ON THE STUDY OF TAIL DEPENDENT TIME SERIES FROM THE PERSPECTIVES OF SPECTRAL DENSITY AND TAIL INDEX

by

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(Under the Direction of Ting Zhang)

ABSTRACT

The dependence latent in time series data especially within the tail part versus on average has drawn increasing attention with the aim of investigating concealed data patterns and underlying structures . The inference on such tail dependence could serve as an insightful tool for uncovering the behaviors of extremal value events, in order to provide informed explanations for observed phenomena. This dissertation's primary objective is to present an analysis of time series sequence from two distinct viewpoints, frequency-domain, and time-domain, both concerning the tail dependence at a high quantile level. To facilitate the analyses, we employ the Tail Adversarial Stability (TAS) framework, which offers a clean and practical framework for studying tail dependent time series.

From the frequency-domain point of view, a novel tool of tail spectral density analysis is considered in the double asymptotic setting, which provides a foundational step toward spectral analysis of tail dependent time series. The asymptotic normality results in an effective tool for constructing confidence interval to gauge the uncertainty of tail spectral density estimator. On the other hand, from the time-domain perspective, we adopt a parametric measurement of tail heaviness, known as the tail index, to study the tail behavior of time series sequence. In particular, we consider estimating the tail index under the TAS framework. Extensive simulation studies are conducted to assess the empirical performance from both perspectives. Besides, some data applications are presented to further illustrate the practical implementation.

INDEX WORDS: Tail Dependent Time Series, Tail Spectral Density Estimation, TAS Framework, Tail Index, Hill's Estimator

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DEDICATION

I dedicate this work to my mom, my dad, and my daughter Lucy.

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Tail Dependent Time Series Analysis

1.1 An introduction to Tail Dependence

Tail dependence, also known as extremal dependence, refers to the dependence between extreme events in a statistical distribution that rarely happened but have significant impacts and consequences. It has been widely used in extreme value analysis and has been vastly developed with implementation in climate science, economics, finance and insurance, and geography fields. It describes the situation where the occurrence of an extreme event increases the probability of another extreme event occurring. Tail dependence analysis usually concerns the dependence structure only in the tail region of a distribution. With the limited amount of data in the tail, relevant statistical inference for tail quantities can be more complicated and challenging.

Extensive studies have been carried out on this topic for past decades, Sibuya et al. (1960) extended the notion of extreme statistics into bivariate distributions and considered the joint distributions of maxima in observed data vectors and its asymptotic properties. In his pioneering work, he was concerned with the sampling and limit distribution of the largest and smallest values and other related statistics in an observed sample and extended these statistics into bivariate context. Then he considered the various cases of bivariate jointly limit distributions and leverage the marginal distributions. For a bivariate sample $(X_i, Y_i), i = 1, \ldots, n$, with marginal distribution $F_X(x)$ and $F_Y(y)$, respectively, the tail dependence function introduces the tail dependence in the upper-right region of a bivariate distribution function that exceeding a limit value, which is later interpreted as

$$\lambda_{up} = \lim_{u \uparrow 1} \mathrm{pr}\left\{X > F_X^{-1}(u) | Y > F_Y^{-1}(u)\right\}.$$

The tail dependence in lower-left region could be defined similarly. This work has shown its impact extensively over a wide range of directions in related research, especially in but not limited to multi-variable extreme value and distribution theories.

Furthermore, De Haan and Resnick (1977) established the necessary and sufficient conditions for the weak convergence of maxima of k-dimensional multivariate distributions with some results well-formulated in the context of multivariates extremal processes; Joe (1993) compared some important properties of bivariate distributions for extreme value inference, including the situations where bivariate families could be extended to multivariate families; Ledford and Tawn (1997) proposed a model with multivariate extreme value threshold for joint tail estimation which could handle the problems of near independent random variables; Coles et al. (1999) comprised an overview of the principal issues through a unified approach that encompasses the situations when multivariate extreme values are dependent or independent, and the development of a novel diagnostic measure for extremal dependence; Embrechts et al. (2002) developed a copula representation of static(non-time-dependent) random vector dependence for risk management; in an extended model , Draisma et al. (2004) discussed the asymptotic properties of two estimators for the parameter that deals with the probability of an extreme event if the component-wise maxima of the observations are asymptotically independent in the setting of bivariate extreme value theory; Poon et al. (2004) presented a general framework for identifying and modeling the joint-tail distribution based on multivariate extreme value theories and provided implications of financial concepts within this framework; to measure nonlinear dependence , Z. Zhang (2008) defined quotient correlation which could be adjusted to generate two new concepts—the tail quotient correlation and the tail independence test statistics, and thus opened up a new field of study; Balla et al. (2014) derived extremal dependence-based systemic risk indicators, motivated by the result that stock returns exhibit strong loss dependence even in their limiting joint extremes, and showed that the proposed systemic risk indicators reflect downturns in the US financial industry very well; Hoga (2018) derived a structural break test for extremal dependence in β-mixing, possibly high-dimensional random vectors with either asymptotically dependent or asymptotically independent components. Meanwhile, some other statistical tools such as the copula methods also have been developed for tail dependence analysis. For example, Joe et al. (2010) expressed recursively the tail dependence function of a vine copula built from a set of bivariate copulas by establishing the interplay of tail dependence and conditional tail dependence functions; followed by Nikoloulopoulos et al. (2012) , which later investigated the asymmetric tail dependence and applied the vibe copula method to financial return data for justification; Durante et al. (2015) employed a copula-based tail coefficient estimated by a non-parametric method to measure the tail dependence behaviors of clustering time series; Kollo et al. (2017) studied the tail behavior of skew t-copula in the bivariate case by comparing the calculated tail dependence coefficients with different estimators for variant skewing parameters, to name a few.

1.2 Tail Dependence in Time Series

The above appreciated works mostly have taken the approaches from bivariate or finite-dimensional multivariate distributions, while in the context of time series data, the assumption of random variables being independent and identically distributed (iid) typically collapses and the processes are inherently dependent on some latent structure, which inspired numerous works to investigate despite the challenges. Along this direction, the first step would be to incorporate the existing results in bivariate or multivariate distribution settings into the time series settings, especially regarding the tail dependence achievements. For this, Z. Zhang (2006) considered a lag-k tail dependence index that summarized the degree of tail dependence at different lags for a time series. The order of lag was determined by the gamma test, and proven to reveal the statistical evidence to study the stock market S&P500, which is a prominent data set for extremal value study, and the lag-k tail dependence has thus become a convenient and valuable tool to study tail dependence in time series. It is defined as follows, for X_i, $i = 1, \ldots, n$ observed from a stationary time series with distribution function $F(x) = \mathrm{pr}(X_i \leq x), x \in \mathbb{R}$, let $\mathcal{U}_F = \lim_{u \uparrow 1} F^{-1}(u)$, the lag-$k$ tail dependence

$$\rho_k = \lim_{x \uparrow \mathcal{U}_F} \mathrm{pr}(X_{k+1} > x \mid X_1 > x).$$

On the other hand, Linton and Whang (2007) proposed a new diagnostic tool called the quantilogram which captures tail dependence by focusing on extremal events that exceed a certain quantile level; R. A. Davis and Mikosch (2009) also defined an analog of the autocorrelation function of extreme values in the sequence, the extremogram, and proposed its estimator as well as associated asymptotic properties under certain conditions; J. B. Hill (2009) established invariance principles for a large class of dependent, heterogeneous arrays which could degenerate tail arrays including sample means and covariances of tail events and exceedances that are popularly handled in the extreme value theory literature ; later J. B. Hill (2010) further developed new extremal dependence measures that's applicable for a wide range of process class with long or short memory, and studied famous Hill's estimator's consistency under near epoch dependence; Han et al. (2016) proposed a cross-quantilogram method to measure the quantile dependence between two time series, which steped into the field of multi-sample analysis for tail dependence in time series. This method provided a test statistic for the existence of predictability of one time series on the other; there are other methods proposed along this path such as Hoga (2018) providing a structural break test for tail dependence in multiple dimensional series under the condition of $\beta-$mixing, which will be introduced in Chapter 2.

T. Zhang (2022) developed an asymptotic theory for sample tail autocorrelations of time series data that can exhibit serial dependence in both tail and non-tail regions without imposing any restrictions on the dependence structure in non-tail regions. In contrast to conventional autocorrelations, the tail autocorrelations considered in the above works have a more intricate asymptotic behavior because they require a double asymptotic approach to capture the tail phenomena. Specifically, unlike conventional autocorrelations that have a universal convergence rate, sample tail autocorrelations do not share the same convergence rate, and their asymptotic behavior can change from one phase to another after the lag index surpasses the point at which tail dependence vanishes. This can pose a challenge in studying their infinite sum, a quantity that appears in the central limit theorem of high quantile regression estimators (T. Zhang, 2021), or more generally their Fourier transforms that leads to the tail spectral density which will be introduced in details in chapter 3.

1.3 Examples

The active research in the time series tail dependence fields as briefly mentioned above has been actively improving the understanding and discoveriess we can obtain from data sets. As more complicated and large data sets have become increasingly accessible, the choice of the methods for analysis of these data sets needs to be more careful and specific. The analysis tools usually come with attached conditions and assumptions, and their applicable scenarios could be limited to answering certain questions, ignoring which could contribute to the failure of capturing the correct data features. We shall here present two simple examples to distinguish tail dependence analysis from the conventional method by employing a familiar measure in time series analysis, the autocorrelation plot, to illustrate the difference. The first set of plots are with Bank of America stock price return upper tail. We download the data set with time period 1973/10/24 - 2023/10/23 from Yahoo Finance (Ticker BAC) and take its log return for autocorrelation analysis.The quantile level is set at 99% level. It can be seen from Figure 1.1 that the dependence structure revealed by the traditional autocorrelation function (ACF) on the left can be very different from the tail autocorrelation analysis of T. Zhang (2022) on the right.

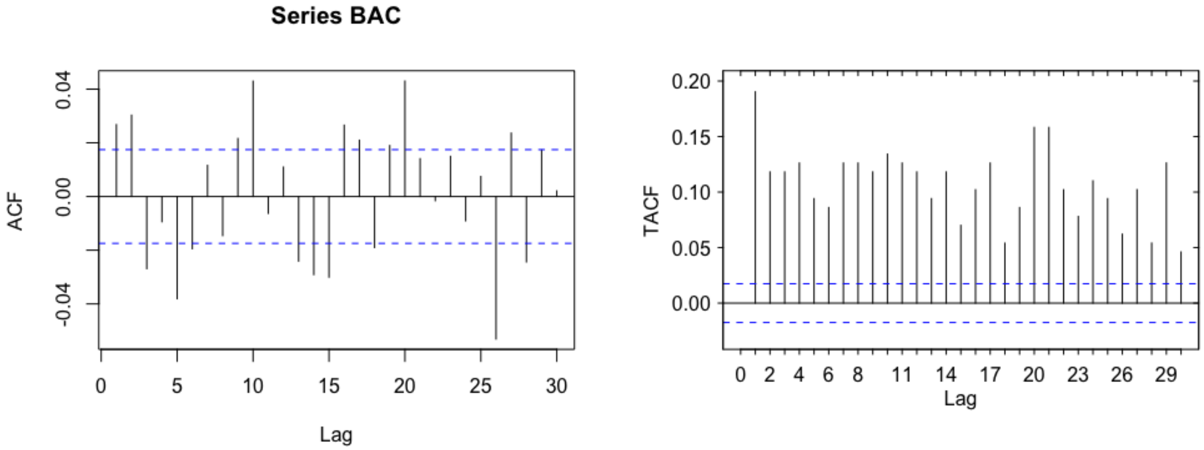

Figure 1.1: Traditional ACF plot (on the left) and tail autocorrelation function (TACF) plot (on the right) for Bank of America Stock Price Log Return

Another data set we use is the monthly average of daily high temperature in the United States for time period 03/1840 to 05/2016. Note that this data set is the anomaly values with seasonal trends being removed, but we still see from Figure 1.2 some seasonal patterns of autocorrelation in the right panel with the tail autocorrelation function(TACF). We can also notice that the biggest difference takes place before lag 10, where traditional ACF plot shows a tendency of decreasing while tail ACF plot shows a pattern similar to rest of the lag ranges. This figure illustrates the different analysis results from two separate methods. We will revisit the investigations into this data set with more details later.

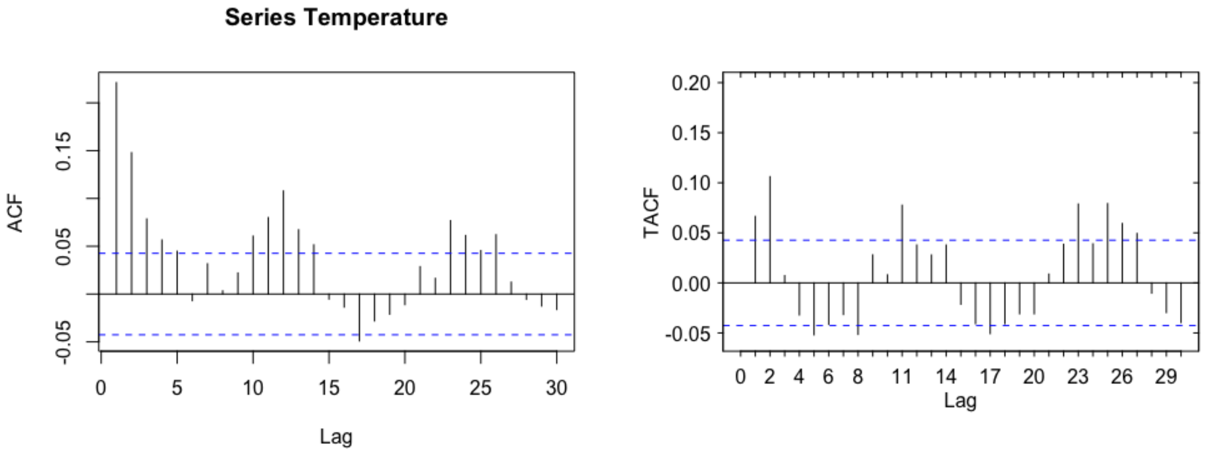

Figure 1.2: Traditional ACF plot (on the left) and tail autocorrelation function (TACF) plot (on the right) for Monthly Averages of Daily High Temperatures in the United States

The intention of these two plots is to show the possibly different conclusions tail dependence analysis methods can achieve that traditional method otherwise cannot. Therefore when dealing with time series data sets, we should be careful about the tools we use. Wrong or incomplete tools may drive us into misguided or partial conclusions, therefore the development of more accurate tools with a better focus on problems of interest is long desired to provide the possibility of uncovering another layer of data's latent structure.

CHAPTER 2

TAIL ADVERSARIAL STABILITY FRAMEWORK

2.1 Dependence Conditions Developed before TAS

Statistical inference is often shaped by formulated limit theorems, whose establishment relies significantly on fundamental assumptions or requirements on the underlying observed data. While we know widely used central limit theorems which usually require independence amongst data points, this assumption usually collapses when handling time series data. Therefore, assumptions on dependence are usually necessary and thus have been described from variant perspectives such as strength, endurance, ranges, and others, and have been proposed and studied comprehensively by numerous works coming before us.

Amongst various conditions, one of many widely celebrated conditions is the strong mixing condition, which was firstly introduced in an influential work of Rosenblatt (1956) , in the same article a central limit theorem under such condition was obtained, which laid the pavement for variants of mixing conditions and these flexible conditions have become a popular assumption when dealing with dependent time series data, which is defined as follows:

Suppose $X := (X_k, k \in \mathbb{Z})$ is a sequence of random variables from probability space $(\Omega, \mathcal{F}, \mathrm{pr})$, for $-\infty \leq a \leq b \leq \infty$, define the σ-field $\mathcal{F}_a^b := \sigma(X_k, a \leq k \leq b, k \in \mathbb{Z})$ and following dependence coefficients for each $n \geq 1$:

$$\alpha(n) := \sup_{j \in \mathbb{Z}} \sup_{\substack{A \in \mathcal{F}_{-\infty}^{j}, \\ B \in \mathcal{F}_{j+n}^{\infty}}} |\mathrm{pr}(A \cap B) - \mathrm{pr}(A)\mathrm{pr}(B)|,$$

then X is said to be "strong-mixing" (or "α-mixing") if $\alpha(n) \to 0$ as $n \to \infty$. Later, Ibragimov (1962) investigated a class of stationary stochastic processes satisfying strong-mixing conditions and established related limit theorems for its summation form; Dehling et al. (1986) explored certain limit conclusions of partial sum of strictly stationary processes satisfying also strong-mixing condition; see also work with ψ- and ρ-mixing conditions in Peligrad (1992) , Fan and Yao (2003) , Chernozhukov (2005) leveraging α-mixing in this work, Bradley (2007) , and references therein for abundant research work for various limit theorems concerning the strong mixing condition and its variants. In an overview work, Bradley (2005) documented the basic properties of strong mixing conditions, including 8 variants, respective flexible properties, and interlaced relations, which makes itself an overall handbook for the strong mixing condition basics that has been widely referred to. However, the strong mixing condition is a way of quantifying the degree of dependence between two variables, and it relies on the strong mixing coefficient. This coefficient is calculated as a supremum over two sigma algebras, and in practical applications, it can be challenging to justify.

Later in a significant work, Wu (2005) introduced a new dependence measure which provided a natural framework for stationary causal processes, and proposed an asymptotic theory that required only mild conditions in the form of an expected norm to measure the underlying dependence strength of time series, which could lead to a martingale approximation; see for some application of this framework in Wu and Zhao (2007) , in which the mean non-stationary models were considered and a test statistic was proposed to capture the simultaneous confidence bands; m-dependent approximation is also heavily employed in Liu and Lin (2009) , which constructed the optimal bound for sums of a class of stationary causal processes under strong invariance principle as well; Wu and Zhou (2011) considered the partial sums of non-stationary processes, and approximated it with sums of independent Gaussian random vectors with invariance principle, which showed great applicability to a wide class of non-stationary time series; T. Zhang (2013) considered the clustered high-dimensional time series and constructed the central limit theorem for a non-parametric trend function for a certain class of nonstationary processes; under the framework of stationary causal processes, Berkes et al. (2014) achieved the approximation of partial sum of the processes by Wiener process with some easily verifiable dependence conditions; under the physical/functional dependence framework, X. Zhang and Cheng (2018) considered a Gaussian approximation to the maximum of the sum of weakly dependent hyper-dimensional time series vectors; Mies and Steland (2023) also considered the partial sum of high-dimensional processes and its Gaussian couplings, and a plausible Gaussian approximation method was proposed to conduct statistical inference on the couplings; to name a few.

2.2 Introduction to TAS

While the strong mixing condition and its variants are widely employed and their broad theoretical utility and capacity are consistently demonstrated to serve as the shaping structure for asymptotic results, they often involve the supreme over two sigma algebras and can be difficult to calculate in general. Therefore, we shall here consider an alternative framework proposed in T. Zhang (2021) . For this, suppose we have observations $X_i, i = 1, \ldots, n$ from a stationary time series with distribution $F(x) = \mathrm{pr}(X_i \leq x), x \in \mathbb{R}$, according to a stationary system

$$X_i = G(\mathcal{F}_i) = G(\ldots, \epsilon_{i-1}, \epsilon_i), \tag{2.1}$$

where $\epsilon_j, j \in \mathbb{Z}$ are iid innovations,and G is a measurable function such that X_i is properly defined. This system could cover a large class of stationary processes. Let $F^{-1}(u) = \inf\{x : F(x) \geq u\}$, then $\mathcal{U}_F = \lim_{u \uparrow 1} F^{-1}(u)$ represents the upper endpoint of the distribution and can go to as large as infinity. Then as x approaches $\mathcal{U}_F$, data points exceeding a certain threshold could be viewed as tailed events. In Z. Zhang (2006) , a lag-k tail dependence index was introduced as

$$\rho_k = \lim_{x \uparrow \mathcal{U}_F} \rho_{k,x}, \quad \rho_{k,x} = \mathrm{pr}(X_{k+1} > x \mid X_1 > x),$$

which smoothly extends the foundational metric of extreme statistics for bivariate distributions in Sibuya et al. (1960) to time series setting. However, though it's a straightforward and easily interpretable expression of tail dependence, it's typically not very useful in deriving asymptotic theory for time series with potential tail dependence. We adopt another framework, tail adversarial stability, proposed in T. Zhang (2021) .

Then let $\epsilon_0^\star$ be an innovation that has the same distribution as ϵ_0 but independent of $(\epsilon_k)_{k\in\mathbb{Z}}$, then $\mathcal{F}_i^\star = (\mathcal{F}_{-1}, \epsilon_0^\star, \epsilon_1, \ldots, \epsilon_{i-1}, \epsilon_i)$ is the coupled shift process and $X_i^\star = G(\mathcal{F}_i^\star)$ represents the coupled output at time i when the innovation at time zero is replaced by its iid copy. T. Zhang (2021) proposed to consider

$$\theta_x(i) = \sup_{z\geq x} \mathrm{pr}(X_i^\star \leq z \mid X_i > z), \tag{2.2}$$

where z is a large enough value, exceeding which a random variable would be considered as in the tail region. $\theta_x(i)$ measures the degree of tail dependence by whether replacing the innovation at time zero with an iid copy affects the output data at the time i being a tail observation. In particular, if X_i does not depend on ϵ_0, then $X_i^\star = X_i$ and $\theta_x(i) = 0$, meaning that ϵ_0 will not have any tail adversarial effect on X_i. On the contrary, if $X_i > z$ while $X_i^\star \leq z$, then changing ϵ_0 to its iid couple $\epsilon_0^\star$ makes the observation at time zero no longer in a tail region, in which case we call ϵ_0 a tail adversarial innovation. $\theta_x(i)$ is called the adversarial tail dependence measure for quantifying such impact. Let

$$\Theta_{x,q}(m) = \sum_{i=m}^{\infty} \{\theta_x(i)\}^{1/q}, m \geq 0, q \geq 1, \tag{2.3}$$

which measures the cumulative tail adversarial effect of ϵ_0 on future observations with gap $i \geq m$, then we say a time series process (X_i) is tail adversarial $q-$stable, or $(X_i) \in \mathrm{TAS}_q$ if

$$\lim_{x\uparrow\mathcal{U}_F} \Theta_{x,q}(0) < \infty. \tag{2.4}$$

To understand (2.4), the current innovation's cumulative tail adversarial effect $\theta_x(i)$ on all future observations being finite shows that the TAS condition is a shortrange condition(SRD). Note this TAS condition only imposes restrictions on the tail part of a distribution, without any further description of the middle or intermediate area. Besides, it's shown in T. Zhang (2021) that it could directly lead to desired asymptotic results for high-quantile regression estimators, unlike strong mixing condition, which usually needs further conditions assumed to bound the degree of dependence in the tail region of a time series distribution (Chernozhukov, 2005)(Chernozhukov & Fernández-Val, 2011).

In addition, we say that the process (X_i) is geometrically tail adversarial stable or $(X_i) \in$ GTAS if there exist some constants $c^\star \in (0, \infty)$ and $\psi \in (0, 1)$ such that

$$\theta_x(i) \leq c^\star \psi^i, \quad i \geq 0,$$

holds for some x that's close enough to its extreme boundary $\mathcal{U}_F$.

T. Zhang (2021) studied asymptotic theory for high-quantile regression estimators with tail dependent time series without setting strongly mixing conditions, but within the newly introduced tail adversarial stability framework as described above. It has been documented that the TAS framework can lead to weaker and cleaner conditions than the conventional strong mixing conditions in certain scenarios.

2.3 Examples under TAS

Here we intend to illustrate the tail adversarial stability framework with two examples from T. Zhang (2021) and T. Zhang and Xu (2023) , respectively.

Example 1 :Let (ϵ_i) be a sequence of independent Fréchet random variables with distribution function $\text{pr}(\epsilon_i \leq z) = \exp(-z^{-\gamma})$ for some $\gamma > 0$. Consider the Moving-Maximum process of Hall et al. (2002) , which is widely adopted in extreme value analysis

$$X_i = \max_{0 \leq l < \infty} a_l \epsilon_{i-l}, \quad i = 1, \ldots, n. \tag{2.5}$$

Above process is well defined if the nonnegative coefficients satisfy $\sum_{l=0}^{\infty} a_l^\gamma < \infty$. Hall et al. (2002) showed that the moving-maximum process is dense in the class of stationary processes whose finite-dimensional distributions are extreme value of a given type, see Z. Zhang and Smith (2004) , Z. Zhang (2006) , and Z. Zhang et al. (2017) for further discussions.

Following we introduce the difference between conditions required for process (2.5) to be well defined under TAS and in existing work. Based on the calculation in T. Zhang (2021) , a sufficient condition for the process to be well defined under TAS framework requires only

$$\sum_{l=0}^{\infty} a_l^{\gamma/q} < \infty,$$

which is relatively mild compared to existing requirement $\sum_{l=0}^{\infty} a_l^{\gamma} < \infty$.

Example 2: Let (ϵ_i) be a sequence of iid regularly varying random variables with index $\gamma > 0$ and

$$X_i = \sum_{l=0}^{\infty} a_l \epsilon_{i-l},$$

where the coefficient (a_l) are chosen from the ARMA equation $\sum_{l=0}^{\infty} a_l z^l = \beta(z)/\psi(z)$, $z \in \mathbb{C}$, with $\beta(z) = 1 + \beta_1 z + \beta_2 z^2 + \cdots + \beta_r z^r$ and $\psi(z) = 1 - \psi_1 z - \cdots - \psi_s z^s$ for some $r, s \geq 0$. Following Mikosch and Zhao (2014) , we assume that $\beta(z)$ and $\psi(z)$ not have common zeros and $\psi(z) \neq 0$ for $|z| \leq 1$, then by the argument in the aforementioned paper the process X_i is also regularly varying with the same index $\gamma > 0$ and we can write $\text{pr}(X_i > x) = x^{-\gamma} L(x)$ for some slowly varying function $L(\cdot)$. For the tail adversarial stability framework, by Bai and Zhang (2022) , it can be shown that under certain regularity conditions, the tail adversarial stability measure satisfies

$$\theta_x(i) \leq c|a_i|^{\gamma'}$$

for sufficiently large x, where $\gamma' < \gamma$ can be chosen arbitrarily close to γ when $\gamma \leq 1$ and $\gamma' = 1$ if $\gamma > 1$.Under the ARMA structure, the coefficients $a_i, i \geq 0$, decay geometrically fast, and thus $(X_i) \in \text{TAS}_q$ for any $q > 0$ and in addition $(X_i) \in$ GTAS. Our consistency results in Theorem1 requires $(X_i) \in \text{TAS}_q$ for $q > 4$ and $n\text{pr}(X_i > x_n) \to \infty$.

Chapter 3

Tail Spectral Density Estimation

3.1 Spectral Density Analysis

In time series analysis, two perspectives of analysis domains are considered to gain insights into the inherent structures of data sets. Though time domain may be the more intuitive and straightforward one, the frequency-domain also lays the pavements for uncovering hidden patterns within sequential data from a wide range of areas, such as in signal processing, engineering, environmental science, and finance. For the research in this direction, the spectral density is a widely used tool for characterizing the frequency-domain behavior of a stationary process.

The methodology of spectral density analysis relates directly to the discrete-time Fourier transformation of autocovariances at different lags, and its estimators and asymptotic theory have been studied in extensive literature. The core of spectral density analysis is that a time series can be decomposed into functions of amplitudes and frequencies, and the contributions of each frequency to the general sequence behavior could be quantified through such functions,see for example Bentkus and Rudzkis (1983) , Rosenblatt (1984) , Dahlhaus (1985) , Velasco and Robinson (2001) , Phillips et al. (2006) and Phillips et al. (2007) , Wu and Shao (2007) , Liu and Wu (2010) , Xiao and Wu (2011) , and references therein. The spectral density measures the amount of variance in the process at different frequencies and can be used to analyze the process's periodicity, seasonality, as well as long-term trends.

However, the spectral density only provides information of mean and long-run variance, if evaluating the process's behavior from near zero frequency, see in Newey and West (1986) , Andrews (1991) , Song and Schmeiser (1995) , Lahiri and Lahiri (2003) , Wu (2009) , Flegal and Jones (2010) , Politis (2011) , T. Zhang (2018) , and references therein. Since the conventional spectral density is constructed using the traditional autocovariances, it mainly concerns the dependence in terms of co-movements with respect to the mean and therefore it may fail to capture precisely the tail behavior of the processes, which could be essential in many applications for revealing initiative and precise conclusions, such as fields demonstrated previously .

3.2 Tail Spectral Density Analysis

Numerous existing work have been paying attention to develop methods aimed to capture tail behaviors by spectral density analysis. R. A. Davis and Mikosch (2009) explored the idea of using the truncated periodogram to estimate the tail spectral density and showed that it's consistent for the estimation. However, they didn't specify the associated convergence rate and asymptotic distribution at the time, leaving this as an unresolved issue. Later, Mikosch and Zhao (2014) developed an asymptotic theory for periodogram ordinates at fixed frequencies, which they used to demonstrate that the smoothed periodogram is a reliable estimator of the tail spectral density. Nonetheless, their Theorem 5.1 only establishes the consistency of the estimator and does not offer any insight into the convergence rate or asymptotic distribution of the smoothed periodogram. Mikosch and Zhao (2015) further investigated the integrated periodogram, but their study does not encompass triangular array weight functions with shrinking support that are necessary for consistent estimation of the spectral tail density. To the best of our knowledge, there is currently a gap in understanding the convergence rate and asymptotic distribution of tail spectral density estimators for a general class of tail dependent time series.

This chapter aims to address this gap, which presents three major challenges. First, unlike the periodogram ordinates in Mikosch and Zhao (2014) or leading diagonal term in Mikosch and Zhao (2015) that can be managed by linear forms, the study of tail spectral density estimators typically requires an asymptotic theory on non-degenerate quadratic forms of tail statistics that has not been extensively researched and can be more challenging to handle. Second, sample tail autocorrelations can exhibit a two-phase asymptotic behavior with dichotomous convergence rates at different lags as shown in T. Zhang (2022) , unlike traditional spectral density estimators, making it difficult to analyze their infinite sums or Fourier transforms in the tail setting that relates to the tail spectral density. Third, existing results in this area were mostly developed under the strong mixing framework of Rosenblatt (1956) , which required additional anti-clustering conditions and regularly varying conditions to handle tail events, leading to complicated conditions that involve the interplay between how fast the strong mixing coefficient decays and how extreme the tail can be, and can result in strong conditions for common extreme value time series models as described in a recent review by T. Zhang (2021) .

We shall here consider a tail counterepart of the traditional spectral density analysis. For a sequence of random variables $X_1, X_2, \ldots, X_n$ drawn from a time series process with common distribution function $F(x)$. Let $x_n \to \lim_{u\uparrow 1} F^{-1}(u)$, consider the tail autocorrelation

$$\rho_{k,n} = \frac{\operatorname{pr}(X_{k+1} > x_n \mid X_1 > x_n) - \operatorname{pr}(X_{k+1} > x_n)}{1 - \operatorname{pr}(X_1 > x_n)}, \tag{3.1}$$

which could be viewed as a standardized pre-asymptotic version of the lag-k tail dependence index of Z. Zhang (2006) . T. Zhang (2022) provided a two-phase asymptotic theory on sample tail autocorrelations and used it to guide the construction of a visualization tool with lines of significance. Let $\iota = \sqrt{-1}$ be the imaginary unit, we focus on the frequency domain of a time series and consider the tail spectral density

$$f_n(\lambda) = \frac{1}{2\pi} \sum_{|k|<n} \rho_{k,n} e^{\iota k \lambda}, \tag{3.2}$$

which naturally extends the conventional spectral density to the tail setting using the tail autocorrelations defined in (3.1).

To estimate the tail spectral density (3.2), we consider the lag-window estimator:

$$\hat{f}_n(\lambda) = \frac{1}{2\pi} \sum_{|k|<n} \hat{\rho}_{k,n} e^{\iota k \lambda} K\left(\frac{k}{B_n}\right), \tag{3.3}$$

where $K : \mathbb{R} \to \mathbb{R}$ is a kernel function and $B_n \to \infty$ is a positive bandwidth sequence. In conventional spectral density analysis, with non-tail setting, lag-window estimators have been extensively studied, see for example Rosenblatt (1984) , Phillips et al. (2006) , Shao and Wu (2007) , Liu and Wu (2010) , the book of Anderson (2011) , and references therein. Although developing a central limit theorem for the conventional spectral density estimator is already a highly nontrivial problem as commented by Liu and

Wu (2010) , achieving it in the current tail setting with the double asymptotic scheme can be even more challenging. T. Zhang (2022) provided an asymptotic theory for sample tail autocorrelations, whose unusual two-phase asymptotic behavior distinguishes them from traditional autocorrelations. We shall here provide an asymptotic theory on their infinite sums that lead to the tail spectral density estimator (3.3). Unlike T. Zhang (2022) , which only focused on individual sample autocorrelations at a fixed lag, the current problem requires the handling of a growing number of sample tail autocorrelations at the same time, which is more challenging and requires a new asymptotic theory on quadratic forms of tail statistics. In the following section is an introduction of a clean and effective framework for capturing some tail behavior and paving the road for deriving such asymptotic theories.

3.3 Asymptotic Theorem for Tail Spectral Density Estimation

In this section we discuss and provide asymptotic results for tail spectral density function estimation based on the estimator in (3.3) under the TAS framework introduced in previous section. Following the notation in Section 1.3, we firstly consider a sample version of autocorrelation (3.1), which is the first key discrepency with conventional spectral density function estimator.

Let $\mathbb{1}(\cdot)$ be the indicator function, $\bar{F}(x) = 1 - F(x)$ and $\hat{\bar{F}}(x) = n^{-1}\sum_{i=1}^{n} \mathbb{1}(X_i > x)$, we have tail autocorrelation estimator

$$
\begin{aligned}
\hat{\rho}_{k,n} &= \frac{\hat{\mu}_{k,n}}{\hat{\mu}_{0,n}}; \\
\hat{\mu}_{k,n} &= \frac{1}{n}\sum_{i=1}^{n-|k|} \left\{ \mathbb{1}(X_i > x_n) - \hat{\bar{F}}_n(x_n) \right\} \left\{ \mathbb{1}(X_{i+|k|} > x_n) - \hat{\bar{F}}_n(x_n) \right\}.
\end{aligned}
$$

In our setting, we would allow $x = x_n$ to approach its upper limit $\mathcal{U}_F$ as $n \to \infty$, which forms a double asymptotic scheme, which distinguish itself from Linton and Whang (2007) which only considered on the level of sample size n approaching infinity.

Throughout this section, we would assume that the kernel function $K \in \mathcal{K}, \bar{\kappa} = \sup_{u \in \mathbb{R}} K(u) < \infty$ and $\kappa = \int_{\mathbb{R}} K^2(u)du < \infty$. We firstly consider a proxy of (3.3) in following formula

$$\tilde{f}_n(\lambda) = \frac{1}{2\pi} \sum_{|k|<n} \tilde{\rho}_{k,n} e^{\iota k \lambda} K\left(\frac{k}{B_n}\right), \tag{3.4}$$

where

$$\tilde{\rho}_{k,n} = \frac{1}{nF(x_n)\bar{F}(x_n)} \sum_{i=1}^{n-|k|} \left\{\mathbb{1}(X_i > x_n) - \bar{F}_n(x_n)\right\} \left\{\mathbb{1}(X_{i+|k|} > x_n) - \bar{F}_n(x_n)\right\}.$$

Following theorem provides the consistency of $\tilde{f}_n(\lambda)$ and quantifies its distance from $\hat{f}_n(\lambda)$.

Theorem 1. *Assume that* $(X_i) \in$ *TAS*$_4$*, if* $F(X_n) \to 1, B_n \to \infty$*, and* $\{n\bar{F}(x_n)\}^{-1}B_n \to 0$*, then for any* $\lambda \in [0, 2\pi)$,

$$\tilde{f}_n(\lambda) - \mathrm{E}\left\{\tilde{f}_n(\lambda)\right\} \to_p 0,$$

and

$$\hat{f}_n(\lambda) = \left\{\tilde{f}_n(\lambda) + \mathcal{O}_p(n^{-1}B_n)\right\}\left(1 + \mathcal{O}_p[\{n\bar{F}(x_n)\}^{-1/2}]\right).$$

Above theorem justifies the consistency of $\tilde{f}_n(\lambda)$ and establishes the relationship with $\hat{f}_n(\lambda)$ in a formula. So that we can further infer that

$$\hat{f}_n(\lambda) - \mathrm{E}\left\{\tilde{f}_n(\lambda)\right\} \to_p 0,$$

where the asymptotic center

$$\mathrm{E}\left\{\tilde{f}_n(\lambda)\right\} = f_n(\lambda) + \frac{1}{2\pi} \sum_{|k|<n} \rho_{k,n} e^{\iota k \lambda} \left\{\left(1 - \frac{|k|}{n}\right) K\left(\frac{k}{B_n}\right) - 1\right\}.$$

Therefore, to approach the asymptotic property, what we need to do is to choose a kernel function that satisfies condition $K(u) = 1$ where $|u| \le c_K$ for some $0 < c_K < \infty$, which is referred to as the class of flat-top kernels in Politis (2011) , then we can derive

$$|\mathrm{E}\left\{\tilde{f}_n(\lambda)\right\} - f_n(\lambda)| \le \frac{1}{\pi}\sum_{k=1}^{\lfloor c_K B_n \rfloor} \frac{|k\rho_{k,n}|}{n} + \frac{1}{\pi}\sum_{\lfloor c_K B_n \rfloor+1}^{n-1} (\bar{\kappa}+1)|\rho_{k,n}| \to 0$$

by the dominated convergence theorem and the proof of Lemma 3 in T. Zhang (2021) .

As discussed in Chapter 2.3, our Theorem 3.1 complements the consistency results in Davis and Mikosch (2009) and Mikosch and Zhao (2014) in the sense that our tail adversarial stability framework can possibly lead to weaker conditions on the degree of tail dependence and allow more extremal tails. This asymptotic consistency result requires weaker conditions on tail dependence and allows more extremal tails as $n\bar{F}(x_n) \to \infty$ v.s. $n^{1/3}\bar{F}(x_n) \to \infty$.

We shall now provide a central limit theorem for the tail spectral density estimator (3.3), which has not been well addressed in the literature.

Theorem 2. *Assume that* (X_i) $\in$*GTAS and* $K \in \mathcal{K}$ *has bounded support and is Lipschiz continuous except for a finite number of points. If* $F(x_n) \to 1$, $B_n\bar{F}(x_n)(\log n)^{-7} \to \infty$ *and* $\left\{n\bar{F}(x_n)\right\}^{-1} B_n(\log n)^8 \to 0$, *then*

(i) for $\lambda \in [0, 2\pi) \setminus \{0, \pi\}$,*where* $f_n(\lambda)$ *is bounded away from zero for all large* n*,*

$$\{\kappa^{1/2} f_n(\lambda)\}^{-1}(B_n^{-1}n)^{1/2}\Big[\tilde{f}_n(\lambda) - \mathrm{E}\left\{\tilde{f}_n(\lambda)\right\}\Big] \to_d N(0,1);$$

and (ii) for $\lambda \in \{0, \pi\}$ *where* $f_n(\lambda)$ *is bounded away from zero for all large* n*,*

$$\{\kappa^{1/2} f_n(\lambda)\}^{-1}(B_n^{-1}n)^{1/2}\Big[\tilde{f}_n(\lambda) - \mathrm{E}\left\{\tilde{f}_n(\lambda)\right\}\Big] \to_d N(0,2),$$

where $\kappa = \int_{u\in\mathbb{R}} K(u)^2 du$.

corollary 1. *Assume conditions of Theorem 2 . If in addition* $K \in \mathcal{K}_{FT}$, *then (i) for* $\lambda \in [0, 2\pi) \setminus \{0, \pi\}$,*where* $f_n(\lambda)$ *is bounded away from zero for all large* n*,*

$$\{\kappa^{1/2} f_n(\lambda)\}^{-1}(B_n^{-1}n)^{1/2}\{\hat{f}_n(\lambda) - f_n(\lambda)\} \to_d N(0,1);$$

and (ii) for $\lambda \in \{0, \pi\}$ *where* $f_n(\lambda)$ *is bounded away from zero for all large* n*,*

$$\{\kappa^{1/2} f_n(\lambda)\}^{-1}(B_n^{-1}n)^{1/2}\{\hat{f}_n(\lambda) - f_n(\lambda)\} \to_d N(0, 2),$$

where $\kappa = \int_{u\in\mathbb{R}} K(u)^2 du$*..*

Theorem 3. *Assume conditions of Theorem 2. Let* $\lambda_1, \dots, \lambda_L$ *be different frequencies in* $[0, 2\pi)$ *with* $(\lambda_l + \lambda_{l'})/\pi \notin \mathbb{Z}$*, and* $(\lambda_l - \lambda_{l'})/\pi \notin \mathbb{Z}$*, for any* $1 \le l \le l' \le L$*. If* $K \in \mathcal{K}_{FT}$*, and* $f_n(\lambda_l)$ *is bounded away from zero foe all large* n *for* $l = 1, \dots, L$*, then* $\{f_n(\lambda_l)\}^{-1}(B_n^{-1}n)^{1/2}\{\hat{f}_n(\lambda_l) - f_n(\lambda_l)\}, l = 1, \dots, L$*, converge jointly to independent normal random variables.*

3.4 Simulation Study

In this part, we would like to explore the applicability of the theoretical results to simulation studies. We plan to try various amount of simulation scenarios based on multiple attributes to help form informed conclusions and observations.

Following T. Zhang (2022) , we firstly consider Moving-Maximum process which is widely used in extreme value relevant analyses with the specific form

$$X_i = \max(\epsilon_i, \epsilon_{i-1}/2), \quad i = 1, \dots, n, \tag{3.5}$$

where distribution of innovations (ϵ_i) is a sequence of independent Fréchet distribution with distribution function $\mathrm{pr}(\epsilon_i \le z) = \exp(-z^{-\gamma})$, for some $\gamma \in \{1, 2, 3\}$. We also consider different sets of sample time points $n \in \{200, 500, 1000, 2000\}$, two quantile thresholds 90% and 95%, and multiple bandwidths as $B_n \in \{10, 15, 20, 25, 30\}$. We devise the study with developed central limit theorem to estimate the tail spectral density of Moving-Maximum process (3.5) and its confidence interval, with its asymptotic variance derived from Theorem 2 and Corollary 1 being formulated as $2n^{-1}B_n\kappa\left\{\hat{f}_n(\lambda)\right\}^2$ for $\lambda = 0$, and $n^{-1}B_n\kappa\left\{\hat{f}_n(\lambda)\right\}^2$ for $\lambda \in \{0.25\pi, 0.5\pi\}$, at confidence interval 90% and 95%, respectively .

We also consider other processes such as autoregression moving average (ARMA) process

$$X_i = \rho X_{i-1} + \epsilon_i + 0.5\epsilon_{i-1}, \quad i = 1, \dots, n,$$

with different values of $\rho \in \{0.2, 0.4, 0.6, 0.8\}$ to examine the finite-sample performance under different dependence strengths, also with Fréchet innovations with $\gamma \in \{1, 2, 3\}$ or Gaussian distribution. We use the developed central limit theorem to construct confidence intervals for the tail spectral density at frequency zero only for ARMA process and the same other sets of scenarios as described above for Moving-Maximum process and with below kernels.

Besides, we also think generalized autoregressive conditional heteroskedasticity (GARCH) process could be informative in simulation study with variant scenarios

$$\begin{aligned} X_i &= \sigma_i \epsilon_i; \\ \sigma_i^2 &= 1 + aX_{i-1}^2 + b\sigma_{i-1}^2, \end{aligned}$$

with $a = b = \rho/2$ so that $a + b = \rho < 1$, and innovation following a standard Gaussian distribution as GARCH process with big correlation coefficient in $\rho \in \{0.2, 0.4, 0.6, 0.8\}$ could produce too many invalid and untrustworthy simulation results with Fréchet innovations. Other than innovation distribution, we consider only the tail spectral density at frequency zero as well for GARCH process and other scenario dimensions are the same as described above for Moving-Maximum process and with below kernels.

We decide to utilize three kernels belonging to the family of flat-top kernels and possessing certain favorable properties from Politis (2011) , Politis and Romano (1995) , McMurry and Politis (2004) , and Parzen (1961) .

- Trapezoidal kernel :

$$K_1(u) = \max[\min\{2(1 - |u|), 1\}, 0]$$

- Infinitely differentiable flat-top kernel (PM04):

$$K_2(u) = \begin{cases} 1, & \text{if } |u| \leq 0.05; \\ \exp\Big\{ \frac{-0.25e^{-0.25/(|u|-0.05)^2}}{(|u|-1)^2} \Big\}, & \text{if } 0.05 < |u| < 1; \\ 0, & \text{if } |u| \geq 1. \end{cases}$$

- A kernel related to what's mentioned in Parzen (1961) (P61):

$$K_3(u) = \begin{cases} 1, & \text{if } |u| \leq 3/7; \\ 1 - 6(1.75|u| - 0.75)^2 + 6|1.75|u| - 0.75|^3, & \text{if } 3/7 < |u| \leq 5/7; \\ 2(1 - |1.75|u| - 0.75|)^3, & \text{if } 5/7 < |u| \leq 1; \\ 0, & \text{if } |u| > 1. \end{cases}$$

The properties owned by above kernels include : as flat-top kernels, like what's mentioned in chapter 3.3, satisfy the condition $K(u) = 1$ and can thus provide good property for the tail spectral density function estimator's asymptotic results derivation; besides, the infinite differentiable kernel (PM04) could be applicable to whatever degree of smoothness of the underlying process has and its tail decays fast enough to avoid edge effects spreading out across the whole interval of $[0, 1]$; furthermore, the Fourier transform of PM04, due to its infinite differentiability, could be very smooth; kernel P61 is considered as a non-negative and truncated type, which could give rise to algebraic estimates with a truncation point. Following we see a plot of the kernels, the differences happen around the transitioning points from 1 and to 0.

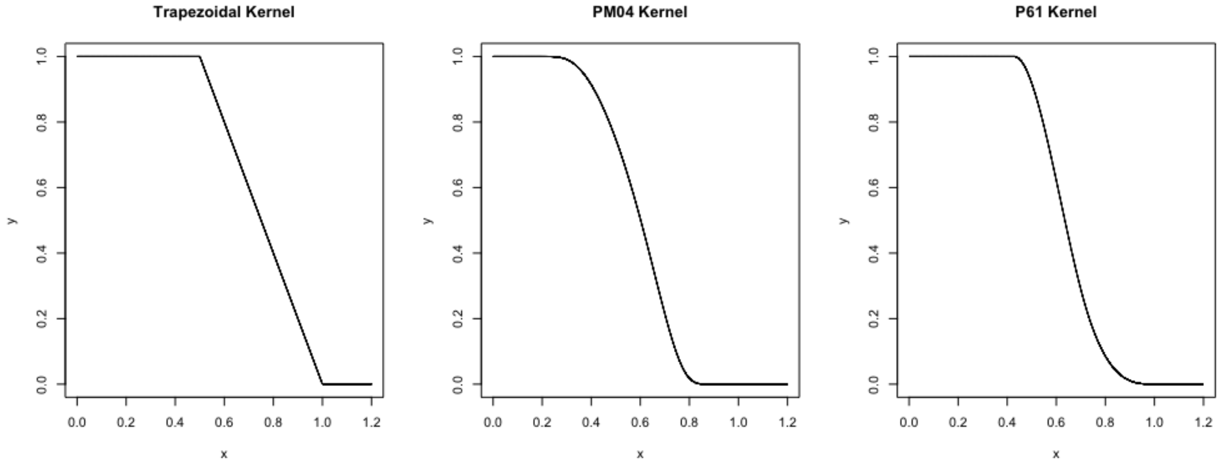

Figure 3.1: Flat-top Kernels Used for Simulation

The simulation results are summarized in following tables, in which we report empirical coverage probabilities and associated confidence interval in the parenthesis.

Following subsection 3.3.1-3.3.3 include the finite-sample simulation performance for Moving-Maximum process, with each specific kernel for a subsection, and a bandwidth value B_n for a table. From the table we can see that the empirical coverage probabilities are mostly close to their nominal levels as long as the sample size is reasonably large. Though the empirical coverage probabilities could be impacted by the level of γ by showing some decreasing trend when γ is larger, especially when sample point $n = 200$.

However, we should note that a sample size of $n = 200$ is in general a challenging case for tail inference as available sample points are fairly limited when quantile level comes close to 1, for which we can also observe a certain degree of size distortion in our numerical results. While as the sample increases, the size distortion generally gets smaller and the results also seem to become more robust to the choice of B_n. For Moving-Maximum process, empirical coverage probabilities are mostly (not for every table) optimized at frequency = 0.25.

Following subsection 3.3.4-3.3.7 include the finite-sample simulation performance for ARMA process, with each innovation distribution for a subsection, and a bandwidth value B_n for a table. It can be seen that the empirical coverage probabilities are mostly close to their nominal levels as well except for some challenging situations (for example when $n = 200, \rho = 0.8$ for small sample size and high process correlation coefficient simultaneously, and the tail threshold x_n is chosen as the 95% quantile) where a certain degree of size distortion can be observed. But the size distortion in such challenging situations could get mitigated by adjusting scenarios to more tolerant cases, such as lowering the tail threshold quantile for x_n to the level of 90% , increasing the sample size n (as can be seen in the tables for cases with larger n), or decreasing ρ for weaker dependence (also can be seen from tables).

Besides, the results can be affected by the bandwidth choice when the sample size is very small, for example when $n = 200$. However, as the sample size increases, the empirical coverage probabilities generally becomes more robust to different choices of the bandwidth and seem to be reasonably stable with large sample points as many as $n = 2000$. Similarly, the empirical coverage probabilities are reasonably robust to different choices of the kernel function when $n = 1000$ or $n = 2000$.

Furthermore, for ARMA process, it's been observed that Fréchet innovation with $\gamma = 3$ performs generally better than smaller $\gamma's(\gamma = 2$ and $1)$, while results from gaussian innovation do not outperform that from Fréchet innovations. On the top of this, Fréchet with $\gamma = 3$ performs more stably when ρ changes from 0.2 to 0.8 than other innovations under the same circumstances, especially when $n = 2000$.

Also for ARMA process with Fréchet innovations, B_n influences empirical coverage probabilities less when $\gamma = 3$ than when $\gamma = 1$. Lastly, average confidence interval lengths are significantly impacted by sample size n and dependence level ρ. With smaller n and ρ, the lengths could be several times of that with larger n and ρ.

Subsection 3.3.8 includes the finite-sample simulation performance for GARCH process, considering Gaussian innovation distribution only, and a bandwidth value B_n for a table. from which we can see that the empirical coverage probabilities are mostly reasonably close to their nominal levels, except for some challenging situations (for example when $\rho = 0.8$ and the tail threshold x_n is chosen as the 95% quantile for the same reason as in ARMA process) where a certain degree of size distortion can be observed. The results generally improve as ρ gets smaller, or as n gets larger, or if we set the tail threshold x_n to be the 90% quantile. Similar to the ARMA case, the empirical coverage probabilities also seem to be reasonably robust to different bandwidth and kernel choices when $n = 1000$ or $n = 2000$. Besides, $\rho = 0.2$ is more sensitive to sample size n than $\rho = 0.8$, generally. And for both ARMA and GARCH processes, $\rho = 0.2$ outperforms 0.8 most of time.

And in the following we summary some patterns that may apply for most of the collected tables :

- Smaller $n(n = 200)$ is more sensitive to kernel difference. For most of cases, a significant improvement in empirical coverage probabilities could be observed when n changes from 200 to 500, which is usually larger than the change between $n = 500$ and 1000.

- Tail quantile threshold level 95% sometimes could be more challenging compared to tail quantile threshold level 90%.

- Bigger the process correlation coefficient ρ, which could be interpreted as dependence level,bigger the size distortion especially when other attributes could also contribute to simulation challenges.

- Kernels do not really make big difference in our simulations, as B_n influences much more, which could be constantly observed under multiple process designs.

- With n increasing, the coverage probabilities grow closer to nominal level, though not strictly when from $n = 1000$ to 2000 all the time, while the average lengths decrease strictly.

3.4.1 Moving-Maximum with Trapezoidal Kernel

3.4.1.1 $B_n = 10$

Table 3.1: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with Trapezoidal kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 10$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.900_{(0.288)}$	$0.936_{(0.343)}$	$0.887_{(0.247)}$	$0.918_{(0.294)}$	$0.869_{(0.214)}$	$0.900_{(0.255)}$
	0.25	$0.897_{(0.198)}$	$0.926_{(0.236)}$	$0.912_{(0.174)}$	$0.942_{(0.207)}$	$0.901_{(0.156)}$	$0.934_{(0.186)}$
	0.50	$0.906_{(0.136)}$	$0.936_{(0.163)}$	$0.916_{(0.136)}$	$0.940_{(0.162)}$	$0.898_{(0.135)}$	$0.931_{(0.161)}$
500	0	$0.918_{(0.194)}$	$0.950_{(0.231)}$	$0.907_{(0.161)}$	$0.942_{(0.192)}$	$0.901_{(0.142)}$	$0.937_{(0.169)}$
	0.25	$0.936_{(0.125)}$	$0.957_{(0.149)}$	$0.919_{(0.109)}$	$0.963_{(0.130)}$	$0.918_{(0.098)}$	$0.957_{(0.117)}$
	0.50	$0.916_{(0.086)}$	$0.950_{(0.102)}$	$0.926_{(0.086)}$	$0.948_{(0.103)}$	$0.913_{(0.086)}$	$0.953_{(0.103)}$
1000	0	$0.908_{(0.138)}$	$0.957_{(0.164)}$	$0.926_{(0.116)}$	$0.964_{(0.138)}$	$0.905_{(0.102)}$	$0.940_{(0.122)}$
	0.25	$0.950_{(0.088)}$	$0.972_{(0.105)}$	$0.928_{(0.077)}$	$0.964_{(0.092)}$	$0.928_{(0.069)}$	$0.960_{(0.083)}$
	0.50	$0.918_{(0.060)}$	$0.960_{(0.072)}$	$0.916_{(0.060)}$	$0.957_{(0.072)}$	$0.927_{(0.060)}$	$0.964_{(0.072)}$
2000	0	$0.898_{(0.099)}$	$0.950_{(0.118)}$	$0.921_{(0.083)}$	$0.957_{(0.099)}$	$0.913_{(0.073)}$	$0.960_{(0.087)}$
	0.25	$0.936_{(0.062)}$	$0.967_{(0.074)}$	$0.918_{(0.054)}$	$0.965_{(0.064)}$	$0.913_{(0.049)}$	$0.962_{(0.058)}$
	0.50	$0.907_{(0.042)}$	$0.955_{(0.051)}$	$0.911_{(0.042)}$	$0.953_{(0.051)}$	$0.924_{(0.042)}$	$0.966_{(0.051)}$
				x_n chosen as the 95% quantile			
200	0	$0.913_{(0.288)}$	$0.947_{(0.343)}$	$0.894_{(0.241)}$	$0.933_{(0.287)}$	$0.885_{(0.217)}$	$0.914_{(0.259)}$
	0.25	$0.892_{(0.197)}$	$0.931_{(0.235)}$	$0.893_{(0.171)}$	$0.912_{(0.204)}$	$0.893_{(0.157)}$	$0.917_{(0.187)}$
	0.50	$0.904_{(0.136)}$	$0.932_{(0.162)}$	$0.926_{(0.137)}$	$0.938_{(0.163)}$	$0.911_{(0.135)}$	$0.937_{(0.162)}$
500	0	$0.911_{(0.191)}$	$0.959_{(0.228)}$	$0.909_{(0.161)}$	$0.940_{(0.192)}$	$0.884_{(0.143)}$	$0.934_{(0.171)}$
	0.25	$0.920_{(0.126)}$	$0.941_{(0.150)}$	$0.918_{(0.109)}$	$0.941_{(0.129)}$	$0.910_{(0.098)}$	$0.935_{(0.117)}$
	0.50	$0.924_{(0.085)}$	$0.952_{(0.102)}$	$0.918_{(0.086)}$	$0.952_{(0.102)}$	$0.914_{(0.085)}$	$0.945_{(0.102)}$
1000	0	$0.927_{(0.139)}$	$0.970_{(0.167)}$	$0.904_{(0.117)}$	$0.951_{(0.139)}$	$0.890_{(0.103)}$	$0.941_{(0.123)}$
	0.25	$0.926_{(0.088)}$	$0.950_{(0.105)}$	$0.913_{(0.077)}$	$0.942_{(0.091)}$	$0.903_{(0.069)}$	$0.938_{(0.083)}$
	0.50	$0.913_{(0.061)}$	$0.948_{(0.072)}$	$0.929_{(0.061)}$	$0.960_{(0.072)}$	$0.918_{(0.060)}$	$0.958_{(0.072)}$
2000	0	$0.920_{(0.099)}$	$0.968_{(0.119)}$	$0.888_{(0.084)}$	$0.934_{(0.100)}$	$0.894_{(0.073)}$	$0.940_{(0.087)}$
	0.25	$0.933_{(0.062)}$	$0.963_{(0.074)}$	$0.921_{(0.054)}$	$0.959_{(0.064)}$	$0.913_{(0.049)}$	$0.959_{(0.058)}$
	0.50	$0.910_{(0.042)}$	$0.959_{(0.051)}$	$0.937_{(0.042)}$	$0.970_{(0.051)}$	$0.933_{(0.042)}$	$0.964_{(0.051)}$

3.4.1.2 $B_n = 15$

Table 3.2: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with Trapezoidal kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 15$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
x_n *chosen as the 90% quantile*							
200	0	$0.861_{(0.346)}$	$0.892_{(0.412)}$	$0.838_{(0.290)}$	$0.867_{(0.345)}$	$0.825_{(0.252)}$	$0.864_{(0.301)}$
	0.25	$0.900_{(0.238)}$	$0.935_{(0.284)}$	$0.883_{(0.213)}$	$0.907_{(0.254)}$	$0.873_{(0.191)}$	$0.905_{(0.228)}$
	0.50	$0.882_{(0.167)}$	$0.916_{(0.198)}$	$0.896_{(0.165)}$	$0.920_{(0.196)}$	$0.895_{(0.167)}$	$0.924_{(0.199)}$
500	0	$0.888_{(0.231)}$	$0.924_{(0.275)}$	$0.884_{(0.194)}$	$0.912_{(0.231)}$	$0.880_{(0.174)}$	$0.920_{(0.208)}$
	0.25	$0.915_{(0.154)}$	$0.950_{(0.184)}$	$0.910_{(0.133)}$	$0.939_{(0.159)}$	$0.908_{(0.121)}$	$0.942_{(0.144)}$
	0.50	$0.907_{(0.103)}$	$0.940_{(0.123)}$	$0.892_{(0.106)}$	$0.932_{(0.126)}$	$0.900_{(0.105)}$	$0.932_{(0.125)}$
1000	0	$0.893_{(0.169)}$	$0.937_{(0.202)}$	$0.901_{(0.140)}$	$0.933_{(0.167)}$	$0.888_{(0.124)}$	$0.921_{(0.148)}$
	0.25	$0.909_{(0.108)}$	$0.949_{(0.129)}$	$0.924_{(0.094)}$	$0.951_{(0.112)}$	$0.914_{(0.085)}$	$0.948_{(0.101)}$
	0.50	$0.908_{(0.075)}$	$0.950_{(0.089)}$	$0.906_{(0.074)}$	$0.957_{(0.088)}$	$0.920_{(0.075)}$	$0.957_{(0.089)}$
2000	0	$0.913_{(0.120)}$	$0.949_{(0.143)}$	$0.919_{(0.101)}$	$0.957_{(0.120)}$	$0.887_{(0.089)}$	$0.937_{(0.106)}$
	0.25	$0.910_{(0.076)}$	$0.947_{(0.091)}$	$0.909_{(0.067)}$	$0.952_{(0.079)}$	$0.920_{(0.060)}$	$0.959_{(0.072)}$
	0.50	$0.900_{(0.052)}$	$0.945_{(0.062)}$	$0.917_{(0.052)}$	$0.952_{(0.063)}$	$0.909_{(0.053)}$	$0.952_{(0.063)}$
x_n *chosen as the 95% quantile*							
200	0	$0.881_{(0.342)}$	$0.910_{(0.408)}$	$0.862_{(0.296)}$	$0.894_{(0.353)}$	$0.877_{(0.249)}$	$0.906_{(0.297)}$
	0.25	$0.905_{(0.243)}$	$0.932_{(0.290)}$	$0.877_{(0.207)}$	$0.913_{(0.247)}$	$0.883_{(0.189)}$	$0.910_{(0.225)}$
	0.50	$0.908_{(0.166)}$	$0.921_{(0.198)}$	$0.900_{(0.164)}$	$0.927_{(0.196)}$	$0.901_{(0.168)}$	$0.927_{(0.200)}$
500	0	$0.914_{(0.232)}$	$0.947_{(0.276)}$	$0.890_{(0.195)}$	$0.936_{(0.233)}$	$0.881_{(0.172)}$	$0.920_{(0.205)}$
	0.25	$0.921_{(0.154)}$	$0.943_{(0.183)}$	$0.884_{(0.134)}$	$0.921_{(0.159)}$	$0.894_{(0.121)}$	$0.932_{(0.144)}$
	0.50	$0.911_{(0.104)}$	$0.941_{(0.124)}$	$0.896_{(0.105)}$	$0.930_{(0.125)}$	$0.922_{(0.105)}$	$0.950_{(0.125)}$
1000	0	$0.888_{(0.171)}$	$0.942_{(0.204)}$	$0.893_{(0.142)}$	$0.940_{(0.170)}$	$0.889_{(0.126)}$	$0.928_{(0.150)}$
	0.25	$0.892_{(0.108)}$	$0.938_{(0.128)}$	$0.896_{(0.094)}$	$0.944_{(0.112)}$	$0.894_{(0.085)}$	$0.932_{(0.101)}$
	0.50	$0.902_{(0.074)}$	$0.945_{(0.088)}$	$0.897_{(0.074)}$	$0.947_{(0.088)}$	$0.891_{(0.074)}$	$0.946_{(0.088)}$
2000	0	$0.898_{(0.122)}$	$0.938_{(0.145)}$	$0.903_{(0.102)}$	$0.955_{(0.121)}$	$0.882_{(0.889)}$	$0.937_{(0.106)}$
	0.25	$0.902_{(0.076)}$	$0.949_{(0.091)}$	$0.896_{(0.067)}$	$0.953_{(0.079)}$	$0.915_{(0.060)}$	$0.955_{(0.072)}$
	0.50	$0.900_{(0.052)}$	$0.946_{(0.062)}$	$0.910_{(0.052)}$	$0.959_{(0.062)}$	$0.928_{(0.052)}$	$0.960_{(0.062)}$

3.4.1.3 $B_n = 20$

Table 3.3: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with Trapezoidal kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 20$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.833_{(0.383)}$	$0.853_{(0.456)}$	$0.837_{(0.328)}$	$0.860_{(0.391)}$	$0.820_{(0.281)}$	$0.842_{(0.335)}$
	0.25	$0.885_{(0.281)}$	$0.906_{(0.334)}$	$0.886_{(0.243)}$	$0.919_{(0.289)}$	$0.872_{(0.219)}$	$0.903_{(0.261)}$
	0.50	$0.879_{(0.191)}$	$0.903_{(0.228)}$	$0.870_{(0.192)}$	$0.904_{(0.228)}$	$0.896_{(0.190)}$	$0.929_{(0.227)}$
500	0	$0.876_{(0.265)}$	$0.911_{(0.315)}$	$0.868_{(0.225)}$	$0.902_{(0.268)}$	$0.856_{(0.196)}$	$0.898_{(0.234)}$
	0.25	$0.916_{(0.176)}$	$0.949_{(0.210)}$	$0.904_{(0.154)}$	$0.934_{(0.183)}$	$0.892_{(0.139)}$	$0.925_{(0.165)}$
	0.50	$0.899_{(0.123)}$	$0.933_{(0.147)}$	$0.916_{(0.122)}$	$0.945_{(0.145)}$	$0.901_{(0.121)}$	$0.937_{(0.145)}$
1000	0	$0.900_{(0.193)}$	$0.935_{(0.230)}$	$0.894_{(0.162)}$	$0.932_{(0.193)}$	$0.897_{(0.142)}$	$0.927_{(0.169)}$
	0.25	$0.912_{(0.124)}$	$0.952_{(0.148)}$	$0.910_{(0.108)}$	$0.949_{(0.129)}$	$0.912_{(0.098)}$	$0.947_{(0.117)}$
	0.50	$0.906_{(0.086)}$	$0.945_{(0.102)}$	$0.900_{(0.086)}$	$0.938_{(0.102)}$	$0.894_{(0.086)}$	$0.946_{(0.102)}$
2000	0	$0.900_{(0.137)}$	$0.947_{(0.163)}$	$0.902_{(0.115)}$	$0.937_{(0.137)}$	$0.905_{(0.102)}$	$0.942_{(0.121)}$
	0.25	$0.922_{(0.088)}$	$0.964_{(0.105)}$	$0.927_{(0.077)}$	$0.959_{(0.092)}$	$0.916_{(0.070)}$	$0.952_{(0.083)}$
	0.50	$0.911_{(0.061)}$	$0.951_{(0.073)}$	$0.907_{(0.060)}$	$0.943_{(0.072)}$	$0.918_{(0.061)}$	$0.951_{(0.072)}$
				x_n chosen as the 95% quantile			
200	0	$0.860_{(0.382)}$	$0.889_{(0.455)}$	$0.827_{(0.328)}$	$0.860_{(0.391)}$	$0.829_{(0.276)}$	$0.862_{(0.329)}$
	0.25	$0.886_{(0.277)}$	$0.913_{(0.330)}$	$0.884_{(0.242)}$	$0.912_{(0.289)}$	$0.875_{(0.225)}$	$0.894_{(0.268)}$
	0.50	$0.886_{(0.196)}$	$0.915_{(0.233)}$	$0.899_{(0.190)}$	$0.923_{(0.226)}$	$0.882_{(0.191)}$	$0.913_{(0.227)}$
500	0	$0.876_{(0.266)}$	$0.914_{(0.317)}$	$0.870_{(0.226)}$	$0.912_{(0.269)}$	$0.867_{(0.196)}$	$0.912_{(0.233)}$
	0.25	$0.893_{(0.175)}$	$0.933_{(0.209)}$	$0.898_{(0.156)}$	$0.927_{(0.186)}$	$0.902_{(0.139)}$	$0.941_{(0.166)}$
	0.50	$0.897_{(0.121)}$	$0.934_{(0.144)}$	$0.893_{(0.120)}$	$0.937_{(0.143)}$	$0.884_{(0.123)}$	$0.926_{(0.146)}$
1000	0	$0.888_{(0.194)}$	$0.929_{(0.231)}$	$0.900_{(0.163)}$	$0.940_{(0.194)}$	$0.866_{(0.143)}$	$0.907_{(0.170)}$
	0.25	$0.892_{(0.125)}$	$0.950_{(0.149)}$	$0.907_{(0.108)}$	$0.942_{(0.129)}$	$0.929_{(0.098)}$	$0.954_{(0.117)}$
	0.50	$0.907_{(0.085)}$	$0.945_{(0.102)}$	$0.896_{(0.085)}$	$0.938_{(0.102)}$	$0.897_{(0.086)}$	$0.942_{(0.103)}$
2000	0	$0.909_{(0.139)}$	$0.952_{(0.166)}$	$0.893_{(0.117)}$	$0.949_{(0.139)}$	$0.912_{(0.102)}$	$0.943_{(0.122)}$
	0.25	$0.912_{(0.088)}$	$0.945_{(0.105)}$	$0.920_{(0.078)}$	$0.962_{(0.092)}$	$0.912_{(0.070)}$	$0.945_{(0.083)}$
	0.50	$0.912_{(0.061)}$	$0.954_{(0.072)}$	$0.919_{(0.061)}$	$0.953_{(0.072)}$	$0.903_{(0.060)}$	$0.949_{(0.072)}$

3.4.1.4 $B_n = 25$

Table 3.4: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with Trapezoidal kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 25$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.828_{(0.423)}$	$0.855_{(0.504)}$	$0.815_{(0.347)}$	$0.836_{(0.414)}$	$0.812_{(0.316)}$	$0.833_{(0.377)}$
	0.25	$0.867_{(0.307)}$	$0.893_{(0.366)}$	$0.862_{(0.272)}$	$0.891_{(0.324)}$	$0.860_{(0.246)}$	$0.888_{(0.293)}$
	0.50	$0.869_{(0.217)}$	$0.899_{(0.259)}$	$0.869_{(0.218)}$	$0.899_{(0.260)}$	$0.860_{(0.212)}$	$0.890_{(0.253)}$
500	0	$0.853_{(0.294)}$	$0.896_{(0.350)}$	$0.867_{(0.245)}$	$0.897_{(0.292)}$	$0.854_{(0.218)}$	$0.899_{(0.259)}$
	0.25	$0.874_{(0.197)}$	$0.915_{(0.234)}$	$0.888_{(0.172)}$	$0.921_{(0.205)}$	$0.888_{(0.157)}$	$0.926_{(0.188)}$
	0.50	$0.892_{(0.137)}$	$0.927_{(0.163)}$	$0.890_{(0.136)}$	$0.922_{(0.162)}$	$0.881_{(0.136)}$	$0.936_{(0.162)}$
1000	0	$0.874_{(0.215)}$	$0.919_{(0.256)}$	$0.877_{(0.178)}$	$0.906_{(0.212)}$	$0.890_{(0.158)}$	$0.919_{(0.189)}$
	0.25	$0.903_{(0.138)}$	$0.949_{(0.164)}$	$0.904_{(0.122)}$	$0.946_{(0.145)}$	$0.897_{(0.109)}$	$0.940_{(0.130)}$
	0.50	$0.888_{(0.095)}$	$0.937_{(0.113)}$	$0.904_{(0.096)}$	$0.940_{(0.115)}$	$0.899_{(0.095)}$	$0.945_{(0.114)}$
2000	0	$0.892_{(0.155)}$	$0.935_{(0.185)}$	$0.890_{(0.130)}$	$0.931_{(0.154)}$	$0.887_{(0.113)}$	$0.926_{(0.135)}$
	0.25	$0.914_{(0.099)}$	$0.951_{(0.118)}$	$0.918_{(0.086)}$	$0.958_{(0.103)}$	$0.911_{(0.077)}$	$0.946_{(0.092)}$
	0.50	$0.902_{(0.068)}$	$0.943_{(0.081)}$	$0.907_{(0.068)}$	$0.948_{(0.081)}$	$0.907_{(0.067)}$	$0.947_{(0.080)}$
				x_n chosen as the 95% quantile			
200	0	$0.846_{(0.397)}$	$0.874_{(0.473)}$	$0.810_{(0.349)}$	$0.844_{(0.415)}$	$0.820_{(0.301)}$	$0.849_{(0.358)}$
	0.25	$0.900_{(0.312)}$	$0.919_{(0.372)}$	$0.870_{(0.273)}$	$0.900_{(0.326)}$	$0.870_{(0.245)}$	$0.901_{(0.292)}$
	0.50	$0.888_{(0.214)}$	$0.914_{(0.255)}$	$0.892_{(0.217)}$	$0.918_{(0.259)}$	$0.874_{(0.216)}$	$0.901_{(0.258)}$
500	0	$0.877_{(0.288)}$	$0.918_{(0.343)}$	$0.866_{(0.246)}$	$0.897_{(0.293)}$	$0.867_{(0.217)}$	$0.902_{(0.259)}$
	0.25	$0.893_{(0.197)}$	$0.918_{(0.235)}$	$0.885_{(0.173)}$	$0.924_{(0.206)}$	$0.892_{(0.157)}$	$0.919_{(0.188)}$
	0.50	$0.898_{(0.137)}$	$0.925_{(0.163)}$	$0.902_{(0.136)}$	$0.940_{(0.162)}$	$0.897_{(0.136)}$	$0.935_{(0.162)}$
1000	0	$0.887_{(0.214)}$	$0.931_{(0.255)}$	$0.880_{(0.181)}$	$0.919_{(0.216)}$	$0.878_{(0.158)}$	$0.920_{(0.189)}$
	0.25	$0.911_{(0.141)}$	$0.939_{(0.167)}$	$0.903_{(0.122)}$	$0.941_{(0.146)}$	$0.887_{(0.108)}$	$0.934_{(0.129)}$
	0.50	$0.888_{(0.096)}$	$0.938_{(0.115)}$	$0.901_{(0.096)}$	$0.947_{(0.114)}$	$0.912_{(0.095)}$	$0.946_{(0.113)}$
2000	0	$0.900_{(0.155)}$	$0.941_{(0.184)}$	$0.892_{(0.130)}$	$0.937_{(0.155)}$	$0.896_{(0.115)}$	$0.938_{(0.137)}$
	0.25	$0.911_{(0.099)}$	$0.949_{(0.118)}$	$0.902_{(0.086)}$	$0.948_{(0.103)}$	$0.911_{(0.079)}$	$0.954_{(0.094)}$
	0.50	$0.915_{(0.068)}$	$0.954_{(0.081)}$	$0.908_{(0.068)}$	$0.948_{(0.081)}$	$0.916_{(0.068)}$	$0.951_{(0.081)}$

3.4.1.5 $B_n = 30$

Table 3.5: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with Trapezoidal kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 30$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.802_{(0.421)}$	$0.836_{(0.502)}$	$0.798_{(0.365)}$	$0.824_{(0.435)}$	$0.791_{(0.312)}$	$0.823_{(0.371)}$
	0.25	$0.861_{(0.340)}$	$0.901_{(0.405)}$	$0.847_{(0.289)}$	$0.881_{(0.345)}$	$0.866_{(0.272)}$	$0.895_{(0.324)}$
	0.50	$0.869_{(0.235)}$	$0.889_{(0.280)}$	$0.860_{(0.232)}$	$0.886_{(0.277)}$	$0.856_{(0.234)}$	$0.892_{(0.279)}$
500	0	$0.879_{(0.318)}$	$0.904_{(0.379)}$	$0.858_{(0.266)}$	$0.888_{(0.317)}$	$0.849_{(0.237)}$	$0.879_{(0.282)}$
	0.25	$0.878_{(0.215)}$	$0.916_{(0.257)}$	$0.880_{(0.186)}$	$0.912_{(0.222)}$	$0.886_{(0.169)}$	$0.919_{(0.202)}$
	0.50	$0.890_{(0.149)}$	$0.925_{(0.178)}$	$0.877_{(0.149)}$	$0.910_{(0.178)}$	$0.884_{(0.148)}$	$0.908_{(0.176)}$
1000	0	$0.878_{(0.233)}$	$0.917_{(0.277)}$	$0.895_{(0.195)}$	$0.923_{(0.233)}$	$0.884_{(0.173)}$	$0.923_{(0.207)}$
	0.25	$0.905_{(0.152)}$	$0.937_{(0.181)}$	$0.913_{(0.136)}$	$0.943_{(0.162)}$	$0.914_{(0.121)}$	$0.952_{(0.144)}$
	0.50	$0.896_{(0.105)}$	$0.939_{(0.125)}$	$0.905_{(0.105)}$	$0.936_{(0.125)}$	$0.894_{(0.105)}$	$0.935_{(0.125)}$
2000	0	$0.909_{(0.168)}$	$0.944_{(0.201)}$	$0.881_{(0.142)}$	$0.931_{(0.169)}$	$0.887_{(0.124)}$	$0.925_{(0.147)}$
	0.25	$0.903_{(0.107)}$	$0.949_{(0.128)}$	$0.907_{(0.095)}$	$0.943_{(0.113)}$	$0.907_{(0.085)}$	$0.951_{(0.102)}$
	0.50	$0.902_{(0.073)}$	$0.956_{(0.087)}$	$0.912_{(0.074)}$	$0.954_{(0.089)}$	$0.906_{(0.074)}$	$0.940_{(0.088)}$
				x_n chosen as the 95% quantile			
200	0	$0.808_{(0.427)}$	$0.839_{(0.508)}$	$0.816_{(0.363)}$	$0.846_{(0.432)}$	$0.803_{(0.313)}$	$0.827_{(0.373)}$
	0.25	$0.881_{(0.342)}$	$0.902_{(0.408)}$	$0.870_{(0.295)}$	$0.889_{(0.351)}$	$0.874_{(0.267)}$	$0.896_{(0.319)}$
	0.50	$0.871_{(0.238)}$	$0.895_{(0.284)}$	$0.885_{(0.236)}$	$0.913_{(0.281)}$	$0.867_{(0.234)}$	$0.889_{(0.279)}$
500	0	$0.855_{(0.311)}$	$0.894_{(0.370)}$	$0.860_{(0.271)}$	$0.891_{(0.323)}$	$0.854_{(0.227)}$	$0.882_{(0.271)}$
	0.25	$0.893_{(0.216)}$	$0.917_{(0.258)}$	$0.886_{(0.190)}$	$0.916_{(0.227)}$	$0.891_{(0.173)}$	$0.917_{(0.206)}$
	0.50	$0.896_{(0.150)}$	$0.926_{(0.179)}$	$0.886_{(0.148)}$	$0.918_{(0.176)}$	$0.883_{(0.148)}$	$0.914_{(0.177)}$
1000	0	$0.882_{(0.230)}$	$0.924_{(0.274)}$	$0.888_{(0.193)}$	$0.925_{(0.230)}$	$0.863_{(0.172)}$	$0.900_{(0.204)}$
	0.25	$0.913_{(0.154)}$	$0.935_{(0.183)}$	$0.899_{(0.133)}$	$0.937_{(0.158)}$	$0.881_{(0.121)}$	$0.930_{(0.144)}$
	0.50	$0.899_{(0.105)}$	$0.945_{(0.125)}$	$0.908_{(0.105)}$	$0.949_{(0.126)}$	$0.891_{(0.106)}$	$0.932_{(0.127)}$
2000	0	$0.896_{(0.169)}$	$0.936_{(0.201)}$	$0.875_{(0.142)}$	$0.919_{(0.170)}$	$0.892_{(0.125)}$	$0.931_{(0.149)}$
	0.25	$0.905_{(0.109)}$	$0.948_{(0.129)}$	$0.895_{(0.095)}$	$0.945_{(0.113)}$	$0.886_{(0.086)}$	$0.933_{(0.102)}$
	0.50	$0.893_{(0.074)}$	$0.943_{(0.088)}$	$0.902_{(0.074)}$	$0.939_{(0.089)}$	$0.907_{(0.075)}$	$0.952_{(0.089)}$

3.4.2 Moving-Maximum with PM04 Kernel

3.4.2.1 $B_n = 10$

Table 3.6: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with PM04 kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 10$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.897_{(0.261)}$	$0.924_{(0.311)}$	$0.887_{(0.221)}$	$0.923_{(0.264)}$	$0.891_{(0.194)}$	$0.925_{(0.231)}$
	0.25	$0.914_{(0.174)}$	$0.939_{(0.208)}$	$0.910_{(0.152)}$	$0.934_{(0.181)}$	$0.914_{(0.139)}$	$0.943_{(0.165)}$
	0.50	$0.912_{(0.120)}$	$0.949_{(0.143)}$	$0.908_{(0.120)}$	$0.939_{(0.144)}$	$0.905_{(0.118)}$	$0.938_{(0.141)}$
500	0	$0.927_{(0.170)}$	$0.956_{(0.202)}$	$0.917_{(0.144)}$	$0.957_{(0.172)}$	$0.895_{(0.126)}$	$0.941_{(0.150)}$
	0.25	$0.939_{(0.110)}$	$0.964_{(0.131)}$	$0.913_{(0.096)}$	$0.956_{(0.114)}$	$0.924_{(0.087)}$	$0.948_{(0.103)}$
	0.50	$0.923_{(0.075)}$	$0.953_{(0.090)}$	$0.924_{(0.075)}$	$0.957_{(0.089)}$	$0.928_{(0.075)}$	$0.959_{(0.090)}$
1000	0	$0.932_{(0.122)}$	$0.969_{(0.145)}$	$0.903_{(0.103)}$	$0.960_{(0.122)}$	$0.919_{(0.090)}$	$0.958_{(0.108)}$
	0.25	$0.946_{(0.078)}$	$0.969_{(0.092)}$	$0.932_{(0.067)}$	$0.964_{(0.080)}$	$0.929_{(0.061)}$	$0.963_{(0.072)}$
	0.50	$0.918_{(0.053)}$	$0.962_{(0.063)}$	$0.930_{(0.053)}$	$0.968_{(0.063)}$	$0.932_{(0.053)}$	$0.968_{(0.063)}$
2000	0	$0.925_{(0.087)}$	$0.965_{(0.103)}$	$0.917_{(0.073)}$	$0.956_{(0.087)}$	$0.903_{(0.063)}$	$0.950_{(0.076)}$
	0.25	$0.944_{(0.054)}$	$0.974_{(0.065)}$	$0.929_{(0.048)}$	$0.970_{(0.057)}$	$0.928_{(0.043)}$	$0.970_{(0.051)}$
	0.50	$0.930_{(0.037)}$	$0.964_{(0.044)}$	$0.935_{(0.037)}$	$0.965_{(0.044)}$	$0.937_{(0.037)}$	$0.962_{(0.044)}$
				x_n chosen as the 95% quantile			
200	0	$0.937_{(0.262)}$	$0.961_{(0.311)}$	$0.886_{(0.218)}$	$0.937_{(0.260)}$	$0.874_{(0.194)}$	$0.922_{(0.231)}$
	0.25	$0.890_{(0.173)}$	$0.932_{(0.207)}$	$0.896_{(0.152)}$	$0.915_{(0.182)}$	$0.897_{(0.137)}$	$0.927_{(0.164)}$
	0.50	$0.918_{(0.121)}$	$0.940_{(0.144)}$	$0.918_{(0.119)}$	$0.940_{(0.142)}$	$0.905_{(0.119)}$	$0.926_{(0.142)}$
500	0	$0.933_{(0.170)}$	$0.970_{(0.202)}$	$0.916_{(0.145)}$	$0.952_{(0.173)}$	$0.911_{(0.125)}$	$0.949_{(0.149)}$
	0.25	$0.927_{(0.111)}$	$0.954_{(0.132)}$	$0.897_{(0.096)}$	$0.932_{(0.114)}$	$0.911_{(0.087)}$	$0.940_{(0.104)}$
	0.50	$0.933_{(0.075)}$	$0.962_{(0.090)}$	$0.930_{(0.075)}$	$0.960_{(0.090)}$	$0.925_{(0.075)}$	$0.951_{(0.090)}$
1000	0	$0.923_{(0.124)}$	$0.966_{(0.147)}$	$0.899_{(0.103)}$	$0.952_{(0.122)}$	$0.894_{(0.090)}$	$0.946_{(0.107)}$
	0.25	$0.933_{(0.077)}$	$0.961_{(0.092)}$	$0.894_{(0.068)}$	$0.939_{(0.081)}$	$0.910_{(0.061)}$	$0.955_{(0.073)}$
	0.50	$0.906_{(0.053)}$	$0.952_{(0.063)}$	$0.925_{(0.053)}$	$0.960_{(0.063)}$	$0.912_{(0.053)}$	$0.952_{(0.063)}$
2000	0	$0.920_{(0.087)}$	$0.965_{(0.104)}$	$0.897_{(0.073)}$	$0.950_{(0.087)}$	$0.898_{(0.064)}$	$0.954_{(0.076)}$
	0.25	$0.917_{(0.054)}$	$0.958_{(0.065)}$	$0.913_{(0.048)}$	$0.955_{(0.057)}$	$0.918_{(0.043)}$	$0.959_{(0.051)}$
	0.50	$0.921_{(0.037)}$	$0.969_{(0.045)}$	$0.933_{(0.037)}$	$0.963_{(0.045)}$	$0.916_{(0.037)}$	$0.961_{(0.044)}$

3.4.2.2 $B_n = 15$

Table 3.7: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with PM04 kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 15$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.895_{(0.308)}$	$0.924_{(0.367)}$	$0.860_{(0.269)}$	$0.891_{(0.320)}$	$0.850_{(0.234)}$	$0.891_{(0.279)}$
	0.25	$0.916_{(0.211)}$	$0.942_{(0.252)}$	$0.896_{(0.185)}$	$0.931_{(0.220)}$	$0.886_{(0.167)}$	$0.918_{(0.199)}$
	0.50	$0.882_{(0.149)}$	$0.913_{(0.177)}$	$0.891_{(0.147)}$	$0.924_{(0.175)}$	$0.894_{(0.146)}$	$0.926_{(0.174)}$
500	0	$0.893_{(0.207)}$	$0.934_{(0.247)}$	$0.893_{(0.173)}$	$0.932_{(0.206)}$	$0.889_{(0.153)}$	$0.933_{(0.183)}$
	0.25	$0.923_{(0.135)}$	$0.953_{(0.161)}$	$0.920_{(0.118)}$	$0.947_{(0.141)}$	$0.914_{(0.106)}$	$0.939_{(0.126)}$
	0.50	$0.911_{(0.093)}$	$0.948_{(0.110)}$	$0.904_{(0.093)}$	$0.944_{(0.111)}$	$0.919_{(0.092)}$	$0.954_{(0.110)}$
1000	0	$0.924_{(0.150)}$	$0.966_{(0.179)}$	$0.897_{(0.126)}$	$0.934_{(0.150)}$	$0.908_{(0.111)}$	$0.951_{(0.133)}$
	0.25	$0.929_{(0.095)}$	$0.962_{(0.113)}$	$0.915_{(0.083)}$	$0.957_{(0.099)}$	$0.922_{(0.075)}$	$0.953_{(0.090)}$
	0.50	$0.921_{(0.065)}$	$0.960_{(0.078)}$	$0.914_{(0.065)}$	$0.949_{(0.078)}$	$0.934_{(0.065)}$	$0.961_{(0.077)}$
2000	0	$0.920_{(0.107)}$	$0.964_{(0.128)}$	$0.907_{(0.090)}$	$0.955_{(0.107)}$	$0.899_{(0.078)}$	$0.944_{(0.093)}$
	0.25	$0.935_{(0.067)}$	$0.968_{(0.080)}$	$0.935_{(0.059)}$	$0.964_{(0.070)}$	$0.918_{(0.053)}$	$0.956_{(0.063)}$
	0.50	$0.916_{(0.046)}$	$0.962_{(0.055)}$	$0.916_{(0.046)}$	$0.958_{(0.055)}$	$0.930_{(0.046)}$	$0.968_{(0.055)}$
				x_n chosen as the 95% quantile			
200	0	$0.913_{(0.304)}$	$0.938_{(0.362)}$	$0.862_{(0.272)}$	$0.909_{(0.324)}$	$0.857_{(0.238)}$	$0.899_{(0.283)}$
	0.25	$0.915_{(0.213)}$	$0.939_{(0.254)}$	$0.894_{(0.194)}$	$0.906_{(0.231)}$	$0.878_{(0.177)}$	$0.912_{(0.211)}$
	0.50	$0.902_{(0.146)}$	$0.932_{(0.174)}$	$0.887_{(0.154)}$	$0.912_{(0.184)}$	$0.901_{(0.156)}$	$0.924_{(0.186)}$
500	0	$0.930_{(0.207)}$	$0.963_{(0.246)}$	$0.903_{(0.176)}$	$0.938_{(0.209)}$	$0.876_{(0.152)}$	$0.919_{(0.181)}$
	0.25	$0.918_{(0.136)}$	$0.948_{(0.162)}$	$0.885_{(0.116)}$	$0.923_{(0.139)}$	$0.903_{(0.107)}$	$0.928_{(0.127)}$
	0.50	$0.899_{(0.093)}$	$0.950_{(0.110)}$	$0.914_{(0.093)}$	$0.940_{(0.110)}$	$0.926_{(0.092)}$	$0.961_{(0.110)}$
1000	0	$0.906_{(0.148)}$	$0.950_{(0.177)}$	$0.898_{(0.126)}$	$0.939_{(0.150)}$	$0.900_{(0.110)}$	$0.944_{(0.131)}$
	0.25	$0.923_{(0.095)}$	$0.955_{(0.114)}$	$0.926_{(0.084)}$	$0.955_{(0.100)}$	$0.904_{(0.075)}$	$0.939_{(0.090)}$
	0.50	$0.907_{(0.065)}$	$0.939_{(0.078)}$	$0.916_{(0.065)}$	$0.952_{(0.077)}$	$0.930_{(0.065)}$	$0.958_{(0.078)}$
2000	0	$0.904_{(0.107)}$	$0.955_{(0.127)}$	$0.881_{(0.090)}$	$0.947_{(0.107)}$	$0.892_{(0.079)}$	$0.944_{(0.094)}$
	0.25	$0.928_{(0.067)}$	$0.961_{(0.080)}$	$0.925_{(0.059)}$	$0.964_{(0.070)}$	$0.883_{(0.053)}$	$0.935_{(0.063)}$
	0.50	$0.914_{(0.046)}$	$0.952_{(0.055)}$	$0.906_{(0.046)}$	$0.947_{(0.055)}$	$0.924_{(0.046)}$	$0.953_{(0.055)}$

3.4.2.3 $B_n = 20$

Table 3.8: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with PM04 kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 20$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.859_{(0.352)}$	$0.889_{(0.419)}$	$0.841_{(0.295)}$	$0.876_{(0.352)}$	$0.834_{(0.261)}$	$0.864_{(0.311)}$
	0.25	$0.887_{(0.242)}$	$0.916_{(0.289)}$	$0.898_{(0.213)}$	$0.931_{(0.254)}$	$0.887_{(0.194)}$	$0.914_{(0.232)}$
	0.50	$0.882_{(0.172)}$	$0.918_{(0.205)}$	$0.881_{(0.172)}$	$0.914_{(0.205)}$	$0.886_{(0.169)}$	$0.916_{(0.202)}$
500	0	$0.886_{(0.238)}$	$0.930_{(0.284)}$	$0.881_{(0.198)}$	$0.924_{(0.235)}$	$0.889_{(0.174)}$	$0.931_{(0.208)}$
	0.25	$0.905_{(0.153)}$	$0.945_{(0.182)}$	$0.913_{(0.136)}$	$0.945_{(0.162)}$	$0.913_{(0.122)}$	$0.950_{(0.146)}$
	0.50	$0.904_{(0.107)}$	$0.936_{(0.128)}$	$0.908_{(0.107)}$	$0.934_{(0.127)}$	$0.893_{(0.106)}$	$0.938_{(0.126)}$
1000	0	$0.891_{(0.171)}$	$0.944_{(0.204)}$	$0.897_{(0.143)}$	$0.929_{(0.171)}$	$0.912_{(0.128)}$	$0.939_{(0.153)}$
	0.25	$0.920_{(0.109)}$	$0.955_{(0.130)}$	$0.897_{(0.095)}$	$0.935_{(0.113)}$	$0.899_{(0.087)}$	$0.940_{(0.104)}$
	0.50	$0.916_{(0.075)}$	$0.947_{(0.090)}$	$0.918_{(0.075)}$	$0.954_{(0.090)}$	$0.909_{(0.075)}$	$0.948_{(0.089)}$
2000	0	$0.904_{(0.123)}$	$0.949_{(0.146)}$	$0.907_{(0.103)}$	$0.946_{(0.123)}$	$0.893_{(0.090)}$	$0.939_{(0.107)}$
	0.25	$0.920_{(0.077)}$	$0.960_{(0.092)}$	$0.916_{(0.068)}$	$0.957_{(0.081)}$	$0.900_{(0.061)}$	$0.944_{(0.073)}$
	0.50	$0.923_{(0.054)}$	$0.961_{(0.064)}$	$0.905_{(0.054)}$	$0.961_{(0.064)}$	$0.920_{(0.053)}$	$0.957_{(0.063)}$
				x_n chosen as the 95% quantile			
200	0	$0.889_{(0.341)}$	$0.920_{(0.407)}$	$0.867_{(0.290)}$	$0.900_{(0.345)}$	$0.856_{(0.258)}$	$0.884_{(0.307)}$
	0.25	$0.911_{(0.248)}$	$0.935_{(0.295)}$	$0.893_{(0.213)}$	$0.911_{(0.254)}$	$0.872_{(0.193)}$	$0.908_{(0.230)}$
	0.50	$0.899_{(0.169)}$	$0.926_{(0.202)}$	$0.894_{(0.168)}$	$0.923_{(0.200)}$	$0.893_{(0.169)}$	$0.930_{(0.202)}$
500	0	$0.888_{(0.239)}$	$0.930_{(0.285)}$	$0.899_{(0.199)}$	$0.935_{(0.237)}$	$0.874_{(0.175)}$	$0.917_{(0.208)}$
	0.25	$0.912_{(0.156)}$	$0.945_{(0.186)}$	$0.890_{(0.136)}$	$0.937_{(0.163)}$	$0.900_{(0.123)}$	$0.941_{(0.147)}$
	0.50	$0.915_{(0.106)}$	$0.942_{(0.127)}$	$0.904_{(0.107)}$	$0.937_{(0.128)}$	$0.907_{(0.107)}$	$0.944_{(0.128)}$
1000	0	$0.912_{(0.172)}$	$0.957_{(0.205)}$	$0.899_{(0.144)}$	$0.938_{(0.172)}$	$0.896_{(0.127)}$	$0.939_{(0.152)}$
	0.25	$0.916_{(0.111)}$	$0.954_{(0.132)}$	$0.892_{(0.097)}$	$0.937_{(0.115)}$	$0.908_{(0.087)}$	$0.948_{(0.104)}$
	0.50	$0.922_{(0.076)}$	$0.950_{(0.091)}$	$0.922_{(0.075)}$	$0.952_{(0.090)}$	$0.900_{(0.075)}$	$0.932_{(0.090)}$
2000	0	$0.922_{(0.122)}$	$0.955_{(0.146)}$	$0.891_{(0.104)}$	$0.943_{(0.124)}$	$0.914_{(0.091)}$	$0.948_{(0.109)}$
	0.25	$0.919_{(0.078)}$	$0.957_{(0.093)}$	$0.899_{(0.068)}$	$0.946_{(0.081)}$	$0.904_{(0.062)}$	$0.953_{(0.073)}$
	0.50	$0.910_{(0.053)}$	$0.962_{(0.063)}$	$0.915_{(0.053)}$	$0.949_{(0.063)}$	$0.905_{(0.053)}$	$0.946_{(0.064)}$

3.4.2.4 $B_n = 25$

Table 3.9: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with PM04 kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 25$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.849_{(0.373)}$	$0.875_{(0.445)}$	$0.818_{(0.319)}$	$0.848_{(0.381)}$	$0.816_{(0.278)}$	$0.846_{(0.332)}$
	0.25	$0.882_{(0.278)}$	$0.907_{(0.331)}$	$0.881_{(0.242)}$	$0.917_{(0.288)}$	$0.873_{(0.218)}$	$0.915_{(0.260)}$
	0.50	$0.869_{(0.191)}$	$0.894_{(0.228)}$	$0.870_{(0.190)}$	$0.890_{(0.226)}$	$0.887_{(0.190)}$	$0.920_{(0.227)}$
500	0	$0.876_{(0.263)}$	$0.909_{(0.314)}$	$0.871_{(0.219)}$	$0.912_{(0.260)}$	$0.861_{(0.192)}$	$0.902_{(0.229)}$
	0.25	$0.912_{(0.175)}$	$0.946_{(0.209)}$	$0.897_{(0.150)}$	$0.930_{(0.179)}$	$0.895_{(0.137)}$	$0.932_{(0.163)}$
	0.50	$0.894_{(0.119)}$	$0.932_{(0.141)}$	$0.902_{(0.120)}$	$0.936_{(0.143)}$	$0.889_{(0.120)}$	$0.932_{(0.143)}$
1000	0	$0.906_{(0.191)}$	$0.938_{(0.227)}$	$0.895_{(0.162)}$	$0.930_{(0.193)}$	$0.892_{(0.139)}$	$0.931_{(0.166)}$
	0.25	$0.923_{(0.122)}$	$0.957_{(0.146)}$	$0.914_{(0.107)}$	$0.954_{(0.128)}$	$0.904_{(0.096)}$	$0.947_{(0.115)}$
	0.50	$0.898_{(0.084)}$	$0.940_{(0.101)}$	$0.914_{(0.085)}$	$0.950_{(0.101)}$	$0.887_{(0.084)}$	$0.936_{(0.100)}$
2000	0	$0.911_{(0.137)}$	$0.959_{(0.163)}$	$0.889_{(0.114)}$	$0.944_{(0.135)}$	$0.895_{(0.100)}$	$0.933_{(0.120)}$
	0.25	$0.916_{(0.087)}$	$0.950_{(0.104)}$	$0.903_{(0.076)}$	$0.951_{(0.090)}$	$0.908_{(0.069)}$	$0.948_{(0.082)}$
	0.50	$0.905_{(0.060)}$	$0.955_{(0.071)}$	$0.907_{(0.060)}$	$0.959_{(0.071)}$	$0.896_{(0.060)}$	$0.940_{(0.071)}$
				x_n chosen as the 95% quantile			
200	0	$0.852_{(0.371)}$	$0.891_{(0.442)}$	$0.848_{(0.305)}$	$0.877_{(0.364)}$	$0.819_{(0.278)}$	$0.856_{(0.331)}$
	0.25	$0.905_{(0.272)}$	$0.923_{(0.325)}$	$0.878_{(0.237)}$	$0.900_{(0.282)}$	$0.878_{(0.217)}$	$0.909_{(0.259)}$
	0.50	$0.893_{(0.191)}$	$0.924_{(0.228)}$	$0.909_{(0.193)}$	$0.929_{(0.230)}$	$0.882_{(0.190)}$	$0.918_{(0.226)}$
500	0	$0.882_{(0.263)}$	$0.920_{(0.313)}$	$0.875_{(0.223)}$	$0.914_{(0.267)}$	$0.875_{(0.192)}$	$0.907_{(0.228)}$
	0.25	$0.907_{(0.173)}$	$0.948_{(0.206)}$	$0.902_{(0.150)}$	$0.932_{(0.179)}$	$0.890_{(0.138)}$	$0.934_{(0.165)}$
	0.50	$0.901_{(0.119)}$	$0.942_{(0.142)}$	$0.895_{(0.119)}$	$0.932_{(0.141)}$	$0.915_{(0.118)}$	$0.942_{(0.141)}$
1000	0	$0.907_{(0.191)}$	$0.940_{(0.228)}$	$0.911_{(0.157)}$	$0.945_{(0.187)}$	$0.881_{(0.141)}$	$0.923_{(0.168)}$
	0.25	$0.923_{(0.123)}$	$0.950_{(0.147)}$	$0.905_{(0.108)}$	$0.951_{(0.128)}$	$0.897_{(0.096)}$	$0.943_{(0.115)}$
	0.50	$0.920_{(0.084)}$	$0.951_{(0.100)}$	$0.902_{(0.084)}$	$0.942_{(0.100)}$	$0.892_{(0.085)}$	$0.933_{(0.101)}$
2000	0	$0.903_{(0.138)}$	$0.939_{(0.165)}$	$0.894_{(0.116)}$	$0.942_{(0.138)}$	$0.884_{(0.101)}$	$0.937_{(0.120)}$
	0.25	$0.913_{(0.088)}$	$0.951_{(0.105)}$	$0.918_{(0.076)}$	$0.948_{(0.090)}$	$0.902_{(0.069)}$	$0.949_{(0.082)}$
	0.50	$0.906_{(0.060)}$	$0.959_{(0.071)}$	$0.895_{(0.059)}$	$0.946_{(0.071)}$	$0.904_{(0.060)}$	$0.952_{(0.071)}$

3.4.2.5 $B_n = 30$

Table 3.10: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with PM04 kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 30$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.809_{(0.401)}$	$0.835_{(0.478)}$	$0.835_{(0.332)}$	$0.868_{(0.396)}$	$0.818_{(0.288)}$	$0.843_{(0.344)}$
	0.25	$0.889_{(0.300)}$	$0.915_{(0.357)}$	$0.881_{(0.268)}$	$0.903_{(0.319)}$	$0.863_{(0.235)}$	$0.901_{(0.280)}$
	0.50	$0.864_{(0.209)}$	$0.902_{(0.249)}$	$0.862_{(0.210)}$	$0.888_{(0.250)}$	$0.857_{(0.209)}$	$0.893_{(0.249)}$
500	0	$0.866_{(0.277)}$	$0.909_{(0.330)}$	$0.879_{(0.235)}$	$0.913_{(0.280)}$	$0.870_{(0.208)}$	$0.904_{(0.247)}$
	0.25	$0.906_{(0.189)}$	$0.939_{(0.226)}$	$0.896_{(0.167)}$	$0.931_{(0.199)}$	$0.892_{(0.150)}$	$0.921_{(0.179)}$
	0.50	$0.888_{(0.131)}$	$0.933_{(0.157)}$	$0.888_{(0.131)}$	$0.922_{(0.156)}$	$0.901_{(0.129)}$	$0.923_{(0.154)}$
1000	0	$0.909_{(0.205)}$	$0.940_{(0.244)}$	$0.873_{(0.175)}$	$0.910_{(0.209)}$	$0.879_{(0.154)}$	$0.917_{(0.183)}$
	0.25	$0.917_{(0.135)}$	$0.946_{(0.161)}$	$0.907_{(0.117)}$	$0.946_{(0.139)}$	$0.897_{(0.106)}$	$0.934_{(0.126)}$
	0.50	$0.892_{(0.092)}$	$0.938_{(0.110)}$	$0.903_{(0.092)}$	$0.934_{(0.110)}$	$0.908_{(0.092)}$	$0.945_{(0.110)}$
2000	0	$0.900_{(0.148)}$	$0.940_{(0.177)}$	$0.884_{(0.124)}$	$0.944_{(0.148)}$	$0.903_{(0.111)}$	$0.947_{(0.132)}$
	0.25	$0.900_{(0.095)}$	$0.947_{(0.112)}$	$0.915_{(0.083)}$	$0.962_{(0.099)}$	$0.903_{(0.075)}$	$0.950_{(0.089)}$
	0.50	$0.901_{(0.065)}$	$0.953_{(0.078)}$	$0.920_{(0.065)}$	$0.965_{(0.078)}$	$0.905_{(0.065)}$	$0.947_{(0.078)}$
				x_n chosen as the 95% quantile			
200	0	$0.849_{(0.399)}$	$0.878_{(0.475)}$	$0.835_{(0.338)}$	$0.866_{(0.403)}$	$0.835_{(0.291)}$	$0.873_{(0.346)}$
	0.25	$0.880_{(0.302)}$	$0.906_{(0.360)}$	$0.871_{(0.260)}$	$0.900_{(0.310)}$	$0.887_{(0.235)}$	$0.913_{(0.280)}$
	0.50	$0.885_{(0.208)}$	$0.906_{(0.247)}$	$0.883_{(0.210)}$	$0.909_{(0.251)}$	$0.875_{(0.203)}$	$0.911_{(0.242)}$
500	0	$0.872_{(0.285)}$	$0.903_{(0.340)}$	$0.870_{(0.237)}$	$0.907_{(0.282)}$	$0.870_{(0.208)}$	$0.912_{(0.248)}$
	0.25	$0.900_{(0.188)}$	$0.930_{(0.224)}$	$0.896_{(0.163)}$	$0.932_{(0.194)}$	$0.907_{(0.150)}$	$0.928_{(0.178)}$
	0.50	$0.898_{(0.131)}$	$0.933_{(0.156)}$	$0.886_{(0.132)}$	$0.916_{(0.157)}$	$0.877_{(0.131)}$	$0.923_{(0.156)}$
1000	0	$0.913_{(0.208)}$	$0.940_{(0.248)}$	$0.880_{(0.173)}$	$0.915_{(0.206)}$	$0.883_{(0.155)}$	$0.928_{(0.184)}$
	0.25	$0.904_{(0.136)}$	$0.940_{(0.162)}$	$0.907_{(0.118)}$	$0.946_{(0.141)}$	$0.904_{(0.107)}$	$0.944_{(0.127)}$
	0.50	$0.900_{(0.093)}$	$0.942_{(0.111)}$	$0.902_{(0.093)}$	$0.949_{(0.111)}$	$0.896_{(0.091)}$	$0.936_{(0.109)}$
2000	0	$0.916_{(0.150)}$	$0.950_{(0.178)}$	$0.884_{(0.126)}$	$0.929_{(0.151)}$	$0.880_{(0.110)}$	$0.931_{(0.131)}$
	0.25	$0.927_{(0.096)}$	$0.956_{(0.114)}$	$0.900_{(0.084)}$	$0.947_{(0.100)}$	$0.904_{(0.076)}$	$0.940_{(0.090)}$
	0.50	$0.906_{(0.065)}$	$0.953_{(0.078)}$	$0.893_{(0.066)}$	$0.941_{(0.078)}$	$0.907_{(0.065)}$	$0.950_{(0.078)}$

3.4.3 Moving-Maximum with P61 Kernel

3.4.3.1 $B_n = 10$

Table 3.11: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with P61Rescaled kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 10$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.900_{(0.274)}$	$0.937_{(0.327)}$	$0.895_{(0.232)}$	$0.929_{(0.277)}$	$0.872_{(0.200)}$	$0.908_{(0.239)}$
	0.25	$0.905_{(0.186)}$	$0.936_{(0.222)}$	$0.899_{(0.161)}$	$0.929_{(0.192)}$	$0.897_{(0.145)}$	$0.930_{(0.173)}$
	0.50	$0.906_{(0.127)}$	$0.933_{(0.152)}$	$0.911_{(0.127)}$	$0.946_{(0.151)}$	$0.911_{(0.127)}$	$0.939_{(0.151)}$
500	0	$0.920_{(0.180)}$	$0.953_{(0.215)}$	$0.901_{(0.151)}$	$0.945_{(0.180)}$	$0.881_{(0.134)}$	$0.920_{(0.159)}$
	0.25	$0.939_{(0.116)}$	$0.958_{(0.138)}$	$0.922_{(0.102)}$	$0.947_{(0.121)}$	$0.934_{(0.092)}$	$0.951_{(0.109)}$
	0.50	$0.912_{(0.080)}$	$0.948_{(0.095)}$	$0.917_{(0.080)}$	$0.954_{(0.095)}$	$0.929_{(0.079)}$	$0.959_{(0.095)}$
1000	0	$0.922_{(0.129)}$	$0.958_{(0.154)}$	$0.921_{(0.109)}$	$0.952_{(0.130)}$	$0.904_{(0.094)}$	$0.940_{(0.112)}$
	0.25	$0.942_{(0.082)}$	$0.970_{(0.098)}$	$0.920_{(0.071)}$	$0.958_{(0.085)}$	$0.912_{(0.065)}$	$0.952_{(0.078)}$
	0.50	$0.924_{(0.056)}$	$0.956_{(0.067)}$	$0.917_{(0.056)}$	$0.959_{(0.067)}$	$0.927_{(0.056)}$	$0.957_{(0.067)}$
2000	0	$0.936_{(0.092)}$	$0.973_{(0.109)}$	$0.907_{(0.077)}$	$0.963_{(0.092)}$	$0.927_{(0.067)}$	$0.969_{(0.081)}$
	0.25	$0.945_{(0.058)}$	$0.975_{(0.069)}$	$0.936_{(0.050)}$	$0.969_{(0.060)}$	$0.926_{(0.045)}$	$0.966_{(0.054)}$
	0.50	$0.922_{(0.040)}$	$0.964_{(0.047)}$	$0.931_{(0.039)}$	$0.967_{(0.047)}$	$0.931_{(0.040)}$	$0.957_{(0.047)}$
				x_n chosen as the 95% quantile			
200	0	$0.927_{(0.275)}$	$0.957_{(0.328)}$	$0.892_{(0.233)}$	$0.923_{(0.278)}$	$0.881_{(0.199)}$	$0.923_{(0.237)}$
	0.25	$0.895_{(0.185)}$	$0.935_{(0.220)}$	$0.898_{(0.162)}$	$0.919_{(0.193)}$	$0.907_{(0.147)}$	$0.926_{(0.175)}$
	0.50	$0.911_{(0.127)}$	$0.928_{(0.151)}$	$0.908_{(0.125)}$	$0.934_{(0.149)}$	$0.920_{(0.127)}$	$0.946_{(0.151)}$
500	0	$0.923_{(0.181)}$	$0.970_{(0.216)}$	$0.898_{(0.151)}$	$0.941_{(0.181)}$	$0.890_{(0.134)}$	$0.933_{(0.160)}$
	0.25	$0.919_{(0.117)}$	$0.946_{(0.140)}$	$0.909_{(0.102)}$	$0.943_{(0.122)}$	$0.913_{(0.091)}$	$0.951_{(0.109)}$
	0.50	$0.920_{(0.079)}$	$0.941_{(0.095)}$	$0.937_{(0.079)}$	$0.964_{(0.095)}$	$0.925_{(0.080)}$	$0.953_{(0.095)}$
1000	0	$0.901_{(0.130)}$	$0.956_{(0.155)}$	$0.902_{(0.110)}$	$0.945_{(0.131)}$	$0.900_{(0.095)}$	$0.948_{(0.113)}$
	0.25	$0.919_{(0.082)}$	$0.943_{(0.098)}$	$0.902_{(0.072)}$	$0.943_{(0.085)}$	$0.917_{(0.065)}$	$0.948_{(0.077)}$
	0.50	$0.926_{(0.056)}$	$0.963_{(0.067)}$	$0.910_{(0.056)}$	$0.944_{(0.067)}$	$0.935_{(0.056)}$	$0.961_{(0.067)}$
2000	0	$0.924_{(0.092)}$	$0.965_{(0.110)}$	$0.905_{(0.078)}$	$0.954_{(0.093)}$	$0.906_{(0.068)}$	$0.952_{(0.081)}$
	0.25	$0.927_{(0.058)}$	$0.955_{(0.069)}$	$0.931_{(0.051)}$	$0.959_{(0.061)}$	$0.914_{(0.046)}$	$0.953_{(0.054)}$
	0.50	$0.918_{(0.040)}$	$0.960_{(0.047)}$	$0.928_{(0.039)}$	$0.967_{(0.047)}$	$0.926_{(0.040)}$	$0.957_{(0.047)}$

3.4.3.2 $B_n = 15$

Table 3.12: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with P61Rescaled kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 15$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.850_{(0.331)}$	$0.899_{(0.394)}$	$0.867_{(0.274)}$	$0.898_{(0.327)}$	$0.849_{(0.243)}$	$0.875_{(0.290)}$
	0.25	$0.900_{(0.223)}$	$0.921_{(0.266)}$	$0.880_{(0.197)}$	$0.914_{(0.235)}$	$0.886_{(0.176)}$	$0.921_{(0.210)}$
	0.50	$0.887_{(0.155)}$	$0.912_{(0.185)}$	$0.897_{(0.155)}$	$0.929_{(0.184)}$	$0.895_{(0.156)}$	$0.924_{(0.186)}$
500	0	$0.898_{(0.219)}$	$0.927_{(0.260)}$	$0.874_{(0.185)}$	$0.922_{(0.220)}$	$0.879_{(0.159)}$	$0.922_{(0.190)}$
	0.25	$0.930_{(0.142)}$	$0.953_{(0.169)}$	$0.913_{(0.123)}$	$0.949_{(0.147)}$	$0.915_{(0.112)}$	$0.941_{(0.133)}$
	0.50	$0.901_{(0.098)}$	$0.935_{(0.117)}$	$0.903_{(0.097)}$	$0.944_{(0.116)}$	$0.905_{(0.097)}$	$0.949_{(0.116)}$
1000	0	$0.890_{(0.158)}$	$0.936_{(0.188)}$	$0.895_{(0.134)}$	$0.945_{(0.159)}$	$0.888_{(0.117)}$	$0.934_{(0.140)}$
	0.25	$0.925_{(0.100)}$	$0.956_{(0.119)}$	$0.920_{(0.088)}$	$0.963_{(0.105)}$	$0.913_{(0.080)}$	$0.948_{(0.095)}$
	0.50	$0.915_{(0.069)}$	$0.951_{(0.082)}$	$0.917_{(0.069)}$	$0.954_{(0.083)}$	$0.923_{(0.069)}$	$0.959_{(0.082)}$
2000	0	$0.903_{(0.113)}$	$0.941_{(0.134)}$	$0.908_{(0.096)}$	$0.953_{(0.114)}$	$0.908_{(0.083)}$	$0.962_{(0.099)}$
	0.25	$0.918_{(0.071)}$	$0.961_{(0.084)}$	$0.914_{(0.062)}$	$0.958_{(0.074)}$	$0.924_{(0.056)}$	$0.958_{(0.067)}$
	0.50	$0.924_{(0.049)}$	$0.956_{(0.058)}$	$0.902_{(0.049)}$	$0.944_{(0.058)}$	$0.914_{(0.049)}$	$0.961_{(0.058)}$
				x_n chosen as the 95% quantile			
200	0	$0.893_{(0.322)}$	$0.933_{(0.384)}$	$0.886_{(0.270)}$	$0.917_{(0.322)}$	$0.842_{(0.243)}$	$0.888_{(0.289)}$
	0.25	$0.909_{(0.223)}$	$0.930_{(0.266)}$	$0.896_{(0.195)}$	$0.920_{(0.232)}$	$0.872_{(0.177)}$	$0.900_{(0.211)}$
	0.50	$0.917_{(0.155)}$	$0.936_{(0.184)}$	$0.907_{(0.157)}$	$0.927_{(0.187)}$	$0.887_{(0.155)}$	$0.912_{(0.184)}$
500	0	$0.906_{(0.221)}$	$0.952_{(0.263)}$	$0.892_{(0.184)}$	$0.928_{(0.220)}$	$0.887_{(0.160)}$	$0.923_{(0.191)}$
	0.25	$0.913_{(0.142)}$	$0.939_{(0.170)}$	$0.904_{(0.126)}$	$0.946_{(0.150)}$	$0.897_{(0.113)}$	$0.935_{(0.134)}$
	0.50	$0.900_{(0.098)}$	$0.943_{(0.117)}$	$0.914_{(0.098)}$	$0.951_{(0.117)}$	$0.923_{(0.098)}$	$0.956_{(0.117)}$
1000	0	$0.914_{(0.158)}$	$0.946_{(0.188)}$	$0.891_{(0.134)}$	$0.924_{(0.160)}$	$0.892_{(0.116)}$	$0.940_{(0.138)}$
	0.25	$0.915_{(0.101)}$	$0.954_{(0.120)}$	$0.899_{(0.088)}$	$0.942_{(0.105)}$	$0.900_{(0.080)}$	$0.938_{(0.095)}$
	0.50	$0.917_{(0.070)}$	$0.953_{(0.083)}$	$0.913_{(0.069)}$	$0.950_{(0.083)}$	$0.915_{(0.069)}$	$0.951_{(0.082)}$
2000	0	$0.897_{(0.113)}$	$0.949_{(0.135)}$	$0.896_{(0.095)}$	$0.941_{(0.114)}$	$0.903_{(0.083)}$	$0.946_{(0.099)}$
	0.25	$0.935_{(0.072)}$	$0.966_{(0.086)}$	$0.905_{(0.062)}$	$0.942_{(0.074)}$	$0.903_{(0.057)}$	$0.947_{(0.068)}$
	0.50	$0.924_{(0.049)}$	$0.953_{(0.059)}$	$0.909_{(0.049)}$	$0.954_{(0.058)}$	$0.916_{(0.049)}$	$0.964_{(0.058)}$

3.4.3.3 $B_n = 20$

Table 3.13: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with P61Rescaled kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 20$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

n	λ/π	$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
		90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.842_{(0.366)}$	$0.873_{(0.436)}$	$0.831_{(0.310)}$	$0.856_{(0.369)}$	$0.842_{(0.268)}$	$0.870_{(0.320)}$
	0.25	$0.897_{(0.264)}$	$0.928_{(0.315)}$	$0.888_{(0.229)}$	$0.914_{(0.272)}$	$0.892_{(0.204)}$	$0.914_{(0.244)}$
	0.50	$0.880_{(0.181)}$	$0.903_{(0.216)}$	$0.864_{(0.179)}$	$0.886_{(0.214)}$	$0.874_{(0.180)}$	$0.904_{(0.215)}$
500	0	$0.872_{(0.252)}$	$0.910_{(0.300)}$	$0.877_{(0.205)}$	$0.911_{(0.244)}$	$0.858_{(0.183)}$	$0.899_{(0.218)}$
	0.25	$0.917_{(0.165)}$	$0.944_{(0.196)}$	$0.911_{(0.143)}$	$0.944_{(0.170)}$	$0.905_{(0.132)}$	$0.933_{(0.157)}$
	0.50	$0.897_{(0.113)}$	$0.940_{(0.134)}$	$0.915_{(0.113)}$	$0.942_{(0.135)}$	$0.893_{(0.113)}$	$0.931_{(0.135)}$
1000	0	$0.894_{(0.181)}$	$0.939_{(0.216)}$	$0.874_{(0.152)}$	$0.926_{(0.181)}$	$0.891_{(0.135)}$	$0.925_{(0.161)}$
	0.25	$0.909_{(0.116)}$	$0.952_{(0.138)}$	$0.922_{(0.102)}$	$0.951_{(0.121)}$	$0.907_{(0.092)}$	$0.944_{(0.109)}$
	0.50	$0.900_{(0.080)}$	$0.944_{(0.095)}$	$0.916_{(0.080)}$	$0.952_{(0.096)}$	$0.905_{(0.080)}$	$0.946_{(0.095)}$
2000	0	$0.911_{(0.129)}$	$0.953_{(0.154)}$	$0.914_{(0.109)}$	$0.955_{(0.129)}$	$0.897_{(0.096)}$	$0.952_{(0.115)}$
	0.25	$0.903_{(0.082)}$	$0.948_{(0.098)}$	$0.912_{(0.072)}$	$0.956_{(0.086)}$	$0.923_{(0.065)}$	$0.962_{(0.077)}$
	0.50	$0.913_{(0.056)}$	$0.950_{(0.067)}$	$0.901_{(0.057)}$	$0.944_{(0.067)}$	$0.925_{(0.056)}$	$0.961_{(0.067)}$
				x_n chosen as the 95% quantile			
200	0	$0.866_{(0.365)}$	$0.906_{(0.435)}$	$0.852_{(0.309)}$	$0.889_{(0.369)}$	$0.832_{(0.262)}$	$0.867_{(0.312)}$
	0.25	$0.904_{(0.257)}$	$0.934_{(0.307)}$	$0.886_{(0.229)}$	$0.911_{(0.273)}$	$0.885_{(0.205)}$	$0.922_{(0.244)}$
	0.50	$0.890_{(0.182)}$	$0.919_{(0.217)}$	$0.887_{(0.182)}$	$0.913_{(0.216)}$	$0.893_{(0.180)}$	$0.933_{(0.214)}$
500	0	$0.905_{(0.252)}$	$0.939_{(0.300)}$	$0.878_{(0.211)}$	$0.927_{(0.252)}$	$0.879_{(0.187)}$	$0.911_{(0.223)}$
	0.25	$0.906_{(0.165)}$	$0.934_{(0.196)}$	$0.898_{(0.145)}$	$0.930_{(0.172)}$	$0.877_{(0.130)}$	$0.922_{(0.155)}$
	0.50	$0.903_{(0.114)}$	$0.932_{(0.136)}$	$0.899_{(0.114)}$	$0.935_{(0.136)}$	$0.895_{(0.113)}$	$0.930_{(0.134)}$
1000	0	$0.892_{(0.183)}$	$0.933_{(0.218)}$	$0.882_{(0.155)}$	$0.918_{(0.184)}$	$0.887_{(0.132)}$	$0.932_{(0.157)}$
	0.25	$0.905_{(0.117)}$	$0.944_{(0.139)}$	$0.901_{(0.102)}$	$0.946_{(0.121)}$	$0.892_{(0.091)}$	$0.932_{(0.109)}$
	0.50	$0.900_{(0.080)}$	$0.941_{(0.096)}$	$0.920_{(0.079)}$	$0.954_{(0.095)}$	$0.898_{(0.080)}$	$0.940_{(0.095)}$
2000	0	$0.901_{(0.130)}$	$0.941_{(0.155)}$	$0.890_{(0.109)}$	$0.938_{(0.130)}$	$0.886_{(0.096)}$	$0.935_{(0.114)}$
	0.25	$0.919_{(0.083)}$	$0.957_{(0.098)}$	$0.905_{(0.072)}$	$0.945_{(0.086)}$	$0.895_{(0.065)}$	$0.939_{(0.079)}$
	0.50	$0.915_{(0.057)}$	$0.951_{(0.067)}$	$0.917_{(0.057)}$	$0.954_{(0.068)}$	$0.901_{(0.057)}$	$0.953_{(0.068)}$

3.4.3.4 $B_n = 25$

Table 3.14: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with P61Rescaled kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 25$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.844_{(0.393)}$	$0.872_{(0.469)}$	$0.832_{(0.327)}$	$0.860_{(0.389)}$	$0.797_{(0.292)}$	$0.820_{(0.348)}$
	0.25	$0.863_{(0.290)}$	$0.888_{(0.345)}$	$0.874_{(0.255)}$	$0.911_{(0.304)}$	$0.888_{(0.232)}$	$0.914_{(0.276)}$
	0.50	$0.873_{(0.205)}$	$0.904_{(0.244)}$	$0.900_{(0.202)}$	$0.926_{(0.241)}$	$0.879_{(0.205)}$	$0.915_{(0.245)}$
500	0	$0.879_{(0.272)}$	$0.911_{(0.324)}$	$0.867_{(0.232)}$	$0.896_{(0.276)}$	$0.852_{(0.201)}$	$0.899_{(0.240)}$
	0.25	$0.904_{(0.183)}$	$0.932_{(0.218)}$	$0.889_{(0.158)}$	$0.925_{(0.188)}$	$0.886_{(0.145)}$	$0.927_{(0.173)}$
	0.50	$0.900_{(0.126)}$	$0.930_{(0.150)}$	$0.898_{(0.127)}$	$0.926_{(0.152)}$	$0.892_{(0.127)}$	$0.925_{(0.151)}$
1000	0	$0.888_{(0.202)}$	$0.919_{(0.241)}$	$0.883_{(0.169)}$	$0.934_{(0.202)}$	$0.892_{(0.149)}$	$0.930_{(0.178)}$
	0.25	$0.913_{(0.132)}$	$0.939_{(0.157)}$	$0.903_{(0.114)}$	$0.943_{(0.136)}$	$0.892_{(0.102)}$	$0.940_{(0.122)}$
	0.50	$0.914_{(0.089)}$	$0.945_{(0.107)}$	$0.903_{(0.089)}$	$0.949_{(0.107)}$	$0.906_{(0.089)}$	$0.938_{(0.106)}$
2000	0	$0.906_{(0.144)}$	$0.948_{(0.172)}$	$0.891_{(0.121)}$	$0.943_{(0.144)}$	$0.899_{(0.108)}$	$0.947_{(0.128)}$
	0.25	$0.914_{(0.092)}$	$0.955_{(0.110)}$	$0.924_{(0.080)}$	$0.963_{(0.095)}$	$0.907_{(0.072)}$	$0.947_{(0.086)}$
	0.50	$0.907_{(0.063)}$	$0.959_{(0.075)}$	$0.893_{(0.064)}$	$0.952_{(0.076)}$	$0.918_{(0.063)}$	$0.952_{(0.075)}$
				x_n chosen as the 95% quantile			
200	0	$0.857_{(0.394)}$	$0.885_{(0.469)}$	$0.829_{(0.329)}$	$0.849_{(0.393)}$	$0.838_{(0.290)}$	$0.863_{(0.346)}$
	0.25	$0.888_{(0.293)}$	$0.903_{(0.349)}$	$0.875_{(0.252)}$	$0.898_{(0.300)}$	$0.882_{(0.228)}$	$0.902_{(0.271)}$
	0.50	$0.881_{(0.202)}$	$0.908_{(0.241)}$	$0.890_{(0.203)}$	$0.921_{(0.242)}$	$0.873_{(0.203)}$	$0.896_{(0.242)}$
500	0	$0.882_{(0.279)}$	$0.911_{(0.332)}$	$0.878_{(0.237)}$	$0.916_{(0.283)}$	$0.851_{(0.205)}$	$0.889_{(0.245)}$
	0.25	$0.900_{(0.184)}$	$0.931_{(0.219)}$	$0.892_{(0.160)}$	$0.924_{(0.190)}$	$0.869_{(0.146)}$	$0.916_{(0.174)}$
	0.50	$0.885_{(0.126)}$	$0.924_{(0.150)}$	$0.886_{(0.126)}$	$0.930_{(0.151)}$	$0.883_{(0.127)}$	$0.913_{(0.151)}$
1000	0	$0.884_{(0.201)}$	$0.924_{(0.239)}$	$0.898_{(0.170)}$	$0.944_{(0.203)}$	$0.877_{(0.148)}$	$0.917_{(0.176)}$
	0.25	$0.889_{(0.130)}$	$0.929_{(0.155)}$	$0.889_{(0.114)}$	$0.932_{(0.136)}$	$0.901_{(0.103)}$	$0.937_{(0.123)}$
	0.50	$0.912_{(0.090)}$	$0.956_{(0.108)}$	$0.904_{(0.089)}$	$0.936_{(0.106)}$	$0.895_{(0.089)}$	$0.944_{(0.106)}$
2000	0	$0.896_{(0.147)}$	$0.939_{(0.176)}$	$0.909_{(0.122)}$	$0.951_{(0.145)}$	$0.898_{(0.108)}$	$0.936_{(0.128)}$
	0.25	$0.914_{(0.093)}$	$0.946_{(0.111)}$	$0.923_{(0.081)}$	$0.954_{(0.096)}$	$0.894_{(0.073)}$	$0.950_{(0.087)}$
	0.50	$0.889_{(0.063)}$	$0.935_{(0.075)}$	$0.917_{(0.063)}$	$0.956_{(0.075)}$	$0.923_{(0.063)}$	$0.959_{(0.075)}$

3.4.3.5 $B_n = 30$

Table 3.15: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the Moving-Maximum process with P61Rescaled kernel, and $n \in \{200, 500, 1000, 2000\}$, $B_n = 30$ when the innovations are Fréchet with $\gamma \in 1, 2, 3$.

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
n	λ/π	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0	$0.801_{(0.421)}$	$0.836_{(0.501)}$	$0.790_{(0.346)}$	$0.826_{(0.413)}$	$0.801_{(0.305)}$	$0.832_{(0.363)}$
	0.25	$0.873_{(0.315)}$	$0.894_{(0.375)}$	$0.871_{(0.280)}$	$0.897_{(0.334)}$	$0.872_{(0.251)}$	$0.903_{(0.299)}$
	0.50	$0.877_{(0.223)}$	$0.911_{(0.265)}$	$0.853_{(0.217)}$	$0.882_{(0.258)}$	$0.869_{(0.221)}$	$0.889_{(0.264)}$
500	0	$0.861_{(0.293)}$	$0.889_{(0.350)}$	$0.855_{(0.252)}$	$0.883_{(0.300)}$	$0.852_{(0.218)}$	$0.882_{(0.260)}$
	0.25	$0.896_{(0.201)}$	$0.928_{(0.239)}$	$0.885_{(0.175)}$	$0.909_{(0.208)}$	$0.892_{(0.159)}$	$0.919_{(0.189)}$
	0.50	$0.890_{(0.140)}$	$0.934_{(0.167)}$	$0.884_{(0.141)}$	$0.916_{(0.168)}$	$0.891_{(0.140)}$	$0.921_{(0.167)}$
1000	0	$0.879_{(0.222)}$	$0.922_{(0.265)}$	$0.894_{(0.183)}$	$0.926_{(0.218)}$	$0.876_{(0.161)}$	$0.910_{(0.192)}$
	0.25	$0.906_{(0.142)}$	$0.940_{(0.169)}$	$0.913_{(0.124)}$	$0.948_{(0.148)}$	$0.899_{(0.111)}$	$0.934_{(0.133)}$
	0.50	$0.885_{(0.098)}$	$0.927_{(0.117)}$	$0.907_{(0.098)}$	$0.938_{(0.117)}$	$0.908_{(0.097)}$	$0.951_{(0.116)}$
2000	0	$0.889_{(0.158)}$	$0.939_{(0.188)}$	$0.909_{(0.131)}$	$0.954_{(0.156)}$	$0.897_{(0.117)}$	$0.939_{(0.139)}$
	0.25	$0.900_{(0.101)}$	$0.951_{(0.120)}$	$0.898_{(0.089)}$	$0.946_{(0.106)}$	$0.903_{(0.080)}$	$0.942_{(0.096)}$
	0.50	$0.910_{(0.069)}$	$0.936_{(0.083)}$	$0.916_{(0.069)}$	$0.947_{(0.083)}$	$0.903_{(0.069)}$	$0.954_{(0.083)}$
				x_n chosen as the 95% quantile			
200	0	$0.830_{(0.411)}$	$0.863_{(0.490)}$	$0.817_{(0.357)}$	$0.849_{(0.426)}$	$0.815_{(0.294)}$	$0.851_{(0.350)}$
	0.25	$0.890_{(0.321)}$	$0.915_{(0.383)}$	$0.848_{(0.275)}$	$0.878_{(0.328)}$	$0.870_{(0.248)}$	$0.900_{(0.296)}$
	0.50	$0.881_{(0.220)}$	$0.912_{(0.262)}$	$0.878_{(0.218)}$	$0.902_{(0.260)}$	$0.874_{(0.220)}$	$0.900_{(0.262)}$
500	0	$0.879_{(0.302)}$	$0.905_{(0.359)}$	$0.858_{(0.253)}$	$0.887_{(0.302)}$	$0.862_{(0.217)}$	$0.894_{(0.259)}$
	0.25	$0.906_{(0.202)}$	$0.940_{(0.241)}$	$0.898_{(0.177)}$	$0.927_{(0.211)}$	$0.901_{(0.159)}$	$0.928_{(0.189)}$
	0.50	$0.888_{(0.138)}$	$0.928_{(0.164)}$	$0.881_{(0.139)}$	$0.917_{(0.165)}$	$0.890_{(0.136)}$	$0.930_{(0.162)}$
1000	0	$0.885_{(0.220)}$	$0.918_{(0.263)}$	$0.876_{(0.185)}$	$0.918_{(0.221)}$	$0.905_{(0.160)}$	$0.943_{(0.191)}$
	0.25	$0.910_{(0.143)}$	$0.948_{(0.171)}$	$0.895_{(0.125)}$	$0.938_{(0.149)}$	$0.903_{(0.114)}$	$0.938_{(0.135)}$
	0.50	$0.897_{(0.098)}$	$0.934_{(0.117)}$	$0.919_{(0.097)}$	$0.950_{(0.115)}$	$0.886_{(0.098)}$	$0.945_{(0.117)}$
2000	0	$0.903_{(0.159)}$	$0.941_{(0.189)}$	$0.890_{(0.132)}$	$0.931_{(0.158)}$	$0.886_{(0.117)}$	$0.934_{(0.140)}$
	0.25	$0.903_{(0.101)}$	$0.945_{(0.120)}$	$0.913_{(0.088)}$	$0.950_{(0.105)}$	$0.917_{(0.080)}$	$0.947_{(0.095)}$
	0.50	$0.915_{(0.069)}$	$0.959_{(0.083)}$	$0.899_{(0.069)}$	$0.943_{(0.083)}$	$0.907_{(0.069)}$	$0.950_{(0.082)}$

3.4.4 ARMA with Fréchet 1 Innovation

3.4.4.1 $B_n = 10$

Table 3.16: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 10$ and different kernel choices when the innovations are Fréchet with $\gamma = 1$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.894_{(0.394)}$	$0.930_{(0.469)}$	$0.908_{(0.360)}$	$0.941_{(0.429)}$	$0.909_{(0.372)}$	$0.944_{(0.444)}$
	0.4	$0.867_{(0.550)}$	$0.906_{(0.656)}$	$0.886_{(0.482)}$	$0.927_{(0.574)}$	$0.886_{(0.512)}$	$0.930_{(0.610)}$
	0.6	$0.860_{(0.788)}$	$0.896_{(0.939)}$	$0.862_{(0.696)}$	$0.893_{(0.830)}$	$0.859_{(0.734)}$	$0.892_{(0.875)}$
	0.8	$0.808_{(1.226)}$	$0.844_{(1.461)}$	$0.851_{(1.005)}$	$0.872_{(1.197)}$	$0.818_{(1.074)}$	$0.856_{(1.280)}$
500	0.2	$0.910_{(0.265)}$	$0.948_{(0.316)}$	$0.923_{(0.230)}$	$0.957_{(0.274)}$	$0.919_{(0.248)}$	$0.954_{(0.295)}$
	0.4	$0.890_{(0.362)}$	$0.947_{(0.431)}$	$0.900_{(0.318)}$	$0.945_{(0.379)}$	$0.901_{(0.338)}$	$0.943_{(0.403)}$
	0.6	$0.903_{(0.546)}$	$0.923_{(0.650)}$	$0.910_{(0.466)}$	$0.946_{(0.555)}$	$0.881_{(0.494)}$	$0.919_{(0.588)}$
	0.8	$0.887_{(0.879)}$	$0.914_{(1.048)}$	$0.919_{(0.697)}$	$0.942_{(0.830)}$	$0.921_{(0.788)}$	$0.942_{(0.939)}$
1000	0.2	$0.926_{(0.189)}$	$0.962_{(0.226)}$	$0.937_{(0.165)}$	$0.971_{(0.196)}$	$0.919_{(0.177)}$	$0.957_{(0.211)}$
	0.4	$0.916_{(0.261)}$	$0.958_{(0.311)}$	$0.914_{(0.229)}$	$0.951_{(0.273)}$	$0.920_{(0.244)}$	$0.957_{(0.291)}$
	0.6	$0.877_{(0.394)}$	$0.930_{(0.469)}$	$0.913_{(0.332)}$	$0.956_{(0.396)}$	$0.902_{(0.358)}$	$0.945_{(0.426)}$
	0.8	$0.921_{(0.649)}$	$0.951_{(0.774)}$	$0.932_{(0.508)}$	$0.960_{(0.606)}$	$0.930_{(0.564)}$	$0.952_{(0.672)}$
2000	0.2	$0.929_{(0.134)}$	$0.975_{(0.160)}$	$0.924_{(0.118)}$	$0.965_{(0.141)}$	$0.910_{(0.125)}$	$0.956_{(0.149)}$
	0.4	$0.913_{(0.187)}$	$0.962_{(0.223)}$	$0.921_{(0.162)}$	$0.961_{(0.193)}$	$0.922_{(0.173)}$	$0.966_{(0.206)}$
	0.6	$0.898_{(0.280)}$	$0.947_{(0.334)}$	$0.929_{(0.237)}$	$0.969_{(0.283)}$	$0.915_{(0.257)}$	$0.964_{(0.307)}$
	0.8	$0.944_{(0.467)}$	$0.963_{(0.557)}$	$0.953_{(0.364)}$	$0.968_{(0.434)}$	$0.953_{(0.404)}$	$0.970_{(0.482)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.907_{(0.382)}$	$0.948_{(0.456)}$	$0.904_{(0.343)}$	$0.956_{(0.409)}$	$0.909_{(0.360)}$	$0.948_{(0.429)}$
	0.4	$0.876_{(0.509)}$	$0.906_{(0.607)}$	$0.879_{(0.458)}$	$0.915_{(0.545)}$	$0.875_{(0.488)}$	$0.912_{(0.581)}$
	0.6	$0.766_{(0.699)}$	$0.830_{(0.833)}$	$0.780_{(0.641)}$	$0.843_{(0.763)}$	$0.769_{(0.665)}$	$0.850_{(0.793)}$
	0.8	$0.753_{(1.105)}$	$0.764_{(1.317)}$	$0.765_{(0.911)}$	$0.775_{(1.086)}$	$0.733_{(0.972)}$	$0.741_{(1.159)}$
500	0.2	$0.887_{(0.256)}$	$0.938_{(0.305)}$	$0.902_{(0.228)}$	$0.944_{(0.272)}$	$0.902_{(0.244)}$	$0.950_{(0.290)}$
	0.4	$0.833_{(0.351)}$	$0.900_{(0.419)}$	$0.830_{(0.311)}$	$0.891_{(0.371)}$	$0.841_{(0.332)}$	$0.911_{(0.395)}$
	0.6	$0.772_{(0.518)}$	$0.839_{(0.618)}$	$0.764_{(0.439)}$	$0.840_{(0.523)}$	$0.806_{(0.470)}$	$0.854_{(0.560)}$
	0.8	$0.767_{(0.831)}$	$0.812_{(0.990)}$	$0.795_{(0.648)}$	$0.831_{(0.772)}$	$0.797_{(0.719)}$	$0.826_{(0.857)}$
1000	0.2	$0.901_{(0.186)}$	$0.945_{(0.222)}$	$0.900_{(0.164)}$	$0.946_{(0.195)}$	$0.875_{(0.174)}$	$0.940_{(0.207)}$
	0.4	$0.860_{(0.254)}$	$0.914_{(0.303)}$	$0.844_{(0.224)}$	$0.916_{(0.267)}$	$0.842_{(0.235)}$	$0.909_{(0.280)}$
	0.6	$0.778_{(0.381)}$	$0.842_{(0.453)}$	$0.805_{(0.323)}$	$0.871_{(0.384)}$	$0.810_{(0.351)}$	$0.878_{(0.418)}$
	0.8	$0.773_{(0.616)}$	$0.832_{(0.734)}$	$0.853_{(0.498)}$	$0.902_{(0.594)}$	$0.816_{(0.553)}$	$0.872_{(0.659)}$
2000	0.2	$0.885_{(0.132)}$	$0.940_{(0.157)}$	$0.887_{(0.116)}$	$0.936_{(0.138)}$	$0.884_{(0.123)}$	$0.942_{(0.146)}$
	0.4	$0.838_{(0.183)}$	$0.908_{(0.218)}$	$0.835_{(0.160)}$	$0.912_{(0.191)}$	$0.840_{(0.170)}$	$0.904_{(0.203)}$
	0.6	$0.793_{(0.273)}$	$0.868_{(0.325)}$	$0.811_{(0.232)}$	$0.870_{(0.276)}$	$0.801_{(0.250)}$	$0.888_{(0.298)}$
	0.8	$0.813_{(0.454)}$	$0.878_{(0.541)}$	$0.864_{(0.359)}$	$0.919_{(0.427)}$	$0.853_{(0.399)}$	$0.905_{(0.475)}$

3.4.4.2 $B_n = 15$

Table 3.17: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 15$ and different kernel choices when the innovations are Fréchet with $\gamma = 1$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.869_{(0.473)}$	$0.901_{(0.564)}$	$0.898_{(0.417)}$	$0.925_{(0.497)}$	$0.876_{(0.447)}$	$0.894_{(0.533)}$
	0.4	$0.868_{(0.637)}$	$0.902_{(0.760)}$	$0.859_{(0.576)}$	$0.911_{(0.686)}$	$0.889_{(0.594)}$	$0.913_{(0.708)}$
	0.6	$0.846_{(0.968)}$	$0.878_{(1.154)}$	$0.862_{(0.844)}$	$0.893_{(1.006)}$	$0.828_{(0.907)}$	$0.872_{(1.081)}$
	0.8	$0.819_{(1.651)}$	$0.851_{(1.967)}$	$0.810_{(1.353)}$	$0.853_{(1.612)}$	$0.784_{(1.500)}$	$0.816_{(1.788)}$
500	0.2	$0.900_{(0.317)}$	$0.935_{(0.378)}$	$0.888_{(0.282)}$	$0.932_{(0.336)}$	$0.901_{(0.299)}$	$0.937_{(0.357)}$
	0.4	$0.879_{(0.443)}$	$0.921_{(0.528)}$	$0.880_{(0.389)}$	$0.924_{(0.464)}$	$0.891_{(0.412)}$	$0.937_{(0.491)}$
	0.6	$0.859_{(0.678)}$	$0.912_{(0.807)}$	$0.872_{(0.592)}$	$0.918_{(0.705)}$	$0.874_{(0.626)}$	$0.913_{(0.746)}$
	0.8	$0.848_{(1.219)}$	$0.891_{(1.453)}$	$0.876_{(0.992)}$	$0.902_{(1.182)}$	$0.883_{(1.093)}$	$0.907_{(1.303)}$
1000	0.2	$0.899_{(0.229)}$	$0.934_{(0.273)}$	$0.906_{(0.203)}$	$0.949_{(0.242)}$	$0.907_{(0.215)}$	$0.956_{(0.256)}$
	0.4	$0.899_{(0.319)}$	$0.947_{(0.381)}$	$0.894_{(0.283)}$	$0.950_{(0.338)}$	$0.895_{(0.299)}$	$0.944_{(0.356)}$
	0.6	$0.882_{(0.492)}$	$0.923_{(0.586)}$	$0.899_{(0.425)}$	$0.944_{(0.506)}$	$0.878_{(0.461)}$	$0.924_{(0.549)}$
	0.8	$0.897_{(0.903)}$	$0.921_{(1.076)}$	$0.905_{(0.737)}$	$0.932_{(0.878)}$	$0.898_{(0.801)}$	$0.928_{(0.954)}$
2000	0.2	$0.904_{(0.164)}$	$0.951_{(0.195)}$	$0.920_{(0.145)}$	$0.966_{(0.172)}$	$0.914_{(0.154)}$	$0.956_{(0.184)}$
	0.4	$0.903_{(0.229)}$	$0.945_{(0.273)}$	$0.891_{(0.202)}$	$0.944_{(0.241)}$	$0.896_{(0.215)}$	$0.948_{(0.256)}$
	0.6	$0.897_{(0.354)}$	$0.944_{(0.422)}$	$0.898_{(0.305)}$	$0.939_{(0.364)}$	$0.898_{(0.329)}$	$0.941_{(0.392)}$
	0.8	$0.887_{(0.651)}$	$0.936_{(0.776)}$	$0.917_{(0.529)}$	$0.958_{(0.631)}$	$0.910_{(0.584)}$	$0.946_{(0.696)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.873_{(0.453)}$	$0.913_{(0.539)}$	$0.884_{(0.400)}$	$0.937_{(0.477)}$	$0.877_{(0.428)}$	$0.916_{(0.509)}$
	0.4	$0.864_{(0.613)}$	$0.899_{(0.730)}$	$0.883_{(0.548)}$	$0.921_{(0.653)}$	$0.863_{(0.573)}$	$0.901_{(0.683)}$
	0.6	$0.791_{(0.867)}$	$0.844_{(1.033)}$	$0.781_{(0.787)}$	$0.838_{(0.938)}$	$0.784_{(0.835)}$	$0.839_{(0.995)}$
	0.8	$0.729_{(1.472)}$	$0.779_{(1.755)}$	$0.749_{(1.247)}$	$0.767_{(1.470)}$	$0.727_{(1.380)}$	$0.782_{(1.644)}$
500	0.2	$0.878_{(0.315)}$	$0.909_{(0.375)}$	$0.884_{(0.274)}$	$0.935_{(0.326)}$	$0.892_{(0.292)}$	$0.933_{(0.349)}$
	0.4	$0.871_{(0.426)}$	$0.911_{(0.508)}$	$0.854_{(0.382)}$	$0.906_{(0.455)}$	$0.858_{(0.399)}$	$0.900_{(0.476)}$
	0.6	$0.787_{(0.646)}$	$0.856_{(0.770)}$	$0.800_{(0.556)}$	$0.857_{(0.663)}$	$0.777_{(0.587)}$	$0.842_{(0.700)}$
	0.8	$0.739_{(1.098)}$	$0.799_{(1.308)}$	$0.748_{(0.922)}$	$0.806_{(1.099)}$	$0.741_{(1.012)}$	$0.795_{(1.206)}$
1000	0.2	$0.879_{(0.226)}$	$0.931_{(0.269)}$	$0.894_{(0.201)}$	$0.938_{(0.239)}$	$0.897_{(0.211)}$	$0.947_{(0.252)}$
	0.4	$0.852_{(0.316)}$	$0.918_{(0.376)}$	$0.845_{(0.277)}$	$0.912_{(0.330)}$	$0.853_{(0.292)}$	$0.914_{(0.348)}$
	0.6	$0.806_{(0.467)}$	$0.864_{(0.557)}$	$0.766_{(0.415)}$	$0.840_{(0.494)}$	$0.789_{(0.441)}$	$0.849_{(0.525)}$
	0.8	$0.757_{(0.843)}$	$0.819_{(1.004)}$	$0.796_{(0.715)}$	$0.835_{(0.844)}$	$0.776_{(0.775)}$	$0.824_{(0.923)}$
2000	0.2	$0.895_{(0.162)}$	$0.946_{(0.192)}$	$0.893_{(0.143)}$	$0.949_{(0.171)}$	$0.895_{(0.151)}$	$0.947_{(0.180)}$
	0.4	$0.852_{(0.225)}$	$0.911_{(0.268)}$	$0.835_{(0.198)}$	$0.897_{(0.236)}$	$0.843_{(0.208)}$	$0.908_{(0.248)}$
	0.6	$0.788_{(0.344)}$	$0.849_{(0.410)}$	$0.785_{(0.297)}$	$0.855_{(0.354)}$	$0.799_{(0.321)}$	$0.871_{(0.383)}$
	0.8	$0.775_{(0.624)}$	$0.840_{(0.744)}$	$0.824_{(0.516)}$	$0.890_{(0.615)}$	$0.779_{(0.568)}$	$0.847_{(0.677)}$

3.4.4.3 $B_n = 20$

Table 3.18: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 20$ and different kernel choices when the innovations are Fréchet with $\gamma = 1$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.836_{(0.510)}$	$0.864_{(0.607)}$	$0.860_{(0.467)}$	$0.887_{(0.556)}$	$0.873_{(0.497)}$	$0.896_{(0.593)}$
	0.4	$0.827_{(0.710)}$	$0.863_{(0.846)}$	$0.858_{(0.643)}$	$0.895_{(0.767)}$	$0.839_{(0.693)}$	$0.876_{(0.826)}$
	0.6	$0.830_{(1.046)}$	$0.861_{(1.246)}$	$0.841_{(0.948)}$	$0.866_{(1.130)}$	$0.833_{(1.036)}$	$0.873_{(1.234)}$
	0.8	$0.793_{(1.871)}$	$0.826_{(2.229)}$	$0.784_{(1.666)}$	$0.810_{(1.985)}$	$0.782_{(1.778)}$	$0.806_{(2.119)}$
500	0.2	$0.869_{(0.362)}$	$0.906_{(0.431)}$	$0.901_{(0.318)}$	$0.939_{(0.379)}$	$0.892_{(0.344)}$	$0.922_{(0.410)}$
	0.4	$0.866_{(0.497)}$	$0.912_{(0.592)}$	$0.894_{(0.453)}$	$0.929_{(0.540)}$	$0.881_{(0.475)}$	$0.916_{(0.566)}$
	0.6	$0.878_{(0.758)}$	$0.909_{(0.903)}$	$0.881_{(0.678)}$	$0.912_{(0.808)}$	$0.869_{(0.713)}$	$0.905_{(0.850)}$
	0.8	$0.814_{(1.444)}$	$0.853_{(1.721)}$	$0.829_{(1.224)}$	$0.864_{(1.459)}$	$0.836_{(1.322)}$	$0.878_{(1.575)}$
1000	0.2	$0.880_{(0.263)}$	$0.919_{(0.314)}$	$0.891_{(0.232)}$	$0.935_{(0.277)}$	$0.903_{(0.247)}$	$0.941_{(0.294)}$
	0.4	$0.898_{(0.369)}$	$0.932_{(0.440)}$	$0.901_{(0.323)}$	$0.946_{(0.385)}$	$0.895_{(0.343)}$	$0.934_{(0.409)}$
	0.6	$0.857_{(0.570)}$	$0.908_{(0.679)}$	$0.886_{(0.497)}$	$0.930_{(0.593)}$	$0.875_{(0.541)}$	$0.922_{(0.645)}$
	0.8	$0.869_{(1.097)}$	$0.907_{(1.307)}$	$0.897_{(0.921)}$	$0.929_{(1.098)}$	$0.874_{(1.015)}$	$0.916_{(1.210)}$
2000	0.2	$0.896_{(0.188)}$	$0.942_{(0.225)}$	$0.918_{(0.166)}$	$0.955_{(0.198)}$	$0.920_{(0.176)}$	$0.949_{(0.210)}$
	0.4	$0.902_{(0.265)}$	$0.944_{(0.315)}$	$0.895_{(0.233)}$	$0.944_{(0.277)}$	$0.895_{(0.247)}$	$0.944_{(0.295)}$
	0.6	$0.890_{(0.410)}$	$0.939_{(0.489)}$	$0.870_{(0.359)}$	$0.930_{(0.427)}$	$0.878_{(0.382)}$	$0.937_{(0.455)}$
	0.8	$0.866_{(0.790)}$	$0.926_{(0.942)}$	$0.894_{(0.663)}$	$0.938_{(0.790)}$	$0.889_{(0.729)}$	$0.927_{(0.869)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.860_{(0.493)}$	$0.881_{(0.587)}$	$0.894_{(0.459)}$	$0.925_{(0.546)}$	$0.855_{(0.480)}$	$0.902_{(0.572)}$
	0.4	$0.854_{(0.671)}$	$0.884_{(0.799)}$	$0.846_{(0.615)}$	$0.874_{(0.733)}$	$0.858_{(0.634)}$	$0.882_{(0.755)}$
	0.6	$0.792_{(0.944)}$	$0.841_{(1.125)}$	$0.790_{(0.873)}$	$0.840_{(1.041)}$	$0.809_{(0.931)}$	$0.854_{(1.109)}$
	0.8	$0.732_{(1.607)}$	$0.759_{(1.915)}$	$0.747_{(1.484)}$	$0.764_{(1.769)}$	$0.747_{(1.538)}$	$0.765_{(1.832)}$
500	0.2	$0.881_{(0.355)}$	$0.923_{(0.423)}$	$0.890_{(0.319)}$	$0.932_{(0.380)}$	$0.889_{(0.331)}$	$0.935_{(0.394)}$
	0.4	$0.859_{(0.494)}$	$0.909_{(0.589)}$	$0.847_{(0.444)}$	$0.901_{(0.529)}$	$0.866_{(0.451)}$	$0.905_{(0.537)}$
	0.6	$0.803_{(0.722)}$	$0.859_{(0.861)}$	$0.804_{(0.652)}$	$0.858_{(0.776)}$	$0.786_{(0.681)}$	$0.839_{(0.812)}$
	0.8	$0.747_{(1.308)}$	$0.798_{(1.558)}$	$0.740_{(1.136)}$	$0.783_{(1.353)}$	$0.730_{(1.221)}$	$0.784_{(1.455)}$
1000	0.2	$0.903_{(0.259)}$	$0.951_{(0.309)}$	$0.880_{(0.229)}$	$0.935_{(0.273)}$	$0.909_{(0.244)}$	$0.938_{(0.290)}$
	0.4	$0.856_{(0.362)}$	$0.911_{(0.431)}$	$0.862_{(0.317)}$	$0.920_{(0.378)}$	$0.855_{(0.332)}$	$0.918_{(0.395)}$
	0.6	$0.812_{(0.542)}$	$0.865_{(0.646)}$	$0.810_{(0.482)}$	$0.868_{(0.574)}$	$0.810_{(0.510)}$	$0.877_{(0.607)}$
	0.8	$0.746_{(1.039)}$	$0.808_{(1.238)}$	$0.750_{(0.866)}$	$0.818_{(1.032)}$	$0.748_{(0.959)}$	$0.806_{(1.143)}$
2000	0.2	$0.891_{(0.187)}$	$0.939_{(0.223)}$	$0.900_{(0.164)}$	$0.947_{(0.196)}$	$0.896_{(0.174)}$	$0.940_{(0.207)}$
	0.4	$0.873_{(0.257)}$	$0.916_{(0.306)}$	$0.862_{(0.227)}$	$0.921_{(0.271)}$	$0.875_{(0.239)}$	$0.927_{(0.285)}$
	0.6	$0.826_{(0.399)}$	$0.890_{(0.475)}$	$0.807_{(0.345)}$	$0.872_{(0.412)}$	$0.785_{(0.370)}$	$0.863_{(0.441)}$
	0.8	$0.756_{(0.762)}$	$0.831_{(0.908)}$	$0.785_{(0.650)}$	$0.847_{(0.774)}$	$0.777_{(0.706)}$	$0.840_{(0.841)}$

3.4.4.4 $B_n = 25$

Table 3.19: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 25$ and different kernel choices when the innovations are Fréchet with $\gamma = 1$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.807_{(0.550)}$	$0.836_{(0.656)}$	$0.838_{(0.523)}$	$0.867_{(0.623)}$	$0.839_{(0.533)}$	$0.877_{(0.635)}$
	0.4	$0.824_{(0.765)}$	$0.854_{(0.912)}$	$0.841_{(0.705)}$	$0.869_{(0.841)}$	$0.831_{(0.734)}$	$0.858_{(0.874)}$
	0.6	$0.803_{(1.124)}$	$0.839_{(1.134)}$	$0.832_{(1.040)}$	$0.867_{(1.239)}$	$0.818_{(1.106)}$	$0.862_{(1.318)}$
	0.8	$0.774_{(2.070)}$	$0.823_{(2.466)}$	$0.788_{(1.832)}$	$0.820_{(2.183)}$	$0.768_{(1.995)}$	$0.796_{(2.377)}$
500	0.2	$0.856_{(0.394)}$	$0.892_{(0.470)}$	$0.882_{(0.354)}$	$0.923_{(0.422)}$	$0.876_{(0.376)}$	$0.906_{(0.449)}$
	0.4	$0.866_{(0.552)}$	$0.901_{(0.658)}$	$0.885_{(0.498)}$	$0.916_{(0.593)}$	$0.860_{(0.522)}$	$0.898_{(0.621)}$
	0.6	$0.841_{(0.842)}$	$0.880_{(1.003)}$	$0.873_{(0.751)}$	$0.904_{(0.895)}$	$0.871_{(0.791)}$	$0.913_{(0.942)}$
	0.8	$0.832_{(1.634)}$	$0.863_{(1.947)}$	$0.845_{(1.418)}$	$0.882_{(1.690)}$	$0.833_{(1.524)}$	$0.876_{(1.815)}$
1000	0.2	$0.891_{(0.289)}$	$0.922_{(0.345)}$	$0.899_{(0.260)}$	$0.931_{(0.310)}$	$0.890_{(0.279)}$	$0.927_{(0.333)}$
	0.4	$0.882_{(0.410)}$	$0.928_{(0.488)}$	$0.890_{(0.362)}$	$0.931_{(0.432)}$	$0.870_{(0.386)}$	$0.916_{(0.460)}$
	0.6	$0.880_{(0.631)}$	$0.931_{(0.751)}$	$0.881_{(0.555)}$	$0.922_{(0.661)}$	$0.884_{(0.586)}$	$0.920_{(0.699)}$
	0.8	$0.850_{(1.246)}$	$0.900_{(1.484)}$	$0.884_{(1.068)}$	$0.910_{(1.273)}$	$0.868_{(1.157)}$	$0.913_{(1.378)}$
2000	0.2	$0.898_{(0.210)}$	$0.940_{(0.251)}$	$0.887_{(0.185)}$	$0.939_{(0.220)}$	$0.907_{(0.197)}$	$0.945_{(0.234)}$
	0.4	$0.894_{(0.291)}$	$0.938_{(0.347)}$	$0.885_{(0.260)}$	$0.939_{(0.310)}$	$0.883_{(0.275)}$	$0.939_{(0.328)}$
	0.6	$0.871_{(0.454)}$	$0.916_{(0.541)}$	$0.893_{(0.399)}$	$0.942_{(0.476)}$	$0.891_{(0.427)}$	$0.937_{(0.509)}$
	0.8	$0.865_{(0.906)}$	$0.921_{(1.080)}$	$0.880_{(0.771)}$	$0.922_{(0.918)}$	$0.884_{(0.842)}$	$0.935_{(1.003)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.831_{(0.542)}$	$0.857_{(0.645)}$	$0.874_{(0.484)}$	$0.907_{(0.577)}$	$0.854_{(0.513)}$	$0.894_{(0.611)}$
	0.4	$0.844_{(0.726)}$	$0.878_{(0.865)}$	$0.871_{(0.671)}$	$0.898_{(0.799)}$	$0.859_{(0.688)}$	$0.894_{(0.820)}$
	0.6	$0.792_{(1.038)}$	$0.882_{(1.236)}$	$0.802_{(0.994)}$	$0.858_{(1.184)}$	$0.806_{(0.994)}$	$0.855_{(1.184)}$
	0.8	$0.770_{(1.800)}$	$0.797_{(2.145)}$	$0.754_{(1.664)}$	$0.776_{(1.983)}$	$0.714_{(1.825)}$	$0.767_{(2.175)}$
500	0.2	$0.869_{(0.384)}$	$0.914_{(0.457)}$	$0.869_{(0.354)}$	$0.904_{(0.422)}$	$0.876_{(0.365)}$	$0.910_{(0.434)}$
	0.4	$0.870_{(0.527)}$	$0.911_{(0.628)}$	$0.859_{(0.478)}$	$0.914_{(0.569)}$	$0.858_{(0.505)}$	$0.905_{(0.601)}$
	0.6	$0.828_{(0.807)}$	$0.871_{(0.962)}$	$0.794_{(0.707)}$	$0.841_{(0.843)}$	$0.810_{(0.742)}$	$0.852_{(0.884)}$
	0.8	$0.739_{(1.485)}$	$0.790_{(1.769)}$	$0.732_{(1.276)}$	$0.796_{(1.521)}$	$0.728_{(1.416)}$	$0.779_{(1.687)}$
1000	0.2	$0.892_{(0.292)}$	$0.932_{(0.348)}$	$0.902_{(0.255)}$	$0.948_{(0.304)}$	$0.880_{(0.269)}$	$0.933_{(0.320)}$
	0.4	$0.864_{(0.395)}$	$0.921_{(0.471)}$	$0.886_{(0.352)}$	$0.925_{(0.419)}$	$0.873_{(0.371)}$	$0.915_{(0.442)}$
	0.6	$0.831_{(0.603)}$	$0.878_{(0.718)}$	$0.801_{(0.532)}$	$0.871_{(0.634)}$	$0.816_{(0.567)}$	$0.872_{(0.676)}$
	0.8	$0.732_{(1.182)}$	$0.799_{(1.408)}$	$0.741_{(1.024)}$	$0.809_{(1.221)}$	$0.743_{(1.075)}$	$0.794_{(1.281)}$
2000	0.2	$0.898_{(0.207)}$	$0.941_{(0.247)}$	$0.886_{(0.183)}$	$0.933_{(0.218)}$	$0.891_{(0.194)}$	$0.934_{(0.231)}$
	0.4	$0.877_{(0.289)}$	$0.938_{(0.344)}$	$0.852_{(0.254)}$	$0.916_{(0.302)}$	$0.868_{(0.269)}$	$0.920_{(0.321)}$
	0.6	$0.837_{(0.441)}$	$0.890_{(0.525)}$	$0.807_{(0.394)}$	$0.864_{(0.470)}$	$0.821_{(0.409)}$	$0.890_{(0.487)}$
	0.8	$0.762_{(0.873)}$	$0.827_{(1.040)}$	$0.741_{(0.758)}$	$0.826_{(0.903)}$	$0.750_{(0.817)}$	$0.834_{(0.973)}$

3.4.4.5 $B_n = 30$

Table 3.20: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 30$ and different kernel choices when the innovations are Fréchet with $\gamma = 1$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.794_{(0.576)}$	$0.825_{(0.686)}$	$0.817_{(0.545)}$	$0.849_{(0.649)}$	$0.798_{(0.561)}$	$0.835_{(0.668)}$
	0.4	$0.779_{(0.804)}$	$0.813_{(0.958)}$	$0.819_{(0.747)}$	$0.849_{(0.890)}$	$0.822_{(0.779)}$	$0.858_{(0.928)}$
	0.6	$0.803_{(1.216)}$	$0.831_{(1.448)}$	$0.820_{(1.135)}$	$0.855_{(1.353)}$	$0.821_{(1.160)}$	$0.863_{(1.383)}$
	0.8	$0.762_{(2.186)}$	$0.808_{(2.604)}$	$0.780_{(2.111)}$	$0.814_{(2.515)}$	$0.786_{(2.173)}$	$0.816_{(2.590)}$
500	0.2	$0.858_{(0.424)}$	$0.890_{(0.506)}$	$0.874_{(0.386)}$	$0.910_{(0.460)}$	$0.866_{(0.417)}$	$0.908_{(0.496)}$
	0.4	$0.865_{(0.589)}$	$0.888_{(0.702)}$	$0.889_{(0.538)}$	$0.913_{(0.642)}$	$0.875_{(0.570)}$	$0.905_{(0.680)}$
	0.6	$0.846_{(0.909)}$	$0.882_{(1.083)}$	$0.882_{(0.812)}$	$0.917_{(0.968)}$	$0.864_{(0.864)}$	$0.908_{(1.029)}$
	0.8	$0.821_{(1.770)}$	$0.850_{(2.109)}$	$0.828_{(1.552)}$	$0.861_{(1.850)}$	$0.847_{(1.687)}$	$0.884_{(2.011)}$
1000	0.2	$0.877_{(0.315)}$	$0.912_{(0.376)}$	$0.885_{(0.282)}$	$0.924_{(0.337)}$	$0.891_{(0.298)}$	$0.924_{(0.355)}$
	0.4	$0.894_{(0.439)}$	$0.937_{(0.523)}$	$0.881_{(0.388)}$	$0.922_{(0.462)}$	$0.893_{(0.414)}$	$0.934_{(0.494)}$
	0.6	$0.878_{(0.686)}$	$0.919_{(0.817)}$	$0.878_{(0.599)}$	$0.924_{(0.713)}$	$0.873_{(0.649)}$	$0.926_{(0.774)}$
	0.8	$0.840_{(1.373)}$	$0.888_{(1.636)}$	$0.863_{(1.180)}$	$0.905_{(1.406)}$	$0.857_{(1.273)}$	$0.897_{(1.517)}$
2000	0.2	$0.889_{(0.230)}$	$0.940_{(0.274)}$	$0.912_{(0.202)}$	$0.945_{(0.240)}$	$0.899_{(0.215)}$	$0.940_{(0.256)}$
	0.4	$0.890_{(0.319)}$	$0.933_{(0.380)}$	$0.882_{(0.282)}$	$0.935_{(0.336)}$	$0.894_{(0.298)}$	$0.938_{(0.355)}$
	0.6	$0.899_{(0.503)}$	$0.947_{(0.599)}$	$0.893_{(0.434)}$	$0.937_{(0.517)}$	$0.884_{(0.465)}$	$0.933_{(0.554)}$
	0.8	$0.859_{(1.007)}$	$0.907_{(1.200)}$	$0.878_{(0.865)}$	$0.926_{(1.030)}$	$0.883_{(0.939)}$	$0.922_{(1.119)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.823_{(0.559)}$	$0.861_{(0.666)}$	$0.862_{(0.523)}$	$0.890_{(0.623)}$	$0.816_{(0.547)}$	$0.844_{(0.652)}$
	0.4	$0.809_{(0.790)}$	$0.850_{(0.941)}$	$0.855_{(0.705)}$	$0.898_{(0.840)}$	$0.845_{(0.728)}$	$0.871_{(0.868)}$
	0.6	$0.779_{(1.108)}$	$0.867_{(1.321)}$	$0.792_{(1.022)}$	$0.850_{(1.218)}$	$0.797_{(1.074)}$	$0.882_{(1.280)}$
	0.8	$0.743_{(1.946)}$	$0.764_{(2.318)}$	$0.741_{(1.758)}$	$0.778_{(2.094)}$	$0.757_{(1.964)}$	$0.773_{(2.340)}$
500	0.2	$0.859_{(0.416)}$	$0.894_{(0.496)}$	$0.885_{(0.372)}$	$0.924_{(0.444)}$	$0.883_{(0.398)}$	$0.925_{(0.474)}$
	0.4	$0.838_{(0.573)}$	$0.879_{(0.683)}$	$0.876_{(0.507)}$	$0.916_{(0.604)}$	$0.861_{(0.555)}$	$0.898_{(0.661)}$
	0.6	$0.829_{(0.863)}$	$0.867_{(1.028)}$	$0.826_{(0.765)}$	$0.863_{(0.912)}$	$0.838_{(0.816)}$	$0.885_{(0.972)}$
	0.8	$0.742_{(1.626)}$	$0.786_{(1.937)}$	$0.745_{(1.442)}$	$0.797_{(1.718)}$	$0.746_{(1.593)}$	$0.787_{(1.899)}$
1000	0.2	$0.873_{(0.307)}$	$0.914_{(0.366)}$	$0.884_{(0.274)}$	$0.931_{(0.326)}$	$0.876_{(0.293)}$	$0.914_{(0.349)}$
	0.4	$0.869_{(0.431)}$	$0.912_{(0.514)}$	$0.865_{(0.385)}$	$0.913_{(0.458)}$	$0.867_{(0.406)}$	$0.920_{(0.484)}$
	0.6	$0.822_{(0.650)}$	$0.880_{(0.774)}$	$0.816_{(0.589)}$	$0.882_{(0.702)}$	$0.849_{(0.616)}$	$0.891_{(0.734)}$
	0.8	$0.757_{(1.294)}$	$0.818_{(1.542)}$	$0.728_{(1.132)}$	$0.807_{(1.349)}$	$0.752_{(1.196)}$	$0.812_{(1.425)}$
2000	0.2	$0.881_{(0.228)}$	$0.930_{(0.271)}$	$0.883_{(0.201)}$	$0.936_{(0.239)}$	$0.874_{(0.213)}$	$0.927_{(0.254)}$
	0.4	$0.866_{(0.314)}$	$0.921_{(0.374)}$	$0.869_{(0.275)}$	$0.921_{(0.328)}$	$0.872_{(0.294)}$	$0.918_{(0.351)}$
	0.6	$0.831_{(0.486)}$	$0.900_{(0.579)}$	$0.838_{(0.426)}$	$0.887_{(0.507)}$	$0.823_{(0.448)}$	$0.885_{(0.534)}$
	0.8	$0.769_{(0.963)}$	$0.829_{(1.148)}$	$0.750_{(0.841)}$	$0.832_{(1.003)}$	$0.708_{(0.900)}$	$0.803_{(1.072)}$

3.4.5 ARMA with Fréchet 2 Innovation

3.4.5.1 $B_n = 10$

Table 3.21: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 10$ and different kernel choices when the innovations are Fréchet with $\gamma = 2$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.890_{(0.332)}$	$0.932_{(0.396)}$	$0.910_{(0.301)}$	$0.941_{(0.358)}$	$0.904_{(0.321)}$	$0.933_{(0.383)}$
	0.4	$0.884_{(0.428)}$	$0.927_{(0.510)}$	$0.889_{(0.384)}$	$0.930_{(0.457)}$	$0.882_{(0.410)}$	$0.921_{(0.489)}$
	0.6	$0.864_{(0.614)}$	$0.907_{(0.732)}$	$0.888_{(0.538)}$	$0.914_{(0.641)}$	$0.883_{(0.582)}$	$0.915_{(0.693)}$
	0.8	$0.833_{(0.983)}$	$0.870_{(1.171)}$	$0.835_{(0.808)}$	$0.858_{(0.963)}$	$0.860_{(0.877)}$	$0.881_{(1.046)}$
500	0.2	$0.920_{(0.225)}$	$0.953_{(0.268)}$	$0.921_{(0.197)}$	$0.953_{(0.235)}$	$0.913_{(0.206)}$	$0.954_{(0.245)}$
	0.4	$0.916_{(0.287)}$	$0.951_{(0.341)}$	$0.925_{(0.257)}$	$0.963_{(0.306)}$	$0.913_{(0.269)}$	$0.950_{(0.321)}$
	0.6	$0.899_{(0.411)}$	$0.941_{(0.490)}$	$0.924_{(0.357)}$	$0.946_{(0.425)}$	$0.899_{(0.382)}$	$0.939_{(0.455)}$
	0.8	$0.889_{(0.680)}$	$0.917_{(0.811)}$	$0.901_{(0.562)}$	$0.928_{(0.670)}$	$0.913_{(0.617)}$	$0.950_{(0.735)}$
1000	0.2	$0.924_{(0.160)}$	$0.957_{(0.190)}$	$0.909_{(0.140)}$	$0.955_{(0.167)}$	$0.920_{(0.149)}$	$0.957_{(0.178)}$
	0.4	$0.903_{(0.207)}$	$0.943_{(0.247)}$	$0.927_{(0.182)}$	$0.961_{(0.217)}$	$0.926_{(0.196)}$	$0.958_{(0.233)}$
	0.6	$0.902_{(0.295)}$	$0.948_{(0.352)}$	$0.927_{(0.254)}$	$0.964_{(0.303)}$	$0.936_{(0.272)}$	$0.967_{(0.325)}$
	0.8	$0.905_{(0.498)}$	$0.938_{(0.593)}$	$0.919_{(0.404)}$	$0.947_{(0.482)}$	$0.918_{(0.446)}$	$0.949_{(0.531)}$
2000	0.2	$0.930_{(0.113)}$	$0.969_{(0.135)}$	$0.916_{(0.101)}$	$0.958_{(0.120)}$	$0.917_{(0.106)}$	$0.965_{(0.127)}$
	0.4	$0.927_{(0.148)}$	$0.961_{(0.177)}$	$0.924_{(0.129)}$	$0.958_{(0.153)}$	$0.939_{(0.138)}$	$0.966_{(0.164)}$
	0.6	$0.920_{(0.212)}$	$0.955_{(0.252)}$	$0.937_{(0.183)}$	$0.974_{(0.218)}$	$0.915_{(0.197)}$	$0.957_{(0.235)}$
	0.8	$0.930_{(0.357)}$	$0.965_{(0.426)}$	$0.922_{(0.290)}$	$0.963_{(0.345)}$	$0.924_{(0.318)}$	$0.962_{(0.380)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.915_{(0.319)}$	$0.944_{(0.381)}$	$0.905_{(0.284)}$	$0.939_{(0.339)}$	$0.918_{(0.299)}$	$0.946_{(0.357)}$
	0.4	$0.912_{(0.405)}$	$0.944_{(0.483)}$	$0.908_{(0.363)}$	$0.949_{(0.433)}$	$0.890_{(0.382)}$	$0.922_{(0.455)}$
	0.6	$0.856_{(0.545)}$	$0.899_{(0.649)}$	$0.842_{(0.478)}$	$0.894_{(0.569)}$	$0.832_{(0.505)}$	$0.881_{(0.602)}$
	0.8	$0.758_{(0.835)}$	$0.785_{(0.995)}$	$0.778_{(0.736)}$	$0.804_{(0.877)}$	$0.767_{(0.762)}$	$0.787_{(0.909)}$
500	0.2	$0.906_{(0.216)}$	$0.950_{(0.257)}$	$0.906_{(0.189)}$	$0.956_{(0.225)}$	$0.903_{(0.203)}$	$0.952_{(0.242)}$
	0.4	$0.873_{(0.276)}$	$0.935_{(0.329)}$	$0.883_{(0.241)}$	$0.928_{(0.287)}$	$0.895_{(0.255)}$	$0.939_{(0.305)}$
	0.6	$0.823_{(0.380)}$	$0.883_{(0.453)}$	$0.842_{(0.336)}$	$0.902_{(0.399)}$	$0.830_{(0.359)}$	$0.892_{(0.428)}$
	0.8	$0.775_{(0.615)}$	$0.826_{(0.733)}$	$0.798_{(0.524)}$	$0.851_{(0.625)}$	$0.788_{(0.558)}$	$0.836_{(0.665)}$
1000	0.2	$0.889_{(0.154)}$	$0.942_{(0.184)}$	$0.901_{(0.135)}$	$0.945_{(0.161)}$	$0.915_{(0.144)}$	$0.965_{(0.172)}$
	0.4	$0.883_{(0.197)}$	$0.940_{(0.234)}$	$0.890_{(0.174)}$	$0.940_{(0.207)}$	$0.892_{(0.185)}$	$0.945_{(0.221)}$
	0.6	$0.854_{(0.279)}$	$0.904_{(0.333)}$	$0.849_{(0.240)}$	$0.909_{(0.286)}$	$0.843_{(0.258)}$	$0.900_{(0.307)}$
	0.8	$0.787_{(0.467)}$	$0.846_{(0.557)}$	$0.811_{(0.381)}$	$0.881_{(0.454)}$	$0.827_{(0.419)}$	$0.878_{(0.499)}$
2000	0.2	$0.902_{(0.110)}$	$0.940_{(0.131)}$	$0.917_{(0.097)}$	$0.960_{(0.115)}$	$0.903_{(0.103)}$	$0.947_{(0.123)}$
	0.4	$0.873_{(0.141)}$	$0.927_{(0.167)}$	$0.898_{(0.124)}$	$0.945_{(0.148)}$	$0.902_{(0.131)}$	$0.946_{(0.156)}$
	0.6	$0.844_{(0.199)}$	$0.908_{(0.237)}$	$0.825_{(0.173)}$	$0.892_{(0.205)}$	$0.835_{(0.184)}$	$0.897_{(0.219)}$
	0.8	$0.822_{(0.338)}$	$0.888_{(0.402)}$	$0.819_{(0.276)}$	$0.879_{(0.330)}$	$0.830_{(0.305)}$	$0.885_{(0.363)}$

3.4.5.2 $B_n = 15$

Table 3.22: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 15$ and different kernel choices when the innovations are Fréchet with $\gamma = 2$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.847_{(0.405)}$	$0.883_{(0.483)}$	$0.873_{(0.352)}$	$0.897_{(0.420)}$	$0.876_{(0.377)}$	$0.910_{(0.449)}$
	0.4	$0.853_{(0.523)}$	$0.892_{(0.624)}$	$0.881_{(0.452)}$	$0.920_{(0.539)}$	$0.869_{(0.486)}$	$0.903_{(0.579)}$
	0.6	$0.862_{(0.721)}$	$0.894_{(0.859)}$	$0.867_{(0.652)}$	$0.909_{(0.776)}$	$0.870_{(0.679)}$	$0.907_{(0.809)}$
	0.8	$0.838_{(1.265)}$	$0.868_{(1.507)}$	$0.833_{(1.074)}$	$0.873_{(1.281)}$	$0.800_{(1.135)}$	$0.845_{(1.353)}$
500	0.2	$0.884_{(0.269)}$	$0.919_{(0.321)}$	$0.909_{(0.239)}$	$0.946_{(0.284)}$	$0.890_{(0.251)}$	$0.930_{(0.299)}$
	0.4	$0.891_{(0.350)}$	$0.934_{(0.417)}$	$0.893_{(0.308)}$	$0.937_{(0.367)}$	$0.911_{(0.326)}$	$0.939_{(0.389)}$
	0.6	$0.881_{(0.492)}$	$0.911_{(0.586)}$	$0.900_{(0.442)}$	$0.939_{(0.527)}$	$0.889_{(0.468)}$	$0.931_{(0.558)}$
	0.8	$0.861_{(0.877)}$	$0.896_{(1.045)}$	$0.869_{(0.749)}$	$0.899_{(0.892)}$	$0.874_{(0.840)}$	$0.902_{(1.001)}$
1000	0.2	$0.894_{(0.195)}$	$0.934_{(0.233)}$	$0.925_{(0.173)}$	$0.960_{(0.206)}$	$0.916_{(0.183)}$	$0.957_{(0.218)}$
	0.4	$0.917_{(0.253)}$	$0.942_{(0.302)}$	$0.925_{(0.221)}$	$0.967_{(0.263)}$	$0.914_{(0.235)}$	$0.954_{(0.280)}$
	0.6	$0.895_{(0.365)}$	$0.935_{(0.435)}$	$0.909_{(0.315)}$	$0.948_{(0.376)}$	$0.913_{(0.343)}$	$0.952_{(0.409)}$
	0.8	$0.887_{(0.658)}$	$0.929_{(0.784)}$	$0.907_{(0.560)}$	$0.948_{(0.667)}$	$0.902_{(0.605)}$	$0.941_{(0.721)}$
2000	0.2	$0.908_{(0.139)}$	$0.952_{(0.166)}$	$0.924_{(0.122)}$	$0.953_{(0.145)}$	$0.910_{(0.130)}$	$0.951_{(0.154)}$
	0.4	$0.897_{(0.180)}$	$0.948_{(0.215)}$	$0.929_{(0.159)}$	$0.967_{(0.189)}$	$0.904_{(0.168)}$	$0.949_{(0.201)}$
	0.6	$0.914_{(0.263)}$	$0.948_{(0.313)}$	$0.898_{(0.228)}$	$0.944_{(0.271)}$	$0.913_{(0.244)}$	$0.956_{(0.291)}$
	0.8	$0.902_{(0.470)}$	$0.944_{(0.560)}$	$0.895_{(0.400)}$	$0.940_{(0.477)}$	$0.917_{(0.432)}$	$0.952_{(0.515)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.877_{(0.376)}$	$0.911_{(0.448)}$	$0.910_{(0.340)}$	$0.946_{(0.405)}$	$0.885_{(0.361)}$	$0.926_{(0.430)}$
	0.4	$0.874_{(0.475)}$	$0.916_{(0.566)}$	$0.902_{(0.429)}$	$0.922_{(0.512)}$	$0.884_{(0.459)}$	$0.916_{(0.547)}$
	0.6	$0.835_{(0.642)}$	$0.870_{(0.766)}$	$0.836_{(0.583)}$	$0.886_{(0.694)}$	$0.851_{(0.621)}$	$0.890_{(0.740)}$
	0.8	$0.770_{(1.065)}$	$0.784_{(1.269)}$	$0.770_{(0.919)}$	$0.796_{(1.096)}$	$0.759_{(0.966)}$	$0.799_{(1.151)}$
500	0.2	$0.884_{(0.259)}$	$0.936_{(0.308)}$	$0.901_{(0.233)}$	$0.947_{(0.277)}$	$0.910_{(0.242)}$	$0.949_{(0.288)}$
	0.4	$0.890_{(0.328)}$	$0.932_{(0.390)}$	$0.913_{(0.293)}$	$0.947_{(0.349)}$	$0.882_{(0.313)}$	$0.938_{(0.373)}$
	0.6	$0.848_{(0.450)}$	$0.897_{(0.536)}$	$0.830_{(0.405)}$	$0.891_{(0.483)}$	$0.851_{(0.431)}$	$0.906_{(0.513)}$
	0.8	$0.779_{(0.783)}$	$0.825_{(0.933)}$	$0.763_{(0.699)}$	$0.812_{(0.833)}$	$0.752_{(0.735)}$	$0.813_{(0.876)}$
1000	0.2	$0.898_{(0.189)}$	$0.948_{(0.225)}$	$0.896_{(0.167)}$	$0.945_{(0.200)}$	$0.891_{(0.175)}$	$0.946_{(0.209)}$
	0.4	$0.887_{(0.239)}$	$0.933_{(0.285)}$	$0.894_{(0.212)}$	$0.936_{(0.253)}$	$0.893_{(0.226)}$	$0.946_{(0.269)}$
	0.6	$0.867_{(0.341)}$	$0.919_{(0.407)}$	$0.851_{(0.297)}$	$0.913_{(0.354)}$	$0.845_{(0.316)}$	$0.903_{(0.377)}$
	0.8	$0.788_{(0.601)}$	$0.851_{(0.716)}$	$0.778_{(0.512)}$	$0.846_{(0.611)}$	$0.784_{(0.562)}$	$0.852_{(0.669)}$
2000	0.2	$0.907_{(0.135)}$	$0.948_{(0.161)}$	$0.906_{(0.118)}$	$0.952_{(0.141)}$	$0.898_{(0.126)}$	$0.948_{(0.150)}$
	0.4	$0.894_{(0.172)}$	$0.941_{(0.205)}$	$0.896_{(0.152)}$	$0.939_{(0.182)}$	$0.886_{(0.161)}$	$0.936_{(0.192)}$
	0.6	$0.857_{(0.245)}$	$0.909_{(0.292)}$	$0.845_{(0.216)}$	$0.904_{(0.257)}$	$0.841_{(0.229)}$	$0.908_{(0.273)}$
	0.8	$0.799_{(0.438)}$	$0.858_{(0.522)}$	$0.783_{(0.374)}$	$0.850_{(0.445)}$	$0.795_{(0.404)}$	$0.866_{(0.481)}$

3.4.5.3 $B_n = 20$

Table 3.23: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 20$ and different kernel choices when the innovations are Fréchet with $\gamma = 2$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.833_{(0.438)}$	$0.856_{(0.522)}$	$0.871_{(0.393)}$	$0.894_{(0.468)}$	$0.839_{(0.420)}$	$0.865_{(0.500)}$
	0.4	$0.860_{(0.569)}$	$0.889_{(0.679)}$	$0.847_{(0.525)}$	$0.896_{(0.625)}$	$0.830_{(0.543)}$	$0.872_{(0.648)}$
	0.6	$0.838_{(0.797)}$	$0.868_{(0.950)}$	$0.856_{(0.729)}$	$0.897_{(0.869)}$	$0.852_{(0.772)}$	$0.883_{(0.920)}$
	0.8	$0.809_{(1.415)}$	$0.851_{(1.686)}$	$0.815_{(1.223)}$	$0.856_{(1.457)}$	$0.808_{(1.311)}$	$0.837_{(1.562)}$
500	0.2	$0.878_{(0.303)}$	$0.916_{(0.361)}$	$0.889_{(0.273)}$	$0.923_{(0.325)}$	$0.866_{(0.295)}$	$0.905_{(0.352)}$
	0.4	$0.862_{(0.400)}$	$0.896_{(0.477)}$	$0.887_{(0.351)}$	$0.927_{(0.419)}$	$0.890_{(0.382)}$	$0.919_{(0.455)}$
	0.6	$0.888_{(0.580)}$	$0.926_{(0.691)}$	$0.891_{(0.507)}$	$0.928_{(0.604)}$	$0.888_{(0.544)}$	$0.923_{(0.648)}$
	0.8	$0.858_{(1.046)}$	$0.894_{(1.247)}$	$0.868_{(0.903)}$	$0.895_{(1.076)}$	$0.863_{(0.961)}$	$0.901_{(1.145)}$
1000	0.2	$0.892_{(0.224)}$	$0.929_{(0.267)}$	$0.897_{(0.196)}$	$0.938_{(0.234)}$	$0.904_{(0.209)}$	$0.941_{(0.249)}$
	0.4	$0.878_{(0.288)}$	$0.930_{(0.344)}$	$0.902_{(0.254)}$	$0.940_{(0.303)}$	$0.896_{(0.272)}$	$0.935_{(0.324)}$
	0.6	$0.906_{(0.417)}$	$0.935_{(0.497)}$	$0.890_{(0.371)}$	$0.930_{(0.442)}$	$0.900_{(0.390)}$	$0.937_{(0.465)}$
	0.8	$0.863_{(0.763)}$	$0.920_{(0.909)}$	$0.891_{(0.671)}$	$0.927_{(0.799)}$	$0.865_{(0.723)}$	$0.909_{(0.862)}$
2000	0.2	$0.894_{(0.161)}$	$0.945_{(0.191)}$	$0.900_{(0.142)}$	$0.942_{(0.169)}$	$0.896_{(0.151)}$	$0.945_{(0.179)}$
	0.4	$0.890_{(0.206)}$	$0.941_{(0.246)}$	$0.906_{(0.182)}$	$0.949_{(0.217)}$	$0.915_{(0.194)}$	$0.950_{(0.231)}$
	0.6	$0.904_{(0.301)}$	$0.955_{(0.358)}$	$0.893_{(0.265)}$	$0.943_{(0.315)}$	$0.899_{(0.283)}$	$0.937_{(0.338)}$
	0.8	$0.886_{(0.566)}$	$0.927_{(0.674)}$	$0.894_{(0.485)}$	$0.936_{(0.578)}$	$0.900_{(0.516)}$	$0.944_{(0.615)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.852_{(0.416)}$	$0.893_{(0.496)}$	$0.891_{(0.378)}$	$0.923_{(0.450)}$	$0.854_{(0.399)}$	$0.904_{(0.475)}$
	0.4	$0.858_{(0.539)}$	$0.897_{(0.642)}$	$0.885_{(0.482)}$	$0.912_{(0.575)}$	$0.890_{(0.508)}$	$0.913_{(0.606)}$
	0.6	$0.844_{(0.748)}$	$0.895_{(0.891)}$	$0.857_{(0.663)}$	$0.898_{(0.790)}$	$0.869_{(0.695)}$	$0.910_{(0.829)}$
	0.8	$0.790_{(1.185)}$	$0.809_{(1.413)}$	$0.766_{(1.058)}$	$0.785_{(1.261)}$	$0.791_{(1.128)}$	$0.801_{(1.344)}$
500	0.2	$0.887_{(0.293)}$	$0.922_{(0.349)}$	$0.891_{(0.263)}$	$0.934_{(0.313)}$	$0.879_{(0.273)}$	$0.920_{(0.325)}$
	0.4	$0.870_{(0.369)}$	$0.911_{(0.439)}$	$0.884_{(0.332)}$	$0.931_{(0.395)}$	$0.890_{(0.354)}$	$0.927_{(0.422)}$
	0.6	$0.857_{(0.526)}$	$0.899_{(0.626)}$	$0.843_{(0.461)}$	$0.896_{(0.549)}$	$0.842_{(0.495)}$	$0.883_{(0.590)}$
	0.8	$0.794_{(0.931)}$	$0.857_{(1.109)}$	$0.787_{(0.813)}$	$0.831_{(0.968)}$	$0.779_{(0.861)}$	$0.826_{(1.025)}$
1000	0.2	$0.905_{(0.217)}$	$0.935_{(0.258)}$	$0.902_{(0.189)}$	$0.944_{(0.225)}$	$0.902_{(0.204)}$	$0.949_{(0.243)}$
	0.4	$0.868_{(0.277)}$	$0.915_{(0.330)}$	$0.901_{(0.246)}$	$0.938_{(0.294)}$	$0.892_{(0.257)}$	$0.932_{(0.307)}$
	0.6	$0.862_{(0.387)}$	$0.912_{(0.461)}$	$0.849_{(0.347)}$	$0.895_{(0.413)}$	$0.847_{(0.368)}$	$0.896_{(0.439)}$
	0.8	$0.789_{(0.694)}$	$0.843_{(0.827)}$	$0.775_{(0.603)}$	$0.845_{(0.719)}$	$0.794_{(0.653)}$	$0.847_{(0.778)}$
2000	0.2	$0.886_{(0.154)}$	$0.939_{(0.183)}$	$0.905_{(0.137)}$	$0.949_{(0.164)}$	$0.887_{(0.145)}$	$0.929_{(0.172)}$
	0.4	$0.881_{(0.197)}$	$0.935_{(0.235)}$	$0.908_{(0.175)}$	$0.943_{(0.208)}$	$0.895_{(0.186)}$	$0.944_{(0.222)}$
	0.6	$0.866_{(0.283)}$	$0.933_{(0.337)}$	$0.854_{(0.248)}$	$0.915_{(0.295)}$	$0.870_{(0.262)}$	$0.921_{(0.312)}$
	0.8	$0.810_{(0.513)}$	$0.873_{(0.611)}$	$0.790_{(0.451)}$	$0.859_{(0.537)}$	$0.784_{(0.480)}$	$0.855_{(0.572)}$

3.4.5.4 $B_n = 25$

Table 3.24: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 25$ and different kernel choices when the innovations are Fréchet with $\gamma = 2$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.820_{(0.467)}$	$0.852_{(0.556)}$	$0.829_{(0.435)}$	$0.859_{(0.518)}$	$0.828_{(0.463)}$	$0.856_{(0.552)}$
	0.4	$0.828_{(0.615)}$	$0.849_{(0.732)}$	$0.842_{(0.570)}$	$0.873_{(0.679)}$	$0.818_{(0.587)}$	$0.854_{(0.699)}$
	0.6	$0.826_{(0.863)}$	$0.865_{(1.028)}$	$0.813_{(0.826)}$	$0.849_{(0.984)}$	$0.825_{(0.852)}$	$0.855_{(1.016)}$
	0.8	$0.795_{(1.504)}$	$0.839_{(1.792)}$	$0.817_{(1.402)}$	$0.846_{(1.671)}$	$0.804_{(1.459)}$	$0.840_{(1.738)}$
500	0.2	$0.870_{(0.331)}$	$0.899_{(0.394)}$	$0.858_{(0.306)}$	$0.909_{(0.364)}$	$0.878_{(0.320)}$	$0.915_{(0.381)}$
	0.4	$0.869_{(0.430)}$	$0.900_{(0.513)}$	$0.877_{(0.386)}$	$0.901_{(0.460)}$	$0.854_{(0.406)}$	$0.885_{(0.484)}$
	0.6	$0.878_{(0.625)}$	$0.908_{(0.745)}$	$0.884_{(0.562)}$	$0.923_{(0.669)}$	$0.895_{(0.593)}$	$0.930_{(0.706)}$
	0.8	$0.836_{(1.145)}$	$0.873_{(1.364)}$	$0.857_{(1.034)}$	$0.891_{(1.232)}$	$0.859_{(1.086)}$	$0.897_{(1.294)}$
1000	0.2	$0.893_{(0.249)}$	$0.930_{(0.297)}$	$0.896_{(0.220)}$	$0.930_{(0.262)}$	$0.888_{(0.232)}$	$0.923_{(0.276)}$
	0.4	$0.864_{(0.316)}$	$0.909_{(0.377)}$	$0.908_{(0.284)}$	$0.941_{(0.338)}$	$0.888_{(0.303)}$	$0.926_{(0.362)}$
	0.6	$0.872_{(0.468)}$	$0.908_{(0.558)}$	$0.896_{(0.408)}$	$0.934_{(0.486)}$	$0.895_{(0.437)}$	$0.933_{(0.520)}$
	0.8	$0.867_{(0.866)}$	$0.909_{(1.031)}$	$0.863_{(0.761)}$	$0.916_{(0.906)}$	$0.883_{(0.821)}$	$0.922_{(0.978)}$
2000	0.2	$0.912_{(0.179)}$	$0.948_{(0.214)}$	$0.901_{(0.157)}$	$0.952_{(0.187)}$	$0.887_{(0.165)}$	$0.939_{(0.197)}$
	0.4	$0.903_{(0.231)}$	$0.946_{(0.275)}$	$0.891_{(0.206)}$	$0.939_{(0.245)}$	$0.893_{(0.218)}$	$0.936_{(0.260)}$
	0.6	$0.914_{(0.337)}$	$0.951_{(0.401)}$	$0.909_{(0.298)}$	$0.939_{(0.355)}$	$0.903_{(0.313)}$	$0.954_{(0.373)}$
	0.8	$0.883_{(0.630)}$	$0.927_{(0.751)}$	$0.884_{(0.554)}$	$0.938_{(0.660)}$	$0.893_{(0.586)}$	$0.944_{(0.699)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.839_{(0.458)}$	$0.863_{(0.546)}$	$0.854_{(0.421)}$	$0.896_{(0.501)}$	$0.858_{(0.428)}$	$0.885_{(0.510)}$
	0.4	$0.816_{(0.562)}$	$0.853_{(0.670)}$	$0.876_{(0.531)}$	$0.909_{(0.633)}$	$0.857_{(0.559)}$	$0.892_{(0.666)}$
	0.6	$0.835_{(0.771)}$	$0.870_{(0.919)}$	$0.856_{(0.702)}$	$0.889_{(0.836)}$	$0.837_{(0.736)}$	$0.883_{(0.877)}$
	0.8	$0.765_{(1.272)}$	$0.791_{(1.516)}$	$0.760_{(1.152)}$	$0.774_{(1.372)}$	$0.782_{(1.241)}$	$0.798_{(1.479)}$
500	0.2	$0.876_{(0.322)}$	$0.911_{(0.384)}$	$0.888_{(0.287)}$	$0.917_{(0.342)}$	$0.880_{(0.302)}$	$0.915_{(0.360)}$
	0.4	$0.845_{(0.409)}$	$0.886_{(0.488)}$	$0.877_{(0.377)}$	$0.925_{(0.449)}$	$0.861_{(0.397)}$	$0.907_{(0.473)}$
	0.6	$0.850_{(0.587)}$	$0.904_{(0.699)}$	$0.849_{(0.510)}$	$0.891_{(0.607)}$	$0.863_{(0.549)}$	$0.904_{(0.654)}$
	0.8	$0.783_{(1.021)}$	$0.828_{(1.217)}$	$0.780_{(0.901)}$	$0.826_{(1.073)}$	$0.768_{(0.973)}$	$0.821_{(1.160)}$
1000	0.2	$0.897_{(0.238)}$	$0.931_{(0.283)}$	$0.898_{(0.212)}$	$0.951_{(0.253)}$	$0.885_{(0.225)}$	$0.933_{(0.268)}$
	0.4	$0.891_{(0.305)}$	$0.936_{(0.364)}$	$0.884_{(0.271)}$	$0.927_{(0.323)}$	$0.881_{(0.286)}$	$0.933_{(0.341)}$
	0.6	$0.873_{(0.428)}$	$0.918_{(0.510)}$	$0.861_{(0.383)}$	$0.908_{(0.457)}$	$0.848_{(0.404)}$	$0.903_{(0.481)}$
	0.8	$0.793_{(0.778)}$	$0.853_{(0.927)}$	$0.782_{(0.697)}$	$0.840_{(0.830)}$	$0.797_{(0.736)}$	$0.858_{(0.877)}$
2000	0.2	$0.886_{(0.172)}$	$0.943_{(0.205)}$	$0.891_{(0.151)}$	$0.942_{(0.180)}$	$0.903_{(0.162)}$	$0.952_{(0.193)}$
	0.4	$0.867_{(0.220)}$	$0.924_{(0.262)}$	$0.895_{(0.194)}$	$0.952_{(0.231)}$	$0.911_{(0.207)}$	$0.953_{(0.247)}$
	0.6	$0.848_{(0.313)}$	$0.900_{(0.373)}$	$0.865_{(0.275)}$	$0.922_{(0.328)}$	$0.874_{(0.293)}$	$0.932_{(0.349)}$
	0.8	$0.790_{(0.575)}$	$0.870_{(0.686)}$	$0.777_{(0.508)}$	$0.869_{(0.605)}$	$0.782_{(0.546)}$	$0.862_{(0.651)}$

3.4.5.5 $B_n = 30$

Table 3.25: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 30$ and different kernel choices when the innovations are Fréchet with $\gamma = 2$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
x_n chosen as the 90% quantile							
200	0.2	$0.782_{(0.493)}$	$0.812_{(0.588)}$	$0.814_{(0.467)}$	$0.845_{(0.557)}$	$0.820_{(0.483)}$	$0.849_{(0.575)}$
	0.4	$0.824_{(0.640)}$	$0.857_{(0.763)}$	$0.842_{(0.596)}$	$0.869_{(0.710)}$	$0.829_{(0.624)}$	$0.856_{(0.743)}$
	0.6	$0.805_{(0.924)}$	$0.832_{(1.101)}$	$0.828_{(0.842)}$	$0.857_{(1.003)}$	$0.816_{(0.852)}$	$0.847_{(1.015)}$
	0.8	$0.778_{(1.668)}$	$0.818_{(1.987)}$	$0.805_{(1.457)}$	$0.855_{(1.736)}$	$0.793_{(1.576)}$	$0.825_{(1.878)}$
500	0.2	$0.856_{(0.364)}$	$0.884_{(0.434)}$	$0.860_{(0.324)}$	$0.890_{(0.387)}$	$0.856_{(0.347)}$	$0.894_{(0.414)}$
	0.4	$0.846_{(0.470)}$	$0.874_{(0.560)}$	$0.883_{(0.424)}$	$0.914_{(0.505)}$	$0.866_{(0.451)}$	$0.894_{(0.537)}$
	0.6	$0.854_{(0.687)}$	$0.888_{(0.819)}$	$0.852_{(0.617)}$	$0.894_{(0.735)}$	$0.860_{(0.645)}$	$0.896_{(0.769)}$
	0.8	$0.847_{(1.267)}$	$0.890_{(1.509)}$	$0.863_{(1.172)}$	$0.908_{(1.396)}$	$0.853_{(1.177)}$	$0.890_{(1.403)}$
1000	0.2	$0.874_{(0.270)}$	$0.919_{(0.322)}$	$0.898_{(0.240)}$	$0.940_{(0.286)}$	$0.875_{(0.252)}$	$0.904_{(0.300)}$
	0.4	$0.875_{(0.355)}$	$0.917_{(0.423)}$	$0.889_{(0.309)}$	$0.918_{(0.368)}$	$0.892_{(0.328)}$	$0.923_{(0.390)}$
	0.6	$0.883_{(0.498)}$	$0.918_{(0.594)}$	$0.893_{(0.447)}$	$0.940_{(0.532)}$	$0.875_{(0.478)}$	$0.926_{(0.569)}$
	0.8	$0.853_{(0.945)}$	$0.901_{(1.126)}$	$0.872_{(0.832)}$	$0.914_{(0.992)}$	$0.866_{(0.891)}$	$0.913_{(1.062)}$
2000	0.2	$0.898_{(0.192)}$	$0.932_{(0.229)}$	$0.888_{(0.172)}$	$0.938_{(0.205)}$	$0.923_{(0.182)}$	$0.956_{(0.217)}$
	0.4	$0.906_{(0.252)}$	$0.943_{(0.301)}$	$0.894_{(0.225)}$	$0.938_{(0.269)}$	$0.903_{(0.238)}$	$0.943_{(0.283)}$
	0.6	$0.905_{(0.370)}$	$0.945_{(0.441)}$	$0.894_{(0.323)}$	$0.939_{(0.385)}$	$0.891_{(0.345)}$	$0.940_{(0.411)}$
	0.8	$0.887_{(0.694)}$	$0.936_{(0.827)}$	$0.895_{(0.612)}$	$0.941_{(0.730)}$	$0.884_{(0.647)}$	$0.925_{(0.771)}$
x_n chosen as the 95% quantile							
200	0.2	$0.828_{(0.469)}$	$0.858_{(0.559)}$	$0.846_{(0.432)}$	$0.881_{(0.515)}$	$0.843_{(0.449)}$	$0.868_{(0.535)}$
	0.4	$0.822_{(0.587)}$	$0.855_{(0.699)}$	$0.847_{(0.566)}$	$0.889_{(0.674)}$	$0.843_{(0.575)}$	$0.867_{(0.685)}$
	0.6	$0.807_{(0.804)}$	$0.845_{(0.958)}$	$0.826_{(0.769)}$	$0.879_{(0.916)}$	$0.821_{(0.806)}$	$0.856_{(0.960)}$
	0.8	$0.767_{(1.400)}$	$0.806_{(1.668)}$	$0.767_{(1.265)}$	$0.785_{(1.508)}$	$0.770_{(1.309)}$	$0.791_{(1.560)}$
500	0.2	$0.875_{(0.344)}$	$0.902_{(0.410)}$	$0.881_{(0.315)}$	$0.916_{(0.375)}$	$0.876_{(0.333)}$	$0.903_{(0.397)}$
	0.4	$0.882_{(0.444)}$	$0.907_{(0.529)}$	$0.882_{(0.404)}$	$0.914_{(0.481)}$	$0.863_{(0.424)}$	$0.898_{(0.506)}$
	0.6	$0.851_{(0.614)}$	$0.888_{(0.732)}$	$0.862_{(0.566)}$	$0.899_{(0.675)}$	$0.839_{(0.596)}$	$0.892_{(0.711)}$
	0.8	$0.794_{(1.138)}$	$0.834_{(1.356)}$	$0.788_{(0.969)}$	$0.832_{(1.155)}$	$0.796_{(1.057)}$	$0.826_{(1.259)}$
1000	0.2	$0.880_{(0.259)}$	$0.920_{(0.308)}$	$0.894_{(0.230)}$	$0.943_{(0.275)}$	$0.886_{(0.246)}$	$0.932_{(0.293)}$
	0.4	$0.886_{(0.331)}$	$0.927_{(0.395)}$	$0.896_{(0.299)}$	$0.939_{(0.356)}$	$0.887_{(0.311)}$	$0.924_{(0.371)}$
	0.6	$0.860_{(0.478)}$	$0.911_{(0.569)}$	$0.869_{(0.416)}$	$0.923_{(0.496)}$	$0.878_{(0.441)}$	$0.928_{(0.525)}$
	0.8	$0.807_{(0.870)}$	$0.863_{(1.036)}$	$0.828_{(0.738)}$	$0.876_{(0.879)}$	$0.797_{(0.801)}$	$0.866_{(0.954)}$
2000	0.2	$0.892_{(0.190)}$	$0.932_{(0.226)}$	$0.903_{(0.168)}$	$0.935_{(0.200)}$	$0.883_{(0.176)}$	$0.930_{(0.210)}$
	0.4	$0.896_{(0.242)}$	$0.943_{(0.288)}$	$0.886_{(0.212)}$	$0.923_{(0.253)}$	$0.892_{(0.226)}$	$0.934_{(0.269)}$
	0.6	$0.862_{(0.342)}$	$0.919_{(0.408)}$	$0.879_{(0.305)}$	$0.934_{(0.363)}$	$0.864_{(0.324)}$	$0.924_{(0.386)}$
	0.8	$0.801_{(0.637)}$	$0.875_{(0.759)}$	$0.812_{(0.554)}$	$0.877_{(0.660)}$	$0.792_{(0.594)}$	$0.872_{(0.708)}$

3.4.6 ARMA with Fréchet 3 Innovation

3.4.6.1 $B_n = 10$

Table 3.26: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 10$ and different kernel choices when the innovations are Fréchet with $\gamma = 3$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n *chosen as the 90% quantile*			
200	0.2	$0.891_{(0.312)}$	$0.923_{(0.371)}$	$0.902_{(0.289)}$	$0.939_{(0.345)}$	$0.893_{(0.303)}$	$0.924_{(0.361)}$
	0.4	$0.894_{(0.408)}$	$0.923_{(0.487)}$	$0.908_{(0.361)}$	$0.945_{(0.430)}$	$0.884_{(0.378)}$	$0.933_{(0.451)}$
	0.6	$0.882_{(0.569)}$	$0.909_{(0.678)}$	$0.885_{(0.497)}$	$0.924_{(0.592)}$	$0.886_{(0.530)}$	$0.915_{(0.632)}$
	0.8	$0.848_{(0.898)}$	$0.884_{(1.070)}$	$0.847_{(0.746)}$	$0.881_{(0.888)}$	$0.827_{(0.802)}$	$0.864_{(0.956)}$
500	0.2	$0.918_{(0.210)}$	$0.947_{(0.250)}$	$0.901_{(0.187)}$	$0.948_{(0.223)}$	$0.908_{(0.200)}$	$0.950_{(0.239)}$
	0.4	$0.895_{(0.268)}$	$0.939_{(0.320)}$	$0.904_{(0.238)}$	$0.947_{(0.283)}$	$0.925_{(0.252)}$	$0.958_{(0.300)}$
	0.6	$0.904_{(0.375)}$	$0.941_{(0.448)}$	$0.909_{(0.327)}$	$0.949_{(0.389)}$	$0.905_{(0.352)}$	$0.939_{(0.419)}$
	0.8	$0.880_{(0.620)}$	$0.914_{(0.739)}$	$0.875_{(0.514)}$	$0.914_{(0.613)}$	$0.901_{(0.564)}$	$0.928_{(0.672)}$
1000	0.2	$0.923_{(0.152)}$	$0.957_{(0.181)}$	$0.914_{(0.134)}$	$0.959_{(0.160)}$	$0.907_{(0.142)}$	$0.960_{(0.170)}$
	0.4	$0.931_{(0.195)}$	$0.956_{(0.233)}$	$0.914_{(0.171)}$	$0.949_{(0.203)}$	$0.919_{(0.182)}$	$0.961_{(0.216)}$
	0.6	$0.914_{(0.272)}$	$0.952_{(0.324)}$	$0.917_{(0.234)}$	$0.951_{(0.279)}$	$0.919_{(0.252)}$	$0.960_{(0.300)}$
	0.8	$0.914_{(0.454)}$	$0.944_{(0.542)}$	$0.924_{(0.371)}$	$0.946_{(0.442)}$	$0.912_{(0.406)}$	$0.946_{(0.484)}$
2000	0.2	$0.913_{(0.109)}$	$0.941_{(0.129)}$	$0.923_{(0.095)}$	$0.960_{(0.114)}$	$0.912_{(0.101)}$	$0.961_{(0.121)}$
	0.4	$0.909_{(0.137)}$	$0.954_{(0.164)}$	$0.915_{(0.121)}$	$0.959_{(0.144)}$	$0.915_{(0.129)}$	$0.955_{(0.154)}$
	0.6	$0.914_{(0.194)}$	$0.952_{(0.232)}$	$0.922_{(0.169)}$	$0.959_{(0.201)}$	$0.911_{(0.179)}$	$0.956_{(0.214)}$
	0.8	$0.892_{(0.324)}$	$0.950_{(0.386)}$	$0.927_{(0.266)}$	$0.964_{(0.317)}$	$0.932_{(0.291)}$	$0.965_{(0.347)}$
				x_n *chosen as the 95% quantile*			
200	0.2	$0.901_{(0.302)}$	$0.940_{(0.360)}$	$0.909_{(0.272)}$	$0.947_{(0.324)}$	$0.921_{(0.285)}$	$0.950_{(0.339)}$
	0.4	$0.903_{(0.378)}$	$0.943_{(0.450)}$	$0.884_{(0.332)}$	$0.927_{(0.395)}$	$0.922_{(0.349)}$	$0.945_{(0.416)}$
	0.6	$0.845_{(0.494)}$	$0.878_{(0.589)}$	$0.828_{(0.440)}$	$0.879_{(0.525)}$	$0.821_{(0.464)}$	$0.863_{(0.553)}$
	0.8	$0.738_{(0.750)}$	$0.767_{(0.893)}$	$0.765_{(0.654)}$	$0.790_{(0.779)}$	$0.766_{(0.704)}$	$0.788_{(0.839)}$
500	0.2	$0.911_{(0.203)}$	$0.951_{(0.243)}$	$0.913_{(0.179)}$	$0.960_{(0.213)}$	$0.906_{(0.188)}$	$0.956_{(0.224)}$
	0.4	$0.892_{(0.250)}$	$0.937_{(0.298)}$	$0.886_{(0.222)}$	$0.934_{(0.265)}$	$0.901_{(0.237)}$	$0.937_{(0.282)}$
	0.6	$0.851_{(0.345)}$	$0.903_{(0.411)}$	$0.848_{(0.300)}$	$0.897_{(0.358)}$	$0.845_{(0.317)}$	$0.904_{(0.378)}$
	0.8	$0.773_{(0.549)}$	$0.832_{(0.654)}$	$0.774_{(0.465)}$	$0.833_{(0.554)}$	$0.770_{(0.510)}$	$0.831_{(0.609)}$
1000	0.2	$0.914_{(0.146)}$	$0.955_{(0.174)}$	$0.925_{(0.129)}$	$0.955_{(0.154)}$	$0.903_{(0.136)}$	$0.950_{(0.162)}$
	0.4	$0.880_{(0.183)}$	$0.932_{(0.218)}$	$0.896_{(0.161)}$	$0.933_{(0.192)}$	$0.888_{(0.171)}$	$0.940_{(0.204)}$
	0.6	$0.866_{(0.248)}$	$0.924_{(0.296)}$	$0.847_{(0.218)}$	$0.909_{(0.259)}$	$0.830_{(0.232)}$	$0.892_{(0.277)}$
	0.8	$0.788_{(0.413)}$	$0.847_{(0.492)}$	$0.800_{(0.341)}$	$0.865_{(0.406)}$	$0.822_{(0.373)}$	$0.875_{(0.445)}$
2000	0.2	$0.905_{(0.104)}$	$0.950_{(0.124)}$	$0.908_{(0.091)}$	$0.963_{(0.109)}$	$0.916_{(0.097)}$	$0.958_{(0.116)}$
	0.4	$0.882_{(0.131)}$	$0.937_{(0.156)}$	$0.882_{(0.115)}$	$0.943_{(0.137)}$	$0.878_{(0.121)}$	$0.936_{(0.144)}$
	0.6	$0.850_{(0.179)}$	$0.913_{(0.214)}$	$0.854_{(0.154)}$	$0.912_{(0.184)}$	$0.849_{(0.166)}$	$0.915_{(0.198)}$
	0.8	$0.794_{(0.297)}$	$0.858_{(0.353)}$	$0.827_{(0.247)}$	$0.890_{(0.294)}$	$0.794_{(0.266)}$	$0.866_{(0.318)}$

3.4.6.2 $B_n = 15$

Table 3.27: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 15$ and different kernel choices when the innovations are Fréchet with $\gamma = 3$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.854_{(0.386)}$	$0.885_{(0.460)}$	$0.885_{(0.347)}$	$0.912_{(0.413)}$	$0.870_{(0.359)}$	$0.905_{(0.428)}$
	0.4	$0.873_{(0.482)}$	$0.908_{(0.575)}$	$0.865_{(0.437)}$	$0.902_{(0.521)}$	$0.873_{(0.456)}$	$0.900_{(0.544)}$
	0.6	$0.845_{(0.649)}$	$0.874_{(0.774)}$	$0.864_{(0.588)}$	$0.907_{(0.701)}$	$0.873_{(0.634)}$	$0.898_{(0.756)}$
	0.8	$0.824_{(1.122)}$	$0.859_{(1.337)}$	$0.817_{(1.000)}$	$0.847_{(1.192)}$	$0.838_{(1.068)}$	$0.872_{(1.273)}$
500	0.2	$0.902_{(0.257)}$	$0.939_{(0.306)}$	$0.893_{(0.228)}$	$0.932_{(0.272)}$	$0.893_{(0.238)}$	$0.931_{(0.284)}$
	0.4	$0.892_{(0.329)}$	$0.931_{(0.392)}$	$0.881_{(0.285)}$	$0.930_{(0.339)}$	$0.904_{(0.304)}$	$0.940_{(0.362)}$
	0.6	$0.892_{(0.454)}$	$0.931_{(0.541)}$	$0.900_{(0.407)}$	$0.931_{(0.485)}$	$0.873_{(0.430)}$	$0.913_{(0.512)}$
	0.8	$0.872_{(0.803)}$	$0.911_{(0.957)}$	$0.870_{(0.683)}$	$0.910_{(0.814)}$	$0.875_{(0.753)}$	$0.916_{(0.897)}$
1000	0.2	$0.902_{(0.185)}$	$0.942_{(0.220)}$	$0.920_{(0.163)}$	$0.952_{(0.195)}$	$0.919_{(0.174)}$	$0.956_{(0.208)}$
	0.4	$0.901_{(0.233)}$	$0.941_{(0.278)}$	$0.910_{(0.210)}$	$0.949_{(0.250)}$	$0.918_{(0.222)}$	$0.947_{(0.264)}$
	0.6	$0.894_{(0.337)}$	$0.932_{(0.401)}$	$0.917_{(0.294)}$	$0.956_{(0.350)}$	$0.891_{(0.311)}$	$0.934_{(0.371)}$
	0.8	$0.895_{(0.590)}$	$0.931_{(0.703)}$	$0.898_{(0.504)}$	$0.939_{(0.601)}$	$0.896_{(0.546)}$	$0.935_{(0.651)}$
2000	0.2	$0.906_{(0.132)}$	$0.947_{(0.158)}$	$0.920_{(0.117)}$	$0.959_{(0.140)}$	$0.903_{(0.124)}$	$0.953_{(0.148)}$
	0.4	$0.923_{(0.169)}$	$0.953_{(0.202)}$	$0.892_{(0.149)}$	$0.947_{(0.178)}$	$0.914_{(0.159)}$	$0.947_{(0.189)}$
	0.6	$0.907_{(0.239)}$	$0.945_{(0.285)}$	$0.903_{(0.211)}$	$0.950_{(0.251)}$	$0.916_{(0.224)}$	$0.955_{(0.267)}$
	0.8	$0.881_{(0.419)}$	$0.933_{(0.500)}$	$0.914_{(0.361)}$	$0.953_{(0.430)}$	$0.919_{(0.392)}$	$0.960_{(0.468)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.887_{(0.357)}$	$0.928_{(0.425)}$	$0.897_{(0.329)}$	$0.926_{(0.392)}$	$0.880_{(0.343)}$	$0.913_{(0.409)}$
	0.4	$0.863_{(0.432)}$	$0.906_{(0.515)}$	$0.879_{(0.396)}$	$0.920_{(0.472)}$	$0.882_{(0.423)}$	$0.925_{(0.504)}$
	0.6	$0.876_{(0.597)}$	$0.898_{(0.711)}$	$0.846_{(0.526)}$	$0.888_{(0.626)}$	$0.867_{(0.549)}$	$0.911_{(0.655)}$
	0.8	$0.762_{(0.940)}$	$0.783_{(1.120)}$	$0.777_{(0.835)}$	$0.796_{(0.995)}$	$0.752_{(0.842)}$	$0.803_{(1.003)}$
500	0.2	$0.871_{(0.245)}$	$0.925_{(0.292)}$	$0.892_{(0.216)}$	$0.943_{(0.258)}$	$0.884_{(0.227)}$	$0.926_{(0.271)}$
	0.4	$0.888_{(0.301)}$	$0.930_{(0.359)}$	$0.863_{(0.274)}$	$0.922_{(0.327)}$	$0.880_{(0.284)}$	$0.925_{(0.339)}$
	0.6	$0.869_{(0.411)}$	$0.908_{(0.490)}$	$0.859_{(0.366)}$	$0.913_{(0.437)}$	$0.854_{(0.391)}$	$0.900_{(0.466)}$
	0.8	$0.777_{(0.705)}$	$0.835_{(0.840)}$	$0.760_{(0.604)}$	$0.829_{(0.720)}$	$0.788_{(0.649)}$	$0.841_{(0.774)}$
1000	0.2	$0.879_{(0.176)}$	$0.930_{(0.209)}$	$0.909_{(0.158)}$	$0.950_{(0.188)}$	$0.913_{(0.165)}$	$0.952_{(0.197)}$
	0.4	$0.877_{(0.221)}$	$0.935_{(0.263)}$	$0.902_{(0.197)}$	$0.948_{(0.235)}$	$0.908_{(0.208)}$	$0.938_{(0.248)}$
	0.6	$0.875_{(0.306)}$	$0.920_{(0.365)}$	$0.843_{(0.272)}$	$0.892_{(0.324)}$	$0.862_{(0.285)}$	$0.919_{(0.339)}$
	0.8	$0.798_{(0.528)}$	$0.866_{(0.629)}$	$0.790_{(0.454)}$	$0.846_{(0.541)}$	$0.787_{(0.490)}$	$0.849_{(0.584)}$
2000	0.2	$0.902_{(0.127)}$	$0.951_{(0.151)}$	$0.899_{(0.113)}$	$0.941_{(0.134)}$	$0.884_{(0.119)}$	$0.937_{(0.141)}$
	0.4	$0.877_{(0.158)}$	$0.941_{(0.189)}$	$0.876_{(0.141)}$	$0.932_{(0.168)}$	$0.887_{(0.148)}$	$0.934_{(0.176)}$
	0.6	$0.853_{(0.221)}$	$0.918_{(0.263)}$	$0.869_{(0.194)}$	$0.921_{(0.231)}$	$0.841_{(0.205)}$	$0.904_{(0.244)}$
	0.8	$0.799_{(0.380)}$	$0.867_{(0.453)}$	$0.771_{(0.327)}$	$0.840_{(0.390)}$	$0.775_{(0.355)}$	$0.856_{(0.423)}$

3.4.6.3 $B_n = 20$

Table 3.28: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 20$ and different kernel choices when the innovations are Fréchet with $\gamma = 3$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.839_{(0.427)}$	$0.873_{(0.509)}$	$0.853_{(0.384)}$	$0.887_{(0.458)}$	$0.864_{(0.399)}$	$0.889_{(0.476)}$
	0.4	$0.833_{(0.531)}$	$0.871_{(0.633)}$	$0.847_{(0.482)}$	$0.883_{(0.574)}$	$0.856_{(0.504)}$	$0.880_{(0.601)}$
	0.6	$0.829_{(0.742)}$	$0.867_{(0.885)}$	$0.872_{(0.680)}$	$0.906_{(0.811)}$	$0.848_{(0.700)}$	$0.880_{(0.834)}$
	0.8	$0.816_{(1.243)}$	$0.849_{(1.481)}$	$0.807_{(1.124)}$	$0.846_{(1.339)}$	$0.836_{(1.210)}$	$0.870_{(1.441)}$
500	0.2	$0.875_{(0.291)}$	$0.905_{(0.346)}$	$0.872_{(0.263)}$	$0.908_{(0.313)}$	$0.871_{(0.277)}$	$0.910_{(0.330)}$
	0.4	$0.876_{(0.371)}$	$0.911_{(0.442)}$	$0.891_{(0.333)}$	$0.932_{(0.396)}$	$0.868_{(0.346)}$	$0.910_{(0.413)}$
	0.6	$0.873_{(0.522)}$	$0.911_{(0.621)}$	$0.893_{(0.467)}$	$0.924_{(0.557)}$	$0.884_{(0.506)}$	$0.911_{(0.603)}$
	0.8	$0.848_{(0.955)}$	$0.886_{(1.138)}$	$0.853_{(0.818)}$	$0.899_{(0.975)}$	$0.845_{(0.882)}$	$0.881_{(1.051)}$
1000	0.2	$0.894_{(0.212)}$	$0.931_{(0.253)}$	$0.911_{(0.187)}$	$0.941_{(0.223)}$	$0.898_{(0.200)}$	$0.937_{(0.238)}$
	0.4	$0.901_{(0.270)}$	$0.932_{(0.322)}$	$0.899_{(0.242)}$	$0.940_{(0.288)}$	$0.891_{(0.252)}$	$0.921_{(0.301)}$
	0.6	$0.895_{(0.385)}$	$0.934_{(0.458)}$	$0.902_{(0.340)}$	$0.945_{(0.405)}$	$0.890_{(0.362)}$	$0.936_{(0.432)}$
	0.8	$0.869_{(0.700)}$	$0.916_{(0.834)}$	$0.881_{(0.607)}$	$0.919_{(0.723)}$	$0.890_{(0.644)}$	$0.928_{(0.767)}$
2000	0.2	$0.908_{(0.153)}$	$0.955_{(0.182)}$	$0.896_{(0.135)}$	$0.943_{(0.161)}$	$0.904_{(0.141)}$	$0.958_{(0.169)}$
	0.4	$0.908_{(0.196)}$	$0.950_{(0.233)}$	$0.908_{(0.172)}$	$0.939_{(0.205)}$	$0.907_{(0.183)}$	$0.953_{(0.218)}$
	0.6	$0.914_{(0.275)}$	$0.950_{(0.328)}$	$0.900_{(0.243)}$	$0.951_{(0.290)}$	$0.907_{(0.261)}$	$0.947_{(0.311)}$
	0.8	$0.885_{(0.506)}$	$0.927_{(0.603)}$	$0.900_{(0.432)}$	$0.941_{(0.515)}$	$0.901_{(0.465)}$	$0.946_{(0.554)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.838_{(0.399)}$	$0.880_{(0.475)}$	$0.864_{(0.354)}$	$0.907_{(0.422)}$	$0.856_{(0.381)}$	$0.883_{(0.455)}$
	0.4	$0.866_{(0.488)}$	$0.884_{(0.581)}$	$0.855_{(0.450)}$	$0.905_{(0.536)}$	$0.849_{(0.473)}$	$0.899_{(0.563)}$
	0.6	$0.840_{(0.670)}$	$0.868_{(0.798)}$	$0.862_{(0.595)}$	$0.903_{(0.709)}$	$0.846_{(0.616)}$	$0.889_{(0.734)}$
	0.8	$0.765_{(1.023)}$	$0.810_{(1.219)}$	$0.767_{(0.971)}$	$0.790_{(1.157)}$	$0.767_{(0.989)}$	$0.794_{(1.179)}$
500	0.2	$0.890_{(0.275)}$	$0.929_{(0.327)}$	$0.877_{(0.248)}$	$0.925_{(0.295)}$	$0.893_{(0.260)}$	$0.927_{(0.310)}$
	0.4	$0.871_{(0.353)}$	$0.921_{(0.420)}$	$0.882_{(0.311)}$	$0.932_{(0.370)}$	$0.866_{(0.330)}$	$0.910_{(0.393)}$
	0.6	$0.843_{(0.471)}$	$0.890_{(0.561)}$	$0.868_{(0.420)}$	$0.927_{(0.500)}$	$0.856_{(0.449)}$	$0.894_{(0.535)}$
	0.8	$0.794_{(0.801)}$	$0.851_{(0.954)}$	$0.774_{(0.719)}$	$0.836_{(0.857)}$	$0.783_{(0.760)}$	$0.833_{(0.906)}$
1000	0.2	$0.906_{(0.203)}$	$0.938_{(0.242)}$	$0.899_{(0.181)}$	$0.936_{(0.215)}$	$0.894_{(0.190)}$	$0.937_{(0.227)}$
	0.4	$0.887_{(0.255)}$	$0.930_{(0.304)}$	$0.884_{(0.225)}$	$0.946_{(0.268)}$	$0.884_{(0.243)}$	$0.933_{(0.290)}$
	0.6	$0.870_{(0.351)}$	$0.920_{(0.418)}$	$0.865_{(0.308)}$	$0.909_{(0.367)}$	$0.861_{(0.334)}$	$0.918_{(0.398)}$
	0.8	$0.795_{(0.619)}$	$0.869_{(0.738)}$	$0.809_{(0.535)}$	$0.875_{(0.638)}$	$0.795_{(0.578)}$	$0.847_{(0.688)}$
2000	0.2	$0.895_{(0.146)}$	$0.941_{(0.174)}$	$0.890_{(0.129)}$	$0.939_{(0.154)}$	$0.893_{(0.137)}$	$0.943_{(0.164)}$
	0.4	$0.890_{(0.183)}$	$0.925_{(0.218)}$	$0.888_{(0.162)}$	$0.941_{(0.193)}$	$0.895_{(0.171)}$	$0.943_{(0.204)}$
	0.6	$0.872_{(0.254)}$	$0.938_{(0.303)}$	$0.866_{(0.224)}$	$0.918_{(0.267)}$	$0.882_{(0.236)}$	$0.926_{(0.282)}$
	0.8	$0.805_{(0.451)}$	$0.871_{(0.538)}$	$0.821_{(0.387)}$	$0.882_{(0.461)}$	$0.823_{(0.418)}$	$0.878_{(0.498)}$

3.4.6.4 $B_n = 25$

Table 3.29: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 25$ and different kernel choices when the innovations are Fréchet with $\gamma = 3$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.813_{(0.453)}$	$0.842_{(0.540)}$	$0.818_{(0.415)}$	$0.854_{(0.494)}$	$0.824_{(0.427)}$	$0.853_{(0.509)}$
	0.4	$0.811_{(0.571)}$	$0.836_{(0.681)}$	$0.832_{(0.524)}$	$0.868_{(0.624)}$	$0.822_{(0.557)}$	$0.856_{(0.664)}$
	0.6	$0.817_{(0.783)}$	$0.850_{(0.933)}$	$0.830_{(0.733)}$	$0.864_{(0.874)}$	$0.818_{(0.772)}$	$0.852_{(0.920)}$
	0.8	$0.793_{(1.437)}$	$0.832_{(1.712)}$	$0.835_{(1.210)}$	$0.863_{(1.442)}$	$0.808_{(1.320)}$	$0.847_{(1.573)}$
500	0.2	$0.845_{(0.320)}$	$0.878_{(0.381)}$	$0.885_{(0.288)}$	$0.927_{(0.343)}$	$0.874_{(0.304)}$	$0.904_{(0.362)}$
	0.4	$0.871_{(0.406)}$	$0.906_{(0.484)}$	$0.877_{(0.369)}$	$0.919_{(0.440)}$	$0.869_{(0.384)}$	$0.901_{(0.458)}$
	0.6	$0.869_{(0.584)}$	$0.902_{(0.695)}$	$0.873_{(0.511)}$	$0.921_{(0.609)}$	$0.858_{(0.553)}$	$0.894_{(0.659)}$
	0.8	$0.859_{(1.057)}$	$0.894_{(1.259)}$	$0.870_{(0.929)}$	$0.905_{(1.107)}$	$0.858_{(0.981)}$	$0.893_{(1.169)}$
1000	0.2	$0.888_{(0.234)}$	$0.934_{(0.279)}$	$0.897_{(0.212)}$	$0.939_{(0.253)}$	$0.897_{(0.220)}$	$0.931_{(0.262)}$
	0.4	$0.875_{(0.298)}$	$0.916_{(0.355)}$	$0.874_{(0.266)}$	$0.923_{(0.316)}$	$0.892_{(0.282)}$	$0.939_{(0.336)}$
	0.6	$0.884_{(0.422)}$	$0.930_{(0.503)}$	$0.893_{(0.383)}$	$0.931_{(0.457)}$	$0.879_{(0.399)}$	$0.925_{(0.476)}$
	0.8	$0.865_{(0.789)}$	$0.908_{(0.940)}$	$0.880_{(0.684)}$	$0.925_{(0.814)}$	$0.869_{(0.738)}$	$0.909_{(0.879)}$
2000	0.2	$0.904_{(0.172)}$	$0.944_{(0.205)}$	$0.900_{(0.150)}$	$0.943_{(0.179)}$	$0.897_{(0.160)}$	$0.937_{(0.191)}$
	0.4	$0.883_{(0.215)}$	$0.925_{(0.256)}$	$0.912_{(0.191)}$	$0.950_{(0.228)}$	$0.897_{(0.203)}$	$0.946_{(0.242)}$
	0.6	$0.908_{(0.304)}$	$0.958_{(0.362)}$	$0.887_{(0.271)}$	$0.929_{(0.323)}$	$0.908_{(0.288)}$	$0.943_{(0.343)}$
	0.8	$0.877_{(0.569)}$	$0.926_{(0.677)}$	$0.909_{(0.497)}$	$0.943_{(0.593)}$	$0.888_{(0.530)}$	$0.934_{(0.632)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.824_{(0.419)}$	$0.852_{(0.499)}$	$0.856_{(0.401)}$	$0.896_{(0.477)}$	$0.841_{(0.408)}$	$0.880_{(0.486)}$
	0.4	$0.851_{(0.534)}$	$0.887_{(0.636)}$	$0.873_{(0.481)}$	$0.902_{(0.574)}$	$0.866_{(0.492)}$	$0.889_{(0.586)}$
	0.6	$0.833_{(0.702)}$	$0.875_{(0.837)}$	$0.848_{(0.656)}$	$0.870_{(0.782)}$	$0.848_{(0.662)}$	$0.886_{(0.789)}$
	0.8	$0.765_{(1.157)}$	$0.793_{(1.378)}$	$0.760_{(1.015)}$	$0.796_{(1.209)}$	$0.772_{(1.095)}$	$0.796_{(1.305)}$
500	0.2	$0.899_{(0.301)}$	$0.923_{(0.359)}$	$0.870_{(0.280)}$	$0.914_{(0.334)}$	$0.878_{(0.287)}$	$0.907_{(0.341)}$
	0.4	$0.881_{(0.376)}$	$0.912_{(0.449)}$	$0.891_{(0.342)}$	$0.924_{(0.408)}$	$0.872_{(0.364)}$	$0.909_{(0.434)}$
	0.6	$0.863_{(0.519)}$	$0.906_{(0.618)}$	$0.881_{(0.465)}$	$0.923_{(0.554)}$	$0.857_{(0.497)}$	$0.899_{(0.593)}$
	0.8	$0.797_{(0.878)}$	$0.836_{(1.046)}$	$0.792_{(0.827)}$	$0.844_{(0.986)}$	$0.809_{(0.840)}$	$0.852_{(1.001)}$
1000	0.2	$0.890_{(0.227)}$	$0.917_{(0.271)}$	$0.885_{(0.199)}$	$0.938_{(0.238)}$	$0.894_{(0.208)}$	$0.934_{(0.248)}$
	0.4	$0.884_{(0.280)}$	$0.930_{(0.333)}$	$0.877_{(0.255)}$	$0.926_{(0.304)}$	$0.896_{(0.267)}$	$0.934_{(0.318)}$
	0.6	$0.891_{(0.381)}$	$0.927_{(0.454)}$	$0.868_{(0.348)}$	$0.916_{(0.415)}$	$0.878_{(0.372)}$	$0.911_{(0.443)}$
	0.8	$0.815_{(0.687)}$	$0.881_{(0.819)}$	$0.784_{(0.602)}$	$0.853_{(0.718)}$	$0.816_{(0.643)}$	$0.881_{(0.766)}$
2000	0.2	$0.897_{(0.163)}$	$0.939_{(0.194)}$	$0.894_{(0.143)}$	$0.942_{(0.171)}$	$0.890_{(0.152)}$	$0.928_{(0.182)}$
	0.4	$0.888_{(0.203)}$	$0.942_{(0.241)}$	$0.902_{(0.179)}$	$0.935_{(0.214)}$	$0.894_{(0.191)}$	$0.936_{(0.227)}$
	0.6	$0.877_{(0.280)}$	$0.917_{(0.334)}$	$0.873_{(0.248)}$	$0.925_{(0.296)}$	$0.876_{(0.266)}$	$0.930_{(0.317)}$
	0.8	$0.799_{(0.506)}$	$0.865_{(0.603)}$	$0.791_{(0.445)}$	$0.867_{(0.530)}$	$0.814_{(0.474)}$	$0.880_{(0.565)}$

3.4.6.5 $B_n = 30$

Table 3.30: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 30$ and different kernel choices when the innovations are Fréchet with $\gamma = 3$.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.813_{(0.485)}$	$0.840_{(0.577)}$	$0.788_{(0.442)}$	$0.831_{(0.527)}$	$0.796_{(0.471)}$	$0.831_{(0.562)}$
	0.4	$0.792_{(0.584)}$	$0.820_{(0.695)}$	$0.808_{(0.558)}$	$0.842_{(0.665)}$	$0.799_{(0.568)}$	$0.837_{(0.677)}$
	0.6	$0.789_{(0.823)}$	$0.833_{(0.981)}$	$0.849_{(0.768)}$	$0.885_{(0.915)}$	$0.812_{(0.802)}$	$0.843_{(0.956)}$
	0.8	$0.802_{(1.440)}$	$0.831_{(1.716)}$	$0.829_{(1.349)}$	$0.857_{(1.608)}$	$0.788_{(1.412)}$	$0.822_{(1.682)}$
500	0.2	$0.855_{(0.341)}$	$0.890_{(0.406)}$	$0.861_{(0.310)}$	$0.891_{(0.369)}$	$0.878_{(0.329)}$	$0.900_{(0.392)}$
	0.4	$0.851_{(0.449)}$	$0.897_{(0.535)}$	$0.883_{(0.403)}$	$0.914_{(0.480)}$	$0.857_{(0.422)}$	$0.896_{(0.503)}$
	0.6	$0.855_{(0.626)}$	$0.887_{(0.746)}$	$0.841_{(0.561)}$	$0.895_{(0.668)}$	$0.862_{(0.590)}$	$0.900_{(0.704)}$
	0.8	$0.857_{(1.114)}$	$0.882_{(1.327)}$	$0.860_{(1.014)}$	$0.901_{(1.208)}$	$0.852_{(1.066)}$	$0.889_{(1.270)}$
1000	0.2	$0.872_{(0.256)}$	$0.909_{(0.305)}$	$0.891_{(0.226)}$	$0.931_{(0.269)}$	$0.882_{(0.242)}$	$0.919_{(0.288)}$
	0.4	$0.883_{(0.330)}$	$0.915_{(0.393)}$	$0.896_{(0.288)}$	$0.930_{(0.343)}$	$0.884_{(0.303)}$	$0.913_{(0.361)}$
	0.6	$0.882_{(0.459)}$	$0.924_{(0.548)}$	$0.883_{(0.408)}$	$0.927_{(0.486)}$	$0.897_{(0.441)}$	$0.928_{(0.525)}$
	0.8	$0.885_{(0.855)}$	$0.914_{(1.019)}$	$0.878_{(0.758)}$	$0.911_{(0.903)}$	$0.876_{(0.790)}$	$0.926_{(0.942)}$
2000	0.2	$0.890_{(0.186)}$	$0.939_{(0.221)}$	$0.898_{(0.163)}$	$0.941_{(0.194)}$	$0.892_{(0.173)}$	$0.934_{(0.207)}$
	0.4	$0.892_{(0.236)}$	$0.935_{(0.281)}$	$0.891_{(0.211)}$	$0.937_{(0.251)}$	$0.899_{(0.221)}$	$0.941_{(0.263)}$
	0.6	$0.879_{(0.334)}$	$0.923_{(0.398)}$	$0.890_{(0.297)}$	$0.937_{(0.354)}$	$0.903_{(0.317)}$	$0.934_{(0.377)}$
	0.8	$0.895_{(0.618)}$	$0.935_{(0.736)}$	$0.883_{(0.546)}$	$0.937_{(0.651)}$	$0.876_{(0.584)}$	$0.925_{(0.696)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.798_{(0.438)}$	$0.832_{(0.522)}$	$0.836_{(0.415)}$	$0.865_{(0.494)}$	$0.842_{(0.428)}$	$0.870_{(0.510)}$
	0.4	$0.805_{(0.539)}$	$0.848_{(0.642)}$	$0.827_{(0.519)}$	$0.859_{(0.618)}$	$0.827_{(0.529)}$	$0.881_{(0.631)}$
	0.6	$0.813_{(0.738)}$	$0.847_{(0.880)}$	$0.843_{(0.678)}$	$0.890_{(0.808)}$	$0.827_{(0.723)}$	$0.858_{(0.861)}$
	0.8	$0.775_{(1.203)}$	$0.802_{(1.432)}$	$0.744_{(1.121)}$	$0.779_{(1.336)}$	$0.765_{(1.163)}$	$0.823_{(1.386)}$
500	0.2	$0.871_{(0.331)}$	$0.898_{(0.395)}$	$0.865_{(0.306)}$	$0.905_{(0.365)}$	$0.862_{(0.310)}$	$0.895_{(0.369)}$
	0.4	$0.856_{(0.408)}$	$0.904_{(0.486)}$	$0.877_{(0.377)}$	$0.914_{(0.449)}$	$0.873_{(0.391)}$	$0.906_{(0.466)}$
	0.6	$0.866_{(0.570)}$	$0.904_{(0.679)}$	$0.863_{(0.493)}$	$0.905_{(0.588)}$	$0.858_{(0.536)}$	$0.899_{(0.638)}$
	0.8	$0.825_{(0.989)}$	$0.851_{(1.178)}$	$0.785_{(0.869)}$	$0.831_{(1.035)}$	$0.796_{(0.921)}$	$0.839_{(1.098)}$
1000	0.2	$0.873_{(0.248)}$	$0.915_{(0.295)}$	$0.870_{(0.220)}$	$0.925_{(0.262)}$	$0.879_{(0.229)}$	$0.922_{(0.273)}$
	0.4	$0.887_{(0.305)}$	$0.919_{(0.364)}$	$0.872_{(0.274)}$	$0.916_{(0.326)}$	$0.870_{(0.289)}$	$0.913_{(0.344)}$
	0.6	$0.883_{(0.422)}$	$0.923_{(0.503)}$	$0.879_{(0.366)}$	$0.919_{(0.436)}$	$0.864_{(0.401)}$	$0.913_{(0.478)}$
	0.8	$0.822_{(0.751)}$	$0.883_{(0.895)}$	$0.813_{(0.668)}$	$0.872_{(0.796)}$	$0.843_{(0.704)}$	$0.895_{(0.839)}$
2000	0.2	$0.880_{(0.177)}$	$0.917_{(0.211)}$	$0.883_{(0.157)}$	$0.936_{(0.187)}$	$0.881_{(0.166)}$	$0.923_{(0.198)}$
	0.4	$0.869_{(0.221)}$	$0.917_{(0.264)}$	$0.892_{(0.196)}$	$0.951_{(0.234)}$	$0.890_{(0.209)}$	$0.936_{(0.249)}$
	0.6	$0.879_{(0.307)}$	$0.926_{(0.366)}$	$0.865_{(0.272)}$	$0.930_{(0.324)}$	$0.876_{(0.288)}$	$0.929_{(0.343)}$
	0.8	$0.819_{(0.551)}$	$0.880_{(0.656)}$	$0.828_{(0.485)}$	$0.889_{(0.578)}$	$0.827_{(0.520)}$	$0.890_{(0.620)}$

3.4.7 ARMA with Gaussian innovation

3.4.7.1 $B_n = 10$

Table 3.31: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 10$ and different kernel choices when the innovations are Gaussian.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.868_{(0.282)}$	$0.908_{(0.337)}$	$0.862_{(0.262)}$	$0.910_{(0.312)}$	$0.882_{(0.265)}$	$0.925_{(0.315)}$
	0.4	$0.872_{(0.360)}$	$0.896_{(0.429)}$	$0.880_{(0.323)}$	$0.916_{(0.385)}$	$0.868_{(0.341)}$	$0.893_{(0.407)}$
	0.6	$0.834_{(0.489)}$	$0.883_{(0.583)}$	$0.847_{(0.436)}$	$0.876_{(0.520)}$	$0.855_{(0.472)}$	$0.884_{(0.563)}$
	0.8	$0.803_{(0.797)}$	$0.836_{(0.950)}$	$0.797_{(0.674)}$	$0.824_{(0.803)}$	$0.816_{(0.721)}$	$0.850_{(0.859)}$
500	0.2	$0.886_{(0.193)}$	$0.928_{(0.231)}$	$0.880_{(0.170)}$	$0.930_{(0.203)}$	$0.904_{(0.178)}$	$0.935_{(0.212)}$
	0.4	$0.881_{(0.247)}$	$0.921_{(0.294)}$	$0.887_{(0.216)}$	$0.920_{(0.257)}$	$0.900_{(0.224)}$	$0.936_{(0.267)}$
	0.6	$0.851_{(0.335)}$	$0.899_{(0.400)}$	$0.864_{(0.291)}$	$0.919_{(0.347)}$	$0.865_{(0.312)}$	$0.915_{(0.371)}$
	0.8	$0.867_{(0.559)}$	$0.899_{(0.667)}$	$0.869_{(0.458)}$	$0.915_{(0.546)}$	$0.885_{(0.497)}$	$0.916_{(0.593)}$
1000	0.2	$0.881_{(0.138)}$	$0.927_{(0.165)}$	$0.900_{(0.122)}$	$0.941_{(0.145)}$	$0.900_{(0.129)}$	$0.954_{(0.153)}$
	0.4	$0.876_{(0.175)}$	$0.934_{(0.208)}$	$0.881_{(0.153)}$	$0.938_{(0.182)}$	$0.880_{(0.163)}$	$0.935_{(0.195)}$
	0.6	$0.856_{(0.243)}$	$0.920_{(0.290)}$	$0.879_{(0.211)}$	$0.931_{(0.251)}$	$0.882_{(0.225)}$	$0.923_{(0.268)}$
	0.8	$0.857_{(0.408)}$	$0.908_{(0.486)}$	$0.890_{(0.332)}$	$0.932_{(0.396)}$	$0.874_{(0.362)}$	$0.928_{(0.431)}$
2000	0.2	$0.902_{(0.098)}$	$0.948_{(0.116)}$	$0.889_{(0.086)}$	$0.939_{(0.103)}$	$0.917_{(0.092)}$	$0.948_{(0.109)}$
	0.4	$0.899_{(0.125)}$	$0.947_{(0.149)}$	$0.893_{(0.108)}$	$0.944_{(0.129)}$	$0.900_{(0.117)}$	$0.940_{(0.139)}$
	0.6	$0.883_{(0.174)}$	$0.932_{(0.207)}$	$0.889_{(0.150)}$	$0.937_{(0.179)}$	$0.889_{(0.161)}$	$0.945_{(0.192)}$
	0.8	$0.874_{(0.287)}$	$0.927_{(0.343)}$	$0.884_{(0.235)}$	$0.933_{(0.280)}$	$0.902_{(0.260)}$	$0.945_{(0.310)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.876_{(0.261)}$	$0.907_{(0.311)}$	$0.856_{(0.237)}$	$0.902_{(0.282)}$	$0.853_{(0.244)}$	$0.898_{(0.291)}$
	0.4	$0.825_{(0.322)}$	$0.872_{(0.384)}$	$0.836_{(0.288)}$	$0.874_{(0.344)}$	$0.846_{(0.305)}$	$0.880_{(0.364)}$
	0.6	$0.789_{(0.422)}$	$0.825_{(0.503)}$	$0.787_{(0.375)}$	$0.833_{(0.446)}$	$0.809_{(0.391)}$	$0.846_{(0.466)}$
	0.8	$0.698_{(0.644)}$	$0.744_{(0.768)}$	$0.711_{(0.570)}$	$0.759_{(0.680)}$	$0.728_{(0.597)}$	$0.761_{(0.712)}$
500	0.2	$0.863_{(0.175)}$	$0.910_{(0.208)}$	$0.863_{(0.155)}$	$0.904_{(0.185)}$	$0.869_{(0.162)}$	$0.908_{(0.194)}$
	0.4	$0.827_{(0.218)}$	$0.895_{(0.260)}$	$0.824_{(0.193)}$	$0.897_{(0.230)}$	$0.825_{(0.205)}$	$0.885_{(0.244)}$
	0.6	$0.790_{(0.294)}$	$0.861_{(0.351)}$	$0.775_{(0.255)}$	$0.852_{(0.304)}$	$0.795_{(0.279)}$	$0.855_{(0.332)}$
	0.8	$0.728_{(0.475)}$	$0.809_{(0.566)}$	$0.706_{(0.395)}$	$0.773_{(0.471)}$	$0.741_{(0.435)}$	$0.809_{(0.518)}$
1000	0.2	$0.859_{(0.127)}$	$0.912_{(0.152)}$	$0.860_{(0.112)}$	$0.921_{(0.134)}$	$0.881_{(0.118)}$	$0.931_{(0.141)}$
	0.4	$0.847_{(0.156)}$	$0.900_{(0.186)}$	$0.843_{(0.136)}$	$0.904_{(0.162)}$	$0.839_{(0.148)}$	$0.902_{(0.176)}$
	0.6	$0.788_{(0.216)}$	$0.859_{(0.258)}$	$0.782_{(0.185)}$	$0.856_{(0.221)}$	$0.785_{(0.199)}$	$0.853_{(0.237)}$
	0.8	$0.729_{(0.354)}$	$0.804_{(0.421)}$	$0.726_{(0.293)}$	$0.796_{(0.350)}$	$0.745_{(0.318)}$	$0.813_{(0.379)}$
2000	0.2	$0.853_{(0.091)}$	$0.930_{(0.108)}$	$0.869_{(0.080)}$	$0.923_{(0.096)}$	$0.849_{(0.085)}$	$0.915_{(0.101)}$
	0.4	$0.839_{(0.112)}$	$0.907_{(0.134)}$	$0.854_{(0.098)}$	$0.911_{(0.117)}$	$0.841_{(0.105)}$	$0.903_{(0.125)}$
	0.6	$0.791_{(0.154)}$	$0.851_{(0.184)}$	$0.787_{(0.134)}$	$0.862_{(0.160)}$	$0.776_{(0.144)}$	$0.859_{(0.171)}$
	0.8	$0.741_{(0.257)}$	$0.821_{(0.306)}$	$0.771_{(0.212)}$	$0.838_{(0.253)}$	$0.742_{(0.230)}$	$0.817_{(0.274)}$

3.4.7.2 $B_n = 15$

Table 3.32: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 15$ and different kernel choices when the innovations are Gaussian.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.836_{(0.343)}$	$0.879_{(0.408)}$	$0.869_{(0.304)}$	$0.897_{(0.363)}$	$0.871_{(0.321)}$	$0.892_{(0.383)}$
	0.4	$0.848_{(0.417)}$	$0.883_{(0.496)}$	$0.855_{(0.387)}$	$0.897_{(0.461)}$	$0.872_{(0.405)}$	$0.901_{(0.483)}$
	0.6	$0.820_{(0.593)}$	$0.852_{(0.707)}$	$0.844_{(0.529)}$	$0.882_{(0.630)}$	$0.842_{(0.572)}$	$0.876_{(0.681)}$
	0.8	$0.799_{(0.991)}$	$0.834_{(1.180)}$	$0.813_{(0.901)}$	$0.845_{(1.073)}$	$0.810_{(0.962)}$	$0.848_{(1.146)}$
500	0.2	$0.871_{(0.230)}$	$0.919_{(0.274)}$	$0.872_{(0.205)}$	$0.910_{(0.244)}$	$0.868_{(0.216)}$	$0.899_{(0.258)}$
	0.4	$0.871_{(0.300)}$	$0.908_{(0.353)}$	$0.874_{(0.260)}$	$0.918_{(0.309)}$	$0.897_{(0.275)}$	$0.934_{(0.328)}$
	0.6	$0.875_{(0.406)}$	$0.907_{(0.484)}$	$0.861_{(0.362)}$	$0.894_{(0.431)}$	$0.854_{(0.389)}$	$0.900_{(0.464)}$
	0.8	$0.826_{(0.724)}$	$0.880_{(0.863)}$	$0.845_{(0.624)}$	$0.886_{(0.743)}$	$0.860_{(0.667)}$	$0.892_{(0.794)}$
1000	0.2	$0.881_{(0.166)}$	$0.927_{(0.198)}$	$0.899_{(0.147)}$	$0.935_{(0.175)}$	$0.896_{(0.159)}$	$0.941_{(0.190)}$
	0.4	$0.882_{(0.213)}$	$0.925_{(0.254)}$	$0.887_{(0.187)}$	$0.939_{(0.223)}$	$0.891_{(0.199)}$	$0.943_{(0.237)}$
	0.6	$0.870_{(0.302)}$	$0.922_{(0.359)}$	$0.876_{(0.266)}$	$0.921_{(0.317)}$	$0.870_{(0.281)}$	$0.928_{(0.335)}$
	0.8	$0.863_{(0.536)}$	$0.908_{(0.638)}$	$0.866_{(0.449)}$	$0.914_{(0.535)}$	$0.873_{(0.485)}$	$0.917_{(0.578)}$
2000	0.2	$0.884_{(0.119)}$	$0.935_{(0.142)}$	$0.903_{(0.105)}$	$0.945_{(0.126)}$	$0.894_{(0.112)}$	$0.945_{(0.134)}$
	0.4	$0.887_{(0.151)}$	$0.940_{(0.180)}$	$0.903_{(0.134)}$	$0.948_{(0.160)}$	$0.906_{(0.142)}$	$0.945_{(0.169)}$
	0.6	$0.877_{(0.216)}$	$0.927_{(0.257)}$	$0.854_{(0.189)}$	$0.922_{(0.225)}$	$0.873_{(0.201)}$	$0.927_{(0.240)}$
	0.8	$0.861_{(0.384)}$	$0.905_{(0.458)}$	$0.881_{(0.324)}$	$0.930_{(0.386)}$	$0.873_{(0.351)}$	$0.920_{(0.418)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.855_{(0.308)}$	$0.887_{(0.367)}$	$0.857_{(0.281)}$	$0.892_{(0.335)}$	$0.865_{(0.290)}$	$0.907_{(0.346)}$
	0.4	$0.827_{(0.380)}$	$0.869_{(0.452)}$	$0.838_{(0.342)}$	$0.881_{(0.408)}$	$0.825_{(0.351)}$	$0.876_{(0.418)}$
	0.6	$0.781_{(0.515)}$	$0.819_{(0.614)}$	$0.795_{(0.455)}$	$0.836_{(0.542)}$	$0.790_{(0.471)}$	$0.831_{(0.561)}$
	0.8	$0.710_{(0.790)}$	$0.743_{(0.941)}$	$0.705_{(0.691)}$	$0.740_{(0.824)}$	$0.735_{(0.757)}$	$0.766_{(0.902)}$
500	0.2	$0.876_{(0.215)}$	$0.917_{(0.256)}$	$0.861_{(0.189)}$	$0.914_{(0.225)}$	$0.874_{(0.200)}$	$0.923_{(0.238)}$
	0.4	$0.832_{(0.258)}$	$0.887_{(0.308)}$	$0.860_{(0.236)}$	$0.920_{(0.282)}$	$0.840_{(0.241)}$	$0.900_{(0.287)}$
	0.6	$0.837_{(0.352)}$	$0.881_{(0.420)}$	$0.793_{(0.318)}$	$0.866_{(0.379)}$	$0.816_{(0.342)}$	$0.881_{(0.407)}$
	0.8	$0.742_{(0.608)}$	$0.804_{(0.724)}$	$0.720_{(0.535)}$	$0.801_{(0.637)}$	$0.743_{(0.569)}$	$0.803_{(0.678)}$
1000	0.2	$0.865_{(0.153)}$	$0.918_{(0.183)}$	$0.862_{(0.137)}$	$0.918_{(0.163)}$	$0.874_{(0.145)}$	$0.923_{(0.173)}$
	0.4	$0.853_{(0.192)}$	$0.913_{(0.229)}$	$0.845_{(0.170)}$	$0.909_{(0.202)}$	$0.847_{(0.180)}$	$0.907_{(0.215)}$
	0.6	$0.828_{(0.267)}$	$0.875_{(0.318)}$	$0.782_{(0.231)}$	$0.858_{(0.276)}$	$0.779_{(0.247)}$	$0.861_{(0.294)}$
	0.8	$0.761_{(0.457)}$	$0.833_{(0.545)}$	$0.726_{(0.386)}$	$0.809_{(0.460)}$	$0.771_{(0.421)}$	$0.832_{(0.502)}$
2000	0.2	$0.885_{(0.111)}$	$0.935_{(0.133)}$	$0.881_{(0.098)}$	$0.925_{(0.117)}$	$0.884_{(0.102)}$	$0.934_{(0.122)}$
	0.4	$0.856_{(0.138)}$	$0.916_{(0.164)}$	$0.840_{(0.122)}$	$0.909_{(0.145)}$	$0.849_{(0.128)}$	$0.910_{(0.153)}$
	0.6	$0.826_{(0.189)}$	$0.891_{(0.225)}$	$0.815_{(0.167)}$	$0.879_{(0.199)}$	$0.800_{(0.180)}$	$0.870_{(0.214)}$
	0.8	$0.749_{(0.336)}$	$0.829_{(0.400)}$	$0.735_{(0.283)}$	$0.805_{(0.337)}$	$0.757_{(0.304)}$	$0.831_{(0.362)}$

3.4.7.3 $B_n = 20$

Table 3.33: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 20$ and different kernel choices when the innovations are Gaussian.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.830_{(0.376)}$	$0.856_{(0.448)}$	$0.830_{(0.337)}$	$0.859_{(0.401)}$	$0.841_{(0.367)}$	$0.875_{(0.437)}$
	0.4	$0.819_{(0.465)}$	$0.848_{(0.554)}$	$0.831_{(0.445)}$	$0.869_{(0.530)}$	$0.834_{(0.448)}$	$0.873_{(0.534)}$
	0.6	$0.792_{(0.675)}$	$0.831_{(0.804)}$	$0.838_{(0.603)}$	$0.872_{(0.719)}$	$0.841_{(0.636)}$	$0.882_{(0.758)}$
	0.8	$0.806_{(1.136)}$	$0.842_{(1.353)}$	$0.794_{(0.987)}$	$0.823_{(1.176)}$	$0.763_{(1.056)}$	$0.809_{(1.258)}$
500	0.2	$0.869_{(0.261)}$	$0.898_{(0.311)}$	$0.878_{(0.232)}$	$0.914_{(0.276)}$	$0.872_{(0.245)}$	$0.914_{(0.292)}$
	0.4	$0.890_{(0.331)}$	$0.918_{(0.394)}$	$0.870_{(0.296)}$	$0.909_{(0.352)}$	$0.865_{(0.314)}$	$0.902_{(0.374)}$
	0.6	$0.860_{(0.478)}$	$0.902_{(0.569)}$	$0.872_{(0.416)}$	$0.917_{(0.496)}$	$0.867_{(0.442)}$	$0.903_{(0.527)}$
	0.8	$0.833_{(0.829)}$	$0.888_{(0.987)}$	$0.844_{(0.737)}$	$0.888_{(0.878)}$	$0.859_{(0.793)}$	$0.900_{(0.945)}$
1000	0.2	$0.888_{(0.194)}$	$0.922_{(0.231)}$	$0.894_{(0.172)}$	$0.934_{(0.205)}$	$0.903_{(0.182)}$	$0.942_{(0.217)}$
	0.4	$0.877_{(0.243)}$	$0.924_{(0.289)}$	$0.894_{(0.214)}$	$0.933_{(0.255)}$	$0.889_{(0.227)}$	$0.929_{(0.271)}$
	0.6	$0.891_{(0.344)}$	$0.929_{(0.410)}$	$0.855_{(0.307)}$	$0.923_{(0.366)}$	$0.875_{(0.322)}$	$0.919_{(0.383)}$
	0.8	$0.855_{(0.626)}$	$0.902_{(0.745)}$	$0.838_{(0.538)}$	$0.908_{(0.641)}$	$0.862_{(0.583)}$	$0.909_{(0.695)}$
2000	0.2	$0.903_{(0.138)}$	$0.949_{(0.164)}$	$0.878_{(0.122)}$	$0.924_{(0.145)}$	$0.892_{(0.128)}$	$0.937_{(0.153)}$
	0.4	$0.873_{(0.176)}$	$0.939_{(0.210)}$	$0.875_{(0.154)}$	$0.924_{(0.183)}$	$0.881_{(0.162)}$	$0.941_{(0.194)}$
	0.6	$0.877_{(0.246)}$	$0.935_{(0.293)}$	$0.867_{(0.220)}$	$0.921_{(0.262)}$	$0.876_{(0.232)}$	$0.925_{(0.276)}$
	0.8	$0.878_{(0.454)}$	$0.931_{(0.541)}$	$0.849_{(0.388)}$	$0.912_{(0.462)}$	$0.852_{(0.421)}$	$0.908_{(0.502)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.842_{(0.333)}$	$0.871_{(0.397)}$	$0.860_{(0.315)}$	$0.890_{(0.376)}$	$0.844_{(0.329)}$	$0.866_{(0.392)}$
	0.4	$0.833_{(0.409)}$	$0.884_{(0.487)}$	$0.839_{(0.385)}$	$0.870_{(0.458)}$	$0.835_{(0.397)}$	$0.872_{(0.473)}$
	0.6	$0.833_{(0.543)}$	$0.864_{(0.648)}$	$0.808_{(0.516)}$	$0.845_{(0.615)}$	$0.785_{(0.535)}$	$0.818_{(0.637)}$
	0.8	$0.753_{(0.893)}$	$0.804_{(1.064)}$	$0.743_{(0.821)}$	$0.768_{(0.978)}$	$0.777_{(0.836)}$	$0.816_{(0.996)}$
500	0.2	$0.858_{(0.240)}$	$0.900_{(0.286)}$	$0.854_{(0.215)}$	$0.909_{(0.257)}$	$0.867_{(0.224)}$	$0.916_{(0.267)}$
	0.4	$0.868_{(0.296)}$	$0.905_{(0.352)}$	$0.858_{(0.270)}$	$0.890_{(0.322)}$	$0.871_{(0.283)}$	$0.902_{(0.337)}$
	0.6	$0.826_{(0.408)}$	$0.881_{(0.487)}$	$0.819_{(0.372)}$	$0.873_{(0.443)}$	$0.820_{(0.389)}$	$0.879_{(0.463)}$
	0.8	$0.761_{(0.721)}$	$0.813_{(0.859)}$	$0.738_{(0.614)}$	$0.799_{(0.732)}$	$0.766_{(0.649)}$	$0.822_{(0.774)}$
1000	0.2	$0.882_{(0.180)}$	$0.935_{(0.214)}$	$0.874_{(0.157)}$	$0.916_{(0.187)}$	$0.879_{(0.168)}$	$0.927_{(0.200)}$
	0.4	$0.854_{(0.220)}$	$0.907_{(0.262)}$	$0.859_{(0.196)}$	$0.918_{(0.233)}$	$0.847_{(0.207)}$	$0.905_{(0.246)}$
	0.6	$0.832_{(0.302)}$	$0.890_{(0.360)}$	$0.812_{(0.268)}$	$0.869_{(0.320)}$	$0.840_{(0.287)}$	$0.899_{(0.342)}$
	0.8	$0.753_{(0.533)}$	$0.829_{(0.635)}$	$0.760_{(0.464)}$	$0.826_{(0.553)}$	$0.749_{(0.498)}$	$0.826_{(0.594)}$
2000	0.2	$0.883_{(0.128)}$	$0.925_{(0.153)}$	$0.889_{(0.112)}$	$0.931_{(0.134)}$	$0.863_{(0.119)}$	$0.914_{(0.142)}$
	0.4	$0.863_{(0.159)}$	$0.911_{(0.189)}$	$0.848_{(0.140)}$	$0.910_{(0.167)}$	$0.860_{(0.148)}$	$0.925_{(0.176)}$
	0.6	$0.826_{(0.219)}$	$0.886_{(0.261)}$	$0.799_{(0.197)}$	$0.874_{(0.234)}$	$0.831_{(0.207)}$	$0.888_{(0.247)}$
	0.8	$0.770_{(0.391)}$	$0.841_{(0.466)}$	$0.723_{(0.341)}$	$0.806_{(0.407)}$	$0.738_{(0.368)}$	$0.811_{(0.438)}$

3.4.7.4 $B_n = 25$

Table 3.34: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 25$ and different kernel choices when the innovations are Gaussian.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.815_{(0.394)}$	$0.843_{(0.469)}$	$0.812_{(0.370)}$	$0.851_{(0.441)}$	$0.824_{(0.397)}$	$0.855_{(0.473)}$
	0.4	$0.805_{(0.498)}$	$0.839_{(0.593)}$	$0.827_{(0.466)}$	$0.867_{(0.555)}$	$0.793_{(0.496)}$	$0.825_{(0.591)}$
	0.6	$0.793_{(0.721)}$	$0.828_{(0.859)}$	$0.806_{(0.641)}$	$0.841_{(0.763)}$	$0.802_{(0.666)}$	$0.842_{(0.793)}$
	0.8	$0.784_{(1.235)}$	$0.813_{(1.472)}$	$0.785_{(1.133)}$	$0.809_{(1.350)}$	$0.786_{(1.216)}$	$0.823_{(1.449)}$
500	0.2	$0.858_{(0.287)}$	$0.899_{(0.342)}$	$0.868_{(0.258)}$	$0.903_{(0.307)}$	$0.861_{(0.276)}$	$0.901_{(0.329)}$
	0.4	$0.846_{(0.363)}$	$0.887_{(0.432)}$	$0.870_{(0.329)}$	$0.903_{(0.391)}$	$0.868_{(0.350)}$	$0.901_{(0.417)}$
	0.6	$0.854_{(0.525)}$	$0.896_{(0.625)}$	$0.858_{(0.463)}$	$0.902_{(0.552)}$	$0.858_{(0.484)}$	$0.904_{(0.577)}$
	0.8	$0.831_{(0.934)}$	$0.871_{(1.113)}$	$0.823_{(1.831)}$	$0.874_{(0.990)}$	$0.846_{(0.890)}$	$0.896_{(1.060)}$
1000	0.2	$0.871_{(0.213)}$	$0.924_{(0.254)}$	$0.892_{(0.190)}$	$0.938_{(0.227)}$	$0.892_{(0.203)}$	$0.925_{(0.242)}$
	0.4	$0.875_{(0.269)}$	$0.913_{(0.320)}$	$0.879_{(0.242)}$	$0.920_{(0.288)}$	$0.871_{(0.255)}$	$0.909_{(0.304)}$
	0.6	$0.866_{(0.386)}$	$0.912_{(0.460)}$	$0.870_{(0.342)}$	$0.911_{(0.407)}$	$0.871_{(0.357)}$	$0.911_{(0.426)}$
	0.8	$0.858_{(0.705)}$	$0.903_{(0.840)}$	$0.847_{(0.629)}$	$0.897_{(0.749)}$	$0.843_{(0.659)}$	$0.890_{(0.785)}$
2000	0.2	$0.872_{(0.154)}$	$0.925_{(0.184)}$	$0.889_{(0.136)}$	$0.937_{(0.162)}$	$0.887_{(0.142)}$	$0.927_{(0.169)}$
	0.4	$0.860_{(0.196)}$	$0.926_{(0.234)}$	$0.891_{(0.172)}$	$0.941_{(0.205)}$	$0.866_{(0.184)}$	$0.931_{(0.219)}$
	0.6	$0.869_{(0.280)}$	$0.922_{(0.333)}$	$0.907_{(0.245)}$	$0.946_{(0.293)}$	$0.850_{(0.261)}$	$0.918_{(0.312)}$
	0.8	$0.872_{(0.513)}$	$0.924_{(0.611)}$	$0.881_{(0.448)}$	$0.924_{(0.534)}$	$0.857_{(0.480)}$	$0.913_{(0.572)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.830_{(0.374)}$	$0.863_{(0.446)}$	$0.819_{(0.343)}$	$0.851_{(0.408)}$	$0.825_{(0.347)}$	$0.857_{(0.413)}$
	0.4	$0.810_{(0.447)}$	$0.855_{(0.533)}$	$0.811_{(0.419)}$	$0.860_{(0.499)}$	$0.821_{(0.429)}$	$0.850_{(0.511)}$
	0.6	$0.781_{(0.591)}$	$0.812_{(0.704)}$	$0.808_{(0.525)}$	$0.840_{(0.626)}$	$0.791_{(0.581)}$	$0.829_{(0.693)}$
	0.8	$0.747_{(0.997)}$	$0.806_{(1.188)}$	$0.730_{(0.876)}$	$0.793_{(1.044)}$	$0.741_{(0.936)}$	$0.782_{(1.116)}$
500	0.2	$0.868_{(0.265)}$	$0.904_{(0.315)}$	$0.863_{(0.242)}$	$0.899_{(0.288)}$	$0.858_{(0.249)}$	$0.902_{(0.296)}$
	0.4	$0.847_{(0.329)}$	$0.890_{(0.392)}$	$0.849_{(0.296)}$	$0.893_{(0.352)}$	$0.864_{(0.304)}$	$0.900_{(0.362)}$
	0.6	$0.836_{(0.461)}$	$0.874_{(0.550)}$	$0.811_{(0.402)}$	$0.860_{(0.480)}$	$0.838_{(0.419)}$	$0.892_{(0.500)}$
	0.8	$0.775_{(0.763)}$	$0.823_{(0.910)}$	$0.765_{(0.697)}$	$0.809_{(0.831)}$	$0.754_{(0.752)}$	$0.794_{(0.897)}$
1000	0.2	$0.886_{(0.200)}$	$0.927_{(0.238)}$	$0.871_{(0.176)}$	$0.918_{(0.209)}$	$0.874_{(0.185)}$	$0.922_{(0.220)}$
	0.4	$0.876_{(0.239)}$	$0.917_{(0.285)}$	$0.852_{(0.215)}$	$0.906_{(0.256)}$	$0.870_{(0.231)}$	$0.923_{(0.275)}$
	0.6	$0.825_{(0.333)}$	$0.894_{(0.397)}$	$0.821_{(0.298)}$	$0.883_{(0.355)}$	$0.833_{(0.311)}$	$0.891_{(0.370)}$
	0.8	$0.785_{(0.602)}$	$0.841_{(0.718)}$	$0.751_{(0.519)}$	$0.816_{(0.618)}$	$0.758_{(0.569)}$	$0.831_{(0.678)}$
2000	0.2	$0.881_{(0.140)}$	$0.940_{(0.167)}$	$0.886_{(0.127)}$	$0.935_{(0.151)}$	$0.896_{(0.134)}$	$0.938_{(0.160)}$
	0.4	$0.859_{(0.175)}$	$0.909_{(0.209)}$	$0.868_{(0.157)}$	$0.918_{(0.187)}$	$0.855_{(0.166)}$	$0.909_{(0.197)}$
	0.6	$0.829_{(0.243)}$	$0.892_{(0.290)}$	$0.828_{(0.217)}$	$0.896_{(0.259)}$	$0.834_{(0.227)}$	$0.894_{(0.270)}$
	0.8	$0.786_{(0.439)}$	$0.860_{(0.523)}$	$0.758_{(0.379)}$	$0.833_{(0.451)}$	$0.779_{(0.413)}$	$0.844_{(0.492)}$

3.4.7.5 $B_n = 30$

Table 3.35: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the ARMA process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 30$ and different kernel choices when the innovations are Gaussian.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.796_{(0.422)}$	$0.819_{(0.503)}$	$0.818_{(0.397)}$	$0.854_{(0.473)}$	$0.810_{(0.410)}$	$0.842_{(0.489)}$
	0.4	$0.793_{(0.544)}$	$0.819_{(0.648)}$	$0.810_{(0.488)}$	$0.847_{(0.581)}$	$0.796_{(0.517)}$	$0.824_{(0.616)}$
	0.6	$0.810_{(0.759)}$	$0.838_{(0.904)}$	$0.806_{(0.715)}$	$0.837_{(0.852)}$	$0.806_{(0.716)}$	$0.833_{(0.854)}$
	0.8	$0.769_{(1.341)}$	$0.804_{(1.598)}$	$0.798_{(1.223)}$	$0.826_{(1.457)}$	$0.771_{(1.293)}$	$0.803_{(1.541)}$
500	0.2	$0.847_{(0.313)}$	$0.876_{(0.373)}$	$0.853_{(0.281)}$	$0.893_{(0.335)}$	$0.864_{(0.291)}$	$0.891_{(0.347)}$
	0.4	$0.856_{(0.393)}$	$0.889_{(0.469)}$	$0.859_{(0.357)}$	$0.894_{(0.425)}$	$0.854_{(0.373)}$	$0.899_{(0.444)}$
	0.6	$0.840_{(0.568)}$	$0.870_{(0.677)}$	$0.842_{(0.496)}$	$0.883_{(0.591)}$	$0.852_{(0.523)}$	$0.892_{(0.623)}$
	0.8	$0.837_{(1.026)}$	$0.873_{(1.222)}$	$0.821_{(0.905)}$	$0.862_{(1.078)}$	$0.846_{(0.953)}$	$0.880_{(1.135)}$
1000	0.2	$0.874_{(0.229)}$	$0.906_{(0.273)}$	$0.868_{(0.204)}$	$0.904_{(0.243)}$	$0.870_{(0.221)}$	$0.910_{(0.263)}$
	0.4	$0.871_{(0.292)}$	$0.897_{(0.348)}$	$0.878_{(0.261)}$	$0.918_{(0.310)}$	$0.881_{(0.279)}$	$0.914_{(0.332)}$
	0.6	$0.862_{(0.414)}$	$0.907_{(0.494)}$	$0.882_{(0.372)}$	$0.929_{(0.443)}$	$0.871_{(0.393)}$	$0.914_{(0.468)}$
	0.8	$0.842_{(0.784)}$	$0.889_{(0.934)}$	$0.859_{(0.687)}$	$0.902_{(0.819)}$	$0.856_{(0.730)}$	$0.906_{(0.870)}$
2000	0.2	$0.896_{(0.167)}$	$0.932_{(0.199)}$	$0.877_{(0.147)}$	$0.935_{(0.175)}$	$0.887_{(0.159)}$	$0.933_{(0.189)}$
	0.4	$0.884_{(0.211)}$	$0.931_{(0.252)}$	$0.876_{(0.190)}$	$0.933_{(0.226)}$	$0.880_{(0.202)}$	$0.925_{(0.240)}$
	0.6	$0.875_{(0.301)}$	$0.922_{(0.359)}$	$0.886_{(0.269)}$	$0.939_{(0.320)}$	$0.895_{(0.286)}$	$0.938_{(0.340)}$
	0.8	$0.862_{(0.567)}$	$0.911_{(0.675)}$	$0.868_{(0.499)}$	$0.917_{(0.595)}$	$0.858_{(0.527)}$	$0.911_{(0.628)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.795_{(0.382)}$	$0.829_{(0.455)}$	$0.816_{(0.349)}$	$0.856_{(0.415)}$	$0.810_{(0.380)}$	$0.848_{(0.452)}$
	0.4	$0.806_{(0.475)}$	$0.830_{(0.566)}$	$0.833_{(0.413)}$	$0.858_{(0.492)}$	$0.809_{(0.449)}$	$0.857_{(0.535)}$
	0.6	$0.818_{(0.609)}$	$0.853_{(0.726)}$	$0.800_{(0.581)}$	$0.826_{(0.692)}$	$0.793_{(0.623)}$	$0.829_{(0.742)}$
	0.8	$0.768_{(0.984)}$	$0.804_{(1.172)}$	$0.751_{(0.940)}$	$0.799_{(1.120)}$	$0.733_{(1.004)}$	$0.792_{(1.197)}$
500	0.2	$0.844_{(0.289)}$	$0.880_{(0.345)}$	$0.850_{(0.256)}$	$0.885_{(0.305)}$	$0.857_{(0.269)}$	$0.885_{(0.320)}$
	0.4	$0.851_{(0.365)}$	$0.874_{(0.434)}$	$0.851_{(0.314)}$	$0.895_{(0.374)}$	$0.848_{(0.329)}$	$0.895_{(0.392)}$
	0.6	$0.821_{(0.492)}$	$0.866_{(0.587)}$	$0.843_{(0.429)}$	$0.884_{(0.511)}$	$0.833_{(0.460)}$	$0.886_{(0.548)}$
	0.8	$0.783_{(0.832)}$	$0.818_{(0.992)}$	$0.759_{(0.771)}$	$0.812_{(0.918)}$	$0.775_{(0.815)}$	$0.828_{(0.971)}$
1000	0.2	$0.888_{(0.208)}$	$0.927_{(0.248)}$	$0.885_{(0.189)}$	$0.936_{(0.226)}$	$0.874_{(0.200)}$	$0.919_{(0.238)}$
	0.4	$0.867_{(0.263)}$	$0.917_{(0.313)}$	$0.872_{(0.232)}$	$0.919_{(0.277)}$	$0.869_{(0.245)}$	$0.915_{(0.292)}$
	0.6	$0.852_{(0.357)}$	$0.897_{(0.426)}$	$0.839_{(0.327)}$	$0.881_{(0.389)}$	$0.845_{(0.340)}$	$0.895_{(0.405)}$
	0.8	$0.786_{(0.662)}$	$0.852_{(0.789)}$	$0.777_{(0.576)}$	$0.836_{(0.686)}$	$0.799_{(0.611)}$	$0.858_{(0.728)}$
2000	0.2	$0.885_{(0.152)}$	$0.928_{(0.181)}$	$0.887_{(0.137)}$	$0.940_{(0.163)}$	$0.874_{(0.145)}$	$0.929_{(0.173)}$
	0.4	$0.873_{(0.191)}$	$0.918_{(0.227)}$	$0.872_{(0.171)}$	$0.923_{(0.203)}$	$0.877_{(0.180)}$	$0.920_{(0.214)}$
	0.6	$0.844_{(0.265)}$	$0.911_{(0.316)}$	$0.842_{(0.235)}$	$0.904_{(0.280)}$	$0.841_{(0.251)}$	$0.900_{(0.299)}$
	0.8	$0.802_{(0.485)}$	$0.868_{(0.578)}$	$0.799_{(0.419)}$	$0.862_{(0.499)}$	$0.776_{(0.450)}$	$0.837_{(0.537)}$

3.4.8 GARCH with Gaussian Innovation

3.4.8.1 $B_n = 10$

Table 3.36: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the GARCH process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 10$ and different kernel choices.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.872_{(0.190)}$	$0.905_{(0.227)}$	$0.890_{(0.169)}$	$0.925_{(0.201)}$	$0.887_{(0.175)}$	$0.911_{(0.209)}$
	0.4	$0.870_{(0.209)}$	$0.896_{(0.249)}$	$0.873_{(0.185)}$	$0.908_{(0.221)}$	$0.898_{(0.197)}$	$0.929_{(0.234)}$
	0.6	$0.846_{(0.236)}$	$0.886_{(0.281)}$	$0.868_{(0.210)}$	$0.900_{(0.250)}$	$0.879_{(0.221)}$	$0.910_{(0.263)}$
	0.8	$0.857_{(0.306)}$	$0.884_{(0.364)}$	$0.865_{(0.260)}$	$0.898_{(0.310)}$	$0.852_{(0.270)}$	$0.891_{(0.322)}$
500	0.2	$0.893_{(0.125)}$	$0.927_{(0.149)}$	$0.899_{(0.110)}$	$0.932_{(0.131)}$	$0.902_{(0.117)}$	$0.939_{(0.140)}$
	0.4	$0.891_{(0.136)}$	$0.934_{(0.162)}$	$0.891_{(0.121)}$	$0.940_{(0.145)}$	$0.890_{(0.129)}$	$0.929_{(0.153)}$
	0.6	$0.884_{(0.159)}$	$0.922_{(0.190)}$	$0.888_{(0.138)}$	$0.928_{(0.165)}$	$0.868_{(0.147)}$	$0.912_{(0.175)}$
	0.8	$0.864_{(0.202)}$	$0.913_{(0.241)}$	$0.878_{(0.173)}$	$0.932_{(0.206)}$	$0.869_{(0.187)}$	$0.914_{(0.223)}$
1000	0.2	$0.898_{(0.089)}$	$0.931_{(0.106)}$	$0.899_{(0.079)}$	$0.946_{(0.094)}$	$0.900_{(0.084)}$	$0.945_{(0.100)}$
	0.4	$0.909_{(0.098)}$	$0.949_{(0.117)}$	$0.893_{(0.086)}$	$0.929_{(0.103)}$	$0.903_{(0.091)}$	$0.942_{(0.109)}$
	0.6	$0.864_{(0.115)}$	$0.922_{(0.137)}$	$0.889_{(0.099)}$	$0.935_{(0.119)}$	$0.888_{(0.105)}$	$0.936_{(0.126)}$
	0.8	$0.856_{(0.150)}$	$0.916_{(0.178)}$	$0.872_{(0.124)}$	$0.921_{(0.148)}$	$0.857_{(0.135)}$	$0.910_{(0.160)}$
2000	0.2	$0.901_{(0.063)}$	$0.957_{(0.076)}$	$0.905_{(0.056)}$	$0.949_{(0.067)}$	$0.924_{(0.060)}$	$0.957_{(0.071)}$
	0.4	$0.904_{(0.070)}$	$0.951_{(0.083)}$	$0.893_{(0.061)}$	$0.947_{(0.073)}$	$0.911_{(0.065)}$	$0.956_{(0.077)}$
	0.6	$0.891_{(0.081)}$	$0.939_{(0.097)}$	$0.870_{(0.071)}$	$0.924_{(0.084)}$	$0.882_{(0.075)}$	$0.939_{(0.090)}$
	0.8	$0.866_{(0.105)}$	$0.915_{(0.125)}$	$0.868_{(0.089)}$	$0.923_{(0.107)}$	$0.884_{(0.096)}$	$0.937_{(0.115)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.909_{(0.190)}$	$0.934_{(0.227)}$	$0.899_{(0.167)}$	$0.937_{(0.199)}$	$0.891_{(0.175)}$	$0.925_{(0.208)}$
	0.4	$0.856_{(0.211)}$	$0.902_{(0.252)}$	$0.875_{(0.185)}$	$0.925_{(0.221)}$	$0.853_{(0.197)}$	$0.911_{(0.234)}$
	0.6	$0.836_{(0.239)}$	$0.877_{(0.285)}$	$0.830_{(0.216)}$	$0.891_{(0.258)}$	$0.826_{(0.233)}$	$0.887_{(0.277)}$
	0.8	$0.804_{(0.316)}$	$0.835_{(0.376)}$	$0.799_{(0.267)}$	$0.844_{(0.318)}$	$0.786_{(0.290)}$	$0.838_{(0.346)}$
500	0.2	$0.879_{(0.125)}$	$0.927_{(0.149)}$	$0.888_{(0.110)}$	$0.929_{(0.132)}$	$0.897_{(0.118)}$	$0.945_{(0.141)}$
	0.4	$0.864_{(0.139)}$	$0.927_{(0.165)}$	$0.860_{(0.122)}$	$0.925_{(0.146)}$	$0.885_{(0.132)}$	$0.933_{(0.157)}$
	0.6	$0.837_{(0.166)}$	$0.893_{(0.198)}$	$0.813_{(0.143)}$	$0.883_{(0.171)}$	$0.846_{(0.152)}$	$0.900_{(0.181)}$
	0.8	$0.788_{(0.218)}$	$0.849_{(0.260)}$	$0.764_{(0.185)}$	$0.834_{(0.221)}$	$0.793_{(0.195)}$	$0.851_{(0.233)}$
1000	0.2	$0.876_{(0.091)}$	$0.933_{(0.108)}$	$0.894_{(0.079)}$	$0.943_{(0.095)}$	$0.904_{(0.084)}$	$0.958_{(0.100)}$
	0.4	$0.866_{(0.101)}$	$0.919_{(0.120)}$	$0.870_{(0.088)}$	$0.916_{(0.106)}$	$0.858_{(0.093)}$	$0.919_{(0.111)}$
	0.6	$0.817_{(0.119)}$	$0.898_{(0.142)}$	$0.834_{(0.102)}$	$0.914_{(0.122)}$	$0.867_{(0.109)}$	$0.914_{(0.131)}$
	0.8	$0.801_{(0.158)}$	$0.865_{(0.188)}$	$0.760_{(0.134)}$	$0.842_{(0.160)}$	$0.793_{(0.145)}$	$0.862_{(0.172)}$
2000	0.2	$0.893_{(0.064)}$	$0.935_{(0.076)}$	$0.891_{(0.056)}$	$0.941_{(0.067)}$	$0.888_{(0.060)}$	$0.944_{(0.071)}$
	0.4	$0.876_{(0.071)}$	$0.928_{(0.085)}$	$0.858_{(0.062)}$	$0.909_{(0.075)}$	$0.882_{(0.066)}$	$0.933_{(0.079)}$
	0.6	$0.825_{(0.085)}$	$0.899_{(0.101)}$	$0.819_{(0.073)}$	$0.900_{(0.087)}$	$0.828_{(0.079)}$	$0.900_{(0.094)}$
	0.8	$0.770_{(0.114)}$	$0.842_{(0.136)}$	$0.769_{(0.096)}$	$0.845_{(0.115)}$	$0.778_{(0.103)}$	$0.849_{(0.123)}$

3.4.8.2 $B_n = 15$

Table 3.37: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the GARCH process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 15$ and different kernel choices.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n *chosen as the 90% quantile*			
200	0.2	$0.853_{(0.226)}$	$0.866_{(0.270)}$	$0.862_{(0.200)}$	$0.892_{(0.238)}$	$0.856_{(0.213)}$	$0.896_{(0.254)}$
	0.4	$0.842_{(0.246)}$	$0.891_{(0.293)}$	$0.854_{(0.216)}$	$0.887_{(0.258)}$	$0.854_{(0.230)}$	$0.891_{(0.274)}$
	0.6	$0.824_{(0.279)}$	$0.859_{(0.333)}$	$0.861_{(0.261)}$	$0.894_{(0.311)}$	$0.849_{(0.266)}$	$0.881_{(0.317)}$
	0.8	$0.816_{(0.378)}$	$0.858_{(0.452)}$	$0.851_{(0.329)}$	$0.884_{(0.392)}$	$0.846_{(0.353)}$	$0.879_{(0.420)}$
500	0.2	$0.874_{(0.150)}$	$0.915_{(0.179)}$	$0.880_{(0.135)}$	$0.928_{(0.161)}$	$0.880_{(0.142)}$	$0.923_{(0.170)}$
	0.4	$0.885_{(0.165)}$	$0.916_{(0.196)}$	$0.878_{(0.148)}$	$0.921_{(0.176)}$	$0.885_{(0.155)}$	$0.921_{(0.185)}$
	0.6	$0.856_{(0.195)}$	$0.896_{(0.232)}$	$0.867_{(0.171)}$	$0.912_{(0.204)}$	$0.887_{(0.184)}$	$0.924_{(0.219)}$
	0.8	$0.869_{(0.255)}$	$0.908_{(0.304)}$	$0.867_{(0.219)}$	$0.905_{(0.261)}$	$0.853_{(0.245)}$	$0.896_{(0.289)}$
1000	0.2	$0.889_{(0.110)}$	$0.942_{(0.131)}$	$0.902_{(0.096)}$	$0.940_{(0.115)}$	$0.908_{(0.102)}$	$0.943_{(0.122)}$
	0.4	$0.892_{(0.120)}$	$0.935_{(0.143)}$	$0.890_{(0.106)}$	$0.938_{(0.126)}$	$0.894_{(0.112)}$	$0.932_{(0.133)}$
	0.6	$0.882_{(0.139)}$	$0.928_{(0.165)}$	$0.886_{(0.124)}$	$0.925_{(0.148)}$	$0.905_{(0.132)}$	$0.941_{(0.157)}$
	0.8	$0.870_{(0.189)}$	$0.916_{(0.225)}$	$0.879_{(0.164)}$	$0.920_{(0.195)}$	$0.874_{(0.176)}$	$0.917_{(0.210)}$
2000	0.2	$0.893_{(0.078)}$	$0.937_{(0.093)}$	$0.896_{(0.069)}$	$0.939_{(0.082)}$	$0.897_{(0.073)}$	$0.939_{(0.081)}$
	0.4	$0.890_{(0.086)}$	$0.943_{(0.102)}$	$0.903_{(0.076)}$	$0.958_{(0.090)}$	$0.896_{(0.080)}$	$0.949_{(0.095)}$
	0.6	$0.904_{(0.100)}$	$0.945_{(0.119)}$	$0.876_{(0.088)}$	$0.928_{(0.105)}$	$0.885_{(0.094)}$	$0.942_{(0.111)}$
	0.8	$0.885_{(0.134)}$	$0.922_{(0.160)}$	$0.865_{(0.115)}$	$0.926_{(0.137)}$	$0.862_{(0.125)}$	$0.919_{(0.149)}$
				x_n *chosen as the 95% quantile*			
200	0.2	$0.855_{(0.234)}$	$0.890_{(0.279)}$	$0.868_{(0.199)}$	$0.913_{(0.238)}$	$0.858_{(0.211)}$	$0.892_{(0.251)}$
	0.4	$0.851_{(0.253)}$	$0.888_{(0.302)}$	$0.860_{(0.222)}$	$0.903_{(0.264)}$	$0.842_{(0.236)}$	$0.889_{(0.281)}$
	0.6	$0.838_{(0.288)}$	$0.882_{(0.343)}$	$0.841_{(0.264)}$	$0.888_{(0.314)}$	$0.859_{(0.266)}$	$0.896_{(0.317)}$
	0.8	$0.777_{(0.387)}$	$0.839_{(0.467)}$	$0.764_{(0.342)}$	$0.824_{(0.407)}$	$0.769_{(0.367)}$	$0.814_{(0.437)}$
500	0.2	$0.887_{(0.150)}$	$0.932_{(0.178)}$	$0.896_{(0.135)}$	$0.936_{(0.160)}$	$0.904_{(0.141)}$	$0.938_{(0.168)}$
	0.4	$0.875_{(0.875)}$	$0.909_{(0.200)}$	$0.871_{(0.149)}$	$0.917_{(0.177)}$	$0.876_{(0.156)}$	$0.922_{(0.185)}$
	0.6	$0.842_{(0.200)}$	$0.900_{(0.238)}$	$0.840_{(0.177)}$	$0.889_{(0.211)}$	$0.842_{(0.188)}$	$0.887_{(0.224)}$
	0.8	$0.780_{(0.279)}$	$0.843_{(0.320)}$	$0.795_{(0.239)}$	$0.856_{(0.284)}$	$0.783_{(0.257)}$	$0.851_{(0.306)}$
1000	0.2	$0.889_{(0.109)}$	$0.937_{(0.130)}$	$0.885_{(0.097)}$	$0.938_{(0.116)}$	$0.877_{(0.104)}$	$0.935_{(0.124)}$
	0.4	$0.872_{(0.122)}$	$0.926_{(0.145)}$	$0.851_{(0.108)}$	$0.903_{(0.128)}$	$0.865_{(0.114)}$	$0.929_{(0.136)}$
	0.6	$0.843_{(0.146)}$	$0.909_{(0.174)}$	$0.848_{(0.129)}$	$0.904_{(0.153)}$	$0.852_{(0.136)}$	$0.898_{(0.162)}$
	0.8	$0.790_{(0.204)}$	$0.852_{(0.243)}$	$0.759_{(0.178)}$	$0.834_{(0.212)}$	$0.759_{(0.189)}$	$0.827_{(0.226)}$
2000	0.2	$0.881_{(0.079)}$	$0.937_{(0.094)}$	$0.916_{(0.069)}$	$0.953_{(0.082)}$	$0.885_{(0.074)}$	$0.948_{(0.088)}$
	0.4	$0.880_{(0.087)}$	$0.936_{(0.104)}$	$0.878_{(0.077)}$	$0.930_{(0.092)}$	$0.894_{(0.082)}$	$0.940_{(0.097)}$
	0.6	$0.834_{(0.105)}$	$0.911_{(0.125)}$	$0.824_{(0.092)}$	$0.896_{(0.110)}$	$0.848_{(0.099)}$	$0.907_{(0.118)}$
	0.8	$0.811_{(0.147)}$	$0.865_{(0.177)}$	$0.777_{(0.128)}$	$0.848_{(0.152)}$	$0.773_{(0.137)}$	$0.849_{(0.164)}$

3.4.8.3 $B_n = 20$

Table 3.38: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the GARCH process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 20$ and different kernel choices.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.825_{(0.240)}$	$0.849_{(0.286)}$	$0.850_{(0.224)}$	$0.875_{(0.267)}$	$0.820_{(0.237)}$	$0.851_{(0.282)}$
	0.4	$0.810_{(0.267)}$	$0.842_{(0.318)}$	$0.819_{(0.244)}$	$0.854_{(0.291)}$	$0.844_{(0.260)}$	$0.872_{(0.310)}$
	0.6	$0.812_{(0.312)}$	$0.849_{(0.372)}$	$0.825_{(0.283)}$	$0.857_{(0.337)}$	$0.828_{(0.300)}$	$0.854_{(0.357)}$
	0.8	$0.798_{(0.428)}$	$0.839_{(0.510)}$	$0.814_{(0.380)}$	$0.864_{(0.453)}$	$0.811_{(0.392)}$	$0.856_{(0.467)}$
500	0.2	$0.870_{(0.171)}$	$0.902_{(0.204)}$	$0.880_{(0.154)}$	$0.917_{(0.183)}$	$0.871_{(0.161)}$	$0.911_{(0.192)}$
	0.4	$0.863_{(0.186)}$	$0.894_{(0.221)}$	$0.889_{(0.167)}$	$0.930_{(0.199)}$	$0.871_{(0.177)}$	$0.908_{(0.211)}$
	0.6	$0.877_{(0.218)}$	$0.907_{(0.260)}$	$0.868_{(0.197)}$	$0.909_{(0.235)}$	$0.872_{(0.206)}$	$0.901_{(0.245)}$
	0.8	$0.849_{(0.292)}$	$0.893_{(0.349)}$	$0.874_{(0.264)}$	$0.915_{(0.314)}$	$0.844_{(0.281)}$	$0.893_{(0.335)}$
1000	0.2	$0.880_{(0.123)}$	$0.926_{(0.146)}$	$0.894_{(0.112)}$	$0.936_{(0.133)}$	$0.881_{(0.117)}$	$0.926_{(0.139)}$
	0.4	$0.880_{(0.136)}$	$0.924_{(0.162)}$	$0.896_{(0.122)}$	$0.939_{(0.145)}$	$0.863_{(0.129)}$	$0.923_{(0.153)}$
	0.6	$0.872_{(0.161)}$	$0.920_{(0.191)}$	$0.878_{(0.142)}$	$0.922_{(0.169)}$	$0.893_{(0.150)}$	$0.928_{(0.178)}$
	0.8	$0.856_{(0.220)}$	$0.902_{(0.262)}$	$0.860_{(0.194)}$	$0.907_{(0.232)}$	$0.890_{(0.208)}$	$0.929_{(0.247)}$
2000	0.2	$0.896_{(0.090)}$	$0.938_{(0.107)}$	$0.898_{(0.080)}$	$0.947_{(0.095)}$	$0.892_{(0.084)}$	$0.947_{(0.100)}$
	0.4	$0.887_{(0.099)}$	$0.944_{(0.118)}$	$0.897_{(0.087)}$	$0.945_{(0.103)}$	$0.897_{(0.092)}$	$0.947_{(0.110)}$
	0.6	$0.886_{(0.115)}$	$0.941_{(0.137)}$	$0.880_{(0.101)}$	$0.926_{(0.121)}$	$0.896_{(0.107)}$	$0.938_{(0.128)}$
	0.8	$0.839_{(0.157)}$	$0.911_{(0.187)}$	$0.862_{(0.138)}$	$0.925_{(0.164)}$	$0.867_{(0.147)}$	$0.919_{(0.176)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.831_{(0.245)}$	$0.870_{(0.292)}$	$0.844_{(0.232)}$	$0.888_{(0.277)}$	$0.856_{(0.232)}$	$0.896_{(0.276)}$
	0.4	$0.831_{(0.276)}$	$0.868_{(0.329)}$	$0.855_{(0.251)}$	$0.885_{(0.299)}$	$0.832_{(0.263)}$	$0.863_{(0.313)}$
	0.6	$0.812_{(0.317)}$	$0.841_{(0.377)}$	$0.821_{(0.298)}$	$0.851_{(0.356)}$	$0.822_{(0.304)}$	$0.851_{(0.362)}$
	0.8	$0.779_{(0.428)}$	$0.822_{(0.510)}$	$0.797_{(0.413)}$	$0.824_{(0.492)}$	$0.776_{(0.417)}$	$0.825_{(0.497)}$
500	0.2	$0.871_{(0.173)}$	$0.906_{(0.206)}$	$0.866_{(0.155)}$	$0.916_{(0.185)}$	$0.885_{(0.166)}$	$0.917_{(0.197)}$
	0.4	$0.862_{(0.193)}$	$0.904_{(0.230)}$	$0.867_{(0.173)}$	$0.922_{(0.206)}$	$0.863_{(0.180)}$	$0.898_{(0.215)}$
	0.6	$0.847_{(0.228)}$	$0.893_{(0.271)}$	$0.828_{(0.207)}$	$0.881_{(0.246)}$	$0.838_{(0.213)}$	$0.883_{(0.254)}$
	0.8	$0.776_{(0.333)}$	$0.833_{(0.397)}$	$0.795_{(0.278)}$	$0.848_{(0.331)}$	$0.789_{(0.296)}$	$0.850_{(0.352)}$
1000	0.2	$0.888_{(0.127)}$	$0.928_{(0.151)}$	$0.895_{(0.111)}$	$0.933_{(0.132)}$	$0.883_{(0.119)}$	$0.936_{(0.141)}$
	0.4	$0.856_{(0.140)}$	$0.905_{(0.167)}$	$0.886_{(0.124)}$	$0.924_{(0.148)}$	$0.878_{(0.132)}$	$0.933_{(0.157)}$
	0.6	$0.852_{(0.168)}$	$0.906_{(0.200)}$	$0.850_{(0.149)}$	$0.908_{(0.180)}$	$0.847_{(0.157)}$	$0.904_{(0.187)}$
	0.8	$0.795_{(0.244)}$	$0.881_{(0.291)}$	$0.779_{(0.207)}$	$0.850_{(0.247)}$	$0.783_{(0.223)}$	$0.845_{(0.266)}$
2000	0.2	$0.895_{(0.091)}$	$0.945_{(0.108)}$	$0.900_{(0.080)}$	$0.946_{(0.095)}$	$0.894_{(0.084)}$	$0.936_{(0.101)}$
	0.4	$0.882_{(0.100)}$	$0.934_{(0.119)}$	$0.892_{(0.089)}$	$0.940_{(0.106)}$	$0.876_{(0.094)}$	$0.927_{(0.112)}$
	0.6	$0.855_{(0.120)}$	$0.919_{(0.144)}$	$0.832_{(0.107)}$	$0.890_{(0.127)}$	$0.849_{(0.112)}$	$0.905_{(0.134)}$
	0.8	$0.804_{(0.173)}$	$0.865_{(0.206)}$	$0.782_{(0.152)}$	$0.847_{(0.181)}$	$0.786_{(0.162)}$	$0.855_{(0.193)}$

3.4.8.4 $B_n = 25$

Table 3.39: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the GARCH process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 25$ and different kernel choices.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.801_{(0.270)}$	$0.831_{(0.322)}$	$0.812_{(0.247)}$	$0.849_{(0.294)}$	$0.808_{(0.257)}$	$0.843_{(0.306)}$
	0.4	$0.793_{(0.281)}$	$0.828_{(0.334)}$	$0.822_{(0.270)}$	$0.855_{(0.322)}$	$0.795_{(0.281)}$	$0.825_{(0.335)}$
	0.6	$0.800_{(0.340)}$	$0.832_{(0.405)}$	$0.802_{(0.311)}$	$0.840_{(0.371)}$	$0.828_{(0.323)}$	$0.852_{(0.385)}$
	0.8	$0.781_{(0.460)}$	$0.817_{(0.548)}$	$0.796_{(0.417)}$	$0.828_{(0.496)}$	$0.801_{(0.454)}$	$0.830_{(0.541)}$
500	0.2	$0.855_{(0.188)}$	$0.897_{(0.224)}$	$0.885_{(0.168)}$	$0.920_{(0.200)}$	$0.861_{(0.179)}$	$0.892_{(0.213)}$
	0.4	$0.861_{(0.207)}$	$0.896_{(0.246)}$	$0.881_{(0.186)}$	$0.910_{(0.221)}$	$0.868_{(0.197)}$	$0.901_{(0.235)}$
	0.6	$0.843_{(0.241)}$	$0.883_{(0.287)}$	$0.866_{(0.219)}$	$0.910_{(0.260)}$	$0.867_{(0.223)}$	$0.892_{(0.266)}$
	0.8	$0.841_{(0.336)}$	$0.888_{(0.400)}$	$0.851_{(0.297)}$	$0.885_{(0.354)}$	$0.821_{(0.319)}$	$0.865_{(0.380)}$
1000	0.2	$0.878_{(0.139)}$	$0.922_{(0.165)}$	$0.872_{(0.124)}$	$0.923_{(0.148)}$	$0.882_{(0.131)}$	$0.920_{(0.156)}$
	0.4	$0.880_{(0.152)}$	$0.917_{(0.181)}$	$0.870_{(0.134)}$	$0.908_{(0.160)}$	$0.874_{(0.142)}$	$0.926_{(0.169)}$
	0.6	$0.879_{(0.181)}$	$0.925_{(0.216)}$	$0.872_{(0.156)}$	$0.918_{(0.186)}$	$0.857_{(0.167)}$	$0.914_{(0.199)}$
	0.8	$0.851_{(0.248)}$	$0.903_{(0.296)}$	$0.854_{(0.216)}$	$0.904_{(0.257)}$	$0.843_{(0.230)}$	$0.900_{(0.274)}$
2000	0.2	$0.900_{(0.099)}$	$0.942_{(0.119)}$	$0.893_{(0.089)}$	$0.935_{(0.106)}$	$0.898_{(0.094)}$	$0.937_{(0.112)}$
	0.4	$0.894_{(0.108)}$	$0.936_{(0.129)}$	$0.911_{(0.097)}$	$0.943_{(0.116)}$	$0.893_{(0.102)}$	$0.942_{(0.121)}$
	0.6	$0.888_{(0.128)}$	$0.947_{(0.153)}$	$0.895_{(0.113)}$	$0.939_{(0.135)}$	$0.878_{(0.120)}$	$0.936_{(0.143)}$
	0.8	$0.861_{(0.179)}$	$0.923_{(0.213)}$	$0.872_{(0.157)}$	$0.930_{(0.187)}$	$0.856_{(0.167)}$	$0.916_{(0.199)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.809_{(0.266)}$	$0.843_{(0.317)}$	$0.840_{(0.242)}$	$0.874_{(0.289)}$	$0.838_{(0.264)}$	$0.872_{(0.315)}$
	0.4	$0.814_{(0.286)}$	$0.835_{(0.341)}$	$0.836_{(0.267)}$	$0.870_{(0.319)}$	$0.823_{(0.281)}$	$0.848_{(0.335)}$
	0.6	$0.785_{(0.357)}$	$0.828_{(0.425)}$	$0.816_{(0.316)}$	$0.854_{(0.377)}$	$0.800_{(0.333)}$	$0.837_{(0.396)}$
	0.8	$0.763_{(0.491)}$	$0.807_{(0.562)}$	$0.800_{(0.428)}$	$0.841_{(0.510)}$	$0.750_{(0.466)}$	$0.803_{(0.555)}$
500	0.2	$0.861_{(0.192)}$	$0.900_{(0.228)}$	$0.886_{(0.168)}$	$0.912_{(0.200)}$	$0.862_{(0.179)}$	$0.900_{(0.213)}$
	0.4	$0.850_{(0.214)}$	$0.879_{(0.255)}$	$0.854_{(0.187)}$	$0.889_{(0.223)}$	$0.864_{(0.198)}$	$0.908_{(0.236)}$
	0.6	$0.838_{(0.245)}$	$0.870_{(0.292)}$	$0.835_{(0.224)}$	$0.892_{(0.267)}$	$0.869_{(0.238)}$	$0.901_{(0.283)}$
	0.8	$0.796_{(0.361)}$	$0.860_{(0.429)}$	$0.785_{(0.320)}$	$0.852_{(0.381)}$	$0.793_{(0.339)}$	$0.844_{(0.404)}$
1000	0.2	$0.888_{(0.138)}$	$0.922_{(0.165)}$	$0.881_{(0.124)}$	$0.917_{(0.148)}$	$0.877_{(0.133)}$	$0.926_{(0.159)}$
	0.4	$0.872_{(0.157)}$	$0.925_{(0.187)}$	$0.876_{(0.136)}$	$0.918_{(0.162)}$	$0.871_{(0.146)}$	$0.922_{(0.174)}$
	0.6	$0.851_{(0.186)}$	$0.909_{(0.221)}$	$0.854_{(0.165)}$	$0.902_{(0.197)}$	$0.854_{(0.175)}$	$0.909_{(0.208)}$
	0.8	$0.813_{(0.264)}$	$0.861_{(0.323)}$	$0.786_{(0.240)}$	$0.847_{(0.286)}$	$0.793_{(0.254)}$	$0.869_{(0.302)}$
2000	0.2	$0.894_{(0.100)}$	$0.941_{(0.120)}$	$0.905_{(0.089)}$	$0.943_{(0.106)}$	$0.879_{(0.094)}$	$0.925_{(0.112)}$
	0.4	$0.880_{(0.111)}$	$0.934_{(0.133)}$	$0.881_{(0.099)}$	$0.939_{(0.118)}$	$0.884_{(0.105)}$	$0.922_{(0.125)}$
	0.6	$0.864_{(0.134)}$	$0.911_{(0.160)}$	$0.840_{(0.119)}$	$0.903_{(0.142)}$	$0.860_{(0.125)}$	$0.912_{(0.148)}$
	0.8	$0.816_{(0.196)}$	$0.883_{(0.232)}$	$0.788_{(0.172)}$	$0.853_{(0.205)}$	$0.805_{(0.181)}$	$0.876_{(0.215)}$

3.4.8.5 $B_n = 30$

Table 3.40: Empirical coverage probabilities (with average lengths in parentheses) of confidence intervals at 90% and 95% nominal levels constructed using the developed central limit theorem for the GARCH process with $n \in \{200, 500, 1000, 2000\}$, $\rho \in \{0.2, 0.4, 0.6, 0.8\}$, $B_n = 30$ and different kernel choices.

		K_1		K_2		K_3	
n	ρ	90%	95%	90%	95%	90%	95%
				x_n chosen as the 90% quantile			
200	0.2	$0.769_{(0.282)}$	$0.825_{(0.333)}$	$0.795_{(0.257)}$	$0.824_{(0.306)}$	$0.798_{(0.261)}$	$0.823_{(0.311)}$
	0.4	$0.773_{(0.309)}$	$0.788_{(0.371)}$	$0.798_{(0.277)}$	$0.825_{(0.330)}$	$0.805_{(0.287)}$	$0.835_{(0.342)}$
	0.6	$0.789_{(0.359)}$	$0.829_{(0.428)}$	$0.801_{(0.331)}$	$0.832_{(0.395)}$	$0.809_{(0.355)}$	$0.843_{(0.422)}$
	0.8	$0.781_{(0.463)}$	$0.827_{(0.553)}$	$0.817_{(0.453)}$	$0.840_{(0.539)}$	$0.785_{(0.473)}$	$0.830_{(0.564)}$
500	0.2	$0.865_{(0.203)}$	$0.892_{(0.241)}$	$0.874_{(0.179)}$	$0.902_{(0.214)}$	$0.849_{(0.191)}$	$0.887_{(0.228)}$
	0.4	$0.844_{(0.219)}$	$0.876_{(0.261)}$	$0.854_{(0.202)}$	$0.884_{(0.240)}$	$0.852_{(0.212)}$	$0.888_{(0.253)}$
	0.6	$0.840_{(0.256)}$	$0.873_{(0.305)}$	$0.852_{(0.233)}$	$0.890_{(0.277)}$	$0.838_{(0.250)}$	$0.884_{(0.298)}$
	0.8	$0.824_{(0.357)}$	$0.862_{(0.425)}$	$0.843_{(0.331)}$	$0.879_{(0.394)}$	$0.841_{(0.338)}$	$0.880_{(0.403)}$
1000	0.2	$0.876_{(0.151)}$	$0.919_{(0.180)}$	$0.875_{(0.134)}$	$0.908_{(0.160)}$	$0.859_{(0.140)}$	$0.890_{(0.167)}$
	0.4	$0.873_{(0.167)}$	$0.912_{(0.199)}$	$0.877_{(0.148)}$	$0.918_{(0.176)}$	$0.871_{(0.158)}$	$0.916_{(0.189)}$
	0.6	$0.856_{(0.192)}$	$0.906_{(0.228)}$	$0.890_{(0.169)}$	$0.923_{(0.202)}$	$0.879_{(0.187)}$	$0.919_{(0.223)}$
	0.8	$0.855_{(0.268)}$	$0.894_{(0.320)}$	$0.873_{(0.237)}$	$0.915_{(0.283)}$	$0.860_{(0.256)}$	$0.892_{(0.305)}$
2000	0.2	$0.889_{(0.110)}$	$0.930_{(0.131)}$	$0.896_{(0.098)}$	$0.934_{(0.116)}$	$0.896_{(0.104)}$	$0.942_{(0.124)}$
	0.4	$0.885_{(0.120)}$	$0.927_{(0.143)}$	$0.889_{(0.107)}$	$0.939_{(0.127)}$	$0.899_{(0.113)}$	$0.941_{(0.134)}$
	0.6	$0.880_{(0.140)}$	$0.923_{(0.167)}$	$0.889_{(0.124)}$	$0.933_{(0.148)}$	$0.892_{(0.132)}$	$0.931_{(0.158)}$
	0.8	$0.843_{(0.198)}$	$0.902_{(0.236)}$	$0.863_{(0.172)}$	$0.920_{(0.205)}$	$0.861_{(0.184)}$	$0.909_{(0.219)}$
				x_n chosen as the 95% quantile			
200	0.2	$0.802_{(0.276)}$	$0.839_{(0.329)}$	$0.847_{(0.258)}$	$0.874_{(0.307)}$	$0.828_{(0.260)}$	$0.856_{(0.310)}$
	0.4	$0.796_{(0.311)}$	$0.828_{(0.371)}$	$0.819_{(0.285)}$	$0.856_{(0.339)}$	$0.801_{(0.301)}$	$0.831_{(0.359)}$
	0.6	$0.793_{(0.358)}$	$0.825_{(0.427)}$	$0.809_{(0.331)}$	$0.843_{(0.395)}$	$0.790_{(0.348)}$	$0.815_{(0.414)}$
	0.8	$0.769_{(0.499)}$	$0.795_{(0.595)}$	$0.777_{(0.460)}$	$0.828_{(0.548)}$	$0.763_{(0.474)}$	$0.806_{(0.574)}$
500	0.2	$0.852_{(0.204)}$	$0.877_{(0.243)}$	$0.866_{(0.185)}$	$0.903_{(0.220)}$	$0.862_{(0.194)}$	$0.898_{(0.231)}$
	0.4	$0.849_{(0.227)}$	$0.880_{(0.271)}$	$0.856_{(0.203)}$	$0.895_{(0.242)}$	$0.860_{(0.219)}$	$0.882_{(0.260)}$
	0.6	$0.825_{(0.274)}$	$0.871_{(0.327)}$	$0.861_{(0.238)}$	$0.895_{(0.284)}$	$0.856_{(0.260)}$	$0.890_{(0.310)}$
	0.8	$0.789_{(0.387)}$	$0.835_{(0.461)}$	$0.807_{(0.340)}$	$0.848_{(0.405)}$	$0.792_{(0.364)}$	$0.855_{(0.436)}$
1000	0.2	$0.877_{(0.152)}$	$0.912_{(0.182)}$	$0.890_{(0.134)}$	$0.924_{(0.160)}$	$0.883_{(0.146)}$	$0.928_{(0.173)}$
	0.4	$0.863_{(0.170)}$	$0.907_{(0.202)}$	$0.874_{(0.149)}$	$0.908_{(0.177)}$	$0.866_{(0.160)}$	$0.906_{(0.190)}$
	0.6	$0.852_{(0.201)}$	$0.896_{(0.240)}$	$0.865_{(0.179)}$	$0.909_{(0.213)}$	$0.860_{(0.190)}$	$0.907_{(0.227)}$
	0.8	$0.797_{(0.292)}$	$0.856_{(0.348)}$	$0.807_{(0.257)}$	$0.867_{(0.307)}$	$0.793_{(0.272)}$	$0.857_{(0.329)}$
2000	0.2	$0.891_{(0.110)}$	$0.923_{(0.131)}$	$0.900_{(0.098)}$	$0.935_{(0.116)}$	$0.898_{(0.104)}$	$0.934_{(0.124)}$
	0.4	$0.885_{(0.124)}$	$0.932_{(0.147)}$	$0.870_{(0.109)}$	$0.932_{(0.130)}$	$0.880_{(0.114)}$	$0.928_{(0.136)}$
	0.6	$0.878_{(0.146)}$	$0.919_{(0.174)}$	$0.853_{(0.129)}$	$0.912_{(0.154)}$	$0.876_{(0.138)}$	$0.927_{(0.164)}$
	0.8	$0.816_{(0.215)}$	$0.868_{(0.257)}$	$0.817_{(0.190)}$	$0.884_{(0.226)}$	$0.813_{(0.201)}$	$0.852_{(0.240)}$

3.5 Implementation Study on Real Data Sets

3.5.1 Application to a Temperature Data

We shall here present the data application results for comparing visualizations of conventional and tail spectral density analysis.

In this section we consider an application to a temperature data that contains the anomaly series of monthly averages of daily high temperatures in the United States from $03/1840$ to $05/2016$. The data and its detailed description are available through Berkeley Earth at its website. We firstly check and compare conventional autocorrelation and the tail autocorrelation visualization tool proposed in T. Zhang (2022) , where a linear trend is detected and removed, and residuals are remained for a tail spectral analysis. In Figure 3.2 is the plots with 95% Quantile as threshold with non-informative lag 0, and in Figure 3.3without non-informative lag 0. Even though the results in Figure 3.2 and Figure 3.3 show no big discrepancy in the absolute values of autocorrelation , tail autocorrelation in both right panels nicely captures the periodic pattern that exists in this temperature sequence, which should have been removed by meteorologists as the data set has been pre-proceeded to remove seasonal pattern at the mean level as it's anomaly data set. Therefore we see the proposed tail autocorrelation method could still capture the seasonal pattern that exists only on the tail level.

Plots in Figure 3.4 provide comparison between the conventional spectral density plot(left) and the tail spectral density plot (right). For both traditional and tail spectral analysis estimators, we use Tropizoidal kernel $K(u) = \max[\min\{2(1-|u|,1\},0]$ from Politis (2011) and bandwidth $B_n = \lfloor n^{1/3} \rfloor = 12$ as the rule of thumb choice from T. Zhang and Wu (2011) . From the plots we can see that the tail spectral density function seems to be quite different from the traditional spectral density function indicating that the dependence structure in the tail can be very different from that in the non-tail regions. In addition, there seems to be a peak around 0.5 for the tail spectral density function in the right panel of Figure 3.4, which relates to a period of 12 months or a yearly cycle. Such a peak does not exist in the conventional spectral density function as plotted in the left panel of Figure 3.4. This is mainly because the data is an anomaly series, where seasonal patterns have already been removed according to certain climate science calibrations. As indicated in Figure 3.3, our analysis shows that, although existing climate science

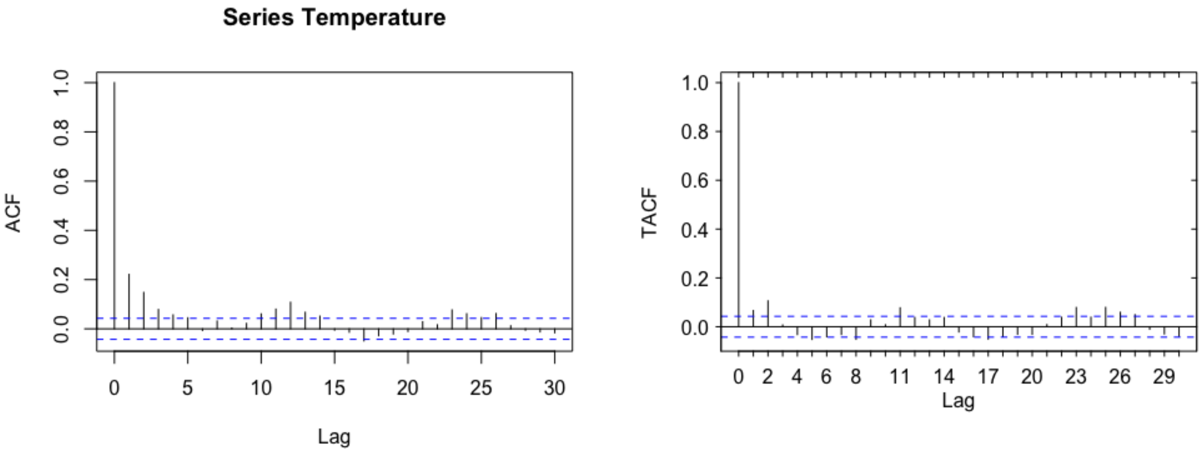

Figure 3.2: Monthly Averages of Daily High Temperatures in the United States, 03/1840 to 05/2016. 95% Quantile as Threshold with Non-informative Lag Zero

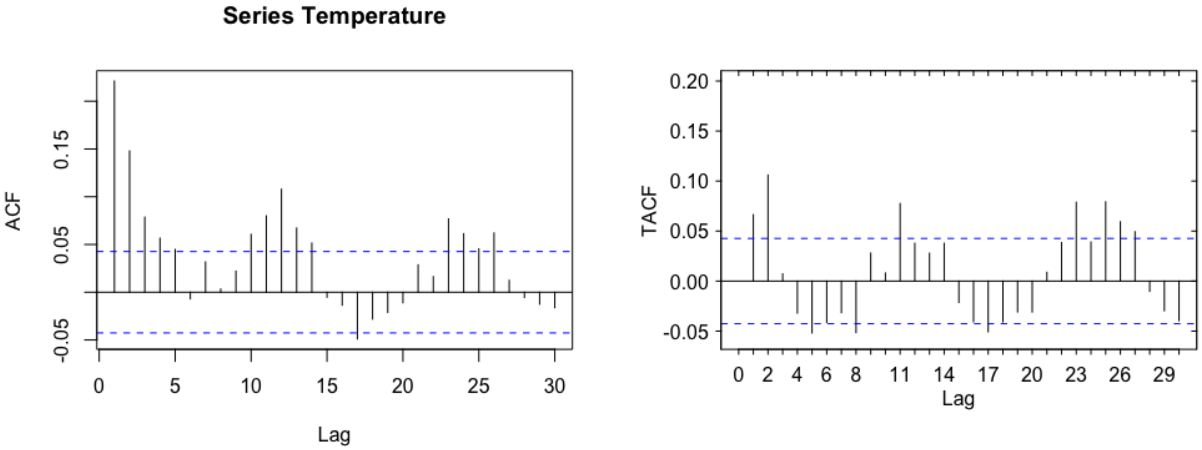

Figure 3.3: Monthly Averages of Daily High Temperatures in the United States, 03/1840 to 05/2016. 95% Quantile as threshold without Non-informative Lag Zero

calibrations are able to remove seasonal patterns in the mean, they may not be able to remove patterns in high quantiles at the same time as the seasonal pattern may not be homogeneous across different quantiles.

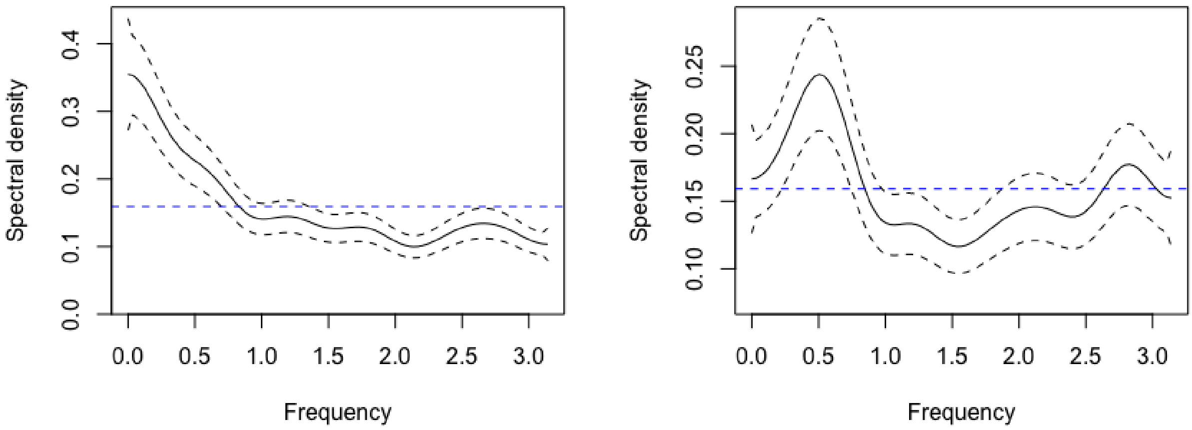

Figure 3.4: Conventional Spectral Density Analysis (left), and Tail Spectral Density Method (right).

3.5.2 Application to a Financial Data

We in this section further illustrate the developed results by considering an application to a financial data that contains the daily adjusted closing price of JPMorgan Chase & Co. (JPM) from 03/17/1980 to 10/15/2021.The data is available through Yahoo! Finance (numbers may subject to change as daily adjustment). We shall here firstly provide a tail spectral analysis on the lower tail part of the log return series to study the tail dependence among big price drops by setting the tail threshold as the 99% quantile of the negative log return series. Again we first conduct a comparison study on conventional and tail autocorrelation, using same Tropizoidal kernel $K(u) = \max[\min\{2(1 - |u|, 1\}, 0]$ from Politis (2011) and bandwidth $B_n = \lfloor n^{1/3} \rfloor = 21$, the estimated autocorrelation function estimation are as shown in Figure 3.5, lower 99% quantile of JPM stock price log return with non-informative Lag Zero, and Figure 3.6,with non-informative Lag Zero.

Comparing to the conventional method on the left panels in Figure 3.5 and Figure 3.6, we see much stronger autocorrelations when using method designed for tail region, which shows some phenomena that could be otherwise overseen. The panels on right side show not only the tail aucorrelation could in fact be much bigger than the conventional method describes, but also has a non-eliminating positive pattern that stands significant for long lags, which means the occurrence of big stock price drop could contribute to another drop even after a long term period. This could provide some information from a fresh perspective in investment as no conclusive advice could be drawn from the plots of traditional autocorrelation function on the left panels.

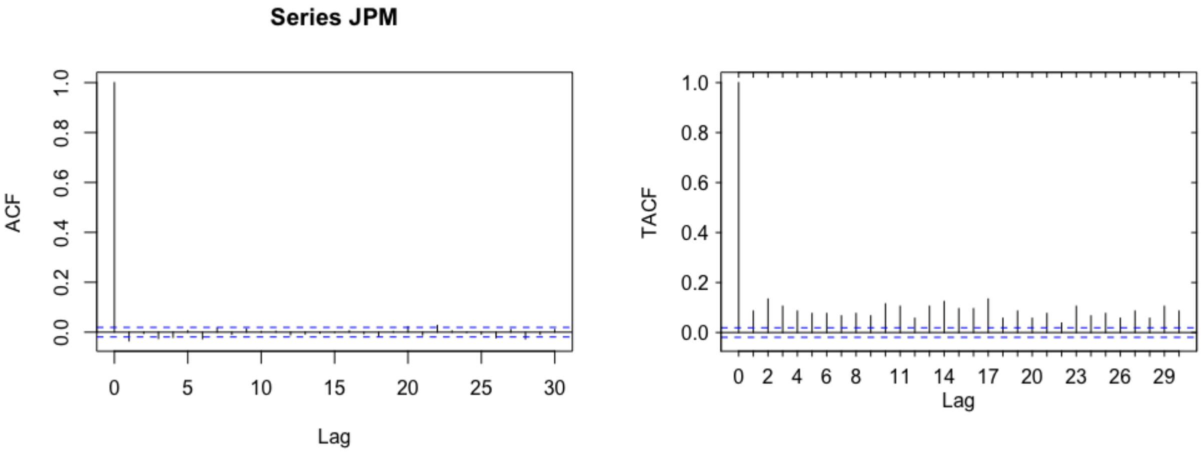

Figure 3.5: Conventional Autocorrelation Function and Autocorrelation Function on Lower Tail 99% quantile of JPM Stock Price Log Return with Non-informative Lag Zero

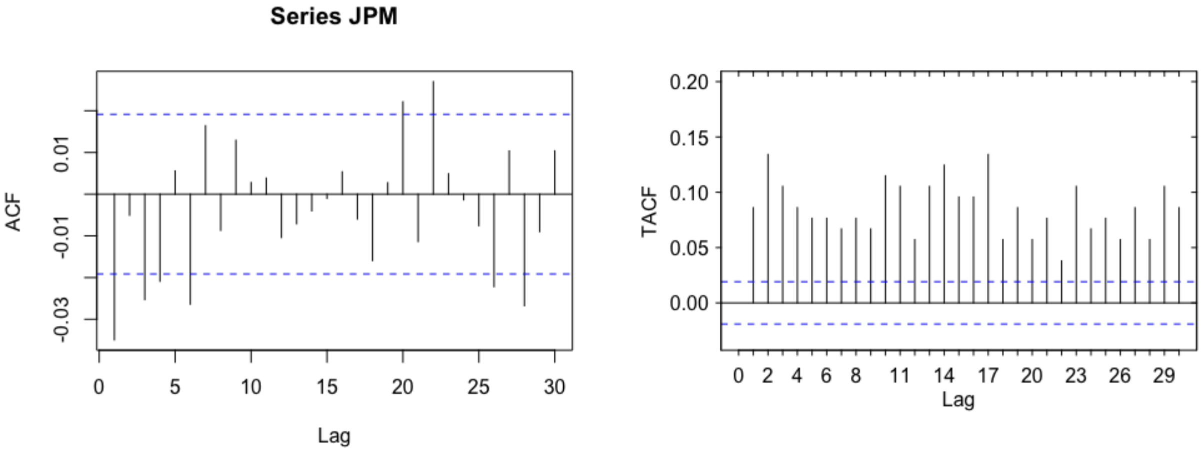

Figure 3.6: Conventional Autocorrelation Function and Autocorrelation Function on Lower Tail 99% quantile of JPM Stock Price Log Return without Non-informative Lag Zero

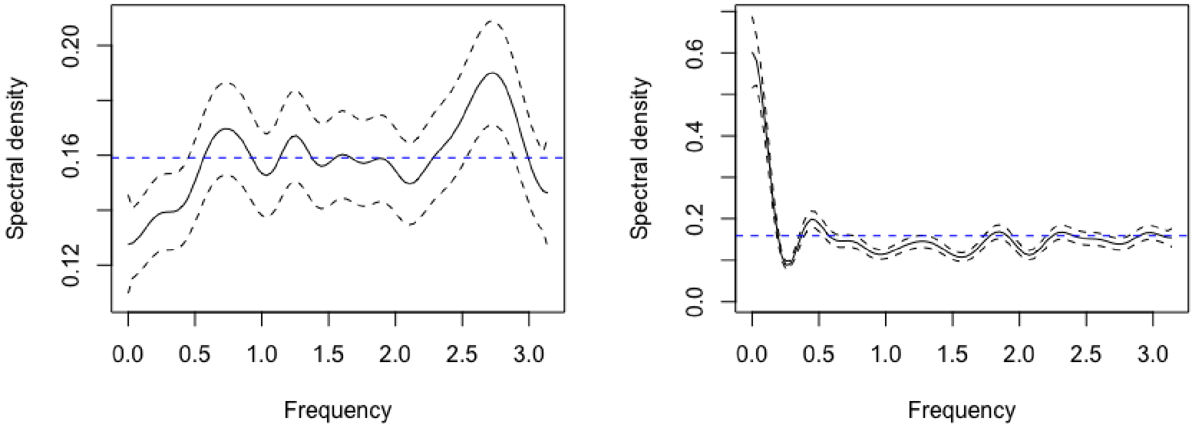

Figure 3.7: Conventional Spectral Density Analysis (left), and Tail Spectral Density Method (right).

Figure 3.7 provides the conventional spectral density plot (left) and the tail spectral density plot (right), from which we can see that the tail spectral density function seems to deviate from a constant more significantly than the conventional spectral density function indicating a higher degree of dependence in the tail. The peak near frequency zero can be related to a strong positive tail dependence, meaning that big price drops are more often associated with further big price drops, which is similarly indicated in above figures. This phenomenon does not seem to be captured by the conventional spectral density function, indicating that the dependence structure in the tail can be different from that in the mean. In addition, there seems to exist another peak around 0.45 for the tail spectral density function, which can be related to a period of 14 days in tail dependence. In contrast, for the traditional spectral density function, it can be difficult to identify any significant peaks around 0.5 given the confidence interval widths. Even if one chooses to ignore the confidence interval or the associated uncertainty,as the upper bound of a valley can be largely bigger than the lower bound of a peak, the first peak of the traditional spectral density estimate seems to be around 0.6 indicating at least a shorter period (if it ever exists), sand actually shows not significant discrepancy from the following turmoil across frequancy $1.25-2.25$ region. Therefore, the proposed tail spectral density estimation and its uncertainty quantification seems to provide the practitioners with a useful tool for analyzing tail dependence in the spectral domain, and can lead to discoveries that otherwise cannot be revealed by the traditional spectral density estimate.

Figures 3.8, 3.9, and 3.10 present a similar analysis on the identical data set of JPMorgan Chase & Co. stock price, but on the upper tail of its log return. We still observe long-lasting and positive dependence captured by tail autocorrelation function in Figure 3.9 and messy dependence information in traditional autocorrelation function; and clear discripancy in Figure 3.10 that the proposed tail spectral density estimator could outline the tail dependence that traditional method otherwise cannot. We are thusly convinced that the tail spectral density analysis with uncertainty quantification could serve as a useful tool for analyzing tail dependence in the spectral domain that complements the traditional spectral density analysis. For this specific stock price data set, the proposed method could be applied to both upper and lower tails.

The asymptotic theory on quadratic forms of tail statistics in the double asymptotic setting, including its application to obtain consistency and the limit theorem for tail spectral density estimators included in this chapter are based on the work of T. Zhang and Xu (2023) and its supplementary material.

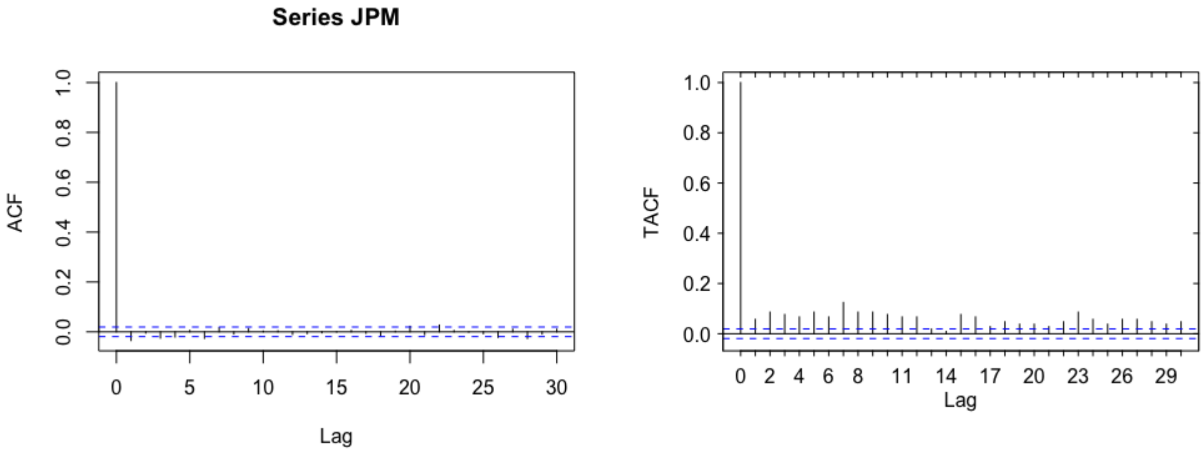

Figure 3.8: Conventional Autocorrelation Function and Autocorrelation Function on Upper Tail 99% quantile of JPM Stock Price Log Return with Non-informative Lag Zero

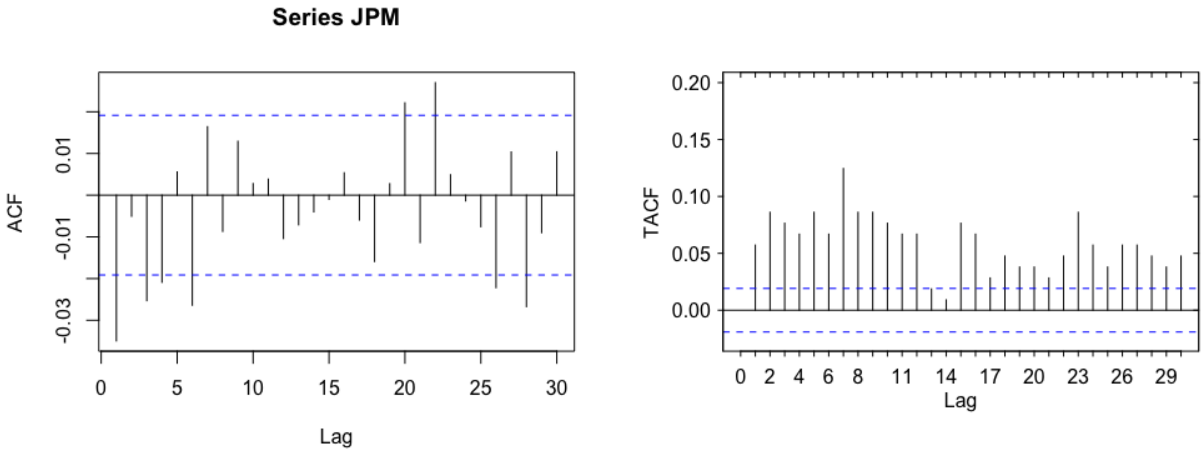

Figure 3.9: Conventional Autocorrelation Function and Autocorrelation Function on Upper Tail 99% quantile of JPM Stock Price Log Return without Non-informative Lag Zero

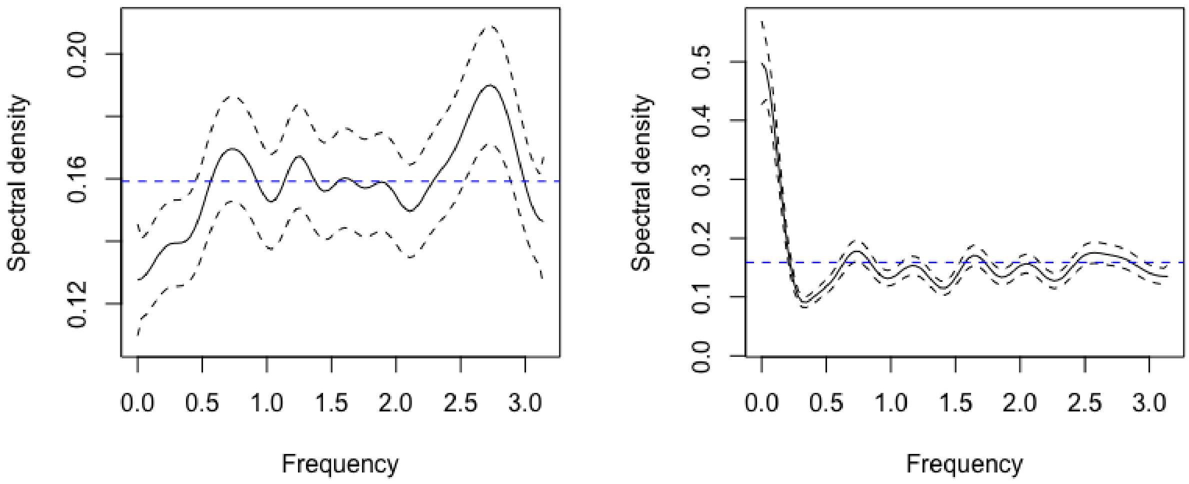

Figure 3.10: Conventional Spectral Density Analysis (left), and Tail Spectral Density Method (right).

CHAPTER 4

TAIL INDEX ESTIMATION

4.1 Introduction to Tail Index and Hill's Estimator

This chapter is based on a working paper (T. Zhang et al., 2023), where we continue to focus on extreme value events which have potentials in causing substantial consequences. When it comes to the tails of probability distribution, a measurement of the thickness or heaviness of this tail is introduced to help quantifying certain aspects of infrequent but influential events. This measurement is concepted as tail index, which provides the comprehensive description of how fast the tail part of a distribution decays. In other words, it serves as a parameter that provides some insights into whether extreme events are likely to occur more or less frequently in order to make informed decisions and manage risks, especially in the research areas mentioned in Chapter 1.The comprehensive exploration of the tail index aims to shed light on its various facets, including its theoretical foundations, practical applications, and estimation performance. Countless literature have been dedicated to capturing the truth of tail index, amongst which a simple general approach was proposed for the inference on the tail behavior of a distribution with the focus on its tail index, which requires assumptions only on its tail region but no particular globally parametric form (B. M. Hill, 1975). The aforementioned article was given rise to by the example of the power laws of Pareto, which was assuming a distribution following $F(x) = 1 - Cx^{-\nu}$ as $x \to \infty$ on its interested tail region, with $F(x)$ being cumulative distribution function of x and C being a constant number, and it was desired to draw inference on the tail index, or the extreme value index, defined as ν^{-1}.

Given samples $X_1, \ldots, X_n$ drawn from a population with tail distribution defined above, let $X_{(1)} \geq X_{(2)} \geq \cdots \geq X_{(l_n)}$ be the l_n largest order statistics, where l_n is given by the threshold decided by either an absolute value or a proportion of sample size. By the Rényi representation theorem (Rényi, 1953),

$$X_{(i)} = F^{-1}\Big[\exp\Big\{-\Big(\frac{e_1}{l_n} + \frac{e_2}{l_n - 1} + \cdots + \frac{e_i}{l_n - i + 1}\Big)\Big\}\Big], \quad \text{for } i = 1, \ldots, l_n,$$

which shows the relationship that the biggest l_n observations could be bridged with independent exponentially distributed random variables e_i in terms of their joint distribution. Then the conditional or pseudo-maximum likelihood estimation method would give Hill's estimator of the tail index in the following form:

$$\hat{H}_n = l_n^{-1} \sum_{j=1}^{l_n} \log X_{(j)} - \log X_{(l_n+1)}, \tag{4.1}$$

The estimator (4.1) has been widely celebrated since its birth and numerous achievements have been made in existing works with its extension and implementation. For example, Mason (1982) firstly justified the consistency of Hill's estimator for iid observations, followed by Deheuvels et al. (1988) topped on it with almost sure convergence, and its asymptotic normality has been established by many works; see Hall (1982) , R. Davis and Resnick (1984) , Csorgo et al. (1985) , Haeusler and Teugels (1985) , Csörgő and Viharos (1995) , De Haan and Peng (1998) , De Haan and Resnick (1998) etc.. Beirlant et al. (1996) introduced a generalized Hill's estimator, and Dekkers et al. (1989) extended Hill's estimator for the index of a regularly varying tail to an extreme-value distribution, and provided its consistency and asymptotic normality proof. Guillou and Hall (2001) proposed a new approach to selecting the threshold when fitting Hill's estimator in extreme value analysis with a soft Pareto assumption. Applications of the methods include insurance claims distribution as in Teugels (1984) ; several pitfalls designed in Diebold et al. (2000) to assess the potential of extreme value theory application in financial risk management with a focus on Hill's estimator; a new hydrological post-processing method using high quantile regression predictions deployed with Hill's estimator in Tyralis and Papacharalampous (2023) ; to name a few.

Hsing (1991) later introduced dependent data to tail index estimation using Hill's estimator and its asymptotic normality with further assumption $1 - F(x) = x^{-\nu}L(x)$, where $L(x)$ is some slowly varying function satisfying $\lim\limits_{x\to\infty} \frac{L(ax)}{L(x)} = 1$ for all $a > 0$. To overcome the obstacle of developing asymptotic theory with dependent data, Hsing (1991) introduced a bypass to cleverly deal with the distance between the jth order statistic and the theoretical α_n level quantile of the process distribution (the threshold). To address this difference, Hsing (1991) primarily leveraged the subtlety around the threshold by further defining two intimately related quantities

$$\tilde{H}_n = l_n^{-1} \sum_{j=1}^{l_n} \{\log X_{(j)} - \log b(n/l_n)\}, \tag{4.2}$$

$$H_n^+ = l_n^{-1} \sum_{i=1}^{n} \{\log X_i - \log b(n/l_n)\}_+, \tag{4.3}$$

where $b(x^{-1}) := F^{-1}(1-x)$, and established the consistency of $\hat{H}_n$, $\tilde{H}_n$, and H_n^+ by taking advantage of well-established properties of slowly varying function and concluded with a general version of asymptotic distributional convergence of Hill's estimator $\hat{H}_n$. It was demonstrated in the same paper that the results could be directly applied to strong mixing processes and l-dependent processes.

Involving Hill's estimator to handle potential dependence in data could lead to further development of new methodologies in relevant directions to capture more complex relationships and patterns that may exist in data, especially in the tail region, instead of on average, and a significant expansion of the estimators' usage across a wider range of practical applications. For example, Rootzén et al. (1990) also provided asymptotic properties of Hill's estimator under a strong-mixing condition amongst others; Hsing (1993) extended his work to the region of extremel tail area under a weakly dependent condition; Resnick and Stărică (1997) examined the asymptotic behavior of Hsing's results by comparing applying the estimator to data and applying to the residuals of an autoregressive model, Resnick and Stărică (1998) further broadened the consistency of Hill's estimator to more models with m-dependent random variables like infinite moving average model with heavy-tail innovation, solution to some stochastic difference equations, ARCH models that cannot be approximated by m-dependent random variabels, and a class of hidden semi-Markov models; for $\beta-$ mixing stationary time series, Drees (2000) established convergence of tail empirical processes and obtained weighted approximations of the tail empirical quantile function; Novak (2002) proposed another estimator under mild mixing conditions and provides its consistency and asymptotic normality; Drees (2003) established the asymptotic normality under mild structure conditions of $\beta-$mixing time series, and applies this theory to a time series of stock index returns; J. B. Hill (2010) studied asympotic properties of Hill's estimator for dependent and heterogeneous processes;

Van Oordt and Zhou (2016) considered the cross sectional systematic tail risk under severely stressed market conditions; and references therein.

4.2 Hill's Estimator : A Revisit

Besides the strong mixing process and the l-dependent process considered in Hsing (1991) , it is expected that the Hill's estimator will continue to work under other dependence frameworks such as the TAS framework introduced in Chapter 2. We in this chapter seek empirical evidence by some Monte Carlo simulations. For this, we consider the moving-maximum process.

By the discussions in T. Zhang (2021) , T. Zhang (2022) , Bai and Zhang (2022) , the TAS framework covers the moving-maximum process of Hall et al. (2002) , which is proven to be dense in the class of stationary processes whose finite-dimensional distributions are extreme-value distributions of a given type (Hall et al., 2002). In particular, let

$$X_i = \max_{0 \le l < \infty} a_l \epsilon_{i-l}, \quad i = 1, \ldots, n, \tag{4.4}$$

which is well defined if the nonnegative coefficients satisfy $\sum_{l=0}^{\infty} a_l^{\gamma} < \infty$. Then let ϵ_j $(j \in \mathbb{Z})$ be independent Fréchet random variables with distribution function $F_\epsilon(z) = \text{pr}(\epsilon_j \le z) = \exp(-z^{-\gamma})$ for some $\gamma = 1, 2, 3$. Given that ϵ_j $(j \in \mathbb{Z})$ are independent, for $x > 0$, we have

$$\begin{aligned} \text{pr}(\max_{0 \le l < \infty} a_l \epsilon_{i-l} \le x) &= \prod_{l=0}^{\infty} \text{pr}(a_l \epsilon_{i-l} \le x) = \prod_{l=0, a_l \neq 0}^{\infty} \text{pr}(\epsilon_{i-l} \le x/a_l) \prod_{l=0, a_l = 0}^{\infty} 1 \\ &= \prod_{l=0, a_l \neq 0}^{\infty} \exp\left\{-(x/a_l)^{-\gamma}\right\} \prod_{l=0, a_l = 0}^{\infty} \exp(a_l) = \exp\left(-\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma}\right) \end{aligned}$$

Therefore, the cumulative distribution function for the moving-maximum process and associated probability distribution function in the expression of X_i satisfy

$$F_X(x) = \exp\left(-\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma}\right), \quad f_X(x) = \gamma \left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma-1}\right) \exp\left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma}\right)$$

Then from the calculation of the tail index in Gabaix et al. (2016) :

$$\begin{aligned}
\nu^{-1} &:= \lim_{x \to \mathcal{U}_F} \frac{d}{dx} \left\{ \frac{\bar{F}(x)}{f(x)} \right\} \\
&= \lim_{x \to \mathcal{U}_F} \frac{d}{dx} \left\{ \frac{1 - \exp\left(-\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma}\right)}{\gamma \left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma-1}\right) \exp\left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma}\right)} \right\} \\
&= \lim_{x \to \mathcal{U}_F} \gamma^{-1}(\gamma+1)x^{-1} \left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma-1} \right)^{-1} \left\{ \exp\left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma} \right) - 1 \right\} - \exp\left(\sum_{l=0}^{\infty} a_l^{\gamma} x^{-\gamma} \right)
\end{aligned}$$

4.2.1 Simulation Study

We generate data according to the moving-maximum process in (4.4). Let a series of $\alpha_n \in (0,1)$ satisfies

$$\alpha_n \to 0, \quad n\alpha_n \to +\infty, \tag{4.5}$$

and let $l_n = \lfloor n\alpha_n \rfloor$. As α_n is allowed not to be fixed, and decays infinitely close to 0 as long as much slower than n does going close to infinity, so we have $l_n \to \infty$ as well.

We consider the order statistics $X_{(1)} \geq X_{(2)} \geq \ldots \geq X_{(n)}$, and employ the Hill's estimator presented earlier with the first l_nth largest order statistics with l_n defined above:

$$\hat{H}_n = \frac{1}{l_n} \sum_{i=1}^{l_n} \{\log X_{(i)} - \log X_{(l_n+1)}\}.$$

For the simulation study, we consider Fréchet distribution's shape parameter as values $\gamma = 1, 2, 3$. Plugging γ values into the above nominal tail index calculation equation and we get the nominal tail index for each distribution is $1, 0.5$, and$\frac{1}{3}$, respectively. Here we provide a simple discussion on the heaviness of the tail. Consider the existence of moments with each γ, we notice when $\gamma = 1$, the first order moment of Fréchet distribution does not exist, which implies it has a heavy tail that decays even slower than n. Similarly, when $\gamma = 2$, the first order moment exists but the second order does not, which indicates a lighter but still heavy tail that decays at a speed slower than $\sqrt{n}$. And when $\gamma = 3$, the first two order moments exist, and Fréchet 3 has the lightest tail among the scenarios considered in our simulation, though it's worth noting that Fréchet still belongs to a heavily tailed distribution family. The following table 4.1

shows the nominal and simulated tail index values, including the average simulated tail index of 1000 paths and associated standard deviation. We can see that the simulation results are reasonably close to their nominal values regarding the simulation average, but need to notice the big standard deviation when sample sizes are relatively small. Besides, when sample sizes increase, we see less improvement in simulated values regarding average, but standard deviations show their tendency of getting smaller. Also we see the simulation averages getting close to the nominal level generally fastest when $\gamma = 3$, which makes sense as mentioned before it has the lightest tail. Then if we look at separate quantile levels, the simulation results get stably close to the nominal values faster when $\alpha_n = 0.01$ than $\alpha_n = 0.05$, with comparably smaller standard deviation, because as mentioned before, higher quantile level could introduce certain difficulties due to limited data points.

Table 4.1: Tail index simulation results with various sample sizes and $\alpha_n = 0.005$ and 0.01. Simulation paths = 1000.

	$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
	nominal = 1		nominal = 0.5		nominal = $\frac{1}{3}$	
n	$\alpha_n = 0.005$	$\alpha_n = 0.01$	$\alpha_n = 0.005$	$\alpha_n = 0.01$	$\alpha_n = 0.005$	$\alpha_n = 0.01$
500	$0.852_{(0.777)}$	$0.925_{(0.536)}$	$0.445_{(0.362)}$	$0.469_{(0.254)}$	$0.316_{(0.229)}$	$0.317_{(0.163)}$
1000	$0.918_{(0.551)}$	$0.951_{(0.399)}$	$0.480_{(0.245)}$	$0.482_{(0.178)}$	$0.323_{(0.156)}$	$0.325_{(0.113)}$
2000	$0.969_{(0.393)}$	$0.979_{(0.292)}$	$0.491_{(0.182)}$	$0.498_{(0.138)}$	$0.326_{(0.114)}$	$0.332_{(0.082)}$
5000	$0.988_{(0.254)}$	$0.991_{(0.182)}$	$0.502_{(0.115)}$	$0.502_{(0.081)}$	$0.333_{(0.075)}$	$0.337_{(0.053)}$
10000	$0.997_{(0.185)}$	$1.001_{(0.129)}$	$0.498_{(0.086)}$	$0.501_{(0.055)}$	$0.332_{(0.051)}$	$0.335_{(0.037)}$
100000	$1.001_{(0.056)}$	$1.001_{(0.040)}$	$0.500_{(0.027)}$	$0.500_{(0.017)}$	$0.333_{(0.016)}$	$0.334_{(0.010)}$

4.2.2 Data Implementation

Next, we provide some tail index estimation with real data sets. As mentioned in previous chapters, it's common that financial data sets present extreme values, therefore we pick several sets of stock price data from multiple sectors. Same as quantile selection in Wang et al. (2012), we test on $\alpha_n = 0.01, 0.005, 0.001$. Table 4.2 shows the tail index estimations on the lower tail of the stock's log return under 3 quantile levels. We pick the lower tail for risk control interest. We can observe that under most scenarios the heaviness of tails are close to the level at Fréchet distribution with $\gamma = 1$ as simulated, except for GGE and BAC show both estimated tail index at quantile level $\alpha_n = 0.005$ and 0.001 greater than 0.4, which we'd look into details later. Some cells are left blank at $\alpha_n = 0.001$ as Hill's estimator is not expected to work well under extremal high quantile with limited sample size in the tail part.

Table 4.2: Tail Index Estimation with Stock Price Data Sets under Different High-quantile Levels

	KMB	NCPL	GGE	BAC	TSLA	LLY
n	10988	4375	3923	12608	3270	12858
$\alpha_n = 0.01$	0.3546	0.1845	0.4658	0.4072	0.3235	0.3075
$\alpha_n = 0.005$	0.3341	0.3779	0.4083	0.4149	0.2573	0.3278
$\alpha_n = 0.001$	0.2852			0.3636		0.4028

We then look further at the price and return plots for GGE and BAC stock in Figures 4.1 and 4.2. First for GGE, we do see a few outstanding fluctuates, when the quantile level is set at $\alpha_n = 0.005$, about 21 values will be included in the tail index calculation, and most of these fluctuates will contribute. And for BAC stock, it's more obvious that large fluctuates happening right before 2010, during the big financial crisis, which directly contribute to its price dump during that time. As BAC stock has a long history and a collected sample size of $n = 12608$, these big jumps happening during this time would contribute to tail index estimation. Another thing we may find from this analysis is that, though for simulation part we see the cases where the nominal tail index value to be 1 and 0.5, from an empirical perspective, a tail index as big as 0.4 could already bring significant impact.

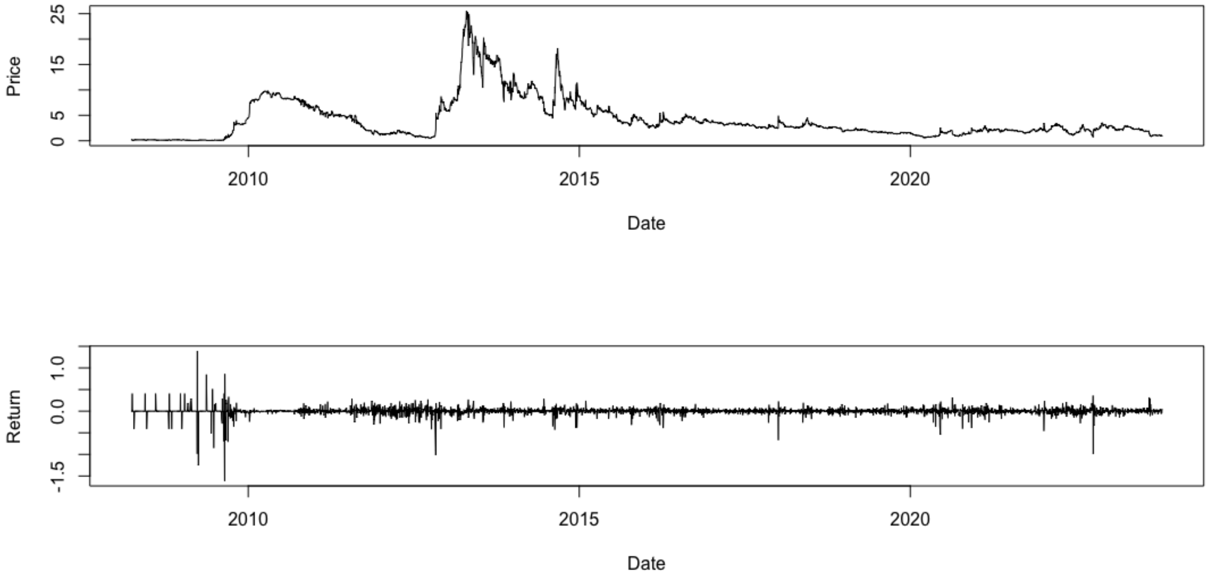

Figure 4.1: Price and Return Plots for GGE Stock

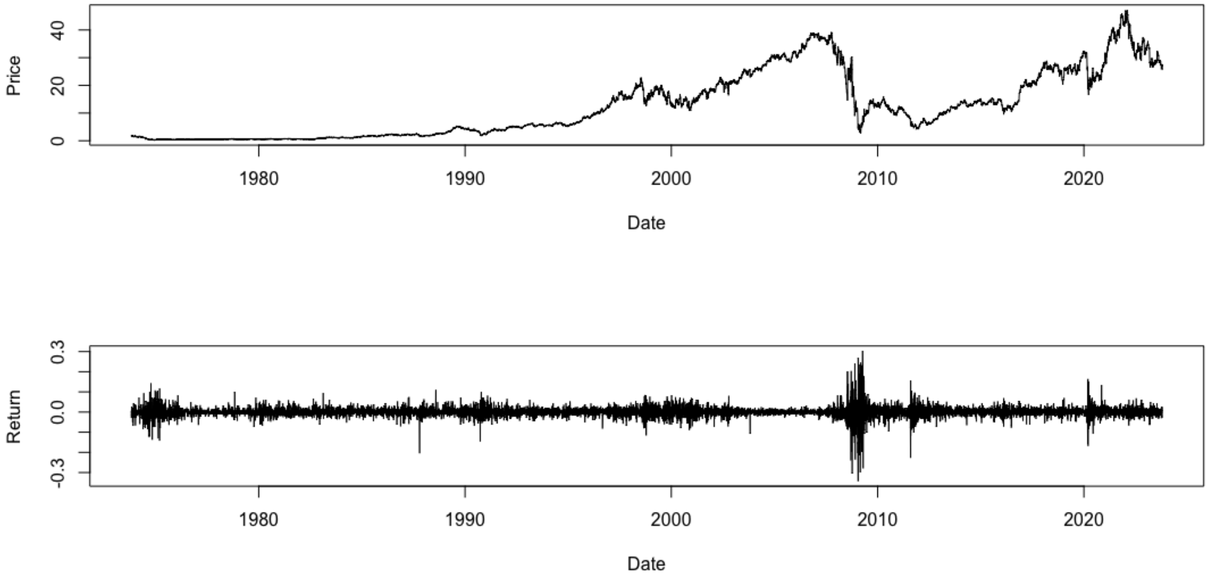

Figure 4.2: Price and Return Plots for BAC Stock

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