INTEGRATING MULTIPLE RISKS IN TIMBERLAND INVESTMENT DECISIONS:

PRICES, BIOLOGICAL GROWTH RATES, AND HURRICANES

by

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(Under the Direction of JACEK SIRY)

ABSTRACT

Timberland investments have attracted institutional investors due to their hedging properties and low correlation with financial assets. These characteristics make timberland suitable for balancing traditional asset portfolios. However, timber price volatility, biological growth uncertainty, long-term nature, and exposure to environmental hazards contribute to risk perception. Over the years, researchers have explored various methods to estimate timberland investment risk, focusing primarily on independent risk analysis. This dissertation explores integrated risk analysis to account for multiple risk sources, offering a more realistic approach for this alternative asset, which faces several risks throughout its maturation. The integration includes timber price volatility, biological growth risk, and hurricane damage. Several mathematical models were used to represent these uncertainties, from multivariate time series and seemingly unrelated regression models to nonlinear programming, to simulate expected value distributions, volatilities, and correlations. These outcomes were integrated to examine how risk interaction affects returns and expected revenue streams. The results indicate that biological growth risk plays a dominant role in shaping expected returns, surpassing the impact of timber price volatility. Hurricane risk proves less influential than the combined effect of biological growth and price uncertainty, even in regions classified as highly hurricane-prone.

INDEX WORDS: Timberland investment risk, Return driver risk, Risk integration, Timber price volatility, Biological growth uncertainty, Hurricane damage, Time series models, Seemingly unrelated regression, Nonlinear programming

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DEDICATION

To God and my family.

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CHAPTER 1

INTRODUCTION

Timberland is a unique asset class that combines biological growth and real estate appreciation. This combination results in inflation hedging and a low correlation with traditional financial assets.

However, due to their long-term nature, timberland assets face various uncertainties throughout their maturity. Timberland investment risks range from price fluctuations to natural hazards, where price uncertainty is often regarded as the primary source of volatility, much like other commodities. In addition, natural disasters such as wildfires have driven environmental risk studies in timberland investments. Researchers have developed sophisticated models to assess environmental risks. These methods range from stochastic programming to spatial analysis. These efforts have enhanced risk assessment and generated new research questions.

Biological growth has received less attention as a source of risk in timberland investment, despite forest biometricians recognizing that determining forest state variables is challenging. Biometric models cannot deliver the precision often expected, as they are built on large datasets drawn from research plots across different geographies. The volatility introduced by a chain of biometric models affects expected value distributions. Moreover, studies that consider multiple sources of risk are also relatively uncommon. Integrating risk models is crucial to reflect the long-term challenges of timberland assets better and reveal interactions that can affect their expected value. As a result, critical gaps remain in capturing multiple risk interactions in timberland risk modeling.

To help fill this integration gap, we propose a multi-source risk assessment that accounts for biological risks, price uncertainty, and natural hazards. Specifically, timber and bare land prices, biological growth, and hurricane damage risks are integrated into a unified framework. This approach allows for a more realistic estimation of financial outcomes by capturing multiple, correlated risk factors that are often analyzed in isolation. By simulating their joint effects, the framework supports more informed decisions related to investment, insurance, and forest management.

We structured this dissertation into seven chapters. Chapter 1 introduces the scope and motivation of the study. Chapter 2 presents a literature review analyzing relevant timber price, biological growth, and natural hazard risk modeling research. Chapter 3 develops multivariate and univariate time series models for timber and bare land prices and corresponding risk algorithms. Chapter 4 fits biological growth models for the main stand state variables and timber products, describing dynamic variance and correlation structures. Chapter 5 combines timber price, land value, and biological growth risks to recalculate the main drivers of timberland returns. Chapter 6 integrates hurricane damage probabilities with timber price and growth models into a risk-adjusted nonlinear programming model for tactical planning. Finally, Chapter 7 discusses the main findings and offers conclusions.

CHAPTER 2

A review of timberland risk modeling[1]

[1] Cabezas, C. To be submitted to a peer-reviewed journal

Abstract

Timberland investment risk has been a significant concern for investors and researchers over the years. This review analyzes the most relevant modeling strategies impacting economic analyses. It considers 101 articles from 1966 to 2024 that model price and natural hazard uncertainty affecting investment cash flows. The review revealed a noticeable trend in risk analysis that has accelerated over the last two decades and is expected to continue in the future. The approaches range from simple distributions to stochastic processes, including sophisticated spatial analyses and optimization algorithms, along with the geometric Brownian motion process prevalent. However, these strategies primarily focus on independent risk assessments, while integration has received less attention. This highlights a gap in integrated risk modeling that must be addressed due to the long-term nature of timberland investment, which is vulnerable to multiple risks over time.

1. Introduction

Timberland investments involve acquiring and managing forests over the long term to generate timber income, deliver environmental and social benefits, and achieve risk-adjusted financial returns (F. Zhang and Chang 2018; D. Zhang and Pearse 2012). Timberland investments require careful planning, spanning from forest regeneration to harvesting. Consequently, managers and investors have relied on capital budgeting based on Fisher's 1907 model to plan irreversible decisions (Duku-Kaakyire and Nanang 2004; Susaeta and Gong 2019; Fisher 1907).

An application of the capital budgeting technique is Wicksell's theory. It states that investments should be realized when the natural rate of return equals the market interest rate to maximize economic gains (Åkerman 1933). Specifically, the Wicksell single rotation approach suggests harvesting trees when their value peaks, considering timber growth rates, market conditions, and discount rates (Wan 1977). Another application is the Martin Faustmann rule, which employs a multiple-rotation method (Faustmann 1995). This approach relies on discounted cash flows with an infinite time horizon, utilizing a suitable hurdle rate to compare the present value with the initial investment cost (Amacher, Ollikainen, and Koskela 2009; D. Zhang 2021; Yin and Newman 1996). These models are central in forest economics, guiding decisions on when and how to harvest trees for maximum economic benefit. However, the timberland cycle comprises extended horizons and inherent risks that challenge these economic assessments and decision-making processes (Lönnstedt and Svensson 2000; Duku-Kaakyire and Nanang 2004; Samuelson 2012).

Risk is measurable uncertainty where the likelihood of different outcomes can be known or estimated (Knight 1921). These outcomes encompass potential gains and losses, although they are generally perceived as adverse (Damodaran 2008). For instance, negative price changes, inaccurate yield forecasts, and natural disasters can lead to financial losses and adverse environmental and social impacts (Prestemon and Holmes 2000).

Long-term forest planning involves various market and biological risks (Brazee and Newman 1999; Kangas and Kangas 2004; Routledge 1980). Therefore, standard financial models based on discounted cash flows underestimate the unpredictable nature of many core variables (Damodaran 2008). Consequently, estimating expected cash flows may prove inaccurate, potentially leading to suboptimal decisions (Brennan and Schwartz 1985; Trigeorgis 1999; Dixit and Pindyck 1994; Routledge 1980). Thus, timberland investors and managers facing multiple risk sources require informed, well-defined management strategies.

Different approaches have been developed to address risk in decision-making in forestry, including probabilistic models that estimate the likelihood and impacts of unforeseen events (Holmes, Prestemon, and Abt 2008; Kangas and Kangas 2004; Pasalodos-Tato et al. 2013). Research has mainly focused on individual risks rather than on integrating risk sources, which pose an extra challenge since real-life situations comprise several sources of uncertainty. Thus, we must further refine risk integration within our investment analysis framework. In other words, it is essential to assess diverse sources of risk to ensure that decision-makers are aware of the most potential outcomes (Kangas and Kangas 2004).

This review aims to consolidate relevant literature concerning risk in timberland decision-making. It encompasses topics such as timber price fluctuations, optimal harvest timing, environmental hazards, and decision-making under risk. This synthesis aims to shed light on various research questions, stimulate further analysis, and contribute a more comprehensive understanding of risk in timberland investments. The review organizes the selected articles into eight sections focusing on timberland risk modeling. Sections 1 and 2 introduce the review and outline the methods used. Section 3 examines the risks associated with timber prices and values. In contrast, Section 4 discusses risks related to timber growth and natural disturbances. Both sections aim to emphasize the application of probabilistic and stochastic processes and decision-making methods. Section 5 outlines the mathematical and simulation approach reviews conducted in timberland risk analysis, alongside assessing the existing literature reviews on the topic. Section 6 discusses the main findings, while Section 7 offers conclusions, summarizing the implications. Finally, Section 8 lists the references incorporated into this document.

2. Methods

A comprehensive literature review was conducted in 2023-2024. It focused on scholarly articles in English that delved into risk in forest economics. This review leveraged the extensive resources available at the University of Georgia library. The sources involved various academic databases and platforms, including ScienceDirect, EBSCO HOST, Project Muse, ProQuest Central, SpringerLink, Oxford Academic, Google Scholar, JSTOR, and Wiley Online Library. This approach ensured a diverse and representative collection of the current state of knowledge in forest economics, making additional contributions to understanding risk in this field.

The search targeted articles on timberland, risk, and investments, using terms like "forest/timberland risk/uncertainty" and "forest/timberland investment" alone or combined. These articles were reassessed, looking for natural disturbances, biological growth, and timber price risk modeling terms within a forest economic analysis or foundational framework. This assessment included gauging titles, abstracts, and keywords based on relevance, methodological rigor, and contribution to the field. The snowball method was also employed to examine the bibliographies of initially identified articles, revealing further pertinent literature.

The timeframe began with the earliest proper publication available via these search terms through the search engine. Therefore, we defined a selection timeframe from December 1966 to April 2024, driven by contemporary perspectives and recent developments in the field. However, it should be noted that this review did not include studies on real options and other risk management strategies. This strategy suggests areas for further enhancement of the review, as real options and other risk management require an in-depth assessment.

The selection process was thorough, initially encompassing a review of 353 articles. These selected articles underwent an in-depth review that identified 185 papers. Upon closer scrutiny, 84 of these papers were deemed less relevant to the primary focus of this study for various reasons, thereby refining the selection to 101 documents included in the review.

The following sections are organized according to the evaluation method used in the selected research, as shown in Table 2.1. This structure offers a systematic way to understand and compare different economic modeling approaches in forest management. It is important to note that this categorization can sometimes be arbitrary, as some studies may explore more than one process or model, allowing for alternative categorizations. The main goal of this arrangement is to facilitate navigation through various studies and methodologies.

3. Timber Price and Forest Value Risk

The stochastic nature of timber prices has been a significant concern for timberland investors (Yousefpour et al. 2012). However, Net Present Value (NPV) calculations and Faustmann rule-based policies typically assume constant timber prices over time, which poses a challenge given price fluctuations. Unexpected price deviations can impact expected returns and complicate irreversible decisions.

3.1. Stochastic Dynamic Programming

The Markov chain or process (MP) is a mathematical model where the future state depends solely on the current state and is memoryless of past states (Norris 1997; Howard 1960). Norstrom (1975) embarked on a pioneering comparative analysis, considering timber prices as a stochastic MP. He examined deterministic and stochastic models for optimal harvest decisions by analyzing reservation values. Specifically, his research explored how deterministic age-dependent growth interacts with the stochastic nature of prices, using dynamic programming. Lohmander (1988) elaborated on Norstrom's work and discussed non-stationary price processes such as martingales and autocorrelations to determine optimal harvesting. He also developed a numerical example to illustrate his considerations and employed stationary prices for Scandinavian forests instead. This example illustrated that reservation prices rise under high-risk conditions.

The Markov Decision Process (MDP) is a discrete and sequential decision-making approach used to assess risk based on MP (Yuzhou Wang 2022; Howard 1960). Lembersky and Johnson (1975) applied the MDP to model risk in management decisions over an infinite horizon. Their model aimed to maximize expected returns under uncertainty in future markets and silvicultural management activities. They developed state transition probabilities to simulate future forest growth and timber market forms based on the current situation and actions taken at each decision point. Thus, they defined 240 states based on average tree size, stocking level, and saw log prices, identifying the optimal expected returns for each possible starting scenario. Also, in an MDP framework, Kaya and Buongiorno (1987) modeled harvesting decisions in uneven-aged stands while accounting for timber price and growth risks. They combined transition probabilities from timber and price state changes to determine forest economic states. Likewise, Buongiorno (2001) used MDP's applications to model a generalization of the Faustmann rule and determine optimal timber production. He assumed stochastic periodic growth and prices to calculate probabilistic distributions and model future stand states. Subsequently, he applied successive approximations based on the dynamic programming optimality principle and linear programming to calculate optimal expected returns. Thus, he confirmed that Faustmann's formula is a special case of an MDP model with transition probabilities of one or zero.

Brazee and Mendelsohn (1988) studied reservation prices for Douglas-fir and loblolly pine forests. They modeled the price process as a series of independent, normally distributed random draws under the assumption of risk neutrality. Accordingly, they developed a dynamic harvesting decision model that adjusts to short-run price fluctuations. Their analysis revealed that this flexible harvest policy increases the expected net present values over the traditional Faustmann model by allowing landowners to delay harvests until favorable prices. Consequently, the findings suggest that price risks can extend rotation ages as landowners wait for prices to rise above average before harvesting.

Clarke and Reed (1989) investigated the optimal harvest rule, factoring risk aversion. They examined the Wicksell and Faustmann harvesting policies using the Hamilton-Jacobi-Bellman equation to incorporate stochastic factors into control problems. They assumed the forest asset value and biological growth processes followed a geometric Brownian motion (GBM) and a Brownian motion (BM) diffusion process, respectively. This assumption implied that the forest asset prices follow a lognormal distribution, and the biological growth follows a normal distribution. Subsequently, Reed and Clarke (1990) delved into stochastic size-dependent growth and stochastic pricing models. Size-dependent assumptions can emulate real-world conditions in natural forests and wildlife populations by accounting for natural disturbances' downsizing effects (Clarke and Reed 1989). Their results showed that delaying or proceeding with the harvest under risk needs a thorough evaluation. Harvest rotation may be extended or reduced based on risks and potential rewards. Later, Yin and Newman (1995) revisited Clarke and Reed's (1989) rule to factor in the decision-maker's opportunity costs associated with land rent and management activities from postponing harvesting. They revealed that opportunity costs are significant considerations in harvesting policies. Optimal harvesting should occur earlier to capitalize on a higher growth rate and offset these alternative costs. In other words, this consideration resulted in a shorter forest rotation than Clarke and Reed's findings could describe.

Haight (1990) developed a continuous feedback thinning model for uneven-age stands, incorporating timber price risks represented as a stationary random process. Among his results, he presented a test case demonstrating that his stochastic model outperformed a deterministic one, providing a higher net present value and smoother timber flow but upholding extended thinning cycles. Teeter and Caulfield (1991) also explored the effects of timber price risk on stand density management decisions. They shaped an MP with future stumpage prices depending solely on the current price. For such analysis, they relied upon stochastic dynamic programming combined with state transition probabilities for expected values to determine optimal stocking levels. They compared economic probabilistic scenarios involving various initial planting densities, thinning levels, discount rates, site indexes, and rotation lengths to a deterministic scenario. Teeter and Caulfield's findings showed that the optimal initial planting density considering price risk is comparable to the deterministic method. Conversely, price risk substantially influences thinning and stand density management decisions.

Haight and Smith (1991) also employed stochastic dynamic programming on thinning and harvesting decisions. They analyzed loblolly pine and mixed loblolly pine-hardwood stands, treating hardwood as competing vegetation in an infinite horizon. Their model employed stationary and randomly fluctuating prices and deterministic age-dependent timber growth. Their findings indicated that price risk had a marginal impact on the early commercial thinning intensity in a rotation regime. Conversely, they found that price risk affected the final harvest time for pure and mixed stands. However, they emphasized these results as contingent upon stationary price assumptions and dynamic programming formulation. For instance, a change to a nonstationary assumption would alter outcomes. Similarly, Brazee and Bulte (2000) examined optimal thinning and harvesting decisions for Scots pine even-aged stands. They assumed risk neutrality and stationary random draw processes for stumpage prices. Brazee and Bulte recognized that the price distribution spread effect on thinning reservation prices was ambiguous due to conflicting impacts on commercial and pre-commercial thinning.

Gong (1999) employed a first-order autoregressive AR(1) process and a random walk as a special case to assess optimal harvest decisions in even-aged stands. He developed a simulation method grounded on stochastic dynamic programming. His model factored in continuous price distributions across multiple decision-making intervals. Gong showed that the AR(1) model type and its autocorrelation coefficients significantly affect the reservation price and harvest age. His results exposed that higher price autocorrelation coefficients reduce optimal reservation prices in older stands while causing fluctuations in younger stands, initially decreasing and then increasing. Additionally, increased autocorrelation reduced the harvest policy's expected net present value.

We can see that different assumptions can lead to varying outcomes. Thus, stationarity or nonstationarity properties in the underlying price processes add complexity when modeling irreversible decisions. Haight and Holmes (1991) analyzed the quarterly sawtimber price series based on initial monthly and monthly average prices. They studied the stationarity implications in the price model definition. They used dynamic programming and Monte Carlo simulations to analyze how stationary autoregressive and random walk price models impact harvest decisions and expected present values. Their findings exposed conflicting outcomes regarding stationarity. The unit root test indicated stationarity for the monthly and quarterly series based on opening month prices but nonstationarity based on average month prices. This divergence resulted in varying performance in expected present value. They emphasized the significance of this research, as different averaging methods resulted in different harvest policies. Afterward, Forboseh, Brazee, and Pickens (1996) extended Haight and Smith's (1991) work and studied stationary and cross-product price risk effects on optimal harvesting in ongoing rotations.

Susaeta and Gong (2019) developed a stochastic dynamic programming model to evaluate optimal reservation prices for even-age stands, incorporating risk-neutrality, age-dependent fire, and timber price risks. They considered discrete harvest decisions that account for timber, including timber salvage income and non-timber benefits. For this model, they assumed that future prices comprise their expected value plus a random error following a normal distribution, characterized by a mean of zero and constant variance. Fina, Amacher, and Sullivan (2001) integrated debt obligations and expected timber price increases in a reservation price model. They used a Poisson process to represent price arrivals from various random offers. Later, they expanded their research to analyze single and infinite rotation models, as well as the effects of debt repayment in future rotations and non-timber benefits. Gong, Boman, and Mattsson (2005) also delved into the optimal rotation, pondering non-timber age-dependent benefits and timber price risks, assuming uncorrelated, normally distributed Scots pine prices.

Lin and Buongiorno (1998) addressed economic and ecological risks in an uneven-aged forest by incorporating natural disturbances into a landscape-level MDP model. They selected an MP to represent ecological features such as species and size diversity of the trees. They characterized sawtimber and pulpwood prices as a random walk and a stationary process. Thus, their study effectively combined economic and ecological benefits to predict forest stand distributions and evaluate management policies.

Yu et al. (2023) studied how carbon credit and timber price risks, along with risk aversion, influenced optimal harvest decisions in China. They developed a dynamic model to maximize a linear version of the land expectation value (LEV) formula, incorporating Monte Carlo simulations and modeling prices as mean-reverting processes. Their results indicated that carbon credit price risk had negligible effects on risk tolerance optima, LEV value, and optimal rotation. Compared to the deterministic version, their dynamic Faustmann model with risk aversion mostly showed shorter optimal rotations and a higher LEV.

3.2. Linear, Stochastic, and Chance-Constrained Programming

Forboseh and Pickens (1996) presented a harvest scheduling model based on linear programming to manage timber price risk and demand constraints in ongoing rotations. Ferguson (2016) analyzed the impacts of uncertainty in tree growth, cost fluctuations, changes in timber prices, and fire risks on harvest scheduling at a pine plantation in Australia. He employed Monte Carlo simulations to model probabilistic effects and used linear programming for the analysis, which included timber salvage activities and the potential impacts of climate change. He assumed that timber growth followed a normal distribution, that correlated prices and costs followed log-normal distributions, and that fire events followed uniform distributions. Ferguson applied a risk-free discount rate when comparing deterministic and stochastic harvest scheduling outcomes to avoid double-counting risk. Buongiorno and Zhou (2017) integrated MP and goal programming to model growth and timber price risks in optimal multi-criteria management decisions for mixed forests. They identified three market states and two growth states based on species, timber products, and their probabilities. Therefore, each future stand state is associated with specific probabilities for transitioning risks, which provides a basis for decision-making. Combining these approaches, they optimized timber revenue, carbon offsetting, tree size and diversity, and old-growth preservation.

Chance-constrained optimization is a mathematical technique that integrates risk probability in the modeling process. This method constrained the optimization problem to satisfy a specified probability (Olson and Wu 2010). Huang et al. (2022) used chance-constrained programming to model stumpage price risk on loblolly pine rotations while integrating different herbicide and fertilizer treatment scenarios. Their model accounted for risk aversion and assumed correlated timber product prices that followed a normal distribution.

3.3. Capital Budgeting-Based Methods

Reed and Haight (1996) studied the present value distribution for a loblolly pine single rotation. They modeled timber price and age-dependent biological growth as GBM processes. They studied the present value distribution for a single rotation and evaluated feedback-cutting policies. Using fiducial probability, they also looked at how system and sampling errors in the model affected present value distributions. Their results showed that fixed rotation ages differed depending on whether the mean or median of the expected present value distribution was used. The mean increased while the median decreased in the positively skewed lognormal distribution from the GBM process. System errors in the price and yield models caused most of the variability, with price system errors being the most significant contributor to present value variation.

Brazee, Amacher, and Conway (1999) also studied the optimal rotation assuming autocorrelated stumpage prices. They analyzed first-order autoregressive coefficients to assess autocorrelation, testing against a null hypothesis of random walk and random draw progressions. Mei, Clutter, and Harris (2013) examined the effects of timber price, biological growth, and land expectation value risk on timberland return drivers. They modeled return distributions using Monte Carlo simulations. They represented the timber price risk as a GBM and an Ornstein-Uhlenbeck mean-reverting process, while land value and tree growth risk were depicted through triangular distributions. Their results showed that biological growth is the most influential factor in timberland returns. They also noted that return distributions varied notably between GBM and Ornstein-Uhlenbeck price processes, recognizing that the GBM process led to a wider spread due to unbounded price progression. Consequently, Mei (2023) revisited the previous timberland returns driver studies by incorporating the risk associated with carbon credit prices amid discussions of climate change and carbon offsets. He assumed the carbon credit price followed a random walk process with a drift rate.

Restrepo and Orrego (2015) conducted a probabilistic LEV calculation using Monte Carlo simulations for a teak plantation in Colombia. They employed logistic models to forecast the probabilities of success under land price risk. Aronow, Washburn, and Binkley (2001) explored the impact of timber price volatility on forest investment performance. They generated future timber prices through Monte Carlo simulations, analyzing how stochastic processes such as random walk and mean reversion govern prices. Their findings underscored the significant effects of the stochastic nature of prices on the timberland asset value, the risk of interest payment default, and the theoretical value of a put option on a property.

3.4. Miscellaneous Methods

Schweitzer, Lundgren, and Wambach (1968) pioneered the development of a Fortran IV-based algorithm to calculate point and interval forecasts for present value and rates of return on timber investments. The program utilized expected estimates of costs, prices, yields, and their variations based on probability distributions.

Intervention analysis is a statistical method used to assess the impact of an event or action (intervention) on time series data, such as policy shifts and environmental changes (Box et al. 2016). Holmes (1991) introduced a market model that combines autoregressive price models with intervention analysis to evaluate the short-term market impacts of the Southern pine beetle epidemic in Texas and Louisiana. He analyzed aggregated damage assessments at the market level. Holmes's study exposed a void in appraisal conventional models because they estimate direct losses from catastrophes but overlook salvaged timber's effects on regional markets. Thus, he provided insights into the price shocks and welfare changes while examining how ecological disasters affect the market's overall dynamics.

As discussed, intervention analysis examines the impact of specific events on data series. In contrast, impulse theory or impulse response explores how variables respond over time to temporal shocks (Enders 2014; Hamilton 1994). Yin and Newman (1999) applied intervention analysis to assess pine and hardwood stumpage price changes following Hurricane Hugo's landfall in South Carolina. They found a gradual price reduction trend, supporting a short-term rather than a long-term effect. Willassen (1998) delved into the impulse control theory, rather than the standard optimal stopping theory, to approach the Faustmann harvesting problem. He analyzed arithmetic, geometric, and logistic Brownian motion processes to demonstrate the theory's applicability to varying stochastic forest value growth specifications. Building on Willassen's work, Sødal (2002) further explored the stochastic Faustmann problem. He favored the markup approach proposed by Dixit, Pindyck, and Sødal (1999), to impulse theory. Dixit, Pindyck, and Sødal defined the markup approach as interpreting optimal investment choices similar to pricing decisions, using a formula to determine the optimal investment "markup." Similar to how firms set prices above marginal costs to account for risk and uncertainty, this approach adds a premium to the base investment cost to reflect the variability of future returns. Sødal considered the more intuitive optimal solutions offered by this approach. Therefore, it should encourage investors to evaluate the trade-offs between immediate investment and potentially higher future values.

Jump effects refer to sudden, significant market changes caused by unexpected events. Although impulse responses can sometimes capture jumps, they generally reflect broader trends and dynamics over time (Aliukov 2023). Modeling jump effects improves prediction and risk assessment by accounting for these unexpected shifts (Dixit and Pindyck 1994). Saphores, Khalaf, and Pelletier (2002) explored the harvest problem under price risk, considering jump effects and autoregressive conditional heteroskedasticity (ARCH) effects. They studied quarterly stumpage price series from species in the Pacific Northwest. They modeled timber prices as a GBM process with jump effects or jump-GBM. They used likelihood ratio tests, statistical bounds, and Monte Carlo simulations to evaluate p-values in small datasets and identify jumps and ARCH effects. Their results provided strong evidence of jump and ARCH patterns observed in the heavy-tailedness of log price changes that violated normality. The findings emphasized that overlooking jump effects in stumpage prices can lead to suboptimal harvest decisions.

Alvarez (2004) combined stochastic calculus and standard nonlinear programming to model the optimal rotation policy. He used a linear diffusion process to represent Faustmann's value growth while analyzing how increased volatility affects optimal rotation. He found that as volatility rises, the optimal rotation age tends to be extended and enhances the value of future harvesting. Using an open-loop control model, Gong and Löfgren (2008) evaluated the effects of risk aversion on optimal rotation. They assumed stochastic prices were normally and identically distributed and studied the discount rate and regeneration cost sensitivities. The derivative of the marginal variance function under risk neutrality showed variable or ambiguous outcomes compared to the deterministic case. Moreover, they identified a monotone and continuous curve delineating two areas influenced by interest rate and regeneration cost factors. This curve determines whether the optimal rotation should be extended or shortened based on the relative costs and economic parameters.

Zhou and Doyle (1998) and Rollins (1999) defined robust control theory as designing systems that preserve performance and stability despite external disturbances. Palma and Nelson (2009) described a robust optimization method for harvesting scheduling problems under timber growth and market demand risks. They compared the robust model's performance with the deterministic Model I from Johnson and Scheurman (1977). Their analysis focused on the level of risk protection offered by the robust model's constraints and its implications on solution outcomes. Zhang and Chang (2018) recognized that landowners' harvesting decisions are not solely based on initial rotation valuations. Instead, landowners decided annually to harvest forests based on their risk aversion and expected timber prices. They examined how risk preferences affect land valuation by analyzing the impact of risk aversion on LEV. They considered variations in the discount rate and lognormally distributed mean-reverting prices. These factors were integrated into a heuristic harvest decision-making model to enhance forest valuation.

The quality of information is fundamental to decision-making. Occasionally, it is only partially observable or too expensive to obtain (Chadès et al. 2021). However, managing partial information is sometimes sufficient. The mixed observable Markov decision processes incorporate variables that are fully observable and others that are only partially observable (H. Nguyen et al. 2022). Sloggy, Kling, and Plantinga (2020) delved into these processes to determine the optimal interaction between partially observable forest inventory information and perfectly observable stochastic prices for economic harvest decisions. Their findings showed that precision inventory decreases uncertainty costs and enhances harvest timing decisions. Additionally, the results demonstrated that the marginal benefits outweigh the marginal costs associated with the higher inventory management expenses.

Data envelopment analysis (DEA) is a method used to measure how efficiently different units perform based on their inputs and outputs (Kao, Chang, and Hwang, 1993). DEA finds the most efficient units and uses them as benchmarks to help less efficient ones improve (Charnes, Cooper, and Rhodes, 1978). Huang and Dwivedi (2023) studied the effects of carbon credit and timber price risks on management decisions considering different silvicultural treatment scenarios. They used DEA and linear programming to gauge production efficiency among 56 scenarios. These settings combined applications of herbicides and fertilizers. Carbon credit and timber price risks were modeled based on their means and covariances for each product. They analyzed how risk aversion influences interpreting and understanding management decisions under risk.

4. Natural Disturbances and Biological Growth Risk

Private forest companies and timber investment management organizations argue that natural disaster losses are historically low (Zinkhan et al. 1992; Hancock Timber Resource Group 2013; Chudy and Cubbage 2020). Conversely, Glauner, Rinehart, and D'Anieri (2012) suggest that institutional investors are hindered by their limited ability to assess timberland risks. Moreover, Lönnstedt and Sedjo (2012) described that timber managers often consider forest investments risky due to natural disturbances. A clear example of this perception is North America's widespread mountain pine beetle infestations, significantly affecting timber harvests and management practices (Schwab et al. 2009; Prestemon et al. 2013). Forest fires can negatively affect timber stands, reducing investment value and producing nonmarket damages (Susaeta and Gong 2019). Additionally, climate change may intensify the frequency and severity of these events (Pau et al. 2023; Weed, Ayres, and Hicke 2013; Wasserman and Mueller 2023; Susaeta, Adams, and Gonzalez-Benecke 2017; Mitton and Ferrenberg 2012).

As described, these risks raise concerns about short and long-term economic sustainability. Implementing proactive forest management practices and diversifying timberland investments can help mitigate some of these risks (West et al. 2021; Mei and Clutter 2023; Zinkhan et al. 1992; Antwi et al. 2024; Thomas, Brunette, and Leblois 2022). Timberland managers can develop robust investment strategies that reduce losses and promote long-term sustainability by evaluating natural risks and incorporating them into financial planning. Thus, we need a thoughtful understanding of the natural risk dynamics for strategic managerial decisions (Duku-Kaakyire and Nanang 2004; Kangas and Kangas 2004; Gaffney 1960).

4.1. Stochastic Dynamic Programming

Hool (1966) introduced an original approach combining dynamic programming with non-stationary MP to represent forest growth risk in forest resource management. He conducted a probabilistic analysis of forest growth and developed adaptive harvesting and thinning strategies over a finite horizon. Miller and Voltaire (1983) assessed stochastic tree growth as a diffusion process to solve the harvest rotation problem. They built upon Brock, Rothschild, and Stiglitz's (1989) optimal stopping rule, which states that an investor should exit the market when the marginal utility of wealth equals the expected marginal cost. They adapted this dynamic decision-making framework to maximize the expected present value during the cycle. In 1979, Van Wagner evaluated the economic impact of individual forest fires at the forest level rather than simply probing the effects on separate forest stands. He pointed out that a harvestable stand will be replaced with a younger one when burned. The study revealed that the real economic loss from a fire corresponds to the overall change in the forest's value due to suboptimal replacements. In other words, he studied potential burned areas and harvest area replacements. He used probabilities and burning rates while factoring discount and value growth rates, gauging harvest rotation and economic effects.

Martell (1980) used probabilistic dynamic programming to assess fire damage, adapted from Wagner's (1969) machine replacement model, extending the Faustmann model in an MDP. He considered conditional fire probabilities to be forest age-dependent and harvesting activity-dependent. His findings indicated an inverse relationship between Faustmann's optimal rotation length and the frequency of fires. Kao (1982) developed and forward-solved another probabilistic dynamic programming model to optimize stock levels and rotation ages. He studied thinning activities and growth prediction for decision-making under risk. He provided a numerical example of Douglas-fir stand management. His findings also showed that increased risk in growth predictions leads to optimal strategies yielding shorter rotations, reduced stocking levels, and lower mean annual increments. Reed and Apaloo (1991) studied the effects of changes in spacing in young plantations and thinning activities on increased fire risk. They hypothesized that these management actions would increase the fuel available on the ground and cause a jump and, later, a smooth decline in fire probabilities. Their analyses assessed the economic impacts of single and continuous rotations and optimal thinning schedules using stochastic dynamic programming models.

Gong (1992) presented a multi-objective dynamic programming for maximizing the timber revenue stream and the non-timber value of forests. He demonstrated that this approach can also be formulated as a multi-objective linear programming problem to enhance computational efficiency. He considered growth risk as a transition probability matrix describing potential forest states and their corresponding harvesting properties based on tree sizes. Couture, Cros, and Sabbadin (2016) analyzed windthrow risk on optimal harvesting strategies in uneven-aged stands management based on MDP across an infinite time horizon. They incorporated climate change and risk aversion into their model and compared joint versus independent management approaches among non-industrial forest owners in northeastern France. They assumed an age-dependent windthrow probability and a conditional probability for trees to be overturned due to wind. Despite the variations in windthrow probability observed in their sensitivity analysis due to climate change, they found a negligible impact on optimal harvesting.

Under an exogenous fire risk, Daigneault, Miranda, and Sohngen (2010) incorporated carbon credit benefits and fuel accumulation rates in optimal forest planning. They modeled fire probabilities as a fixed ratio, developing a stochastic model that maximizes expected future forest value, while determining thinning schedules and harvest age. Thus, they evaluated how landowners could be incentivized to adopt different thinning and rotation practices to balance timber production and carbon sequestration. They noted that fire risk influences management decisions, mostly leading to shortened rotation ages due to higher fire risks. Al Abri, Grogan, and Daigneault (2023), leveraging on Daigneault, Miranda, and Sohngen's (2010) study, presented an economic model that optimizes forest management activities, factoring in fuel load levels and fire risk. They used an MDP model that relied on stochastic dynamic programming, incorporating timber and non-timber benefits to define optimal thinning, fire prevention decisions, and rotation age. Their work novelty studied exponential, logistic, and concave fuel accumulation processes to represent different management scenarios. They defined fire risk as a non-homogeneous Poisson process. They tested different fire arrival rates that were a function of the historical average rates, age, and fuel accumulation. Their results underlined the importance of fuel considerations when facing fire management assessments.

Three approaches address risk in capital budgeting: the certainty equivalent, risk-adjusted discounting, and the method based on probability distributions or the Hillier-Hertz approach (Perrakis, 1975). The certainty equivalent method can approximate a stochastic model by replacing random variables with expected values and risk-adjusted discounting (Hull 2017). Kuusela and Lintunen (2020) described that accounting for natural disturbances and different tree age classes in forest financial models can make them highly complex or unsolvable. They further explored this complexity by examining stochastic and deterministic methods in harvesting decisions amid disturbances within a market-level model. They defined the deterministic method as the certainty equivalent for comparison with the stochastic method and applied dynamic programming to solve the model. This approach incorporated a straightforward age structure classified as young and mature forest stand age classes. Their results showed that the deterministic certainty equivalent method yielded similar outcomes to the stochastic method. Moreover, their results exposed that disturbance risk increases effective discount rates and shorter rotations at the market level.

Building upon Martell's (1980) work, Caulfield (1988) refined his fire risk model for optimal harvesting by integrating stochastic dominance to accommodate risk aversion. He compared stochastic dominance against alternative decision-making methods like mean-variance and mean-coefficient rules. He demonstrated the superior efficacy of first and second-degree stochastic dominance in identifying rotations suited to various risk aversion levels. Stevens (1986) used stochastic dynamic programming for harvest optimization in forest stands at fire risk. He assessed lognormal fire risk through Monte Carlo simulations while optimizing an even timber flow and penalizing for reduced yields. Stevens showed that, under fire risk, smoother harvest volumes required significant reductions in harvest levels. However, this is recommended only if the smoothness benefits offset the costs.

4.2. Linear, Stochastic, and Chance-Constrained Programming

Reed and Errico (1986) studied stochastic fire risk across multiple-stand levels in British Columbia, employing linear programming to optimize harvest schedules. Specifically, they applied a general form of Model II (Johnson and Scheurman 1977). They incorporated fire probabilities and ignored timber salvaging and demand fluctuations for such a model. They found that even minimal fire rates could significantly impact harvest scheduling, and overlooking these negligible fire rates could lead to timber overestimations. Finally, concurring with Van Wagner's (1983) observations, Reed and Errico determined that the fire impact is better assessed at the forest level rather than by the sole timber burned. Later, Gassmann (1989) drew on Reed and Errico's

(1986) study. He used stochastic programming to maximize timber harvest value over a single horizon. However, he introduced flexible timber flow constraints with penalties in the objective function for breaches. He placed shaping assumptions. First, biological growth is assumed to be age-dependent and deterministic. Second, fire loss probabilities are defined by a stationary probability discretization.

Boychuk and Martell (1996) studied fire risk in the forest-level timber supply. They grounded their modeling on multistage stochastic programming. They assumed random fire losses as a discrete two-point probability distribution and non-salvage activities. This distribution represented high and low burn rates. Their findings suggested ensuring an even timber flow by reducing annual cuts to account for fire risk when timber surplus is available. Armstrong (2004) evaluated the combined impacts of harvesting rate and wildfires on sustainable timber harvesting in Alberta's mixed boreal forests. He employed sequential Monte Carlo simulations and linear programming to integrate timber harvesting, random fire, and reforestation rates. He assumed a lognormal continuous distribution represented fire. This probabilistic model suggested that integrating fire risk into harvest scheduling requires reducing harvest levels to ensure an even wood flow. Armstrong underlined that these results were consistent with prior findings (Reed and Errico 1986; Stevens 1986; Gassmann 1989; Boychuk and Martell 1996; Van Wagner 1983).
Nguyen (2012) employed multi-stage stochastic programming, pondering fire risk and spatial considerations. In a formulation similar to Model III for harvest scheduling (Gunn 2007), he modeled optimal harvest subject to adjacency constraints and random fire events. He integrated even flow timber production and mature forest core areas preservation under fire risk. He assumed fire probabilities were age-dependent and events occurred after management activities.

The sample average approach in optimization involves generating a random sample to estimate the expected value function using an average derived from the sample (Kleywegt, Shapiro, and Homem-de-mello 2001). Eyvindson and Kangas (2015) studied inventory data and growth model risk implications in assessing management decisions through the sample average approach. Thus, they explored different scenario set sizes to validate the optimality gaps in management models based on stochastic programming. They found that as the number of scenarios rose, the optimality gaps decreased. Subsequently, Eyvindson and Kangas (2016) studied risk aversion using stochastic programming in forest management planning. They aimed to minimize the negative deviations from goals in decision-making by accounting for risk preferences and imperfect information risk.

Robinson, McLarin, and Moss (2016) incorporated deviations in timber yield predictions into their harvest scheduling to maximize volume. They analyzed differences between historical yields and predictions to calculate risk rates, minimize prediction variances, and set future yield goals as constraints in portfolio strategy optimization. Building upon this work, Eyvindson and Kangas (2017) evaluated the incremental trade-off costs required to minimize timber yield deviations. They integrated stochastic programming, stochastic average approximation, and conditional value at risk (CVaR) methods to manage downside risk effectively.

The decomposition technique means breaking down large problems into independent subproblems and penalizing deviations from averages (Rockafeller and Wets 1991; Tian et al. 2024; De Pellegrin Llorente, Hoganson, and Windmuller-Campione 2022). Combining multi-stage stochastic programming with decomposition techniques, de Pellegrin Llorente, Hoganson, and Windmuller-Campione (2022) modeled growth risks in large-scale forest planning amid climate change. Hof, Kent, and Pickens (1992) investigated optimization in natural resources using chance constraints and chance maximization, integrating stochastic yield parameters. They used an example of forest land allocation with random timber product volumes to explain their findings better.

4.3. Capital Budgeting-Based Methods

Routledge (1980) extended Faustmann's discrete-time rotation analysis to encompass probabilities of catastrophic events, factoring in timber salvage. Van Wagner (1983) introduced a model to evaluate the long-term impact of forest wildfires on timber supply. He assumed an annual fire with a constant, age-independent rate and random selection of stands for burning. His results showed that annual fires caused a greater reduction in maximum sustainable harvest than the prompt volume lost. Furthermore, his findings exposed that lowering harvesting rates below the allowable yearly cut made them relatively insensitive to fire risk effects. Reed (1984) examined optimal rotation in a continuous-time outline under fire risk. He assumed fire risks followed a constant-time-independent Poisson process, resulting in complete stand destruction. He found that the optimal rotation shortens as the likelihood of fire increases. Moreover, fire risk adds a premium to the discount rate. He later expanded his analysis by relaxing the total destruction constraint, incorporating a non-homogeneous Poisson process, and adding timber salvage rates.

Expanding upon Reed (1984), Yin and Newman (1996) examined forest investment decisions with catastrophic events. They modeled timber prices and biological growth as a GBM process, while total destruction from natural disturbances was modeled as a time-dependent Poisson jump process. After such a devastating event, they explored the abandon option against investing in perpetuity in the forest project. In a non-reinvestment setup, catastrophic risks reduce the investment value and raise the premium needed for investment, which deters investment appeal. However, they discovered that if the owner could continue to invest after such an event, the adverse impacts on investment premiums and project values would be moderated. Englin, Boxall, and Hauer (2000) adopted a multiple-use management approach, modeling fire risk as a Poisson process and integrating wilderness recreation amenities into the Faustmann rule. Haight, Smith, and Straka (1995) investigated the stand-level effects of comparable storms to Hurricane Hugo on North Carolina's loblolly pine plantations. Thus, expected present values are analyzed by factoring in age-dependent stem sweep damage and salvage actions in a single rotation. Susaeta et al. (2016) also elaborated on Reed's (1984) work and explored a generalized model to assess fire risk in optimal slash pine management. They integrated stumpage prices, regeneration cost, timber volume, fire events, and timber salvage variations. This integration allowed them to study LEV optimization under increasing and steady fire risk rates.

Xu, Amacher, and Sullivan (2016) assessed the effects of multiple disturbances in forest stands, assuming that catastrophic events follow a homogeneous Poisson process. Their model explored whether to harvest or wait following a disturbance. Findings suggested that harvest decisions and land rent values are not optimal, as multiple disturbance probabilities are not accounted for. However, they also noted that optimal forest management and rotation strategies significantly vary along regional factors, such as disturbance types, species, and forest recovery capabilities. Loisel (2011) developed a population dynamics model to study the effects of natural hazards on Faustmann's policy for optimal thinning decisions. Taking some of Reed's (1984) considerations, they modeled natural risk as a non-stationary Poisson process. The population dynamics model simulated the evolution of stem density, basal area, and natural tree mortality state variables considered grounds for thinning evaluations. He conducted sensitivity analyses that probed total destruction, no destruction, and partial destruction coupled with salvage action scenarios. His results indicated that risk could imply a decrease in the optimal rotation age. However, Loisel’s findings also showed that thinning activities extended the stand rotation age compared to no thinning cases. Additionally, with density-dependent growth, thinning occurred early in the presence of risk, independent of total or partial destruction considerations. He underscored that this early thinning could be considered self-insurance to face risk. Knoke et al. (2021) examined the economic impacts of natural disturbances in Norway’s spruce forests through forest growth modeling and Monte Carlo simulations. Using a previous disturbance probability study in Germany (Brandl et al. 2020), they predicted age-dependent damage risks associated. They assessed the impact of significant disruptive events using the CVaR. Their findings indicated shorter rotations and lower LEV, which is consistent with previous studies. However, they also discovered that salvage harvesting could mitigate these adverse effects.

4.4. Miscellaneous Methods

Dixon and Howitt (1980) used the Linear-Quadratic-Gaussian method for harvest scheduling optimization under timber growth risk. Reed (1987) relied on optimal control theory to model the optimal protection schedule against fire for single and infinite rotations. While studying the protection expenditures, he considered constant and age-dependent fire probabilities. Using cointegration and intervention analyses, Prestemon and Holmes (2000) analyzed Hurricane Hugo's short and long-term consequences on stumpage prices. Thus, they identified deviations from market equilibrium and timber prices’ responsiveness to new information. They found that southern United States timber markets are interlinked, supporting intertemporal arbitrage. Finally, severe disturbances caused an immediate supply inflow from salvage, decreasing prices, followed by a sustained increase in timber prices for remaining forest resources, aligning with Kuusela and

Lintunen's (2020) findings. Peter and Nelson (2005) examined the spatial effects of fire disturbances on timber harvesting in northeastern British Columbia. They presumed fire occurrence patterns following a Poisson distribution and size fire variation following an exponential distribution. Their analysis evaluated potential economic effects and timber deficits across different harvesting intensity levels, timber salvage actions, fire control states, and risk aversion. For this purpose, they grounded their simulations on the FPS-ATLAS polygon-based harvest scheduling simulator (Nelson 2003) and factored in adjacency constraints. Also supported by spatial disturbance modeling systems, Blennow and Sallnäs (2005) outlined an active decision-support risk management approach for forest owners in Sweden. They presented a windthrow risk analysis based on the WINDA system developed by Blennow and Sallnäs (2004) to determine wind damage probabilities. WINDA estimated this likelihood on a stand-by-stand basis across a landscape under different climates and forestry management. WINDA based its wind simulations on meteorological data series to determine free-stream friction velocities and probable directions. Bettinger (2010) studied tabu search heuristics applications to incorporate spatial wildfire models into harvest scheduling. This model reassigned forest management prescriptions and timber harvest schedules, factoring in fire events and their severity in the management horizon. Helmes and Stockbridge (2011) investigated timber growth risk using nonlinear optimization to determine optimal thinning and harvesting. They examined the Wicksell and Faustmann policies. They developed models combining stochastic forest growth, characterized by a mean-reverting stochastic process, with deterministic price models.

Compromise programming is a decision-making method that minimizes the maximum deviation from the ideal solution in multi-criteria decision-making (Kashfi, Hatami, and Pedram, 2010). Diaz-Balteiro et al. (2014) adopted a multi-criteria technique to ascertain the forest rotation while integrating carbon offset and stochastic fire losses. They utilized Pareto frontiers to analyze and quantify the trade-offs between NPV and carbon sequestration, pursuing increased social welfare because of fire risk reduction. Thus, they combined multi-objective and compromise programming to contextualize Faustmann's policy by incorporating constant fire probability, carbon offsetting, and non-salvage harvesting.

In an integrated view, Mei, Wear, and Henderson (2019) explored financial and physiological growth risks to evaluate investment decisions. They combined stochastic growth and stumpage price models, age-dependent natural disturbance risks, and climate change effects to predict expected harvest areas. Climate change effects on growth were modeled using 3PG (physiological processes predicting growth) by Landsberg and Waring (1997). Natural disturbance effects were derived from random draws from Monte Carlo simulations. Timber harvest volumes and prices were modeled as geometric Brownian motion processes. They demonstrated that this real options approach is a powerful tool for making investment decisions, particularly in evaluating risks intensified by climate impacts.

Rinaldi and Jonsson (2020) modeled forest growth risk based on macroeconomic concepts from Hanse and Sargent's (2008) work. Their model integrated robust control theory and Knight's uncertainty (Knight 1921) for informed decision-making in forestry. This method showed that landowners aim to maximize revenue and stand volume, with their goals shaped by uncertainty, risk aversion, and information quality. As new information reduces uncertainty, landowners may shift their focus between making money and growing the forest. Halbritter, Deegen, and Susaeta (2020) examined the economics of thinning and rotation age related to natural risks in even-aged forests. They combined optimal control theory with age-dependent and stock-density varying disturbance modeling, described as a Poisson non-stationary process, to evaluate the impacts on human well-being and infrastructure.

Restrepo et al. (2022) expanded a 3PG growth model to incorporate a stochastic climate spatial generator, called the stochastic spatial 3PG or 3PGS2 model. Consequently, this tool can predict the risk of forest growth under varying conditions and identify areas more vulnerable to climate change across the Southeastern United States. Henderson et al. (2022) examined the impacts of hurricanes on forest markets by integrating spatial considerations. They used remote sensing, forest inventory data, and salvage harvesting to measure the effects of Hurricane Michael and evaluate changes in producer welfare and carbon storage. They analyzed short- and long-term impacts, considering how age distribution changes post-hurricane. Their findings showed forest carbon increases in all hurricane scenarios after 40 years, although none reached the original carbon levels. They initially found a price decline due to salvaged timber flows, followed by a subsequent increase as prices tended to revert to the mean. Consequently, Henderson et al. proposed extending the rotation age to maximize producer welfare benefits in light of observed price fluctuations and the long-term recovery process.

A Bayesian network approach is a graphical model representing random variables and their conditional dependencies through a directed graph without feedback loops (Rao and Rao 2014; Heckerman 2008). Nepal et al. (2023) investigated wildfire risk in the southeastern United States using a Bayesian network model that integrated biophysical, socio-ecological, and socioeconomic data. They examined how fire risk varies between different management options and how fire occurrence and severity interact with socio-ecological exposure. The findings emphasized the importance of evidence-based strategic prescribed fire planning to reduce risk and enhance community and landscape resilience.

The Ricardian method (Ricardo 1817) is an economic approach that posits land value is determined by competitive demand, with farm rent reflecting the productivity and net revenue of land, regardless of specific crops (Mendelsohn and Massetti 2017). In 1994, Mendelsohn, Nordhaus, and Shaw presented a Ricardian approach adapted to assess the effects of climate change on agricultural land values and farm revenues. Similarly, Wang and Lewis (2024) integrated the Ricardian economic approach and Reed's (1984) framework. They incorporate spatial factors in this hybrid approach to examine how drought stress and fire risks impact timberland value on the West Coast of the United States. To support this integration, they relied on machine learning techniques to evaluate the economic impacts of wildfires and drought across different regions and climate conditions.

5. Previous Articles on Risk Modeling Methods and Literature Reviews

Mei, Clutter, and Harris (2010) delved into the stochastic nature of sawtimber prices. They employed various time series models to simulate real pine prices in 12 southern regions of the United States. The selected areas covered circa 90% of the total annual pine harvest in the south of the United States. Discrete univariate models, continuous GBM and Ornstein-Uhlenbeck processes, and generalized autoregressive conditional heteroskedasticity (GARCH) models were evaluated. They assessed the accuracy of these discrete and continuous-time models while examining interrelationships in regional markets. Their results established that southern regional timber markets are cointegrated. Thus, they should not be evaluated in isolation for short-run forecasting. Additionally, this study determined that the bivariate GARCH model effectively captured the conditional variance and covariance for the southern regions of Georgia and South Carolina. Almeida et al. (2022) studied different stochastic processes to depict timber prices in

Santa Catarina's pine market in Brazil. Specifically, they tested Fractional Brownian Motion (FBM) and GBM processes based on time series data. They found that FBM was the most suitable process for price modeling.

Risk management can be intricate and challenging. It can push the standard boundaries to complex theoretical methods (Wolke 2017). Kangas and Kangas (2004) reviewed fundamental theories, including classical and Bayesian probability methodologies, for managing risk in forestry and their applications in decision-making. They explored approaches such as frequentist probability theory, Bayesian probability theory, evidence theory, fuzzy set theory, and possibility theory. Hildebrandt and Knoke (2011) also conducted a comprehensive review of probability valuation methods for financial decisions under risk in forest management. They assessed various modeling strategies for long-term economic decision-making. Strategies that included stochastic dominance, downside risk assessment, mean-variance analysis, option pricing models, and robust optimization techniques. Other researchers also conducted comprehensive reviews focusing on integrating ample risk concepts into forest management decisions, factoring in forest planning, decision support systems, and climate change considerations (Brumelle et al., 1990; Von Gadow, 2000; Hanewinckel, Hummel, and Albrecht, 2011; Yousefpour et al., 2012; Pasalodos-Tato et al., 2013).

Brumelle et al. (1990) investigated risk modeling and classified articles based on whether problems are structured or unstructured and whether risk aversion is known or unknown. Von Gadow (2000) analyzed risk modeling in forest planning, focusing on exogenous hazards and cumulative survival rates. Hanewinckel, Hummel, and Albrecht (2011) focused on timberland risk management to incorporate climate factors to address tangled hazards effectively, highlighting future research needs to explore these complexities further. For instance, the linkages among hazards like storms, insect outbreaks, and fire damage. They concluded that risk models must evolve to adapt to the dynamic interrelations shaped by climate change.

Yousefpour et al. (2012) showed that there has been a significant increase in risk modeling publications over two decades in forestry. Their analysis revealed that the GBM method was predominant, used in 19% of the studies reviewed, particularly for modeling stochastic price changes and NPV. This method was preferred for its simplicity and effectiveness in addressing price risk. In contrast, autoregressive models, including vector autoregressive models, were employed in 7% of the research, indicating a lesser but still significant use in pricing models. Pasalodos-Tato et al. (2013), like Kangas and Kangas (2004), reviewed classic and innovative methods for risk modeling in forest planning, covering stand and forest-level assessment. They analyzed linear and nonlinear models, optimization methods, spatial factors, and real-world impacts. They concluded that choosing the right risk model is intricate and depends on the planning scale, problem type, and uncertainty.

6. Discussion

The intrinsic stochastic nature of timberland investing challenges modeling considerations for capital budgeting, adding layers of risk to expected outcomes. However, building on the seminal work of Knight (1921) and Samuelson (2012), which delved into risk assessment, research has since expanded to include a variety of risk models, methods, and applications of stochastic processes. Early studies by Hool (1966), Norstrom (1975), Miller and Voltaire (1983), Martell (1980), Reed (1984), and Lohmander (1988), and others helped build the foundation for modeling the complex relationship between uncertain growth and timber prices. Together, they highlighted the value of using a probabilistic approach over traditional methods, marking a key shift in forest economics.

Traditional economic models and policies, like Wicksell and Faustmann rules, have been adapted to address stochastic considerations. Thus, advanced mathematical structures are increasingly employed to optimize timberland management and provide a way out of myopic deterministic frameworks. Some applications have shown us that timber price risks can increase expected values, as flexible management rooted in feedback-based models encourages landowners to capitalize on future timber price increases (Brazee and Mendelsohn 1988; Lohmander 1988). However, timber price risk tends to shorten harvesting rotations at the stand level as we ponder the opportunity cost of land rent (Yin and Newman 1995). In harvest and thinning scheduling under price risk, optimizations exposed changes in expected benefits and produced smoother timber flows (Teeter and Caulfield 1991; Haight 1990). Furthermore, risk influences vary when considering different types of thinning (Brazee and Bulte 2000) or when salvaging activities are considered risk effects relativized (Xu, Amacher, and Sullivan, 2016; Knoke et al., 2021).

Pricing model considerations can also lead us to different outcomes. Stationarity, model system error, and coefficient implications can guide us to different outcomes (Haight and Holmes, 1991; Haight and Smith, 1991; Reed and Haight, 1996; Brazee, Amacher, and Conway, 1999; Gong, 1999). Hence, these findings lead us to critical modeling considerations. Likewise, stochastic process assumptions add another layer of complexity. For instance, a GBM or Ornstein-Uhlenbeck underlying process can generate different outcomes and expected distributions of our models, altering the probabilistic calculations and the understanding of our results (Reed and Haight 1996; Clarke and Reed 1989; Reed and Clarke 1990; Mei, Clutter, and Harris 2010; 2013; Aronow, Washburn, and Binkley 2001).

Risk in timberland comprises several risk sources, not only market variables. Growth and natural disturbances add intricacy to our risk assessment when determining management policies. Different process assumptions for disturbance representation, such as Poisson processes and GBM, have been proposed. Random walks, MP, BM, FBM, and diffusion processes, among others, have also been hypothesized in hazard modeling. Fire events have been intensively studied among disturbances (Yousefpour et al. 2012). Wildfire probabilities will likely shorten stand rotations, such as adding a premium to the applied discount rate (Reed 1984; Loisel 2011; Knoke et al. 2021; Martell 1980). However, natural hazards impact rotation age in various ways, particularly when considering thinning. They lead to extended rotation periods and cause thinnings to happen earlier (Loisel 2011). Age, harvest activities, fuel accumulation, and silvicultural treatments, among other variables, have been considered to calculate fire arrival probabilities and severity (Kao 1982; Reed and Apaloo 1991; Daigneault, Miranda, and Sohngen 2010; Al Abri, Grogan, and Daigneault 2023). At the forest level, negligible fire rates could significantly impact harvest scheduling (Reed and Errico 1986). Suboptimality issues have been addressed, such as burnt stands being replaced by young plantations in forest planning (Van Wagner 1979). Likewise, storms and windthrow have been addressed at the stand and regional levels. Furthermore, multiple disturbance effects have been studied, meaning that over time, more than one catastrophe may occur (Xu, Amacher, and Sullivan, 2016). Devastating large-scale occurrences, like hurricanes, contributed to altering market equilibrium regionally. Furthermore, market cointegration characteristics extend implications (Prestemon and Holmes, 2000; Mei, Clutter, and Harris, 2010). Timber supply inflows from salvage harvesting temporarily flood the market, initially driving down prices and subsequently enhancing the value of standing timber due to anticipated shortages (Prestemon and Holmes, 2000; Kuusela and Lintunen, 2020; Henderson et al.,2022).

Technological changes have improved simulations from Schweitzer, Lundgren, and Wambach's (1968) probabilistic model. Spatial features in hazard modeling have been used to mimic destruction rates, mitigation or control responses, and socio-ecological effects (Peter and Nelson 2005; Bettinger 2010; Blennow and Sallnäs 2005; Nepal et al. 2023; Yuhan Wang and Lewis 2024). These features provide a new state-of-the-art where topographic and adjacency constraints are incorporated to represent ecological and social considerations in timber management. Moreover, climate and physiology-based models have been developed to address climate change, disturbances, and growth risks (Mei, Wear, and Henderson, 2019; Restrepo et al., 2022). Thus, environmental variability and physiological considerations enhance our understanding of how disturbances may be interconnected rather than occurring independently. This insight enables improved predictions of forest evolution and adaptation strategies, thereby supporting sustainable forest management under changing climatic conditions (Couture, Cros, and Sabbadin 2016; De Pellegrin Llorente, Hoganson, and Windmuller-Campione 2022; Restrepo et al. 2022; Yousefpour et al. 2012).

Timberland risk modeling has also delved into other variables affecting the business analysis, such as carbon credit price risk, different amenities, species-age diversity, silvicultural treatments, and debt obligations (Gong, Boman, and Mattsson 2005; Fina, Amacher, and Sullivan 2001; Lin and Buongiorno 1998; Haight 1990; Huang et al. 2022; Mei 2023; Englin, Boxall, and Hauer 2000). Amenities such as wilderness recreation, old-growth core areas, and carbon sequestration have been incorporated into optimization models as spatial or linear constraints or multi-objective functions (Diaz-Balteiro et al. 2014; Halbritter, Deegen, and Susaeta 2020; Henderson et al. 2022; Nepal et al. 2023; Gong 1992; D. T. Nguyen 2012; Englin, Boxall, and

Hauer 2000). These last variables enhance conventional evaluations of the forest asset model, making them more holistic.

Risk-averse assumptions shift the model's outcome interpretations toward stability and reduced exposure to adverse scenarios. Conversely, risk neutrality or risk-seeking preferences might result in different valuations and a higher tolerance for risk. Thus, risk aversion and various stochastic process choices in risk modeling have been extensively studied, enabling further exploration of price forecasting and its effect on managerial assessments.

Different methods and considerations help manage risk by offering more detailed and realistic simulations. Advanced approaches provide deeper insight into forest management risks and support better decision-making. Stochastic programming, based on Dantzig's methods (Hillier and Liebermann 1980), offers broad applications for modeling expected variables and their joint distributions. It provides a probabilistic framework for investment analysis and has been widely used in risk modeling. For instance, Gassmann (1989) and Boychuk and Martell (1996) demonstrated stochastic programming's versatility. Other researchers have explored additional approaches, such as multistage stochastic programming, compromise programming, and chance-constrained programming (Helmes and Stockbridge 2011; Boychuk and Martell 1996; D. T. Nguyen 2012; Diaz-Balteiro et al. 2014; Hof, Kent, and Pickens 1992; Huang et al. 2022).

Other advanced methods were also explored. The classical and Bayesian probability theories have evidenced application opportunities for decision-making (Kangas and Kangas 2004), accommodating the complexities of real-world scenarios. The robust control theory, markup approach, impulse control theory, nonlinear programming, the linear-quadratic-Gaussian method, and the Bayesian network have been explored beyond traditional borders (Dixon and Howitt 1980; Rinaldi and Jonsson 2020; Nepal et al. 2023; Willassen 1998; Sødal 2002). Thus, all methods described have characterized the progression of research in the field. Aligned with Yousefpour et al. (2012) review, we expect the trend toward increasingly sophisticated risk research to continue, mainly as climate change introduces further complexities to forest management.

Despite advances in stochastic and probabilistic methods, gaps persist in integrating economic and timber growth risks, natural disturbances, and climate change impacts into forest management analysis. These gaps may widen as timberland businesses face increasing scrutiny from a more aware and conscientious public about environmental matters. Thus, this public's evolution underlines the need for models that balance financial returns with environmental stewardship amid risks. Several articles have integrated or partially integrated ecological disturbances, economic, and biological growth risks (Clarke and Reed 1989; Reed and Clarke 1990; Susaeta and Gong 2019; Lin and Buongiorno 1998; Ferguson 2016; Yin and Newman 1995; Mei, Clutter, and Harris 2013; Mei 2023). Future research should continue to develop further comprehensive models that enable robust and adaptive timberland management strategies. Such integration may further explore the complexity of decision-making in forest management, where even minor methodological changes can have significant policy impacts.

Machine learning algorithms could enhance economic and ecosystem modeling by modeling financial and timber growth risk assessments and accurately simulating natural disturbances and climate impacts (Aziz et al., 2022; Munro, Montes, and Gandhi, 2022; Estrada et al., 2023; Lamichhane, Mei, and Siry, 2023; Pereira Martins Silva et al., 2023). Therefore, these algorithms could improve the correctness of risk evaluations by analyzing large datasets to identify complex patterns and predict outcomes (Jung 2023).

Integration and advanced simulation methods are fundamental in refining models to determine the most accurate outcomes, from rotation ages and harvesting strategies to complex asset valuations. This combination enhances timberland management's economic adeptness more straightforwardly. Building upon a comprehensive exploration within the timberland risk modeling becomes manifest and offers new avenues for further investigation to account for market-driven and ecological risks. This research endeavor was to delve into state-of-the-art timberland risk assessment techniques and modestly shed light on diverse research questions. Thus, we can encourage deeper analysis and improve timberland risk understanding and, consequently, informed decision-making.

7. Conclusions

Timberland investments encounter risks from market forces, biological growth, and natural disturbances because of their long-term, challenging decision-making. This review aims to inspire research questions and further method exploration in timberland investment risk management. The review consolidates comprehensive literature, covering different stochastic processes and evaluation methods.

Early studies, such as Martell (1980) and Reed (1984), have foundational enlightened risk modeling, creating a turning point in forest economics. Over recent decades, forest economics research has increasingly shifted from deterministic methods to probabilistic approaches due to the former's failure to capture unexpected outcomes. Research has modeled market and environmental biological changes using various strategies, including GBM and Poisson processes. These processes have been integrated into several mathematical methods, with stochastic dynamic models and stochastic programming methods being the most utilized.

Research must continue pushing the boundaries of current knowledge in timberland risk modeling. Climate change and growing public scrutiny create a sensitive focus on sustainability and environmental responsibility, and will drive more sophisticated and comprehensive approaches to risk management. Thus, we expect this uptrend in risk research to continue. Therefore, further research into mathematical methods such as classical and Bayesian probability methodologies, machine learning, spatial modeling, and probabilistic-stochastic processes remains necessary.

Despite current advances and ongoing methodology exploration, gaps must be addressed in effectively integrating various risk sources. The stochastic processes are intricate and involve several interrelationships. Thus, risk integration and advanced simulation techniques are needed to refine asset valuations that assess market risks and biological hazards with greater precision. Such integration will allow decision-makers to significantly enhance their economic assessments, moving beyond risk-specific analyses and enabling a game-changer strategy.

Table 2.1. Approaches for evaluating timberland risk

	Timber Price and Forest Value Risk		Natural Disturbances and Biological Growth Risk	
Methods	**Articles**	**Prevalent Stochastic Process**	**Articles**	**Prevalent Stochastic Process**
Stochastic Dynamic Programming	21	GBM and Markov Process	13	Poisson Process
Linear, Stochastic, and Chance-Constrained Programming	4		11	
Capital Budgeting-Based Methods	6		10	
Miscellaneous Methods	12		15	
Subtotal	43		49	
Total	**92**			
Previous Articles on Risk Modeling Methods and Literature Reviews	9			
Grand Total	**101**			

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CHAPTER 3

A multivariate approach to timber price risk modeling[2]

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Abstract

Price forecasting significantly determines future income stream estimations, impacting economic assessments based on discounted cash flows. Furthermore, the risk associated with fluctuating prices adds complexity to revenue modeling. Timberland investments present significant challenges in projecting price risk due to their long-term horizon. This research compares multivariate autoregressive models for timber price forecasting and univariate time series models for land value forecasting. Thus, this multivariate approach accounts for cross-relationships while capturing historical dependencies. A vector error correction model, a multivariate generalized additive model, and random walks with a drift rate models are compared for timber prices. Land value modeling is considered through autoregressive integrated moving average with and without exogenous variables, evaluated against a random walk with drift. The results indicate that the multivariate generalized additive model and the autoregressive integrated moving average with exogenous variables outperformed other models. However, timber price residuals exhibited autoregressive conditional heteroskedasticity effects. This finding prompted us to model conditional heteroskedasticity using a dynamic conditional correlation Glosten-Jagannathan-Runkle multivariate generalized autoregressive conditional heteroskedasticity model with structural breaks, successfully capturing these effects and modeling price risk.

1. Introduction

The attractiveness of alternative asset investments depends on the expected returns and unique characteristics of these ventures (Tinic and West 1979). Several calculations are necessary to assess their investment appeal and revenue potential. For timberland investment, wealth generation relies on three main return drivers: biological growth, price changes, and bare land price appreciation (Caulfield, 1998). Timberland investments have a long investment span. Thus, the calculation of the stream of cash flows is challenging, as we need to forecast prices over a long time. Although biological growth remains the primary return driver in timberland investment, timber and bare land prices still significantly influence profitability.

One of econometrics' best tools for future forecasting is historical information, since past prices often explain future behavior. Time series models have demonstrated effectiveness in capturing historical dependencies for prediction. However, several price series exhibit interdependencies that make univariate models less precise, complicating expectations.

Multivariate time series modeling provides a method for accounting for these cross-relationships. From their inception, traditional vector autoregression (VAR) and vector error correction (VECM) models have helped with expected price forecasting, accounting for such interdependencies and modeling cointegration properties (Sims 1980; Johansen 1991). These models allow a more accurate representation of real-life situations where prices are not entirely independent or when markets interact, behaving as one, affecting price trends.

Machine learning models have improved modeling from financial data, influencing algorithmic trading, risk management, and price forecasting (Aziz et al. 2022). Timber prices are not the exception; for instance, Lamichhane, Mei, and Siry (2023) assessed the predictive capacity of artificial neural networks against classical econometric models in timber price forecasting.

Generalized additive models (GAM) are interpretable machine-learning algorithms (Molnar 2025). They can represent relationships between predictors and the response as a sum of smooth functions, showing how each one influences the prediction (Hastie and Tibshirani, 1986; Wood, 2017). The multivariate GAM (MGAM) enhances the properties of GAM by extending smooth functions into an equation system. This multi-equation approach makes the model capable of modeling both correlation structures and the dynamic behavior of data series.

As we have seen, applied econometrics has addressed the challenges of modeling future prices and interdependencies. There is still a lack of precision because there is uncertainty about future rewards (Dixit and Pindyck 1994). Uncertainty can be observed as the heteroskedasticity phenomenon in expected price model residuals. It is common to find tendencies of volatility to cluster over time, known as autoregressive conditional heteroskedasticity (ARCH) effects.

In 1986, Bollerslev introduced the generalized autoregressive conditional heteroskedasticity (GARCH) models to extend the ARCH framework proposed by Engle (1982) by modeling past conditional variances as a function of past squared errors and past variances. GARCH models can be widely used in volatility modeling since almost every asset class price presents volatility clustering (Engle 2002b). Furthermore, an extension of GARCH is the multivariate GARCH model (MGARCH), which captures volatility dynamics, correlation transmission, and spillover effects across multiple series. This method makes it more suitable for cases where standard GARCH models fall short due to the impact of correlations (Silvennoinen and Teräsvirta 2008).

Therefore, we propose to study the stumpage prices of the primary pine timber products in South Georgia: pulpwood (PULP), chip-and-saw (CNS), sawtimber (SAW), and bare land prices. We analyzed different univariate and multivariate time series models while evaluating volatility patterns. For timber prices, we chose the VECM, MGAM, univariate random walks with drifts (RWD), and MGARCH models. We modeled bare land prices using an autoregressive integrated moving average model (ARIMA), an autoregressive integrated moving average model with exogenous variables (ARIMAX), and an RWD. To later analyze the different model forecasting performances using mean absolute percentage error (MAPE), mean absolute error (MAE), root mean square error (RMSE), and Bias criteria, in order to identify the model that best fits the timber price series.

2. Data

The TimberMart-South (TMS) is a market analysis organization that provides timber price reporting, among other services (Norris Foundation 2022). It started publishing stumpage price data in 1976 and covered the most important pine products: pulpwood (PULP), pine chip-and-saw (CNS), and sawtimber (SAW).

This study utilizes the TMS quarterly stumpage price series for pine in South Georgia (GA2) to develop price forecasting models. The data covers the period from the first quarter of 1992 to the fourth quarter of 2023, which includes several extreme volatility market shocks. Periods of significant market changes sharply affected the timber supply and drove up price volatility. Specifically, the analysis begins in 1992, when a clear structural break emerged in the pine timber market (Misztal et al. 2024).

Notable events include the early 1990s housing boom (Baker 2002) and the logging restrictions to protect the northern spotted owl habitat in the mid-1990s (Murray and Wear 1998). Specifically, the analysis begins in 1992, when a clear structural break emerged in the pine timber market .

The subprime mortgage crisis of 2007-2008 profoundly altered global markets (Hodges et al., 2011; Keegan et al., 2011), including construction and commodity trading. In recent years, the market dynamics of the COVID-19 pandemic have significantly distorted market trends as factories stopped operations and logistics were interrupted, as shown in Figures (3.1) and (3.2).

Similarly, we used TMS bare land prices (LND) to develop forecasting models. In this instance, the data covers a shorter period from 2000 to 2022. However, we did not use direct data. We averaged land transactions to determine nominal yearly prices and complete an annual series. This averaging method combines large transactions, over 20,000 acres, with medium transactions of 5,000 to 20,000 acres and small transactions of less than 5,000 acres. We acknowledge this method can be arbitrary since large, medium, and small transaction prices are sometimes not comparable. Furthermore, averaging these land prices cannot reflect true prices because these are location-specific and depend on factors such as infrastructure, site quality, productive area, and alternative land use (Guiling, Doye, and Brorsen 2007). However, we believe this land price approximation method is adequate for providing an indication and insight into the variability that can be found in the market.

3. Methods

3.1. Inflation and Seasonality

The TMS's nominal data was adjusted to real prices using the Producer Price Index (Federal Reserve Bank of St. Louis 2024), which uses 1982 as the base year (1982=100) for a nuanced understanding of time series temporal patterns.

Afterward, we conducted seasonality tests on the selected timber product categories. We applied the Webel-Ollech (WO) seasonality algorithm. This procedure integrates results from the

Quade-Serfling (QS) test and the Kruskal–Wallis (KW) test on the residuals of an automatic nonseasonal ARIMA model (Webel and Ollech 2019; Ollech 2022). A time series is classified as seasonal if the QS test p-value is below 0.01 or the KW test p-value is below 0.002.

The QS test is a rank-based nonparametric test that detects seasonal patterns in a time series. It accounts for autocorrelation while examining ranked residual deviations to determine the presence of seasonality as described in equation (1):

$$QS = \frac{\sum_{i=1}^{s}\left(\sum_{j=1}^{n_i} R_{ij} - \frac{n_i\,(N+1)}{2}\right)^2}{\sum_{i=1}^{s}\frac{n_i\,(N^2-1)}{12}} \tag{1}$$

where R_{ij} is the rank of the observation j in season i. The term s represents the number of seasons. The components n_i is the number of observations in season i and N is the total number of observations.

The KW test is also a rank-based nonparametric method to test whether the medians of multiple groups differ significantly to detect seasonality. Equation (2) depicts the KW test as follows:

$$KW = \frac{12}{N\,(N+1)}\sum_{i=1}^{s}\frac{R_i^2}{n_i} - 3\,(N+1) \tag{2}$$

where R_i^2 is the squared sum of ranks for group i. The element n_i is the number of observations in the group i and the component N is the total number of observations.

3.2. Stationarity and Cointegration

We evaluated stationarity using two tests: the augmented Dickey-Fuller (ADF) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests (Dickey and Fuller 1979; Kwiatkowski et al. 1992). The ADF test assesses a unit root presence indicating non-stationarity, while the KPSS test assesses stationarity. We also analyze cointegration among these non-stationary variables using the Johansen test to look for long-run equilibrium among the price series (Johansen 1991).

The number of lags for autoregressive modeling was initially determined based on the Bayesian Information Criterion (BIC). However, we also explored different numbers of lags as the maximum likelihood convergence or model performance was compromised.

The BIC criterion can be described using the following equation (3):

$$BIC_j = -2 \, Log \, L_j + k_j \, Ln(n) \tag{3}$$

where $Log \, L_j$ represents the maximized likelihood for model j, k_j is the number of parameters and $Ln(n)$ is the natural logarithm of the number of observations.

Like the method used for timber product prices, we deflated land prices using the Producer Price Index (PPI), conducted ADF and KPSS stationarity tests, and used BIC to guide the autoregression lag number.

3.3. Timber Prices Models

Time series data means observations for a single variable over successive time intervals. Time series differ from cross-sectional data because the sequential order of observations matters (Brockwell and Davis 2002; Box et al. 2016; Lastrapes 2023). Time series analysis studies temporal dependencies, which can be used to develop autoregressive models for forecasting future trends based on past values (Box et al. 2016; Hamilton 1994; Enders 2014).

VECM, MGAM, and MGARCH are time series models that can accommodate interdependencies among variables and heteroskedasticity. They ponder multidirectional impacts among variables and facilitate robust forecasting in complex scenarios (Montgomery, Jennings, and Kulahci 2015; Tsay 2014).

We defined two competitive multivariate models for timber price mean forecasting: VECM and MGAM, a cointegrated vector autoregressive model, and a multivariate interpretable machine learning autoregressive model. We also fitted an RWD model as a benchmark to evaluate predictive performance. This comparison allowed us to evaluate the multivariate models' forecasting capability. Given the stochastic nature of the RWD model, if prices exhibit purely random behavior, the RWD should outperform. Furthermore, considering the autoregressive conditional heteroskedasticity (ARCH) effects on the residuals of price models, we addressed heteroskedasticity clustering using MGARCH models that result in a price volatility forecasting tool.

3.3.1. VECM

VECM allows for direct modeling of relationships, maintains stability, and captures both short-term changes and long-term interactions in multivariate time series (Tsay 2014; Pfaff 2008).

VECM uses error correction terms to guide system coefficients toward a long-term equilibrium where variables drift upward jointly (Greene 2011). Mathematically, it can be written as equation (4):

$$\Delta Y_t = \alpha\, \beta'\, Y_{t-1} + \sum_{l=1}^{p-1} \Gamma_l\, \Delta Y_{t-l} + C + \varepsilon_t \tag{4}$$

where Y_t represents the vector of endogenous variables at time t, ΔY_t is the difference operator denoting $Y_t - Y_{t-1}$, α is the matrix of coefficients for the error correction term, which represents describing the rate at which a dependent variable returns to its equilibrium value. The term β' represents the equilibrium relationships between the variables in levels. Thus $\beta'\, Y_{t-1}$ can characterize the distance between variables and their equilibrium. Γ_l are the matrices of short-term coefficients, p represents the lag denoting the maximum number of lagged differences, C is the intercept or constant term matrix, and ε_t is the vector of error terms at time t (Mohr 2019; Box et al. 2016).

3.3.2. MGAM

GAMs can be considered part of an interpretable machine-learning model array because of their transparent nature in modeling relationships between variables (Molnar 2025). They depict nonlinear relationships and analyze patterns within exponential family models and likelihood-based frameworks (Hastie and Tibshirani 1986). Furthermore, GAM model the intricate between covariates or predictors and a response variable by employing smooth functions (Wood 2017).

The MGAM expands the GAM and facilitates the simultaneous analysis of multiple response variables while depicting complex nonlinear interactions. These characteristics prompted us to evaluate an autoregressive MGAM as one of the competing models for timber price series forecasting. MGAM is formulated as described in equation (5):

$$Y_t = \alpha_i \, \gamma + \sum_{j=1}^{p} f_j \left(Y_{t-j} \right) + \varepsilon_t, \qquad Y_t \sim EF(\mu_i, \theta) \tag{5}$$

where Y_t represents the vector of endogenous variables at time t. The terms α_i and γ characterize i^{th} row in a parametric model matrix and the corresponding parameter vector, respectively. The smooth functions of lagged covariates Y_{t-j} capturing the non-linear influences from past values are denoted by f_j. The expression ε_t represents the vector of error terms at time t, and the element $EF(\mu_i, \theta)$ embodies an exponential family distribution with mean μ_i and scale parameter θ (Wood 2017).

Specifically, our MGAM was configured to capture multiple correlated continuous response variables representing prices using thin plate regression splines. This configuration allows for capturing smooth, nonlinear relationships between predictors and responses. The MGAM also assumed a multivariate normal distribution for the joint modeling of the responses, allowing for their potential correlation. The model employed penalized smoothing with automatic term selection to retain only significant effects and used restricted maximum likelihood for stable and unbiased estimation of smoothing parameters.

3.3.3. RWD

Random walks are stochastic processes that simulate variables moving randomly without the influence of their past. Over the long term, they do not tend to revert to a long-term mean or trend (Tsay 2005). RWD is a specific case of this process that introduces a directional component or the drift to a random walk. RWD results in a variable that, on average, tends to move upwards or downwards over time, in addition to its random fluctuations the drift term determines the direction and speed of this trend. The following equations (6 and 7) can represent an RWD (Tsay 2005):

$$Y_t = \mu + Y_{t-1} + \varepsilon_t \tag{6}$$

where μ represents the expected value of $(Y_t - Y_{t-1})$ or $E(Y_t - Y_{t-1})$, Y_{t-1} depicts lagged covariates and ε_t denotes a white noise or random noise series.

The following equations (7) can represent the evolution of RWD:

$$\begin{gathered} Y_1 = \mu + Y_0 + \varepsilon_1 \\ Y_2 = \mu + Y_1 + \varepsilon_2 = 2\mu + Y_0 + \varepsilon_1 + \varepsilon_2 \\ . \\ . \\ . \\ Y_t = t\,\mu + Y_0 + \varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_1 \end{gathered} \tag{7}$$

where the constant term μ of the model above still symbolizes the time trend of the series Y_t or the model's drift. The term t is the time index or period in the random walk sequence. To see this, assume that the initial variable is Y_0 (Tsay 2005; Lastrapes 2023).

3.3.4. MGARCH

GARCH models, from their inception (Bollerslev 1986), have been one of the most influential models and widely used in financial engineering for capturing the conditional heteroskedasticity phenomena (Xing et al. 2021; Engle 2002b; Engle and Kroner 1995). GARCH's innovation in the variance equation incorporates past squared observations and past conditional variances, with variance forecasts being linear combinations of these variables based on previous conditional variances (Engle 2002b; Bollerslev 1986).

MGARCH models extend GARCH to analyze multiple series, enhancing understanding of market dynamics, asset correlations, time-varying volatility, and interdependencies among time series (Engle and Kroner 1995; Engle 2002a; De Santis and Gerard 1997). Initially developed in the late 1980s and early 1990s, MGARCH models experienced a period of calm in the late 1990s, followed by rapid expansion from the 2000s onward (Bauwens, Laurent, and Rombouts 2006). MGARCH models have been used to model volatility interdependencies across financial markets. They have also been applied to compute dynamic hedge ratios by estimating conditional variance-covariance matrices, enhancing risk management strategies and economic policies (Bauwens et al., 2006; Diebold, 2007; Enders, 2010; Herwartz and Roestel, 2022; Lien and Tse, 2002). The general equations for the MGARCH model are as follows in the equation system (8):

$$r_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = D_t z_t \tag{8}$$

$$D_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j D_{t-j}^2$$

The term r_t is the vector of values at time t. The μ_t is the conditional mean or expected value vector at time t. The ε_t is the residuals or error term vector at time t. The element D_t represents the diagonal matrix of the error term's conditional standard deviation or volatility at time t. The z_t represents the standard normal random variables vector, implying $z_t \sim N(0, I)$. The expression D_t^2 is the conditional variance matrix of the error terms at time t. The term ω is the vector of constant terms representing the volatility baseline level. The element α_i represents the coefficients vector for the lagged squared residuals or ARCH terms. It captures past shocks' influence on current volatility. The component ε_{t-1}^2 depicts the vector of lagged squared residuals from the value equations, denoting past shocks. The term β_i describes the coefficients vector for the lagged conditional variances or GARCH terms that capture the persistence of past volatility in current volatility. Finally, the expression D_{t-1}^2 corresponds to the matrix of lagged conditional variances or the past levels of volatility.

MGARCH extensions introduce different features to represent specific volatility patterns in the data. For instance, the threshold MGARCH (TMGARCH), exponential MGARCH (EMGARCH), and Glosten-Jagannathan-Runkle MGARCH (GJR-MGARCH) enhance time series modeling with asymmetric volatility dynamics. The TMGARCH allows for modeling different responses to positive and negative shocks. The EMGARCH incorporates the variances' logarithms to ensure positive values. The GJR-MGARCH addresses adverse shocks that have a more pronounced impact on future volatility than positive shocks (Engle, 2002b).

The GJR component adds an asymmetric term to the univariate GARCH models within the multivariate framework. Precisely, each conditional variance D_t^2 in the MGARCH model is adjusted to the equation (9):

$$D_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j \, D_{t-j}^2 + \sum_{i=1}^{q} \gamma_i \, \varepsilon_{t-1}^2 \, I(\varepsilon_{t-i} < 0) \tag{9}$$

where D_t is the diagonal matrix of conditional standard deviations from the univariate GJR-GARCH models. The term ω represents the vector of constant terms. α_i and β_j depict the coefficient vectors for lagged squared residuals and conditional variances. The concept γ_i characterizes the impact of negative shocks or leverage effect and $I(\varepsilon_{t-i} < 0)$ corresponds to an indicator that equals 1 if $\varepsilon_{t-i} < 0$.

Moreover, several GARCH and MGARCH extensions delve into correlation dynamics, including the Baba-Engle-Kraft-Kroner (BEKK), Constant Conditional Correlation (CCC), and Dynamic Conditional Correlation (DCC) models (Engle and Kroner 1995; Bollerslev 1990; Engle 2002a). The first, BEKK, advocates for a positive and definite covariance matrix that prompts invertibility and positive eigenvalues. The second, CCC, prompts for a constant correlation, easing the estimation process and the model's interpretation. The third, DCC, incorporates time-varying correlations to better represent interlinked covariate movements.

We initially tested several BEKK- and CCC-MGARCH models, but these were discarded due to poor performance or ARCH effects in the residuals. Thus, we focused on DCC-type GARCH equations.

The DCC-MGARCH models the dynamic volatility and conditional correlation structure in multivariate time series. It captures the dynamic interdependencies among multiple series. The conditional covariance matrix H_t is decomposed as depicted in equation (10):

$$H_t = D_t R_t D_t \tag{10}$$

where D_t is a diagonal matrix of conditional standard deviations of univariate GARCH models, and R_t is the time-varying correlation matrix. Thus, the dynamic correlation structure is given by the equations (11):

$$\begin{aligned} Q_t &= (1 - a - b)\ \bar{Q} + a\,(\varepsilon_{t-1}\,\varepsilon_{t-1}^{T}) + b\,Q_{t-1} \\ R_t &= diag(Q_t)^{-1/2}\, Q_t\, diag\,(Q_t)^{-1/2} \end{aligned} \tag{11}$$

where the term Q_t is the conditional covariance matrix of standardized residuals. The $\bar{Q}$ is the unconditional covariance matrix of standardized residuals. The expressions a and b are scalar parameters controlling the dynamics of the correlation structure.

Structural breaks in price modeling refer to sudden and significant changes in the underlying relationship or behavior of a price time series due to economic or external shocks. Thus, another refinement to price variance modeling is to include structural breaks. Recognizing these breaks prevents misinterpretation of time series dynamics and enables models to adapt accordingly (Bai and Perron 2003). Thus, the MGARCH model and structural break considerations enhance the representation of volatility dynamics, leading to more accurate and robust risk forecasts.

Therefore, considering all the refinement strategies exposed, we chose a DCC-GJR-MGARCH-t for volatility modeling in this study. Such a model also considers t-distribution for heavy-tail behavior in asset returns. It stands out as a model for our variances for its ability to simultaneously represent volatility and correlations of multiple residual series and external shifts. The DCC structure allows the model to capture time-varying correlations, and the GJR addresses volatility asymmetries caused by negative and positive shocks.

Additionally, we tested structural breaks or shocks using the Bai-Perron methodology to identify multiple structural breaks in our residual series (Bai and Perron 2003). This technique specifies a linear regression model where coefficients can change at unknown breakpoints. The procedure minimizes the sum of squared residuals through a dynamic programming algorithm and uses sequential F-tests along with the BIC to determine and select the optimal number of breakpoints (Bai and Perron 1998; Zeileis et al. 2002). Therefore, we incorporated the structural breaks as external regressors in the DCC-GJR-MGARCH-t models to capture the data's volatility dynamics. As a result of this model's extension, parameters can better accommodate the complexities of our series, leading to more accurate forecasting.

We evaluated nine DCC-GJR-MGARCH-t models with structural breaks, using MGAM residuals. The structural breaks are represented in the model by incorporating dummy variables as external regressors in the mean equation. All these volatility models were assessed based on economic rationale and implications, fit goodness, and parsimony using BIC to select the most suitable configuration.

3.4. Bare Land Price Models

Modeling bare land prices involves several complexities because of land characteristics and the dynamics of forest markets. Significant variations in geographical location, soil quality, land use, market demand, non-timber considerations, and conditions of sale determine the asset value (Mei and Clutter 2023; Guiling, Doye, and Brorsen 2007), making accurate price forecasting challenging.

We based our bare land price modeling on time series models incorporating exogenous variables to improve the model's predictive accuracy. Timber prices intrinsically correlate with land values, reflecting how historical timber price fluctuations impact future cash flow on land. Thus, we chose an autoregressive integrated moving average model (ARIMA) and an autoregressive integrated moving average model with exogenous variables (ARIMAX) as competing models to represent bare land prices in this study.

3.4.1. ARIMA

The ARIMA model is often used in time series analysis and forecasting (Box et al. 2016). Their three model's components, autoregression (AR), differencing (I), and moving average (MA), represent the correlation between an observation and its lagged values, the level difference fitting, and the relationship between an observation and past residual errors, respectively.

The following equation (12) can characterize the ARIMA model:

$$Y_t = \beta_0 + \sum_{j=1}^{p} \phi_j \ Y_{t-j} + \sum_{k=1}^{q} \theta_k \ \varepsilon_{t-k} + \varepsilon_t \tag{12}$$

where Y_t represents the dependent variable at time t. The intercept coefficient is symbolized by β_0. The terms ϕ_j are autoregressive parameters, and Y_{t-j} represents the dependent variable at lag j. The θ_k vector embodies the moving average parameters, and ε_t corresponds to the error term at time t.

3.4.2. ARIMAX

The ARIMAX model extends ARIMA by incorporating exogenous variables influencing the price series, enhancing forecast accuracy. Like stumpage price models, the competing models are contrasted against an RWD model as the naïve model.

The ARIMAX model can be represented using equation (13) as follows:

$$Y_t = \beta_0 + \sum_{i=1}^{m} \beta_i \, X_{i,t-i} + \sum_{j=1}^{p} \phi_j \; Y_{t-j} + \sum_{k=1}^{q} \theta_k \; \varepsilon_{t-k} + \varepsilon_t \tag{13}$$

where Y_t represents the dependent variable at time t, $X_{i,t-i}$ are exogenous variables at lag i. The intercept and the coefficient vector associated with the exogenous variables are represented β_0 and β_i. The terms ϕ_j are autoregressive parameters, and Y_{t-j}represents the dependent variable at lag j. The θ_k vector embodies the moving average parameters and ε_t is the error term at time t.

This study compares an ARIMA, an ARIMAX, and an RWD model to characterize bare land prices. This strategy mirrors the timber price modeling strategy described earlier. The autocorrelation and ARCH effect tests indicated that our ARIMA and ARIMAX models for bare land price time series showed no significant residual autocorrelation, non-normality, or ARCH effects, making conditional heteroskedasticity modeling dispensable.

3.5. Models Performance

We compared various types of time series models for price prediction as described. We employed a cross-validation approach to select the most suitable models. This approach considered a subset of the training and validation price data sets. The training data was used for an initial model fitting. Then, the validation data was used to assess predictive performance over 6 and 2 periods, corresponding to 5% and 9% of the data for timber and bare land prices, respectively.

Cross-validation residuals allow us to evaluate prediction performance. We utilized the Diebold-Mariano test, mean absolute percentage error (MAPE), mean absolute error (MAE), root mean squared error (RMSE), and bias criteria (Bias) as leading performance indicators. The Diebold-Mariano permits us to determine predictive differentiation among competing models (Diebold and Mariano 2002), while MAPE, RMSE, MAE, and Bias assess predictive accuracy. These criteria can be expressed through the following equations (14):

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\hat{y}_i - y_i}{y_i}\right|100$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|\hat{y}_i - y_i| \qquad (14)$$

$$Bias = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)$$

where the term n denotes the total number of observations. Expressions $\hat{y}_i$ and y_i are the predicted value and the actual value for the i-th observation.

Moreover, time series models require a complete residual analysis, including autocorrelation, normality, and autoregressive conditional heteroskedasticity effects (ARCH effects). Depending on the model type, we conducted univariate or multivariate tests. To identify correlations in residuals, we studied autocorrelation functions (ACF), partial autocorrelation functions (PACF), and cross-correlation functions (CCF).

ACF computes the correlation between the residuals of a time series model at various lags. PACF isolates the correlation between residuals at two points in time, adjusting for correlations at all shorter lags. CCF evaluates the relationship between the residuals of two time series models as they vary in time relative to each other.

Specifically for timber prices, we evaluated four different tests for ARCH effects, general autocorrelation, and for identifying other model inadequacies. The first was the Lagrange multiplier (LM) test, which used the Q(m) statistic for the squared series for ARCH effects (Engle 1982). The second is the rank-based test for serial correlation in residuals, a non-parametric approach to detecting autocorrelation (Ling, Tsay, and Yang 2021). The third is the Q statistic test statistic or multivariate Portmanteau Q statistic for squared residuals, a variation of the Q statistic used for examining autocorrelation in squared residuals (Box and Pierce 1970). Fourth, robust test statistics assess the model parameters' significance while accommodating violations of standard assumptions like homoscedasticity and normality of errors (Huber 1981).

For univariate bare land price models, we used the Shapiro-Wilk test to assess normality, the Breusch-Godfrey test, ACF, and PACF to check for serial correlation. Additionally, we applied the LM test to assess ARCH effects and rank-based tests to provide a non-parametric alternative for detecting serial correlation in residuals.

Finally, we must acknowledge that our time series models assume no timber market regional cointegration, a condition previously documented for Georgia (Mei, Clutter, and Harris 2010; Yin, Newman, and Siry 2002). Additionally, we recognize that the bare land price forecasts assume no changes in highest and best-use decisions, which could alter land values as the assumption is relaxed.

4. Results

4.1. Seasonality, Stationarity, and Cointegration

The seasonality test revealed seasonal trends in the PULP price series. Thus, we adjusted seasonality by applying seasonal differentiation to the specific series (Tsay 2005).

The ADF and KPSS tests, considering a constant and a trend, strongly indicated non-stationarity for the timber price series, as reported in Table (3.1). The Johansen test (Johansen 1991) revealed at least one cointegrating relationship among timber prices.

Like timber prices, the ADF and KPSS tests indicated non-stationarity in the bare land price series as indicated in Table (3.2), suggesting the need for integrated time series models to avoid spurious regression issues.

Considering stationarity and cointegration in timber prices, we fitted one integrated and one non-integrated model: a VECM and an MGAM, respectively. This strategy allows us to compare how each model captures the short- and long-term dynamics under different integration and model structure assumptions.

4.2. Timber Price Models

The cross-validation process involved testing the VECM and MGAM with up to four lags. Our results showed that the one-lag models consistently outperformed the others regarding MAPE, MAE, RMSE, and Bias. The VECM(1), MGAM(1), and the univariate RWD models consistently tracked the validation data trends from Q1 1992 to Q2 2022, as we can observe in Figure (3.3).

The VECM(1) and MGAM(1) models consistently outpaced the naive univariate RWD models across all timber products outside the training dataset, Figure (3.3). This outcome indicates that time series models, using past data, predict future prices more accurately than this stochastic process. Further, the RWD models lacked precision in tracking timber product prices, particularly for CNS and SAW. For instance, the RWD model predicted a SAW price of \$9.5 per ton for the fourth quarter of 2023, significantly underestimating the actual validation data price of \$12.6 per ton by 24.6%.

Figure (3.3) also demonstrates the effectiveness of VECM(1) and MGAM(1) in capturing overall trends in timber price data, including during the validation period from Q3 2022 to Q4 2023. This validation period encompasses complexities arising from the COVID-19 pandemic, which challenges the models' robustness in maintaining predictive accuracy amid irregular economic conditions.

Regardless of the similar effectiveness of the VECM(1) and MGAM(1), the Diebold-Mariano tests indicated a significant difference in predictive accuracy between them for all timber prices. The p-values found were 0.02, 0.04, and 0.001 for PULP, CNS, and SAW, respectively.

Since these p-values are all below the significance level of 0.05, the null hypothesis that the two models have equal predictive accuracy is rejected.

Table (3.3) shows that no multivariate model excels over the other in all predicting criteria. The VECM(1) performs well for CNS in terms of MAPE and RMSE. This model has the advantage of capturing long-term relationships among variables, which makes it particularly valuable in scenarios where understanding the underlying dynamics and long-term dependencies is essential. MGAM(1), in particular, excels for all products regarding the bias indicator and SAW in the remaining criteria.

Overall, we regarded MGAM(1) as the best model, balancing all criteria due to its strong performance across most metrics and lower bias values. Thus, we selected the MGAM(1) model for forecasting timber prices and its residuals for variance modeling.

The ACF, PACF, and CCF analyses indicate that the VECM(1), MGARCH(1), and RWD models do not exhibit autocorrelation or cross-correlation among their residuals. For example, the MGAM(1) cross-correlation function shows no significant cross-correlation effects among the price series pairs PULP-CNS, PULP-SAW, and CNS-SAW, as shown in Figure (3.4). Each chart displays the correlation coefficient at lags ranging from -15 to 15. The blue dotted lines indicate that no significant correlations exceed the confidence bounds. Most correlation coefficients are within ±0.1, suggesting no strong linear relationships between the pairs at different lags. This result implies that the time series pairs do not exhibit strong linear dependencies on each other at different times.

The Lagrange Multiplier rejected the null hypotheses of no ARCH effects for VECM(1) and MGAM(1), indicating conditional time-varying volatility, where periods of high volatility tend to cluster, potentially affecting the models' forecast accuracy. Additionally, rank-based tests identified issues with autocorrelation and non-normality. The multivariate Portmanteau tests for squared residuals in the VECM(1) and MGAM(1) models indicate that the residuals are not white noise, implying autocorrelation or that the models may not fully capture all dynamics in the data. Moreover, the robust tests indicated significant autocorrelation or other model inadequacies that must be addressed in the VECM(1) but not in the MGAM(1).

Furthermore, the Bai-Perron method identified five meaningful structural breaks in the residuals of the MGAM(1) model. These structural breaks suggest significant and lasting changes in the MGAM(1) error terms rather than temporary shifts in the underlying market behavior. When structural breaks are overlooked, simple first-order and higher-order autoregressive models do not accurately capture the dynamics of the series (Hillebrand 2004). Consequently, it requires us to integrate such breaks in the variance modeling for more reliable and precise variance predictions and to improve forecasting accuracy.

4.3. Timber Price Variance Models

Given the evidence above, MGAM(1) residuals exhibit some autocorrelation, fluctuating volatility clustering, and structural breaks that must be addressed. We opted for DCC-GJR-MGARCH-t equations with structural breaks to model residuals and conditional volatility. These models allow us to depict dynamic asymmetric volatility, conditional correlations, and structural shock effects of multivariate residual series. Consequently, they adeptly manage complex volatility interdependencies that simpler MGARCH models often overlook. We used the student t-distribution because it is more robust to outliers, accounts for heavy tails, and better captures extreme variations than the normal distribution (Ardia and Hoogerheide 2010; Hall and Yao 2003).

We fitted various model specifications with structural breaks from DCC(1,1)-GJR-MGARCH(1,1)-t to DCC(3,3)-GJR-MGARCH(3,3)-t. Our DCC(3,1)-GJR-MGARCH(1,1)-t model configuration for MGAM residuals outperformed other model configurations. The DCC(3,1)-GJR-MGARCH(1,1)-t residual analysis based on the ACF, PACF, CCF, LM, Rank-based, multivariate Portmanteau, and Robust tests showed no autocorrelation, cross-correlation, or ARCH effects. It also demonstrated superior overall performance regarding the BIC criterion compared to other specifications.

Therefore, the DCC(3,1)-GJR-MGARCH(1,1)-t with structural breaks equation is preferred model, which represents the MGAM(1) residual patterns well. This decision underlines how critical it is to balance model fit, complexity, and economic implications, ensuring that the model captures the true dynamics of the data without overfitting.

DCC(3,1)-GJR-MGARCH(1,1)-t with structural breaks effectively handles time-varying correlations and captures the dynamic variance structure in our multivariate data, as described in Figure (3.5). It identifies critical volatility points in all timber products that align with significant events in the timber industry. These include environmental policy shifts in the late 1990s, the subprime mortgage crisis in 2008, and the COVID-19 pandemic-related disruptions from 2020 to 2022. However, Figure (3.5) also shows pronounced volatility peaks for PULP from 1998 to 2000. These peaks may be linked to the model's underperformance or a combined effect resulting from the significant decline in global pulp and paper prices, which increases volatility rates. The price decline was likely caused by increased recycling, shifts in industrial production, and economic challenges following the Asian financial crisis. Despite some of the model's limitations, the DCC(3,1)-GJR-MGARCH(1,1)-t with structural breaks models can still serve as a foundation for understanding volatility patterns in our stumpage prices, even if it does not capture every nuance of heteroskedasticity.

4.4. Land Price Models

Land value is determined by the net returns from its best alternative use and the discount rates used to discount those returns (Belongia 1985). Forest bare land values have slightly decreased over time, likely influenced by ongoing timber price declines due to high inventory, slow sawmill expansion, industry shifts, and reduced harvesting. Figure (3.6) displays that the ARIMAX(5,1,3) model with two lags of timber prices as exogenous variables, or ARIMAX(5,1,3;2), and the RWD model depicts this slight downward tendency well. The RWD outperformed the ARIMA(5,1,3) model in forecasting accuracy for the validation dataset, showing the highest values across all criteria, as shown in Table (3.4). Nonetheless, RWD estimation did not accurately reflect the training data like the ARIMA(5,1,3) model did, Figure (3.6).

The ARIMAX(5,1,3;2) model better captures the trend of the validation data than ARIMA(5,1,3) and RWD. It overperformed by exhibiting lower MAPE, RMSE, MAE, and the Bias criterion values. However, it underestimates the land values in the validation data set between 2021 and 2022, contrary to the ARIMA(5,1,3) and RWD models, which overestimate those values. The ARIMAX(5,1,3;2) model predicted land values of $436 per acre for 2021 and $456 per acre for 2022, contrasting with the observed values of $493 per acre for 2021 and $537 per acre for 2022. This results in an underestimation of 11.7% for 2021 and 15.1% for 2022, respectively.

Timber stumpage prices as exogenous variables significantly enhanced the accuracy of the ARIMAX(5,1,3;2). The timber price lags captured delayed price fluctuations, offering more profound insight into the economic factors influencing land value and improving the model's ability to reflect its dynamics. Similarly to timber price model testing, the Diebold-Mariano tests showed a significant difference in predictive accuracy between ARIMA(5,1,3) and ARIMAX(5,1,3;2) and RWD bare land price models. The test revealed p-values below the significance level of 0.05, indicating unequal predictive accuracy.

The ACF and PACF analyses show that the ARIMA(5,1,3), ARIMAX(5,1,3;2), and RWD models do not exhibit significant autocorrelation in their residuals. The Breusch-Godfrey test results, with p-values of 0.99, 0.63, and 0.14, also support the absence of serial correlation. The Shapiro-Wilk test suggests that the data are likely drawn from a normal distribution, with p-values of 0.16 and 0.53, and 0.77 for ARIMA(5,1,3), ARIMAX(5,1,3;2), and RWD, respectively.

Moreover, the Lagrange Multiplier and Rank-based tests found no ARCH effects or identified issues with autocorrelation and non-normality, with p-values of 0.62 and 0.98, and 0.83 and 0.24, 0.077, and 0.85, respectively, for ARIMA(5,1,3), ARIMAX(5,1,3;2), and RWD. This result indicates stable variance over time. Given the stability in the residuals and the absence of significant autocorrelation, serial correlation, and ARCH effects, developing dynamic variance models, as done for timber prices, may not be necessary.

The standard error of the ARIMA model was 168.34, compared to 66.45 and 61.14 from the ARIMAX(5,1,3;2) and RWD models. These values indicate the volatility or risk associated with such models, which will influence the land risk forecast.

5. Discussion

Our results indicated that autoregressive models remarkably outperformed stochastic processes. Although timber prices tend to be less volatile than lumber prices, extreme events like the COVID-19 pandemic still introduced significant shocks that stochastic models were better suited to capture. The autoregressive model performance implies that timber market dynamics remain more stable and predictable than other markets during large economic shocks. Timber markets can exhibit a degree of predictability even during financial turmoil, and autoregressive models can still capture predictive power in the data.

Past research efforts in timber price volatility modeling have explored different methods; the main effort has been focused on stochastic processes (Yousefpour et al. 2012). Norstrom (1975) and Lohmander (1987) modeled timber prices as a stochastic Markov chain to identify the optimal harvest. Clarke and Reed (1989) and Reed and Clarke (1990) studied a geometric Brownian motion process to represent price behavior. Mei, Clutter, and Harris (2013) represented price risk using geometric Brownian motion (GBM) and Ornstein-Uhlenbeck mean-reverting processes, while they used triangular distributions to depict land value risk.

The comparison of VECM, MGAM, and RWD performance offered essential insights into systematic-predictable and random patterns. The multivariate models provided significant predictive enhancements.

The MGAM(1) demonstrated strong performance across all metrics and exhibited low bias. This interpretable multivariate machine learning model successfully captured the trends in TMS data across the validation data. The MGAM(1) spline function captures the nonlinear relationships present in the price series, providing interpretable insights into the nonlinear influences in timber prices that linear models with rigid structures cannot represent. Additionally, the MGAM(1)'s multivariate configuration effectively incorporates co-movements in the market, making this strategy a high-performance choice in econometric modeling.

However, the expected price modeling is not the only component in timberland investments. The unpredictable nature of prices reflected in the heteroskedasticity found in timber price models is seen as a significant source of risk for timberland investments, as it directly impacts both the expected revenue and the long-term value of these assets (Mei, Clutter, and Harris 2013; Zinkhan et al. 1992). This risk has become a substantial concern for timberland investors and economists in recent decades (Yousefpour et al. 2012; Mei and Clutter 2015).

The residual analysis of the MGAM(1) model, including ACF, PACF, and CCF tests, showed no autocorrelation or cross-correlation. However, our selected ARCH effect tests failed to reject the null hypothesis of no ARCH effect, indicating the presence of conditional heteroskedasticity and clustering among the residuals. This suggests that the residuals are not white noise, and there remains information within the residuals that the model cannot fully capture.

Our dynamic variance findings are consistent with those of Mei et al. (2010), who identified volatility clustering in the GA2 sawtimber price series when comparing various time series models. This finding prompted us to develop variance and correlation models to forecast MGAM(1) volatility.

We opted for DCC-MGARCH-t models with structural breaks, as they can capture volatility patterns that other ARCH and GARCH models cannot. Expressly, we selected a DCC(3,1)-GJR-MGARCH(1,1)-t model with structural breaks. This model effectively captures asymmetrical volatility, where adverse shocks have a more significant impact than positive ones, and identifies critical volatility points due to external market shocks.

This particular asymmetric volatility model depicted the nuances of the GA2 region. The timber market dynamics, extended biological growth cycles, and landowners' decision-making processes contributed to distinctive volatility patterns in timber prices in the Southern United States, particularly when large shocks arose. Institutional investors and real estate investment trusts (REITs) often delay harvesting when prices are low, waiting for better conditions, while small landowners, under economic pressure, may harvest abruptly during downturns. These dynamics suggest that adverse shocks can quickly increase volatility as small landowners rush to harvest while large landowners strategically reduce supply. Positive shocks, however, lead to more gradual adjustments. This model captured the more substantial impact of adverse shocks and the persistence of volatility over time, making it a suitable tool for modeling the complex dynamics of the timber industry.

This finding indicates that stochastic or probabilistic models with fixed volatilities might not be appropriate for this dataset, as they could produce inaccurate assessments of variability. The volatility backcast from our models accurately reflected the standard deviation patterns for CNS and SAW prices. Our model characterized critical events, including the environmental policy shift in the Northwestern United States, the subprime crisis, and the COVID-19 pandemic. First, restrictions on supply caused by conservation efforts for the spotted owl in the Northwest restricted and drove up prices in the Southeast. Then, the subprime crisis led to drastic timber price volatility, which caused market activities to slow down. The COVID-19 pandemic increased volatility further by disrupting supply chains and involving rapid changes in demand. All these events significantly impacted the United States' timber market dynamics, which our model identifies. The variability models provide a solid baseline for forecasting, analysis, and probabilistic assessment that should outperform fixed-volatility approaches.

The ARIMAX(5,1,3;2) bare land price model performed better than the ARIMA(5,1,3) and RWD models. This finding suggests that even two lags of timber prices affect bare land value, which was slightly unexpected, as timber price changes are generally known to influence bare land value over the long term rather than in the short term. This finding may also represent structural changes in the real estate component of timberland investment. This result can mean that land values are becoming more responsive to short-term fluctuations in timber prices.

6. Conclusions

Our results indicate that the MGAM(1) model predicted prices accurately and outperformed competing models. They also suggest that interpretable machine learning algorithms hold increasing potential for time series econometrics as their capacity to model nonlinearity and temporal dependencies continues to evolve. More sophisticated interpretable machine learning algorithms, such as Bayesian Additive Regression Trees (BART) and Explainable Boosting Machines (EBMs), offer new approaches for further investigation.

The DCC(3,1)-GJR-MGARCH(1,1)-t model with structural breaks effectively captured the dynamics of the MGAM(1) model's variability, highlighting a major market shock from the late 90s while accounting for correlations. Our results indicate that timber price series involve complex relationships that univariate or simple GARCH models cannot adequately capture.

The ARIMAX(5,1,3;2) model, which uses stumpage prices as exogenous variables, effectively captures land price dynamics. The two lags of timber prices, acting as exogenous variables, influenced the representation of bare land prices, indicating that timber prices have a short-term effect on bare land prices and may signal market changes for land.

Our research has made modest advances in forest econometrics while also highlighting limitations that future studies can address. We relied on a fixed validation dataset rather than using rolling validation, which could strengthen model performance testing. We also tested only a limited set of MGARCH specifications, which remain a rich source of possible refinements. Additionally, future work could incorporate a Kalman filter framework to enhance the estimation of time-varying parameters and further improve model flexibility.

Table 3.1. Stationarity test p-values for timber product prices

Variable	ADF test	KPSS test	α	Non-Stationarity
PULP	0.26	< 0.01	0.05	✓
CNS	0.31	< 0.01	0.05	✓
SAW	0.18	< 0.01	0.05	✓

Table 3.2. Stationarity test p-values for bare land prices

Variable	ADF test	KPSS test	α	Nonstationarity
LND	0.44	< 0.01	0.05	✓

Table 3.3. Timber price model's predictive accuracy

Model	MAPE (%)			RMSE ($/ton)		
	PULP	CNS	SAW	PULP	CNS	SAW
VECM	16.77	6.84	9.13	1.17	0.85	1.20
MGAM	19.08	9.20	6.66	1.14	0.87	0.91
RWD	21.93	17.98	19.22	1.42	1.78	2.36

Model	MAE ($/ton)			Bias ($/ton)		
	PULP	CNS	SAW	PULP	CNS	SAW
VECM	1.03	0.66	1.11	-1.03	-0.66	-1.11
MGAM	0.86	0.69	0.51	0.37	-0.05	-0.21
RWD	1.08	1.51	2.01	-1.33	-1.68	-2.32

Table 3.4. Bare land price model's predictive accuracy

Model	MAPE (%)	RMSE
	LND	LND
ARIMA	60.26	371.04
ARIMAX	13.43	70.57
RWD	19.80	106.24

Model	MAE	Bias
	LND	LND
ARIMA	318.01	318.01
ARIMAX	69.59	-69.59
RWD	100.32	100.32

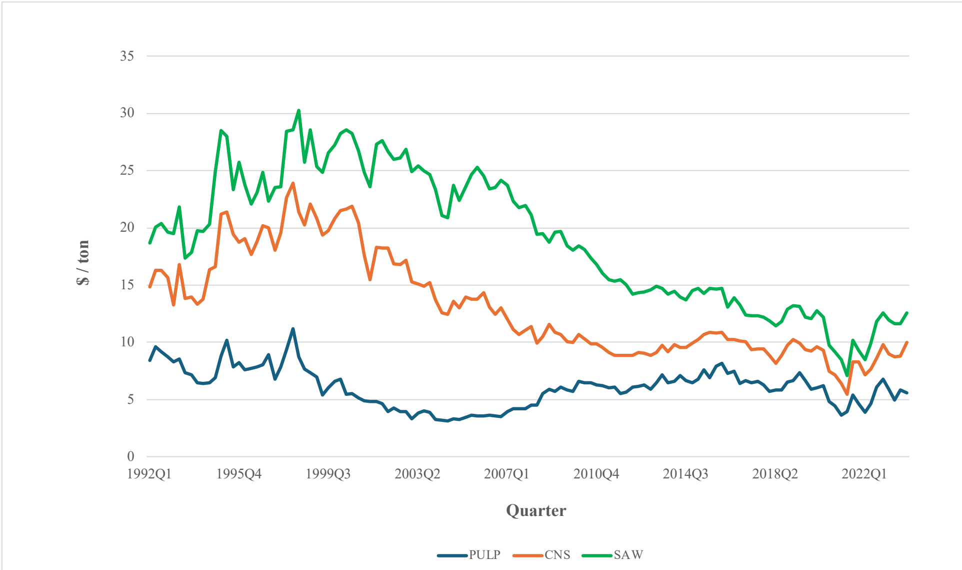

Figure 3.1. Timber real prices in the South Georgia region (GA2)

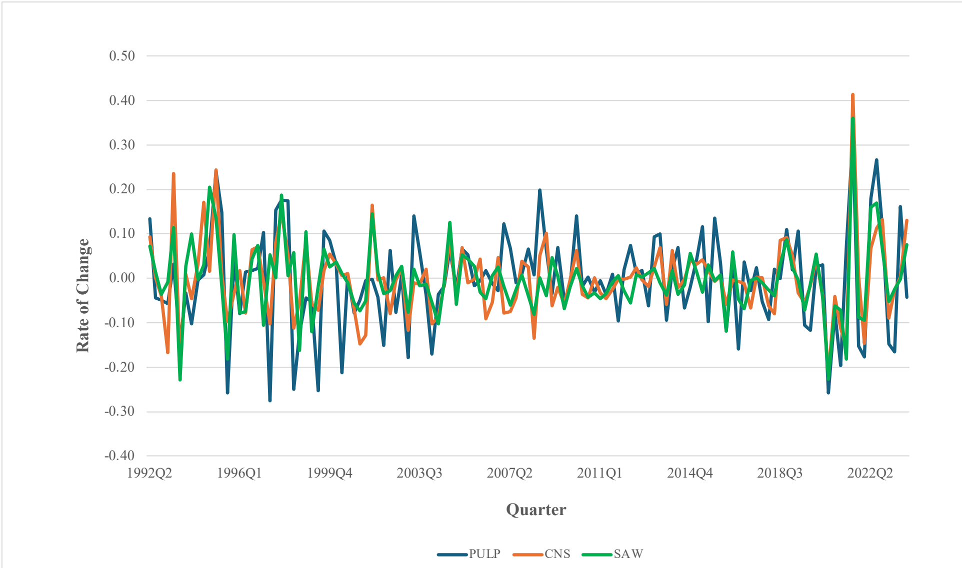

Figure 3.2. Timber real price rates of change in the South Georgia region (GA2)

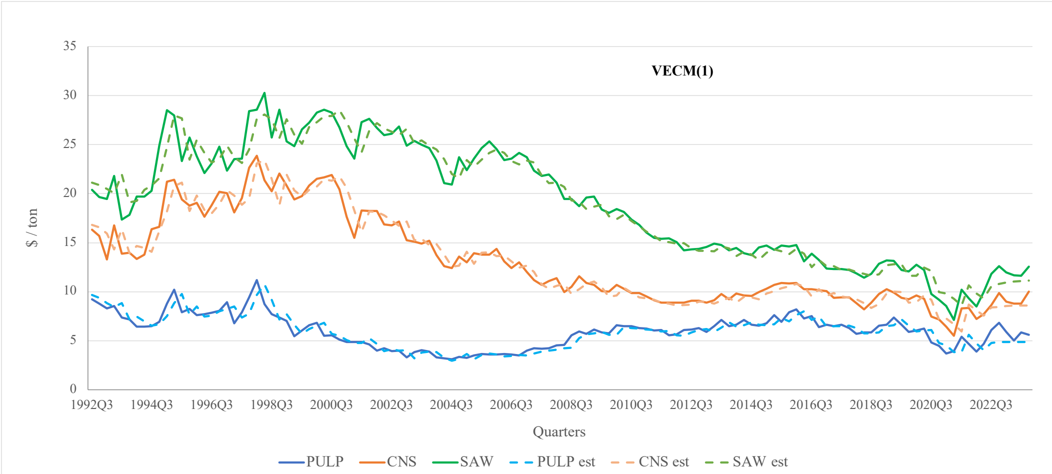

Figure 3.3. Timber real price models' prediction

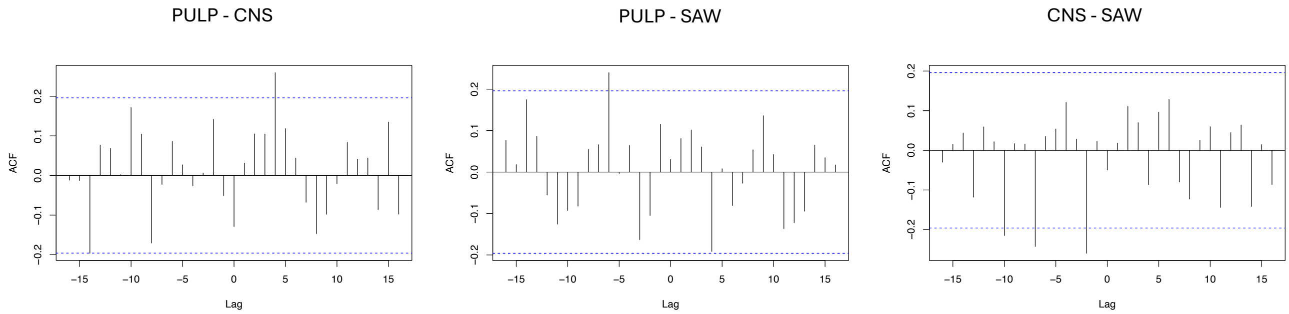

Figure 3.4. MGAM(1) cross-correlation functions

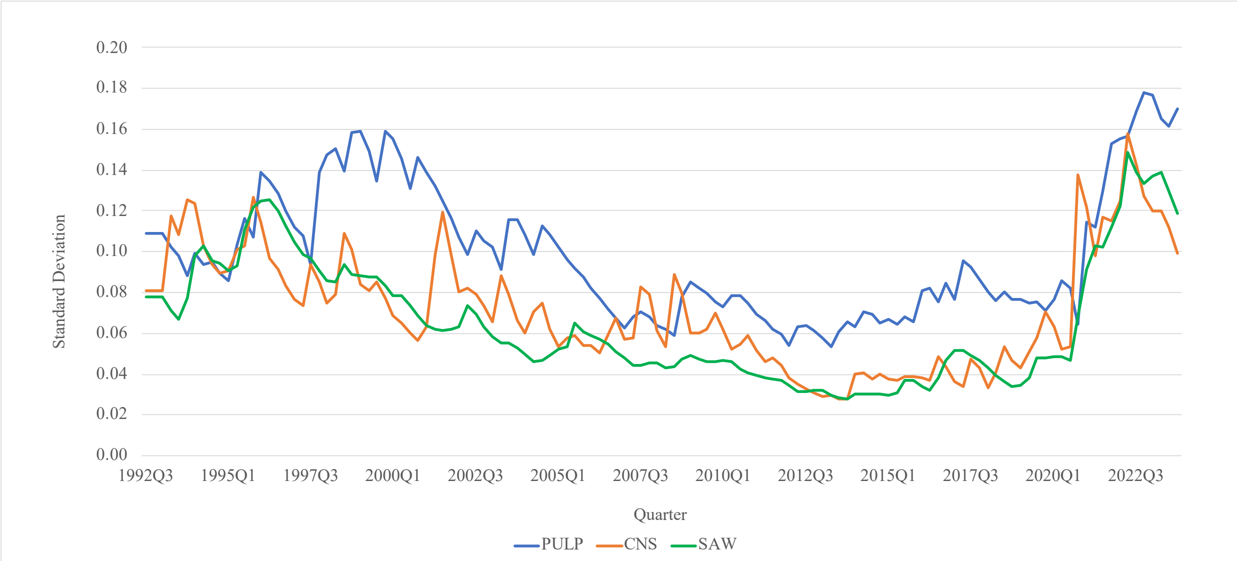

Figure 3.5. The backcasted standard deviation of timber real price rate changes, modeled using a DCC(3,1)-GJR-MGARCH(1,1)-t approach with structural breaks

Figure 3.6. Bare land real price models' prediction

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CHAPTER 4

Biological growth risk modeling: A seemingly unrelated regression approach[3]

[3] Cabezas, C. To be submitted to a peer-reviewed journal

Abstract

Timber growth modeling involves developing various functions that represent the state of the forest. Over the years, forest biometricians have developed sophisticated models that provide information for timber estimates. However, most of these models are univariate, disregarding interdependencies among variables. Furthermore, they fail to account for variability in the models, which can change over time, making timber growth forecasting challenging due to a lack of understanding regarding the variability of predictions. This research proposes the development of two seemingly unrelated regression equation systems to model forest state variables and timber products while accounting for dynamic variability and correlations. Given the complexity of the 23- and 41-parameter equation systems, we needed to combine the maximum likelihood method with automatic differentiation to overcome convergence issues. The state variable and timber product model fits exhibited high adjusted coefficients of determination, exceeding 0.89, indicating strong explanatory power for all models. The mean absolute percentage error suggests that the state variable models outperformed the timber growth models, making them more reliable in prediction. Moreover, the performance of the timber growth models indicates inaccurate forecasting power. While more advanced techniques could enhance our models, they sufficiently meet this dissertation's prediction and volatility modeling requirements.

1. Introduction

General timber growth models have been used in economic studies to approximate its risk. Miller and Voltaire (1983) investigated forest growth using a diffusion process. Clarke and Reed (1989), Reed and Clarke (1990), and Yin and Newman (1996) examined biological growth as a geometric Brownian motion (GBM) process affecting economic decisions. Most assumed that risk or growth uncertainty would be fixed over time, which is not necessarily applicable to timberland investment projects.

Moreover, the quantity and quality of timber products determine the value of forests, necessitating the development of more precise timber product models for revenue forecasting. Timber product distribution changes as the forest matures. In the beginning, younger forests have more low-value products; as forests grow, higher-value timber increases the overall value. Consequently, estimating timber volume throughout the rotation has been a significant concern in forest biometrics.

Different mathematical expressions have been developed to estimate tree volume based on simple measurements (Burkhart, Avery, and Bullock 2019; Prodan et al. 1997). However, determining the volume or weight of timber products remains challenging due to variations in tree form at different heights and the relocation of the timber product as years pass. Amateis, Burkhart, and Burk (1986) developed a stand-level ratio model to allocate total stand merchantable yield, demonstrating the effectiveness of such an approach. Harrison and Borders (1996) built on the work of Amateis, Burkhart, and Burk by developing yield prediction equations for a whole-stand growth system in loblolly pine plantations located in the Southeastern United States.

However, different equation fitting methods fail to account for forest variable interactions, violating the independent errors assumption (LeMay 1990). In 1962, Zellner proposed the seemingly unrelated regression approach (SUR), a system of simultaneous regression equations with correlated error terms. He estimated equations jointly to improve efficiency and obtain more accurate parameter estimates. His novel study found that the SUR regression coefficient estimators obtained were asymptotically more efficient than those obtained by an independent equation application. SUR is suitable for forestry systems because multivariate equations are prone to correlated errors that challenge model development, providing a more effective and unbiased method to improve forest modeling (Borders 1989; LeMay 1990).

Based on Zellner's SUR systems, Borders (1989) utilized a series of related equations to model the growth and yield of forest variables, including timber volume. Fang, Borders, and Bailey (2000) developed a segmented stem taper model based on variable-form differential equations. They also used simultaneous equation estimation techniques with SUR to fit a taper model, as well as equations for merchantable volume and total volume. Sandoval et al. (2021) developed a SUR system to model forest biomass. They integrated variance and correlation functions to model heteroskedasticity and interactions among variables. Their approach showed an efficient parameter assessment, controlled interdependency, and improved the precision of the estimates as risk is modeled. Sandoval, Montes, and Bullock (2024) incorporated uncertainty estimators while modeling basal area yield, introducing variance-covariance matrices, and testing multiple error structures. They found the best performance when prediction and projection errors were combined. Thus, when an unequal variance situation arose in timber growth, variance modeling needed to be implemented (Clutter et al. 1983).

The heteroskedasticity modeling added an important inside of model risk for decision-making, allowing a better understanding of predictions and their variability. This chapter refitted the Lundqvist (1957), Gallagher et al. (2019), and Harrison and Borders (1996) models for dominant height, mortality, basal area, and timber green weight, respectively, using SUR models. For this purpose, we developed two SUR equation systems to analyze the state variables: dominant height, mortality, basal area, and timber weight variables: total green weight, pulpwood, chip-and-saw, and sawtimber green weight. The state variables provided the necessary input variables for the green weight set of equations. Like Sandoval et al. (2021) and Sandoval, Montes, and Bullock (2024), we fitted the equation systems simultaneously with the variance and correlation models. This strategy is recommendable because many regression models in forestry face issues related to correlation and heteroskedasticity in pairs of variables (Borders 1989; LeMay 1990). The proposed approach enables us to capture the volatility and interactions among the variables. Then, we can forecast the forest variables and understand how the uncertainty evolves, which is essential for developing the following chapters in this dissertation.

2. Data

For decades, the Plantation Management Research Cooperative (PMRC) at the University of Georgia has helped forest companies simulate timber yield and evaluate forest management practices in the United States (PMRC 2024). Its extensive field-based systems and regional trials across the Southern United States have provided in-depth insight into forest plantation development and improved forest operations planning.

This study used loblolly pine yield data from PMRC. Specifically, it focused on data from loblolly pine culture density studies conducted across eight installations in the Eastern Coastal Plain Region of South Georgia and North Florida. The PMRC's data collection process involved measurements taken at intervals of 2 to 5 years since 1997. This database offers a valuable understanding of southern loblolly pine plantation development, underpinning timber yield forecasting model fitting. The dataset variables include diameter at breast height, initial planting density, number of trees, quality categorization, total height, total green weight (TOT), pulpwood (PULP), chip-and-saw (CNS), and sawtimber (SAW) green weight of individual trees. These state variables will be utilized for modeling forest products at the stand level, as equation systems are described in the subsequent sections of this study.

3. Methods

3.1. Seemingly Unrelated Regression Model

Harrison and Borders (1996) specified equations for estimating dominant height (HD), tree survival rate (TPA), and basal area (BA), seeking flexibility in modeling various physiographic regions using two decades of PMRC data. They also developed stand-level ratio models for total timber and product class yield. For such product models, they defined proportions characterized by a top diameter and a diameter at breast height (DBH) threshold limit grounded on Amateis, Burkhart, and Burk (1986) work.

This study refitted the Harrison and Borders models to estimate and forecast BA, TOT, PULP, CNS, and SAW product green weight for our data. The timber product models consider proportion equations, PULP, CNS, and SAW, which factor the TOT using a ratio-predicting approach (Amateis, Burkhart, and Burk 1986). We modeled HD and TPA using the Lundqvist height growth model (Lundqvist 1957) and the Gallagher et al. (2019) proposed tree survival equation. We split the state variable and timber product equation systems into two batches to facilitate the fitting process and ensure the model's convergence.

These equation systems were fitted using the SUR approach through the maximum likelihood estimation (MLE). This method combination allows the simultaneous equation fitting while ensuring compatibility. Moreover, as discussed previously, the emerging heteroskedasticity phenomenon and capturing hidden correlations are critical analytical considerations. Therefore, we modeled heteroskedasticity and correlations while predicting forest stand variables. Thus, our SUR equation systems simultaneously fit the coefficients of stand-state variables and timber product models, integrating dynamic variance and correlation models.

Zellner (1962) and Sandoval et al. (2021) described the generalized equation (1) for SUR models, along with its corresponding matrix notation, as follows:

$$Y = \beta * X + \varepsilon, \qquad \varepsilon \sim N(0, \Sigma) \tag{1}$$

$$\rho = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \rho_3 \\ & 1 & \rho_4 & \rho_5 \\ & & 1 & \rho_6 \\ & & & 1 \end{pmatrix}$$

$$\Sigma_1 = \begin{pmatrix} \sigma^2_T & \sigma_T\ \sigma_P\ \rho_1 & \sigma_T\ \sigma_C\ \rho_2 & \sigma_T\ \sigma_S\ \rho_3 \\ & \sigma^2_P & \sigma_P\ \sigma_C\ \rho_4 & \sigma_P\ \sigma_S\ \rho_5 \\ & & \sigma^2_C & \sigma_C\ \sigma_S\ \rho_6 \\ & & & \sigma^2_S \end{pmatrix}$$

where the term β is the parameter vector for forest state variables and timber products, the component X is the matrix of predictors, and the element ε represents the error term, assuming a multivariate normal distribution. The normal density function can be described as $N(0, \Sigma)$, meaning zero error mean with Σ variance-covariance matrix. This probability density function comprehensively represents the intricate relationships between diverse allometric components. The matrix ρ represents the correlation matrix among variables. The terms ρ_1 to ρ_n represent the correlation between two state or product variables. In contrast, the matrix Σ_1 contains variances σ^2 and differential covariances σ_T, σ_P, σ_C, and σ_S, indicating interactions among variables T, P, C, and S that represent state variables or timber products in the matrix.

Initially, we used the model parameters of Gallagher et al. (2019) and Harrison and Borders (1996) as starting points for fitting the TPA and BA nonlinear models. This opening stage evolves from independent models with fixed variance and correlation to multivariable models with dynamic variances and correlations. Subsequently, we incorporated variance and correlation models in the SUR system as the parameter search and convergence were met. Thus, this evolution ensures model accuracy.

MLE seeks to maximize the likelihood function to represent the probability of the observed data given a model. However, our model sets involve many parameters for state variables and timber product equation systems, challenging MLE convergence. Thus, we combined MLE using the quasi-Newton algorithm Broyden–Fletcher–Goldfarb–Shanno (BFGS) and the gradient automatic differentiation approach to assess our multivariate SUR models. Automatic differentiation calculates gradients rather than depending on numerical approximations, improving optimization and resulting in accurate parameter estimates. Thus, MLE using BFGS along the automatic differentiation improves convergence speed and precision by approximating second-order derivatives by a Hessian matrix (Nocedal and Wright 1999).

We addressed the model performance using the mean absolute percentage error (MAPE), root mean squared error (RMSE), mean absolute error (MAE), and bias criteria. Thus, these metrics collectively assess model performance, identify prediction accuracy, and detect potential systematic error inaccuracies. However, the MAPE calculation for the CNS and SAW models excludes zero observed values to avoid skewing the results with undefined or disproportionately large percentage errors. MAPE can be less reliable when observed values are zero (Makridakis, Wheelwright, and Hyndman 1998). This adjustment helps us to provide a more precise model performance evaluation since zero values usually correspond to young stands with no timber production for such products.

3.1.1. State Variable Models

The state variables are depicted by the anamorphic projection equations (2), (3), and (4) as follows:

$$HD_2 = HD_1 \, e^{\beta_1 \left(\frac{1}{AGE_2^{\beta_2}} - \frac{1}{AGE_1^{\beta_2}} \right)} \tag{2}$$

$$TPA_2 = \left(TPA_1{}^{\beta_{11}} \, e^{\beta_{12}} \left(\left(\frac{HD_2}{10} \right)^{\beta_{13}} - \left(\frac{HD_1}{10} \right)^{\beta_{13}} \right) \right)^{\frac{1}{\beta_{11}}} \tag{3}$$

$$BA_2 = BA_1 \, e^{\left(\frac{\beta_{22}}{AGE_2} - \frac{\beta_{22}}{AGE_1}\right)} \left(\frac{TPA_2{}^{\left(\beta_{23} + \frac{\beta_{24}}{AGE_2}\right)}}{TPA_1{}^{\left(\beta_{23} + \frac{\beta_{24}}{AGE_1}\right)}} \right) \left(\frac{HD_2{}^{\left(\beta_{25} + \frac{\beta_{26}}{AGE_2}\right)}}{HD_1{}^{\left(\beta_{25} + \frac{\beta_{26}}{AGE_1}\right)}} \right) \tag{4}$$

where *HD* represents dominant height, *TPA* implies tree density or tree survival. *BA* represents the basal area, AGE represents age in years, and β_n corresponds to the model's coefficients.

Variance and correlation models, σ and ρ, are integrated into the multivariate equation system to capture variability and correlation patterns. The following equations (5) and (6) represent variance and correlation models:

$$\sigma = exp(\varphi_1) \frac{Z_t{}^{\varphi_2}}{Z_{t-i}{}^{\varphi_2}} \tag{5}$$

$$\rho = \frac{2}{1 + e^{(\delta_1 + \delta_2 * t)}} - 1 \tag{6}$$

Terms φ_1 and φ_2 represent the σ model's coefficients. The expression Z corresponds to the state variables. The δ_1 and δ_2 are the correlation model ρ coefficients. The expression t represents age and i the number of years prior, making $t - i$ the initial age.

3.1.2. Timber Product Models

The following equations (7) and (8) characterize timber product yield models:

$$TOT = e^{\beta_{15}} TPA^{\left(\beta_{16} + \frac{\beta_{17}}{t}\right)} HD^{\beta_{18}} BA^{\left(\beta_{19} + \frac{\beta_{20}}{t}\right)} \tag{7}$$

$$W = TOT\, e^{\left[\beta_{21} \left(\frac{k}{DBH}\right)^{\beta_{22}} + \beta_{23} TPA^{\beta_{24}} \left(\frac{K}{DBH}\right)^{\beta_{25}}\right]} \tag{8}$$

TOT, *W*, *HD*, *DBH*, and *BA* denote the total green weight, timber product's green weight, dominant height, diameter at breast height, and basal area, respectively. The β_n terms represent the coefficients. The expressions k and K denote each timber product's top diameter limit and *DBH* threshold.

Like the state variable SUR system, individual variance models σ and correlation models ρ are integrated into the equation system to address heteroskedasticity and correlations. These models can be depicted as equations (9) and (10):

$$\sigma = exp(\lambda_1) + \lambda_2 Q \tag{9}$$

$$\rho = \frac{2}{1 + e^{(\mu_1 + \mu_2 * t)}} - 1 \tag{10}$$

The model's coefficients are represented in the elements λ_1 and λ_2. The element Q represents the timber product green weight variables. The terms μ_1 and μ_2 denote the correlation model ρ coefficients, where the subscript t corresponds to age.

4. Results

The study refitted equations for HD, TPA, BA, and timber product yield models proposed by Lundqvist (1957), Gallagher et al. (2019), and Harrison and Borders (1996), respectively. The fitting processes involved two SUR equation systems of 23 and 41 parameters for state variables and timber product yields, respectively. Automatic differentiation facilitated this computation by evaluating gradients directly.

Although the automatic differentiation structural improvements aim to develop advanced models, the residuals from the PULP and SAW models indicate underestimation in timber product forecasts. Specifically, the PULP model shows a consistent tendency to underestimate compared to other timber product models. Conversely, residuals from the CNS model suggest overestimation, Figure (4.2).

The HD, TPA, BA, PULP, CNS, and SAW yield models exhibit more evident heteroscedasticity than TOT, as shown in Figures (4.1) and (4.2). This means that the standard deviations of the errors are not constant across estimated variable values. Additionally, in the residual plots, we observe a fan-shaped pattern, where the variance of the residuals increases along with the fitted values, particularly for the HD, PULP, and SAW models. Hence, the models make more accurate predictions for small values with fewer errors, but exhibit higher errors for large values.

The Mardia, Henze-Zirkler, and Doornik-Hansen tests for multivariate distributions showed normality, multivariate skewness, and kurtosis issues for all model residuals. We addressed heteroskedasticity effects through the dynamic volatility and correlation models integrated into the SUR equation systems. This strategy allows for more accurate variance prediction within a probabilistic analysis.

All MAPE values for state variables were under 10%, indicating a generally good model performance. The HD model exhibited lower MAPE values than TPA and BA, Table (4.1). The high TPA’s RMSE and MAE suggested that the model's variation from the actual data is extensive. The models' adjusted R^2 values indicated solid explanatory power for each HD, TPA, and BA model, implying that most of the variability in the dependent variable is captured. The HD, TPA, and BA’s bias values were low, but HD and TPA showed underestimation features in contrast to BA’s.

The MAPE for timber product models was high, exceeding 30.0%. TOT and PULP showed MAPE values of 30.0% and 32.4%, respectively, indicating reasonable estimation accuracy. In contrast, CNS and SAW had MAPE values of 55.0% and 48.9%, suggesting poor predictive performance. However, we must recognize that MAPE can overstate errors, particularly for CNS and SAW, as these models predict values close to zero for young plantations. In such cases, even minor absolute errors can result in a large percentage of errors.

The variation explained by the models, represented by the adjusted coefficient of determination or adjusted R^2, Table (4.2), gave us moderately high values. The R^2 for total green weight, PULP, CNS, and SAW reached 0.99, 0.90, 0.91, and 0.92, respectively. Thus, we can consider that the adjusted coefficients of determination represent a good model’s ability to predict or explain outcomes. The total green weight’s R^2 was very significant, meaning the proportion of total variation in outcomes explained by the model is very high. The RMSE indicated higher PULP and CNS prediction error values, which showed more significant deviations. The SAW’s MAE performed better with the smallest value. Bias indicated CNS overestimating and TOT, PULP, and SAW underestimating. However, all timber product bias metrics suggested the models' reasonable accuracy and reliability.

Therefore, the models displayed different levels of performance. TOT performs well, with low RMSE, MAE, and minimal bias, supported by a high adjusted R^2 of 0.99. PULP and CNS had higher RMSE, MAE, and MAPE values, indicating more significant prediction errors, although their adjusted R^2 values (0.90 and 0.91) suggested a reasonable fit. SAW showed good MAE and bias but a high MAPE, likely due to small values affecting percentage errors. Overall, the biometric models were adequate for prediction, but PULP and CNS could benefit from further refinement to reduce error.

5. Discussion

We have developed two equation systems to model biometric variables. These equation systems were constructed assuming the state variable model's errors from HD, TPA, and BA were noncorrelated with the timber product yield models. Thus, we divided the biometric models into two SUR equation systems. This assumption can generate some debate. This study hypothesized this split to address convergence issues during the fitting process. Despite the stated assumption, the fitting process remained challenging because the maximum likelihood method had to converge on high-dimensional models with extensively parameterized systems. The SUR models must capture dynamic and complex relationships among variables, managing 23- and 41-parameter equation systems.

Convergence in such complex systems is often contingent on choosing or estimating appropriate starting values at the beginning of the iterative process. These starting values and the choice of suitable optimization algorithms usually prove critical in obtaining stable and accurate parameter convergence (Nash 2014). Therefore, such convergence complexity and the SUR model-fitting approach were only feasible by adopting the automatic differentiation technique, performing efficient equation derivatives by parameters. This procedure allowed for a precise and fast evaluation of a gradient in maximum likelihood estimation, enhancing parameter optimizations among complex interdependencies within our SUR formulation.

One of the most significant advantages of variability accounting in our SUR systems is the time-varying changes in variance and correlation representation in growth and mortality caused by environmental factors. The dynamic variance and correlation models put risk in growth patterns in place along with their interactions. This dynamic system increases the robustness of long-term forecasting with better accuracy for forest stand projections. These growth equation systems with dynamic variability models enable better-informed decisions in forest management and resource allocation as we gauge uncertainty. This adaptability approach accurately depicts biological processes compared to static SUR models.

Nonetheless, our SUR dynamic variance and correlation equations are simple. This approach may accommodate further improvement, such as testing advanced or sophisticated equations within the SUR system. Thus, future research can focus on refining these models to better represent variance and correlation structures while capturing complex interactions. This enhanced modeling should increase predictive accuracy, offering profound insights into system variability and relationships.

The HD, TPA, BA, and timber yield models appropriately mimicked the stand growth patterns. All models presented significant adjusted coefficients of determination, adjusted R^2, all over 0.89, and low bias. The MAPE criterion for HD, TPA, and BA models indicated lower error percentages than timber yield models. The state variables ranged from 4.3% to 9.1% compared to 23.0% to 55.0% in timber yield models. The timber product models were inaccurate since the residual dispersion is vast compared to the state variables. However, these volatility patterns are captured in variability modeling for forecasting.

Regardless of the model's sophistication, timber yield models are very volatile forecasting tools, underlining the complexity of the timber product fitting and projecting process. The variance transmission from state variables to timber product models challenges achieving accurate forecasts, and it becomes more significant when variance changes over time (Clutter, 1963). Though often implicit, these challenges are widely acknowledged within forest operations when projecting biological growth.

6. Conclusion

The SUR models utilized 23- and 41-parameter equation systems for state variables and timber product yields, respectively. Automatic differentiation facilitated the MLE's convergence by directly evaluating the model's gradients, improving parameter estimation and the model's stability.

The residual analyses displayed normality issues for all the models based on the multivariate normality tests. All model residuals also exhibited heteroskedasticity patterns. However, since heteroskedasticity and conditional correlation were considered during modeling, the equation systems can represent state variables, green weight growth, and their dynamic volatility patterns. Therefore, our biological growth modeling strategy advances risk modeling from previous studies by accounting for variances and correlations among forest variables.

The models that refitted Lundqvist, Gallagher, and Harrison and Borders' proposed equations for HD, TPA, and BA, as well as timber yield models, exhibited a high adjusted R^2 with values over 0.89. The MAPE values were consistently lower for state variables than green weight variables, indicating a better model performance. Moreover, the CNS and SAW's MAPE exceeded 48%, indicating an inaccurate forecasting ability compared to state variables.

Independently, the sophistication of timber product models often results in unstable forecasts due to the complexity of modeling timber products and their interrelations. This instability arises from the impact of variability in key inputs on projections, especially when that variability shifts over time. A challenge well recognized in forest operations. We acknowledge that our models can be enhanced by incorporating more advanced modeling techniques into this approach. However, we believe these models fulfill the requirements of this dissertation for volatility modeling.

Table 4.1. Statistics of state variable models

Model	MAPE (%)	RMSE	Adjusted R^2
HD	4.26	3.03	0.95
TPA	8.86	79.24	0.89
BA	9.08	16.15	0.89

Model	MAE	Bias
HD	2.12	-0.04
TPA	50.79	-1.24
BA	11.01	1.22

Table 4.2. Statistics of timber product models

Model	MAPE (%)	RMSE	Adjusted R^2
TOT	30.03	5.39	0.99
PULP	32.35	11.97	0.90
CNS	54.97	11.12	0.91
SAW	48.86	4.48	0.92

Model	MAE	Bias
TOT	4.41	-1.00
PULP	9.65	-4.15
CNS	4.82	3.08
SAW	1.54	-0.54

* MAPE for CNS and SAW models excludes zero values to prevent distortion in large percentage errors

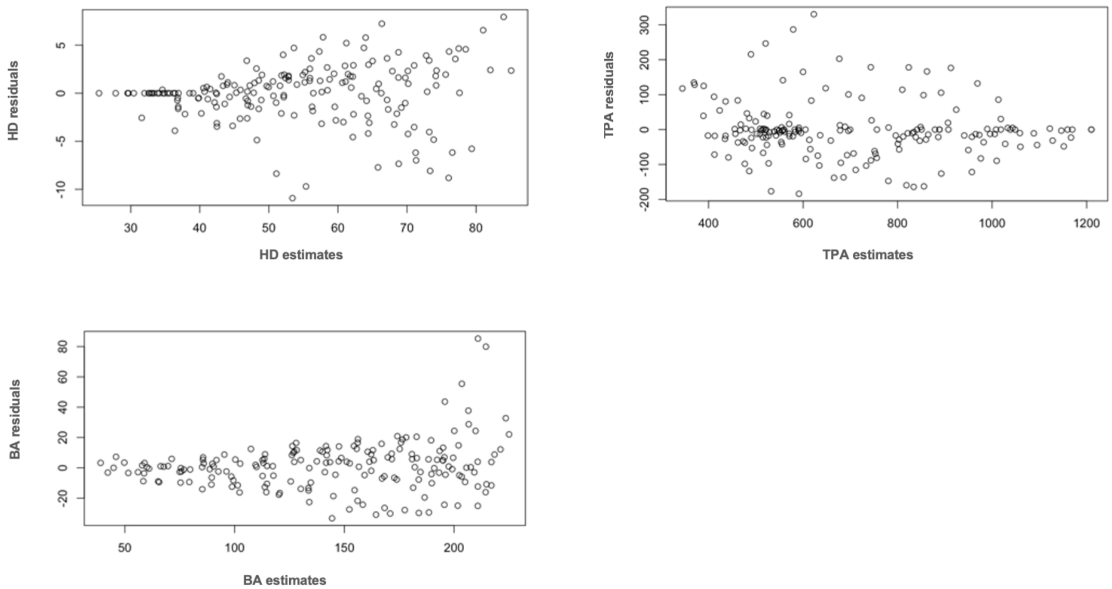

Figure 4.1. State variable model residuals

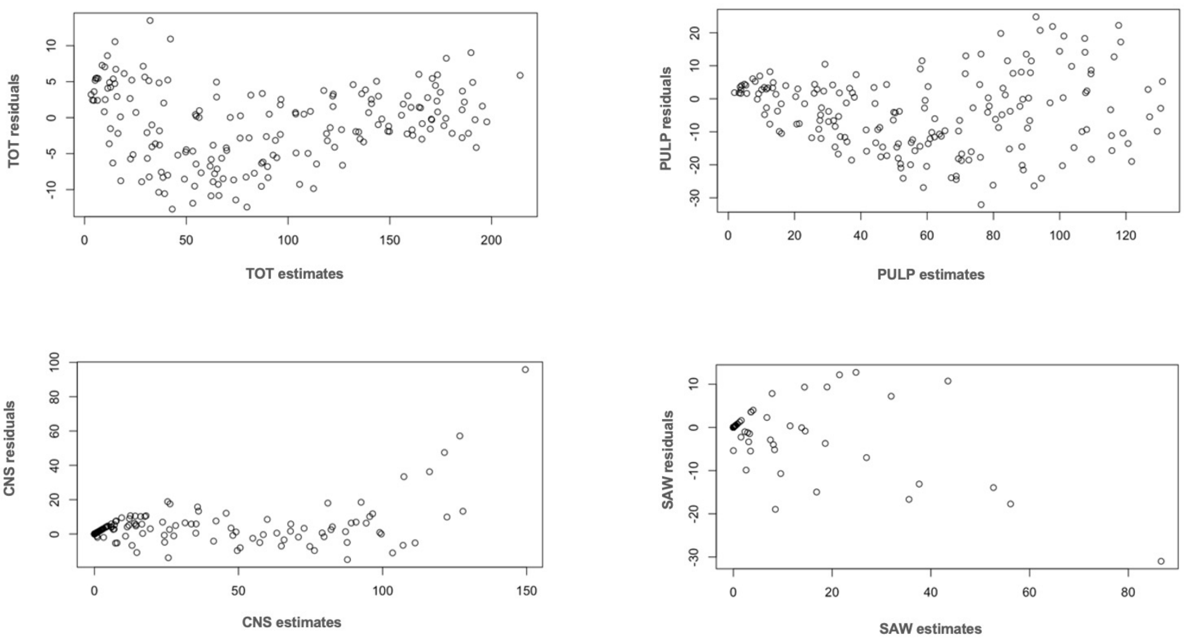

Figure 4.2. Timber product model residuals

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CHAPTER 5

Revisiting timberland return drivers: Integrating risk models for timber price, bare land value, and biological growth[4]

[4] Cabezas, C. To be submitted to a peer-reviewed journal

Abstract

The distinctive characteristics of timberland as an asset class are widely acknowledged as significant factors for enhancing asset portfolios. The primary return drivers for this alternative asset class include biological growth, timber price fluctuations, and land value appreciation. However, deviations from expected returns are common and pose a significant concern for investors. This study revisits the timberland return drivers using a probabilistic approach, considering timber price, biological growth, and land value volatilities. Monte Carlo simulations were employed to model dynamic volatility and correlation patterns over a 15-year horizon. This strategy showed that integrated risk sources, including biological growth, timber prices, and land value variances, expand the expected return distribution. The return's standard deviation increased from 0.9% to 12.8% when comparing only timber price and land value risks to biological growth, timber price, and land value integration. These findings indicate that the biological growth risk is significant and must be accounted for in investment decisions.

1. Introduction

Since the 1980s, U.S. institutional timberland investments have transformed significantly, growing to over $100 billion in value (Chudy and Cubbage 2020; D. Zhang 2021). Unlike conventional investments in publicly traded forest product companies, timberland investments offer direct access to this unique asset class (Mei and Clutter 2023). These timberland investments provide inflation protection and low correlation with traditional markets, making them attractive for diversification (Mei and Clutter 2023; Washburn and Binkley 1993; Waggle and Johnson 2009). The returns from timberland investments are driven by timber biological growth, timber price changes, and land value appreciation (Caulfield 1998). Among these, biological growth has been well-documented as the most significant return driver since it steadily increases the timberland asset's worth through natural forest maturation independently from financial cycles (Caulfield 1998; Mei, Clutter, and Harris 2013; Mills and Hoover 1982; Fu 2016; Mei 2023).

Caulfield’s (1998) foundational work, which described the timberland return drivers, assumed merchantable timber growth with certainty and no price volatility during the harvest cycle. However, deviations from the expected outcomes are not uncommon and continue to be a significant concern for timberland investors (Mei, Clutter, and Harris 2013; Thomson 1992). Moreover, these deviations, influenced by unpredictable factors, can create dynamic volatility patterns that impact forecasting accuracy. Therefore, return driver modeling must address this variability to gauge the inherent investment risk (Brazee and Newman 1999).

Commodity price volatility, including stumpage price volatility, plays a central role in predicting prices and estimating expected returns (Alquist, Kilian, and Vigfusson 2013; Pindyck 2004). However, timber price is not the unique risk source in timberland investments. The other return drivers also exhibit volatility patterns, adding different layers of risk to expected returns.

Deviations in timber product yield and bare land price predictions can alter precise return forecasting. Timber product yields and prices often show cross-correlation dynamics, and land prices present fluctuations that complicate modeling. Thus, an accurate assessment of financial outcomes requires advanced approaches to capture most interactions and variability among variables (Brennan and Schwartz 1985; Lohmander 2000; F. Zhang and Chang 2018; De Pellegrin Llorente, Hoganson, and Windmuller-Campione 2022; Sims 1980).

Previous studies on forest growth risk have explored various methods to evaluate multiple effects. Clarke and Reed (1989) and Reed and Clarke (1990) modeled timber growth as a geometric Brownian motion process. Ferguson (2016) modeled timber growth risk by incorporating random variation into planned wood flows, utilizing a normal distribution based on assumed unbiased inventory and growth data. Mei, Clutter, and Harris (2013) modeled timber growth as triangular distributions with the expected value as the most likely value and maximum and minimum values set at 10% above and below the expected value. Sandoval et al. (2021) developed a seemingly unrelated regression (SUR) system to model forest biomass dynamic volatility and correlations.

A multiple risk assessment allows for more accurate representations of this variability and offers insights into price and growth patterns. We propose revisiting the timberland return drivers while integrating risk models for timber price, bare land value, and timber growth. We utilized Monte Carlo simulations, which are based on the principle that a random variable's potential outcomes are defined by their probability distribution (Glasserman 2004). Thus, the return distribution estimation through thousands of scenario simulations describes the risk associated with the outcome (Metropolis and Ulam 1949).

Nonetheless, our approach differs from previous Monte Carlo probabilistic analyses by Mei, Clutter, and Harris (2013) and Mei (2023) that assumed constant variances. We utilized timber price and growth models developed by Cabezas (2025) to simulate dynamic variabilities. He based timber price and uncertainty models on multivariate generalized additive models (MGAM) and a multivariate Glosten-Jagannathan-Runkle generalized autoregressive conditional heteroskedasticity (DCC-GJR-MGARCH) model to capture correlation and dynamic volatility. He models bare land prices using an autoregressive integrated moving average model with external regressors (ARIMAX). Finally, he derived timber growth models by refitting the Lundqvist (1957), Gallagher et al. (2019), and Harrison and Borders (1996) models for dominant height (HD), mortality (TPA), basal area (BA), and timber green weight. These timber growth models were fitted using SUR, which accounted for dynamic variances and correlations using PMRC data. This approach allows us to assess the return's risk using price time series and direct field information from the forests.

By integrating price and timber growth models, we create a combination that captures interactions among variables and variability dynamics. This research addresses challenges in calculating the return drivers of a loblolly pine plantation by incorporating multi-modeling strategies over a 15-year horizon. Our analysis does not explicitly consider environmental hazards or potential disruptions from climate change. This modeling framework replicates previous strategies (Mei, Clutter, and Harris 2013; Mei 2023), which enables us to understand how risks jointly influence overall expected return distributions.

2. Models

2.1. Price Models

Cabezas (2025) compared a vector error correction model, a multivariate generalized additive model (MGAM), and a univariate random walk with a drift rate for modeling the expected timber prices. Similarly, he developed autoregressive integrated moving average and autoregressive integrated moving average with exogenous variables (ARIMAX) models for bare land prices to compare with univariate random walk models. He found that the MGAM(1) and ARIMAX(5,1,3;2) models outperformed the competing models based on economic rationales and performance criteria, such as mean absolute percentage error and mean absolute error.

He also found that MGAM(1) residuals exhibit autoregressive conditional heteroskedasticity (ARCH) patterns. Therefore, he tested several DCC-GJR-MGARCH models to address this volatility, ultimately selecting the DCC(3,1)-GJR-MGARCH(1,1)-t as the final model. This configuration enabled him to forecast variances over time while capturing correlations among the residuals. In contrast, the ARIMAX(5,1,3;2) model showed no significant ARCH effect, autocorrelations, and normality issues in their residuals, indicating that the constant volatility assumption over the years was valid. Thus, we relied on MGAM(1) and DCC(3,1)-GJR-MGARCH(1,1)-t for timber price and ARIMAX(5,1,3;2) for bare land price forecasting.

These models projected expected prices and volatilities over a 15-year horizon for the timberland return driver calculations and analysis, assuming steady market conditions. This last assumption has limitations, as it may not account for market integration or unexpected shocks, such as new manufacturing facilities or future mill shutdowns.

2.2. Timber Growth Models

We defined a synthetic loblolly pine plantation for the analysis. It was initially characterized by a Site Index of 75 feet and an initial density of 601 trees per acre at year 10, consistent with PRMC data used in the model’s construction. This contrasts with previous studies (Mei, Clutter, and Harris 2013; Caulfield 1998; Mei 2023) that utilized different site indexes and initial densities. As expected, all yield forecasts differed from previous studies due to variations in initial stand characteristics and simulation methods.

For simulation, we used the SUR models developed by Cabezas (2025) for HD, TPA, and BA prediction. He refitted the Lundqvist (1957), Gallagher et al. (2019), and Harrison and Borders (1996) models, respectively. This approach included dynamic variance and correlation estimation, illustrating the risk associated with each model.

Comparably, Cabezas (2025) refitted the ratio models developed by Harrison and Borders (1996) to estimate the green weight, which uses state variables mentioned before as independent variables. Total green weight (TOT), pulpwood (PULP), chip-and-saw (CNS), and sawtimber (SAW) were estimated along with variability and correlations. As a result, this SUR system allowed us to calculate the state variables and green weights, which can predict forest growth and volatility.

2.3. Return Drivers and Monte Carlo Simulations

Our study explored an investment strategy to acquire an established middle-aged loblolly pine plantation in South Georgia. We considered no thinning treatments. According to current industry standards, a stand with no thinning is considered a non-intensively managed plantation. The non-intensive management stand yields timber products dominated by low-value products, typically used to produce pulp, paper, wood particle boards, and fiber panels (Mei, Clutter, and Harris 2013). We assumed that clear-cutting and divesting were implemented at age 25. Property taxes, administration costs, and hunting lease incomes were excluded from the analysis, differing from previous studies.

We assumed the predicted second-quarter stumpage prices represent a stand-in for annual pricing. Additionally, we hypothesized a future yearly inflation rate of 2.0% to convert real prices into nominal ones. Thus, we forecasted the expected timber prices and their risk over time. Likewise, using the SUR models described, we simulated the biological growth of our stand, determining the timber product weight in the 15-year horizon. We must note that this procedure accounts for error propagation from state variables to timber green weight, as reflected in a chain variance accounting, which is captured later in the Monte Carlo simulations.

Consequently, we integrated prices, timber growth, and volatility forecasts from the models described above to calculate all stand values. Specifically, we summed the PULP, CNS, and SAW values to determine the stumpage value. We added the land value to the stumpage value to calculate the total or annual forest value. Percentage changes determined annual changes in total value, and the geometric mean of these changes was used to calculate the annual investment return. These considerations mimic Caulfield (1998), Mei (2023), and Mei, Clutter, and Harris (2013) assumptions and calculations.

We used @RISK (Lumivero 2025) for Monte Carlo simulations. This software emulates risks associated with predictions, as we assumed lognormal and normal distributions for all prices and biological growth. Our probabilistic analysis incorporated 10,000 simulations and explains the return variations. We also conducted a sensitivity analysis in the Monte Carlo simulation framework to isolate the effect of timber yield volatility on returns. This last analysis considers the timber yield deterministic and compares the outcome return distribution to gauge the effects of volatility and correlation when timber growth and price fluctuations are included. Therefore, we performed a detailed assessment of investment returns to better understand the asset's risk profile.

3. Results

3.1.1. Model's Forecast

We simulated our hypothetical 10-year-old loblolly pine stand, projecting it 15 years into the future by integrating biometric models with timber and land price models to estimate expected values. The variance models we developed defined our dynamic volatility and correlation patterns used in the Monte Carlo probabilistic analysis.

3.1.2. Timber and Land Price Models

The autoregressive MGAM(1) model forecasted a slight upward trend for all timber product prices in nominal terms, explained by an annual inflation rate of 2.0%, as shown in Table (5.1). PULP price increased from the inception of $15.51 to $16.05 per green ton. CNS and SAW prices also increased from $23.71 and $30.01 to $24.98 and $32.00 per green ton, respectively, from inception until year 25. However, the real PULP, CNS, and SAW prices declined by 23.2%, 21.7%, and 20.7%, respectively.

The DCC(3,1)-GJR-MGARCH(1,1)-t model with structural breaks captured dynamic correlations and volatilities between PULP, CNS, and SAW stumpage prices over time, considering asymmetric effects. Moreover, since our volatility model used t-distributions, the model also captured potential tail risks relevant to extreme price movements. The nominal volatility forecast, Figure (5.1), indicates that the PULP standard deviation will consistently be higher than the other stumpage prices. It will remain relatively stable over the forecast horizon, varying from $2.46 to $2.54 per green ton. Conversely, the CNS and SAW volatilities showed decreases. SAW sharply declined when its standard deviation decreased from $2.72 to $1.78 per green ton during the simulation period. The CNS exhibited a more gradual decay, from $2.24 to

$2.07 per green ton. Moreover, the timber price conditional correlations forecast from the DCC(3,1)-GJR-MGARCH(1,1)-t model with structural breaks shows slight changes over the years for all prices.

In nominal terms, the ARIMAX(5,1,3;2) bare land forecast displayed evident fluctuations over the 15-year horizon. The results varied between $912 and $2,320 per acre throughout the forecasting period. In real terms, it did not indicate a significant upward or downward trend. Instead, it demonstrated a more mean-reverting pattern. As explained previously, the ARIMAX (5,1,3;2) variability was projected as a constant variance of $66 per acre in real terms for the probabilistic analysis, and no variance model was fitted.

3.1.3. State Variable and Timber Product Models

The HD and BA models yielded from 50.4 feet to 75.0 feet high and from 120.6 to 195.2 square feet per acre between years 10 and 25. Table (5.2). The mortality model forecasted a detriment of 160 trees/acre from the project inception. Over 15 years of simulation, the timber product models have yielded a total green weight of 177.3 ton/acre. CNS model by year 25 generated 84.9 green ton/acre. PULP and SAW yielded 48.7 and 15.9 green ton/acre, respectively. Our findings are consistent with (Mei 2023) and Mei, Clutter, and Harris (2013), who found that CNS is the dominant overall product, followed by PULP and SAW in non-intensive silvicultural practices.

As the simulation progressed, the state variable, timber product volatility, and correlation models identified variance-changing patterns for all variables, Table (5.3). The simulation's variance-covariance and correlation matrices evolve yearly, describing how the biological risk can affect expected outcomes. Our correlation models display an increasing pattern in state variables and timber products as the plantation matures. Most forest stand variables also show a trend of increased volatility. PULP yield was the only one to show a decreasing volatility pattern after year 17.

3.2. Return Drivers and Monte Carlo Simulations

After incorporating the expected values for biological growth and prices from the selected models, we found that biological growth accounted for 58.9% of the total return in nominal terms. Meanwhile, changes in timber prices and land appreciation accounted for 4.4% and 36.8%, respectively, as shown in Table (5.4). In real terms, our calculations also indicated that biological growth is the major contributor. However, biological growth accounted for 102.3% of the total return, while changes in timber prices reduced returns by 32.8%, and land appreciation added 30.5%.

The study shows that the nominal stumpage value increased from $897 per acre in year 10 to $2,927 per acre in year 25. Our bare land model shows nominal land values increased from $1,139 to $2,320 per acre. Together, stumpage and land values resulted in a combined timberland or forest value ranging from $2,035 to $5,246 per acre from year 10 to year 25, as shown in Table (5.5).

The Monte Carlo simulations displayed a nominal median of 6.3% and a mean of 5.4%, a return highly influenced by PULP and CNS green weight, which dominate the timber products due to non-intensive management. The simulations also showed a left-skewed return distribution and a standard deviation of 12.7% for the annualized return from inception at year 25, Figure (5.2). This standard deviation indicated significant volatility due to biological growth and price risks. This level of volatility denotes a 61.7% probability of achieving annual returns between 0% and

7.3%. The likelihood of reaching a return between 5.3% and 7.3% is 28.7%. The probabilities of a negative return and a return above the last 3-year average are 3.2% and 35.1%, respectively.

Monte Carlo simulations also indicated that PULP stumpage prices exhibited more significant fluctuation among timber prices at year 25, with a coefficient of variation of 15.8%. This contrasts with the CNS and SAW stumpage prices of 8.3% and 5.6%, respectively. Bare land prices displayed a variation of 10.3%, surpassing the variability observed in CNS and SAW prices.

Timber yield exposed the highest volatility compared to prices. The coefficients of variation for PULP, CNS, and SAW yield reached 72.6%, 118.9%, and 158.1%, respectively. The SAW yield showed the most extensive volatility, standing out from the other timber product yields and timber and bare land price distributions.

Our sensitivity analysis indicates that the biological growth volatility considerably widens our return distribution, making our investment riskier, as shown in Figure (5.3). Considering the sole timber price and land price volatility, the simulation presents a more normal distribution shape than the left-skewed distribution when biological risk is included. The biological growth risk amplified the annual return standard deviation by 14-fold while decreasing the expected return median from 6.6% to 6.2%.

The simulation without timber yield risk shows no probability of obtaining a negative return and a lower likelihood of achieving higher returns above 7.3%, Figure (5.3). It also indicates a higher likelihood of obtaining returns between 5.3% and 7.3%. This last result aligns with Mei's (2023) findings, as he demonstrated only nonnegative return probabilities.

4. Discussion

We forecasted price and biological growth variables and developed a Monte Carlo probabilistic outline, including dynamic price and forest growth model volatilities. For the probabilistic analysis, we conducted 10,000 simulations that integrated expected timber stumpage, land prices, state variables, and the green weight of timber products. We assumed that timber and land prices followed a lognormal distribution while the loblolly pine growth variables followed a normal distribution. This forecasting strategy enabled us to calculate the timberland return drivers and assess the expected return confidence interval, thereby gauging risks.

The MGAM(1) forecasted an upward trend in all stumpage prices in nominal terms. The price of PULP increased 3.5% per green ton in year 25. In parallel, CNS and SAW prices also experienced gains, rising 3.2% and 6.6% per green ton, respectively. Nonetheless, the MGAM(1) forecast indicated that real stumpage prices constantly decline. The real PULP, CNS, and SAW prices declined from inception until year 25 by 23.2%, 21.7%, and 20.7%, respectively. The model's price decline reflects a possible oversupply effect, consistent with economic outlooks. For instance, Bennett (2019), Parajuli (2021), and Li and Campbell (2024) described that pine pulpwood prices in the Southern United States will remain under pressure, with limited growth due to increased sawmill byproducts competing in the market and structural shifts in the paper industry toward recycled materials. Sawtimber prices are expected to remain low across the Southern United States (Li and Campbell 2024; Sanquetta, Cabezas, and da Silva 2024; Lang 2020). This surplus, equivalent to decades of removals, limits opportunities for price recovery in many areas. Furthermore, our model prediction is consistent with Forisk's analysis, which indicates that sawtimber inventories will exceed demand into the next decade (Lang 2023). While no new sawmills are being built, the excess supply continues suppressing prices throughout the region.

In nominal terms, the DCC(3,1)-GJR-MGARCH(1,1)-t showed that the standard deviation of PULP will stay relatively constant, varying by -3.3% per green ton over the forecast horizon. This forecast suggests that global factors, rather than domestic ones, may explain the long-term uncertainty reflected in the TMS data. Conversely, the forecasted volatilities of CNS and SAW decreased. SAW experienced a sharp decline, dropping its standard deviation by 34.5% during the simulation period. The CNS showed a more gradual decrease, close to 7.9%. This pattern indicates a diminishing reaction to past shocks, notably the COVID-19 pandemic. Therefore, the volatilities of CNS and SAW may relate to improvements in domestic market conditions associated with the economic stabilization period following the pandemic.

The timber price conditional correlation forecast from the DCC(3,1)-GJR-MGARCH(1,1)-t model did not predict pronounced value changes over the years. This behavior suggests that the underlying correlations may remain resilient even when structural breaks occur as timber prices adapt to evolving market conditions.

The ARIMAX(5,1,3;2) bare land price forecast did not exhibit significant upward or downward trends in real terms, which considers a constant model's standard error of $66 per acre. However, the nominal bare land prices forecast fluctuated over the 15-year horizon, ranging from $912 to $2,320 per acre. Despite the ARIMAX(5,1,3;2) model's effectiveness in representing bare land value trends, it has limitations in precise forecasting, as evidenced by Cabezas (2025). The model faces challenges in accounting for variability because the data reflects transaction averages, which include large, medium, and small property sales. Thus, it can partially capture size-dependent and location-specific factors. Transactions with differing size characteristics should be modeled using differentiated algorithms. Nevertheless, given the study's objectives and the limitations of the data series, we believe this bare land price model still provides valuable insight into the general trend of the forest land market. Future studies could enhance bare land price forecasting accuracy by incorporating larger transaction datasets that detail characteristics influencing land value fluctuations.

Caulfield (1998), Mei (2023), and Mei, Clutter, and Harris (2013) relied on the North Carolina State University's Loblolly Pine Growth & Yield model, the open-source PMRC forest growth and yield simulator, and Forestech International's SiMS simulator, respectively, for biological growth forecast. Unlike the previous studies, we did not depend on external closed-system simulators. The study used an open SUR system of equations (Cabezas 2025) to forecast forest state variables, timber yield, and biological risk.

The SUR model yielded an MAI of 1.64 feet/year and 5.00 square feet/acre/year in HD and BA, respectively. The TPA model forecast showed remarkable mortality within the 15-year simulation horizon. It resulted in 10.7 trees/acre/year annual mortality or a mortality rate of 26.6%. The timber total green weight, PULP, CNS, and SAW yielded 177.3, 48.7, 84.9, and 15.9 green ton/acre, respectively. The PULP and SAW weights at year 25 made up 32.6% and 10.6% of the commercial volume, respectively. The CNS weight at year 25 accounted for 56.8% of the commercial green weight, which is the study's dominant timber product. This CNS dominance occurs because our theoretical loblolly pine stands did not undergo thinning treatments, producing lower-value products as the stand matures.

The SUR equation system forecasted an upward trend for state variables, TOT, CNS, and SAW volatilities. As growth increases, upward trends are expected due to the nature of our specific volatility models in our equation systems and because our variance models are based on predicted values. The PULP yield was the only one to show a decreasing volatility pattern beyond year 17. We must acknowledge that volatility equations are straightforward. Future studies can expand on this by investigating more sophisticated volatility models for enhancement.

Findings regarding the volatility of price and timber growth dynamics suggest that using fixed-volatility models on this dataset may lead to inaccuracies in probabilistic outcomes analysis, as Clutter (1963) described. For example, rigidity in volatility can lead to overly committed decisions when caution is necessary or premature withdrawal, causing one to miss a valuable long-term opportunity since the fixed-variance model does not account for expected risk. Time-varying variances must be modeled accordingly. The DCC(3,1)-MGARCH(1,1)-t and SUR systems account for dynamic variances and provide an improved risk forecast. This framework offers volatility information to adjust risk aversion, minimize adverse exposure, or analyze potential favorable market movements.

Our calculations indicated that the expected biological growth accounted for 58.9% of the future total nominal return, remaining the most substantial driver. The timber price change and land value appreciation comprised 4.4% and 36.8%, respectively. In real terms, biological growth and land appreciation accounted for 102.3% and 30.5% of the total return. Conversely, timber price changes reduced real returns by 32.8%. These last findings align with previous studies, which identified the negative contribution of changes in timber prices.

The nominal annualized return since inception has dropped from 33.7% to 6.5%. The reduced yield projections for SAW, the highest-value product, make it susceptible to substantial declines in annual value growth. When comparing the study's nominal return with the NCREIF timberland index for the Southern States, we find that the NCREIF timberland index indicates timberland returns have averaged 7.3% annually over the last three years and 5.3% over the previous ten years (Roth 2022). Thus, our expected return of 6.5% aligns with the ten- and three-year average returns of the NCREIF timberland indexes.

In contrast, our nominal return from Monte Carlo simulations displayed a median of 6.3%, a mean of 5.4%, and a standard deviation of 12.7% at year 25. The median aligned more closely with our expected return calculation of 6.5% and the NCREIF timberland indexes. Therefore, the median is a better representation of the probable or typical return we can anticipate based on our return distribution. Our nominal return standard deviation indicates significant volatility due to biological growth and price risk integration. This degree of volatility suggests a 61.7% chance of achieving annual returns ranging from 0% to 7.3%.

Additionally, the probability of obtaining a return within the narrower range of 5.3% to 7.3% stands at 28.7%. Meanwhile, the likelihood of surpassing the average return over the past three years of the NCREIF timberland index is 35.1%. These probabilities highlight the potential for moderate gains and the underlying risks inherent in return fluctuations, providing a nuanced view of performance expectations.

The study's nominal return distribution resulted in a left-skewed distribution, with most values clustering towards the right and a tail extending to the left. This result contrasts with the return distributions of Mei (2023) and Mei, Clutter, and Harris (2013), which presented a more normally distributed shape pattern. Furthermore, Mei (2023) outcomes exhibit a zero percent probability of obtaining negative returns from his timberland investment. This last outcome can be considered unusual in timberland investments because biological factors randomly affect the forest stand. However, our study assigned a low probability of loss, yet there is still a 3.2% chance of negative returns, which is low but realistically possible.

Timber price variability has been the primary risk concern in timberland investment research over the years (Yousefpour et al. 2012). However, when analyzing the individual factors driving timberland return probabilities in this research, it becomes evident that timber yield volatility was considerably higher than the variations in timber and land prices. The variation of PULP, CNS, and SAW yields was 72.6%, 118.9%, and 158.1% in terms of the coefficient of variation. In contrast, the effect of timber price change rates on the overall return distribution is relatively minor. PULP and bare land prices stand out in price variability statistics by the end of the rotation, with coefficients of variation of 15.8% and 10.3%, respectively. Hence, our timberland return risk findings highlight the critical role of biological growth variability in determining return outcomes.

Nonetheless, we must emphasize that our timber product models depend on dominant height, tree mortality, basal area, and total green weight as inputs to determine the green weight. The inherent variability of biological factors broadens the potential outcomes for green weight. This error propagation significantly impacts the distribution spread of timber growth and the returns distribution. This process is known as forest inventories, and growth models rely on this interconnected series of models.

Furthermore, the biological growth volatility effect can be broader if the general models are used because they tend to be more volatile when modeling timber products than location-specific models. General models attempt to accommodate most conditions and variations for various regions and are less precise predictors. Conversely, local models fitted to single conditions within a particular area usually generate more stable and accurate forecasts. This situation implies that solid dependence on general models can increase uncertainty in forecasting trends for forest products, which translates into the expected return driver risk.

The influence of biological growth is significant in our study; thus, we conducted a sensitivity analysis to isolate this effect concerning changes in expected returns. Our sensitivity analysis considered a deterministic timber yield as input and reran the Monte Carlo simulations. The results displayed an expected return in a more normal distribution shape from this probabilistic simulation. This finding aligns with Mei (2023) and Mei, Clutter, and Harris (2013) work, which modeled timber yield as triangular distributions and found this type of shape. This timber yield deterministic consideration in our analysis remarkably narrows the expected deviation of the returns, making them look less risky. The standard deviation dropped from 12.8% to 0.9%, eliminating the probability of negative returns. This result indicates that we cannot overlook the biological growth effect on returns since it is variable in nature.

5. Conclusions

We studied timberland return drivers by applying several advanced price and biometric models within a probabilistic framework. These models captured cross-correlations and dynamic variability to calculate timberland return risk, which allowed us to estimate the risks more precisely than traditional models.

The nominal annualized return since inception reached 6.5%. Our Monte Carlo simulations aligned with this previous result, showing a median return of 6.3% and a mean of 5.4% in a left-skewed distribution. It also aligns with the three-year and ten-year NCREIF timberland indexes for the Southern States of 7.3% and 5.3% annually, respectively.

Our study was consistent with previous studies and demonstrated that the return contribution from biological growth is the most important. Nominal timber prices showed a diminished influence on returns in the simulation. However, overall, this timber price contribution was favorably compared to Mei (2023) and Mei, Clutter, and Harris (2013), who described negative contributions.

The Monte Carlo simulations exposed broad expected return volatility in a left-skewed distribution. The sensitivity analysis exhibited that a deterministic timber yield assumption dropped the return's standard deviation from around 12.7% to 0.9%. The timber yield variability significantly exceeds timber prices and land value, making it the primary contributor to return risk. This volatility not only reflects the variance of the timber growth model on its own but also accounts for error propagation as the state variable models input the green weight calculations. Thus, biological growth risk is not negligible and must be pondered in any timberland investment assessment.

Risk analysis is one of the cornerstones of strategic planning and informed decision-making in timber investments. We can simulate possible dispersions of returns by identifying multiple risks and simulating the expected return volatility. This study does not aim to provide specific solutions to such a complex analysis. Instead, it seeks to illustrate how price and biological volatilities interact and affect expected returns in a forecasting scenario. Our analysis excludes explicit potential impacts from environmental hazards and disruptions related to climate change so that they can be included in future research. We hope these findings promote the development of more sophisticated modeling approaches and future research questions, with risk modeling integration as their central component.

Table 5.1. Forecast of nominal prices for timber and bare land

Year	Pulpwood ($/green ton)	CNS ($/green ton)	Sawtimber ($/green ton)	Bare Land ($/acre)
10	15.51	23.71	30.01	1,139
11	15.54	23.81	30.17	1,628
12	15.58	23.89	30.30	1,606
13	15.62	23.97	30.43	2,015
14	15.65	24.06	30.55	2,276
15	15.69	24.14	30.68	1,952
16	15.73	24.22	30.81	1,978
17	15.76	24.30	30.94	1,659
18	15.80	24.39	31.07	1,108
19	15.84	24.47	31.20	1,113
20	15.87	24.55	31.34	919
21	15.91	24.64	31.47	912
22	15.94	24.72	31.60	1,428
23	15.98	24.81	31.73	1,616
24	16.02	24.89	31.87	1,938
25	16.05	24.98	32.00	2,320

Table 5.2. Timber yield forecast

Age (years)	Dominant Height (feet)	Tree Density (trees/acre)	Basal Area (ft^2/acre)	Green Weight Total (ton/acre)	Pulpwood	CNS	Sawtimber
10	50.4	601	120.6	61.1	44.7	7.2	0.0
11	53.2	592	131.4	73.8	50.4	12.3	0.0
12	55.8	581	140.9	85.8	54.6	18.4	0.0
13	58.0	570	149.1	97.1	57.5	24.9	0.1
14	60.1	558	156.3	107.4	59.5	31.7	0.2
15	62.0	546	162.6	116.9	60.6	38.4	0.4
16	63.8	535	168.0	125.6	61.0	45.0	0.8
17	65.4	523	172.8	133.6	60.8	51.3	1.3
18	66.9	511	176.9	140.8	60.2	57.2	2.2
19	68.3	500	180.6	147.5	59.1	62.8	3.3
20	69.6	489	183.8	153.6	57.7	67.8	4.7
21	70.8	478	186.6	159.2	56.1	72.3	6.4
22	72.0	468	189.1	163.3	61.9	71.9	4.8
23	73.0	459	191.4	169.1	52.4	79.7	10.8
24	74.0	450	193.4	173.4	50.5	82.6	13.3
25	75.0	441	195.2	177.3	48.7	84.9	15.9

Table 5.3. Timber yield volatility expressed as standard deviation

Age (years)	Dominant Height (feet)	Tree Density (trees/acre)	Basal Area (ft^2/acre)	Green Weight (ton/acre) Total	Pulpwood	CNS	Sawtimber
10	0.4	33	11.3	9.7	18.2	3.6	0.5
11	0.5	35	14.5	11.0	19.7	6.9	0.5
12	0.6	37	16.9	12.3	20.8	11.2	0.5
13	0.7	39	19.1	13.4	21.6	16.2	0.5
14	0.8	42	21.2	14.5	22.2	21.6	0.6
15	0.8	45	23.1	15.5	22.5	27.2	0.8
16	0.9	48	24.8	16.4	22.6	32.9	1.1
17	1.0	52	26.4	17.2	22.6	38.5	1.6
18	1.1	56	27.8	18.0	22.3	43.9	2.3
19	1.2	60	29.0	18.7	22.0	49.0	3.2
20	1.2	64	30.2	19.3	21.5	53.7	4.4
21	1.3	69	31.2	19.9	20.9	58.0	5.9
22	1.4	74	32.1	20.3	22.5	57.0	4.5
23	1.5	79	32.9	20.9	19.5	65.2	9.5
24	1.5	84	33.7	21.3	18.8	68.0	11.7
25	1.6	90	34.4	21.7	18.0	70.3	13.9

Table 5.4. Contribution of drivers to timberland returns in nominal terms

Year	Total value ($/acre)	Biological growth ($/acre)	Timber price changes ($/acre)	Land price changes ($/acre)	Stumpage value ($/acre)
10	2,035	897		1,139	897
25	5,246	2,786		2,320	2,927
Change	3,211	1,890	140	1,181	2,030
Relative importance	**100.0%**	**58.9%**	**4.4%**	**36.8%**	

Table 5.5. Stumpage prices, land and forest values, and returns from the inception

Year	Stumpage value ($/ac)	Land value ($/ac)	Total value ($/ac)	Annual value change (%)	Annual return from inception (%)
10	897	1,139	2,035		
11	1,092	1,629	2,721	33.7%	33.7%
12	1,282	1,606	2,888	6.1%	19.1%
13	1,463	2,015	3,477	20.4%	19.5%
14	1,633	2,276	3,909	12.4%	17.7%
15	1,793	1,952	3,745	-4.2%	13.0%
16	1,943	1,978	3,920	4.7%	11.5%
17	2,082	1,659	3,742	-4.6%	9.1%
18	2,213	1,109	3,321	-11.2%	6.3%
19	2,335	1,113	3,448	3.8%	6.0%
20	2,450	919	3,368	-2.3%	5.2%
21	2,557	912	3,469	3.0%	5.0%
22	2,626	1,428	4,054	16.8%	5.9%
23	2,754	1,616	4,369	7.8%	6.1%
24	2,843	1,938	4,781	9.4%	6.3%
25	2,927	2,320	5,246	9.7%	6.5%

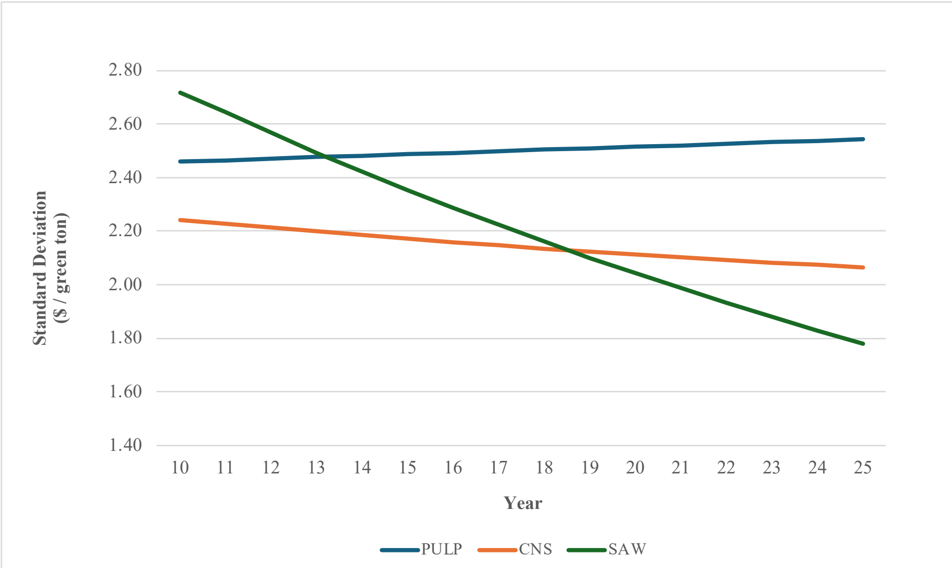

Figure 5.1. Forecast of the standard deviation of timber nominal price using a DCC(3,1)-GJR-MGARCH(1,1)-t model with structural breaks

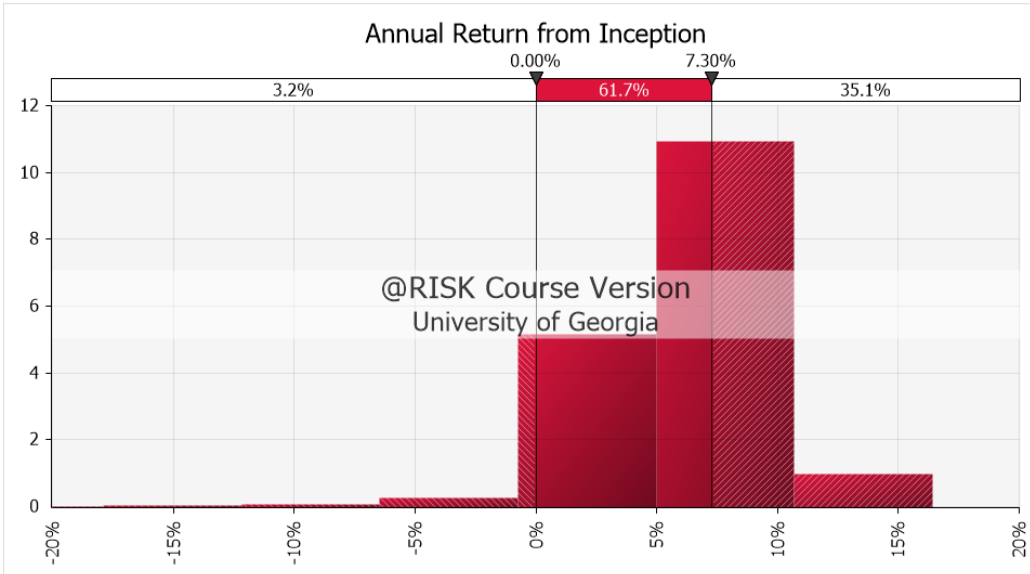

Figure 5.2. Probability density distribution of nominal annual returns from Monte Carlo simulations

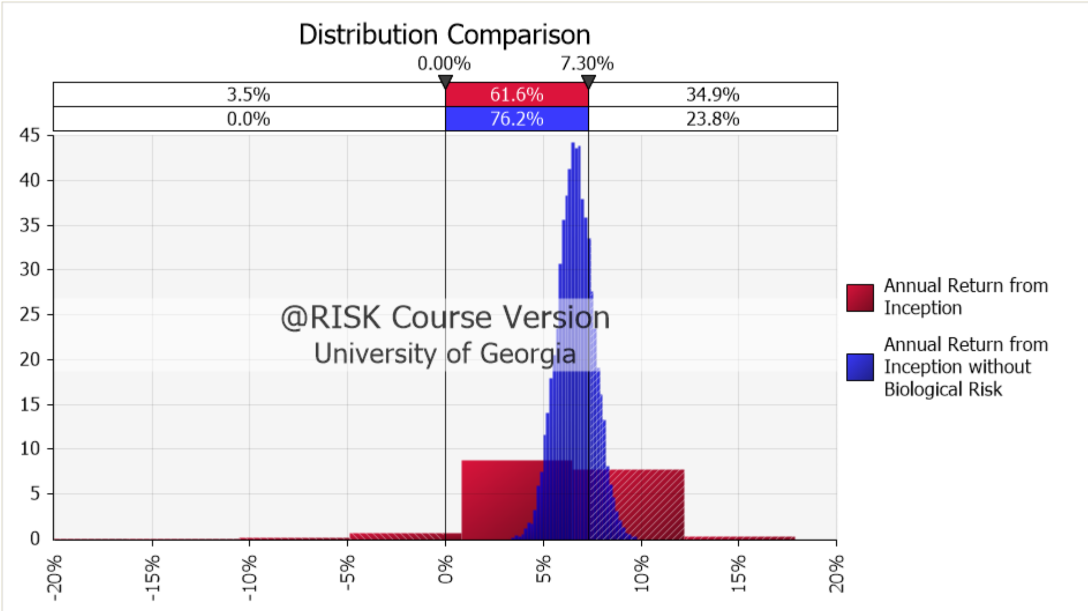

Figure 5.3. Probability density distribution comparison from Monte Carlo simulations with and without timber yield risk

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CHAPTER 6

Forest tactical planning at risk: Timber prices, biological growth, and hurricane uncertainties[5]

[5] Cabezas, C. To be submitted to a peer-reviewed journal

Abstract

Forest planning is typically categorized into strategic, tactical, and operational levels, based on the scope and time frame of decisions. Different optimization models are used at each level to maximize expected benefits over the planning horizon. However, various sources of uncertainty can impact the expected outcomes of optimized planning, challenging the decision-making process. This research integrates multiple risk factors by incorporating uncertainties related to timber prices, biological growth, and potential hurricane damage into a tactical plan. Specifically, it uses nonlinear programming algorithms and Monte Carlo simulations to study revenue uncertainty while accounting for hurricane damage and timber salvage. The results showed that revenue risk aversion restrictions significantly affected the optimized net present value, reducing it by 22.2%. The isolated effects of hurricanes had a minor impact on the maximized net present value. However, integrating revenue and hurricane risk demonstrates an additive effect that decreases the expected net present value by 29.7%. Timber wood flows exhibited a front-loaded distribution in the early years, affecting expected cash flows as risks were incorporated. These findings highlight the importance of integrating risks into investment decision-making.

1. Introduction

Forest resource management employs management science for decision-making, extending from strategic timber production to ecosystem protection (Kangas et al. 2015; Buongiorno and Gilless 2003). Various optimization tools are widely used to address complexity, such as solving scheduling problems and assigning limited resources among competing activities (Hillier and Liebermann 1980; Leuschner 1990; Bettinger et al. 2009).

Optimization techniques have been widely used in forestry in timber production planning. The hierarchy of these planning problems has been classified into strategic, tactical, and operational levels. This categorization varies in time horizons and how to balance economic, ecological, and operational factors (Bettinger et al. 2009). However, regardless of its hierarchy, timber planning faces financial challenges due to fluctuating prices, uncertainties in biological growth, and disturbances. Over the years, research has developed different approaches to tackle uncertainty in forestry. A wide range of methods and algorithms has been applied. However, these approaches mainly focus on individual risk sources (Cabezas 2025).

Historically, timber price risk modeling has often concentrated on stochastic processes rather than autoregressive ones, with geometric Brownian motion models (GBM) being the most prevalent (Yousefpour et al. 2012). Some examples of GBM approaches in risk price modeling are the studies by Clarke and Reed (1989) and Reed and Clarke (1990) in a stochastic dynamic programming framework. Other approaches, such as Brazee, Amacher, and Conway (1999) and Gong (1999), examined autoregressive processes. The first autoregressive framework studied price risk in stumpage markets using first-order autoregressive processes, analyzing random walks and mean reversion while assessing management strategies. The second analyzed optimal harvest decisions under price risk, modeling timber prices with a first-order autoregressive process influencing harvest adaptive decisions.

Biological growth uncertainty has received less attention than prices (Yousefpour et al. 2012). However, its contribution to expected return risk is significant (Cabezas 2025). Clarke and Reed (1989) and Clarke (1990) integrated biological growth risk into their stochastic programming algorithm using a Brownian motion approach for such risk. Reed and Haight (1996) modeled biological growth risk using a GBM variant incorporating age-dependent growth, capturing uncertainty in timber yield over time. Mei, Clutter, and Harris (2013) utilized triangular distributions to model the risk of biological growth while modeling timber prices as GBM and Ornstein-Uhlenbeck mean-reverting processes.

Additionally, natural disasters lead to an increase in uncertainty in the cash flows. Storms, wildfires, and diseases can lead to significant economic losses and disrupt market equilibrium. Extensive literature in risk analysis modeling approaches to disasters primarily focused on wildfires and less on storm hazards (Yousefpour et al. 2012). Storm phenomena are responsible for significant losses and damages in forest plantations. For instance, in 1954 and 2005, Hurricane Edna and Katrina caused losses between 15 and 40% of timber volume in the affected area (Merry, Bettinger, and Hepinstall 2009; Stanturf, Goodrick, and Outcalt 2007). Some storm studies, such as Haight, Smith, and Straka (1995), studied storm damages from Hurricane Hugo in South Carolina and considered the risk of damage and stocking reduction due to age-dependent tree mortality and economic impact. Knoke et al. (2021) used forest growth models and Monte Carlo simulations to predict age-dependent natural disasters in spruce forests, assessing economic impacts using conditional value at risk (CVaR). Yin and Newman (1999) used intervention analysis to determine the regional effect of Hurricane Hugo on stumpage prices. Likewise, Henderson et al. (2022) modeled the impact of hurricanes on the timber market, addressing timber salvaging activities and their effects on the market.

This research proposes examining a harvest scheduling problem under multiple sources of risk at the tactical level. Specifically, it integrates price uncertainty, timber growth risk, age-dependent hurricane damage, and timber salvage into the economic analysis.

Therefore, it defined a harvesting plan that used a synthetic portfolio of 26 loblolly pine properties in South Texas and South Louisiana, with a 16-year horizon, regions identified as hurricane-prone areas. We used this time horizon as it aligns with forest companies' tactical planning practices and the extended lifespan of timberland investment funds.

We developed multivariate autoregressive models to forecast timber prices, correlations, and volatilities. Then, we applied the seemingly unrelated regression (SUR) models from Cabezas (2025) to estimate timber product growth along with its correlations and uncertainties. Next, we used nonlinear programming to optimize the portfolio and maximize net present value (NPV). The optimization model balanced wood flow stability while incorporating timber price risk, biological growth volatility, and hurricane impacts. Finally, we assessed the contribution of different risk factors and analyzed how risk aversion affects timberland investment decisions.

2. Data

2.1. Prices

Since 1976, TimberMart-South (TMS) has collected data on stumpage prices to better represent timber market dynamics in the Southern United States (Norris Foundation 2022). This study utilized TMS's quarterly stumpage price series for pine in the South Texas and South

Louisiana markets across three key pine product categories: pulpwood (PULP), chip-and-saw (CNS), and sawtimber (SAW) from Q1 1992 to Q4 2023, Figure (6.1).

The TMS nominal price series from the Southern Texas (TX) and Southern Louisiana (LA) regions, also known as TX2 and LA2, showed significant fluctuations from 1992 to 2023. The general trend indicates a substantial increase in the mid-1990s, likely due to harvesting regulations regarding the spotted owl in the western United States, which strengthened the timber market in the Southeast United States. This upward trend was followed by periods of volatility and decline in the late 1990s in the CNS and SAW series, while the PULP series displayed a milder decline. Subsequently, prices fell sharply after 2008, increasing volatility, likely due to the financial crisis that affected the housing and construction markets and the timber markets. Prices stabilized after 2014, showing moderate fluctuations, before becoming volatile again during the COVID-19 pandemic, Figure (6.2).

Although CNS TX and CNS LA show similar price patterns, CNS TX exhibits a slightly steeper decline. PULP TX and PULP LA have consistently represented the lowest-priced category throughout the study period. While prices remain relatively stable, slight fluctuations and falls persist, with PULP TX showing slightly higher fluctuations than PULP LA.

2.2. Synthetic Loblolly Pine Portfolio

We developed a synthetic portfolio of loblolly pine plantations ranging from 8 to 24 years old. The portfolio was divided between TX and LA, with each region containing 13 stands. TX covered 2,226.6 acres, while LA included 2,139.6 acres. Thus, the total area amounted to 4,366.2 acres. The age distribution had a significant portion in the middle-aged range, indicating that most plantations will develop over the tactical planning horizon rather than consisting of mature plantations ready for harvest, Table (6.1).

The average stand size for TX and LA was 171.3 and 164.6 acres, respectively. The average age of properties in TX and LA was approximately 15 and 14 years, and the average Site Index was 75.1 feet for both regions. The loblolly pine plantation densities ranged from 355 to 888 trees per acre, representing plantations without thinning, Table (6.1). These tree densities indicated that all stands produce primarily PULP and CNS rather than SAW.

2.3. Hurricanes

The National Oceanic and Atmospheric Administration (NOAA) is a federal agency that studies climate, weather, oceans, and coasts nationwide (NOAA 2025b). Within NOAA's structure, we find the National Weather Service (NWS) and the National Hurricane Center (NHC). The NWS monitors severe weather and issues public warnings to protect lives and property. In contrast, the NHC observes weather phenomena and specializes in tropical hazards like hurricanes.

The NWS Lake Charles Weather Station (LCWS) is located at the Lake Charles Regional Airport in Louisiana and provides comprehensive local weather data and forecasts from the late 1800s (NOAA 2025b). Specifically, it monitored extreme weather conditions, such as hurricanes and tropical storms, to broadcast alerts, weather predictions, and essential public communications. The LCWS includes the counties of Jefferson, Orange, Jasper, Newton, Tyler, and Hardin in Southeast Texas. In Louisiana, this encompasses the parishes of Cameron, Vermilion, Iberia, St. Mary, Upper and Lower St. Martin, Calcasieu, Jefferson Davis, Acadia, Lafayette, Beauregard, Allen, Evangeline, St. Landry, Vernon, Rapides, and Avoyelles (Roth 1997). Our research hypothetically located the synthetic portfolio of loblolly pine properties in the northern region of the LCWS County Warning Area to simulate hurricane severity effects on our tactical plan, Figure (6.3).

The Saffir-Simpson Hurricane Wind Scale represents severity using a 1-to-5 rating system. This scale categorizes hurricanes based on their sustained wind speeds and potential damage (Bettinger, Merry, and Hepinstall 2009; NOAA 2025c). Category 1 represents minimal damage, while Category 5 indicates catastrophic damage with extreme winds and storm surge (NOAA 2025c).

We analyzed storms' historical frequency and severity using the Saffir-Simpson Hurricane Wind Scale. We used the LCWS data, which includes a wealth of historical records on hurricanes and tropical storms, enhancing our understanding of hurricane frequency in the area. Our hurricane data horizon spans from 1886 to 2024.

We best approximated the hurricane category by gathering information from NOAA agencies' reports (Roth 1997; NOAA 2025a; 2010) to reflect the severity of each phenomenon in our hypothetical study area. Initially, we identified inconsistencies in severity classification on the NOAA website when comparing general information with more detailed reports. These discrepancies arose because a hurricane's category can change as it moves from the ocean to land. Therefore, we estimated the severity of the cyclone affecting the study area using the most precise data available from NOAA's databases and detailed reports.

Over the past 138 years, 42 hurricanes have affected the LCWS County Warning Area, averaging about 0.30 hurricanes yearly, or roughly one every three to four years. Notably, there were four years in which two hurricanes struck in the same year, with three of these double events occurring in the last 53 years, Table (6.2). Considering that this double-impact phenomenon averages only 0.03 occurrences per year, such events are exceptionally rare.

3. Methods

3.1. Price Inflation, Seasonality, Stationarity, and Cointegration

We constructed our price models by adjusting the nominal price data to real prices using the Producer Price Index (Federal Reserve Bank of St. Louis 2024). Index that uses 1982 as the base year (1982=100).

Next, we conducted seasonality and stationarity tests on the selected timber product categories. For seasonality, we supported our analyses on the Webel-Ollech (WO) seasonality procedure, which integrates the Quade-Serfling (QS) test and the Kruskal–Wallis (KW) test on the residuals of an automatic non-seasonal ARIMA model (Webel and Ollech 2019; Ollech 2022). For stationarity, we used three different tests: the augmented Dickey-Fuller (ADF), Kwiatkowski-Phillips-Schmidt-Shin (KPSS), and Lee-Strazicich Unit Root (LS) tests (Dickey and Fuller 1979; Kwiatkowski et al. 1992; Lee and Strazicich 2003). The ADF test assesses the presence of a unit root, indicating non-stationarity, while the KPSS test directly assesses whether a time series is stationary. The LS test improves stationarity analysis by evaluating stationarity while considering structural breaks. The LS test extends traditional unit root tests like the ADF test by incorporating multiple structural breaks in the time series. This is crucial because conventional stationarity tests may incorrectly conclude non-stationarity when structural breaks exist. As a result, we analyzed our data series using conventional ADF and KPSS tests, along with the LS test, to more accurately define the stationarity properties of the series.

Additionally, we used the Johansen test (Johansen 1991) to conduct cointegration analyses on the price series for the TX and LA regions. We assessed series cointegration separately by region and in combination to look for long-run equilibrium.

3.2. Price Models

3.2.1. Vector Error Correction Model

We selected a vector error correction model (VECM) for price prediction. This multivariate model cointegrates levels, capturing the long-run equilibrium relationships and the short-run dynamics, including price correlations and market interdependencies between timber products. VECM enables straightforward estimation and analysis while ensuring stability and capturing directional relationships. By incorporating error correction terms, the model guides system coefficients toward a common long-term equilibrium, where variables trend upward or downward jointly (Tsay 2014; Pfaff 2008; Greene 2011).

Our time series model did not consider market cointegration beyond the TX2 and LA2 regions. We must acknowledge that regional cointegration has been reported in the timber market, including several southern areas of the United States. For instance, Yin, Newman, and Siry (2002) studied cointegration among the regions of the Southern United States. They found that there was no full integration among the regions. Still, some regions form a single market, such as Southern Arkansas (AR1), Southern Texas (TX2), and the Northern and Southern Louisiana (LA1 and LA2) regions. Nonetheless, we still believe that our autoregressive price model is valid for representing timber price evolution in the TX2 and LA2 regions within a tactical plan, despite considerations of partial market cointegration.

We initially selected the number of lags for VECM autoregression based on the Bayesian Information Criterion (BIC). Subsequently, we adjusted the number of lags as needed to ensure the model's performance and to provide a robust representation of temporal dependencies.

3.2.2. Price Model Performance

After developing our VECM, we conducted a residual analysis, combining autocorrelation, normality, and autoregressive conditional heteroskedasticity (ARCH effects) assessments.

We assessed autocorrelation through autocorrelation functions (ACF), partial autocorrelation functions (PACF), and cross-correlation functions (CCF). ACF measures the correlation between residuals across different lags, while PACF refines this by isolating direct correlations at specific lags, removing the influence of shorter lags. Finally, CCF examines how the residuals of two time series interact over time.

To assess ARCH effects and model inadequacies in the residuals, we utilized the Lagrange multiplier, rank-based, multivariate Portmanteau Q statistic, and robust test statistics. The Lagrange multiplier (LM) test employed the Q(m) statistic for the squared series to detect ARCH effects (Engle 1982), while the rank-based test applied a non-parametric approach to identify serial correlation in the residuals (Ling, Tsay, and Yang 2021). The multivariate Portmanteau Q statistic analyzed autocorrelation in squared residuals (Box and Pierce 1970), while the robust test evaluated the significance of the model's parameters, considering standard assumption violations such as the homoscedasticity and normality of errors (Huber 1981).

3.2.3. Multivariate Generalized Autoregressive Heteroskedasticity Model

Autoregressive conditional heteroskedasticity (ARCH) effects demonstrate changes in residual variance over time, emphasizing periods of clustered volatility. Standard modeling strategies do not account for these volatility patterns, as they assume constant variance (homoskedasticity) and do not capture volatility clustering. Bollerslev (1986) introduced generalized autoregressive conditional heteroskedasticity (GARCH) equations to depict conditional heteroskedasticity. GARCH improves variance modeling by integrating past squared observations and conditional variances, making variance forecasts linear functions of prior conditional variances (R. Engle 2002b; Bollerslev 1986; Hamilton 1994).

Financial volatilities tend to move together across assets, making multivariate models a better alternative to separate univariate analyses (Bauwens, Laurent, and Rombouts 2006). MGARCH models extend GARCH to capture these volatility dynamics across multiple time series, enabling a more comprehensive analysis of market behavior. This form of model helps improve portfolio risk modeling, interdependencies among financial assets, and co-movements in economic variables (Silvennoinen and Teräsvirta 2008).

We modeled the price volatility using dynamic conditional correlation multivariate generalized autoregressive conditional heteroskedasticity (DCC-MGARCH) models. These models estimate time-varying volatility and capture volatility spillovers and co-movements across multiple time series.

Thus, our approach effectively captured volatility patterns, spillovers, and co-movements across multiple time series, resulting in more accurate and robust risk forecasts. Additionally, we incorporated a t-distribution and accounted for structural breaks in the residual modeling. The t-distribution more accurately captures the heavy tails commonly seen in the residuals of financial price series, which arise due to extreme price movements and deviations from model expectations, leading to the specification of a DCC-MGARCH-t model. Accounting for structural breaks in the residuals helps prevent the misinterpretation of time series dynamics, as such breaks can obscure the true underlying behavior of the series. We supported this consideration by conducting an analysis based on the method developed by Bai and Perron (2003) for structural breaks. This method defines a linear regression model with coefficients that shift at unknown breakpoints. Once we identified structural breaks, we captured abrupt shifts in the data caused by economic shocks, incorporating dummy variables as external regressors in the mean equation for structural break representation. This meant we lastly adopted a DCC-MGARCH-t with structural breaks as the final structure for volatility simulation.

We evaluated several DCC-GJR-MGARCH-t models on our VECM residuals. Our final selection was guided by economic rationale, implications, and the BIC to balance goodness of fit and parsimony.

Finally, after modeling and forecasting the timber prices and volatilities over the planning horizon, we converted real prices to nominal prices using a 2.0% annual inflation rate while incorporating seasonality. After obtaining the quarterly nominal prices, we represent annual prices by averaging these quarterly prices to serve as input for our dynamic risk-constrained nonlinear model in the tactical plan.

3.3. Timber Product Modeling

We based our timber projection on the models fitted by Cabezas (2025) for forest state variables and timber products. These models were grounded in equations developed by Lundqvist (1957), Gallagher et al. (2019), and Harrison and Borders (1996). They were fitted using the seemingly unrelated regression (SUR) method, which employed maximum likelihood estimation. The Cabezas (2025) refitted models effectively represented forest growth patterns and depicted heteroskedasticity and conditional correlations with the 23- and 41-parameter equation systems.

Therefore, we simulate our 26 stands in TX and LA over 16 years. First, we simulated dominant height, mortality, and basal area. Later, using the results from these equations, we forecasted the timber products based on the timber product equation system, which provides us with the timber product weights, their dynamic correlation, and variance-covariance matrices for each period.

This procedure assumes that the Cabezas (2025) SUR equation systems are suitable not only for loblolly pine stand simulation in Georgia but also for plantations in TX and LA. We acknowledge that a more precise estimate of timber products for the study area can be achieved using locally fitted models. However, we believe that the results obtained from these models are reasonable enough for risk modeling purposes.

We utilized only the variance and correlation information from timber products derived from the SUR model. Nonetheless, we also recognized that the HD, mortality, and BA models introduce a significant source of uncertainty in forecasting product green weight, and these risks were not considered in this study.

3.4. Hurricane Probability Estimation

The Poisson distribution is a discrete probability distribution. It represents the probability of observing a specific number of independent events within a fixed time or space interval, assuming they occur at a constant average rate (Siegel 2016). The Poisson distribution is commonly used to approximate a binomial distribution when the number of trials is large and the probability of success in each trial is small (Ross 2014).

The probability mass function of a Poisson-distributed random variable X with mean λ can be described by equation (1) as follows:

(1)

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

where $P(X = r)$ denotes the probability of exact r occurrences. The term λ is the expected number of occurrences (mean rate of events) and $r!$ is the factorial of r.

Thus, we evaluated hurricane occurrence probabilities by fitting a homogeneous Poisson model. We verified that the mean and variance of the counts were approximately equal, consistent with Poisson behavior and distinct from the negative binomial distribution, where these parameters differ (Massey 1951; Stephens 1986). We also fitted a generalized additive model, with linear and exponential functions, to the count data over time to assess whether the rate parameter λ varies.

Our analysis confirmed that the Poisson distribution provided a reasonable fit to the hurricane counts, with no significant trends in frequency over time. This supports the assumption of a homogeneous Poisson process. Consequently, we used this model to estimate annual hurricane occurrence probabilities and severity levels using NOAA hurricane records.

3.5. Hurricane Damage and Timber Salvage Rates

With hurricane severity indicated by the hurricane category on the Saffir-Simpson scale, we assessed the level of damage and the potential timber that could be salvaged once a hurricane strikes. To achieve this, we utilize age-dependent theoretical standing timber damage and recovery rates. This approach is similar to that used by Haight, Smith, and Straka (1995), where damage levels are more significant in older than younger ages. While Haight et al. defined a fixed timber recovery rate, we adopted an age-dependent salvage rate, assuming that younger plantations are more resilient to damage than older ones (Sharma et al. 2021; Merry, Bettinger, and Hepinstall 2009; Haight, Smith, and Straka 1995).

Therefore, we defined two age ranges to gauge damage and severity: plantations younger than 15 years and those older than that. These age categories and hurricane severity gave us the damage and timber salvage rates to include in our optimization model, Tables (6.3) and (6.4).

We need to underline that the damage accounted for in these rates reflects only the average direct impact of hurricanes. It does not consider any spatial damage differentiation related to hurricane trajectory or proximity to the eyewall. This direct impact excludes losses occurring in subsequent years due to internal damage, which may result in delayed tree mortality or trigger disease outbreaks commonly observed in the years following a hurricane strike (Merry, Bettinger, and Hepinstall 2009).

This study focuses solely on hurricanes as natural disturbances to be modeled. It does not consider damage from tropical storms or other minor weather events. The analysis assumes that hurricanes are independent events and does not consider climate change effects affecting hurricane frequency and severity. Furthermore, the probability analysis conducted in this study is limited to the impact of a single hurricane, as the likelihood of multiple hurricanes occurring within the same year is sufficiently low to be considered negligible.

3.6. Dynamic Risk-Constrained Nonlinear Programming Model

We estimated the annual revenue for each stand projection by multiplying the green weight of each product by the corresponding timber nominal price, as projected by the VECM and SUR models over the years. Stand revenues were discounted at a nominal 5% annual hurdle rate to calculate the expected NPVs. We did not account for administrative costs, additional revenue streams, or their associated uncertainties. All financial analyses were conducted on a pre-tax basis.

To address stand-level variability, we estimated revenue risk associated with NPV by integrating uncertainties in timber prices and product growth. The DCC-MGARCH-t and SUR models were used to generate time-varying variance-covariance and correlation matrices required for this analysis. Risk calculations incorporated numerous matrices compiled over the 16 years, reflecting correlated timber prices and product growth across TX and LA. Accordingly, we approximated the NPV variance using 10,000 Monte Carlo simulations in @RISK software, assuming that timber prices and biological growth follow a normal distribution. This assumption led us to hypothesize that the NPV outcome was also normally distributed.

Chance-constrained programming is a stochastic optimization technique used for decision-making under uncertainty. Most applications of chance-constrained programming employ fixed probability thresholds based on the context. These criteria ensure that the constraints are satisfied with a specified probability, which helps manage uncertainty and supports risk-aware decision-making.

For this tactical plan, we maximized the NPV from harvesting various loblolly pine stands, ensuring an annual wood flow between 20,000 and 50,000 tons. Like chance-constrained programming, we also restricted uncertainty in our optimization model by incorporating a dynamic risk constraint using the coefficient of variation from NPV. This allows for dynamic variance calculation within the optimization instead of setting it in advance.

We embedded our optimization algorithm within a nonlinear programming technique. This approach enables us to include variances, standard deviations, and coefficients of variation in the constraints, facilitating nonlinear calculations. Furthermore, we optimized the decision variable regarding stand selection as mixed integer solutions, preventing the algorithm from harvesting small portions of the stands and creating an atomized set of stands within the properties. Thus, the nonlinear optimization algorithm was implemented using LINGO software (LINDO Systems 2025).

We structured four optimization algorithms that we identify as base case, revenue-risk, hurricane-risk, and hurricane-revenue-risk models.

The base case corresponds to a nonlinear programming model that restricts the optimization only to our operational green weight constraint.

The revenue-risk model corresponds to the algorithm that incorporates revenue risk into the constraints, establishing a risk aversion restriction. This implies that revenue risk is limited to not exceeding a 10% optimized NPV coefficient of variation along the operational wood flow constraint described above.

The hurricane-risk model was an algorithm that considers the operational wood flow constraint and evaluates age-dependent hurricane damage and timber recovery probabilities in the objective function, as described in Tables (6.3) and (6.4). For this model, we assumed that the TX and LA markets would not change their current trend or undergo structural changes after a hurricane strikes. We also hypothesized that the salvaged timber would be sold entirely at 50% of the expected PULP prices because of breakage and other damage caused by a storm, and that there would not be total destruction of the stands.

The hurricane-revenue-risk model integrated all the features described for the revenue-risk and hurricane-risk models, producing a comprehensive representation of various risk sources that can impact the timberland portfolio. Adding revenue risk and hurricane damage helped us understand the combined risk effects on NPV, represented by timber price risk, biological growth uncertainties, and hurricane damage. Moreover, this risk integration enabled us to better understand the risks in timberland investment, providing a more realistic comparison to individual risk analysis.

Therefore, we optimized the NPV for 416 decision variables while computing their variances in a nonlinear programming model. We implemented an operational constraint on the annual wood flow. We developed various model modifications with differing risk integration levels, which consider revenue risk, hurricane damage, and their combinations. These dynamic risk-constrained nonlinear programming models can be represented as follows:

3.6.1. Decision Variable

Let:

$$x_{it} = \begin{cases} 1, & if\ stand\ i\ is\ harvested\ \ in\ year\ t, \\ 0, & if\ stand\ i\ is\ not\ \ havested\ in\ year\ t \end{cases}$$

where x_{it} is a binary decision variable indicating where stand i is harvested in year t.

3.6.2. Objective Functions

Objective function, equation (2) that represents the total NPV (ϕ) maximization over the planning horizon:

$$Max\ \phi = \sum_{t=1}^{T} \sum_{i=1}^{n} \sum_{j=1}^{m} x_{it} A_{it} P_{ijt} Q_{ijt} \tag{2}$$

where the term A_{it} is the area harvested from stand i in year t. The component P_{ijt} denotes price per unit of harvested product from product j from stand i in year t. The element Q_{ijt} represents the weight of each product j from stand i in year t. The expression T is the horizon length in years. The terms m and n depict the total number of stands and products.

The objective function that accounts for hurricane damages can be expressed as equation (3):

$$Max\ \phi = \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{j=1}^{m} x_{it}A_{it}K_{ijt}S_{ijt}\Psi_t + \sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{j=1}^{m} x_{it}A_{it}P_{ijt}Q_{ijt}\,(1-\Psi_t) \tag{3}$$

where the element Ψ_t corresponds to the annual hurricane probability. The term K_{ijt} is the price for salvaged timber after a hurricane strikes. The component S_{ijt} is the salvaged weight to be harvested from stand i in year t adjusted by plantation age.

3.6.3. Operational Constraints

The annual weight to harvest constraints are represented as follows in equation (4) and (5):

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{it}A_{it}Q_{ijt} \geq W_{Min}, \qquad \forall\, t \tag{4}$$

And

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{it}A_{it}Q_{ijt} \leq W_{Max}, \qquad \forall\, t \tag{5}$$

where W_{Min} and W_{Min} represent the lower and upper bounds for harvested weight at year t.

The harvest exclusivity constraint to ensure that no stand is harvested more than once over the planning horizon is denoted as equation (6):

$$\sum_{j=1}^{m} x_{it} \leq 1, \qquad \forall\, i \tag{6}$$

The risk constraint that limited the NPV variability must not exceed a given threshold α is expressed as follows:

3.6.4. Revenue Risk Constraint

To control the economic risk, we restrict the dynamic coefficient of variation at the NPV level, which is represented as follows in equation (7):

$$\frac{\sigma_\phi}{\mu_\phi} \leq \alpha \tag{7}$$

where σ_{ϕ} is the total NPV standard deviation. The component μ_{ϕ} denotes the expected total NPV and α represents the NPV's coefficient of variation threshold.

This risk assessment can be instrumental in timber harvest scheduling because it dynamically factors harvested quantity and market price risks. By incorporating risk constraints, forest managers analyze stable harvest volumes over multiple periods while accounting for stochastic changes in yields and prices. For instance, this approach can help forest managers calculate deterministic equivalents to assess volume targets with higher confidence despite variability (Birge and Louveaux, 2011). Alternatively, they can approximate the risk tolerance based on the expected shortfall.

Supported by the NPV normality assumption, we calculated the expected NPV shortfall represented by the Value at Risk (VaR) and the Conditional Value at Risk (CVaR). VaR estimates the maximum potential loss at a given confidence level, setting a threshold beyond which extreme losses occur (Jorion 2007). Conversely, CVaR represents the average severity of worst-case outcomes beyond VaR's threshold (Serraino and Uryasev, 2013). These risk aversion tools provide a straightforward approximation of downside risk.

The equation (8) represents the VaR formula:

$$VaR_{\propto} = \mu_{\phi} + Z_{\propto}\,\sigma_{\phi} \qquad (8)$$

The CVaR can be described as follows in equation (9):

(9)

$$CVaR_{\propto} = \mu_{\phi} + \frac{\varphi(Z_{\propto})}{1-\propto}\sigma_{\phi}$$

where the term $Z_{\propto}$ represents the inverse cumulative function (quantile function) of the standard normal distribution at $1-\propto$, which determines the critical value for the confidence level. The element $\varphi(Z_{\propto})$ represents the probability density function of the standard normal distribution $Z_{\propto}$. The expression $1-\propto$ denotes the probability of exceeding the value at risk, representing the likelihood of extreme losses in the tail of the distribution.

Finally, we calculated the loss-based VaR and CVaR for nonlinear programming NPV outcomes at a 95% confidence level. These values are calculated by subtracting the VaR and CVaR from the mean NPV.

4. Results

4.1. Price Inflation, Seasonality, Stationarity, and Cointegration

The inflation adjustment provided a more accurate basis for understanding and modeling timber price trends. The seasonality test revealed seasonal trends in the PULP and SAW price series. To address seasonal effects, we applied seasonal differencing to these time series (Tsay 2005).

The ADF and KPSS stationarity tests revealed conflicting results concerning the non-stationarity properties of all timber price series. The LS identifies structural breaks but supports the hypothesis of non-stationarity in the series, prompting us to develop non-stationarity models for the price series. The Johansen test revealed at least one cointegrating relationship among all timber prices. This finding prompted us to adjust a VECM that combines all price series.

4.2. Price Modeling

We tested several VECMs with lags ranging from 1 to 4. The selected VECM configuration utilized four lags, denoted as VECM(4). It demonstrated an ability to capture the most historical trends for the real timber prices of TX and LA while considering the correlation among the series. The model effectively addressed the economic disturbances, including the COVID-19 pandemic from 2020 to 2023, Figure (6.4). Moreover, it exhibited more rational forecasting as we extended the 16-year horizon and identified the correlation between the two regions' series. The overall trend for all prices in real terms indicated a decreasing pattern, Figure (6.5). This decreasing trend tends to disappear once a forecasted 2% inflation rate is incorporated to estimate nominal prices for each. region.

The VECM(4) residuals showed no evidence of autocorrelation or cross-correlation based on the ACF, PACF, and CCF tests. However, the residuals exhibited ARCH effects and model anomalies, as evidenced by the Lagrange Multiplier, rank-based, Portmanteau, and robust tests, indicating strong evidence of clustered conditional heteroskedasticity. This prompted us to develop several DCC-MGARCH-t models to better accommodate this conditional variability.

We tested different configurations, ranging from DCC(1,1)-MGARCH(1,1)-t to DCC(2,2)-MGARCH(2,5)-t models with structural breaks, which were compared based on BIC and economic logic. The best MGARCH model that captured clustered heteroskedasticity was the DCC(2,2)-MGARCH(2,2)-t, which is economically sound and whose residuals did not exhibit significant ARCH effects or model anomalies according to the specialized tests applied, Table (6.5). Additionally, the ACF, PACF, and CCF did not show evidence of autocorrelation and cross-correlation from the DCC(2,2)-MGARCH(2,2)-t. Therefore, we used this model to forecast the variance-covariance and correlation matrices for our VECM(4) over 16 years, which is used as input for our dynamic risk-constrained nonlinear programming model variance calculations.

4.3. Timber Product Modeling

The SUR model from Cabezas (2025) enabled us to simulate the 26 stands in TX and LA over the 16-year horizon. The state variables HD, mortality, and BA were modeled, and their results were utilized to determine timber products' green weight. The SUR models also produced 836 variance-covariance and correlation matrices that depict the volatility dynamics of each product in the stands over time.

Figure (6.6) illustrates the total commercial weight forecast for stands 104 and 107 in the TX region and 203 and 205 in LA, over the planning horizon. It also shows the selected stands' initially reduced upward green weight trend, which strengthens after period two. The tree density function influences the early trend. During the first two years, the mortality rate exceeds the product growth rate but recovers thereafter, resulting in steadier growth.

4.4. Hurricane Probability Calculation

The hurricane probability modeled from a homogeneous Poisson distribution indicates that the annual chance of one hurricane impacting properties is 22.4%. Given that a hurricane occurs, the probability of it being a Category 1 or 2 is 66.7%, a Category 3 is 21.4%, and a Category 4 or 5 is 11.9%. These category probabilities represent the conditional likelihood of each hurricane intensity relative to the total occurrence of hurricanes.

These hurricane landfall and severity probabilities, encompassing age-dependent damage and timber salvage, Tables (6.3) and (6.4), give us the information needed to be incorporated into the objective function of our dynamic risk-constrained nonlinear programming model.

4.5. Dynamic Risk-Constrained Nonlinear Modeling

Our nonlinear model utilized 416 decision variables, representing 26 stands over 16 years. It used the nominal timber price and biological growth forecasts generated from the VECM and SUR models, respectively. Additionally, the nonlinear programming model incorporated NPV variances calculated from the variance-covariance and correlation matrices of timber prices and biological growth simulated through the DCC(2,2)-MGARCH(2,2)-t, SUR models, and @RISK computations.

The base model with only an operational constraint demonstrated an optimized NPV of $4,587,276 with a standard deviation of $611,138. This model harvested all 4,366 acres, and the annual timber cut rate varied between 145.8 and 401.7 acres. The yearly wood flow ranged from 24,207 to 48,114 tons, Figures (6.7) and (6.8), totaling 591,343 tons over the entire horizon. Over time, the distribution of wood flow remained relatively uniform due to harvesting rather than accumulating in an early-edge manner.

The present value (PV) distribution displayed two peaks at years 1 and 5, exceeding $500,000, and two low points below $140,000 at years 15 and 16. In the remaining years, the cash flow stabilizes between $200,000 and $340,000, Figure (6.9).

The model that constrains revenue risk below the 10% coefficient of variation exhibits an optimal NPV of $3,572,440 and a standard deviation of $357,052. In the planning horizon, this model harvests only 3,952 acres and 498,731 tons.

The revenue-risk model prioritizes higher harvest levels in Periods 2 to 5, peaking at Period 5. It also displays a subsequent decline from periods 6 to 10 as it allocates for harvesting areas in early periods. The model then adjusts with a secondary increase in periods 11 through 13 before stabilizing toward the end of the period, Figures (6.10) and (6.11).

The PV distribution over the planning horizon also showed a front-loaded revenue strategy, Figure (6.12). The PV concentrates from periods 1 to 5, with two significant peaks in periods 2 and 3, where the highest present value exceeded $450,000. The PV profile also indicated early revenue generation due to the risk aversion constraint. After period 5, there is a steep decline in PV, leveling off at a declining rate from periods 7 through 15, with one peak in period 16 of $279,062.

The nonlinear programming model, which did not include the risk aversion constraint and incorporates hurricane damage probabilities or a hurricane-risk model, yields an optimal NPV of $4,499,868. The hurricane-risk model exhibits a standard deviation of $621,822 while harvesting 4,366 acres. The harvested acreage, wood stream, and PV distribution exhibited the same shape as the base case, mainly because the hurricane damage probabilities did not substantially affect the optimization decisions, Figures (6.13), (6.14), and (6.15).

However, when accounting for hurricane damage and limiting revenue aversion to a standard deviation of 10% of the NPV. The optimized NPV significantly declined to $3,224,100 and an NPV standard deviation of $324,307 as the model cut only 3,830 acres.

The harvested area and wood flow also showed intensive harvesting in the early years. However, they presented a more stable pattern compared to the revenue-risk model. Particularly, there was a noticeable spike in acreage and wood harvested in period 10, reaching 423 acres and 47,820 green tons, respectively, in Figures (6.16) and (6.17).

Figure (6.18) also illustrates early-stage revenue maximization, with the upper and lower bounds at $119,626 and $336,035, respectively. After period 5, the present values remarkably decline, stabilizing at lower levels between periods 7 and 9. However, a distinct spike in period 10 indicates a temporary increase in revenue, likely due to a strategic harvesting decision of harvesting stands 104 and 105 in Texas, which account for $254,751. In the later periods from 11 to 16, the present values remain relatively stable but gradually increase toward period 16.

4.6. VaR and CVaR

The base model reached a 95% confidence VaR of $3,582,057, indicating that under the normal distribution assumption, there is only a 5% probability that the NPV will fall below this threshold. This model also yielded a positive Conditional Value at Risk (CVaR) of $3,326,676, indicating that in the worst 5% of cases, the average NPV remains positive. The revenue-risk and hurricane-revenue-risk models yielded VaRs of $2,985,141 and $2,710,649, and CVaRs of $2,835,943 and $2,575,134, respectively, in Table (6.6).

The loss-based VaR and CVaR for the base model at a 95% confidence level are $1,005,233 and $1,260,603, respectively. Loss-based VaR indicates how much lower the NPV could be than the mean NPV in the worst 5% cases, representing the potential loss relative to the mean in these scenarios. Similarly, the loss-based CVAR represents the expected shortfall beyond the VaR, which resulted in a loss of $1.26 million relative to the mean NPV. This implies that the model's worst-case scenarios are less favorable than the mean NPV, but the loss is not catastrophic, as shown in Table (6.7). For revenue-risk and hurricane-revenue risk models, the loss-based VaR values are $587,299 and $533,438, respectively, and the loss-based CVaR values are $736,497 and $668,953.

5. Discussion

The VECM(4) for price forecasting effectively fitted the timber real price series from 1992 to 2023 for the TX and LA regions. The model aligned well with market shifts, such as environmental restrictions on western harvesting that affected the southern timber markets, the Subprime crisis, and the COVID-19 pandemic. Since the TX and LA markets are not independent, this model also addresses the cointegration properties, capturing their long-run interaction.

However, the VECM(4) residuals exhibited ARCH effects, leading to the development of multivariate heteroskedasticity models to account for the clustering of variability. We tested several configurations of DCC-MGARCH-t models with structural breaks and selected a DCC(2,2)-MGARCH(2,2)-t as the final configuration. This selected model demonstrated the ability to account for volatility clustering by mitigating ARCH effects in the residuals. Therefore, VECM and DCC(2,2)-MGARCH(2,2)-t models were used for timber price mean forecast, variance, and correlation modeling over a 16-year planning horizon.

The VECM(4) predicted a downward trend for timber product prices in TX and LA in real terms. Specifically, the model forecasted similar negative slopes for the TX and LA regions due to a long price declining trend from the mid-1990s and cointegration properties. The model also differentiated an accelerated rate of decline for CNS and SAW prices and a mild decrease in PULP prices. This feature can indicate a differentiated market behavior due to domestic influence on the lumber markets and a more significant international influence in the pulp and paper markets. The PULP real price model forecast indicates a price decrease in real terms from \$2.83 to \$1.93 per ton over the planning horizon, reflecting approximately a 31.8% decline for the TX and LA regions. CNS predictions declined from \$5.99 to \$3.37 per ton and from \$7.33 to \$4.31 per ton, representing decreases of 43.7% and 41.2% for TX and LA, respectively. The SAW price estimation also significantly reduced from \$12.28 to \$7.00 per ton and from \$11.24 to \$6.50 per ton, or 43.0% and 42.2% for TX and LA, respectively.

The DCC(2,2)-MGARCH(2,2)-t with structural breaks model predicted 32 variance-covariance and correlation matrices that capture the dynamic variances and correlations among the six timber prices. Afterward, we transformed the real expected prices and variances into nominal non-seasonally adjusted values to represent the observed market prices. Thus, the VECM and our DCC(2,2)-MGARCH(2,2)-t outputs in nominal terms generated the expected price and risk information for revenue calculations within the optimization model. Additionally, the SUR model produced estimations of the green weight of timber by simulating 26 loblolly pine stands in TX and LA, resulting in yields of PULP, CNS, and SAW.

Later, we calculated the annual hurricane probability using a homogeneous Poisson distribution for the Lake Charles area. Our calculations indicated a 22.4% probability that this storm could hit our properties annually. Assuming a hurricane occurs, there is a 66.7% chance it will be Category 1 or 2, a 21.4% probability of reaching Category 3, and an 11.9% likelihood of escalating to Category 4 or 5. These figures represent the conditional probabilities of hurricane intensities that trigger various damage levels and age-dependent timber recovery rates. The assumption is based on the premise that loblolly pine plantations that are 15 years old or younger are less affected by cyclones and have higher timber salvage rates than those older than 15 years.

The Lake Charles area, where our properties are hypothetically located, can be considered a high hurricane occurrence area. In the last 138 years, 42 hurricanes have made landfall, meaning an occurrence rate of 0.30 hurricanes yearly. This rate approximates the average for the Gulf Coast states of Texas and Louisiana, the second and third most susceptible to hurricanes after Florida (NOAA 2010).

We determined the expected cash flow and variances using the timber price and growth models. We conducted 10,000 Monte Carlo simulations to estimate the revenue variances, assuming that timber price and biological growth were normally distributed. Next, the revenues were converted into present values using an annual discount rate of 5% for the optimization models. Finally, the present value estimates were adjusted to incorporate the hurricane occurrences and their damage effects into the nonlinear problem's objective function to complete our analysis.

Therefore, we ran four different nonlinear programming models that maximized the NPV in a tactical harvesting plan. These models incorporated 416 binary decision variables in a mixed integer framework for optimization. The decision variables represent the harvesting possibilities for 26 stands over the 16-year planning horizon. First, the base model was only restricted to an operational constraint of 20,000 to 50,000 green tons of timber harvested annually. Second, a revenue-risk constrained model accounting for the timber weight operational constraint mentioned above, along with an NPV coefficient of variation of less than 10%. Third, we developed a hurricane damage risk model constrained by operational annual green weight, similar to the base model, while incorporating hurricane damage probabilities and salvage activities into the objective function. Fourth, we created a hurricane-revenue-risk model that considered all the restrictions of the second model but also included hurricane damage probabilities.

The base model without risk constraints yielded \$4.6 million in optimal NPV. The NPV standard deviation for this model was around \$611 thousand, indicating a coefficient of variation of 13.3%. This model harvested the entire planning area, accounting for 4,366 acres. The model totaled 591 thousand green tons over 16 years and a relatively homogeneous wood flow that ranges from 24,207 to 48,114 tons a year. Conversely, the PV distribution through the planning horizon showed two spikes in years 1 and 5 over $500 thousand. The remaining years stabilized between $200 and $340 thousand annually.

The revenue-risk model that accounted for keeping an NPV coefficient of variation below 10% reached a maximized NPV of $3.6 million with a standard deviation of $357 thousand. This variability represents a coefficient of variation of 9.9%, which meets the revenue risk constraint. Conversely to the base model, the revenue-risk model harvested only 3,952 acres and 499 thousand green tons of wood. The acreage distribution and wood flow concentrate in the early years, resulting in front-loaded graphic representations. The acreage and harvested wood distributions reveal initial peaks, followed by a decline during periods 6 to 10, ending with a mild oscillation. The PV distribution also displays this front-loaded pattern, but it is even more pronounced. This array illustrates how the present values concentrate, with the highest peaks occurring within the first few years, particularly around periods 2 and 3, surpassing $450 thousand. Like the acreage and wood stream, the PV distribution declines sharply and stabilizes at a lower level, but it exhibits a late peak at year 16 that accounts for around $279 thousand. This pattern signifies that most financial or profits are made in the initial years, followed by a low and consistent stream in later years to compensate for risk aversion.

The hurricane-risk model that excluded any risk aversion constraint and incorporated hurricane damage probabilities yields an optimal NPV of $4.5 million and a standard deviation of $621 thousand while harvesting 4,366 acres. The structure of harvested acreage, wood flow, and PV distribution stayed consistent with the base case, as the hurricane damage probabilities had little influence on the optimization outcomes.

The hurricane-revenue-risk optimization model conjugated the adverse effects of hurricanes and timber salvage activities in its objective function alongside the NPV variation in the constraints. The model's NPV achieved an optimal $3.2 million and an NPV standard deviation of $324 thousand, representing a coefficient of variation of 10.0%. This indicates a 29.7% and 9.7% decline in NPV compared to the base and revenue-risk models.

This hurricane-revenue-risk model harvested partly the planning area, cutting 3,830 acres. Though milder than the revenue risk-constrained model, this integrated model also displayed an early concentrated distribution of the harvested acreage and wood stream. It presented concentrated spikes between periods 1 and 4, along with a significant spike in year 10, attributed to intensive harvesting from two large stands in TX, covering 423 acres of harvesting and 48 thousand green tons of wood. The present value stream enhanced the front-loaded shape of its figure's acreage and wood flow, indicating notable early income generation. In the first four years, the present values averaged $318 thousand per year, a trend that declined over time, with a final spike in period 16.

Consequently, the revenue-risk model reduces the maximized NPV by 22.2% compared to the base model, decreasing from $4.6 million to $3.6 million. This $1.0 million difference in NPV means around $0.3 million for each percentage point change in the NPV coefficient of variation, reflecting the risk aversion price for each point reduction. Furthermore, when comparing the base case to the hurricane-revenue-risk model, this difference amplifies by 7.5%, reaching 29.7% as the NPV decreases to $3.2 million, showing a standard deviation of $324 thousand, representing 10% in the NPV coefficient of variation. The difference in NPV between the base case and the revenue-hurricane-risk model corresponds to a $1.4 million difference, or approximately $0.4 million for each one percent in coefficient of variation.

Interestingly, the catastrophic hurricane event regarding NPV does not significantly reduce the optimized value. The decrease from $4.6 million to $4.5 million represents a change in value of approximately 1.9%, which is negligible in economic terms. This finding seems counterintuitive given the massive natural disaster affecting commercial plantations in a high-hurricane-influence area, but we must recall that the probability of a giant tropical cyclone even is still low even in this particular region. However, this finding also suggests a multiplicative effect when risk sources are combined, as observed when comparing the hurricane-revenue-risk and the hurricane-risk models against the base case. Still, we recognize that our study fails to consider the indirect damage, which can be even more severe than the direct effects.

These previous results also indicate that multiple sources of risk analysis are significant, and standalone risk analysis can miss hidden effects. Therefore, integrating risk into economic analyses is crucial in decision-making, as forest plantation risks are numerous throughout the decades-long rotation cycle. We also observe that revenue risk, which includes timber price and growth uncertainties, surpasses the effect of hurricane risk.

Furthermore, from Cabezas (2025), we recognized that the risk of timber growth exceeds the price risk, at least for the series studied, due to the uncertainty embedded in several state variables that mold the timber product, generating an accumulation of uncertainty throughout the modeling chain.

Based on the maximized NPV and its standard deviation, the base model had a VaR of \$3.6 million at a 95% confidence level, meaning there is only a 5% probability that NPV will fall below this threshold under normal distribution assumptions. The revenue-risk and hurricane-revenue-risk models yielded VaRs of \$3.0 million and \$2.7 million, respectively. In other words, these results indicate that, even in the most adverse scenarios incorporating price fluctuations and timber growth uncertainty, there is a 95% probability that NPV will remain positive.

In absolute terms, the downsizing threshold for the base model, optimized for NPV relative to its VaR, is \$1.0 million, indicating a 5% likelihood of exceeding this loss as the worst outcome. The revenue-risk model showed an absolute value of \$0.6 million, and the hurricane-revenue-risk model displayed \$0.5 million. In percentage terms, the base model's VaR represents a 21.7% NPV fall in worst-case scenarios, while revenue-risk and hurricane-revenue risk models showed 16.7% and 15.6%.

The base model achieved a CVaR of \$3.3 million, representing the expected average NPV once the VaR threshold is breached and reflecting potential losses in extreme scenarios. The revenue-risk and hurricane-revenue-risk models obtained CVaRs of \$2.8 million and \$2.6 million, respectively. Similar to the VaR findings, the CVaR demonstrated a less severe decline in extreme scenarios for the optimizations that incorporated non-risk considerations. In absolute terms, the differences amount to \$1.3 million, \$0.7 million, and \$0.7 million for the base case, revenue-risk, and hurricane-revenue-risk optimization models, respectively. In percentage terms, the difference between optimized NPV and its CVaR was 28.3%, 22.2%, and 18.7% for each model, respectively.

The VaR and CVaR findings highlight a clear trade-off between maximizing profits and minimizing financial risk. While achieving the highest NPV of \$4.6 million, the base model is also more exposed to significant potential losses under adverse conditions. Although they provide more financial stability, the revenue-risk and hurricane-revenue-risk models reduce overall profitability by 22.2% and 29.7% in NPV.

The risk-constrained nonlinear programming models provided a simple approach to ponder risk. Their outcomes revealed critical strategic implications for the tactical plan studied. For instance, to reduce the NPV coefficient of variation by 3.4 percentage points, the model eliminates 22.2% of profit and reduces our CVaR by 28.8% as we focused solely on revenue risk.

There is no straightforward answer when analyzing risk aversion, as it depends on a firm's risk aversion and context, which determine whether to pursue greater returns or adopt more resilient strategies. For instance, if a forest manager's primary goal is to reduce financial downturns while significantly increasing profits, the hurricane-revenue-risk model can guide the analysis in estimating a more predictable financial outcome, even under extreme conditions. Conversely, if the forest manager aims to achieve maximum long-term returns, potential losses become a secondary concern. The base model remains the best choice, as it achieves the highest NPV, albeit with greater NPV volatility.

This risk-constrained nonlinear programming approach provides an integrative perspective on risk within an operational context. It incorporates revenue risk by merging uncertainties regarding timber prices and growth into an optimization model designed to maximize NPV across 26 loblolly pine stands. Subsequently, the model integrates revenue risk and hurricane damage probabilities to better understand their combined effects. Although the study approach accounts for multiple risks in the tactical plan, it remains applicable. Forest companies routinely employ optimization models, develop biometric timber growth projections, and estimate future prices for budgeting and strategic planning while acknowledging the model uncertainties. Our approach maintains simplicity since forest companies understand these foundational elements and do not require sophisticated software or algorithms that use massive computing resources. Thus, they can adopt risk-constrained strategies that enable decision-makers to make more informed choices when considering multiple risk factors. This approach helps shape their analysis toward potential business opportunities.

Our approach addressed only timber prices and growth risks while considering the direct damage effects of hurricanes. We acknowledge that other sources of risk can also be integrated, such as wildfires and diseases. Furthermore, the hurricane damage consideration can be enhanced by simulating the long-lasting effects of mortality beyond the hurricane landfall and the interaction with insect outbreaks years later. We can simulate the risk of tornadoes that can precede hurricane passage or storm surge on the coastal forest and incorporate it into our optimization model. Our approach has provided a modest contribution to multi-risk frameworks. Nevertheless, future research can delve deeper into this integration, allowing for a more complete risk analysis that represents the potential additive effects when risks interact.

6. Conclusions

We integrated the revenue stream and its uncertainty into our nonlinear programming algorithm. The result for the base model, which constrained green weight, yielded a maximized NPV of \$4.6 million and an NPV coefficient of variation of 13.3%.

The revenue-risk model, restricted to a coefficient of variation of less than 10%, exhibited a 22.2% decrease in NPV, resulting in an optimized NPV of \$3.6 million. The hurricane-risk model showed a negligible change in the maximized NPV, reaching \$4.5 million, or a 1.9% decrease compared to the base model.

The fully integrated risk or hurricane-revenue-risk model revealed that the combined risk approach had a multiplicative effect compared to the risk standalone analyses. The optimal NPV dropped about 29.7%, displaying an NPV of \$3.2.

The base and hurricane-risk models harvested all 4,366 acres and exhibited a more homogeneous distribution than the risk-constrained ones. The revenue-risk and hurricane-revenue-risk models also showed a lower harvested area that declined by about 9.5% and 12.3%, respectively, and displayed more front-loaded distributions of the harvesting area and PV.

The expected shortfall analysis revealed that considering revenue risk and hurricane damage effectively lowered the thresholds for catastrophic economic scenarios. In percentage terms of the optimal NPV, it first decreased by 22.2% and 29.7% compared to the base model. Likewise, the VaR and CVaR also dropped in percentage terms. The difference between the base model's CVaR-to-mean ratio of 28.3% declined to 22.2% and 18.7% compared to the revenue-risk and hurricane-revenue-risk models, respectively. In absolute terms, the CVaRs fell from $1.3 million to $0.8 million and $0.6 million, respectively.

The revenue-risk model offsets the direct impact of a hurricane strike on NPV loss. This finding may seem counterintuitive because these tropical cyclones are massive and inflict significant damage. However, the likelihood of destruction remains minimal, even in regions most vulnerable to hurricanes.

This study offers a multifaceted risk assessment relevant to forest planning and evaluating multiple sources of risk. We hope it encourages new research questions and supports the integration of diverse risks into timberland investment analyses.

Table 6.1. Overview of the synthetic loblolly pine portfolio in Texas and Louisiana

TX	Area (acres)	AGE (years)	TPA (trees/acre)	DQ (inches)	HD (feet)	SI (feet)	BA (ft^2/acre)	PULP	CNS (tons/acre)	SAW
Min.	87.5	8	435	4.9	44.2	69.7	95.0	46.8	0.0	0.0
Mean	**171.3**	**15**	**612**	**6.9**	**58.5**	**75.1**	**157.5**	**76.8**	**31.7**	**2.4**
Max.	258.2	24	889	9.3	77.6	82.4	230.4	140.7	112.5	22.3
SD	60.7	6	143	1.4	11.4	4.3	48.7	25.7	41.9	6.2

LA	Area (acres)	AGE (years)	TPA (trees/acre)	DQ (inches)	HD (feet)	SI (feet)	BA (ft^2/acre)	PULP	CNS (tons/acre)	SAW
Min.	88.2	8	355	4.9	42.3	61.0	89.5	32.7	0.0	0.0
Mean	**164.6**	**14**	**652**	**6.8**	**57.4**	**75.1**	**155.4**	**75.2**	**27.1**	**1.8**
Max.	278.8	24	860	9.7	74.5	83.6	212.2	126.8	113.7	12.1
SD	62.8	5	194	1.6	10.5	6.4	38.8	33.2	37.1	4.4

Table 6.2. Hurricanes that impacted the LCWS County Warning Area from 1886-2024

Hurricane Name	Year	Category	Hurricane Name	Year	Category
Number 9	1886	3	Hilda	1964	2
Number 3	1888	1	Betsy	1965	3
Number 1	1891	1	Fern	1971	1
Number 8	1893	2	Edith	1971	2
Number 2	1897	1	Carmen	1974	3
Galveston	1900	4	Babe	1977	1
Number 2	1915	1	Alicia	1983	3
Number 1	1918	3	Danny	1985	1
Number 2	1920	2	Juan	1985	1
Number 3	1923	1	Bonnie	1986	1
Number 3	1926	3	Chantal	1989	1
Number 2	1932	4	Jerry	1989	1
Number 5	1934	1	Andrew	1992	3
Number 2	1938	1	Lili	2002	1
Number 2	1940	1	Rita	2005	3
Number 1	1942	1	Humberto	2007	1
Number 1	1943	1	Gustav	2008	2
Number 4	1947	3	Ike	2008	2
Audrey	1957	4	Harvey	2017	4
Debra	1959	1	Laura	2020	4
Cindy	1963	1	Delta	2020	2

* Category based on the Saffir-Simpson Hurricane Wind Scale.

Table 6.3. Standing timber damage rate by hurricane severity and plantation age

Hurricane Severity	Loblolly Pine Age Class	
	15 years old and younger	older than 15 years old
Category 1-2	7%	11%
Category 3	25%	38%
Category 4-5	40%	60%

Table 6.4. Timber salvage rate by hurricane severity and plantation age

Hurricane Severity	Loblolly Pine Age Class 15 years old and younger	 older than 15 years old
Category 1-2	30%	25%
Category 3	25%	20%
Category 4-5	20%	15%

Table 6.5. ARCH effects and model anomalies tests for the DCC(2,2)-MGARCH(2,2)-t

Test	p-value
Lagrange Multiplier	0.809
Rank-based	0.222
Portmanteau	0.061
Robust Test (5%)	0.163

Table 6.6. VaR and CVaR values

Model	Base	Revenue-Risk*	Hurricane-Revenue-Risk*
Value at Risk	3,582,046	2,985,141	2,710,649
CVaR	3,326,676	2,835,943	2,575,134

* Constrained to a NPV coefficient of variation lower than 10%

Table 6.7. Loss-based VaR and CVaR values

Model	Base	Revenue-Risk*	Hurricane-Revenue-Risk*
Loss-based VaR	1,005,233	587,299	533,438
Loss-based CVaR	1,260,603	736,497	668,953

* Constrained to a NPV coefficient of variation lower than 10%

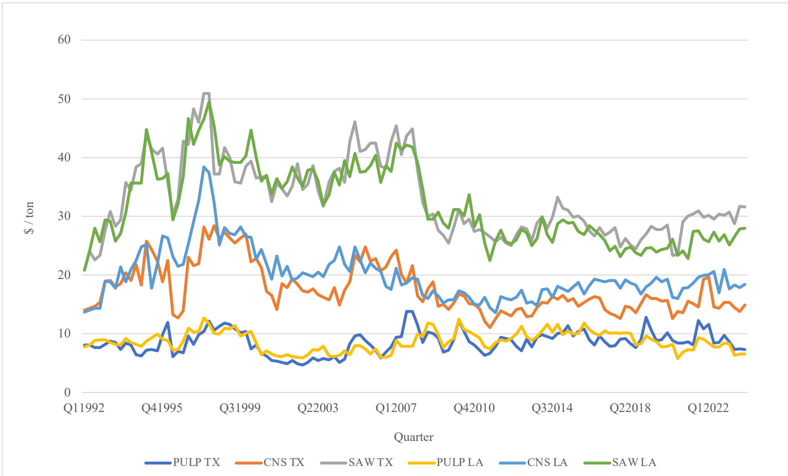


Figure 6.1. Timber nominal price time series for Texas and Louisiana

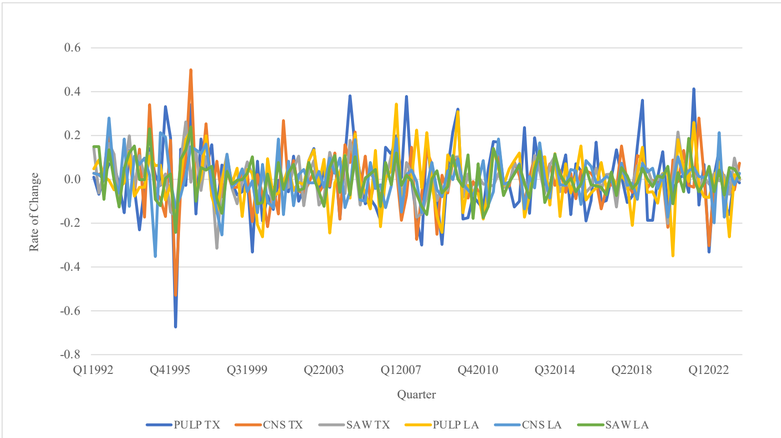
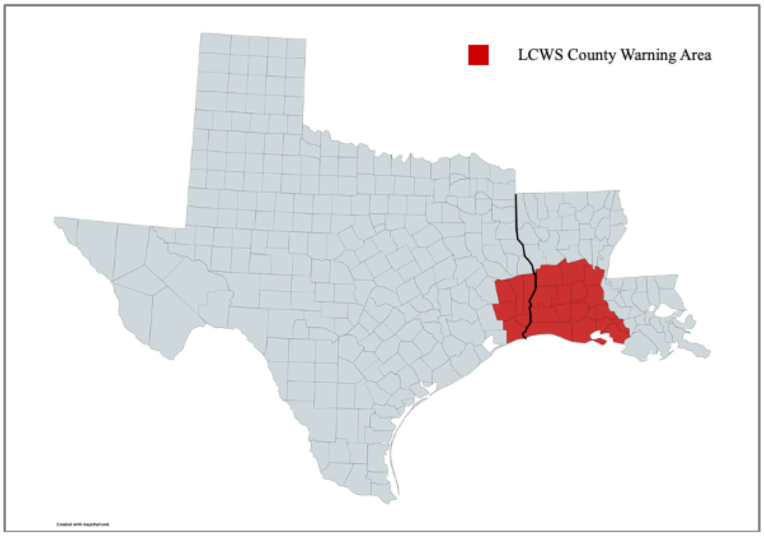


Figure 6.2. Timber nominal price rates of change for Texas and Louisiana



Source: www.mapchart.net.

Figure 6.3. Lake Charles Weather Station's County Warning Area in Texas and Louisiana

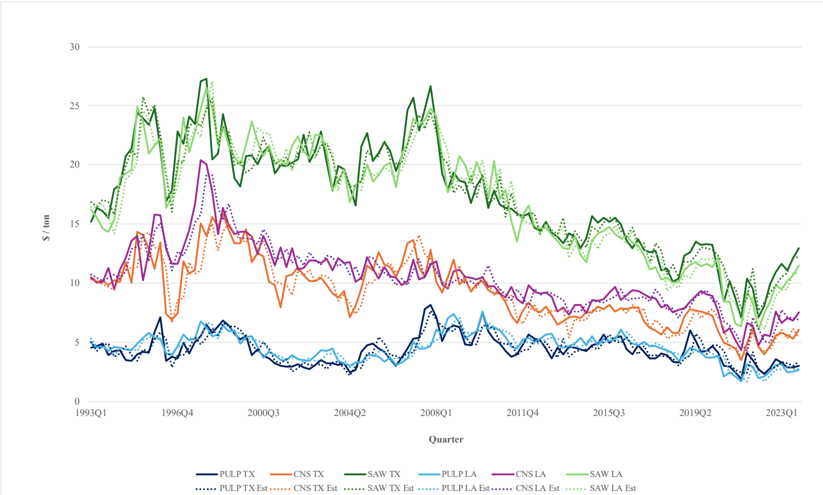


Figure 6.4. Timber price series and VECM(4) estimated prices in real terms for Texas and Louisiana

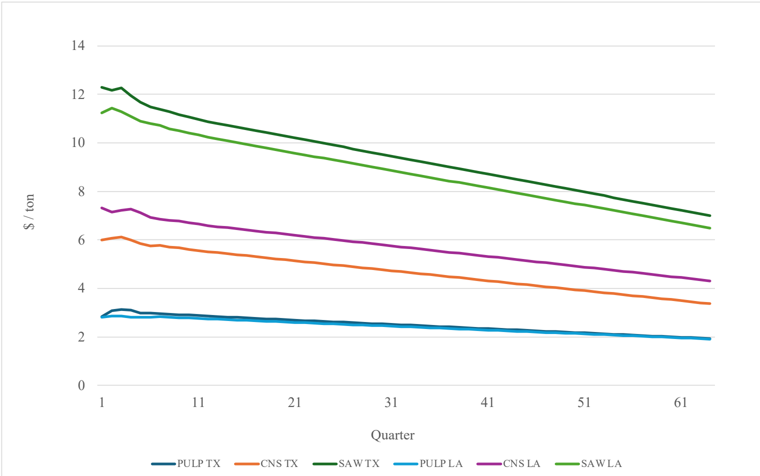


Figure 6.5. VECM(4) forecasted prices in real terms for Texas and Louisiana over a 16-year horizon

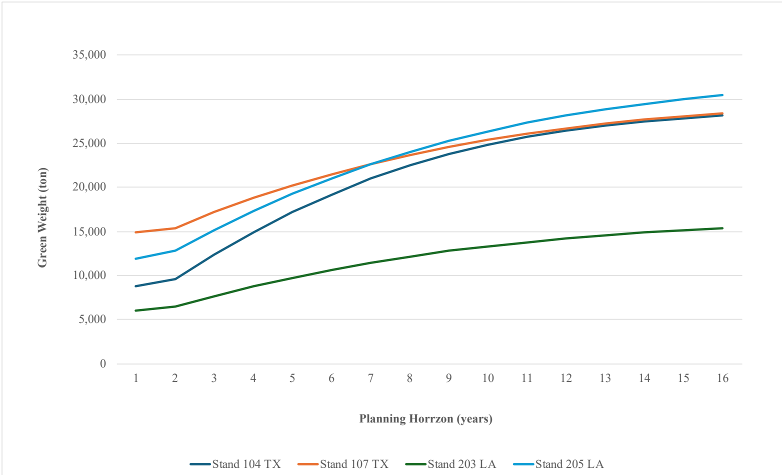


Figure 6.6. PULP weight forecasted in green tones for stands 1 and 2 in Texas and stands 19 and 21 in Louisiana

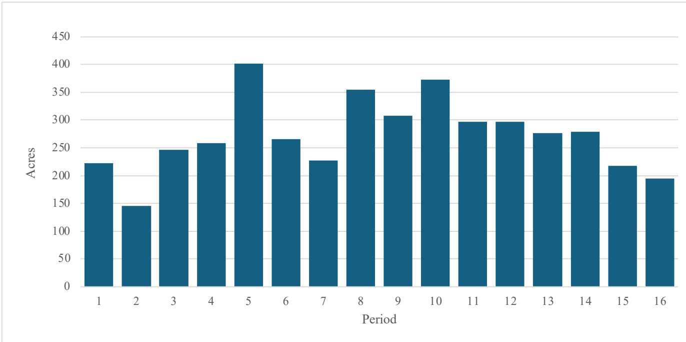


Figure 6.7. Harvested area distribution for the base model

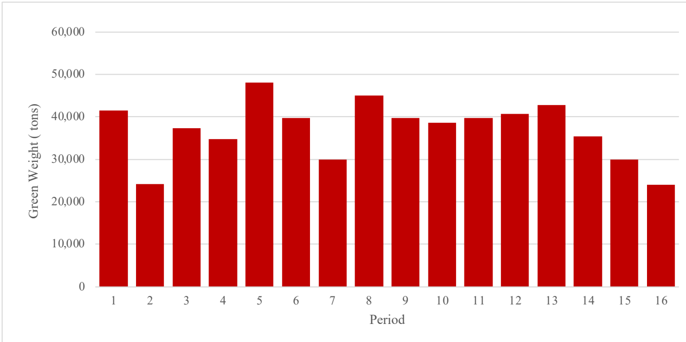


Figure 6.8. Wood flow for the base model

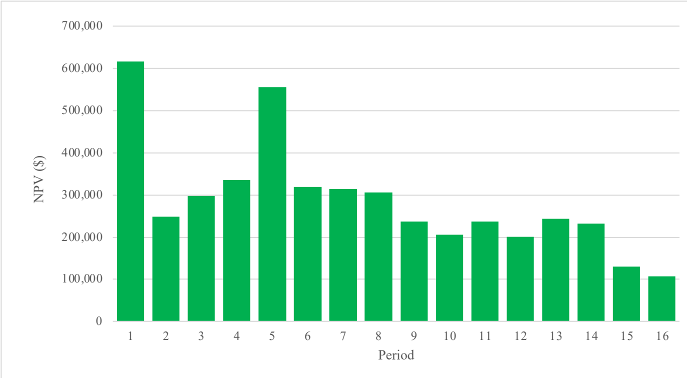


Figure 6.9. PV distribution for the base model

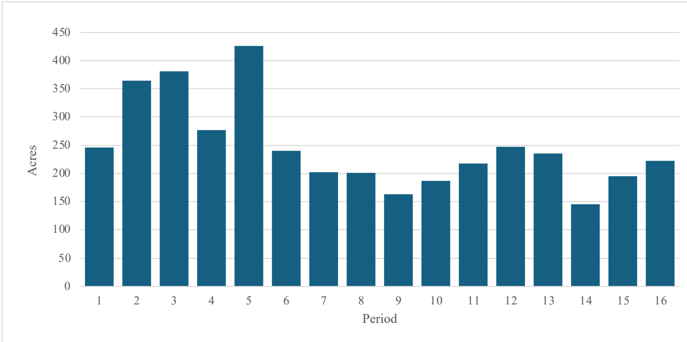


Figure 6.10. Harvested area distribution for the revenue-risk model

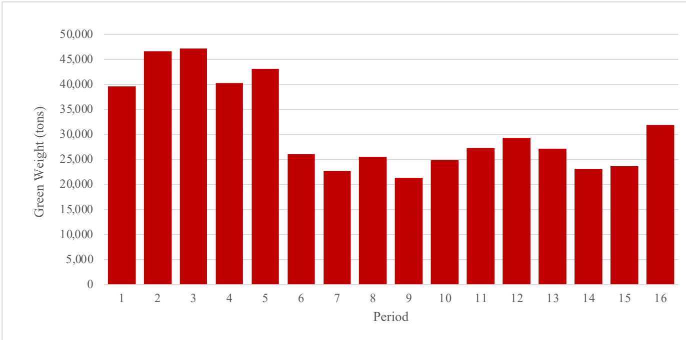


Figure 6.11. Wood flow distribution for the revenue-risk model

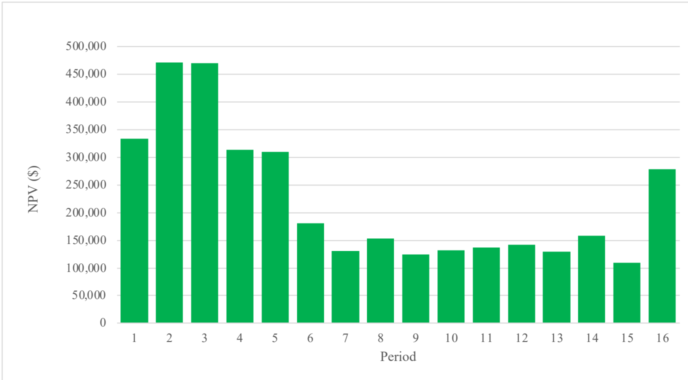


Figure 6.12. PV distribution for the revenue-risk model

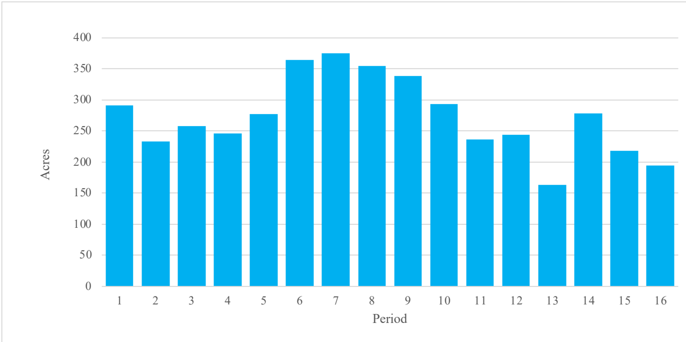


Figure 6.13. Harvested area distribution for the hurricane-risk model

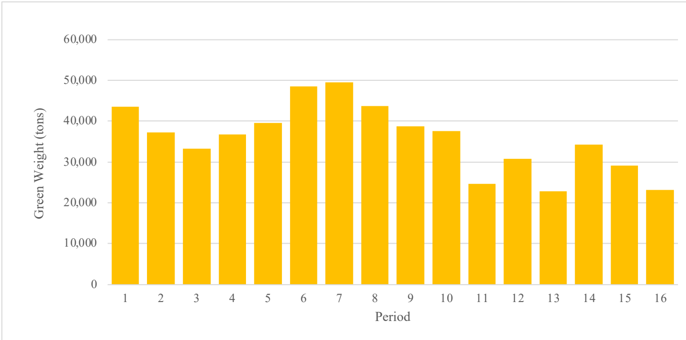


Figure 6.14. Wood flow for the hurricane-risk model

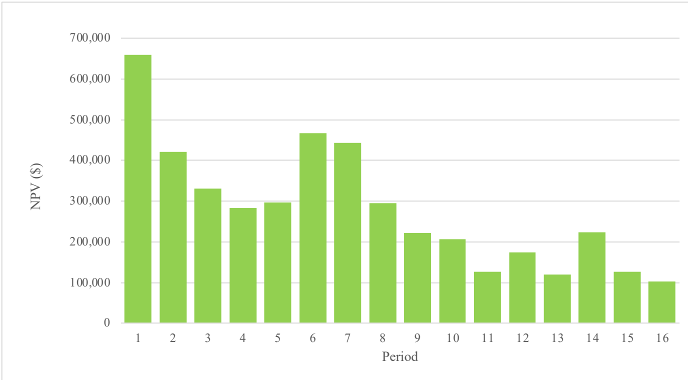


Figure 6.15. PV distribution for the hurricane-risk model

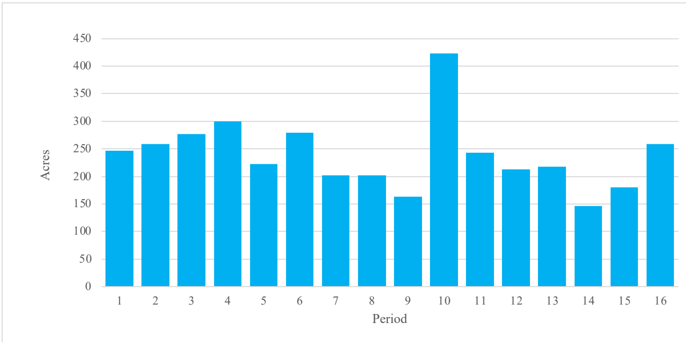


Figure 6.16. Harvested area distribution for the hurricane-revenue-risk model

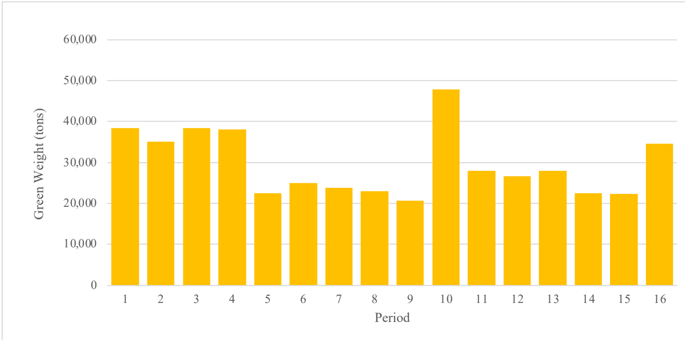


Figure 6.17. Wood flow for the hurricane-revenue-risk model

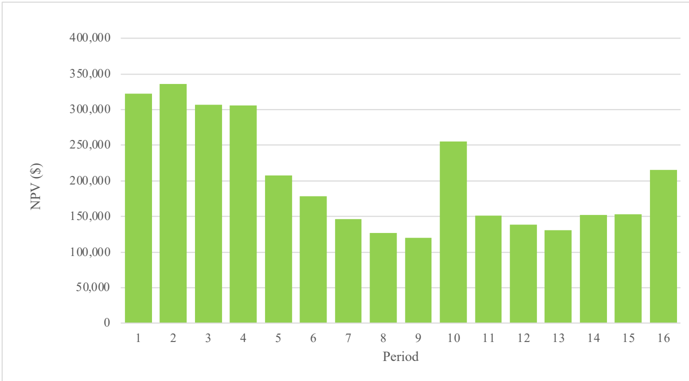


Figure 6.18. PV Distribution for the hurricane-revenue-risk model

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CHAPTER 7

CONCLUSIONS

Timberland is an appealing alternative asset class that derives its return fundamentals from biological growth, price change, and land appreciation. The long-term nature of these assets makes them vulnerable to various sources of risk that impact each return driver. Over the past few decades, timberland risk modeling has progressed from simple approaches to sophisticated methods to better represent the uncertainty associated with timberland investments. However, this effort has primarily focused on price risk, with less focus on the uncertainties surrounding biological growth uncertainty and storm damage risk. Moreover, these initiatives have primarily targeted individual risks, leaving many interactions among multiple risks unexamined. This dissertation explored various approaches related to timberland risk modeling, structured as follows: (i) a literature review of timberland risk modeling, (ii) the development of multivariate autoregressive timber and bare land price and risk models, (iii) the fitting of SUR models to represent biological growth and its uncertainty, (iv) the timberland return driver analysis that incorporates price and timber growth risks within a probability framework, and (v) the development of a nonlinear programming model that considers timber price and biological growth risks alongside hurricane damage.

Chapter 2 analyzed various univariate and multivariate autoregressive models. The results indicate that the MGAM(1) outperformed the competing models, making it more suitable for timber price forecasting. The ARIMAX(5,1,3;2) model outperformed its competing models for bare land prices. The MGAM residual showed ARCH effects. This last finding prompted us to select the DCC(3,1)-MGARCH(1,1)-t with structural breaks model to represent price dynamic volatilities and correlations. This model not only captured the volatility dynamic and external market shocks but also their asymmetric effects.

Chapter 3 refitted the Lundqvist (1957), Gallagher et al. (2019), and Harrison and Borders (1996) models for dominant height, mortality, basal area, and timber green weight, respectively, using SUR models. Volatility and correlation models were also incorporated into the SUR equation system to capture biological growth risk over time. The SUR systems performed well for the forest state variables but did not excel in estimating timber product green weight modeling. However, they effectively captured the volatility and correlation trends for all stand variables that affect the risk prediction.

Chapter 4 used the models described in the above paragraphs to forecast prices and biological growth along with their volatilities. Monte Carlo simulations helped in the probabilistic analysis to observe the effect on the expected returns. The main finding reveals that biological growth risk significantly surpassed price risk and widened the standard deviation of the expected returns 14-fold. This result demonstrates that biological growth must be considered in any timberland risk assessment.

Chapter 5 integrates expected values and volatilities from timber prices and biological growth while considering hurricane damage into a tactical harvesting plan that maximizes the NPV. Various combinations of risks were proposed to observe their effects on the NPV when combined. The revenue-risk model, which incorporates price and biological growth risks, significantly impacted the NPV as its coefficient of variation was limited to below 10%. The revenue-risk model caused the NPV to drop by 22.2%, while the standalone hurricane-risk model affected the NPV by only 1.9%. The additive effect of combining revenue and hurricane risks, the hurricane-revenue-risk model, increased the NPV drop to 29.7%. The base and hurricane-risk models harvested all available areas with more uniform green weight distributions. In contrast, the revenue-risk and hurricane-revenue-risk models resulted in 9.5% and 12.3% in the harvested area, respectively, exhibiting more front-loaded harvest area and NPV distributions. These findings indicate that multiple risk analyses are crucial when designing risk management strategies since there are effects that can be hidden when the risks are analyzed independently.

Finally, these multiple risk analyses provide an integrative view of timberland investment risk. However, we recognize there are more interactions to explore. For instance, our hurricane damage assumption only considered the direct impact of the hurricane, which makes this approach incomplete. It is known that hurricanes increase tornado probabilities as the storm passes or that the hurricane's impact lasts several years beyond landfall and correlates to disease risk. We hope this work inspires new questions that lead to better-informed decisions through multiple risk analyses.

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