has long been seen as a problem area. The U.S. Department of Education's Institute of Education Sciences released a report in 2010 on effective K-8 fractions instruction as part of its What Works Clearinghouse to address this gap in conceptual knowledge (Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., and Wray, J., 2010). The report outlined that half of the 8th graders could not order a list of three fractions from least to greatest on the 2004 National Assessment of Educational Progress (NAEP) in math. Siegler et al., reported that fewer than 30 percent of 17-year-olds could convert 0.029 into a fraction. The NAEP 2019 test results indicated that fourth and eighth graders are low performing the same as 30 years ago. The lack of proficiency in mathematics and the gaps in math knowledge gaps are evident when teachers attempt the licensure exam required to be certified. States administer licensure exams for teachers entering the profession to measure their minimum required content knowledge to teach. Licensure exams are common in many professions. Teacher certification tests report a high failure rate in basic math skills. "Fifty-four percent of those who take the Praxis test on elementary-level content fail on their first try. Twenty-five percent never manage to pass" (Wexler, 2019, para.2). Similarly, according to Putman and Welsh (2019), the National Council of Teacher Quality (NCTQ), reported that 46% of teachers fail licensure exams on their first attempt.

A local news anchor, Belcher, (2011, WSBTV), reported on an investigation of the Georgia Assessment for Certification of Educators (GACE) called "The Disgrace of the GACE." This coverage drew attention to the high failure rate on the teacher certification test in Georgia.

Belcher reported that over twelve thousand teachers had failed at least one section of the test on their first attempt. Moreover, a report by the National Council on Teacher Quality listed two factors as the probable cause of teacher licensing exam failure; the lack of alignment between program coursework and the required content knowledge that the states deemed as necessary for elementary teachers to be effective. Putman and Welsh (2019) reported that only one in four teacher preparation programs covers the breadth of mathematics content necessary for elementary grades (p.3, Putman and Welsh, 2019).

However, the problem is not centered around licensure tests; in-service teachers admitted to struggling with the subject knowledge they are required to teach. Putman and Welsh also reported that surveys conducted by the National Science Foundation (NSF) and U.S. Department of Education indicated that two thirds of new teachers admitted to not having a strong grasp of elementary subjects**.** Furthermore, many teachers are left to learn on the fly, often barely covering content or omitting it altogether in their classrooms**.**

The insufficient understanding of mathematical knowledge and pedagogy leads to inadequate K-12 mathematics instruction. Developing teachers' mathematical content knowledge for teaching is central to improving students` mathematical proficiency and mathematics education as a whole (Ball, 1990). An essential factor in education reform is teacher quality and teacher learning. Education reform is synonymous with teacher professional development (Desimone, 2009). The problem is mathematical content knowledge of elementary school teachers needs to be addressed through professional development (PD) interventions to improve student achievement in mathematics.

Conceptual Framework

The concept that frames this study is mathematical knowledge required for teaching mathematics (MKT) Figure 1 is adapted from the (Ball, Thames, & Phelps, 2008) MKT framework. The framework is divided into two main paradigms: subject matter knowledge and pedagogical content knowledge. The framework is broad and further divided into sub domains; common content knowledge, knowledge at the mathematical horizon, specialized content knowledge and, knowledge of content and students, knowledge of content and teaching and, knowledge of curriculum.

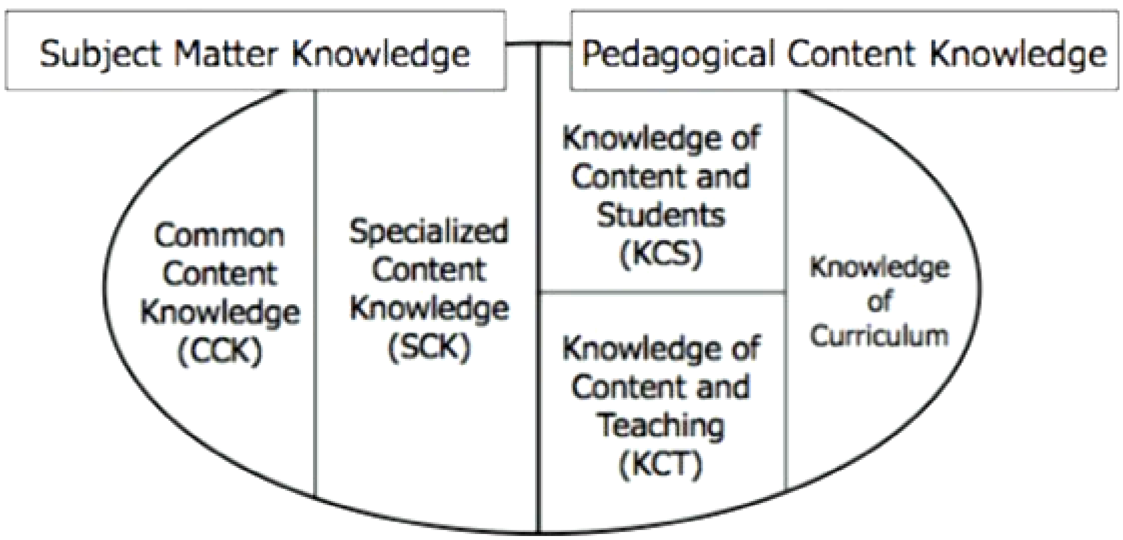

Figure 1. Framework for Mathematical Knowledge for Teaching (Adapted from Ball, Thames, and Phelps, 2008)

The next section introduces common content knowledge which is a subconstruct of subject matter knowledge and the motivation to develop teachers' content knowledge as outlined by Shulman (1987) and Ball (1990) in the field of teacher knowledge development. The two major frameworks, subject matter knowledge and pedagogical content knowledge were developed consecutively over a thirty-year period, but they share a common emphasis.

Developing Teachers' Content Knowledge. Teachers' content knowledge is defined as the practice-based theory that describes the content knowledge involved in teaching a subject. Shulman (1987) advocated for developing teachers' subject matter knowledge and pedagogical content knowledge (PCK) to impact student learning. Ball (1990) criticized previous studies that focused on teachers' beliefs about learning more than content knowledge. Ball addressed content knowledge and built on Schulman`s recommended framework for pedagogical content knowledge (PCK). Shulman argued that there needed to be more focus on mathematics content knowledge. Ball refined the popular concept of PCK by Shulman to the broader concept of content knowledge (CK) for teachers. Shulman expressed concern that little research was done to examine content knowledge and its role in instruction and dubbed it the missing paradigm in research on teaching and teaching knowledge (1986). Ball, Thames, and Phelps (2008) refined the framework of mathematical knowledge for teaching to include four other domains: 1) common content knowledge (CCK), 2) specialized content knowledge (SCK), 3) knowledge of content and students (KCS), and 4) knowledge of content and teaching (KCT).

Common content knowledge is defined as the mathematical knowledge and skills used to correctly solve math problems. Teachers need to know the mathematics they teach. They need to be able to correct students who have misconceptions and also recognize textbook errors. Common content knowledge includes knowledge of the subject and the subject`s organizing structures. Teachers need to know what certain math facts mean and why certain rules apply. Ball (1988) developed interview questions that revealed the inadequacies of teacher math knowledge needed for teaching. Video recordings in subsequent studies reveal teachers' misconceptions and errors in teachers' content knowledge (Ball, Thames & Phelps, 2008). Ball et al. (2008) continued to promote the importance of content knowledge to improve teaching.

Addressing teachers' content knowledge for in-service educators is best addressed through professional development interventions. Defining Professional Development (PD) is complex because there are many types of offerings considered to be professional development. Due to the variety of types of interventions considered to be PD, measuring effective PD is challenging. DeSimone, (2009) proposed a conceptual framework that lists the main features of effective PD associated with changes in knowledge, practice, and ultimately student achievement. Desimone proposed such a framework to study the impact of critical features of professional development. Desimone derived the PD framework`s critical features based on a consensus from research on PD literature concerning what features constitute effective professional development. According to Desimone, content focus, active learning, coherence, duration, and collective participation, which are the core features of PD, lead to increased knowledge and skills and improved student learning depicted in the interactive linear model in Figure 2, adapted from (Desimone, 2009). The scope of addressing all the core features and phases in the framework was too large for the current study.

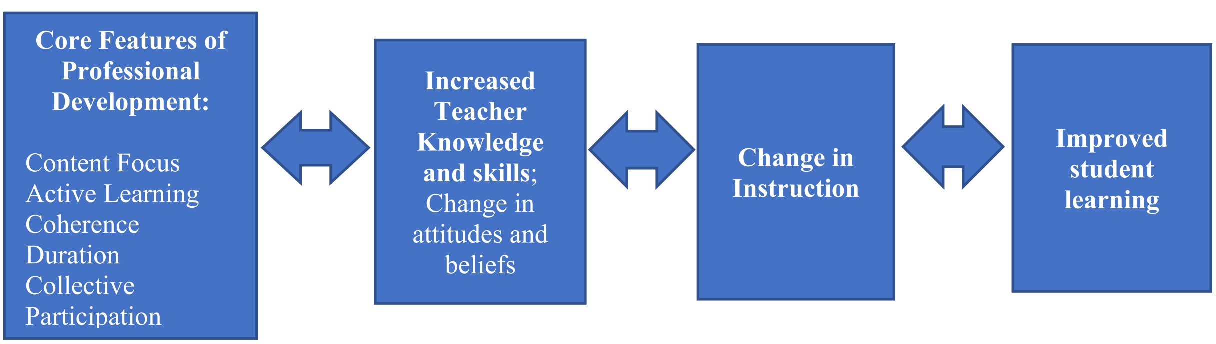

Figure 2. Framework for Effective Professional Development.

The following core features of Desimone's core PD framework were addressed: content focus, active learning, and collective participation. Desimone described content focus in the model as the most influential feature of teacher learning. DeSimone stated that, "A compilation of evidence in the past decade points to a link between activities that focus on subject matter content and how students learn, with increases in teacher knowledge and skills, improvements in practice, and increase in student achievement" (Desimone 2009, p.182). Giving teachers the flexibility to develop their content knowledge in an online learning community will provide accessibility to just in time learning. Teacher learning models continue to evolve as a result of empirical research and external factors that influence the education landscape. Technology advancement is an example of an external factor that can influence daily life.

The increased use of technology in daily routine activities is connected with our professional and social lives. Mishra and Koehler (2006) recognized that routines and daily practices have changed due to technology. The influx of developing technologies incited anticipation of dramatic changes in learning and teaching. However, Mishra and Koehler noted that education is slower than industry in adopting technology. Exposing teachers to online learning opportunities will help teachers develop their comfort level with technology. The resources available to teachers through the World Wide Web and internet provides opportunities for anywhere, anytime, just in time, and continuous opportunities for virtual professional development. Learning on demand is becoming a type of lifestyle in our modern society. Learners take advantage of digital and networked technologies to seek out new information, find answers to questions, and participate in just in time learning. Learners are, therefore not passive information consumers but rather active information seekers or contributors. Modern learners engage in personal learning as needed. Personal learning can serve as a platform to integrate formal and informal learning and foster self-regulated learning (Dabbagh and Kitsantas, 2012). The professional development intervention for this study was designed and delivered in an online format which offered informal and formal learning with technology, with the intention of directly building content knowledge. Teachers engage in self- directed informal learning as they plan lessons or adopt new textbooks, curriculums, and access the myriad of online resources available to them. Developing teachers' reflective learning skills will allow them to evaluate learning processes and resources. The next section describes the theory of reflective learning as it pertained to this study.

Theoretical Framework

Reflective Learning Theory. Reflective learning theory informs this study and is included in the study intervention design to enhance the development of content knowledge for teachers. John Dewey (1933) is regarded as the founder of the theory of reflection as it relates to personal learning. Dewey defined reflection as an "active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and further conclusions to which it leads" (Dewey 1933 pg. 118). Dewey described reflection within a learning context as not a passive recall of an event but rather a deliberate, active process. Dewey explained that reflection should include recalling an event that was confusing and then pose questions to explore and try to make sense of the event. Schon (1983) built upon Dewey's theory of reflection and identified two types of reflection: reflection which is done retrospectively after a learning event and reflection in action which he described as thinking on your feet. Schon associated reflection with professional development and professional practice. Schon further explained that reflecting retrospectively can help educators build upon previous learning event experiences and become the experts in the subject. Both Dewey and Schon advocated for a proactive learner centered form of reflection in which the educator is the learner and leads his or her own reflection process. Teacher reflection has been viewed as a tool for enhancing teacher effectiveness since Dewey introduced the reflection learning theory. Kipp, Artiles, and Torres (2003), discussed discourse communities in teacher reflection. Kipp et al.

critiqued Dewey and Schon as suggesting levels of reflection by labeling reflection as either in the moment of learning or after the moment of learning. Kipp et al. explained that teachers produce artifacts as a result of reflection at either a primary, secondary, or tertiary level. Kipp et al. describe primary artifacts as items used in direct production such as words, curriculum materials, and writing instruments. Secondary artifacts are produced from using primary artifacts, such as beliefs and norms. Tertiary artifacts are described as social spaces of reflection and action. Primary and secondary artifacts are for the teachers to reflect on their own practice, whereas tertiary artifacts are negotiated and created to transform teacher learning. An example of tertiary artifacts is after school inquiry groups where teachers study and reflect on their own professional practice for the purpose of learning. Japanese lesson study (Yoshida, 1999) is an example of a teacher reflection practice used to improve content knowledge in mathematics. The current study utilizes teacher reflection at the primary and secondary level in the intervention with reflection exercises for teachers to reflect on their own learning.

Reflective inquiry for educators allows for ongoing adjustments to be made to their practice in order to maximize student learning and outcomes. Providing various platforms for professional development (PD), especially online, will make PD more accessible to educators and have a greater reach. Vrasidas and Glass, (2004) stated that as a result of teacher participation in effective PD, “Students receive an extra one third to one half of a months’ worth of learning growth for every standard deviation rise on their test scores” (p.8). The value of quality accessible PD should be emphasized more.

Significance of the Study

Improving student achievement in mathematics is inextricably dependent on teacher preparation and development in subject-matter content knowledge. The focus on teacher quality can be traced back to No Child Left Behind (NCLB) requiring all teachers to be highly qualified. The teacher quality initiative was the Race to the Top Initiative, which ended in 2015 (Pierce, 2016). A report by the National Center of Education Statistics (NCES, 2017) on the certification status of U.S. public schools indicated that certification levels still needed to be addressed due to large numbers of teachers not passing the certification tests. Forty-three states have initial competency test requirements for teachers, which include basic general knowledge skills in math and reading and then a second content test in the subject they wish to teach. The concern is that the data shows a large number of prospective teachers not passing these competency tests even after graduating with their Bachelor's degree in education. Some states will still employ non-certified teachers (National Center of Education Statistics, 2017). This report was based on data from 2013 to 2015. The need for enhanced subject matter knowledge for teachers will help improve mathematics education. The purpose of this study was to implement and test the use of an online professional development intervention to develop mathematical content knowledge for teaching. Participants included in-service teachers and paraprofessionals who teach mathematics in first through eighth grade. Both local and non-local educators were invited to participate in the online intervention. The school system involved with this study has 13 elementary schools and 4 middle schools. Participants were recruited from all 17 schools. All the local schools are categorized as Title 1 schools. Middle school teachers were included in the sample because the intervention curriculum is geared to grades K -8. A pretest posttest research design was used to measure the impact of the intervention and interviews and surveys were used to analyze the teacher's experiences with online PD. The results of this study will be used to add to the existing literature on effective online teacher professional development features that promote the development of content knowledge in fractions, decimals, and percentages.

Improving student achievement in mathematics in the U.S. is critical to increasing graduates for the STEM industry to meet national STEM objectives. Teacher knowledge and teacher quality play a major role in student achievement. By specifically addressing teacher content knowledge for teachers through effective online professional development, this will foster online learning in a just in time delivery format. Furthermore, addressing teacher content knowledge in areas supported by unsatisfactory student testing data should be a priority for in-service teacher development.

Research Questions

Due to the need for further investigation required on developing content knowledge for mathematics teachers and the unfolding literature on developing effective PD, this study sought to address the following research questions:

1. How does the online Math and Parent Partnership (MAPPS) curriculum offered as a professional development enhance content knowledge in fractions, decimals, and percentages for teachers?
2. How do online manipulatives, videos, and discussions help teachers develop content knowledge for fractions, decimals, and percentages?

CHAPTER 2:

REVIEW OF RELATED LITERATURE

I conducted a computerized search of recent reports from National Government Reporting Agencies, including Government Reports of evaluation studies, peer reviewed journals, National Center for Education Statistics (NCES), The Institute of Education Sciences (IES), What Works Clearinghouse (WWC), National Skills Development Corporation (NSDC), and database searches on ERIC and PsycINFO. The keyword search descriptors used were these: teacher knowledge, teacher learning, professional development (PD), online professional development, math professional development, math content knowledge, in-service teachers, and para-professionals. The following were the criteria for selecting studies for the review:

1. Development of teacher knowledge models 2. PD studies directed for K – 8 math educators 3, Studies on fractions PD for teachers 4, and Recent studies on online PD. I followed a historical overview for the review outline of the approaches to teacher PD initially and then later narrowed my focus to math PD. The following questions were addressed:

1. What models and approaches have been used for teacher's PD in math studies?
2. What frameworks address teacher learning?
3. What methodologies and theories do the fraction studies address?

The review of related literature in this next section provided the background information necessary to interpret the relationship between the variables and informed the conclusions to the research questions in the study. Salient points from the literature that inform my study are summarized under these main section headings: Teacher Knowledge Development; Approaches to professional development (PD); Online Teacher PD; Content Studies in Fractions.

Teacher Knowledge Development

Teacher knowledge development is broad and complex. There have been many opinions for what should comprise teacher knowledge in order to become an effective teacher. The National Education Association (NEA) 2016 policy lists subject matter content knowledge, pedagogical content knowledge, and assessed classroom-based performance as the main components of a teacher knowledge framework that included important teacher required knowledge components. Over the years, various scholars like (Schulman, Ball, and Ma) developed frameworks on what should be included in teacher knowledge development. The research literature on teacher knowledge development can be split into two subgroups: pre-service teacher development and in-service teacher development. Some pre-service literature is included in the review to better understand the trends for in-service teachers' knowledge development. Pre-service and in-service professional development is addressed in the following review of related literature. Each of these sub groups, pre-service and in-service teacher development, is complex and detailed in its own fields. While there are literature topics generic to both pre-service and in-service teacher knowledge development, an attempt was made for literature related to the variable in the study, especially in-service teachers. This section will address teacher knowledge development from a historical perspective to better understand the current frameworks on teacher knowledge development and what critical components teacher knowledge should include.

Teacher Characteristics and Resources Impact on Knowledge Development. It is necessary to review some of the previous influences on teacher knowledge development in order to understand how each time period emphasis influenced the contemporary approaches to teacher knowledge development. Dating back to the 1960s, teacher learning research focused on the impact of teacher characteristics, behaviors, and beliefs on student achievement. The Coleman report presents the literature on the studies describing the relationship between teacher behavior and student achievement as the process-product paradigm on teaching and teacher professional development (Coleman, 1966).

Studies on how teachers' beliefs and behaviors impact student learning continued to dominate teacher learning in the late 1970s, and, scholars had accumulated evidence that only certain teaching behaviors did affect student achievement gains. These types of studies were also known as process-product studies. The biggest criticism of process-product studies was the lack of attention given to subject matter and pedagogy and how it influenced their findings (Hill, Rowan, & Ball, 2005).

The second type of knowledge development that dominated teacher learning in the 1960s was professional development that focused on the relationship between education resources and the outcomes on student achievement, also known as production- function knowledge. The Coleman report (1966) referred to these types of studies as education production function studies. The goal of the Coleman research study was to predict student achievement on standardized tests from the resources possessed by the students, teachers, school, and others (Greenwald, Hedges & Laine, 1996; Hanushek, 1981). The production function studies followed two approaches; the first approach focused on teacher preparation and experience as a predictor of student success. The factors that were considered in production- function studies were these: teacher education level, certification status, and the number of post-secondary subject matter courses taken and the number of years of classroom experience.

Begle (1972, 1979), and Greenwald et al. (1996) disputed the extent to which variables such as teacher preparation and experience contributed to student achievement (Hill, Rowan & Ball, 2005). Begle (1979) reviewed production function studies that measured teacher knowledge and found that the studies did not show positive effects for teachers who took more mathematics courses. Boyle (1979) and Hanushek (1981) measured teacher knowledge based on their performance on certificate exams or other subject matter competency tests. By examining teacher knowledge and teacher performance, the researchers assumed a relationship between teacher content knowledge and teacher performance that leads to improved student achievement. The process product and education production period of studies which were largely behaviorist contributed to the rise of the cognitive scientists view of learning. Behaviorist scholars focused on teacher knowledge from the perspective that if teachers took more math courses and methods courses, they would be more knowledgeable. The results of these studies on subject matter knowledge did not show main effects on student achievement (Begle, 1979). However, later research on teaching and on teacher knowledge revealed that teachers' understanding did affect their students' opportunities to learn (Grossman, 1990; Lampert 1989; Leinhardt and Smith, 1985).

Frameworks of Teacher Knowledge. During the early 1980s, the research paradigm began to shift from behaviorist focus to the cognitive science perspective in education psychology. Cognitive scientists who differed with the behaviorist approach to learning began to become prominent in the late 1980s in the field of teacher knowledge and what it should encompass. Teacher knowledge has continued to be the focus of many research studies on teacher education. New ideas about teacher learning developed. Researchers began to examine the knowledge base unique to teaching. Lee Shulman’s research in 1986 on the “missing paradigm," was in response to the behaviorist theorist view of teacher knowledge and characteristics. Shulman contended that there needed to be more attention given to subject matter knowledge rather than teacher characteristics. Over the years, Shulman, Ball, and colleagues, have developed various frameworks about teacher learning.

Shulman recommended that three types of knowledge were necessary for teaching: content knowledge, pedagogical content knowledge, and curricular knowledge. In his description of content knowledge, Shulman stated that, "The teacher need not only understand that something is so; the teacher must understand why it is so and on what grounds its warrant can be asserted and under what circumstances our belief in its justification can be weakened and even denied" (Shulman,1987, p.9). Shulman defined pedagogical content knowledge (PCK) as subject matter knowledge for teaching (p.9). Teachers should be able to present the materials to students to help them understand concepts and help the students understand where their mistakes are. PCK is a combination of subject knowledge and knowledge of teaching that is unique. The third type of knowledge Shulman describes is Curricular Knowledge, which is essentially knowledge of the curriculum. Shulman felt it is important that teachers know what the learner's prior knowledge should be and how their current knowledge helps them understand future concepts on curriculum map.

Shulman considered lack of focus on knowledge of the content as missing link in the research on study of teaching. Shulman contented that hardly any attention was given to the specific content knowledge and pedagogy that teachers had. Shulman`s framework ignited many major studies on teacher knowledge growth. Shulman`s work has been cited extensively in teacher education and professional development frameworks. Shulman`s presidential address (1986) and the related Harvard Education Review article (1987) have been cited in more than

1,200 refereed journal articles (Ball, Thames, & Phelps, 2008, p. 392) in a large variety of educational topics. Shulman and his colleagues responded to criticism of his earlier model and focused on making subject matter knowledge at the forefront of teacher knowledge (Grossman, 1990, 1991; Grossman, Wilson, & Shulman, 1989; Shulman, 1987; Wilson et al., 1987). Much of this research was on PCK and subject matter knowledge for teaching during the late 80s but continued into the early 90s.

A group of researchers at Stanford called their project *Knowledge Growth in a Profession*. Grossman (1990) developed Shulman's idea of subject matter knowledge to include four areas: content knowledge for teaching, substantive knowledge, syntactic knowledge, and beliefs about subject matter. The depth of the teachers' subject matter knowledge will determine how they present the material and interact with the students' questions. The substantive knowledge of discipline provides teachers with the knowledge base required to facilitate student learning in the discipline. Syntactic knowledge is overall knowledge of the field and how new knowledge and ideas relate to the field. Both of these types of knowledge are important to teachers' knowledge development. Grossman addressed teachers' beliefs or orientations in her framework. Grossman explained that if teachers believed that mathematics was about following the steps and finding the correct answer, this is how they would teach math.

Hiebert and Lefreve (1986) published a book that addressed conceptual and procedural knowledge. Similar to Grossman's idea of orientations, Hiebert believed that teachers' knowledge orientations are either conceptual or procedural. Hiebert & Lefreve (1986) describe conceptual knowledge as a web of connected information and ideas. Procedural knowledge is described as recognition of forms and knowledge of rules, algorithms, and procedures. Hiebert & Lefreve associated conceptual learning with meaningful learning. Procedural knowledge lends itself more to memorization of procedures which does not promote making connections between concepts and enhance understanding. Wilson (1987), Star (2005), Baroody, Feil, and Johnson (2007) agree that deep procedural knowledge does exist and that deep procedural knowledge cannot be achieved without having a conceptual knowledge base. Recently there has been more emphasis on pedagogical content knowledge (PCK) rather than content knowledge (CK) or a blend of both in teacher professional development through the influences of work of Deborah Ball on Mathematical Knowledge of Teaching (MKT).

Ball (1988) examined teacher learning in mathematics. Ball identified two areas important for teacher learning: knowledge of mathematics and knowledge about mathematics. In the early 1990s, Ball further examined the mathematical knowledge needed in teaching and how this knowledge was developed. Later, Ball and colleagues developed measures to test for different types of mathematical knowledge for teaching. The framework they developed both included and expanded Shulmans' categories.

Ball and her colleagues (e.g., Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill & Ball, 2004; Hill, Ball, & Schilling, 2008; Hill, Schilling, & Ball, 2004) expanded and defined the mathematical knowledge for teaching. Ball, Thames, and Phelps claimed that Shulman's PCK lacked definition and empirical foundation which limited its usefulness (p.389). Ball and colleagues stated that thousands of scholars cited Shulman's PCK without direct attention to content area, instead making general claims about teacher knowledge, teacher education, and or policy. Ball et al. stated that, "scholars have used the concept of PCK as though its theoretical foundations, conceptual distinctions and empirical testing were already well defined and universally understood" (p.394). Ball et al. suggested that Shulmans category of subject matter knowledge should include two main sub categories of common content knowledge (CCK) and specialized content knowledge (SCK). Pedagogical Content Knowledge (PCK) from Shulman's framework comprises two sub categories: knowledge content for teaching (KCT) and knowledge content of students (KCS). The framework is fairly new (2008), and the researchers continue to develop each category. Ball et al., explain how each domain is distinct but works together to encompass teacher knowledge: recognizing a wrong answer is common content knowledge (CCK), while sizing up the error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK). In contrast, familiarity with common errors and deciding which of several errors students are most likely to make are examples of knowledge of content and students (KCS). Knowledge content for teaching combines knowing about math content and knowing about teaching. Ball et al. gives an example of the KCT domain; "during a class discussion, they have to decide when to pause for more clarification, when to use a student's remark to make a mathematical point, and when to ask a new question or pose a new task to further students' learning" (Ball et al., 2008, p.401). The new framework developed by ball and colleagues which extends and combines with Shulmans work has been dubbed the "egg" framework. Davis and Simmit (2006) have criticized this framework as being confusing and difficult to follow, but they agree that mathematic knowledge for teaching is important and a central focus of most research on mathematics education.

Borko (2004) utilized a framework that focused on subject matter and how students learn the subject matter by engaging teachers as learners in strong professional learning communities to develop teacher knowledge. Wilson and Berne (1999) suggested three features to effectively develop teacher knowledge: communities of learning, teachers playing an active role, and critical colleagueship in a trusted environment. The National Research Council (2001) recommended five strands for learning mathematics: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. Conceptual understanding can be compared to what Ball describes as content knowledge. Strategic competence is defined as the ability to formulate, represent, and solve math problems. Adaptive reasoning is defined as the capacity for logical thought, reflection, explanation, and justification. Productive disposition is defined as the habitual inclination to see math as sensible, useful, and worthwhile (NRC, p.5).

Similarly, the National Council for Teachers of Mathematics (NCTM) published the process standards for learning that would develop and increase mathematical understanding (2000). These standards have been adopted into many teacher preparation programs. According to the NCTM, the process standards highlight the mathematical processes students draw on to acquire and use their mathematical content knowledge. The process standards are as follows: problem solving, reasoning and proof, communication, connections, and representation. Problem solving is defined as the process of applying a variety of appropriate strategies-based on information provided, referenced, recalled, or developed. Students require frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of effort. The reasoning and proof standard require making and investigating mathematical conjectures and develop arguments and proof. Students with this skill, note patterns and structure mathematically and in real-world scenarios. The communication standard is defined as organizing mathematical thinking coherently and clearly to peers, teachers, and others, using the correct math language to express ideas precisely. Connections refer to understanding how mathematical ideas interconnect and build on one another to produce a coherent whole. Lastly, representations refer to creating multiple representations to organize, record, and communicate mathematical ideas. These standards were published for students. The standards are included in this review as the intervention activities are designed around the NCTM standards in my study. These process standards apply to teachers as students learning math content. Additionally, the intervention in my study includes the process standards already previously designed into the learning activities.

The literature on teacher learning frameworks share common features recommended for effective development of teacher knowledge. Two main evident categories are as follows: subject matter knowledge and pedagogical content knowledge. Many synonyms have been used for subject knowledge and pedagogical content knowledge. Almost thirty-one years later, we can still trace the influence of Shulman's framework to current models. Ball et al. who expounded on Shulmans framework, also lead in their sphere of influence on teacher knowledge development. While the frameworks discussed can be generic to teacher learning, Ball and colleagues focus purely on mathematics teachers. The frameworks specific to mathematics education continue to be investigated as the need for further research still exists in mathematics education for educators.

Content Knowledge Development on Fractions

This next section provides a review of studies on fractions as my study seeks to develop teachers' content knowledge in fractions, decimals, and percentages (FDP). I chose to address FDP in my study because current testing data reveals this domain is an area of concern for elementary and middle school students. Professional development workshops in fractions are frequently sought out as educators seek ways to help students learn fractions. Charalambous and Pitta-Pitanzi (2007) mention that fractions are among the most complex mathematical concepts that children encounter in elementary education. Charalambous & Pitta-Pitanzi explain that one of the main reasons for this complexity is that fractions comprise a multifaceted concept encompassing five interrelated sub-constructs: part whole, ratio, operator, quotient and measure. Another obstacle to students learning of fractions is the instructional approaches they receive on fractions (Lamon, 1999). Students struggle to demonstrate understanding and teachers find teaching this domain challenging (Stewart, 2005). Fraction numbers sense includes determining the size of the fractions, the order of fractions, the equivalence of fractions, and making connections to establish reasonable answers and compare them to a benchmark (Lamon 2012, p.136). Proficiency in fractions also includes being able to perform an operation with fractions and thinking of fractions as part of the number system. Teachers need to exhibit proficiency in fractions content knowledge in order to teach it. It is important that we look at research already done in developing teachers' content knowledge in fractions. While numerous studies of fractions exist, the majority of the studies cover pre-service teachers' knowledge, teacher's knowledge of multiplication and division of fractions, and or pedagogy studies on fractions. I selected studies in this review that provided a sampling of studies on content knowledge of fractions for teachers over a time period and from different countries where possible. My intention was to report on the prevailing emphasis of the studies and direction of recommended future research for developing content knowledge of teachers. This next section will summarize a sampling of fraction content studies and their emphasis.

Earlier Fraction Content Studies. Kieren (1976) in a study on perspectives of rational numbers, proposed a thorough understanding of fractions and that the knowledge of fractions is comprised of several sub constructs. In order to understand fractions, each construct and how each one relates had to be understood. Kieren identified the four subconstructs that explained the meaning of fractions as follows: measure, quotient, operation, and ratio. Kieren considered part -

whole the main construct on fractions. The part - whole construct is seen as fundamental to fractions, and it seems to be the focus of most curriculums and text books. However, it is the relationship between all the constructs that is lacking in the instruction of fractions. Understanding all the sub constructs is required to problem solve fractions. Kieren recommended that a study be done on teachers' concepts of rational numbers since rational numbers are complex. Similar studies by Lamon 2007, 2012, Kieran 1976, and Ball 1990, agreed that to gain a deep understanding of rational numbers, it is necessary to understand different interpretations of fractions.

Leindhardt and Smith (1985) conducted a study on the relationship between expert teachers' classroom behavior and their subject matter knowledge on fractions. The teachers' fraction knowledge was explored in depth as it occurred in natural teaching settings. Data collection involved interviews, activities, and video recording of lessons. Leindhardt & Smith recommended in-service support, including marginal notes in teachers' manuals to help develop conceptual understanding. This study investigated teachers' knowledge of fractions, but more specifically equivalent fractions. The results revealed that expert teachers did perform better on the test for content knowledge than novice teachers, but a surprise result was that the expert teachers lacked deep understanding of equivalent fractions and did not perform any better than the novice teachers in demonstrating deep understanding.

Graeber and Tirosh (1989), examined developing content knowledge of fractions by using representations of the partitive and quotative models. The partitive or sharing model involves dividing the total amount by the number of groups in order to find the number in each group (Greer, 1992). Greer defined the quotative model as measurement or repeated subtraction model of division which involves grouping. Graeber & Tirosh found that developing an understanding of the partitive and quotative models of division affected the choice of operations when solving problems. The teachers tended to pick partitive for division and quotative for multiplication. In an earlier study by Graeber & Tirosh, it reported that some teachers invert division (to divide a larger number by a smaller number) on problems such as the following: "Twelve friends together bought 5 pounds of cookies. How many pounds did each get if they each got the same amount?" (Graeber & Tirosh, 1988, p. 265).

Ball (1988) examined prospective teachers' knowledge and reasoning, both mathematical and pedagogical, when they began their teacher education. One aspect of the study focused on teachers' substantive knowledge of mathematics specifically with division of fractions. The study also examined what they believed makes something true or reasonable in math and what counts as mathematical justification (p.132). Contextual questions were used to examine the teachers' knowledge of division with fractions and other topics. Responses were coded for correctness and the nature of the justifications provided. The participants in this study were prospective teachers. Ten were enrolled in general elementary education, and nine were enrolled in secondary education as math majors. In an example where participants were asked to solve 13/4 divided by ½, 17 of the 19 prospective teachers were able to solve the problem correctly. Only 5 were able to generate an appropriate representation with justification. The study's outcome reinforced that relying on prospective teachers' pre-college math knowledge is unlikely to provide adequate subject matter knowledge for teaching mathematics with understanding. The results of this research contributed to further studies in developing teachers' mathematical knowledge for teaching in an in-depth manner. Two years later, Ball 1990 surveyed 252 prospective teachers to examine their mathematical understanding as they entered formal teacher education. The results revealed that the teacher candidate's mathematical knowledge was rule bound and shallow. Ball challenged three assumptions about learning to teach elementary and secondary math: (1) that traditional school mathematics content is not difficult, (2) that precollege education provides teachers with much of what they need to know about mathematics, and (3) that majoring in mathematics ensures subject matter knowledge.

Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992) examined a student teacher whose lesson failed because this person was unsuccessful in providing conceptually based justification for the division of fractions algorithm. The mathematics methods course that the student was enrolled in did not require the student to examine their knowledge base and beliefs about good mathematics teaching. Borko et al. suggested that math education programs should reconsider how they provide subject matter knowledge and opportunities to teach it to pre-services teachers.

Khoury and Zazkis (1994) conducted a study where pre-service teachers were asked to solve two fraction problems with different symbolic representations. Numeration systems with bases other than ten were used to represent fractions. The study results revealed that understanding of fraction operations is not robust. The participants struggled to understand different interpretations of fractions in general and identifying equal parts if the numeric value was changed. The results from this study also revealed that most of the participants were generally comfortable with performing algorithms when solving fractions problems. However, they struggled to explain why the algorithms work (Ball,1990, Borko et al.,1992). Participants tended to overgeneralize rules for whole numbers and tended to apply them to fractions as well (Graeber et al., 1989).

Behr 1997 interviewed 32 in-service teachers to identify and classify their strategies in solving rational number multiplication and division problems. The participants use of intuitive models influenced how they solved the problem. Another finding was that participants were unable to create word problems to represent the fraction problem to be solved. Harel and Behr (1995) found that only teachers who connected the ratio and proportion concepts could solve the problems correctly. Teachers also used basic keywords to identify the operation they should use to solve the problem. Keywords for subtraction are giving away or less than. A keyword for division is sharing, and for multiplication the keyword is of following the word percent or fraction. A later study by Behr and colleagues (1997) analyzed pre-service teachers' problem-solving strategies in answering the question "How many piles of sticks are in three-fourths of eight bundles of four sticks?" Behr and colleagues found that the pre-service teachers commonly used two strategies to solve the problem: the number of units in a unit of units and the unit's size in a unit of units. The pre-service teachers who displayed this knowledge successfully demonstrated more specialized knowledge rather than common content knowledge because a strong knowledge of units is required to represent fraction multiplication situations successfully.

Ma (1996) conducted a study on the profound understanding of elementary school teachers' math, specifically on fractions. Ma compared subject knowledge of teachers in the U.S. and teachers in China, her native country. In the 1996 study, Ma developed a hierarchical framework for teachers' knowledge of fundamental mathematics. The first tier of the framework she called procedural understanding. Ma noted that most teachers in the study could use algorithms to solve math problems. Ma explained that using procedures to solve math is what many people associate with "doing math" (p.218). However, this procedural fluency is what laymen and teachers possess. Ma contends that teachers should have more than just procedural knowledge to teach math. The second tier of her framework is conceptual understanding (Ma, 1996). Ma believed that conceptual understanding is built on the foundation of procedural understanding. Conceptual understanding is deeper than procedural understanding, and is the type of knowledge that is unique to teachers. Ma points out that most teachers can provide a brief explanation of the procedure they are following when solving math problems when they are teaching students; however, "laymen usually do not tend to make an explicit explanation about the procedure of solving a math problem when they are teaching you" (p.92). Ma calls the third-tier knowledge packages. Ma explains that, "a knowledge package refers to a relationship between and among a group of mathematical ideas that specifically connect to the present topic which the teachers were addressing" (p.226). The fourth tier of Ma`s framework is called the structure of math. Ma defines the structure of math as an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough. Ma constructed a knowledge package of fundamental math comprised of associative, commutative, and distributive concepts. Ma described these as the breadth of teachers' subject matter knowledge. The depth of knowledge she described is linking ideas such as the concepts in her knowledge package. The idea of thoroughness which Ma described as being one's capability of passing through all parts of the field was the most important. Olanoff (2011) describes this thoroughness as the glue that holds one's mathematical knowledge together and makes it complete. Teachers within the United States were lacking in their profound understanding of math compared to Chinese teachers. Ma concluded that her research revealed that teachers' subject matter knowledge of a topic is one of the major cornerstones on which pedagogical content knowledge is built. Before one can make anything teachable, one first has to understand the topics. Ma warns that it should not be assumed that teachers know all they need to know at the start of teaching and recommends that the education should be continuing. Ma later published a book in 1999 on knowing and teaching elementary math. Based on this study, Ma developed hierarchical framework for teachers' knowledge of fundamental mathematics, which signaled the sustained and still necessary attention to developing teachers' content knowledge, particularly in fractions. Ma`s and other scholars' work mentioned earlier in this review has contributed greatly to the earlier studies on mathematics knowledge development for teachers.

The earlier studies on fractions centered around rational numbers, multiplication and division with fraction, and decimals and sought to understand how teachers understood these concepts. The teachers' conceptual knowledge development was largely influenced by their beliefs on learning. The studies revealed that problem solving with algorithms was the traditional approach most teachers defaulted to, and conceptual understanding was vague. This next section will highlight a few studies from 2000 to some of the most current on developing teachers' content knowledge in fractions.

Later Fraction Studies for Teacher Content Knowledge Development. Chinnapan 2000 investigated the transfer of understanding fractions to a computer environment called Java Bars. The use of technology was increasing and similar studies had reported technology to aid students in the understanding of transferring whole number knowledge to fractions (Oliver, 2000; Tirosh 2000). Computer programs for learning increasingly became an acceptable alternative to traditional learning approaches. Chinnapan examined a group of Australian preservice primary mathematics teachers' understanding of fractions. Results from this study suggest that while the participants showed acceptable levels of knowledge of fractions, they may not have integrated the knowledge sufficiently with their knowledge about the computer software. Chinnapan reported that the results suggested that teacher education programs need to analyze the mathematical content and software interface carefully. Immersing the teachers in the general learning activities allows the teachers to explore the software as learners. Such an approach should aim to generate learning activities in which preservice teachers could explore the connections between their own mathematical knowledge and how that knowledge could transform within the computer environment. By doing so, teachers can expect a better understanding of children's own learning difficulties (p.252).

Domoney 2002, conducted a study in Great Britain on procedural and conceptual knowledge. He found that participants had limited conceptions of fractions dominated by part whole constructs. Participants had a stronger preference for the part-whole meanings of fractions over the meanings of quotient and rational. Performance on the conceptual items was less than satisfactory. The participants had difficulty with number lines. When asked to locate 3/5 on the number line of 5 units with 0-5 labeled, one participant placed 3/5 on the unit labeled 3.

In the learning mathematics knowledge for teaching (LMT) report, Hill (2007) examined the mathematical knowledge of a random national sample of approximately 600 middle school teachers. Hill investigated a mix of teachers' common content knowledge and specialized content knowledge in numbers and operations and pre-algebra and algebra. The numbers and operations domain covered an extensive selection of concepts: whole numbers and operations, rational numbers and operations, integers, ratios, proportions, percentages, and radicals (p.99). The study results indicated that teachers with more mathematics course work, subject-specific certifications, and teaching experience scored higher than teachers who did not. The learning mathematics instrument to measure teachers' content knowledge was developed as a result of this study.

Green, Piel, and Flowers (2008) examined the impact of manipulatives-based instruction on two independent cohorts of preservice elementary teachers. Green et al. investigated problem-solving with whole numbers and fractions using concrete and representational manipulatives.

The intervention was held over five sessions. The post-test survey results showed significant improvement in fraction knowledge and a decrease in arithmetic misconceptions. Thus, the use of manipulatives in problem-solving had proven effective. Evans (1990) supported the idea of unconventional approaches to concept development and said that, "Meeting a familiar concept in an unfamiliar situation forces the teachers to re-examine their subject-matter knowledge, overcome difficulties, and construct a better, deeper, and more articulated notion" (p.541). The use of manipulatives has increased with common core mathematics, but it has not fully been adopted by the more traditional teachers who still teach algorithms.

Li and Kulm 2008 investigated 46 pre-service middle school teachers on their knowledge of fraction division and on their own perceptions about their knowledge preparation on the topic. Li & Kulm argued that for teacher preparation and professional development to be effective, it is required that information on teachers' knowledge deficiencies and teachers own perceptions about their needs be considered. The study's results revealed a wide gap between the participants' general perceptions or self-confidence and their limited math knowledge needed for teaching division of fractions conceptually. The results suggested that teachers need to develop a sound and deep understanding of mathematics knowledge to build their confidence for instruction. Li et al. stressed the importance of conceptualizing the notion of teachers' knowledge in mathematics to include teachers' perceptions. In this study, Lin (2010) investigated web-based instruction versus traditional lectures to help pre-service teachers learn procedural and conceptual knowledge of fractions. The results showed that the web-based instruction was more effective for learning procedural and conceptual knowledge of fraction operations.

Lo and Luo (2012) compared Taiwan and U.S. teachers on a variety of foundational fraction knowledge topics: part-whole, quotient constructs in different orientations, and their concepts of equivalence and meanings of fractions operations. The results showed that the Taiwanese teachers outperformed the U.S teachers. The difference between the two groups was statistically significant for 12 out of the 15 test items. Lo et al. recommended that U.S. teachers be more familiar with fractions in the area and linear models. The study results suggested that the pre-service elementary teachers from both countries needed a deeper understanding of the meaning of fraction division and multiplication operations.

Putra and Zetra (2016) investigated in-service teachers' fraction content knowledge in Indonesia. Putra & Zetra used a variety of technologies for the multiplication of a fraction with a whole number and also the multiplication of a fraction by a fraction. The participants displayed several mistakes when designing a contextual problem of a fraction by multiplication. Putra et al. speculated that the participants generalized the meaning of multiplication from the context of integers to that of fraction operations. The results showed that the in-service teachers who had insufficient knowledge for multiplying fractions needed further content knowledge development.

The following study, conducted by Garet, Heppen, Walters, and Parkinson (2016) focused on math knowledge and the impact of content-intensive teacher professional development. The intervention was designed around content knowledge, instructional practice, and student achievement. The study was 93 hours long, including 80 hours' direct math instruction and 13 hours' participation in a math teaching community to analyze students' work, reinforce math content, and help teachers apply the content to improve their instruction. The content that was covered included whole numbers, fractions, rational numbers, ratio, proportion, rate and linear equations, and functions. The teachers were assessed on content and three dimensions of practice: the richness of the math, student participation, errors and imprecision. The results reported from the study indicated that PD had a positive impact on teacher knowledge. The treatment group had an average 21% higher score than control teachers in 2014. The PD had a positive impact on some aspects of instructional practice, particularly the richness of math. While the PD did have an impact on some of the teacher outcomes, it did not have an impact on student achievement. The status of math improvement is still a high priority. 2015 NASSE reported on the progress of education. The report stated that 60% of 4th graders scored below proficiency level. Garet et al. suggested that elementary school teacher will especially benefit from content focused professional development (PD) because they are less likely to study math in college than secondary school teachers specializing in a subject. There is limited evidence of impact of content focused PD. Because of this, the scholars in this study recommend further research (Garet et al., 2016).

In summary, research studies conducted in the 90s and earlier about math knowledge and the impact of content intensive teacher professional development reported that teachers showed limited understanding of fractions and rational numbers and resorted to algorithms to solve math problems (Behr 1997; Ball 1990; Borko et al 1992; Ma, 1996). Olanoff, Lo, and Tobias (2014) conducted an extensive review of the literature on prospective elementary teachers' content knowledge in fractions. Olanoff et al. noted that over the last few decades, an accumulating body of research has documented constraints on teachers' understanding of multiplication and division with fractions and decimals. Olanoff et al. agree that although most teachers can quickly execute algorithms to determine the product of two fractions or decimals, many studies have reported constraints on U.S. teachers' capacities to reason about such products when numbers are embedded in problem situations. Olanoff et al. cited several studies that reported constraints on in-service and preservice teachers' performance when explaining products of two fractions or decimals (e.g., Armstrong & Bezuk, 1995; Ball, Lubienski, & Mewborn, 2001; Eisenhart et al., 1993; Sowder, Philipp, Armstrong, & Schappelle, 1998; Tirosh & Graeber, 1990). The more recent studies align with a standards-based approach to learning and align with the NCTM standards or the national common core to develop conceptual understanding of fractions. The quest to improve understanding of fraction concepts is ongoing as the issue of poor mathematical performance is prevalent and concerning.

Approaches to Teacher Professional Development

This section addresses teacher professional development (PD) approaches and what is deemed effective teacher professional development. Teacher PD is defined as experiences that allow in-service teachers to enhance their knowledge attitude and skills (Borko, 2004). The types of TPD experiences are varied, with very few considered to be effective. Borko stated that teacher professional development (TPD) has been fragmented and superficial and mostly comprised of just short seminars with no after training support (p.3). Hill (2007) criticized PD programs as being one-time instructional events. Banilower, Smith, Weiss, Malzahn, Campbell, & Weis, (2013) reported that ninety-one percent of teachers indicated that they attended a single-day workshop on math. Many PDs are not high quality sustainable or transformative and lack ongoing support to implement new curricula or pedagogy (Borko 2004; Dede 2009). Teachers struggle to implement new strategies from PD workshops in their classrooms. As a result, teachers often become frustrated with PD programs because they are ineffective, irrelevant, based on assumptions, demand large investments of their time, or lack ongoing support (Dede et al., 2009). Dexter noted that the PD programs were not learner centered (2014). Dexter explained that 36 teachers reported that PD did not meet their needs in specific content areas and schedules. Furthermore, a different 2012 survey results reported that less than fifty percent of the teachers' responses indicated experiences that reflected best practices in professional development (Banilowe et al., 2013). Therefore, the need for effective professional development became critical.

Numerous studies have addressed the inadequate PD practices by making recommendations for effective professional development (Desimone, 2009). As mentioned earlier, Desimone recommended critical core features of effective PD: content focus, active learning, coherence, duration, and collective participation (2009). Scher and O`Reilly (2009) conducted a meta-analysis of effective professional development. The result was three components of effective PD interventions: duration, content, and form. There is consensus for content and duration as the recommended components for effective PD between Scher and Oreilly and Desimone.

Avalos (2011), in a review of 111 studies on teacher PD that impacted student learning, reported the majority of the studies focused on collaboration, mentoring, reflection and tools for teaching. This is consistent with the literature on effective PD by the National Center of Education Evaluation from Darling and Hammond and Ball and Cohen. Darling- Hammond stated that, "Teachers learn best by studying, doing, and reflecting; by collaborating with other teachers; by looking closely at students and their work; and by sharing what they see" (p.2, 2000).

Mentoring. Designing professional development interventions should include effective approaches from the related literature. Mentoring has proven to be an effective approach in teacher professional development. Learning communities incorporate the features of mentorship. The idea of induction and mentoring new employees has been in existence for many decades. It is not unique to the teaching profession. Mentoring is the personal guidance usually provided by seasoned veterans to beginning teachers in the schools (Ingersoll, 2011). The support and guidance of new teachers positively impact classroom instructional practices and student achievement. The idea of induction has been in existence across industries to orient new employees. The most common PD was and still is mentoring new teachers. While this is still a common and ongoing type of PD, the studies' emphasis was on new strategies to mentor teachers more effectively in order to retain new teachers in the profession. The Organization of Economic Co-operation and Development (OECD) recognized that beginning to teach is a particular difficult stage of teacher learning. Therefore, mentoring new teachers has been a focus of study for many researchers. The following studies investigated supporting new teachers and training and mentoring best practices, (Harrison, Dyemoke & Pell, 2006; Devos, 2010; Rajuan, Tuchin, & Zuckermann, 2011). Polly and Hannafin (2011) studied 2 elementary teachers during a yearlong project found that cooperative planning lessons with an experienced teacher increased the consistency between the goal of the PD, in this case, the implementation of learner centered instruction and teachers enacted practices. Retaining new teachers has also become an area of focus as many new teachers are resigning as educators within their first five years, (Fantilli & Mc Dougal, 2009; Wiebke, & Bardin, 2009, Odell, & Ferraro, 1992; Feiman-Nemser, 2003). Thus, the need for well-designed professional development remains an ongoing challenge.

Professional development through collaboration. Another focus of teacher PD has been collaboration between schools and local universities and schools and materials resource developers. The collaborative type PD has been either structured, semi structured, or an informal process that stimulated teachers to alter or reinforce their teaching practices. In most studies, the external collaborator like the university professors or researchers play a key role. Darling-

Hammond (2000); Bartholomew and Sandholtz (2009), and Gravani (2008) advocate for the importance of partnerships in professional development. The above-mentioned researchers believe that it bridges the gap between different perspectives of PD and encourages mentoring at different levels.

Teacher co-learning is another type of collaborative professional development. The importance of understanding how teachers work together and share practices with learning purposes is reflected in studies that look at teacher networks, communities of practice, and communities of learning as well as professional learning communities (Banilower et al., 2013). Lesson study is the Japanese teacher co-learning experience through mutual collaboration and feedback. The following studies link its effects to the improvement of instruction and collaboration. The following studies reviewed the conditions that impact lesson study effectiveness (Fernandez, & Yoshida, 2012; Lewis, Perry, & Murata, 2009; Watanabe, 2002; Yoshida, 1999). Lesson study has been documented as an effective collaborative and reflection type of professional development for math educators.

The terms, professional learning communities (PLC), communities of practice (CoP), and communities of learning, have all been used interchangeably to mean the same thing. Stoll, Bolam, McMahon, Wallace, and Thomas (2006) state that, “There is no universal definition for PLCs, but there is a broad international consensus that the definition suggests a group of people sharing and critically interrogating their practice in an ongoing, reflective, collaborative, inclusive, learning oriented, growth promoting way” (p.223). Stoll et al. further explained that the concept of PLC seems to have emerged from a variety of sources. The idea of an effective PLC is not new; certain key features were evident in education writers’ work in the early part of the last century. Dewey (1933), for example, was committed to the view that: educational practices provide the data and the subject matter, which form the problems of inquiry (Stoll et al., p.223).

Stoll, Bolan, McMahon, Wallace, and Thomas (2006); Cochran-Smith, and Lytle (1999); DuFour (2010); Du Four and Eaker, (2005); Hord (1997); Vescio, Ross, and Adams (2008) addressed the benefits of professional learning communities on school reform, on teaching practices, and student learning. A common focus in these studies was that professional learning communities (PLC) are regarded as a viable change agent in school reform. Stoll et al. noted that international literature reviews of PLCs advise that developing PLCs appears to hold considerable promise for capacity building for sustainable improvement. The following two studies refer to PLCs and communities of practice (Lave & Wagner, 1998 and Eckert, 2006) and explain that a community of practice defines itself by its members, function, and the capability it has produced. Lave and Wagner (1998) define a community of practice as a collection of people who engage on an ongoing basis in some common endeavor. Communities of practice emerge in response to common interest and play an important role in forming what their members participate in. Lave et al. argue that communities of practice are everywhere and we are in a number of them at work, home, and school. Participants' levels of involvement differ from community to community (Lave et al., & Eckert). Lieberman and Miller (2008) informed by research and observed experiences are convinced that teachers learn best within their own work environment. Lieberman and Miller focus on five items of practice within communities of practice: context, capacity, content, commitment, and challenge.

Vescio, Ross, and Adams (2008) in their literature review of ten studies reported that collective results suggest that well developed PLCs have positive impact on both teaching practice and student learning. Hord (1997) described the benefits of PLCs as a reduction in teacher isolation, increased commitment to the mission and goals of the school, shared responsibility for student development, powerful learning that defines good teaching and classroom practice, and enhances understanding of course content and teacher roles. Vescio et al. reported that teachers participating in PLCs also expressed higher job satisfaction and morale and had lower rates of absenteeism. Mc Laughlin and Talbert (2006), in their study provide extensive evidence that school-based teacher learning communities improve student outcomes. Mc Laughlin and Talbert explain that learning communities provide an agenda to develop and sustain a collaborative professional culture that helps to change the school and teaching contexts. However, Jaworski (2006) addresses the challenges of developing teaching as learning in practice and use of inquiry in math learning. He expounds on math teachers' roles and goals in education in such communities, being both distinct and deeply intertwined. The inquiry task in the communities is a tool where practice leads to better understanding. Dougiamas and Taylor (2003) conducted an action research study testing a virtual learning community. The results showed improved processes with communities of reflective inquiry. While the task of reflection is contained in the learning cycle of professional communities and self-regulated learning, it is also recognized in the literature as a stand-alone effective approach to professional development.

Reflection as a form of professional development. In its simplest form, reflective practice is thinking about or reflecting on what one does. Moon (2002) defines reflection as "a form of mental processing which we use to fulfill a purpose or to achieve some anticipated outcome. It is applied to relatively complicated or unstructured ideas for which there is not an obvious solution and is largely based on the further processing of knowledge and understanding and possibly emotions that we already possess" (p. 2). Reflection is closely linked to the concept of learning from experience, as one thinks about what he or she did, what happened, and decide from that what he or she would do differently next time. Calderhead and Gates (2003) in their book, recognize that reflection has become a crucial element in the professional growth of a teacher. Calderhead & Gates examined the nature of reflection in teachers' professional development and discussed its value in teachers' ongoing development. Calderhead et al. explain that reflection allows teachers to adopt an analytical approach to their teaching. Reflection is also used as a narrative tool for self-assessment in qualitative studies. Self-assessment is a necessary skill in self-regulated learning; therefore, reflection types of activities for the discussion board lend themselves well to the design of my study intervention. Teachers are encouraged to write reflective essays about their learning in PD workshops (Shank, 2006).

Virtual communities as professional development. Virtual communities are popular, especially with social media, and are another effective approach in professional development design. A learning community is made up of a group of people who have a common cause of learning and shared values and experiences. Members of a learning community share common goals, respect, support for each other, and have a commitment to learn from each other. However, a virtual community consists of a group of people with shared common goals who interact online regularly. Lock (2006) explained that an online community consists of people who interact socially online but performed specific roles. Lock stated that their shared purpose provided the motive for the existence of the community. They also have policies in place that guide their interactions. Lastly, they use personal computers and the internet to create the gathering place for the group. The partnerships and interactions, with a shared purpose, define the community and not the technology.

Professional development needs to transition from the low-impact traditional approaches used to a community of learning model. A learning community will foster a culture of sharing and provide sustained professional development support for teachers. The learning process within a community is a social constructivist approach, and members construct meaning from experience and interpretation in their social interactions. Teachers can share their interpretations of problems and collaboratively explore issues and best practices to gain new insights from each other that may benefit their students. The affordances of technology allow for that learning community to be virtual, including teachers from all over the world, making the collaboration dynamic and spontaneous. The resources available to teachers through the World Wide Web and internet provide opportunities for anywhere, anytime, just-in-time and continuous collaboration for virtual professional development.

Technology emphasis professional development. Training teachers to use tools for teaching has been a long-standing practice. The tools now include various forms of technology for both educators and learners. Several articles deal with the uses of technology in professional development. A poll on educational technology usage among U.S. teachers by Harris Interactive in 2013 reported that 86% of teachers think that it is important to use technology in the classroom; 96% of teachers think that technology in the classroom increases student engagement in learning; 89% of teachers think that education technology improves student outcomes. However, only 14% of teachers actually use digital curricula weekly; only 19% of teachers use subject specific tools weekly; and only 11% of teachers implement bring your own device (Bates, 2016). While there are numerous studies on educational technology tools, there are obviously more interventions needed to equip teachers to use technology in their lessons. The US Department of Education has made strides to increase the use of technology in the classroom by including technology standards in the curriculum for all subjects, but the implementation process has been slow. Most teachers are not yet equipped and skilled to do so. Schrum (1999) reviewed the literature on efforts to increase pre-service and in-service teachers' use of technology. Schrum reported that pre-service programs made considerable effort but much work was still needed for in-service PD in technology. However, the surge of technology usage in general has led to an increase in studies involving technology and has created a focus on online teacher professional development (OTP).

Online Teacher Professional Development

Online teacher professional development (OTPD) has the potential to address geographic and resource limitations and to increase the levels of support and reflection through synchronous and asynchronous interactions. Online teacher professional development provides teachers with resources that are not locally available and can facilitate large scale teacher professional development (TPD) with online resources that are vetted (Dede, Ketelhut, Whitehouse, Breit, & McCloskey, 2009). Technology and web based mediated PD has many different titles; OTPD, Professional Learning Communities, Professional Learning Networks, Online learning and eLearning. According to Trust (2012), statistic reports show large and ever-increasing numbers of OTPD platforms. Edmodo 6.5 million users, Classroom 2.0 and E4ducator TV have over 72, 000 members combined. There has been an increase of Massive Open Online Courses (MOOCs). Some large and renowned universities offer free OTPD through platforms like Coursera. Universities such as Johns Hopkins, Vanderbilt, University of Washington, and the University of Virginia and Oregon offer free OPTD through platforms like Coursera. Due to the rise in OTPD, scholars have seen a need for further research to address the effectiveness of offerings. Jones and Dexter (2014) noted that teachers have increasingly started to use online resources to engage in informal conversations with colleagues and independent learning activities but school districts do not recognize and build upon these types of professional development. Because of the increased use of online resources, I designed my study to direct teachers to online resources that have been validated.

According to Hill, Biesengel, and Jacob (2013), the field of teacher PD has reached a consensus over the last two decades on design elements that maximize teacher learning in a PD program. This consensus includes strong content focus, inquiry orientating learning approaches, collaborative participation, and coherence with school curricula and policies (p.476). Pape et al. designed their online PD study on fractions to incorporate the recommended elements and reported gains in teacher content knowledge development in the first six months of their study. My intervention design is informed by the current literature. Each study in this review seeks to improve teacher PD based on recommendations from prior studies. However, not all PD interventions observe satisfactory outcomes. Teachers' satisfaction rates and their perceived professional growth vary extensively. An example of a major e-learning initiative sponsored by the US Department of Education, with 30000 teachers from nine states participating, which revealed high levels of attrition in OTPD programs (O` Dwyer et al., 2010). During the first 3.5 years of the project, only 74 % enrolled teachers completed the OPD courses (Reeves and Pedulla, 2011). The findings of high attrition suggest that current OTPD programs may not be responding appropriately to teachers' needs and expectations.

Online teacher professional development studies. The majority of the studies compared the effects of face-to-face versus online professional development programs. Fischer, J. B., Schumaker, J. B., Culbertson, J., and Deshler, D.D (2010) focused on multiple variables: PD methods, teaching methods, online teacher professional development models, participation interactions, and teacher's perceptions.

Fischer et al. (2010) compared a computerized PD program without human facilitation to a face-to-face program based on Kilpatrick's four evaluation levels: teacher and student learning, reaction, behavior, and results. Fischer et al. used a randomized control trial with a sample size of N=50 teachers and N=152 students. The results show that both were equally effective. The face-to-face program results reported higher satisfaction. The teachers in both programs gained similar knowledge. The next study by Reeves and Pedulla (2011), involved an eLearning initiative sponsored by the US Department of Education. Their objective was to identify factors that co- vary with online PD course satisfaction among educators. These factors were related to participant background and OPD design and implementation. They conducted a multiple regression analysis with a sample of 3998 teachers from nine states. The variables with the strongest relationship to teacher satisfaction of the OPD were these: beneficial discussion topics, quality of learner interactions, ease of content transferability, course organization, adequacy of compensation, and the clarity of participation expectations.

Tsai, Laffey, and Hanuscin (2010) sought to understand how teachers' participation in a community of practice (CoP) enhanced their teaching and examined teachers' perceptions of their social experience and professional growth. The CoP focused on teaching methods for K-8 science. A mixed methods study design was used with t-tests and content analysis as well as sample size N= 92 in-service teachers. Teachers reported that CoP was significant in supporting their teaching. Tsai et. al also reported significant changes in teachers' perceptions of social navigation, ease of use, usefulness and satisfaction of community of practice. Holmes, Spyner, and Mcleod (2011) described the teachers' perspective on an online PD program, the value of online presence, and the factors that affect PD quality and satisfaction. Various online courses were offered to in service teachers, N = 95. A mixed method's approach was used. Holmes, Spyner, and Mcleod reported that social and teacher presence was the most important factors related to teachers learning and satisfaction.

Mc Aleer and Bangert (2011) explored the professional growth of mathematics teachers after they participated in a mentoring program. The intervention used an electronic mentoring for student success model (EMSS). The study sample N =43 comprised of math mentor teachers. Dash, Magidin de Kramer, O Dwyer, Masters, and Russell (2012) reported that the results revealed that the EMSS program promoted individual and social knowledge construction. The observed patterns of engagement gave rise to the perceived growth. Dash et al. evaluated the effects of OPD in math on 5th Grade teachers and students. The intervention was eLearning for Educators (EFE), an initiative sponsored by the US Department of Education. They conducted a randomized control trial N=79 math teachers and N=1438 students. Results: Teachers in the experimental group exhibited significant gains in PCK and pedagogy practices. There was no difference reported between student math achievements. Fisherman et al. (2013) identify a difference in teacher knowledge and beliefs, teacher classroom practice, and student learning outcomes as a result of online versus face-to-face PD. Fisherman conducted a yearlong study for high school teachers and used a randomized controlled trial with a sample of 49 teachers. There was no difference between the two programs.

Francis and Jacobsen (2013) in this study, examined the formation of a professional learning community to improve math teachers' pedagogical strategies. This research was a qualitative study that involved a sample of 13 teachers. The professional learning group, made up of teachers, discussed mathematical tasks and learning activities that promote creativity among their students. The online synchronous environment allowed teachers to learn about collaborative mathematical problem-solving and improve their teaching practices. For optimal learning to take place for the teachers, the appropriate discussion question and task had to be selected.

Hunt, Powell, Little, and Mike (2011), in their study, evaluated the effects of participation of novice special education teachers in an online mentoring program on their perceived teacher preparedness and knowledge of professional standards. The electronic mentoring for the student success model (eMSS) initially was developed for math and science teachers; however, it was later expanded to mentor novice special education teachers. A mixed methods design was employed with a sample size N=22 teachers for the quantitative study and N=10 teachers for the qualitative study. The results showed a statistically significant difference in teachers' levels of perceived preparedness and knowledge of standards after they participated in the online mentoring program. There were no effects reported for perceived teacher knowledge.

Pape, Prosser, Griffin, Deans, and Algina Bu (2104) conducted an OTPD study on improving mathematical content knowledge among teachers of grades, 3 to 5. This yearlong study had a sample of 23 teachers in general education and special education. The data results indicated growth in content knowledge of teachers' initial growth in knowledge among the students. Knowledge growth is evident from the pretest to the mid-test, but not after that because participation dropped off significantly. The online intervention was comprised of math modeling, ongoing practitioner focused journals and websites, developer constructed materials, classroom implementation, reflection, and discussion. Pape et al. (2014) believed all these carefully planned components resulted in increased mathematical content knowledge. Various features recommended by previous scholars for effective OTPD were included in the design of the Pape and colleagues' study. The features of math modeling, reflection, and discussion are contained in the framework used to guide my current study. However, I use slightly different language when referring to them: content focus, active learning, and collective participation.

Dede (2009), in a review of nearly 400 studies on online, face-to-face, and hybrid teacher PD programs, identified only forty studies were methodically sound. Most of the studies in their review on OTPD were lacking in various areas. A summary of the related literature suggests further research is needed in OTPD. The agenda calls for blended research or mixed methods to evaluate approaches to inform the design and extend the understanding of models that affect teacher learning and behavioral change. The rationale is that a mixed method's design was necessary since knowing that a program was effective under certain criteria was not enough. Understanding why these programs worked and for whom and under what circumstances is equally important. The research findings inform the design of future OTPD programs.

Summary of Related Literature Review

The review of related literature combines summarized information from approximately seventy-one publications: eleven articles on teacher knowledge development, thirty-two studies on fractions, eighteen studies on types of professional development, and thirteen studies on online teacher professional development. In response to questions asked at the beginning of the literature review, I will summarize the main points to each question*: What frameworks address teacher learning? What models and approaches have been used for teachers' PD in math studies? What methodologies and theories do the fraction studies address?*

There needs to be further investigation of PD program's actual content in mathematics and how this content may influence teacher knowledge. This recommendation is due to the teacher professional development studies' findings that inform the need for a focus on content in mathematics PD. Hill, Rowan, and Ball (2005) advised that when teachers are engaged in math analysis, it influences positive learning. Hill et al. further explain teachers engaged in math analysis as exploring alternate representations, communicating explorations, and sharing connected ideas with other teachers and PD facilitators. Ball and Cohen (1999) suggested that effective PD for math can be characterized as engaging participants in a collaborative examination of curriculum, participating in investigations of math content, investigating students' responses to math content, reflecting on their efforts, implementing instructional strategies in the classroom, developing collaboration between practitioners and university personnel, and recognizing teachers as professional.

Testing data at the state level still indicates that students lack understanding of fractions. In 2015, the NASSE reported on the progress of education. The report states that 60% of 4^{th} graders scored below proficiency level (Garet et al., 2016). Another issue is that 58% of teachers reported to have taken two or fewer of the five college level math courses recommended by NCTM which include numbers and operations, algebra, geometry, probability and statistics (Banilower, p.18).

The supporting evidence makes it an imperative that teachers develop sound content knowledge on fractions in order that they are better equipped to handle their students' misconceptions around the concept and develop their students' mathematical understanding. It is clear from the literature that further research is needed in math content studies. In terms of the design of online PD, the literature indicates the biggest impact is evident with learning communities and collaboration features, but there are still concerns about attrition rate among participants, duration and engagement. Desimone and Garet (2015) in a recent study, tested the best practices of professional development: content focus, coherence collaboration, active learning, and duration. Desimone et al. recommended that more work needs to be done to test the specific features of the best practices in order to achieve effective professional development. Desimone et al. also discovered that content knowledge changes were more difficult to achieve than pedagogy or instructional changes. A trend in professional development (PD) is to link PD to teacher evaluations in an effort to enforce some of the best practices (Desimone & Garet, 2015). More studies are needed to test for effective strategies for online teacher professional development (OTPD); moreover, the second question in the current study sought to explore the impact online resources on the development of teachers' content knowledge in fractions, decimals, and percentages.

CHAPTER 3:

RESEARCH DESIGN

This chapter describes the research design that was used to study the impact of online teacher professional development on content knowledge in fractions, decimals, and percentages. Specifically, this chapter describes the components of the research design: participants, context, data collection tools, data collection procedures, and data analysis plan. A mixed method's design was used, which included a pretest and posttest survey to measure content knowledge and qualitative surveys to evaluate and understand the experiences of the participants. The following research questions guided the study:

1. How does the online MAPPs curriculum develop content knowledge in fractions, decimals, and percentages for teachers?
2. How do online manipulatives, videos, and discussions help teachers develop content knowledge for fractions, decimals, and percentages?

Overview of Pilot Study

The pilot study was conducted to test the survey instruments, the online intervention platform, and data collection processes. A sample of teachers and paraprofessionals who teach math from grades K - 8 were solicited. All six participants were employed at schools with Title 1 designation. In the treatment group, all three participants were school teachers, two were elementary teachers, and one was a middle school teacher. One of the teachers was fairly new to the profession and had under two years' experience, but she was also actively pursuing her master's degree in math education. The other two participants each had more than 10 years' experience each, teaching math. One participant taught at an elementary school for 15 years and had taught grades 3, 4, and 5, while the other had 11 years' experience with her service time between elementary and middle school teaching grades 4 and 6. The veteran teacher in this sample had completed her specialist degree in math education. The third participant has a bachelor's degrees in education. In the control group, the three participants included two paraprofessionals and a veteran teacher with 23 years' experience. Of the two paraprofessionals, one was employed in an elementary school and the other in a middle school. Both paraprofessionals have been employed for under five years as paraprofessionals but were pursuing their bachelor's degrees part time. The third control group participant taught at an alternative school for over 10 years and taught various levels of mathematics.

Data Collection Tools. The survey instrument was used to measure content knowledge of teachers on fractions, decimals, and percentages. The Learning Mathematics for Teaching Instrument was developed by The University of Michigan, School of Education with funding from an NSF grant (Hill, 2007). The content specific instrument was piloted with two thousand teachers in the testing for validity phase of development. It is regarded reliable and valid. It is a pre-approved measurement instrument from learning mathematics for teaching project that has been used extensively in mathematics education research. Figure 3. is a sample of the test items.

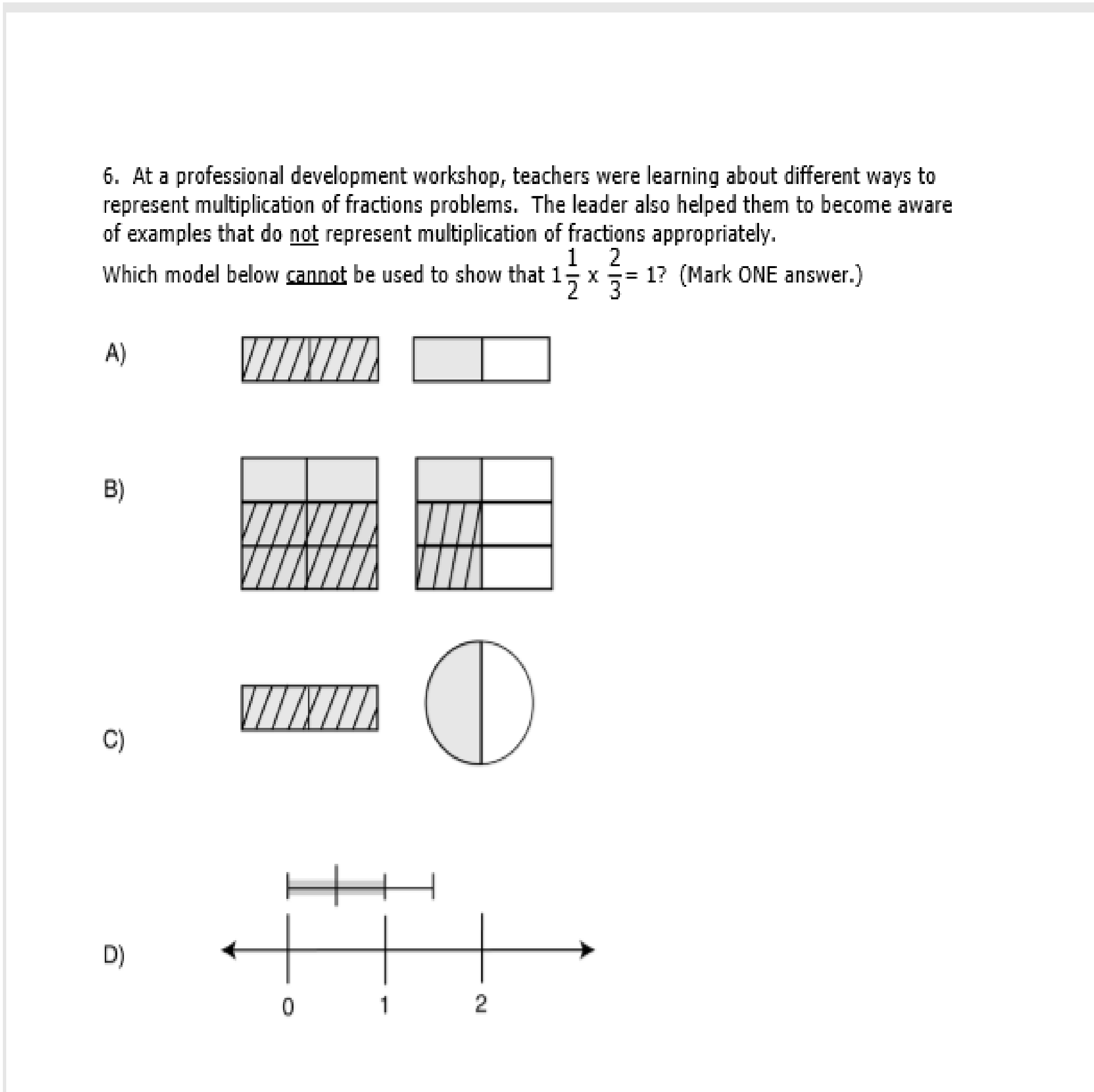

Figure 3. Sample Content Knowledge Survey Item

I also administered surveys and interviews to assess the participants online learning experiences. These surveys included a Likert item and some qualitative type questions. Figure 4. below is a sample of those survey items. The interview questions are listed below.

Fractions, Decimals, and Percentages Online Professional Development Survey

Session Activities	Place a check mark if you tried the activity	Comments/ Feedback on activity
Session One		
Measuring Map **BLM 3, 4, 5, 6, 7**	☐ BLM 3 ☐ BLM 4 ☐ BLM 5 ☐ BLM 6 ☐ BLM 7 ☐ Measuring Map	
Session Two		
List of Locations **BLM 11, 12, 13,**	☐ BLM 11 ☐ BLM 12 ☐ BLM 13	

Rate the videos on the scale: 1 –being not helpful and 5- being very helpful in understanding and completing activities / BLMs. Place a check mark in the box you choose.

1 – Not Helpful	2- Slightly Helpful	3 - Neutral	4 – Helpful	5-Very Helpful

Additional Comments:

____-----------

Rate the manipulatives on the scale: 1 –being not helpful and 5- being very helpful in understanding and completing activities / BLMs. Place a check mark in the box you choose.

1 – Not Helpful	2- Slightly Helpful	3 - Neutral	4 – Helpful	5-Very Helpful

Additional Comments:

Figure 4. Survey Sample Items

The interview questions used in the interviews were semi structured to allow for variation. Below is the list of interview questions used to guide the interviews. The interview questions were grouped into content related questions, experiences with the online platform, and general questions related to the professional development.

Questions related to teachers' content knowledge:

1. Did you learn anything new about fractions, decimals, and percentages from this professional development?
2. Has your content knowledge in fractions, decimals, and percentages improved? If yes, how?
3. Can you give an example of a concept that was confusing or intimidating before, that you now understand more clearly?
4. How do you feel about mathematics in general?
5. Do you feel math is important to learn and why?
6. Do you have any questions about the content that you did not get answered?

Questions about the online platform:

7. How would you describe your online learning experience?
8. Was the professional development interface user friendly?
9. What would you change about the online professional development?

General Questions:

10. What was most useful about this experience?
11. What was not useful?
12. What can be improved in this intervention?

Pilot Study Procedures and Research Methodology. The pilot study participants were solicited either in person or via email. All surveys and intervention materials were placed online or communicated via email. Surveys were conducted through a third-party online host to ensure anonymity. Participants were provided with the URL to the intervention for the pilot study and access instructions via email. All responses were collected in the online platforms.

Pilot Study Data Analysis. Pre and posttest designs are used as the preferred method to compare participant groups and to measure the degree of change as a result of treatment or intervention. The analysis included the pretest analysis between groups and item analysis of the treatment group, the Likert item analysis, and the qualitative responses analysis. I conducted a simple comparison of means between the pre and posttest of the treatment and control groups. Results of the pre and posttest analysis can be seen in Figures 5 and 6.

Because my sample was very small in the pilot study, I used non- parametric tests to analyze the surveys. Jim Frost (2013) recommends using non-parametric tests for analysis when the sample size is small, for ranked or ordinal data, the data is non - normal, and data is skewed. There are limitations in using this analysis with a small sample size as it is difficult to detect significant effect. I also conducted a simple comparison of means between the pre and posttest of the treatment group. The results are shown in Figure 7.

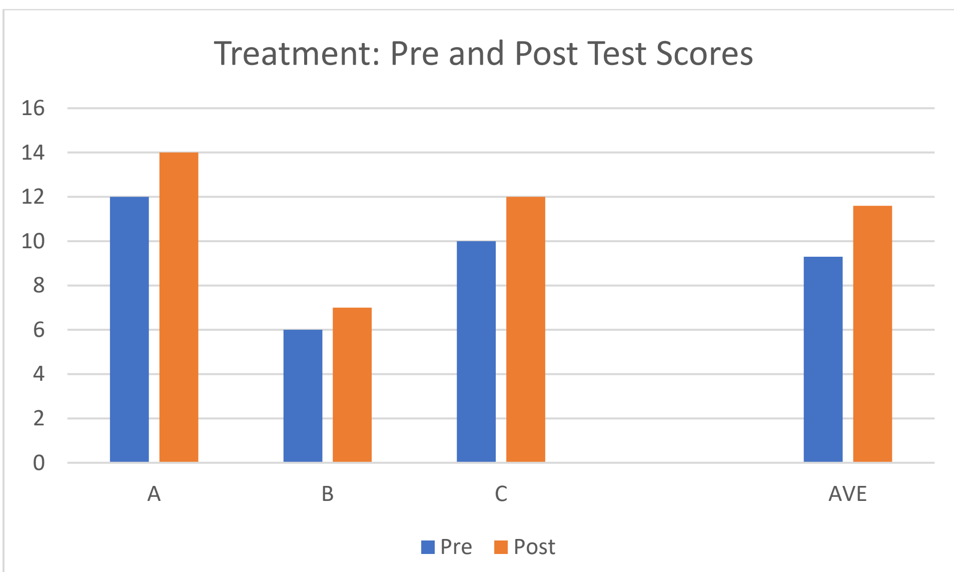

Figure 5. Treatment group scores

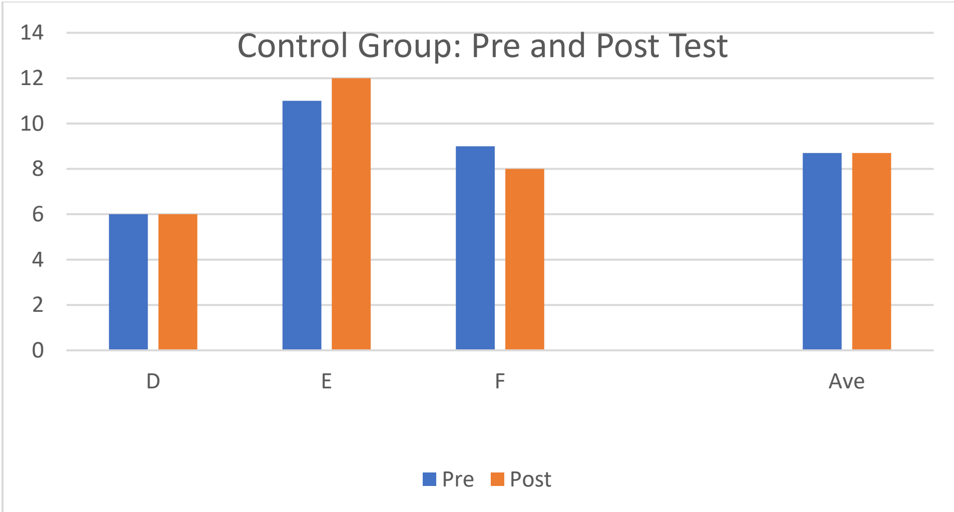

Figure 6. Control Group Test Scores

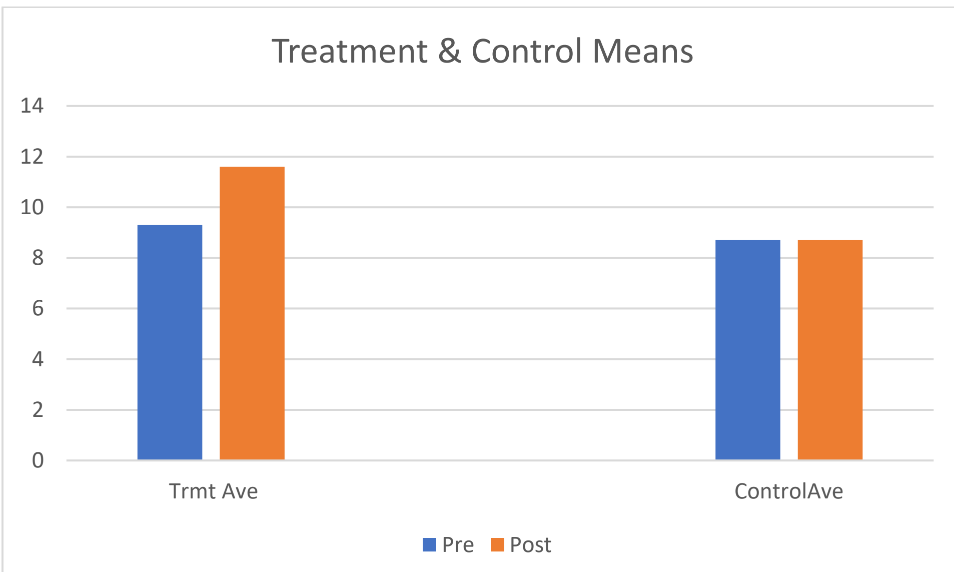

Figure 7. Comparison of means

Looking at the basic mean scores only, one can see the treatment group has slightly higher mean averages than the focus group. When the means between the pretest and posttest of each group are compared, there is an increase in treatment averages, but the control averages are the same for this group. The results of this pilot study data were not generalized to a larger population.

Other analysis items included a Likert scale item surveying the effectiveness on the online videos and manipulatives. All scores recorded were for 5, most helpful except for one participant who selected neutral as a response. Testing the Likert scale item was valuable since, I realized from all the clarifying questions, that the scale was a little confusing to users. I was therefore able to adjust it for clarity. One of the participants from the treatment group was interviewed. While the responses were captured, they were only analyzed for key words and not coded for theme trends but used primarily for preliminary feedback on the participants' experiences with the platform.

Pilot Study Analysis of Qualitative Responses. Qualitative analysis interview items are consistent with phenomenology as a research method. The results of the analysis were used to adjust the professional development intervention and actual study. Question one is better suited to a qualitative analysis: How does online learning in fractions, decimals, and percentages develop teachers' content knowledge?

The data was analyzed for key words in this initial interview. The key words would help inform themes from the data in the actual study. The key words were these: unclear, checklist, engagement, interesting, critical thinking, challenging, helpful information, relevant and manageable, and discussion. The Likert scale additional comments were stated: "The online resources were needed to complete activities." "Videos were helpful." "Likert scale item is confusing."

Even though the qualitative data was limited at this stage, the data was still useful in interpreting participants' experiences. The responses also helped me to make modifications to the surveys. Several modifications were made to data collection tools for the full study. I included a checklist of all items to be completed during the intervention; pre, during, and post. The items to be completed seemed unclear to participants based on the clarifying questions received and incomplete items. The qualitative survey, Likert scale, and the intervention assignment list were all adjusted for clarity. The participants did indicate that the online platform interface was easy to navigate. One valuable finding was that the online platform was most compatible with Google Chrome.

The pilot study provided an opportunity to test the online platform and adjust for unforeseen challenges that may arise during the actual study. Details of the actual study are explained in the next section.

Present Study

Background on the Intervention Curriculum. In 1999, the University of Arizona received a National Science Foundation (NSF) grant to develop the Math and Parent Partnership Program. Math and Parent Partnerships in the Southwest (MAPPS) is an NSF-funded project on parental involvement in mathematics education. The project activities involve leadership development for teams of parents, teachers and administrators, Math for Parents' mini courses, and Math Awareness Workshops are open to all parents and family members in the district community. Although the focus is on parents, parental involvement is unlikely to succeed without teachers and school administrators being part of the process and supporting it.

Several University Mathematics Education Departments collaborated on the development of the curriculum. The content is based on the NCTM standards-based curriculum and process standards for developing mathematical understanding and comprises courses in five domains: Numbers and Operations, Fractions, Decimals and Percentages, Geometry, Algebra, and Data. The NCTM Principles and Standards for School Mathematics (PSSM) are guidelines produced through a consensus process by the National Council of Teachers in 2000. These guidelines form a national vision of standards-based mathematics education in the United States. The guidelines include six principles for learning the curriculum standards and the process standards.

The six principles are these: equity, curriculum, teaching, learning, assessment, and technology. The process standards are: Problem Solving, Reasoning and Proof, Communication, Connections, and Representations.

Context and Intervention Description. In 2003, MAPPS Programs were in place in 12 districts in 9 states around the country. In 2007, the MAPPS program was implemented for the first time at UGA – Griffin campus in partnership with local schools in a face to face format. In 2008, teachers and paraprofessionals were invited to participate in the learning community and the model produced many positive outcomes. As a University of Georgia employee at the time, I had previously been involved with offering math professional development to teachers at the local schools. I already had a working relationship with the local schools for several years. The demand for the face to face format still existed, but scheduling was always an issue, and seats were limited.

The study's intervention was an adaption of the face-to-face program into an online asynchronous offering. This was done in an effort to test the viability of an online offering that would ultimately make the curriculum accessible to more teachers. The online intervention was titled: Fractions, Decimals, and Percentages Content Professional Development. I adapted the program to run for eight weeks, comprised of six modules with a week at the beginning and the end for surveys. My role in the intervention delivery has been both researcher and facilitator. The online intervention was set up to require minimal involvement from me. As the facilitator, I monitored the discussion boards and responded to a few posts to keep participants engaged in the discussions. I also read the reflection paragraphs and provided feedback to the participants.

The intervention modules are: Fraction Concepts I, Fraction Concepts II, Fraction Concepts III, Decimal Concepts, Connecting Decimals and Fractions, Connecting Decimals, Fractions, and Percentages. Overview of session one: Explore fraction representations. Overview of session two: Participants use color tiles to enhance their understanding of equivalence, simplest form, and common denominators. Overview of session three: Concepts formed in session two are reinforced but using pattern blocks instead. Pattern blocks are not uniform in size, thus requiring spatial – visual reasoning to compare sizes. Session four: Participants use Cuisenaire rods to explore equivalence, unit and non-unit fractions, and reciprocals. Cuisenaire rods are based on length and measurement models for fraction reasoning. Session five: The participants use base ten blocks and paper strips to explore addition and subtraction with decimals and to compare decimals. Session six: Participants use paper strips and calculators to examine the relationship between fractions and decimals. Session seven: The participants explore percentages using a hundred square percent grids, color tiles, and number lines. Session eight: The participants use numbers strips and Cuisenaire rods to deepen their understanding of percentages and the relationship between fractions, decimals, and percentages. The module outlines are presented in Figure 8 below.

Module Outlines	**Checklist**
Module One: Developing Fraction Concepts I • NCTM Learning Principle • Math Class Web • Tangram Pattern Sheets (BLM 3-8) • Discussion	• Read NCTM Learning Principles Page • Discussion board – Math web • Complete the tangram activities • Use virtual / actual manipulates to solve • Watch session videos for help • Post your reactions to the tangram activities
Module two: Developing Fraction Concepts I • NCTM Problem Solving Standard • Color Tile Activity One & Two – (BLM 11) • Fraction Problems Set I	• Post to discussion board how you have implemented the problem-solving standard • Complete Tile Activity One and Two and Post a few of your responses
Module three: Developing Fraction Concepts II • NCTM Number & Operation Standard • Division Activity – (BLM 16) • Pattern Block Activity – (BLM 17) • Fraction Problems II – (BLM 19)	• Post your interpretation of the standard on the discussion board. • Share some of your responses and how you problem solved • Complete the reflection activity 1
Assessment: Reflection Activity 1	Write a reflection on how you would change your lesson plans to include something new that you have learned.
Module four: Developing Fraction Concepts III • NCTM Communication Standard • Cuisenaire Rod Activity II (BLM 22) • Cuisenaire Rod Relationships (BLM 23) • Fraction Problems III	• Post your interpretation of the standard. • Use virtual / actual manipulates to solve • Watch session videos for help • Post your reactions to the activities to discussion board prompts.
Module Five: Decimal Concepts • NCTM Technology Principle • Decimals Activity I, Decimal Problems I Connecting Fraction & Decimals • NCTM Reasoning & Proof Standard • Fractions – Decimal Summary Chart	• Post to discussion board how you have implemented the technology standard • Complete Decimal Activities and Post a few of your responses • Post your responses to Connecting Decimal and Fractions on discussion board prompts
Module Six: Developing Percentage Concepts • NCTM Number & Operation Standard • Percent Estimation and Color Tiles • Percent Number Lines Connecting Fractions, Decimals & Percentages • Percent Strip & Cuisenaire Rod (BLM 48)	• Post your interpretation of the standard on the discussion board. • After you have completed the activities share some of your responses and how you problem solved • Complete the reflection Activity 2
Assessment: Reflection Activity 2	Now that you have completed several activities, write a reflection on how you would change your lesson plans to include something new that you have learned or write a reflection on any of the activities that you have tried and the learning that has occurred.

Figure 8: Intervention Activity Outline and Checklist

Present Study Participants

Teachers and paraprofessionals who teach math or assist students with math in the classroom were recruited for the study. After obtaining permission from the local school administrators, participants were recruited via bulk email sent through the school system professional development department. Flyers were also distributed to the schools and placed in the teachers' lounges. Announcements were also posted in community and school system social media platforms. An initial survey and research agreement letter were given to volunteers to gather background information on them in order that they can be matched and placed in balanced groups in terms of their background education and experience. In this initial survey, I collected information on their education background, years of experience teaching math, and their attitudes towards math. Mixed groups consisting of teacher and paraprofessionals were created, with an attempt to balance the groups by matching teacher backgrounds and years of experience in each group. The paraprofessionals were selected and placed in groups based on their number of college credits achieved thus far and years of experience. Paraprofessionals with sixty or more college credits and the highest number of years with classroom experience assisting with math were given priority in placing them in groups. An equal number of teachers and an equal number of paraprofessionals were placed in each group. Paraprofessionals were included because some are working towards their teaching degrees and certifications.

The paraprofessionals in the local school district are required to have 30 college credits before they are employed as paraprofessionals. This has not always been the case, but many school districts are now adopting this practice as they realize the value of well-trained paraprofessionals. The study's volunteers were recruited from the local schools in south east Georgia. Volunteers were selected and placed in two groups: treatment and control.

Present Study Data Collection Tools

The primary data collection tool was a Learning Mathematics for Teaching (LMT) survey instrument. The various measures developed in the LMT project were piloted with five thousand teachers and were used to study how teachers learn mathematical knowledge for teaching (Hill & Ball, 2004). The instrument used was a domain-specific multiple-choice Content Knowledge for Teaching Mathematics (CKT-M). It measures the pre- and posttests knowledge of participants and corresponds to the content for online PD in fractions, decimals, and percentages (Hill et al., 2004). Reliability on the CKT-M measures instrument was previously established for in-service elementary and middle school teachers (Hill, 2007). An initial survey for demographic information, a Likert scale survey with interview questions were also used. The intervention platform collected responses to the activities, discussion posts, and reflection exercises.

Present Study Data Collection Procedures

An approved IRB exists for this study from both the University and a local school system (see Appendix A). I approached school administrators of the elementary and middle schools and explained the purpose of the study. I then requested permission to communicate with teachers in their schools either via email or flyers. I appealed to teachers from grades one to eight who are required to teach fractions, decimals, and percentages. I appealed to teachers who enjoy teaching this concept, those who struggled to teach it, those who were seeking new methods to help their students who struggle with understanding the concept. This included new teachers, veteran teachers, teachers who dislike teaching math, and teachers who admit they enjoy math. I wanted the sample to represent varied backgrounds, experiences, and attitudes. A volunteer general survey regarding their backgrounds and attitudes towards math helped to ensure a diverse group. The sample was drawn from the volunteers. The data was collected in this order:

1. Volunteer and consent form were administered prior to start of PD.
2. The Pretest was administered prior to PD.
3. Qualitative question and Likert scale item surveys were administered during the study.
4. Discussion Boards from PD intervention.
5. The Posttest was administered after the study.

Present Study Data Analysis Plan

Once all data was collected, it was then screened for a completed data set. The survey response data set was automatically created in the online platform and exported for the analysis in SPSS. The descriptive statistics was calculated for all independent and dependent variables in the study which included the means and standard deviations of the variables.

The dependent variable in terms of this study is the teachers' math content knowledge while the independent variable is the online PD on fractions, decimals, and percentages. A pre and posttest design is the preferred method to compare participant groups and measure the degree of change occurring as a result of treatment or interventions. I used a two group pre – post design where both the treatment and the control group received both tests. The pretest – posttest is able to measure how much of a difference the intervention made by comparing the treatment and control groups.

A one-way ANOVA was used to compute the difference between the pretest and post test scores and then analyze the difference using treatment versus control as the only factor. If the main effect is significant for the treatment group, then the change from pretest to post test is not the same in the two groups. The data can also be analyzed by looking at the interaction effect in a 2 x 2 analysis of variance with treatment (treatment vs. control) as a between subject's factor (Becker, 2000)

The treatment group participated in the Fractions, Decimals, and Percentages intervention in the first offering. To keep the control group engaged they were offered a chance to participate in the intervention at later offering. The second intervention group results will not be used in the dissertation study. The intervention was offered in Fall beginning the third week in August and completed the end of October. This allowed for each offering to run for the planned eight weeks, with a week's break in between to accommodate the school break.

A mixed method's study integrates quantitative and qualitative methods in one study. Gay (2009) explained that mixed method's studies build on the synergy that exists between quantitative and qualitative methods to understand a phenomenon more fully than is possible using qualitative and quantitative alone. The research design included a pretest, posttest design to measure impact of professional development (PD) on content knowledge and a survey with interview questions to evaluate online learning experiences. The latter survey and the reflection exercises contained comments on their experiences. The comments and surveys required a qualitative analysis. Pretest – posttest design is consistent with measuring impact or intervention or changes in knowledge. Desimone stated that, "Survey questions are best used to answer defined, discrete questions about frequencies and trends, specific features of professional development, and instructional time spent on specific content and practices" (Gay 2009, p.190). Survey data is broad and produces statistics, quantitative, systemic numerical descriptions of events, behavior, or practice. Teacher surveys that ask behavioral and descriptive, not evaluative questions about the teachers' PD experiences, have shown to have good validity and reliability (Desimone 2009, Jacobson 2004). Desimone further validates those critical features of professional development (PD), especially content focus and active learning which can best be measured with surveys. The survey data was used to address the first research question: How does online learning develop content knowledge in fractions, decimals and percentages for teachers?

Analysis of Qualitative Data Responses. Research question two is better suited to a qualitative analysis: How do online manipulatives, videos, and discussions help teachers develop content knowledge for fractions, decimals, and percentages? Qualitative Analysis of interview, discussion, and reflection items are consistent with phenomenology as a research method and are able to capture interaction details that a quantitative analysis can sometimes miss. The results of the analysis will inform the development of future professional development interventions and contribute to the field of online professional development.

Analyzing and making sense of the interviews is considered the data analysis process (Meriam, 2009) for qualitative data. All interviews are recorded and then transcribed. The results are reported and inform the overall study as they are the direct experiences of the participants. Recommendations and changes can, therefore, be made to future interventions. Constant comparison analysis is the most common analysis type for qualitative data, commonly known as coding. The fathers of grounded theory, Glaser and Strauss (1967) created the method of constant comparison analysis. Leech and Onwegbeuzie (2007) explained that constant comparison can be undertaken inductively, deductively, or abductively. Deductively is when codes are identified prior to analysis and then looked for in the data. Inductively is when codes emerge from the data and abductively is when codes emerge iteratively from the data. In order to conduct a constant comparison analysis, the researcher first reads through the entire set of data then divides the data into smaller meaningful parts into descriptive subsets. These descriptive titles are known as codes. Once all data is coded, the data with similar codes is grouped into themes.

Merriam (1998) advocates for member checking codes after the coding process by going back into the field and undertake member checking by asking participants of the study if the themes and assertions developed from the codes accurately describe their statements. This practice leads to descriptive validity. Leech et al. explain that the idea of constant comparison analysis was originally developed to analyze data that was collected over a series of rounds. The various rounds of data analysis led to theoretical sampling which involved additional participants and activities in order to identify conceptual boundaries. Constant comparison analysis has since been modified to analyze data from single round interviews.

Qualitative data analysis can be time consuming, but the use of computer assisted qualitative data analysis software (CASQDAS) programs like NVIVO, QSR and ATLAS can perform the constant comparison analysis. Atlas TI was used to analyze the data in this study. In conclusion, quantitative research is helpful in answering questions of who, where, how many, how much, and questioning the relationship between variables. Qualitative research answers the why and how questions. Qualitative research can handle process orientated questions. Leech et al. (2007) describe many positive features of qualitative data. Leech et al. describe it as natural occurring information that tends to be collected in close proximity for direct observation. As a result, the data contains richness and wholeness that is contextualized. Leech et al. also state that most times the data is collected over a long period of time allowing for longitudinal analysis. Finally, qualitative data can strengthen quantitative research design. The results of the analysis are discussed in the next chapter and will inform the development of future online professional development interventions specifically in fractions, decimals, and percentages.

CHAPTER 4:

RESULTS

The quantitative and qualitative data analysis and results are reported in response to the research questions in this chapter. This study is in response to the overarching issue of poor math performance in our nation's schools. It has been established in previous studies that providing effective professional development (PD) to teachers and enhancing teachers' content knowledge is one way of improving students' performance in mathematics (Hill, Rowan and Ball, 2005). Moreover, according to benchmark testing data for students, fractions, decimals, and percentages is the one domain in math with the lowest scores reported. Improved performance in mathematics could be influenced by many variables. One such variable is teachers` content knowledge. More specifically, developing the mathematical content knowledge for educators in fractions, decimals, and percentages through online PD interventions to make PD accessible to educators. The framework for effective professional development by Desimone (2009) was used to guide this study.

The impact of developing educators' mathematical content knowledge through online PD is being measured in this study. This chapter represents an overview of the research questions, data analyses, and research results. The research results are presented in tables with brief summaries of the data. The questions asked in this study were these:

1. How does the online Math and Parent Partnership (MAPPS) program offered as a professional development enhance content knowledge in fractions, decimals and percentages for teachers?
2. How do online manipulatives, videos, and discussions help teachers develop content knowledge for fractions, decimals and percentages?

Participants

A total of 81 participants responded to the solicitation emails, flyers, and announcements. Seventy-three returned the initial information surveys and consent agreements. Sixty-two completed all survey items and the intervention requirements.

Demographics. Figure 9. is a summary of the participants characteristics, their education, and their experience. Participants were 46% African American, 42 % Caucasian, and 12 % other. A quasi-experimental design was used due to the participants' self-election to join the study. The 62 participants were split into matched treatment and control groups.

Participants Category	N	Assoc. or Bachelors	Masters Degree +	< 5 yrs. experience	>5<10 yrs. experience	>10<30yrs. experience
	Treatment Group					
Paraprofessional	6	6	0	4	2	
Educator K -3	13	5	1	1	7	3
Educator Gr 4-8	12	7	12	1	2	5
	Control Group					
Paraprofessional	6	6	0	5	6	
Educator K-3	13	5	1	2	8	1
Educator Gr 4-8	12	6	13	1	3	3
Total	62	35	27	21	29	12

Figure 9: Summary of Study Sample Characteristics

Data Analyses and Presentation of the Findings

The instrument used was a domain-specific multiple-choice Content Knowledge for Teaching Mathematics (CKT-M) survey which measures as pre- and posttests corresponding to the content for online PD in fractions, decimals, and percentages (Hill et al., 2004). Reliability on the CKT-M measures instrument was previously established for in-service elementary and middle school teachers (Hill, 2007). An independent samples t-tests was conducted on the scaled scores from the CKT-M tests generated through Item Response Theory (IRT) to assess for improved domain-specific knowledge.

The primary quantitative result was a comparison of means for the pretest for both control and treatment groups. Both the control and treatment pretest data sets were first checked for normality using the Anderson – Darling test (p=.127, p=.224 respectively). A large p value p> alpha =0.05 indicates normality. With both p values > .05 the data sets can be accepted as normal. A t test was used to compare the pretest scores for both the control and treatment group was conducted. The means were not significantly different with p >.05. Control (n=31, M=61.29) and the Treatment (n=31, M = 64.19 in Table 1).

Table 1

Comparative analysis of test means for pretest scores

Source	N	Mean	SD Deviation	df	t	Sig.(2-tailed)
Control Pretest	31	61.29	16.624	60	0.633	.528
Treatment Pretest	31	64.19	19.285			

Note: The 95% confidence interval for the groups' means were [6.68,11.08] and [5.85, 9.07] An independent samples t –test was used to compare the treatment and control groups pretest scores. There was no significant difference in the scores for pretest for the treatment group (n=31, M =64.19 SD= 19.285) and control (n=31, M= 61.29 SD=16.624) ;(t = 0.633, p > 0.05)

The post test scores for both control and treatment groups were also compared. There was a significant difference between the post test scores for the two samples. Control (n=31, M= 64.04) and treatment (n=31, M=85.93). The difference between pre and post test scores for both groups was also compared. The pre and post test scores were analyzed in SPSS. The results of the second t test comparing the posttest means for both the treatment and control groups is presented in Table 2.

Table 2

Comparative analysis of the test means for the post test scores

Source	N	Mean	SD Deviation	df	t	Sig.(2-tailed)
Control Posttest	31	64.04	17.383	60	4.357	.000
Treatment Posttest	31	85.93	19.933			

Note: The 95% confidence interval for the groups' means was [6.12, 9.49] and [6.92,8.05]
An independent samples t –test was used to compare the treatment and control groups post test scores. There was a significant difference in the scores for posttest between the treatment group (n=31, M =85.93 SD= 19.933) and control (n=31, M= 64.04 SD=17.383) ;($t = 4.357$, $p < 0.05$)

Next, an ANOVA between subjects' test was conducted to see if there is a difference between groups. The difference between the means is listed as the dependent variable and the fixed factor is the group. A within subjects' design was used to find out if the statistic is significantly different between all participants for the pre and posttest. A within subjects' design eliminates the error of using another participant. The pre and post variables are highly correlated at .780 with significance at .000. The difference in test means is significant at < .001. Other descriptive tests included the profile plot and test for homogeneity. There is a statistically significant difference between the pretest and the post test scores $p < .05$. The results of the ANOVA are presented in Table 3.

Table 3

Analysis of Variance Descriptive Statistics: Dependent Variable (DIFF)

GROUP	Mean	Std. Deviation	N	Mean Square	F	Sig.	Partial Eta squared
1 (control)	5	12.503	31				
2 (treatment)	22.18	8.059	31				
Group				4726.563	37.82	.00	.093
Total			**62**				

Summary of quantitative results. The results show a significant difference in pre and post test scores of the CKT -M survey for the treatment group. We can conclude that the online MAPPS professional development enhanced the participants content knowledge in fractions, decimals, and percentages. The difference between the pre and post test scores for the control group was not significant. The CKT-M surveys were designed such that a well-prepared elementary teacher would get 50% of the questions correct, which would be an Item Response Theory (IRT) scaled score or standard deviation of 0.

Item #	**Description** Elementary Content Knowledge (CK) Items	**Answer**	**Construct**	**Elementary or Middle School**
1a	0 is even	1	NCOP/CCK	EL
1b	0 is not a number	2	NCOP/CCK	EL
1c	8 is .008	1	NCOP/CCK	EL
2	371 is prime	3	NCOP/CCK	EL
3a	3.5 x 25 Method A	1	NCOP/SCK	EL
4	Divisibility rule for 4	2	NCOP/SCK	EL
5	5/8 total of 2 pizza	3	NCOP/SCK	EL
6	Model 1 ½ x 2/3	3	NCOP/SCK	EL
7	Divide 1 ¼ pies with 2 families	2	NCOP/SCK	EL
8	983 X 16 Alternative method	3	NCP /SCK	EL

Figure 10: Sample of Released Learning Mathematics for Teaching Items and Construct Codes

A closer look at the test item results revealed that the treatment group scored higher on the items coded as specialized content knowledge after the intervention. Figure 10. provides a sample of the released LMT survey items with their associated construct codes. The common missed items for the control group in the posttest were the same items as on the pretest with the exception of one additional item. These results were not surprising since the control group did not participate in the intervention.

The most incorrect items were coded SCK -EL and are all division and multiplication of fractions type items. This is not a surprising outcome, but is consistent with prior research on the division and multiplication of fractions. As mentioned earlier under fraction studies in the review of literature, Ball, 1990; Fischbein et al.,1985; Graeber & Tirosh, 1989, Ma, 1999) all focused on the complexity of multiplication and division of fraction in their studies on developing teachers' math knowledge. The above-mentioned researchers agree that one reason for the complexity in understanding this concept is that the two concepts of fractions and division and fractions and multiplication intersect and that teachers never learned these topics conceptually. Ball, Lubienski, and Mewborn, (2001), agreed that the mathematics that is required for teachers to know even at the elementary level is more complex than what was originally considered.

Olanoff (2011), in a study on multiplication and division of fractions, analyzed three math tasks and the knowledge required by them. The study's results revealed that the components of the framework needed for teaching teacher's division of fractions were understanding multiple representations of fractions multiplication and division, and how these representations relate to each other and to whole number ideas and to algorithms. These three components needed to understand division of fractions listed by Olanoff are consistent with the curriculum design and content of the intervention in this study and the NCTM process standards. The online MAPPs professional development has proven to be effective in enhancing teachers content knowledge in fractions, decimals, and percentages. The qualitative data analysis results will provide details on the participants learning experiences within the intervention.

Qualitative Data Analysis and Results

To analyze the response to the second research question, the reflection, discussion posts, and the interviews were qualitatively and quantitatively analyzed for development of content knowledge for teaching, evidence of participants' understanding, and other factors related to participants experiences in the online PD. All discussion posts and reflections were captured in the online platform. Seven interviews were conducted and audio recorded. All interviews were transcribed, and the data grouped with the other qualitative data. Discussion assignment posts, interview data, and reflections were analyzed together using open coding. The list of open codes revealed by the data at level one analyses were compressed and clustered into themes and then further compressed into primary and secondary codes at level two. Once all data was coded, cross - case analysis was conducted to establish the final list of codes and frequencies as a third level of analyses (Coffey & Atkinson, 1996). The frequencies were calculated quantitatively in Table 4.

Table 4

Aggregate Results from Qualitative Data

Code	Freq
Primary:	
Math tools- abstract to concrete	51
Knowledge of Factions (CK)	26
Rigor	6
Enjoyment of/Valuing Learning	7
New approaches /PCK	18
Learning Community	6
Selection of strategies	10
Secondary:	
Challenge thinking and reasoning	4
Discussion	7
Problem solving and process standards	4

The data analyses results are discussed in response to research question two: How do the use of manipulatives, videos and discussion boards enhance content knowledge in fractions, decimals, and percentages? As part of the process to measure impact from the FDP intervention, I looked at quantitative coding tallies (see Table 4). Top codes are related to math tools, knowledge of fractions, new approaches, and strategies that attributed to improved domains of content knowledge and knowledge of content and teaching (KCT). I then looked at the related data with a qualitative lens to analyze how these constructs may have prompted improved content knowledge (CK).

The data was first coded for evidence of use of manipulatives, videos and discussion boards and their impact on participants experiences within the online intervention. There were fifty-one instances of use of manipulatives, no mention of the use of videos and seven codes for the discussion boards. The codes for the manipulatives were compressed into a theme, use of math tools. The theme, use of math tools includes any type of learning aid used from within the intervention in the learning fractions, decimals, and percentages content. The 51 total instances of the use of manipulatives were linked to either new uses, gains in pedagogical knowledge, and or gains in math content knowledge for either the participants or their students.

Use of math tools. In this section, I quote some instances from the data that support the codes listed in table 6 under the use of math tools. These are quotes from the interviews, discussion posts and reflections. This first quote is from a participant when questioned on the use of manipulatives in an interview.

Interviewer: I know you said that you use lots of manipulatives already, but were any of the manipulatives new to you in the online MAPPs PD?

Participant C: Two. Tangrams. I had never heard of tangrams until we did the activity and also the Cuisenaire(?) rods. I had heard of them but I had never used them until we did something…we were measuring distances and we were comparing them to each other, the different colors. You know like the rods are like ten different colors and they go in order.

Interviewer: You are correct they are called Cuisenaire rods.

This quote is evidence of a veteran teacher being exposed to new manipulatives to learn fractions. Even though the participant was familiar with using manipulatives, the intervention content exposed her to new manipulatives that she had not encountered before. Tangrams and Cuisenaire rods have been in use for several years in teaching math concepts, but this participant had not been exposed to them. The online intervention provided convenient access and exposure to math tools for this participant and the other participants and helped to improve their pedagogical content knowledge (PCK).

The idea that manipulatives were limited for use among the lower elementary grades seemed to be prevalent in the data responses. This quote is from a participant who expressed that idea when asked about the use of manipulatives.

Participant K: I`ve always used manipulatives when I taught the lower grades in elementary school. I had not used them as much in 4rd grade and 5th grade. I did not really know how to use them with the older grades. I have a much better idea after doing these tasks as most involved manipulatives. I had pretty much just done the computation and not too much with the students with the manipulatives so that was nice to be able to do that differently.

The above quote is evidence of participants being exposed to new uses of the manipulatives through the intervention for higher elementary grade levels.

Gains in pedagogical content knowledge. The use of manipulatives revealed the theme of gains in pedagogical knowledge that emerged from the data. This participant who taught math at the middle school level, indicated that using manipulatives at the middle school level was new to him but found value in the use of manipulatives for at risk students. Below is a quote from the participant when asked about the learning experience in the online professional development on content knowledge.

Interviewer: Did you learn anything new about fractions, decimals or percentages in this professional development?

Participant D: I would not say new but more a different way of presenting the concepts is what I learned. It's mainly helped with the at-risk students about giving me more options and more tools to show them, to put in front of them to make the math make sense to them. Incorporating the standards into learning the concepts and seeing that connection.

This participant expresses the gains in their pedagogical content knowledge (PCK) to help at risk students as a result of the exposure to the manipulatives. The participant also makes the connection between the process standards and the concepts being learned. This enhanced knowledge can be classified as specialized content knowledge in the mathematical knowledge for teaching framework. There were 18 instances specifically referring to gains in PCK

Gains in Content Knowledge. The use of manipulatives and the evidence of the gains in content knowledge and PCK as a result of the use of the manipulatives is interconnected in the data. The instances were frequent enough in the data to be condensed and recognized as two major themes of gains in content knowledge and gains in pedagogical content knowledge. There were 26 codes for gains in content knowledge which also emerged as a major theme. This quote describes how the manipulatives enhanced their understanding of decimals that the participant had previously struggled with.

Interviewer: Can you give me an example from the PD tasks on decimals that you remember working through that have helped your understanding?

Participant A: The base ten blocks manipulatives that we used in those decimals problems really helped me to visualize the value of the decimal and compare it. You see I know base ten blocks, we use those already in first grade, so to use them to help me learn decimals with was different, something I would have never considered before.

Participant A improved her conceptual understanding of decimals with the help of the manipulatives as a visual aid. Another participant commented on a discussion boards post that the manipulatives helped them with understanding fraction equivalency and finding the lowest common denominator.

"This activity was a creative way of learning equivalent fractions with color tiles. Being able to see the equivalent groups with different denominators I another way to find the LCD. I hope to try this activity with my 4th graders."

Not only were there several instances of gains in content knowledge, but the data also revealed instances of the participants being challenged by the content of the PD. This quote embodies the rigor of the content from a participant.

"The most powerful part was when we had to change the blocks and not let them be their traditional hundreds, tens and ones. Moving it to the thousandths, ten thousandths, tenths, etc."

Here is another quote from an interview, where the participant describes the specialized content knowledge that they gained.

Interviewer: Did you learn anything new about fractions, decimals or percentages in this professional development?

Participant B: Oh yes. One of the tangram activities where we were given a tangram with a specific area and then we determined the value of the picture; it was awesome. I liked this task because it connected fractions, decimals, and percentages with area in a problem-solving style. It forces you to reason. It's really a great end of unit project. Seeing all three concepts and how they relate is neat. It's like they should not be separated. They should be taught together or in sequence at least so the connections can be made.

The participant states that the activity forced them reason which indicates the level of rigor with in the curriculum tasks. Evidence of application of the process standards and the connections between the concepts is what Ball describes as horizon content knowledge (HCK) which was added alter to framework on mathematical knowledge for teaching.

Design of Professional Development. Another theme that emerged was the design of the professional development. The data was surveyed for evidence of effectiveness of the design. There were 17 codes for the design of the professional development. The characteristics of effective professional development (PD) framework by Desimone was used a guide in the design of the intervention. One of the characteristics of the design was content focus with a cohesive and engaging design. The codes for content knowledge discussed earlier are evidence of the content focus of the design. A following quote by a participant indicated several results from the data about the design.

Interviewer: Did you use any of the resources available like the manipulatives, videos and discussion boards?

Participant B: Yes, I used all the manipulatives that were associated with the tasks. You almost could not complete the tasks without them. Videos I did not. The discussion boards I did not really follow up on those, I just posted what I was required to post but I did not join in any discussions. I read a few but that was about it and it was good to see similar responses to mine, so at least I was on the same page as some of the other teachers, we were reasoning the same you know, so that was good to see.

The statement on the manipulatives being closely aligned with the tasks is evidence of cohesiveness of the design and the focus on content. This comment also is evidence of engagement with content. Other participants in a previous comment discussed how the concepts built on each to culminate in the connection between fractions, decimals and percentages. The lack of use of videos and the few codes on the discussion boards is evidence that the community engagement aspect of the design could be improved. When asked how the intervention can be improved, participants did request grade specific discussion boards and some requested for the intervention to be accessible beyond the 8 weeks of the PD for access to their learning community and resources. There was evidence of participants at the same school being more engaged and decided to complete the PD together. The math coach decided to dedicate the planning time in the week to participants from the same school to meet and discuss the intervention tasks. Below is a quote from a discussion board post made by a math coach.

> "It's hard to select just one activity to discuss. Talking about operating and learning in the Standards of Mathematical Practices (SMPS)--our PLC truly learned through perseverance, modeling with mathematics, constructing arguments, using math tools, finding patterns and noting how they were playing a part and definitely reasoning abstractly and quantitatively."

This professional learning community (PLC) evidence was anectodical but has proven to be beneficial to those participants. The PLC formed organically as a result of the intervention. This model of engagement will be considered for future interventions and included into the design of online professional development sessions. This idea is consistent with the recommendation from the literature on the need for follow up with the participants during and after professional development workshops (Yoon, 2007). There is also evidence of strong connections to the process standards that emerged as a result of the intervention.

Value and Enjoyment. Lastly, the theme of value and enjoyment also emerged from the data. This quote speaks to the value of the program.

> "We really enjoyed all the activities but the percentage lessons/activities were one of our favorites. Unit 7 and Unit 8 percentage activities sort of tied everything we've learned in the previous units together. We could see how decimals, fractions, and percentages connected/ related to each other. We discovered multiple ways to reaching the answers."

The participant speaks collectively of their group enjoying the activities. Besides the theme of value and enjoyment this is also evidence of the community of learning. Evidence of enhanced content knowledge and connections to the process standards also exist within this quote. The discovery of new approaches for solving to incorporate multiple representations is evidence of the interactive nature of the curriculum. She also mentions that her group saw the connections between fractions, decimals, and percentages and how each module built on each concept idea and tied all concepts together in the end. It is assumed that teachers already possess this knowledge. Ball considers this type of knowledge as horizon content knowledge (HCK) which teachers should possess for mathematical knowledge for teaching (MKT). Here is another quote from a discussion board post on value and enjoyment of the intervention.

> We really feel like going through this program will help us in many ways when it comes to FDP: 1) Be more confident in our level of comfort with FDP. 2) Taking these standards and being able to teach and reinforce with more of a concrete and representational way. 3) And breaking the mold on manipulatives. Who says that manipulatives have to just stand for what it traditionally means (a whole, a half, etc.) ...rock their world and blow their mind by changing the value and talking about rigor kicking in.

There are many major themes in this quote, but what stands out is the use of manipulatives and the community of learning that participant references. The participant mentions learning from other participants. There are instances of the link to the standards of math practice and multiple ways of solving challenging tasks in her experience with the intervention. There is evidence of value in increasing their comfort level with fractions, decimals and percentages and also considering advanced activities to for their students as a result of their participation in the intervention.

The activities in the intervention integrated the use of manipulatives in the tasks intentionally as mentioned on previous interactions with participants. I discovered that many of the participants owned math manipulatives but did not know how to use them to solve various types of math problems. The tasks in the intervention thus exposed the teachers to the use of the manipulatives as learners. The direct quotes have referenced the value of the manipulatives. An added benefit is that teachers would hopefully start to use the manipulatives with their students as a result of their learning experience.

Another discussion board quote from a participant who indicates evidence of reflective learning.

> "It involves using the pattern blocks to compare fractions. Again, for the visual learner and hands on student, this manipulative is great to build understanding. By using these manipulatives, students are also being introduced to geometric concepts. How could this activity be extended to challenge the students thinking and make connections?"

This quote shows evidence of use of manipulatives and enhanced knowledge. There is also evidence of reflective learning as the participants thinks about their students learning from their learning experience, and they also pose a question about extensions of the activity as they reflect.

The qualitative data was instrumental in capturing the experiences of the participants that the quantitative data was not able to capture. The evidence from the data has provided insight into what the participants valued most from the intervention. The evidence was varied and widespread enough to capture several major themes as discussed earlier. There was no evidence from the data that the participants classified as veteran educators where more advanced in the knowledge than the novice educators. There was evidence that participants with more advanced degrees did exhibit stronger specialized content knowledge by the end of the intervention compared to the participants without advanced degrees. The qualitative results provided information not only to find answers to the second research question, but also to make connections consistent with the frameworks that informed the entire study.

Chapter 4 Summary

The research questions in this study were used to determine if the online professional development in fractions, decimals, and percentages had an impact on the content knowledge (CK) of the participants and how their content knowledge was enhanced. A t- test analysis results showed that there was a significant improvement in the post test scores for the treatment group. The evidence from the qualitative data showed that the participants content knowledge in fractions, decimals and percentages was enhanced from their participation in the intervention. The major themes that were evident through the data are: gains in content knowledge, gains in pedagogical content knowledge, active learning, coherence, and collective participation. All of these themes align with the core features of the framework for effective professional development proposed by Desimone (2015). The gains in content knowledge is further evidence of the fact that assumptions are made that teachers are already aware of this content when in fact some teachers still require this professional development. This evidence also confirms findings in the review of literature for the continued need for content knowledge development in mathematics for teachers. Chapter Five contains a discussion of the results, limitations of the study, and recommendations for future research.

CHAPTER 5:

DISCUSSION

This study aimed to explore the impact of online professional development on fractions, decimals, and percentages content knowledge for teachers. The study enquiry had three main focus points: 1) developing content knowledge for teachers in fractions, decimals, and percentages, 2) exploring and implementing effective professional development strategies, and 3) examining online professional development.

In Chapter One, the study drew attention to the overarching issue of poor math performance in the United States and how the U.S. compared unfavorably with other developed nations around the world. The issue of poor math performance can be attributed to numerous factors, but the study sought to bring awareness to one factor: the need to improve elementary and middle school teachers' content knowledge in mathematics, particularly in fractions, decimals and percentages. The study built a case for developing teachers' content knowledge through effective professional development practices in an online platform.

The review of related literature examined teachers' knowledge development frameworks and how teachers developed mathematical knowledge. Literature on fraction knowledge development studies for teachers for three decades was explored to understand the emphasis of previous studies and recommendations for future studies. Lastly, the review also explored effective professional development strategies to inform the design of this study.

Quantitative and qualitative data was collected to find responses to the research questions discussed in the design research chapter. The results of the data analysis were presented in

Chapter Four. This chapter discusses the results, implications, limitations, and recommendations for future research.

Summary of Results

Research Question 1. To examine the impact of content knowledge in an online professional development in fractions, decimals, and percentages, a pre and posttest survey was administered to test the knowledge growth of the participants. A domain specific measure from the learning mathematics for teaching project (LMT) was used as the survey instrument. Results showed a significant improvement for the scores of the treatment group but not the control group.

Research Question 2. To examine how the online intervention and support resources helped participants enhance their content knowledge in fractions, decimals, and percentages, qualitative data from discussion board posts, reflections, and interviews were analyzed to capture the participants experiences. The data was coded for frequency of major themes. The results showed evidence of enhanced use of manipulatives and improved content knowledge, active learning, coherence, reflective practice and collective participation. The results were consistent with the recommended frameworks for effective professional development strategies discussed in the review of related literature.

Discussion of Results

Research over the past few decades has focused on teacher knowledge and its effect on student success. At the forefront has been teachers' subject matter knowledge also known as content knowledge and pedagogical content knowledge because it has shown to affect teachers' instructional practice and student learning in mathematics (Hill, Rowan and Ball, 2005). After reviewing the related literature and fraction knowledge, there is consensus for developing teachers mathematical content knowledge and effective online professional development (PD). Recent studies and reports still show overall math performance as needing improvement and for teacher content knowledge to be improved.

After examining past research on developing teachers' content knowledge (CK), this study investigated developing teachers' content knowledge specifically in fractions, decimals, and percentages in an eight-weeks' intervention for in-service teachers and para professionals. The teachers' content knowledge development was assessed by knowledge tests, discussion posts' and reflection exercises in an online professional development environment.

Ball (2008) advocated for content knowledge (CK) and pedagogical content knowledge (PCK) to be developed together. Shulman (1986) regarded CK as prerequisite to PCK. Common content knowledge is at different levels for educators based on the background and preparation level of the teacher (Grossman, 1990). Teachers gain their knowledge for teaching from various sources. Grossman lists the sources of teachers' subject matter knowledge as the teachers own k -12 learning experience, pre-service education and professional development, and their teaching experiences. Formal and non-formal learning opportunities contribute to the development of CK and PCK, and that teaching experience alone is insufficient. Measurement of CK across different population groups showed big differences even within the sample population. The sample in this study was comprised of para professionals and certified teachers. Some teachers had specialist degrees. The varying backgrounds in CK was evident in test scores and the responses to activities within the intervention.

The eight-weeks' intervention comprised of basic fraction concepts initially, then progressed to decimal concepts, and lastly the integration of percentages. The progression each week challenged thinking from continuous wholes to discreet wholes. Manipulatives were carefully chosen to help deepen the understanding on complex concepts. Fractions have traditionally been taught algorithmically with no deep understanding. Previous studies and current standardized testing still reveal the lack of understanding in fractions, decimals, and percentages. Algorithmic methods are still being used to teach fractions and are still preferred. Bulgar suggests that algorithms will not make sense to even competent students. The conceptual challenges prove that algorithms do not necessarily build understanding. Yeatts (1991, p. 7) suggested that "manipulatives assist students in bridging the gap from their own concrete sensory environment to the more abstract levels of mathematics." In this study the participants were the students and responded positively to the use of manipulatives. The participants expressed in their reflections the difference the manipulatives made on aiding their understanding from abstract to concrete. Some participants expressed that exposure to the use manipulatives was a positive experience that they would like to incorporate in their classrooms in the future, particularly in teaching fractions concepts.

Content Knowledge. The results showed overall improvement in content knowledge (CK) for the treatment group but not the control group. Further review of the most common items scored incorrect on the test results, matched to construct codes, CCK – EL common content knowledge for elementary school teachers, CK- MS content knowledge for middle school teachers, and SCK – EL specialized content knowledge for elementary school teachers. The missed item codes on CCK represented both the treatment and control groups. These results reinforce the need to develop teachers' content knowledge that still exists. Evidence of common content knowledge codes confirms the issue of varying levels of math instruction during the formal school years and the knowledge gaps that are evident in teachers' knowledge test scores. The focus on the issue of poor math performance has resulted in lots of research around math education. While many researchers have made advancements in the field of mathematics education over the past two decades, there is still a glaring need to keep addressing the issue, especially for teachers' content knowledge development. The scoring of incorrect items revealed lacking conceptual understanding in place value of decimals, as well as multiplication and division of fractions and ratio. Years of experience did not appear to have any impact on the outcomes of the test scores. Participants with advanced degrees did show a slightly deeper knowledge level, but they still regarded some of the intervention knowledge as new with regard to their existing knowledge on fractions, decimals, and percentages. The value of standards-based learning was more evident in the reflections among participants with graduate degrees than the other participants.

Effective Online Professional Development. This study also explored effective characteristics of online professional development (PD) and incorporated the features of effective PD as much as possible. Effective professional development has been deemed a necessary strategy to improve teacher quality. The Wenglinsky (2000) report documented the positive effects of professional development. The report provided evidence that students whose teachers had received professional development in content outperformed their peers on mathematics assessments by more than a full grade level. Moreover, students whose teachers had received professional development in higher order thinking skills out performed their peers on mathematics assessments by 40 % of a grade level. Further research indicates that teachers who receive substantial professional development, approximately 49 hours per year, can boost their students' academic development by approximately 21 percent. Despite the effort and guidelines to improve teacher quality, the Wenglingsky report (2000), indicates that few teachers receive content focused professional development for a sufficient duration. Although almost all elementary teachers reported participating in content focused PD, only 8% received more than 24 hours of PD in math. Consistent with the Wenglinsky report, the teachers in this study received more than 24 hours of PD in fractions, decimals, and percentages.

Professional development is most effective if characterized by focus on content knowledge, active learning, coherence with other learning activities, sufficient duration, collective participation, and a reformed rather than traditional approach (Desimone, 2015). Evidence from the qualitative data confirm that the intervention adequately included these characteristics in its design. The unavailability of flexible PD that meets teachers' needs is a big obstacle faced by teachers. Online PD has been championed as the anytime, anywhere option. Due to the busy schedules of teachers' online PD also provides access as well as ongoing support to vetted resources that might not otherwise be affordable or available. Online PD also allows for connection between teachers from different schools, thereby fostering an online community. Ultimately online PD provides the potential means of meeting ambitious goals of NCLB regarding teacher quality, whereby a large number of teachers can be provided with quality ongoing professional development in mathematical content at a given time. As mentioned earlier, the study results showed evidence of effective professional development strategies having an impact on enhancing content knowledge.

When comparing this online offering of the Math and Parent Partnership (MAPPs) curriculum as a professional development to the a past study of a face to face offering of MAPPs in Arizona, the results are typical for teachers (Bernier, Allesxsaht-Snider, and Civil, 2003). There are lots of similarities in the qualitative data results between the online version and the face to face version of MAPPs. The standards based curriculum encourages engagement and active learning and the impact of enhanced content knowledge is evident for both the online group and the face to face offerings.

The Limitations of the Study

Mathematical knowledge for teaching (MKT) is broad and comprises several related constructs. Attempting to single out content knowledge (CK) from the MKT framework, and the need to develop CK only in this study, has been a challenge. The existing literature has focused more on the development of pedagogical content knowledge together with content knowledge. This study sought to bring attention to the issue of poor content knowledge particularly in fractions, decimals, and percentages and how it needed to be addressed at the educator level. This study did not measure if the teacher's knowledge gains impacted their student understanding in fractions, decimals, and percentages. The scope of this study and the time constraints limited the number of research questions that could be addressed in a manageable time frame.

Another limitation of the study was the lack follow up with participants to encourage continued engagement in the online learning community after the intervention had ended. Follow up and continued engagement is recommended by the research literature for effective professional development. A revised intervention should include the continued engagement to encourage communities of learning amongst educators. Teachers need to have an ongoing learning community for support purposes and continued learning. The study intervention did not plan for opportunity to observe and encourage the ongoing practice of using reflection in self evaluations and lesson planning. The anecdotal evidence of an unplanned professional learning community forming at a particular school with several participants from the intervention seemed to work very well for engagement. While the study was limited in planning for this occurrence, it worked well, and the idea of inviting participants from the same schools to participate in future online professional development offerings is a model to consider in the design of the intervention. Another limitation was the intervention worked well with a cohort to foster engagement in discussion boards and among participants. Ideally if the intervention design could work without the cohort restrictions, I think that more teachers would participate with a continuous enrollment design.

Results and Implications of the Results for Practice

Consistent with the models of effective PD (Yoon, 2007) this study was based on the theory that online professional development (PD) will increase teachers' content knowledge (CK), increased CK will lead to improved knowledge for instruction and handling of student's misconceptions, improved knowledge will result in improved students' achievement. The results show improved content knowledge as a result of the intervention for both the paraprofessionals and the veteran in-service teachers with advanced degrees. This means that CK for in-service teachers can still be developed and improved. Strong CK allows teachers to understand the misconceptions that their learners may have about the content. Mohr's statement that, "the connection between staff and learning and student learning is direct and intense and can't be overestimated. And in order for teachers to be able to offer learning opportunities to kids they need to experience it themselves first" (2004, p.5). Because the study participants engaged in and with mathematics it has been a dialogic process the participants have been able to reflect on their pedagogy concerning what mathematical thinking is and the construction of mathematical knowledge. A closer look at the missed items in the survey revealed items that were coded as common content knowledge (CCK) in the content knowledge construct. CCK is described as the common knowledge in math that all adults should possess, not just teachers. The missed items coded as CCK, draw possible connections to poor performance on the basic arithmetic items on the teacher certification exams. The value of engaging in standards-based learning and knowing how concepts are related and build upon each helped some of the participants score more correct items coded as specialized content knowledge (SCK) in the post test.

Recommendations for Future Research

The analysis process raised additional questions that could serve as opportunities for future research. While the Mathematical Knowledge for Teaching (MKT) framework is comprehensive, it may not have been ideally suited to focus on content knowledge development as a stand-alone variable due to interconnectedness of the other items in the framework. The questions raised as a result of the analysis were:

1. Can content knowledge (CK) be developed as a stand-alone framework to provide in depth conceptual knowledge for pre-service teachers? Math education programs have made major advancements in pre-service preparation, but many programs offer more methods courses than conceptual courses.
2. How does the MKT framework incorporate planning and reflection into knowledge development for teachers? Responses from this enquiry could form the basis of future professional development emphasis.

The results of this study add to the literature on developing content knowledge for teachers by confirming the need still exists to develop content knowledge of teachers particularly in fractions, decimals, and percentages. The participants in this study had a wide range of credentials and teaching experience, yet still displayed evidence at all levels of improving their conceptual knowledge on fractions, decimals, and percentages through the intervention. The design of the study incorporated many of key features recommended for an effective online professional development environment to help foster reflection learning skills for educators. While the data suggests that the intervention was mostly successful in attaining its objective in the design, the results also indicate the need to improve the intervention design to incorporate and foster engagement in the learning community beyond the tasks of the intervention.

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APPENDICES

APPENDIX A: APPROVED IRB

Check One
New Application: ☒
Resubmission*: ☐ **Revision** ☐ *(All changes must be highlighted)*

Human Subjects Office
University of Georgia
612 Boyd GSRC
Athens, GA 30602-7411
(706) 542-3199

**NOTE: A new application is required every five years.*

IRB APPLICATION

MAIL 2 COPIES OF APPLICATION TO ABOVE ADDRESS

(Check One) **Dr.** ☐ **Mr.** ☐ **Ms.** ☒
(Check One) **Faculty** ☐ **Undergraduate** ☐ **Graduate** ☒

Racheal Landers — 8101045251
Principal Investigator — **UGA ID – last 10 digits only**

Learning, Design & Technology, Rivers Crossing
Department, Building and + Four
(Include department even if living off campus or out of town)

1475 Gloria Street, Griffin Ga 30224
Mailing Address (if you prefer not to receive mail in dept.)

404-694-7749
rlanders@uga.edu
Phone Number (s) — **E-Mail (REQUIRED)**

****Signature of Principal Investigator**

(Check One) **Dr.** ☐ **Mr.** ☐ **Ms.** ☐
(Check One) **Faculty** ☐ **Undergraduate** ☐ **Graduate** ☐

Co-Investigator — **UGA ID – last 10 digits**

Department, Building and + Four

Mailing Address (if you prefer not to receive mail in dept.)

Phone Number (s) — **E-Mail**

Signature of Co-Investigator *(use additional cover sheets for more than one Co-Investigator)*

UGA Faculty Advisor: Dr. Robert Branch (**Name**) — CIS, Rivers Crossing (**Department, Bldg+ Four**) — Rbranch@Uga.Edu (**E-Mail (REQUIRED)**) — 706-255-1542 (**Phone No.**)

****Signature:** ______ **Date:** ______ **UGA ID – last 10 digits only** ______

*****Your signature indicates that you have read the human subjects guidelines and accept responsibility for the research described in this application.***

If funded:
******Sponsored Programs Proposal#** ______ **Name of Funding Agency** ______
******By listing a proposal number, you agree that this application matches the grant application and that you have disclosed all financial conflicts of interest (see Q6a)***

TITLE OF RESEARCH: Developing Mathematical Content Knowledge in Fractions, Decimals and Percentages for Elementary Educators

NOTE: SUBMIT 4-6 WEEKS PRIOR TO YOUR START DATE

APPROVAL IS GRANTED ONLY FOR 1 YEAR AT A TIME

CHECK ALL THAT APPLY:

Investigational New Drug ☐ **Exceptions to/waivers of Federal regulations** ☐
If yes to the above, provide details:

Data Sets ☒ **Existing Bodily Fluids/Tissues** ☐ **RP Pool** ☐ **Deception** ☐
Illegal Activities ☐ **Minors** ☐ **Moderate Exercise** ☐ **Audio/ Video taping** ☒
MRI/EEG/ECG/NIRS/Ultrasound/ Blood Draw ☐ **X-RAY/DEXA** ☐ **Pregnant Women/Prisoners** ☐

IRB.PF (www) 3/07 Page number 1

IRB Application-Disserta

APPENDIX B: CONSENT LETTER

January 2017

Dear Participant:

I am a graduate student under the direction of Dr. Branch in the Department of Learning Design and Technology at The University of Georgia. I invite you to participate in a research study entitled Developing Content Knowledge in Fractions, Decimals and Percentages. The purpose of this study is to develop content knowledge in Fractions. Decimals and Percentages in an online offering. As the researcher I will need to look at your responses to the surveys in order to determine in changes in content knowledge.

Your participation will involve coursework in an asynchronous online program and should only take approximately 16 hours total for eight sessions. Your involvement in the study is voluntary, and you may choose not to participate or to stop at any time without penalty or loss of benefits to which you are otherwise entitled. At the beginning and the end of the 16-hour online intervention, you will be asked to complete a short survey about knowledge and attitudes towards mathematics. You also may be asked to participate in a short audio - recorded interview about content on fractions. Your online coursework data may be used. All data including audio tapes will be kept confidential and any identifiers removed. Data will be kept in a locked filing cabinet and restricted to the researcher. After the data analysis is complete all codes will be destroyed and your identity will be protected at all times. Your identity will not be used in written reports. If you decide to stop or withdraw from the study, the data collected from you or about you up to the point of your withdrawal will be kept as part of the study and may continue to be analyzed. The results of the research study may be published, but your name or any identifying information will not be used. In fact, the published results will be presented in summary form only.

The findings from this project may provide information on improved content knowledge. There are no known risks or discomforts associated with this research. You may omit any questions in the survey and interviews you are not comfortable responding to. The results of the research study may be published, but your name or any identifying information will not be used. In fact, the published results will be presented in summary form only. If you would like the results of the study or have any questions during the study, please contact: Racheal Landers, Learning Design and Technology – UGA, 221 Rivers Crossing. 850 College Station Road, Athens, GA 30602 or email me rlanders@uga.edu

Questions or concerns about your rights as a research participant should be directed to The Chairperson, University of Georgia Institutional Review Board, 609 Boyd GSRC, Athens, Georgia 30602; telephone (706) 542-3199; email address irb@uga.edu.

By completing and returning this questionnaire in the envelope provided, you are agreeing to participate in the above described research project.

Thank you for your consideration! Please keep this letter for your records.

Sincerely,

Racheal Landers

APPENDIX C: VOLUNTEER FORM

Sample Volunteer Recruitment Survey

Name: ____________________________________

Email: ____________________________________ Contact #: ____________________________

Current Employer: ____

# of years teaching experience and grade levels: ____

____

List the subjects that you teach: ____

____

Teaching Certificate: __________Yes __________No __________Eligible

Subjects / Areas certified to teach: ____

College Education / Degrees:

____

____

If you answered no to teaching certificate, are you a Paraprofessional ______Yes _______No

Years of Experience: ______________________________

Are you required to help students with math? ________Yes _______________ No

# of College Credit Hours: __________________

Do you plan to become a certified educator? _________Yes ___________No

Attitudes towards math: Check the one that describes you

________I enjoy math ________ I find math challenging _________ I dislike math

APPENDIX D: INSTRUMENT

Testing Instrument – Page 1

Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of LMT. Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

lmt
learning mathematics for teaching project

Learning Mathematics for Teaching
University of Michigan
School of Education
610 E. University #1600
Ann Arbor, MI 48109-1259

Testing Instrument – Page 2

of base ten blocks available to her:

When teaching place value with whole numbers, the use of the blocks seems simple. But for decimals, it seems more complex, and she asks Mrs. Carroll next door what she thinks the values of the blocks should be. How should Mrs. Carroll reply? (Mark ONE answer.)

a) Ones "cubes" become wholes; tens "rods" become tenths; hundreds "flats" become hundredths.

b) Hundreds "flats" become wholes; tens "rods" become tenths, and ones "cubes" become hundredths.

c) Either use of the blocks will work.

d) Neither use of the blocks will work.

e) I'm not sure.

Testing Instrument Page 3

accept as a correct explanation? (Mark ONE answer.)

a) $\frac{5}{9}$ is greater than $\frac{3}{7}$ because 5 is greater than 3.

b) They are equal because each is missing four pieces from the whole.

c) They are equal because adding two to the numerator in $\frac{3}{7}$ and two to the denominator in $\frac{3}{7}$ produces $\frac{5}{9}$.

d) $\frac{3}{7}$ is greater because the pieces are bigger.

e) $\frac{5}{9}$ is greater because it is more than one-half, while $\frac{3}{7}$ is less than one-half.

f) I'm not sure.

3. Mr. Hosko was wondering what it meant to say that division by 0 is *undefined*. He asked his colleague, Mrs. King, what she thought. Which of the following best explains this? (Mark ONE answer.)

a) Division by 0 is undefined because you cannot do it.

b) Division by 0 is undefined because you cannot make 0 groups of something.

c) Division by 0 is undefined in school curricula because college-level mathematics is needed to do this calculation.

d) Division by 0 is undefined because there is no single answer that when multiplied by the divisor 0 gives the original number.

e) Division by 0 is undefined because every number divided by 0 equals 0.

f) I'm not sure.

Testing Instrument Page 4

fractions by dividing the numerator and the denominator by a common factor. One of her students asks, "Why doesn't the fraction's value get smaller when we divide the numerator and the denominator by the same number?" Below are responses to the question from other students in this class.

Although none of these is a complete explanation, which one provides the **best** evidence that the student understands **why** reducing a fraction produces an equivalent fraction? (Mark ONE answer.)

a) This works because you divide the top and bottom by the same number, so the new fraction has to be the same amount.

b) This works because you are really just dividing the fraction by 1, so the new fraction is the same amount.

c) This works because you are making the numerator and denominator smaller by the same amount.

d) This works because, for example, $\frac{3}{4}$ is the same amount as $\frac{12}{16}$, only with smaller numbers.

e) I'm not sure.

Testing Instrument – Page 5

$$\begin{array}{r} 0.23 \\ \times\ \ 95 \\ \hline \end{array}$$

First I ignore the decimal point and do the multiplication, which gives me 2185. Then I use estimation to place the decimal point. I know that 0.23 is about 1/4 and 95 is about 100 and 1/4 of 100 is 25, so my answer would be 21.85.

Which of the following is most appropriate to say about Nathaniel's approach? (Mark ONE answer.)

a) It happens to work in this case, but will not work for most problems.

b) It only works if one of the numbers is a whole number.

c) It works for any numbers, but some examples are harder to estimate.

d) It works equally well for all problems.

e) I'm not sure.

Testing Instrument – Page 6

Begin with your age. Multiply by 4, add 20, divide by 2, and subtract 10. Describe the number you get.

Her students agreed that the number was "2 times your age." But one student wanted to know why this was true. The students agreed that multiplying by 4 and then dividing by 2 was the same as multiplying by 2, but wanted to know why the addition and subtraction cancelled one another out. Mrs. Earl wanted to let students come to the answer on their own. What's the most important thing for students to realize? (Mark ONE answer.)

a) Both the 20 and the 4 are divided by 2.

b) Multiplication comes before addition in the order of operations.

c) Addition and subtraction are inverse operations.

d) All of the above.

e) I'm not sure.

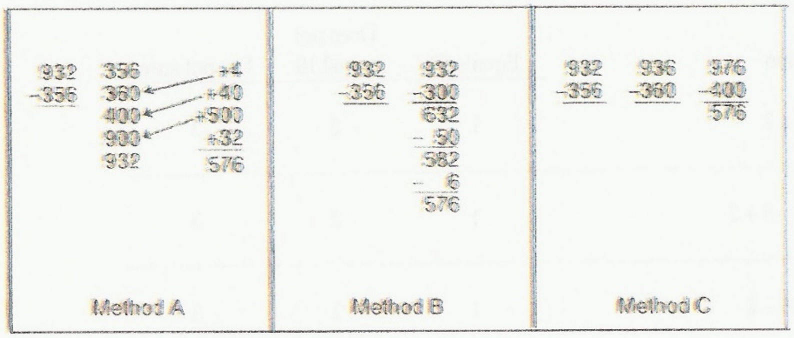
Testing Instrument – Page 7

mathematical ideas or procedures. Sometimes, however, these handy memory devices are not actually true, or they are not true for all numbers. For each of the following, decide whether it is true all of the time or not. (Mark TRUE FOR ALL NUMBERS, NOT ALWAYS TRUE, or I'M NOT SURE.)

	True for all numbers	Not always true	I'm not sure
a) You cannot subtract a larger number from a smaller number.	1	2	3
b) You can't subtract a number from 0.	1	2	3
c) If the first of two numbers is smaller than a second, and you add the same number to both, then the first sum is smaller than the second.	1	2	3
d) Dividing a number makes it smaller.	1	2	3

Testing Instrument – Page

your students' papers, you notice that some have displayed their work in the following ways:



Which of these students is using a method that could be used to subtract any two whole numbers? (Mark ONE answer.)

a) A only

b) B only

c) A and B

d) B and C

e) A, B, and C

f) I'm not sure.

Testing Instrument – Page 10

answer of 10. After reviewing her students' work, she was pleased that many of her students used more than one operation in their expressions. However, she was concerned that their notation was not mathematically correct. Determine if each expression, as it is written below, equals 10.

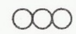
For each expression, mark EQUALS 10, DOES NOT EQUAL 10, or I'M NOT SURE.

Expression	Equals 10	Does not equal 10	I'm not sure
a) 2 + 3 x 2	1	2	3
b) 8 + 9 – 5 + 2	1	2	3
c) -3 + 15 – 2	1	2	3
d) 80 ÷ 4 x 2	1	2	3
e) $-3^2 + 1$	1	2	3
f) 6 + 2 x 2	1	2	3

Testing Instrument Page 11

many circles she should draw to start, and to her surprise her students made different proposals.

Asa: I would draw <u>four</u> circles because the denominator tells you what the whole is.



José: I was thinking that fractions mean divide, and <u>three</u> circles is the whole thing. I would start with three circles, then divide them up.

Mina: I would draw <u>one</u> circle. One is the whole, and you break the whole up into four parts.

Ms. Kelly had planned to draw one circle, but now she was unsure. Which of these students is using a correct interpretation of fractions?

a) Only Asa.

b) Only José.

c) Only Mina.

d) Both Asa and Mina, but not José.

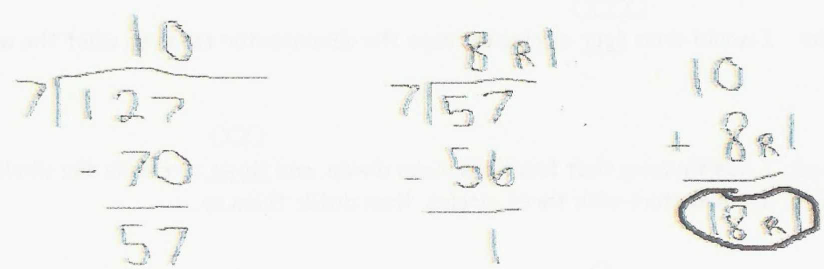
e) Asa, Jose, and Mina.

f) I'm not sure.

Testing Instrument Page 12

the following non-standard approach to divide 127 by 7:

127 divided by 7



What is true about Chad's approach?

a) His approach is not mathematically valid; it is a coincidence that his answer is correct.

b) His approach is not mathematically valid because he subtracted 70 from 127 instead of subtracting 7 from 12.

c) His approach is mathematically valid, but could be inefficient with large dividends.

d) His approach is mathematically valid, but only works with single-digit divisors.

e) I'm not sure.

Testing Instrument Page 13

the following problem(s) could she use as a word problem for $\frac{1}{2} - \frac{1}{3}$? (Mark YES, NO, or I'M NOT SURE for each one.)

	Yes	No	I'm not sure
a) I have $\frac{1}{2}$ of a pizza left. My brother comes in and eats $\frac{1}{3}$ of my leftover pizza. How much pizza is left?	1	2	3
b) Farmer Brown has plowed up $\frac{1}{2}$ of a field. He wants to plant $\frac{1}{3}$ of that half in corn. What fraction of the entire field will be planted in corn?	1	2	3
c) Mom has $\frac{1}{2}$ of a cup of sugar. She needs to use $\frac{1}{3}$ of a cup of sugar to make some brownies. How much sugar will Mom have left?	1	2	3
d) Jon has $\frac{1}{3}$ of a foot of licorice. Francisco has $\frac{1}{2}$ a foot of licorice. How much more licorice does Francisco have than Jon?	1	2	3

Testing Instrument Page 14

for subtraction (pictured below), and he wondered whether it would always work. He showed it to Ms. Braun, next door, and asked her what she thought.

$$\begin{array}{r} 37 \\ -19 \\ \hline -2 \\ 20 \\ \hline 18 \end{array}$$

How do you think Ms. Braun should respond? (Mark ONE answer.)

a) She should tell Mr. Lewis the procedure works for this problem but would not work for all numbers.

b) She should tell him this does not make sense mathematically.

c) She should let Mr. Lewis know that this would work for all numbers.

d) She should say that this procedure only works in special cases.

e) I'm not sure.

14. Mr. Lewis asked his students to divide $\frac{6}{8}$ by $\frac{1}{2}$. Charlie said, "I have an easy method, Mr. Lewis. I just divide numerators and denominators. I get $\frac{6}{4}$, which is correct." Mr. Lewis was not surprised by this as he had seen students do this before. What did he know? (Mark ONE answer.)

a) He knew that Charlie's <u>method</u> was wrong, even though he happened to get the right <u>answer</u> for this problem.

b) He knew that Charlie's <u>answer</u> was actually wrong.

c) He knew that Charlie's <u>method</u> was right, but that for many numbers this would produce a messy <u>answer</u>.

d) He knew that Charlie's <u>method</u> only works for some fractions.

e) I'm not sure.

APPENDIX E: Sample Intervention Activity

Fraction Problems I

Solve using color tiles, other objects, or a picture.

1. Carla spent $\frac{2}{5}$ of her allowance on a CD and $\frac{1}{3}$ of her allowance on a new book. What part of her allowance does she have left?

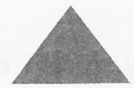
2. A large bag of flour is $\frac{2}{3}$ full. The pastry chef used $\frac{3}{4}$ of this flour to bake a cake. What part of a full bag did she use?

3. Rita has read 360 pages in her book. The book has 570 pages in it. Is she more or less than $\frac{2}{3}$ of the way through her book?

Sample Activity 2

Pattern Block Chart

Complete each row of the chart by naming values for each block. The value of each block must relate to the given value on that row.

			
1			
			1
		1	
	1		
$\frac{1}{2}$			
	$\frac{1}{3}$		
		$\frac{1}{4}$	