ESSAYS IN BAYESIAN FINANCIAL ECONOMETRICS

by

MYUNG D. PARK

(Under the Direction of Jeffrey H. Dorfman)

ABSTRACT

Even though the relation between asset return and its risk is a fundamental of finance, the empirical evidence using the generalized autoregressive conditional Heteroskedasticity in mean (GARCH-M) model has been conflicted. This dissertation focuses on the risk-return tradeoff in the U.S. equity market. The first study investigates the factors causing empirical results of the risk-return tradeoff in stock market and finds that the risk-return tradeoff is hidden by market aggregation. The second study concentrates on the relation between risk and return in agribusiness stock portfolios and finds supporting evidence of a positive risk-return tradeoff and suggests multivariate GARCH-M specifications to produce better results. The third study employs multivariate GARCH-M models and examines Merton (1973) intertemporal capital asset pricing model (ICAPM). In this study, robust estimates of a positive risk-return tradeoff for individual portfolios and the market portfolio is revealed.

Keywords : Asset pricing; Bayesian econometrics; GARCH-M; Risk return tradeoff

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DEDICATION

I dedicate this study to my family.

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CHAPTER 1

INTRODUCTION

The risk return tradeoff is fundamental to modern finance. Since the intertemporal capital asset pricing model (ICAPM) was introduced by Merton(1973) numerous studies have explored the tradeoff between market expected return and conditional volatility. Following Engle (1982)'s model of autoregressive conditional heteroskedasticity (ARCH), many modified models (for example, GARCH(1986); TARCH(1993); EGARCH (1991)) have been developed and they have become a popular workhorse to estimate the evolution of volatility as a proxy of portfolio risk. My main research focus is on empirical studies with Bayesian inference using the GARCH-in-mean (GARCH-M) framework which describes the linear relation between expected risk and expected return designed by Engle, Lilien, and Robins (1987) and Bollerslev, Engle, and Wooldridge (1988).

Previously, empirical results with the GARCH-M framework have faced two major econometric difficulties in estimating the risk-return tradeoff precisely. First, even though the belief that riskier assets must have higher expected returns is widely accepted, some studies have found negative risk-return tradeoffs (cf., Nelson, 1991 and Glosten, Jagannathan, and Runkle, 1993). Another difficulty is that most of those results are not statistically significant. To solve those problems, possible solutions have been suggested ranging from longer data series, the specification of the volatility evolution, and the presence of an intercept. My research has focused on resolving past econometric difficulties and estimating the risk-return tradeoff accurately through Bayesian inference.

In the first study, I provide a new solution to estimate a positive, significant risk-return tradeoff. The aggregation of different companies into the U.S. total market hides the true company-level relation between risk and return. Using disaggregated portfolios by market cap, book to market ratio, dividend yield, momemtum and industry, I find most of them follow ICAPM except the largest market cap stock portfolio regardless of the volatility specification and the largest 10% market cap stocks are the main cause of inaccurate past results. Especially, higher market cap stock portfolios with lower book to market ratio and higher momemtum show very weak risk-return tradeoff.

My research interest expands the univariate GARCH-M model to a multivariate GARCH-M framework in the second study. Multivariate GARCH models have been commonly employed to estimate time-varying conditional covariances between asset returns (for example, Bollerslev (1990); Engle and Kroner (1995); Engle (2002)). I investigate the risk-return tradeoff in agribusiness stocks, specifically those in the agricultural production and food manufacturing industries. The expected positive relation between stock return and its risk holds for both industries, but the posterior probability of a positive tradeoff is lower for the food manufacturing industry. The sign of the risk-return tradeoff is not sensitive to volatility specification. A positive, significant risk-return tradeoff for the total U.S. market portfolio is estimated in the bivariate GARCH-M framework. This implies that a multivariate GARCH-M framework should be employed to demonstrate Merton's ICAPM as it may offer improved empirical results.

In the third study, using bivariate GARCH-M models, I find strong evidence of a positive relation between expected return and time-varying (co)variance for individual portfolios formed by market size, book to market ratio, dividend yield, momentum and industry and the market portfolio. I also construct a robust estimate for the positive risk-return tradeoff across model specifications using Bayesian model averaging. A positive risk-return tradeoff is estimated with high posterior probability and Merton's ICAPM is empirically supported.

CHAPTER 2

IS AGGREGATION HIDING THE RISK-RETURN TRADEOFF?

2.1 INTRODUCTION

Since Merton's (1973) seminal study it has been widely accepted throughout economics and finance that investors expect higher returns in exchange for holding riskier assets. While this theory has remained undisputed for almost forty years, the empirical evidence of this relationship has been spottier than would be expected for so widely held a belief. Researchers have encountered two major difficulties in estimating the risk-return tradeoff for U.S. and U.K. stock market portfolios (the most common empirical examples). In some studies, the econometric models find the expected positive relationship between risk and returns but the relationship is not statistically significant. In other studies, researchers have even estimated negative risk-return tradeoffs (cf., Nelson, 1991 and Glosten, Jagannathan, and Runkle, 1993).

Researchers have rarely doubted the theory at the heart of the issue, but rather have searched for an econometric answer to the dilemma. Most empirical research has utilized either ARCH (Engle, 1982) or GARCH (Bollerslev, 1986) models to represent the stock market excess return data and the evolution over time of the portfolio risk (measured commonly by the conditional volatility of the excess returns). Thus, poor empirical results have spurred researchers to invent new, more flexible variance specifications resulting in the introduction of such models as EGARCH, TARCH, QGARCH, and NAGARCH to name some of the most common varieties (Nelson, 1991; Rabemananjara and Zakoian, 1995; Glosten, Jagannathan, and Runkle, 1993; Campbell and Hentschel, 1992; Sentana, 1995; Engle and Ng, 1993). Other researchers have investigated the potential role of the data span (Lundblad, 2007), that is the length of time for which data is collected, the data frequency (Anderson and Bollerslev, 1998 and Bali and Peng, 2006), and the role of an intercept term in the relationship linking excess returns to risk (Scruggs, 1998; Lanne and Saikkonen, 2006; Lanne and Luoto 2008). While many of these approaches have yielded some improvement in empirical results, all such improvements seem fragile rather than being robust across specification, data sample, or data source.

In this study we investigate a different cause for these empirical difficulties: the aggregation of many individual companies into the stock portfolios examined and, simultaneously, whether investors use the same measure of risk for all types of stocks. If investors in different types of stocks all have the same risk-return tradeoff (demand the same expected excess return for an additional amount of risk), that would tend to make aggregation of the stock returns into a total market portfolio acceptable. However, a second condition is neccesary; the measure of risk must also be the same across the different stocks or at least economists must use the correct measure for each stock in constructing the aggregate variables to use in research. If either of these conditions do not hold, then empirical research performed with aggregate, total stock market data will be prone to yield false conclusions from econometric studies. In this paper, we contend that the second condition does not hold and this difference in risk measures causes the empirical confusion in trying to estimate the risk-return tradeoff.

Merton's theory says expected excess returns should be a linear function of the risk of an investment. Econometricians searching for empirical evidence to support his theory have, at least since the introduction of GARCH models (Bollerslev, 1986); traditionally employed the conditional volatility of excess returns as the measure of risk. Implicitly or explicitly, this is an assumption; that is, conditional volatility is a proxy for the investor's expected risk of that investment. Depending on the investor's time horizon, knowledge base, risk attitudes, and investment goals, conditional volatility might be a good or even excellent proxy for risk, or it might be a very poor one. If risk is not well proxied by conditional volatility, then the model estimated suffers from an error-in-variables (or measurement error if you prefer) problem and the estimators obtained will not provide consistent estimates of the risk-return tradeoff. Ludvigson and Ng (2007) point out that too few conditioning variables create insignificant results which implies the time-varying volatility may not be an ideal proxy for the risk and Ghysels, Santa-Clara, and Valkanov (2005) propose a different volatility estimator to find the positive risk-return tradeoff. Risk would also not be well-proxied for by conditional volatility under Campbell and Vuolteenaho's (2004) two-beta model, where cash-flow risk and discount-rate risk have different prices. Campbell and Vuolteenaho find different stocks have different mixes of these two betas and thus command different expected excess returns.

In this chapter, we will examine whether aggregation of stock return data into market aggregate portfolios is a factor of the empirical shortcomings that have been experienced in the estimation of the risk-return tradeoff. In searching for aggregation issues we will employ data on U.S. stock returns that are disaggregated from the total market in multiple dimensions as we search for the dimension in which aggregation might be causing the problems. Thus, we will set up less aggregated portfolios that are deconstructed along market capitalization, book-to-market ratios, dividend yield, momentum and industry lines. If risk attitudes, or investors' perceived risk measures vary across these different dimensions our empirical results should uncover those differences and allow us to create improved estimates of the risk-return tradeoff.

The remainder of the chapter is organized as follow. Section 2 describes the mean equation and the variance equation in the GARCH-M framework and explains our Bayesian computation method. Section 3 provides the data description. Section 4 discusses the empirical results from estimating the risk-return tradeoff for total U.S. stock market returns and a variety of less aggregated portfolios sorted by the different market characteristics mentioned above. Section 5 looks at the aggregation issue and what might be causing it in more depth. Section 6 concludes.

2.2 ECONOMETRIC METHODOLOGY

2.2.1 PREVIOUS WORK AND BASIC MODELS

The GARCH in mean (GARCH-M) model (Engle, Lilien, and Robins, 1987 and Bollerslev, Engle, and Wooldridge, 1988) has been employed and modified to examine the risk return tradeoff in numerous previous works. The GARCH-M model consists of the mean equation and the volitility specification. The mean equation in the GARCH-M framework describes the linear relation between expected return and expected variance. The mean equation which has been widely used in empirical literature can be written as

$$r_t = \mu + \lambda h_t + \epsilon_t, \qquad \epsilon_t \sim N(0, h_t) \tag{1}$$

where r_t is the excess return in period t, λ is the coefficient of relative risk aversion, and h_t is the conditional volatility of returns which is the proxy of the risk of the portfolio.

The volatility specification characterizes the progress of the conditional variance of the error from the mean equation as a function of past conditional variances and lagged errors. Previously, the conditional variance specifications have been considered one of the most critical causes for the confounding empirical results. To remove this suspicion and demonstrate the consistent results across the conditional volatility specifications, we employ three popular alternative specifications in this study. They are as follows: GARCH (Bollerslev, 1986), TARCH (Rabemananjara and Zakoian, 1995; Glosten, Jagannathan, and Runkle, 1993), QGARCH (Campbell and Hentschel, 1992; Sentana, 1995). Since the GARCH specification imposes a symmetric response to return shocks in conditional volatility, numerous refinements of the GARCH model such as EGARCH, NGARCH, TARCH, and QGARCH have been developed by researchers to better reflect the feature of the equity market which is commonly called the leverage effect. The TARCH and QGARCH specifications are popular modifications of the GARCH model to capture an asymmetric response with negative return errors having a bigger effect on conditional volatility than positive return errors. These different models for the evolution of volatility are as follows:

$$\begin{aligned}
&\text{GARCH (1,1)}: \quad && h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} \\
&\text{TARCH (1,1)}: \quad && h_t = \omega + \alpha\epsilon_{t-1}^2 + \gamma I_{t-1}\epsilon_{t-1}^2 + \beta h_{t-1} \\
&\text{QGARCH(1,1)}: \quad && h_t = \omega + \alpha(\epsilon_{t-1} - \gamma)^2 + \beta h_{t-1} \qquad (2)
\end{aligned}$$

where I_{t-1} in the TARCH model is an indicator function that equals one when ϵ_{t-1} is negative and zero otherwise.

Campbell and Hentschel (1992) point out that the sign of γ is expected to be positive because a positive γ creates a negative relation between a market or portfolio error, ϵ_{t-1}, and conditional volatility the next period, h_t.

In Merton's work, the expected excess return is a linear relation with its conditional variance and covariance between the market portfolio and the hedging instrument. Generally, time varying volatility, h_t, is considered to proxy the market risk, however, investers' expected risk could be different from the estimated conditional volatiltiy. If conditional volatility is a suitable as a risk proxy, the risk-return tradeoff should be positive and significant.

2.2.2 THE BAYESIAN ESTIMATION ALGORITHM

We utilize a simple Bayesian estimation approach because of its benefits in the presence of inequality restrictions (Geweke, 1986). The hope is that improving estimation of the volatility equation will yield better estimates of conditional volatility. If this provides a better proxy for risk, it might help resolve past empirical difficulties. The prior distribution and the likelihood function are two key components in a Bayesian analysis of a statistical model. The posterior distribution results from the product of the prior and the likelihood and is an optimal combination of those two information sources (Zellner, 1971).

We assume that the error term follows a normal density. The log likelihood function constructed by summing the log normal densities can be expressed as

$$L(r|\theta) = \sum_{t=1} L_t(r|\theta); \quad L_t(r|\theta) = -1/2 \cdot \log h_t - \epsilon_t^2/2h_t. \tag{3}$$

The prior distribution summarizes the information in the researcher's subjective beliefs about model parameters prior to seeing the data. Our prior beliefs are simply that the conditional volatility should be positive in all the time. Instead of following past practices of imposing positivity on the individual parameters of variance specifications in equation (2), we impose positivity directly on volatility. This opens the possibility that the coefficients in equation (2) could be negative if negative parameters are more appropriate than positive ones. Independent normal prior densities are chosen for all individual parameters and represented by $N(mean, variance)$. All prior means are set to zeros. Given the magnitude of the variances, those independent normal priors are relatively diffuse except for the information from the indicator function, $I(h_t)$ and the assumed lack of correlation between coefficients. This prior can be represented as

$$p(\mu, \lambda, \omega, \alpha, \gamma, \beta) = I(h_t) \cdot N_\mu(0, 5) \cdot N_\lambda(0, 5) \cdot N_\omega(0, 5) \cdot N_\alpha(0, 5) \cdot N_\gamma(0, 5) \cdot N_\beta(0, 5). \quad (4)$$

An indicator function ($I(h_t)$) in the prior density equals one if a parameter vector generates positive and finite conditional volatility in all time periods and zero otherwise.

The posterior density is proportional to the likelihood function times the prior distribution. When a simple analytical formula for the posterior density doesn't exist, posterior simulation is required to calculate posterior results. This arises most commonly from complicated prior density. Since our prior density is truncated by inequality constraints and a nonlinear function, it is not feasible to compute the posterior distribution analytically.

Previously, various posterior simulations have been employed in Bayesian studies on GARCH models. Geweke (1989) and Kleibergen and van Dijk (1993) used importance sampling to formulate exact predictive densities and to explore the nonstationarity of

GARCH specifications, respectively. Bauwens and Lubrano (1998, 2002) employed the Griddy-Gibbs sampler and compared the performances of Bayesian simulations using an asymmetric GARCH specification (TARCH). Nakatsuma (2000) proposed a new Markov chain Monte Carlo (MCMC) method for the ARMA-GARCH model. Vrontos et al. (2000) demonstrated full Bayesian inference for GARCH and EGARCH models and Osiewalski and Pipien (2004) performed Bayesian inference on a multivariate GARCH model with the Metropolis-Hastings algorithm. In our study, the Random Walk Chain Metropolis-Hastings algorithm (Koop, 2003) is employed to estimate the posterior distribution. Formally, the Random Walk Chain Metropolis-Hastings algorithm generates candidate draws according to

$$\theta^* = \theta^{(s-1)} + z \tag{5}$$

where z is called the increment random variable. Equation (5) implies that candidates are generated by a random walk and the current candidate is drawn randomly by addition of the increment random variable, z, to the previous parameter vector. The coefficients from maximum likelihood estimation (MLE) can be used as the starting value, $\theta^{(0)}$. Each draw is accepted with the acceptance probability

$$\alpha(\theta^*|\theta^{(s-1)}) = \min\left[\frac{p(\theta = \theta^*|r)}{p(\theta = \theta^{(s-1)}|r)},\ 1\right] \tag{6}$$

where $p(\theta|y)$ is the posterior distribution. If a current draw is not accepted, the previous draw is reused.

We want a high proportion of the candidate draws to be from the areas of high posterior probability but also want to adequately sample from the regions of low posterior probability. The acceptance probability accomplishes this by favoring draws from areas of high posterior probability while still sometimes keeping draws from low probability regions according to acceptance probability in eq. (6). The candidate generating density for the Random Walk Chain Metropolis-Hastings algorithm is determined by the density of the increment random variable, z. The multivariate normal distribution is a common and convenient choice of density for z and we employ it here. The $var(\hat{\theta}_{ML})=\widehat{\Sigma}$ from MLE is used in the candidate generating density along with c, a tuning constant to adjust the acceptance rate. The candidate generating density then can be written as

$$q(\theta|\theta^{(s-1)}) = N(\theta^{(s-1)}, c \cdot \widehat{\Sigma}). \tag{7}$$

There is no general rule for an optimal acceptance rate. However, as Koop (2003) points out, too small an acceptance rate implies that the chain will not move enough to get information about the entire posterior density because candidate draws are almost always rejected and the region where the chain explores stays too close to the initial value, $\theta^{(0)}$. On the other hand, too high an acceptance rate (close to unity) will result in needing an unfeasibly large number of candidate draws to ensure that the chain collects information about the entire posterior density. Koop suggests that an acceptance rate of roughly 0.5 works best empirically. The value of c for each estimation is set to around 0.05 ($c \approx 0.05$), in each case making the acceptance rate approximately 0.45. 50,000 draws are generated and the first 10,000 draws discarded to remove the effect of initial value for each prior, saving 40,000 draws for computations. Each accepted draw weights equally in the Metropolis-Hastings algorithm and the simple average is the estimated posterior mean. In other words, the average value of any general function of the model parameters, $g(\theta)$, of the S draws from the posterior distribution is the estimated posterior mean of the function $g(\theta)$. This can be expressed as

$$\widehat{g_S} = \frac{1}{S}\sum_{r=1}^{S} g(\theta^{(s)}). \tag{8}$$

The posterior mean is usually employed as the point estimator of the posterior density, and we follow that convention here.

2.3 DATA DESCRIPTION

We use monthly return data for the period 1927 $\sim$ 2008 (T=984) compiled by the CRSP (Center for Research in Security Prices). The value-weighted CRSP index of NYSE, AMEX, and Nasdaq is employed for the U.S. total market returns. The excess return on the market, the difference between market return and risk free asset return, is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson Associates). To investigate the risk-return tradeoff of decomposed market return portfolios, monthly returns of 10 decile portfolios formed on market capitalization, book to market (BM; A ratio used to find the value of a company by comparing the book value of a firm to its market value), dividend yield, momentum and 17 industry-specific portfolios are used. Two subsets of five-by-five quiltile portfolios formed by market cap and book to market ratio and market cap and momentum are also employed to investigate in additional detail. The S5B5 (biggest market cap, highest BM) and S5M1 (biggest market cap, lowest momemtum) portfolios have missing observations. For the S5B5 portfolio, we use the period of data after the missing observations (from 1931). There are 4 missing data point in the S5M1 portfolio, so we replace the missing values with the average of S5M1 portfolio. The sample period of the returns formed on market capitalization, book to market, and industry is 1927 ∼ 2008 (T=984). The data on portfolios formed by dividend yield are from the period 1928 ∼ 2008 (T=972). In this study, we created value weighted lower 50% to 90% portfolios based on market cap to demonstrate the effect of aggregation. All returns were obtained from the Kenneth R. French on-line data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data _ library.html).

2.4 EMPIRICAL RESULTS OF UNIDIRECTIONAL DISAGGREGATION

Table I shows the posterior results of the risk return tradeoff for the U.S. total market return. Even though the estimated λs are positive across all three volatility specifications, they are not particularly statistically significant. The symmetric, standard GARCH model performs best, with a 91% posterior probability of a positive risk-return tradeoff. The other two specifications have below an 80% posterior probability of positive λ. These results are fairly standard for this type of analysis (for example, Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993; Campbell and Hentschel, 1992).

2.4.1 DISAGGREGATING BY MARKET CAPITALIZATION

To investigate the aggregation effect of total market portfolio, we look at the portfolio disaggregated by market cap first. Table II provides the empirical estimates of the risk-return tradeoff, λ in equation (1), for 10 decile portfolios formed by market cap for three different volatility specifications. If aggregation across firms of different market size is leading to the empirical difficulty in estimating the risk return tradeoff, these results should reveal it. The results in table II confirm that removing the effect of aggregation greatly improves the estimation of the risk return tradeoff. Regardless of the variance specification, a positive relation between expected return and expected volatility is estimated for each size portfolio. However, while λ is always positive, it is not always statistically significant. The results in table II show a clear pattern of more statistically precise estimates for smaller cap portfolios, with the probability of positive risk-return tradeoff highest for stocks with smaller market cap.

An important result from Table II is that the biggest size portfolio (decile 10) does not have a statistically significant risk-return tradeoff for any of the three volatility specifications. With the symmetric specification, GARCH, we find a significant, positive λ through decile 1 to 9. For portfolios of decile 1 to 9, the probability of a positive λ exceeds 95% in all cases, but the probability drops to the 85% for decile 10 portfolio. However, the probability of a positive λ drops sooner for the relatively larger size portfolios with the asymmetric specifications, TARCH and QGARCH. The TARCH model results show the estimated λ is strongly positive (probability $> 90\%$) for portfolios from decile 1 to 8 while the first 9 size portfolios show a probability of being positive that exceeds 90% for the relation between return and conditional variance with the QGARCH specification. The aggregation of market cap portfolios appears to hide the expected behavior of smaller size portfolios. It is only the largest market cap stocks that cause the statistical difficulties. The estimation issues in earlier studies using the total market portfolio are possibly due to the market portfolio data being dominated by companies with the larger market caps.

Figure 1 shows the time varying conditional volatility of the portfolios from deciles 2 to 10 based on market cap from the GARCH variance specification. As the market cap of the portfolio increases, the estimated mean and standard deviation of conditional volatility decreases. The figure clearly shows that h_t, the proxy for risk, declines as the market cap of firms in the portfolio increases. The figure raises the possibility that the much lower volatility in large cap stocks is contributing to the econometric difficulty in estimating the risk-return tradeoff.

Table III shows the empirical results of the market-cap-weighted, cumulative portfolios containing from 50% to 90% of total firms and market portfolio. These results confirm the phonomenon revealed in the previous results on individual decile portfolios and show again that the larger market cap stocks are the cause of the difficulty in estimating the risk-return tradeoff. For all three variance specifications, a positive risk-return tradeoff is estimated with a high probability of being positive for all portfolios except the total market portfolio. Interestingly, the probability of positive λ drops dramatically across all three volatility specifications when we move from the 90% cumulative market cap portfolio to the total U.S. market portfolio. The largest 10% of stocks by market cap severely degrades the empirical results when estimating the risk-return tradeoff. Figure 2 displays the posterior distributions of λ for the low 60%, low 80%, and the total market portfolio from the TARCH specification. The posterior distributions of the risk return tradeoff move toward zero as bigger stocks are added and the inclusion of the largest decile stocks moves the posterior distribution strongly toward zero and significantly increases the posterior standard deviation.

Is it aggregation in general that matters or is it the firm size (as measured by market cap) that matters? To investigate this further, we look at disaggregated portfolios in four other dimensions.

2.4.2 DISAGGREGATING BY BOOK TO MARKET (BM)

First, we look at portfolios comprising deciles of the total U.S. market decomposed by book to market ratio (BM). The empirical results for the 10 decile portfolios formed on book to market (BM) are provided in table IV. The revealed pattern on these portfolios is not as clear as the results of the 10 decile portfolios based on market cap. Generally, a positive sign for λ is estimated, but two negative signs of the estimated λ also are found in the lowest book to market portfolios in the asymmetric specifications (TARCH and QGARCH). Depending on the variance specification, we find the probability of a positive risk return tradeoff exceeds 90% for 7 (GARCH), 5 (TARCH), or 4 (QGARCH) of the 10 portfolios. There is no clear trend to the probabilities as BM increases. Figure 3 shows the time varying conditional volatility of deciles 2 to 10 formed on book to market from the GARCH specification. Unlike in figure 1, no clear pattern is revealed. These results suggest that the risk-return tradeoff does not vary systematically by book to market ratio. However, the portfolios from deciles 7 to 10 show a statistically positive risk-return tradeoff regardless of volatility specifications, so we can say that a significant and positive λ is usually revealed on higher book to market ratio portfolios.

2.4.3 DISAGGREGATING ON DIVIDEND YIELD

To further determine if market cap is the key characteristic, we also looked at portfolios disaggregated by dividend yield. Table V provides the empirical results of the risk-return tradeoff with 10 decile portfolios formed by dividend yield. Except for the decile 1 portfolio with the TARCH model, a positive relation between risk and return is found on all portfolios. However, the number of portfolios with statistically strong evidence is fewer than that of portfolios formed by market cap and book to market (BM). With the GARCH and QGARCH specifications, five portfolios (decile 6 through 10) have probabilities greater than 90% of a positive λ. However, for the TARCH specification, only 4 deciles match that standard. No clear pattern is revealed across all deciles but the stocks with higher dividend yield seem to produce a stronger positive risk return tradeoff across three volatility specifications. In examining the estimated mean and standard deviation of h_t, no general tendency emerges. While the smaller market cap portfolios have relatively larger estimated mean and standard deviation than the bigger market cap portfolios and the bigger book to market portfolios have relatively larger estimated mean and standard deviation than the smaller book to market portfolios across the volatility specifications, the estimated mean and standard deviation of h_t for the portfolios formed by dividend yield show no particular pattern (see figure 4). Even though there is no clear pattern from decile 1 to 10 as in the market cap portfolios, we can find a very similar trend to the portfolios formed on book to market. The statistically positive risk-return tradeoff is easily estimated on the portfolios from higher dividend yield portfolios (decile 8 to 10) across volatility specifications.

2.4.4 DISAGGREGATING ON MOMENTUM

Table VI shows the empirical results of the risk-return tradeoff for 10 decile portfolios formed by momentum for three variance equations. The results of 10 decile momemtum portfolios are close to those of 10 decile market cap portfolios; the significant, positive risk-return tradeoff is revealed on smaller momemtum portfolios across volatility specifications. With the GARCH specification, the probability of a positive λ exceeds 90% for deciles 1 to 7 portfolios while portfolios for deciles 1,2,3,4, and 6 have even more statistically accurate estimates (probability $>$ 95%). The probability of positive λ decreases sooner for the higher momentum portfolios with asymmetric volatility specifications, TARCH and QGARCH. This pattern is very similar to that of market cap portfolios. For the first 4 decile portfolios, a significant, positive λ is estimated on TARCH and QGARCH but the probability of positive λ drops with higher momemtum portfolios.

Another interesting result of disaggregating portfolios on momentum is that a negative risk-return tradeoff is estimated on the decile 10 portfolio with all volatility specifications. With GARCH and QGARCH specifications, the probability of negative λ exceeds 90%. The highest momemtum portfolio plays the same role in causing the econometric difficulty in estimating the precise risk-return tradeoff as the largest market cap portfolio does. Figure 5 displays the time varying conditional volatility of the portfolios from decile 2 to 10 based on momentum from the GARCH volatility specification. These figures show a similar pattern to that of the market cap portfolios; as the momemtum increases, the estimated mean and standard deviation of conditional volatility decreases.

2.4.5 THE MONTHLY RISK RETURN TRADEOFF OF THE PORTFOLIOS FORMED BY INDUSTRY

As a final test of the impact of disaggregation on estimation of the risk-return tradeoff, we examine seventeen industry-specific portfolios. If certain industries behave differently, this should reveal a pattern. Table VII shows the results of the estimation of λ for the 17 industry-specific portfolios. A negative risk return tradeoff is estimated for both the oil and utility industries with all three specifications and for the two asymmetric specifications the mining industry portfolio has a negative estimated risk-return but none of these negative estimates are significant. Strongly positive risk-return tradeoffs are difficult to find. Out of the seventeen industries, the results reveal either nine (GARCH), two (TARCH), or seven

(QGARCH) estimated λs with greater than 90% probabilities of being positive. Six of the seven industries with significant results for QGARCH are also significant for the GARCH specification and the two industries with good results under the TARCH variance model also have significant λs under the other two specifications, so the industries that have good estimates are quite consistent, but not many industries show positive and significant estimates of the risk-return tradeoff. Clearly, disaggregation by industry does not improve empirical results when estimating the risk-return tradeoff.

2.5 SEARCHING FOR A CAUSE: BIDIRECTIONAL DISAGGREGATION

Given the empirical results from the previous section, we know much more about the causes of past statistical difficulties in estimating the risk-return tradeoff. First, the volatility specification is not a major issue as our results are generally robust across three variance specifications. Second, insufficient data span and the presence of an intercept are not the main culprits to the empirical difficulties. Instead, we find that the key is aggregation of stock data into a total market portfolio. By disaggregating the data in many dimensions, we have uncovered the role of market capitalization as perhaps the most useful direction in which to disaggregate the data. Some of the other dimensions of data disaggregation also reveal insights, particularly momentum, but none are as clear as when the data is disaggregated by the stocks' market caps.

In this section, we try to narrow down our focus on the cause of the empirical difficulties in estimating the risk return tradeoff by disaggregating the total market portfolio in two new ways. The new portfolio sets contain 25 portfolios each, with the market disaggregated into quintiles in the directions of market cap versus book to market ratio or versus momentum. The hope is that these five-by-five portfolios will allow us to pinpoint more precisely why the very large market cap stocks behave differently with respect to risk and expected returns.

2.5.1 BIDIRECTIONAL EXAMINATION OF MARKET CAP AND BOOK TO MARKET RATIO

Table VIII presents the empirical results of the risk-return tradeoff on the five-by-five quintile portfolios formed by market cap and book to market ratio. For the sake of simplity, we characterize all portfolios by capital letters and numbers, for example, S1B1 represents the portfolio formed by smallest market cap with lowest book to market ratio. Regardless of the volatility specifications and book to market ratio, the smallest market cap portfolios don't have any problem in estimating statistically significant and positive λ. All of their probabilities of positive λ exceed 95%. For quintile 2 market cap portfolios, all of estimated λs are stastically significant and positive with the symmetric GARCH specification and only S2B1 shows insignificant results from the asymmetric specifications. A pattern of difficulty in estimating λ from the portfolios formed with larger market cap and lower book to market ratio becomes clear as the rest of Table VIII is examined. For portfolios with large market caps and low book to market ratio, we have difficulty estimating a significant, positive risk-return tradeoff. This results suggest that the difficulty lies not just with the largest market cap stocks, but with those large cap stocks with lower book to market ratios.

2.5.2 BIDIRECTIONAL EXAMINATION OF MARKET CAP AND MOMENTUM

Table IX shows the empirical results of the risk-return tradeoff on the five-by-five quintile portfolios formed by market cap and momentum and the results confirm that the portfolios formed by larger market cap stocks with higher momentum are another cause of the conundrum in previous research. The smallest market cap portfolios have significant positive λs (probability $> 95\%$) across the volatility specifications regardless of the momentum quintile. However, as market cap increases failures in estimating a significant, positive λ begin to emerge. These failures are particularly focused on the high-momentum quintiles (the lower right part of each five-by-five block of results). The combination of top quintile market cap with top quintile momentum yields a portfolio with a negative estimated risk-return tradeoff for all three volatility specifications. Clearly large cap, high momentum stocks are a subset to be explored in finding the behavioral cause of these confounding empirical results.

With the cause narrowed down to large cap stocks that also have either low book to market ratios or high momentum, the question is why do these stocks cause the empirical difficulty? The most likely answer is that investors perceive the risk of these stocks differently. Large caps stocks may be seen as safer than represented by numerical measures such as volatility, thus causing investors not to demand as much return to hold them. Low book to market and high momentum stocks are unlikely to be perceived as safe, but high momentum stocks certainly have classes of investors who favor them. The demand by investors to "get on the bandwagon" of these high momentum stocks may be a contributing factor by bidding up price now thereby lowering future expected returns at the same time risk is likely increasing. Following Campbell and Vuolteenaho (2004) one would postulate that these large cap/high momentum/low book to market stocks have a different mix of good and bad betas, leading to very different risk-return tradeoffs in the aggregate. Trying to estimate a single λ for the total U.S. stock market would not be appropriate.

In summary, the behavior of investors in a small subset of the stock market is distorting the empirical finding based on total market portfolio. It may be that risk of these stocks is not well represented by conditional volatility, or that investors' perceived risk measure differs from actual risk.

2.6 CONCLUSION

Even though there is general agreement that the risk return tradeoff is positive, the evidence from the total U.S. equity market from GARCH-M models in previous research has been mixed and often statistically insignificant depending on volatility specifications. In this paper, we find that the choice of portfolio is the key cause of those problems. First, larger market cap stocks degrade the empirical results of the risk-return tradeoff with the inclusion of the largest 10% of stocks by market cap dramatically weakening the statistical relation between risk and return in the U.S. total market portfolio. Our finding shows that an aggregation with larger market cap stocks can hide the relation between risk premium and return which exists in all our portfolios of smaller cap stocks. Second, higher momentum stocks play the same role as larger market cap stocks: the risk premium is easily revealed on lower momentum stocks. A negative risk premium is even uncovered with the highest 10% momentum portfolio. Third, other disaggregations by book to market, dividend yield, and industry don't show any clear pattern to the estimation results for the risk-return tradeoff.

We also explore the risk premium with bidirectional disaggregated portfolios and find that the larger market cap portfolios with lower book to market ratio and higher momemtum are the main causes creating econometric difficulty in estimating the risk premium of the total U.S. equity market. The difficulty is not the data span, the presence of an intercept, or the volatility specification, all of which have been previously suspected. Instead it appears to be either that conditional volatility is not the correct proxy for risk for a subset of stocks (e.g., large caps) or that investors in some market segments are misperceiving the riskiness of their investments. Disaggregation of the total market portfolio solves most of econometric difficulty yet it is not enough to solve the larger market caps puzzle. Possible avenues for further exploration of the remaining puzzle have been proposed recently. As Ludvigson and Ng (2007) suggest, more conditioning variables might be needed to resolve this problem for large market cap portfolios. Alternatively standard conditional volatility in GARCH-in-mean models could be replaced by separate measures of cash flow and discount rate risk for portfolios of large market cap stocks in accordance with the two beta theory advanced by Campbell and Vuolteenaho (2004).

Table 2.1

The Monthly Risk-Return Tradeoff of U.S. Market Return

	GARCH	TARCH	QGARCH
μ	0.0056	0.0053	0.0053
	(0.0019)	(0.0022)	(0.002)
λ	1.0772	0.7447	0.5766
	(0.7978)	(0.9113)	(0.7574)
$\omega \times 10^2$	0.0078	0.0105	0.0078
	(0.0026)	(0.0032)	(0.003)
α	0.1418	0.0726	0.134
	(0.0236)	(0.0278)	(0.0216)
γ		0.1178	0.0186
		(0.0453)	(0.0057)
β	0.8388	0.832	0.8253
	(0.0214)	(0.0232)	(0.0225)
Prob($\lambda > 0$)	0.9091	0.7950	0.7827
Mean(h_t)	0.0031	0.0030	0.0029
S.E.(h_t)	0.0044	0.0037	0.0039

Note: Standard errors are reported in parentheses.

Table 2.2

The Monthly Risk-Return Tradeoff of 10 Decile Portfolios formed on Size

	GARCH	TARCH	QGARCH
Decile 1	0.7238 (0.9929)	1.5182 (1.0000)	0.8145 (0.9902)
Decile 2	0.9733 (0.9981)	1.1352 (0.9970)	0.9242 (0.9876)
Decile 3	1.0093 (0.9846)	1.1565 (0.9934)	1.0295 (0.9819)
Decile 4	1.2025 (0.9937)	1.1677 (0.9926)	0.9948 (0.9770)
Decile 5	1.2333 (0.9852)	1.2788 (0.9739)	1.2059 (0.9720)
Decile 6	1.1499 (0.9710)	1.1325 (0.9696)	1.0416 (0.9345)
Decile 7	1.2278 (0.9770)	0.9255 (0.9113)	0.9053 (0.9109)
Decile 8	1.3754 (0.9771)	1.2332 (0.9494)	1.2276 (0.9451)
Decile 9	1.1814 (0.9735)	0.8330 (0.8851)	0.9198 (0.9261)
Decile 10	0.9869 (0.8588)	0.3781 (0.6607)	0.5255 (0.7278)

Note: Numbers without parentheses are the estimated λ and probability of positive λ is reported in parentheses.

Table 2.3

The Monthly Risk-Return Tradeoff of Portfolios formed on Size from low 50% to 90%

	GARCH	TARCH	QGARCH
50%	1.2436 (0.9944)	1.2339 (0.9925)	1.0670 (0.9763)
60%	1.2447 (0.9966)	1.2092 (0.9724)	1.0785 (0.9686)
70%	1.2380 (0.9855)	1.1478 (0.9677)	0.9827 (0.9579)
80%	1.2254 (0.9726)	1.0982 (0.9554)	1.1411 (0.9486)
90%	1.2644 (0.9697)	0.9598 (0.9250)	1.0359 (0.9314)
Total market	1.0772 (0.9091)	0.7447 (0.7950)	0.5766 (0.7827)

Note: Standard error is reported in parentheses

Table 2.4

The Monthly Risk-Return Tradeoff of 10 Decile Portfolios formed on Book to Market

	GARCH	TARCH	QGARCH
Decile 1	0.4989 (0.7061)	-0.2456 (0.3722)	-0.1304 (0.4564)
Decile 2	1.3058 (0.9561)	0.9215 (0.8352)	0.8440 (0.8350)
Decile 3	1.1964 (0.9310)	0.9990 (0.9035)	0.6719 (0.7881)
Decile 4	0.6833 (0.8934)	0.2290 (0.6246)	0.5487 (0.7693)
Decile 5	0.9478 (0.9167)	0.4843 (0.7083)	0.6614 (0.8214)
Decile 6	0.6984 (0.8802)	0.6669 (0.8261)	0.6394 (0.8032)
Decile 7	1.0031 (0.9604)	1.1674 (0.9745)	1.0010 (0.9389)
Decile 8	1.2115 (0.9760)	0.9205 (0.9372)	1.1730 (0.9861)
Decile 9	1.2456 (0.9977)	1.0048 (0.9706)	1.0012 (0.9688)
Decile 10	0.6963 (0.9948)	0.7276 (0.9748)	0.6363 (0.9554)

Note: Numbers without parentheses are the estimated λ and probability of positive λ is reported in parentheses.

Table 2.5

The Monthly Risk-Return Tradeoff of 10 Decile Portfolios formed on Dividend

	GARCH	TARCH	QGARCH
Decile 1	0.3469 (0.6782)	-0.0876 (0.4519)	0.1059 (0.5382)
Decile 2	1.0973 (0.8852)	0.7583 (0.8424)	0.8993 (0.8714)
Decile 3	0.3867 (0.6854)	0.2725 (0.6639)	0.3152 (0.6468)
Decile 4	0.8312 (0.8499)	0.1000 (0.5260)	0.2422 (0.5908)
Decile 5	0.9608 (0.8998)	0.4653 (0.7402)	0.8590 (0.8411)
Decile 6	1.4573 (0.9683)	1.2015 (0.9326)	1.3083 (0.9419)
Decile 7	1.3445 (0.9381)	0.6370 (0.7430)	1.2805 (0.9128)
Decile 8	1.4247 (0.9893)	0.9385 (0.9347)	1.1736 (0.9373)
Decile 9	1.6685 (0.9957)	1.2067 (0.9648)	1.2874 (0.9789)
Decile 10	1.4196 (0.9959)	0.9604 (0.9813)	1.5670 (1.0000)

Note: Numbers without parentheses are the estimated λ and probability of positive λ is reported in parentheses.

Table 2.6

The Monthly Risk-Return Tradeoff of 10 Decile Portfolios formed on Momentum

	GARCH	TARCH	QGARCH
Decile 1	0.8596 (0.9876)	0.8373 (0.9780)	0.7176 (0.9638)
Decile 2	1.0380 (0.9947)	0.5174 (0.9174)	1.1420 (0.9938)
Decile 3	1.0307 (0.9869)	0.6202 (0.9220)	0.9704 (0.951)1
Decile 4	1.4243 (0.9943)	0.9947 (0.9629)	1.2766 (0.9705)
Decile 5	0.9918 (0.9360)	0.7887 (0.8846)	0.8235 (0.8734)
Decile 6	1.1541 (0.9536)	0.8629 (0.8814)	1.0168 (0.9109)
Decile 7	1.2367 (0.9100)	1.0831 (0.8882)	1.2090 (0.9135)
Decile 8	0.8452 (0.7917)	0.6416 (0.7132)	0.8826 (0.799)1
Decile 9	0.2792 (0.6032)	0.2421 (0.5863)	0.3631 (0.6287)
Decile 10	-1.4733 (0.0897)	-1.0241 (0.1586)	-1.3292 (0.0798)

Note: Numbers without parentheses are the estimated λ and probability of positive λ is reported in parentheses.

Table 2.7. The Monthly Risk-Return Tradeoff of Industry Specific Portfolios

	GARCH	TARCH	QGARCH
Foods	1.2974	0.3788	1.1438
	(0.9134)	(0.6187)	(0.9282)
Mines	0.2077	-0.1141	-0.1339
	(0.6094)	(0.4348)	(0.4389)
Oil	-0.0113	-0.1493	-0.1350
	(0.4817)	(0.4108)	(0.4187)
Clothes	1.3905	0.8455	0.5340
	(0.9720)	(0.8727)	(0.7311)
Durable	0.8578	0.6098	0.6582
	(0.9550)	(0.8599)	(0.9076)
Chemical	0.9061	0.6986	0.6675
	(0.8759)	(0.8221)	(0.7812)
Consumer	1.3817	0.7944	0.8545
	(0.8659)	(0.7126)	(0.7938)
Construction	0.8190	0.7096	0.7926
	(0.8789)	(0.8541)	(0.9231)
Steel	0.7479	0.5143	0.6402
	(0.9126)	(0.8560)	(0.9176)
Fabricated	1.4287	1.1064	1.3573
	(0.9667)	(0.9353)	(0.9724)
Mechanical	0.7127	0.2712	0.5003
	(0.8717)	(0.6555)	(0.7887)
Cars	0.9931	1.1448	0.9413
	(0.9593)	(0.9677)	(0.9608)
Transportation	0.9941	0.8058	0.6047
	(0.9502)	(0.8953)	(0.8213)
Utilities	-0.0625	-0.2232	-0.0845
	(0.4799)	(0.3528)	(0.4424)
Retail	0.8427	0.6462	0.9192
	(0.9004)	(0.7856)	(0.9070)
Finance	0.8427	0.5519	0.8018
	(0.9518)	(0.8626)	(0.9400)
Others	0.4696	0.3346	0.4676
	(0.6848)	(0.6770)	(0.6927)

Note: Numbers without parentheses are the estimated λ and probability of positive λ is reported in parentheses.

Table 2.8

The Monthly Risk-Return Tradeoff of Portfolios formed by 5 $\times$ 5 Size-Book to Market

GARCH	B1	B2	B3	B4	B5
S1	0.6234	0.6338	0.9804	1.0744	0.9079
	(0.9702)	(0.9870)	(0.9941)	(0.9963)	(0.9976)
S2	0.9687	1.0130	1.1584	1.2081	1.0120
	(0.9597)	(0.9813)	(0.9926)	(0.9840)	(0.9955)
S3	0.6206	1.2579	1.4012	1.4273	1.0665
	(0.8834)	(0.9742)	(0.9914)	(0.9877)	(0.9967)
S4	0.4107	1.1662	0.9686	1.3488	0.9739
	(0.7097)	(0.9603)	(0.9257)	(0.9951)	(0.9916)
S5	0.6212	0.6784	0.6165	0.9207	1.2266
	(0.7400)	(0.7960)	(0.8236)	(0.9594)	(0.9824)

TARCH	B1	B2	B3	B4	B5
S1	0.7087	0.8467	0.9490	1.2808	1.7755
	(0.9903)	(0.9935)	(0.9929)	(0.9922)	(1.0000)
S2	0.6303	1.2130	1.1574	1.3980	1.0359
	(0.8537)	(0.9792)	(0.9807)	(0.9923)	(0.9889)
S3	0.3752	1.0066	1.1211	1.2778	0.9680
	(0.7546)	(0.9265)	(0.9619)	(0.9783)	(0.9839)
S4	0.0389	0.9289	1.0992	1.2950	0.8342
	(0.5160)	(0.8909)	(0.9266)	(0.9831)	(0.9839)
S5	0.2009	0.2556	0.0969	0.7211	1.1589
	(0.5991)	(0.6178)	(0.5596)	(0.9136)	(0.9831)

QGARCH	B1	B2	B3	B4	B5
S1	0.5730	0.6508	0.8950	0.9877	0.9455
	(0.9555)	(0.9808)	(0.9941)	(0.9865)	(0.9943)
S2	0.7383	0.9996	1.0804	1.1606	0.8063
	(0.8940)	(0.9740)	(0.9691)	(0.9792)	(0.9571)
S3	0.4057	1.0315	1.2163	1.0999	0.8901
	(0.7505)	(0.9403)	(0.9687)	(0.9548)	(0.9722)
S4	0.1001	0.9636	0.7797	1.2313	0.8846
	(0.5655)	(0.9037)	(0.8509)	(0.9816)	(0.9863)
S5	0.3761	0.2259	0.4972	0.9496	1.0336
	(0.6571)	(0.6034)	(0.7476)	(0.9526)	(0.9489)

Note: Numbers without parentheses are the estimated λ. Numbers in parentheses are prob(λ >0).

Table 2.9

The Monthly Risk-Return Tradeoff of Portfolios formed by 5 × 5 Size-Momentum

GARCH	M1	M2	M3	M4	M5
S1	0.9879	1.0269	0.9570	0.9735	0.9277
	(0.9994)	(0.9986)	(0.9949)	(0.9956)	(0.9708)
S2	1.0589	1.4062	1.1408	1.0886	0.8282
	(0.9994)	(0.9994)	(0.9841)	(0.9705)	(0.9135)
S3	1.0004	1.2532	1.1936	1.0321	0.7965
	(0.9950)	(0.9964)	(0.9818)	(0.9261)	(0.8274)
S4	0.6370	1.1609	1.5064	1.5361	0.2794
	(0.9822)	(0.9887)	(0.9906)	(0.9788)	(0.6088)
S5	0.8021	1.1555	0.9389	0.8984	-1.2216
	(0.9864)	(0.9789)	(0.9211)	(0.8271)	(0.1387)

TARCH	M1	M2	M3	M4	M5
S1	1.5305	1.3700	1.1559	1.0738	1.0049
	(1.0000)	(1.0000)	(0.9963)	(0.9887)	(0.9881)
S2	1.1180	1.5200	1.1333	1.2338	0.7509
	(0.9975)	(0.9997)	(0.9777)	(0.9792)	(0.8919)
S3	0.9913	1.0549	1.0609	0.9568	0.7926
	(0.9920)	(0.9879)	(0.9631)	(0.8992)	(0.8250)
S4	0.7328	0.7360	1.2392	1.3408	0.4333
	(0.9790)	(0.9326)	(0.9676)	(0.9605)	(0.6633)
S5	0.0983	0.6289	0.7557	0.8573	-0.7640
	(0.6084)	(0.8789)	(0.8607)	(0.7901)	(0.2191)

QGARCH	M1	M2	M3	M4	M5
S1	1.0595	1.0089	0.9354	0.9470	0.9351
	(0.9998)	(0.9966)	(0.9924)	(0.9930)	(0.9797)
S2	0.9545	1.2717	1.0280	0.9786	0.7099
	(0.9764)	(0.9942)	(0.9718)	(0.9481)	(0.8458)
S3	0.8884	1.1056	1.0137	0.9165	0.6348
	(0.9793)	(0.9871)	(0.9548)	(0.8883)	(0.7856)
S4	0.6559	1.1539	1.2221	1.4670	0.2323
	(0.9652)	(0.9817)	(0.9697)	(0.9765)	(0.5809)
S5	0.8038	1.1138	0.7354	0.8466	-1.2071
	(0.9704)	(0.9711)	(0.8354)	(0.7874)	(0.1007)

Note: Numbers without parentheses are the estimated λ. Numbers in parentheses are prob(λ >0).

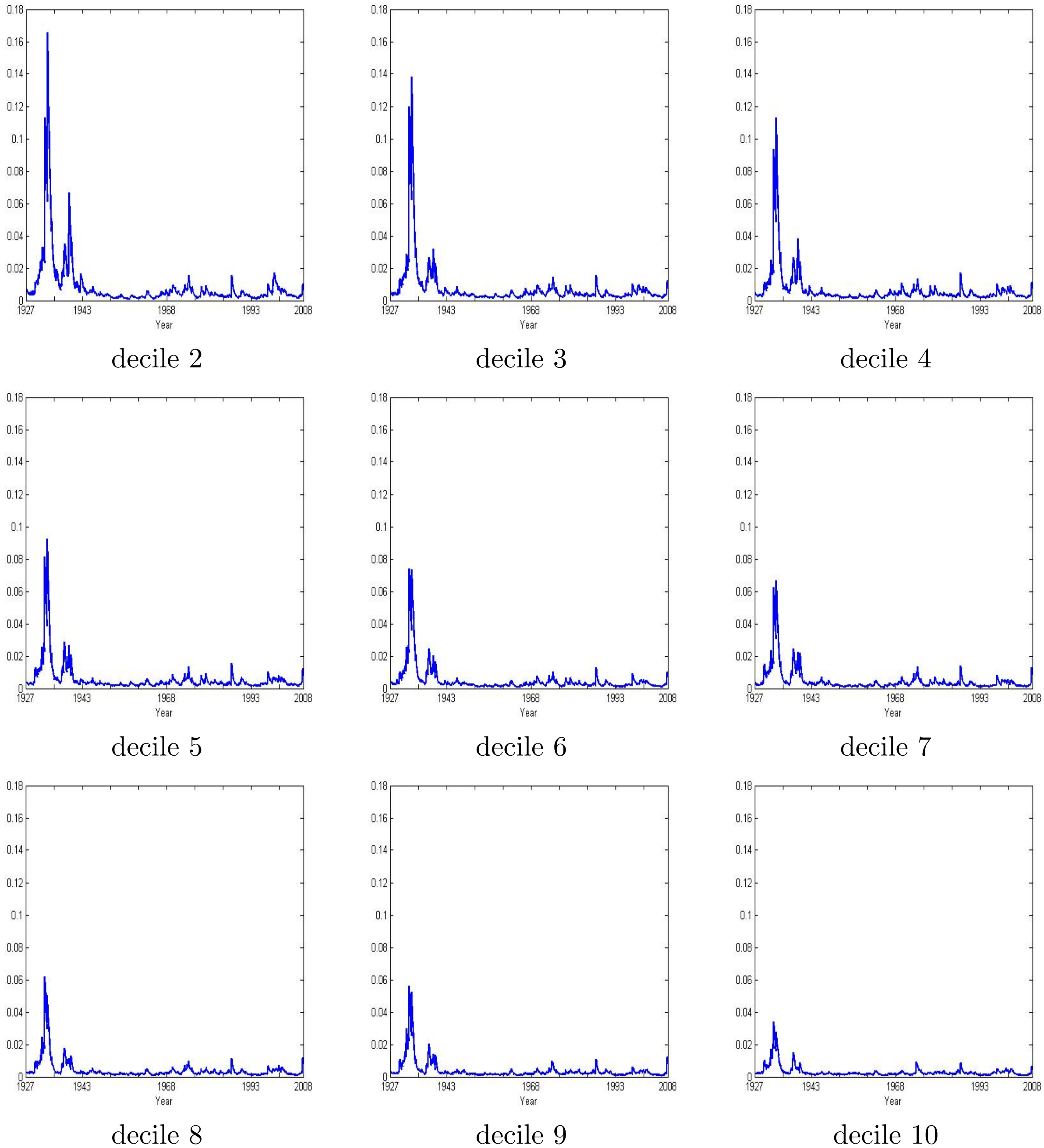


Figure 2.1 Conditional volatilities of the portfolios formed on size from the GARCH model

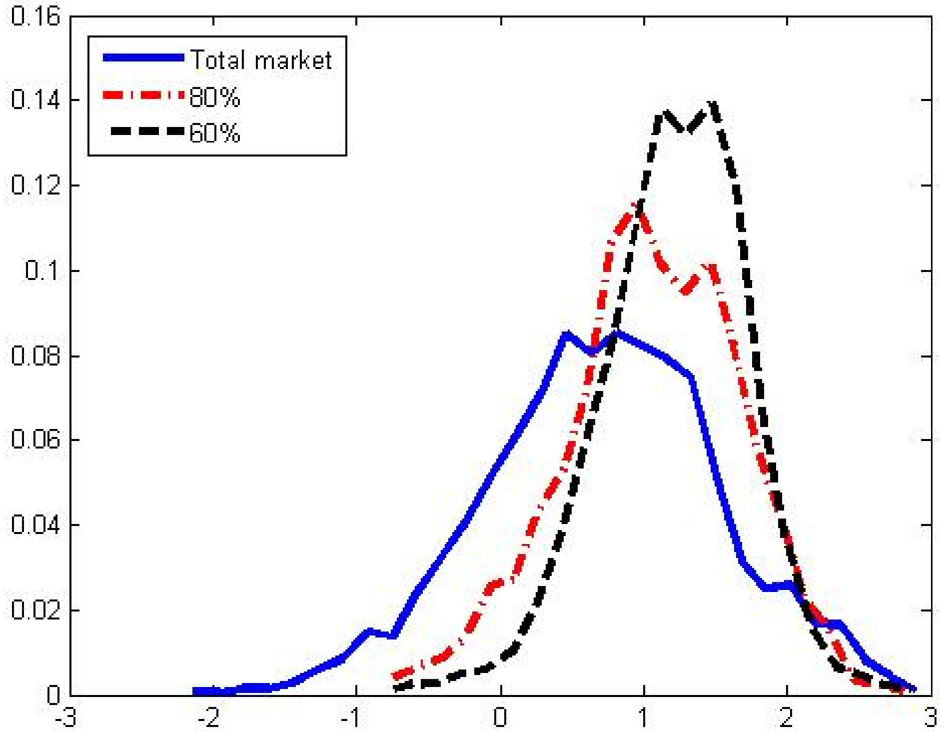


Figure 2.2 Marginal posterior distributions of the risk return tradeoff of the portfolios formed on size from the TARCH model

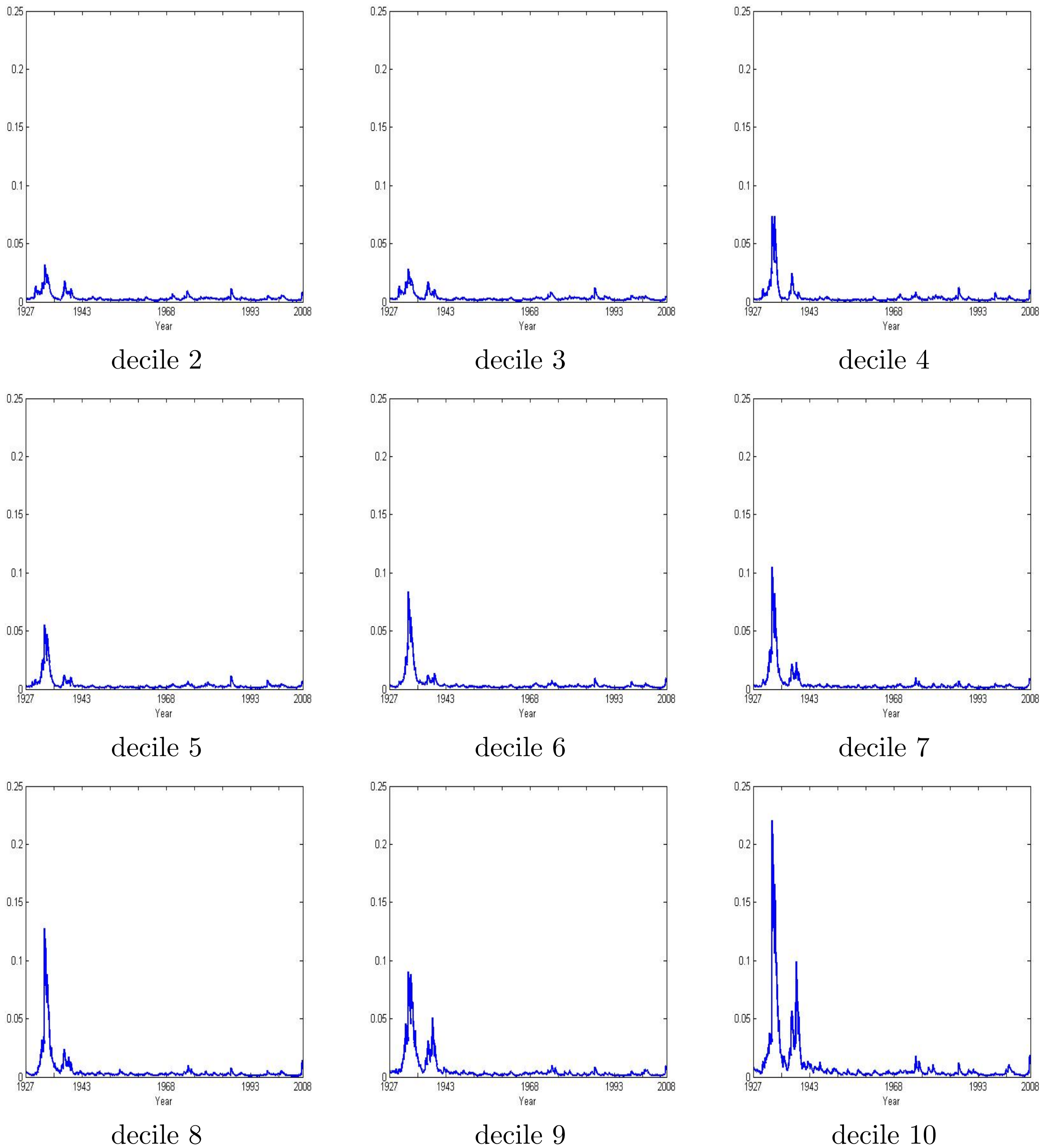
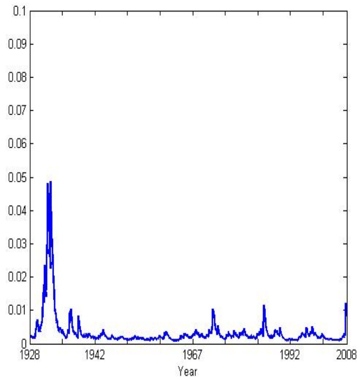


Figure 2.3 Conditional volatilities of the portfolios formed on book to market from the GARCH model



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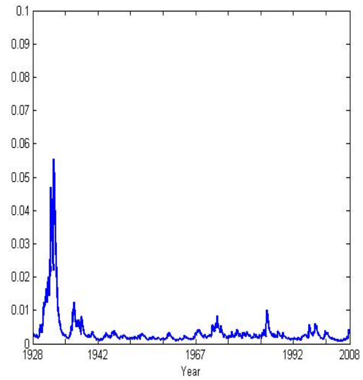

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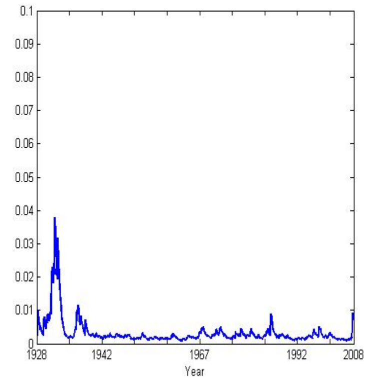

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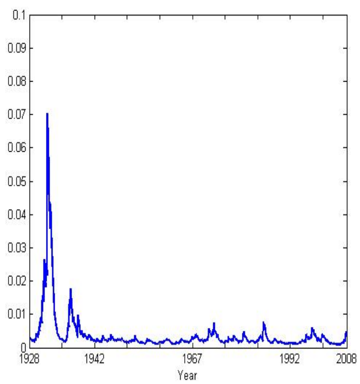

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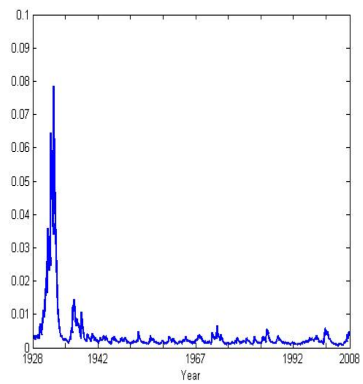

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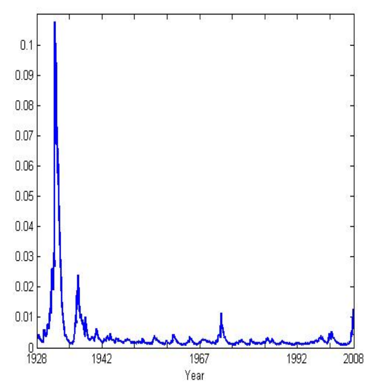

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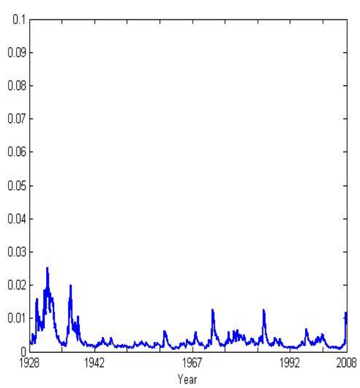

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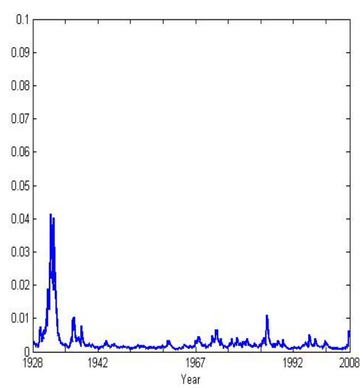

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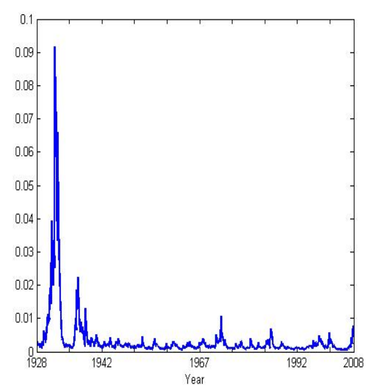

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Figure 2.4 Conditional volatilities of the portfolios formed on dividend yield from the GARCH model

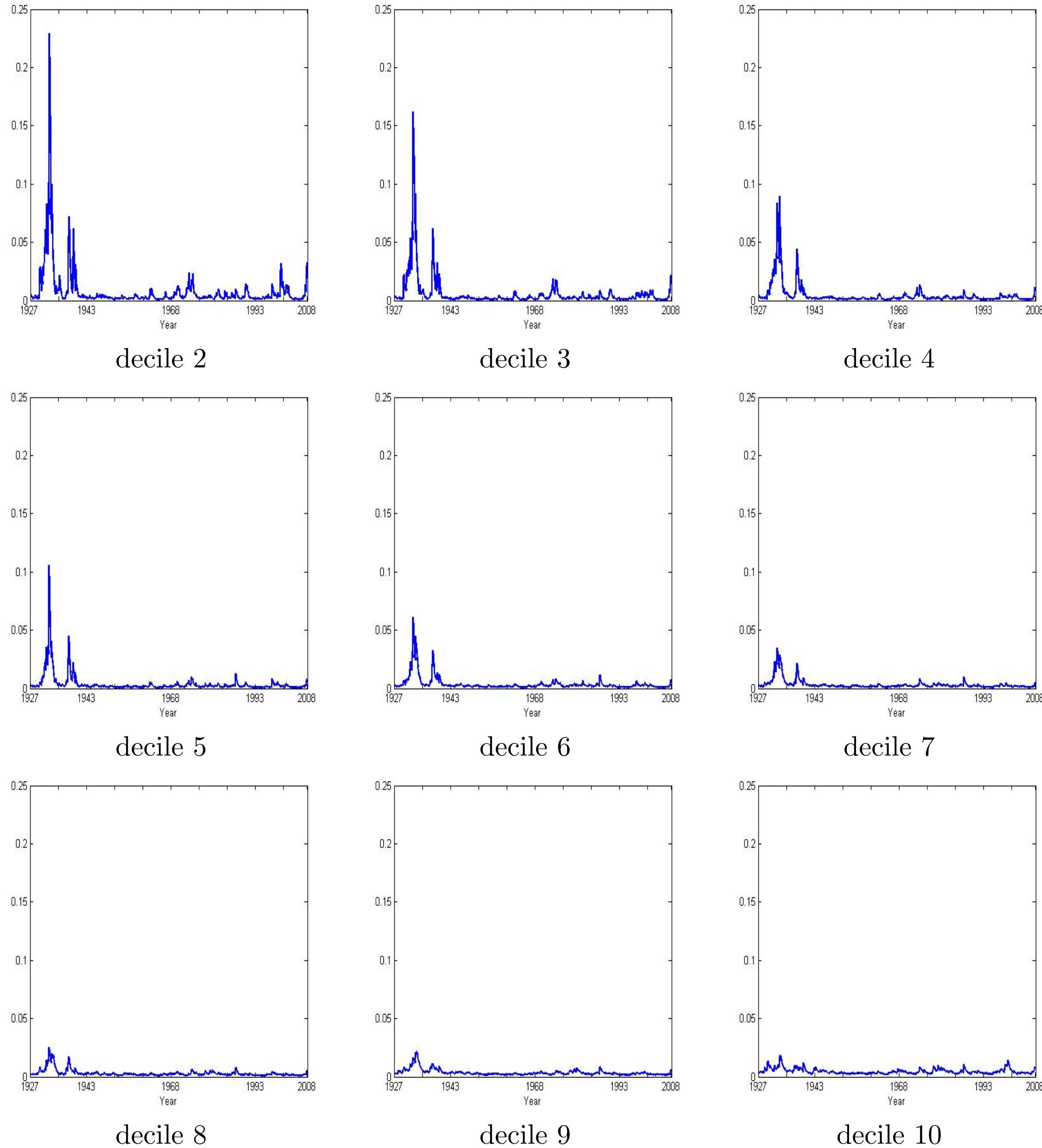


Figure 2.5 Conditional volatilities of the portfolios formed on momentum from the GARCH model

CHAPTER 3

ESTIMATING THE RISK RETURN TRADEOFF IN AGRIBUSINESS STOCKS: LINKAGES WITH THE BROADER STOCK MARKET

3.1 INTRODUCTION

With the recent turmoil in world financial markets and the additional volatility in agricultural commodity markets enhanced by the impact of biofuel policies, it is vital to better understand the volatility in, and linkages between, the agricultural sector, the food manufacturing sector, and the broad stock market. This paper examines the stock returns and volatility of those returns for the U.S. agricultural and food manufacturing industries. We study the relationship between volatility and returns for each industry and the role of correlation with volatility in the U.S. stock market. Improved knowledge of these links is needed for investors, farmers, and food processors to properly diversify their asset holdings and manage their risks.

If consolidation continues, both in agricultural production and in food manufacturing, more and more of these industries will consist of publicly traded companies. Investors interested in buying such stocks want to understand the volatility of such assets and how their returns are correlated with the broader stock market. Similarly, agricultural producers and food manufacturers who have most of their assets (and income) within those sectors need to understand the same facts so that they can diversify their holdings and reduce the risk they face from their asset concentration in a single sector.

In this study, we investigate the relation between risk and return for two industry-specific portfolios, agricultural production and food manufacturing, and the role of time-varying covariance between each portfolio and the total U.S. market motivated by Merton's (1973) intertemporal capital asset pricing model (ICAPM).

The remainder of the paper is organized as follow. Section 2 describes the univariate and bivariate generalized autoregressive heteroskedasticity-in-mean (GARCH-M) framework and explains our Bayesian estimation approach. Section 3 provides the data description. Section 4 discusses the empirical results from estimating the risk-return tradeoff for our two industry-specific portfolios and the role of a time-varying covariance between the market and the portfolio. Section 5 concludes.

3.2 ECONOMETRIC METHODOLOGY

In this study, we employ univariate and bivariate GARCH-M models introduced by Engle, Lilien, and Robins (1987) and Bollerslev, Engle, and Wooldridge (1988) to investigate the risk-return tradeoff. Univariate GARCH-M is used to estimate the risk-return tradeoff for each portfolio return and then bivariate frameworks are applied to investigate the role of the covariance between the agricultural production and food manufacturing industry-specific portfolios and the total market return.

3.2.1 UNIVARIATE GARCH-M MODELS

In previous work, the univariate GARCH-M framework has usually been employed to estimate the risk-return tradeoff for the U.S. total market. In this study, we apply this model to the portfolio returns of the agricultural production and food manufacturing industries. The expected portfolio return is assumed to follow a linear relation with its time-varying variance. The mean equation can be written as

$$r_{i,t} = \mu + \lambda_i h_t + \epsilon_{i,t}, \qquad \epsilon_{i,t} \sim N(0, h_t) \tag{1}$$

where $r_{i,t}$ is the excess return of a portfolio or asset in period t, λ_i is the coefficient of relative risk aversion, and h_t is conditional volatility.

Since Engle (1982) and Bollerslev (1986) introduced the ARCH and GARCH specifications, numerous modifications have been developed because the volatility specifications have been considered one of the major causes of counter-intuituve empirical results often found when estimating the risk-return tradeoff. We employ three different variance specifications in a univariate GARCH-M framework to protect against sensitivity of our results to the volatility specification. They are as follows: GARCH (Bollerslev 1986), TARCH (Rabemananjara and Zakoian 1995; Glosten, Jagannathan, and Runkle 1993), and QGARCH (Campbell and Hentschel 1992; Sentana 1995). These volatility specifications are as follows:

$$\begin{aligned}
\text{GARCH (1,1)}: &\quad h_t = \omega + \alpha\epsilon_{i,t-1}^2 + \beta h_{t-1} \\
\text{TARCH (1,1)}: &\quad h_t = \omega + \alpha\epsilon_{i,t-1}^2 + \gamma I_{t-1}\epsilon_{i,t-1}^2 + \beta h_{t-1} \\
\text{QGARCH(1,1)}: &\quad h_t = \omega + \alpha(\epsilon_{i,t-1} - \gamma)^2 + \beta h_{t-1}
\end{aligned} \tag{2}$$

where I_{t-1} in the TARCH model is an indicator function that equals one when $\epsilon_{i,t-1}$ is negative and zero otherwise. The TARCH and QGARCH specifications are generalized to allow an asymmetric response to positive versus negative return shocks (the so-called leverage effect).

3.2.2 BIVARIATE GARCH-M MODELS

In a bivariate GARCH-M framework, our model is motivated by Merton's ICAPM: any asset or portfolio return is the function of the time-varying covariance between that portfolio and the total market return while the total market return is explained by its conditional volatility. The mean equations of a bivariate model can be described as

$$r_{i,t} = \mu_{i,t} + \lambda_{im} h_{im,t} + \epsilon_{i,t}$$

$$r_{m,t} = \mu_{m,t} + \lambda_m h_{m,t} + \epsilon_{m,t} \tag{3}$$

where $r_{i,t}$ is the asset or specific portfolio excess return, $r_{m,t}$ is the total market excess return, $h_{im,t}$ is the time-varying covariance between the total market return and the specific asset or portfolio return, $h_{im,t} = COV(r_{i,t}, r_{m,t})$, and other parameters and error terms are the logical extensions from equation (1).

Multivariate GARCH models have been commonly employed to estimate time-varying conditional covariances between asset returns (for example, Bollerslev 1990; Engle and Kroner 1995; Engle 2002). In this study, we employ the VECH specification introduced by Bollerslev, Engle, and Wooldridge (1988). The specifications of the variances and covariance are

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

$$h_{m,t} = \omega_m + \alpha_m \epsilon_{m,t-1}^2 + \beta_j h_{m,t-1}$$

$$h_{im,t} = \omega_{im} + \alpha_{im} \epsilon_{i,t-1} \epsilon_{m,t-1} + \beta_{im} h_{im,t-1}. \tag{4}$$

Theoretically, the relation between a return and its conditional volatility is expected to be positive and the sign of the relation between asset returns and conditional covariance is also expected to be positive.

3.2.3 THE BAYESIAN ESTIMATION ALGORITHM

In Bayesian inference, the posterior density summarizes all the information available from the likelihood function and the prior density (Zellner 1971). That is, a researcher summarizes prior information and beliefs in a prior density, the information in the data is summarized by the likelihood function, and those two information sources are optimally combined into the resulting posterior distribution according to Bayes Theorem.

We assume that the error term follows a normal density (for example, Bali 2008; Bollerslev 1990) and the log-likelihood function is the summation of the log normal densities. The log-likelihood function (ignoring normalizing constants) for a univariate model can be described as

$$L(r|\theta) = \sum_{t=1} L_t(r|\theta); \quad L_t(r|\theta) = -1/2\log(h_t) - 1/2(\epsilon_{i,t}^2/h_t) \tag{5.a}$$

where θ is a vector of the unknown parameters. For the bivariate model, the log-likelihood function (again ignoring normalizing constants) can be written as

$$L(r|\theta) = \sum_{t=1} L_t(r|\theta); \quad L_t(r|\theta) = -1/2\log|\mathbf{H_t}| - 1/2\varepsilon_{\mathbf{t}}'\mathbf{H_t^{-1}}\varepsilon_{\mathbf{t}} \tag{5.b}$$

where $\varepsilon_{\mathbf{t}}$ and $\mathbf{H_t}$ denote the error vector and time varying covariance matrix, respectively,

$$\varepsilon_{\mathbf{t}} = \begin{bmatrix} \epsilon_{i,t} \\ \epsilon_{m,t} \end{bmatrix} \quad \text{and} \quad H_t = \begin{bmatrix} h_{i,t} & h_{im,t} \\ h_{mi,t} & h_{m,t} \end{bmatrix}. \tag{6}$$

The prior distribution reflects any belief or information the researcher has before seeing the data. Our prior beliefs are that the conditional volatility should be positive and finite in all time periods without any other restrictions. Commonly, inequality constraints are imposed on the coefficients in equation (2) and (4) to keep the time-varying volatility positive. We impose positivity directly on volatility in the process of estimation, leaving the individual parameters free within the parameter space that satisfies the positive volatility constraints. For the bivariate GARCH-M model, we add one more piece of prior information: the time-varying covariance doesn't need to be positive but the covariance matrix, $\mathbf{H_t}$ has to be positive definite. This prior should be an improvement over more restrictive inequality-restricted maximum likelihood estimation.

Independent normal densities are selected for all individual parameters and represented by N(mean, variance). The prior mean and variance for each parameter in our application are set to zero and five, respectively. This prior information is informative but due to the relatively large prior variances, the information from the independent normal densities is quite diffuse and most of the prior information is from the indicator function, $\mathbf{I}(\mathbf{H_t})$. These prior densities for univariate and bivariate models can be represented as

$$p(\theta) = \mathbf{I}(\mathbf{H_t}) \times \prod_{i=1}^{K} N_{\theta_i}(0, 5) \tag{7}$$

where θ is the (K $\times$ 1) vector of parameters and θ_i indicates a ith component of the parameter vector. The indicator function, $\mathbf{I}(\mathbf{H_t})$, in the prior density equals one if the parameter vector generates positive and finite conditional volatilities and a positive definite covariance matrix in all time periods and zero otherwise.

In Bayesian inference, one of the difficulties is that simple analytic results for the posterior are not easily derived when a prior density is a very complicated or a nonlinear function. Since our prior is a nonlinear function of θ truncated by inequality constraints, we need to use posterior simulation to compute posterior results. Our choice is the Random Walk Chain Metropolis-Hastings algorithm which is very useful when a good approximating density for the posterior cannot be found (Koop 2003). In the Random Walk Chain Metropolis-Hastings algorithm, candidates are generated by a random walk as follows:

$$\theta^* = \theta^{(s-1)} + z \tag{8}$$

where z is the increment random variable, superscript numbers in parentheses index draws from the posterior simulator, and the coefficients of MLE are used as $\theta^{(0)}$.

For the increment random variable, the multivariate normal distribution has been a common and convenient choice. The candidate generating density can be written as

$$q(\theta^*|\theta^{(s-1)}) = N(\theta^{(s-1)}, c\widehat{\Sigma}) \tag{9}$$

where $\widehat{\Sigma}$ is the covariance matrix from MLE and c is a tuning constant to adjust the acceptance rate. Each candidate is accepted selectively using an acceptance probability which ensures the algorithm converges to the posterior distribution. The acceptance probability is formed as

$$\alpha(\theta^*|\theta^{(s-1)}) = \min\left[\frac{p(\theta = \theta^*|r)}{p(\theta = \theta^{(s-1)}|r)},\ 1\right] \tag{10}$$

where $p(\theta|r)$ is the posterior distribution. If accepted, θ^* becomes $\theta^{(s)}$. If the current draw is rejected, the previous draw is reused, so $\theta^{(s)} = \theta^{(s-1)}$.

A proper acceptance rate is required to explore the entire posterior density and arrive at accurate results. Exact guidance for the optimal acceptance rate does not exist, but Koop (2003) suggests that around 0.5 is appropriate. We generate 55,000 draws and discard the first 5,000 draws to remove the effect of the initial value. The acceptance rate is tuned to approximate 0.45 for each estimation. All accepted draws are weighted equally, so the estimated posterior mean is the simple average of accepted draws. Thus, the average value of the S draws from the posterior simulator for a function of the model parameters, $g(\theta)$, is the estimated posterior mean of $g(\theta)$. This can be expressed as

$$\widehat{g_S} = \frac{1}{S}\sum_{s=1}^{S} g(\theta^{(s)}). \tag{11}$$

The posterior mean is employed here as the point estimator of the posterior density.

3.3 DATA DESCRIPTION

Monthly return data for the period 1927 $\sim$ 2008 (T=984) compiled by the Center for Research in Security Prices (CRSP) were obtained from the Kenneth R. French on-line data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data _ library.html). The value-weighted CRSP index of NYSE, AMEX, and Nasdaq is employed for the U.S. total market returns. The two industries are part of the industry-specific returns from the 48 industry dataset. Agricultural production contains firms producing crops, livestock, commercial fishing, feeds for animals and agricultural services. Food manufacturing is industries such as food and kindred products, meat products, dairy products, etc. Exact details of the two industries are on the French data library webpage. All asset returns used in this study are the excess return which is the difference between the market return and the risk free asset return. A one-month Treasury bill rate (from Ibbotson Associates) is employed as the risk-free asset return.

3.4 EMPIRICAL RESULTS

3.4.1 UNIVARIATE MODELS

Table 1 shows the posterior results of the univariate GARCH-M model with three volatility specifications for returns of the agricultural production industry for the period 1927 $\sim$ 2008. A positive posterior risk-return tradeoff (represented by λ_i) is estimated and the posterior probability of a positive λ_i exceeds 95% regardless of volatility specfication. Since γ reflects the effect of a negative portfolio error on conditional volatility, a positive sign for γ is expected (Campbell and Hentschel 1992) and the posterior mean of γ follows the expectation. Thus, we confirm the leverage effect for this industry.

The posterior results for the food manufacturing industry are presented in table 2. Although positive λ_is are estimated across the volatility specifications, lower probabilities of a positive risk-return tradeoff are estimated for the two asymmetric specifications, TARCH and QGARCH. Negative and insignificant λ_is have been estimated with asymmetric specifications previously when looking at the total U.S. stock market (Glosten, Jagannathan, and Runkle 1993; Nelson 1991); thus these results are an improvement. Using the symmetric specification, GARCH, the probability of a positive λ_i exceeds 95%. We also again find strong posterior support for leverage effects.

Figures 1 displays the conditional volatility for the returns of the agricultural production and food manufacturing industries. The conditional volatility of the food manufacturing industry is significantly smaller than that of the argicultural production industry. It seems that investors recognize the food manufacturing industry is safer than the agricultural production industry.

3.4.2 THE MONTHLY RISK RETURN TRADEOFF OF INDUSTRY PORTFOLIOS FROM A BIVARIATE MODEL

The results for the bivariate GARCH-M models are presented in table 3. Positive risk-return tradeoffs represented by λ_{im} and λ_m are estimated in the bivariate model for agricultural production and the total U.S. market return (shown in the first column of table 3). Previously, several authors (French, Schwert and Stambaugh, 1987; Campbell and Hentschel, 1992) have estimated a positive but insignificant λ_m for the total U.S. market return in univariate models. However, in this bivariate portfolio, the risk-return tradeoff has over a 99% posterior probability of being positive for the total U.S. market. This suggests that multivariate models might be gainfully employed to investigate the relation between stock market risk and return more precisely.

The second column in table 3 shows the empirical results of the bivariate model for food manufacturing and the total market. Both risk-return tradeoffs are positive but the posterior probabilities of positive λ_{im} and λ_m are lower than for the agricultural production bivariate model.

The coefficient of relative risk aversion for both industry-specific portfolios is bigger than that of the total U.S. market ($\lambda_{im} > \lambda_m$). Dorfman and Park (2009) point out that the risk-return tradeoff is easily estimated for smaller market cap portfolios while bigger market cap portfolios are less likely to show a significant risk-return tradeoff. Our finding is consistent with their results. When considering smaller portfolios and the total market simultaneously, investors think the industry-specific portfolio (smaller portfolio) is likely to be more volatile than the total market (bigger portfolio).

The sign of the parameter on the conditional covariance, λ_{im}, is positive for each industry-specific portfolio in our results. The estimated time-varying covariance for both portfolios with the total market is almost always positive which means that the time-varying correlation is also positive for both assets (see figure 2). Thus, these two industry-specific portfolios and the total market generally move in the same direction. The time-varying covariance appears to be a good proxy of risk for each portfolio.

Figure 3 displays the conditional volatilities for agricultural production and the total U.S. market return and their time-varying covariance. The conditional volatility for agricultural production is much larger than that of the total U.S. market and the conditional covariance is similar in magnitute to the conditional volatility of the total U.S. market. Figure 4 shows the same series for the food manufacturing model. In contrast to agricultural production, the conditional volatility of food manufacturing is smaller than that of the total U.S. market and the conditional covariance is also slightly smaller than the conditional volatility of the total U.S. market.

3.5 CONCLUSION

In this study, we investigate the risk-return tradeoff in agribusiness stocks, specifically those in the agricultural production and food manufacturing industries. The expected positive relation between stock return and its risk holds for both industries, but the posterior probability of a positive tradeoff is lower for the food manufacturing industry. The sign of the risk-return tradeoff is not sensitive to volatility specification. A positive risk-return tradeoff for the total U.S. market portfolio is estimated in the bivariate GARCH-M framework with very strong posterior support. This implies that the multivariate GARCH-M framework may offer improved empirical results to demonstrate Merton's ICAPM.

The positive sign on the covariance between each industry and the total market, combined with the positive covariance itself, suggests that periods where agribusiness returns are more tightly correlated with the broader market are correctly perceived by the stock market as riskier periods for holding those assets. With correlations between both agricultural production and food manufacturing portfolios and the total market between 0.7 and 0.9 for most of the sample period, investors looking to diversify holdings won't find much to like here.

Table 3.1 The monthly risk-return tradeoff of agricultural production industry

	μ	λ	$\omega \times 10^2$	α	γ	β	Prob($\lambda > 0$)
GARCH	0.0015	1.4853	0.0304	0.1324		0.8127	0.9850
	(0.0033)	(0.7071)	(0.0095)	(0.0222)		(0.0308)	
TARCH	-0.0031	2.1165	0.0473	0.0382	0.1659	0.7832	0.9960
	(0.0039)	(0.8414)	(0.0127)	(0.0230)	(0.0426)	(0.0381)	
QGARCH	-0.0012	1.7119	0.0367	0.1292	0.0279	0.7771	0.9823
	(0.0039)	(0.8306)	(0.0116)	(0.0243)	(0.0093)	(0.0409)	

Note: Posterior standard deviation is reported in parentheses.

Table 3.2 The monthly risk-return tradeoff of food manufacturing industry

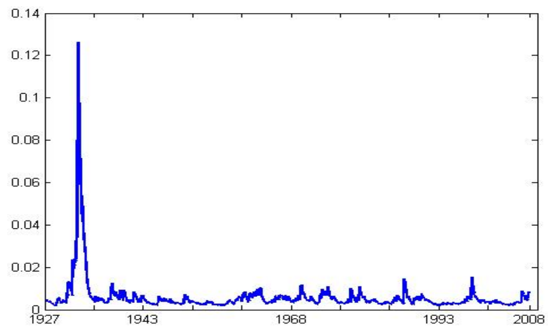
	μ	λ	$\omega \times 10^2$	α	γ	β	Prob($\lambda > 0$)
GARCH	0.0042	1.5999	0.0053	0.1112		0.8731	0.9587
	(0.0021)	(0.9665)	(0.0020)	(0.0173)		(0.0182)	
TARCH	0.0055	0.5920	0.0071	0.0556	0.1115	0.8628	0.7215
	(0.0021)	(1.0139)	(0.0025)	(0.0188)	(0.0369)	(0.0194)	
QGARCH	0.0044	1.2071	0.0052	0.1053	0.0109	0.8711	0.8944
	(0.0021)	(0.9790)	(0.0021)	(0.0160)	(0.0046)	(0.0179)	

Note: Posterior standard deviation is reported in parentheses.

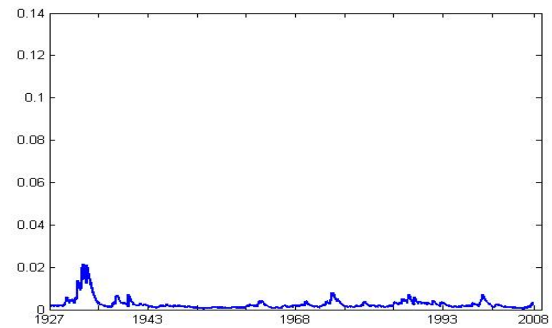
Table 3.3 The risk-return tradeoff of bivariate GARCH-in-mean model

	AG - Market	Food - Market
μ_i	-0.0012	0.0044
	(0.0034)	(0.0018)
$\lambda_{i,m}$	4.7804	1.1673
	(1.5494)	(0.9283)
$\omega_i \times 10^2$	0.0485	0.0062
	(0.0104)	(0.0018)
α_i	0.1598	0.1037
	(0.0249)	(0.0135)
β_i	0.7508	0.8736
	(0.0338)	(0.0166)
μ_m	0.0031	0.0048
	(0.0022)	(0.0021)
λ_m	2.2859	0.8601
	(0.9275)	(0.8508)
$\omega_m \times 10^2$	0.0127	0.0085
	(0.0030)	(0.0023)
α_m	0.1127	0.1103
	(0.0192)	(0.0129)
β_m	0.8388	0.8621
	(0.0223)	(0.0164)
$\omega_{i,m} \times 10^2$	0.0154	0.0066
	(0.0029)	(0.0018)
$\alpha_{i,m}$	0.0943	0.1036
	(0.0146)	(0.0123)
$\beta_{i,m}$	0.8328	0.8671
	(0.0174)	(0.0169)
Prob($\lambda_{i,m} > 0$)	0.9999	0.8935
Prob($\lambda_m > 0$)	0.9984	0.8478

Note:Posterior standard deviation is reported in parentheses

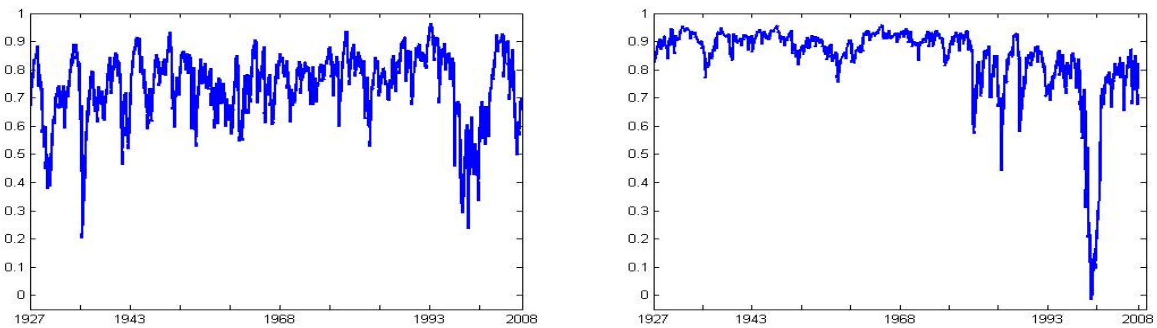


(A) Agricultural production



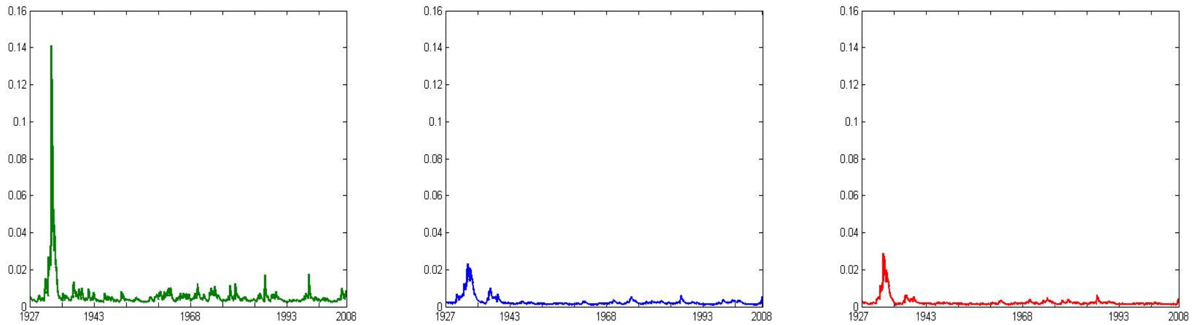
(B) Food manufacturing

Figure 3.1 Conditional volatility from univariate GARCH models



(A) Agricultural production (B) Food manufacturing

Figure 3.2 Time-varying correlation with total stock market

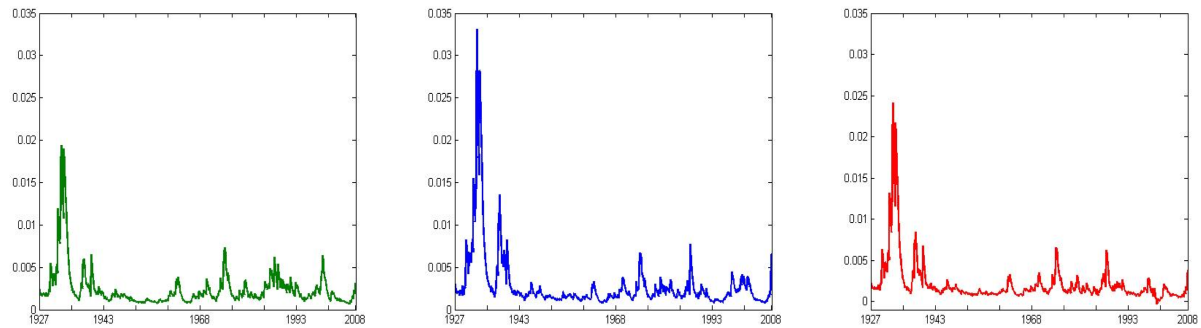


Conditional Volatility of Agriculture

Conditional Volatility of Total Market

Conditional covariance between Ag. and Market

Figure 3.3 Conditional volatilities and covariance from the agricultural production bivariate model



Conditional Volatility of Food

Conditional Volatility of Total Market

Conditional covariance between Food and Market

Figure 3.4 Conditional volatilities and covariance from the food manufacturing bivariate model

CHAPTER 4

SMALLER PORTFOLIO RETURNS AND THE RISK RETURN TRADEOFF FOR THE WHOLE MARKET

4.1 INTRODUCTION

Since Merton's (1973) pathbreaking article deriving the intertemporal capital asset pricing model (ICAPM), the relation between expected return and risk has centered on this foundation of modern finance theory and numerous studies have explored this risk-return tradeoff in attempts to estimate the magnitude of the tradeoff itself. Researchers have mainly focused on the nature of the relation between the market portfolio and its conditional variance using the popular generalized autoregressive heteroskedasticity-in-mean (GARCH-M) framework which estimates the expected return and conditional volatility jointly. However, some difficulties of estimating a risk-return tradeoff have emerged. Surprisingly, the empirical evidence has been somewhat mixed. Just naming a few studies and confining the listing to studies using some form of GARCH-M models, we find confirmation of the positive risk-return tradeoff in French, Schwert, and Stambaugh (1987), Chou (1988), Baillie and DeGennaro (1990), and Campbell and Hentschel (1992), while a negative risk-return tradeoff was estimated by Nelson (1991) and Glosten, Jagannathan, and Runkle (1993). Also, most empirical studies report a statistically insignificant coefficient of the risk-return tradeoff. This leads researchers to investigate possible causes and solutions of this difficulty. For instance, Lundblad (2007) finds that a very long data span is required to discover a strong relation between expected return and risk. Anderson and Bollerslev (1998) and Bali and Peng (2006) show that high frequency data dramatically improves conditional volatility estimation. Ghysels, Santa-Clara, and Valkanov (2005) propose a different volatility estimator and Ludvigson and Ng (2007) suggest that more conditioning variables might be needed to resolve this problem. Despite these suggestions, a robust answer for the risk-return tradeoff is still being investigated.

In this paper, we jointly investigate the nature of the risk-return tradeoff for the market portfolio and individual assets using bivariate GARCH-M models. We also employ Bayesian inference to resolve the difficulty of maximum likelihood estimation (MLE) which has usually been employed to estimate conditional volatility in GARCH specifications and take advantage of the existence of prior information. Lanne and Saikkonen (2006) report that high correlation between maximum likelihood estimates exists which leads statistical tests for a positive risk-return tradeoff to have low power. Employing Bayesian estimation can help avoid possible effects from the problem of high correlation. Using the prior density, inequality constraints on the GARCH-M model volatilities are imposed properly in comparison with some previous studies. With five different categories of portfolios made up of tenths of the market divided by sorting on measures such as market capitalization, book to market ratio, dividend yield, momentum factor and industries, we estimate high probabilities of positive risk aversion coefficients for the time-varying covariance and conditional volatility for each asset and the market portfolio. Thus, by utilizing the additional, less aggregated, information in the decile and industry portfolios, we get improved estimates of the risk-return tradeoff for the market portfolio.

Since previous empirical results seem to be very sensitive to variance specification, we also consider symmetric and asymmetric volatility specification and, with two possible assumptions of the risk-return tradeoff, we find that Merton's theory is proper to explain the relation between return and risk. A Bayesian model averaging technique is also employed to construct a robust estimate across models under consideration. The results are in general agreement that the risk-return tradeoffs of the market and individual portfolios are positive. However, a sufficient data span remains a critical factor to estimate this relation. With a shorter data span, we still have problems estimating a positive risk-return tradeoff except for market capitalization portfolios. As Lundblad points out, since the explanatory power of conditional volatility and covariance is extremely low, a long data period is demanded and our posterior results are consistent with his findings.

The remainder of the chapter is organized as follows. Section 1 describes Merton's ICAPM and the mean and variance equations in the GARCH-M framework. Section 2 explains Bayesian inference and our Bayesian computation method, the Random Walk Chain Metropolis-Hasting algorithm. Section 3 provides the data description. Section 4 discusses the empirical results and section 5 explains Bayesian model averaging and presents these empirical results. Section 6 concludes.

4.2 ECONOMETRIC METHODOLOGY

4.2.1 THEORETICAL MODEL

The seminal work of Merton, in the ICAPM, describes the relation between return and its own variance and covariance with the state variables of the investment opportunity set. Without the state variables, this relation focuses on the risk-return tradeoff. For any risky asset or portfolio, this relation can be written as

$$r_i - r_f = \lambda_{im}\sigma_{im} \tag{1}$$

where r_i and r_f represent a return of a risky asset and a risk free asset return, respectively, and σ_{im} is the covariance between the return of the risky asset or portfolio i and the market portfolio m.

For the market portfolio, the return can be described by a linear relation to its own risk:

$$r_m - r_f = \lambda_m\sigma_m^2 \tag{2}$$

where σ_m^2 is the variance of return on the market portfolio m and r_m is the return of the market portfolio. λ_{im} in Eq. (1) and λ_m in Eq. (2) are the coefficients of relative risk aversion for a ith asset and the market portfolio, respectively, and λ_m, the market risk-return tradeoff, is expected to be positive.

4.2.2 EMPIRICAL FRAMEWORK

To investigate this relation empirically, GARCH in mean models (GARCH-M: Engle, Lilien, and Robins, 1987; Bollerslev, Engle, and Wooldridge, 1988) have usually been employed. Most previous studies have been focused on the risk-return tradeoff for the market portfolio in Eq. (2) using an univariate GARCH-M model (for instance, French, Schwert, and Stambaugh, 1987; Chou, 1988). A bivariate GARCH-M framework has been used to investigate the time-varying covariance between the return of the market portfolio and bond yield as a state variable of the investment opportunity set (for instance, Scruggs, 1998; Scruggs and Glabadanidis, 2003). Bali (2008) employed the bivariate GARCH specification to estimate the covariance between the return of the market portfolio and an individual asset. We employ bivariate GARCH-M frameworks and examine an ith asset or portfolio and the market portfolio simultanuously.

GARCH-M consists of mean and variance equations. In our framework, we use two different mean equations and sets of volatility specification. The first set of mean equations we employ can be written as follows:

$$
\begin{aligned}
r_{i,t} - r_{f,t} &= \mu_i + \lambda_{im} h_{im,t} + \epsilon_{i,t} \\
r_{m,t} - r_{f,t} &= \mu_m + \lambda_m h_{m,t} + \epsilon_{m,t}
\end{aligned} \tag{3}
$$

where $h_{im,t}$ is the time-varying covariance between the returns of the market portfolio and the ith asset and $h_{m,t}$ is the conditional volatility of the return of market portfolio. The second set of mean equations used are:

$$
r_{i,t} - r_{f,t} = \mu_i + \lambda_i h_{i,t} + \lambda_{im} h_{im,t} + \epsilon_{i,t}
$$

$$r_{m,t} - r_{f,t} = \mu_m + \lambda_m h_{m,t} + \epsilon_{m,t} \tag{4}$$

where $h_{i,t}$ is the conditional volatility of the return of ith risky asset.

The mean equation in Eq. (3) describes Merton's ICAPM, but in Eq. (4) we assume that an individual risky asset is affected by not only the time varying covariance with the market portfolio but also by its own conditional volatility.

A volatility specification is the key component of a GARCH-M model because covariances and volatilities are critical factors for investigating the movement of risky assets. Since the autoregressive conditional heteroskedasticity (ARCH) specification was introduced by Engle (1982), many researchers have designed refinements of ARCH or similar models used to investigate the conditional volatility. A partial list includes Bollerslev (1986), Baillie and DeGennaro (1990), Nelson (1991), Campbell and Hentschel (1992), Glosten, Jagannathan, and Runkle (1993), Sentana (1995), Hentschel (1995), Rabemananjara and Zakoian (1995), and Lundblad (2007). Various specifications of time-varying covariance have been developed and employed in other studies. For example, Bollerslev, Engle, and Wooldridge (1988) introduce the VECH (vector-GARCH) specification and Engle and Kroner (1995) suggest a BKKK model. The factor-ARCH covariance structure is developed by Engle, Ng, and Rothschild (1990). Bollerslev (1990) and Engle (2002) design new specifications for multivariate GARCH frameworks, the constant conditional correlation (CCC) and dynamic conditional correlation (DCC) models, to estimate a conditional covariance conveniently.

In this study, we employ both symmetric and asymmetric VECH specifications (Kroner and Ng, 1998) to model the conditional variances and covariances. These specfications can be described as follows.

Symmetric VECH (Standard GARCH):

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

$$h_{m,t} = \omega_m + \alpha_m \epsilon_{m,t-1}^2 + \beta_m h_{m,t-1}$$

$$h_{im,t} = \omega_{im} + \alpha_{im} \epsilon_{i,t-1} \epsilon_{m,t-1} + \beta_{im} h_{im,t-1} \tag{5}$$

Asymmetric VECH (TARCH):

$$h_{i,t} = \omega_i + \alpha_i \epsilon_{i,t-1}^2 + I \gamma_i \epsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$$

$$h_{m,t} = \omega_m + \alpha_m \epsilon_{m,t-1}^2 + M \gamma_m \epsilon_{m,t-1}^2 + \beta_m h_{m,t-1}$$

$$h_{im,t} = \omega_{im} + \alpha_{im} \epsilon_{i,t-1} \epsilon_{m,t-1} + \gamma_{im} I \epsilon_{i,t-1} M \epsilon_{m,t-1} + \beta_{im} h_{im,t-1} \tag{6}$$

where I and M are indicator functions for $\epsilon_{i,t-1}$ and $\epsilon_{m,t-1}$ respectively. If $\epsilon_{i,t-1}$ or $\epsilon_{m,t-1}$ is negative, the respective indicator function, I or M, equals one, and otherwise, they equal zero. This model is designed to allow negative errors to increase volatility in the next period more than positive errors. Typically this phonomenon is called the "leverage effect." The strength of the asymmetric response to previous errors is reflected in the parameter γ in each volatility specification. As Campbell and Hentschel (1992) point out, we expect γ, the coefficient for negative errors, to be positive since a positive γ makes the conditional variance and covariance in next period increase.

Following previous studies (for instance, Lundblad, 2007; Bali, 2008), we assume that the error term follows a normal density and the log-likelihood function is the summation of the log normal densities. The log-likelihood function (ignoring normalizing constants) for the bivariate model can be written as

$$L(r|\theta) = \sum_{t=1} L_t(r|\theta); \quad L_t(r|\theta) = -1/2 \cdot \log|\mathbf{H_t}| - 1/2\varepsilon_{\mathbf{t}}' \mathbf{H_t^{-1}} \varepsilon_{\mathbf{t}} \tag{7}$$

where θ is a vector of the unknown parameters and $\varepsilon_{\mathbf{t}}$ and $\mathbf{H_t}$ denote the error vector and time varying covariance matrix, respectively,

$$\varepsilon_{\mathbf{t}} \quad = \quad \begin{bmatrix} \epsilon_{i,t} \\ \epsilon_{m,t} \end{bmatrix} \qquad \text{and} \qquad H_t \quad = \quad \begin{bmatrix} h_{i,t} & h_{im,t} \\ h_{mi,t} & h_{m,t} \end{bmatrix} \tag{8}$$

where $h_{im,t}$, the time-varying covariance, doesn't need to be positive but the covariance matrix, $\mathbf{H_t}$, has to be positive definite.

4.3 BAYESIAN INFERENCE

In this study, we employ Bayesian inference to estimate the GARCH-M model because of difficulties with maximum likelihood estimation (MLE). As discussed in Lanne and Saikkonen (2006), high correlation typically exists between ML estimators of the intercept and risk aversion coefficient expressed as μ and λ in the mean equation on the univariate GARCH-M model. This creates a problem similar to multicollinearity in a normal regression model and leads to low power for statistical tests of whether λ is positive. The effect from this high correlation between parameters in the mean equation can exist in the multivariate GARCH-M framework. Further, imposing positivity on conditional variances without restricting individual parameters of equation (5) and (6) is easier in the Bayesian framework than with MLE.

The benefit of the prior distribution is one of the reasons for using Bayesian. In Bayesian inference, the posterior distribution is proportional to the likelihood function times the prior distribution. The prior distribution, a key component of Bayesian estimation, reflects the researcher's subjective beliefs for the parameters of the model before seeing the data. Therefore, the posterior distribution summarizes all available information from the likelihood function and the prior information (Zellner, 1971).

Our prior beliefs are related to the natures of the time-varying variance and the covariance matrix. First, the conditional volatility over time needs to be positive without imposing any restrictions on the variance equation to manipulate a positive volatility. In the prior distribution, we allow the parameters on the variance equations to be negative because of the absence of inequality constraints. This opens the possibility that the coefficients in Eq. (5) and (6) could be negative if negative parameters are more proper than positive ones and removes the chance of possible bias being introduced into the parameter estimates through unneeded nonnegativity constraints on the parameters in the (co)variance equations. Second, the conditional covariance can be negative but the time-varying covariance matrix must be positive definite over time. Using an indicator function, the positivity constraints are imposed on the conditional volatility directly instead of the parameters to keep the time-varying variance positive. Our prior distribution can be described as below:

$$p(\theta) = \mathbf{I}(\mathbf{H_t}) \cdot \prod_{i=1}^{K} N_{\theta_i}(0, 10^2) \tag{9}$$

where θ is the (K $\times$ 1) vector of parameters such as μ, λ, ω, α, and β and θ_i indicates the ith component of the parameter vector. The indicator function, $\mathbf{I}(\mathbf{H_t})$, in the prior density equals one if the conditional variances for each time-series and covariance matrix in all time periods fulfill the conditions of positivity and zero otherwise.

The prior densities for the individual parameters are set to independent normal densities with zero means and variance $= 10^2$. This prior distribution is informative but because of the large prior variances, our prior distribution can be considered a diffuse prior. Most of the information for the researcher's prior beliefs is created by the indicator function, $\mathbf{I}(\mathbf{H_t})$. As Bauwens and Lubrano (1998) did, the initial variances $h_{i,0}$, $h_{m,0}$, and $h_{im,0}$ are treated as a known constant. We can write our posterior distribution as below:

$$p(\theta|r) \propto p(r|\theta)p(\theta) \tag{10}$$

where $p(\theta|r)$ denotes the posterior density and $p(r|\theta)$ and $p(\theta)$ are the likelihood funtion and the prior distribution, respectively.

4.3.1 THE BAYESIAN ESTIMATION ALGORITHM

Previously, numerous studies have employed Bayesian inference to investigate the nature of GARCH processes (for example, Geweke, 1989; Kleibergen and van Dijk, 1993; Bauwens and Lubrano, 1998 and 2002; Nakatsuma, 2000; Vrontos, Dellaportas, and Politis, 2000; Osiewalski and Pipien, 2004; Lanne and Luoto, 2008). Most of the studies with Bayesian inference employ posterior simulators such as the Markov Chain Monte Carlo (MCMC) algorithm and Monte Carlo integration to estimate the posterior distribution because it is not feasible to compute the posterior analytically when the prior distribution is nonlinear or complicated. In this study, we employ the Random Walk Chain Metropolis-Hastings algorithm because of its benefits in the absence of a good approximating density for the posterior distribution (Koop, 2003).

In the Random Walk Chain Metropolis-Hastings algorithm, candidate draws are generated by a random walk process,

$$\theta^* = \theta^{(s-1)} + z, \tag{11}$$

where z is called the increment random variable. θ^* and $\theta^{(s-1)}$ are a candidate and previous draw from the posterior simulation, respectively. For an initial value of candidate draws, $\theta^{(0)}$, coefficients of maximum likelihood estimation (MLE) are used[1].

The distribution for z, the increment random variable, becomes the candidate generating density and the multivariate normal distribution is chosen in this study due to its convenience and our assumption of normality for the error term in Eq. (7). The candidate generating density can be described as follows:

$$q(\theta^*|\theta^{(s-1)}) = N(\theta^{(s-1)}, c \cdot \widehat{\Sigma}) \tag{12}$$

where $\widehat{\Sigma}$ is the covariance matrix from MLE and c is set to achieve an optimal acceptance rate.

The candidate draws are accepted or rejected with an acceptance probability that is computed as

$$\alpha(\theta^*|\theta^{(s-1)}) = \min\left[\frac{p(\theta = \theta^*|r)}{p(\theta = \theta^{(s-1)}|r)},\ 1\right] \tag{13}$$

where $p(\theta|r)$ is the posterior distribution. If the current draw is accepted, $\theta^{(s)}$ is $\theta^{(*)}$. If rejected, the previous one is reused ($\theta^{(s)} = \theta^{(s-1)}$).

[1] The results of λ_i, λ_{im}, and λ_m from MLE are mostly statistically insignificant.

The acceptance rate of generated draws is critical for an accurate numerical approximation to the true distribution. Suppose that this rate is too high. In this case, the estimated posterior distribution will be very similiar to the candidate generating density. If the acceptance rate is extremely low, it implies that the random walk chain is not moving enough to explore the entire posterior density and the estimated posterior mean may not be much different from initial values of coefficients. In both cases, it is highly doubtful that the posterior simulator worked well and the estimated posterior distribution is likely inaccurate. Unfortunately, there is no general rule for the optimal acceptance rate. The rule of thumb often considered is that the acceptance rate for candidate draws should be around 0.5. If you achieve roughly 0.5 as the acceptance rate, the posterior simulation is likely to approximate the posterior density correctly (Koop, 2003). To follow Koop's suggestion, our acceptance rates for all estimations are calibrated to roughly 0.45 by choice of c in Eq. (12).

The posterior mean is commonly used as the point estimator of the posterior distribution. The simple average of all accepted candidate draws is the posterior mean because in the Metropolis-Hastings algorithm, each accepted candidate draw is weighted equally. The posterior mean, $\widehat{g_S}$, can be written as

$$\widehat{g_S} = \frac{1}{S} \sum_{s=1}^{S} g(\theta^{(s)}). \tag{14}$$

where $g(\theta^{(s)})$ denotes any general function of the model parameters and S is the number of accepted draws.

The posterior simulation is executed as follow. We gather 55,000 accepted draws and discard the first 5,000 accepted draws as the initial burn-in to eliminate the effect of initial values. If a candidate draw does not satisfy the condition of positivity for the variance and covariance matrix, a draw is regenerated until it satisfies the researcher's subjective belief (this is an accept-reject step within our posterior simulator to handle the truncation of the posterior distribution due to the indicator function in the prior for H_t). The previous accepted draw remains the mean of candidated generating density until a new candidiate draw has a positive conditional variance and positive definite covariance matrix for the entire period. The marginal likelihood is computed to compare different models by simple averaging of all posterior densities of accepted draws. For each estimation, we perform Geweke's (1992) diagnostic to check the convergence of our Metropolis-Hastings algorithm. Let S_A and S_C denote first 10% and last 40% accepted draws. The test statistic for Geweke's convergence diagnostic (CD) can be written as

$$CD \;=\; \frac{\widehat{g_{S_A}} - \widehat{g_{S_C}}}{\frac{\widehat{\sigma_{S_A}}}{\sqrt{S_A}} + \frac{\widehat{\sigma_{S_C}}}{\sqrt{S_A}}} \quad \longrightarrow \quad N(0,1) \tag{15}$$

where $\widehat{g_{S_A}}$ and $\widehat{g_{S_C}}$ denote the posterior means of S_A and S_C, respectively. The terms $\frac{\widehat{\sigma_{S_A}}}{\sqrt{S_A}}$ and $\frac{\widehat{\sigma_{S_C}}}{\sqrt{S_C}}$ are the numerical standard errors of these two estimates.

In the MCMC algorithm, the posterior standard errors are different than the numerical standard errors (NSE) since the draws are correlated and a typical central limit theorm does not work. We compute the numerical standard errors using the formula suggested by Koop, Poirier, and Tobias (2007). The formula for the NSE is:

$$NSE(\widehat{g_S}) \;=\; \sqrt{\frac{\sigma^2}{m}\Big[1 + 2\sum_{j=1}^{m-1}\big(1 - \frac{j}{m}\big)\frac{\sigma_j}{\sigma^2}\Big]} \tag{16}$$

where σ_j is the covariance between vectors $[\theta_1 \; \theta_2 \; \cdots \; \theta_{m-j}]$ and $[\theta_{j+1} \; \theta_{j+2} \; \cdots \; \theta_m]$ and σ^2 denotes the posterior variance of each parameter. Typically, $\sigma_j > 0$ and the numerical standard error is bigger than the posterior standard errors.

A posterior model probability is used to compare model specifications. A posterior model probability is computed by the product of marginal likelihood and prior model probability. Let M_i denote I different considered models for $i = 1, \cdots, I$ and $p(r|M_i)$ and $p(M_i)$ are the marginal likelihood of and a prior model probability of M_i , respectively. A posterior model probability can be described as

$$p(M_i|r) \propto p(r|M_i)p(M_i). \tag{17}$$

We set equal prior weights for all considered models, thus posterior model probabilities are proportional to the marginal likelihood values from the considered models. The model that has highest posterior model probability is considered the best specification.

4.4 DATA DESCRIPTION

We use monthly return data on individual portfolios, the market portfolio, and a risk free asset. All portfolio returns for portfolios are the value-weighted CRSP index of NYSE, AMEX, and Nasdaq and the one-month Treasury bill rate is employed as a risk free asset return. Five different categories of individual portfolios (10 decile portfolios formed on market capitalization, book to market (BM) ratio, dividend yield, momentum and industry) are used to investigate the time-varying covariance between the market portfolio and each portfolio. The sample period is 1927 $\sim$ 2008 (T=984) for portfolios formed on market capitalization, book to market (BM) ratio, momentum and industry and the market portfolio. For the portfolios formed on dividend yield, the sample period is 1928 $\sim$ 2008 (T=972). All return data were obtained from Kenneth R. French on-line data library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data _ library.html). With 50 different disaggregated portfolios to pair with the market portfolio and four possible model specifications, we thus have 200 models to estimate.

4.5 EMPIRICAL RESULTS OF BIVARIATE FRAMEWORK

Since the researchers' interest is focused on the coefficients of relative risk aversion, we report only the results related to parameters λ_i, λ_{im} and λ_m in the tables for the simplicity. Hereafter, we call the combination of Eq. (3) and Eq. (5) model (A) and the combination of Eq. (3) and Eq. (6) model (B). Models (C) and (D) will be the combination of Eq. (4) and Eq. (5) and the pairing of Eq. (4) and Eq. (6), respectively. Bayesian posterior simulation for each estimation satisfies Geweke' convergence diagnostic (test statistics available from the author).

4.5.1 RELATION BETWEEN THE MARKET PORTFOLIO AND THE SIZE PORTFOLIOS

Table 1 provides the empirical results of bivariate GARCH-M models between the market portfolio and 10 decile market capitalization portfolios. In the results of model (A), most of the coefficients of relative risk aversion have greater than a 95% posterior probability of being positive. The coefficients for the time-varying covariance, λ_{im}, have greater than a 95% posterior probability of being positive for all portfolios. The risk-return tradeoff for the market portfolio, λ_m, is strongly positive for all market cap portfolios except the decile 9 portfolio. A similar pattern of results is revealed with model (B). In each estimation, we discover positive λ_{im}s and λ_ms with high posterior probability.

The difference between the mean equation in Eq. (3) and Eq. (4) is the existance of λ_i, the risk aversion coefficient for the time-varying variance of the return on individual portfolio. The posterior distribution for λ_i in model (C) does not show strong evidence of being positive or negative for any individual portfolio. For portfolios of decile 5, 8, and 10, the posterior probability of positive λ_{im} is less than 90%. but the risk aversion coefficient for the market portfolio, λ_m, has over a 95% posterior probability of being positive for all deciles. The results from model (D) are similar to those of the framework with model (C). The risk-return tradeoff for the market portfolio has greater than a 95% of posterior probability of being positive for all deciles. The probability of positive λ_{im} is under 90% only for the portfolios of decile 2, 5, and 9. The posterior distributions for λ_i do not strongly support either sign for all deciles.

Across the four different models, there are some consistent results. First, the risk aversion coefficient for the market portfolio, λ_m has over a 95% posterior probability of being positive for all deciles regardless of model specification. Thus, we have very robust evidence of the positive risk-return tradeoff on the market portfolio. Second, most coefficients of the time-varying covariance for each portfolio, λ_{im}, have greater than a 95% posterior probability of being positive. This implies that individual portfolios formed by market cap are affected by the conditional covariance between a ith asset or portfolio and the market portfolio positively. The risk aversion coefficient of conditional volatility of each portfolio, λ_i in model (C) and (D), is not a strong explanatory variable for all deciles. Finally, the estimates of λ_{im} are usually higher than these of λ_m and estimated λ_{im} decreases as the market cap of the portfolio increases (from S1 to S10). This implies that risk-averse investors perceive the risk of portfolios by their market cap easily because the smaller market-cap portfolios are considered a relatively riskier asset compared to the bigger market-cap portfolios. Figure 1 shows the time-varying covariance with the market portfolio for decile portfolios 1, 4, 8, and 10 from model (A). The estimated conditional covariance decreases from decile 1 to 10 portfolio, so risk averse investors expect higher return when they hold smaller market cap portfolios and the risk aversion coefficient increases.

Based on the posterior model probabilities for four different frameworks, Merton's ICAPM employed in models (A) and (B) is heavily favored relative to the mean equation in models (C) and (D). In previous studies, the volatility specification has been considered as a suspect in the difficulty of precisely estimating the risk-return tradeoff of the market portfolio but the results of table 1 shows a strong and positive risk aversion coefficient, λ_m, regardless of variance equation. The symmetric specification (GARCH) produces a higher posterior model probability than the asymmetric model (TARCH) in most estimations.

In the results of models with the highest posterior model probability, nearly all of the λ_{im} and λ_m have greater than a 95% posterior probability of being positive. For seven portfolios (decile 3, 4, 5, 6, 7, and 10), model (A) has the highest posterior model probability and all λ_{im}s and λ_ms are strongly positive with over 95% posterior probability. Model (B) has the highest posterior model probability for portfolios of decile 1,2, and 9 and all posterior means for risk aversion coefficients are strongly positive. The risk aversion coefficient for the portfolio of decile 8 in model (C) does not show strong and positive results. Model (D) never has the highest posterior model probability.

4.5.2 RELATION BETWEEN THE MARKET PORTFOLIO AND THE BOOK TO MARKET PORTFOLIOS

The posterior empirical results of four different bivariate GARCH-M frameworks between the market portfolio and 10 decile book to market portfolios are provided in table 2. In the results of model (A), risk aversion coefficients, λ_{im}, for all BM portfolios except the decile 1 portfolio have greater than a 95% posterior probability of being positive. For the risk return tradeoff of the market portfolio, λ_m is positive with high posterior probability for all BM portfolios except the decile 1 portfolio. Neither risk aversion coefficient for the decile 1 portfolio has a strong posterior probability of being positive, but all other portfolios have excellent results with model (A). In the results of model (B), six of the risk aversion coefficients of the time-varying covariance, λ_{im}, have greater than a 95% posterior probability of being positive coeffieicents (for portfolios of decile 2, 3, 6, 8, 9 , and 10) and four of the risk-return tradeoff for the market portfolio are a strongly positive (for portfolios of decile 6, 8, 9, and 10). With the asymmetric volatility specification, the difficulty of estimating positive risk aversion coefficients is increased.

In the results of model (C), even though many of λ_i and λ_{im} do not have greater than a 95% posterior probability of being positive, the risk-return tradeoff of the market portfolio, λ_m, is a strongly positive with over a 95% posterior probability for all portfolios except decile 2 and 3 portfolios. The posterior probability for λ_i in the decile 1 and 10 portfolios, shows strong evidence of being negative but the posterior distirbutions of other portfolios do not provide strong posterior support for being positive or negative. Only four risk aversion coefficient for the time-varying covariances, λ_{im} have greater than a 95% posterior probability for the portfolio of decile 1, 7, 8, and 10. Strong posterior evidences of positive risk aversion coefficients are discovered for some portfolios with model (D). Only one λ_i has a greater than a 95% posterior probability of being positive (a decile 4). Three portfolios (decile 1, 8, and 10) have a strong posterior probability of a positive risk aversion coefficient for the time-varying covariance and the risk-return tradeoff of the market portfolio has greater than a 95% posterior probability of being positive for portfolios of decile 3, 6, 8, 9, and 10.

In comparing models by posterior model probability, model (A) is favored over other models for most portfolios and model (B) has the highest posterior model probability for portfolios of decile 7 and 10. This is consistent with the results of market cap portfolios and proves the conditional volatility of an asset or portfolio might not be a proper explanatory variable. The posterior means of risk aversion coefficients in the model with highest posterior model probability, the risk-return tradeoff of the market portfolio, λ_m, and the coefficient of the time-varying covariance, λ_{im}, have strong posterior evidence of being positive for eight portfolios. Only for portfolios of decile 1 and 7 do both coefficients not have greater than a 95% posterior probability of being positive. Thus, the robust posterior evidence in favor of a positive risk aversion coefficient for the market portfolio and the relation between expected return of individual portfolios formed on market cap and the time-varying covariance is repeated for portfolios formed on BM deciles

Regardless of model specfication, most of the estimated risk aversion coefficients of λ_{im} and λ_m have greater than a 95% posterior probability of being positive for portfolios of deciles 8, 9, and 10. This implies that the higher BM portfolios and the market portfolio are affected by each other. Investors who bought higher BM portfolios perceive the risk of the market portfolio very easily and it leads the risk aversion coefficient of the time-varying covariance with higher BM portfolios to increase. The coefficients for the time-varying covariance do not show any pattern clearly across model specifications.

4.5.3 RELATION BETWEEN THE MARKET PORTFOLIO AND THE DIVIDEND PORTFOLIOS

In table 3, the posterior results of bivariate GARCH-M models between the market portfolio and 10 decile dividend yield portfolios are presented. Like the results of the market cap and BM portfolios, the risk aversion coefficients of $h_{im,t}$ and $h_{m,t}$ have mostly greater than a 95% posterior probability of being positive except for the decile 1 portfolio in the results of model (A). In the results of model (B), the risk aversion coefficient of the time-varying covariance has a strong posterior probability of being positive for portfolios of decile 2, 3, 6, 7, 8, and 9 and the positive risk-return tradeoff of the market portfolio has strong posterior support for portfolios of decile 3, 5, 6, 7, and 8. In the posterior results of model (C) and (D), the majority of risk aversion coefficients for h_i and h_{im} do not have strong posterior evidence of being positive. However, many of the risk-return tradeoff parameters of the market portfolio have greater than a 95% posterior probability of being positive in both models. Specifically, a positive λ_m with strong posterior evidence is revealed in all portfolios in model (C).

Model (A) has the highest posterior model probability for portfolios of decile 1, 2, 3, and 7 and model (B) is the most likely model for the rest of them. This is consistent with the results of other categories of portfolios. Merton's ICAPM is heavily supported by all posterior evidence. In the results of the model with the highest posterior model probability, the risk-return tradeoff of the market portfolio has over a 90% posterior probability of being positive for all portfolios except the decile 10 portfolio. The risk aversion coefficient of the time-varying covariance shows a strong posterior probability of being positive with over a 90% posterior probability for eight portfolios except decile 1 and 10 portfolios. The results of dividend portfolios are relatively weak compared to those of the market cap and BM portfolios but the results still show robust evidence of positive parameters for the risk aversion coefficients, λ_{im} and λ_m.

4.5.4 RELATION BETWEEN THE MARKET PORTFOLIO AND THE MOMENTUM PORTFOLIOS

The results in table 4 present the posterior results of bivariate GARCH-M models between the market portfolio and 10 decile momemtum portfolios. In the results of model (A), all risk aversion coefficients of h_{im} and h_m have greater than 95% posterior probability of being positive. In the results of model (B), the coefficient of the time-varying covariance has over 95% posterior probability of being positive for portfolios of deciles 1, 2, 3, 5, 6, 7, and 10 and positive λ_m is estimated for portfolios of decile 3, 5, 6, 7, and 10. Many risk aversion coefficients of Model (C) and (D) do not have strong posterior probability of being positive. In the smaller momentum portfolios, we find over 95% posterior probability of a positive λ_{im} for portfolios of decile 1, 3, and 4 in model (C) and decile 2 and 3 in model (D).

Merton's ICAPM described in model (A) and (B) is favored relative to the mean equation (4) in model (C) and (D) based on the results of posterior model probability. The results of momentum portfolios are also relatively weak compared to those of the market cap and BM portfolios. In the results of models with the highest posterior model probability, some risk aversion coefficients don't have greater than a 95% posterior probability of being positive but the risk aversion coefficient of the market portfolio has mostly a strong posterior evidence of being positive.

4.5.5 RELATION BETWEEN THE MARKET PORTFOLIO AND THE INDUSTRY PORTFOLIOS

The posterior results of bivariate GARCH-M models between the market portfolio and industry portfolios are provided in table 5. Model (A) has the highest posterior model probability for 9 portfolios and model (B) has one portfolio with highest posterior model probability. This is consistent with previous results: Eq. (3) that describes Merton's ICAPM is mostly favored over Eq. (4). The coefficients of relative risk aversion of h_{im} and h_m in models with highest posterior model probability have greater than a 95% posterior probability of being positive for the portfolios of Durable, Manufacturing, Hi-technology, Telecommunications and others. Some of risk aversion coefficients of the rest of portfolios do not have a strong posterior evidence of being positive. But, overall, the majority of risk aversion coefficients are strongly positive.

4.5.6 EMPIRICAL RESULTS FOR THE POSTWAR PERIOD

Lundblad (2007) shows that a sufficient data span is a critical factor for estimating a positive risk-return tradeoff of the market portfolio. With a long data span, Lundblad estimates a positive and statistically significant risk-return tradeoff of the market portfolio regardless of volatility specification (GARCH, TARCH, QGARCH, and EGARCH). In this section, we discuss empirical results of all portfolios and model specifications already presented but with data for only the postwar period (1950 $\sim$ 2008). Our results partially confirm what Lundblad found. Except model (A) with market cap portfolios, other risk aversion coefficients don't have greater than a 95% posterior probability of being positive (not reported). Table 6 provides the posterior results of bivariate GARCH-M with model (A) for market cap portfolios. All risk aversion coefficients of h_{im} have greater than a 95% posterior probability of being positive except the portfolio of decile 10. A positive risk-return tradeoff of the market portfolio is revealed for portfolios of decile 3, 4, 5, 8, and 10. Compared to table 1, the number of positive risk aversion coefficients with a strong posterior evidence decreases. This is likely from the lack of sufficient data period.

These results imply that the market capitalization is a convenient measure to perceive the risk of an individual portfolio to risk averse investors. Compared to other categories such as a book to market ratio, dividend yield, and momentum factor, the market size of individual asset or portfolio can be considered representative of its portfolio risk. Thus, our results support a positive risk-return tradeoff of portfolios and the need of a sufficient data period. The coefficients of relative risk aversion for the time-varying covariance decreases from smaller to bigger portfolio. This is consistent with the results for the longer period ($1927 \sim 2008$).

Figure 2 presents the time-varying covariance with the market portfolio for portfolios of decile 1, 4, 8, and 10. The estimated conditional covariance decreases from decile 1 to 10 portfolio and this is consistent with the results of figure 1.

4.5.7 SUMMARY OF EMPIRICAL EVIDENCE

Using 50 pairs of an individual portfolio with the market portfolio, we estimate two hundred risk aversion coefficients for time-varying covariance and conditional volatility. We find that λ_m, a risk-return tradeoff of the market portfolio, has greater than 95% of posterior probability of being positive in 142 out of 200 models and a risk aversion coefficient of conditional covariance, λ_{im}, has posterior probability of being positive of 95% in 105 out of 200 models. The results with market cap portfolios are better than those with other individual portfolios. The risk-return tradeoff of the market portfolio has greater than 95% of posterior probability of being positive in 49 out of 50 models and postive λ_{im} with over 95% posterior probability is estimated in 41 out of 50 models. Only 4 risk aversion coefficients for conditional volatility, λ_i have greater than 95% of posterior probability of being positive. In the results of highest posterior model probability, 34 risk aversion coefficients of time-varying covariance and 36 risk aversion coefficients of the risk-return tradeoff of the market portfolio have posterior probability of being positive of 95% out of 50 models. Especially, with market cap porfolios, only one λ_{im} does not have over 95% posterior probability of being positive and the risk-return tradeoff of the market portfolio has greater than 95% of posterior probability of being positive in all estimations.

There are some consistent empirical results across model specifications and variant data combinations. First, the mean equation of Eq. (3) in models (A) and (B) is favored over Eq. (4) in models (C) and (D) for most estimations. With the conditional volatility of individual portfolios, most risk aversion coefficients of h_{im} have a low posterior probability of being positive and the posterior model probability for Eq. (4) in models (C) and (D) is usually lower than that of Eq. (3) in models (A) and (B). This implies the conditional volatility of an individual asset or portfolio might be an inappropriate explanatory variable and strongly supports Merton's ICAPM. Second, the difficulty of estimating positive risk aversion coefficients increases with an asymmetric variance equation. With (co)variance equations in Eq. (6), the number of positive coefficients of relative risk aversion is decresed. These results are relatively consistent with previous studies since the negative risk aversion coefficients are discovered with asymmetric specification such as TARCH and EGARCH (Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993). Third, in the U.S. equity market, researchers have confirmed the existence of the leverage effect and this has led them to develop lots of refinements to the symmetric GARCH specification. However, in the bivariate GARCH-M models, the symmetric variance equations in Eq. (5) are favored to the asymmetric variance equations in Eq. (6) in most estimations by the posterior model probability. Fourth, as Lundblad said, a sufficient data period is required to estimate a positive risk-return tradeoff and this argument is also proved by our posterior results for bivariate GARCH-M models with postwar periods. Some of risk aversion coefficients do not have a strong posterior probability of being positive. This also becomes the evidence of needing long data period to estimate the risk-return tradeoff. Finally, in the posterior results for the models with the highest posterior model probability, most of risk aversion coefficients have a strong posterior probability of being positive. This is empirical evidence of the positive risk-return tradeoff in the equity market. Especially, in the results of market cap portfolios, the risk aversion coefficient of the time-varying covariance, λ_{im}, shows a decreasing trend moving from smaller to bigger market cap portfolio. Thus, investors perceive that the smaller market cap portfolio is riskier than the bigger market cap portfolio.

4.6 BAYESIAN MODEL AVERAGING

Bayesian model averaging (BMA) is a technique designed to address the problem of model uncertainty by averaging over models under consideration rather than by selecting a single model. Previously, BMA has been employed to investigate stock return predictability and its model uncertainty (for example, Avramov, 2002; Cremers, 2002). Here we use it to address our uncertainty over specification of the mean and volatility equations for each of the 50 different portfolios. Simply speaking, BMA is a weighted averaging of a set of considered models by the posterior model probabilities. Let $p(\theta|r, M_i)$ and $p(M_i|r)$ be the posterior density of θ and a posterior model probability for M_i, respectively. The posterior density of BMA is computed by the summation of a posterior density times a posterior model probability of each considered model. It can be described as

$$p(\theta|r) = \sum_{i=1}^{I} p(\theta|r, M_i)p(M_i|r). \tag{18}$$

The sum of posterior model probabilities under consideration should be equal to one in the BMA technique. This can be written as

$$\sum_{i=1}^{I} p(M_i|r) = 1. \tag{19}$$

We consider four different bivariate GARCH-M models for each estimation, so the sum of posterior model probabilities for the four models must equal one. As mentioned before, we set equal prior weights for all considered models, thus no prior preference for a certain model exists.

In table 7, the results of λ_{im} and λ_m using BMA are reported for the period 1927 $\sim$ 2008. A risk aversion coefficient of the market portfolio, λ_m, in the averaged model has greater than a 95% posterior probability of being positive at 35 out of 50 models. A risk aversion coefficent of time-varying covariance, λ_{im} in the averaged model has posterior probability of being positive of 95% at 28 out of 50 models.

In the posterior results of market cap portfolios, all risk aversion coefficients, λ_{im} and λ_m have greater than a 95% posterior probability of being positive except λ_{im} for decile 8 portfolio. The risk aversion coefficient of the time-varying covariance decreases from decile 1 to decile 10 portfolio. This intuitively makes sense because risk averse investors expect higher returns from the smaller market cap portfolio because it is typically considered a riskier asset. It seems that market capitalization is a very easy index to perceive the risk of individual portfolio or asset. The pattern of the decreasing risk coefficients of the time-varying covariance is only revealed from market cap portfolios. The risk-return tradeoff of the market portfolio represented by λ_m is positive for all portfolios and the estimated coefficient for each estimation is relatively unchanged. The results of the portfolios of BM, dividend, momentum, and industry portfolios are weak compared to market capitalization portfolios. However, most estimated risk aversion coefficients have greater than a 90% posterior probability of being positive. This implies that the relation between expected return and risk would be positive and support Merton's theory.

4.7 CONCLUSION

Since Merton's intertemporal capital asset pricing model was introduced, the relation between expected return and risk has been of central importance. However, the existing empirical studies lack consistent evidence that the risk-return tradeoff is positive. In this article, we employ bivariate GARCH-M models to investigate the nature of the intertemporal risk-return tradeoff between the market and individual portfolios and find the following general results.

First, a positive risk-return tradeoff of the market portfolio is estimated in general. This implies that the empirical results of ICAPM might be improved in multivariate GARCH-M models. Second, the risk aversion coefficient of the time-varying covariance between the market and individual portfolios is also positive in portfolio-level analysis. Using five different categories of individual portfolios, we discover the relation between individual portfolio returns and the time-varying covariance with the market portfolio have a strong posterior probability of being positive in most estimations. Third, as Lundblad (2007) claimed, we find that the data span is another key factor to estimate positive risk-return relation. With a shorter data span, the postwar period, the posterior probability of a positive relation between expected return and risk is lower. Finally, the market capitalization is an easy measure of the risk of individual portfolios to risk averse investors. The risk aversion coefficient increases from larger to smaller market cap portfolios. This implies that the smaller portfolios are considered riskier assets and investors expect higher returns from portfolios composed of smaller market cap stocks even if the volatility remains constant.

A robust answer for the risk-return tradeoff is provided using the Bayesian model averaging technique. Summarizing all available information into a posterior density, the evidence of a positive risk-return tradeoff for the market and individual portfolios is empirically estimated with very high posterior probabilities in support. Combining the evidence from four different possible bivariate GARCH-M model specifications allows the posterior density to show very strong support for the positive risk-return tradeoff both for the market as a whole and for decile portfolios disaggregated by market capitalization.

Table 4.1 The Bivariate GARCH-M with Market cap Portfolios

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
					Model(A)					
λ_{im}	6.663	4.988	3.958	3.892	2.839	3.161	2.527	3.270	1.815	1.616
	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(0.994)	(0.995)	(1.000)	(0.954)	(0.977)
λ_m	2.871	2.465	2.455	2.527	2.042	2.476	1.843	2.457	1.607	1.716
	(1.000)	(0.998)	(0.995)	(0.997)	(0.992)	(0.984)	(0.974)	(0.986)	(0.931)	(0.982)
Prob.	0.000	0.307	**0.971**	**0.979**	**0.977**	**0.969**	**0.957**	0.000	0.371	**0.933**
					Model(B)					
λ_{im}	5.155	4.084	3.459	2.974	2.356	2.609	2.214	2.496	1.931	1.763
	(1.000)	(1.000)	(1.000)	(1.000)	(0.991)	(0.993)	(0.987)	(0.997)	(0.980)	(0.980)
λ_m	2.652	2.332	2.179	1.955	1.705	2.068	1.537	1.796	1.696	1.869
	(0.998)	(1.000)	(0.992)	(0.993)	(0.961)	(0.981)	(0.948)	(0.978)	(0.972)	(0.986)
Prob.	**0.976**	**0.656**	0.001	0.001	0.002	0.008	0.001	0.001	**0.605**	0.000
					Model(C)					
λ_i	-0.607	-0.317	-0.525	-0.143	0.387	-0.733	-1.098	-0.884	-1.611	0.408
	(0.142)	(0.337)	(0.286)	(0.450)	(0.619)	(0.277)	(0.203)	(0.326)	(0.158)	(0.569)
λ_{im}	8.677	5.661	4.861	4.136	2.404	4.091	4.233	3.884	3.799	1.180
	(1.000)	(1.000)	(0.996)	(0.975)	(0.857)	(0.977)	(0.961)	(0.899)	(0.957)	(0.761)
λ_m	3.094	2.493	2.370	2.588	2.128	2.418	2.043	2.155	1.712	1.650
	(1.000)	(0.993)	(0.996)	(0.997)	(0.985)	(0.991)	(0.975)	(0.988)	(0.972)	(0.992)
Prob.	0.000	0.007	0.027	0.020	0.021	0.022	0.043	**0.999**	0.014	0.067
					Model(D)					
λ_i	0.095	0.802	0.056	0.215	0.587	-0.248	-1.101	-1.451	-0.607	-1.806
	(0.558)	(0.853)	(0.531)	(0.628)	(0.721)	(0.383)	(0.237)	(0.240)	(0.387)	(0.177)
λ_{im}	5.006	2.203	3.347	2.669	1.380	2.991	3.756	4.463	2.630	4.130
	(0.995)	(0.877)	(0.965)	(0.946)	(0.777)	(0.953)	(0.926)	(0.931)	(0.837)	(0.986)
λ_m	2.442	1.928	2.115	1.987	1.549	2.127	1.720	2.281	1.712	2.487
	(1.000)	(0.985)	(0.985)	(0.983)	(0.962)	(0.996)	(0.966)	(0.989)	(0.969)	(0.994)
Prob.	0.024	0.031	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.000

This table reports the posterior means of risk aversion coefficients for the market portfolio and 10 decile market capitalization portfolios in four bivariate GARCH-M models for 1927 $\sim$ 2008. Model (A), (B), (C), and (D) are the combination of Eq. (3) & Eq. (5), Eq. (3) & Eq. (6), Eq. (4) & Eq. (5), and Eq. (4) & Eq. (6), respectively. λ_i, λ_{im}, and λ_m denote the coefficients of relative risk aversion of h_i, h_{im}, and h_m, respectively. Numbers in parentheses are a posterior probability of positive λ and Prob. denotes posterior model probability.

Table 4.2 The Bivariate GARCH-M with Book to Market Portfolios

	BM1	BM2	BM3	BM4	BM5	BM6	BM7	BM8	BM9	BM10
					Model(A)					
λ_{im}	0.903	2.436	1.836	1.926	1.809	1.574	2.319	2.437	2.864	3.030
	(0.853)	(0.988)	(0.986)	(0.966)	(0.970)	(0.964)	(0.990)	(0.990)	(1.000)	(1.000)
λ_m	0.957	1.914	1.365	1.949	1.833	1.747	2.063	2.196	2.346	2.016
	(0.895)	(0.958)	(0.957)	(0.976)	(0.982)	(0.982)	(0.988)	(0.987)	(0.999)	(0.989)
Prob.	**0.761**	**0.945**	**0.980**	**0.927**	**0.846**	**0.965**	0.000	**0.527**	**0.765**	0.338
					Model(B)					
λ_{im}	0.367	2.040	1.433	1.226	0.851	1.502	1.022	1.708	2.214	2.517
	(0.656)	(0.988)	(0.956)	(0.875)	(0.792)	(0.953)	(0.876)	(0.994)	(0.999)	(0.999)
λ_m	0.441	1.511	0.899	1.329	1.082	1.758	0.979	1.605	1.853	1.635
	(0.710)	(0.944)	(0.878)	(0.897)	(0.878)	(0.973)	(0.894)	(0.989)	(0.992)	(0.966)
Prob.	0.021	0.025	0.003	0.000	0.106	0.001	**0.973**	0.434	0.210	**0.626**
					Model(C)					
λ_i	-3.495	2.307	0.529	2.292	1.181	0.538	-0.925	-0.837	0.229	-1.397
	(0.003)	(0.900)	(0.657)	(0.955)	(0.799)	(0.698)	(0.222)	(0.154)	(0.606)	(0.048)
λ_{im}	4.977	-0.044	1.106	-1.127	0.356	0.910	3.865	4.229	2.165	7.509
	(1.000)	(0.494)	(0.716)	(0.304)	(0.574)	(0.684)	(0.957)	(0.991)	(0.842)	(1.000)
λ_m	1.665	1.677	1.125	1.694	1.744	1.877	2.175	2.533	2.020	2.650
	(0.985)	(0.933)	(0.904)	(0.957)	(0.966)	(0.989)	(0.984)	(0.999)	(0.982)	(0.997)
Prob.	0.214	0.029	0.017	0.073	0.043	0.034	0.000	0.016	0.017	0.000
					Model(D)					
λ_i	-3.729	2.509	0.053	2.762	1.255	0.300	-0.595	-0.863	0.271	-0.785
	(0.005)	(0.935)	(0.529)	(0.956)	(0.812)	(0.607)	(0.318)	(0.117)	(0.613)	(0.112)
λ_{im}	4.513	-0.621	1.707	-2.114	-0.912	1.104	1.872	3.126	1.723	4.138
	(0.991)	(0.365)	(0.875)	(0.172)	(0.332)	(0.740)	(0.805)	(0.972)	(0.841)	(0.998)
λ_m	0.952	1.330	1.279	1.273	0.737	1.757	1.007	1.674	1.780	1.689
	(0.854)	(0.915)	(0.954)	(0.899)	(0.769)	(0.979)	(0.869)	(0.974)	(0.983)	(0.978)
Prob.	0.004	0.001	0.000	0.000	0.006	0.000	0.027	0.023	0.009	0.035

This table reports the posterior means of risk aversion coefficients for the market portfolio and 10 decile book to market portfolios in four bivariate GARCH-M models for 1927 $\sim$ 2008. Model (A), (B), (C), and (D) are the combination of Eq. (3) & Eq. (5), Eq. (3) & Eq. (6), Eq. (4) & Eq. (5), and Eq. (4) & Eq. (6), respectively. λ_i, λ_{im}, and λ_m denote the coefficients of relative risk aversion of h_i, h_{im}, and h_m, respectively. Numbers in parentheses are a posterior probability of positive λ and Prob. denotes posterior model probability.

Table 4.3 The Bivariate GARCH-M with dividend Portfolios

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
					Model(A)					
λ_{im}	1.259	2.203	2.305	1.661	1.626	2.306	2.189	2.982	2.698	2.004
	(0.865)	(0.973)	(0.988)	(0.979)	(0.952)	(0.999)	(0.981)	(1.000)	(0.999)	(0.996)
λ_m	1.539	1.643	2.483	1.656	1.792	1.435	1.635	2.249	1.769	1.585
	(0.933)	(0.955)	(0.989)	(0.986)	(0.967)	(0.989)	(0.956)	(0.999)	(0.972)	(0.986)
Prob.	**0.823**	**0.959**	**0.964**	0.391	0.462	0.298	**0.689**	0.006	0.001	0.000
					Model(B)					
λ_{im}	1.139	1.832	1.804	1.275	1.236	2.207	1.387	2.259	1.976	0.865
	(0.863)	(0.977)	(0.980)	(0.921)	(0.933)	(0.999)	(0.964)	(0.991)	(0.997)	(0.874)
λ_m	1.242	1.343	1.922	1.303	1.455	1.316	1.170	1.716	1.221	0.690
	(0.898)	(0.939)	(0.991)	(0.937)	(0.955)	(0.961)	(0.953)	(0.981)	(0.944)	(0.823)
Prob.	0.131	0.002	0.000	**0.534**	**0.478**	0.245	0.029	**0.920**	**0.922**	**0.949**
					Model(C)					
λ_i	2.474	-1.467	1.384	2.757	1.674	-1.056	1.911	1.009	0.581	0.363
	(0.856)	(0.202)	(0.812)	(0.935)	(0.834)	(0.270)	(0.836)	(0.752)	(0.645)	(0.649)
λ_{im}	-1.293	3.815	0.737	-1.364	-0.401	3.623	0.267	1.663	1.744	1.456
	(0.334)	(0.962)	(0.634)	(0.255)	(0.459)	(0.965)	(0.528)	(0.776)	(0.771)	(0.791)
λ_m	1.698	1.871	2.211	1.253	1.704	1.743	1.655	2.207	1.566	1.484
	(0.957)	(0.978)	(0.987)	(0.947)	(0.970)	(0.988)	(0.966)	(0.999)	(0.958)	(0.971)
Prob.	0.033	0.039	0.036	0.035	0.021	**0.440**	0.025	0.000	0.000	0.000
					Model(D)					
λ_i	2.261	-1.617	1.229	2.758	1.253	-1.167	0.383	1.638	1.675	1.584
	(0.895)	(0.174)	(0.824)	(0.965)	(0.796)	(0.264)	(0.561)	(0.850)	(0.975)	(0.910)
λ_{im}	-0.949	3.656	0.683	-1.665	-0.317	3.251	0.606	0.214	-0.429	-1.594
	(0.353)	(0.961)	(0.553)	(0.185)	(0.440)	(0.951)	(0.623)	(0.513)	(0.297)	(0.209)
λ_m	1.569	1.454	2.073	1.029	1.323	1.272	0.675	1.704	0.914	0.497
	(0.954)	(0.950)	(0.983)	(0.902)	(0.932)	(0.965)	(0.778)	(0.973)	(0.840)	(0.727)
Prob.	0.012	0.000	0.000	0.041	0.039	0.017	0.257	0.074	0.077	0.051

This table reports the posterior means of risk aversion coefficients for the market portfolio and 10 decile dividend portfolios in four bivariate GARCH-M models for 1928 $\sim$ 2008. Model (A), (B), (C), and (D) are the combination of Eq. (3) & Eq. (5), Eq. (3) & Eq. (6), Eq. (4) & Eq. (5), and Eq. (4) & Eq. (6), respectively. λ_i, λ_{im}, and λ_m denote the coefficients of relative risk aversion of h_i, h_{im}, and h_m, respectively. Numbers in parentheses are a posterior probability of positive λ and Prob. denotes posterior model probability.

Table 4.4 The Bivariate GARCH-M with momentum Portfolios

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
					Model(A)					
λ_{im}	3.770	1.886	2.174	2.612	2.381	2.147	2.485	1.450	1.740	3.096
	(0.999)	(0.991)	(0.996)	(0.998)	(0.998)	(0.993)	(1.000)	(0.947)	(0.975)	(0.998)
λ_m	2.520	1.356	1.969	1.944	2.753	2.005	1.932	1.753	1.890	1.936
	(0.988)	(0.957)	(0.991)	(0.985)	(0.999)	(0.985)	(0.987)	(0.980)	(0.995)	(0.997)
Prob.	0.000	0.000	0.000	0.000	0.000	**0.927**	0.000	**0.953**	0.000	0.000
					Model(B)					
λ_{im}	1.946	1.110	1.332	1.347	1.881	1.635	2.638	1.223	0.662	2.333
	(0.976)	(0.963)	(0.970)	(0.921)	(0.982)	(0.971)	(0.981)	(0.885)	(0.752)	(0.998)
λ_m	1.235	0.784	1.279	1.086	2.020	1.531	2.309	1.384	0.925	1.862
	(0.904)	(0.875)	(0.956)	(0.868)	(0.986)	(0.970)	(0.990)	(0.939)	(0.862)	(0.987)
Prob.	**0.991**	**0.538**	0.402	**0.978**	**0.957**	0.018	0.000	0.028	**0.971**	0.000
					Model(C)					
λ_i	-0.399	-0.274	-1.559	-1.036	0.529	1.594	-0.070	0.486	0.980	-0.039
	(0.299)	(0.342)	(0.021)	(0.221)	(0.659)	(0.830)	(0.486)	(0.608)	(0.746)	(0.455)
λ_{im}	4.874	2.528	5.216	4.433	1.794	0.041	2.791	0.989	0.475	0.978
	(0.986)	(0.931)	(0.998)	(0.964)	(0.806)	(0.508)	(0.870)	(0.685)	(0.597)	(0.704)
λ_m	2.663	1.460	2.434	2.240	2.623	1.762	2.657	1.784	1.595	1.830
	(0.999)	(0.954)	(0.998)	(0.989)	(0.997)	(0.985)	(0.994)	(0.973)	(0.953)	(0.970)
Prob.	0.000	0.000	0.000	0.000	0.000	0.055	**0.992**	0.018	0.000	0.000
					Model(D)					
λ_i	0.207	-1.482	-2.271	0.087	0.324	-1.932	0.361	1.153	1.420	0.367
	(0.627)	(0.029)	(0.007)	(0.521)	(0.615)	(0.198)	(0.592)	(0.768)	(0.863)	(0.688)
λ_{im}	1.512	3.777	5.095	1.238	1.410	3.141	1.661	0.052	-1.041	1.187
	(0.822)	(0.987)	(0.999)	(0.706)	(0.762)	(0.802)	(0.765)	(0.504)	(0.267)	(0.825)
λ_m	1.351	0.769	1.515	1.085	1.999	0.650	2.066	1.253	0.608	2.286
	(0.910)	(0.736)	(0.984)	(0.872)	(0.985)	(0.714)	(0.980)	(0.908)	(0.763)	(0.998)
Prob.	0.009	0.462	**0.598**	0.022	0.043	0.000	0.008	0.001	0.029	**1.000**

This table reports the posterior means of risk aversion coefficients for the market portfolio and 10 decile momentum portfolios in four bivariate GARCH-M models for 1927 $\sim$ 2008. Model (A), (B), (C), and (D) are the combination of Eq. (3) & Eq. (5), Eq. (3) & Eq. (6), Eq. (4) & Eq. (5), and Eq. (4) & Eq. (6), respectively. λ_i, λ_{im}, and λ_m denote the coefficients of relative risk aversion of h_i, h_{im}, and h_m, respectively. Numbers in parentheses are a posterior probability of positive λ and Prob. denotes posterior model probability.

Table 4.5 The Bivariate GARCH-M with industry Portfolios

	Nodur	Durbl	Manuf	Energy	HiTech	Telcm	Shops	Health	Utility	Others
					Model(A)					
λ_{im}	1.209	3.748	1.905	1.056	1.928	1.745	1.441	1.715	1.258	2.104
	(0.918)	(1.000)	(0.983)	(0.842)	(0.987)	(0.973)	(0.951)	(0.910)	(0.894)	(0.990)
λ_m	1.012	2.329	1.671	1.006	1.974	1.405	0.998	1.588	1.460	2.310
	(0.890)	(0.990)	(0.972)	(0.867)	(0.987)	(0.964)	(0.894)	(0.955)	(0.948)	(0.997)
Prob.	**0.967**	**0.863**	**0.967**	**0.946**	**0.804**	**0.973**	**0.944**	**0.792**	**0.963**	0.077
					Model(B)					
λ_{im}	0.797	3.197	2.011	0.541	1.138	1.170	1.243	0.626	0.722	2.141
	(0.797)	(0.997)	(0.984)	(0.676)	(0.955)	(0.934)	(0.943)	(0.712)	(0.769)	(0.983)
λ_m	0.604	1.981	1.713	0.728	1.223	1.088	0.945	0.624	1.240	2.409
	(0.766)	(0.978)	(0.971)	(0.778)	(0.964)	(0.931)	(0.879)	(0.761)	(0.870)	(0.996)
Prob.	0.000	0.110	0.002	0.000	0.002	0.002	0.000	0.159	0.000	**0.889**
					Model(C)					
λ_i	1.218	-0.194	1.983	-1.867	0.676	0.316	1.122	-2.681	0.529	0.112
	(0.753)	(0.443)	(0.862)	(0.127)	(0.737)	(0.609)	(0.716)	(0.068)	(0.623)	(0.533)
λ_{im}	-0.036	3.980	-0.438	3.132	0.482	1.592	0.735	4.827	0.417	1.970
	(0.488)	(0.938)	(0.432)	(0.939)	(0.589)	(0.849)	(0.640)	(0.977)	(0.559)	(0.850)
λ_m	0.816	2.225	1.794	1.183	1.510	1.620	1.463	1.771	1.144	2.260
	(0.844)	(0.989)	(0.971)	(0.926)	(0.954)	(0.975)	(0.957)	(0.966)	(0.918)	(0.997)
Prob.	0.033	0.022	0.031	0.054	0.194	0.025	0.056	0.037	0.037	0.002
					Model(D)					
λ_i	-1.241	1.419	2.986	-2.104	0.614	0.194	1.226	-2.739	0.578	0.451
	(0.157)	(0.885)	(0.862)	(0.112)	(0.715)	(0.550)	(0.717)	(0.054)	(0.620)	(0.643)
λ_{im}	1.696	1.049	-1.196	2.753	0.159	1.117	-0.126	3.616	0.268	1.479
	(0.881)	(0.658)	(0.390)	(0.907)	(0.543)	(0.795)	(0.498)	(0.951)	(0.570)	(0.795)
λ_m	0.657	2.006	1.871	0.789	1.023	1.014	0.801	0.610	1.320	2.369
	(0.787)	(0.996)	(0.904)	(0.813)	(0.829)	(0.907)	(0.843)	(0.769)	(0.954)	(0.990)
Prob.	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.012	0.000	0.033

This table reports the posterior means of risk aversion coefficients for the market portfolio and 10 industry-level portfolios in four bivariate GARCH-M models for 1927 $\sim$ 2008. Model (A), (B), (C), and (D) are the combination of Eq. (3) & Eq. (5), Eq. (3) & Eq. (6), Eq. (4) & Eq. (5), and Eq. (4) & Eq. (6), respectively. λ_i, λ_{im}, and λ_m denote the coefficients of relative risk aversion of h_i, h_{im}, and h_m, respectively. Numbers in parentheses are a posterior probability of positive λ and Prob. denotes posterior model probability.

Table 4.6 The Bivariate GARCH-M with market cap Portfolios for the postwar period

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
λ_{im}	7.681	8.205	7.011	7.619	7.781	5.732	4.895	5.125	3.467	3.050
	(0.991)	(0.997)	(0.997)	(0.999)	(0.999)	(0.995)	(0.996)	(1.000)	(0.953)	(0.927)
λ_m	2.409	3.225	3.896	3.701	4.255	3.468	2.585	3.198	2.525	3.806
	(0.885)	(0.921)	(0.967)	(0.962)	(0.962)	(0.946)	(0.880)	(0.957)	(0.893)	(0.978)

Table 6 presents the posterior means of risk aversion coefficient for 10 decile market capitalization portfolios and the market portfolio in bivariate GARCH-M model of model (A) for 1950 $\sim$ 2008. Numbers in parentheses are a posterior probability of positive λ.

Table 4.7 The results of Bayesian model averaging for Bivariate GARCH-M models

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
λ_{im}	5.152	4.314	3.982	3.895	2.829	3.177	2.600	3.882	1.921	1.587
	(1.000)	(0.996)	(1.000)	(0.999)	(0.997)	(0.993)	(0.994)	(0.899)	(0.969)	(0.963)
λ_m	2.647	2.362	2.452	2.528	2.043	2.471	1.852	2.154	1.664	1.711
	(0.998)	(0.999)	(0.995)	(0.997)	(0.992)	(0.984)	(0.974)	(0.988)	(0.957)	(0.983)
	BM1	BM2	BM3	BM4	BM5	BM6	BM7	BM8	BM9	BM10
λ_{im}	1.776	2.351	1.822	1.703	1.630	1.552	1.045	2.166	2.706	2.748
	(0.881)	(0.973)	(0.981)	(0.917)	(0.931)	(0.954)	(0.874)	(0.991)	(0.996)	(0.999)
λ_m	1.097	1.896	1.360	1.930	1.743	1.752	0.980	1.933	2.233	1.766
	(0.910)	(0.956)	(0.956)	(0.974)	(0.969)	(0.982)	(0.894)	(0.988)	(0.998)	(0.974)
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
λ_{im}	1.132	2.265	2.248	1.215	1.323	2.877	1.711	2.110	1.792	0.739
	(0.841)	(0.973)	(0.975)	(0.890)	(0.913)	(0.983)	(0.877)	(0.955)	(0.943)	(0.840)
λ_m	1.505	1.651	2.473	1.428	1.611	1.539	1.375	1.718	1.198	0.680
	(0.929)	(0.956)	(0.989)	(0.955)	(0.960)	(0.981)	(0.910)	(0.981)	(0.936)	(0.818)
	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
λ_{im}	1.942	2.342	3.581	1.344	1.860	2.021	2.782	1.434	0.613	1.187
	(0.975)	(0.974)	(0.987)	(0.916)	(0.973)	(0.966)	(0.869)	(0.941)	(0.738)	(0.825)
λ_m	1.236	0.777	1.420	1.086	2.019	1.983	2.652	1.743	0.916	2.286
	(0.904)	(0.811)	(0.973)	(0.868)	(0.986)	(0.985)	(0.994)	(0.979)	(0.859)	(0.998)
	Nodur	Durbl	Manuf	Energy	HiTech	Telcm	Shops	Health	Utility	Others
λ_{im}	1.168	3.680	1.833	1.167	1.645	1.740	1.402	1.679	1.227	2.117
	(0.903)	(0.997)	(0.966)	(0.847)	(0.910)	(0.970)	(0.933)	(0.881)	(0.881)	(0.977)
λ_m	1.005	2.287	1.675	1.016	1.882	1.410	1.024	1.430	1.448	2.400
	(0.889)	(0.989)	(0.972)	(0.870)	(0.981)	(0.964)	(0.897)	(0.922)	(0.947)	(0.996)

This table presents the posterior means of risk aversion coefficients by averaging of Bayesian model averaging technique for four bivariate GARCH-M models for individual portfolios. Numbers in parentheses are a posterior probability of positive λ.

Figure 4.1 Time-varying covariance and variance with size portfolio for 1927 $\sim$ 2008

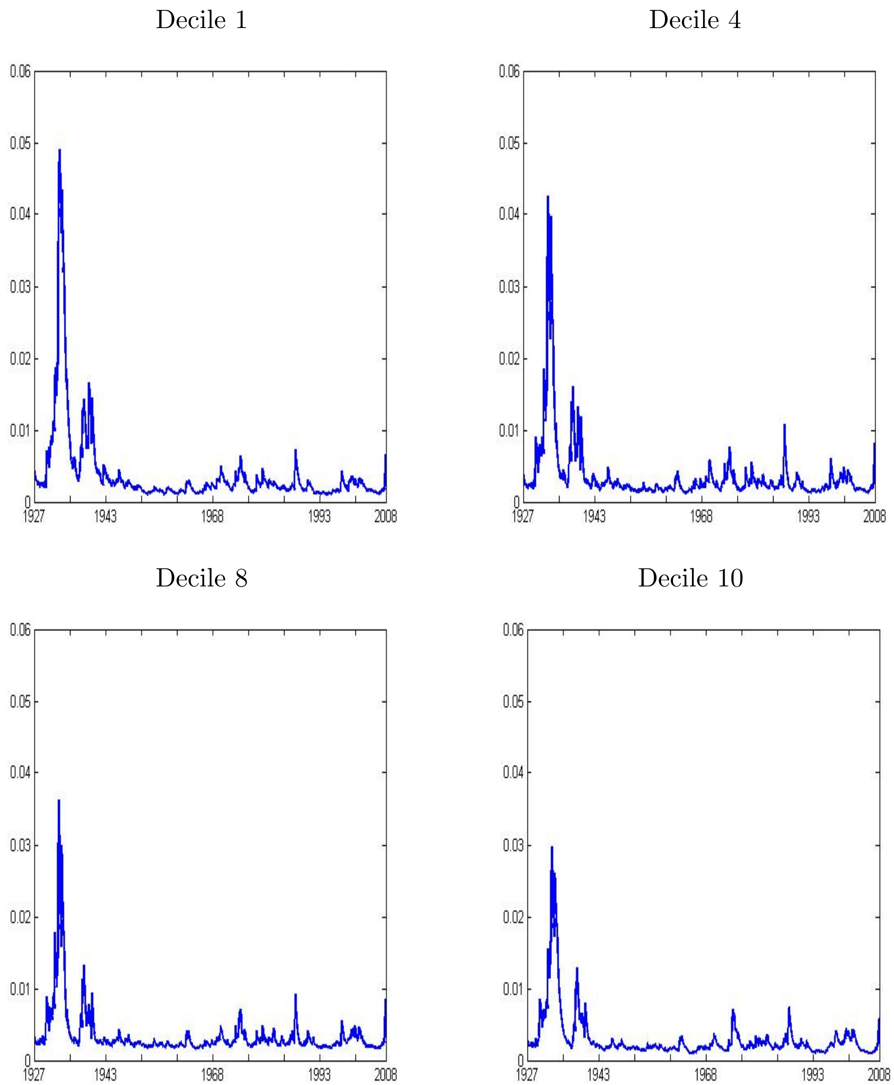
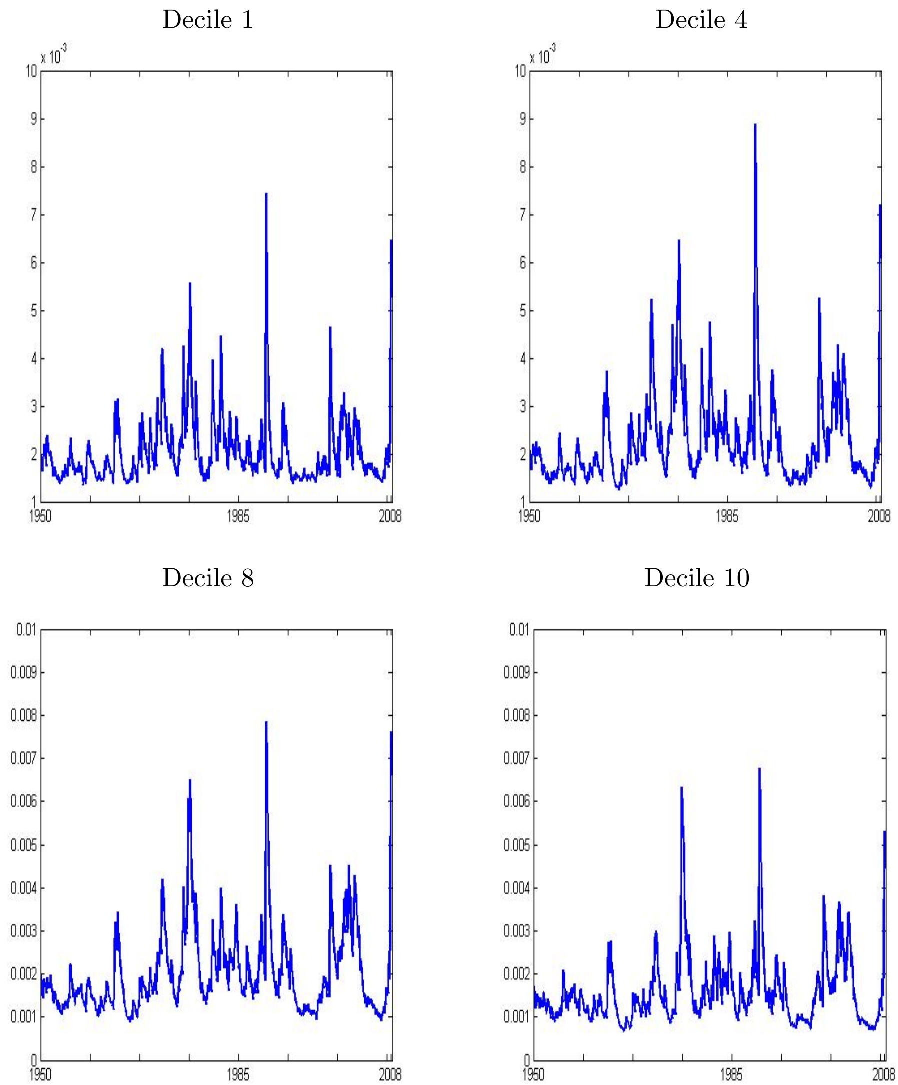


Figure 4.2 Time-varying covariance and variance with size portfolio for 1950 $\sim$ 2008



CHAPTER 5

CONCLUSION

This dissertation is composed of three essays. The first essay shows that the choice of portfolio is a key cause of econometric difficulty in estimating the risk-return tradeoff. Larger market cap stocks degrade the empirical results of the risk-return tradeoff with the inclusion of the largest 10% of stocks by market cap dramatically weakening the statistical relation between risk and return in the U.S. total market portfolio. Higher momentum stocks also play the same role as larger market cap stocks and the risk premium is easily revealed on lower momentum stocks. Our findings show that market aggregation can hide the relation between risk premium and return.

The second essay investigates the risk-return tradeoff in agribusiness stocks (agricultural production and food manufacturing industries). The expected positive relation between stock return and its risk holds for both industries, but the posterior probability of a positive tradeoff is lower for the food manufacturing industry. The sign of the risk-return tradeoff is not sensitive to volatility specification. A positive risk-return tradeoff for the total U.S. market portfolio is estimated in the bivariate GARCH-M framework with very strong posterior support. This implies that the multivariate GARCH-M framework may offer improved empirical results to demonstrate Merton's ICAPM.

In the third essay, we employ bivariate GARCH-M models to investigate the nature of the intertemporal risk-return tradeoff between the market and individual portfolios and find the following general results. First, a positive risk-return tradeoff of the market portfolio is estimated in general. This implies that the empirical results of ICAPM might be improved in multivariate GARCH-M models. Second, the risk aversion coefficient of the time-varying covariance between the market and individual portfolios is also positive in portfolio-level analysis. Using five different categories of individual portfolios, we discover the relation between individual portfolio returns and the time-varying covariance with the market portfolio have a strong posterior probability of being positive in most estimations. Third, we find that the data span is another key factor to successfully estimate a positive risk-return relation. With a shorter data span, the postwar period, the posterior probability of a positive relation between expected return and risk is lower. Finally, the market capitalization appears to be a good proxy of the risk of individual portfolios to risk averse investors.

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