THE DESCRIPTION AND APPLICATION OF A NEW MEASURE OF FINANCIAL DECISION-MAKER RISK AVERSION

by

MARTHA N. FULK

(Under the Direction of John E. Grable)

ABSTRACT

The primary research objective of this dissertation was to associate the concept of risk tolerance typically used by financial planners in research and practice to the concept of risk aversion used by economists by introducing a new measure of financial decision-maker risk aversion that can be used in practice to facilitate financial and investment decision-making. This dissertation advances the financial planning, financial risk-aversion, and investment management literature by describing a mathematical transformation procedure that can be used to transform responses to the proposed risk-aversion measure (i.e., certainty equivalent amounts) into coefficients of relative risk aversion (i.e., gamma coefficients) in a way that matches theory related to constant relative risk aversion (CRRA). Based on data collected from 500 adults using a Qualtrics questionnaire, it was determined that the single-item question effectively and successfully combines elements from revealed-preference and propensity measurement techniques in a way that matches traditional CRRA estimation procedures. The new measure was validated using tests of association with existing propensity measures, revealed-preference tests, and other well-established assessments of risk tolerance and risk aversion (e.g., the Survey of

Consumer Finances (SCF) risk-aversion item) that are widely used in financial planning research and practice. Findings from the study confirmed the concurrent and construct validity of both the transformation procedure and the proposed new measure of risk aversion using a combination of correlational and regression tests. Scores from the proposed measure were found to correlate with the other measures of risk aversion, as well as with indicators of risk-taking. The results showed that the new direct measure can be used to identify the appropriate risk-aversion level for a financial decision-maker. Results further showed that resulting gamma coefficients can be used to estimate a financial decision-maker's expected utility—the certainty equivalent for any gamble or financial risk—and overall describe a financial decision-maker's behavior. Findings were also used to illustrate the use of gamma coefficients obtained from the proposed measure in the context of Modern Portfolio Theory.

INDEX WORDS: Risk, Uncertainty, Risk Aversion, Constant Relative Risk Aversion, CRRA, Risk-Aversion Measurements, Propensity Measures, Revealed-preference Tests

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by

MARTHA FULK

B.S., University of Illinois at Urbana-Champaign, 2003

M.S., University of Illinois at Urbana-Champaign, 2004

M.S., Georgia State University, 2010

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

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by

MARTHA FULK

Major Professor:	John E. Grable
Committee:	Pamela R. Turner
	Kenneth J. White

Electronic Version Approved:

Ron Walcott

Vice Provost for Graduate Education and Dean of the Graduate School

The University of Georgia

August 2021

DEDICATED

to

JOY DESTA FULK

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CHAPTER 1

INTRODUCTION

Risk is an unescapable part of everyone's daily life. Risks can be physical, emotional, psychological, financial, or legal. Risks can be real or imaginary or purely hypothetical. Given its importance in the daily lives of individuals and households, risk has been the focus of numerous studies. In fact, the study of risk has a long and diverse history. The study of risk has its roots in the work of Renaissance mathematicians and theorists (Stahel et al., 2017). The study of risk in the context of household finance, personal finance, and financial planning is a much more recent event. Until the 1950s, the concept of risk in the management of securities and portfolios was based on subjective probability estimates. Beginning in the 1950s, risk emerged from the back corners of academia to take a prime spot in all types of models designed to predict human behavior. The modeling of risk was quickly integrated into the domains of economics and finance. Today, the notion of risk is a cornerstone of nearly all household finance and personal finance, particularly in relation to financial decision-making in the domain of financial planning (e.g., insurance management, investment selection, consumer choices, etc.) and portfolio management. Day-to-day household decisions rely on accurately assessing and understanding financial risk and risk aversion.[1] Understanding and utilizing the concept of risk aversion is as

[1] The theoretical opposite of risk aversion is risk tolerance. While this dissertation focuses on the measurement and application of risk aversion, readers whose training is in financial planning or financial counseling can conceptualize the notion of risk aversion as the inverse of risk tolerance. As such, someone who exhibits high risk aversion— i.e., avoids risks—would be said to have a low risk tolerance.

critical as understanding the role of money itself.

In the context of this dissertation, and as defined by The Palgrave Encyclopedia of Strategic Management (Augier & Teece, n.d.), *risk* refers to decision-making situations under which all potential outcomes and their likelihood of occurrences are known to the decision-maker prior to the engagement in a behavior. Risk differs from uncertainty. *Uncertainty* refers to situations under which either the outcomes and/or the probability of occurrence is unknown to the decision-maker. In the domains of personal finance and financial planning, the concept of uncertainty dominates decision-making models. In the context of portfolio management, risk tends to dominate asset allocation modeling. The reason is that very few household financial decisions are made with full knowledge of potential outcomes or known probabilities. Nearly all personal, family, and household financial decisions require a decision-maker to first subjectively assess the probability of gains and losses. It is this subjective evaluation that links risk and uncertainty together, and that also makes obtaining a clear and valid assessment of a household financial decision-maker's unwillingness to take financial risk of critical importance. Uncertainty can arise as the result of the lack of perfect information, the lack of background information, information asymmetry (i.e., incongruent information), or as a result of inexperience. This dissertation focuses on modeling how risk, as a consequence of uncertainty, can be applied in a financial planning setting.

Risk and Financial Planning

Nearly all daily decisions involve various outcomes that are uncertain and unpredictable. Household financial decision-makers are constantly faced with making choices where the decision outcome is both indeterminate and potentially negative. Financial decisions involve anticipating future events, estimating uncertainty, and measuring risk, as well as risk acceptance and avoidance levels (for an individual). As a result, the study of financial risk-taking behaviors, risk aversion, risk analysis, risk measurement, and risk management are all at the core of the financial planning process and financial decision-making for households. In general, understanding what risk is and how household financial decision-makers deal with risk is the first step to effectively managing risk. Risk analysis is applied to household decision-making in a variety of ways, including weighing options about taking on college student loans, a mortgage loan for a house, saving for retirement, purchasing life insurance, buying car insurance, and many other day-to-day life decisions.

Financial planning, which is often conceptualized as a sub-discipline of personal finance (Schuchardt et al., 2007), is typically categorized into six primary areas: (a) household financial management (e.g., cash flow, net worth, financial ratio, and spending plan management), (b) risk management and insurance, (c) investment planning, (d) tax planning, (e) retirement planning, and (f) estate planning. The concept of risk—and thus a household financial decision-maker's risk aversion—plays an integral role in each of the above-stated areas of personal finance and financial planning. For instance, financial planners work with their clients to build emergency savings (i.e., an emergency fund), which ensures the client's family is prepared for unexpected life events with sufficient liquid assets and the ability to meet financial obligations, as well as maintain the family's lifestyle during a crisis. Although this is a simple example of mitigating short-term risk, similar examples can be outlined for each of the content areas of financial planning. It also goes to show the ubiquitous integration and references to risk and risk analysis in financial planning.

For those who are active in the financial planning profession, including a discussion of risk and risk aversion in their conversation with clients is not only a "good-to-do" but a mandated practice by state, federal, and self-regulatory governing bodies. Financial planning professionals must actively seek to determine the risk aversion[2] of their clients before offering investment advice or creating a financial plan for a client (Grable et al., 2019). Accurately measuring financial risk aversion is one of the most essential tasks a financial planner faces when providing advice to a client.

Overall, the reasons stated above clearly indicate that the study of risk and risk-aversion measurements to be fundamental to the study and practice of personal financial planning and related disciplines (e.g., financial counseling, portfolio management, etc.). Moreover, as will be discussed next, many personal finance topics directly, as well as tangentially, relate back to the concept of risk, including risk tolerance, risk aversion, risk prediction, risk management, risk preference, risk perception, personal risk-taking behavior, and risk capacity.

Purpose of Study

The purpose of this dissertation is multifaceted. The first purpose is to illustrate a technique that can be used to transform certainty equivalent amounts—derived from a question designed to elicit a financial decision-maker's unwillingness to take financial risk (i.e., risk aversion)—into gamma coefficients. The terms certainty equivalent amount and gamma coefficient will be defined in detail later in this chapter. The second purpose is to validate resulting gamma coefficient estimates as descriptors of financial decision-maker behavior.

[2] This mandate is sometimes referred to as measuring a current or prospective client's risk preference, risk aptitude, or risk tolerance. The mandate falls under "Know Your Client" (KYC) standards of practice.

At its core, the purpose of the dissertation is to introduce a procedure that can be used to transform certainty equivalent dollar amounts into gamma coefficients, which are inputs into constant relative risk-aversion models. In order to illustrate this procedure, this dissertation proposes a new and direct measure of financial decision-maker risk aversion. The proposed measure prompts a financial decision-maker to indicate their preferred level of investment associated with an investment opportunity. The final aspect of the dissertation involves evaluating the validity of both the transformation procedure and the proposed measure of risk aversion using a combination of correlational and regression tests. Specifically, this dissertation was conceptualized around the following outcomes:

1. To associate the concept of risk tolerance typically used by financial planners in research and practice to the concept of risk aversion used by economists by introducing a new measure of financial decision-maker risk-aversion;

2. To provide a mathematical transformation procedure that can be used to convert dollar certainty equivalent amounts to gamma coefficients—defined as the coefficient of relative risk aversion—in a way that matches theory related to constant relative risk aversion (CRRA);

3. To validate the transformation process and proposed measure of financial decision-maker risk aversion using construct and concurrent validity tests; and

4. To demonstrate the use of gamma coefficients obtained from the proposed measure in the context of Modern Portfolio Theory concepts and applications.

Significance of the Study

This dissertation adds to the existing body of literature by helping advance the work of researchers, financial service practitioners (e.g., financial planners, financial counselors, financial therapists, money management professionals, and others), and household financial decision-makers in relation to measuring risk aversion with more precision and ease. The dissertation also provides a blueprint that can be used to create a customized investment portfolio according to a household financial decision-maker's specific level of risk aversion. In this regard, the study shows those who are interested in the dual concepts of financial risk aversion and portfolio management the process to transform risk-aversion assessment scores into model inputs that correspond to the efficient frontier and an optimal portfolio.

This dissertation advances the practice of financial planning by helping financial service professionals move beyond simple heuristics and professional judgment when developing portfolio allocation decisions. In this regard, and as noted by Hubble et al. (2020b), the majority of those who provide financial and investment advice to others do so without regard to theory. This can, and often does, lead to inappropriate asset allocation recommendations (Hubble et al., 2020a). Doing so may result in problematic goal achievement and significant opportunity costs. The ramifications of misaligning a client's portfolio with the client's level of risk aversion can be devastating. Hence, it is crucial that financial planners, for example, avoid creating a low-risk low-return portfolio for a client who is willing to take on higher risk for the possibility of a better return. While it is difficult to have a conversation with a client when there is the possibility of a significant loss due to a high-risk investment, it can be worse to have a conversation with a client who was either misunderstood about their desire to be in high-risk investments or a client who comes to regret not being more aggressive in their portfolio. Of course, the opposite is also true. Clients who are positioned in portfolios that clearly exceed their level of risk aversion often sell at a loss during periods of market volatility, which places goal achievement out of reach (Hubble et al., 2020b).

Without a theoretical foundation to determine a client's portfolio that aligns with a client's risk aversion, the possibility of misallocation is always present and likely. One outcome of this dissertation is to provide the tools needed to implement a valid and reliable way to measure risk aversion that can be used in accordance with portfolio theory to match a client's risk-aversion level while maximizing expected return. The historical context discussion that is presented later in this chapter will highlight the significance of measuring a financial decision-maker's risk-aversion level accurately, while the applied topics of this dissertation will focus on the importance and the need to measure risk aversion as one of the pillars of a well-designed financial and investment plan. Finally, the results from this dissertation, in future chapters, will also help shift the way financial planners conceptualize recommendation decisions away from professional judgment to a foundation of theoretical clarity and precision when assessing a client's risk-aversion level.

Positioning of Dissertation in the Literature

The academic and commercial marketplace for financial risk-aversion and risk-tolerance tests is large and growing. On the academic side, researchers use a variety of testing procedures to evaluate research respondents' willingness to take risks in surveys and experiments. On the commercial side, numerous firms offer tools, tests, and questionnaires that are purported to help financial advisors uncover the risk aversion of their clients. Much of the commercial growth in risk-aversion test offerings have been directly related to state and federal regulations that mandate such tests.

Although different measures are currently used to assess risk-aversion levels, propensity measures and revealed preference methodologies dominate the marketplace. Researchers and financial service professionals have struggled, and at times have been at odds with each other, over how to most accurately and consistently measure a household financial decision-maker's risk-aversion level. Psychometric tests and scaling tools, which are sometimes referred to as propensity measures or elicitation assessments (Cardak & Martin, 2019), are often used by investors and financial service professionals to evaluate risk aversion.[3] An advantage associated with propensity measurement approaches is that a well-designed measure can account for a financial decision-maker's deeply held feelings of fear, regret, anxiousness, and greed. Propensity measures do a good job of uncovering a financial decision-maker's subjective evaluation of investment outcomes. The following examples, adapted from Grable and Lytton (1999), represent the type of question that is often used in propensity measures:

(1) *When you think of the word "risk," which of the following words comes to mind first?*

a. Loss; b. Uncertainty; c. Opportunity; d. Thrill

(2) *You have just finished saving for a "once-in-a-lifetime" vacation. Three weeks*

[3] A propensity measure, which is sometimes referred to as a self-report or a stated preference assessment, places a decision-maker on a scale ranging from high risk aversion to low risk aversion. While single-item measures are occasionally used, nearly all propensity measures are presented as a questionnaire. Such questionnaires are typically developed using classical test theory procedures, although Rausch modeling and Item Response Theory tests have grown in use over the past decade.

before you plan to leave, you lose your job. You would:

> *a. Cancel the vacation; b. Take a much more modest vacation; c. Go as scheduled, reasoning that you need the time to prepare for a job search; d. Extend your vacation, because this might be your last chance to go first-class.*

A fundamental problem associated with the use of propensity measures is that "scores obtained from questionnaires are difficult to map to portfolio choices in a traditional mean-optimization framework" (Grable et al., 2020, p. 2). This difficulty, coupled with the general lack of mathematical precision associated with the way propensity tests are developed and used, has led many researchers and those who provide advice to investors to opt for assessment tools designed around the concept of the economics definition of risk aversion, or more precisely, the notion of constant relative risk aversion (CRRA). Revealed preference tests represent the primary way CRRA is measured. Some have argued that revealed preference risk-aversion models provide the only rigorous theoretical way to link risk aversion to optimal portfolios (Hanna & Lindamood, 2004). Revealed preference measurement approaches assume investors are rational and they have the requisite cognitive skills needed to make risk and return (i.e., gain and loss) estimations (Grable & Chatterjee, 2016; Kahneman et al., 1991).

As noted above, as a traditional economic concept, CRRA is most often assessed using revealed preference methodologies. A typical CRRA revealed preference test asks decision-makers to choose between two options in which the outcome probabilities are known prior to the decision. As such, a revealed preference test places the assessment in the domain of risk rather than uncertainty. In reality, financial decision-makers and those who provide advice to investors make decisions in environments where probabilities are generally never known in advance with certainty (Bird & Yeung, 2012). Thus, the use of CRRA estimates assumes that risk and uncertainty are equivalent. In other words, it is necessary to assume that objective probability framing is similar to a financial decision-maker's subjective probability estimation. A majority of the CRRA based tools available in the marketplace assume users have a deep familiarity with the principles of economics. As a result, some revealed preference choice scenarios place a high cognitive load on the decision-maker. The types of questions asked to elicit estimates of risk aversion may simply be too complex for the average investor to answer with care and honesty (Guiso & Sodini, 2013). On the flip side, some research related to CRRA concepts over-simplify the concept of risk aversion and fail to provide any mathematical context for the types of questions asked.

It is difficult to find information on the concept of CRRA that is accessible (both mathematically and practically) to those who need to know how this economics tool can be used in practice. In some ways, CRRA is so embedded in economic thought that the actual way in which CRRA is conceptualized, measured, and applied has been lost among non-economists. In fact, very few household financial decision-makers or their advisors understand the connection between CRRA scores and expected financial decision behavior or financial decision making. Consider the typical research project that incorporates CRRA estimates. Most studies apply an assumed "average" CRRA score as an input into financial planning and investment models. Investors and those who provide guidance and advice to investors are then left to ponder what the score represents in a practical sense.[4]

[4] It is not uncommon to find papers referring to a CRRA model input of, say, 2 or 3. What these numbers mean in practice is seldom discussed or explained.

CRRA tests typically employ choice scenarios that require a household financial decision-maker to choose between two options, one with a certain outcome and the other with a probability of success or failure (Barsky et al., 1997). When CRRA measures are applied in a Modern Portfolio Theory (MPT) framework, tradeoffs can be estimated and mapped to a financial decision-maker's utility function. This leads to a direct pathway to portfolio selection. The following example, adapted from Hanna and Lindamood (2004), represents the type of questions often used to derive estimates of CRRA:

> *Suppose that you are about to retire, and have two choices for a pension. Pension A gives you an income equal to your preretirement income. Pension B has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be 20% less than your preretirement income. You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after-tax. Which pension would you choose?*

This dissertation was conceptualized to reposition the way CRRA is thought of, considered, and used by introducing a simple and direct way to measure the risk-aversion level of a financial decision-maker (i.e., investor). Unlike previous studies, the measure presented in this study takes away the presumption that someone's level of risk aversion is consistent across various lifestyle scenarios. Rather than assume, for example, that someone who is willing to participate in a sky diving adventure is also likely to invest their money aggressively, the risk measurement introduced in this study avoids indirect questioning and linking investment choices to other lifestyle choices. Instead, the proposed measure was designed to give household financial decision-makers a direct say and the ability to determine their desired financial risk level. In other words, the assessment technique avoids the fallacy that might exist in extrapolating non-financial risk-taking behavior to estimate financial and investment risk-taking choices.

As described in more detail in Chapter 3, the measure of CRRA presented and tested in this dissertation builds on a previous study by Grable et al. (2020). However, the methodology applied in this dissertation goes a step further by providing an open-ended continuous scale of certainty equivalent (*CE*) values, which can then be linked directly to risk premium amounts. Rather than using a series of skip-pattern questions (i.e., a technique widely used when evaluating revealed preferences), the assessment technique presented in this dissertation takes a more radical approach by asking respondents to input their own *CE* value on a continuum instead of the conventional approach that asks respondents to select from pre-determined amounts, typically listed in a chart or table.

Summary

To summarize, the assessment tool on which this dissertation is premised makes a significant contribution to the existing financial planning and financial risk-aversion measurement literature in the following ways:

(1) Decision-makers are allowed to select any amount on a continuum as their desired *CE* value;

(2) The question is a direct investment question, instead of an unrelated lifestyle choice question;

(3) No prior knowledge of economics or mathematics is needed to answer the question in a valid manner;

(4) The question does not induce cognitive overload;

(5) The manner in which the question is presented, decision-makers are less likely to establish a baseline answer; and,

(6) Decision-makers are provided with free range to select any dollar amount that matches their unique personal and financial situation; this avoids the suggestive nature of a pre-determined table or list of values that might influence a decision-maker's response.

How the Assessment Advances the Literature

The measure of CRRA introduced in this dissertation solves many of the problems associated with other measurement techniques. However, providing a continuous scale to decision-makers as a response option creates unique challenges, both mathematical and practical. Nearly all revealed preference tests use a series of skip-pattern questions to make estimates of CRRA easier to convert and apply. For this reason, single-item revealed preference measures are seldom seen in the literature—answers to an open-ended single-item question[5]—like the one proposed in this dissertation do not map easily to CRRA. It can be challenging to convert responses, which are framed as dollar amounts, into estimates of the economic term called gamma (γ) primarily because CRRA estimates are traditionally described as whole numbers ranging from 1.0 to 10.0. It is important to note, however, that CRRA scores can range far beyond 10.0 (i.e., extreme risk aversion). Additionally, CRRA scores can theoretically be

[5] See Chapter 3 for a detailed description of the question.

defined as intervals rather than whole numbers.

This dissertation provides a blueprint to deal with both of these issues. Furthermore, this dissertation shows readers how random *CE* values can be converted to meaningful gamma values that can then be applied across various scenarios. This dissertation also illustrates how continuous gamma coefficients can be used to describe a household financial decision-maker's utility function and how an indifference curve can be developed and applied in the context of Modern Portfolio Theory when choosing a portfolio on the efficient frontier. Finally, this dissertation adds to the existing literature by showing how nominal dollar amounts—as a proxy for certainty equivalent (*CE*) values—can be mapped to a value that can be used in portfolio management tools and other financial planning models.

Theoretical Framework

The theoretical foundation of this dissertation is a mathematical framework. As described previously in the chapter, the proposed measure of risk aversion, which serves as the basis of the study, asks a decision-maker to report a dollar amount that they are willing to invest in a scenario that provides detailed information about potential returns (both losses and gains). Given the way the question is asked and the manner in which decision-makers respond, it is important to understand how the certainty equivalent amounts (i.e., the nominal dollar amounts that represent each decision-maker's investment choice), and the resulting risk premiums, are estimated.

In order to estimate CRRA, it is useful to first conceptualize what is meant by relative risk aversion (RRA). In this regard, assume, for example, that a household financial decision-maker is faced with a one-time period decision in which the person will use all of their wealth to purchase or consume goods and services immediately. Also, assume that the utility of wealth is a twice differentiable function. Under these assumptions, RRA (Pratt, 1964) is defined using the derivation of a Utility function, $U' and\ U''$, and can be estimated as follows:

$$RRA(U', U'') = -W\frac{U"(W)}{U'(W)} \qquad \text{(Equation 1.1)}$$

where W is wealth, $U'(W)$ signifies marginal utility of wealth (the first derivative of total utility), and $U''(W)$ means the rate of change of marginal utility with respect to wealth (the second derivative of total utility). It can also be simplified as:

$$RRA = -W\frac{U"(W)}{U'(W)}. \qquad \text{(Equation 1.2)}$$

Suppose further that this individual is risk-averse and has the following power utility function, which exhibits a diminishing absolute risk aversion:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma} \qquad \text{(Equation 1.3)}$$

with γ indicating the degree of relative risk aversion and W being equal to wealth, where $W > 0$, $\gamma > 0$, and $\gamma \neq 1$. This utility function cannot be applied for $\gamma = 1$, since the output will result in a division by 0, thus being undefined at 1. Instead, for $\gamma = 1$, a separate utility function can be used to indicate the same outcome as all other γ values to provide complete proof. Thus, the CRRA utility function for any $\gamma > 0$ and $\gamma \neq 1$ is equivalent to Equation (from above. In the instance of $\gamma = 1$, the following logarithmic utility function can be used:

$$U(W) = \ln(W) \qquad \text{(Equation 1.4)}$$

As stated above:

$$RRA(U', U'') = -W\frac{U"(W)}{U\prime(W)} \quad \text{(Equation 1.5)}$$

As such, the first and second derivatives of Equation 1.7 can be found using differential calculus with respect to W when:

$$U(W) = \ln(W) \quad \text{(Equation 1.6)}$$

$$\frac{d}{dW}[U(W)] = \frac{d}{dW}[\ln(W)] \quad \text{(Equation 1.7)}$$

$$U'(W) = \frac{1}{W} \quad \text{(Equation 1.8)}$$

and

$$\frac{d}{dW}[U'(W)] = \frac{d}{dW}[\frac{1}{W}] \quad \text{(Equation 1.9)}$$

$$U''(W) = -\frac{1}{W^2} \quad \text{(Equation 1.10)}$$

Converging these findings results in:

$$RRA(U', U'') = -W\frac{U"(W)}{U\prime(W)} \quad \text{(Equation 1.11)}$$

$$RRA(U', U'') = -W\frac{[-1/W^2]}{[1/W]} \quad \text{(Equation 1.12)}$$

$$RRA(U', U'') = 1 \quad \text{(Equation 1.13)}$$

$$RRA = 1. \quad \text{(Equation 1.14)}$$

This process is the first step to proving that RRA is constant for all gamma values. The above derivation illustrates how RRA is constant at $\gamma = 1$. This first half completes the case to show $\gamma = 1$ and $RRA = 1$; it follows that $RRA = 1 = \gamma = CRRA$.

In the same manner, the second step of this proof shows RRA to also be constant for all other values of gamma where $\gamma \neq 1$.

Recall from above that the utility function that is used for $\gamma > 0$ and $\gamma \neq 1$ is:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma} \qquad \text{(Equation 1.3)}$$

with γ indicating the degree of relative risk aversion and W being equal to wealth, where $W > 0$. Taking the first derivative of this equation with respect to W, and using the power rule for differentiation (not the same as the power utility function), results in:

$$U'(W) = \frac{(1-\gamma)\, W^{1-\gamma-1}}{1-\gamma} \qquad \text{(Equation 1.15)}$$

$$U'(W) = W^{-\gamma}. \qquad \text{(Equation 1.16)}$$

The second derivative of this utility function is given differentiating the first derivative equation obtained above (Equation 1.17) with respect to W, as follows:

$$\frac{d}{dW}[U'(W)] = \frac{d}{dW}[W^{-\gamma}] \qquad \text{(Equation 1.17)}$$

$$U"(W) = -\gamma\, W^{-\gamma-1} \qquad \text{(Equation 1.18)}$$

which gives the first and second derivative of the utility function to be substituted into the definition of RRA as follows:

$$RRA = -W\frac{U"(W)}{U'(W)} \qquad \text{(Equation 1.19)}$$

Using substitution for the first and second derivatives in Equations (Equation 1.8) and (Equation 1.10) from above, the RRA function (Equation 1.19) can be rewritten as:

$$RRA = -W \frac{-\gamma W^{-\gamma-1}}{W^{-\gamma}} \quad \text{(Equation 1.20)}$$

This can further be simplified as follows:

$$RRA = \gamma \quad \text{(Equation 1.21)}$$

$$RRA = \gamma \ = CRRA \quad \text{(Equation 1.22)}$$

$$\therefore CRRA = \ \gamma. \quad \text{(Equation 1.23)}$$

The above calculation provides proof that RRA is, in fact, the constant value γ for all values when $\gamma > 0$, $\gamma \neq 1$. Hence, given conditions in Equations 1.14 and 1.21, for all values $\gamma > 0$ and $\gamma \ \neq 1$, one can conclude that CRRA = γ.

Based on this methodology, it is possible to estimate certainty equivalent amounts associated with the proposed measure of risk aversion, which is shown below:

Assume that your financial advisor approaches you with the following investment opportunity. You have an opportunity to make an investment that will return either $25,000 or $50,000. Your financial advisor estimates that the probability of receiving $25,000 is 50% and the probability of receiving $50,000 is also 50%. Your financial advisor tells you that demand for this investment is high and that only those who make the largest investments will be given shares. Based on this information, how much would you be willing to invest, assuming you had the *money?*

The proposed measure presents a household financial decision-maker with two possible outcomes: receive $25,000 or $50,000, each with a probability of 50%. The expected value of this investment, which represents the anticipated average value, is $37,500, where expected value (*EV*) is calculated as follows:

$$EV = \sum_{i=1}^{n} W_i P_{Wi} \qquad \text{(Equation 1.24)}$$

where W_i is a discrete random variable (with a possible outcome in a numerical form) with *n* outcomes, and P_{Wi} indicates the probability of W_i . In this example,

$$EV = (\$25{,}000 * 50\%) + (\$50{,}000 * 50\%) \qquad \text{(Equation 1.25)}$$

$$EV = \$37{,}500. \qquad \text{(Equation 1.26)}$$

The certainty equivalent (*CE*) is the amount a household financial decision-maker with a relative risk aversion equal to γ would be willing to accept instead of taking a chance with an uncertain outcome. In other words, *CE* is the amount that a household financial decision-maker is indifferent to when making a decision between a choice scenario offering outcomes with probabilities and an immediate outcome with certainty. The *CE* can also be conceptualized as the lowest amount that a household financial decision-maker is willing to pay to avoid a gamble that has risky outcomes.

To calculate *CE*, the following expected utility *E(U*) function can be applied. According to utility theory, a financial decision-maker's possible level of wealth that maximizes the expected value of a financial decision-maker's utility can be estimated by a utility value using the decision-maker's utility function (Myerson & Zambrano, 2019). *EU* for wealth, assuming wealth has a discrete distribution, thus, can be calculated as follows:

$$E(U) = \sum_{i=1}^{n} U(W_i)P_{W_i} \qquad \text{(Equation 1.27)}$$

where $U(W_i)$ means utility of W_i from the investment choice with n outcomes, and P_{W_i} is the probability of W_i. Given two possible outcomes of wealth levels where $n = 2$, this can be reduced to:

$$E(U) = U(W_1) * P_{W_1} + U(W_2) * P_{W_2} \qquad \text{(Equation 1.28)}$$

where $U(W_1)$ denotes utility of the first wealth outcome amount, $U(W_2)$ is the utility of the second wealth outcome amount, P_{W_1} indicates the probability of W_1, and P_{W_2} signifies the probability of W_2.

It follows that the utility obtained from a wealth outcome with certainty must at least equal the *E(U)* of a gamble, as shown in Equation (Equation 15). The expected utility of the risky choice associated with the investment must be less than or equal to that associated with avoiding the risky choice (or an amount offered with certainty). Stated another way, the *CE* amount will be greater than or equal to:

$$U(W_{CE}) \geq E\big(U(W)\big) \qquad \text{(Equation 1.29)}$$

$$E(U(W_{CE})) \geq E\big(U(W)\big) \qquad \text{(Equation 1.30)}$$

Conceptually, this means that the utility from wealth offered by an investment with certainty must equal (or be greater than) to the expected utility obtained from making a choice between two possible investments containing two wealth outcomes with similar risk probabilities.

Continuing with the scenario using the logarithmic utility function shown in Equation (Equation 1.4) when $\gamma = 1$, $E(U)$ is defined by:

$$E\ (U(W)) = \ln(\$25{,}000) * 50\% + \ln(\$50{,}000) * 50\% \quad \text{(Equation 1.31)}$$

$$E(U(W)) = 10.47 \quad \text{(Equation 1.32)}$$

Therefore,

$$U(W_{CE}) \geq E(U(W)) \quad \text{(Equation 1.33)}$$

$$U(W_{CE}) \geq 10.47 \quad \text{(Equation 1.34)}$$

$$\ln(W_{CE}) \geq 10.47 \quad \text{(Equation 1.35)}$$

$$W_{CE} \geq \$35{,}355.34 \quad \text{(Equation 1.36)}$$

Solving this for the expected wealth amount at *CE* is equivalent to $W_{CE} = \$35{,}355.34$.

This suggests that for a household financial decision-maker with a relative risk-aversion score of γ = 1, the *CE* amount the financial decision-maker would be willing to invest is \$35,355.34. This decision-maker is willing to accept a risk premium of \$2,145. In other words, a household financial decision-maker with a CRRA of γ = 1 will value the risky *EV* of \$37,500 as equivalent to \$35,355.34 with certainty and therefore be willing to accept a risk premium of \$2,145. As the γ value increases, the risk premium also increases, thus lowering the *CE* amount.

Now consider cases were γ > 1. The following formula from Equation (Equation 1.3) can be used to estimate the utility of a more risk-averse investor, where γ = 2:

$$U(W) = \frac{W^{1-2}}{1-2} \quad \text{(Equation 1.37)}$$

$$U(W) = -\frac{1}{W} \quad \text{(Equation 1.38)}$$

The *EU,* using Equation (Equation 15), is given by:

$$E(U(W)) = \left(-\frac{1}{\$25{,}000}\right) * 50\% + \left(-\frac{1}{\$50{,}000}\right) * 50\% \quad \text{(Equation 1.39)}$$

$$E(U(W)) = -0.00003 \quad \text{(Equation 1.40)}$$

Using the formula from Equation 1.29), the *CE* value can be estimated as follows:

$$U(W_{CE}) \geq E(U(W)) \quad \text{(Equation 1.41)}$$

$$U(W_{CE}) \geq -0.00003 \quad \text{(Equation 1.42)}$$

$$-\left(\frac{1}{W_{CE}}\right) \geq -0.00003 \quad \text{(Equation 1.43)}$$

$$\frac{1}{W_{CE}} \leq 0.00003 \quad \text{(Equation 1.44)}$$

$$\frac{1}{0.00003} \leq W_{CE} \quad \text{(Equation 1.45)}$$

$$W_{CE} \geq \$33{,}333. \quad \text{(Equation 1.46)}$$

As stated above, at γ = 2, $W_{CE} \geq$ \$33,333. Stated another way, the *CE* amount for a household financial decision-maker with a CRRA score of $\gamma = 2$ is ≥ \$33,333. This indicates a household financial decision-maker who has a risk premium of $4,167.

Now consider a case where γ = 5. Using the formula shown in Equation (Equation 1.3),

$$U(W) = \frac{W^{1-5}}{1-5} \qquad \text{(Equation 1.47)}$$

$$U(W) = \frac{1}{-4W^4} \qquad \text{(Equation 1.48)}$$

Thus, *EU* can be calculated as follows:

$$E(U(W)) = \left(-\frac{1}{4*\$25{,}000^4}\right) * 50\% + \left(-\frac{1}{4*\$50{,}000^4}\right) * 50\% \qquad \text{(Equation 1.49)}$$

$$E(U(W)) = -1.14 \times 10^{-7} \qquad \text{(Equation 1.50)}$$

Continuing with the estimate,

$$U(W_{CE}) \geq E\,(U(W)) \qquad \text{(Equation 1.51)}$$

$$U(W_{CE}) \geq -1.14 \times 10^{-7} \qquad \text{(Equation 1.52)}$$

$$-\frac{1}{4{W_{CE}}^4} \geq -1.14 \times 10^{-7} \qquad \text{(Equation 1.53)}$$

Solving for W_{CE}, one can estimate W_{CE} as $30,285. The risk premium at this point is $7,215.

$$U(W) = \begin{cases} \frac{W^{1-\gamma}}{1-\gamma} \ ; \gamma > 0 \; and \; \gamma \neq 1 \\ \ln(W)\, ; \; \gamma = 1 \end{cases} \qquad \text{(Equation 1.54)}$$

Theoretical Estimations in Practice

Table 1.1 shows only selected 10 *CE* amounts and the corresponding gamma coefficient associated with the risk-aversion question used in this dissertation. The amounts shown in the second column of the table correspond to whole number gamma levels ranging from 1.0 to 10.0. Keep in mind, however, that given the open-ended nature of the current measure is presented, a decision-maker's response could fall outside of or between these whole numbers.

Table 1.1 *CE* Amounts and Corresponding γ Coefficients for the Measurement Item

γ	*CE* Amount
1	$35,355.34
2	$33,333.33
3	$31,622.78
4	$30,285.34
5	$29,282.98
6	$28,541.26
7	$27,989.13
8	$27,571.57
9	$27,249.41
10	$26,995.64

An approximation of *EV* and *CE* allows for the estimation of a financial decision-maker's risk premium, which can be defined as:

$$Risk\ Premium = EV - CE \qquad \text{(Equation 1.55)}$$

where *EV* denotes the expected value and *CE* is the certainty equivalent. Those with low risk aversion will have a lower risk premium compared to those with a higher aversion to risk. Table 1.2 shows 10 risk premiums, matched to the dollar amounts in Table 1.1, associated with the risk-aversion questions proposed in this dissertation and an $EV = \$37{,}500$.

Table 1.2. Risk Premium Amounts and Corresponding γ Coefficients

γ	Risk Premium
1	$2,144.66
2	$4,166.67
3	$5,877.22
4	$7,214.66
5	$8,217.02
6	$8,958.74
7	$9,510.87
8	$9,928.43
9	$10,250.59
10	$10,504.36

The risk premium column in Table 1.2 represents the difference between the expected value of the investment, $\$37{,}500$, and the certainty equivalent of a household financial decision-maker's investment. Risk premium amounts can provide the same level of information as the *CE* values. As the RP amount goes up, the risk-aversion level goes up, indicating a direct relationship between risk premiums and risk aversion. The opposite is true for risk-aversion levels and certainty equivalent amounts. These have an inverse relationship, whereas as the risk-aversion level increases, the *CE* value decreases. In fact, some readers might prefer to associate risk aversion relative to risk premiums given the direct nature of their relationship.

Summary

Figure 1.1 illustrates how the theory was applied in this dissertation. To summarize the theoretical foundation of this discussion, the figure below shows an individual with a risk-aversion coefficient of 10**:**

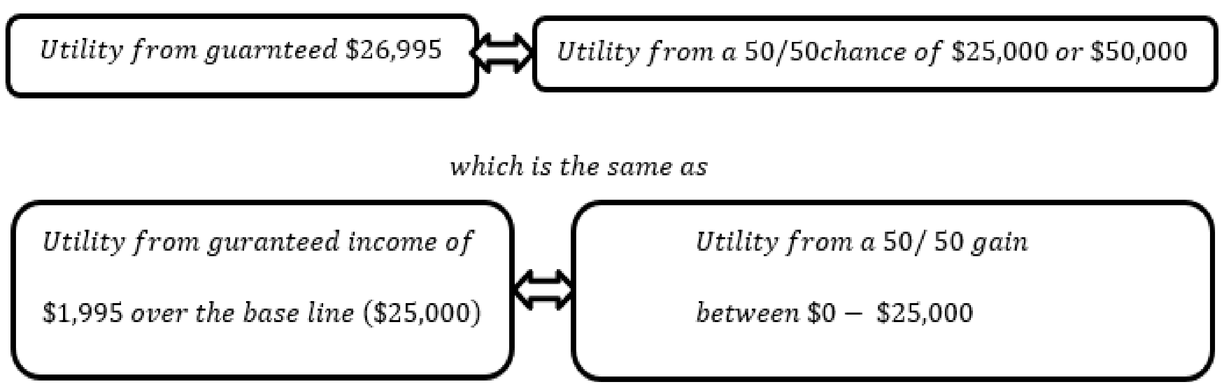

Figure 1.1. Illustration of Theoretical Foundation of Dissertation

As stated earlier, any decision-maker who is willing to avoid making an investment for less than the expected outcome of $37,500 is, by definition, risk-averse. Hence, a decision-maker with a score of 1.0, is considered to be only slightly risk-averse since their certainty equivalent (*CE*) amount is $35,355 because their *CE* value is slightly lower than the expected outcome. This places their risk premium at only $2,144. This individual is willing to forgo the potential for an expected payoff of $37,500 and give up $2,144 and in exchange lock in $35,355. It is worth remembering, however, that there is still a chance the payoff associated with the investment could be higher than the average expected value and with a small probability of the outcome landing somewhere between $37,500 and $50,000.

To further illustrate this concept, a decision-maker with a score of 10 is considered so risk-averse that they would receive more utility out of a guaranteed amount of $26,995 than a gamble with a 50/50 chance of winning a $25,000 or $50,000. This individual is so risk-averse they are very close to being willing to walk away with just the $25,000 without any need to 'gamble', while they have nothing to lose and everything to gain by taking the 50/50 gamble that has a guaranteed expected outcome greater than $25,000. This type of decision-maker would likely be too scared to leave their house for fear of some type of calamity, even those events with a minuscule probability. For this reason, researchers generally do not explore CRRA scores that are greater than 10. On the opposite side of the scale, a decision-maker who exhibits a *CE* amount near or above $37,500 is not considered risk-averse.

Research Questions

The following questions were tested in this dissertation:

(1) Can an investment choice question be used to elicit response from investors in a valid manner?

(2) Is it possible to estimate CRRA (gamma) scores from self-reported investment preferences using an investment choice question?

(3) Is it possible to develop a mathematical model that can convert any *CE* value to a

gamma score?

(4) Can a mathematical model be developed to show how CRRA scores can be used to design an efficient portfolio in a Modern Portfolio Theory (MPT) context for any continuous level of gamma?

Definition of Terms

It is important that readers be familiar with several terms and concepts used throughout this dissertation. The following definitions are provided as a foundation in the terminology used in this dissertation. The definitions are primarily based on the research publications of Arrow (1965, 1971), Carr (2014), Dalton and Dalton (2004), Markowitz (1952), Nobre and Grable (2015), Pratt (1964), Tversky and Kahneman (1974, 1979, 1992), and Kahneman and Tversky (1979).

Financial Risk. Financial risk involves a financial decision that involves a degree of uncertainty related to the decision outcome. Financial risk could arise as a result of the lack of perfect information, the lack of background information, information asymmetry, or as a result of the uncertainty of an outcome. This dissertation focuses on risk as a consequence of uncertainty. It is difficult to define risk based on a specific level of uncertainty primarily because what is risk or risky differs from person to person. However, the use and application of risk in this dissertation rely on risk as a result of uncertainty. This is not to say risk is equivalent to uncertainty but rather that risk should be considered to be a result of uncertainty.

Risk Tolerance. Risk tolerance represents a household financial decision-maker's willingness to take on a degree of uncertainty when making a risky decision. Thus, financial risk tolerance is the maximum level of uncertainty a household financial decision-maker is willing to accept when making a financial decision or the highest level of risk they are willing to engage in when the outcome of a decision is both unknown and potentially negative.

Risk Aversion. Risk aversion is defined as a household financial decision-maker's preference to avoid a risky decision outcome. Risk aversion is a household financial decision-maker's lack of willingness (i.e., unwillingness) to take on a particular level of risk and the desire to avoid uncertainty (Carr, 2014). Risk aversion represents a household financial decision-maker's level of risk avoidance, which is the opposite of risk tolerance.

Risk Preference. Risk preference describes a household financial decision-maker's general predilection that one risky choice is better than another, regardless of whether this feeling is factually true or false.

Expected Return (EV). Expected value or expected return is a mathematical calculation that can be used to determine the average outcome of a trial if the trial is repeated several times. In other words, expected return predicts the average expected outcome of a trial. For example, assume a fair coin is tossed where a person has a 50% chance of winning $1,000 if the coin lands on tails and a 50% chance of winning $0 if it lands on heads. The average predicted winning amount can be calculated as follows:

$$EV = p_1 w_1 + p_2 w_2 \qquad \text{(Equation 1.56)}$$

$$where$$

$$p_1 \text{ is the probablity of getting tails on a toss,}$$

$$w_1 \text{ is the dollar amount if tails,}$$

$$p_2 \ is\ the\ probablity\ of\ getting\ heads\ on\ a\ toss, and$$

$$w_2 \ is\ the\ dollar\ amount\ if\ heads.$$

$$Expected\ return = (50\%)(\$1{,}000) + (50\%)(\$0)$$

$$Expected\ return = \$500.$$

This can be shown using the above coin toss with different scenarios as shown in Tables 1.3, 1.4, and 1.5:

Table 1.3. Case 1: Number of Trials = 10

Coin Toss	Outcome	Winnings
1	H	\$0
2	H	\$0
3	H	\$0
4	T	\$1,000
5	T	\$1,000
6	T	\$1,000
7	T	\$1,000
8	T	\$1,000
9	T	\$1,000
10	H	\$0

Case 1 average winnings over 10 tosses = $\frac{\$6{,}000}{10} = \$600.$

Table 1.4. Case 2: Number of Trials = 10

Coin Toss	Outcome	Winnings
1	T	$1,000
2	T	$1,000
3	H	$0
4	H	$0
5	H	$0
6	T	$1,000
7	H	$0
8	T	$1,000
9	T	$1,000
10	H	$0

Case 2 average winnings over 10 tosses = $\frac{\$4{,}000}{10} = \$400$.

Table 1.5. Case 3: Number of Trials = 10

Coin Toss	Outcome	Winnings
1	H	$0
2	H	$0
3	H	$0
4	T	$1,000
5	T	$1,000

6	T	\$1,000
7	T	\$1,000
8	H	\$0
9	T	\$1,000
10	H	\$0

Case 3: average winnings over 10 tosses = $\frac{\$5{,}000}{10} = \$500$.

The average winnings between the three trials fluctuates as shown above, but the average of the average winnings for the three cases is:[6]

$$= \frac{\$600 + \$400 + \$500}{3} = \$500$$

The above example uses a small n-value. Hence, the average winnings of a gamble with enough repeated trials is given by: $EV = p_1w_1 + p_2w_2$. The expected value of a scenario is not the outcome one would expect to occur every time; it is the average of all outcomes. Rational economic theory states that (a) the assumption of rational behavior implies a household financial decision-maker would rather take actions that will benefit them versus actions that are either neutral or harmful in some way (Tversky & Kahneman, 1979); (b) one can assume that a decision-maker will behave in accordance with the expected values of the uncertainty of a decision; and (c) the decision-making process is based on making choices that result in the

[6] This can be mathematically proven using mathematical induction as $n \to \infty$, but this is outside the scope of this dissertation.

optimal level of benefit or utility for that individual. In short, expected value is just the sum of all the possible outcomes of a gamble, multiplied by their respective probabilities.

Utility. Utility is the value or the level of satisfaction a household financial decision-maker acquires from an item, money, or a choice between competing priorities. Economic theories, based on rational choice assumptions, argue that household financial decision-makers will strive to maximize their utility based on their personal preferences. Under definitive circumstances, it is not surprising that household financial decision-makers strive to maximize utility, but even in a position of uncertainty household financial decision-makers still make decisions to maximize expected utility. For example, if one compares two household financial decision-makers who are offered $10 in exchange for an hour's worth of work, Person A might decline the work offer because they receive more utility from the one hour than they would from the $10. On the other hand, Person B might be willing to sacrifice their time in exchange for the money. For Person B, the $10 provides more utility compared to the one free hour.

Figure 1.2 illustrates how utility can be graphed for three different types of investors depending on their degree of risk aversion. The red curve represents the utility function for a risk-averse financial decision-maker. The function is concave down, indicating a decreasing marginal utility when wealth increases in value. For a risk-averse decision-maker, as wealth goes up, the amount of utility obtained from wealth decreases. For example, the first $1,000 provides a lot more value and comfort than the 50th $1,000 for a risk-averse individual.

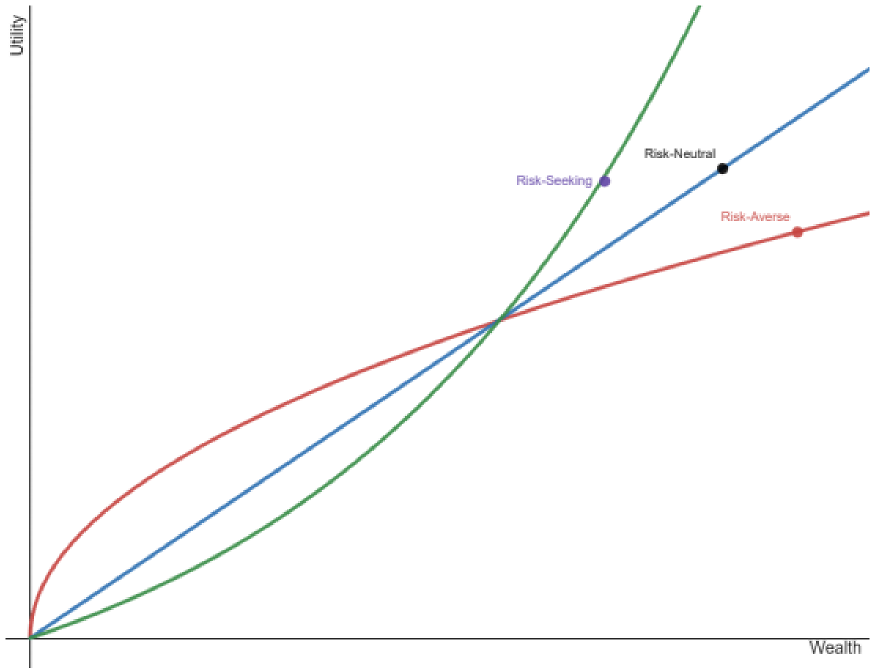

Figure 1.2. Graph of Utility as a Function of Wealth $U(W)$

The blue line in Figure 1.2 represents the utility function for a risk-neutral individual. The function is a straight line, indicating a constant rate of change or constant marginal utility. For a risk neutral decision-maker, the value of \$1,000 remains the same as the 50th \$1,000. That is to say a wealth increase from \$10,000 to \$11,000 brings them the same level of 'happiness' as an increase in wealth from \$50,000 to \$51,000.

The green curve in Figure 1.2 is the utility function for a risk-seeking decision-maker, where utility is increasing at an accelerating rate—concave up—indicating an increasing marginal utility as wealth increases. For a risk-loving or risk-seeking financial decision-maker, the 50th \$1,000 provides more value than the first \$1,000. That is to say an increase in wealth by a \$1,000 when they have \$50,000 brings them more excitement, happiness, and drive compared to the same amount of wealth increase if there were at \$10,000.

Utility Theory. Utility theory is a notion based on the concept of utility and personal preference. Instead of assuming that household financial decision-makers want to maximize the mathematical expected value of a choice, utility theory instead assumes that each individual has a personal utility that assigns a utility value to every possible income level that the individual might receive, such that the individual always wants to maximize their own utility based on their (risk) preference. This is not to say household financial decision-makers entirely ignore considerations of expected value, instead they will use some mental math to consider and weigh their options based on their personal preferences and circumstances (i.e., individual household decision-makers will make decisions to maximize their own expected utility) (Tversky & Kahneman, 1979).

In 1944, von Neumann and Morgenstern presented a method supporting this idea and showing that any consistent rational decision-maker will choose among risky gambles according to utility theory. Since then, decision analysts have developed techniques to assess household financial decision-makers' utility functions. Such assessments can be difficult to apply in practice because household financial decision-makers exhibit difficulty thinking about decisions under uncertainty, and because there are many possible utility functions. But one may simplify the process of assessing a household financial decision-maker's personal utility function if it is possible to assume that the utility function is in some mathematically natural family of utility functions.[7] To summarize, different financial decision-makers may have different preferences for accepting and avoiding risks, hence, different utility functions. Once a decision-maker's utility

[7] In fact, based on economic theory, one can assume that a decision-maker will behave in response to the expected values of the uncertainty of a decision they face. According to Finke and Guillemette (2016), household financial decision-makers are not only wealth maximizers but they also account for the probability and magnitudes of different acceptable outcomes when approaching financial decisions.

function has been identified, then this information can be distilled into the argument to introduce a mathematical function that can be used to predict a decision-maker's preference over all possible monetary gambles.

Indifference Curves. According to utility theory, different household financial decision-makers hold different preferences for taking and avoiding risks because they have different utility functions. Arrow (1971) concluded that each person has their own utility function based on their risk-aversion level(s). Arrow called these corresponding curves indifference curves. Indifference curves represent the graph for a household financial decision-maker where the utility is constant along that line or curve. The curve represents a set of possible wealth or consumption values between which the individual is indifferent.

Figure 1.3 demonstrates the concept of risk indifference graphically. Each of the indifferent curves in Figure 1.3 represent various decision-makers according to their risk-aversion coefficient (i.e., gamma (γ) score). For illustrative purposes, the graph below, while keeping the risk-free rate (R_f) constant at 4%, compares the different expected investment return values that would make each decision-maker indifferent based on the investment's level of risk (i.e., its' standard deviation). For instance, based on Figure 1.3, a decision-maker with $\gamma = 3$ would be indifferent between a risk-free return of 4% and an investment that has $SD = 12\%$ and $ER = 6\%$. Simultaneously, it can be observed from the same graph, a decision-maker with $\gamma = 7$ would require an investment with $SD = 12\%$ to provide an expected rate of return of 9% in order to be indifferent from the risk-free return offer of 4%. This shows a financial decision-makers with a higher γ score is more risk-averse and require a higher return rate to be indifferent compared to a decision-maker with lower gamma score.

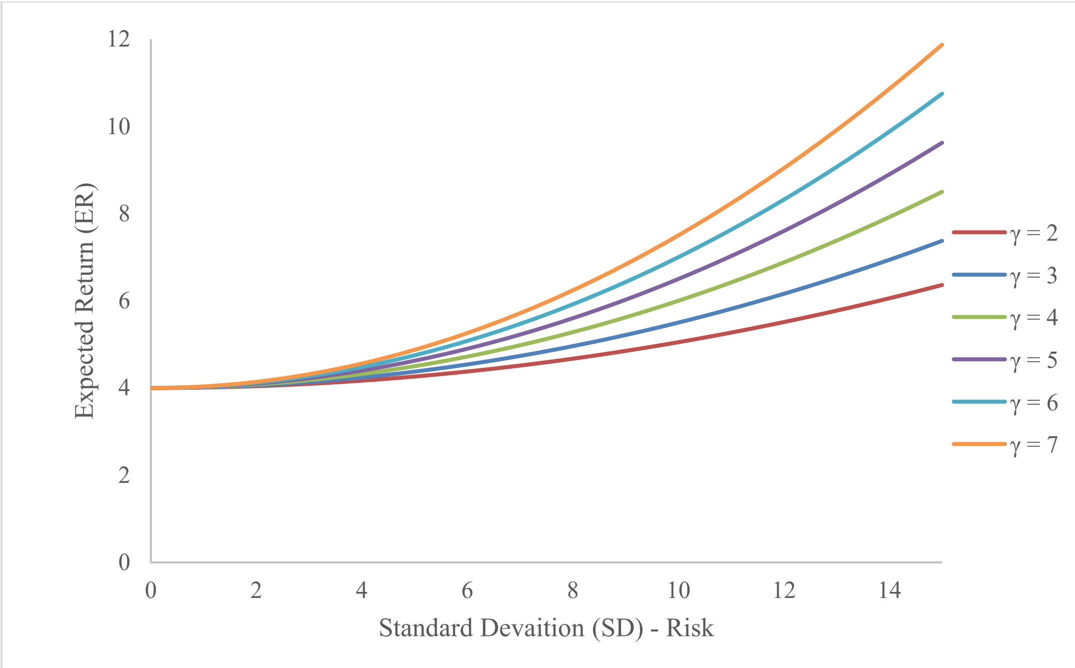

Figure 1.3. Risk Indifference Curves for Decision-Makers with Different Gamma (γ) Values

Decision-making Under Uncertainty. Modern decision-making theory encompasses the study of the behavior of individuals when faced with uncertainty. It is the study of how and why individuals make decisions when faced with uncertain outcomes and measuring those decisions into quantifiable values as well as the application and use of those parameters. As many researchers in this branch of economics have shown, when household financial decision-makers make decisions under uncertainty, they will seek out as much certainty as possible (Tversky & Kahneman, 1979; Mather et al., 2012; Bird & Yeung, 2012). This leads directly to the study of risk, risk measurement, risk management, and risk aversion of individuals.

Risk-Neutral. A household financial decision-maker is deemed to be risk-neutral if they are willing to base their decisions purely on maximizing the expected value of their income or wealth (i.e., the certainty equivalent amount for a risk neutral individual is the same as the expected value of a risk), where:

$$CE = EV$$

$$Utility\ from\ CE = Utility\ from\ EV$$

$$U(CE) = U(EV)$$

In fact, the criterion of maximizing expected value is so simple to work with that it is often used as a convenient guide in modern decision-making theory to define and study many risk-taking behaviors.

Risk Aversion. As mentioned earlier in the chapter, risk aversion is the *unwillingness* of a person who is asked to make a choice with an unknown payoff to agree to a more predictable but lower payoff. In this case, household financial decision-makers have a certainty equivalent amount that is less than the expected value of a risky situation. The certainty equivalence amount can vary, indicating the varying level risk aversion for household financial decision-makers, as follows:

$$CE < EV$$

$$Utility\ from\ CE > Utility\ from\ EV$$

$$U(CE) > U(EV)$$

Figure 1.4 graphically illustrates the various levels of risk aversion graphs for utility from wealth for a risk-averse financial decision-maker. The certainty equivalent between different risk-averse individuals will vary, hence resulting in different utility functions and graphs to represent their choices. Based on Figure 1.4, an individual whose utility function is represented by the curve 'Risk-Averse 1' is less risk-averse hence more risk-taking than the individual whose utility function is 'Risk-Averse 2'. This applies for all the other graphs in Figure 1.4, where the curve 'Risk-Averse i' represents more risk taking than the 'Risk-Averse j' curve, where $i < j$. In other words, the more curved a concave utility function is, the more risk-averse the decision-maker (i.e., a lower certainty equivalent and a higher their risk premium). The flatter or straighter the utility function is for an individual (i.e., getting close to the risk-neutral linear line shown in Figure 1.4), the higher (or closer) the certainty equivalent will be to the expected value of the gamble, and the smaller the risk premium, indicating a lower risk aversion relative to the other curves.

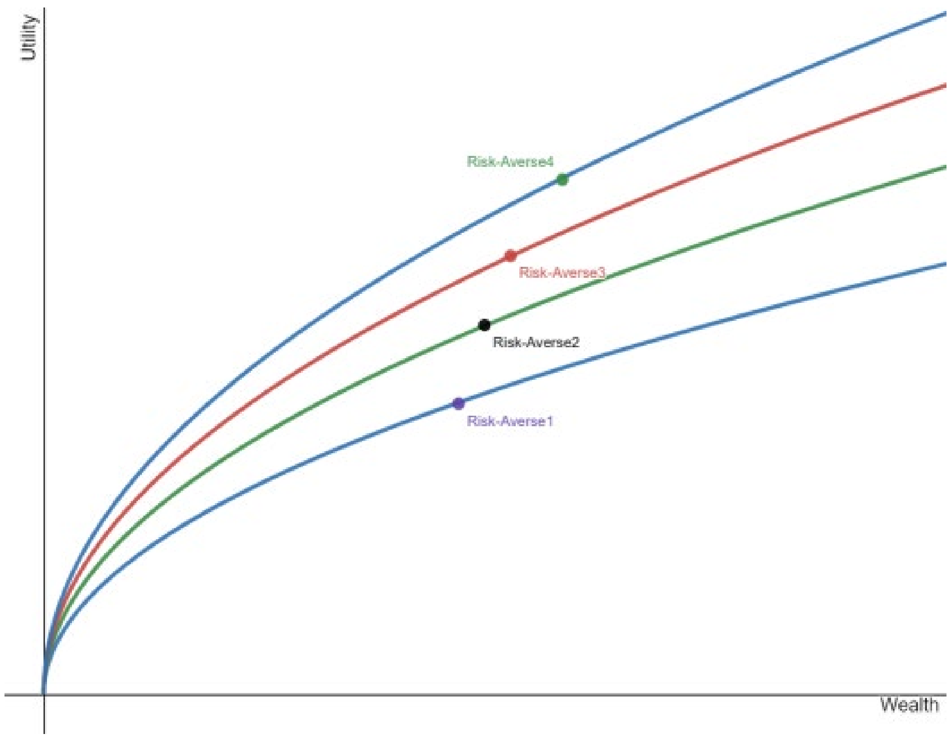

Figure 1.4. Utility Functions for Different Levels of Investor Risk Aversion

Risk-loving. This term describes household financial decision-makers who are only willing to accept a certainty equivalent offer if it is greater than the expected value. A risk loving individual will exhibit an increasing marginal utility from wealth, which indicates that their utility from wealth increases as wealth increases. To denote this in similar notation as the other risk categories described above, consider the following:

$$CE > EV$$

$$Utility\ from\ CE < Utility\ from\ EV$$

$$U(CE) < U(EV)$$

CRRA. In modern decision theory, the most convenient utility functions are those that have a special property called constant relative risk aversion (CRRA). Von Neumann and Morgenstern (1944) formulated the use of CRRA in their study of classical economic theory. CRRA implies that if one changes a gamble by adding a fixed additional amount of money to the financial decision-maker's payoff in all possible outcomes of a gamble, then the certainty equivalent of the gamble should increase by the same percentage amount. Using CRRA, it is possible to show that household financial decision-makers exhibit a constant relative risk-aversion. Hence one only needs to measure this one risk-aversion level for the decision-maker. By asking a decision-maker to subjectively assess their personal certainty equivalent using a gamble, it is possible to obtain enough information to compute the risk aversion that accounts for this personal assessment. Once the appropriate risk-aversion level has been identified, it is possible to estimate the decision-maker's expected utility and certainty equivalent for any gamble or financial risk, no matter how complex.[8]

Modern Portfolio Theory. Modern Portfolio Theory (MPT) was introduced by Harry Markowitz in 1952. MPT is a portfolio construction theory that determines the minimum level of risk for an expected return. MPT assumes that investors will favor a portfolio with a lower risk level over a higher risk level for the same amount of return. A central tenant of MPT is how one security (or asset class) in a portfolio impacts the risk and return profile of an entire portfolio. The efficient frontier, a cornerstone of MPT, shows the set of portfolios that provide the highest level of return for the lowest level of risk. When a portfolio falls to the right of the efficient frontier, the portfolio is thought to possess greater risk relative to return (i.e., an inefficient portfolio). Conversely, when a portfolio falls beneath the slope of the efficient frontier, the portfolio is thought to offer a lower level of return relative to risk.

Study Limitations and Delimitations

As with any study, this dissertation has a set of limitations and delimitations that need to be acknowledged. To begin with, given the exploratory nature of the study, a convenience sample of financial decision-makers (N = 500) was used to gather data about the proposed risk-aversion question. Related to this limitation is the notion that the sample described in Chapter 3 is not generalizable to the U.S. population. The sample frame was delimited to include primarily financial decision-makers who were engaged in the financial planning process and to those who

[8] Barsky et al. (1995) are credited with developing a modern set of questions that can be used to measure CRRA. Their methodology uses a set of questions with various risk potential outcomes to mathematically calculate a CRRA score.

were generally better educated. In this regard, respondents were more representative of high-income U.S. households with a middle-aged head of household. While the results may not be generalizable to the entire population, the results can, nonetheless, help financial planners and other financial services professionals who are actively seeking to recommend an optimal portfolio based on their clients' risk-aversion level, particularly among better educated and high-income clients.

When evaluating the methods and findings from this study, it is worth considering that there is some evidence to suggest that the dollar amount presented in risk-aversion measures can influence the way decision-makers respond to questions. Those with high incomes may perceive a given dollar threshold as trivial, whereas the same dollar amount may seem impossibly high to a low-income decision-maker. As a result, the amount of investment presented in the proposed question studied in this dissertation may have influenced respondent's answers.

Finally, given that the proposed measure tested in this study is new, follow up studies are needed to determine the reliability of question scores across time. Further tests will also be needed to validate the concept that a single item can be used in a valid manner to estimate CRRA with different samples and populations.

Chapter Summary

This chapter provided an overview of the study, justification for the dissertation, an introduction to the terminologies and concepts at the core of this dissertation, the rationale for the study, the theoretical framework that underpins the study, the purpose of the study, as well as the research questions it seeks to answer, and finally the limitations as well as the delimitations of the study. The reminder of this dissertation is organized in the following manner. Chapter 2 provides a literature review that will discuss previous studies and their findings in the areas of risk aversion, expected utility theory, and MPT. Chapter 3 introduces the sample, the mathematical procedure, and statistical methods used for validation related to the dissertation's objectives. Chapter 4 presents the statistical results of validity tests used to validate the proposed risk-aversion measure. The dissertation concludes with a summary of findings, with implications for research, practice, and policy.

CHAPTER 2

REVIEW OF LITERATURE

The purpose of this chapter is to provide a contextual overview of this dissertation and to provide a review of relevant literature used to address the research questions presented in Chapter 1. This literature review is divided into three sections. The first section explores the historical context for the study of risk aversion. The second section introduces the variables that are commonly used by researchers in studies designed to better understand financial risk aversion. The final section provides a historical context of Modern Portfolio Theory as a tool that can be used to guide the application of risk-aversion estimates in practice.

A Brief History of Risk-aversion Conceptualization and Measurement

Risk aversion was defined, for the purposes of this dissertation, in Chapter 1 as a financial decision-maker's *unwillingness* to take financial risk or a decision-maker's reluctance to take on a particular level of risk due to their desire to avoid uncertainty (Carr, 2014). It is important to acknowledge that this behavioral preference can result from a natural or environmental inclination to avoid risk. Risk aversion can also arise from biopsychosocial factors unique to the decision-maker (Kuzniak & Grable, 2017). Some of these factors or variables will be examined in detail in the second section of this literature review. The following discussion provides a concise review of the history of the study of decision-making under uncertainty.

History of the Study of Decision-making Under Uncertainty

Decision-making is a complex process that includes the analysis of different and, in most cases, competing priorities of a situation to reach a decision (Tzeng & Huang, 2011). There are various steps involved in a typical decision-making scenario, each of which needs to be considered, analyzed, and overcome. One of the most important steps in the process is the sub-decision is the decision to accept or avoid risk. When decision-makers are faced with making decisions under uncertainty, they will typically seek to avoid or reduce risk by eliminating as much uncertainty as possible (Fishburn, 1982; Machina, 1987; Kochenderfer, 2015). Assessing all possible outcomes of a situation and the level of risk associated with each outcome both provide an element of preparedness that may allow decision-makers to feel that their choices are not reckless. That is, the identification and weighing of outcomes is one way decision-makers conduct due diligence to avoid or eliminate risk. In fact, decision-makers will consider various complex factors and weigh their options based on their personal risk preferences and circumstances when assessing a risk situation and probabilities (Machina, 1987; Tzeng & Huang, 2011). These factors may include the average outcome of a risky decision (i.e., its expected value), a decision-maker's financial standing or capacity to endure a possible loss, and/or the decision-maker's intrinsic needs. When viewed from a normative economics perspective, decision-makers are expected to make decisions that maximize their own expected utility (Fishburn, 1982; Machina, 1987).

The most widely accepted framework for analyzing decision-making when the outcomes are uncertain and potentially negative is expected utility theory (Bajtelsmit & Bernasek, 1997). The history of utility theory goes back to the 18^{th} century to a now famous puzzle called the St.

Petersburg Paradox. In 1738, the Swiss mathematician, Nicholas Bernoulli, asked his cousin, Daniel Bernoulli, how it could be explained that people feel unable to make decisions if they are uncertain of the outcome(s). The St. Petersburg Paradox was framed as a gamble based on the outcome of a coin toss, where the expected value of the behavior was infinity (Machina, 1987). However, the Bernoullis found it hard to find anyone, wealthy or not, to wager any substantial amount of money to participate in a gamble that had infinity as an expected value (Machina, 1987). Based on this experiment, Nicholas Bernoulli's central proposition was the value decision-makers place on an item—in this case, money—is quite often different than the item's monetary value. The utility or the subjective, internal value decision-makers attach to an additional unit of money or the satisfaction they obtain is determined by how much money they already have (Machina, 1987). This, in turn, triggers a risk-aversion response. For example, the value that a homeless person would attach to a $100 bill is far more than the value a CEO for a multi-national corporation would place on the same $100 bill. The Bernoullis also hypothesized that a person's utility function can be best modeled using a logarithmic function indicating a decreasing marginal utility, which is the same function used in the CRRA theoretical framework of this dissertation (see Chapter 1).

In 1944, mathematicians John von Neumann and Oskar Morgenstern (Fishburn, 1982; Machina, 1987) extended Bernoulli's (1738) concept by deriving a theory of expected utility that shows that decision-makers have a subjective utility or value derived from money that is different from the economic monetary value assigned to it. Von Neumann and Morgenstern (1953) defined an expected utility function over lotteries or gambles and proposed a set of assumptions constraining a decision-maker's preference in a way that can be used to construct a utility function unique to that person. In some cases, those who apply the theory must assume that decision-makers in a poorer financial state will value their money more and, therefore, will be more risk-averse as they perceive their potential loss to be greater. Von Neumann and Morgenstern also affirmed the Bernoullis finding that people maximize their expected utility from a gamble, rather than the expected value of the gamble. In short, decision-makers care primarily about the satisfaction or utility they gain from the outcome of some uncertain event and not about the dollar value of the outcome(s).

Another foundation of the von Neumann and Morgenstern (1953) utility theory is the assertion that a decision-maker's preferences can be represented using a decision-maker's subjective utility function. The utility function can be captured in the specific form of an expected utility function that will determine the decision-maker's indifference between various choices, which leads to the discussion of indifference curves and risk premiums.

Depending on any given risk-taking scenario, von Neumann-Morgenstern (von Neumann & Morgenstern, 1953) utility functions for decision-makers can be represented as either: (a) a risk-averse decision-maker's utility function, which is a concave down utility graph; (b) a risk-neutral decision-maker's utility function, which is a straight-line utility graph; or (c) a risk-seeking decision-maker's utility function, which is a concave-up utility graph. These graphed functions are shown in Figure 1.2. Furthermore, the degree of risk aversion is related to the curvature of the decision-maker's utility function. The more curved a concave utility function is, the lower is the decision-maker's certainty equivalent and thus the higher their risk premium. The flatter the utility function is, the closer the certainty equivalent will be to the expected value of the gamble and the smaller the risk premium.

Expected utility theory was extended by developing two measures of risk aversion to compare decision-makers in terms of their attitudes toward risk. Arrow (1971) and Pratt (1964) independently created a widely used measure of risk aversion, called the Arrow-Pratt measure of risk aversion. Using utility theory, they developed what has become to be known as relative risk aversion and later the notion of constant relative risk aversion (CRRA). Since then, their findings have been used to advance the development of many theoretical and empirical models for measuring financial decision-maker risk aversion (Eeckhoudt, 2012).

The CRRA function designed by Arrow (1971) and Pratt (1964) is a measure of risk-aversion postulated on the potentiality that as the amount of wealth increases, the proportion of wealth held in risk assets for investors remains the same. Stated another way, the percentage of wealth in risky assets remains constant. Arrow and Pratt used two levels of the derivatives of the utility functions to formulate the function used to measure risk aversion. The theoretical framework of this dissertation, as described in Chapter 1, relies on the following derivation of CRRA:

$$The\ Arrow/Pratt\ measure\ of\ relative\ risk - aversion\ = -\frac{U"(W)}{U'(W)}$$

$$The\ Arrow/Pratt\ measure\ of\ constant\ relative\ risk - aversion\ = -W\frac{U"(W)}{U'(W)}$$

where,

$W\ =\ Level\ of\ wealth, U(W) =\ utility\ from\ wealth$

$U'(W) =\ First\ derivative\ of\ utility\ to\ wealth$

$U'(W)$ *measures how utility changes as wealth changes*

$U''(W) =$ *Second derivative of utility to wealth, and*

U'' *measures how the change in utility itself changes as wealth changes.*

The theory of CRRA implies that if a decision-maker changes a gamble by adding a fixed amount of money to the financial payoff, in all possible outcomes of a gamble, then the certainty equivalent of the gamble should increase by the same percentage amount. Using CRRA, it is possible to show that household financial decision-makers have a constant relative risk-aversion level. Hence, one only needs to measure this one risk-aversion level in order to draw conclusions across different features of a decision scenarios. By asking a decision-maker to subjectively assess their personal certainty equivalent using one gamble (or a set of gambles), it is possible to obtain enough information to compute the risk aversion that accounts for this personal assessment (Myerson & Zambarno, 2019). Once the appropriate risk-aversion level has been identified, it is then possible to estimate a decision-maker's expected utility and certainty equivalent for any gamble or financial risk, no matter how complex.

Based on the above discussion, it is evident that the study of utility theory, relative risk aversion, and then the study of CRRA all heavily rely on one important factor: the risk-aversion level of a decision-maker. At the root of all models developed to evaluate how risk aversion influences decision outcomes are the following questions:

- What is the risk-aversion level of the decision-maker?
- How do researchers measure the risk aversion of decision-makers?
- Is risk aversion being measured accurately?

- What are the factors that potentially influence the risk aversion of decision-makers?

The remaining sections of this literature review provide some preliminary answers to these questions.

The History of Risk-Aversion Measurement

Although different measures (i.e., tools and techniques) are currently used to measure risk-aversion, propensity measures and revealed preference methodologies dominate the marketplace. Researchers and financial service professionals have struggled, and at times been at odds with each other, over how to most accurately and consistently measure a household financial decision-maker's risk-aversion level. Psychometric tests and scaling tools, which are sometimes referred as propensity measures or elicitation assessments (Cardak & Martin, 2019), are often used by investors and financial service professionals to evaluate risk aversion. An advantage associated with propensity measurement approaches is that a well-designed measure can account for a financial decision-maker's deeply held feelings of fear, regret, anxiousness, and greed. Propensity measures do a good job of uncovering a financial decision-maker's subjective evaluation of investment outcomes. A propensity measure, which is sometimes referred as a stated-preference assessment,[9] places a decision-maker on a scale ranging from high risk aversion to low risk aversion. While single-item measures are occasionally used, nearly all propensity measures are presented as questionnaires with scores resembling either an index or a scale.

[9] Typically, a stated-preference assessment is a single-item measure, whereas a propensity measure is designed around several items that can then be scored either as an index or scale.

Revealed preference tests represent another way researchers evaluate a financial decision-maker's risk-aversion level. Revealed preference assessments differ from propensity measures and stated-preference assessments. A revealed preference assessment or test is most often constructed around a set of questions organized on a skip-pattern sequence. Almost all revealed preference methodologies are based on the concept of pure risk rather than uncertainty (Elsner et al., 2015). Questions are framed in such a way the decision-maker (i.e., decision-maker) knows the exact probabilities of success or failure prior to making a decision. Each subsequent question builds upon the previous answer in a way that can be used to categorize someone into categories of risk aversion, ranging from risk-seeking to risk-avoiding. Some have argued that revealed preference risk-aversion models provide the only rigorous theoretical way to link risk aversion to optimal portfolios (Hanna & Lindamood, 2004). Revealed preference measurement approaches assume investors are rational and that they have the requisite cognitive skills needed to make risk and return (i.e., gain and loss) estimations (Grable & Chatterjee, 2016; Kahneman et al., 1991).

Irrespective of how financial risk aversion is assessed, the current literature coalesces around the importance of identifying and analyzing investor characteristics that may impact the degree to which someone is willing or unwilling to accept financial risk. The following discussion highlights some of the individual and household characteristics, factors, and variables typically thought to be associated with financial risk aversion.

Review of the Variables Associated with Risk-Aversion

The following discussion provides a broad overview of some of the most commonly used demographic and socioeconomic characteristics, either individually or in combination, as descriptors or predictors of financial risk aversion. The extant literature indicates that factors such as age, gender, education, marital status, household income, race/ethnicity, employment status, and homeownership all have an association with a person's degree of risk aversion, risk tolerance, and overall risk-taking behavior. According to Grable et al. (2020), these factors represent some of the most frequently analyzed financial decision-maker characteristics that are also used as control variables or moderating factors in studies that describe decision-maker's risk-taking behaviors. The following narrative discusses each of these factors in detail by focusing on the direction of their association (positive/negative), the extent/impact of their influence on a decision-maker’s risk aversion (when available in the literature), their use in previous propensity and revealed preference measurement studies, and finally, the justification for their use in the validity measures applied in this dissertation.

Gender

According to much of what has been published, there are risk-aversion differences by gender, especially in the financial behaviors of women and men (Fisher, 2019; Ganguli et al., 2011). In fact, the differences in how women and men engage in financial risk are often quite pronounced. Fisher’s (2019) research on the correlation between gender and financial risk found that females are more likely to be risk-averse and hence invest less aggressively. Because of this, women typically tend to choose investments that are less rewarding due to the lower probability of significant loss (Bajtelsmit & Bernasek, 1997; Hanna & Lindamood, 2004; Sunden & Surette, 1998; Watson & McNaughton, 2007).

Fisher and Yao (2017) found gender differences in risk aversion stem more from differences in the environment and socialization (e.g., learning) than in legitimate brain wave/chemical differences between genders. This indicates that the relationship between financial risk aversion and gender is likely a product of correlation, not causation (the correlating factors being economic differences, demographic characteristics, and expectations) (Fisher & Yao, 2017). The authors further argued that what appear to be gender disparities do not result from gender per se because there are likely moderating variables between gender and risk aversion, such as net worth and income/income uncertainty. A study by Eisenhauer and Ventura confirmed this notion in a 2003 study where they found "their higher average income gives men greater relative risk aversion" (p. 1480). According to Fisher and Yao (2017), income uncertainty moderates the relationship between degrees of risk aversion and gender. Economic, social, and cultural dispositions affect the genders differently. As such, income uncertainty likely has a positive impact on risk-aversion levels for women but a negative effect on men as they tend to exhibit lower risk aversion than women.

Recent research has shown that women are also typically more gambling averse than their male counterparts (Wieland et al., 2014) and often provide a lower valuation for gambles compared to men (Sarin & Wieland, 2016). This is relevant in understanding the risk aversion of women and the differences that show up when comparing the genders given the standard for measuring financial risk-taking behavior is at sometimes based on gamble related questions as a proxy for general financial risk aversion (Holt & Laury, 2002). This leads to the question of how much of the observed gap is an aversion to gambling versus an aversion to risk-taking in financial matters is aligned. Furthermore, when examining the effect of income uncertainty, for most women, this factor has been shown to have a negative effect on risk aversion. The causes of gender differences in regard to risk-aversion levels appear to partly be associated with income uncertainty and net worth (Fisher & Yao, 2017).

Although the gender education gap is closing in the United States, American women are still disadvantaged as a gender income gap still exists (Bobbitt-Zeher, 2007). A survey conducted by Bobbitt-Zeher (2007) revealed that college-educated men in their mid-20s were employed and earning approximately $7,000 more per year than women in the same category. Moreover, recent studies confirm the income gap persists throughout parenthood and into later ages. According to Rad et al. (2016), later life parenthood increases the income gap for women. Rad et al. showed that the percentage difference between income earned by men and women increases by 32% 15 years after the birth of a first child.

As these studies suggest, the relationship between gender and risk aversion should not be considered in isolation from other socio-economic factors affecting the lives of women and their role in society. The findings show a difference in risk aversion by gender should be viewed as a warning that unless women are given equal access to education and equal pay, they may remain risk-averse. This is an important message because of the relationship between risk aversion and portfolio choice. As financial planners and researchers well know, higher risk aversion tends to be associated with allocating household assets to low-risk investment assets, which, by definition, provide low long-term historical returns. The cycle between risk aversion and portfolio choice behavior may help explain persistent wealth gaps observed in the United States, particularly among female head of households. While women are now closing the education and income gap, the gap in risk aversion appears to be entrenched with the narrowing of this gap relatively far off.

This disparity must be addressed for multiple reasons, but the primary one is risk aversion, as noted above, is one of the main factors that determine the appropriate composition of assets in a portfolio, which must be optimal in terms of risk and return relative to the needs of the decision-maker (Droms, 1987). Risk-averse decision-makers (i.e., in this case, women) generally invest in much more conservative assets over their lifespan. Differences in investment strategy can lead to a gender wealth gap, especially later in life during retirement. According to Gottschalck (2008) and the US Bureau of the Census (2017), women spend up to five more years than men in retirement due to a longer life expectancy. This should concern both policy makers and financial planners as 'running out of money' is a real and legitimate concern for many women. It is possible that a narrowing of the gender risk-aversion gap may help address this issue in the future.

Age

Age is among the most widely studied personal characteristics thought to be associated with risk aversion. There are decades of research findings documenting the link between age and risk aversion among decision-makers and households. A few studies have found the association to be negative, while a majority of studies have concluded that the correlation between these two variables is positive (i.e., older decision-makers exhibit greater risk aversion).

There is, however, some evidence that younger decision-makers may also be risk-averse. According to Wang and Hanna (1998) and Grable (2000), younger investors may be more risk-averse because it is harder to endure losses with the limited financial resources available at the early stage of their wealth accumulation phase (i.e., they lack the financial capacity to incur large losses). Based on this reasoning, as age increases, the level of risk aversion should go down, indicating that decision-makers might be less likely to avoid risk and more likely to accept risk as they age. A major indicator of this trend is the proportion of net wealth invested in risky assets—younger decision-makers hold a significantly smaller percentage of their wealth in risky assets compared to older decision-makers.

On the other hand, the majority of recent research has concluded that risk aversion increases with age (e.g., Gollier & Zeckhauser, 2002; Hallahan et al., 2004; Hariharan et al., 2000). The extant literature indicates that older adults are more likely to avoid risk, while younger decision-makers may be more inclined to exhibit risk-taking behaviors. Based on this literature, it is reasonable to hypothesize that the willingness to take risk declines with age. While contrary to the reasoning that wealth acts to build capacity against losses, this hypothesis can be explained by the realization that younger investors have a greater (expected) number of years to recover from any loss that may be incurred as a result of risky investments. In fact, Brooks et al. (2018) found that not only does risk aversion increase as age increases, but it increases at an increasing rate. This is a non-trivial finding that leads to a postulation that a positive relationship between age and risk aversion is present among financial decision-makers. For example, assume a decision-maker has a risk-aversion coefficient of 1 at age 20, and further assume that after a decade, at age 30, the same decision-maker has a risk-aversion coefficient of 2 (i.e., slightly more risk-averse). Given these assumptions, it is reasonable to expect that the same decision-maker after another 10 years, at age 40, will have a risk-aversion coefficient greater than 3 (i.e., much more risk averse).

Education

Formal attained education is a personal characteristic for which there is little ambiguity in its relationship to risk aversion and the direction of its association. Researchers have shown in many studies, across several decades, that an increased level of formal education is associated with a lower level of risk aversion (Grable, 2000; Grable & Joo, 2004; Hallahan et al., 2004; Larkin et al., 2013; Pinjisakikool, 2017; Wong, 2011). This provides inferential evidence that risk aversion is therefore inversely related to education, and financial decision-makers with high levels of education are less risk-averse (Outreville, 2015). Stated another way, as level of education goes up, level of risk-aversion goes down.

In the majority of studies published in personal finance and financial planning journals, the explanation for the association tends to follow this premise: education enhances human capital, increases someone's exposure to and understanding of capital markets, and allows for more critical thinking. When combined together, these factors act to reduce the fear associated with uncertainty, thus reducing risk aversion. The "fear of the unknown" is a real effect, impacting the less educated to a greater extent than it does a well-educated decision-maker, and making those with less formal education more hesitant and less confident, therefore risk-averse (Outreville, 2015). Further cementing this point, Calvet et al. (2009) found education was positively correlated with an increased financial sophistication.

Marital Status

Unlike formal education, there is no clearly established link between marital status and risk aversion. Anbar and Eker (2010) stated outright that there is no relationship between marital status and risk aversion. However, more than one study has shown that married decision-makers exhibit increased risk aversion relative to their non-married counterparts (e.g., Grable & Joo, 2004; Koekemoer, 2018). There are also studies that indicate single decision-makers exhibit lower levels of risk aversion in their investment behavior (e.g., Hallahan et al., 2004; Wong, 2011).

These sorts of mixed findings make it difficult to conclusively state whether a clear link exists between marital status and risk aversion, and if such an association does exist, in which direction it points. There are likely confounding or moderating variables that make it difficult to extract a meaningful correlation, such as transitions to a new stage of life (i.e., getting married), age, and changes in income, among other factors. That said, it has been asserted among some researchers that risk aversion is lowest for single males, followed by married males, unmarried females, and married females, respectively (Yao et al., 2011). Therefore, despite some research failing to find a significant relationship between the two variables (e.g., Haliassos & Bertaut, 1995; McInish, 1982; Masters, 1989), other available evidence does suggest that unmarried investors are less risk-averse and more risk-tolerant (Roszkowski et al., 1993).

Household Income

Household income has been studied extensively in regard to its association to risk-taking behaviors and risk preferences. Many researchers have found income is among the most significant factors influencing the risk aversion of decision-makers, with a clear strong negative association in the existing literature (Faff et al., 2009; Fang et al., 2020; Grable, 2000; Grable & Joo, 2004; Pinjisakikool, 2017; Wong, 2011). Research studies in this area have been consistent in reporting that as income increases, risk aversion decreases. This indicates that high income earners are more willing to take risk and invest in riskier assets with potentially higher returns. For instance, income has been found to be a positive factor in relation to stock ownership (Lei, 2018). Meanwhile, low-income decision-makers often prefer to invest in safer investment options that historically generate lower returns. This could end up leading to a further increased financial inequality and wealth, with decision-makers who have higher incomes continuing to accumulate wealth at a faster rate than low-income households.[10] The primary reason underlying the observed negative association between household income and risk aversion is that income is thought to provide a buffer to cushion potential investment and financial losses (i.e., income builds financial capacity).

Race/Ethnic Background

Race and ethnicity are personal characteristics that have been frequently excluded from risk-aversion studies. In recent decades, however, racial disparities in wealth and income have forced many academicians to examine the role of race and ethnicity across economic issues, including risk aversion and portfolio allocation frameworks.

The research findings in this area are mixed and fall short of telling the full story of the association between race/ethnicity and risk aversion. According to the current literature, there is some evidence to suggest those who identify as White exhibit more risk aversion than those who self-identify as Black (Dickason & Ferreira, 2018; Fisher, 2019).[11] Self-identified Asians are thought to exhibit more risk aversion than self-identified Blacks (Dickason & Ferreira, 2018), whereas self-identified Hispanic/LatinX households are thought to exhibit higher level levels of investor risk aversion compared to all other racial/ethnic groups (Coleman, 2003). Furthermore,

[10] As nearly all financial planning practitioners and researchers understand, decision-makers with lower incomes are implicitly taking on higher risks to obtain higher returns. However, the sense or the lack of financial security that results from low-income levels is a factor that likely discourages households from investing in risky assets that also provide higher returns.

[11] Fisher (2019) reported that a smaller proportion of Black households report having lower risk aversion than similar White households, after controlling for income and education. However, the source of this distinction was not explained, beyond reference to racial differences in human capital and other financial characteristics.

minority women are known to sometimes exhibit less risk aversion compared to other women (Hsiao et al., 2018).

As evidenced by findings from the existing literature, it is difficult to arrive at a hypothesis regarding the association between race/ethnicity and financial risk aversion. Previously published research results are mixed and inconclusive at best. Many of the above studies noted that race or ethnicity is not a determining factor in and of itself for observed differences in risk-taking behavior. Instead, there are likely moderating variables that underly the observed outcomes. Similar to the case of the association between gender and risk aversion and gender and risk-taking behavior, it seems advisable to encourage women and minorities to invest more aggressively in assets that carry more risk but also more returns. However, in practice, a lack of opportunity, financial illiteracy, lack of social safety nets, and savings deficits likely discourage this behavior. This can often perpetuate the racial and gender wealth gap. Fisher (2019) suggested financial literacy programs that are more culturally or socially relevant and accessible to various groups in their content and language might provide a feasible solution to some of the above-stated challenges.

Employment Status

With respect to the relationship that may exist between risk aversion and employment status, the current literature has primarily focused on the employment status categories of unemployed, fully employed, and self-employed. Other employment statuses (i.e., part-time or underemployment) have not typically been used or studied in this context. According to Shtudiner (2018), self-employment is correlated with lower risk aversion and a higher propensity for selecting riskier investment assets compared to other decision-makers with different employment status. Shtudiner (2018) also suggested that this relationship may be explained by understanding that those with an entrepreneurial inclination likely have a lower risk aversion; in effect, self-employment is a form of revealed preference (i.e., business owners are willing to take on the risk of starting a business to be self-employed). A study by Sung and Hanna (1996) corroborates this possibility, with self-employed and heads of households exhibiting lower risk aversion in the study. Sung and Hanna (1996) also argued perhaps the rational expectation might be for self-employed decision-makers to be more risk-averse given the lower predictability and inconsistency of income relative to those employed by corporations or institutions. However, both studies found self-employed persons to be generally less risk-averse than those who are in salaried or working in wage employment positions.

Homeownership

Homeownership is another variable thought to be associated with risk aversion. Similar to some of the other variables discussed thus far, homeownership has mixed findings in the extant literature when examining its correlation with risk aversion. Sung and Hanna (1996) and Larkin et al. (2013) reported no relationship between homeownership and a decision-maker's level of risk aversion. However, some studies have observed a negative association between homeownership and risk aversion (e.g., Grable & Joo, 2004; Yang, 2004; Jianakopolos & Bernasek, 2006). According to these studies, homeowners are more risk-averse and tend to avoid risk to a greater degree than those who do not own property. The reason may be homeowners believe that they have more to lose in the event of a financial loss, whereas non-homeowners may feel as if they have "nothing to lose" by engaging in financial risk-taking behavior.

Summary

The assessment of financial decision-maker risk aversion is a complicated process that touches on all aspects of the decision-maker's existence, including socio-economic, demographic, and biopsychosocial characteristics. This section described what the current literature shows to be the associations between risk aversion and age, gender, education, marital status, household income, race/ethnicity, employment status, and homeownership. These variables represent factors that are most frequently cited as having the most association and possible influence on the decision-maker's level of risk aversion either directly or as descriptors, control variables, or moderating factors. Based on the above discussions, education and income have a clear negative correlation with risk aversion. Two variables—gender and employment status—also have a clear association with risk aversion where men and self-employed decision-makers have been shown to exhibit consistently lower risk aversion relative to their counterparts. Homeownership historically has been shown to have a mixed relationship with risk aversion where it has either a negative correlation (i.e., homeowners are more risk-averse than non-owners) or no association. The three variables that appear to have the most nuanced association with risk aversion are age, marital status, and race/ethnicity. The outcomes of previous studies indicate more research is needed to uncover and decompose the associations between and among these variables.

Finally, taking a look at the statistical models developed from the studies above, Grable (2000) determined the best factors for differentiating financial risk aversion (or risk tolerance) are primarily a combination of education, income, and occupational status. These variables can explain as much as 22% of a decision-makers risk-taking behavior. This conclusion was supported by Quy and Van (2018) who reported education, wealth/income, and investment background knowledge are important indicators of financial risk-taking behavior.

In summary, as the review of literature indicates, gauging a decision-maker's degree of financial risk aversion has typically been evaluated through studies that include a variety of personal and household characteristics. In some studies, variables such as gender, age, and income have been used to describe, and in some cases, even predict risk aversion. In other studies, these and other variables have been used as control variables in the context of assessing financial and investment behavior in which risk aversion is an independent variable. It is important to note that regardless of the role of the variables discussed above, the literature is relatively silent on the topic of endogeneity issues—that is, the notion of causality is seldom addressed. Instead, as described in this chapter, variables of interest tend to be described in terms of correlations and associations with risk aversion.

The remainder of this chapter describes how estimates of risk-aversion, and some financial decision-maker characteristics, relate directly to portfolio development theory. It is this theoretical link that will be explored in more detail in Chapter 5.

The History of Modern Portfolio Theory

In 1952, Harry Markowitz introduced Modern Portfolio Theory (MPT). MPT takes a quantitative approach using the mean-variance of assets to design a diversified portfolio that maximizes returns while minimizing risk. Historically, financial planners, portfolio managers, and investors used traditional portfolio management tools and techniques in an attempt to build portfolios with a variety of assets (such as stocks and bonds) from different companies, industries, and asset classes, but the approach was more qualitative than quantitative. Before

MPT, the portfolio development process was done in a non-quantitative manner. To accomplish this, financial service professionals used their knowledge and experience to pick and assemble a portfolio with the goal of maximizing returns and minimizing risk. The asset selection process focused on various factors related to the client's needs, such as risk preference measured qualitatively, risk capacity and time horizon, as well as market related factors. Markowitz (1952) showed this approach to investment management was inefficient and prone to human and systematic errors.

Markowitz (1952) argued that a better way to assemble a portfolio of investment assets requires one to first know how to calculate the returns and risks of a portfolio and how to minimize risks through diversification. MPT advances the portfolio creation by removing the guesswork out of the portfolio development and management by modeling efficient portfolios given a set of quantitative parameters. These parameters include an accurate estimate of a decision-maker's desired risk level and the mean-variance of assets that could potentially be used in a portfolio. The risk-return characteristics of an investment asset is one of the inputs into MPT calculations which is readily available from various reputable sources. However, another input—an investor's preferred risk level—is a parameter that is difficult to measure. In fact, the assessment of an investor's risk appetite, risk preference, and/or aversion to risk (the opposite of which is risk tolerance) has hampered the wide scale use of MPT among those who provide financial advice to all but the wealthiest of households.[12] Despite the clear advantages of MPT, very few risk-aversion assessments generate scores that can be easily mapped to the efficient frontier, and even when a score is mappable. In fact, very few financial services professionals

[12] MPT is widely used when managing institutional portfolios and portfolios of high net-worth households because these types of investors tend to have very long (indefinite) time horizons, which minimizes or eliminates the need to account for specific levels of individual or household risk aversion.

have the skills or resources necessary to convert risk-aversion estimates into utility functions that can be used in accordance with MPT.

MPT uses a statistical approach based on the standard deviations of assets (i.e., securities) and the correlation between the selected assets to design a diversified portfolio that maximizes returns while minimizing risk. It is the correlations between and among investment assets that drives the identification of efficient portfolios. Investment portfolios for households are constructed to consist of a vast number of assets with an equally broad number of proportions. These portfolios can be transposed to create many different types of portfolios with varying levels of risk-return. If one were to take this large number of portfolio possibilities with varying proportions and plot them on a graph that compares the expected return with the expected risk, each plotted point would represent a feasible portfolio. The portfolio with the highest return for the lowest risk is called an optimal portfolio. Theoretically, two investors with the same level of risk aversion should choose the same optimal portfolio on the efficient frontier, holding all other factors constant.

During the construction of a portfolio, Markowitz (1952) showed that it is possible to add risky but low correlating assets to a portfolio, thereby increasing returns without, at the same time, increasing risk. In practice, this should decrease the risk associated with the portfolio. This approach capitalizes on the notion that a lower covariance/correlation between portfolio securities results in a lower overall portfolio standard deviation (i.e., lower portfolio risk). This allows investors and those who provide advice to investors to adjust a portfolio to the maximum possible point where risk can be reduced without reducing returns. This is a normally difficult task under most circumstances since a reduction in risk generally means a reduction in returns

(Merton, 1972). The MPT process also allows for a more diversified set of holdings and better asset choices that have little to no relation to one another within a portfolio. This has been shown to substantially reduce the overall risk of a portfolio (Markowitz, 1952). MPT provides the theoretical justification for the argument that diversification not only reduces risk but also enhances returns over the long run.

The main objective of MPT is to apply economic principles—in this case, utility theory—to portfolio management using statistics and mathematics to identify an *optimal portfolio*, which is a portfolio that yields the highest return for a specific risk, or stated in another way, the lowest risk for a given return. An optimal portfolio designed under the axioms of MPT provides the mechanism to combine the best possible assets into a portfolio that can maximize returns for a given investor's level of risk aversion.

Markowitz (1952) used a graphical representation of these concepts where each feasible portfolio is plotted on a graph that compares the expected return with the expected risk. Figure 2.1 shows how this graphing process allows an efficient frontier for an optimal portfolio to be identified. The efficient frontier, a cornerstone of MPT, shows the set of portfolios that provide the highest level of return for the lowest level of risk. When a portfolio falls to the right of the efficient frontier, the portfolio is thought to possess a greater risk relative to expected returns. Conversely, when a portfolio falls beneath the slope of the efficient frontier, the portfolio is thought to offer a lower level of return relative to risk. Portfolios above the efficient frontier curvature (i.e., to the left of the red curve on the figure below) are considered unattainable and hence not considered for portfolio design. MPT is the process of identifying and using investment asset combinations (portfolios) that are either on or near the efficient frontier curve.

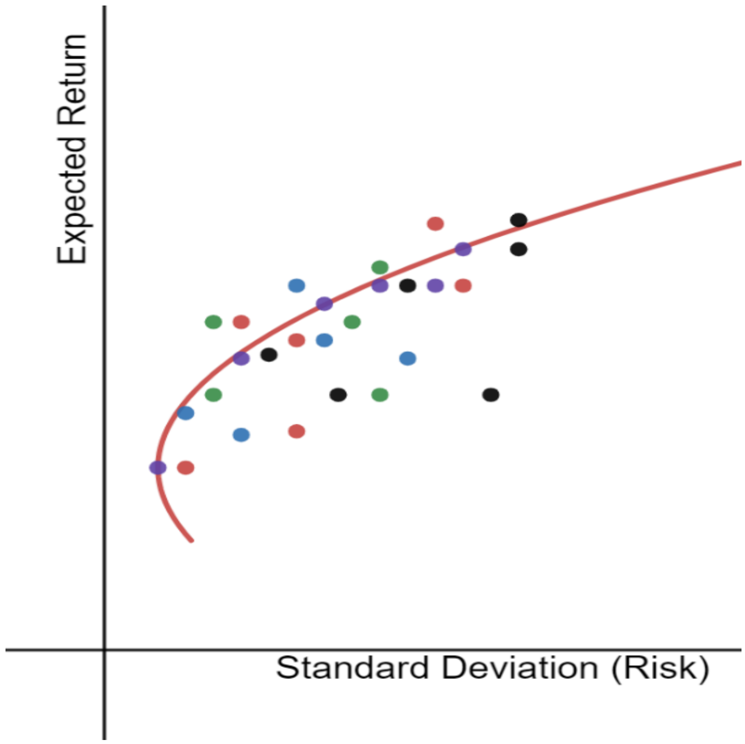

Figure 2.1. The Efficient Frontier

The efficient frontier also consists of the set of all efficient portfolios that yield the highest return for each level of risk on the curve. The efficient frontier can then be combined with an investor's utility function to find an investor's optimal portfolio. This offers an individualized, unique portfolio offering the greatest return for the risk that the investor is willing to accept (i.e., their utility function). This is shown in Figure 2.2. In this illustration, the tangent point where the utility function meets the efficient frontier is the financial decision-maker's optimal portfolio. It is important to remember, in the context of this dissertation, that the primary way a utility function is estimated is through the valid and reliable measurement of the decision-maker's aversion to risk, which is conceptualized as constant relative risk aversion (CRRA).

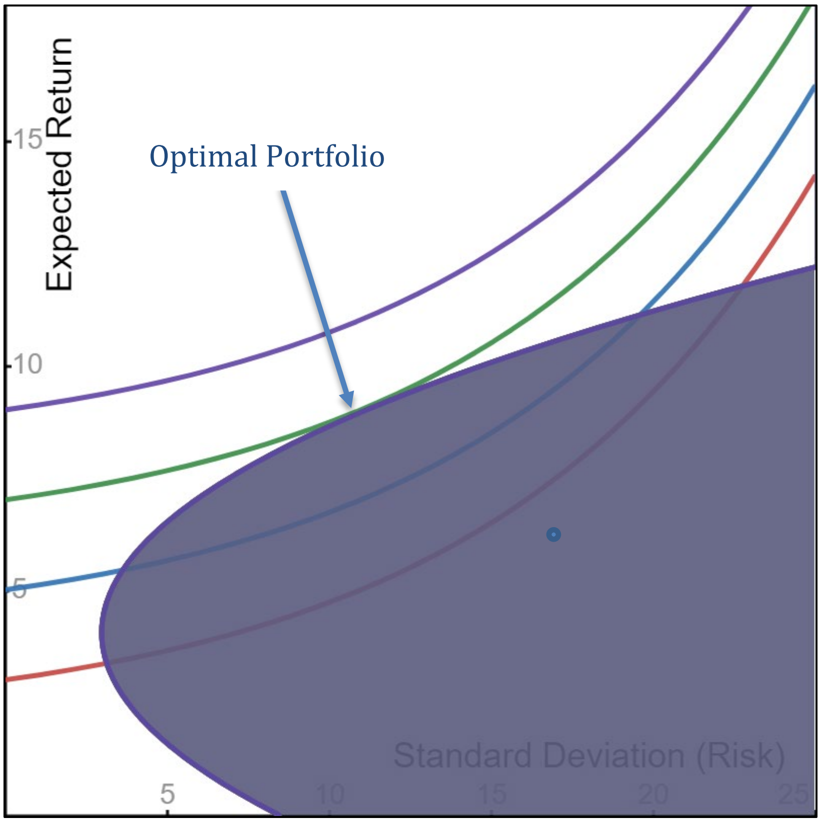

Figure 2.2. Mapping of Riks Indifference functions to Efficient Frontier

As shown in Figure 2.1 and Figure 2.2, each plotted point is a feasible portfolio. Given the principles of MPT, it is essential to map estimates of a decision maker's risk aversion to what is known as the efficient frontier. Successful optimization of the return versus risk paradigm places an optimal portfolio along the efficient frontier curvature. Portfolios that lie below the efficient frontier are sub-optimal because they do not provide enough return for the level of risk. Stated another way, portfolios that cluster to the right of the efficient frontier are sub-optimal because they have a higher level of risk for the defined rate of return. The efficient frontier curve shows the set of investment portfolios that are expected to provide the highest returns at a given level of risk. A portfolio is said to be efficient if there is no other portfolio that offers a higher return for a lower or equal amount of risk. As noted above, where portfolios are located on the efficient frontier depends on a financial decision-maker's degree of risk aversion.

Summary

In summary, Markowitz (1952) demonstrated that it is possible to not only develop efficient portfolios, but it is also possible to identify optimal portfolios for financial decision-makers (i.e., investors) based on their unique risk-aversion profile. To date, the link between portfolio optimization and portfolio selection has tended to be primarily theoretical. That is, while almost all financial service professionals use some type of portfolio optimization modeling or software, the actual selection of portfolios on the efficient frontier tends to be based on professional judgment and previous experience. Very few financial services professionals estimate investor utility functions based on measures of risk aversion. Even fewer map estimates of risk aversion to the efficient frontier. One outcome associated with this dissertation is to provide a pathway to make this step in the MPT process more accessible to investors and those who provide advice to investors.

Chapter Summary

This chapter provided a literature review on three core tenants of the dissertation. The chapter began by providing an expanded background on the historical context for the study of risk aversion and utility theory. This was followed by a detailed account of variables historically used in the study and measurement of decision-maker risk aversion. The chapter concluded with a review of the historical context of Modern Portfolio Theory with an emphasis on showing how risk aversion can be used to select optimal portfolios.

To summarize, this chapter has shown the significance of:

(1) Conceptualizing financial decision-maker risk aversion in the context of utility theory and MPT;

(2) Measuring investor risk aversion accurately and consistently, as this concept serves as the launching point for the development and management of portfolios;

(3) Identifying and analyzing financial decision-maker characteristics that can be used to describe the degree to which someone is willing or unwilling to accept financial risk (i.e., factors used to validate risk-aversion measures); and

(4) Understanding how financial risk aversion is an integral aspect of MPT and the selection of optimal portfolios on the efficient frontier.

The remainder of this dissertation describes, in Chapter 3, the sample for the study and the mathematical procedure and statistical methods used for validation related to the dissertation's objectives. Chapter 4 presents the results of tests to validate the proposed risk-aversion measure. Lastly, the dissertation concludes with Chapter 5 that provides a detailed discussion on the application of the findings in relation to MPT concepts as well as a summary of findings, with implications for research, practice, and policy.

CHAPTER 3

METHODOLOGY

The purpose of this dissertation is to (a) illustrate a technique that can be used to transform certainty equivalent amounts—derived from a question designed to elicit a financial decision-maker's risk-aversion level —into gamma coefficients, and (b) validate the resulting gamma coefficients as descriptors of financial decision-maker behavior. This chapter introduces the sample, the mathematical procedure, and the statistical methods used tests to validate the proposed risk-aversion measure.

The chapter begins by revisiting the purpose of the dissertation and associated research hypotheses. This is followed by the presentation of the proposed financial decision-maker risk-aversion measure. The mathematical derivation of the certainty equivalent conversion procedure is then presented. This is followed by a description of variables used to validate the resulting gamma coefficients and descriptive statistical analysis of the validity tests that were run to confirm the robustness of the transformation process. This chapter is followed by a presentation of results in Chapter 4 and a summary of findings, with implications for research, practice, and policy, in Chapter 5.

Purpose of Dissertation

The purpose of this dissertation is multifaceted. At its core, the purpose of the dissertation is to introduce a procedure that can be used to transform certainty equivalent dollar amounts into gamma coefficients, which are inputs into constant relative risk-aversion models. In order to illustrate the procedure, this dissertation adapts and builds upon a new measure of financial decision-maker risk aversion. The measure used in this dissertation prompts financial decision-makers to indicate their preferred level of investment risk associated with an investment opportunity. The second aspect of the dissertation involves evaluating the validity of both the transformation procedure and the measure of risk aversion using a combination of correlation and regression estimations. Specifically, the dissertation was conceptualized around the following outcomes:

1. To associate the concept of risk aversion used by economists to the notion of risk tolerance typically used by financial planners;
2. To provide a mathematical transformation procedure that can be used to convert dollar certainty equivalent amounts derived from a measure of financial risk aversion to gamma coefficients—defined as the coefficient of relative risk aversion—in a way that matches theory related to constant relative risk aversion (CRRA);
3. To validate the transformation process and measure of financial decision-maker risk aversion using construct and concurrent validity tests; and
4. To demonstrate the use of gamma coefficients obtained from the measure in the context of efficient frontier modeling as described in Modern Portfolio Theory (MPT).

The following hypotheses relate specifically to the validity test outcomes (outcomes three and four from above):

H_1: Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of financial risk aversion are significantly (negatively) correlated with scores from a propensity measure of financial decision-maker risk aversion (validity measure one).

H_2: Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are significantly (negatively) correlated to a decision-maker's stated risk preference (validity measure two).

H_3: Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are significantly (negatively) correlated to scores from other types of revealed preference risk-aversion measures (validity measures three and four).

H_4: Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are significantly (negatively) correlated to the likelihood of gambling income (validity measure five).

H_5: Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are positively associated with cash holdings, and negatively associated with equity portfolio holdings when controlling for factors known to be associated with financial risk aversion and portfolio choices (validity measure six).

The remainder of this chapter is organized as follows: The discussion begins by describing the sample. The discussion then centers on the presentation of the proposed direct measure of financial decision-maker risk aversion. This is followed by a description of the certainty equivalent transformation process and an explanation of the variables used to validate the resulting gamma coefficients and the financial decision-maker risk-aversion measure used in this dissertation. The chapter concludes with a summary of the validity testing procedures used in the study.

Sample and Data

The data for this study were collected in late 2019 using Qualtrics, an online survey tool. The survey from which data were collected was developed for a larger research project. Questions (and associated data) directly applicable to this dissertation were culled from the larger survey. The survey was distributed by Dynata. The primary delimiting factor was that those included in the sample needed to be a financial decision-maker who was age 18 or older at the time of the survey. In this regard, the sample frame was not intended to be nationally representative. The goal of the sample selection process was to obtain a representation of individuals who were, at the time of the survey, in a position to invest money in a variety of securities and products (i.e., a sample that represented a cross-section of clientele that a financial planner or financial counselor might deal with on a day-to-day basis). Dynata distributed the Qualtrics survey link to a pool of adults living in the United States. The final sample included 527 individuals. The survey included questions related to a respondent's financial situation and demographic profile. The survey took approximately 15 minutes to complete. Upon completion of the survey, respondents were compensated with a $3 incentive. The survey and procedure were first approved by the University of Georgia's Institutional Review Board prior to distribution.

Measures

As discussed in Chapter 1, nearly all measures of financial decision-maker risk aversion infer a certainty equivalent (*CE*) value and gamma coefficient based on a financial decision-maker answering a series of skip-pattern questions in which the financial decision-maker is tasked with choosing a preferred gamble or lottery between two options. These measures are known as revealed preference tests, which differ in context from a propensity or stated preference measure. The resulting gamma score obtained from a revealed preference test typically ranges from 1 to 10 with higher scores representing greater risk aversion (Mehra & Prescott, 1985). Several issues associated with traditional revealed preference tests dampen the validity of resulting scores when viewed from a financial planning and investment management perspective. For instance, revealed preference tests tend to be cognitively demanding for a financial decision-maker to complete (Guiso & Sodini, 2013; Kahneman et al., 1991). Further, resulting scores are not always related to investment and financial planning outcomes. The measure of risk aversion used in this dissertation was developed to address these issues and other weaknesses associated with traditional revealed preference methodologies while allowing the derivation of gamma coefficients that can be used in the context of building efficient portfolios.

A version of the risk-aversion measure used in this study was first conceptualized by Grable et al. (2020). When developing the original measure, Grable et al. combined attributes from propensity scales with those from revealed preference tests. More specifically, the measure was written to provide a financial decision-maker with an applied and true-to-life scenario investing scenario, which is similar to assessment techniques employed with propensity measures. Additionally, Grable et al. borrowed scaling techniques similar to what is used within propensity assessments. Rather than have a financial decision-maker choose between static probability choices, the measure asks a financial decision-maker to provide a dollar amount that matches their revealed preference for making an investment in the context of the scenario. The measure also borrowed from revealed preference methodologies in framing the scenario in a way that is consistent with the derivation of constant relative risk aversion (CRRA).

The original Grable et al. (2020) question was modified for this dissertation in a way that was thought to place less cognitive stress on a financial decision-maker, compared to traditional revealed preference tests, by providing an easy point of reference and an open-ended response approach. The assessment question used in this dissertation is as follows:

> *Assume that your financial advisor approaches you with the following investment opportunity. You have an opportunity to make an investment that will return either $25,000 or $50,000. Your financial advisor estimates that the probability of receiving $25,000 is 50% and the probability of receiving $50,000 is also 50%. Your financial advisor tells you that demand for this investment is high and that only those who make the largest investments will be given shares. Based on this information, how much would you be willing to invest, assuming you had the money?*

In the survey, respondents were asked to indicate how much, in U.S. dollars, they would be willing to invest. An interval scaling approach (see Figure 3.1) was used to record respondent answers. It was possible for a survey respondent to indicate any dollar amount between zero and $50,000 (represented as 50 in the survey). Theoretically, a financial decision-maker with high risk aversion would have indicated their risk preference by opting to invest less, whereas a financial decision-maker with a low level of risk aversion would have been willing to invest more. In this dissertation, the values selected by a survey respondent were assumed to be an indicator of each respondent's degree of risk aversion. Based on this assumption, it was thought that these dollar amounts could then be used to derive direct estimates of gamma, as described later in this chapter.

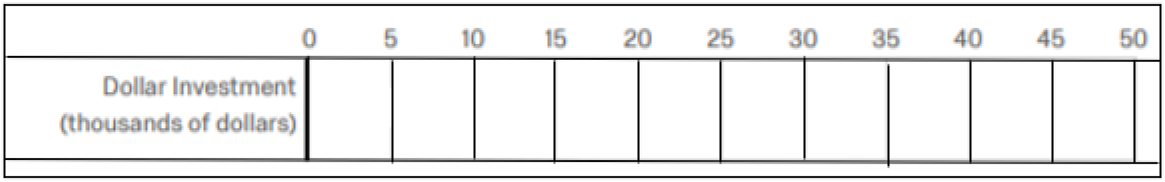

Figure 3.1. Scale Associated with the Measure of Financial Decision-Maker Risk Aversion

The Gamma Estimation Process

While the measure of financial decision-maker risk aversion used in this study solves several of the confounding issues associated with nearly all revealed preference tests, a significant shortcoming is that financial decision-maker responses are presented as dollar amounts. While it is true that these dollar amounts can be thought of as indicators of *CE*, the open-ended question response format produces various responses that are not easily referenced in *CE* charts or functions used by economists, investment managers, and financial planners. Stated another way, while the dollar amounts indicated by respondents when answering the question serve as a nuanced indicator of the degree to which a respondent is willing to take investment risk, the dollar amounts are not directly transferrable into CRRA models. Therefore, in order to estimate a financial decision-maker's CRRA score, it is necessary to transform the dollar amounts into gamma coefficients. The following discussion illustrates how this transformation process was accomplished in this dissertation.

Recall that the risk-aversion measure asks a financial decision-maker to indicate their willingness to take risk by indicating the certainty equivalent amount that represents their investment preference. The way the question used in this dissertation was conceptualized, these dollar amounts can be transformed in a way that results in a direct measure of CRRA. As discussed in Chapter 1, the expected value of the investment presented to those in this study, which represents the anticipated average value, is \$37,500, where expected value (*EV*) is calculated as follows:

$$EV = \sum_{i=1}^{n} W_i P_{Wi} \qquad \text{(Equation 3.1)}$$

where W_i is a discrete random variable (with a possible outcome in a numerical form) with n outcomes, and P_{Wi} indicates a probability of W_i. For the measure used in this study,

$$EV = (\$25{,}000 * 50\%) + (\$50{,}000 * 50\%) \qquad \text{(Equation 3.2)}$$

$$EV = \$37{,}500.$$

The certainty equivalent (*CE*) is the amount a financial decision-maker with a relative risk aversion equal to γ would be willing to accept in lieu of taking a chance with an uncertain outcome. In other words, *CE* is the amount that a financial decision-maker is indifferent to when deciding between a choice scenario offering outcomes with probabilities and an immediate outcome with certainty. The amount of payoff that a decision-maker would have to receive to be indifferent between that payoff and a given gamble and the gamble's *EV* (i.e., certainty equivalent) is called the risk premium. The risk premium can also be conceptualized as the lowest amount that a financial decision-maker is willing to pay to avoid a gamble that has risky outcomes.

Transforming Certainty Equivalent Amounts to Gamma Coefficients

Traditionally, *CE* amounts are reported as whole numbers ranging from 1 to 10. That is, nearly all revealed preference tests attempt to classify financial decision-makers into one of 10 γ categories. As an example, Table 3.1 shows how *CE* amounts ranging from a γ of 1 to a γ of 10 map back to the measurement question used in this dissertation. Decision-makers who are very risk-averse are represented by a γ of 9 or 10, whereas those who are slightly risk-averse are indicated by a γ of 1 or 2. However, in this study, it was possible for a respondent to select a dollar amount that either fell between γ levels or beyond a γ of 10. For instance, a respondent could have indicated a willingness to invest, say, \$28,000, \$34,000, \$42,500, or any other dollar figure. As such, although Table 3.1 shows only a select 10 levels of γ. In actuality, it was possible for the CE to be recorded as any positive integer between 0 and 50 (i.e., \$50,000), and therefore the corresponding γ value to be any integer (including decimals).

In order to estimate a financial decision-maker's CRRA score, it is necessary to transform the *CE* dollar amounts into gamma coefficients (γ). The process involves numerous interrelated mathematical steps (see Chapter 1 for a theoretical discussion related to this point). A significant contribution of this dissertation is the presentation of these steps in a way that can be replicated in other studies and in practice by investment managers and financial planners. These steps are described below.

A Mathematical Model for Converting any CE Value to γ

The following conversion model was developed using the mathematical software Wolfram Alpha. The given function uses the certainty equivalent amount as an input and produces its corresponding gamma coefficient. The function can be adjusted accordingly based on a given measure's probabilities and dollar values.[13] The input/output process and resulting estimation for the proposed measure are as follows:

$$Input = \ CE\ value\ in\ dollars$$

$$Output = \ \gamma\ level\ on\ a\ continuum\ [1{,}10]$$

$$Constant\ values = \ \mu_i$$

$$\gamma = \ \gamma(\sigma\ (CE)) \qquad \text{(Equation 3.3)}$$

Intermediate calculation:

$$Intermediate\ Input\ \&\ output = \ \sigma$$

$$\gamma(\sigma) = \sigma + \frac{-2.986105029259}{\sigma} + 14.2778954039378 \qquad \text{(Equation 3.4)}$$

where,

$$\sigma\ (CE) = \ (((\mu_4 * 2CE + \mu_3)^2 + \mu_2)^{\frac{1}{2}} + \mu_1 * 2CE + \mu_0)^{\ \frac{1}{3}} \qquad \text{(Equation 3.5)}$$

and

$$\mu_4 = \ 0.0720160163620389$$

$$\mu_3 = -3804.92944067349$$

[13] This function is unique to the question used in this dissertation; however, the process of converting *CE* amounts to gamma estimates can be extended to include scenarios in which the dollar outcomes and probabilities are different from those used in this dissertation.

$$\mu_2 = 26.6265707391952$$

$$\mu_1 = -0.0720160163620389$$

$$\mu_0 = 3804.92944067349$$

This model can be adjusted and used for any *CE* dollar value associated with the question as an input to return a γ value, with the γ value indicating a financial decision-maker's risk-aversion level on a scale [1,10]. Table 3.1 shows the conversion based on the model developed for this dissertation and the corresponding gamma coefficient values (for 10 whole numbers representing γ), as well as the expected output.[14]

[14] Given the robustness of the modeling technique, it is possible to classify financial decision-makers into categories of risk aversion that fall outside the boundaries of 1 (not risk averse) to 10 (very risk averse). For simplicity and applicability, those who indicated a willingness to invest more than $36,700 were coded as 0.10, indicating a substantial willingness to take risk. Conversely, those who indicated a willingness to invest less than $26,700 were coded 11.0, suggesting a very high degree of risk aversion. Effectively, someone with a risk-aversion of 11 or higher should not be investing in assets that exhibit price volatility. On the other hand, someone with a risk-aversion score of 1.0 or lower likely has the attitudinal disposition to invest in assets that exhibit substantial price volatility.

Table 3.1. ***CE*** **Amounts with Corresponding Estimated and Actual Gamma Coefficients**

CE Amounts	Estimated γ *(New Model)*	Actual γ *(CRRA Model)*
$35,355.34	1	1
$33,333.33	2	2
$31,622.78	3	3
$30,285.34	4	4
$29,282.98	5	5
$28,541.26	6	6
$27,989.13	7	7
$27,571.57	8	8
$27,249.41	9	9
$26,995.64	9	10

The γ values shown in column two of Table 3.1 represent the γ estimates using the modeling procedure from above. The γ values in column three of the table are the actual levels of γ value generated in the calculus-based economic CRRA model discussed in Chapter 1. Essentially, the information in Table 3.1 shows that the modeling approach, as described above, results in a robust and valid estimation that matches the actual γ values. In this particular case, a $CE = \$35{,}355$ produces a gamma coefficient of 1. As can be seen above, of the 10 values, the only diversion of the estimate from the actual γ value occurred at the highest risk-aversion level, although the difference was relatively minor and the result of rounding errors. This shows that the model developed is “safe” to use for all other values of *CE* (i.e., values that fall between or above the 10 levels shown in the table). A list of many other *CE* amounts between this range, and the corresponding estimated gamma coefficients, based on the equations from above, in increments of $500, is provided in Appendix 3C.

Given the robustness of the model, the γ-value estimation process was expanded from 10 to 100 levels of *CE* based on increments of 0.1, as shown in the theoretical model. To demonstrate the calculation that can be completed using the model, if a financial decision-maker states their *CE* as, say, $34,000 (i.e., a dollar amount that falls between a γ value of 1 and 2), a more precise estimate can be made. Using the model from above shows that:

$$\sigma\ (\$34{,}000) = 0.23$$

$$\gamma(0.23) = 1.53$$

$$\gamma(\sigma(\$34{,}000)) = 1.53$$

$$\therefore \gamma = \ 1.5\ when\ CE\ amount\ is\ \$34{,}000.$$

It can be concluded that a financial decision-maker whose stated value of *CE* is $34,000 has a relative risk-aversion level of 1.5. This process can be repeated for any value of *CE* between the interval [0, 50 (i.e., $50,000)]. A list of various *CE* amounts between this range, and the corresponding estimated gamma coefficients, based on the equations from above, in increments of $500, is provided in Appendix 3C.

Validity Tests

As discussed above, with more details in Chapter 4, it is possible to convert *CE* dollar amounts into gamma coefficients across a wide spectrum of dollar amounts. Even so, this does not necessarily mean that these estimates are valid or useful as an input into investment management processes or financial planning models. It is essential that the resulting estimated gamma coefficients exhibit validity in terms of financial decision-maker behavior. Validity tests can also help provide evidence regarding the value of the measure of financial decision-maker risk aversion used in this study as a financial planning tool.

In this study, construct and concurrent validity tests were applied to test the robustness of the gamma coefficients and the risk-aversion measure. *Construct validity* refers to the accuracy of a measure or score when describing the concept of interest (Messick, 1980; Shepard, 1993). In other words, construct validity in this study indicates the degree to which the gamma coefficients and the measure of risk aversion describe a financial decision-maker's willingness to take risk as well as actual risk-taking behavior. Pearson product moment correlation coefficients were used to assess construct validity.

Concurrent validity tests are typically used to assess the outcome of one measure against another measure known to be associated with an outcome (in this study, the outcome represented financial decision-maker behavior) or how closely a measure relates to another existing similar measure (Messick, 1980; Shepard, 1993). A strong relation with other criteria known to be valid can signal that the new measure in question is also valid. In the current study, as stated in H_5, those who reported low *CE* dollar amounts on the risk-aversion measure were hypothesized to be risk-averse and hence more likely to invest less of their current financial portfolio in equities and a larger portion of their portfolio in cash or cash equivalent holdings. Concurrent validity was evaluated using Tobit regression models, with respondent portfolio equity and cash holdings, respectively, as the outcome variables and gamma coefficients derived from the transformation process as the independent variable, controlling for other factors.

The Tobit model is a censored normal regression model in which the outcome variable, typically y, is observed for values greater than zero but censored (not observed) for values that are less than or equal to zero (Sigelman & Langche, 1999). A Tobit model is generally used when the dependent variable contains negative values that have been censored to zero or are non-negative but clustered at zero. A Tobit model is designed to deal with biases introduced by censoring. When applied under the specific conditions for which it is appropriate, the Tobit model provides a useful supplement to a standard regression (OLS) model.

In this study, the Tobit model was operationalized as follows:

$$y_i = \begin{cases} y_i^* & if\ \ y_i^* > 0 \\ 0 & if\ y_i^*\ \leq 0 \end{cases} \qquad \text{(Equation 3.6)}$$

$$where$$

$y_i^* = \ x_i\,\beta + \ \varepsilon_i$ and y_i^* is the uncensored y or the true observed value of y, y_i is the censored outcome/dependent variable, x_i represents the independent variables, β represents coefficients, and ε_i is the error term. The standard Tobit model assumes that the dependent variable is censored at zero. If no censoring has occurred or if censoring has occurred but not at zero, then the standard Tobit specification is inappropriate (Sigelman & Langche, 1999). The error term, ε_i, is assumed to be independently and normally distributed.

Validity Assessment Variables

The validity tests described above were used in this study to determine whether the gamma coefficients derived from the transformation process (and by association, the measure of financial decision-maker risk aversion) offer financial decision-makers and financial planners who provide advice to financial decision-makers a robust insight into investing and financial preferences. First, construct validity tests were made to ensure that responses associated with the measure were related to other indicators of financial decision-maker risk aversion and risk-taking behavior. Second, concurrent validity was tested by evaluating the significance of item scores in differentiating among varying levels of risky asset ownership. The following discussion describes the validity measures used for these tests.

Validity Measure 1: Propensity Measure. The Grable and Lytton (1999) financial risk-tolerance assessment scale was used as a proxy propensity measure. The scale is comprised of 13 multiple-choice questions. The summed score from the scale is intended to classify financial decision-makers from low risk tolerance (high risk aversion) to high risk tolerance (low risk aversion). The scale has been widely used by researchers as a valid and reliable measure of financial decision-maker risk attitudes (Kuzniak et al., 2015). It was hypothesized that scores on the propensity measure would be negatively associated with gamma coefficients estimated from the measure of financial decision-maker risk aversion. The 13 items comprising the scale, in addition to the scoring methodology, are presented in Appendix 3A. Scale scores can theoretically range from 13 to 41, with higher scores being indicative of low risk aversion or high risk tolerance, respectively. The mean score for respondents was 24.85 with a standard deviation of 5.53. Scores on this scale were used to establish construct validity of the gamma coefficients (H_1).

Validity Measure 2: Survey of Consumer Finances (SCF) Risk-Aversion Item. The SCF is a triennial randomized cross-sectional survey of U.S. households sponsored by the Federal Reserve Board in cooperation with the Department of the Treasury. The survey includes comprehensive and nationally representative information on families' balance sheets, pensions, income, and demographic characteristics with detailed information about the structure of household finance and investment portfolio. The survey also includes a single-item risk-aversion question that has been widely used by researchers (e.g., Grable & Lytton, 2001; Grable & Schumm, 2010; Gilliam, Chatterjee & Grable, 2010; Yao, Hanna, & Lindamood, 2004). This item was used to establish construct validity of the gamma coefficients (H_2). The SCF asks:

Which of the following statements on this page comes closest to the amount of financial risk that you are willing to take when you save or make investments?

1. Take substantial financial risk expecting to earn substantial returns

2. Take above average financial risks expecting to earn above average returns

3. Take average financial risks expecting to earn average returns

4. Not willing to take any financial risks

For the purposes of this study, answers were reverse coded so that a score of 4 represents high risk tolerance (low risk aversion), whereas a score of 1 suggests a low level of risk tolerance (high risk aversion). It was hypothesized that scores on the SCF would be negatively associated with gamma coefficients derived from the measure of financial decision-maker risk aversion. The modal response on this measure was 3.0 (i.e., take average financial risks expecting to earn average returns), with 32.9% of respondents choosing this option. It is worth noting that about

26.0% of respondents indicated being not willing to take any financial risks, which indicates that the majority of respondents—approximately 59%—were risk averse.

Validity Measure 3: Revealed Preference Test 1. A seven-question revealed preference test developed by Hanna and Lindamood (2004) was used as a proxy for CRRA revealed preference measures. The Hanna and Lindamood questions were designed to measure retirement income gambles. Scores were coded so that 7 represents a high tolerance for risk (extremely low risk aversion), whereas a score of 1 represents a low level of risk tolerance (extremely high risk aversion). It was hypothesized that scores on the revealed preference measure would be negatively associated with gamma coefficients derived from the measure of financial decision-maker risk aversion. The questions and assessment procedure associated with the measure are shown in Appendix 3B. Test scores for this measure can range from 1 to 7. The modal response categories in this study were 1.00 and 4.00, which were both at 25.50%. Scores on this test were used to establish the construct validity of the gamma coefficients (H_3).

Validity Measure 4: Revealed Preference Test 2. A second revealed preference measure developed by Barsky et al. (1997) was used to validate the measure of financial decision-maker risk aversion. The Barsky et al. test asks respondents to indicate their willingness to gamble income on the probability of various increases or cuts to their family income. The questions have been included in recent versions of the Health and Retirement Survey, as well as other national surveys of household financial behavior. A risk-aversion score ranging from 1 (low risk tolerance/high risk aversion) to 4 (high risk tolerance/low risk aversion) can be estimated based on the following three items. It was hypothesized that scores on this revealed preference measure would be negatively associated with gamma coefficients derived from the measure of financial decision-maker risk aversion. The question pattern in the Barsky et al. procedure is as follows:

> *Suppose that you are the only income earner in the family, and you have a good job guaranteed to give you your current (family) income every year for life. You are given the opportunity to take a new and equally good job, with a 50–50 chance it will double your (family) income and a 50–50 chance that it will cut your (family) income by a third. Would you take the new job?*

If the answer to the first question is "yes," the questioning continues as follows:

> *Suppose the changes were 50-50 that it would double your (family) income, and 50-50 that it would cut it in half, would you still take the job?*

If the answer to the first question is "no," the questioning continues as follows:

> *Suppose the chances were 50-5- that it would double your (family) income and 50-50 that it would cut it by 20 percent, would you then take the new job?*

This test has been widely used by researchers who are interested in matching risk-aversion scores to health and retirement outcomes at the household level (Grable et al., 2020). Test scores for this measure can range from 1 to 4. In this study, the modal response category was 1.00 (43.80% of respondents). Test scores were used to establish construct validity (H_3).

Validity Measure 5: Willingness to Gamble Income. The following single item willingness to gamble question (see Blais & Weber, 2006) was included as a validity measure: *How likely is it that you would bet a day's income at a casino?* Answers to the question were scored as 1 = very unlikely to 10 = very likely. It was hypothesized that scores on this question would be negatively associated with gamma coefficients derived from the measure of financial decision-maker risk aversion. The modal response category was very unlikely (50.47% of respondents). Test scores were used as a validity measure (H_4).

Validity Measure 6: Portfolio Holdings. Two variables were used to assess the concurrent validity of the gamma coefficients derived from the proposed measure of financial decision-maker risk aversion. Respondents were asked to indicate the percent of their entire portfolio (excluding personal assets and their primary residence) held in cash and equities. The question was asked as follows: *Suppose that you were to take a snapshot of your current financial position. Approximately what percent of your total savings and investments are in cash or cash equivalents* and *equities, not including your home?* Data obtained from this question were used to determine the degree to which gamma coefficients were able to differentiate between and among risk-taking behavior. As stated in H_5, risk-aversion scores were expected to be negatively associated with equity portfolio holdings and positively associated with cash holdings. Tobit regression models were used to establish the concurrent validity of the measure.

Control Variables for the Validity Tests

An important element associated with the construct and concurrent validity tests, as described in more detail later in this chapter, was the use of personal characteristics and other variables, used as controls, when estimating the association between gamma coefficient scores and portfolio holdings. Several demographic characteristics, introduced in Chapter 2, were included in the validity test models.

A respondent self-identified gender was coded 1 = male and 2 = female. The sample was approximately evenly disturbed between male and female respondents with 51.2% of respondents being female, which was close to the current national gender demographic of 50.8% female (U.S. Census Bureau, 2019).[15] The racial demographic distribution was close to the current racial national distribution in the United States (U.S. Census Bureau), with the sample being self-identified as 62% Caucasian/White, 13.5% Black/African American, 10.7% Hispanic/LatinX, 6.2% Asian, 2.6% Native American, and 4.9% other. Because of sample size limitations, the other category was combined with the Native American category. The modal category was Caucasian/White (62.03%).

Age was measured in years. The mean age of respondents was 45.08 years with a standard deviation of 16.53 years. Marital status was assessed using four categories: (a) never married, (b) single but living with a significant other, (c) separated/divorced/widowed, and (d) married. Approximately 49% of respondents indicted being married at the time of the survey. The mean response was 2.57 people. Employment status was coded as 1 = full-time, 2 = part-time, or 3 = other. Nearly 42% of respondents were employed on a full-time basis. Retirement status was coded 1 = retired, otherwise 0. Approximately 18% of respondents indicted being currently retired.

Home ownership was coded dichotomously with those owning a home, with or without a mortgage, coded 1, otherwise 0. Approximately 60% of respondents were homeowners. Household income was assessed using 11 categories ranging from 1 = none to 11 = above $100,000. The modal category was $100,000 or above (23.50%). Formal attained education was measured using the following six categories: (a) some high school or less, (b) high school graduate, (c) some college/trade/vocational training, (d) Associate degree, (e) Bachelor’s degree,

[15] Although respondents were given a choice to select ‘non-binary’ or ‘other’ as a gender choice, no one self-identified as other than male or female.

and (f) graduate or professional degree. The modal education category was a bachelor's degree at 27.7%. Because of sample size limitations, the some high school or less and high school graduate categories were combined in the final analysis.

Chapter Summary

This chapter provided detailed information about the purpose of the dissertation and the manner in which the research hypotheses were tested. The chapter also presented the proposed measure of financial decision-maker risk aversion that was tested in this study. A key feature of the chapter was the presentation of the process used to transform *CE* dollar amounts to gamma coefficients. A discussion of the way each variable used in the analyses was operationalized was also presented. The chapter concluded with an overview of the methods used to test the validity of the gamma coefficients as descriptors of financial decision-maker behavior. The remaining chapters in this dissertation include Chapter 4, which presents the study results, and Chapter 5, which provides implications and application of this study's findings.

CHAPTER 4

ANALYSIS AND RESULTS

The primary goal of this dissertation is to expand the financial planning and financial risk-aversion (risk-tolerance) assessment body of knowledge by introducing a direct measure of investor risk aversion that utilizes CRRA principles and methodology. With this goal in mind, the first purpose of the dissertation was to combine the concept of risk tolerance typically used by financial planners in research and practice with the concept of risk aversion used by economists by introducing a new measure of financial decision-maker risk aversion. The second purpose of the dissertation was to provide a mathematical transformation procedure that can be used to convert certainty equivalent (*CE*) amounts to risk aversion coefficients in a way that matches the theoretical framework described in Chapter 1.

This chapter describes the results of tests used to validate the proposed risk-aversion measure and to demonstrate the value of gamma coefficients obtained from the proposed measure in describing risk-taking behavior. This chapter also provides the demographic characteristics of the study sample, discusses the representativeness of the sample, and reports on the statistical results of the study, including the validity test findings.

Currently, researchers and financial service professionals use propensity measures and revealed preference methodologies to measure a household financial decision-maker's risk-aversion level. As discussed in chapter 1, both approaches offer unique advantages but with notable limitations. The new proposed measure builds on the advantages of these methods while solving many of the problems associated with other measurement techniques. The validity tests of the proposed new measure and the transformation procedure are not only a significant part of this chapter but also of the dissertation. Hence, it is critical that the findings of the validity tests support the application of the new measure.

Demographic Characteristics of the Sample

The data for this study were collected in 2019 using Qualtrics and distributed by the data platform, Dynata. The sample was approximately evenly disturbed between male and female respondents with 51.2% of respondents being female, which was close to the current national gender demographic of 50.8% female. The racial demographic distribution was also near the current racial distribution in the United States (U.S. Census Bureau, 2019). The mean age among respondents was 45.08 years with a standard deviation of 16.53 years. Table 4.1 provides a detailed description of the sample. The sample frame was delimited to include primarily financial decision-makers who were engaged in the financial planning process and those who were generally better educated. In this regard, respondents were more representative of high-income U.S. households with a middle-aged head of household. While the results may not be generalizable to the entire population, the results can, nonetheless, help financial planners and other financial services professionals who are actively seeking to build an optimal portfolio based on their clients' risk-aversion level.

Table 4.1. Descriptive Analysis: Sample Demographic Information

Variable	Percentage	*M*	*SD*
Gender			
Male (coded 1)	48.8%		
Female (coded 2)	51.2%		
Age (in years)		45.08	16.53
Marital Status			
Never Married	27.3%		
Not Married/Living w/Sig. Other	10.7%		
Married	49.3%		
Separated	1.7%		
Divorced	8.2%		
Widowed	2.8%		
Employment Status			
Part-Time	17.7%		
Full-Time	41.5%		
Retired	18.2%		
Not Employed	17.1%		
Other	5.5%		
Racial/Ethnic Background			
Caucasian/White	62.0%		
Black/African American	13.5%		

Variable	Percentage	*M*	*SD*
Hispanic/LatinX	10.7%		
Native American	2.6%		
Asian or Pacific Islander	6.2%		
Other	4.9%		
Housing			
Own without a Mortgage	29.5%		
Own with a Mortgage	31.0%		
Rent	29.9%		
Live with Relative	9.0%		
Other	0.6%		
Household Income			
$0	3.4%		
Less than $20,001	11.7%		
$20,001 - $30,000	9.1%		
$30,001 - $40,000	7.6%		
$40,001 - $50,000	9.1%		
$50,001 - $60,000	8.3%		
$60,001 - $70,000	7.0%		
$70,001 - $80,000	6.1%		
$80,001 - $90,000	8.1%		
$90,001 - $100,000	6.1%		
Above $100,000	23.5%		

Variable	Percentage	*M*	*SD*
Education			
Some High School or Less	4.0%		
High School Graduate	18.6%		
Some College/Trade/Vocation Training	22.8%		
Associate Degree	10.1%		
Bachelor's Degree	27.7%		
Graduate or Professional Degree	16.9%		

Report of Statistical Results for Proposed New Measure

The proposed financial risk-aversion measure asked decision-makers to indicate their preferred level of investment associated with an opportunity as described in the following question:

> *Assume that your financial advisor approaches you with the following investment opportunity. You have an opportunity to make an investment that will return either $25,000 or $50,000. Your financial advisor estimates that the probability of receiving $25,000 is 50% and the probability of receiving $50,000 is also 50%. Your financial advisor tells you that demand for this investment is high and that only those who make the largest investments will be given shares. Based on this information, how much would you be willing to invest, assuming you had the money?*

Respondents were then instructed to indicate their preferred level of investment using a scale that ranged from 0 and 50, with 50 indicating $50,000. Table 4.2 shows the mean value indicated by respondents. This degree of risk aversion represents what one would expect of a valid assessment instrument; that is, the mean of 25.74, as shown below in Table 4.2, was close to the center of the scale, indicating that the typical respondent exhibited an average level of financial risk aversion.

Table 4.2. Descriptive results for the Proposed New Measure

	M	***SD***
Investor response in Thousands of Dollars	25.74	14.16

As a reminder, the nature of the question allowed respondents to enter any value without the bias of a scale, chart, or predetermined dollar amount. The use of an open-ended response choice set solved what is a typical problem with many existing risk-assessment tools—providing a baseline notion of what is acceptable to a decision-maker by constraining choice options. However, providing a continuous open scale to decision-makers as a response option creates unique challenges, both mathematical and practical. The open-ended nature of the question makes it difficult to convert respondent-provided dollar amounts into gamma estimates. The methodology developed in Chapter 3 provides a mathematical procedure to convert responses into gamma coefficients.

Hence, to proceed with the next step of validating the measure, responses were converted to the appropriate dollar value (i.e., thousands) and then transformed into a constant relative risk-aversion (CRRA) risk-aversion (γ) coefficient for each respondent. Table 4.3 shows a sample of the conversion process that was performed for the values received from decision-makers.

Table 4.3. Samples of investor responses and their conversion to Gamma Coefficients

Respondent Answer	Equivalent *CE* Value	Equivalent γ Value
29	$29,000	5.5
32	$32,000	2.8
33	$33,000	2.1
34	$34,000	1.5
35	$35,000	1.0

The gamma values (γ) shown in the third column of Table 4.3 were then used in the validity tests of the measure. It is important to note, at this point, that the respondent provided estimates, as shown in the first column of Table 4.3, could have also been used in the validity tests; however, it was decided that the γ measure provided a more accurate estimate of CRRA. Additionally, it the estimate of γ, rather than a respondent-provided dollar answer, that is most relevant in the context of Modern Portfolio Theory (MPT). Specifically, it is each respondent's γ estimate that is used in the efficient portfolio modeling procedure.

Report of Statistical Results for the Validity Measures

Two types of validity measures were used to confirm the appropriateness of the proposed measure and the results of the gamma transformation process. Table 4.4 provides a summary of the descriptive statistics for the propensity- and revealed preference measures, tests, and items that were used in the construct validity tests. Respondents consistently exhibited high-risk aversion across the measures. Scores on the Grable and Lytton (1999) propensity scale fell into the average range. Scores on the Barsky et al. (1997) test indicated that investors were also risk-averse with a means score of 1.6. Respondents fell into the average category in terms of the single-item SCF risk-aversion item, with a significant percentage of respondents indicating that they were not willing to take much, if any financial, risk. Scores on the Hanna and Lindamood (2004) revealed preference test also indicated high-risk aversion with a mean score of 2.9. The mean score on the Blais and Weber (2006) item asking about the likelihood of betting a day's income at a casino also reflected above-average risk aversion with a value of 3.48. In terms of risk-taking behavior, it was determined that respondents held approximately 51% of their portfolios in cash and cash equivalents and 17% of their portfolios in riskier equity assets.

Table 4.4. Descriptive Statistics for the Validity Measures

Variable	**Percentage**	***M*** (***SD***)
Proposed New Measure		25.74 (14.16)
Grable and Lytton Propensity Scale		24.85 (5.53)
SCF Risk-Assessment Item		2.31 (1.02)
Not Willing	25.8	

Average	32.9	
Above-Average	26.0	
Substantial	15.3	
Hanna and Lindamood Revealed		2.90
Preference Test		(1.96)
0.00	11.0	
1.00	25.5	
2.00	5.7	
3.00	11.2	
4.00	25.5	
5.00	13.3	
6.00	3.7	
7.00	4.2	
Barsky et al. Revealed Preference Test		1.60
		(1.05)
0.00	11.0	
1.00	43.8	
2.00	26.3	
3.00	12.2	
4.00	6.7	

Blais and Weber -Casino Gambling	3.48
	(3.14)
Portfolio Cash Holdings	50.91
(in percent)	(39.97)
Portfolio Equity Holdings	16.99
(in percent)	(25.42)

Construct Validity Test Results

The first of the validity tests were used to evaluate construct validity. The tests were made to measure the correlation of the new measure with other measures that are already known to assess the same underlying construct, in this case, risk aversion. For the purposes of this study, concurrent validity refers to the accuracy of the proposed new measure of risk aversion and the degree to which the gamma coefficients describe a financial decision-maker's willingness to take a risk. The first five validity-check measures were evaluated using Pearson correlation coefficients. The sixth validity test (i.e., concurrent validity) was evaluated using Tobit regression models.

Table 4.5 shows the correlation coefficient estimates for the variables of interest in this dissertation. The proposed measure was found to be statistically significantly associated with six of the seven measures of financial risk-aversion. The propensity scale, the SCF item, the Hanna and Lindamood scale, the Barsky et al., and the casino gambling item were coded so that higher scores indicated lower risk aversion. As such, it was expected that a high score on the proposed measure would be negatively associated with scores on these validity measures. The only non-significant relationship was between the proposed measure, and the Barsky et al. revealed preference measure. These findings were not surprising because it corresponds with what others have reported when validity tests using the item have been noted (e.g., Grable et al., 2020). When reviewing the coefficients in Table 4.5, a high score on the proposed measure was expected to be negatively related to equity ownership but positively associated with cash holdings. These associations were confirmed.

Table 4.5. Construct Validity Correlation Coefficients

	Direct CRRA Measure	Propensity Scale	SCF	Hanna & Lindamood	Barsky et al. Scale	Casino Gambling	Cash	Equities
Direct CRRA Measure	1.00							
Propensity Scale	-0.31**	1.00						
SCF	-0.34**	0.52**	1.00					
Hanna & Lindamood	-0.06*	0.21**	0.08	1.00				
Barsky et al. Scale	-0.05	0.21**	0.04	0.49**	1.00			
Casino Gambling	-0.32**	0.43**	0.53**	-0.05	-0.09*	1.00		
Cash Holdings	0.15**	-0.34**	-0.32**	-0.02	-0.01	-0.17**	1.00	
Equities	-0.11**	0.24**	0.16**	0.10*	0.03	-0.00	-0.55**	1.00

Note: ***$p < .001$ **$p < .01$ *$p < .05$

Concurrent Validity Test Results

Concurrent validity was evaluated using Tobit regression models, with cash and equity holdings, respectively, as the outcome variables and gamma coefficients derived from the transformation process as the independent variable, controlling for variables known to be associated with risk-taking attitudes and behavior as noted previously. Table 4.6 shows the results from the multivariate Tobit regression that was designed to test the concurrent validity of the proposed measure in relation to investor portfolio *cash* holdings. The relationship was found to be statistically significant and positively associated, which means high scores on the measure were directly and positively associated with greater cash holdings. These findings were consistent with the financial postulate that suggests risk-averse financial decision-makers should be more likely to hold assets, such as cash or cash equivalents, in their portfolio.

Table 4.6. Tobit Regression Results for Cash Holdings

Variable	**Coefficient**	**Standard Error**	***P* value**
Constant	55.45	12.00	0.00
Marital Status			
Never Married	6.37	4.92	0.20
Live w/Sig. Other	-2.35	6.35	0.71
Sep/Div/Wid	11.94	5.60	0.03
Employment Status			
Part-Time	5.96	4.96	0.23
Retired	-7.89	5.87	0.18

Others	6.73	5.02	0.18
Racial/Ethnic			
Black/African	-7.87	5.44	0.15
Hispanic/LatinX	-5.41	5.83	0.35
Asian	0.24	7.35	0.97
Other Race	14.04	6.88	0.04
Education			
Some College	-6.11	5.18	0.24
Associate	0.78	6.51	0.90
Bachelor	-11.90	5.35	0.03
Graduate Degree	-10.60	6.34	0.09
Gender	9.20	3.77	0.01
Age	0.09	0.14	0.55
Own Home	-14.37	4.20	0.00
Household Income	-0.73	0.68	0.29
Proposed Measure	-0.28	0.12	0.03

Additionally, other variables were significantly associated with the level of cash assets held by decision-makers. Compared to those who were married, those who were separated, divorced, or widowed were more likely to hold cash in their portfolios. In comparison to Caucasians/Whites, those who self-identified as 'Other' as a racial/ethnic identity were found to hold more cash. Cash assets were lower among those holding a bachelor’s degree compared to those with a high school diploma or less. Women reported holding more cash in their portfolios, whereas homeowners were less likely to hold cash compared to those who did not own a home at the time of the survey.

Concurrent validity was also evaluated using another Tobit regression model using portfolio equity holdings as the outcome variable. Similar to the above model, a decision-maker's gamma coefficient, derived from the transformation process, was the independent variable in the model using the same control variables shown in Table 4.6. Table 4.7 shows the results from the multivariate Tobit regression. The relationship between equity holdings and gamma estimates was found to be statistically significant. The relationship, as expected, was negative, indicating high scores on the new measure were inversely associated with the percentage of equity assets held in a decision-maker's portfolio. This is consistent with the extant literature that suggests risk-averse individuals are less likely to hold risky assets, such as equities, in their portfolios.

Table 4.7. Tobit Regression Results for Equity Holdings

Variable	**Coefficient**	**Standard Error**	***P* value**
Constant	-20.28	13.95	0.14
Marital Status			
Never Married	3.69	5.63	0.51
Live w/Sig. Other	-1.59	7.59	0.84
Sep/Div/Wid	-24.37	6.95	0.00
Employment Status			

Part-Time	7.36	5.73	0.20
Retired	12.22	6.43	0.06
Others	15.84	6.31	0.01
Racial/Ethnic			
Black/African American	-1.57	6.36	0.81
Hispanic/LatinX	-12.89	7.31	0.08
Asian	-6.10	8.24	0.46
Other Race	-16.91	9.29	0.07
Education			
Some College	14.84	6.51	0.02
Associate	7.98	8.14	0.33
Bachelor	23.85	6.45	0.00
Graduate Degree	25.96	7.36	0.00
Gender	-13.79	4.29	0.00
Age	0.15	0.16	0.36
Own Home	1.90	4.98	0.70
Household Income	2.00	0.78	0.01
Proposed Measure	0.32	0.14	-0.02

The results from Table 4.7 confirm the robustness of the proposed measure but additionally also highlight other significant associations between variables in the study. As shown in Table 4.7, women reported holding portfolios with fewer equities, which is in line with the discussion of this variable in the literature review. Compared to married individuals, those who were separated, widowed, or divorced were less likely to invest in equities. Those with some college, a Bachelors' degree, or a graduate degree were more likely to hold a greater proportion of their portfolio assets in equities than those with a high school education or less. This result is also consistent with current literature as discussed in the literature review. Finally, a positive association between household income and equity asset ownership was observed, which also aligns with established literature on the subject.

Overall, the findings from the statistical tests confirmed the validity of the proposed measure of financial risk aversion on both counts of construct and concurrent validity. Tobit results provided support for the concurrent validity of the proposed measure, with four out of the five construct validity measures quantitively being related to gamma coefficient estimates. On the whole, the test results reported above substantiate that the proposed measure assesses what it claims to measure—individual-level financial risk aversion.

Report of Hypothesis Results

The following hypotheses relate to the validity test outcomes described in Tables 4.5 through 4.7:

Support for the hypothesis that stated, "Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of financial risk aversion are significantly (negatively) correlated with scores from a propensity measure of financial decision-maker risk aversion (validity measure one)" was found. The gamma coefficients obtained from the new measure of financial risk aversion were significantly and negatively correlated with scores from validity measure one, which was the financial risk-tolerance assessment scale/propensity measure developed by Grable and Lytton (1999).

Support was also noted for the second hypothesis, which stated, "Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are significantly (negatively) correlated to someone's stated risk preference (validity measure two)." Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion were significantly and negatively correlated to validity measure two, which was the SCF item.

Partial support was noted for the third hypothesis, which stated, "Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are significantly (negatively) correlated to scores from other types of revealed preference risk-aversion measures (validity measures three and four)." Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion were significantly and negatively correlated to scores from the Hanna and Lindamood (2004) test, but not significantly associated with scores from the Barsky et al. (1997) revealed preference test.

Support was noted for the fourth hypothesis, which stated, "Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are significantly (negatively) correlated to the likelihood of gambling income (validity measure five)." Financial decision-maker CRRA scores (i.e., gamma coefficients) obtained from the proposed measure of risk aversion were significantly and negatively associated with scores from the willingness to gamble question developed by Blais and Weber (2006).

Finally, support for the fifth hypothesis, which stated, "Financial decision-maker CRRA scores (gamma coefficients) obtained from the proposed measure of risk aversion are negatively associated with equity portfolio holdings, and positively associated with cash holdings, when controlling for factors known to be associated with financial risk aversion and portfolio choices (validity measure six)" was obtained. Financial decision-maker CRRA scores (i.e., gamma coefficients) from the proposed measure of risk aversion were significantly and negatively associated with equity portfolio holdings and positively associated with cash and cash equivalent holdings when controlling for factors known to be associated with financial risk aversion and portfolio choices.

In summary, findings from the validity tests indicated that scores from the new proposed measure were associated with known personal characteristics and risk-taking behaviors of financial risk-takers. The findings from the tests confirm that the proposed measure, and the mathematical transformation process, appear to be valid.

Summary

Table 4.8 summarizes the results from this study in relation to the research questions proposed in Chapter 1.

Table 4.8. Summary Comments Related to Dissertation Research Questions

Research Question	Comments
Can an investment choice question be used to elicit a revealed preference response from financial decision-makers in a valid manner?	Given the validity findings from the regression and Tobit tests, the dissertation has shown that a well-designed investment choice question can be used to elicit responses to measure financial decision-maker (and investor) risk-

	aversion in a valid manner.
Is it possible to estimate CRRA (gamma) scores from self-reported investment preferences using an investment choice question?	The results of the tests reported in this chapter have shown that it is possible to use CRRA measures to estimate gamma scores from a self-reported investment preference using an investment question. The certainty equivalent amounts were successfully converted into gamma scores; this confirms that it is possible to estimate CRRA gamma coefficients from self-reported investment preferences using this particular risk-aversion question.
Is it possible to develop a mathematical model that can convert any *CE* value to a gamma score?	The findings from the tests reported in this chapter have shown that it is possible to develop and successfully convert any *CE* value to a gamma score. The validity tests confirmed the transformation process from *CE* values to gamma scores.
Can a mathematical model be developed to show how CRRA scores can be used to design an efficient portfolio in an MPT context for any continuous level of gamma?	The validity tests reported in this chapter suggest that CRRA scores derived from the proposed measure can be mapped to an MPT efficient frontier. The proof of

	this assertation will be presented in Chapter 5.

Chapter Summary

This chapter began by revisiting the purpose of the dissertation and providing a brief review of the methodology used to test the validity of the proposed risk-aversion measure and the score transformation process. This chapter presented (a) the demographic characteristics of the sample, (b) a report of the statistical results used to validate the proposed risk-aversion measure, (c) a report of the results of the research hypotheses, and (d) a report of answers to the dissertations broader research questions. As noted in this chapter, support related to the validity of the proposed measure and the score transformation process was obtained. Based on these results, the dissertation will conclude in Chapter 5 by discussing how the findings can be applied in practice. Chapter 5 will also present findings in the context of implications for research, practice, and policy.

CHAPTER 5

DISCUSSION AND CONCLUSION

The purpose of this chapter is to summarize the findings from this study and to show how risk-aversion gamma coefficients—derived from the risk-aversion measure introduced in this dissertation—can be used to select optimal portfolios. This chapter ties together elements that are typically used to build optimal portfolios, including utility theory, constant relative risk aversion (CRRA), and risk indifference curves (in the context of Modern Portfolio Theory). Additionally, the chapter provides suggestions and recommendations for future studies in the context of study limitations. This chapter concludes this dissertation through integration with the first four chapters, which were presented as follows:

The first chapter in this dissertation provided an overview of the study and established a justification for the study, as well as the purpose of the dissertation and the rationale for the subsequent analytical work presented later in the dissertation. This was followed by the theoretical framework for the study and a presentation of the research questions used to guide the analyses. The second chapter provided a review of the relevant literature by focusing on the three core tenants of the dissertation. These were (a) an expanded background on the historical context for the study of risk aversion and utility theory, (b) a detailed account of variables historically used in the study and measurement of decision-maker risk aversion, and (c) a review of the historical context of Modern Portfolio Theory (MPT) with an emphasis on showing how estimates of risk aversion can be used to select optimal portfolios for individual financial decision-makers. The third chapter introduced the sample and data used in this dissertation. The chapter also provided a detailed account of how the hypotheses were tested. The chapter presented the proposed measure of financial decision-maker risk aversion that was tested in this study. A key feature of the third chapter was the presentation of the process used to transform certainty equivalent (*CE*) dollar amounts to gamma coefficients. A discussion of the way each of the variables were used in the analyses and operationalized was also presented. The chapter concluded with an overview of the methods used to test the validity of the gamma coefficients as descriptors of financial decision-maker behavior. The previous chapter (Chapter 4) presented the demographic characteristics of the sample, a report of the statistical results used to validate the proposed risk-aversion measure, a report of the results of the research hypotheses, and a report of answers to the dissertation's broader research questions. As a key takeaway from Chapter 4, the validity tests confirmed the construct and concurrent validity of the proposed risk-aversion measure. The current chapter extends the discussion of risk aversion by showing how the findings presented in Chapter 4 can be applied in practice.

Summary of Findings

As noted throughout this dissertation, risk is an inescapable part of daily life. The concept of risk serves as a cornerstone of nearly all areas of personal finance decision scenarios and financial planning practice activities. Day-to-day financial decisions heavily rely on accurately assessing and understanding financial risk and a financial decision-maker's willingness to take financial risk—a concept encompassed in the notions of risk aversion, risk tolerance, and risk preference. In the context of this dissertation, risk was defined to include decision-making situations under which all potential outcomes, with associated probabilities, are known to a decision-maker prior to engagement in the decision-making process. The notion of subjective probability assessment connects risk with uncertainty. It is a decision-maker's subjective evaluation of probabilities that links risk and uncertainty together and shines a light on a financial decision-maker's risk-taking behavior, their assessment of risk, personal preferences for taking or avoiding risk, and degree of unwillingness they are likely to exhibit when taking a financial risk when faced with uncertainty.

This dissertation, in the course of seeking to answer the study's research questions, confirmed that a well-designed single-item investment choice question can effectively and validly be used to elicit responses to measure a financial decision-maker's (e.g., an investor's) risk-aversion. The validity results from the correlational, regression, and Tobit tests (see Chapter 4) confirmed this conclusion. In addition, the results of the validity tests added support to the argument that the procedure developed to convert estimates of a financial decision-maker's certainty equivalent amount to a personal risk-aversion coefficient (i.e., gamma score) is both possible and compelling. Analytical results from Chapter 4 suggest that it is possible to estimate CRRA gamma coefficients from a self-reported direct measure of risk-aversion investment preference question. The following discussion highlights the implications of this study.

Research and Practice Implications

Results from this study show that it is possible to use an algebraic mathematical model when not only estimating gamma values from *CE* values but also when calculating *CE* amounts from gamma values[16]. The mathematical model developed using Wolfram Alpha can be used to convert gamma values, even those that include decimals, to their *CE* equivalent. The process

[16] Although this dissertation was focused on describing a process for converting *CE* amounts to gamma values, this dissertation shows that the opposite transformation can also occur.

described below offers a relatively easy and straightforward alternative to the methodology discussed in the theoretical framework of this dissertation.

The model is polynomial of degree two that uses gamma as an input value resulting in *CE* values as the output. The model is as follows:

$$CE(\gamma) = 114.23\gamma^2 - 2{,}122.4\gamma + 37{,}101 \qquad \text{(Equation 5.1)}$$

where $CE(\gamma)$ represents the certainty equivalent for an individual based on their risk-aversion level, γ. The algebraic function above works for any discrete value of γ as well as for any other γ on a continuum and can be used to calculate the same values shown in Table 1.1 for the given investment question used to design the proposed measure. However, unlike the calculus model used in the theoretical model, this process can simply be inserted into an ordinary scientific calculator.

For example, for $\gamma = 1$,

$$W_{CE}(1) = 114.23(1)^2 - 2122.4(1) + 37101 = \$35{,}092.83.$$

The process can be repeated for $\gamma = 2$,

$$W_{CE}(2) = 114.23(2)^2 - 2122.4(2) + 37101 = \$33{,}313.12.$$

In addition, this equation can be used for any continuous value of γ. For example, $\gamma = 1.1$,

$$W_{CE}(1.1) = 114.23(1.1)^2 - 2122.4(1.1) + 37101 = \$35{,}143.68,$$

or for $\gamma = 2.5$,

$$W_{CE}(2.5) = 114.23(2.5)^2 - 2122.4(2.5) + 37101 = \$32{,}432.20.$$

Table 5.1 shows a side-by-side comparison of values from [1,10] demonstrating the proximity of each of the values based on the algebraic model compared to the CRRA model.

Table 5.1. Gamma Coefficients and Comparison of Their Corresponding *CE* Values

γ	*CE* values based on CRRA Model	*CE* values based on Algebraic Model
1	$ 35,355.34	$ 35,092.83
2	$ 33,333.33	$ 33,313.12
3	$ 31,622.78	$ 31,761.87
4	$ 30,285.34	$ 30,439.08
5	$ 29,282.98	$ 29,344.75
6	$ 28,541.26	$ 28,478.88
7	$ 27,989.13	$ 27,841.47
8	$ 27,571.57	$ 27,432.52
9	$ 27,249.41	$ 27,252.03
10	$ 26,995.64	$ 27,300.00

Table 5.1 compares the calculation based on the algebraic model shown in Equation 5.1 to the CRRA model explained in the theoretical framework in Equations 1.1 through 1.23. The simplified model (Equation 5.1) replicates results that are only 0.43% points away from the more precise but complex CRRA model. The use and application of a simpler mathematical model for

converting gamma coefficients to *CE* values allow anyone with a calculator to use the results presented in this dissertation to find *CE* values with relative ease and without the need to delve into sophisticated economic concepts. In this way, researchers (and financial planners and investment managers) can more easily apply estimates of *CE* when evaluating the risk attitudes, preferences, and behaviors of financial decision-makers.

Another important implication to arise from the results of this dissertation relates to applying stated *CE* values from the risk-aversion measure to gamma estimates. Specifically, this study provides a pathway for those who are interested in the dual concepts of financial risk aversion and portfolio management to transform risk-aversion assessment scores into Modern Portfolio Theory (MPT) inputs that correspond to the efficient frontier and an optimal portfolio.[17] The theoretical foundation of this process was detailed in Chapters 1 and 2. In practice, this can be done by first obtaining a financial decision-maker *CE* value using the proposed risk-aversion measure and converting the *CE* into a gamma value as shown in Table 4.3.[18]

The following example demonstrates the application of MPT using one financial decision-maker's answer when responding to the new risk-aversion question (i.e., a *CE* of $33,000), which is equivalent to a gamma (γ) value of 2.1 (i.e., the risk-aversion gamma coefficient).

The gamma (γ) value can be used as an input to create a risk indifference and utility

[17] In this regard, the dissertation makes a significant contribution to the existing literature by helping advance the work of researchers, financial service practitioners (e.g., financial planners), and household financial decision-makers when identifying and applying estimates of preference with more precision and with more ease.
[18] As described in Chapter 3, the input/output process estimation can be completed using Equations 3.3 – 3.5. This model can be used for any *CE* dollar value obtained from the question used in this dissertation as an input and return a γ value, with the γ value indicating a financial decision-maker's risk-aversion level on a scale [1,10].

curve for the financial decision-maker in the following equation.

$$U = ER - (.005 * SD^2 * \gamma) \qquad \text{(Equation 5.2)}$$

$$where$$

$$U = Utility$$

$$ER = Expected\ Return$$

$$SD = Standard\ Deviation$$

$$\gamma = Risk\ Aversion\ Coefficient$$

$$and, 0.005\ is\ a\ normalizing\ factor\ to\ reduce\ the\ size\ of\ the\ standard\ deviation.$$

The utility obtained by an asset can be calculated using Equation 5.2 by including the value of gamma, the risk (or standard deviation (SD)) coefficient of the asset, and the asset's expected return. In this case, when the risk-aversion coefficient of the decision-maker is 2.1, the equation becomes:

$$U = ER - (.005 * SD^2 * 2.1) \qquad \text{(Equation 5.3)}$$

To simplify the example, assume that the risk-free rate is equivalent to the expected rate of return ($R_f = 4\%$), which means $SD = 0\%\ and\ ER = 4\%$. For this risk-free asset, Equation 5.2 becomes $U = 4\%$ for all investors. In other words, or $U = ER = 4\%$ for all investors at the risk-free rate. Stated another way, the 4% denotes a *CE* value to which all other investments and assets can be compared. Using this example, Equations 5.3 can be adjusted to show the expected rate of return for an asset (or portfolio) for this investor, where γ = 2.1, as follows:

$$ER = 4 + (.005 * SD^2 * 2.1) \qquad \text{(Equation 5.4)}$$

In the context of Equation 5.4, an additional input is needed: the standard deviation of the asset (or portfolio) to be compared to the risk-free asset. For this example, assume the asset (or portfolio) has a standard deviation (*SD*) of 10. Given this assumption, the expected return of the asset (or portfolio) is calculated as follows:

$$ER = 4 + (.005 * 10^2 * 2.1) \qquad \text{(Equation 5.5)}$$

$$ER = 5.05\%$$

The result 5.05% represents the minimum return needed to be compelled to invest in the asset (or portfolio). Stated another way, the 5.05% return provides the same degree of utility as a risk-free rate of return equal to 4%, for a financial decision-maker with a γ of 2.1.

Now assume, for the same financial decision-maker with risk-aversion coefficient of 2.1, that the individual has an opportunity to acquire a higher risk-free return rate equal to 5% (i.e., $R_f = 5\%$). Given the new opportunity with a higher R_f, the same decision-maker will require a higher return rate from other investment assets (or investment portfolios) to be compelled to invest elsewhere instead of the risk-free investment. To determine the new requirement for the expected return, for the same investment with $SD = 10\%$, Equations 5.2 through 5.5 can be modified as follows:

$$ER = 5 + (.005 * SD^2 * 2.1) \qquad \text{(Equation 5.6)}$$

$$ER = 5 + (.005 * 10^2 * 2.1) \qquad \text{(Equation 5.7)}$$

$$ER = 6.05\%.$$

In the same manner, the result of 6.05% represents the minimum return needed to be compelled to invest in the asset (or portfolio) for the same financial decision-maker with γ = 2.1. The new investment with $SD = 10\%$ and $ER = 6.05\%$ provides the same degree of utility as a risk-free rate of return equal to 5% for a financial decision-maker with a γ of 2.1.

Depending on the level a risk-free return rate offers, it is possible to use the same logic and calculation from above to estimate different utility curves for the same financial decision-maker, with the only difference in the curves being the assumed risk-free return rate. Figure 5.1 shows the results of the estimation process based on different risk-free rates of return for a financial decision-maker with a $\gamma = 2.1$. For interpretation purposes, each utility curve represents a set of choices, based on a fixed risk-free return rate, to which an asset (or portfolio) will be compared. The resulting expected rate of return, based on a $\gamma = 2.1$ and a given risk-free rate of return, is the value that makes a financial decision-maker indifferent between the risk-free rate of return and another investment (or portfolio) with a given risk and expected return.

To select an optimal portfolio, the risk-indifference curves shown in Figure 5.1, need to be overlayed onto the efficient frontier. The point at which one of the indifference curves intersects the efficient frontier at a single point is the portfolio that will yield the best or optimal risk-return trade-off given the risk-aversion level of the financial decision-maker.

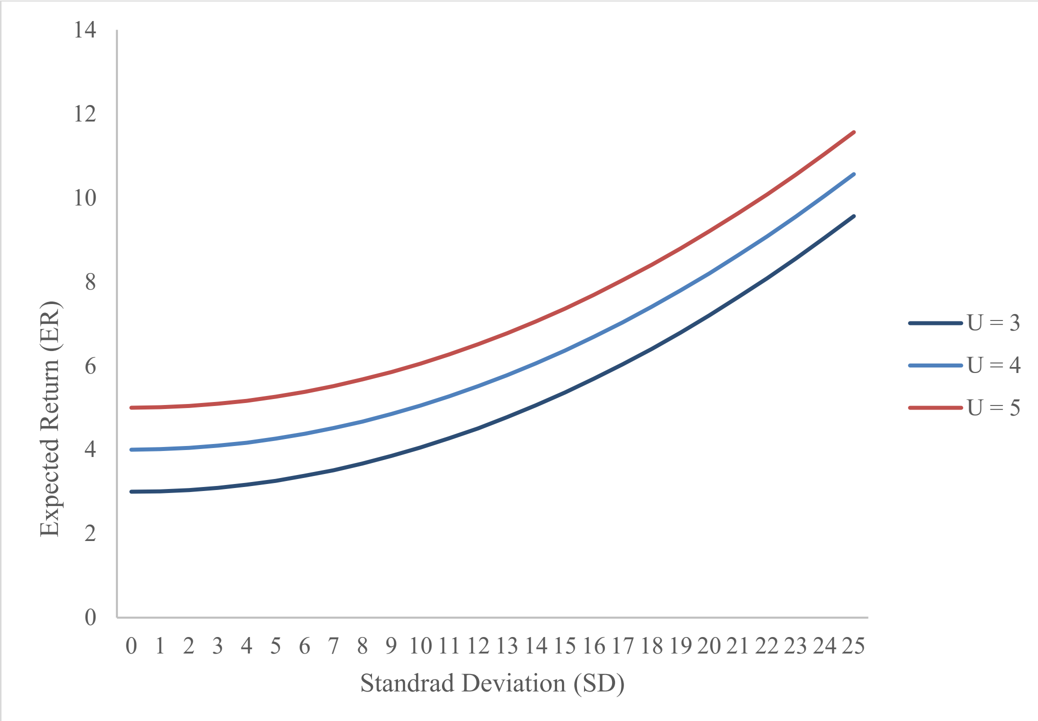

Figure 5.1. Risk Indifference Curves for a Financial Decision-maker with a $\gamma = 2.1$

The application of this methodology requires that investment options first be identified. For example, a simple two or three asset efficient frontier can be estimated using cash, fixed income, and/or equity assets (i.e., composite measures or index funds). Modern Portfolio Theory employs a calculation that incorporates the mean and variance of different investment asset options. As such, the estimation of the efficient frontier, as described in the following examples, requires the following inputs: (a) historical rates of return for each asset, (b) the standard deviation of returns for each asset, and (c) the percent of invested in each asset.

As an example, assume two assets are in consideration for a financial decision-maker with a γ = 2.1. Furthermore, assume one asset (A_1) has a standard deviation (SD_1) of 3% and an expected return (ER_1) of 8%, whereas the second asset (A_2) has a standard deviation (SD_2)

of 20% and expected return (ER_2) of 12%. The efficient frontier that combines these two investments to construct various portfolios uses the following mean-variance calculation:

$$ER_p = \rho\, ER_1 + (1-\rho)ER_2 \qquad \text{(Equation 5.5)}$$

$$SD_P^2 = \rho * SD_1^2 + (1-\rho) * SD_2^2 + 2 * \rho * (1-\rho)\, SD_1^2 SD_2^2 SD_{1*2}^2 \qquad \text{(Equation 5.6)}$$

$$where$$

$$ER_p = the\ expected\ return\ rate\ of\ the\ combined\ portfolio$$

$$ER_i = the\ expected\ return\ rate\ of\ Asset\ i$$

$$SD_P = the\ standard\ deviation\ for\ the\ combined\ portfolio$$

$$\rho = the\ percent\ of\ Asset\ 1\ in\ the\ portfolio$$

$$SD_i = the\ standard\ deviation\ of\ Asset\ i$$

$$SD_{1*2} = the\ correlation\ between\ the\ two\ assets$$

Table 5.2 shows the portfolio calculation based on different percentages invested by combining Assets A_1 and A_2.

Table 5.2. Investment Portfolio Risk and Return Rate Based on Percentage of Investment in Asset A_1 and Asset A_2

Amount of A_1	Amount of A_2	Standard Deviation	Expected Return
0%	100%	3.0%	8.0%
10%	90%	3.4%	8.4%
20%	80%	4.7%	8.8%
30%	70%	6.4%	9.2%
40%	60%	8.2%	9.6%

50%	50%	10.1%	10.0%
60%	40%	12.1%	10.4%
70%	30%	14.0%	10.8%
80%	20%	16.0%	11.2%
90%	10%	18.0%	11.6%
100%	0%	20.0%	12.0%

Using values from Table 5.2, it is possible to create a graph of the efficient frontier juxtaposed with the graph of risk indifference curves from Figure 5.1. Figure 5.2 illustrates the combination of the graphs. The location of the arrow in Figure 5.2 provides the point at which utility can be maximized for this financial decision-maker; this is, in other words, the optimal portfolio for someone with a $\gamma = 2.1$.[19]

[19] The efficient portfolio is indicated by the *tangent* point of the left furthest most utility curve. The portfolios indicated by the *intersection* of the middle utility curve and the right furthest most utility curve are less efficient because these portfolios either expose the financial decision-maker to too much risk or less risk than the decision-maker is actually willing to take.

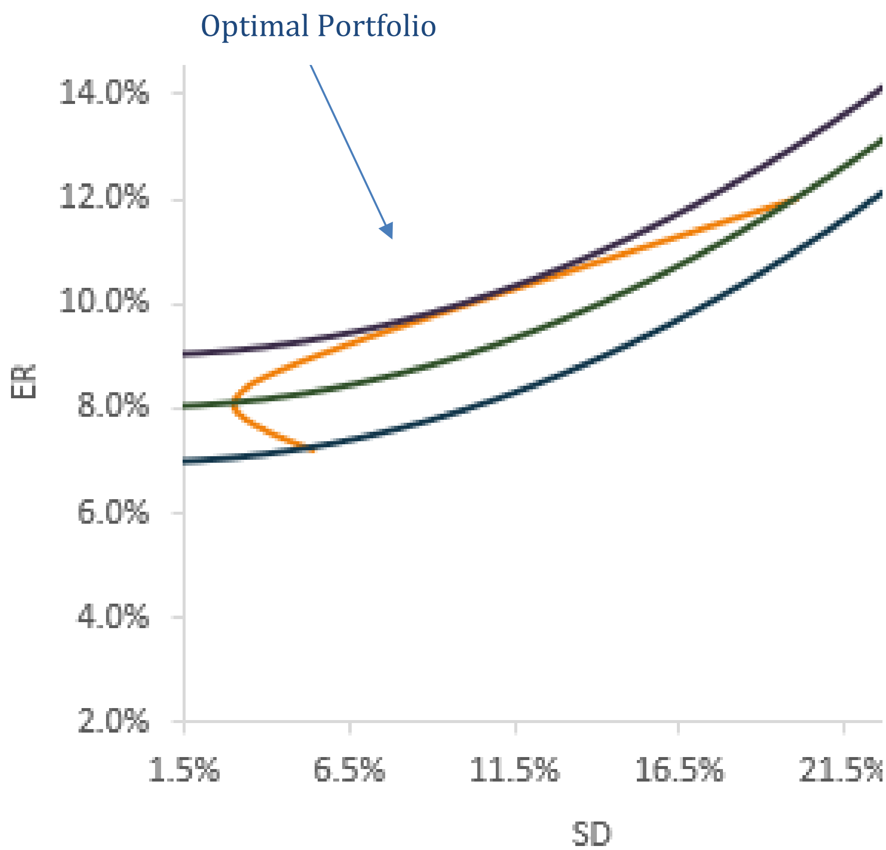

Figure 5.2. Mapping of Risk Indifference Functions to the Efficient Frontier

Interpretation of Figure 5.2 is relatively straightforward. The efficient frontier curve shows the set of investment portfolios that are expected to provide the highest returns at a given level of risk. A portfolio is said to be efficient if there is no other portfolio that offers higher returns for a lower or equal amount of risk. Portfolios that lie below the efficient frontier are sub-optimal because they do not provide enough return for the level of risk. Based on the financial decision-maker's level of risk aversion, the point identified with the arrow (the tangent point of the utility curve with the efficient frontier) shows that a portfolio with 90% allocated to Asset 1 (A_1) and 10% to Asset 2 (A_2) will provide the optimal return for the risk taken. The process described here can be used for any level of gamma obtained from the proposed risk-aversion measure.

Summary

Currently, researchers and financial service professionals most often use propensity measures and revealed-preference methodologies to assess a household financial decision-maker's risk-aversion level. Both of these approaches offer unique advantages but with notable limitations. The proposed measure of risk aversion used in this study was built using the best attributes from both methods while solving many of the problems associated with each measurement technique. The primary benefit associated with using the proposed measure is a financial decision-maker's answer to the question can be mapped directly to an MPT efficient frontier. This is not currently possible with traditional propensity measurement scores. The process presented in this dissertation is also simpler and conceptually easier to implement compared to traditionally used revealed-preference tests.

As illustrated above in Figure 5.2, scores from the proposed risk-aversion measure provide an accessible means for selecting portfolios. The process outlined in this dissertation, and illustrated in this chapter, elevates the practice of financial planning and portfolio management for individuals and households from being primarily based on professional judgment or unrealistic risk-aversion tests to one that is founded on theoretically and empirically proven processes. In this way, this dissertation makes a significant contribution to expanding the financial planning and financial risk-aversion (risk-tolerance) assessment body of knowledge.

Study Limitations

A unique strength of the methodological approach described in this dissertation is the estimates of financial decision-maker risk aversion were based on a measure of financial risk-aversion that combines the best features of propensity and revealed-preference methodologies.

Scores from the measure were found to exhibit robust validity in relation to other widely used assessment methodologies and in terms of describing risk-taking behavior. Furthermore, it was shown that scores could be mapped to the efficient frontier through the estimation of financial decision-maker utility curves. However, with any research study, the value of the results from this dissertation needs to be assessed in the context of certain limitations. These are described below.

First, given the exploratory nature of the study, a convenience sample of financial decision-makers (N = 500) was used to gather data about the proposed risk-aversion question. Related to this limitation is the notion that the sample described in Chapter 3 was not intended to be generalizable to the U.S. population. In addition, the sample frame was delimited to include primarily financial decision-makers who were engaged in the financial planning process and to those who were generally better educated. In this regard, respondents were more representative of high-income U.S. households with a middle-aged head of household. Future studies are needed to replicate the findings from this study with a more generalizable sample. Even so, while the results may not be generalizable to the entire U.S. population, the results can, nonetheless, help financial planners and other financial services professionals who are actively seeking to recommend an optimal portfolio based on their clients' risk-aversion level, particularly among better educated and high-income clients.

Furthermore, the reliability of the quantitative analysis was limited by the use of a single-item measure. Some readers might argue that multiple-item scales produce better reliability estimates, with longer indexes and scales ensuring the construct of interest is more appropriately captured. The evidence from this study, however, contradicts this notion. While scores from the proposed measure were not perfectly correlated with the other longer measures of risk aversion and risk-tolerance assessed in this study, scores were nonetheless positively associated with these other measures. Furthermore, the effect size of the associations was robust. Even so, follow-up studies are needed to determine the reliability of question scores across time. Further tests are also needed to validate the concept that a single item can be used validly to estimate CRRA with different samples and populations. In this regard, specialized studies with underrepresented samples are needed to determine whether variation in scores is systematic or associated with the demographic profile of a financial decision-maker.

When evaluating the methods and findings from this study, it is worth considering that there is some evidence to suggest that the dollar amount presented in risk-aversion measures can influence the way decision-makers respond to questions. Those with high incomes may perceive a given dollar threshold as trivial, whereas the same dollar amount may seem impossibly high to a low-income decision-maker. As a result, the amount of investment presented in the proposed question studied in this dissertation may have influenced respondent's answers. In addition to the dollar value amount, the concept of a hyperbolic discounting bias may have played a role in the choice of certainty equivalent amounts. Hyperbolic discounting is a cognitive bias where individuals prefer a smaller immediate reward over a larger delayed one (Dasgupta & Maskin, 2005). In this case, hyperbolic discounting may have prompted some survey respondents to report a willingness to accept a lower amount (i.e., closer to a guaranteed certainty equivalent amount) instead of a higher investment with the expected value of $37,500 and a 50% probability of a $50,000 investment return. Future tests are needed to determine whether if the dollar thresholds used in this and similar measures, in fact, alter choice outcomes.

Future Research Directions

Researchers are encouraged to adapt and expand the work presented in this dissertation. An immediate need involves expanding the generalizability and reliability of the measure proposed in this dissertation. Although the current sample provided a good representation of racial diversity, the income and age profile of decision-makers was less representative of the general U.S. population. A study with a larger sample size, with sample diversity weights—not only in terms of racial background, but also in income, age, educational attainment, and other control variables used—can provide evidence of item validity, reliability, and justification for a wider application of the measure. Future studies should also consider including a financial knowledge and/or a cognitive functioning assessment as a control variable.

In addition, to address the concern of a potential hyperbolic discounting bias, the risk-aversion measure could be modified or additional questions added to create a scale. These changes might include the following: (a) asking financial decision-makers their certainty equivalents for an investment proposition that will be revealed immediately, (b) disclosing the expected value of the investment in addition to the probability, and (c) using varying dollar outcome amounts. Another suggestion involves limiting dollar choices associated with the question to a narrower range, say between $27,000 and $40,000. This change will reduce outliers and extreme risk-aversion estimates.

Conclusion

At its core, this dissertation was conceptualized to bring greater clarity around the importance of accurately and consistently measuring financial risk aversion using a theoretically established methodology. The results from this study show the critical importance of transferability and applicability that allow the use of risk-aversion measurements to guide the development of advice and products offered to financial planning and/or investment management clients. To that end, the dissertation confirmed that financial risk aversion can be measured in a way that combines elements from propensity and revealed-preference methodologies. In this respect, the approach presented in this dissertation should appeal conceptually to financial planners, investment managers, and researchers. Furthermore, the proposed direct measure of risk aversion was shown to be firmly grounded in modern economic theory, valid, and applicable in the context of MPT.

The most significant outcome associated with this dissertation was the application of the proposed measure in the context of MPT. Currently, while almost all financial service professionals use some type of portfolio optimization modeling program or software, the actual selection of portfolios on the efficient frontier tends to be based on professional judgment and previous experience. Very few financial services professionals estimate utility functions based on measures of risk aversion. Even fewer map estimates of risk aversion to the efficient frontier. A clear gap exists between normative MPT modeling and actual practice. The results from this dissertation help shift the way financial planners conceptualize recommendation decisions away from professional judgment to a foundation of theoretical clarity and precision when assessing/measuring a financial decision-maker's risk-aversion level. A significant outcome associated with this dissertation is the ability to make the MPT process more accessible to financial decision-makers and those who provide advice to investors. In this regard, the dissertation advances the practice of financial planning by helping financial service professionals move beyond simple heuristics and professional judgment when developing portfolio allocation decisions.

Overall, the study described in this dissertation makes a significant contribution to the existing financial planning and financial risk-aversion measurement literature by introducing a risk-aversion measure that is easy to understand, easy to implement, and easy to apply. Although follow-up studies are be needed to enhance the current study, this dissertation opens a new chapter to the study of financial risk-aversion measurement and its applications.

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APPENDICES

Appendix 3A

Grable & Lytton 13-Item Risk Tolerance Scale

1. In general, how would your best friend describe you as a risk-taker?

    a. A real gambler

    b. Willing to take risks after completing adequate research

    c. Cautious

    d. A real risk avoider

2. You are on a TV game show and can choose one of the following, which would you take?

    a. $1,000 in cash

    b. A 50% chance at winning $5,000

    c. A 25% chance at winning $10,000

    d. A 5% chance at winning $100,000

3. You have just finished saving for a "once-in-a-lifetime" vacation. Three weeks before you plan to leave, you lose your job. You would:

a. Cancel the vacation

b. Take a much more modest vacation

c. Go as scheduled, reasoning that you need the time to prepare for a job search

d. Extend your vacation, because this might be your last chance to go first-class

4. If you unexpectedly received $20,000 to invest, what would you do?

a. Deposit it in a bank account, money market account, or an insured CD

b. Invest it in safe high quality bonds or bond mutual funds

c. Invest it in stocks or stock mutual funds

5. In terms of experience, how comfortable are you investing in stocks or stock mutual funds?

a. Not at all comfortable

b. Somewhat comfortable

c. Very comfortable

6. When you think of the word “risk,” which of the following words comes to mind first?

a. Loss

b. Uncertainty

c. Opportunity

d. Thrill

7. Some experts are predicting prices of assets such as gold, jewels, collectibles, and real estate (hard assets) to increase in value; bond prices may fall, however, experts tend to agree that government bonds are relatively safe. Most of your investment assets are now in high interest government bonds. What would you do?

a. Hold the bonds

b. Sell the bonds, put half the proceeds into money market accounts, and the other half into hard assets

c. Sell the bonds and put the total proceeds into hard assets

d. Sell the bonds, put all the money into hard assets, and borrow additional money to buy more

8. Given the best and worst case returns of the four investment choices below, which would you prefer?

a. $200 gain best case; $0 gain/loss worst case

b. $800 gain best case; $200 loss worst case

c. $2,600 gain best case; $800 loss worst case

d. $4,800 gain best case; $2,400 loss worst case

9. In addition to whatever you own, you have been given $1,000. You are now asked to choose between:

a. A sure gain of $500

b. A 50% chance to gain $1,000 and a 50% chance to gain nothing

10. In addition to whatever you own, you have been given $2,000. You are now asked to choose between:

a. A sure loss of $500

b. A 50% chance to lose $1,000 and a 50% chance to lose nothing

11. Suppose a relative left you an inheritance of $100,000, stipulating in the will that you invest ALL the money in ONE of the following choices. Which one would you select?

a. A savings account or money market mutual fund

b. A mutual fund that owns stocks and bonds

c. A portfolio of 15 common stocks

d. Commodities like gold, silver, and oil

12. If you had to invest $20,000, which of the following investment choices would you find most appealing?

a. 60% in low-risk investments, 30% in medium-risk investments, 10% in high-risk investments

b. 30% in low-risk investments, 40% in medium-risk investments, 30% in high-risk investments

c. 10% in low-risk investments, 40% in medium-risk investments, 50% in high-risk investments

13. Your trusted friend and neighbor, an experienced geologist, is putting together a group of financial decision-makers to fund an exploratory gold mining venture. The venture could pay back 50 to 100 times the investment if successful. If the mine is a bust, the entire investment is worthless. Your friend estimates the chance of success is only 20%. If you had the money, how much would you invest?

a. Nothing

b. One month's salary

c. Three month's salary

d. Six month's salary

Scoring for Grable & Lytton Scale

1. a = 4; b = 3; c = 2; d = 1

2. a = 1; b = 2; c = 3; d = 4

3. a = 1; b = 2; c = 3; d = 4

4. a = 1; b = 2; c = 3

5. a = 1; b — 2; c = 3

6. a = l ; b = 2 ;c = 3 ;d = 4

7. a = 1; b = 2; c = 3; d = 4

8. a = 1; b = 2; c = 3; d = 4

9. a = 1; b = 3"

10. a = 1; b = 3

11. a = 1; b = 2; c = 3; d = 4

12. a = 1; b = 2; c = 3

13. a = 1; b = 2; c = 3; d = 4

The survey instrument can be accessed on a scale ranging from 13 to 47. Higher scores are descriptive of increased financial risk tolerance.

Appendix 3B

Hanna and Lindamood (2004) Revealed Preference Risk-Aversion Questions

1. Suppose that you are about to retire, and have two choices for a pension

Pension A gives you an income equal to your preretirement income.

Pension B has a 50% chance your income will double your preretirement income, and a 50% chance that your income will be 20% less than your preretirement income.

You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after tax.

Which pension would you choose?

If A, go to #2.

If B, go to #5.

2. Suppose that you are about to retire, and have two choices for a pension

Pension A gives you an income equal to your preretirement income.

Pension C has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be 10% less than your preretirement income.

You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after tax.

Which pension would you choose?

If A, go to #3 on the next page

If C, your subjective risk tolerance is Moderate

3. Suppose that you are about to retire, and have two choices for a pension

Pension A gives you an income equal to your preretirement income.

Pension B has a 50% chance your income will be double your preretirement income, and

a 50% chance that your income will be 8% less than your preretirement income.

You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after tax.

Which pension would you choose?

If A, go to #4

If D, your subjective risk tolerance is Low

4. Suppose that you are about to retire, and have two choices for a pension

Pension A gives you an income equal to your preretirement income.

Pension B has a 50% chance your income will be double your preretirement income, and a 50%

chance that your income will be 5% less than your preretirement income.

You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after tax.

Which pension would you choose?

If A, your subjective risk tolerance is Extremely Low

If E, your subjective risk tolerance is Very Low

5. Suppose that you are about to retire, and have two choices for a pension

Pension A gives you an income equal to your preretirement income.

Pension F has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be one third less than your preretirement income.

You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after tax.

Which pension would you choose?

If A, your subjective risk tolerance is Moderately High

If F, go to #6

6. Suppose that you are about to retire, and have two choices for a pension

Pension A gives you an income equal to your preretirement income.

Pension G has a 50% chance your income will be double your preretirement income, and a 50% chance that your income will be half of your preretirement income.

You will have no other source of income during retirement, no chance of employment, and no other family income ever in the future. All incomes are after tax.

Which pension would you choose?

If A, your subjective risk tolerance is Very High

If G, your subjective risk tolerance is Extremely High

Appendix 3C

Additional CE amounts and corresponding gamma coefficients using Equations 3.3 – 3.5

CE Amounts	Estimated γ
$35,000	1.0
$34,500	1.2
$34,000	1.5
$33,500	1.8
$33,000	2.1
$32,500	2.5
$32,000	2.8
$31,500	3.2
$31,000	3.6
$30,500	4.0

$30,000	4.5
$29,500	5.0
$29,000	5.5
$28,500	6.2
$28,000	7.0
$25,000	7.9
$27,000	9.3