VALUING RECREATION: METHODS AND APPLICATIONS

by

ASHLEY S. BARFIELD

(Under the Direction of J. Scott Shonkwiler)

ABSTRACT

This dissertation comprises three manuscripts which examine the recreational travel cost literature and consider new approaches to data management and demand modelling that improve the statistical efficiency and accuracy of standard travel cost methods and applications. Together, these papers provide valuable insights and recommendations for future econometric applications of the travel cost model.

Revealed preference methods require survey data on past resource use, and numerous studies have found reported recreation frequency to be overestimated and concentrated on prototype values. Our first paper develops two approaches to treat extreme values and rounded responses. We illustrate how, when modeling single-site trip data using a negative binomial (NB) distribution, employing the incomplete beta function simplifies the incorporation of censored intervals. We show the NB's fit is improved by either reassigning rounded responses to censored regimes where reported trip numbers define the intervals' upper bounds, or by mixing the NB with a continuous distribution at a cut-point where response behavior begins to exhibit rounding.

Much of the travel cost literature uses mixed logit (MXL) models to evaluate recreational site choice data. Multinomial probit (MNP) models are less common, as they have been difficult to work with historically. Our second paper compares these models' performances and explores implications for welfare analysis in the case of multi-site trip data. Utilizing a new, more efficient approach (dubbed the Delta Method Approximation) for estimating the distribution of the mean benefit from policy implementation in MNP models, we discuss the merit of increasing MNP models' prevalence in non-market valuation studies.

North Carolina's beaches are imperiled by coastal erosion, sea level rise, severe storms, and oceanfront development. Proposed solutions to these problems include beach replenishment, coastal retreat, and shoreline armoring. These policies affect the quality and value of coastal resources and recreation, and assessing these welfare impacts is necessary for benefit-cost-analysis of these alternatives. Our third paper analyzes multi-site trip data for North Carolina households using travel costs and site attributes. We employ a MXL model in our recreation demand analysis and discuss the advantages of incorporating a Kuhn-Tucker generalized corner solution model in future extensions of this analysis.

INDEX WORDS: Censoring, Extreme responses, Incomplete beta function, Kuhn-Tucker, Mixed logit, Multinomial probit, Negative binomial distribution, Non-market valuation, Recall bias, Recreation demand, Revealed preference, Rounded responses, Travel cost

VALUING RECREATION: METHODS AND APPLICATIONS

by

ASHLEY S. BARFIELD

A.B.J., The University of Georgia, 2010

B.S.E.S., The University of Georgia, 2010

M.S., The University of Georgia, 2012

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2016

VALUING RECREATION: METHODS AND APPLICATIONS

by

ASHLEY S. BARFIELD

Major Professor:	J. Scott Shonkwiler
Committee:	Craig E. Landry
	Greg Colson
	John C. Bergstrom

Electronic Version Approved:

Suzanne Barbour
Dean of the Graduate School
The University of Georgia
December 2016

ACKNOWLEDGEMENTS

It is my great privilege to acknowledge and thank everyone who contributed to the completion of my dissertation research. Their support was invaluable, and I am deeply grateful to have had so many helpful, positive sources of encouragement and inspiration.

Firstly, I would like to thank my advisory committee for providing me with such a solid academic, professional and personal foundation for my graduate studies. They helped me with important decisions, guided me to valuable resources, and provided plenty of advice and constructive criticism to strengthen the basis of this research. For helping me network with brilliant researchers at professional conferences, graciously accommodating my increasingly frequent office visits, writing recommendation letters with little to no notice, revising and re-revising drafts of papers, abstracts and cover letters, and responding to my pleas for help at all hours, I can never thank them enough.

Dr. Shonkwiler, it has been an honor to work with you. When I was graduating from the Master's program and getting ready to leave the department, Dr. Ramirez told me I should stay because I would have the opportunity to learn from one of the finest minds in our field if I did. I am so glad that I took his advice and that you were willing to chair my Doctoral committee. Econometrics was never my strong suit, but your patience as a teacher and your ability to take complex methodologies and explain them succinctly and with profound clarity have made all the difference in my education. Your extensive knowledge and deep understanding of technique have been such incredible resources. Thank you for sharing them and your time with me these last four years. I hope we continue to work together in the future.

Dr. Landry, I feel so fortunate that you joined our faculty just as I was beginning to focus on my research and my goals as a Ph.D. student. You have been so insightful and you have given me so many opportunities to grow, to broaden my experience, and to find inspiration. Thank you so much for your friendship, for being so supportive, for always listening, and for helping me to reach further and to strive to be more innovative and independent.

Dr. Colson, there is a reason why so many of the students in our department want to work with you. You go out of your way to make yourself available to us, and you never fail to lend a helping hand. We are always welcome in your office, and if you don't know the answer, you know where we can look next. Knowing you have a dedicated, gifted and reliable teacher in your corner makes you feel like you can meet every challenge. You have been my rock throughout this program - I would not have finished it without you. Thank you, thank you, thank you.

Dr. Bergstrom, I wouldn't be here at all if it weren't for you. You noticed my potential as an undergraduate and encouraged me to apply to graduate school. Under your leadership, I earned my Master's degree and published my first manuscript in a peer-reviewed journal. You have been my advisor, my mentor, and a dear friend for nearly a decade now. I am so incredibly thankful for everything you have taught me and for every moment you have spent helping me develop and refine my skills and interests. Your trust and confidence in me has meant everything.

My research, and indeed my entire doctoral program of study, would not have been possible without the faculty, staff and administration of the Department of Agricultural and Applied Economics in the College of Agricultural and Environmental Sciences, and of the Department of Economics in the Terry College of Business, to whom I am forever indebted. I would also like to thank the staff and administration of the UGA Graduate School for their support and resources.

Finally, I would like to thank my family for loving me and supporting me throughout my entire academic career here at UGA.

Most importantly, I would like to thank my husband, Brandon Barfield, for his unerring patience and encouragement; for seeing me through a decade of degrees and research at UGA; and for always reminding me of what really matters and why I do what I do, especially when it has been difficult for me to maintain my focus. Thank you for always believing in me, for helping me to believe in myself, and for never letting me give up.

I would also like to thank my parents, Karin and Jim Dronenburg, my brothers, Jeremy and Zach Dronenburg, my in-laws, Kathy and Randy Barfield, and the entire Barfield family, for always telling me how proud they are of me and how sure they are that I will succeed and do great things. Thank you all for inspiring me to become the person you feel I can be.

TABLE OF CONTENTS

Page

ACKNOWLEDGEMENTS iv

CHAPTER

1 INTRODUCTION AND LITERATURE REVIEW 1

2 SURVEY RESPONSE DATA: PATTERNS AND PROBLEMS 7

3 MULTINOMIAL PROBIT AND MIXED LOGIT MODELING OF RECREATION DEMAND: A COMPARATIVE ANALYSIS 31

4 BEACH EROSION, SITE QUALITY, AND RECREATION DEMAND: APPLICATION OF MIXED LOGIT AND KUHN-TUCKER GENERALIZED CORNER SOLUTION MODELS 55

5 CONCLUSIONS 83

REFERENCES 87

CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

The travel cost model is one of the corner-stones of non-market valuation, and is frequently applied in the context of recreational site choice data. This dissertation is organized as a collection of three separate (but related) manuscripts which address the following topics: (1) Best practices for correcting patterns of response rounding and overstatement of recreational behavior in survey data; (2) Comparative analysis of mixed logit and multinomial probit random utility models' performances in the case of multi-site trip data; and (3) Application of the mixed logit model to the welfare analysis of beach erosion, site quality, and recreation demand, and examination of research extensions incorporating the Kuhn-Tucker generalized corner solution method.

The estimation of economic values for environmental amenities is critical to making informed resource management decisions and accurately assessing environmental damages. Revealed preference methods for measuring economic values require data on observed behavior. Typically such behavior is elicited by surveys of target populations regarding their past use of a natural resource. Since recall periods are generally over a past season or year, the issue of recall accuracy needs to be considered. Systematic biases in reporting past behavior may compromise the methods used to derive values from revealed preference data.

Numerous studies have found that respondents tend to both overestimate their recreation frequencies and round off their responses to end in a zero or five. Tourangeau et al. (2000) report that open-ended questions which require a numerical response may manifest these characteristics: i) the larger the number to be reported the more likely it will be a round value; and ii) the distances between successive rounded values increase as the numbers increase. They also state that "by reporting their answers as round values, respondents may be consciously attempting to communicate the fact that their answers are at best approximations."

Vaske and Beaman (2006) describe how respondents may answer recall questions about frequencies (such as days of participation) using episode enumeration, formula-based multipliers, and prototypes, which cause their responses to deviate from what occurred in reality. All of these approximations can manifest in the data as response "heaping" – where reported numbers occur more often than chance would suggest. As the authors explain, heaping is likely related to number (or digit) preference: "numbers that a person has a disposition to use or avoid." Indeed, Huttenlocher et al. (1990) find that respondents tend to overuse both multiples of 5 and 10 and numbers associated with calendar events such as weeks or months (7, 14, 21, 30 and 60, for example). Another manifestation of recall error is response "leaping" – where response heaping increases with reported magnitudes. "For responses under 15, several studies have found limited 0-5 heaping…Above 100, responses may fall largely on 150, 200, 300, and 500 with gaps widening as responses move into the thousands," (Vaske and Beaman 2006).

Several papers discuss front-end reduction (prevention) of response bias and suggest strategies to improve survey design and implementation (Pudney 2008; Schaeffer and Presser 2003; Miller and Anderson 2002; Tarrant and Manfredo 1993; Chu et al. 1992). With regard to back-end reduction (correction) of response bias, Evans and Herriges (2010) provide a recent example. Using generated data experiments and a latent class model which assumes respondents are members of either a rounding or non-rounding class, they find that rounding bias can have significant impacts on parameter estimates and resulting welfare measurements. Chapter 2 of this dissertation focuses on the data portion of travel cost modelling by presenting two new approaches to treat the presence of extreme values and rounded responses in survey count data.

In terms of modelling strategies used to evaluate recreational site choice data, Random Utility Models (RUM), which divide recreational seasons into multiple discrete choice occasions in which respondents either take or do not take a trip, have historically been quite popular. To date, much of the travel cost literature has focused on the use of multinomial logit (MNL) and mixed logit (MXL) (random parameters logit) models. Multinomial probit (MNP) models are less common, as they have been more difficult to work with historically. While MNL models are relatively simple to work with and can model site selection decisions (substitution effects), they cannot model decisions about total trips taken over the course of a recreational season (participation effects). For this reason, while they can provide welfare estimates on a per-trip basis, MNL models cannot provide estimates of seasonal welfare impacts. To achieve this, it is necessary to link the site choice model to a participation decision model. This has typically been accomplished through some sort of nested logit model.

Standard logit models cannot represent random taste variation, they adhere to the Independence of Irrelevant Alternatives (IIA) axiom (which results in restricted, unrealistic substitution patterns among similar alternatives), and they cannot accommodate correlation in unobserved factors over time. While generalized extreme value models (the family of models to which nested logits belong) relax the IIA constraints, they remain plagued by the problems of random taste variation and serial correlation (Train 2009). MNP and MXL models are equipped to deal with these challenges, however. Compared to standard MNL and nested logit models, MNP and MXL models are more flexible, and their respective simulation methods are capable of handling a wider variety of datasets.

The MNP model is advantageous in that it is not bound by the IIA axiom and it can incorporate both random taste variation and temporally correlated error terms. It also captures correlations in utilities between alternatives when the error term covariance matrix, Ω, is normalized. However, the MNP model is challenged both by its inherent assumption that all unobserved components of utility are normally distributed[1] and by its lack of a closed form expression for expected maximum utility (Train 2009). Like the MNP model, the MXL model is advantageous in that it does not hinge on the IIA axiom and can incorporate both random taste variation and correlation in unobserved factors across time. Additionally, the MXL model is not limited by the assumption of normality made by the MNP model (Train 2009).

Chapter 3 of this dissertation focuses on the methodology portion of travel cost modelling by comparing the performance of MNP and MXL models and by employing an innovative, analytical approach for calculating expected maximum utility in the MNP context which may provide theoretical verification of standard simulation procedures while demonstrating a computational advantage. In sum, chapter 3 offers insights as to the merit of increasing MNP models' prevalence in the non-market valuation literature.

In recreation demand studies, it is frequently the case that a researcher's dataset will consider respondents' socio-demographic characteristics and reported seasonal visitation to a large number of alternatives (perhaps a dozen sites or more), for which there is an accompanying site attribute index. Often, a respondent will visit a subset of sites multiple times, and other sites

[1] This assumption may hold in most cases, but particularly in the case of price coefficients, it may lead to estimates that are not theoretically desirable. The normal density has mass on either side of the mean of zero, implying that some members of the population would have a positive price coefficient where we would (almost always) anticipate a negative (Train 2009).

not at all. “To consistently derive welfare measures for price and attribute changes with such data, structural econometric models that behaviorally and statistically account for the mixture of corner solutions (unvisited sites) as well as interior solutions (sites with one or more trips) are required,” (von Haefen and Phaneuf 2005). Chapter 4 of this dissertation presents a timely application of the travel cost model which utilizes the MXL framework examined in chapter 3; it further describes the advantages and challenges of employing a Kuhn-Tucker generalized corner solutions model to evaluate the same dataset.

North Carolina's beaches are imperiled by a number of forces including coastal erosion, sea level rise, storm events of increasing frequency and severity, and oceanfront development. Three primary solutions to these problems have been proposed: beach replenishment, coastal retreat, and shoreline armoring. Each of these management approaches induces changes in the quality of coastal resources, affecting the distribution of beach and dune sediments, presence and location of hardened structures, and configuration of buildings and infrastructure. These changes, in turn, affect the economic value of coastal recreation. We consider the use values associated with North Carolina (NC) beaches and how these values could be influenced by the implementation of the aforementioned management policies. The accurate assessment of such welfare impacts is, naturally, a critical component of the benefit-cost-analysis of these alternative proposals. Our primary research goal is to identify and characterize preferences for beach width.

To this end, we analyze revealed preference beach site choice data for a random sample of NC households (data collection funded by East Carolina University and NC Sea Grant in 2013). Through the use of the NC Department of Environmental Quality's Coastal Geographic Information Systems (GIS) files, a traveler’s manual for NC beaches (Morris 2005), and a host of Outer Banks tourism websites, we create a site-attribute matrix for NC beaches that includes information regarding travel costs and beach length, width, and accessibility. We employ a MXL model in our analysis of recreation demand and the impact of site characteristics (many of which can be influenced by coastal policy and erosion management) on site choice and intensity of beach recreation. Our research therefore represents an important contribution to the understanding of people's preferences and support (willingness to pay, WTP) for different erosion management scenarios.

CHAPTER 2

SURVEY RESPONSE DATA: PATTERNS AND PROBLEMS[2]

[2] Barfield, A.S. and J.S. Shonkwiler. To be submitted to *American Journal of Agricultural Economics*

Abstract

Revealed preference methods require survey data on past resource use, and numerous studies have found reported recreation frequency to be overestimated and concentrated on prototype (rounded and calendar-based) values. This paper develops two approaches to treat extreme values and rounded responses in survey datasets and thereby improve model fit and resulting welfare estimates. We illustrate how, when modeling single-site trip data using a negative binomial (NB) distribution, employing the incomplete beta function simplifies the incorporation of censored intervals. We show the NB's fit is improved by either reassigning rounded responses to censored regimes where reported trip numbers define the intervals' upper bounds, or by mixing the NB with a continuous distribution at a cut-point where response behavior begins to exhibit rounding. We feel these methods will be useful for recreation demand research and may have broad applicability to the general analysis of count data.

Introduction

The estimation of economic values for environmental amenities is critical to making informed resource management decisions and accurately assessing environmental damages. Revealed preference methods for measuring economic values require data on observed behavior. Typically such behavior is elicited by surveys of target populations regarding their past use of a natural resource. Since recall periods are generally over a past season or year, the issue of recall accuracy needs to be considered. Systematic biases in reporting past behavior may compromise the methods used to derive values from revealed preference data.

Numerous studies have found that reported recreation frequency has been overestimated. For instance, Connelly and Brown (1995) find that reported angling trips on Lake Ontario are over-estimated by roughly 44% as compared with diary data, with recall bias increasing with user avidity. Hoehn et al. (1996) similarly find recall bias to be associated with respondents' over-statement of Michigan angling trips. Explanations for this bias are concerned mainly with the saliency of the resource, the respondent's strategic behavior (real or imagined), and the respondent's self-delusion (or effort to impress the surveyor) if the activity can be considered glamourous or healthy. Another dimension of recall bias is respondents' tendency to round off responses to end in a zero or five. Tourangeau et al. (2000) report that open-ended questions which require a numerical response may manifest these characteristics: i) the larger the number to be reported the more likely it will be a round value; and ii) the distances between successive rounded values increase as the numbers increase.[3]

[3] Tourangeau et al. (2000) further clarify that if respondents round fairly (i.e., if they always round their responses to the nearest round value), due to the uneven spacing of round values, the net effect of rounding will actually be a downward bias in the data. However, if respondents are not rounding fairly but are characteristically rounding up or

To illustrate the patterns observed in recreation survey data, consider the following table of reported trips in three different recreation demand studies:

Table 2.1. Trips reported in recreation demand studies.

Trips	Ozuna and Gomez (1995)	Moeltner (2006)	Parsons et al. (1999)		Trips	Ozuna and Gomez (1995)	Moeltner (2006)	Parsons et al. (1999)
0	417	469	287		**16**	1	0	0
1	68	24	60		**17**	0	0	2
2	38	22	38		**20**	3	1	11
3	34	9	36		**25**	3	2	4
4	17	7	25		**26**	1	0	0
5	13	10	24		**28**	0	0	1
6	11	5	17		**30**	3	0	2
7	2	4	4		**35**	0	0	1
8	8	2	5		**40**	3	0	3
9	1	1	0		**50**	1	0	2
10	13	4	34		**88**	1	0	0
11	2	1	0		**100**	0	1	1[a]
12	5	0	0		**N**	**659**	**563**	**565**
13	0	1	0					[a]Also 150,200, 250
14	0	0	1					
15	14	0	4					

The Ozuna and Gomez (1995) study collects data from a random sample of registered boat owners about boating trips to a popular lake in Texas. Moeltner (2006) collects data from a random sample of fishing license holders regarding fishing trips to the trophy section of a local Nevada river. Parsons et al. (1999) collect data from a random sample of Delaware residents regarding their visits to a popular beach.

down, systematic error is introduced into the model in the direction of the rounding. The evidence in the recreational survey response literature generally finds that respondents do not round fairly – they overstate their participation.

It is noteworthy how many properties these datasets share. First, there is a large proportion of zeros (indicating a lack of involvement in the recreational activity being studied) which suggests what Sarker and Surry (2004) refer to as a "fast decay" process. Second, there is a disproportionate number of rounded responses and some evidence of rounding to the half-dozen and dozen. Third, there are some very large reported values which are almost all rounded to the nearest 10.

This paper develops two different approaches to treat both the presence of extreme values and rounded responses that we feel will be of interest to recreation demand modelers and that may have broad applicability to the analysis of other types of count data.

Theoretical Background

A common problem in recreation survey response data is a preponderance of zeros due to non-participation. This excess-zero problem may be addressed by considering a negative binomial (NB) estimator for the recreation demand model or by employing some of the alternative count data estimators suggested by Sarker and Surry (2004).[4] The remaining question is how to treat the rounded responses. Schaeffer and Presser (2003) have claimed that "estimation strategies lead to heaping at common numbers, such as multiples of 5 or 10…these strategies can be considered techniques for 'satisficing'…conserving time and energy and yet producing an answer that seems good enough for the purposes at hand." Similarly, Tourangeau et al. (2000) state that "by reporting their answers as round values, respondents may be consciously attempting to communicate the fact that their answers are at best approximations."

[4] If the zeros are generated by a different process than the non-zero responses (i.e., if some inherent, behavioral difference between users and non-users of a resource is readily identifiable), then hurdle count data models may need to be estimated (Haab and McConnell 2002).

> "There are no established conventions for rounding survey responses. Hence, researchers cannot be sure how much rounding there may be in survey data. Nor can researchers be sure whether respondents round to simplify communication or to convey partial knowledge," (Manski and Molinari 2010).

Vaske and Beaman (2006) provide a summary of their research on the topic of survey response, describing how respondents may answer recall questions about frequencies (such as days of participation) using episode enumeration, formula-based multipliers, and prototypes, which cause their responses to deviate from what occurred in reality. With low participation and episode enumeration (the recall and counting of specific occurrences), episode omission or telescoping may cause response errors. With greater participation and formula-based multipliers (the recall of a frequency rule applied to a time frame), misestimation of the rule or failure to recall exceptions to it may result in response error. With the use of prototypes (a single number used to represent a range of values), response clusters can occur, commonly around 0's and 5's.

All of these approximations can manifest in the data as response "heaping" – where reported numbers occur more often than chance would suggest. As Vaske and Beaman (2006) explain, heaping is likely related to number (or digit) preference: "numbers that a person has a disposition to use or avoid." Indeed, Huttenlocher et al. (1990) find that respondents tend to overuse both multiples of 5 and 10 and numbers associated with calendar events such as weeks or months (7, 14, 21, 30 and 60, for example).[5] These patterns (heaping, rounding and digit

[5] Huttenlocher et al. (1990) also describe the occurrence of forward bias, which results both from response leaping and from response "bounding": the imposition of an upper boundary (self-imposed or otherwise) on reports. This phenomenon (and more generally, response contraction bias) is further explored in Tourangeau et al. (2000).

preference) have all been observed and studied in the demographic, epidemiological and historical literatures (Pudney 2008).

Another manifestation of recall error is response "leaping" – where response heaping increases with reported magnitudes. "For responses under 15, several studies have found limited 0-5 heaping…Above 100, responses may fall largely on 150, 200, 300, and 500 with gaps widening as responses move into the thousands," (Vaske and Beaman 2006).

A number of papers discuss front-end reduction (prevention) of response bias and suggest strategies to improve survey design and implementation (Pudney 2008; Schaeffer and Presser 2003; Miller and Anderson 2002; Tarrant and Manfredo 1993; Chu et al. 1992). With regard to back-end reduction (correction) of response bias, Evans and Herriges (2010) provide a recent example. They employ a latent class model which assumes respondents are members of either a rounding or non-rounding class. Using generated data experiments, they find that rounding bias can have significant impacts on parameter estimates and resulting welfare measurements, with consumer surplus loss due to site closure being overstated by 5-37 percent.

Methodology

The Censored Regime Method:

Vaske and Beaman (2006) also propose some methods for reducing the bias that heaping may generate. Their approach attempts to smooth out the heaps by distributing the values over an interval whose shape is related to the underlying distribution of the un-heaped data. Since the recreation demand models most frequently used for the analysis of single-site visitation data entertain discrete distributions, this smoothing can be accomplished by assigning the heaped observations to intervals. Thus, we can view the resulting estimator as a count data model with censored regimes – outcomes are assigned to occur in a particular region or segment of the distribution. This is consistent with Manski and Molinari's (2010) interpretation of rounded reported numerical values as interval data.

Our statistical approach employs the NB distribution (see Cameron and Trivedi 2013), which is capable of handling large numbers of zeros and extreme values.[6] Its probability mass function is:

(1)
$$\frac{\Gamma(y+\alpha^{-1})\left(\frac{\mu}{\mu+\alpha^{-1}}\right)^{y}\left(\frac{\alpha^{-1}}{\mu+\alpha^{-1}}\right)^{\alpha^{-1}}}{\Gamma(y+1)\Gamma(\alpha^{-1})}$$

where Γ is the gamma function, y is the number of trips to a site, $\mu = E(y)$, and α is a scale parameter capturing overdispersion. Note that if $\alpha = 0$, the NB distribution collapses to a Poisson distribution.

To define the intervals to which the heaped data will be assigned, we impose a structure that is informed by the findings of previous studies. If we assume that rounded data signal approximations and if we subscribe to the notion that respondents tend to exaggerate their participation, then it follows that the intervals will include values no greater than the heaped value. Further, the larger the heaped response, the larger the interval to which it should be assigned. Essentially, this is an empirical issue. Specification tests such as Pearson's chi-square statistic or the deviance statistic can help guide model specification.

Implementation of a count data estimator with numerous censored regimes does pose the complication that sums of probabilities comprise each of the intervals. In the case of the NB distribution, however, the incomplete beta function can be used to compute cumulative probabilities by representing the cumulative distribution function (cdf) of the NB probability

[6] For additional discussion of the NB distribution’s strengths in this context, please see Sarker and Surry (2004).

mass function (pmf).[7] This greatly simplifies the censored estimation. The incomplete beta function is:

(2) $$I_x(z,w) = \frac{\Gamma(z+w)}{\Gamma(z)\Gamma(w)} \int_0^x t^{z-1}(1-t)^{w-1}dt$$

where $x = \left(\frac{\alpha^{-1}}{\mu+\alpha^{-1}}\right)$, $z = \alpha^{-1}$, w is the upper-bound on the regime (i.e., the heaped response we are reassigning to the regime), and t is the argument of integration.

In our application, to determine the probability that a response, *y*, falls within a regime with lower-bound, *k*, and upper-bound, *w*, the incomplete beta function will calculate Pr [$k \leq y \leq w$] = Pr[y<(w+1)]-Pr[y<(k+1)]).

We use the generalized Pearson X^2 statistic (McCullagh and Nelder 1989) to evaluate our model fit. This statistic is chi-square distributed with degrees of freedom equal to the number of observations (i.e. respondents) minus the number of parameters estimated:

(3) $$X^2 = \sum_{i=1}^{n} \{y_i - E(y_i;\hat{\theta})\}^2 / V(y_i;\hat{\theta})$$

where *n* is the number of respondents, y_i is the number of trips reported by person *i*, $\hat{\theta}$ is a vector of estimated parameters, and *V* is variance. This form of the Pearson statistic is preferred to the form based on observed and expected frequencies as it does not require the assignment of data to groups ("bins"). The null hypothesis of this statistic is that the model fits the dataset well; specifically, that the model's predicted values accurately reproduce the dataset's first two moments (the mean and variance). Thus, a low p-value for this statistic indicates that the model fits badly – there is a low probability of error in rejecting this null hypothesis – and vice versa.

The Distribution Transition Method:

[7] "The sum of a number of negative binomial terms can be expressed in terms of an incomplete beta function ratio, and hence as a sum of binomial terms," (Johnson et al. 1992, pg. 209).

Both the Evans and Herriges (2010) latent class model and our Censored Regime Method have their merits and shortcomings. The Evans and Herriges (2010) approach requires the identification of two regimes: rounders and non-rounders. As the mean number of rounders' trips will likely greatly exceed the mean number of non-rounders' trips, a stochastic specification to identify class membership must be employed. It is unclear how to precisely formulate this, and there is no obvious reason why rounders and non-rounders should have different conditional means. While our Censored Regime Method allows for the same conditional mean formulation for all respondents and may improve model fit, it suffers from the fact that increasing the size of the intervals will necessarily increase the value of the log likelihood. Since there is no statistical penalty from this approach, the selection and size of the censored regimes can only be based on a reasonableness criterion.[8] The researcher must consequently justify the sizes and positions of multiple intervals.

An alternative approach is to assume that at some cut-point, the distribution of responses changes from a discrete to a continuous distribution. The selection of this single cut-point, where a transition from non-rounding to rounding behavior can be assumed, is again based on a reasonableness criterion. For a discrete distribution, a given integer outcome has a unique probability associated with it, and though the term "count data regression" has become commonplace, it is somewhat misleading. In truth, the count data model is based on a probability mass function with a conditional mean – it is not, in fact, a regression. There is no underlying

[8] Increasing the width of the intervals allows each regime to encompass larger sums of probabilities. Attempting to parameterize the bounds on these regimes will therefore result in the model selecting bounds at the minimum and maximum responses. To avoid this collapsing of regimes, the researcher must determine and impose what they deem to be appropriate interval bounds for their particular dataset.

distribution of outcomes associated with a conditioning variable. By contrast, a regression model with a continuous response variable has a distribution of responses associated with a conditioning variable because the probability of a given response is very small.

For smaller responses where rounding is less likely to have occurred and where an outcome of zero is meaningfully different from outcomes such as one or two, the (less flexible) discrete distribution can reasonably be applied to calculate an exact probability for each response. For larger responses where rounding is more likely to have occurred and there is less certainty that outcomes are exact, the (more flexible) continuous distribution calculates probabilities around each response.[9] Under this formulation, the area of responses defined by the discrete distribution is right truncated, and the area of responses defined by a continuous distribution is left truncated. The form that the likelihood function takes in each of these partitions is therefore determined by the specific mixture of distributions chosen, and will be illustrated with regard to our specific application.

Application

Using data from Parsons et al. (1999),[10] we consider day-trip visits to a single site (Cape Henlopen State Park) in our estimation. The numbers of trips reported by the respondents in this survey are shown below in table 2.2. Again, we see a large number of zeros, possible heaping at rounded numbers and the half-dozen and dozen marks, some extreme values, and increasing

[9] If a respondent has engaged in rounding behavior, the researcher may observe that the respondent took, for example, 50 trips to a recreational resource. In fact, there is some distribution of trips around this 50 trip response that better represents the respondent's true pattern of visitation.

[10] The Parsons et al. (1999) survey was conducted in October, 1997, asking respondents about recreational trips to 62 Mid-Atlantic beaches during 1997 to date.

distances between the larger numbers of reported trips ("leaping"). There is also evidence of overdispersion, with an observed mean of 2 and a variance of over 48.

Table 2.2. Reported trips to Cape Henlopen State Park, 1997.

Trips Reported	Respondents	Proportion		Trips Reported	Respondents	Proportion
0	378	0.6690		**15**	4	0.0071
1	54	0.0956		**18**	1	0.0018
2	24	0.0425		**20**	4	0.0071
3	26	0.0460		**25**	3	0.0053
4	11	0.0195		**30**	1	0.0018
5	21	0.0372		**35**	1	0.0018
6	10	0.0177		**40**	1	0.0018
8	6	0.0106		**50**	1	0.0018
10	11	0.0195		**72**	1	0.0018
12	5	0.0088		**100**	1	0.0018
14	1	0.0018		**Descriptive Statistics**	**n=565**	**s^2=48.73 mean=2.09**

In the application of our Censored Regime Method, we fit a number of variations of the NB distribution and assess our models' success by employing the Pearson statistic, which follows equation (3) where *n*=565 and the variance is defined by the specific form of the NB distribution being estimated. In the application of our Distribution Transition Method, we consider a mixture of the generalized NB distribution and the lognormal distribution, again utilizing the Pearson statistic to assess model performance.

Results

Censored Regime Method:

We first fit a standard NB distribution to the data, with variance defined as:

(4) $$V(y_i) = \hat{\mu}_i(1 + \alpha\hat{\mu}_i) = \hat{\mu}_i + \alpha\hat{\mu}_i^{\,2}$$

where $\hat{\mu}_i = E(y_i; \hat{\theta})$, $\hat{\theta} = \{\hat{\beta}, \hat{\alpha}\}$, and β is a vector of explanatory variables (travel cost and socio-demographic information: the natural log of age and dummy variables for having a child under 10 years old, being retired, or being a student).

Our results from this specification are summarized in table 2.3 below.

Table 2.3. Estimates: Standard negative binomial distribution.

Variable/ Parameter	**Coefficient**	**Robust Std. Error**	**z-Value**
Constant	0.2411	1.6121	0.1496
Trip Cost	-0.0333	0.0041	-8.0644
ln(Age)	0.4313	0.4249	1.0150
Child <10	0.5370	0.2546	2.1091
Retired	-0.6952	0.3369	-2.0636
Student	1.0942	0.3230	3.3879
Dispersion: α	4.3993	0.4943	8.9003
Pearson Statistic: 729.0, p=0.000	**Log Likelihood = -797.76**		

Our Pearson statistic indicates a poor model fit, which leads us to our subsequent specification – the generalized NB distribution. This model is a more flexible form of the standard NB distribution and estimates an additional variable, Φ, to be included in $\hat{\theta}$. Its variance is defined as:

(5) $$V(y_i) = \hat{\mu}_i + \alpha\hat{\mu}_i^{\,2-\Phi}$$

Our results from this specification are summarized in table 2.4 below.

Table 2.4. Estimates: Generalized negative binomial distribution.

Variable/Parameter	Coefficient	Robust Std. Error	z-Value
Constant	1.4250	1.4686	0.9703
Trip Cost	-0.0354	0.0045	-7.9368
ln(Age)	0.1283	0.3926	0.3269
Child <10	0.7206	0.2090	3.4486
Retired	-0.5810	0.3190	-1.8216
Student	0.9522	0.3252	2.9278
Dispersion: α	6.4913	0.8720	7.4443
Φ	0.4765	0.1033	4.6141
Pearson Statistic: 621.5, p=0.030	**Log Likelihood = -785.11**		

Our Pearson statistic improves slightly, indicating a somewhat better fit. We therefore use this model in our subsequent specifications which incorporate censored regimes and reassignment of the heaped observations. In our first censored, generalized NB model, we impose the following, dual-regime structure: the pmf is fit to observations with fewer than 50 reported trips; the single observation of 50 trips is assigned to a regime of 36-50[11] trips; and the two remaining observations are assigned to a regime of greater than 50 trips. Our results from this specification are summarized in table 2.5 below.

[11] While the upper-bound of 50 trips is predetermined by our assumption that the intervals contain no more than the reported number of trips, the lower-bound is somewhat arbitrary. In this case, 36 was selected given that the majority of the observations occur at 35 or fewer reported trips.

Table 2.5. Estimates: Dual-censored generalized negative binomial distribution.

Variable/ Parameter	Coefficient	Robust Std. Error	z-Value		
Constant	1.5521	1.4316	1.0842		**Censored Regimes**
Trip Cost	-0.0345	0.0044	-7.8536		>50
ln(Age)	0.0795	0.3812	0.2087		50 » 36 to 50
Child <10	0.6967	0.2010	3.4660		
Retired	-0.5626	0.3101	-1.8145		
Student	0.9313	0.3212	2.8995		
Dispersion: α	6.4054	0.8516	7.5216		
Φ	0.5105	0.1048	4.8735		
Pearson Statistic: 433.8, p=0.999	**Log Likelihood = −774.73**				

Our Pearson statistic[12] indicates a significant improvement in the fit of the model, which supports the incorporation of censored regimes and reassignment of heaped observations in our estimation procedures. In our second censored, generalized NB model, we impose the following, multiple-regime structure: the pmf is fit to observations with fewer than 20 reported trips; the four observations of 20 trips are assigned to a regime of 16-20 trips; the three observations of 25 trips are assigned to a regime of 21-25; the observation of 30 trips to a regime of 26-30; the observation of 35 trips to a regime of 31-35; the observation of 40 trips to a regime of 31-40; the

[12] Computing the Pearson statistic for a censored observation requires calculation of said observation's expected value and variance, given that it falls in an interval: $E(y_i|a<y_i<b)$ and $V(y_i|a<y_i<b)$.

observation of 50 trips to a regime of 36-50[13]; and the two remaining observations to a regime of greater than 50 trips. Our results from this specification are summarized in table 2.6 below.

Table 2.6. Estimates: Multi-censored generalized negative binomial distribution.

Variable/ Parameter	Coefficient	Robust Std. Error	z-Value		
Constant	1.5154	1.4168	1.0696		**Censored Regimes**
Trip Cost	-0.0341	0.0044	-7.7450		>50
ln(Age)	0.0803	0.3777	0.2126		50 » 36 to 50
Child <10	0.6942	0.1989	3.4901		40 » 31 to 40
Retired	-0.5535	0.3067	-1.8045		35 » 31 to 35
Student	0.9171	0.3180	2.8843		30 » 26 to 30
Dispersion: α	6.3425	0.8437	7.5177		25 » 21 to 25
Φ	0.5267	0.1083	4.8641		20 » 16 to 20
Log Likelihood = −755.69					

Our log likelihood[14] indicates that the fit of our model has improved yet again by incorporating the additional regimes. This provides further evidence that respondents are over-reporting their visitation and rounding up to multiples of 5 and 10. In the context of over-reporting and extreme values, a common practice in the survey response literature is simply to

[13] The increasing range of the intervals to which observations are reassigned accounts for the increasing distance between rounded values as numbers of reported trips themselves increase.

[14] Due to the number of regimes and reassigned observations in this model, the Pearson statistic becomes computationally-difficult to calculate. The two-regime model has been shown to reproduce the first two moments of the dataset; by incorporating additional censored regimes, we are increasing the model's flexibility, and would not expect the Pearson statistic to suffer as a result. The improved log likelihood value lends support to this assumption.

exclude larger values from the dataset on the basis that they are likely to be unrepresentative of the general (or target) population. To this end, we consider how our results would be affected if we truncated the data at 50 (thereby losing three, extreme-value observations) and estimate a generalized NB model. Our results from this specification are summarized in table 2.7 below.

Table 2.7. Estimates: Truncated, generalized negative binomial distribution.

Variable/ Parameter	Coefficient	Robust Std. Error	z-Value
Constant	2.0370	1.3548	1.5036
Trip Cost	-0.0309	0.0041	-7.5161
ln(Age)	-0.1108	0.3549	-0.3123
Child <10	0.6409	0.1897	3.3783
Retired	-0.5312	0.2958	-1.7961
Student	0.8711	0.3123	2.7891
Dispersion: α	5.9586	0.7362	8.0940
Φ	0.5994	0.1131	5.2982
Pearson Statistic: 488.2, p=0.979	**Log Likelihood = -757.72**		

We have lost information in estimating this model (by eliminating data points), and as a result, the fit is not quite as good as when we incorporate this information under uncertainty. Estimated per-trip consumer surplus[15] moves from $29.33 (standard error of 3.78) in the multi-censored distribution to $32.36 (standard error of 4.29) in the truncated distribution, and we observe changes in all of the parameter estimates. While these changes are not statistically different, we have only lost three observations in this particular example. In datasets with large

[15] Calculated as the inverse of the estimated coefficient on trip cost, the marginal utility of income.

numbers of extreme values to either truncate or model under uncertainty, the disparities in these parameter estimates could become significant with regard to policy implications.

Distribution Transition Method:

Following the results of our Censored Regime Method application, we propose the use of the generalized NB for the discrete distribution. For the continuous distribution, we propose the use of the lognormal distribution. The lognormal distribution has the advantage of a long right tail, and the NB and lognormal distributions follow the same conditional mean process. This makes the transition from the discrete to the continuous distribution smoother than in the case of latent class models where there are different conditional means depending on class membership.

For observations at or below the cut-point, *c*, the likelihood is:

(6) $$\frac{\Gamma(y+a)\left(\frac{\mu}{\mu+a}\right)^{y}\left(\frac{a}{\mu+a}\right)^{a}}{\Gamma(y+1)\Gamma(a)\Pr(y\leq c)}\Pr(y\leq c)$$

where $a = \frac{1}{\alpha}\mu^{\Phi}$, μ is the conditional mean, α is the dispersion parameter, and *y* is the number of trips to a site up to the cut-point, c (i.e., *y* = 0,1,…,*c*). This is the result of multiplying the right truncated generalized NB distribution by the probability of being in that regime—hence, the two probabilities will cancel out.

For observations above the cut-point, the likelihood is:

(7) $$\frac{\exp(-.5(\frac{(\log(y)-\log(\mu))}{\sigma})^2)(1-\Omega(x,a,c+1))}{(\sigma\sqrt{2\pi})(\Phi\left(\frac{\log(\mu)-\log(c+1)}{\sigma}\right))}$$

where *y* is the number of trips to a site beyond the cut-point (i.e., *y* = *c*+1,…), $\Omega(x,a,c+1)$ is the sum of probabilities of the generalized NB distribution from 0 to c, and $\Phi(z)$ is the standard normal cumulative distribution function. In this case, $\Phi(z)$ is the probability that the lognormal distribution is above *c* to account for the left truncation. The term $(1-\Omega(x,a,c+1))$ accounts for the probability of being above the cut-point and can be computed using the incomplete beta function (note that x is now defined as $(a/(a+\mu))$.

To calculate the Pearson statistic, we must obtain the conditional means and variances of this mixed distribution model.

For the right truncated generalized NB distribution, we refer to the recent work by Shonkwiler (2016), which corrects the second moments as reported by Gurmu and Trivedi (1992) and Cameron and Trivedi (2013). The formulas are as follows for the conditional mean:

(8) $$E[Y|Y \leq c] = \mu - \frac{(c+1)pmf(c+1)}{x\,\Pr(Y \leq c)} = \mu^0$$

and conditional variance:

(9) $$V(Y|Y \leq c) = \mu + \mu^2/\mathrm{a} + (c+1)(\mu^0 - \mu) - (\mu^0 - \mu)^2 - (\mathrm{a} - 1)(\mu^0 - \mu)\mu/\mathrm{a}$$

where $pmf(c+1)$ represents the generalized NB probability mass function evaluated at c+1.

For the left truncated lognormal distribution[16], the conditional moments can be written as:

(10) $$E(y|y > c) = \exp(\log(\mu) + .5\sigma^2)\ \frac{\Phi(\sigma + (\log(\mu) - \log(c))/\sigma)}{\Phi((\log(\mu) - \log(c))/\sigma)}$$

(11) $$E(y^2|y > c) = \exp(2\log(\mu) + 2\sigma^2)\ \frac{\Phi(2\sigma + (\log(\mu) - \log(c))/\sigma)}{\Phi((\log(\mu) - \log(c))/\sigma)}$$

This permits straightforward computation of the Pearson statistic.

In our application of the Distribution Transition Method, our model was fit to the data with a cut-point set at 19, as we believe responses of 20 reported trips or more could exhibit rounding behavior. The results we obtained (table 2.8 below) are remarkably similar to those

[16] These formulas are based on the work of Bebu and Mathew (2009) (note what they report as the variance is actually $E(y^2)$).

found in tables 2.5 and 2.6, and display similar improvements as compared with the simple truncated distribution summarized in table 2.7.

Table 2.8. Estimates: Mixed generalized negative binomial-lognormal distribution.

Variable/Parameter	Coefficient	Robust Std. Error	z-Value
Constant	1.5680	1.4055	1.1156
Trip Cost	-0.0340	0.0044	-7.7660
ln(Age)	0.0653	0.3728	0.1752
Child <10	0.6897	0.1980	3.4833
Retired	-0.5750	0.3091	-1.8602
Student	0.9255	0.3198	2.8940
Dispersion: α	6.2178	0.7927	7.8438
Φ	0.5168	0.1077	4.7985
σ	1.0608	0.1884	5.6306
Pearson Statistic: 477.5, p=0.993	**Log Likelihood = -741.04**		

Discussion

The survey response literature has established that respondents tend to over-report their recreational activities, and correcting for “heaps and leaps” in survey response data is largely an empirical issue. This paper illustrates how, when modeling single-site recreational trip data using a negative binomial distribution, employing the incomplete beta function to represent the cdf of the NB distribution simplifies the incorporation of censored intervals. We further provide evidence that the NB model’s fit is significantly improved by either (1) reassigning heaped responses to censored regimes where reported trip numbers determine the intervals’ upper-bounds, or (2) mixing the NB distribution with a continuous distribution at a cut-point where it is supposed that response behavior begins to exhibit rounding.

We find two socio-demographic variables to be significant: Child<10 and Student. We hypothesize this is because Cape Henlopen State Park is a popular vacation destination for those who are, or have children who are, out of school during the summer. Our analysis did not find a statistically significant difference in the parameter or per-trip consumer surplus estimates when extreme values were either truncated or incorporated under uncertainty. However, only three observations were truncated in our particular application, which may not have provided a significant enough loss of information to impact the overall estimation. As we expand this research, we may examine other sites in the Parsons et al. (1999) dataset (or different datasets entirely) where the impacts of truncation may be more extensive.

References

Bebu, I. and T. Mathew. 2009. "Confidence intervals for limited moments and truncated moments in normal and lognormal models." *Statistics & Probability Letters* 79(3): 375-380

Cameron, A. C., and P. K. Trivedi. 2013. *Regression analysis of count data, 2nd ed.* Cambridge, NY: Cambridge University Press

Chu, A., D. Eisenhower, M. Hay, D. Morganstein, J. Neter and J. Waksberg. 1992. "Measuring the recall error in self-reported fishing and hunting activities." *Journal of Official Statistics* 8(1): 19-39

Connelly, N. A., and T. L. Brown. 1995. "Use of angler diaries to examine biases associated with 12-month recall on mail questionnaires." *Transactions of the American Fisheries Society* 124(3): 413-422

Evans, K., and J. A. Herriges. 2010. "Rounding in recreation demand models: A latent class count model." Iowa State University. Working paper

Gurmu, S. and P. K. Trivedi. 1992. "Overdispersion tests for truncated Poisson regression models." *Journal of Econometrics* 54: 347-370

Haab, T. C., and K. E. McConnell. 2002. *Valuing Environmental and Natural Resources: The Econometrics of Non-Market Valuation*. Cheltenham: Edward Elgar

Hoehn, J. P., T. Tomasi, F. Lupi and H. Z. Chen. 1996. "An economic model for valuing recreational angling resources in Michigan." Michigan State University. Report to the Michigan Department of Environmental Quality and to the Michigan Department of Natural Resources

Huttenlocher, J., L. V. Hedges, and N. M. Bradburn. 1990. "Reports of elapsed time: Bounding and rounding processes in estimation." *Journal of Experimental Psychology: Learning, Memory, and Cognition* 16(2): 196-213

Johnson, N. L., S. Kotz and A. W. Kemp. 1992. *Univariate Discrete Distributions*. 2nd ed. New York, NY: Wiley

Manski, C. F., and F. Molinari. 2010. "Rounding probabilistic expectations in surveys." *Journal of Business & Economic Statistics* 28(2): 219-231

McCullagh, P. and J.A. Nelder. 1989. *Generalized Linear Models*. 2nd ed. New York, NY: Chapman & Hall/CRC

Miller, C. A., and W. L. Anderson. 2002. "Digit preference in reported harvest among Illinois waterfowl hunters." *Human Dimensions of Wildlife* 7(1): 55-65

Moeltner, K. 2006. "Anglers' Fishing Choices on the Truckee, Carson, and Walker Rivers and the Threat of the New Zealand Mud Snail." Report prepared for the Pacific States Marine Fisheries Commission and the U.S. Fish and Wildlife Service, June 27, 2006

Ozuna, T. Jr., and I. A. Gomez. 1995. "Specification and testing of count data recreation demand functions." *Empirical Economics* 20(3): 543-550

Parsons, G. R., D. M. Massey, and T. Tomasi. 1999. "Familiar and favorite sites in a random utility model of beach recreation." *Marine Resource Economics* 14(4): 299-315

Pudney, S. 2008. "Heaping and leaping: survey response behaviour and the dynamics of self-reported consumption expenditure." University of Essex. Institute for Social and Economic Research. Working paper

Sarker, R., and Y. Surry. 2004. “The fast decay process in outdoor recreational activities and the use of alternative count data models.” *American Journal of Agricultural Economics* 86(3): 701-715

Schaeffer, N. C., and S. Presser. 2003. “The science of asking questions.” *Annual Review of Sociology* 29: 65-88

Shonkwiler, J. S. 2016. “Variance of the truncated negative binomial distribution.” *Journal of Econometrics* forthcoming

Tarrant, M. A., and M. J. Manfredo. 1993. “Digit preference, recall bias, and nonresponse bias in self reports of angling participation.” *Leisure Sciences* 15(3): 231-238

Tourangeau, R., L. J. Rips, and K. A. Rasinski. 2000. *The Psychology of Survey Response.* Cambridge: Cambridge University Press

Vaske, J. J., and J. Beaman. 2006. “Lessons learned in detecting and correcting response heaping: Conceptual, methodological, and empirical observations.” *Human Dimensions of Wildlife* 11(4): 285-296

CHAPTER 3

MULTINOMIAL PROBIT AND MIXED LOGIT MODELING OF RECREATION DEMAND: A COMPARATIVE ANALYSIS[17]

[17] Barfield, A.S., G. Colson, and J.S. Shonkwiler. To be submitted to *Environmental and Resource Economics*

Abstract

Much of the travel cost literature uses mixed logit (MXL) models to evaluate recreational site choice data. Multinomial probit (MNP) models are less commonly used, as their relatively cumbersome simulation procedures have made them more difficult to work with historically. This paper compares model performance and explores implications for welfare analysis in the case of multi-site trip data. We calculate estimates of average expected maximum utility (pre and post policy implementation), as well as willingness to pay estimates for site quality improvements and the distributions of these estimates. Our results display parallel patterns of inference across both models. We also utilize a new, more efficient approach to estimate the distribution of the mean benefit from policy implementation in MNP models (the Delta Method Approximation), and illustrate this approach's advantages over traditional simulation procedures. Given our findings, we discuss the merit of increasing MNP models' prevalence in the non-market valuation literature.

Introduction

The travel cost method is well-established within the field of economic valuation. Its use in the non-market valuation of natural resources and amenities is particularly prevalent. To date, much of the travel cost literature has focused on the use of multinomial logit (MNL) and mixed logit (MXL) (random parameters logit) models to evaluate recreational site choice data. Multinomial probit (MNP) models are much less common, as they have been more difficult to work with historically. Compared to standard MNL and nested logit models, MNP and MXL models are more flexible, and their respective simulation methods are capable of handling a wider variety of datasets. This study compares the MNP model to its popular and natural competitor (the MXL model) and employs an innovative, analytical approach for calculating expected maximum utility which is less computationally demanding than standard simulation methods.

By comparing MNP and MXL models, we explore the extent to which logit specifications capture correlations in utilities for different alternatives, which may be significant in terms of welfare estimates. These correlations are necessarily accounted for in MNP models. If logit models capture the majority of these correlations, their results should be quite similar to those of MNP models. If our results indicate that the two models provide significantly different pictures of these correlations, we may confront the claim that MXL models can approximate any random utility model (including a probit).[18] By comparing our approach for expected utility estimation with typical simulation methods, we provide an alternative procedure for welfare

[18] For example, if MXL models do not actually provide an excellent approximation of MNP models, and MNP analysis provides a good fit for a researcher's data, the results of said MNP analysis would provide different information and conclusions for policy analysis than a MXL analysis would.

analysis in MNP models that may provide theoretical verification of these simulation procedures while demonstrating a computational advantage. In sum, this study offers insights as to the merit of increasing MNP models' prevalence in the non-market valuation literature.[19]

Theoretical Background

While MNL models are relatively simple to work with and can accurately model site selection decisions (substitution effects), they cannot model decisions about total trips taken over the course of a recreational season (participation effects). For this reason, while they can provide welfare estimates on a per-trip basis, MNL models alone cannot provide estimates of seasonal welfare impacts resultant from, for example, changes in site quality or quantity. To achieve this, it is necessary to link the site choice model to a participation decision model. This has typically been accomplished through some sort of nested logit model.

Recent advances in computational power have made the MNP and MXL models applicable to studies with a large number of alternatives to evaluate. Both approaches have advantages over the traditional MNL and nested logit models. Logit models cannot represent random taste variation, they adhere to the Independence of Irrelevant Alternatives (IIA) axiom (which results in restricted, unrealistic substitution patterns among similar alternatives), and they cannot accommodate correlation in unobserved factors over time. While generalized extreme value models (the family of models to which nested logits belong) relax the IIA constraints, they remain plagued by the problems of random taste variation and serial correlation (Train 2009). The MNP and MXL models are equipped to deal with these challenges, however.

[19] This discussion is particularly relevant given that recent increases in computational power makes MNP analysis feasible for around a dozen alternatives.

The MNP model is advantageous in that it is not bound by the IIA axiom and it can incorporate both random taste variation and temporally correlated error terms. It also captures correlations in utilities between alternatives when the error term covariance matrix, Ω, is normalized.[20] However, the MNP model is challenged both by its inherent assumption that all unobserved components of utility are normally distributed[21] and by its lack of a closed form expression for expected maximum utility (Train 2009). Like the MNP model, the MXL model is advantageous in that it does not hinge on the IIA axiom and can incorporate both random taste variation and correlation in unobserved factors across time. Additionally, the MXL model is not limited by the assumption of normality made by the MNP model (Train 2009).

However, there is no implicit guarantee that a researcher's data are generated by random utility model process, and if they are not, a flexible mechanism to describe allocation choices is needed. Flexibility in MXL models can only be achieved through a random parameters specification. MNP models are (perhaps) more flexible, in that you may impose any structure on Ω (unlike in MXL) while also being able to specify random parameters (as long as they are normal).

Beyond the initial step of estimating and signing the parameters of interest, to place our study in context with the rest of the travel cost literature, it is necessary that we also provide

[20] The researcher must normalize the error term covariance matrix to ensure identification of the parameters affecting utility. This normalization occurs automatically in logit models but must be done manually in probit models (Train 2009).

[21] This assumption may hold in most cases, but particularly in the case of price coefficients, it may lead to estimates that are not theoretically desirable. The normal density has mass on either side of the mean of zero, implying that some members of the population would have a positive price coefficient where we would (almost always) anticipate a negative (Train 2009).

comparable welfare estimates that measure responses to changes in environmental quality. To do this, we must estimate expected maximum utilities before and after implementing the proposed policy. This is a relatively straightforward process in logit models, but probit models require more complicated methods.

Methodology

For both the MNP and MXL models, we consider utility to be composed of observed and unobserved components such that:

(1) $$U_{nj} = V_{nj} + \varepsilon_{nj}$$

where there are *n* respondents and *j* alternatives; V_{nj} is the observed portion of utility which may be expressed in terms of explanatory variables x_{nj} and coefficients β such that, in the linear case, $V_{nj} = \beta_n' x_{nj}$; and the distribution of ε_{nj} depends on the model chosen (i.i.d type I extreme value in logit, generalized extreme value in nested logit, normal in probit, etc.).

The Multinomial Probit Model:

MNP choice probabilities take the form:

(2) $$P_{ni} = \int I(V_{ni} + \varepsilon_{ni} > V_{nj} + \varepsilon_{nj} \ \forall \, j \neq i) \emptyset(\varepsilon_n) d\varepsilon_n$$

where *I(·)*is an indicator variable for the truth of the statement in parentheses; ε_n is a vector of error terms [$\varepsilon_{n1}, \ldots, \varepsilon_{nJ}$]; and $\emptyset(\varepsilon_n)$ is the normal density of ε_n:

(3) $$\emptyset(\varepsilon_n) = \frac{1}{(2\pi)^{J/2}|\Omega|^{1/2}} \, e^{-\frac{1}{2}\varepsilon'_n \Omega^{-1} \varepsilon_n}$$

where Ω is the (JXJ) covariance matrix[22] of ε_n:

(4) $$\Omega = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1J} \\ \vdots & \ddots & \vdots \\ \sigma_{J1} & \cdots & \sigma_{JJ} \end{pmatrix}$$

[22] For simplicity, we omit the subscript *n* on Ω, but each respondent is likely to have their own, unique Ω.

The integrals in equation (2) are therefore J-dimensional over all values of ε_n. But, as only differences in utility matter in this context, it is also possible to express the MNP choice probabilities as (J-1)-dimensional integrals over differences between errors:

(5) $$P_{ni} = \int I(\tilde{V}_{nji} + \tilde{\varepsilon}_{nji} < 0 \; \forall j \neq i) \emptyset(\tilde{\varepsilon}_{ni}) d\tilde{\varepsilon}_{ni}$$

where $\tilde{V}_{nji} = V_{nj} - V_{ni}$; $\tilde{\varepsilon}_{nji} = \varepsilon_{nj} - \varepsilon_{ni}$; $\tilde{\varepsilon}_{ni}$ is a vector of error term differences over all alternatives but i [$\tilde{\varepsilon}_{n1i}, \ldots, \tilde{\varepsilon}_{nji}$]; and $\emptyset(\tilde{\varepsilon}_{ni})$ is the normal density of $\tilde{\varepsilon}_{ni}$:

(6) $$\emptyset(\tilde{\varepsilon}_{ni}) = \frac{1}{(2\pi)^{1/2(J-1)} |\tilde{\Omega}_i|^{1/2}} e^{-\frac{1}{2}\tilde{\varepsilon}'_{ni}\tilde{\Omega}_i^{-1}\tilde{\varepsilon}_{ni}}$$

where $\tilde{\Omega}_i$ is derived from Ω[23] (Train 2009).

Discrete choice models must be normalized so that only economically significant information is preserved in the covariance matrix of the error term – specifically, so that elements of the covariance matrix dealing with the irrelevant concepts of level and scale of utility (which do not affect behavior) are removed. This is an issue of parameter identification. In this sense, the reduction in the number of parameters is not a restriction, but rather a "correction" of sorts.[24] A critical difference between logit (and nested logit) and probit models is that this normalization occurs automatically in logit models, whereas it must be manually imposed in probit models. An unrestricted, unnormalized model will have $J(J+1)/2$ covariance matrix parameters; an unrestricted, normalized model will have $[(J-1)J/2]-1$ covariance matrix parameters (Train 2009). Train (2009) provides a "procedure[25] that can always be used to

[23] For a "straightforward" way to derive $\tilde{\Omega}_i$ from Ω, please see Train (2009) pg. 99-100.

[24] Additional restrictions on the error term covariance matrix may be imposed at the researcher's discretion, but their structure may or may not achieve the necessary normalization on their own (Train 2009).

[25] Please see Train (2009) pg. 101-102.

normalize a probit model and assure that all parameters are identified…either by itself or as a check on another procedure."

Probit probabilities require numeric simulation to evaluate. The most widely used simulation method for probit models is the GHK simulator,[26] which is employed in the context of utility differences (P_{ni} is simulated based on U_{ni} having been subtracted from all other utilities). Recall that with utility differences against alternative i, $\widetilde{U}_{nji}$ = $\tilde{V}_{nji}$ + $\tilde{\varepsilon}_{nji}$; $\tilde{\varepsilon}_{ni}$ is a (J-1)X1 vector of error term differences over all alternatives but i: [$\tilde{\varepsilon}_{n1i}$,…,$\tilde{\varepsilon}_{nji}$]; and $\tilde{\varepsilon}_{ni}$ ~ N(0, $\widetilde{\Omega}_{\mathrm{i}}$) where $\widetilde{\Omega}_{\mathrm{i}}$ is derived from Ω (Train 2009).

Now define L_i to be the lower-triangular Cholesky decomposition matrix of $\widetilde{\Omega}_{\mathrm{i}}$ such that $L_i L_i'$=$\widetilde{\Omega}_{\mathrm{i}}$.

$$L_i = \begin{pmatrix} c_{11} & 0 & \dots & \dots & 0 \\ c_{21} & c_{22} & 0 & \dots & 0 \\ c_{31} & c_{32} & c_{33} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \tag{7}$$

And define the vector η_n'=[η_{1n},…,$\eta_{(J\text{-}1)n}$] as a vector of i.i.d. standard normal deviates (obtained by taking (J-1) draws from a random number generator for a standard normal distribution) such that η_{nj}~N(0,1) ∀ j. Using these definitions, we can see that $\tilde{\varepsilon}_{ni}$ = $L_i\eta_n$, because Cov($\tilde{\varepsilon}_{ni}$) = E[$\tilde{\varepsilon}_{ni}\tilde{\varepsilon}'_{ni}$]= E[$L_i\eta_n(L_i\eta_n)'$]= L_i E[$\eta_n\eta_n'$] L_i'= L_i I L_i'=$\widetilde{\Omega}_{\mathrm{i}}$. Therefore, we can express the model as $\widetilde{U}_{n1i}$ = $\tilde{V}_{n1i} + c_{11}\eta_1$, $\widetilde{U}_{n2i}$= $\tilde{V}_{n2i} + c_{21}\eta_1 + c_{22}\eta_2$, etc. (Train 2009). The choice probabilities now are:

[26] The GHK simulator is known to be extremely reliable, particularly given that it is unbiased for any number of replications and given that its estimates display smaller variances than any of its competitors' (Chen and Cosslett 1998).

(8) $P_{ni} = Prob\left(\tilde{U}_{nji} < 0 \,\forall\, j \neq i\right)$

$$= Prob\left(\eta_1 < \frac{-\tilde{V}_{n1i}}{c_{11}}\right) X\ Prob\left(\eta_2 < \frac{-(\tilde{V}_{n2i} + c_{21}\eta_1)}{c_{22}} \middle| \eta_1 < \frac{-\tilde{V}_{n1i}}{c_{11}}\right) X \ldots$$

With this structure in mind, the GHK simulator is calculated by:

1) First, calculating $\left(\eta_1 < \frac{-\tilde{V}_{n1i}}{c_{11}}\right) = \Phi\left(\frac{-\tilde{V}_{n1i}}{c_{11}}\right)$, where $\Phi\left(\frac{-\tilde{V}_{n1i}}{c_{11}}\right)$ is the standard normal cumulative distribution evaluated at $\left(\frac{-\tilde{V}_{n1i}}{c_{11}}\right)$.
2) Then, drawing a value of η_1, labeled ${\eta_1}^r$, from a standard normal distribution truncated at $\frac{-\tilde{V}_{n1i}}{c_{11}}$. To take a draw from a truncated normal is a two-step process:
    a. Take a draw from a standard normal distribution labeled ${\mu_1}^r$
    b. Calculate ${\eta_1}^r = \Phi^{-1}\left({\mu_1}^r\, \Phi\left(\frac{-\tilde{V}_{n1i}}{c_{11}}\right)\right)$
3) Then, calculating $Prob\left(\eta_2 < \frac{-(\tilde{V}_{n2i} + c_{21}\eta_1)}{c_{22}} \middle| \eta_1 = {\eta_1}^r\right) = \Phi\left(\frac{-(\tilde{V}_{n2i} + c_{21}{\eta_1}^r)}{c_{22}}\right)$
4) Continuing this process for all alternatives but *i*.
5) Then, calculating the simulated probability for the *r*th draw of η_1, η_2, etc., as:

   $\hat{P}_{ni}^r = \Phi\left(\frac{-\tilde{V}_{n1i}}{c_{11}}\right) X\, \Phi\left(\frac{-(\tilde{V}_{n2i} + c_{21}{\eta_1}^r)}{c_{22}}\right) X \ldots$
6) Repeating steps 1-5 *R* times.
7) Then, calculating the overall simulated probability as $\hat{P}_{ni} = \frac{1}{R}\sum_{r=1}^{R} \hat{P}_{ni}^r$ (Train 2009).

When using the GHK simulator in maximum likelihood estimation, there are a few things we must consider. Since the GHK simulator uses utility differences that are taken against the alternative we are calculating the probability for, we must take different utility differences for respondents who choose other alternatives. Also, since once respondent might choose alternative *i* (where we would use the covariance matrix $\tilde{\Omega}_i$), and another might choose alternative *j* (where we would use the covariance matrix $\widetilde{\Omega}_j$), we must ensure that all *J* possible covariance matrices are derived from the same original matrix Ω (and that they are positive definite). Naturally, the matrix Ω must also have been normalized so that parameters are identified, as previously discussed (Train 2009).[27]

The Mixed Logit Model:

MXL choice probabilities take the form:

$$P_{ni} = \int L_{ni}(\beta) f(\beta) d\beta \tag{9}$$

where $L_{ni}(\beta)$ is the logit probability evaluated for the parameters β:

$$L_{ni}(\beta) = \frac{e^{V_{ni}(\beta)}}{\sum_{j=1}^{J} e^{V_{nj}(\beta)}} \tag{10}$$

and $f(\beta)$ is a density function (a.k.a. a mixing distribution). In the linear utility case, then, the MXL probability is:

$$P_{ni} = \int \left(\frac{e^{\beta' x_{ni}}}{\sum_{j=1}^{J} e^{\beta' x_{nj}}}\right) f(\beta) d\beta \tag{11}$$

In this sense, the MXL probability "is a weighted average of the logit formula evaluated at different values of β, with the weights given by the density $f(\beta)$," (Train 2009). The density functions can be either discrete or continuous, but in practice, they have typically been specified as the latter. The normal and lognormal densities are commonly used,[28] but gamma, uniform, and other densities can also be employed. In estimating a MXL model, there are two sets of parameters to be concerned with – the β's, which evaluate the logit formula, and the parameters in θ (mean, μ, and covariance, Σ) which describe the density function. Often, the parameters of

---

[27] Train (2009) illustrates a procedure that satisfies these requirements on pg. 129-130.

[28] The lognormal distribution is most useful in the case where a coefficient is expected to have the same sign for every individual in the sample (positive for income, negative for cost/price, etc.).

interest are those describing the density function,[29] more accurately written as $f(\beta|\theta)$. For this reason, the parameters β are integrated out of the MXL probabilities: $P_{ni} = \int L_{ni}(\beta) f(\beta|\theta) d\beta$ (Train 2009).

The approach to the MXL model outlined above is known as the Random Coefficients approach and is the most direct, most commonly used method. Each individual in the sample knows their own β_n's and ε_{nj}'s for all *j* alternatives, and will select alternative *i* only when $U_{ni} > U_{nj}$ for all $j \neq i$ (we only observe the x_{nj}'s, however). Integrating $L_{ni}(\beta_n) = (\frac{e^{\beta_n' x_{ni}}}{\sum_{j=1}^{J} e^{\beta_n' x_{nj}}})$ over all possible β_n results in equation (11). This approach is most useful when patterns of taste are the primary research interest and the number of explanatory variables is small (estimating the distribution of a large number coefficients can become quite difficult and impractical) (Train 2009).

An alternative (but formally equivalent) approach to the MXL model is known as the Error Components approach, which uses dual error terms that create correlations in the utilities for different alternatives. In this specification, utility is expressed as:

(12) $$U_{nj} = \alpha' x_{nj} + \mu'_n z_{nj} + \varepsilon_{nj}$$

where x_{nj} and z_{nj} are vectors of observables on alternative *j*, α is a vector of fixed coefficients, μ_n is a vector of zero mean random terms, and ε_{nj} is once again i.i.d. extreme value. Therefore, the random portion of utility is $\eta_{nj} = \mu'_n z_{nj} + \varepsilon_{nj}$, which can be correlated across alternatives

[29] If we also want the values of the β's to be interpreted in their typical sense (as coefficients indicating individual preferences), Train (2009) offers a description of how to obtain this information using the data and estimates of θ (pg. 259-281).

depending on how we specify z_{nj}. When z_{nj} is not zero, utility will be correlated over alternatives as follows:

(13) $$Cov\ (\eta_{ni}, \eta_{nj}) = E(\mu'_n z_{ni} + \varepsilon_{ni})(\mu'_n z_{nj} + \varepsilon_{nj}) = z'_{ni} W z_{nj}$$

where W is the covariance matrix of μ_n. Therefore, even when W is diagonal (the error components are independent) utility will be correlated across alternatives. Any number of correlation patterns (and therefore, substitution patterns) can be achieved depending on which variables are selected to enter as error components. For instance, it is possible to specify a MXL model using the Error Component approach so that it is analogous to a nested logit model.[30] This approach is most useful when prediction of substitution patterns is the primary research goal and the number of explanatory variables is large (Train 2009).

Simulation methods are easily applicable to the MXL model. First, we specify a functional form for $f(\beta|\theta)$. We then 1) for each person in the sample, draw a value of β from $f(\beta|\theta)$, labeled β_{D1} for draw 1; 2) calculate $L_{ni}(\beta_{D1})$; and 3) repeat steps (1) and (2) R times and average the results yielding:

(14) $$\hat{P}_{ni} = \frac{1}{R} \sum_{r=1}^{R} L_{ni}(\beta_{DR})$$

This is an unbiased estimator for P_{ni}, the probability that person n visits alternative i.[31] We obtain n of these estimates for each of the j alternatives and calculate the simulated log likelihood (SLL):

(15) $$SLL = \sum_{n=1}^{N} \sum_{j=1}^{J} d_{nj}\ ln\hat{P}_{ni}$$

[30] For more on this particular specification of the MXL and on the Error Components approach's formal equivalence with the Random Coefficients approach, please see Train (2009) pg.139-140.

[31] $\hat{P}_{ni}$ is also strictly positive, is twice differentiable in the parameters θ and the variables x, and sums to 1 over all j alternatives (Train 2009).

where d_{nj} is an indicator variable taking the value of 1 if person n chose alternative j (and 0 if they did not). The value of θ (i.e., the mean, μ, and covariance, Σ, of the distribution of β) which maximizes the SLL is the maximum simulated likelihood estimator (MSLE) (Train 2009).

Welfare Analysis:

In the MNP context, utilities are elements of a multivariate normal random vector. If we let $X=[X_1, X_2,\ldots,X_M]$ be a normally distributed random vector (of utilities) with mean μ and covariance Σ, and we define maximum utility to be $X_{(m)}=\max_{1\leq g\leq M}\{X_g\}$, then the probability density function of $X_{(m)}$ is (from Arellano-Valle and Genton 2008[32]):

(16) $$f_{X_{(m)}}(x)=\sum_{g=1}^{M}\frac{exp\left(-(x-\mu_g)^2/2\Sigma_{gg}\right)}{\sqrt{2\pi\Sigma_{gg}}}\,\Phi_{M-1}((i_{M-1}\otimes x)-\mu_{-gg};\Sigma_{-g-gg})$$

where: Σ_{gg} is the variance for the gth alternative; μ_g is the mean for the gth alternative; i_{M-1} is a unit vector with (M-1) rows; $\Phi_{M-1}(.)$ is the (M-1)-dimensional standard normal cumulative density function; I_g is an M-dimensional identity matrix with the gth row deleted; r_g is the gth row of an M-dimensional identity matrix; $\mu_{-g} = \mu I_g'$, $\Sigma_{-gg} = r_g\Sigma I_g'$, and $\mu_{-gg} = \mu_{-g} + (x - \mu_g)\Sigma_{-gg}/\Sigma_{gg}$; $\Sigma_{-g-g} = I_g\Sigma I_g'$, and $\Sigma_{-g-gg} = \Sigma_{-g-g} - \Sigma_{-gg}'\Sigma_{-gg}/\Sigma_{gg}$.

From this equation we arrive at the general result that the expected value of the maximum, $E(X_{(m)})$ is:

(17) $$E(X_{(m)}) = \int_{-\infty}^{\infty} x f_{X_{(m)}}(x)\,dx$$

While relatively few transformations or constructions are required for this method, evaluating this integral is challenging when μ and Σ vary across observations. And unless Σ has

[32] The following notation is adapted from Corollary 4 (equation [5]), page 31.

been scaled, the range of integration can be sizeable. Afonja (1972) [33] develops an approach to approximate the evaluation of this integral using the moment generating function:

$$(18) \qquad MGF_{X_{(m)}}(t)=\sum_{g=1}^{M} exp\left(t\mu_g+\tfrac{1}{2}t^2\Sigma_{gg}\right) \Phi_{M-1}\left([(a_g - s_g)']:[\infty i_{M-1}];R_g\right)$$

where:[.]:[.] are the lower and upper limits of integration; S_g is an (M-1)xM–dimensional matrix obtained by inserting $-i_{M-1}'$ in the g[th] column of an (M-1)–dimensional identity matrix; $\Omega_g=S_g\Sigma S_g'$; a_g is an (M-1)x1 vector defined such that $a_g=S_g\mu./\sqrt{diag(\Omega_g)}$; R_g is a correlation matrix defined such that $R_g(i,j)=\Omega_g(i,j)/\sqrt{\Omega_g(i,i)\Omega_g(j,j)}$, i,j=1, 2, …, M-1; v_g is an (M-1)x1 vector of the form [1 2 3 … M]' with the g[th] element deleted; and s_g is an (M-1)x1 vector defined such that $s_g(i)=\left(\Sigma_{g,g}-\Sigma_{g,v_g(i)}\right)t/\sqrt{\Omega_g(i,i)}$, i=1, 2, …, M-1.

It follows, then, that expected maximum utility is defined as:

$$(19) \qquad \partial MGF_{X_{(m)}}(t)/\partial t|_{t=0} = E(X_{(m)})$$

While this method provides for a bounded upper limit on the integral and is faster than integrating the entire density as described in equation (17), it requires the construction of many, perhaps observation-variant, matrices and vectors.

To estimate welfare impacts associated with changes in environmental quality, Chen and Cosslett (1998) employ the GHK simulator in their application – a recreational site choice study using data on Michigan salmon anglers in the early 1980’s. They use an unbiased frequency simulator to estimate expected maximum utility and mean benefit of policy implementation. They also estimate the distribution of this mean benefit using both the Krinsky-Robb and bootstrap procedures.

[33] The following notation is adapted from Equation 3.2, page 255.

Chen and Cosslett (1998) estimate expected maximum utility of making a choice from available alternatives, $\hat{\bar{U}}_m$, as follows:

(20) $$\hat{\bar{U}}_m = \frac{1}{R}\sum_{r=1}^{R}\sum_{j=i}^{J}\left(p_j\alpha + x_j\beta + u_j{}^r\right) I\left(p_j\alpha + x_j\beta + u_j{}^r \geq p_l\alpha + x_l\beta + u_l{}^r, \forall l\right)$$

where there are J alternatives, p_j is the travel cost to site j (and α can therefore be considered the marginal utility of income), x_j is a vector of site attributes, u_j is a normally distributed error term drawn randomly for each of the R replications run, and $I(\cdot)$is an indicator variable for the truth of the statement in parentheses.

They then calculate the expected benefit of the policy measure as:

(21) $$EW(x^1|x^0) = E(B_i|x^0 \rightarrow x^1) = \frac{\bar{U}_{im}(x^1) - \bar{U}_{im}(x^0)}{-\alpha}$$

where x^0 and x^1 are the vectors of attributes pre and post policy change, respectively, and B_i is the benefit for the i^{th} observation. The numerator calculates the change in expected maximum utility caused by the policy and the denominator monetizes this impact. The estimation of expected maximum utility and the expected benefit of policy implementation in the MXL context is identical to these simulation processes (except that the error term, u_j, is instead distributed i.i.d type 1 extreme value).

The distribution of this expression, as calculated by the Krinksy-Robb procedure, involves, for both x^0 and x^1: 1) taking D draws from the asymptotic normal distributions of the parameter estimates, 2) calculating $\bar{U}_{im}(x^{\cdot})$ over R replications, and 3) calculating equation (21) and its standard error to determine a distribution. The distribution of this expression, as calculated by the bootstrap procedure, involves for both x^0 and x^1: 1) resampling the original data to create S new datasets, 2) calculating new parameter estimates for each of these S datasets, 3) calculating $\bar{U}_{im}(x^{\cdot})$ over R replications, and 4) calculating equation (21) and its standard error to determine a distribution.

Both of these approaches are data intensive, and Chen and Cosslett (1998) find they returned very similar results. Furthermore, Chen and Cosslett (1998) find that the log likelihood values and parameter estimates tended to stabilize after 100 replications, but they report results for up to 2,000 replications as well. The "ideal" number of replications is a subject that is still debated in the literature.

We propose another method for estimating the distribution of the mean benefit from policy implementation. This approach is based on the application of the delta method to the moment generating function of expected maximum utility (Afonja 1972), and conveniently side-steps the issue of determining the optimum number of replications. In addition, it may also be more computationally efficient than the Chen and Cosslett (1998) procedures in practice. If we define θ as a vector of the parameter estimates, θ = [α β], where β itself contains the parameters on all explanatory variables but travel cost (trip price), we can use the delta method to approximate the distribution of the mean benefit as follows:

(22) $$\Delta_i = \frac{\partial \frac{\bar{U}_{im}(x^1) - \bar{U}_{im}(x^0)}{-\alpha}}{\partial \theta}$$

(23) $$V\big(E(B_i | x^0 \rightarrow x^1)\big) \approx \Delta V(\theta) \Delta'$$

where $\Delta = \sum_{i=1}^{n} \Delta_1$. In practice, we find the estimates achieved using this Delta Method Approximation to be nearly identical to those obtained following Chen and Cosslett (1998)'s procedures.

Application

We compare the results of the MXL and MNP models when they are applied to a common recreational site choice dataset. In our analysis we use a subset of the Callaway et al. (1995) data that was collected via a 1993 survey of a sample of Pacific Northwest residents. The survey questionnaire focused primarily on Columbia River reservoirs, of which we select four significant examples for our study: Lake Roosevelt (site 1), Dworshak (site 2), Lower Granite (site 3), and Lake Pend Oreille (site 4).

Respondents reported their visits to each of the reservoirs during the summer months. For the four reservoirs considered in this analysis, the randomly sampled respondents reported a total of 1396 trips. In addition to travel cost (price), we use the monthly average deviation of each reservoir's water level away from its full pool level (e.g., negative ten means ten feet below full pool) as the right hand side variables driving visitation on the left hand side. Summary statistics for these explanatory variables are provided in table 3.1 below.

Table 3.1. Reservoir trips: Travel costs and average water level deviations from full pool.

Variable	Mean	Minimum	Maximum	Std. Dev.
Travel Cost (1993 dollars)	63.5199	1.2500	235.5500	38.7634
Deviation from Full Pool (ft.)	-9.9079	-57.6000	0.6000	17.3864

In both our MNP and MXL specifications, our model allows price to be random and takes the form:

(24) $$U_{gi} = \beta_i p_{gi} + \gamma dev_{gi} + \delta_1(1LowerGranite) + \delta_2(1PendOreille) + \varepsilon_{gi}$$

where U_{gi} is the utility of the gth site for the ith observation, i=1,2,…,n; p_{gi} is travel cost and dev_{gi} is deviation from full pool for the gth site and ith observation; $\beta_i = (\beta + v_i)$ and $v_i \sim N(0, \omega^2)$. Alternative specific constants are included for the third and fourth sites because of their fundamental differences from the first two sites.

In our MNP specification, $\varepsilon_i \sim N(0, \Omega)$ and $E(\varepsilon_i v_i) = 0$; therefore $(v_i p_i + \varepsilon_i) \sim N(0,\Sigma)$ where $\Sigma = \Omega + \omega^2 p_i' p_i$ and is a variance-covariance matrix which introduces correlation across prices. Following Train (2009), we restrict Ω to be a diagonal matrix such that:

(25) $$\Omega = \begin{pmatrix} \omega_{11} & 0 & 0 & 0 \\ & \omega_{22} & 0 & 0 \\ & & 1 & 0 \\ & & & \omega_{44} \end{pmatrix}$$

In our MXL specification, however, $\varepsilon_i \sim EV(0, 1)$, and we cannot collapse the error terms into a single vector as in the MNP specification.

Results

For our MNP model, we employ the GHK simulator using 1,000 replications for the maximum likelihood estimation. The results of our initial estimation suggest that $\omega_{11} = \omega_{22} = \omega_{44}$, an assumption imposed in subsequent simulations.[34] Our results from this specification are reported in table 3.2 below.

Table 3.2. Estimates: Multinomial probit model maximum likelihood analysis.

Variable/Parameter	**Estimate**	**Std. Error (R)[1]**	**Std. Error[2]**	**Z (R)**
Travel Cost: β	-0.0432	0.0035	0.0035	-12.4060
Deviation from Full Pool: γ	0.0048	0.0020	0.0020	2.4000
ASC on Lower Granite: δ_1	-0.7687	0.0798	0.0809	-9.6328
ASC on Pend Oreille: δ_2	0.2582	0.0479	0.0501	5.3960
SD on Travel Cost: ω	0.0200	0.0021	0.0021	9.6030
Error Term Variance: ω_{11}	0.1871	0.0431	0.0467	4.3411
[1]*Robust standard error* [2]*Conventional standard error*	**Log Likelihood = -716.14**			

[34] A Likelihood Ratio test of this hypothesis yielded $X^2 = 2.404$ with 2 degrees of freedom (p=0.301).

For our MXL model, we employ a Random Coefficients simulation approach using 5,000 replications (Halton draws) for the maximum likelihood estimation. Our results from this specification are reported in table 3.3 below.

Table 3.3. Estimates: Mixed logit model maximum likelihood analysis.

Variable/Parameter	**Estimate**	**Std. Error (R)[1]**	**Std. Error[2]**	**Z (R)**
Travel Cost: β	-0.0877	0.0059	0.0059	-14.8820
Deviation from Full Pool: γ	0.0059	0.0042	0.0042	1.3912
ASC on Lower Granite: δ_1	-1.3042	0.1913	0.1691	-6.8172
ASC on Pend Oreille: δ_2	0.6042	0.1001	0.1097	6.0363
SD on Travel Cost: ω	0.0320	0.0051	0.0051	6.2691
[1]*Robust standard error* [2]*Conventional standard error*	**Log Likelihood = -730.79**			

To test the performance of the welfare analysis methods previously described, we first calculate the average of expected maximum utility.[35] For the MNP model, we use the Probability Density Function Approach (equations [16] and [17]), the Moment Generating Function Approach (equations [18] and [19]), and the Simulation Approach using 1,000, 2,000, 5,000, and 10,000 replications (equation [20]). For the MXL model, we use the Simulation Approach (equation [20]) using 1,000, 2,000, 5,000, and 10,000 replications. Our results from these calculations are reported in table 3.4 below.

[35] Note that the signing of utility is not intuitive in this context. Utility is an ordinal measure.

Table 3.4. Estimates: Average of expected maximum utility.

	Estimates	
Approach	**MNP Model**	**MXL Model**
Probability Density Function Approach	-0.9968	-
Moment Generating Function Approach	-0.9967 (true[36])	-
Simulation Approach (R=1000)	-1.0174	-2.5812
Simulation Approach (R=2000)	-0.9972	-2.5826
Simulation Approach (R=5000)	-1.0032	-2.5817
Simulation Approach (R=10000)	-0.9979	-2.5815

We then calculate the expected benefit (i.e., effect on expected maximum utility) of increasing the total travel cost to each individual site by $5 as described in equation (21). For the MNP model, we employ both the Moment Generating Function and Simulation Approaches (using 1,000, 2,000, 5,000, and 10,000 replications). For the MXL model, we employ the Simulation Approach (using 1,000, 2,000, 5,000, and 10,000 replications). Our results from these estimations are reported in table 3.5 below.

[36] Both the Probability Density Function and Moment Generating Function approaches for the MNP model are based on theory and the evaluation of a "true" formula (rather than simulation). However, because the Probability Density Function approach requires integration over a cumulative density function, it has the potential to introduce more "noise" into its estimation than the Moment Generating Function approach (which reduces noise by evaluating derivatives). For this reason, we consider Moment Generating Function approach's estimate to be the "true" value.

Table 3.5. Estimates: Average benefit from increasing individual site prices by $5.

	Estimates								
	True (MGF)	Simulation (R=1000)		Simulation (R=2000)		Simulation (R=5000)		Simulation (R=10000)	
Site Price Change	**MNP**	**MNP**	**MXL**	**MNP**	**MXL**	**MNP**	**MXL**	**MNP**	**MXL**
P1 + $5	-2.0810	-2.0950	-2.0606	-2.0740	-2.0388	-2.0860	-2.0636	-2.0810	-2.0650
P2 + $5	-0.4140	-0.3960	-0.4022	-0.4030	-0.3882	-0.4070	-0.3996	-0.4120	-0.4103
P3 + $5	-0.7640	-0.7870	-0.7512	-0.7590	-0.7286	-0.7730	-0.7469	-0.7640	-0.7443
P4 + $5	-1.5120	-1.5120	-1.5680	-1.5000	-1.5482	-1.5190	-1.5424	-1.5070	-1.5522

Lastly, for both the MNP and MXL models, we calculate the distribution of the mean benefit of a universal one foot increase in deviation from full pool using traditional (simulation) methods and the Delta Method Approximation approach (equations [22] and [23]). This measure can be interpreted as the consumer (recreator) per-trip willingness to pay (WTP) for a one foot increase in water levels at all four reservoirs. Our results from these calculations are reported in table 3.6 below.

Table 3.6. Estimates: Mean per-trip benefit of a universal one foot increase in water levels.

	Estimates (1993 dollars)			
	MNP Model		MXL Model	
Approach	**Mean Benefit**	**Robust Std. Deviation**	**Mean Benefit**	**Robust Std. Deviation**
Simulation[37]	$0.1132	$0.0487	$0.0668	$0.0505
Delta Method Approximation	$0.1108	$0.0486	$0.0668	$0.0489

Discussion

It is unsurprising that point estimates of average expected maximum utility (table 3.4) differ between the model specifications – error terms are being drawn from different distributions and the two models are estimating a different number of parameters (recall that an "extra" parameter is estimated in the MNP context to account for scale and level of utility). Nevertheless, it is clear that the patterns of sign and relative significance of these parameter estimates persist across both specifications, and the models achieve similar log likelihoods at convergence (tables 3.2 and 3.3). Furthermore, the inference both models provide about the welfare impacts of a policy change (a $5 travel cost increase) is nearly identical, regardless of the number of replications employed in the simulation (table 3.5). In terms of WTP measures and their distributions, the MNP model predicts the average benefit of a universal one foot increase in water levels to be roughly twice that of the MXL model (table 3.6). This is a result of the

[37] In the MNP estimation, we employed a Krinsky-Robb procedure using 1,000 draws. In the MXL estimation, we employed a bootstrapping procedure using 1,000 iterations.

aforementioned difference in the original parameter estimates.[38] Interestingly, however, both models provide a very similar picture of the distribution around this WTP estimate (a standard deviation of approximately $.05 across the board).

With regard to methodology, we find that the Delta Method Approximation provides a computational advantage over both of the more traditional simulation procedures (Krinsky-Robb and bootstrapping) – welfare estimates for both models could be calculated in a matter of minutes as opposed to a matter of hours. The Delta Method Approximation also eliminates the need for the researcher to run multiple simulations to determine the optimal number of replications to report.

While the MXL and MNP models performed similarly overall, the MNP model did fit slightly better in terms of log likelihood achieved at convergence. Given this result and the advantages and simplifications that the Delta Method Approximation presents for welfare analysis in the MNP context, we believe the MNP model merits greater consideration in the recreational travel cost (and non-market valuation) literature.

[38] The MXL model estimates the coefficient on price to be roughly twice what the MNP model estimates. As the WTP for a one foot increase in deviation in the MXL context is the negative of the ratio of the coefficient on deviation to the coefficient on price (i.e., $-\gamma/\beta$), the MXL model will necessarily estimate the WTP measure to be roughly half what the MNP model will.

References

Afonja, B. 1972. "The Moments of the Maximum of Correlated Normal and t-Variates." *Journal of the Royal Statistical Society. Series B (Methodological)* 34: 251-262

Arellano-Valle, R.B., and M.G. Genton. 2008. "On the Exact Distribution of the Maximum of Absolutely Continuous Dependent Random Variables." *Statistics & Probability Letters* 78: 27-35

Callaway, J.M., S. Ragland, S. Keefe, T.A. Cameron, and W.D. Shaw. 1995. "Columbia River Systems Operation Review Recreation Impacts: Demand Model and Simulation Results." RCG/Hagler Bailly, Inc. Final report prepared for Matthew Rae and the Recreation Work Group, U.S. Army Corps of Engineers, Portland, Oregon

Chen, H.Z., and S.R. Cosslett. 1998. "Environmental Quality Preference and Benefit Estimation in Multinomial Probit Models: A Simulation Approach." *American Journal of Agricultural Economics* 80: 512-520

Train, K. 2009. *Discrete Choice Methods with Simulation.* New York: Cambridge University Press

CHAPTER 4

BEACH EROSION, SITE QUALITY, AND RECREATION DEMAND: APPLICATION OF MIXED LOGIT AND KUHN-TUCKER GENERALIZED CORNER SOLUTION MODELS[39]

[39] Barfield, A.S. and C.E. Landry. To be submitted to *Journal of the Association of Environmental and Resource Economics*

Abstract

North Carolina’s beaches are imperiled by coastal erosion, sea level rise, severe storms, and oceanfront development. Proposed solutions to these problems include beach replenishment, coastal retreat, and shoreline armoring. These policies affect the quality and value of coastal resources and recreation, and assessing these welfare impacts is necessary for benefit-cost-analysis of these alternatives. In this paper, we analyze multi-site revealed preference trip data for North Carolina households using travel costs and beach site attributes (beach width, beach length, number of access points, parking area, ferry-only access, and presence of lifeguards) as explanatory variables. We employ a mixed logit model in our recreation demand analysis and discuss the advantages of incorporating a Kuhn-Tucker generalized corner solution model in future analyses. The welfare estimates we obtain have immediate policy relevance and contextualize future research efforts utilizing this Sea Grant dataset (of which the revealed preference responses are but a subset).

Introduction

North Carolina's beaches are imperiled by a number of forces including coastal erosion, sea level rise, storm events of increasing frequency and severity, and oceanfront development. Three primary solutions to these problems have been proposed: beach replenishment, coastal retreat, and shoreline armoring. Each of these management approaches induces changes in the quality of coastal resources, affecting the distribution of beach and dune sediments, presence and location of hardened structures, and configuration of buildings and infrastructure. These changes, in turn, affect the economic value of coastal recreation. This paper considers the use values associated with North Carolina (NC) beaches and how these values could be influenced by the implementation of the aforementioned management policies. The accurate assessment of such welfare impacts is, naturally, a critical component of the benefit-cost-analysis of these alternative proposals. Our primary research goal is to identify and characterize preferences for beach width.

To this end, we analyze revealed preference beach site choice data for a random sample of NC households (data collection funded by East Carolina University and NC Sea Grant in 2013). Through the use of the NC Department of Environmental Quality's Coastal Geographic Information Systems (GIS) files, a traveler’s manual for NC beaches (Morris 2005), and a host of Outer Banks tourism websites, we create a site-attribute matrix for NC beaches that includes information regarding travel costs and beach length, width, and accessibility. We employ a Mixed Logit (MXL) model in our analysis of recreation demand and the impact of site characteristics (many of which can be influenced by coastal policy and erosion management) on site choice and intensity of beach recreation. Our research therefore represents an important contribution to the understanding of people's preferences and support (willingness to pay, WTP) for different erosion management scenarios.

Theoretical Background

In recreation demand studies, it is frequently the case that a researcher's dataset will consider respondents' socio-demographic characteristics and reported seasonal visitation to a large number of alternatives (perhaps a dozen sites or more), for which there is an accompanying site-attribute index. Often, a respondent will visit a subset of sites multiple times, and other sites not at all. "To consistently derive welfare measures for price and attribute changes with such data, structural econometric models that behaviorally and statistically account for the mixture of corner solutions (unvisited sites) as well as interior solutions (sites with one or more trips) are required," (von Haefen and Phaneuf 2005).

Random Utility Models (RUM), which divide recreational seasons into multiple discrete choice occasions in which respondents either take or do not take a trip, have historically been quite popular in this context. Much of the recent travel cost literature has focused specifically on the use of MXL models to evaluate recreational site choice data and estimate recreation demand. MXL models are more flexible than standard multinomial logit models, and their simulation methods can accommodate a greater variety of datasets. Additional advantages MXL models have over simpler logit formats include: (1) They are not bound by the Independence of Irrelevant Alternatives (IIA) axiom that yields unrealistic substitution patterns among similar alternatives; (2) They can incorporate random taste variation and temporally correlated error terms; (3) Through various specifications and impositions on structural form, they may be able to approximate any RUM process (Train 2009).

Methodology

Following the classic RUM framework, we assume utility is composed of observed and unobserved elements such that:

(1) $$U_{nj} = V_{nj} + \varepsilon_{nj}$$

where there are *n* respondents and *j* alternatives (i.e., sites); V_{nj} is the observed portion of utility, which is expressed in terms of explanatory variables (i.e, site attributes and travel cost) x_{nj} and coefficients β such that, in the linear case, $V_{nj} = \beta_n' x_{nj}$; and ε_{nj} is the unobserved portion of utility (an error term) distributed i.i.d type I extreme value.

MXL choice probabilities take the form:

(2) $$P_{ni} = \int L_{ni}(\beta) f(\beta) d\beta$$

where $L_{ni}(\beta)$ is the logit probability evaluated for the parameters β:

(3) $$L_{ni}(\beta) = \frac{e^{V_{ni}(\beta)}}{\sum_{j=1}^{J} e^{V_{nj}(\beta)}}$$

and $f(\beta)$ is a density function (i.e., a mixing distribution). In the linear utility case, the MXL probability is:

(4) $$P_{ni} = \int \left(\frac{e^{\beta' x_{ni}}}{\sum_{j=1}^{J} e^{\beta' x_{nj}}}\right) f(\beta) d\beta$$

As such, the MXL probability "is a weighted average of the logit formula evaluated at different values of β, with the weights given by the density $f(\beta)$," (Train 2009). Density functions can be discrete but are typically specified to be continuous - the normal and lognormal densities are common,[40] though other densities can also be used. This is known as the Random Coefficients approach and is the most direct, most commonly used MXL method. Each respondent knows their own β_n's and ε_{nj}'s for all *j* alternatives, and will select alternative *i* only when $U_{ni} > U_{nj}$ for all $j \neq i$ (we only observe the x_{nj}'s, however) (Train 2009).

[40] The lognormal distribution is most useful when a coefficient is likely to have the same sign for all respondents.

Simulation methods are easily applied to MXL models. We first specify a functional form for *f(β).* We then 1) for each respondent, draw a value of β from *f(β)*, labeled β_{D1} for draw 1; 2) calculate $L_{ni}(\beta_{D1})$; 3) repeat steps (1) and (2) *R* times; and 4) average the results yielding:

(5) $$\hat{P}_{ni} = \frac{1}{R}\sum_{r=1}^{R} L_{ni}(\beta_{DR})$$

This is an unbiased estimator for P_{ni}, the probability that person *n* selects alternative *i*.[41] We obtain *n* of these estimates for each of the *j* alternatives and calculate the simulated log likelihood (SLL):

(6) $$SLL = \sum_{n=1}^{N}\sum_{j=1}^{J} d_{nj}\, ln\hat{P}_{ni}$$

where d_{nj} is an indicator variable which takes a value of 1 if person *n* chose alternative *j* (and 0 otherwise) (Train 2009).

Calculating the expected benefit of (i.e., WTP for) a change in site attributes, as might result from policy implementation, involves first calculating the difference in expected maximum utility under initial and altered conditions, and then monetizing this impact:

(7) $$EW(x^1|x^0) = E(B_i|x^0 \rightarrow x^1) = \frac{\bar{U}_{im}(x^1)-\bar{U}_{im}(x^0)}{-\alpha}$$

where $\bar{U}_{im}$ is expected maximum utility; x^0 and x^1 are vectors of site attributes pre and post policy change, respectively; α is the coefficient on travel cost and can therefore be considered the marginal utility of income; and B_i is the benefit for the i^{th} observation. The distribution of this expression can then be calculated using either Krinsky-Robb or bootstrapping procedures.

In logit models which are linear in parameters, this WTP measure can conveniently be obtained by taking the negative of the ratio of the estimated coefficient on the attribute of interest to the estimated coefficient on travel cost:

---

[41] $\hat{P}_{ni}$ is also strictly positive, is twice differentiable, and sums to 1 over all *j* alternatives.

(8) $$WTP = -\frac{\hat{\gamma}_x}{\hat{\alpha}}$$

where $\hat{\gamma}_x$ is the estimated coefficient on any site attribute, x, other than travel cost, and $\hat{\alpha}$ is the estimated coefficient on travel cost.

Application

Data

In our application, we analyze revealed preference beach-site-choice data gathered from a random sample of NC households through a 2013 internet survey funded by NC Sea Grant and East Carolina University.[42] Roughly 1,000 respondents provide socio-demographic information and number of trips to 20 sets of Outer Banks beaches[43] over the previous year (41 beaches grouped from north to south based on assumed similarity of associated travel costs). For the purposes of this study, we introduce a no-go option (a 21st alternative) and consider the preferences of respondents who report taking 52 day trips a year or less in order to achieve a weekly repeated discrete choice format. This formatting assumes respondents take a maximum of one trip per week with 52 choice occasions (where timing of the trips throughout the year is irrelevant), and reduces our sample size to 259 respondents.

Table 4.1 below reports the total number of day trips taken by these 259 respondents to each of the 20 sets of beaches.

[42] This revealed preference data is a subset of the (much larger) dataset, which has dense revealed preference, stated preference, and contingent valuation components.

[43] Respondents were also offered two write-in choice options where they could self-report beaches they visited but were not listed in the survey. The majority of those self-reported beaches were, in actuality, subsumed by the options specifically offered in the survey, and we recode the data accordingly.

Table 4.1. Distribution of day trips taken to NC beaches.

Beach	Total Trips	Proportion of All Trips
Corolla/ Duck	41	0.0263
Kitty Hawk/ Kill Devil Hills/ Nags Head	254	0.1627
Pea Island	6	0.0038
Rodanthe/ Waves/ Salvo/ Avon	5	0.0032
Buxton/ Frisco/ Hatteras	14	0.0090
Ocracoke	18	0.0115
Cape Lookout/ Core Banks	5	0.0032
Fort Macon/ Atlantic/ Pine Knoll Shores/ Salter Path/ Indian/ Emerald Isle	207	0.1326
Hammocks Beach/ Bear Island	14	0.0090
North Topsail	78	0.0500
Surf City/ Topsail	67	0.0429
Figure 8 Island	10	0.0064
Wrightsville	220	0.1409
Masonboro Island	8	0.0051
Carolina/ Kure/ Fort Fisher	194	0.1243
Bald Head Island	11	0.0070
Oak Island/ Caswell/ Yaupon/ Long	137	0.08776
Holden	113	0.0724
Ocean Isle	45	0.0288
Sunset	114	0.0730

To supplement the data collected in the NC Sea Grant survey, we create a site-attribute index which characterizes these groups of NC beaches in terms of beach length, beach width (minimum, maximum, quartiles, average and standard deviation), number of access points, total area of parking lots, the presence of lifeguards, and whether a boat or ferry ride is required to access beaches within the group. Data on these attributes is gathered from the NC Department of Environmental Quality's (NCDEQ) Coastal GIS files, a traveler's manual for NC beaches (Morris 2005), and a variety of North Carolina tourism websites.[44]

The presence of lifeguards and the requirement of a ferry or boat ride to access a beach are coded as percentages. For example: if a choice set contains one beach, if lifeguards are (not)present, (0)100 percent of the beaches in that set have lifeguards, and the lifeguards attribute is set equal to (0)1. If a choice set contains multiple beaches and lifeguards are present at some, but not all of the beaches, the lifeguards attribute is set equal to whatever proportion of the total length of the choice set the beaches with lifeguards represent (i.e., combined length of beaches with lifeguards/total length of all beaches in the choice set).

Length and width are measured in meters. Width measurements are taken every 100 meters, from the edge of the water inward to the edge of the sand (i.e., the width of the beach as it would appear to a respondent walking along it), along 50 meter transect gridlines. Access points are those officially demarcated in the "NC Beach and Waterfront Access" layer of the

[44] (Cape Lookout National Seashore 2016), (Fort Caswell 2016), (Outer Banks: Lifeguard Locations Information 2016), (Outer Banks North Carolina Rentals 2016), (Outer Banks Vacation Guides 2016), (The Official Travel and Tourism Website for North Carolina 2016), (The Outer Banks 2016).

NCDEQ's GIS data.[45] Parking area is associated with these access points (i.e., parking areas adjacent to access point but not part of clearly private or commercial property) and is measured in square meters (rounded to the nearest 50).

Figure 4.1 below provides a screen capture image of the NCDEQ's interactive GIS mapping applet. It displays the 50 meter transects layer used to measure beach length and width, as well as the beach and waterfront access layer used to identify access points and associated parking lots. The blue and orange icon indicates an official access point.

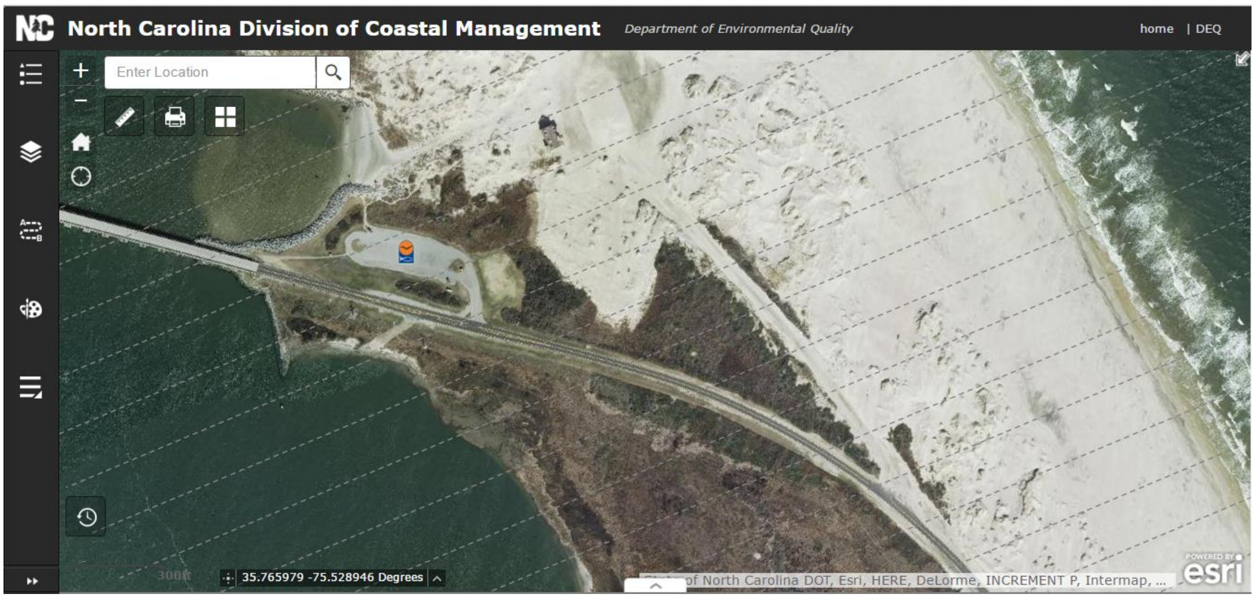

Figure 4.1. Screen capture of satellite imagery used to construct NC site-attribute index.

Source: North Carolina Department of Environmental Quality

Our final site-attribute index is shown in tables 4.2 and 4.3 below.

[45] There are beach access points which exist but are not officially demarcated in this layer (including some piers), however, and many of these may have public parking available. Unfortunately, it is difficult to delineate exactly what constitutes an access point by looking at the satellite imagery alone, which is why we currently rely on officially recognized points. Future revisions to this dataset may be able to incorporate more comprehensive access point counts.

Table 4.2. NC beach attributes: Width measurements (meters).

Beach	Min Width	Q1 Width	Q2 Width	Q3 Width	Max Width	Avg Width	SD Width
Corolla/Duck	14.00	28.00	32.00	36.00	45.00	31.84	5.38
Kitty Hawk/Kill Devil Hills/Nags Head	11.00	39.50	56.00	78.00	410.00	63.71	38.02
Pea Island	16.00	51.00	85.00	117.00	385.00	92.26	56.49
Rodanthe/Waves/Salvo/Avon	8.00	52.00	62.00	76.00	168.00	65.91	22.90
Buxton/Frisco/Hatteras	17.00	50.25	71.50	132.75	420.00	94.88	62.42
Ocracoke	30.00	65.00	80.00	122.75	1355.00	162.44	236.76
Cape Lookout/Core Banks	5.00	52.00	70.00	102.00	1154.00	97.34	115.28
Fort Macon/Atlantic/Pine Knoll Shores/Salter Path/ Indian/Emerald Isle	30.00	48.00	55.00	65.00	432.00	61.00	30.17
Hammocks Beach/Bear Island	31.00	47.00	52.00	113.00	474.00	112.07	122.61
North Topsail	8.00	26.00	29.00	34.00	187.00	30.89	14.17
Surf City/Topsail	12.00	22.00	29.50	66.00	356.00	48.74	43.94
Figure 8 Island	18.00	60.50	72.00	84.00	396.00	79.77	45.03
Wrightsville Beach	46.00	55.00	64.00	71.00	148.00	67.07	18.82
Masonboro Island	15.00	63.50	80.00	97.00	160.00	80.58	24.34
Carolina/Kure/Fort Fisher	0.00	42.00	54.00	68.00	287.00	60.60	34.62
Bald Head Island	19.00	39.00	58.00	93.00	242.00	69.45	41.80
Oak Island/Caswell/Yaupon Beach/Long	7.00	24.00	36.00	52.00	391.00	42.03	33.23
Holden	12.00	23.00	31.50	41.00	360.00	39.50	36.71
Ocean Isle	0.00	32.00	39.00	57.00	216.00	46.41	29.54
Sunset	0.00	24.50	36.00	45.00	252.00	41.85	32.32

Table 4.3. NC beach attributes: Length (meters), access points (#), parking area (sq. meters), ferry access (%), lifeguards (%).

Beach	Length	Access Points	Parking Area	Ferry	Lifeguards
Corolla/Duck	33900	16	11950	0.00	0.81
Kitty Hawk/Kill Devil Hills/Nags Head	38950	86	64000	0.00	0.89
Pea Island	15550	6	9100	0.00	0.00
Rodanthe/Waves/Salvo/Avon	42500	6	8850	0.00	0.00
Buxton/Frisco/Hatteras	26200	7	12250	0.00	0.46
Ocracoke	26200	0	0	1.00	1.00
Cape Lookout/Core Banks	88700	0	0	1.00	0.00
Fort Macon/Atlantic/Pine Knoll Shores/Salter Path/ Indian/Emerald Isle	39750	96	48100	0.00	0.71
Hammocks Beach/Bear Island	6000	0	0	1.00	1.00
North Topsail	17850	38	21700	0.00	0.00
Surf City/Topsail	17900	51	10050	0.00	0.00
Figure 8 Island	13200	0	0	0.44	0.00
Wrightsville Beach	7400	43	15050	0.00	1.00
Masonboro Island	12900	0	0	1.00	0.00
Carolina/Kure/Fort Fisher	18900	47	23450	0.00	1.00
Bald Head Island	13850	27	1500	1.00	0.00
Oak Island/Caswell/Yaupon Beach/Long	20950	67	25500	0.00	0.27
Holden	13100	20	2950	0.00	0.00
Ocean Isle	9200	24	13600	0.00	0.00
Sunset	6350	33	6900	0.00	0.00

For each respondent, we also generate round-trip travel costs that are the sum of estimated mileage costs, time costs, and fees. Mileage costs are calculated using the AAA 2013 per mile cost of $0.608 over round-trip driving distances. Time costs are calculated to be one-third the hourly wage rate (determined by reported income) and assume that respondents are driving 55 miles per hour. Ferry fees are also included where applicable.

Specifications

To keep the number of explanatory variables feasible for estimation, we include site attributes (alternative-variant variables) only. Each of our models allows the beach width term to be a random normal parameter while all other parameters are held fixed (constant). Our estimations compare the measures of central tendency with regard to beach width – mean and median – as the standard deviation on beach width is often large and one measure may provide a better representation of recreator preferences than the other. We also consider nonlinear transformations of the width measurements (quadratic, natural log, inverse) and an interaction term which introduces one respondent-variant variable – a dummy for concern about beach width – into the estimation.[46] These specifications therefore take the following forms:

$$U_{gi} = \beta_i W_{gi} + \alpha P_{gi} + \gamma_1 L_{gi} + \gamma_2 A_{gi} + \gamma_3 K_{gi} + \gamma_4 F_{gi} + \gamma_5 G_{gi} + \varepsilon_{gi} \tag{9}$$

$$U_{gi} = \beta_i W_{gi} + \alpha P_{gi} + \gamma_1 L_{gi} + \gamma_2 A_{gi} + \gamma_3 K_{gi} + \gamma_4 F_{gi} + \gamma_5 G_{gi} + \gamma_6 WC_{gi} + \varepsilon_{gi} \tag{10}$$

where U_{gi} is the utility of the g^{th} site for the i^{th} observation, i=1,2,…,n; W_{gi} is beach width, which may be median width, average width, the natural log of either of these measures, the inverse of either of these measures, or the square of either of these measures; P_{gi} is travel cost; L_{gi} is beach

[46] Respondents answered the question "how concerned are you about the width of developed beaches along the North Carolina shoreline?" on a scale of 1-4 with 1 being "not concerned at all" and 4 being "very concerned." We create a dummy variable for this concern set equal to 1 if respondents' answers were either a three or a four.

length; A_{gi} is number of access points; K_{gi} is parking area; F_{gi} is the proportion of beaches in the gth site requiring boat or ferry access; G_{gi} is the proportion of beaches in the gth site with lifeguards; WC_{gi} is an interaction term multiplying the width measure W_{gi} by the dummy variable for concern about beach width, C_{gi}; $\beta_i = (\beta + v_i)$ and $v_i \sim N(0, \omega^2)$; and $\varepsilon_{gi} \sim EV(0, 1)$.

To evaluate our models' fits, we employ the Akaike/Bayesian Information Criterion (AIC and BIC, respectively). The AIC and BIC can be used to compare the success of non-nested models – the lower the criterions' values, the better the models are performing.

Results

We estimate a total of sixteen MXL maximum likelihood specifications each using 1,000 Halton draws - eight for each measure of central tendency on beach width, where the first four models (one with the raw width measurement, three with the nonlinear transformations) do not include the interaction term, and the last four models do include the interaction term. None of our specifications find the interaction term to be statistically significant, and the best fitting models for both average and median width (as indicated by the AIC and BIC values) employ the log transformation of the width measurement. We therefore use these log-width models in our subsequent welfare analysis. Results from these estimations are provided in tables 4.4 and 4.5 below.

Table 4.4. Estimates: MXL maximum likelihood analysis on log-median-width.

Variable/Parameter	Estimate	Std. Error	Z
ln(medwidth): β	-0.5722	0.0554	-10.3276
travel cost: α	-0.0219	0.0008	-27.0397
length: γ_1	-0.0000	0.0000	-2.7064
access points: γ_2	0.0068	0.0030	2.2866
parking area: γ_3	0.0000	0.0000	5.0630
ferry: γ_4	-0.9356	0.1597	-5.8565
lifeguards: γ_5	0.8219	0.0803	10.2341
SD on ln(medwidth): ω	0.4924	0.0361	13.6307
Log Likelihood = -6120.58	**AIC = 12257.16**	**BIC = 12341.58**	

Table 4.5. Estimates: MXL maximum likelihood analysis on log-average-width.

Variable/Parameter	Estimate	Std. Error	Z
ln(avgwidth): β	-0.5553	0.0534	-10.4051
travel cost: α	-0.0218	0.0008	-27.0290
length: γ_1	-0.0000	0.0000	-3.0140
access points: γ_2	0.0095	0.0031	3.0968
parking area: γ_3	0.0000	0.0000	4.0937
ferry: γ_4	-0.7922	0.1616	-4.9023
lifeguards: γ_5	0.7758	0.0788	9.8387
SD on ln(avgwidth): ω	0.4733	0.0344	13.7421
Log Likelihood = -6116.30	**AIC = 12248.59**	**BIC = 12333.01**	

To contextualize these results, we calculate point estimates and 95% confidence intervals for consumer (household) per-trip WTP for (mean benefit of) a unit increase in each of the site attributes using a Krinsky-Robb procedure with 1,000 replications. These measures can be interpreted as, for example, the per-trip benefit a respondent would experience from the provision of an additional beach access point, square meter of parking area, meter of beach width, etc. Our results are provided in table 4.6 below.

Table 4.6. Estimates: Per-trip WTP for unit increases in NC beach site attributes.

	Estimates (2013 dollars)			
	Log-Median-Width Model		**Log-Average-Width Model**	
Variable	**WTP**	**95% CI**	**WTP**	**95% CI**
width[47]: β	-\$0.4773	(-\$0.5936, -\$0.3639)	-\$0.3666	(-\$0.4558, -\$0.2799)
length: γ_1	-\$0.0005	(-\$0.0010, -\$0.0001)	-\$0.0006	(-\$0.0010, -\$0.0002)
access points: γ_2	\$0.3113	(\$0.0312, \$0.5902)	\$0.4357	(\$0.1451, \$0.7252)
parking area: γ_3	\$0.0011	(\$0.0007, \$0.0016)	\$0.0009	(\$0.0005, \$0.0014)
ferry: γ_4	-\$42.6308	(-\$57.0380, -\$27.8941)	-\$36.3062	(-\$50.1055, -\$21.4101)
lifeguards: γ_5	\$37.4512	(\$29.7899, \$46.2706)	\$35.5516	(\$27.9166, \$44.0323)

Discussion

The different specifications on width perform very similarly and provide nearly identical pictures of the influence of the explanatory variables on trip demand, though there are noticeable

[47] Our WTP estimates for the width parameters are evaluated at the means for both measures and have been adjusted to account for the log transformation, which is nonlinear, and therefore prevents the straightforward application of equation (8) in this context.

differences in the parameter estimates for lifeguards, ferry access, and number of access points. As a result, the confidence intervals around some site attributes' WTP estimates do not always overlap between the models. Additionally, in the log-average-width specification (which fits slightly better overall), all of the parameters are highly significant, whereas number of access points strays towards bordering on insignificance in the log-median-width specification.

Generally speaking, the signs on the parameters are what we expect. Travel costs and requiring the arrangement of a boat or ferry ride to access a beach have a negative influence on trip demand. Length also has a (small) negative influence, perhaps because the walking distance between beach amenities such as restrooms and beachside attractions is greater on longer beaches. Number of access points, the amount of parking area, and the presence of lifeguards all have positive impacts on trip demand.

Somewhat surprising, however, is the result that beach width has a negative influence on recreation demand. This could be indicative of significant heterogeneity in preferences for beach width, or the fact that the value respondents place on beach width is dependent upon activities they engage in which we don't have data on. Our finding could also indicate that the balance of recreational beach activities favors easy water access. Furthermore, a negative demand for beach width could also reflect respondents' crowding concerns – because more people go to wide beaches where there is more space, respondents may be entangling their preferences for beach width and congestion. It is also possible that including alternative specific constants for each of the beach choice sets and introducing more sociodemographic information into the model through interaction variables could help to better identify preferences for beach width. To ensure that this negative result is valid, we will continue to explore measurement and specification issues.

Nevertheless, while beach erosion and sea level rise directly impact beach width, they also threaten beach access in general. In certain sea level rise scenarios, the number of beach access points could be reduced and the proportion of beaches accessible only by ferry or boat could increase. Given the results of our welfare analysis, these consequences could have substantial economic impacts and should be considered in future natural resource management decisions for the Outer Banks region.

Overall, this paper provides insights that we feel will be of interest to fellow researchers and to coastal management authorities. We analyze a very recent, highly disaggregated and detailed revealed preference dataset, as well as a unique and diverse site-attribute index. With these high quality data, we evaluate a significant number of alternatives using an advanced econometric model that is particularly well-suited to this valuation context. The welfare estimates we obtain have immediate policy relevance, and provide context for future work with this dataset (of which the revealed preference responses are but a subset).

Future Work

While RUM models like the MXL have historically been the work-horse estimation procedures used in recreation demand studies, to calculate demand and welfare impacts at the seasonal level, RUM models require an additional estimation process to determine the season-wide implications of per-trip outcomes. The Kuhn-Tucker Generalized Corner Solution (KT) models originally developed by Wales and Woodland (1983) provide a utility-theoretic[48]

---

[48] A significant appeal of the KT model is the "unified and internally consistent" framework it offers to characterize the nature of corner solutions. The KT model is explicitly derived from the utility function and utility maximization theory, which necessarily ensures that the restrictions of this theory are satisfied and that the behavioral implications of corner solutions are accounted for (Phaneuf et al. 2000).

alternative to RUM models, though their reputation for being computationally laborious has hindered their adoption in the literature. In KT models, it is assumed that individual preferences are distributed randomly across the population. As a result, the classic Kuhn-Tucker conditions associated with utility maximization become likewise randomly distributed, allowing for construction of the probabilities that corner solutions will occur and of the likelihood function (Phaneuf et al. 2000).

> "These 'Kuhn-Tucker' models… consistently account for both the extensive (which sites to visit) and intensive (how many trips to take) margins of choice and can be used to recover a coherent representation of an individual's seasonal preferences. As such, the KT framework has a significant conceptual advantage over discrete choice approaches for modeling seasonal recreation demand," (von Haefen and Phaneuf 2005).

Fortunately, KT models have become more accessible as a result of recent advances in computational power and simplifications of the algorithms used to estimate the models. For this reason, we plan to incorporate a KT model in a second-stage analysis of our NC beach data, employing the framework established and described by Phaneuf and Siderelis (2003) and von Haefen and Phaneuf (2005) as follows.[49]

We first consider that a respondent's direct utility function takes the form:

$$U(x, z; Q, \varepsilon, \beta) \tag{11}$$

[49] As far as we are aware, the KT model presented in von Haefen and Phaneuf (2005) represents the most comprehensive and refined approach currently available in the literature. Sanchez et al. (2016), for instance, utilize this approach in their analysis of the recreational values of the San Jacinto Wilderness. They furthermore acknowledge Phaneuf for providing them with the MATLAB source code employed in their estimation.

where x is an M-dimensional vector of trips; Q is an $L \times M$ matrix of site attributes; z is a numeraire representing spending on all other goods (price normalized to one); β is a vector of structural parameters to be estimated; and ε is a matrix (or vector) of unobserved heterogeneity – components of utility known to the respondent but unknown (random) to the researcher. This framework is therefore consistent with random utility maximization theory.

Respondents maximize their utility over x and z subject to budget and non-negativity constraints:

(12) $$max_{x,z}\, U(x, z; Q, \varepsilon, \beta) \quad s.t. \quad y = p'x + z, \quad x_j \geq 0, \; j = 1, \dots, M$$

where U has the typical curvature properties (is continuously differentiable, quasi-concave, etc.); p is an M-dimensional vector of travel costs (including access fees); and y is income. The non-negativity constraint ensures that the first order conditions are Kuhn-Tucker conditions. Assuming $z > 0$, the Kuhn-Tucker conditions defining the optimal consumption bundle (x^*, z^*) are:

(13) $$\frac{\partial U/\partial x_j}{\partial U/\partial z} \leq p_j, j = 1, \dots, M$$

and

(14) $$x_j \times \left(\frac{\partial U/\partial x_j}{\partial U/\partial z} - p_j\right) = 0, j = 1, \dots, M$$

These equations can be interpreted thusly: the marginal rate of substitution between trips to a visited site (an interior solution) and other goods is equal to the travel cost to the site, and the marginal rate of substitution between trips to an unvisited site (a corner solution) and other goods is less than the travel cost to the site. For corner solutions, then, travel cost exceeds the respondent's reservation price. For interior solutions, allowing $g_j(x, y, p, Q, \beta)$ to represent the solution to equation (14), equations (13) and (14) can be rewritten as:

(15) $$\varepsilon_j \leq g_j(x, y, p, Q, \beta)$$

and

(16) $$x_j \times \left(\varepsilon_j - g_j(x, y, p, Q, \beta)\right) = 0$$

Given distributional assumptions about the form of ε, we may define the probabilities of observing both corner and interior solutions, and through maximum likelihood estimation, we may recover that parameters in β that will define x^*.

In terms of welfare analysis, Hicksian compensating surplus (CS^H) resulting from a change in prices (travel costs) and/or site attributes from (p^0, Q^0) to (p^1, Q^1) can be expressed through either indirect utility functions (equation [17]) or expenditure functions (equation [18]) as:

(17) $$v(p^0, Q^0, y, \beta, \varepsilon) = v(p^1, Q^1, y - CS^H, \beta, \varepsilon)$$

or

(18) $$CS^H = y - e(p^1, Q^1, U^0, \beta, \varepsilon)$$

where $U^0 = v(p^0, Q^0, y, \beta, \varepsilon)$. There are computational challenges with either approach.

In either of these scenarios, respondents switch between membership in either the non-visitation (corner solution) or visitation (interior solution) regimes to maximize their utility/minimize their expenditures. These regimes correspond to the 2^M combinations of interior and corner solutions possible for the *M* sites. When *M* is large, solving equations (17) or (18) can be a daunting task. Additionally, because CS^H is a random variable (as it is partially defined by ε), the researcher can only compute measures such as $E(CS^H)$ through simulation methods. Fortunately, advances made by Phaneuf et al. (2000), von Haefen (2004), and von Haefen et al. (2004) have allowed for the estimation of KT welfare measures even in cases where *M* is sizable.

Empirically, the three KT specifications examined by von Haefen and Phaneuf (2005) rely on the concept of additive separability[50] – i.e., that $U = \sum_j^M u_j\left(x_j\right) + \ u_z$ – and are variations of the direct utility function:

(19) $$U = \sum_{j=1}^{M} \Psi_j \ln\left(\Phi_j x_j + \theta\right) + \frac{1}{p} z^p,$$

$$\Psi_j = \exp\left(\delta' s + \mu \varepsilon_j\right),$$

$$\Phi_j = \exp\left(\gamma' q_j\right),$$

$$\rho = 1 - \exp(\rho^*),$$

$$\theta = \exp(\theta^*),$$

$$\mu = \exp(\mu^*)$$

where *s* is a vector of individual characteristics; *(δ, ϒ, θ*, ρ*, μ*)* are structural parameters; ε = $(\varepsilon_1, \dots, \varepsilon_M)$ is unobserved heterogeneity where each element is distributed i.i.d EV; and $\rho < 1$. This utility function format ensures that weak complementarity holds – changes in q_j (site attributes of site j) have no impact on utility when $xj = 0$. Weak complementarity necessarily implies that estimated welfare effects will represent only use values.

This particular utility structure implies the following likelihood of observing a specific outcome *x*, conditional on *(δ, ϒ, θ*, ρ*, μ*)*:

(20) $$L(x|\delta, \gamma, \theta^*, \rho^*, \mu^*) = |\boldsymbol{J}| \prod_j [\exp(-g_j(\cdot))/\mu)]^{1(x_j>0)} \times \exp[-\exp(-g_j(\cdot))]$$

where $|\boldsymbol{J}|$ is the determinant of the Jacobian; $1(x_j > 0)$ is an indicator variable; and $g_j(\cdot)$ is the right hand side of equation (15) in the particular context of this utility function:

[50] Additive separability eliminates the possibility of inferior goods and Hicksian complementarities between goods, and implies that wealthier respondents will visit more sites more frequently than other respondents. This implication may be either plausible or bothersome depending on the specific application.

$\frac{1}{\mu}[-\delta' s + ln\frac{\rho_j}{\Phi_j} + ln\,(\Phi_j x_j + \theta) + (\rho - 1)\; ln(y - \rho' x)]$, for all j.

The three KT specifications examined by von Haefen and Phaneuf (2005) differ in their treatment of the structural parameters as either fixed (more restrictive) or random (more general) across the population. A fixed parameters classical model, random parameters Bayesian model, and random parameters classical model are described. As we progress with our application, we will determine which of these specifications is most appropriate.[51]

Conducting a welfare analysis in the KT framework involves a two-step procedure to calculate CS^H at each iteration in a simulation procedure. First, the unobserved heterogeneity must be simulated such that they are consistent with the choices observed under baseline conditions. Second, CS^H must be solved for conditional on these simulated elements of unobserved heterogeneity. The approach advocated by von Haefen and Phaneuf (2005) is a conditional approach which, by the law of iterated expectations, should represent the expectation of unconditional welfare estimates (so long as the data-generating process is correctly specified), and which has been shown to provide significant time savings as compared with the standard, unconditional approach.

> "…we simulate the unobserved heterogeneity such that our model perfectly predicts observed behavior at baseline conditions and use the model's structure to predict how individuals respond to price, quality, and income changes…this conditional approach to welfare measurement differs from the more traditional unconditional approach where the structural model is used to predict both behavior at baseline conditions and responses to price, quality, and income changes," (von Haefen and Phaneuf 2005).

[51] See von Haefen and Phaneuf (2005), pages 141-146, for details.

To simulate the unobserved heterogeneity, we must first draw from the joint distribution $[f(\beta_t, \varepsilon_t|x_t)]$ of an individual t's structural parameters $[\beta_t = (\delta_t, \gamma_t, \theta_t^*, \rho_t^*, \mu_t^*)]$ and i.i.d EV draws conditional on said individuals observed trips. Note that:

(21) $$f(\beta_t, \varepsilon_t|x_t) = f(\beta_t|x_t)\, f(\varepsilon_t|\beta_t, x_t)$$

which illustrates that we may first simulate from $f(\beta_t|x_t)$ and then from $f(\varepsilon_t|\beta_t, x_t)$. Not every specification of the KT model will require simulation from $f(\beta_t|x_t)$, however. If we decide to use the random parameters classical specification, this simulation will require the use of an algorithm described in von Haefen and Phaneuf (2005), pages 147-148.

To compute values of CS^H, von Haefen and Phaneuf (2005) recommend the use of an expenditure function approach developed by von Haefen (2004), as it has been shown to be significantly faster than the utility function approach develop by von Haefen et al. (2004).[52] These computational savings arise from the fact that von Haefen et al. (2004)'s method requires the researcher to solve multiple constrained maximization problems, whereas von Haefen (2004)'s method requires a solution to only one constrained minimization problem.

Recall that under the assumption of additive separability, a respondent's Kuhn-Tucker conditions for expenditure minimization can be stated as:

(22) $$\frac{\partial u_j(x_j)/\partial x_j}{\partial u_z(z)/\partial z} \leq p_j, j = 1, \ldots, M$$

and

(23) $$\overline{U} = \sum_j^M u_j\left(x_j\right) + u_z(z)$$

[52] This approach is nevertheless detailed on pages 149-150 of von Haefen and Phaneuf (2005).

Therefore, if the researcher can solve for the optimal value of *z*, equation (22) can be used to solve for all optimal values of *x*. An iterative algorithm that identifies these solutions is as follows:

1. At iteration i, set $z_a^i = (z_l^{i-1} + z_u^{i-1})/2$. Initially, set $z_l^0 = 0$ and $z_u^0 = u_z^{-1}(\bar{U} - \sum u_j(0))$.
2. Conditional on z_a^i, solve for x_i using (22) and $\tilde{u}^i = U(x^i, z_a^i)$ using (23).
3. If $\tilde{U}^i < \bar{U}$, set $z_l^i = z_a^i$ and $z_u^i = z_u^{i-1}$. Otherwise set $z_l^i = z_l^{i-1}$and z_a^i = z_u^i.
4. Iterate until $|(z_l^i - z_u^i)| \leq c$ where c is arbitrarily small.

The general approach to solve for estimates of CS^H can be summarize as follows:

1. On each iteration, first simulate β_t from $f(\beta_t|x_t)$ and then ε_t from $f(\varepsilon_t|\beta_t, x_t)$. Recall that simulation from $f(\beta_t|x_t)$ is not required in the fixed parameter classical model, and is automatically generated at each step of the Bayesian random parameters model. The procedure necessitated by the random parameters classical model is detailed on pages 147-148 of von Haefen and Phaneuf (2005).
2. Conditional on the simulated values (β_t, ε_t), compute values of CS^H resulting from changes in travel cost and site attributes according to one of two methods:
    a. The indirect utility function approach provided by von Haefen et al. (2004), which utilizes a numerical bisection method to determine the necessary change in income required to equate baseline and altered utility levels. Each iteration uses an algorithm to solve the respondent's constrained optimization problem.

b. The expenditure function approach provided by von Haefen (2004), which uses an algorithm to determine the minimum necessary expenditure required to achieve baseline utility levels under altered conditions.

3. Average the computed values of CS^H to determine *E(CS)*, the expected value of a respondent's Hicksian surplus.

We anticipate that the KT model will outperform the MXL model's results and will provide statistically different welfare estimates and policy inference. Given how large our choice set is (20 alternatives and hundreds of respondents), if dimensionality becomes an issue with regard to convergence, we may need to aggregate the dataset up to the county level.

References

Cape Lookout National Seashore. 2016. “Cape Lookout National Seashore | BEACHES.” National Park Planner. Accessed December 2015. http://npplan.com/parks-by-state/north-carolina-national-parks/cape-lookout-national-seashore-park-at-a-glance/cape-lookout-national-seashore-beaches/

Fort Caswell. 2016. “Coastal Retreat and Conference Center.” Accessed December 2015. http://www.fortcaswell.com/

Morris, Glenn. 2005. *North Carolina beaches*. 3rd ed. Chapel Hill: The University of North Carolina Press

NC DEQ: Coastal Maps and Data. 2016. “Coastal Maps and Data.” North Carolina Department of Environmental Quality: Division of Coastal Management. Accessed December 2015. http://deq.nc.gov/about/divisions/coastal-management/coastal-management-data/coastal-maps-data

Outer Banks: Lifeguard Locations Information. 2016. “Outer Banks: Lifeguard Locations.” Brindley Beach Vacations & Sales. Accessed December 2015. http://www.brindleybeach.com/concierge/results.html?cat=Lifeguard+Locations+

Outer Banks North Carolina Rentals. 2016. “Outer Banks of North Carolina Information & Resources.” Sun Realty of Nags Head, Inc. Accessed December 2015. https://www.sunrealtync.com/outer-banks-of-north-carolina-info-resources

Outer Banks Vacation Guides. 2016. Accessed December 2015. http://www.outerbanks.com/

Phaneuf, D.J., and C. Siderelis. 2003. “An Application of the Kuhn-Tucker Model to the Demand for Water Trail Trips in North Carolina.” *Marine Resource Economics* 18(1): 1-14

Phaneuf, D.J., C.L. Kling, and J.A. Herriges. 2000. “Estimation and Welfare Calculations in a Generalized Corner Solution Model with an Application to Recreation Demand.” *The Review of Economics and Statistics* 82(1): 83-92

Sanchez, J.J., K. Baerenklau, and A. Gonzalez-Caban. 2016. “Valuing Hypothetical Wildfire Impacts with a Kuhn-Tucker Model of Recreation Demand.” *Forest Policy and Economics* 71: 63-70

The Official Travel and Tourism Website for North Carolina. 2016. Economic Development Partnership of North Carolina. Accessed December 2015. https://www.visitnc.com/

The Outer Banks. 2016. “The Outer Banks of North Carolina.” Outer Banks Visitors Bureau. Accessed December 2015. https://www.outerbanks.org/

Train, K. 2009. *Discrete Choice Methods with Simulation.* New York: Cambridge University Press

von Haefen, R.H. 2004. “Empirical Strategies for Incorporating Weak Complementarity into Continuous Demand System Models.” Working paper. Department of Agricultural and Resource Economics, University of Arizona

von Haefen, R.H., and D.J. Phaneuf. 2005. “Kuhn-Tucker Demand System Approaches to Non-Market Valuation.” *Application of Simulation Methods in Environmental Economics*, 135-157. Netherlands: Springer

von Haefen, R.H., D.J. Phaneuf, and G.R. Parsons. 2004. “Estimation and Welfare Calculation with Large Demand Systems.” *Journal of Business and Economic Statistics*, 22(2): 194–205

Wales, T.J., and A.D. Woodland. 1983. “Estimation of Consumer Demand Systems with Binding Non-Negativity Constraints.” *Journal of Econometrics* 21(3): 263-285

CHAPTER 5

CONCLUSIONS

This dissertation presents three studies which examine the recreational travel cost literature and consider new approaches to data management and demand modelling that improve the statistical efficiency and accuracy of standard travel cost methods and applications. Together, these papers provide valuable insights as well as recommendations for future econometric applications of the travel cost model.

The survey response literature has established that respondents tend to over-report their recreational activities, and correcting for “heaps and leaps” in survey response data is largely an empirical issue. Our first paper develops two approaches to treat the presence of extreme values and rounded responses in survey datasets and thereby improve model fit and resulting welfare estimates. We illustrate how, when modeling single-site trip data using a negative binomial (NB) distribution, employing the incomplete beta function simplifies the incorporation of censored intervals. We show the NB’s fit is improved by either reassigning rounded responses to censored regimes where reported trip numbers define the intervals’ upper bounds, or by mixing the NB with a continuous distribution at a cut-point where it is supposed that response behavior begins to exhibit rounding. Our analysis did not find a statistically significant difference in the parameter or per-trip consumer surplus estimates when extreme values were either truncated or incorporated under uncertainty. However, only three observations were truncated in our particular application, which may not have provided a significant enough loss of information to impact the overall estimation. We feel these results will be useful for recreation demand research and may have broad applicability to the general analysis of count data.

Much of the travel cost literature uses mixed logit (MXL) models to evaluate recreational site choice data. Multinomial probit (MNP) models are less commonly used, as their relatively cumbersome simulation procedures have made them more difficult to work with historically. Our second paper compares these models' performances and explores implications for welfare analysis. In our application using multi-site trip data, we calculate estimates of average expected maximum utility (pre and post policy implementation), as well as willingness to pay (WTP) estimates for site quality improvements and the distributions of these estimates. We find that while point estimates of average expected utility (unsurprisingly) differ between the MXL and MNP models, the patterns of sign and relative significance of our parameter estimates persist across both specifications, and the models achieve similar log likelihoods at convergence. Furthermore, our results display consistent, parallel patterns of inference across both models. In terms of WTP measures and their distributions, we find that the MNP model predicts the average benefit of a universal one foot increase in water levels to be roughly twice that of the MXL model. Interestingly, however, both models provide a very similar picture of the distribution around this WTP estimate.

With regard to methodology, we find that the Delta Method Approximation provides a computational advantage over both of the more traditional simulation procedures (Krinsky-Robb and bootstrapping) – welfare estimates for both models could be calculated in a matter of minutes as opposed to a matter of hours. The Delta Method Approximation also eliminates the need for the researcher to run multiple simulations to determine the optimal number of replications to report. While the MXL and MNP models performed similarly overall, the MNP model did fit slightly better in terms of log likelihood achieved at convergence. Given this result and the advantages and simplifications that the Delta Method Approximation presents for welfare analysis in the MNP context, we believe the MNP model warrants greater consideration in the recreational travel cost (and non-market valuation) literature.

Our third paper aims to identify and characterize preferences for beach width among North Carolina households by analyzing multi-site revealed preference trip data using travel costs and beach site attributes (beach width, beach length, number of access points, parking area, ferry-only access, and presence of lifeguards) as explanatory variables. We employ a MXL model in our recreation demand analysis and discuss the advantages of incorporating a Kuhn-Tucker generalized corner solution model in future extensions of this analysis.

The different specifications on the beach width measurement that we compare (log of average width and log of median width) perform very similarly and provide nearly identical pictures of the influence of the explanatory variables on trip demand, though there are noticeable differences in the parameter estimates for lifeguards, ferry access, and number of access points. As a result, the confidence intervals around some site attributes' WTP estimates do not always overlap between the models. The signs on the parameters are generally what we expect. We find that travel costs and requiring the arrangement of a boat or ferry ride to access a beach have a negative influence on trip demand. Length is also found to have a (small) negative influence, perhaps because the walking distance between beach amenities such as restrooms and beachside attractions is greater on longer beaches. Number of access points, the amount of parking area, and the presence of lifeguards are all found to have positive impacts on trip demand. It is somewhat surprising, however, that we find beach width has a negative influence on recreation demand, and we discuss possible justifications for this result.

Nevertheless, while beach erosion and sea level rise directly impact beach width, they also threaten beach access in general. In certain sea level rise scenarios, the number of beach access points could be reduced and the proportion of beaches accessible only by ferry or boat could increase. Given the results of our welfare analysis, we find that these consequences could have substantial economic impacts and should be considered in future natural resource management decisions for the Outer Banks region. Overall, this paper provides insights that we feel will be of particular interest to coastal management authorities. The welfare estimates we obtain have immediate policy relevance, and provide context for future work with this Sea Grant dataset (of which the revealed preference responses are but a subset).

REFERENCES

Afonja, B. 1972. “The Moments of the Maximum of Correlated Normal and t-Variates.” *Journal of the Royal Statistical Society. Series B (Methodological)* 34: 251-262

Arellano-Valle, R.B., and M.G. Genton. 2008. “On the Exact Distribution of the Maximum of Absolutely Continuous Dependent Random Variables.” *Statistics & Probability Letters* 78: 27-35

Bebu, I. and T. Mathew. 2009. “Confidence intervals for limited moments and truncated moments in normal and lognormal models.” *Statistics & Probability Letters* 79(3): 375-380

Callaway, J.M., S. Ragland, S. Keefe, T.A. Cameron, and W.D. Shaw. 1995. “Columbia River Systems Operation Review Recreation Impacts: Demand Model and Simulation Results.” RCG/Hagler Bailly, Inc. Final report prepared for Matthew Rae and the Recreation Work Group, U.S. Army Corps of Engineers, Portland, Oregon

Cameron, A. C., and P. K. Trivedi. 2013. *Regression analysis of count data, 2nd ed.* Cambridge, NY: Cambridge University Press

Cape Lookout National Seashore. 2016. “Cape Lookout National Seashore | BEACHES.” National Park Planner. Accessed December 2015. http://npplan.com/parks-by-state/north-carolina-national-parks/cape-lookout-national-seashore-park-at-a-glance/cape-lookout-national-seashore-beaches/

Chen, H.Z., and S.R. Cosslett. 1998. “Environmental Quality Preference and Benefit Estimation in Multinomial Probit Models: A Simulation Approach.” *American Journal of Agricultural Economics* 80: 512-520

Chu, A., D. Eisenhower, M. Hay, D. Morganstein, J. Neter and J. Waksberg. 1992. “Measuring the recall error in self-reported fishing and hunting activities.” *Journal of Official Statistics* 8(1): 19-39

Connelly, N. A., and T. L. Brown. 1995. “Use of angler diaries to examine biases associated with 12-month recall on mail questionnaires.” *Transactions of the American Fisheries Society* 124(3): 413-422

Evans, K., and J. A. Herriges. 2010. “Rounding in recreation demand models: A latent class count model.” Iowa State University. Working paper

Fort Caswell. 2016. “Coastal Retreat and Conference Center.” Accessed December 2015. http://www.fortcaswell.com/

Gurmu, S. and P. K. Trivedi. 1992. “Overdispersion tests for truncated Poisson regression models.” *Journal of Econometrics* 54: 347-370

Haab, T. C., and K. E. McConnell. 2002. *Valuing Environmental and Natural Resources: The Econometrics of Non-Market Valuation*. Cheltenham: Edward Elgar

Hoehn, J. P., T. Tomasi, F. Lupi and H. Z. Chen. 1996. “An economic model for valuing recreational angling resources in Michigan.” Michigan State University. Report to the Michigan Department of Environmental Quality and to the Michigan Department of Natural Resources

Huttenlocher, J., L. V. Hedges, and N. M. Bradburn. 1990. "Reports of elapsed time: Bounding and rounding processes in estimation." *Journal of Experimental Psychology: Learning, Memory, and Cognition* 16(2): 196-213

Johnson, N. L., S. Kotz and A. W. Kemp. 1992. *Univariate Discrete Distributions*. 2nd ed. New York, NY: Wiley

Manski, C. F., and F. Molinari. 2010. "Rounding probabilistic expectations in surveys." *Journal of Business & Economic Statistics* 28(2): 219-231

McCullagh, P. and J.A. Nelder. 1989. *Generalized Linear Models*. 2nd ed. New York, NY: Chapman & Hall/CRC

Miller, C. A., and W. L. Anderson. 2002. "Digit preference in reported harvest among Illinois waterfowl hunters." *Human Dimensions of Wildlife* 7(1): 55-65

Moeltner, K. 2006. "Anglers' Fishing Choices on the Truckee, Carson, and Walker Rivers and the Threat of the New Zealand Mud Snail." Report prepared for the Pacific States Marine Fisheries Commission and the U.S. Fish and Wildlife Service, June 27, 2006

Morris, Glenn. 2005. *North Carolina beaches*. 3rd ed. Chapel Hill: The University of North Carolina Press

NC DEQ: Coastal Maps and Data. 2016. "Coastal Maps and Data." North Carolina Department of Environmental Quality: Division of Coastal Management. Accessed December 2015. http://deq.nc.gov/about/divisions/coastal-management/coastal-management-data/coastal-maps-data

Outer Banks: Lifeguard Locations Information. 2016. "Outer Banks: Lifeguard Locations." Brindley Beach Vacations & Sales. Accessed December 2015. http://www.brindleybeach.com/concierge/results.html?cat=Lifeguard+Locations+

Outer Banks North Carolina Rentals. 2016. “Outer Banks of North Carolina Information & Resources.” Sun Realty of Nags Head, Inc. Accessed December 2015. https://www.sunrealtync.com/outer-banks-of-north-carolina-info-resources

Outer Banks Vacation Guides. 2016. Accessed December 2015. http://www.outerbanks.com/

Ozuna, T. Jr., and I. A. Gomez. 1995. “Specification and testing of count data recreation demand functions.” *Empirical Economics* 20(3): 543-550

Parsons, G. R., D. M. Massey, and T. Tomasi. 1999. “Familiar and favorite sites in a random utility model of beach recreation.” *Marine Resource Economics* 14(4): 299-315

Phaneuf, D.J., and C. Siderelis. 2003. “An Application of the Kuhn-Tucker Model to the Demand for Water Trail Trips in North Carolina.” *Marine Resource Economics* 18(1): 1-14

Phaneuf, D.J., C.L. Kling, and J.A. Herriges. 2000. “Estimation and Welfare Calculations in a Generalized Corner Solution Model with an Application to Recreation Demand.” *The Review of Economics and Statistics* 82(1): 83-92

Pudney, S. 2008. “Heaping and leaping: survey response behaviour and the dynamics of self-reported consumption expenditure.” University of Essex. Institute for Social and Economic Research. Working paper

Sanchez, J.J., K. Baerenklau, and A. Gonzalez-Caban. 2016. “Valuing Hypothetical Wildfire Impacts with a Kuhn-Tucker Model of Recreation Demand.” *Forest Policy and Economics* 71: 63-70

Sarker, R., and Y. Surry. 2004. “The fast decay process in outdoor recreational activities and the use of alternative count data models.” *American Journal of Agricultural Economics* 86(3): 701-715

Schaeffer, N. C., and S. Presser. 2003. “The science of asking questions.” *Annual Review of Sociology* 29: 65-88

Shonkwiler, J. S. 2016. “Variance of the truncated negative binomial distribution.” *Journal of Econometrics* forthcoming

Tarrant, M. A., and M. J. Manfredo. 1993. “Digit preference, recall bias, and nonresponse bias in self reports of angling participation.” *Leisure Sciences* 15(3): 231-238

The Official Travel and Tourism Website for North Carolina. 2016. Economic Development Partnership of North Carolina. Accessed December 2015. https://www.visitnc.com/

The Outer Banks. 2016. “The Outer Banks of North Carolina.” Outer Banks Visitors Bureau. Accessed December 2015. https://www.outerbanks.org/

Tourangeau, R., L. J. Rips, and K. A. Rasinski. 2000. *The Psychology of Survey Response.* Cambridge: Cambridge University Press

Train, K. 2009. *Discrete Choice Methods with Simulation.* New York: Cambridge University Press

Vaske, J. J., and J. Beaman. 2006. “Lessons learned in detecting and correcting response heaping: Conceptual, methodological, and empirical observations.” *Human Dimensions of Wildlife* 11(4): 285-296

von Haefen, R.H. 2004. “Empirical Strategies for Incorporating Weak Complementarity into Continuous Demand System Models.” Working paper. Department of Agricultural and Resource Economics, University of Arizona

von Haefen, R.H., and D.J. Phaneuf. 2005. “Kuhn-Tucker Demand System Approaches to Non-Market Valuation.” *Application of Simulation Methods in Environmental Economics,* 135-157. Netherlands: Springer von Haefen, R.H., D.J. Phaneuf, and G.R. Parsons. 2004. “Estimation and Welfare Calculation with Large Demand Systems.” *Journal of Business and Economic Statistics*, 22(2): 194–205

Wales, T.J., and A.D. Woodland. 1983. “Estimation of Consumer Demand Systems with Binding Non-Negativity Constraints.” *Journal of Econometrics* 21(3): 263-285