THE STORY OF ENGAGEMENT IN COLLEGE PRECALCULUS

by

AIDA ALIBEK

(Under the Direction of ANNAMARIE CONNER)

ABSTRACT

Precalculus is a critical gatekeeper course that determines undergraduate students' academic trajectories, yet student engagement within it remains poorly understood. This hermeneutic phenomenological study addresses this gap by asking: What is the story of engagement for students in college Precalculus? Through a series of four interviews with 12 undergraduates, incorporating methods like engagement artifacts and graphing, this research study captured the lived experiences of engagement across the course of a semester. The findings reveal that students do not experience engagement as a series of static states, but rather as a dynamic, interconnected system, akin to a pinball machine. Students' engagement is in constant flux, shaped by a web of interactions with instructors, peers, course materials, and assessments. A single disruption, such as a perceived misalignment between homework and a test, can create a ripple effects, shattering confidence and forcing a complete recalibration of strategies. A central contribution of this dissertation is the proposal of two distinct orientations of engagement: reactive engagement, which is a direct response to external course pressures, and proactive engagement, which is self-initiated and driven by the student's own intentionality. This nuanced, student-centered perspective challenges simplistic categorizations of students as "engaged" or "disengaged." The study concludes that the story of engagement is one of continuous adaptation and sense-making, providing a foundation for designing more supportive and effective learning environments in high-stakes gatekeeper courses.

INDEX WORDS: student engagement, mathematics engagement, precalculus, undergraduate mathematics, phenomenology

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This dissertation is a culmination of a very long story that spanned decades, crossed oceans, soared to the highs of connecting with students through mathematics, and dove into the depths of inner despair and doubt. None of us journey alone, and every human we encounter has an impact, whether it is noticed or unnoticed. If I left someone out, it is because I could not recall every single person who has entered and left an imprint on my lifeworld, and because my heart is simply too full with gratitude and overwhelming happiness at reaching this major academic milestone.

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Finally, to our cat, Whiskey “Whiskers” Wittgenstein: you are the best boy. Thank you for cheering me up in this last stretch and soaking my tears in your fur.

PREFACE

On the surface one might think that mathematics and I go way back: I have a bachelor's and a master's degree in mathematics, I worked at a mathematics research institute, I pursued doctoral education in mathematics at one of the top institutions in my area of mathematics, and I have published articles in mathematics research journals. However, the truth is my positive emotions towards mathematics have been waning since elementary school all the way through the end of high school. I was still a good student and got overall good grades in the subject, but I did not believe I was good at math. This might not have been an objective assessment, because I was enrolled in a STEM magnet school starting in grade 6, where the STEM curriculum was highly challenging. I was not comparing myself to an "average" student, I was comparing myself to students who were participating in Mathematics Olympiads.

My academic journey took a hard turn when the only scholarship I got to attend college back home in Kazakhstan was for a mathematics major. It is a long story to explain how I got a scholarship I did not even apply for, but I was too dejected about not being able to pursue my dream of becoming a NASA astronomer one day, so I thought to myself that I might as well take advantage of a semester or two of free tuition in college. I did not realize at the time that my entire life path would be changed that year.

Pure mathematics clicked with me: the processes of argumentation, proving, and thinking about abstract concepts with other people captivated my mind. I was never fast at cracking math problems, I could not—and still can not—do mental math to save my life. But to my surprise "real math" wasn't like most of the things I've seen in school or even the tasks I've seen in the Mathematics Olympiad problem sets that we would occasionally all be subjected to in my school.

This situation wasn't true for all the other students in the mathematics major at my university: many were struggling with making sense of mathematical concepts and writing proofs. My peers did not always connect with things that our professors (most of whom were research mathematicians) were saying. Instead of focusing on just my own learning, my friends and I organized study groups and mutual tutoring sessions, where I could connect to their experiences as someone who wasn't quite always getting the math. Similarly to me many of my peers were in this major for the scholarship and tuition, but unlike me they were not falling in love with the mathematics. They wanted to get a degree and move on with their lives, so the coursework was really just a set of hoops to pass.

I did not realize at the time that I was already beginning my journey as a mathematics educator; in particular, an educator who was good at helping struggling students in a way that connects with them (because I was once a struggling student myself), and good at recognizing and being mindful of the financial and academic pressures that college kids are facing as they are making sense of the courses that they have to somehow pass. So when I came to Chicago to pursue a math PhD and started teaching developmental math courses to former Chicago Public School students, I was ready to support them where they were and respect the struggles that they were facing.

On this teaching journey I couldn't help but feel frustrated that college mathematics instructors (including myself) weren't always engaging their students in meaningful learning and were potentially misunderstanding what engagement might look like for these students. I was frustrated that the financial and academic pressures weren't always taken into account when considering why the students were in our classrooms in the first place and why they were choosing to engage with the tasks and activities in some ways over others.

When I took the time to talk to the students themselves, I learned some of the stories behind these moments of seeming (dis)engagement. One of my students always ate lunch in my Linear Algebra for Business class when they were supposed to work in groups. After talking to him I learned that he was coming to my classroom directly from his first job and would go directly to his second job after he was done with classes for the day. He was still paying attention and wished he could afford to quit one of the jobs to participate more actively in my lessons. One of my students in a summer bridge program would often hesitate to engage her male group mates in discussion and go as far as erasing her correct solution to the task. When I asked her why she did not challenge their solutions or answers, she admitted that she just assumed her work was wrong because the boys sounded really confident, and she wasn't good at Algebra. Finally, during my brief stint as a Graduate Assistant at the Disability Resource Center, I worked with a disabled mathematics graduate student who needed certain accommodations that were often unmet by the department, the university, and even individual mathematicians visiting the university[1]. He wanted to participate in the mathematics community, but the socially acceptable ways of engagement were inaccessible to him.

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[1] A world-famous mathematician refused to share his guest lecture notes. When asked if he would make an exception for a disabled graduate student, he told the student that he should just sit down and write his own notes, because the invited speaker himself has never met another mathematician who couldn't write.

The students I've met throughout my journey are why I persevered in doing the work: to listen to and elevate student voices, to understand how students engage with mathematics and how we as educators can support them, to help all mathematics teachers find better ways of providing and supporting opportunities to engage, to create better mathematics learning spaces, where different modes of engagement are acknowledged and students are respected and provided equitable supports. This dissertation is for the students.

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INTRODUCTION

College Precalculus is a critical gatekeeper course that determines students' academic trajectories. Most often the students in this course are not intended mathematics majors, with a majority of those who do pass the class never enrolling in Calculus I afterwards (McGowen, 2006). Precalculus is frequently a site of frustration and unexpected setbacks, even within reformed, active-learning classrooms. A primary intent behind incorporating active learning pedagogies is to improve student engagement compared to the more passive student participation in lecture-based classes. However, I posit that without a clear understanding of what engagement means and how it is experienced in a course such as Precalculus, it is difficult to assess whether these reforms are genuinely improving students' lived realities.

The K-12 mathematics education literature provides a robust foundation for engagement research, acknowledging engagement as a complex, multidimensional phenomenon. In their *Compendium* chapter, Middleton, Jansen, and Goldin (2017) describe engagement as occurring when students participate in mathematics activities with some level of affective and cognitive investment. They acknowledge the usefulness of the three dimensions of engagement—behavioral, affective, and cognitive—proposed by Fredricks et al. (2004). However, they crucially warn that engagement is far too complex to be divided into three neat boxes. This complexity is especially exacerbated in mathematics, a subject notorious for eliciting negative emotional reactions, beliefs about inherent cognitive difficulty, and socially normalized avoidant behaviors.

When engagement is studied in college mathematics, it is often through quantitative measures or is a tangential finding in studies of instructional methods. Qualitative studies that delve deeply into the student perspective are rare. This creates a critical gap in the RUME literature: a lack of deep, qualitative understanding of engagement as a unified, multidimensional lived experience from the student's own perspective. We know that students disengage in courses like Precalculus, but we understand less about *how* the various dimensions of engagement manifest, intertwine, and are experienced by the students themselves. Moreover, is it even fair to label all reluctant behavior as disengagement? To truly support students, we must move beyond these simplistic categorizations and uncover the nuanced story of their engagement.

Overview of the Study

This dissertation addresses this gap through a hermeneutic phenomenological study of students' lived experiences in Precalculus at the University of Georgia. The central research question of the dissertation is: *What is the story of engagement for UGA students in Precalculus?* The initial sub-questions at the beginning of data collection and data analysis were:

RQ1: What is the lived experience of engagement as seen through student descriptions of their engagement in Precalculus across the semester?

RQ2: How do the four proposed dimensions (affective, behavioral, cognitive, academic) of engagement manifest and interact with each other throughout the semester?

RQ3: What is the interaction between student engagement, topics in the Precalculus curriculum, and participants' perception of how these topics are taught?

To guide this inquiry, I adopted a four-dimensional framework (behavioral, cognitive, affective, and academic) of engagement to holistically capture the phenomenon. I agree that engagement is a meta-construct (Fredricks et al., 2004), but the three original dimensions are insufficient for this context. In a gatekeeper course like Precalculus, the academic dimension—encompassing grades, homework completion, and credit attainment—is extremely salient for students' motivation and is a critical part of their engagement story (Christenson & Thurlow, 2004; Yazzie-Mintz, 2007). This framework allows for a holistic consideration of engagement that recognizes how these dimensions are interwoven in the student's lifeworld.

Guided by this conceptual framework together with the methods of hermeneutic phenomenology of van Manen (2014) and reflective lifeworld research approach of Dahlberg et al. (2001, 2008), I conducted a semester-long study with 12 undergraduate students. I engaged in a series of four in-depth interviews with each participant, using innovative techniques like engagement graphs and artifacts to elicit rich, pre-reflective descriptions of their experiences, supplemented by classroom observations.

My analysis revealed that engagement is not a static state but a dynamic, interconnected system. Students' engagement is in constant flux, shaped by a web of interactions with instructors, peers, course materials, and assessments. A single disruption—such as a perceived misalignment between homework and a test can create a ripple effect, shattering a student's confidence and forcing a complete recalibration of their strategies.

As the process of data analysis and writing unfurled, the initial research sub-questions were revised to better represent the findings of the study. The core question "What is the story of engagement in college Precalculus?" remained the same; research sub question 2 was removed from the written findings; research sub questions 1 and 3 were rewritten to better represent the essences of the phenomenon as gleaned from the research process. Here are the revised versions marked with asterisks to make them distinct from the original questions:

RQ1*: How does the lived experience of engagement in Precalculus (as seen through student descriptions of their engagement) unfold throughout the semester?

RQ3*: How do elements of the Precalculus course at UGA (e.g., assessments, instruction, and mathematical topics) shape students' experiences of engagement throughout the semester?

My analysis revealed that the story of engagement is not a static checklist of behaviors or dimensions, but a dynamic, longitudinal narrative of adaptation. Students' engagement is a journey marked by pivotal moments—like the first exam—that force a recalibration of their strategies and their very relationship with the course. It is a story of continuous sense-making, where students actively decipher the 'rules of the game' in response to the course's elements. Therefore, to honor this finding, the initial research sub-questions were revised to better reflect the temporal, experiential nature of the phenomenon.

A central contribution of this dissertation is to propose the concepts of reactive and proactive engagement. This distinction captures a richer portrait of the full depth of students' experiences, differentiating between engagement that is a direct response to external course pressures and engagement that is self-initiated and meaningful. This nuanced, student-centered perspective, grounded in lived experience, challenges simplistic models and provides a foundation for designing more supportive and effective learning environments in high-stakes gatekeeper courses.

Structure of the Dissertation

This dissertation is organized into six chapters to guide the reader through this exploration:

Chapter 2: Literature Review reviews the extant literature on the multidimensional construct of student engagement, its specific manifestations in mathematics education, the context of college gatekeeper courses, and the theoretical foundations of hermeneutic phenomenology.

Chapter 3: Methods provides a detailed account of the research methodology, including the hermeneutic phenomenological approach, participant recruitment, data collection procedures, and the iterative process of data analysis.

Chapter 4: Phenomenological Description presents the thick descriptive findings of the study, organizing the participants' lived experiences around the core elements of their engagement lifeworld.

Chapter 5: Hermeneutic Analysis interprets the findings from Chapter 4, drawing out the essential meanings and structures of the engagement experience.

Chapter 6: Discussion considers the findings of the study in the context of extant research, as well as discussing implications for mathematics education research theory and the teaching practice of Precalculus.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews the extant literature relevant to the dissertation study, which sought to understand the lived experiences of engagement among students in a college-level Precalculus course. The review is organized into four primary sections. First, it explores the evolution and current conceptualizations of student engagement as a multidimensional construct, culminating in the four-dimensional framework (behavioral, cognitive, affective, academic) that guides this inquiry. Second, it examines the body of research concerning student engagement in mathematics, tracing the developments in K-12 research and identifying space for growth in the undergraduate mathematics education (RUME) literature that this study aims to address. Third, I will provide an overview of research on college mathematics gatekeeper courses and initiatives towards shifting these courses to include more active learning. This section will provide more nuance to understanding the context of the study's setting (flipped classroom). Finally, this chapter culminates with the theoretical and methodological foundations of hermeneutic phenomenology, drawing specifically on the works of Max van Manen and the Reflective Lifeworld Research approach of researchers like Dahlberg, Drew, Nyström, and Dahlberg to establish the philosophical and procedural underpinnings of this inquiry.

Defining Student Engagement

The concept of student engagement has shifted significantly from its early narrow definitions like "time-on-task" (Carroll, 1963) to a recognized multifaceted construct crucial for academic achievement and student learning. Despite its importance, the field grapples with a "conceptual haziness" (Reschly & Christenson, 2012), where the term 'engagement' describes different concepts, core ideas are called by different names, and an abundance of models measure essentially different ideas (Wong & Liem, 2022). To address these discrepancies Christenson, Reschly, and Wylie edited the *Handbook of Research on Student Engagement* (2012) in order to open a conversation among engagement and motivation researchers about explicating definitions, conceptualizations, and models of engagement. Each chapter's authors were requested to provide definitions of engagement and motivation as well as an explicit explanation of how they view the distinction between the two.

In their commentary on Part I of the *Handbook* (Christenson et al., 2012), Eccles & Wang (2012) observe that the definitions provided in Chapters 1-5 of the book shed light on the problem of overgeneralization of the term engagement. Some definitions in literature are so broad that the lines between other terms and theories begin to get blurred and it becomes hard to draw a clear line between engagement and motivation (a term that also has issues of definition). On one hand this broadness causes problems, especially when it comes to measuring student engagement and inferring precursors and mediators of engagement; on the other hand, a broader conceptualization can serve a purpose of building more general theories of engagement and persuading policymakers to promote impactful reforms (Eccles & Wang, 2012).

Moreover, different researchers utilize different models of engagement in their scholarship. In order to understand the broad conceptual variety of engagement in educational research, let us consider some key overview publications on the term, starting with the first chapter of the *Handbook* (Christenson et al., 2012) that traces the evolution of engagement in the research literature (Reschly & Christenson, 2012). The authors note that many of the studies on the topic since the 1980's (including their own) were initially rooted in the work towards reducing drop-out rates in secondary schooling. However, over time the concept of engagement has gained traction among the broader education research community working towards better understanding of student achievement, behavior, and learning. This rising interest prompted multiple overviews of scholarship on student engagement.

For instance, in 2004, Fredricks, Blumenfeld, and Paris published an extensive review of literature on the topic, painting a contemporary picture of how researchers defined, measured, and framed the term engagement up to that point in time. In the article the authors argued for the potential of studying engagement as a "meta-construct" that encompasses three different, yet overlapping dimensions: *behavioral* engagement (positive conduct, participation, involvement in academic and social activities), *affective* engagement (emotional reactions such as interest, enjoyment, boredom, anxiety, and a sense of belonging), and *cognitive* engagement (investment in learning, including the use of self-regulation and a willingness to exert effort in comprehending complex ideas).

Appleton, Christenson, and Furlong (2008) also published an analysis on the lack of consensus in the research community on the definition of engagement itself, as well as which facets of engagement must be included in defining the concept. They joined the call of (Fredricks et al., 2004) to seriously consider the benefit of studying students' engagement as a complex multi-dimensional construct. A more recent review of literature on engagement by (Wong & Liem, 2022) also addressed the many issues surrounding the definition of the concept, as well as the main scholarly perspectives to date on studying student engagement.

Conceptualizing Engagement

As mentioned above, the conceptualization of engagement as a three-dimensional meta-construct (Appleton et al., 2008; Fredricks et al., 2004) is a widely acknowledged and referenced model of engagement in education research. In fact, the article by Fredricks, Blumenfeld, and Paris from 2004 has framed a great deal of research in understanding students' school engagement: according to Google Scholar, this publication has been cited an astounding 19,779 times. Appleton et al. (2008) also has an impressive 3,862 reported citations on Google Scholar. This model of engagement also shows up in a great deal of mathematics education research literature on the topic (Cevikbas & Kaiser, 2022; Goldin et al., 2007, 2011; Middleton et al., 2017; Watt et al., 2017).

Subsequent scholars have proposed expanding this model. Some have added an *academic* dimension (Christenson & Thurlow, 2004; Yazzie-Mintz, 2007), focusing on observable indicators like number of credits earned and homework completion. Others suggest a *social-behavioral* dimension (Fredricks et al., 2016; Linnenbrink-Garcia et al., 2011) to account for the influence of group work, or an *agency* dimension (Reeve & Tseng, 2011), where students actively influence their learning environment. In mathematics education specifically, (Goldin, 2019) has proposed a *conative* dimension dealing with human needs and wants, even though in much of the research this is subsumed within motivation concepts. Other researchers have pushed for broader perspectives on engagement: ones that better capture the complexity of the concept as well as address some of the concerns raised in Fredricks et al. (2004), Appleton et al. (2008), and Reschly & Christenson (2012).

Social-ecological model of engagement. Lawson & Lawson (2013) point out the importance of students' engagement with extracurricular and youth-community spaces, in addition to their school engagement and classroom academic engagement. Hence, they propose a social-ecological framework that considers the interaction between students' engagement in schooling as part of the broader ecological system of students' lives (or phenomenologically speaking, the students' lifeworlds). The following diagram (Fig. 2.1) demonstrates a simplified version of their proposed model of engagement.

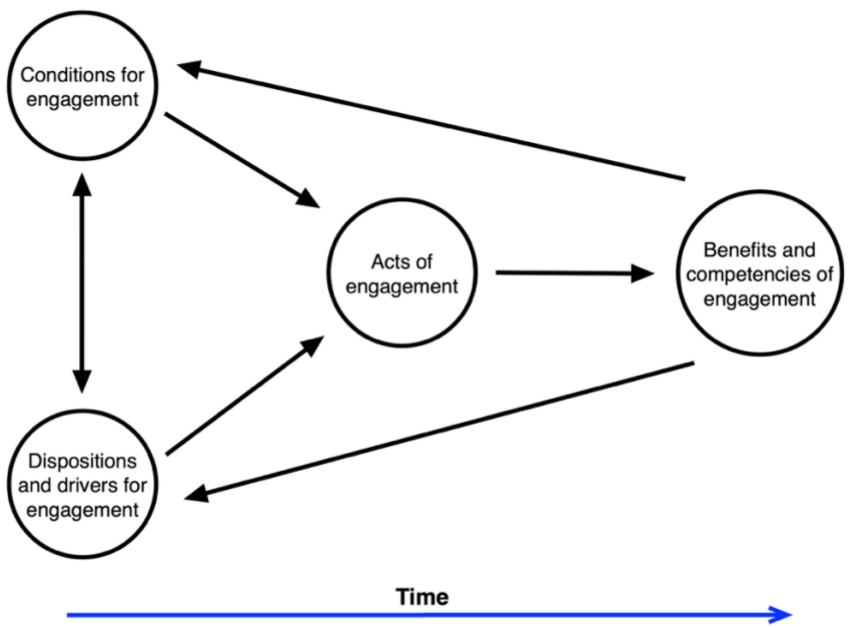

Figure 2.1: Transactional model of student engagement (adapted from Lawson & Lawson, 2013, p. 443)

The central piece of the model is the phenomenon of engagement itself, the acts of engagement. According to Lawson & Lawson (2013) they are "the states of experience of individuals as they participate in discrete activities at particular moments in time" (p.

442). As the authors note, these states can range from disengagement to states of high engagement in an activity, and they expand beyond students' affect to include behavioral and cognitive dimensions of engagement.

The two precursors of acts of engagement are the conditions for engagement (which were noted earlier in this review as the environmental and contextual factors of engagement) and the dispositions and drivers for engagement (which are shaped by the students' previous affective, cognitive, and behavioral experiences). Lawson & Lawson (2013) observe that there is a mutual mediation between these contextual factors and students' dispositions for engagement, signified by a two-way arrow in Figure 2.1.

Finally, the benefits and competencies that students receive (or do not receive) from each act of engagement are influenced by the quality of the engagement, and these benefits and effects can further impact students' dispositions (Lawson & Lawson, 2013). Moreover, these benefits can occur along the different dimensions of engagement: socio-cultural, affective, cognitive, and behavioral.

The blue arrow denoting the passage of time indicates the typical order of conditions and cause for engagement, leading to the acts of engagement themselves, while the resulting benefits and competencies of engagement can lead to another cycle of engagement. Thus, outcomes (indicator) of one act of engagement can end up as a process variable in another engagement act.

Dual component framework of student engagement. Aside from conducting a most recent review of literature on engagement, Wong & Liem (2022) offer a perspective on studying engagement that counters an issue they see in the Fredricks et al. (2004) meta-construct: the fuzziness of the distinction between engagement in learning and engagement with the school community. In their model of engagement, they consider student engagement as an umbrella term that has two separate components: learning engagement and school engagement (see Fig. 2.2). Here learning engagement is defined as “students’ psychological state of activity that affords them to feel activated, exert effort, and be absorbed during learning activities” (Wong & Liem, 2022, p. 120), while school engagement captures the connection to school community (both through its people and communal activities).

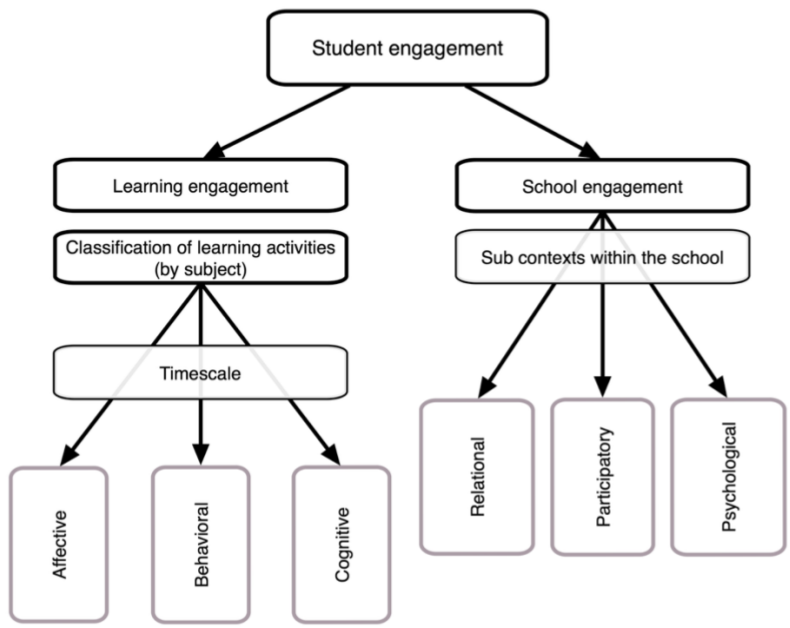

Figure 2.2: Dual component framework of student engagement (adapted from Wong & Liem, 2022, p. 119)

Some key points that Wong & Liem (2022) made in their development of the dual component model is that learning engagement is subject/discipline-specific, and when looking at its dimensions they also rely on the three dimensions outlined in Fredricks et al. (2004). On the right side of the diagram Wong and Liem note that unlike learning engagement, school engagement is sub-context-specific, with relational dimension (students' emotional connections to people in the school community), participatory dimension (compliance, cooperation, and participation in school activities), and psychological dimension (identification with the school).

Expectancy-value theory. Another theory that touches on student engagement is the Expectancy-value theory (EVT). Initially developed to understand the differences in STEM enrollment differences between men and women, over time Eccles and her colleagues expanded it to consider broader behavior, achievement, and persistence (Eccles & Wang, 2012). Although engagement is a part of EVT, this model mainly focuses on the behavioral and cognitive dimensions of the concept. As Eccles says herself: "I (Eccles) would put emotional engagement as either an antecedent of behavioral/cognitive engagement (i.e., anticipated positive or negative emotional arousal resulting from engagement) or an emotional reaction to being engaged in doing the task" (Eccles & Wang, 2012, p. 142).

EVT offers a map of how beliefs, behaviors, dispositions, and previous experiences (influenced by the socio-cultural contexts) influence and motivate decision-making in achievement-related choices. The parts of EVT most connected to research on engagement are: (1) affect as the factor and consequence of engagement; (2) perceived task value as it relates to fulfilling the students' short-term needs (e.g., enjoyment of engaging in the task) and long-term needs (e.g. succeeding in the course as a result of engaging in the task) (Eccles & Wang, 2012); and (3) perceived abilities, as shaped by previous learning experiences and societal expectations (Watt & Goos, 2017). For an example of the use of EVT in mathematics education research see Weber et al. (2020).

Self-determination theory. Another theory that concerns itself with the learner's needs is the self-determination theory (SDT), which focuses on the learner's intrinsic motivation to engage in tasks and how that motivation is shaped by socio-cultural contexts (Reeve, 2012). SDT posits that a prerequisite of engagement is satisfaction of students' key needs: the need for autonomy, belonging, and competence (Watt & Goos, 2017). In the context of the three-dimensional model (Fredricks et al., 2004), SDT embraces the three dimensions but also adds the agentic engagement as the fourth dimension (Reeve, 2012).

There is some literature in mathematics education research that has taken up SDT as a theory for understanding student motivation and engagement. For instance, Duffin et al. (2020) used the self-determination model to explore college algebra students' experiences in different learning environments (direct instruction vs flipped instruction, and social support integration): this framework helped them understand whether the students' needs were met in those classrooms. In another mathematics example, Durksen et al. (2017) studied six middle grades teachers to understand how the interactions with their students promoted high levels of engagement and motivation. Although SDT was only one of several theories that informed the researchers in this article (as they used Martin's (2007) Motivation and Engagement Wheel), it was one of the more impactful framings for understanding how teachers satisfied students' needs for relatedness in one-on-one interactions.

The fuzziness of definition and the wide range of conceptualizations of engagement (Fredricks et al., 2004; Reschly & Christenson, 2012) make it especially difficult to define the phenomenon beyond highlighting the idea of multiple dimensions contained within or taking on the more pragmatic approach of restricting to the task at hand. Thus, researchers studying student engagement tend to focus on conceptualizations that fit within their contexts and the purposes of their research inquiries.

Students' relationships with family, peers, teachers/instructors, schools/colleges, and schooling are not uniform across school systems (or universities); even within the same classroom (whether K-12 or college) these conditions and contexts are not uniform among students. In addition to the variability of relationships and attitudes, there is a great range of ways academic environments (both physical and not) can directly impact and sometimes even dictate the kinds of engagement that are "desired" in these particular educational settings. These are all considerations identified by researchers as relevant to the choice of conceptualization of engagement.

Building on examples in Appleton et al. (2008), one can consider the following sample environmental and contextual factors in a college mathematics course: attendance and participation requirements set by the instructor or institution for passing the course, access to course-appropriate tutoring for students (possibly through a department-facilitated mathematics tutoring center), academic advising, social capital for success in a post-secondary education setting, etc. Fredricks et al. (2004) note in their review that a greater amount of research-based evidence shows connections between school-level factors and students' behavioral engagement, with less evidence showing connections to the affective and cognitive engagement.

Considering the learning processes, it is also valuable to note the specific contexts that can draw student engagement or have the opposite effect of disengagement: use of tools or technologies (e.g., students engaging with tasks across disciplines, as long as they involve technology use), task and activity types (e.g., laboratory assignments), specific disciplines (e.g., mathematics), interactions with other humans (e.g., preference for group work or individual work), and learning setting (e.g., community-based vs. school-setting) (Lawson & Lawson, 2013; Pekrun & Linnenbrink-Garcia, 2012). Lawson & Lawson (2013) call this kind of engagement, *attentional engagement*.

For this study, conducted in the high-stakes context of a college gatekeeper course like Precalculus, I adopted a four-dimensional framework was adopted. This includes the original behavioral, affective, and cognitive dimensions from Fredricks et al. (2004), but explicitly incorporates the academic dimension as introduced by Christenson & Thurlow (2004). This was critical because in a course like Precalculus, aspects like assignment grades, homework completion, and credits earned are extremely important for students' motivation, persistence, and ultimate success (Ganga & Mazzariello, 2018). This synthesized framework of behavioral (active participation), cognitive (deep processing), affective (emotional responses and identity), and academic (strategic management of learning tasks) engagement provided a holistic lens to capture the richness of students' lived experiences.

The Role of Motivation in Engagement

Motivation is personal investment reflected in action and exhibited affect (Maehr & Meyer, 1997), which is distinct from engagement. However, when considering motivation in educational contexts, the actions taken towards learning and the affect displayed in connection with learning immediately overlap with some of the dimensions of engagement as considered by the various frameworks mentioned above (affect, behavior, cognition). Thus, it is not surprising that some research on motivation touches on issues of student engagement in learning (Appleton et al., 2008; Fredricks, 2011; Fredricks et al., 2004, 2016; Lawson & Masyn, 2015).

As Fredricks et al. (2004) point out, motivation research has had a longer history than engagement research and thus a lot of the engagement literature is directly impacted by how motivation research has conceptualized and studied different aspects of the term. Nevertheless, there is consensus that the two are distinct, albeit connected, concepts (Appleton et al., 2008; Fredricks et al., 2008; Reschly & Christenson, 2012; Wong & Liem, 2022). There is empirical research demonstrating some connections between engagement and motivation, e.g., engagement depends on both students' motivation and the environment (Lawson & Masyn, 2015); Reeve & Tseng (2011) have found an association between agentic engagement and motivation.

Two of the models for studying engagement presented above directly involve motivation as a factor: EVT (Eccles & Wang, 2012) and SDT (Reeve, 2012). Indeed, Reeve (2012) notes that the two concepts are inherently connected, and the focus will depend on who is doing the research: "those who study motivation are interested in engagement mostly as an outcome of motivational processes, whereas those who study engagement are interested mostly in motivation as a source of engagement" (p. 151). Hence, within this dissertation's focus on students' mathematics engagement we can frame motivation as an impetus for the phenomenon of interest, a state of motivated action (Wong & Liem, 2022).

SDT distinguishes between two types of motivation: *intrinsic* and *extrinsic* motivation, where the former relates to motivation stemming from inherent interest, while the latter deals with motivation connected to external factors (Deci & Ryan, 1985).

Middleton & Spanias (1999) considered how these relate to students' learning of mathematics, e.g., the connection between students' work on mathematics tasks as extrinsically motivated by the drive to successfully complete the task, or how enjoyment of mathematics can establish stronger intrinsic motivation for learning disciplinary content.

Although much of research literature tends to frame intrinsic motivation as more conducive to learning and extrinsic motivation as somehow "worse," (Ryan & Deci, 2000) cast light on the different types of extrinsic motivation and how some of them are valuable to the learning process. In particular, they highlight the importance of understanding how these two types of motivation interact with students' agency. As we will see in the findings chapter, this is indeed an important factor, not just in understanding students' motivation; it might also have a direct impact on how students engage in learning.

Students' mathematics engagement

Engagement is inherently subject-specific (Wong & Liem, 2022), and mathematics education researchers argue that mathematical engagement possesses unique, discipline-specific features (Middleton et al., 2017). It is well-established that student engagement is context-dependent (Fredricks et al., 2004). The "context" is profoundly shaped by the academic discipline itself, which is particularly true in mathematics, where the forms and objects of engagement are tied to uniquely disciplinary practices and epistemologies. In mathematics education research, this has led to a focused inquiry into what constitutes mathematical engagement, distinct from general school engagement.

Researchers posit that the very nature of mathematical work—centered on problem-solving, reasoning, abstract representation, and a community standard of argumentation and proof—creates a unique engagement landscape. For instance, the cognitive dimension of mathematical engagement is often characterized by persistent problem-solving and navigating multiple solution pathways, a focus that differs from the cognitive engagement required in a literature or history class (Schukajlow et al., 2017). Similarly, affective engagement in math is intensely connected to experiences of confusion, frustration, and the eventual "aha" moment of insight, a well-documented emotional trajectory in mathematical learning (Goldin, 2014). Furthermore, the social-behavioral dimension in a math classroom is frequently manifested through collaborative work on shared problems and explaining one's mathematical thinking, practices that are central to the discipline's discourse (Yackel & Cobb, 1996). Therefore, to understand the lived experiences of students in a college Precalculus course, this study must be framed through the specific lens of mathematical engagement and its unique disciplinary features.

Learning from K-12 Mathematics Education Research

In the *Compendium for Research in Mathematics Education* (Cai, 2017) there is a chapter on student engagement (Middleton et al., 2017), where the authors note that they believe mathematical engagement possesses unique features that are specific to the discipline of mathematics. This is aligned with some of the conceptualizations of engagement considered in the educational psychology literature: that learning contexts impact engagement (Fredricks et al., 2004), and learning engagement is subject-specific (Wong & Liem, 2022). In one study (Plenty & Heubeck, 2013), researchers compared mathematics motivation and engagement to general academic motivation and engagement among high school students. They found that mathematics-related motivation and engagement decline more rapidly than general academic interest at the beginning of high school, especially among female students.

Similar to their educational psychology peers, mathematics education scholars have grappled with defining mathematics engagement as a construct. In one of the earlier publications in the field, Peterson & Fennema (1985) studied sex-related differences in learning mathematics between 4th grade boys and girls and whether the students' engagement in the classroom played a role in the discrepancy in achievement. Citing some previous evidence of teacher behavior as a mediator for student achievement, the authors wanted to better understand sex-differences in students' mathematics engagement in the classroom. Even though Peterson & Fennema (1985) did not define engagement, we can infer from the objects of measurement in Table I in the article (pp. 316-317) that engagement was mainly seen by them as time on task, although there is some consideration for the kinds of engagement occurring (e.g., solving high or low cognitive-demand tasks, helping someone, being helped by teacher or student, etc.). We can see that even if there is some distinction along the social and cognitive dimensions of engagement, behavioral engagement (measured as proportion of class time) took center stage in this research.

As scholarship on the topic progressed, so have the ways mathematical engagement was measured and conceptualized. A key shift in this direction built on the field's growing interest in the affective aspects of learning, highlighted by a handbook chapter written by McLeod (1992) where he proposed reconceptualizing how the mathematics education research community considers and measures various affective constructs (beliefs, attitudes, motivation, etc.). Some of these were later linked to students' mathematics engagement in various capacities. Towards the end of the 20th century, researchers like Valerie DeBellis, Gerald Goldin, and James Middleton were leading the charge to take on the difficult task of studying the complex affective ideas connected to the learning of mathematics. One of the key concepts proposed at the time was affective pathways, which represent sequences of affective states interacting with cognitive configurations (Goldin, 2000). This concept was highly useful at the time to address the non-trivial interactions between affect and cognition during mathematical problem-solving and the "state-like" affective states of students that shift over time (unlike the "trait-like" affect that is seen as more stable and harder to change).

Research on affective pathways had an impact on concepts like motivation (Middleton & Spanias, 1999) and attitude (Hannula, 2002). It also directly led to the development of the idea of engagement structures (Goldin et al., 2007, 2011). Engagement structures refer to idealized theoretical profiles of students' in-the-moment motivations, affective dispositions, behaviors, cognitive processes, and other aspects influencing engagement (Goldin et al., 2011). One example of such a structure is 'Look how smart I am,' which is characterized by the student's motivating desire to demonstrate to others their mathematical skill by constructing other people in the social surrounding as either admirers of their talent or competitors. Behavioral engagement might involve effort towards solving mathematical tasks faster and/or better than others, and affective engagement includes feelings of satisfaction when their skill and knowledge are acknowledged (Goldin et al., 2011). Other examples of engagement structures include

‘Get the job done,’ “Check this out,’ ‘I’m really into this,’ ‘Don’t disrespect me,’ ‘Stay out of trouble,’ It’s not fair,’ ‘Let me teach you,’ Pseudo-engagement,’ etc. (Goldin et al., 2007, 2011).

Throughout the evolution of research on mathematics engagement, scholars in our field have embraced it as a complex meta-construct encapsulating students’ behaviors, cognitive processes, and affective dispositions towards and related to instances of mathematics teaching and learning (Middleton et al., 2017). Despite the difficulty of wrangling multiple dimensions and attending to the fast-changing in-the-moment manifestations of mathematical engagement, mathematics education researchers have built by now a rich body of knowledge on the topic, especially in the K-12 literature.

Quite a large share of the K-12 research literature attends to the role of the teacher in students’ engagement, as well as how teachers perceive what engagement looks like. In some studies teachers were observed to achieve increased engagement through high-quality instructional interactions (Durksen et al., 2017). Hettinger et al. (2023) found that student motivation and engagement correlated to perceived emotional support from teachers, even when teachers did not necessarily perceive themselves as supportive as students perceived them. In other situations, teachers provided the cognitive support, whether through goal setting in mathematical reasoning (Barnes, 2019) or using student-centered instruction that emphasizes both cognitive and emotional engagement with mathematics (Lin et al., 2018). Klem & Connell (2004) also noted that “[s]tudents who perceive teachers as creating a caring, well-structured learning environment in which expectations are high, clear, and fair” were more likely to report being engaged in school (p. 270).

Teacher enthusiasm, which can also be thought of as teachers' modelling of emotional engagement, was considered as a factor across engagement profiles by Watt et al. (2017). Although there was a correlation between student engagement and perceived teacher enthusiasm, when considering the compliant students (neither engaged, nor disengaged) the levels of perceived teacher enthusiasm were similar to that among engaged students. Thus, it is yet unclear what role teachers' enthusiasm (as modelling emotional engagement) can have in impacting student engagement.

Another way that mathematics teachers impact student engagement is through providing in-the-moment feedback to students as they are actively doing mathematics tasks. For example, in Cevikbas & Kaiser (2022), students in a flipped secondary mathematics class noted the value of having opportunities to ask for feedback and support as they were working on mathematics tasks in the classroom. This availability of the teacher to walk around and provide in-the-moment answers to student questions increased levels of engagement among more students. Lazarides & Rubach (2017) found a similar pattern: students' engagement and mastery disposition towards learning mathematics increases when teachers are able to provide them with individual information on how they can improve their mathematics skills.

When discussing teachers' roles in influencing student engagement we must be careful to account for teachers' own self-efficacy, beliefs about engagement, and content knowledge, because they can play a role in their ability to influence student engagement. In an intervention study, Bobis et al. (2016) embarked on supporting three teachers in reshaping their beliefs about student engagement and found that these three factors had a great impact on the teachers' journeys in the intervention. The authors note that beliefs about student engagement also included “the degree to which they acknowledge[d] responsibility for student engagement” (2016, p. 53).

But what do teachers perceive as engagement in the mathematics classroom? In one study Mohammad Mirzaei et al. (2023) juxtaposed teachers’ assessment of student engagement with the students’ assessment. Because engagement is complex, it is no surprise that the findings demonstrated a wide range of agreement between teachers and students. In particular, the researchers looked for agreement across three separate concepts: valence (engaged or not engaged), dimensions of engagement (how students engaged), and features (what supports/constraints students’ engagement/disengagement). Here the dimensions of engagement align with the previously described conceptualization (due to Fredricks et al., 2004), with the addition of concepts like social engagement (Fredricks et al., 2016) and the instrumentality of mathematics (Gaspard et al., 2015).

The educator plays a big role in shaping students’ engagement, including shaping a learning environment where students are more likely to engage with mathematics and learning. However, the learning space is also shaped by greater societal, environmental, and social aspects of learning mathematics that exert a good amount of influence on how students choose to engage in the mathematics classroom. From the physical space of the classroom, its whiteboards and desks, all the way to the influence of social and cultural norms that affect students’ personal and school lives, students engage in mathematics as part of their lifeworld within their specific school environment. A good term for these constraints and influences is environmental press, or the constraints of the social environment that affect students’ participation in activities of learning and fulfilling their psychological needs (Middleton et al., 2017).

One study that explored the effects of environment on engagement (Watt et al., 2017) showed that there is some correlation between classroom goal structure and students' engagement levels and profiles. Here classroom goal structure indicates how students perceive the goal of studying math in the classroom: conceptual learning mastery, learning oriented towards performance, getting good grades and/or better grades than other students in the class. Similarly to the outcome on teacher enthusiasm noted in the theme prior, although higher perception of mastery environment correlated with higher level of engagement, it was not significant enough between the engaged and compliant student profiles in the study. Thus, it cannot be confirmed that promoting such a learning environment will immediately have an impact of increasing student engagement.

It must be noted that there are also certain physical environment constraints that might be pertinent to students' engagement, especially for those students who identify as disabled (or as people with disabilities). This is not a typical point of research in mathematics education; however, there has been at least one study on the physical and technological learning environment in a secondary school in Australia: Imms and Byers (2017) found that flexible and open classroom arrangements in tandem with one-on-one technological improvements had a positive correlation with student engagement and quality of instruction. However, authors suggest that more research needs to be conducted to explore causality between the physical space and engagement.

Mathematics engagement in RUME

Although there is a growing need for research on affective aspects of college-level mathematics (Melhuish et al., 2022), much of the RUME work on engagement is tangential, often embedded in studies of active learning, flipped classrooms, or inquiry-based learning (e.g., Mazur & Taylor, 2022). Nevertheless, there are some studies that make explicit arguments about college students' engagement with mathematics courses.

For instance, although learning and persistence in college can be a self-motivated, students still care about who their instructor is and how they are teaching them. In studying students who were retaking a college calculus course, Dibbs (2019) found that students thought having a "better" instructor in their repeat class was a significant factor for success and engagement. Students defined a "better" instructor in terms of their own affective engagement: as someone who cared about their learning and success and made it safe to ask questions. Their affective relatedness to a caring instructor also prompted students to be more motivated and put more effort into the course, leading to improvement in behavioral engagement as well. A "better" instructor also returned graded work in a timely manner, which was directly related to their cognitive engagement because they could use the feedback to prepare for exams.

One might imagine that a good instructor would also be able to notice engagement patterns in the classroom and stimulate participation from less engaged students. Reinholz et al. (2019) conducted a study in an active learning classroom, where the researchers helped a college instructor use the EQUIP tool to analyze student participation in the classroom and make sense of the analytics to inform future teaching practices. Some of the findings indicated that engagement is a complex phenomenon that is not easy to process as one is teaching, but using tools like EQUIP can help instructors and teachers notice patterns that can be addressed in future classes.

Although College Algebra is not the focus of the proposed research, it is tangentially relevant because it is another entry-level mathematics gatekeeper course for many students in US higher education. There was a notable quasi-experimental study in a College Algebra course (Duffin et al., 2020) that manipulated two aspects of instruction: mode of delivery (inquiry-based vs. direct instruction) and availability of social support interventions. The authors found positive impact from instruction that provided social support on students' affective engagement: in particular, students in both in inquiry-based and direct instruction classes with built-in social support reported higher needs-satisfaction levels than in groups without social support; students in the inquiry-based class with social support expressed feeling more competent than the students in the direct instruction group with social support. In fact, there is a plethora of RUME studies on active learning, inquiry-based learning, and other student-centered modalities of instruction that either address engagement directly (e.g. Mazur & Taylor, 2022; Stanberry, 2018) or tangentially through signifiers of engagement.

Dedicated learning spaces within higher education institutions, like tutoring rooms and study halls have been shown to positively impact student engagement. Waldock et al. (2017) found that an accessible and welcoming space to engage in mathematics learning outside of the classroom provided students with the environment where they could have their need for belonging and support met. Moreover, through high levels of attendance of the space and its usefulness in learning, they witnessed high levels of behavioral and cognitive engagement among the interviewed students.

Aside from the pedagogical and physical structures of learning spaces, classrooms are also spaces that can reify existing socio-cultural norms and stereotypes of society (e.g., Asians are good at math, or girls don't do well in math). If unaddressed (and sometimes despite being addressed and countered), these norms and stereotypes can affect students' engagement in the subject of mathematics. For example, gender differences in engagement can be seen in normalizing behavioral engagement among women, while also impacting affective engagement through harmful gender stereotypes about mathematics ability (Ní Fhloinn et al., 2016) or persistence in the Calculus sequence (Ellis et al., 2016). Microaggressions and educator's expectations are another factor impacting marginalized students' experiences and engagement in the classroom (Maloney & Matthews, 2020; Roberts & Almeida, 2023).

The latest initiatives in US tertiary education have skewed strongly towards increasing student success, graduation rates, and producing more STEM specialists (Roksa et al., 2009; Seymour & Hewitt, 1996; Seymour & Hunter, 2019), especially among groups of people traditionally minoritized in STEM (Ellis et al., 2016). In this context, mathematics is one of the gatekeeper subjects for student success as well as STEM student retention, especially for minoritized students (Bressoud, 2014). One of the ways mathematics departments have adapted and changed to promote student learning, especially in entry-level college mathematics courses, is through reforming gatekeeper classes through initiatives like course coordination, active learning, flipped classrooms, inquiry-based learning, etc. (Apkarian et al., 2019; Bressoud et al., 2015). However, as researchers have found, these reforms have not necessarily resulted in an equitable increase in engagement (Johnson et al., 2020). Finally, on the opposite side of the engagement spectrum Trenholm et al. (2019) have found that overreliance on recorded lecture videos led to undergraduate students' cognitive disengagement, which resulted in more surface level understanding of course content, as well as behavioral disengagement (lower levels of live lecture attendance).

Affect can play a role in undergraduate mathematics students' engagement too, which is partially due to emotional responses to mathematics that are established by college. Visscher & White (2020) studied responses of undergraduate pre-service elementary teachers and found a strong affective anxiety response among students who had to engage in mathematical problem solving and explanation of their work. Martínez-Sierra & García-González (2016) push for studying emotional responses when doing mathematics beyond problem-solving. They considered emotional engagement among undergraduate linear algebra students in their day-to-day mathematical learning and noted the value of using appraisal theory for studying affect in mathematics education. When teachers implement some form of student-centered learning environment (flipped classroom, inquiry-based learning, etc.), there are opportunities for students to engage in more open-ended mathematical tasks with the teacher's support, which can offset some math anxiety.

The Gatekeepers: Entry-Level College Mathematics Courses

Representatives of institutions of higher education state that they care about the students they serve and are aware of the difficulties faced by students embarking on their undergraduate studies journey. Mathematics coursework at the tertiary level plays a key role, because it acts as a gatekeeper: an obstacle that must be overcome to pursue the desired major and earn the undergraduate degree. However, students entering universities and colleges in the US are often underprepared to succeed at these checkpoints, which explains the emphasis on so-called "developmental" courses (e.g., Ganga & Mazzariello, 2018). There are various research manuscripts that provide insights rooted in higher-education-perspectives on "developmental" course design (e.g., Boatman, 2021). Somewhere in-between research articles and policy reports are also research reports for college systems like the Roksa et al. (2009) report for the Virginia Community College system on increasing passing rates in English and Mathematics for students needing "remediation." Reports like these are constantly produced to provide colleges and universities with information on the state of their students' success, as well as to report on the effectiveness of various reforms towards increasing success (as measured by institutions, which is not always the same measure as for content instructors).

Because mathematics is the main gatekeeper for STEM majors, many higher education researchers are invested in improving college mathematics instruction to increase the number of US-produced STEM specialists. Two notable longitudinal studies, foundational to the topic of college retention and STEM persistence, are "Talking about leaving" (Seymour & Hewitt, 1997) and "Talking about leaving revisited" study (Seymour & Hunter, 2019). Both large-scale studies aimed to empirically explore factors affecting attrition among STEM undergraduates in the United States

The first study provided an important shift in the way that STEM retention was viewed by college administrators and educators. By pointing out the harms and ineffectiveness of the "weed out" processes, Seymour and Hewitt prompted the community to challenge the existing structures and educational practices and shift educators' "objectives from selection to education" (1997, p. 394). An important finding of that study was that there was no significant difference between STEM-switchers and STEM-persisters: they all faced and pointed out similar issues in their education trajectories. The main difference was in the attitudes and coping actions that persisters developed to survive.

Although much had changed by the time Seymour & Hunter (2019) published the follow up study, the main factors affecting students did not change. However, the number of overall factors increased and the weight of their effect on switching was evaluated differently by the students in the second study. Students in the second study much more often reported that they found aptitude for non-STEM majors, experienced a loss of confidence in their skills and abilities, and/or disliked the competitive climate in their STEM courses.

College Precalculus

Precalculus is one of the more common gatekeepers to entering STEM majors in universities and colleges; researchers—especially in the RUME community—have studied this course as a bridge into the entire Calculus sequence. In the last 14 years, groups of researchers in the US RUME community have conducted large-scale national studies of the college-level Precalculus-Calculus sequence (Apkarian & Kirin, 2017; Bressoud et al., 2015; Rasmussen et al., 2019). The findings from these studies produced a plethora of insights into multiple aspects of teaching and learning in these mathematics courses, including valuable census statistics (Bressoud et al., 2015; Apkarian & Kirin, 2017), findings on departmental change around these courses (Apkarian et al., 2021; Voigt et al., 2017), concerns around issues of equity (Ellis et al., 2016; Leyva et al., 2021; Tremaine et al., 2022), and many others. These projects also organized a national conference on the Precalculus to Calculus sequence (Apkarian et al., 2021), which shed light on the many concerns faced by practitioners and departments in supporting students enrolled in these courses.

Aside from these larger studies, during this time other smaller-scale studies have also been conducted on the topic in mathematics education, allowing the field to paint a larger and more nuanced picture of students' experiences with college Precalculus and Calculus courses (Battey et al., 2022; Hauk & Hsu, 2022; Leyva et al., 2021; Sonnert & Sadler, 2014; Wang et al., 2017; Wu et al., 2022).

As mentioned earlier in this chapter, there are a variety of ways mathematics departments work on improving student success and engagement in college mathematics courses. One of the ways is through introducing a flipped classroom format, which is the format offered at the University of Georgia. Collins (2019) found a meaningful difference between student performance in a traditional Precalculus class and those in the flipped classroom format. Collins conjectured one of the reasons for better performance might have been due to increased student engagement in the flipped classroom. However, Collins only considered performance variables and did not gather data on student engagement.

Mkhatshwa (2021) also conducted a study of a flipped Precalculus course and found that the students in the flipped classes performed at least as well as the students in the traditional lecture classes on exams throughout the semester and even outperformed on the final exam. This study considered students' perceptions of the format, finding that a majority of students' attitudes towards different aspects of the course changed throughout the semester: some for better and some for worse. For instance, non-traditional students preferred traditional lecture over flipped format due to the amount of time that one had to engage in the course outside of the classroom to succeed. Overall students appreciated the opportunity to work on problems with their peers in class and felt more comfortable discussing course content with their peers in the flipped format classes. Another benefit of the flipped Precalculus instruction in this instance was the decrease of mathematics anxiety among students about taking the Precalculus course towards the end of the semester.

Aside from the choice of instructional format within a Precalculus course, there are also different ways that one can structure the Precalculus course itself. Voigt et al. (2020) conducted a survey of US tertiary institutions' Precalculus to Calculus II sequences for these course variations. One of the choices that a department has is having a stand-alone Precalculus course or have the content broken up into several course offerings (e.g., College Algebra, Trigonometry). When it comes to Precalculus's purpose of serving the Calculus sequence, aside from the usual approach of passing Precalculus, one option is infusing Precalculus content into a Calculus I course for students to have an "as needed" content support; another option is to offer a concurrent class for Calculus I students that offers insights into Precalculus content as it comes up in Calculus I.

In a more systemic recalibration of a Precalculus to Calculus II sequence, researchers at San Diego State University implemented a plethora of reforms and shared their experiences with the process (Apkarian et al., 2018). Because Precalculus content is essential for Calculus success, the department implemented ALEKS software as both a placement and a learning tool for students ahead of the semester. Students had the ability to familiarize themselves with the software ahead of their official placement test and they had access to learning modules that allowed them to brush up on topics they might know but had forgotten. They also restructured the Precalculus course to have three lectures a week with the instructor and one active learning lab with an undergraduate instructional student assistant.

Due to the gatekeeping role of Precalculus in post-secondary education in the United States, understanding student engagement can provide insight into improving student success; to succeed in this work, our research field needs to first better understand students' experiences of engagement in a class like this. Thus, this dissertation study's main purpose was to understand the phenomenon of engagement as perceived by students. Beyond the conceptual framework of engagement this line of inquiry required a theoretical framework that would provide powerful tools needed to explore the phenomenon in all its complexity.

Theoretical Framework: Hermeneutic Phenomenology

Having established the multidimensional and context-dependent nature of student engagement, this study required a methodological approach capable of uncovering its essence as a lived experience. Hermeneutic phenomenology, as both a philosophy and a research practice, is uniquely suited to this task. It moves beyond measuring frequencies or correlating factors to seek a rich, interpretive understanding of what it means for students to be engaged or disengaged in their Precalculus classroom. This approach does not seek to produce generalizable laws but to illuminate the essential structures of this phenomenon within its specific context.

This study builds on the *interpretive* approach of van Manen (2014) with the influence of the *reflective lifeworld* approach of Dahlberg, Drew, Nystrom, and Dahlberg (2001; 2008). Linda Finlay (2013) says that "the value of returning to such concrete experience helps the research focus on discovering dynamic processes rather than seeing phenomenon as fixed." Because engagement is a complex and dynamic phenomenon, it makes sense that by exploring students' rich descriptions of lived experience, we can get to the dynamic essence of engagement, as opposed to capturing a static image at some point in time.

Phenomenology

Phenomenology is known as both a branch of philosophy and a set of methodologies for conducting qualitative research. It was born into the world at the beginning of the twentieth century from the philosophical work of Edmund Husserl, in which he tried to get to the essence of lived experiences (*Erlebniswesen*) and thus to get to know the unknowable. His famous call—"Back to the things themselves!" (zu den Sachen)—called for understanding the transcendental through the only outlet that he believed could be accepted without doubt and with certainty, our experiences (Husserl, 1976 – as cited in Dahlstrom, 2018). He posited that as humans experience the world; our grasp is limited to the things we directly experience ourselves. Husserl believed that to gain insight into these essences one needs to suspend the natural, everyday attitude by "bracketing" away or suspending our assumptions about the phenomena (a process called *epoché*).

Husserl had initially hoped that phenomenology would grow to become a rigorous science of all transcendental phenomena, a "science of essences" (Husserl, 1983 – as cited in van Manen, 2014, p. 90), but as this branch of philosophy developed through his and his students' work, the idea of turning phenomenology into a science was eventually rejected. Thinkers like Martin Heidegger and Maurice Merleau-Ponty shifted the focus from consciousness in isolation to the human being concretely situated in the world. In particular, Heidegger's analysis of *Dasein* ("Being-in-the-world") and Merleau-Ponty's work on embodied consciousness argued that understanding is always grounded in our practical engagement and bodily presence.

Hermeneutic Phenomenology

Although Husserl's transcendental phenomenology sought to bracket preconceptions to arrive at the pure essence of a phenomenon, the hermeneutic tradition, influenced heavily by Hans-Georg Gadamer, argues that such bracketing is impossible. Instead, it acknowledges that understanding is always interpretive and is shaped by the researcher's own historical and cultural context—their historically effected consciousness (Gadamer, 2006). For van Manen (2014), this means the researcher is not a neutral observer but an active interpreter, engaged in the 'hermeneutic circle,' where the meaning of the whole (e.g., engagement) is understood through its parts (e.g., specific lived experience descriptions), and vice versa.

For a study of engagement, which is inherently meaningful only within a specific socio-cultural and educational context, this interpretive stance is not a limitation but a necessity. Unlike approaches that seek to bracket the researcher's preconceptions to describe the essence of a phenomenon (for which Husserl or Giorgi argued), hermeneutic phenomenology acknowledges that understanding is always an act of interpretation. The researcher enters a 'hermeneutic circle' with the data, where their own experiences as an educator and the participants' descriptions dialogue to create a deeper, co-constructed understanding. This is fitting for engagement, a phenomenon whose meaning is inextricable from the context I, as a researcher, am also seeking to understand.

Phenomenology as a research method still relies on the core concepts developed by philosophers. One of them is the *lifeworld* (*lebenswelt* in German), first coined by Husserl and built upon by philosophers like Gadamer and Merleu-Ponty, who perceived it as the tacit world in which we are immersed, and which precedes knowledge (Dahlberg et al., 2001). Dahlberg is one of the people who have developed a direction of inquiry called reflective lifeworld research (Dahlberg et al., 2001), which focuses on bringing the phenomenological approach more fully into social science research: bridging philosophy and practice.

Lifeworld offers an exceptionally good fit to the proposed study: in their Introduction to the *Reflective Lifeworld Research* book Dahlberg, Drew, and Nyström (2001) talk about students' (and patients') lifeworlds in terms of caring relations. Indeed, if we want to truly effect change in better supporting students, we need to understand teaching (as caring for students) and as Dahlberg and colleagues (2001) note, to understand teaching we need to understand the lived experiences and lifeworlds of students. We often hear what instructors perceive as difficult topics and concepts to teach students or what instructors perceive as students' shortcomings, but there is great value in listening to students' voices when it comes to how they live through the phenomenon (Dahlberg, 2006).

The concept of the lifeworld is central. A student's engagement does not occur in a vacuum but is embedded within their unique lifeworld—a world of prior math experiences, cultural beliefs about intelligence, social relationships with peers and instructors, institutional pressures, and personal aspirations. The Precalculus classroom is a distinct 'lifeworld slice' where these broader horizons fuse. A hermeneutic-phenomenological inquiry seeks to understand how engagement manifests within this specific lifeworld. For instance, a student's decision to remain silent (behavioral disengagement) may be inextricably linked to a lifeworld shaped by previous humiliating math experiences (affective history) and the perceived competitive climate of the classroom (social horizon).

Lived experience (*erlebnis* in German) is the prereflective experience of human existence (van Manen, 2014). To grasp someone's lived experience phenomenologically means to attempt to capture their existence as "lived through" in the moment. In the context of Precalculus and students' mathematics engagement, the lived experiences of interest represent a wide range of possibilities: from smallest instances of students deciding to act and engage in the moment within a class period in the Precalculus classroom (something as simple as raising their hand and asking a question) all the way to a more holistic experience of engagement in a course as a whole for the student in a semester. Van Manen states that "the notion of 'lived experience' announces the intent to explore directly the originary or prereflective dimensions of human existence" (2014, p. 57). Thus, the lived experience description (henceforth LED) is the narrative that focuses on describing the experience of the event as it happened without reflecting on it and making evaluative statements.

Considering the complex nature of engagement and its many contextual aspects, we can benefit from hearing about students' lived experiences while acknowledging that students' lifeworlds are an integral (albeit potentially implicit) part of their engagement experiences. For example, as Finlay points out, engaging participants in dialogue beyond their reflection on events that transpired allows us as researchers to see the more holistic picture of the experience, which “cannot be uncoupled from the wider lifeworld” (2012, p. 181). Although gathering rich phenomenological data is not an easy task, it is a rewarding endeavor that can uncover for us aspects of student engagement that are usually hidden and hard to grasp otherwise (van Manen, 2014).

In summary, hermeneutic phenomenology provides the philosophical foundation and methodological tools to answer the core research question of this dissertation: 'What is the story of engagement in a college Precalculus course?' It allows us to treat engagement not as a set of variables to be measured, but as a profoundly human phenomenon to be understood. By prioritizing the students' lifeworld and interpreting their lived experience descriptions, this study aims to produce a nuanced, context-rich, and ultimately actionable understanding of what it means to engage with mathematics at this critical academic gateway.

CHAPTER 3

METHODS

To focus on understanding students' experiences of engagement, I used qualitative research methods to elicit and analyze data that would provide deep insight into the various aspects of engagement in a college Precalculus course. The emphasis on lived experience together with my own positioning lent themselves well to using phenomenology as both a qualitative research approach and an overarching theoretical framework, which have shaped the study top to bottom: from research questions to the smallest analytical decisions.

This chapter will start out with a positionality statement that provides the context for many research design choices throughout the study. Afterwards I will provide an overview of hermeneutic phenomenology methods and elicitation tools used in the study, as opposed to its use as a theoretical framework described in Chapter 2. Then I will describe the context of the study and the data collection process, followed by an explanation of the data analysis processes and procedures.

Recall that the central research question of the dissertation is: What is the story of engagement for UGA students in Precalculus? The sub-questions were:

RQ1: What is the lived experience of engagement as seen through student descriptions of their engagement in Precalculus across the semester?

RQ2: How do the four proposed dimensions (academic, affective, behavioral, cognitive) of engagement manifest and interact with each other throughout the semester?

RQ3: What is the interaction between student engagement, topics in the Precalculus curriculum, and participants' perception of how these topics are taught?

Researcher Positionality

In this journey I am an outsider: a foreigner in the United States who has never herself taken a Precalculus course, a person who has never taught Precalculus at UGA, and someone who is new to the state of Georgia. At the same time, I am an insider: I have taught Precalculus a handful of times in the US (outside UGA), I have a perspective on mathematics as a former mathematician and mathematics education as an education researcher, and I have the privilege of knowing some aspects of the course that might not be known to the students. The tension of being both outside and inside of the space where the phenomenon resides allows me to naturally take on the phenomenological position of non-interference and wonder (Finlay, 2013). At the same time, as someone who cares deeply about mathematics education and student success in gatekeeper courses, I can be an empathetic ear (Finlay, 2013, pp. 182-184) for students to share their experiences. This could be a way to pay back and honor the participants' stories, because as Linda Finlay says, "[i]n addition to knowing that one's perspective is witnessed, being listened to opens up potentially transformative space and time, allowing the person to make sense of their experience, perhaps going beyond previous understandings" (2013, p. 181).

My experiences so far also shed light on the unique shifts in my horizons of the phenomenon. Recall that Gadamer (2006) describes things we know and believe as our horizons. Our horizon moves with us. In my journey from Kazakhstan to the United States my horizon shifted not just geographically, but I also encountered the subject of Precalculus. Engaging in teaching this subject has moved my horizon further to a place of frustration: this course did not make coherent sense to me as a mathematician, which made it hard to motivate my students to persist in my class. Given the title of the course, I assumed Precalculus was a precursor to Calculus. However, by talking to the Precalculus coordinator at the University of Illinois at Chicago, I learned that students pursuing a nursing degree were required to take Precalculus but were not expected to pass Calculus. This situation is similar at UGA, where certain majors require students pass Precalculus but not Calculus I. It feels as if I have traveled in a circle and came back to the same spot, but at the same time my horizon has grown. As my horizon expanded, my journey took me to wonder if we can do better for our students.

This positionality also fits within the idea of rejecting subjectivity and accepting that my previous experiences and horizons will inform my work (Finlay, 2013). I believe my journey and its questioning and wonder led me to choose phenomenology as both a philosophical framework and a methodology. As van Manen says:

> Phenomenology is primarily a philosophic method for questioning, not a method for answering or discovering or drawing determinate conclusions. But in this questioning there exist the possibilities and potentialities for experiencing openings, understandings, insights – producing cognitive and noncognitive or

> pathic perceptions of existentialities, giving us glances of the meaning of phenomena and events in their singularity. (2014, p. 29)

Hermeneutic Phenomenology as a Research Method

The specific method used in the proposed study is hermeneutic phenomenology, which focuses on the human experience of a phenomenon as well as the meaning of that experience (Friesen et al., 2012). The process of interpreting meaning is valuable in education research because it allows for contextualizing both the participants' experiences within the broader socio-cultural environment where learning happens and the individual learner's personal disposition and interaction with education. In his 2014 book *Phenomenology of Practice* Max van Manen describes hermeneutic phenomenology as "a method of abstemious reflection on the basic structures of the lived experience of human existence" (p. 26). As educators and education researchers, we are constantly trying to understand and describe students' learning experiences; however, there is a chasm between our perspective as "experts" and the students' experiences in moments of learning as it is happening. This chasm cannot be bridged without intentionality in our methods of thinking about the phenomenon, gathering data, and analyzing it. Thus phenomenology, and hermeneutic phenomenology in particular, allows us to explore, describe, and reflect on phenomena while centering the students' lived experience.

Phenomenology as a qualitative research method relies on the core concepts proposed by phenomenological philosophers (as described in Chapter 2), while supporting researchers in their inquiry into understanding various phenomena. One of these concepts is the *lifeworld* (*lebenswelt* in German), first coined by Husserl and built upon by philosophers like Gadamer and Merleu-Ponty, who perceived it as the tacit world in which we are immersed, and which precedes knowledge (Dahlberg et al., 2001).

Lifeworld offers an exceptionally good fit to the proposed study: in their Introduction to the *Reflective Lifeworld Research* book, Dahlberg, Drew, and Nyström (2001) talk about students' (and patients') lifeworlds in terms of caring relations. Indeed, if we want to truly effect change in better supporting students, we need to understand teaching (as caring for students) and as Dahlberg and colleagues (2001) note, to understand teaching we need to understand the lived experiences and lifeworlds of students. We often hear what instructors perceive as difficult topics and concepts to teach students or what instructors perceive as students' shortcomings, but there is great value in listening to students' voices when it comes to how they live through the phenomenon of engagement (Dahlberg, 2006).

Lived experience (*erlebnis* in German) is the prereflective experience of human existence (van Manen, 2014). To grasp someone's lived experience phenomenologically means to attempt to capture their existence as "lived through" in the moment. In the context of Precalculus and students' mathematics engagement, the lived experiences of interest represent a wide range of possibilities: from smallest instances of students deciding to act and engage in the moment within a class period in the Precalculus classroom (something as simple as raising their hand and asking a question) all the way to a more holistic experience of engagement in a course as a whole for the student in a semester. Van Manen states that "the notion of 'lived experience' announces the intent to explore directly the originary or prereflective dimensions of human existence" (2014, p. 57). Thus, the lived experience description (henceforth LED) is the narrative that focuses on describing the experience of the event as it happened without reflecting on it and making evaluative statements. In the proposed study these will be solicited from the students during the interview process.

Considering the complex nature of engagement and its many contextual aspects, we can benefit from hearing about students' lived experiences while acknowledging that students' lifeworlds are an integral (albeit potentially implicit) part of their engagement experiences. For example, as Finlay points out, engaging participants in dialogue beyond their reflection on events that transpired allows us as researchers to see the more holistic picture of the experience, which "cannot be uncoupled from the wider lifeworld" (2012, p. 181). Although gathering rich phenomenological data is not an easy task, it is a rewarding endeavor that can uncover for us aspects of student engagement that are usually hidden and hard to grasp otherwise (van Manen, 2014).

Context of the Study

The study was conducted at the University of Georgia, a 4-year public land-grant research university. It is the flagship of the University System of Georgia (USG), classified as an "R1: Doctoral Universities: Very High research activity" institution under the Carnegie Classification, and it is considered to have "Very High" undergraduate admissions standards. For instance, although USG requires all its college applicants to complete the Required High School Curriculum, including 4 Carnegie units of college preparatory mathematics (USG, 2022), in recent years 98% of UGA freshmen have pursued an honors or higher-level track coursework (UGA, n.d.). This usually involves taking classes up to Calculus in high school.

Nevertheless, the number of UGA students enrolling in Precalculus (about 1700) is roughly similar to the number of students enrolled in Calculus 1 in the same year (Gleason, 2020). Although students are placed into Precalculus based on their test scores and placement test results, a high number of them have already taken Precalculus in high school. So, the content of the course might not be entirely new for many students.

The Precalculus course at UGA (MATH 1113) satisfies the “Quantitative reasoning” graduation requirement; it is also a required class for several majors at the university, including all of the majors at the Terry School of Business. Because of this requirement, a majority of the participants recruited for this study were intended Business majors or minors.

MATH 1113 at UGA is offered every semester, and although there is an online version of the course (MATH 1113E), this study focused on the more “traditional” college student, adults between ages 18-23 (Baruah et al., 2022; Forbus et al., 2011), staying in Athens (all UGA freshmen are required to live on campus), and attending the course in person. Most of the in-person MATH 1113 course sections are taught using the flipped classroom format, but there are usually some instructors who chose to teach via lecturing. The UGA mathematics department received a grant to support a small class size initiative, so each section of in-person Precalculus is limited to at most 19 students. The classes meet either 3 times a week (Monday, Wednesday, Friday) for 50 minutes per session or 2 times a week (Tuesday and Thursday) for 1 hour and 15 minutes per session.

Students in all sections of Precalculus (lecture and flipped) must purchase access to ALEKS (Assessment and Learning in Knowledge Spaces), an adaptive educational platform, where students complete regular online assignments throughout the semester.

At the beginning of the course each student must complete an initial knowledge check test to identify their strengths and weaknesses, then throughout the semester it guides students across the curriculum by filling these individual knowledge gaps. The system presents explanations and practice problems, and students must demonstrate mastery of each topic before new concepts are unlocked. The amount of work that each student must complete in ALEKS throughout the semester varies from student to student and adapts to their knowledge of Precalculus content.

Students in the flipped classroom sections have access to playlists of pre-recorded class videos for each segment of the course, and they are expected to watch them all before each class. Additionally, prior to each session (aside from test days), students have short pre-class work that is due early morning the day of class. During the in-class sessions in the flipped classroom, students are expected to work on in-class worksheets, ideally in groups. Both the pre-class and in-class worksheets are available as a purchasable packet of printed worksheets from the university bookstore. All of the flipped classroom materials for MATH 1113 (videos, worksheets) are the same across all the sections of the course.

The MATH 1113 course and the mathematical content therein are divided into three parts. The first covers functions (linear, quadratic, polynomial), their properties and representations: equations and graphs, range and domain, and function operations. In the second part students learn about inverse, exponential, and logarithmic functions. The third and final part of the semester is dedicated to trigonometric functions. At the end of each part students complete a mid-term test, and they have a cumulative common exam at the end of the semester.

Data Collection

All the data for this study has been collected during the Spring 2024 semester at the University of Georgia. The study proposal was reviewed by the UGA Institutional Review Board and determined as Exempt under the following ID: PROJECT00008676. At the beginning of the semester, I contacted all instructors of record who were teaching a section of MATH 1113, asking them to share the recruitment materials with the students in their classes. Within the first two weeks of classes I was able to recruit the target number of participants (12) for the study.

Throughout the semester I met at most four times with each participant for in-person interviews, every 3-4 weeks on average. In addition to interviews, I also attended several MATH 1113 lessons throughout the semester, making sure to attend each participant's section at least once. During these observations, I took field notes and gathered some course materials used during the lessons. Finally, I wrote researcher memos throughout the data collection and analysis process to supplement the process of analyzing data.

Participants

At the start of the study I recruited 12 undergraduate students, only 9 of whom completed all 4 interviews. The other three participants withdrew from the course at various points in the semester and thus could not complete four interviews.

Table 1

Participant information

No	Pseudonym	Pronouns	Year in college	Intended major (at the time)	Interviews completed	Class format
1	Ashley	she/her	Freshman	Business	4	flipped

2	Beth	she/her	Freshman	Sports management (Business minor)	4	flipped
3	Camila	she/her	Freshman	Marketing/ Entertainment media studies	4	flipped
4	Destiny	she/her	Freshman	Marketing/ International business	4	flipped
5	Gloria	she/her	Freshman	Undecided/ considering Health promotion	4	lecture
6	Jordan	she/her	Sophomore	Accounting	4	flipped
7	Maggie	she/her	Freshman	Undecided/ considering Finance	4	flipped
8	Penelope	she/her	Freshman	Accounting	4	flipped
9	Tori	she/her	Freshman	Marketing (Interior design minor)	4	flipped
10	David	he/him	Freshman	Finance	3	lecture
11	Robin	they/them	Freshman	Business/Art (Film studies minor)	2	flipped
12	Elena	she/her	Freshman	Risk management	1	flipped

The 12 participants attended 8 different sections of Precalculus with 4 different instructors. David and Gloria, the only participants enrolled in a lecture-based format, were taught by the same instructor but were in two different sections. Ashley, Camila, and Robin had the same instructor; Ashley and Camila were in the same section. Elena and Tori had the same instructor and were in the same section and were even in the same classroom group. Finally, Beth, Destiny, Jordan, Maggie, and Penelope had the same instructor; Destiny, Jordan, and Penelope were in the same section and in the same group, while Beth and Jordan were in two other separate sections.

As previously mentioned, almost all the participants had a major or minor in the Terry School of Business and, according to them, Precalculus is considered a “weed out” (or gatekeeper) course for Business school majors at UGA. Multiple participants shared that their academic advisors and other UGA students said that when applying for a major in Terry, the top two things that the school will look at in their application are their grades in Accounting and Precalculus.

One of the participants – Elena – unfortunately had to withdraw both from the study and the Precalculus course due to health issues, and we did not get a chance to talk as much about her engagement with the course. In contrast, by coincidence the two participants who also withdrew from the course before the end of the semester—Robin and David—withdrew around the time of their next scheduled interview. They both consented to doing one more interview, where we got a chance to talk not just about their engagement but also their choice to withdraw. As such I was able to obtain a better picture of their engagement in the course compared to Elena. Due to these factors, while David’s and Robin’s interviews remained part of the data analysis set, and Elena’s interview has been set aside.

Phenomenological interviews

The key difference between a phenomenological interview and interview as part of most other qualitative traditions is in its aim: the goal here was to elicit participants’ prereflective descriptions of their lived experiences as they happened to them. Many qualitative research methods specialists point out that it is a very challenging task to get interviewees to share these (van Manen, 2014), compared to asking participants to share their opinions on a topic. Phenomenological interviews aim to elicit rich descriptions of human experience, so the interviewer needs to straddle a delicate balance between openness and preparedness. On one hand, one must remain open to the dialogue happening between the interviewee and interviewer and the stories of experiences as shared by the participant in the moment. On the other hand, this doesn't mean that the researcher enters the interview with no plan: on the contrary, interviews need to be extremely well-thought-out beforehand. The greatest responsibility of the researcher is to keep their focus on the phenomenon of interest, which will sometimes involve shifting the interview flow on the spot.

In this study, the phenomenon of interest was students' mathematics engagement in a college Precalculus course, so the aim of the series of in-depth interviews was getting the participants to share their personal life experiences with engaging in this course. As Dahlberg et al. (2001) point out, "[t]he focal point of the interview is the way that the interviewee experiences the phenomenon and expresses its meaning" (p. 155). Connecting this to the concepts and ideas presented in Chapter 2, this involved the interviewer's careful attentiveness to the glimpses into participants' lifeworlds through the openings in the narratives elicited and shared during the interview process. The interview protocols offered the interviewer and interviewee opportunities to partake in joint exploration of the participants' lived experiences. Several important research-informed considerations shaped the design of the interviews in this study, some of which are unique to the methods of hermeneutic phenomenology.

First, one of the top priorities for conducting a phenomenological interview is establishing an interview space where the participants feel safe and comfortable enough to be vulnerable about their learning experiences in MATH 1113 (van Manen, 2014). The interview should not feel rushed, and the researcher must be personable enough to instill a sense of trust in the participants of the study. Because a phenomenological interview probes into lived experience as it happened, participants will talk not just about events and their order, but also about their affective reactions and behavior, which can be sensitive information for some participants. In this study, all the interviews took place in reserved closed spaces in Aderhold Hall, which houses the UGA College of Education. It is a 10-minute walk south from Boyd Hall, which houses the UGA Mathematics Department.

Second, despite the emphasis on the openness of the interview process, the interview need not necessarily be unstructured. In fact, both van Manen (2014) and Dahlberg et al. (2001) warn their readers about the dangers of delving into open-ended research interviews without a game plan for the sake of emphasizing interview as a dialogic conversation. In particular, Dahlberg et al. (2001) point out that despite researchers' desires to conduct an interview as a dialogue, the researcher needs to remember that due to the research nature of the inquiry, "the researcher and the interviewee do not participate under the same conditions" in the process (p. 157). At the same time the researcher must make sure that they are not rigidly following the protocol if the conversation is not producing recollections of lived experiences pertaining to the phenomenon of interest. The interviewer needs to follow the line of questioning that leads the interviewee back to the phenomenon and not shy away from moments of discomfort like pauses, silence, or instances when one needs to bring the participant back into the prereflective space (van Manen, 2014). All four in-person interviews were semi-structured. They were organized into subsections with separate, clearly-stated goals, a selection of suggested questions, as well as exact wording for specific key points in the interview (see Appendices A-D for the interview protocols).

Third, something that might set the phenomenological interviewing process distinctly in its own category is its non-focus on validity in the traditional research sense. As described in Chapter 2, this study approaches the phenomenon of engagement as something that is shaped by each individual within their socio-cultural experiences in the classroom. Hence, there was no capital "T" Truth that I attempted to distil through the conversation. Instead, the focus was on exploring and teasing out the different aspects of the experience of engagement among Precalculus mathematics students. The interview data was not collected to juxtapose with student grades or instructor interviews. Participants were positioned as authorities in their own experiences of engagement, and I did not seek out to corroborate their descriptions against some "subjective" information.

Finally, as the data collection rounds progressed, the re-reading and/or re-listening of participants' LEDs of engagement during the prior rounds of interviews led me to identify intermediate themes and narratives. These in turn informed not necessarily the core questions of the following interview rounds, but the follow up questions in the existing protocols (Fig. 3.1).

Engagement artifacts

Although asking participants to share and reflect on their experiences of engagement was a central feature of the interviews, it is critical to note that each participant might have their own conceptualization of "mathematical engagement," which might not align with the conceptualization used in this thesis. Hence it was important to get each participant to explicate how they think about engagement in order to analyze and understand their responses about their self-reported engagement in Precalculus.

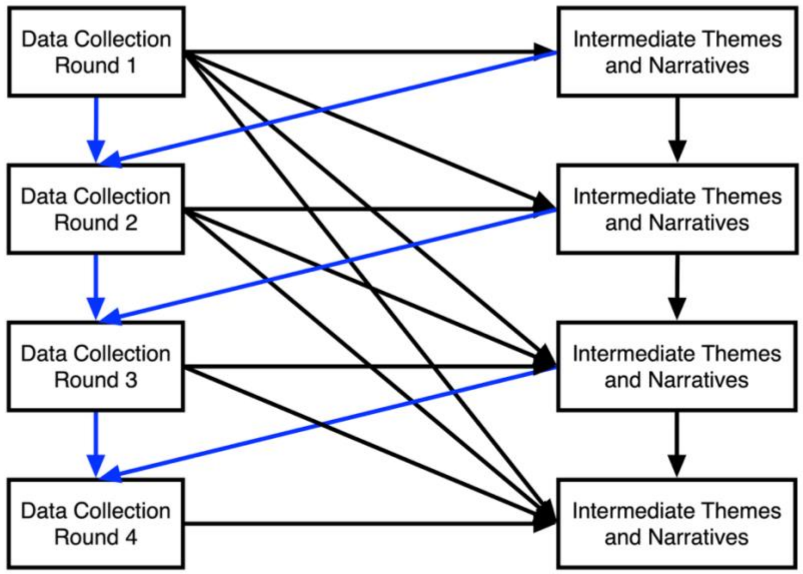

Figure 3.1: Data collection and analysis flowchart
Note: blue arrows indicate that one aspect of the process informed the other.

However, it is a difficult task for participants to come up with a definition of "mathematical engagement" on the fly; in fact, during practice rounds of interviews this issue came up immediately. If one takes into account that engagement researchers themselves are still struggling with coming to a consensus on the conceptualization of engagement (Wong & Liem, 2022), it is a tall order to ask freshmen to provide a coherent definition during an interview.

Wellborn (1992) states that "[e]ngagement refers to actions that are organized around the performance of some specified task or set of tasks" (p. 35). Using this definition, if one narrows the question down to the different actions and tasks involved with being a college Precalculus student, it might be a more reasonable question to ask during a one-hour interview. Thus, the final version of the question in the protocols for

Interview 1 was: “What are some ways you think a college student can be engaged in their Precalculus class?” Most of the responses that participants shared during the study resembled lists (Fig. 3.2), while few chose less conventional depictions (Fig. 3.3).

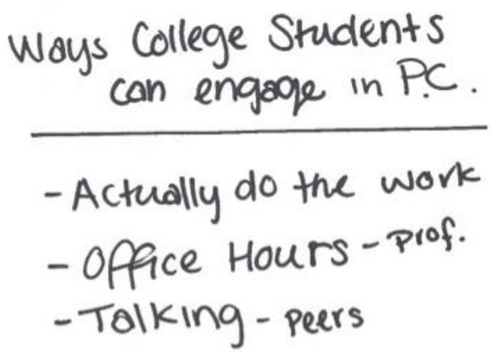

Figure 3.2: Destiny’s engagement artifact from Interview 1

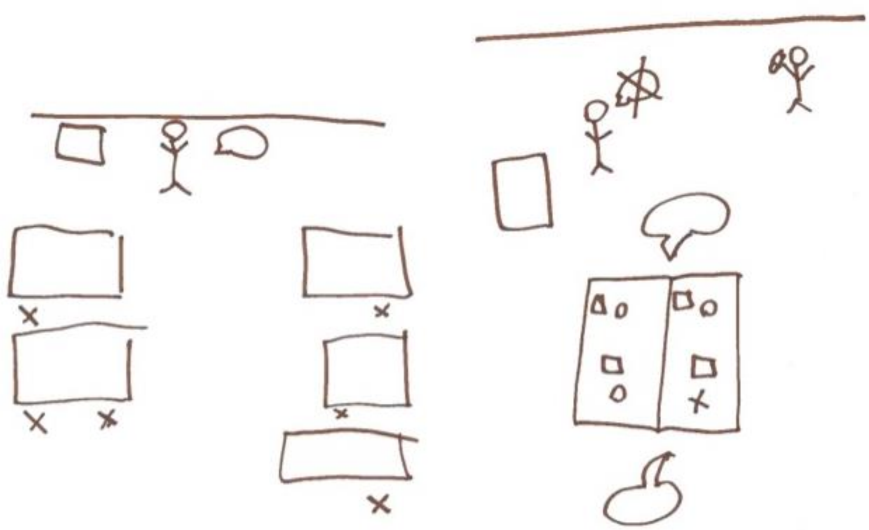

Figure 3.3: Ashley’s engagement artifact from Interview 1

In Ashley’s case she was contrasting ways she saw students engage in two different UGA MATH 1113 sections that she was in. On the left she described a lecture-based section she was enrolled in only for the Monday of Week 1 of the Spring semester, before she switched to a flipped classroom section, depicted on the right. During the interview Ashley described how there were very few ways one could engage in the lecture-based class, because the instructor was talking at the students, and all they could do was listen and take notes from their rows of tables. In contrast, Ashley noted that in her flipped-classroom section students could engage by talking to each other at their group tables, where they sat facing each other. She also noted that students could go to the board to show and explain to the whole class how they solved a problem from the worksheet. Unlike in the class on the left, the instructor was not talking "at" the students, so we can see that Ashley distinguishes instructor-focused engagement in one class and peer-focused engagement in the other.

During Interview 2, participants were given their artifacts and asked to review and edit them as they wanted, as well as marking the ways that the participants engaged in the course themselves within the week leading up to the interview. While some participants' artifacts did not change much, some added a lot to their previous list (e.g., see Destiny's response in Fig. 3.4)

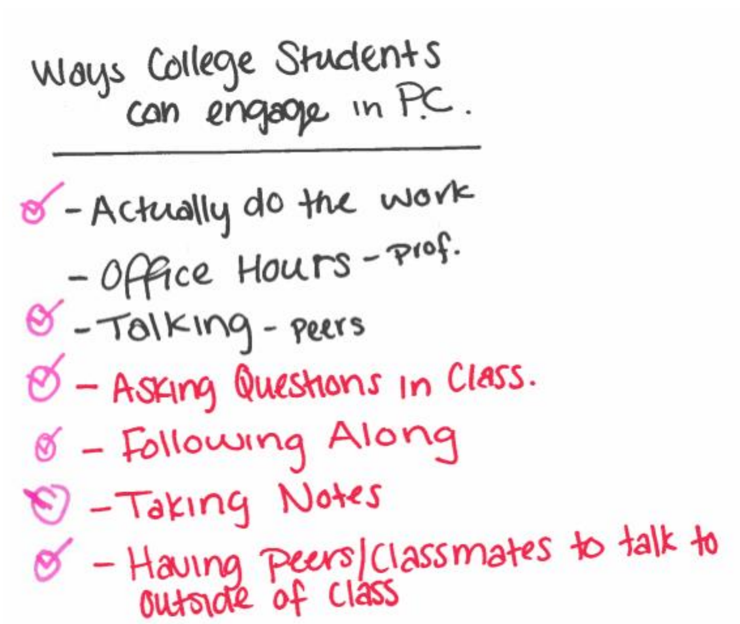

Figure 3.4: Destiny's engagement artifact from Interview 2

Because engagement is a concept that has both similar manifestations across different disciplines and is also context-specific (Middleton et al., 2017), it was important to also understand what sets mathematical engagement apart from general university engagement or engagement in other college courses. In order to capture this boundary, during Interview 3 the participants were asked to produce an engagement artifact for another class that they are enrolled in during that semester and then to compare the two artifacts. For example, in Figure 3.5 we see the two artifacts that Tori compared during her third interview.

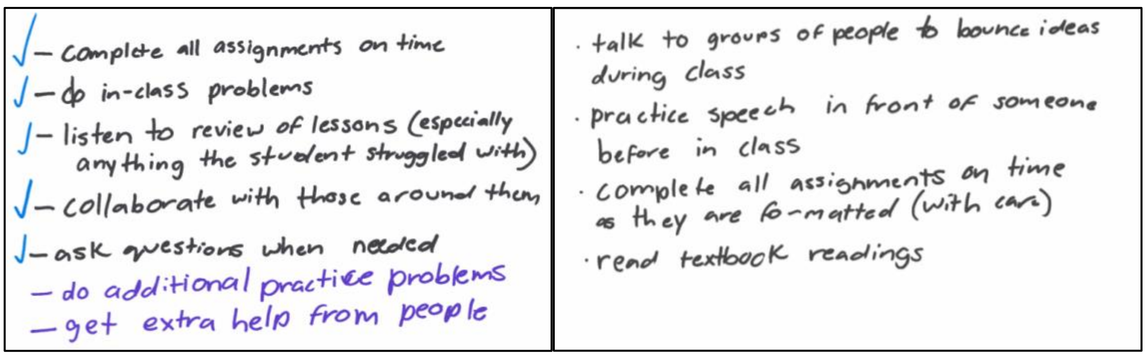

Figure 3.5: Tori's engagement artifacts for Precalculus (left) and Communications (right)

Engagement graphs

In a recent publication, Satyam and colleagues (2022) conducted an analysis of three mathematics education research articles that used graph representations as a tool to study students' affect when learning mathematics. In their review of literature, they note several benefits of asking students to graph their mathematics-related affect. First, as a non-verbal medium, it circumvents the difficulty of verbally conveying affective experiences, which can lead to potential miscommunication and misinterpretation of data. Second, it allows one to consider affective states as they shift and change over time, instead of capturing a potentially static portrait of affect as a rather stable trait. These strengths allowed authors of the articles considered by Satyam et al. (2022) to explore two fluctuating affective phenomena: mathematics-related confidence and emotion. Similarly, Riske et al. (2021) used a graphing tool to study how high school students perceived their mathematical engagement throughout one semester. In this case the authors used the idea of being “into math” as a proxy for engagement and asked students to define what it meant for them to be “into math,” provide an example of the phenomenon, and sketch a graph of their self-assessed engagement over the course of the semester.

All the studies in these articles used a two-dimensional coordinate plane, where the x-axis represented time values and the y-axis represented the values corresponding to the affective phenomenon studied. However, some other properties of the planes varied among the manuscripts: different ways to mark/scale time, some restricted their graphs to the first quadrant, others used two (first and fourth quadrant of the plane). Although there is no single right way to use an elicitation tool of this kind, Satyam et al. (2022) point out the importance of considering the phenomenon studied, as well as the goals and context of the study, when making these design choices.

In this study, I conducted interviews with the same participants 4 times throughout the 15-week long Spring 2024 semester at UGA and I implemented an elicitation strategy similar to the one described in Riske et al. (2021), where students were asked to sketch a graph of their levels of engagement in the mathematics class as a function of time. During each interview the participants graphed their engagement up until the time of the interview: first few weeks during the first interview, the weeks between the first and second interview during our second meeting, etc. Thus, each graphed segment covered around 3-4 weeks of the semester. Moreover, the participants were able to review their sketches from previous interviews and build on them. In order to express themselves in whatever way was most comfortable for them, I provided alternative color markers to show how they would revise previously sketched graphs or letting them sketch the new segment in a different color.

During Interview 1 each participant was given a blank coordinate plane. As one might note in Figure 3.6, there were no labels and tick marks on either the x-axis or the y-axis. This was an intentional choice, as each participant was given the opportunity to choose their own interpretation of the plane and scales of quantities in order to represent their own unique experiences of engagement in the Precalculus class both for the scale of time and levels of engagement. While they were not directed to do so, all participants chose the x-axis to represent time and the y-axis to represent the level of engagement. It is important to note that I never aimed to measure students' self-assessed affect against each other in this study.

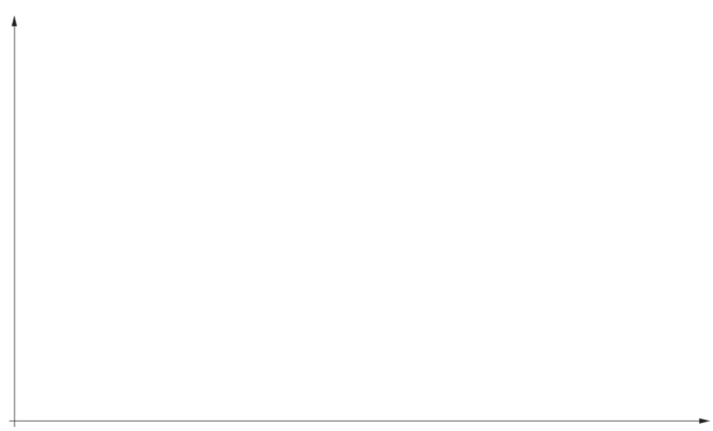

Figure 3.6: Handout used for sketching engagement

One of the ways that the process of sketching complimented phenomenological interviewing was through its power of evocation and revocation (van Manen, 2014). As participants drew and re-drew their sketches, they were looking back over their experiences from previous weeks, and through the process of careful interviewing, these vivid experiences were brought up more naturally and participants were able to better recall experiences to be phenomenologically reflected upon. The graphing tool also provided a medium that relied on the mathematical language of graphing in addition to the English language and could thus help participants express their stories in more than one way.

Although most participants drew continuous functions for their engagement graphs (Fig. 3.7), one chose to use bar charts (Fig. 3.8), and one drew multiple graphs (Fig. 3.9).

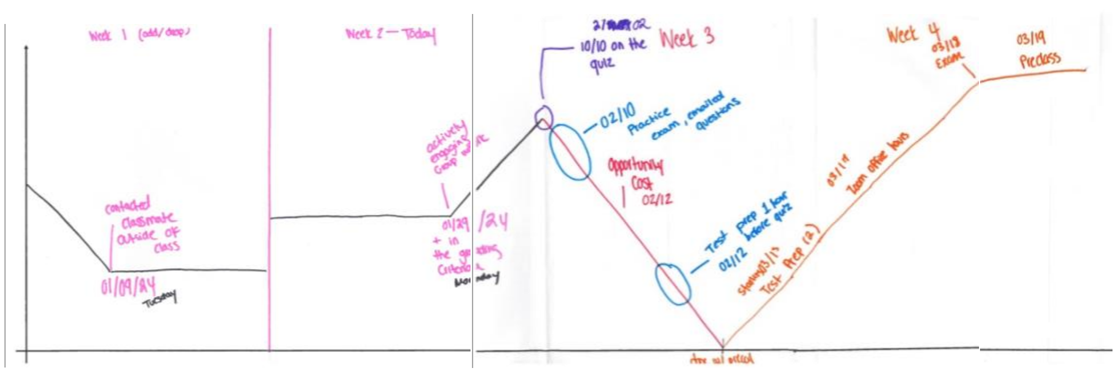

Figure 3.7: Penelope’s engagement graph at the end of Interview 3

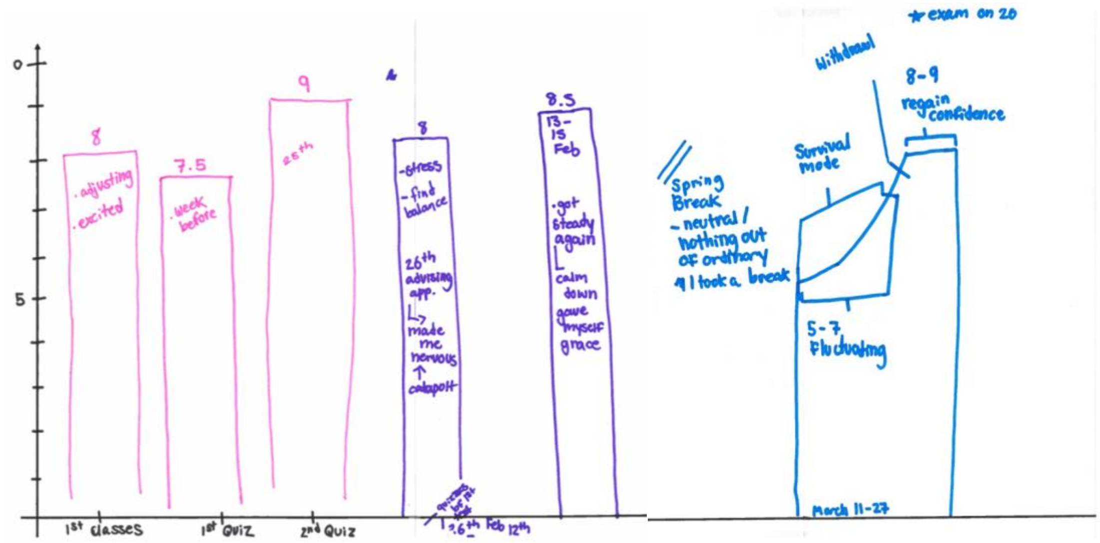

Figure 3.8: Camila's engagement graph at the end of Interview 3

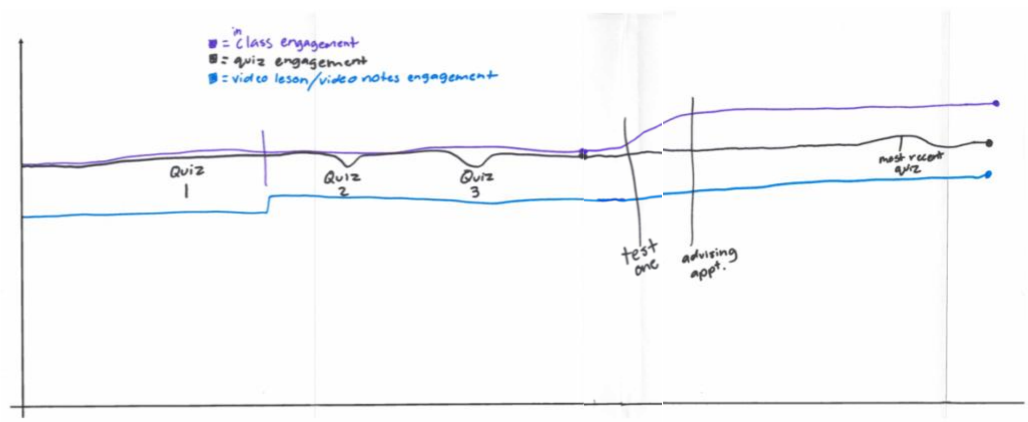

Figure 3.9: Tori's engagement graph at the end of Interview 2

Classroom Observations

Throughout the semester I conducted 18 classroom observations across 8 different sections of Precalculus. The four instructors were contacted ahead of time and informed that someone from the study was enrolled in certain sections that they were teaching. They were never given any names or indication of who the participants were. The instructors allowed me to observe in all the indicated sections. The participants were informed ahead of time (during the consent process) that I would not acknowledge them in class to avoid identifying them as participants in the study.

Sections taught by the same instructor were observed 4 to 5 times each throughout the course of the entire semester, making sure to rotate between the sections attended by the participants. During this time, I sat at the back of the room and observed the class and took notes using the IRB-approved Classroom Observation Guide (see Appendix E). Most of the time the instructor and students would ignore my presence; occasionally instructors would share a copy of the course materials for that day with me (quizzes, review sheets, etc.). I also obtained an electronic copy of the MATH 1113 worksheet packet at the beginning of the semester, so I could look at the problems that the students were working on in the session that I was observing.

In *Reflective Lifeworld Research* Dahlberg et al. (2001; 2008) draw parallels between observations in phenomenology to field work in anthropology and ethnography. When an anthropologist or ethnographer studies a culture that they are themselves connected to, it is not possible to fully “remove” oneself from that bond, so bridling can play an especially valuable role during field observations. If you recall the positionality statement at the beginning of this chapter, I have always felt like a slightly awkward insider/outsider trying to do research on college Precalculus. So going into the classrooms—aside from taking some more formal notes on the topic, teaching and student work modalities, as well as opportunities to engage—I also tried to engage with the class session myself.

I attempted to solve some of the problems posed by the instructor or those from the worksheet. I also listened to the explanation of a solution by one of the students. In the lecture-based classroom, I sat and copied the notes from the board. Such an approach helped me contextualize the phenomenon the way it might have manifested to the participants in this study. After all, I might have handed out worksheets in my own Precalculus classroom, but I never tried to make my own way through a 5-page in-class worksheet on trigonometry at UGA (In-class activity 22, MATH 1113 packet, pp. 211-215). Indeed, there were too many problems and there was no way I would make it through all of them.

It is one experience to bristle at the barrage of coordinated quizzes as an instructor, but it is radically different to come to a class to observe a flipped session with a 15-minute quiz at the end, only to be told upon my arrival that they will be doing a new quiz format. That day the quiz lasted the entire class session and students worked on it in groups. I chose to stay to observe and record students' engagement with the quiz and one another throughout the entire time (Fig. 3.10).

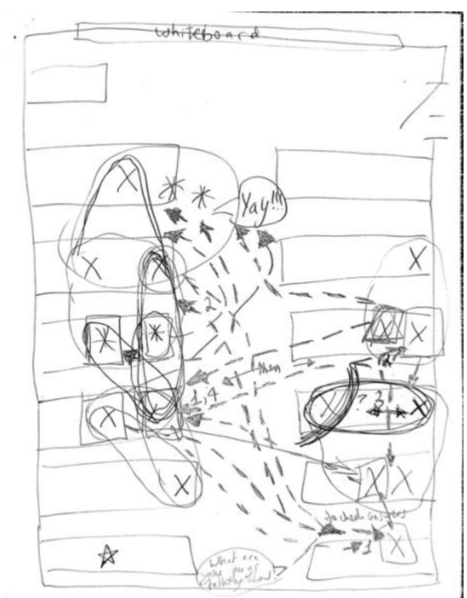

Figure 3.10: Group quiz observation diagram

The classroom observations within this study were never meant to verify students' accounts of self-reported experiences of engagement. Rather they helped me as a researcher to expand my horizon in order to bridle my pre-conceptions and developing understandings, as well as contextualize the phenomenon within the setting of the UGA MATH 1113 course. In that sense, these observations more than anything else were meant to support the process of data analysis.

Data Analysis

The data analysis process was fundamentally hermeneutic. My understanding of the whole (the phenomenon of engagement) was continuously revised through an iterative dialogue with its parts (individual transcripts, artifacts, and memos), and vice-versa. This cyclical interpretation is central to uncovering meaning in hermeneutic phenomenology. The process of phenomenological analysis involved multiple rounds and steps throughout the processes of data collection, data analysis, and writing. Writing is purposefully included here because unlike some qualitative research methodologies, where writing is the final step of reporting the findings of the study, writing in phenomenological research is an integral part of the entire data analysis process (van Manen, 2014). Oversimplified, phenomenological writing is part of the cycle of phenomenological inquiry, wherein through constant drafting and re-writing one gets deeper into the essence of the phenomenon. Writing (together with reflection on themes) is a process that started at the beginning of this study and kept going until the end, because "phenomenological reflection is writing" (van Manen, 2014, p. 365).

Experiential and thematic draft writing (van Manen, 2014) are two practices that go well together when practicing phenomenological writing. This involves both succinctly describing themes discerned from data analysis in form of compelling narratives, and it requires drafting and incorporating experiential material from participant LEDs within those narratives. As the writing is drafted, it needs to always focus on the more concrete instances of the experience of the phenomenon of interest, with the assumption that over rounds of data analysis and re-writing these accounts will be sharpened to the point of a compelling narrative that paints a recognizable picture of the phenomenon. More specifically, experiential draft writing usually culminates in written anecdotes within the narrative of the findings. Anecdotes are not simply examples of participants' lived experiences, they are focused narratives that go beyond just compelling: they must connect abstract concepts and ideas to reality (van Manen, 1990).

With each round of reading and writing my intermediate meaning-making of the data, it became clear that the story of engagement is very complex (which was anticipated), but it is difficult to describe in a simple narrative that captures the "one" essence of the phenomenon (which I should have anticipated). In this subsection I first outline the intended plan and then expand on each step and how the process needed to be adapted to truly reflect the complexity and depth of the phenomenon.

Initial Data Analysis Plan

When first proposing the study in the Prospectus, the data analysis plan seemed straightforward (Table 2). For RQ1: identify participants' LEDs of engagement in each interview, synthesize those into phenomenological anecdotes, and reiterate this process at each round of interviews. For RQ2: code identified experiences of engagement for the conceptualized dimensions of engagement at each round and work on more phenomenological anecdote writing. For RQ3: analyze the identified LEDs and code with the content topic progression across the MATH 1113 curriculum. If this seems too good to be true, you are right: a couple of things happened that prompted me to shift the gears for the process of data analysis.

Table 2

Research sub-questions and corresponding data analysis

Research sub-question	Collected data	Data analysis
RQ1: What is the lived experience of engagement as seen through student descriptions of their engagement in Precalculus across the semester?	Interviews (1-4) Engagement artifacts (1-4) Engagement graph Researcher memos	Participants' LEDs from interviews (1-4). Phenomenological synthesis of LEDs and memos produced after rounds of data analysis and writing. Phenomenological anecdote writing.
RQ2: How do the four assumed dimensions (academic, affective, behavioral, cognitive) of engagement manifest and interact within the accounts of students?	Interviews (1-4) Engagement artifacts (1, 2, 4) Engagement graph Researcher memos	Analysis of participant LEDs. Coding of engagement experiences for the four conceptualized dimensions of engagement. Phenomenological anecdote writing.
RQ3: What is the interaction between student engagement, topics in the Precalculus curriculum, and participants' perception of how these topics are taught?	Classroom observation & field notes Interviews (1-4) Engagement artifacts (1, 2, 4) Engagement graph Researcher memos	Analysis of participant LEDs. Cross-coding of students LEDs of engagement with topics of the Precalculus curriculum. Cross-coding of students LEDs of engagement with students' perceptions of pedagogy. Phenomenological anecdote writing.

First, soliciting LEDs ended up being somewhat complicated. As is stated in research, it is not easy to elicit pre-reflective descriptions of phenomena: most of human interaction involves some level of interpretive processing. However, despite this knowledge and this study's emphasis on the hermeneutical phenomenological approach, it proved to be difficult to capture longer chunks of data that focused on the experience itself. In some sense it only provided another argument for the complexity of the phenomenon of engagement. Students in this study did not experience engagement as blank slates: their experiences of engagement were felt through layers of prior experiences, beliefs, contexts, emotions, etc. Engagement is not a fully independent variable. In fact, it is quite dependent on many factors both expected and sometimes unexpected. The synthesis of data across all participants often seemed beyond reach.

Second, the interviews ended up being much richer in data than I had anticipated. Being open to the phenomenon as it occurs in the data whether it is the interviews, or artifacts, or graphs, or observations can be an overwhelming experience. Each participant's narrative provided what feels like infinite directions for exploration and understanding, so multiplying that sensation times 41 has proven to be difficult and at times felt unfeasible.

Finally, I lost my way in the weeds of specificity, and phenomenological anecdote writing did not come as easily as I had hoped. The goal is to describe the essence of the phenomenon; to write a narrative that makes one immediately connect to the experience of engagement in a mathematics course. However, so much of the data seemed idiosyncratic to the participants and their contexts that the unified story of engagement felt out of reach.

Actual data analysis process

Staying true to phenomenological analysis, wherein one analyzes the data in whole-parts cycles (Dahlberg et al., 2008; van Manen, 2014), the very first cycle of the analysis involved (Fig. 3.11):

- wholistic reading of all interviews from a single participant, while taking notes and identifying salient ideas and wonderings about the phenomenon;
- noting key ideas relative to the phenomenon across interviews;
- reviewing segments of interviews that pertain to the identified key ideas and questions;
- repeat the cycle to revisit ideas that are still not quite fleshed out.

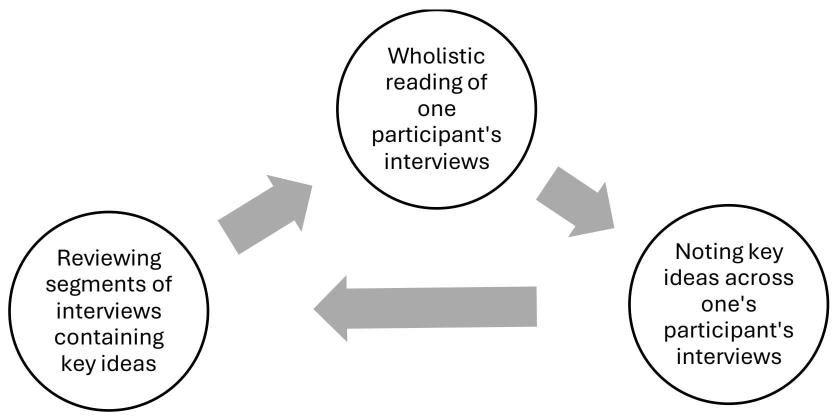

Figure 3.11: Reading and analysis process for participant's interviews

This cycle was repeated for each participant until there were some key ideas that come forth across multiple participants. These key ideas were cross-referenced with the ideas identified during preliminary data analysis and the participant interview member-checking segments (see end of this Chapter). After the general ideas (or as Dahlberg calls them, *patterns of meanings*) were noted, I moved on to selective reading, wherein I went through each participant's interview transcript and recording together with their produced engagement artifacts and graph sketches. During this time certain aspects came to the forefront and were considered in the subsequent rounds of data analysis.

Engagement profiles. At some point when the rounds of data reading and analysis were reaching a level of saturation, I switched to the process of producing each participant's engagement profile (Fig. 3.12). These were written as phenomenological descriptions of each participant's unique experiences of the phenomenon. The intention was to produce a unified written narrative (as opposed to a simple bullet list of commonalities): a phenomenological anecdote that would capture the essence of engagement in UGA Precalculus.

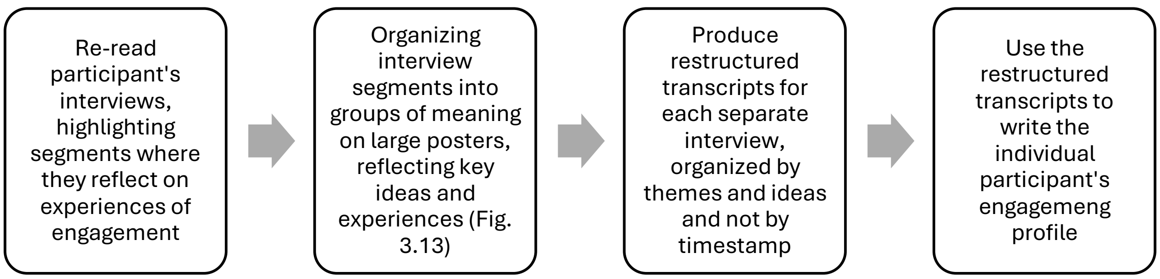

Figure 3.12: Producing participants' engagement profiles

During this process it became abundantly clear that certain aspects of students' lived experiences of engagement varied too much between the participants on a surface level, which did not allow an immediate production of that desired unified narrative that would be applicable and reflective of the spectrum of engagement stories shared by all 11 participants. Although the narratives in the individual engagement profiles were not the same across the participants, there were certain common aspects of their experiences. But how does one convey that a certain type of engagement is happening while acknowledging the spectrum of experiences rooted in each participants' own lifeworld? This is where diagrams came into play.

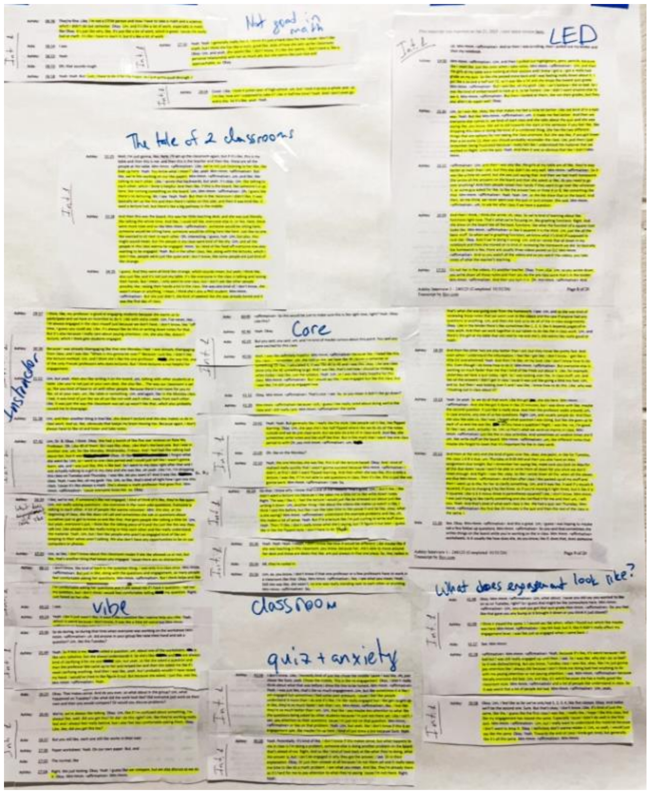

Figure 3.13: Ashley's Interview 1 poster

Engagement diagrams. Initially I was attempting to create individual engagement profile diagrams for each participant indicating the complexity of each participants' lived experiences (Fig. 3.14). However, at the end of the day this was not aligning with the intent of describing the essence of the phenomenon. I was committing the ultimate phenomenological sin: focusing too much on each individual case rather than the experience and essence of the phenomenon itself. This was no longer a phenomenological diagram; it was now turning into a diagram in a case study.

Instead, I shifted the process of diagraming to reflect the common experiences in a way that could provide the range and nuance of lived experiences. Figure 3.15 contains a conceptual diagram draft in progress, which was part of the writing process. The purpose of arrows and blocks will be explained in more detail in Chapters 4 and 5. But as one can see from the shared drafts of diagrams, the complexity of the phenomenon played a big role in the difficulties of writing and to some extent analysis as well.

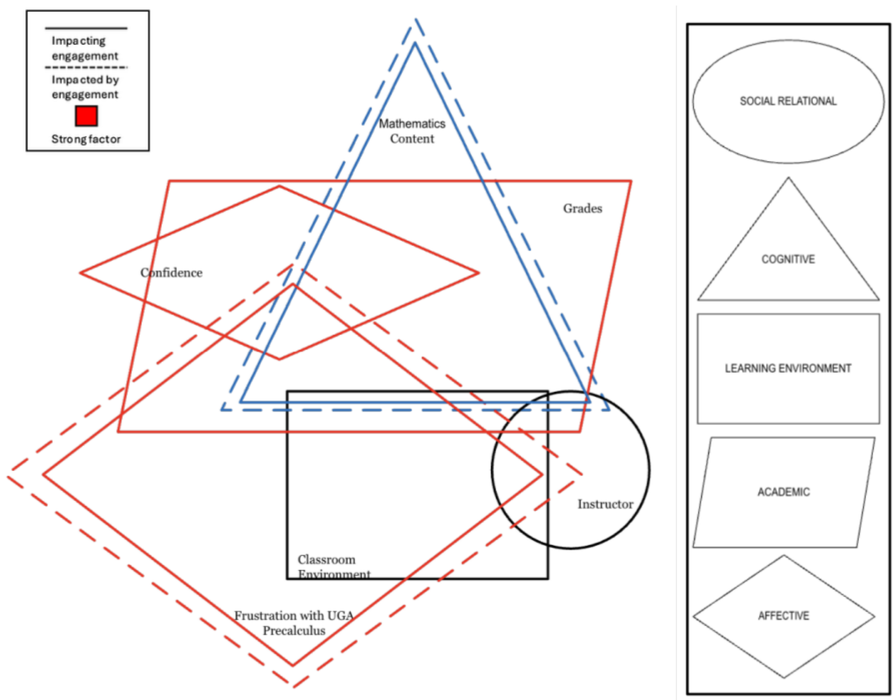

Figure 3.14: Beth’s draft engagement diagram

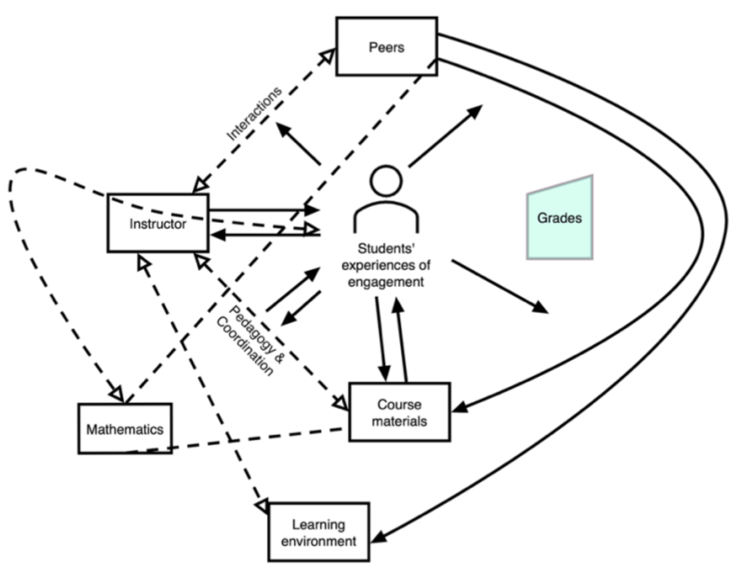

Figure 3.15: The messy process of diagraming engagement

The final iteration of the diagrams in many ways helped to convey a more complete picture of what engagement looked like for the participating students throughout the semester in their Precalculus course, while attending to the complexities of the phenomenon. Moreover, these final diagrams connected to some of the central themes that were identified through the multiple rounds of phenomenological reading, memoing, and analysis.

Phenomenological Vignettes

As a result of producing individual participant engagement profiles and engagement diagrams, it became clear that although the diagrams were useful to capture lived experiences of engagement during the entire semester, they did not always reflect the journeys and progressions of the phenomenon as the nature of the participants' experiences evolved over the course of time. To reflect the dynamic nature of engagement and stay true to the phenomenological practices of writing, the engagement profiles were used to produce a series of vignettes that captured composite lived experience descriptions not just at one static moment in time, but at several points when a shift in experiences was happening for the participants.

The participants' engagement profiles against the backdrop of their engagement graphs often showed a timeline where certain "chunks" of time reflected similar trends in students' engagement experiences. As a result of analyzing the profiles, the five produced anecdotes reflected the following 5 segments of the semester:

- *Vignette 1: Day 1* reflects the first impressions that students had of the course and their instructors, which often set the tone for the entire semester by having a direct impact on participants' affective engagement.

- *Vignette 2: Finding the Flow* covers the time period before the first test, where all the students were acclimating to the demands of the course, establishing their behavioral engagement, and beginning to cognitively engage with the mathematical content in earnest.
- *Vignette 3: This Was Not What I Expected* focuses on Test 1 and its immediate aftermath, because the first higher-stakes assessment of the semester proved to be a stress test to the students' engagement patterns: testing not only the students' knowledge of functions but also the students' studying approaches and strategies. This was a critical point in the semester for all participants.
- *Vignette 4: Bouncing Back* reflects how participants in the study adjusted their engagement as a direct response to their assessment outcomes and the shifts in course after Test 1.
- *Vignette 5: I Think I Am Getting It* captures the aftermath of students' adjustments to their engagement approaches mentioned in the previous vignette. Not many fundamental aspects of participants' engagement patterns changed much after this point, so this is the last vignette described in the dissertation.

To provide more detail into the process of writing these anecdotes, let us focus on Vignettes 2 and 4. According to the engagement graphs, the first part of the semester included at least one moment of growth in engagement for all the participants in the study, followed by a plateau in engagement as they were finding a workflow that fit them and their learning approach for this course. Students were making sense of the way the course was conducted, how their instructor approached teaching, what the different assignments were like, as well as gauging their level of knowledge of the mathematics content (some of which was at least partially familiar for most of the students from high school). This gradual ramp up and stabilization in engagement was captured in Phenomenological Vignette 2 named "Finding the Flow," reflecting and capturing the narratives of multiple participants during this period of the course.

However, not all parts of the engagement profiles aligned cleanly into a single narrative like the vignette described above. Some segments of the semester showed more dramatic discrepancies between participants' lived experiences of engagement. When creating Vignette 4, I found that students' experiences diverged so much, they could not be consolidated. Instead, I incorporated all three clusters of engagement profiles as exemplified by a participant from each of the categories (Ashley, Beth, and David). In this anecdote the three participants are the hypothetical anecdote author's friends, and the author shares descriptions of their friends' narratives to reflect the spectrum of engagement trends in this moment of the semester.

Research Sub Questions Revisited

As mentioned briefly in the Introductory Chapter, a direct consequence of the data analysis process and the shifts in my understanding of the phenomenon of engagement in Precalculus at UGA, the sub questions for this dissertation had to be adapted in order to reflect the richness of the data and highlight the novel findings that have the potential to impact how the field of mathematics education research views student engagement in mathematics classes.

After the findings of the study started coming together, it became clear that although the initial research questions were useful in designing the study and guiding the entire process of inquiry, the process of data analysis was unfurling a story of engagement that was not merely a catalog of interactions neatly organized by dimensions of engagement and Precalculus topic. The initial questions, which treated engagement as a composite of dimensions and topic interactions, proved insufficient to capture the rich, temporal narratives that were emerging from the engagement profiles and graphs. The creation of the phenomenological vignettes, which trace the experience across the semester, was the methodological response to this discovery, and the research questions needed to be revised to align with this new representation.

Recall that the initial sub questions for this dissertation study were:

RQ1: What is the lived experience of engagement as seen through student descriptions of their engagement in Precalculus across the semester?

RQ2: How do the four proposed dimensions (affective, behavioral, cognitive, academic) of engagement manifest and interact with each other throughout the semester?

RQ3: What is the interaction between student engagement, topics in the Precalculus curriculum, and participants' perception of how these topics are taught?

The lived experience of engagement was not a static entity that was measured at different points in time like a temperature reading. The four dimensions of engagement showed up and interacted with each other, but their manifestations were not groundbreaking findings on their own. Students' engagement also interacted with the topics and how the content was taught as was hypothesized, but these on their own were not necessarily contributing to understanding the core research question: *What is the story of engagement in college Precalculus?*

The stories shared by the participants were all pointing towards a powerful, longitudinal narrative of how engagement in the course unfolds, changes, and how it is experienced and lived through by the students over the course of the semester. As a result of grappling with the data and analysis, here are the revised sub questions:

RQ1*: How does the lived experience of engagement in Precalculus (as seen through student descriptions of their engagement) unfold throughout the semester?

RQ3*: How do elements of the Precalculus course at UGA (e.g., assessments, instruction, and mathematical topics) shape students' experiences of engagement throughout the semester?

Although the four-dimensional framework remained essential for initial analysis, the findings demonstrated that these dimensions are so deeply intertwined and context-dependent that isolating their interactions as a separate question provided a much more superficial representation of findings. The manifestations of these dimensions are instead woven throughout the longitudinal narrative presented in Chapters 4 and 5, as they are in the lived experience itself. Thus, it made the most sense to remove the initial sub question RQ2 to maintain the focus of the dissertation on the wholistic understanding of engagement.

Intermediate Data Analysis Validation

Another important step in the data analysis process occurred while still in the process of data collection. To validate some of the research findings of the study, I incorporated a form of member-checking into the fourth and final interview with the participants. The participants were asked to read a short write up on the experience of the phenomenon produced as a result of limited preliminary data analysis and ask whether they agreed with the way the phenomenon was described. The idea behind the activity is that good phenomenological writing should connect with the reader and evoke feelings of experiencing the phenomenon, as well as eliciting reflection and wonder (van Manen, 2014).

Due to the limited amount of time during the semester, instead of an evocative narrative I limited the writing of preliminary findings to a bullet list of key ideas that have been noted across interviews in the process of transcript reviews (see Appendix F). Despite the brevity of the segment, all participants who reflected on the list expressed a connection to many of the ideas. Some participants noted that separate items were not salient for them in this course during that semester, some participants noted that they would add some ideas they felt were important. Overall, the preliminary findings evoked reflection in participants and conversation during the interview, which achieved the main goal of the activity and provided a form of validation for the findings.

In summary, this study leveraged the hermeneutic phenomenology framework and research methods to develop robust data collection tools, which resulted in a rich set of data. Although the data analysis process has encountered some methodological obstacles, the core issue was in the complexity of the phenomenon and the depth of information present in the collected data. This complexity is reflected in the research findings, resulting in the two following chapters. One provides an overview and description of the students' lived experiences and the other delves into the results of the hermeneutic analysis.

CHAPTER 4

PHENOMENOLOGICAL DESCRIPTION

The participants in this study shared a rich and bountiful tapestry of their lived experiences with engagement in a college Precalculus course. The amount of information captured during all 41 interviews is quite large, so to make sense of the results of data analysis, it is important to first understand the overall picture of the “data.” To achieve this goal, I provide in this chapter a *somewhat* organized overview of these experiences with relevant participant quotes and assigned dimensions of engagement. Many of these accounts are directly shaped by the participants’ own beliefs, attitudes, and prior life experiences. This is why there will be examples that might seem contradictory but are understandable within the participants’ lifeworlds.

This chapter addresses RQ3* by mapping the specific elements of the Precalculus course at UGA—including its assessments, instructional practices, and mathematical topics—and analyzing how they actively shape students' experiences of engagement. This detailed examination of the course ecosystem provides the essential context for understanding the overarching patterns of how engagement unfolds across the semester, which is the focus of Chapter 5. By understanding the landscape students must navigate, we can better appreciate the emergent modes of engagement—reactive and proactive engagement—that they develop in response.

Accompanying the descriptions throughout this chapter will be diagrams depicting participants’ experiences with the phenomenon. The participants and their lifeworld will always be the focal point of these diagrams, represented by a circle with a simple outline of a person. Each new subsection will present new element(s) of the Precalculus course (depicted by rectangles) that the students engaged with throughout the semester; in these depictions the connecting lines will typically represent the phenomenon of engagement, unless stated otherwise. Later on, more complex elements will be added to the diagrams, but they will be explained in the corresponding subsections.

Engaging with the Instructor

As expected, all participants mentioned their instructors at some point in the interviews when sharing their experiences of engagement (Fig. 4.1). In many situations students' engagement in the course involved direct interactions with their instructor (asking questions, answering instructor requests, etc.); there were also many ways that the instructors' actions prompted student engagement (explanation of material, advice, etc.).

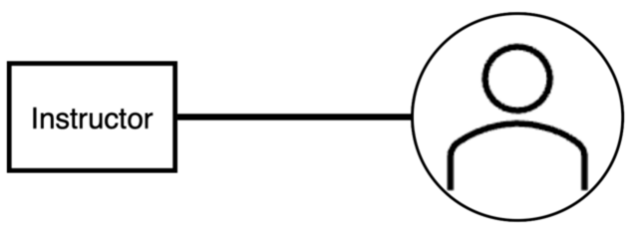

Figure 4.1: Students' experiences of engagement with the course instructor

In some instances, the impact of the instructor on engagement was less direct. For example, some students shared how their own perception of their instructor had an impact on their engagement with the course, or how certain course structure decisions by the instructor (as perceived by the students) shaped how these participants engaged. These indirect impacts will be discussed later in this chapter.

Questions and Answers in Class

Asking questions, getting answers, hearing questions, responding or thinking about responding are integral parts of the learning process. Naturally, the participants in this study often brought up the acts of asking and answering questions as ways that they engaged in the course and got help from the instructor with understanding and/or making sure they truly understand the material. This demonstrates asking/answering as behavioral engagement, driven by their cognitive engagement with the mathematics content.

Participants in the flipped classrooms said that their instructor regularly asked if students had questions, especially about the mathematical content of the pre-recorded videos, the pre-class worksheets (which were due the morning of the lesson), or the material covered in the upcoming quiz. In this setting, they shared that when students started working on in-class worksheets (behavioral and cognitive engagement), their instructors circled around the room and did brief check-ins with groups of students and occasionally with individual students as well. These moments were a common recurring point of direct engagement with their instructor throughout the semester: "[T]he professor walks around in case any one of us has questions and usually people do. … [S]he goes up to each of us and she was like, Ashley, do you have a question?" (Ashley, Interview 1, ~19:19).

The group check-ins sometimes provoked group level conversations. In many cases individual students within the group asked questions that prompted others to listen in, and sometimes this encouraged other students in the group to ask questions as well. Instructors actively asked if other students in the group had questions, soliciting and opening up potential opportunities for engagement:

> Especially when [the instructor] goes to our little groups. [I say,] I need help on this. Ask a question. Or if the girls ask another question, then [I'd say,] okay, to follow up that, blah, blah, blah. Or sometimes I even try to help them. So he's explaining and they're not getting it. [I tell them:] I think this is what he's talking about. (Destiny, Interview 2, ~37:21)

> [A]ll of us [in the group sometimes had the same questions, so we would ask [the instructor]. And I think also even when we have help from each other, we still want to ask the professor for clarification. (Tori, Interview 2, ~12:56)

In fact, some of the participants noted that having the instructor come up to the group alleviated some of the anxiety of raising your hand to ask a question (affective engagement), because the instructor was already there, ready to check in and expecting the people in the group to engage with them:

> [E]ven if you don't ask her anything, she can come up to you and say: How are you doing? Do you understand? And that makes me feel better, 'cause even if I might feel pressured sometimes to ask, I don't have to, 'cause she is gonna come to me. Today she was helping a girl right next to me … and I needed her help right after. So she just came right over to me. (Camila, Interview 1, ~13:10)

> I had the confidence [to ask the instructor] because I also was working in a group. They were asking questions that I knew [the answers to], and so I knew that if I wasn't going to judge them for asking, and I'm helping them, they're not going to judge me for asking another question. (Destiny, Interview 2, ~20:11)

In some instances when participants were not part of a group that worked together during class, they would instead engage with the instructor for checking their work and confirming their solutions:

> I had [the instructor] come over a few times just to make sure I was doing everything right. Recently I've been struggling with mixing up numbers on the graph, so I was trying to make sure I was doing everything. And she did help me because I did mix up a number. (Robin, Interview 1, ~10:09)

On occasion participants shared moments when their instructor redirected certain questions towards the whole class. Participants felt that the instructor may have wanted everyone in the classroom to hear the question and engage with the material (behavioral and cognitive engagement), because they deemed it important enough or wanted to get other students in the class engaged:

> Yesterday she was asking about questions. … I went out to the board, I drew [a graph of a function] out [and asked:] can you do a constant domain, or do you need a union? And she asked the class, and then the class was [saying] no, yes. And she was like: "Yes." And that was probably helpful for other people, because clearly some people thought "No," but it was actually, "Yes". … If I went up to her and I had a question, she would tell the whole class. Most likely [there are] people who wouldn't understand or also had the same question but didn't want to ask. She really wants people to ask a question from the whole class or that the whole class understands. (Ashley, Interview 2, ~51:09)

Moreover, some participants said that their instructor regularly opened up the floor for students to show how they solved a certain problem on the in-class worksheet:

> She'll wait a few minutes and then she'll ask if anybody's done with this the first question. We can come do it on the board. … She'll have the student explain how they got their answers so we can understand it in a way that a student understood it. So, I think that's also pretty helpful. (Robin, Interview 1, ~12:51)
>
> [A]s class went along, to help us she asked students to get up and write their answers on the board once they do [the problem]. Since she thinks that way we'll be hearing solutions in a different perspective, not just her perspective. She's big on that, which is nice. (Camila, Interview 1, ~07:53)

In some instructors' classes this was done as an extra credit opportunity (academic engagement): "Sometimes I'll go up to the board and solve some of the questions because we get extra points on our pre-class assignments if we do that" (Robin, Interview 1, ~10:09). "[Y]ou get extra credit if you raise your hand and do stuff on the board." (Ashley, Interview 1, ~21:56).

These experiences lead one to consider that in addition to students' direct engagement experiences with instructors, there are also indirect engagement experiences with the instructor: for instance, through the teacher's interactions with their class peers (Fig. 4.2). In this diagram the student-participant's engagement (new solid line) was with the interactions occurring between the instructor and their peers (dotted arrow). The reason the arrow is two-sided is because the participants have seen both the instructor and their peers initiating the interactions.

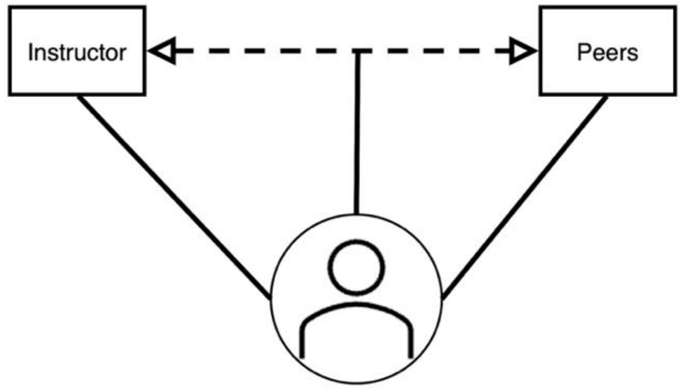

Figure 4.2: Students' experiences of engagement with the instructor, peers, and their interactions

In rare cases these interactions caused negative emotional responses (affective engagement) and sometimes even disrupted students' own cognitive engagement in a task:

> One girl who's in my group, during the test she spent the whole time standing next to the teacher asking him questions, which is fine, but he was standing right next to my desk. … The professor was sitting on my desk in order to talk to the girl, so that was frustrating. I felt like I couldn't really focus. (Beth, Interview 2, ~10:30)

> Sometimes it is like I am engaged, but sometimes I feel some peer pressure. 'Cause I feel like people understand it more than I do. And I'm really engaged, but I get caught up in [thinking] they're so much faster than I am. I feel like they're so much better than I am. That I pay maybe less attention to what the questions being asked by other students [are], because I'm just not there yet. I can't pay attention to their questions 'cause I'm just not on that question or on that problem. (Ashley, Interview 1, ~43:59)

There were also instances when participants shared moments of disengagement with their instructor when it came to asking and/or answering questions. One of the participants, Destiny, shared at the beginning of the semester that she wanted to put work into a task (cognitive engagement) before asking the instructor for help (behavioral engagement). She noted that not asking the instructor more questions lowered her overall engagement in the course at that moment in time. But she grew more comfortable and confident over the course of the semester (affective engagement):

> I'll ask if I really need help with something. I'll struggle a little bit. That's a fatal flaw of mine, but I'll struggle a little bit and see if I can work through that struggle. But if I don't, I'll give up and be like, Hey, what'd you get for this? (Destiny, Interview 1, ~31:46)

Another participant, Beth, shared that she noticed early in the semester that her instructor would often give a longer answer to what she perceived was a short and simple question. Her main purpose for engaging the instructor with a question was to check her answer to a problem on the worksheet and her expectation was that the instructor would provide a brief "Yes" or "No" answer. In our second interview I asked Beth how often she asked the instructor questions in class and she said:

> Not very often. And when I do, I'll usually just ask him to check something for me. So it's a quick 30 second question and he'll turn it into a five-minute explanation of what I already know. Just like, can you check this, make sure I get this right? And then he goes on to explain everything. So I feel like just a lot of time is being wasted. (Beth, Interview 2, ~14:17)

Here we see that Beth's cognitive engagement was eclipsed by her growing negative emotions have led to both behavioral and cognitive disengagement.

How students asked and answered questions looked a bit different in the lecture-based sections of Precalculus. Most of the questions from the instructor were integrated into the lecture through some practice problems and involved asking for students' input on the next step in a solution. Both Gloria and David noted that few people raised their hands and volunteered during this point in the lesson, demonstrating overall lower behavioral engagement:

> [The instructor] gave us a second to work through it on our own, try to figure it out, and she'd ask questions. Do you know how to do the next step? And then she eventually goes through it with us. … when she's going over it, she asks us if we know how to start and asks for volunteers. And then normally nobody [responds], I think everyone's shy. (Gloria, Interview 1, ~14:13)
>
> No one really raises hands. The class is pretty much: she asks something, the first person to respond she let answer them. (David, Interview 1, ~17:48)

Because other students in the class were not surveyed, we can not make the assumption that not raising their hands as absence of behavioral engagement necessarily implies the absence of cognitive engagement. Moreover, as Gloria hypothesized this might be due to feelings of anxiety or fear, which is related to the affective engagement of the students in the classroom.

In contrast to how students in the flipped classroom were sometimes asked to come up to the board and show and explain their work to the class, participants did not mention this happening in their lecture-based class. Instead, students would “explain it at their desk” (Gloria, Int 1, ~15:23).

Course materials and content

Participants brought up engaging with the course and its mathematics content not just through the instructor, but also through the various course materials, which were often viewed as closely connected to and/or shaped by their instructors. The way students interacted with the content and materials depended on the modality of the course (flipped vs lecture section), because within the flipped classroom sections most of the course materials were engaged outside the classroom (videos, pre-class worksheets, practice tests). In fact, instructors’ in-class teaching was intended to supplement the content presented in the pre-class videos and instead focus on students’ groupwork on in-class worksheets, as well as conducting in-person assessments (quizzes and tests).

Videos and pre-class worksheets

The assigned videos in all flipped sections of Precalculus were pre-recorded by a single instructor; for some of the participants the person teaching them in class and the person in the video were not the same:

> [T]here are usually three videos posted on eLC. And so you watch all the videos and as you watch the videos, you take notes of what the teacher's teaching. It's not [my instructor] in the videos, it's another teacher from UGA. (Ashley, Interview 1, ~16:59)
>
> With our flipped classroom he gives us another professor's videos. I forgot her name, but it's the same professor. (Destiny, Interview 1, ~27:17)

When the participants' instructor was different from the instructor in the video, some students felt like they preferred their own teacher's explanations, but some liked the way the teacher in the video explained the content. Either way they all mentioned some benefit of seeing a different way of solving problems:

> We have videos for our pre-class assignments, and it's by a different professor, and that's uploaded. And then when [our instructor] comes in, he does [the problems]. And he was like: "You can definitely do it [the way it is in] the video if you like that better. But this is how I choose to do it." And I feel like it's easier, but also, he gives us options of how we can solve it. (Destiny, Interview 2, ~14:38)

> And I do think the videos are pretty engaging. Not [my instructor] that's doing them, it's this one person. They're all the same for every topic we do, which I think is also a pretty good thing because I like the way she teaches. I like the way the teacher in the video teaches, so I take the notes the way she does, and then I go on to do my homework. (Robin, Interview 1, ~23:50)

Here the students' behavioral engagement (watching the videos) is leading to cognitive engagement with the mathematics content (making sense of the concepts). Students' affective engagement is also somewhat impacting their cognitive engagement, because of students' attitudes towards the way the same material is presented to them by a different educator. A lack of familiarity can lead some to experience enhanced cognitive engagement (it is helpful to see the material presented a different way) or lead to feelings of frustration throughout their engagement (wishing it was their instructor in the video).

Participants shared a wide variety of ways they engaged with those videos, and a lot of those interactions worked in tandem with the pre-class worksheets. Some students dedicated separate time to viewing the videos and taking notes, before they would work on the pre-class assignment:

> … the videos, I just sit down and I'm going to take the notes. And then I try to take them as best as possible, so I don't have to go back to the videos when I'm doing the pre-class assignment. (Robin, Interview 1, ~23:50)
>
> [W]hen I do the homework, and I look at the videos, I obviously am able to take my time at like really pausing, understanding, doing the work, looking back in my notes. (Ashley, Interview 1, ~23:39)

Some students viewed the videos as they were working on the pre-class worksheets:

> [W]hat I like to do is while [watching] the video, I like to do my homework assignment and kind of do the two at the same time. … I'm watching the video, getting the information, and then doing the worksheet right after I know the principle of how to do the problem. (Jordan, Interview 1, ~25:21)
>
> [I]f I'm stuck on a [pre-class worksheet] problem or I'm not a hundred percent confident, I'll go and watch a video and try to find some problem that she's doing that's similar. And then I go and I'm like, okay, I see how she's doing that. (Destiny, Interview 1, ~28:36)

One of the participants—Beth—stopped engaging with the videos at some point in the semester, because she did not perceive them as helpful:

> I tried to watch the videos to learn, but I just felt like it really slowed me down and wasn't helpful. (Interview 3, ~07:50)

> [I stopped watching the videos] pretty early on. I started to a little at first; I noticed they were very slow. They weren't done by our teacher at first. So there was a little bit of disconnect. The teacher since started making them, but I haven't even watched one. For the first test, I watched them while I reviewed, and I didn't do very well on that test. So after the first test, I was like, this is not going to help me at all. And I was right by getting a 96 on the second test. (Interview 4, ~07:45)

As we can see, students in the flipped classroom sections engaged with the videos in a variety of ways (a spectrum of behavioral engagement), but all of them considered the videos in the context of their own instructor's teaching (Fig. 4.3). Some preferred their own instructor's explanations, some preferred the video instructor's explanation, some appreciated the different perspectives. But all of them (save for Beth) viewed these videos as helpful specifically to learn and master the mathematics material of the course, which aligns with the purpose of the videos within the flipped classroom model.

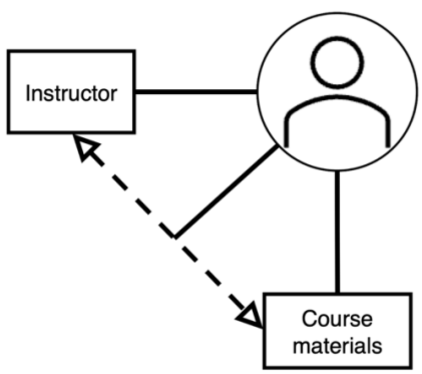

Figure 4.3: Students' experiences of engagement with the instructor, course materials, and their interaction

Students also acknowledged the importance of engaging with the covered content during their in-class sessions with their instructors. Explanations in the flipped classroom sessions were less common or would take much less time at the whole class level than they would in a lecture-based class:

> I would say a third of the class is just [the instructor] going over questions from the pre-class. And then … she sort of reteaches it in 5-10 minutes. She reteaches what was 20-30 minutes so we can refresh it. I think that's also the point of the flipped classroom, at least how she does it. That we already know of the concept, but we'll still probably be a bit confused, unsteady. So we already have experience going into class rather than we're already in class and we don't know what's going on immediately. So I appreciate that format. (Camila, Interview 2, ~35:08)

ALEKS

Videos and pre-class worksheets were not the only course materials that provided students with opportunities to learn and practice the content. Another part of the course that students had to work on was the ALEKS platform, as described in Chapter 3. However, although it was only one of multiple types of homework assignments for the students in the flipped classrooms, it was the only homework that students in the lecture sections had assigned to them ("All her homework is strictly on ALEKS." David, Interview 1, ~14:14). The participants in the study expressed a spectrum of attitudes towards ALEKS. Some positive:

> … I like [ALEKS] better than the book. It shows you a work[ed]-through [problem] to understand it. So you can tell where you went wrong. So it'll tell me every time. So you look through how it works and you say, okay, I missed a negative. Okay, I missed this multiplication. It tells exactly where I missed it. (David, Interview 1, ~38:41)

> I like how [in ALEKS] it's broken down first and I can basically just copy what I see. I think I can learn better by seeing exactly how they do a similar problem and then I can instantly transfer that to a different problem. (Beth, Interview 1, ~24:36)

> I feel like the ALEKS has helped with [trigonometry], 'cause it's prompting you to know [the trigonometric identities], and know the reciprocals, and the Pythagorean theorem … It just makes you practice more than what the pre-class did. … redoing similar problems and getting it into your brain. So I feel like that helped me on this one. … as annoying as the ALEKS is to do, it's helpful in just practicing what you're doing. (Maggie, Interview 4, ~4:28)

Some much less positive:

> ALEKS explanations are not really that good. You can see it, it gives an example, which is kind of good now, but it's mainly you're [doing] trial and error. If you get it wrong, you have to start over all the way again. So it's definitely about like repetition. But it's good that it's repetition, 'cause I need that repetition. (Destiny, Interview 3, ~6:24)

> … there's practice there, but ALEKS only asks you two or three questions per topic. So the only way I'm actually able to really practice is getting it wrong. I don't know. I don't love ALEKS, but it's also not terrible. … I don't think anyone really loves it. (Ashley, Interview 2, ~25:05)

One of the participants—Tori—noted the time-consuming nature of the platform:

> Yes. I'd say that's the most time consuming thing. It takes up a lot of time. I think this week's I probably spent without distraction, without breaks. I probably spent 8 hours on it total on this week's ALEKS. (Tori, Interview 1, ~32:26)
>
> I feel like doing additional practice problems or spending time going to office hours would help me much more than the ALEKS assignments, which is really frustrating because I need to devote so much time to those, but I feel like those aren't benefiting me as much as something else could, so it's not a better use of my time. (Tori, Interview 2, ~25:38)

ALEKS was also occasionally used as a review tool for quizzes and tests:

> When you go into ALEKS, there's a review section. And it'll just take a concept and ask you a question from that concept. (David, Interview 2, ~8:52)
>
> [ALEKS] helped me figure out what I was weak at. Because I think I had to do like five questions to do [a concept] and I kept on getting 'em wrong and I just like did not understand at all. (Destiny, Interview 3, ~47:01)

Although some students found some benefit to deeper cognitive engagement with the ALEKS platform, many students talked about this element of the course as a time-consuming chore that must be done (behavioral and academic engagement).

Relating back to the students' engagement not just with the course materials and the instructor but the interaction between the two (Fig. 4.3), one of the participants mentioned that her instructor was not fond of the platform: "He hates it. He doesn't [like ALEKS]. He was like: If I could teach my class how I wanted, y'all would not be doing ALEKS. But he was like: it is what it is" (Destiny, Interview 1, ~17:23).

Textbook

Finally, unlike their flipped classroom counterparts, the students in the lecture sections mentioned their Precalculus textbook as another course material that they engaged with. However, the participants did not engage with the book consistently throughout the semester (neither cognitively, nor behavioraly). David found the practice problems in the book not as helpful as the problems on ALEKS:

> We have a book, but it's an issue with the book itself. It's a little confusing at times, I feel compared to the ALEKS website. I've looked at it and some of my other classmates have looked at it. There's practice problems in there, but there's just so many and then they're not really… it's not exactly the most helpful, the ones in the book. (David, Interview 1, ~19:43)

While Gloria enjoyed reading and taking notes from the textbook (affective engagement), over the course of the semester she tapered off her note taking because it took too much time. She also found the textbook practice problems helpful in the beginning, but that also waned towards the end of the semester, because she did not find them as helpful in preparing for course tests (cognitive engagement):

> Well, we're supposed to read the textbook before class each time. So what I normally do is I'll take notes while I'm reading the textbook. (Gloria, Interview 1, ~24:57)
>
> So I've kind of stopped taking notes exactly from the textbook just because it's very time consuming. It is nice and it is helpful to a degree when I get to class, but I don't think [it's worth] all that extra time. (Gloria, Interview 2, ~18:29)
>
> [Preparing for the test] I didn't check the textbook this time as much. (Gloria, Interview 3, ~37:10)

Notably all UGA Precalculus students have access to the course textbook built into their ALEKS platform interface and the course website on the mathematics department references this specific course textbook. Yet not a single participant from the flipped classroom mentioned hearing about or using the textbook.

Peers and in-class collaboration

The in-class worksheets were a major way that participants in the flipped sections of Precalculus engaged with the mathematics content and their instructor, but how they went about solving those depended on each student's particular context. Those that worked in a group with peers who interacted with one another experienced more engagement not just with the mathematics tasks, but also with their peers, and sometimes even the instructor.

> We're always talking. If I'm confused about something, I'm always like, wait, did you get this? Or did I do this right? They're working really fast [on the worksheet] and I always feel really behind, but I also feel comfortable asking them. (Ashley, Interview 1, ~26:41)

> …if [the instructor is] explaining a problem to [another] group and it's the same [worksheet] problem I'm on, I'll listen in, because I can hear it and I can watch. Even though it's not directly to me, I'll listen in because I'm not going to ask him the same question that he just did. (Maggie, Interview 1, ~21:53)

Figure 4.4 builds on previous diagrams (Figs. 4.2 and 4.3), expanding the engagement captured in the data showing not just engagement with separate aspects of the course (instructor, peers, course materials), but also the engagement with the complex interactions between these separate parts. Indeed, the data of this study shows that engagement is most often complex and ties together different aspects of the learning environment.

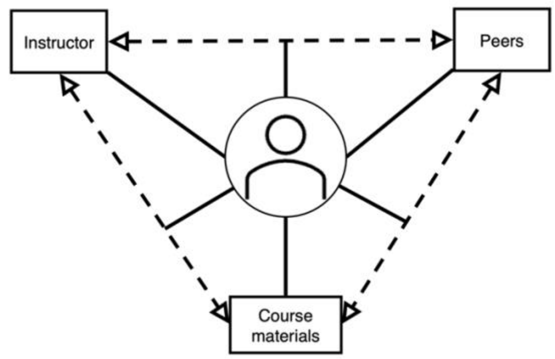

Figure 4.4: Students' experiences of engagement with the instructor, peers, course materials and the interactions between all three.

However, not all students had positive experiences of engagement with the peers in their groups:

> My table is pretty quiet, so we don't really work with each other, but I do see the other students. There's this one group of guys: they work together on every problem. So I do think [group work] does work. I just don't know if it works with everybody. I do find working with others very helpful, but I think my table doesn't really like working with others. (Robin, Interview 1, ~21:04)

> I know that me and my group members don't talk a lot simply because if … [my tablemate asks] I'll just hand her my worksheet—which is usually already done—and be like: yeah, this is what I did. So, there's not a lot of working and collaborating and I'd say learning together, at least not from what I can say. (Beth, Interview 1, ~19:51)

For some participants in-class worksheets ended up being more of a solo endeavor, so the engagement with other peers' work on course materials was not as strong as it was for others. This can be represented by a paler connection in Figure 4.5 compared to Fig. 4.4. Although these types of engagement were not as strong for students like Beth and Robin, it was still present. Sometimes they presented more as passive interactions with other students' work on course materials (closer to behavioral rather than cognitive engagement).

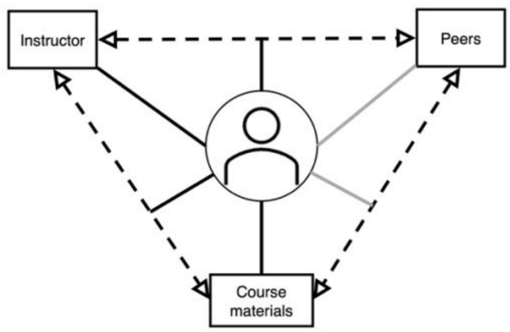

Figure 4.5: Weaker engagement with peers and their interaction with course materials

Finally for the two participants in lecture-based classrooms, there was next to no interaction between peers during class and outside of class, especially when compared to their flipped classroom peers:

> We don't have time to talk with each other. That's not happened in the class, but in little side moments, if [the instructor] writing down something. (Gloria, Interview 1, ~30:04)

Neither David nor Gloria mentioned study groups or GroupMe[2] for their Precalculus class, so the engagement diagram for them so far might look more like Fig. 4.6 rather than Fig. 4.4. The grey lines demonstrating engagement with the instructor-

[2] A messaging platform used by many college students across the United States to communicate with instructors and fellow students.

peers and instructor-course materials interactions denote a weaker form of engagement, as we've witnessed in previous sections.

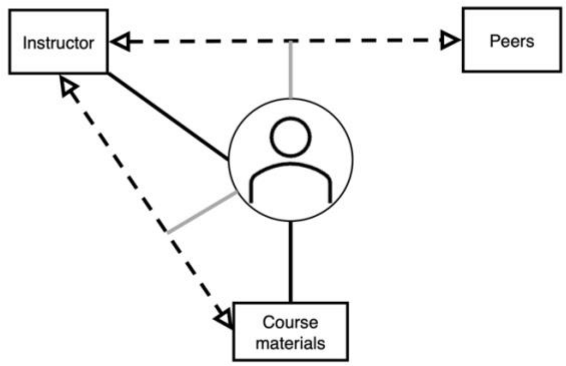

Figure 4.6: Lecture section students' engagement with the instructor, peers, course materials, and their instructors' interaction with the two components

Engaging across course materials and assessments

As discussed in previous subsections, there were many instances of students engaging not just with each course material, but with the interactions between the materials as well (Fig. 4.7). For example, we've already learned about how students engaged with the connections between videos and their pre-class worksheets. We've heard from participants about their engagement with ALEKS as complementary to their learning from other course materials. The in-class worksheets were also engaged with in the context of students' understanding of pre-class videos and worksheets.

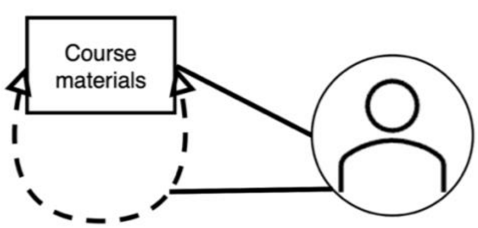

Figure 4.7: Student engagement with the course materials and the interaction between the different course materials

This is captured in detail in one fascinating instance, when during one of the interviews Ashley shared a lived experience description of attending a class without having seen the corresponding videos or doing all the assigned pre-class worksheets, because she did not see them in the course learning system:

> I just thought there was a pre-class assignment I didn't do … I tried to do the work, but I didn't have notes, so it was hard to understand. … I was just kind of annoyed I guess, because I wish I had seen it. … it's not like that half a class is just wasted or anything like that. But I wish I had the practice before, 'cause then I feel like I could really use the time now to practice rather than learn it. (Ashley, Interview 4, ~15:15)

This description is fascinating for two reasons: on one hand it is a clear instance of variation in the phenomenon, where Ashley's engagement was altered by a single and clearly defined change in her regular Precalculus learning routine. I got to capture a rare instance where a participant could communicate the nuanced shifts in her affective and cognitive engagement during class in response to not completing a pre-class assignment (a mix of behavioral and academic engagement).

On the other hand, this lived experience description further demonstrates that the course materials are not engaged with separately in a vacuum; instead, students engage with the complex interactions between different course materials (Fig. 4.8). Furthermore, students actively engage with the mathematical content as it connects across the different course materials (Fig. 4.9).

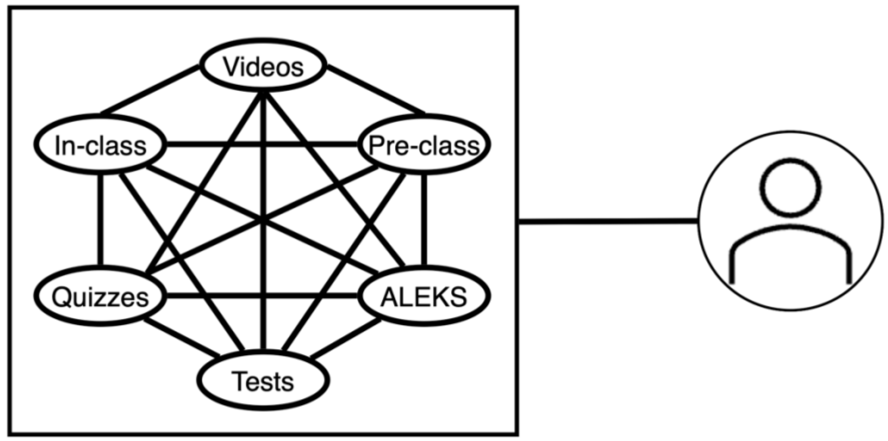

Figure 4.8: Students' experiences of engagement with the complex interactions between different course materials

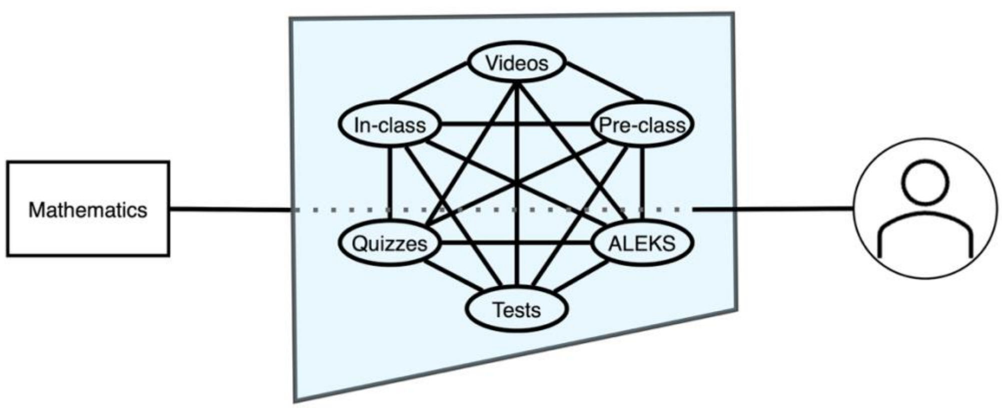

Figure 4.9: Students' experiences of engagement with the mathematics content through their engagement with the course materials

From this perspective, students' engagement with various forms of course assessments helped them and their instructors understand students' understanding of the content (cognitive engagement). The participants in both the flipped classroom and lecture-based sections shared a great deal about their experiences with MATH 1113 course assessments, especially as these assessments related to other course materials.

Quizzes and Tests

Quizzes were a frequent type of assessment that students engaged with on an almost weekly basis. These were usually administered once a week during class time either at the beginning or at the end of the class for 15-20 minutes (aside from that time the instructors decided to conduct a group quiz that lasted an entire class period). As for tests, there were a total of three midterm tests that were also administered during class time.

Almost all the participants shared increasing their behavioral and cognitive engagement levels leading up to these assessments:

> Whenever there is a quiz or a test, I feel like that should be a[n] [engagement] peak like this … [regular assignments are] not like a quiz or a test where I have to [have] my engagement at all time high. (Jordan, Interview 1, ~29:47)
>
> That was the peak [engagement] then: the 10 out of 10 quiz. I studied for that really well. (Penelope, Interview 2, ~28:47)
>
> My overall engagement did not change … I'm trying to illustrate [that] my engagement specifically in the quizzes changed, not my engagement in the class. (Tori, Interview 1, ~41:09)

In the last quote above, one of the participants—Tori—saw her engagement specifically for quizzes as being separate and more variable compared to engagement with the rest of the course. Tori felt strongly about this distinction, leading her to draw a separate engagement graph for quizzes vs in-class and video/video notes engagement (Fig. 4.10).

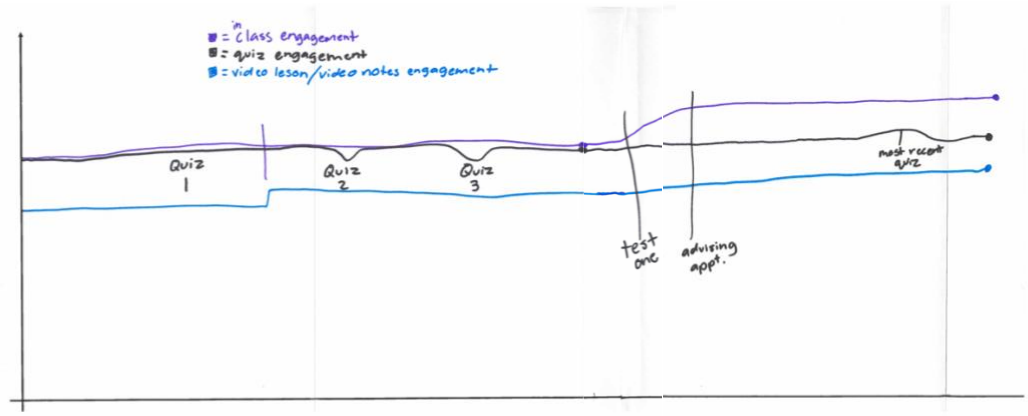

Figure 4.10: Tori's engagement graph depicting her quiz engagement as separate from other experiences of engagement

In a surprising turn of events however, instead of drawing peaks like some other students, Tori depicted "dips" for Quizzes 2 and 3, because: "I studied for this quiz more than I did for these two. And the reason for that is because the first quiz, we did not have an ALEKS assignment due at midnight the night before, and these two we did." (Tori, Interview 1, ~41:09). This further supports the diagrammed complexity of students' engagement with the different forms of course materials simultaneously (Figs. 4.7-4.9).

The participants in this study expressed a great deal of emotions when discussing their engagement with the quizzes and tests, hinting at experiences of affective engagement:

> I knew I got a really bad grade on my quiz. So [the instructor] passed mine back and I was feeling really down about it. I got like a 6.5 out 12, so it was like a 54 and she drops the lowest quiz grade. But I was like, oh my gosh. I can't believe I did so bad. So I was kind of embarrassed to look at it, to be honest. I didn't want anyone else to see it. (Ashley, Interview 1, ~14:50)

> The first quiz was I wasn't as prepared for. The second one felt much better. I got a better grade on that one. (Robin, Interview 1, ~34:37)

One of the participant's experiences with engagement were always intertwined with her affective engagement in the course. Camila explicitly shared how her cognitive engagement increased or decreased based on her emotions about her experiences in the class:

> … the first quiz, when that came around, maybe [my engagement] went down a little bit. Not 'cause I wasn't- like not paying attention or anything. But I felt a little frustrated 'cause I got a little bit of a lower grade than I wanted to on the quiz. (Camila, Interview 1, ~34:30)
>
> …seeing that I did well on the [second] quiz [made my engagement increase]. I think I got a 100 on it. … By studying a little bit, I can do well and feel good about it. It just made me wanna keep on working hard. So I think that's what made me feel happier. (Camila, Interview 1, ~40:11)

This connection between engagement and affect was true for Camila both in relation to assessments and to the overall course as well. During one of the interviews, when asked how tightly her engagement was interwoven with her emotions, she pretty much said they are inseparable for her.

Some of the emotional aspects changed throughout the semester both for quizzes and tests, as well as the ways students engaged with them over time. All the participants shared how the first assessments of any kind were more stressful than the subsequent ones. Some attributed this to level of preparation, some to becoming more familiar with the format of the assessment, and some mentioned both:

> I didn't realize how the quizzes were going to be. I didn't study as much as I thought I should have [for] the first quiz. And then the second quiz, I did study more. So I think I did better because I understood the concepts and how we were going to actually take the quiz. (Robin, Interview 1, ~35:38)

> I think I felt confident [the first] test and then I opened it and I was like, this isn't what I thought it was gonna be. So then for [the second] test I did a lot more studying. I practiced the study guides, the practice test, did all the work on ALEKS, … the pre-class assignments, asked [instructor] questions during class, would go to his office hours after class. I just prepared for it for a lot, a lot more than I did for Test 1. (Jordan, Interview 3, ~24:11)

Jordan's experience was a common one among the participants, wherein students adapted their behavioral and cognitive engagement in preparation for a quiz or a test after a negative experience with the previous assessment.

Among the different emotions there was also a great deal of frustration with the students' perceived misalignment between types of questions and levels of difficulty across different kinds of course materials. For instance, many felt like the quiz and test questions were significantly harder (or at least significantly different) than the textbook, ALEKS, and the pre-class and in-class worksheets:

> [S]he does put some harder questions on the quiz. … the quizzes are different variations of the questions that are in the in class and pre-class work. (Ashley, Interview 1, ~23:39)

… the example she gave [during lecture] are pretty simple. … But then she asked a really confusing worded one [on the] quiz. … It's different than any example she does [in class]. (David, Interview 1, ~33:09)

I can do an example problem, like an inverse function perfectly fine. But then she wants weird examples on the quizzes and stuff. (David, Interview 2, ~21:18)

I felt like I understood better [doing math on ALEKS], but then she asked different stuff on quiz and tests. (David Interview 3, ~31:41)

There's technically a lot of practice in the [worksheet packet] book that everyone has to get because there's in class and then additional practice, but the questions get so hard, they're not realistic for the quiz, so I can't really practice with those. (Ashley, Interview 2, ~23:02)

Participants also noted the differences between quiz and test questions, especially in the context of how to gauge their grasp of the material (metacognitive aspect of cognitive engagement):

The quizzes I feel are just a little easier [than the test]. When it came to the test it was like: oh my goodness, what is this? I dunno. Maybe it's the way they word it. … I'd say that the textbook questions and the quizzes are kind of more similar to each other. (Gloria, Interview 2, ~04:54)

The test was much harder, much harder than the quizzes. Usually, the quizzes went over one tiny concept: today's quiz was just over 3.1. We went over a lot of things in 3.1, and it just gave me one problem: find the inverse of this equation, and that was it. I know that the test is going to be much harder because the 3.1 [unit] in class was much harder. So I mean, I feel like the quizzes are a little useless right now. I mean, I could do this one problem, but I'm probably not prepared for the test of 3.1. (Beth, Interview 2, ~24:32)

Some of these tensions led participants to question the usefulness of the quizzes in the overall scheme of the class. This clearly has been noticed by the instructors. During one of the interviews, Beth received an email from her teacher, soliciting feedback on the quiz format. She read the email out loud in our interview:

> Hello, class. Over the next five days, I'd like to spend some time reevaluating the way in which we conduct our quizzes. In particular, I have noticed that these assignments appear to be causing even more stress than the tests, and I would very much like to make them a more pleasant or at least tolerable part of your classroom experience. To that end, I'm soliciting your opinions. To be clear, this is not a vote or anything, but a way for me to gather sentiments across the board to better inform how I can improve this class. If you are able, please fill out the following 5 question multiple-choice survey. (Beth, Interview 2, ~39:01)

Other participants shared similar responses to the survey:

> [S]o [the instructor] kind of asked us: do you want the quizzes to be more like the tests or the workbook that they give us, that we have to buy. And I think we all were kind of like: oh, we should take it from the previous tests. (Maggie, Interview 2, ~11:22)

Furthermore, Maggie had a hypothesis for the reason behind the survey, as well as a very pragmatic response for the potential changes to the quiz format:

> I think why he changed it was because the test averages were lower than he expected. … the quizzes had never been that hard to me personally, but maybe

with a change they might be a little bit harder. But I'm fine with that, considering if it's the questions that we would expect to go on the test. (Maggie, Interview 2, ~20:08)

Grades

Figure 4.9 illustrates participants' engagement with the mathematics content occurring through students' engagement with the course materials. But while conducting rounds of data analysis, I noticed an additional aspect of the phenomenon: students' experiences of engagement were often mediated by the grades they got in the course in general, but especially on the quizzes and tests:

> I feel better when I do better on quizzes and tests, and it's like I see my work that I've put into the class come out in my grades. (Maggie, Interview 1, ~41:25)

> The three quizzes we've had for this section, I've done well. So I hope... that translates to the test too. (Ashley, Interview 4, ~2:17)

Although it is common for educators to talk about the role of formative assessment in giving feedback about student learning and their own teaching, it was interesting to see how the study participants talked about adapting their engagement using grades as feedback. This point seems related to the idea of comfort with the content: for students their grades are an indicator of sorts whether they grasp the content or not. This informed their next steps: lower grades signaled that they needed to do more, pay more attention; higher grades signaled that they have some level of understanding and thus can shift their energy towards engaging in other things. This was also highly related to the students' busy college schedules, when they are juggling multiple demands on their time and energy towards engagement. To highlight how this mediation played out we will consider simultaneously two examples where grades informed students' engagement. In the lead up to Test 1, Ashley and Beth had diametrically opposing problems.

Ashley was not doing well on quizzes in the class so far. In fact, her first quiz she got a 6.5 out of 12 and the instructor told her class that anyone who got a 6 or below on the quiz should consider withdrawing from Precalculus or enrolling in a support class. Ashley's ongoing struggles with quizzes impacted how she felt about her understanding of the material. She even shared that she thought she knew the content well enough before the first quiz, but the grade proved otherwise. These struggles led Ashley to increased engagement and a lot of preparation for the first test of the semester, resulting in her getting a 90.5 out of 100 on it. The highest point in the entire engagement graph for Ashley occurred in preparation for Test 1.

Beth had quite the opposite situation compared to Ashley. She felt confident in her knowledge of the material, averaging 95% on her quizzes. Moreover, she would often be done with the quizzes within only a couple of minutes, and as illustrated previously, Beth was not as engaged with the course videos or group work as her peers. Thus, she was not as engaged in the course leading up to Test 1, on which Beth earned a 77 out of 100.

Building on Fig. 4.9, we would not remove the line of engagement with the mathematics through course materials, because these occurred even without relying on grades. Instead, there were additional "academic-hued" instances of engagement (depicted in red in Figure 4.11), where students used grades as an additional lens to assess their own knowledge of the mathematics. So in the example discussed previously, Ashley's engagement with the grades informed more cognitive and behavioral engagement. She believed poor quiz performance was indicating her unsatisfactory engagement with the material. Beth, in contrast, used the grades to decrease her engagement (not by a lot; she still prepared for the test), which to an extent involved a false sense of comfort and safety that were not supported by her grade on Test 1.

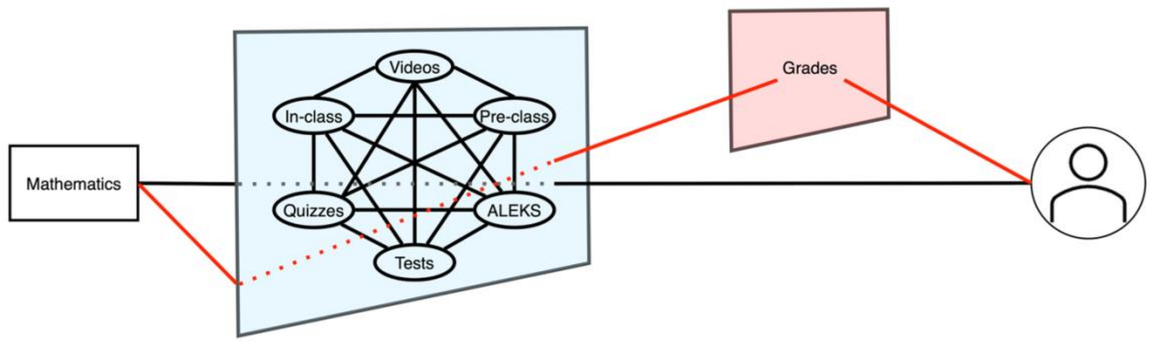

Figure 4.11: In addition to engaging with mathematics through course materials, students' engagement was mediated by the grades that they got on their assessments.

Summary

This chapter has provided a panoramic view of the complex, multi-faceted tapestry of engagement as experienced by students in a college Precalculus course. Through their own words, we have seen that engagement is not a singular state but a dynamic phenomenon, constantly shaped and reshaped by interactions with instructors, peers, course materials, and assessments. These descriptions reveal a system of interconnected experiences, where a change in one element—like a missed pre-class video—ripples through a student's entire engagement ecosystem.

Additionally, participants' lived experience descriptions have demonstrated the interwovenness of the four dimensions of engagement. As demonstrated in this chapter, almost every single instance of engagement shared by the students included at least two dimensions occurring either simultaneously, in sequence, or in a complex relationship with each other. As I hypothesized, the academic dimension of engagement was present, but wasn't always at the forefront of students' immediate engagement experiences. Rather, it often played the role of a motivator, which will be explored in the next chapter.

In answer to RQ3*, the analysis in this chapter demonstrates that course elements are not a neutral backdrop but powerful forces that structure and constrain student engagement. The data show that students do not respond to these elements in a uniform way. Instead, they develop characteristic modes of interacting with this ecosystem. As the next chapter will reveal, the most salient pattern to emerge from analyzing the longitudinal data for RQ1* was a fundamental dynamic between reactive and proactive modes of engagement. Chapter 5 will trace how this dynamic unfolds across the semester, providing a cohesive narrative of the lived experience of engagement in this course.

CHAPTER 5

HERMENEUTIC EXPLANATION

This dissertation's findings are divided into two chapters. The previous chapter focused on descriptions of the phenomenon as shared through students' lived experience descriptions; this chapter delves into the hermeneutic side of things and makes sense of the essence of the phenomenon as interpreted by me and the participants through our conversations and the process of data analysis. Having laid out a rich descriptive foundation, we will now undertake hermeneutic analysis to interpret the deeper meanings and essential structures within these lived experiences.

In a way, Chapter 4 provided the "What?" of the phenomenon, and Chapter 5 answers the "So what?" of the phenomenon. We have witnessed the diversity and complexity of students' lived experiences of engagement; we have walked around the forest and looked at the trees in it, so to speak. Now is the time to fly and take in the essences of the forest as one complex whole.

This chapter presents the core findings of this study by directly addressing RQ1*: How does the lived experience of engagement in Precalculus unfold throughout the semester? The phenomenological analysis revealed that the students' stories were not best characterized by a simple aggregation of discrete manifestations of engagement dimensions. Instead, the most salient and powerful pattern to emerge was a fundamental dynamic that shaped their entire journey: a continual navigation between reactive and proactive modes of engagement.

Reactive engagement describes experiences where students feel like they are simply responding to the external demands of the course—a grade, a deadline, a specific instruction. They are like a pinball, bouncing through the semester based on the bumps and flippers of the course ecosystem detailed in Chapter 4. *Proactive engagement* describes experiences where students act as active agents of their own learning. Their engagement is directed by their own goals and strategies. They anticipate challenges, seek out resources, and attempt to control their learning path, becoming the player rather than the ball. This reactive-proactive dynamic serves as the central lens for understanding how the lived experience of engagement unfolds over time. The composite phenomenological vignettes presented in this chapter trace this narrative arc across the semester, illustrating how students move between these modes and how these shifts are shaped by the specific course elements analyzed in Chapter 4.

This chapter is organized in the following way: first, I will provide the research findings through a series of phenomenological vignettes and supporting analysis; then I will delve into the major novel contribution of the dissertation by proposing new engagement concepts of reactive and proactive engagement orientations.

Experiencing Engagement in Precalculus

The difficulty in understanding the core essence of the phenomenon of students' engagement lays in the fact that although the study participants' experiences of the phenomenon shared a great deal in common, there were also certain combinations of factors that led to different sequences of engagement or different perceptions and reactions to similar situations. Consider for a second the following analogy suggested to me by my advisor, Dr. AnnaMarie Conner, after listening to me explaining the emerging complexity of the phenomenon in the process of data analysis. Suppose the Precalculus course is a pinball machine, where the course materials, assessments, learning environments, mathematical concepts, etc. are all elements inside the machine's field. Some act more like obstacles, some boost the pinball's speed, some lead the pinball in specific directions.

In this analogy the pinball is the student and the ball's interactions with the elements of the field are the students' engagement. But the pinballs themselves might be different, the speed at which they enter the machine varies, and the angle at which they hit different elements change all the time as they zoom across the field. So even though the field (the course) is the same, we can not expect the path of each ball (the patterns and experiences of engagement) to be the exact same each time for the different pinballs. Similarly, while there are common overarching pathways, there are also instances where these experiences will diverge from each other.

In this sense, the tension of writing up findings of a phenomenological study with 11 participants and 41 interviews is very tricky. On one hand there is a great deal of these experiences that demonstrate the essence of engagement across all participants (which was also confirmed during Interview 4 when participants gave feedback on the intermediate findings of data analysis); on the other hand, there were also experiences that must be highlighted to explain ways in which the essence of engagement diverged.

In a way we have already seen all the pinballs bouncing around in Chapter 4. Now, in Chapter 5, we are attempting to understand the physics of the machine and interpret the laws of motion that govern those bounces. To demonstrate these, we will "experience" engagement together through a series of phenomenological vignettes –

written anecdotes that represent amalgamations of multiple participants' narratives. The vignettes will take us on a journey through the entire semester, showing small stretches of the pinball's (students') journeys within the machine (the course) and exploring what elements of the field (factors) were shaping the trajectory of the pinball (the experiences of engagement) at that particular moment in time. After each vignette we will delve into making sense and contextualizing the dynamics behind the experiences of engagement.

Phenomenological Vignette 1: Day 1

> *It is the first day of classes of my second semester at UGA, and this semester I am taking some required courses for applying to Terry. I am most worried about Precalculus, because I heard some students say that it is not an easy class, and I am already not that great at mathematics. I did take Precalculus in high school, but I didn't do well enough on my placement test in the summer. But I am feeling motivated to do my best and hopefully pass Precalc. I looked into the instructors on Rate My Professor and Prof. X seems to have good reviews, so I feel good about going to my first class.*
>
> *When I enter the class, the instructor is already there and I find a spot, ready to take notes or whatever. When the time comes, Prof. X goes over course stuff, and it seems like I'll need to balance a lot of work: different types of homework and I will need to watch videos, because this section of Precalc is flipped. According to the instructor, we will be working in groups on math problems from the packet we got at the bookstore. I am already thinking about my schedule and how I need to fit in watching the videos by the next class session.*

Undergraduate students come into college Precalculus with feelings, habits, beliefs, and approaches colored by their prior mathematical experiences. If, for instance, you have had a strained relationship with mathematics in school, your engagement is tinged with a certain level of apprehension: “I think it's just a hard class for me 'cause I don't like math” (Ashley) or “I think I was pretty nervous to take a math class. … I wouldn't say that I'm not good at math, but sometimes I might get into my head about mistakes” (Camila).

Some of the participants were retaking the college course after not passing in Fall (e.g., Gloria), some were taking it in college after having taken it in high school (e.g., Ashley, Beth), and some were taking it for the first time (e.g., Destiny). Whether or not they took Precalculus in high school, all of the students were placed into the course via a placement test in summer.

As mentioned in Chapter 3, most of the participants were either intended business majors or minors, where Precalculus was a required prerequisite course to apply to the Terry School of Business.

A great majority of students also shared being very strategic about their choice of Precalculus section in which to enroll. Almost all the participants shared using the website Rate My Professor to read students’ reviews of the instructors at UGA before they chose their section of MATH 1113. Many shared that highly rated instructors were the first to go, and they were monitoring openings in some of the more sought out instructors’ sections.

When it came to modality, some participants knew what they wanted in in terms of choosing a lecture vs flipped class section, and some adapted based on previous experiences. For example, Gloria enrolled in a flipped classroom section in Fall but was not able to pass the course at the time, so she was retaking the class in a lecture-based section in Spring. Ashley attended one class in a lecture-based section on Monday before realizing the environment would not be conducive to her learning and sought out opportunities to switch to a flipped classroom section by the next day. Similarly, Penelope switched within flipped classroom sections because her first instructor did not seem available to her, because he did not schedule office hours, so Penelope assumed that she wouldn't be able to get timely help from him throughout the course.

As we can see, some form of strategic engagement was happening before the course even truly began; the students were seeking out a good instructor, a modality that would work for them, and they were doing so in very strategic ways. They were highly motivated by a desire to achieve a clearly set goal: pass the class and get into their major/minor. Thus, students' motivation and engagement for this gatekeeper course was initially directly dictated by academic requirements. Only Ashley noted the importance of the social element in her classroom from the beginning; she perceived that because her peers were quiet and not asking questions in their first lecture-based section class, they must not be engaging. She wanted to switch to a class where she could ask questions and engage, and so would her peers.

After they settled into their chosen section during the add/drop week, the participants shared beginning to settle into a rhythm within the course and taking their first tentative steps towards juggling all the assignments and mastering the mathematical content of the course.

Phenomenological Vignette 2: Finding the Flow

It's been a few weeks since the semester began, and I feel like I've settled on a routine for my Precalculus class. Since I had Precalculus in high school, the first few classes I tried to do my pre-class worksheets without watching the videos or maybe watching them only when I was stuck on my assignment. But now that the topics are getting harder, I started making a point to set aside time to watch videos, take notes, and then immediately do my pre-class homework.

I usually go to a study hall in the library the day before my Precalc class and I sit down, take out my nice notebook and colorful pens, and start watching the videos. There are usually three videos for each lesson, and there is a professor teaching in those. It's not my instructor, but she's ok. Aside from teaching, she does example problems in the videos. Sometimes I pause the video and try to see if I can solve the problem on my own. It's nice to be able to check right away if I got it right. Once I am done with the video, I try to solve the pre-class worksheet so I can submit it that night. They are due in the morning on the day of class, but I usually do it the evening before for an hour or so.

Once a week I also make a point to hunker down and do my ALEKS homework. ALEKS isn't too bad so far, the topics are manageable. That might be because I did ok on the initial test on the website, or at least for the topics that I remembered from school. Plus if you get a couple of problems correct in a row, it marks the topic you are doing as done, so I've been able to knock those out relatively quick.

When I am doing ALEKS I also try to review for our weekly quiz The first time we had a quiz it was stressful: I wasn't sure what to prepare for, so I tried to review *on ALEKS and re-do the pre-class worksheets, but the problems were a bit different than that. I've now started reviewing our in-class worksheets, which has been helping. The quizzes are also really short: just a couple of problems for 15 minutes of class time. It can be hard to solve everything in such a short time. My first quiz didn't go too well, but now that I know what to expect from them, I feel better about it.*

During our classes, we usually start by discussing the pre-class worksheets, then the instructor teaches a bit, and then we get into our groups to work on the in-class worksheet that we have in our books. It's usually a lot of problems and we don't manage to get through all of them. The instructor walks around and checks in with us. You can ask questions during that time or just check if you got a problem right. I am not usually comfortable asking questions in a math class, but Prof. X is very nice and when she stops by our group, she asks me how I'm doing. So I feel less intimidated about asking her a question.

Now that I've found a flow, I feel better. It's still not easy, because it is math after all. But I've found a flow that works for me right now, and I feel pretty engaged, so I hope I can do well in this class. The first test is coming up, so I guess we will see how I do there…

In the first vignette, the pinball is launched: it begins moving around the field and bouncing off the elements that it encounters (engaging with the elements of the course the student encounters). Here in the second vignette, the pinball finds a stable, looping path (a routine for engagement). It's not a high-score pinball frenzy (peak engagement), rather it's a sustainable orbit. The student learned the physics of this specific machine and is now playing with it, not just being played by it.

At this point in time the students are no longer the passive pinballs launched into the course; the students are beginning to use the controls. They are starting to manage their own engagement and building self-regulated strategies that respond and handle the demands of the course, while also juggling the demands of other classes and other social and academic responsibilities. This is a crucial and often overlooked state of engagement—the sustainable, routine practice that lies between anxiety and mastery.

We saw in Chapter 4 how students ritualized their engagement with the course throughout the semester: whether it was watching and taking notes before attempting pre-class worksheets (e.g., Robin, Ashley) or doing both simultaneously on a just-in-time basis (e.g., Jordan, Destiny). However, the rituals were not necessarily maintained throughout the semester; some rituals were adapted later in the semester. We see a hint at this in the second vignette, where we learn how the ALEKS platform directs students' engagement with the content topics and how one can advance through the platform tasks.

The initial test in ALEKS ends up dictating a great deal of the workload for the rest of the semester. So for some students ALEKS could feel not as difficult and cognitively demanding at the beginning, because those are familiar, "easier" topics. However, this structure also runs the risk of creating a deferred cognitive load, forcing students into a much larger workload when they encounter unfamiliar topics later in the semester. Maggie's experience vividly illustrates this shift from manageable routine to heavy load:

> “[F]or most of the first part of ALEKS until Test 2 I had a decent amount of topics for each section done, just 'cause I've done stuff like that before or it was kind of simple. And so for these, Chapter 4 and 5, I didn't know how to do those. … Because in the beginning it was slope and stuff like that. I knew how to do that. So I would have [only] a couple of topics of ALEKS to do. … some of it was just very simple like algebra or addition or stuff where obviously I know how to do that. So it definitely got more challenging as time went on and I would have more topics to do.” (Maggie, Interview 4, ~56:05)

Another aspect of engagement that sees shifts for students is related to the changes in their comfort and familiarity with the course itself (and the instructor’s style) as well as understanding what is expected of them for things like assessments. We saw some of these mentioned in Chapter 4 when students discussed becoming familiar with the format of quizzes and tests. Something not mentioned before, but also relevant to the students, was a developed familiarity with the instructors’ expectations for the level of detail in student solutions, as well as how instructors might grade submitted work.

For example, in one instance Beth expressed that she was a bit frustrated with her instructor, because of an unexplained quiz expectation, which she tried to advocate for:

> “I got a 70 on a quiz that we did … [I]t was a two-part problem and the first part was to show a formula and then the second part was to solve the formula. So on the first part I kind of showed half the formula and then got the right answer on the second. So I was trying to prove that I did everything that I was supposed to because if I got the right answer, well then I must have the right formula. But no, you have to [show work], and I asked him, so I can't just know what I'm doing, I

have to show that I know what I'm doing? And he said yes." (Beth, Interview 4, ~30:50).

Finally, at the end of this vignette we witness a callback to how students were building a relationship with their instructor during in-class interactions in Chapter 4. Some participants (e.g., Destiny, Robin) grew more comfortable with their instructors, as they persisted in walking around the room, checking in with students, and consistently maintained the practice of being open and friendly towards students' questions. This growing comfort seemed to improve students' confidence and lowered the affective barrier to engagement between the student and instructor.

The vignette ends on a cliff-hanger: the first test is looming on the horizon and all the routines that students developed are about to be stress-tested with a higher stakes assessment.

Phenomenological Vignette 3: This Was Not What I Expected

I just got my first test back and while I didn't completely bomb, I didn't do as well as I believed I would right before the test. I still feel so frustrated! And I put a lot of work into preparing, too! I reviewed my notes, I tried to re-solve all the pre-class worksheets again, I went over in-class worksheets as well, I looked at my quizzes. I even checked out the review section in ALEKS. But then I went into the test, and the problems just felt different.

Some of them weren't hard but the wording seemed unusual, so I wasn't always sure what I needed to do. There were problems that also felt way harder than anything we've done in Precalc so far. One of them… I don't even know where that one came from. It talked about a square on a coordinate plane and then there *is some line and the square touches this line. But then it asked about area. In the moment I felt completely lost. I've never seen a problem like this in any part of this course. During the test Prof. X ended up giving us some hints so we at least had a picture, but I ran out of time before I could figure that problem out.*

Once we got our tests back, our instructor answered our questions and showed some of the solutions we were confused about. I made some silly mistakes that brought my overall score down, but I feel like I am just frustrated overall because this test… It just wasn't what I expected it to be.

Just as the participants felt like they figured out a way to manage this class, the first test felt like a huge disruption to their feelings of comfort and the brief period of time where they thought they had the quizzes figured out and everything would be okay. In our pinball analogy, the test was a bumper minefield and one of the problems felt like a hidden trap door in the machine. The briefly gained sense of control is shattered: the engagement strategy needs to be revisited. The "physics of the machine" turned out to be less predictable than they were led to believe.

As demonstrated in Chapter 4, students experienced higher levels of engagement leading up to quizzes and even higher increases before the tests ("I have to pay more attention because there is a test soon and it's going to be on this stuff" – Ashley; "When it comes to quizzes and tests … I think [engagement will be] increasing over time because I want to do well" – Maggie). Despite the higher level of engagement, the first test still threw many students off and many were not too happy with their grades.

The core of this vignette also ties in the theme of course material misalignment from the previous chapter. We see that the students have developed a system of engagement and preparation for themselves (reviewing notes, worksheets, ALEKS), which was built on the expectation that the assessments would reflect course materials. They felt like their preparation should have been successful, but the test's unusual wording and seemingly out-of-nowhere problems threw in a wrench in the plan. This misalignment violated a psychological contract between the student and the course. They held up their end (did all the assigned work) and expected the course to hold up its end (test on that work). The vignette is about the frustration of that broken contract.

This disruption was especially profound because it broke the students' trusted feedback loop, where they used grades on smaller assessments (quizzes) to predict their performance on larger ones (tests). As Beth's experience demonstrates, doing well on the first few quizzes lulled her into a sense of preparedness leading up to the test:

> "I was out of the classroom in three minutes [after the quiz], and I know that's not going to prepare me for the test. Because if I got a 95 average on the last quizzes and 20 points lower on the test, then I feel like I'm not adequately prepared and can't use that to gauge how I'm going to do on the test. Because I thought I was going to do great on the test. I studied for eight hours, I had a 95 average on the quizzes, I've been to every class. And then I was completely, honestly, caught off guard." (Beth, Interview 2, ~23:39).

Minimum/maximum problem. This vignette mentions an actual problem that the students encountered in the first test, the "square on a coordinate plane" was actually a problem of maximizing an area of a rectangle given a constraint (Fig. 5.1). So, at the core, students had to write down two equations and find the vertex of a parabola. Here is a problem that emulates the wording and key concept tested in that question:

“Consider a rectangle in which the sides are aligned with the x and y axes in the first quadrant of the coordinate plane. One of the corners of the rectangle must be a point on the line $y = 5 - 6x$*. What size should the rectangle be to give it the greatest possible area?”*

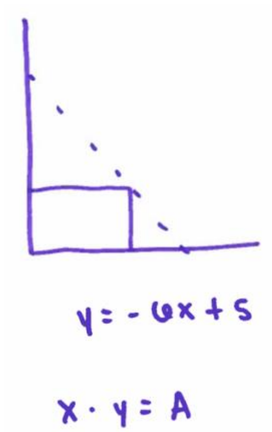

Figure 5.1: Maggie’s recollection of the rectangle problem on Test 1

Multiple participants brought up this problem as one they did not expect:

“This one was, I mean, completely almost out of left field. We've seen similar questions, but not to this extent, to this difficulty, I feel like. … We had never had something like that be introduced: the fact that a point lies on this line, but it's also the rectangle. That, we had just never had a problem like that” (Beth, Interview 2, ~26:41)

“I think [this problem] had an average of 43%, which is not that good. Yeah, it might've been lower than that, but that was just a difficult question. The rest of the test was pretty straightforward. But this one, I think it was worth 15 points maybe. It just got a lot of people, [it] got me. I just didn't understand it.” (Maggie, Interview 2, ~51:47)

Destiny contrasted this problem with most of the minimization/maximization problems they've seen throughout the semester (Fig. 5.2) that other students also mentioned, and one I saw myself as well during classroom observations. In the word problem scenario in class, a farmer has a certain length of fence (e.g., 30 feet in Destiny's picture) and they are trying to maximize the area of the farm enclosure. Students knew that it is usually a problem with two equations, a substitution of a variable, and finding the vertex of a parabola for the resulting quadratic function.

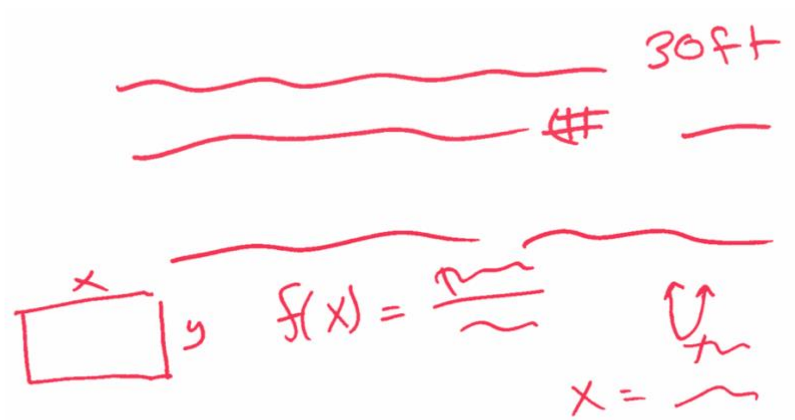

Figure 5.2: Destiny's depiction of how they usually approached similar maximization word problems in class

So far students experienced an emotional rollercoaster in Precalculus: from anxiety about taking a math course (vignette 1) to confidence gained through practice and hard work (vignette 2). But this first test was a large pitfall for many, where the students' affective engagement turned sour; making participants experience feelings of being let down and frustrated by the course. Experiences like this led students to start questioning what went wrong. Was it the fault of the person in the math department who put poorly worded problems on the test? Was it their fault for not putting in enough work or not understanding the material well enough? Or was their flow not good enough to help them succeed?

This third vignette is a clear example of how a cognitive challenge ("I don't even know where that one came from") directly triggered a powerful affective response ("I still feel so frustrated!"). As we will see next, experiences like this can lead students to adapt their overall engagement across all the dimensions of engagement.

Phenomenological Vignette 4: Bouncing Back

> *After the first test, I knew I had to reevaluate my strategy for Precalc, because clearly not everything was working. I put in the time and effort into test prep, but my grade wasn't the best. The worst part of it was believing that I was grasping the concepts and then realizing that I might not have actually understood them. As I am re-evaluating what to do next, I was talking to three of my friends who are also in Precalculus. They are not in the same section as me and we all have different instructors.*
>
> *One of them is Ashley, who has been really rocking it lately! She was so worried about the test because her quiz grades aren't that good. She says she gets anxious about the time and being slow at math. Which is fair: quizzes are super short. But then she told me that she got some extra practice packets from a tutoring company, and she did extra practice tests that her instructor gave them to work on.*
>
> *Honestly, her instructor sounds great! She seems to really help students understand the math, not just memorize how to do the problems. Ashley says she is excited that she finally gets the why behind the math. I wish I could say the*

same, but this test really shook my confidence. Anyway, Ashley said she felt like all her hard work paid off and she thinks she will keep doing all the practice problems all the time now. This makes me feel like I need to find some more practice tests as well. But when I talked to my friend Beth, she had a completely different take on the whole thing.

Beth is also in a flipped class like Ash and me. But she is really annoyed with the course. Honestly, I don't know what's going on there. I think Beth is good at math and she can crack the problems like nuts, but then she got almost the same score on the test as me! When I talked to her she told me her whole perspective on the situation: that the math department is really to blame for the test being so hard and all the recent changes to the class. Like, we now have mandatory groupwork as part of the grade, or our quizzes are gonna look different now. She also thinks that her instructor doesn't believe in the flipped classroom format.

Anyway, Beth doesn't bother watching videos anymore and she says she kinda just pretends like she is doing groupwork, she just does her problems on her own. Yesterday she was literally texting me during class! I was like: what are you doing, why aren't you paying attention? But she just said she wants to just survive Precalc this semester and never take a math class ever again.

I also talked to David. He is actually in a lecture class, and they don't do half of the stuff we need to do all the time, but we have the same tests though. Last time I talked to him he was actually considering withdrawing from the class and just earning Precalculus credit at a community college over the summer. I am a bit sad he is in that situation, but I also get it. I keep hearing people say UGA makes

Precalculus way harder than it needs to be. My friends from other places in GA say this course is notorious in the state: for example, my brother heard someone at Georgia Tech say that UGA Precalculus is harder than Calculus I!

I feel like I am now at a crossroads: I can't keep engaging the same way I did before Test 1. But how do I recalibrate? Do I throw myself into more work like Ashley, do I just exert strategically minimal effort just to pass like Beth, or is it bad enough for me to just give up this semester?

The worst part is that the semester is only going to get harder not just in Precalculus, but in aaall of my other courses as well. This will not be an easy choice. My academic advisor says I need a good grade for Terry, so I'm stuck between wanting to learn like Ashley and just needing to survive like Beth.

In this long vignette we explore the different engagement pathways that students took as the course continued after a test. Every time there was an assessment, as discussed in Chapter 4, participants used the grades as a temperature check for their cognitive engagement and self-correction of behavioral engagement. Their affective reactions to the grade often drove their motivation to engage further. However, what form that further engagement took varied depending on the students' disposition and a variety of other factors.

Here we see three distinct engagement pathways emerging from the disruptive event of the first test. For Ashley, her frustration with the smaller bumpers (quizzes) prompted her to tackle the bigger bumper (Test 1) head-on. The pinball of her engagement was propelled with more speed and purpose as she gained confidence in her control of the machine. For Beth, however, the test felt like a trap; the machine seemed to have been tampered with, changing it to an unfair game. She stopped trying to master it and focused purely on survival, her path becoming a solitary, strategic ricochet just to stay in play. Finally, David considered the ultimate form of disengagement: withdrawing his ball from the game entirely, questioning if the payoff was worth the effort. Consequently, the first test was not solely an assessment of math knowledge; it was a catalyst that forced students to fundamentally reassess their relationship with the entire course.

Another aspect that comes to light is the systemic role of Precalculus, a perception shared not only by the participants but by the broader UGA community. The course's notorious reputation preceded it. Jordan shared that people pitied her for taking it; a participant from the pilot study heard from Georgia Tech friends that UGA Precalculus was considered harder than Calculus I. This reputation is inextricably linked to its function as a filter course. It became clear during analysis that MATH 1113 is not just a prerequisite for Terry Business School—it is a gatekeeper. Multiple participants received advice that Terry admissions would scrutinize their grades in Precalculus, Accounting, and Economics.

Learning this information was a pivotal moment, often causing a noticeable shift in students' engagement graphs. This was the academic dimension of engagement manifesting in its most potent form. For some, the pressure caused engagement to spike. For others, like Camila, it triggered a drop (see Fig. 5.3). She noted this dip coincided with her advising appointment, where she learned that merely passing was insufficient; she needed a good grade.

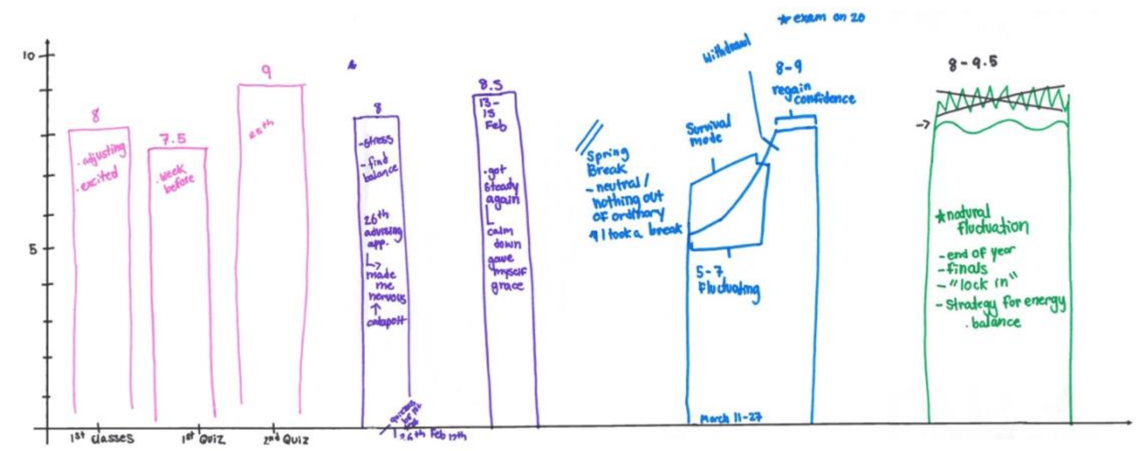

Figure 5.3: Camila's engagement graph, where the 4th bar shows the timestamp of the advising appointment

> "I had to calm down from that afterwards. I told you before that I get a little nervous with all these academic expectations. … [A]t the [advising] meeting I was doing pretty good, and then directly—I'm a little embarrassed—but directly afterward I cried in the stairwell for five minutes." (Camila, Interview 2, ~54:34)

These instances shared by the participants further emphasize the complexity of the phenomenon of engagement: engaging in the Precalculus course is not an experience isolated from other aspects of the students' lives. The complexity of the phenomenon is once again on full display, where the different dimensions keep interacting and triggering each other. The participants are navigating the pushes and pulls of their emotions, the cognitive demand of the mathematics content, along with managing time and self-regulation.

The fourth vignette reflected the inner monologue of a student at a crossroads after Test 1. To continue our journey through the research findings, let our hypothetical student persist in the course. Recall from Chapter 3 that the Precalculus curriculum in MATH 1113 consists of three distinct parts and each midterm test corresponds to one of them. In the fifth and final vignette we witness the shifts in engagement that are attributed to the content of Parts 2 and 3 in addition to the student's response to Test 1.

Phenomenological Vignette 5: I Think I Am Getting It

> *It's past Spring Break and I gotta admit, I didn't think I would feel better about Precalc after Test 1… And here I am, actually feeling better. The first part of the class had all the function stuff that I am not good at, but after we switched to exponents and logarithms, I started feeling better. It's ramping up again because we are starting trigonometry, so I'll see how it goes.*
>
> *I think the laws of logs and exponents are just easier. In functions you have to think about what's going on, especially when it asks about domain and range. But here there are some rules and tricks, and if you know them, you can kinda figure it out. With functions I was constantly second-guessing myself. If I got an answer that wasn't a whole number or a graph that looked a little off, I'd panic. Especially if I couldn't check the answer. And so much stuff we have in this class has no answer key. The practice exams and in class worksheets don't have answer keys. I need to either ask the instructor or use an app on my phone that can tell me the answer.*
>
> *I have a GroupMe chat with the girls in my group and sometimes I would text: what did y'all get for this worksheet? And we all have different answers. Same after Test 1: we were trying to compare answers, and it just felt scary to hear people say something different than what you got. I feel like in math some answers just look "right" and I get really anxious if mine don't. But with logs Prof. X says we can just leave our answers in log form. And even if other people have other*

answers that's ok, because maybe I used the natural log and another person used log 10. So both answers are actually correct, even if they don't look the same.
Now that we are learning trig I am getting a bit anxious again, because I really struggled with it in high school. But I think I'm actually starting to get it. In school I didn't understand why the triangle stuff works… You know, SOHCAHTOA? I was afraid I would have to memorize the unit circle but we don't. Plus, I actually understand why it's a unit circle now and how it connects to the trig identity. I can't believe I didn't learn this in school!!!
I also feel like ALEKS is coming in handy now: it gives me lots of problems to solve. I feel like with topics like logs and trig, it's helpful to just do a bunch of problems with all the formulas. It helps memorize things, which I think will be helpful on the test. I know it helped me on the second one!

Vignette 5 provides a compelling answer to the third research question, revealing that mathematical topics are not a passive backdrop but active elements that dynamically reshape the landscape of student engagement. The story of engagement is one of constant recalibration with each new unit, marked by a central tension: the cognitive desire for conceptual understanding ("the why") versus the affective safety and strategic value of procedural fluency.

Returning to the pinball analogy, the machine is not homogeneous. Instead, the landscape changes as the semester unfolds. Each topic can present an entirely different terrain to each student based on their previous experiences with the mathematics content or the prerequisite knowledge needed to understand the new concepts. In this particular anecdote the functions were a cluster of bumpers introducing anxiety and the bouncing between ambiguity and disparate answers from multiple directions; the logarithms were a calm and smooth ramp that allowed the ball to glide more confidently; trigonometry at first appeared on the horizon as a scary drop, but the student's cognitive engagement shifted it into a ball saver that allowed the student to stay in the game.

The shift from the perceived ambiguity of functions to the safe rule- and formula-based world of logarithms shifted some of the students' behavioral engagement from a persistent need for solution validation to a more sustained and confident cognitive engagement. Many participants would mention how important checking their answer was to them. For example, Beth just wanted the instructor to tell her if her answer was correct or not instead of discussing the concept; Ashley shared that she found an app that solves math problems, but she mainly used it to check her work. Some of that anxious engagement is due to a strong drive to be ready to "do well" on the assessments: getting the right answers, getting a good score. This instance is also an example of a socio-cultural norm that the students might have internalized from previous mathematics classes: some answers just look more "right" than others. Although this is a valuable observation, this is not within the scope of the study.

Another point connecting engagement to the mathematical content of the course is the participants' cognitive engagement and conceptual understanding of ideas. Here trigonometry transforms from a source of math anxiety and trauma to a space of understanding and authentic conceptual learning. Here the vignette references Ashley's newfound appreciation for the conceptual beauty of the unit circle and moving beyond rote behavioral memorization she had to engage with in high school to deeper cognitive engagement.

Sadly, memorization is commonplace in mathematics learning and teaching, especially in gatekeeper mathematics courses. In the fifth vignette the student acknowledges the benefit of understanding; yet we immediately witness their behavioral engagement with ALEKS, where practice problems tend to lean towards repetition and formula application. This doesn't automatically mean no cognitive conceptual engagement is happening, but the repetitive practice problems are not good at clearly eliciting this type of thinking.

Recall the "weird" Test 1 problem in Vignette 3. Maggie shared that she saw one problem like this on one of the practice tests, but it was only one such problem for the minimum/maximum concept compared to a greater number of "fence"-themed word problems in the rest of the practice tests. When asked why she did not review that problem, Maggie pointed out a very pragmatic and cognitively interesting approach to engagement:

> I think that there's just more practice problems of [the fence and area type of problems]. And since this [coordinate type problem] was only on it once, there's more multiples of different problems that were essentially the same thing, just different numbers. That [other type of problem] wasn't on the test, but this was. I feel like they prepared us in the wrong way. Obviously, you shouldn't know which problems are going to be on a test, but I wish we had more practice on these, because also I think it was at the very end of a worksheet, so it's like not everyone gets to it." (Maggie, Interview 2, ~54:38)

Students are considering not just mathematical concepts when preparing for assessments, they are also considering the odds of one type of problem appearing on the test with different numbers. On one hand, this shows that some of the learning that students are engaging in might be surface level and very procedural; on the other hand, educators might need to consider the drawback of providing learning materials that present the same type of problem, and which do not promote conceptual understanding.

The purpose of these vignettes was two-fold: (1) to provide a phenomenological narrative that captures the essences of the engagement experience, and (2) to connect individual lived experiences to the broader patterns of meaning within the data. Together, they portray a story of student resilience and adaptability within the complex system of a gatekeeper course. This analysis naturally leads to question: What drives this adaptability? The data suggest it is the relentless interplay between the student's lifeworld—their past trauma, their academic pressures, their need for safety—and the ever-changing demands of the mathematical "pinball machine." Engagement, therefore, is not just student's effort, but continuous negotiation of meaning, strategy, and survival within a high stakes learning environment.

Reactive and proactive engagement

During the process of data analysis, I noted many instances when, in addition to the impact of various factors and the relationality of engagement, another aspect was salient to students' experiences of the phenomenon. It was more than what or whom the students were engaging with within MATH 1113, or mere directionality of the activity (engagement of the instructor with the student vs engagement of the student with the instructor). There was also a great element of intentionality behind it. Indeed, we know from literature that engagement and motivation are tied closely together (e.g., Eccles, 2005), and one could say that motivation is often the impetus to engage. But aside from motivation, students' lived experiences in this study pointed to something more specific about the nature of engagement itself.

In any organized learning process, it is safe to assume that certain educational events occur: the teacher assigns homework to the students, students could be asked questions directly by the teacher or their peers, students are expected to complete class assessments, etc. In the context of higher education, all of these activities and student expectations are outlined within a course syllabus. It is in a way a "contract" established with the instructor-student rights and responsibilities. One could also reason that there is always a socio-cultural contract in place that dictates social and academical norms for any particular classroom of how everyone is expected to act and behave.

In this context, it is no wonder that once a homework assignment is set by the instructor, the student is expected to complete and turn in their work on the assignment to the instructor in a timely manner to fulfill their side of the agreement. Even when such an interaction is mandated and the student might feel obligated to abide, this is nevertheless an instance of engagement. The student makes the time, gathers the necessary materials, potentially meets with a study group, works on understanding the mathematical content needed to complete the assignment, and then presents it to the instructor in an acceptable format within an assigned timeframe. The entire process carries within instances of different kinds of engagement: cognitive, affective, behavioral, social, etc.

However, there are certain aspects of the process that can demonstrate slight variation in the way engagement occurs. A large part of the students' engagement in this scenario is a reaction to a course requirement, a formal query by the instructor. Students are highly motivated to fulfill this requirement to uphold their side of the contract and –

probably more importantly to the student – to ensure they can pass the course, fulfilling an academic need. This is still authentic, meaningful engagement often involving cognition and affect in some way, but the impetus is most likely to be a reaction to a course requirement. That is, one could consider: had this assignment not been required or asked, would the students work on it? I propose to call instances like these *reactive engagement*.

Consider instances when Beth sits in a group and helps her peers with the worksheet problems. She is truly engaging with the content and her fellow classmates, but that engagement is not authentic and not actively sought out. She is checking off a box of mandatory group participation, because it has a tangible grade value (15%). She is reacting to a requirement that she is explicitly not happy about.

Consider another scenario to explicate the concept. A student is attending a lesson in a lecture-based section of MATH 1113. The instructor is at the board, writing down some mathematical content and providing a solution to a sample problem. Typical ways that students like David and Gloria engage in these instances are listening to the instructor, following along, taking notes. At the basic level these are usually expected by the student-teacher social contract (and most likely the course syllabus). Students are expected to take notes in a lecture-based course because no pre-recorded videos are provided. Students are expected to listen to the instructor and not talk to each other, so as not to disrupt the lesson for other students. Depending on how the student is approaching these activities, this could in fact be another example of reactive engagement.

In contrast, if a student in this scenario is not only following along with the lecture and taking notes, but also actively thinking about the mathematical concepts, thinking of questions to ask the instructor, or trying to think of the next step in the solution for sample problems, this represents a different kind of engagement (definitely cognitive and likely affective and behavioral as well). I propose that we call instances of such kinds of student engagement *proactive engagement*. While very desirable and probably often expected (or at least deeply hoped for) by instructors, this type of engagement requires a certain level of intentionality, self-motivation, and drive from the student.

Throughout the semester Destiny shared on many occasions that she was constantly looking for opportunities to increase her engagement and break out of her comfort zone. At first, she sought to gather the courage to ask the instructor questions without fearing wasting his time. Later she moved on to actively going to his office hours with other students. Finally, by the end of the course she got to a point of comfort with contacting the instructor with questions via email as she was preparing for tests. The progression of her engagement was intentional and planned. She wanted to do more and be a more active participant in the course, which is a perfect example of a proactive form of engagement.

The proposed concepts are not novel in their core nature per se. Educators often talk about what we consider to be desirable forms of student engagement, but there are currently no exact terms describing this particular kind of dichotomy of students' engagement orientations that captures the distinction found in this data set analysis. This refinement came about as a direct result of the phenomenological framework in the study. When considering the nature of the experiences of the phenomenon, I kept stumbling on the differences in participants' backgrounds and prior lived experiences that have shaped how their stories described moments of engagement in the course. To acknowledge this diversity, I was thinking of each participant and their lifeworld as being a unique bubble that has its own color, size, texture, elasticity. As Merleau-Ponty says, our lifeworld "signifies how we are to the world, that is, how we relate to and interact with the world." Thus, each human's background literally colors the bubble of self and one's lifeworld. This analogy is not fixed and does not capture every aspect of the lifeworld, because the lifeworld can obviously shift and change much more than an actual soap bubble. But it did help me think of the phenomenon in the moment.

If the student and their lifeworld are indeed a bubble, what happens when a phenomenon is encountered? Experienced? Engendered? If a "bubble" encounters an outside force, it is natural for that bubble to engage in a somewhat reactive way. If a wind blows our bubble in some direction, our bubble will likely fly in that direction. If an instructor asks us a direct question, we feel obliged to answer (even if we say that we do not know). However, if the bubble is reaching out towards an experience, if it is moving towards something and trying to engulf it, feel it, experience it within the bubble, it is a much more proactive stance. The participant is moving the bubble, and the phenomenon is produced rather than reactively provoked.

Figure 5.4: Students' lifeworlds imagined as colorful floating bubbles

As a result of raw phenomenological analysis, the bubble analogy gave way to the two engagement orientations proposed here. However, to explain how these internal orientations manifest within the complex, dynamic system of the Precalculus course, we must return to the pinball machine. The pinball is the student-in-action, and their 'bubble' is the internal lens they bring to the game. Within this machine, reactive engagement is the pinball's path being determined by its collisions with the fixed elements of the game—bouncing off the weekly quiz bumper, being diverted by the slingshot of a tough test question. Proactive engagement, however, is the student wielding the pinball launcher and flippers. It is the strategic, intentional act of propelling oneself toward a specific target, whether that be using office hours to clarify a confusion or seeking out extra practice problems to prepare for a test. In reactive engagement, the participant's engagement is determined by the system; in proactive engagement, the participant's engagement is being purposefully determined by the participant.

The presence of these engagement orientations does not end there. In the previous Chapter we saw and discussed a variety of engagement diagrams (Figs. 4.1-4.10). However, the lines in those diagrams were simply connecting the students with the elements of the course that they engaged with. In the context of the proposed concepts, these lines are in fact actually two-way or one-way arrows, where one direction demonstrates the reactive engagement and the other direction represents proactive engagement (Fig. 5.5).

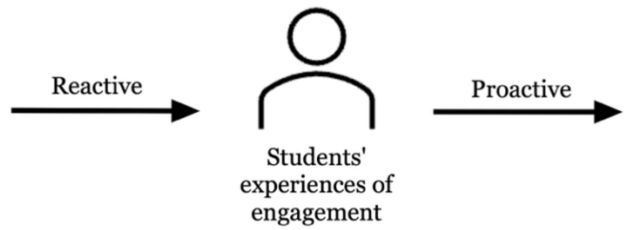

Figure 5.5: The directions of arrows demonstrate the specific orientations of engagement: pointing in is reactive, while pointing out is proactive

The arrows pointing towards the student represent situations when the phenomenon of engagement is moving from the "outside" toward the student's lifeworld. In the pinball analogy, the arrow represents the force pushing the student/pinball to move and react. The phenomenon in this case is reactive in nature: outside events, people, and stimuli interact with the participant and their lifeworld, prompting them to engage.

The arrows pointing away from the student represent situations when the phenomenon was proactively engendered by the participant towards various parts and activities within their Precalculus course. Here the student is propelling the ball themselves, they are the one moving it in a desired direction. Here the proactive nature and disposition of the phenomenon means that the students themselves are deciding on what they engage with and how.

The reactive and proactive engagement could be thought of as modes and orientations of engagement, when considered in tandem with Husserl's theory of intentionality. "Intentionality refers to the relationship between a person and the object or events of her/his experience or more simply, one's directed awareness of an object or event" (Husserl, 1907; 1998; as cited in Dahlberg et al. (2001)). The engagement of the participants in the study was always in tandem with the awareness of the multiple events and components of the course. In this light, reactive engagement reflects an intentionality directed by the system's prompts, while proactive engagement reflects an intentionality directed from within the student's own goals and curiosity. In many ways this is what makes engagement a difficult concept to study, because it is natural to focus on the events and objects to which the students' engagement is attached to. To study engagement as a phenomenon of its own means to understand not only what provokes it but also the intentionality and the meaning that students assign to engagement itself.

It is crucial to note that proactive engagement is not merely a function of student effort. It also requires an accurate mental model of the 'pinball machine.' When the system's demands are misaligned or opaque—such as when test questions bear little resemblance to course materials—even the most proactive strategies can fail, leading to frustration and a potential reversion to reactive modes.

Similarly, reactive engagement should not be dismissed as inherently negative. It is a fundamental and often adaptive response to the environment. A student reacting to a quiz deadline by studying is demonstrating responsiveness. The critical distinction lies in the origin of the impetus. Because we aim to create classrooms that engage all learners, we must be mindful of those who may default to a reactive orientation, especially in gatekeeper courses like Precalculus that can profoundly impact their academic trajectories.

Modes of Engagement in the Phenomenological Vignettes

Circling back to the phenomenological vignettes in the first part of this chapter, we could consider the shifts in engagement across the vignettes through the lens of reactive and proactive engagement. Starting with Vignette 1, we have reactive engagement with the placement test as well as proactive engagement with identifying the best possible section of Precalculus for their learning needs. The placement test was just part of the college requirement: a hoop they had to jump through during the summer before their freshman year. They did not prepare, review, or ask questions about it. In contrast, participants shared being very active and determined to find an instructor and a section of the course that would fit their needs as a learner.

There are more instances of both kinds of engagement in Vignette 2. For example, the student shares a very reactive attitude towards pre-class worksheets, where initially the videos were viewed on a "as needed" basis, before shifting into a more proactive routine of note taking. However, a decent amount of the actions are still somewhat rooted in the reactive attitude of completing tasks, even when meta-cognitive engagement is happening wherein the student is reflecting on their learning practices.

Vignette 3 is the moment when we really see the student encountering a tangible consequence of their reactive engagement. Their preparation for the test focused much more on reviewing things they've already done (notes, previous quizzes, ALEKS), not actively seeking out and attending to gaps in conceptual understanding. This is a moment of reckoning and reassessing, which nicely sets up the next story.

Vignette 4 demonstrates a spectrum of ways students in this course adjusted their engagement. From Beth leaning heavier into reactive engagement, to Ashley feeling rewarded for proactive engagement and continuing in that vein, to David deciding that neither is something he wants to keep doing.

Finally, Vignette 5 is much more ambiguous, reflecting the ways that at the end of the day participants in this study settled into patterns of engagement that would most often contain a mix of both reactive and proactive engagement.

It is important to close this chapter with a critical caveat about any kind of "findings" from the hermeneutic phenomenological lifeworld perspective used in this dissertation. The interpretations and experiences of the phenomenon that I as a researcher and the UGA Precalculus students as the participants have assigned to engagement in this study is just an instance of meaning. But meaning is contextual, fluid, and rooted in its relation to the lifeworld (Dahlberg et al, 2001). As educational landscapes shift in our rapidly-changing world, so will the lifeworld, and so will the meaning of the phenomenon of engagement in a college Precalculus course. This is only one story of engagement.

CHAPTER 6

DISCUSSION

This dissertation set out to answer a deceptively simple question: What is the story of engagement in a college Precalculus course? Guided by a hermeneutic phenomenological framework, the preceding chapters have provided a rich, nuanced account of this story, revealing engagement not as a static trait or a simple checklist of behaviors, but as a dynamic, complex, and often contested phenomenon. Chapter 4 laid out the descriptive tapestry of students' experiences, illustrating the intricate ecosystem of interactions with instructors, peers, course materials, and assessments. Chapter 5 provided the hermeneutic explanation, offering the pinball machine as a central analogy and introducing the novel concepts of reactive and proactive engagement to explain the orientations students take within this system.

This concluding chapter discusses these findings by situating them within the extant literature reviewed in Chapter 2. I argue that the present study addresses critical gaps in the research on student engagement, particularly in undergraduate mathematics education (RUME), by offering a phenomenological, rather than solely psychometric and quantitative, perspective. The discussion proceeds in four parts. First, I revisit the conceptual "haziness" of engagement (Reschly & Christenson, 2012; Fredricks et al., 2004) and demonstrate how the pinball machine analogy and the four-dimensional framework provide a cohesive, context-rich model. Second, I discuss how the findings around the instructor's role, peer interactions, and course materials deepen our understanding of mathematical engagement specifically (Middleton et al., 2017). Third, I position the concepts of reactive and proactive engagement as a meaningful theoretical contribution that clarifies the intentionality behind student actions. Finally, I consider the implications of this study for instructors and departments navigating the high-stakes environment of a mathematics gatekeeper course (Seymour & Hunter, 2019; Apkarian et al., 2021).

Beyond Conceptual Haziness: The Pinball Machine as a Dynamic Model

A primary challenge in engagement research has been its "conceptual haziness" (Reschly & Christenson, 2012) and the tendency for broad definitions to blur the lines with related constructs like motivation (Eccles & Wang, 2012). This study confronts this haziness not by proposing a stricter universal definition, but by offering a dynamic, systems-oriented model—the pinball machine—that embraces the complexity earlier scholars identified (Fredricks et al., 2004; Lawson & Lawson, 2013).

The pinball machine analogy resonates strongly with Lawson & Lawson's (2013) social-ecological model (Fig. 2.1), which posits a transaction between environmental conditions and student dispositions. In this study, the machine's layout—the bumpers, flippers, and traps—represents the conditions for engagement: the institutional structures, the flipped vs. lecture modality (Collins, 2019; Mkhatshwa, 2021), the ALEKS platform, and the high-stakes testing culture. The pinball itself, with its unique entry velocity and angle, embodies the students' dispositions and drivers—their prior mathematical experiences, beliefs, and strategic choices, often shaped by external pressures like admission to the Terry School of Business.

This model allows us to see engagement not as a fixed state but as a trajectory, a series of "acts of engagement" (Lawson & Lawson, 2013, p. 442) shaped by the constant interaction between the student and the system. For example, the "broken contract" students felt when test problems misaligned with course materials (Vignette 3) was not merely a cognitive disappointment; it was a systemic event that altered the physics of the machine, forcing a recalibration of strategies (Vignette 4). As David lamented, “the examples she gave [during lecture] are pretty simple... But then she asked a really confusing worded one [on the] quiz,” a sentiment echoed by Ashley who found the additional practice problems “not realistic for the quiz.” This finding extends models like the Expectancy-Value Theory (Eccles et al., 1983; Watt & Goos, 2017) by showing how "task value" and "perceived ability" are not stable precursors but are constantly reassessed through lived experiences within the course's specific ecosystem.

Furthermore, the adopted four-dimensional framework (behavioral, cognitive, affective, academic) proved essential for capturing the holistic nature of these experiences in a gatekeeper course. The academic dimension, in particular, was not a peripheral indicator but a central mediator of engagement, as seen in the starkly different trajectories of Ashley and Beth. Ashley's poor quiz grade served as a 'wake-up call,' triggering a surge in cognitive and behavioral engagement that led to test success. Conversely, Beth's high quiz grades created a 'false sense of security,' leading to a reduction in engagement and a disappointing test score (Fig. 4.10). This validates the decision to include this dimension for this context, addressing the call for conceptualizations that fit specific research inquiries (Appleton et al., 2008).

The Unique Landscape of Mathematical Engagement in College

This study firmly establishes that engagement is subject-specific (Wong & Liem, 2022) and that mathematical engagement possesses unique disciplinary features (Middleton et al., 2017). The findings provide a college-level, lived-experience corroboration and extension of principles previously explored more often in K-12 settings.

The Instructor's Role: Beyond "A Better Teacher"

The data vividly illustrates that the instructor's impact is multifaceted and extends far beyond content delivery. Participants' experiences align with K-12 findings that teachers influence engagement through caring relationships (Klem & Conell, 2004; Dibbs, 2019), in-the-moment feedback (Cevikbas & Kaiser, 2022; Lazarides & Rubach, 2017), and shaping the learning environment. However, this study adds nuance. The instructor's physical movement through the classroom during group work, for instance, was not just a teaching strategy; it was a powerful affective intervention that lowered the barrier to asking questions. As Camila noted, "even if you don't ask her anything, she can come up to you and say: How are you doing? ... that makes me feel better." This echoes Dibbs's (2019) finding that a "better" instructor makes students feel safe, but it specifies a concrete pedagogical practice that enacts that safety.

Conversely, Beth's experience with her instructor's overly lengthy explanations highlights how even well-intentioned actions can provoke disengagement if they misalign with a student's immediate strategic academic needs. Her frustration—"I'll usually just ask him to check something for me... and he'll turn it into a five-minute explanation of what I already know"—illustrates that the quality of interaction is defined by its responsiveness to the student's perceived need, not just its existence.

Peer Interactions: The Social Dimension of a Flipped Classroom

The flipped classroom format, as studied by Collins (2019) and Mkhatshwa (2021), was designed to foster collaboration. This study reveals the starkly different realities of that intention. For students like Ashley and Destiny, the group was a source of rich social-behavioral and cognitive engagement, a finding that supports the benefits of student-centered modalities. Ashley described her group as a safe space: "If I'm confused about something, I'm always like, wait, did you get this?... I always feel really behind, but I also feel comfortable asking them." However, for students like Beth and Robin, the group was a site of quiet, solo endeavor or perfunctory compliance. Robin observed: "My table is pretty quiet, so we don't really work with each other," while Beth's collaboration consisted of handing her completed worksheet to a tablemate. This divergence underscores that simply structuring a classroom for collaboration does not guarantee the relational dimension of school engagement that Wong & Liem (2022) describe. The engagement diagrams (Figs. 4.4-4.6) powerfully visualize this spectrum, showing that the strength and quality of peer connections are highly variable and critically shape the overall engagement ecosystem.

Course Materials: An Interconnected Web, Not a Checklist

A significant finding was that students do not engage with course materials in isolation. They engage with the interactions between materials (Figs. 4.7-4.9). Ashley's experience of attending class without doing the pre-class work was a powerful lived experience description of this principle; her entire engagement was altered because the materials' intended synergy was broken. She described feeling 'annoyed' and that the class time felt somewhat 'wasted' because she couldn't use it to 'practice rather than learn it.'

This complicates the view of engagement as simply "time-on-task" (Carroll, 1963) and aligns with Goldin's (2019) more complex conative perspective, where engagement is tied to fulfilling needs and wants within a specific system. It also adds a layer of understanding to findings like those of Trenholm et al. (2019), who noted disengagement from over-reliance on videos; here, we see that disengagement can also stem from the absence of expected material interactions.

This interconnectedness also manifested in students' strategic use of materials. Some, like Robin, dedicated separate time to watching videos and taking notes, while others, like Jordan, “like to do my homework assignment and kind of do the two at the same time.” The spectrum of attitudes towards ALEKS—from David's appreciation of its worked examples to Tori's frustration with its time-consuming nature—further demonstrates that engagement with a single material is always filtered through its perceived role within the broader web of course requirements and personal learning strategies.

A New Dichotomy: Reactive and Proactive Engagement

Perhaps the most significant theoretical contribution of this study is the proposed dichotomy of reactive and proactive engagement. This construct emerged directly from the phenomenological data as a way to describe the intentionality or orientation behind students' engagement, addressing a nuance that is often glossed over in multidimensional models.

This distinction helps to clarify the "fuzziness" between engagement and motivation. Reactive engagement is engagement that is prompted by the external environment—a bumper in the pinball machine. It is the student answering a direct instructor question, completing an ALEKS assignment because it is due, or participating in group work because it is graded. This is authentic engagement—it involves cognition, behavior, and affect—but its impetus is external. Proactive engagement, in contrast, is driven from within the student's lifeworld. It is the student wielding the flippers: seeking out office hours, forming study groups, or diving into extra practice problems. This aligns with Reeve's (2012) concept of agentic engagement but sharpens it by contrasting it directly with the more common reactive mode.

This construct provides a powerful lens for re-interpreting the data. Beth's strategic, minimal-effort approach after Test 1 was a form of reactive engagement—a calculated response to a system she perceived as unfair. Ashley's deep dive into extra practice, however, was proactive—a purposeful taking of control. Even within classroom interactions, we see this spectrum: Destiny gaining the 'confidence' to ask questions because her group was already engaged represents a proactive move within a reactive structure, while a student merely listening to a peer's question represents a more passive, reactive form of cognitive engagement. The directional arrows in Figure 5.5 provide a visual language for this, suggesting that engagement is not just a connection but a vector with a specific origin point.

This concept also enriches our understanding of constructs from the literature. For example, Goldin et al.'s (2011) engagement structure of "Get the job done" could be seen as primarily reactive, while "I'm really into this" might represent a proactive orientation. By introducing this dichotomy, this study offers a new tool for researchers and practitioners to describe not just if a student is engaged, or even how, but why—shedding light on the underlying motivational dynamics and the student's sense of agency within the learning environment.

A natural question about the proposed constructs is how they relate to and/or differ from the intrinsic/extrinsic dichotomy of motivation (Ryan & Deci, 2000; Wilkie & Sullivan, 2018). It is true that, in many instances, moments of reactive engagement mapped to some form of extrinsic motivation (e.g., getting into the Terry Business School, getting a good grade, etc.) and moments of proactive engagement were directly connected to intrinsic motivation where participants were eager to learn and understand mathematics (e.g., actually understanding the "why," enjoying working with group peers). However, in many instances even those study participants who were overall more proactively engaged were extrinsically motivated by grades and major requirements.

Although Ashley's and Destiny's proactive engagement orientations carried intrinsic value to them, a lot of it was nevertheless motivated by the external academic demands of their chosen majors. Robin really enjoyed learning from their instructor and seemed to believe in the value of grasping the underlying mathematics content, but their engagement often leaned towards a reactive orientation.

In another example, even though some of the participants listed instructor's office hours in their engagement artifact, despite their intrinsic motivation, they might have faced some forms of resistance to follow through and take proactive steps towards attending this particular engagement opportunity. They saw the value and understood the integral role of office hours in their college experience, but when it came to the action (engagement), it did not manifest in real life.

When conceptualizing motivation as an impetus to action, it is appealing to hope that intrinsic motivation would lead to proactive engagement. But it is not always the case, which is why there is value in understanding these two modes of engagement in their own right and not just as an expected result of intrinsic and extrinsic motivation.

Implications and Conclusion

The story of engagement in college Precalculus, as told by the participants of this study, is one of complexity, dynamism, and constant negotiation. It is a story where a single disruptive event, like a challenging test, can shatter a student's sense of control and alter their entire trajectory (Vignette 3). It is a story where the same classroom structure can foster profound connection for some and isolation for others. The implications of this story are significant.

Implications for Instructors and Course Designers

Transparency and Alignment are Key. The pervasive frustration around assessment misalignment ("the test was much harder, much harder than the quizzes" - Beth) suggests a critical need for greater transparency. Instructors should strive to ensure that quizzes, tests, and other high-stakes assessments are clearly aligned with the practice opportunities provided in class and through other materials. The instructor's decision to survey students about quiz format, as reported by Beth and Maggie, is a commendable step in this direction.

Let us consider this in light of the minimum-maximum problem in Chapter 5. Transparency does not imply that the instructor should hint at which problems will be on the tests and alignment does not mean that all the problems should look the same when assessing the same mathematical concept. Instead, we need to consider the repertoire of practice problems and exercises that we give when supporting students in mastering any mathematical concept. If all problems for any given topic look and sound the same to our students (e.g., the farmer's fence), eventually the students are lulled into reactive engagement wherein they feel like memorizing the procedure is enough and being able to reproduce the procedure is a sign that they have grasped the concept.

By giving ten "fenced in area" problems in a row, we might be unintentionally shutting off our learners' meta-cognitive reflection on their understanding of the core idea behind a minimization/maximization problem. Instead, they are reflecting on their ability to solve a "fenced in area" problem. Then, when assessment time comes around and we prompt them with a different kind of minimization/maximization problem, the frustration about the misalignment is somewhat justified because we did not test their ability to solve that procedure. In this scenario, giving a task in a different context on the test is too little too late: students needed to encounter more variety of problems before the high stakes assessment.

To fix this, we as instructors could provide students with a varied spectrum of tasks on the same concept, so our students could explore and engage with the same concept across different contexts. We could also lean into transparency by guiding them towards recognizing that these problems that "look" different are actually addressing the same mathematical idea. Thus, transparency and alignment might potentially lead students to a more proactive engagement orientation, wherein they look for key ideas that appear across the different contexts and recognize when they need to grapple more with the concept itself.

Design for Proactivity. Although reactive engagement is inevitable and necessary, we should design learning environments that invite proactive engagement. This can include offering choice in assignments, creating low-stakes opportunities for student-directed inquiry, and explicitly encouraging and valuing student-led initiatives. Building in mechanisms for students to explore different solution paths, as Destiny's instructor did by acknowledging the video professor's methods, can foster this proactive stance.

Acknowledge the Systemic Pressure. Instructors should be aware of the immense external pressures students face, from the course's gatekeeper status to the institutional reputation of its difficulty. Acknowledging this shared challenge can build rapport and mitigate the affective toll.

Implications for Researchers

Embrace Phenomenology. This study demonstrates the value of phenomenological approaches for uncovering the essence of complex educational phenomena. Future research in RUME could leverage this methodology to explore other under-examined lived experiences. What is notable about the affordances of the phenomenological method of analysis in the study, is that the process of analysis elevated the spectrum of experiences that reflects the diversity of how engagement plays out for students in a college Precalculus course. It was not simply about including certain experiences in the classroom and course design: it is also about the quality of those experiences that will interact with the students' previous learning experiences and life situations.

Using Elicitation Tools. Graphing as an elicitation tool is not a novel approach for either education research more broadly (Bravington, 2023) or mathematics education

(Riske et al., 2021; Satyam et al., 2022). In fact, graphing is especially useful when dealing with constructs like engagement that involve affect, are malleable, and can significantly shift and change over time. By leveraging a wider range of creative tools within our qualitative data collection methods we can obtain rich qualitative data and thus, gain a deeper insight into more complex concepts in mathematics education.

Explore the Dichotomy. The concepts of reactive and proactive engagement warrant further study. Future research could investigate how to reliably identify these orientations, what factors promote a shift from reactive to proactive engagement, and how these modes manifest in other disciplinary and institutional contexts.

Additionally, the engagement constructs proposed and the engagement conceptualizations applied to this data set show that engagement is indeed a very complex phenomenon. Moreover, understanding how students experience engagement shows that students who might seem engaged to researchers and instructors might not be actually engaged at the levels perceived or might be much more reactively engaged even when we might believe that they are actually proactively engaged.

We need to conduct more research that involves the students' voices and experiences, because we have more than just teaching at stake. Many disabled students[3], especially those whose behaviors might not look similar to the behaviors of able-bodied students, might present as disengaged, while they might actually be cognitively and affectively engaged, and potentially even behaviorally engaged but in ways that are not considered normative in our society. Including student voices, especially voices of

[3] The "disabled" descriptor is used in consultation with members of the disability community and their current linguistic preferences. This language aims to destigmatize disabled existence as opposed to detaching it from the human experience.

marginalized students is critical if we want to reflect the diversity of experiences of engagement and ensure access to more equitable engagement opportunities.

Investigate the Role of Grades. The finding that students like Ashley and Beth used grades as a primary feedback mechanism to regulate their engagement opens a new line of inquiry. Research should explore how to help students interpret grades more formatively and how assessment design can better support proactive, rather than purely strategic, engagement.

In conclusion, this dissertation has listened to the voices of students navigating a critical academic gateway. By situating their lived experiences within the broader scholarly conversation, we have moved beyond conceptual haziness toward a more vivid, nuanced, and ultimately more human understanding of engagement. It is not merely about doing the work; it is about the dynamic, intentional, and often emotional relationship a student builds with the work, the people, and the system that constitute a college Precalculus course. It is the story of Ashley's redemption through proactive effort, Beth's strategic recalibration, Camila's emotional rollercoaster, and Tori's meticulous but frustrated time management. This story, while specific to one context, illuminates universal truths about learning, agency, and the complex interplay between individuals and the educational structures they inhabit.

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APPENDICES

A-D: INTERVIEW PROTOCOLS (1-4)

[see next page]

Interview 1 Protocol

Part of the interview	Interview script and guiding questions	Purpose
1. Introduction [5-10 minutes]	- Tell me a bit about yourself: When did you start your studies at UGA? - What is your current or potential major at UGA? - Which courses are you enrolled in right now? - Which section of MATH 1113 are you in right now? Who is teaching it? Where is your classroom located?	Get to know the participant. Help them feel a bit more relaxed and comfortable with the interviewer.
2. Entering the phenomenological disposition and practicing providing lived experience descriptions [10-20 minutes]	Disclaimer on the phenomenological nature of the interview: "The series of interviews that we will share this semester might not look like a typical interview. I will sometimes ask you to describe your experiences of some events as if you were walking me through them as they happened." Provide an example of an LED similar to the following script: "If I was prompted to tell you about my morning as it happened, I could say something similar to: When I woke up today, it took me a moment to realize that it was brighter outside than usual. I reached for my phone and saw that I overslept. As that realization hit me, I suspect I literally jumped out of bed and ran to my wardrobe: it happened so fast, I am not 100% sure. I sighed in relief because I remembered that I already prepared my outfit the day before. I was hoping to quickly brush my teeth, grab my backpack, and rush out the door, but my roommate was taking a shower. I felt very anxious and jumpy as I paced the living room, waiting for her to come out. I couldn't help but fidget with my phone; I was so nervous, I even dropped it a few times. I kept listening for the sound of the bathroom door opening." Check with participant if they have any questions.	Introduce the participant to the idea of providing a LED.
	Ask for a description of their latest lived experience of being in their MATH 1113 classroom: "Now that you heard my example, can you please take a moment to remember your last MATH 1113 class that you attended?"	Solicit a LED of the participant's experience with being in their MATH 1113 classroom.

	Questions: - When did it happen? - Can you please describe to me that class as it happened, from the moment you came to the classroom, until the moment you left the classroom?	
3. What does it mean to engage in Precalculus? [10-15 minutes]	Place an empty sheet of paper in front of the participant and give them the following prompt: "Here is a blank sheet of paper and some markers: you can use any color you like. I will give you a prompt right now and I would like you to jot down ideas you associate with the prompt. You can organize it on paper in any way you want; it can be a list, a word cloud, a concept map, whatever is most comfortable to you." Give the participant an opportunity to get settled and get a marker before giving them the prompt: "Are you ready? What are some ways you think a college student can be engaged in their Precalculus class?" If the student asks me for my definition of engagement in a math class, give them the following: "To be engaged in a math course for me, means taking steps towards participating in course-related tasks and activities."	Prompt the participant to share their ideas of engagement in a more low stakes setting, where they don't need to produce a coherent definition. This artefact will be used in consequent interviews to dive deeper into their ideas of engagement.
	Once the student starts producing the artefact, listen attentively and ask clarifying questions similar to: - You wrote down… Do you mind explaining what does it mean? - I see you have… here. Is that something that usually happens in class or outside of class? - Do you think you engage in all these? Which ones you do and which ones you don't?	Ask follow-up questions in order to understand their list and interpret their personal meaning of presented ideas.

4. Graphing engagement tool [20-30 minutes]	Introduce the graphing tool: "In the second part of the interview we will do some sketching as we talk. The purpose of this sketch will be to give us something to discuss. Remember that there is no right or wrong response here: I am not here to judge, I am very interested in your story and your experiences. Here is a sheet of paper with an empty plane. Could you please sketch your *engagement* in MATH 1113 until now?" If the participant has questions about axes, scales, units: "There is no rule here. Sketch what makes sense to you. If you want to change your sketch at a later point in time, you can absolutely do that. Like I mentioned before: there are no right or wrong answers in this interview."	Introduce the participant to the graphing tool and give them the prompt. Answer any questions they might have about the tool.
	As the participant is sketching or when the participant is finished sketching, ask relevant follow up questions: - Can you tell me a bit about your sketch: what do the x-values represent? What about y-values? - Can you pick a point on this graph and tell me a bit about your engagement in that moment in this class for you? Just like you did earlier in the interview? - I notice … in your sketch. Can you tell me more about this moment? - Now that you talked about some points on the graph, can you tell me a bit about the scales of your graph. Did you have any specific timestamps in your mind as you were sketching? Tell me about it please. - As we wrap up our interview, is there anything else you would like to share?	Ask follow up questions in order to understand their sketch and interpret their meaning of the sketch.

Interview 2 Protocol

Part of the interview	Interview script and guiding questions	Purpose
1. Introduction [5-10 minutes]	"Thank you so much again for meeting me for a second interview. Your help is truly appreciated!" - Before we start, I want to check in with you. How are you? How is the semester going so far overall? - How are you feeling about MATH 1113 right now?	Help the participant feel a bit more relaxed and comfortable with the interviewer.
2. What does it mean to engage in Precalculus? [10-15 minutes]	Place the artefact produced during the previous interview before the participant and give them time to recall what they jotted down last time. Once they had time to familiarize themselves, ask them if they want to edit anything on the page (adding, removing, providing more details, making any connections, etc.): "Here is the sheet of paper from our last interview where you noted some ways that a college student can be engaged in their Precalculus class. Take a moment to look it over. … Would you add, remove, or change anything?" When the participant is ready, give them the following prompt: "Which ones on this paper can you recall engaging in this past week in MATH 1113? You can circle/mark them with a pen or marker. Can you please describe … instance as it happened? The same way you described your class to me last time."	Remind the participants of the artefact they produced last time and prompt them to recall instances of engagement from their last week of MATH 1113 classes. Solicit participant's LEDs.
	Once the student starts reflecting on the artefact, listen attentively and ask clarifying questions similar to: - You mentioned… Do you mind explaining what does it mean? - I see you noted… here. Can you recall an instance when it happened and describe it to me as it happened please? - What about this one?	Ask follow-up questions in order to understand their list and interpret their personal meaning of presented ideas.
3. Graphing engagement tool [30 minutes]	Bring back the graphing tool:	Remind the participant of the graphing tool and give them the opportunity to edit.

	"As you may recall, last time you sketched your engagement in MATH 1113 on this sheet of paper. Before I give you the prompt, take a look at your sketch. Is there anything you would like to change before we go on?" Once the participant is ready, ask them: "As you can see, I added some more paper on the right. Could you please use a marker to continue your sketch and depict your engagement in MATH 1113 between our last interview and today? Remember that there is no right or wrong response here: I am not here to judge. The purpose of this sketch is to give us something to discuss, just like last time."	Answer any questions they might have about the tool. Ask them to continue their sketch to reflect the time between Interview 1 and Interview 2.
	As the participant is sketching or when the participant is finished sketching, ask relevant follow up questions: - Can you pick a point on this new section of your sketch and tell me a bit about your engagement in that moment in this class for you? Just like you did last time we met. - I notice … in your sketch. Can you tell me more about this moment? - Since you continued your sketch from last time, did you think about the scales in the new segment compared to the scale you had in our last interview? - I am trying to understand these two points. Do they have different y-coordinates or roughly the same y-coordinates? What does that mean? Can you say a bit more? - As we wrap up our interview, is there anything else you would like to share?	Ask follow up questions in order to understand their sketch and interpret their meaning of the sketch.

Interview 3 Protocol

Part of the interview	Interview script and guiding questions	Purpose
1. Introduction [5-10 minutes]	"Thank you so much again for meeting me for a third interview. Your help is truly appreciated!" - Before we start, I want to check in with you. How are you? How is the semester going so far overall? - Can you please remind me which courses you are taking right now? - How are you feeling about MATH 1113 right now?	Help the participant feel a bit more relaxed and comfortable with the interviewer. Checking about which courses the participant is currently enrolled in to prepare for the next section of the interview.
2. What does it mean to engage in a math class? [15-20 minutes]	Provide the participant with a blank sheet of paper and markers and give them the following prompt: "You mentioned earlier that you are currently in … class. Could you please jot down some ways that a college student can be engaged in … class? Similar to your map from the first interview for MATH 1113." Once the student starts producing the artefact, listen attentively and ask clarifying questions similar to: - You wrote down… Do you mind explaining what does it mean? - I see you have… here. Is that something that usually happens in class or outside of class? - Do you think you engage in all these? Which ones you do and which ones you don't?	Ask participant to produce an engagement artefact for another class that the participant is enrolled in this semester.
	Produce the engagement artefact from the first two interviews and place it next to the artefact produced in this interview for another course. Ask the participant some follow up questions: - Can you please take a look at these two side by side? Are there any ways you can engage in Precalculus but not in this other course? What about vice versa?	Compare engagement in Precalculus compared to another college course through a dialogue with the participant.

	- What are some instances from these two courses that you can recall that show how you engage differently in the two classes? Can you describe those moments as they happened?	
3. Graphing engagement tool [30 minutes]	Bring back the graphing tool: "As you may recall, last time you sketched your engagement in MATH 1113 on this sheet of paper. Before I give you the prompt, take a look at your sketch. Is there anything you would like to change before we go on?" Once the participant is ready, ask them: "As you can see, I added some more paper on the right. Could you please use a marker to continue your sketch and depict your engagement in MATH 1113 between our last interview and today? Remember that there is no right or wrong response here: I am not here to judge. The purpose of this sketch is to give us something to discuss, just like last time."	Remind the participant of the graphing tool and give them the opportunity to edit. Answer any questions they might have about the tool. Ask them to continue their sketch to reflect the time between Interview 2 and Interview 3.
	As the participant is sketching or when the participant is finished sketching, ask relevant follow up questions: - Can you pick a point on this new section of your sketch and tell me a bit about your engagement in that moment in this class for you? Just like you did last time we met. - I notice … in your sketch. Can you tell me more about this moment? - Since you continued your sketch from last time, did you think about the scales in the new segment compared to the scale you had in our last interview? - I am trying to understand these two points. Do they have different y-coordinates or roughly the same y-coordinates? What does that mean? Can you say a bit more? - As we wrap up our interview, is there anything else you would like to share?	Ask follow up questions in order to understand their sketch and interpret their meaning of the sketch.

Interview 4 Protocol

Part of the interview	Interview script and guiding questions	Purpose
1. Introduction [5-10 minutes]	"Thank you so much again for meeting me for a fourth interview. Your help is truly appreciated!" - Before we start, I want to check in with you. How are you? How is the semester going so far overall? - Now that the end of semester is approaching, how do you feel about MATH 1113? - Do you feel satisfied with your engagement in this class? Why?	Help the participant feel a bit more relaxed and comfortable with the interviewer. Soliciting some end-of-semester thoughts and feelings on the Precalculus course.
2. What does it mean to engage in a math class? [15-20 minutes]	Provide the participant with the artefact they produced during Interviews 1 and 2. Give them some time to familiarize themselves and recall what they wrote. "Since this is our final interview, I was hoping that we can take one last look at this paper together. As the semester is coming to an end, think back to your engagement in MATH 1113 and pick one way of engaging from your map/list that you believe best represents your engagement in the class this semester." Participants might experience discomfort with choosing just one. They can talk about at most two, but not more. If they are struggling, ask them to pick one that was more meaningful for them this semester. Ask any follow up questions, including soliciting examples (LEDs)	Ask participant to summarize their engagement in Precalculus this semester and essentialize their experience.
	Share my own preliminary findings with the participant: "I appreciate you sharing your thoughts on this! I've been working hard this semester talking to Precalculus students and trying to learn from all of you about your experiences. I jotted down a brief description of my understanding so far. If I share it with you, would you mind giving me some feedback?"	Member checking of preliminary conceptualization of students' experience of engagement.

	Share the printed conceptualization with the participant and read it out loud to them. Once done, ask some follow up questions: - Does it sound like an experience of engagement you recognize and can relate to? - Why? Why not? - What aspects speak to you? - Does it connect to anything in your map/list? - Can you recall an instance this semester when you experienced something like this? Do you mind sharing it?	
3. Graphing engagement tool [30 minutes]	Bring back the graphing tool: "Last time you sketched your engagement in MATH 1113 on this sheet of paper. Before I give you the prompt, take a look at your sketch. Is there anything you would like to change before we go on?" Once the participant is ready, ask them: "As you can see, I added some more paper on the right. Could you please use a marker to continue your sketch and depict your engagement in MATH 1113 between our last interview and today? Remember that there is no right or wrong response here: I am not here to judge. The purpose of this sketch is to give us something to discuss, just like last time."	Remind the participant of the graphing tool and give them the opportunity to edit. Answer any questions they might have about the tool. Ask them to continue their sketch to reflect the time between Interview 3 and Interview 4.
	As the participant is sketching or when the participant is finished sketching, ask relevant follow up questions: - Can you pick a point on this new section of your sketch and tell me a bit about your engagement in that moment in this class for you? Just like you did last time we met. - I notice … in your sketch. Can you tell me more about this moment?	Ask follow up questions in order to understand their sketch and interpret their meaning of the sketch.

	- Since you continued your sketch from last time, did you think about the scales in the new segment compared to the scale you had in our last interview? - I am trying to understand these two points. Do they have different y-coordinates or roughly the same y-coordinates? What does that mean? Can you say a bit more? - As we wrap up our interview, is there anything else you would like to share? "Thank you so much for being so generous with your time this semester! I really appreciate it! I hope your stories can help improve college math instruction."	

APPENDIX E: CLASSROOM OBSERVATION GUIDE

Observation Guide

Date	
Time	
Pseudonyms of participants enrolled in this section (not attendance)	

Precalculus topics covered in class
Class setting (lecture vs flipped instruction)
Individual vs group work opportunities
Observed opportunities to engage in learning mathematics during class

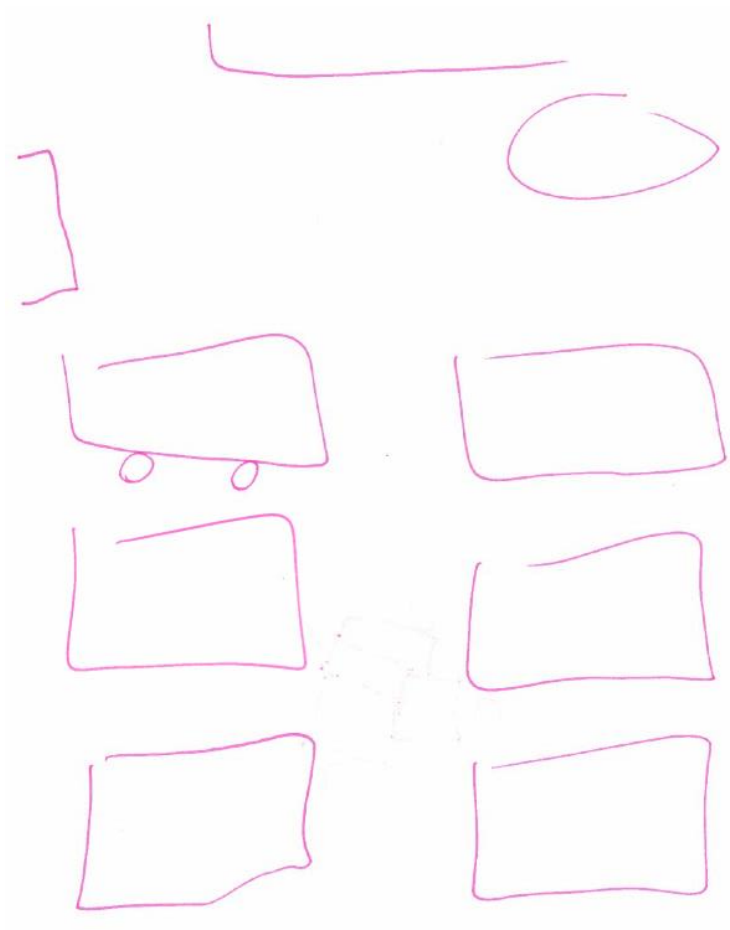
Other notes

APPENDIX F: MEMBER CHECKING INTERVIEW 4

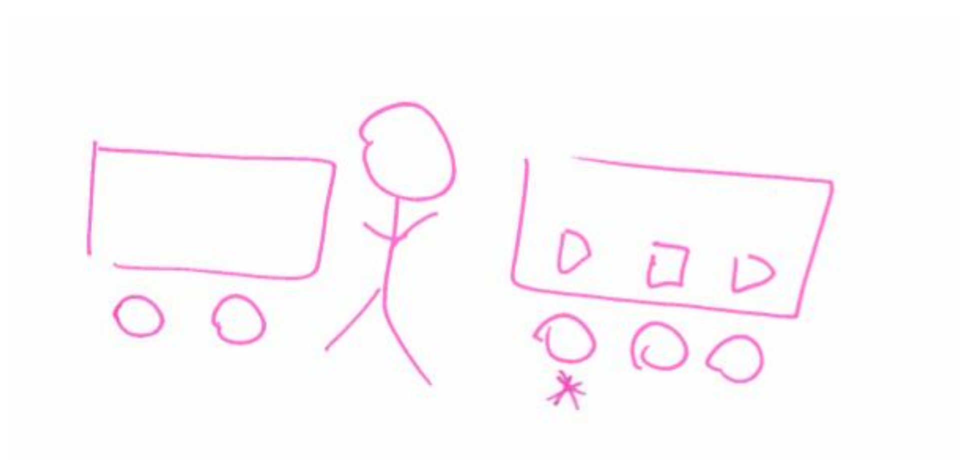
- Motivation to engage in Precalculus plays a role in your level of engagement (major requirements, working on getting a specific grade, etc.)
- Engagement depends on many factors, including:
  - Time constraints, balancing multiple demands on your time.
  - How you feel (emotionally, physically, etc.).
- Comfort is important for your engagement:
  - Comfort with your instructor;
  - Comfort with the topic and how it is taught;
  - Comfort with your group-mates.

APPENDIX G: BETH'S ENGAGEMENT GRAPH AND ARTIFACTS

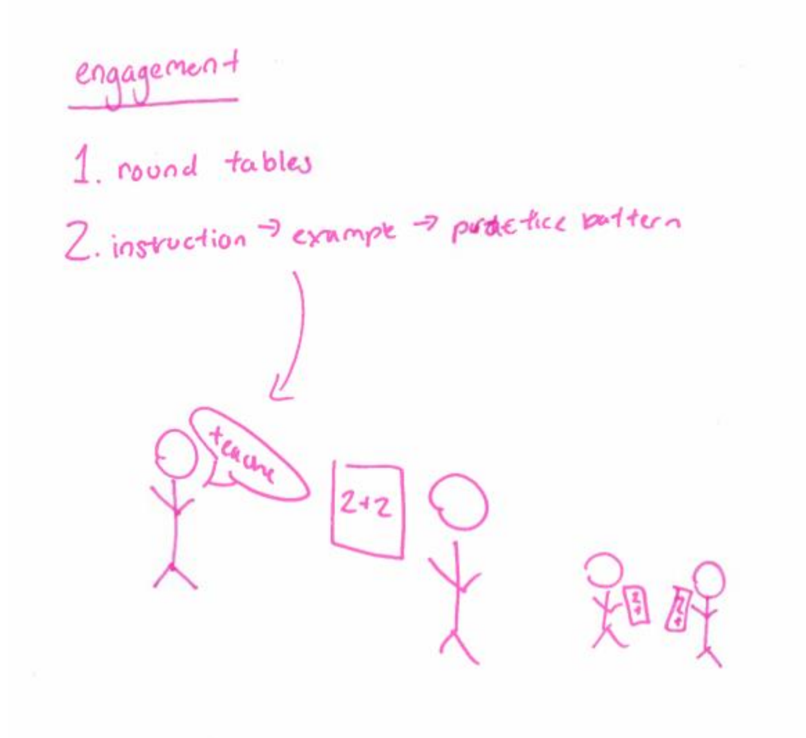
Interview 1 Artifact 1



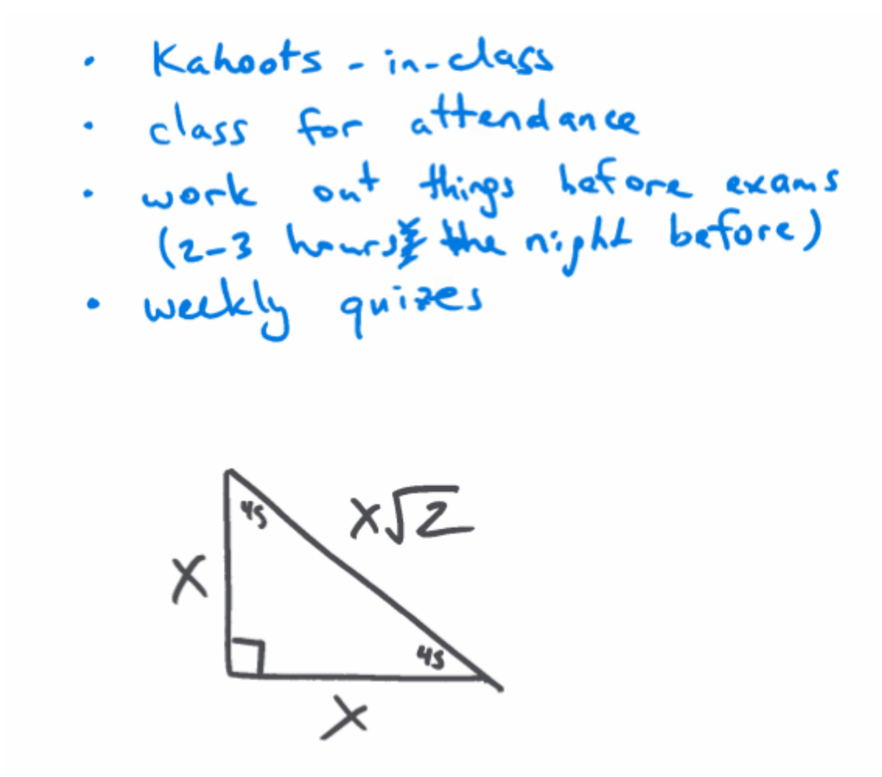
Interview 1 Artifact 2



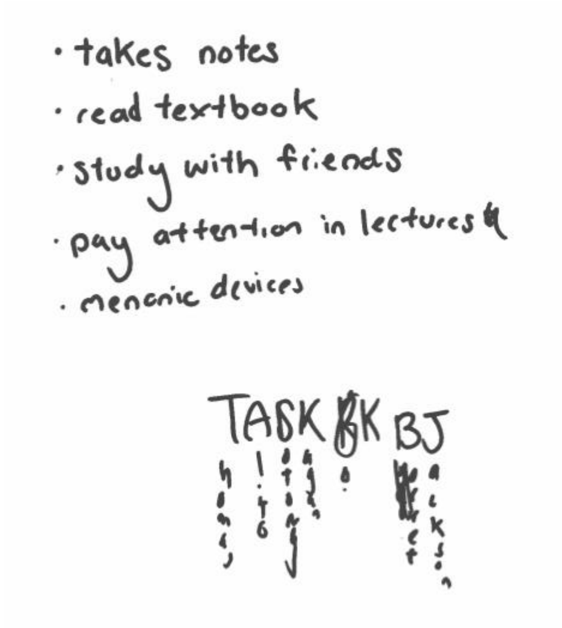
Interview 1 Artifact 3



Interview 3 Artifact 1



Interview 3 Artifact 2

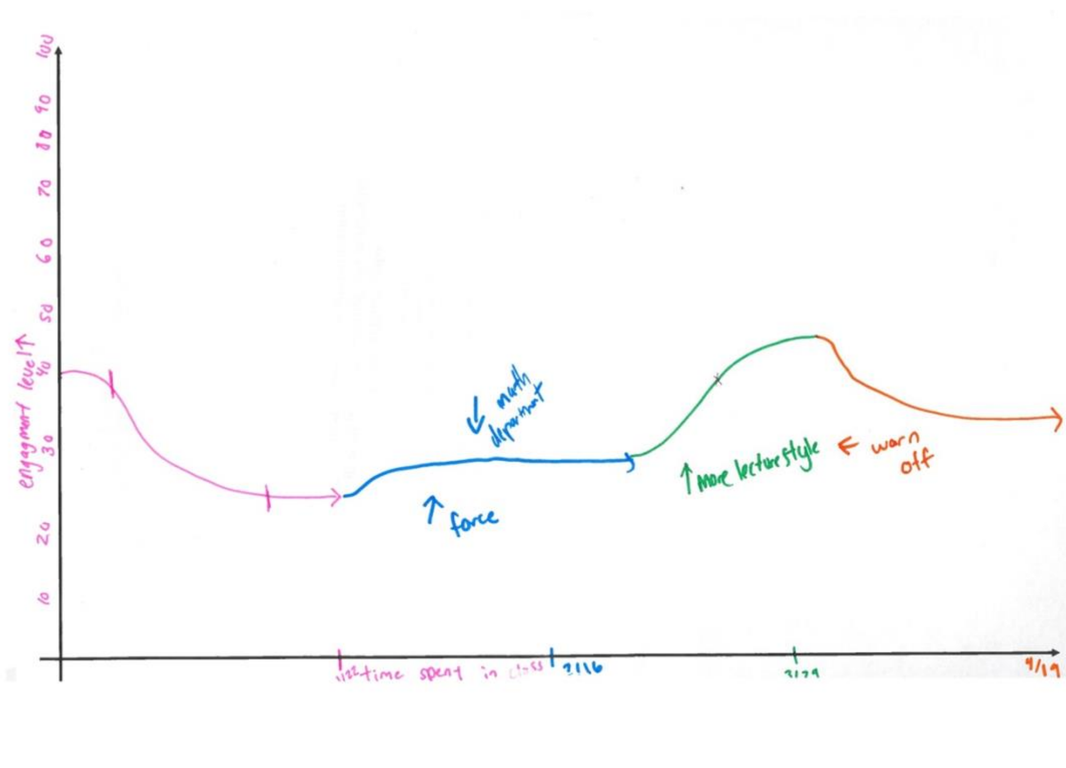


- takes notes
- read textbook
- study with friends
- pay attention in lectures
- menonic devices

TASK

BJ

Final Engagement Graph



APPENDIX H: BETH'S ENGAGEMENT PROFILE

Beth was a freshman student at UGA, a sports management major and considering a business minor. She had previously taken Precalculus as a Junior in high school and later even AP Calculus. However, she did not pass her AP exam and placed into Precalculus in the UGA mathematics placement test. Recognizing that the placement test did not necessarily reflect her level of mathematics knowledge, Beth chose to take Precalculus, hoping it would secure an easy good grade for her. Since she had to take a math class anyway, she thought that she might as well enroll in the "easier" one, as a "GPA-booster."

When the semester and the course began, Beth was determined to pay attention, because she expressed a concern that college Precalculus might be different than high school Precalculus, but once she grew assured that she understood the material, her engagement has gone down significantly. This doesn't mean she did not complete her assignments or stopped coming to class. However, she was not engaging cognitively neither with her instructor and peers, nor the pre-class videos within the flipped classroom setting. She wasn't paying much attention to the examples presented by her instructor in class.

From the start Beth contrasted her less engaging college Precalculus experience with her engaging high school mathematics experience. She would often recall the physical setting in her high school class, where she would be sitting facing her classmates and engage in discussion with them. In contrast she felt like the college classroom she was in had fixed tables and people weren't facing each other: engagement with groups of students felt more artificial for her as opposed to the way the social interaction occurred in her high school Precalculus class.

Another aspect of her engagement with the course that was radically different between her secondary and post-secondary experience was the way mathematics instruction was organized. Her high school Precalculus teacher followed a “I do, we do, you do” pattern, which Beth really enjoyed. However, she was enrolled in a flipped classroom section of Precalculus at UGA, where students were expected to watch pre-recorded videos on the new material, do the pre-class assignment, and work on in-class worksheets in groups during class time (a sort of “you do, we do” pattern). Beth would go on for the entire semester barely watching the videos, she would often do the pre-class assignment last minute and still get good grades, and she had a tense relationship with the in-class worksheets.

One of the reasons to Beth’s particular patterns of engagement was her comfort with the material in the class. She did well in her high school Precalculus (she had an A in that class), so: (a) she did not care for the videos, (b) the pre-class assignments were too easy for her, and (c) even the in-class work wasn’t always challenging enough for her. She shared doing the pre-class worksheet first and then since it was easy for her, she wouldn’t bother watching the videos.

She would do extremely well on quizzes (averaging 95%) and pre-class worksheets prior to Test 1 only to earn a 77% on it. The pre-class assignments were easy for her, while tests were much harder. She would try to get quick feedback on her answer to problems that she felt she understood, only to get a long instructor lecture on things she already knew. One thing that Beth appreciated in the UGA Precalculus curriculum were the assigned ALEKS modules. She found them helpful and productive, because the system would break everything down for her and the structure helped her practice at her own pace, repeating similar prompts only when she did not get a type of problem right. Throughout the semester Beth would repeat that ALEKS kept her engaged.

Beth was often ahead of her peers in the class while solving problems. She was the only participant who on occasion would finish the entire in-class worksheet and would browse on her computer for the rest of the lesson; other participants shared that the in-class worksheets were too long for them, and they never got through them entirely. Part of the reason is due to the fact that Beth used to work on the in-class problems, while the instructor was going over examples. While other students listened and took notes, Beth shared that she would half-listen to the instructor, occasionally looking up if something seemed relevant to her. Otherwise, she was focused on getting through the worksheet problems.

Beth did not experience much collaboration in class, most of her group work was very unidirectional: her helping others, sharing her solutions. She did not really ask her peers for help. When groupwork was expected of her, Beth found herself spending a lot of time explaining things to her struggling groupmates. The way that groupwork was organized in her classroom felt forced to Beth and the only reason she participated is because 15% of her grade in the course depended on that activity. Instead of experiencing meaningful cognitive and social engagement, Beth felt forced to work in a group while "looking engaged," which she would later label as "fake" engagement.

A large factor impacting how Beth experienced the course was shaped by her frustrations with the organization of the course, which she actively attributed to the mathematics department. There were too many changes to the syllabus, course structure, and communication about course policies. Many things did not make sense to her, and she constantly felt like the course was flip-flopping. Some of her frustrations involved:

- quizzes cancelled at the last moment, showing a disregard for student planning and time
- changes to the grading breakdown, where group participation became virtually mandatory through a 15% weight in the course grade; for Beth this forced her to only pretend to look engaged, while feeling held back from working on problems at her pace
- changing decisions on whether the tests would have been graded on a curve or not, especially once she found out her section's Test 1 average was 62 (out of 100): Beth found it unfair that the promise of a curve was reverted; moreover, she believed the class average score was speaking more to the shortcomings of the course rather than its students
- instructional shifts, wherein the instructor would go from pushing for flipped classroom instruction to almost abandoning it halfway through the semester

By the end of the semester, Beth tied the "flip-flopping" to a belief that aligned with a narrative she had heard reiterated by other students as well: "UGA makes Precalculus way harder than it needs to be." Other participants in this study have shared similar stories! For example, Elena shared that while in High School, some of the alumni who went on to be UGA students came to share their experiences and gave advice to avoid taking Precalculus at UGA. A handful of other participants shared how fellow UGA students would express condolences upon hearing that someone was enrolled in this course. Even during the pilot interview, Charlie shared that his friends at Georgia Tech have heard rumors about UGA Precalculus being harder than UGA's Calculus I course.

What solidified the math department's responsibility for Precalculus's notorious reputation for Beth was her observation that most of the negative comments she had heard were associated with the course itself rather than any particular instructor. She had shared that whenever she heard of a class with a bad reputation on campus, there was always an instructor named mentioned in the conversation. She said she did not hear similar things for MATH 1113.

Mid-way through the semester the instructor started implementing more lecturing during class sessions, which immediately increased Beth's self-assessed engagement levels. She admitted that it was easier for her to pay attention when her instructor was lecturing at the board, and she was no longer forced to work in a group. Not because the requirement was changed, but there was simply no time for that anymore with the lectures. Both changes have had an impact on Beth's frustration with the course: she felt less annoyed in class. Removing the forced groupwork and teaching the material in class increased her engagement in a way that felt more authentic to Beth, as opposed to faking and pretending because she felt obligated to look physically engaged.

After the first test Beth worked harder, which resulted in higher grades and feeling better about her (cognitive) engagement with the material on her own. She was self-aware enough to notice since high school that she learned best by "doing" and relying on her memory. This served her particularly well in the second and third modules of the course, when she really put the time into memorizing and applying logarithmic, exponential, and trigonometric rules and identities, as well as working out problems and practice tests. She acknowledged that she did not understand some of the mathematical ideas and concepts, but she could practice and memorize enough to be successful in tests and assignments for the rest of the semester. In contrast, the first module of MATH 1113 focused on functions and did not have as many rules that could have been memorized. Beth also felt that there were more word problems in the beginning, where she would struggle to figure out rules to apply.

As her confidence in solving math problems grew towards the end, the novelty of the instructor's lecturing wore off and Beth was ready to be done with Precalculus and mathematics forever. By the last interview she expressed less frustration and more apathy: she did not feel better about the class, but she did not feel worse either. Beth no longer cared to understand the how or the why behind the material. At this point in the semester, she was often relying on her prior mathematics knowledge from high school.

When asked to compare her engagement in other courses during that semester, Beth noted that she had to some extent similar levels of cognitive engagement in her Accounting class, but she did not feel the need to behave a certain way. Additionally, she felt more comfortable with the more "traditional" lecture structure of the class, because to her flipped classrooms did not feel like learning for her. She could also memorize and apply rules like she did in Precalculus without the added stress of constant changes to the course structure.

Finally, when discussing her most engaging class—Legal Studies—Beth reflected that engagement was about interactions with other people: the instructor, fellow students, learning within the classroom space, as well as collaborating with peers outside of it. She acknowledged that her engagement in Precalculus was not optimal and abysmally lower than her level of engagement in Legal Studies. In the end, Beth believed that she was merely studying to survive college Precalculus, not really learning or understanding it.