PRESERVICE SECONDARY MATHEMATICS TEACHERS' BELIEFS ABOUT

TEACHING WITH TECHNOLOGY

by

KEITH RIGBY LEATHAM

(Under the Direction of Thomas J. Cooney)

ABSTRACT

This study investigated preservice secondary mathematics teachers' (PSTs) beliefs about teaching mathematics with technology, the experiences in which those beliefs were grounded, and how those beliefs were held. Beliefs were defined as dispositions to act. Coherentism and the metaphor of a belief system provided a conceptual framework through which the PSTs' beliefs were seen as sensible systems. Coherentism was posited as an alternative way of interpreting apparent inconsistencies between teacher's beliefs and their practice. Through the qualitative research methodology called ground theory, four PSTs were purposefully selected and studied. Data stories were written that demonstrated the organization and structure of the PSTs' belief systems. From an analysis of the PSTs' experiences with technology, a theory was posited that focused on the PSTs' ownership of learning mathematics with technology. Experience, knowledge, and confidence were the primary factors that constituted ownership. The primary dimensions of the PSTs' core beliefs with respect to technology, referred to as their beliefs about the nature of technology in the classroom, were the availability of technology, the purposeful use of technology, and the importance of teacher knowledge of technology. The PSTs envisioned technology playing a multitude of roles in their classroom. Motivational roles of technology were nonmathematical in nature and were closely tied to the PSTs' beliefs that effective teachers motivated their students to learn and used a variety of teaching methods. Procedural roles involved using technology to execute calculations or procedures that could also be (and often were) done by hand. Conceptual roles facilitated the visualization and exploration of mathematics. The more PSTs wanted to focus on conceptual understanding and wanted students to take responsibility for that understanding, the more they were concerned about their own ability to facilitate such learning and the need for technology availability. The more PSTs focused on procedural understanding in mathematics and on teacher-centered lessons, the more they were concerned about students misusing technology and failing to learn the procedures.

INDEX WORDS: Technology, Beliefs, Mathematics education, Preservice teacher education

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TABLE OF CONTENTS

CHAPTER 1: INTRODUCTION

The technological revolution has produced a marvelous array of devices: televisions, cellular telephones, VCRs, video games, copy machines, overhead projectors, calculators and computers, to name a few. We have computerized everything from pencil sharpeners and notepads to bicycles and golf courses. Many of these technological advances have found their way into our schools and our mathematics classrooms. Nevertheless, despite increases in mathematics-specific software, access to calculators and computers (U.S. Department of Education, 2000), and research supporting the use of educational technology in the classroom (e.g., Epper & Bates, 2001; Laborde, 2001; Passey, 2000), mathematics teachers still seldom teach with technology (Cuban, 1993; Hannafin, 1999; Hodas, 1993; Jakobsson, 2001; Jones, 1998). Kaput's 1992 prophecy appears to be coming true: "Major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social structures" (p. 515).

Perhaps the old social structures have contributed to confusion with respect to the function of technology in the classroom. "Teaching with technology," like so much of educational phraseology, is not well defined. The mere presence of calculators and computers in the school may be perceived by some as evidence teachers are teaching with technology; yet it is clearly important to consider not just whether technology is available, but whether and how it is used and for what purpose. Some may consider the use of computers to minimize paperwork and calculate grades as teaching with technology; others may view teaching with technology as using overhead projectors or watching videos. Although some of the preceding examples are potentially valuable uses of technology, these views could more appropriately be labeled "teaching near technology." There are often misunderstandings between educational researchers who advocate the use of technology in reform-oriented teaching and preservice and inservice

teachers who may see "teaching with technology" as the latest wave in an endless tide of research-touted recommendations for change. Calls for change, when decontextualized, can seem like calls for change for change's sake.

In 1991, the National Council of Teachers of Mathematics (NCTM) stated in their *Professional Standards for Teaching Mathematics* that mathematics teachers should "help students learn to use calculators, computers, and other technological devices as tools for mathematical discourse" (p. 52). This position was a weak though admirable endorsement for the use of technology in the teaching of mathematics. By contrast, the NCTM's *Principles and Standards for School Mathematics* (2000) devoted one of its six overarching principles wholly to technology. The four principles addressing curriculum, teaching, learning, and assessment have long been pillars of their recommendations for educational reform (e.g., NCTM, 1961, 1980, 1989, 1991, 1995). The other two, addressing equity and technology, are not new to NCTM's vision, but their prominence is. The Technology Principle states: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 11). The statement "technology is essential" is strong language. That technology can enhance learning is commonly accepted, although less commonly translated into practice; the claims that technology might influence the very mathematics that is taught and that it is essential in the classroom is essentially not mainstream thinking in U.S. mathematics classrooms. The following excerpt further illustrates NCTM's (2000) considerably stronger commitment to a reform-oriented approach to teaching with technology:

> Students can learn more mathematics more deeply with appropriate use of technology (Dunham and Dick 1994; Sheets 1993; Boers-van Oosterum 1990; Rojano 1996; Groves 1994).[1] Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster

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[1] Note that the five sources cited in support of these statements were published since the release of the 1989 NCTM *Standards*.

those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics. (p. 25)

If mathematics teaching is going to reflect this vision, preservice teachers (PSTs) need to exit teacher education programs with training and beliefs compatible with the vision. Old social structures, which include the "school mathematics tradition" (Gregg, 1995), have been home to most PSTs for the last 15 years of their lives. Yet, as a result of living in the computer age, PSTs have also grown up with new technological structures. Many come from schools where the presence of television sets, VCRs, graphing calculators, and computers have become as common in the classroom as overhead projectors and scientific calculators (U.S. Department of Education, 2000). As students, PSTs have listened and observed, learning not just educational content but educational beliefs and philosophies. They have absorbed various ideas of what it means to be a good or a not-so-good teacher; they have accumulated more than mere lists of teaching characteristics and methodologies—they have faces, names, and experiences to go with them (see Dwyer, Ringstaff, & Sandholtz, 1990; Pajares, 1992; Vacc & Bright, 1999).

A Personal Rationale

As a graduate student, I too listened and observed in the capacities of student, teacher, and novice researcher. As a consequence, I became convinced I, and the rest of the mathematics education community, could benefit greatly from an increased understanding of PSTs' beliefs with respect to technology. The more mathematics educators know about what PSTs might believe about teaching mathematics with technology and the ideologies with which those beliefs are associated, the better able they will be to provide the kinds of educational experiences likely to help those views expand and mature. In order to create teacher education programs informed by PSTs' beliefs, however, we first need to know what PSTs believe and how they believe it. Several intersecting experiences led me to draw this conclusion and, as such, to undertake this study.

One of my first experiences upon arriving at the University of Georgia was to assist John Olive, both as a teaching assistant in his Technology and Secondary School Mathematics course and as a research assistant on a research project[2]. The purpose of the research project was to explore how the Math Forum (TMF) Web site could be used with PSTs. Throughout the 1998 academic year we experimented with various activities involving the use of the site. Through classroom observations, questionnaires, and interviews, we developed a picture of how the PSTs used the Internet, TMF, and technology in general. We discovered that, despite what we believed to have been a focus on pedagogical implications of the technology, during student teaching the PSTs were much more likely to use technology, particularly the Internet, on their own than with their students. The primary use of the Internet was to browse for possible lesson ideas. The PSTs were looking for ideas, sometimes on TMF but more often anywhere common search engines took them. Thus, the PSTs saw the Internet more as a pedagogical resource than as a pedagogical tool. As so often happens, the experience of conducting this research raised new questions for me. I began to wonder about the origins of these PSTs' beliefs about technology and about the influence their teacher education experiences had had on the formation of those beliefs.

In the fall of 1999 I was given the opportunity to teach the Technology and Secondary School Mathematics course. As I taught this course, I became increasingly aware of my desire to know more about the background and beliefs of my students. The course was designed to give PSTs experiences exploring mathematics with technology, primarily with computers. In addition, I tried to design activities to allow the students to explore some of the pedagogical implications of what they were learning. As I reflected on the experience, however, I realized I did not actively or explicitly explore the role my students believed technology should play in their current or future classrooms. I did not

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[2] The Internet Support for Pre-Service Mathematics Education Project was a subcontract of the Math Forum project at Swarthmore funded by the National Science Foundation.

have a good sense of how they would interpret classroom activities because I did not know how they interpreted past experiences with technology. Thus, although I was interested in how my teaching was influencing the PSTs' beliefs, I was also interested in something more fundamental: the beliefs they currently held.

As my interest with respect to PSTs' beliefs about teaching with technology grew, I began to look for related research literature. I looked for research on beliefs, on technology, and on beliefs about technology. I found a great deal of the research in the first two categories to be unsatisfying. Much of the research I found on beliefs seemed surface-level, the result of minimal interaction with participants and of isolated use of Likert-scale surveys—surveys that failed to capture many nuances of individual meaning making. Much of the literature on technology was not research literature, as it addressed technology possibilities more so than findings. There was a great deal about what could be done (i.e., technology can revolutionize teaching) and about what was not being done (i.e., teachers do not use it) but little about how teachers actually thought about the use of technology in their teaching. The intersection of these sparse domains was, as might be expected, sparser still. I was convinced, however, that this area of research was important, particularly because there seemed to be ever-increasing emphasis on the use of technology in the teaching of mathematics.

As mentioned earlier, this ever-increasing emphasis is illustrated by the NCTM's *Principles and Standards* (2000) and its claim of the essentiality of technology in the mathematics classroom. Emphasis on the use of technology to explore mathematics was also part of the vision of the Department of Mathematics Education at the University of Georgia (UGA). Thus, technology played an integral role in many of the mathematics education courses PSTs took as part of their undergraduate program at UGA. In addition, most of them took a technology-rich geometry course from the Mathematics Department. Consequently, the PSTs had, at the very least, heard of this vision; they may or may not have had this vision themselves, however, a phenomenon that has not escaped attention in the literature on reform (see Frykholm, 1999).

The Research Questions

With this background, what beliefs about technology and its place in the mathematics classroom do PSTs take with them as they enter the profession? Is the ship ready to sail or is it sinking before it leaves the harbor? Are there holes whose patches will come off in the calmest of waters or the first mild storm? Teacher education should not be about patching holes in the hulls of PSTs' beliefs. Teacher education should be about presenting students with ample and appropriate experiences that enable their educational ideologies to grow and expand. The hulls should be thought of not so much as porous as incomplete. So, although we need to know what PSTs believe with respect to the use of technology in teaching mathematics, we also need to know how they hold these beliefs and with what other beliefs they might be connected—or clustered (Green, 1971). To address these needs, I conducted a study designed to explore PSTs' beliefs about teaching mathematics with technology. Two principal research questions guided my inquiry:

1. What are PSTs' beliefs about teaching mathematics with technology, in what experiences are those beliefs grounded, and how are those beliefs held?

2. What relationships exist between PSTs' beliefs about teaching mathematics with technology and their beliefs about mathematics, teaching, and learning?

There are three primary components to the first question—what PSTs believe, their influential experience, and how PSTs believe what they believe. In asking what PSTs' beliefs are about teaching with technology, I wanted to emphasize what they considered technology to be and what they considered teaching with technology to be. I wanted to design a study that would allow me to get at the individual meaning making of PSTs—a study that was grounded in their experiences and language. There were two primary reasons I felt compelled to explore the PSTs' experiences. First, knowing something about their experiences could lend insight into what may have been influential in forming certain beliefs. Second, any recounting of those experiences would necessarily

be influenced by their current system of beliefs and intervening experiences. Therefore, PSTs' discussions of past experiences would lend insight into what they currently believed, as "we cannot avoid framing and understanding our recollections in terms of the concepts we have at present" (Von Glasersfeld, 1995).

There is yet a third aspect to this first research question: how PSTs believe what they believe. Although we humans have beliefs about many things, certain beliefs influence our words and actions more than others. We have beliefs about what is, what might be, and what should be. Some beliefs are open for debate, and others are set in stone. Beliefs may have strong emotional, intellectual, cultural, or spiritual ties. In consideration of these various dimensions, I thought it important to study not just what PSTs believe but how they believe it.

The second research question brings beliefs other than about technology into the picture. Not surprisingly, research on teacher beliefs often discusses relationships between pedagogical and content-specific beliefs (Kagan, 1992; Thompson, 1992). Several researchers have posited such relationships with respect to mathematics (e.g., Copes, 1982; Ernest, 1988; Thompson, 1984). Primarily these beliefs were categorized as beliefs about mathematics, teaching, and learning. I wondered whether PSTs' beliefs about teaching mathematics with technology were related in significant ways to their beliefs about teaching, about learning, and about the nature of mathematics. Knowing that beliefs about mathematics, teaching, and learning had been studied a great deal, I wanted to see how those beliefs might interact with beliefs about teaching with technology. This desire was the impetus for my second research question.

<div align="center">Addressing the Questions</div>

This dissertation reports the genesis, conduct, and results of the study I conducted to answer these research questions. As I began to consider how I might go about answering them, I was drawn toward qualitative research methodologies and, in particular, the grounded theory research tradition (Creswell, 1998), through which a theory of some state of affairs is developed as it emerges from and is grounded in the

words and experiences of those involved in that state of affairs. Through the underlying philosophy of grounded theory, Green's (1971) metaphor of a belief system, and the philosophical notion of coherence, I began to develop a framework for exploring PSTs' beliefs.

CHAPTER 2: REVIEW OF THE LITERATURE ON BELIEFS

This chapter is divided into three parts. The first part discusses the ways I have come to conceptualize beliefs and their structure. I refer to these ways as my conceptual framework, the lens through which I was looking as I organized and conducted this study. The next part discusses theoretical literature that has influenced my thinking about research on preservice teachers' (PSTs') beliefs. I wanted to ground a theory of PSTs' beliefs about teaching with technology in the words and experiences of PSTs. Thus, this theoretical literature plays a different role than the conceptual framework. These theories gave me food for thought, but my desire never was to place these theories onto what I found or to pigeonhole my participants therein. Rather, this theoretical literature influenced the way I was thinking about PSTs' beliefs as I began the study. The last part of the chapter reviews the empirical literature on teachers' beliefs so as to orient the study within the context of similar studies as well as to highlight its rationale and purpose.

A Conceptual Framework for Studying Beliefs

Numerous studies in the 1970s and 1980s focused in one way or another on describing, exploring, and explaining teachers' beliefs and possible relationships between those beliefs and the practice of teaching. In 1992, Kagan, Pajares, and Thompson each published a synthesis of research on beliefs. These three syntheses, although from slightly different vantage points, each tried to accomplish a similar goal: to portray various research agendas and the resultant research on teachers' beliefs. Kagan (1992) and Pajares (1992) discussed educational research on beliefs across disciplines; Thompson (1992) discussed primarily research in mathematics education. Pajares (1992) focused primarily on the underlying definitions of belief and belief systems necessary for quality research on teacher beliefs; Kagan (1992) focused on the variety of methodological underpinnings and implications of such research. Thompson's (1992) synthesis spanned both of these while focusing on mathematics education. All three essentially concluded

that research on teacher beliefs, although fraught with pitfalls to avoid and difficulties to surmount, had great potential to inform educational research and practice and was therefore worth the effort.

These reviews were a call for more and better research on teacher beliefs and, in particular, on PSTs' beliefs, with Pajares (1992) postulating that "unexplored entering beliefs may be responsible for the perpetuation of antiquated and ineffectual teaching practices" (p. 328). I was happy to see calls for the kind of research I wanted to conduct and cognizant of the need to address not just the "more" but the "better" portion of this request. Pajares (1992) alludes to at least two reasons research on PSTs' beliefs is particularly difficult and has often been avoided: minimal time and minimal action. First, it is difficult to do longitudinal research with PSTs, in part because they are not official PSTs for very long. Students usually do not declare their intention to become a teacher until at least midway through their undergraduate program. In states where fifth year programs are in place, many students do not become PSTs until after they have received their Bachelor's degree and are one year away from entering the ranks of teaching.

Second, PSTs are by definition not yet serving as teachers. Thus, research on their beliefs must be done in the absence of observing them as full-time teachers. Most of the actions one can observe of a PST are not teaching actions but learning actions. Granted, there are some opportunities (e.g., early field experiences and student teaching) to learn through doing. But these are brief and in many ways "unreal," as the students are not their students and the classroom is not their classroom. Often the PST does not plan the lessons or arrange the desks or make up the test. And quite often, the PST does not assign a grade. It is possible some researchers have chosen not to study PSTs' beliefs because of these difficulties. Too, when researchers choose to study PSTs' beliefs without adequately addressing these difficulties, they risk producing superficial and incomplete accounts of PSTs' beliefs.

The title of Pajares' 1992 article is telling: "Teachers' Beliefs and Educational Research: Cleaning Up a Messy Construct." One of the reasons research on beliefs is so

difficult to compare and apply is that *belief* is seldom well defined. Terms such as *belief*, *attitude*, *view*, *notion*, *conception*, and *knowledge* are often used interchangeably, with little or no attempt to clarify what is meant. Over the next few pages, and in an attempt to let the reader know my thoughts when I conceived of and carried out this study, I review the literature that has most influenced the way I have come to think about *belief*.

Pajares (1992) suggests the defining of belief to be of utmost importance: "It will not be possible for researchers to come to grips with teachers' beliefs … without first deciding what they wish *belief* to mean and how this meaning will differ from that of similar constructs" (p. 308). I have taken this admonition more as a plea for internal than external consistency when defining *belief*. Different philosophical and epistemological orientations should naturally lead to different definitions of fundamental concepts. For a researcher's attempt to "come to grips with teachers' beliefs" to become more credible, they need to make explicit these fundamental definitions. What follows is a description of what I "wish *belief* to mean" as well as a framework for conceptualizing the structure and organization of beliefs. This conceptual framework is grounded primarily in the writings of Rokeach (1968) and Green (1971).

<div align="center">*Definition of* Belief</div>

The word *conception* has been used by some (e.g., Lloyd & Wilson, 1998; Thompson, 1992) as a general category containing constructs such as beliefs, knowledge, understanding, preferences, meanings, and views. Educational researchers generally agree with this broad category; it is when we get down to distinguishing the members of this set that there is considerable variation (Pehkonen & Furinghetti, 2001). In particular, the relationship between belief and knowledge has been viewed in extremely different ways, although this disagreement may be more semantic that substantive (Pajares, 1992). Some choose to view knowledge as a subset of beliefs; others view beliefs as a subset of knowledge. The desire to distinguish these two constructs and yet maintain a strong relationship between them stems primarily from a desire to make this definition consistent with our everyday usage of these terms. We speak of these constructs

similarly, yet differently. If there is something we claim to know for certain, such as that there are 50 states in the United States of America, it would seem odd to make the statement, "I believe there are 50 states." Somehow, in this instance, knowing is stronger than believing (Rokeach, 1968). But that does not mean beliefs need be seen as a subset of knowledge. I have found it more useful to consider those conceptions to which we assign some truth value as beliefs, and then to refer to as knowledge a certain subset of those beliefs. How do we define that subset? Knowledge is a belief we take as fact. We may learn some conceptions as knowledge, or fact, from the beginning. Other conceptions may start out as belief and become knowledge over time. When we say we know something, we no longer state we "merely" believe it. Despite the inclusion of one within the other, it is most common in our everyday language to speak of beliefs and knowledge as separate constructs, and I will continue to do so. Although knowledge is a subset of beliefs, we tend to refer to the compliment of knowledge, rather than to the set within which it resides, as beliefs. When I use the term *belief* in this study, I am referring to the subset of beliefs we do not refer to as knowledge.

In defining *belief*, I wish to pay particular attention to the notion that what one believes influences what one does. I adopt Rokeach's (1968) description: "All beliefs are predispositions to action" (p. 113). This description does not imply, however, the person holding a belief must be able to articulate the belief, nor even be consciously aware of it. It thus makes sense to discuss uncovering, discovering, and exploring one's own beliefs. In addition, a belief "speaks to an individual's judgment of the truth or falsity of a proposition" (Pajares, 1992, p. 316), but the belief may exist independently of the proposition. Finally, in this study I assumed that beliefs are the fundamental elements that make up our personal philosophies and ideologies.

Belief Structure

Having described my use of the term *belief*, I now consider possible relationships between beliefs—what is often referred to as a belief system. A primary goal of research on beliefs is to describe how the beliefs we hold influence (and are influenced by) our

actions. My desire was to develop a framework that would facilitate my exploration of how PSTs' belief systems were structured. This framework reflects my attempt to articulate the way I conceptualize (metaphorically) individuals' belief systems. The works of Rokeach (1968) and Green (1971) and the notion of coherence theory were particularly influential in shaping and articulating these ideas.

I have chosen to view an individual's beliefs system as a sensible system. That is, I assume individuals develop beliefs into organized systems that make sense to them. This view is closely related with what is referred to as coherentism:

> Our knowledge is not like a house that sits on a foundation of bricks that have to be solid, but more like a raft that floats on the sea with all the pieces of the raft fitting together and supporting each other. A belief is justified not because it is indubitable or is derived from some other indubitable beliefs, but because it coheres with other beliefs that jointly support each other…. To justify a belief … we do not have to build up from an indubitable foundation; rather we merely have to adjust our whole set of beliefs … until we reach a coherent state. (Thagard, 2000, p. 5)

In coherentism, beliefs become viable for an individual when they make sense with respect to their other beliefs. This viability via sense making implies an organization or system of beliefs—what I refer to as a sensible system. To discuss what this sensible system might look like, I turn to the works of Rokeach (1968) and Green (1971).

Green (1971) suggested three dimensions one can consider as a metaphor for visualizing a belief system. One dimension, referred to as "psychological strength" (p. 47), describes the relative importance a person might ascribe to a given belief. Both Rokeach (1968) and Green (1971) describe this dimension as varying from central to peripheral. Assuming "the more central a belief, the more it will resist change" (Rokeach, 1968, p. 3), Rokeach introduces the idea of connectedness as a means of exploring the central or peripheral nature of a belief. Beliefs can vary with respect to the degree to which they are existential, shared, derived, or matters of taste. Existential beliefs are those we associate with our identity—with who we are and how we fit into our world. They have a high degree of connectedness and are thus more strongly held—more

central. We also tend to hold more centrally those beliefs we think we share with others. If, however, a belief is derived from an association with a group, then it may be less connected and thus more peripheral in nature. Finally, "many beliefs represent more or less arbitrary matters of taste" (p. 5). These beliefs, as implied by the use of the word *arbitrary*, are less connected and thus more peripheral in nature. I find it helpful to visualize the placing of beliefs along this dimension (and each of the other dimensions as well) as a sense-making activity. Beliefs naturally go where they make the most sense to us—where they fit in.

A second dimension considers the quasi-logical relationships that may exist between an individual's beliefs (Green, 1971, p. 44). Consider the following statements:

A: Students need to learn their times tables.
B: Students should not use calculators.

Some teachers maintain there is a logical relationship between these statements. That is, for some, A implies B: IF you want students to learn their times tables THEN they should not be allowed to use calculators. And if a person believes that A implies B, and they believe that A is true, then B is seen as true because it is the logical conclusion from knowing that A is true. Green (1971) refers to the relationship as quasi-logical. Whether B does in fact follow from A is not at issue. In this person's belief system, A implies B; that is how they hold these beliefs. In this case, belief B is referred to as *derivative*,[3] and belief A is referred to as *primary*. This quasi-logical relationship need not correlate directly with the central-peripheral dimension. That is, the same person described in the preceding example may hold belief B considerably stronger than belief A, even though belief A is a primary belief. Belief B may be much more important to the person than belief A. One of the reasons we may posit such a quasi-logical relationship is a desire to make two beliefs more coherent when considered in tandem.

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[3] Note the difference between the previous notion of a belief being derived from association with a group and this notion of a belief being seen as derivative of another belief.

A third dimension of beliefs is the extent to which beliefs are clustered in isolation from other beliefs (Green, 1971, p. 47). Beliefs seen as contradictory to an external observer are not likely to be seen as contradictory to the one holding those beliefs. In one sense, this dimension allows for the contextualization of beliefs; a person may believe one thing in one instance and the opposite in another. There are often exceptions to rules. One need not, however, be consciously aware of these beliefs. Consequently, seemingly contradictory beliefs may exist in different belief clusters with no explicit exception or delineation of context. Although not all beliefs are based on evidence (for instance, matters of taste), even those based on evidence are based on what is *seen* as evidence by the one holding the belief. In this same light, the same evidence may be used to bolster different beliefs, beliefs clustered in isolation. Thus, defining a belief to be a "conviction of the truth of some statement or the reality of some being or phenomenon especially when based on examination of evidence" (*Merriam-Webster online dictionary,* 2000) is more specific than I have chosen to be in my definition of belief. Whether a belief is "based on examination of evidence" is a question of how a belief is held; it is a question of structure.

The assumption that belief systems are sensible systems does not allow contradictions. Whenever beliefs that might be seen as contradictory come together, the person holding those beliefs finds a way to resolve the conflict within the system—to make the system sensible. Now, as observers, we may not find the resolution sensible. It may not seem logical, rational, justifiable, or credible. But our incredulity does not diminish another's coherence. It is often difficult as researchers, however, to look beyond the beliefs we assume must have been (or should have been) the predisposition for a given action. I have developed a conceptual framework minimizing the extent to which I make these assumptions. In essence, when beliefs are viewed as sensible systems, observations of seeming contradictions are, in the language of constructivism, perturbations, and thus an opportunity to learn.

Educational Ideologies

The intent of the present study was to infer PSTs' beliefs with respect to educational issues in general and technology issues in particular. In addition, I used the conceptual framework just outlined to infer a structure of those beliefs. My desire was to formulate a picture of what their individual belief systems looked like given the metaphor of a sensible system. There exist delineations of belief systems, however, intended to describe not individual, but group systems of beliefs. These generalizations, often referred to as ideologies, are formed as researchers and philosophers synthesize possibilities and research findings with respect to education. Although my participants might or might not have associated themselves with a common educational ideology, I thought knowing some of these ideologies might shed light on what the PSTs believed and how they believed it. In a sense, knowledge of these various general ideologies gave me a running start—a dimension and a language I could consider in exploring my participants' individual belief systems.

Meighan and Siraj-Blatchford (1997) defined ideologies as "competing belief systems" (p. 180) and discussed various dichotomous approaches that have been used to describe educational ideologies. Examples of these approaches include teacher-centered versus child-centered, traditional versus progressive, and meaning receiving versus meaning making. Meighan and Siraj-Blatchford argued that the value in these dichotomous approaches is limited, primarily because, although the first halves of the examples above (teacher-centered, traditional, and meaning receiving) are often seen as correlated, the second halves of the dichotomies (child-centered, progressive, and meaning making) are not always seen as conceptually correlated. It thus becomes difficult to constructively use several of the dichotomies in tandem. Meighan and Siraj-Blatchford suggested "the notion of a network of ideologies" (p. 189) as a more constructive way to compare the various dimensions of one's belief systems. They discussed eleven theories that might make up someone's ideology of education:

1. A theory of discipline and order

2. A theory of knowledge, its content and structure
3. A theory of learning and the learner's role
4. A theory of teaching and the teacher's role
5. A theory of resources appropriate for learning
6. A theory of organization of learning situations
7. A theory of assessment that learning has taken place
8. A theory of aims, objectives and outcomes
9. A theory of parents and the parent's role
10. A theory of locations appropriate for learning
11. A theory of power and its distribution. (p. 191)

The purpose of the present study was to explore PSTs' beliefs about teaching mathematics with technology and the relationships these beliefs might have with PSTs' other educational beliefs. Although it might have been nice to address each theory explicitly in my data collection and analysis, I was not ready to do that. Although I had the various theories in mind, I focused on certain ones as I designed my data collection procedures. In particular, Theories 5 and 6 seemed relevant to the use of technology, although many other theories might prove closely related. Given the nature of my study and my research questions, I focused on the relationships that might exist between Theories 5 and 6 and between Theories 2, 3, and 4, where I took Theory 2 to be, more specifically, a theory of the nature of mathematics. I next describe some dimensions specific to mathematics education that researchers have ascribed to these various theories. These dimensions helped guide the formulation of my research questions as well as the collection and analysis of my data.

Theories of the Nature of Mathematics

Studying teachers' beliefs naturally leads one to study beliefs about the content they teach. I too wanted to explore PSTs' beliefs about the nature of mathematics. But using the sensible system framework, I did not want to assume the PSTs' beliefs about mathematics were necessarily related in strong ways to their beliefs about teaching, learning, or technology. Thus, the question was not (or at least I did not believe it should be) *whether* a PSTs' beliefs about mathematics were related to their beliefs about teaching mathematics or learning mathematics (with or without technology). Instead, my

focus was on *how* these beliefs were related. I recognized from the outset the relationships might be tenuous. This observation brings up a question for the mathematics education community. Once the presumption that certain beliefs about mathematics necessarily correlate with certain beliefs about teaching or learning (or teaching with technology) is removed, the question then becomes, "Which belief correlations are more or less valuable? Which beliefs 'should' be correlated?" These questions, particularly because they are value laden, are certainly philosophical; I do not know if they are researchable.

That said, there are various ways philosophers of mathematics have talked about theories of knowing mathematics, and familiarity with those theories influenced the way I thought about and inferred the PSTs' beliefs about the nature of mathematics. For instance, Ernest (1988) wrote about three different conceptions of, or beliefs about, mathematics. According to Ernest, one can have an instrumentalist, Platonist, or problem-solving view of mathematics. For the instrumentalist, mathematics is a set of tools. Knowing mathematics is knowing what tools you have and how to use them. Mathematics teachers have this knowledge and impart it to others. The Platonist view focuses more on the whole toolbox, asking how the various tools work together and what makes them work. The key, however, is that all the tools fit in the toolbox. With a problem-solving view, mathematics is seen as an ever-growing, ever-changing field. Mathematics, being a human endeavor, is not discovered; rather, it is created.

Mura (1993, 1995) surveyed mathematicians and mathematics educators, asking them to define mathematics. The 14 themes emerging from that data provide a much wider range of beliefs about the nature of mathematics than Ernest's (1988) three philosophies. In particular, definitions that would be classified as either instrumentalist or Platonist are almost nonexistent. Nevertheless, several of the themes can be aligned with Ernest's problem-solving view:

1. The creation and study of formal axiomatic systems, of abstract structures and objects, of their properties and relationships

    4. Design and analysis of models abstracted from reality; their application. A
      means of understanding phenomena and making predictions
    6. Problem-solving
12. Culturally determined content (ethnomathematics). (Mura, 1995, pp. 389-390)

In a sense the alignment of these themes serves to provide a broader view of the problem-solving philosophy of mathematics. Other themes describe dimensions not so easily encapsulated by Ernest's (1988) three philosophies:

    7. The study of patterns
    8. Inductive thinking, exploration, observation, generalization
    9. An art, a creative activity, a product of the imagination; harmony and beauty.
    (p. 390)

It is the diversity of beliefs amongst mathematicians and mathematics educators I find most intriguing about Mura's (1993, 1995) studies. In discussing some of the possible implications of her findings, Mura (1993) pointed out that, given the diversity amongst her participants, we should neither be surprised nor judgmental of the diversity existing among schoolteachers (and I add PSTs). Once aware of this diversity, "one can hardly consider a particular belief inadequate among school teachers, when a similar belief is also present among university teachers, unless it is known to lead to less effective teaching" (p. 384). She further stated the following:

> We may count on most of our colleagues' support when we criticize the instrumentalist or Platonist conceptions of mathematics, but this is no longer true when it comes to formalism and constructivism. And if there is no consensus among either mathematicians or our own professional community, how can we maintain that one belief is more desirable than another? (1995, p. 396)

My intent was to explore possible connections between PSTs' beliefs about mathematics and their beliefs about teaching mathematics with technology. I took Mura's (1995) statements as warnings against judging the value of my participants' beliefs based on relationships I might or might not find.

Theories of Teaching and Learning Mathematics

In addition to setting forth three philosophies of mathematics, Ernest (1988) discussed how those philosophies might be related to philosophies about the teaching and

learning of mathematics. I have summarized these posited relationships in Table 1. These theories were valuable to me not as a means of categorizing my participants but as a means of looking for possible threads of coherence.

Table 1

Paul Ernest's (1988) three philosophies of mathematics

	Instrumentalist	Platonist	Problem solving
Nature of mathematics	An accumulation of facts, rules, and skills	A static but unified body of certain knowledge	A dynamic continually expanding field of human creation and invention
Teacher's role	Instructor	Explainer	Facilitator
Intended outcome	Skills mastery with correct performance	Conceptual understanding with unified knowledge	Confident problem posing and solving
Use of curricular materials	Strict adherence to a text or scheme	Modification of the textbook approach, enriched with additional problems and activities	Teacher, student, or school construction of the mathematics curriculum

Copes (1979, 1982) adapted Perry's (1968) Development Scheme to mathematics, using it as "a metaphor for learning and teaching mathematics" (Copes, 1982, p. 38). As I worked with PSTs, the age group with which Perry (1968) worked, this metaphor helped me to think about the ways the PSTs might be thinking (and expressing those thoughts) about teaching and learning mathematics. Copes (1982) described how students at the four main stages of Perry's (1968) scheme—dualism (absolutism), multiplism, relativism, and dynamism (commitment)—might approach learning mathematics:

> *[Dualism (absolutism)]* Every question has an answer, … there is a solution to every problem, and … the role of an authority is to know and deliver those answers and deliver those truths.

> *[Multiplism]* Everybody has a right to his [or her] own axiom system, and they're all equally good since mathematics is only, after all, a collection of meaningless strings of marks.

[Relativism] Not all opinions are equally good. First, there are standards such as validity, internal consistency, and consistency with observed data. Second, validity depends upon context.

[Dynamism (commitment)] My knowledge of mathematical functions is different from yours, and that's OK, although I happen to be enthusiastic about my viewpoint enough to share it with you. (pp. 38-39)

Copes (1982) further discussed how a teacher's awareness of these various stages at which students might be in their thinking about mathematical learning could inform their theory of teaching mathematics. As such, a given teacher may simultaneously subscribe to quite a range of approaches to teaching mathematics. Perhaps more importantly, this approach emphasizes the importance of aligning, in some way, the teacher's theory with the perceived learner's theory. Beliefs about teaching and learning may include a variety of levels because of the interconnectedness with beliefs about learning. Mathematics teaching is, after all, "an intentional activity" (Pearson, 1989)—an activity from which the teacher intends students to learn mathematics. And, as Pearson states, truly understanding this activity "requires one to understand the beliefs and intentions of the actor" (p. 66). My research focused on the beliefs and intentions of PSTs. Their beliefs, however, are shared as students; they have formed the majority of their educational beliefs as the taught rather than as the teacher.

Theories of Technology

Table 1 also connects theories about use of curricular materials with theories of mathematics and teaching. By viewing technology as curricular material, one can get a picture of possible theories of teaching with technology connected with these other philosophies. An instrumentalist-oriented teacher might focus primarily on the computational facilities of technology. If the textbook being used for the class did not explicitly call for the use of technology, use of technology would likely be extremely rare. The Platonist might supplement given curricular materials by creating step-by-step worksheets to accompany and dictate computer activities or focus. It is likely these

activities would be designed with very specific mathematical objectives in line with a given curriculum. In conjunction with a problem-solving theory of mathematics, one might expect a teacher to use technology for open-ended exploration, with broad curricular objectives that encompass a diverse range of mathematical topics. Ernest (1988) did not specifically talk about technology, but I found it valuable to use his theories as a springboard for considering possibilities as to the beliefs a PST might have about teaching mathematics with technology and possible connections to their beliefs about mathematics, teaching, and learning.

The previously mentioned work on the Internet Support for Pre-Service Mathematics Education Project (see Olive & Leatham, 2000) also helped me think about several factors specifically related to PSTs' use of technological resources. The research team followed a cohort of PSTs through their undergraduate mathematics education program including their student teaching. In addition to the technology integrated in several of their courses, special efforts were taken to introduce the PSTs to Internet resources such as those offered at The Math Forum (http://www.mathforum.com). As stated earlier, we found the PSTs viewed the Internet (and technology in general) more as a resource for their own preparation and exploration than as a tool for teaching mathematics and they were much more likely to use it themselves than to incorporate it into their classroom lessons. Related to this notion, Jones (1998) found that the beliefs and skills involved in learning with technology and teaching with technology were quite different. Olive and Leatham (2000) concluded that "providing rich opportunities for pre-service teachers to use technological tools for learning and doing mathematics in their college courses is not sufficient preparation for their use of these same tools with their students when teaching" (p. 8). As mentioned in the introductory chapter, my decision to conduct the research described herein stemmed, in part, from my experience working on this Project. I desired a deeper understanding of PSTs' beliefs about teaching mathematics with technology at the time they leave campus and start student teaching.

The theories I have described to this point in the chapter are of two primary types. The first set, related to the definition and structure of beliefs, provided a conceptual framework for the design and implementation of data collection strategies and the analyses of the data. The second set represent some of the educational ideologies related to mathematics, its teaching and learning that provided a theoretical background for my thinking about these beliefs. My intent was not to situate my participants within these ideologies; rather it was to allow a theory of PSTs' beliefs, grounded in their words and experiences, to emerge. But I recognized that I neither could (nor in my opinion should) embark on such research assuming I would be neutral in the research process. Not only do I have my own educational ideologies, not all of which I myself am explicitly aware, but I also know of the work others have done in their own attempts to understand teacher beliefs. My objective in conducting this grounded theory, then, was not to minimize the understandings with which I started the study, but rather to try to maximize the extent to which the understandings I came out with in the end, although influenced my subjectivity, were grounded in the PSTs' words and experiences and in my attempt to infer sensible systems of beliefs.

<div align="center">Empirical Literature on Teacher Beliefs</div>

This last part of the chapter reviews empirical literature on teachers' beliefs. One of the difficulties in comparing research findings on teacher beliefs stems from the varied and often minimally described conceptual frameworks used. Thus, as I discuss empirical literature on teachers' beliefs, I will compare their frameworks to the conceptual framework I have chosen—one that focuses on beliefs structure and the inherent coherency of that structure. In essence, I chose studies whose research questions were similar in nature to those of this study. I hope to situate my study in this body of literature through a discussion about the theoretical approaches used to explore these questions and the findings that were then reported.

Based on participant population, there are two primary dimensions that distinguish between studies on teacher beliefs—preservice versus inservice, and

elementary versus secondary. The variation in experience, position, and responsibility of these various subsets certainly warrant research focusing on each of the four intersections of these dimensions. In addition, some studies have explored teachers' general educational beliefs and others have focused on particular content areas like mathematics. One can further distinguish between studies by considering their theoretical and methodological approaches. In orienting my study in a larger body of literature, it was not always reasonable or desirable to stay in the smallest subset of these various dimensions containing my own research questions. I have divided this part of the chapter into two pieces: Research on beliefs about mathematics, teaching, and learning, and research on beliefs about teaching with technology. Additionally, each of these pieces is subdivided along the preservice/inservice teacher dimension.

Beliefs About Mathematics, Teaching, and Learning

Inservice Teachers' Beliefs

Raymond's (1997) study on "relationships between a beginning elementary school teacher's beliefs and mathematics teaching practices" (p. 550) used a framework allowing her to claim a teacher's beliefs can be inconsistent with their practice. She cited studies that have found inconsistencies between beliefs and practice, and studies that have found consistency. As my framework does not allow for this type of inconsistency, I explored some of the research Raymond cited as having found in-consistencies. I first discuss one of these cited studies and then discuss Raymond's study.

Raymond (1997) referred to the case of Fred (Cooney, 1985) as an example of a study that found inconsistencies between beliefs and practice. Based on coherentism, I believe that there is another valid and valuable way to interpret the findings of this study. Perhaps Cooney found the meanings Fred attached to such concepts as "problem solving" and "the essence of mathematics" were different than the meanings Cooney had originally supposed. Although there is little question as to the struggle Fred had as a beginning teacher, it does not appear to be a struggle of belief. In fact, with respect to belief, the biggest struggle in this case study seemed to be similar to what others have

found—the difficulty, despite an incredible amount of quality research, to get into Fred's mind and characterize the structure of his beliefs. There is some evidence in the case of Fred to suggest that Fred's core belief about mathematics was that mathematics is interesting in its own right. I am not sure what Fred thought "problem solving" meant, but I think it was a catchword he came to associate with what he enjoyed about doing mathematics. In this sense, motivating students to engage in mathematics was getting them to "problem solve"—just not in the exact same sense the researcher thought of problem solving. Thus Fred seems to have constructed a meaning for "problem solving" that differed from the intended meanings he had been taught and these two meanings differed in important ways. With this interpretation, Fred's core beliefs are indeed manifested by his actions. Thus, the inconstancy is not between Fred's beliefs and his practice. The inconsistency is between Fred's practice and the beliefs Cooney thought would most likely influence that practice. Perhaps teacher educators assume that one's beliefs about mathematics must be the core belief that influences one's practice.

Raymond (1997), in the case of Joanna, stated the following with regard to the relationships between Joanna's beliefs and practice:

> Joanna's model shows factors, such as time, constraints, scarcity of resources, concerns over standardized testing, and students' behavior, as potential causes of inconsistency. These represent competing influences on practice that are likely to interrupt the relationship between beliefs and practice. (p. 567)

It seems, from this wording, the author is asserting Joanna's beliefs about mathematics are the only beliefs that could possibly influence her actions in the classroom. The factors of time, resources, standardized testing, and students' behavior are simply described as influences; there is no mention of Joanna's beliefs with respect to these factors. Certainly Joanna has beliefs about how she should use the amount of time she is given, beliefs about the importance of the test, and beliefs about what must be done in order to keep students' behavior in check. That these beliefs were more strongly held than her beliefs about learning mathematics through group work was interpreted as an inconsistency. It is more useful to me to talk about Joanna's beliefs about learning mathematics as beliefs

about some strategies for learning mathematics that were better than others, given the necessary circumstances. It is a large jump from "students can learn this way" to "students should learn this way" to "students must learn this way."

The inconsistencies I saw in Raymond's (1997) study were primarily inconsistencies in the way the author referred to some influences as beliefs and other influences as peripheral factors. One need not interpret the case of Joanna as a case of beliefs being inconsistent with practice. When one defines beliefs as predispositions to act, and then views a system of beliefs as a sensible system, certain beliefs have more influence over certain actions in certain situations. If Joanna chose to keep her students working quietly in their desks rather than working in groups, she did so because she had a belief about classroom management that far outweighed her belief about group work. She was predisposed to deal with issues of behavior management OVER issues of group work. In this view, there is nothing inconsistent about it.

This reinterpretation of the cases of Fred and Joanna is not meant to call into question the value of the research. I only mean to point out the to take into account the conceptual framework for beliefs when interpreting the findings of research on beliefs. Raymond's (1997) model only defined mathematics beliefs. These were defined as "personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics" (p. 551). Note the relationships between certain beliefs and actions implied by this definition. In addition, Raymond's model placed Joanna in the position of being able to explicitly state her beliefs as well as the relationships between her beliefs and her teaching practice. In this sense, Raymond believed a person can not only verbally articulate their own beliefs about such complex issues as the nature of mathematics, but a person can also verbally articulate the relationships existing between their various beliefs and their teaching practices. The assumption someone can simultaneously articulate their own beliefs AND be inconsistent in their actions with respect to those beliefs is not an assumption I am willing to make. I assume, rather, when Joanna was asked to articulate

her beliefs, Joanna simply took her best shot at it. I am convinced not only is it insufficient to ask someone what their beliefs are, it may be impeding. As Kagan (1992) said, "A direct question such as 'What is your philosophy of teaching?' is usually an ineffective or counterproductive way to elicit beliefs" (p. 66). Participants may try so hard to figure out what they are supposed to believe that their responses get in the way of sufficiently revealing what they do believe.

Skott (2001) attempted to solve the problem of viewing beliefs and practice as inconsistent by limiting the type of beliefs he studied. He did this by focusing his research on the beliefs he described as "teachers' explicit priorities" (p. 6)—beliefs of which teachers are explicitly aware and that they can articulate. His purpose was then to study the relationships that might exist between these priorities and what takes place in the classroom. Skott focused on finding what made these explicit priorities and practices consistent rather than inconsistent. This approach is illustrated through the case of a novice teacher referred to as Christopher.

Christopher's explicit priorities with respect to teaching mathematics were that mathematics should be about experimenting and investigating, so teaching mathematics should be about inspiring students to learn independently. Much of Christopher's teaching (action) that Skott (2001) observed seemed consistent with these priorities. Christopher was seldom the center of attention at the front of the classroom and his students spent a significant amount of time working on open-ended problems in small groups. There were actions, however, that initially appeared to be inconsistent with Christopher's priorities. In particular, as Christopher moved about from group to group, Skott observed he would often use what Skott described as mathematics-depleting questioning. This kind of questioning would often replace rather than facilitate students' mathematical explorations. Rather than viewing this apparent inconsistency as something needing to be fixed, Skott tried to make sense of it. His analysis revealed there were other related yet competing priorities Christopher was attempting to manage. In particular, Christopher's priorities with respect to student learning focused on his ability to interact

with as many students as possible and on each student feeling successful and good about themselves. In light of these other priorities, Skott stated that the teaching he observed

> should not be seen as a situation that established new and contradictory priorities, but rather as one in which the energizing element of Christopher's activity was not mathematical learning. He was, so to speak, playing another game than that of teaching mathematics. (p. 24)

It turns out, as has been previously postulated, the apparent inconsistency with respect to the case of Christopher was in the researcher initially assuming Christopher's beliefs about mathematics would have the strongest influence on his pedagogical decisions. The more centrally held belief for Christopher was his belief in the importance of individuals and their need to feel successful. The importance of this belief meant mathematical beliefs sometimes took a back seat. The way Skott (2001) describes the consistency between beliefs and practice has important implications for teacher education and for future research on teachers' beliefs. The intent in this study was to search for consistency in the participants' accounts—to view their beliefs as sensible systems—systems that help them to make sense of and operate in the world around them.

PSTs' Beliefs

Inconsistency in beliefs has also played a role in frameworks used to explore preservice teachers' beliefs. In a report of their study on elementary teachers' changing beliefs, Vacc and Bright (1999) stated that "preservice teachers may acknowledge the tenets of CGI [*Cognitively Guided Instruction*] and yet be unable to use them in their teaching" (p. 89). This acknowledgment was taken as belief and this "inability" as inconsistency between beliefs and practice. The sensible system model provides a different view of this apparent inconsistency. Through the model, I interpret this "inability" as a consequence of some beliefs being more strongly, centrally held than others.

The presumption that inconsistency exists when certain beliefs do not appear to by the primary impetus for action is further illustrated in Vacc and Bright's (1999) discussion of the case of Andrea:

> When Andrea began student teaching, she appeared to believe that children are able to find their own solutions to problems and that their sharing of solution strategies provides helpful information for planning instruction. Although these beliefs seemed fairly stable throughout Andrea's student-teaching experience, as evidenced by her belief scores and journal entries, such beliefs were not evident in her instruction. Indeed, the relationship between her beliefs and instruction seemed to become more divergent while she gained teaching experience. (p. 105)

I wonder to which of Andrea's beliefs the authors were referring when they stated that "the relationship between her beliefs and instruction seemed to become more divergent." Vacc and Bright are speaking of the beliefs that they had assumed were most closely related to Andrea's pedagogical decisions. What diverged was the relationship between the researchers' model of the beliefs that most influenced Andrea's practice and the beliefs that were, in actuality, most influential. Beliefs and practice are always consistent—it just is not always apparent which beliefs are "winning out" in given situations.

Research emanating from the project RADIATE (Research and Development Initiatives Applied to Teacher Education) currently provides the most substantial inquiry into teacher beliefs that matches both my population—preservice secondary mathematics teachers—and my theoretical orientation—a focus on not just *what* but *how* beliefs are held(Cooney, Shealy, & Arvold, 1998; Cooney, Wilson, Albright, & Chauvot, 1998). I will review some of the findings of the RADIATE research and then discuss similarities and differences between this research and my study.

In project RADIATE, a cohort of PSTs was followed from the time they were admitted to the mathematics education program through their student teaching experience and graduation. In addition, several of the participants were followed through their first year of teaching (Chauvot, 2000; Shealy, 1994). Their beliefs concerning mathematics, its teaching and learning were explored with the intent of conceptualizing how those beliefs were structured. Although not the only theoretical resource, Green's (1971) metaphor of a belief system was used. The researchers found that a majority of the PSTs' "core beliefs centered around caring for students" (Cooney, Wilson et al., 1998, p. 14). In

addition, their strongest beliefs about teaching were most often about teaching in general rather than mathematics teaching in particular. With respect to mathematics, Cooney, Wilson, et al. (1998) found that the PSTs "saw mathematics as a body of knowledge that builds on previous knowledge" (p. 14). Their beliefs specific to mathematics learning and teaching seemed to be derived from this belief about mathematics.

Case studies of several PSTs provided evidence to support various conclusions as to how PSTs individually held their beliefs (Cooney, Wilson et al., 1998). For example, one PST—Kyle—held core beliefs that the teacher's role was "to make mathematics/learning interesting" and that "mathematical knowledge builds from a strong foundation" (p. 15). The authors then posited another of Kyle's beliefs—a belief in the importance of using problem solving in mathematics teaching, provided the basics had previously been covered—derived from these core beliefs. Although technology was not mentioned explicitly, Kyle's profile was similar to a belief seen in other teachers with regard to technology—the common belief that technology should be used, but only after the concepts have been mastered.

Shealy (1994) followed two of the PSTs from the RADIATE project through their first year of teaching. His findings indicated the PSTs' belief structures had considerable influence on their continued development as first-year teachers. The more evidentially held the beliefs and the less isolated the belief structures, the more amenable a teacher was to adapt and change their beliefs—to be, in a sense, teachable. Because of the seeming influence belief structure had on change in beliefs, Shealy (1994) called for further research on the structure of preservice teachers beliefs. This call stemmed from viewing mathematics teacher education as "a process of enabling the teacher to strengthen the evidential base of his or her belief structures and create and redefine connections between beliefs" (p. 176).

As outlined in the research agenda, the RADIATE research was more intent on conceptualizing the developmental nature of the PSTs' beliefs than on conceptualizing how those beliefs were held. Learning how beliefs change and how one might influence

that change is, in the end, the goal of educational research on beliefs (Cooney, 1999; Green, 1971; Rokeach, 1968; Thompson, 1992). The present study, however, was not designed to study changes in PSTs' beliefs. Rather, it was designed to infer just what their beliefs might be and how they are held as they prepare to graduate and begin their teaching careers. As such, I view this research as preliminary to studying the development or change in these beliefs. I wish first to look in depth at *how* PSTs' beliefs are held, particularly those related to teaching mathematics with technology.

Weaknesses in the Research

Although other literature on teacher beliefs does exist, I take issue with some of their theoretical and methodological approaches. But, as is often the case in mathematics, nonexamples are as important as examples. I feel that I would be doing a disservice if I did not in some way mention the studies that made me think, "No, there has got to be a better way than that," or "That does not seem credible. How could one credibly answer that question?" For instance, I am not convinced that several researchers (e.g., Frank, 1990; Lasley, 1980; Zollman & Mason, 1992) credibly inferred their participants' beliefs. The use of Likert scales surveys cannot delve into *how* beliefs are held, and because of the difficulty inherent in accounting for individual meaning making, I doubt their value when used exclusively to determine *what* beliefs are held. Additionally, it is rare for the researchers in such studies to define what they mean by a belief.

Also, the most common approach to research on inservice teacher beliefs is to look at the relationship between beliefs and practice. Because of poorly defined constructs, and often implied or non-existent conceptual frameworks, there is often the claim that beliefs and practice are "inconsistent." One partial remedy has been to provide caveats such as "professed" or "explicit" (Skott, 2001; Tirosh & Graeber, 1989) when referring to self-reported beliefs, but these delineations are rare. The framework used in my study is one where beliefs are assumed to be both internally consistent and consistent with practice. Perceived inconsistencies come from trying to interpret what someone else is doing and *why* they are doing it. The "professed" notion is taken care of by viewing

many beliefs as subconsciously held. Thus, I believe what is often reported as belief is a misrepresentation. It is often surface level and "professed"—spur of the moment opinions, or rhetoric. Beliefs are not easy to get at. Thus, although several other studies on PSTs' beliefs have been conducted, the quality of the research is disappointing. For instance, Lasley (1980) wrote an article claiming to be about PSTs' beliefs when, in fact, the data were taken from a study done with first year teachers. It seems highly unlikely the turbulent and highly influential nature of a teacher's first year of teaching could have been factored out when trying to infer what those teachers might have believed when they were still PSTs. It seems sufficiently challenging to infer what someone currently believes; it seems nearly impossible to infer what someone *used* to believe, particularly given they may not have been consciously aware of some of those beliefs. The PSTs in my study were asked to reflect on their previous experiences using technology with mathematics. Their responses, however, even when referring to what they "used to think," were taken as inference for their current beliefs.

Beliefs About Teaching Mathematics With Technology

One of the main objectives of the study described in this dissertation was to delineate the various beliefs PSTs have concerning using technology to teach mathematics. Through the use of grounded theory methodology, a theory evolved out of the experiences of the PSTs involved in the study. I did not begin this inquiry into PSTs' beliefs, however, in the absence of preconceptions with respect to the beliefs PSTs might have about teaching mathematics with technology. Several studies influenced and guided me as I developed my own framework for viewing these beliefs and then went on to collect and analyze data. Although general research has been done in the area of technology integration in the classroom (e.g., Cadiero-Kaplan, 1999; Flake, 1990; Hannafin, 1999) as well as teachers' beliefs about educational technology (e.g., Lowther & Sullivan, 1994; McKenzie, 1994), research on beliefs about technology in the mathematics classroom is sparse. Research on preservice mathematics teachers' beliefs about teaching mathematics with technology is rarer still. This last portion of this review

will discuss this research first on inservice teachers and then on PSTs' beliefs about teaching with technology.

Inservice Teachers' Beliefs

Brill (1997) located three positions along "the continuum of beliefs about technology use in the mathematics classroom" (p.20): exploratory, premastery, and postmastery. Teachers with exploratory beliefs believe technology can and should be used to introduce and explore mathematical concepts and procedures. Teachers with postmastery beliefs about technology believe that technology should be used only after mathematical concepts and procedures have been learned by hand. Teachers with premastery beliefs have begun to use technology before their students have attained full mastery of the mathematical content, but that technology use is either rare or unproductive. Use of technology before content mastery has, however, begun to find a place in their teaching. Teachers who have exploratory tendencies while still holding, for example, instrumentalist views of mathematics (Ernest, 1988) might be classified as having premastery beliefs. Although Brill's (1997) research was done with inservice elementary teachers rather than preservice secondary teachers, it was the best example I have seen of delineating beliefs about teaching mathematics with technology. It seemed likely to me, however, there are more dimensions to PSTs' beliefs about teaching with technology than that of mastery. I saw potential in applying and elaborating on this model as it might apply to PSTs' beliefs about the use of technology in the teaching of mathematics.

The Brill (1997) continuum primarily addressed teachers' beliefs about *when* to use technology in their teaching of mathematics, somewhat assuming that *when* implies *how*. As I set out to design data collection strategies, I sought situations that would allow me to explore PSTs' beliefs about how, not just when, technology would be used in their teaching. In addition, I believe a teachers' decision of when to use technology is greatly affected by the teachers' knowledge about technology and its use in the teaching of mathematics, as well as their beliefs about their own knowledge—their confidence in

their knowledge. A framework that might be modified to study and help delineate PSTs' beliefs about technology, particularly their confidence to teach mathematics with technology, comes from the Apple Classrooms of Tomorrow (ACOT) research project.

In their work resulting from the ACOT project, which placed computer technology in K-12 classrooms across the curriculum and then helped teachers incorporate it into their classroom instruction, Sandholtz, Ringstaff, and Dwyer (1997) discussed five phases in what they termed a "model of instructional evolution" (p. 34) with respect to teaching with technology. These five phases were entry, adoption, adaptation, appropriation, and invention. Their model outlined the phases teachers went through over several years' involvement in the project. In the entry phase, teachers learned the rudiments of using the technology themselves. During this phase they often expressed concerns about having sufficient time to learn how to use the technology so as to be able to use it with their students. Once teachers became more comfortable with the technology, they moved into the adoption phase, wherein they started to focus on how they could use the technology as part of their instruction. They sought out programs and strategies to reinforce and support the teaching methods they were already comfortable with in their classrooms. This use of technology led teachers into the adaptation phase. In this phase teachers began to recognize the benefits being provided by using technology with their students. Increased productivity was a common theme—teachers became convinced their students were benefiting from classroom use of technology.

The adaptation phase led to the appropriation phase, described by Sandholtz et al. (1997) as "less a phase in instructional evolution and more a milestone" (p. 42). They went on to explain, "Appropriation is the point at which an individual comes to understand technology and use it effortlessly as a tool to accomplish real work" (p. 42). According to the model, many teachers, because of lack of knowledge or lack of access to technology, never reach this phase. Because of the intervention of ACOT, however, access to and knowledge of technology was not a hindrance to the teachers in that project. Those who reached the appropriation phase, like most of the teachers in the ACOT study,

were then primed to move into the final phase: invention. In this phase the teachers started to seek out new ways to use technology, often ways involving teaching methods that differed from their usual teaching approach. Many of these approaches involved the use of interdisciplinary and cooperative group projects. The teachers began to attempt more elaborate uses of technology, often despite setbacks or lack of collegial support. As I began the present study, it seemed likely I could compare the phases teachers might go through when implementing technology in their classrooms with the beliefs PSTs might have about the use of technology in the teaching of mathematics before they are actually in an implementation position.

Doerr and Zangor (1999, 2000) studied the interactions of one secondary mathematics teachers' beliefs about, knowledge of, and use of graphing calculators. There are two aspects of their research of particular interest to me. The first was their theoretical framework, "in which the meaning of a tool for teaching and learning (such as the graphing calculator) is constituted by particular cultural practices within which it is used for some purpose" (1999, p. 269). Doerr and Zangor's focus on individual meaning making (and, in their case, on the collective norms that emerged there from) was in line with the focus of my study. The second aspect was their findings. They found the teachers' beliefs about the use of technology in her classroom "were reflected in a particular set of pedagogical strategies" (1999, p. 269). For example, the teacher's confidence in her knowledge of the use of the graphing calculator led to flexible, open-ended use of the tool. At the same time, her beliefs about the limitations of the tool led to a classroom norm wherein students were expected to justify their solution strategies.

PSTs' Beliefs

A study by Turner and Chauvot (1995), conducted in conjunction with project RADIATE, focused primarily on PSTs' beliefs about technology. They followed two preservice secondary mathematics teachers through the four quarters of their undergraduate training program and found that both PSTs held the belief that successful exploration of a mathematical topic using technology required previously acquired knowledge of the

mathematical topic. The PSTs believed they would teach their students how to work mathematics by hand before they turned to technology. Both PSTs would be classified as postmastery on the Brill (1997) continuum. Turner and Chauvot (1995) concluded knowledge of these PSTs' belief structures could inform researchers and teacher educators as to ways to challenge PSTs' belief structures. As mentioned previously, it seemed likely that there are more dimensions to PSTs' beliefs about teaching with technology than simply whether technology should be used before, during, or after mastery of mathematical concepts or procedures. The present study was designed to explore whether these other dimensions existed and what they were.

The fact that I inferred PSTs' beliefs about the use of technology in the teaching of mathematics, many of which might not have been consciously held, cannot be overemphasized. Not all qualitative research on beliefs has taken the inference approach. Take, for example, the recent article entitled "Technology: Preservice Teachers' Beliefs and the Influence of a Mathematics Methods Course" (Quinn, 1998). Although I was initially excited by the prospect of reading an article whose title so closely matched my research interests, I became disenchanted when I read about Quinn's methodology. The only reported method for exploring the students' beliefs was to ask them what their beliefs were. As implied by the title of the article, the study was designed to look at how PSTs' beliefs changed through the process of taking a methods course. The initial data were collected as follows:

> At the beginning of the semester, each participant wrote for eight minutes in response to the question "What are your beliefs concerning the use of technological aids (calculators, computers, etc.) in the teaching of mathematics? (Quinn, 1998, p. 375)

Two of the four questions used for the final interview were as follows:

> What are your current beliefs concerning the use of technological aids in the teaching of mathematics?

> How have your beliefs concerning the use of technological aids in the teaching of mathematics changed since the beginning of the semester? (p. 375)

These questions sound much more like research questions than interview questions.

The present research was conducted under the premise that beliefs must be inferred. You cannot merely ask someone what their beliefs are (or whether they have changed) and expect them to know or know how to articulate the answers. Similarly, *how* one believes something is not simply a matter of whether one's statement of *what* one believes ends in an exclamation point! One infers the structure of beliefs about a given topic through analysis of what a person says, intends, and does (Pajares, 1992) with respect to that topic; the context is crucial. It is not simply a matter of asking someone what they believe. The person must be observed in situations where they (and the researcher) can uncover and infer the person's beliefs.

On a related note, questioning one's own beliefs has more to do with exploring what and how one believes than it does with questioning the validity, rationality, or evidentiality of a belief about which one is already aware. It is a question of discovery long before it is a question of change. In teacher education, we want our students to explore what they believe in a context where they are likely to discover and question those beliefs we think are critical to successfully teaching mathematics. Becoming aware of these beliefs in the right environment is what can lead to beliefs being acquired, solidified, altered, or changed—being moved to a more appropriate or desirable place within a belief system.

Summary

The conceptual framework described at the beginning of this chapter had its origins in my own experiences and in my desire to build on the literature on teachers' beliefs. My review of the literature further convinced me of the value of continued research in this area. Some researchers were asking questions similar to mine, but the methodological and conceptual frameworks seemed insufficient to provide credible answers. Other studies, although not specific to PSTs' beliefs about teaching with technology, provided methodological and theoretical foundations. In the conceptual framework I have chosen for this research endeavor, I define *belief* as a predisposition to

action. Beliefs are described using several different dimensions including the degree to which they are consciously or subconsciously held. For visualizing belief structure, beliefs are also seen as varying with respect to psychological strength, having pseudo-logical relationships, and existing in contextual clusters. Through the theory of coherentism, beliefs are viewed as sensible systems; any attempt to infer individuals' beliefs and the structure of those beliefs necessitates the search for this sensibility. This search requires a close look at the experiences of the individual and attention to the meanings ascribed by those individuals. Examples of how this framework specifically influenced the design and implementation of this study are discussed in the next chapter.

CHAPTER 3: METHODOLOGY

> Beliefs cannot be directly observed or measured but must be inferred from what people say, intend, and do—fundamental prerequisites that educational researchers have seldom followed. (Pajares, 1992, p. 207)

In order to infer a person's beliefs with any degree of believability, one needs numerous and varied resources from which to draw those inferences. In the present study I wanted to infer what PSTs' beliefs were and how they were held, and I wanted a way to uncover the PSTs' individually constructed and highly contextual meanings. As I learned about interpretivist methodologies, I identified various methods and strategies that would allow me to make these inferences. As Howe (1998) explained, "Interpretivists hold that human beings are self-creating…. It is not as if human beings are simply pushed to and fro by existing social arrangements and cultural norms. Instead, they actively shape and reshape these constraints on behavior" (p. 16). As I mentioned in chapter 1, PSTs have been a part of the "social arrangements" and "cultural norms," or, as Kaput (1992) put it, "old social structures" of our education system for many years. They actively construct their beliefs based on their experiences within the system. For this and several other reasons explained below, I was drawn to the grounded theory research tradition. Grounded theory appeared to have the flexibility and power to address my research questions.

Grounded Theory

Grounded theory methodology is the intentional search for and development of theory. Strauss and Corbin (1998) note that "theory consists of *plausible* relationships proposed among *concepts* and *sets of concepts*" (p. 168). Specifying these *concepts* as *beliefs* provides a nice restatement of my research objective for the present study. I wanted to posit plausible relationships among PSTs' beliefs about teaching mathematics with technology, and grounded theory provided the framework for designing a study in order to meet this objective. The purpose of grounded theory is to develop a theory grounded in the experience of the participants. Because I perceived a general lack of

information regarding PSTs' beliefs about teaching with technology, I desired to develop a theory, grounded in the PSTs' experiences, of what those beliefs might be.

Theory has also been defined as that which "makes sense of a series of observations, statements, events, values, perceptions, and correlations" (Schensul, Schensul, & LeCompte, 1999, p. 10). In a sense, all research, by this definition, seeks to develop theory. Of course, "to be human is to be a theory builder" (Wolcott, 1988, p. 25). But this definition of theory illustrates another reason why grounded theory fit my purposes: It provided a methodological structure that complemented my conceptual framework. In order to view PSTs' beliefs as sensible systems, I needed a sensible way to view them. Participants' individual meanings are central to grounded theory, as is their natural environment. As such, context is critical. In addition, with respect to research on teachers, "sense-making is the heart of the matter, the medium of teaching and learning that is also the message" (Erickson, 1986, p. 127).

There is one aspect of grounded theory methodology that can be particularly misleading in the context of studying beliefs. In grounded theory, through constant comparison, data collection ceases only when the emergent categories have become "saturated" (Strauss & Corbin, 1998)—when further data collection reconfirms but no longer adds to or takes away from the generated theory. In the exploration of the content and structure of individuals' beliefs, one can never truly saturate the categories, in part because individuals' beliefs are not static. The theories I present in chapter 8 are not meant to be exhaustive. I have no doubt there were valuable beliefs and connections between beliefs that I was not able to infer in this study. With respect to beliefs, I do not think it is possible for the participants to have said everything relevant. Beliefs are too complex a phenomenon for that. In addition, there are myriad belief clusters that *might* be connected to those of interest in this study. The possibilities seem endless, leaving some to posit that current research on beliefs is plagued by a "fragmented perspective" (Sztajn, 2001, p. 3). What did emerge from this study were beliefs that seemed likely to influence the PSTs' technology use in their classrooms.

Although some qualitative researchers (e.g., Creswell, 1998; Jacob, 1987, 1988) have advocated strict adherence to specific traditions, others have argued that "the idea that traditions represent distinctive and comprehensive approaches is factually inaccurate" (Atkinson, Delamont, & Hammersley, 1988, p. 233). My research relied heavily on the grounded theory research tradition, but did not stay neatly within it. The theory I developed concerned individual belief structures and, as such, required a deep understanding of the individual. The collective case study tradition provided further insights into focusing on the individual and using a wide range of data collection strategies in order to explore PSTs' "bounded systems" (Creswell, 1998, p. 61) of beliefs.

Participant Selection

As discussed in chapter 1, teaching the Technology and Secondary School Mathematics course greatly influenced my desire to conduct this study. Given my experience and relationship with the PSTs in that class, and the fact that they were at the end of their program at the time I started my study, these former students of mine seemed the perfect candidates for participants. It is most common to define a PST as someone who has declared themselves in some way as preparing to become a teacher. Thus, a PST could likely be classified as such for several years. The intent of this study was to describe the content and structure of PSTs' beliefs at the cusp of their preservice experience. I wanted a picture, in essence, of the PSTs' beliefs as they finished their program. "Time is a factor in the determination of meanings and perceptions" (S. J. Ball, 1984, p. 80), and these PSTs were involved in a methods course closely connected in time and context with their upcoming student teaching experience. Through project PRIME[4] the PSTs completed a two-week early field experience (FE) with the same

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[4] PRIME (Partnerships in Reform in Mathematics Education) was a teacher enhancement project at the University of Georgia, supported by national Eisenhower funding. Details of this project can be found elsewhere (Wilson, Anderson, Leatham, Lovin, & Sanchez, 1999).

cooperating teacher with whom they would later do their student teaching. This experience provided the PSTs with specific classrooms and students to which they could refer in their methods course as well as in my research.

The PSTs had just begun their methods course at the time I began to select the participants for this study. Based on my experience as their teacher, I paid particular attention to a group of 10 students I had targeted as possible participants. I had targeted them because they were former students of mine who were reflective and had a working knowledge of and an appreciation for common mathematics education software like Geometer's Sketchpad (GSP; Jackiw, 1996) and spreadsheets. They were also students who had demonstrated an ability to communicate well and with whom I believed I had a positive relationship. My intention in purposefully selecting my participants was primarily to avoid what I had found to be a pattern in the research on beliefs about technology. Too often this research reports the beliefs that seem to influence why teachers choose *not* to teach with technology. I wanted to formulate a theory of PSTs' beliefs about teaching with, not without, technology. Consequently, I wanted to ensure, as much as possible, the participants I selected did not fall into the category of saying, "I do not plan to use technology in my teaching."

Having isolated 10 candidates for participation in the study, I began observing the methods class these PSTs were attending. I observed how the PSTs were interacting with the teacher and their classmates. The curriculum and methods used in this course were intended to create an atmosphere wherein the PSTs would view teaching as problematic. I looked for evidence of PSTs who had recognized this vision—PSTs who seemed to see teaching as problematic and who were primed to reflect on it. I wanted to choose these PSTs for two related reasons. First, I hoped these PSTs would *want* to participate in my study, as they would view participation as a means of furthering their opportunities to reflect on their beliefs about teaching. Second, it seemed that PSTs in this mindset would be able to provide the richest data in the limited available timeframe.

Of the 10 PSTs, 6 were initially chosen in hopes of maintaining, despite unforeseen attrition, a minimum of 4 participants. I asked the 6 PSTs if they would participate in a study about their beliefs about mathematics, teaching, and learning. I informed them that over the course of the rest of the school year, I would be sending them email surveys and that we would then schedule an interview in which we would further discuss the topic of and their responses to the email. I purposely did not tell them I was focusing on the relationship between their beliefs about mathematics, teaching, and learning and their beliefs about the use of technology. I wanted to avoid, if possible, the PSTs feeling "I love technology" was the right answer. One male PST, recognizing he "had too much on his plate," withdrew before data collection began. The remaining 2 male and 3 female PSTs participated for the duration of the study. Although data from all 5 were analyzed, only 4 (Ben, Jeremy, Katie, and Lucy) were chosen for discussion in this dissertation. These 4 sufficiently represented the theory that was developed and, as such, Kara's story seemed to be covered by those of the others.

The PSTs were between 20 and 21 years of age at the time of the study. Each had entered college directly following high school graduation. Both Ben and Jeremy had attended two-year colleges before transferring to the UGA. Jeremy had attended a college near his small hometown; Ben had attended a college on a baseball scholarship. Katie, Lucy and Ben each grew up in the Atlanta metropolitan area; Jeremy grew up in a rural Georgia town. Ben had a 45-minute commute each day to campus; the others lived nearby. Lucy worked part-time throughout her schooling and was only able to take off work for the student teaching experience. Katie secured a part time job part way through her senior year and continued to work some nights even during her student teaching experience. Ben enjoyed hunting and baseball. Jeremy was an amateur pilot and was very active in his religion. Katie excelled at school and her interests were eclectic. Lucy had always wanted to be a teacher and was very dedicated to teaching as a career. She struggled in her mathematics classes more so than the others.

Participants' Perceptions of the Research and the Researcher

Although I focused a great deal on technology throughout data collection, it was not until the final group interview that I told the PSTs I had been primarily interested in their beliefs about technology. I first reminded the PSTs I had given them a consent form at the beginning informing them of the nature of the study. I then asked them what they believed the focus of the study had been. Lucy chimed in immediately with, "I felt like we talked a lot about technology. I don't even remember what the form said, personally," to which everyone added, "I don't remember either." I explained that I had purposefully left technology off of the consent form, to which Katie immediately responded, "Because that would bias our answer, yeah."

Lucy went on to express she felt the focus was "the way that we wanted to teach, and the way that we actually did teach, and just different aspects of what we feel is important in the mathematics education field. But technology really sticks out for me." She "assumed the whole thing was about technology" because she knew me well enough to know my interests and because I had taught her course on technology. Because of this knowledge, "it made sense that you were going to do your dissertation on technology." Jeremy, too, thought it just made sense that I would talk about technology: "It doesn't surprise me at all that this survey—Oh, I've got 84 technology questions on this one."

Katie added that she initially thought, "It was more like just the views and philosophies of preservice teachers." Near the end of the study, Katie came across my curriculum vita on the Internet and noticed that the proposed title of my dissertation included the word *technology*. She stated that she started to "see how it all gradually built up to pure technology…. Like how did we see that being involved in our teaching? But initially I just thought it was our view on education all together." Although Ben figured, "Yeah, technology was a part of it, I guess, and it did get brought up about everything probably," he felt there was a wider focus: "We kept talking about things that we thought should be implemented in education versus things that are actually implemented in education, and the whole realization that theory doesn't meet the real world."

Given that my participants were former students of mine and that the course they took from me focused on technology in mathematics, I knew I needed to address the inherent researcher bias and participant bias in this study. From the beginning of the study, I felt it likely the PSTs would associate me with technology, and I figured they certainly were aware I viewed technology as a valuable tool in exploring mathematical concepts. I attempted to address these issues throughout the data collection process, and I will describe the measures taken as I discuss each data collection strategy. I was willing to risk these biases, however, because I believed the advantages far outweighed the disadvantages. I believed my familiarity with the PSTs would encourage their participation and allow them to share freely their thoughts and feelings. In addition, I took this acknowledgement of possible bias as a challenge. I endeavored in the planning and carrying out of my data collection to "go deeper" in my probing to ensure that the PSTs were not simply saying what they thought I would want to hear. I also knew the participants well enough to have a fairly good idea about what they thought I would want to hear.

During the final group interview, I asked the PSTs to discuss how they viewed my role during the research process. Although they played off each other's words a great deal during this discussion, in the end each fairly concisely described their perception of our relationship:

> Lucy: I don't think your role affected, like, me personally. I knew you were somebody I could come talk to at any time, and I knew you were going to give me an honest answer…. I guess it made it a little easier for me to open up to you and tell you what I was really feeling, that we had had that relationship before.

> Ben: I don't know that I actually put a role on you…. I mean, yeah, at one point you were our teacher and at one point … you were in this research process, but I'd still come and talk to you about— it doesn't even have to be school, it could be anything. So in that aspect you're a friend. So, I mean, I don't really put a role on you…. I think you did get more out of me because you had played all those roles than if you had not. Because if you had not, I would have been more searching for, "What does he really want?" than what's really in my mind.

Jeremy: When you were our teacher, I think obviously then we saw you strictly as teacher. But through that, I think, we gained a lot of respect for what you had been doing, and we realized you had a lot of credibility in that…. I saw you more after that as a student who just had a lot of respect and a lot of credibility. You had a lot of credibility and I had a lot of respect for you as a student—someone who was a lot more experienced than I was.

Katie: But even when you were a teacher, I didn't perceive you as a traditional teacher in any respect. You still seemed like one of us. And the way you taught, you seemed to bring it down— like I said, you were one of us, even though you really were not and you knew so much more. But you didn't let on and you were just like … when they were talking about Hooten and how he would have a conversation with you and you didn't even know but he was teaching something. And like, that's how you were as a teacher. And I admired that. I thought that was really cool. So, that's my perspective.

From my participation in this discussion, and my perception of my relationship with these PSTs, I believe my initial assumptions were corroborated. I had a good relationship with these PSTs. Even in my role as a teacher they viewed me as someone to whom they could relate, with whom they could come and talk. They viewed me as more than just a teacher, but as a fellow student and as a friend. Most important, they all believed this relationship strengthened their ability to be forthright and uninhibited throughout the research process.

<div align="center">Data Collection</div>

Pajares' (1992) admonition that beliefs must be inferred from what people say, what they intend, and what they do influenced my choice of data collection strategies and my desire to have variety in those strategies. What follows is a description of those strategies, each of which was intended to provide a context from which I could both capture what PSTs said and guide inferences about what PSTs intended with respect to teaching mathematics with technology. The variety of strategies implemented provided the foundation for both data source triangulation and methodological triangulation (Janesick, 1998). The former focuses on different contexts from which to make inferences; the latter focuses on bringing up information from one context in those other contexts. The primary difference is that data source triangulation provides different contexts in the hopes of seeing similarities. Methodological triangulation brings up the

same topic in different contexts in hopes of further understanding that topic. In addition, as qualitative research is largely (although not completely) inductive, the multiple contexts helped me see the data collection as "progressive problem solving, in which issues of sampling, hypothesis generation, and hypothesis testing go hand in hand" (Erickson, 1986, p. 140). Table 2 provides an overview and timeline of the data collection strategies I used.

Table 2

Overview and timeline of the data collection strategies

Data Collection Strategy	Strategy Description	Time of Data Collection
Methods class observations	Observed the PSTs' interactions in their methods course and took field notes	Throughout fall 2000 semester
Secondary data	Consisted primarily of the PSTs' written assignments from the methods course	Throughout fall 2000 semester
Initial email & Initial interview	Focused on beliefs about mathematics, teaching, and learning	Oct. 17-18, 2000
PBS 1 email & PBS 1	Surrounding the topic of quadratic functions	Oct. 23-25, 2000
PBS 2 email & PBS 2	Surrounding the topic that would be taught during the two-week early field experience (FE)	Nov. 1-3, 2000
Observation & Observation interview	The PSTs were observed teaching a class once during their FE. The interview was conducted directly following the observation.	Nov. 9-20, 2000
Tech email & Tech interview	Questions focused explicitly on technology for the first time	Dec. 1-5, 2000
PBS 3 email & PBS 3	Surrounding a topic that was chosen so as to capitalize on the PSTs' beliefs about teaching mathematics with technology	Dec. 8-15, 2000
Final email & Final interview	Further explored emerging themes both within and across the PSTs	Apr. 12-19, 2001
Group interview	Allowed the PSTs a chance to reflect on the research experience	Apr. 26, 2001

Response Mediums

Except for the observation and group interviews, an email survey preceded each interview. The PSTs' responses to the survey were then used as the foundation for the interview that followed. The observation interview followed immediately after I observed the PSTs teach a lesson. The group interview had no specific pre-interview data collection. I combined email surveys with interviews for several reasons. First, it seemed this combination would provide supporting data, as the interviews always gave me opportunities to ask for clarification or elaboration. Second, I was concerned about the PSTs being put "on the spot" in trying to discuss things they may not have tried to articulate before. The email surveys gave the PSTs as much time as they wished to think about their responses; the interviews allowed them to reiterate what they meant and gave them a chance to "think on their feet." The combination of strategies worked well. In addition, the participants knew they could contact me easily through email. This open line of communication even facilitated some spontaneous data collection. On two separate occasions Katie sent me additional email messages in which she reflected further on interviews we had recently conducted.

Although it had not been the reason for combining email surveys and interviews, some of the participants preferred one medium to the other. Lucy preferred the email surveys to the interviews. She liked to have a chance to think through her responses. In addition, the interplay between email messages and interviews caused Lucy to really think about her survey responses (the other PSTs indicated agreement with this statement):

> When I was answering some of the questions, I was sitting there thinking, "Golly, what's he going to ask from this?" And I'd try to make it as clear as possible so that you wouldn't ask me something that I couldn't answer. So I went back, and I'd read over it again and think, "Is he going to ask me something that I don't know how to do?" (Lucy, Group interview)

Ben, on the other hand, was somewhat self-conscious of his writing ability. He had had me proofread papers for him (for classes in addition to the one he took from me), and his

writing was very similar to his talking: flow of consciousness with little attention to proper grammar. Because of his difficulty with writing, Ben preferred the interviews to the email surveys. For example, one of Ben's responses in the PBS 1 email consisted of just 6 words. When I mentioned the question during PBS 1 (making no allusion to his minimal response), Ben interrupted me and elaborated for several minutes in response to the email prompt (278 words). In the group interview, after others had expressed how much they liked the email medium, Ben responded,

> And see, I'm just the opposite. I'm better at talking than I am writing. I may have a million things running through my head, but only two or three are coming out on the paper. But if I can talk about them … most of them will come out. (Group interview)

> Jeremy also expressed, "It's easier to talk than write for me, I guess" (PBS 1). He

elaborated somewhat on Lucy's statement about the desire to be clear in the emails in order to avoid difficult follow-up questions in the interview:

> I started expecting you to go deeper, after the first couple of interviews, than what you had asked about on the emails. I would go in thinking, "Okay, this is what he's thinking. He's going to try to go a step further on this question. I think he's going to probe this question." That's what I figured on some of your questions. And some of them— that's why I wouldn't answer very much. I'd just, you know, put a sentence down and, "Well, okay. He's going to talk about this one. So I'm just going to put, 'See in interview.'" (Group interview)

Ben echoed this statement with "I did too." Thus, in some ways, the email surveys served as an advanced organizer for the PSTs. Although they were not always either able or willing to write down what they were thinking, the questions got them thinking. Then in the interview they felt comfortable talking through their thoughts—brainstorming—and reacting to my probing questions. As Jeremy put it, "I knew that was coming. I was just sitting here waiting for you to ask" (PBS 2).

Katie had mixed feelings about these two mediums. On the one hand, as she put it, "I'm not very good with presenting my thoughts verbally…. I never talk well" (Initial interview); this view was tempered, however, with her preference, as will be further discussed in her data story, for spontaneity:

> My thoughts tend to come out more clearly on paper … because I have to think about it, because I'm writing about it. But I really don't enjoy the whole writing process, so it's a constant struggle for me…. I like interviews because you don't have to come in prepared, which is how I do all my interviews for schools at this point. With writing it does take a little more thought, which is an important thing, of course. (Group interview)

The interplay between email surveys and follow-up interviews was an important means of giving the PSTs varied contexts in which we could explore their beliefs. It allowed them to take advantage of those mediums with which they were most comfortable and often more capable of expressing their ideas. I now provide a more detailed description of each data collection strategy and the associated rationale.

Methods Class Observations and Secondary Data

After selecting my participants, I continued to attend the methods course throughout the remainder of the semester. Through these visits, I was able to observe my participants grapple with important issues as the instructor provided the PSTs with opportunities to question their beliefs about what it means to be a mathematics teacher. The classroom, in many ways, represented the "natural environment" of my participants. It soon became clear that classroom discussions had prompted them to think about issues of teaching and learning mathematics they had not thought about before. The PSTs often referred to experiences from class as they tried to answer my questions and to elaborate on those answers. Sharing the experience with the PSTs provided me a common context and vocabulary for our discussions. I took field notes and referred several times in our interviews to classroom activities and student comments from class. I took opportunities to listen to various groups during cooperative learning activities, sometimes participating in their discussions. In addition, as I knew everyone in the class quite well, the students, and sometimes the instructor, occasionally included me in whole class conversations. In general, however, I chose to sit to the side and observe.

An example further illustrates the value I saw in observing the methods course. One day during a class break, and after Ben had participated in a classroom activity, he

made the following comment to me: "I saw the point of the activity and figured that a lot of the others in the class did not see it." On one other occasion, Katie expressed a similar sentiment; she felt she got what was going on and some of the others did not. This acknowledgement may seem like conceit, but I thought that it demonstrated these students' capabilities and involvement in the class. In addition, these comments indicated to me they were reflecting about the purpose of their methods course and were reasonably confident in their own abilities.

Just as with the classroom activities, many of the assignments for the course were designed to encourage the PSTs to reflect about what it meant to teach and learn mathematics. I collected copies of these reflection papers as well as the PSTs' other assignments. In addition to providing valuable data in and of themselves, these *secondary data* (Schensul et al., 1999) often were the impetus for further personalization of the interview protocols. I followed the PSTs' assignments closely so I could refer to them from time to time. The classroom observations gave me the context of the PSTs' assignments and allowed me to follow the PSTs' fairly hectic and ever-changing schedule, thus enabling me to plan the timing of my data collection activities. Finally, I was able to be present for a classroom discussion in which the PSTs talked about their recent FEs. This discussion lent further insights into my own observations of those teaching experiences.

Initial Email and Interview

I sent an initial email survey (see Appendix A) to each PST. The survey elicited background information about the PSTs educational beliefs and provided contexts in which they could discuss mathematics, teaching, and learning. I then adapted the protocol for the initial interview to the individual based on their initial email response. This strategy primarily allowed me to infer the PSTs' beliefs based on what they said. Although this initial email and interview gave me the desired initial information regarding the PSTs' beliefs about mathematics, teaching, and learning, one aspect of this strategy surprised me: Questions designed to elicit beliefs about one area of beliefs often

elicited more information about another area. For instance, the responses to the question, "How can you tell whether a student is learning?" ended up revealing more about the PSTs' beliefs about the nature of mathematics than their beliefs about learning.

This initial interview, as well as each subsequent interview, was audio-recorded and transcribed. I transcribed the audiotapes of each interview before the next data collection session. I followed this pattern throughout the data collection process and found it facilitated the constant comparison of data. This transcription process will be discussed in more detail in the later section on analysis.

<p align="center">*Pedagogical Brainstorming Sessions*</p>

Determining what PSTs *intended* with respect to teaching mathematics with technology was more difficult than determining what they said. To reveal the PSTs' intentions, I developed a data collection strategy I refer to as *pedagogical brainstorming*. The main purpose of the pedagogical brainstorming was to put the PSTs in a context in which they considered various approaches they might use to teach a mathematical concept. This activity is, in essence, what PSTs do: they think about possibilities for future teaching.

There are two main phases in the strategy: I first sent a preparatory email to the PSTs, prompting them to brainstorm about how they might teach a particular mathematical topic. As prompts, I used general questions one might consider when teaching almost any mathematical topic (see Appendices B and C). The PSTs responded to the email and a follow-up interview—referred to as a Pedagogical Brainstorming Session (PBS)—was scheduled within the next several days. The protocol for this interview was developed to reflect the response to the PBS email. The PSTs were asked to clarify and provide examples of their responses. Some of the questions in the interview protocols were designed to encourage the PSTs to explore and discuss with me the processes they went through in responding to the PBS email. Other questions elicited further brainstorming on the part of the PST, particularly in the areas in which the PST had indicated technology might play a role.

Three PBSs were conducted with each PST. The PSTs were in the process of preparing for their FE when the first two PBSs were conducted. To take advantage of these preparations and possibly increase the PSTs' motivation to participate by providing some obvious connections between their schoolwork and the "work" I was asking them to do, the topic of the first two PBSs was directly linked to the topic for the forthcoming FE. As the PSTs had recently observed the classrooms in which they would have their FE, I asked them to consider the topic being taught at that time—the topic to be taught just prior to their FE—as the topic for PBS 1. The PBS 1 email and the corresponding protocol for PBS 1, along with sample follow-up questions, are in Appendix B. The topic for PBS 2 was the topic the PSTs had been given (by their cooperating teacher) to prepare for the FE. Because the PSTs were preparing detailed lesson plans for the FE, they gave me copies of their lesson plans in lieu of completing a PBS 2 email.

PBS 3 was conducted after the PSTs returned from the FE and after an email survey and interview focusing on technology were conducted (the technology email and interview are discussed below). The topic for PBS 3 was chosen in light of my analysis of the PSTs' beliefs about technology up to that point. I wanted to ensure the PSTs had an opportunity to brainstorm about a mathematical topic for which, when teaching, they believed they were likely to use technology. Two PSTs (Katie and Ben) indicated the technology with which they were most comfortable was graphing calculators. These PSTs were given the topic of polynomial functions for PBS 3. The other three PSTs (Lucy, Kara, and Jeremy), who expressed greater comfort with computer software (in particular, Geometer's Sketchpad), were given the topic of polygon similarity and congruence. I chose polynomial functions and polygon similarity and congruence based on my own experience using technology with these topics and because I knew the PSTs also had technology experience with them. In addition, each PST was asked to focus their brainstorming on a subtopic they felt had "high potential" for use of technology. This focus allowed me to have a similar discussion with each participant in which they

explained what it meant for topics to have different degrees of potential with regards to technology. Appendix C contains the PBS 3 email and a corresponding PBS 3 protocol.

Teaching Observation and Observation Interview

Inferences about what PSTs believe based on what they do seems problematic given that, by definition of "preservice," they are not yet teaching. However, as mentioned previously, these PSTs had a two-week pre-student teaching field experience. During the FE, the PSTs taught one class per day. I observed each participant once during this FE. PST teaching observations were not intended to be a focus of this study. The goal was to study their beliefs before they started teaching, and certainly this one observation did not and could not provide significant evidence to infer the PSTs' beliefs based on what they *do*. Learning, not teaching, is a PSTs' primary activity (whether that ought to be the case is another matter). Given that the PSTs would be teaching in another teacher's classroom, and for only a few days, little could be inferred from the observation. Nevertheless, the FE presented a valuable context through which to explore the PSTs' beliefs during an interview conducted directly following the teaching observation. That the PSTs continued to reflect on the FE became apparent during remaining interviews, as they often referred to the experience in general and the lesson I had observed in particular. This shared context was invaluable. I used an observation protocol adapted from one used by Akujobi (1995, pp. 192-195) when he observed teachers' use of technology in the classroom. I had several prepared questions for the observation interview protocol, but I primarily developed the interview questions as I took field notes during the observation (see Appendix D).

Technology Email and Interview

As stated earlier, although the PSTs had been told my research focus was their beliefs about teaching and learning mathematics, I did not tell them I was particularly interested in their beliefs about technology. After PBS 2, however, I sent the PSTs an email and conducted a follow-up interview focusing solely on their beliefs about and experiences with technology. From this point on, although I still never stated it explicitly,

my questions more openly focused on technology. I personalized the technology interview in light of not only the email but also my understanding of the PSTs' beliefs. In addition, the results of this email and interview greatly influenced the focus of my remaining data collection opportunities—in particular, the topic of focus for PBS 3 and many of the questions in the final email and interview (see Appendix E for the technology email and a sample interview protocol). This interview (as well as all other interviews) was conducted in a room where a computer was easily accessible. In addition, the PSTs were always invited to bring their graphing calculators to the interviews.

Final Email, Final Interview and Group Interview

The majority of the data collection was completed by the end of fall semester 2000. Several months later, after the PSTs had completed their student teaching, they came back to campus for a 5-week seminar. During this time I sent each PST a final email survey and then conducted a final interview (see Appendix F). Numerous themes had emerged to this point and this email and interview allowed me to obtain further information about the strength of those themes. Finally, just before the end of the school year, I conducted a group interview with all five PSTs (see Appendix G). During this interview I asked the PSTs to reflect on the experience of being involved in the research study and to provide feedback as to the purpose of the study and what they had gained from it. I learned, as has been described by Goldman and McDonald (1987), "the value of the group interview, however, ultimately derives from the temporary social structure that evolves over the course of several hours and the way in which that social structure provokes and facilitates information flow" (p. 66). The group interview allowed the PSTs to share how they felt about the experience and to respond to each other. I found this interaction valuable as they played off what each other said. Throughout the data-collection process I had inferred numerous similarities and differences among the PSTs. During the group interview I was able to observe as the PSTs confirmed that they agreed or disagreed with what others said. The group interview added to my data triangulation.

The primary purpose of the final email and interview was to further pursue the themes I saw emerging from the data. I wanted to further tease out similarities and differences across the participants. For instance, the phrase "playing around with mathematics" had occurred at various times with all of the participants, and it seemed to be connected to their beliefs about what it meant to explore mathematics. I asked the PSTs to discuss what that phrase meant for them, and then we further discussed it in the interview. I also wanted to explore with the others some of the themes I had seen for individual PSTs. I wanted to see to what extent these themes were unique to a given PST; I believed that others' responses would increase my understanding of the PST from whom the theme had initially emerged. For example, Lucy and Ben had often talked of the importance of applications in mathematics. I wanted to give them a chance to revisit this idea (saturate, if possible, this category for them) and at the same time see whether I was correct in inferring that applications were not as important to Katie and Jeremy. Similarly, Katie's notion of "winging it" seemed quite different from the others' notions of being prepared, so I wanted to explore further the notion of preparation in order to draw a more accurate distinction.

Analysis

Analyses associated with the grounded theory research tradition use the constant comparative method. This method of coding and recoding data throughout the data-collection process informed that process and led me to categories, themes, and eventually theories to help describe and explain PSTs' beliefs about teaching with technology. Through comparing and contrasting the PSTs' belief systems, I developed theories—"plausible relationships" (Creswell, 1998, p. 56) among PSTs' beliefs about teaching mathematics with technology. In this section, I describe and provide illustrative examples of the analysis techniques I employed. Wolcott (1994) expands the traditional use of the term *analysis* to include description and interpretation, preferring to refer to the whole as *data transformation*. In so doing, he highlights the continuous and expansive nature of transforming qualitative data. The constant comparative method of analysis

employed in this study further blurs the lines between data collection and data transformation. Despite the "constant" nature of this approach to analysis, there were still somewhat discrete periods of my analysis. I have termed these periods collecting data, coding data, and writing about data. I discuss these periods individually and, in general, in the order in which they occurred. These periods, however, often coincided.

Collecting Data

A significant amount of analysis occurred while I was in the process of collecting data. For instance, as each interview was preceded by an email response or the collection of secondary data, I needed to read through that material in order to decide how it would influence the interview. With little data in hand, this process was initially fairly straightforward. At this point statements, for the most part, seemed to make sense. I compared what I was seeing in the data with the literature in order to help make decisions with respect to follow-up questioning. As I continued with the data collection, the analysis process became more complex. Comparison became possible, first across and then within participants' data. Questions such as, "How does that fit with what they said yesterday?" or "How does what Lucy said relate to what Katie said?" were important in building a sensible description of the individual and in developing individual and overall theories of the PSTs' beliefs. The analysis questions that I asked myself evolved over time from questions of clarification to questions of connections with what I understood to that point with respect to the PSTs' beliefs. For example, early in the data collection process I asked questions like, "In what ways do you think you might 'integrate technology in your classroom'?" (Initial interview) simply because I did not understand what Jeremy had meant by that phrase. As the data collection progressed, however, I asked questions such as, "Would the students be using technology when working on these problems?" (PBS 3) in order to explore further Jeremy's beliefs about technology integration in the classroom.

Interviewing, for me, was a part of the analysis process. I was constantly making decisions about which follow-up questions to ask. I always prepared more questions than

I would have time to ask, so I had to decide which questions to pursue and which to let go. The same decision had to be made with each PST's response; I had to decide whether to seek further clarification. This "groping for coherence about what is being said" (Seidman, 1998, p. 78) was an important aspect of the analysis process. The interviews and PBSs were audio-recorded and then I transcribed them (prior to the next data collection session) for analysis purposes. This activity was invaluable, as it provided an opportunity not only to relive the experience but also to reflect momentarily on each word my participants (as well as I) said. Throughout the transcription process I noted directions I wanted to go in future data collection sessions. In addition, this personal transcribing allowed me to reflect on my own interviewing strategies. I noticed right away that I was talking too much during the interviews and made a concerted effort to be brief with my own comments. I also noticed I was saying words like *right* and *correct* in an attempt to acknowledge the PSTs' responses. In an effort to avoid having these acknowledgements construed as approval, I forged the habit of simply nodding or saying "Okay," "I see," or "Next question."

According to Freeman (1996), "the study of what people know generally turns on an analysis of what they say they know" (p. 733). This critique of traditional, often static, forms of data collection and analysis can also be applied to the study of beliefs. People's beliefs are inferred largely from what they say, as is true with the bulk of my data-collection strategies. Freeman suggested the need, when using language data, to focus not just on the presentational data (i.e., what they say) but also on the representational data (i.e., how they say what they say). He claimed this additional dimension of analysis has the potential to greatly strengthen a study:

> With the addition of presentational analysis, language itself becomes the locus of study. Such analyses can show evidence of the processes of teachers' self-definition, learning, and change. The integration of representational and presentational analyses reveal not only *what* is being learned or is changing, but also *how* it is being learned or is changing. (p. 735)

My transcription of the interviews allowed me to focus not just on *what* my participants were saying but also on *how* they were saying it. This focus in turn helped my inferences about both *what* they believed and *how* they believed it.

The transcription process forced me to think through exactly *what* was said and often caused me to reflect on *why* I thought it was said, both by the PSTs and by me. As I transcribed, I wrote in brackets any ideas I had as to what I remembered thinking at the time or what had occurred that would not be captured on tape. Here are several examples of these bracketed comments:

> [He goes over and gets his laptop. He used his laptop throughout the lesson, as he had it hooked up to an overhead projector. He also had his lesson plan on there and had referred to it numerous times during the lesson. He goes to this electronic version of his lesson plan now.] (Jeremy, Observation interview)

> [She gives me one of those looks that says, "Wow, I can't believe that's what I was saying."] (Lucy, PBS 2)

I also took note of ideas that seemed to warrant further discussion or clarification in later interviews. The transcription process allowed me to re-experience the interviews.

Coding Data

I initially developed very broad codes derived from my research questions and from my review of theoretical literature. As I coded data I continually revised and reorganized the codes, creating new codes as themes emerged and dropping other codes as it became apparent they were anomalies. I chose to work through one PSTs' data at a time, adding codes as I went. I would then take time to reorganize the codes before turning to the next PST. After having gone through this process several times using paper printouts, I entered my codes and data into the computer program NUD*IST (Richards & Richards, 1997). The process of coding and of reorganizing codes was simplified dramatically with this program. An example of the progression of codes under one category is provided in Appendix H. At one point early in the coding process, I had a colleague code several transcripts. Because this exchange was somewhat informal, it

cannot be used to measure coding reliability. Our discussions, however, did lead me to reorganize my codes and to look for several nuances I had not been looking for before.

Writing About Data

Once I had coded my data across the participants, I turned to finding themes for each of the participants. I used the writing of data stories to facilitate the generation and refinement of these themes. As Wolcott (1994) pointed out, "Development of the descriptive material is every bit as much an interactive process as is any subsequent analysis or interpretation" (p. 21). I began this phase of analysis by writing the complete data story of one PST. I first "sorted" the data coded under mathematics, both electronically and, several times, by printing out copies and arranging them into various piles. I then pieced together the themes that emerged into a coherent picture of the PST's beliefs about mathematics. I did this process in turn for beliefs about teaching and learning, searching for consistencies and apparent inconsistencies with what I had previously written. I then turned to technology. NUD*IST allowed me to search my coding in multiple ways, as well as search my documents for words or phrases that emerged as particularly meaningful.

I received feedback from several colleagues as I revised the data stories. The readers looked for apparent inconsistencies and unjustified statements. Throughout this process, I tried to make sense of what the PSTs were saying by finding sensible ways to interpret their various statements and the beliefs they implied. I often needed to return to previously written sections to reassess how well they fit the rest of the story. If a statement or a theme did not seem to correspond, I searched for something I might have missed to make it cohere. Through this distilling process I was able to infer the PSTs' beliefs about mathematics, teaching, learning, and teaching with technology. I collected a lot of data and found, as Wolcott (1988) stated, that the difficult "task is to strike a balance between extremes of telling too little and telling too much" (p. 27). Having others read my data stories helped me to strike this balance, as they were able to recognize, more than I, places where I was being either vague or redundant.

Member Checking

To lend credence to my inferences, I sent each PST their data story and asked them to read it and give me feedback. The two primary reasons I wished to do this member checking are expressed well by Stake (1995): "The actor is asked to review the material for accuracy and palatability" (p. 115). I wanted to know whether the PSTs believed my representation of their beliefs "rang true" to them. In addition, I wanted to know whether they felt uncomfortable with the way they had been represented. I had tried very hard to avoid using value-laden language in the data stories, and I wanted their opinion on how well I had accomplished that. Thus, having stated I did not wish to misrepresent them, I asked the PSTs to "take note of anything that seems 'not quite right' or that needs clarification. I would appreciate you filling in any information you think would be relevant" (Member check email).

Lucy, Ben, and Jeremy were very pleased with what they read and offered no clarifications or elaborations. Jeremy stated, "I felt that you did a good job of describing our interviews and my views on teaching math" (Data story reaction). Although Katie was also quite pleased with her data story, there were several places where she wished to make further clarifications. For example, in describing Katie's beliefs about mathematics, I had initially written the following:

> Mathematics, for Katie, was a way of thinking which does not come naturally to people. She said, "It's not like people already know some form of mathematical thought and all they're trying to do is define it in more mathematically accepting terms. Mathematics actually creates a new way of thinking" (Initial email). This mathematical way of thinking was a way of explaining the world. (Data story draft)

In response, Katie stated,

> I don't think the second sentence correctly describes my view. I think it said, "It's not like people already know some form of mathematical thought." Anyway, I do think everyone already has mathematical thought. It's just that we have to learn how to define it in accepted mathematical terms. (Reaction email)

These statements and the further elaboration she provided gave me a better idea of what Katie had meant in her earlier statements. Through this reaction I was able to revise the above statements about Katie's beliefs about mathematics as follows:

> Mathematics, for Katie, was a way of thinking. Although "everyone already has mathematical thought" (Reaction email), the language used to communicate those thoughts does not come naturally to people: "The basic idea has always been in people's minds, it's just that we've had to learn how to communicate it better (Reaction email). Thus, as students learn of these ways of communicating, "mathematics actually creates a new way of thinking" (Initial email). This mathematical way of thinking was a way of explaining the world. (Data story)

I was also able to use secondary data as a means of checking my inferences in the data stories. For instance, after I had developed Jeremy's section on his beliefs about the nature of technology in the classroom, I found an electronic copy of a paper he had written that I had not read before. In that one short paper I found Jeremy had articulated three main themes that had emerged in that portion of his data story.

After having written and revised the data stories, I went back and added a summary at the end in which I described in my own words the PSTs' beliefs and connections between those beliefs. I also created a table in which I recorded brief statements representing each PST's beliefs. I used this table as I began to articulate theories emerging across the PSTs. I sought out literature that helped me to understand what I was seeing and what it might mean. Through many experiences asking questions of the PSTs as well as asking questions of my data, I inferred the content and structure of the PSTs' beliefs. The PSTs' beliefs about mathematics and the learning of mathematics were often intertwined with their beliefs about the teaching of mathematics. This intertwining is not surprising given that the majority of activities of their teacher preparation program were designed to encourage reflection on the teaching of mathematics. The language the PSTs used to express their beliefs was primarily the language used to describe teaching as opposed to the language associated with epistemology or philosophies of mathematics.

Chapters 4 through 7 contain descriptions of each PST's beliefs about mathematics, the teaching of mathematics, the learning of mathematics, and the teaching of mathematics with technology. In each chapter, the portion on teaching mathematics with technology is broken into four sections. The first, in partial response to the first research question, describes the PSTs' experiences with technology. The second portion—referred to as the nature of technology in the classroom—describes the PSTs' overall view of how technology fits into their classroom. Describing the PSTs' beliefs about the nature of technology in the classroom came about as a direct result of data analysis and the desire to describe the PSTs' centrally held beliefs with respect to teaching mathematics with technology. The third portion describes the various roles each PST believed technology played in the classroom, and the fourth describes concerns they had related to using technology. The last two portions, in essence, emerged in response to the remainder of the first research question. These portions describe what the PSTs believed about teaching with technology and how those beliefs were held.

Although the organization of the data stories is the result of data analysis—in particular an effort to view each PST's beliefs as part of a coherent belief system—the PSTs' own words are tied together to tell their story. The final section of each of these chapters provides a brief description of connections I have inferred among each PST's various beliefs. The intent of this section is to make explicit the belief structure implied by both the content (what I chose to include) and the organization (how I chose to present it) of the data stories.

CHAPTER 4: BEN'S BELIEFS ABOUT MATHEMATICS EDUCATION

Beliefs About Mathematics

Mathematics, for Ben, was "not just a set of rules" (Initial email)— it was more:

> There's ways of doing things, but there's concepts behind mathematics. If it was just a set of rules, then nobody would have any problems with mathematics. You just do this, this, and this, and that's it. But that's not what math is. If it was like that, then math would not be powerful. (Initial interview)

When Ben came across a problem in mathematics, he went to his mental toolbox and picked "a tool to work on a problem in order to solve it" (Final email). The "set of rules" was in this toolbox, as were the concepts behind those rules. In addition, a set of problem-solving tools brought power to the procedural tools. He described these problem-solving tools primarily in terms of logical thinking.

The distinction Ben drew between logic and reasoning lends clarity to how he viewed these problem-solving tools. He stated, "In a lot of mathematics you're teaching logic, and in a lot of other subjects you're teaching reasoning, and I think there's a big difference" (Initial interview). Reasoning was the kind of subjective thinking used in literature, whereas logic was the more objective thinking of mathematics. With logic, if two people started with the same assumption and proceeded logically, they would end up at the same place. They need not take the same logical path, however, to arrive at the same conclusion. Ben described how in high school he had frequently used his own logical path to solve mathematical problems. His teachers were often frustrated with him because his method was different from the one they had taught. But Ben liked this characterization of mathematics: there were different approaches each equally logical and valid.

Ben believed the problem-solving tool of logical thinking played an important role in making real-life decisions. This belief was exemplified in a discussion pertaining

to a project, which Ben was planning to use in his Technical Algebra teaching unit, focused on the Space Shuttle Challenger:

> The overall problem that I've been thinking about is this Challenger shuttle problem—and it's a "launch or don't launch" problem. But a lot of things are like that. In the business world we're saying, "Do I—?" Like on Wall Street you're doing, "Do I buy or do I sell?"—decision-making things, I guess, or a decision question. Not necessarily just a yes or no thing, but it's, "Do I want to go with this product or do I go with that product?" Things where they have to make a clear decision and it's not a, "Well, I could do this or I could do [this]." (PBS 1)

Ben believed that applying the logic of mathematical problem solving to real-world situations where decisions needed to be made was a powerful use of the tools in his toolbox. When asked what he most wanted his students to learn, Ben responded,

> I want my students to learn the necessary foundations for practical living. That is, through mathematics, student life, home life, and just life in general, I want them to begin to see that life is a bunch of decisions, and how they make these decisions is going to determine the road or course their lives take. This is broad, I know. Being specific, I want them to apply what they learn to their lives. (Tech email)

With respect to decision making in mathematics, Ben recognized he had not always used his toolbox in powerful ways. He remembered having solved problems without appealing to logical thinking. He had solved "problems without that thinking process. I mean, I did before I came here. But the thinking makes the problem solving easier—another tool in the toolbox" (Final interview). Ben had been used to going to the toolbox and indiscriminately choosing a tool based on whether he knew how to use it. As he acquired more problem-solving tools, his utilization of the toolbox became more judicious:

> Before this I went to the toolbox and I found a tool that I knew how to use, whether it was a formula, or procedure…. Now it's more like, "Well, I need this, and I don't need this. This is junk, this is going to help me get to where—." And I'm going to set a strategy, or I'm going to set a way that I think I'm going to get to where I want to go. And if I don't get there, well, I start back; I do it again. I come up with another strategy to go that way. Now I may use formulas and stuff in my strategy, but now it's not just, "Well, I do this, this, and this to get to this answer." (Final interview)

Ben did not feel equally competent with his mathematical tools. When asked how strong he felt he was mathematically, he responded, "Procedural or conceptual?" He felt, given a procedure, "[I can] apply it to just about anything, and I can give it to you. I can do it" (Final interview). Although he was less confident with the conceptual tools of mathematics—"the way something is viewed and the way it works" (Final interview)—than he was with the procedural tools of mathematics, Ben believed his conceptual understanding was increasing. In particular, these conceptual tools became more powerful when he started to see "a foundation" for mathematics:

> I used to see [mathematics] as this huge thing that was hard to comprehend and it was kind of something I was reaching for, you know? And now, it's kind of like—. It's still this big thing, but its not so complicated as it was. I mean, it has complicated parts, but it's kind of a foundation you learn…. There's a foundation that explains a lot of the processes and laws. (Initial interview)

Beliefs About Teaching Mathematics

Ben believed his primary role as a teacher was to create an environment where students were motivated to learn. There were several ways he believed he could accomplish this goal. One strategy was to teach mathematics in ways to interest students, which meant he had to be flexible when it came to planning and carrying out lessons. After I observed his teaching, Ben commented, "You can tell there's been a lot of adaptation going on here" (Observation interview). He believed students have different needs and if they were going to be motivated to learn, his responsibility was to address those needs even if it meant changing his lesson plan. For example, the day Ben had planned to introduce the "launch or don't launch" project he had planned surrounding the shuttle Challenger was the day after the 2000 Presidential election. The students were talking about the indecision, so he decided to reorganize the project around the election. He recalled making this decision "as class was going on" and thinking, "I'm going to change the project to that, 'cause it's something they're interested in. It's something that's really going on right now, that they can put a hand on" (Observation interview).

Ben's flexibility applied not only to changing *what* he did in the classroom but also to changing *how* he did it. He believed part of being flexible in teaching was using a variety of approaches, and he wanted to base this decision on what would most likely bring about conceptual understanding. He provided the following analysis:

> There are certain things you can conceptualize better when you do it with paper and pencil. I think there are certain things you conceptualize better when you do it in an activity; I think there are certain things you conceptualize better when you do them on the computer…. You may do a lesson one time and realize, "Well, that didn't go so well. Well, let's try it this way." And I think when you realize which lessons work with that, then you can maximize their learning. (Initial interview)

One approach Ben believed would both create interest and facilitate conceptual understanding in his students was letting them feel they were teaching themselves. He described this approach as one in which students were "asking questions, and I'm asking questions that make them answer their questions—kind of letting them guide their own adventure—but with my direction" (PBS 3). Ben did not stay in the front of his class when he taught. He wandered around and would sometimes sit in a student desk during a classroom discussion. He believed this sitting back helped students feel they were teaching themselves. His role in these situations was to move things in the direction he thought would accomplish the mathematical goal for the day. He believed "the teacher doesn't have to be the teacher up there in front of everybody all the time" (PBS 1). With this approach, he said, the students

> didn't even really realize I was there. I was just kind of sitting back watching them and then, when they would start to drift somewhere else where I didn't want them to go I'd kind of pull them back and make a probing question or something, or just say a little comment to get them back on the right direction. And they were teaching themselves; they didn't even know that they were teaching themselves but they were. (PBS 1)

To help the students teach themselves, Ben believed it was important to know them well. He explained how he came to know his students:

> I'm a people watcher. I watch everybody. I watch eyes, I watch motions and faces and [pause] there's a certain air about someone who has confidence. And it's a different air than someone that's cocky. I mean, you can see it. And there's a

certain—. When somebody doesn't understand—…. I don't want to say it's a judgment call, but it kind of is. I mean, it's not like you're like, "Oh, that person's stupid over there because they just don't get it." I don't know how to explain it. It's just kind of an air they put out. (Initial interview)

Ben seldom talked about a teaching situation without using the names of his students. After a short time in the classroom, he was able to make very specific statements about the students in his class. For example, he described one student as someone who "normally pays attention to every syllable that comes out of your mouth. He may not understand it but he pays great attention to you." He described another as "the one in there that does not need to be in that class. He can do it all." A third was "a big NBA fan and I knew that." And a fourth was one who "will answer you every time" (Observation interview). Ben actually felt guilty for focusing so much on getting to know his students. He felt there was "so much more of me watching how kids react to certain things and how I can adjust it to them. It seems like I'm more focused on that than I am [on] actually teaching the lesson" (Observation interview).

Ben believed knowing his students allowed him to get on their level. From that vantage point a teacher could "find what's going on with you and figure out how to get this mathematics to apply to you" (Initial interview). Through getting to know his students and getting on their level, Ben believed he could focus on reaching them, wherever they were, and he recognized he would spend perhaps his entire career improving in this area. He wanted to "continually get better at engaging the students and finding ways to reach the students, and I think that affects every aspect of the teacher" (PBS 2).

Beliefs About Learning Mathematics

Ben believed that, although the process may be more difficult for some than for others, everyone could learn mathematics. He compared being good at mathematics to being good in baseball:

A lot of talent in baseball is learned. There are some people who have this talent, and it's easy for them to learn. And then there's some people who don't, and it's hard, and they have to work at it. So, I don't think it's necessarily just a talent that you have, although some people do. (Initial interview)

Students did not need to have natural mathematical talent in order to learn mathematics. What they needed more than anything was "a confidence thing." He described this thing as a need "to have a foundation in it somewhere to start their confidence from, and it's got to be built—. But you've got to find the level in each one of them, where to start it from" (Observation interview).

Part of gaining confidence came from understanding that "you're not going to know everything, but that's okay. That it's a process…. It has to be a feeling that, 'Yes, I understand it, but you're going to have to go through a whole lot to get there'" (Initial interview). Instilling confidence and understanding in his students created a learning environment "conducive to positive criticism" (Initial email), where "you can say what you need to say—you say what you think. But, you also know that, somewhere along the line, it can be criticized" (Initial interview). In this sort of environment, students could explore and conjecture without fear of being ridiculed for their ideas. Ben described an environment where students were making and testing their own conjectures as one in which "they were talking about mathematics, doing their own thing…. And they were teaching themselves" (PBS 1).

Beliefs About Teaching Mathematics With Technology

Experience With Technology

Ben's earliest experiences using technology with mathematics were on computers. Although he remembered going to the computer lab in middle school, he did not recall how the computers were used. In 9th grade, Ben had taken geometry and been introduced to GSP. He described this experience as one where he and his classmates "used GSP in the computer labs. Basically we had a worksheet to complete using GSP and the teacher went around helping us throughout the lab" (Tech email). The following "year, in Algebra II, we did something with a program there, but I don't remember what the program's called…. It would graph, it would solve equations…. It was kind of a multi-purpose program" (Tech interview). Later, in his high school calculus class, Ben recalled using GSP "a little bit, but I used it more on my own than I did with a class. I had it at

home, so I could use it when I wanted to with that. It was pretty neat" (Tech interview). Ben's high school experiences using computers in the mathematics classroom were isolated, and he felt they had minimal impact on him. Although he recalled occasionally going to the computer lab during high school, "we were playing games. I never did anything meaningful" (Group interview). Thus, although he "had played Number Muncher and stuff like that, as far as actually using it for real learning, I had probably never done that" (Group interview).

It was not until his college Calculus III teacher used the computer to "to draw these graphs of things that you can't picture—you cannot see them in your mind"—that Ben felt he experienced "technology really used in a math class" (Initial interview). And it was in the Technology and Secondary School Mathematics course that he first "really saw that you could teach with it, and it wasn't just a side product" (Tech interview). Ben stated he loved technology and that, after taking this course he "realized how to use many forms of technology to further my interests in mathematics. I also learned how to use it in my teaching" (Final reflection paper).

Ben's early experiences with graphing calculators had been considerably more memorable and rewarding than those with computers. He did not feel his mathematics teachers' use of technology had been worth the effort, in general, "until we got to Calculus and [used] the calculators. We used the TI-85s very extensively in Calculus" (Tech email). In fact, his most positive experience using technology involved using graphing calculators in his high school calculus course:

> In my AP Calculus class in high school, me and [another student]—. [Our teacher] was sitting there talking to us and told us, one day after class, "There's a way to do this on the calculators, but I don't have the programs anymore…." As a matter of fact, all these programs—he gave to us. That Calculus Toolkit you saw that one time—he programmed the whole thing. He did it all on the calculator. It's not something you buy; it's something you put on there. Anyway, and he said, "You know, we ought to program this. I think we could do it." And he showed us how to program a Calculus Toolkit. And he showed us the way, and he wrote it all out so he wouldn't lose it. And we started programming the different things for the integrals and all that stuff into the calculator. And that was pretty good. So it

was more or less a challenge to us. It wasn't nothin' with the class; he was trying to challenge us, personally. (Tech interview)

Ben had been asked to do something with the graphing calculator that, although related to the mathematics he was studying, was not part of classroom instruction. He saw the experience as a challenge—one that helped to increase his knowledge of and appreciation for the graphing calculator.

Ben described his teacher's classroom use of the TI-85 as using the calculators "to speed things up" (Tech interview). First, his teacher showed the class "how to do it by hand;" then "he showed us how to do it on the calculator." Students were "allowed to do it by hand if we wanted to" or "to do it on the calculator—either way" (Tech interview). Ben described his calculus teacher's philosophy about technology use:

> His philosophy was—he was 60 or 70 years old when he was teaching us this course—he said his philosophy was, "Well, I had to do everything pencil and paper back in the old days." He said, "That's the old days. We have the technology. We're using it. What's the use in having it if you're not going to use it?" So, he very much used technology and wanted us to use it. (Tech interview)

Despite the positive impact the experience of using graphing calculators in calculus had on Ben, he did not believe any of his high school experiences, including those in calculus, constituted teaching with technology. He saw it as "more, 'Hey we have this. It would be neat to give you an experience with it.' But they didn't really use it for teaching that much. I really don't think they knew how to, to be honest with you" (Tech interview).

Ben felt more comfortable with graphing calculators than with any other sort of technology. He said, "As you can tell, from what I'm talking [about, I'm most comfortable with] the graphing calculators. I have every one from the 81 through the 89…. I have every one of them" (Tech interview). Being most comfortable with graphing calculators did not mean, however, Ben felt uncomfortable with computers. He stated, "[Although comfortable with] GSP and stuff like that, I just don't use it as much. I don't find myself going to my computer [and] flipping up something on GSP as much as I find myself grabbing a calculator and playing with it" (Tech interview).

Whether he was using graphing calculators or computer programs, Ben was confident that if he did not know how to do something, he would be able to figure it out. For instance, with respect to graphing calculators, Ben said, "If you know how to use a scientific calculator, it's not that far a stretch to go to the graphing calculators" (Tech interview). He was confident in his ability to generalize his technological knowledge. Although he "had no idea about Graphing Calculator"[5] when he started taking the Technology and Secondary School Mathematics course,

> that didn't bother me at all. It wasn't that hard to learn. I don't really look at technology as something I can't learn. If I don't know it, it's not something I can't figure out. It just takes a minute to sit down and figure it out. So I don't really think I'm uncomfortable with any of it. (Tech interview)

This confident approach to using technology was particularly manifested in PBS 3. Ben had indicated he wanted to introduce the idea of solving systems of equations by "solving a system of equations with a cool 3-d graph from my calculus class" (PBS 3 email). During PBS 3, he used Graphing Calculator to demonstrate the introduction strategy he was considering. Although Ben did not remember how to get the program to do everything he wanted it to do, he was confident that with a little bit of trial and error he would get what he wanted. As he experimented with the program, Ben said a number of things, often speaking directly to the program, indicative of the ease with which he approached this activity. Four such statements follow:

- Oh, that's fine; I see what you're doing.

- Let's do this a little differently, just because I don't know what that's doing.

- I'll beat ya somehow.

- Wow. Well, we just created an egg—a three dimensional ellipse. And it's enclosed my other one, because, well, my range is wrong. So let's change this. See if we can make that still enclosed in there. (PBS 3)

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[5] Graphing Calculator (Avitzur, 1999) is a computer program that allows the user to graph and dynamically manipulate algebraic relations.

Ben felt comfortable just playing around with Graphing Calculator. Initial uncertainty about how to accomplish an objective or even about the nature of the objective itself did not overly concern him.

Ben recognized that he was most comfortable using technology in settings that were familiar to him. As such, he supposed his use of a classroom set of graphing calculators would depend on the course he was teaching.

> I think it a little bit depends on the course, but I also think that in an algebra class or this technical class that I was teaching—something like that, or calculus—yeah, I think you'd bring it out every day. I mean, I think there's situations where you can use that every day. If you're teaching a geometry class it may not be used as often. But there again, there may be ways to use [it that] I'm just not recognizing. (Tech interview)

Whereas Ben had used graphing calculators frequently with algebra and calculus, he had not used them with geometry. He had few ideas of how he would use them in that class and was less comfortable considering their use in that context. Ben was confident, however, that his abilities would improve as his experience increased: "I think that's another thing with the comfort issue—the more you use it the more flexible you're going to be" (Tech interview).

The Nature of Technology Use in the Classroom

As part of PBS 3, Ben categorized a list of mathematical topics according to whether they had high, medium, or low potential for technology use. After the categorization, he explained that he considered a topic to have high potential if technology "can be used and it's very advantageous." Ben created a medium-high category for those topics with which "I most of the time use technology, but sometimes it's easier for me to do it by hand—by looking at it and by hand." Medium potential meant: "I use technology more than I don't." And low potential meant that, although technology could be used with that topic, "I don't use technology for that at all. I do that in my head" or "by hand." For every topic Ben recognized a way technology *could* be used; his categorization then hinged on how advantageous he found that use. Ben

believed that it was advantageous for his students to use technology when "it's a benefit and an aid to what they're doing. It's not a crutch to what their doing, and it's not a reliance for what they need to do. But it can aid them and it can benefit them" (Tech interview). Thus, although technology was by no means necessary for teaching mathematics—"you could teach any mathematics without the use of technology, heck they did it for years"—technology was often "a huge help, and will enhance the teaching" (Tech email).

When asked whether there were situations in which he definitely would not use technology in his mathematics teaching, Ben stated, "I'm really not sure, right off hand. A calculator is always available and useful, but I guess when I wanted the students to learn something the old [fashioned] way, I would not use technology" (Tech email). He explained,

> In some situations I know [calculators] are not always available, but in my personal experience, they have always been there if I needed them. And when I say useful—I've never seen a place where the calculator itself is the problem. Granted, there's things that I would like for students to do what I call the old [fashioned] way, or paper-and-pencil way, but I don't think the calculator hinders that. I believe your teaching is what's going to hinder that…. It's how you use the calculator. But I think if you use it correctly that it's going to pretty much always be useful. (Tech interview)

For Ben, to teach mathematics with technology meant "to use technology in a way that helps students to maximize their potential" (Initial email). To accomplish this goal, he felt technology "should be used in a fashion to further education and to help students move on to greater things" (Final email), which meant using technology, although always a viable choice, was not always the best pedagogical choice. As discussed in the section on Ben's beliefs about teaching, he believed that "certain things" were better conceptualized using "paper and pencil," others "in an activity," and yet others "when you do them on the computer" (Initial interview). He went on to explain that, for a given mathematical topic,

> when you realize which lessons work with that then you can maximize their learning. And when you force technology into a lesson which it really doesn't

need to be in, that energy that it takes to figure out how it works is taking the time away from the concepts that they really need to learn. (Initial interview)

In this sense, advantageous use of technology was in part judicious use of technology. Ben believed that, although technology "can be used just about everywhere, the emphasis changes from class to class"—even "from topic to topic" (Tech interview).

Ben believed technology was an appropriate pedagogical choice as long as it was used "correctly." This statement meant that in class, on homework, and on tests technology use needed to be consistent and aligned. Ben felt this alignment was missing in the classroom in which he did his field experience. He stated, "[Students] can use calculators on the test, but they don't ever use them in class. And see, I have a strong feeling on that" (Observation interview). These strong feelings were connected to the experience he had using MAPLE in his linear algebra class: The teacher was "teaching paper-pencil and saying, 'This technology can help you.' But he [was] not telling us how to use the technology" (Tech interview). Ben suggested, "If he wants to use the MAPLE program so bad, [he should] use it during class—use it when you're teaching it—and show how it reflects what this section is doing" (Tech interview). He expressed further frustration with respect to alignment:

> And then, we'd come in on test day, and we'd have to go to the lab and take the test—our test was all on MAPLE—and he hadn't taught anything about MAPLE. I guess it goes along with what you teach needs to align with what you test. Because if he would have taught MAPLE, I would have been fine. Or if he would have just done everything handwritten, I would have been fine. But the fact that he was teaching one [way] and testing another really messed me up. It was very frustrating. (Tech interview)

Ben summed up how he felt about the need for this overall alignment:

> If you're not going to use [calculators] in class, don't use them on tests. If you want to use them in class, then use them on the test. With technology, it needs to be a streamline where if you use them, you need to use them all the way through. If you're not going to use them, don't use them on anything. (Observation interview)

For Ben, technology *could* almost always be used but *should* be used only when it was seen as an advantageous pedagogical choice. When it was used, this use needed to be

fully aligned—that is, technology needed to be integrated into classroom instruction, individual work, and assessment.

<div align="center">Roles of Technology</div>

Ben's notion of the possible roles technology could play in the classroom had evolved over the course of his teacher education program. He characterized how he and others in his class felt about technology when they first entered the mathematics education program:

> We felt like we could do that math with our hand…. "Why do we need to use technology?" And I think by the end of that technology class we saw, "Hey, wait a minute. This is a little bit more efficient, a little bit easier to learn. We can actually use it—." I think we saw an importance for it in that class, where to start with, we didn't have an importance for it. (Group interview)

Ben saw this efficiency that technology had to offer as something of a default role for technology: "If nothing else, you check your work from it or you speed something up with it" (Tech interview). Although, at the very least, technology could be used to verify or expedite mathematical procedures, there were more powerful roles that Ben wanted technology to play in his classroom:

> After [the Technology and Secondary School Mathematics] class I realized how to use many forms of technology to further my interests in mathematics. I also learned how to use it in my teaching. The technology is for more than speeding up the process. This was a big lesson learned. (Final reflection paper)

The "teaching" roles that made up this "big lesson"—exploration and visualization—are discussed in the next two sections. A final section contains a discussion Ben and I had related to the interplay of these roles.

Facilitating Exploration

Ben stated, "I think technology brings discovery and this leads to learning," (Final email) and went on to say the biggest advantage of using technology in the classroom was "personal discovery" (Final email). It was through exploration that Ben saw his students making personal discoveries and, as a consequence, learning mathematics. When Ben discussed teaching with technology, he often did so in terms of the explorations he

would do with technology. For example, when I gave him various scenarios regarding access to technology, Ben discussed how he would take advantage of each situation in terms of how exploration would be hindered or facilitated. He stated if he only had five computers, he would put his students in groups and give them "questions to explore," but he did not "know how good that is because sometimes you still have those one or two that straggle off somewhere" (Tech interview). Ben was much happier with the scenario of having a computer lab with about half as many computers as he had students:

> I think in that situation, you have a lot more room to allow students to explore, because you do have two people really conversing about ideas. And with that they have a little bit more room to explore. Where before you had 5 people, you had different people, maybe one straggling off, you had 3 or 4 people just yelling out things or, you know, giving ideas. You can't explore all the ideas. So I think, with two people, you can explore more. And I think your assignment can be a little bit more extensive, a little bit more in detail. Mainly because—the time at the computer, where the two people are there—together they actually get more what I call computer time than they would if it was in a big group. I think they can make a little bit more exploration. (Tech interview)

As mentioned in the section on Ben's beliefs about mathematics, he believed an important aspect of mathematics was the logic behind problem solving and decision making, particularly with respect to real-world situations. Thus, when it came to exploration, Ben often focused on exploring a real-world situation. In this context, he saw exploration via technology as one possible approach to making decisions and solving problems. For example, what follows is an account Ben gave of a lesson he taught on descriptive statistics:

> I'm not really worried about them finding the mean, the median, [and] the mode…. I'm wanting them to see how the organizing of the data can get them to that. And how you can do it faster, with organized data, than you can with just erratic data everywhere. And I think [one student] got that when he said, "Well, we just put it in the computer now." And I said, "Exactly. A computer organizes it for you." I said, "It's still better than looking at a sea of data that you don't know what it is, isn't it?" And he picked up on it then. (Observation interview)

Ben believed that once the computer organized the data, Ben believed he and his students would be able to focus on exploring how that organization could be helpful in describing and making decisions concerning the data.

In another interview Ben described a real-life situation he wanted to use with his students when studying systems of equations. The discussion of this situation—aimed at exploring rifle trajectories—follows. It illustrates what Ben considered an exploratory problem and the role technology played in that exploration:

> Ben: The graph of a bullet … goes down, up, and then down…. But we hear people say, "My line is an inch high at a hundred yards," or "It's dead-on at a hundred yards," or something like that. Well, if it's dead-on at a hundred yards, is it dead-on going up, or is it dead-on going down? And that's the first question I always ask, and they're like, "Huh?" Most of the time it's dead-on going up, which means it's dead-on again at three hundred, or something. I mean, it's way on down. So the specific application I wanted to do with this is, when we say dead on we mean our line of sight. So we're talking about a straight line with our eyes. And if we're talking about bullet trajectory, then you have a curved graph, where that line of your sight's going to intersect in one or two spots. So, you know, two equations, two unknowns, you solve them. Actually, those equations could be very, very complex. But, it could be simplified.
>
> Keith: Where do you get those equations?
>
> Ben: The equation for your line of sight is going to be a parallel line. When I say parallel, I mean parallel with the ground. But it depends on the ground. Actually, I don't think your line of sight's parallel, because at some point it reaches the horizon. It needs to intersect somewhere. But at any rate—.
>
> Keith: A straight line.
>
> Ben: Yeah. That equation for the bullet trajectory—I guess you have to get from a chart, because there's a lot of things that go into that. There's the weight of the bullet, the speed of the bullet—. It wouldn't be that difficult to develop the equation…. The easiest way to develop that equation, though—. It just has like distances. It starts at 50 yards and goes all the way out to 500 yards. And it has two columns. One's the speed of the bullet, and one's the height of the bullet. So it wouldn't be that complex to come up with an equation for that…. Just to use Excel to input the information given off the chart, and then allow it to develop the equation. Or you can allow the students to play with it, on Excel, and then let Excel come up with the equation. Let them guess something. (PBS 3)

As can be seen in this example, the real-world concept being explored here was much more important to Ben than the fact that he might use technology to explore it. Although he did see technology as playing a role in the exploration, he did not see it as necessary. Technology was simply one tool that might help him accomplish the exploration. And in the end, the purpose of the exploration was to make a decision: With

a given rifle, if someone says their line of site is dead on, do they mean dead-on going up or dead-on going down? Ben went on to explain how the trajectory problem could be an example of what he called "an Excel problem." An Excel problem was one that involved

> having a list of data or something and then coming up with the solutions. Or just go back to my bullet trajectory problem. Say I have two different rifles with two different trajectories. And then at some point, the trajectories are going to cross, if they're both sighted in at the particular point. So through the computer, I want to find out which one's the better gun? You decide based on the equations that you come up with and based on the information you're doing on the computer problem. (PBS 3)

In both of the above instances, Ben thought technology was used as a tool for exploration; the purpose of the exploration was to enable students to make a decision.

Visualization

Ben saw technology as playing a vital role in helping students visualize mathematical concepts. One of the things he liked most about technology was that with technology, mathematics "is not just words on a piece of paper; it's something real" (Group interview). This visual aspect of technology had a number of advantages. One was that, particularly as an introductory technique, technology could provide "general pictures to grab attention" (PBS 3 email). More importantly, Ben believed with technology he could help students visualize the mathematics they were talking about in class. For instance, if he was talking about solving systems of equations, Ben thought it would be beneficial to use Graphing Calculator to graph several three dimensional figures. He wanted to "let the computer graph it, and show that you can rotate the graph. Show where the intersection points are, and show why these particular points, or why this point is the solution to the problem" (PBS 3). Ben graphed a system of equations in three variables in order to demonstrate (see Figure 1). Using technology in this way provided his students with "a visual interpretation" (PBS 3) of the mathematics they were discussing in class.

$$\blacksquare \quad \frac{(y^2 + 2x^2 + 3z^2)}{4} = 25$$

$$\blacksquare \quad \frac{y^2}{3} + \frac{x^2}{4} + z^2 = 16$$

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x : -10 ... 10
y : -10 ... 10
z : -10 ... 10
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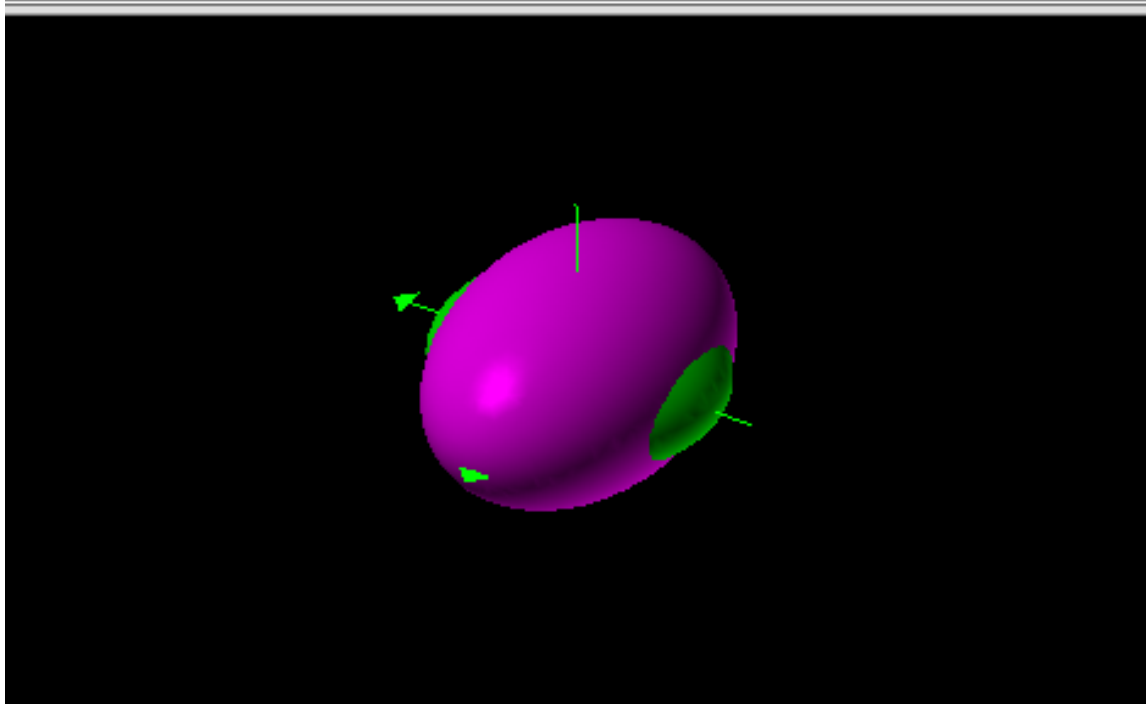

Figure 1. The three-dimensional figures Ben created to illustrate how he would use Graphing Calculator to help students visualize the concept of a solution to a system of equations.

Often when Ben talked about the visualization role of technology, it was to help visualize mathematics he felt was practically impossible to see otherwise. He recalled being frustrated in his Calculus III class because "you try to draw these graphs of things that you can't picture—you cannot see them in your mind," but "on the computer or graphing calculator, you can see it" (Initial interview). The use of technology to visualize "helps you to understand things better. You're not frustrated by the fact that you can't

draw it" (Initial interview). This was one situation that, without the visual aid of technology, Ben said, "no way I know would get the concept" (Initial interview). In essence, Ben believed technology would allow his students to "see how things work" (PBS 3). Technology helped both by drawing things you could not draw otherwise and by making it easier to understand the things you could draw by hand. The advantage to using technology was, even if you could draw it yourself, "with the technology [you] can see it" (Final interview).

Different Roles for Different Levels of Students

Ben indicated, although he wanted to use technology with all his students, he had not always thought that way:

> I used to think it depended on the course because there weren't programs and stuff that I knew about to use for certain courses. And I also thought that there were certain students that it would be harder for them to actually have to go through learning the technology than it would be to just learn the mathematics. (Tech interview)

Ben came to believe technology could be used in meaningful ways with all students. The role technology played, however, and "the amount of stress put on it," (Tech email) would depend on both the course level of the students and on the mathematical topic he was teaching. He stated, "I think [technology] should be mentioned in any situation, and I think it can be used just about everywhere. But the emphasis on how much you use it is different for different classes" (Tech interview). Ben went on to explain he foresaw this emphasis changing from "topic to topic, course to course, sometimes day to day" (Tech interview). As for the emphasis changing from topic to topic, he used factoring as an example, stating that, "I would show [my students] that you could use [technology], and how to use it, but my emphasis on it will not be a whole lot, because I want them to do [factoring] by hand" (Tech interview). In this instance, technology took on a supplemental role.

As indicated above, Ben felt the emphasis he placed on technology depended on the course. With respect to using technology to explore mathematics, Ben felt students in

advanced classes differed in the quality of questions they asked and thus in the quantity of exploration of which they were capable:

> For the advanced-level kids, there may be a lot more exploring done because I'm going to be prompting more questions. In the lower-level classes I think that they're going to have questions, but they're not going to know what to ask. So I think you're going to be spending more time asking them prompting questions to get to the right question. So, it's really going to mean less exploring. You may explore one thing a little bit more, but you may not get to explore a whole wide range of things. (Tech interview)

Additionally, Ben's purpose for using technology differed according to the class:

> In the advanced level, they know the theories and stuff, and you may want to use [technology] to speed things along. And you may end up getting their concepts out of it too, but I think that the lower-level classes, you're giving them the technology, so that in their mind it's not so cluttered—or try not to be as cluttered—so you can get to further concepts. (Tech interview)

Ben believed students in advanced-level classes could understand mathematical concepts by executing related mathematical procedures by hand. Thus, although technology may further conceptual understanding, its primary role was to expedite the related procedures. Students in lower-level courses, however, often struggled with carrying out mathematical procedures and understanding the underlying concepts. In this case, technology played a different role. By allowing technology to execute certain procedures, Ben believed he would be able to foster more conceptual understanding in these students.

Concerns About Using Technology

Ben had a number of concerns about using technology in his classroom. These were not reasons or excuses, however, for not using technology. Rather, they were issues Ben was considering—issues he felt he needed to address if he was going to effectively use technology to teach mathematics. These concerns were focused primarily on the need to teach students how to use technology and his uncertainty concerning the appropriate approach to choose when using technology.

Teaching the Technology

In response to the email question, "Do you have concerns about using technology in your classroom?" Ben responded, "The only concern I have is do the students know enough about the technology to use it without taking the time to teach the technology" (Tech email). He later stated "the biggest disadvantage" to using technology in the classroom "would be if you have to teach the technology" (Final email). The need to teach the technology was something Ben felt teachers often used as an excuse not to use it in their classrooms. He indicated those who use this excuse say something like, "I don't want to have to teach the technology. If they know it, then I would love to use it, but if they don't know it—." He explained that these people "kind of shy away from it, because they don't want to teach the technology. They don't want to take the time to do it, I guess" (Tech interview). Ben explained how he felt about this excuse:

> I understand where they're coming from, because even when I was out there, it seemed like there are certain things that have to be done at certain times—even though I kind of threw out my schedule and went to another one. But if you have to stop and take a day to completely teach the technology, that interrupts your flow of your unit. I can kind of see what they're saying. But at the same time, if you don't teach it to them, the next teacher says the same thing, the next teacher says the same thing. When are they ever going to learn it? When is it ever going to be useful to them? (Tech interview)

Although Ben was concerned about the need to teach students how to use technology, he did not see this concern as an excuse not to use technology with his students. He was concerned because he recognized this as a problematic area when it came to teaching mathematics with technology.

The issue of teaching how to use technology became problematic for Ben as he prepared for his FE. He had previously observed the class he would be teaching and was "worried a little bit about it because I don't know how much they know how to use these calculators, because I haven't seen them brought out yet in class" (PBS 1). In part because of the short amount of time he would be there, Ben did not "want the class to

become, 'Well, let's learn how to use the TI-82'" (PBS 1). He decided he would try to

evaluate how and how much the students were used to using calculators in their class:

> I'm going to let them play with the calculators and see if they really use the calculator. How much playing are they doing on the calculator and how much work are they doing with it, or are they confused with it? And I may not be able to see how good everybody is with the calculator, but I can assess maybe overall, "Is it a friend to them, or do they hate it?" (PBS 1)

When it came to computers, Ben planned to teach his students how to use

"different programs on the computer" when he needed to use them. With graphing

calculators, however, he felt his students "should learn them earlier, even as far as late

elementary school" (Tech interview). He said,

> Even though they may not know how to use [it], or know even what the rest of it's for, they're graphing stuff in elementary school. They may not be graphing equations or anything—they may be graphing sets of data and stuff like that—but it's the same principle. If they have a feel for how to use it, it's easier to expand on that. (Tech interview)

Thus, although he was willing to teach his students how to use technology, Ben believed

it would be better if they had already had technology experience before high school.

From his observation, this trend had begun:

> I know first graders who are playing on computers and stuff. I mean, I have a six-year-old cousin that can do more with the computer than I can, and I'm not computer illiterate. So I think it has started, where they're doing it at a younger age. So when they get to the middle school and high school, they're more comfortable with it—they're more willing to explore it. (Tech interview)

He contrasted this experience with the experience he had using technology in middle

school and high school: "We didn't know how to use it, and we didn't want to play with

it because we didn't know anything. So we were kind of scared of it" (Tech interview).

Knowing and Choosing the Appropriate Technology Approach

In addition to his concern about the need to teach students how to use technology,

Ben was concerned about knowing and choosing which technology strategies he should

teach. He recognized there were often multiple ways to use technology to accomplish a

given mathematical objective, and he was concerned about knowing when certain approaches were most appropriate:

> We learn all these shortcuts along the way, but when do you teach shortcuts? Or when is that confusing? When is it helpful, and when is it confusing? There's certain places where it's good to teach them, then there's certain places they don't need to know it yet…. When I'm using a piece of technology—taking the individual steps all the way out so that they see it better, or so that they can understand how to use it—my concern is that I don't do that. I do all the shortcuts and, as a matter of fact, a lot of times, I've forgot the long way. (Tech interview)

In the following excerpt Ben gives a context wherein he contrasts the short and long ways of approaching a mathematical task with technology:

> For instance, on the TI-85 you can solve for a system of equations by graphing them, or you can solve it through the equation solver, or whatever. Equation solver is what you're actually doing, but you have to plug in all this stuff. And then you have to interpret what it gives you back, because it gives you back several different things. And you have to know which numbers to look at and stuff like that. And to me that's the long way. And the shortcut way for me is just put the two equations in, graph it, and find the intersection. And I think you can teach both ways. (Tech interview)

Although part of Ben's concern dealt with knowing when to teach the shortcut, he was primarily concerned that he would not know how to carry out the procedure the long way: "My concern is somebody's going to come up and ask me how to do that equation solver, and I'm going, 'Ah, I don't know'" (Tech interview). This statement led to a discussion surrounding Ben's concern about not knowing how to answer a question about technology:

> And really, not knowing is not a bad thing, it's just that, when you're using the technology at the time, I think [the students] need to have confidence that you know what you're doing. Because if they don't have that confidence that you know what you're doing, they're not going to have confidence in what they're doing. Sometimes with math it's okay for you not to know things, and you go back and find things. But I really think with technology, you kind of need to have a good understanding of it. And, if not, know how to get there so you bring it right back to them. I think the quickness of bringing back what you don't know is important.
>
> Keith: And do you feel like in mathematics you can do that?
>
> Ben: Right.

Keith: But in technology maybe not all the time?

Ben: Well, I think in mathematics, you have a longer period of time. When you don't know something, I think you have a longer period of time to find your answer and bring it back to them. But I'm saying, with technology I think that time is restricted. I think you can not know something, but you need to be able to find what it is and how to do it pretty quick and bring it back to them. I think with math that you can bring a subject back up and deal with it at a different time and they'll come back to you. I think with the technology, it seems like if you get to that point where, "Well, I don't know the rule," you lose them. They don't continue to do other stuff like they do in a regular class. Or at least that's what I've seen. (Tech interview)

Although Ben did not believe, as the teacher, he always needed to have an immediate answer to students' questions, he saw a distinction in how he needed to respond between questions about mathematics and questions about technology. With mathematical questions, Ben was willing to allow students to see when he had uncertainty and to wait for a while before the answer was found. With technological questions, however, he felt that not knowing the answer right away would be detrimental to his students' confidence. He was concerned that, in such a situation, his uncertainty would hinder his ability to effectively use technology to teach mathematics.

Some Connections Among Beliefs

Decision making was the thread that tied together Ben's beliefs about mathematics, teaching, and learning. He saw mathematics as problem solving, and he saw problem solving as making decisions. These decisions could be within mathematics or applications of mathematics. As the teacher, Ben wanted to motivate and engage students. He could do that by being adaptive and flexible in his teaching decisions, which meant he had to know his students and then ask the kinds of questions that would allow them to teach themselves. He also believed that all students can learn mathematics if they decide that they can; the important ingredient is having confidence. Confidence allows students to take risks—it allows them to be willing to make and test conjectures and take positive criticism. Consequently, doing, learning, and teaching mathematics was not about knowing exactly what to do; it was about gaining the confidence so that you could

make a decision in the absence of certainty. One had to be willing to simply decide on something, try it out, and make adjustments as necessary. One then reflects on the experience in order to decide on possible changes for the next time around.

This theme of confident decision making continued to be seen in Ben's beliefs about teaching with technology. His core belief about the nature of technology use in the classroom was that technology should be used judiciously and consistently; his responsibility as the teacher was to make the kinds of decisions to ensure such use. Thus, Ben believed technology, although always a viable option, was not always the best option; it was often, but not always, advantageous. This belief did not mean, however, that Ben wanted to limit his students' access to technology. It was his pedagogical decisions with respect to technology use rather than the presence of technology that made the difference in the classroom. Ben felt there were times when teachers forced the use of technology, which was not worthwhile. If technology was going to be used in the classroom for a given topic, it should be used consistently—you should teach with it and the students should have access to it during instruction, while working on their homework, and when they are assessed on the topic.

Ben was concerned about needing to teach the technology to his students before he could use it. But he also recognized that at some point they would have to learn it if they were going to use it. He believed he would need to make the decision based on the situation, that is, sometimes it would be worth the time it took to teach it and sometimes it would not be worth it. Furthermore, Ben was concerned about not being able to handle questions about technology when they arose. These questions were more likely to be procedural than technical. He worried he would only know one way to do something and that might be a shortcut method and possibly not the best method for the situation. Ben was confident he would eventually be able to devise a method. He just felt that uncertainty with technology needed to be addressed more quickly than with mathematics. This difference is an example of Ben seeking coherence in his beliefs. It is different but there is a reason for the difference, so there is not conflict. The difference in this instance

was time. With mathematics, it was okay not to know the answer immediately and to let answers to questions hang out there for a while. On the other hand, Ben did not believe this time was good with technology. He believed that students expected immediate answers with technology; if they did not get those answers they lost interest or got frustrated much more quickly than they did with mathematics.

There are strong connections between Ben's experience with and concerns about technology in the classroom and his core beliefs about the nature of technology in the classroom. His belief about the judicious use of technology is directly tied to his beliefs about making informed decisions and then adjusting as you go. His experience with technology had, for the most part, prepared him to do make these decisions, but he was concerned about his ability to make these decisions at all times. Ben's desire for consistent technology use was deeply rooted in a negative experience in which a teacher had not used technology in a consistent way. Because he did not view that example as good teaching, he did not want to emulate it. Rather, he wanted to emulate what he saw as a viable solution to the problem, namely, if you use technology for a given topic, use it in all aspects of the class for that topic.

Further connections can be seen between Ben's core beliefs about the nature of technology and his beliefs about the role of technology in his teaching. For instance, although technology could expedite procedures, he viewed this role as something of a default role for technology. He believed if students have access to technology, they can at the very least use it to expedite procedures. Technology, however, could and should be used for much more than expediting procedures. Not only were there other roles (like exploration and visualization), but there was more to this expediting role as well. Just how technology was used was a decision Ben and his students needed to make. He used the example of different course levels, explaining that with students in higher-level courses, technology might be used to expedite procedures, learning those procedures in the process. By contrast, with students in lower-level courses, technology might be used

to expedite a procedure they were not yet ready to learn so as to gain access to a concept they were ready to learn.

Ben believed the expediting role of technology took on different meanings based on who the students were; it was his responsibility to make that decision. He drew a similar distinction with exploration. Students in higher-level courses were more likely to ask the kinds of questions that facilitated continued exploration in a given activity. Students in lower-level courses were less likely to ask the kinds of questions (or to know what questions to ask) that would allow them to continue the exploration independently. As such, he would need to intervene more in lower-level courses when technology was playing an exploratory role. Because of this distinction, Ben believed the exploration would not go as deep for these students as it would for the more advanced students. Ben's belief about the exploratory role of technology was also very much connected with his beliefs about the nature of mathematics and mathematics learning. He wanted his students to use technology to explore mathematics problems—or to mathematize situations—so as to make an informed decision.

CHAPTER 5: JEREMY'S BELIEFS ABOUT MATHEMATICS EDUCATION

Beliefs About Mathematics

Jeremy likened doing mathematics to cooking, with recipes being mathematical procedures and theorems. Some of these recipes "have been handed down, that we use, that I have no idea who wrote. There are some that are handed down with people's names on them, but I don't know how they got them" (Initial interview). But Jeremy did not see mathematics as just what had been handed down:

> All kinds of people write recipes. Sometimes Keith [speaking of the author] writes them, sometimes I do. And once you're finished cooking you can tell whether or not it is any good. Students can write recipes. Anybody can write a recipe; it just may turn out like garbage. (Initial interview)

Mathematics was something created by the individual, even though someone else may have done it before. Take, for instance, the writing of proofs. For Jeremy, "finding proofs—that's definitely problem solving" (Final interview). Creating a proof, like writing your own recipe, involved

> knowing where you are, where you need to end up, [and] filling in the gap. You came up with it yourself. It's a valid argument. It works. It may be that somebody else came up with [it], but you didn't know that. You came up with it. (Initial interview)

Jeremy saw mathematics as a human endeavor, one that had great impact on humankind. He credited mathematics with enabling many of the great discoveries and inventions of the 20th century. In response to the question, "If mathematics were an animal, what animal would it be?" Jeremy responded, "A dog, because it is man's best friend" (Initial email). He later added that although mathematics was a great benefit to humankind, like a dog it could turn around and bite you. Mathematics gained its power from the fact that "when you apply it, the formulas and all hold together," but that did not imply mathematics was "the best thing that's ever happened to mankind, because it's not." Mathematics and its application were always limited by the fact that they "could

falter under certain situations that [were not] accounted for" (Initial interview). The value in mathematics did not necessarily come just from its application in the real world. Some mathematics was just "a good exercise in reasoning—in connections" (PBS 3).

Beliefs About Teaching Mathematics

Jeremy was determined to create a classroom environment that would "make students desire to become engaged in mathematics" (Initial email). This determination existed, at least in part, because he had seen that good things could happen with students when they were mathematically engaged. For example, he wanted to incorporate real-world applications into his lessons because "such applications will help students to see value in the problems they are learning about and hopefully make them more interested in learning about them" (Final email). Jeremy believed that "you don't have determination to do something until you see value for it. You're not going to be determined to engage your students unless you see the value and the impact it can have on them" (Initial interview). Once he saw value in something, it became his responsibility to use it with his students. For Jeremy, "anything that can be used to help students learn is necessary for good learning" (Tech interview). If he believed something would help students learn mathematics, then he believed it was necessary that he use it in his classroom. Jeremy's sense of responsibility was connected to his belief that teaching was his calling. In fact, Jeremy implied that for teachers to be good teachers, teaching had to be their calling:

> I think that every teacher that is considered to be a good teacher, if you asked them why they're teaching, they would say, "Well, it's my calling. That's what I need to do." And I think that if you get a teacher who can't say that that's that person's calling, I don't know how they can just sit in there and do that. I've seen plenty of teachers who I think their calling is not teaching. (Initial interview)

Jeremy considered it his responsibility to provide the type of environment in which he could engage students in learning mathematics. He wanted to set up situations in which students became interested in the mathematics and then be patient and see what came of it, even if it took awhile. He approached these lessons thinking, "'Let's just see what happens.' I want to hang out there for a little while, talk with them about that—get them to talk with me about if they see something happening" (PBS 2). During these interactions, Jeremy's role was to "listen to their questions, and then I think I try to pull questions out of them. I like to hear them talk more than—I would rather them talk than me" (Final interview). Jeremy believed that, in this way, his students would be engaged in learning and he would be able to "find out how they're thinking about the mathematics" (Final email). This desire to engage students had the potential to take precedence over completing all that Jeremy had planned for a given day's class. If he felt students were engaged, he did not have a problem

> just camping out. I mean, if I have other stuff, I can push stuff back. That's not a big deal. I don't have a problem if I feel it's legitimate. And if it's a legitimate, "I don't understand," then, "Hey, I want to make you—. I will make sure you understand and do what I can to help you." (Observation interview)

Despite the immense responsibility Jeremy felt for creating an engaging learning environment, he still believed a certain amount of responsibility rested with the students. He indicated a mathematics teacher was like "an orchestra conductor, because the conductor has to direct the group in order to keep everyone together, but yet it is the orchestra that determines how well the performance will be" (Initial email). So, although the ultimate responsibility for learning lay with the student, it was Jeremy's responsibility as a teacher to provide an environment wherein students were most likely to be engaged in learning mathematics.

<div align="center">Beliefs About Learning Mathematics</div>

When asked to choose a metaphor that did *not* describe learning mathematics, Jeremy chose learning to talk. He chose that metaphor because, although there are some people with physical disabilities who cannot talk, "learning math is not limited to people

with 'normal' health. Anyone can learn to do some type of mathematics" (Initial email).
Jeremy talked about the process of learning mathematics in terms of it being a struggle.
He said,

> There's a lot of good things, though, that come out of something if you struggle
> and struggle and struggle. It's better in the long run. It may not be more exciting,
> but it's better as far as learning goes. I think you learn more that way. (Initial
> interview)

This struggle in learning mathematics was akin to the problem-solving process. Jeremy
defined problem solving as "a process of realizing that a question or series of related
questions exists that one does not have an answer to" (Final email). Learning at all levels
of mathematics, even as elementary as how "to get past 10 on the number line," drew on
this problem-solving process:

> It involves a process. You just don't automatically know that eleven comes next.
> After awhile you start seeing, "Oh, this goes zero through nine, and then it goes
> zero through nine again, and each time I pass nine this one in the tens unit goes up
> a number. Oh, I see the deal here. That's what going on." I think that's a process,
> and it could be problem solving. (Final interview)

Jeremy believed that learning mathematics took a lot of hard work but that "once
you plow through it and struggle through it and get to the end, it's more concrete in your
mind" (Initial interview). This struggle was an individual one. Although he saw value in
students conversing with each other about mathematics, Jeremy felt that in the end they
needed to do it themselves. He felt "the more experience you get with [mathematics] by
yourself," the better. Although you could "watch somebody else play with it, it's no fun
to watch somebody else play. You want to do it yourself. And you can learn more about
it that way" (PBS 2). Thus, just telling students mathematics was not a very effective way
for them to learn; students learned mathematical concepts and procedures much better,
Jeremy said, "if they derive them themselves instead of me telling them to look at the
'blue box' on page 'whatever'" (PBS 1). Jeremy expressed concern that teachers often try
to tell students mathematics even though that was not how they themselves had learned it.
He said,

We learn something and we go, "Oh, that's cool. That's how that works." And then we get this procedure built down because we like to do things that way; we like to simplify things so we can understand them. So we make a procedure for it, and then we say, "Hey students, here's this procedure." And they're just looking at us and we wonder why they're looking at us with their mouth on the floor. And it's because we're not teaching like we learned it. (Group interview)

For Jeremy, there was more to learning mathematics than learning procedures, because "people may be fine with each step of working the problem, but that does not imply that the person working the problem understands the problem" (Initial email). Jeremy believed one could tell whether a "student has learned information when he or she can apply that information to problems and make use of that information in other circumstances" (Initial email).

<div align="center">Beliefs About Teaching Mathematics With Technology</div>

<div align="center">*Experience With Technology*</div>

Jeremy's earliest experience using technology in a mathematics classroom, "(other than a four function calculator) was using the graphing calculator in the 9th grade in Alg 2" (Tech email). For this class the teacher had a classroom set, but the calculators were only handed out periodically, and "it was very rare when we would all use it together" (Tech interview). Jeremy recalled "the first time we used it—the graphing calculator—it was a TI-81 and we thought that was the biggest thing. 'Whoa, what is this? This calculator has a big screen on it'" (Tech interview). The graphing calculators intrigued Jeremy, but he believed,

> The time we spent on it was really not enough for me to catch on [to] how the calculator works. So I went and got one—an 85—and just played with it until I got it to do what we got it to do in class…. It really fascinated me, so that's why I wanted to learn more about it. (Tech interview)

Although his teacher had not used the calculator a great deal in class, she had introduced Jeremy to the technology. He considered that initial experience with the graphing calculators to have been "worthwhile—for me it was—because I wanted to know how to use it. I thought, 'If I can use this on my test, man, this will help'" (Tech interview). Thus, it was not *how* his teacher used graphing calculators but *that* she used them that

Jeremy found valuable. The teacher used graphing calculators just enough to spark Jeremy's interest. He then saw possibilities, and because these intrigued him, he went on to explore the calculator's abilities on his own.

Jeremy continued to learn how to use his graphing calculator into the next school year. When asked to describe his most positive experience using technology, Jeremy said it was "when I used my calculator effectively to make a 101 on a test in Alg 3. This was a turning point for me in all of my mathematics" (Tech email). I asked Jeremy to describe this experience in more detail:

> Jeremy: The test was dealing with roots of polynomials and synthetic division and all that stuff where you have to divide and have all of these possibilities, and you have to go find the right one: "Well, that one didn't work. Try again." I thought, "Look on this. I am not doing that. I'm going to get a good idea, and then I'm gonna show it. Because," I thought to myself, "how in the world can I get all this done and make a good grade?" Because there was a lot of problems if I remember right. Well, it took a lot of time in a 50-minute class to do that, "and I need time to work on some of these other problems. I need to spend more time on those. So if I can cut some of this out—." And I could, man. I plugged [the function] into the calculator and got an idea of where [the graph] crossed, and that really sped up the process and freed up some time to go do some other stuff on the test…. And that was the first time I had really done well on a test. And when the teacher gave it back to me she was very excited. She said, "Do you really want to see this?" And I said, "Yeah." And she said, "Well, here you go." She was all smiling and she said, "Congratulations." And I thought, "Man, that's cool." That was the first time I had ever done really well on a math test. And then I thought, "I can do this," after that. That felt really good…. I had been what I considered to be not a very good math student. I don't know what I consider myself now, but the possibility then existed, in my head, that maybe I could be a good math student.

> Keith: Now you hadn't necessarily had bad experiences in math, you just hadn't had great ones?

> Jeremy: Right, yeah…. I just didn't seem to be able to do stuff like that. I didn't have very much confidence. (Tech interview)

That this experience was a defining moment for Jeremy became even clearer in the final interview when I asked Jeremy who or what had had the greatest influence on him as a teacher. He responded,

> The "what" part I can definitely say is technology, because I've learned a lot through it over the years. I think I said it before that when I had a class one time I

used the graphing calculator, and it just really helped me to just tear up a test. And that made a big impact on me because I didn't think I could make an A on a trig test and I did. So that's the "what"—technology. (Final interview)

Jeremy was encouraged to use his graphing calculator throughout his Algebra III/Trigonometry course as well as the following year in the calculus course, both of which were taught by the same teacher. With respect to technology use in those courses, Jeremy said, "We used the graphing calculators a lot; we used them a good bit. She encouraged us to use them. But that's all we used—that's all we had" (Tech interview).

Jeremy stated that until his senior year in high school, "in the math class we never used computers at all" (Tech interview). Although he was aware of mathematics students who used the computer lab, at the time he had a fairly limited concept of the mathematics that might be done there:

> I think our labs were used for all lower-level students in my high school…. It wasn't necessarily a math lab, but if there were math students in there, they were the lower level math students, like the folks who had failed out of Algebra I in the textbook; they were there to take that Algebra I in the computer lab. And they were going to do that program, or whatever it was, on there. I never even considered that the math class would go in there…. "What are we doing in the computer lab? That's what they type papers on. Why are we going in there? You go, you type papers, you get on the Internet, and that's all. Why do I need to go in there?" (Group interview)

During his senior year of high school, Jeremy took a mathematics course at the local junior college. It was in that class that he had what he recalled as his first experience using "a computer for mathematics" (Tech interview). He described this one-time computer activity:

> We did an exploration on polar graphs on MAPLE. But it was all typed out, what we needed to do: "Do this. Do this. Do this." Okay. [He makes quick scribble sounds] And then I was done. All I remember is, "Oh, that looks cool. Okay, next?" It wasn't anything special. (Tech interview)

Although Jeremy had had other experiences using computers with mathematics, these had been on his own rather than as part of a mathematics class. He said, "[I had] used formulas on Excel" and seen "a graphing calculator program on a Macintosh before and I played with it some, but that was still on my own" (Tech interview). Despite these

experiences, Jeremy thought it strange when he started the teacher education program his junior year in college that anyone would use computers in a mathematics classroom. He explained,

> I think most of us in our class, when you were coming at us with technology, we were bringing out the guns saying, "I don't think so." At least there were several people in there doing that—some more extreme than others. I think that [one student] actually said, "I really did not like this at all. I hated it. But now I think it's great." And I think a lot of us had that same opinion because we just didn't know how to use it; we didn't know what it was good for. And like I said, my definition of using technology was, "The little red underline on Microsoft Word when you get a spelling error." That was technology; that was using technology to help me out. And somehow there was this distinction between using that, and that not being a problem, and using technology a lot in math. "Oh, if you use it in math, now that's a problem. But if I can use it consistently and spell check, that's not a problem." (Group interview)

Once Jeremy started the mathematics education program, he felt that for the first time, he was using technology for "real live learning" (Group interview). When asked whether there had been times when he had wished teachers had used technology, Jeremy responded,

> Looking back on things, I would like to have seen a lot more technology, but during the time I didn't know that it existed, so I didn't have anything to compare it to. I would like to have seen GSP. I don't know if it was around when I was in 10th grade. It probably was—they probably had some dynamic geometry. I'd like to have seen that and had some experiences with that…. In fact, my calculus teacher at my two-year school, he would say he's getting out multimedia when he brought out the multi-colored chalk—that was his multimedia. So I'd like to have seen technology there. (Tech interview)

In general, once Jeremy was introduced to a computer program, he quickly became both comfortable and proficient with it. For instance, with respect to his knowledge of GSP before entering his teacher education program, Jeremy stated that he "didn't know GSP from that, you know, brown cat outside the barn" (Group interview). He felt that one of the most influential aspects of his teacher education programs was the knowledge he had gained about technology and how to use it in his teaching: "Teaching more technology was definitely a big impact. That really helped a lot. Learning GSP was a great influence. I feel like I can use that well—well enough to get my class involved with it" (Final

interview). There was one computer program with which Jeremy was not very comfortable—MAPLE—and this discomfort was associated with a negative experience he had had with it. This experience is discussed in the next section. For now, suffice it to say, Jeremy did not like MAPLE because "the commands get strange and well, you leave out a comma or—. I don't have time for that. I want to do some math, you know? I don't have time to get the syntax right" (Tech interview). When asked with what type of technology he was the most comfortable, he first asked whether I wanted him to compare computers versus calculators or compare different computer programs. I responded that he could define it however he wished and he said,

> Well, when you say "math technology," the first thing that comes to mind is the graphing calculator—like the TI-85—because that's what I've always had and what I've always used. So that's the first thing that comes to mind. That's technology to me. I'm getting more comfortable with GSP. I really like GSP. (Tech interview)

The Nature of Technology Use in the Classroom

Jeremy believed that to teach mathematics with technology meant "to integrate technology as a tool to teach mathematics in a classroom" (Initial email), and he stated that he planned to "involve all students in technology" (Tech email). Jeremy wanted "to require [his] class to have graphing calculators from Algebra 1 up. Good ones. Not blue—more of a dark color"[6] (Initial interview). He wanted his students to have access to graphing calculators because he believed it was necessary that he use technology in the teaching of mathematics. The following section describes Jeremy's beliefs about the necessity of technology use in the classroom.

The Necessity of Technology Use

When asked whether there were mathematical topics with which he felt it was necessary to use technology to teach, Jeremy replied he thought it was "necessary to use

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[6] The blue graphing calculators to which Jeremy referred were TI-81s; the darker colored ones were the black TI-85s and the dark gray TI-83s, both of which were considerably newer than the TI-81.

technology in all mathematics above and including at least Algebra I" (Tech email). This opinion seemed contradictory, given that he had stated the desire to use technology with all of his students and that he would likely be teaching some courses that came before Algebra I. When asked to clarify his response, Jeremy explained that although he knew he would be teaching courses

> like pre-algebra and the general math, I don't know much about that. I don't have very much exposure…. So, I guess what I'm saying is, no matter what level I'm teaching, it doesn't matter; I would like to use it…. So, in that sense, it doesn't depend on what level I'm teaching. And then, "Are there topics where you think that it is necessary?" I think it's necessary above Algebra I. (Tech interview)

I asked Jeremy to clarify how he was using the word *necessary* in his last statement. His response, although used previously in the discussion of Jeremy's beliefs about teaching, is worth repeating, as it was given in this context: "I think in my class I will consider [technology] necessary, because I've seen how it can help you learn and I think that anything that can be used to help students learn is necessary for good learning" (Tech interview). Thus, for instance, because Jeremy had seen how valuable GSP was in exploring geometric concepts, he believed that GSP "is very necessary in geometry" (Tech interview) and he planned to use it in his classroom. Jeremy stated that his view when it came to technology really boiled down to the following: "I would like to teach with it" (Tech interview).

Jeremy felt so strongly about the value of using technology in the classroom that if he did not use technology, he figured it would likely be because he "just didn't feel like taking the time and the energy to put in—just, you know, lazy" (Tech interview). When asked whether there were mathematical topics that could only be taught with technology, Jeremy took a more conventional approach to the idea of necessity:

> I don't want to say that it can't be done. Would I want to teach it without it? Maybe some calculus things, some integrals—I wouldn't want to teach that without technology. And some integration techniques that are already in the calculator that I don't want to play with…. Definite integrals—all of hundreds of millions of formulas—I wouldn't want to teach without it. Could it be done? Probably. (Tech interview)

When Jeremy used the conventional definition of *necessary*, he recognized that there were very few topics, and perhaps none at all, for which technology was necessary in that you absolutely could not teach that topic without technology. But this interview excerpt highlights Jeremy's personal use of the term *necessary*. Jeremy used *necessary* in much the same way people often use the word *must*—*necessary* indicated a strong desire and commitment to technology use, not a logical absolute.

I observed Jeremy use GSP as part of a lesson he was teaching during his pre-student teaching field experience. Because he had created a number of GSP sketches in advance, I asked him whether he felt it had taken him more time than usual to prepare for class. He answered, "Average—probably on the shorter side of average. It didn't take a long time" (Observation interview). I then rephrased the question as follows: "So, the fact that you were using GSP for today's lesson didn't necessarily make it take more time to prepare than it would have if you hadn't been using GSP?" Jeremy responded, "If I hadn't been using GSP then I wouldn't have had much of a lesson" (Observation interview). Jeremy saw his use of GSP as not just integrated into his lesson but essential to it. He had designed the lesson so as to take advantage of the visual and dynamic nature of GSP, and the lesson would have been very different had he not used technology. Furthermore, Jeremy had had to go considerably out of his way to use GSP in this classroom. Because there was no classroom computer, Jeremy brought his own laptop and then took several days locating an overhead projection device. I asked him what he would do if he ended up not having GSP available where he took his first teaching position. He responded,

> Like what I did the day you came. I threw it up on the projection machine…. But then, if I don't have one of those projectors in my school—which is unlikely—I don't know what I'd do. I may take huddle groups: "Come look at my laptop, folks. You five, come on. Okay." (Tech interview)

Jeremy considered it essential that he use technology with his students, and he was willing to go out of his way to do so.

Jeremy's belief that it was necessary to use technology in his classroom was connected to both mathematical and nonmathematical objectives. What he most wanted his students to get from his class, related to technology, were the abilities "to use technology effectively as a tool for learning mathematics as well as apply technology in any other areas that are possible (which may include things I have not thought of yet)" (Tech email). Thus, Jeremy wanted his students to learn how to use technology both within and outside of the context of mathematics:

> Related to technology, I don't want them to say when they get out of my class, or when they get out of high school that, "Man, I wish I'd had more experience working with technology, because everything's going that way and I can't do it." It's important because, get some exposure to it, and they become more comfortable with it. So that's what I mean. Being able just to use Excel, for whatever they need. Then they can think of, "Hey, I need to organize this a little bit better. Oh, maybe I could use Excel. I do know how to use Excel because I learned it in his class." (Tech interview)

Jeremy recognized it would likely take a significant amount of time devoted to learning technology in order to meet these objectives. He had heard teachers use the excuse that technology "takes too much time to learn," claiming, " [I] would have to teach them GSP instead of teaching the lesson that I want to teach. I would have to teach them the TI-83 instead of teaching them how to do inverse matrices" (Tech interview). But Jeremy did not accept that excuse:

> Well, I hope that I'm not guilty of those excuses, if I'm teaching. I think that if GSP is available at my school, then I would like to have [my students] in [the computer lab].... If I'm doing Geometry class and say, "Hey, we're going to take some time and we're going to learn how to do this. And we're going to come in fairly regularly and do something on this." So, I may take a few days at the beginning and say, "All right, let's do some introductory stuff—learn how to use this." (Tech interview)

For Jeremy, technology use was valuable enough to be worth the extra time and trouble. In addition, he wanted to use available technology on a regular basis. Jeremy wanted each of his students to have their own graphing calculator, and he planned to encourage them to use it "a lot.... When they didn't use the calculators, it would be because I told them they couldn't, which would be rare" (Tech interview).

During one PBS, I asked Jeremy to categorize a list of mathematical topics according to whether he felt they had low, medium, or high potential for technology use in his teaching. No topic ended up being categorized as "low potential," so I asked Jeremy what kind of topic he would consider putting there. He responded,

> Low potential—. I don't know. I would just like to think I could always find something that I could use technology to help with. And low, to me, sounded like, "Well, you can't really use much technology with it." I like technology so much that I would like to think I could find something that would do what I want to do. (PBS 3)

Jeremy liked technology and thought of it as a necessary tool. He was committed to using it in his classroom. When asked whether he was concerned about technology replacing his students' understanding or becoming a substitute for thinking, Jeremy stated that he felt "this concern exists when someone doesn't understand the value of technology, and has been misled [as to] how it is used. Technology enhances understanding in the classroom, it doesn't replace it" (Final email). He then quoted the following excerpt from the NCTM (2000) *Principles and Standards*: "Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions" (p. 25). Given that this quotation stated that technology *should* not be used to replace understanding, I asked Jeremy whether he felt it *could* be used that way. He responded, "I guess there's certain ways that you could use it that would not be beneficial, but I don't know how to get an example of that" (Final interview). In fact, the only disadvantage of using technology in the classroom that he could think of was that he "may have a technologically rich classroom and send a student on to another level only to have the student find out that technology isn't valued in another classroom (this may be a stretch)" (Final email).

An "Unnecessary" Experience With Technology

As was the case with several of the other preservice teachers (PSTs), Jeremy had had a fairly poignant negative experience with technology involving the computer program MAPLE. He took a Linear Algebra course in which the students used "an

interactive textbook with special linear algebra packages that we loaded onto MAPLE"
(Tech interview). Jeremy was disappointed with the way MAPLE was used. Despite the
interactive textbook, he did not believe that the professor had integrated the program into
the course. There were two main problems. First, homework and assessment were not
aligned; second, the program was used simply as a means of executing procedures rather
than as a means for understanding linear algebra. Because of this lack of integration,
Jeremy thought the use of MAPLE was unnecessary, if not detrimental.

With respect to the lack of alignment between homework and assessment, Jeremy
explained, "[On the homework], we were assessed on whether or not we could play with
MAPLE. We were not assessed on what we knew about linear algebra" (Group
interview). Furthermore, students "would use MAPLE on everything and then have to do
tests by hand, no technology" (Tech interview)—"not even a four function calculator"
(Group interview)—"which really makes sense, you know? 'Test what you teach.' I don't
know, call me crazy" (Tech interview). Jeremy went on to explain that, although the
teacher finally let students use the computer for the final exam, they were not allowed to
use the "linear algebra package for MAPLE that had all these nice extra commands.... It
was just the plain MAPLE program. So that gets you an invalid assessment" (Group
interview).

In addition to feeling frustrated over the disjunction between how technology was
being used on homework and on tests, Jeremy did not believe using MAPLE helped him
understand linear algebra. Consequently, he felt that this use of MAPLE was pointless.
He gave the following example to illustrate how he and his classmates used MAPLE in
fulfilling their homework assignments:

> We looked in that book, and we would find a command that was close to what we
> wanted—eigenvalues or eigenvectors—and it was supposed to give us this long
> equation. And we were excited when we punched in the command, and it gave us
> an equation. It's like, "A blue equation popped up—score!" And they were right,
> and we had no idea what they meant. And so we would get a good grade on our
> assignment and that's what kept us passing. (Group interview)

What bothered Jeremy the most about his use of MAPLE was that he did not understand what he was accomplishing when he used it. He described this situation as "ridiculous because we were just doing stuff just to try to get partial credit for something just so we could pass the class. We weren't learning a bit of linear algebra" (Tech interview). When asked whether the technology could have been used in a positive way in that class, Jeremy responded,

> There has to be some way where that could have been beneficial. But the fact is, I don't know how I could have better understood the mathematics. I understood the MAPLE commands … but I didn't know what that was telling—. I didn't understand, "Well, what is this?" (Tech interview)

Thus, in this experience, Jeremy viewed using MAPLE as unnecessary because it did not facilitate his understanding of linear algebra. He explained,

> If I get an answer, I think I need to know what it means…. Maybe I had a minus sign where there shouldn't be a minus sign at all, and it gives me an answer and I don't know how to tell whether or not that answer makes any sense based on whether or not I made a typo. And I ought to be able to tell that, I think. Maybe that's a bad illustration, but I think I should be able to interpret my answer and know where it came from. (Group interview)

Roles of Technology

A number of the roles Jeremy saw technology playing in his classroom involved having technology do something that students would also know how to do by hand. In each case, however, technology added something to the learning experience that Jeremy determined had made using technology worth the effort. For instance, one of the reasons Jeremy was pleased with a lesson he had taught using GSP was that using the program in front of his class had caught his students' interest and attention:

> At first they were like, "Oh, that's pretty neat." And they all stopped for a minute and were like, "What's going on up there?" It was something out of the ordinary they could look at and go, "Hey, that's cool. What are you going to do with that?" (Observation interview)

Thus, one advantage Jeremy saw to using technology was that it had the potential to motivate his students to be more engaged in the lesson. In general, however, this potential was only a side benefit. Jeremy was not using technology *so that* his students would be

motivated; he simply recognized that using technology captured their attention. The primary roles he envisioned technology playing are discussed in the following sections.

Same As by Hand

Jeremy stated he wanted "technology to be able to free up 'dirty work'" (Tech email). He used a number of terms for "dirty work": "'dirty' calculations" (Tech email), "'dirty' paper-and-pencil arithmetic" (Final email), "nitty-gritty calculations or drawings" (Final interview) and "long grueling process" (Group interview). In addition, Jeremy gave a number of reasons why he wanted technology to play this role. For instance, he wanted technology to take care of these calculations so that students could "do without worrying about making silly errors" and "concentrate on more interesting ideas" (Tech email)—"concentrate on more in-depth stuff" (Final interview). Jeremy believed that when technology took care of the dirty work, it freed "up students to gain more understanding" and allowed them "to work more freely with problems that may not be as feasible to work on with only paper and pencil" (Final email).

When I asked Jeremy to talk more about this idea of technology taking care of the dirty work, he provided a number of specific examples. The first example came from the experience discussed previously—when Jeremy used his calculator to "ace" an Algebra III test:

> Finding and checking all those possibilities for the roots of the polynomial—that's dirty work. If you can know that, "Hey, negative three is a root," then I can check negative three in synthetic division and say, "Here's the synthetic division. I know how to do synthetic division, here's the result." And that way I don't have to show a hundred different possibilities—that's dirty work. (Tech interview)

In this example, Jeremy emphasized that he knew "how to do synthetic division." Because he knew how to do synthetic division, and because the method involved a significant amount of trial and error, he was able to use the calculator to replace the dirty work of guessing which numbers to try.

The next example Jeremy gave involved GSP. During PBS 3, he had mentioned that he would use GSP when teaching a lesson on the area of polygons. He wanted to do that because "in GSP, constructing an interior polygon region—measure the area of it, and you can measure the area on GSP—boom, like that" (PBS 3). This was another example of allowing the calculator to do dirty work:

> If you're playing around with area of something, and you're changing the figure, and you have the area right in front of you—you see it and it says, "Area of 'the thing' is 'blob'"—then you can test different ideas that you have. Like maybe I want to know if [the] areas of two things are the same, or … how it changes when I change the radius of a circle, "Well, how does the area change?" You know, I can see it and I don't have to calculate it—each time it's there. That's kind of dirty work, after a point. I think students should know where the area of a circle comes from—how to find it. But after that, then, "Hey, we know that, but let's move on and try other things." (Tech interview)

Again, it was important to Jeremy that students understand the concept around which the activity was centered, but there were peripheral procedures and calculations that technology could execute so as to allow them to focus on more important aspects of that concept. The purpose of using technology to do the dirty work was for students to better understand the central mathematical concept. The absence of this dual purpose was one of the reasons Jeremy had struggled with the use of MAPLE in his linear algebra course. With respect to using MAPLE to find eigenvalues Jeremy stated, "We had to do some by hand. That is a long, grueling process. And why would you want to do that by hand? You shouldn't. But I should know what's going on there" (Group interview). Even when Jeremy wanted students to know how to do a mathematical procedure by hand, there came a "point to move on and look at some technology…. After a certain point, when they get a really nasty one, maybe they can stick it in there and go for it" (Tech interview).

One other example of this role of technology came from a discussion about solving equations in an Algebra I class Jeremy had observed. He described the situation as follows:

> There was an Algebra I class I was observing where they were working on slope and slope-intercept—and they had four pages of problems to do during one class period. And [the teacher] would just kind of sit up there and talk and … I was thinking, "Man, what are you—? You're killing them! You're giving them a hundred problems to do"—he didn't say there were a hundred problems, but that's about what they had to do. (Tech interview)

There were two reasons why Jeremy felt that this teacher should have used technology in this situation. The first was similar to the reason given in the preceding example. Jeremy felt that it was necessary, "at some point, to move on and look at some technology…. Some [calculators] have equation solvers, and after a certain point, when [students] get a really nasty one, maybe they can stick it in there and go for it" (Tech interview). The second reason Jeremy wanted technology to be used in this situation was for variety:

> I thought, "You know, you could break out the graphing calculator here, at least, and do something with that. We could talk about *x*-intercepts. We could get them in the lab and use Excel and say, 'Hey, let's find out when two lines intersect. Let's see if we can do something to figure it out in some other way—just to let them know there's another way out there than just, 'take it to the other side, change the sign, divide' or whatever. (Tech interview)

For an assignment during his post-student teaching seminar, Jeremy wrote a letter responding to a hypothetical parent's concern regarding the requirement to purchase a graphing calculator for their child's use in Jeremy's class. The three main points made in his letter offer a fitting summary of the role of technology discussed in this section and its relation to Jeremy's beliefs about the nature of technology in the classroom. These ideas were as follows: (1) technology is a tool that can allow students to avoid careless mistakes; (2) technology enhances learning because it provides variety; and (3) technology does not replace thinking.

Dynamic Illustration

Jeremy viewed technology as a means to dynamically illustrate mathematical concepts. For instance, Jeremy planned to use GSP when teaching a unit on transformations because one of his objectives was for his students "to be able to visualize moving things around in the plane" (PBS 2). Having spent several days and taught one

lesson in the classroom where he would do his pre-student-teaching field experience, Jeremy was aware early on he would have very little available technology in that setting. Although he was willing to bring in his own laptop computer, there was only "a small little TV screen in front of the classroom" to which he could attach it, which was insufficient because he did not think everyone could see the small TV well enough for the technology to "do what [he] wanted it to do" (Initial interview). I asked him what it was he had wanted the technology to do for him in that lesson. He responded, "I could have animated that thing to show them that the angles are summing to the exterior angle out here" (Initial interview). I invited Jeremy to show me in GSP how he would have used it to make this point. The following discussion occurred as Jeremy constructed what is shown in Figure 2:

> I want angle EGB to stay the same, I think. But, either way, I could still show, if I did an animation here, that these [angles A and E] are going to sum to give me that [angle EGB]. So, if I did that animation—. That's the way I was giving it to them. I think it was 145 maybe, or something like that. Yeah, because they wanted to say that these two were 72.5 and this was 145. And I was thinking, "You know what? If I could just snatch that on up a little bit, I could show them, 'Hey, that's not always—if I just do this a little bit, I change the angle. It's not 72.5 anymore.'" I think it would have been powerful. (Initial interview)
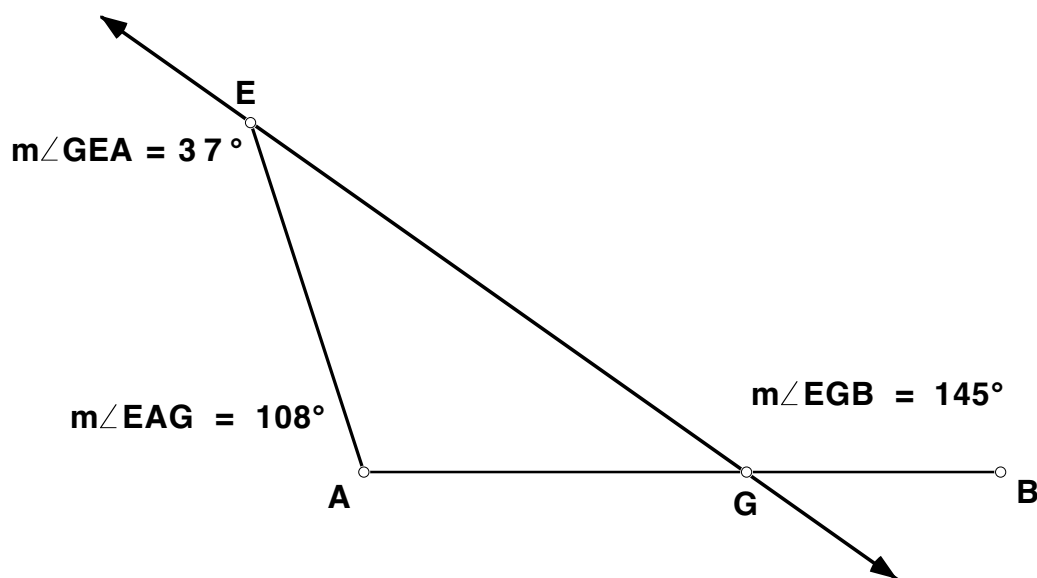
Figure 2. The GSP sketch Jeremy used to discuss the relationship between an exterior angle of a triangle and the sum of the opposite interior angles.

Another time Jeremy had designed several GSP sketches to help him teach vectors. He wanted to use GSP because it would allow him to illustrate how "the vector makes the figure [move]…. I could mark vector on GSP and translate by marked vector and show them that, 'Hey, you know what? The vector just moved all the points.' And the relations would be the magnitude" (Observation interview).

On another occasion, Jeremy indicated he wanted to "use GSP to help measure ratios" (PBS 3 email). When I asked him to show me what he was thinking about, he got a disk out of his backpack and inserted it into my computer. He opened GSP and opened a sketch I had seen him use during the lesson on vectors described in the previous paragraph. Jeremy then proceeded to create a ratio on the screen and animate the sketch (see Figure 3). He explained his reason:

Figure 3. The first GSP sketch Jeremy used to describe how he would use the dynamic nature of GSP to explore the concept of ratio.

> This is just the distance ratio, but it is a ratio anyway. It's the ratio of the length from B to B' of [the] image and the pre-image to the distance between the two parallel lines. [He clicks the animate button] And, okay, that's very slow, but the ratio is not changing when I animate the script. The ratio is the same here. (PBS 3)

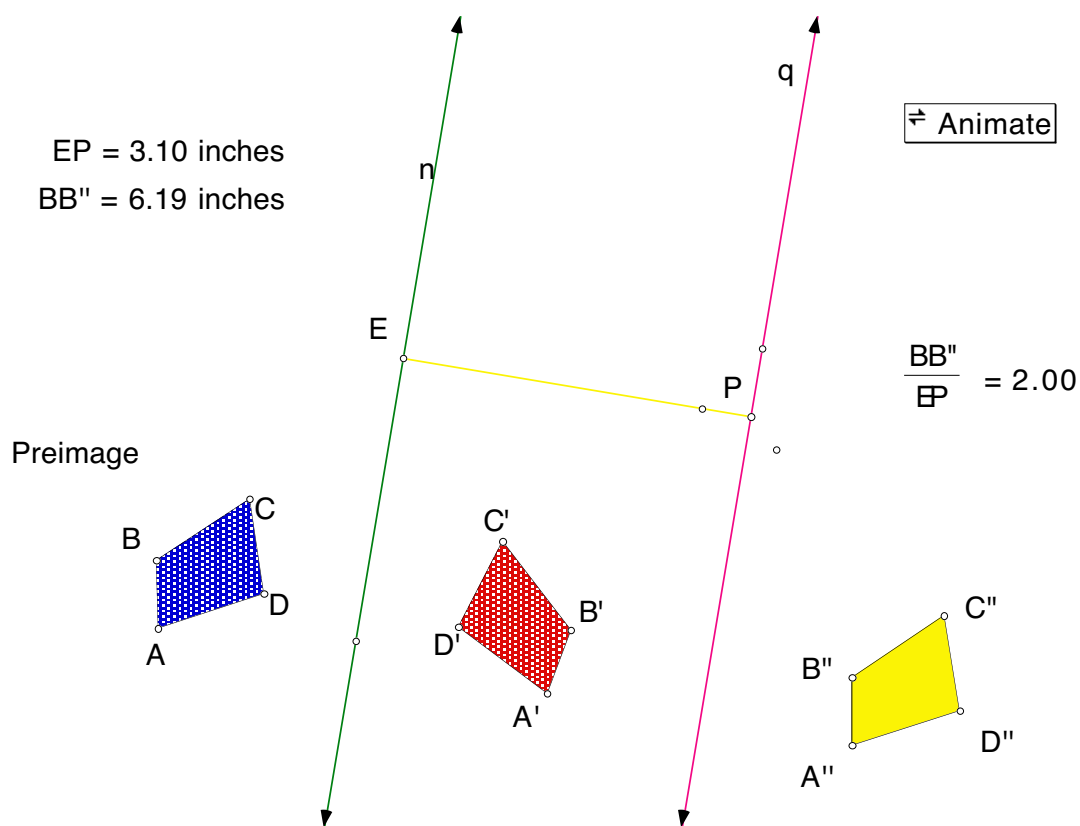Once again, when Jeremy considered using GSP to illustrate this concept, he thought about the dynamic nature of GSP and how animation would help students understand the concept. He then opened another sketch, constructed a rectangle, and proceeded to set up the ratio of the length and the width (see Figure 4):

> I'm measuring some lengths here. [He measures lengths and then uses the calculator to set up a ratio of the lengths.] There's a ratio and I can change it. There's a ratio of two sides of the rectangle. And now, if I move it, and I get one close to the 1.61 ratio, which looks about like a credit card and I have the ratio. (PBS 3)

$$m \overline{FE} = 2.93 \text{ inches}$$

$$m \overline{EA} = 1.82 \text{ inches}$$

$$\frac{m \overline{FE}}{m \overline{EA}} = 1.61$$
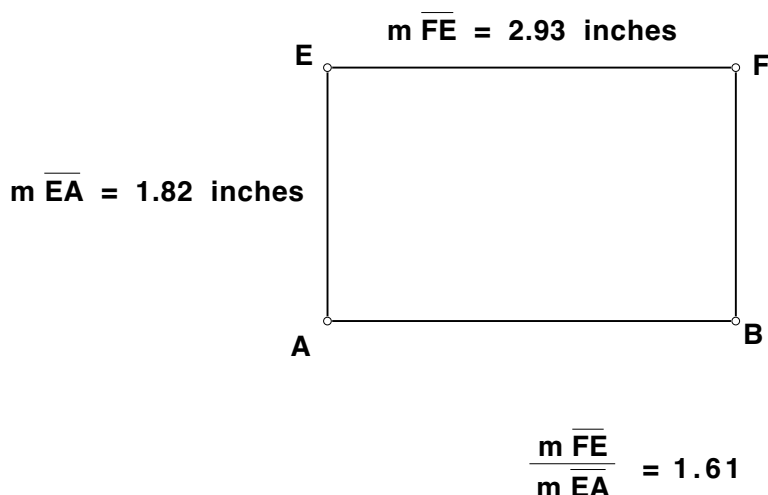
Figure 4. The second GSP sketch Jeremy used to describe how he would use the dynamic nature of GSP to explore the concept of ratio.

I had asked Jeremy to show me how he would use GSP to talk about ratio. He set up the ratio of the two sides of a rectangle and then seemed to wonder what he was going to do with it. What he decided to do was take advantage of the dynamic nature of GSP to approximate a golden rectangle. Jeremy had had a positive and confirming experience using GSP in this way with his students. He had asked his students to fill out evaluation forms after his pre-student-teaching field experience and was pleased when "lot's of them

commented saying, 'We really like the geometry program. It was easy for us to see … up on the screen, what you were talking about. And that made sense'" (Tech interview).

Facilitate Exploration

In addition to illustrating concepts, Jeremy saw the dynamic nature of technology as facilitating the exploration of concepts. Using GSP as an example, he explained that, when "doing geometry, I can have it animate, and watch the figure change. And we can make conjectures about what's going on there and test them out" (Initial interview). When asked what he saw as the difference between exploration with technology and exploration in a pencil and paper environment, Jeremy said, "I guess I'm thinking mostly in terms of geometry" and then went on to explain, "I think it would actually be easier on technology—in something like GSP it would be definitely easier—it's dynamic. I mean, you can change it real easy. You don't have to erase—waste paper or time" (Final interview).

Jeremy believed both the dynamic nature of GSP and its ability to expedite calculations facilitated mathematical exploration. The interaction between these roles can be seen in the following excerpt, as Jeremy described how he used GSP in a lesson I observed him teach:

> I had a GSP sketch already planned out. And I had put in this plan for today, the lectures I wanted to do, and the questions that I wanted to ask…. Like in the first one I did, where we had the two parallel lines…. They had this big long theorem and it was like half the page in the book and I thought, "You know what? They aren't even going to look at that…." So I said, "Maybe I could do something here where we can make the measurements." And I had it do an animation where it would go. I wanted them to be able to come up with that theorem in a better-looking form than what it said in there. And I was happy that they did that. (Observation interview)

Jeremy used the dynamic nature of GSP, as well as its ability to calculate and track measurements, to facilitate classroom exploration. From this exploration he and his class were able to derive the theorem he wanted the students to know and understand—an understanding he did not think they could get from reading and discussing the theorem from the textbook.

It was not only the dynamic nature of technology that Jeremy saw as facilitating exploration, nor did he see technology as the only way to explore. But technology was the tool to which Jeremy planned to turn first. Jeremy defined "playing around with mathematics" as "exploration, investigation" and "making conjectures" (Final email). He gave the following examples as situations in which he would have his students play around with mathematics: "With patterns or pattern related problems. For example, triangular numbers, Fibonacci sequences (using a spreadsheet), or towers of Hanoi (using manipulatives—pegs and disks)" (Final email). Jeremy also found technology fascinating because it allowed him to explore connections:

> I liked when we got up here, and we started using the GSP stuff—the way algebra and geometry are very much connected…. When we used GSP, learning how to construct the square root of 2, which I used to think, "Oh, that's an algebraic deal, the square root of 2." But actually, the construction of it, that's using geometry. (Initial interview)

Although Jeremy wanted to use technology to explore mathematical concepts, he did not believe technology was sufficient for that exploration. For instance, he believed technology could help you make conjectures, but it could not prove those conjectures:

> GSP does not really give you a proof, although it can support your conjectures. So you can find out things on GSP that may very well be true and that can be proved, but just because you did several examples doesn't mean that you've proven that particular issue. (PBS 3)

Concerns About Using Technology

Although, as mentioned earlier, Jeremy was not concerned with technology replacing his students' understanding, there were several related issues about which he was concerned. In particular, Jeremy was concerned about his ability to *see* what his students were thinking:

> When using graphing calculators, if students just give you an answer, you can't see what they're thinking. Like if they're giving you an assignment, and they just give you an answer and they're using a calculator—there's no explanation of where it came from—I can't see a thing. And the thing that you hear a lot, when teachers are using graphing calculators, "Show me the buttons you push if you use it. And just tell me how you got the answer, so I can see what you're thinking." I

think that that helps me to see what they're thinking, if they comply by that. If they don't then I don't have a clue as to what they're thinking. (Final interview)

Jeremy felt the need to see what his students were thinking so he could assess their understanding. Thus, although he believed technology could foster his students' understanding, he was concerned that it could potentially impede his ability to assess that understanding. He said, "I want to know that students haven't just learned how to get answers with technology, such as graphing calculators, but that they understand the mathematics behind what they are doing" (Tech email). Recall that this was one of the major concerns Jeremy had about the way he had used MAPLE in his linear algebra course—he could get the program to give him an answer, but he did not know what the answer meant. Jeremy wanted to ensure not only that his students knew how to use technology to give them an answer, but also that the answer had meaning.

One reason Jeremy was concerned about students' understanding what their answers meant was connected to his awareness that technology could often be misleading without that understanding. He described using technology without understanding as using "technology inappropriately" (Tech email). He explained,

> Like when the calculator may be not telling the whole picture…. Like if I have a fifth degree polynomial, and in my window I see it crossing twice and think, "Well, that's the only time…." Or if I see it crossing once and say, "Well, that's the only time it crosses," and don't even know if it's possible that it crosses some more…. Or on the TI-85, if you graph x to the two-thirds, I don't think that it really graphs x to the two-thirds. I think that it has an error going on there. I think you have to graph x to the one-third and square it to make it do right, or you have to just graph it twice and do two sides of the graph, If I remember correctly…. You have to do something to fix it. (Tech interview)

Jeremy posited that one way he might assess whether his students understood mathematical concepts when they had been using technology was to see what they could do without it. Thus, although his students would almost always have access to technology, "the exception would be when I would say, 'Okay, I'm not going to let you do calculators this time because I want to see your whole thought process. I just want you to pour out your brain on the paper, and I don't want to see the calculator' (Tech

interview). In general, however, Jeremy believed that if he taught the right way, he would be able to assess his students' understanding as they were using technology, because "technology enhances understanding in the classroom, it doesn't replace it" (Final email).

Access to Technology

Because Jeremy believed it was necessary he teach with technology, he was "concerned that it won't be readily available" (Tech email). This concern was manifested primarily when he expressed the desire to use technology frequently with his students. For example, although Jeremy recognized that most mathematics *could* be taught without technology, "not considering schools' money situation and all that, yeah, it should be" (Tech interview) taught with it. He wanted to require his students to have their own graphing calculators "depending on [their] economic status" (Initial interview). While brainstorming about the kinds of problems he would have his students work when learning about polygon similarity and congruence, he explained that if his students had technology, "they would use it as much as possible" (PBS 3).

Jeremy was also concerned that, although graphing calculators and computers would likely be available in his school, specific computer programs might not be available. Although he wanted to use GSP with his students "if GSP is available" (Tech interview), he recognized the possibility that he might not have a computer lab, let alone GSP. He resolved this concern, however, fairly easily. When asked what he would do if his school did not have GSP, Jeremy simply referred to the time during his FE when he had brought his own laptop and "threw it up on the overhead" (Tech interview). Because Jeremy believed it was necessary that he use technology in his classroom, he was not frustrated by his concerns about assessing student understanding and having access to technology. He viewed these concerns as challenges to overcome so that technology could be used in his classroom rather than as excuses why technology could or would not be used.

Some Connections Among Beliefs

Individual responsibility is the theme that cuts across Jeremy's beliefs about mathematics education. Individuals construct mathematics for themselves; it is hard work, but anyone can do it if they just put forth the effort. Jeremy believed that, as a teacher, it was his responsibility to give students the best shot possible—to do everything in his power to help them learn. This meant taking advantage of every teaching strategy he knew to be helpful. In order for any teacher, including himself, to live up to these high expectations, teaching needed to be a calling; Jeremy believed that teaching was his calling. Although Jeremy recognized that the ultimate responsibility for learning lay with the individual students, he believed that his responsibility was to do everything in his power to help students learn. The primary theme here is individual responsibility and hard work; learning is a struggle and takes place through the problem-solving process.

Jeremy's core belief about the nature of technology in the classroom was that technology was a necessary tool. He used the term *necessary* to mean he should teach with technology as much as possible, not that he could not teach without it. He believed that technology should be used and, furthermore, that it should be used to foster understanding. Jeremy felt that he could always find a valuable way to use technology. He believed that the way to integrate technology into his teaching was to make it an integral part of his instructional approach. In this approach, it should be used to foster conceptual understanding. Otherwise, it was a waste of time.

Jeremy believed that his calling as a teacher implied that it was necessary for him to use technology. If he did not, then he was not being a good teacher. Jeremy's negative experience with MAPLE brought out his desire to use technology to facilitate mathematical understanding. He wanted this use to be aligned with his objectives and instructional approach. Not surprisingly, Jeremy's biggest concern with technology had to do with availability. If someone believes that it is necessary to use technology, naturally they would be concerned about the availability of technology.

Jeremy believed there was value in using technology in almost any situation as long as its use facilitated understanding. His definition of technology as necessary did not mean that technology was the only thing that could facilitate understanding, nor did it mean that technology should always be used. Sometimes he would not use technology, sometimes other things like manipulatives were helpful, and sometimes, once technology had filled its role (like helping to form a conjecture), the other work (proving the conjecture) needed to be done separately.

CHAPTER 6: KATIE'S BELIEFS ABOUT MATHEMATICS EDUCATION

Beliefs About Mathematics

Mathematics, for Katie, was a way of thinking. Although "everyone already has mathematical thought" (Reaction email), the language used to communicate those thoughts does not come naturally to people: "The basic idea has always been in people's minds, it's just that we've had to learn how to communicate it better (Reaction email). Thus, as students learn of these ways of communicating, "mathematics actually creates a new way of thinking" (Initial email). This mathematical way of thinking was a way of explaining the world—something Katie wanted her students to see:

> Math actually makes sense and explains something in the world…. I really hope I get to do some word problems with them 'cause they don't even see it. They just think math is numbers and doing things that somehow work out and, "look, you got an answer, and that's what we get graded on." And I want them to be able to see that they actually explain and represent things…. I want them to realize that math really does explain things and things make sense. (PBS 2)

Not only did mathematics explain the world around her, but everything in mathematics was also explained. Katie liked that "with mathematics, everything is plain—it's not just like a freak of nature. Everything's explained and it's pretty cool" (Initial interview).

Viewing everything in mathematics as explained did not mean, however, that it was simplistic. When asked, "If mathematics were an animal, what animal would it be?" Katie described mathematics as "a chameleon, because it is a complex creature (all creatures are complex) that is ever changing in the eyes of the beholder" (Initial email). When asked to discuss this response further, she added,

> Who knows if your purple is the same color of purple as mine? And then it can be changing—there's just so much happening…. Everybody takes a different meaning in mathematics, too, depending on what your job or your everyday experiences are, what you learn, the way you perceive…. It's just constantly—. Everything is just changing. (Initial interview)

Katie saw mathematics as fascinating and constant. People's understanding and perceptions of mathematics, however, as well as their ability to see mathematics, were not constant; they changed and grew. This understanding also differed from person to person, and that made it challenging and exciting. Mathematics was about patterns and describing complex situations with simplicity. It was a game, a puzzle—and "who doesn't like puzzles" (Initial interview)? When asked whether someone who did not like puzzles could still enjoy mathematics, Katie responded, "Oh, yeah. Definitely there is still a possibility that they could enjoy mathematics. 'Cause when you're explaining, math is everywhere anyways, and so it's not like they find it as a game the same way that I do" (Initial interview). Mathematics was everywhere, however, because it explained everything that was out there, not because mathematics itself was out there. Katie clarified that mathematics, after all, was "all in your head" (Initial interview).

<div align="center">Beliefs About Teaching Mathematics</div>

The motivation of students was one of Katie's primary objectives as a teacher. She wanted her students to be excited about mathematics and she felt that the key was that she be excited. She stated, "If the teacher is excited about the subject being taught, then this excitement spreads to the students, and motivates the students to learn the material" (Initial email). Unfortunately, viewing mathematics as fun and exciting was something Katie felt she had lost somewhat while in college, and she wanted to get that back again:

> Sometimes, when I'm teaching, I feel like I don't give off a "fun persona" which is not what I want to do. Sometimes I feel like I'm too "mathematical" and boring in my teaching. I used to be "fun" in front of class, but now I feel like college has made me a "dull" person. I strive to be "fun" again, because I think me enjoying math will rub off on the students to enjoy math. (Katie's additional emails)

Katie believed student motivation was requisite for learning. She recognized that in order to motivate all students, she would need to take on multiple roles in the classroom—roles that were determined by the needs of her students. Some students "might need to be motivated more than other students and that might require more like

coaching" while other students "can't focus very long so you need to be more of a performer for them … and keep their attention going" (Initial interview). Although she felt there were multiple roles required of her in order to motivate students to learn, once students wanted to learn, Katie's role became very specific:

> A bad math teacher will tell students if they're right when they ask if their answer is correct. If this teacher were good, they would lead the student to determine on their own if their answer is correct. Also, a bad teacher just tells a student how to do something, or what the answer is, rather than helping the student to do it on their own. (Initial email)

Katie did not want to be one of those teachers who told students everything they needed to know. In particular, rather than just telling them whether their answers were correct, she wanted to lead them to discover that on their own. Katie and I discussed this role in our first interview and later that night she sent an email with the following clarification:

> Ideally, I think a teacher should not just give a student the answer or the method for doing something; rather, a teacher should help students discover it on their own. But many times I think it is okay for a teacher to give answers to students, or for teachers to tell students directly if they are correct, because in certain cases, doing this will increase a student's self esteem and confidence level. I know if the students are able to figure out they're correct on their own (or with the teacher's help in directing), that this can be more rewarding in the long run for the student, but many times students are not motivated enough to do this. So, in certain cases, I think a teacher can increase students' interest in the subject by reassuring them that they are correct. (Katie's additional emails)

Given a certain level of student motivation, Katie thought of alternatives to simply telling students whether their answers were correct. She did not want to be the only authority in the classroom. She expressed the desire to "have another student actually do the correcting…. So it's not like I'm going to be the sole mediator for showing the correct way if they do it wrong. I'd like to have another student show the correct way of doing it" (PBS 1).

Katie believed that "a classroom with a motivated intelligent teacher is environment enough for optimally learning mathematics" (Initial email). To be a great mathematics teacher one needed to have "a great knowledge of the material being taught

because then he/she can play off of the students more naturally" (Initial email). Katie had excelled in both secondary and undergraduate mathematics and believed she had an excellent understanding of the secondary mathematics curriculum. Knowing the mathematics was by far the most important aspect of being prepared to teach any given lesson. She did not feel that she needed to write out a lesson plan, or even necessarily plan ahead of time what she was going to do in class. This preference made it difficult at times for her to respond to the PBS emails. In Katie's PBS 1 email, she indicated that for her lesson on the Cartesian coordinate system, she would use "random examples." When asked to clarify what this term meant she explained, "some map up on the grid, or locations of things, or have students suggest things. I don't know. I just don't want it to be some boring thing I've already come up with. I don't know" (PBS 1). Katie often used the phrase "I don't know" in our interviews; she used the phrase 44 times during PBS 1 alone[7]. Although this phrase seemed to be a part of her regular speech pattern, it was often used in the context of describing how she might teach a given mathematical concept. In these situations, the phrase did not seem to mean she had no idea what she would do. Rather it meant she did not want to think about the specifics at that moment. She preferred to just wait and see when the time came:

> I don't know. I just don't like planning stuff. I don't like to sit down and write everything out. I just like to get up and do it, and I do well usually. Like tutoring—I like to tutor. It's just like you don't know what you're going to be doing—I don't know. I think if you know math, you should be able to come up with something to do. I just don't like thinking about it. (PBS 2)

Katie's notion of being prepared was based on knowing the mathematics, not on knowing beforehand exactly how she would teach the mathematics. In many ways, extensive planning took the fun out of the teaching activity, an aspect of teaching she needed if she was going to be excited enough about the mathematics to motivate her students.

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[7] For the sake of clarification and readability, the phrase "I don't know" has been removed from a majority of Katie's quotations outside of this section.

Beliefs About Learning Mathematics

Katie compared learning mathematics to learning one's native language, drawing similarities between the stages of these learning processes. She indicated both mathematics and language "are difficult in the beginning because you have to memorize the basics of the language." Eventually, after you practice manipulating the basics, you become better at using both mathematics and language to "explain more precisely the world around you" (Initial email).

Katie believed repetition and motivation were the primary factors that enabled her students to learn the basics. Katie's FE was with a small Pre-Algebra class consisting primarily of students who were repeating the class. With respect to how these students learned mathematics, Katie "figured they needed that repetitiveness" (PBS 2). To learn the basics, she believed it was important "to do repetitious stuff … to get it in their heads" (PBS 1). Katie described the basics as the things students "have to know and understand" before they "can start to grow and see things differently and on their own" (Initial email). Katie described this stage of the learning process:

> I want to give adequate time for the students to work on their own so they can begin thinking on their own. This is why during the lesson I want to give the students time to work independently on problems before actually giving the solution because this is training them how to do it on their own (learning is taking place). (PBS 1 email)

Katie believed that, although it was more productive for students to figure things out on their own, "many times students are not motivated enough to do this" (Katie's additional emails). She wanted students to "try and think and get their brain cells working" rather than "just sitting there watching the teacher" (PBS 2). She wanted them to recognize that "you have to spend time thinking on your own to get it" (PBS 2).

Katie also believed students "always have different ways of figuring things out on their own" (Initial interview). She indicated that "some people are visual learners, verbal learners, or even hands-on learners" (Initial email); some learn well when they are shown "step-by-step how to work a specific problem" and others "can conceptualize procedures

more naturally than others" (Katie's additional emails); some "might need to be motivated" and others "can't focus very long so you need to keep their attention going…. There's just all different types of students and people, and different things work different with different types" (Initial interview).

Katie indicated that when it came to her students learning mathematics, "they can all do it and understand it." The key was whether they were "motivated—it's all about that" (PBS 2). The combination of different learning styles and levels of motivation was the big difference between how the students in her Pre-Algebra class and students in more advanced mathematics classes learned. Although students in higher-level courses could "get the concept doing only a couple" of problems, the students in lower-level courses needed the "repetitive stuff" (PBS 2). They needed

> repetitiveness … because just for them to do one or two problems—they might not even do those two problems on their own, you know. They might end up just copying it or having a neighbor—. Having more, they can't just sit down and have their neighbor do all of them for them, you know. And this would make them be held more accountable to do it on their own and practice. (PBS 2)

By contrast, Katie believed she could expect "a lot more" from the more advanced mathematics students. For one, these students could "pick up on certain things better, with more understanding, and quicker;" they were also "more motivated to work certain problems or talk about mathematics … and stuff like that" (PBS 2).

Beliefs About Teaching Mathematics With Technology

Experience With Technology

Katie had quite a bit of exposure to computers in her high school mathematics classes. She recalled that, when she was in high school, "all of our math teachers—pretty much once every two weeks we at least went to the lab" (Group interview). Katie remembered using some sort of graphing program as well as a spreadsheet program in the computer lab, but she could not recall how she had used them. She explained,

> So I was exposed to computers and how to use stuff, it's just that I don't recall any specific program I used. But we did stuff. And my senior year we even had to come up with our own lab. That was one of our assignments: Pretend we're the

teacher and we had to make our own lab with the program and stuff like that. So apparently I understood how to use the programs at the time. (Tech interview)

Although Katie could not remember the details of this early use of technology, she did remember that she "never felt confident with my ability at that time" (Tech email).

Early in her college career Katie took a mathematics class in which they did a considerable amount of work using MAPLE. This work primarily consisted of doing "out-of-class projects using Maple. We also had to take a few tests, including part of our final, using this program" (Tech email). Katie's teacher used MAPLE for demonstration during class, but he did not spend class time teaching them how to use the program. Katie spent quite a bit of time in the teacher's office learning the necessary commands and earned an A in the course. She indicated that MAPLE was the technology with which she was the least comfortable, stating that, "I still don't like MAPLE too much, just because they do use different syntax stuff" (Tech interview). Once Katie was admitted to the mathematics education program, she had considerably more exposure to using computers in mathematics. She had three courses that were held in a computer lab: Technology and Secondary School Mathematics, Euclidean Geometry and Non-Euclidean Geometry. Katie summed up her college experiences with technology as follows:

> I have learned how to use certain computer programs that I was never exposed to before. For example, I now know how to use the graphing calculator on the Macintosh, the Excel spreadsheet, the maple program, GSP, power point, equation editor, and how to create a web page on the Macintosh. Overall, my effective use of technology has been greatly increased since I came to [college]. (Final paper—methods course)

Katie's experiences using graphing calculators was more extensive, positive, and memorable than her experiences using computers. Her first experience using graphing calculators came in 9th grade, when her teacher infrequently used a classroom set of graphing calculators. Katie characterized this infrequent use as follows:

> We had the class set and she'd pass them out and then take them back up…. I think we did use them a couple of times over the year…. I'm sure I started figuring out a little bit, but I remember that first time I was just like, "Okay, what am I doing?" All it was were some commands I'd never heard of.. I had no idea

what they were doing and what they stood for. I didn't know anything. It was more like, "These are the commands that work for this calculator. Now do it this way." It just didn't make sense. (Tech interview)

Although when she first started high school the school only had classroom sets of graphing calculators, they soon "had enough of them to assign to each student to take home. I then mastered the TI-calculators and have been dependent on them ever since" (Tech email); "I'm so dependent on it. For anything, I break out my calculator" (Tech interview). Katie went so far as to say she was "always thinking about the whole calculator thing because I'm totally dependent upon it" (Initial interview). In each of the above instances, Katie went on to explain that being dependent did not mean she could not "totally do everything by hand, too" (Initial interview). She clarified, "Well, I can do the math usually on my own … but I'd rather have access to my calculator" (Tech email).

Although her initial experience with calculators in high school was negative, Katie's subsequent experiences were very positive. She explained that, with the graphing calculator, "all of our teachers used it. It was in their hand all the time" (Tech interview). Katie described learning how to use these calculators as her most positive experience with technology. She stated, with respect to learning "how to operate the TI-89 and TI-92PLUS calculators," that "they can do EVERYTHING!!! I LOVE THEM" (Tech email). Katie's experience using the TI-92s actually came at the end of her senior year, after she and her classmates had taken the AP Calculus exam and the teacher was "just trying to waste class time" (Tech interview). Katie described the experience as follows:

We broke out the new technology so we'd all be exposed to it. So we were pretty privileged students. And it was really cool just to be like, "Yeah. This is what we've all seen in the papers and stuff and then we're getting to use them ourselves, so it's kind of cool." (Tech interview)

By the time Katie graduated from high school she was "pretty fluent" with the graphing calculator. She felt this fluency had really helped her in college, and was surprised she had classmates "that have never even used graphing calculators" (Tech interview). Katie stated that she was "most comfortable with the graphing calculator….

Because I've used it a lot and I know like everything on there. I've used it for a long time" (Tech interview). I asked her how she would feel if TI came out with a new calculator and she was given a classroom set of them to use with her students. She replied,

> I'd feel fine. I mean, once you really learn how to use certain technology, everything else becomes more natural in that aspect too, because you just learn how computer programmers think, or something. It's easier after you learn one thing really good to jump onto another thing of similar type.... So I'd be fine. (Tech interview)

She later said, "I love the graphing calculator. I have one in my book bag right now.... Even though I have no math classes, I would still have it in my book bag—that's how much I love it" (Group interview).

The Nature of Technology Use in the Classroom

Katie believed to effectively use technology in the classroom, "technology should be a natural part of classroom instruction" (Initial email). There were three primary dimensions to this natural inclusion of technology in the classroom: 1) Students needed to have constant access to the technology; 2) technology needed to be used frequently across the curriculum; and 3) the teacher needed to have both knowledge of how to use technology and confidence in that knowledge. These dimensions are discussed in the sections that follow. A final section relates several of Katie's experiences with technology in the classroom wherein these dimensions were absent.

Constant Access

For technology to be a natural part of the learning environment, Katie believed that "students should feel free to use calculators whenever they want" (Initial email). So as to enable students to feel this way, technology needed to be constantly available. Part of this constant availability came "with having the technology in the classroom all the time" (Tech interview). Katie felt this way about graphing calculators more so than computers. She stated, "The main piece of technology I'd use in my classroom would be the graphing calculator" (Tech interview). In addition, when asked what concerns she had

about technology use she said, "I think my only concern right now is that not every student in my class will have access to a graphing calculator" (Tech email). With respect to graphing calculators, however, this constant access went beyond the classroom. Katie believed that graphing calculators, by their very nature, were more accessible than computers. She liked that "you can carry it in your hand—it's easy to bring with you anywhere" (Tech interview). Katie expressed concern about using graphing calculators every day if the students could not take the calculator home with them:

> I don't want to do it every day because then whenever they get home for homework it will be like, "I can't do this without my graphing calculator!" So I'd have to not let them get too dependent, but let them be comfortable with them. So that would require a lot of work each week … because you do still have to assign homework, even if they don't have the calculators. It would be hard to have to everyday break out the calculators … and have them return it. (Tech interview)

If the students had their own calculator, however, then Katie said that she would "use it all the time. I'd let them use it on their homework—all the time" (Tech interview). This desire for constant access was connected to her previous experience. Recall that Katie viewed her early high school experience using graphing calculators as negative primarily because they only "occasionally used the graphing calculators" (Tech email) and that the turning point for Katie, when she "mastered" the calculators and "felt more confident," was when she gained constant access to them.

Constant access to computers was not as important to Katie as constant access to graphing calculators. Yet, she hoped "the school I teach at has a math computer lab for us to use frequently," and she wanted "to have a permanent computer in my classroom that can be hooked up to the overhead projector for class discussions and explorations and lectures," (Tech email). As with the graphing calculators, this belief was connected to a negative experience with technology. As previously stated, although Katie remembered being "one of the best when it came to doing the computer assignments" in high school, the computers "still felt like a foreign object to me. This may be because we did computer work once every couple weeks" (Tech email). When I asked Katie how she

would feel about teaching a mathematics class in a computer lab where every student had constant access to a computer, Katie responded that she liked that environment because it allowed her to "feel free just to break off and do things on the computer when we find it necessary, but we don't always do that" (Tech interview). She also felt that, in this environment, the technology was "more accessible for the students and they know how to use it so it's just a natural part of your classroom" (Tech interview).

Frequent Utilization

Having access to technology was only one dimension of making technology a natural part of the classroom. Katie wanted her students to "use [technology] all the time" (Tech interview) and she personally wanted to use technology all the time as well. For technology to "be a real regular use in [her] classroom" Katie believed that she needed "to use technology (i.e., computers and calculators) throughout the curriculum to either introduce a concept, support a concept, build off of a concept, explore a concept" (Initial email), "teach it, or to show an application use of it" (Tech interview). In essence, this meant that technology could and should be used in most any aspect of instruction as well as with "every aspect of mathematics" (Tech email). Katie did not want the use of technology "to be an interruption or a foreign type of object to" (Initial interview) her students. She went on to explain:

> I remember towards the beginning of my high school career we'd get out the calculators and it was just like a whole new thing…. I want it to be more like a pencil and paper. I want it to be more of just this natural part of your learning environment rather than, "Okay, let's break out the calculators and do this." And it's kind of more separated from the actual task of learning—it's just like a different part. (Initial interview)

Katie looked forward to when she would be a fulltime teacher because she would then have "my own classroom and my own schedule and then I can use it as frequently as I want so that it won't be something foreign to my students to be exposed to…. I want to use technology on a regular basis" (Group interview).

Katie described natural use of technology in the classroom as using technology as "a continuous, ongoing part of their learning so it will be more natural, like simple arithmetic" (Tech interview). Just as arithmetic becomes taken as given in later mathematics courses, for Katie, technology needed to be treated the same way. A teacher achieved this natural state by viewing technology as one of the several ways one could represent mathematics:

> If you're given a word problem, there's all different ways of representing it. You can write it out as an equation, as a t-chart, as a graph—I mean, it's just all inter-related. And holistically it's all one thing. I don't see calculators as just something that's separate. I see it as explaining the problem at hand. I think it can be used with everything, every problem you have…. I think it should be used frequently. (Initial interview)

Katie wanted to turn to technology in her teaching just as she would turn to making a table or writing an equation. When exploring a mathematical topic, it was important to her to use these various inter-related ways of representing mathematics; similarly, Katie wanted technology to be one of the various strategies to which she turned when she taught mathematics.

Teacher Knowledge

Katie believed that, as part of this ongoing use of technology, she needed to make it clear to her students why they were using technology. She did not want her students to have the same experience she initially had of being confused and even not liking technology because they did not see the point in using it. This meant that Katie needed to have a strong understanding of the technology herself; she needed to be familiar with and understand the commands and capabilities of the calculator. As stated in the section on Katie's beliefs about mathematics, Katie believed that, to be an excellent teacher, a teacher had to "have a great knowledge of the material being taught because then he/she can play off of the students more naturally" (Initial email). This knowledge was important with respect to technology because Katie believed that "the teacher should naturally want to use [technology] … but at their discretion" (Initial interview). She

believed that to be able to act "naturally" in front of the students, she needed to really know both the mathematics she was teaching and the technology she was using.

Katie believed her knowledge and confidence with graphing calculators would allow her to use them in a natural way; she was not as comfortable with using computers in the classroom and this made her feel less confident in her ability to make computers a natural part of the learning environment: "I'm not really up to date with using the computer in the front of the classroom. And so, as I get more comfortable with that, I would see myself maybe trying to use it more in a natural way" (Tech interview). We see here a connection between Katie's comfort level and the way she wanted technology to be a natural part of the classroom. She felt she was more capable of making calculators a natural part of the classroom because (1) they are handy and can be pulled out and used at any time in the class, and (2) she was very comfortable with them and so using them came naturally to her. On the other hand, she felt that the less-accessible nature of computers, and her lack of experience with them would hinder her from making computers a natural part of the classroom.

Several Not-So-Natural Experiences

Katie's desire for technology to be a natural part of the classroom was strongly connected to her past experiences. The experiences Katie described as being negative, with regards to technology, were classified as such because the technology was not used in a natural way. Consider her early high school experience with calculators and computers. Katie classified this as a negative experience, primarily because it failed to meet the "natural" criteria of continuous, ongoing use and constant access. Katie recalled, "After we were finished with the graphing calculators, my teacher picked them back up. Throughout the rest of the semester, we occasionally used the graphing calculators" (Tech email). This occasional use left Katie feeling "lost and confused. I had no idea what I was doing" (Tech email). Similarly, because they only "did computer work once every couple weeks," Katie "never felt confident with [her] ability" (Tech email). In addition, Katie was able to do very well in class, even though she remembered being

"clueless. I don't even know if I was figuring it out right" (Tech interview). She was able to do well because they did not use calculators on the tests. In reference to this unnatural use of technology, Katie stated, "I was just disturbed by the fact I'm using them. I was not wanting to. After that I was like, 'Well, this is pointless. Why are we doing this?'" (Tech interview). She wished the teacher could have found a way to "teach the calculators to be almost natural" (Tech interview).

There were similar characteristics in the negative experience Katie had using MAPLE in college. The program was used infrequently, primarily outside of class, and the connection between class content and the assignments done with MAPLE was not made explicit. Katie described these problems as follows:

> The reason it was sooooo bad was because our class was NOT in the computer lab…. We went to the computer lab maybe once at the beginning of the semester, but that was it. To do well with a new program, you need a lot of practice. Our professor expected us to be experts with the program with no in-class, teacher-supervised practice. (Tech email)

Because Katie believed that, to effectively use technology in the teaching of mathematics, technology needed to be a natural, regular part of the classroom, she had a real dilemma when it came to her FE. She wrote the following note on the cover page of the first draft of the unit she was preparing for the field experience:

> No technology will be used in this unit. I debated about using graphing calculators when teaching how to solve systems of linear equations; however, since the students NEVER use the calculators and can't use them on the test, I decided we shouldn't use them for this lesson. (Original lesson plan)

Knowing that Katie had thought a great deal about this decision and had a number of reasons for having made it, I later asked her to explain more about it. Although being aware that students never used calculators in that class was the primary reason behind her decision, there was more. Since they never used calculators in their class, Katie would have had to spend time teaching students how to use them. Given that she would only have ten days with them, she decided "the whole calculator thing would just throw them off and it would be more of a hassle rather than helping them see the actual concept take

place" (PBS 2). She had considered taking a full day to get the students acquainted with the calculators, but decided "that's kind of pointless for them because they never use the calculators" (PBS 2). She discussed this issue again after she had completed the FE: "I didn't use technology with my little class because I was only teaching for 10 days, and so I didn't feel as though I had enough time to introduce them to a whole new way of thinking" (Tech interview).

In the end it was the combination of "the time frame I have, and them never being exposed to calculators" (PBS 2) that made the decision clear for Katie. She could not see a way to use the calculators in a natural way with those students because, in the short amount of time she had with them, the calculator would be nothing more than an "occasional supplement" (Tech interview). Katie had had the opportunity to observe a classroom where the technology was used as an "occasional supplement." Although she wasn't pleased that the teacher simply "broke out the graphing calculators one day," she still felt that that use of the technology was "worth the effort" because at least some of the students were exposed to the technology and they knew "it's out there … in case they ever have the desire to learn" (Tech interview). However, based on the reasons previously stated, Katie chose not to use technology, even in this limited way, during her pre-student teaching field experience.

Roles of Technology

One of the reasons Katie loved technology, in particular graphing calculators, was because "they can do everything" (Tech email). She remembered being "amazed at how it could do everything" (Tech interview) she was doing in her high school mathematics classes. She believed that "pretty much all operations can be facilitated using technology, that's why I think technology should be used as much as possible," (Tech email) "with everything—every problem you have" (Initial interview). There were two primary roles Katie saw technology playing in her classroom: enhancing mathematics procedures and facilitating conceptual understanding. The following sections describe these roles.

Enhancing Mathematical Procedures

There were a number of ways Katie saw technology as a tool for doing the same procedures she would have the students do by hand. Allowing the calculator to do these procedures made "everything so much more simple" (Initial interview). Katie envisioned enacting this role of technology in her classroom by "first learning stuff" by hand "and then [using] it to see … how to do it much simpler and quicker. So then it would facilitate that when we do the next thing. So we could do that quicker" (Tech interview). In essence, once a procedure had been learned by hand, it was then time to learn how to do it on the calculator. From that time on, rather than doing it by hand, using the calculator became "a shortcut to do something" (Tech interview). Katie believed this role of technology could "make math appear simpler and more applicable to my students" (Tech email). She further explained that, with technology, mathematics

> can be applied easier…. It's less work. You can just put something in and you understand the outcome, so that's all you wanted to know. So it's easier for you to apply it to something, rather than worrying about, "Did I make a mistake here?" or something like that. (Tech interview)

Knowing the calculator could quickly do the same procedures she was teaching her students to do by hand allowed the calculator to play a verifying role. Katie appreciated using technology to "check your work," partly because "it's fun to see what I'm doing is actually correct, even just on the calculator" (Initial interview). Katie remembered technology having played this role for her when she was taking calculus in high school. Although in her class they were learning "how to do derivatives and all this other complicated stuff," her friends "who were taking AP physics at the time" were "working with derivatives as well, but they learned the shortcuts…. And so they knew how to use the functions on the calculator and they showed me and I was like, 'Yes!' so I could check my work every time" (Tech interview). Whether technology was doing the procedure for her or providing a check for her manual calculations, technology "helped things go by quicker" and "saved a lot of time" (Tech interview).

There was another reason Katie wanted to use technology to execute the same calculations students were doing by hand. While Katie was describing ideas for teaching about Cartesian coordinates, the following interaction occurred:

> Katie: I wish I could bring them into some computer lab that has a program where they have to answer certain questions dealing with coordinates. That would be more fun for them, or just hands-on for them.

> Keith: Tell me what you were envisioning there, with this program that would ask them questions about coordinates. What are you picturing?

> Katie: I don't know, just like the same general questions you could have on worksheets, but have them on computers, but maybe have more fun, and innovative in some sense…. And cool graphics, nice colors. (PBS 1)

The role Katie saw technology playing in this example was non-mathematical in nature. The computer program would present the same thing the students would have seen on a worksheet. Technology would make it more fun, however, because there would be "cool graphics" and "nice colors."

Katie believed technology played an important role when it came to accuracy. She stated that, with technology, "you get more accurate pictures, especially with geometry" (Tech interview). More important still was the ability of technology to provide accurate calculations. When asked whether there was any mathematical topic that required technology, Katie said,

> Well yeah, I mean, go with really big numbers for graphing, or when you have messy numbers, or even the square root algorithm. I'd much rather use a calculator to get an accurate decimal representation of that if for some reason I needed a decimal representation…. There's a lot of little things…. We can develop our own estimate of what things could be—an estimated graph, an estimated something—but it won't be accurate. So when we want an accurate reading of something complex, well yeah, technology should be necessary. (Tech interview)

Katie saw technology as capable of doing things that she and her students could not do by hand, e.g., it could execute calculations much more accurately. In addition, technology could execute calculations with big or messy numbers—calculations that were

impractical, if not impossible, to do by hand. If one wanted to do those calculations, technology was necessary.

Facilitating Conceptual Understanding

Katie often used phrases such as "see what's happening" and "see what's actually taking place" when she spoke of using technology in the classroom. Sometimes the phrase referred to literal visualization, other times to figurative visualization. As for the literal meaning, Katie recalled being "amazed at how [the graphing calculator] could do everything that we were doing and it was … right there in front of you" (Tech interview). She believed that technology could help her students "visualize things that they can't draw better. Or where they can move the pictures—that will help. They can visualize certain things better" (Tech interview).

Most of the time, however, Katie used these phrases in a more figurative sense referring to understanding. She talked of using technology as means of seeing "how what you're doing by hand really is happening" (Initial interview). So, in addition to technology making it quicker and easier for students to do mathematics, Katie believed that technology made it quicker and easier for students to understand mathematics. She stated, "Technology can expedite the process of mathematical understanding" (Initial email). Technology expedited this process because students could "see whatever we're doing taking place a lot more quicker and easier in front of them" (Tech interview). The mathematics was "more understandable" because the technology allowed her students "to do so much more in a short period of time" (Tech email). For instance, Katie wanted to use the graphing calculators with her pre-algebra students because

> then they can see what they've been doing and they can see more the effects of—.
> You can graph more equations in a given period of time, and that means you can
> do more and get the concept across more. (PBS 2)

Katie believed her students were more likely to understand a concept if they were able to see numerous examples in a short period of time than if they got bogged down doing each example by hand. With the calculators the students "can see what's happening and then

it's not like you have to spend 20 minutes trying to graph the little graph. They'll be able to totally understand just by looking on the calculator" (Initial interview).

Katie talked of two different kinds of understanding. As part of the methods course she had read an article by Skemp (1976) wherein mathematics understanding is discussed as being either instrumental or relational. The class often interchanged instrumental with procedural and relational with conceptual in their class discussions as well as in their written assignments. As these were the terms they had used when discussing mathematical understanding, I asked Katie to describe the difference she saw between procedural and conceptual understanding. She replied,

> Procedural is where you know how to do things step by step and what to do. And then conceptual is, you just understand all those steps together, in a sense. I mean, you know what the results are supposed to be. Procedural understanding, it's just knowing how to plug and chug and stuff like that. And then conceptual is realizing what's happening and what the solution represents…. Conceptual is being able to understand everything that's taking place and how it holds to the whole big picture—where it falls with everything else, not just the little things…. To me, that's what conceptual is. And people are always striving to get that, I guess. But we're stuck with procedural. Which is why technology might help emphasize the conceptual more. (Tech interview)

I went on to ask Katie the role technology played in helping students to gain conceptual understanding. She said,

> Some people don't seem to have the skills to do certain things. It seems like it's harder for them to learn a process and do it by hand than just to do it on computers. And they see the end results and they see what's taking place more…. I think technology really does help. Because sometimes I think if you do it by hand, then you see what's actually taking place step by step. So in a way it lets you see the whole picture, and then you see the final picture with technology. But a lot of people can't take all the little steps to put them together to see the final picture on their own and so, for the majority of people, technology helps facilitate all of that into one big picture for them. (Tech interview)

Katie saw technology as facilitating students' conceptual understanding in that it allowed them to "see the big picture." Notice Katie used the phrase "the end results." She believed that, with "technology, the result of the pictures that you got to see was totally beneficial" (Tech interview). If students could not do certain procedures by hand then let

the technology do it for them and use the result. Katie saw conceptual understanding in mathematics as the "big picture" of what one was accomplishing when doing mathematical procedures. The best way to gain that conceptual knowledge was to do it by hand, but "a lot of people can't" do it by hand *and* see the big picture. Thus, Katie felt that technology could take care of doing the procedures and allow the students to focus on seeing the big picture.

Concerns About Using Technology

I asked Katie to share some excuses she had heard teachers give for not using technology in the classroom and then tell me how she felt about those excuses. She first mentioned that some teachers think technology "will throw my students off…. It'll confuse the students. 'I'm learning the book stuff plus I need to learn how to do the calculator stuff'" (Tech interview). To avoid this, teachers need to make technology "a natural part of their learning environment" (Tech interview). Another excuse she listed was, "Financial, maybe—'Not all students can get that technology'" (Tech interview). This excuse did not hold water because "they could get the funds these days—and so that's a poor excuse" (Tech interview). The final excuse Katie said she had heard people use was that they "don't understand it themselves" (Tech interview). To this Katie said, "That's a poor excuse too because they should be learning stuff right now" (Tech interview). With regards to her list of excuses, Katie said:

> They're not good. I said all the excuses that don't count because there are no excuses that do count…. I mean, those are all poor excuses … because all of them are false. I mean that there's no excuse. Pick out each one I said and it's just wrong. All students are capable of using it, and some students are more capable of using technology than doing stuff by hand. (Tech interview)

Katie discussed one other excuse she had heard for teachers not using technology, one about which she felt somewhat less critical: "Some people don't want [students] to become dependent on it…. I'm so dependent on it. I have to say that. For anything, I break out my calculator" (Tech interview). Although Katie did not see "dependency" as a valid excuse for not using technology, she did see it as a concern—it was something she

felt she needed to be aware of and address. As previously mentioned, Katie had declared her own dependence on calculators on several occasions, always following up these statements with the caveat that she still knew how to do it all by hand. Katie's biggest concern about using technology was that students would fail to learn how to do the mathematics by hand. The next section describes this concern and is followed by a section describing how Katie reconciled this concern with her beliefs about the nature of technology use in the classroom.

By Hand or Not By Hand

Katie was concerned that using technology would take away much of what she considered to be "fun" in mathematics. She said, "But I like math, I don't want computers doing all the work…. It's hard to let go of things you like" (Technology interview). I asked her to give an example of the kind of mathematical "work" that she enjoyed doing herself:

> I just enjoy being able to do all the algebraic simplification problems. I remember I just loved to do that stuff…. Whenever I was done I was like all happy with myself for doing it correctly and stuff. But then computers can do it [snaps fingers] like that and then, there you go, no enjoyment. I mean, it's right there. So, it's cool to be able to see that you can see what you need to substitute in complex trig things and how to simplify things and other people are clueless and I feel like, "Yeah, I can do it." I just enjoyed being able to do things that other people couldn't. And so now computers do it and everybody can get the right answer. (Tech interview)

Katie felt that people were really missing out on something if they let technology do everything for them. She said, "I think a lot of people who let computers do everything miss out on letting their minds develop in that sense too. Because it opens up your eyes to see things differently" (Tech interview).

As mentioned in the section on visualization, "seeing things" often meant "understanding things" for Katie, and this was one of those instances. When Katie talked about doing mathematics "by hand" she was usually referring to understanding mathematics; sometimes this understanding was procedural, other times is was conceptual. In other words, doing mathematics "by hand" did not necessarily mean doing

mathematics using paper and pencil. Sometimes "by hand" meant understanding how to do by hand the procedures the calculator executed; other times it meant understanding the concept behind what the technology was doing.

Consider how Katie used "by hand" in reference to wanting her students to understand what technology was doing when it executed prescribed mathematical procedures. Although she thought technology could be "used with everything—every problem you have—then again, you want students to pick up some skills that you need to do by hand so that you understand what's going on" (Initial interview). She later stated,

> I know that over time the students probably will forget, for the most part, the way to calculate certain things on their own but that they will still remember how to use the calculator to solve it, but I want the students to at some point understand how to calculate it because then they have imbedded in their minds the concepts taking place, so then when they use the technology they have some idea if the output is valid (you never know when you type something incorrectly into the program being used, but you should have some idea if the output is correct—this comes from understanding the operations taking place). (Tech email)

Now consider the more conceptual meaning Katie ascribed to the phrase "by hand." For example, I asked Katie whether she felt it was necessary for her students to use a calculator when finding the square root of a number. She responded first of all by saying "I don't think it's bad because you know what a square root stands for" (Tech interview). She went on to clarify what she meant was that it was not bad *if* students understood the concept of square root:

> However, a lot of people I've tutored, whenever I tell them to take the square root they don't realize the square root is just the opposite of the square. And so I want them to get that idea, so I let them check using their calculator, but I want them to be able to give me an estimate of what's going on in their head. I don't want them to just put it in their calculator and have no idea why that's the square root of it. "Because if you multiply it two times, by itself, then there you go." I want them to know what that square root means and I don't want them to get lost and just, "Oh, let me get out my calculator." And that's why I feel it's necessary to still do things by hand…. Well, not completely, but constantly ask them, after they figure it out on the calculator, "Now, what does that mean?" But I don't know. It's frustrating. (Tech interview)

Katie was in turmoil over the issue of doing computations "by hand" versus doing computations with technology. She still wanted her students to be able to do mathematical procedures by hand because she associated this ability with understanding, yet she recognized that she herself did not always do mathematical calculations that way. I wondered what Katie meant when she said, "it's necessary to still do things by hand…. Well, not completely," so I asked her to clarify whether she was saying that it was necessary to be able to find the square root of any number by hand. She responded,

> No, but to be able to find the square root of four—I mean, they should be able to do that in their head, and some people can't because they don't really understand what that square root means. And so, stuff like that is what I mean. I want them to be able to do things by hand using variables, maybe, if that's what I'm talking about. Just the general meaning of what is taking place—the concept. And so that's where a lot of people lose with not just technology, but with learning in general. It's all about the teaching method. (Tech interview)

Students did not need to know a "by hand" algorithm for finding square root in general; what they needed was an understanding of the concept of square root. Katie mentioned two ways she could tell if they understood the concept: one, if they could find the square root of a small square number in their head; two, if they could take the square root of an expression involving variables.

Reconciliation with Nature

Katie needed to reconcile her desire to make technology a natural part of the classroom and her belief that students needed to know how to do mathematical procedures by hand. I asked Katie whether there were any situations when she definitely would not use technology in her teaching. She replied,

> I'd like to say that I would definitely use technology in all my mathematics teaching. But I do see certain situations when I would not allow my students to use calculators. For instance, I may want to give my students a memorization quiz dealing with theorems or definitions. Or I may give them a quiz where I want them to do certain math using only paper and pencils (i.e. adding/multiplying fractions, simplifying algebraic or trig functions [since the more advanced graphing calculators can now do this, I may not allow my students to use them on tests]) because there are certain things I want students to be able to do without being totally dependent on technology. (Tech email)

In general, Katie's solution was not to prohibit her students from using technology; she had other strategies to allow her to verify that her students understood. One such strategy was to allow students to use the calculator to execute mathematical procedures, rather than doing them by hand, and then require students to describe the steps they had taken with the calculator:

> I'd let them use it on their homework. But sometimes I'd have them show some steps on their homework. And if they do use their calculator on their test, I'd tell them to write down the commands they used to do certain things, if that's how they figured it out. We had to do that even on our calculus exam if we used our graphing calculator to do something—to graph a derivative or solve something with the derivative or antiderivative—we had to then tell them the commands, exactly, that we punched in, to let them know that we knew what we were doing. (Tech interview)

In addition, although Katie was willing to allow her students to have constant access to technology, this did not mean that she would constantly be showing them how to use technology to execute the mathematical procedures they were learning. She planned to show students how to do mathematical procedures by hand first, and then later demonstrate how to do the procedures with technology. I asked Katie what she would do if a student discovered the calculator procedure before she was to that stage of instruction. Here is how Katie thought she would handle that situation:

> Then I'll say, "Yeah, that's good. But I expect you to do your homework the same as everybody else right now. And then, when we get to that later, you can introduce it later if you want to. So you keep expanding with it." And I'll tell him that, "You can check your work that way. It's a good way to check your work, but it's not acceptable at this point to just do it like that." But if someone came up to me, or even in class said, "Yeah, I know you can do it on the calculator with this."—Depending on where I'm at in my lesson I may say, "Well, hold on." or something. Then work on it. At the right point I'd have them go up and teach the class how to do it. See how well they really knew it. (Tech interview)

There is a connection here between several roles technology can play and the limitations Katie placed on those roles. During the time when students were supposed to know how to do mathematical procedures by hand, the calculator could be used to check their work. There then came a point when students understood the procedures enough that they were allowed to let the calculator do the procedure instead. At this latter stage,

technology played an efficiency role, making mathematical procedures quicker, easier and more applicable. Initially, in the above example, Katie would let the student use the calculator to check their work. When the time was right she would let the student help introduce the calculator's capabilities to the class. With this method Katie was able to manage the way her students used technology without discouraging them from continuing to use the technology. This allowed her to see technology as a natural part of the learning environment without compromising her instructional goals.

Katie recognized that she was just beginning to develop strategies that would help reconcile her desire to use technology with her belief that students needed to know how to do mathematics by hand:

> It's definitely a tool to help enhance the concepts they're learning and exploration and stuff like that. But yeah, I do want them to have an idea of what they're typing in—how it applies. Duh, I mean, that's the whole point of doing it…. But the extent I would expect them to know how to do something, I don't know that at this point. That's something I'll learn when I'm teaching—what's truly important. But I want to use technology on a regular basis…. But I did used to always think you had to know how to do it by hand, and then maybe you could use the computer. But you definitely had to know how to do it by hand. But I don't know how much I feel about that now. (Group interview)

Although the issue of doing things by hand was still present, Katie's concerns with regard to computers were quite different from those surrounding graphing calculators. As previously mentioned, Katie did not see herself using computers as frequently as she would use graphing calculators, and her concerns about computer use were related to this belief. Katie realized that the only constant access she would likely have to computers would be a single computer in her classroom. In light of this concern, I presented a number of other scenarios she might encounter involving access to more than just one computer. I first asked her what she thought she might do if she had five permanent computers in her classroom:

> I think, with five computers, I would try and involve it, but I don't know how frequently I would do that. I mean, it depends on the type of class, I guess, and what we're covering, and the program. I mean, how long will it take them to

figure it out on a good enough term to not confuse them? That might be like once every two weeks or a once a week thing. (Tech interview)

I then asked her how she thought she would take advantage of a computer lab with approximately half as many computers as she had students:

Okay. Pair up. I would give them—. Ooh yeah, I would have them—. Oh my goodness, sometimes it almost seems like it takes away from class time—like it's a burden on the teachers just to go—. I'd like to say once a week, but I do not see that. I see more like once every two weeks. I don't know what kind of projects I would give them, but I wouldn't want one person doing all the work…. But I couldn't do that everyday. (Tech interview)

Katie perceived this use of a computer lab as something that would only be occasional and, as such, would not feel natural. Similar to her decision not to use graphing calculators with students during her pre-student teaching field experience, Katie had serious doubts whether it was worth the time. In addition, she was concerned about students working together and failing to evenly distribute the workload.

Finally I asked Katie what she would do if she were to teach in a classroom that had a computer for every student. She responded as follows:

Oh, my goodness. I don't know! It depends on the class I'm teaching. I don't know. Technology scares me. Is it taking over everything? Because that's like our classroom is here and we feel free just to break off and do things on the computer when we find it necessary, but we don't always do that…. I have had geometry in that room and, with geometry, most of the time we did break off and go to the computers. But second semester geometry, with spatial stuff, we didn't go to the computers hardly ever, because we had stuff like our cubes and tetrahedrons that we made—so it was more hands-on things. But I would definitely feel more confident in just saying, "Okay, go to your computers and do this." You know, because they would know how to do it, because it's much more natural for them. I mean that's very idealistic, because then you don't have to worry about teaching them commands or explaining what commands you have to do to do this new assignment you have. It's more accessible for the students and they know how to use it so it's just a natural part of your classroom. I wouldn't use it all the time. I'd be much more confident to use it whenever I wanted to, which is a lot better than just one lab for all math teachers. So I'd feel comfortable, like the graphing calculator, but not as comfortable all the time. (Tech interview)

Katie felt a classroom with constant access to graphing calculators was desirable, if not necessary, to her ability to make technology a natural part of the classroom. Although she

would "like to have a class set of computers for my students to use any/all the time," Katie explained the reason she "would not use them ALL THE TIME would be because I don't think we need/should use them all the time at this stage in our human history" (Reaction email). Katie's beliefs about using computers differed from her beliefs using graphing calculators, in part because of a lower level of confidence when it came to computers, and in part because of the less accessible nature of computers.

Some Connections Among Beliefs

For Katie, mathematics was a way of thinking that explained the world around her. She saw mathematics as a fascinating, exciting puzzle but recognized that others might see it differently. She believed if she knew the mathematics she was teaching then lesson plans did not need to be particularly detailed. In fact, she preferred to come up with a general plan and then just see what happened. One of her primary roles as the teacher was to motivate students. She felt she could do this only if she herself was excited—that excitement breeds excitement. Too, in order to foster student motivation, she needed to vary her teaching strategies according to her students' learning styles. She approached the learning of students in lower- and higher-level courses differently, very much in line with her belief that in mathematics one needs to first memorize the basics and then move on from there. Students in lower-level courses needed extra help acquiring the basics, so repetition and practice were integral to teaching at that level. Students in higher-level courses did not need as much practice to get these basics down, so she could have a greater focus on questioning and exploration. Katie needed to be highly motivated in order to engage herself and she thought of her students as not unlike herself.

Katie's experiences with computers differed from her experiences with graphing calculators. Although she had considerable exposure to computers in her early years, it was not until college that she started to see them play a role in learning mathematics. Aside from her experience using MAPLE, she came to value numerous other computer programs. Katie was introduced to graphing calculators early on as well and, once she gained constant access to them, she fell in love with them. She used them extensively in

both high school and college—the latter on her own rather than as a required part of the classes she was taking. She felt confident in her knowledge of and ability to use graphing calculators.

Katie's core belief about the nature of technology was that technology should be a natural part of the classroom. There were three criteria necessary for this natural use to take place: Technology needed to be constantly available, it needed to be frequently used, and she the teacher needed to be knowledgeable of and confident with it. When technology was a natural part of the classroom, rather than a foreign object or novelty, it became a natural part of her repertoire of teaching strategies—just one of the approaches she could choose to use most any time. Katie cited several excuses she had heard people use for why they did not use technology, excuses she believed evaporated when technology was made to be a natural part of the classroom. As such, these were not of great concern to her. Katie was concerned, however, about her students' ability to be able to do mathematics by hand in the presence of technology. Algebraic manipulation was an aspect of mathematics that Katie had always excelled in and enjoyed. She associated understanding of mathematics as much, if not more, with the ability to do these procedures as with knowing what they meant. It was difficult to infer how Katie reconciled her belief that technology should be constantly available and frequently used with her belief that students needed to know how to do mathematical procedures by hand. She felt students needed to learn how to do things by hand before becoming dependent on calculators, but believed it was necessary this learning be done in the presence rather than in the absence of calculators.

The primary role Katie saw technology playing in her classroom was that of enhancing mathematical procedures. This enhancement could come through checking a procedure that was done by hand, expediting the calculation of the procedure, or doing procedures that would other wise be inaccurate or impossible to do by hand. Although sometimes quite limited, there was almost always an enhancement role for technology to play—it was this role, primarily, that enabled the constant availability and frequent use of technology.

 Technology could also be used to facilitate conceptual understanding, and this role was very much related to the previous role. For example, technology could produce

numerous examples in a short amount of time and, through this, students could understand concepts more quickly. Conceptual understanding for Katie was the big picture of what was going on in the procedures. Also, technology could sometimes help someone understand something that they would not otherwise understand. In essence, the best way to understand mathematics was to initially learn how to do it by hand. In some situations, however, certain students are unable to understand how to do it by hand. In these situations, you can use technology so they can understand. They will not understand as much, but it is the next best thing.

CHAPTER 7: LUCY'S BELIEFS ABOUT MATHEMATICS EDUCATION

Beliefs About Mathematics

Lucy's beliefs about mathematics were primarily focused on the interplay between seeing mathematics as a set of skills and procedures and finding value in mathematics by applying it to the real world. She believed that "everyone uses some part of mathematics every day. Maybe not the same thing, but everyone uses some. It is everywhere" (Initial interview). She wanted her students to have both "application problems, as well as problems that simply test their skills. I want my students to have a balance of real-world application problems and book problems" (PBS 1). Although it was important to Lucy that her students practice mathematical procedures, she did not want them to just "'go through the motions' of solving a problem and find the correct answer." She wanted her students to "understand the importance or the reasoning of the steps within the problem. I feel that understanding the problem is as important as getting the correct answer" (Initial email). Understanding the problem meant seeing both the logic behind the steps needed to find the solution—"Why does this step go here? Why is this step three and not step two?" and the purpose in doing the procedure—"What are you accomplishing with these steps? What is it getting you?" (Initial interview).

Lucy believed understanding a problem was what brought the skills and procedures of mathematics together with real-world applications—understanding a problem was problem solving. But not all problems were problem-solving problems. Lucy saw a distinction between merely working problems, whether they were "real-world application problems" or "book problems," (PBS 1) and problem solving:

> Students would venture to say that all math is problem solving, but I would have to disagree. I feel as though some of the problems we, as teachers, have students work are "practice" problems. These problems allow students to use the method that we are currently teaching and practice that method. I feel that problem solving takes several concepts into account and asks students to think critically about the tools they have and how to use those tools. Problem solving involves

more than just "regurgitating" material that was just covered; it allows students to use brainpower and a great thought process…. In order to consider mathematics as problem solving, I think that the students have to think!! (Final email)

This combination of knowing skills and procedures and then being able to apply them to real-world situations allowed students "to think about what they've just done and how they can use that" which in turn helped students "find meaning for the math. And I think that's the biggest part of problem solving" (Final interview).

Beliefs About Teaching Mathematics

Lucy believed "that the teacher/professor makes the difference in the feelings about mathematics" (Initial email). She believed it was her responsibility as a teacher to motivate students to enjoy and want to learn mathematics and she intended to accomplish this by making students comfortable, using a variety of teaching approaches, and demonstrating the applicability of mathematics. Lucy wanted her "students to know that it is okay to struggle in mathematics and that it is okay to not understand a topic" (Tech email). She believed

> many people are intimidated by mathematics and you want to depict an environment conducive to learning for these people. You want students to feel comfortable expressing ideas and conjectures openly. Even if a conjecture is incorrect, you want students to feel comfortable expressing these conjectures. (Initial email)

One reason Lucy wanted her students to be comfortable in the classroom was so they could handle challenge. She wanted them to learn that "challenge is good and that giving up is not an option" (Tech email). Lucy recognized, however, that it was not necessarily going to be easy to challenge students, primarily because of her desire to bail them out when she saw them struggling:

> That is one of the toughest things. It is really difficult because you really need to let students struggle with things but it can't be too hard. On the other hand, I hate it when teachers just say, "Here's what you need to know" and then you just give it back to them word for word. There needs to be some challenge to it. This is one of those things I think is really difficult. (Initial interview)

Lucy believed she could make students comfortable enough to be challenged if she tried to build on their previous knowledge. She described a mathematics teacher as being like a carpenter, building a foundation of mathematics and moving on from there. She designed her lessons to build on what students already knew, and she wanted students to be aware of this connection so they realized that what they were learning was not completely new. Lucy believed this awareness would make students comfortable and give them the confidence to explore new ideas. She could then allow "students to guide me in the exploration of new ideas. As a facilitator I found that students like to feel as though their work is valued and important, even if their idea is not on the 'right' path" (Final email).

Along with her desire to make students comfortable, Lucy believed teachers needed to "know what they are teaching and know several ways of expressing concepts and ideas" (Initial email). It was important to Lucy to use a variety of approaches in her teaching and strive "to find different ways to present the material so that I could capture each student's attention and understanding" (Final email). This desire was tied to her belief that students have varied learning styles. She stated, "Since students learn differently, it is important to find different ways to relate the math to them…. I had to find different ways to present the material so that I could capture each student's attention and understanding" (Final email). Lucy did not "want everything coming from here or there. I want it to be a combination, so that they see different ideas and different types of problems" (PBS 3).

An important source of different ideas and approaches for Lucy came from seeing where mathematics could be applied in the real world. Paramount in her role as teacher, and connected to her beliefs about the nature of mathematics, was the responsibility Lucy felt to illustrate the applicability of the mathematics she was teaching. Lucy wanted her students to see that the mathematics they were learning in class was useful outside of class. She believed that a teacher could "best prepare for a lesson by discovering real-world examples" (Final email). When at one point she struggled to come up with a real-

world application, she recognized that she was "going to have to research more and more…. That's going to be part of my job once I start student teaching and actually teaching is researching where in the world this can be used" (PBS 3). Lucy indicated that real-world applications would be important regardless of the mathematical topic. When pressed as to what she would do if she could not find a real-world application, she responded, after considerable pause, "I don't know. I know there's probably going to be some things where it's going to be more difficult…. But, I'm sure there's something out there—hopefully" (PBS 3).

<p style="text-align:center">Beliefs About Learning Mathematics</p>

Lucy viewed learning mathematics as hierarchical—as a process of making connections between new and previously learned ideas. She compared learning mathematics to learning Spanish:

> Learning mathematics is like learning Spanish because there are many similarities between English and Spanish. In many words there are relationships between English and Spanish. And, in many cases people already know some Spanish without knowing. Knowing some of these words creates a "bank" with which to build upon. (Initial email)

She went on to explain that similarly, in mathematics, students know some "mathematics and have created a bank with which to build more knowledge" (Initial email). Lucy wanted students to "use previous knowledge to build on new knowledge" (Initial email). The way students built onto their previous knowledge was by making connections. Often students already had "ideas, but they don't have them all connected" (PBS 1). Learning consisted of making these connections. When she introduced a new topic, Lucy wanted to explain to students, "Look, you already know some of this stuff, so what we're going to do is just build on what you know." She felt that if they "started with what they knew" they would not "feel like they were doing anything more difficult" (PBS 1).

To be able to learn in this manner, students needed to remember the mathematics they had previously learned and then "relate one concept to another" (Final email). Lucy believed that if students "don't make those connections, it's hard to understand what's

going on" (Tech interview). As such, Lucy thought it extremely important to review previously learned concepts. As Lucy talked about how she might teach a given topic, she referred to going over ideas students had previously learned, stating that it was "important to kind of review that and get that drilled in their mind (PBS 2). This belief was related to Lucy's belief that students needed to have what she called "practice" problems. She explained that these were problems that "allow students to use the method that we are currently teaching and practice that method" (Final email). Lucy believed "most of the people who do their homework will do fine" (Observation interview). She described a time when she had given students a quiz on solving systems of linear equations and a number of them had done poorly:

> I told them three or four days in advance that they were having a quiz and that they needed to go ahead and prepare for it, and it was material that we had covered—that we had gone over and done on homework and gone over multiple examples. And throughout the quiz they were all sitting there whining about it. And after I took the quiz back up I asked them, "Who did every single homework problem I assigned?" And like three people raised their hand, and I said, "I'm sorry, I don't feel sorry for you. If you've done all that, you should be able to do this stuff…." To me that was a little disappointing because I felt like they should be able to do this stuff. It was just the same stuff they had seen before—not the same examples, not the same problems, but it was generally the same concepts. (Observation interview)

Lucy wanted her students to practice, but she did not want them to memorize. She believed that mathematics could not be memorized and that "many people try to memorize math and fail" (Initial email).

In general, Lucy believed students needed to learn mathematics "on their own" (PBS 1), stating that "each student has to think for himself" (Initial email). Students learned mathematics for themselves through exploration. Lucy believed "that allowing students to discover mathematics by exploring allows the students to find answers on their own" (Final email). She did not "want to give them any information because they should learn to be detectives too" (PBS 3). This "grappling with it on your own" (Initial interview) was ideally carried out without too much reliance on the teacher—"I don't want them to be dependent on me. I don't want to just hand them everything" (Tech

interview), on other students—"I think that teachers have to be careful not to allow students to become dependent on each other. Each student has to think for himself" (Initial email), or on technological tools—"it is basically traditional learning; it is giving an opportunity to understand it on your own before you have technology do it for you" (Initial interview). However, Lucy believed getting students to learn in this independent manner was not always realistic. One way she felt she could make it more realistic was to give some structure or guidance to mathematical explorations. Lucy had "found that even if students are exploring, they need some type of guidance or they tend to get off track" (Final email). She struggled to know just how much guidance she should give:

> I feel that "bad" teachers are on both ends of the spectrum. For example, a teacher who gives students information is as bad as a teacher who gives only theory and no examples of how to use that theory. I do not like teachers who tell students what is on a test or how to work every single problem. Who is learning??? I also do not like when teachers are so complicated that students do not understand how to apply information. Who is learning??? There needs to be a balance somewhere between these two extremes. (Initial email)

Even a balanced approach to exploration was not always realistic for Lucy. At one point she was struggling with how she would teach the procedure for completing the square. She was not happy with what she had come up with at that point because she felt that she was "kind of just giving them the stuff they need to know. And I don't like doing that, but I just can't think of another way to do it" (PBS 1). She went on to explain,

> I've tried to think of a way to teach this so that they can develop completing the square on their own, but I can't come up with it. To me it's a difficult topic and, at this point, I've kind of gone instrumentally with it, which is probably a bad thing to do. But I've thought and thought and thought and I can't come up with a way to teach it so that they can derive it on their own. (PBS 1)

Lucy believed the way she taught determined the way students understood. Whether students gained either relational or instrumental understanding was determined primarily by her pedagogical decisions as the teacher.

Lucy wanted her "students to feel as though they have accomplished learning concepts through their own way of thinking and not *my* way of thinking" (Initial email).

This feeling of independence was important to her because "students learn differently" (Final email) and "each person represents mathematical thinking differently. No two people learn in the same manner" (Initial email). Lucy believed that there were "a lot of visual people" (PBS 3), whom she described as a "large portions of students who can find meaning in mathematics once visualization has been made." There were some students "who needed more structure" and others "who needed less" (Final email). Some "need more or different explanations" (Initial email). There were yet other students who learned in a manner similar to Lucy, in that "somebody can tell me how to do something a thousand times, but until I actually do it for myself and I look at it and I see somebody do it, I don't really grasp the concept" (Tech interview).

Beliefs About Teaching Mathematics With Technology

Experience With Technology

Using graphing calculators in Algebra II during her junior year of high school was the earliest experience with technology in a mathematics classroom Lucy could recall. She remembered being impressed with what the teacher was showing them on the overhead, but not having any idea what she was actually doing on the calculator and, consequently, merely mimicking what her teacher had done when she tried it herself. She did not recall using computers in mathematics until she took pre-calculus in college. She described using "the computers for modules and taking tests and stuff, but I wouldn't consider that technology with mathematics" (Tech interview). The first time Lucy remembered thinking she was actually using technology with mathematics was her junior year in college when she took the Technology and Secondary School Mathematics course for which I was the instructor. Since that time, (one year before the data collection phase of this study), Lucy described using technology a great deal. In addition to using it in the technology course, she used technology extensively in a problem-solving course and in two Geometry courses. In fact, Lucy stated that she had used technology—in particular, GSP—"in every course since I started using technology" (Tech interview).

Although Lucy felt she had started to use technology a great deal, she still believed her knowledge and experience were both limited and limiting. She expressed concern that she hadn't "had much time with technology" (Tech interview). This lack of experience put Lucy in a situation where she did not "really know enough about technology to be able to use it consistently" (Tech interview). I asked Lucy with what technology she felt the least comfortable. She responded,

> Anything new. Over the past couple of years I've played with different programs—different applications—and each time I'm introduced to something new it freaks me out. So anytime I'm introduced to a new program or a new area within a program, I kind of feel uneasy about it. (Tech interview)

Lucy felt she did not have "a really good grasp on what technology I should use and when I should use it. I do not feel as though I am qualified to use some forms of technology because I do not have a strong background" (Tech email). For example, Lucy felt uncomfortable with her knowledge and experience with graphing calculators. Speaking of the students she would be teaching during her FE, she said, "I'm still in TI-82 world. They're in TI-83 world, so I have to get lessons on that" (PBS 2). When we spoke near the end of the FE, Lucy expressed that she could have used graphing calculators more than she did. She went on to explain,

> But I'm not proficient with it. I don't know a whole lot about graphing calculators and how they can work, so I have to sit down and go play with that. But I know I didn't use them when I could have. And I had one lesson where I used them and didn't really know what I was doing. And that kind of fell apart, so I was kind of weary [sic] of using them again. I have a TI-82 personally and they're using TI-83s, and there's a big jump between those. The TI-83 does a whole lot more than the TI-82. So, I have to kind of take one of those home and work on it and figure out what I can do with it. (Observation interview)

Lucy believed she lacked knowledge about how graphing calculators worked and that that hindered her from knowing how they could be used in her classroom.

The area where Lucy had the strongest background was using GSP. When asked with what technology she was most comfortable, she responded, "GSP. I feel like I've had the most work in that program…. To me it makes the most sense. It's easy to use. It's

easy to learn" (Tech interview). Regardless of the technology, however, Lucy still felt that her knowledge and experience limited her. Although she was most comfortable with GSP, she still expressed concerns about her ability to use it in the classroom. While brainstorming how she might introduce polygon similarity and congruence, she stated, "I know that I have used GSP in order to simulate these concepts, but I would have to take a refresher course on how to use GSP to do this" (PBS 3 email). When asked what she meant by "a refresher course" she explained, "I know that there's a way that you can copy that angle somewhere else, but I cannot remember how to do it…. So, I mean, I'll have to play with that a little bit more" (PBS 3). Whereas with graphing calculators Lucy felt she lacked knowledge of their capabilities, with GSP she felt that she knew what it could do, she just needed to review how to do it.

The Nature of Technology Use in the Classroom

Lucy described technology as a "learning tool" (Tech email) that was "wonderful" (Initial email), "handy" (Tech interview), "great," "powerful" and "strong" (Group interview). Each time Lucy used one of these adjectives to describe this tool, she added a disclaimer: "technology is a wonderful tool, if used in the correct setting" (Initial email); technology is "a handy tool, but you have to know the math before you should be able to use the technology" (Tech interview); "technology is such a strong tool, but you have to be able to use it the right way" (Group interview). Furthermore, technology was just one of the tools you could use in the mathematics classroom. Technology was a subset of a set of tools she described as manipulatives. In addition to technology and algebra tiles, using manipulatives involved "using just different—I don't like the word 'things' but—mathematical 'things' so they can see what's going on within the concept" (Tech interview). Lucy indicated that the compass and protractor fit into this category as could dice or a deck of cards. She added, "It depends on how you use it" (Tech interview).

Just as mathematical "things" were considered manipulatives for Lucy depending on how they were used, using technology in the classroom was considered teaching with

technology depending on how it was used. For instance, Lucy did *not* believe that using "computers for modules and taking tests" should be considered "technology with mathematics" (Tech interview) because this use of technology was not tied directly to the learning of mathematics. Regardless of the reason Lucy was using technology, it was critical to her that the technology be used as an enhancement to the learning process. Although this belief meant that technology use in her classroom would be connected to the learning of mathematics, it also meant that technology could not be "the primary source of learning" (Initial email); rather, technology needed "to become an added tool of learning mathematics" (Tech email).

I gained further insight into how Lucy defined teaching mathematics with technology through exploring the distinction she drew between the way technology was used in her high school mathematics classes and the way it was used in her mathematics education program. She characterized the former usage as one in which technology was "pretty much a supplement to what we were doing" (Tech interview). This meant it was used "to check graphs—make sure our graphs looked right" and to "perform operations so that we didn't have to do the paper and pencil style. We could just plug it into the calculator" (Tech interview). As such, she felt it was mostly used for efficiency-sake and as "just kind of an added interest" (Tech interview). There did not seem to be a good rationale for using the technology. In contrast, Lucy felt technology "was used completely differently" (Tech interview) in her mathematics education program. There technology "was used to really enhance and incorporate new ideas…. I believe that it's saved me time and it's helped me to form conjectures about particular types of mathematics" (Tech interview). In this latter experience Lucy saw a reason for using technology.

In describing her high school experience, Lucy used the words "supplement" and "added interest." She often used these same words when discussing how she wanted to use technology in her classroom, but she included words like "enhance" and "advantage." The distinction seemed to be that in her high school experience, Lucy did not think that

the technology was really helping her learn the mathematics—she did not see any advantage to using it. Lucy wanted to use technology in a supplemental, yet relevant way so that, although technology was not the primary source of learning, the students were "using the technology to their advantage and learning something from it" (PBS 1). For Lucy, teaching mathematics with technology meant using technology as an enhancement to mathematics teaching. Lucy described what she meant by enhancement: "I don't want it to become my whole lesson, and I want to teach based with technology. I want to teach, and then use technology within that" (Tech interview).

Just when a lesson should be supplemented with technology was of utmost importance to Lucy. For technology to enhance the learning of mathematics, it primarily needed to be used after students had mastered mathematical concepts. Lucy believed it was her responsibility as a teacher "to monitor the students and make sure that they are learning before the technology is available" (Final email). She stated that students

> need to understand the concept before they use technology. Technology is something that can come afterward to help see it better or understand it more. You need to be careful that the technology doesn't replace the understanding. Technology is an extra thing that you can add on. (Initial interview)

Referring to her observation of the Honors Algebra II class she would later teach during the FE, Lucy described seeing technology used in a way she believed was not advantageous:

> I would rather them not use calculators, especially at the level that they are. They're Honors Algebra II, so they should be able to do it without calculators, in my personal opinion. So maybe, for something like [graphing], I wouldn't allow them to use the calculators, just because that's defeating the purpose of, "Can you graph this function?" (PBS 1)

This view caused some difficulty for Lucy in that her mentor teacher allowed the students to use the calculators all the time. I asked Lucy when she would allow students to use calculators. She indicated that it would be hard to outline all the times when they could so I then asked, "When can't they?" She indicated that she did not want students to

use calculators "when they're trying to graph something" on quizzes and tests. She went on to explain,

> That's the part I have to work out. If I ask them to graph something, I don't want them to go to the graphing calculator and put it in and just find the graph and just redraw it. I want them to show me how they got the points, how they got the vertex, that kind of thing. I just have to figure out how I'm going to do that. That's going to be the hard part. And what my mentor teacher had told me is, maybe the last two minutes of the quiz, let them use their calculators. Because I'm going to be walking around while they're taking the quiz or the test, and if they don't have their graphs done when I give out the calculators they don't get credit for it. Somehow I'll make a mark or something, or maybe I'll do it in two pages and say, "When you finish this, then you can come get the graphing part, but you've got to put your calculator up." So, I don't know. I haven't decided how that's going to work yet. (PBS 2)

When discussing graphing in a subsequent email, Lucy stated, "students should be expected to graph equations, but I feel that once they have mastered this, they should then be able to use the calculator" (Final email). Lucy classified this approach to learning mathematics as "paper-pencil type learning" (Initial email). When I asked her what paper-pencil type learning was, she responded, "It is basically traditional learning. It is giving an opportunity to understand it on your own before you have technology do it for you" (Initial interview).

This is not to say that Lucy would never consider using technology before students understood a concept. Although she "would not teach a topic by using only technology," Lucy was willing to "introduce a topic with technology and then begin explaining by using a paper and pencil style of teaching" (Tech email). However, for Lucy, teaching mathematics with technology primarily meant using technology as an enhancement after students had learned a mathematical concept. In fact, knowing the mathematics was in many ways a necessary prerequisite to using technology in an enhancing way: "If you know the mathematics, you can use the technology to enhance it, and you can use the technology to actually perform the mathematics for you, but you have to know first how to do the mathematics" (Tech interview). I now turn to the

various enhancement roles Lucy believed technology could play in her teaching of mathematics.

<div style="text-align:center">*Roles of Technology*</div>

Lucy believed that technology could be used in a variety of ways to enhance the teaching of mathematics. These roles are discussed in the following sections.

A Nonmathematical Role

Lucy believed that technology "keeps the students interested" (Tech interview) in her classroom. This enhancing role of technology as a motivator was fairly nonmathematical in nature. She believed that students were motivated to learn when they used technology because they liked using it. She had "found that [students] are more engaged when they're on the computers. They're actually trying to work through problems, and they're trying to figure out what's going on, because they like the technology" (PBS 3). This view of technology as a motivator was connected to Lucy's belief that she should use a variety of approaches in her teaching. She went on to explain that technology gave students "a different approach to it, and they're more interested in it" (PBS 3). Lucy believed this enhancing role of technology was particularly valuable with her lower-level courses. She believed that she

> would use technology more with students on lower levels of study because they do not want to use paper and pencil style of learning. Lower level students are usually more interested in using manipulatives than sitting and working problems. These students do not like to do the same thing day after day (not that higher level students do). (Tech email)

In general, using manipulatives, of which technology is a subset, provided variety to Lucy's teaching and, as such, was more motivating and interesting to the students.

Recall that Lucy believed one of her primary roles as a teacher was to prove the applicability of the mathematics to her students. She saw technology as enhancing her ability to fulfill that role. Quite aside from mathematics, Lucy believed that it was her responsibility to teach her students technology as a life skill: "As we talk about real-

world applications, we must consider that we are in a technologically advancing world where students must also learn how to use technology" (Final email).

Real-world Application

Just as Lucy valued real-world applications of mathematics, she valued real-world applications of technology and, as with mathematics, this was a rationale for using technology in her classroom. Technology was not just a life skill; it also represented a real-life application of mathematics:

> Lucy: There's big companies out there who don't necessarily know that they're using mathematics, because they're using a computer to do the mathematics. And I think that a lot of students are so, "I don't see where I'm able to use this." If I can use that computer, that Excel program, or whatever, to show them that, "Look! If you're in the business world, you can use this. You're using mathematics with this computer. But if you don't understand what's going on, and you type something in incorrectly, then how's the computer going to know? You need to have the knowledge to know what you're doing is right—to tell that computer how to do it." So I think the big correlation there is that they can actually use mathematics in the real world with technology.
>
> Keith: How would you make that correlation explicit to them?
>
> Lucy: I think by example. There's going to be a time where, if you're in Excel, you're going to type in an incorrect number—incorrect word—something that's going to throw the whole entire problem or situation off. If you can't figure out what, mathematically, you did wrong, then it's going to be an experience-type thing, where you have to see it to understand how it's in relation to each other. (Tech interview)

In this interaction, Lucy described two benefits for her teaching related to the fact that technology is used a great deal in the real world. The first was that it gave her an additional way to prove the applicability of the mathematics she was teaching, something that has previously been established as important to her. Secondly, Lucy used the technology-in-the-real-world context to show the importance of knowing how to do mathematics by hand, not just on the computer, thus providing a rationale for learning how to do things by hand.

Visualization

Lucy saw technology as playing a role in helping students visualize mathematics. She had "found that technology enhances the visualization of mathematics" (Final email) and she described this role of technology as helping "students see what's going on" (PBS 1). Lucy believed that "some people are visual learners…. So actually seeing how it works, and seeing what's going on, the technology can enhance that—make it a little clearer" (Tech interview). In fact, technology had the potential to help students see things that they otherwise could not see. At one point Lucy was teaching her class "about the unit circle and the graphs and how they relate" (Final group interview). She explained,

> Until I took them to the lab and got them to construct the unit circle, construct the graph and just play with it on GSP they didn't understand what the connection was. They couldn't see how if you went counterclockwise that was making the sine wave. It was such a powerful tool. (Final group interview)

Lucy believed the mathematics she was teaching had "been around for a long time, but with technology you can look at it another way" (Initial interview). She went on to explain, "It may be the same mathematics, but we look at it in different ways" (Initial interview). These different ways of visualizing mathematics had the potential to help students understand mathematics more clearly.

As Lucy stated above, technology could help students "understand what the connection was" (Final group interview) with the mathematics she was teaching. Lucy believed technology could help "tie several concepts together, so the student can actually see…. It ties things together so that students can actually make the connection because, if you don't make those connections, it's hard to understand what's going on" (Tech interview). Technology enhanced her students' understanding of mathematics by allowing them to make connections they might not have seen otherwise.

Lucy also recognized the dynamic aspect of visualization through technology. For instance, when talking about using GSP to visualize similar polygons, Lucy said she wanted her students to "see what happens when they move around the points. When I move these around they all stay proportionate to one another. I think it's important for

them to see that" (PBS 3). What Lucy most liked about GSP was that "it's visual. You can see it; you can see what happens if you move it. Does it stay the same or does it completely change" (Final group interview)? She wished she had had access to the computer program Graphing Calculator "so that you can look at how a graph changes as you change one little thing. That's important, personally, for me to be able to see how … you can compare graphs" (Tech interview).

Although in general Lucy believed technology should be used after a concept had been understood, visualization was one role for technology she thought could take place before instruction. Lucy believed that there were "some concepts that are easier to see first, than to actually just hear somebody talk about them…. I think [technology] can be used to enhance the lesson—to enhance the understanding" (Tech interview). This belief was exemplified in a statement Lucy made when she was brainstorming about how she would teach similarity. She stated that she was choosing "a visual way to approach this topic" and explained that she recognized that "for the first time, I am basing the first knowledge of a concept on computers. I find this interesting because I said that I would like to introduce the topic first. You got me!!" (PBS 3 email).

Exploration

Lucy often talked about having her students explore mathematics with technology. Exploration was more than just "sitting there punching it in the calculator and making a guess" (PBS 1). Rather, exploration was when students "still have questions as to what's going on and why they're doing this" (PBS 1). As with visualization, exploration was a role technology could play before the students knew how to do something by hand. Exploration, for Lucy, entailed investigating a mathematical concept when students "don't have a whole lot of background" (PBS 1). Exploration allowed students to get a feeling for what was going on. For example, Lucy discussed having the students use technology to explore "what a root is, or what a zero is" (PBS 1). Once students had a feel for what a root was, then they could move on to the paper and pencil style learning of "how you find it and what's the importance of it" (PBS 1).

Lucy characterized the way she wanted to use exploration with her students as "guided exploration so as to enhance their knowledge" (Final email). There were several things Lucy needed to do in order for guided exploration to be successful. First of all, the exploration needed to be set up. For instance, if Lucy took her students to the computer lab, she needed to "have the program up, and have everything in some sort of order, and have it set for them" (PBS 3). This initial setup was there because Lucy did not want the students "to get in GSP and just start drawing all sorts of figures and shapes and not have a clue as to what they're supposed to be doing" (PBS 3). In addition to the initial setup, Lucy wanted to give the students "a guideline" whereby she could "tell them what I expect them to do" (PBS 3). This guideline consisted primarily of questions that, when answered, would take the students through certain steps, but still allow them "the chance to find relationships" (Final email) and "explore some of the scenarios themselves" (PBS 1 email).

Lucy also played a role during the guided exploration, one where she would "walk around and answer questions—leave them to what they're trying to do. Push them to go further, push them to make conjectures" (PBS 1). Lucy wanted students think through the mathematics without her telling them exactly what to do, but she wanted to guide them through the process. Lucy preferred this guided exploration, in part, because she usually had something very specific in mind that she wanted the students to discover through their exploration. I asked Lucy what some of these guiding questions might be in the context of a lesson she was planning on similar polygons. She first described the kinds of guiding questions she would place within GSP sketches as part of the initial setup:

> I'd probably have questions up there pertaining to, "What do you notice about the sides?" "What do you notice about the angles?".… I want to prompt them, but I don't want to give them the answers.… I want them to explore several different types of examples similar to this so that they can see that if it's similar then it's going to have the same properties. (PBS 3)

I then asked her what kind of instructions she would give. She indicated that initially she would

> just let them explore and move around points, see what happens when they move around the points…. Then maybe on the second and third—however many sketches I have—prompt them as to, "What do you notice about the sides of the similar triangle?" I don't want to be specific because I want them to kind of explore it on their own and try to figure out what they need to explore to begin with. And then after they've tried to figure something out, maybe prompt them along—guide them a little more—try to make formal definitions. (PBS 3)

Concerns About Using Technology

Lucy stated her "biggest concern about using technology in the classroom is that my students will know more about the technology than I will" (Tech email). She worried this disparity would cause them to view her as incompetent. Several times students had come to her with questions about the calculators for which she did not have the answer. She lamented, "I just feel like that makes me look like I'm so stupid—like I have no clue what's going on" (Tech interview). Lucy also expressed concern about the unpredictability of technology, stating, "It's kind of like you never know what's going to happen" (Tech interview). This instability had two meanings for Lucy. First of all, there was the unpredictability of the computer: "A computer could not work, or it could freeze up or—. You name it and it's happened to me, I'm sure" (Tech interview). Lucy was concerned that the technology wouldn't do what she was expecting it to do. Second, she was concerned that, when she used technology with her students, the students wouldn't do what she expected them to do:

> I'm scared to use it just for the simple fact of—What happens if I can't get the students to do this? If they don't get involved or engaged in it, what am I going to do with them? How am I going to get them back into what I want them to be doing? (Tech interview)

Not only did Lucy feel limited by her concerns about technology, she also seemed to see technology as playing a limited role in her classroom. I observed an interaction between Lucy and a student where the student was having difficulty graphing a function on the calculator. Lucy did not know how to answer his question about the calculator.

What she did, however, was point out the directions for the problem. She said, "He thought the question was, 'Graph this' and it was actually 'Solve this.' So he was pretty much wasting his time with the graph to begin with" (Observation interview). She did not appear to see the value in a student looking at a graph on the calculator when her intentions were that the student apply a procedure for solving the problem by hand.

Some of Lucy's concerns about using technology with her students were allayed because of a positive event that took place during her student teaching. She characterized her attitude at the beginning of the study and her subsequent change in attitude as follows:

> At the beginning I feel like I was kind of—not against technology, but I was like, "I don't know how I'm going to use it in the classroom. I don't know what I'm going to do with it. I don't know that I'll ever take my kids to the lab." And by the end of the survey it was like, "You know this is really powerful, and my kids really learned a lot from this." Until you actually get to have the opportunity to use it in the classroom you don't realize how powerful it is. My view completely changed from the beginning to the end. I feel like I was completely honest at the beginning when I was like, "I just don't know if I'm going to use it." (Group interview)

What changed for Lucy was her belief about *whether* to use technology in her classroom; she became convinced that she should use it. Although Lucy felt that, because of her lack of experience with technology, using it in her classroom would require extra time on her part, she was now convinced that it was worth the effort. When asked what excuses she had heard people give for not using technology in the classroom, Lucy said teachers often feel that it takes too much time—too much time to prepare and too much time away from instruction time. This was not a valid excuse for Lucy. She explained,

> If you're using it to the best of your ability and the best way for your class, then it's not going to take time away from your class and it's not going to take up more time. It's going to actually enhance what you're doing…. As far as too much work for a teacher—I don't agree with that because it's my job to help my students learn. And if it takes me two days to get this technology lesson written out and done and planned for, it's going to take me two days. I don't see how that can be an inconvenience because that's my job. That's what I'm here to do, is to help these students learn and understand. So I don't agree with taking too much of my time. (Final interview)

Thus, despite her inexperience, Lucy became convinced that using technology in her classroom was valuable and important. This conviction brought up another concern for Lucy, however, and that was that technology, if not used appropriately, might replace students' thinking. This concern was related to her insistence that technology not be the primary source of learning and that whether it was used for visualization or for exploration, technology was there to enhance paper and pencil style learning. Early on in the data collection process Lucy made the following statement:

> Well, take for instance constructions in geometry. I think you should use a compass and a ruler and straight edge and learn how to do the constructions by hand. Then, after you know how to do that, you could go to something like GSP. You need to be able to figure it out yourself. (Initial interview)

Lucy seemed to be struggling with how the relationship between her beliefs about how and when to use technology. At one point Lucy stated, "You can do almost anything with regards to geometry by using technology" (Tech email). In the subsequent interview I referred to this statement, placing emphasis on the word "can," and asked, "Should you?" A fascinating conversation ensued—one that will be discussed in greater detail in chapter 8. For now, suffice it to say, Lucy was concerned that her students would not understand mathematics, in general, unless they first learned how to do that mathematics by hand. She seemed to believe this primarily "because that's the way I learned it, was by paper and pencil. And, to me, it's easier to see it that way. For some students it might be easier for them to see it this way—in GSP" (Tech interview). Thus, it eventually came down to an assumption that, because she had learned it a certain way, that must be the way one actually came to understand it. That had been her experience—"the way we've been doing it our whole life" (Group interview).

Some Connections Among Beliefs

Mathematics, for Lucy, was a set rules and procedures that logically fit together. The value in mathematics lay primarily in its application to the real world. There were times in mathematics when more thinking and bringing together of ideas was required—that was called problem solving. The teacher's responsibility in the classroom

was to motivate students to learn. This responsibility meant the teacher could really make the difference as to whether students learned or not. In order to motivate students to be engaged in the mathematics, the teacher needed to ensure that the students were in a comfortable learning environment, use a variety of teaching strategies, and demonstrate the real-world applicability of the mathematics. Lucy believed that mathematics and the learning of mathematics is hierarchical. You start with the knowledge people already have, build on that foundation, and then make conceptual connections. The teacher's role is to guide students in this direction, but the teacher cannot do it for them—students have the responsibility, in the end, to make these connections for themselves. Effective teaching is a constant struggle to find the right balance between telling all and telling nothing; sometimes this balance is compromised because there just does not seem to be any other way to get things across.

Making connections is the common theme across Lucy's beliefs about mathematics and its teaching and learning. Mathematics itself consisted of mathematical concepts that were logically connected to each other and connected to the real world. Learning mathematics involved forging connections between previously and newly acquired concepts. Teachers motivate students by making them comfortable enough to be challenged, by providing enough variety to meet many students' learning styles, and by demonstrating real-world applications. The demonstration of real-world applications gives students a reason to learn mathematics by connecting the mathematics to the real world and it connects new concepts to the mathematics students already know.

Lucy had had very little experience with graphing calculators and no experience with computers in mathematics before coming to college and limited experience in her first few years of college. It was not until her junior year, the year leading up to the study, that she felt she had really used technology with mathematics. This lack of experience limited her knowledge of how to use technology herself and it limited the ways she felt she could use it with her students. She was uncomfortable when exposed to a new computer program or a different graphing calculator. Despite this lack of comfort, she had several experiences that influenced her belief that technology was a valuable resource and worth the extra effort it took to use it with students.

Lucy's core belief about the nature of technology in the classroom was that technology was one of many tools (what she called manipulatives) a teacher could use in the classroom. It was important to her that technology be one of the manipulatives that she used, but that it never come across as the primary source of learning. In addition, when technology was used, it needed to be used in the right way. This meant that technology should be used to enhance rather than replace the learning process. Primarily this meant that the students needed to have some basic understanding of the mathematics before having the technology "do" it—in order to enhance mathematical understanding, some mathematical understanding needed to be there to be enhanced. Besides being concerned about her own lack of knowledge and confidence with technology, Lucy was concerned that, if used incorrectly, technology would replace students' thinking.

There are two primary factors of Lucy's experiences with technology that seem to have influenced her beliefs about teaching with technology. On the one hand, Lucy had been in mathematics classes where technology had been used in supplemental ways that Lucy did not find relevant to the teaching objectives. On the other hand, the positive experiences she had using technology (primarily) in her mathematics education courses were of a somewhat different nature. In these classes technology was used to explore mathematics for which Lucy believed she already had a reasonable foundation. These factors result in a core belief about the nature of technology in the classroom that emphasizes periodic use of technology primarily AFTER students learn the underlying mathematical concepts. Lucy's major concerns about using technology were her lack of experience with and knowledge of technology (she was worried about her students knowing more than she did) and that technology would replace students' thinking. These two concerns are directly linked to the two major aspects of her belief about the nature of technology in the classroom. When technology is used as a supplement, Lucy can use technology for those topics with which she is also more comfortable with the technology, and this use allows Lucy to spend the time necessary to prepare herself to use it. Secondly, if students come to some understanding of the mathematics before using technology, she believed the technology is more likely to enhance rather than replace their thinking.

There were several nonmathematical roles Lucy saw technology playing in her classroom. She believed that technology itself was motivating to students and that they would consequently be more interested and engaged in mathematics. The supplemental use of technology also provided variety, something that was important to student motivation. Lucy saw technology as a means of demonstrating real-world applications of mathematics and as a means of discussing real-world applications that require students to know how to do mathematics by hand and not just with technology. Technology could also be used to help students more easily visualize various connections and relationships in mathematics. It could then be used to explore these mathematical concepts. In general, this use was guided exploration, where students begin with some basic understanding of the concepts, use technology to get a better feel for what is going on or to see some specific aspect of the mathematics, then students leave the technology and return to paper and pencil to learn how to do the mathematics and what it all means.

CHAPTER 8: AN ANALYSIS OF PRESERVICE TEACHERS' BELIEFS ABOUT

TEACHING MATHEMATICS WITH TECHNOLOGY

The data stories that comprise chapters 4 through 7 represent the data collected for this study in order to answer two research questions:

1. What are preservice teachers' (PSTs') beliefs about teaching mathematics with technology, in what experiences are those beliefs grounded, and how are those beliefs held?

2. What relationships exist between PSTs' beliefs about teaching mathematics with technology and their beliefs about mathematics, teaching, and learning?

The intent of these chapters is to present the PSTs' beliefs about mathematics, teaching, learning, and teaching with technology—their educational ideologies—as internally consistent belief systems, that is, as sensible systems. Through the process of coding the data, searching for consistency within PSTs' beliefs systems, and writing the data stories, themes emerged. Thus, although the larger sections covering mathematics, teaching, learning, and teaching with technology were predetermined by the research questions, the four subsections into which the section on beliefs about teaching with technology was divided were direct results of the analysis that addressed the research questions. As the overall intent of this study was to ground a theory of PSTs' beliefs about teaching mathematics with technology, this chapter is organized around the themes that emerged from the PSTs' beliefs about teaching mathematics with technology. The chapter expands on each of those four subsections—experience with technology, the nature of technology in the classroom, roles of technology, and concerns about technology—as a means of answering more explicitly the research questions.

Experience With Technology

In this study the PSTs and I often used the word *technology* in a general sense, not delineating the specific type of technology to which we were referring. When asked how they were defining technology, all the PSTs responded, in essence, they meant computers

and graphing calculators used in the mathematics classroom. I now compare the PSTs' experiences learning with these two types of technology. As discussed in chapter 2, when a PST described how their teachers had used technology in their mathematics classroom, I looked at the answer more as a window into the PST's current beliefs about teaching mathematics with technology than as a window into their past beliefs. That is to say, when a PST said, "I don't think my teachers really taught with technology," I did not take this statement as evidence of what the PST's beliefs about technology had been when they were in that teacher's class. Rather, I used the reflection on past experience as a means of inferring current beliefs and the influence of experience on those beliefs.

A Comparison of Experience With Graphing Calculators and With Computers

Lucy had had minimal exposure to the use of graphing calculators in the mathematics classroom. Katie, Jeremy and Ben, on the other hand, each had had considerable experience using graphing calculators. The latter two also described their experiences as minimal, however, but in a different sense. Although they had had experience using graphing calculators, they did not feel their teachers had used the calculators to teach mathematics. When faced with minimal exposure to teaching with graphing calculators, Jeremy and Ben chose to use graphing calculators anyway. They each had had experiences that convinced them of the value in using graphing calculators and so they pursued further use on their own. Jeremy was intrigued by what one of his teachers was showing the class using the graphing calculator. But since there was only a classroom set of graphing calculators to which he did not have constant access, he purchased his own. He then taught himself how to use the graphing calculator with the mathematics they were learning in class. Acing a test because of expeditious calculator use reinforced his desire to continue to take advantage of the graphing calculator on his own. Although Ben's calculus teacher had used graphing calculators in class, Ben did not believe his teacher *taught* with them. Near the end of the course, and most likely because he recognized Ben's interest in the graphing calculator, the calculus teacher came up with a special project for Ben and a classmate. They were challenged to learn how to input

programs into the calculators and then teach the others in the class how to use them. These autonomous experiences instilled in Ben a great deal of confidence in his knowledge of calculators and in his ability, if he did not know how to do something, to learn how.

In fact, all the PSTs but Lucy felt extremely comfortable with graphing calculators. Katie had had considerable exposure to graphing calculators, and by the 10th grade had been provided one for her constant use. By the time she graduated from high school she had had four years of using various graphing calculators and seeing her teachers frequently use them. As far as Katie was concerned, if you had seen one graphing calculator, you had seen them all. This view is in stark contrast to that of Lucy, who was concerned about her ability to use the graphing calculator during student teaching because it was slightly different from the one on which she had learned. Lucy did not have autonomous experiences with the graphing calculator and did not have confidence in her ability to use it.

Although Ben and Katie had had experience using computers in the mathematics classroom before college, for all the PSTs most such experience came once they started college. The PSTs' experiences with computers, therefore, were more recent and more homogenous than their experiences with graphing calculators. All the PSTs had had both positive and negative experiences using computers to learn mathematics, and they all felt they had passable knowledge that would support using the computer in a variety of ways. Although their early experiences with computers constituted limited exposure to what the PSTs considered teaching mathematics with computers, they each described multiple experiences in college when teachers had used computers to teach mathematics.

It was only in their high school mathematics classrooms that the PSTs observed teachers using graphing calculators. In addition, each expressed the belief that, although teachers had exposed them to the use of graphing calculators, these teachers had seldom actually used the technology to teach mathematics. Except for a brief (approximately one week) unit in the Technology and Secondary School Mathematics course, they had no

formal experience using graphing calculators in college or having a teacher use the graphing calculator to teach mathematics. Despite this contrast with respect to exposure to using these technologies in teaching, the PSTs who had autonomous experiences using graphing calculators remained more comfortable learning, and thinking about teaching, with graphing calculators than with computers. They seemed to generalize what they observed to be teaching mathematics with computers to beliefs about teaching mathematics with technology in general and then, more specifically, to beliefs about teaching with what they were most comfortable, namely, graphing calculators. Thus, Ben, Jeremy, and Katie each indicated that they felt more comfortable with graphing calculators because they had been using them for a longer period of time. They were comfortable with computers but not *as* comfortable, because they were less used to them. Although Lucy had had experience with graphing calculators before having had experience with computers, the lack of autonomous usage of graphing calculators at those early stages resulted in little use thereafter. The more recent, positive experiences with computers were far more empowering to her than the rather unimpressive experiences she had had with graphing calculators.

Ownership of Learning with Technology

As I collected data for this study and, in particular, as I wrote the data stories for each PST, patterns emerged with respect to the PSTs' experiences with technology. The cogent factors seemed to be experience, confidence, and knowledge. The experiences involved learning with (both in the classroom and on their own) and being taught with technology. I became further convinced over the course of the study that it was profitable to view PSTs' confidence as a belief—an individual's belief in their ability to act on their knowledge. The PSTs' knowledge of technology and its applicability to mathematical concepts, their beliefs (in particular, their attitudes and feelings) about technology, and their level of comfort or confidence with technology seemed closely connected to their experiences with technology. That is, from these various experiences, the PSTs formed

closely related beliefs about their personal knowledge of and confidence with technology. I use "ownership" as a label for this belief cluster.

As previously mentioned, PSTs' educational experiences are primarily in the role of student rather than teacher. As such, their experiences are primarily experiences learning or being taught mathematics with technology. Although it is premature to discuss PSTs' ownership of teaching with technology, one can discuss their ownership of learning with technology; my supposition is that the latter will be a foundation for the former. As reviewed in chapter 2, researchers involved in project ACOT located five phases of evolution for technology integration in the classroom: entry, adoption, adaptation, appropriation, and invention (Dwyer et al., 1990; Sandholtz et al., 1997). Although not delineated by these authors, nor applied directly to mathematics, the five phases can be seen as varying along a continuum involving experience with, knowledge of, and confidence in using technology with mathematics. Teachers advance through these phases as they increase in experience, knowledge, and confidence, that is, in ownership.

I now discuss a theory similar to that posited by Sandholtz, Ringstaff, and Dwyer (1997) but epistemological rather than instructional. This is a theory based on the experiences of the PSTs in this study of how students come to gain ownership of learning with technology. Its purpose is to explain the similarities and differences I saw in the PSTs' experiences with technology. Although it came about as a result of an analysis of the data stories, it is also extrapolated from my own experience with learners of technology. In addition, I have chosen to use the terms from the ACOT model in describing the phases in this theory. My intention, however, is not to imply that I studied these PSTs as they progressed through these phases. This study was designed to explore PSTs' beliefs, not their changes in beliefs. I am reporting the evidence I have for the phase at which I inferred the PSTs were at the time of the study as well as evidence that they had passed through the previous phases. My theory is that students go through these phases as they learn how to learn mathematics with technology. Examples from the

PSTs' data stories will be used to illustrate this evolution. Because this discussion refers to the participants primarily as mathematics students rather than PSTs, I often speak of students in general rather than using the term PST.

Entry

Although students may have some experience with technology outside mathematics, in the entry phase students are introduced for the first time to technology in a mathematical context. For the PSTs in this study there was considerable variance with respect to when this introduction occurred, particularly when it came to computers. As was outlined in the data stories, Ben, Jeremy, and Katie were introduced to graphing calculators in 9th grade. Lucy was introduced to them during the 11th grade. Ben went through the entry phase for computers in middle school. Having been introduced to them in school, he then used one at home. Katie remembered going to the computer lab several times a week during high school but never really knowing what was going on. Jeremy had limited exposure to computers during his senior year and felt it was strange to use computers in a mathematics classroom until he started college. It was not until Lucy's junior year in college, when she took the Technology and Secondary School Mathematics course, that she was introduced to computers in a mathematics environment.

During this entry phase, students may be getting acclimated to the hardware, the software, or both (in the case of graphing calculators, in many ways the hardware and software are the same from the users perspective). Students may have previous experience using computers, but they may never have used specific mathematics program like GSP or MAPLE. Or students may have experience using a spreadsheet, but never have used one in mathematics. As Ben described it, "Most of the stuff I had used for technology was, 'Well, you use a computer to write a paper'" (Group interview). In the entry phase, learning with technology is a novel experience, one that could be positive, negative, or somewhere in between. Jeremy's description of how he felt when he first heard that he would be using computers to learn mathematics is a nice example of the entry phase: "We didn't know how to use it; we didn't know what it was good for"

(Jeremy, Group interview). When Katie was first introduced to the TI-92 she recalled thinking, "They can do EVERYTHING!!! I LOVE THEM!" (Tech email). She did not yet know *how* to do everything, but she was extremely enthusiastic at the prospect of learning how. The PSTs in this study had all experienced the entry phase before the time of the study, in high school when it came to graphing calculators and in college, primarily, when it came to computers.

Adoption

As students gain knowledge of and confidence with technology, they move from the entry phase to the adoption phase. In this phase they start to see how technology can help them accomplish mathematical objectives. These connections between the mathematics they are studying and technology's capabilities are usually made by the teacher or by a worksheet. Because of the origin of these connections, in this phase students are often very dependent on the teacher or on memorized procedures. Katie described it in this way:

> All it was were some commands I'd never heard of. I had no idea what they were doing and what they stood for. I didn't know anything. It was more like, "These are the commands that work for this calculator. Now do it this way." It just didn't make sense. (Tech interview)

With respect to graphing calculators, Lucy was at this phase when she left high school and did not show evidence of having left it. In fact, there was considerable evidence she was still at the adoption phase at the time of the study. When her teacher used graphing calculators in Algebra II during her junior year of high school, she remembered being impressed with what the teacher was showing the students on the overhead (entry phase) but not having any idea what she was actually doing on the calculator. Consequently, she merely mimicked what her teacher had done when she tried it herself (adoption phase). She remained uncomfortable with her knowledge and experience with graphing calculators.

If students never internalize the connections between the mathematics they are learning and the capabilities of available technology, if they never see what the

technology is accomplishing for them, then they remain at this adoption phase—possibly appreciating what technology can do for them when shown by someone else but never making these connections on their own. With respect to graphing calculators, Lucy remained at the adoption phase. She used graphing calculators in a limited way and was not inclined to explore new ways to use them. These factors, along with the limited exposure she had been given to the use of graphing calculators, likely inhibited her from moving beyond this phase.

Adaptation

The primary difference between the adoption and the adaptation phase is internalization. At the adaptation phase, students themselves start to make connections between the mathematics they are studying and the capabilities of the technology. The teacher might still initially make these connections, but the student is then able and willing to expand on them. Thus, in the adaptation phase students start to seek out their own ways to use technology in order to learn the mathematical concepts the teacher is teaching (or that are behind what the teacher is teaching). Or they see technology as a way to execute procedures so that they can then focus on the concept behind the procedure. This phase of ownership is characterized by a desire to use the technology for more than what a teacher or textbook might be prescribing. Technology is seen as a tool to which the student can turn, not just to calculate but also to explore.

The hallmark of this phase of ownership is actually the more traditional use of the word *ownership*. For the PSTs in this study, it was often their personal acquisition of the technology that facilitated their ability to internalize mathematics and technology connections. The PSTs' desire to spend time with the technology "on their own" was previously discussed. Although the graphing calculator initially intrigued Jeremy, he believed.

> The time we spent on it was really not enough for me to catch on [to] how the calculator works. So I went and got one—an 85—and just played with it until I got it to do what we got it to do in class…. It really fascinated me, so that's why I wanted to learn more about it. (Tech interview)

Similarly, for Katie, the turning point was when the school acquired enough graphing calculators "to assign to each student to take home. I then mastered the TI-calculators and have been dependent on them ever since" (Tech email).

All four PSTs demonstrated they had become capable of autonomously making connections between the mathematics they were studying and the capabilities of the computer. Thus, with respect to the use of computers, all four seemed to have moved from the adoption phase to the adaptation phase. Lucy was the only one, as would be expected from earlier descriptions of ownership, who was at a later phase with respect to learning mathematics with computers than with learning with a graphing calculator. Her higher level of confidence with respect to computers and her ability to envision ways to explore mathematics on her own would seem to put her in the adaptation phase. This ownership with computers was primarily with GSP, a program that she came to value so much that she could state, with respect to her geometry course, "I honestly believe that I could not have made it through that class without GSP" (Group interview).

Appropriation

As in the ACOT theory (see chapter 2), I refer to appropriation as more of a milestone than a phase. When students reach appropriation, they view technology as indispensable, not because they cannot live without it but because they do not want to live without it. Katie expressed this belief when she stated that, with respect to her graphing calculator, she was "totally dependent upon it," (Initial interview) and explained, "I can do the math usually on my own, but I'd rather have access to my calculator" (Tech email). Jeremy went so far as to say it was necessary to use technology, not because he could not teach mathematics without it but because once he had seen how powerful it was, he believed he should use it. Thus, at this phase, technology becomes indispensable because it is viewed as extremely valuable.

As was the case in the adaptation phase, in the appropriation phase students are confident that, even if they do not know how to do something, they will be able to figure it out. But, in addition, at the appropriation phase, they seek out ways to get the

technology to do what they envision without assuming a built-in algorithm to accomplish this. This is the difference between "I'm sure this is in the menu somewhere. What if I look here?" and "I'm sure I can make it do this. What if I try this?" Compare Ben's reaction to not remembering how to accomplish something with Graphing Calculator—"I'll beat ya somehow" (PBS 3)—to Lucy's reaction to not remembering how to accomplish something with GSP—"I know that there's a way, but I cannot remember how to do it" (PBS 3). Appropriation does not mean, however, that students are dependent on technology to think mathematically. What it means is that technology becomes like a piece of paper and a pencil, a whiteboard, or a compass. Technology becomes a tool that they turn to both powerfully and naturally. This allows students to enter the final phase of learning mathematics with technology, that of invention.

Invention

In this phase, students start to explore mathematics *because* of technology. They see mathematics they might not otherwise have been exposed to, and they seek to explore mathematics that is not necessarily being introduced in the classroom. Opportunities to use technology with mathematics (i.e., experiences) are sought out, knowledge of how to use the technology to learn mathematics is broad, and the students' confidence in their ability is high. Students become willing and able to help other students use technology to learn mathematics. In addition, students feel their knowledge of current technology is easily transferable to other technology (e.g., different programs, makes, models, versions, platforms, languages). They have developed ways of thinking about learning mathematics that generalize across technologies.

Ben and Katie each showed evidence that they were in the invention phase at the time of the study. As Ben stated,

> I don't really look at technology as something I can't learn. If I don't know it, it's not something I can't figure out. It just takes a minute to sit down and figure it out. So I don't really think I'm uncomfortable with any of it. (Tech interview)

Katie expressed a similar sentiment:

> Once you really learn how to use certain technology, everything else becomes more natural in that aspect too, because you just learn how computer programmers think, or something. It's easier after you learn one thing really good to jump onto another thing of similar type. (Tech interview)

Contrast this with Lucy's ownership of graphing calculators. She struggled to see how she could use graphing calculators, and compared herself to her students by stating, "I'm still in TI-82 world. They're in TI-83 world, so I have to get lessons on that" (PBS 2). She believed that when it came to these two graphing calculator models, "there's a big jump between those" (Observation interview).

Figure 5 shows the phases in which, according to the data I had collected, I was able to categorize the PSTs at the time of the study.

	Graphing Calculators	Computers
Ben	Invention	Invention
Jeremy	Appropriation	Appropriation
Katie	Invention	Appropriation
Lucy	Adoption	Adaptation

Figure 5. The PSTs' phases of ownership.

The Nature of Technology in the Classroom

Having laid forth a theory of ownership with respect to learning with technology, I now turn to another aspect of PSTs' beliefs about technology. It became apparent in the data analysis that there were some overarching beliefs at play in the way the PSTs thought about teaching mathematics with technology. Although they talked of the various roles they wanted technology to play and the expected outcomes from those roles, as well as their concerns about teaching with technology, there were beliefs that seemed to encompass and significantly influence all of these. I refer to these centrally held beliefs as beliefs about the nature of technology in the classroom. In a sense, beliefs about the nature of technology in the classroom are overarching beliefs about the prominence of technology in the classroom. These are the centrally held beliefs about which other beliefs about technology were clustered. Teacher's beliefs about when technology is used, whether before, during, or after instruction, as discussed by Brill (1997), are an

example of beliefs about the nature of technology in the classroom. Beliefs about the nature of technology in the classroom are described in terms of a broader dimension within which Brill's continuum fits, as well as several other dimensions that emerged in this study.

Because of their central location in the PSTs' belief systems, beliefs about the nature of technology in the classroom can be thought of as, in essence, the PSTs' "definition" of teaching mathematics with technology. These beliefs are connected to more specific beliefs about the roles technology should and should not play in the classroom (these roles and concerns make up the last portion of this chapter). In sum, no PST believed that technology should be *the* driving force behind the teaching of mathematics. Each PST, however, viewed technology as a force to be reckoned with; one that in many ways could be beneficial but that also needed to be controlled lest it compromise their overall educational objectives. Their beliefs about the nature of technology in the classroom reflected both their desires to use technology in meaningful ways *and* to control or predict in some way the outcome of that usage. The dimensions of beliefs about the nature of technology in the classroom that emerged from this study—availability, purposeful use, and teacher knowledge—are discussed below.

Availability

The primary dimension Brill (1997) found with respect to elementary teachers' beliefs about using technology in their teaching was *when* they felt it was appropriate to use technology. She posited a continuum on which, on the one end, teachers did not choose to use technology until after the students already knew how to do the mathematics by hand (postmastery) and, on the other end, teachers chose to use technology in an exploratory way before students fully understood the mathematics (exploratory). She further described a middle ground in which teachers were beginning to use technology before full understanding of a concept was assumed (premastery). One of the intents of this study was to explore whether these dimensions of postmastery, premastery, and exploratory were also valuable in describing PSTs' beliefs about technology. I

purposefully chose participants who had significant experience with technology in hopes of further delineating that dimension, as well as finding similar dimensions. I believe that, through this research, other dimensions have indeed emerged. In particular, I found it more valuable to think of Brill's categories as part of a broader dimension which I refer to as the availability of technology to students. I start with a discussion of Brill's categories and then expand on this dimension to discuss further the notion of availability.

Postmastery, Premastery, and Exploratory

Brill's (1997) exploratory dimension nicely described both Ben and Jeremy. They made little distinction between whether students were allowed to use technology before or after they had mastered mathematical content. They believed that technology could and should be used to facilitate mathematical understanding and thus could be used profitably at most any stage of the learning process. Each expressed the belief that it was not when technology was made available to students but their decisions as teachers that made the difference. Katie seemed to be in the premastery category. Because of her focus on students' ability to do things by hand, she felt there were times when she might not let them use technology, but she expressed the desire for this restriction to occur only rarely. In general, she too wanted to take the responsibility herself, in the presence of technology, to ensure that her students were gaining the understanding she desired. Thus, she felt confident in using technology before her students understood mathematical concepts or in order that they might come to that understanding.

The beginning stages of premastery best describe where Lucy was with respect to when she believed technology should be used with her students. For the most part, Lucy believed that technology should only be used after her students had gained a certain level of understanding of any given mathematical concept. This format was very important to Lucy, as, in order to enhance instruction, she believed technology needed to come afterwards. Students needed some understanding already, and then the technology could be used to enhance that understanding. This central belief is illustrated well through her beliefs about students' learning of graphing. Lucy wanted her students to know how to

graph by hand. She did not mind them using technology after they had demonstrated mastery of this technique. There were several times, however, when Lucy expressed (somewhat to her own amazement) that there were times when she would use technology before her students understood how to do things by hand. Lucy envisioned using the visual capabilities of technology in order to introduce a new topic. She felt this use of technology had significant potential for motivation. Still, closely connected to her belief that technology should always be supplementary, Lucy primarily saw technology as being used in a postmastery way. This belief had less to do with *when* and more to do with *how often* technology was used in the classroom. I now turn to further elaboration on beliefs about whether technology is constantly or periodically made available to students.

Constant Versus Periodic

Whether technology should be constantly or periodically available was one dimension of the nature of technology use in the classroom that emerged from this research. Katie believed strongly that technology should be constantly available to her students. Constant access, however, did not equal constant use. The constant availability was intended, rather, to provide constant choice to both teacher and student. In this environment the teacher could turn to technology whenever she deemed it valuable or appropriate. Similarly, during certain classroom activities, students could be given the option of whether to use technology. Lucy did not want constant access. In fact, it was important to her that technology not be constantly available. She did not want technology to be perceived as the primary source of instruction. It was a supplement. This notion of supplement did not fit Ben's belief about the nature of technology availability in the classroom. It was all or nothing for him. It was okay if you did not use technology; sometimes it just was not available or feasible. But, if you were going to use technology, then you had better use it in all aspects of instruction (e.g., in class, on homework, and on tests). Jeremy, on the other hand, was willing to use technology in whatever ways were possible. He wanted his students to have constant access to graphing calculators; constant access to computers would be great, but he did not deem it likely. He would be satisfied

if he at least had access to a computer. He wanted to use whatever technology he had as much as possible. His students would be allowed and encouraged to use technology at all times unless he told them otherwise, something he figured he would do only rarely.

Both Lucy and Katie felt strongly that students needed to know how to do things by hand, not just with technology. Their beliefs about the nature of technology, however, were very different, although in the same vein. In essence, Katie's solution was to make technology constantly available to her students but to use it in ways that supported her desire to have the students learn the procedures by hand. Lucy's solution was to make technology only periodically available.

The PSTs in this study believed that, as teachers, their own access to technology depended greatly on the factors (such as school funding and school priorities) that would be, at least initially, out of their control. They also recognized, however, that given these constraints, they as teachers needed to make decisions about their students' access to technology. They believed that the nature of technology use in the classroom differed significantly based on how available the technology was. When technology is periodically present, its use by both teacher and students must be premeditated. Teachers are likely to make technology available only on days when they have specific plans to use technology. One of the implications of periodic availability for student use is similar to that which is implied by so-called problem-solving exercises relegated to final section in many mathematics textbook problem sets (or marked with a star to warn the unsuspecting). Their very context, separated from what is considered to be "normal" mathematics, and sometimes labeled as an application, often redefines the problem for the students as, "Let's see. This problem comes in the chapter on solving systems of linear equations, so they must want me to set up a system of linear equations to solve this problem." This is not to say that in either of these situations, there is not potential for valuable use. The context, however, has limited that potential. So, although periodically available technology may serve quite valuable roles in learning, the nature of its use, despite the role, puts students in a different position. When technology is periodically

available, students associate appropriate use of technology with when it is made available. When technology is constantly available, students must choose when to use it. Of the PSTs in this study Lucy desired most to control her students' access to technology. Ben and Jeremy wanted the students to have significant access and for that access to be aligned with their instruction. Katie wanted constant access so that technology was always an option.

When technology is constantly available to teachers and students, the choice of whether to use technology can be based on the needs and circumstances that arise in daily mathematical discourse. On the other hand, if technology is only available periodically, someone (most likely the teacher) will decide ahead of time when to make technology available—when to hand out the calculators or when to go to the computer lab. In these situations, it is fair to assume (as an outside observer, certainly, but more importantly, as a student) that the teacher expects the students to use technology that day. This expectation gives a different flavor to a students' choice not to use technology that day. With periodic access, one can envision a student saying, " But look, I can do it without technology"; with constant access a student might say "But look, I can do it with technology." The nature of technology availability in the classroom is influenced by teachers' decisions and greatly influences both their and students' decisions with respect to technology use. That this use should have some purpose is the next dimension of belief about the nature of technology in the classroom. Before describing that dimension, however, I discuss briefly the notion of alignment.

Alignment

Another aspect of technology availability dealt specifically with alignment. The PSTs were committed to the idea that their use of technology in the classroom be aligned across their classroom activities. As was the case with so many of these beliefs about the nature of technology in the classroom, the PSTs had had classroom experiences in which they believed technology use had not been well-aligned and they were determined to do otherwise in their own classrooms.

Jeremy's experience using MAPLE in his college Linear Algebra course convinced him of the need to fully integrate technology use in his classroom. He was primarily concerned about the lack of alignment between homework and tests. Ben's belief about the need for technology alignment was also closely connected to his MAPLE experience in linear algebra. That experience, along with another experience he had observing in a high school classroom, convinced Ben that it was critical that technology use be fully aligned across classroom instruction, individual work, and formal assessment. He was adamant that if his students were allowed to use calculators at all, they should be allowed to use them in all situations.

Katie was concerned that her students might not have constant access, in which case she did not believe that she could really use technology in a natural way because technology would not be available to them, for instance, when they were doing their homework. It needed to be used in every part of instruction. She wanted to turn to it like she might turn to a piece of paper or turn to a graphical representation. This issue came up when it came time for her FE. The students in that classroom were not allowed to use technology, so she knew that even if she used it in class, the students would not have technology available to them when they went home, and they would definitely not have technology available to them after her two weeks were over. Although she wanted to use technology with them, she decided that it was impossible to give them sufficient access and experience in such a short time in order to make technology a natural part of the classroom. She did use it herself, however, for demonstration and discussion. As one might imagine, alignment of technology use was not a major concern for Lucy. She believed that technology should be used periodically in order to enhance her teaching in various ways. In many ways, her core beliefs about the periodic availability of technology precluded the possibility of technology being fully aligned.

Belief about the proper alignment of technology across classroom activities is an example of how important availability was to these PSTs. Beliefs about alignment are concerned with both when and how technology can be used in conjunction with formal

and informal assessment. The connection between alignment and availability, however, can look quite different for different teachers. For one teacher, constant access might mean access except during formal assessment. Other teachers might use graphing calculators all the time in their classrooms, and yet others might use them infrequently; but all might prohibit students from using calculators on their tests. Or, a teacher might use the computer a lot in their own instruction, but students are never given a chance to use it on their own even though they are required to know certain things about it. The PSTs in this study demonstrated an awareness of these issues and a desire that their own use of technology be aligned with their instructional practice. Regardless of whether technology is made constantly or periodically available, the PSTs in this study believed that it should serve a useful purpose.

Purposeful Use

The PSTs believed that, although there are many reasons one might choose to use technology in the classroom, at least one of those reasons should be present when you use it, namely, that technology should be used purposefully and not just for the sake of using it. The reasons teachers choose to use technology in their teaching are diverse (and many of these roles of technology will be discussed later in this chapter). In addition, what one person deems purposeful may not be deemed purposeful by another. As mentioned previously, these central beliefs about the nature of technology in the classroom seemed to be strongly related to poignant, often negative, experiences learning with technology. Although at times the PSTs expressed sentiments such as, "I loved this so I want to do it too," their discussions were more likely to have this flavor: "That experience was horrible, so I plan to do this instead so that my students do not have to go through what I went through." Several of these experiences will be revisited as I outline how the PSTs defined purposeful use of technology.

For Ben, purposeful use was when he could envision some advantage to using technology. He believed that there almost always would be such an advantage. It was his responsibility as the teacher to ensure the presence of that purpose. For Jeremy, although

he believed he could always find some way to use technology, technology use was purposeful if it was made integral to the lesson. It was important for Jeremy to integrate technology whenever possible because he felt it was necessary to use it. If he did not use it, he believed that he would not be being true to himself. His beliefs about the purposeful use of technology were also influenced by his experience in a Linear Algebra course, as he felt that the use of MAPLE in that class had not been beneficial at all. For technology use to be purposeful, he believed that the technology needed to help you better understand the mathematics, not just execute some calculations that you did not understand. For Jeremy, it was necessary to link technology use with understanding or else it was pointless. Katie defined purposeful use as having a point to using technology. In order to use technology in a natural way, there needed to be some purpose to the use and the teacher needed to know what that purpose was. Lucy, perhaps more than any of the PSTs, believed that, if you were going to use technology, there had better be a good reason. For Lucy, that reason needed to enhance student learning. In order for technology to enhance learning it needed to be used in supplemental yet relevant ways. This belief was connected to her postmastery beliefs about technology. For example, using technology before students understood how to graph by hand defeated her purpose of the students knowing how to graph by hand. Using technology after students understood how to graph something by hand did not defeat this purpose; in fact, it could enhance their learning as it could expedite the process so that they could focus on some other related mathematical concept.

There are certainly other variations on purposes that did not fully emerge from this study but that one might imagine. A teacher might believe, for instance, that one reason to use technology is because you have it. The purpose is to take advantage of an available resource so that it does not go to waste. A teacher might believe that, in order to be purposeful, technology use in the mathematics classroom must have pedagogical purpose; another might be less lenient, restricting pedagogical purpose to mathematical roles of technology. In addition, one can imagine a teacher having a purpose with neither

mathematical nor pedagogical ties. For instance, a teacher might choose to use technology because they were told to: "Here is a set of graphing calculators. The district has spent a lot of money on these, so you had better use them." The teacher who has no vision as to why technology should be used is not likely to use technology with purpose. As far as the theme of purposeful use that emerged from these PSTs, they believed that, to be purposeful, there must be something important that the technology might accomplish. It seems, then, that purposeful use is necessarily connected to an awareness of the possible roles of technology. If a teacher gives an assignment and says, "Feel free to use your calculators," the teacher may not have a specific purpose in mind in allowing that use. That the PSTs in this study were aware of multiple roles technology could play in their classrooms will be discussed shortly. There is one other aspect of this purposeful use, and what the PSTs considered not so purposeful, that came out of this study, namely, the distinction between using technology as a tool or as a crutch.

The PSTs, in general, defined tool and crutch in this way: Technology is being used as a tool if it is being used either with or to gain understanding; it is used as a crutch if it us used without or in order to avoid understanding. Note that this dimension is not the same as Brill's (1997) categories, although many connections between these dimensions seem clear. In part, however, this dimension explains why the Brill continuum was limited in describing these PSTs' beliefs about the nature of technology use in the classroom. It is likely that someone who is classified as *postmastery* would view any use of technology before students have understood a concept as using technology as a crutch. Those who are *premastery* have started to recognize that there are some ways that technology can be used as a tool before students have completely understood a concept. Those classified as *exploratory* likely see many ways to use technology as a tool both before and after students understand a given concept.

Teacher Knowledge

The PSTs recognized their own knowledge about technology greatly influenced the possibilities for the nature of technology in their classroom. It influenced the ways

they did and could think about technology use. Although connected to the PSTs' ownership of technology, this belief has to do with their personal awareness of that ownership. This dimension is defined by the knowledge the PSTs recognized in themselves and how important they believed that knowledge was to their successful use of technology in the classroom. All the PSTs felt the responsibility to know what they were doing and why they were doing it when it came to using technology in their classrooms. These beliefs are compared in the paragraphs that follow.

Jeremy was confident in his knowledge of technology and how he could use it in classes above and including Algebra I. In a sense, he took this knowledge for granted. When he considered teaching a General Mathematics or Pre-Algebra course, however, he recognized that, until he gained further knowledge of that curriculum, he would not know how to integrate technology into his teaching. Ben, too, was confident in his knowledge of how to use technology; but he was mostly confident in his abilities to explore mathematics himself. He recognized that he still had much to learn about teaching with technology. Ben's confidence in his own teaching abilities and in his knowledge of technology, however, seemed to be sufficient for him to learn as he went. This uncertainly was somewhat problematic, however, as he believed that uncertainty with respect to technology was more disconcerting to students than uncertainty with respect to mathematics. He wanted to ensure that he always had the right answer when it came to questions about technology whereas he was more willing to let students constructively flounder with the mathematics.

Katie believed a teacher must be knowledgeable with respect to technology in order to use it effectively in the classroom. This knowledge was critically important to Katie. In order for her to naturally use technology, she really needed to understand what she could do with it. Then, as the opportunities presented themselves, she could use technology "in the moment." Without that knowledge and confidence, she did not believe that she could use technology in a natural way in the classroom. In many ways, Lucy recognized this same need. The difference, however, was that Katie believed that she had

the knowledge that she needed to use technology, and Lucy believed that she did not have that knowledge. This lack of knowledge would limit the possibilities for her use of technology. Lucy recognized that she had a limited understanding of technology, which fit in very well with her notion of supplemental use. She could take the time to learn the things she wanted to do with technology, or she could simply bring it out when she came across something where she already had the knowledge.

Roles of Technology in the Classroom

It became clear from the outset of this study that the PSTs believed that technology had the potential to play multiple roles in their mathematics classrooms. Over the course of the study, roles that emerged from the data seemed to fall into three main categories: motivational, procedural, and conceptual. A section follows for each of these categories. The category itself is first defined briefly, and then the individual roles are discussed in more detail.

Motivational Roles

The PSTs often spoke of using technology for reasons only indirectly related to mathematics or to their students' ability to better understand mathematics. These roles were very much connected to the PSTs' beliefs about their role as teacher. Given that each PST placed emphasis on their role as a motivator, it came as no surprise that these nonmathematical roles had student motivation as the primary objective. The use of technology in their teaching was seen as a means of motivating students partly because the PSTs believed students simply liked technology. Students were interested in technology, so they would be more interested in mathematics if technology were involved. In this sense, the use of technology encouraged student involvement and provided variety to classroom activities. For all of these roles, the PSTs saw the objectives (i.e., motivation, variation, real-world applications) as important aspects of their own roles as teachers in general. Technology could have played this same role regardless of the subject they were teaching. Nevertheless, these roles were specific to their responsibilities as teachers. In addition, the PSTs did not include organizational

roles for technology (such as keeping grades or creating tests) as important aspects of teaching mathematics with technology. These roles were seldom mentioned and, when they were, were mentioned merely to point out that that was *not* what they were talking about.

Each of the PSTs believed that one of their primary roles as a teacher was to motivate students to learn. Connected to this belief was the belief that the use of technology would be motivational to students in that students would be more interested in learning mathematics when technology was involved in the process. This belief did not necessarily mean, however, that the students themselves needed to be using the technology. For instance, Jeremy was quite pleased when he had used Geometer's Sketchpad as part of a classroom discussion. His satisfaction stemmed, in part, from the perception that his students had been more motivated to learn because of the technology. At other times, however, the PSTs indicated that the motivation stemmed not just from the use of technology but also from students' individual use of technology. This individual use of technology was viewed as "more fun or just hands-on" (Katie, PBS1 interview), something that "keeps the students interested" (Lucy, Tech interview). Several of the PSTs referred to individual students' use of technology as a "hands-on approach to learning," quite a different meaning than the common phrase "by hand."

It was common for the PSTs to speak of variation when they talked of the characteristics of a good teacher. There were two main reasons they felt variation was important: the desire to not be boring and the desire to address students' various learning styles. Several PSTs felt that the use of technology provided an alternative to the standard way of doing things. Katie, on the other hand, did not refer to it in quite the same way. For Katie, using a variety of teaching approaches, one of which was technology, *was* the standard way. Thus technology could be thought of as part of the standard variation repertoire or as "something out of the ordinary [students] could look at and go, 'Hey, that's cool'" (Jeremy, observation interview).

Ben, Katie and Jeremy recognized that an important aspect of technology use was that it could provide students with both variation and motivation, but this role seemed to be more of a side benefit. So, although these roles were not seen as valid reasons in and of themselves to use technology, these PSTs recognized that, when they were using technology for other purposes, it additionally motivated their students. These nonmathematical roles were much more important for Lucy. They fit easily into the periodic, enhancement use of technology she envisioned for her classroom.

Another reason to use technology, primarily discussed by Lucy, was that knowing how to use technology was an important life skill. Students need to know how to use technology to be productive members of society so using technology, in any number of different ways, helped students develop technological skills. Lucy had a strong belief that the motivation of her students depended on her ability to show them real-world applications of mathematics. Technology was something students needed to use in the real world. Thus, if her students could use technology in connection with doing or learning mathematics, then they were seeing a real-world application. But notice that this application was actually a mathematical application of technology. No matter—the quasi-logical relationship for Lucy was that this was a real-world application. It satisfied that need for her.

Lucy also made a distinction between the motivation derived from using technology with students in higher- and lower-level courses. Students in lower-level courses did not like doing things by hand, so they were more motivated when they were using technology. In essence, she believed that students in higher-level courses, by their very nature, are more motivated. It is also interesting that Lucy would use technology *more* with students in lower-level courses because of this role, whereas Ben and Katie would use technology *less* (or at least differently) with these same students. These quasi-logical relationships were based on the primary roles the PSTs intended technology to play. Lucy saw technology as being particularly motivating and, because she viewed students in lower-level courses as lacking in motivation, she wanted to use technology

with them in order to motivate them to learn. The objective was motivation. Ben and Katie, on the other hand, had other roles for technology in mind when they distinguished between students in lower- and higher-level courses. Thus, variation in the primary role of technology resulted in different conclusions as to the emphasis technology would receive with students in lower- and higher-level courses.

Procedural Roles

Procedural roles of technology were often discussed in the context of saying that the result of using technology in these ways could also be accomplished by hand without technology. It may have been that students already know how to do the mathematical procedure by hand, that they are in the process of learning how to do the procedure, or that the teacher did not expect the students (at this stage or ever) to know how to do the procedure by hand. The following roles are included here: checking, expediting, calculating, solving, improving accuracy, organizing data, getting beyond the basics, and making things "easier." Often these roles were described as roles that enabled the teacher to focus on an aspect of the mathematics that was considered to be beyond the procedure being executed by the technology. In this sense, some of these roles took on more of a "facilitating teaching and learning" role.

Jeremy focused a great deal on procedural roles for technology. Technology could expedite procedures, it could free you from making mistakes (accuracy), it made calculations quicker (as opposed to long and grueling), and you could do calculations with technology that you might not be able to do otherwise. He saw these as important roles for technology. You often arrive at a place in mathematics where you understand what you are doing, and you need to do calculations or "dirty work." Technology could take care of this for you and allow you to get on with it.

Katie also focused on the procedural roles of technology. She wanted students to first learn how to do things by hand and then use technology to expedite the procedures. Her reasons for doing using technology in this way varied. Technology made things quicker and more accurate, it allowed you to verify that what you did by hand was

correct, and it could do things that you either could not or would not want to do by hand. These roles of technology were closely connected to her belief about the nature of technology in the classroom—the notion of constant access. The combination of her beliefs about the importance of learning mathematical procedures and her love of using technology to expedite these procedures resulted in a desire to have technology available all the time. These procedural roles of technology simply could not be carried out with mere periodic technology availability.

For Ben, the procedural roles were partly default roles. He felt that he had always known that technology could be used for verification or to execute calculations more quickly, but there were so many more powerful ways that it could be used. At the very least, technology would be used to expedite, but the goal was to use it to do much more than that. Like Ben, Lucy did not speak much of these procedural roles for technology, but for a different reason. She saw technology as a means of showing her students real-world applications of the mathematics they were learning to do by hand. The need to be able to recognize a mistake on the computer was a further rationale for the need to know how to do the mathematics by hand and not just with technology. There was no intention, however, for students to move on to using technology, in general, to execute these calculations. In this way, Lucy's intentions differed significantly from the other PSTs' intentions. Katie wanted her students to learn how to do the mathematics by hand, but she also really wanted her students to move on to the technology. Lucy, on the other hand, did not really intend for her students to move on to using technology. They needed to learn how to do mathematical procedures by hand and, once they knew those procedures, there was really no need for the technology. They could use it if they wanted to, but Lucy did not intend to emphasize it. As technology was primarily a supplement—an add-on—it could not play procedural roles very efficiently.

The use of technology to execute procedures was highly connected to the PSTs' intentions in teaching procedures. There is something of a continuum here. Lucy had the procedures themselves as goal. As such, there was little need to move on from there with

the technology. Katie and Jeremy's objectives were often close to but slightly beyond the procedures. For them, the expediting role was very important. Ben's focus was often beyond the mathematical procedures. With this focus, he seemed to take the expediting role for granted.

Conceptual Roles

Conceptual roles primarily focused on using technology to facilitate the teaching and learning of mathematical concepts. Technology was seen as the means through which the students would come to understand a mathematical concept. These roles included demonstration, illustration, visualization, and exploration, as well as making connections to other mathematics and to the real world. Although the nonmathematical and procedural roles were somewhat peripheral to what was being taught, either in purpose or in deed, these roles have the understanding of the mathematics itself as the objective. In a sense, they are "closer" to the mathematics. That is, they are more intentional with respect to learning mathematics. Being closer the mathematics itself, these roles have the potential to be significantly more powerful than the other roles. Beliefs about the visualization and exploration roles of technology were most common to the PSTs and will be the primary focus of this section.

Visualization

For Ben, visualization with technology made the mathematics "real" and had the potential to really capture students' attention. Technology allowed students to visualize mathematics (his example was systems of equations). He believed that it was practically impossible to visualize certain mathematics without technology. In general, pictures could help illustrate mathematical concepts that might otherwise seem extremely abstract. Lucy also appreciated the power technology had for allowing students to visualize mathematics—to see things that they might not otherwise see. She valued this role for technology because she believed that some students were visual learners and would particularly benefit. The visualization role of technology provided increased variety, namely several different ways to represent things. In this sense, technology could really

enhance the mathematics. It also allowed students to see connections that they might not otherwise see. Because of the visual nature of technology, Lucy recognized that if she wanted a visual approach to introducing a mathematical concept, she might actually use technology beforehand (and she recognized that this use was different from the norm for her). Lucy also liked the dynamic nature of technology. For her, the dynamic nature of technology was part of how technology enabled students to see things. Through technology, students could watch for what changed and what did not change.

This dynamic nature of technology was one of the more important roles of technology for Jeremy. He spoke of and demonstrated numerous ways of using technology to dynamically illustrate mathematical concepts. This role of technology allowed students to visualize relationships between mathematical objects. There were two aspects of this role. On the one hand, technology could be used to visualize a concept; on the other hand, it could be used to visualize a relationship between mathematical concepts. Both Jeremy and Ben believed that they could use technology to enhance these ways of visualizing mathematics.

Katie recognized and appreciated the visual capabilities of technology, but she usually was referring to understanding when she referred to "seeing things" with technology. Technology sped up understanding as well as calculations. Technology helped students get beyond procedures and see the big picture, which, for Katie, was conceptual understanding. Although this notion was discussed primarily in Katie's data story—as it came up by far more often with her than with the others—it was something that was common to all. All four PSTs referred both literally and figuratively to using technology to "see what is going on." Thus, the PSTs frequently used "see" to mean "understand." They believed that technology could take what might be a long, drawn out process of showing numerous examples and show them very quickly, thus allowing students to see the big picture. In this sense, technology simply carried out numerous procedures in a short amount of time so that rather than focusing on the procedures, students could focus on relationships between the procedures. To show multiple

examples in a brief amount of time so as to see the connections between them is, in essence, the definition of *dynamic*. Thus dynamic does not always mean a graph or a sketch. Tables and multiple algebraic procedures, when done multiple times in close proximity, become dynamic representations of "the big picture." The intent of this use of technology is either to allow students to "discover" or allow the teacher to "show" a mathematical relationship.

Exploration

For Jeremy, technology facilitated exploration because it facilitated the making of conjectures, primarily because of the dynamic nature of technology—both visually and through expediting procedures so patterns could then be recognized. He referred to exploration as "playing around with mathematics" and emphasized that, although you could explore and conjecture by way of technology, in the end you needed to prove those conjectures by hand—the technology could not do that for you.

Exploration, for Ben, was all about personal discoveries. In fact, using technology in general for Ben involved two things: It was personal, and it was about discovery. Exploration differed for Ben based on the level of class with which he was working. For Ben, exploration often involved exploring real-world situations in order to make some decision (solve problems). Lucy defined exploration as primarily a means of introducing a mathematical topic so that students can get a feeling for what is going on. She described it as guided exploration. It was guided in that she would have everything set up so that she could control what they did (and so that they did not mess around too much). There would be guiding questions so that, as they answered those questions, they would then see the relationships that she expected them to see. Her job was to prompt and prod, leading the students toward the mathematics that they would later really get into and understand through doing it by hand. Exploration did not emerge as a role of technology for Katie. Although at one point Katie mentioned that technology could be used in numerous ways, one of which was "to explore a concept," that was the only time that she ever used the term *explore* in our interactions, nor did synonymous terms emerge.

For the most part, the PSTs' core beliefs about the nature of technology in the classroom did not preclude any of the major roles technology could play. The central beliefs, however, influenced both the definition of these roles and the degree to which the roles would be incorporated into the classroom. For instance, Lucy believed that technology should be supplemental to her teaching, never becoming its main focus. In contrast, Katie wanted technology to be constantly available and the use of technology to be as natural and everyday as possible. Despite these very different core beliefs about the nature of technology use in the classroom, both PSTs believed that technology was a valuable tool for expediting, visualizing, and exploring. But the ways they described them in their classroom were quite different.

Concerns About Using Technology

I now turn to the PSTs' concerns about using technology. Much has been written about why teachers choose not to use technology in their teaching (e.g., Barnes, 1994; Cuban, 1993; Hodas, 1993; Jones, 1998). I purposefully chose participants who expressed interest in using technology so that I could see variations in beliefs about teaching with technology rather than reasons for not doing so. Still, the PSTs in this study did have concerns about their use of technology, and several themes emerged therefrom. I have categorized these concerns into two main areas: teacher responsibility and student responsibility. The more a PST wanted to focus on conceptual understanding and wanted students to take responsibility for that understanding, the more the PST was concerned about their own ability to facilitate such learning and on the need for technology availability now and in the future. The more a PST focused on procedural understanding in mathematics, and on teacher-centered lessons, the more the PST was concerned with students misusing the technology and failing to learn the procedures. The focus, as far as teacher understanding is concerned, was more on knowing how to use the technology themselves, and less on knowing how to use it to teach. I now discuss each of these levels of concerns.

Teacher Responsibility

Both Ben and Jeremy expressed concerns about the need to teach their students how to use technology in order for them to use technology to learn mathematics. They also both pointed out that, although the need to teach students to use technology was a concern of theirs, it was not an excuse for not using technology. They believed that the time and effort for this additional learning needed to be factored into their teaching activities. Somewhat related to this concern about teaching students to use technology, Jeremy was concerned that his students would become comfortable using technology in his class and then move on to another class where they were not allowed to use technology. Once again, this concern was not seen as a reason not to use technology in his classroom, but he did see it as important to consider that the nature of technology in other classrooms would likely be different than that of his own.

Somewhat aside from these primarily logistical concerns, the PSTs expressed other concerns about their own responsibility when it came to teaching with technology. Ben worried that he had become so comfortable using multiple approaches to learning with technology that he would not be able to delineate the pedagogically appropriate method for his students. Jeremy worried that he would have difficulty assessing his students' understanding when they were using technology. He was concerned that he would not be able to see what they were thinking and was in the process of developing ideas about how he could get at seeing that understanding. Katie, on the other hand, was concerned because she wanted her students to perceive mathematics as fun and, in her experience, doing mathematics by hand had always been fun. She worried that, if technology was allowed to carry out all of those mathematical procedures, she might be taking away students' opportunity to enjoy that aspect of mathematics.

Student Responsibility

Very much related to her concern about students not being given the opportunity to enjoy doing mathematics by hand, Katie was concerned that the students themselves would not have the self-discipline to keep themselves from becoming dependent on

technology. Although Katie considered herself to be dependent on technology, she felt that this dependency was okay because she knew how to do the mathematics without technology but simply chose not to. In a similar vein, Lucy's primary concern about technology use was that the technology would replace her students' thinking. The influence of Lucy's beliefs about the nature of mathematics is clear. Mathematics was primarily a highly connected and hierarchical body of procedures. Understanding and performing those procedures was the essence of mathematical learning. Thus, if technology performed those procedures, it would literally be replacing the mathematical thinking that was the primary objective of her teaching.

The PSTs' concerns were strongly connected to their experiences and to their core beliefs about the nature of technology in the classroom. It is interesting to note that there were concerns about students having access to technology before they came to them (Ben and Jeremy), while they had them (Katie and Lucy), and after they left them (Jeremy). Often, their concerns could be summed up as follows: They feared that the conditions conducive to their beliefs about the nature of technology in the classroom would not be met. These fears could be either internal or external in nature. For example, an external concern would be the concern that the school would not have sufficient technology resources. If so, then the students could not have constant access to technology. The more integral constant access was to the PSTs' beliefs about the nature of technology in the classroom, the more concern there was over sufficient resources. Internal concerns had more to do with fears about their own inadequacies as a teacher. For instance, Jeremy's core belief about the nature of technology in the classroom was that because he was convinced that technology was a valuable learning tool, he should use it in his classroom. He believed he needed to take advantage of what he knew to be a valuable resource. As one might imagine, his concern was that he might not know how to take advantage of it. He was also concerned that he would not have it, but he showed he was willing to bring the technology in himself if it was not there. He had no problem with that.

In summary, the PSTs' core beliefs about the nature of technology in the classroom were strongly tied to their experiences. These experiences, along with the knowledge and confidence derived therefrom, can profitably be described in terms of phases of ownership vis-à-vis technology. Although the roles of technology fit into very similar categories across the PSTs, their beliefs about the nature of technology in the classroom greatly influenced just what these roles meant to them and how they envisioned they would come together in their classroom. Beliefs about the nature of technology in the classroom were, in turn, influenced by beliefs about teaching and learning mathematics. Similarly, the PSTs' concerns about the use of technology in the classroom were very much connected to their beliefs about the nature of technology in the classroom.

CHAPTER 9: SUMMARY AND IMPLICATIONS

Summary

This study investigated preservice secondary teachers' (PSTs') beliefs about teaching mathematics with technology. My initial interest for the study stemmed from an awareness of increasing availability of and educational emphasis on technology. In addition, through teaching a course for PSTs on the use of technology in secondary mathematics, I realized how little I knew about my students' experiences with and beliefs about technology. I felt greater knowledge of these beliefs could influence the technology education of future teachers. Two research questions guided this inquiry:

1. What are PSTs' beliefs about teaching mathematics with technology, in what experiences are those beliefs grounded, and how are those beliefs held?

2. What relationships exist between PSTs' beliefs about teaching mathematics with technology and their beliefs about mathematics, teaching, and learning?

For the purposes of this study, I adopted Rokeach's (1968) definition of belief: "All beliefs are predispositions to action" (p. 113). In addition, beliefs, whether consciously or subconsciously held, speak "to an individual's judgment of the truth or falsity of a proposition" (Pajares, 1992, p. 316). I used coherentism and Green's (1971) metaphor of a belief system to form a conceptual framework for thinking about how the PSTs held their beliefs. In coherentism, beliefs become viable for individuals when those beliefs make sense with respect to their other beliefs. It is only when beliefs become viable that they are considered part of an individual's belief system. Green's (1971) three dimensions of psychological strength, quasi-logical relationships, and clustering were used to visualize what a sensible system of beliefs might look like.

In order to provide context and vocabulary for the discussion of the PSTs' beliefs, I discussed various theories from the literature with regard to beliefs about the nature of mathematics (Ernest, 1991; Mura, 1993, 1995), teaching and learning mathematics (Copes, 1979, 1982; Ernest, 1988), and technology (Brill, 1997; Olive & Leatham, 2000).

This was followed by a review of literature on mathematics teachers' beliefs. I argued that coherentism provides an alternative way of interpreting apparent inconsistencies between teacher's beliefs and their teaching practice. Actions are always a result of some belief; the belief upon which the action is based, however, may not always be what the researcher or the individual expected. Although some research on teachers' beliefs about technology was reviewed (e.g., Brill, 1997; Doerr & Zangor, 1999, 2000; Sandholtz et al., 1997), the scarcity of such literature, particularly with respect to PSTs' beliefs, further compelled me to conduct this study.

Qualitative research methodologies, in particular the grounded theory research tradition, were chosen as the most effective way of answering the research questions. Four PSTs were purposefully chosen to participate in the study. Data collection strategies included classroom observations (both where the PSTs were students and where they were student teachers), interviews, email surveys, and secondary data. The interviews and email surveys were conducted in tandem. Three of these, referred to as Pedagogical Brainstorming Sessions (PBSs), required the PSTs to explore how they envisioned teaching specific secondary mathematics topics. The constant comparative method of analysis took place while data was being collected, through an iterative coding process, and through writing the PSTs' data stories. NUD*IST (Richards & Richards, 1997) qualitative research software was used extensively to organize, categorize and synthesize the data.

Data stories were written about each of the four participating PSTs. These were written so as to represent the PSTs' beliefs as sensible belief systems. Their beliefs about mathematics, teaching, and learning were first described. Then their beliefs about technology were discussed in four parallel sections: experiences with technology, the nature of technology in the classroom, roles of technology, and concerns about technology. Brief summaries of the connections among these beliefs are given at the end of each data story. The research questions were answered on an individual basis through

these data stories. The common themes that emerged were then analyzed in order to answer the research questions in a more general sense.

The first research question asked, in part, in what experiences the PSTs' beliefs were grounded. From an analysis of the PSTs' experiences with technology, a theory was posited concerning the PSTs' ownership of learning mathematics with technology. Experience, knowledge, and confidence were the primary factors making up ownership. A theory from the ACOT project (Sandholtz et al., 1997) was adapted in order to describe the various observed phases of ownership: entry, adoption, adaptation, appropriation and invention. At the time of the study the PSTs were located at phases ranging from adoption through invention.

According to Green's (1971) metaphor of a belief system, those beliefs most strongly held are referred to as centrally held or core beliefs. I inferred the PSTs' core beliefs with respect to technology and termed these beliefs their beliefs about the nature of technology in the classroom. These beliefs had strong connections to the PSTs' other beliefs about technology as well as their beliefs about mathematics, teaching, and learning. The primary dimensions of their beliefs about the nature of technology in the classroom were the availability of technology, the purposeful use of technology, and the importance of teacher knowledge of technology.

The PSTs envisioned technology playing a multitude of roles in their classroom. The roles that emerged as common to all were divided into three categories: motivational, procedural, and conceptual. Motivational roles of technology were nonmathematical in nature and closely tied to the PSTs' beliefs that effective teachers motivated their students to learn and used a variety of teaching methods. Procedural roles involved using technology to execute calculations or procedures that could also be (and often were) done by hand. Through the use of technology, those procedures were expedited, simplified, or made more accurate. Conceptual roles facilitated the visualization and exploration of mathematics. When using technology in these ways, the PSTs' intent was on developing students' conceptual understanding.

The PSTs' concerns about teaching with technology were categorized into two main areas: teacher responsibility and student responsibility. The more PSTs wanted to focus on conceptual understanding and wanted students to take responsibility for that understanding, the more they were concerned about their own ability to facilitate such learning and the need for technology availability. The more PSTs focused on procedural understanding in mathematics and on teacher-centered lessons, the more PSTs were concerned with students misusing the technology and failing to learn the procedures.

<div align="center">Implications for Research</div>

Many research projects have focused on introducing technology in the classroom and then working with teachers to help them use that technology in their teaching. In terms of ownership, much seems to be done for teachers at the entering and adoption phases. Although this approach has merit and is certainly still important, I believe that there is a need for a different focus. Many schools have acquired a sizeable amount of technology through induction-oriented projects. Eventually teacher support in these projects diminishes, and teachers are left to their own devices. In addition, despite the potential of technology to significantly impact mathematics teaching and learning, not all mathematics teachers are going to make technology an integral part of their classroom. Although all teachers deserve continued support and encouragement in their use of technology to teach mathematics, those who demonstrate a desire to use technology in significant ways in their classrooms deserve more attention than they have received in the past. I propose future professional development and associated research that concentrates specifically on teachers who already have a commitment to technology use in their classrooms. The focus of such programs would be one of augmentation rather than initiation. Researchers need to find ways to identify these teachers and then identify their technology and training needs. Teacher educators, along with teachers, can then design and offer professional development courses that address these needs.

Some past research has had as its "ultimate goal to move the performance of all teachers into the expert range" (Leinhardt, 1989, p. 53). The research agenda proposed

above seeks instead to provide assistance to those who have expressed the desire "to move". The section on implications for teacher education will discuss possibilities for connecting beliefs about learning with technology to beliefs about teaching with technology. These connections are critical if teachers are going to use technology in meaningful ways with their students. The extent to which this connection is critical is not known and could be the focus of future research. Such research would also need to focus on finding ways to make these connections.

Reflections on the Study

This exploratory study into PSTs' beliefs about technology suggests that future studies focus on PSTs over a more extended period of time. In particular, following PSTs from the time they begin their teacher education program through their first few years of teaching could provide valuable insights into their belief systems and the evolution thereof. How would the PSTs from this study use technology in their own classrooms? In positing a picture of the belief systems of the PSTs, I have inferred which beliefs are likely to influence their use of technology in the teaching of mathematics. These inferences could be further explored and tested, with coherence as the ultimate criterion, by following the PSTs through their first few years of teaching.

Although this study did not focus on PSTs' mathematical knowledge, the Pedagogical Brainstorming Sessions (PBSs) did provide a context wherein some inferences about the PSTs' mathematical knowledge could have been made. There were instances in which the PSTs' mathematical knowledge seemed to be severely lacking and other instances in which the PSTs demonstrated significant mathematical insight and understanding. Future research should explore the relationships between PSTs' mathematical knowledge and their beliefs about teaching with technology. In particular, it would be valuable to explore the connections between mathematical knowledge and ownership of technology. There were many times during data collection when the PSTs seemed either extremely certain or extremely uncertain about mathematical concepts. These PBS moments could give rise to inferences on PSTs' ownership of mathematics

and, in these contexts, possible connections with their beliefs about teaching with technology.

Researchers have noted that inservice and preservice teachers' beliefs about mathematics are often not the beliefs that most influence their teaching practice (Cooney, Wilson et al., 1998; Skott, 2001). As stated earlier, perhaps teacher educators assume that teachers' beliefs about mathematics must constitute the core beliefs that influence teachers' practice. It seems, however, that research on teachers' beliefs needs first to infer the beliefs that most influence teachers' practice, regardless of the domain of those beliefs. Informal discussions with Jeremy revealed that his religious beliefs greatly influenced his beliefs about teaching and learning. Perhaps in general there are circumstances or beliefs from outside the domain of mathematics education that significantly influence teachers' teaching of mathematics. Were I to conduct this study again, I would incorporate into the data collection a means for exploring the PSTs' beliefs about areas outside of education that were of particular value and interest to them. Indeed, future research on mathematics teachers' beliefs should seek to cast a broader net, exploring other educational beliefs (e.g., beliefs about curriculum, authority, the purpose of secondary schooling, assessment, the role of parents) as well as beliefs outside of education (e.g., beliefs about society, religion, and equity). Such research would increase the possibilities of accurately inferring teachers' centrally held beliefs in general and, in turn, those beliefs that are most likely to influence teachers' practice.

Building on the Study

Brill (1997) classified inservice elementary teachers' beliefs about teaching mathematics with technology as exploratory, premastery, or postmastery. The present study found that these classifications could be seen as just one of several dimensions of PSTs' beliefs about the desired availability of technology in the classroom. In addition to availability, pedagogical alignment and teacher knowledge constituted a set of beliefs referred to as beliefs about the nature of technology use in the classroom. Future research should seek to further delineate and extend these dimensions of beliefs about the nature

of technology in the classroom, and should explore the applicability of these constructs to both preservice and inservice teachers. Similarly, the implications these beliefs have on teachers' beliefs about the roles or technology in their classrooms need to be further explored. For instance, although some variation in the PSTs' beliefs about exploratory and visualization roles of technology are discussed in this dissertation, evidence for these particular roles was not explicitly sought in data collection. Contexts designed to elicit PSTs beliefs and actions with respect to these specific roles of technology could provide deeper understanding of the meanings teachers' ascribe to these roles.

Pajares (1992) stated, "Beliefs cannot be directly observed or measured but must be inferred from what people say, intend, and do—fundamental prerequisites that educational researchers have seldom followed" (p. 207). This study was designed in an attempt to satisfy these "fundamental prerequisites." Data collection strategies were designed in order to facilitate inferences based on a variety of contexts surrounding the typical experiences of PSTs. As discussed in chapter 3, the combination of email surveys and follow-up interviews, particularly in the form of PBSs, provided opportunities for triangulation that would not have been possible otherwise. As mentioned previously, PBSs could be designed in order to facilitate the inference of PSTs' mathematical knowledge or their beliefs about more specific roles of technology. PBSs could also serve to facilitate the exploration of PSTs' beliefs about other areas, such as pedagogical content knowledge or beliefs about assessment, cooperative learning, curriculum, or reform.

The notion of consistency is an overlooked theoretical assumption in much of the research on teacher beliefs. Not only is the definition of a belief often glossed over, the idea of a belief system, of how beliefs are related to each other and to practice, is often ignored. Thus, researchers claim beliefs impact practice, then call "foul" when the beliefs they thought would most influence practice do not do so. The challenge for teacher education is not merely to influence *what* preservice teachers believe—it is to influence *how* they believe it. When it comes to making pedagogical decisions, there are certain

desirable beliefs (Brouseau & Freeman, 1988) teacher educators want PSTs to hold in such a way that those beliefs strongly influence practice. Coherence theory is a constructive approach to viewing teachers' belief systems and changes in those systems. By way of coherentism, teachers are seen as complex, sensible people who have reasons for the many decisions they make. When teachers' belief systems are viewed in this way, we have a basis for constructing the types of teacher education advocated above. But this perspective requires more than simply administering a survey or observing a class. Multiple data collection strategies and opportunities are needed in order to make such inferences.

Research on teachers' beliefs should focus on building coherent models of teachers' belief systems. As Jeremy stated, "If I say it's necessary [to use technology], but then didn't do it? That would say I'm not making very much sense" (Tech interview). The process of exploring and explaining apparent inconsistencies rather than pointing out inconsistencies lends itself to developing a deeper understanding of the nature of beliefs and how they are held. This understanding, in turn, provides a different kind of information for teacher educators. Although change is still the fundamental goal of teacher education, it is the connections more than the beliefs that we desire to change. One of the goals of mathematics teacher education should be to influence teachers' beliefs about mathematics such that those beliefs strongly influence their teaching. In order to have this influence, however, teacher educators and the teachers themselves need to become aware of the beliefs that are currently filling that "most influential" role. Teacher educators need to find ways to present their values in such a way that preservice and inservice teachers see those values as beliefs they wish to adopt. From this perspective, teachers' belief systems are not simply "fixed" through a process of replacing certain beliefs with more desirable beliefs. Rather, teacher educators should seek to challenge teachers' beliefs in such a way that teacher educators' beliefs are seen by teachers as important beliefs with which to cohere.

Implications for Teacher Education

The philosophy of teacher education can be quite different when beliefs are viewed as sensible systems. Through the lens of coherentism, teacher education is not a matter of making teachers' beliefs and practice more aligned; beliefs and practice are aligned by default. Rather, with this view the goal of teacher education (or professional development) is to discover and then affect those beliefs most influencing the action of teaching. When discussing the relationship between beliefs and practice, educational researchers must be careful not to assume which beliefs precipitate a given action. The PSTs' core beliefs about the nature of technology in the classroom mediated whatever role the PSTs' envisioned for the use of technology in the classroom. We cannot assume, for example, that when teacher educators discuss exploring mathematics with technology, students are thinking about exploration in the same ways or with the same purposes as each other or as the teacher. Thus, when teacher educators teach PSTs about the use of technology to explore mathematics, they must also attend to variations on exploration that exist based on the nature of technology use. The variations in these natures need to be addressed and discussed. PSTs and teachers need opportunities to discuss what their classroom might (or does) look like and the kinds of activities that could be (or are) taking place.

Learning to Teach Mathematics With Technology

To make the implications of this study more concrete, it has been profitable for me to think about how this information could have helped me in teaching the Technology and Secondary School Mathematics course. At the time, I assumed my students already knew technology could be used to expedite procedures, and I did not think it would take much effort to convince them of its value for visualization. I focused on the use of technology for increasing understanding (reconceptualization), but primarily I was interested in them realizing that technology could be used to explore mathematics in ways that allowed them to learn new mathematics. What I did not address, however, was the idea that exploration looks very different in classrooms in which teachers have

different beliefs about the nature of technology. I was unaware of any dimensions of the nature of technology in the classroom other the premastery, postmastery continuum (Brill, 1997). In fact, I assumed anyone who would be using technology to explore mathematics would be doing it in the same way I would.

There is a parallel here to the idea that teaching for understanding looks very different in a classroom in which the teacher has instrumental beliefs about mathematics than in a classroom in which the teacher has problem-solving beliefs about mathematics. I assumed because the ideas were parallel (underlying philosophies or ideologies influencing teaching and learning) the categories would be parallel as well. I also assumed that someone who viewed mathematics instrumentally would view the use of technology quite differently than one who held a problem-solving perspective. I no longer believe this assumption. When viewed from Ernest's (1988) conceptions, my participants had very similar beliefs about the nature of mathematics (although the subtle differences are still quite interesting). Their beliefs about the nature of technology in the classroom, however, were quite different. They did not fit into this nice set of parallel dichotomies. In retrospect, I would have done things differently in the Technology and Secondary School Mathematics course, including providing opportunities that allowed the PSTs to explore their beliefs about the nature of technology in the classroom. I would have constructed activities designed to help them recognize explicitly what these beliefs were and then to reflect on them.

Connections Between Learning With Technology and Teaching With Technology

As a result of studying five secondary mathematics classrooms over the course of 3 years, Goos, Galbraith, Renshaw, and Geiger (2000) posited metaphors for viewing teachers' and students' interactions with technology. They apply the four metaphors of *master*, *servant*, *partner*, and *extension of self* both to students' learning with technology and to teachers' teaching with technology. With technology as *master*, the user is "subservient to the technology and is able to employ only such features as are permitted either by limited individual knowledge or force of circumstance" (p. 307). The user

seldom questions the validity of the result of the technology; rather, it is taken as given. When technology takes on the role of *servant*, the user likely has considerable knowledge of the technology, but the technology neither dictates nor significantly influences the choice of activity. Technology is used in order to carry out preferred or established strategies more quickly or accurately than could be done without technology. The authority of the technology is questioned only so far as the user checks to make sure they did what they thought they did: "Did I give the correct command? Then my answer must be correct." When the user views technology as a *partner*, the user is extremely comfortable with technology, seeks out new ways to use technology, and questions whether the results are mathematically or technologically reasonable. With technology as an *extension of self*, "powerful and creative use of both mathematical and communications technology" (p. 308) becomes as prominent as any other mathematical learning or teaching strategy. Not only are the results of technology questioned, opportunities to test such limitations are sought out.

Although Goos et al. (2000) drew no connection between the students' and teachers' similarly categorized interactions (i.e., technology as *servant* for student and technology as *servant* for teacher), my study allows me to posit some connections. PSTs find themselves in both the student realm and the teacher realm. Consequently, their beliefs about both learning and teaching are based in experience as learners. It is becoming increasingly rare to find PSTs who have not been exposed to technology. In general, PSTs have had considerable experience learning mathematics with technology as students, before becoming teachers. At the same time, as was discussed in the review of the literature, learning how to learn mathematics with technology does not appear to be sufficient to enable PSTs to teach mathematics with technology (Jones, 1998; Olive & Leatham, 2000).

There are similarities between the four metaphors from Goos et al. (2000) and the theory I posited for PSTs' ownership of learning with technology. With the theory of ownership, PSTs build on rather than replace their ownership from the previous phases.

Similarly, with these metaphors, the roles technology can play in the earlier phases continue to be valuable roles in the more advanced phases. There is nothing intrinsically wrong with using technology as a means of expediting procedures or checking for accuracy. This use could easily take place when a user views technology as an extension of self. But this user would also exhibit many other uses of technology, uses that take advantage of some of the more pedagogically sophisticated roles of technology.

Now suppose one were able to categorize a PST as primarily operating at one of these metaphorical levels as a student. What implications would this categorization have for the PST's technology use as a teacher? It seems likely that PSTs would not be able to operate as a teacher at a more advanced phase than they are operating as a student. In addition, it seems reasonable to assume that technology would initially play a somewhat lower role for a PST as a teacher than it would for a PST as a student. This relationship is really the notion of tacit versus explicit ways of knowing—the common phenomenon of being able to do something yourself but not being able to explain that something to others (D. L. Ball, 1991). A number of the PSTs had tacit knowledge of using technology to learn mathematics but were uncertain about how to use it to teach.

The Goos et al. (2000) metaphors provide a step towards making the connections between the phases students go through in learning mathematics with technology and the phases they might go through in teaching with technology. Take, for example, the classifications of roles of technology (e.g., motivation, expediting procedures, visualization, exploration) that emerged in this study. These roles represent a cross-section of the possibilities for technology use in the classroom and they seem to cut across the metaphors discussed by Goos et al. (2000). Consider the role of exploration. As was previously discussed, exploration did not have the same meaning for each of the PSTs in this study. For Ben and Jeremy, their beliefs about exploration treated technology as an *extension of self*. Lucy's notion of exploration treated technology as *master*. Exploration, for Lucy, was fairly closed-ended with little room for variation from the prescribed destination and known technology capabilities. In general, the procedural

roles parallel the metaphor of using technology as *servant*. But, once again, the PSTs did not define these procedural roles in the same way and this can make a big difference. Technology is servant if it serves only to speed up a procedure. But if the purpose in speeding up is to facilitate access to mathematical content beyond the procedure, then technology is acting more as a *partner*.

The theory is that PSTs' beliefs about teaching with technology are directly linked to their beliefs about learning with technology. These beliefs grow out of their experiences, naturally, but also are linked to their knowledge and confidence. Thus, the kinds of experiences PSTs have, and the opportunities they have to reflect on those experiences are critical. A course like the Technology and Secondary School Mathematics course can provide a valuable learning experience for PSTs. It can provide them opportunities to experience learning mathematics with technology and to reflect on their beliefs about the use of technology in the mathematics classroom. Additionally, PSTs need contextual experiences in which to ground their beliefs about learning and teaching with technology. They need opportunities to observe teachers teaching with technology. With these experiences in place, PSTs have a chance of enacting their beliefs, whatever those beliefs might be, when they enter the classroom. These experiences likely will not convince every PST to use technology in their teaching. But those PSTs who are going to use technology in their teaching need these kinds of experiences in order to have constructed sensible belief clusters around technology use in the classroom strongly connected to their beliefs about mathematics, teaching, and learning. Meaningful use is a direct result of meaningful connections among these beliefs.

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APPENDIX A: INITIAL EMAIL AND INTERVIEW PROTOCAL

Italicized words were used for my own organization or to indicate possible interview questions and did not occur in the actual email survey. The majority of the questions in the interview were follow-up questions to the participants' responses to the survey, asking the participants to elaborate or clarify their responses.

Mathematics

1. a) What do you like most about mathematics?
   b) What do you dislike about mathematics?

2. Is it possible to get the right answer to a mathematics problem and still not understand the problem? Explain.

3. If mathematics were an animal, what animal would it be? Why?

Learning

4. Describe the optimal learning environment for learning mathematics.

5. How can you tell whether a student is learning?

6. Consider the following similes: Learning mathematics is like

learning to ride a bike	learning archaeology	learning Spanish
learning to talk	learning to sing	learning to dance
learning history	learning to draw	learning to throw a baseball
learning to cook	learning to weld	learning a new software program

   a) Choose the simile that you believe best describes learning mathematics and explain your choice.
   b) Choose a simile that you believe does not describe learning mathematics very well and explain your choice.

Teaching

7. a) What mathematics subject do you most look forward to teaching? Why?
   b) What mathematics subject do you least look forward to teaching? Why?

8. a) List three qualities of an excellent teacher. Are any of these qualities optional (i.e., can someone still be an excellent teacher and not have this quality)?
   b) What makes someone a "bad" teacher (aside from not having the qualities you mentioned in part a)?

9. a) List three qualities specific to being an excellent <u>mathematics</u> teacher.
      Can these qualities be learned or acquired or are they intrinsic—natural gifts or talents?
      Which of these qualities do you feel is your strongest?
      Which of these qualities do you feel is your weakest?
      What do you see as the major differences between teaching mathematics and teaching other subjects?
   b) What makes someone a "bad" <u>mathematics</u> teacher?

10. Consider the following similes: A mathematics teacher is like

a news broadcaster	a missionary	a coach
a doctor	an entertainer	a social worker
a gardener	an orchestra conductor	a carpenter

    a) Choose the simile that you believe best describes a mathematics teacher and explain your choice.
    b) Choose a simile that you believe does not describe a mathematics teacher very well and explain your choice.

Resources, Techniques and Tools

11. Give an example of how you would use cooperative learning in your classroom.

12. What does it mean to teach mathematics with technology?

13. Consider the following question:

    Joachim's teacher asks him to find a fraction between $\frac{1}{2}$ and $\frac{3}{4}$. Joachim says $\frac{2}{3}$ will work. His teacher asks him to explain how he got his answer and why he thinks his method works. Joachim explains that he chose 2 for the numerator because 2 is between 1 and 3. He chose 3 for the denominator for the same reason. Does Joachim's method always work? Explain your reasoning.

    In what way and in what context would you use this item in your classroom?

APPENDIX B: PBS 1 PROTOCOL FOR LUCY

The protocol is organized around the PBS 1 email and the PST's responses. The outline

is the text of the actual email. The bold lettering in the left-hand column underneath each

question is the PST's written response; in the right-hand column are my possible

interview questions.

Here is the email that I promised you. I have listed some questions that might get you started, but do not feel like you must follow them exactly—use them as a springboard for your ideas. You are welcome to use any resources at your disposal, and you may assume that technological and other educational tools are available. Please bring your graphing calculator to the session. We will have a computer available for our use as well. Let me know if there are other tools/supplies you think you might want to use.

I. The topic being covered just prior to my teaching unit is: **Quadratic Equation and concepts surrounding it.**

    A. List the kinds of mathematics you think fall under this category (these need not come directly from the chapter just previous to the one you are teaching, or even from the book, for that matter).

Graphing, plotting, finding intercepts, relating linear equations and other types of equations, square roots, quadratic formula, completing the square and the purpose of completing the square, factoring, finding roots and what roots signify.	• What is being covered before you get there and what will you be covering?
	• Tell me the connection you see between quadratic equations and these topics:
	  • relating linear equations and other types of equations
	  • square roots

    B. How familiar are you with this topic? Have you had recent experience with it?

My sophomore year in college, I researched this topic for Dr. Oppong. I have tutored several people regarding this subject matter. As far as teaching a whole unit on quadratic, I am having difficulties producing a couple lessons.	• What kind of project did you do? Have you referred to any of that in planning your current unit? • What kinds of problems were the people you tutored having with quadratics? • With what lessons are you having difficulties? What kind of difficulties?

C. Where else might this topic come up in the secondary mathematics curriculum?

**I am not sure**, **but maybe with other graphing topics and solving equations. ???**	• Do you think your students well have had any prior experience with quadratics?

D. What would you like students to know about this topic?

I think that students should understand that this quadratic equations are everywhere. Quadratics, though semingly [sic] complicated at first, are important to several different types of applications. I would like for my students to make connections within other disciplines of mathematics (graphing, finding intercepts, etc.). I want my students to learn several different ways to solve quadratic equations and several ways in which they can be used.	• You say quadratic equations are everywhere—where are they? • What "different types of applications" are you talking about? • Why do you want them "to learn several different ways to solve quadratic equations?" • Why do you want them to know these things? Do you have some sort of mental checklist?

Now choose one (or several) of the ideas you listed in IA and explore how you might approach teaching it to your students.

II. Planning the Lesson

A. How might you introduce the topic?

GRAPHING QUADRATICS: **I was given several different resources [sic] to use in my quest for teaching this lesson. Many of the lesson I plan on teaching will intail [sic] using class starters. The one that I have chosen to use for graphing quadratics has the students explore several graphs using a graphing calculator, and they are to discover patterns with each type of equation given. This allows the students to make conjectures without teacher intervention.**	• Can you recall what you went through in trying to decide which of your original list of topics to choose for this section? • Let's try this out. What kinds of equations are we talking about here? • What kinds of patterns might the students find? • How are they going to record, test and share their conjectures? • What kind of time frame are we talking about for a class starter? • Are the students working individually or together? • Will the students be using the graphing calculators to graph quadratic equations at other times? For other purposes?

B. What types of examples/non-examples might you use?

I believe that the students will know some concepts about quadratic equations. I plan on asking them what they know about quadratics before the lesson begins, which will tell me where they stand on with this subject matter. I plan on using the class starter to help ask questions to the students about the subject.	• Let's get more specific. Walk me through how you might start this part of the lesson. Does it come after the class starter? • What are some questions you might ask? What will you do with the responses?

C. Are there specific applications or problems you want to use?

[Nothing]	• You mentioned earlier that there were many applications surrounding quadratic equations, but you didn't list any for this subtopic of graphing. Why?

D. What types of problems would you have the students work on?

I think that it is important that the students know how to graph the equations, as well as recognize a quadratic equation and graph. I also think that students should work on manipulating quadratic equations so that they are recognizable and they have the ability to graph and explain the graph and equation.	• How do you want the students to graph equations? What will they be doing to learn this? • Explain what you mean be "manipulating quadratic equations so that they are recognizable." • What do you mean by "explain the graph and equation?"

E. What might their homework look like?

I would like for the students to have application problems, as well as problems that simply test their skills. I want my students to have a balance of real-world application problems and book problems. I also want my students to relate this subject matter to other subject matters within mathematics.	• Any specifics? • Give me an example of an application problem. • Give me an example of a "simply test their skills" problem. • What is the distinction you are making between "real-world application problems" and "book problems?" • Give an example of what the students might do to relate graphing quadratics to another subject within mathematics.

III. Coordinating the lesson

A. How would you want to spend your class time exploring this topic?

I believe that class discussion and student exploration are vital tools to learning. I want to have feedback from my students and I want them to explore ideas on their own. Some topics are difficult to do this with, but the class starter that I have chosen will help me and the students learn on their own.	• Are there other topics in this unit you are thinking of that might be more difficult to do this with?

B. What would you do?

I have several types of questions that I plan on asking the students so that they can lead the discussion. For example, "What is a vertex? What is another name for a vertex?", "How can you tell if a parabola opens up or down? When is parabola wider or narrower than a standard parabola?"	• What is another name for a vertex?

C. What would your students do?

To try to answer some of these questions, my students will be equipped with a graphing calculator and explore some of the scenarios themselves. I want my students to facilitate the learning and exploring and lead the class discussions, as mentioned earlier.	• No questions

IV. Skills and Resources

A. How might cooperative learning, technology, practice, writing, etc. be used to teach this topic?

I believe that this question has been answered previously, but through graphing calculators, class discussions, student lead [sic] discussions, and other means of communication. I will encourage the students to explain conjectures to the class and when necessary have the students approach the board for further explanation.	• Who is doing what for each of these?

B. What kind of formative evaluation might you use?

I think that the best assessment I could give with this type of concept is questioning students and observation within class discussions. I would also like to check on homework and give a quiz.	• No questions

C. What are some questions you might ask on quizzes or tests?

The first question that comes to mind is giving the students an equation and having them go through the steps to graph it. This might entail manipulating the equation, solving for graphing purposes, and graphing. By the end of the chapter, they will have several methods of doing this type of question.	• What are the steps to graphing a quadratic equation? • Explain the distinction or connection between the three things you mention here: "manipulating the equation, solving for graphing purposes, and graphing." • By the end of the chapter, can they choose which method they want to use?

APPENDIX C: PBS 3 PROTOCOL FOR JEREMY

The protocol is organized around the PBS 3 email and the PST's responses. The outline is the text of the actual email. The bold lettering in the left-hand column underneath each question is the PST's written response; in the right-hand column are my possible interview questions.

In thinking about how you might go about teaching this, you are welcome to use any resources at your disposal, and you may assume that technological and other educational tools are available. Please bring your graphing calculator to the session. We will have a computer available for our use as well. Let me know if there are other tools/supplies you think you might want to use.

I. The category: **Polygon Similarity and Congruence**

    A. List the kinds of mathematics you think fall under the category Polygon Similarity and Congruence.

I have used the "Discovering Geometry" textbook to help me answer some of these questions. You would have received "slim pickins'" had I not used this book. I'll bring it in tomorrow with me. **Area, Proofs of similarity & congruence, ratios of similar parts of a polygon, proportions, Golden Ratio, similar triangles**	• Why would I have received "slim pickins" had you not used the book? • Tell me how you came up with this list. What did you add? • Was "Polygon Similarity and Congruence" a unit in the book?

    B. How familiar are you with this topic? Have you had recent experience with it?

Pretty familiar with it. **Yes, within the past year.**	• Describe the experience you have had with this "within the past year." • In what ways did your [experience with this topic] influence your brainstorming? • Of the things you listed above, which have you actually observed being taught? • Have you taught or tutored any of these topics yourself?

C. Where else might this topic come up in the secondary mathematics curriculum?

Geometry and/or algebra, trig, calculus	• Talk to me a little bit about how this topic comes up in Algebra, Trig, Calculus.

D. What would you like students to know about this topic?

How to do or knowledge of the list under part A.	• Could you be a little more specific? For instance, what do you want your students to know about area? The Golden Ratio? • Why do you want them to know these things? Do you have some sort of mental checklist?

Now choose one (or two) of the ideas you listed in IA and explore how you might approach teaching it to your students. Choose one that you think has a high potential for technology use.

Area or ratios and proportions

- Can you recall what you went through in trying to decide which of your original list of topics to choose?
- What were some of the criteria you used? What said "do this" or "don't do this" to you? What was there about your initial thoughts on [chosen topic] that led you to choose it?
- Do card sort activity:

High Potential for Technology
Medium Potential for Technology
Low Potential for Technology

- Area
- Proofs of similarity & congruence
- Ratios of similar parts of a polygon
- Proportions
- Golden Ratio
- Similar triangles

II. Planning the Lesson

A. How might you introduce the topic?

Introduce this by telling students I can find out how tall the flagpole is with some math and a mirror.	• Why did you decide to introduce the topic this way? • Have you observed or taught this idea before? • What happens after you tell them this. • Could you role-play this demonstration for me now? • How might this discussion of _____ go? What do you think the students might come up with?

B. What types of examples/non-examples might you use?

The mirror and the flagpole problem, a golden ratio problem	• What are these examples of? • Can you think of a non-example of a ratio? Something that someone might think is a ratio but it really isn't? How about a proportion?

C. Are there specific applications or problems you want to use?

Activities on Golden ratio (Discovering Geometry, P.475-478—I will bring book)	• What do you like about these activities? Is there anything you don't like? • Where would these applications show up if you were teaching a unit on similarity? • Choose one and explore it, talking about the specifics of what they would do.

D. What types of problems would you have the students work on?

Derivation of the Golden Ratio at some point, solving problems with similar triangles, proportions and area (p. 506), Volume and proportions	• Would the students be using technology when working on these problems? • Talk to me about the "volume and proportions" problems. What kinds of problems are you talking about?

E. What might their homework look like?

Some journal writings, some investigative work (such as find other examples that have the golden ratio), calculating some ratios, and some proofs are good candidates for homework	• What might you ask the students to write about in their journals? • How would you expect the students to find other examples that have the golden ration? What would you do with the examples they find? • How would the students be calculating ratios?

III. Coordinating the lesson

    A. How would you want to spend your class time exploring this topic?

Work an activity where we measure ratios of lengths of different parts of our bodies; e.g., body height vs. height of navel above floor, and comparing ratios.	• How much time do you think this activity would take? How long would the measuring take? The discussion? Then what?

    B. What would you do?

Circulate the room and participate with the class	• What kind of directions would you give?

    C. What would your students do?

Work together and compare their answers	• So they would be in groups?

IV. Skills and Resources

    A. How might cooperative learning, practice, writing, etc. be used to teach this topic?

It may be used a lot, especially with group work and comparison of data. Writing in journals would be used a lot so I can see how they are thinking about the mathematics involved. Practice will be used as well.	• No questions

    B. How integral is technology to your lesson?

It will be integral. Possibly use GSP to help measure ratios, or use Excel to work with the Golden ratio.	• What are some ratios you might measure on GSP? [Maybe have him show me on GSP]

    C. What kind of formative evaluation might you use?

Formative evaluation: journals, oral comments	• [If I haven't already asked this in IIE] Can you give me a specific example of a journal prompt for this topic?

    D. What would the end-of-unit assessment include?

Perhaps a project, written test, oral test (one on one with me)	• Talk to me about this idea of an oral test

APPENDIX D: OBSERVATION AND OBSERVATION INTERVIEW PROTOCOL

Adapted from Akujobi, 1995, pp. 192-195

Class Description

- What is the class size? Gender? Race?
- Do students work in groups?
- What role did the mentor teacher play during the class period?

Narrative Description

- Describe the lesson—the topics and the tasks.
- Compare with the lesson plan.

Instructional Materials

- The type of software available (number of computers, calculators, television, VCR, overhead projector, display, etc.) and how they are used during the lesson.
- Is the teacher telling or moving around watching how the kids interact with the technology?
- How is the technology used?
- Is the role of technology emphasized? Is it used to un-pack a complex mathematical idea, solve mathematical algorithms, construct graphs, find relationships, etc.? In what ways?
- Are students comfortable with the technology?
- Are they familiar with the software?
- Are they exploring new mathematical ideas or are they looking for right or wrong answers with the technology?
- Are they interested in computer games rather than using it to achieve the goal/objectives of the lesson?
- What do the students do with the computer that seems okay to the teacher?

Mathematics Instruction

- What is the goal? Are the mathematical meanings of the content emphasized in the lesson? the procedures? in what ways?
- What kinds of questions does the teacher ask? the students?
- Could the students have moved faster or understood better if they had (or had not) used technology?

Post-Observation Interview Protocol

- How do you feel the class went?
  - How did things compare with what you had expected?
  - Did anything surprise you?
  - Was there anything with which you were particularly pleased?
  - Did anything disappoint you?
  - Upon reflection, is there anything you would do differently if you were to teach this lesson again?
  - What was your biggest challenge today?

- I noticed you said _____. Why did you do that? Does it have any particular advantages?

- I noticed _____. Why did that occur? Was that intentional?

APPENDIX E: TECHNOLOGY INTERVIEW PROTOCOL FOR KATIE

The protocol is organized around the technology email and the PST's responses. The outline is the text of the actual email. The bold lettering in the left-hand column underneath each question is the PST's written response; in the right-hand column are my possible interview questions.

1. Describe your earliest experience learning mathematics with technology. What did you do? What did the teacher do?

My earliest experience learning mathematics with technology occurred when I think I was in ninth grade. In ninth grade, we were allowed to use calculators. I also think we went to the math computer lab once every two weeks. My first experience with the graphing calculator was probably in tenth grade in my Algebra II class. I remember my teacher passing out the calculators to everyone. Then I remember that she gave us specific commands to follow for opening up certain menus and selecting certain operations. I really don't remember what we used the calculators for—maybe graphing? All I remember about that experience was that I was lost and confused. I had no idea what I was doing, but I acted like I did by keeping busy with the calculator and not asking questions while the teacher was walking around. I'm sure my teacher assumed I followed everything because I always excelled in her class. After we were finished with the graphing calculators, my teacher picked them back up. Throughout the rest of the semester, we occasionally used the graphing calculators. I think this was because we only had a class set and because my teacher probably had to share it with other teachers. We also went to the computer lab once every one/two weeks. I really don't recall what programs we used

when in the lab. I think we used a graphing program (I know we definitely used one my senior year in calculus). I think we used a spreadsheet program at some point. I know that I definitely used the spreadsheet program my senior year in my statistics class, but I don't think I truly understood how to operate it. All this information I've told you is questionable for when it actually occurred. I remember that I was one of the best when it came to doing the computer assignments in the lab, but I never felt confident with my ability at that time. The computer still felt like a foreign object to me. This may be because we did computer work once every couple weeks. I felt more confident with the graphing calculators after our teachers had enough of them to assign to each student to take home. I then mastered the TI-calculators and have been dependent on them ever since! (Well, I can do the math usually on my own, but I'd rather have access to my calculator.)

- You mentioned being lost in 9th grade when you were using the calculators. Did this cause any problems for you as far as accomplishing the teacher's objectives?
- Thinking back on the various ways your teachers used technology in the classroom, do you think it was worth the effort? Do you think it took a lot of extra effort? Would you have done it differently?
- Have there been times when you wish you could have used technology in a class? Have you ever thought, "The teacher really should be using technology right now?"

2. What is the most positive mathematical experience you have had using technology? The most negative?

The most positive mathematical experience I've had using technology is when I learned how to operate the TI-89 and TI-92PLUS calculators. They can do EVERYTHING!!! I LOVE THEM! My senior year in high school the TI-92PLUS came out and my school had a class set of them. My calculus teacher first taught us the basics and then made us get into small groups, pick a feature (most of which we had never heard of), learn how to use it effectively, and present it to the class. Last year (I think), my brother bought the TI-89 and I was soooo amazed by its capabilities. It seems to be able to do everything that the TI-92 does, but it's the same size as the previous TIs. The most negative experience had to be in my diff-eq class here at Georgia (I think that was the class). My professor made us do out-of-class projects using Maple. We also had to take a few tests, including part of our final, using this program. The reason it

was sooooo bad was because our class was NOT in the computer lab. Our professor always brought his laptop to class and used Maple and projected it onto the overhead screen. Since I sit in the back of class, this was not very good. We went to the computer lab maybe once at the beginning of the semester, but that was it. To do well with a new program, you need a lot of practice. Our professor expected us to be experts with the program with no in-class, teacher-supervised practice. I then found out that the reason why my teacher was so into the "computer thing" is because he receives a lot of grant money for such programs to be used. This experience (and my very first experience with the TI- in tenth grade) is what I would consider my most negative, I think?

- Aside from the activities you did to learn about the capabilities of the TI-92, how did your calculus teacher use the calculators in class?
- Can you think of a more effective way the technology could have, or even should have been used in the diff-eq class?

3. What are some situations when you definitely would not use technology in your mathematics teaching?

I'd like to say that I would definitely use technology in all my mathematics teaching. But I do see certain situations when I would not allow my students to use calculators. For instance, I may want to give my students a memorization quiz dealing with theorems or definitions. Or I may give them a quiz where I want them to do certain math using only paper and pencils (i.e. adding/multiplying fractions, simplifying algebraic or trig functions [since the more advanced graphing calculators can now do this, I may not allow my students to use them on tests]) because there are certain things I want students to be able to do without being totally dependent on technology. I know that over time the students probably will forget, for the most part, the way to calculate certain things on their own but that they will still remember how to use the calculator to solve it, but I want the students to at some point understand how to calculate it because then they have imbedded in their minds the concepts taking place, so then when they use the technology they have some idea if the output is valid (you never know when you type something incorrectly into the program being used, but you should have some idea if the output is correct—this comes from understanding the operations taking place).

- What role, if any, do you think technology plays in helping students gain conceptual understanding?
- What are some excuses you have heard teachers or preservice teachers give for not using technology in the classroom? How do feel about those excuses?

4. Do you have concerns about using technology in your classroom?

I think my only concern right now is that not every student in my class will have access to a graphing calculator. The main piece of technology I'd use in my classroom would be the graphing calculator. I hope the school I teach at has a math computer lab for us to use frequently, but I doubt there would be a class set of computers in my classroom at all times. I would like to have a permanent computer in my classroom that can be hooked up to the overhead projector for class discussions and explorations and lectures.

- Is availability the reason graphing calculators would be "the main piece of technology" you would use in your classroom? Are there any other reasons?
- With what technology are you the most comfortable? Why do you think that is?
- With what technology are you the least comfortable? Why do you think that is?
- I am going to give you various situations in which you might find yourself. Describe how (and how often) you envision you might use technology to teach mathematics in these settings:
  - You have a classroom set of graphing calculators.
  - Each of your students has a graphing calculator they can take home with them.
  - You have one computer in your classroom.
  - You have five computers in your classroom.
  - You have a computer lab with enough computers for students to pair up.
  - You have a computer lab with enough computers for each student to have their own.
  - Your classroom has enough computers for each student to have their own.

5. What is the most important thing your students get out of being in your class related to life in general? related to being a student? related to mathematics? related to technology?

I want my students to (1) see math as a way to explain real-world situations, and (2) enjoy mathematics with understanding. I want my students to see that math can be understood by anyone. Math is just a way to look at the world around you. I also want my students to learn to work cooperatively with others. I want students to realize that everyone has a different way to look at the same situation but they can all be right at the same time. I want technology to be "natural" for my students. I don't want technology to intimidate them like it did me in high school (I really don't remember if I was intimidated, or if I just think I was because I know so much more now). Technology should make math appear simpler and more understandable and more applicable to my students because it allows us to do so much more in a short period of time. Also, I want my students to understand that the quick way to do things is not always the best way to do certain things. Some math problems take time, just like writing a paper for English takes time.

- Talk to me a little more about what you mean by "enjoy mathematics with understanding."
- In what ways do you see technology as making math appear "more applicable" to your students?
- What is the most important thing your students get out of being in your class related to being a student? related to life in general?

6. Do you think your use of technology will depend on the course you are teaching?

I hope my use of technology will NOT depend on the course or level of students I'm teaching because I feel every student has the ability and therefore should have the privilege to use technology. I also think technology is applicable to every aspect of mathematics.

- Do you recall ever having thought differently? Why did you think that?

7. Do you think your use of technology will depend on the level of students you are teaching?

[combined this with response to #6]

- Talk to me about the various levels of stress you might put on technology in different courses. How about the way it is used in these different courses?

8. Are there any mathematical topics with which you feel it is necessary to use technology to teach?

Well... pretty much all operations can be facilitated using technology, that's why I think technology should be used as much as possible.

- Okay, so all mathematics <u>can</u> be facilitated using technology, and it <u>should</u> be, but is there any mathematics where the technology is <u>necessary</u>?

APPENDIX F: FINAL INTERVIEW PROTOCOL FOR BEN

The protocol is organized around the final email and the PST's responses. The outline is the text of the actual email. The bold lettering in the left-hand column underneath each question is the PST's written response; in the right-hand column are my possible interview questions.

1. What role do you want real-world applications to play in your classroom?

I would want real-world applications to be a focus of a classroom. This seems to be where we drop the ball for students. They always want to know where they are going to use something, or what good is it doing to learn something. Well this is a great way to show them and give them a tool to solve problems for math and for life.	• How do you define "real-world application?" • In what ways do you want your students to be able to apply the mathematics they learn? Are you talking about inside your classroom or outside your classroom? • What do you think about the phrase students often use: "When are we ever going to use this?" • Have you ever asked this yourself? What are some answers you've heard people give? • What if you come across a concept where you just can't find any real-world applications? What would you do about that?

2. We talk a lot about problem solving in mathematics. What does problem solving mean?

Problem solving to me means to be able to take a problem that you are faced with and break it down into the good stuff and the junk, and then be able to set a strategy to solve the problem, and if it does not work pick a different avenue to try and solve the problem. For me mathematics gives me a tool box and I have to pick a tool to work on a problem in order to solve it.	• Is there more to mathematics than problem solving? Are there activities you think are an important part of mathematics that you would not consider problem solving? • How strong would you say you are mathematically? What are your strengths or weaknesses when it comes to mathematics?

3a. What does it mean to "play around with mathematics?"

To play around with mathematics simply means to take what you know and see what you can come up with from there. how would this effect this problem or can I use this here, etc... it is a way to discover things in mathematics.

- Why would someone do this?

3b. Describe a situation where you would have students play around with mathematics.

Area of Regular Polygons. In this case they already know hoe to find the area of triangles and they could break any regular polygon into triangles, but can they figure out a formula to fins the area of any regular polygon? I did this

- You did this exploration yourself or you had your students do it? If students, talk about how it went. Where did the students struggle? Did they find things other than what you were expecting? Were they drawing the figures themselves or did you give them to them?
- Go to questions in number 8.
- Do you play with mathematics? Do you want your students to play with mathematics in the same way as you? Can they?
- Also, with technology—What does it mean to play around with technology? Would you want your students to do that?

4. What does it mean to get down on the level of your students?

To me it means to relate what I am teaching to a area or situation in their lives. This is one of my strengths I believe.

- What level are the students on? What level are you on?
- How important is it to use a variety of approaches when you are teaching? Why is it important to do this? What are some of the various approaches you would use most often?

5. What or who has had the greatest influence on you as a teacher? Explain.

This would probably be [my methods course teacher] or you because I kind of knew how to get my ideas into a teachable lesson from my parents but ya'll should me how to organize and perform lessons in a way that gains conceptual understanding and the way in which each of you teach helped me to find my on style. And I guess everyone I have ever seen teach has impacted me.

- Describe your teaching style.
- You state that probably everyone you have ever seen teach has impacted you. Why do you say that? In what ways do you think they impacted you?
- Does this change if we change it from "teacher" to "mathematics teacher?"

6. What does it mean to be fully prepared to teach a mathematics lesson?

That is just about impossible. I guess to know a subject well enough to be able to handle as many questions or tangents as possible and how to get it back to the point I was trying to teach. This is hard.	• What do you mean here when you say "subject?" For a given day's topic, what do you do to feel prepared? • You talk about getting it back to the point you were trying to teach. How do you prepare to do that? • Does that change when manipulatives/technology are/is involved? • How do you feel about "winging it" in the classroom? • What did you think about the textbooks you used while you were out teaching?

7. How well do you have to know students to be able to effectively teach them?

I think you can effectively teach students that you don't know if they had a respect for learning and education, now in the real world I think you need to know them pretty well so that they know your expectation and you know how to handle them.	• So is discipline the deciding factor here in what you are saying? Aside from this, how important is it to know them? • What are some learning styles your students might have? What do you think your learning style is? • How important is it to you to have students figure things out on their own? • What does it mean to figure out mathematics on your own?

8. Describe a lesson you can see yourself doing with high school students that you would classify as "exploring mathematics with technology."

The GSP lesson of finding the formula for a any regular Polygon.	• Describe where you might have students exploring mathematics without technology. • What are the students exploring? • What does it mean to explore mathematics using technology? • What if, when exploring, students find things other than what you had in mind?

9. One concern expressed by many about using technology in the mathematics classroom is that the technology may "replace understanding" or become a "substitute for thinking." How do you feel about this concern and what will you do as a teacher to try to make sure that it doesn't happen in your classroom?

Well I think the technology brings discovery and this leads to learning so it does not replace thinking or understanding, but it should be used in a fashion to further education and to help students move on to greater things.

- What is that fashion? What is not?
- What do you see yourself doing to address that concern?
- We have talked quite a bit about technology in these discussions. When you use the word technology with respect to the mathematics classroom, just what do you mean by that term? What does "technology" entail?

10. What is the biggest advantage to using technology in your classroom? The biggest disadvantage?

the biggest advantage to me is personal discovery and the biggest disadvantage would be if you have to teach the technology.

- What do you mean by "personal discovery?"
- You say "if you have to teach the technology." When do you and when don't you?
- Does using technology in a mathematics classroom replace the thinking of students? What thinking is it replacing?
- What kind of thinking would you like your students to be doing in your class?

APPENDIX G: GROUP INTERVIEW PROTOCOL

Feelings about the research experience

- Did you get anything out of this experience? Did you learn anything about yourselves?
- What was the most difficult aspect of being involved in this research? What did you find easy?
- I used several strategies for collecting data—email, brainstorming, interviews—with which medium did you find it easiest/hardest to express yourself? Why?
- Did the email prompts help you to feel more prepared for the interviews?

Me and my questions

- I feel like I have played several roles with most of you—your teacher, counselor, friend, and now researcher—was that ever difficult? In what role did you primarily think of me during the interviews?

The focus of the study

- What do you feel like the focus of this study was?
- Was there ever anything you felt like was the "right answer?" Did you ever feel like technology was the "right answer?"
- Here was my main question: What relationships exist between preservice secondary mathematics teachers' beliefs about teaching mathematics with technology and their beliefs about mathematics, it's teaching and learning?
- What do you think I would say about your beliefs about these areas—first, mathematics? teaching? learning? teaching with technology?

Miscellaneous—get them talking

- Who in this group is
  - most likely to use technology on the first day they teach?
  - most likely to ask more questions than they give answers in the classroom?
  - most likely to get a masters degree in mathematics education?
  - most likely to be teaching high school mathematics in the year 2011?

APPENDIX H: SAMPLE CODING TRANSFORMATION

Example 1:

A. Role of Technology

Example 2:

A. Role of Technology (How they use it)—could be with any stage of the timeline

i. Demonstration
ii. Exploration
iii. Motivation

iv. Visualization
v. Practice
vi. Reward

vii. Distraction
viii. Discipline

Example 3:

A. Role of Technology

i. Demonstration
ii. Exploration
iii. Motivation
iv. Visualization
v. Practice

vi. Reward
vii. Distraction
viii. Discipline
ix. Check
x. Supplement

xi. Expedite
xii. Crutch
xiii. Replacement
    for knowledge

Example 4:

A. Role of Technology

i. Demonstration, illustration
ii. Exploration, experimentation, conjecture, discovery, investigation
  a. Individually
  b. As a class
iii. Motivation
iv. Visualization
v. Practice
vi. Reward
vii. Distraction
viii. Discipline
ix. Check
x. Supplement
xi. Expedite
xii. Calculate, solve

xiii. Improve accuracy
xiv. Roll and grades
xv. Dynamic—track change
xvi. Provide alternative/another way
xvii. Life skill
xviii. Organization
xix. Real world
xx. Tutorial/test
xxi. Involve Students
xxii. Get students thinking
xxiii. Make connections
xxiv. Get beyond basics
xxv. Make easy
xxvi. Do things you couldn't otherwise

Example 5:

A. Role of technology
    i. Not mathematical
        a. Benefit to students
        b. Motivation
           (1) Reward
           (2) Discipline
        c. Roll and grades
        d. Life skill
        e. Involve students
    ii. Procedural
        a. Check
        b. Expedite
        c. Calculate, solve
        d. Improve accuracy
        e. Tutorial/test
        f. Get students thinking
        g. Make easy
    iii. Conceptual
        a. Demonstration, illustration
        b. Exploration
        c. Play
        d. Visualization
        e. Dynamic-track change
        f. Organization
        g. Do thing you couldn't do otherwise
        h. Model
    iv. What it gets you
        a. Supplement
        b. Alternative-another way
        c. Real world
        d. Make connections
        e. Get beyond basics