

TEACHERS' USE OF DIGITAL COGNITIVE TECHNOLOGY:
CONNECTIONS TO BELIEFS

by

LEIGHTON MCINTYRE

(Under the direction of James Wilson)

ABSTRACT

In this study, I investigated three secondary mathematics teachers' use of digital cognitive technologies. I also examined their beliefs about teaching mathematics and about the use of technology for teaching mathematics. I used qualitative case study methodology to examine possible relationships and commonalities among their technology use, beliefs and other factors. For each teacher, I collected data via eight in-depth interviews and classroom observations. I analyzed the data using thematic analysis and drawing upon works of Pea (1987), Ernest (1989) and Goos, Galbraith, Renshaw, & Geiger (2003). I presented findings through a case for each teacher, as well as comparisons across cases. Results indicated these teachers' uses of digital cognitive technologies appeared more aligned to their beliefs about teaching mathematics *with* technology than beliefs about teaching mathematics in general. The teachers' uses of digital cognitive technologies did not correspond directly to their expressed factors affecting choice and use of technology. Recommendations were made for teachers' focus on specific ways of teaching with digital cognitive technology and developing strong beliefs about teaching mathematics *with* technology.

INDEX WORDS: Digital Cognitive Technology, Secondary Mathematics Teachers, Beliefs, Teaching Mathematics with Technology, Use of Technology, Amplifier, Reorganizer, Partner, Servant, Facilitator

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A Dissertation Submitted to the Graduate Faculty of the University of Georgia in Partial
Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2017

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DEDICATION

I dedicate this dissertation to everyone who perseveres.

ACKNOWLEDGEMENTS

I would like to say thank you to all persons who had an input in the shaping of this dissertation. I acknowledge my wife, Valsie, and my three children, Jabari, Jemila, and Jamal. Thank you for hanging in there and praying with me during this writing. I also thank my mother, father, sisters and brothers who, from across the miles, encouraged me. I thank members of my church for prayers and support.

I express my gratitude to my advisor and dissertation committee. My advisor, Jim Wilson, was always encouraging and questioned my ideas, providing direction on ways to proceed with the dissertation. His unwavering support kept me going even at times I would have given up. Committee member, Ryan Smith encouraged me to hone my skills in his research group and my independent research, providing useful feedback on organizing my research. Committee member Dorothy White, read and edited my drafts, and encouraged me to persevere to the end.

I thank Claudette, Dario, and Angel who wrote with me in my invited writing sessions. I thank Ashley, Jaime, and AnnaMarie for reading my drafts and providing feedback. I thank Kirsten, who critiqued my initial coding and analysis.

I thank Ryan Smith and Les Steffe for allowing me to observe their teaching with technology. I am grateful to Pierre, Nick and others who allowed me to test the interview instruments with them. I thank Torrey and others for hours of listening to changing versions of my study.

I thank the principals of Yellow and Green high schools. Finally, I would like to say a big thank you to Beth, Ruth, and Sam, for the many hours spent talking with me about this study and allowing me to observe the use of digital cognitive technology in your mathematics classrooms. Without you, this dissertation would not have been possible.

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CHAPTER 1: INTRODUCTION

1.1 Statement of the Problem

In this study, I sought to provide descriptive and interpretative insights about teachers who are considered frequent users of digital cognitive technologies in the classroom and how their use of technology appeared to be impacted by their beliefs about teaching mathematics and about the use of technology for teaching mathematics.

Teachers determine how they want to use mathematics specific technology in a lesson, when the technology should be used, such as after a topic has been taught, when students should use technology, and the number of times that technology is used in the classroom (Leatham, 2006; Schultz, 2009; Zbiek, Heid, Blume, & Dick, 2007). Moreover, the teacher is crucial in establishing the modes of technology use in the classroom, such as for computation or conceptual development (Doerr & Zangor, 2000; Drijvers, 2015). Hence, teachers play a pivotal role in the way mathematics is taught and learned with technology (Judson 2006; National Council of Teachers of Mathematics (NCTM), 2000; Schultz, 2009; Sherman, 2011).

I use the term digital cognitive technologies to refer to technologies of digital nature, specific to mathematical thinking. Examples of digital cognitive technologies include dynamic technologies such as GeoGebra, Desmos graphing calculator technology and TI-Nspire Navigator (Figure 1). Some technologies are cognitive but not digital, such as the abacus. In the same vein, some technologies are digital but not cognitive, such as the Smart Board.

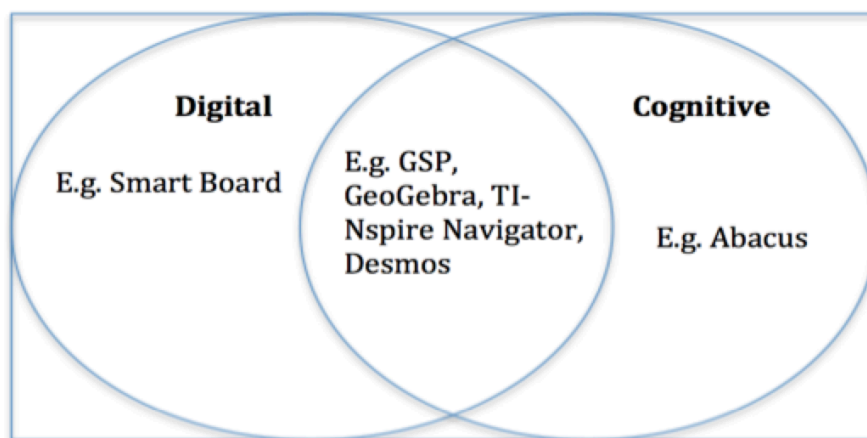


Figure 1. Digital technologies and cognitive technologies

Researchers (e.g. Ertmer, 2005; Heid, 1997; Olive & Lobato, 2008; Niess, 2009) indicated the use of technology potentially creates beneficial effects to the classroom, such as focusing on problem-solving, reflective thinking, conceptual understanding, and meaningful mathematics. Additionally, many organizations (e.g. NCTM, 2000, 2014; International Society for Technology in Education (ISTE), 2007, 2016) have recommended incorporating technology into mathematics lessons on the basis that technology may be used to enhance teaching and learning. Furthermore, resources have been used for developing technology, providing professional technology training for teachers, and making technology increasingly available in classrooms (Kaput, 1992; Koehler & Mishra, 2009).

Even though technology is increasingly available in the classroom and has the potential to be used in ways that positively impact teaching and learning, teaching with technology remains a complex endeavor (Brush, Glazewski, Rutowski, Berg, Stromfors, Van-Nest, & Sutton, 2003; Cuban et al., 2001; Koehler & Mishra, 2009). Furthermore, Koehler & Mishra, (2009) indicated some teachers and researchers hold the notion that there is no 'one best way' to include technology, particularly newer digital technologies,

in the classroom. Thus, teachers have developed varying ways of using technology in their lessons. Some teachers make limited use of the technology such as using technology only as an add-on to traditional ways of teaching. Traditional teaching is usually associated with teaching based on the transmission of abstract rules and ways of teaching (Cuban et al., 2001). For example, teachers may use technology to reinforce procedures exactly as they did without the use of technology. Hence, they may provide little or no access for students to use technology. Additionally, they may not display evidence of designing, selecting or implementing activities for teaching and learning with technology in ways other than by drill and practice (Cuban, Kirkpatrick, & Peck, 2001; Doering, Hughes, & Huffman, 2003). On the other hand, some teachers may make extensive use of technology, incorporating digital technologies into their mathematics activities on a regular basis, not just for practicing mathematical procedures but also for conceptual development (Ottenbreit-Leftwich, Glazewski, Newby, & Ertmer, 2010).

Teachers who teach mathematics as an exploration of mathematical concepts may find it easier to incorporate technology into a mathematics lesson. This suggests the way teachers incorporate digital cognitive technologies may be impacted by their pedagogical practices and their views about using digital cognitive technology in their classrooms. Pea (1985) indicated the use of cognitive technologies include some digital ones. Some cognitive technologies are not digital e.g. abacus

Cognitive technologies allow persons to transcend the limitations of their minds in problem solving activities and mathematical thinking. Sherman (2014) indicated although much research has been carried out regarding the ways that teachers use technology in the classroom, less research has been done on teachers' use of digital

cognitive technologies. Furthermore, there has been limited research on teachers' use of these technologies based on their views about their effective ways of teaching with technology (Ertmer and Ottenbreit-Leftwich, 2010; Shafer, 2008; Srisurichan, 2012). Even when the views of teachers are included, the focus has been on nature of the mathematics or of the mathematics pedagogy (Thompson, 1992; Kastberg & Leatham 2005) and not about teaching mathematics with technology (Leatham, 2002).

1.2 Use of Digital Cognitive Technology

Teachers may use digital cognitive technology in the classroom in a variety of ways. Researchers (e.g. Goos, Galbraith, Renshaw, & Geiger, 2003; Pea, 1985, 1987) have used metaphors to describe teachers' use of cognitive technology in the classroom. For instance, Pea (1985, 1987) indicated teachers' use technology as either an Amplifier or Reorganizer. As an amplifier, teachers use technology for making more efficient, accurate or displaying the same mathematics as done with pencil and paper. Teachers may also use technology in ways that the technology serves as a cognitive tool, that is, as a Reorganizer of mathematical thinking. As a Reorganizer of knowledge, technology is used to shift the mathematical thinking in ways that may be difficult or impossible to do without technology. In this regard, technology may be used for directing thinking beyond mere calculations, drawing, and measuring. The technologies available to teachers in the 1980s were very different from those in this study, so the uses of technology shared some similarities and some differences.

Goos et al. (2003) investigated the use of technology in mathematics classrooms and found some users may do so in ways that the mathematical task was expedited. For these users, they were able to determine whether the output of the technology made

sense. In that case, Goos et al. (2003) described the use of technology as Servant.

Another use of technology was for obtaining correct results, where users depended on the technology to provide the correct solution without the users' validation of the answer, and this use was described as Master. The third use of technology was that of a Partner, in which the users worked with the technology as a tool for exploration and not just to get expected results. The users are in control of the mathematics but recognize the technology has some authority, and there should be a balance between the results produced by the technology and the mathematics produced without technology. The final use is technology as Extension of Self, an indispensable natural part of the users' mathematical thinking. The technology was used as an extension of the users' abilities, as part of their mathematical repertoire and as part of their argumentations (Nzuki, 2010).

Table 1: Uses of Digital Cognitive Technologies (DCTs)

Uses of DCT (Goos et al., 2003)	Uses of DCT (Pea, 1987)
Master	Amplifier
Servant	
Partner	Reorganizer
Extension of Self	

Looking at the categories of technology proposed by Pea (1987) and Goos et al. (2003) some connections may be made. It may be possible as teachers use technology as Amplifier they may also be working in the Master or Servant categories of using technology (Table 1). When using technology as Reorganizer teachers may also be directing the use of technology as Partner or Extension of Self.

1.3 Beliefs

Researchers (Cooney & Wilson, 2008; Ernest, 1989; Ertmer, 1999; Wilkins, 2008) have suggested teachers' practices with technology in the mathematics classroom

appear to be influenced by their beliefs. For instance, Ertmer (1999) indicated many teachers' reluctance to use technology in the most beneficial ways for mathematical thinking might be due to barriers, including their pedagogical beliefs. Other researchers (e.g. Manoucherhri, 1999; Norton, McRobbie & Cooper, 2000) have indicated teachers' non-use or limited use of technology may be associated with their views about the relevance of technology for teaching mathematics. Some researchers (Leatham, 2006; Ertmer & Ottenbreit-Leftwich, 2010) have theorized teachers' use of technology may also include the interplay of pedagogical beliefs, beliefs about mathematics, and non-mathematics beliefs such as beliefs about time and culture. Kastberg and Leatham (2005) provided a meta-analysis of research studies which indicated teachers' use of technology in ways not deemed most beneficial to learning appeared to be associated with teachers' beliefs about the nature of mathematics and what doing mathematics in school means. In fact, the practices of teachers who use technology in the classroom could be associated with a wide range of beliefs including beliefs about assessment, self-efficacy, and students' characteristics.

Researchers (e.g. Judson, 2006; Hermans, Tondeur, van Braak, & Valcke, 2008; Palak and Walls, 2009; Ertmer, Ottenbreit-Leftwich, Sadik, E. Sendurur, & P. Sendurur, 2012) have examined the relationships between teachers' pedagogical beliefs and their classroom practices with technology. For instance, Palak and Walls (2009) investigated teacher technology practices and found teachers with teacher-centered or traditional beliefs were likely to use highly controlled technology practices, such drill and practice in their lessons, while teachers with constructivist beliefs, even though more likely to use constructivist technology practices, may not incorporate technology in their lessons. In

the same vein, Ertmer et al. (2012) investigated the relationship between espoused and enacted beliefs of teachers who were recognized for their technology use. They found teachers with espoused constructivist beliefs tended to enact constructivist activities with technology. On the other hand, Judson (2006) found no significant relationship between teachers' constructivist beliefs and the enactment of constructivist practices with technology. Varied (and sometimes conflicting) research results for the relationships between beliefs held by teachers and their practices with technology suggested the need for valid research in this area.

Some researchers (e.g. Kastberg & Leatham, 2005; Thompson, 1992) indicated that much research on beliefs and technology use in the mathematics classroom focused on beliefs about mathematics. For instance, (Thompson, 1992) focused on variations in teacher beliefs about the nature of mathematics, and on inconsistencies between beliefs and practices. Thompson (1992) however, did not examine teachers' beliefs about teaching with technology and their practices, particularly with digital cognitive technologies. Furthermore, Schafer (2008) posited research on teacher practices with cognitive technologies did not place much emphasis on the viewpoints of the teacher. Sherman (2014) suggested within the context of cognitive technologies in the classroom, teachers' beliefs might influence the different uses of these technologies. He contrasted whether technology is used at the level of Amplifier or the level of Reorganizer of mathematical knowledge without specifying any particular beliefs.

I wondered if the inclusion of more specific beliefs, such as beliefs about teaching mathematics with technology could provide deeper understanding of teachers' use of digital cognitive technology in practice. I also wondered if this understanding could be

gained by finding the commonalities among beliefs of teachers who routinely use digital cognitive technologies. In light of this, I focused on beliefs about teaching mathematics and using technology for teaching mathematics, which may be related to teachers' use of digital cognitive technologies. Thus, I sought to provide deeper insight of the way teachers hold these beliefs in relation to their use of technology.

Various beliefs frameworks may be considered to be suitable for conducting my study. In fact, researchers (e.g. Cooney, Shealy, Arvold, 1988; Ernest, 1989; Mura, 1995) have devised categories of teachers' beliefs, which appeared to be adaptable to my research. I found Ernest's (1989) beliefs categories useful for my study as an initial point of reference for beliefs because these categories involved pedagogical beliefs that may influence what types of technology a teacher may want to include in a lesson, and how the teacher may direct students to use technology. Furthermore, Ernest's (1989) beliefs categories are specific to mathematics. Ernest's (1989) categories include beliefs about teaching mathematics, which also seem applicable to teachers who use technology to teach mathematics.

Ernest's (1989) categories do not specifically include beliefs about technology, so I adapted categories proposed by Goos et al. (2003) in this study to indicate teachers' views about the use of technology. Even though Goos et al.'s (2003) categories were intended to look at ways teachers use or allow students to use technology, they seemed suitable for describing the nature of the beliefs teachers hold about technology. Furthermore, Goos et al. (2003) indicated pedagogical strategies evidenced during use of technology in the classroom are influenced by the underlying beliefs of teachers.

Smith, Kim, and McIntyre (2016) previously adapted Ernest's (1989) and Goos et al.'s (2003) categories to examine the beliefs of prospective teachers. These researchers used Goos et al.'s (2003) categories to indicate beliefs about teaching and learning with technology and Ernest's (1989) categories to indicate beliefs about teaching and learning mathematics. However, whereas these researchers connected prospective teacher's beliefs to technological pedagogical and content knowledge (TPACK), my research connected the beliefs of in-service teachers to their use of digital cognitive technologies. Knowing the particular beliefs exhibited by teachers and connections to their use digital cognitive technologies in the classroom may allow us to be able to recommend ways of challenging or supporting particular teachers' beliefs, particularly about using technology for mathematics activities in the classroom.

1.4 Research Questions

The research questions guiding this study are:

For secondary mathematics teachers who routinely use technology into their classrooms:

1. What factors do they consider in choosing technology for use?
2. What digital cognitive technologies do they use?
- 2b. How do they use digital cognitive technologies to teach mathematics?
3. What are the teachers' beliefs about teaching mathematics, technology and teaching mathematics with technology?
- 3b. What commonalities are observed among teachers' beliefs and use of digital cognitive technologies?

1.5 Purpose of the Study

Although there are many recommendations about how to use technology in the classroom (e.g. Angeli & Valanides, 2009; Mishra & Koehler, 2006; Wilson & Hollebrands, 2011), there is limited research on teaching mathematics with technology particularly based on cases of teachers who incorporate technology. The purpose of the study was to provide descriptive and interpretative insights about teachers who are considered frequent users of cognitive technologies in the classroom and how their use of technology appear to be impacted by their beliefs about teaching mathematics and teaching mathematics with technology.

Furthermore, since researchers (e.g. Cooney et al., 2008; Ernest, 1989) indicate what teachers do in the mathematics classroom may be a product of their beliefs, teachers may not use technology in ways considered most beneficial, and amid the background that teachers' viewpoints on their effective use of technology are limited in research, I wondered what commonalities may exist among the beliefs about teaching mathematics, and teaching mathematics with technology and the practice of teaching with cognitive technologies. Finding commonalities in these areas may be important to fine-tuning the beliefs that are predominantly influential to teaching mathematics with technology. Thus, understanding the commonalities among the beliefs of teachers who incorporate these technologies into their lessons and their selection and use of the technologies may add to research literature on effective use of technologies in the teaching and learning of mathematics.

Additionally, my study seeks to add to existing literature on beliefs and technology by addressing factors are involved in teachers' use of technology for

mathematics and beliefs of teachers that may be more specific to their use of technology. By identifying ways teachers select and use digital cognitive technologies for particular mathematical activities, and providing opportunities for teachers to reflect on their selection and use of technology, my research may help teachers to focus on the type of teaching that takes place with their use of technology for mathematics teaching. In other words, help teachers to determine whether they use technology in ways to enhance the mathematical understanding of their students (NCTM, 2014).

In the next chapter, I review the relevant literature on teaching with cognitive technologies and on teacher beliefs, in particular beliefs about teaching mathematics, and teaching mathematics with technology.

CHAPTER 2: REVIEW OF LITERATURE

In this chapter, I situate my study in literature relevant to how teachers use cognitive technology and their beliefs about teaching mathematics and teaching mathematics with technology. First, I describe the use of technology as posited by various organizations and researchers, and the factors that impact teachers' selection and use of technology. Then, I examine the definitions of beliefs, teachers' beliefs about teaching mathematics and using technology to teach mathematics, and make comparison of beliefs to similar constructs. At the end of the chapter, I indicate the research paradigms pertinent to my study.

2.1 Technology and Teaching

In this section, I discuss the literature on the use of technology for teaching in the classroom, culminating with a focus on cognitive technologies.

Teaching with technology. Educational organizations such as National Council for Teachers of Mathematics (NCTM, 2000, 2014), International Society for Technology in Education (ISTE, 2007), Conference Board of Mathematical Sciences (CBMS, 2012), National Council for Education of Teacher Educators (NCATE, 2001) have acknowledged the importance of using technology for mathematical activities. The NCTM (2000) noted that technology use is one of the six principles of high quality school math. In fact, this organization stated, "technology is essential in teaching and learning mathematics: it influences the mathematics that is taught and enhances students learning" (NCTM, 2000, p.24).

The International Society for Technology in Education (ISTE, 2007) indicated technology could support effective teaching and learning environments. The ISTE has created technology standards for learners, teachers, coaches, and administrators. The standards are interlinked and thus work together to support those most responsible for technology use in education. They promote teachers' use of their knowledge of subject matter, pedagogy and technology to support the learning of students with digital age technology. The ITSE indicated the standards make use of 'current research and the wisdom of practice'. (p.4.) Other educational organizations (e.g. NCATE) have adopted the ITSE standards. The National Council for Education of Teacher Educators (NCATE) is the organization responsible for granting accreditation to an educational unit such as colleges that offer teacher education training, after comprehensive evaluation and determination the educational unit meets the national standards of the teaching profession. All NCATE accredited programs since 2001 require that technology is integrated into the instruction, practice, and experience of the teachers. Hence the use of technology in education is seen as an important part of learning and teaching.

According to the NCTM (2000), teachers should use technology to enhance the learning of the students by selecting or creating tasks that technologies can do well (p.25). Thus, when technology is available to the teacher, it is the teacher who determines how, when, and for what tasks technology may be used in the classroom. The teacher is positioned as the one who has control over what happens with the technology within the context of the classroom. Therefore, the teacher is a key agent in determining the effective use of educational technology when engaging with mathematics activities (NCTM, 2000).

Even though the teachers may be key agents on the classroom particularly with respect to the incorporating technology, the majority of research on technology in education has focused the number of times technology is used, or whether using technology produced higher achievement among learners (Sherman, 2011). Few studies, however, have focused on how the teachers use the technology in the classroom (Judson, 2006) or teacher actions and roles with while teaching with technology (Zbiek & Hollebrands, 2008, Marshall, 2012). Additionally, Peressini & Knuth, (2005) lamented the lack of focus on teacher actions with technology to understand types of instruction teachers need to develop in order to use technology more meaningfully.

Many studies have indicated teachers may not make the most appropriate use of technology because they continue to teach in ways they did before the introduction of technology (e.g. Cuban et al. 2001; Manoucherhri, 1999; Koehler & Mishra, 2009; Russell, Bebell, Dwyer & O'Conner, 2003). In particular, teachers may use technology in ways to maintain ineffective teacher-centered practices. Ineffective teacher-centered practices with technology include teaching skills in isolation (of context), drill and practice procedures, and providing basic technology literacy (Ertmer, Ottenbreit-Leftwich, Sadik, E. Sendurur, & P. Sendurur, 2012). On the other hand, effective teacher-centered practices may incorporate technology practices beyond drill and practice and technology literacy (Wilson, October 2016, personal communication). Many learner-centered practices incorporate technology for exploration of knowledge construction, skills taught in context and applications, and collaboration (Ertmer et al., 2012). Some learner-centered practices, however, may not incorporate technology for exploration or knowledge construction. Ertmer (2005) and Ertmer and Ottenbreit-Leftwich (2010)

indicated in many classrooms technology is not generally used in ways most powerful for learning. Likewise, Chen (2008) indicated teachers in general could make more appropriate use of technology. Hence, teachers could use technology to better support students in productively engaging with mathematical tasks with technology as opposed using technology to sustain practices that already existed prior to its introduction.

Jonassen, Wilson, and Peck (1999) writing from a constructive perspective identified two types of instruction that may occur the presence of technology: (1) technology-as-teacher and (2) technology-as-partner. Jonassen et al. (1999) propose the use of technology should shift from the traditional use of technology-as-teacher to that of technology-as-partner for teaching to be meaningful. In the capacity of technology-as-partner, the teacher may use technology for supporting construction of knowledge, communicating, exploration of mathematics, learning-by-doing activities and reflecting. Jonassen, Wilson, and Peck (1999) did not conduct research on these different uses of technology but suggested categories based on what the available technology at the time could be used for.

Jonassen et al.'s (1999) idea of technology-as-teacher is similar to Goos et al.'s (2003) idea of technology as master, while their idea of technology as partner is somewhat similar to Goos et al.'s (2003) technology as partner. Jonassen et al. (1999) indicate the teacher and technology serve as intellectual partners to help students construct mathematics, similar to Goos et al. (2003). However, whereas Jonassen et al. (1999) were more focused on the particular meanings and personal constructions of knowledge the users made with the technology, Goos et al. (2003) was more focused on

the role technology played in the process of helping users understand mathematical concepts.

Marshall (2012) summarized past research on the use of technology and found most research on the use of technology positioned the technology as a tool or mediator. As a tool, technology is something to be handled by the user; wherein the way the user interacts with the technology determines the outcome of the educational activities with technology. Peressini and Knuth (2005) indicated five modes in which teachers may use technology as a tool in the classroom. These modes are, as communication tool, management tool, assessment tool, motivational tool, and cognitive tool. Peressini and Knuth (2005) differentiated among the roles of technology in the ways the technology helps teachers versus how technology helps students. The fourth and fifth roles were directed towards helping students, the second and third roles for helping teachers, and the first role for helping both students and teachers. In the fifth role, Peressini and Knuth (2005) suggested harnessing technology as a cognitive tool to help students to engage with mathematics, carry out exploratory activities, and better understand mathematics. Even though this use was directed at student thinking, the teacher played an active role in the process of 'harnessing' the technology in efforts to direct the technology to play this role in the classroom (Sherman, 2011).

The idea of technology as a cognitive tool is an underlying assumption that guided my research because my research focused on cognitive technologies. In particular, my study looks into ways the technology is used by the teacher personally and ways the teacher directs the use of technology to help students engage in exploratory activities geared towards a better understanding of mathematics. Furthermore, my research

develops ideas on how cognitive technology may be effectively used by looking into teachers' beliefs on the use of cognitive technology for mathematical thinking.

Teaching Mathematics. Kuhs and Ball (1986, cited in A. Thompson, 1992) proposed the existence of four models of mathematics teaching. These are (1) learner focused, wherein the learner personally constructs the mathematics, (2) content-focused with emphasis on understanding of the concepts, wherein the teaching is driven by the mathematics content, but the focus is on the conceptual understanding of the given mathematical content, (3) content-focused with emphasis on procedures, wherein the teaching is driven by the content but the teacher emphasizes mastery of rules, procedures and skills and, (4) classroom focused wherein, the emphasis is on what the teacher knows and thinks about effective classrooms, but not based on content, such as having routines.

Thompson (1992), however, described three ways of teaching she observed in mathematics teachers. In the first case, a teacher taught mathematics in a prescriptive manner with emphasis on teacher demonstrations of procedures and rules. In the second case, a teacher emphasized mathematical meaning of concepts and the logical meaning of mathematical procedures. In the third case a teacher emphasized the involvement of students in activities in 'generative processes of mathematics' (p. 134). The ways teachers displayed their teaching of mathematics in Thompson's (1992) study appear to be similar in nature to the roles of a teacher described by Ernest (1989) wherein the roles played by the teacher, include, (1) instructor, where the teacher emphasizes giving the students skills and procedures in such a way that the students become competent at the procedures without necessarily understanding them, (2) Explainer, where the teacher explains a

concepts of a unified body of knowledge called mathematics and, (3) Facilitator, wherein the teacher allows to students to explore and develop their own ideas about mathematics.

Other researchers, however, have articulated the teaching of mathematics as somewhat of a dichotomy: such as traditional versus reformed teaching (e.g. Goos, Stillman, & Vane, 2007); behaviorist versus constructivist teaching (Handal, 2003); instrumental versus relational teaching (Skemp, 1978). I must note that researchers who possess an inclination to one side of an argument wrote many of the dichotomy presentations, and generally put forward evidence to support their biases. These presentations were written here for comparison reasons. According to Goos, Stillman, and Vane (2007), who appeared to hold a more reform oriented perspective, the more traditional direct instruction approach includes teacher-centric, skills oriented, formulaic, drill and practice, procedural, and mastery of a predetermined body of knowledge. In contrast, the more reformed approach to teaching involves having students engage in creating, problem solving, reasoning and sense making, using real world applications, and evaluation of mathematical ideas (Goos, Stillman, & Vane, 2007).

Handal (2003), who appeared to hold a constructivist perspective, summarized the differences between the constructivist ideas of J. Piaget and behaviorist ideas of B. F. Skinner. He posited the constructivist classroom is one in which the learners are allowed to construct their own learning, personally and socially. Handal (2003) argued strategies associated with constructivist learning include group-work and exploration. By contrast the behaviorist classroom is one in which the transmission of content takes place from teacher to students and the value of the teaching and learning in showing the correct procedures and product (Handal, 2003). Skemp (1978), who appeared to hold a more

relational understanding perspective, indicated two types of understandings the teacher may focus on in a mathematics classroom, relational understanding and instrumental understanding. Teaching for Instrumental understanding means the teacher focuses on the understanding of carrying out a mathematical procedure such as knowing the steps to get to an answer. In contrast, teaching for relational understanding means the teacher focuses on the relationships between the steps, why the steps work and relationships between one mathematical representation and another (Skemp, 1978).

Thompson (1992) likened a learner focused classroom to the constructivist classroom wherein the teacher acts as facilitator. In this scenario, the teacher actively supports the students to actively engage in explorations, investigations, and development, and formalization of mathematics. This learner-focused classroom may also be seen as a mark of reformed teaching (Goos, Stillman, & Vane, 2007). The second, content-focused understanding of concepts model is akin to the relational model (Thompson, 1992). In this model the teacher has predetermined content, which is given to the students, as the teacher and students to connect ideas in mathematics. Thompson (1992) noted this model most closely resembles the teacher as Explainer proposed by Ernest (1989). The content-based with emphasis on procedures model is akin to the Instrumental model of Skemp (1978). In this model, the mathematics content again is predetermined, and the teachers give the students this content in the form of rules or procedures. The teacher serves as the instructor, demonstrator, and giver of procedures while the students listen and try to reproduce what the teacher tells them. This model is mostly akin to Ernest's (1989) teacher as Instructor model. It is also more closely related to what is called 'traditional teaching' (Goos, Stillman, & Vane, 2007).

Using cognitive technology for teaching mathematics. Research on mathematics teacher actions with technology indicated teachers used digital cognitive technologies as an Amplifier or a Reorganizer (e.g. Heid, 1988, 1997; Judson, 1990; Lee & Hollebrands, 2008; Manoucherhri, 1999; Pea, 1985, 1987; Sherman, 2011). Some researchers (e.g. Heid, 1988; Judson, 1990) have used technology to reorganize an entire mathematics course. For instance, Heid (1988) investigated the impact of the use of technology while she taught calculus concepts. She used technology, such as computer algebra, table tools, and graphing tools, at the beginning and throughout the course to develop concepts in calculus. The computational aspects of the mathematics were briefly dealt with toward the end of the course. The results of this reorganization were noteworthy in that students in the reorganized course performed just as well in the computation aspects of an examination, as a control group of students enrolled in a 'traditional' course. Additionally, they outperformed the control group on conceptual tasks. Although the use of technology to reorganize a course was not the primary focus of my study, it served as a useful indication of how technology had been used as a Reorganizer in mathematics.

Pea (1985, 1987) indicated some technologies such as some algebra software are designed to free users from hand calculations. In that way, the technology makes the work quicker and in some cases may eliminate the need to for cumbersome mathematical procedures such as long division. This frees up the classroom time to 'explore problem solving'. In addition to computer software, Pea's (1987) conception of cognitive technologies was broadly based on many writing and notation systems. In each of these systems, technology could be used as an Amplifier or Reorganizer. Pea (1987) termed the use of technology as an Amplifier when the technology was used making the

mathematics quicker. When technology was used as a means of freeing up the mind to focus on more problem solving, Pea (1987) termed this use as Reorganizer.

Pea (1987) provided the examples above based on technologies available in 1987, and as such, it appeared the use of technology for amplification was close to reorganization. It seemed doing multiple calculations quickly allowed for conceptual development. Because technology and its affordances have changed much since the 1980s many of the cumbersome mathematical procedures such as inverting a 5 by 5 matrix could be easily performed by modern technologies. Additionally, the technology could provide in the moment display of all students' work. Thus, the dichotomy forwarded by Pea (1987) on technology use needed to be revisited.

Some researchers (Sherman 2011; Manoucherhri, 1999) have studied the use of cognitive technologies the grain size of mathematical activities. For instance, Manoucherhri (1999) studied middle and high school teachers' use of technology in which the teachers used technology on mathematical activities students could do with pencil and paper. She found technology was an Amplifier only in these classes because the technology was only used to speed up the time spent on the task. Also, the technology was used to further support the drill and practice purposes with individual students.

Sherman (2011) investigated the use of cognitive technology at the grain size of a mathematical task in indicated several instances teachers used technology as an Amplifier and or a Reorganizer. One example from this research in which technology was used as an Amplifier was when the students used calculators to input numbers for the formula for finding the sum of the interior angles in a 16-gon, $180^\circ \cdot (n-2)$. On the other hand, an example in which technology was used as a Reorganizer occurred during an activity on

concurrencies of a triangle. In this activity, students used a dynamic geometry environment to drag a triangle after they constructed the orthocenter, centroid, and circumcenter. Then the students made conjectures and provided reasoning about the points of concurrency in a triangle to observe the orthocenter, centroid, and circumcenter all lie on a line. Thus, the teacher and students were making use of the technology as a Reorganizer of knowledge during the mathematical activity. Technology here was the Reorganizer because the dragging actions the students performed to get to the conjecture making could not be done without the technology. Sherman (2011) alluded to the beliefs of the teachers in his study and suggested the beliefs of the teachers may influence the use of technology for higher-level thinking.

Some studies have indicated teachers using digital cognitive technology as Master, Servant, Partner, or Extension of Self (Goos et al., 2003; Nzuki, 2010). In a study of a secondary classroom, Goos et al. (2003) indicated activities in which teachers and students engaged in mathematical activity as master, servant, partner, and extension of self. An example from this research of technology use as Master involved an episode where the teacher included technology activities only as required by a syllabus and would not allow student explorations with technology because of the teacher's lack of technological knowledge. An example from this research of technology as Servant by a teacher was for display of the mathematics that could already be done with pencil and paper. Additionally, the teacher used the technology to support aspects of teachers' usual form of instruction such as first showing matrix rotation with manipulatives only and then using a calculator for computations. As partner, a teacher used technology to extend to new approaches to a task, such as writing and executing programs to simplify

calculations or to initiate group mathematical discussions. As Extension of Self the teacher may integrate multiple technological resources easily into the lessons and also write course materials that support an integrated teaching program. Goos et al. (2003) investigated the use of digital technology, such as graphing calculators and computers along with projection devices. Since technology for teaching mathematics is continuously evolving, particularly in digital cognitive technology spaces, there appears the need for research to provide insight on how teachers adapt these technologies during the mathematics activities they enact in the classroom.

Using technology as Amplifier appears synonymous to using technology as Servant, while a Reorganizer use of technology appears synonymous to that of using technology as Partner (Sherman, 2014). For my study when technology was used as an amplifier, the teacher uses and instructs students to use technology for speed, accuracy, and display regardless of knowledge of technology, but as Servant the user's knowledge of the technology is relevant. Using technology as Servant, the user has some knowledge of the technology and the output of the technology must be aligned with what the user knows the technology should do, and the user is aware of the technological reasons why output may not be aligned. When a teacher uses and directs students to use technology as Master, the user has limited knowledge of the technology and tends to accept of the output of the technology without questioning.

As Reorganizer of knowledge, the technology is used in ways the mathematical activity extends beyond what can be done using pencil and paper only, as Partner the technology is also used to initiate group discussions or whole class discussion in addition to making individual conjectures. As Extension of self, the teacher encourages and allows

the students to freely explore with multiple technologies and go in multiple directions with their mathematical activities. My study seeks to extend the Amplifier Reorganizer analogies not only by investigating what teachers do at the grain size of mathematical activity but additionally how teachers' technology specific pedagogical beliefs come into play as they use technology as Amplifier or Reorganizer of mathematical thinking in the classroom.

My study involved secondary teachers who may use technology as a Reorganizer at the level of mathematical activities rather than of the entire course. In particular, by carefully observing teachers actions with technology and interviewing teachers about their use of technology, my study examined ways teachers focused on the use of technology at and beyond mere faster computations and displays to focus on higher order thinking and conceptual understandings at the level of mathematical activity. Whereas Pea (1985, 1987) focused on general-purpose hardware such as computers, I investigated the teachers' beliefs and use of mathematical software such as Geometer's Sketchpad (GSP) (Jackiw, 2010), GeoGebra, Desmos, used on computer or interactive whiteboard and TI-Nspire Navigator technology. Hence, cognitive technologies for my research are cognitive mathematical technologies.

2.2 Factors Affecting Selection of Technology

In this section, I discuss the main factors identified in research on technology in the classroom as having an influence on the technologies teachers choose to use. Researchers have posited a multitude of factors that may impact teachers' decisions about what technologies to incorporate into their lessons and how to incorporate these technologies. Some of these factors include access to technology, training and support

(Ertmer, 1999; Aygei & Vogt 2012; Cuban, Kirkpatrick, & Peck, 2001; Wong, Li, Choi, & Lee, 2008); knowledge of technology and its changes (Ertmer, 1999; Koehler & Mishra, 2009); mathematical considerations (Goldenberg, Scher, & Feurzeig, 2008; Koehler & Mishra, 2009); pedagogical considerations (Lin 2008; Russell et al., 2003; Niess, 2012), learning considerations (Niess, 2012), and time for planning lessons and teaching with technology (Ertmer, 1999; Wepner, Ziomek, & Tao, 2003). Each factor is discussed below.

Access to technology and training. Researchers (e.g. Goos & Bennison, 2008; Cuban et al., 2001) have indicated teachers with limited access to technology (e.g. lack of one-one access, and limited laboratory time for technology use) may not be able to incorporate technology into the classroom in meaningful ways. Goos and Bennison, (2008) found access to technology was significantly related to use of technology in the classroom, while Cuban et al. (2001) found many classrooms had only one computer, which may have led to the limited instruction with technology. Likewise, Judson (2006) conducted research in 32 classrooms and indicated teachers with less access to technology tended to underuse technology more than teachers with more access to technology. Furthermore, teachers who have access to technology also need access to ongoing training and support.

C. Kim, M. Kim, Lee, Spector, & DeMeester (2013), who conducted research on technology integration among 22 teachers, suggested that much technology training provided to teachers may involve no more than learning specific technology skills, such as learning to use particular computer software and or a particular tool. The lack of technology and training may be insufficient to lead to the use of technology in the classroom in appropriate ways (Koehler & Mishra, 2009).

Knowledge of technology. Ertmer, (1999) indicated even with access to technology teachers may lack knowledge of the technology itself and how to incorporate it into their lessons. Having knowledge of technology requires the teachers understand the technology in a broad enough way to productively apply it at work and in everyday life, and understand how the technology may assist or impede the accomplishment of a task (Koehler & Mishra, 2008). Niess (2009) suggested teachers should consider unique features of the technology. Knowing about the features of the technology may pertain to the ease of use, clarity of the visual presentation, and feedback provided to the user. Furthermore, knowledge of technology entails continuing to learn about changes in the technology. Given that digital cognitive technology is ever changing some teachers may have difficulties staying abreast of these changes and may underuse technology in the classroom (Cuban et al. 2001).

Mathematical considerations. Sherman, (2011) indicated teachers should consider how the mathematics is represented with various digital technologies. Additionally, teachers may also have to be aware that software developers face constraints regarding pixels and number of decimal places (Sherman, 2011), as these constraints could influence the ways students interpret the mathematical output from technology. Goldenberg et al. (2008) posited previously known mathematics might be changed with technology. Hence, teachers should consider how the technology reflects the properties of mathematical objects or possesses mathematical fidelity (Zbiek, Heid, Blume & Dick, 2007; Dick, 2008).

Pedagogical considerations. Teachers make pedagogical considerations when incorporating technology because this influences what and how to teach with technology.

Teachers should consider the capabilities of the technology and how these capabilities may be useful in helping them to develop the mathematical learning of the students. Having knowledge of the technology and content alone is not sufficient to teach effectively with the technology (Lin, 2008; Russell et al., 2003). Teachers also need “knowledge of instructional strategies and representations for teaching subject matter topics with technologies” (Niess, 2012, p.5). The presence of technology may change the way teachers think and communicate mathematics (Zbiek et al., 2007). It may not be easy for teachers to factor technology into their teaching since there is no single best way to integrate the technology into teaching (Koehler & Mishra, 2009).

Learning considerations. Teachers may also make learning considerations when incorporating technology because these indicate how students' use of technology influence learning with technology. Students may likely take certain actions with technology and teachers should consider how students think and learn with technology (Niess, 2012). The teacher should be cognizant of how the technology reflects the actions of students as they work with technology, that is, the cognitive fidelity of the technology (Bos, 2011; Zbiek, Blume, Heid, & Dick, 2007). Teachers who do not make the requisite learning considerations may not be able to use technology in appropriate ways.

Time. Many researchers (Becker, 1994; Cuban et al., 2001; Koehler et al., 2009) indicated the time for planning lessons and teaching with technology was one of the key factors determining whether or not teacher chose to incorporate technology in the classroom. For instance, Becker (1994) found time spent learning to use technology and time learning to teach well with technology, including planning for teaching with technology, was significantly related to teachers' making effective use of technology.

2.3 Beliefs

In this section, I discuss definitions of beliefs from various researchers' perspectives, make comparisons among the definitions and describe the definition I adapt for this study. Given some of the difficulties in defining beliefs emerge from ideas the beliefs construct is closely related to other constructs such as knowledge and conceptions, I first identify what some researchers have attempted to provide as definitions of beliefs and then differentiate beliefs from other closely related constructs. Finally, I detail particular beliefs researchers have described about technology and why I chose to study particular beliefs as the most relevant in this study.

Beliefs definitions. Pajares (1992) indicated researchers defined beliefs in a variety of ways. Definitions vary by researchers' areas of study, among other things (Pajares, 1992). In particular, because construct of beliefs is not well defined in research this results in what Pajares (1992) termed a 'messy' construct. Pajares (1992) further indicated a definition of beliefs is useful to distinguish it from other closely related constructs such as conceptions, and knowledge. Additionally, Philipp (2007) noted distinctions among beliefs and the related constructs are necessary because some researchers use these constructs interchangeably. Researchers (Furenghetti & Pehkonen, 2002; Philipp, 2007) have also indicated the construct of beliefs is not agreed upon by researchers and as such a researcher should determine what definition(s) of beliefs are relevant in his or her study. In the following discussion, I provide definitions some researchers in mathematics education and other fields have proffered. Some of these definitions were useful in shaping what I considered to be a definition of beliefs for my research.

Dewey (1933) posited belief is the third meaning of thought, which "... is marked by acceptance or rejection of something as reasonably probable or improbable" (p. 2). Hence, a belief has some degree of certainty. This distinction is important when trying to differentiate beliefs from similar constructs, such as knowledge. Abelson (1979) also defined beliefs in a way such that it is closely aligned with knowledge. He stated humans might manipulate a stored body of structured knowledge (set of propositions, concepts) under some contexts and for particular purposes; this may be considered beliefs in some circumstances. Hence, beliefs have particular characteristics related to knowledge in particular circumstances.

Some researchers (e.g. Thompson, 1984; Philipp, 2007) have described beliefs as a component of conceptions, which includes other components like concepts, meanings, rules, propositions, and even knowledge. On the other hand, some researchers (e.g. Pehkonen, 2004; Singletary, 2012) regarded beliefs encompassing conceptions as the cognitive component. Thus, the distinction between conceptions and beliefs is not clear in the literature. In the first set of definitions, beliefs is a subset of conceptions, and in the second set of definitions, conceptions is a subset of beliefs. To make the comparison between the constructs even less lucid, sometimes researchers (e.g. Thompson, 1984) used the terms beliefs and conceptions interchangeably. In doing so, these researchers seemed to think there was not a very big difference between these constructs.

Some researchers (Goldin, 2002; McLeod, 1992; Nespor, 1987) indicated belief is component of affect. Goldin (2002) indicated the affective domain includes beliefs, emotions, attitudes, and values. He proposed beliefs are constituted of aspects of feelings and moods, and not purely of cognition. Other researchers (e.g. McLeod, 1992; Nespor,

1987) shared the view of beliefs being a component of affect and posited there are some distinctions between beliefs and the other components of affect. Goldin (2002) described beliefs as having “multiple encoded cognitive/ affective configurations, usually including (but not limited to) prepositional encoding to which the holder attributes some truth-value” (p. 64). Thus, Goldin (2002) alluded to a critical distinction between beliefs and the other subdomains of affect; wherein beliefs have more cognitive component, the other subdomains have less of a cognitive component. In having more of a cognitive component, beliefs is more like knowledge, and in this regard, much research on beliefs do not include belief as a subdomain of affect (Philipp, 2007).

Some researchers (Cooney, Shealy, & Arvold, 1998; Leatham, 2006; Philipp 2007; Rokeach, 1968) considered beliefs as related to the actions of the individuals. For instance, as Rokeach (1968) described a belief as "any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase, 'I believe that . . . '" and indicated beliefs were predispositions towards action. Other researchers have considered beliefs as dispositions towards action and had the power to motivate an individual (Cooney, Shealy, & Arvold, 1998; Leatham, 2006). Philipp (2007) defined belief as “psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action” (p. 259).

The definitions of beliefs as related to actions and the distinctions between beliefs, knowledge, and conceptions are pertinent to my research because I carried out observations as a part of my study. Many inferences I drew from the observations about

the teachers were reflected as their beliefs. For my research study, I include a cognitive as well as an affective component in beliefs and therefore my perspective is more consistent with Goldin's (2002) that the beliefs comprise part cognitive and part affective aspects. I agree with Philipp's (2007) idea of beliefs being lenses through which one views the world and with Rokeach's (1968) as predispositions towards action. As I conducted research via interviews and observations, I held beliefs may be inferred from what a person says or does; being the thoughts or ideas a person holds to be 'true' without needing consensus.

Belief Systems. Thompson (1992) stated a belief system is “a metaphor for examining and describing how an individual's beliefs are organized” (p. 130). Some researchers (Green, 1971; Leatham, 2006; Rokeach, 1968) have made suggestions of how belief systems may be organized. For example, in a belief system, there exist central (or core) beliefs and peripheral beliefs (Rokeach, 1968; Green, 1971). The more central a belief is, the harder it is to change the belief in light of new evidence challenging it. Beliefs that tend to be more central are ones shared by others and about the existence of a person (Leatham, 2006). On the other hand, the more peripheral a belief is the easier it is to change. Beliefs that tend to be more peripheral are those arbitrarily a matter of taste and those derived from association with a group (Leatham, 2006). Green (1971) represented ideas of central and peripheral beliefs as concentric circles with the more central beliefs or core beliefs in the center. Leatham (2006) indicated the centrality of a belief relate to the psychological strength of the belief, so the more a belief is the stronger it is psychologically.

Beliefs are also classified according to their quasi-logical strength (Greene, 1971). In this regard, some beliefs are termed primary beliefs and others derivative beliefs (Green, 1971; Thompson, 1992). For example, consider the following two beliefs: 'students should be able to sketch graphs' and 'students should not use graphing calculators.' It is entirely possible there may be a logical relationship between these two beliefs. Thus, a teacher wants students to be able to sketch graphs, and the students should not be allowed graphing calculators. In such case 'students should be able to sketch graphs' may be said to be the primary belief and 'students should not use graphing calculators' is the derivative belief. The primacy and centrality of beliefs are two different properties. A derivative belief may be more central than the primary belief (Philipp, 2007). For example, in the case of the graphing calculators belief 'students should not use graphing calculators' and the sketching of graphs belief 'students should be able to sketch graphs', whereas the second belief 'students should not use graphing calculators' is a derivative belief, it may be more central than the first belief. So despite if there is some sketching of graphs to be done, the teacher may never allow the students to use graphing calculators.

Researchers (e.g. Green, 1971; Leatham, 2006) posited beliefs are held in clusters in a beliefs system. For instance, Leatham (2006) found this clustering helps understanding how teachers' beliefs and practice may appear inconsistent, or that a teacher may appear to hold contradictory beliefs at the same time. Further, Leatham (2006) used clustering of beliefs to help explain how a teacher may act one way in one context and a different way in another context. This clustering of beliefs idea may be important to how teachers in my study act under various contexts when teaching with

technology. Leatham (2006) argued a teacher's beliefs are based on sensible belief systems. This means teachers have the tendency to resolve conflicts in their belief system, which seem inconsistent to the observer.

Reasoning about the centrality and primacy of beliefs was important to my study of the commonalities of various beliefs teachers hold and their practice, because the ways teachers hold particular beliefs may be displayed in their teaching and discussions with the interviewer. Because I investigated commonalities among beliefs, I had to decide whether or not the beliefs held by the teachers should be assumed to form sensible systems. I identified the beliefs of the participants and shared the apparent beliefs with them so they could point out which beliefs appeared not to fit with what they were doing. Thus, I gave participants the opportunity to respond and verify whether beliefs they held in their particular contexts were line with what I stated.

Two types of teachers' beliefs I investigated in this dissertation are: (1) beliefs about teaching mathematics (Cooney et al., 1998; Spangler, 1995; Conner, Edenfield, Gleason, & Ersoz, 2011; Ernest, 1989; Raymond, 1997), including beliefs about learning (Ambrose, Clement, Philipp & Chauvot, 2004; Cooney et al., 1998; Ernest, 1989), and (2) beliefs about teaching mathematics with technology, using metaphors developed by Goos et al. (2003). For my research, the beliefs teachers' hold may make sense, to them, in relation to the other beliefs and thus teachers were assumed to have a coherent or sensible system of beliefs systems in line with Leatham's (2006) ideas.

Beliefs and practice. Thompson (1992) suggested characterizing a teacher's conception of mathematics by examining the teacher's professed beliefs, the instructional setting, the teacher's practice, and the relationship between the teacher's professed views and

practice. She noted two types of beliefs directly related to practice are beliefs about mathematics, and beliefs about teaching mathematics. Pajares (1992) furthered the argument on relationships between beliefs and practice by stating belief is one of the strongest predictors of teacher practice, even stronger than knowledge. Wilkins (2008) supported this idea in a study to uncover the association between teacher practice and other variables such as mathematical content knowledge and beliefs about inquiry teaching. He found the strongest predictor of teacher practice was teacher beliefs.

Despite beliefs being described as a predictor of practice, much research on teachers' beliefs about teaching and learning, and their practice has shown inconsistencies between teachers' professed beliefs about teaching and learning and their practice (Cooney & Wilson, 1995; Ertmer, 2005; Raymond, 1997; Richardson 1996; Thompson, 1992). In Raymond's (1997) study, one of the participants, Joanna, held beliefs about mathematics teaching and learning such that students should learn by working in groups, talking to each other, and solving problems. However, Raymond (1997) observed Joanna invariably would not allow her students to work in groups or speak with each other, but instead led a rather teacher-directed class. Thus, the practices of Joanna appeared inconsistent with her espoused beliefs about teaching and learning. Her practices may have been more related to her beliefs about some other idea not specified in the study, such as the time factor for teaching content, or a deeper aspect of mathematics pedagogy, than simply about teaching and learning mathematics in general.

Greene (1971) posited beliefs are held in clusters with varying degrees of strength, and this idea could account for the apparent inconsistencies between some teachers' beliefs and practice. Building on this idea, Leatham (2006) proposed no

inconsistencies exist if all the beliefs about the factors that pertain to the observed teacher practices are considered. For example, in the case of Joanna in the Raymond (1997) study, if the beliefs about other factors that may have influenced actions, as well as all mathematics-related beliefs had been taken into consideration by the researcher, then what are seen as apparent inconsistencies may not be so if other beliefs are held more strongly than beliefs about teaching mathematics. For my research, the idea that beliefs are held in clusters and the teachers resolve inconsistencies in their belief systems are included. In this light, I investigated mathematical and technology related beliefs held by teachers on the basis these beliefs likely influenced teaching mathematics with digital cognitive technologies. Additionally, I allowed teachers to member check the ideas I held about their beliefs.

Beliefs about teaching mathematics. Adapting Rokeach's (1968) definition of beliefs, pedagogical beliefs about mathematics may be defined as teachers' consciously or unconsciously held assumptions about the way mathematics should be taught to their students. Many researchers (e.g. Ernest, 1989; Ertmer & Ottenbreit-Leftwich, 2010) have indicated the importance of teachers' pedagogical beliefs in teaching mathematics. For instance, Ertmer and Ottenbreit-Leftwich (2010) acknowledged teachers' pedagogical beliefs are key to the enactment of activities in the mathematics classroom. Ernest (1989) suggested three categories of teacher pedagogical beliefs that may influence their teaching of mathematics. His model of teacher beliefs about teaching mathematics included beliefs about the role of a teacher being instructor, facilitator, or explainer.

Other researchers (Stipek, Givvin, Salmon, & MacGyvers, 2001; Wood, Cobb & Yackel, 1991) have indicated a dichotomy of teachers' beliefs about mathematics. For

instance, Stipek, Givvin, Salmon, and MacGyvers (2001) indicated beliefs about teaching mathematics ranged from beliefs the teacher should be in control of the mathematics given to the students to beliefs the students should develop the mathematics by inquiry. Similarly, Wood, Cobb, and Yackel (1991) ascribed beliefs ranging from transmission of mathematics content to initiating and guiding students' mathematical knowledge. Beliefs that the teacher should be in direct control and transmission of mathematics beliefs would be akin to beliefs about the teacher's role of an instructor. Beliefs the teacher should be as guide in more inquiry-oriented mathematical classroom situations is akin to the beliefs in the role of teacher as facilitator. The belief of teacher in the role of explainer may be aligned either with beliefs the teacher should be in direct control of all the mathematical activities or with beliefs the teacher should initiate activities and guide students' knowledge in mathematics.

Beliefs about teaching mathematics with technology. Beliefs about teaching mathematics with technology may be defined as teachers' consciously or unconsciously held assumptions about when to use, how to use and why particular technology should be used in teaching mathematics. Leatham (2007) suggested many teachers hold beliefs about whether technology should be included in mathematics lessons before or after the mathematical concepts are taught by hand (without technology). Teachers, who hold beliefs technology should only be used after the mathematics is taught may also hold beliefs that technology should be assimilated with traditional ways of teaching; hence teachers may hold beliefs technology should serve as add-on to their prior instructional strategies (Palak & Walls, 2009). This scenario indicated these teachers may also hold

beliefs about teaching with technology in ways that technology serves as Master or Servant (Goos et al., 2003).

Teachers who hold beliefs technology should be used before teaching mathematics by hand also tend to hold beliefs technology should support teaching more conceptually. Hence these teachers may hold beliefs technology should be used in ways such that it helps to develop deeper conceptual understanding of mathematics. This scenario would indicate the teachers also hold beliefs about teaching with technology in ways the technology is used as partner or extension of self (Goos et al., 2003). Leatham (2002) indicated one might hold beliefs that technology also plays non-mathematical roles, such as motivation, in addition to beliefs pertaining to the development of conceptual understandings, and the quick completion of procedures.

Summary of included beliefs. From the literature, it appeared teachers beliefs about teaching mathematics with technology occur in relation to their teaching with mathematics in general. As such teachers may decide to include technology before or after they teach mathematics without technology (e.g. Leatham, 2006). Teachers beliefs may include teaching with technology the same content they taught without technology, as well as use technology to reinforce their prior practices without technology (Palak & Walls, 2007). It is also likely that some teachers may include different content with technology than content without technology or teach differently with technology than without technology. I developed a diagram (Figure 2), which showed possible intersections of beliefs about teaching mathematics and beliefs about teaching mathematics with technology.

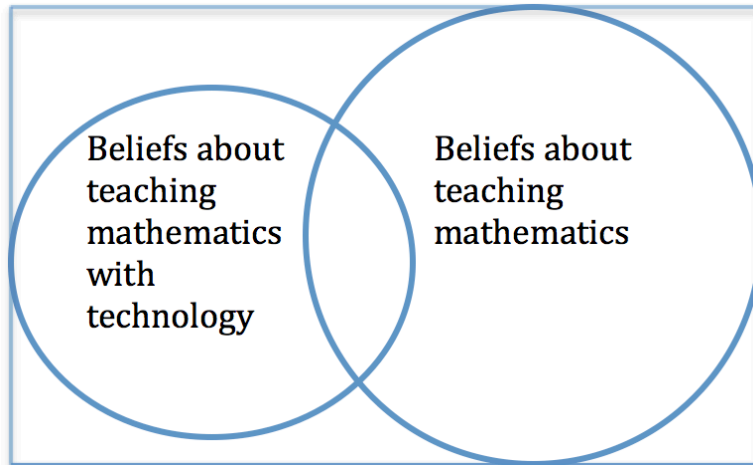


Figure 2. Beliefs about Teaching Mathematics and Teaching Mathematics with Technology

2.4 Knowledge of Teaching and Technology

In this section, I discussed research on knowledge because the knowledge construct is often intertwined with beliefs and is a component of many frameworks for teaching with technology (e.g. Technological, Pedagogical and Content Knowledge (TPACK)). Here, I outlined some perspectives on teacher knowledge that have become important in mathematics education. These included Pedagogical Content Knowledge (PCK), Mathematical Knowledge for Teaching (MKT) and Mathematical Understanding for Secondary Teaching (MUST). Finally, I addressed knowledge for teaching with technology.

Teacher Knowledge. Researchers (e.g. Mishra & Koehler, 2006; Niess, 2005; Pierson, 2001; Zhao, 2003) have suggested teacher knowledge is important for successfully integrating technology into the classroom. These same researchers have indicated knowledge for teaching with technology is linked in ways similar to Shulman's (1986) way of using the term Pedagogical Content Knowledge (PCK). PCK refers to the special

kind of knowledge teachers need to teach effectively. Since PCK was not sufficient to describe knowledge for teaching mathematics effectively, however, Ball and colleagues (Ball, 2007; Ball & McDiarmid, 1990; Ball, Hill, & Bass, 2005; Ball, Thames & Phelps, 2008) introduced the term Mathematical Knowledge for Teaching (MKT) to describe the knowledge for teaching mathematics effectively. Their research then set the stage for other researchers (e.g. Silverman & Thompson, 2008; Heid et al., 2015) to subsequently refine the forms of knowledge and understandings needed for teaching mathematics. For instance, Heid, P. Wilson, J. Wilson, Kilpatrick, Blume, and Zbiek (2016) forwarded a framework, Mathematical Understandings for Secondary Teachers (MUST) which indicated the knowledge and understandings needed to teach mathematics effectively at the secondary level.

Given that MUST and many other knowledge for teaching mathematics frameworks, did not have a technology focus, other researchers (e.g. Angeli & Valanides, 2005; Niess, 2005; Mishra & Koehler, 2006), which incorporate knowledge of teaching with technology, are discussed here. For instance, Angeli and Valanides (2005) introduced the acronym ICT-related PCK, which included content, pedagogy and technology components. Niess (2005) and Mishra and Koehler (2006) introduced 'technological, pedagogical, content knowledge,' these ideas which have over time merged to become Technological Pedagogical and Content Knowledge (TPACK).

Koehler and Mishra (2009) argued for teachers to teach effectively with technology, they should have an integrated knowledge of technology, content, and pedagogy, which is a new kind of knowledge: Technology, Pedagogical, and Content Knowledge (TPACK). Smith, Kim, and McIntyre (2016) investigated connections

between teachers' knowledge (TPACK) and their beliefs about teaching and learning with technology. A related finding of their study to this one was the existence of relationships between higher-level beliefs and advanced levels of TPACK.

Teacher knowledge versus beliefs. I make attempts to differentiate between knowledge and beliefs in this research because data collected included include these two very closely intertwined constructs (Smith, Kim, & McIntyre, 2016). Because the data collected on beliefs likely included knowledge components it was useful to distinguish between the two constructs for clarity of analysis. I distinguished beliefs from knowledge in three ways: degree of conviction, consensus, and affective components.

Researchers (e.g. Abelson, 1979; Pajares, 1992; Thompson, 1992) posited beliefs may be held with varying degrees of conviction whereas knowledge is not held with varying degrees of conviction. For example, one might say he strongly believes constructivism is the best epistemology, but would not say he strongly knows a multiplication fact. Beliefs are not consensus driven, meaning one can believe something about which there is no general agreement, whereas knowledge typically requires consensus from a community (Pajares, 1992). Beliefs are generally associated with more evaluative components, whereas knowledge is associated with more cognitive components (Abelson, 1979; Nespor, 1987; Philipp, 2007). Hence, beliefs tend to consist of categories of thinking that may be alluded to in one way or another as themselves 'good' or 'bad' or leading to something 'good' or 'bad'. However, this is not so in the case of knowledge (Pajares, 1992).

CHAPTER 3: RESEARCH DESIGN AND METHODS

This chapter details the methods I used to answer the research questions. The research design involved qualitative case study methodology. The data collection methods included interviews, observation field notes, and supporting documents including teachers' lesson plans, teacher technology files, and mathematical task activities. Member checking was used to corroborate the information garnered by the interviews. What follows are the reasons I chose to use particular methods, how these methods helped to answer the research questions of the study, and the frameworks related to specific research questions (see Table 2).

3.1 Research Design

Denzin and Lincoln (2003) indicated a study of a qualitative nature is beneficial when the researcher seeks to provide rich illustrations and to detail processes in context. Furthermore, a qualitative design is applicable when the researcher seeks to understand the meanings participants hold about their experiences within a particular context and the process by which the participants' actions take place (Maxwell, 2005). Because my study involved the beliefs of teachers, a qualitative research design helped me to gain an understanding of the meanings teachers hold concerning their espoused views about teaching mathematics, and using technology to teach mathematics. Furthermore, because I investigated the teachers' use of technology, qualitative research methods are ideal for detailing their use of technology in the context of the mathematics classroom. One type of qualitative research design I use is the qualitative case study.

Table 2: Research Questions and Data Collection

Research Questions	Data Sources	Frameworks
1. What factors do teachers consider in choosing technology for use?	Participant Selection Observation Beliefs & Technology Interview Prelesson Interview Lesson Plans	Pea (1987) Goos et al. (2003)
2) What digital cognitive technologies do teachers use? 2a) How do teachers' use digital cognitive technologies?	Participant Selection Observation Observation Field notes Postlesson Interview Mathematical Task Activities	Pea (1987) Goos et al. (2003)
3) What are teachers' beliefs about teaching mathematics, and teaching mathematics with technology? 3a) What commonalities are observed among teachers' beliefs and their use of digital cognitive technologies?	Beliefs & Technology Interview Observation Field Notes	Ernest (1989) Goos et al. (2003)

For this research, I used multiple case study format (Yin, 2003) with one case for each of three teachers. Using three cases instead of a large number of cases helped to provide for greater depth of information. Yin (2003) indicated multiple case studies provide more compelling evidence or robustness to the conclusions drawn from the data, than single case studies. I chose to study three teachers because of the limited resources I had when dealing with the study. For example, time was a limiting factor. A small number of cases is substantiated by research (e.g. Yin, 2003) indicating that too many cases may not provide additional evidence to answer the research questions. Yin (2003) indicated selecting two or three cases for analysis is sufficient when the researcher is not interested in ensuring certainty (such as in a causation), or if the rival theories would lead to different results. In my study there certainty or causality was not sought, so using three studies should be sufficient to infer a correlation among the participants' beliefs about

teaching mathematics, about the use of technology for teaching mathematics, and the actual use of technology in the classrooms investigated.

The first and second research questions in this study concerned the way cognitive technologies are selected and used in the classroom by the teachers. Yin (2003) pointed out case studies were useful for research questions that inquire into ‘how’ something is done. Because this study is concerned with how teachers use technology in practice, case study is applicable for examining and documenting evidence of teachers’ actions in their naturalistic setting as they undertake the challenging and dynamic process of teaching mathematics with technology.

In the third research question, I sought to identify specific beliefs exhibited by teachers. Some researchers (Leatham, 2006; Pajares, 1992; Richardson, 1996) indicated beliefs should be elicited by multiple sources because beliefs are not visible, and are difficult to access. The semi-structured interview used in a case study is one of the best ways to access a teacher's views, and hence their beliefs (Patton, 1980). I used multiple interviews of varying types and durations in my study. The interviews were used at the beginning of the study, before and after lessons, and at the end of the study. Additionally, I conducted classroom observations in part to check whether teachers’ espoused beliefs are displayed in practice. I also conducted member checks to allow the participants to corroborate my interpretation of their beliefs. Because my research focused on commonalities among beliefs, it was necessary to know how each teacher interprets my statements of their beliefs, that is, whether I interpreted their data in the way they desired.

The third research question examined the commonalities among teachers’ beliefs, technology use and choice of technology. This examination entailed looking for patterns

in the data across the cases of teachers and searching for similar results, so the use of more than a one case study was beneficial for detailed comparison (Yin, 2003). For my study, the evidence provided by patterns of beliefs and technology use in data across participants is what I hold to be the commonalities. Hence, I used a multiple case study design to assimilate depth of information about the participants and to examine commonalities among the participants. The use of multiple cases and finding the commonalities lead to an overall pattern for all the cases. Yin (2003) identified this characteristic as replication logic. Replication logic is the idea that each case can be regarded as a single experiment. When an important result was achieved in one case, then the next step is to check for this result in subsequent cases, to find comparable results. Replication increased the possibility of attaining certain results over a number of cases (Yin, 2003).

3.2 Researcher Perspective

Merriam (1998) indicated the researcher is the primary instrument of data collection and analysis in a qualitative study, and therefore the researcher must consider how his or her perspective influences the research. I was cognizant my biases may influence the process of data collection and the analysis of data. In fact, my beliefs may have impacted the data I collected through observations and interviews. Thus, I found it necessary to consider my beliefs related to teaching mathematics and using technology for the teaching of mathematics.

As a mathematics teacher and user of cognitive technologies, I was intrigued by the different uses of cognitive technologies in mathematics. I believed technology is used in a more powerful way when it is used as a tool for thinking rather than just a tool to

calculate procedures or to play games. I believed that whenever possible cognitive technologies should be used as exploratory tools to ‘play with the mathematics’ (Cuoco, Goldenberg, & Mark 1996) and as a means for making sense of mathematics. In my classroom, I encouraged the use of digital cognitive technologies. I have seen mathematics appear to ‘come alive’ for learners, particularly when using sliders and animations with dynamic mathematics software. These learners tried to make better sense of what was happening with the mathematics while working with the technology, as I played the role of a guide. I held the idea that teacher’s beliefs impact when and how digital cognitive technology is used in the mathematics classroom.

As a researcher I was interested in how teachers used cognitive technologies in their lessons with their students and, the specific beliefs that appear to impact their use of cognitive technology and the meanings teachers hold about the use of cognitive technologies. I understood the beliefs I held may act as filters through which I saw the participants’ beliefs, and their use of technology. Hence there existed the possibility for me to combine my own beliefs with the data I collected from the participants. Because of this possibility, I was careful when interpreting the beliefs of the participants in my study. As a result, I asked participants to check my written narratives concerning their beliefs about teaching mathematics with technology and about teaching mathematics.

3.3 Participant Selection

I initially emailed eight high school principals in the northeast Georgia area to request their schools’ involvement in this study. In this email I introduced myself as a researcher, informed the principals of the topic and nature of my study, indicated how my study may impact mathematics teaching, and requested the principal’s permission for the

school be included in my study (See Appendix G). Of these eight principals, two agreed to allow the study to be conducted in their schools. These principals asked the heads of the mathematics departments to arrange for a meeting with me to discuss the study. After my meeting with the department heads, they contacted all their mathematics teachers and created a list of teachers for me to contact. I emailed 15 teachers and eight of them agreed to let me observe them teach a mathematics lesson that included their use of technology.

From these initial observations, I chose three teachers to participate in this study. I based this decision on two factors relating to the participant observation worksheet that I had developed prior to the study: the number of yeses marked, and on the statements noted in the evidence column of the worksheet during the observation. The worksheet was designed such that the more yes boxes checked the more there was an indication of use of technology tending towards a higher level such as partner (Goos et al., 2003) or Reorganizer (Pea, 1987). The evidence displayed by the teacher was some indication of the strength of the level of technology use (See Appendix A). I also considered their willingness to continue in the study as a factor in determining the final selection of participants.

3.4 Development of the Participant Observation Worksheet

Before I conducted the study I had developed a Participant Observation Worksheet (Table 3). The purpose of the worksheet was to serve as a guide to identifying possible uses that teachers might make of some digital cognitive technologies in their classrooms. In order to develop this worksheet I observed three lessons of each of two teachers (Teacher A and Teacher B), who used digital cognitive technologies in their mathematics teaching and taught content similar to secondary level mathematics. Teacher

A used Geometers Sketchpad (GSP) and Teacher B used GeoGebra. I chose them because I knew they incorporated technology in their lessons and other teachers and students recommended them as being avid users of technology. I consulted with them before the observations to find out which days they would be teaching technology integrated lessons and would be willing to have me observe their lessons. I observed both their classes with an open mind and with the intent of capturing as much as possible their actions with technology.

I wrote field notes focusing on the teachers' actions during their teaching with technology for all of the six lessons. In my writings, I noted when and how the teachers personally used technology and prompted students to engage with technology either individually, in groups, or as a whole class. I also made notes about how the teacher allowed students to engage in the mathematical content with technology and any discussions regarding the content with technology. Also, I noted whether the teacher or students did presentations with technology, how mathematical discussions occurred around these presentations with technology, and who critiqued these presentations with technology.

Upon cross comparison of the two teachers' actions with technology, I highlighted twelve important actions that both teachers took over the three observations. I used these 12 actions as the means to later determine which actions involving technology would be amenable to high use which is the use of technology by the teacher at the level of Reorganizer or at least as Partner during a mathematical activity. The converse, that of the low use, which is the use of technology as an Amplifier and up to the level of at most

Servant, would also be captured in the evidence provided in the participant observation worksheet.

During the development of the participant observation worksheet I identified aspects of technology use which might determine whether the teachers' use could be regarded as amplifier, including as Master or Servant on the one hand or as Reorganizer, including as partner or extension of self on the other hand. This was done because the teacher could use the same technology in various modes in one lesson. For example, since the dynamic geometry environments used could be for display or making quicker the computations of mathematics, my observation included whether the teachers focused on using technology for multiple representations, making connections among representations, interpretation of the mathematics and for focusing discussion centered on a mathematical understanding of the concepts. Thus, my observations indicated whether the teacher focused on the affordance of technology as a Reorganizer of mathematical knowledge (Pea, 1987) or as partner (Goos et al., 2003) versus whether the teacher focused on the use of technology as an Amplifier (Pea, 1987), Servant or Master (Goos et al., 2003).

In addition to identifying actions in which teachers appeared to use technology as Amplifier or Reorganizer (Pea, 1987) and as servant, master, or partner (Goos et al., 2003), I also identified teacher actions that indicated whether teachers intended to have a technology-incorporated lesson. For example, in the Participant Selection Observation worksheet, in the first teacher action namely, 'verbally directs students to use technology'; the action of the teacher was not specific to any particular use of technology

but served as an indicator to an observer that the teacher had intentions to have students engage with technology in the lesson.

Table 3: Observation Worksheet (partial)

Teacher Action	Yes	No	Evidence/Reason
1. Verbally directs students to use technology	√		E.g. Teacher instructs students to go to GeoGebra, create a triangle and label Points A, B, C
2. Allows students to construct their own figures (shapes, graphs, spreadsheets, etc.)	√		E.g. Teacher waits a few minutes while students open GeoGebra and construct triangles. Teacher asks whether students are having difficulty opening GeoGebra and constructing the triangle

If the teacher gave students specific directions concerning what should be done with the technology, I noted this in the evidence section of the observation sheet. I corroborated the information gathered from the observation of the teachers' lessons with research on teaching with cognitive technology (Goos et al., 2003; Pea, 1987). Thus, the corroboration of the in-class observations and readings of research on technology use resulted in the participant observation selection instrument (partially shown in Table 3).

3.5 Interviews

In the book *Research Design; Quantitative, Qualitative and Mixed Methods Approaches*, Creswell (2013) indicated that “ the data collection in a case study is typically extensive, drawing on multiple sources of information such as interviews, observations, documents and audiovisual materials” (p.185). I conducted four types of interviews: a belief and technology selection interview, a pre-lesson interview, a post-lesson interview, and a final beliefs interview. The frequencies for each of these interviews varied. For instance, the beliefs and technology selection interview was

conducted once with each of the three participants. The pre-lesson interview was conducted before each lesson with each teacher, so there were three interviews with each teacher over three lessons. Thus, there were a total of nine pre-lesson interviews. The Postlesson interviews followed the same pattern of the Prelesson interviews and but were conducted after each lesson observed. The final beliefs interview was conducted once for each teacher. A summary of the data sources can be seen in Table 4.

Table 4: Data Sources

Name	Focus of Data Collection			Frequency
	Factors	Technology Use	Beliefs	
Participant Selection Observation		X		Once
Beliefs and Technology Selection Interview	X		X	Once
Prelesson Interview	X		X	Before each lesson observed
Observation Field Notes		X		During each lesson observed
Postlesson Interview	X	X	X	After each lesson observed
Final Beliefs Interview			X	Once

I used the interviews as a data collection method because of the complex nature of teachers' beliefs and use of digital cognitive technologies. Furthermore, interviews allowed for the capture of the depth of information that would enable the commonalities among particular beliefs and the teachers' choice and use of technology to be determined. Another benefit of using the interview as a method of data collection is that I gain first-hand information about the way a teacher selects and uses technology in the mathematics classroom. Although my classroom experiences may have become enmeshed with the

experiences of the participant, I was able to gather meaningful information about the participants' experiences of teaching mathematics with cognitive technologies.

3.5.1 Beliefs and Technology Selection Interview. The purpose of this interview was to explore the beliefs that the participants held about teaching mathematics and teaching mathematics with technology. Furthermore, this interview delved into the factors that affected the participants' selection of technology as well as the experiences that they had teaching mathematics with technology. This interview was used mainly to answer research questions 1 and 3 and was conducted before the first prelesson interview and the observations. This interview was semi-structured and lasted approximately 60 minutes. It was longer in duration than subsequent interviews because it involved gathering more in-depth information about the teacher. Furthermore, some information in this interview was necessary to collect only once per teacher. The interview was video recorded. Sample questions from the interview are provided in Figure 3, and the protocol for this interview can be seen in Appendix B.

Consider the following similes¹ (I will provide the words as cut outs):
A mathematics teacher is like:
News broadcaster, Entertainer, Doctor, Orchestra conductor, Gardener, Coach
a. Which simile that you believe best describes a mathematics teacher? Explain your choice.
b. Which simile that you believe does NOT describe a mathematics teacher very well? Explain your choice.

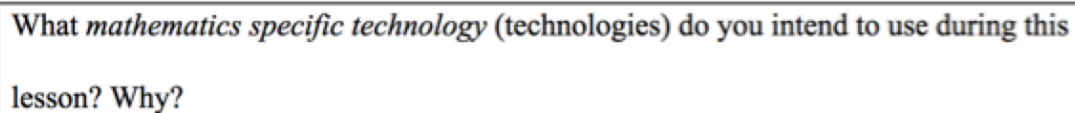
Figure 3. Sample question from Beliefs and Technology Selection Interview

The interview questions for the beliefs about teaching were based on the works of Cooney, Shealy, Arvold, (1998) and Raymond (1997). Cooney, Shealy, and Arvold (1998) investigated teachers' beliefs about mathematics, teaching, and learning. Raymond

(1997) investigated beginning teachers' beliefs and mathematics teaching practices. The beliefs about technology and teaching with technology sections of the interview instrument were based on the work of Landry (2010) who examined teachers' beliefs about technology.

The interview questions for technology selection section of the first interview were developed from aspects of the works of Sherman (2014) and Goos et al. (2003). Sherman examined the relationship between cognitive demands of mathematical tasks in the presence of digital cognitive technology. Goos et al. (2003) examined the ways that teachers and students used cognitive technology in the classroom.

3.5.2 Pre-lesson Interview. In the pre-lesson interview, I gathered information pertaining to the teachers' selection and intended use of technology for the lesson I observed. This semi-structured interview was video recorded and lasted approximately 10 minutes. The protocol for this interview can be seen in Appendix C. This interview was designed to gather information on the role the teacher intended to play and the expected role of the students when they used technology to engage with various mathematical activities. The information served as guide for the post-lesson interview with the teacher. The questions for the pre-lesson interview were developed based on works of Pea (1987) and Goos et al. (2003). Sample questions from the prelesson interview are provided in figure 4:



What *mathematics specific technology* (technologies) do you intend to use during this lesson? Why?

Figure 4. Sample question from Prelesson Interview

3.5.3 Post-lesson Interview. In the second interview, which occurred after each lesson, I focused on clarifying ideas about the use of technology during the mathematical activities. This interview was approximately 10 minutes in duration and was semi-structured. This interview was video recorded. The protocol for this interview is found in Appendix D. During this interview, I asked participants questions about how they thought the lesson went with regard to the way technology was used during the lesson. The participants described ways technology helped to shape the lesson. The questions for the post-lesson interview were developed based on Pea's (1987) and Goos et al.'s (2003) frameworks. Sample questions from the post-lesson interview are provided in Figure 5.

How could you or your students use (name of digital cognitive technology) in different ways to help the students with concepts they may not have grasped?
- How would your role have changed?

Figure 5. Sample question from Postlesson Interview

3.5.4 Final Beliefs Interview. The purpose of this interview was to gather more in-depth information about the teachers' beliefs based on their enactment of their mathematics lessons with technology. The information in this interview was necessary to collect only once per teacher. The final beliefs interview (see Appendix E) with each teacher was semi-structured, lasted approximately 45 minutes and was video recorded. Similar to the first beliefs interview, the interview questions for the beliefs section of this interview were based on the ideas from Cooney, Shealy, and Arvold (1998) and Raymond (1997), while the beliefs about technology section was based on the work of Landry (2010).

Sample questions from this interview are shown in figure 6 below.

In what ways do you think technology may have helped students' thinking during the lessons?

- Why do you think so?

Figure 6. Sample question from Final Beliefs Interview

3.6 Observation Field Notes

During each of the lesson observations, I made jottings (Emerson, Fretz, & Shaw, 1995) about the events that unfolded in the classroom. These notes were about elements of the lesson important to the research, such as teacher actions with technology. The field notes were developed from written jottings that focused on the way teachers used and facilitated students' use of digital cognitive technologies during the mathematical activities. I recorded sequences of events, the nature of the mathematical activities, the pedagogical moves of the teachers concerning using technology, and facilitating students to use technology in the classroom. Examples of written jottings included notes about the teacher demonstrating a solution to a problem using technology. The jottings were written in a shorthand form so I could remember the events as they occurred when I wrote a much more detailed form of the notes after each of the lessons.

The field notes were included in conjunction with the other data collection techniques when interpreting data. Including field notes in the collection gave me multiple means of gathering data for the research, and thus multiple sources from which to interpret to ensure that the interpretation was fair. Thus, the field notes served the

purpose of triangulation of data sources, that is, to supplement and validate the data gathered from interviews.

3.7 Supporting Documents

The supporting documents I collected for this study include teachers' lesson plans (very limited), mathematical activities, and teacher technology files. The lesson plans served as verification of the teachers' choice of technology, and a basis for connecting the ideas they reflected upon after they taught the lessons with digital cognitive technologies. The lesson plans also serve as a means for triangulation of data collected by the other instruments.

The mathematical activities included tasks the teachers provided for the students to engage with each lesson. They included the instructions on electronics and (sometimes paper copies) the students should relate to in order to do the mathematics the teacher expected of them. These served as a basis for understanding how the teachers used and allowed students to use technology in the lesson. The teachers' views about use of technology and their use of technology may also be associated with the mathematical activity, so the mathematical activity may be used to give insight about the teachers' moves in the classroom with technology. Teacher technology files included previously saved files on computer that the teacher used from time to time and these served as a basis for the way teachers use technology in the classroom. The participants emailed links to mathematical activities and some technology files either before or after the lessons taught. If instructions were also provided on a paper worksheet the participant provided me a copy.

3.8 Timeline for Data Collection

Table 5: Timeline of Data Collection

Date In 2016	Item	When item was done
March 25	Beliefs & Technology Selection Interview for Beth	After school
April 8	Prelesson Interview 1, Beth, Field notes 1, Beth, Postlesson Interview 1, Beth	Before, during and after Beth's first lesson respectively
April 12, 15	Beliefs & Technology Selection Interview for Sam Sam - Prelesson Interview 1, Field notes 1, Postlesson Interview 1	After school Before, during and after Sam's first lesson respectively
April 19, 21, 22	Beliefs & Technology Selection Interview for Ruth Ruth - Prelesson Interview 1, Field notes 1, Postlesson Interview 1 Beth - Prelesson Interview 2, Field notes 2, Postlesson Interview 2	After school Before, during and after Ruth's first lesson respectively Before, during and after Beth's second lesson
April 26	Ruth - Prelesson Interview 2, Field notes 2, Postlesson Interview 2 Sam - Prelesson Interview 2, Field notes 2, Postlesson Interview 2	Before, during and after Ruth's second lesson respectively Before, during and after Sam's second lesson respectively
May 2	Beth - Prelesson Interview 3, Field notes 3, Postlesson Interview 3	Before, during and after Beth's third lesson respectively
May 12	Ruth - Prelesson Interview 3, Field notes, Postlesson Interview Sam - Prelesson Interview, Field notes, Postlesson Interview	Before, during and after Ruth's third lesson respectively Before, during and after Sam's third lesson respectively
May 22	Final interviews	

The data for this study were collected from March through May 2016 and are listed in Table 5. For some days, I was able to observe two teachers lessons with technology, but for the most part, only one teacher was available on a particular day. The initially agreed upon dates for observations had to be changed many times before getting to the dates displayed.

3.9 Method of Analysis

Merriam (1998) indicated that an analysis of data is done to make sense of the data collected. I used thematic analysis (Braun & Clarke, 2006) for the analysis of the data in my study. In thematic analysis, the information is organized into themes based on codes that were developed from the data. Braun and Clarke (2006) stated, “A theme captures something important about the data in relation to the research question, and represents some level of patterned response or meaning within the data set” (p. 82). Hence, the basis for determining the themes is how the data answered my research questions. Furthermore, because my research intended to capture commonalities in participants’ ideas, thematic analysis was done because the nature of thematic analysis is the search for commonalities. As Braun and Clarke, (2006) stated, “TA is a way of identifying what is common to the way a topic is talked or written about and of making sense of those commonalities.”

After I completed video recording each of the interviews, I fully transcribed the recorded data. I listened again to the recordings and checked the transcriptions were correct. This was done because accuracy of transcription meant a more accurate analysis. I regarded the transcription phase as the first phase of analysis. My transcription of the data set the stage for the coding aspect of the analysis. A section of the organization table

for dealing with the analysis of the interviews can be seen be is shown in Table 6, while the full table is shown in Appendix F.

I used this tabular organization as a basic guideline for all the interviews. Thus, in the first section, after I inserted the entire transcript, I highlighted areas of the transcript or the episodes that pertain to particular beliefs about teaching mathematics and teaching mathematics with technology. In the summary section, I gave a brief account of what transpired in the area highlighted in the transcript.

Table 6: Coding of data (partial)

Transcript (Beth)	Summary of Episode	Code	Comments/ Relation to Research Question
P1: I mean, appropriately and effectively it means that students are engaged with the technology, they are trying things on their own, technology is really good for trial and error, it's like move this angle over here, see what happens with the triangle, you know, its them, trying and discovering is good engagement,	When asked what she thought about effectively teaching with technology Beth indicated that teachers should allow learners to use the technology as a means of engaging with mathematics, for trial and error, for discovering properties of mathematical objects rather than giving students precise information about what should be done and what the outcomes should be.	Use of technology as exploratory and engagement tool with respect to thinking about mathematics	This relates to how a teacher should direct students to use technology. It is not an observed use so it relates to what the teacher believes about the way technology should be used and thus is a belief about teaching mathematics with technology RQ3

Next, I developed a code that appeared to capture what the highlighted transcript was about. Then I looked for codes that appeared to be related to the first code developed and grouped these codes into a common and emerging theme, based on similarities of the codes. In particular, I made codes on what the teacher believed about each of the major areas in the interview, teaching mathematics, technology, and the use of technology to teach mathematics. A similar approach was applied to the selection of technology section

of the interview where the codes involved the factors influencing teachers' selection of technology.

3.9.1 The Participant Selection Observation

I wrote notes during the initial observation of the all the teachers' lessons. The notes were written with a focus on the elements of the participation observation sheet, which can be seen in Appendix A. I analyzed the observation data by combing through the notes to find teachers who have a high use of technology in a single lesson. A high use of technology on the observation was determined by the number of yeses recorded in the observation sheet in addition to the quality of evidence displayed during the lesson. The quality of the evidence was given greater consideration than the number of yeses in determining whether the teacher used technology at a high level. For the initial selection of participants, when teachers had the same number of yeses, the level of evidence was used to determine which teacher was chosen to participate further in the study. The intent was to identify three teachers, who scored highly on this single observation and who were willing to continue participating in the study until the study was completed. The criteria for the selection of future participants in the study were as follows: First, the teacher used cognitive technology in mathematics lessons at a level that would be synonymous with (up to the level of) a partner (Goos et al., 2003). Second, the teacher was willing to be interviewed further. Third, the teacher was willing to be observed further.

The rationale for this method was that I wanted to choose teachers who appeared to make the most appropriate use of technology in the initial observations. The most appropriate use was ways determined as teachers used technology in a way such to encourage reorganization of mathematical thinking (Pea, 1987) or used technology as at

least as a partner (Goos et al., 2003) occurs. Hence the more of the teachers' actions on the observation sheet that were checked 'yeses' and the stronger the evidence in support these actions, the more likely the teacher may be classified as tending to use technology more appropriately.

I purposely selected three participants, Beth, Ruth, and Sam, on the basis that they exhibited use of technology up to the level of a partner and or that of a Reorganizer during the initial observation. For example, Beth used technology at the level of partner when she had the students determine quadratic functions through three points using GeoGebra activity. Ruth used technology at the level of a partner when she had students construct quadratic expressions for a Desmos marble slides activity, and Sam used technology at the level of a partner when she had students interpret the results of their own distance rate time graphs investigations with Desmos. I chose these three teachers with the view that the opportunity to determine the commonalities the commonalities among their technology use, factors involved in their selection of technology and their beliefs is greatest when their actions with technology are deemed as highly appropriate for teaching mathematics.

3.9.2 Beliefs and Technology Selection Interview

After I fully transcribed the entirety of the interviews, I reread the interview transcripts and checked the written transcripts against the recordings to ensure the transcripts were correct. Then I read the transcripts to identify codes in the document, more specifically episodes where the participant appeared to be talking about his or her beliefs, checking to make sure that all the relevant episodes were highlighted. After completing all the initial coding for an interview, I assigned themes to a set of codes by

relevance in the sense the codes are closely associated or that they ‘live together’ under a particular theme. To develop a theme for the belief about technology I looked for other episodes throughout the entire transcript where the participant indicated similar ideas. The analyses of the prelesson interview and the postlesson interview were carried out in a similar way to the analysis of the beliefs and technology selection interview. The initial themes generated during the analysis process continued to be refined until a final set of themes is made for the written report.

3.9.3 The Three-day Observations

I used the same observation instrument during the three-day observations as I used for the initial selection observation. During the three-day observations, however, the idea of the observations was not focused on comparison among the teachers but on detailing how each teacher used technology for teaching mathematics. Despite the fact that for the initial observations it was not necessary to develop themes for technology use, in the three-day observations, it is necessary to connect the teachers’ use of technology in one lesson to their use in another lesson. To detail these actions with technology I wrote field notes during each lesson. The written notes were read, and parts of the transcripts highlighted for teacher actions with technology. I coded episodes of teacher actions with technology. I referenced Pea’s (1987) and Goos et al.’s (2003) frameworks to shape my views on how the teachers used and directed the class to use technology in an observable way. This level of coding was the start of breaking the data into thematic groups for deeper analysis. At a higher level after the themes for each participant’s use of technology are developed, and comparisons were made across the cases.

3.9.4 Validity

Throughout the process of data collection and analysis, I used several strategies to ensure the inferences that I drew about the data met the requirements for valid study. Yin (2003) emphasized a case study should possess construct validity, which he defined as “the correct operational measure of the concepts.” Validity was achieved in my study because of the use of multiple sources of evidence. These sources include the data collection observations, interviews, and artifacts. Additionally, I ensured that the study was valid by having participants’ member checking narratives of their beliefs and having other persons such as my advisor and peers read the draft reports of the cases.

CHAPTER 4: FINDINGS

In this chapter I describe how teachers in this study select and use digital cognitive technology and the factors relevant to their choice and use of this technology (research questions 1 and 2). I also describe each teacher's beliefs about teaching mathematics, and beliefs about teaching mathematics with technology. I detail the commonalities among teachers' beliefs and their practices while using cognitive technology to teach mathematics (research question 3).

4.1 The Case of Beth

Research question 1

About Beth and her prior experiences with technology

Beth was a second-year mathematics teacher at Yellow High School. She stated she enjoyed teaching mathematics and loved to experiment with different technologies for teaching mathematics. Beth was considered one of the most avid users of technology by the head of the mathematics department at her school and was one of the first teachers at her school recommended for my study. When I met Beth at the school, she was busy preparing for a mathematics lesson with technology.

Beth informed me she engaged with numerous cognitive technologies for the teaching and learning of mathematics during her college years. Some of these technologies included Geometers Sketchpad and GeoGebra. Beth was more confident with using Geometers Sketchpad as she had more exposure to it, while she did not have much exposure to GeoGebra. For the lessons I observed, however, Beth chose to use

GeoGebra for reasons that will be explained in the section on factors affecting the use of technology. Another technology Beth engaged with during her college years was the Desmos online graphing calculator. She stated Desmos is the software she used most often in her classroom to teach mathematics. She would select premade materials online from persons who have developed Desmos related activities to use in her mathematics lessons. Of all the technologies Beth chose to use, Desmos was at the top of her list for reasons that will be discussed in the section on factors affecting her use of technology.

In high school, Beth had access to and used the handheld TI-84 graphing calculator. Because of her prior experiences with the TI-84, Beth would like to use this type of graphing calculator more in her classes on a regular basis, particularly when the lessons were amenable to graphing calculator use, as the students were involved in mathematical tasks. The situation at her school, however, limited her usage of this technology. The school provided a small number of graphing calculators, and this set number of calculators had to be shared by all the mathematics teachers. The teachers who had the first preference for the use of the graphing calculator were the Calculus and Precalculus teachers. Beth taught Algebra 1 and Geometry and thus had to wait her turn to use the TI-84 graphing calculators when other teachers were not going to need them.

Beth acknowledged the need to have some large display screen in her classroom. The screen took the form of a Smart board. In my study, the Smart board is not treated as a cognitive technology but only as a display screen on which users see the mathematics of the cognitive technologies. This display screen served as a medium for which the work of all the students, a single student, or of the teacher could be displayed when needed by the teacher. Beth also expressed the need to use other technologies not considered as

cognitive technologies for my study, such as Khan Academy, on which the students may watch videos and practice related content before and after particular mathematics lessons.

When I observed Beth teaching with technology, she used Desmos and GeoGebra (Table 7). She used these technologies because both technologies were free to anyone with access to compatible hardware, such as laptops. Yellow high school did not have a one-to-one technology policy. The students had access to the laptops only during the class. This access was based on administrators' prioritization of which class needed the laptops most.

Table 7: Digital Cognitive Technologies Beth Used

Date	Topic	Technology Used
April 8	Quadrilaterals	GeoGebra
April 22	Circles	Desmos
May 2	Functions	Desmos

Factors affecting Beth's Use of technology

There were several factors Beth indicated determined how or whether she used technology in her classroom. The factors included the cost of technology, access to technology by the students, time for learning mathematics with the technology, currentness of the technology, visual appearance of the technology, and availability of premade materials amenable to the technology.

The cost factor was important to Beth because the more the technology cost, the less likely the parents, the school, or whoever was paying for the technology would be able to afford. Therefore, a key reason Beth used GeoGebra and Desmos for her mathematical lessons was they are free software. When asked what technologies she used specifically for teaching mathematics she said, "I'll have them use Desmos, they love

Desmos; it helps a little bit, it's good that the technology is becoming freer, they don't have to pay for it, so it's good.”

The next factor Beth indicated would determine her use of a technology was students’ access to the technology, particularly outside the classroom. Access to technology was necessary since she wanted her students to continue to engage with mathematics with technology on their own after they left the classroom. In this situation, GeoGebra and Desmos were readily available to anyone who had Internet access and computer or other hardware compatible with these technologies. Beth indicated the lack of students having access to technology outside of the classroom might lead to less technology-incorporated lessons and vice versa. The access to technology inside the school was also a bit of an issue. Lack of access was an issue because even though the GeoGebra and Desmos were free; not all students had access to a laptop all the time. At Yellow High school there was not one-to-one access, so teachers had to sign out class sets of laptops for the students to use when needed. Beth indicated even though she had signed out the laptops for the classroom observation, if the school administration deemed another teacher to be more in need of the laptops, that teacher may just sign them out as well and come and get them.

Time for using the technology to learn and do mathematics is another reason Beth indicated could determine whether or not she would use technology. According to Beth, if technology could save time helping to get students to the level they needed to be in mathematics, then that was a real incentive for her to use technology in her lessons. In her situation, she looked to technology to save time in eliminating learning gaps in the students who came to high school but were still functioning at the lower middle-grade

level. In such a situation Beth felt she could use technology to get the students to the appropriate mathematical level quickly so the class could move on to the mathematical material at the current grade level.

Another factor Beth indicated she used to determine whether or how she used technology in the classroom was the visual appearance of the mathematical objects shown with the technology. In one lesson when Beth used GeoGebra, she noticed when her students were using GeoGebra some students had a hard time seeing the visual on their screens. In the interview following the lesson, Beth was asked if there was another technology she might have used. She indicated she might have used something with better visuals such as Desmos.

Beth stated currentness of the technology determined whether or not she used technology in her lesson. An example of this factor was displayed when Beth was asked why she was using GeoGebra. Beth indicated she used GeoGebra because some other technologies were available but not as up to date. She mentioned some applets made to run on older versions of Java were not up to date, and would not run on later versions of Java, but GeoGebra could run on all updated computing platforms.

The availability of supporting or premade materials amenable to the technology she wanted to use in the classroom is another factor determining how and whether Beth used the technology. Beth explained that both Demos and GeoGebra had plenty of premade materials, so it was easy to choose to use these technologies in her classroom. In fact, in all the lessons observed, the materials Beth used were premade for use with the technologies she chose to use.

Research question 2

Cognitive Technology Beth used

In the lessons where I observed Beth teaching with cognitive technologies, the ones she opted to use were the Desmos graphing calculator software and the GeoGebra online graphing software. GeoGebra is described as a cognitive technology for my study because the user may use it to create mathematical objects (algebraic and geometric), manipulate the mathematical objects, and conduct mathematical procedures.

Beth used GeoGebra in a lesson where students were required to discover the properties of quadrilaterals because one of the features of this technology is useful for dragging various parts of the mathematical objects. This feature is the move tool. When using the move tool, the user only has to click on a movable part of the mathematical object, hold down the cursor and drag parts of the mathematical object across the screen. The user can then observe what happens to the various measures of angles, side lengths, etc. to determine what conjectures may be made about the mathematical object. The move feature also facilitated the user being able to see which parts of the object remained unchanged after dragging.

The measure feature of GeoGebra allows the user to get precise measures of the sizes of the angles and the measures of the side lengths of the quadrilaterals. By using the measure features of GeoGebra, the user can tell which side lengths and which angles are congruent to other side lengths and angles. Thus users would be able to develop an insight of the different properties of mathematical objects or in the case of Beth's lesson, quadrilaterals while using GeoGebra. Beth did not display an awareness of possible misconceptions that may arise while using the measure feature, such as students using the

measuring as proof. It is possible that she may have been aware of possible misconceptions but did not focus on these because of her lack of experience with GeoGebra. In fact, she mentioned that if she were to do the topic in another way she would have the students do constructions of quadrilaterals, but with another technology, not GeoGebra.

Beth used Desmos graphing calculator because Desmos has affordances for users to explore mathematical concepts dealing with conic sections, the affordance to color code mathematical objects, the ability to represent mathematical concepts in various ways, such as graphing or algebraically. Desmos graphing calculator may be accessed online and can be used on iPads, laptops, and even mobile phones. In the graphing calculator area of Desmos, the user may make use of sliders and may just type a numeric they wanted Desmos to display. Beth used Desmos in two of the lessons observed.

Uses of cognitive technologies in Beth's classroom

During the lessons in which Beth taught mathematics with cognitive technology Beth used and encouraged her students to use technology in many different ways. These included, using technology as checking tool, for play and exploration, for the development of conceptual understanding of mathematics, for teacher demonstrations, presentation of student work, and providing an entry into problem posing with technology.

When using technology as a checking tool, Beth displayed the work of students on the screen or the names of all the students on the slide. In this way, the technology was used to show which of the students were engaged in the mathematical task and how far along on the task the students were. Beth indicated this aspect of using technology was

crucial to her because she could see at a glance which students were struggling, what question they were struggling with, and where to go as a teacher to help the students.

Checking students' work in this way increased the efficiency in the classroom compared to when she had to go to each student to find out where they were at on the task. The use of technology to show students progress during the task also provided the opportunity for all the students in the class to know where each student was at on the task and served as motivation for some students to work harder on the section of the task they engaged. For example, during the first observation Beth noticed that some students had not yet begun their work. She displayed the work of one student to the rest of the class. When the students who did not appear to be initially engaged with in the task, saw the student's work, they then began to do some work on their own. In fact, in her first interview Beth noted that some students are willing to engage more when they see what other students are doing on their mathematical problems. It was possible some students may simply have copied the work displayed but Beth ensured that after the initial display the students completed the rest of the task on their own. During the periods of time she was checking on students' work Beth had plenty manipulation of the technology.

Beth directed the use of technology for play and exploration when she provided opportunities for the students to use technology in ways for them to carry out investigations of mathematical concepts without being given specific procedures for carrying out the activities. One example where this use of technology was explored was the second observation Beth had the students engage in a task which the students were required to write the equations of given circles. The students did not appear to know what an equation for a circle looked like so they had to use trial and error methods to write in

various equations until they could find one that fit. The teacher did not give them an example of what an equation of a circle looked like before they started the investigations. During the play and exploration phase, Beth had little manipulation of the technology.

Beth directed the use of technology for the conceptual development of mathematics where the focus was for students to come up with conjectures and make arguments for why the solutions they arrived at with the use of technology were correct. For example, on the mathematical task with the circles, after the students got the circle correct the students were then required to write an explanation to a friend telling why the answer made sense. In this case, then the technology was used to get students to develop a more robust understanding of the circle as the distance of a set of points on the circumference to the center. The explanation then would count for whether or not the students knew underlying meaning of the numbers and variables in the equation and thus for the conceptual understanding of the mathematical concept. During the conceptual development phase, Beth had some manipulation of the technology. In this case, Beth showed examples where one equation resulted in a line and another in an oval to get students to see what equations would not result in a circle.

Beth used technology for the presentation of students' work to highlight the works that made the most robust solutions to the mathematical activity and for an example the class could aspire toward. In the same vein, she could highlight the works that were not as robust and have the class discuss ways in which those solutions could be improved. Invariably, she could also give her ideas on whether the answers were correct or incorrect and moved along with each solution in much the same way. In one example with the first lesson Beth highlighted the work of some students who first did not do the correct

equations of the circle. The class knew the solutions were not right because the equations did not produce circles matching the original circle. Beth continued to show students' work on the equations circles until she found one that was the correct solution for the initial circle given. During the presentation phase, Beth had much manipulation of the technology. Sometimes the students would use the technology to show what they did on their task, but for the most part, Beth was the only one who made use of the technology here.

Beth used technology for the demonstration of the mathematics she wanted the students to be engaged with and for showing the technical skills required to engage in the mathematical activities. As she did the demonstrations, the students would watch and later attempt to carry out the same procedures with the technology. Beth carried out a demonstration in lessons with the GeoGebra technology and with the Desmos technology. Beth used GeoGebra for a demonstration of mathematics when she demonstrated the dragging of parts of a parallelogram before the students engaged with the mathematical activity in the first lesson observed. The dragging actions demonstrated by the teacher allowed the students to observe not only how to carry out the actions but also observe what happened to the mathematical object, regarding measures of angles and side lengths as various parts of it was dragged.

The next use of technology Beth encouraged was for students to use technology to pose their mathematical problem situations after they had a good grasp of the mathematical concept. The students should pose problems for their other classmates to solve. The problems did not necessarily need to be a replica of the same problems they had recently engaged with, but could be as elaborate as they wanted as long as good

mathematical questions were asked. For example, at the end of the task concerning the circles, Beth required the students to make their problems and pose to the class. Two students tried to do that section of the task but did not get to complete the requirement. During the problem-posing phase, Beth had no personal manipulation of the technology.

Research question 3

Beth's beliefs about teaching mathematics

I found Beth espoused several beliefs about teaching mathematics. These beliefs were espoused during the first interview and in the prelesson and postlesson interviews. I developed three major themes concerning Beth's beliefs about teaching mathematics. Beth's beliefs about teaching mathematics included motivation, real-world practices, and student-centeredness.

Beth espoused the belief about motivation during the first interview I conducted with her. Beth clearly indicated in that interview and throughout many of the follow-up interviews that one of the most important things during teaching was whether or not the students were motivated to learn mathematics. She likened the job of teaching to that of a sports coach. She used the idea that just as a sports coach had to be motivating the players to compete in sports, in similar manner, a mathematics teacher should also motivate the students in mathematics. When given a list of occupations including doctor, news broadcaster, gardener, coach, to choose one which best represented a mathematics teacher, Beth was initially torn between a doctor and a coach. Even though she weighed heavily towards coach because of the motivations aspect Beth also argued that the mathematics teaching was also about finding out what difficulties the students had with the mathematics and to use strategies to help students alleviate those mathematical

difficulties. Beth made more arguments, however, in support of the mathematics teacher being represented as a coach. She said, “coach, because of the motivation, I think it’ll (mathematics) be easier to teach if you didn’t have the motivational aspect so I think we are more of a coach because coach is there for the motivational aspect.” She claimed the biggest thing in the classroom was how the teacher might motivate the students. She said, “that (motivation) is the catch-all for education is like how we can motivate the kids.” She stated teachers should develop a good rapport with the students. Beth believed teachers are role models for the students and students look up to what the teachers do and say. She indicated teachers should do and say things in the classroom to help motivate students who come to the classroom not wanting to learn mathematics. Hence, teachers could motivate students by providing opportunities for them to want to learn mathematics.

The second important belief permeating the interviews with Beth concerning mathematics was the importance of using real-world practices in the classroom. Beth’s point of view concerning the term ‘real-world practices’ did not indicate the mathematical problems should include word problems about real world applications. Neither did the term ‘real world’ mean the mathematics problems should include word problems about the students’ lives or applications of mathematics to their lives. Rather, ‘real world practices’ from Beth’s point of view were the actual practices carried out by the students and teacher in the classroom reflecting the way the world works in real life outside of the classroom. For example, when the students worked in groups to brainstorm ideas to solve a new mathematics problem, this was a reflection of real world work in teams to solve real-world problems. In a scenario when Beth was discussing the

usefulness of her teaching strategies during the first interview, She said, “to me its real world, like having them work in groups and giving a problem its real world.” She compared the way the students worked in the mathematics classroom and the way people do work in real life. She continued, “ like, whenever you work for some company, like say I am in a board meeting my CEO is not going to come and teach me how to graph our most watched shows, you know like you are going to have to sit with four or five people and figure it out.”

The third belief about teaching mathematics Beth displayed was that the teaching of mathematics should be student-centered. In this regard, Beth held the teaching of mathematics should lead students to be actively engaged in constructing the mathematics the teacher expected them to learn instead of the teacher simply telling the students what to do. In this scenario students should be working in groups, solving their mathematical problems, and presenting their solutions. Beth also indicated she believed students should be able eventually to teach mathematics they have understood, as their teaching helps them to learn more. The mathematics teacher should be guiding the students while they work. She said, “I want them (students) to be in groups. I want them to be talking, what I visualize is, I mean wouldn’t it be great? I noticed at university the way you were given a problem, they work on it and they come up with a solution and they work on it together, and they come up to the board, and I like when students are teaching, they teach because you can see well that kid got it, I can be the teacher. I tell the kids, you know you are the teachers, I don’t let it click in your brain you know your fast tricks, I like kids work on a problem, one of them figure it out and then they can show everyone.” Here Beth indicates good mathematics teaching involves allowing students to eventually be the teachers of

the mathematics they developed while working on solving mathematical problems. Beth indicated her visualization of the teaching in this more student - centered way was what she would call good teaching.

Beth indicated the role of a teacher in the classroom in the mathematics lessons should be one of a facilitator of the mathematics the students are constructing mathematics by themselves. In this way, the teacher serves as more of a guide to help students stay with doing their mathematics. Beth said of her teaching, "I am a guider, I guide them." She indicated while the students were doing their work the teacher should be walking around to the various groups and guiding them on with their mathematics. She said, " I am usually moving around, I am answering questions, and usually one kid who just feel that they could do it, they could help them." Thus, the teacher facilitates the students' work, and when some students understand the material, the teacher should encourage these students to help other students to come to an understanding of the mathematics. She also indicated the teacher might sit with a group of students and work along with them as they try to solve their mathematics problems. She said, " I like when kids can figure out a problem, one of them figure it out and they can go up and show everyone, and everyone else can figure it out, or ask questions you know. And I will do it; I will sit with a group." Beth believes there can exist some lecturing in the classroom but not for an extended period. She stated good teaching is not supposed to be a lecture but should be more activity-based.

Beth's beliefs about teaching being student-centered appear to be aligned with Ernest's (1989) category of facilitator. Ernest (1989) described the facilitator role as on in which the teacher believes the goal of instruction is for students to become confident

problem solvers. Beth's ideas of the role of teachers facilitating the students' discovery of mathematics, encouraging group work, problem solving and explanation of understanding align with Ernest's (1989) facilitator category.

Beth's beliefs about teaching mathematics with technology

Beth displayed many beliefs about teaching mathematics with technology. These included ideas such as technology should be a checking tool of students' progress and solutions, an exploratory tool for playing with mathematics, a conceptual tool for students mathematical understanding, a procedural acceleration tool for skills practice, a presentation tool, and a tool for developing the real meaning of mathematics, a display tool for good visualization of the mathematics.

One of the first beliefs Beth displayed about teaching with technology was technology should be used as a means of checking students' work. Here the teacher used the technology as means of displaying the mathematics engaged in by the students, as opposed to using the technology as a means of checking correct answers only. Beth held this belief because in her ideal way of teaching she would like to respond more quickly to a student's solutions to the mathematical problems. Beth indicated she would like the idea of seeing what each student was doing on his or her mathematical activities in real time. As such she indicated one of the things teachers should be able to do is to easily check the mathematics of the students while they were working. Beth also indicated the technology helped the teacher to see where the students were at. Students could also see other students' work and perhaps somehow be motivated to work harder to get the mathematical task to completion. She said, "other people (students) can look at it and it can help them (to do their work)." She also indicated the display of the students work

may not always have a positive response because some students may feel embarrassed by the work displayed, particularly if their work was not mathematically robust.

The next belief Beth appeared to hold was technology is a means of playing with the mathematics. Beth indicated doing mathematics was a process of trial and error and as such the available technology enabled users to engage in many trial and error activities. Beth appeared to define trial and error as having students engage with the mathematical activity in a way that they discovered the mathematical ideas for themselves, instead of the teacher doing things for them and they simply follow along. Hence, Beth appeared to believe that trial and error is a deliberate and purposeful rather than random. She said, “they (students) do the discovering and they find the answers.” Beth used the terms engagement and discovery to describe this trial and error method of using technology to do mathematics. Beth indicated she believed the appropriate teaching with technology involved teachers allowing students to engage with technology and to think about the mathematics being developed while they were playing with the mathematics.

Beth's meaning of engagement is not the same as motivation, even though she noted a slight motivational role technology played in helping to get students to engage with the mathematics. Beth's explanation of engagement is not the same as skill practice or of conceptual understanding even though the act of engaging with technology may lead towards the discovery of some important mathematics concepts and practice of some mathematical procedures. According to Beth, the engagement was more like students trying things on their own and coming up with something for themselves.

In the first lesson I observed of Beth teaching with technology she indicated in her lesson plans one of the goals of the lesson was for the students to know the differences between various quadrilaterals, such as squares, rectangles, trapezoids, and kites. When asked how technology was going to help them to reach those goals, Beth said, “technology, I let them do a lot of trial and error.” Thus Beth indicated she believed by engaging with technology by trial and error methods students would be able to reach the mathematical goals of the lesson eventually.

The next belief was displayed by Beth was technology as a tool for developing mathematical concepts. Here developing the mathematical concepts imply the underlying meaning of the mathematical ideas in the lesson, not the process of finding the solution. Beth believed students could use the technology to develop the concepts with some help from the teacher. She indicated while the teacher may provide some basic procedures, the students should develop relationships and the connections among the mathematical ideas they investigated, and provided explanations for why the mathematics worked, where necessary. She said “you are not gonna tell where, when and how to do all the mathematics, you are just gonna give them the basics.” In the first prelesson interview when asked what she intended for the students to be doing with GeoGebra, Beth indicated the students should play with the technology and also ‘notice the difference between the shapes’, that is, contrast the shapes. This statement was not a direct instruction to the students but more an indication to the interviewer of what she wanted the students to accomplish during the lesson. In this situation she intended for the technology to be used as a visualization tool to help students to develop the concepts of quadrilaterals and characteristics the students noticed as they dragged edges and saw the

measurements of various parts of the shapes. Furthermore, Beth indicated the students should use technology in such ways to be able to make statements about whether each of their initial conjectures was always true, or only sometimes true.

The next belief Beth appeared to hold was the use of technology for acceleration of mathematical procedures and skills. Here the technology may be used to make the mathematical work more easy and quick. When speaking about using technology to solve problems involving graphing, Beth indicated the user could plug in the points, use technology to plot the points, or show it on a table of pull up the graph of the when needed. She said, “I mean it's less work, it (technology) makes the work easier.” In using technology for speed, the learning gaps that occur may be bridged by redoing with technology the prerequisite concepts the students need to learn. She gave an example of some students she taught in the freshman year who did not have a good understanding of what they should have learned in the 6th grade. She indicated technology could be used as a means of quickly getting the students to review previous concepts so they can move on to engaging with the concepts at the new grade level. When speaking about how technology may be used to quickly review concepts, she said, “you know some of them do not have the middle foundations; they can punch it in the right table, figure it out if they get a growth or decay. So 6th grade is not holding back and, it (technology) bridges that gap.” Thus she indicated technology should be used to quickly go over concepts students had previously learned.

The next belief Beth espoused was using technology as a means of presenting the mathematical procedures to be used to create mathematics and the solutions during a lesson. In particular, the teacher should be in charge of the presentations. Even though

Beth would like for the students to do their presentations with the technology she has realized invariably the students do not want to do so. Despite this, she realized presenting the work to the class has a real impact in getting those students who do not understand to engage with the mathematics. Additionally, the teacher demonstrations may give the students an entry point into how to engage in the mathematical tasks with technology. One example of Beth's apparent espousal of this belief was when asked about what she would be using GeoGebra to do in the first lesson with technology. Beth indicated she would use GeoGebra to give the students an entry point into doing the mathematical activity by demonstrating the dragging actions of the technology. The students could look at what she did and then follow afterward in their activities. She said, "show them how they can move things around and change it, like how they are not all the same, like a shape is not like exactly that way [used thumb and forefinger on both hands to form the shape of a quadrilateral and tilt it] you know what I am saying, like tilting." Thus she indicated by providing an entry point for the students she would be allowing them to engage more easily with the technology.

The belief teachers should present the students' work appeared to be in contrast to the belief concerning student-centeredness Beth espoused on earlier in the interview. Thus it appears Beth held beliefs at both ends of the teacher-centered and student-centered spectrum. The way these beliefs play out in her use of technology was shown in this research.

The next belief about the use of teaching with technology Beth espoused was the teacher should use technology as a means of getting the students to the real meaning of doing mathematics. From Beth's viewpoint, the real meaning of mathematics is all trial

and error at the various mathematical tasks until appropriate solutions are found. She indicated this involved the students trying and discovering on their own things, for example by moving parts of mathematical objects, observing what happens and making their logical conclusions, and possibly make numerous mistakes, rather than the teacher telling them exactly what to do and the expected outcomes. It appears that Beth held the belief the trial and error process was purposeful and deliberate one, not a random type. The technology could be used to ease the burden of a cumbersome and tedious trial and error process. Technology also allowed students to be engaged with the mathematics in a fun environment. In such ways, by using technology, the students will be able to get a better grasp of what doing mathematics means.

Possible Relationships among Beth's Beliefs Factors and Uses of Technology

The relationships between the beliefs and uses of cognitive technology are inferred using the definition of beliefs in this study. Specifically, I used the definition of beliefs teachers hold to be their dispositions towards action. For this study, a relationship was said to exist if the teacher espoused a specific belief and then the observed action with technology in the classroom appears to hold some similarity to the belief espoused. I compared the beliefs they held to their uses of technology and to the factors they indicated affected their use of technology (Table 8) and drew some connections from the data.

Table 8: Beth's Use of Technology, Beliefs and Factors

Beth Uses of Technology	Beth Beliefs about teaching mathematics	Beth Beliefs about teaching Mathematics with Technology	Beth: Factors
U1: Checking students' work	B1: Motivation	B11: Checking students' work.	F1: Cost
U2: Play and exploration	B2: Real-world practices	B12: Play and exploration	F2: Access
U3: Development on conceptual mathematics	B3: Student-centeredness	B13: Generating mathematical concepts	F3: Time to learn with technology
U4: Presenting and critiquing students' mathematics		B14: Acceleration of mathematical procedures and skills	F4: Visual appearance
U5: Teacher demonstrations		B15: Presenting mathem	F5: Currentness of technology
U6: Posing problems of mathematical situations		B16: Real meaning of doing mathematics	F6: Supporting, Premade materials

Beliefs about teaching mathematics

Motivation (B1) was not visible or easily measured by the observation of the actual use of technology. In particular, because the focus of the study was on teaching and not student learning then the measure of the impact of motivation on the use of technology or vice versa appeared to be backgrounded in this study. Real world practices (B2) as defined by Beth was somewhat evident and could be seen by the students working in groups and asking each other questions, but was not related to any particular use of technology observed. Student-centeredness (B3) was evident for the same reasons (B2) was evident, but was not directly related to any of the observed uses of technology.

Beliefs about teaching mathematics with technology

Checking students' work (B11) is somewhat linked to the use of technology checking the mathematical solutions of the students (U1). Belief about technology as a tool to play explore with the mathematics (B12), is apparently related to the use of technology for play and exploration (U2). The belief technology is useful for generating mathematical concepts (B13) appears to be related to the development of conceptual mathematics (U3). The belief technologies should be used for the acceleration of mathematical procedures and skills (B14) do not appear to be directly related to any particular use of technology displayed during the lessons observed. The belief technology should be used to develop the real meaning of doing mathematics (B16) appears to be related to the observed use of technology for play and exploration (U2), and development of conceptual mathematics (U3). The belief technology should be for presenting mathematics (B15) appeared to be related to the use of technology for teacher demonstration of what students should do with technology (U5) and to the presentation of students' work (U4).

Factors and Use of Technology

The factor, cost of technology (F1) did not appear to have a connection to any particularly observed use of technology in the lessons taught but determined the technology used. Access to technology (F2) did not appear to have a direct relation to any particular use of technology but determined the particular technology to be used. Time to learn with technology (F3) was one factor that may relate indirectly to the use of technology even after the technology has been chosen. Time may be related to the students' practice, the teachers learning time, and to the students' exploration if the

students are not familiar with the technology or have difficulties manipulating the technology. For the observed lessons, however, the students appeared to have already learned to use the cognitive technologies. The factor, visual appearance of the technology (F4) may be related to what the teacher may use technology to demonstrate and to the material that may be taught with the technology, but did not appear to be directly related to any observed use of technology in the mathematics classes. The currentness of technology (F5) appears to be related to whether or not the teacher may use the technology deal with particular mathematical concepts. The final factor, availability of premade materials amenable to technology (F6) appears to determine whether or not the teacher uses the technology for procedural or conceptual understandings or both.

Summary of Beth's Beliefs, Technology use, and Factors

In general, it appears the observed use of technology in Beth's classroom is more closely related to her beliefs about teaching mathematics with technology than to her other beliefs about teaching mathematics only, or to the factors associated with her choice and use of technology (Figure 7). In particular, it appears her beliefs technology is best used to convey the real meaning of doing mathematics, and the technology should be used for presentation purposes were related to the most uses of technology observed in her mathematics classroom. Of six belief themes about teaching mathematics with technology, Beth espoused, five of them were somewhat related to some uses of technology observed in Beth's classroom. Of the three other beliefs Beth espoused about teaching mathematics only, none of them appeared to be directly related to any of the uses of technology observed in Beth's classroom.

One use of technology observed in Beth's classroom did not appear to be related to any of any of her espoused beliefs. This was the limited use of technology for problem posing activities. If Beth held this belief, then it may have been espoused and may perhaps have been more clearly displayed during the observations.

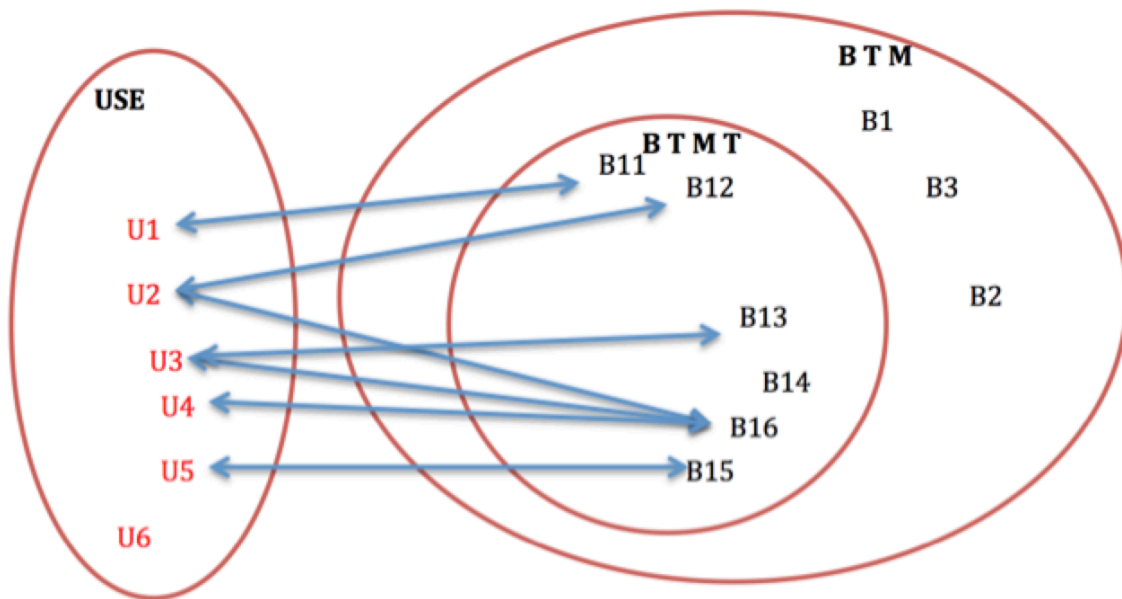


Figure 7. Possible relationships in Beth's uses of technology and beliefs

In figure 7 the bidirectional arrows indicate a possible two-way influence between beliefs and use of technology.

BTM – Beliefs about Teaching Mathematics

BTMT – Beliefs about Teaching Mathematics with Technology

USE – Observed use of technology during the teaching of a mathematics lesson

4.2 The Case of Ruth

Research question 1

About Ruth and her prior Experiences with Technology

About Ruth and her prior Experiences with Technology

Ruth was a second-year mathematics teacher at Green High School. She had taught at other schools before teaching at this school. She has been exposed to technologies such as Calculus in Motion and Desmos online graphing calculator software during her college years. She indicated she loved to teach mathematics using Calculus-in-Motion because she had more experience with it than other technologies. She stated she loved to learn about the use of new technologies in the classroom and took every opportunity she got to attend professional development meetings that include technology. She had limited experience with Geometers Sketchpad and no experience with GeoGebra. She expressed an interest in learning to do some mathematical activities with GeoGebra.

At Green high, she had experience with TI-Nspire Navigator software. She still has not had much experience with that software but was willing to have me observe a few lessons with her teaching using TI-Nspire technology. Ruth suggested the neat thing about the technologies at Green high school is that the school appears to be a one-to-one school as the mathematics department has provided TI-Nspires and laptops to all the students in the school. The students were allowed to carry the laptops and TI-Nspires home so they would have access to technology all the time. When I observed Ruth teaching with technology she used Desmos and TI-Nspire Navigator (Table 9). She indicated that she used these technologies often because the students had access to hardware and Desmos was free.

Ruth said she loved to teach with technology and was one of the teachers recommended by her head of department as an avid user of technology. She stated she wanted to use technology as much as possible in her lessons and shared she liked technologies that allowed for writing mathematical explanations as well as doing calculations. Thus, she loved to use digital cognitive technologies. She also expressed the desire to have a nice display screen such as a Smartboard to show up the work of students and the teacher.

Table 9: Digital Cognitive Technologies Ruth Used

Date in 2016	Topic	Technology Used
April 21	Derivatives and Integrals	Desmos
April 26	Riemann Sums	TI-Nspire
May 12	Derivatives and Integrals 2	Desmos

Factors Affecting Ruth's Use of Technology

In this section, I list all the factors Ruth indicated would affect her decisions whether or not to use technology in a lesson. These factors include time for students to learn to use the technology, whether or not the topic is relevant to technology use, proximity to testing days, availability of support or premade materials for use with the technology, and technical difficulties associated with the technology.

Regarding the time factor, Ruth indicated the time students spend to learn to use the technology in class is a crucial factor in determining whether she would use the technology in class. In particular, if the technology contained many features time-consuming to learn then she would not use the technology. On the other hand, if the technology were easy to learn to use then the technology would often be used in her classroom. When asked what were some things she considered when deciding whether or

not to use technology Ruth said, “if the student is going to spend a lot of time learning the different features then that is not gonna be worth it, that's what I like about Desmos, everything is very easy to implement so I like the learning curve to be manageable.” Ruth indicated the learning to use technology would take some time and the shorter the learning curve was for a particular technology, then the easier it was to incorporate the technology into her lessons. Hence the shorter the length of time lost at the beginning of the implementation of the technology, the better it was to use the technology in the classroom.

Ruth indicated another factor determining her use of technology in her classroom was the amenability of the technology to the mathematical topic she was to teach to the students. If the topic were geared to more lecture type lessons, then the technology would not be used as much, but if the topic were not geared towards lecture type lessons, then she would use technology heavily in those lessons. An example of this was, when asked what she considered when deciding whether or not to use technology in her lessons, she said, “It really depends on the topic, sometimes I feel like I am using it (technology) really heavily, sometimes, move back to kind of like more lecture type lessons, again it just depends on the subject matter.’

Ruth was also very concerned that tests, especially end-of-course tests were not technology dependent and as such the proximity of testing days determined how heavily technology was used in her lessons. The closer to the testing days the lesson was, then the less likely technology would be used and vice versa. She said, “ and the class relation sometimes reflect that you know, sometimes if you have a test coming up, the days going towards that maybe I don’t use that (technology) very much.”

The availability of support materials or premade materials was another factor that determined the use of technology in Ruth's classroom. This also included the flexibility changing the materials to enable the addition of her activities for use with the students. For example, when asked why she chose to use Desmos in the first observation lesson; Ruth indicated Desmos came with many premade activities suitable for her classes and the fact that just about anyone could make their activities with Desmos was one of the reasons. She said, "I was looking for some activities to do with them this one was created so like on Desmos you can create something on your own, or you can search for something that was prepared by either by the Desmos team or faculty or other math teachers." Ruth mentioned premade materials also exist for the TI-Nspire technology.

Another factor determining how or whether Ruth used technology in her classroom was her (and the students') preparation for technical difficulties with the technology. In this scenario, ideas such as the loss of charge for TI-Nspire calculators are expected. An example of Ruth taking this factor into consideration was when asked if she had any difficulties when using technology before this lesson in which she was going to use the TI-Nspire Navigator system. Ruth indicated that sometimes the TI-Nspires powered off because of inadequate charging, so she had to encourage the students to ensure that the units are fully charged when the lessons began.

Another example occurred while investigating the topic of Riemann sums, the slider on the TI-Nspires' software began to behave a bit tricky and difficult to drag to the positions indicated by the numbers on the screens. The TI-Nspires would display a different number for n on the screen, such as $n = 2$ when the number of rectangles was 5 (and the display should be $n = 5$). Because the technology displayed incorrect results, the

teacher and or students needed to have a strong understanding of the mathematics in order to know what were good conjectures to make.

The difference between the mathematics displayed by the TI-Nspire and the mathematics, which should be displayed, was of concern to Ruth. This was because inconsistencies between the correct mathematics and the incorrect one displayed by the technology may lead students to make less than robust or incorrect conclusions. Here the teacher knowledge was very important. She said, “I would change the sliders, I did not like the sliders because it was a little bit tricky to get it, and when n was at 2, it was showing you n equals 5, so it was a little bit finicky, and I told the kids beforehand because we were doing this yesterday (in another class) so I told them beforehand just to be careful, make sure that you have it’s kind of matching so that you can get the right answer.”

The final factor Ruth shared about her decision to teach with technology was apparently linked to the fact she chose to use more than one cognitive technology in the classroom on a regular basis. In this scenario, Ruth provided ideas to indicate the use of one (new) particular technology might be affected by the use of prior technologies the students had used. An example of this factor occurred when in her second observed lesson with cognitive technology Ruth planned to use the TI-Nspire calculator to do the mathematical activities. When asked why she chose to use the TI-Nspire technology over all other technologies, Ruth indicated the TI-Nspire was available to all the students because the mathematics department has given all students the technology. She also indicated the students were very familiar with Desmos and the TI-Nspire technology

shared similar features with Desmos. Because the students have also used Desmos, the transition should not be tough.

Research question 2

Cognitive Technology Ruth used

The cognitive technologies Ruth used were Desmos online graphing calculator technology and TI-Nspire Navigator technology. The Desmos technology is a cognitive technology for the purpose of my research. The TI-Nspire Navigator technology is based on a wireless connection of individual TI-Nspire graphing calculators to an access point in on the teacher's desk. The students must put 'hats' on their calculators to send and received signals to the teacher's laptop. The TI-Nspire must be charged before the lessons to work throughout the lesson. The users may manipulate sliders on the TI-Inspire. There are many premade activities in the TI-inspire on multiple grade levels in mathematics. The TI-Nspire technology also contains lessons and suggestions for teachers to follow if needed. It also gives indicators as to when during a lesson would be good times to do certain activities.

The TI-Nspire is considered a cognitive technology for my study because it not only does calculations but also can be used to investigate many mathematical concepts such as Riemann sums, derivatives, maxima and minima. Thus it can be a tool to help users move beyond mere procedural skill practice to more robust mathematical thinking. Even though Ruth did not have any issues accessing the TI-Nspire for use with students whenever she wanted, she still had an affinity for using Desmos more than the TI-Nspire because the TI-Nspire is a bit more cumbersome.

Uses of cognitive technologies in Ruth's classroom

During the lessons in which I observed Ruth teaching mathematics with cognitive technology, she used and encouraged her students to use technology in many different ways. These ways of using technology included, checking students' progress, play and exploration, conceptual development of mathematics and debunking mathematical misunderstandings, presenting and critiquing students' solutions, and eliciting mathematical discussions.

Ruth's first indicated use of technology involved the checking the students' progress regarding whether they had logged in to Desmos or into the TI-Nspire software and how far along they had progressed in their designated mathematical task. In the Desmos technology, the checking screen showed the list of students' names with a bar located to the right of each name containing a dot in the bar representing the progress of the students. The further to the right on the bar the dot was indicated more completion of the work. The TI-Nspire technology showed the names of the students who were logged in and working on the activity. In general, this use of technology involved Ruth displaying the names of the students and how far they had progressed with their mathematical activity. For example, when working on the mathematical task 'derivatives and integrals', Ruth instructed students to log onto the mathematical task with the online tool Desmos.com by inserting a code she provided to them. She kept track of the students who had logged in by displaying the information on her screen and looking at her screen as they logged in and began to work on their mathematical task. The students could also see their names on Ruth's screen displayed on the board, as well as how far each student had progressed on the task. Here Ruth was using the technology as a means of keeping

track of the students' progress. This feature of newer digital technologies is one that Ruth used very often during her teaching because it allowed her to keep track of what her students were doing at any moment during the lesson without her having to be physically near to her students.

The next use of technology was for play and exploration. In this scenario, the mathematical task was also designed to incorporate students' play and exploration of mathematical concepts. When using the TI-Nspire technology, Ruth encouraged the students to click and move sliders and mathematical objects and to observe what happens as the technology dynamically changes all the parameters affected by the objects that the user changes. For example during the second observation, after Ruth had ensured that the students had logged into the system, she told the students they should play around with the sliders and vary the number of rectangles to get an understanding of what happens when they vary the number of rectangles. On the students' TI-Nspires the task was first set up such that there existed a curve sloping upward from left to right. Under the curve was a single rectangle. On the screen to the right existed a slider with the letter n below it. On the same screen there existed the letters L , R , M and the word actual. Students could play with the sliders and any other changeable icons on the screen and observe what happened.

The next use of technology was the focus on the development of conceptual mathematics and debunking mathematical misunderstandings. In this scenario, Ruth encouraged the students to use technology to develop the mathematical concept such as the meaning of derivatives or area under a curve or to make conjectures based on their investigations. For example, after the students had completed the section of the

mathematical task about finding the estimates of the areas under the curve from the left rectangles, Ruth pulled up and displayed the work of one student who had only been changing her number of rectangles from two to three and vice versa. Ruth questioned the student's reasons for using only a small number of rectangles and encouraged the student to make more rectangles and make a conjecture about the estimate of the area of the rectangles to the left of the actual area under the curve.

In line with developing the useful conceptions in mathematics was the use of technology for debunking mathematical misconceptions. One misconception Ruth indicated the students had was there existed only one correct answer to a mathematical problem. Ruth stated technology was used in the lesson to help eliminate this misconception. Desmos was used to illustrate there could be multiple solutions to a problem. The overview feature of Desmos, which allows the teacher to show multiple student answers, also helped the students to think about why multiple correct answers may exist. She said, “especially with like the graph one, it was very nice, to see which two kids got like the different answers and you can say this one although different it still is a correct answer.”

The next use of technology occurred where Ruth used technology for the purpose of presenting students' solutions. In the Desmos software, Ruth would click on a student's name and display the solution arrived at by the student. Ruth would then speak about the student's work and state whether or not the solution provided by the student was correct or incorrect and how robust the solution was.

The next use of technology was of prompting mathematical discussions and of eliciting the students' understanding. Here Ruth would display the students' work and

then ask verbal questions about the conjectures the students wrote. For example, sometimes after the students had completed their work on the Riemann sums Ruth pulled up and displayed the students' work on the screen and then began a discussion with the students on how they came up with the conclusions they made about the areas under the curve.

In another scenario, Ruth used the quick poll feature of the TI-Nspire software to pose a question to the whole class. This gave a quick picture of the general ideas the students had of the estimates of the areas under the curve, such as whether the left Riemann sum underestimated or overestimated the area under the curve. Ruth would then discuss with the students which of the answers were correct and why.

Research question 3

Ruth's Beliefs about Teaching Mathematics

There were several beliefs Ruth espoused about the teaching of mathematics. These beliefs included motivation, teachers' mastery of mathematical knowledge, relating the mathematics to students' real life experiences and interests, professional growth, and building on students' knowledge.

One of the main beliefs Ruth espoused very early in the first interview was the idea students needed to be motivated to engage in the mathematics. For the most part, this motivation appeared to be extrinsic because the students do not come to class ready and willing to learn mathematics. In the absence of technology, the teacher himself or herself provided the extrinsic motivation in the form of praise or some other incentive, such as providing fun activities, to get students to engage in the mathematics. An example of the way Ruth indicated mathematics teachers should motivate their students was when asked

what simile, best described a mathematics teacher, Ruth indicated the mathematics teacher is more like a coach because of the motivation and celebratory aspects. She indicated just as a coach provided motivation to the athletes in the form of encouragement, the mathematics teacher also encouraged students to do mathematics. As a part of motivating the students, the teacher should also celebrate with students when they achieved some of their learning goals in mathematics.

Ruth indicated the mathematics teacher should possess strong mathematical knowledge and experience in doing mathematics to be a good teacher of the subject. When asked why she thought the simile 'coach' best represented a mathematics teacher, Ruth explained one of the reasons she chose coach was because just as the coach should Master the knowledge of his sport so too should the teacher possess a mastery of the mathematics he or she teaches. She said, "I feel that a good coach first has to know the craft if it like a sports, they usually have to have played it ... have some experience playing it, so as a teacher you have an experience of doing it yourself you have to have an experience of learning it."

The next belief Ruth espoused was the mathematics teacher should relate the mathematics taught in the classroom to the real-life experiences of the student and the students' interests. Ruth espoused an example of this belief when asked what are some of the things a good mathematics teacher does. She indicated she believed a good mathematics teacher related the material to real life, so the material was interesting to the students. She said, "when or how are we gonna use this?" Like in real life, and even if they are not gonna use this in real life you have to make this interesting, so they are interested in learning it and are interested in going along with you." Thus the

mathematics teacher should use mathematics material related to the students' experiences or may apply to students' future lives, so the students want to stay interested in the mathematics being taught.

Ruth also indicated teaching mathematics also involved the teacher being up to date with developments in his or her field. Thus the teacher should continuously seek professional growth opportunities. The teacher should seek to know of the latest developments in technology and pedagogy in the area of mathematics. An example of this belief occurred when Ruth was asked what are some of the things a good mathematics teacher does. Ruth said, 'you should keep yourself up to date, the professional development is very important.' In keeping up to date with the latest developments, Ruth indicated teachers should learn from their colleagues and other professionals who were trying to help others improve teaching and learning. She indicated professional development was a continuous process and newer methods were always being developed and tested. Hence the onus was on teachers to seek out the latest developments from which they may be able to include strategies that helped them to become better teachers. In another scenario, when asked a what teachers should do with their older teaching practices Ruth indicated teachers should have the flexibility to accept changes and not be too rigid in holding to their old practices. However, she indicated the teachers should also determine which of their older practices are still viable and not be willing to throw out the useful practices. She said, "they (teachers) should be able to change, not saying that they have to throw away everything but if there are better practices they should adopt them."

Ruth indicated good teaching of mathematics included building on mathematics students already know. She felt the teacher should teach new mathematics based on using mathematics the students had previously encountered. She stated if students are already familiar with content somewhat connected to the topic being taught then the new material will be easier for the teacher to get across to the students. In the second postlesson interview, when asked if she thought the goals of the lesson were met, Ruth indicated because the topic Riemann sums was a nice, and quick topic and the students already knew about integrals, about the making of area estimates using approximation, then the follow up with Riemann sums was beneficial for the students.

Ruth's Beliefs about Teaching Mathematics with Technology

Ruth espoused many beliefs about teaching mathematics with technology. These included teaching with technology as a means for checking the correctness of students' work, enrichment, teacher demonstrations, feedback, and making sense of the mathematics (See Table 10).

Ruth appeared to believe the technology should also be used to check students' solutions with the intent to making corrections to the students' work. In this scenario, the teacher used technology to direct students towards good mathematical work. For example, when asked how teachers may effectively use technology to teach a mathematics lesson Ruth indicated teachers might use technology effectively to direct students toward correct mathematical solutions. She said, "if I go and see there are a couple of kids having problems with a certain, question I can say, look at what so and so did, and I can project their answer." In this manner, the teacher may direct the students needing help to correct solutions provided by their peers, or demonstrate paths to the

correct solutions with technology. The teacher may also guide students who have difficulties with the task to come up with their correct solutions.

Table 10: Ruth's Use of Technology, Beliefs and Factors

Ruth: Use of Technology	Ruth: Beliefs about Teaching Mathematics	Ruth: Beliefs about Teaching Mathematics with Technology	Ruth: Factors
U1: Checking students working	B1: Motivation	B11: Checking, instantly correct students' direction of mathematical work	F3: Time
U2: Play and exploration	B4: Mastery of mathematical knowledge and experience	B17: Use technology for enrichment and fun	F7: Topic related technology use
U3: Development of conceptual mathematics and debunking misconceptions	B2: Real life experiences/ student interests	B15: Technology should be used for presenting mathematics.	F8: Testing days proximity
U4: Presenting and critiquing students' solutions	B5: Keeping up to date/ professional growth	B18: Technology should provide useful feedback	F6: Availability of Premade materials amenable to technology
U7: Prompting discussions and eliciting students' understanding	B6: Building on mathematics that students already know	B13: Think about and make sense of the mathematics through the visualizations.	F9: Ease of transition to using other technologies

The next belief Ruth displayed was the teacher should use technology for enrichment. In this regard, the teacher should use technology for providing further mathematical work for students after they have been learning some mathematical concepts. The work with technology could be in the form of fun games, which are related to the mathematics topics. For example, when asked if some of the activities she did with technology were for enrichment, Ruth indicated some of the activities were more to make up for extra time. She had chosen some Desmos activities for students to do as an

enrichment of the mathematics topic on quadratic functions. One type of Desmos activity Ruth liked for her students to do after a mathematical topic was completed was marble slides. In this type of activity, the students would complete specific types of equations, such as a quadratic equation, to match the pattern of a partially solid curved graph. After entering this equation, a dot (marble) would be released which slid along the graph hitting some dots (marbles) marked on the graph. If the equation typed by the student matched the graph, then the moving marble would strike all the other dots (marbles) along the path of the graph and score points. This game would then move to the next usually more complex level. Ruth indicated she also used Jeopardy games for enrichment over a mathematics topic she had already taught to the students. In these enrichment activities, the students could have fun while doing mathematics at the same time.

The next belief Ruth espoused was the teacher should be the one to conduct demonstrations of the mathematics the students should learn where necessary, and to present the work of the students during the mathematical tasks. When doing demonstrations with technology the teacher provided an entry point to the students for good use of the technology and to provide procedures for doing the mathematical task. For example, when asked how technology was used in her classroom Ruth indicated she believed teachers should demonstrate the mathematics to the students and the students may follow what the teacher had demonstrated. This was particularly useful when the students were new to the technology and needed to learn the various keystrokes involved when solving a particular mathematics problem. Thus the teacher demonstrations would give the learners an idea of what the keystrokes looked like.

The teacher could demonstrate how to use technology to answer particular mathematics problems, and this would give the students an entry point into how to solve a mathematical problem with technology. This would then give students an idea of what to do when they have to engage with a mathematical task on their own. They would be able to visualize what to do, and this will make their learning much easier. She said, “the teacher can pull it (software) up, and let's say the kids are having a problem and I can do it on my laptop and they can see what the keystrokes are and what the end result should look like, that's easier to kind of like ‘oh, I am not getting that answer, what am I doing wrong' and then you can kind of answer it.”

Ruth indicated the technology should be used as a checking tool regarding providing feedback to the solutions the students provide. The technology may provide feedback indicating whether or not the student had a correct solution. Ruth discussed what she liked about using technology, in particular, dynamic technology, in her classroom. She stated features such as instantly displaying student work and the capability of the technology to provide feedback are what she liked about the Desmos and TI-Nspire technologies she used.

Ruth indicated she used the feedback features of Desmos and the learning support aspect of TI-Nspire technology. She talked about an activity in which students worked in pairs on a mathematical activity in which one student chose a mathematical object while the partner tried to figure out what the student first chose by asking mathematical questions. For each round, the partner tried to ask better questions and so on. There was a benefit to both teacher and students in Desmos provided feedback to the students about whether the solution (mathematical object eventually chosen by partner) was correct.

The next belief espoused by Ruth was the visual displays of the technology should help students to make sense of the mathematics. For example, when asked if the technology (Desmos) helped the students in thinking about the mathematics. Ruth indicated she believed the technology helped learners to think more deeply about mathematics because the visual aspect of technologies stimulated their thinking. For example, the students could be able to see better, the changes in the slopes of the graph with technology, rather than make a connection between the concept of derivative and slope in another way. She said, “I believe in the visual aspect of each problem like you know when the students are asked like you know what is the derivative of a function mean? They might like not understand it, but if you give them a graph of a function and then ask Umm, on what intervals is the derivative positive, negative, then they know uh, I know the graph, I know the derivative is the slope, so I can find where it is gonna be negative, or positive. So the visual aid is umm, very important.”

Possible Relationships among Ruth’s Beliefs Factors and Uses of Technology

The beliefs Ruth held were inferred from her statements in the interviews. The actions with technology observed in the classroom are related to the beliefs Ruth held based on the apparent similarity of the actions with technology to the particular beliefs espoused. The relationships between actions with technology and espoused beliefs are determined based on the idea beliefs are predispositions to action.

Beliefs about teaching mathematics

Motivation (B1) does not appear to be related to any particularly observed use of technology. Whereas Ruth provided encouraging words in the form of praise to the students, using words such as ‘good job’, and whereas this may apparently help students

to engage more with the task with technology, the measure of whether the encouragement provided by the teacher to how the students was related to the how the students used the technology is unclear. The belief teachers should possess a mastery of mathematical knowledge and experience (B4) was noticeable when Ruth indicated the students needed to be cognizant of the output of the TI-Nspire, and could, therefore, be related to the use of the technology for prompting mathematical discussions and eliciting students' understanding (U7). The knowledge here may be the knowledge of content and technology held by the teacher. Ruth demonstrated knowledge of the technology when she indicated the technology was particularly difficult to move sliders to the correct positions indicated by the numbers on the screen.

Ruth's belief the mathematics should be related to the students' real life experiences (B2), and student interests appeared to be related to the activities or topics chosen with the technology but not related to any particular use of the technology. The beliefs teachers should be keeping up to date and of professional growth (B5) does not appear to be related to any particular classroom use of technology observed, but may be related to the teacher's knowledge of using the technology before classroom use. Finally, the belief the mathematics being taught should be building on mathematics students already know (B6) does not appear to be related to any use of technology observed.

Beliefs about teaching mathematics with technology

The belief technology should be used for the checking, and instantly correct students' direction of mathematical work (B11) appears to be related to the use of the technologies for checking whether students were working or progressing on their work (U1). The belief technology should be used for enrichment and fun (B17) may be related

to the activities chosen for the lessons but does not appear to be related to any particular use of technology observed. Ruth's belief technology should be used for teacher demonstration and presentation (B15) appears to be related to the use of technology for presenting and critiquing the students' solutions (U4). The belief technology should provide useful feedback (B18) might be indirectly related to the observe use of technology for play and exploration (U2). The belief technology should help uses to think about and make sense of the mathematics through the visualizations (B13) appears to be related to the observed use of technology for play and exploration (U2) and for the development of conceptual mathematics and debunking misconceptions (U3).

Factors and Use of Technology

The factor, time to learn with the technology (F3) does not appear to be related to any particularly observed use of technology. It might be related to the exploration phase of the lesson if the students were not already familiar with the technology. However, in the lessons observed, the students were already familiar with the technologies. The mathematical topic factor (F7) was useful for Ruth to consider when deciding what technology to use but does not appear to relate to the observation of how the technology was used.

The proximity of testing days factor (F8) was related to the number of days that would be dedicated to technology use, but not to a particular observed used of technology in the classroom. The availability of premade materials amenable to technology factor (F6) appears to be related to the whether the technology may be used and the procedures involved in the materials may indicate to the user how to use the technology. The ease of

transition to using other technologies factor (F9) may determine whether or not the new technology is used but not related to any particular observed use of technology.

Summary of Ruth's Beliefs Technology Use and Factors

In general, it appears for Ruth most of the observed uses of technology were more related to her beliefs about teaching mathematics with technology than to her beliefs about teaching mathematics only or to other factors related to her choice and use of technology (Figure 8). In particular, it appears her belief pertaining conceptual understanding with technology was the most important when looking at the relatedness to the observable uses of technology in her classroom, as it appeared to be related to two observed uses of technology. Overall, of the five uses of technology observed, four of these uses were related to Ruth's beliefs about teaching mathematics with technology, while one use was related to her beliefs about teaching mathematics only.

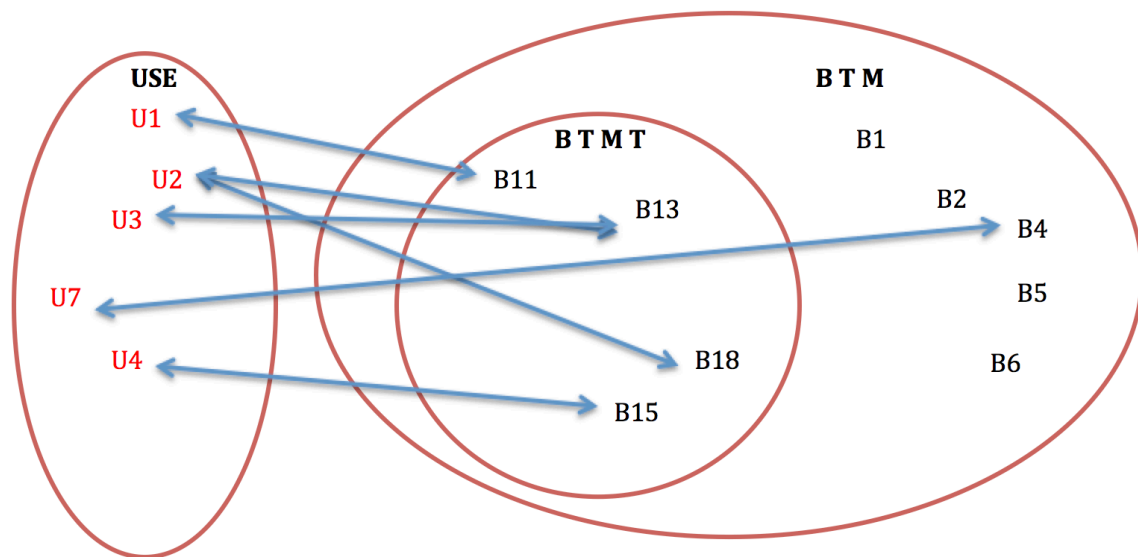


Figure 8. Possible relationships in Ruth's uses of technology and beliefs

In figure 8 the bidirectional arrows indicate a possible two-way influence between beliefs and use of technology.

BTM – Beliefs about Teaching Mathematics

BTMT – Beliefs about Teaching Mathematics with Technology

USE – observed use when teaching mathematics with technology

4.3 The Case of Sam

Research question 1

About Sam and her Prior Experiences with Technology

Sam was a tenth year mathematics teacher at Green high school. She spent most of her teaching career at the high school level, working in both the private and public school systems. Because of her experiences, she was one of the first teachers recommended by her principal to participate in my study.

In college, Sam majored in mathematics and obtained both a Bachelors degree and Masters degree in mathematics. She had also taken some mathematics education courses and was familiar with many catch phrases in mathematics education such as 'facilitator' and 'productive struggle'. Sam stated she had taught all the mathematics courses in a high school classroom, from Algebra 1 all the way up to Multivariate Calculus. When I conducted observations in her classrooms, she taught Precalculus and Calculus lessons with technology.

When I met Sam, she had just finished teaching a mathematics class. She was very exuberant about teaching mathematics. Because she had developed a strong love for mathematics, she wanted to inspire her students to love mathematics and see mathematics as being useful to them. She was not always great at mathematics, but her parents hired a tutor to help her during high school. Later in her college years, she found professors who loved doing mathematics and their passion for mathematics somewhat influenced her to love mathematics as well. She stated all teachers could do the same for their students.

She indicated she loved to teach mathematics with technology. She has been exposed to many technologies such as calculus in motion most recently, algebra in

motion, Better Lesson, Geometers Sketchpad, GeoGebra, Desmos, and TI-Nspire Navigator technology. She was apparently an expert with the TI-Nspire and stated she used it a lot in her mathematics classes. Desmos was her second most liked technology she used in her lessons. She was not comfortable with GeoGebra, with which she had not had much experience. When I observed Sam teaching with technology she used Desmos and TI-Nspire Navigator (Table 11). She used these technologies because of the ease of access to these technologies by the students. Desmos was free software available to all who have access to hardware. Green high school had a one-to-one technology policy. The school administrators ensured every student has access to a laptop all the time. The administrators also ensured all students have access to a TI-Nspire all the time.

Table 11: Digital Cognitive Technologies Sam Used

Date in 2016	Topic	Technology Used
April 15	Displacement	Desmos
April 26	Riemann Sums	TI-Nspire
May 12	Shifts in Functions	Desmos

Factors that affect Sam's Use of Technology

There were many factors Sam indicated determined whether or not she would use technology in her classroom. These factors include time for learning technology, ease of use of the features of the technology, and the support students needed when to be engaged with technology.

Sam indicated the use of technology might be affected by the time it took for the teacher to learn to use the technology. When asked what she wanted in technology she would use in the classroom, she stated what was needed most of all in the technology she was using to be simple, that is, not too complicated to learn to use. She said, “ (for our

technologies) we want simple, all teachers need simple because we are very busy and if somebody develops a product that is umm, my training must take 10 minutes... I don't want to spend hours and hours and hours to do (learn) that because that is pretty overwhelming, you know.”

Another factor Sam indicated determined whether or how she would use technology was the ease of use of the features of the technology. When asked why she used a particular technology, Desmos in her lessons, Sam explained she used Desmos in the investigation because of its ease of use (simplicity), and quality of the software to do graphing of equations given certain information. This simplicity was required because the ease of use was a great thing for teachers and students who have to manipulate the technology. She said, “Desmos is one of those hot (technologies) right now and the reason why it is so popular, is it is so simple to use.” She compared Desmos to GeoGebra. When asked if she considered any other technology to use in her lessons Sam said she had considered GeoGebra, but she did not use it because it was not as easy to use as Desmos. She said, “GeoGebra I think, but no its not intuitive, you really have to have some previous knowledge of it to use GeoGebra but Desmos, anybody can use Desmos to jump into it, it may take 5 minutes to obtain a lesson activity, you can 5 minutes to build your own it is very, very simple.”

Another factor Sam indicated affected her use of technology in the classroom was the type of mathematical support the students needed to learn the material. She indicated sometimes the students needed to focus on conceptual understanding and at other times they needed to focus on the applications of the mathematics. An example of this belief was displayed where Sam was asked at what point would she decide not to use

technology. Sam indicated she believed technology should not be used if the teacher felt the students needed practice in the mathematics without the use of technology. She stated this was necessary in cases where the technology may hide the procedures of doing the mathematics, while simultaneously students lacked basic understandings of the hidden process. She indicated she would remove the technology if the students needed practice of some mathematical skills to be reinforced. She said, “if I identify there is a certain skill we need to reinforce, there will be some time to reinforce certain skills ... if they cannot do, you know thinking about dividing 20 by 5 conceptually, you say ok what is a 20 divided by 5, that is a 5 that you can place in 20 four times... mathematically, you know what I am saying, so at some point in time you need that mastery to do more complex things.”

Research question 2

Cognitive Technologies Sam Used

Similar to Ruth, the cognitive technologies Sam used during the classroom observations were Desmos online graphing calculator and the TI-Nspire Navigator calculator technology. Both these technologies are considered as cognitive technologies for my research. Sam used the slider features the Desmos graphing calculator as well as the slider features of the TI-Nspire calculator technology.

Sam used premade activities in both Desmos and TI-Nspire.

Uses of Cognitive Technologies in Sam’s Classroom

Sam displayed several uses of technology during the observation of her lessons with cognitive technology. These uses included checking and displaying students work on the mathematical tasks, exploration, developing conceptual understanding of

mathematics, presenting students' solutions, prompting mathematical discussions and eliciting students' understanding, and problem-posing.

Sam used technology for checking and displaying students' work when she wanted to see what students had done in the mathematical task. Displaying of the mathematical work was one of the uses of cognitive technology. In the Desmos technology, the screen showed the list of student names with a bar to the right of each name containing a dot in the bar representing the progress of the students. In the TI-Nspire technology showed the names of the students who were logged in and working on the activity. Sam used the technologies for checking and displaying of mathematics students created or were supposed to engage with. When Sam wanted to give students and idea of how to get to the mathematics they were supposed to engage in, she displayed the login page and the mathematical task on the screen for the entire class to see. When Sam wanted the students to see an approach to the mathematical task by one of the students she would look on the display of the students' names to see which student was working on a particular problem and then by clicking on the name of the student, she would display the mathematical work of the student. During this checking and display stage, Sam had much manipulation of the technology.

Sam encouraged students to use the technology as a means of exploration and manipulating mathematical objects to better understand the problems they were attempting. An example of this occurred in the activity with displacement problem featuring two runners going along a zigzagging pattern on a graph and required students to indicate which runner would be ahead in 40 seconds. The users of the technology were encouraged to manipulate the sliders on the graph to get an idea of where each runner

would be at what point in time. The technology helped to illustrate more easily and more accurately the idea of which runner went over the greater distance. Another example of exploration occurred during the third observation when during the activities on the transformation of functions the students were required on the mathematical task to write equations depicting the transformation of the functions illustrated in the activities. There was no specific direction given by the teacher on how to write the function, but the activity contained a few examples for the users to follow. The mathematical examples in the task were very general, and the students had to come up with the specific functions to match the ones needed for the activity. During this stage, Sam did very little manipulation of the technology itself.

Sam focused the use technology in ways students were able to conduct inquiries into a mathematical concept such as the ‘area under a curve’ or the ‘transformation of functions’ and develop a conceptual understanding of the mathematics involved. An example of this occurred on the third observation in which Sam allowed the class to engage in an activity on finding shifts in functions and making conjectures about the shifts. Sam indicated the topic was not covered in class before the observation, but the students knew what the next topic would be so when they wanted some background information they could read ahead on their own. During the activity, Sam did not provide an entry point for the students via demonstrations nor did the activity provide examples the students could follow a set procedure to get to an answer. The activity required the students to make conjectures and explain their answers to shifts which were not particularly intuitive, such as horizontal shifts. During this stage, Sam had little manipulation of the technology.

Sam used the technology as a means of promoting mathematical discussions of the mathematical work students were doing by engaging them in a discussion on the mathematics they created with technology. Sam used the technology to display a student's work and engaged in a discussion of the solutions the student had submitted. Sometimes Sam took a look at the answers of the entire class to engage them in a mathematical discussion. At times when the students were stuck in a solution path, Sam would ask them questions such as, “ what do you need to do to come up with that solution?” This helped the students to think with the technology when no direct approach was given. In the lesson on the area under the curve Sam used the quick poll feature of the TI-Inspire to check for the students understanding of the mathematical concepts involved in the investigations, they had done with the technology concerning the area under the curve via Riemann sums. The quick poll captured the number of student responses that indicated which type of estimate indicated over or underestimated of the actual area. During this stage, Sam had some manipulation of the technology.

The final use of technology observed in the lesson Sam taught was for mathematical problem posing. This was done after students had completed the investigations. Sam delineated types of problems the students should want to create focusing on problems not only geared to the simplest of mathematical ideas such as using linear functions but also included more elaborate ideas such as what would be most suitable for the students' own stories. An example of this use occurred in the last section of the Desmos activity on displacement. Sam encouraged students to create their activity about velocity with Desmos based on their experiences with the concepts they had just investigated. She said, “think about your story; graphs cannot be linear.” She encouraged

them to look into the meanings of what was happening in their graphs. Not only were they to look into the mathematical meanings but also to critique the results regarding the implications for the happenings in real life. During this stage, Sam did not manipulate the technology much.

Research question 3

Sam's Beliefs about Teaching Mathematics

Sam espoused numerous beliefs about teaching mathematics. These beliefs include motivation, finding and diagnosing student problems, active involvement, productive struggle, strong mathematical knowledge, and seeing mathematics as useful in life.

One of the first beliefs Sam espoused was the idea the students needed to be motivated to engage with the mathematics. For the most part, this motivation is extrinsic usually from the teacher. Sam did not refer to technology as something motivating for the students. Sam shared ideas the students are themselves not motivated to do the mathematics in class, so there was no intrinsic motivation on the part of the students. For example, when asked to explain her views about the craft of teaching mathematics Sam indicated the job of teaching clearly included motivating the students to learn because the students lack interest in doing mathematics on their own. Sam indicated the use of motivation techniques by the teacher was more essential when students lack interest in mathematics or were prone to be disruptive in the classroom. In essence, motivation was more essential for students considered difficult to teach. She said, “ teaching mathematics including motivation, umm, motivation ok, what are you going to do with the students

who are saying I don't care I mean like you know which was in many (public) schools was the big issue, I don't care, just don't, you know, how you motivate the students.”

Sam believed the teaching of mathematics involved finding and diagnosing students' learning problems. She expressed ideas teaching mathematics was mainly about diagnosing and remedying students' mathematical deficiencies. She stated students come to class with some mathematical deficiencies and it was up to the teacher to identify those student needs and decide on what are the most suitable and relevant strategies for dealing with mathematical deficiencies. The diagnosis was accomplished in an interaction, wherein the teacher and the students communicate with each other. For the remedying, the teacher must use all necessary materials, including technology, to help the students get to a level where they are mathematically proficient. At this level of mathematical proficiency, the students can calculate, analyze, create and ask good mathematical questions.

When given a list of similes to indicate which one of the similes best described a mathematics teacher, Sam indicated the job of a teacher was to find out what the students' needs are and use strategies to remedy the needs of the students. The needs of the students are on an individual basis, and the teacher should get at each deficiency and isolate methods to deal with these on an ongoing basis until the deficiencies are resolved. The deficiencies are identified by the ways students struggle with particular concepts. She said, “I am choosing doctor, is ... that is the part of a teacher who identifies students who struggle in mathematics, so they diagnose they can diagnose every student with a specific deficiency and umm, prescribe umm, a remedy kind of.”

Sam believed active involvement in the classroom by both teachers and students was important. She promoted ideas the teaching of mathematics cannot be dictated solely from one direction. It must be from teacher to student and also from student to teacher. This leads to a continued two-way interaction in which both the teacher and students are actively engaged in the mathematics created in the classroom. An example of this belief was illustrated when Sam was asked what simile least represented what a mathematics teacher does. Sam indicated a news broadcaster was the least because the news broadcaster was a one-way interaction. Thus the students and the teachers should be actively involved in the whole process of mathematics teaching. She said, “we do not want one-way interaction from the teacher, as news broadcaster ...its not teaching, it's not giving information to the board when meaningful interaction with the kids without giving the students opportunity or you audience opportunity to actually participate.” Here she indicated the students and teacher should be developing the mathematics in the classroom, not the teacher alone.

Sam believed the teacher should have students engage in a productive struggle in the mathematics classroom. In this regard, Sam indicated the teacher should have students engage in mathematical tasks and problems in which students struggle to get to a solution. In these kinds of problems the solution path and the answers are not immediately apparent and so the students may have to develop their solution paths for these problems. Sam opined giving the students time and space for productive struggle was a useful idea in mathematics education and time for productive struggle is a very useful means of getting students to meaningfully engage with the mathematics. She

indicated productive struggle allows students to experiences more ‘aha’ moments in mathematics.

Sam indicated she believed for one to be a good mathematics teacher he or she should demonstrate strong knowledge of what mathematical content to teach and what strategies to use to teach mathematics. When asked what she thought about good teaching Sam indicated the teacher should possess knowledge and ability for a wide range of mathematical content. At the high school level, this included teaching content from Algebra 1 through to multivariate calculus. Hence a person with good mathematical teaching should have good content knowledge in the grade level he or she taught and also for all the other grade levels. She said, “There are two things a mathematics teacher must be able to do. 1) Teach better mathematics, and 2) teach mathematics better ...all the way from algebra 1 to multivariable calculus.”

The next belief Sam espoused was the teacher should be purposeful in getting students to see how mathematics can be useful in their daily lives and the world around them. The teacher should include students in activities either he or she selects or allow the students to select based on their life experiences involving mathematics. An example of this belief can be displayed when Sam was asked what she wanted to see in her mathematics class. Sam stated she wanted her students to see the usefulness of mathematics. She indicated this usefulness should be seen in their lives and the world around them. She opined when students see the usefulness of mathematics they would be fascinated and more interested in learning more mathematics. She said, “I want them to be fascinated with how useful mathematics is in their life.” Any evidence of beliefs

pertaining to the usefulness of mathematics in the lives of the students in the observations was discussed in the section on the relationships between beliefs and uses of technology.

Sam's Beliefs about Teaching Mathematics with Technology

Sam espoused numerous beliefs about teaching mathematics with technology. These include frequent usage, problem posing with technology, real life activities, enrichment, facilitating conceptual understanding, and facilitating student inquiry (see Table 12).

Sam believed technology should be incorporated in the classroom on a frequent basis. In fact, she said she tried to use technology the classroom on a daily basis. She used a variety of technologies and strived to use varied representations with technology in her teaching. She said, “ I try incorporating technology as much as the resources allow umm in everyday classrooms...GeoGebra and Desmos and many, many other others. Umm, I believe that mathematics must be presented in an umm, absolutely graphical, numerical and analytical umm approaches.”

Sam also indicated the teacher should have the students engage in technological activities in which they are challenged to develop good mathematical questions. Not only should the activities enable students to ask good mathematical questions, but they should also encourage them to develop solutions to these questions as well. The use of technology to generate questions and come up with solutions to these questions was very powerful as it gave the students power and ownership over the work. When asked about the activities she used with technology, Sam indicated she liked activities in which she can have students develop questions based on what they do observe. She indicated that students may use technologies to engage with a mathematical situation and are then

required to ask some good questions and solutions regarding the situations presented in the activities with technology. She said, “they (students) should determine what was going on there to put in that in a perplexing mode and then they have to generate a question and come up with a solution to verify it.”

Sam also appeared to hold the belief the teacher should have students use technology to engage with real life situations, in particular, situations involving their lives. An example of this belief was displayed when Sam was asked about how the technology may be used effectively in a lesson. Sam indicated activities involving students engaging in real world activities were the most effective. She described a scenario where she had students engaged in an activity with trigonometric functions wherein they were required to find an activity in their lives involving trigonometry and use technology in such a project. She had the students used some technologies to investigate some real life phenomenon in their lives. The students used technology to record, measure, pose problem scenarios and make solutions. She said, “I was teaching geometry ... I did a project with the kids where they had to ... we did trigonometry unit they had to go somewhere in an activity in their lives they had to (use technology in) a certain activity in their lives that involve trigonometry, and they had to come up with all this development of trigonometry solution.” She noted projects like these using technology give satisfaction to the students as well as ownership of the mathematics, as it related to their lives. Any evidence of the beliefs pertaining to real life situations in the observations was discussed in the section on the relationships between beliefs and observed uses of technology.

Another belief espoused by Sam was technology should be used for validation of concepts taught without technology or use technology for enrichment. In this scenario, the topic was first taught without technology, and then technology was used as a form of support for the work done using pencil and paper methods. She also proposed idea validation of work done while using technology may depend on the topic more and the grade level of the students. For example, when asked to compare pencil and paper methods to work done with technology Sam indicated the technology was mostly used to validate work done by hand and not the other way around. She said, “you do have to do certain things by paper and pencil, but validate it with the technology and the technology should be used carefully so that it is not substituting the thinking process, the cognitive process, right, it should be supplement to the, should enhance, enrich I would say enrich.”

Sam appeared to believe the teacher should use technology in the classroom in ways to promote conceptual understanding of mathematics whenever possible. As such technology was not simply for practicing what was already learned before the use of technology but technology can be the starting point of getting to understand why the mathematics works. For example, Sam talked about two things that happen with mathematics in the classroom. The first thing was applications of mathematics and the second thing was the understanding (conceptual) of mathematics. When asked if technology was more useful for applications or for understanding mathematics Sam indicated technology was more useful for understanding. In particular, technology was useful when considering how to develop an understanding of the new mathematics concepts. She said, “for understanding, for when you are teaching new concepts it is very

nice to get the students to understand that, or students who applied that already and they know that they can visualize the results.”

Table 12: Sam's Use of Technology, Beliefs and Factors

Sam: Uses of Technology	Sam: Beliefs about Teaching Mathematics	Sam: Beliefs about Teaching Mathematics with Technology	Sam: Factors
U1: Checking students' work	B1: Motivation	B19: Frequently and multiple representational uses.	F3: Time for learning technology
U2: Use of technology for exploration	B7: Diagnosis and Remediation	20: Questioning, problem posing with technology	F10: Ease of use of features
U3: developing conceptual understanding of mathematics	B8: Active teacher and student engagement	B12: Involve students in exploring real life activities with technology	F7: Mathematical support for what students will be learning with the technology
U7: Prompting mathematical discussions	B9: Engage in productive struggle	B17: Technology for validation of concepts taught without technology, enrichment	
U6: Questioning, problem posing	B4: strong knowledge	B13: Technology should be for conceptual understanding	
	B10: Usefulness in life	B21: the role of the teacher is to facilitate the students' investigations in tasks involving technology	

The final belief espoused by Sam concerned the role of the teacher with technology. Sam forwarded ideas the teacher's role with technology should be consistent

with the teacher's role in situations without technology. She stated the teacher should be a facilitator of student investigations with the technology. When asked what should the teacher be doing while the students are working with the technology, Sam indicated the teacher should work with each group of students, moving from group to group, making sure they understood the process of getting the activities done. She said, "I am going to monitor the groups while they are working, answer their questions while they are doing the operations and I will take every group that is finished with the Desmos activity."

Possible Relationships among Sam's Beliefs Factors and Uses of Technology

I defined espoused beliefs as the teacher's espoused depositions towards action. Hence, I describe actions observed in the classroom related to the teacher's beliefs as those actions, which appeared to be similar in nature to these beliefs.

Beliefs about Teaching Mathematics

The belief concerning the motivation of students (B1) did not appear to be related to any particular action with technology Sam's mathematics classroom. The belief concerning diagnosis and remediation of student deficiencies (B7) appeared to be somewhat related to the use of technology for checking the students' work (U1), particularly the diagnosis part of the belief. The belief about an active teacher and student engagement (B8) appear to somewhat related to some uses of technology. The active teacher engagement may be seen in cases where the teacher manipulated the technology, and the active student may be seen when student manipulate the technology. The active teacher and student engagement do not, however, appear to relate to a specific use of technology but may be related to technology use in more general ways. The belief concerning students' engagement in productive struggle (B9) may be related to the type

of activity chosen but may not be directly related to a particular use of technology observed. In fact engaging in productive struggle may not depend on how the technology was used.

The belief mathematics teachers should possess strong mathematical knowledge (B4) may be related to the observed use of technology for prompting mathematical discussions and eliciting students' understanding (U7). The belief mathematics teachers should help students to see mathematics as useful in life (B10) was related to the type of mathematical activities chosen to include real life situations such as cops chasing criminals in cars, but not to any specific use of technology.

Beliefs about Teaching Mathematics with Technology

The belief technology should be used frequently and for multiple representational (B19) uses does not appear to be related to any particularly observed use, but may be related to the way users could see and manipulate the multiple representations given by the technology during their explorations. The belief technology should be used for questioning and problem posing (B20) appears to be related to the use of the technology for posing problems and solutions to those problems (U6). The belief mathematical activities should involve students in exploring real life activities with technology (B12) appear to be related to observed use of technology for exploring mathematics (U2). Sam's belief technology should be used for enrichment (B17) does not appear to be related to a specific observed use of technology, but from the interviews, some of the concepts may have been taught to the students on days before their work with the technologies. The belief technology should be used for conceptually understanding mathematics (B13) appears to be directly related to the observed use of technology for

developing conceptual understanding of mathematics (U3). The belief the role of the teacher was to facilitate the students' investigations in tasks involving technology (B21) does not appear to be directly related to a particular observed use of technology.

Factors and Use of Technology

The factor time for learning technology (F3) may be related to the students' practice, the teachers learning time, the students' exploration if the students were not familiar with the technology or had difficulties manipulating the technology. However, for the lessons observed the teacher and students appeared to have been already familiar with the technologies. The factor, ease of use of features of the technology (F10) may be somewhat related to the use of technology for carrying out mathematical activities but in particular, when the students use technology to carry out explorations (U2). The factor concerning what students will be learning with the technology (F7) may be related to the task chosen but not directly related to a particular use of technology observed. The factor, availability of premade materials amenable to technology (F6) may somewhat determine whether or not the teacher encourages students to use technology for the development of conceptual understanding.

Summary of Sam's Beliefs Technology Use and Factors

In general, it appears for Sam the beliefs about teaching mathematics with technology are more related to the use of technology for teaching mathematics than are the general beliefs about teaching mathematics only or the other factors the Sam indicated had an influence on her choice and use of technology (Figure 9). Of the five uses of technology observed in the mathematics lessons Sam taught, three of the uses appeared more related to her beliefs about teaching mathematics with technology, while

two of the observed uses appeared to be slightly related to her other beliefs about teaching mathematics.

Although four of the more general beliefs about teaching mathematics did not appear to have a relationship with the observed uses of technology, only two of the beliefs about teaching with technology did not appear to be related to the observed uses of technology. Hence the actual use of technology in Sam's mathematics classroom appeared have more connections to the beliefs about teaching mathematics with technology than beliefs about teaching mathematics in general.

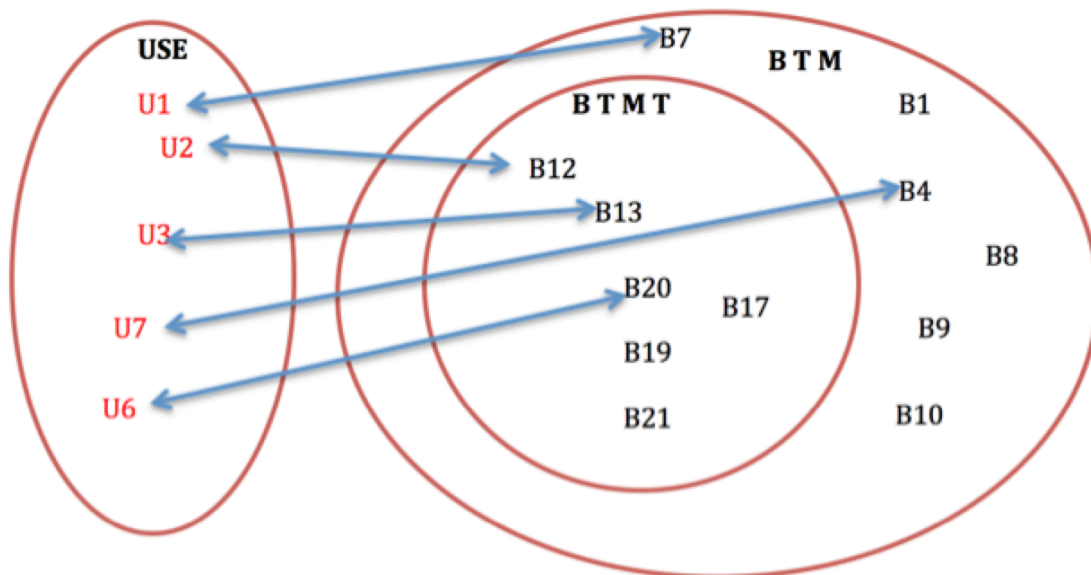


Figure 9. Possible relationships in Sam's uses of technology and beliefs

In figure 9 the bidirectional arrows indicate a possible two-way influence between beliefs and use of technology.

BTM – Beliefs about Teaching Mathematics

BTM – Beliefs about Teaching Mathematics with Technology

USE – observed use when teaching mathematics with technology

Commonalities among Factors for Beth Ruth and Sam

Whereas the factors influencing the choice and use of technology may not have had a direct influence on the observed uses of technology in the mathematics classrooms, they do however have importance in the technologies the participants chose to use. There existed many commonalities among participants as shown in Table 13. Details pertaining to the teachers indicated factors affecting their choice and use of technology and how this related to the literature on factors that affect cognitive technologies are discussed in this section.

Table 13: Commonalities among Factors

Beth: Factors	Ruth: Factors	Sam: Factors
F3: Time to learn with technology	F3: Time	F3: Time for learning technology
F6: Supporting, Premade materials	F6: Availability of Premade materials amenable to technology	F6: Availability of Premade materials amenable to technology
F1: Cost	F7: Topic related technology use	F7: What students will be learning with the technology
F2: Access	F8: Testing days proximity	F10: Ease of use of features
F4: Visual appearance	F9: Ease of transition to using other technologies	
F5: Currentness of technology		

Researchers (Cuban, 2001; Koehler & Mishra, 2009) have indicated the time factor was very important to teachers deciding whether to use technology. In this study, the time factor (F3) was a prevalent factor among all the three teachers in the study. The participants noted the time factor was important for learning to use the technology for both teachers and students. Also, the participants indicated time was important when

thinking about how much material needed to be covered in the lessons. Even though the participants indicated time was a factor in their choice and use of technology, they spoke about time in slightly different ways. This indicated there were other possible considerations interacting with the time factor. Beth seemed focused on how time affected coverage of the material in class to get students to an expected level of mathematics. Ruth and Sam were, however, more focused the time it took to teachers and students learn how to use the technology. Perhaps because the students in Beth's school had limited access to technology Beth spoke about the time factor in a different way than Sam and Ruth who both taught in a school with plenty access to technology.

The other factor common among all three of the participants in the study was the availability of premade materials (F6) to use in the mathematics classes. The third most prevalent factor was the technology was amenable to the mathematical topic to be taught. This factor was indicated by two of the three participants. In this factor, the technology was related to what technology was to be used for but not directly to how technology was used.

The other factors mentioned by the participants did not have common ground to what other participants in the study indicated. For example, Beth mentioned access to technology, which was not a factor indicated by Ruth and Sam. This may have been because Ruth and Sam taught in a school with one-to-one access to the cognitive technologies they used with their students. Furthermore, the students did not have a difficulty accessing the new technologies because they were allowed to take the technologies home. Beth, on the other hand, was teaching in a school where there was not

one-to-one access in the classroom, and the students' access to technology outside of class was limited.

Some researchers (Aygei & Vogt 2012; Cuban et al., 2001) have indicated that access was an inhibiting factor to technology use in classrooms. In my study, from the participants' point of view, technology was becoming freer. Therefore as schools moved to giving students more access to technology, teachers would have less of an issue with the access factor to using cognitive technologies in the classroom. However, Desmos and GeoGebra needed support remain free to users and be of good quality. In fact, good quality software free to users was a cost to someone else. Desmos had some support to remain free, and GeoGebra seemed to be a bit lagging in support. In the short term, there did not seem to be a problem with the software, but not much was known about long-term support for software of good quality, which were also free.

Commonalities among Technology, Beliefs and Uses of Technology

Common technologies used by Beth, Ruth and Sam included Desmos online graphing software (Table 14). Common technologies used by Ruth and Sam also included TI-Nspire Navigator. Beth used GeoGebra, which Sam and Ruth did not use. Sam and Ruth may have used the same technologies because they taught at the same school and also taught similar courses, Precalculus and Calculus. Beth on the other had taught at a different school and different courses, Algebra 1 and Geometry, from both Ruth and Sam.

Table 14: Commonalities among Digital Cognitive Technologies

Beth	Ruth	Sam
Desmos	Desmos	Desmos
GeoGebra	TI-Nspire Navigator	TI-Nspire Navigator

Common uses of technology among Beth, Ruth, and Sam include checking and displaying the students' mathematics (U1), play and exploration (U2) and development of conceptual mathematics (U3). Common uses of technology between Ruth and Sam only include Prompting mathematical discussions and eliciting students' understanding (U7).

There were no common uses for Beth and Sam only or between Beth and Ruth only (Table 15).

Table 15: Commonalities among Participants' Uses of Technology

Beth: Uses of Technology	Ruth: Uses of Technology	Sam: Uses of Technology
U1: Checking students' work	U1: Checking students' work	U1: Checking students' work
U2: Play and exploration	U2: Play and exploration	U2: Use of technology for exploration
U3: Conceptual understanding	U3: Development of conceptual mathematics and debunking misconceptions	U3: Developing conceptual understanding of mathematics
U4: Presenting students' mathematics	U4: Presenting and critiquing students' solutions	U7: Prompting mathematical discussions
U5: Teacher demonstrations	U7: Prompting mathematical discussions and eliciting students' understanding	U6: Questioning, problem posing
U6: Problem posing		

Common beliefs about teaching mathematics among Beth, Ruth, and Sam include motivation (B1). Common beliefs about teaching mathematics between Ruth and Sam only include mastery of mathematical knowledge (B4). Common beliefs about teaching

mathematics between Beth and Ruth only include real world experiences for the students (B2). There were no common beliefs about teaching mathematics for Beth and Sam only indicated in the data (Table 16).

Table 16: Commonalities among Beliefs about Teaching

Beth: Beliefs about teaching mathematics	Ruth: Beliefs about Teaching Mathematics	Sam: Beliefs about Teaching Mathematics
B1: Motivation	B1: Motivation	B1: Motivation
B2: Real-world practices	B2: Real life experiences/ student interests	B7: Diagnosis and Remediation
B3: Student-centeredness	B4: Mastery of mathematical knowledge and experience	B8: Active teacher and student engagement
	B5: Keeping up to date/ professional growth	B9: Engage in productive struggle
	B6: Building on mathematics that students already know	B4: Strong knowledge
		B10: Usefulness in life

Common beliefs about teaching mathematics with Technology among Beth, Ruth, and Sam include generating mathematical concepts (B13). Common beliefs about teaching mathematics with Technology between Beth and Ruth only include checking and correcting students' direction of mathematical work (B11) and presenting the mathematics (B15). Common beliefs about teaching mathematics with Technology between Beth and Sam only include only B12, play and exploration. Common Beliefs about Teaching Mathematics with Technology between Ruth and Sam only are not captured by the data (Table 17).

Table 17: Commonalities among Beliefs about Teaching Mathematics with Technology

Beth: Beliefs about Teaching Mathematics with Technology	Ruth: Beliefs about Teaching Mathematics with Technology	Sam: Beliefs about Teaching Mathematics with Technology
B11: Checking students' work.	B11: Checking, instantly correct students' direction of mathematical work	B19: Frequently and multiple representational uses.
B12: Play and exploration	B17: Use technology for enrichment and fun	20: Questioning, problem posing with technology
B13: Generating mathematical concepts	B15: Technology should be used for presenting mathematics.	B12: Involve students in exploring real life activities with technology
B14: Acceleration of mathematical procedures and skills	B18: Technology should provide useful feedback	B17: Technology for validation of concepts taught without technology, enrichment
B15: Presenting mathematics	B13: Think about and make sense of the mathematics through the visualizations.	B13: Technology should be for conceptual understanding
B16: real meaning of doing mathematics		B21: the role of the teacher is to facilitate the students' investigations in tasks involving technology

Putting it all Together

Overall beliefs about teaching mathematics with technology appear to be espoused more in common between two or more of the teachers than beliefs about teaching mathematics.

Whereas all three teachers shared one belief about teaching mathematics, motivation (B1), and a belief about teaching mathematics with technology, generating mathematical concepts (B13), in common, only the belief about the teaching with technology is related to a common use of technology, play and exploration (U2), held by the teachers (see figure 10).

From the commonalities noted in the observed practices when teaching with technology, participants' espoused beliefs pertaining to teaching mathematics and teaching mathematics with technology, the beliefs about teaching mathematics with technology appear to have a stronger influence on the participants' use of cognitive technology. This was apparent by the number of more direct relationships, on the use of cognitive technology in the mathematics classroom compared to the beliefs about teaching mathematics only.

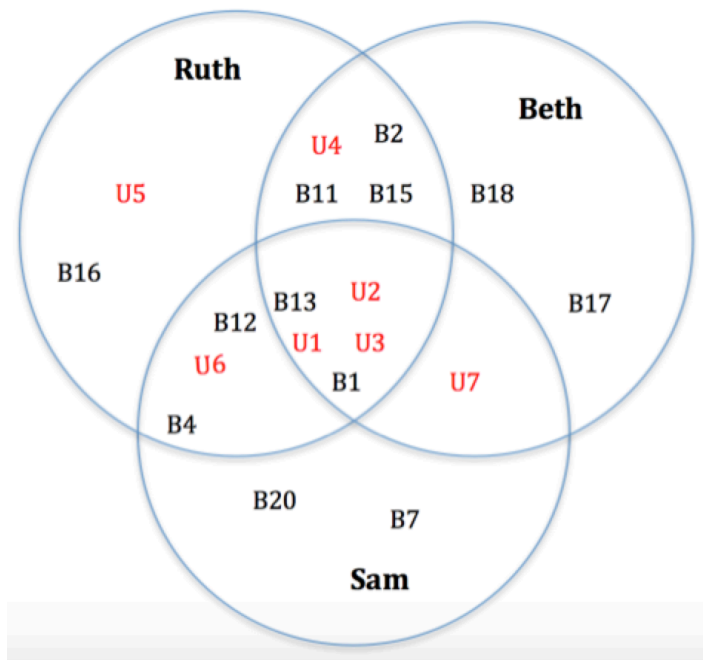


Figure 10. Commonalities among beliefs about teaching mathematics, beliefs about teaching mathematics with technology, and uses of technology

Further Analysis of the Use of Cognitive Technologies

In the uses of technology exhibited by the three participants, the teacher appeared to be the chief manipulator of the technology when the technology was used to demonstrate to the students what to do with the mathematics or to show what mathematics the students had done and the solutions they produced. Because the technology here was used for display purposes and the mathematics did not change simply as a result of using the technology then these uses of the cognitive technology appeared to be in line with Pea's (1987) notion of technology being an amplifier. The teachers' intention was to demonstrate the mathematical steps (procedures) for the students to follow. The teacher did not make conjectures expected of the students. Even when the task was set up as one where students would be required to make conjectures, the use of technology at the demonstration and presentations phases would not be classified Reorganizer. The phases of demonstration and presentation also appear to be in line with the category of Servant proposed by Goos et al. (2003) because the teachers used their knowledge of the technology and content when deciding whether to accept the output of the technology. Use of technology as Master was not evident, perhaps because the teachers in this study were avid users of technology. Hence, for teachers who were avid users of technology, the most basic level of cognitive technology use was at the level of Servant and Amplifier categories, henceforth referred to as Servant-Amplifier.

Other aspects of the use of technology included the use of technology for explorations, development of conceptual mathematics, and promoting mathematical discussions. All three participants appeared to focus on these uses of technology use for extended periods of time while teaching mathematics with technology. Whereas the

students manipulated the technology in these situations, the teacher would sometimes intervene to help students who were having difficulties fall in the Reorganizer category (Pea, 1987) use of technology at the grain size of a mathematical task. These uses of technology also appear to be consistent with the Partner category of technology use (Goos et al., 2003). The uses of the technology being similar to both Partner and Reorganizer are henceforth referred to as Partner-Reorganizer. Although the teacher had less personal manipulation of the technology at this stage, it was he or she who tended to legitimize the results of the mathematics the students produced.

Table 18: Major Themes for Uses of Digital Cognitive Technologies (bolded)

Pea (1987)	Goos et al. (2003)	From my study (2017)
	Master	DPPS
Amplifier	Servant	
Reorganizer	Partner	
	Extension of Self	

There were other uses of technology noticeable in the observations of the lessons taught with cognitive technologies, which did not appear to align with the categories defined by Pea (1987) or Goos et al. (2003). One such use was the use of technology for the purpose focusing on the problem posing aspect of mathematics. This use appears to be a different level for using cognitive technology in the mathematics classrooms. I describe the problem posing and solving level use of technology as the use beyond understanding how or why the mathematics works to being able to develop the user's own problems or tasks with the cognitive technology in a sensible mathematical way. I grouped the observed uses of technology into three major themes: Servant-Amplifier, Partner-Reorganizer and Digital Problem Posing and Solving (DPPS) (See Table 18).

Participants did not necessarily use cognitive technology in a linear manner from Servant-Amplifier, through to Partner-Reorganizer to DPPS levels. The use of cognitive technology was observed to take on different directions in the classroom. Sometimes the participant may have used a Servant-Amplifier, then a Partner-Reorganizer to DPPS use in some sort of methodical way. Sometimes the tasks chosen were set up to be carried out in a way such that at the very beginning there was some teacher demonstration or some teacher checking of initial student work, so the technology was used as an Servant-Amplifier. Then the participant would encourage some exploratory activity, consistent with Partner-Reorganizer, and finally in some limited ways may have encouraged some DPPS level of using the cognitive technology. Alternately, there may be some exploratory work, aligned with Partner-Reorganizer, then some teacher presentations of student work Servant-Amplifier and perhaps some DPPS level technology use.

Summary of General Connections

Beliefs of the participants appear to pertain either to teacher-focused actions with technology and student-focused actions with technology or both. There were also instances where beliefs about teaching mathematics with technology existed, but these pertained more to what the technology could be used to do rather than whether teachers or students were responsible for actions on the technology, such as the technology may be used to provide feedback.

Beliefs about teaching with technology pertaining to teacher-focused actions with technology (BTFAT) included checking (B11) and presenting (B15). Beliefs pertaining to student-focused actions only with technology (BSFAT) included exploration (B12), real meaning of doing mathematics (B16), and problem posing (B20). Beliefs pertaining

to BTFAT and BSFAT together included generating mathematical concepts (B13), and acceleration of mathematical procedures and skills (B14). Beliefs that pertained to neither teacher focused actions with technology or to student-focused actions with technology included useful feedback (B18), multiple representational uses (B19), and enrichment (B17). The relevant belief about teaching mathematics with technology falling under these themes that are related to the observed uses of cognitive technologies in the mathematics classrooms of the participants are shown in Figure 11.

All three participants displayed some beliefs appearing to be either the teacher-focused or student-focused actions with technology. They held beliefs such that students and teachers should be active in developing the correct mathematical concepts with technology. Beth and Ruth displayed some beliefs pertaining to teacher-focused actions only with technology. These were beliefs teachers should use technology to check the mathematics of the students and critique these mathematical solutions. They also held beliefs teachers should demonstrate mathematics the students were to do. Beth and Sam some displayed beliefs pertaining to student-focused actions with technology only. They displayed beliefs students should explore on their own mathematics with the technology and come to their own conclusions without outside help. Sam also displayed beliefs students could develop their own mathematical problems and solutions.

The data presented by the participants indicated some beliefs about teaching with technology did not pertain to any focus on teacher- focused or student- focused actions with technology, but rather pertained to what technology might be used to do. These beliefs were not discussed or analyzed because the nature of the research, which

investigated only how the technology was used. Thus, these beliefs were not included in further discussions in this paper.

It appeared the more the participant displayed beliefs tended toward teacher-focused actions with the technology (BTFAT), the more likely the observation of the use of technology as Servant – Amplifier when teaching with technology. In the same vein, the more the participant displayed beliefs that tended to wards student-focused actions only with technology (BSFAT), the more likely there existed observed use of technology as Partner- Reorganizer or DPPS use technology when teaching with technology. When participants’ beliefs appeared to include both teacher-focused and student-focused actions with technology the more there existed observed use of technology as Partner-Reorganizer.

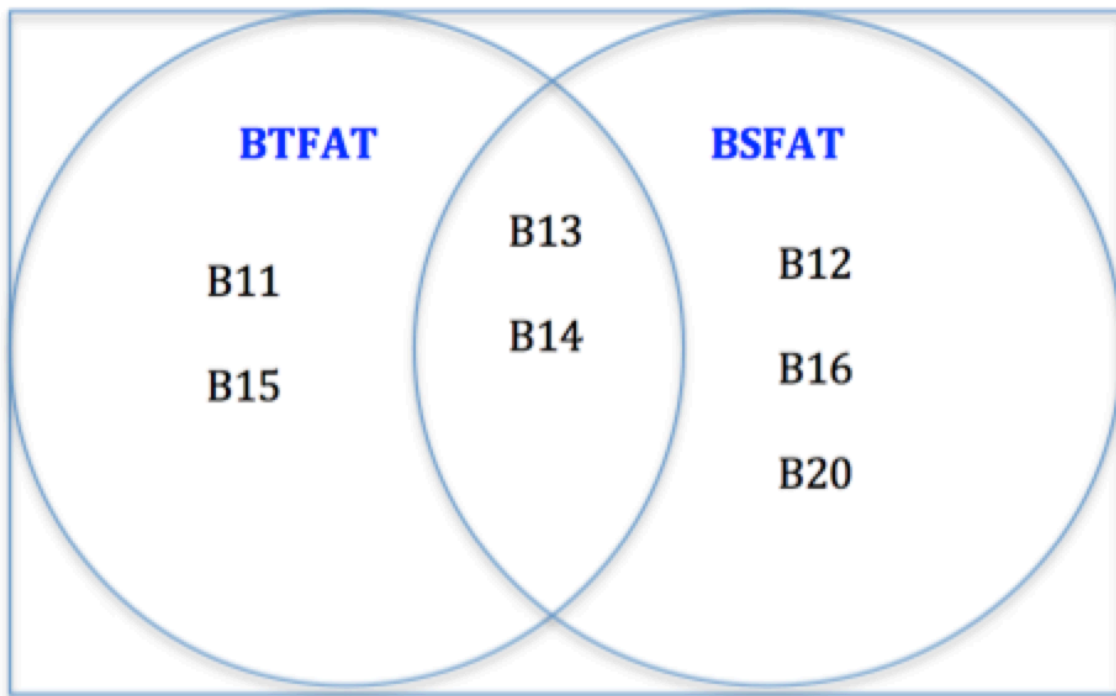


Figure 11. Major themes for beliefs about teaching mathematics with technology related to uses of technology.

CHAPTER 5: DISCUSSION AND IMPLICATIONS

In this chapter, I restate the research questions of the study, discuss the relevant findings, the implications and limitations of the study. The findings of the study were mainly based on individual cases. The themes from the cases are built into more comprehensive ones concerning the participants' use of cognitive technology, the factors related to their choice and use of technology, and beliefs about teaching mathematics and use of technology for teaching mathematics. The focus of the discussion in this chapter mainly surrounds the individual teachers' use of digital cognitive technologies and their beliefs about teaching mathematics with technology. Beliefs about teaching with technology appear to be the major influencers of the use of cognitive technology in the mathematics classroom.

5.1 Discussion

This study extends the metaphors proposed by Pea, (1985, 1987) and Goos et al. (2003) for the use of technology specifically to the context of digital cognitive technologies. Digital cognitive technologies are defined as technologies of an electronic nature, specific to mathematics, and used for more than calculation purposes. The factors participants mentioned are also compared to factors mentioned in the current research literature and indicate how factors related to cognitive technologies only, compared to factors related to technologies in general. Furthermore, the beliefs espoused by participants were analyzed for possible relationships to the uses of technology, and the results of the participants compared.

The research questions of the study were as follows:

For secondary mathematics teachers who routinely use technology into their classrooms:

1. What factors do they consider in choosing technology for use?
2. What digital cognitive technologies do they use?
- 2b. How do they use digital cognitive technologies to teach mathematics?
3. What are the teachers' beliefs about teaching mathematics, and teaching mathematics with technology?
- 3b. What commonalities are observed among teachers' beliefs and use of digital cognitive technologies?

The first and second research questions concerned the way digital cognitive technologies are selected and used. They are typical of 'how' questions suitable for case study methodology. To answer the third research question, I investigated the beliefs of teachers concerning teaching mathematics and using technology to teach mathematics. I also noted the commonalities between beliefs, factors, and use of technology of the participants. The use of case study methodology to answer these types of research questions is consistent with the recommendations of Yin (2003).

To answer the research questions I used various data collection methods. These included interviews, observation field notes, and supporting documents such as lesson plans, teacher technology files, and mathematical task activities. Using multiple sources of data collection concerning participants' beliefs is recommended by a number of researchers in the field (e.g. Leatham, 2006; Pajares, 1992; Richardson, 1996). Furthermore, the use of multiple sources is common in case studies. I used various types of interviews; Beliefs and Technology selection interview, Prelesson interviews,

Postlesson interviews and a Final beliefs interview. The interviews were semi-structured and used to gather detailed information about the participants' views pertaining to teaching mathematics, and teaching mathematics with technology.

I observed nine lessons of all the participants using digital cognitive technologies to teach mathematics. The technologies the participants used included Desmos, GeoGebra, and TI-Nspire Navigator. The classroom observations I conducted were means of describing how the participants used technology and later to determine the espoused beliefs of the participants evident in their teaching with technology.

I collected supporting documents from the participants such as their lesson plans, technology files from prior lessons, and the mathematical tasks they used in their lessons with technology. I used aspects of the supporting documents to help to create a picture of how the teachers choose and used technology in their lessons in general and to determine if the picture I painted of the teacher was consistent with what the teacher did in their lessons in general.

To ascertain the meanings I developed concerning the participants' beliefs and use of technology were true to what the participants themselves held, I conducted member checks with each of the participants. The participants agreed with most of the assertions I made about their beliefs and made very few changes where necessary, which I updated in my writing. The member checking aspect of the process served as a means of checking the accuracy of my interpretation of the statements of the participants to enable me to make a reasonably good analysis of their beliefs and use of technology.

Factors

In this research, I found there were several factors the participants claimed as having influenced their choice and use of technology. Beth was concerned with factors such as access to technology, the cost of technology, time for learning mathematics with the technology, currentness of the technology, and visual appearance of the technology. Ruth was concerned with factors such as time for students to learn to use the technology, whether or not the topic was relevant to technology use, proximity to testing days, availability of support or premade materials for use with the technology, and technical difficulties associated with the technology. Sam was concerned with factors such as time for learning technology, ease of use of the features of the technology, and the support students needed when to be engaged with technology.

Commonalities among Factors

In this study, two factors appeared to dominate the scenarios given by the teachers. These were the time factor and the opportunity to make use of premade materials for the technology. The time factor was prevalent as teachers indicated they had so much to do and so much material to cover so learning new technologies should not be time-consuming. The availability of premade materials was important because this would free up the teacher to do other things apart from having to design material for the lessons. Whereas these factors may have been related to the way technology was chosen to be used, it appeared there was no direct observed use of technology related to these factors.

The consideration of the time factor is consistent with the findings of other researchers in the field. Researchers (Cuban, 2001; Ertmer, 1999; Koehler & Mishra, 2009; Wepner, Ziomek, & Tao, 2003) have indicated the time factor is very important to

teachers deciding whether to use technology. Whereas the research literature pointed mostly the time teachers take to learn and teach with the new technologies, the participants in my study while strongly indicating the learning of teaching by teachers should not take too much time, were also concerned about the learning curve with the technology for the students.

Two of the three participants indicated the topic to be taught with the technology was important to the technologies they chose to use. This factor may be subsumed under pedagogical considerations factors (Niess, 2012; Zbiek et al., 2007) in the related literature, which indicates knowledge of the way the technology can be used to deal with subject matter topics is essential for being able to effectively teach with a particular technology. The topic factor did not appear to be directly related to the observed use of technology during in the lessons but may have been related to what technologies the participants chose to use in their lessons.

Despite the fact there were no other common factors present among the participants' descriptions, I discuss the access factor, as it appeared to be dependent on the school one of the participants taught. In the case of Beth, there was limited access to the technologies provided by the school in that there was one class set of technological hardware such as chrome books or iPads on which the mathematical software could be run. Access to hardware had some bearing on what alternative was chosen to be used, or to when the technology could be used.

Some researchers (Ertmer, 1999; Aygei & Vogt 2012; Cuban, Kirkpatrick, & Peck, 2001; Wong, Li, Choi, & Lee, 2008) have indicated access was an inhibiting factor to technology use in classrooms. Whereas participants who taught in a school providing

the teachers and students with one-to-one access to laptops or TI-Nspire calculators did not indicate access to technology as a problem, the participant in the school where such access was not provided cited this factor as a problem. Moreover, because software was becoming more freely available, the access to hardware appears to be the factor limiting the instances of the mathematical software the teacher and students would use to do mathematics.

The findings of my study did not indicate any direct relationships between the factors listed by teachers and their observed uses of digital cognitive technologies while teaching mathematics. It appears some of the factors may have had some effect in the technologies chosen to be used. After choosing the technologies, the considerations given to these factors by the participants bore little relationship if any to their actual use of technology.

Uses of Technology

In this research, I found participants used technology in several different ways. Beth exhibited six uses of technology while teaching mathematics lessons with technology. These uses included using technology as checking tool, for play and exploration, for the development of conceptual understanding of mathematics, for teacher demonstrations, presentation of student work, and providing and entry into problem posing with technology. Ruth exhibited five uses of technology while teaching mathematics with technology. These uses included checking students' progress and solutions, play and exploration, conceptual development of mathematics and debunking mathematical misunderstandings, presenting and critiquing students' solutions, and eliciting mathematical discussions. Sam exhibited six uses of technology while teaching

mathematics with technology. These uses included checking students' solutions on the mathematical tasks, exploration, developing conceptual understanding of mathematics, presenting students' solutions, prompting mathematical discussions and eliciting students' understanding, and problem-posing.

The participants shared many uses of technology. All three participants used technology in ways to check on the mathematical work of the students and display the work of the students to the class, for play and exploration, and for the development of conceptual mathematics. In using technology as a checking and display tool the teacher was focusing the technology more as an Amplifier of the mathematics (Pea, 1987). Other researchers have posited the use of technology for checking and display wherein this type of checking may be seen more as a management tool function of technology (Peressini & Knuth, 2011). Whereas arguably the display aspect of technology use was not inherently cognitive, the fact cognitive technologies are used for this purpose was considered as use of digital cognitive technology in my study.

The use of technology for play and exploration is a cognitive use and this being common to all the participants and was more consistent with the use of technology as a Reorganizer of mathematical thinking (Pea, 1987) given the non-existence of explicit procedures. The use of technology for play and exploration appeared to be a use of technology as a Partner (Goos et al., 2003). Other researchers have proposed ideas about the use of technology for play such as the use of technology as a cognitive tool (Peressini & Knuth, 2011).

Similar to the use of technology for exploration purposes, all three participants focused the use of technology for the development of conceptual mathematics. This use

of technology appeared similar to the use of technology for as cognitive tool (Peressini & Knuth, 2011), as a Reorganizer of mathematical thinking (Pea, 1987) and Partner (Goos et al., 2003).

Two of the participants used technology for prompting mathematical discussions and eliciting student understanding. These uses of technology are more in line with the Reorganizer (Pea, 1987) and of partner (Goos et al., 2003). Two of the participants used the technology for the presentation of students work and for teacher demonstrations. In these cases, the participant took on the role of a mediator wherein the participant validated the output of the technology. The use of technology for the display of mathematical work and procedures was more aligned with Amplifier of the mathematics (Pea, 1987). This use was mainly also in line with Servant (Goos et al., 2003) because the participant was knowledgeable of the output of the technology, and the output of the technology had to be aligned with the output the participant expected. In instances where the output of the technology did not align with the participant expected output, the participant would not accept the technology's output.

On further analysis of the uses of technology exhibited by the three participants, I posit the observed uses of technology for all the participants should be grouped into three major categories: Servant- Amplifier, Partner-Reorganizer, and Digital Problem Solving and Posing (DPPS). In the first category, the teacher appeared to have the more active role using the technology. In this situation, teacher knowledge of expected output took precedent while the technology output was backgrounded. The uses of technology in this category included checking and display of students' mathematics, presentation and explanation of student mathematics by the teacher, and when the technology was used to

demonstration of mathematics students should engage with. I classified this group of uses of technology as Servant- Amplifier because of the observed role of teacher knowledge and the use of technology for display and speed purposes.

In the second category, Partner-Reorganizer, the teacher still had somewhat of an active role, the students were active, and the technology was used more as a source of co-intelligence to get students to think mathematically and make the correct conjectures wherein the teacher and students legitimized the output of the technology. Even in cases where students were unable to validate the output of the technology the teacher knowledge could be called upon. These uses of technology included: using technology for play and exploration, using technology for the development of conceptual mathematics, and prompting mathematical discussions. I classified this group of uses as Partner-Reorganizer because the uses were more for focusing on why the mathematics works in the given situations.

The third category of technology use, DPPS, included the use of technology for the purpose of problem posing and solving novel mathematical situations. This appeared to be using cognitive technology beyond reorganization of mathematical thinking about given situations. This use of technology was beyond reasoning about how or why the mathematics works in given problems and the development of conjectures for given problems, to being able to develop their own problems or tasks with the cognitive technology in a sensible mathematical way. This group of uses I classified as digital problem posing and solving (DPPS) because of the focus on the creation of the users own novel (and good) mathematical problem situations and solutions with technology.

Beliefs about Teaching Mathematics

In this research, I found the participants exhibited several beliefs about teaching mathematics. Beth exhibited three beliefs about teaching mathematics. These beliefs included motivation, real-world practices, and student-centeredness. Ruth exhibited five beliefs about teaching mathematics. These beliefs included motivation, teachers' mastery of mathematical knowledge, relating the mathematics to students' real life experiences and interests, professional growth, and building on students' knowledge. Sam exhibited six beliefs about teaching mathematics. These beliefs included motivation, finding and diagnosing student problems, active involvement, productive struggle, strong mathematical knowledge, mathematics as useful, and problem posing.

All three participants indicated belief student motivation was a key component of effective teaching of mathematics. In fact, they all indicated the teacher should provide the motivation, and it was the job of the teacher to inspire the students to want to learn mathematics. The motivation aspect has been forwarded by researchers (e.g. Leatham, 2002) as relating to the teaching of mathematics. Motivation does not appear to be seen as a belief held about teaching mathematics, however, but more as something teachers or students should possess or develop in general. Another major belief about teaching mathematics indicated by two of the participants was the teacher mastery of mathematical knowledge. The third most common belief forwarded by two of the participants included teaching mathematics should involve real world experiences of the students. Overall, it appeared all the participants in this study held beliefs (such as student-centeredness, active teacher and student involvement), that would be classified in the category of facilitator (Ernest, 1989) because participants appeared to be more interested in ways the

teacher could be a guide to the students while the students make inquiries in mathematics related to the students interests and experiences.

Beliefs about Teaching Mathematics with Technology

In this research, I found the participants exhibited several beliefs about teaching mathematics with technology. Beth exhibited seven beliefs about teaching mathematics with technology. These beliefs included a checking tool of students' progress and solutions, an exploratory tool for playing with mathematics, a conceptual tool for students mathematical understanding, a procedural acceleration tool for skills practice, a presentation tool, and a tool for developing the real meaning of mathematics, a display tool for good visualization of the mathematics. Ruth exhibited five beliefs about teaching mathematics with technology. These beliefs included technology as a means for checking the correctness of students' work, enrichment, teacher demonstrations, feedback, and making sense of the mathematics. Sam exhibited eight beliefs about teaching mathematics with technology. These beliefs included frequent usage, problem posing with technology, real life activities, enrichment, facilitating conceptual understanding, and facilitating student inquiry with technology.

All three participants exhibited the common belief the technology should be used for the development of conceptual mathematics in the students. Prior researchers (e.g. Leatham, 2006) had indicated this was a belief teachers may hold. This prevalent belief among avid uses of technology for my research appeared to be likely to hold relationships to their use of technology.

Two of the participants held beliefs pertaining to checking and correcting students' mathematical work with technology. Although this belief appeared to be more

administrative in nature, it gave credence to how participants wanted control over the use of digital cognitive technology in the classroom.

Two participants held beliefs pertaining to presenting the mathematics of the students or for demonstrating mathematics to the students. These beliefs appear to be consistent with procedural beliefs (Leatham, 2006) and to beliefs regarding using technology as Servant (Goos et al., 2003).

Two participants held beliefs pertaining to play and exploration. This appeared to be consistent with beliefs about the use of technology as Partner (Goos et al., 2003). Even though the students themselves may use technology as Master during explorations, it was unlikely the teacher would want the students to have as the intended use. These teachers knew all the technologies may display output leading users to incorrect mathematics.

Relationships among Beliefs and Technology Use

The participants in this research espoused more beliefs about teaching mathematics with technology than beliefs about teaching mathematics in general. Also, the participants held more beliefs in common about teaching mathematics with technology than they did about teaching mathematics in general.

I found the participants in this research exhibited some beliefs about teaching mathematics, in general, appeared to be related to some observed uses of technology. These beliefs included teaching mathematics was about diagnosing, and remediation and teachers should have strong mathematical knowledge to teach mathematics effectively. These beliefs appeared to be related to the observed uses of technology for checking and displaying students' mathematics and for eliciting mathematical discussions respectively.

Some observed uses of digital technologies in the classrooms of these participants appeared to be related to some beliefs about teaching mathematics with technology. The belief technology should be used for checking and displaying student work was related to the observed use of technology for displaying the mathematics of the students. The belief the technology should be used for the conceptual development of mathematics was related to the observed use of technology for developing conceptual understanding of mathematics in the classroom. The belief technology should be used for questioning and problem posing was related to the observed use of technology for posing mathematical problems and developing solutions to the problems.

Each of the three participants espoused more beliefs in their interviews about teaching mathematics with technology than beliefs about teaching mathematics in general. Furthermore, the beliefs about teaching mathematics with technology appeared to be more directly related to the observed uses of technology for each participant. Beth held five beliefs about teaching mathematics with technology related to observed uses of technology but no beliefs about teaching mathematics in general directly related to uses of technology. Ruth held three beliefs about teaching mathematics with technology related to observed uses of technology but one belief about teaching mathematics in general directly related to an observed use of technology. Sam held three beliefs about teaching mathematics with technology related to observed uses of technology but two beliefs about teaching mathematics in general related to uses of technology.

Beliefs about teaching with technology commonly held by the participants were more related to uses of technology than the commonly held beliefs about teaching mathematics in general. For instance, all the participants shared the belief about

motivation, a belief about teaching mathematics in general. There were no observed uses of technology that appeared to be directly related to the motivation belief. Participants also shared the common belief; technology should be used as a means of conceptual development of the mathematics, a belief about teaching mathematics with technology. Two uses of technology: for play and exploration, and for the conceptual understanding of mathematics were related to this belief. Furthermore, beliefs held by two participants only about teaching mathematics with technology were more related to their observed uses of technology than beliefs about teaching mathematics in general.

In summary, participants exhibited more beliefs about teaching mathematics with technology than with teaching mathematics in general. There also appeared to be more direct relationships between participants' beliefs about teaching mathematics with technology and observed uses of technology, than relationships between teaching mathematics in general and observed uses of technology. Furthermore, there were more commonalties among the participants in the relationships between beliefs about teaching mathematics with technology and observed uses of technology than beliefs about teaching mathematics in general. Thus the observed uses of digital cognitive technologies participants' beliefs about teaching mathematics with technology appeared to have more connections to beliefs about teaching mathematics with technology than to beliefs about teaching mathematics in general.

Beliefs about teaching mathematics, in general, were related to the uses of technology in the mathematics classroom (Ertmer and Ottenbreit-Leftwich, 2010; Leatham, 2006). Researchers (e.g. Ernest, 1989; Thompson, 1992) indicated the beliefs about the nature of mathematics and about teaching mathematics were very related to the

use of technology for teaching mathematics in the classroom. My study extended this literature by finding deeper relationships on beliefs and technology wherein the beliefs about teaching mathematics with technology appeared to be more related to the observed uses of digital cognitive technologies in the classroom for teaching mathematics. Furthermore, my study extended on the literature on ideas about the use of cognitive technologies.

I further classified beliefs about teaching mathematics with technology into two major groups. In order to develop these categorizations, I looked at the categories provided by Ernest (1989) and uses by Goos et al. (2003) as a guide toward the shaping of the themes pertaining to beliefs about teaching of mathematics with technology.

For the most part, the participants' beliefs about teaching mathematics were related to Ernest's (1989) category of facilitator. The facilitator category, however, was not ideal for the diverse beliefs teachers held about teaching mathematics with technology. Thus, I looked at Goos et al.'s (2003) four categories namely master, servant, partner and extension of self for ideas. The participants' beliefs about teaching with technology could only be classified into two categories of Goos et al.'s (2003) categories, Servant or Partner. But few of the beliefs the teachers espoused fit perfectly into any individual category of Goos et al.'s (2003) categorizations. For example, the belief technology should be used for teacher demonstrations appeared to be of the partner category with one participant and Servant category with another participant. On further examination of the other beliefs about technology espoused by the participants, it appeared all beliefs about teaching mathematics with technology could either fall into more than one of Goos et al.'s (2003) categories simultaneously or none of these

categories at all. Thus, I found it more useful to categorize the beliefs into ways that appeared to relate to the uses of technology displayed by the teachers.

There were 11 belief themes among the participants generated during the interviews. I classified the participants' beliefs about teaching mathematics with technology into the following groups: (1) beliefs pertaining to teacher-focused actions with technology, (2) beliefs pertaining to student-focused actions with technology, or (3) beliefs pertaining to both teacher-focused actions and student-focused actions with technology, and (4) beliefs pertaining to no action with technology.

General Relationships: Uses of Technology and Beliefs

The more the participant espoused beliefs pertaining to actions with technology the more likely the action was observed in the participant's classroom. Because beliefs pertaining to no actions with technology were related to any observed uses of technology, then further discussion does not include these beliefs. For the beliefs pertaining to some actions with technology, most of the participants' espoused beliefs pertained to a mix of teacher-focused actions and student-focused actions with technology. Furthermore, most of the observed uses of technology in the classrooms were related to beliefs pertaining to a mix of teacher-focused and student-focused actions with technology.

Uses of technology at the level of the Servant-Amplifier level appeared to be associated with beliefs pertaining to teacher-focused actions with technology.

Uses of technology at the partner-reorganizer levels appeared to be related to beliefs pertaining to mixed teacher-focused and student-focused actions with technology. Uses of technology at the DPPS level appeared to be related to beliefs pertaining to more student-focused actions with technology.

The findings of this study provide three main contributions to the literature. First, it provides an extension to the use of cognitive technologies by providing an additional aspect of using cognitive technologies. Second, it deepens the literature on beliefs about teaching with cognitive technologies by taking into account the beliefs of teachers about teaching with cognitive technologies. Third, it provides the idea teachers' beliefs about teaching mathematics with technology appears influential to teaching mathematics with digital cognitive technologies.

5.2 Implications

Findings of this research have implications for practice, technology design and for further research. These are dealt with in the following subsections:

Practice

The findings of my study indicate many uses of digital technologies among teachers who are avid users of these technologies. All the uses were further grouped into three broad categories Servant-Amplifier, Partner-Reorganizer, and DPPS. These findings of the uses support trends in mathematics education such that mathematics teachers should seek to develop various uses of digital cognitive technology and find ways to apply these uses in their lessons. They should provide mathematical activities for use with digital cognitive technologies amenable to the various ways of using these technologies. Literature in mathematics education mainly focuses on teaching mathematics with technology from Servant- Amplifier level, up to the Partner-Reorganizer level and is in part based on older technologies. The results of my study are based on newer digital cognitive technologies, which provide for an increased number of

uses. As a result of my study teachers, where possible, should also place emphasis on their students' DPPS technology use.

The findings of my study indicate technology use does not necessarily proceed in some linear fashion, from Servant-Amplifier to Partner-Reorganizer to DPPS. An implication of this finding is teachers should use technology in ways supporting meaningful learning. The finding supports current trends in mathematics education. Whereas having students gain proficiency at mastering the procedures and skills, and getting correct mathematical answers with technology is necessary, the technology should not be used for masking the underlying mathematical understandings of those mathematical procedures. Teachers should also provide students with technology activities allowing them to engage in conceptual development of the mathematics, such as making conjectures. As a result of this study, teachers should also make provisions for DPPS by providing mathematical activities that encourage students to move beyond merely solving given mathematical problems, and making conjectures. This provision for DPPS should be done after teachers have ensured students had opportunities to develop conceptual understating of mathematics. At the DPPS level mathematics teachers should guide students in developing good mathematical questions, problems and solutions with technology.

Even though this study did not investigate professional development, one finding of this study was the belief professional development was vital to good mathematics teaching. Professional development, which includes technology integration, is valuable to teachers who use technology in the classroom. The professional development could focus on the various ways teachers may use digital cognitive technology in the classroom.

As a result of this study, professional development could be designed around teachers' needs or interests to use a particular technology such as GeoGebra, which two of the participants were aware of but did not know how to use. This professional development could be a three-part process wherein the first is developing the teachers' knowledge of the technology, in particular focusing on the teachers' technology pedagogical and content knowledge (TPACK). This is consistent with research (e.g. Smith, Kim, & McIntyre, 2016). The next part is designing activities based on the uses of technology from the findings of this study, namely Servant-Amplifier, Partner-Reorganizer, and DPPS. The final part is the focus on beliefs of the teachers' that help them to integrate the technology and in their classrooms.

The findings from my study indicated beliefs about teaching mathematics with technology appear to be influential to teachers' use of digital cognitive ways than beliefs about teaching mathematics in general. As a result of my study and consistent with trends in mathematics education, professional development programs should not only focus on teachers' beliefs about teaching mathematics but also, and perhaps more so, on teachers' beliefs about teaching mathematics with technology.

This study found various factors affect teaching with technology. As a result of this study, professional development could be designed around the factors affecting choice and use of digital cognitive technologies. One factor indicated in the study was that of lack of access to technology. Professional development could be designed on how teachers may effectively navigate teaching with technology in the contexts of the limitations of their schools such as lack of one-to-one access and where there was low priority emphasis on the use of technology in the school. For example, professional

developers could provide and develop ways for teachers to integrate limited technology into their mathematics teaching. This professional development could also hold for cases where there is one-to-one access to technology.

This study supports the current trends in mathematics education where mathematics teacher educators should focus on prospective teachers being proficient users of technology and on developing beliefs about teaching mathematics with technology. Mathematics teacher education programs tend to include the use of technology for speed and accuracy up to the getting students to develop an understanding why the procedures for the mathematics work for given mathematical situations. The inclusion of technology use beyond speed, accuracy and why the mathematics works in given mathematical situations may challenge prospective teachers to develop more robust beliefs about teaching mathematics with digital cognitive technologies.

The findings of this study about the uses of technology and the factors affecting the choice and use of technology have several implications for designers of digital cognitive technologies. For one the mathematical fidelity of the technology is of concern to the teachers. For example, the representations in the TI-Nspire technology wherein the number of rectangles displayed did not match the numerical display on the screen was of concern to the teachers. Designers should be cognizant of the mathematical fidelity of the technologies they design. This recommendation is consistent with research (e.g. Dick, 2008). Based on the results of my study designer should also design technologies to incorporate all the uses of the technology the teacher may make, up to the DPPS level.

Further Research

This study indicated several uses exist for digital technologies and teachers indicated various ways they viewed the use of digital cognitive technologies. This study did not investigate the effectiveness of these uses of technology. Future research could investigate whether these uses found in this study were effective. For example, these uses could be investigated in the future concerning whether or not they are effective in promoting student learning.

The findings of my study indicated some school-based factors such as prioritizing who gets to use technology were evident in what technology was chosen to be used. From the findings of this study, school-based research on digital cognitive technology could investigate the effects of more school-based influencers such as limitations on technology use like prioritizing particular practices, the school schedules, the curriculum, and the effects of testing on the use of digital cognitive technologies in the mathematics classroom.

The findings of this research were based on teachers who were avid users of technology and on commonalities among the teachers. This research did not investigate differences in time teachers spend on the various uses of digital cognitive technologies or differences in teachers' levels of use of the technologies. Further research could investigate the amount of time teachers spend on the kinds of activities with the various technology uses and whether this is an effective use of classroom time. Future research could also investigate differences between teachers with varying levels of use of digital cognitive technologies. It could also delve into the interactions that occur between

students, teachers, and technology that take place in the classroom when engaging with technology at the various levels of technology use.

This study included the use of several theoretical perspectives in the collection and analysis of the data. These included, Pea's (1985, 1987) model of technology uses, Ernest's (1989) and Goos et al.'s (2003) categories. Whereas these frameworks were useful for investigating what these and other researchers (e.g. Nzuki, 2010; Sherman, 2011) studied, there appeared to be some level of mismatch between these frameworks and the findings of my study. For example, the finding U6 for the use of technology did not fit into the existing frameworks, so I provided a new category for this use of technology, DPPS.

Whereas my research found relationships appear to exist between particular uses of digital cognitive technology and beliefs, the domain of research pertaining to these relationships is still open and in need of further research. For example, what are some methodologies that may be used to establish some causality in the relationships? Research is also needed on the purpose and quality of the various uses found. For example, is there a need for teachers to focus on digital problem solving and posing (DPPS)? Furthermore, what kind of research is needed to develop the use of technology for DPPS?

Findings from this research indicate the need for the development of a perhaps more robust framework for the investigation of beliefs and digital cognitive technologies. Whereas the partial model generated in this research may be adequate for indicating the relationships found here, the development of a perhaps more comprehensive model and expanded theory is possible in future research.

The findings of my research were based on an interview instrument on beliefs about teaching mathematics and about teaching mathematics with technology. To date there exist few instruments that exist to measure teacher beliefs about teaching mathematics with technology, in particular with newer digital cognitive technologies. As a result of this study, there seems to be the need for more research on developing ways to reliably and validly measure the beliefs about teaching mathematics with technology.

Limitations of the Study

There were a number of limitations to this study including school rules on distribution of technology, limited theoretical foundations for the study, some possible bias in the selection of participants, inadvertent bias in data collection, possible bias in the participants' providing answers they think the interviewer wanted, limited number of technologies observed, and generalizability of the study.

The findings of the study indicated some school-based factors involved in the technology chosen. One factor was access to technology based on school policy on technology distribution. School rules on the distribution of technology interfered with the use of technology by one participant. The participant did not get to include the use of particular technology in the observed mathematics lessons. If the access to technology was not limited by the rules of the school, there may perhaps have been other observed uses of technology.

In this study, the instruments for identifying beliefs of teachers were limited to interviews and observations. Whereas, these instruments allowed for deeper levels of capturing beliefs, there may be potentially some beliefs, which may not have been captured by these instruments. The instrument for collecting data on technology use

included observation field notes only, and this instrument was limited in scope to the uses caught by the eye of the researcher. It is possible there may have been missed uses of technology pertinent to the study.

The theoretical foundations for grounding this research appeared to be somewhat limited in scope and thus seemed to be an insufficient fit for the findings. An extension of the theoretical base for the findings could perhaps lead to a more clear view of the uses of technology and beliefs. As a result, the study had to propose an extension to frameworks and theory, which may be useful for future research.

There may have been inadvertent bias in the selection of the participants on the part of the researcher. The researcher thought of the type of participants needed for the study and so purposefully chose participants that were thought to be of best fit to the study. This results obtained from this group of participants could perhaps align with the results the researcher expected to find.

There may have been inadvertent bias in the responses to the interview questions given by the participants. The participants were chosen from a larger pool, and they knew the requirements for being chosen in the study. Thus they may have perhaps given certain responses to the researcher based on what they thought the researcher expected them to say and do, after their being selected.

There may have been inadvertent bias in the observations, based on the researcher's perspective, and on the fact some of the participants taught in the same school or had similar backgrounds. Inadvertent bias on the researcher's perspective could have led to the foregrounding of some uses of technology. The fact some of the teachers taught in the same school and perhaps had similar training with the technologies could

have influenced similarly observed uses of technology. Perhaps the results would not be so similar if all participants had taught at different schools, which perhaps had different training programs and restrictions on technology use.

The findings of the study were based on a limited number of digital cognitive technologies. Even though the participants indicated they were exposed to multiple digital cognitive technologies and had access to more technologies than they exposed to, they used few digital cognitive technologies during the observation of their lessons. I observed each participant use two digital cognitive technologies. IN total I observed all the participants use three digital cognitive technologies, namely, Desmos, GeoGebra, and TI-Nspire Navigator. Additionally, there was no use of multiple digital technologies in one lesson.

Even though the number of participants (three) was sufficient for my research, this number may be a limitation in a different study. Whereas in a different study with more participants, the results may be more generalizable, the results of my study may likely not be as generalizable.

Conclusion

The findings indicated many uses of digital cognitive technologies. These uses are more than in literature on technology in mathematics education. The larger number of uses could primarily be because the focus of my study is on newer digital technologies. Uses of cognitive technologies continue to change or increase with improvements in newer digital technologies. Some of these uses include display of all student work and digital problem solving and posing (DPPS). The results of my study indicated that

mathematics teachers should be cognizant of various ways to use newer digital cognitive technology and determine the best ways of using technology in the classroom.

The findings of this study also indicated teachers beliefs about teaching mathematics and about teaching mathematics with technology are somewhat related to the observed uses of digital cognitive technologies for the teaching of mathematics. This research found the connections between beliefs of teachers and use of digital cognitive technologies that indicate beliefs about teaching mathematics with technology are more related to the uses of current digital cognitive technologies than beliefs about teaching mathematics in general.

The findings of the study indicate even though there are relationships between teaching mathematics with technology and uses of technology and between mathematics in general and the uses of technology, the beliefs about teaching mathematics with technology appeared to be more connected to the highest levels of use of digital cognitive technologies. Hence, teachers should develop knowledge newer digital cognitive technologies, practice in the uses of these technologies and strong beliefs about teaching mathematics with technology.

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APPENDIX A: PARTICIPANT OBSERVATION WORKSHEET

Teacher Action	Yes	No	Evidence/Reason
1. Verbally directs students to use technology			
2. Allow students to construct their own figures (shapes, graphs, etc.)			
3. Allows students to do further explorations with technology before providing assistance to students			
4. Encourages students to make conjectures about results provided by technology			
5. Demonstrates solution steps with technology after students attempt a solution			
6. Encourages students to provide steps and answers prior to seeing the teacher's results with technology in demonstration.			
7. Allow students to use technology before doing written work.			
8. Does not provide step-by-step assistance to students during the task with technology			
9. Encourages student-to-student interaction with technology			
10. Encourages student presentation of results with technology			
11. Encourages (whole class) critiquing of students' results with technology			
12. Encourages (whole class) critiquing of teacher's own results with technology			

APPENDIX B: BELIEFS AND TECHNOLOGY SELECTION INTERVIEW

Beliefs Questions

Beliefs about teaching mathematics: *This section deals with the way the participant thinks that mathematics should be taught. The next three questions were created based on the third category of Ernest (1989) beliefs framework and the work of Cooney et al. (1998). This section was developed to answer research question 3 of my study.*

1. Consider the following similes¹ (I will provide the words as cut outs):

A mathematics teacher is like

- news broadcaster
- entertainer
- doctor
- orchestra conductor
- gardener
- coach

a. Which simile that you believe best describes a mathematics teacher? Explain your choice.

b. Which simile that you believe does NOT describe a mathematics teacher very well? Explain your choice.

2. What are some things that a *good* mathematics teacher does (I will make a list) and which of these things do you think is most important?

Possible follow up prompts:

- Why do you think this is the most important thing?
- If you had to continue in order of importance what thing would be next and why?

3. Mathematics teaching should involve *mainly* _____

- Giving students lots of drill and practice
- Providing for collaboration between students
- Focus on independent thinking and discovery
- Covering material the pace of the students
- Other (state)
 - o Why do you think so?

Beliefs about technology and teaching with technology: *This section deals with the way the participant thinks about the nature of technology and how mathematics should be taught with technology. The next four questions were created based on Goos et al.'s (2003) technology framework. This section was developed to answer research question 3 of my study.*

3. Which of the mathematics lessons that I observed your teaching with technology do you think was particularly effective?
- Why do you think the lesson was effective?

4. Think about the technologies that you used in the lessons in which I observed you. Which technologies help the understanding of mathematics better than using pencil and paper only? Which technologies do not help the understanding of mathematics better than using pencil and paper only?

Possible follow up prompts:

- Why do you think these technologies help with the understanding of mathematics?
- Why do you think these technologies get in the way of the understanding of mathematics?

Selection of Technology Questions

This section deals with the way teachers choose technology for their mathematics lessons. The next three questions were created based on Pea's (1987) technology framework and Goos et al.'s (2003) technology framework. This section was developed to answer research question 1 of my study.

5. Of the technologies that you used to teach mathematics, what features of these technologies do you and your students use most often?

Possible follow up prompts:

- Why did you use these features of the technologies?

6. What are some things you think about when deciding whether (or not) to use a particular technology for a mathematics lesson?

Possible follow up prompts:

- Why do you think that focusing on this particular factor is important?
- Do you think of things that may inhibit the use of technology?
- Do you think of things that may support the use of technology?

7. What do you consider to be your major role in the lessons?

Possible follow up prompts:

- Why do you think so?
- What role will the students play?

APPENDIX C: PRELESSON INTERVIEW

This interview deals with how the teacher intends to use technology during a particular lesson. The questions are created based on Pea's (1987) technology framework, and Goos et al.'s (2003) technology framework. This interview focuses on question 1 of my study.

1. What are your goals for this lesson?

Possible follow up prompts:

- How do you see technology helping in reaching those goals?

2. What *mathematics specific technology* (technologies) do you intend to use during this lesson and why?

Possible follow up prompts:

- Are there any other factors that influenced your decision to use this particular technology (technologies)?

3. How do intend you use the technology for implementing the mathematical activity?

Possible follow up prompts:

- What will the students be doing with the technology?
- What will you be doing with the technology?

APPENDIX D: POSTLESSON INTERVIEW

This interview deals how the participant views the way technology was used during the lesson. The questions were created based on Pea's (1987) technology framework and Goos et al.'s (2003) technology framework. This was developed to answer research question 2 of my study.

1. Regarding the use of technology in the lesson, how did you think the lesson went?

Possible follow up prompts:

- How did your students use the technology?
- How did you use the technology?
- Was this the way you intended to use it?

2. How could you or your students' use (name of digital cognitive technology) in different ways to help the students with concepts they may not have grasped?

Possible follow up prompt:

- How would your role have changed?

3. In what ways do you think technology may have helped students' thinking during the lesson? Possible follow up prompt:

- Why do you think so?

4. Is there any way that the mathematical activity using (name of digital cognitive technology) could be done by hand (pencil and paper only)?

Possible follow up prompt:

- Why do you think so?

APPENDIX E: FINAL BELIEFS AND TECHNOLOGY SELECTION INTERVIEW

Beliefs Questions

Beliefs about teaching mathematics: *This section deals with the way the participant thinks that mathematics should be taught. The next three questions were created based on the third category of Ernest (1989) beliefs framework and the work of Cooney et al. (1998). This section was developed to answer research question 3 of my study.*

1. Consider the following similes¹ (I will provide the words as cut outs):

A mathematics teacher is like

- news broadcaster
- entertainer
- doctor
- orchestra conductor
- gardener
- coach

a. Which simile that you believe best describes a mathematics teacher? Explain your choice.

b. Which simile that you believe does NOT describe a mathematics teacher very well? Explain your choice.

2. What are some things that a *good* mathematics teacher does (I will make a list) and which of these things do you think is most important?

Possible follow up prompts:

- Why do you think this is the most important thing?
- If you had to continue in order of importance what thing would be next and why?

3. Mathematics teaching should involve *mainly* _____

- Giving students lots of drill and practice
- Providing for collaboration between students
- Focus on independent thinking and discovery
- Covering material the pace of the students
- Other (state)
 - o Why do you think so?

Beliefs about technology and teaching with technology: *This section deals with the way the participant thinks about the nature of technology and how mathematics should be taught with technology. The next four questions were created based on Goos et al.'s (2003) technology framework. This section was developed to answer research question 3 of my study.*

3. Think of your technology wish list for your classroom. If you had the opportunity to have all the technologies that you desire to have in your classroom, what technologies would you want to have?

Possible follow up prompts:

- Why do you want to have these technologies?
 - Which of these technologies would be most important for you to have?
4. Think about the technologies that you used during the lessons I observed.
- Describe how a teacher could use each of these technologies to effectively teach mathematics.
- Possible follow up prompts:
- Why do you think so?
5. Describe a mathematics lesson that I observed you teach that you think was particularly effective, in which the students used technology.
- Why do you think the lesson was effective?
- How is your teaching with technology similar or different from the way you were taught mathematics with technology?
6. In what ways do you think technology may have helped students' thinking during the lessons I observed?
- Possible follow up prompt:
- Why do you think so?
7. Think about the technologies that you have access to teach with. Which technologies help the understanding of mathematics better than using pencil and paper only? Which technologies do not help the understanding of mathematics better than using pencil and paper only?
- Possible follow up prompts:

- Why do you think these technologies help with the understanding of mathematics?
- Why do you think these technologies get in the way of the understanding of mathematics?

Selection of Technology Questions

This section deals with the way teachers choose technology for their mathematics lessons. The next question was created based on Pea's (1987) technology framework and Goos et al.'s (2003) technology framework. This section was developed to answer research question 1 of my study.

8. Of the technologies that you and your students used during my observations of your lessons? What features of these technologies did you and your students use most often?

Possible follow up prompts:

- Why do you use these features of the technologies?

APPENDIX F: PARTIAL INITIAL ANALYSIS DOCUMENT

Transcript (P1-Beth)	Summary of Episode	Code	Comments/ Relation to Research Question
I: OK so think about effective or appropriate uses of technology, (OK)... let's say a teacher was using technology effectively or appropriately, what do you think or describe what that would be like P1: I mean, appropriately and effectively it means that students are engaged with the technology, they are trying things on their own, technology is really good for trial and error, it's like move this angle over here, see what happens with the triangle, you know, its them, trying and discovering is good engagement, it's not umm, I think bad engagement is like what I was saying with the [some technology] when like listen exactly as I say, get it out exactly as I say, it's just it's the same thing as notes I am telling you exactly what to do, good engagement is me not telling you exactly what to do it's like hey here is this [technology] and like some notes go along with it like when you move the triangle, what happens to the triangle, like properties kind of discovery kind of things, well it should be. Well these	When asked what she thought about effectively teaching with technology Beth indicated that teachers should allow learners to use the technology as a means of engagement with the mathematics, for trial and error, for discovering properties of mathematical objects rather than giving students precise information about what should be done and what the outcomes should be.	Use of technology as exploratory and engagement tool with respect to thinking about mathematics	This relates to how a teacher should direct students to use technology. It is not an observed use so it relates to what the teacher believes about the way technology should be used and thus is a belief about teaching with technology RQ3

kids (yeah) in my ... period is kind of loud, they don't really get quiet for me I: so back to the question of effective use, why do you think that is effective use versus, just telling them what to do P1: I think it is effective because it shows them that they have thing in their hands and can discover stuff about.	When asked again why she thought what she said about effectively teaching with technology Beth indicated that it is effective because it shows the students what math they can figure out with the technology		
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APPENDIX G: SCHOOL RECRUITMENT EMAIL

Dear (Name of Principal),

My name is Leighton McIntyre, a doctoral candidate in Mathematics Education at the University of Georgia, Athens (UGA) and former mathematics teacher. My research interest is in use of technology in mathematics education; particularly digital technologies dedicated to helping students learn mathematics. I am also interested in the beliefs of teachers who use technology or allow their students to use technology.

This communication serves to request permission to conduct research in (Name) High School and participation in my study by teachers of your school. I intend to interview mathematics teachers about their use of technology to teach mathematics, and to observe some classes in which teachers use technology to teach mathematics.

In return for your permission to conduct research, I am willing to volunteer, helping students and teachers in your school. We can have a discussion about multiple ways of doing this. I am willing to sit down with you for a meeting if you so desire.

If you need additional details or would like to see a brief proposal, please do not hesitate to contact me.

Best,

Leighton McIntyre