MULTIMODAL MEANING-MAKING RESOURCES IN TEACHING MATHEMATICS TO MULTILINGUAL LEARNERS IN HIGH SCHOOL ESOL CLASSROOMS

by

KHANH N. BUI

(Under the Direction of RUTH HARMAN)

ABSTRACT

Human interaction is multimodal. ESOL K-12 teachers use multimodal learning resources (e.g., gestures, visual displays, digital teaching materials) to assist multilingual learners in understanding disciplinary knowledge, including mathematics. The purpose of this dissertation is to examine the perception and practices of ESOL teachers about the use of multimodal meaning-making resources in teaching mathematics to multilingual students. Additionally, from the social semiotic perspective, this dissertation uses the methodological framework of systemic functional multimodal discourse analysis in analyzing the meaning-making process of gestures and other digital teaching materials. The data for this dissertation includes semi-structured interviews with five ESOL teachers, classroom observations, field notes, and video recordings of a focal ESOL Algebra I class at a high school in the Southeastern United States These findings indicate the need for promoting awareness of the importance of using multimodal meaning resources in professional development, including the meaning-making processes of multimodal resources, and developing a methodological framework for choosing multimodal (including digital) teaching materials.

INDEX WORDS: social semiotic perspectives, systemic functional multimodal discourse analysis (SF MDA), systemic functional linguistics (SFL), multimodality, multilingual learners, English to Speakers of Other Languages (ESOL), mathematics, digital teaching materials, high school.

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DEDICATION

In the true spirit of my father's memory, I dedicate the completion of this dissertation to him.

The work is also dedicated to my mother and my entire family, who constantly inspire and motivate me.

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CHAPTER 1

This dissertation is a journey of my personal, professional, and academic involvement in teaching disciplinary knowledge to multilingual learners. In this chapter, I provide details about my personal background and experiences, which inform how I developed the study. This chapter also includes the statement of the problem, the purpose of the study, and the research questions.

Personal Background of the Problem

My current concentration in disciplinary pedagogical practices can be traced back to my working experiences in Vietnam and higher education in the United States. After graduating from Ho Chi Minh City University of Education, I was invited to become a lecturer focusing on pre-service teacher education. Since then, I started working on developing pre-service teacher training programs, giving lectures, and conducting workshops to provide English teaching strategies to pre-service and in-service teachers, especially in four language domains, listening, speaking, reading, and writing. With advances in economy and globalization, many schools in Vietnam developed curricula using English as a medium to teach science and mathematics. Since this is relatively new in Vietnam, I started reading books. I developed an interest in content-based instruction and different approaches to teaching disciplinary knowledge to learners in the English as a Foreign Language (EFL) context.

To further my education, I applied for a Fulbright scholarship and went to the United States, where my interest in content-based language teaching grew stronger. When taking one of my courses in my Master's program, I was invited by my advisor to tutor mathematics to three eighth-grade bilingual learners from Mexico. They spoke mainly in Spanish, while my primary language was English. During my tutoring, I realized that with low levels of English language proficiency, those bilingual learners struggled with mastering different concepts in mathematics, even though they had developed literacy in their first language. I began talking to them to gain insights into their challenges. It turned out that they found it difficult to grapple with knowledge in class because most of the teachers whose primary language is English focused mainly on teaching mathematical concepts, then asked them to do work without fully explaining the conceptual knowledge. I then talked to mathematics teachers. They also shared the same concerns about their students. However, they did not have much training in teaching disciplinary subjects to bilingual learners. Therefore, they also struggled with transferring knowledge to those students.

Coincidently, in 2015, I met a mathematics instructional coach at a high school in Dawn County[1], the Southeastern United States. This was one of the most linguistically and culturally diverse schools I had ever encountered. Students from 69 countries spoke 49 different languages. The instructional coach also raised the issue of the need for more training for ESOL teachers in teaching mathematics to multilingual learners[2]. She allowed me to observe and work with teachers at her school to know more about teachers' challenges and concerns. There I had the opportunity to meet an ESOL mathematics teacher, Mr. Shawn, who finally became my focal teacher. In his class, with multiple semiotic resources such as gestures, visual displays, and drawings, his students from different countries became very interested in lessons. Within this context, I was curious about the class instruction and developed a particular interest in

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[1] All names in the dissertation are pseudo names.

[2] *Multilingual learners*, inspired by WIDA, are used here to refer to students' home languages and dialects rather than to idealize a particular language. Today, thanks to technological advances, students can easily learn other languages through music, poetry, and apps. Also, I wanted to break down the binary barrier between English and another language, so the term *bilingual* is not used.

multimodal communication and effective ways to facilitate multilingual learners in learning mathematics. With my curiosity, I explored the current situation of teaching and learning mathematics to multilingual learners. This will be explained in the next section.

Statement of the Problem

A common assumption among mathematics teachers is that mathematics is a universal language with the same system of numbers and mathematical symbols used across countries (de Araujo et al., 2018; Farsani et al., 2022). Therefore, students need to know the numbers, so they will be able to solve problems in mathematics. This assumption, however, does not account for the role of critical thinking and meaning-making practices such as defining, explaining, and justifying the solutions to a particular problem in US school mathematics (Moschkovich, 2015). For example, there is a question in Algebra I: *the function $f(x)$ represents the number of pages a student can read after x hours, and $f(x) = 4x + 7$. Explain the meaning of $f(4)$*. Answering this question requires students to understand the context and meaning of function. Hence, learning mathematics can be challenging for some students (Murphy, 2016), including multilingual learners whose population is increasing in the U.S. classroom (McFarland et al., 2017). Schools in the United States are becoming more culturally and linguistically diverse. In the Fall of 2019, 10.4% (or 5.1 million) of public schools in the United States were multilingual learners. Compared to Fall 2010, there were 4.5 million multilingual learners, which accounts for 9.2% of public-school students (National Center for Education Statistics [NCES], 2022). The enrollment growth of multilingual learners is 3.5 times faster than the national average from 2000-2017. In Fall 2019, 6.3% of the total student population in Georgia was in the ESOL program (Owens, 2020). Schools within the United States are predominantly English-speaking schools whose teachers' primary language is English. This entails challenges for students who still struggle with the English language to learn disciplinary subjects, including mathematics.

Secondly, teachers' pedagogical practices in the classroom can significantly impact the level of understanding of multilingual learners. In 2020, the National Assessment of Educational Progress reported that only 20% of high school students had in-depth knowledge of the expected mathematical concepts. Recent research shows that most high school students still underperform in classrooms, evidenced by assessment scores, course-taking patterns, and high school graduation rates (Obuon, 2019). Therefore, teachers need to adapt their teaching practices to meet the growing needs of multilingual learners. Multilingual learners, however, cannot wait until they are proficient in language skills to master disciplinary knowledge. Therefore, the concurrent teaching of language and making meaning of mathematics should be considered (Brinton et al., 1989). Despite the increasing number of multilingual learners, there has been little change in how mainstream mathematics teachers are prepared to address the academic needs of those students (Coady et al., 2019). To address this issue, teachers need to find ways to help multilingual learners make sense of mathematical knowledge, construct viable arguments (Moschkovich, 2015), and use multimodal tools (such as drawings, gestures, or visual displays) strategically to support disciplinary meaning making (Harman et al., 2021).

Thirdly, as an observer at the time of conducting this study and a current ESOL teacher, I noticed a mismatch between teaching and learning mathematics in ESOL classrooms. When teaching multilingual learners, most teachers believe it is sufficient to give the students the formula and instruct them to plug the numbers in the formula. As mentioned in the previous example (i.e., *The function $f(x)$ represents the number of pages a student can read after x hours, and $f(x) = 4x + 7$. Explain the meaning of $f(4)$),* the students were instructed that for the function

$f(x) = 4x + 7$, to find $f(4)$, just replace x with 4. Students, therefore, needed help explaining the meaning of $f(4)$. Coupled with that, disciplinary teachers just translate the materials into students' native language. However, in my observation, even though multilingual learners can translate all the word problems into their native language, they cannot solve them. In addition, multilingual learners are not allowed to use accommodations such as dictionaries in the end-of-course tests. As a result, the problem does not solely stem from their inability to understand English. With the current format, the end-of-course tests require students to show their logical reasoning about abstract mathematical objects. For example, in the assessment, students are supposed to answer the following questions:

> *A rumor is spreading over social media about the day high schoolers get to return to school. One person starts the rumor and spreads it so that the amount of people who hear it doubles every day, d. Use this equation to find R(3). (Please show your work to receive full credit.) R(3) = __________________ What does R(9) = 512 mean in context?*

In this problem, students must understand the problem to translate it into linear or exponential functions. Then they need to explain what the value 512 means in the context of the question. That said, the question does not require students to calculate but explain the logical meaning. However, in class, teachers ask students to plug the numbers into the given formula or use technology to find the results. It is unclear what the formula means in specific contexts. Due to this mismatch between instruction and assessment, multilingual learners cannot achieve high academic results on their final exams. Therefore, teaching mathematics is to help multilingual learners understand mathematical concepts and then perform mathematical operations. This process can be made through available meaning-making resources such as visual displays or gestures, not just translating from one language to another. In the following section, I will discuss the important aspects of conducting the study.

Significance of the Study

To discuss the motivations and significance of my study, I offer an anecdote from my ethnographic field site. On a beautiful day in 2016 at a high school, I went with an instructional coach to observe mathematics classes. We had a small debrief with one of the teachers when the bell rang. The teacher asked me what I liked most about his lesson. Without hesitation, I told him that I was interested in the way he used a variety of gestures to explain mathematical concepts to multilingual learners. He told me he liked using gestures because he himself was a kinesthetic learner. Then the instructional coach added that using gestures or other manipulatives was very common in a mathematics class. However, even though the teacher's use of gestures was spontaneous, what interested me was that the multilingual learners replicated the teacher's gestures when discussing with each other in the work session after the mini-lesson. It was my AHA moment. I wondered whether gestures, when used intentionally could help construct knowledge in teaching mathematics to multilingual learners.

In teaching mathematics to multilingual learners, previous researchers have paid attention to the linguistic challenges of mathematical problems. However, as mathematics is abstract by nature, this subject can be approached using multiple semiotic resources (Arzarello, 2006; Bui & Harman, 2019; O'Halloran, 2005). This helps to gain attention to the multimodal nature of pedagogical practices in teaching disciplinary knowledge to multilingual learners. Semiotic resources are not used only as visual aids in the classroom to support multilingual learners' understanding, but also as a means of knowledge construction in teaching disciplinary knowledge (Grapin, 2019). This focus on the explicit use of multimodality marks a turn in studies about the use of semiotic resources in teaching disciplinary knowledge to multilingual learners. Several recent studies focus on exploring the application of semiotic resources in content-based classrooms, for example, in science classrooms (Buxton et al., 2019; Jewitt, 2002; Kress et al., 2001), in the English classroom (Jewitt, 2002; Kress et al., 2005), and in the history classroom (Derewianka & Coffin, 2008). In my recent publication with colleagues, we argued that the availability of multiple modes enhanced multilingual youth's civic engagement and leadership, which contributed to social change (Harman et al., 2022). In the multimodal turn, multiple semiotic resources have also been studied in the mathematics classroom (O'Halloran, 2005). However, most studies place emphasis on analyzing the nature of meaning-making of language, symbols, and diagrams, which are considered multiple representations in mathematics education (see, for example, O'Halloran, 2008) or manipulatives, which are defined as physical objects designed to concretely represent abstract mathematical ideas (see, for example, Moyer, 2001). While using multiple representations or manipulatives has been considered an effective teaching method for many years (Haylock, 1984), the complexity of using semiotic resources (i.e., gestures, body movement, or pictures, hereafter addressed as *modes*) to generate meaning has not been acknowledged as often (Kress, 2010). Therefore, to discuss the meaning-making potential of multiple semiotic recourses, this study uses a social semiotic approach (which will be explained further in the next section) as a conceptual framework to explore how multiple modes can be used to make meaning in teaching mathematics to multilingual learners.

Not only focusing on the use of multiple modes in teaching mathematics in face-to-face classes, but this study also explores how meaning can be made in virtual contexts. When this study was conducted, Covid-19 broke out in the United States. All the schools were closed for two weeks, and instruction was conducted online. This context entailed challenges to ESOL teachers and multilingual learners. However, informed by our previous work together using multiple modes in teaching mathematics before the Covid-19 period, Mr. Shawn, the mathematics teacher, intentionally used technological tools and other semiotic resources such as visual displays or color coding to help multilingual learners learn mathematics. With multimodal ensembles, the combination of words, verbal language, visual displays, and mathematical symbols, mathematical concepts can be explained to multilingual learners (Danielsson & Selander, 2021). To understand the affordances or the potentials for meaning making of each mode, the data of this study was analyzed through the methodological framework of Systemic Functional Multimodal Discourse Analysis (Harman et al., 2022), which will be explained further in Chapter 3. Therefore, this study can provide a pedagogical and methodological resource for analyzing multimodal data.

With a focus on the meaning-making of multiple modes in mathematics, my study responds to the need for interdisciplinary research in applied linguistics and mathematics education. This study can support mathematics educators to consider classrooms for multilingual learners as a site for the orchestration of multimodal discursive practices and meaning-making resources. In the same vein, Gutierrez and her colleagues (2010) argued that "this view of mathematics as a multimodal and multi-semiotic activity, we believe, moves us toward a more expansive way of approaching mathematical communication and learning." (p. 32). In other words, the co-deployment of multiple semiotic resources engages multilingual learners in learning. To understand the terms, the next section will provide an overview of key terms in the social semiotic approach.

Social Semiotics: Key Terms

This dissertation defines multimodality in terms of the social semiotic tradition. Semiotics is generally the "study of signs" (Chandler, 2007). Signs refer to the connection between the signifier and the signified (Saussure, 1983). A signifier is the form which the sign takes and signified is the concept the sign represents. For example, the signifier is the word *house*, and the signified concept is the place where people live. Then Halliday (1978) redefines semiotics as "the study of signs" rather than "the study of sign systems," which emphasizes a system of signs orchestrating together to make meaning. In other words, language is not the only meaning-making resource, but other resources are used to make meaning, such as visual displays, mathematical symbols, and gestures. These are considered "semiotic resources," which are "the actions, materials, and artifacts we use for communicative purposes" (van Leeuwen, 2005, p. 285). For example, the mathematical symbols + is used for solving addition which is the joining of two or more sets.

From a social semiotic approach, multimodality has been defined as "the idea that communication and representation always draw on a multiplicity of semiotic modes of which language may be one" (Kress, 2003, p. 67-68). Within this approach, multimodalities in the teaching and learning process in mathematics involve a wide range of actions and productions used by multilingual learners and by teachers, such as words (orally or in written form), non-verbal modes of expression (gestures or glances), different types of inscriptions (drawings, sketches, graph), and various instruments (from the pencil to technological devices); and so on. All these often function as *modes*. From the social semiotics perspective, Kress (2001, 2004) and Jewitt (2007, 2008) prefer the term "mode" to "semiotic resource." Jewitt (2008) distinguishes a mode as "an organized set of resources for making meaning" (p. 17). The meaning of key terms used in this dissertation can be found in Table 1.1.

Table 1.1

Definition of key terms

Term	Definition
Semiotic resource	Materials used for communication. E.g.: pen, gestures, or computer hardware
Mode	a resource was socially and culturally shaped in society for making meaning. E.g.: Colors in the traffic light
Multisemiotic	Combining different semiotic resources such as language, image, and music in communicative act
Multimodality	The use of multiple modes to make meaning.

In mathematics, all these modes are activated and used by students and teachers simultaneously to grasp mathematical ideas (Arzarello, 2006). Therefore, the study of how modes can be used to enhance mathematical thinking and communication should receive more attention in research, as they serve as an essential bridge between the formal, symbolic expression of mathematical ideas and linguistic features of mathematics. So how does my study of the use of multiple modes contribute to the current research of mathematics education and applied linguistics? In the next section, I will discuss the organization of the dissertation.

Organization of the Dissertation

This three-article dissertation consists of seven chapters. Following this introduction, Chapter 2 provides a literature review of theoretical perspectives on teaching mathematics to multilingual learners. Chapter 3 describes the general methodology for the entire dissertation. Chapter 4, which is the first paper, explores ESOL teachers' perceptions of using multiple semiotic resources in teaching mathematics to multilingual learners. Chapter 5, which is the second paper, investigates how a high school teacher used gestures to teach mathematics concepts to multilingual learners. Chapter 6, which is the third paper, delves into how digital teaching materials contribute to constructing mathematical knowledge for multilingual learners. Finally, Chapter 7 concludes the dissertation by discussing the implications, limitations, and recommendations for future research.

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CHAPTER 2

Human interaction is always multimodal. In classroom discourse, we see the simultaneous use of speech and writing, the physical space of students, and the gestures that teachers use to direct and maintain students' attention. In mathematics instruction, there is a whole semiotic system of notation, signs, and symbols that come into play with gestures, diagrams, and visual displays (Ginsberg, 2015). With that being said, we should address more general questions about how learners, especially multilingual learners understand and communicate mathematically with various semiotic resources, which is not limited to language. This brought my attention to language and other semiotic resources in teaching and learning mathematics, especially in studies to develop pedagogical practices in teaching multilingual learners (Morgan, 2006). To confront this challenge, based on my ontology, epistemology, and axiology, I draw on the socio-semiotic theory and one of its analytic tools i.e., systemic functional linguistics. Then I discuss how mathematics has been taught, and the contribution of the socio semiotic approach in teaching mathematics to multilingual learners. So the next section, I will explain my research paradigm.

Social Semiotic Theory in Teaching Mathematics to Multilingual Learners (MLs)

To consider the idea of multimodal teaching and learning in supporting multilingual learners' creating mathematical meanings, this study is situated within social semiotic theory. Previously, cognitivist perspectives were common in mathematics education in the 1970s and 1980s, reflecting the origins of the disciplinarity of mathematics education as a domain at the intersection of mathematics education and educational psychology (Barwell et al., 2017). Most researchers from cognitivist perspectives, interested in language diversity in mathematics education, usually conducted studies on student performance (Peng & Kievit, 2020; Wild & Neef, 2023). Within this perspective, researchers often focused on individual learners. Learning mathematics was generally considered to be the acquisition or individual construction of mathematical knowledge. Through means of tests or interviews, researchers investigated the acquisition of mathematical knowledge by individual learners (Abedi, 2009). Informed by this perspective, language is considered in cognitive terms, as a system that is learned and stored in the individual mind. However, learning not only occurs through cognitive development but also through relationships, interpersonal meaning, attitudes, and beliefs. This is helpful to multilingual learners as they need social interaction to motivate their learning. Within this context, a social semiotic perspective focuses on meaning-making in social contexts and that language is functional within those contexts. Also, the social semiotic approach focuses on the range of functions performed using language and other semiotics. The social semiotic theory proposes that various multimodal forms, such as verbal language, gestures, symbols, and concrete objects, act as semiotic resources from which students can generate meaning (Lemke, 1990). In the same vein, Kress and van Leeuwen (2006) emphasize that to help make meaning in the classroom, full repertoire of meaning-making resources should be used in different contexts (action, visual, spoken, gestural, written, three-dimensional, and others, depending on the domain of representation). Therefore, it is important that teachers use a combination of modes with language and other semiotics to make meaning.

Having seen that meaning can reside in each mode, we are dealing with a methodological question: how can we describe the affordances of each mode? In a methodological overview, Halliday's systemic functional linguistics can be used as a tool to investigate the metafunction of each mode in making meaning. In the next section, I discuss how systemic functional linguistics can be used to explore the meaning of language.

Systemic Functional Linguistics and the Applications

Halliday's theory of language as social semiotic (Halliday, 1978) provides some powerful ways to analyze language. This offers powerful tools to develop knowledge about the uses of language. Halliday's (1978) systemic functional linguistics (SFL) centers on the interpretation of language in the context of its function. This social semiotic perspective of teaching and learning disciplinary knowledge was first developed in Australia (e.g., Green & Lee, 1994; Macken-Horarik et al., 1989; Rose & Martin, 2012). Australia's expanding industrial economy at the time influenced changes in student demographics, and many teachers struggled to understand and respond to the educational needs of those students. SFL became the foundation of a social justice tool of literacy teaching to "democratize the outcomes of education systems" (Rose & Martin, 2012, p. 3). Then this functional approach gained attention in the United States (e.g., de Oliveira & Iddings, 2014) and within the European Union through the Content and Language Integrated Learning Project (CLIL) (e.g., Llinares et al., 2012). This approach has also been used in many college-level world language departments (e.g., Byrnes et al., 2010).

SFL examines how language is used and the purpose it serves within contexts. Halliday (1978) notes that one of the purposes of pursuing language analysis within a subject area is to "establish general principles relating to the use of language" (p. 22). Therefore, Halliday's functional grammar provides a method of language analysis that enables researchers to discuss how language is used in a particular subject area. Halliday's functional grammar analysis is based on three elements: ideational function, interpersonal function, and textual function. He refers to these as the three meta-functions of language. The ideational function looks at the content or ideas construed in a text. Halliday (1978) describes it as “the categories of one’s experience of the world and how they interpret this experience” (p. 38). The interpersonal function examines the social and personal relationships between the author and the audience. Halliday (1978) defines his interpersonal function as “including all forms of the speaker’s intrusion into the speech situation and speech act” (p. 41). The textual function makes the language relevant for its intended purpose. Halliday (1978) points out that the textual function “distinguishes a living message from a mere entry in a grammar or a dictionary” (p. 42). In other words, the textual grammar focuses on organizing experiential, logical, and interpersonal meanings into coherent and cohesive texts. These three meta-functions operate simultaneously, that is, “speakers and writers simultaneously present content, negotiate role relationships, and structure texts through particular grammatical choices that make a text the kind of text it is” (Schleppegrell, 2001, p. 432). The social interpretation of language can be extended to “other semiotic resources such as …the mathematical symbolism and diagrams found in the discourse of mathematics” (O’Halloran, 2008, p. 7). Therefore, systemic functional linguistics has been applied in studies in other content-based areas such as science and mathematics. To discuss how social semiotic theory and systemic functional linguistics can contribute to the current teaching context of mathematics to multilingual learners, in the next section, I will describe how mathematics has been taught in previous studies.

Perspectives on Teaching Mathematics to Multilingual Learners

Focusing on Language

Previous studies have emphasized the importance of analyzing linguistic features of mathematical registers in teaching and learning mathematics to English learners (see, for example, Huang et al., 2005; Schleppegrell, 2007). Those studies focus on unpacking the linguistic features of mathematics such as dense and long noun phrases, the use of relational verbs, or how technical terms are different from everyday language. Within this context, teacher education programs place emphasis on analyzing linguistic features of mathematics and how teachers can incorporate technical terms in teaching and learning mathematics (Moschkovich, 1999; Veel, 1999). Multiple theoretical perspectives have been used to frame the research. The analyses of the relationship of learners' language proficiency and their mathematics achievement stem from cognitive perspectives on learning (Clarkson, 2007). Technical terms are necessary for enhancing students' disciplinary literacy. However, teaching and learning mathematics does not just focus on introducing technical terms. Students, especially multilingual learners, can use other modes to make meaning. In this sense, social semiotics have significantly contributed to the recognition of the functions performed by using language and other semiotic resources (Kress & van Leeuwen, 2006).

Language and Culture

Language and culture have a profound connection to each other. When teaching mathematics in multilingual classrooms, besides investigating students' linguistic resources, teachers need to pay attention to students' cultural resources, which might support their learning. Researchers have examined the effectiveness of tasks that draw on students' fund of knowledge (Dominguez et al., 2014) to engage MLs' in meaning-making process in the mathematics classroom.

Considering multilingual learners' cultures is also emphasized in previous studies in teaching and learning mathematics. Moschkovich (2015) argues that early studies of bilingual students' learning mathematics often focused on, for example, translating from the language of instruction to mathematical symbols in solving word problems. Therefore, when having a word problem with many new vocabulary items in a context that is not familiar to multilingual learners, they find it difficult to solve it by themselves without teachers' support. Being aware of the culture and how it interacts with learning is especially essential in the mathematics classroom because mathematics is typically considered a universal language and a subject that is culture free (Nasir et al., 2008). Barwell et al. (2016) argue for the importance of recognizing that "language and multilingualism, in particular, interact with learning mathematics" (p. 15). In the same vein, White (2016) argues that pre-service teachers need to be aware of the relationship between language and culture and how those are embedded in mathematics education. Research calling for multicultural learning communities also highlights the importance of creating inclusive, transformative classrooms to engage students in learning (Nieto, 1999). Research demonstrates that when teachers incorporate historically accumulated knowledge and skill bases of their students and the communities from which they come (their "funds of knowledge"), learning is enhanced (González et al., 2005). Therefore, multilingual learners should be able to use their full repertoire of meaning making resources to enhance disciplinary meaning-making (Poza, 2018) in a dialogic way (Flecha, 2000). This allows multilingual learners to articulate their understanding of disciplinary knowledge as all participants should equally value their knowledge (Gutiérrez, 2002), and they have equal opportunities to learn mathematics (White, 2002).

To recap, a social semiotics approach helps educators and researchers explore the use of multiple modes (i.e., gestures, body movements, or visual displays) besides language or the concept of "multiple representations" frequently used in mathematics education. Also, a social semiotic approach recognizes the importance of culture embedded in how multilingual learners perceive each mode and how they strategically use each mode to make meaning.

Multiple Semiotic Resources in Teaching Mathematics

In multilingual and multicultural mathematics learning contexts, being considerate and conscious of available semiotic resources becomes a necessary pedagogical tool for teachers. Multilingual learners might find it challenging to communicate mathematical ideas when instruction is done solely through language (Adler, 2016). They might refrain from expressing and developing their own ideas as they cannot express them to the fullest in a language they cannot master (Poza, 2018).

Also, through examining students' speech, gestures, gaze and actions, Evans et al. (2011) demonstrate that students are more likely to discuss and articulate their ideas throughout problem-solving processes. According to Kress and van Leeuwen (2006), to help make meaning in the classroom, entire repertoire of meaning-making resources should be used in different contexts (action, visual, spoken, gestural, written, three-dimensional, and others, depending on the domain of representation). Therefore, it is essential that teachers use gestures together with language and other semiotics to make meaning. This approach can help students to articulate their understandings.

Besides using gestures, and other semiotic tools, teaching and learning mathematics to bilingual learners needs to include kinesthetic tools. This approach allows the interaction between students and objects to make meaning in mathematics classrooms. Giving students opportunities to move around while learning new concepts helps them understand further information (Young & Marroquin, 2008). Young and Marroquin in their study found that the lessons incorporating real-life drawings supported multilingual learners in creating a stronger connection to mathematical concepts and disciplinary vocabulary. From a social semiotic perspective, those objects which are the representation of mathematical concepts help multilingual learners understand abstract concepts (Kress, 2003). In the same vein, White (2001) suggests that one way to help children develop this understanding is by exploring the different ways that mathematics is used in, for example, the designs of fabrics from many different cultures. In other words, the understanding of mathematical knowledge can be enhanced through the interaction with cultural objects.

Research on multimodality focuses on how multilingual learners use language, gestures, and interactions with diagrams to communicate while they engage in an activity in class. In their study, Radford et al. (2009) point out that when students involved themselves in an activity, different modalities, (e.g., tactile, perceptual and kinesthetic) became integral parts of their cognitive processes. In line with this, Arzarello et al. (2009) discuss the use of gestures in students' meaning making. Findings from those studies argue for an expanded view of communication for multilingual learners. In other words, from a sociocultural perspective in teaching and learning mathematics, focusing on speech alone is not enough to fully capture one's competence in mathematical communication. Researchers from those studies argue that assessment practices that put an emphasis on linguistic features of the mathematical discourse tend to neglect students' demonstration of mathematical learning through non-linguistic means (e.g., gestures and diagrams). Previous studies demonstrate that researchers often focused more on the cognitive aspects of nonlinguistic tools (i.e., gestures, or visual displays) and how they could be used in the classroom as a supplementary section to teach in class (e.g., Arzarello et al., 2009; Chen & Herbst, 2013; Radford, 2009). However, those available resources can also be used as meaning making resources, either dependent on or independent of language used in the classroom. To confront this challenge, i.e., to explore a multimodal classroom and how different semiotic resources could contribute to knowledge construction in the classroom, socio semiotic theory and its analysis tool, systemic functional linguistics can help me answer my questions.

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CHAPTER 3

METHODOLOGY

Research Paradigm

Research paradigm is defined as the summary of responses given to three fundamental questions: (1) Ontological question: What is the form and nature of reality; (2) Epistemological question: What is the basic belief about knowledge; and (3) Axiology question: What methods can be used to answer those questions (Guba & Lincohn, 1994). Those three components of philosophical assumptions interact with each other to form a guiding framework for a coherent system of thought and action (Patton, 2015). To my belief in teaching and learning, the constructivism paradigm is also reflected within three components.

Ontology

Ontology is defined as "a formal, explicit specification of a shared conceptualization. 'Conceptualization' refers to an abstract model of phenomena in the world by having identified the relevant concepts of those phenomena. '*Explicit*' means that the type of concepts used, and the constraints on their use are explicitly defined. '*Formal*' refers to the fact that the ontology should be machine readable. '*Shared*' reflects that ontology should capture consensual knowledge accepted by the communities" (Gruber, 1993, as cited in Ding and Foo, 2002, p. 2). In other words, ontology refers to how researchers perceive the reality.

As for constructivists, there is no single reality or truth. Schwandt (2000) describes what he calls "everyday" constructivist thinking in this way:

> In a fairly unremarkable sense, we are all constructivists if we believe that the mind is active in the construction of knowledge. Most of us would agree that knowing is not passive—a simple imprinting of sense data on the mind—but active; mind does something with those impressions, at the very least forms abstractions or concepts. In this sense, constructivism means that human beings do not find or discover knowledge so much as construct or make it (p. 197)

In other words, within a constructivist paradigm, knowledge is constructed through social interaction and active participation in the research process (Schwandt, 2000). Therefore, in this study, I attempted to explore the phenomenon from the point of view of participants. However, there is no single reality. Thus, my goal is to understand the multiple social constructions of meaning and knowledge. This process of integrating multiple mental constructions may lead to conflict, therefore, perceptions of reality may be changed throughout the process of the study.

Epistemology

The epistemological stance explores the nature of the relationship between researchers and audience and what can be done (Paul, 1993). Therefore, constructivists lean towards a more personal, interactive mode of data collection. Data, interpretations, and outcomes should be based on contexts and people actively participating in those contexts. In this study, I used case study to learn about the teachers in the specific context of their classrooms.

Axiology

The axiological stance focuses on how researchers find out the reality (Morgan, 1998). In other words, this informs the common methods used by researchers to explore the world. As for constructivists, truth is relative and dependent on one's perspective. This paradigm "recognizes the importance of the subjective human creation of meaning but doesn't reject outright some notion of objectivity. Pluralism, not relativism, is stressed with focus on the circular dynamic tension of subject and object" (Miller & Crabtree, 1999, p. 10). Therefore, this approach requires a close coloration between the researcher and the participant, which enables participants to tell their stories (Miller & Crabtree, 1999). This process allows researchers to better understand participants' actions and their views of reality. Informed by the constructivist paradigm, reality should be investigated through social interaction. Therefore, this study was conducted through the lens of social semiotic theory.

Case study

In formed by my research paradigm, to explore participants' thoughts and experiences, a case study approach was my main study design. Case studies allow researchers to explore or describe a phenomenon in context using a variety of data sources (Yin, 2003). According to Yin (2003), a case study design should be taken into consideration when: (a) the purpose of the study is to answer "how" and "why" questions; (b) researchers cannot control participants involved in the study; (c) researchers' purpose is to provide contextual conditions because those might be relevant to the findings; or (d) there are no clear boundaries between the phenomenon and context. These thoughts and perspectives can be explored within their context using a variety of data resources. This process ensures participants' perspectives are explored through multiple lens.

In particular, case study was used in my study because of the following reasons: (1) my goal was to explore how different modes are used as tools to make meaning of mathematical concepts; (2) I wanted to provide the audience with context of the study (i.e., multilingual and multicultural classrooms), so that ESOL teachers might find that using different semiotic resources helpful for multilingual and multicultural classroom in problem solving activities.

Specifically, among different typologies of case study design, an exploratory case study, which was used to explore situations without clear or single set of outcomes (Yin, 2003), was applied in this study.

Research Questions

This dissertation addresses the following questions:

1. What were ESOL teachers' perceptions of using multiple semiotic resources in teaching mathematics to multilingual learners?

2. How were gestures used to make meaning by the teacher? How did the teacher avail of the use of gestures?

3. How could digital teaching materials contribute to knowledge construction in Mathematics?

Research Site

This study was conducted at Dawn High School in Dustin County in the Southeastern United States. Dustin County is encountering an influx of immigrants with a variety of cultures and languages. Dawn High School is in central Dustin County where the community is most diverse. According to Graphiq (2016), 975 students, or 63.1% of the student population at Dawn High School are categorized as Black[3], making up the largest proportion of the student body. Other ethnics are 3.0 % White, 3.4 % Hispanic, 29.5 % Asian, 0.8 % Two Races and 0.1 % Native American. Dawn High School is considerably different from a typical school in the state of Georgia, which comprises 36.9% of Black students on average. Students at Dawn High School come from more than 54 countries and speak 47 languages.

[3] The term "Black" is used to include both African Americans and students from Africa and the Caribbean.

This study took place in a 9th grade ESOL Coordinate Algebra Class. Mr. Shawn, the teacher of this class, is a White male ESOL teacher with 6 years teaching experience and 4 years teaching ESOL. The class, however, was highly diverse. The total number of students was 25 (Males: 11, Females: 14). They were from different countries including Congo, Nepal, Thailand, and Myanmar. They spoke a variety of languages including Arabic, Burmese, French, Karen, Nepali, Spanish, and Thai. The class had an average ACCESS[4] score of 2.8 (out of 6). This was such a highly multicultural and multilingual classroom that it caused a lot of challenges to the teacher to communicate and each concept and procedure to students, especially when helping them move toward technical vocabulary in academic subjects such as mathematics.

Access

My acquaintance with the school's instructional coach helped me gain access to the school. While taking a Discourse Analysis class in 2016, I was introduced to an instructional coach at Dawn High school. I was extremely interested in the diversity of the school. Therefore, I asked her to give me opportunities to visit the school. With her support, I could visit different classrooms and observe different teachers. When working with Mr. Shawn, I was interested in his class. After two years at the school, I was able to build up my rapport with Mr. Shawn and become more familiar with the school settings. Therefore, I conducted this study in Mr. Shawn's class.

Data Collection

For this study, I first visited Mr. Shawn classes in one semester, i.e., Fall 2016. I stayed there three times a week to observe and build relationships with students. Each class lasted 90

[4] Every year, multilingual learners in kindergarten through grade 12 in WIDA Consortium member states take ACCESS, which is the name for WIDA's suite of summative English language proficiency assessments. This assessment helps to gauge the student's proficiency in the English language and provides valuable insights into their progress and areas of improvement (https://wida.wisc.edu/assess/access).

minutes. I stayed with the class for a whole year. Then in 2017, I applied for IRB at the University of Georgia and Dustin County. I went back to the school in Fall 2017 and asked the teacher for the permission to video record the class and conducted a pilot study with the teacher. I published a book chapter from this pilot study (Bui & Harman, 2019). I stayed another year with the teacher to conduct some activities with him. Unfortunately, Covid-19 broke out at the end of 2019, and all the schools closed. Since the school transitioned to an online platform, I could not implement any activities with the teacher, so I observed the class online, and had the permission to record his lessons on Microsoft Teams.

The data collection was divided into three stages. At the first stage, I conducted semi-structured interviews with five ESOL teachers to know about their perceptions of using multiple semiotic resources in teaching mathematics to multilingual learners. During this stage, I also went to Mr. Shawn's class, found a seat at the back of the class, and took field notes of all events in the Coordinate Algebra. My purpose was to observe students in the classroom and let students feel comfortable with my presence in the class. In the second stage, I walked around the Mr. Shawn's class, talked to students, and helped them if they had any questions. At this stage, I video recorded the class and analyzed the lessons for my pilot study (Bui & Harman, 2019). I planned some sections of the lesson with Mr. Shawn and developed the lesson with him. At the final stage, when Covid-19 broke out, the classroom was transitioned to online platform on Microsoft Teams. I joined his class on Microsoft Teams and observed how the instruction was modified in an online environment to help support multilingual learners. I also recorded the lesson on Microsoft Teams.

Data Sources

Classroom Observations

Observation benefits researchers in different ways: (1) researchers become less dependent on prior assumptions of the context; (2) researchers can observe routine activities which are ignored by participants, therefore, researchers can discover things which are not mentioned in interviews; (3) researchers can learn things which participants are unwilling to share with in interviews; and (4) being in a setting regularly helps build rapport between researchers and participants, so researchers can draw on personal experiences from participants. This fosters the quality of the interpretation of data analysis (Patton, 2015).

As a case study, classroom observation gave me opportunities to observe and explore the participant's engagement in using multiple modes in the classroom (see Appendix B for Observation Guidelines). Moreover, I also figured out the pattern of implementation of modes (e.g., how gestures were used in what stages of a lesson). Classroom observations were also used to triangulate the data from the interview. The teacher that I observed provided many examples of his instruction in the interview. Therefore, classroom observation helped me deepen the data provided in the interview. In 2016, I started coming and observing Mr. Shawn's Coordinate Algebra I. I went there twice a week. Each time I came, I observed a 90-minute class. I wrote field notes every time I came. The first purpose of the observation was to know about the curriculum and the teacher. Then in 2017 and 2018, I went to his class three times a book. Each time I came, I observed two classes. Each class lasted 90 minutes. Then in 2019, before Covid-19 broke out, I came to Mr. Shawn's class three times a week. Since Spring 2019, I joined his class on Microsoft Teams every day.

Video Recordings

My purpose was to see how Mr. Shawn used gestures and other semiotic resources, so I used video recordings as a method of data collection. Video recordings can not only increase the quality of field observation but also allows me to view the interactions repeatedly (Patton, 2015). This process supplements what might be missing in the field observation. This is applicable for my case study which requires multimodal discourse analysis. The thick description of data ensures the transferability of data analysis. Finally, videos originally done for research can be used for future training or program development (Patton, 2015). In the same vein, Heath et al. (2010) explain that videos "enable us to record naturally occurring activities as they arise in ordinary habitats, such as home, the workplace or the classroom. These records can be subject to detailed scrutiny. They can be repeatedly analyzed, and they enable access to the fined details of conduct and interaction that are unavailable to more traditional social science methods" (p. 2). Therefore, for multimodal analysis, video recordings were used to help me capture the focal teacher's gestures. For data analysis, I watched the video recordings and analyzed the gestures used in each frame. In 2017, I started video recording his class. At first, I just set up a camera at the end of the class with a wide angle. The purpose was to record Mr. Shawn's instructions. Then in 2018, I had one camera with a tripod at the end of the class to record the whole class and Mr. Shawn. I also had a handheld camera and moved from one table to another table to record the students' interactions.

Semi-structured interview

Semi-structured interviews are designed to include a set of themes and sample questions that allow researchers to make changes according to participants' responses (Suzuki et al., 2007). In other words, they ensure the flow of conversation between researchers and participants. Semi-structured interviews include broad and open-ended questions used to explore participants' experiences and perspectives of lessons using multiple representations (Patton, 2015). To conduct them, I prepared Interview Guides (see Appendix A) which included a set of questions based on my literature review, research questions, and classroom observation. This allowed me to be prepared and appear competent during the interview.

In this study, I prepared a set of questions to ask five ESOL teachers (see Appendix A). However, to allow participants to have the freedom to express their views from their own perspectives, during the interview I made some changes or reorder the questions. The purpose of this interview was to identify characteristics of multimodal semiotic pedagogy, which helped consolidate my interpretations of the observation in class. Each interview lasted about 45 minutes.

The use of each method can be summarized in Table 3.1.

Table 3.1

Research questions

Research Question	How Research Questions are aligned with study's methodology
RQ 1 What were ESOL teachers' perceptions of using multiple semiotic resources in teaching mathematics to multilingual learners?	Four semi-structured interviews from this study were used to explore what five ESOL teachers think about the use of multiple modes in teaching linear functions. Then thematic data analysis was used to analyze the data.
RQ2	Data from video recordings of the classroom observations was analyzed by SFMDA

How were gestures used to make meaning by the teacher? How did the teacher avail of the use of gestures?	(O'Halloran, 2006) to answer this question. I captured the gestures and identified the metafunctions of gestures. Through the videos, I was able to analyze gestures that the focal teacher used in his instruction.
<u>**RQ3**</u> How could digital teaching materials contribute to knowledge construction in Mathematics?	Using social semiotic approach to multimodal resources coupled with system functional multimodal discourse analysis, I explored how digital teaching materials could make meaning in comparison to printed texts.

Data Analysis

To understand the meaning potentials of semiotic resources and how they were used together to make meaning, data from this study were analyzed through Systemic Functional Multimodal Discourse Analysis (O'Halloran, 2011). Systemic Functional Multimodal Discourse Analysis (SF MDA) supports the analysis of how the teacher and bilingual learners will use modal resources (e.g., gesture and movement) to make meaning of mathematical concepts. SF-MDA developed by Kress and Leeuwen (1996) and O'Toole (1994) is an extension of the Systemic Functional Linguistics Theory (SFL) developed by Halliday (1978). SFL examines the meanings made in language through the systems choices oriented around the ideational, interpersonal and textual metafunctions (Halliday & Matthiessen, 2004). According to Halliday (1985), linguistics is a "kind of semiotics" because language is viewed as "one among a number of systems of meaning that, taken all together, constitute human culture" (p. 4). In other words, Systemic Functional Theory is a theory of meaning, initially applied to language and more lately through SF-MDA to the other semiotic resources. Therefore, Djonov (2005) describes SF-MDA as "an analytic practice which tests the application of the key principles of Systemic Functional Linguistics to the analysis of semiotic systems other than language and their interaction with each other and with language in semiosis" (p. 73). Subsequent research has built upon these two approaches and extended them into new domains. For example, contextual approaches have been developed for speech, sound and music (van Leeuwen, 1999), scientific texts (Lemke, 1998), action and gesture (Martinec, 2000), educational research (Jewitt, 2006) and literacy (Kress, 2003).

As illustrated in Figure 3.1, SF-MDA focuses on three fundamental principles: (1) Identify the meaning potential of semiotic resources into three distinct metafunctions: ideational meaning including experiential meaning (i.e., representation and portrayal of experience in the world) and logical meaning (i.e., construction of logical relations in that world), interpersonal meaning, and textual meaning; (2) analyze the inter-semiotic expansions of meaning among semiotic resources (e.g., diagrams and text); and (3) analyze the resemioticization of multimodal phenomena as social practices. These three principles will be applied for analyzing the data of this study.

Figure 3.1

Principles of Systemic Functional Multimodal Discourse Analysis (reproduced from O'Halloran, 2011)

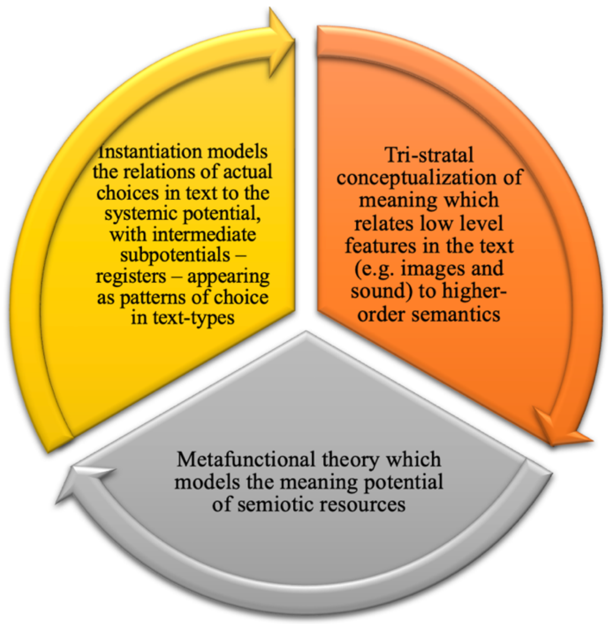
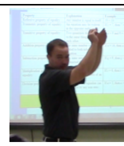


Table 3.2 below demonstrated how I analyzed the data:

Table 3.2.

An example of analyzing gestures

Phase	**Giving instruction to students**
Visual Frame	
Mr. Shawn's speech	In your group, discuss those properties for me. Don't write… don't write. Do not write.
Gestures	He raised both of his hands high in the air. The left hand was turned wide open to represent a notebook or paper. He imitated the action of writing with his right hand.
Metafunctions	Ideational meaning: participants: represent the action of writing, process: writing. Interpersonal meaning: engagement

To analyze the data, first I watched the videos of Mr. Shawn in Coordinate Algebra. Then I captured the gestures he used in the class. Informed by three metafunctions of systemic functional linguistics, I coded the gestures he used. For example, as shown in Table 3.2., while telling the students to discuss the properties of equality, Mr. Shawn raised both of his hands high in the air. The left hand was turned wide open to represent a notebook or paper. He imitated the action of writing with his right hand. This gesture represents the action of writing.

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CHAPTER 4

ESOL Teachers' Perception of Multiple Semiotic Resources in Teaching Mathematics to Multilingual Learners

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Abstract

In the context of the increasing number of multilingual learners in the US, the traditional instruction focusing on memorizing formulas and key vocabulary needs to shift into learning through dynamic multiple semiotic resources. This paper aims to explore how ESOL mathematics teachers perceive the use of multiple semiotic resources (e.g., gestures, visual displays, or graphs) in fostering multilingual students' understanding of mathematical concepts. Data for this study includes four semi-structured interviews with five ESOL mathematics teachers at a high school in the Southeast region of the US. Informed by the social semiotic approach, the thematic analysis of the interviews of the teachers reveals that (1) language is not the only means of making meaning in ESOL classroom; (2) multilingual learners use meaning-making resources to learn mathematics, and (3) pattern recognition is considered as a principle in using multiple semiotic resources. The findings of this study raise ESOL teachers' awareness of using multiple semiotic resources in teaching mathematics to multilingual learners and also inform how teacher educators should consider incorporating the training of using multiple semiotic resources.

Key words: ESOL, mathematics, multilingual learners, semiotic resources, perception.

ESOL Teachers' Perception on Multiple Semiotic Resources in Teaching Mathematics to Multilingual Learners

A common assumption among mathematics teachers is that mathematics is a universal language with the same numbers and mathematical symbols used across countries (de Araujo et al., 2018). This assumption does not account for the role of linguistically complex practices such as defining, explaining, and justifying in school mathematics (Moschkovich, 2015). Hence, learning mathematics can be a challenge for some students (Murphy, 2016), especially multilingual learners whose population is increasing in the U.S. classroom (McFarland et al., 2017). In addition, teachers' mathematics practices in the classroom can make a significant impact on the level of understanding for students. Recent research through evidence such as assessment scores, course-taking patterns, and high school graduation rates shows that this population of students still underperforms (Menken, 2013). There is a need for different methods, strategies, curricula, and professional development that support multilingual learners in learning mathematics. Given these factors, traditional ESOL instruction that focuses exclusively on memorizing formulas and key vocabulary needs to shift into learning through dynamic multiple semiotic resources that can mediate abstract reasoning and concrete application (Driscoll et al., 2012).

Making sense of mathematical concepts is the first of the eight standards for mathematical practice (Common Core State Standards Initiative, 2010, p. 6). The steadily increasing number of multilingual and multicultural students in Mathematics classes whose low language proficiency can impede their conceptual understanding (Beal et al., 2010) complicates this process. However, mathematics teachers did not have much training on working with multilingual learners. Usually, they said they used a range of manipulatives in their instruction, but what they thought about using those manipulatives is still something that we need to discuss. Therefore, the purpose of this interview project is to explore how ESOL Mathematics teachers perceive the use of multiple semiotic resources (i.e., drawing tables, diagram, and patterns) in fostering multilingual and multicultural students' reasoning development in teaching and learning mathematics. Informed by theories of multimodality and social semiotics (Bezemer & Jewitt, 2009; Kress, 2009), this interview project addresses the following questions: What are the teachers' perceptions of using multiple semiotic resources in teaching mathematics to English learners?

Multimodality in Multilingualism Mathematics Education

Educators and researchers working on equitable mathematics teaching usually recognize the critical role of language in creating inclusive pedagogical practices and effective teaching (e.g. Bartell et al., 2017; Celedón-Pattichis et al., 2018). However, the impact of diverse language practices on educational contexts cannot be ignored. Educators must recognize and adjust to the fact that students come from diverse backgrounds and speak different languages (Blommaert & Rampton, 2011). Yet many people still view monolingualism as the standard when it comes to languages. This can lead to unfair treatment of individuals who are multilingual or speak languages that are not commonly spoken in their community. Additionally, this mindset can intersect with other forms of oppression and result in inaccurate views of students' linguistic abilities. This calls for a need to recognize and celebrate linguistic diversity, rather than trying to force everyone to conform to one particular standard (Rosa, 2019). In formal education, there exist limited perceptions of language which often result in narrow pathways to learning. This can act as a major barrier to success, especially for students who are part of minoritized communities. Educators should recognize the impact of language on student success and work towards creating more inclusive learning environments (García, 2017).

Research shows a relation between multilingual learners' language proficiency and mathematical performance (Verzosa & Mulligan, 2013; Vukovic & Lesaux, 2013). Particularly, research has shown that multilingual learners with high language proficiency in both L1 and L2 perform better on mathematical assessments than if they have low proficiency in both languages (Clarkson, 2007; McDermott & Honigsfeld, 2021). Therefore, it can be understood that supporting multilingual learners to improve their language proficiency can foster their mathematical understanding. In line with this, Sigley and Wilkinson (2015) point out that as students advance to higher grades in school, the language becomes more complex as presented in sophisticated mathematical solutions.

Linguistic challenges for multilingual learners also come from the language used to explain tasks, curriculum materials or both. Several studies found that multilingual learners find it difficult to comprehend words in the instruction of activities, unfamiliar problem contexts, grammar, and syntax (Jhagroo, 2015; Novotná & Moraová, 2005; Zahner, 2012). For example, Lager (2006) demonstrated that the use of terminology such as *show* or *pattern* affected multilingual learners' performance on tasks. In this study, students were confused about the word *pattern* in the instruction. Similarly, when they were asked to show their answers, students interpreted *show* as *draw a picture* rather than writing the explanations to the questions. In other words, only when a student fully understands the meaning of the language, can they perform well in tests. Kazima (2007) points out that students who literally translated terms from first to second language often interpreted the terms differently. In the same vein as Moschkovich (2002), Kazima (2007) argues for instruction with focus on developing the meaning of mathematical terminology and supporting students with understanding of the relevant concepts. In addition to the challenges within tasks, the exclusive use of English for mathematics instruction can lead to multilingual learners' negative feelings of their mathematical abilities due to the fact that they had difficulty demonstrating their mathematical understandings in English (Jhagroo, 2015).

Despite the relationship between language proficiency and mathematical performance, researchers emphasized not putting multilingual learners' mathematical learning on hold while they are still learning the language (Hudson, 2020). Research shows that while students are learning English, multilingual learners can still engage in mathematical activity and take part in some forms of discourse in mathematical classrooms (Barwell, 2005). For example, Barwell (2005) investigated the ways in which students paid attention to language and mathematics while they worked on word problems. He found that regardless of the level of language proficiency, attention to telling stories while working on word problems showed that students developed shared meaning of the task context and developed social relationships with other students. In other words, if we just read the word problems to students, and asked them to find out the solution, students might feel discouraged and get lost. When teachers provided a clear context for students, and engaged them in the story, they could make sense of the word problems, and identify the calculation to solve the problems. This attention to narrative experience, i.e., telling stories, contributed to students' abilities to "work together to write and solve word problems with relative ease and with little sign of language being a major issue" (p. 345). His findings indicate that putting students' mathematical learning on hold while they are learning English is unnecessary because disciplinary language can be developed along with the mathematical learning.

According to Barwell et al. (2017), educators consider students' mathematical ideas beyond just their ability to communicate in conventional ways. They suggest that educators should also pay attention to ideas that are expressed through various meaning-making practices. This approach can help to provide a more comprehensive understanding of students' mathematical thinking and ultimately support their learning. Multilingual learners have unique approaches to learning mathematics, for example, non-verbal modes of representation to support their understanding and communication of complex ideas (Domínguez, 2005). Therefore, to effectively support multilingual learners, educators should acknowledge and respond to students' diverse language practices and multiple modes of communication. By recognizing the social constructs that create artificial barriers between languages, we can create an inclusive learning environment that promotes the success of all children, regardless of their linguistic background.

In other words, multimodality can support multilingual learners across various disciplines use to convey their thoughts and communicate effectively. Through this practice, they integrate different linguistic and semiotic resources to create meaning that is easily understood by their audience (García, 2011). For mathematics, understanding and mastery comes from being able to grasp abstract concepts in a variety of ways and communicate effectively using different modes of expression (Radford et al., 2017). In other words, multiple modes of mathematics are often touted as being more authentic, contextual, and culturally diverse when it comes to conveying knowledge and practice. For instance, according to Takeuchi's (2015) ethnographic study, the use of multimodal resources was found to be crucial for the involvement of diverse and multlingual learners in the process of learning mathematics. In the same vein, Karsli-Calamak and Allexsaht-Snider (2020) discovered that incorporating multimodality and embodiment into the teaching process can make a significant difference in empowering multilingual learners and providing them with an authentic learning experience when it comes to mathematical concepts. By allowing for multiple modes of learning and engaging the body in the process, we can disrupt any power imbalances that may exist in formal teaching and create a more inclusive and supportive environment for all learners. Despite the well-established importance of multimodality for learning, there has been limited research into its relation to multilingualism, even in fields beyond mathematics. This highlights the need for further exploration and investigation into the potential benefits of incorporating multimodality and embodiment into the learning process for multilingual individuals (Blackledge & Creese, 2017). To understand the mechanism of how multimodality can make meaning, this study based on the social semiotic approach to multimodality.

A Social Semiotic Approach to Multimodality

To consider the idea of multimodal teaching and learning in supporting bilingual learners' creating mathematical meanings, this study aligned with the approach of Kress and his colleagues (e.g., Kress et al., 2001). As observed by Kress et al. (2001), meaning-making resources in mathematical classrooms include the use of verbal, visual, mathematical, and actional languages, which can refer to a "multimodal ensemble" (p. 1). From multimodal perspectives, language needs to be used in accordance with other semiotic resources and make meaning through the orchestration of these modalities and resources. In addition, to serve communicative purposes, each mode is "shaped through their cultural, historical and social uses to realize social functions" (Bezemer & Jewitt, 2009, p. 183). In other words, representations become meaning-making resources to mediate students' understanding when they give meaning to events in a particular context and are perceived as such by individuals.

In the past mathematics teaching and learning centered on inert graphical representations (Lemke, 2002; Sfard, 2008); however, recent research now recognizes the importance of multimodality. Lemke (2002) states:

> The point of these observations is that the total activity is an integrated whole with respect to meaning-making. Again and again it would not be possible to get a complete and correct meaning just from the verbal language in the activity, nor just from the mathematical expressions written and calculations performed, nor just from the visual diagrams, overheads, and chalkboard cues, nor just from the gestures and motor actions of the participants. It is only by cross-referring and integrating these thematically, by operating with them as if they were all component resources of a single semiotic system that meanings actually get effectively made and shared in real life (p. 236).

In other words, the meaning making of mathematical concepts is mediated through a configuration of semiotic resources (e.g., gestures, visual displays, symbols, and language) for specific purposes and audiences. In the same vein, Kress and van Leeuwen (2006) emphasize that to help make meaning in the classroom, different repertoires of meaning-making resources should be used in different contexts (action, visual, spoken, gestural, written, three-dimensional, and others, depending on the domain of representation). Therefore, teachers use a combined range of modes to make meaning. Within this theoretical framework, previous studies have focused on how multimodal forms can support multilingual learners in learning mathematics. Therefore, the next section demonstrates what kind of knowledge in mathematics multimodality can support multilingual learners.

Multimodality and Types of Knowledge in Mathematics

Many educators have long believed that using a variety of teaching materials like manipulatives or diagrams can help students learn better. However, there is a need to delve deeper into how different types of learning resources can impact the kind of knowledge that students acquire in the classroom. There are two types of knowledge in mathematics: conceptual knowledge and procedural knowledge. According to Rittle-Johnson and Koedinger (2009), concepts are derived from specific cases, then are generalized into a domain, for example, fractions. Also, “fractions” can become conceptual knowledge when this knowledge is linked to other knowledge, such as a part of the whole. Therefore, conceptual knowledge can be demonstrated as a connecting map of relationships (Rittle-Johnson & Schneider, 2015). Procedures, on the other hand, are a series of steps and/or actions used to achieve a task (Rittle-Johnson, 2017). Procedural knowledge in mathematics involves a series of steps to solve problems or achieve a particular goal. According to Canobi (2009), by mastering procedural knowledge, students can tackle complex mathematical problems and achieve their desired outcomes.

Researchers have identified what kind of knowledge students can get through the use of multimodality. As students try to make sense of fractions, Cramer and Henry (2002) found that the use of manipulatives can greatly enhance their development of fraction sense. They also highlighted the importance of allowing students to invent their own strategies. The authors identified four key beliefs: firstly, students construct their knowledge by using different representations and models. Secondly, manipulatives should be used over time to allow for the development of “mental images” that will help with the conceptual understanding of fractions and reasoning with fraction operations. Buisson and Quinton (2010) referred to these mental images as "internalized activities." Thirdly, students learn by sharing and distributing their knowledge with their teacher and peers. Lastly, it is crucial to focus on developing students' conceptual knowledge of fractions before introducing them to rules and procedures for fraction operations.

Besides linguistic resources, researchers have placed an emphasis on the importance of encouraging multilingual learners' use of nonverbal communication such as gestures (Shein, 2012). In line with this, Dominguez (2005) investigated the ways in which multilingual learners used gestures to communicate their mathematical reasoning. He concluded that teachers should pay more attention to multilingual learners' nonverbal communication along with speech to have a complete picture of understanding of multilingual learners' mathematical knowledge. Several studies have built on the advantages of multimodal communication for multilingual learners in designing activities and curricula to enhance multilingual learners' mathematics learning (Cho et al., 2015; Warren & Miller, 2015). For example, in a project named *Help With English Language Proficiency (HELP) Math*, a technology-based mathematics curriculum was designed to support middle-grades Spanish speaking multilingual learners with multimodal learning opportunities (Crawford, 2013). In this project, students interacted with different modes such as solving math with pictures and performing counting with music. Students were able to count the numbers and understand the concepts that teachers wanted to teach them. Findings from those projects point out that the use of curricula that integrates multimodal communication might be beneficial for multilingual learners.

Research on multimodality focuses on how multilingual learners use language, gestures and interactions with diagrams to communicate with each other while they engage in an activity in class. In their study, Radford et al. (2009) pointed out that when students involve themselves in a word problem, different modalities (e.g., tactile, perceptual and kinaesthetic) became integral parts of cognitive processes. In line with this, Arzarello et al. (2009) discussed the use of gestures in students' meaning making. Findings from those studies argued for an expanded view of communication for English learners. Francaviglia and Servidio (2011) also found that gestures play an important role for supporting learners in acquiring and solving mathematical problems. In other words, from a sociocultural perspective in teaching and learning mathematics, focusing on speech alone is not enough to fully capture one's competence in mathematical communication. Researchers from those studies argued that assessment practices which put an emphasis on linguistic features of the mathematical discourse tended to neglect students' demonstration of mathematical learning through non-linguistic means (e.g., gestures and diagrams). It can be seen that previous studies focused on how multimodality could be used in the classroom, but there are not many studies focusing on mathematics teachers' perceptions in teaching mathematics to multilingual learners. Therefore, there is a need to explore teacher's perceptions on teaching mathematics to multilingual learners.

Methodology

Participants

In this study, I invited ESOL teachers who had experience working with multilingual and multicultural students to participate in the interviews. There were five ESOL teachers who participated in the interviews. They were all mathematics teachers who received ESOL Endorsement and were assigned to teach multilingual learners in a separate classroom. One of the teachers, Mr. Shawn, had been teaching mathematics, particularly Coordinate Algebra I, for more than 10 years, and teaching ESOL students for more than 8 years. He was White and only spoke English, but he was trying to learn other languages such as Nepalese. In other words, Mr.

Shawn was very interested in other cultures. Ms. Smiths was also a White mathematics teacher. She had just started teaching ESOL students for one year at the time of the interview. She also received ESOL endorsement. Ms. Adam was a Black mathematics teacher. She had been teaching ESOL students for 3 years. She also received ESOL Endorsement. Ms. Thompson was a Black mathematics teacher. She had been teaching ESOL students for 5 years, and also got ESOL Endorsement. Finally, Mr. Hilton was a Black mathematics teacher. He had been teaching ESOL students for 8 years. They were invited to this study because they were teaching multilingual learners and they were very interested in exploring different pedagogical practices in teaching them.

Researcher Positionality

As an international graduate student, I felt fascinated by the student population and teachers when visiting Dustin High School. The school was highly diverse. Vietnam, where I came from, was a monocultural county. Moreover, English is a foreign language in Vietnam, therefore, all the students are required to learn English. They are not supposed to learn mathematics in English. However, in the US, multilingual students are supposed to learn mathematics in a language that is different from their mother tongue. Noticing the diversity of ESOL classrooms, I was so curious as to how monolingual teachers could teach disciplinary subjects to those multilingual students. Therefore, I attended ESOL classes at Dustin High school to explore the interactions between the teachers and students. After the class, I had the opportunity to discuss the teaching techniques with ESOL teachers. During class, I also questioned and wondered how multilingual learners could learn mathmatics if they did not have enough language competence. Also, I wondered at teachers' undertanding about waiting or not until multilingual learners mastered English to teach them mathematics. I also questioned my identity as a bilingual student speaking both languages (English and Vietnamese), and how my knowledge of mathematics in Vietnamese could support my understanding of mathematics in English. Not coming from mathematics education, I really wanted to see and listen to ESOL teachers to find out how ESOL teachers support multilingual learners and know more about their challenges while teaching those students, so as an applied linguist, I can figure out what I can do to help them.

Research Context

This study was conducted at Dawn High School in Dustin County, an immigrant hub in Southeast of the United States. Dustin County is experiencing an influx of immigrants with a variety of cultures and languages. Dawn High School is in central Dustin County with a highly diverse community. When multilingual learners arrived at Dawn County, they were directed to the International Center. This center was specifically designed to help families of international students navigate the registration process. There, the students received valuable information on survival language and culture. After being tested, the students were then sent to satellite schools based on where they were living. The multilingual learners were taught in sheltered classrooms, which meant that they were grouped into classes where they were taught mathematics and language by a content teacher. The purpose is to make sure that multilingual learners received the individualized attention and support they needed to succeed.

Data Collection

The purpose of this paper is to obtain in-depth knowledge about ESOL teachers' perception of teaching mathematics to multilingual learners, so qualitative semi-structured interviews have been used. This qualitative study consisted of four 45-minute semi-structured interviews with mathematics teachers in a high school in the Southeastern of the United States.

In this study, purposeful sampling (Patton, 2015) was used for the identification and selection of information-rich cases. This process involves choosing individuals that are especially knowledgeable about or experience with a phenomenon of interest (Cresswell & Clark, 2011).

Data Analysis

I transcribed interview data and uploaded it to One Drive (an online storing system). I named all audios in the following convention: Nameoftheinterviewee_date.mp3. To preserve all the data, I archived all the data and saved them in a password-protected computer. The data was then uploaded to NViVo (data analysis software). To analyze the interviews, I followed the inductive approach. I read all the data files and made notes about preliminary codes and possible themes. I developed the main codes, such as *challenges, gestures, and language*. Then I developed sub-codes such as *challenges_test and gestures_difficulty* (See Appendix D for the Codebook). Then I read all the codes again, recognized the patterns, and summarized them into three findings (See Appendix C for theme development). In the next section, I describe the findings from the interviews and provide examples from the data.

Findings

The interviews revealed three findings: (1) Translated words confused multilingual learners; (2) Multimodality enhances the understanding of conceptual knowledge; and (3) Repeated use of multimodality fosters multilingual learners' conceptual knowledge.

Translated words confused multilingual learners

The interviews discussed the challenges of teaching mathematics to multilingual learners. Ms. Smith, Ms. Adam, and Ms. Thompson agreed that teaching and learning vocabulary in mathematics could be a challenge for multilingual learners. At the time of the interview, Ms. Smith just had one year of experience in teaching multilingual learners. Ms. Adam and Ms.

Thompson had been teaching multilingual learners longer than Ms. Smith. Despite the differences in their teaching experience, they recognized that focusing on language in teaching mathematics to multilingual learners could be a challenge. For example, Ms. Adam and Ms. Thompson mentioned the language barrier as one of the challenges.

> Researcher (R): So can you name some difficulties that ESOL students may have when they study mathematics?
>
> Ms. Adam: The main thing is the language barrier. You know […] they (multilingual learners) understand when you're writing the numbers on the board. But as you're giving the academic vocabulary, they may not understand what you're saying.
>
> (personal interview, 2018)

Similarly, in other interviews, when discussing the challenges of teaching multilingual learners, teachers usually thought of language at first. For example, in the interview, Ms. Smith discussed the difficulties of translating the vocabulary into the students' home language.

> R: So do you have any kind of struggles or do you have like any kinds of challenges when you teach vocabulary to them?
>
> Ms. Smiths: […] I would love to be able to communicate with them in English, so I have to try different things to be able to get them to understand without just translating the word. Yeah. Not all words can be easily translated. For example, the word like *sequence* can be translated into *secuencia*, but the word *difference* can't be translated well. In English we can say that if you look at *the difference between two numbers*, you *subtract* them. yes. Yeah that's a common word, but it's not common in Spanish language. So I have to make sure that what they know when they say that word it means to subtract or find the gap between sort of things like that.

(personal interview, 2018)

In other words, using direct translation from other languages to English, and focusing on teaching vocabulary without context could cause students' confusion. For example, the excerpt from the interview with Ms. Smith shows that the word *difference* means subtract in math, while it can have different meanings in daily contexts. Ms. Thompson also gave an example of the difference between vocabulary in a daily context and in mathematics.

R: So you also mentioned, uh, uh, you also mentioned academic vocabulary. So what do you mean by that?

Ms. Thompson: Let's just say I mentioned *line*. So like for geometry *line* means something different versus going to the cafeteria and standing in the *line*. So they have to be able to distinguish.

(personal interview, 2018)

As we can see from the excerpt, ESOL teachers have highlighted that the direct word-for-word translation method is not an effective approach to translating the vocabulary into the students' home language. Furthermore, with a diverse range of students in the classroom, it could be quite challenging for ESOL teachers to decide which language to use to support their students. As an example, Ms. Adam and Ms. Thompson were unable to count the number of languages in their classes, let alone translate the materials into the languages of their students.

R: So I may ask like how many languages a day in your class?

Ms. Adam: A lot.

Ms. Thompson: Oh in my class? Oh I can't count (spoken at the same time as Ms. Adam). I know I probably have over, I know well over 12.

(personal interview, 2018)

In schools, a large number of languages were spoken, and teachers could not translate all materials into students' first languages or provide interpreters to assist all of them simultaneously.

Also, there was also the possibility of losing the meaning of a word, especially mathematical concepts, during translation. In response to this, Ms. Smiths gave two examples: in mathematics, *difference* in "the difference between the two numbers" means "subtracting a number from another number". The meaning of *difference* in this context is different from the meaning of this word in daily language, for example, "my friend's house is different from mine". In the same vein, Ms. Smiths also mentioned the concept *common*. In an arithmetic sequence, the concept *common* means the difference between two terms in a sequence. Ms. Smiths asked a Spanish teacher to help translate the word in Spanish, but the Spanish teacher told it was not easy to translate this term, as there might not be an equivalent in Spanish. To sum up, three teachers in this interview realized that some vocabulary in mathematics may be different from daily life. Therefore, just simply translating the word in mathematics into the students' home language does not actually help them in understanding mathematical concepts. Teachers need to unpack the concepts with concrete vocabulary.

Multimodality enhances the understanding of conceptual knowledge

To aid multilingual learners in learning mathematics, ESOL teachers in the interviews emphasized the use of multiple semiotic resources. For example, Ms. Smiths emphasized the use of pictures in teaching vocabulary.

> R: So what other strategies that you used to teach them besides using like similar vocabulary?

Ms. Smiths: Or I can show them pictures of what we're about to do. […] I'm putting pictures so they understand. For example, they don't know *a piggy bank*. I'm going to put a piggy bank, so you know what that is.

(personal interview, 2018)

By using the picture of "piggy bank", and the action of putting the money and talking the money out of the piggy bank, Ms. Smith could help students understand the concept of "withdraw". In this scenario, the picture helped the students to know the concept.

In another interview, Mr. Shawn usually used gestures to help multilingual learners understand mathematical concepts. He discussed his lesson with me in the interview.

R: Yeah how about the instruction in the classroom? How do you scaffold learning?

Mr. Shawn: Definitely, students should be able to see it, hear it, maybe even do it. I'm very like into kinesthetic learning. Yeah I want the students to like to get up and do something. And I think that's a very like a great way to learn a lot of things. […] Yeah a good example is like instead of having students graph by hand, they can walk out and graph.

(personal interview, 2018)

In the interview, Mr. Shawn discussed the benefits of movements in helping multilingual learners understand the definition of *slope*. In this context, the use of gestures helped foster multilingual learners' understanding of conceptual knowledge of the concept "slope".

In the interview, Ms. Smith also compared the way she taught ESOL students and her students in general education. In the school, Ms. Smith taught three classes: 1 ESOL sheltered class and 2 general education classes. So she could see the differences in her instruction. Ms. Smith said, "I don't use as many pictures and things like that do with students who are not ESOL students.” (personal interview, 2018). With that being said, teachers are more aware of linguistic competence of ESOL students and the use of multiple semiotic resources can help students understand mathematical concepts better. In this vein, different modes of a word problem can support students understand the meaning of word problems, which can help them solve the problems.

Repeated use of multimodality fosters multilingual learners’ conceptual knowledge

According to the interviewees, semiotic resources should be used multiple times in different lessons to enable students to recognize patterns and understand concepts. Frequent and repetitive use of multiple semiotic resources fosters multilingual learners’ pattern recognition. As in the interview, Mr. Hamilton mentioned:

> R: So do you notice how do they communicate with each other?
>
> Mr. Hamilton: Like right now if you come to my classroom you see like a whole lot of gesturing […] you’ll see little kids said use x-axis and they extended their arms.
>
> (Mr. Hamilton, personal communication, September 24, 2018)

Mr. Hamilton realized that the students could comprehend his gestures and use the same gestures in the discussion with their friends. To some extent, if a gesture is used repeatedly, students can recognize it, comprehend it, and use it to communicate with each other. Ms. Adam and Ms. Thompson discussed the notion that multilingual learners have the potential to acquire patterns and understand mathematical concepts.

> R: How did the student react to the gestures that you use?
>
> Ms. Adam: So we use hand movements for that, for slope, positive, negative, zero or undefined. It's funny to them. Yeah. Yeah, yeah. It engages them. Yeah. And they get it.
>
> (Ms. Adam, personal interview, August 28, 2018)

In conclusion, when teachers use consistent gestures or visual displays, it can greatly benefit multilingual learners in comprehending mathematical concepts. This pattern can guide them to navigate through the materials, allowing a deeper understanding of the mathematical concepts.

Discussion and Conclusion

This paper reports on ESOL teachers' perceptions of teaching mathematics to ESOL students to meet the school standards and support their Math and linguistic knowledge in the classroom. Mathematical concepts are believed to be more accessible to multilingual learners because they are based on numbers. In the interviews, ESOL teachers discussed challenges multilingual learners confronted in learning mathematics in and English-only instructional setting. Therefore, educators should understand that students come from a variety of backgrounds and may speak different languages (Blommaert & Rampton, 2011). Previous studies that only focus on language in teaching mathematics may need to be directed toward this realization. Unfortunately, many people still view monolingualism as the norm when it comes to languages. It's important for educators to recognize and adjust to the diversity in their classrooms. Therefore, teachers should be aware of promoting students' meaning-making as interdisciplinary moves from concrete examples to abstract explanations (Buxton et al., 2019). Instead of focusing on mathematics vocabulary in isolated contexts, teachers should draw attention to how disciplinary vocabulary and concrete vocabulary differ from each other (Herbel-Eisenmann et al., 2014). For example, before discussing the features of quadratic equation graphs, teachers can ask students to observe the pathway of a ball falling from the top of a building. In this scenario, students can observe the pathway using their concrete vocabulary. Then teachers can scaffold them with disciplinary vocabulary.

In this study, ESOL teachers discussed using multiple semiotic resources in teaching mathematics to multilingual learners. Findings from the interviews revealed that ESOL teachers acknowledged the role of multiple semiotic resources in their pedagogical practices. This is similar to what was discussed in previous studies, for example, Karsli-Calamak and Allexsaht-Snider (2020) highlights the crucial role of incorporating multimodality and embodiment in teaching multilingual students. The study unequivocally proved that this approach can effectively empower these learners and provide them with an authentic learning experience, particularly when it comes to mathematical concepts. It's exhilarating to witness how innovative teaching methods can significantly enhance students' learning outcomes.

Furthermore, the findings from the interviews of all teachers can demonstrate what kinds of knowledge multimodality can bring to the classroom. For example, Ms. Smith shared a helpful tip with her class recently. She showed them a picture of a piggy bank while explaining the term "withdraw". This really helped the multilingual learners in the class to understand that they could use a piggy bank to store their money. When they took money out of the piggy bank, it meant they were "withdrawing" the money. Using a picture like this is a great way to help students understand the concept of "withdraw", which means "take out the money from the piggy bank". By activating their procedural knowledge in this way, students are more likely to remember that when withdrawing money, they need to "subtract". As shown in this example, Ms. Smith used pictures to help explain the concept to multilingual learners. This can scaffold their understanding of procedural knowledge. Therefore, there should be more professional developments on the meaning-making of multiple semiotic resources. For example, teachers should be aware of their aims when choosing an image or using gestures in their class. Multiple semiotic resources can be a source of knowledge construction, not just supplementary materials

(Grapin, 2019). The new Common Core State Standards also emphasize multilingual learners across grade levels are expected to "make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations" (NGA & CCSSO, 2010, p. 22). In other words, there should be more discussion on the meaning-making potential of each mode. In that way, teachers can be more strategic and more confident with their multimodal instruction deliveries (DaSilva Iddings & Rose, 2012). This can help engage multilingual learners in fostering their disciplinary knowledge (Grapin, 2019).

Finally, findings from this study reveal that multilingual learners recognize the pattern of semiotic resources. ESOL teachers realized that their students used the same gestures when they communicated mathematically with each other. This can contribute to the field of multimodality that language is not the only means of communication among multilingual learners. By incorporating multiple semiotic resources in communication, multilingual learners can convey their meaning if they cannot express it in the purely verbal language (Arzarello et al., 2009). Due to the pandemic, I could not interview the students to explore multilingual learners' perceptions of the use of multiple semiotic resources. Future research should focus on how the students think about multimodal instruction in the classroom.

In conclusion, due to a growing population of multilingual learners, it has been recognized by ESOL mathematics teachers that multiple semiotic resources are essential in teaching mathematics to these learners. To achieve this, it is important to emphasize the multimodal content standards. It is recommended that teachers receive more training, professional development, and collaboration in designing a multimodal curriculum that supports multilingual learners. By doing so, the full potential of a multimodal curriculum can be realized, engaging multilingual learners in disciplinary practices. The use of multiple semiotic resources can be a crucial part of constructing disciplinary knowledge for multilingual learners.

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CHAPTER 5

The Interplay between Gestures and Mathematical Concepts in an ESOL Classroom

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Abstract

Standards for Mathematical Practice in the Common Core State Standards (CCSSI, 2010) establish expectation that students develop reasoning skills rather than memorizing formulas and vocabulary. This issue creates a challenge for ESOL teachers. With this set of demands, the traditional ESOL instructional focus on language needs to shift into teaching/ learning through a dynamic multiple representational process (e.g., visual displays, and diagrams). The purpose of this qualitative case study is to investigate the interplay between gestures, body movements and mathematical concepts enacted by a White teacher in a highly multilingual Grade 9 ESOL Coordinate Algebra. Informed by theories of Systemic Functional Multimodal Discourse Analysis (O'Halloran, 2008) and social semiotic approach to multimodality (Bezemer and Jewitt, 2009; Kress, 2009; van Leeuwen, 2005), the researcher analyzed three 90-minute recorded videos of classroom instruction to explore how the teacher used both gestural and verbal modalities and examine how such multimodal interactions construct mathematical meaning. The results showed that the teacher's gestures could be endowed with meanings and mathematical concepts to enhance multilingual learners' understandings. Implications include (1) expansion of use of gestures and body movement in multimodal pedagogic practices for mathematics teachers; (2) principles and contribution to the reconceptualization of professional development for pre- and in-service Mathematic teachers in the context of multilingualism and multiculturalism in the United States.

Key words: Systemic Functional Multimodal Discourse Analysis, ESOL Mathematics, Multimodal Pedagogic Practices, Gestures.

Georgia's K-12 Mathematics standards emphasize that students need to understand how different concepts connect and apply them in real life. This will help them face challenges and achieve their goals in school and work (Common Core State Standards, 2010). Learners, in other words, should be instructed to understand the meaning of mathematical concepts without conceiving mathematics as a series of manipulating formal calculations. The new standards have created more opportunities for multilingual learners to understand standard-based mathematical concepts fully. To scaffold multilingual learners in making mathematics concepts more comprehensible and accessible, Georgia's K-12 standards emphasize the use of visual representations (Georgia Department of Education, 2022). Using such meaning-making resources helps multilingual learners access mathematical knowledge through the connection between visual representations and abstract symbols (Van de Walle et al., 2009). Similarly, the "multimodal turn" (Jewitt, 2009, p. 4) in the standards acknowledges that language can be co-deployed with other semiotic resources in mathematics, and meaning is made multimodally because of the orchestration of these resources. This aligns with advocates such as O'Halloran (2007), who argue for developing a theoretical framework and pedagogical practices to promote effective teaching strategies for ESOL teachers. Within the context of this study, when assisting a mathematics ESOL high school class in an urban area in the Southeastern United States as a participant observer, I realized that the teacher's gestures functioned as part of his instructional strategies on different mathematical topics. This made me wonder whether gestures could help multilingual learners make meaning in the classroom. Gestures in mathematics education have been studied for a long time (Sfard, 2009). Still, few studies focus on the meaning-making process of gestures to identify their functions in interaction. To contribute to the new dynamic focus on multimodal disciplinary instruction, this study investigates how gestures help construct and communicate mathematical understandings in a multilingual mathematics classroom.

Gestures as Semiotic Resources

Using gestures can aid in a student's learning process. Incorporating gestures into learning can reduce the cognitive load required to understand and retain new information (Goldin-Meadow, 2011). The students could produce more information in gesture, which is surprinsgly correct and different from the information they produced in speech (Broaders et al., 2007). Gestures are defined by McNeill (1992) as movements of the hands and arms in which "speech and gesture are elements of a single integrated process of utterance formation in which there is a synthesis of opposite modes of thought" (p. 35). In other words, gestures and speech are part of the same meaning-making process. Gestures are produced when meanings are made. Therefore, gestures do not precisely carry the producers' initial understandings and thoughts in different contexts. Gestures in mathematics education have been studied for a long time, centering on various aspects of gestures, such as their contribution to students' visual engagement (Alibali & Nathan, 2007; Farsani et al., 2021), and explaining concepts in geometry (Chen & Herbst, 2013). Alibali and Nathan (2007) argued that the use of gestures together with teachers' speech fosters students' understanding of mathematics. In their study, the teachers used gestures to connect verbal language with physical referents such as objects, actions, and diagrams in a sixth-grade mathematics lesson centered on algebraic relations. They concluded that using gestures could support students in learning the lesson content. However, gestures in this study were used as a form of "grounding" (Alibani & Nathan, 2007) to direct learners toward the referents in teachers' speech.

Gestures can also provide additional spatial information to complement verbal communication in mathematics classrooms (Williams & Ryan, 2019). This helps enhance classroom conversation. Research has found that students' mathematical repertoires of mathematics terminology were increased as their understanding was built on multimodal resources (Arzarello et al., 2009). By using diachronic (i.e., looking at what semiotic resources are used in a particular instance) and synchronic (i.e., looking at all the semiotic resources used in an activity), Arzarello and his team showed that the use of gestures and other semiotic resources could help students understand the definition of slope. Gestures could help students know the direction of the graph that they drew. Using gestures in teaching mathematics is particularly helpful for students whose English is not their first language (Krause & Farsani, 2021). For example, Latin@ students in Los Angeles could use gestures in math classes to understand mathematical terms such as *perpendicular* (Castellón, 2007). In their study, Castellón (2007) suggested that students, especially multilingual learners, could communicate their thoughts using gestures rather than terminology such as *vertical, horizontal, parallel, and perpendicular*. The students did not need to have a direct translation of the word, but they still could understand the concept. However, in those studies, the researchers did not explicitly analyze the metafunction of gestures and what knowledge gestures could contribute to multilingual learners' understanding of mathematical concepts. Gestures, as a mode of conveying meaning, are a sign of meaning-making (Bui & Harman, 2019). Therefore, in this study, I consider how specific gestures contribute to producing and interpreting mathematical concepts in an ESOL classroom. To do this, I used the systemic functional multimodal discourse analysis (SF-MDA) as a methodological framework to explore the meaning-making process of gestures in supporting ESOL students in understanding mathematical concepts. SF-MDA is, in fact, an extension of systemic functional linguistics (SFL) (Halliday, 1978). Before discussing the critical tenets of SF-MDA, in the next section, I will provide the key points of systemic functional linguistics.

Systemic Functional Linguistics

Informed by SFL, the analysis of written and spoken texts is conducted within the context of the situation, surrounded by the context of culture (Halliday & Hassan, 1985). For example, the pandemic might influence the pedagogical practices of mathematics instruction. A mathematics lesson, as a context of a situation, is also influenced by how mathematics is described in spoken and written language and in gestures. At the same time, the lesson is also affected by the context of a culture in which teachers and students are involved. The context of culture in mathematics is influenced by national curricula and the specific group of students being taught. However, it is assumed that there are common understandings within mathematics education across countries (Meaney, 2018). Therefore, mathematics education has its own context of culture. Within a multilingual setting in ESOL classrooms in the US, this notion is essential in identifying the meaning of metaphorical gestures of mathematics concepts so multilingual learners can understand the meaning of mathematics concepts.

Within a context of situation and context of culture, Halliday (1978) notes that one of the purposes of pursuing language analysis within a subject area is to “establish general principles relating to the use of language” (p. 22). Therefore, Halliday’s functional grammar provides a method of language analysis that enables researchers to discuss how language is used in a particular subject area. Halliday’s functional grammar analysis is based on three elements: ideational function, interpersonal function, and textual function. He refers to these as the three meta-functions of language. The ideational function, comprised of an experiential and logical entity, looks at the content or ideas construed in a text. Halliday (1978) describes it as "the categories of one's experience of the world and how they interpret this experience" (p. 38). The interpersonal function examines the author's and audience's social and personal relationships. Halliday (1978) defines the interpersonal function as "including all forms of the speaker's intrusion into the speech situation and speech act" (p. 41). The textual function makes the language relevant for its intended purpose. Halliday (1978) points out that the textual function "distinguishes a living message from a mere entry in a grammar or a dictionary" (p. 42). In other words, the textual grammar focuses on organizing experiential, logical, and interpersonal meanings into coherent and cohesive texts. These three meta-functions operate simultaneously: "Speakers and writers simultaneously present content, negotiate role relationships, and structure texts through particular grammatical choices that make a text the kind of text it is" (Schleppegrell, 2001, p. 432). However, the social interpretation of language can be extended to "other semiotic resources such as …the mathematical symbolism and diagrams found in the discourse of mathematics" (O'Halloran, 2005, p. 7). Within this context, SFL has been extended to systemic functional multimodal discourse analysis (SF-MDA), which will be discussed further in the next section.

Systemic Functional Multimodal Discourse Analysis (SF-MDA)

Systemic Functional Linguistics examines the meanings made in language through the choices of the system oriented around the ideational, interpersonal, and textual metafunctions (Halliday, 1985, 1994; Halliday & Matthiessen, 2004). In other words, Systemic Functional Theory is a theory of meaning, which is initially applied to language through Systemic Functional Linguistics, and more recently through SF-MDA to the other semiotic resources. Therefore, Djonov (2005) describes SF-MDA as "an analytic practice which tests the application of the key principles of Systemic Functional Linguistics to the analysis of semiotic systems other than language and their interaction with each other and with language in semiosis" (p. 73). In this paper, the SF-MDA, originating from Systemic Functional Theory, is applied to investigate multimodal pedagogic discourse, or gestures in this case, to point out how teachers can make meanings of mathematical concepts through those gestures. The following section focuses on defining gestures and what kinds of meanings gestures can convey.

From SF-MDA perspectives, researchers classify gestures according to their realizations of ideational, interpersonal, and textual metafunctional meanings (Martinec, 2004; Hood, 2007, 2011). For the purpose of this study, to explore the representation of gestures, I only focus on the ideational meaning of gestures.

Ideational Meanings in Gesture

According to Martinec (2000), gestures realize their metafunctional meanings based on the type of gestures. Martinec (2000) proposes three kinds of actions with distinctive systems to realize ideational meanings: presenting, representing, and indexical action. Presenting action realizes ideational meaning through a transitivity process similar to language. According to Martinec (2000), presenting action does not serve a semiotic function. Examples of such actions in the classroom include picking up a pen and writing on the board. Presenting action can, therefore, be "seen as part of our experience of reality, formed in our interaction with it by means of our perceptions and motor actions" (Martinec, 2000, p. 247). To identify the meaning making of gestures, this paper only focuses on the representing action. Representing action realizes ideational meaning through its representations of participants, processes, and circumstances. That is, the elements or participants involved in the action process and how they are inscribed. Martinec (2000) defines representing action as gestures with a conventional signifying function.

These gestures are either comprehensively identifiable to all communities or within a semiotic community. The ideational meanings made by representing action are classified as participants, processes, and circumstances. Examples of these gestures are placing one's arm up at an angle of forty-five degrees to represent the concept *increasing* and putting one finger on one's mouth to represent *silence*. As Hood (2011) explains, "At times, however, the representation of ideational meaning is made only in gestures and is not expressed in the verbiage. In other words, the teachers' gestures carry the full ideational load" (p. 41). The ideational meanings in the representing action relate to their linguistic forms. For example, representations such as *triangle* and *square* are annotated as participants, representations such as *switching* and *canceling* are coded as processes, and representations such as *in front of* and *up* are noted as circumstances. This classification couples with three metafunctions of ideational meaning in language proposed by Halliday (1978).

Systemic functional multimodal discourse analysis has been used to investigate the association of language and other semiotic resources. For example, Lim (2021) studies the mechanisms of the way gesture and language was used to make meaning. His study provided the way for teachers to develop an effective way of learning through multimodality. In addition, Lim (2019) also studies the function of two EFL teachers' use of gestures in a language classroom. Purwaningtyas (2020) used SF-MDA to explore the effective use of visual images in textbooks. The finding from this study suggested images in textbooks could foster the understanding of the written content in the book. In other words, SF-MDA has been used to study the relationship between language and other semiotic resources. There are not many studies focusing on the association of gestures to the disciplinary knowledge such as mathematics. Therefore, the purpose of this study is to explore the use of gestures in mathematics. This study aims to answer the following question: How were gestures used to make meaning by an ESOL mathematics teacher?

Methodology

Research Site and Participants

This study was conducted at Dawn High School[5] in Dustin County, the third largest county in the Southeastern of the US. Dustin County is encountering an influx of immigrants from various cultures and languages. According to 2015 United States Census, the racial components of the county were 54.7% Black or African American, 36.4% White, 6.2% Asian, 0.5% American Indian, and 2.1% from two or more races. Those of Hispanic or Latino origin made up 8.8% of the population. This makes Dustin the most diverse county in Georgia.

Dawn High School has the most diverse community in central Dustin County. [6]In 2018, 975 students, or 61% of the student population at Dawn High School are categorized as Black, making up the most significant number of the student body. The following ethnicities constitute the rest of the student population at Dawn High School: 3.0 % White, 3.0% Hispanic, 32.0 % Asian, 1.0 % Two Races, <1.0% Native American, and <1% Hawaiian Native. Dawn High School is considerably different from a typical school in the state, which comprises 36.9% of Black students on average. Students at Dawn High School come from more than 54 countries and speak 47 languages.

Through purposeful sampling (Patton, 2015), in this article, I focus on data collected during the second year of the study. Data was collected in Mr. Shawn's Coordinate Algebra I. This is a sheltered class. Students in this class were classified as multilingual learners. Mr.

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[5] All are pseudo names

[6] Retrieved from https://www.greatschools.org

Shawn, a White male, has worked as an ESOL teacher for six years. The class, however, is highly diverse. The total number of students is 25 (Males: 11, Females: 14). They are from Congo, Nepal, Thailand, and Myanmar. They speak various languages, including Arabic, Burmese, French, Karen, Nepali, Spanish, and Thai. The class has an average ACCESS[7] score of 2.8, which is considered relatively low. Mathematics instruction was in English. In such a multicultural and multilingual classroom, the teacher faces many challenges in communicating and transferring knowledge to students, especially in helping them move towards technical vocabulary in academic subjects, such as mathematics.

Data Selection

The data in this study comes from a corpus of three video-recorded lessons. Based on the data collected from three video-recorded lessons, this study aimed to explore the pattern of gestures used by Mr. Shawn during instruction. The recordings were part of a larger data collection over four years, and after analyzing the videos, the critical moments of using gestures were chosen to illustrate in this paper. The researchers focused on three 90-minute lessons, where Mr. Shawn introduced the properties of equality, instructed students to solve one-step equations, and worked with students to solve a word problem. By examining the use of gestures in these lessons, this study aims to provide insights into the role of gestures in effective teaching practices.

Data Analysis

I investigated gestures and oral language that the teacher produced and used while he presented mathematical knowledge to students. Therefore, the analysis relied on transcripts and

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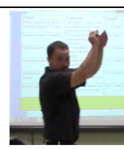
[7] ACCESS is a computer-based, adaptive test that responds to student performance and may be administered in group or individual settings. This assessment is given annually to students in Grades 1-12. It tests students' language in four domains: Listening, Reading, Speaking, Writing (https://wida.wisc.edu/assess/access).

images produced from the video recordings (Zack & Graves, 2001). To capture the teacher's uses of gestures in his interactions, I paused and print-screened the videos. Then I compiled all the gestures and verbal instructions in an EXCEL file, classified the different categories of gestures, and looked at the transcript to associate the verbal instruction with the gesture to see how the teacher used his gestures to make meaning to students. Doing this could help me identify the metafunction of gestures. Table 5.1 shows an example of my transcript. In this example, Mr. Shawn raised his hands high in the air. His two hands represented a notebook. He showed the action of writing and told the students to discuss and not write down. In this example, he used gestures to illustrate the action of writing.

Table 5.1.

An example of analyzing gestures

Phase	**Giving instruction to students**
Visual Frame	
Mr. Shawn's speech	In your group, discuss those properties for me. Don't write… don't write. Do not write.
Gestures	He raised both of his hands high in the air. The left hand was turned wide open to represent a notebook or paper. He imitated the action of writing with his right hand.
Metafunctions	Ideational meaning: participants: represent the action of writing, process: writing. Interpersonal meaning: engagement

Findings

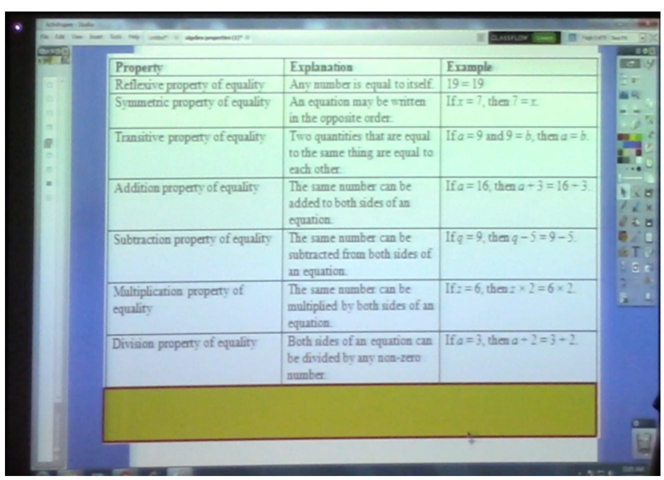
The teacher used a range of representation actions in his multilingual classroom. As described earlier, these actions represented concrete participants (i.e., concrete persons or objects), processes (i.e., activities), and circumstances (manner or context). In the findings, I demonstrate three transcripts in which Mr. Shawn used gestures to convey the mathematical concepts to the multilingual learners in class.

Transcript 1

This first transcript comes from a lesson on the properties of equality. In this lesson, Mr. Shawn explained the seven properties of equality, including the reflexive property of equality, symmetric property of equality, transitive property of equality, addition property of equality, subtraction property of equality, multiplication property of equality, and division property of equality. At first, he asked the students to read aloud the definitions of each property on the board (see Figure 1 for the text provided to the students).

Figure 5.1

The definitions of properties of equality



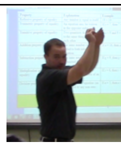
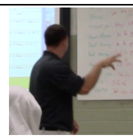
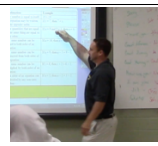
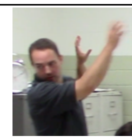
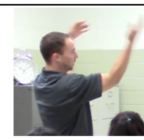
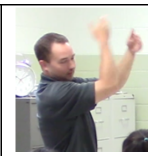
Property	Explanation	Example
Reflexive property of equality	Any number is equal to itself.	$19 = 19$
Symmetric property of equality	An equation may be written in the opposite order.	If $x = 7$, then $7 = x$.
Transitive property of equality	Two quantities that are equal to the same thing are equal to each other.	If $a = 9$ and $9 = b$, then $a = b$.
Addition property of equality	The same number can be added to both sides of an equation.	If $a = 16$, then $a + 3 = 16 + 3$.
Subtraction property of equality	The same number can be subtracted from both sides of an equation.	If $q = 9$, then $q - 5 = 9 - 5$.
Multiplication property of equality	The same number can be multiplied by both sides of an equation.	If $z = 6$, then $z \times 2 = 6 \times 2$.
Division property of equality	Both sides of an equation can be divided by any non-zero number.	If $a = 3$, then $a \div 2 = 3 \div 2$.

Afterward, he realized the students were confused when they read the definition. Aside from that, he also pointed out to me that there were many words in the definitions. To make sure students understood each property, he explained it to them. When explaining the definition, he wrote examples on the board for students to notice: a x b = b x a. The concepts were also explained through accompanying gestures. The gestures he used in this section of the lesson are listed in Table 1. In the first stage of the lesson, he provided instructions to the students. In the context of situation, he wanted the students to discuss the meaning of each property. Therefore, he told the students, “You can speak in Nepalese or English.” Also, as seen in frames 1, 2, and 3 of Table 1, Mr. Shawn also incorporated gestures into his instruction. In the instruction, Mr. Shawn said, “Don’t write. Don’t write. Do not write. Discuss. Speak”. Mr. Shawn wanted to emphasize the material processes (writing and speaking) in this utterance. Additionally, the gestures accompanying the presentation convey a sense of the material process. For example, as seen in frame 1, he opened one of his hands to represent a notebook while using the other hand to imitate the writing action. The goal of the gesture is to draw students’ attention towards his instruction, i.e., discussing with each other and not writing.

After giving instruction to the students, he approached a group of students and discussed the property with them. As seen in frames 4, 5, and 6 of Table 1, when explaining to students the *symmetric property of equality,* to indicate the participant, i.e., *an equation*, he moved his left hand to the left and his right hand to the right. This represented two parts of an equation. Then to visualize the definition of the *symmetric property of equality* (*the symmetric property of equality states that if two variables a and b exist, and a = b, then b = a)*, he switched his two hands and said, “What we have, **one side** and **the other side,** we can switch.” With the action of switching two hands, he helped students visualize the definition of the symmetric property of equality with his action of Representation Action to identify this process.

Table 5.2

Gestures of explaining the symmetric property of equality

Stage	Explaining the symmetric property of equality					
Phase	Giving instructions to students			Explaining the property to students		
Salient Visual Frame						
	Frame 1	Frame 2	Frame 3	Frame 4	Frame 5	Frame 6
Mr. Shawn's speech	In your group, discuss those properties for me. Don't write… don't write. Do not write.	Discuss.. speak… about what each property means. It tells you right there, but I want you to say. So read through each one, then I want you to say about what each property means.		You can discuss it in Nepalese or English. What about one side of the equation and the other side? Then we can switch. That's all that I am asking you right now.		
Gestures	He raised both of his hands high in the air. The left hand was turned wide open to represent a notebook or paper. He imitated the action of	He made the shape of a mouth with his right hand. Then he opened and closed the hand to illustrate the action of speaking.	He stretched his arm over the board and moved his arm as he said, "Each property."	He put his two arms one by one into the air.	He kept his two arms high in the air.	He switched his two arms in the air.

	writing with his right hand.					
Metafunctions	Ideational meaning: process: the action of writing. Interpersonal meaning: engagement	Ideational meaning: process: the action of speaking Interpersonal: engagement	Interpersonal meaning: engagement	Ideational meaning: Participants: represent the two parts of the equation	Ideational meaning: process – switching the two parts of the equation	Ideational meaning: Participants: the two parts of the equations after being switched.

In SFL, for expressing ideational meaning (what is being discussed), language choices are affected by the context. In this instance, the teacher's explicit goal was for the students to memorize and understand how to use the symmetric property of equality. As seen in Table 5.3, the gestures used in Mr. Shawn's instruction allow the students to do symmetric property of equality. In contrast, the definition written on the board, "An equation may be written in the opposite order," focused more on the manner, i.e., the position of the equation. So, gestures in this situation provide the students with the visualization of the process of symmetric property of equality. With that being said, students could know that if they want to use the symmetric property of equality, they will need to switch the two parts of the equation.

Table 5.3.

Comparison between the verbal language and gestures

Stage	Symmetric Property of Equality	
Modes	Verbal Explanation	Gestures
Transcript	An equation may be written in the opposite order.	He put his two arms one by one into the air. Then he kept his two arms high in the air, and he switched his two arms in the air.
Metafunctions	Ideational meaning: Participants: an equation Process: may be written Manner: in the opposite order	Ideational meaning: Material process: switching the two hands

In short, in this transcript, Mr. Shawn used a range of gestures and verbal explanations on the board to help students understand the symmetric property of equality. The gestures can help student imagine the conceptual knowledge of the symmetric property of equality. Mr. Shawn also used his body to explain the meaning of word problems to students, as shown in Transcript 2.

Transcript 2

One of the significant challenges in teaching mathematics to multilingual learners is to help them comprehend word problems. Transcript 2 demonstrates a lesson on helping multilingual learners understand word problems before they solved the problems. The word problem in that day's lesson was:

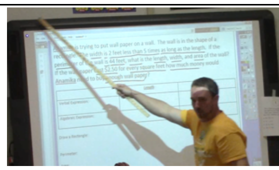
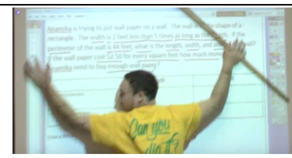
> *Anamika is trying to put wallpaper on a wall. The wall is in the shape of a rectangle. The width is 2 feet less than 5 times as long as the length. If the perimeter of the wall is 44 feet, what is the length, width, and area of the wall? If the wallpaper cost $2.50 for every square feet how much money would Anamika need to buy enough wallpaper?*

In this activity, instead of asking students to underline the keywords and write down the expression as I saw in other classes, Mr. Shawn asked students to go to the board and figure out the meaning of *perimeter, area, length, and width*. As seen in the problem, he chose the name *Anamika* in the word problem. As I talked to him, he wanted to ensure that the students were familiar with the name in their culture.

At the beginning of solving the problems, he asked one of the students to go to the board to show him the perimeter of the board. He noted that he had shown them the perimeter and area of the board the previous day. After the students showed him the perimeter and the area, he showed the students the perimeter of the board and pointed to the length and the width. Then he asked the students how they could calculate the perimeter of the board. The students answered that they needed to find the sum of all four sides of the board. He pointed to the sides of the board with a ruler and dragged along the length and width of the board to show students what the length and width were. Then he showed students the area of the board. As seen in frame 3, he moved his hands higher, and his whole body covered the board to indicate the area of the board.

Table 5.4.

Gestures to explain the meaning of perimeter and area

Stage	Explaining the perimeter and area in a word problem	
Phase	Explaining the perimeter to students	Explaining the area to students
Salient Visual Frame		
	Frame 1	Frame 2
Mr. Shawn's speech	It said that the perimeter is 44ft. So first, show me what the perimeter is.	Show me the area.
Gestures	He moved his hands along the sides of the board while explaining the concept of the perimeter.	His face is facing the board, and he raises his hand high to cover the board to represent the area of the rectangle.
Metafunctions	Interpersonal meaning: engagement Ideational meaning: participants: the lengths and widths of the board.	Ideational meaning: areas

In this scenario, gestures were used as a meaning-making process in explaining the perimeter and the area of a rectangle. The pointing gestures with the action of dragging the ruler visualize the length and the width of the board. This could implicitly tell the students how to find the perimeter of the board. The gestures in this scenario make the concept *perimeter* vivid to the students. Instead of providing the definitions, Mr. Shawn provided the meaning of *perimeter* and *area with his gestures*. Also, in this transcript, gestures, which were used spontaneously or drawn from a repertoire of past teaching experiences, not only provided a metaphoric image for helping students to come up with the answer but were also employed as a teaching tool that could provide students with a learning strategy (use of gestures themselves). This process could help them understand the context and provide the conceptual understanding of the perimeter of a rectangle. It is interesting to notice that when Mr. Shawn asked students to show the area of a rectangle, one student came to the board and imitated Mr. Shawn's gestures when explaining the concept of area to her classmates. This process provided an evolving potential for the internalization of embodied learning. The students not only observed the teachers but also used those gestures again in their explanation of the area of the rectangle when communicating with their peers in the class.

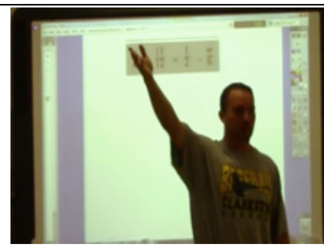
In short, the embodiment aided students in visualizing perimeters and areas of a rectangle, fostering their understanding and helping them solve word problems. For example, when Mr. Shawn asked one student to show the perimeter of the board, the student used a ruler and dragged it along the sides of the board. This process aided multilingual learners in the class in understanding the concept of perimeter and how it is calculated without having to access a lot of words defining the perimeter of a rectangle in textbooks. Not only by focusing on the action but the gestures used in the class also defined the mathematical concepts as shown in Transcript 3.

Transcript 3

In this lesson, Mr. Shawn explained the process of adding and subtracting fractions to the students. In this scenario, verbal language and gestures were used to conceptualize a fraction and convey the meanings of *denominators* and *numerators.*

Table 5.5

Collection of gestures to explain the denominator and the numerator

Phase	Explaining the denominator and numerator
Salient Visual Frame	
Mr. Shawn's speech	T: What operation is that? (*T points at each fraction*) T: Division, and you say a good word, denominator. What is my denominator here? (*T points at denominator on screen*) T: denominator bottom, numerator top (*T shows gestures*)
Gestures	He raised his hands higher to show the position of the numerator, and he lowered his hands to indicate the position of the denominator.
Metafunctions	Ideational meaning: participants: the position of the denominator and numerator.

As seen in Table 5.5, when the teacher raised his arm higher and said "denominator" and pointed at the number above the line in the fraction. Table 5.5 shows the interaction of all these semiotic resources. Instead of using long and dense noun phrases to define the denominator and numerator, the teacher can use his hand movement. The textbook shows that the denominator is "the number placed below the horizontal line of a fraction"; the numerator is "the numerator is the number written on the top of a fraction." The SFL analysis of the definition highlights the circumstance (e.g., manner or location) of the position of the numerator and denominator in a fraction. The hand gestures also show the position of the numerator and denominator. Hand gestures can visualize the positions of the numerator and denominator in a fraction.

Discussion and Implications

This study aimed at delving into the various meanings that gestures could convey in a mathematics classroom, particularly when it comes to supporting multilingual learners. To do this, I applied the analytic framework of systemic functional multimodal discourse analysis (O'Halloran, 2005; Lim, 2011). Through the analysis, I aimed to determine whether these meanings were consistent or varied between gestures and spoken language and how these differences could aid multilingual learners in grasping mathematical concepts.

The findings revealed that gestures and speech could effectively convey semantic intent, either separately or together, to help multilingual learners better understand the context (e.g., the perimeter of a rectangle), and position of a fraction. This can later help them understand mathematical concepts. In terms of ideational meaning, for instance, I found that gestures were particularly helpful in illustrating the operational process of doing symmetric property of equality or finding the perimeter. During the data analysis, I observed how Mr. Shawn used gestures to help multilingual learners visualize how to calculate perimeter and area. In another scenario, I noticed how he combined gestures and verbal language to explain the position of the denominator and numerator in a fraction. For instance, he raised his hands to the top to indicate the numerator. This study emphasizes the importance of incorporating gestures in a math classroom, especially for multilingual students who may benefit from different modes of communication.

In terms of interpersonal meaning, gestures served as a means of communication that drew the multilingual learners' attention, making it easier for them to comprehend the material being taught. For example, in the findings, gestures are often used to help multilingual learners understand concepts like *perimeter* and *area*. Overall, gestures are an effective tool for facilitating interaction and promoting learning of mathematical concepts in the classroom.

In all three examples, the gestures and the verbal language carried the same ideational meaning, the intended knowledge the teacher wanted the students to learn about. This observation has led me to incorporate more gestures into my teaching practice, which has helped me better communicate with multilingual learners. For example, I learned that in the classroom, gestures could be just as effective as verbal language in helping multilingual learners understand concepts and complete activities. By using gestures to communicate mathematically, I was able to help my students understand how to find the median of a series of numbers. I found that my students were able to replicate these gestures the next day, showing that gestures can contribute to knowledge construction. It's clear that language is not the only resource in the classroom when it comes to communicating with students, and I plan to continue using gestures to help my students learn. As explained by Jewitt (2008), the way that teachers and students use their gaze, body posture, and the distribution of space and resources can have a significant impact on literacy development in the classroom. Multimodal research in education is important because it allows for more intentional use of semiotic resources that can enhance teaching and learning. Humans create meaning through various forms of communication and often do so unconsciously. Teachers can better orchestrate their use for more effective teaching and learning by becoming more aware of the range of semiotic resources available. This paper proposes a theoretical framework for identifying and analyzing teachers' use of gestures, which, when combined with the analysis of other semiotic resources, can provide insights into how teachers orchestrate multimodal resources to support literacy development.

Conclusion

This study aimed to collect and analyze a corpus of gestures the ESOL teacher used to construct the meaning of mathematical concepts. Working with multilingual learners requires teachers to use other semiotic resources such as gestures, visual displays, or graphs rather than depend solely on language. Though gestures cannot be applied in all instances of teaching and learning, findings from this study show that teachers can use them when applicable, i.e., in teachable moments, to foster multilingual learners' understanding of mathematical terms, which then help them understand the concepts. We believe that the embodiment of mathematical thinking, instruction, and communication has the potential for learning to occur and cognitive development. Nowadays, it has become easier to gather data about how teachers are using different resources to enhance their pedagogy and create a meaningful learning experience for their students. However, analyzing this multimodal data presents a challenge for researchers. In this paper, I hope to add to the ongoing conversation on this topic by proposing an approach informed by systemic functional multimodal discourse analysis.

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CHAPTER 6

Digital Teaching Materials in Teaching Mathematics to Multilingual Learners

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Abstract

The pandemic has brought about a lot of changes in the way we live and work, and this includes the classroom environment. Teachers have had to adapt and come up with new ways of using teaching materials to make sure that their students continue to learn and thrive. One of the ways they have done this is by incorporating digital teaching materials. Digital teaching materials have provided new potentials in teaching, especially through animations and interactive features. This study explores the evaluation of digital teaching materials used by a high school ESOL mathematics teacher in Georgia. Informed by the social semiotic approach, this study explores the three metafunctions of digital teaching materials and how they can make meaning to scaffold multilingual learners in understanding mathematical concepts. The data for this study was collected from online lessons delivered through Pear Deck and Desmos. It was discovered that digital teaching materials provide students with multiple semiotic resources, which helps them to comprehend the materials. Additionally, the process of doing mathematics was emphasized, not just showing the relation between mathematical identities. Finally, the relation between image and language was also highlighted to support the students' understanding of mathematical concepts.

Key words: digital teaching materials, social semiotic approach, multilingual learners, mathematics

As school was functioning normally in 2019, the pandemic broke out. Students and teachers across the United States were unsure what to do. A two-week shutdown was imposed on the entire state of Georgia. During those days, teachers and students were required to stay at home, avoiding the daily routines of their school lives. Physical interaction was prevented in the event of a lockdown and social distancing. Schools then resumed with students having to join classes online. Students and teachers had to rethink their approach to work, learning, and play. However, guidelines about how to meet the needs of multilingual learners varied between states, and most guidelines were offered late (Villegas & Garcia, 2022). Therefore, ESOL teachers faced additional challenges in scaffolding multilingual learners' understanding of their materials (Wong et al., 2022). In this context, digital technology has assumed greater prominence than ever before. To create digital learning experiences effectively, teachers who had previously been reluctant or resistant to adopting digital technology have had to understand how to design them. This was coupled with the "multimodal turn" (Jewitt, 2016) to enlighten how materials were developed to scaffold multilingual learners' understanding of disciplinary materials.

This context led some teachers to embrace multimodal literacy in digital teaching materials (Kim et al., 2021). Digital teaching materials enable opportunities to include new multimodal resources and to organize mathematical information in new ways by linking to explanations, definitions, examples, and tasks that can be shown or hidden (O'Halloran et al., 2018). In other words, the pandemic forced teachers to think of new ways of using materials in the classroom. Digital teaching materials then provided new potentials in teaching, for example, through animations and interactive features. In other words, digital teaching materials could support new forms of individualized learning and interaction with new types of resources (Utterberg et al., 2019). This forces us to expand our understanding of literacy to encompass other semiotic resources beyond written text (Harste, 2010). “Multimodality” describes texts that include two or more semiotic systems or modes of communication, such as still images, moving images, writing, speech, sound, gestures, layout, and spatial orientation (The New London Group, 1996; Jewitt & Kress, 2003). In response to this change, based on comparing mathematics text in digital environments and texts in print, Usiskin (2018) concludes that the digital environment is well suited for displaying several different representations.

Technology, however, is only a vehicle for change. Changes must focus on making meaning more accessible and available to learners (van Leeuwen, 2017). This understanding can enable multilingual learners to adapt to new ways of interacting with their teachers, their peers, and the screen. Moreover, the knowledge of the meaning-making affordances of digital teaching materials can help teachers guide students in working with digital materials (Lim & Toh, 2020). It is also expected that students acquire the digital literacy skills necessary to explore the complex digital multimodal knowledge intended to engage their multiple senses through multimedia. As a result, multilingual learners can access digital multisemiotic resources in a more straightforward manner (Wang & Hemchua, 2022). In other words, a deepened understanding of the meaning-making mechanism of digital teaching materials in teaching mathematical concepts to multilingual learners should be considered. To do this, I have relied in this study on a social semiotic perspective (O’Halloran, 2005). An important point within this perspective is that the chosen multimodal expressions direct the realized meaning (Bergvall & Dyrvold, 2021). By studying the meaning potential of digital teaching materials, we can gain an understanding of the meaning offered to readers. From social semiotic perspectives, previous studies show that gestures and other semiotic resources (e.g., postures and wooden blocks) can facilitate multilingual learners’ construction of disciplinary knowledge (Bui & Harman, 2019;

Harman et al., 2021). In the same line of inquiry, digital teaching materials can generate further interest in this field due to the development of new modes of representation. To be more specific, it is necessary to explore the different ways of meaning-making in digital learning, including what kind of knowledge is presented in the materials, the interaction between the learners and the materials, and how learning is organized through the affordances of multiple semiotic resources used in digital teaching materials (Lim, 2021). Therefore, this paper aims to reflect on the different ways of meaning-making digital teaching materials in Mathematics offered in virtual learning experiences. To understand the meaning potential digital teaching materials can bring to the class, systemic functional linguistics to multimodal discourse analysis (Halliday, 1978) should be used.

Social Semiotic Perspectives on Mathematics Digital Multimodal Resources

The advent of technology has impacted meaning-making processes and simultaneously has provided a platform for the continued development of disciplinary knowledge and practices. From a social semiotic perspective, the meaning potential of each semiotic system differs (Kress & van Leeuwen, 2001). For example, O'Halloran (2005) argues that visual modes such as graphs (e.g., the parabola) demonstrate the representation of "graduations of different phenomena" (p. 132), while algebraic symbolisms represent the relation between elements of an equation. This dynamic raises questions regarding the kinds of meaning that can be derived from digital teaching materials.

A social semiotic perspective suggests that communication comprises three metafunctions: ideational, interpersonal, and textual (Halliday & Matthiessen, 2014). The ideational function looks at the content or ideas construed in a text. Halliday (1978) describes it as "the categories of one's experience of the world and how they interpret this experience" (p.

38). The interpersonal function examines the author's and audience's social and personal relationships. Halliday (1978) defines his interpersonal function as “including all forms of the speaker’s intrusion into the speech situation and speech act” (p. 41). The textual function makes the language relevant for its intended purpose. Halliday (1978) points out that the textual function “distinguishes a living message from a mere entry in a grammar or a dictionary” (p. 42). In other words, the textual grammar focuses on organizing experiential, logical, and interpersonal meanings into coherent and cohesive texts. These three meta-functions operate simultaneously: “Speakers and writers simultaneously present content, negotiate role relationships, and structure texts through particular grammatical choices that make a text the kind of text it is” (Schleppegrell, 2001, p. 432). As Kress and Bezemer (2003) and O’Toole (1994) point out, the notion of metafunctions of language extends not only to language itself but also to other semiotic resources such as diagrams, displays, and gestures. It is possible to gain a deeper understanding of the meaning offered to the reader by studying the metafunctions of each mode. Different studies use a wide range of terminology to identify metafunctions. This paper uses the metafunctional terms from Systemic Functional Linguistics (i.e., ideational, interpersonal, and textual). The metafunctions of SFL also play an essential role in analyzing other semiotic systems. For example, Kress & van Leeuwen (1996) and O’Toole (1994) applied this aspect of SFL in analyzing visual texts. Within this inquiry, Systemic Functional Multimodal Discourse Analysis (SF-MDA) was developed as an extension of systemic functional linguistics to explore multimodal resources.

SF-MDA offers theoretical and practical approaches for analyzing printed and digital materials where semiotic resources (e.g., language, visual displays, mathematical symbolism, gesture, and other meaning-making resources) combine to make meaning (O’Halloran, 2008). As

SF-MDA is built on Halliday's (1978) metafunctional principle, the framework explains how semiotic resources interact to produce meaning (Kress & van Leeuwen, 2006). The terminology of metafunctions of SFL is different among studies. For example, Kress and van Leeuwen (1996) used representational, interactive, and compositional, referring to ideational, interpersonal, and textual. However, in this paper, due to the analysis of both language and other semiotic resources, the terms from Halliday (i.e., ideational, interpersonal, and textual) are used. Within this framework, I investigate three metafunctions of digital teaching materials in mathematics (see examples in Table 6.1): (1) ideational meaning (i.e., the disciplinary knowledge the tools can provide); (2) interpersonal meaning (i.e., the interaction between the multilingual learners and the tools), and (3) the textual meaning (i.e., the organization of the content within a text and across the modes).

Ideational meaning

The ideational meaning refers to the knowledge of mathematics brought to the audience through the system of Participants (i.e., things or elements), Processes (e.g., operational or relational process), and Circumstances (i.e., setting of the situation). In mathematics, O'Halloran (2005) classified Processes into two categories: operational process and relational process. Operational processes show mathematics as constructed by "doing," for example, *arrows or adding + symbol*. Relational processes demonstrate mathematics as a system of relationships between objects, for example, hierarchal order or multiplying (*) in 8x = 64.

Interpersonal meaning

The interpersonal meaning focuses on the relation between the reader, in this case, the students, and the materials. In mathematics, the relationship between the reader and the texts can be based on 'modality marker' and 'coding orientation' (Kress and van Leuween, 2006). Kress and van Leeuwen (2006) describe modality markers as ranges of colors, background detail, and spectrum of light and shading. With coding orientation, Kress and van Leeuwen refer to the needs of specific groups within a particular context. Therefore, the representation of modality depends on the purpose of the representation goals of the users in a specific genre. They propose four coding orientations: (1) The naturalistic is the communication between the object of representation and how the naked eye would view it; (2) technological emphasizes the effectiveness of representation as a 'blueprint' for a user; (3) sensory refers to the ability of image that can awaken the sensories of the users; and (4) abstract usually used in art or science refers to the abstract ideas of the objects. In mathematics, natural coding orientation and technological orientation are usually used. For example, naturalistic can refer to images depicting real situations, or the absence of mathematical notions, while technological orientation refers to the mathematical notation or the disciplinary language (Bergvall & Dyrvold, 2021).

The textual meaning

The textual meaning discussed which semiotic resources, modes, and dynamics are used to express the central aspects of the target mathematical concept. As described in O'Halloran (2005), three semiotic resources are discussed: language, symbols, and images. The logico-semantic relation between semiotic resources can be classified through elaboration, extension, and enhancement (Martinec & Salway, 2005). Martinec and Salway present exposition and exemplification as the two variations of an elaborative logico-semantic image–writing relation. Exposition means that image and text have the same generality, while exemplification means one is more general. The extension means that either writing or image adds new, related information to the composition. Enhancing the logico-semantic relation between image and writing means one qualifies the other circumstantially.

Table 6.1

Three metafunctions of analyzing digital teaching materials

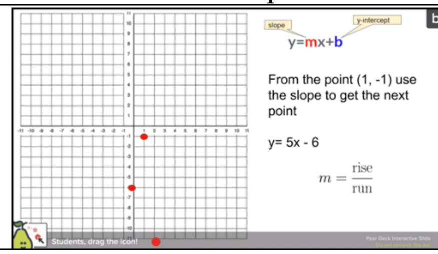
Metafunctions		Examples
		
Ideational meaning	- Participant - Process: Operational process or relational Process - Circumstances	- Slope, the dot, rise, run, y-intercept. - Relational process: m = rise/run. - Operational process: the movement of the dots. - Circumstances: the graph on Pear Deck.
Interpersonal meaning	- Coding orientation: naturalistic, technological, sensory, and abstract.	- Technical: use a lot of mathematical notations.
Textual meaning	- What semiotics are used and how are they related to each other?	- Verbal language, graphing, and formula. - Extension: the dot shows the position of the y-intercept and the shapes of the equation.

Table 6.1 shows these three metafunctions are closely intertwined and interact in making meaning. Identifying these metafunctions can assist in explaining how each mode contributes to constructing mathematical knowledge. Digital teaching materials, however, use a combination of

modes to create meaning. Due to their interconnections, modes used in digital teaching materials should be regarded as part of an intersemiotic system.

The Intersemiosis Across Digital Multimodal Semiotic Resources

Digital teaching materials are multimodal in nature. This calls for an investigation of how a range of semiotic resources co-exists to produce meaning. Within this line, O'Halloran (2005) introduces the concept *intersemiosis* to describe "the meaning arising across semiotic choices" (p. 159). To contribute to this dynamic, Royce (1998) offers the construct of "intersemiotic complementarity" where "visual and verbal modes semantically complement each other to produce as single textual phenomenon" (p. 26). In the same vein, Lemke (1998) suggests that the combination of modes used in scientific textbooks can provide more profound knowledge. With that being said, the variety of semiotic resources can create a deeper meaning than the meaning made by an individual semiotic resource. Thus, this study also focuses on the relationship between semiotic resources in digital teaching materials.

The theoretical framework of intersemiosis has been applied in previous studies to explore the meaning-making process across the modes. Royce (1998), for example, investigated the ideational intersemiotic metafunction of verbal language and images in the article *Mountains* of the Economist magazine. In this study, he proposed that the visual and verbal modes convergently produced the meaning. Similarly, Cheong (2004) conceptualized the expansion of ideational meaning across visual and linguistic means in print advertisements. In teaching disciplinary knowledge, Hsu and Yang (2007) investigated the impact of integrating science text and images on students' reading comprehension. While the two texts used in their study contained similar scientific concepts, there were differences in how they were presented in terms of visual displays and texts. The findings highlighted how images used in the texts enhanced students' reading comprehension. In other words, reading comprehension is improved when print and images are integrated. In the theory of intersemiosis, mathematics discourse is often characterized as multimodality. It "involves language, mathematical symbolism, and visual images ... [and] mathematical printed texts are typically organized in particular ways which simultaneously permit segregation and integration of the three semiotic resources" (O'Halloran, 2005, p. 11). In other words, each mode of mathematical discourse contributes to the learners' knowledge construction. Previous studies focused on analyzing ideational meaning across the verbal and visual images of printed materials. However, the development of digital teaching materials has also incorporated animated pictures into the design of the lessons. Therefore, this study focuses on identifying the content, the interaction between the materials and learners, and the organization of the materials in supporting multilingual learners' understanding of mathematical concepts. This study was conducted to address the following question: How could digital teaching materials contribute to knowledge construction in Mathematics?

Methodology

Research Site and Participants

This study was conducted at Dawn High School[8] in Dustin County, the third largest county in Georgia. Dustin County is encountering an influx of immigrants from various cultures and languages. In the school year 2020-2021, the total population of students was 93,674, comprising 59% African American, 20% Hispanic, and 7% of Asian. This makes Dustin the most diverse county in Georgia.

Dawn High School has the most diverse community in central Dustin County. According to US News, 95.2% of the students are minority enrollment. Regarding demography, 4.8% of the

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[8] All names used in this study are pseudo names.

students are white, 57.7% are African American, 32.2% are Asian, and 1% are Hispanic. Dawn High School is considerably different from a typical school in Georgia. Students at Dawn High School come from more than 54 countries and speak 47 languages.

Through purposeful sampling (Patton, 2015), this study takes place in a ninth-grade ESOL Coordinate Algebra class. Mr. Shawn is an ESOL teacher who identifies as a White male. He has been teaching for nine years. However, the class is highly diverse. The total number of students is 23 (Males: 10, Females: 13). They are from Congo, Nepal, Thailand, and Myanmar. They speak various languages, including Arabic, Burmese, French, Karen, Nepali, Spanish, and Thai. The class has an average ACCESS[iii] score of 2.8, which is considered relatively low. I have been working with Mr. Shawn for four years. When collecting data for this study, I was a graduate student and a researcher interested in different semiotic resources in teaching Mathematics. Introduced by an instructional coach at Dustin County, I observed Mr. Shawn's class. His spontaneous gestures helped multilingual learners understand mathematical concepts, which interested me. Since then, we discussed how different semiotic resources could support multilingual learners in understanding mathematical concepts. During the pandemic, he was confused about what we could do to help multilingual learners. Our discussion about what to do during the pandemic led us to use digital tools to scaffold the understanding of multilingual learners of mathematical concepts. With the available resources provided by the county, we decided to use Pear Deck and Desmos[9] in designing the lessons. As discussed in my previous studies, gestures and other semiotic resources (e.g., postures and wooden blocks) could facilitate multilingual learners' construction of disciplinary knowledge (Bui & Harman, 2019; Harman et al., 2021). Coupled with other semiotic resources, digital teaching materials such as Pear Deck and Desmos have generated further interest in this field due to the development of new modes of representation.

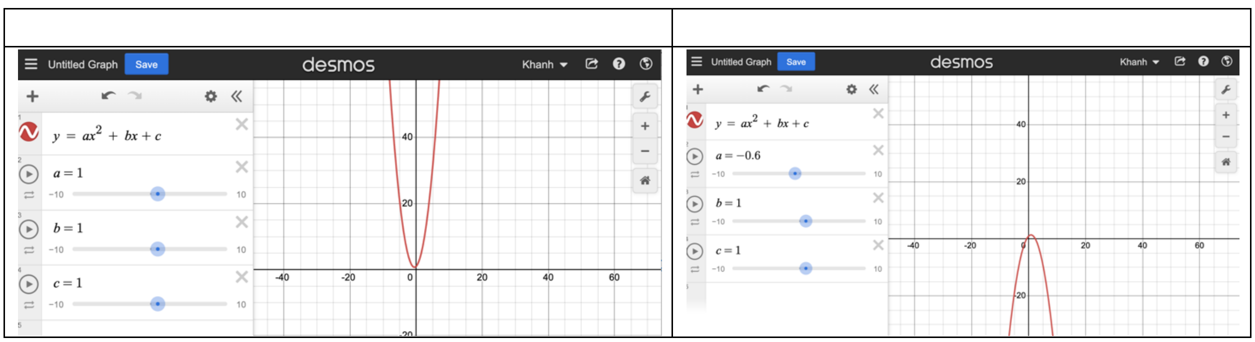
The Digital Multisemiotic Environment

As mentioned earlier, Mr. Shawn and I used digital tools during the pandemic to scaffold multilingual learners' mathematical concepts. In 2019, schools were disrupted by the outbreak of COVID-19. Then schools resumed online instruction. I could not observe Mr. Shawn's class at that time as everyone did not know what to do to support the students. Starting in the Fall of 2020, students were still learning online. Therefore, digital teaching materials were still prominent tools. I started contacting and working with Ms. Shawn to discuss using digital tools and other semiotic resources to support multilingual learners. At the time of the experiment, September 2020, the students were studying linear functions, so I worked with him on designing lessons on the linear function. The lessons were conducted in five weeks. My study explored teaching experiments involving a multi-semiotic interactive learning environment. Mr. Shawn, the focal teacher in this study, conducted the teaching experiments. The lessons were designed to provide multiple linked representations to enhance students' understanding of concepts of linear functions and quadratic equations. In our discussion, Mr. Shawn and I aimed to find out how semiotic resources and demonstrations could be used to assist students in identifying elements in linear and quadratic formulas and figuring out how to write them in mathematical symbols.

The interactive learning environment of the class consisted of Pear Deck and Desmos. These two platforms allowed the multilingual learners to make observations, construct the answers, and record their answers on the slides. Desmos (https://www.desmos.com/) is a tool that can be used to engage students innovatively in mathematics. Desmos is a free online graphing calculator which can be accessed via a browser or as a mobile app. In Desmos, students can graph equations and inequalities and perform operations. To start using Desmos, visit www.desmos.com/calculator and simply type any expression. For example, when students enter y = 5x + 8, it returns immediate feedback with a graph. In Desmos, students can add a slider to change the parameters instead of having constants to observe the change in the graph. For example, as shown in Figure 1, when students change the value of a, the shape of the graph will change. Therefore, students can observe the change in the graph. Another use of Desmos is that teachers can access students' responses. In that way, teachers can support multilingual learners in enhancing their vocabulary.

Figure 6.1

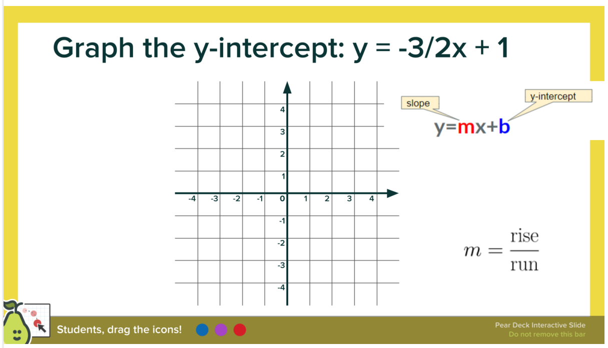
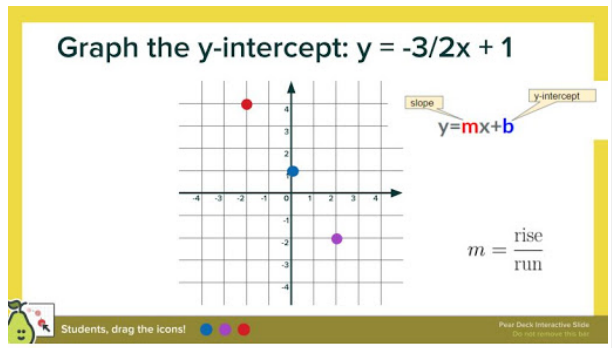
Demonstration of Desmos using the slider



Besides Demos, we used Pear Deck to enhance students' interaction and engagement. Pear Deck is an online application used on smartphones, tablets, laptops, and computers. To use it, teachers need to create lessons on google slides and import them into Pear Deck. Teachers can design interactive questions like open answers, multiple choice, and drawing. This allows the students to become more self-motivated. For example, as shown in Figure 2, one of the activities we asked the students to do in class was dragging the dots with different colors to identify the components, i.e., the slope and y-intercept of a linear equation. Teachers could observe the students while they were working on the activity so that teachers could provide immediate feedback.

Figure 6.2

The activity on Pear Deck

<table>
<tr><th>Slide 3</th><th>Your Response</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

Most of the lessons were conducted on Pear Deck. The focal unit in this study is the linear function on Pear Deck. I chose to work on linear equations, an integral unit in their curriculum. Most of the End-of-course questions are related to linear equations. Also, Mr. Shawn worked on linear functions with the students during the observation. The objective of the unit was to identify elements of linear functions (i.e., slopes and y-intercept), graphing the linear functions, and writing down a linear expression based on word problems. At the end of the curriculum, for the unit on quadratic functions, we used Desmos, so students could observe how to graph a parabola on Desmos. This unit focused on identifying the components of quadratic expressions, graphing quadratic functions, and solving quadratic questions using the area models and factoring methods.

Data Selection and Analysis

The data in this study comes from a corpus of twenty video-recorded lessons. As I mentioned earlier in this paper, the students attended virtually, and I also had to observe the lessons virtually. The lessons were conducted through Microsoft Teams. There were three blocks a day, but I observed the first block. Each block lasted 60 minutes, which was shorter than the regular schedule. I joined the class every morning in the second semester of 2020-2021. When I joined the class, I recorded the lessons. At the time of the observation, the students were working on linear functions.

The purpose of the study is to explore the metafunctions of each mode in digital teaching materials. Therefore, they relied on transcripts and images produced from the video recordings (Zack & Graves, 2001). After each observation, I captured the modes used in each lesson. Then I applied the systemic functional multimodal discourse analysis framework in analyzing each mode's metafunctions (as shown in Table 6.1). Studying the metafunctions of digital teaching materials provided findings that may inform the role of visuals in mathematics learning.

Findings

The analysis of the metafunctions of the lessons on Pear Deck and Desmos and their comparison with lessons on printed texts provided insight into how meaning was made in those digital teaching materials. The lessons were conducted mainly on Pear Deck. Then with the quadratic equation, we switched to using Desmos. During the observation and data analysis, I noticed the common themes analyzed below from the class, which informs the role of visuals in supporting multilingual learners with mathematics learning.

The Moving Points

This section compares the printed text from the students' textbook and a presentation on Pear Deck. The lesson highlighted how to create a table of values and plug the ordered pairs in the coordinate plane. In this section, I compare a static page of the lesson with no dynamic elements with the same materials on Pear Deck, including moving points for the multilingual learners to observe.

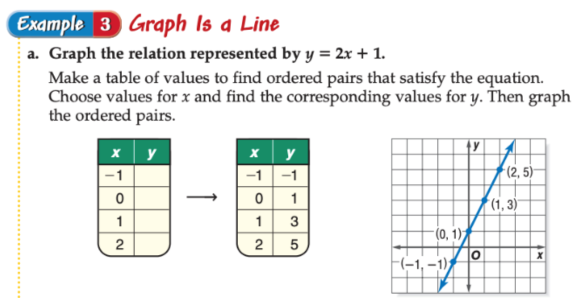
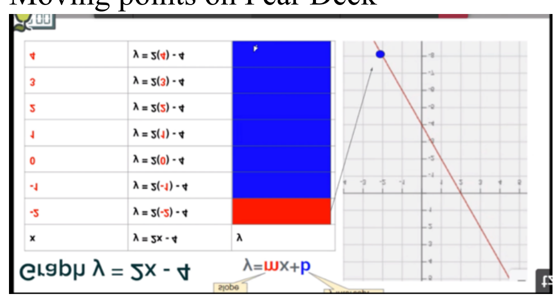
Regarding ideational meanings, the graph on the paper indicated a higher level of abstraction. The table shows the y values in relation to the x values. In addition, there is no relation between the graph and the table of values. On the Pear Deck, the arrow provides the operation process of plugging the ordered pairs from the table values and how to put them in the coordinate plane with the dynamic change of the points.

Regarding interpersonal meanings, the printed text and the slides on Pear Deck used technical coding with mathematical notation with no contexts. Students just see the expression and the numbers.

The textual meanings highlight dynamic aspects unique to the digital teaching material on Pear Deck (see Table 6.2). In the printed text, the students saw two tables of values and how the values, i.e., ordered pairs, were represented in the graph. While on the Pear Deck, there was an arrow to show the relationship between the ordered pairs and the points on the graph. The arrow moved to the next points when the slide showed the following points. The sequentially occurring images and color marks highlighted important details for the understanding of graphing linear functions. However, in the printed text, two semiotic resources are used, i.e., language and visual display. The relation between the text and the image shows that the image illustrates the language used in the printed material. Table 6.2 visualizes a simplified representation of the elements in the textbook and the movement on the Pear Deck. The analysis of the three metafunctions shows that the visuals in digital teaching materials could support multilingual learners with the procedure of doing mathematics. With the moving points, multilingual learners could see how they could find the table values, and how they could graph the points in the coordinate plane. Those activities could not be easily shown in static visuals in printed texts.

Table 6.2

The printed text and the slide on Pear Deck

Metafunctions	Printed text 	Moving points on Pear Deck 
Ideational meaning	- Participant: table, graph - Relational process: show the corresponding y values of x	- Participant: graph, table - Operational process: plug the numbers in the formula and arrow to show the connection between the ordered pairs in the table and point in the graph.
Interpersonal meaning	- Technical coding: the mathematical notion	- Technical coding: the mathematical notion
Textual meaning	- Semiotic resources: language (step-by-step instructions) and visual display. - Static on printed materials	- Semiotic resources: visual display, color - Dynamic: change over time

	- Still content	- Content is hidden behind a click

(The printed text retrieved from:

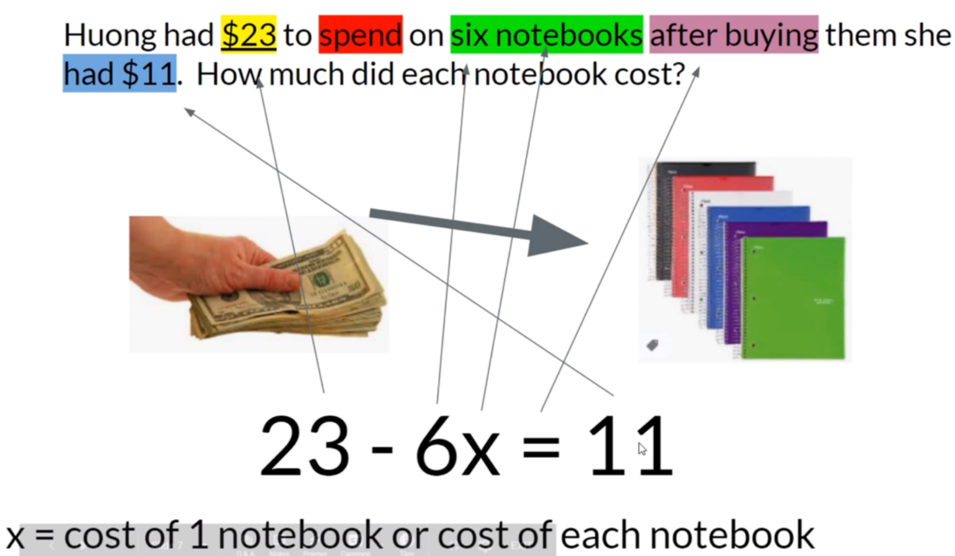
https://math24seven.weebly.com/uploads/4/8/2/8/4828203/chapter_2.pdf)

Intersemiosis Between Pictures and Text in Word Problems

Findings from the analysis shows the association between pictures and language in word problems. For word problems, we used pictures to illustrate the meaning. Our students were asked to solve the following word problem: *Huong had $23 to spend on six notebooks after buying them she had $11. How much did each notebook cost?* To demonstrate the word problem, we used an image of money, an arrow, and a picture of six notebooks (see Figure 6.3).

Figure 6.3

Word problem and the demonstration

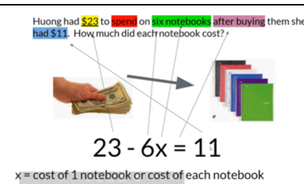


As shown in Table 6.3, in analyzing the verbal language of word problems, when it comes to ideational meanings, two participants are being tracked in the word problem given to the students, namely, the amount of money and the notebooks. The word problem consists of two phases. A time marker “after” indicates the amount of money that Huong has left after purchasing the six notebooks. In this word problem, the process moves from static (had) into action (spend) and back to static (had) after the time marker (after). A linear function word problem usually involves change, so time markers are usually incorporated into the word problem.

The visual display shows two participants, namely the money and the notebook. This process is indicated by an arrow which refers to the action *spend*. The visual display only introduces the participants and the action *spend*, which is the first phase of the problem.

Table 6.3

The intersemiosis between the verbal language and the visual display

<table>
<tr><th></th><th>Printed text</th><th>Pear Deck</th></tr>
<tr><td></td><td>Huong had $23 to spend on six notebooks. After buying them she had $11. How much did each notebook cost?</td><td>
</td></tr>
<tr><td>Ideational meaning</td><td>Processes: action (spend, buying)
Participant: Huong, $23, six notebooks
Time markers: after</td><td>Participants: picture of money, picture of 6 notebooks.
Process: operational process: change (arrow)</td></tr>
<tr><td>Interpersonal meaning</td><td>Natural coding: problem related to real life.</td><td>Natural coding: image of real-life context (money and books)</td></tr>
<tr><td>Textual meaning</td><td>Semiotic resource: language</td><td>Semiotic resources: language and visual display</td></tr>
</table>

		Use of colors The dynamic movement of arrows shows the relationship between information in the word problem and the equation

Upon further examination, it becomes evident that the visual display only illustrates a portion of the word problem. The analysis demonstrates the association between visual displays and language in supporting multilingual leaners in understanding word problems.

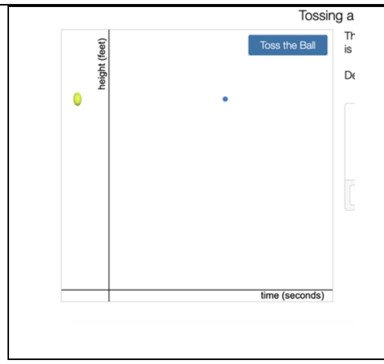
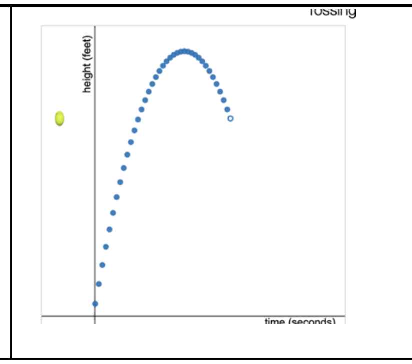
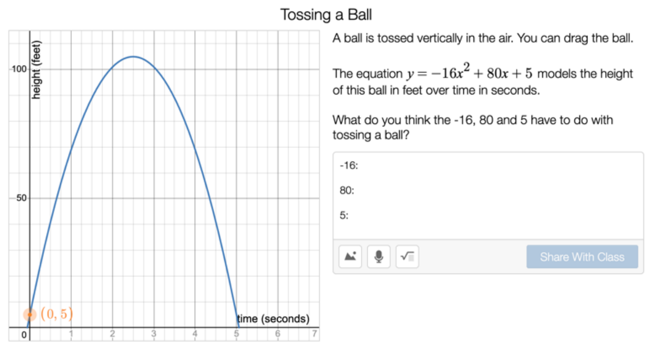
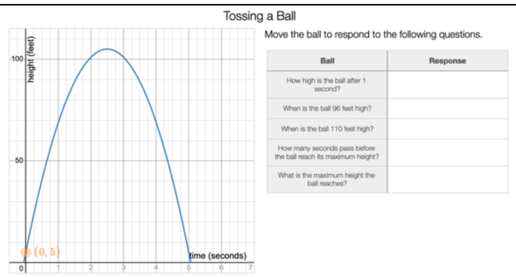
Interactive Tools Enhance Multilingual Learners' Experiences

In the unprecedented time of the pandemic, Mr. Shawn and I tried everything that we thought could be helpful to multilingual learners. This analysis was derived from a lesson on the introduction of quadratic expressions. To assist multilingual learners in comprehending three terms of quadratic expressions, i.e., the quadratic term, the linear term, and the constant time, we conducted the lesson on Desmos. The lesson was adapted from the Desmos library[10] so that multilingual learners could understand and explain the components of the quadratic formula in a real-world context. The stages of the lesson are summarized in Table 6.4. The lesson began with students observing a ball being tossed and describing its path (see Table 6.4). The students were then asked why the ball did not travel in a straight line. As a next step, they needed to identify what might impact the ball's trajectory. Mr. Shawn explained the quadratic expression and its terms. As a final step, the students were asked to move the ball, observe the graph, and answer the questions.

[10] The lesson was adapted from https://teacher.desmos.com/activitybuilder/custom/5e8b99ceb517907cb906171e

Table 6.4

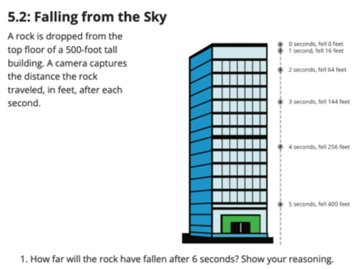
Stages of the lesson

Stages	Demonstrations
Students hit the button "toss the ball" and observed the pathway of the ball. Then they needed to write down what they noticed about the height, speed, and time in the air of the ball as it was tossed.	 
Mr. Shawn introduced the terms and asked students to identify the meaning of each term	
Students moved the ball and answered the questions	

At the first stage of the lesson, the students had to hit the button "toss the ball" and write down what they could observe and share in the formation with the whole class. The analysis of this stage is shown in Table 2. In this section, I compare the opening stage of the lesson in the printed text and on Demos. In terms of interpersonal meaning, both used natural coding on which the images are related to real life examples, i.e., a building and a rock in the printed text and a ball in Desmos. Regarding ideational meaning, the printed text shows no process, while Desmos shows the operational process. Students can hit the button "Toss the ball," and then they can observe the pathway of the ball and write down their observations (see Table 6.5). The printed text can tell students exactly the height of the rock at each second, but the students cannot see the pathway of the rock. In terms of textual meaning, the printed text provides the static image of the building and the height of the rock at different seconds. In Desmos, students can see the movement of the ball. The image is dynamic, not static. Also, when students hit the "Share with Class," they can read their classmates' answers. Mr. Shawn and I could also read their answers and provide immediate feedback to the students. The same feature can be used in the third stage of the lesson, when students can move the points to locate the ball at different times to find out the height of the ball at other times.

Table 6.5

The ball tossed, and its path

<table>
<tr><td>Metafunction</td><td>Printed text[11]</td><td>Desmos</td></tr>
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td>Ideational meaning</td><td>- Participant: rock, the building.
- Process: No processes
- Circumstances: the rock falling from the building.</td><td>- Participant: the ball, coordinate plane, the point.
- Process: operational process (the point moves)
- Circumstances: the coordinate plane</td></tr>
<tr><td>Interpersonal meaning</td><td>- Naturalistic: showing examples from real life.</td><td>- Naturalistic: showing examples from real life.</td></tr>
<tr><td>Textual meaning</td><td>- Semiotic resources: language and visual display.
- Static
- Content visible to students</td><td>- Semiotic resources: language and visual display
- Dynamic
- Content was hidden from students</td></tr>
</table>

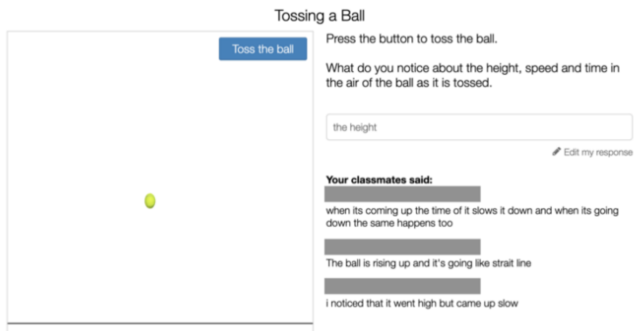
As shown in Table 6.5, Desmos supports a multi-semiotic environment, linking symbolic representations (equations) using mathematical notations with animated models and graphs. In this lesson, multilingual learners were provided multiple semiotic resources, including a visual

[11] The printed text extracted from the textbook used: https://im.kendallhunt.com/HS/teachers/1/6/5/preparation.html

display of the graph's shape, the mathematics equation, and interaction with the materials. Multilingual learners can construct meanings for mathematical objects and concepts based on various semiotic systems available. As seen in Figure 6.4, after observing the ball, the students wrote their observation, for example, "the ball is rising up and it's going like strait line" (*strait* can be rewritten as *straight*). They then could share their responses with their classmates.

Figure 6.4

Students' responses



Discussion and Implications

Analyzing the metafunction of each mode revealed how mathematics knowledge can be constructed using digital multimodal semiotic resources. Printed texts only provide students with the relational processes, e.g., the formula for quadratic expressions or a graph that shows the shape of the function (O'Halloran, 2018). This is a high level of abstraction (Bergvall & Dyrvold, 2021). Digital teaching materials, however, incorporate operational processes in their materials, which is beneficial to multilingual learners. For example, they can observe the ball's movement, so they can know that the graph of a quadratic equation is different from a graph of a linear equation. Focusing on meaning-making can aid multilingual learners in answering the reasoning questions (e.g., *What is the height of the ball when it hits the ground?*) commonly seen in the end-of-course tests.

The analysis also suggests that different semiotic resources should be used when teaching mathematics to multilingual learners since they can express different mathematical meanings. Based on the findings, the equation and formula describe the relationship between elements of the components to describe the phenomenon. We can use it to determine the exact location of the ball at a certain point in time. A graph can assist in visualizing the path of the objects. Digital teaching materials, on the other hand, are dynamic and can stimulate students' interaction, allowing them to retain the material more easily and understand the meaning of the formula (Bergvall & Dyrvold, 2021). Educators can know this meaning-making dynamic when selecting materials for multilingual learners. Also, digital teaching materials include dynamic images and provide them with immediate feedback. This can enhance their self-motivation. These might consist of using vocabulary words a teacher wants to introduce or examples of students noticing characteristics about the figures or graphs. (Caniglia et la., 2017). Based on the textual analysis of the materials, it is evident that students can be active participants in creating mathematics. This can contribute to the discussion of Systemic Functional Multimodal Discourse Analysis. In the previous paper, gestures were discussed to emphasize the operation process (Bui & Harman, 2019).

Digital teaching materials allow multilingual learners to learn mathematics by interacting with the materials. Students can gain real-life experience through this process in mathematics. Multilingual learners can also deepen their knowledge of mathematics using digital teaching materials since they can interact with the materials. This aligns with the Common Core Standards (CCSSI, 2010) state that mathematically proficient students "are able to use technology to explore and deepen their understanding of concepts" (p. 7). NCTM's Principles to Actions states, "An excellent mathematics program integrates mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking" (2014, p. 78). Teachers can use digital technology to promote constructivist student-centered teaching and learning (Wang, 2010).

As for the textual metafunction, digital teaching materials contribute to the organization of the learning experiences designed by the teacher. The design of the materials can assist teachers in understanding the holistic organization and stages of a lesson (Lim, 2021). It would be helpful to have a more explicit link between the pictures and the word problem so that multilingual learners could understand the context and solve the problem. From the analysis of the intersemiosis, the semiotic resources used can result in convergence of meaning (co-contextualizing relations of parallelism) or divergence of meaning (re-contextualizing) in multimodal texts (O'Halloran, 1999). To aid multilingual learners in understanding the context of a question, test designers or material developers might incorporate appropriate visual representations to demonstrate the problems to students.

Conclusion

This article describes a multimodal analysis method for digital teaching materials in mathematics based on social semiotic theory. This model is proposed as a tool to analyze the mathematical meanings contained in digital teaching materials. When used in the classroom, digital teaching materials are a means of communicating ideas and knowledge to students. For this reason, teachers should be aware that digital teaching materials can be used as part of their repertoire of resources for designing meaningful learning opportunities. Educators and developers can use this approach to evaluate which educational apps and technological software to use in their schools. This study provides pedagogical tools and practices for teachers when working with multilingual learners. Nevertheless, due to the pandemic, I could not reach out to teachers or students to inquire about their experiences using these digital materials. Additionally, to avoid creating unnecessary tension for the teacher during such a sensitive time, I did not have a lot of opportunities to interact with the teacher. As part of a future study, I would like to interview the teacher to understand his perspective on digital teaching materials even after the pandemic.

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CHAPTER 7

CONCLUSION AND IMPLICATIONS

This study aimed to explore the perception of ESOL mathematics teachers on using multiple semiotic resources in teaching mathematics and, in particular, how a high school mathematics ESOL teacher used numerous semiotic resources (including digital teaching materials) to support multilingual learners in a sheltered class to understand mathematical terms. For this purpose, this study consisted of three papers, each addressing a research question. The first paper explored ESOL teachers' perspectives regarding using multiple semiotic resources. The paper addressed the question: What were ESOL teachers' and multilingual learners' perceptions of using multiple semiotic resources in teaching mathematics to multilingual learners? In the second paper, I explored how gestures might contribute to multilingual learners' understanding of mathematical concepts. The second paper aimed at answering the question: How were gestures used to make meaning by the teacher? How did the teacher avail the use of gestures? The third paper discussed the meaning-making potential of digital teaching materials. This paper answered the following question: How could digital teaching materials contribute to knowledge construction in mathematics?

Overall, the three studies led to some key findings useful for researchers and educators working in second language literacy and mathematics disciplinary discourse. First of all, through the interviews, I have discovered that ESOL mathematics teachers have used a range of semiotic resources to effectively teach mathematics to multilingual students. During interviews, the majority of these teachers acknowledged that employing multiple semiotic resources can help explain mathematical terms. There is a need for further exploration and understanding of the ways in which different semiotic resources can be effectively used to support multilingual learners. The implementation of new content standards in the United States requires that multilingual learners are able to take full advantage of these opportunities and overcome these challenges. It is essential that educators at all grade levels should reexamine the current practices and explore new ones, i.e., consider incorporating multimodal component *strategically* in their curriculum design to enhance students' engagement and understanding.

The second paper highlighted the significant role of gestures in the classroom in constructing mathematical knowledge. The study found that gestures, alongside verbal language and mathematical symbols, can aid multilingual learners in comprehending the mathematical concepts. By analyzing three SFL metafunctions of gestures in relation to mathematical concepts, we gain a better understanding of how this contributes to knowledge construction. While the other modes (e.g., mathematical symbols) show the relational process, which is difficult for multilingual learners to access, gestures can provide the operational process. Knowing how to perform mathematical operations can benefit multilingual learners. This can contribute to the development of systemic functional multimodal discourse analysis. SF-MDA, a branch of discourse analysis, studies language in conjunction with other modes such as visual displays, body language, and sounds. As a result, SF-MDA can be used as an effective analytical framework because it includes the analysis of metafunctions that contribute to creating meaning. Therefore, it can be concluded that SF-MDA can be used to explore the multimodality of communication.

The third study found that using digital teaching materials can benefit multilingual students in the process of learning mathematics. By analyzing and comparing printed texts and digital teaching materials, the study showed that digital teaching materials allow learners to understand the process of doing mathematics, rather than just seeing the relationship between symbolic notations. This enhances the interaction between students and materials and helps students understand the relationship between real-world context and mathematical operations. These findings contribute to the study of different representations of semiotic resources discussed earlier, including gestures.

The analysis of metafunctions of digital teaching materials provides a comprehensive framework that can assist educators in selecting appropriate digital teaching materials for multilingual learners. According to O'Halloran's (2005), language, mathematical symbols and visual displays play a crucial role in creating relational mathematical knowledge. This study also revealed that digital teaching materials can aid in knowledge construction, which can have a positive impact on pedagogical discourse in mathematics. Educators can use the systemic functional multimodal discourse analysis framework to select appropriate materials for their students, considering their linguistic and cultural backgrounds, individual learning styles, and preferences. This approach can ensure that the digital teaching materials are not only effective but also culturally responsive, leading to better learning outcomes for all students. Findings from this paper help me design a comprehensive Table 7.1 that features an array of questions that educators can refer to when selecting digital teaching materials.

Table 7.1

Sample framework for analyzing (digital) teaching materials

Metafunctions	Questions to Consider
Ideational meaning	- What types of process (operational or relational process) are included in the material?

Interpersonal meaning	- Is the language related to everyday contexts? - Does the image depict real life situations? - Are there any mathematical notations?
Textual meaning	- What are the semiotic resources used in creating the materials? - What modes are used in the materials? - Does it provide static or dynamic images?

This table can serve as a resource for educators who wish to make informed decisions and ensure that their students receive the best possible learning experience. There are three important elements in choosing digital teaching materials:

- The first aspect, focusing on ideational meaning, addresses the question: What types of process (operational or relational process) are included in the material? In this question, operational process provides the multilingual learners with the action of doing mathematics. Relational process refers to the relation between mathematical identities, e.g., $2x = 8$.
- The second aspect, focusing on the relation between the learners and the materials, address the questions: Is the language related to everyday contexts? Does the image depict real life situations? Are there any mathematical notations? The purpose is to explore whether the material is too abstract for multilingual learners.
- The third aspect, focusing on the elements of the materials, addresses the question: What are the semiotic resources (e.g., language, images) used in creating the materials? What modes (e.g., audio, video or animation) are used in

the materials? Does it provide static or dynamic interaction (e.g., interaction, immediate feedback)?

In conclusion, findings from this dissertation provide suggestions for interdisciplinary collaboration between applied linguistics and mathematics education in (1) providing professional development for in-service teachers, (2) curriculum designing in educating pre-service teachers, and (3) the collaboration between applied linguists and material developers.

Firstly, it may be useful to incorporate a social semiotic approach to multiple semiotic resources into professional development for in-service teachers. This perspective emphasizes how individuals use semiotic resources to construct and exchange meaning. By incorporating the social semiotic framework into mathematics education, teachers can better address the complexities of teaching mathematics and support the State Standards. Additionally, using semiotic resources can be especially beneficial for multilingual learners when implemented through social semiotic perspectives. Teachers can scaffold learners by preparing, evaluating, and revising lessons which focus on linguistic, gestural, and diagrammatic tools. By using multiple semiotic resources, teachers can assist multilingual learners in making sense of mathematical concepts and help them communicate disciplinary knowledge within the classroom, aligning with state standards such as Georgia Standards of Excellence (GSE) and WIDA.

Secondly, for multilingual learners, multimodal pedagogical practices should be incorporated into teacher education programs for pre-service mathematics teachers, especially those who are going to work with multilingual learners. It is becoming more and more important for teacher educators to equip pre-service teachers with the skills necessary to effectively use multiple semiotic resources in constructing disciplinary knowledge. As the number of multilingual learners in the United States continues to grow, teachers should communicate and connect with them in a way that allows for a deep understanding of the subject matter. By preparing pre-service teachers to use a variety of resources, we can ensure that they are able to meet the needs of all students and create a more inclusive learning environment.

Finally, to material developers, digital teaching materials should provide multilingual students with more intellectual challenges. Also, material developers can consider incorporating more activities to show the process of doing mathematics to support multilingual learners' understanding of mathematical concepts. This can help further the definition of affordances of semiotic modes developed by Kress (2015), "different modes allow you to do different things, and not only allow you to do different things but insist that different things are done" (p. 88). Developers should consider motivating and scaffolding multilingual learners to enhance their understanding of mathematical concepts. By motivating and scaffolding multilingual learners, we can shift the dynamic of the classroom from teacher-centered to student-centered, ultimately leading to better outcomes for multilingual learners.

In my future research, I am interested in exploring how multilingual learners engage in a multimodal environment where teachers use various semiotic resources such as gestures, visual displays, and digital teaching materials. However, I must acknowledge that there are certain limitations in my dissertation. Due to the pandemic, I was unable to physically attend class and interact with the students or conduct interviews to gain a deeper understanding of their perceptions regarding the use of multimodal materials. Additionally, I was unable to design a complete unit with the focal teacher to explore how students perceive the use of multiple semiotic resources as I planned. With that being said, this study just focused on exploring the teacher's use of gestures. However, educators and researchers should explore the impact of using gestures on developing multilingual learners' conceptual and procedural knowledge. Therefore, researchers should work with multilingual learners to explore how multimodality can help them develop conceptual and procedural knowledge. By doing so, we can gain a more comprehensive understanding of how multilingual learners acquire and apply knowledge, allowing us to develop more effective teaching strategies and educational programs in mathematics. Also, there should be an interdisciplinary collaboration between applied linguistics and mathematics education in developing research and analyzing data. This can provide more insights into the data analysis and drawing conclusions.

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APPENDIX A

INTERVIEW GUIDELINES

Before the Interview

- Explain the purpose of the interview, the overall topic, and the process for protecting confidentiality (all identifying information deleted or use of pseudonyms during transcription
- Set a time/date in a quiet location
- Review practices of good interviewing
- Prepare informed consent document (download consent form and fill highlighted spaces with your information)
- Procure recording equipment (OIT media services, Aderhold 232), check battery, practice using it

During Interview

- Thank the individual for participating
- Remind participant about overall topic and that this is for a class assignment
- Go over consent document, leave a copy with the participant
- Ask permission to audio-record
- Remind participant that he/she does not have to answer any questions he/she doesn't want to and can withdraw participation at any time
- Set up equipment and begin recording
- Actively listen to understand
- Probe or ask for clarification when needed
- Keep aware of the time
- End on time
- Thank your participant for his/her time and participation

After Interview

- Download interview into password protected folder
- Delete the interview from digital voice recorder
- Transcribe interview following outline in rubric (questions bolded, responses not bolded, spaces provided between speakers, pages and lines numbered)
- During transcription delete or replace any names of people or locations with pseudonyms
- Delete recording before the end of the semester

Interview questions

Hi. My name is Khanh Bui and I am a doctoral student in the Department of Language and Literacy Education at the University of Georgia. I am conducting a research project on exploring teachers' perceptions of using multiple modes in teaching Coordinate Algebra to multilingual and multicultural students. Multiple modes include different meaning making resources such as gestures, visual displays, diagrams, and so on. This interview project is a part of larger study. Before the intervention, I would like to learn from teachers' perspectives about multiple modes in teaching Mathematics. Therefore, your response will significantly contribute to the success of my research. Thank you very much for your participation in my interview today.

Here is the consent form. As mentioned in this form, your information will be kept confidential and your participation is voluntary. Please feel free to skip any questions if you feel uncomfortable to answer them. Data from this study may be presented in conferences. The interview may last about 45 to one hour. Do you have any questions before we start?

(If there are no questions)

Thank you. I would like to check my equipment first. We can start our conversation now.

Interview questions

1. Could you tell me a little bit about yourself? How long have you been teaching ESOL students?
2. From your teaching experiences, can you name some difficulties that ESOL students may have when they study mathematics?
3. What are challenges you might have when teaching mathematics to ESOL students?
4. Do you have different instructional designs for ESOL and general education students? If so, how differently do you teach ESOL students?
5. What are the accommodations you have when teaching ESOL students?
6. Have you heard about the use of multiple modes before?

   Probing questions: If not, what do you think about using visual displays, diagrams, and gestures in the classroom? If yes, do you think multiple representations are effective in teaching a mathematical concept to ESOL students? Why do you think so?

   I'm sorry I need to check my equipment again to see if it is still working properly.
7. I now may turn to word problems. What strategies do you often use when teaching word problems to ESOL students?
8. What do students often do when they solve word problems?
9. What are your opinions about using different procedures such as drawing tables, pictures, diagrams when teaching word problems?
10. What are the differences between ESOL students and general education students when they solve word problem?
11. Does language barrier cause any problems when you communicate with ESOL students? Why or why not?
12. What are the suggestions for other teachers when using multiple modes in the classroom?
13. Can you tell me some difficulties when using multiple modes?
14. Do you think that using multiple modes can cause abundancy in teaching and take a lot of time?

APPENDIX B

OBSERVATION GUIDELINES

Before the Observation

- Check class time and date for the observation
- Review practices of good observation
- Prepare notebook for the observation

During the Observation

- Get into class on time
- Get a seat at the back of the class
- Sit quietly and take note activities in class
- Do not interrupt teacher and students in class

After Interview

- Secure the written note in a safe place
- Type the note using computer and save it in a safe place with password protected.

What to observe in the classroom

When observing students and the teacher in the classroom, researchers will focus on the following areas:

1. The teacher’s gestures when teaching Mathematics to students.
2. Materials the teacher will use when teaching students.
3. Activities the teacher will conduct in class.
4. How the teacher interacts with students.
5. How students interact and communicate Mathematics with each other in groups.

APPENDIX C

THEME DEVELOPMENT

1. **THEME: LANGUAGE IS NOT THE ONLY MEANS OF MAKING MEANING IN ESOL CLASSROOM**

Definition: Verbal communication is not effective in multilingual and multicultural classroom.

Characteristics: using language, identify key words, direct translation, knowing the language.

Conditions under which code operates: It is not effective when teachers focus on direct translation, identifying key words, and teaching vocabulary.

Proposition: Within a context of multilingual and multicultural classroom, using direct translation from other languages to English, asking students to memorize key words, and focusing on teaching vocabulary sometimes cause students' confusion and do not foster their Mathematical understandings.

Illustration:

- So like I said you know learning and learning languages doesn't really help you because like what language to learn [Language_Translation]
- That's a very easy translation here but to teach them what the word difference means and it doesn't translate well [Language_Translation]
- Yeah that's a common word it's not common in Spanish language. So I have to make sure that when they know when they say that word it means to subtract or the gap between sort of things like that [Language_Translation]

Negative case: Students need language to do well in the test.

- So they have to know words like domain and range and things like that.

2. **<u>THEME: GESTURES USED AS SEMIOTIC RESOURCES IN EXPLAINING MATHEMATICAL CONCEPTS</u>**

Definition: Hand movements to express meanings in Mathematics

Characteristics: meaning-making, movement, walk, kinesthetic learning, get up, show meanings, recognized by students

Conditions under which code operates: Gestures are considered as meaning-making resources when a mathematical concept is attached to them and are recognized by students.

Proposition: Hand movements are considered as semiotic resources when they are produced by teachers, carry a mathematical concept and are recognized by students.

Illustration:

- The kids know to go up three and walk around [Gesture_enactment]
- I enjoy coming in and having the students get up and do things like I can see like they enjoy it too [Gesture_enactment]
- Arm can be up here with a partner. Okay show me what reflection is [Gesture_enactment]

Negative case: Any gestures are accidentally made by teachers, which do not convey any meanings. In addition, gestures are very difficult to implement.

- So I don’t want the students to like physically do something and a lot of teachers maybe find it difficult difficult for a math classroom to do that [Gesture_difficulty]

3. **<u>THEME: PATTERN RECOGNITION AS A PRINCIPLE IN USING MULTIPLE REPRESENTATIONS</u>**

Definition: Frequent and repetitive use of multiple representations fosters ESOL students' pattern recognition.

Characteristics: frequently, repetitive, same procedure, everyday

Conditions under which code operates:

Proposition: Different meaning resources, e.g., visual displays, gestures or tables, should be used multiple times in and across lessons so students can recognize patterns and understand concepts.

Illustration:

- So I drew an arrow showed that that name plus three wrote next to it that that is the common different to we've done what we've done over the last few weeks we've just written that over and over again [Principle_pattern]
- You know the more they work on this the easier it is [Principle_pattern]

Negative case: Single use of multiple representations could not help ESOL students to understand the concepts.

Hypothetical negative case: *We just show them the table and instruct them once they can understand the concept.*

APPENDIX D

CODEBOOK

Name	Description	Files	References
Challenges		0	0
Challanges_test	Difficulties of using multiple representations in tests	3	2
Challenges_teacher bias	Teachers' perceptions about multiple representations	2	4
Gestures		0	0
Gestures_difficulty	Difficulties of using gestures	2	1
Gestures_enactment	application of gestures in the classroom	3	11
Language		0	0
Language_English only	Students are asked to use English only in class	3	1
Language_key words	Focusing on key words when teaching Maths	3	1
Language_Translation	Translating every word does not help	3	3
Using table			1
Using table_pros	The advantage of using table in teaching Maths	2	1
Using table_cons	The disadvantage of using table in teaching Maths	2	
Visual displays			6
visual display_pros	The advantage of using visual display in teaching Maths	2	2
Visual displays_cons	The disadvantage of using visual display in teaching Maths	2	4
Visual displays_general English	Using visual displays to teach general English besides Mathematic vocabulary	3	2
Principles	Suggestions for using multiple	3	2

Name	Description	Files	References
	representations		
Learning Styles		0	0
Learning styles_students	catering for different learning styles in the classroom	2	2
Learning styles_teacher	catering for teachers' styles	2	2
Scaffolding	Strategies to support students	3	1
Technology	Using technology to support learning	3	1
Gen_Ed_students	How general education students learn Maths	3	2