ESTIMATION OF DEMAND SYSTEMS AND ITS APPLICATION TO FOOD ASSISTANCE PROGRAMS

by

SHAONAN WANG

(Under the Direction of Chen Zhen)

ABSTRACT

This dissertation aims to estimate a comprehensive food demand system and apply the analysis to food assistance programs. The primary objectives of this study involve estimating different food demand systems and examining the price and income elasticities of households enrolled in the Supplemental Nutrition Assistance Program (SNAP) and the Special Supplemental Nutrition Program for Women, Infants, and Children (WIC). Moreover, this study develops a novel approach to efficiently estimate a large non-linear demand system accounting for price endogeneity.

In Chapter one, we examine whether there was a structural change in WIC households' demand when the revision was implemented. We find that demand for skim/low-fat milk and whole grain became less price elastic post revision. The model also detects a spillover effect to demand for carbonated beverages that are not included in WIC food packages. These results are consistent with the hypothesis that the 2009 revision increased WIC households' preferences for healthier foods.

In Chapter two, we investigate the change in food demand of SNAP households during the benefit month and how food demand responds to changes in food prices. By using two-way

Exact Affine Stone Index demand, we compare the difference in food demand sensitivity between households in the first two weeks of the SNAP benefit month and the last two weeks of the benefit month.

In Chapter three, we derive the asymptotic properties for a tractable iterated three stage least squares (I3SLS) estimator and apply it to estimate a large quadratic almost ideal demand (QUAID) system with endogenous prices. Then we calculate an exact price index from the estimates and compare it with several commonly used price indexes.

INDEX WORDS: Cost of Living Index, Demand Elasticity, Food Assistance Program, Food Demand System, Iterated Three-stage Least Squares

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SHAONAN WANG

BS, Nanjing Agricultural University, China, 2015

MS, Nanjing Agricultural University, China, 2018

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SHAONAN WANG

Major Professor: Chen Zhen
Committee: Travis A. Smith
Mateusz Filipski

Electronic Version Approved:

Ron Walcott
Vice Provost for Graduate Education and Dean of the Graduate School
The University of Georgia
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DEDICATION

This dissertation, the culmination of years of exploration, dedication, and diligence, is dedicated to my family and my friends.

To my family. Your constant love and encouragement have kept me steady on this path. Your unflinching sacrifice and resilience are the underpinnings of my success. Each page of this dissertation carries a part of you. This work is yours as much as it is mine.

To my friends, wherever you may be, your presence has enlivened my journey. Your uplifting words and radiant smiles served as the guiding stars through my longest nights, inspiring me to persist toward the dawn of accomplishment.

For all this and more, this dissertation is a tribute to you. You have been and will always be the quiet heroes of my story.

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CHAPTER 1

THE IMPACT OF THE 2009 WIC FOOD PACKAGE REVISION ON PARTICIPANTS' FOOD AND BEVERAGE PURCHASES

1.1 Introduction

The Special Supplemental Nutrition Program for Women, Infants, and Children (WIC) is the third largest nutrition assistance program, after the Supplemental Nutrition Assistance Program (SNAP) and the National School Lunch Program, in the United States. In 2019, WIC served 6.4 million low-income pregnant and postpartum women, infants and children up to age five at a cost of $5.2 billion, of which $3.2 billon is food cost (FNS 2020[1]). Unlike SNAP food benefits that participants use to purchase retail foods with few restrictions, the WIC food packages prescribe specific supplemental foods to participants. Since the inception in 1972 and until 2009, the WIC food packages remained largely unchanged. In 2005, National Academy of Medicine (formerly called the Institute of Medicine, 2006) recommended changes to the WIC food packages with the goal of aligning WIC supplemental foods with current scientific evidence regarding the nutritional needs of WIC participants, the 2005 Dietary Guidelines for Americans, and infant feeding practice guidelines of the American Academy of Pediatrics.

The US Department of Agriculture (USDA) Food and Nutrition Service (FNS), which administers WIC, largely adopted the recommended changes and state WIC agencies implemented the food package revisions in 2009. Some of the prominent revisions include the

[1] Food and Nutrition Service (FNS). WIC Data Table. Retrieved June 1st, 2022, from https://www.fns.usda.gov/pd/wic-program.

introduction of fruit and vegetables (F/V) issued in $6.00–$10.00 cash value vouchers, and whole-grain cereals and bread. The revision restricted milk fat content and reduced maximum quantities or elimination of milk, eggs, juice, and cheese from some food packages. For example, the maximum quantity of 100% juice prescribed to children aged 1 to 4 years was reduced from 288 fl oz before to 128 fl oz after the revision. FNS also gave discretion to states in allowing food substitutes. For example, soy products may be prescribed as substitutes for milk, and brown rice, bulgur, oatmeal, barley, tortillas or whole wheat pasta as substitutes for whole wheat bread. Overall, the goal of the 2009 revision was to promote a healthier diet among the participant population while recognizing the diverse cultural eating patterns of the participants.

The objective of this study is to evaluate the effect of the 2009 food package revision on WIC household preferences for food and beverage. We attempt to contribute to the WIC literature in three ways. First, we leverage the between-state difference in the timing of the revision in 2009 to identify its effect on preferences. This allows us to avoid problems associated with the well-documented under-reporting of participation in WIC (Bitler, Currie and Scholz 2003) and other safety net programs (Meyer, Mok and Sullivan 2009) in self-reported surveys. Second, we use a flexible demand system to examine whether there has been a structural change in the preference parameters when the food packages were revised. Structural modeling allows us to detect not only changes in the level of purchases but also potential shifts in the price coefficients. This secondary effect, if exists, will help us better understand the nature of the preference change induced by the 2009 revision. Third, the demand system approach allows us to explore changes in preferences for sugary drinks, which were never in the WIC food packages, at the time of the revision. Analyses of these distal outcomes complement the existing literature (Schultz et al. 2015), which is largely concerned with the more proximal outcomes (such as whole grain intake) directly targeted by the revision. As WIC households may compensate the revised food packages by increasing purchases of unhealthy non-WIC items, examining the distal measures is important for detecting any unintended consequences of the 2009 revision.

We estimate WIC household preferences using Nielsen Homescan data from households self-reporting to be WIC recipients. Preferences are approximated by a flexible functional form demand system where we account for zero purchases, price endogeneity and other unobserved heterogeneity. Our results indicate that the 2009 WIC food package revision led to increases in healthier foods encouraged by WIC and decreases in less healthy food and beverage WIC seeks to limit. In addition to changes in the level of demand, we also present a pattern of shifting price elasticities that are consistent with a change in preferences toward healthier foods among WIC households.

The rest of this study is organized as follows. The next section reviews previous evaluations of the 2009 revision, followed by a description of the scanner data. We then discuss the strategy for identifying the effect of the WIC food package revision on recipient households' preferences. Afterward, the demand system model and econometric considerations are discussed. The last two sections present the empirical results and conclude.

1.2 The Literature on the 2009 Food Package Revision

The bulk of the scientific literature on WIC lies in public health. A cursory Google Scholar search of articles since 2000 based on the phrase "women, infants and children program WIC" returned 17,300 results! The majority of this largely descriptive literature examines the associations of WIC enrollment with birth and breastfeeding outcomes, dietary intake, obesity, and health outcomes. Of the fewer economics studies that paid close attention to the role of unobservable in the non-random selection into WIC, several found WIC improved birth outcomes (Bitler and Currie 2005; Hoynes, Page and Stevens 2011). Still, the lack of significant policy changes prior to 2009 made it difficult to establish causation out of many observed WIC-diet associations.

The 2009 food package revision, being the most significant change to WIC since its inception, offers a rare policy experiment with which causal inferences are possible. Schultz et al. (2015) reviewed twenty studies of the 2009 food package revision, including nine that collected purchase quantities or dietary intake. The review concluded that the 2009 policy change was associated with healthier amounts of intake or purchase of the food groups targeted by the revision. Among these, Ishdorj and Capps (2013) analyzed food frequencies collected from two cross sections of Native American WIC participating children before and after the food package revision. They found that the revision was associated with healthier eating patterns including increased frequencies of fruit, vegetable, whole grain and reduced-/low-fat milk consumption. The authors later conducted a similar analysis for Texas WIC children aged 2 to 4 (Ishdorj and Capps 2017). They found the amount of reduced/low-fat milk increased to partially offset the reduction in whole milk. They also found frequencies of 100% juice and diet drinks declined but the frequency of SSBs increased after the food package changes.

Several studies used scanner data to track changes in purchases of WIC households before and after the 2009 revision. Andreyeva and colleagues acquired loyalty card-linked scanner data from a supermarket chain. Using a pre-post design, the authors found that 1) the reduced maximum allowance of 100% juice was associated with net reductions in 100% juice and SSB purchases (Andreyeva et al. 2013); 2) the introduction of whole-grain bread and brown rice was associated with a significant increase in the share of whole-grain products in total bread and rice purchases (Andreyeva and Luedicke 2013); and 3) the reduced maximum allowance of milk and restriction of milk fat content for women and children aged 2+ years were associated with a net reduction of saturated fat from milk and cheese (Andreyeva et al. 2014).

There are a few studies based on the Nielsen Homescan household scanner data. Oh, Jensen and Rahkovsky (2016) were interested in the effect of the 2009 revision on whole-grain purchases. Using propensity score matching to form a control group, the authors estimated that WIC participation was associated with higher whole-grain expenditures, and the 2009 revision doubled the magnitude of this association. In a unique study, Frisvold, Leslie and Price (2020) asked the question: how long do purchase patterns shaped by WIC food packages last after a household ages out of the program? The authors exploited the variation in the length of WIC eligibility after the food package revision to identify the long-run effect of WIC on purchase habits. They found that, although the 2009 revision increased whole grain demand among WIC households, habits over whole grains formed by WIC dissipated after the household losing eligibility.

Of these evaluations of the 2009 revision, three design issues remain. First, most use the pre-post design that lacks a control group. Second, for the one study (Oh, Jensen and Rahkovsky 2016) that uses eligible nonparticipating households as the control group, there are significant pre-revision differences in preferences between WIC and control households as evidenced by WIC households' higher whole grain purchases before the revision. Third, underreporting of WIC status causes Frisvold, Leslie and Price (2020) to focus on estimating the intention-to-treat effect among WIC eligible households. Over the last decade, WIC participation continues to decline (Oliveira 2017). It can be argued that estimates of the effect of WIC on participants are more useful to policymakers than intention-to-treat estimates. We address these issues by leveraging the different timing of the food package revision across states to identify its effect on households self-reporting as WIC participants.

1.3 Data and Descriptive Statistics

Household purchase data come from Nielsen Homescan household scanner data over the 2006–2013 period. Our sample consists of 1,143 WIC-eligible households who self-reported as participating WIC participants. WIC eligibility is met if household income is below 185% of the federal poverty guideline and at least one household member is less than 5 years old.[2] Nielsen ScanTrack retail scanner data provide price information that we use to create food group price indexes and their instruments. We focus on purchases of select groups of packaged food and beverage. Items are identified at the Universal Product Code (UPC) level. We rely on the (post-revision) federal requirements for WIC-eligible foods[3] to categorize items into WIC-eligible food groups and ineligible groups. We focus on food and beverage that are prescribed in significant quantities, barring infant formula, and their less healthy counterparts. This leads to four WIC-eligible groups: skim/reduced-fat milk, whole grain cereal, whole grains, and 100% juice; and five less healthy substitutes: whole milk, other sugary cereal, refined grains, juice drinks, and carbonated beverage.

Table 1.1 shows the descriptive statistics for WIC households and, for comparison, eligible nonparticipating households from the 2006–2013 period. The 1,143 WIC households provided

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[2] A low-income household without a child < 5 years old could be eligible for WIC if it has a pregnant household member. Because only birth year is available for the Kilts Nielsen data, we could not determine with confidence whether a household has a pregnant member. So some self-reported WIC households with pregnant members may have been mistakenly dropped from our sample.

[3] Food and Nutrition Service (FNS). WIC Food Packages - Regulatory Requirements for WIC-Eligible Foods. Retrieved June 1st, 2022, from https://www.fns.usda.gov/wic/wic-food-packages-regulatory-requirements-wic-eligible-foods.

20,643 household-months of purchase data. The average household income is $2,364.17 per month and the average household size is 4.77 persons. Among the 5,001 eligible households who reported not in WIC, their average income is slightly higher than that of WIC households at $2,614.19 and their average household size is also slightly higher at 4.87 persons.

WIC households on average purchase more food and drinks than non-participants. WIC households purchase 24.88 oz whole grain bread per month, which is about 3.62 oz more than purchase of eligible nonparticipants. Similar patterns also appear in the purchase of skim/reduced-fat milk (357 v. 313 oz) and 100% juice (70 v. 61 oz). Because WIC vouchers can be considered as additional income for low-income households, after households receive the prescribed food in kind, the freed-up income may be spent on other foods and goods. Table 1.1 shows WIC households purchased more WIC-ineligible food and beverage than eligible nonparticipants, including carbonated beverage (603 v. 537 oz). This pattern is also consistent with eligible households with higher food preferences self-selecting into WIC.

Table 1.2 shows the change in purchase of WIC participant after WIC package revision. WIC participant's purchase of whole grain bread increased from 19.43 oz to 30.43 oz, which is consistent with previous studies (Oh, Jensen and Rahkovsky 2016; Ng et al. 2018; Frisvold, Leslie and Price 2020). WIC participants also purchased more skim/reduced-fat milk (from 352.51 oz to 360.60 oz) and 100% juice (68.63 oz to 71.40 oz). Changes are not limited to WIC food groups. After the revision, WIC households purchased 104.11 oz less of carbonated beverages. These observations motivate us to take a more formal examination of the direct and spillover effects of the revision on WIC household preferences.

1.4 Identification Strategy

We leverage the difference in the timing of the revision across states to identify the effect of the revision on WIC households' preferences. This affords us not having to use eligible nonparticipants, who as we have shown likely have different preferences from WIC households, as the control group. Instead, WIC households in state that implemented the policy earlier than the federal deadline of October 1, 2009 can be considered to be in the treatment group while those in state that implemented the policy later than the treatment group can be considered to be in the control group until themselves are treated with the revised food packages. For example, New York state revised the WIC packages in January 2009. New York WIC households can be classified as the treatment group who received the treatment in January 2009. In comparison, WIC households in Georgia, which did not implement the revision until the October deadline, serve in the control group until they started to receive the revised food packages in October 2009.

Of the 48 contiguous states and Washington, DC, 16 states (figure 1.1) implemented the revision before October 2009. New York and Delaware were the first to implement the revision in January 2009. An inspection of figure 1.1 reveals no obvious regional differences in timing of the implementation. Nevertheless, our identification strategy would be in jeopardy if timing was determined by state-level socio-economic conditions that were also determinants of WIC households' preferences. That is, there is a possibility that the change in WIC households' preferences is caused by shocks other than the food package revision. To examine the exogeneity of a state's timing decision, we use a logit model to regress a dummy variable for October implementation (st10) on state characteristics. st10 is equal to 1 if the revision is implemented in

October, and 0 if earlier. We also estimate an ordered logit model, where the dependent variable st_imp=(1,5,6,7,8,9,10) indicates the month of implementation.

Table 1.3 reports the logit and ordered logit results. These regressions find no significant correlation between the choice of timing and state characteristics. In particular, there is no evidence that poor economic conditions as measured by high unemployment rate and high poverty rate induced a state to move up implementation of the revision. Demographics and education are also not associated with the timing of implementation. These results indicate that a state's decision on the timing of revision was unlikely to be based on the state's economic situation, thus minimizing the concern about endogeneity of the revision policy.

1.5 The Demand System

To detect the preference change, including behavior related to prices and income, we need to estimate a structural demand system with a flexible functional form. We choose the Exact Affine Stone Index (EASI) demand system developed by Lewbel and Pendakur (2009) for this purpose. Compared with the Almost Ideal Demand (Deaton and Muellbauer 1980) and its quadratic extension (Banks, Blundell and Lewbel 1997), EASI has three advantages. First, EASI allows the Engel curves take arbitrary shapes as it is not restricted by the rank three limitation of Gorman (1981). Second, the EASI Hicksian (as well as Marshallian) demand varies with income while conventional demand systems only allow the Marshallian demand to vary with income through the income effect. Third, the regression errors in the EASI estimating equations can be interpreted as unobserved preference heterogeneity. By contrast, the regression residuals in most other demand systems do not have such an economic underpinning.

We specify the two-way EASI demand system with structural change as:

$$w_{iht} = \alpha_i + \sum_{r=1}^{R} b_{ir} y_{ht}^{r} + \sum_{j=1}^{n} A_{ij} \log(p_{jht}) + \sum_{j=1}^{n} B_{ij} \cdot y_{ht} \cdot \log(p_{jht}) + \boldsymbol{c}_i \boldsymbol{Z}_{ht} + \boldsymbol{d}_{i1} \boldsymbol{Year} + \boldsymbol{d}_{i2} \boldsymbol{Month} + \boldsymbol{d}_{i3} \boldsymbol{Region} + \boldsymbol{d}_{i4} \cdot \boldsymbol{Region} \cdot \boldsymbol{time} + \boldsymbol{\delta}_{i1} \boldsymbol{MM}_t + \boldsymbol{\delta}_{i2} \boldsymbol{ST} + \delta_{i3} Revise + \sum_{j=1}^{n} \gamma_{ij} \cdot Revise \cdot \log(p_{jht}) + \theta_i \cdot Revise \cdot y_{ht} + \epsilon_{iht} \qquad (1.1)$$

where w_{iht} is budget share of household h for food category i ($i = 1, \dots, n-1$) at time t. $n = 10$ is the number of goods in the demand system that includes four WIC food groups, five non-WIC food groups, and a numéraire representing all other goods and services. y_{ht} is equal to household income deflated by a stone price index. p_{jht} is the price index of food category j for household h at time t. $\boldsymbol{Z_{ht}}$ contains a series of demand shifter including household demographic characteristics. $\boldsymbol{Year}$, $\boldsymbol{Month}$ and $\boldsymbol{Region}$ are year, calendar month, and Census region dummies, respectively. $\boldsymbol{time}$ is a linear time trend. $\boldsymbol{Region} \cdot \boldsymbol{time}$ captures region-specific secular trend in demand. $\boldsymbol{MM}_t$ is a 7×1 vector of indicators with elements $MM_{t,T} = 1[t \geq T]$, where $1[\cdot]$ is the indicator function and $T = \{\text{Jan, May, June, Jul, Aug, Sept, Oct}\}$ of 2009. These are the months in 2009 when at least one state started prescribing the revised food packages. For example, for the March 2009 purchases of a household, $\boldsymbol{MM}_t = \{1,0,0,0,0,0,0\}$. The role of $\boldsymbol{MM}_t$ is to control for national demand shocks occurring at the time when some states implemented the revision. $\boldsymbol{ST}$ is a vector of seven state group dummies, where we classify states into seven groups based on their months of implementation. We use $\boldsymbol{ST}$ to control for unobserved preference heterogeneity common across states that implemented the revision in the same month. $Revise$ is an indicator equal to 1 if WIC household h made the purchase after the revision was implemented in its state, and 0 otherwise. The α, b, A, B, c, d, δ, γ, and θ's are structural preference parameters to be estimated. ϵ_{iht} is the residual.

We use the interaction terms $Revise \cdot \log(p_{jht})$ and $Revise \cdot y_{ht}$ to detect preference changes as related to the price and income effects, respectively. The existing literature focuses on changes in the level of demand and overlooks potential shifts in how WIC households respond to prices and income. Under our specification, the effect of the food package revision on the budget share of food category i is measured by $\delta_{i3} + \sum_{j=1}^{n} \gamma_{ij} \cdot \log(p_{jht}) + \theta_i \cdot y_{ht}$.

To account for the zero purchases, the budget share w_iht is modeled as a censored dependent variable. We apply the extended Amemiya's generalized least squares (AGLS) estimator developed by Zhen et al. (2014) to the system of eq. (1) to estimate the structural parameters. The extended AGLS estimator expands the original AGLS estimator (Amemiya 1979; Newey 1987) from a single Tobit regression, with endogenous explanatory variables, to a system of Tobit regressions. In the case of a demand system, the extended AGLS estimator offers the option of imposing the cross-equation restrictions of homogeneity, symmetry and adding up.

The price coefficient estimates A, B, and γ could be biased if endogeneity in p_{jht} is not corrected for. There are three sources of endogeneity. First, there is the unit value bias (Cox and Wohlgenant 1986; Deaton 1988). Within each food group, a number of products are available at very different prices. Much of the within-group price difference is driven by product quality, either real or perceived. If we use the average price of products household h purchased as p_{jht}, the price coefficient estimates will likely be biased measures of the household's quantity response to price changes. This occurs because the average price is a function of the household's (mostly unobserved) preferences over quality that also enters the residual ϵ_{iht}. To correct the unit value bias, we follow the approach of Zhen et al. (2011) by using the Törnqvist index as p_{jht} in eq. (1).

The Törnqvist price index is constructed as

$$p_{jht} = \exp(0.5 \cdot \textstyle\sum_v (s_{v0} + s_{vht}) \cdot \ln{(\pi_{vht}/\pi_{v0})}) \tag{1.2}$$

where π_{vht} and s_{vht} are the price and budget share (within food group j) of UPC v for household h in period t, respectively; and π_{v0} and s_{v0} are base price and base budget share of UPC v. We set the base at the national average over the same period. When a household did not purchase anything in food group j in period t, we replace the household index with the average retail Törnqvist index of stores shopped by household. We calculate the retail Törnqvist index using barcode-level prices and sales from the ScanTrack retail scanner data. The household and retail price indices for each food group use the same base such that the index numbers are comparable. The Törnqvist price index is a superlative in the sense that it is exact for the translog cost function (Diewert 1976). This allows us to build a cost-of-living index for each food group that accounts for barcode-level quality differences without explicitly estimating a demand system for products differentiated at the barcode level.

The second source of price endogeneity relates to unobserved preference heterogeneity over quantity. For households with above-average preferences for food group j, they are likely to pay below-average prices even after accounting for quality because savings from employing cost minimization strategies such as intensive comparison shopping are greater when demand is higher. In this case, the causation is reversed: higher demand leads to lower prices paid. The final source of bias comes from the familiar supply-demand simultaneity. This could be caused by local demand shocks that are common across households and to which retailers respond through price actions. We instrument the endogenous p_{jht} by the weighted average retail Törnqvist index of all counties in a 100-mile radius from the household's home county, excluding the home county. The weight for each county is its inverse distance to the home county. Hausman (1997) proposed using surrounding-area prices as instruments in demand systems where supply-side variables lack the specificity to identify the price effects for highly disaggregated goods. This strategy was later popularized in Nevo's (2001) analysis of brand-level demand for breakfast cereal. The identification assumption is that local demand shocks are spatially uncorrelated after controlling for observed demographics, or are not responded to by chain retailers. We account for broader demand shocks at the national, regional, and state level and over time by the rich set of fixed effects in eq. (1). In terms of firms' pricing decisions, there is strong evidence that chain retailers do not price to local demand (DellaVigna and Gentzkow 2019).

1.6 Empirical Results

The highest order of income polynomial R in eq. (1.1) determines the shape of the Engel curves. However, too high of a value of R will cause severe multicollinearity. We test the joint significance of the b_{iR}'s, starting at $R = 2$, increase R by one and re-test if the last test is significant. This testing procedure leads to our determination that $R = 3$, a rank four demand, is appropriate given the narrow range of y_{ht} owing to the low income of WIC households.

Table 1.4 presents the income elasticities for the ten food groups and the numéraire. Almost all income elasticities lie between 0 and 1 indicating that the food groups covered in the demand system are necessities for WIC households. The income elasticities shifted significantly for some food groups post revision. Among those that changed by 50% or more, skim/reduced-fat milk and CSD experienced an increase while whole grain cereal, other sugary cereal, refined grains, and 100% juice experienced a decline. The income elasticities are identified by within- and between-household income variations. A decomposition of the variance of y_{th} finds that 90.74% of the variance comes from between-household variation with within-household variation accounting for the rest. Figure 1.2 shows that the impact of WIC package revision on the sensitivity of household food demand in relation to income is relatively limited. At the 5% significance level, carbonated beverages exhibit increased income elasticity, whereas 100% juice demonstrates decreased income elasticity. Despite the inclusion of whole grain bread in the revised WIC package, the income effect on demand has not changed substantially.

The last column of table 1.4 also shows how the budget share for ten food groups including numéraire changed after WIC package revision. As a complement of previous studies, our structural model accounts for price and income effects driven by the WIC package revision. Our result is consistent with some previous studies (Andreyeva et al. 2013; Odoms-Young et al. 2013; Oh, Jensen and Rahkovsky 2016; Ng et al, 2018; Frisvold, Leslie and Price 2020). After WIC package revision, WIC participants tended to consume more skim/reduced-fat milk and whole grain and less whole milk, which is consistent with the recommendations of Dietary Guidelines for Americans (DGA). Under interim rule, whole grain/wheat bread and other whole grain food were added in Food Packages III, IV, V and VII (FNS, 2007), leading to an increase in purchase of whole grain products. The revision has reduced the maximum allowances of juice for women and children in Food Packages IV-VII (FNS, 2007[4]) since excessive intake of fruit juice has increased the risk of obesity in children (Wojcicki and Heyman 2012; Shefferly 2016) and juice provides no additional nutritional benefit over whole fruit (FNS 2007; Heyman et al. 2017). Therefore the purchase of juice has decreased after package revision. We also find a decrease in the purchase of carbonated soft drinks, suggesting a spillover effect of WIC package revision to the food groups not covered by the packages.

Table 1.5 and Table 1.6 report the pre- and post-revision Marshallian price elasticities, respectively. Regarding cross-price effects, skim/reduced-fat milk and whole milk turned from

[4] For children, the maximum monthly allowance of juice reduced from 288 fl oz to 128 fl oz. For pregnant and partially breastfeeding women, the maximum monthly allowance of juice was reduced from 288 fl oz to 144 fl oz; for postpartum women from 192 fl oz to 96 fl oz; and for fully breastfeeding women, from 336 fl oz to 144 fl oz.

substitutes before the revision to complements after the revision. This shift in the cross-effect between milk types is consistent with the change in WIC guidance on the prescription of milk. Before the revision, there was few restrictions on milk fat content. So a WIC agency may very well treat milk of different fat content as substitutes. After the revision, whole milk may only be prescribed to children aged 12 months to 2 years and skim/reduced-fat milk to older children and women (FNS, 2014[5]). A WIC household in the post-revision period is more likely to have both skim/reduced-fat milk and whole milk than the pre-revision period, thus creating a situation of complementarity at the household level.

The reduction in skim/reduced-fat milk own-price elasticity and increase in whole milk own-price elasticity (both in absolute value) are also consistent with the programmatic change to milk prescription. In theory, WIC households are not very price elastic for the prescribed food. As the amount of prescribed skim/reduced-fat (whole) milk increases (decreases), price sensitivity will decline (rise). Similarly, with the introduction of whole grain in the WIC package, the WIC household's demand for whole grain has become less price-sensitive. The package revision also increased WIC participants' price sensitivity to beverages through spillover effects. This finding implies that as the WIC program has improved, WIC households have become more concerned about a healthy diet, and hence the price effect on their demand for healthy food has decreased. However, a food group being WIC eligible does not imply the price

[5] Food and Nutrition Service (FNS). Special Supplemental Nutrition Program for Women, Infants and Children (WIC): Revisions in the WIC Food Packages. Retrieved June 1st, 2022, from https://www.federalregister.gov/documents/2014/03/04/2014-04105/special-supplemental-nutrition-program-for-women-infants-and-children-wic-revisions-in-the-wic-food.

elasticity should be zero. There are seven WIC food packages post revision.[6] Not all households are prescribed the same type and amount of foods. Those who do not receive a food type or not in sufficient amount (for the entire household) must purchase additional using other cash or other forms of payment. As the demand system estimates price responses at the margin for an average household, the price elasticities will not be zero for WIC-eligible foods. For example, after the revision households may choose cheese, tofu/soy-based beverages as substitutes for milk. Those who choose milk substitutes in their WIC packages would have to pay for milk and are likely sensitive to changes in milk price.

1.7 Conclusion

This study estimates a WIC-household demand system for WIC food groups and several closely related food groups. We leverage the 2009 WIC food package revision as a policy experiment to test whether there has been a structural change in WIC households' preferences post revision. Our results indicate that the 2009 revision led to increases in healthier foods encouraged by WIC and decreases in less healthy food and beverage WIC seeks to limit. In addition to changes in the level of demand, we also present a pattern of shifting price elasticities that are consistent with a change in preferences toward healthier foods among WIC households. The finding of spillover effects is new and not explored by previous studies.

Our findings provide informative insights for nutrition policy and research. Our study avoids the issue of underreporting participant numbers in the HomeScan. Researchers using the Difference-in-Differences (DID) method to assess the effects of WIC package revisions should

[6] Food and Nutrition Service (FNS). Special Supplemental Nutrition Program for Women, Infants and Children (WIC): Revisions in the WIC Food Packages. Retrieved June 1st, 2022, from https://www.federalregister.gov/documents/2007/12/06/E7-23033/special-supplemental-nutrition-program-for-women-infants-and-children-wic-revisions-in-the-wic-food.

be cautious when using WIC-eligible non-participants as a control group, as this may lead to an underestimation of the impact due to potential misclassification of WIC participants. Our results illustrate the importance of understanding how multiple food policies may interact with each other to amplify or dampen the effect of each policy. For example, if participants become more responsive to price changes in sugar-sweetened beverages (SSB) after the WIC package revision, then implementing an SSB tax at a constant rate would result in a more substantial effect on SSB demand than before the revision. This indicates that the WIC package revision could potentially enhance the influence of the SSB tax.

Our study also has some limitations. The elasticities we estimate did not exclude the impact of WIC in-kind food vouchers. Since WIC mainly offers quantity-based vouchers (except fruit and vegetable cash-value vouchers), households consuming fewer WIC-eligible foods may exhibit complete insensitivity to price variations in such products (price elasticity is 0). Therefore, it is essential to bifurcate demand into two components: price-invariant demand and typical demand. The price elasticities we present only measure the aggregate of these components rather than considering them individually. One possible solution is to estimate a generalized demand considering pre-committed quantity (Piggott and Marsh 2004). Second, heterogeneous effects across different units and time periods (Callaway and Sant'Anna 2021) can be incorporated into the demand model while the estimation would be much more complicated.

Table 1.1: Descriptive Statistics for WIC Participants and Nonparticipants

	WIC participant		WIC non-participant		
	Mean	STD	Mean	STD	Difference
Budget Share (% of income)					
Skim/reduced-fat milk	0.531	0.013	0.380	0.009	0.152
Whole milk	0.355	0.009	0.267	0.008	0.088
Whole grain breakfast cereal	0.081	0.005	0.062	0.003	0.018
Other cereal	0.593	0.013	0.437	0.010	0.156
Whole grains	0.173	0.005	0.133	0.005	0.041
Refined grains	0.362	0.008	0.333	0.007	0.029
100% juice	0.224	0.007	0.145	0.005	0.079
Juice drink	0.550	0.015	0.407	0.011	0.142
Carbonated beverage	0.864	0.020	0.671	0.016	0.193
Purchase (OZ)					
Skim/reduced-fat milk	356.5	481.9	312.8	444.4	43.7
Whole milk	191.2	372.3	158.6	318.9	32.6
Whole grain breakfast cereal	9.3	24.4	8.5	23.5	0.7
Other cereal	58.5	71.2	53.3	70.4	5.2
Whole grains	24.8	43.6	21.2	40.6	3.6
Refined grains	77.0	97.3	75.9	95.4	1.0
100% juice	69.9	125.6	60.6	125.5	9.3
Juice drink	240.1	333.6	220.4	330.5	19.6
Carbonated beverage	602.8	823.7	536.6	802.2	66.1
Characteristics					
Household size	4.775	1.503	4.870	1.462	

Number of children	1.522	0.681	1.361	0.602
Income ($/month)	2364	1007	2614	1062
White	0.789	0.408	0.781	0.414
Black	0.106	0.307	0.113	0.317
Asian	0.020	0.141	0.028	0.164
College	0.740	0.439	0.771	0.420
Number of unique households	1143		5001	

Notes: we use UPC description and refer to the federal requirements for WIC-eligible foods to classify items into food and beverage groups encouraged by the revised food packages and those not WIC eligible. Some of the federal requirements are: (1) Skim/reduced-fat milk: should be skim, low-fat, or reduced fat (2% reduced); should contain vitamin A and D. (2) Whole grain cereal: Should be plain, whole grain or whole wheat; should be instant ready-to-eat or instant cereal (e.g. oatmeal). (3) Whole grain: whole wheat/grain bread, buns and rolls; whole wheat tortillas; brown rice, bulgur, oatmeal without added sugar, oil and salt. (4) 100% Juice: 100% unsweetened juice. (5) Carbonated beverage: carbonated regular soft drinks and diet soft drinks.

Table 1.2: Budget Share, Purchase and Expenditure for WIC Participants Before and After Package Revision

	Pre		Post		
	Mean	STD	Mean	STD	Difference
Budget Share (% of income)					
Skim/reduced-fat milk	0.547	0.015	0.516	0.011	-0.031
Whole milk	0.402	0.010	0.307	0.008	-0.094
Whole grain breakfast cereal	0.078	0.003	0.083	0.006	0.006
Other cereal	0.563	0.012	0.624	0.014	0.061
Whole grains	0.119	0.004	0.229	0.006	0.110
Refined grains	0.376	0.007	0.349	0.008	-0.027
100% juice	0.230	0.006	0.218	0.007	-0.012
Juice drink	0.542	0.014	0.558	0.016	0.016
Carbonated beverage	0.881	0.020	0.847	0.020	-0.034
Purchase (OZ)					
Skim/reduced-fat milk	352.5	495.2	360.5	467.9	8.0
Whole milk	222.7	432.0	159.3	296.4	-63.4
Whole grain cereal	9.5	22.9	9.0	25.9	-0.5
Other cereal	58.7	69.2	58.2	73.1	-0.5
Whole grains	19.4	37.8	30.4	48.1	10.9
Refined grains	88.8	104.0	65.0	88.4	-23.8
100% juice	68.6	132.4	71.3	118.3	2.7
Juice drink	237.4	328.5	242.8	338.6	5.3
Carbonated beverage	654.5	860.4	550.3	781.1	-104.1
Expenditure ($)					

No-fat/low-fat milk	9.090	12.807	9.080	11.703	-0.010
Whole milk	6.473	11.297	4.993	8.488	-1.480
Whole grain cereal	1.247	2.925	1.290	3.911	0.043
Other sugar cereal	9.525	10.663	10.359	12.516	0.834
Whole grain	2.015	3.978	4.043	6.407	2.028
Refined grain	6.386	6.967	5.601	6.729	-0.785
100% juice	3.713	6.592	3.396	5.448	-0.317
Juice drink	8.094	10.811	8.515	11.498	0.421
Carbonated beverage	14.149	19.787	13.489	19.668	-0.660

Table 1.3: Determinants of State's Implementation Timing for WIC Package Revision

	imp10		imp_all	
variable	Coeff	std.err	Coeff	std.err
Unemployment rate	-0.206	0.215	-0.187	0.202
Poverty rate	-0.041	0.168	-0.047	0.162
Education (% of bachelor degree)	-0.041	0.063	-0.067	0.070
Proportion of Asian	17.301	23.229	11.977	22.521
Proportion of Black	10.679	14.190	7.859	13.447
Proportion of White	8.329	15.433	5.454	14.687
Number of observation	48		48	

Notes: imp10 indicates whether the state implemented the WIC package revision on Oct 1st, 2009. We run logit model to examine whether the choice of states is affected by some economic factor. imp_all is an ordinal variable indicating which month (how earlier) the state would choose to implement the package revision before the last date of mandatory. We run ordered logit model to examine whether the economic factor would affect the state's choice.

Data source:

Unemployment rate: U.S. Bureau of Labor Statistics, https://www.bls.gov/lau/rdscnp16.htm

Poverty rate: U.S. Census Bureau. Small Area Income and Poverty Estimates (SAIPE) Program. https://www.census.gov/programs-surveys/saipe.html.

Education: FRED. Educational Attainment, Annual. https://fred.stlouisfed.org/release/tables?rid=330&eid=391444&snid=391485#.

Race: Bridged-Race Population Estimates. CDC. U.S. Census Bureau and NCHS. https://wonder.cdc.gov/bridged-race-population.html.

Table 1.4: Income Elasticities and Change in Budget Share

	Income Elasticities		Change in Budget share
	Pre	Post	
Skim/reduced-fat milk	0.06083	0.51832	0.00135
	(0.19291)	(0.19797)	
Whole milk	0.421	0.35788	-0.00121
	(0.08290)	(0.08915)	
Whole grain cereal	1.13271	0.69709	-0.00201
	(0.25361)	(0.25674)	
Other cereal	0.50049	0.12022	0.0007
	(0.15055)	(0.14834)	
Whole grains	0.77985	0.6152	0.00146
	(0.16242)	(0.13751)	
Refined grains	0.88934	0.40148	-0.0033
	(0.17154)	(0.18274)	
100% juice	0.80737	0.12324	-0.00086
	(0.13934)	(0.13726)	
Juice drink	0.88859	0.7744	-0.0025
	(0.12322)	(0.13050)	
Carbonated beverage	0.26408	0.68985	-0.00212
	(0.12538)	(0.12897)	
Numéraire	1.04285	1.04786	0.00849
	(0.00236)	(0.00245)	

Note: The values in parentheses are standard errors.

Table 1.5: Price Elasticities Before the WIC Package Revision

Pre	Skim/reduced-fat milk	Whole milk	Whole grain cereal	Other cereal	Whole grains
Skim/reduced-fat milk	**-0.927**	0.461	-0.06	0.948	-0.241
	(0.247)	(0.079)	(0.088)	(0.147)	(0.06)
Whole milk	0.586	**-0.616**	-0.048	-0.368	0.269
	(0.09)	**(0.069)**	(0.056)	(0.076)	(0.033)
Whole grain cereal	-0.038	-0.047	**-2.145**	-0.829	0.177
	(0.314)	(0.163)	**(0.24)**	(0.264)	(0.133)
Other cereal	0.996	-0.291	-0.4	**-1.901**	-0.315
	(0.15)	(0.068)	(0.075)	**(0.166)**	(0.054)
Whole grains	-0.52	0.535	0.202	-0.653	**-1.081**
	(0.192)	(0.092)	(0.123)	(0.168)	**(0.093)**
Refined grains	-0.05	0.003	-0.651	0.155	0.149
	(0.233)	(0.127)	(0.115)	(0.192)	(0.086)
100% juice	-0.01	-0.306	-0.33	-0.96	0.055
	(0.165)	(0.085)	(0.098)	(0.15)	(0.056)
Juice drink	-0.712	-0.251	-0.042	0.04	0.022
	(0.148)	(0.073)	(0.071)	(0.097)	(0.038)
Carbonated beverage	-0.329	-0.169	0.523	0.178	0.264
	(0.153)	(0.057)	(0.067)	(0.111)	(0.041)
Numéraire	0.002	-0.001	0.008	0.011	-0.003
	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)
Pre	Refined grains	100% juice	Juice drink	Carbonated beverage	Numéraire
Skim/reduced-fat milk	-0.008	0.004	-0.793	-0.447	1.038
	(0.134)	(0.085)	(0.177)	(0.224)	(0.376)
Whole milk	0.027	-0.18	-0.351	-0.293	0.569
	(0.086)	(0.054)	(0.104)	(0.101)	(0.226)
Whole grain cereal	-0.798	-0.316	-0.12	1.465	1.509
	(0.231)	(0.187)	(0.292)	(0.37)	(0.756)
Other cereal	0.092	-0.448	0.065	0.266	1.462

	(0.12)	(0.084)	(0.119)	(0.173)	(0.284)
Whole grains	0.211	0.075	0.055	0.738	-0.327
	(0.163)	(0.094)	(0.145)	(0.189)	(0.387)
Refined grains	**-2.155**	-0.992	0.279	1.841	0.536
	(0.307)	(0.128)	(0.156)	(0.259)	(0.489)
100% juice	-1.159	**-1.1**	0.636	1.161	1.203
	(0.131)	**(0.112)**	(0.117)	(0.175)	(0.393)
Juice drink	0.146	0.259	**-0.791**	-0.282	0.728
	(0.078)	(0.054)	**(0.175)**	(0.142)	(0.314)
Carbonated beverage	0.761	0.397	-0.214	**-1.377**	-0.269
	(0.095)	(0.062)	(0.112)	**(0.164)**	(0.252)
Numéraire	0.003	0.005	0.006	-0.015	**-1.061**
	(0.003)	(0.002)	(0.004)	(0.003)	**(0.011)**

Note: The values in parentheses are standard errors.

Table 1.6: Price Elasticities After the WIC Package Revision

Post	Skim/reduced-fat milk	Whole milk	Whole grain cereal	Other cereal	Whole grains
Skim/reduced-fat milk	**-0.743**	-0.383	0.333	0.134	0.658
	(0.262)	(0.072)	(0.091)	(0.143)	(0.07)
Whole milk	-0.502	**-1.293**	-0.474	-0.158	-0.28
	(0.091)	**(0.077)**	(0.049)	(0.078)	(0.041)
Whole grain cereal	0.754	-0.714	**-1.272**	0.255	-0.018
	(0.326)	(0.152)	**(0.172)**	(0.217)	(0.095)
Other cereal	0.185	-0.08	0.15	**-0.864**	-0.112
	(0.155)	(0.068)	(0.065)	**(0.153)**	(0.051)
Whole grains	1.273	-0.402	-0.016	-0.182	**-0.801**
	(0.171)	(0.084)	(0.071)	(0.121)	**(0.067)**
Refined grains	1.241	-0.576	-0.496	0.129	-0.614
	(0.236)	(0.129)	(0.089)	(0.184)	(0.088)
100% juice	-0.368	0.202	0.399	-0.258	-0.042

	(0.19)	(0.09)	(0.065)	(0.142)	(0.056)
Juice drink	0.254	0.221	-0.585	0.377	-0.497
	(0.143)	(0.064)	(0.07)	(0.096)	(0.051)
Carbonated beverage	-0.195	-0.115	-0.384	1.62	-0.133
	(0.163)	(0.062)	(0.06)	(0.144)	(0.049)
Numeraire	-0.017	0.014	0.014	-0.031	0.007
	(0.004)	(0.002)	(0.002)	(0.002)	(0.001)
Post	Refined grains	100% juice	Juice drink	Carbonated beverage	Numeraire
Skim/reduced-fat milk	0.731	-0.188	0.273	-0.206	-1.103
	(0.128)	(0.099)	(0.162)	(0.221)	(0.403)
Whole milk	-0.364	0.142	0.33	-0.18	2.449
	(0.085)	(0.062)	(0.095)	(0.11)	(0.231)
Whole grain cereal	-0.577	0.462	-1.38	-1.217	3.013
	(0.173)	(0.127)	(0.299)	(0.299)	(0.639)
Other cereal	0.066	-0.083	0.427	2.37	-2.143
	(0.107)	(0.082)	(0.117)	(0.22)	(0.287)
Whole grains	-0.661	-0.048	-1.064	-0.42	1.714
	(0.12)	(0.077)	(0.151)	(0.168)	(0.286)
Refined grains	**-1.349**	0.065	-0.621	2.811	-0.966
	(0.243)	(0.112)	(0.165)	(0.297)	(0.387)
100% juice	0.1	**-1.03**	-0.488	0.115	1.284
	(0.116)	**(0.143)**	(0.121)	(0.198)	(0.399)
Juice drink	-0.299	-0.243	**-1.827**	-1.548	3.369
	(0.079)	(0.058)	**(0.145)**	(0.155)	(0.327)
Carbonated beverage	1.115	0.035	-1.224	**-2.736**	1.33
	(0.111)	(0.077)	(0.125)	**(0.219)**	(0.24)
Numeraire	-0.009	0.002	0.037	0.017	**-1.083**
	(0.002)	(0.002)	(0.003)	(0.003)	**(0.009)**

Note: The values in parentheses are standard errors.

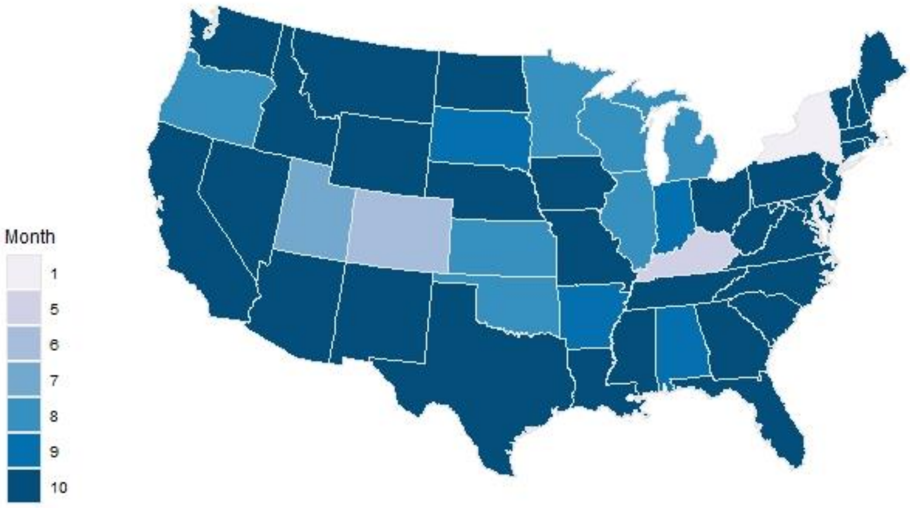

Figure 1.1: Month of Implementation of the 2009 WIC Food Package Revision

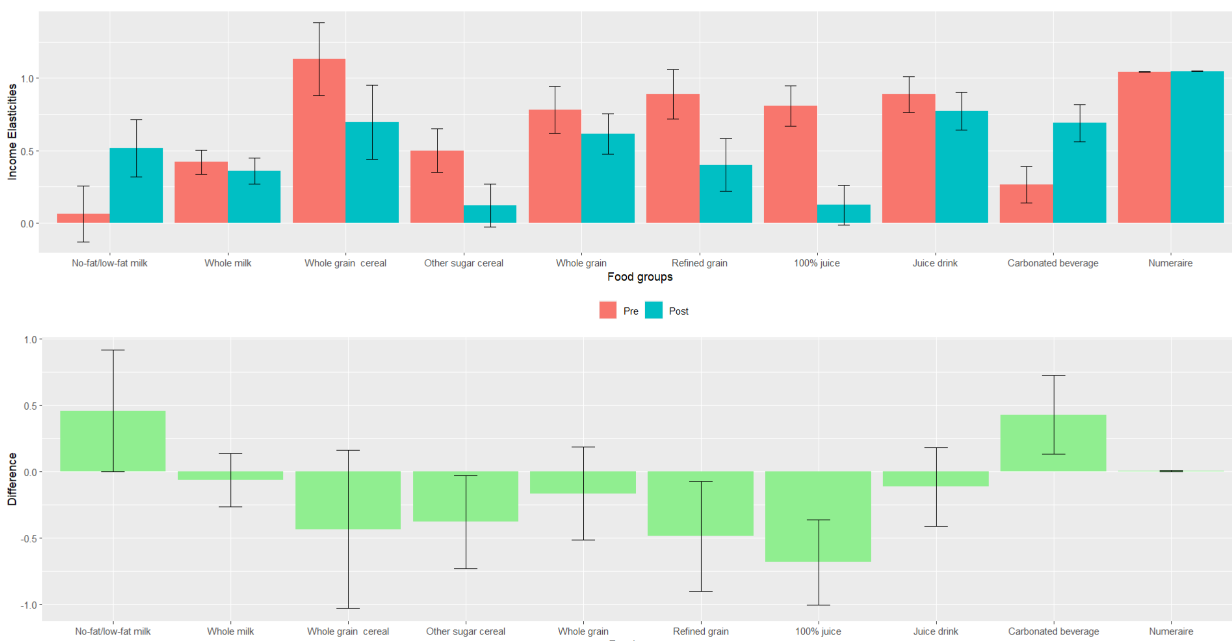

Figure 1.2: Income Elasticities of Food Groups

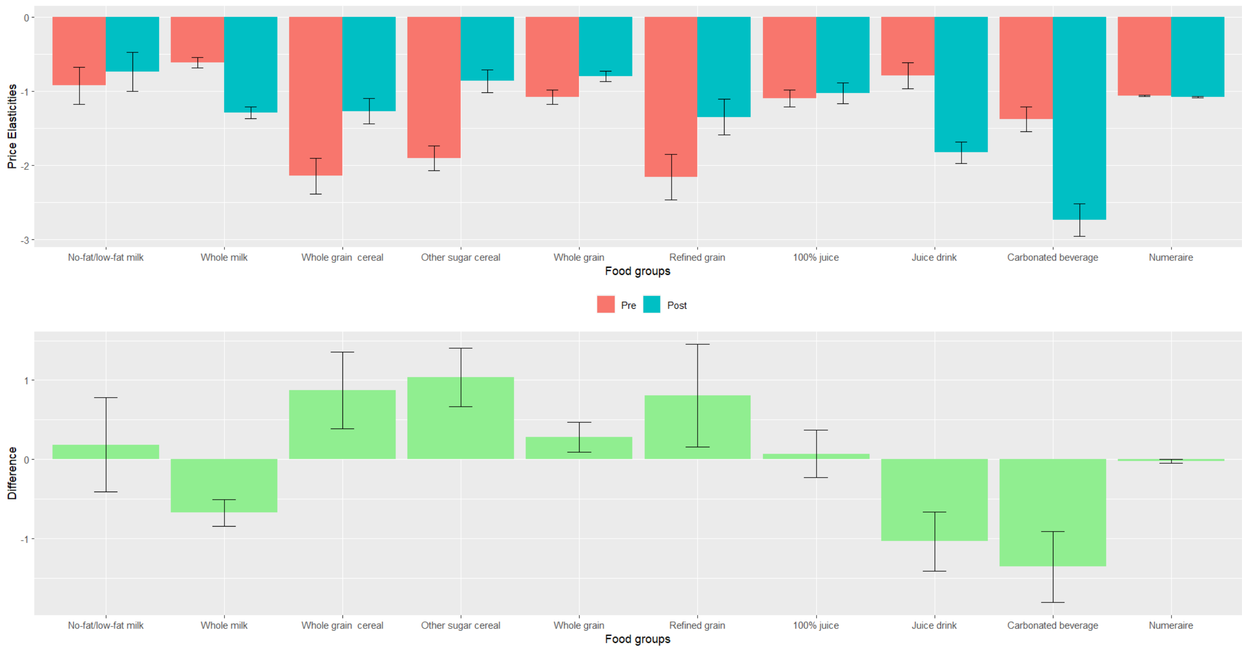

Figure 1.3: Own-price Elasticities of Food Groups

CHAPTER 2

EXAMINING THE CONSISTENCY OF DEMAND ELASTICITY IN SNAP HOUSEHOLD FOOD DEMAND DURING SNAP BENEFIT MONTHS

2.1 Introduction

The Supplemental Nutrition Assistance Program (SNAP), formerly known as the Food Stamp Program, is a federal initiative with the primary objective of alleviating food insecurity and enhancing nutritional standards among low-income households. As the nation's largest food and nutrition assistance program, SNAP has served more than 41 million low-income participants with average monthly benefits of $217.9 per person in fiscal year 2021[7]. It also functions as a significant socioeconomic smoother by mitigating income disparities and providing a financial buffer to low-income households, promoting stability and reducing the adverse effects of economic shocks on vulnerable populations. Figure 2.1 shows that the change in SNAP participation is positively correlated with poverty and unemployment levels.[8] During periods of financial crisis and pandemics, the number of participants increased with escalating unemployment and poverty rates. Researcher from public health and economics have paid a lot of attention to the effect of SNAP on household dietary quality, health outcomes, consumption,

[7] Food and Nutrition Service. SNAP Data Tables. Retrieved May 10th, 2023, from https://www.fns.usda.gov/pd/supplemental-nutrition-assistance-program-snap.

[8] Food and Nutrition Service. Yearly trends in SNAP participants, unemployment, and poverty. Retrieved May 10th, 2023, from https://www.fns.usda.gov/yearly-trends.

and benefit (e.g., Leung et al. 2012; Collins and Klerman 2017; Kim, Rabbitt and Tuttle 2020). In these studies, economists have noticed that the volume and frequency of food purchases made by low-income households vary over time after they receive the benefit receipt. Those households that are impatient in the short run will increase their instantaneous consumption in the next few days after receiving the check (Smith et al. 2016; Wilde and Ranney 2000), thus exhibiting a cyclical consumption pattern (Huffman and Barenstein 2005; Stephens 2006). Although there is still no consensus on whether to support or reject the Life‑Cycle/Permanent Income Hypothesis (LCPIH), some empirical papers have examined that the cyclical consumption pattern is prevalent among SNAP households, which may lead to cyclical food insecurity (Wilde and Ranney 2000; Shapiro 2005; Hastings and Washington 2010; Smith et al. 2016; Valizadeh, Smith and Ver Ploeg 2021). Therefore, it is necessary to conduct a comprehensive study on the food consumption patterns of SNAP households and explore potential explanations for cyclical consumption in order to smooth expenditure during their benefit month.

The objective of this study is to examine the change in food demand of SNAP households during the benefit month and investigate how food demand responds to changes in food prices. First, we examine the food consumption patterns of SNAP households during the benefit month. Diverging from prior literature that mainly focused on total food expenditure, our study disaggregates food into ten categories, facilitating a more nuanced exploration of household consumption variations in specific food groups throughout the benefit month. We break the benefit month into two segments: the first two weeks of the benefit month and the last two weeks of that, and compare the demand for foods during the two periods. Second, we assume that most SNAP households are patient and present rational preferences (Hoderlein 2011; Dorfman et al.

2019) so they rationally allocate their food budgets over these two time periods. Dorfman et al. (2019) showed a small share (39%) of SNAP recipients are impatient. With this assumption, a two-way EASI demand system is estimated to investigate the parametric change in utility function. After that, the expenditure and price elasticities are calculated from ESAI. We compare the difference in food price sensitivity between households in the first two weeks of the month and the last two weeks.

Eligible foods that SNAP households can buy are fruits and vegetables, meat, poultry, and fish, dairy products, breads and cereal and accessory foods (such as snacks, spices and seasonings, juices and beverages).[9] And households are not allowed to buy liquor, cigarettes, pharmacy items and nonfood items. We use 2010-2016 Nielsen scanner data to aggregate Universal Product Code (UPC) level items to several food groups. SNAP-eligible households are chosen by 130% poverty line. Two self-reported variables are used to mark whether the household participated in the program during the month.[10] In most states, SNAP benefits are randomly distributed to households on different dates of the month (Shapiro 2005; Valizadeh, Smith and Ver Ploeg 2021). In our study, we only focus on the states that distribute benefits to all recipients on the same single date (Hastings and Washington 2010). Based on SNAP distribution schedule, 16 states are chosen in our analysis.[11] In the end, biweekly household purchase data combined with demographics is created in our analysis.

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[9] Food and Nutrition Service. What Can SNAP Buy?. Retrieved May 10th, 2023, from https://www.fns.usda.gov/snap/eligible-food-items.

[10] Two variables are "whether household participated in SNAP" and "how long has the household been participating". These two variables are not recorded in Nielsen consumer panel. We obtained it from Nielsen.

[11] Economic Research Service. SNAP Policy Data Sets. Retrieved May 10th, 2023, from https://www.ers.usda.gov/data-products/snap-policy-data-sets/. These states are Connecticut, Idaho, Maine, Montana, Nebraska, Nevada, New Hampshire, New

We estimate a flexible functional form demand system where we account for zero purchases, price endogeneity and other unobserved heterogeneity. Our investigation into the demand elasticity of SNAP households across ten food groups reveals considerable sensitivity to both price and income variations throughout the SNAP benefit month. The lower price sensitivity of staple foods and the increased price elasticity of seafood after SNAP households receive the benefits highlight the effect of SNAP on households' purchasing decisions. The substitution effect is further confirmed by the positive cross-price elasticity observed between meat and seafood. Moreover, our analysis identifies a significant increase in income elasticity for certain food groups, such as fruits and vegetables, and meat, over the benefit month. This pattern underlines the pivotal role of budget constraints in shaping the food consumption patterns of SNAP households.

Our study makes the following contributions. First, we use Nielsen household scanner data to confirm the consumption pattern of SNAP households during the benefit month. More disaggregated food groups are discussed in our analysis. Further, we examine changes in the level of food consumption and changes in potential market behavior driven by prices. We are the first to examine the change in the instantaneous utility function during the SNAP benefit month, which provides a potential explanation of the SNAP cycle.

Jersey, New York, North Dakota, Oklahoma, Rhode Island, South Dakota, Vermont, Virginia, and Wyoming. Some of these states distributed the benefit uniformly in the first week (e.g., Nebraska uniformly issues the benefits from the 1st to the 5th of the month). We include these states in our analysis to increase our sample size, tolerating some errors. We exclude Alaska since we don't have the data.

The rest of the paper is structured as follows. The next section is a literature review, then a description of the scanner data. We then discuss the EASI model and its setting. The last two sections are the empirical results and conclusion.

2.2 Literature Review

Researchers have studied SNAP from various aspects, such as consumption and benefit, retail supply, economic cost, nutrition and health. Verghese, Raber, and Sharma (2019) conducted a systematic review to examine the variety and impact of interventions targeting SNAP beneficiaries, specifically regarding their diet and nutrition-related outcomes. They identified 12 unique interventions from 16 articles and categorized them into three types: monetary incentives, nutrition education, and a combination of both. Their review suggested that the most effective strategy to enhance the dietary habits of households benefiting from SNAP is a combination of financial incentives and nutritional education. Schanzenbach (2023) has highlighted many economic questions related to SNAP and done a detailed review of food consumption, administrative burden, and benefit adequacy. Researchers and policymakers are interested in questions about the adequacy of SNAP benefits. They often begins with how SNAP affects participants' consumption by comparing the consumption of participants and non-participants with different characteristics, then evaluating the SNAP. For example, Kim (2016) used the Consumer Expenditure Survey (CEX) to compare the difference between the consumption patterns of households participating in the SNAP and those of eligible non-participants. This study used a difference-in-differences (DID) approach to identify a significant effect of SNAP benefits on food expenditure. Furthermore, food availability, food insecurity, and dietary quality are often discussed by health economists to evaluate the effect of SNAP (e.g., Pak and Kim 2020). Then, some studies identified a cyclical expenditure pattern throughout the benefit month.

Food expenditures displayed an initial increase in the early days following benefit receipt, followed by a non-linear decline towards the end of the benefit month (Wilde and Ranney 2000; Todd and Gregory 2018). This finding implies that low-income households, despite receiving SNAP benefits, are still experiencing cyclical food insecurity. For example, Hastings and Washington (2010) find that the reduction in the quantity of food purchased, not the quality, is the main factor contributing to the decrease in food expenditure. This unsmooth consumption results in a decrease in calorie intake and an increase in the risk of food insecurity at the end of the benefit month (Shapiro 2005; Todd 2015; Todd and Gregory 2018; Kuhn 2018; Gregory and Smith 2019).

Uncovering the explanation of SNAP benefit cycle is the key to minimize the utility loss due to cyclical consumption. First, liquidity constraints affect participants' behavior, exhibiting an imperfect consumption smoothing over the SNAP month (Todd and Gregory 2018). Todd and Gregory (2008) show that increasing SNAP real benefits can help reduce participants' health costs. The second source is income fungibility (Smith et al. 2016). The marginal propensity to consume food when using SNAP benefit is higher than that when using cash income no matter households are constrained or unconstrained (Fraker, Martini, and Ohls 1995; Breunig and Dasgupta 2005). Third, the time-inconsistency of preferences of SNAP households, induced by hyperbolic discount functions (Laibson 1997), is the most discussed contributor to the benefit cycles. An impatient household (or a consumer with self-control problems) tends to consume more rather than save when owning an adequate budget, then suffer nutritional risk at the end of the benefit month (Dorfman et al. 2019; Mastrobuoni and Weinberg 2009). Our study provides a new perspective on this question, examining the impact of SNAP households' perceptions of food prices on their food demand. This may provide some support to policymakers on how to arrange the distribution of SNAP benefits to smooth the food consumption. For example, adjusting the distribution time (Cotti, Gordanier and Ozturk 2021).

2.3 Data Description

We use Nielsen Homescan Consumer scanner data over the year 2010 to year 2016. Homescan data consists of a series of purchase information provided by over 40,000 households each year. Every recorded purchase information contains the Universal Product Code (UPC) for each item, quantity purchased and price paid. They also recorded the item information, which tells us the type of product, product module and brand. The Food and Nutrition Service (FNS) presents a list consisting of six SNAP-eligible food groups[12]. Based on the FNS list and module description provided by scanner data, we construct ten food groups: fruits and vegetables, milk, bread and cereals, juices, cheese, eggs, seafood, soy base products, non-carbonated and carbonated beverages and meat. These ten groups cover most food items commonly purchased by households. The food groups are aggregated from UPC level data

Nielsen Homescan further provides two variables enabling the identification of SNAP households and the duration of their participation in 2015 June/July and 2016 January/February. These variables are used to determine households that have participated in SNAP during a specific month. Different states have set up different distribution schedules. Certain states, like Rhode Island, adopt a practice of disbursing benefits to all SNAP households on a fixed day of the month. Conversely, other states adopt a randomized approach, distributing benefits to SNAP households across a range of days. For instance, Wyoming uniformly distributes SNAP benefits from the 1st to the 4th of each month, while New Mexico extends the distribution period from

[12] They are (1) fruits and vegetables, (2) meat, poultry, and fish, (3) dairy products, (4) breads and cereals, (5) other foods such as snack foods and non-alcoholic beverages, and (6) seeds and plants, which produce food for the household to eat.

the 1st to the 20th. Meanwhile, certain states, such as North Carolina, adhere to a schedule for the allocation of SNAP on specific days of the month. A state may also change distribution schemes. For example, Oklahoma changed from a fixed day distribution to a random distribution in February 2010. Due to the unavailability of precise information regarding the dates of benefit reception, our study only focuses on households living in states that distribute benefits on a fixed single day[13]. Finally, our data consists 689 households and 31632 biweekly-household observations.

Table 2.1 and Table 2.2 show the unweighted and weighted mean consumption of ten food groups for SNAP participants.[14] They also show changes in purchases from SNAP participants between the first two weeks after reception and the last two weeks before the new reception. We observe significant variations in several food groups. The mean consumption of fruits and vegetables exhibits a statistically significant decrease of 5.49 OZ (unweighted) / 5.32 OZ (weighted), indicating a potential decline in their intake in the last two weeks of the benefit month. Similarly, bread and cereals demonstrates a decrease in consumption, with a mean difference of -4.50 OZ (unweighted) / -6.20 OZ (weighted). SNAP households also purchase more meat (weighted mean 9.05 OZ) and seafood (weighted mean 7.78 OZ) after receiving SNAP benefits and spend less at the end of the month (6.34 OZ and 6.3 OZ). Conversely, milk consumption experiences a notable increase, with a mean difference of 13.17 OZ (unweighted) / 14.60 OZ (weighted), suggesting a higher consumption in the latter period. This spending pattern is also observed in non-carbonated and carbonated beverages. On the other hand, the weighted

[13] To increase the sample size, we also include the states that uniformly distribute benefits in the first week (e.g., Wyoming).

[14] To ensure representation of the total population, we use the "projection factor" as a weight when calculating the weighted mean consumption.

mean consumptions of juice, eggs, and soy-based products display no statistically significant differences within the benefit month. Compared to the decline in total food spending, our study highlights notable variations in dietary habits across different food groups over the observed time frame.

2.4 Price Index and Price Endogeneity

Since each category consists of a large number of UPC level food or beverages, a categorical price index needs to be calculated. We first unify the units of goods in the same category (in OZ), and aggregate all eligible UPC level food or drinks within the same category. The Törnqvist price index for category j is calculated by

$$P^{jt} = \exp\left(0.5 \cdot \sum\nolimits_{v} \left(s_v^0 + s_v^{jt}\right) \cdot \ln\left(p_v^{jt}/p_v^0\right)\right)$$

where p_v^{jt} are s_v^{jt} price and share of UPC item v for in period t. p_v^0 and s_v^0 are base price and base share at the national average.

Due to the biweekly-household level of our data, not all households made purchases across all food categories within a single temporal unit. Therefore it is likely to observe zero purchases and missing price data for certain households during bi-weekly intervals. We employ ScanTrack retail scanner data to construct Törnqvist retailer price index to impute missing prices. To achieve this, we track the household's grocery store visits using the "Trip file" and the "Purchases file". Then, for the imputation of missing prices within a specific food category, we compute the average price index based on all visited stores pertaining to that particular category. In cases where there are still missing prices due to the absence of price information for that food category at local grocery stores, we adopt the average county-retailer price index to impute the missing prices in a similar way.

The demand estimate may be biased if ignoring price endogeneity. There are three sources of price endogeneity. The unit value bias (Cox and Wohlgenant 1986; Deaton 1988), unobserved preference heterogeneity over quantity, and common demand shock (Zhen et al. 2014). Hausman instrumental prices (1997) are constructed to instrument endogenous household prices. The instrument for household prices is calculated as the weighted average of county-level store price indices of the same food category from other counties, excluding the county where households live. We calculate the Euclidean distance between the centroids of each pair of two counties. Then the inverse of the Euclidean distances between the county targeted household lives in and neighboring counties within 100 miles is used as weight.

Table 2.3 shows the mean price indices of ten food groups. We don't not find that food prices changed significantly after SNAP households received the benefits. This result is similar to the study of Goldin, Homonoff and Meckel (2022). In their study, they employed Nielsen Retail Scanner data to examine the potential impact of SNAP issuance on retailers' pricing strategies. However, their empirical model did not discern any significant correlation between SNAP issuance and retailer price response.

2.5 EASI Demand Estimation

Our study uses the Exact Affine Stone Index (EASI) demand system developed by Lewbel and Pendakur (2009) to examine the variations in price elasticities and income elasticities across ten food and beverage categories through the benefit month. One of the reasons we use a structural model instead of a reduced form is that equations from the EASI model are directly derived from the expenditure functions based on utility theory by solving optimization problems. It can capture more complex utility/preference forms and gives us a Hicksian demand with a lot of desired properties. EASI model, by including a higher order of income term, allows more flexible Engel curves, and the error terms absorb unobserved household heterogeneity (Zhen et al. 2014).

Our two-way EASI model is specified as follows:

$$w_{iht} = \alpha_i + \sum_{r=0}^{R} b_{ir} y_{ht}^{r} + \sum_{j=1}^{n} A_{ij} \log(p_{jht}) + \sum_{j=1}^{n} B_{ij} \cdot \mathrm{y}_{ht} \cdot \log(p_{jht}) + \boldsymbol{c_i Z_{ht}} + \boldsymbol{d_{i1} Year}$$
$$+ \boldsymbol{d_{i2} Month} + \boldsymbol{d_{i3} Region} + \delta_i D + \sum_{j=1}^{n} \gamma_{ij} \cdot D \cdot \log(p_{jht}) + \theta_i \cdot D \cdot y_{ht}$$

where w_{iht} is budget share of household h for food category i at time t. y_{ht} is real households income which is household income deflated by stone price index. p_{jht} is the price of food category j paid by household h at time t. $\boldsymbol{Z_{ht}}$ is a vector of demand shifter including household size, age of household head, gender of household head, education, and race dummy. $\boldsymbol{Year}$, $\boldsymbol{Month}$ and $\boldsymbol{Region}$ are year fixed effect, month fixed effect and region fixed effect. The variable of interest, denoted as D, takes the value of one if household h has made a purchase in the final two weeks of the SNAP benefit month; otherwise, it takes the value of zero. Two-way EASI includes interaction terms $\gamma_{ij} \cdot D \cdot \log(p_j)$ and $D \cdot y$ to capture the price effect and income effect, which reflect changes in price and income sensitivity.

Because of zero purchases, dependent variable w_{iht} is censored at 0. Zhen et al (2014) extended Amemiya's generalized least squares (AGLS) estimator from a single equation (Amemiya 1979; Newey 1987) to simultaneous equations that allows cross-equation restrictions (Homogeneity, Symmetry, and Adding-up) required by demand theory. A system of IV Tobit regressions is used to estimate the demand equations.

2.6 Results

Compared with the Almost Ideal Demand (Deaton and Muellbauer 1980) and its quadratic extension (Banks, Blundell and Lewbel 1997), EASI allows more flexible functional form and Engel curve. We have tried a series orders of polynomial of income in the budget share equations ($r = 2,3,4,5$) and rerun the model . By testing the joint significance of b_{ir}, we finally choose a rank five demand with $r = 4$.

Table 2.4 shows price elasticities across the two periods within the SNAP benefit month. The demand for most food items among SNAP households exhibits high sensitivity (with an elasticity greater than 1) to variation in food prices, indicating the heightened vulnerability of low-income households to price shocks. As the food stamp is a cash-value voucher, SNAP participants will allocate the benefits by prevailing food prices. Variations in prices exert a substantial impact on their food demand patterns of them. Juice, cheese, and eggs are three commodities that display higher price elasticity, as evidenced by their respective price elasticities greater than 2. The price elasticities of other food groups are between 1 and 2. According to Figure 2.2, we find a noticeable difference in demand elasticity for some commodities across the two periods within the SNAP benefit month. There are five commodities whose demands are more price sensitive at the end of the month than at the beginning. Among them, the price elasticity of juice changes more obviously, from -2.02 to -4.06. Similarly, bread and cereals, commonly considered staple foods, typically serve as primary calorie sources and offer the advantage of easy storage. Therefore, SNAP households exhibit relatively lower responsiveness to price changes concerning these items. Our findings indicate that the demand for both eggs and cheese also displays an increase in price elasticity as the benefit month progresses to the end. However, we don’t find a significant change in the price elasticities of vegetables and fruits (-

1.43 and -1.43) , milk (-1.23 and -1.24), beverages (-1.23 and -1.10) and meat (-0.99 and -1.26). On the other hand, due to the low unit prices of milk, vegetables and fruits, and beverages, slight price changes at the beginning and end of the month are less likely to significantly influence consumer behavior or attract substantial attention. Conversely, we find that the price elasticity of seafood and soy products decreases at the end of the benefit month. The price elasticity of seafood changed from -1.83 to -1.55, which is less price-elastic. Seafood is often viewed as a "luxury" food - more expensive than many other protein sources. Thus, although it can provide high-quality protein, SNAP households might search for other alternatives based on unit prices. For example, the cross-price elasticity between meat and seafood is positive, which indicates they are substitutes. SNAP households may purchase cheaper meat products (such as poultry) when seafood price is high.

The table 2.4 also shows the change in income elasticity. In the first two weeks of the benefit month, most foods have income elasticities close to 1 (between 0.9 and 1.1). The demand for beverages is the most income elastic, with an elasticity of 1.6. In the last two weeks of the benefit month, the income elasticities become more variant. Among them, the income elasticity of vegetables and fruits, beverages, and meat is greater than 1, indicating luxury goods. Bread and cereals, juice, and soy base products are less income elastic with elasticities less than 1. We find that compared to the beginning of the month, the income elasticity of vegetables and fruits, meat products changed significantly at the end of the benefit month. Figure 2.3 shows a significant increase in the income elasticity of fruits and vegetables and meat products when comparing the beginning and end of the SNAP benefit month. Specifically, the income elasticity of fruits and vegetables increased from 1.07 to 1.35, while meat products presented a more significant increase from 1.01 to 1.75. When households allocate their budgets, food adequacy is considered, and when they receive SNAP benefits, they tend to buy more food for emergencies. Meat product has a higher income elasticity in the last two weeks, indicating more income sensitivity when households do not have a sufficient food budget. After receiving their SNAP benefits, households may be more willing to purchase meat products, regardless of price variations. This may be because meat product is more easily refrigerated. SNAP households are more intended to stock up on necessary food for the rest of the month, leading to lower income sensitivity in the initial weeks. Meat is an essential source of protein in many diets. The demand for meat may remain relatively stable in the short term following benefit receipt, as households will prioritize their nutritional needs with enough food budgets.

2.7 Conclusion

Our study estimates a censored food demand system for SNAP households, accounting for endogenous prices. We leverage the states' SNAP issuance schedules to compare the difference in price and income elasticities between the first two weeks and the last two weeks of the SNAP benefit month. While numerous studies have delved into cyclical consumption, few have focused on the responsiveness of demand to fluctuations in food prices. Goldin, Homonoff and Meckel (2022) finds SNAP beneficiaries shopping during the weeks in which all benefits are issued exhibit marginally higher price sensitivity compared to that shopping in other weeks of the month. Our study delineates distinct purchasing patterns among SNAP households concerning various food categories. For the majority of food groups, our analysis shows an amplification in household consumption after the receipt of SNAP benefits and a decrease as households approach the end of the benefit cycle, anticipating new issuance. Two exceptions are milk and beverages, whose consumptions increase as the benefit month ends. At the same time, we did not find that the price changed significantly during the benefit month. The elasticities estimated from the demand system show that SNAP households respond differently to changes in the prices of different foods. While this study does not provide a specific economic theory, it discerns variations in price and income elasticities throughout the SNAP benefit month among SNAP households. This observed divergence may contribute to a partial explanation of the cyclical consumption patterns of SNAP beneficiaries. One policy implication of our study is the welfare of SNAP households may be affected by the SNAP issuance schedule. Our elasticity estimates can provide evidence for the design of appropriate issuance schedules to improve household welfare.

Our research requires attention to the following caveats. The scope of this study is narrow to examining variations in the demand elasticities of SNAP households within the benefit month. Unlike evaluations of the SNAP program which typically employ a Difference in Differences (DID) framework, our study abstains from using such a design to identify the effect in question. That is, we do not set up a treatment group and a control group for this study. Due to the endogenous selection into program participation and the systematic under-reporting of participation status (Kreider et al. 2012), the estimate of the effect of the SNAP program is usually biased without valid instruments. Moreover, our study does not eliminate the joint impacts of multiple policy measures. It is often the case that households participating in SNAP concurrently participate in other food assistance programs, such as WIC. Our study does not delineate the potential interactive effects.

Table 2.1: Unweighted Mean Purchases for SNAP Participants (OZ)

	First two weeks		Last two weeks		
Food Group	Mean	STD	Mean	STD	Mean Difference
Fruits and vegetables	72.96	122.81	67.47	123.39	-5.49***
Milk	95.59	175.92	108.76	202.83	13.17***
Bread and cereals	52.98	126.14	48.49	87.13	-4.50***
Juice	34.32	89.42	33.99	90.47	-0.34
Cheese	14.49	28.08	12.99	28.28	-1.50***
Eggs	13.35	29.72	12.61	29.57	-0.74**
Seafood	6.50	20.53	5.51	17.89	-0.99***
Soy base products	7.02	33.42	7.02	35.26	0.00
Non-carbonated and carbonated beverages	185.17	399.28	196.97	437.70	11.80**
Meat	8.61	42.28	6.35	39.55	-2.26***
Observations	15816		15816		

Table 2.2: Weighted Mean Purchases for SNAP Participants (OZ)

	First two weeks		Last two weeks		
Food Group	Mean	STD	Mean	STD	Mean Difference
Fruits and vegetables	72.70	123.80	67.38	124.04	-5.32***
Milk	91.70	160.65	106.30	188.10	14.60***
Bread and cereals	60.77	157.84	54.57	104.08	-6.20***
Juice	37.65	95.72	38.75	100.51	1.09
Cheese	13.96	27.31	12.42	26.29	-1.54***
Eggs	14.25	32.90	13.64	33.38	-0.61
Seafood	7.78	23.81	6.30	20.07	-1.49***
Soy base products	7.36	36.30	6.61	34.81	-0.74**
Non-carbonated and carbonated beverages	180.30	384.30	198.88	433.28	18.58***
Meat	9.05	42.42	6.34	38.57	-2.70***

Table 2.3: Mean Price Index of Food Groups

Food Group	First two weeks Mean	First two weeks STD	Last two weeks Mean	Last two weeks STD	Mean Difference
Fruits and vegetables	1.13	0.45	1.12	0.47	0.00
Milk	1.15	0.36	1.15	0.36	0.00
Bread and cereals	1.31	0.44	1.29	0.42	-0.01**
Juice	1.26	0.35	1.25	0.28	-0.01*
Cheese	1.29	0.40	1.30	0.40	0.00
Eggs	1.34	0.48	1.35	0.49	0.01
Seafood	1.13	0.30	1.14	0.31	0.00
Soy base products	1.15	0.38	1.15	0.39	0.00
Non-carbonated and carbonated beverages	1.28	0.37	1.27	0.37	-0.02***
Meat	1.16	0.44	1.17	0.42	0.00

Table 2.4: Median Own-price and Income Elasticities

Food groups	Own Price Elasticity: The first two weeks	Own Price Elasticity: The last two weeks	Income Elasticity: The first two weeks	Income Elasticity: The last two weeks
Fruits and vegetables	-1.434	-1.426	1.073	1.352
	(0.067)	(0.072)	(0.067)	(0.080)
Milk	-1.227	-1.242	1.009	1.018
	(0.060)	(0.069)	(0.054)	(0.056)
Bread and cereals	-1.211	-1.378	1.000	0.978
	(0.046)	(0.072)	(0.069)	(0.081)
Juice	-2.023	-4.061	1.012	0.864
	(0.175)	(0.314)	(0.253)	(0.312)
Cheese	-2.084	-2.411	0.998	1.009
	(0.058)	(0.074)	(0.052)	(0.062)
Eggs	-2.179	-2.651	1.114	0.990
	(0.107)	(0.127)	(0.137)	(0.173)
Seafood	-1.825	-1.548	0.981	0.955
	(0.106)	(0.116)	(0.172)	(0.201)
Soy base products	-1.554	-0.611	0.943	0.63
	(0.173)	(0.192)	(0.223)	(0.309)
Beverages	-1.227	-1.095	1.612	1.67
	(0.074)	(0.077)	(0.054)	(0.062)
Meat	-0.993	-1.257	1.011	1.747
	(0.157)	(0.165)	(0.133)	(0.186)
Numéraire	-0.808	-0.893	0.880	0.924
	(0.054)	(0.066)	(0.031)	(0.037)

Table 2.5: Cross-price Elasticities and Income Elasticities in the First Two Weeks of the SNAP Benefits Month

	Fruits and vegetables	Milk	Bread and cereals	Juice	Cheese	Eggs
Fruits and vegetables	**-1.434**	-0.498	0.129	0.473	-0.088	-0.118
	(0.067)	(0.059)	(0.020)	(0.074)	(0.014)	(0.036)
Milk	-0.364	**-1.227**	0.060	0.077	0.017	0.400
	(0.045)	(0.060)	(0.018)	(0.055)	(0.011)	(0.033)
Bread and cereals	0.227	0.135	**-1.211**	-0.214	0.257	0.219
	(0.030)	(0.034)	(0.046)	(0.042)	(0.023)	(0.023)
Juice	0.512	0.137	-0.146	**-2.023**	-0.042	0.185
	(0.093)	(0.091)	(0.039)	(0.175)	(0.018)	(0.063)
Cheese	-0.271	0.107	0.660	-0.148	**-2.084**	0.625
	(0.051)	(0.052)	(0.057)	(0.051)	(0.058)	(0.040)
Eggs	-0.253	1.270	0.370	0.428	0.429	**-2.179**
	(0.097)	(0.128)	(0.045)	(0.141)	(0.034)	(0.107)
Seafood	0.669	-0.424	-0.773	0.709	-0.374	0.630
	(0.103)	(0.107)	(0.092)	(0.132)	(0.054)	(0.075)
Soy base products	0.325	0.056	0.210	0.588	0.109	-0.265
	(0.081)	(0.076)	(0.026)	(0.100)	(0.014)	(0.047)
Beverages	-0.142	0.152	-0.064	-0.151	0.055	-0.168
	(0.035)	(0.04)	(0.015)	(0.062)	(0.009)	(0.024)
Meat	0.238	-0.181	-0.316	-0.220	-0.174	-0.207
	(0.085)	(0.089)	(0.029)	(0.151)	(0.015)	(0.065)
Numéraire	0.007	0.006	0.006	0.007	0.002	0.002
	(0.002)	(0.002)	(0.002)	(0.002)	(0.000)	(0.001)

	Seafood	Soy base products	Beverages	Meat	Numéraire	Income
Fruits and vegetables	0.171	0.244	-0.247	0.134	-0.128	**1.073**
	(0.025)	(0.062)	(0.080)	(0.045)	(0.123)	(0.067)
Milk	-0.089	0.009	0.354	-0.072	-0.017	**1.009**
	(0.021)	(0.045)	(0.068)	(0.036)	(0.092)	(0.054)
Bread and cereals	-0.301	0.204	-0.071	-0.241	0.002	**1.000**
	(0.032)	(0.023)	(0.046)	(0.022)	(0.122)	(0.069)
Juice	0.194	0.483	-0.135	-0.109	0.007	**1.012**
	(0.040)	(0.091)	(0.160)	(0.090)	(0.447)	(0.253)
Cheese	-0.370	0.312	0.578	-0.348	0.008	**0.998**
	(0.049)	(0.037)	(0.073)	(0.029)	(0.094)	(0.052)
Eggs	0.406	-0.549	-0.848	-0.284	-0.182	**1.114**
	(0.050)	(0.097)	(0.142)	(0.090)	(0.243)	(0.137)
Seafood	**-1.825**	-0.119	0.113	0.468	0.030	**0.981**
	(0.106)	(0.068)	(0.145)	(0.071)	(0.305)	(0.172)
Soy base products	-0.038	**-1.554**	-0.026	-0.158	0.137	**0.943**
	(0.023)	(0.173)	(0.127)	(0.092)	(0.407)	(0.223)
Beverages	-0.003	-0.049	**-1.227**	0.197	-0.972	**1.612**

	(0.017)	(0.044)	(0.074)	(0.034)	(0.111)	(0.054)
Meat	0.225	-0.277	0.807	**-0.993**	-0.013	**1.011**
	(0.032)	(0.131)	(0.145)	(0.157)	(0.233)	(0.133)
Numéraire	0.001	0.002	0.009	0.001	**-0.808**	**0.880**
	(0.000)	(0.001)	(0.003)	(0.000)	(0.054)	(0.031)

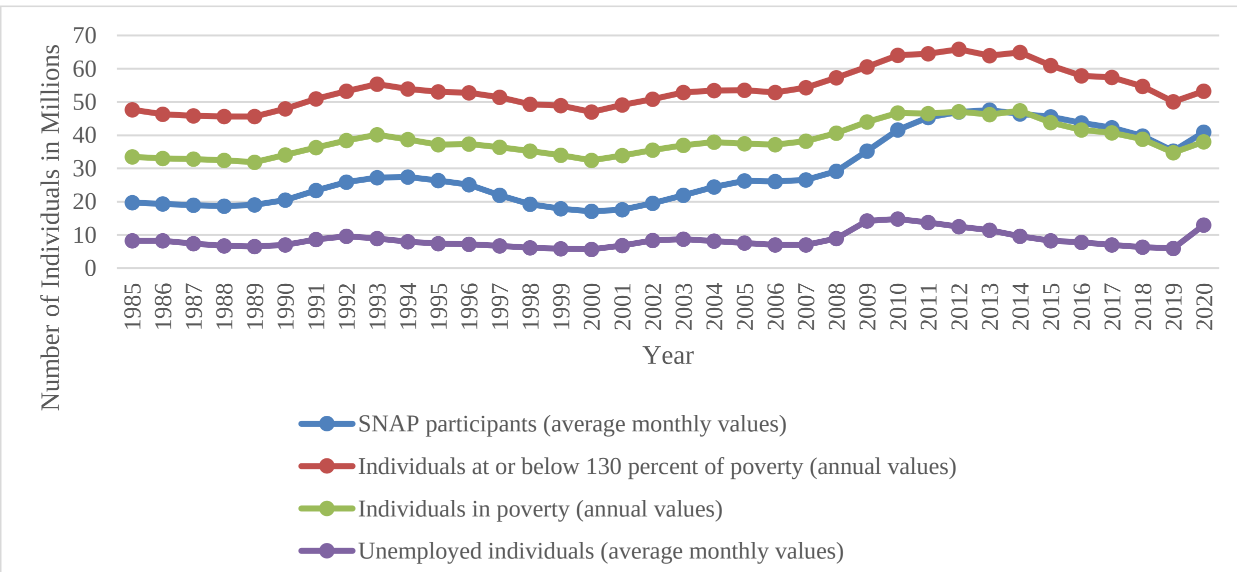
Note: values in parentheses are standard errors

Table 2.6: Cross-price Elasticities and Income Elasticities in the Last Two Weeks of the SNAP Benefits Month

	Fruits and vegetables	Milk	Bread and cereals	Juice	Cheese	Eggs
Fruits and vegetables	**-1.426**	-0.743	0.144	0.887	-0.089	0.021
	(0.072)	(0.065)	(0.023)	(0.085)	(0.013)	(0.036)
Milk	-0.488	**-1.242**	0.021	0.413	0.026	0.494
	(0.044)	(0.069)	(0.019)	(0.060)	(0.012)	(0.030)
Bread and cereals	0.273	0.081	**-1.378**	-0.322	0.479	0.232
	(0.032)	(0.038)	(0.072)	(0.047)	(0.032)	(0.024)
Juice	1.046	0.694	-0.181	**-4.061**	0.055	-0.080
	(0.128)	(0.123)	(0.046)	(0.314)	(0.023)	(0.069)
Cheese	-0.259	0.160	1.226	0.134	**-2.411**	0.518
	(0.050)	(0.065)	(0.078)	(0.067)	(0.074)	(0.040)
Eggs	0.103	1.767	0.428	-0.234	0.356	**-2.651**
	(0.092)	(0.143)	(0.049)	(0.157)	(0.032)	(0.127)
Seafood	0.577	0.453	-0.038	-0.729	0.350	-0.041
	(0.098)	(0.126)	(0.094)	(0.161)	(0.06)	(0.068)
Soy base products	-0.066	-0.745	0.122	1.344	-0.046	-0.087
	(0.087)	(0.097)	(0.033)	(0.129)	(0.017)	(0.051)
Beverages	-0.159	-0.037	-0.062	0.263	-0.049	-0.078
	(0.037)	(0.036)	(0.018)	(0.071)	(0.009)	(0.026)
Meat	0.158	-0.076	-0.471	-0.24	-0.163	-0.162
	(0.098)	(0.096)	(0.033)	(0.18)	(0.019)	(0.075)
Numéraire	0.004	0.004	0.004	0.005	0.001	0.001
	(0.002)	(0.003)	(0.002)	(0.002)	(0.000)	(0.001)
	Seafood	Soy base products	Beverages	Meat	Numéraire	Income
Fruits and vegetables	0.142	-0.141	-0.326	0.115	-0.562	**1.352**
	(0.024)	(0.066)	(0.086)	(0.051)	(0.150)	(0.080)
Milk	0.079	-0.491	0.036	-0.004	-0.037	**1.018**
	(0.022)	(0.048)	(0.058)	(0.034)	(0.096)	(0.056)
Bread and cereals	-0.025	0.042	-0.054	-0.297	0.039	**0.978**
	(0.035)	(0.027)	(0.055)	(0.022)	(0.141)	(0.081)
Juice	-0.198	1.090	0.910	-0.079	0.226	**0.864**
	(0.050)	(0.127)	(0.203)	(0.099)	(0.561)	(0.312)

Cheese	0.330	-0.185	-0.229	-0.286	-0.006	**1.009**
	(0.057)	(0.044)	(0.079)	(0.035)	(0.111)	(0.062)
Eggs	-0.033	-0.204	-0.315	-0.177	0.009	**0.990**
	(0.046)	(0.097)	(0.153)	(0.096)	(0.303)	(0.173)
Seafood	**-1.548**	-0.180	0.115	0.213	0.072	**0.955**
	(0.116)	(0.083)	(0.165)	(0.072)	(0.357)	(0.201)
Soy base products	-0.055	**-0.611**	-0.616	0.313	0.667	**0.630**
	(0.029)	(0.192)	(0.142)	(0.089)	(0.575)	(0.309)
Beverages	-0.006	-0.292	**-1.095**	0.055	-1.061	**1.670**
	(0.019)	(0.044)	(0.077)	(0.034)	(0.127)	(0.062)
Meat	0.092	0.447	0.160	**-1.257**	-1.206	**1.747**
	(0.037)	(0.132)	(0.154)	(0.165)	(0.329)	(0.186)
Numéraire	0.001	0.002	0.006	0.001	**-0.893**	**0.924**
	(0.000)	(0.000)	(0.004)	(0.000)	(0.066)	(0.037)

Note: values in parentheses are standard errors



Note: Data from USDA: https://www.fns.usda.gov/yearly-trends.

Figure 2.1: Number of SNAP Participants

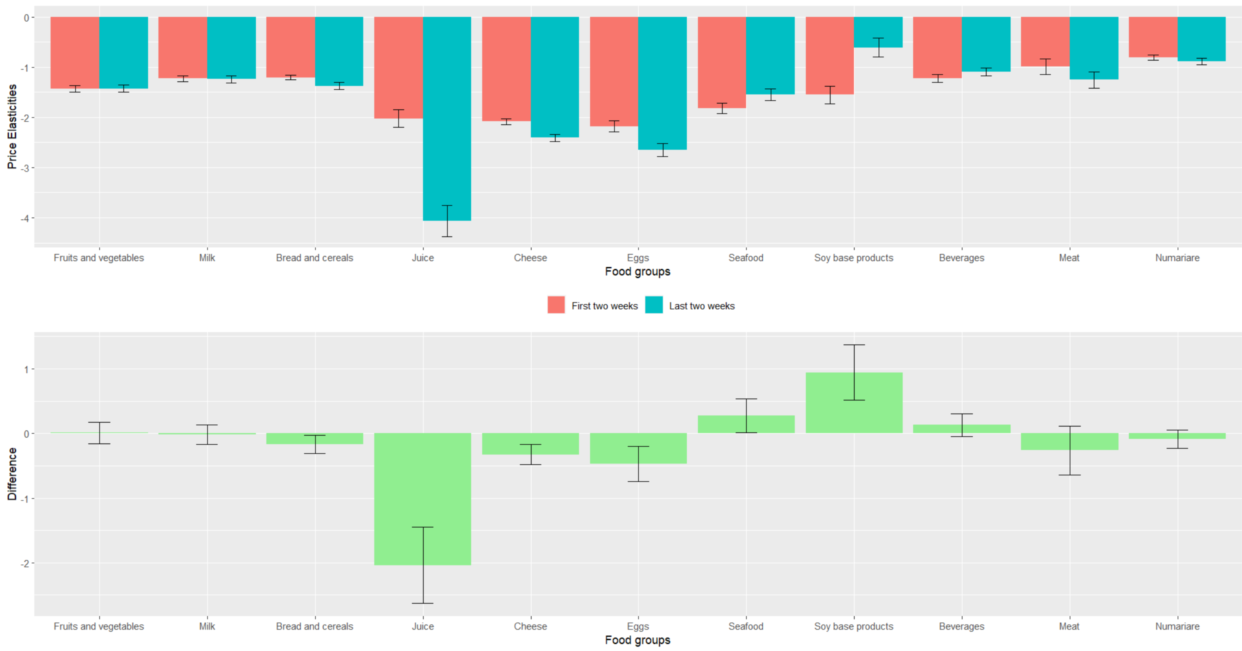


Figure 2.2: Own-price Elasticities of Two Benefit Periods

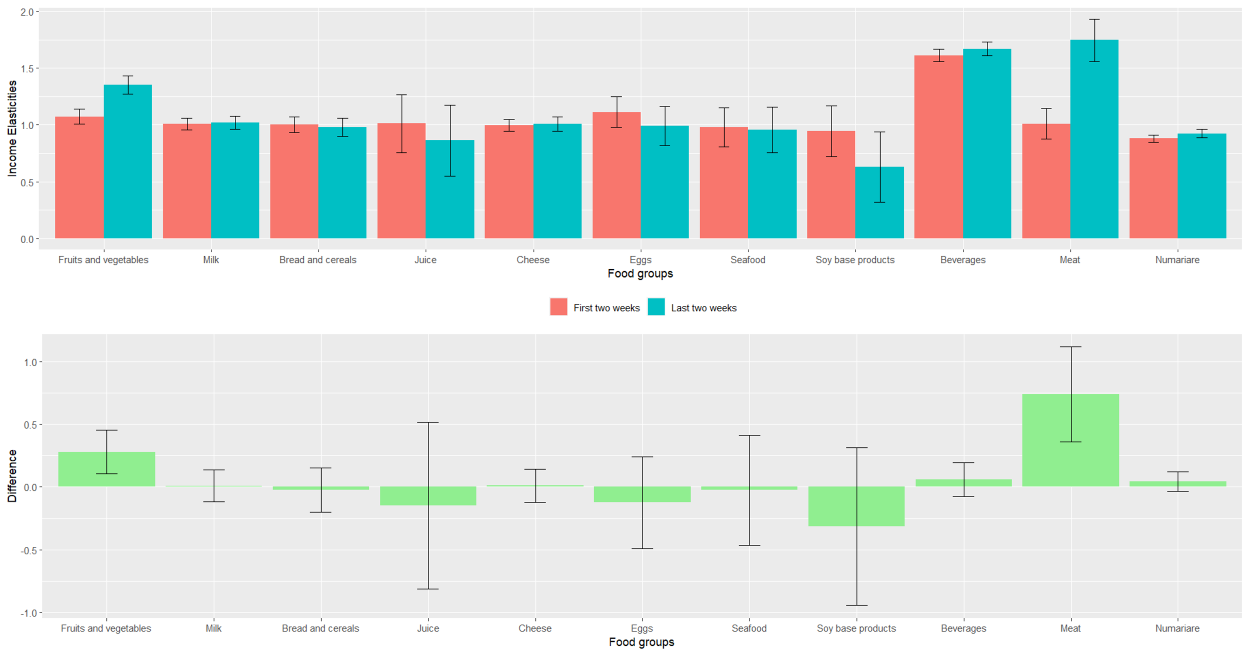


Figure 2.3: Income Elasticities of Two Benefit Periods

CHAPTER 3

AN EFFICIENT APPROACH TO ESTIMATING A LARGE NON-LINEAR DEMAND SYSTEM WITH ENDOGENOUS PRICES

3.1 Introduction

The increasing availability of retail scanner data for academic research is a gift to consumer demand research. The product, temporal and geographical granularity of these data means not only that it is possible to estimate demand for narrowly defined food groups, but also that demand can now be modeled in a large system of many goods.[15] Despite the new opportunities afforded by scanner data, estimation of demand systems with more than a couple dozen goods is rare even when a large system may offer unique insights. We argue that this is mainly due to the perceived computational complexity of estimating a large demand system.

The objective of this study is threefold. First, we develop a feasible approach to estimating large conditionally linear demand systems (i.e., flexible demand systems with many goods). Following the suggestion of Lewbel and Pendakur (2009), we derive the asymptotic properties of the iterated linear three stage least squares (I3SLS). Second, we use retail scanner data in

[15] The fear of multicollinearity and small sample size have often been the reasons for not pursuing a large demand system. The precision with which retail scanner data register the transaction prices, the geographical price variation, and large scanner sample mean that multicollinearity between prices of closely related foods may be less severe than if prices were derived from, for example, national disappearance data. A classic example of disappearance data-based demand analysis is the enormous literature on the meat demand system using US disappearance data (e.g., Piggott and Marsh 2004; Bryant and Davis 2008), where consumption is calculated as (production + inventory + import – export).

conjunction with I3SLS to estimate a quadratic almost ideal demand (QUAID) system of 19 fresh fruits and vegetables and apply the utility-theoretic demand parameter estimates to the construction of an exact price index for fruit and vegetables. Third, the exact price index is compared to other popular indexes that do not require econometric estimates of consumer preferences to assess the temporal and geographical variation in the cost of living index of fresh fruit and vegetables. We find the I3SLS to be straightforward to apply and converge rapidly. The superlative Törnqvist and Fisher ideal price indexes based on homothetic preferences provide good approximation to the exact index based on the much more flexible QUAID demand. The price indexes reveal substantial spatial and seasonal variations in the cost of consuming fruit and vegetables.

The remainder of this article is organized as follows. We first briefly review existing methods for estimating demand systems with many goods. Next, we describe the I3SLS approach to estimating a conditionally linear demand system. This is followed by an application to fruit and vegetable demand and the derivation of the exact price index based on the parameter estimates. The final section concludes.

3.2 Approaches to Estimating Large Demand Systems

As the true preference structure is unknown, the specification of flexible demand systems that provide second-order approximation to any preferences at a point (i.e., local flexibility) has attracted some of the best minds of economics. Prominent examples of flexible demand systems include Christensen, Jorgenson and Lau's (1975) Translog demand, Deaton and Muellbauer's (1980a) original almost ideal demand (AID), Banks, Blundell and Lewbel's (1997) quadratic AID (QUAID), and Lewbel and Pendakur's (2009) Exact Affine Stone Index (EASI) demand. To attain flexibility, each own- or cross-price effect requires a dedicated parameter, leading to a total of $n(n-1)/2$ price coefficients where n is the number of goods. Since the number of parameters to be estimated increases exponentially in the number of goods (i.e., the curse of dimensionality), large demand systems can quickly exceed the computational power of even modern computing power, especially if the demand system itself is non-linear.

There are three approaches to reducing dimension of the parameter space. The most common approach is to invoke weak separability of preferences to compartmentalize the consumption decision into two or more budgeting stages (Edgerton 1997). Under multistage budgeting, the higher stages are concerned with the allocation of total expenditures to aggregate categories and the lower stages focus on more disaggregate demand given the category budget allocated in the previous stage. The main drawback of multistage budgeting is that the cross-price effect between two goods from two different categories is severely restricted.

The second approach is to reduce the number of dimensions by aggregating goods into fewer groups guided by Lewbel's (1996) the generalized composite commodity theorem (GCCT). Lewbel (1996, p. 528) showed that a demand system of GCCT-consistent aggregates produces unbiased estimates of aggregate demand elasticities that would be obtained from estimation of a (larger) disaggregate demand system. Empirical tests of GCCT frequently find that goods can be consistently aggregated to a few broad categories (Davis 2003; Reed, Levedahl and Hallahan 2005). Nevertheless, when the research begs estimates of disaggregate price elasticities, GCCT-consistent aggregate estimates are not useful. For example, comparing the effect of a sugar-based soda tax to that of a volume-based one requires estimates of demand for beverages differentiated by sugar content (Zhen, Brissette and Ruff 2014). An estimate of overall demand for sugar-sweetened beverages is not enough for this purpose.

The third approach casts the large number of price coefficients onto a small number of observed product attributes (Rojas and Peterson 2008; Pinkse and Slade 2004). The own-price effect is specified as a function of product attributes and the cross-price effect as a function of the two goods' closeness in observed attributes. Although one can build unobserved product heterogeneity into the distance metric (Zhen, Brissette and Ruff 2014), the cross-price effect is largely determined by observed attributes. This is a major weakness of the distance metric approach.

Of course, one can always try estimating the large demand system using a powerful computer and canned software. For example, the PROC MODEL statement in SAS 9.4 can estimate nonlinear systems of equations using a variety of estimators. Problems arise, however, when using instrumental variables to account for price endogeneity. As is the case for flexible demand systems such as Translog, AID and EASI, each budget share is a function of all price coefficients across all budget share equations. PROC MODEL requires the number of instruments for each budget share equation to be at least equal to the total number of price coefficients ($n(n-1)/2$), rather than the number of goods (n), in the system. This artificial under-identification makes 2SLS, 3SLS and GMM unusable for nonlinear demand system estimation under PROC MODEL. Although one can employ the full information maximum likelihood estimator in this situation by fully specifying the price generating processes, it is also more computationally demanding. In our experience, we could not accommodate more than two dozen goods even on a workstation with 192 GB of RAM.

3.3 The Iterated Three Stage Least Square Estimator in Demand System

In their discussion of estimation methods, Lewbel and Pendakur (2009) suggested that their new EASI demand system, which belongs to the class of conditionally linear demand systems, could be estimated by iterating the 3SLS estimator until convergence. This approach is similar in spirit to Blundell and Robin's (1999) iterated least squares estimator for this class of demand systems. Blundell and Robin used the control function to account for endogeneity. Several demand system studies have applied the Blundell and Robin estimator to correct for endogeneity in total expenditures (e.g., Allais, Bertail and Nichèle 2008; Dauphin et al. 2011). The advantage of the control function approach is that it generates a simple regression-based Hausman test of exogeneity as a byproduct (Wooldridge 2015). Its potential drawback lies in the independence assumptions, which are much stronger than the exogeneity assumption of standard instrumental variables estimators when the endogenous explanatory variable appears as polynomials (Wooldridge 2007). This would be the case for total expenditures in the QUAID and EASI systems. In addition, as the control function approach adds as many control variables as there are endogenous regressors to each demand equation, a nuisance may occur when estimating a large system with endogenous prices: the total number of coefficients nearly doubles. By contrast, I3SLS is more robust to misspecification and more parsimonious in large demand system applications.

Let the class of conditionally linear demand system be represented by the following equation system:

$$y_{it} = g(x_t, \theta)'\theta_i + \varepsilon_{it} \tag{3.1}$$

where $i = 1, \dots N$ and $t = 1, \dots T$ index the good and observation, respectively. y_{it} measures demand for good i at observation t. x_t is a $K \times 1$ vector of explanatory variables including total expenditure and prices. $g(.)$ is a $K \times 1$ vector function with the ith element being $g_i(x_t, \theta)$. θ_i is a $K \times 1$ vector of parameters directly related to demand for good i, and $\theta' = (\theta_1', \dots, \theta_N')$. Conditional on the values of $g(x_t, \theta)$, eq. (1) is linear in θ_i. Let

$$G(\theta) = \begin{pmatrix} g(x_1,\theta)' \\ \vdots \\ g(x_T,\theta)' \end{pmatrix} \text{ and } Z = \begin{pmatrix} z_1' \\ \vdots \\ z_T' \end{pmatrix}$$

where z_t is a $K \times 1$ vector of instruments for x_t. Under the regularity assumptions, we have $E\{Z'[y_i - G(\hat{\theta})\hat{\theta}_i]\} = 0$ for each i, where y_i is a $T \times 1$ vector with y_{it} being the tth element. Then $\hat{\theta}$ weakly converges to the population θ and

$$\sqrt{T}(\hat{\theta} - \theta) \rightsquigarrow N(0, J^{-1} \cdot (\Sigma \otimes E[z_t z_t']) \cdot J'^{-1}) \tag{3.2}$$

where $\Sigma = E[\varepsilon_t \varepsilon_t']$ is the variance-covariance of the residuals of demand system (1), ε_t is $N \times 1$ with ε_{it} being the ith element, $J = I_n \otimes E(z_t \cdot g(x_t,\theta)') + E[(\Theta' \frac{\partial g(x_t,\theta)}{\partial \theta'}) \otimes z_t]$, and $\Theta = (\theta_1, \theta_2, \dots, \theta_N)$. Proof: see the Appendix.

We implement the I3SLS in the following steps:

Step 1: Set initial value $\theta^{(0)}$.

Step 2: Use $\theta^{(0)}$ and x to calculate $g(x,\theta)$.

Step 3: Holding $g(x,\theta)$ fixed, estimate system of N conditionally linear equations by 3SLS to get the new coefficients $\hat{\theta}$ and set $\theta^{(1)} = \hat{\theta}$.

Step 4: Use $\theta^{(1)}$ and x to update $g(x,\theta)$ and repeat step (3) to get $\theta^{(2)}$.

Step 5: Repeat step (4) until $|\theta^{(L)} - \theta^{(L-1)}|$ is sufficiently small.

As shown in the proof in Appendix, the asymptotic variance-covariance matrix of $\hat{\theta}$ is calculated as

$$Asy.\hat{V}(\hat{\theta}) = \hat{J}^{-1} \cdot \hat{\Sigma} \otimes \widehat{K} \cdot \hat{J}'^{-1} \tag{3.3}$$

where $\hat{\Sigma} = \frac{1}{T}(Y - G(\hat{\theta})\widehat{\Theta})'(Y - G(\hat{\theta})\widehat{\Theta})$ and $Y = (y_1, y_2, \dots, y_N)$. $\hat{J} = \frac{1}{T}(I_N \otimes Z') \frac{\partial [I_N \otimes G(\hat{\theta})]\hat{\theta}}{\partial \theta\prime}$ is the estimator of J in equation (2) and $\widehat{K} = \frac{1}{T} Z'Z$ is the estimator of $E[z_t z_t{}']$.

When imposing restrictions, equation (3) becomes $\{[(I_N \otimes Z) \cdot A]' \cdot \frac{\partial [I_N \otimes G(\hat{\theta})] \cdot A \cdot \hat{\theta}_r}{\partial \theta_r'}\}^{-1} \cdot [A' \cdot (\hat{\Sigma} \otimes Z'Z) \cdot A] \cdot \{[(I_N \otimes Z) \cdot A]' \cdot \frac{\partial [I_N \otimes G(\hat{\theta})] \cdot A \cdot \hat{\theta}_r}{\partial \theta_r'}]\}^{-1\prime}$, where vector θ_r only contains unrestricted parameters. A is a transformation matrix such that $(I_N \otimes Z) \cdot A \cdot \theta_r$, $\left(I_N \otimes G(\hat{\theta})\right) \cdot A \cdot \theta_r$ are the same as $(I_N \otimes Z) \cdot \theta$ and $(I_N \otimes G(\theta) \cdot \theta$.

3.4 Exact Price Indexes

An interesting use case for the I3SLS estimator is to provide the preference parameters for constructing exact price indexes because of the large number elementary products that go into a price index. We assume the representative consumer has the following indirect utility function that underlies the rank-three QUAID system (Banks, Blundell and Lewbel 1997)

$$u = \left\{\left[\frac{\ln X - \ln a(p)}{b(p)}\right]^{-1} + \lambda(p)\right\}^{-1} \tag{3.4}$$

where X is total expenditure, $\ln a(p) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln(p_i) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \gamma_{ij} \ln p_i \ln p_j$, $b(p) = \prod_{i=1}^{N} p_i^{\beta_i}$, $\lambda(p) = \sum_{i=1}^{N} \lambda_i \ln p_i$, p_i is price of good i, and α, β, γ, and λ are parameters. We suppress the observation subscript t for succinctness. Applying Shephard's lemma yields the following estimating equation:

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln(p_j) + \beta_i \ln \frac{X}{a(p)} + \frac{\lambda_i}{b(p)}\left[\ln(\frac{X}{a(p)})\right]^2 + \epsilon_i \tag{3.5}$$

where w_i is the budget share of good i, and ϵ_i is the residual. Conditional on $a(p)$ and $b(p)$, the demand system in (3.5) is linear in parameters.

The exact price index of goods in the demand system is calculated as

$$P_{QUAID}(p^t, p^0; u^0) = e(p^t, u^0)/e(p^0, u^0) \tag{3.6}$$

p^t and p^0 are N-dimensional vectors of prices at observation t and the base 0, respectively. u^0 is utility achieved by optimal consumption at base prices. $e(p^0, u^0)$ and $e(p^t, u^0)$ are minimum expenditures needed to achieve u^0 when prices are at p^0 and p^t, respectively. u^0 can be calculated using equation (3.4), base total expenditure, and base prices. Then $e(p^t, u^0)$ can be derived using the following log cost function implied by the indirect utility (3.4)

$$\ln e(p^t, u^0) = \ln a(p^t) + \frac{u^0 b(p^t)}{1 - u^0 \lambda(p^t)} \tag{3.7}$$

P_{QUAID} is an exact price index because it is derived from a utility function. As the QUAID system accounts for consumer behavioral changes when faced with price changes, so does P_{QUAID}. These include reducing consumption and switching to substitutes when the price of a good increases. When we impose the restriction $\lambda_i = 0 \; \forall \; i$, P_{QUAID} is reduced to P_{AID}—the exact index for the AID functional form. A further restriction of $\beta_i = 0 \; \forall \; i$ reduces P_{QUAID} to a homothetic-AID (HAID) price index P_{HAID}.

We also calculate four other price indexes that do not require econometric estimates of demand. To distinguish from the exact indexes based on demand system estimates, we call these arithmetic indexes. The Laspeyres index is defined by $P_L(p^t, p^0; q^0) = p^t \cdot q^0 / p^0 \cdot q^0$, where q^0 is the N-dimensional vector of quantities demanded at base. As such, Laspeyres makes no attempt to adjust for consumer responses to price changes. Therefore, it does not represent the minimum cost needed to maintain the baseline standard of living (i.e., utility). The Paasche index is defined by $P_P(p^t, p^0; q^t) = p^t \cdot q^t / p^0 \cdot q^t$, where q^t is the N-dimensional quantity vector at observation t, which can be a time period for a time series index, a location for a spatial index, or a unique pair of time and location for a panel index. Using current quantities as weights fully accounts for demand responses to price changes but fails to hold utility constant between the base and t. In other words, Laspeyres and Paasche are not exact indexes because they are not derived from utility maximization or its dual. Konüs (1924) showed that Laspeyres and Paasche form the upper and lower bounds of the true but unobserved cost of living if preferences are homothetic.

The Törnqvist index is given by $\log\left(P_T(p^t, p^0)\right) = \sum_j (w_{j0} + w_{jt}) \log\,(p_{jt}/p_{j0})$, w_{j0} and w_{jt} are budget shares of good j at base and t, respectively. The Fisher ideal index is calculated as $P_F(p^t, p^0) = \sqrt{P_L P_P}$. Törnqvist and Fisher ideal are superlative indexes in the sense that they are derived from (i.e., exact to) flexible functional forms providing second-order approximations to an arbitrarily twice differentiable linear homogenous cost function (Diewert 1976). Fisher ideal and Törnqvist are exact to the quadratic mean of order two unit cost function and translog (unit or total) cost function, respectively (Diewert 1976). The advantage of superlative indexes is that they do not require demand estimation. The potential limitations are twofold. First, the Fisher ideal index is only exact to homothetic preferences, which put implausible restrictions on the shape of the Engel curves. Second, while the Törnqvist index is exact to the nonhomothetic total translog function, it measures the change in cost of living from the base to t holding the reference utility at $u^* = \sqrt{u^t u^0}$, rather than u^0 (Diewert 1976, p. 122). So the more general cost function that the Törnqvist index is consistent with is of little use in constructing a constant-utility index, such as that needed for comparing the cost of living across more than two observations.

3.5 An Application to Fruit and Vegetable Price Indexes

We apply the I3SLS estimator to QUAID for fruit and vegetables and compare the QUAID price index with exact price indexes based on more restricted functional forms and with the arithmetic indexes. Retail fruit and vegetable sales are provided by Nielsen. Our sample covers 19 fresh fruits and vegetables (i.e., N = 19) from 2006 January to 2017 December. Table 3.1 reports the mean monthly dollar sales share and unit value by commodity. We estimate demand for the 19 commodities.

Each fruit or vegetable j contains barcode-level products. To reduce unit value bias (Cox and Wohlgenant 1986; Deaton 1988), which is one source of price endogeneity, we create each p_j in eq. (3.5) as a Törnqvist index whose elementary prices are the unit values at the barcode level. To further reduce unit value bias and other sources of endogeneity, we instrument p_j in each county by the average contemporaneous price in all other counties in the same designated market area (DMA) weighted by the inverse of their distances to the instrumented county. This approach of using neighboring county prices as instruments originated from Hausman (1997) and has seen wide applications since Nevo (2000). Common supply shocks ensure strong correlation between p_j and its instrument. Identification requires that demand shocks predictable by suppliers be uncorrelated across markets. When demand shocks are spatially correlated and are anticipated and reacted to by suppliers with price actions, the Hausman instruments will not account for this simultaneity bias. We include census region fixed effects, year fixed effect and month fixed effects in eq. (3.5). We instrument total expenditure X by its DMA average. The I3SLS estimator converges in 10 iterations with the convergence criterion set at 1E-10. Besides QUAID, we also estimate the rank-two standard AID and a homothetic demand system by setting $\lambda_i = 0$ and $\lambda_i = \beta_i = 0$, respectively.

3.5.1 Price and Expenditure Elasticities

Table 3.2 reports the QUAID median Marshallian price elasticities. All 19 own-price elasticities are negative and statistically significant at the 5 percent level. 11 fruits and vegetables have median own-price elasticities less than unity in absolute value, which indicates inelastic demand. Of the remaining, 2 vegetables and 6 fruits are price elastic. All fruits are price elastic. Of the

342 cross-price elasticities, 179 are positive indicating substitution and 163 are negative suggesting complementarity.

In our system, potatoes are the most consumed commodity by budget share. The own-price elasticity of potato is −0.59, which is also the most price inelastic commodity. Table 3.2 shows that 1 out of 6 fruits and 4 out of 13 vegetables are substitutes to potatoes. Strawberries are the most consumed fruit in our demand system[16]. The own-price elasticity of strawberry is −1.44. 4 out of 6 fruits and 10 out of 13 vegetables are substitutes to strawberries.

Table 3.2 also reports the median expenditure elasticities. All expenditure elasticities are significantly positive. 3 fruits and 8 vegetables have their expenditure elasticities less than unity, which means most vegetables and one half of fruits are necessities. The expenditure elasticity of potato is 0.89, indicating a one percent increase in total expenditure increases purchases by 0.89 percent for potatoes. The most expenditure-elastic commodity is garlic (1.22) and the most inelastic one is radish (0.81).

3.5.2 Fruit and Vegetable Price Indexes

We construct four arithmetic indexes: Laspeyres, Paasche, Törnqvist and Fisher ideal, and the QUAID exact indexes. The base quantity and price were set at the national average for the entire sample, defining the base (national average) price index as 1.

Figure 3.1 shows the spatial variation in the cost of fruit and vegetables at the county level. The range of cost of living is about 0.60 points. The county with the highest cost of fruit and vegetables is New York County, NY and the lowest is Accomack County, VT. For example, using the QUAID exact index with u^0 as reference utility, one dollar worth of fruit and

[16] In fact, bananas are the top fruit consumers purchase. However, since our samples don't include random-weight fruits and vegetables. Bananas are not included in the demand analysis.

vegetables at the national average costs 0.81 dollars in Accomack County, VT but 1.41 dollars in New York County, NY. Overall, the Northeast and the Southeast rank the highest in cost of fruit and vegetables, and the Southwest rank the lowest. The index numbers in the East are on average higher than that in the West. Most of the counties with high price levels are in Pennsylvania, New York and New Jersey. The cost of fruits and vegetables in California exhibits a greater variability, with the Central and Northern regions surpassing the national average, whereas the Southern being lower than the national average. The cost of living index in Florida ranges from 1.02 to 1.09, which is slightly higher than the national average.

Figure 3.2 plots the nationally average Laspeyres, Paasche, Törnqvist, and QUAID exact index over time. Within a year, the index series is U-shaped: relatively low in the early-mid of the year but relatively high at the end of the year. For example, we choose three states (New Jersey, Florida, and California) from three different areas (Northeast, Southeast, and West) and plot the exact price indexes. Figure 3.3 reveals a high cost of fruit and vegetables in New Jersey compared to the other two states. A seasonal trend is observed across all three states, with a marked increase in the cost during the winter months. Overall, the spatial and temporal patterns exhibited in our indexes are consistent with those of Çakır et al. (2018, p. 699), who found the spatial price difference to be more significant than the between-market difference in seasonality.

Figure 3.2 also shows that Laspeyres and Paasche bind Törnqvist and Fisher ideal indexes. Most of the time, they also bind the exact price index. Given the formula of exact price index, $\mathrm{P}(p^t, p^0; u^0) = e(p^t, u^0)/e(p^0, u^0)$, the inequalities of these price indices are demonstrated by $P_L(p^t, p^0; q^0) = p^t \cdot q^0/p^0 \cdot q^0 \geq e(p^t, u^0)/e(p^0, u^0) = P(p^t, p^0; u^0)$ and $P_P(p^t, p^0; q^t) = p^t \cdot q^t/p^0 \cdot q^t \leq e(p^t, u^t)/e(p^0, u^t) = P(p^t, p^0; u^t)$. However, the inequalities don't always guarantee Laspeyres and Paasche being the upper bound and lower bound of cost of living index

(Deaton and Muellbauer 1980). At the county level, the roles of Laspeyres and Paasche indexes are switched in 9.8% of our samples. This can occur in practice when demands for some fruits and vegetables are not downward sloping at these data points, especially using micro-level data. For example, suppose the price of apple is higher at the tth observation than at the base while all other prices are constant. For simplicity, assume all cross-price elasticities are zero. In this case, both Laspeyres and Passche at t will be higher than their base value of one. If demand for apple is downward sloping at t, the Passche index will increase by less than the Laspeyres index because quantity of apple demanded is lower at t and apple takes a smaller importance in the Passche index than in the Laspeyres index. The relative position of the two indexes reverses at t if the quantity of apple demand does not decline.

Figure 3.4 plots the QUAID indexes using the three alternative reference utility levels. For either the QUAID or AID exact index, differences in index numbers among the choices of reference utility u^0, u^t, u^* are negligible.[17] Recall that Törnqvist uses the geometric mean of u^0 and u^t as its reference utility, which varies from one observation to another. The Törnqvist index's close tracking of the three constant-utility QUAID index series suggests that its theoretical inability to fix reference utility across observations appears to be an empirical red herring.

3.6 Conclusion

In this study, we derive the asymptotic properties of the I3SLS estimator that was suggested by Lewbel and Pendakur (2009) as a practical solution to estimating conditionally linear demand systems. A 19-good QUAID system is used to demonstrate the ease at which the I3SLS estimator

[17] The HAID exact index is invariant to the choice of reference utility because of homotheticity. The cost function of HAID is in form of $e(p,u) = ua(p)$, which gives its exact index as $a(p^1)/a(p^0)$ independent of reference utility.

can be applied to demand systems with many goods. A number of applications of QUAID used the linearized version of the model where $\ln a(p)$ is replaced by the Stone price index and $b(p)$ is replaced by $\sum_i(\bar{w}_i - \bar{w}_i^0)\log(\frac{P_i}{P_i^0})$ (Matsuda 2006). Note that although linearization simplifies estimation, apart from the bias caused by linearization (Pashardes 1993), the linearized AID-type of models are no longer integrable even if common restrictions are imposed (LaFrance 2004). This is especially problematic for welfare analysis, which is often the reason to estimate a utility-theoretic demand in the first place. Given the simplicity of I3SLS, there is really no justification for linearizing the AID family of models moving forward.

The main motivation for estimating utility-theoretic demand system is to facilitate welfare analysis. We apply the parameter estimates from the 19-good fruit and vegetable demand to create exact price indexes—indexes that track the cost of consuming fruit and vegetables while holding the standard of living fixed. We find that the superlative Törnqvist and Fisher ideal price indices, which only require arithmetic operations, approximate the QUAID exact price index well.

Finally, several caveats are in order. First, we have focused on demand estimated by aggregate data. In micro data applications, the ubiquity of zeros in the purchase data and the need to address these corner solutions econometrically means that the I3SLS is not going to work for micro data. Methods such as the system-extended Amemiya generalized least squares (Zhen et al. 2014) can be used to estimate large systems with many zeros and endogenous prices. However, the budget share equations will have to be linearized. Second, relaxation of homotheticity has been shown to be important for calculating price indexes stratified by income (Handbury 2021), which require household-level data on purchase quantity and price. For random-weight foods, including much of fruit and vegetables, household scanner data do not collect the quantity information. Therefore, until reliable micro data become available, it appears that indexing the cost of consuming random-weight foods would have to lean on market-level data. In the latter case, we have shown that relaxing homotheticity may be secondary to accounting for substitution and complementarity. Lastly, we do not address the question of variety. It has been comprehensively researched elsewhere (Feenstra 1994; Broda and Weinstein 2010; Handbury and Weinstein 2015) using constant elasticity of substitution demand. Generalizing the approach to flexible functional form demand is nearly computationally impossible owing to the sheer number of new and disappearing goods that must be modeled as corner solutions. Fortunately, Handbury and Weinstein (2015) found that the variety effect is negligible compared to the effect of product heterogeneity in their ranking of city cost of living.

Table 3.1: Descriptive Statistics

Name	Dollar sales share		Price	
	Mean	Std	Mean	Std
Apples	0.067	0.030	1.036	0.116
Sprouts	0.003	0.002	1.052	0.180
Herbs	0.020	0.012	1.014	0.104
Mushrooms	0.076	0.018	1.008	0.080
Carrots	0.106	0.024	1.033	0.115
Cauliflower	0.022	0.011	1.149	0.290
Celery	0.050	0.018	1.027	0.174
Cranberries	0.008	0.011	1.045	0.229
Garlic	0.004	0.003	1.024	0.139
Grapefruit	0.007	0.005	1.111	0.188
Kiwi	0.002	0.002	1.069	0.189
Lettuce	0.107	0.025	1.079	0.164
Onions	0.054	0.018	1.032	0.157
Oranges	0.039	0.023	1.088	0.161
Potatoes	0.183	0.057	1.043	0.150
Radishes	0.003	0.002	1.038	0.167
Spinach	0.031	0.016	1.028	0.083
Strawberries	0.129	0.062	1.293	0.306
Tomatoes	0.089	0.023	1.039	0.108

Table 3.2: Median Marshallian Price and Expenditure Elasticities

	Apples	Sprouts	Herbs	Mushrooms	Carrots
Apples	**-1.454**	-0.032	-0.006	-0.068	0.124
	(0.075)	(0.359)	(1.404)	(1.446)	(0.147)
Sprouts	-0.893	**-0.798**	0.437	0.29	-0.946
	(0.039)	**(0.025)**	(0.108)	(0.114)	(0.018)
Herbs	-0.024	0.054	**-0.67**	-0.446	0.121
	(0.039)	(0.026)	**(0.015)**	(0.022)	(0.006)
Mushrooms	-0.058	0.009	-0.105	**-1.044**	-0.077
	(0.029)	(0.02)	(0.016)	**(0.014)**	(0.005)
Carrots	0.083	-0.021	0.025	-0.053	**-0.916**
	(0.014)	(0.016)	(0.022)	(0.027)	**(0.019)**
Cauliflower	0.269	0.017	0.023	0.071	-0.046
	(0.016)	(0.013)	(0.016)	(0.017)	(0.02)
Celery	0.127	-0.004	0.014	0.125	-0.125
	(0.039)	(0.069)	(0.14)	(0.183)	(0.109)
Cranberries	0.275	-0.007	-0.056	0.146	-0.203
	(0.097)	(0.105)	(0.187)	(0.187)	(0.173)
Garlic	-0.513	0.052	0.349	-0.658	-0.345
	0.038	0.051	0.095	0.113	0.084

Grapefruit	0.335	0.034	0.039	0.093	0.931
	(0.242)	(0.212)	(0.461)	(0.435)	(0.373)
Kiwi	0.557	0.455	0.228	-0.639	0.175
	(0.01)	(0.009)	(0.009)	(0.01)	(0.014)
Lettuce	0.023	0	0.038	0.102	-0.046
	(0.019)	(0.013)	(0.014)	(0.017)	(0.023)
Onions	-0.237	-0.007	0.043	-0.113	0.021
	(0.021)	(0.016)	(0.019)	(0.025)	(0.033)
Oranges	0.026	0	0.152	0.061	0.359
	(0.017)	(0.013)	(0.009)	(0.011)	(0.02)
Potatoes	0.129	0.007	-0.071	0.019	-0.134
	(0.129)	(0.112)	(0.239)	(0.237)	(0.191)
Radishes	0.748	-0.009	-0.1	-0.74	-0.532
	(0.055)	(0.042)	(0.041)	(0.042)	(0.07)
Spinach	0.428	0.02	-0.128	0.398	-0.063
	(0.011)	(0.008)	(0.007)	(0.008)	(0.013)
Strawberries	-0.073	-0.004	0.012	0.041	0.042
	(0.009)	(0.019)	(0.05)	(0.054)	(0.014)
Tomatoes	-0.11	-0.001	-0.004	-0.124	0.067
	(0.012)	(0.023)	(0.037)	(0.039)	(0.01)

	Cauliflower	Celery	Cranberries	Garlic	Grapefruit
Apples	0.083	0.092	0.055	-0.032	0.033
	(0.375)	(0.058)	(0.184)	(0.119)	(0.125)
Sprouts	0.151	-0.094	-0.04	0.095	0.099
	(0.036)	(0.01)	(0.022)	(0.018)	(0.014)
Herbs	0.02	0.03	-0.042	0.08	0.013
	(0.011)	(0.004)	(0.009)	(0.008)	(0.007)
Mushrooms	0.017	0.079	0.026	-0.035	0.008
	(0.008)	(0.004)	(0.007)	(0.006)	(0.005)
Carrots	-0.011	-0.06	-0.026	-0.013	0.06
	(0.046)	(0.017)	(0.034)	(0.026)	(0.023)
Cauliflower	**-1.161**	-0.226	0.01	0.088	0.004
	(0.012)	(0.008)	(0.014)	(0.014)	(0.01)
Celery	-0.097	**-0.828**	-0.02	0.005	-0.011
	(0.129)	**(0.046)**	(0.138)	(0.113)	(0.085)
Cranberries	0.014	-0.075	**-1.036**	0.034	0.063
	(0.17)	(0.11)	**(0.059)**	(0.068)	(0.049)
Garlic	0.437	0.044	0.108	**-0.792**	-0.311
	(0.106)	(0.052)	(0.038)	**(0.079)**	(0.032)
Grapefruit	0.011	-0.08	0.127	-0.191	**-2.014**
	(0.363)	(0.199)	(0.175)	(0.199)	**(0.481)**
Kiwi	-0.006	-0.106	0.083	-0.231	-0.304
	(0.01)	(0.015)	(0.009)	(0.011)	(0.013)
Lettuce	-0.06	-0.088	0.003	0.01	-0.024
	(0.021)	(0.023)	(0.013)	(0.016)	(0.022)
Onions	-0.043	-0.053	-0.011	-0.021	0

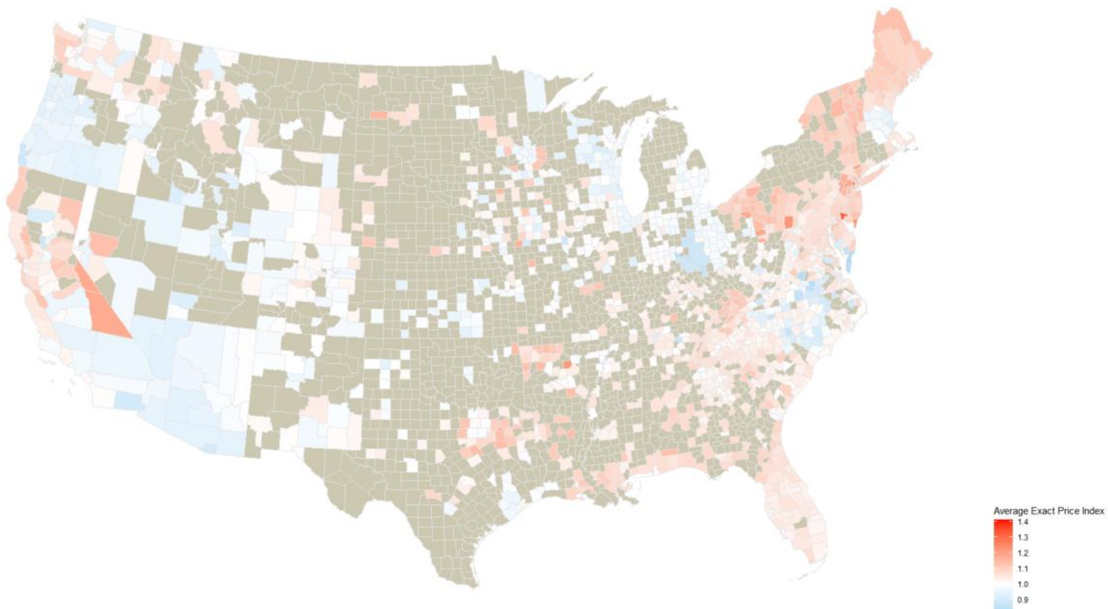
	(0.029)	(0.029)	(0.02)	(0.021)	(0.028)
Oranges	0.025	-0.021	0.101	-0.001	0.013
	(0.014)	(0.022)	(0.012)	(0.015)	(0.017)
Potatoes	-0.022	-0.033	-0.032	0.003	-0.009
	(0.211)	(0.148)	(0.094)	(0.108)	(0.141)
Radishes	-0.159	-0.129	-0.038	0.002	0.329
	(0.054)	(0.086)	(0.044)	(0.053)	(0.059)
Spinach	-0.014	-0.061	-0.012	0.024	0.081
	(0.009)	(0.013)	(0.009)	(0.011)	(0.012)
Strawberries	0.059	0.099	0.004	0.007	0.003
	(0.03)	(0.011)	(0.03)	(0.027)	(0.021)
Tomatoes	0.061	0.015	-0.002	0.007	-0.017
	(0.015)	(0.007)	(0.016)	(0.012)	(0.01)

	Kiwi	Lettuce	Onions	Oranges	Potatoes
Apples	0.015	0.031	-0.193	0.013	0.327
	(0.494)	(0.466)	(0.386)	(1.457)	(0.158)
Sprouts	0.356	0.003	-0.174	-0.002	0.473
	(0.053)	(0.038)	(0.035)	(0.129)	(0.016)
Herbs	0.023	0.198	0.117	0.319	-0.747
	(0.013)	(0.01)	(0.01)	(0.022)	(0.007)
Mushrooms	-0.015	0.139	-0.083	0.029	0.018
	(0.01)	(0.008)	(0.009)	(0.019)	(0.005)
Carrots	0.003	-0.046	0.008	0.134	-0.255
	(0.07)	(0.058)	(0.062)	(0.17)	(0.024)
Cauliflower	0	-0.291	-0.109	0.049	-0.196
	(0.022)	(0.023)	(0.023)	(0.05)	(0.011)
Celery	-0.004	-0.183	-0.058	-0.018	-0.133
	(0.484)	(0.395)	(0.398)	(1.257)	(0.126)
Cranberries	0.011	0.021	-0.043	0.297	-0.431
	(0.241)	(0.177)	(0.198)	(0.554)	(0.064)
Garlic	-0.101	0.226	-0.277	-0.016	0.08
	(0.175)	(0.126)	(0.126)	(0.404)	(0.049)
Grapefruit	-0.082	-0.374	-0.002	0.078	-0.266
	(1.006)	(0.915)	(0.851)	(2.375)	(0.302)
Kiwi	**-2.101**	0.339	0.537	0.24	-0.094
	(0.009)	(0.01)	(0.01)	(0.02)	(0.007)
Lettuce	0.006	**-0.772**	-0.043	0.043	-0.092
	(0.02)	**(0.013)**	(0.018)	(0.039)	(0.011)
Onions	0.019	-0.08	**-0.716**	-0.064	0.184
	(0.026)	(0.024)	**(0.029)**	(0.06)	(0.016)
Oranges	0.011	0.119	-0.087	**-1.898**	-0.043
	(0.012)	(0.011)	(0.013)	**(0.01)**	(0.008)
Potatoes	-0.001	-0.042	0.056	-0.007	**-0.598**
	(0.24)	(0.211)	(0.232)	(0.549)	**(0.09)**
Radishes	-0.176	0.286	-0.028	0.004	-0.651
	(0.067)	(0.057)	(0.051)	(0.069)	(0.041)

Spinach	-0.027	-0.73	0.112	0.217	-0.414
	(0.01)	(0.009)	(0.01)	(0.013)	(0.011)
Strawberries	0.001	0.061	0.057	0.03	-0.072
	(0.062)	(0.052)	(0.073)	(0.151)	(0.048)
Tomatoes	0.011	0.035	-0.056	-0.028	-0.147
	(0.021)	(0.017)	(0.016)	(0.035)	(0.013)

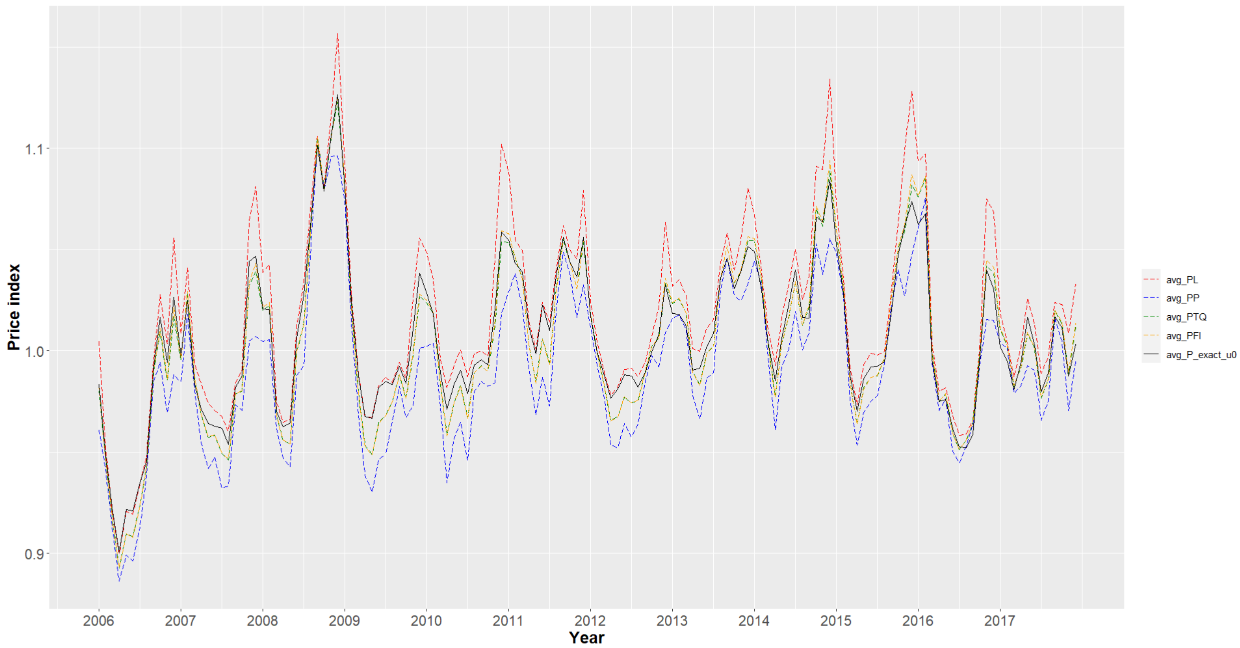
	Radishes	Spinach	Strawberries	Tomatoes	**Expenditure**
Apples	0.035	0.204	-0.121	-0.146	**1.036**
	(0.826)	(0.61)	(0.356)	(0.083)	**(0.004)**
Sprouts	-0.014	0.283	-0.154	-0.015	**0.949**
	(0.071)	(0.05)	(0.092)	(0.033)	**(0.029)**
Herbs	-0.018	-0.219	0.079	-0.025	**1.138**
	(0.015)	(0.011)	(0.059)	(0.025)	**(0.006)**
Mushrooms	-0.032	0.17	0.079	-0.139	**1.018**
	(0.013)	(0.008)	(0.046)	(0.033)	**(0.002)**
Carrots	-0.017	-0.015	0.066	0.065	**0.992**
	(0.105)	(0.073)	(0.058)	(0.02)	**(0.002)**
Cauliflower	-0.025	-0.017	0.35	0.273	**0.92**
	(0.034)	(0.021)	(0.054)	(0.241)	**(0.006)**
Celery	-0.009	-0.036	0.246	0.034	**0.977**
	(0.866)	(0.474)	(0.313)	(0.271)	**(0.004)**
Cranberries	-0.009	-0.027	0.043	-0.011	**0.997**
	(0.342)	(0.251)	(0.66)	(0.122)	**(0.034)**
Garlic	0	0.182	0.203	0.138	**1.221**
	(0.254)	(0.199)	(0.359)	(0.544)	**(0.026)**
Grapefruit	0.154	0.386	0.067	-0.214	**0.97**
	(1.433)	(1.005)	(1.529)	(0.013)	**(0.011)**
Kiwi	-0.308	-0.462	0.063	0.506	**1.044**
	(0.017)	(0.011)	(0.052)	(0.021)	**(0.054)**
Lettuce	0.008	-0.218	0.082	0.036	**0.993**
	(0.034)	(0.019)	(0.087)	(0.026)	**(0.002)**
Onions	-0.002	0.069	0.136	-0.085	**0.961**
	(0.04)	(0.029)	(0.163)	(0.013)	**(0.003)**
Oranges	0	0.176	0.105	-0.051	**0.952**
	(0.012)	(0.008)	(0.074)	(0.329)	**(0.006)**
Potatoes	-0.012	-0.068	-0.022	-0.056	**0.894**
	(0.467)	(0.48)	(1.62)	(0.061)	**(0.003)**
Radishes	**-0.774**	0.4	0.564	0.199	**0.811**
	(0.02)	(0.019)	(0.337)	(0.01)	**(0.037)**
Spinach	0.04	**-0.969**	-0.056	0.07	**1.09**
	(0.005)	**(0.009)**	(0.084)	(0.016)	**(0.007)**
Strawberries	0.015	-0.015	**-1.441**	0.061	**1.109**
	(0.12)	(0.11)	**(0.063)**	(0.023)	**(0.004)**
Tomatoes	0.006	0.027	0.086	**-0.891**	**1.066**
	(0.028)	(0.017)	(0.021)	**(0.425)**	**(0.005)**

Note: The numbers in parentheses are standard errors



Notes: This map illustrates the average QUAID index number for each county over the 2006–2017 period. Gray area means no data matched

Figure 3.1: The Spatial Variation of Fruit and Vegetable Cost



Note: avg_PL indicates Laspeyres; avg_PP indicates Paasche; avg_PFI indicates Fisher Ideal; avg_PTQ indicates Törnqvist; avg_P_exact_u0 indicates the QUAIDS exact index at reference u^0

Figure 3.2: National Price Indexes Over Time

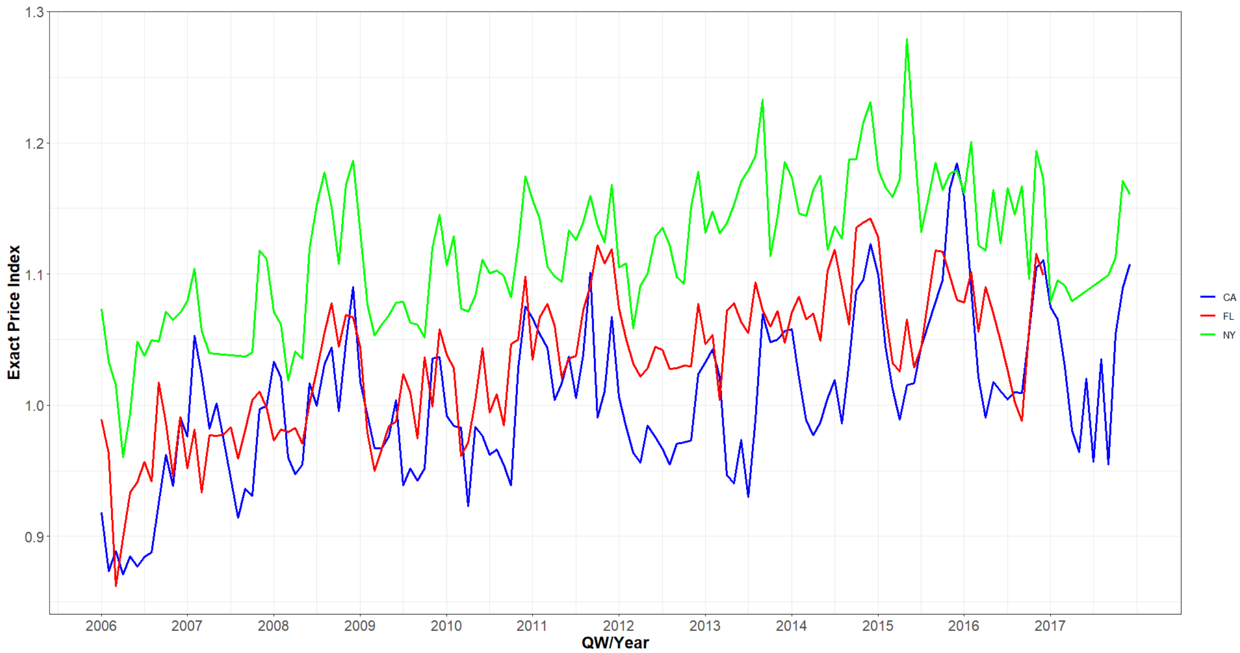


Figure 3.3: Fruit and Vegetable Cost Over Time for Three States

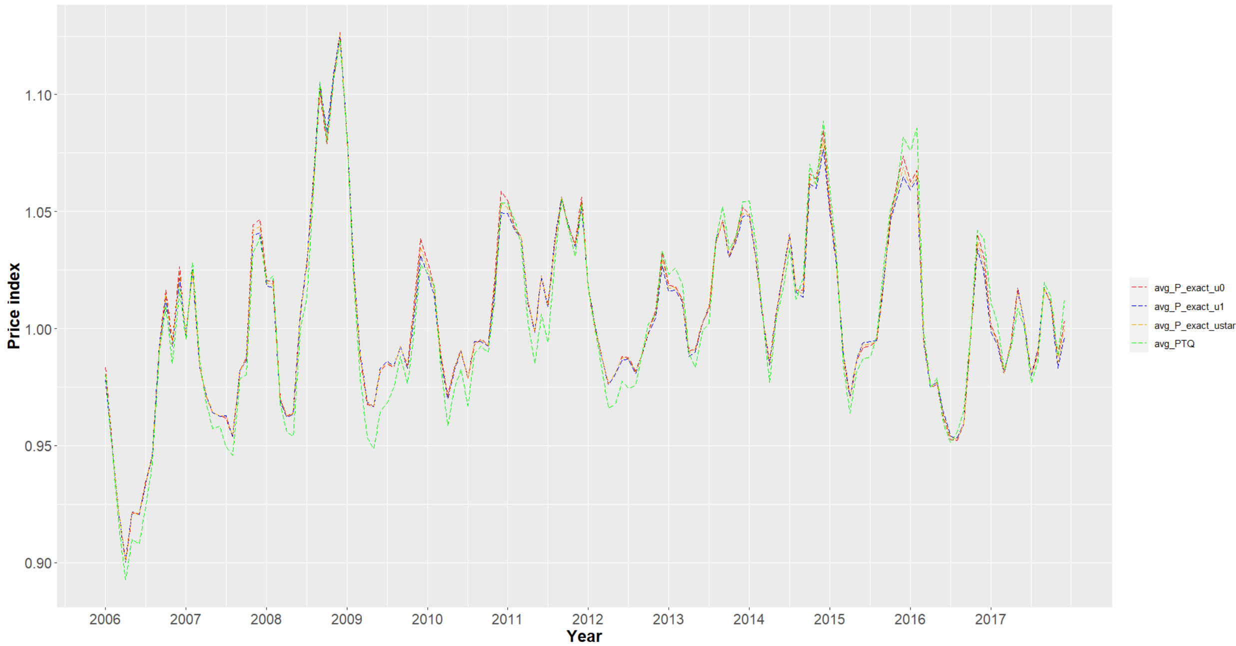


Figure 3.4: QUAID Indexes Using Different Reference Utility and the Törnqvist Index

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APPENDIX A

PROOF OF ASYMPTOTIC DISTRIBUTION OF I3SLS ESTIMATOR

Regularity assumption: following Blundell and Robin (1999), we have

Assumption 1: For all $t \in \{1,2,\dots,T\}$, $E[\varepsilon_t|z_t] = 0$, $[\varepsilon_t\varepsilon_t'|z_t] = \Sigma_0$

Assumption 2: For all t, $g(x_t, \theta)$ is continuous w.r.t θ. And $\partial g(x_t, \theta)/\partial\theta'$ is continuous.

Assumption 3: The matrices $\frac{1}{T}\sum_{t=1}^{T} z_t g(x_t, \theta)'$, $\frac{1}{T}\sum_{t=1}^{T} z_t z_t'$ and $\frac{1}{T}\sum_{t=1}^{T} g(x_t, \theta) g(x_t, \theta)'$ converge to $E[z_t g(x_t, \theta)']$, $E[z_t z_t']$, and $E[g(x_t, \theta) g(x_t, \theta)']$ respectively. They are non-singular.

Assumption 4: The matrix $\frac{1}{T}\sum_{t=1}^{T} \frac{\partial}{\partial\theta'}\{z_t[y_{it} - g(x_t, \theta)'\theta_i]\}$ converges to $E\left[\frac{\partial}{\partial\theta'} z_t[y_{it} - g(x_t, \theta)'\theta_i]\right]$. For all θ, the matrix whose ith row is $E\left[\frac{\partial}{\partial\theta'} z_t[y_{it} - g(x_t, \theta)'\theta_i]\right]$ is non-singular.

Assumption 5: The estimator $\hat{\theta} = (\hat{\theta}_1', \hat{\theta}_2', \dots, \hat{\theta}_N')$ solves $\hat{\theta}_i = [Z'G(\hat{\theta})]^{-1}Z'y_i$, $i = 1,2,\dots,N$

We provide the proof that IV estimator $\hat{\theta}_{IV}$ of ILSE weakly converges to true θ.

Let's set

$$G(\theta) = \begin{pmatrix} g(x_1, \theta)' \\ \vdots \\ g(x_T, \theta)' \end{pmatrix}$$

Our IV is

$$Z = \begin{pmatrix} z_1' \\ \vdots \\ z_T' \end{pmatrix}$$

Considering that we have K independent variables, so for each $g(x_t,\theta)'$, we have $g(x_t,\theta)' = (g_1(x_t,\theta), g_2(x_t,\theta), \dots, g_K(x_t,\theta))$ and $z_t' = (z_{t1}, z_{t2}, \dots, z_{tK})$.

Recall our basic assumption of IV, we need $[\varepsilon|Z] = 0$, then we can get $E[Z'\epsilon] = 0$.

Equivalently we can obtain that:

$$\frac{1}{T}\sum_{t=1}^{T} z_t[y_{it} - g(x_t,\theta)'\hat{\theta}_i] = 0 \text{ for each equation i} \tag{A.1}$$

By the theorem of intermediate values, we get

$$0 = \frac{1}{T}\sum_{t=1}^{T} z_t[y_{it} - g(x_t,\theta)'\hat{\theta}_i]$$

$$= \frac{1}{T}\sum_{t=1}^{T} z_t[y_{it} - g(x_t,\theta)'\theta_i]$$

$$+\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'\theta_i^*]\}\big(\hat{\theta} - \theta\big)$$

$$= \frac{1}{T}\sum_{t=1}^{T} z_t\varepsilon_{it} + \frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'\theta_i^*]\}\big(\hat{\theta} - \theta\big)$$

$$\xrightarrow{a.s.} 0 + \frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'\theta_i^*]\}\big(\hat{\theta} - \theta\big)$$

$$= \frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'\theta_i^*]\}\big(\hat{\theta} - \theta\big) \text{ (A.2)}$$

θ^* is some intermediate value between $\hat{\theta}$ and θ. Also $\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'] \neq 0$

Let $B_i\big((y_{it}, x_t), \theta^*\big) = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'\theta_i^*]\}$, then $B\big((y,x),\theta\big)$ is an asymptotic non-singular matrix whose ith row is $\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta\prime}\{z_t[y_{it} - g(x_t,\theta^*)'\theta_i^*]$.

Hence, $\hat{\theta} \rightsquigarrow \theta$.

When $T \to \infty$, $\hat{\theta} \rightsquigarrow \theta$. Since θ^* is between $\hat{\theta}$ and θ, then $\theta^* \rightsquigarrow \theta$. In that case,

$$0 = \frac{1}{T}\sum_{t=1}^{T} z_t[y_{it} - g(x_t,\theta)\hat{\theta}_i']$$

$$= \frac{1}{T}\sum_{t=1}^{T} z_t[y_{it} - g(x_t,\theta)'\theta_i]$$

$$+\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta'}\{z_t[y_{it}-g(x_t,\theta)'\theta_i]\}\big(\hat{\theta}-\theta\big)$$

$$=\frac{1}{T}\sum_{t=1}^{T}z_t\varepsilon_{it}+\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta'}\{z_t[y_{it}-g(x_t,\theta)']\,\theta_i\}\big(\hat{\theta}-\theta\big) \qquad \text{(A.3)}$$

Then

$$\frac{1}{T}\sum_{t=1}^{T}\frac{\partial}{\partial\theta'}\{z_t[y_{it}-g(x_t,\theta)'\theta_i]\}$$

$$=\frac{1}{T}\sum_{t=1}^{T}\frac{\partial z_t}{\partial\theta'}\{[y_{it}-g(x_t,\theta)'\theta_i]\}-\frac{1}{T}\sum_{t=1}^{T}z_t\frac{\partial g(x_t,\theta)'\theta_i}{\partial\theta'}$$

$$=0-\frac{1}{T}\sum_{t=1}^{T}(z_t\theta_i{}'\cdot\frac{\partial g(x_t,\theta)}{\partial\theta'}+z_t\cdot 1_i'\otimes g(x_t,\theta)')$$

$$=-\frac{1}{T}\sum_{t=1}^{T}z_t\theta_i{}'\cdot\frac{\partial g(x_t,\theta)}{\partial\theta'}\ -\frac{1}{T}\sum_{t=1}^{T}z_t\cdot 1_i'\otimes g(x_t,\theta)' \quad \text{(A.4)}$$

where $1_i'$ is the ith column of the N by N identity matrix.

By the strong law of large numbers, for each i:

$$\frac{1}{T}\sum_{t=1}^{T}z_t\theta_i{}'\cdot\frac{\partial g(x_t,\theta)}{\partial\theta'}\ \to E[z_t\theta_i{}'\cdot\frac{\partial g(x_t,\theta)}{\partial\theta'}] \qquad \text{(A.5)}$$

$$\frac{1}{T}\sum_{t=1}^{T}z_t\cdot 1_i'\otimes g(x_t,\theta)'\ \to E[z_t\cdot 1_i'\otimes g(x_t,\theta)'] \qquad \text{(A.6)}$$

By the central-limit theorem,

$$\begin{pmatrix}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}z_t\varepsilon_{1t}\\ \vdots\\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T}z_t\varepsilon_{Nt}\end{pmatrix}=\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\boldsymbol{\varepsilon_t}\otimes z_t\to N(0,\Sigma\otimes E[z_t z_t{}']) \qquad \text{(A.7)}$$

Notice that $0=\frac{1}{T}\sum_{t=1}^{T}z_t\varepsilon_{it}-[\frac{1}{T}\sum_{t=1}^{T}z_t\theta_i{}'\cdot\frac{\partial g(x_t,\theta)}{\partial\theta'}\ +\frac{1}{T}\sum_{t=1}^{T}z_t\cdot 1_i'\otimes g(x_t,\theta)']\big(\hat{\theta}-\theta\big)$

for all i.

Then

$$\sqrt{T}(\hat{\theta}-\theta)\rightsquigarrow N(0,J^{-1}\cdot\Sigma\otimes K\cdot J'^{-1}) \qquad \text{(A.8)}$$

where

$$K = E[z_t z_t{}']$$

$$J = I_n \otimes E[z_t \cdot g(x_t, \theta)'] + E[(\Theta' \frac{\partial g(x_t, \theta)}{\partial \theta'}) \otimes z_t]$$

$$\Theta = (\theta_1, \theta_2, \dots, \theta_N)$$

The estimator of the asymptotic variance-covariance matrix of $\hat{\theta}$ is proved as follow:

Let's start from:

$$[I_N \otimes Z'] \cdot \left[y - \left(I_N \otimes G(\hat{\theta})\right) \hat{\theta}\right] = 0 \qquad \text{(A.9)}$$

where $y = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}$ and $\hat{\theta} = \begin{pmatrix} \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_N \end{pmatrix}$

Then

$$0 = [I_N \otimes Z'] \cdot \left[y - \left(I_N \otimes G(\hat{\theta})\right) \hat{\theta}\right]$$

$$= [I_N \otimes Z'] \cdot \left[y - \left(I_N \otimes G(\theta)\right) \theta\right]$$

$$+ [I_N \otimes Z'] \cdot \frac{\partial}{\partial \theta'} \left[y - \left(I_N \otimes G(\theta)\right) \theta\right] \cdot \left(\hat{\theta} - \theta\right) + R(\hat{\theta})$$

$$= [I_N \otimes Z'] \cdot \varepsilon - [I_N \otimes Z'] \frac{\partial [I_N \otimes G(\theta)] \theta}{\partial \theta'} \left(\hat{\theta} - \theta\right) + R(\hat{\theta}) \text{ (A.10)}$$

When sample size is very large, $R(\hat{\theta})$ goes to 0.

$$Asy.\hat{V}\left(\hat{\theta}\right) = [[I_N \otimes Z'] \frac{\partial \left[I_N \otimes G\left(\hat{\theta}\right)\right] \hat{\theta}}{\partial \theta'}]^{-1} \cdot [I_N \otimes Z'] \cdot \hat{\varepsilon}\hat{\varepsilon}' \cdot [I_N \otimes Z']'$$

$$\cdot [[[I_N \otimes Z'] \frac{\partial \left[I_N \otimes G\left(\hat{\theta}\right)\right] \hat{\theta}}{\partial \theta'}]']^{-1}$$

$$= [[I_N \otimes Z'] \frac{\partial \left[I_N \otimes G\left(\hat{\theta}\right)\right] \hat{\theta}}{\partial \theta'}]^{-1} \cdot \left(\hat{\Sigma} \otimes Z'Z\right) \cdot [[I_N \otimes Z'] \frac{\partial \left[I_N \otimes G\left(\hat{\theta}\right)\right] \hat{\theta}}{\partial \theta'}]]^{-1\prime}$$

$$= \hat{J}^{-1} \cdot \hat{\Sigma} \otimes \hat{K} \cdot \hat{J}'^{-1} \qquad \text{(A.11)}$$

However, when imposing cross equation restrictions (especially symmetry), $\hat{J}$ may become singular. In that case, we redefine the $I_N \otimes Z$ and $I_N \otimes G(\hat{\theta})$ by a transformation matrix A and rearrange θ such that $(I_N \otimes Z) \cdot A \cdot \theta_r$, $\left(I_N \otimes G\left(\hat{\theta}\right)\right) \cdot A \cdot \theta_r$ are the same as $(I_N \otimes Z) \cdot \theta$ and $(I_N \otimes G(\theta) \cdot \theta$ (Wooldridge 2002, pp 167), where vector θ_r only contains unrestricted parameters . Then (A.11) becomes $\{[(I_N \otimes Z) \cdot A]' \cdot \frac{\partial\left[I_N \otimes G\left(\hat{\theta}\right)\right] \cdot A \cdot \hat{\theta}_r}{\partial \theta_r'}\}^{-1} \cdot [A' \cdot \left(\hat{\Sigma} \otimes Z'Z\right) \cdot A] \cdot \{[(I_N \otimes Z) \cdot A]' \cdot \frac{\partial\left[I_N \otimes G\left(\hat{\theta}\right)\right] \cdot A \cdot \hat{\theta}_r}{\partial \theta_r'}]\}^{-1\prime}$.

DISCLAIMER

(i) Researcher(s)' own analyses calculated (or derived) based in part on data from Nielsen Consumer LLC and marketing databases provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. (ii) The conclusions drawn from the NielsenIQ data are those of the researcher(s) and do not reflect the views of NielsenIQ. NielsenIQ is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.