

EXPERIENCED COLLEAGUES' OBSERVATION FEEDBACK ON MATHEMATICS  
TEACHING PRACTICES OF FIRST-TIME COLLEGE CALCULUS INSTRUCTORS

by

JADONNA BREWTON

(Under the Direction of James W. Wilson)

ABSTRACT

Having experienced colleagues observe new mathematics instructors' teaching and provide feedback is an integral part of professional development for new teachers. This descriptive multiple-case study investigated the nature and influence of experienced colleagues' feedback with the following questions: (a) What mathematical areas or mathematics instructional practices do experienced colleagues notice and address in their feedback to new instructors, and (b) In what ways do new instructors describe how they plan to modify their mathematics instructional practices based on experienced colleagues' feedback? Categories of mathematics instructional practices were identified using the Mathematical Knowledge for Teaching (MKT) and Mathematical Understanding for Secondary Teaching (MUST) frameworks. Four new instructor/experienced colleague mentor pairs (eight participants) from the mathematics department at an undergraduate military institution were the subjects of this cross-case synthesis. During one semester, each pair completed one or two observation cycles consisting of a pre-observation meeting, classroom observation, and feedback session. I witnessed each portion of the cycle and separately interviewed each member after the feedback session. I analyzed meeting and interview data and journal writings. Findings suggest the

primary feedback focus was on the new teachers' knowledge of content and students (KCS) and teaching (KCT). Teachers' reflection on the mathematics of practice and instructional actions primarily focused on mathematical representations and meaning, accessing and understanding the mathematical thinking of learners, assessing the mathematical knowledge of learners, and developing learners' mathematical reasoning skills. This research suggests how to foster meaningful feedback and potentially establish a sustainable and effective form of professional development for mathematics teachers.

**KEYWORDS:** college calculus teaching, feedback, mathematical knowledge for teaching (MKT), mathematical understanding for secondary teaching (MUST), mathematics teaching, mentoring, peer or colleague observation of teaching, professional development, reflective practice

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## DEDICATION

I honor those who invested in me with genuine unconditional love & encouragement:

To my Mama: Ardrey Anderson Brewton, who is unrelenting with a stubborn belief in the best of me—in spite of my many flaws and failures—and tirelessly nudged me along this marathon. Know with swelling pride, anything good I accomplish is because everything you have given me is more priceless than infinite quantities of platinum gold.

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The completion of this endeavor **Proves He Delivers!**

Bless the Lord, oh my soul, and all that is within me!

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## CHAPTER 1

### INTRODUCTION

The purpose of this chapter is to provide the motivation for this study. The first section gives the research questions. The next section describes my background, association with the research site, and interest for the study. The final section presents the rationale and significance of the study.

#### **Research Questions**

As socially cognitive beings, we often learn from others who have more experience or expertise in an activity we want to master (Bruning, Schraw, & Norby, 2011). While we engage in the activity, we learn from our own actions by assessing how well we perform. This assessment is often a combination of others' and our own evaluation. Having the relatively unbiased insight of someone with more experience stimulates self-reflection and postures us to consider ways to help us improve our own performance (Bandura, 1977). Learning to teach mathematics more effectively should be no different, particularly for new instructors. That is, having experienced colleagues observe new mathematics instructors' teaching and provide feedback should be an integral part of professional development. Using this idea as a premise, my study investigated the following question: In what ways do experienced colleagues' observations and feedback influence how new instructors reflect upon and plan to modify their mathematics instructional practices? My supporting questions were as follows:

- What mathematical areas or mathematics instructional practices do experienced colleagues notice and address in their feedback to new instructors?

- In what ways do new instructors describe how they plan to modify and/or begin to modify their mathematics instructional practices based on experienced colleagues' feedback?

### **Background**

As an assistant faculty development director for 4 years in the mathematics department at a four-year undergraduate military institution, I helped evolve a faculty mentorship and colleague observation program. The goal of the faculty development program was to prepare new instructors for the classroom and continue an ongoing development effort to establish and maintain effective instructional practices throughout the department. Colleague observation and feedback for all faculty members throughout the entire department from newest to most senior faculty was a standard practice in this department. In particular, each new instructor was paired with a more experienced colleague mentor with a doctoral degree and more than 10 years teaching experience. The experienced colleague observed the new instructor in the classroom and provided feedback during informal, one-on-one sessions.

My role was to keep a pulse of the department. I regularly observed instruction and mentally noted the effectiveness of our program. The more I saw and heard, the more I wanted to better understand how effectively and in what ways our program influenced instructional practices. Much of my work led me to believe the program was extremely effective and worth advocating for other mathematics departments. The most compelling evidence was the confidence with which our new instructors embraced their role in the classroom. Having listened to their informal accounts of how being observed and observing others helped them prepare for, execute, and reflect upon instruction; I was convinced our observation program was a valuable faculty development mechanism. In an interview, one new instructor said the following:

I think it's a perspective from having someone else see you from the outside. Having those experiences of being observed lets me reflect and take a little bit of a pause to think,

“What are we really doing? How is this really going?” It lets me self-reflect. It creates a space in my day to say, “How do I think I’m doing? How do other people perceive what I’m doing?” I have gotten a lot of good feedback! (First-year calculus instructor interview)

### **Rationale**

The traditional culture of United States schools has not promoted openness to classroom observations because observations are often negatively associated with formal and punitive measures of performance conducted by administrators who offer little or no feedback fostering effective pedagogy. Teachers are more receptive, however, to informal collaborative efforts that are teacher-initiated and teacher-owned. Fortunately, there is growing advocacy for professional development using collaboration among teachers to promote effective teaching (Kardos, 2004). Gosling (2009) cited a collection of literature supporting the use of peer observation of teaching as a teacher development tool. His work closely examined the defining characteristics needed for successful implementation of the practice. My study went beyond description to implementation of the practice of peer observation of teaching by highlighting the practice among experienced colleagues who observed new instructors’ teaching of college calculus and provided feedback. More attention on teaching and the creation of professional development resources is needed to help new instructors improve their mathematics teaching. Research of this sort is a valuable resource for people designing professional development opportunities for novice college teachers (Speer, Smith, & Horvath, 2010).

In Chapter 2, I briefly discuss research in collegiate mathematics. My study was situated in college calculus courses because more attention on teaching practices and development for new instructors of college mathematics is needed to better understand potential ways to improve instruction. Reflective practice is the theoretical basis supporting the premise of the study—reflection is the catalyst for analysis, change, and improvement of mathematics instruction.

Literature supporting colleague observation and feedback as a professional development tool to facilitate reflection is also presented. Mathematical knowledge and understanding for teaching are frameworks providing a foundation to focus observations, a descriptive language about mathematics instruction to guide discussions, as well as a vision of effective instruction setting a standard to which a new teacher can aspire to improve his or her practice.

## CHAPTER 2

### LITERATURE REVIEW

The purpose of this chapter is to summarize literature relevant to my research questions. This study was situated in college calculus courses because more attention on teaching practices and development for new instructors of college mathematics is needed to better understand potential ways to improve instruction in this pivotal course. First, I give an overview of research in collegiate mathematics teaching in general and in calculus. Then I describe the theoretical basis situated within the conceptual framework of reflective practice. It continues with a review of reflective practice in mathematics education. The next section highlights research on colleague observation and feedback as a professional development tool. The final section describes the frameworks of mathematical knowledge and understanding for teaching.

#### **Collegiate Mathematics Teaching**

Speer et al. (2010) conducted an exhaustive review of published peer-reviewed literature on the scholarship of collegiate mathematics teaching and concluded there was very little empirical research at the collegiate level describing and analyzing what mathematics teachers do and think daily in class as part of their classroom practices. The systematic review by Speer et al. (2010) spanned the previous 10 years and is described below:

- (1) They searched electronic databases (ERIC, JSTOR, and RUME) and Google Scholar using the following search words: “mathematics, college, collegiate, undergraduate, university, teaching, teacher, education” (p. 102).
- (2) They manually searched journals specific to the field of study, including but not limited to the following: *American Mathematical Monthly*, *College Mathematics Journal*, *Educational Studies in Mathematics*, *For the Learning of Mathematics*, *International Journal of Mathematics Education in Science and Technology*, *Journal of*

*Mathematical Behavior, Journal of Mathematics Teacher Education, Journal for Research in Mathematics Education, Mathematical Thinking and Learning, Notices of the American Mathematical Society, School Science and Mathematics, and Research in Collegiate Mathematics Education.* They also examined books published in the *Mathematical Association of America Notes* series and volumes in Erlbaum's *Studies in Mathematical Thinking and Learning* series (p. 102).

(3) They asked colleagues who had conducted empirical research on collegiate mathematics education to identify articles focusing on collegiate teachers' classroom practice. Publications satisfied four criteria: (a) reported disciplined inquiry framed by research questions, (b) contained empirical data collected and analyzed on teachers and teaching, (c) included descriptions of teachers' classroom practice, and (d) reflective research carried out in mathematics courses taught to a wide range of students (including majors) rather than to specific populations such as pre-service teachers (p. 102).

Some studies were mathematicians' memoirs or reflections about past teaching experiences recalling classroom events and judgments about the success of teaching practices. Others focused on issues of teaching or learning in repeated cycles of teaching particular or innovative collegiate courses. They analyzed aspects of their teaching and their students' learning. Others researched the impact of instructional activities on student learning and engagement. Activities included computer-based lab work and small group cooperative learning or problem-solving. Research generally examined the effect on students' engagement and achievement attributed by these instructional methods in contrast to lecture alone. Although this research is empirical and descriptive, it was not focused on teachers' practices.

Studies targeting classroom practices of precollege level mathematics teachers are more prevalent. According to Speer et al. (2010),

Such studies have been productive in understanding the choices and acts of teaching, the factors shaping them, and the design and practice of teacher education. Similar research at the collegiate level holds equal promise for understanding teachers' choices (and rationales for them) and for aiding beginners by informing the design of professional development. (p. 100)

The study of mathematical practices at the secondary level has been productive in helping mathematics educators better understand how teachers make pedagogical decisions, enact their

instructional practices, and engage students. More research at the collegiate level is imperative for acquiring the same understanding at this level. In the interim, one could justify applying some knowledge from the more robust body of work at the secondary level. For a list of the specific studies, see Speer et al.'s *Collegiate Mathematics Teaching: An Unexamined Practice* (2010).

**College calculus teaching.** The President's Council of Advisors on Science and Technology (PCAST) (2012) discussed the economic rationale for finding ways to improve the teaching of introductory science, technology, engineering, and mathematics (STEM) courses—Calculus 1 in particular. There are an increasing proportion of STEM occupations, as well as other jobs, requiring STEM disciplinary knowledge. In order to have a sufficient number of candidates in the United States for these careers, universities must produce, over the next decade, about one million more STEM graduates than predicted by current graduation rates. PCAST (2012) reported the number of STEM majors is decreasing at an alarming rate. Less than 40% of students who enter college planning to be a STEM major actually complete a degree in a STEM field. Some students who leave STEM majors “describe the teaching methods and atmosphere in introductory STEM classes as ineffective and uninspiring” (p. 5). There is a dire need for better teaching of Calculus 1, because it is commonly considered a “gateway course for many other STEM fields” (p. 27). PCAST (2012) urged researchers to “launch a national experiment in postsecondary mathematics education to address the mathematics-preparation gap” (p. 27).

Seymour (2006) had previously reported findings similar to PCAST (2012). Based on exit interviews with students who changed from STEM major tracks and graduating seniors in STEM tracks, she discovered poor learning experiences were the most common complaint. Students attributed these poor learning experiences to lack of discussion of conceptual material,

faculty's implicit or explicit dislike for teaching, courses attempting to cover too much material too fast, and becoming bored with introductory courses even when incoming interest was strong. Seymour (2006) suggested, "The balance of status and rewards has, over time, tipped heavily towards research and away from teaching" (p. 2). The result is many faculty members no longer engage in interactive teaching functions such as tutorials, seminars, and individual mentoring and advising of students. "Straight lecturing"—a seemingly less effective pedagogical strategy for learning mathematics—has largely become faculty's dominant mode of contact with students (p. 2). The overall quality of undergraduate education has suffered as a result. Seymour recommended institutions place greater value on teaching, and implement systems of professional development for faculty. Productive solutions to the problem of STEM major retention may lie in the direction of improving the teaching of Calculus 1 by first identifying and documenting good teaching practices.

In 2010, the Mathematical Association of America (MAA) began a 5-year study of Calculus 1 programs. The study included case studies of successful Calculus 1 programs conducted by multidisciplinary research teams. A team of National Science Foundation (NSF)-sponsored researchers conducted a national study, under MAA, titled *Characteristics of Successful Programs in College Calculus (CSPCC)* (2012). The study surveyed 160 institutions across the United States to identify the demographic makeup of students in Calculus 1, to measure the impact of the various characteristics of calculus classes believed to influence student success, and to identify successful programs. These programs became the focus of follow-up studies to determine what institutional factors contributed to their success. The following sections describe these studies and their findings.

Dawkins (2014) conducted a study at a large (25,000), suburban institution in the southwestern United States. The mathematics department strongly coordinated the teaching of calculus by having a common text, common examinations, and shared homework lists. Instructors taught using a lecture/recitation structure. Each week students attended 150 minutes of large lecture (two to three sessions, class size  $\leq 60$ ) and 100 minutes of recitation (two meetings, class size  $\leq 40$ ). The data presented for this study came from two of the large lecture sections (four recitation sections) made up of 135 total participants. The study investigated interactions between calculus learning and problem-solving. Students' problem-solving abilities were assessed during focused recitation sessions with small-group problem-solving, small- and large-group discussions, student and group presentations, constant group and instructor feedback, and reflection upon in-depth mathematical tasks. Overall findings indicated blended reform/traditional instruction improved students' problem-solving—providing some affirmation of reform for improving problem-solving. The calculus instruction in this study also significantly improved students' performance on non-routine problems. Dawkins (2014) advocated for greater efforts to make calculus courses more concept-driven and multi-representational to better align with the vision for calculus reform and NCTM (2000) Standards.

Sonnert, Sadler, Sadler, and Bressoud (2015) conducted a national multivariate analyses study of 3103 students at 123 United States colleges and universities to track changes in students' attitudes toward mathematics in a calculus course. Data [two student surveys (one at the beginning and the other at the end of the calculus course), two instructor surveys (again one at the beginning and the other at the end of the calculus course), and one survey of course coordinators at participating institutions] was drawn from the CSPCC project. Instructors characterized as having “good teaching” practices had the most positive impact on students.

These practices included asking questions to check students' understanding, listening carefully to students' questions, acting as if students were capable of understanding calculus, and providing understandable explanations of key ideas.

Bagley's (2015) study described four different Calculus 1 classes taught at a large and highly diverse public university in the southwestern United States. The study examined the impact of different instructional approaches on students' attitudes, dispositions and beliefs about mathematics, and their conceptual and procedural achievement in calculus. The data collected came from the following sources: enrollment data, surveys, measures of achievement, classroom observations, and focus group interviews. Data from classroom observations was used to identify "theoretically-plausible explanatory relationships between the opportunities for learning provided by the structure of each class and student persistence, attitudes, and achievement" (Bagley, 2015, p. 46). Four to ten students per class session participated in focus group semi-structured interviews. Data was analyzed using a mixed methods approach. A number of positive impacts were reported. An instructor's non-threatening demeanor and relatively small class size encouraged students to feel comfortable asking questions. Asking students to provide feedback on how to solve problems also helped students stay involved. Classes with more affordances for students' engagement and participation in classroom activities were seen as presenting more opportunities for student learning. The opportunity to work problems with other students in class is a prime example. According to Bagley (2015), working with the instructor during active-learning activities is also a rich opportunity for engaging students with challenging material while the instructor is physically present to provide scaffolding and support.

Burn, White, and Mesa (2016) presented the community college case study findings and highlighted the benefit of the multidisciplinary research teams. Their study identified factors

jointly contributing to Calculus 1 program success—including high-quality instruction, academic and social support for students, loose course coordination, a culture of faculty autonomy and trust, and attention to course placement and transfer policies. Faculty professional development to improve interactive lectures was a primary focus area. Data from faculty interviews, student focus groups, and classroom observations, revealed features of high-quality instruction.

Effective instructors were described as being approachable and available, possessing abundant content knowledge, and having high expectations for students' mathematical learning.

Instructors created opportunities for students to practice skills, used substantial amounts of questions, and created assignments to increase conceptual understanding and integrate ideas.

Burn et al. (2016) noted the influence of conventional resources (e.g., class size, technology, and textbooks) to positively shape instruction. The instructors' ability to build relationships with students, to create integrative assignments, and to use technology effectively to help students visualize calculus concepts were noted as instructors' personal resources having a positive effect. Faculty members enjoyed much freedom and latitude in teaching and were trusted by their colleagues and administration to do the best for their students. Informal peer communication was the main mechanism for professional support. Coordination across sections was sought through informal faculty collaboration as well as more formal Calculus 1 committees providing coordination through common course outlines and/or common textbooks.

The body of college calculus teaching research cited above laid the groundwork for my study. There is a resounding call for research designed to better understand the characteristics and professional development needed to improve calculus instruction. My study answers this call. It was positioned to capture the evolution of instructional practices by investigating new instructors who engaged with experienced colleagues in reflection on their practices. Through

this critical two-way lens, they envisioned how to improve their work of teaching using a collaborative developmental activity. The process of improvement is a cycle in which they exchanged perspectives, experimented with instructional modifications, assessed the results, received more feedback, enacted more modifications, and repeated the cycle. The core element in the journey to improved practice is reflection.

## **Reflective Practice**

### **Theoretical Foundation for Reflective Practice**

An important outcome for receiving feedback is to stimulate reflection about one's teaching. Critical reflective teaching involves thought and action. It raises teachers' consciousness of their actions and causes them to scrutinize their beliefs and knowledge of the subject and their practice. The ability to reflect on one's teaching is necessary for effective instruction. Teachers gain a better understanding of their teaching and ultimately improve their instructional practices when they engage in critical self-reflection. As such, the conceptual framework of *reflective practice* was used as the theoretical basis for this study. The theory and research applications about reflection and reflective practice are based on the work of Dewey (1933) and Schön (1983; 1987).

Dewey was among the first American educational theorists to view teachers as reflective practitioners. He believed the ideal teacher engages in an ongoing process of reflection to continuously improve instruction. Rodgers (2002) distilled four criteria from Dewey's writing characterizing Dewey's concept of reflection and its purposes:

- Reflection is a meaning-making process which moves a learner from one experience into the next with deeper understanding of its relationship with and connections to other experiences and ideas
- Reflection is a systematic, rigorous, disciplined way of thinking, with its roots in scientific inquiry
- Reflection needs to happen in communities, in interaction with others

- Reflection requires attitudes valuing the personal and intellectual growth of oneself and of others (p. 845)

Like Dewey, Rodgers (2002) regarded reflective thinking as essential for teachers' learning. She summarized the process as follows: Teachers reflect on their work by relating their understanding of events from their classroom to the rationale for their lesson plans. These reflections lead to changes in their practices and efforts to acquire resources for planning follow-up lessons.

Schön (1983; 1987) first used the terms *reflection-in-action* and *reflection-on-action* to describe teachers' thinking in their classroom practice. The term *reflection-in-action* describes teachers' reflection while they are teaching (e.g., Are students engaged in the task? Are they bored? Should I move on to a new topic?). The term *reflection-on-action* is retrospective thinking after an event. Lee and Tan (2004) found Schön's ideas to be especially beneficial to teacher educators because they are directly associated with daily classroom situations and the kinds of thinking processes accompanying teachers' work. According to Procee's (2006) philosophical analysis, reflective practitioners view their classroom experiences as opportunities to learn. They are concerned about the contexts of their practices and the implications for action; that is, they reflect on their assumptions in conjunction with theories of practice and take action.

### **Reflective Practice in Teaching**

Sowder (2007) believed teachers who engage in reflective practices increase their learning and improve their practice. They plan more effectively, have greater ability to anticipate students' difficulties, and know how to scaffold knowledge to assist students in developing understanding. Hill, Sleep, Lewis, and Ball (2007) asserted teachers who can describe, explain, and reflect on their work are potentially better teachers. Hill et al.'s rationale is as follows: "The ability to articulate one's practice is an indicator of deliberateness, and the

ability to write cogent reflections is an indicator of analytic capacity” (p. 145). This view echoes Darling-Hammond (1998) who stated, “Teachers need to be able to analyse and reflect on their practice, to assess the effects of their teaching and to refine and improve their instruction” (p. 198).

A teacher’s reflection can be enhanced by engaging in reflective dialogue with colleagues (Pollard, 2002; York-Barr, Sommers, Ghere, & Montie, 2006; Zeichner & Liston, 1996). York-Barr et al. (2006) believed reflecting on practice with a colleague can greatly enrich a teacher’s understanding by promoting awareness of fixed assumptions discovered by getting another’s perspective. This is a crucial element in the process of analyzing and improving one’s practice. Farrell (2004) suggested collaborative activities such as group discussions and colleague observations enhance teacher reflection. Teachers who productively engage in reflection on their practice may potentially acquire deeper understanding of their teaching, growth in professional knowledge, greater effectiveness as a teacher, and ultimately improve instructional practices.

Vidmar (2006) described a model for reflective peer coaching in higher education as “a formative model for improving teaching and learning by examining intentions prior to teaching, then reflecting upon the experience” (p. 135). The goal is to promote self-assessment and collaboration for better teaching and ultimately better learning. At the core of the process is a nonthreatening, collegial relationship facilitating conversation and collaboration between peers. When instructors reflect on their classroom experiences with a colleague, they learn about the intended results in comparison with the actual lesson by acknowledging both accomplishments and frustrations. “By making the collegial conversations part of instruction, instructors build upon the everyday classroom experiences, complementing class time with the conversations before and after teaching” (Vidmar, 2006, p. 136). Teachers become more conscious in the

classroom and use the thinking in concert with performance to manage their actions. They continually address and self-monitor their instructional practices and ultimately learn through critical reflection on their experiences. The two main elements of reflective peer coaching are outlined as follows:

1. Planning conference
  - Clarify intentions: What are the lesson goals and objectives?
  - Teaching strategy and procedures: What will the teacher do?
  - Student achievement: What will the students do to indicate success?
  - Data to support self-assessment: What is important to the teacher?
2. Reflective conference
  - Assessment of lesson: How did the lesson go?
  - Recall data to support reflections
  - Compare intentions with the actual lesson: What was different and why?
  - Effect on future lessons: New learning, discoveries, insights
  - Comment on the coaching process and refine as needed (Vidmar, 2006, p. 146)

In the model Vidmar (2006) described, it is not necessary for the peer to be more experienced or observe the other instructor in the classroom. The model resembles the one used in this study because it includes reflective discussions both before and after a classroom session. However, it lacks the real-time observation by a more experienced colleague component serving as the primary activity for discussion in the feedback session.

### **Reflective Mathematics Teaching Research**

**Collegiate.** Breen, McCluskey, Meehan, O'Donovan, and O'Shea (2014) reported on a professional development project study aimed at developing sustained reflection on and critique of their teaching practice using Mason's (2011) Discipline of Noticing:

The discipline of noticing is a collection of techniques for (a) *pre-paring* to notice in the moment, that is, to have to come to mind appropriately and (b) *post-paring* by reflecting on the recent past to select what you want to notice or be sensitized to particularly, in order to *pare*, that is, to notice in the moment. (p. 35)

The five authors were university-level mathematics lecturers in Ireland with doctoral degrees in either pure or applied mathematics. Four of five of them had more than 12 years teaching experience; the fifth had 8 years. They kept written accounts of moments or critical incidents from their teaching, periodically provided them to the others, and held discussions as a group every 6 to 8 weeks. At the end of the academic year, each lecturer reflected on her experiences with the process and discipline of noticing. Their reflections highlighted the “benefits of collaboration, challenges and benefits of writing brief-but-vivid accounts, and challenges of noticing in-the-moment” (p. 293). They particularly valued the opportunities to discuss their classroom practices with each other and learning from what others do in practice.

Two common themes emerged from the accounts: promoting student engagement and gauging student understanding. Although the participants engaged in collaborative reflection as they shared their accounts, the actual accounts focused solely on individuals’ isolated perspectives of unobserved events in their classroom. Furthermore, the motivation for selecting particular events about which accounts were written is confined to each author’s personal preferences. There might be concern because of some measure of subjectivity; however, this is not necessarily a negative consequence. This subjectivity became a means by which the lecturers identified specific practices to improve. The study differs from my study because the lecturers had more mathematics teaching experience than the new instructors in my study.

Jaworski and Matthews (2011) reported on the analysis of data from a series of 10 video recorded seminars presented by university mathematics teachers [four from established mathematicians (who do research in mathematics), two from former mathematicians (who now do research in mathematics education), three from mathematics educators (who do research in mathematics education) and one from a university teacher (who does no research)] on the topic

“How We Teach.” The two-and-a-half year study took place in a school of mathematics which included a Mathematics Education Centre and focused on research into university mathematics learning and teaching. The seminars were each presented by a mathematician or a mathematics educator to an audience of teachers of mathematics or engineering students. The researchers intended to reveal how mathematics teachers talk about their teaching in the following ways: “what teachers say they do, how and why, and the related discussion between seminar participants, covering approaches to teaching, reasons behind these approaches, and issues arising for the presenter, or in discussion” (p. 1). The authors highlighted issues emerging from discussion around various approaches to teaching, use of resources, and interaction with students:

- *Strategies/Approaches to teaching*: use of questions, tests, examples, group work, animations, video, weblinks, electronic voting systems, and computer assisted assessment
- *Provision of resources for students*: notes, problem sheets, solutions, and a series workbooks focusing on basic mathematical concepts (developed for engineering students, used with various groups of students in mathematics and engineering, provided free to students)
- *Approaches to/relationships with students*: different ways in which lecturers and students meet each other and kinds of interactions that result (p. 4)

Discussions addressed being intentional in providing instruction more likely to encourage student attendance; the role, nature, and production responsibility of lecture notes; how students come to understand mathematics; ways to promote understanding; use of technology in lecture presentations; and teaching and learning resources. The authors characterized the sessions as follows: “In the main, the discourse was meta-mathematical, assuming the basis of mathematics but talking about processes and practices rather than focusing *in* the mathematics” (p. 9). This study also lacks an observation component in which a second pair of eyes witnessed classroom

interactions. Instead, accounts were based on a single teacher's perspective of generalizations of instructional issues rather than specific practices during particular lessons.

McAlpine and Weston (2000) analyzed the reflective processes of six exemplary university professors (three were mathematicians, three were mathematics educators) in their day-to-day planning, instructing, and evaluating of learners. They analyzed only the professors' cognition about their teaching rather than their in-class actions. Each professor was videotaped in one-third of each of the 39-hour courses. Participants were interviewed before and after class and reviewed a video of class sessions during the post class interviews to stimulate recall about their reflections during teaching. When the researchers finished their analysis, they held a symposium to present the results and the model representing the metacognitive process of reflection. The following is the authors' summary of the model:

In the process of reflection we documented, experience is the anchor, both the grounds on which the reflection is based and the *action* that results when decisions resulting from reflection are enacted in teaching activity. We found that these professors *monitored* their teaching actions to achieve their *teaching* and *learning goals*, prior to, concurrent with and retrospective to instruction.

When monitoring they attended to and evaluated a multitude of *cues*, the most salient being student responses to their teaching. We contend that this attention to student cues results from their recognition of the link between their instruction and the learning process and that external student cues are the primary vehicle for assessing what is happening in terms of student learning.

When *decision-making*, professors were deciding in relation to their goals to adjust or modify teaching actions dependent on where the cues fell in relationship to the *corridor of tolerance*; such changes were mostly to method and content. Ongoing use of the processes of monitoring and decision-making link knowledge and action, and are essential for building and accessing *knowledge*. Increasing knowledge increases one's ability to reflect effectively and develop as a teacher. (p. 371)

By the end of the symposium, the professors used the language of the model to discuss how they evaluated their teaching. They also had considerable knowledge about learners and used this knowledge to reflect on the impact of their teaching. The authors reported the following:

- Professors attended most to goals related to instructional methods (33%), next to student understanding (26%), and then to content (24%)
- More than 70% of all cues monitored were related to students' verbal and non-verbal cues
- Most modifications were made to instructional methods (52%) and to content (43%).
- Professors drew most heavily on pedagogical knowledge (34%) to articulate their rationales for monitoring and decision making as well as knowledge of learners (20%). These were followed by pedagogical content knowledge and content knowledge (pp. 368-370)

The authors highlighted an important observation: “Skilled teachers have developed knowledge enabling them to be more pointed in their reflection, to monitor and evaluate the responses to their teaching, and to make decisions to enhance their instruction” (p. 375). Subsequently, “without specialized training or support from experienced teachers who can model their own ways of reflecting, inexperienced professors may find it hard to develop their knowledge bases and improve their ability to reflect” (p. 375). This observation is significant because it advocates involving experienced colleagues in the process of reflective practice for improving new instructors' teaching. The study falls short, however, because it does not actually demonstrate partnerships between new instructors with more experienced colleagues in the process of reflection.

**Secondary.** I expanded my search to research in secondary schools and describe them in this section. Posthuma (2011) investigated the nature of five South African secondary mathematics teachers' reflective practice through lesson study and offered the following definitions of *teacher reflection* and *reflective practice*:

Teacher reflection is an interrogation of practice before, during and after the act of teaching (reflection-for-practice, reflection-in-practice and reflection-on-practice), asking questions about the effectiveness of the teaching and learning experience and how these might be refined to meet the needs of the learner. The teacher is reflectively aware of the context in which he/she teaches as well as his or her own beliefs, knowledge and values regarding not only mathematics, but also the learners in the class. (p. 60)

The study mostly focused on secondary mathematics teachers' understanding of reflection; the content and level of their reflections before, during, and after teaching a lesson; and the contextual factors influencing their reflections. The following were notable findings:

They all verbally *reflected-on-action* and in writing. Three of the five *reflected-in-action* while teaching. They all reflected on a recall level of reflection (R1) and rationalization level of reflection (R2). Three teachers reflected critically on their learners' understanding of mathematics and their own teaching of concepts by the end of the research project. (p. iv)

They reflected on pedagogical matters (classroom management, time management, teaching style, learners' understanding of mathematics), personal issues (language, shortcomings as a teacher), external factors (curriculum, interferences while teaching, class size), and critical issues (learners' needs, learners' thinking). (p. 131)

The lesson study component of this study makes it similar to my study—as experienced colleagues observed and provided feedback:

The lesson study context experience proved to be a positive influence on all the teachers' reflective practice. All teachers reported positively on the cooperative planning of a lesson, revealing they learned much from the experience of planning with the goal to improve learners' understanding of concepts.

They reported they were teaching with more confidence as a result of watching themselves as teachers and learning from watching their colleagues. They also reported a sense of increased and deeper awareness of their learners' needs and the importance of involving learners in their lessons. Two of the teachers regarded the lesson study experience as self-research enabling them to compare themselves with their colleagues and observe their own actions critically while watching the videos.

The results of this study show that when mathematics teachers are made aware of their reflections on their practice in a context of working cooperatively, they are encouraged to reflect at a more critical level. (p. 134)

The most significant connection between Posthuma's work and this study is the increased awareness of students' thinking through reflection:

By relating each teacher's level of reflective thinking to his or her observation lesson, it seems that a possible relationship might exist between a teacher's reflection and his or her instructional decisions. Teachers who were reflecting on a critical level of reflection seemed to pay more attention to their learners' thinking about mathematics and how their

own instruction of mathematics might influence their learners' understanding of mathematics. (p. 131)

The study does a thorough job of analyzing and categorizing the nature of teachers' reflective practice habits: "The voices of the five mathematics teachers were heard, explaining their understanding of reflection, their reflection-for-action, reflection-in-action and reflection-on-action, as well as the contextual factors perceived to influence their reflections" (p. 128). The voices of the observers are not included and there is no evidence of a consistent ongoing relationship between a teacher and a particular colleague with whom they collaboratively reflect. Finally, its analysis does not focus on specific instructional practices.

Luwango's (2008) case study investigated the critical reflective practice by three mathematics teachers (each with more than 4 years teaching experience) at secondary schools in Namibia and how it shaped their classroom practices. Participants reflected extensively on their classroom practice during and after lessons in the following areas: learners' understanding, ability and performance; learners' attitude towards mathematics, lesson objectives, lesson delivery and flow, time management, teaching methods, feedback, and scope of work. The following is a summary of how teachers reflected on particular aspects of their instruction during and after lessons:

- Highly involved all learners through class activities
- Observed learners' participation in discussions; posed questions to slower learners
- Checked learners' work and detected problems as they worked
- Assessed learners' understanding of class work and homework
- Asked learners to explain why they failed to do homework
- Assessed learners' performance and common errors on tests
- Evaluated learners' reactions towards mathematics in terms of difficulty
- Created an atmosphere for learners to ask questions
- Verified lesson objectives and planned work were achieved
- Considered lesson effectiveness: what happened and what went wrong,
- Considered how to finish the scope of work
- Reflected together with learners on good aspects of the lesson
- Considered the degree of clarity provided on a specific topic

- Determined appropriateness of teaching methods
- Considered new problem-solving strategies (Luwango, 2008)

The author grouped the common aspects using Kilpatrick, Swafford, and Findell's (2001) five strands of teaching mathematics for proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These aspects later became part of the mathematical understanding for secondary teaching (MUST) framework used in my study.

Luwango concluded reflection on practice helped the teachers analyze and evaluate their teaching, identify weaknesses in their classroom practices, and plan alternative approaches in line with effective mathematics teaching. She contended "a teacher needs to be critically reflective to evaluate his or her teaching in line with learner-centered education. Through critical reflection teachers can look inside their classrooms to identify learners' needs or problems experienced in learning mathematics" (p. 3). The study is limited because the only data collected was from interviews and document analyses; no classroom observations were conducted. The perspectives were based solely on the teachers' recall without any collaborative reflection with a colleague. The study also differs from my study because the teachers had more mathematics teaching experience than the new instructors in my study.

Elias, Tee, Mahyuddin, and Uli (2008) conducted a descriptive correlational study to identify reflective thinking among 147 mathematics teachers from 19 secondary schools in Malaysia. Participants in the study responded to a set of questionnaires made up of six sections: demography, reflective thinking practices, time constraints, teachers' perceptions about mathematics learning, learning orientation, and problem-solving inventory. The objectives of the study were as follows:

1. To identify the level of reflective thinking practices among mathematics teachers in teaching and learning
2. To examine the relationship between reflective thinking practices of mathematics teachers and time constraint, external and internal learning orientation, teachers perception on mathematics learning and problem-solving skills
3. To identify factors which influence reflective thinking practices of mathematics teachers in teaching and learning (p. 146)

The researchers used four constructs for reflective thinking: retrospective and predictive thoughts, critical inquiry, problem-solving skills, and acceptance/use of feedback. The study revealed mathematics teachers employed retrospective and predictive thoughts more than the other constructs. The results of this study showed a positive and significant relationship between the four factors in objective two and teachers' reflective thinking practice. No data about specific instructional practices was gathered and none of the teachers were observed in the classroom setting. Again, this is a limitation because the data represented only the participants' recollection without the context of teaching episodes. Instead, the study highlighted characteristics of thinking practices. Although this study has the least in common with my study, it is important for understanding how teachers approach reflection. This is worthwhile because it potentially leads to more productive reflective thinking practices to help teachers improve their instructional practices. Such information can guide professional development incorporating reflective practices as its core component.

### **Colleague Observation and Feedback**

Literature about the practice of colleague observation describes the model, how it is implemented, the benefits and impact, and effectiveness as professional development. Most studies at post secondary schools involve disciplines other than mathematics education. These studies are relevant because they emphasize universal characteristics of colleague observation of

teaching. The studies specific to mathematics education take place in both post secondary and secondary schools.

### **General Higher Education**

Peer observation within the same content area has been the focus of various international studies in education. Martin and Double (1998) enacted an action-based approach to develop teaching skills in higher education across multiple departments and unspecified content areas in the United Kingdom through peer observation and collaborative reflection. They suggested models of development firmly based in existing practice directly benefitted teachers and enhanced collegiality. Teachers subsequently gained insights into their personal practice and improved their ability to apply pedagogical knowledge to discipline-specific action.

Hammersley-Fletcher and Orsmond (2004) gathered data about the peer observation process from the School of Sciences and School of Law (content courses were not specified) within a university in the United Kingdom. They asserted it is vital for peer observation of teaching in higher learning institutions to be integrated into departmental learning and teaching strategies as it provides an opportunity for scholarly discussion.

Donnelly (2007) studied professional development at an Irish higher education institute offering a peer observation of teaching scheme as part of an accredited Postgraduate Certificate in Teaching for academic staff. It was used to explore 90 participants' perceptions of its impact. Participants completed evaluation forms, participated in semi-structured interviews, and provided their peer observation components of their teaching portfolios for analysis. The author did not specify the specific content areas the participants taught. He summarized the program of observation was formative, developmental, collaborative, reflective, and enabled personal exploration of practice. New teachers highly valued the level of their observers' expertise, found

feedback to be helpful, and developed a sense of confidence in their teaching. The program encouraged all teachers to share insights to discover new ways of discussing theory and practice of teaching. Teachers reported the following results for helping them improve their instructional practices: having a deeper understanding for the “the why” behind the content they delivered, acquiring “tricks of the trade” to employ in their classroom, providing alternate ways to explain topics to students, revealing information about student understanding which was previously unnoticed, motivating changes to presentation skills, prompting more activity-based lesson approaches, incorporating more productive ways to use video and audio clips to enhance instruction, and introducing more peer learning and cooperative activities.

Australian higher education developers generally view peer observation of teaching in all content areas as an extremely effective form of teaching development. Bell (2002) described the use of peer observation of teaching at Australian universities as a development and training model using the following cyclic peer observation process: pre-observation meeting, observation, , and reflection. Bell found anecdotal evidence peer observation creates a positive environment of collegiality, develops reflective practice, and improves teaching. She outlined numerous strengths:

- A supportive and constructive, practical, collegial activity
- Motivates and affirms all those involved
- Develops awareness of problems in teaching shared by others and solutions found with others
- Provides new ideas and skills
- Builds awareness of the value of/skills in critical reflection and reflective practice
- Stimulates discussion about teaching and learning within departments
- Develops a sense of collegiality and an environment which values the sharing of experiences and ideas through teaching discourse
- Promotes self-assessment
- Benefits the observer by providing insights and ideas
- Opens up the private teaching space to others
- Supports continual improvement
- Reassures highly self-critical teachers

- Changes are made based on evidence
- Can be a significant turning point (Bell, 2002, p. 7)

Atkinson and Bolt's (2010) action research highlighted a teacher observation process in an Australian business school. The researchers were coordinators for the school's teaching and learning development program; one of them acted as the observer in the teachers' classroom. They recruited 10 staff members to participate for two semesters; one observation each semester. The teachers taught adult learners with diverse backgrounds ranging from undergraduate to postgraduate educational level, local to international groups, and full-time students to experienced professional full-time workers. The authors described the process as follows:

A cyclic approach was taken whereby after observation participants received personalized oral and/or written feedback from the observer. After each semester there was a group debriefing session to facilitate general feedback and inform the next cycle of teaching observations. At the end of each semester the observer summarized the feedback comments and reported the aggregated and anonymous strengths, weaknesses and ideas for consideration to the whole group. They identified trends and practices common across the group. (p. 7)

The teachers felt it fostered strong intrinsic motivation and noted the following positive aspects: individual observation by an independent person with teaching qualifications and opportunity to share the experience and feedback in the group debrief. Atkinson and Bolt's (2010) expressed the following limitation:

As peer review of teaching was not already embedded in the culture of the university and there were no direct rewards to encourage teachers to participate, the researchers suspect that it could be difficult, in the short term, to access high numbers of participants for further research. However, the results of this initial research indicate there is an appetite amongst participants to extend the program and include others in the systematic voluntary peer review of teaching. (p. 8)

My study directly counters this concern because peer review of teaching was fully embedded in the department's culture and the practice was already systematically enacted by members in the

department. Experienced colleagues within the department were involved in contrast to an outside observer in the dual role as researcher-participant.

Hendry and Oliver (2012) found the benefits of peer observations in teaching at a large multi-campus comprehensive Australian university included enhancing teacher confidence for learning and implementing new instructional strategies. Participants included nine graduates from the institute's Foundations of University Learning and Teaching program. They participated in semi-structured interviews in which they described their understanding of the usefulness of the peer observation process for their teaching and how they applied their experiences. Four themes emerged: "learning how to use new teaching strategies by watching, affirmation of current teaching practice by watching, seeing things as too difficult to do, and learning from feedback given by the observer" (p. 3). The authors cited the following limitation: "results were obtained in the context of a mandatory program that encouraged staff to generate goals for improving their teaching through peer observation that they otherwise might not have developed" (p. 7).

The above studies broadly affirm peer observation of teaching as a productive developmental activity across higher education academic disciplines; however, they do not render any specifics concerning feedback specific to course content. Additionally, the studies provide only the perspectives of the teachers and not the observer. My study extended these studies by focusing on mathematics and mathematics teaching content. It also included the interactions between and both perspectives of the observer and instructor.

### **Mathematics Education**

**Collegiate.** Colgan and DeLong (2015) reported on the success of their teaching polygon within the nine-person mathematics department at a small, private, liberal arts college. A

teaching polygon is a structured program of peer observation combined with collaborative reflection. Each faculty member visited a different faculty member's class once each month for an entire academic year. The goal for the observer was focusing on positive attributes of the teaching and reflecting on his or her own teaching. Every observer used the same observation sheet consisting of the following three prompts and space to capture the observer's notes: (a) strengths of this class session; (b) what I have learned about how this course fits into the major and/or how the instructor approaches the course; and (c) an idea I might try. The entire group met at the end of each semester to discuss observations from the teaching polygon. The faculty reported they gained valuable insights about colleagues and students, pedagogical strategies, teaching and learning, and the department's curriculum. Overall, being in each other's classrooms strengthened their sense of community as a department, reenergized enthusiasm for teaching, and encouraged reflection on ways to improve one's own teaching. The paper is not a report of a formal study and lacks specific data regarding feedback about individual classroom observations; however, it provokes worthwhile discussion for my study.

**Secondary.** Kensington-Miller (2012) described a peer mentoring model piloted as professional development for 3 (there were 11 in the entire sample) senior mathematics teachers in low socio-economic, under resourced, underachieving secondary schools in New Zealand. The model was content-focused, school-based, incorporated into day-to-day work, collaborative, and teacher driven. In this study, peer-mentoring was defined as "a relation where both partners are at comparable levels of experience and may be both mentor and protégé simultaneously, as they work together to facilitate growth and development in each other" (p. 294). Five features were identified as essential to make peer mentoring effective and practical: "recognizing each relationship is unique and working within those bounds, taking time to establish a relationship,

having effective communication, being committed, and each pair designing a structure to suit their own situation” (p. 303). Teachers agreed the professional development was beneficial for sharing resources, exchanging pedagogical ideas, and getting a new perspective of one’s teaching. They particularly valued the support from the group talks with other colleagues about mathematics applications and the content itself. Peer mentors within the same school were most accommodating for meeting times, travel, access, and organization. The study is limited and not generalizable because of the small number of participants with similar high experience levels. The under resourced, underachieving site created administrative constraints that challenged peer-mentoring efforts—particularly the ability to meet on a convenient or consistent basis. The study focused only on whether the peer mentoring modeling was a feasible and sustainable professional development. It did not capture any classroom interactions or feedback exchanges between partners.

Gellert and Gonzalez’s (2011) one-year case study focused on the benefits of teacher collaboration for two new secondary mathematics teachers who were alternatively certified teaching fellows participating in a mentoring program in New York City. “This mentoring program consisted of four components: a) a rigorous mentor selection process, b) mentors whose full-time job was mentoring, c) intensive mentor professional development, and d) regional rather than school-based assignments” (p. 5). The study used observations (twice a month) and interviews to gain insight into the various influences on the teachers’ mathematical practices. The researchers considered how these influences informed future developmental practices to support new teachers. They concluded teachers learned better by collaboratively reflecting on their teaching with experienced teachers with whom they regularly worked more than external mentors (district coaches or university mentors). The collaboration provided an

opportunity to work, communicate, generate, explore and reason, discover, plan, and build on others' ideas. Additionally, professional development on integrating reform teaching and accommodating common planning times with other teachers was useful. This study did not capture real-time interactions between the mentors and the new teachers; however, the teachers referred to the types of exchanges they had with their mentors and how the information helped them make adjustments in their instruction.

### **Advancing the Research**

The studies above overwhelmingly support collaborative, reflective practice among colleagues as a valuable form of professional development. The most prominent feature lacking in the studies, however, are the voices of experienced colleagues who partner with new instructors, conduct observations of classroom interactions, and provide feedback on mathematics instructional practices. Observers can provide a more objective perspective and assist instructors whose reflection-in-action ability is less developed. Ultimately, experienced colleagues' contribution in reflective practice helps new instructors reflect more critically. Additionally, few studies provide insights into the new instructors' intentions to adjust their instructional practices based on the feedback they received. My study both captured those voices and intensely focused on the substance of those exchanges with a mathematical knowledge, understanding, and instructional lens. In Chapter 9, I will discuss how some features from the above empirical studies potentially motivate or have similar findings as this study. I will address how this study extended beyond the scope of these studies.

A critical element determining the influence of colleague observation and feedback on instructional practice is the quality of the feedback. Brinko (1993) reviewed literature pertaining to feedback in the fields of education, psychology, and organizational behavior. She

extrapolated feedback-giving practices effective in helping post secondary teachers improve their teaching. She acknowledged feedback is more effective when the source of the information is perceived as credible, knowledgeable, and well-intentioned. Brinko also found the content of the feedback is more effective when it is descriptive rather than evaluative, contains models for appropriate behavior, and creates cognitive dissonance. Experienced colleagues' feedback is invaluable because it stimulates reflection beyond an instructor's own perspective, which may overlook important features of their practice. Quality feedback in mathematics can potentially aid in strengthening a teacher's foundational mathematics knowledge and foster effective mathematics instructional practices. The next sections describe the mathematical knowledge and understanding necessary for effective mathematics teaching.

### **Mathematical Knowledge for Teaching (MKT)**

Mathematical Knowledge for Teaching (MKT) is a construct describing the mathematical knowledge and skills necessary for a teacher to carry out the work of teaching mathematics (Ball, Thames, & Phelps, 2008). Ball and her colleagues asserted MKT is not simply limited to mental information; rather, it includes the ability to use mathematical knowledge at multiple levels to meet mathematical demands embedded within tasks involved in teaching. These tasks include activities across the spectrum of teaching: planning lessons; engaging in the real-time interactions of teaching (explanation, demonstration, questioning, discourse, interpreting/responding to students' inputs, etc.); writing/grading assessments to evaluate students' understanding; attending to concerns of equity; and even negotiating curricular concerns with parents and administrators. Ball and her colleagues took a "job analysis" approach focusing on the work of teaching to form the foundation of their practice-based framework of MKT. Their framework uses two of Shulman's (1986) categories—subject matter

knowledge and pedagogical content knowledge—as the major domains of MKT. Within the subject matter domain, three subdomains are identified: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge. The three subdomains of pedagogical content knowledge are: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum. The following outline summarizes these subdomains:

- Subject matter knowledge (SMK):
  - Common content knowledge (CCK): ability to
    - correctly calculate and solve mathematics problems
    - use mathematics in a variety of settings
  - Specialized content knowledge (SCK): ability to strategically
    - unpack mathematics knowledge
    - parse knowledge out to students in comprehensible pieces
    - create opportunities for students to make connections among concepts
    - help students repack knowledge and understanding gained during instruction
  - Horizon content knowledge: awareness of how topics are related across a span of mathematics in a given curriculum
- Pedagogical content knowledge (PCK):
  - Knowledge of content and students (KCS): ability to
    - anticipate students' thinking and areas of confusion
    - predict what will interest and motivate students
    - interpret students' emerging thought processes being expressed in their language
    - decipher students' conceptions and misconceptions about mathematical content
  - Knowledge of content and teaching (KCT): ability to make strategic decisions about
    - designing instruction
    - sequencing content and examples
    - choosing most appropriate representations
    - which student inputs to address, ignore, or pursue further development
  - Knowledge of content and curriculum: Shulman's (1986) curricular knowledge

### **Mathematical Understanding for Teaching**

The Mathematical Understanding for Secondary Teaching (MUST) framework was designed to reflect a more dynamic view of MKT with the notion understanding requires application of a teacher's knowledge (Heid, Wilson, & Blume, 2015). The framework has three overlapping components: mathematical proficiency, mathematical activity, and mathematical

context of teaching. Mathematical Proficiency (MP) includes aspects of mathematical knowledge and ability teachers must deeply and thoroughly understand in order to guide students. Mathematical Activity (MA)—simply put, “doing mathematics”—includes activities (mathematical noticing, reasoning, and creating) teachers employ and want students to learn. Mathematical Context of Teaching (MC) involves the integration of a teacher’s content and process knowledge to increase students’ mathematical understanding. The following outlines the MUST framework (Kilpatrick et al., 2015):

- Mathematical Proficiency (MP)
  - Conceptual understanding
  - Procedural fluency
  - Strategic competence
  - Adaptive reasoning
  - Productive disposition
  - Historical and cultural knowledge
- Mathematical Activity (MA)
  - Mathematical noticing
    - Structure of mathematical systems
    - Symbolic form
    - Form of an argument
    - Connecting within and outside mathematics
  - Mathematical reasoning
    - Justifying/proving
    - Reasoning when conjecturing and generalizing
    - Constraining and extending
  - Mathematical creating
    - Representing and defining
    - Modifying/transforming/manipulating
- Mathematical Context of Teaching (MC)
  - Probe mathematical ideas
  - Access and understand the mathematical thinking of learners
  - Know and use the curriculum
  - Assess the mathematical knowledge of learners
  - Reflect on the mathematics of practice (p. 14)

## **Framework for Observation Feedback**

The MUST framework was developed from mathematical practice; therefore it can be considered in developing mathematics instructional practices. When colleagues provide observation feedback, the framework could serve as a well-structured mechanism for focusing the observer's attention to instructional behaviors closely tied to effective mathematics teaching. In the absence of any guidance, feedback may vary depending on the observer's preference for attending to particular instructional areas. The focus may be motivated by any number of factors: a personal area of interest or expertise, an instructor's strength or weakness, what is convenient to target, etc. If there is no intentional or consistent observational focus, the quality of the feedback is likely to be less effective.

The MUST framework can guide the observer's attention while providing a solid foundation for critiquing instruction —consequently improving the quality of feedback. Since the framework is grounded in practice-based research, it validates feedback when instructional behaviors are tied to characteristics in the framework. For example, an observer may discuss the mathematics-creating component of MUST to address an episode in which the instructor struggled to adequately clarify students' misconceptions by offering alternative explanations. Feedback focused on the elements of MUST can facilitate rich and productive dialogue about effective mathematics instructional practices. Likewise, the new instructor is given a foundational basis for effective mathematics instruction upon which to reflect and can plan modifications based on the framework's elements.

## **Conclusion**

The scarcity of empirical research about reflection on collegiate mathematics teaching practices has restricted our understanding of learning the work of teaching and the development

of beginner teachers. Many studies emphasize the process and/or implementation of peer observations as a faculty development tool but fail to analyze characteristics of feedback. Neither do they examine how new instructors use experienced colleagues' feedback to re-examine and adjust their instructional practices. This study observed a well-established, consistent collaborative professional development activity to analyze the interactions and feedback content between colleagues with the goal of revealing its potential to improve mathematics instruction. Although the practice may occur at other universities, it is not likely to be as consistent as it is within the mathematics department at this institution. Standardization and consistency are hallmarks of military institutions. As such, the study focused more on the influence of this type of faculty development rather than whether it is pervasive enough to have an impact on instructional practices.

In the next chapter, I describe the study: the culture of the research site and its participants, research design and methodology, and data collection and analysis. The subsequent chapters summarize the four case studies, present cross-case analysis and discussion, and conclude with findings, limitations, and implications for future research.

## CHAPTER 3

### DESCRIPTION OF THE STUDY

The purpose of this study was to analyze experienced colleagues' feedback and investigate the influence on new instructors' intentions to adjust their mathematics instructional practices. The following supporting research questions were used: (a) What mathematical areas or mathematics instructional practices do experienced colleagues notice and address in their feedback to new instructors? (b) In what ways do new instructors describe how they plan to modify and/or begin to modify their mathematics instructional practices based on experienced colleagues' feedback? A qualitative research design is best to answer these questions because it is a systematic approach used to describe life experiences, give them meaning, and gain insight about particular phenomenon inherent to the experiences (Glesne, 2010). The most effective way to do this was with a case study design and its appropriate methods— observations, interviews, artifacts, and documents. “A case study investigates a contemporary phenomenon (the case) in its real-world context” (Yin, 2014, p. 2). I felt a single case was not sufficient to address my research questions because I wanted multiple perspectives from a variety of new instructors and experienced colleagues. Having evidence from multiple cases would make my study more compelling, and the overall study would be considered more robust (Yin, 2014). I needed to qualitatively examine—using observations, interviews, and review of participant written responses for multiple cases—the characteristics of the feedback focusing on new instructors' mathematical knowledge, understanding, and instructional implementation; then capture new instructors' intentions to modify their mathematics instructional practices.

To achieve my research goals using the multiple-case study design, I had to either select multiple sites or a single site with a structure to facilitate the design. The site (an undergraduate military service academy) was a good fit for this multiple-case study because its mathematics department had in place a well-structured, intentional culture of observation and collaborative reflection designed to help instructors provide quality instruction for their students. The process of assigning mentors to new instructors was a part of the faculty development program with a prescribed implementation plan for observations and feedback. A good structure, therefore, was in place to facilitate a multiple-case study.

In this chapter, I discuss the research site and methods for collecting and analyzing data for my study. First, I describe the research site and the participant selection process. Then I give a brief overview of the research design. Finally, I present the methods for collecting data, identify the types of data, and explain my approach for data analysis.

### **Research Site**

The site was the United States Air Force Academy (USAFA), a four-year undergraduate military institution composed of approximately 4000 students pursuing Bachelor of Science degrees in 32 academic disciplines. The institution's unique integrated mission is to educate, train, and inspire students to become officers of character, motivated to lead the Air Force in service to the nation. Other than being a military school attracting some of the top students in the nation and internationally, the school is quite similar to a relatively small university where there is more emphasis on teaching than research. There is, however, a rapidly growing emergence of pedagogical research which began in 2009 through the Center for Educational Excellence's (CEE) Scholarship of Teaching and Learning (SoTL) program designed to enrich student learning by "linking new approaches to published good practices, to collect data on the

effectiveness of those approaches, and to share the findings with colleagues” (Scharff, 2016).

The CEE’s vision permeated the academic culture with the goal of promoting a student-centered, active learning environment in every classroom. High student engagement in the classroom was a standard throughout all departments.

### **Mathematics Department**

The mathematics department offered the same undergraduate mathematics courses as most typical universities. A list of courses offered is available on the department’s website: <http://www.usafa.edu/df/dfms/courseOfferings.cfm?catname=dfms>. There were 40 faculty members in the mathematics department of which 8 were permanent civilian professors with doctor of philosophy degrees in a mathematical discipline. The remaining 32 were active duty military members with at least a master’s degree in mathematics, statistics, engineering, operational research, or some other mathematically based subject—12 military faculty members had doctoral degrees. These active duty members normally served 4 years in a teaching assignment at USAFA. In a given year there was turnover of 5 to 10 military members. Since the assignment was a special duty outside of one’s normal military career field, most new military faculty had limited (if any) academic classroom teaching experience upon arrival—although some may have been instructors in some other military capacity. Class sizes in the mathematics department ranged from about 15 to 23 students in the core classes (calculus and statistics). The sizes were less in major courses.

Most new instructors each taught four sections of differential and integral calculus in sequence for the first two semesters. In the second year they would become a course director for calculus or choose to teach statistics or an upper level course. New instructors with a strong background in academic mathematics or specialization in an advanced mathematics area may

teach an upper level course in their first two years. In the third and fourth years, they could also become a course director in some course or continue with upper level courses. Instructors with more than 2 years teaching experience also taught core calculus courses. Some professors who taught fewer sections in upper level courses occasionally taught one section of a core calculus course. The new instructors audited the lessons of the more experienced instructors and professors. Lesson auditing is explained in the next section.

### **New Instructor Training**

The mathematics department had a well-established new instructor training program to prepare new faculty and continue development to provide quality instruction. In the summer prior to their first fall semester, new instructors participated in a 2-week calculus review course. A variety of faculty members taught different sections of the course. This gave the new instructors their first opportunity to observe more experienced colleagues. Then they had a 4-week new instructor training (NIT) program in the department. During NIT, they were oriented to the departmental administrative matters, classroom issues, pedagogical topics, and prepared for two full 53-minute class period practice lessons. Only the faculty development director and assistant and the other new instructors observed the first practice lesson. The second lesson was open to the entire department. Other faculty members acted as students during the lessons. After each lesson, the new instructors received feedback from the audience.

When the semester began, new instructors audited lessons taught by experienced colleagues in their course a day prior to teaching. The class scheduling structure facilitated this with a rotating A-Day and B-Day where multiple sections of the course were offered on both A and B days. More experienced faculty typically taught on the A-Day, and new instructors taught on the B-Day. There were 40 lessons per semester; all sections stayed on the same pace outlined

in the syllabus designed by the course director. For example, a new instructor might observe an experienced colleague teach Lesson 13 on derivative rules for polynomials in Section 4.1 on Wednesday (A-Day). The new instructor would teach the same lesson on Thursday (B-Day).

### **New Instructor and Mentor Relationship**

Each new instructor was paired with a more experienced colleague who served as a mentor. Typically, mentors were civilian professors or senior military faculty members with a doctor of philosophy degree and at least 10 years of collegiate teaching experience. Mentors were rarely in the same course as the new instructor since they mostly taught upper level and mathematics major courses. They were not in the supervisory chain of the new instructors. This arrangement eliminated any power issues or intimidation with regard to formal evaluation. The department's operation instruction (OI) described the mentor as follows:

A person a new faculty member approaches with developmental skills questions (lesson planning, teaching, research, etc.). The mentor also serves as the support/feedback system for the new faculty member, allowing their apprentice to experiment in the classroom, reflect rationally on the outcome, improve the experiment, and try the experiment again if necessary. (USAFA/DFMS, 2007, p. 1)

A mentor was encouraged to informally interact with the new instructor as much as desired; however, the mentor should observe the new instructor at least twice per semester for an entire class period and provide feedback during an informal one-on-one session. The pair was encouraged to meet prior to the observation to establish goals or focus areas and to discuss the overall lesson plan with desired outcomes for the lesson. During the lesson, the mentor took notes without interrupting or interacting in the lesson. There was no official form used for observations. Mentors were given guidance from the faculty development team, however, about pedagogical areas to notice: instructional strategies, lesson phasing, mathematical knowledge (including errors and misconceptions), emphasis of mathematical value, and teacher beliefs. The

feedback session between the experienced colleague and new instructor usually occurred the same day or within one day of the classroom observation. The feedback session remained confidential between the new instructor and mentor; all notes and comments were kept between the two and not submitted in any formal evaluation documents. New instructors were encouraged to reflect on the feedback in a way most effective for them and make adjustments as they deemed necessary in their teaching.

### **Observation and Feedback Culture**

In general, there was a very comfortable culture of observation where colleagues frequently observed and provided informal feedback throughout the entire department from newest to most senior faculty. “The faculty observation program is designed to foster a positive atmosphere where observer and observed can learn from a variety of other instructors both inside and outside of the department” (USAFA/DFMS, 2007, p. 3). Informal colleague observations are a standard practice. The following describes observer roles:

- **Mentor:** Each observation should focus on various pedagogical areas determined by mentor and mentee. In addition to observing a mentee, a mentor should facilitate sessions to discuss topics including but not limited to those addressed in the mentee’s teaching portfolio.
- **Faculty Development Team (FDT):** Maintains a post-summer new instructor training (NIT) relationship for the new instructor’s first two years. Also maintains awareness of department-wide teaching; tracks the “pulse” of the department. Observations focus on pedagogical areas determined by instructor and FDT.
- **Rater, Course Director (CD), & Division Chief:** Use observations to support requirements prescribed by the assigned duty relationship. Observation focus areas are as follows:
  - **Rater:** Exhibiting professionalism/military role model, upholding classroom standards, and properly managing classroom
  - **CD:** Instruction is well aligned with course vision and objectives
  - **Division Chief:** Instruction is well-aligned with course and division objectives. A division chief may also choose to observe instructors of courses in the division (p. 8)

The following focus areas were suggested guidelines to help facilitate observation discussions:

“Teacher Pedagogy Competency, Discourse between Teacher and Students, Verbal Behavior of Students, Task Orientation of Students” (p. 9). Teacher Pedagogy Competency was the primary observation focus area for the faculty development team and mentors. The following is a description of this area:

- Note the instructional strategies used by the teacher, such as cooperative learning, students explaining their work at the board, teacher lecturing, etc.
- Note the phases of the lesson: initiation, development, closure
- Record any mathematical errors, misconceptions or misrepresentations you notice
- Suggest how errors could be corrected
- Comment on the mathematical knowledge of the teacher
- Describe each time the teacher explicitly pointed out the value of the mathematics the students were learning
- Discuss other opportunities the teacher could have used to get the students to appreciate and understand the value of the mathematics they were studying
- Make a conjecture regarding the teacher’s goals for what the students would learn in the lesson and what these goals reflected about their beliefs about the value of the mathematical content of the lesson
- Describe how the teacher tried to motivate the students to want to learn
- How could the motivation for this lesson have been improved?
- What are your conjectures about the teacher’s beliefs about the motivations?
- Make conjectures regarding the teacher’s beliefs about students, how they learn mathematics, and their ability to learn math taught in this class (p. 10)

There was an organized system facilitating interactions between new instructors and colleagues in functional roles within the department. Figure 1 is the department’s observation plan providing observation and auditing requirements with timing constraints:

<i>≤ L25 = observe on/before lesson 25 each semester</i>	Observed				
Observers	NIT (1Y)	2Y	3Y	4Y	5Y+
Mentor	≤ L15 ≤ L35	≤ L25	≤ L30	≤ L35	N/A
Faculty Development Team (FDT)	≤ L15 ≤ L35	≤ L20	Once per year		
Rater	≤ L30	≤ L30	≤ L30	≤ L30	≤ L30
Course Director (CD)	Observe each instructor in course once during semester				
Division Chief	Observe each CD in division once during semester				
Annual Auditing/Observation Plan					
NIT (1Y)	Audit: ≥ 20 lessons of 1st time course(s) Observe: 3 Math faculty at varying experience levels				
2Y	Audit: ≥ 20 lessons of 1st time course(s) Observe: 3 Math faculty at varying experience levels				
3Y	Observe: 3 Math faculty at varying experience levels & 1 non-Math faculty in another discipline				
4Y	Observe: 3 Math faculty at varying experience levels & 1 non-Math faculty in another discipline				
5Y+	Observe: 2 Math senior (5Y+) faculty (1 mil/1 civ) & 1 non-Math faculty in another discipline				

Figure 1. Observation and auditing plan (p. 7).

Colleagues frequently engaged in group conversations about teaching in the break room, at course meetings, as well as one-on-one in the hallway or each other's offices. New instructors were particularly comfortable among themselves exchanging ideas covering a range of topics: lesson planning, classroom experiences, their observations of experienced colleagues, student results and behavior, reactions to feedback, and so on.

### Participants

I first contacted the faculty development director in the mathematics department during the fall semester of 2013 to introduce my research interest and request potential participants for

some pilot interviews. I asked him to identify new instructors and their mentors who were willing to do interviews following their next observation and feedback session. I conducted four pilot interviews with two new instructor/mentor pairs. The pilot interviews were beneficial for refining my interview approach and focusing on topics to best address my research questions. This activity also gave me the opportunity to plant the seeds for future data collection by reconnecting with the department and began a dialogue about my research. The faculty development director at the time was extremely cooperative and helpful by bringing me up to date on the status of new instructor training and faculty development; especially covering how the program evolved since I was a member of the department. He also facilitated the videoconferencing medium through which I conducted pilot interviews. Finally, he informed me his duties would transition to another senior faculty member in the spring semester. The new director would be in charge of the summer new instructor training.

I contacted the new faculty development director in the spring to introduce my research interest and express my desire to have the incoming new instructors and their mentors participate in my study. He provided the summer new instructor training schedule and informed me there would be five new instructors. I visited the department for a week during the portion of the summer training when the new instructors taught their practice lessons in front of the faculty development team and other department members. I observed the new instructors' lessons and group feedback sessions. I became familiar with potential participants and got a feel for their initial instructional practices. I gave an overview of my study to the new instructors and solicited their interest for participating in the study during the fall semester.

I arrived in the department after fall semester began. I formally presented details about my study to each of the new instructors and offered them the opportunity to participate. Four of

the five new instructors agreed to participate. I informed their mentors, presented details of the study, and asked them to also participate; they agreed. All eight participants (four new instructor and mentor pairs) were formally briefed about all aspects of the study, data collection procedures, and the consent process. They signed consent forms and were eager to participate.

The four new instructors—Carl Pappus, Kevin Joule, Laverne Pascal, and Colt Jacobian—were first-year instructors in the mathematics department with no mathematics teaching experience. Three of the new instructors—Kevin, Laverne and Colt—taught differential calculus for freshmen. Carl taught integral calculus for freshmen who were advanced placed in the course as a result of high scores on the department’s mathematics placement test given the summer prior to fall semester. Two mentors—Dr. Walter Bethune and Dr. Felipe Ignacio—were civilian professors with doctoral degrees. Two mentors—Dr. Arnold Paine and Dr. Brian Tougaloo—were senior military faculty members with doctoral degrees.

### **Research Design and Methods**

This was a qualitative, descriptive multiple-case study designed to understand how the practice of colleague observation and feedback influenced new mathematics teachers’ instructional intentions and practices. The research goals are as follows:

- To describe interactions between new instructor/experienced colleague mentor pairs
- To analyze the nature of feedback about mathematics and mathematics instruction
- To capture new instructors’ interpretations of the feedback
- To investigate new instructors’ intentions to modify their instruction
- To document new instructors descriptions of their actual instructional modifications

In order to accomplish these goals, it was necessary to embed myself within the everyday contexts of the participants’ environment and practices. The most effective way to do this was with a case study design and its appropriate methods— observations, interviews, artifacts, and documents. “A case study investigates a contemporary phenomenon (“the case”) in its real-

world context, especially when the boundaries between the phenomenon and context may not be clearly evident” (Yin, 2014, p. 2). It “allows the investigators to focus on a case and retain a holistic and real-world perspective” (Yin, 2014, p. 4). Since the study focused directly on teaching practices and feedback about those practices, I considered classroom instructional activities and the feedback sessions as the real-world, in-the-moment experiences from which data was generated. One of the key components of the case study research design is the *unit of analysis*—the actual single “case” or “phenomenon” to be analyzed (Yin, 2014). The top level unit of analysis was the new instructor/experienced colleague mentor pair and their interactions. The next level was each observation cycle and its embedded phases. To acquire information and narrow the scope of investigation, I analyzed the content of their interactions focusing on mathematics or mathematical instructional practices.

When conducting case study research, it is important to decide if a single case is sufficient to address the research questions. If the purpose is to investigate a unique or extreme phenomenon, then a single case may be sufficient. However, according to Yin (2014):

The evidence from multiple cases is often considered more compelling, and the overall study is therefore considered being more robust. Each case must be carefully selected so that it either (a) predicts similar results (*a literal replication*) or (b) predicts contrasting results but for anticipatable reasons (*a theoretical replication*). (Yin, 2014, p. 57)

As I developed my research questions, I envisioned ways to acquire rich data to strongly support my findings. I was convinced I needed multiple cases—at least three cases—to produce a compelling study because the cases would produce both similar and contrasting results. I expected to discover by cross-case synthesis overarching similarities among the cases; furthermore, each case would naturally differ in significant ways because multiple factors affected the dynamics of each pairing. For example, each new instructor/experienced colleague pairing had distinct characteristics based on personalities, experiences, teaching beliefs,

philosophies, etc. I used four cases to gain insight about the influence of this developmental activity with the goal of making generalizations to be broadly applicable in mathematics education. According to Yin (2014), “case studies are generalizable to theoretical propositions and not to populations or universes. The goal will be to expand and generalize theories (analytic generalizations) and not extrapolate probabilities (statistical generalizations)” (p. 21).

### **Data Collection**

My data collection methods included researcher observations of classroom instruction and pair interactions (pre-observation meetings and feedback sessions) and participant interviews. I audio recorded pair interactions and interviews and collected participant written artifacts: lesson plans, instructional notes, feedback session notes, and new instructor responses to a reflective prompt. I arrived in the department about one month into the fall semester of 2014—the courses were on Lesson 13. I immediately began observing instructors’ classes in both calculus courses. I observed the new instructors as well as instructors of various experience levels who were not participants in my study. My goal was to attend every lesson of each course while rotating through various instructors. The purpose was to stay on pace with the content of the courses and get a feel for the instructional styles of all the instructors in the courses. This gave me insight about any potential influences on the new instructors’ practices.

I witnessed observation cycles for each pair. An observation cycle consisted of the following: pre-observation meeting between the experienced colleague and new instructor; an entire classroom lesson where the experienced colleague observes the new instructor; feedback session between the experienced colleague and new instructor; and interviews with each participant (separately). Because of the timing of my arrival, I was unable to witness the first complete observation cycle for two of the pairs. One pair (Case 1) had already accomplished a

pre-observation meeting, observation, and feedback session prior to my arrival. For Case 1, I collected information about all the events solely from each of the interviews. Another pair (Case 2) completed an observation (with no pre-observation meeting) but had not conducted their feedback session. I observed their feedback session and collected information about the actual lesson in the interviews.

I witnessed observation cycles with pairs who had not conducted an observation before I arrived. For one of these pairs (Case 3), the mentor casually dropped in to the new instructor's classroom and stayed for part of the period. He did not consider it a full observation but still gave very generic feedback. When I conducted an introductory interview with the new instructor, she described this casual observation and feedback. At my request, the participants notified me when they planned observations. Sometimes these notifications were short notice based on course events or personal schedule preferences. I also made informal visits to new instructors' offices to check in, get a feel for how things were going in their classrooms, and inquire about casual interactions with their mentors. Figure 2 is a record of all the events between the pairs. It lists participants and events accomplished in each observation cycle.

Case	Role	Name	Pre-obs Mtg	Obs 1	FB 1	Int 1	Pre-obs Mtg	Obs 2	FB 2	Exit Interview
1	New Instr	Carl	Y*	L4*	Y*	Y	Y	L24	Y	Y
	Mentor	Dr. Bethune				Y				
2	New Instr	Kevin		L5*	Y	Y		L20	Y	Y
	Mentor	Dr. Ignacio				Y				
3	New Instr	Laverne		L16	Y	Y				Y
	Mentor	Dr. Paine				Y				
4	New Instr	Colt		L15	Y	Y				Y
	Mentor	Dr. Tougaloo				Y				

Figure 2. Participant events matrix.

Italicized font with an asterisk (\*) indicates the event took place, but I was unable to directly witness it. Two pairs did not accomplish two full observations during the semester. This was attributed to a number of possible reasons including: mentors only did casual “pop-ins”; they were satisfied with the adequacy of the new instructor’s teaching practices based on the first observation; or they simply did not have the opportunity to do a second observation because of time constraints or unexpected schedule conflicts. This situation was not completely uncommon. After all, there was no formal penalty for not completing two observations.

### **Researcher Role**

**Nonparticipant observer.** In a descriptive case study, “a researcher has little or no control over behavioral events” (Yin, 2014, p. 2). As such, I was a nonparticipant observer during all meetings between the new instructor and mentor as well as during the classroom observations. The purpose of the study was to describe, analyze, and explain a practice as it is existed; not influence or alter it. My role was to simply record the interactions without interference; therefore, I simply observed and did not influence these exchanges. The interviews provided opportunities for me to get clarification of interactions during these events.

**Pre-observation meeting.** I was a nonparticipant observer during the pre-observation meeting. I audio recorded each pre-observation meeting and took notes to capture various insights: the participants’ expectations for an observation; what instructional areas concerned the participants; what areas a new instructor viewed as important to receive feedback from the mentor. These areas potentially provided a preliminary view into the new instructors’ reflective priorities (Posthuma, 2011).

**Classroom instruction.** I was a nonparticipant observer during full classroom periods of instruction. My observations in the classroom served as secondary data used as reference in

subsequent feedback sessions and interviews. For example, I recorded the sequence of events in the classroom, the topics covered, significant discourses, and significant instructor physical nuances. I used these notes as frames of reference during the feedback session and interview discussions. I also used them as needed to solicit more detail or points of clarity or reflection during the interview. Immediately after the lesson, I made preliminary analysis of how I thought the lesson flowed in terms of instructional strategies and results. The purpose was to have a point of reference for the feedback session, as well have some points to address during the interviews.

**Feedback session.** The feedback session between the experienced colleague and new instructor usually occurred the same day or within one day of the classroom observation. In one case, it was a few days afterwards. Again, I was a nonparticipant observer and simply took notes. I began pre-analysis by attempting to categorize the various types of feedback significant to my research questions. Immediately after the feedback session, I wrote in my researcher notes how I viewed the exchange based on the focus of the feedback. I began to create categories and rudimentary coding of feedback based on my observations (Glesne, 2010; Saldana, 2013).

## **Interviews**

I conducted interviews after the feedback session. Through the interview with each participant, I gained insight into the participant's perceptions of the feedback session and its effectiveness for applying what was learned to future instructional practices. I used experienced colleague interviews to understand how they made choices for noticing and addressing practices they observed. I used new instructor interviews to gain insight about their reflection on their mathematics teaching and on feedback they received from experienced colleagues. I asked the new instructors how they intended to implement or adjust some instructional practices based on

what they learned from feedback. I conducted semi-structured interviews using interview guides (Appendices A and B). The questions were open-ended with follow-up probes based on the participant's response (Roulston, 2010). Participants often elaborated and self-analyzed their recollections. After each interview, I reflected on the session in my researcher notes. As some preliminary analyses were completed, I noticed potential themes (Glesne, 2010). The exit interviews (Appendix C ) solicited the new instructors' reflective analyses of their overall experiences and worthiness to their development as instructors based on how much impact the program had on their instructional practices (Roulston, 2010).

### **Reflection Prompt**

I initially asked the new instructors to maintain a journal recording their thoughts about how they intended to modify or had adjusted some instructional practices based on what they learned during feedback sessions. Documenting how one reflects upon instructional practices in a journal enhances reflection because it is an additional source of critical self-reflection (Artzt & Armour-Thomas, 2002; Farrell, 2004). The intended purpose of the journal was to provide a link to understanding how the participants self-reflected upon feedback, reexamined approaches, mentally intended to make adjustments, and self-assessed attempts to adjust instructional practices. I wanted to have a realistic and productive way to get this information. I discussed this issue in the beginning with the instructors to get their opinion of what would work best for them. They agreed receiving a reflection prompt would be most effective, productive, and convenient for them. I prompted their reflection (Appendix D) 2 weeks after the midpoint of the semester. This was the only journal-like reflection I collected.

## Data Analysis

Data sources included audio recordings, transcriptions, new instructor responses to the reflective prompt, participant inputs during all interviews, instructional plans, and my observation notes from the pair interactions. The interview and transcript were the primary sources of data. My goal was to identify different types of feedback and then categorize the data through thematic analysis (Maxwell & Miller, 2008; Roulston, 2010). I developed coding schemes to decipher topics and description of actions described during the interview (Saldana, 2013). I also used the codes in Figures 3 and 4 based on the MKT (Ball et al., 2008) and MUST (Kilpatrick et al., 2015) frameworks:

MKT	Code
<b>Subject Matter Knowledge</b>	<b>SMK</b>
<i>Specialized content knowledge</i>	SCK
<i>Common content knowledge</i>	CCK
<i>Horizon content knowledge</i>	HCK
<b>Pedagogical Content Knowledge</b>	<b>PCK</b>
<i>Knowledge of content and students</i>	KCS
<i>Knowledge of content and teaching</i>	KCT
<i>Knowledge of content and curriculum</i>	KCC

Figure 3. MKT codes.

MUST	Code
<b>Mathematical Proficiency</b>	<b>MP</b>
Conceptual Understanding	MP_CU
Procedural Fluency	MP_PF
Strategic Competence	MP_SC
Adaptive Reasoning	MP_AR
Productive Disposition	MP_PD
Historical and Cultural Knowledge	MP_HC
<b>Mathematical Activity</b>	<b>MA</b>
Mathematical creating	MA_CR
Mathematical reasoning	MA_RS
Mathematical noticing	MA_NO
<b>Mathematical Context of Teaching</b>	<b>MC</b>
Access and understand the mathematical thinking of learners	MC_LR
Assess the mathematical knowledge of learners	
Reflect on the mathematics of practice	MC_RP
Know and use the curriculum	MC_CUR
Probe mathematical ideas	MC_ID

Figure 4. MUST codes.

I read line by line through the hard copy of the transcript and blocked off complete phrases. I maintained complete integrity of the data by blocking off sentences and circling key words and phrases to capture the essence of the participants' thoughts. I jotted down summary ideas in the margins. Then I went to the electronic version, turned on track changes, highlighted the blocked sections, and added comments. My comments were thematic summaries of the ideas I wrote in the margin of the hard copy. My initial coding stayed close to the data and was captured as action phrases about what was being described. This method followed Charmaz's (2014) coding process:

Line-by-line coding with gerunds is a heuristic device to bring the researcher into the data, interact, with them, and study each fragment of them. It helps to define implicit meanings and actions, gives researchers directions to explore, spurs making comparisons, and suggests emergent links. (p. 121)

I also maintained the sequence of each interview and noted themes throughout each interview.

I created in Excel a data analysis matrix with the following headings to capture 263 blocks of data from all interviews: Participant, Topic, Block, Theme, MKT, and MUST. The participant was the primary speaker in the data block. Topics were based on the interview content as well as topics related to my research questions. The topics were broad categories assigned to data chunks during the coarse-grained phase of analysis (Butler-Kisber, 2010). I continued with an "iterative process of expanding and reducing categories to provide conceptual and interpretive understandings grounded in the data" (Butler-Kisber, 2010, p. 32).

Mentoring and reflection on practice were broad contexts for this study and were expected to prevalent in all cases. The research questions also significantly narrowed the scope of the data to be analyzed. Participants covered a variety of topics in their discussions; however, the research questions required data specific to mathematics or mathematical teaching practices. As I analyzed the cases, I made connections among the data to consolidate down (reduction) to

fewer thematic topics with more concise, meaningful titles (Maxwell & Miller, 2008). The 15 emerging themes based on discussions and reflections among all cases are described as follows:

**General practice:** general statements about teaching (statements such as “I take more time to lesson plan”) not exclusive to some facet of mathematics or mathematics teaching

**Mentoring:** generic aspects of mentoring (statements such as “We have a great relationship”) not exclusive to some facet of mathematics or mathematics teaching

**Assessing student progress:** informally assessing students’ understanding and work

**Choosing examples:** strategically selecting examples to meet lesson

**Representing mathematical ideas/notations:** using explanations and notations to represent concepts

**Questioning:** using questioning techniques to assess students’ understanding

**Making meaning:** using definitions, notations, and explanations to clarify meanings

**Anticipating student responses:** accounting for possible student responses and planning for planning productive responses

**Modeling problem-solving:** demonstrating problem-solving techniques and representing required standards for students’ written work

**Connecting across curriculum:** strategically linking current lesson content to prior and future curriculum concepts

**Using correct notation:** correctly representing notation during

**Timing and pacing:** strategically using time and pacing during instruction

**Persevering through student questions:** sufficiently addressing student questions

**Emphasizing learning:** helping students reflect on learning and develop their own critical thinking skills and requiring students to use critical thinking

**Extending problems:** investigating extensions and generalizations and creating variations of foundational concepts or basic examples

I enabled filtering of all headings and tracked the counts of the topic, theme, MKT, and MUST categories. The filtering capabilities allowed me to view the data from any focus area I

desired and consolidate or arrange in meaningful ways. I performed a reiterative analytical sweep through the data using the filtering capability to refine my descriptions and coding in the spirit of constant comparative analysis (Butler-Kisber, 2010). My spreadsheet completely maintained the integrity of the data because data blocks preserved verbatim excerpts from the interviews. To conduct my overall analysis, I filtered and maintained a frequency count for each theme. I recorded which cases referenced each theme. I designated the term *prevalence* to mean the number of cases referencing a given theme and sorted themes by prevalence then frequency. Finally, I recorded the frequencies of assigned MKT and MUST categories within a theme. To keep a convenient organization of filtered data I maintained three data workbooks consisting of the following: original data and categories, sorted data separated by cases, and data sorted by themes. The analysis results are provided in Chapter 8. I used member checks to verify I had appropriately captured the participants' words and ideas. All interviews served as sources to help establish consistency or triangulation for my interpretations. The analysis provided an efficient and organized way to identify (a) specific mathematical areas or mathematics instructional practices experienced colleagues noticed and addressed in their feedback to the new instructors and (b) ways in which the new instructors described how they planned or began to modify their mathematics instructional practices based on the feedback. These were the goals of the supporting research questions.

## CHAPTER 4

### CASE 1: CARL PAPPUS AND DR. WALTER BETHUNE

#### **New Instructor 1 – Carl Pappus**

**Carl’s background.** Carl was a first-time instructor teaching integral calculus for undergraduate freshmen placed in the course by having relatively high scores on the mathematics department’s placement test. Carl graduated from the United States Air Force Academy (USAFA) with a double major in economics and operations research (OR). He was one of the top students among the OR majors in the mathematics department. As such, he was competitively selected and fully sponsored to attend graduate school for two years at a top-rated technical institution where he earned a master’s degree in OR. His research focused on applying various optimization methods to maintenance scheduling problems for the military. In his first job, he used design of experiments and statistical significance to determine sufficient sample size in analysis for aircraft flight tests and various data analysis techniques. After 3 years he was selected to teach in USAFA’s mathematics department.

**Carl’s mathematics learning and understanding.** Carl appreciated his graduate school experience because it helped evolve his way of learning mathematics. Carl focused on memorizing equations, formulas, and processes as an undergraduate. He believed his conceptual knowledge of mathematics improved in graduate school: “Why do I learn and understand better at a conceptual level when I’m just a few years removed from being a student? It is having more experience—your ability to learn matures and grows with experience” (Interview 1, October 1, 2014, Lines 102 – 108). The change in the way Carl studied mathematics was motivated by the

way his graduate mathematics courses were taught. He summarized the prevailing instructional approach in his graduate mathematics courses as follows: “They’ll teach you the base theory. Then my homework problems would be extensions/variations of what we did in class; that made it more difficult. They expect you to think on your own and really figure it out” (Lines 148 – 152). As a result, Carl delved deeper into his studies by reading, digesting, and rereading. Carl greatly valued the instructional approach he experienced in graduate school because it forced him to become a better independent learner and critical thinker.

**Carl’s belief about teaching mathematics.** Carl’s graduate school experience influenced his philosophy of teaching mathematics. He believed one of his key responsibilities was to “teach students how to learn and how to teach themselves” (Lines 820 – 821). One instructional practice Carl used was introducing a concept and intuitive ideas along with a basic example. Then he posed a problem requiring an extension of the concept and left it up to the students to develop a solution path on their own. This tactic was reminiscent of his graduate courses. Carl’s goal was to structure lessons in a way requiring students to think critically and extend basic concepts to solve more complicated problems; to teach students how to learn. Carl’s beliefs about teaching reflected the way he was taught in graduate school. This is consistent with research (Brown & Borko, 1992) suggesting teachers' beliefs about mathematics and how to teach mathematics is influenced by the way they were taught. Likewise, he wanted a mentor who was like-minded in this philosophy of teaching.

### **Mentor 1 – Dr. Walter Bethune**

**Dr. Bethune’s background.** Dr. Walter Bethune was a civilian professor with 18 years teaching experience in the mathematics department. He had a doctor of philosophy degree in biometrics, a master’s degree in mathematical and computer sciences, and a bachelor’s degree in

geophysical engineering. Dr. Bethune was a teaching assistant in graduate school and a research assistant while working on his doctorate. All of Dr. Bethune's collegiate teaching experience was at USAFA; however, in his prior military career, he taught basic physics (advanced high school to undergraduate level) to personnel preparing to attend a specialized nuclear power operations training school. As the most senior civilian professor in the department, Dr. Bethune held a variety of positions in the department: engineering mathematics division chief, academics director, and interim department head. He was a vital contributor in birthing the vision for and molding the department's faculty development program

**Dr. Bethune's beliefs about teaching and learning mathematics.** Dr. Bethune taught mostly upper level courses for mathematics majors. During this semester, he taught advanced probability. Dr. Bethune's driving motivation in his teaching was holding students accountable for their own learning by setting high expectations from the beginning and holding students to those expectations. For example, he expected his students to read and comprehend the mathematics textbook prior to coming to class. He approached a lesson assuming students already knew or had struggled with definitions and foundational justifications for the concepts. In class he required students to communicate their understanding of concepts and demonstrate their ability to apply this knowledge for more complicated problems.

Dr. Bethune told his students his instructional decisions were made with considerations of what best benefited their long-term learning. He admitted some students were not accustomed to his style of teaching, often complained at first, or experienced lower grades than they were accustomed to. In the end, they expressed their appreciation for being challenged and having gained better learning skills. He suggested, "More experienced teachers hold students more

accountable, which leads to short-term failure but leads to long-term better study habits”

(Interview 1, Lines 960 – 962).

**Dr. Bethune’s approach to mentoring.** Dr. Bethune told those instructors he mentored, “I can be as much hands-on or as little hands-on as you need; it’s up to you” (Lines 470 – 471). Based on his experiences, he found most new instructors appreciated a hands-off approach without over-involvement. He described himself as follows:

I do mentoring differently depending on the personality of the person I’m mentoring. I consider: What do they need? What things can I help them with? What do I perceive they’re struggling with or what do I perceive their strengths are? I adjust based on that. The mentor gives a new instructor someone senior who’s taught awhile to be a sounding board. (Lines 476 – 484)

Dr. Bethune also believed getting better at teaching required “just doing it, getting down there and experiencing it” (Lines 625 – 626), and teachers should self-assess and identify areas to work on to improve their instructional practices. He expected the new instructor he mentored to initiate discussions by self-identifying areas for improvement. “I like to do the couple of observations and then ask them to tell me about areas of concern and things they’re struggling with and just talk to them” (Lines 634 – 637). Dr. Bethune also stressed mentors and new instructors should focus on one area at a time. His rationale: If the new instructor is told in one feedback session to remember too many things to improve instruction, he may become overwhelmed and not be able to give due diligence to any one particular area at all.

### **Carl and Dr. Bethune’s Mentoring Relationship**

Carl specifically requested Dr. Bethune as his mentor. Carl was recommended for and selected to work on a project with Dr. Bethune when he arrived in the department. They built a positive professional rapport during the project. Carl asked and the faculty development director agreed to assign Dr. Bethune as Carl’s mentor. Regarding mentoring relationships, Dr. Bethune

expressed: “Like anything else, it’s a relationship and you’re doing it ad hoc. Sometimes the personalities mesh well and sometimes they don’t. I think Carl Pappus and I actually mesh pretty well” (Lines 637 – 640).

**Frequent interaction.** Carl and Dr. Bethune engaged frequently beyond the required two observation cycles. Carl observed Dr. Bethune’s course each lesson. This sparked much discussion on a daily basis. Carl described the way Dr. Bethune responded when he asked him to be his mentor:

He said, “Sure, but I’ll tell you one thing: I’m not going to come to you and chase you around to give you feedback. If you want feedback, mentoring, and advice; come see me.” I’ve spent hours talking to him. He’s very open to talking. If I have questions, I go to him. I appreciate that. (Interview 1, Lines 864 – 874; 883 – 884)

Dr. Bethune would never ask me if I’m ready for a lesson or, “How do you feel about tomorrow’s lesson?” He would say, “That’s your job. You do that on your own. What I can provide to you is teaching philosophy and some insight into learning development and curriculum.” That’s the way he approaches mentoring—not a day to day kind of thing; but a bigger picture. (Exit Interview, Lines 898 – 909)

Dr. Bethune described Carl as “a very thoughtful guy. He thinks and reflects quite a bit; he’s very hard on himself. He takes students’ failure very personally” (Interview 1, Lines 826 – 830; 987 – 988). Carl discussed other facets of teaching such as holding students to high expectations and the pressure he felt when his students’ performance was below the performance of students taught by other instructors who differ in their expectations. Overall, Dr. Bethune was pleased with Carl’s proactive approach to being mentored and inquisitive pursuit of improving his craft of teaching.

**Mentor as a model.** Dr. Bethune shared his teaching philosophies with Carl during their interactions and influenced Carl’s goals for teaching. Carl noticed many of Dr. Bethune’s teacher characteristics from planning to classroom practices:

He's very meticulous in how he preps; he preps a ton! He knows exactly what he's doing – he knows the content inside out: What questions are going to be asked, what questions the book asked, what page gave the example for a topic, where the students can get the resources to solve the problems, and problems similar or related to ones in the book. He's probably read four books on it for the same lesson. (Interview 1, Lines 1248 – 1258; 1262)

Carl regarded Dr. Bethune as a model and set some of his instructional goals based on Dr.

Bethune's teaching behaviors:

I don't understand all the nuances such as how/what lessons tie together. He knows exactly what he wants them to develop at certain times—when they need to know certain theorems and principles and how they're going to use them later. If I could do that – by lesson planning ahead – I try to. (1279 – 1280; 1286 – 1289)

Carl noticed Dr. Bethune wrote very little on the board and allowed more space and time for students' own problem-solving. Carl gave an example of how Dr. Bethune began a lesson:

He said, "All right, here's the problem. You guys did the reading. Try solving this problem." Then he'll say, "Here are the general concepts we're talking about: conditional probabilities and how we do these things. Code it in R [a programming language and software environment for statistical computing and graphics]." He didn't tell them how to do it in R. He just said to do it in R. He expects them to figure it out on their own and learn it. (Lines 1169 – 1176)

This particular instructional practice appealed to Carl. One of his goals was "to give a simple example in class and then give a hard example to work on their own" (Lines 1292 – 1293).

**Conducting observations and feedback.** For an observation cycle, Carl identified one focus area and a good lesson for addressing it. Carl and Dr. Bethune had a pre-observation meeting to discuss the lesson and Carl's goals. Dr. Bethune observed the lesson and had a feedback session with Carl. In the feedback session, Dr. Bethune let Carl evaluate himself based on his goals and focus area identified in the pre-observation meeting. Then they engaged in a discussion about how Carl met his objectives, reasons why he fell short of his goal, potential corrections, and ways to improve for future lessons.

## Observation Cycle 1

### Pre-Observation

Carl and Dr. Bethune had a very informal pre-observation meeting. Carl told Dr. Bethune he was ready to be observed. Once they agreed on a lesson, Dr. Bethune asked Carl for his focus area. Carl described the conversation as follows:

I said, “I thought you just come in and watch me and tell me what I need to be better at.” Dr. Bethune said, “No, I can’t look at everything. You tell me exactly what you think you’re weak at and I will evaluate that part and give you focused feedback on that one area.” (Interview 1, Lines 1356 – 1361)

Dr. Bethune wanted Carl to identify “the one thing preventing him from having the classroom performing at peak level; I wanted him to think about it in terms of performance” (Interview 1, Lines 682 – 687). Carl responded:

Time management in the classroom; I want to make sure I’m not rushing things. I don’t want to just do things for the sake of doing it. I want to be very intentional and purposeful with the time I have and make sure it’s productive. (Interview 1, Lines 1364 – 1369)

Dr. Bethune gave Carl the following advice:

Plan to do three big things during your class but anticipate you’re only going to get to two of those. Put them in the order of most important—understanding you may get to the third; but you probably won’t. (Interview 1, Lines 746 – 752)

This pre-observation meeting was a short, casual conversation lasting a few minutes.

### Observed Lesson

**Topic:** Lesson 4 – “Approximate Integration” with the following objectives from the course syllabus (Appendix E) and notes to instructor (Appendix F): (a) Use Simpson’s Rule to estimate definite integrals; (b) Use the Trapezoid Rule to estimate definite integrals; (c) Explain when Trapezoid and Midpoint rules give overestimates or underestimates and determine numerical upper and lower bounds for various definite integrals.

**Instructor resources:** Carl created and used a PowerPoint presentation (Appendix G) and his own handwritten notes (Appendix H).

**Summary/notable events:** Class began with a review of the definite integral definition, Riemann sums, and definite integral properties. Carl displayed on the overhead projector two review examples of definite integrals (see slide 5 in Appendix G) for the students to work at their seats. After all students completed the problems, Carl asked two students to write their solutions on the board. According to Dr. Bethune's observation notes timeline (Appendix I), Carl spent approximately 14 minutes on the review portion of the lesson. Carl then moved on to the new material for the lesson: left-hand (LHS), right-hand (RHS), and midpoint (MID) Riemann sums to estimate the value of a definite integral. He displayed illustrations of LHS, RHS, MID, and TRAP Riemann sums (see slide 7 in Appendix G) to facilitate a whole-class discussion.

In the discussion, he talked students through the derivation of the LHS ( $L_n$ ), RHS ( $R_n$ ), and MID ( $M_n$ ) estimates. He wrote derivations for Riemann sums (see page 1 of Carl's notes in Appendix H). He then drew a Riemann sum methods comparison table (see page 1 of Carl's notes in Appendix H). His discussion focused on whether each method under or over estimates the definite integral based on the concavity of the graph and whether the graph was increasing or decreasing over an interval. He engaged the students in discussion while having them determine what to fill in the blanks. He wrote a derivation of the TRAP Riemann sum and completed the remainder of the table (see page 2 of Carl's notes in Appendix H). Finally he illustrated a comparison of MID and TRAP Riemann sums (see slide 8 in Appendix G). According to Dr. Bethune's observation notes timeline (Appendix I), Carl spent approximately 18 minutes on this portion of the lesson. Then he spent six minutes on a derivation for Simpson's Rule (see page 2 of Carl's notes in Appendix H).

Carl revisited the solution (see slides 10 and 11 in Appendix G) to an application problem introduced in the previous lesson. This discussion took about five minutes. Carl added the displayed application problem extension questions (see slide 12 in Appendix G). He sent students to the board to use the estimation methods to answer the new questions. He gave the students about 5 minutes in groups to work their solutions before he displayed and discussed for 5 more minutes the solutions to the application problem extension questions (see slide 13 in Appendix G). The purpose of this activity was to use the formal estimation methods and emphasize they are tools providing more precise determinations for the scenario. These solutions concluded the lesson.

### **Feedback Session**

**Timing and pacing.** Dr. Bethune focused on Carl's pacing. He kept a detailed timeline of events during the lesson because he thought it would be most effective in the feedback if he could refer to exact moments for specific instructional activities. Dr. Bethune pointed out too much time was spent on the two students at the board as part of the review. He noticed the rest of the class was not engaged since they had already completed the problems for homework. The remaining students were off-task. He asked Carl, "What was the purpose? Is there a way to speed it up if you want to show student work? Could you have them submit it, put it on an overhead, and have students review it" (Lines 759; 765 – 766). He explained, "Two students were learning because they were presenting the problems. But there was no accountability for the rest. It was a large waste of time because it takes a while for students to write on the board" (Lines 771 – 774).

***Efficient assessment of student progress.*** Dr. Bethune believed having a student write a solution to a problem most of the students have already accomplished for homework was a waste

of time because it left a huge opportunity for disengagement. He wanted Carl to consider more efficient ways to check homework progress. There was also a broader issue about expectations for completing homework: If students know someone will work out the homework problems in class, it potentially sent a message there was no expectation to complete homework before class because a solution would be provided. Dr. Bethune did not think the homework problems were complicated enough to warrant the time taken to cover them. Since the purpose was review, Carl should have spent much less than 14 minutes on them. Dr. Bethune addressed the teaching dimension of time allocation. According to Speer et al.(2010),

This dimension of practice is non-trivial because collegiate teachers likely do not allocate time based solely on the content they must speak and write on the board. They likely also consider the difficulty of particular elements based on prior teaching and the likelihood and duration of students' questions. (p. 108)

Dr. Bethune drew on his own experience when he advised Carl in the pre-observation meeting to identify three big things to accomplish and anticipate only completing two of them. His insight was based on an anticipation of students' reaction to instruction and the need to adjust time based on those reactions.

Dr. Bethune and Carl discussed the principles behind what Carl was trying to achieve, envisioning what it looks like in the classroom, and translating it into enacting instructional activities. Carl recalled the discussion as follows:

Dr. Bethune: Well what's the principle?

Carl: You learn a lot by explaining things.

Dr. Bethune: Okay, well maybe all the students do a pair/share: turn to each other, one explains it to the other or explains it back, then one person stands up and presents. Now the whole class is engaged; they're all presenting information. If that's your principle—the students learn by presenting—think of a way to get them all involved instead of just one or two.  
(Interview 1, Lines 829 – 837)

Dr. Bethune reasoned Carl had the right principle in mind but struggled with executing it in the classroom. Dr. Bethune acknowledged, “Carl’s principle to hold students accountable for homework by randomly selecting anyone to present was well-intentioned, but it cost more time and most of the students were disengaged because there was no accountability for them” (Interview 1, Lines 843 – 845). He encouraged Carl: “Go back to the principle you’re trying to achieve and then think about how you can make sure this is the most effective” (Lines 849 – 850). Dr. Bethune said Carl’s pacing was good for the remainder of the lesson. He was satisfied Carl had given all the students an opportunity to work at the boards on new problems using the new material. Board work occurred at the end of class and warranted more time.

**Looking forward.** At the end of the feedback session, Dr. Bethune included a few notes on other things he “wanted to use as a springboard. If we felt good about time management, this would give us a conversation for the next thing to work on: redundant questions and anticipating common mistakes” (Lines 699 – 707).

**Questioning.** Dr. Bethune said he believed “question asking is a critical part of teaching” (Line 861 – 862) and was worth addressing for a future observation. He explained Carl often used questions in trivial ways not requiring students to think critically. For example, in some instances, Carl asked, “What do I do next?” in a parenthetical way and then answered his own question. Or he threw out a simple question, such as the result of an operation or step in a solution, just to get an off-task student’s attention. Other times, he genuinely asked a question challenging students to think critically. Dr. Bethune said it was ineffective for “getting any sense of the feedback because most of the questions were easily answered or he didn’t even wait for an answer” (Lines 864 – 866). Dr. Bethune told Carl the inconsistency with which he asked questions demonstrated a “mixing of pedagogical styles” (Line 902) leaving the students

confused because they could not decipher what thinking level was expected of them. Dr. Bethune's points were consistent with research (Cotton, 1989; Gall & Rhody, 1987) suggesting higher cognitive questions are most effective for higher ability students; and longer wait-time is needed for higher cognitive questions resulting in more thoughtful responses from students (Ingram & Elliott, 2016; Rowe, 2003; Tobin, 1987).

*Anticipating student responses.* Dr. Bethune also noticed Carl showed some uncertainty when students responded in unanticipated ways. The way in which Carl responded to students' incorrect responses could be perceived as Carl being dismissive and unhelpful for student learning. Carl often responded with "No, that's not right. Here's what you need to do" (Line 908) or just went to the next student without addressing why the first student was wrong. Dr. Bethune said Carl needed to consider a way to acknowledge a wrong answer but should also be able to redirect the student to help him or her give a better response. He suggested Carl try to anticipate incorrect student responses and say, "Let's see what went wrong here" (Line 910). Carl acknowledged he would be more aware of it in the future.

Knowledge of content and students (KCS) (Ball et al., 2008) is required to anticipate how students might make errors. Dr. Bethune referred to this mathematical knowledge domain with his feedback when he suggested Carl consider how to use students' incorrect answers as launching points for learning. Allowing students to pursue a solution path based on their own knowledge is beneficial because they create connections to their prior knowledge and build upon this knowledge. If there is an error in the student's solution path, the teacher must redirect the student to construct the knowledge differently. If the teacher simply tells the student the correct answer, it is less meaningful than if the student is required to rethink his or her own solution path. The new knowledge is better constructed if the student understands what was lacking in

his or her original solution path and corrects the thought on his or her own. The experience tends to leave a lasting impression for the learner.

### **Feedback Impact**

**Carl's response to the timing feedback.** Carl was receptive to Dr. Bethune's feedback. He acknowledged time was not efficiently used at the beginning. He summarized and agreed with Dr. Bethune's advice:

He said, "If you want to be more efficient, you should have someone do it on the board while everyone's still working on paper. Don't have everyone do it on paper first and then pick two students to write their answers on the board. It's inefficient because no one's doing anything." He said "Have someone work it simultaneously on the board so they can see what they're doing and see a different perspective in how they are doing it – develop that way." (Lines 1446 – 1454)

**Carl's instructional modification for use of time.** When asked how he incorporated Dr. Bethune's feedback on use of time, Carl said he no longer used a student to present a problem everyone already completed. Instead, he allowed students time to work on a problem. He monitored their progress and helped struggling students. If a student finished early, Carl sent him or her to the board. He used the work at the board to help struggling students. He said it was beneficial because having students work simultaneously at their desks and on the board saved time and kept everyone on-task. Struggling students could "watch someone working it out while they worked. I also helped them and got more time with the struggling" (Lines 1496 – 1499). Carl was satisfied the feedback helped him achieve his goal of using time more efficiently. He felt it allowed him to be more interactive with the students who needed it most and provided more "tailored instruction" (Line 1532) to meet their needs.

**Carl's response to the feedback on questioning.** Carl acknowledged he asked many questions as he worked through example problems. However, he admitted most of the questions were low-level, next-step or next-operation type questions. He admitted he was "not really

helping them with those questions. Someone's just telling what they already know" (Lines 1394 – 1395). He recalled Dr. Bethune's encouragement to think strategically about questioning; consider ways to ask more productive questions requiring critical thinking. Carl interpreted this encouragement to mean "questions should lead to some kind of learning" (Line 1399).

***Carl's planned instructional modification for questioning.*** Carl's planned modification was to be more intentional about asking more conceptual questions. He referred to his observation of Dr. Bethune's style of questioning: "When Dr. Bethune asked the question in his class, he's thought about the question he's asking. He asks very conceptual questions like: How does this relate to this concept? What are we trying to do here? Interpret this" (Lines 1384 – 1385; 1389 – 1391). Carl wanted to get better at asking questions in different ways requiring students to explore other aspects of a problem and expected these types of questions to promote better learning.

Dr. Bethune noticed and addressed Carl's attempt to assess and assess students' understanding by having students demonstrate their work and through questioning. Dr. Bethune also noted Carl's reaction to students' incorrect responses and encouraged Carl to improve his ability to anticipate student misconceptions while considering ways to redirect student misconceptions. Carl agreed with the feedback and described how it helped him adjust his instructional approaches. He incorporated time for task execution, which gave him opportunities to work with individual students and address common issues as they arose. Carl also intended to improve his questioning techniques by asking deeper questions. These are aspects of the mathematical context of teaching (MC) (Heid et al., 2015) in which Carl sought to increase his knowledge of content and students (KCS) (Ball et al., 2008).

## Observation Cycle 2

### Pre-Observation

**Following up on progress.** Dr. Bethune followed up with Carl about Carl's new timing strategy and use of students at the board. By this time, Carl had reduced the amount of time he used individual students at the board. He preferred walking around and addressing issues individually. Having one student at the board was not effective because students were not paying attention since they were focused on their own work: "I let them work at their desks on their own. I'll walk around and work with them" (W. Bethune & C. Pappus, Pre-Observation Meeting 2, October 22, 2014, Lines 49 – 52). Walking around gave Carl a chance to notice issues he could address on the spot. He sometimes went to the board to address the common issue. Even then, he did not provide a whole solution: "I'll let them struggle. Later I'll put something on the board and leave that one step so they can finish it from there" (Lines 76 – 79).

**Focus on extending students' learning.** Dr. Bethune referred to his notes (Appendix I) from the previous feedback session and recalled the two areas he had written as potential focus areas for the second observation: redundant questions and anticipating common mistakes. Remaining true to his mentoring strategy, however, he left the decision up to Carl: "We were going to do one of those two things, but we can change to something if there's something more pressing in terms of what you think" (Lines 83 – 85). Carl wanted to explore a way to help his students learn how to learn by tackling extensions of foundational concepts or problems. The conversation went as follows:

Carl: I want to be able to show them a very basic example or the foundational concept of what we're doing, then give them an extension of the problem and let them work it on their own. The big question is, "How do you deal with this small extension or small variation of the problem?"

Dr. Bethune: So, you haven't tried that?

Carl: I've done it before, and I'm trying to get better at it without students saying, "This is unfair! We haven't seen this problem before." I say, "That's the point; it's not supposed to be something you've seen before. If I teach you a concept, the concept still applies. The mechanics might be a little different, but the concept still applies." (Lines 123 – 126)

It's a fine balance between anticipating mistakes and letting students explore them on their own. I don't want to jump in and say, "Be careful because in these cases you might get a situation where you have to do this. In this case you might get this..." and explain all 10 of these random occurrences. (Lines 134 – 151)

Carl summarized two objectives he had in mind: "I want them to learn how to learn" (Line 191) and "I want them to know this is different; I guess facilitate that shift from high school learning to college learning" (Lines 195 – 197).

**Student defined cases or extensions.** Dr. Bethune offered another approach: "Have the students come up with their own cases or extensions of problems. You're trying to teach them how to learn" (Lines 205 – 207). The idea sparked the following jubilant dialogue:

Carl: So, you would ask them to come up with their own extension to the problem and solve it themselves?

Dr. Bethune: No, but in the process of them having to think about it, they're going to have to think about how to solve it.

Carl: Ask, "What could change in this case?"

Dr. Bethune: Yeah! "What could I change and why?" Of course a lot of things will be superficial and obvious: "Well, I changed that two to a three." Say, "Ok. Why?" Have them explain. (Lines 213 – 234)

Carl was concerned about how to approach the next topic: sequences, series, and limits. He struggled to figure out an extension for those ideas: "I show them the basic idea of a sequence. We treat it like a function to take the limit. There are different types of functions I could use; more advanced functions" (Lines 362 – 365). He was not sure how he could have students come up with extensions for those topics. Dr. Bethune encouraged him to look at objectives and

homework problems associated with those topics. He also assured Carl it was fine if he was not able to come up with extension ideas. He even suggested Carl rely on the students to come up with ideas. He warned Carl to prepare to sacrifice an extra 10 – 15 minutes.

Students gain a deeper level of understanding when they consider ways to vary problems. In the process of creating or varying problems, students must understand the structure and underlying principles of a problem. They must also know reasonable constraints to fit the context of the problem. Finally, they must know how to correctly execute the process to solve the problem. In some cases, they might reverse engineer or deconstruct the problem. If students are able to do this activity, their conceptual understanding is greatly increased and enables them to tackle extensions or variations of a problem.

***Discovering variations of advanced problems.*** Carl described an instructional practice he had used in a recent lesson:

Recently, I've been giving them handouts and say, "Here are four more advanced problems. You can have the last 20 minutes of class to work with each other but I'm not going to tell you how to do it."

I'll post the solution later, maybe. But I don't go over it in class with them. I don't have them write and solve their own problem. But I think they could have a dialogue about it and say, "Here are four variations you could do." (Lines 383 – 390)

Dr. Bethune warned group work and posting solutions had the potential to give the students a false sense of understanding—a “delusion of competency” (Line 519)—if they did not take the time to work through the problems on their own. However, it was a trade-off of time to assess their understanding during the next lesson. He stressed it was important to motivate students to acquire understanding. It was a time commitment decision Carl would have to make to meet his objective.

**Reading and comprehending the textbook.** Carl gave another example of how he attempted to teach students how to learn:

The other one I did was partial fractions decomposition. There are four cases. I presented the first case and I said, “Read the book for the last three cases. I explained number one and what it all means. So you should be able to read number one and match it up with what I explained in class and figure out what the terminology means. Then you should learn how to read your book.” (Lines 431 – 436)

Carl stressed to students the value and importance of comprehending a mathematics textbook because the students often complained the book was not useful. He justified, “Following the book’s examples and explanations is beneficial to learning and I’m trying to build that skill set” (Line 451). Dr. Bethune agreed.

**Productive studying.** The session moved to a discussion about productive studying from *10 Rules of Good Studying; 10 Rules of Bad Studying* (Oakley, 2014). Dr. Bethune made the guide available to his students on his course website and addressed its tenets in class to help his students identify areas in which they needed to improve. He suggested Carl try to take a little time each lesson to discuss a good and bad habit. He acknowledged it was a time decision Carl had to make. Carl agreed to try it.

**Focus area for the observation.** Dr. Bethune asked Carl to specify the focus area for the next day’s observation. The discussion went as follows:

Dr. Bethune: What do you want me to look at tomorrow; one thing?

Carl: Do I give them the opportunity to learn how to learn or am I just spoon-feeding them; even though I think I’m challenging them? Is it just delusional competency?

Dr. Bethune: No, it’s alright because I struggle with that too. Even though giving examples is a delusion of competency, I try to tell them why I do this step. I try to give them insights they don’t get when reading the book. I say, “Here’s what I was thinking about there.”

I don't have any research on it, but I think there's value in doing examples where I break it down for them and say, "Here's the key step and this is the way I think about it."

I've talked about how the key is to understand the "Why." I'll tell them, "This is the key step. Why would I do that? What's the big idea? Where did that come from?" I try to give them a sense of what they don't necessarily get with their book.

I think there's value in doing examples. But I think if you tie it back to the learning skills and you say, "I'm trying to get you to think about this. I'm trying to help you with recall." Or, "I'm trying to help you with....," whatever the idea.

Say, "Doing this helps you learn. This is what I'm trying to do in this example." And just see how that goes.

Carl: Sure.

Dr. Bethune: Then we'll see how you get them to reflect on their own learning. We'll look for the learning moments and do feedback on that. (Lines 540 – 600; 747 – 748)

Students benefit from having problem-solving modeled for them. It exposes them to metacognitive and decision-making processes required to solve problems. Self-questioning techniques are also important for critical thinking. Students must not be intimidated with unknowns and must learn to ask themselves leading questions to help them progress in problem-solving.

***Standards, leadership, and accountability.*** The session wrapped up with a discussion of setting and maintaining expectations and holding students accountable for doing their part. Dr. Bethune gave an example of one of his past students who took personal responsibility and improved tremendously because she identified where she was weak in her study habits. He suggested to her one of the good study habits: making note cards with the objectives. Dr. Bethune reiterated the value of the study habits guide and making students aware of their own actions. Carl liked the example. Finally, they agreed on a time for the observation.

## Observed Lesson

**Topic:** Lesson 24 – “Sequences” with the following objectives from the course syllabus (Appendix E) and notes to instructor (Appendix J): (a) Explain what it means for a sequence to converge or diverge; (b) Compute the limit of a convergent sequence; (c) Calculate bounds for a given sequence; (d) List terms of a sequence given the definition of the sequence (recursive sequences as well).

**Instructor resources:** Carl used his own handwritten notes (Appendix K)

**Summary/notable events:** Carl returned a test at the beginning of class and discussed common errors. He identified a problem most students had incorrectly evaluated because they either did not recognize the need to use or made errors while attempting to use partial fraction decomposition. He then told them if they had questions about point deductions to come talk to him. He also discussed the upcoming fundamental integration skills quiz and told them to practice to get better. Carl used 11 minutes of class time at this point.

Carl then moved onto the new material: infinite sequences. He started with the definition of a sequence and notation for the  $n$ th term ( $a_n$ ), and gave examples. He asked the students to think of ways to describe and generate the sequences. He introduced the terms, *explicit* and *recursive*, and had the students figure out both the explicit and recursive formulas for the example sequences. They practiced and verified the formulas for explicitly and recursively defining sequences (see page 1 of Carl’s notes in Appendix K). When Carl added the explicit formula  $a_n = \frac{(-1)^n}{2^n}$  for the sequence  $\left\{\frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16} \dots\right\}$ , he told the students to determine the recursive formula. A student gave an incorrect response. Carl checked it by substituting numbers, showed it was incorrect, and provided the correct recursive formula:  $a_n = \frac{-a_{n-1}}{2}$ .

They had a whole-class discussion about the pros and cons of explicit and recursive formulas.

Figure 5 summarizes the discussion:

	<b>Explicit</b>	<b>Recursive</b>
<b>PRO</b>	Easy to define $a_n$	Easy to derive a generating formula
<b>CON</b>	Sometimes hard to derive a generating formula	Relies on generating $(n-1)$ terms to get the $n$ th term.

Figure 5. Summary of explicit and recursive formulas from Carl's notes.

Carl showed another example to emphasize the Fibonacci sequence, whose recursive definition is easily derived but has a complicated explicit generating function:

$\{1, 1, 2, 3, 5, 8, 13, 21, \dots\}$  where  $a_1 = a_2 = 1$  and  $a_n = a_{n-2} + a_{n-1}$  where  $n \geq 3$ . He showed the

explicit generating formula:  $a_n = \frac{(1+\sqrt{5})^{n+1} - (1-\sqrt{5})^{n+1}}{2^{n+1}\sqrt{5}}$ . Carl added as a fun fact, the Fibonacci

sequence was mentioned in the movie *Good Will Hunting*, a loosely based depiction of the life of William James Sidis whose work related to operations research.

The next part of the lesson focused on determining the limit of a sequence. Carl's example was the sequence generated by  $a_n = \frac{n}{n+1} \Rightarrow f(x) = \frac{x}{x+1}, x \in \mathbb{Z}^+$ . He graphed it to give the students a visual representation. The students reviewed and explained a theorem from their textbook (Stewart, 2012): If  $\lim_{x \rightarrow \infty} f(x) = L$  and  $f(n) = a_n$  where  $n$  is an integer, then  $\lim_{n \rightarrow \infty} a_n = L$  (p. 693); and asked for clarifications. Carl gave them an example to determine if the sequence converged based on the theorem and definition:  $\lim_{x \rightarrow \infty} r^x = 0$  if  $|r| < 1$  (p. 696). The students completed the problem and determined the limit was zero which meant the sequence converged to zero. A student asked, "What happens when  $r$  is negative?" Carl replied they should think about the graph and notice  $r^x$  oscillates between positive and negative values but still settles around zero as  $x$  approaches infinity.

Carl's final example was  $a_n = \sqrt{\frac{n+1}{9n+1}}$ . Some students immediately tried to recall a simplification shortcut rule they learned using the highest degree of the polynomial either in the numerator or denominator. The students could not correctly remember it. Carl admonished them for simply trying to memorize and emphasized the importance of understanding a concept rather than relying on memorization of a shortcut without understanding why it worked. Class ended with this example.

### Feedback Session

Dr. Bethune asked Carl how he felt the lesson went in terms of his focus on student learning. Carl's response was:

I spent a lot of time thinking about how I would focus on learning and get them to focus on learning how to learn. I had a hard time with the lesson planning on this one—trying to figure out a way to do that. (W. Bethune & C. Pappus, Feedback Session 2, October 23, 2014, Lines 5 – 8)

Dr. Bethune responded: "First of all, this was your first attempt at trying to do this student learning focus. I think you should continue to work on it" (Lines 67 – 70).

**Principles (not points) of learning.** Dr. Bethune addressed the student learning issue throughout the lesson. He referred to his notes (Appendix L) and started from the beginning:

We started by giving the test back. You said, "If you have any questions on why you lost points, come talk to me." You set the mode: it was about points and not about learning. You have this principle about learning. If you focus on that being the principle of your lesson, then all conversations revolve around that. (Lines 70 – 97)

Dr. Bethune emphasized discussions about how assessment should focus on fundamental reasons why students gave incorrect responses. Carl should not have dwelled on point values but should have had students evaluate their own thought processes and assess themselves based on correct concepts.

Have conversations about improving on the test; not about why I lost points: “What idea did I not convey?” I still struggle with these things; I’m still in a fight. I try to make learning the currency of exchange. Do I always win? No. I just don’t focus on points because I’m trying to tell them what my principle is. We talked about this. This is your word; you’re about learning. So keep it about learning (Lines 421 – 445; 450 – 453)

Dr. Bethune addressed assessment from the perspective of Carl’s learning focus principle. This approach illustrated his commitment to observing Carl’s instructional practices through the lens of student learning and exhibited his discipline to stick to one observation focus area.

**Specifying learning strategies.** Dr. Bethune told Carl to give the students specific study strategies when preparing for a test.

The other thing I think you struggled with is not having some of the tangible things to tell them. Again, when you talk at the abstract level it’s very hard for people. You said, “You know we have a fundamental integration skills quiz coming up. You should practice.” They know that. But what does it mean to practice?

You have to give them tangible things to think about because in the abstract they agree; they just don’t know how to execute it. And they’ll continue to pull off the poor habits without the tangible. (Lines 99 – 147; 398 – 401)

Dr. Bethune offered Carl specific alternate ways to have more productive learning focused dialogue with his students.

**Diagnosing faulty memory and misunderstanding.** The next issue Dr. Bethune and Carl discussed was students’ faulty memorization of a short-cut process without conceptual understanding. They discussed the following:

Carl: The idea was a sequence represented as explicit or recursive. The students asked, “Can I just write the answer in recursive form?” I explained the explicit form and responded, “I could write it in recursive form as well.” I said, “How did you do that on your own then?”

Dr. Bethune: They had a discussion on memorizing results. You could see some of the stuff they were struggling with.

Carl: Some of the students said, “If you take a limit, if you have the highest degree in the polynomial and the top and the bottom are the same then it’s just the ratio of the coefficients.” I said, “You memorized that; but there

has to be a reason why you show it. It's pretty simple. It's not that hard to show why it's true."

Dr. Bethune: That was it. So that's one where you were actually getting to some of the points of learning. (Lines 19 – 43)

Dr. Bethune recalled another discussion about a question on the test Carl had returned:

You went through the questions and asked, "Why don't you recognize partial fractions? What's going on? Why aren't you picking these up?" You can conjecture; you can have a conversation. Again, this will cost you time. But if you're going to do it without having a conversation you could say, "These are the typical reasons why people don't get it right. They memorized homework problems. They didn't practice in context." Go through the reasons why they didn't recognize it.

Those things popped up later in your lesson. When you talked about sequences, there was this golden opportunity. People kept throwing series out there and saying, "Oh, I know this harmonic series diverges." And you said, "No, I'm not asking about the series. I'm asking about the sequence. And  $\frac{1}{n}$  actually converges."

There was a golden opportunity to say, "Look, you're not thinking. You're going to a memory of high school calculus and just pulling out that memory and loosely associating it with this idea. That will cause you to fail. Slow down and think about what's being asked. When you get very good, you can start making these jumps. But right now, these jumps are wrong because you're going off faint memories. Let's go back to definitions and build on those things."

This is what learners do. They keep on it and keep seeing it; it's alive for them and it makes sense. (Lines 149 – 205; 393 – 398)

Dr. Bethune recalled the study habits (Oakley, 2014) discussed in the pre-observation meeting:

"This comes from looking at those ten rules and thinking about learning. Translate it into practical things instead of high level discussions. Give them details" (Lines 207 – 214).

**Requiring thorough explanations.** Dr. Bethune was enthusiastic and complimentary of Carl's effort in challenging a student's explanation:

This was a great conversation. I think they were trying to come up with a recursion. The student said, "Oh, I worked it through." And you said, "I don't know what that means. It would have been nice to tie that to learning. You could say, 'I let you off the hook by just saying 'Work it through.' You're not explaining it.

Explaining and teaching are great ways to learn something. I am very deliberate when I explain things; I do a lot of learning. I'm not going to accept 'Work it through' because that's impeding your learning. Don't use, 'Work it through.' Tell me what you're thinking. Teach me because you'll learn."

That would have been a great opportunity. It was somewhat there, but you didn't tie it back to learning. This is an area to keep working on, right? The principle is learning. What helps you learn?—practice. What does practice mean?—writing test questions and teaching. How is a student going to do these things? (Lines 216 – 244)

It is important for students to communicate their processes. Not only does it give a teacher insight into student thinking, it also helps students clarify meanings and be more precise with their mathematical vocabulary.

**Teaching from mistakes.** Dr. Bethune addressed the opportunity to use a student's mistake as a teaching moment:

Dr. Bethune: The other one that came up is when the student had, " $(a_{n-1})^2(-1)$ " for the alternating geometrics series,  $\left\{\frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16} \dots\right\}$ . It's a mistake, right? This is a perfect learning moment. You should have said, "Tell me how you thought about that."

Carl: Did I not ask him that?

Dr. Bethune: You said, "Let's check it." You demonstrated how to learn. But you didn't point out, "This is one of the things I should do; I should check my result. Let's put a number in there. It doesn't work. Student (whatever his name was), tell me how you came up with this? Let's debug it because there's something flawed in your learning here. There's something to be gained we can understand."

But instead you said, "This doesn't make sense." He said, "Oh, yeah; it's two times that. No, it's a half." There's another moment where you can say, "Look, you're just guessing now. Let's slow down and think our way through because that doesn't help learning. You keep repeating numbers until you get the right thing. Stop. What do we need to do here?"

There was some laughter. Nothing against laughing but say, "We learn from mistakes. He's offering up and learning quite a bit. You can laugh, but he's actually moving ahead of you guys." Celebrate the failure.

Carl: That's something I talked about early on in the semester and hopefully I think it stuck with them. I said, "If you don't know how to do something,

I want you to be on the board and I want you working it. You guys aren't even trying. If he fails, at least he's getting something out of it. You're not even working on it because you don't know how to do it. It's better to at least work on it and fail." (Lines 246 – 303)

We learn best from our own mistakes. Launching discussions from students' mistakes is a valuable strategy to improve students' understanding. These discussions are most beneficial when students are forced to assess their own thought sequences, identify breakdowns in logic, and make corrections based on enlightened understanding (Bruning et al., 2011).

**Allowing students to think and discuss.** Dr. Bethune pointed out an instance where Carl did not allow students to do their own thinking:

Dr. Bethune: You asked the question, "How would you think about this alternating series?" You threw it out there and started answering. Why not let them discuss it for a second? This is a learning moment. Let them discuss and then have someone explain it; otherwise, you get the first person who responds and the other ones will be passive.

Carl: Let them discuss it amongst themselves and take the time.

Dr. Bethune: Take a second and say, "When you discuss in a group, one person typically takes charge. The rest of you may not know. So I will ask someone to explain it. (Lines 305 – 328)

Once again, Dr. Bethune addressed the important issue of wait-time. Giving students sufficient time to process a question and think before answering fosters more thoughtful responses from students (Tobin, 1987; Rowe, 2003; Ingram & Elliott, 2016).

**Interpreting the text.** Next in the feedback session, Dr. Bethune recalled a quick discussion Carl had with the students about reading and interpreting meaning from the textbook. Carl mentioned his students found it difficult to comprehend the textbook. During the lesson, they read and explained a definition. Dr. Bethune made the following observation:

This was a good job of linking learning. You said, "You guys said you can't read the book, but you picked this definition and no one asked any questions about it. That means you can read it."

The questions started flowing once you did that: “I don’t know what  $L$  is. I don’t know what  $f(x)$  is.” You said, “Great, let’s break this thing down and talk about it. That was a good way to go back to learning. The only way you will learn is to take some time and read and start asking questions.” (Lines 330 – 339)

Mathematics textbooks are valuable resources because they provide definitions and demonstrate solution techniques. Although students often have difficulty deciphering the formal language, it is extremely beneficial for students’ learning to interpret and inculcate meanings and processes.

***Generalizing concepts.*** At one point in the lesson, students pondered the effect on the limit of an alternating function:

Dr. Bethune: The students asked some very good questions about generalizing. You were on the far board and the student asked about the graph of an alternating function.

Carl: Right, the alternating geometric.

Dr. Bethune: There were some questions about  $r$ .

Carl: Right, because I said “The absolute value of  $r$ .” They said, “What if  $r$  is negative?” I said, “Well, the limit still applies.”

Dr. Bethune: Emphasize to them, “This would be a great thing to do: Ask yourself, ‘What happens when  $r$  is this? What will it look like?’ If you can explain it to yourself it means you understand it. That’s a very good learning strategy.”

Carl: So we can ask, “Why is the absolute value of  $r$  less than one?”

Dr. Bethune: Yeah. “Why is it absolute value?”

Carl: Consider why it is not simply “ $r$  is less than one.”

Dr. Bethune: Tell them, “This is how you teach yourself. You ask these questions. You create your own examples. You’re reinforcing those types of ideas.” (Lines 341 – 373)

Dr. Bethune reiterated the importance of allowing students to explore and reason about variations of functions. When students engage in this activity, they search for meaning and gain understanding about the general behavior of functions and effects of variations.

**Summarizing the feedback and providing encouragement.** Finally, Dr. Bethune gave Carl some overarching thoughts on his progress and going forward:

Dr. Bethune: You have the principle upfront, but you still haven't thought about how you're going to have a conversation. What's hard is there's no if/then map we can go through that says, "If they say this, do this." You must have these principles in your mind; so when these things pop up you have some way to address them.

Carl: Okay. (Lines 375 – 391)

Dr. Bethune: The hardest thing in learning is having the principles and main ideas. Once the problem comes up, then categorize it into one of those areas and help students see it through that lens. (Lines 455 – 459)

This is a very difficult pedagogy to master. I commend you on trying this so early. It's worth continuing. (Lines 467 – 469)

Carl: Absolutely; it makes sense for a lot of things I hadn't thought about. I really haven't slowed down at all to talk to them about study habits or test preparation. I have only said "Make sure you're ready." But I don't really give them tangible skills or study habits.

Dr. Bethune: You've learned quite a bit from your experience in graduate school. You had to learn on your own. You're trying to help them make that transition. They're going to have their failures. Help them without trying to figure it out for them. They'll struggle, but some will pick it up.

Carl: Absolutely. I really appreciate it. It makes sense and is a lot to work on.

Dr. Bethune: I have setbacks too: "Oh, I should not have said..." There is no perfect teacher. All you can do is keep thinking of these things and pressing on. Eventually you get better because you start getting more familiar. It makes it a little easier. (Lines 471 – 517)

Dr. Bethune reiterated the principle of teaching students to learn how to learn and encouraged Carl to continue working at it because it is a pedagogical skill requiring time and practice to

master. Carl demonstrated he understood by acknowledging what areas in his instructional practices needed improvement.

### **Feedback Impact**

**Fostering independent learning.** After the semester was over, Carl discussed in his exit interview the overall impact of Dr. Bethune's mentoring and feedback:

Dr. Bethune's mentorship has impacted the way I teach and the way I see the classroom on a daily basis. When you come into teaching with no background in teaching, you have no real knowledge or experience in pedagogy and these different methods and pedagogical concepts.

The one big idea Dr. Bethune and I talked about early on was this idea of giving students a chance to develop their learning skills – learning how to learn. Giving them the opportunity to learn how to learn is critical. Understanding the course material itself is likely to fade over time. I don't think my kids will remember the formulas years from now. But the ability to learn is a lifelong skill. I focus heavily on this now. (Exit Interview, Lines 17 – 33)

At the time of the exit interview, Carl was four weeks into the next semester and had taught nine lessons of integral calculus again. He reflected on his revised instructional approach based on his previous semester experiences and feedback:

This semester, I'm focusing more on giving them a chance to learn how to learn. I don't do as many examples in class. I do a lot of principle concepts. I want them to take a concept and apply it on their own. I tell my students, "I gave you the concepts; now use the book and put it together to apply it to a problem. Here's your workout problem." Very rarely do I give them answers in class. (Lines 73 – 77)

He allowed students to work together on problems he assigned during class. When students asked for examples he responded, "Figure it out. I assure you, it's the same. The concepts are general and broad enough; they will apply to this problem. It's up to you to figure out the nuances; the small adjustments you need to make" (Lines 85 – 88). Carl's strategy mirrored his graduate school professors' instructional approach:

I had to learn on-the-fly in grad school where they don't help you out. They say, "Go apply this to this problem. You learned the concepts in class. The assignment is due next

lesson.” They don’t give you the same example problems they worked in class. They give you concepts and expect you to learn, read the book and figure it out. (Lines 103 – 108)

Carl believed students must have opportunities to wrestle with problems and make mistakes to improve their learning. His instructional approach aligned with his belief. Dr. Bethune and Carl’s discussions focused on their shared belief of the importance of giving students enough opportunities to “make decisions; to fail and learn on their own” (Line 173). As a result, Carl was more intentional in incorporating instructional practices designed to help students become more critical and independent learners.

I asked Carl to describe his approach with material often treated as a mechanical process. His example was the  $u$ -substitution method for solving integrals:

It can be mechanical. Some instructors teach: “Pick your  $u$ , pick your  $du$ . Solve for  $du$ , and plug it back in and cancel the terms out”. I just say, “It’s the chain rule backwards. Think backwards. What happens when you execute chain rule? You take the derivative of the outside times the derivative of the inside. So  $u$ -substitution should be a method that undoes it.” If you understand what’s going on with the chain rule, you could do it in a conceptual way: “What function would have to be the anti-derivative to get that derivative?”

I might use a very basic example:  $y = e^{2x^2} * 4x$ . Or I might use a complex example they would never do on their own and say, “We will apply this to something very difficult. Now, work from this advanced example backwards on your own or work your way up from a very basic example on your own.” (Lines 202 – 214; 217 – 229)

Carl described a concept and allowed students to collaborate on problems. He told them, “If you get stuck, I might help you; but I won’t do any problems as an example for you. You must work them on your own” (Lines 263 – 266).

**Teaching with more foresight.** Carl said having one semester under his belt helped him better frame his teaching to make it “easy to relate to what’s coming in the future” (Lines 347 – 348). He recalled a specific example of how he approached one concept differently than he did the previous semester:

You can teach Riemann sums and definite integrals as areas under a curve. It's almost exclusively how I taught it last semester. It was areas between a curve and the  $x$ -axis or  $y$ -axis. But when we got to volumes last semester, they struggled with the general idea of an integral as a cumulative sum.

If you add up rectangles, you get an area. That's what we did with Riemann sums; took the integral of  $f(x)dx$ . If you form what's inside the integral as a volume of a characteristic slice and integrate, you get total volume. Or if you integrate a small piece of work or work over a small distance— $(Force * Distance)/(small\ distance)$ —and then sum it all up; you get total work. The integral is regarded as a cumulative sum rather than an area under a curve.

Now when I teach Riemann sums, I don't just say, "It's an area under a curve." I say, "It is an area under a curve because we're taking the cumulative sum of areas of small rectangles or thin rectangles. In general, the integral itself is a cumulative sum. It is area here; but not always." I gave them some foresight by saying, "Not always." We're using  $f(x)dx$ , where  $f(x)$  is the height and  $dx$  is the width of the rectangle; that's what you're doing now.

It lends itself much easier for the upcoming application problems: force, work, hydrostatics, and solids of revolution problems. It makes it much easier. I say, "You just do little force times a little distance several times and add them all up." I'm trying to press that earlier: just knowing what's coming and changing what I'm doing now to prepare for that. (Lines 384 – 407; 426 – 430)

Carl's revised approach was consistent with the NCTM (2000) Connections Standard to "enable students to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole" (p. 354).

**Comfortably challenging students to be independent learners.** I asked Carl what he felt most comfortable about in his teaching. He responded, "I take pride or am confident in the fact that I will not give in to what I consider a lower level of learning by spoon-feeding examples to students and having them just give it back to me" (Lines 469 – 473). He intentionally forced students to collaborate rather than rely on him as the only source of knowledge. "I'm very comfortable with how far I go with concepts, at which point I turn them loose on their own, and knowing it is the right thing to do. I'm thinking with bigger and long-range perspective" (Lines 475 – 488).

It can be difficult for a mathematics teacher to step back and allow students to have control of their learning because students are often comfortable with passive learning where the teacher performs show-and-tell examples and students mimic the process with limited understanding. Teachers may also be complacent with this traditional strategy because it allows them to maintain control and account for what content is covered—often fulfilling a requirement to document objectives achieved according to some mandate (e.g., Common Core State Standards). When teachers abandon this traditional strategy, students may become resistant because they are forced from their comfort zone. Because of his graduate school experiences, Carl became comfortable with this approach, was convinced it reaped higher learning gains, and taught him how to be an independent learner. As such, Carl was willing to apply this instructional approach because he held the belief it was most beneficial for his students' learning development.

### **Summary**

Carl and Dr. Bethune's mentoring relationship demonstrated what makes mentoring productive: role modeling, good rapport, common goals, frequent and informal interaction, and collegial sharing of ideas about learning principles and learner-focused instructional strategies — both philosophical and practical. They often discussed a wide variety of topics: what Carl observed in Dr. Bethune's classes, education at large, philosophy, student responsibility and development, leadership, the latest news or research in random subjects, etc. Carl observed Dr. Bethune's teaching and noted strategies he believed made teaching more effective for student learning. Dr. Bethune modeled the instructional suggestions he offered and made feedback a two-way exchange. This positively influenced Carl's reflection on teaching. Their ongoing communication shaped a common instructional vision to which Carl aspired. This aligns with a

common belief: collaborative reflection of practice with more experienced colleagues fosters a positive collegial environment and helps new teachers improve their practice (Bell, 2002; Gellert & Gonzalez, 2011; Rodgers, 2002).

Figure 6 summarizes the frequency for areas Carl and Dr. Bethune discussed and reflected upon. The themes are ordered in decreasing order based on the sum of the frequencies of each participant's contribution to the data.

Themes	Carl	Dr.B	Sum
Emphasizing learning	12	20	32
Assessing student progress	7	4	11
Timing/pacing	4	6	10
General practice	1	8	9
Mentoring	4	5	9
Extending problems	7	1	8
Questioning	5	3	8
Anticipating student responses	1	3	4
Modeling problem solving	0	4	4
Connecting across curriculum	3	0	3
Choosing examples	1	0	1
<b>Total</b>	<b>45</b>	<b>54</b>	<b>99</b>

Figure 6. Case 1 data matrix summary of themes.

Carl identified time management and instructional techniques to engage students in independent, critical thinking as his focus areas. Figure 6 captures Carl's interest in student learning (12 of 45) and student progress (7 of 45)—together they are approximately 42.2% of his data blocks. Dr. Bethune consistently noticed and addressed student learning and progress at roughly the same rate ( $24 \text{ of } 54 \approx 44.4\%$ ) as Carl. Carl reflected on ways to emphasize learning and efficiently assess student progress. Dr. Bethune's feedback helped resolve timing issues early the semester. Dr. Bethune suggested ways to help students self-develop their learning through productive studying habits and creating extensions of problems. Carl became more

intentional to approach material more conceptually and require students to explore extensions and generalizations of problems once they have a basic foundation.

Figures 7 and 8 summarize frequencies for MKT and MUST categories, respectively, assigned to Carl and Dr. Bethune's interactions and interviews data.

MKT	Code	Carl	Dr.B	Sum
<b>Subject Matter Knowledge</b>	<b>SMK</b>			
<i>Specialized content knowledge</i>	SCK	0	4	4
<i>Common content knowledge</i>	CCK	3	0	3
<b>Pedagogical Content Knowledge</b>	<b>PCK</b>			
<i>Knowledge of content and students</i>	KCS	20	26	46
<i>Knowledge of content and teaching</i>	KCT	18	24	42
<i>Knowledge of content and curriculum</i>	KCC	4	0	4
	<b>Total</b>	<b>45</b>	<b>54</b>	<b>99</b>

Figure 7. Case 1 data matrix summary of MKT categories.

MUST	Code	Carl	Dr.B	Sum
<b>Mathematical Proficiency</b>	<b>MP</b>			
Conceptual Understanding	MP_CU	3	0	3
<b>Mathematical Activity</b>	<b>MA</b>			
Mathematical reasoning	MA_RS	7	4	11
Mathematical creating	MA_CR	0	1	1
<b>Mathematical Context of Teaching</b>	<b>MC</b>			
Access and understand the mathematical thinking of learners	MC_LR	20	24	44
Assess the mathematical knowledge of learners	MC_LP	10	25	35
Reflect on the mathematics of practice	MC_RP	5	0	5
Know and use the curriculum	MC_CUR	5	0	5
	<b>Total</b>	<b>45</b>	<b>54</b>	<b>99</b>

Figure 8. Case 1 data matrix summary of MUST categories.

Carl and Dr. Bethune mostly engage about areas pertaining to the KCS and KCT domains and devote most of their reflection on contexts for accessing and assessing student learning.

## CHAPTER 5

### CASE 2: KEVIN JOULE AND DR. FELIPE IGNACIO

#### **New Instructor 2 – Kevin Joule**

**Kevin's background.** Kevin was a first-time instructor teaching differential calculus for undergraduate freshmen. The summer after his sophomore year of high school, Kevin told his father (an adjunct professor of physics at a local college) he was not planning to return to high school. When fall semester came, his father took him to register at the college where he taught and told him to take calculus, physics, and something which interested Kevin. Taking advantage of his high school's post secondary option during his junior and senior years, he attended community college and earned both a diploma with a two-year associate's degree. He went to a large, major public university in the Midwest and earned a bachelor's degree in electrical engineering on a Reserve Officer Training Corps (ROTC) scholarship.

After college, he joined the military and worked for two years as an electronic warfare officer before deploying to Saudi Arabia in support of Operation Iraqi Freedom. After completing training as an aircraft pilot, he flew state-side search and rescue missions. To be more competitive for promotion in rank, Kevin considered what master's degree program interested him. His wife had a degree in mathematics and taught high school mathematics; so he applied at a major public university in the south central part of the country where he earned his master's degree in mathematics. He noticed annual job openings in the mathematics department at USAFA, but the positions were not open to his career field initially. When the positions

opened up to his career field, he applied and was hired. He was assigned as an instructor for differential calculus for freshmen.

**Kevin's instructor experiences.** Prior to teaching calculus, Kevin's only other teaching experience was six months as an instructor pilot. Kevin said experience as an instructor pilot influenced his calculus instructional practices in some ways because of the general commonalities in a learning setting. The difference in the goals and purpose of the subjects and the environment helped Kevin contrast his experiences and instructional approach in the academic classroom.

**Preparation and time.** Kevin said the most obvious skill transfer from being an instructor pilot was in preparing for instruction: "What are my objectives for this lesson? How am I going to teach them? How am I going to evaluate them? How am I going to prepare the student? Those all translate very well I think" (K. Joule, Interview, October 9, 2014, Lines 132 – 136). Kevin also felt the pressure for time as an instructor pilot was greater than in the academic classroom. In the pilot world, a number of unexpected external events such as weather conditions or a student's mistakes in flight execution could easily hinder instructional time. Since the classroom environment schedule was well-coordinated and strictly adhered to, external factors were controlled to protect academic time. Kevin typically planned what he wanted to accomplish in 53 minutes and adjusted instruction in real-time depending on students' responses. He said he had no trouble staying on pace with course objectives and made time for students' questions because "when students are asking good questions, they are learning" (Line 140). He considered the time well-spent even if it meant he did not cover something he originally planned.

**Interaction with students.** Kevin believed the pilot training environment required a unique teaching approach and type of interaction with students: "The teaching methodology

with the student interaction is very different” (Lines 136 – 137). The pilot training environment was extremely competitive with little room for error. Pilot candidates must work well under stressful conditions. Part of Kevin’s role as the instructor pilot was to create pressure during instruction. He humorously added, “Here, they took away my three best instructing tools: fear, sarcasm, and public ridicule [*laughs*]” (Lines 111 – 113). Another difference was the frequent opportunity to work individually with students. “One-versus-one is instructor-versus-student in the flying world. My student-teacher interaction is me versus the student pilot only; whereas here, there are 18 to 19 of them against one of me” (Lines 175; 181 – 184). Kevin was more confident knowing a student pilot’s level of understanding and ability than a calculus student’s because he could interact one-on-one more in the pilot world.

***Procedural versus conceptual learning.*** Kevin contrasted the types of learning expected of the students from the different environments:

Here we’re trying to teach at a conceptual level. We want to develop thoughtful people; critical thinkers. With flying, much of it is “monkey see, monkey do”—task oriented. I don’t want my lieutenants to be overly thoughtful when they’re flying. I want them to make a decision in 700 milliseconds. I don’t want it to be a good decision that takes five seconds. I want it to be an adequate one. If you keep making adequate decisions you’ll bring my lieutenant and airplane back and get the mission done. (Lines 117 – 125)

He also described the instructional method best which complimented learning in each setting. Student pilots learned mostly from demonstration and individual execution: “In flying, I demo and then you do. Then I shadow you because I don’t quite trust you yet” (Lines 145 – 146). The method in calculus class was similar because the demonstration was in the form of example problems. Kevin explained: “When I do an example problem, it’s a collaborative effort where I deliberately make them use what they already know” (Lines 149 – 151).

***Real-world versus abstract context.*** Kevin highlighted the fact that pilot training takes place in an environment where physical factors must be accounted for with real-time execution.

He described it as follows:

Context is much more abstract here. We use the context of an artificial problem. In flying the context is, “We are doing this right now.” As a mission instructor, flying a live sortie over Afghanistan context is very real. It’s very apparent and they wouldn’t get there without knowing the context. They’re deliberately here because they don’t understand the context yet. That part is very different. (Lines 152 – 158)

He acknowledged the calculus course often had contrived problems giving students foundational understanding to later help them to solve problems with real-world contexts.

***Cost versus benefit of student errors.*** Kevin acknowledged while errors were launching points for learning; in pilot training, execution errors had real-time physical consequences.

There is much less tolerance for mistakes in flying. At 300 miles an hour there is not a lot of time to recover from a mistake; and the consequences can be fatal. I will not let them get off the plan very far. Here, the consequence of a mistake is generally learning something. Letting them make a mistake is good and healthy here. So that’s very different. When flying, I’m deliberately anticipating what they’re going to do and guarding against it. Whereas in the classroom I’ll say, “Ah that’s an interesting approach” then I see if they can figure it out. (Lines 158 – 168)

Kevin’s belief alluded to the construct of learning growth emerging from recognizing and fixing mistakes (Bruning et al., 2011).

***Student motivation and engagement.*** Kevin confirmed the majority of student pilots are motivated. In contrast, he found many of his calculus students disengaged for various reasons:

It’s not the same people every time; but the ones who already understand or think they already got it are disengaged. There are a few who don’t care. Some days they’re tired, some days they’re hungry, some days they’re studying for the chemistry test in 53 minutes. Maybe that’s a better use of their time than paying attention. They just don’t have the maturity to blow off what’s happening in other classes or what’s happening in the rest of their lives. You won’t make it through pilot training if you can’t focus on the mission. (Lines 242 – 250)

Kevin admitted:

I'm not making much of an effort and not succeeding much in engaging the ones who are disengaged. I don't know if that's something I need to fix or not. I want them more engaged; but it's challenging. There are a lot more things to fix than I have time to fix. (Lines 257 – 261)

Kevin worried if he was not reaching students because he could not always tell if they were fully mentally engaged:

It's the ones who don't appear engaged. You don't get any feedback. I have no idea. Are they getting it? Do they think they're getting it but they're wrong? Are they lost but not showing it or are they just not there at all? There's a huge difference between a student who's not engaged and a student who does not look engaged. There are some who go, "Oh, okay" and have no idea what I'm talking about. There are a few of those, but there are a lot of students who are okay and not responding because they're tired or not showing any response. They're still engaged but not fully engaged. If I don't get any feedback, I can't tell. Communication is a big part of learning. (Lines 559 – 572)

***Using board work to assess students' understanding.*** Kevin used board work to informally assess students' understanding.

From across the room it's very hard to tell their level of participation. It's really hard to tell whether they're just writing what their teammates tell them or whether they're not writing anything at all. When someone else is writing, do they understand? When I'm right there, I can tell. I do pay attention to each group. When someone dominates the marker, I move it around the group.

In some groups the weak student is aggressive about grabbing the marker: "Let me do this." In some of the groups the strong students are very aggressive and grab the marker and then just do it to get it done. Some are really engaged in leading their teammates. That's great. I've got one table where they all do it independently; then one of them will grab the marker and write on the board. I don't get any individual feedback from that. There's some opportunity with board work, but it's also very hard to tell the individual piece versus the best of the group or the worst of the group.

**Influences on Kevin's mathematics teaching approach.** Kevin was familiar with the education environment because of his wife. He also realized his instructional approach was influenced by his instructor pilot experience:

I've already set this pattern in class: we discuss, they try, and then we recap. That flows back to flying where we pre-brief, fly, and de-brief. I hadn't thought about it, but that model is so engrained in me; I naturally do that in class. (Lines 320 – 324)

Kevin used the course director's notes to instructor (Appendix N) as a helpful guide for structuring a lesson.

Kevin borrowed some of his teaching style from one of the experienced instructors (Dr. Sarah Brown) in the course whose class he audited.

Our styles are not that similar, but the gross methodology is. I do my lesson plan and then I watch Sarah teach. It's very helpful to draw on her experience on what students have trouble with. At least I think she's using her decade or so of experience to shape her lesson plans. I don't go to her first lesson. As her lesson plans evolve through interactions with her students, I'm getting a better one later in the day.

There are certainly things I should do—gaps my students have that I don't address because she didn't address them. Whether or not her students had that gap; it's hard to tell because I'm not grading her. But I do see a weakness with that. But I'm getting better and better at addressing that. It's still very reactive but I don't know that we could be much better than reactive. (Lines 274 – 291)

Sometimes I'll make adjustments; then sometimes I'll go, "Oh I guess I'll do it this way." She does a lot of board work too. That gives me, at a group level, a good idea how well they're getting it. (Lines 625 – 626; 640 – 641)

During his audits of Dr. Brown's class, Kevin paid close attention to how her students responded. He made adjustments to his lesson plans. For example, if her students struggled with something, he took a different approach. He used her students' reaction as a gauge for predicting his students' reactions.

## **Mentor 2 – Dr. Felipe Ignacio**

**Dr. Ignacio's background.** Dr. Felipe Ignacio was a civilian assistant professor with more than 10 years teaching experience. After he earned an undergraduate degree in mathematics, he began graduate school but was deployed as a guardsman. When he returned, he taught middle school mathematics for a year and was deployed again. After this deployment, he

returned to graduate school where he was a teaching assistant who was given plenty of teaching opportunities in a wide variety of classes (algebra, trigonometry, calculus). He recalled in his interview (F. Ignacio, Interview, October 10, 2014) the strong emphasis on teaching: “I liked the program. They encouraged their graduate students to do a lot of teaching there. They take teaching very seriously” (Lines 62 – 64). He was given full reign to run his own classes as an instructor of record and a course convener. He accepted a two-year post-doctoral position at a liberal arts college which emphasized teaching more than research. When he began his search for full-time teaching positions, USAFA was attractive to him because its mission gave priority to teaching. He took the position and had been in the department for two years.

**Dr. Ignacio’s teaching and mentoring foundation.** When Dr. Ignacio was a graduate student he was assigned a faculty mentor who discussed teaching in general and gave specific feedback about his teaching. Dr. Ignacio affirmed, “I had some good teaching mentors there” (Lines 70 – 71). He also enjoyed a fun opportunity to participate in a teacher education program sponsored by a National Science Foundation (NSF) grant to work with inservice middle school mathematics teachers who wanted to improve their mathematical knowledge for teaching. Dr. Ignacio summarized his graduate school experience as follows: “I had a lot of good teaching experiences. I’m glad I went to school there because I wanted to teach. I enjoy doing mathematics research, but that’s not the end-all, be-all of mathematics. I’m more interested in teaching” (Lines 98 – 102).

His graduate teaching experiences fed his passion for teaching and directly influenced his postdoctoral decision. He chose the liberal arts college because of its commitment to excellence in teaching, great reputation for mathematics teaching, and access to mentorship from nationally recognized mathematics teachers. The mathematics department had an established informal

mentoring program pairing him with a mentor who observed him and offered feedback as well as helped him navigate various administrative things during teaching. This experience followed a similar construct as his graduate program. In his graduate experience, the mentorship came from a peer—making the experience exactly similar to USAFA’s mathematics department. Feedback communications from observations were informal, between the mentor and observed colleague only. They never influenced any formal job evaluation.

**Dr. Ignacio’s approach to mentoring.** When Dr. Ignacio arrived at USAFA, he was paired with a colleague mentor who had attended the same university where he earned his doctor of philosophy degree. They shared mathematical foundations shaped by formative academic courses taught by many of the same mathematicians. Since Dr. Ignacio already had more than 10 years of teaching experience, his mentoring focused mostly on adjusting to the peculiarities of USAFA’s culture and its unique instructional dynamics. Dr. Ignacio and the mentor were a natural fit because they also shared similar perspectives of teaching and mentoring: “A mentor can point out things you do in the classroom; but ultimately the person who is doing the teaching has to figure out what they need to do and what will work for them” (Lines 267 – 269).

Even though he was certain he would have eventually become a mentor in the department, Dr. Ignacio did not expect to so soon after his arrival. He welcomed the opportunity because he viewed it as an important role. “It’s a nice experience; and it’s actually pretty good. You get to stay in touch with what’s going on with people who have just arrived and maybe haven’t had a chance to teach before” (Lines 311 – 316). He added, “Mentoring is beneficial to me because I get to see things about teaching through new eyes” (Lines 316 – 320) and can consider other possibilities. Dr. Ignacio described his observation and feedback approach as follows:

I'm not very much into a prescriptive view of a mentor. I don't go about it saying, "I'm going to identify what's wrong and fix his teaching." It's his job to figure out what works and doesn't work. It's my job to watch what's happening with a neutral eye and just make note of it.

Occasionally something will puzzle me and I'll make a comment in my notes to ask about this or ask what was going on. Then when it comes time I'll say, "Hey, this is what I saw" and just try to have a discussion and ask, "Was that intentional?" I let him draw his own conclusions based on what observations I saw. I'll be that second pair of eyes looking at it from another point of view. (Lines 325 – 351)

According to Dr. Ignacio, being a second set of eyes was necessary because "it's difficult for the teacher to get an overall picture of what's going on in the classroom. That's what I hope to provide as an observer; to just watch and give that other perspective" (Lines 363 – 365; 367).

### **Kevin and Dr. Ignacio's Mentoring Relationship**

The faculty development director noticed their good rapport and assigned Dr. Ignacio as Kevin's mentor about 2 weeks into the semester. Kevin said, "When Brian assigned him as my mentor, it was because a mentoring relationship was already there" (Lines 386 – 387). Kevin and Dr. Ignacio had offices across the hall from each other. This arrangement allowed for frequent informal communication. According to Kevin their relationship evolved naturally by just talking to each other: "He's very approachable. We have similar teaching philosophies; similar sense of humor. Of course being geographically close helped" (Line 384). Kevin was comfortable with Dr. Ignacio's mentoring and feedback style: "We've done a lot of feedback informally. We have very similar teaching philosophies" (Line 385). Dr. Ignacio (2014) also felt the informality of the mentoring relationship made it most effective: "Most of the mentoring really has very little to do with the observation feedback" (Lines 372 – 374). He described as follows:

A lot of it is just casual conversations. I'll drop by and say, "How are things going?" And we'll have a short conversation. That's where most of the mentoring is taking place; in more of these less structured, less formal interactions. I prefer it that way because the

whole mentoring thing was supposed to be an informal process; this isn't an evaluative process. This is about helping and making sure new instructors have resources available.

Kevin also has no problem popping in. I've told him, "If my door is open, come on in and sit down. We'll talk." He does a couple times a week usually. Sometimes it's about teaching. Sometimes it's not. Sometimes it turns into something else like sports or politics or weather or something. That's fine too. (Lines 398 – 416)

Kevin used Dr. Ignacio to review quizzes and discuss assessment results. When Dr. Ignacio reviewed a quiz, he would comment on its length and difficulty or ask, "What were you trying to get at with this problem?" (Lines 396 – 398). Kevin (Joule, 2014) affirmed, "That feedback has worked well" (line 475).

Kevin described their pairing as "a very good relationship that helps me substantially" (Lines 480 – 481) and valued Dr. Ignacio's wealth of teaching experience: "He has a lot more teaching experience than everybody else in this hallway probably combined" (Lines 394 – 402). Kevin appreciated his expertise to help him improve: "It's always good to know some balance checking is occurring to make sure I'm not way off in left field or grossly divergent from what they need to learn" (Lines 534 – 536).

**Conducting observations and feedback.** Dr. Ignacio allowed Kevin to decide when he wanted to be observed. For the first observation, Dr. Ignacio looked for things with which he believed new instructors typically had issues, precision of language and representation. He also encouraged Kevin to identify issues he wanted him to notice in future observations. Kevin also liked deliberate, critical feedback because it was the way in which feedback was delivered in the pilot world. Dr. Ignacio obliged Kevin with this approach.

## Observation Cycle 1

### Pre-Observation

Kevin and Dr. Ignacio did not have any pre-observation meeting. Dr. Ignacio approached Kevin and told him he wanted to observe him to see how things were going. Kevin selected a day and Dr. Ignacio agreed to observe then. This agreement took place a few days prior to the observation.

### Observed Lesson

**Topic:** Lesson 5 – “Continuity” with the following objectives from the course syllabus (Appendix M) and notes to instructor (Appendix N): (1) Assess the continuity of a function at a point using the limit definition of continuity, (2) Assess the continuity of a function at a point from the left or right using the limit definition of continuity, (3) Explain whether or not a function is continuous over an interval based on the type of function it is.

**Instructor resources:** Kevin’s handwritten notes for Lesson 5 (Appendix P) and PowerPoint presentation of the Quiz: Limit Review (Appendix Q)

**Summary/notable events:** Class began with a five-minute quiz reviewing limits. Kevin spent about 10 minutes afterwards going over the answers to the quiz and discussing the meanings of “a limit does not exist” and “the limit is infinite.”

### Feedback Session

Dr. Ignacio and Kevin’s feedback session (F. Ignacio & K. Joule, Feedback Session 1, October 3, 2014) occurred about a week after the observed lesson because their schedules were busy right after the lesson. The session was held in Kevin’s office and lasted 35 minutes.

**Clarifying meaning for students.** Dr. Ignacio referred to his notes (Appendix R) and first addressed Kevin’s handling of students’ questions about the difference between a limit

which does not exist and an infinite limit. The question was about the  $\lim_{x \rightarrow 0} \frac{1}{x^2}$ . Dr. Ignacio stated, “It was a pretty tough question because they’re looking at it as ‘Infinity is a number.’ I think you did a good job maintaining this distinction between the limit being infinity, as opposed to the limit not existing” (Lines 47 – 53). He continued:

***Making subtle distinctions in meanings.*** Try to make sure they understand when you say “A limit is infinity,” you are saying it doesn’t exist. But you’re saying more in some sense. You definitely maintain a distinction between those because somebody asked, “Well, how do I know if the limit doesn’t exist or if it’s infinity?” There was a discussion going on there. At some point, you expressed the idea infinity is not a value; it’s about a behavior.

This discussion took a while. Certainly, there was a lot of back and forth. You would come up with something and evidently it didn’t seem to answer their questions. That discussion went around for a little while and that was good. That’s an important concept and being willing to devote time to stick to it until the question is answered is valuable.

***Persevering through tough student questions and closing the deal.*** It’s not about coming up with the right or perfect description or the perfect explanation right away. It’s keeping at things until the problem gets solved. At the end of this discussion that had gone off in all these different directions, you went back to the original student who asked the question and verified the question was answered. That’s something I like to make sure I do. I was glad to see you do that because it’s very important. (Lines 55 – 76; 94 – 108)

To clarify meaning for students, a teacher must first understand their questions then comprehend disconnects in their understanding. This requires knowledge of content and students (KCS) (Ball et al., 2008) and falls under the mathematical context of teaching aspect of accessing the thinking of learners (Kilpatrick et al., 2015). Dr. Ignacio alluded to the idea of instructional perseverance. A teacher’s instructional perseverance is analogous to the principle of students engaging in productive struggle (NCTM, 2014). Just as it is a goal for students to exhibit perseverance in working through mathematical problems, teachers should persevere in working through the art of decoding students’ mathematical understanding during instruction.

**Precision of language and notation.** Dr. Ignacio pointed out some issues with Kevin's precision with mathematical representation and meanings.

I have a quibble. You wrote, "The function  $\left(\frac{1}{0}\right)^2$  does not exist." Be careful because this isn't a function. In fact, it's not even a numerical value. It's easy to fall into the language the students use. The student just before said, "Well what about the function  $\left(\frac{1}{0}\right)?"$  It's easy when you're engaged in the discussion with them. It's something to watch out for because it might end up confusing somebody—the mixing of the evaluation of the function versus the function itself.

For a lot of our students, it's a subtle point they don't understand at all. To them, the function at a point is the same thing as the function. You must say, "No. There's a significant difference between how a function behaves at one point versus the function itself." That's one of the things we're trying to get them to think of in calculus. It's the jump from working with numbers—and they might be thinly veiled or cloaked numbers that you're working with in algebra—to thinking about functions as abstract objects themselves. (Lines 114 – 147)

Dr. Ignacio highlighted another example in which Kevin corrected himself real-time. He commended Kevin for using his error as a learning point for students:

I did have another quibble. I'll write it up here [on the board] and you're going to cringe. But then I'm going to say it had a happy ending. But it almost didn't have a happy ending because you wrote this: " $\lim_{x \rightarrow -3} = DNE$ " without a function. I was cringing and saying, "Ah, this is the bad thing!"

You started having a discussion with a student and then you fixed it because you stuck a function in there. But you did fix it and I said, "Thank goodness!" because this is a bug-a-boo we're constantly fighting with them. You probably saw it on the first exam. They wrote that 37,000 times; every chance they could and the limit stands alone. I'm saying, "Oh, no! I hope he doesn't just let that stand." You didn't so that was a good catch. (Lines 441 – 471)

**Intuitive definitions.** Dr. Ignacio's next topic was about Kevin's use of intuitive versus formal definitions. He recalled the discussion about limits and continuity:

You started with the question, "What happens if we graph the function and there was an open hole or a hole where the point is moved?" The students seemed to struggle a bit. So you said, "Well, let's go back to the definition: the function continues at this point if the limit exists and agrees here." It made it easier for them.

You then said, “I’ve got this picture. I want to ask about continuity and check that the limit as  $x$  goes to...” I think it was a number there. I can’t remember exactly what your picture said, but it was this sort of question. That’s something we definitely want them to do since we don’t formally focus on the rigorous definition of a limit in this calculus course. All they have is a very non-rigorous definition for continuity. For us to have anything to work with, we have to go back to some statement they can actually check.

Continuity is a tough subject. We teach such a fuzzy calculus course. This becomes really tough because it’s a very intuitive concept. If you don’t have any intuition and you don’t have any kind of a rigorous definition—your *epsilons* and *deltas* to fall back on—then you’ve really got nothing to work with. You know our instructors really have to push that intuition because it’s the only hope our students have with the expectations we have. (Lines 477 – 502, 682 – 690)

Dr. Ignacio emphasized the importance of using clear formal language and notations as well as the effectiveness taking time out to help students have an intuitive and visual understanding of a concept traditionally treated with greater rigor.

**Group work and student engagement.** Dr. Ignacio asked Kevin how he felt group work was going in the classroom. He observed Kevin walking around to engage with students as they worked in small groups at the boards. Their dialogue went as follows:

Kevin: I like it. I perpetually struggle with the different speeds of the groups.

Dr. Ignacio: Clearly there were very different ability levels between the groups.

Kevin: I’m getting better at asking a follow-up question to the faster groups. I’ve finally accepted some groups will not finish.

Dr. Ignacio: And you have to be ok with that. You’re taking a risk sending them to the boards and turning them loose on a problem. I mean you’re risking time and you’re risking some sense of control over what they’ll see. The payoff is: if it works, it’s better than what you could have shown them.

Kevin: Well, I’m pleasantly surprised with how many of the weaker students take it seriously. Occasionally I’ll send them in groups of two. Do I seem to be reaching most of them?

Dr. Ignacio: I didn’t notice anybody obviously off-task. They seem to pay attention. When they were at the boards, they all put in effort to work on the problems. From an external observation, they were engaged. Now I have no idea from one moment to the next who’s tracking and who’s not. But if

I knew that, I'd be a mind-reader and I'd be doing a different job and making a lot more money than I am now! (Lines 542 – 564; 699 – 707)

Dr. Ignacio's emphasized group work was risky in terms of time and control but had huge payoff when students remained on-task and was facilitated with good questioning holding every student accountable for understanding. He viewed board work as better for learning than simply watching the instructor do examples.

**Shifting from conceptual to procedural learning.** Dr. Ignacio noted the course was starting to introduce more procedural content than the previous more conceptual lessons. He allowed Kevin to express his thoughts on that:

Dr. Ignacio: But now you're into derivatives, right? It's a very different game now. You've stepped into a procedural part of the course; not very intuitive.

Kevin: I don't like the very procedural focus we have now. There's no "Why?" to it and I don't like that.

Dr. Ignacio: But you've already discussed it. You led with the "Why?" in the sense that, "This is why we care about the derivative; this is what it measures." Do you feel it's been distanced in the students' minds, at least in their experience? Is it disconnected and now they're doing something different?

Kevin: I've gone from preaching and grading/teaching on, "What's the concept? What are we thinking about?" Now I've got tiers: "If you see this then do that. What does that look like? It looks like they do that. Ok, that looks like multiplication. That looks like a product rule, so do the product rule."

Dr. Ignacio: In spite of the fact that you dislike it, do you find the students are more comfortable with this because it's very procedural?

Kevin: Yeah. They want the right answer. They have never before in math class been rewarded for understanding. They've been rewarded for the right answer and they think math should be about the right answer instead of speaking this foreign language.

Dr. Ignacio: Yeah, that's the other thing. Your view on it is, "Hey, I'm a language teacher! Conjugate these Latin verbs over and over again. Here are the irregular ones."

Kevin: There's a lot more to it. You can't express the beauty of calculus if you can't understand the language. It's not opera if you can't appreciate an Italian opera. You can't appreciate calculus if you can't understand the words. (Lines 711 – 784)

Kevin enjoyed the conceptual aspects of mathematics and viewed mathematics as a language whose beauty can only be appreciated by understanding the meanings of its words. Dr. Ignacio acknowledged students liked performing procedural mathematics over struggling to make meaning of concepts. Kevin agreed students were more comfortable because procedural learning characterized most of their previous mathematics learning experiences.

**Looking ahead.** Dr. Ignacio wrapped up the session by asking if Kevin had a preference for the lessons he wanted him to observe next. Kevin said related rates was coming up and would be good lessons to observe. Dr. Ignacio agreed because the topic required students to use conceptual understanding of the problems, successfully execute procedural steps, and interpret results. Dr. Ignacio asked Kevin what issues he wanted him to watch for in the next observation. Kevin said he wanted Dr. Ignacio to notice if his language correctly represented the mathematics and if he carefully attended to all students. He expressed, “I always worry I’m missing students because I don’t know the names of the ones who do the board work and don’t answer or ask questions” (Lines 843 – 845). Dr. Ignacio agreed to do so.

### **Feedback Impact**

**Kevin’s response to feedback and instructional adjustment.** Kevin (Joule, 2014) recalled his biggest take-away from Dr. Ignacio’s feedback session: “Getting nailed on being precise with the verbiage is the big one. I did not make it clear that the phrase *limit is infinity* really means *limit does not exist*; the behavior approaches infinity” (Lines 343, 366 – 368). Kevin admitted another experienced instructor, Kreighton Dillard, gave him similar feedback:

“Watch the little details; make sure your language is proper in the details too” (Lines 547 – 550).

Kevin described how he attempted to adjust his approach as a result of the feedback:

I try to slow down before I answer questions and think through what I’m going to say. It’s still challenging. Putting distinct effort into being deliberate about what I say helps; particularly when answering board work questions. On the other hand, it’s very hard to see what you said wrong. Sometimes it’s obvious. You’ll say something and know it wasn’t right. But it’s hard for me to judge. A good thing about the feedback is someone else catches that. That’s the big takeaway for me. I think it’s helped but it’s hard to tell. (Lines 498 – 506)

Kevin appreciated having an observer’s second set of eyes and ears to catch what he was unable to notice real-time during his teaching. He was aware of his tendency to lack precision of language and notation during instruction and became cognizant to be more careful.

## **Observation Cycle 2**

### **Pre-Observation**

Kevin and Dr. Ignacio had no formal pre-observation meeting. In the previous feedback session, they had agreed Dr. Ignacio would observe a lesson on related rates. Kevin requested Dr. Ignacio pay attention to any particular issues with clarity in communication, written or oral.

### **Observed Lesson**

**Topic:** Lesson 20 – “Related Rates” with the following objectives from the course syllabus (Appendix M) and notes to instructor (Appendix O): (1) Solve a related rates problem by applying implicit differentiation; (2) Apply the chain rule to determine the rate of a change of a composition of functions

**Instructor resources:** Kevin created an example problem to introduce the lesson and assigned some book problems as board work for the students.

**Summary/notable events:** Kevin opened the discussion with the diagram depicting a piston engine crank shaft in Figure 9 below:

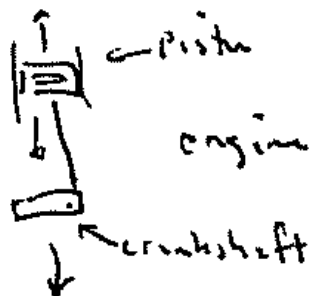


Figure 9. Dr. Ignacio's sketch of Kevin's piston-crank shaft.

He posed the question, "How fast is the piston moving if the crankshaft is moving at a certain speed? Specifically, find the speed of the piston at a given position." Kevin simplified the diagram into the following generic triangle in Figure 10 below:

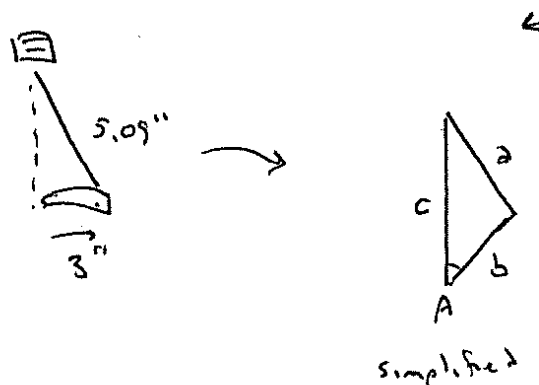


Figure 10. Dr. Ignacio's simplified diagrams of piston-crankshaft.

He told the students to use the law of cosines as their equation for differentiation:  $a^2 = b^2 + c^2 - 2bc \cos A$ . He identified which quantities were fixed ( $a$  and  $b$ ) and which were changing ( $c$  and  $A$ ). For the changing quantities, he emphasized they could be represented as functions of time and re-wrote the equation accordingly:  $a^2 = b^2 + [c(t)]^2 - 2b[c(t)] \cos[A(t)]$ . The class spent the rest of the period discussing this scenario.

### Feedback Session

Dr. Ignacio started the feedback session (F. Ignacio & K. Joule, Feedback Session 2, October 14, 2014) by allowing Kevin to assess how the lesson went. Kevin felt most of the

lesson went well. He stated, “I’m not sure I got across the idea that all variables are a function of time and why that tied to the chain rule. I don’t think I communicated adequately” (Lines 10 – 14). He also noted the students’ uneasiness about not getting the answer. After confirming Kevin would continue the topic next lesson, Dr. Ignacio asked Kevin how he planned to follow up. Kevin responded, “My plan is to finish and hand out the solution the way I want to see their homework look: drawing, goal, setup, work, and conclusion. There’s going to be a huge challenge with doing that on the homework” (Lines 21 – 24). Dr. Ignacio recommended Kevin also give them opportunities to do some practice problems on their own; otherwise, they would not do well with correctly executing such problems.

**Being explicit about the implicit.** Dr. Ignacio commented, “I like your piston engine crank shaft example. I don’t know if all appreciated how complicated that system is and the fact that we’re computing something an engineer might want to know. You smuggled in the Law of Cosines” (Lines 80 – 87). Dr. Ignacio praised Kevin for emphasizing why the equation required implicit differentiation by representing the variable quantities as functions of time. Referring to his notes (Appendix S), Dr. Ignacio’s discussion was as follows:

You talk about using implicit differentiation. I like what you did. You made it very explicit because you wrote it out. You said, “Okay how would I rewrite this now with appropriate things in terms of  $t$ ?” I agree, probably not all of them appreciated what was happening – even though you were very explicit about writing it. A lot of people who teach this aren’t careful about that. Their students wonder what’s going on.

You might have started by saying, “The  $c$  was the explicit function of  $t$ ” then ask, “What else is an explicit function of  $t$ ?” and try to get to the angle. Many of them missed some of your discussion on what was going on with the angle though. They failed to appreciate it was also a function of  $t$ .

That’s to be expected but it’s a crucial piece of implicit differentiation. I refer to it as step zero: if you’re going to do implicit differentiation, you have to start by assuming everything is a function of some same variable that may not explicitly appear anywhere.

Sometimes you have a function involving  $y$  and  $x$ , and you'll say, "We're going to assume  $y$  is a function of  $x$ ." In this case,  $t$  appeared nowhere in this thing. Then you said these things are a function of time; that's a difficult concept. If you do the handout, you probably want to explicitly say that and hopefully reinforce it to them because this variable didn't appear originally. (Lines 105 – 139)

Dr. Ignacio emphasized the subtleties imbedded in implicit differentiation problems students easily miss or simply don't bother to comprehend. He commended Kevin for writing the variables as functions of time but reiterated the importance of taking time to discuss the reasons why to help students better understand the purpose from the very beginning.

**Effective and consistent use of notations.** Dr. Ignacio pointed out Kevin's tendency to mix notation in the middle of differentiation. The discussion went as follows:

Dr. Ignacio: At the point where you started taking derivatives, you explicitly wrote, " $c$  is a function of  $t$  and  $A$  is a function of  $t$ ." Then you started mixing two different kinds of notations. You kept  $c$  and  $A$  as functions of  $t$  inside the chain rule wrappers. Then you had things like  $\frac{dA}{dt}$  and  $\frac{dc}{dt}$  outside instead of  $A'$  and  $c'$ , respectively.

There's nothing wrong with mixing Leibniz and functional notation; it's a conscious choice. Is that just something you did? Because I'm afraid occasionally some students may wrestle with that. They get wrapped up around notation like it's some kind of magic talisman or something – as if different notations mean different things.

Kevin: I want to make it very obvious  $\frac{dA}{dt}$  is a rate with respect to time.

Dr. Ignacio: Yep, it certainly does. That notation is very good for that reason.

Kevin: I don't want to say I've used up the value of the  $f(x)$  notation; but I have applied it as I needed. I wanted to keep  $f(x)$  notation for the product rule so it's clear there are two functions.

But in the big picture know this is a rate with respect to time; but also to avoid—both for me and the model—losing the prime in  $f'$ . It's very easy to lose that little dash.

Dr. Ignacio: Yeah it's easy to lose the prime. The number one problem with implicit differentiation is they forget to take the derivative of the inside function. They forget there is an inside function and then they have problems. You

should take a second to point back to that and make sure all of them are tracking, “I’m mixing different notations and it’s actually on purpose.”

It’s okay if you do that too. The key is we don’t use notation as if they are magic symbols we move around on the paper. We want notation to actually indicate something. “What does it look like to you when you have  $\frac{dc}{dt}$ ?” It’s clearly a rate of change with respect to time. It would help some of them if you noted that.

They struggle mightily with notation in our calculus 1 and 2 classes and it carries over. I see it in calculus 3 where if you use an alternate notation for something, they feel it has some tremendous significance to the problem—when sometimes it’s just because I feel like using this notation. Oftentimes I use different notations just to mess with them and help them to see how different notations can be used. (Lines 161 – 214)

Dr. Ignacio recognized a common struggle students have with variations on mathematical notation. Kevin had mixed forms of notation for derivatives throughout the example. Dr. Ignacio was not opposed to this mixing; however, his main concern was Kevin’s motivation for doing so and encouraged Kevin to be more transparent to his students about why he chose particular notations at given stages in the process. He agreed certain forms of notation tied more closely with conceptual meanings and may have been more beneficial for students’ understanding. He suggested Kevin provide a carefully written handout intentionally demonstrating notations he preferred the students to model in their work.

**Thinking about thinking in problem-solving.** Dr. Ignacio pointed out an instructional move capturing his curiosity about Kevin’s intentionality:

Dr. Ignacio: Then you said “Okay, stop” and had your meta-discussion again: “Set-up on these problems is the challenging part.” Then you looked at your problem and said, “In terms of set-up: if I were to do this I would do it differently.” Then you went back to, “Ok, we have to state our goal and give our set-up.”

Was that intentional? As if to say, “I’m going to dive in. I’m going to muddle my way through. Then I’m going to hit a point and then I’m going to say, ‘You know what, this would’ve been a lot easier if I had done it by this method’” Or did that just occur to you and you said “Okay

now I'm going to recover and I'm going to give them an object lesson of how you can clean up?"

Kevin: I really wanted to give them a lesson: "Set-up is important and helps. If you don't do your set-up, you get stuck."

Dr. Ignacio: Okay, so that was intentional. I had to ask: "Was that intentional? Was that an on-the-fly adjustment?" If it was on-the-fly adjustment, it was a descent transition to a nice recovery.

It's also a great technique to say, "Look, I've done all kinds of stuff. Do I feel any closer to the answer? How could I have made this better?" These meta-discussions are really important in terms of how we teach them problem-solving skills in addition to techniques. That was interesting.

At that point it was time to roll into group work. We were largely through the class and you had them getting into board work. Did that accomplish your goals?

Kevin: Yeah, it didn't move as fast as I had hoped. I wanted them to get through the set-up and then discuss it. But it was just much slower than I anticipated. (Lines 223 – 257)

Dr. Ignacio: Were you going to let it stand or were you going to try to do anything with that? Sometimes it just doesn't work and you just say, "Okay I'm going to write off that five minutes and just move on." But I didn't know if you had any ideas about preferences for that.

Kevin: I'm going to re-attack. Tomorrow, I'm going to say, "Alright let's do the picture. Okay everybody has a picture. Now let's write our goal and set-up and then let's solve."

Dr. Ignacio: If they had trouble with that, they've got two more classes to get a handle on it. Some of them definitely still weren't tracking. I was looking around at them. I tried to keep from interfering with them, but I saw some interesting things. Some of them just cranked it out right away; and then some of them didn't. (Lines 278 – 294)

Dr. Ignacio was particularly interested in how Kevin modeled the trial and error and recovery aspect of problem-solving. Kevin admitted he was trying to emphasize the importance of planning a goal and setting up before pursuing some course of action. Dr. Ignacio offered suggestions for this approach and how Kevin might continue for the next lesson.

**Choosing examples.** Dr. Ignacio asked about other examples Kevin had planned to use in the lesson if there had been more time. Kevin mentioned problems involving a sliding ladder, moving shadow, and draining water from a trough. Dr. Ignacio agreed these were neat, more traditional problems. He admitted the example Kevin had chosen was especially challenging because it required a formula students do not use often (law of cosines). However, he was fine with using it. He offered the following advice regarding examples:

When you pick examples, try to pick a good variety of problems. Pick one or two that are fairly easy. Also pick a few for which they have must work to include constraints because they often really struggle with that. As you said, a set-up is the challenging part. The actual execution is usually something they can muddle their way through. But I liked your choice of example, actually. The ladder problem is my usual go-to; but I like the piston and engine. It is pretty neat. (Lines 333 – 342)

Kevin discussed how he planned to have the students provide set-ups for the homework problems. He pondered how he might help them in class without working the problems for them. Dr. Ignacio suggested he use parameters and give them hints about the approach but allow them to do the execution. The session wrapped up with Kevin describing the remaining topics in the course: linear approximation and optimization.

### **Feedback Impact**

In his final interview (K. Joule, Exit Interview, January 30, 2015), Kevin concluded after a semester of teaching and receiving feedback, he was more comfortable reacting to students by being less mechanical and less scripted with fewer pages of notes: “I’m going in with more of a big picture. I’m here to talk about this and I’m going to do an example, not a detailed plan. It lets me respond more to the students” (Lines 30 – 40). He credited Dr. Ignacio with encouraging him to “think more deliberately about where the students are. He’s helped me gauge my expectations lower” (Lines 77 – 80). Kevin regarded Dr. Ignacio’s years of experience as being helpful to make him consider: “Where are they, what are they thinking, what misconceptions do

they already hold” (Lines 85 – 87)? Dr. Ignacio and Kevin often discussed how students thought about concepts differently than instructors. Kevin admitted students often struggled with things intuitive to him. Dr. Ignacio’s feedback helped him be more intentional in anticipating students’ understanding.

### Summary

Kevin and Dr. Ignacio’s open and informal mentoring relationship formed because their offices were located next to each other. This conveniently allowed multiple interactions on a daily basis. They built good rapport and shared common ideas about teaching and expectations for students. Kevin used Dr. Ignacio as a sounding board for ideas, sanity check for quizzes he created, and a second pair of eyes and ears in the classroom. Dr. Ignacio’s observation approach was to notice the general flow of a lesson and identify issues potentially needing attention. He used feedback to provide a perspective and have a dialogue about what he noticed in concert with understanding Kevin’s intentions compared to what was enacted.

Figure 11 summarizes the areas Kevin and Dr. Ignacio addressed in their discussions and reflections.

Themes	Kevin	Dr.I	Sum
Making meaning	0	9	9
General practice	6	2	8
Assessing student progress	3	3	6
Mentoring	1	4	5
Representing mathematical ideas/notations	0	4	4
Persevering through student questions	1	2	3
Using correct notation	0	3	3
Anticipating student responses	2	0	2
Modeling problem solving	0	2	2
Choosing examples	0	1	1
Connecting across curriculum	0	1	1
<b>Total</b>	<b>13</b>	<b>31</b>	<b>44</b>

Figure 11. Case 2 data matrix summary of themes.

Dr. Ignacio noted ways Kevin could improve on clarifying meanings, being more consistent with mathematical notations and language, assessing individual students during group work, and selecting appropriate examples to achieve instructional aims. Kevin intended to modify his instruction with better planning to anticipate student responses, model problem-solving, and be more consistent with notation.

Figures 12 and 13 summarize frequencies for MKT and MUST categories, respectively, assigned to Kevin and Dr. Ignacio's interactions and interview data.

MKT	Code	Kevin	Dr.I	Sum
<b>Subject Matter Knowledge</b>	<b>SMK</b>			
<i>Specialized content knowledge</i>	SCK	0	11	11
<i>Common content knowledge</i>	CCK	0	3	3
<b>Pedagogical Content Knowledge</b>	<b>PCK</b>			
<i>Knowledge of content and teaching</i>	KCT	7	11	18
<i>Knowledge of content and students</i>	KCS	6	5	11
<i>Knowledge of content and curriculum</i>	KCC	0	1	1
<b>Total</b>		<b>13</b>	<b>31</b>	<b>44</b>

Figure 12. Case 2 data matrix summary of MKT categories.

MUST	Code	Kevin	Dr.I	Sum
<b>Mathematical Activity</b>	<b>MA</b>			
Mathematical creating	MA_CR	0	17	17
Mathematical reasoning	MA_RS	0	2	2
<b>Mathematical Context of Teaching</b>	<b>MC</b>			
Reflect on the mathematics of practice	MC_RP	7	6	13
Access and understand the mathematical thinking of learners	MC_LR	6	5	11
Assess the mathematical knowledge of learners				
Know and use the curriculum	MC_CUR	0	1	1
<b>Total</b>		<b>13</b>	<b>31</b>	<b>44</b>

Figure 13. Case 2 data matrix summary of MUST categories.

Kevin worked to improve his KCT and KCS while Dr. Ignacio emphasized building SCK, engaging more in mathematical creating, and reflecting on the mathematics of practice to be more effective in responding to unexpected student questions or confusion. Accessing and assessing student learning was also priority.

## CHAPTER 6

## CASE 3: LAVERNE PASCAL AND DR. ARNOLD PAINE

**New Instructor 3 – Laverne Pascal**

**Laverne's background.** Laverne was a first-time instructor teaching differential calculus for freshmen. She had a master's degree in meteorology and spent the early part of her military career doing practical operational forecasting for Army units. Next she did computer weather forecasting and computer weather research. She explained in her introductory interview this job required "computer modeling stuff. It was actually using mathematics and calculus" (L. Pascal, Introduction Interview, September 29, 2014, Lines 54 – 55). Later she took on jobs requiring "traditional algorithm development, pseudo code where you're writing down the derivatives you want your program to solve – so some serious mathematics in that job" (Lines 60 – 62). When her family relocated to the area, Laverne inquired about a position at USAFA as a good fit for her educational background. Since she had three calculus classes and two statistics classes at the graduate level, she was placed in a temporary position in the mathematics department.

**Laverne's teaching experiences.** Laverne had no formal academic training or teaching experience prior to teaching calculus. However, Laverne taught introductory science topics to preschoolers and believed this experience combined with her experience as a weather data analyst allowed her to transfer some teaching skills to her calculus instruction:

I don't want to call it "dumbing it down" because that just insults everybody. But I'm taking some technical information and distilling it to a level a 4-year-old can understand. I think that's a skill that transfers here. It's a skill I learned from elsewhere.

For example, you learn how to tell weather to a General Officer. I'm seeing a lot of technical stuff; looking at numbers and derivatives in my head. I can read a map and see a plot of a rate of change of temperature; it's a derivative. I'm not telling the General the rate of change of temperature. I'm just going to tell the General it's getting warmer or colder. We get heavily trained in our career field in that skill – taking tons of information and boiling it down to what the General cares about and can understand.

For a 4-year-old, I have to boil it down to what he/she can understand. In the case of this calculus, I have to boil it down to what my 18/19-year-olds from all over the United States and 10,000 different pre-calculus programs can understand. And I'm watching that happen and taking in what's working and what's not. (Lines 182 – 206)

Regarding helping students who have trouble, Laverne expressed the following:

I think I've had a couple different approaches in my back pocket. And that's a large part from sitting in on everyone's different classes; thinking of different approaches. (Lines 218 – 221)

I've only had a couple students come in and say, "I have no clue." They're very specific when they come in for help. It's easy to find a different way to explain it. I don't think I've had a problem with that. But for a couple of them, no matter what I do it's really hard for them. A lot of my one-on-one instruction has not been about the calculus. It is algebra, pre-calculus skills, laws of exponents, laws of logarithms. (Lines 229 – 240)

**Influences on Laverne's mathematics teaching approach.** Laverne audited

experienced instructors in the course to help her prepare her own instruction:

The material is not what I'm going in there to gather. I'm going in there to gather the pedagogy. How is Yvonne presenting it? How is Dr. Brown presenting it? How is Colt presenting it? They're all very different. Some of the new instructors sit in on the same instructor over and over. I haven't been doing that because I'm not sold on any one particular instructor's style. I've been mixing it up for my students.

The thing I do the most every single time is use PowerPoint. I have the PowerPoint on the white board. I write on the slides. That's Olivia's method. She's not even a calculus instructor; but she came in the summer and presented to us and I liked that style. It's very similar to how a weather person gives a weather brief.

I'm gathering techniques and watching the instructor's time management. If there are four objectives, do they cover them? Yvonne wrote the objectives. She is very clear: "Okay, we're done with this objective. We're now going to cover the next objective." I like that clarity: tell them what you're going to tell them and then tell them what you told them. She's pretty good about that. But she's the course director. She's got an advantage with that because she wrote the objectives.

I have seen every calculus instructor, except one because he teaches the same time I do. But I have officially made those rounds. (Lines 336 – 379)

Laverne explained how auditing affects how she planned for her lessons:

Especially for topics stretched over two or three lessons. It gives me an idea of where to imagine a break point. I can't hard-program a break point because my sections are moving at different rates; but I can mentally tell myself. For example, implicit differentiation: based on observing Yvonne, I can probably get all the way through definitions; maybe one or two practice problems.

The next lesson I'll probably go into the next piece of that objective: "I hand you points and slopes. Then you tell me the equation of a line based on implicit differentiation."

In my mind I say, "Don't expect to do that second part until the second of the two lessons." I'm getting more of the time management perspective. I may have my slide deck going, but sometimes I have all the material on the slide deck. Sometimes I might have a handout. Sometimes it might be as simple as, "Let's all work through the example on page 209 together" and I'm writing it on the board.

I have the board work in there. Some lessons lend themselves well to board work more than others. For implicit differentiation, I'll probably have to do a lot of talking on Tuesday then more board work Thursday.

I'm gathering those sorts of things. I know implicit differentiation; I certainly don't have to pick up that part. But like I said, down the road, I may be taking in related rates or optimization. That one's going to be crazy for me. But that's where I'm going to sit there and go, "This will help me feel more comfortable with the material and how to present it." (Lines 427 – 463)

Laverne used the course director's notes to instructors (Appendix N) as a guide for identifying good examples and specific points to give particular emphasis. The notes warned instructors about topics or concepts with which students struggled. Laverne used the notes as well as her observations of how students responded in the class she audited to give her instructional ideas.

### **Mentor 3 – Dr. Arnold Paine**

**Dr. Paine's background.** Dr. Arnold Paine was a military assistant professor with more than 10 years teaching experience. He graduated from USAFA with a double major in mathematics and general engineering then went directly to graduate school to earn a degree in

applied mathematics. In the military, he served as a pilot (including time as an instructor pilot) for about ten years and then taught in USAFA's mathematics department for three years. He taught differential, integral, and multivariable calculus as well as discrete mathematics for computer science before the mathematics department sponsored him for a doctoral degree. He began his research in applied mathematics before switching to harmonic analysis; however, his dissertation focused strictly on harmonic analysis of function space. When Dr. Paine returned to USAFA, he taught calculus and became the calculus division chief. After two and half years in the calculus division, he became the applied mathematics division chief for a few years until he took on instructor pilot duties in addition to his mathematics teaching responsibilities. At the time of the study, he taught differential equations courses.

**Dr. Paine's teaching philosophy.** Dr. Paine described his view of teaching and parallels between pilot instruction and teaching mathematics:

Teaching is trying to motivate people to do what you know they need to succeed. The calculus division struggled with getting students to learn and maintain fundamental skills of derivatives and integrals. We knew they needed lots of fundamental skills practice problems and repetition. There is no substitute for doing that, right? The teaching aspect is in how we translate what we know to getting students to do what they need to do. It's almost like being a parent.

This is true in teaching as well as flying. For example in the powered flight course, students chair-fly in between flights; that is, they sit in the chair and they think about the flight. They go through the whole mission; they practice doing the checklist. They practice doing those things to be more relaxed in the plane. I can teach them about flying the plane. However, if they don't do anything in between flights and are worried about the next radio call or this or that, they're going to be nervous or not listening to what I tell them and won't get as much out of it.

Teaching is trying to motivate people. None of us can just impart knowledge on somebody. We can guide them to that knowledge but ultimately you must be able to understand a problem in the way you think. A teacher helps you get it straight in your mind so it makes sense.

There's a misperception that teaching mathematics is demonstrating something and having students repeat it: if you can repeat it, then you've learned it. That's just not true.

You truly learn something if you can demonstrate it even if I change the problem because you understand the concept and can adjust. You must use problem-solving skills for that.

Many times we get students who look for us to teach them and think they don't have to do much because they've done a lot of mimicking. I spend a lot of time with freshmen discussing, "How should you study? How should you think about a problem? What do you need to do on a daily basis?" (A. Paine, Interview, October 17, 2014, Lines 469 – 540)

**Dr. Paine's mentoring experiences.** This semester was the first time Dr. Paine was formally assigned to mentor a new instructor, Laverne. However, Dr. Paine had a wealth of informal mentoring experiences in his roles as division chief, instructor pilot, and supervisor.

I did a lot of mentoring with course directors on test building: "What makes a good problem? How do you word a problem without ambiguity? How do you test specific objectives? How do you build meaningful objectives and assess evaluations of objectives in a meaningful way?"

As an instructor pilot you're a mentor all the time; especially on a crew aircraft. As an aircraft commander, you're constantly teaching co-pilots how to do the job as well as transition to be a commander.

As a supervisor I did career mentoring: "What do you want to do with your career and how do you want to get there?" That's an interesting dynamic because it's dependent on an individual's goals.

In a lot of ways, most people mentor in some form or another throughout life, we just happen to do it in a more formal sense. I have always talked to instructors about good ways to teach topics. I've frequently sat in on new instructors' practice lessons. It's just having the experience of teaching. I've taught this material enough to know the hang-ups students have and where they are confused so it's just a natural thing. (Lines 567 – 617)

### **Laverne and Dr. Paine's Mentoring Relationship**

The faculty development director assigned Dr. Paine as Laverne's mentor after the semester began. Laverne remarked, "He asks how things are going and do I have any questions. He checks in on me at least once a week. Sometimes it's twice" (Introduction Interview, Lines 974 – 980). Laverne appreciated the informality of their relationship and knew Dr. Paine was available as much as she wanted to utilize him: "I haven't actively gone to him with any

information because he has taken the initiative to come by” (Lines 1086 – 1088). She reiterated after her feedback session, “I like his casual laid-back, not so military approach” (Interview 1, Line 11). When they discussed her teaching, Dr. Paine focused on Laverne’s teaching style and the classroom. He described the interaction as follows:

It’s pretty informal. I just talk to Laverne and see how she’s doing so far. I’m not too worried about her; she’s mature. If she’s got an issue she’ll use the people in her course and say, “I’m not sure how to do this.” She’s always free to ask me a question if she’s got an issue.

As long as somebody is not struggling and I’m just tweaking things around the edges, I’m fine with being informal. If somebody’s having serious issues it needs to become more formalized. (Lines 866 – 881)

I don’t see big deficiencies with Laverne. I see a new instructor doing what she needs to do. Most of the mistakes she makes are minor and just things most people at that point do. It’s just a lack of experience. You don’t want to take away people’s mistakes. You let them have a few mistakes and say, “Well, that didn’t work; why not?”

It goes back to the idea, “I could try and teach you how to do something but until you try it and figure out your mistakes, nothing happens until you figure it out.” Part of learning is failure. You have to learn how to deal with failure. I’ve done it. For a particular lesson plan I thought, “This will be awesome,” and it just went down in flames. I said, “Well, that didn’t work.” How do you learn from that and adjust? (Lines 897 – 912)

**Conducting observations and feedback.** Dr. Paine allowed Laverne to decide when she wanted to be observed: “I tend to observe a more challenging lesson to see how they deal with it” (Lines 782 – 784). Laverne preferred Dr. Paine to observe lessons in which she was “imparting new knowledge” (Interview 1, Line 419)—instructor-led discussions introducing new information. Dr. Paine’s feedback ideally focused on how well she explained material. He described his approach as follows:

I usually tell people before I observe them. I’d rather observe them on something they’ve fully prepared. I sit back, observe, and see what they do. I want to get an impression of where the instructor is, how they manage the classroom, and how they interact with their students. I try to note the good and bad; what seemed to work really well, what didn’t work well. I try to notice things a new instructor isn’t necessarily going to key on because of less experience. For example: not noticing when students are struggling but

don't want to say anything.

I try to use those opportunities to point out, "You may not realize it, but most of your class is confused at this point. It's not you, it's just this material." Then point out ways they can draw that out of their students. It's hard to do the first time; you don't know they don't understand until later when you start looking at homework or a test.

If they run into a pitfall, I'll try and guide them how to do it better the next time; how to avoid the pitfall. I look for different things progressively. As a mentor, my goal is to say, "Why didn't that work? Where were they confused?" Then try and help new instructors adjust. (Lines 619 – 741)

### **Observation Cycle**

#### **Pre-Observation**

Laverne and Dr. Paine had a very informal pre-observation meeting. Dr. Paine approached Laverne and told her he wanted to see how things were going. Laverne selected a lesson for Dr. Paine to observe. He asked if there were any questions or issues for him to notice. According to Dr. Paine, "I think she had one or two little things. But I was curious to see what she would do" (Lines 791 – 792).

#### **Observed Lesson**

**Topic:** Lesson 16 – "Implicit Differentiation" with the following objectives from the course syllabus (Appendix M) and notes to instructor (Appendix N): (1) Explain the difference between implicit and explicit functions; (2) Use the chain rule in implicit differentiation; (3) Differentiate an implicit function

**Instructor resources:** Laverne's PowerPoint presentation for Lesson 16 (Appendix T) and practice worksheet (Appendix U).

**Summary/notable events:** Laverne began class reminding students of upcoming assessments. Then she returned and reviewed a quiz covering derivatives. She pointed out the most common issue students had: correctly executing the product and chain rules.

## Feedback Session

Dr. Paine and Laverne's feedback session occurred two days after the observation. The session was held in Laverne's office and lasted 43 minutes. Dr. Paine began with positive comments about Laverne's instruction: "Overall, I thought it was a good lesson. It was well-organized and the flow was about right. You had really good control of the class. It was good when you took questions. You gave good responses to their questions" (A. Paine & L. Pascal, Feedback Session, October 2, 2014, Lines 7 – 11).

**Let students fix mistakes.** Dr. Paine referred to his notes (Appendix V) and addressed Laverne's review of common errors on the derivative quiz. One problem was to give the derivative of  $f(x) = \frac{1}{4x^6}$ . He commented, "You pointed out the common error, but tell them to factor out  $\frac{1}{4}$  then differentiate. You talked about it, but actually show it on the board" (Lines 45 – 48). He revisited this problem later in the session:

I laughed when you put that up. If you had asked me, "What's the problem they're going to miss the most?" It's  $f(x) = \frac{1}{4x^6}$ . In calculus 2, we give them the same problem for integrals. It's not a derivative thing; it's an algebra thing. They can't factor out the fraction as a constant. Even though I've seen it for so long, it continually amazes me. That's the hardest problem for them to do. (Lines 335 – 348)

He pointed out another example:

Then you gave a chain rule review to get the derivative of  $f(v) = \left(\frac{v}{v^3+1}\right)^6$ . That was a good challenging problem with the chain rule and quotient rule. One of the hardest things to do, as an instructor, is step back and let them do it. You did to a point.

Even as long as I've taught, I don't like the uncomfortable silence. You gave them a little bit of time but then jumped in, "Hey, that's how we do that." You could've given them a little bit more time to sit there and work on it. (Lines 51 – 63)

Dr. Paine suggested more wait-time to allow students ample time to think and respond. When students are given more time to respond, they typically provide better responses and get more

from the experience than if the teacher had simply given them the answer (Tobin, 1987; Rowe, 2003; Ingram & Elliott, 2016).

**Generalizing the chain rule.** Dr. Paine highlighted Laverne's generalization of the chain rule as an introduction for implicit differentiation and encouraged her to consistently apply the concept when executing implicit differentiation:

You said, "Hey, let's do a new kind of chain rule." You wrote  $f(x) = x^2$ . You differentiated the "outside" ( $x^2$ ) then the "inside" function ( $x$ ); which I liked. You talked about including  $\frac{dx}{dx}$ , "because I'm going to get the derivative of that inside function"; which was good. You emphasized the derivative is a mathematical operator.

The only thing I didn't like is you did the work in front of your PowerPoint slide. When you wanted to go to the next slide, you erased it. If you had done the work elsewhere it would have been good to leave as review of the chain rule because we're going to use it a lot going forward. That was just a little organization thing. I think you realized it right after you did that. (Lines 86 – 95)

Dr. Paine also liked Laverne's explanation of the difference between explicit and implicit:

You used  $y = \pm\sqrt{25 - x^2}$  as the explicit function and  $x^2 + y^2 = 100$  for the implicit equation; which was good. The only comment I had there is you might want to just use  $x^2 + y^2 = 25$  to show the relationship between the two. You had a few questions on what implicit and explicit functions are.

You gave them your handout (Appendix U). You did the first two as a group:  $y = (\sin x)^5$  and  $y^{1/5} = \sin x$ . You differentiated the first one with the chain rule; you used implicit differentiation for the second one. You took the derivative with respect to  $y$  but never emphasized, "This is the chain rule. When I have  $y^{1/5}$  and want to take the derivative with respect to  $x$ , what do I have to do since this is implicitly a function of  $x$ ?" It would have been good if you would have put that on the board as  $\frac{d}{dy}\left(y^{1/5}\right)$  times the derivative of the inside; which is  $\frac{dy}{dx}$ ." On that one, you wrote  $\frac{dx}{dx}$ . On the other side you showed  $\frac{dx}{dx} = 1$ .

After that, you stopped using  $\frac{dx}{dx}$ . I would have continued using it because some of them were confused about when you need  $\frac{dy}{dx}$  and when you need  $\frac{dx}{dx}$ . It's almost better when they first start doing it. Always do the derivative of the outside times the derivative of the inside. You get  $\frac{dy}{dx}$  and you get  $\frac{dx}{dx}$ . Then after you do that say, "Now we know

$\frac{dx}{dx} = 1$ .” Write it on the board so they know we’re doing the same on both sides.

Explain, “What we don’t actually know is the exact expression for  $\frac{dy}{dx}$ .” Then you had them differentiate  $\sin y = xy^5$ . Again, keep  $\frac{dx}{dx}$  in there. (Lines 122 – 157)

Later in his interview, Dr. Paine rationalized why he liked her strategy and preferred the notation she used:

Leibniz notation lends itself to the chain rule. Using Leibniz notation when you do implicit differentiation with the chain rule is more obvious. When I teach chain rule I show them Leibniz notation because I know when they do implicit differentiation, that notation will help them. Later in multivariable calculus, they’ll use chain rule. With Leibniz notation, it’s easier to see where you end up. That is just an experience thing. I think she did okay with that. (Lines 830 – 847)

Making choices about symbolic representations to facilitate clearer understanding for students is an important mathematical activity (MA)—specifically, the mathematical creation strand emphasizing “the need to represent mathematical entities in ways that reflect given structures or properties.... Teachers must be able to fluently construct representations that underscore key features of mathematical entities” (Kilpatrick et al., 2015).

**Assessing and enhancing student understanding.** Dr. Paine noticed how Laverne attempted to determine if students understood by using overhead questions:

You did a lot of overhead questions when doing examples: “Does everybody see and understand that?” You’re not going to get them to answer back with overhead questions. It’s better to call on somebody or have him/her do the examples.

Say, “Hey, does anybody have the solution to this?” after they work on it. Or say, “Hey, why don’t you put it up at the board?” Now you have somebody to directly ask questions. A lot of times students are more comfortable asking another kid a question: “Hey, how’d you do that?” A lot of times, I’ll have a student do that. (Lines 164 – 171)

Students solidify their understanding when they are required to verbally communicate what they know. It is also beneficial for students to hear other students verbalize information because students are typically more receptive to ideas expressed in the shared vernacular of their peers

(NCTM, 2000). Dr. Paine also suggested enhancing students' understanding by graphically representing the derivative of an implicit function:

Then you had  $\frac{dy}{dx} = \frac{y^5}{\cos y - 5x^4}$ . I would have liked you to step back and say, "Now, what does this mean?" The assumption is that it's the derivative of  $y$  with respect to  $x$ . We all understand that. But students really don't understand because it's a different process to them. They're going, "What does that mean? I didn't start with  $y$  as an explicit function of  $x$ ."

It would have been good to graph it and say, "Look, when I'm on this curve, that implicit function is the relationship between  $y$  and  $x$ . So now, what does this derivative mean? At any point on this curve, if I take an  $x$ -value and a  $y$ -value and plug them in, it gives me the derivative at that point." I know this comes in the second lesson, but it would have been good to spiral forward as the introduction to it. (Lines 193 – 203)

He summarized his main points:

- Give them that uncomfortable wait-time (Lines 246 – 247)
- Use less overhead questions (Line 256)
- Use students' work to drive discussions, especially after they've worked a few examples; let students put work on the board (Lines 260 – 262)
- Explicitly show them the chain rule: take the derivative with respect to  $y$  (the outside function) then take the derivative of the inside with respect to  $x$  (Lines 273 – 281)

This is confusing material to them. I don't know why; it's just a different way of looking at something. Part of the problem is they have been so tuned to " $y$  is a function of  $x$ ." Now we're saying, "No,  $y$  is not explicitly a function of  $x$ ." That upsets them. In some of these problems the equation is not even a function but defines a relationship between  $x$  and  $y$ . They're like, "I don't quite get that. What does this mean to be an implicit function?" There is anxiety in this lesson that's hard to get rid of. (Lines 283; 295 – 315)

There are a lot of places where different people are going to be confused in this lesson; that's why it's tough. You have to present it, get them working on it, and keep them thinking about it. (Lines 390 – 392)

Dr. Paine advocated the following mathematical practices: eliciting and using student thinking "to assess progress toward mathematical understanding" and connecting mathematical representations to deepen understanding of mathematics concepts and procedures" (NCTM, 2014, p. 3). These practices help assess and enhance the mathematical knowledge of learners

(Kilpatrick et al., 2015) and require the teacher's knowledge of content and students (KCS) (Ball et al., 2008).

**Additional topics.** The remainder of the session was dedicated to other topics Laverne wanted to discuss. Below are excerpts from the conversation.

***Balancing student board work and time.***

- Laverne: I try to move things along by doing examples myself. You saw a day where they weren't getting out of their seats. I vary that; approximately one lesson out of every three is one where I just don't feel like there's space for them to come up.
- Dr. Paine: You can have them do part of it. If they start to struggle, you say, "That's good. Where do we go from here?" Let them sit and you take over.
- Laverne: I like board work a lot; it reaps huge dividends. They're learning from each other; it's really cool. I try to do it when I can; but last lesson was an example of introducing a lot of stuff.
- Dr. Paine: Honestly, it's good; but not every lesson lends itself to board work. It is a good tool, but it's not the end-all, be-all. In my upper level class I give them the option: "If you want to work at your desk, go ahead. If you want to work as a group at the board, go ahead." Usually I have one or two groups at the board and I have a lot of people working at their desks.
- Laverne: I've done that many times. You see the same people that prefer the board.
- Dr. Paine: And the same people like working at their desk. You just have to walk around and make sure they're actually working on it.
- Laverne: Tuesday, you caught a non-board work day. Today they did board work and they had a great time doing it.
- Dr. Paine: And that's the thing. It was the same material over two lessons because it's tough material they struggle with. The first day is to introduce it, get questions back and forth, and talk about the concepts. The second day is applying it and usually lends itself to board work. (Lines 479 – 555)

***Variables.***

- Laverne: Today I told them, "Open your mind. These variables are not rigid. You will go on to some courses and they become Greek; they become nu ( $\nu$ ) and mu ( $\mu$ ) and pi ( $\pi$ ) and theta ( $\theta$ ).

- Dr. Paine: I tell them, “Look, use variables that describe the problem. If I’m going to have a variable for time, let it be  $t$  so when I’m looking at it, I don’t have to wonder what means. If I’m trying to figure out the perimeter, use  $p$ . Use variables with meaning for the problem.”
- Laverne: I guess that will come up at about Lesson 20. That’s related rates where you start introducing techniques for solving word problems.
- Dr. Paine: When you start doing optimization and modeling tell them, “I’m going to give you a problem. I’m not going to give you the variables; you have to turn this into a mathematical statement. Part of that is deciding what variables to choose. Get away from  $x$  and  $y$ .” (Lines 578 – 602)

They wrapped up on a positive note. Laverne expressed she was enjoying the teaching experiences both in class as well as during individual help sessions outside of class time. It gave her opportunities to get to know her students and help them understand the material at a level and pace best suited for them. Dr. Paine affirmed by saying she was doing well.

### Feedback Impact

**Laverne’s response to feedback.** In her interview after the feedback, Laverne addressed the main topics from the session:

**Assessing student understanding.** One item I’m trying to fix: I present something and then say, “Does anyone have any questions?” Brian Tougaloo is great about it. He points right at somebody and says, “Do you have any questions?” The student looks stunned and says, “No, Sir.” Sometimes he’ll ask and it forces the person to think, “Do I have a question? Yes, I do.” I also need to be more specific with questions or be more specific by looking for cues from my audience. (Lines 27 – 36; 63 – 65)

**Notation.** He likes the  $\frac{dx}{dx}$  notation. I could have improved my explanation. The explanation I’m giving is weak; in part because this is a new way for me to think of implicit differentiation. I was taught procedurally, not with mathematical rigor. Now I need to know this stuff solid. I really dug in and I now understand: you’re taking the derivative of every term and using the chain roll. Since  $y$  is implicitly defined in terms of  $x$ , you need  $\frac{dy}{dx}$ . I understand it better now.

Explaining it to the students is still hard for me. I leaned more toward the procedural way of teaching it instead of being mathematically sound. I’m trying to get better about that. The students were getting more confused but they understand it procedurally.

They don't understand the  $\frac{dx}{dx}$ . They ask, "Why is this one  $\frac{dy}{dx}$  but over here it might be  $\frac{dv}{dt}$ ?" Their brains want to think, "Well, anytime it's not  $x$  we've got to have a  $\frac{dy}{dx}$  or  $\frac{dv}{dx}$ ."

I tell them it's not always  $x$ ; it might be  $t$ . I need to be comfortable enough to offer three different ways to explain the same thing so they might catch the reasoning somewhere. It's a work in progress.

He was okay introducing  $\frac{dx}{dx}$  with, "Think of it like this:  $\frac{dx}{dx} = 1$  just like  $\frac{3}{3} = 1$ . (Lines 86 – 131)

Laverne demonstrated a common deficiency in new teachers' specialized content knowledge (SMK) (Ball et al., 2008). Teachers with an incomplete mathematical foundation may have been influenced by previous teachers who lacked a strong mathematical foundation and subsequently teach concepts the same way they were taught. Fortunately, the struggle she experienced to help her own students understand made Laverne recognize the need to gain a deeper understanding for herself to enable her to be more flexible in her explanations.

***Wait-time and managing independent work.*** In response to Dr. Paine's feedback about students working examples without her taking over, Laverne expressed, "They do more work at their seats. I allow the silence. It's hard. I'm not used to the silence but it is fine. I can't let them go on forever obviously, but 30 – 45 seconds is enough time" (Lines 134 – 151; 170 – 172). Laverne described the gains and how students became more confident:

They're doing more on their own instead of just watching me do it; they need to be doing that. As for setting up, I'm wondering if I could say, "Okay, we now have everything for you to finish setting up your equation. Then give me an answer." I'm hoping to head in that direction.

Related rates will take three lessons. Now I set them up to a certain point. But over time I should be doing very little. At the end of the third lesson I should be saying, "Here's what's given. Answer this question." Silence; see what happens. (Lines 179 – 200)

As an example, Laverne discussed her most recent lesson during which she gave students a bucket and tasked them to find the rate of change of volume,  $\frac{dv}{dt}$ , from the water fountain. She remained quiet and gave few directions to allow the students an opportunity to discover an approach for the task then set up and solve the problem. She described the lesson as follows:

Each of the four sections used a different approach to answer the question; which is very interesting. None of them were wrong. They took different approaches to measure the vessel. They should have been able to apply the measurements to an implicit derivative. They did and it worked out well. (Lines 256 – 262)

In three of the classes, somebody divided the responsibility:

[Student A] “We’re going to go measure. You guys start working on the implicit derivative and let me know what you need me to measure.”

[Student B] “You need to be measuring the height.”

[Student A] “Okay. I’m going to go measure height.”

Then they come back with the bucket:

[Student A] “All right, here are my 24 heights, the times, and the heights.”

That was really outstanding what they did. We found the change in volume wasn’t constant. I said, “All right. What are we going to do now?”

[Students] “Let’s pick a time.”

[Laverne] “At that time what was the rate of change of the volume?”

I guess the water wasn’t coming out at a constant press. You press a button on a water fountain and it’s coming at different rates.

It was a great discussion and it was fun. They were out of their seats and out of the classroom walking the halls. It was less structured. It’s hard to get 17 people all evenly involved. I only had one bucket.

But this got them thinking about relevant and irrelevant data. They were stressing over a rate of change of radius of a cylinder. I asked, “I wonder what that rate of change in radius is? It might be zero. If it’s zero what can we do with that part of the implicit derivative?” They responded, “Oh!” So they could see it in practice. I love that. (Lines 266 – 294; 306 – 314)

Laverne admitted one of her challenges was mitigating the different levels of task engagement because some students worked more quickly or were off-task:

They were all over the place! Half the kids were out in the hallway, half of them all over the white boards and on their tablets; it was kind of a mess. Those who were actively collecting data took about twice as long as those who stayed behind to do a very simple implicit derivative. It was mainly one section who did 24 measurements. *[Laugh]* They were out of the classroom at least 10 minutes.

What do I do with them after they finish the implicit derivative? They want me to tell more stories about deployments. I'm trying to manage two groups of people doing two different things in two different places. I didn't anticipate the big gap. (Lines 324 – 360)

**Laverne's intended instructional adjustments.** Laverne reflected on what feedback was most impactful to her throughout the semester.

During Dr. Paine's observation, he noticed I omitted part of the explanation of why implicit differentiation is necessary and why it works. He said I needed to explain the concept properly: "Even though we are performing a derivative operation on a dependent variable ( $y$ ), we need to always remember the dependent variable ( $y$ ) is implicitly defined in terms of the independent variable ( $x$ ). He didn't think I was being clear.

That concept is difficult for me to fully understand. Procedurally, I can do it perfectly well. Imparting the concept on others is very difficult. Although I haven't had to formally teach implicit differentiation lesson again, I've had to help folks with the topic and I'm trying to improve that explanation. I think I've taken an improved approach to implicit differentiation: Now, students simply include  $\frac{d<variable>}{dx}$  after every derivative operation as a constant reminder the independent variable is  $x$ . When we're finished,  $\frac{dx}{dx} = 1$ .

Doing this new method helped me understand implicit differentiation much better. This is a correction I think will have the most impact when I teach implicit differentiation in future semesters. (Journal Entry, November 20, 2014)

Laverne acknowledged she was not well prepared for topics with which she was less familiar. Prior to the implicit differentiation lesson, she was able to "wing it" because she was very comfortable with the material. However, her own weak conceptual understanding of implicit differentiation caused confusion for her students. In her exit interview, she resolved to prepare more thoroughly:

For most of everything after implicit differentiation, I needed a greater level of preparation. I worked out every single example in the book. I worked completely through problems by myself and showed every step. I was trying to find real-world examples to be able to answer the question, “Why do we need to care about implicit differentiation?” We do a lot of examples. I try to turn them into real-world examples. (L. Pascal, Exit Interview, January 30, 2015, Lines 140 – 145, 153 – 154)

Laverne also hoped to get better at detecting students’ lack of understanding real-time rather than much later (i.e., too late) on a graded event:

I have to better manage that because I was blowing off students when I needed to slow down. A lot of them won’t say, “Slow down.” I need to draw the feedback out of them. It’s a communication thing I really needed to keep an eye on. Managing that feedback from students is as important as making sure they’re all okay before I just plow forward not paying attention to the looks on their faces. (Lines 191 – 198; 214 – 217)

Laverne alluded to mathematical context (MC) of teaching whereby the teacher accesses and understands the mathematical thinking of students and assesses their mathematical knowledge (Kilpatrick et al., 2015). She hoped to improve her mathematical knowledge of content and students (KCS) (Ball et al., 2008).

### **Summary**

Dr. Paine made frequent casual office drop-ins and made himself available if Laverne needed to discuss anything. Although Laverne had advanced mathematical knowledge for meteorology, she recognized some weaknesses in her understanding of conceptual foundations for some topics necessary to teach her students. Dr. Paine focused on Laverne’s treatment of implicit differentiation using the chain rule and commended her for using of Leibniz notation to connect back to the chain rule and encouraged her to be consistent with notation during discussion and in examples. Laverne reflected on how to better manage instructional time, balance the varying speeds of students’ progress during group work, and questioning techniques to better solicit individual student responses. She also committed to better preparation for less

familiar topics. Figure 14 summarizes frequency for topics Laverne and Dr. Paine discussed and reflected upon.

Themes	Laverne	Dr.P	Sum
Representing mathematical ideas/notations	6	9	15
Assessing student progress	4	5	9
General practice	3	5	8
Questioning	3	3	6
Making meaning	4	1	5
Choosing examples	1	0	1
Mentoring	0	1	1
Timing/pacing	1	0	1
<b>Total</b>	<b>22</b>	<b>24</b>	<b>46</b>

Figure 14. Case 3 data matrix summary of themes.

Figures 15 and 16 summarize frequencies for MKT and MUST categories, respectively, assigned to Laverne and Dr. Paine's interactions and interview data.

MKT	Code	Laverne	Dr.P	Sum
<b>Subject Matter Knowledge</b>	<b>SMK</b>			
<i>Specialized content knowledge</i>	SCK	4	1	5
<b>Pedagogical Content Knowledge</b>	<b>PCK</b>			
<i>Knowledge of content and teaching</i>	KCT	11	15	26
<i>Knowledge of content and students</i>	KCS	7	8	15
	<b>Total</b>	<b>22</b>	<b>24</b>	<b>46</b>

Figure 15. Case 3 data matrix summary of MKT categories.

MUST	Code	Laverne	Dr.P	Sum
<b>Mathematical Proficiency</b>	<b>MP</b>			
<i>Conceptual Understanding</i>	MP_CU	2	0	2
<b>Mathematical Activity</b>	<b>MA</b>			
<i>Mathematical creating</i>	MA_CR	7	10	17
<b>Mathematical Context of Teaching</b>	<b>MC</b>			
<i>Access and understand the mathematical thinking of learners</i>	MC_LR	7	8	15
<i>Assess the mathematical knowledge of learners</i>				
<i>Reflect on the mathematics of practice</i>	MC_RP	6	6	12
	<b>Total</b>	<b>22</b>	<b>24</b>	<b>46</b>

Figure 16. Case 3 data matrix summary of MUST categories.

Laverne focused on KCT and KCS and mathematical creating by strategically using notation to connect facilitate meaning and connect topics.

## CHAPTER 7

### CASE 4: COLT JACOBIAN AND DR. BRIAN TOUGALOO

#### **New Instructor 4 – Colt Jacobian**

**Colt's background.** Colt was a first-time instructor teaching differential calculus. As an undergraduate, Colt spent two years at a college on a Reserve Officer Training Corps (ROTC) scholarship. He took basic core science, technology, engineering, and mathematics (STEM) classes required for electrical engineer majors; i.e., chemistry, physics, calculus, etc. He then applied and was accepted to USAFA. His previous two years in college gave him an advantage in USAFA's robust technical core curriculum. He achieved a high GPA and was competitive for medical school. He earned a pre-medical basic sciences degree; however, he chose a pilot career. He later earned a master's degree in aerospace engineering.

After he graduated from USAFA, he attended pilot training for a year and remained there as an instructor pilot for training aircraft for almost 3 and 1/2 years. For his next assignment he flew aircraft used for aerial refueling, cargo and personnel transport. Because of his expertise, he was upgraded immediately to aircraft commander, instructor pilot, and a check ride evaluator. He did this for about three and one-half years before he moved to his next duty station where he flew Navy airlift planes. Once again he was upgraded to an instructor pilot but was deployed numerous times. Next he served for 3 years in an overseas assignment as commander of a staff detachment where he coordinated fighter aircraft movement between the United States, Africa, Middle East and Europe.

Within his strategy division during his deployment, Colt worked with three operational analysts who were mathematics instructors at USAFA. When they realized he had an engineering degree, they encouraged him to consider being an instructor at USAFA because there was a need for subject area instructors who were also pilots. The opportunity was appealing to Colt because he spent much time away from home as a pilot. An assignment at USAFA would allow him to be home more. After serious thought and discussion with his wife, he applied and accepted the job in the mathematics department.

**Colt's teaching experiences.** Colt had no formal academic training or teaching experience in calculus. His wealth of teaching experience as an instructor pilot, however, gave him foundational skills he believed translated well to his calculus teaching. He described them:

What has translated to the classroom is the ability to recognize the look on somebody's face when they don't understand. I think I'm pretty good at that. A lot of times students just give you the automatic head nod like, "Oh yeah, I get it." And then you see on their faces they don't get it. That's the same with training student pilots.

I might lack the in-depth mathematical background of a mathematics professor, but I think I've made up for it with my understanding of instructor duties. If you haven't done good prep and done your homework on possible questions you'll get asked, students catch you with a curveball. In my class the other day, I messed up with exponentials; but I recovered quickly. In pilot training you have to come back quickly or you lose credibility. You may make mistakes, but as long as you do your homework and catch up students appreciate it. (Lines 49 – 75)

Interpreting students' nonverbal cues is a universal skill for facilitating learning because it helps a teacher know when deeper or alternative explanations are needed (Galloway, 1972).

Anticipating questions also guides a teacher's instruction. Both skills require knowledge of content and students (KCS) (Ball et al., 2008). Colt's prior experiences made him more intentional in noticing students' expressions of understanding and preparing for student responses to instruction. Colt used mathematical context of teaching (MC) to access and understand students' thinking (Kilpatrick et al., 2015).

**Colt's teaching approach.** As an instructor pilot, Colt insisted his students truly understood all material. He described himself as follows:

I wouldn't let them get by with knowing only half the material. They kind of hated me for it; but my classes gave me the Best Instructor Pilot Award. They realized when they got to their check ride: being hard, sticking on them to do the right thing, and working until they got the right answer ended up getting them really good results.

I learned I wanted to be the guy who didn't accept students understanding half the material; even if it means I have to ask 50 questions over the same thing until they get it, re-attack the concept, teach the concept a different way. (Lines 173 – 185)

Colt set high expectations for his students' learning and held them accountable by informally assessing their understanding and making instructional adjustments to help them better understand concepts.

***Instructional flexibility with meaningful examples:*** Colt valued the ability to reach students by using various techniques or explaining concepts in alternate ways:

There are a bunch of techniques to get to the right answer. Sometimes a student doesn't always grasp the schoolhouse solution or technique. It's just not how they think. You have to put yourself into their mind of how they get to a solution. Maybe it means one of the other three techniques is better for them. (Lines 189 – 194)

The book gives two ways with two different notations of how to do the chain rule. One of my students just cannot grasp one of the notations, but for some reason can really grasp the other one. Even though it's not the one I prefer, it's what I prefer to teach her because it was what she understood. I'm still working on recognizing the best technique to get to them; which means I have to know all of them. (Lines 197 – 202)

What seems intuitive to me and 90 percent of the class may not be intuitive to the other 10 percent. Being able to recognize the 10 percent who are not getting it and trying to flip the instruction style so they can actually get it is what I'm trying to get better at. Multiple views solidify the concept for me as well. Coming up with some good examples that show students different concepts helps me too. (Lines 218 – 222; 248 – 250)

Colt's development of specialized content knowledge (SCK) enhanced his ability to use multiple perspectives and be flexible with his instruction. His selection of examples most appropriate for his students' perspectives demonstrated his knowledge of content and teaching (KCT) as well as

knowledge of content and students (KCS) (Ball et al., 2008). The combination of Colt's knowledge and application in the classroom exhibit the three MUST perspectives: mathematical proficiency (MP), mathematical activity (MA), and mathematical context of teaching (MC) (Kilpatrick et al., 2015).

***Engaging students.*** Colt set a standard for full engagement in his classroom. He encouraged his students to teach each other and explained: "If you can get to the point where you can teach your buddy the material, it will help you understand it so much more. The understanding is so much greater when you can actually teach" (Lines 230 – 234). His students spent much time at the boards in groups and were expected to communicate their work:

During board work I want to see everybody get involved. I walk around and ask everybody questions. But when your groups are more than three, there's always somebody in the shadows. Even though I try to recognize it and make sure I'm asking that person the questions, I know I miss some. With two, it's easy to ping back and forth and say to each, "You answer one question." When you start getting to four it's difficult. (Lines 364 – 373)

Colt cultivated an environment where students actively applied concepts they learned and communicated with each other as well as with Colt when he informally assessed their understanding through questioning. He exemplified NCTM's (2000) Communication Standard to "establish a classroom climate conducive to the respectful exchange of ideas" (p. 351).

**Instructional influences through auditing.** Colt took advantage of opportunities to observe a variety of instructors within his course as well as a future course he would teach:

I chose Dr. Tougaloo first block. I wanted to start observing senior instructors' teaching methods, examples they use, and how they work the classroom. This block I'm auditing Yvonne who is in her second year to see how a younger instructor interacts with students. For the third block I'll audit Dr. Sarah Brown, a civilian professor. There's something to learn from each because they bring something different to the table.

I'm always a fan of a good example as well as a bad example because you learn just as much from either. Dr. Tougaloo gave a problem we talked about afterwards. He was like, "Yeah, I probably shouldn't have given that problem." It didn't do what he wanted

it to. We recognized immediately it was a bad example. I focus heavily on the objectives and want to make sure my examples truly tie into the lesson objectives. That's mainly what I take note of when I'm auditing.

I also watch the students to notice completely lost looks on their faces. I try to alleviate some of that confusion in my class after I notice it from auditing. (Lines 733 – 786)

I've also been auditing Olivia's differential equations class. Although I'm not a big fan of that level of PowerPoint use, she makes it work. There's a lot of conceptual stuff: What's a homogenous equation? What's a particular solution? There are a lot of terms and concepts. The way she uses the slides totally works. I'm thinking of using her model to hit home some of those concepts. (Lines 810 – 817)

Colt was intentional in his observation of other instructors. He specifically looked for the effectiveness of examples in terms of how the students responded and how well they covered objectives. Colt also noticed students' nonverbal communication to determine how well they understood. Since Colt chose to observe his mentor, Dr. Tougaloo, they often discussed what Colt observed and referred to it when it related to something discussed during feedback sessions. Colt's intentionality in auditing was driven by what he deemed important focus areas in his own instruction: student understanding, effective examples, and meeting lesson objectives.

#### **Mentor 4 – Dr. Brian Tougaloo**

**Dr. Tougaloo's background.** Dr. Brian Tougaloo was a military assistant professor with more than 10 years teaching experience. He attended USAFA and graduated with a Bachelor of Science degree in astronautical engineering. At the start of his career he worked on satellite and launch vehicle testing and began graduate school in astronautical engineering; however his interest shifted and he completed his master's degree in operations research. For his next assignment, he performed modeling and simulation of air-to-air combat for fighter jets at a base which often hosted tours for senior faculty from USAFA. During one of these tours, he met Colonel Davidson who was then the head of USAFA's mathematics department. When Colonel Davidson learned Tougaloo had a master's degree in OR, he asked if he was interested in

teaching at USAFA. Tougaloo said yes and taught for two years before he went to school to pursue a doctor of philosophy in industrial engineering. Upon completion of his degree he was assigned an operational job running statistical models for mobility equipment procurement before returning to USAFA's mathematics department where he was for the last seven years. His roles in the department included division chief of operations research, statistics, and calculus as well as deputy director for academics for two and half years. He had recently become the faculty development director in charge of the new instructor training program.

**Dr. Tougaloo's teaching and mentoring foundation.** One of Dr. Tougaloo's responsibilities as faculty development director was assigning mentors to new instructors. Other than trying to avoid assigning multiple first-year instructors to a mentor, he had no strict strategy for matching pairs. He typically asked available qualified mentors (civilian professors and senior military faculty with doctor of philosophy degrees and more than five years teaching experience) if they were willing to mentor a new instructor and had any preferences. Sometimes there was an existing relationship (former student or colleague from a prior military assignment) or an emerging interest from interactions during the new instructor training activities. For example, a new instructor was influenced by a mentor's instructional style exhibited during the calculus crash course; or a mentor was influenced by a new instructor's performance during their practice lessons. This semester was the first semester Dr. Tougaloo was a mentor to one of the new instructors, Colt.

### **Colt and Dr. Tougaloo's Mentoring Relationship**

Dr. Tougaloo assigned most of the new instructor/mentor pairs before he assigned himself as Colt's mentor. When the semester began, Colt audited Dr. Tougaloo's class. They frequently talked about the lessons. Colt also felt comfortable discussing various non-teaching

work topics. The pairing seemed a good fit since they were the same military rank and had become comfortable with each other during the summer new instructor training. Dr. Tougaloo described Colt as “not going to be too difficult. He had a higher starting point. He was very comfortable in front of the students. He was good to begin with. I think he was more advanced from the beginning” (Lines 340 – 343). They maintained an informal mentoring relationship, in which they discussed lessons, shared lesson materials, reviewed assessments, or gave observation feedback. Dr. Tougaloo often shared his lesson ideas with Colt and included challenges he experienced:

I might say, “Hey, I did this problem, and this is why” and he would give me some comments back like, “Yeah, I probably wouldn't use that but something else; a coefficient or something less ambiguous to bring a point across better.” We talk back and forth about various things I plan to do. There are some times when we dive into the mathematics behind some things. (Lines 456 – 463)

Colt confirmed, “I’ve had a lot of interaction with him. He went through a problem he was a little confused on. When I give a quiz, I give it to him first so he can tell me if it is reasonable” (Lines 413 – 420).

In general, Colt and Dr. Tougaloo had a collegial relationship with mutual respect for each other’s instructional strengths and perspectives. Dr. Tougaloo welcomed Colt’s feedback about his classroom dynamics and often used Colt as a critical pair of eyes for his own instructional practices. Colt was comfortable speaking candidly to Dr. Tougaloo. This level of comfort was likely attributed their equivalence in rank. Overall, their mentoring relationship worked well and was mutually beneficial.

**Conducting observations and feedback.** When Dr. Tougaloo wanted to observe Colt, he dropped by his office to let him know. He asked if there was anything he wanted him to notice. If there was nothing specific, Dr. Tougaloo noted Colt’s strengths and areas for

improvement: “That's how I do my observations: I'll go through and write down good things, bad things, things to improve on, or things that are awesome” (Lines 375 – 381). Dr. Tougaloo took detailed notes for their feedback session. Colt (Jacobian, 2014) was comfortable with Dr. Tougaloo's impromptu style because he received feedback in this way for his practice lessons during new instructor training. Colt said, “Dr. Tougaloo is very good about hitting not only content but delivery” (Lines 398 – 399).

### **Observation Cycle**

#### **Pre-Observation**

Colt and Dr. Tougaloo had a short informal pre-observation meeting. Dr. Tougaloo stopped by Colt's office and said, “Hey, I'm observing you. Is there anything weird you want me to look for?” Colt said, “No, nothing out of the normal” (Lines 384 – 386) and noted he would give a 10-minute quiz at the start of class. Dr. Tougaloo was satisfied with the topic and the planned events for the lesson.

#### **Observed Lesson**

**Topic:** Lesson 15 – “Chain Rule” with the following objectives from the course syllabus (Appendix M) and notes to instructor (Appendix N): (1) Decompose complicated functions; (2) Determine the derivative of a function using the Chain Rule

**Instructor resources:** Colt's PowerPoint presentation (Appendix W) and written notes (Appendix X) for Lesson 15

**Summary/notable events:** Colt began class reminding students of upcoming assessments and administered a 10-minute fundamental derivative skills quiz. In addition to covering the lesson material, Colt emphasized the importance of using parentheses to group terms appropriately. The students worked in groups at the board.

## Feedback Session

Dr. Tougaloo and Colt's feedback session occurred later the same day of the observation in Colt's office and lasted 23 minutes. Dr. Tougaloo referred to his notes (Appendix Y) and began with positive comments about Colt's effective use of the first few minutes to cover administrative details for upcoming events. Since the first activity of the class was a quiz, Dr. Tougaloo addressed administrative details for the quiz: giving all clarifications and reminders before the quiz starts instead of in the middle of the quiz and reiterated the academic security policy to not discuss the quiz with students who had not taken it.

**Notation and organization.** Dr. Tougaloo encouraged Colt to be more consistent with modeling meaningful notation and logical organization of processes:

I would like to see you model what you expect from them. When the  $f(x) = 7^x$  problem came up I noted: "Good accepting responsibility for errors"; however you just wrote  $7^x$ . It's a function, so make sure you communicate there's actually a function there. Be tighter on use of equations and what you're trying to say, because eventually you will see board work or on a test where they're bouncing all over the place and there's no organization. Make sure you keep good organization because you want them to communicate that way. What they see is what they'll do. (B. Tougaloo & C. Jacobian, September 26, 2014, Lines 31 – 44)

Dr. Tougaloo referenced a specific comment Colt made in class: "Parentheses will keep you out of trouble on quizzes and tests" (Line 79). Dr. Tougaloo challenged, "Yes, that's true, but why?" Colt provided context for his statement:

Let me give you some background. That is something I've been harping on a lot. I gave them a bunch of examples last time of how not putting parentheses around things can get you in trouble when you're in a hurry. I told them, "It's communicating to you and it communicates to me. It helps you solve a problem and it helps me understand what you're trying to communicate when I'm grading."

The problem is that our book doesn't do that at all. I'm trying to teach them a way of communicating to help them. (Lines 84 – 103; 111)

Dr. Tougaloo agreed but cautioned Colt not to focus solely within the context of a graded event but to emphasize in general, “We want to be mathematically correct. We want to be unambiguous” (Lines 107 – 109). He also noted Colt occasionally omitted parentheses. Colt responded, “I know and I’ve told them if they see it, point it out. A couple of them have done it before” (Lines 120 – 123). Dr. Tougaloo concluded, “It’s not easy, especially in your first semester when you’re just trying to get material across. I’m better now than I used to be with explicitly writing what I want them to see. That will come with time” (Lines 126 – 129).

Dr. Tougaloo commended Colt on his effective use of composite function notation to provide a solid foundation for introducing implicit differentiation and acknowledged he would adjust his own instructional approach based on what he observed:

I like the way you introduced “ $F$  circle  $G$ ” ( $F \circ G$ ), and “ $G$  circle  $F$ ” ( $G \circ F$ ), the composite functions. You got them to see the inside and outside functions. I never even put on the board a composition symbol – I went right to the chain rule. I like what you’ve done because now they have a better basis for implicit differentiation.

I’ll have to back-track a little bit to help my students understand what we are doing. I think my guys will know how to do the chain rule fairly well; but it’s not necessarily going to bode well when I teach them implicit differentiation. I’ll have to do some more explaining to get them there. You’ve already laid more groundwork; I like that. (Lines 131 – 144)

Colt reasoned, “To understand decomposition you have to understand how functions are composed” (Line 146). Colt’s specialized content knowledge (SCK) (Ball et al., 2008) enabled him to show students the connection between composite functions and implicit differentiation to give the students a stronger foundation. This is important because, “when students can see connections across different mathematical content areas, they develop a view of mathematics as an integrated whole...and become increasingly aware of the connections among various mathematical topics” (NCTM, 2000, p. 354).

Dr. Tougaloo advocated using function notation to distinguish the parts of a composite function and facilitate more clarity in performing implicit differentiation:

When you're using Leibniz's notation you may be better off writing your original equation in function notation. You wrote  $y = \ln(x^3)$ . Then you wrote  $y = \ln(u)$ . Be careful; these are two different  $y$  equations! Which one am I using? I would write  $y(u) = u^3$  then figure out the derivative of  $y$  with respect to  $u$ ,  $\frac{dy}{du}$ .

Have  $y$  as a function of  $x$  [i.e.,  $y(x)$ ] as the first one and let  $u = x^3$ . Now create  $y$  as a function of  $u$  [i.e.,  $y(u)$ ] as opposed to what you had. You wrote  $y = \ln(x^3)$  and  $y = \ln(u)$ . If you put  $y$  as a function of  $u$  [i.e.,  $y(u)$ ], you can go back when trying to figure out  $\frac{dy}{du}$ . You can say, "Oh, I need to go to the one that has  $u$  as the input"; whereas you don't necessarily see that with the way you wrote it.

I think this is a difficult concept for them. Some of them don't understand function notation very well, so this is another place you can insert it. I think it's a little bit easier for them to find  $\frac{dy}{du}$  or  $\frac{du}{dx}$  from the respective equation and be able to get the derivative.

During that problem,  $y = \ln(x^3)$  was projected there [*Uses white board in the office to illustrate*] and you went off the projection and put  $u = x^3$  here. Then you came back down here and did some more stuff. Model the organization you want to see. Too often we'll see people do it wrong because they have stuff shot gunned all over the place. Model organization: What are we doing next? What are the steps you want them to do? Have it in an organized method they can do every time. (Lines 159 – 202)

Colt responded, "I was trying to illustrate creating terms in relation to  $u$  and  $x$ , then getting  $\frac{du}{dx}$  and  $\frac{dy}{du}$ . You can put all the work on the side, plug and chug, and you're done. Maybe I just wasn't communicating well" (Lines 204 – 213). Dr. Tougaloo supported Colt's intentions and further emphasized:

I think they should be explicitly writing that; especially the less experienced students. Sure, you've got some confident ones who say, "Oh I've done the chain rule already. I know how to do it." And they can get  $\frac{dy}{dx} = \frac{3x^2}{x^3}$  right away because they have the process down in their head. But I focus the lesson presentation on the ones who have never done a chain rule before and ask, "How do I want them to do it?" I want them to write down  $u$  as a function of  $x$ ; that is,  $u = x^3$  and  $\frac{du}{dx} = 3x^2$ . I want to see them lay it out so they can be more comfortable with the chain rule process because chain rule is certainly not the easiest thing we do. Modeling good organization is better for them. (Lines 159 – 224)

This discussion was important because it emphasized the NCTM (2000) Representation Standard: students must understand different representations of functions and appropriately operate with them. This feedback referred to Colt's use of specialized content knowledge (SCK) (Ball et al., 2008) to perform the mathematical activities (MA) categorized under mathematical noticing: representing mathematical objects and connecting mathematical concepts (Kilpatrick et al., 2015).

**Board work and communication.** Dr. Tougaloo was pleased with how well engaged the students were as they worked at the boards: "Everyone seemed very engaged. They were all working together and talking" (Lines 226 – 229). Colt attributed this positive effect to his board work rule: "The person who writes isn't the person who talks, so they all have to pay attention. If I have time, I make the non-writers brief the group's solution" (Lines 231 – 235). Dr. Tougaloo agreed, "That's working great because they were all engaged" (Line 237). He then focused on the quality of the students' written communication on the board:

When they are writing on the board do you want them to write the formal equation for the chain rule? Some of them had your equation up there but just wrote the answer " $f' = \dots$ " They did the chain rule [*sound of fingers snapping*] like that. These are the hotshots who know how to do it. But does the other guy standing around know what's really happening if he is asked, "Where did that come from?"

I would push them and say, "Write what you're doing." They're going to be afraid to write  $F'(x) = f'(g(x)) * g'(x)$  because they wonder, "Is the prime in the right place? Do I need a prime here? Where do I put  $g(x)$ ? Where do I put the parentheses?" They're afraid because they're not confident with it. They don't really know what the notation means. So I would push them to do that.

I would like to see them show what they're putting into  $F'(x)$  instead of skipping from " $F(x) = \dots$ " to " $F'(x) = \dots$  (the answer)." It happens to be the derivative of the outside times the derivative of the inside. Make them communicate that step. It will be better for them and the grader; especially with more difficult problems because they may not be able to do that in their heads. I write them down just to keep it straight. It's easier to take it piecemeal than in one swoop. Communication lacked in some cases on the boards for the difficult problem. Model good communication and make sure they're doing it; keep on them. (Lines 239 – 288)

The student engagement was productive because Colt required all students to play a role in task completion and communicate by writing or explaining. Colt exemplified NCTM's effective teaching practice of facilitating "discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 3).

Dr. Tougaloo's final notation comment was: "They wrote  $F'(x) = \frac{1}{2}(1 - 2x)^{-1/2} \cdot -2$ . Get them out of that habit because having ' $\cdot -2$ ' is a NO-GO! Tell them to use parentheses when multiplying by a negative number. Stay on them to make sure they do well with the notation" (Lines 329 – 333). Dr. Tougaloo's feedback was consistent with the NCTM (2000) Communication Standard for students to "use mathematics language and symbols correctly and appropriately, be good collaborators who work effectively with others, and communicate their mathematical thinking coherently and clearly to peers and teachers" (p. 348).

**Clarifying task and purpose.** Dr. Tougaloo questioned Colt about the purpose of an example he used:

Dr. Tougaloo: For  $f(x) = \sin(\sin(x))$ , you asked, "Isn't this  $f$  composed of  $f$  [i.e.,  $f \circ f$ ]? It's not  $f$  composed of  $g$  [i.e.,  $f \circ g$ ]."

What was the point there?

Colt: My challenge problem at the end was a similar problem. In decomposition, our objective is to decompose multiple functions. Well this was just one function composed of its self.

I didn't want it to be a shock when they got to the challenge problem. I wanted to emphasize a function with other functions as well as with itself; that was the point. Maybe I just didn't communicate it very well.

Dr. Tougaloo: You didn't finish the statement. You just said "This is  $f$  composed with its self." Follow up with, "We can do this, and you'll see it later on our challenge problem."

Finally, Dr. Tougaloo reminded Colt to specify the task when presenting a problem. He referenced Colt's challenge problem:  $f(x) = \ln(\ln(\ln(x)))$  and commented, "Where's the question? Is it: Find the derivative? I assume that's what you wanted them to do; so no big deal because you mentioned it earlier; but give clear instructions" (Lines 354 – 356). This feedback was also consistent with the NCTM (2000) Communication Standard for Colt to give a "clear sense of the mathematical goal for the students" (p. 351).

### Feedback Impact

**Colt's response to feedback.** In his interview (C. Jacobian, Interview 1 September 29, 2014), Colt admitted he expected feedback on modeling good notation: "Those were very similar to comments we had during the summer. I already knew what he was going to make comments on. And even knowing, I still made some mistakes" (Lines 399 – 405). He continued, "When I did  $u$ -substitution, I could have been more orderly. I was doing it so fast, I didn't realize I wasn't clear with my function  $y$  as a function of  $u$ . I could have been clearer" (Lines 475 – 479).

Colt further clarified his parentheses admonishment to his students which piqued Dr. Tougaloo's interest in the feedback:

Since the beginning of this block I have been harping on, "make sure you group your terms appropriately"; particularly with the logarithm or cosine of a function. For instance:  $\ln(x^2 + 2x)$ .

If you don't use parentheses, it becomes logarithm of  $x^2$  then add the  $2x$  later [i.e.,  $\ln(x^2) + 2x$ ]. That's not what you're supposed to do; you're supposed to add the quantity then take the logarithm. I've been getting on them about putting parentheses around the appropriate quantity: "Don't lose track of that when you're in a hurry."

For cosine of five  $x$  plus three: Was it  $\cos(5x) + 3$  or was it  $\cos(5x + 3)$  [cosine of the quantity  $5x+3$ ]? It's just one of these techniques I use, but I think it'll save them some hassle in the end. (Lines 584 – 616)

Colt recalled what stuck with him most:

The structure of the notation and making sure I'm very clear; particularly with this next lesson on implicit differentiation. It would help them to see a good example from me. That was his point: "They model everything you do," so if I can give them a good example of exactly how it looks, they will have a good model. (Lines 631 – 640)

Every once in a while I get in a hurry and won't use parentheses. I've told them to say something to me: "Even though you're a student, I don't care. Say something to me because I want to model good behavior." I have to talk the talk and walk the walk. (Lines 648 – 655)

That's the primacy of learning. The first thing they see sticks the most. If you don't get it right the first time and they see it, they'll fight to do it right because they never got their good picture. That's why I re-attacked a mistake; to make sure they understood exactly how it was supposed to be. I wanted the correct thing to be stuck in their head; not the mistake. (Lines 685 – 691)

Colt correctly referred to Thorndike's (1927) learning theory of primacy: what is learned first makes the strongest impression in the mind and is difficult to unlearn.

**Colt's intended instructional adjustments.** Colt identified the most impactful feedback for future instruction: "During my last extra help session, there was a girl who missed class. I was trying to be clear. Without the feedback, I would have been in a hurry and scribble-scrabbled through it. The feedback was good" (Lines 481 – 487). He also expressed some concern for the next lesson based on students' confusion:

Honestly I'm a little worried about next lesson because I know implicit differentiation is tough. I'm worried one lesson on the chain rule is not enough and I'll have to spend a little bit longer on review. You can't even think of implicit differentiation until you completely understand chain rule.

A couple of my students came for extra help today. They're really close but there's still some conceptual understanding needed to help them get the idea, "I've got to do the derivative of one and then I've got to do the derivative of the inside. It's the same way when you have a function with  $y$  in it. You have to take the derivative with respect to  $y$  and then the derivative of  $y$  with respect to  $x$ ." I'm not sure they get that yet.

You're never going to get everybody; I realize that. But does a majority of my class truly understand how to get through the chain rule? I'm giving a quiz before class; unfortunately, I can't truly assess it until I grade the quiz. (Lines 501 – 527)

Colt intended to be more careful with clarity of notation and communication. The feedback made him realize how his instruction was viewed from a student's perspective and gave him insights about how to adjust for student understanding. Colt resolved to improve his mathematical activity in the areas of communication and symbolic representation as well as his ability to discern students' understanding. This required effective use of common and specialized content knowledge (CCK and SCK), knowledge of content and students (KCS) (Ball et al., 2008), and execution of mathematical context of teaching to access student thinking (Kilpatrick et al., 2015).

### **Colt's Reflection**

#### **Auditing Effect**

In his exit interview (C. Jacobian, Exit Interview, January 30, 2015), Colt summarized the impact of observation feedback and auditing other instructors.

**Preparing for student questions.** Colt described how auditing motivated him to pay attention to student questions to gain insights for preparing his instruction. Seeing other students' reactions was a huge advantage for anticipating his students' questions:

It forced me to do extra preparation beyond hitting the course objectives because certain students ask deeper questions. I see that in my differential equations class this semester. I have a couple of students who really understand the concept and dig for more.

I have to put in extra time preparing course materials to be prepared to field questions that may or may not apply. I've spent a lot of time preparing; probably because I'm new and I'm not sure exactly what level of fidelity I need to be able to respond to their questions on the spot as opposed to coming back to address them later. (Lines 21 – 36)

**Effective examples.** Auditing also highlighted the importance of using examples at the appropriate level of complexity. Colt adopted a basic approach for selecting examples:

I would never give a complex problem to start off. I would start with a baby step problem and throw an extra thing to make it a little more complex. Then work up to the more complex problem so they get the building blocks, the foundation; not the specific

concept but the methodology of how to attack a problem. Break a problem down into smaller parts and then attack the smaller parts. Watching good examples from the more senior instructors is very helpful. (Lines 95 – 128)

Selecting appropriate examples most beneficial for developing concepts for students' understanding requires knowledge of content and teaching (KCT) as well as knowledge of content and students (KCS) (Ball et al., 2008).

### **Instructional Strengths**

Colt provided aspects of his teaching with which he felt most confident and comfortable:

**Engaging students with questions:** I think students respond to my presentation style. I'm loud, so it keeps them awake. I hate to say it, but that's a big deal here because they are so sleep-deprived. Seriously though, not just being loud; I try to ask every student a question each time. We're lucky we have small classes so I can do that. I think that's a good thing. They know they will get asked a question at some point. (Lines 178 – 190)

Colt used MUST's mathematical context of teaching (MC) to access student thinking through questioning. Questioning is a powerful instructional mechanism for engaging students, deciphering students' understanding, and promoting higher level thinking (Clegg, 1987; Cotton, 1989; Gall & Rhody, 1987; Hughes, 1974).

**Noticing confusion through non-verbal communication:** I've also become better at identifying non-verbal communication from students. The "I'm not sure I understand what's going on, but I'm not going to say anything because I don't want to hold up class" look. I'm pretty good at recognizing that look and trying to address it on the spot. I don't want them to leave without understanding. (Lines 192 – 199)

Having a keen awareness of students' understanding in the absence of verbal communication is an aspect of mathematical context (MC) of teaching in which a teacher accesses and understands the mathematical thinking of learners (Kilpatrick et al., 2015). Improving this instructional practice was a goal Colt stated in his initial interview.

**Allotting time for thoughtful student responses:** One thing Dr. Brown mentioned in my initial new instructor training was, "When you ask a question, make sure you take enough of a pause before you jump right in and give them the answer." The painful wait; I think I'm pretty good at it.

The other day I waited 20 – 30 seconds. I was willing to do it because I wanted them to break a problem down and come up with a good idea. I eventually got a good idea. It wasn't what I was thinking, but I could tell they thought about it. I've developed in this area. Instead of just jumping right in and moving on; taking time to let them internalize the question, come up with an answer, and not be afraid to answer. (Lines 205 – 232)

Wait-time affords time for students to process what is being asked and engage in deeper thinking before responding (Ingram & Elliott, 2016; Rowe, 2003; Tobin, 1987). Colt planned to be more intentional in giving students sufficient time to think and respond to his questions.

### **Observation Program Evaluation**

Colt evaluated the observation and feedback program. In general, he believed new instructor training was extremely valuable: “Having gone through what we do, I felt really prepared on day one and subsequently knew how to prepare for day one for my new class second semester” (Lines 254 – 257). He also acknowledged being observed and getting feedback was good but had concerns about the front-end frequency:

I would have appreciated if it was more spread out. For instance, we're learning a lot of good stuff from our mentors, course directors, and assistant course directors early on. You hope those skills you're learning are maintained. But towards the end of the semester it would have been nice to have been validated. For example have someone say, “Hey, we talked about these things early and then I observed you Lesson 35 and you did everything we talked about” or, “You picked up new skills that were good.”

I think we're always learning and progressing. I really like it. I would have rather had, instead of four early observations from multiple people in a row, maybe three lessons spaced out. Then you're looking at a good 15 lessons worth of instruction to progress based on the feedback.

You would get a chance to practice some of the things you talked about, put them into action and see if you truly retained those skills. For me as a teacher, that's what I think might have been more beneficial. (Lines 279 – 301)

Colt's evaluation of the program captured all of the participants' thoughts about the program.

Overall, they found it extremely beneficial for their development. They all agreed its informal nature was what made it most appealing and garnered people's buy-in.

### Summary

Having the same professional rank, prior feedback interactions, an informal approach, and teaching the same course fostered a comfortable and productive mentoring relationship between Colt and Dr. Tougaloo. Colt relied on Dr. Tougaloo's expertise—Dr. Tougaloo also solicited Colt's perspective about his instruction, including his own challenges. Discussing pedagogy with colleagues is a form of professional development involving reflection of practice while valuing the novice's perspective builds confidence (Garet, Porter, Desimone, Birman, & Yoon, 2001; Gellert & Gonzalez, 2011; Kensington-Miller, 2011; 2012).

Figure 17 summarizes the frequency for topics Colt and Dr. Tougaloo discussed and reflected upon.

Themes	Colt	Dr.T	Sum
Assessing student progress	14	4	18
Representing mathematical ideas/notations	6	12	18
Using correct notation	6	6	12
General practice	9	2	11
Choosing examples	5	0	5
Questioning	3	0	3
Persevering through student questions	2	0	2
Anticipating student responses	1	0	1
Connecting across curriculum	1	0	1
Making meaning	1	0	1
Mentoring	0	1	1
Modeling problem solving	0	1	1
<b>Total</b>	<b>48</b>	<b>26</b>	<b>74</b>

Figure 17. Case 4 data matrix summary of themes.

Dr. Tougaloo reminded Colt to model correct and consistent mathematical notation, representations, and process. He also highlighted Colt's strategic competence, ability to show connections between mathematical topics in anticipation for future topics, and high level of student engagement. Colt became more intentional with modeling good notation and process. He also honed his questioning skills to consistently target all students with deeper level questions

and longer wait-times. Colt intended to improve the following areas: anticipating student questions and selecting effective examples.

Figures 18 and 19 summarize the of MKT and MUST categories, respectively, for Colt and Dr. Tougaloo's interactions and interview data.

MKT	Code	Colt	Dr.T	Sum
<b>Subject Matter Knowledge</b>	<b>SMK</b>			
<i>Common content knowledge</i>	CCK	6	6	12
<i>Specialized content knowledge</i>	SCK	3	1	4
<b>Pedagogical Content Knowledge</b>	<b>PCK</b>			
<i>Knowledge of content and teaching</i>	KCT	19	15	34
<i>Knowledge of content and students</i>	KCS	19	4	23
<i>Knowledge of content and curriculum</i>	KCC	1	0	1
	<b>Total</b>	<b>48</b>	<b>26</b>	<b>74</b>

Figure 18. Case 4 data matrix summary of MKT categories.

MUST	Code	Colt	Dr.T	Sum
<b>Mathematical Activity</b>	<b>MA</b>			
Mathematical creating	MA_CR	15	18	33
Mathematical reasoning	MA_RS	0	1	1
<b>Mathematical Context of Teaching</b>	<b>MC</b>			
Access and understand the mathematical thinking of learners Assess the mathematical knowledge of learners	MC_LR	19	4	23
Reflect on the mathematics of practice	MC_RP	13	3	16
Know and use the curriculum	MC_CUR	1	0	1
	<b>Total</b>	<b>48</b>	<b>26</b>	<b>74</b>

Figure 19. Case 4 data matrix summary of MUST categories.

Colt continually reflected on his practice to make his classroom interactive and to fully engage every student. He had a good foundation for pedagogical knowledge and was strong in implementing ways to access and assess student learning.

## CHAPTER 8

### ANALYSIS AND DISCUSSION

#### **Cross-Case Synthesis**

This chapter describes the results of the analysis and interpretation of the four case studies. The first section briefly summarizes each case. The next section summarizes the similarities among the cases. The following sections describe the thematic categories emerging from the data. The chapter ends with a discussion of the overarching themes within the contexts of MUST and the goals of the observation program.

#### **Case Characteristics**

**Case 1 – Carl and Dr. Bethune.** Carl and Dr. Bethune’s mentoring relationship was the strongest in terms of amount of discussion about instruction and learning. They had a prior existing relationship as they worked together on a project a few months before the semester. Since Carl had been a top student in USAFA’s mathematics department, he and Dr. Bethune both knew of each other’s reputations. Carl specifically requested to be paired with Dr. Bethune and observed all of Dr. Bethune’s lessons in an upper level probability course. They engaged daily about teaching and learning principles along with a myriad of intellectual topics. Carl’s rigorous graduate school experience shaped his philosophy: a teacher’s role is to challenge students to learn how to learn through critical thinking and independent learning experiences. Since Dr. Bethune’s philosophy was similar, Carl desired to model his instructional practices.

Dr. Bethune differed from the other mentors because he required Carl to initiate interactions between them instead of checking up on him. His approach to giving feedback was

also more targeted than the other mentors because he focused on one instructional area for an observation rather than taking a general overview. They were also more intentional and structured than the other pairs with conducting pre-observation meetings. Case 1 focused on student learning, assessing students' progress, and instructional timing and pacing. Other notable emerging themes included having students explore extensions or create problems and using higher level questioning techniques to foster students' critical thinking.

**Case 2 – Kevin and Dr. Ignacio.** Since their offices were across the hall, Kevin and Dr. Ignacio were closest in proximity than the other pairs. They had the greatest amount of general contact time because they informally engaged every day. Of the mentors, Dr. Ignacio had the most prior participation in structured colleague mentoring and observation programs during his graduate school and postdoctoral experiences. Dr. Ignacio was Kevin's sounding board for instructional ideas and quizzes, and was an additional set of eyes and ears in the classroom when he observed. Dr. Ignacio approached observations with no particular focus; rather he took in the lessons and made note of any potential issues. He used feedback sessions to learn Kevin's motivation and rationale for his instructional decisions.

Case 2 focused mainly on clarifying meaning, being more consistent with proper mathematical notations and language, connecting intuitive and formal definitions, assessing individual students during group work, selecting appropriate examples to achieve instructional aims, and having instructional tenacity to address students' questions. Kevin intended to modify his instruction by better planning to anticipate student responses, model problem-solving, and be more consistent with notation.

**Case 3 – Laverne and Dr. Paine.** Laverne and Dr. Paine's relationship was very casual. They had no prior existing connections. Dr. Paine usually dropped by Laverne's office once a

week to check on her and allowed her to decide when she was ready to be observed. He looked for general instructional flow and how well Laverne explained material. Dr. Paine favored Laverne's representation of implicit differentiation using the chain rule and encouraged her to be consistent with notation. Their feedback also focused on reading students' cues for understanding, giving sufficient wait-time to allow student responses or questions, calling on specific students rather than using overhead questions addressed to the entire class, allowing students to fix their own mistakes, managing time and the varying speeds of students' progress during board work or group tasks. Laverne intended to do a better job of noticing students' cues about understanding, increase wait-times, and better prepare for less familiar topics.

**Case 4 – Colt and Dr. Tougaloo.** Colt and Dr. Tougaloo's relationship was informal and collegial. Since they were the same rank, they viewed each other more as equals than any of the other pairs did. Dr. Tougaloo shared instructional materials and exchanged instructional ideas with Colt. He openly praised Colt for his effective instructional practices and incorporated them into his own teaching—admitting Colt was advanced for a new instructor.

Colt spent much time during his operational military career as a top instructor pilot. He translated fundamental teaching skills from this wealth of instructor experience; particularly his ability to read students' nonverbal language and cues about their understanding. He was extremely comfortable with managing an instructional environment where students worked in groups at varying levels, and he used built-in strategies to informally assess individual student progress through questioning and assigning specific roles for students to communicate their understanding. Case 4 focused mainly on modeling proper notations, completing the chain rule process, holding students accountable for complete written communication, high level of student engagement, and the importance of defining the task and goal when presenting an example.

## Similarities

To extract the most common emerging themes among the cases I considered two factors:

(a) the frequency with which themes emerged (b) the prevalence of themes across the cases.

Figure 20 summarizes the frequency of emerging themes from all cases:

Themes	Count
Assessing student progress	44
Representing mathematical ideas/notations	37
General practice	36
Emphasizing learning	32
Questioning	17
Mentoring	16
Making meaning	15
Using correct notation	15
Timing/pacing	11
Choosing examples	8
Extending problems	8
Anticipating student responses	7
Modeling problem solving	7
Connecting across curriculum	5
Persevering through student questions	5
<b>Total</b>	<b>263</b>

Figure 20. All cases data matrix summary of themes.

At first glance, it might seem easy to characterize the highest frequency themes as being the most noticed, addressed, and reflected upon. It is not sufficient, however, to simply consider frequency alone because a single case might have several data blocks categorized in a particular theme. One particular case could dominate the frequency and give a false impression the theme is most prevalent among all the cases. To clarify the saliency of a theme among the cases, I created an additional matrix to account for frequency and prevalence. Frequency is defined as the total number of data blocks assigned in a category. Prevalence is the number of cases (from 1 to 4) in which the theme was assigned for at least one data block by any participant in a case. Figure 21 below summarizes the prevalence and frequency of all themes among all cases.

Prev	Theme	Frequency				
		Freq	Case 1	Case 2	Case 3	Case 4
4	Assessing student progress	44	11	6	9	18
	General practice	36	9	8	8	11
	Mentoring	16	9	5	1	1
	Choosing examples	8	1	1	1	5
3	Representing mathematical ideas/notations	37	0	4	15	18
	Questioning	17	8	0	6	3
	Making meaning	15	0	9	5	1
	Anticipating student responses	7	4	2	0	1
	Modeling problem solving	7	4	2	0	1
	Connecting across curriculum	5	3	1	0	1
2	Using correct notation	15	0	3	0	12
	Timing/pacing	11	10	0	1	0
	Persevering through student questions	5	0	3	0	2
1	Emphasizing learning	32	32	0	0	0
	Extending problems	8	8	0	0	0
Totals		263	99	44	46	74

Figure 21. Prevalence and frequency summary of themes.

### Emerging Themes

Emerging themes are based on discussions and reflections among all cases. Figure 22 shows the ordering of the themes by prevalence and frequency and links them to primary components of MUST (Kilpatrick et al., 2015) and subdomains of MKT (Ball et al., 2008).

Prev	Theme	Freq	MUST	MKT
4	Assessing student progress	44	MC_LR	KCS
	General practice	36	MC_RP	KCT
	Mentoring	16	MC_RP	KCT
	Choosing examples	8	MA_CR	KCT
3	Representing math. ideas/notations	37	MA_CR	KCT
	Questioning	17	MC_LR	KCS
	Making meaning	15	MA_CR	SCK
	Anticipating student responses	7	MC_LR	KCS
	Modeling problem solving	7	MA_RS	SCK
	Connecting across curriculum	5	MC_CUR	KCC
2	Using correct notation	15	MA_CR	CCK
	Timing/pacing	11	MC_RP	KCT
	Persevering w/student questions	5	MC_LR	KCS
1	Emphasizing learning	32	MC_LR	KCS
	Extending problems	8	MA_RS	KCT
Totals		263		

Figure 22. Themes sorted by prevalence with MUST and MKT.

**Assessing student progress.** The most prevalent and frequent theme among the participants focused on ways to engage students to determine their understanding and ability to perform mathematical tasks. Discussions included students presenting work on the board, addressing individual issues, gauging students' reactions to instruction while auditing other instructors' lessons, identifying common misconceptions or struggles students had with certain material, using student mistakes to launch discussions, managing varied competence levels during group work, recognizing nonverbal cues for confusion, holding students accountable to justify and communicate their work, and adjusting instruction to facilitate students' understanding.

**General practice.** All participants discussed teaching practices in general terms and not specifically associated with facets unique to mathematics or mathematics instruction. This theme covers discussions in broad terms. For example, an instructor may have discussed writing lesson plans, arranging the room for an activity, a general teaching philosophy, or learning strategies. Often, these discussions ultimately led to a topic being placed in another category. These discussions were also categorized under reflection on practice and knowledge for content and teaching (KCT) (Ball et al., 2008).

**Mentoring.** All participants discussed their mentoring relationship. Discussions included strategies for observing and providing feedback, how decisions are made to watch a lesson or be observed, the mentor's approach, other instructional activities beyond observations, the dynamics of the mentoring relationship, or being observed by other colleagues. Mentoring conversations are also categorized as reflection on practice.

**Choosing examples.** All participants acknowledged the importance of effective choices for examples. Discussions emphasized the value of using multiple resources to collect a variety

of examples ranging in difficulty levels, using both traditional as well as nontraditional examples to illustrate fundamental principles behind concepts, using real-world examples to motivate topics and engage students' interests, and linking examples to course objectives.

**Representing mathematical ideas/notations.** This theme was the most frequent of those prevalent in three cases. Discussions included representing concepts in multiple ways (graphical, numerical, tabular, etc), selecting notational forms of mathematical operators or objects best reflecting a concept (e.g. Leibniz vs. prime or functional notation for derivatives), selecting meaningful variables, and modeling structure and technique in problem-solving.

**Questioning.** This theme was the second most frequent for those prevalent among three cases. Discussions included asking higher level or more conceptual questions to foster critical thinking, giving longer wait times to solicit more meaningful responses, directing questions to specific students, and attempting to query each student during a lesson.

**Making meaning.** This theme was the third most frequent of those prevalent in three cases. Discussions included elaborating on definitions or meaning of notations, interpreting definitions, linking intuitive meanings with formal definitions, clarifying when variables are implicitly defined, and increasing one's own conceptual understanding in order to better explain concepts to students.

**Anticipating student responses.** This theme was tied for fourth most frequent of those prevalent in three cases. Discussions included appropriately responding to incorrect responses with redirection, modeling how and encouraging students to evaluate their own responses for reasonableness or validity, and considering multiple ways students might respond and planning how to use those responses as learning points.

**Modeling problem-solving.** This theme was tied for fourth most frequent of those prevalent in three cases. Discussions included demonstrating an organized and logical problem-solving framework when working through examples, providing rationales for why particular mathematical decisions were made during problem-solving, engaging students in metacognitive discussions to help them reason, being aware of students' delusion of competency when they only watch but do not practice problem-solving, and emphasizing clear written and oral communication during problem-solving.

**Connecting across curriculum.** This theme was the fifth most frequent of those prevalent among in cases. Discussions included strategically structuring lesson content and emphasizing particular concepts to better prepare students for future concepts and using multiple curriculum resources to provide a variety of approaches or problem types.

**Using correct notation.** This theme was the most frequent of those prevalent in two cases. Discussions included distinguishing between the evaluation of a function at a specific value and the general function, including all parts in a limit expression, and appropriately including parentheses in mathematical expressions.

**Timing and pacing.** This theme was the second most frequent of those prevalent in two cases. Discussions included prioritizing and apportioning time for lesson activities, planning breakpoints for multi-lesson topics, and minimizing wasted time when students tend to disengage.

**Persevering through student questions.** This theme was the third most frequent of those prevalent in two cases. Discussions included planning for possible questions students may have, deciphering students' questions, connecting the fundamental concepts necessary to address

students' questions, being prepared with multiple approaches to address students' questions, and ensuring students' questions are answered with specificity and completeness.

**Emphasizing learning.** This theme was the most frequent of those prevalent in only one case (Case 1). Discussions included realizing how deeper conceptual learning best occurs through experience in investigating problems, teaching students how to learn with tangible and specific strategies, modeling mathematical logic and perseverance in problem-solving, setting high expectations and holding students accountable for their own learning, requiring students to read and interpret mathematical texts, requiring students to communicate (in writing and orally) their understanding of concepts and thought processes, requiring students to demonstrate their ability to apply their knowledge to solve more complicated problems, facilitating appropriate collaborative opportunities to fully engage all students in independent thinking and knowledge presentation, and helping students reflect on learning and develop their own critical thinking skills.

**Extending problems.** This theme was the second most frequent of those prevalent in only one case (Case 1). Discussions included requiring students to investigate extensions of foundational concepts or basic examples and allowing students to create their own variations of problems.

## **Discussion**

### **Prevalence versus Frequency**

I have chosen to weigh prevalence slightly above frequency. The rationale is prevalence suggests common interest. Frequency simply records how many data blocks were labeled accordingly. Although the frequency could somewhat suggest the passion or intensity with which participants expressed their interests, a single instance of the presence of a theme—

regardless of the number of data blocks in which it emerges—is just as significant multiple appearances of the theme in data blocks within a single conversation. By this standard, themes with prevalence in four or three cases were regarded as being more significant than those with lower prevalence; therefore, the first eight themes described above were the most significant, with “Assessing student progress” as the most significant theme.

### Overarching Themes

Grouping themes in overarching MUST categories is another consideration for interpreting the data because there may be related principles connecting themes differing in prevalence or frequency. The above grouping would not capture this simply by the numbers. Grouping themes by MUST and MKT categories and sorting by the average prevalence of themes within the groupings yields the data shown in Figure 23 below.

Avg Prev	Prev	Theme	Freq	MUST	MKT
3.33	4	General practice	36	MC_RP	KCT
	4	Mentoring	16	MC_RP	KCT
	2	Timing/pacing	11	MC_RP	KCT
3	4	Choosing examples	8	MA_CR	KCT
	3	Representing math. ideas/notations	37	MA_CR	KCT
	3	Making meaning	15	MA_CR	SCK
	2	Using correct notation	15	MA_CR	CCK
3	3	Connecting across curriculum	5	MC_CUR	KCC
2.6	4	Assessing student progress	44	MC_LR	KCS
	3	Questioning	17	MC_LR	KCS
	3	Anticipating student responses	7	MC_LR	KCS
	2	Persevering w/student questions	5	MC_LR	KCS
	1	Emphasizing learning	32	MC_LR	KCS
2	3	Modeling problem solving	7	MA_RS	SCK
	1	Extending problems	8	MA_RS	KCT
Totals			263		

Figure 23. Themes sorted by average prevalence and MUST categories.

I sorted by MUST because the framework is better suited to categorizing classroom practices in action rather than knowledge alone. The subthemes are discussed below and organized in

descending order of average prevalence for themes within MUST aspects. Note, when more than one aspect occurs, the first one listed is the dominant aspect with the highest frequency.

**Reflecting on practice (MC\_RP).** Participants dedicated attention to various aspects of instruction. One major theme was timing and pacing of a lesson with the goal of making overall instruction most productive for student learning. Other topics about general practice and mentoring were also discussed. The following describes the reflection on practice aspect:

Teachers should be able to analyze and reflect on their mathematics teaching practice in a way that enhances their mathematical understanding. There are many ways to reflect on one's practice, and one of the most important is to use a mathematical lens. Thoughtful reflection of problems of practice can be reconsideration of a lesson just taught or it can be part of the planning for a future lesson. It may occur as the teacher interprets the results of a formal assessment, or it may be prompted by a textbook treatment of a topic.

Teachers often reflect on their teaching as they teach—as they are making split-second decisions. A teacher's decisions about how to proceed after accessing student thinking depend on many factors, including the mathematical goals of the lesson. It is valuable to revisit these quick reflections and decisions when there is time to think about the mathematics one might learn from one's practice. (Kilpatrick et al., 2015, p. 29)

Instructors used the mentors as sounding boards for broader instructional discussions: classroom management, student motivation and engagement, lesson planning, sequencing instruction, conveying concepts in multiple ways, recovering from instructional shortfalls, and gauging assessment instruments.

**Mathematical creating (MA\_CR).** Four themes were categorized by this aspect: using correct notation, representing mathematical ideas and notations, choosing examples, and making meaning. The following describes the mathematical creating aspect:

Inherent in mathematical work is the need to represent mathematical entities in ways that reflect given structures, constraints, or properties. The creation of representations is particularly useful in creating and communicating examples. Teachers need to be able to assess what features of the mathematical entity each form captures and what features it obscures. Representing involves choosing or creating a useful form that conveys the crucial aspects of the mathematical entity that are needed for the task at hand. Teachers

need to be able to create representations of those common forms in ways that reflect conventions. (Kilpatrick et al., 2015, pp. 23-24)

Participants emphasized making intentional decisions about representing function and derivative notations to facilitate better understanding of concepts, to set up and organize problems, and to exhibit logical structure during problem-solving. Participants also discussed choosing real-world examples to motivate topics, gradually increasing difficulty levels of examples to build students' skill level, and ensuring examples are closely tied to course objectives. Participants believed students' ability to interpret definitions was a significant part of their learning; therefore, they emphasized formal definitions with intuitive meanings. "Teachers need to be able to appeal to a definition to resolve mathematical questions, and they need to be able to reason from a definition. Teachers need to create definitions and assess the definitions students create or propose" (Kilpatrick et al., 2015, p.24).

**Know and use the curriculum (MC\_CUR).** One theme was categorized by this aspect: connecting across the curriculum. The following describes the curriculum aspect:

Mathematical understanding related to the mathematics curriculum requires a teacher to identify foundational or prerequisite concepts that enhance the learning of a concept. The teacher needs to understand how the concept being taught can serve as a foundational concept for future learning. The teacher needs to know how the concept fits learning trajectories. The teacher also needs to be aware of common mathematical misconceptions and how those misconceptions may sometimes arise at particular points in this trajectory. A teacher proficient in the mathematical work of teaching understands that a curriculum contains not only mathematical entities but also mathematical processes for relating, connecting, and operating on those entities. (Kilpatrick et al., 2015, p. 27)

Participants were often concerned with preparing students for upcoming topics within the course and beyond. For example, they paid careful attention to laying a foundation for implicit differentiation when they taught the chain rule and referenced the benefits of using Leibniz's notation later in multivariable calculus.

**Student learning (MC\_LR).** Five themes were categorized by this aspect: assessing student progress, questioning, anticipating student responses, persevering through student questions, and emphasizing learning. The following describes the student learning aspect:

Mathematics teachers should understand how their students are thinking about particular mathematical ideas. A competent teacher uncovers students' mathematical ideas in a way that helps them see the mathematics from a learner's perspective. Teachers can gain some access to students' thinking through the written work they do in class or at home, but much of that information is highly inferential. Through discourse with students about their mathematical ideas, the teacher can learn more about the thinking behind the students' written products. Classroom interactions play a significant role in teachers' understanding of what students know and are learning. (Kilpatrick et al., 2015, p. 26)

Participants devoted a significant amount of instructional time for students collaboratively working at the boards or their desks. Instructors were intentional in making the rounds to informally assess students' written work and used questioning to probe their understanding or required them to clarify their work.

**Mathematical reasoning (MA\_RS).** Two themes were categorized by this aspect: modeling problem-solving and extending problems. Teachers model reasoning when they demonstrate problem-solving from start to finish. Students witness the logical thinking as well as mathematical perseverance necessary to execute the problem-solving process. The following describes a particular dimension of the reasoning aspect:

Fundamental to mathematical reasoning is constraining, extending, or otherwise altering conditions and forms. Constraints can be removed, altered, or replaced to explore resulting new mathematics. Mathematical relationships and properties can be extended. Teachers engage in mathematical reasoning when they consider the effects of constraining or extending mathematical objects. The mathematical reasoning involved in constraining and extending enables teachers to create extensions to given problems or questions. (Kilpatrick et al., 2015, p. 22)

Having students investigate extensions of problems was one of Carl's (Case 1) major objectives. Engaging in this practice is a productive way to improve students' reasoning and critical thinking skills while developing greater capacity for independent learning.

## Conclusion

These themes represent the most noticed and addressed instructional practices discussed during feedback and reflected upon by the new instructors among the four cases. The major MUST categories were used to capture overarching themes representing the mathematical areas or mathematics instructional practices most noticed and addressed during the feedback sessions. The end goal of USAFA mathematics department's mentoring program was to foster a culture of continual reflection on and improvement of mathematics instruction. Ideally, improved instruction ultimately yields better student learning. To facilitate better student learning, teachers must have a solid foundation of mathematical content and pedagogical knowledge with deep understanding. The five prevailing aspects above target critical elements of mathematical understanding. A teacher's growth in these areas is imperative to achieve the end goal—assuming this is a goal for mathematics teachers, at all levels, who engage in similar developmental activities. One's growth in practice begins by evaluating performance through reflection. Reflection is enhanced when it is done in collaboration—especially with someone who possesses expertise. The core of the work of teaching resides in a teacher's ability to create multiple representations of mathematical concepts, fluidly weave together ideas throughout the curriculum, know where students are on the path of mathematical understanding and interpret students' mathematics, and model good problem-solving through critical thinking (Kilpatrick et al., 2015). The results of this study reveal the successful nature of the program in addressing the core competencies of effective mathematics instruction. Chapter 9 summarizes these results in the context of the research questions and makes connections with the literature on reflective practice and colleague observation of teaching.

## CHAPTER 9

### CONCLUSIONS AND IMPLICATIONS

In this chapter I give a brief overview of the study and the research questions. I address the research questions using the thematic results from the previous chapter and reference the literature introduced in Chapter 2. Next, I discuss the limitations of the study along with potential ways the study could have been improved. I suggest implications for further or future studies and conclude with final thoughts pertaining to professional development supporting new teachers.

#### **Overview**

For this study, I analyzed experienced colleague mentors' feedback on new calculus teachers' instructional practices and new teachers' intended instructional modifications. There were eight participants forming four cases. Each case included a new instructor/experienced colleague mentor pair engaging in one or two observation cycles consisting of the following: pre-observation meeting, a complete classroom lesson where the experienced colleague observed the new instructor, a feedback session between the experienced colleague and new instructor, and interviews with each participant. I witnessed the interactions between the pairs as a nonparticipant observer.

The mentoring relationships were open and informal. New instructors had comfortable and unrestricted access to their mentors, with no formal supervisory roles between them. The strongest relationship occurred when the pair had an existing relationship (Case 1). In this case, the new instructor (Carl) specifically requested his mentor (Dr. Bethune). In all cases, the pairs

had frequent interactions beyond their prescribed observation cycles. During these interactions their discussions covered a wide variety of topics: lesson planning, assessment instrument creation, results of student performance, pedagogy, issues in the classroom, instructional techniques, learning theory, effective study strategies, mathematics, educational theory, current life events, common personal interests, and so on. All participants regarded their relationships as trusting and collegial—meeting the goal of the department’s faculty observation program “designed to foster a positive atmosphere where observer and observed can learn from a variety of other instructors” (USAFA/DFMS, 2007, p. 3).

There was no formal structure for determining observations. The experienced colleague allowed the new instructor to choose a lesson to be observed. Pre-observation meetings were typically short, casual interactions in which the new instructor was given an opportunity to identify any particular areas on which he or she wanted the experienced colleague to focus. The experienced colleagues served as a second pair of eyes and noted particular instructional activities warranting discussion. Case 1 (Carl and Dr. Bethune) was more structured than the others in this process because Dr. Bethune required Carl to present only one focus area for an observation cycle. His observation lens and feedback were strictly confined to this focus area. Feedback sessions usually occurred within a day of an observation; however, in a few cases the time in between was slightly longer. I used participants’ dialogue and written reflection as sources of data. I created data blocks from the transcripts of all the pair interactions and interviews as well as the new instructors’ written reflections. I created thematic categories and assigned appropriate corresponding mathematical knowledge for teaching (MKT) (Ball et al., 2008) and mathematical understanding for secondary teaching (MUST) (Kilpatrick et al., 2015) labels to each block. To capture emerging themes throughout the data, I arranged the data by

prevalence and frequency. Finally, I used the major MUST categories to capture overarching themes representing the mathematical areas or mathematics instructional practices most noticed and addressed during the feedback sessions.

## **Research Questions**

My research question was the following: In what ways do experienced colleagues' observations and feedback influence how new instructors reflect upon and plan to modify their mathematics teaching practices? The supporting questions and answers are provided below:

(a) What mathematical areas or mathematics instructional practices do experienced colleagues notice and address in their feedback to new instructors?

### **Answer:**

#### **Mathematical representation and creating:**

- using correct notation
- representing mathematical ideas and notations
- choosing effective examples
- making meaning of mathematics
- conveying concepts in multiple ways

#### **Knowing and using the curriculum:**

- connecting across the curriculum
- preparing students for upcoming topics within the course and beyond

#### **Student learning:**

- assessing student progress and understanding
- questioning and wait-time
- anticipating student responses
- persevering through student questions
- emphasizing independent thinking and learning
- student motivation and engagement

#### **Mathematical reasoning:**

- modeling problem-solving
- solving extensions of problems
- creating extensions of problems
- investigating generalizations and special cases of problems

**Reflecting on practice:**

- lesson planning
- timing and pacing of a lesson
- sequencing instruction
- recovering from instructional shortfalls
- gauging assessment instruments classroom management

(b) In what ways do new instructors describe how they plan to modify and/or begin to modify their mathematics instructional practices based on experienced colleagues' feedback?

**Answer:**

The experienced colleagues either discussed strategies for improving in these areas by sharing their own experiences or allowing the new instructors to brainstorm about ways to adjust their own practices. The new instructors were receptive and in many cases expected the feedback. They discussed being more intentional in preparing their instruction in ways to improve in these areas.

The following are the most prevalent modifications the new instructors planned to make:

- Assessing student progress by having students present their work on the board or through probing questions as they worked
- Choosing most appropriate examples by considering the main objectives for the lesson, nuances or important consequences resulting from variations of fundamental examples, and most common student errors
- Representing mathematical ideas and notation in multiple ways while emphasizing the advantages of particular representations to enhance meaning in various scenarios
- Generating higher level questions requiring critical thinking and allowing more wait-time for more thoughtful student responses
- Anticipating student responses when planning lessons and preparing productive ways to use student responses as launching points for additional learning opportunities
- Modeling problem-solving in a more deliberate and structured manner; however, also being transparent in revealing the inherent thought processes and struggle involved in problem-solving
- Being more intentional in connecting past and future content to current lessons and topics
- Structuring examples and tasks in ways requiring students to use more critical thinking and independent learning

**Comparison with Previous Studies**

These practices are consistent with characteristics identified in previous studies for effective college calculus teaching (Bagley, 2015; Burn et al., 2016; Dawkins, 2014; Sonnert et al., 2015). Seymour's (2006) study identified in calculus courses a lack of discussion of

conceptual material. Participants (Carl and Dr. Bethune in particular) in my study placed high value on requiring students to grapple with concepts and engage in critical thinking to build on foundational concepts. The department's as well as USAFA's institutional emphasis on learner-focused instruction demonstrates efforts to meet Seymour's (2006) recommendation for instructors to engage in more interactive teaching functions other than straight lecturing. The department's well-structured mentoring program also satisfies Seymour's (2006) call for institutions to place greater value on teaching and implement systems of professional development for faculty.

USAFA's mathematics department was similar to departments in Dawkins's (2014) study: they strongly coordinated the teaching of calculus by having a common text, common examinations, and shared homework lists. Their blended reform/traditional instruction improved students' problem-solving was also similar to USAFA's mathematics new instructors' practices. The new instructors' emphasis on representing mathematical ideas and notation in various ways also illustrates answers Dawkins's (2014) suggestion to be multi-representational in calculus courses.

Sonnert et al. (2015) identified the following good teaching practices: asking questions to check students' understanding, listening carefully to students' questions, acting as if students were capable of understanding calculus, and providing understandable explanations of key ideas. The participants in my study emphasized these behaviors as a means to access and informally assess student understanding—enabling instructors to adjust instruction in the moment and future to better facilitate student learning. Bagley (2015) identified small class size as one way to make students feel comfortable asking questions. USAFA's class sizes are small and meet this standard. Bagley (2015) also encouraged instructors to ask students and to keep them involved.

A prevailing theme among the instructors at USAFA was effective questioning. USAFA's overall priority to have high levels of student engagement, especially in collaborative activities, also meets Bagley's (2015) suggestion for increased opportunities for student learning: opportunity to work problems with other students in class and working with the instructor during active-learning activities instructor. The new instructors were all committed to circulating the room to address group work as well as individual issues—allowing the instructors to scaffold and support students' learning.

Burn et al. (2016) identified high-quality instruction and academic and social support for students the following factors jointly contributing to Calculus 1 program success. They touted a culture of collegiality where faculty members were trusted by their colleagues, administration and given autonomy to do the best for their students, and informal peer communication and collaboration was the main mechanism for professional support. This culture definitely existed in USAFA's mathematics department. Effective instructors in Burn et al.'s study (2016) were described as being approachable and available, possessing abundant content knowledge, having high expectations for students' mathematical learning, created opportunities for students to practice skills, and used substantial amounts of questions. USAFA instructors must be available between the hours of 7:30 am to 3:30 pm for office hours (this is called extra instructional time at USAFA) when they are not teaching or have other mandatory duties. All military instructors had degrees in areas requiring advanced mathematical skills and knowledge. They also worked in professional fields for many years directly applying these skills; they brought a wealth of content knowledge as well as innovative application ideas to the classroom. High expectations are a hallmark of USAFA as an institution; this expectation is carried over into the classroom. USAFA instructors incorporated activities to requiring students to present work at the board or

apply skills in application based activities. A prevailing theme among the instructors at USAFA was effective questioning.

This study also contributes information not often discussed in collegiate mathematics teaching research. It focuses on collaborative reflective practices born out of experienced colleague observations of instruction. Previous collegiate studies (Breen, McCluskey, Meehan, O'Donovan, & O'Shea, 2014; Colgan & DeLong, 2015; Jaworski & Matthews, 2011; McAlpine & Weston, 2000) reported the benefits of reflective practice as a productive professional development activity to stimulate discourse among colleagues about mathematics instruction; however, there was no real-time observation of classroom interactions. The studies also involved experienced mathematics teachers or mathematicians as opposed to new instructors. The studies suggested reflections of more experienced professors are typically more critical and provide more informed perspectives than reflections of novice teachers. My study capitalized on this finding by using the expertise of experienced colleagues to reflect in partnership with new instructors. This developmental activity also answers Seymour's (2006) call for institutions to place greater value on teaching, and implement systems of professional development for faculty.

McAlpine and Weston (2000) found professors relied on pedagogical content knowledge as well as knowledge of learners to guide their monitoring and decision-making when attending to students. They advocated for further studies exploring how reflection on teaching affects students' learning experiences. Breen et al. (2014) identified emerging themes of promoting student engagement and gauging student understanding. Colgan and DeLong's (2015) report on teaching polygons also suggested an emphasis on pedagogical methods enhancing student learning. My study found high prevalence and frequency among participants for instructional

practices targeting student learning and understanding. My study's results are therefore consistent with previous findings.

Secondary level studies (Gellert & Gonzalez, 2011; Kensington-Miller, 2012; Posthuma, 2011) added evidence supporting reflective practice and colleague observation of teaching. Posthuma (2011) suggested less experienced teachers' have limited understanding of reflection and limited ability to notice subtle events during instruction. The lesson study context afforded the opportunity for collaborative reflection with more experienced colleagues. Similar to the studies cited above and the present study, teachers also reported greater awareness of learners' understanding and the importance of incorporating more learner-focused activities. Gellert and Gonzalez (2011) demonstrated new teachers have richer reflections in collaboration with experienced colleagues with whom they regularly work and who periodically observe their teaching. The construct of Gellert and Gonzalez's (2011) study was most like mine because it paired new instructors with experienced colleagues. It came close in describing the types of exchanges pairs had during feedback and how the new teachers adjusted instruction; however, this information was given solely from the teacher's perspective. I witnessed, recorded the experienced colleagues' reflections on, and analyzed the content and context of those exchanges. In doing so, I believe my study strengthens implications from previous similar studies lacking those details.

My study achieved more than the previous studies by documenting the classroom activities during the experienced colleague's observation, actual interactions between the pairs, and both the new instructor as well as the mentor colleague's reflections about the classroom activities and feedback sessions. Although past studies have incorporated colleague observation of teaching and collaborative reflection, they fall short of connecting reflection to intended

modifications of instructional practices. My study launched from prior findings by analyzing the content of feedback sessions, identifying specific instructional practices, and revealing new instructors' intended modifications based on feedback from experienced colleagues.

### **Conclusions**

I analyzed experienced colleagues' feedback and the influence on new instructors' intentions to adjust their mathematics instructional practices. I also examined the characteristics of feedback focusing on new instructors' mathematical knowledge, understanding, and instructional implementation. I captured new instructors' intentions to modify as well as their perceived improvements in their mathematics instructional practices. My study highlights a department successfully implementing an effective collaborative professional development activity because its new instructors have constant access to expert perspectives and engage in continual reflection to help them improve their teaching. The characteristics of the department making the program successful include having a culture of collegial relationships to encourage department-wide observations among colleagues with low-threat informality; on-going open dialogue about pedagogy, learning theories, and instructional practices; and a commitment by members at all levels to sustain the program.

By Martin and Double (1998) and Bell's (2002) standards, this department exemplifies a positive environment of collegiality to develop reflective practice and foster improved teaching. This study also offers an interesting perspective about teacher learning as a two-way growth opportunity between colleagues benefitting from collaborative reflection. New instructors relied on the expert perspectives of their mentors. The new instructors also had strong mathematical backgrounds and had used advanced mathematics in their previous professions; therefore, they also possessed expertise of their own. Mentors often gained enlightened perspectives when new

instructors' practices motivated a new a way of thinking about the mentors' own approaches to topics or instruction. This study suggests how to foster meaningful feedback and potentially establish a sustainable, effective form of professional development to improve teachers' mathematics instruction.

### **Limitations**

The participants represented a variety of educational backgrounds and professional experiences; however, two new instructors had at least 2 years of professional experience providing instruction in other classroom settings. They were not purely first-time instructors and did not typically struggle with general classroom management (rapport with students, discipline, movement, etc) issues along with new calculus teacher issues; subsequently, they could devote more reflective energy to the mathematics instruction. The new instructors may also be considered a rather homogeneous group since they resided in a department influencing a substantial amount of standardization in the course. The course was more structured and standardized than at a typical undergraduate institution—giving the instructors very similar resources and approaches.

The new instructors' teaching practices or intentions to adjust were likely equally influenced by observing some of the same instructors in the course—the auditing effect. The opportunity to consistently audit a lesson taught by an experienced teacher prior to teaching the same lesson is a unique component. New instructors had an advantage by witnessing an actual real-time lesson to help them reflect ahead of time on instructional strategies as well as get a preview of students' responses. Most schools do not have a scheduling structure to accommodate this practice. The scheduling at the research site also facilitated plenty of concurrent planning time for faculty to engage. Scheduling is a significant challenge to

collaborative reflection at the secondary level; therefore, the execution of such a developmental practice may not easily translate to the secondary level.

Intentions to modify and actually modifying instruction were subjectively expressed by the new instructors. Although I did make an effort to observe all the new instructors at least two or three times beyond the observation cycle lessons, one semester is not enough time to document actual changes in practice. The duration of this study was too short to prove evidence of change. Any noticeable changes may have been attributed to patterning after those the new teachers audited more than the influence of the mentors' feedback. The reflective component of the study was not as strong as I had intended. Other than the interviews, the only other reflective data captured was one written reflective prompt submitted by only two of the new instructors. Although I am confident the interviews provided solid data, a more frequent written source might have contained more thoughtful reflections because the participants would have had time to collect their thoughts as ongoing reflective practice.

If I had to adjust my methodology to improve the study, I would have included a session at the end of the semester with all the mentors (beyond the four in my study) in the department to discuss observation and feedback strategies of mentors across the department. This is similar to the group feedback session conducted at the end of each semester as described in Atkinson & Bolt's (2010) study. USAFA's mathematics department did not have a particular forum such a discussion. Some mentors said this arrangement might be a good feature to add to the program. Nonetheless, all participants felt this was not a significant void because all mentors possessed a wealth of knowledge and experience to make them effective. Another change I would have made is being onsite at least 1 week before the semester began. I was delayed in arriving because of some commitments before beginning my study. An earlier arrival would have

allowed me to witness more of the observation cycles in person rather than relying on the accounts of the participants for the portions I missed. I was still able to witness at least one full observation cycle for every case.

### **Implications for Further Research**

Because of the unique features of this department and institution, more research at traditional institutions is needed. The sample of new instructors should include people who have formal training for academic careers being nonacademic career professionals placed in a teaching job for a short period of time and not likely to remain in academia. The implications of this study are intended to be useful for career mathematics teachers. Studies with longer duration are also necessary to capture this professional development activity's long-term influence on new teachers' instructional practices. Studies in which mentor colleagues actually use a framework such as MUST as their observational lens may also be useful. This study was descriptive without any suggested modifications or treatments. Finally, future studies should do a better job of capturing how new instructors reflect and adjust their teaching practices. Such data could inform the creation of a developmental continuum through which new teachers grow as they improve their practice. Such a construct could potentially influence how to make mentoring and other collaborative reflective efforts more productive.

### **Final Thoughts**

This study and other literature about similar practices strongly support experienced colleague observation and feedback as a rich and productive form of professional development, particularly for new teachers. Teachers benefit in a number of ways by gaining opportunities to engage in reflective dialogue about mathematics and mathematics instruction; expert insight, support, and validation from an experienced colleague; and on-going development of self-

reflective analysis of one's teaching leading to improvement in instructional practice. According to Brown and Smith (1997), good mathematical pedagogy is integral for effective mathematics instruction. A good model of professional development encompasses a cycle of planning, instruction, and reflection with mathematical knowledge and understanding at its core. Having an experienced colleague observe a lesson and give feedback supports teacher learning through instruction and systematic, deliberate, and critical reflection to improve one's teaching. As teachers regularly engage in this supported activity, they develop self-sustaining growth; what Franke et al. (1998) defined as *self-sustaining generative change*. "We learn by doing; we enhance learning by reflecting upon the doing. These reflections impact future classroom experiences as part of the decision-making process and instructional design" (Vidmar, 2006, p. 139). Every day in the classroom is a learning experience for the teacher as well as the students.

Experienced colleague observation of teaching and mentoring provide the opportunity for another set of eyes to reflect on teaching as well as to support the teacher in their practice. The experienced colleague mentor can identify instructional issues not easily apparent to an instructor in the act of teaching. Engaging in collaborative dialogue can help a teacher sift out causes of issues and consider adjustments to make instructional practices more effective. Experienced colleague mentors can also recognize and validate good practices while reinforcing the use of effective instructional strategies (Kedzior & Fifield, 2004; Zachary, 2000). Engaging in fruitful feedback and focused reflection is a worthwhile professional development activity because it fosters collegiality, builds trust, and creates a safe environment for ideas to be exchanged and challenged. It is also sustainable because it is site-based, on-going, accessible, inclusive, and part of a teacher's everyday practice.

Rogers et al. (2007) presented a summary of effective professional development criteria:

(a) direct application to the classroom; (b) involve teachers in various roles as a learner (modeling research-based teaching, reflecting on instructional practice, engaging in mathematical tasks in the role as students); (c) cultivate collegiality; and (d) emphasize improvement of teachers' content knowledge and ability to help students understand the subject matter. Experienced colleague observation of teaching coupled with quality feedback captures all of these elements. Establishing a culture in which teachers take the lead in their own development gives them a greater sense of being a part of a professional community. Colleagues should own the process and maintain it as an informal developmental tool for progressive improvement of instructional practice.

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## **Appendix A – Interview Guide: Colleague Observer**

Please reflect on your approach as the Observer. That is, think about how you made decisions concerning how and what you focused on, what you saw, and your reactions to what you observed. Reflect on the feedback session you had with the instructor you observed. Think about how you approached giving him/her feedback based on what you observed. Use the following questions as a guide.

1. Walk me through your observation process from beginning to end.
  - Did you meet with the instructor to be observed prior to the observation? If so, what was discussed?
  - As you prepared for your observation, what things did you consider regarding what you wanted to focus on?
  - Did you make any sort of checklist prior to the observation? If so, did you follow it or deviate from it? Why?
  - What things did you make note of?
  - Are you able to categorize things you observed? What are your categories?
2. Describe the mathematics instruction you observed.
3. Tell me about the mathematics content covered in the lesson.
4. Walk me through how you gave feedback.
  - What notes did you make regarding mathematics and mathematics instructional practices?
  - Describe the types of feedback you gave.
5. Compare this observation to any prior observations with this instructor (changes in approaches, knowledge, improvements, etc)
6. Is there anything else you would like to add or share?

### **Appendix B – Interview Guide: Observed New Instructor**

Please reflect on your thoughts going into the observation. That is, think about how you felt going into the observation and you what you expected from being observed. Reflect on the feedback session you had with the observer. Think about how you feel the feedback session went and what you learned. Finally think about how you would apply what you learned to your own practices in your classroom. Use the following questions as a guide.

1. Walk me through your observation process from beginning to end.
  - Did you meet with your observer prior to the observation? If so, what was discussed?
  - As you prepared for your observation, did you want and/or express to the observer to focus on any particular areas? If any, what were they and why?
  - Did you experience any intimidation about being observed? Explain.
  - Did you make any particular notes of things to do because you were being observed? Why/why not? Describe.
2. Describe the mathematics instruction you gave.
3. Tell me about the mathematics content covered in the lesson.
4. Walk me through your feedback session.
5. Describe your observer's approach to giving you feedback.
  - Do you feel your observer was completely candid while giving feedback? Why or why not?
  - How well did the observer match what you perceived about things which were positive and negative?
  - Describe the types of feedback your observer gave you.
  - What type(s) of feedback from your observer were most useful to you? Explain why.
6. What did you learn from the feedback session? What was most helpful?
7. What instructional practices do you plan to adjust or implement based on the feedback you received?
8. Compare this observation & feedback to any prior observations with this colleague (changes in approaches, knowledge, improvements, etc)
9. Is there anything else you would like to add or share?

### **Appendix C – Exit Interview Guide**

Please reflect on all of your observations throughout the semester and the feedback sessions you had. Also use your journal as a memory jogger. Think about what types of feedback have been most effective for you and in helping you evolve as an instructor in the ways you implement instructional practices. Use the following questions as a guide.

1. What do you find to be most valuable about observations and feedback sessions?
2. What types of feedback are most useful to you? Why?
3. What mathematics instructional practices do you intend to/have adjust(ed)? Describe how.
4. Describe any adjustments you made to your instruction and the result.
5. What suggestions do you have to improve colleague observations and feedback sessions?
6. Is there anything else you would like to add or share?

### **Appendix D – Reflection Prompt**

**DIRECTIONS:** Please respond to the following journal reflection prompt in any manner best facilitating an authentic and thoughtful response. Submit your response in the most convenient way to you. Here are some options:

- A. In writing via email or any Word/Text document
- B. As an audio/voice file (e.g., voice recorder, I-Phone, etc.)

### **JOURNAL PROMPT**

You've been observed by and received feedback from your mentor at least once this semester.

You've probably also had a few impromptu, informal discussions with your mentor independent of an observation. Please take some time to reflect on your teaching as a result of these experiences with your mentor. Discuss the following topics using specific examples:

- **Describe any feedback/discussions which have impacted the way you approach teaching calculus.**

**Based on any feedback/discussion with your mentor:**

- **Describe any adjustments to your teaching**
  - a. **You have made and the effect (how it went/how's it going?)**
  - b. **You intend to try in the future; why?**

## Appendix E – Calculus 2 Syllabus

### Math 152 Syllabus: Fall 2014 (Stewart Early Transcendentals)

	Lsn	Section	Topic(s)	Lesson Objectives	Required HW	Additional Problems	Graded Event
Homework Set 1 - Estimating Integrals	1	4.9	Antiderivatives	Calculate antiderivatives of a function as a family of functions Calculate antiderivatives of functions given in first column of table on page 345 along with $a^x$ Use antidifferentiation to calculate velocity from acceleration and position from velocity while applying initial conditions Graph an antiderivative, $F(x)$ , given the rate of change function and a particular value of $F(x)$ Identify the critical points and inflection points of an antiderivative, $F(x)$ , given a graph of the rate of change function, $f(x)$	23, 25	3,5,7,8,10,11,13,14,20,29,40,41,51,52,65,75	
	2	5.1	Areas and Distances	Explain how to interpret area between the curve and the axis as the limit of a sum of areas of thin rectangles Use sigma notation to express Riemann sums (left, right, general) and be able to calculate it Determine upper and lower Riemann sums to bound area estimates Estimate distance travelled given velocity and time data	2,13	5,8,14-18,25	
	3	5.2	The Definite Integral	State the definition of the definite integral Express the relationship between the definite integral and Riemann sums Estimate definite integral given various areas relative to curves Use the Midpoint Rule to estimate definite integrals Use properties of definite integrals to estimate various combinations of definite integrals	5,8	6,7,9,11,17,18,26,34,43,47,49,52	
	4	7.7	Approximate Integration	Use Simpson's Rule to estimate definite integrals Use the Trapezoid Rule to estimate definite integrals	2,11	1,10,30,31,37,40, 43	
	5	5.3	The Fundamental Theorem of Calculus	Explain the Fundamental Theorem of Calculus, Parts 1 & 2, in the context of a real world scenario Apply the FTC to evaluate definite integrals	2,21,37	3,12,20-26,29,36,37,40,43,45,46,53,54,67,68	Deriv Review
Homework Set 2 - Integration Techniques	6	5.4	Indefinite Integrals and the Net Change Theorem	Compute the total change of a function from its rate of change function by applying the Net Change Theorem, to include applications Explain the difference between indefinite and definite integrals	49,51	20,21,24,28,31,52,53,54,58,63,64,65, 66,70	HW Set 1 and FSPA 1/FSQ1
	7	5.5	The Substitution Rule	Recognize when the substitution rule is an appropriate method of evaluating an integral Show that the substitution rule is "undoing" chain rule derivatives Evaluate a given integral (definite or indefinite) using the substitution rule Apply properties of symmetric functions (even or odd) to calculate definite integrals	3, 17, 33	5,7,8,9,13,18,22,28,53,59,80,82,85,86	

### Appendix E – Calculus 2 Syllabus cont.

Homework Set 2 - Integration Techniques	8	6.1	Areas Between Curves	(For 6.1-6.5,8.1,8.3) Understand main concepts: a. Divide an object into thin slices b. Calculate the quantity of interest (i.e. area, volume, force, work, etc) for one slice c. Add up the results from step b for all slices using Riemann sums d. Take the limit of the Riemann sum as number of slices go to infinity to obtain an integral Use integration to calculate the area between curves Recognize when to define regions by regarding x as a function of y	13,29	1,2,3,7,8,30	
	9	6.2	Volumes	Use the volume formula to calculate volumes using integration Calculate volumes of revolution using the disk and washer methods	7, 47	1,2,5,10,21,23,40,41,69	
	10	6.3	Volumes by Cylindrical Shells	Identify when using cylindrical shells is appropriate to a given volume problem Use the method of cylindrical shells to calculate appropriate volumes	11,47	2,3,5,9,13,20	FSPA 2/FSQ2
	11	GR #1		Demonstrate proficiency on material from sections 4.9, 5.1-5.5, 6.1-6.3, 7.7			HW Set 2 and WW 1
Homework Set 3 - Applying Integrals	12	7.1	Integration by Parts	Show that Integration by Parts (IBP) is derived from reversing product rule derivatives Recognize when IBP is an appropriate method for evaluating an integral and be able to evaluate integrals using IBP Evaluate a given integral using IBP in context of real world scenario	5,12,67	3,4,6,26,66	
	13	7.4	Integration of Rational Functions by Partial Fractions	Use partial fractions to divide a proper rational function into a sum of partial fractions Recognize when integration of rational functions by partial fractions is an appropriate method, using long division if necessary Evaluate integrals by using partial fractions and properties of integration	1,7,19	2,8,12,17,66	
	14	7.8	Improper Integrals	Determine if an integral is improper and justify why Rewrite an improper integral as a limit of proper integrals and evaluate Calculate whether an improper integral converges or diverges and justify	5, 13	1,2,7,18,28,42,68	
	15	6.4 /6.5	Work/Average Value of a Function	Compute work done by a constant force over a constant distance Identify when an integral is required to calculate work Construct and compute a definite integral representing the work done lifting objects by applying the first objective of lesson 8	Sec 6.4: 15,16,19,21 Sec 6.5: 7,9	Sec 6.4: 3-5,9,15,17,20,30 Sec 6.5: 2,16,18	FSPA 3/FSQ3 (L15)
	16			Construct and compute a definite integral representing the work done pumping liquids Use an integral to calculate the average value of a function over a closed interval			

## Appendix E – Calculus 2 Syllabus cont.

Homework Set 3 - Applying	17	8.3	Applications to Physics and Engineering	Compute hydrostatic pressure and force at a fixed depth	1,3,7,14	4,15,26	
	18			Identify when an integral is required to calculate hydrostatic force Construct and compute a definite integral representing hydrostatic force on an object by applying the first objective of lesson 8			
Homework Set 4 - Differential Equations	19	9.1/9.2	Modeling with Differential Equations/Direction Fields	Recognize what makes an equation a differential equation Calculate the general solution and find particular solution to a differential equation by applying given condition(s) Determine if a function is a solution to a differential equation and/or initial condition Determine the nature of the solution from the slope given in the differential equation Calculate equilibrium solutions for differential equations and describe their meaning in context of a given problem	3,5	4,9,10,14	HW Set 3
	20	9.3	Separable Equations	Determine if a differential equation is separable Solve separable differential equations by using separation of variables and applying initial conditions Model and solve mixing problems using separable differential equations	2,11,43,46	3,5,10,19,21,46	FSPA 4/FSQ4 (L20)
	21			Formulate population models using differential equations and solve for given conditions			
	22	9.4	Models for Population Growth	Translate statements involving rates into differential equation models and solve model applying given conditions	3,9	10	
	23	GR#2		Demonstrate proficiency on material from sections 7.1,7.4, 7.8,6.4,6.5,8.3,9.1, 9.3			HW Set 4 and WW 2
Homework Set 5 - Series/Convergence Tests	24	11.1	Sequences	Explain what it means for a sequence to converge or diverge Compute the limit of a convergent sequence List terms of a sequence given the definition of the sequence (recursive sequences as well)	17,25	1,3,5,9,11,15,28,65,67	
	25	11.2	Series	Explain what it means for a series to converge or diverge Describe the difference between a sequence and a series Calculate the partial sum of a series Recognize a geometric series and compute the sums of both finite and infinite geometric series Model real world scenario using series Apply the test for divergence to determine if a series is divergent	17,27	1,3,4,18,28,57,69,71	
	26	FSE		Demonstrate Proficiency on Fundamental Integration Skills			
	27	11.7/11.3	Strategy for Testing Series/The Integral Test and Estimates of Sums	Identify appropriate convergence test for a given series Recognize when to apply the integral test for convergence and use the test to show convergence or divergence of appropriate series Know the definition of a p-series and be able to demonstrate when the p-series converges or diverges	Sec 11.3: 7,15	Sec 11.3: 1,16,27 Sec 11.7: 1-6	

### Appendix E – Calculus 2 Syllabus cont.

Homework Set 6 - Taylor Series/Power Series	28	11.4/11.5	The Comparison Tests/Alternating Series	Understand difference between Comparison and Limit Comparison Tests	Sec 11.4: 1,5,17 Sec 11.5: 1,11	Sec 11.4: 7,28,37 Sec 11.5: 3,9,23	
				Use Comparison, Limit Comparison and Alternating Series Tests to show convergence/divergence of series			
				Explain why 'the tail' of the series determines convergence/divergence			
	29	11.6	Absolute Convergence and the Ratio and Root Tests	Explain what it means for a series to be absolutely convergent	1,7,35	8,12,19	HW Set 5
				Recognize how and when to apply the Ratio and Root Tests			
				Use Ratio and Root Tests to show convergence/divergence of series			
	30	11.8/11.9	Power Series/Representation of Functions as Power Series	Identify what makes a series a power series	Sec 11.8: 5,15 Sec 11.9: 5,13	Sec 11.8: 2,7,10,33 Sec 11.9: 3,6,7,11	
				Calculate interval and radius of convergence for given series			
				When applicable, represent certain functions by power series			
				Use differentiation, integration, and known power series to calculate power series expansions of given functions			
	31	11.10	Taylor and Maclaurin Series	Calculate the coefficients for a power series expansion of a function	9,13	3,7,15,17,39,44	Project Due
				Understand that a Maclaurin series is a Taylor series centered at zero			
				Explain and calculate Taylor series expansions for a given function			
	32	11.11	Applications of Taylor Polynomials	Describe the relationship between the nth-degree Taylor polynomial of a function and the Taylor series expansion of the function	13a, 31	5,8,23,25	
				Know how to approximate functions by polynomials			
Use Taylor polynomials to estimate functions in context of modeling scenarios							
33		GR #3	Demonstrate proficiency on material from sections 11.1-11.11			HW Set 6 and WW 3	

## Appendix E – Calculus 2 Syllabus cont.

Homework Set 7 - Parametric Equations and Vector Operations	34	10.1	Curves Defined by Parametric Equations	Sketch a curve identified by parametric equations	7, 11	2,9,14,19,20,46a	
				Find parametric equations for given Cartesian curves			
				Find a Cartesian equation of a curve from parametric equations			
				Describe the motion of a particle with position (x,y) as the parameter t varies given equations for x and y in terms of t			
				Apply parametric equations to real world situations			
	35	8.1/10.2	Arc Length/Calculus with Parametric Curves	Calculate the length of a curve defined either by parametric or Cartesian equations	Sec 8.1: 3,7 Sec 10.2: 5,41	Sec 8.1: 6,11,19,39 Sec 10.2: 1,3,13,45	
				Calculate slopes and equations of tangent lines to curves defined by parametric equations			
	36	10.3	Polar Coordinates	Describe points in two space using polar coordinates	6,33	3,4,11,12,28,40	
				Convert points in polar coordinates to Cartesian coordinates and vice versa			
				Sketch curves or regions in a plane given in polar coordinates			
				Recognize when polar coordinates are more useful than Cartesian coordinates			
	37	10.4	Areas and Lengths in Polar Coordinates	Calculate areas enclosed by polar curves using polar coordinates	1,45	2,5,9,10,47	
				Calculate arc length of curves defined in polar coordinates			
	38	12.1/12.2	Three-Dimensional Coordinate Systems/Vectors	Explain how to locate points in three dimensions	Sec 12.1: 10,33 Sec 12.2: 5,19	Sec 12.1: 4,6,27,35,37 Sec 12.2: 3,5,17,24,25,30,31	
				Describe and sketch three-dimensional surfaces/regions from equations			
				Determine distances in three dimensions			
				Describe equations of spheres and calculate the center of the given sphere			
				Apply the definition of vectors, vector addition, and scalar multiplication			
				Given two points, determine the vector from one point to the other			
				Calculate the magnitude of a given vector			
				Explain the concept of basis vectors (i,j,k) and unit vectors; calculate a unit vector in same direction of given vector			
				Determine resultant forces using vectors and apply to application problems			

### Appendix E – Calculus 2 Syllabus cont.

Homework Set 7 - Parametric Equations and Vector Operations	39	12.3	The Dot Product	Calculate the dot product of two vectors, understanding that the result is a scalar	7, 17, 23	3, 5, 18, 24, 39, 49, 50	Block Quiz
				Use the dot product to calculate the angle between two vectors			
				Use the dot product to determine if two vectors are orthogonal			
				Calculate scalar projections and the vector projection of one vector onto another			
				Use dot product properties to solve work problems when the applied force is not in direction of distance travelled			
	40	12.4	The Cross Product / Review	Calculate the cross product of two vectors and understand that the result is a vector	14, 17, 39	3, 5, 15, 18, 19, 35	HW Set 7 and WW 4
				Use $\mathbf{a} \times \mathbf{b}$ to calculate an orthogonal vector to both $\mathbf{a}$ and $\mathbf{b}$ and determine its general direction			
				Use the Cross Product to calculate torque			

## Appendix F – Calculus 2 Notes to Instructor for Lesson 4

Math 152 Fall 2014 NTI

Lesson 4 (21, 22 August)

Section	Topic(s)	Lesson Objectives	Required HW	Additional Work	Graded Event
7.7	Approximate Integration	Use Simpson's Rule to estimate definite integrals	2,11	1,10,30,31, 37,40, 43	
		Use the Trapezoid Rule to estimate definite integrals			
		Explain when Trapezoid and Midpoint rules give overestimates or underestimates and determine numerical upper and lower bounds for various definite integrals			

### Admin

- Derivative Review due 0600 25/26 August (M4 / T4)

### Lesson Suggestions

- Students have been introduced to midpoint rule previous lesson. Expand during the review to add the trapezoid rule (graphically and as combination of LHS and RHS) to whatever review you do in the first 15 minutes. CalcTool is very useful again.
- Introduce Simpson's rule with the basic idea but don't go crazy into the detail

### Key Points

- Any data can be integrated numerically but the least error you can achieve is restricted by the nature of the data
- Concavity determines whether Mid/Trap rules are over/under estimates, increasing/decreasing for LHS, RHS
- By splitting integrals we can determine upper and lower bounds for our integral with combination of LHS/RHS/Mid/Trap
- Project will be along the lines of this class - estimating integrals from raw data

## Appendix F – Calculus 2 Notes to Instructor for Lesson 4 cont.

### Common Problems

- Cadet will try to do midpoint rule by interpolating data at a midpoint if not given and not realize this is simply the trapezoid rule

### Suggested Problems for class

- 10 is a good problem to tie in Mid/Trap and Simpson's but make sure to point out that the book has an error in the integral since  $dx$  has been omitted. Have them write out the solutions in terms of the function evaluation and have a discussion as to the error limits for each rule.
- 31 is a good problem with tabular data

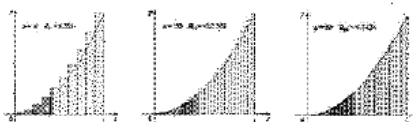
### Extras

- In many cases we can determine the magnitude to the error for our estimates. LHS and RHS are easy if the function is strictly increasing or decreasing. The error bounds for Mid/Trap/Simpson's are in the text. I wouldn't spend too much time other than pointing out these formulas to the cadets and noting that in general Simpson's is better than Mid which is better than Trap.

## Appendix G – Carl's Presentation for Lesson 4

### Definition of Definite Integral

- The area beneath a curve can be approximated to within any desired degree of accuracy by a Riemann sum.



- Suppose  $f$  is continuous for  $a \leq t \leq b$ . The **definite integral** of  $f$  from  $a$  to  $b$  is written as

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$

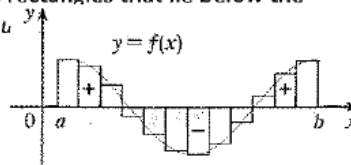
Limits of Integration       $\int_a^b$        $f(x)$        $dx$        $\lim_{n \rightarrow \infty}$        $\sum_{i=1}^n$        $f(x_i^*)$        $\Delta x$

Integrand

- The **definite integral** is the limit of the left-hand or right-hand sums with  $n$  subdivisions of  $a \leq t \leq b$  as  $n$  goes to infinity.

### Definite Integral - Riemann Sums

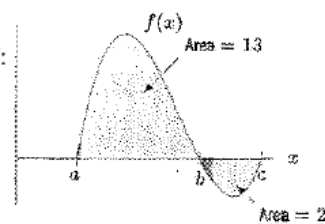
- If  $f$  takes on both positive and negative values, as in the Figure, then the Riemann sum is the sum of the areas of the rectangles that lie above the  $x$ -axis and the *negatives* of the areas of the rectangles that lie below the  $x$ -axis (the areas of the blue rectangles *minus* the areas of the red rectangles).



- A definite integral can be interpreted as a **net area**, that is, a difference of areas.

- Example: Use the figure to find the values of:

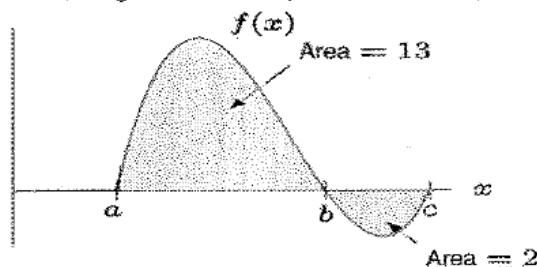
(a)  $\int_a^b f(x) dx$     (b)  $\int_b^c f(x) dx$   
 (c)  $\int_a^c f(x) dx$     (d)  $\int_a^c |f(x)| dx$



## Appendix G – Carl's Presentation for Lesson 4 cont.

## Definite Integral

- A definite integral can be interpreted as a **net area** or the difference of areas



- Calculate the following definite integrals:

$$\int_a^b f(x) dx$$

$$\int_a^c f(x) dx$$

$$\int_b^c f(x) dx$$

$$\int_a^c |f(x)| dx$$

3

## Integral Properties

$$\int_a^b [f(x) \mp g(x)] dx = \int_a^b f(x) dx \mp \int_a^b g(x) dx$$

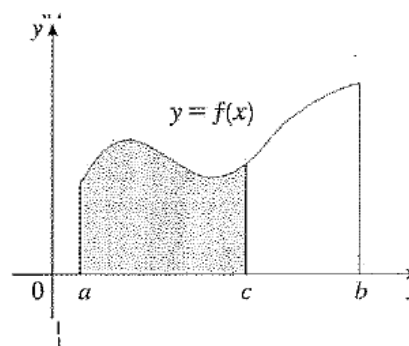
$$\int_a^b c f(x) dx = c \int_a^b f(x) dx$$

$$\int_a^b c dx = c(b - a)$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^a f(x) dx = 0$$

$$\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx$$



4

## Appendix G – Carl's Presentation for Lesson 4 cont.

## Examples

47. Write as a single integral in the form  $\int_a^b f(x) dx$ :

$$\int_{-2}^2 f(x) dx + \int_2^5 f(x) dx - \int_{-2}^{-1} f(x) dx$$

49. If  $\int_0^9 f(x) dx = 37$  and  $\int_0^9 g(x) dx = 16$ , find  $\int_0^9 [2f(x) + 3g(x)] dx$ .

5

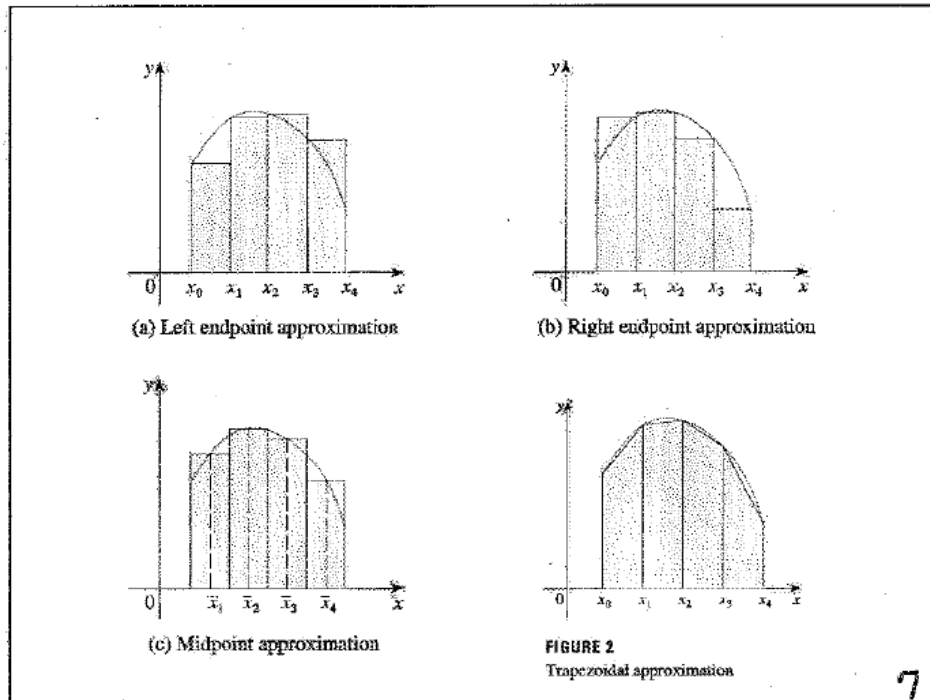
## Math 152 – Lesson 4

Lesson	Topic	Reading	Homework
M3	The Definite Integral	5.2	5,8
M4	Approximate Integration	7.7	2,11
M5	The Fundamental Theorem of Calculus	5.3	2,21,37

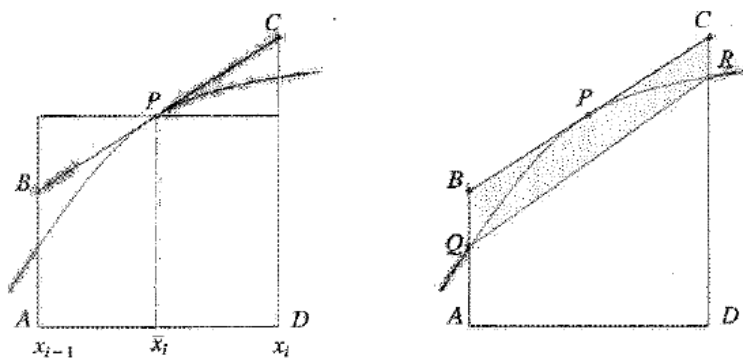
Admin	
25 Aug	Derivative Review (WeBWork)
27 Aug	FSQ#1/FSPA #1 (WeBWork)
27 Aug	HW Set #1 (1.1 - 1.5)

6

# Appendix G – Carl's Presentation for Lesson 4 cont.

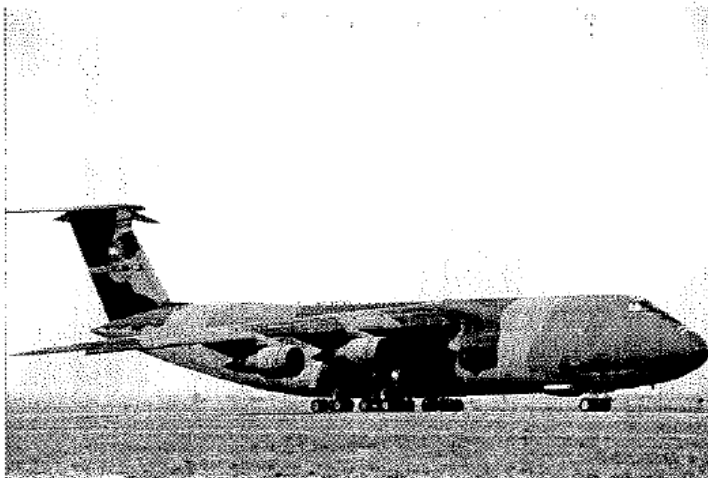


## Midpoint vs Trapezoid



# Appendix G – Carl's Presentation for Lesson 4 cont.

## Takeoff Roll Example



9

- **C-5 on Takeoff Roll**
- -- Aircraft's velocity given by:

t, (sec)	0	7	14	21	28	35	42
v, (ft/sec)	0	41	78	113	145	173	200

- Aircraft starts takeoff roll from the beginning of the runway
- Runway is 5200 feet long
- At 42 sec aircraft takes off
- Aircraft velocity is increasing

Determine:

- Lower estimate for total distance traveled during first 42 seconds of the takeoff roll.
- Upper estimate for the total distance traveled during first 42 seconds of the takeoff roll.
- Did aircraft takeoff before reaching the end of the runway?

10

### Appendix G – Carl's Presentation for Lesson 4 cont.

## Answers

(a)  $(0*7)+(41*7)+(78*7)+(113*7)+(145*7)+(173*7) = 3850$  ft

(b)  $(41*7)+(78*7)+(113*7)+(145*7)+(173*7)+(200*7) = 5250$  ft

(c) Can't tell. At 42 seconds, the aircraft has travelled somewhere between 3850 ft and 5250 ft.

11

- **C-5 on Takeoff Roll**
- -- Aircraft's velocity given by:

t, (sec)	0	7	14	21	28	35	42
v, (ft/sec)	0	41	78	113	145	173	200

- Aircraft starts takeoff roll from the beginning of the runway
- Runway is 5200 feet long
- At 42 sec aircraft takes off
- Aircraft velocity is increasing
- Aircraft velocity is concave down

Determine:

- Best lower estimate for total distance traveled during first 42 seconds of the takeoff roll.
- Best upper estimate for the total distance traveled during first 42 seconds of the takeoff roll.
- Did aircraft takeoff before reaching the end of the runway?

12

# Appendix G – Carl's Presentation for Lesson 4 cont.

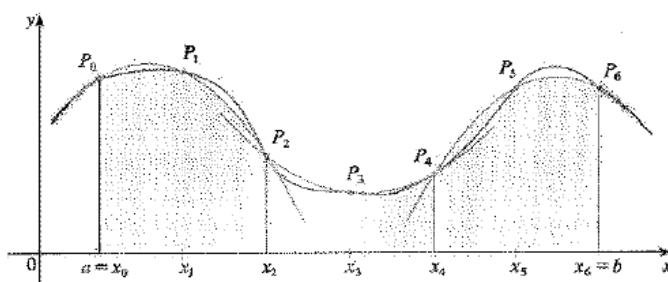
## Answers

$L_6 = 7 * (0+41+78+113+145+173) = 3850$  ft (underestimate since  $v(t)$  is increasing)  
 $R_6 = 7 * (41+78+113+145+173+200) = 5250$  ft (overestimate since  $v(t)$  is increasing)  
 $M_3 = 14 * (41+113+173) = 4578$  ft (overestimate since  $v(t)$  is concave down)  
 $T_6 = 7/2 * (0+2*41+2*78+2*113+2*145+2*173+200) = 4550$  ft (underestimate since  $v(t)$  is concave down)  $= \frac{1}{2} L_6 + \frac{1}{2} R_6$   
 $S_6 = 7/3 * (0+4*41+2*78+4*113+2*145+4*173+200) = 4559.33$  ft (don't know if this is an underestimate or an overestimate)  $= 1/3 T_3 + 2/3 M_3$   
 Note that the shortcut formula for  $S_6$  requires that you find  $L_3$  &  $R_3$  to determine  $T_3$

- The best lower bound for takeoff distance is 4550 ft.
- The best upper bound for takeoff distance is 4578 ft.
- Since the runway is 5200 ft, the C-5 will takeoff before running out of runway.

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## Simpson's Rule



14

## Appendix G – Carl's Presentation for Lesson 4 cont.

## HW Problem

40. The table shows values of a force function  $f(x)$ , where  $x$  is measured in meters and  $f(x)$  in newtons. Use Simpson's Rule to estimate the work done by the force in moving an object a distance of 18 m.

$x$	0	3	6	9	12	15	18
$f(x)$	9.8	9.1	8.5	8.0	7.7	7.5	7.4

## Appendix H – Carl's Notes for Lesson 4

## Lesson 4

Review Problem: (NOT FOR T4A)

$$\text{If } \int_0^9 f(x) dx = 37 + \int_0^9 g(x) dx = 16$$

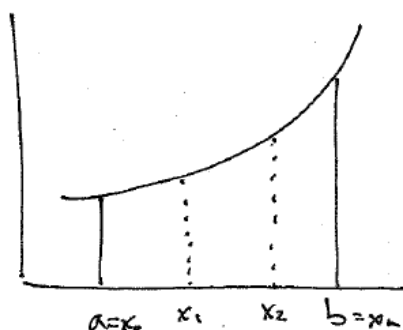
$$\text{find } \int_0^9 2f(x) + 3g(x) dx$$

$$\begin{aligned} \int_0^9 2f(x) + 3g(x) dx &= 2 \int_0^9 f(x) dx + 3 \int_0^9 g(x) dx \\ &= 2(37) + 3(16) \\ &= 122 \end{aligned}$$

$$\int_a^b f(x) dx \approx L_n = \Delta x (f(x_0) + f(x_1) + \dots + f(x_{n-1}))$$

$$R_n = \Delta x (f(x_1) + \dots + f(x_{n-1}) + f(x_n))$$

$$M_n = \Delta x (f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)) \text{ , where } \bar{x}_i = \frac{1}{2}(x_i + x_{i-1})$$



	$f(x)$ conc	$f(x)$ con	$f(x)$ incr	$f(x)$ decr
LHS	—	—	underestimate	overestimate
RHS	—	—	overestimate	underestimate
Mid	underestimate	overestimate	—	—

## Appendix H – Carl's Notes for Lesson 4 cont.

~~the~~ Trapezoid

$$\int_a^b f(x) dx \approx \frac{L_n + R_n}{2} = T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$L_n = \Delta x (f(x_0) + f(x_1) + \dots + f(x_{n-1}))$$

$$R_n = \Delta x (f(x_1) + \dots + f(x_{n-1}) + f(x_n))$$

$$L_n + R_n = \Delta x (f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n))$$

$$\frac{L_n + R_n}{2} = T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$$

SHOW SLIDE

- Complete over + under estimate chart

Simpson's Rule  $\rightarrow n$  must be an even number

$$\int_a^b f(x) dx \approx S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$$S_{2n} = \frac{1}{3} T_n + \frac{2}{3} M_n$$

$$\text{so... } S_6 = \frac{1}{3} T_3 + \frac{2}{3} M_3$$

## Appendix I – Dr. Bethune's Observation Notes for Carl's Lesson 4

TIMING		what is best? 3 intervals or 6 intervals	How to differentiate two trapezoids	CONCISE EXAMPLES WERE BOTH INC & CORRECT	Not case it is counter-example	Mid method	- Future Areas: redundant questions - Common mistakes anticipating
1330	Review, 3 min to work on GRAPHER						
	Questions: Does anyone want to come up						
1336	Two students at board, what do other students do? Why not all at board?						
1338	Good catch on stopping while					from explaining was writing	
1344	Now LH Riemann & RH Riemann						
1347	$(L+R)/2 \approx T_n$ and finishing table						
1358	Midpoint method vs Trapezoid rule						
1402	SIMPSON'S RULE and ideas/formulas						
1407	Another formula for Simpson's Rule						
1408	BOARD WORK - DIDN'T HAPPEN SINCE PROBLEM WAS DISCUSSED IN PREVIOUS LESSON SO WENT OVER PROBLEM AGAIN						
1413	BOARD WORK						
1418	STOP BOARD WORK						
1423	STOP CLASS						

## Appendix J – Calculus 2 Notes to Instructor for Lesson 24

Math 152 Fall 2014 NTI

Lesson 24 (22, 23 October)

Section	Topic(s)	Lesson Objectives	Required HW	Additional Work	Graded Event
11.1	Sequences	Explain what it means for a sequence to converge or diverge	17, 25	1,3,5,9,11, 15,28,65,6 7	
		Compute the limit of a convergent sequence			
		Calculate bounds for a given sequence			
		List terms of a sequence given the definition of the sequence (recursive sequences as well)			

### Admin

### Lesson Suggestions

- Define a sequence and then give several examples. Emphasize that a sequence is a list of numbers. Give them some sequences and have them try to find a pattern (just pick a simple function for  $n$  to develop examples). I would then discuss the two definitions of limits on page 692. This group can handle a discussion on the second definition with the epsilon. Theorem 3 also brings an interesting discussion between a function and associated sequence. A good illustration of Theorem 3 is  $f(x) = \frac{1}{x}$  with the associated series as the harmonic series. Review properties of limits. Finally, a discussion of increasing/decreasing, monotonic, and boundedness would wrap it up well with the culmination of the Monotonic Sequence Theorem.

### Key Points

- A sequence is a list a numbers with a definite order
- Be able to find whether a sequence converges or diverges
- Be able to list terms in a sequence either by definition or recursively

### Common Problems

- Emphasize that a sequence is a list of numbers as cadets will later confuse sequences with series.
- Cadets struggle with limits and how to compute them so several examples will be needed

### Suggested Problems for class

- The additional problems are good. Sequences with  $a_n = n^2, \frac{1}{2n}, \frac{2}{2-\frac{1}{n}}, \frac{-n}{n^2}$  are decent examples. The power point presentation for this lesson also has several good examples for class.

# Appendix K – Carl's Notes for Lesson 24

## Lesson 29 – Chapter 11.1, Sequences

What is a sequence?

- An ordered list of numbers
  - Different from a set because order matters
  - $\{a_1, a_2, a_3, \dots\}$  where  $a_1$  is the 1<sup>st</sup> term,  $a_n$  is the  $n^{\text{th}}$  term

$$\{a_n\}_{n=1}^{\infty}$$

- Examples:

$$\{1, 3, 5, 7\} \rightarrow \text{odd numbers from 1-7}$$

$$\{2, 4, 6, \dots\} \rightarrow \text{all even numbers}$$

$$\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty} \quad \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots\right\}$$

Explicit

$$\{1, 3, 5, 7\} = \{2n-1\}_{n=1}^4$$

$$a_n = 2n-1$$

Recursive

$$a_1 = 1$$

$$a_n = a_{n-1} + 2, n > 1$$

$$\{2, 4, 6, \dots\} = \{2n\}_{n=1}^{\infty}$$

$$a_n = 2n$$

$$a_1 = 2$$

$$a_n = a_{n-1} + 2, n > 1$$

$$\left\{-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots\right\} = \left\{\frac{(-1)^n}{2^n}\right\}_{n=1}^{\infty}$$

$$a_n = \frac{(-1)^n}{2^n}$$

$$a_1 = -\frac{1}{2}$$

$$a_n = \frac{(-1)^n}{2^n} = \frac{-a_{n-1}}{2}, n > 1$$

$$\{1, 1, 2, 3, 5, 8, 13, 21, \dots\} = ?$$

?

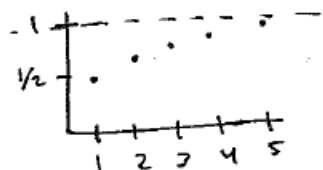
$$\left[ A_n = \frac{(1+\sqrt{5})^{n+1} - (1-\sqrt{5})^{n+1}}{2\sqrt{5}} \right]$$

$$a_1 = a_2 = 1$$

$$a_n = a_{n-2} + a_{n-1}, n \geq 3$$

## Appendix K – Carl's Notes for Lesson 24 cont.

$$a_n = \frac{n}{n+1} \Rightarrow f(x) = \frac{x}{x+1}, x \in \mathbb{Z}$$



$$x - \frac{n}{n+1} = \text{difference} = \frac{1}{n+1}$$

✱ If  $\lim_{x \rightarrow \infty} f(x) = L$  and  $f(n) = a_n$  when  $n$  is an integer, then  $\lim_{n \rightarrow \infty} a_n = L$

$$\lim_{n \rightarrow \infty} \frac{n}{n+1} = \cancel{\frac{n}{n+1}} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{1}{n}} = 1$$

## Appendix K – Carl's Notes for Lesson 24 cont.

$$17) \left\{ \frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6} \right\}$$

$$a_n = \frac{n^2(-1)^{n+1}}{n+1}$$

$$25) a_n = \frac{3+5n^2}{n+n^2}$$

$$\lim_{n \rightarrow \infty} \frac{3+5n^2}{n+n^2} = \lim_{n \rightarrow \infty} \frac{\frac{3}{n^2} + 5}{\frac{1}{n} + 1} = 5$$

$$28) a_n = \frac{3^{n+2}}{5^n}$$

$$\lim_{n \rightarrow \infty} \frac{3^{n+2}}{5^n} = \lim_{n \rightarrow \infty} \frac{\cancel{3^{n+2}}}{\cancel{5^n}} = \lim_{n \rightarrow \infty} \frac{3^n 3^2}{5^n}$$

$$\begin{aligned} & \lim_{n \rightarrow \infty} \left( \frac{3}{5} \right)^n 9 \\ & = 0 \end{aligned}$$

$$30) \lim_{n \rightarrow \infty} \sqrt{\frac{n+1}{9n+1}}$$

$$\begin{aligned} &= \sqrt{\lim_{n \rightarrow \infty} \frac{n+1}{9n+1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{1+\frac{1}{n}}{9+\frac{1}{n}}} \\ &= \sqrt{\frac{1}{9}} \\ &= \frac{1}{3} \end{aligned}$$

## Appendix L – Dr. Bethune's Observation Notes for Carl's Lesson 24

$-\left(\frac{1}{1+\frac{1}{n}} - 1\right) < \epsilon \quad \forall \epsilon > 0 \exists N \forall n > N$   
 $\frac{1}{1+\frac{1}{n}} < \epsilon + 1 \quad \left| \frac{1}{1+\frac{1}{n}} - 1 \right| < \epsilon$   
 $\frac{1}{n} > 2 - \frac{1}{\epsilon + 1} \quad 2 + \frac{1}{n} \quad \text{KEEP CHANGING SERIES SOA WHEN}$

23 OCTOBER T24 Calc II

CHECK ON LEARNING

$n > \frac{1}{\frac{1}{1+\frac{1}{n}} - 1} = \frac{1}{\frac{1}{1+\frac{1}{n}} - 1} = \frac{1}{\frac{1}{1+\frac{1}{n}} - 1}$

- COME SEE ME ABOUT POINTS - HOW DOES IT RELATE TO LEARNING
- NOT RECOGNIZING PARTIAL FRACTION - WHY DID THEY DO THIS
- YOU CAN SEE CONVERSATION WAS ABOUT POINTS NOT LEARNING

YOU NEED TO ANSWER HOW TO GET BETTER

HOW DO YOU STUDY FOR F30, AGAIN DISCUSSION IS ON POINTS

1240 - HAD CONVERSATION ABOUT PRACTICE, BUT WAS HIGH LEVEL

"WORK IT THEN" YOU PUSHED CADOT ON THIS IDEA FROM A LEARNING STANDPOINT BEING SPECIFIC MEANS YOU UNDERSTAND

GOLDEN LEARNING MOMENT for  $a_n = (-1)^n / 2^n$

STUDENT GAVE  $a_n = (a_{n-1})^2 (-1)$  WE QUICKLY MOVED ON LAUGHING ABOUT THE MISTAKE, HOWEVER, WHAT WAS THOUGHT PROCESS

WHY NOT PLOT

"HOW WOULD YOU THINK ABOUT THIS?" WHY NOT

LET THEM DISCUSS AND THEN EXPLAIN

ONE STUDENT JUMPED TO SERIES, WHY? THEY ARE GOING OFF MEMORY, LOST LINKS NOT THINKING 1300, YOU DID PLOT

GOOD JOB LINK READING BOOK TO YOUR DEFINITION IT GENERATED QUESTIONS

CADETS ASKED GOOD QUESTIONS ABOUT GENERALIZING ABOUT THINKING ABOUT SERIES, AGAIN CHANGED TO TAKE

JICE JOBS DISCUSSING  
 IN MEMORIZING  
 RESULTS

ABOUT  
 POINTS

## Appendix M – Calculus 1 Syllabus

MATH 141 - CALCULUS I - FALL 2014 SYLLABUS				
Lsn	Topic	Before Class Introduction	Section and Reading	Objectives
1		Read Course Letter	INTRO	1) Review algebra skills. 2) Familiarize yourself with Math 141 SharePoint, the textbook and online textbook resources. 3) Install graphing application CalcTool.
2	Limits	<a href="#">KA Video Secant Line Slope</a>	2.1	1) Understand relationships between secant and tangent lines and their slopes. 2) Express the concept of a limit with limit notation and with words. 3) Evaluate limits where the input value approaches a finite value.
3		Ex 7 pg. 92	2.2	1) Evaluate infinite limits. 2) Evaluate one-sided limits. 3) Determine relationship between limits and vertical asymptote.
4		Stewart Algebra Review (SAR) Factoring pg. 3 and Radicals pg. 6	2.3	1) Evaluate limits directly, algebraically, numerically and graphically using Limit Laws. 2) Apply simplification techniques of difference of squares and rationalizing the numerator. 3) Recognize direct substitution properties of polynomial and rational functions.
5	Continuity	<a href="#">KA Video Limits and Continuity</a>	2.5	1) Assess the continuity of a function at a point using the limit definition of continuity. 2) Assess the continuity of a function at a point from the left or right using the limit definition of continuity. 3) Explain whether or not a function is continuous over an interval based on the type of function it is.
6	Limits at Infinity		2.6	1) Evaluate limits directly, algebraically, numerically and graphically where the input value approaches infinity or negative infinity. 2) Find the horizontal and vertical asymptotes of a function using limits.
7		Ex 11 pg. 137		
8	Limit Definition of the Derivative	Fig 6 pg. 145	2.7	1) Compute the derivative of a function at a point using the limit definition of the derivative. 2) Estimate the derivative of a function using average rates of change. 3) Interpret the units and meaning of a derivative in the context of a scenario. 4) Explain how the derivative (instantaneous rate of change) is related to the average rate of change.
9				
10	Derivative as Function	<a href="#">KA Practice Visualizing Derivatives</a>	2.8	1) Compute the derivative of a function at a point using the limit definition of the derivative. 2) Determine where a function is differentiable using the limit definition of differentiability. 3) Explain how the differentiability of a function is affected by its continuity. 4) Explain how a function is non-differentiable and how that affects the graph of the derivative. 5) Sketch the graph of a function's derivative given a graph of the function.
11				
12	GR1 Section 2.1-2.3, 2.5-2.8			

### Appendix M – Calculus 1 Syllabus cont.

13	Derivative Shortcuts	Stewart Algebra Review (SAR) Exponents pg. 7-8	3.1/3.3/3.6	1) Determine the derivative of a constant multiple of a function. 2) Determine the derivative of the sum or difference of functions. 3) Determine the derivative of a power function. 4) Determine the derivative of trigonometric functions. 5) Determine the derivative of exponential and logarithmic functions.
14		pg. 188 Note	3.2	1) Determine the derivative of a function using the Product Rule. 2) Determine the derivative of a function using the Quotient Rule. 3) Determine the derivative of a function with parameters.
15		1.3: 31, 37, 43, 47	3.4	1) Decompose complicated functions. 2) Determine the derivative of a function using the Chain Rule.
16	Implicit Differentiation	<a href="#">KA Video Implicit Differentiation</a>	3.5	1) Explain the difference between implicit and explicit functions. 2) Use the chain rule in implicit differentiation. 3) Differentiate an implicit function. Evaluate the derivative at a point (if the point is on the curve). 4) Determine where horizontal tangent lines exist using implicit differentiation. 5) Determine where the slope of the curve is equal to a specific value using implicit differentiation.
17		SAR Arithmetic and Fractions pg. 1-2		
18	Exponential Modeling	1.6: 51, 53	3.8	1) Use the Laws of Exponents to simplify various expressions. 2) Understand and apply the properties of exponential functions. 3) Create an appropriate exponential model for a scenario (growth/decay, heating/cooling). 4) Solve exponential equations for variables or constants using the properties of logarithms. 5) Compute the growth/decay rate, compute half-life/doubling-time.
19		<a href="#">Stewart Video Exponential Model Ex1</a>		

### Appendix M – Calculus 1 Syllabus cont.

20	Related Rates	<a href="#">Stewart Video Related Rates Ex1</a>	3.9	1) Solve a related rates problem by applying implicit differentiation. 2) Apply the chain rule to determine the rate of change of a composition of functions. 3) Explain whether the method of related rates should be applied to a particular problem.
21		Ex 2 pg. 245		
22				
23	Tangent Line Approx.	Fig 1 pg. 250	3.10	1) Create a linear approximation for a function and use it to estimate the output of the function at an input. 2) Explain the process of linearization and why linearization is useful. 3) Compute the output differential of a function given the input differential.
24				
25				
26	GR2 Section 3.1-3.6, 3.8-3.10			
27	L'Hospital's Rule	<a href="#">KA Video l'Hopital's Rule</a>	4.4	1) Determine if a limit is in an indeterminate form. 2) Explain what it means for a limit to be in indeterminate form. 3) Use l'Hospital's Rule to evaluate a limit in an indeterminate form.
28				
29	Absolute Extrema	Fig 3 pg. 274	4.1	1) Use graphs to find local and absolute extrema. 2) Use the first derivative to determine the location of critical numbers. 3) Find the absolute maximum and minimum of a function on a closed interval. 4) Find the absolute maximum and minimum of a function with parameters on a closed interval.
30				
31				
32	Local Extrema	<a href="#">KA Video Inflection Point</a>	4.3	1) Find local maximums and minimums by finding critical numbers and applying an appropriate derivative test. 2) Use the first derivative tests to determine whether a local maximum and minimum occurs at a critical number. 3) Use the first and second derivatives to determine the location of inflection points and critical numbers. 4) Determine properties about a function given information about its first and second derivative. 5) Find the absolute maximum and minimum of a function on an open interval. 6) Find the absolute maximum and minimum of a function with parameters on an open interval.
33		Fig 3 pg. 291		
34				

### Appendix M – Calculus 1 Syllabus cont.

35				FSE
		Thanksgiving Break		
36	Optimization	Ex 1 pg. 326	4.7	1) Create a function and interval that represent a given scenario. 2) Apply appropriate objectives from sections 4.1 and 4.3 to find an optimal solution to a problem. 3) Interpret the solution to a problem in the context of a scenario. 4) Apply the process of optimization on both closed and open intervals.
37		<a href="#">KA Video Optimizing</a>		
38				
39	GR3 Section 4.1, 4.3, 4.4, 4.7			
40	Review			Review
			Final	Sections 2.1-2.3, 2.5-2.8, 3.1-3.6, 3.8-3.9, 3.10, 4.1, 4.3-4.4, 4.7

## Appendix N – Calculus 1 Notes to Instructor for Lessons 5, 15, 16-17

### Lesson 5

#### Continuity (2.5)

- HW Set 1 is due
- Students struggle with the 3-part limit definition of continuity
  - Second condition-limit existing (3 ways limit DNE-review from L3, Section 2.2)
  - Keep with students intuitive definition of continuity and apply to different functions (polynomials, piece-wise, rational functions)
- Theorem 4 and 7 is higher emphasis than Intermediate Value Theorem

Lesson Plan: Fig 1, Definition 1, Ex 1 and Ex 2, Definition 2, Definition 3 as an idea (less concerned if they can apply it), Theorem 4, 5, 7

Suggested In-class Problems: Section 2.5 #4, #11, #19, #39, #44

### Lesson 15

#### Derivative Shortcuts Chain Rule (3.4)

- FS Quiz #1 (first 10 minutes of class)
- Decomposition practice (Section 1.3 pg. 40, Ex 9 pg. 41)
- Chain Rule
  - Practice identifying “inside function” and “outside function”
  - Students often don’t see that the chain rule must be applied because the inside function looks TOO simple. Example:  $f(x) = 7^{3x}$
  - $f(x) = \cos^2(x^3 - 6x)$  Many students have never seen this notation before; rewrite the function as  $f(x) = (\cos(x^3 - 6x))^2$  before taking the derivative. This notation also applies to all trig functions and logarithmic functions.

Lesson Plan: Sect 3.4 Ex 1-7, Suggested In-class Problems: 3.4 #1-6

### Lesson 16-17

#### Implicit Differentiation (3.5)

- Review FS Quiz #1 and common mistakes
- Should understand the difference between  $\frac{d}{dx}$  (like a verb “take the derivative” of what comes next) and  $\frac{dy}{dx}$  (like a noun “the derivative or slope”).
- Encourage the students to write  $\frac{dy}{dx}$  instead of  $y'$  because it’s easy to miss the prime.
- Students find using the product rule on the term  $xy$  very difficult when using implicit differentiation
  - Example: Find  $\frac{dy}{dx}$  for the curve  $x^2 + xy = y^2$
- Students should understand the application of finding  $\frac{dy}{dx}$ ; as an equation where the output is the slope of the tangent line to the function and can be used to find slope at a point if the coordinate point is given and on the function.
- Students should also be able to work backwards, find a coordinate point on the curve when the slope is given. They will need help seeing the original function as a constraint to solve for the coordinate point.
- Students should feel comfortable sketching circles and ellipses, do not need to know shape of other implicit functions.

Lesson Plan: Sect 3.5 Ex 1 & 2

Suggested In-class Problems: Sect 3.5 #5, #8, #11, #27, set-up of #76 or #80

**Appendix O – Calculus 1 Notes to Instructor for Lesson 20**

## Lesson 20

**Review & Related Rates (3.9)**

- Review FS Quiz #2 and common mistakes
- 3 Lessons for this material, you have time to answer derivative practice questions, review implicit differentiation, and answer HW questions before diving into related rates.
- Practice reading word problems and identifying key information, problem-set up, labeling and identifying rates of change as derivatives. Use HW instructions from 3.9 #11-#14 (parts a-e) as structure for in-class problems.
- Problem Solving Strategy on pg. 246

Lesson Plan: Ex 1 (students should have watched **Ex 1 Video** before coming to class)

Suggested In-class Problems: Sect 3.9 **#5** (how radius is NOT changing with respect to time), #8, #12

## Appendix P – Kevin's Notes for Lesson 5

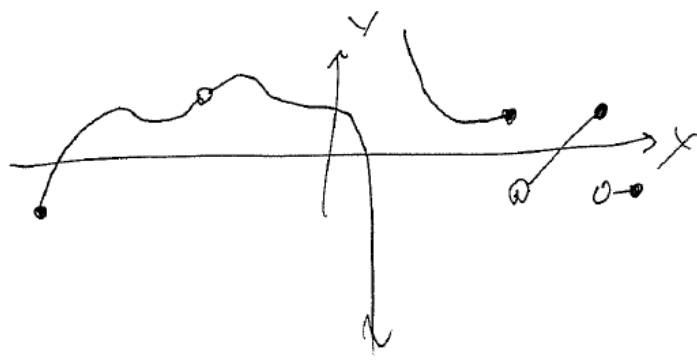
D. S. S. S.

5

Quiz

Rotate to grade

Draw function



Discuss continuity @ interesting points

Def 1 p 118  $\lim_{x \rightarrow a} f(x) = f(a)$ 

Def 7 p 123 continuous in domain

$$\frac{x^2 - 4}{x - 2}$$

lead into trig function

sin/cos

Boardwork: where is sin/cos discontinuous, graph

HW 19, 21, 22

## Appendix Q – Kevin's Quiz: Limit Review

## Quiz Answers

1. Estimate the slope of the tangent line of this function at (1,1).

The slope at (1,1) is -2.

(accept numbers less than -1)

2. Evaluate  $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$

3. True or False: This graph contains two (2) vertical asymptotes.

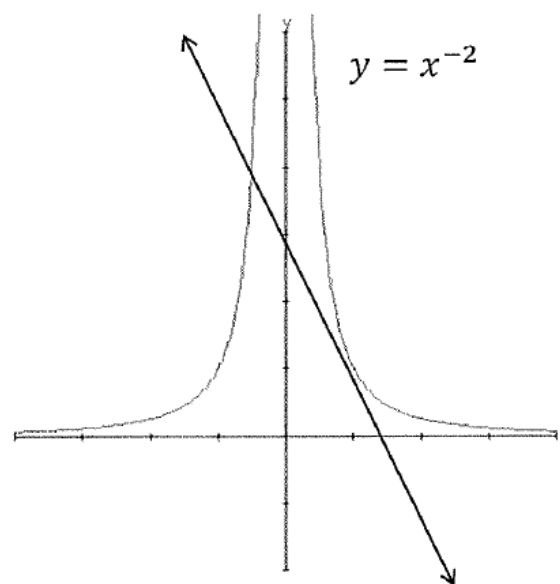
There is one asymptote at  $x=0$

4. Use the limit definition of continuity from the reading to prove that this function is or is not continuous at  $x = 0$ . Any one:

1.  $\lim_{x \rightarrow 0} \frac{1}{x^2} \neq \frac{1}{0^2}$

2. The function  $\frac{1}{0^2}$  DNE.

3.  $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$



## Appendix R – Dr. Ignacio's Observation Notes for Kevin's Lesson 5

- ~~Awesome~~ Awesome start – right to business  
"course letter → quiz." diff: mil vs. civ.  
5:00 quiz.

9:45 segue into topic du jour: Continuity.

Text out; referenced.

To the definition. ← force them to engage with them!

9:50 ∞ discussion – tough question.

*notation  
what do we  
mean?* Good, but maintaining distinction between "∞", "one",  
number, etc. "Behavior vs. value"!

focus on communication, keep at it until solved, ←

general: check back, verify with student that question answered  
→ good!

quibble: on quiz solution:

"The fraction  $\frac{1}{\infty}$  or DNE."  
↑  
value of fact?

9:55 good "note" discussion of quiz – lessons learned.

10:00 interactive notations of continuity.

10:00 "what will we be like?" Ans: look to the homework.

10:05 "lim ~~not~~ DNE"  
 $x \rightarrow 3$  missing fix. fixed. good catch.

# Appendix R – Dr. Ignacio's Observation Notes for Kevin's Lesson 5 cont.

10:07 "holes" problems – good discussion; back to def's again.

10:10 left limit vs. two-sided limit for right endpoint.  
good to include subtleties/limiting cases.

note: is Seta a particular project of yours?  
He seems to have difficulty with some concepts, but  
has a good interaction with you and wants to engage...  
Good patience working with him... good awareness of the  
rest wandering → send to boardwork.

20-22 related notes  
→ watch for little writing tips & problems.

# Appendix S – Dr. Ignacio's Observation Notes for Kevin's Lesson 20

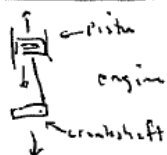
0930 - 0943

Mid-course Feedback

start/stop/continue framework.

0943: Quiz returned. PSQ discussion

0947:



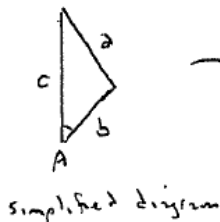
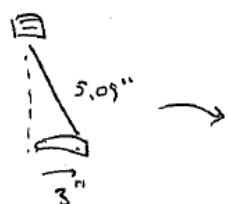
motivation

"How fast is the piston moving if crankshaft is moving at a certain speed?"

Model Solution Method

1. Draw a picture

or:



simplified diagram

groups from students

Law of Cosines

→ to the text: ref R2.

used in this homework

emphasis on communication &amp; consistency: cys for angles, L.C. for lengths,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

back to physical meanings in picture: a, b fixed, c changing:

$$a^2 = b^2 + c(t)^2 - 2bc(t) \cos(A(t))$$

## Appendix S – Dr. Ignacio's Observation Notes for Kevin's Lesson 20 cont.

$$a^2 = b^2 + c(t)^2 - 2bc(t)\cos(A(t)) \rightarrow \text{use implicit diff.}$$

you've made it  
explicit? not really.  
good reminder

$$\frac{d}{dt}(a^2) = \frac{d}{dt}(b^2 + c(t)^2 - 2bc(t)\cos(A(t)))$$

good notation.

$$\left(\frac{d}{dt}(a^2) = 0 \text{ discussion}\right).$$

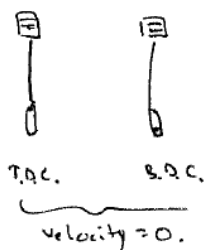
$$0 = 0 + 2c(t)\frac{dc}{dt} - \frac{d}{dt}(2bc(t)\cos(A(t))) \quad \left(\frac{d}{dt}c(t)^2 = 2c(t)c'(t) \text{ discussion}\right)$$

product discussion, plus 2b constant multiple.

$$\underbrace{c(t)}_f \underbrace{\cos(A(t))}_g$$

$$0 = 2 + c(t)\frac{dc}{dt} - 2b \left[ \frac{dc}{dt} \cos(A(t)) + c(t)(-\sin(A(t))\frac{dA}{dt}) \right]$$

discussion: why are lengths, angles still both involved.



side b 2r

$$f = c(t)$$

$$f' = \frac{dc}{dt}$$

$$g = \cos(A(t))$$

$$g' = -\sin(A(t)) \frac{dA}{dt}$$

question:  
mix of notations?

$$\text{unit conversion: } 3500 \text{ rpm} = 3500 \frac{\text{rev}}{\text{min}} \cdot 2\pi \frac{\text{radians}}{\text{rev}}$$

$$= 7000\pi \frac{\text{radians}}{\text{min}} = A'(t)$$

discussion →  
rate of change  
at A.

now, back to solving for c from L.O.C.

Step → step is challenging part.

Back to problem solving technique: State Goal.

intuitive, or  
on-the-fly  
adjustment?

# Appendix S – Dr. Ignacio's Observation Notes for Kevin's Lesson 20 cont.

Goal: Find speed at the piston

Setup: A crankshaft angle is radians

$$\frac{d\theta}{dt} = \text{rpm}$$

$C$  = piston height.

→ Boardwork problem 12, p. 248

Picture

Goal

Setup

distraction:  
 student ~~unconcerned~~ concerned  
 about prog grade.  
 Extra credit? ?

## Appendix T – Laverne's Presentation for Lesson 16

## Math 141 Lesson 16: ★ Implicit Differentiation

- 1) Explain the difference between implicit and explicit functions
- 2) Use the chain rule in implicit differentiation.
- 3) Differentiate an implicit function. Evaluate the derivative at a point (if the point is on the curve.)
- 4) Determine where the horizontal tangent lines exist using implicit differentiation.
- 5) Determine where the slope of the curve is equal to a specific value using implicit differentiation.

### On the Horizon

- FSPA #2 is due \_\_\_\_\_
- FS Quiz #2 is \_\_\_\_\_
- Homework #3 is due \_\_\_\_\_
- GR #2 is \_\_\_\_\_

## Appendix T – Laverne's Presentation for Lesson 16 cont.

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**FS Quiz Common Errors**

- 1.) 73 out of 74 of you got this one correct. Great job!
  - 2.)  $y(t) = e^t \cdot 7t^3$ 
    - Product Rule! The solution should have been two terms added together
  - 3.)  $s(x) = 7^x + 3\ln(x)$ 
    - It's  $\ln(7)$  (or whatever constant you're working with), not  $\ln(x)$
    - The derivative of  $7^x$  is not  $x7^{x-1}$ . Not the power rule!
- 

---

**FS Quiz Common Errors**

- 4.)  $\frac{d}{dx} \left[ \frac{1}{4x^6} \right]$ 
    - What is the constant in this equation? (This was the most common error, with about 55% of you getting this one wrong)
    - Do you really need to use the Quotient Rule?
    - Simplification – is  $\frac{-18}{36}$  good enough?
  - 5.)  $m(r) = \frac{1}{3} \cos(r)$ 
    - $\frac{1}{3} - \sin(r)$  ... what did you really mean here?
    - FINAL WARNING on this one!
-

## Appendix T – Laverne’s Presentation for Lesson 16 cont.

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**Chain Rule Review**

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- Perform the following derivatives:

$$G(y) = \frac{(y-1)^4}{(y^2+2y)^5}$$

$$F(v) = \left( \frac{v}{v^3+1} \right)^6$$

---

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**A New Approach to the Chain Rule**

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- Consider what “taking the derivative” really means
  - Even taking the derivative of  $f(x) = x^2$  uses the chain rule.
  - How?
-

## Appendix T – Laverne's Presentation for Lesson 16 cont.

## Notation Notes

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- Please get into the habit of regarding:
    - $\frac{d}{dx}$  as a mathematical operator, NOT the derivative itself
    - $\frac{dy}{dx}$  as a derivative, NOT a mathematical operator. How do we say this?
  - Start to think about how to take the derivative with respect to the equation's *independent variable* each time you take the derivative.
- 

## Implicit Differentiation

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- Consider an equation in which you cannot easily isolate one variable in terms of the other.
- How do we take the derivative?
- Remember we are taking the derivative with respect to *one* of the variables in this case.
  - Get into the habit of thinking about

$$\frac{d}{dx}$$

...or whatever variable you are taking the derivative with respect to

---

## Appendix T – Laverne's Presentation for Lesson 16 cont.

### Implicit Differentiation Procedure

- 
- Take the derivative of both sides of the equation with respect to  $x$  (or whatever the independent variable is). This will create terms with  $\frac{dy}{dx}$  whenever the Chain Rule is applied.
  - Isolate terms with  $\frac{dy}{dx}$  on one side of the equation and terms without  $\frac{dy}{dx}$  on the other side. Use good algebra skills.
  - Factor out the  $\frac{dy}{dx}$
  - Divide both sides of the equation by the factor multiplied by  $\frac{dy}{dx}$
- 

### Next Lesson

#### TO DO

- Take Home Quiz!!! Due Lesson 17!
- Homework #3: Section 3.5 #1, 7, 15, 23, 31, 76 are now assigned

#### Next Time

Lesson 17: Implicit Differentiation Continued – Using Implicit Differentiation to find slopes of tangent lines to complicated functions.

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**Appendix U – Laverne’s Practice Worksheet for Lesson 16****Explicit vs. Implicit Differentiation**

Determine whether each equation is implicit or explicit. Then take its derivative.

$$y = [\sin(x)]^5$$

$$y^{\frac{1}{5}} = \sin(x)$$

$$y = x^6 + 3 \sin(x) + \cos(x) + e^x + 4^x + 2 \ln(x) + 16$$

$$x^2 + y^2 = \ln(y) + y + 1$$

$$y = x \sin(x)$$

$$\sin(y) = xy^5$$

$$y = \frac{\ln(x)}{x^2}$$

$$3e^y = \frac{x}{\sqrt{y}}$$

# Appendix V – Dr. Paine's Observation Notes for Laverne's Lesson 16

Observation of Lt Col Patricia Vollmer by Lt Col Paul Harris re Lesson 16 - Implicit Differentiation (T4) Math 141 - Calc I

- used slides - objectives on first slide
- Nice fill in the blank for upcoming due dates
- FS Quiz review
  - said 'chain rule' in  $e^{7x^3}$  / later said product rule
  - actually got solution of your review
    - i.e.  $\frac{d}{dx}[7^x] = \ln(7)7^x$  - talked about rule but never stated actual derivative
    - same for  $\frac{d}{dx}[\frac{1}{4x^6}]$  had answer near  $4x^{-6}$  but never showed actual answer

Chain Rule Review (#30 Section 3.4)

$$F(v) = \left(\frac{v}{v^2+1}\right)^6 = 6 \left(\frac{v}{v^2+1}\right)^5 \left[ \frac{(v^2+1) - v(2v)}{(v^2+1)^2} \right]$$

$$= \frac{6v^5(-2v^2+1)}{(v^2+1)^7} = \frac{-12v^7+6v^5}{(v^2+1)^7}$$

- Give them a little more time before holding

inside  $\frac{v}{v^2+1}$        $F'(v) = 6 \left(\frac{v}{v^2+1}\right)^5 \cdot \frac{\frac{dv}{dv}(v^2+1) - \frac{d}{dv}(v^2+1)v}{(v^2+1)^2}$

outside  $( )^6$

said 'v three' a few times

$$= 6 \frac{v^5}{(v^2+1)^5} \cdot \frac{(v^2+1) - (2v^2)v}{(v^2+1)^2}$$

$$= \frac{6v^5}{(v^2+1)^5} \cdot \frac{1-2v^2}{(v^2+1)^2} = \frac{6v^5(1-2v^2)}{(v^2+1)^7}$$

Now Revert to Chain Rule:

$$f(x) = y^2$$

$$\text{outside } ( )^2 = f(y)$$

$$\text{inside } y = g(x)$$

$$f'(x) = f'(g(x))g'(x) = 2y \cdot \frac{dy}{dx} = 2xy'$$

Erased relatively quick - do to side of projection board and leave up

## Appendix V – Dr. Paine's Observation Notes for Laverne's Lesson 16 cont.

$\frac{d}{dx}$  is mathematical operator

referred to  $\pm 4 \frac{dy}{dx} = -\frac{4}{7} x^{-7}$  (i.e. correct!) be careful  
to  
o. FSQwiz  $\left[ \frac{d}{dx} \left[ \frac{1}{4} x^0 \right] = \frac{1}{4} \frac{d}{dx} (x^{-6}) = -\frac{6}{4} x^{-7} \right]$

Explicit: Scarface, Snow Day song

$$y = \pm \sqrt{25 - x^2}$$

Implicit: Snow Day / covered up (Involved)  
 $y^2 + y^2 = 100$  (just use 25 to have same)  
as above

referred Chain Rule example: note: this is explicit function of  $x$

A few questions on I/E functions

- Snow Day it - first two as given

$$y = (\sin(x))^5$$

$$\frac{dy}{dx} = 5(\sin(x))^4 \cos(x)$$

$$\frac{d}{dx} (y^{1/5} = \sin(x)) \text{ Procedure on slide}$$

$$\frac{1}{5} y^{-4/5} \cdot \frac{dy}{dx} = \cos(x) \frac{dx}{dx}$$

- this is

$$\frac{d}{dx} (y^{1/5}) \text{ be explicit this is } \frac{dy}{dx} \text{ w.r.t } y$$

$$\frac{dy}{dx} = 5 y^{4/5} \cos(x)$$

$$\frac{dy}{dx} = 5 (\sin(x))^{4/5} \cos(x)$$

alternatively write next / on

$$\left[ 5 \sin^{4/5}(x) \cos(x) \right]$$

(11:09) here

Give 5 minutes to work on board

## Appendix V – Dr. Paine's Observation Notes for Laverne's Lesson 16 cont.

$$\sqrt{x+y^2} = \ln(y) + y + 1$$

$$\left[ \begin{aligned} 2x \frac{dx}{dy} + 2y \frac{dy}{dy} - \frac{1}{y} \frac{dy}{dy} + \frac{dy}{dy} &= \left( \frac{1}{y} + 1 \right) \frac{dy}{dy} \\ 2x &= \left( \frac{1}{y} + 1 - 2y \right) \frac{dy}{dy} \Rightarrow \frac{dy}{dx} = \frac{dy}{\frac{1}{y} + 1 - 2y} \end{aligned} \right]$$

11:14 - worked on

$$\frac{d}{dy} (\sin(y)) = y^5$$

Ans is  
to sin, a constant

$$\cos(y) \frac{dy}{dx} = 5y^4 \frac{dy}{dx} + y^5 \frac{dy}{dx}$$

$$\cos(y) \frac{dy}{dx} - (5y^4) \frac{dy}{dx} = y^5$$

$$\frac{dy}{dx} (\cos(y) - 5y^4) = y^5 \quad \text{lots of overhead}$$

Everyone saw that

$$\frac{dy}{dx} = \frac{y^5}{\cos(y) - 5y^4}$$

Let them tell you what to do  
next (you ask but then fill in)

allow the uncomfortable silence  
rather than uncomfortable silence

once here explain  
what this means → confusing at this point for calculus

11:49

$$\frac{d}{dx} (3e^y) = y y^{-1/2}$$

$$3e^y \frac{dy}{dx} = \left( \frac{1}{2} y^{-3/2} \frac{dy}{dx} \cdot x \right) + y^{-1/2}$$

Gr = fake home op in all repl

# Appendix V – Dr. Paine's Observation Notes for Laverne's Lesson 16 cont.

Good:

- Organization and flow were about right
- Good control of class

Could improve: overall explanation once you have derivative

- less overhead questions - more direct
- let students answer and derive examples
- better explanation use of chain rule:

$$\frac{d}{dx}(xy^5) = \frac{d(y^5)}{dy} \cdot \frac{dy}{dx} \cdot x + y^5 \frac{dx}{dx}$$

Why?

This was confusing to some:  
Need to take derivative w/rt y

**Appendix W – Colt's Presentation for Lesson 15****Lesson 15 – Derivative Shortcuts**

- **Admin –**
  - Attendance
  - 5M Target    - FSPA / FS Quiz #1 on **TODAY!**
  - 10M Target   - FSPA / FS Quiz #2 on \_\_\_\_\_
  - 25M Target   - HW Set #3 on \_\_\_\_\_
- **Review –**
  - Product and Quotient Rules

**FS Quiz**

- Clear desk of everything but pencil.
- You have 10 minutes to answer the 5 questions.
- Remember to simplify your answer like the examples on the front page.

**Appendix W – Colt's Presentation for Lesson 15 cont.**

## Lesson 15 – Objectives

- 1) Decompose complicated functions.
- 2) Determine the derivative of a function using the Chain Rule.

## Composite Functions

- $f(x) = x^2$        $g(x) = \cos(x)$
- $f \circ g$  is also written as  $f(g(x))$
- What is  $f \circ g$  ?
- What is  $g \circ f$  ?
- Identify inside and outside functions

### Appendix W – Colt's Presentation for Lesson 15 cont.

#### How do we compose functions?

If: $f(x) = \sin(x)$ $g(x) = x^5$  $f \circ g =$  $g \circ f =$ <hr/>	If: $f(x) = \sqrt{x}$ $g(x) = x + 1$  $f \circ g =$  $g \circ f =$ <hr/>
If: $f(x) = \ln(x)$ $g(x) = x^3$  $f \circ g =$  $g \circ f =$	If: $f(x) = x^2 + 3$ $g(x) = e^x$  $f \circ g =$  $g \circ f =$

#### Derivative of composite functions

**The Chain Rule** If  $g$  is differentiable at  $x$  and  $f$  is differentiable at  $g(x)$ , then the composite function  $F = f \circ g$  defined by  $F(x) = f(g(x))$  is differentiable at  $x$  and  $F'$  is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if  $y = f(u)$  and  $u = g(x)$  are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$F(x) = \sin(x^5)$$

$$G(x) = \ln(x^3)$$

**Appendix W – Colt's Presentation for Lesson 15 cont.**

Examples	
$F(x) = \sin(x^5)$ $F'(x) = f'(g(x)) \cdot g'(x)$	$G(x) = \ln(x^3)$ $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Boardwork	
<ul style="list-style-type: none"> <li>• <u>Group 1</u>  <math display="block">F(x) = (4x - x^2)^{100}</math> </li> <li>• <u>Group 2</u>  <math display="block">y = \sin(\sin x)</math> </li> <li>• <u>Group 3</u>  <math display="block">F(x) = \sqrt{1 - 2x}</math> </li> </ul>	<ul style="list-style-type: none"> <li>• <u>Group 4</u>  <math display="block">y = xe^{-kx}</math> </li> <li>• <u>Group 5</u>  <math display="block">F(x) = (x^4 + 3x^2 - 2)^5</math> </li> <li>• <u>Group 6</u>  <math display="block">f(x) = \ln(x^2 - 2x)</math> </li> </ul>

**Appendix W – Colt's Presentation for Lesson 15 cont.**

### Challenge Problem

$$f(x) = \ln \ln \ln x$$

Review!!!

Applicable problems from Chapter 3 review

## Appendix X – Colt's Notes for Lesson 15

$$7^{7^x} \quad (7^7)^x = (7^7)^x$$

$$a^x \ln(a) = 7^7 \ln(7^7)$$

$$=$$

$$F(x) = \sin(x^5)$$

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$f(u) = \sin u \quad g(x) = x^5$$

$$F'(x) = \cos(u) \cdot 5x^4$$

$$F'(x) = 5x^4 [\cos(x^5)]$$

$$G(x) = \ln(x^3)$$

$$y = \ln(u) \quad u = x^3$$

$$\frac{dy}{du} = \frac{1}{u} \quad \frac{du}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot 3x^2$$

$$= \frac{1}{\ln(x^3)} \cdot 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{\ln(x^3)}$$

$$F(x) = (4x - x^2)^{100}$$

$$u = 4x - x^2$$

$$du = 4 - 2x$$

$$F'(x) = 100(u)^{99} \cdot (4 - 2x)$$

$$F'(x) = 100(4x - x^2)^{99} (4 - 2x)$$

$$y = \sin(\sin x)$$

$$u = \sin x \quad y = \sin u$$

$$du = \cos x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos(u) \cdot \cos x$$

$$= \cos(\sin x) (\cos x)$$

## Appendix X – Colt's Notes for Lesson 15 cont.

$$F(x) = \sqrt[3]{1-2x}$$

$$u = 1-2x \quad y = u^{\frac{1}{3}}$$

$$du = -2 \quad dy = \frac{1}{3} u^{-\frac{2}{3}}$$

$$F'(x) = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{3} u^{-\frac{2}{3}} \cdot -2$$

$$= \frac{1}{3 \sqrt[3]{1-2x}} \cdot -2$$

$$= -\frac{2}{3 \sqrt[3]{1-2x}}$$

$$y = x e^{-kx}$$

$$y' = x e^{-kx} \cdot -k + 1 \cdot e^{-kx}$$

$$= -kx e^{-kx} + e^{-kx}$$

$$y' = e^{-kx} (1 - kx)$$

$$F(x) = (x^4 + 3x^2 - 2)^5$$

$$F'(x) = 5(x^4 + 3x^2 - 2)^4 \cdot (4x^3 + 6x)$$

$$f(x) = \ln(x^2 - 2x)$$

$$= \frac{1}{(x^2 - 2x)} \cdot (2x - 2)$$

$$f'(x) = \frac{2x-2}{x^2-2x} = \frac{2(x-1)}{x(x-2)}$$

$$f \circ f \circ f$$

$$f'(x) = \frac{1}{\ln(\ln(x))} \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

$$u = \ln(\ln(x))$$

$$du = \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

## Appendix Y – Dr. Tougaloo's Observation Notes for Colt's Lesson 15

- Like you putting the onus on your section member to take role.
- 5m, 10m, 25m targets
- I'd like to see you model what you expect from them — good accepting responsibility for errors. As opposed to  $7^{7^x}$ , write  $f(x) = 7^{7^x}$  instead.
- Good reminder of simplification... I like to add a comment of authorized resources (read it verbatim).
  - Once you start the quiz, I'd refrain from giving them extra instruction (simplify, check) or good test taking techniques.
  - Name on paper — ensure beforehand if possible.
  - Academic security
- Parentheses will keep you out of trouble on the GR/ESQ... try mathematically unambiguous/correct.
  - Model those parentheses yourself. You used them most of the time, but occasionally you left them out.
- I liked the way you introduced the  $f \circ g$ ,  $g \circ f$  and got them to see what the examples would be for inside and outside functions.
- When you're using Leibniz's notation, you may be better off writing your original equation ( $g(x) = \ln(x^3)$ ) as  $y = \ln(x^3)$  since you let  $y = \ln(u)$ .
  - I'd also write them as  $y(u) = \ln(u)$  and  $u(x) = x^3$ . That should enable them to see where they're getting  $\frac{dy}{du}$  or  $\frac{du}{dx}$  from (which equation to pull from).
  - I'd model more organization... you were putting different parts in "random" locations.
- Boardwork: Everyone seemed very engaged.
  - Do you want them writing equations for chain rule? I think doing so will guide them (and a grader), especially when they get more difficult.
  - Communication lacked in some cases. Make sure you do your best to correct early and often.
  - fct: you pointed out that all they really did was fof... you made a point of it, but so what?
  - " $F(x) = \frac{1}{2}(1-2x)^{-\frac{1}{2}} \cdot -2$ " Get them out of  $\bullet \rightarrow 2$  habit.
  - Hard problem: Correct their notation (no equal signs); organization and writing out correct math-ese is most important in these difficult problems.
- Challenge Problem:  $f(x) = \ln \ln \ln(x)$  what about it? Take derivative.
- Great rapport with students, it seemed. They were comfortable with you — laughing is good.
- No eating in class.