PRESERVICE TEACHERS' QUESTIONING IN ELEMENTARY STUDENTS'

MATHEMATICAL PROBLEM SOLVING

by

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(Under the Direction of DENISE A. SPANGLER)

ABSTRACT

The quality of teacher questioning affects the extent of its utilization, particularly in a dynamic teacher-student interaction. I first analyze 13 preservice teachers' questioning moves while elementary school students were engaged in mathematical problem solving. I then examined six preservice teachers' questioning practices on whole-number arithmetic tasks in their single-student mathematical field experiences (SSMFE) and focused on the construction and functioning of teacher-student interactional turns to describe the functions of interactional turns and patterns.

Data were collected in the form of observations, video recordings, and course assignments and analyzed using theme-based coding from an integrated framework including categories of questioning moves and interactional patterns. The findings regarding questioning moves revealed four influential features in preservice teachers' questioning: (a) flexibility in the setup, (b) limited extent of inquiry, (c) non-specific probing questions, and (d) neglect of the child's unexamined but valuable strategies. The analyses show that 1) Task Clarification (TC) moves were successes when teachers provided flexible support in questioning, and 2) Procedural Understanding (PU), Making Connections (MC), Rationale Behind a Strategy (RA), and

Alternative Strategy (AS) moves resulted in deviated from the contextual features and mathematical relationship in problem solving and indicate that the preservice teachers' functional moves have potential to elicit multidimensional facets of students' mathematical thinking and yet may not be enacted competently in mathematical problem solving.

These findings not only detail the conditions for enacting functional moves but also contribute to better document the successes and struggles a functional move prompted. Based on the findings, suggestions for developing a practice-based training curriculum to enhance the cultivation of teacher questioning are presented. In addition, how to effectively utilize student-produced discourse to inform teacher questioning strategies in early field experiences has important implications for curriculum designers, teacher education programs, and for teacher educators and researchers.

INDEX WORDS: Teacher questioning, mathematics methods courses, field experience, mathematical problem solving

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A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2019

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DEDICATION

This dissertation is dedicated to my family in Taiwan and all friends for the support throughout the entire doctorate program. I would also like to thank my husband, Perry Wu, for all the wonderful meals he cooked and standing by me along the way.

ACKNOWLEDGEMENTS

My research is situated in a community that the meaning of instructions among people was co-constructed. Therefore, I believe that every individual involved in this research community contributed to this study and I view this work as "our" work.

I would like to thank Dr. Denise Spangler for her guidance, time, support and love, which helped me grow as a better researcher and educator. I would also like to express my gratitude to mt committee members (Dr. Beckmann and Dr. Bishop) for their suggestions and help on my research. I also wish to thank the course instructors and all preservice teachers and students who participated in my study. I could not have completed the whole study without their participation. In addition, I would like to thank other individuals (colleagues, librarians, OIT staff, and writing editors) who helped and encouraged me through my years of graduate school. This process could be a difficult journey without them.

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CHAPTER 1

INTRODUCTION

In the Chinese language, the concept of knowledge is expressed as an amalgamation of learning and questioning. The two-component ideographs are "學 (xué)" and "問 (wèn)". The ideograph for questioning shows an open mouth, presumably in the midst of evoking a question. The Chinese language therefore explicitly manifests the central role that questions have in the development of knowledge. Rather than being merely a passive accumulation of facts, knowledge comes from actively questioning during the learning process (Lauer, Peacock, & Graesser, 1992). Therefore, asking questions could be a means to accomplish the goal of receiving information and constructing knowledge in a dynamic interaction between the questioner and the respondent.

Although asking questions is taken for granted in the usage of language in society, asking questions plays a considerable role in the field of education, particularly when one plans to learn something from others. Learning is the main goal in every classroom, regardless of subject matter. On the one hand, students learn how to construct knowledge through seeking relevant information, and, on the other hand, teachers apply instructional moves to learn about their students in order to scaffold students' learning. This type of learning of teachers is also described as a process of learning to teach (Ball & Bass, 2000; Frank & Kazemi2001; Mewborn & Stinson, 2007; Shulman, 1987). For most teachers, the most common technique used to access and assess students' knowledge is enacting questioning, as Wassermann (1991) underscored: "Questions are the building blocks of the instructional process" (p. 257).

According to Shulman (1987), there are, at minimum, seven categories of the teacher knowledge base. One of them is the knowledge of learners and their characteristics, which plays a vital role in teachers' teaching practices (Raymond, 1997). However, the process of constructing the knowledge of learners could be extremely challenging for novice teachers, who have relatively limited experience working with students. Therefore, I am interested in learning how questioning is enacted to construct the knowledge of learners in preservice elementary teachers' initial learning-to-teach stage.

The Importance of Teacher Questioning

Although researchers have conceded that students rarely ask questions in their learning (Dillon, 1987, 1988; Flamer, 1981; Graesser, Person, & Huber, 1992), they have acknowledged that teachers frequently employ questioning as a tool in their teaching (Floyd, 1960; Moyer, 1967; Stevens, 1912). Aschner (1961) called the teacher "a professional question maker" and asserted that teachers "probably devote more time and thought to ask[ing] questions than anybody since Socrates" (p. 44). Additionally, scholars have suggested that the ability to enact questioning considerably influences students' learning opportunities (Hackenberg, 2005; Martino & Maher, 1999; van Zee & Minstrell, 1997; Webb, Nemer, & Ing, 2006). Nevertheless, teachers' questioning techniques seem oriented toward particular types in their practice. For example, Boaler and Brodie (2004) found that teachers using traditional curricula asked 95% of their questions to gather information or lead students through a method, and even the experienced teachers, who were using reform curricula, enacted 60% to 75% of the same type of questions in their teaching.

A century ago, Stevens (1912) rightly asserted that the commonly used question-and-answer type of recitation is more fruitful for the teaching process compared with repeating facts, rote testing of facts, and lecture. In addition, she warned that a teacher might "foster in her pupils negative habits of work, poor associations, and careless impression" (Stevens, 1912, p. 4) if she is not an expert in using the right questions in the right place to teach her pupils to construct knowledge. Equally important, when the intention of questioning is implicit, the questioning move per se could confuse students, particularly younger children, about teachers' motivations for using questions in teaching. The following vignette described by Wragg and Brown (2001) is an example:

> A 5-year-old girl returned from her first day at school and announced that her teacher was no good because she did not know anything. When asked why she thought this, she replied that "the teacher just kept on asking us things." (p. 5)

The *Professional* Standards *for Teaching Mathematics* (NCTM, 1991) has emphasized that teachers are expected to ask and stimulate students to ask questions in order to help students: (a) "work together to make sense of mathematics;" (b) "rely more on themselves to determine whether something is mathematically correct;" (c) "learn to reason mathematically;" (d) "learn to conjecture, invent, and solve problems;" and (e) "connect mathematics, its ideas, and its applications" (NCTM, 1991, pp. 3-4). That is, effective classroom discourse is crucial in developing mathematical skills and literacy, and "this development cannot be achieved without teachers' asking a variety of questions that challenge students' thinking" (Vacc, 1993, p. 91).

However, Mehan (1979) warned that the questions asked in classrooms contain unique features, and "[t]eachers are sometimes not aware that the child's display of knowledge is constrained by the structure of the task, the organization of discourse, and the physical parameters of the teaching-learning situation" (p. 294).

Teacher questioning in the mathematics classroom involves multi-faceted knowledge, including knowledge of mathematics, knowledge of students, and knowledge of the pedagogy of mathematics (Lappan & Theule-Lubienski, 1994) and is "a practical matter [that can]not [be learned] by talking about it, but by doing it" (Fitch, 1879, p. 78). Most importantly, teacher educators should keep in mind that learning to enact questioning requires "shifting the practices and beliefs of the individuals engaged in those interactions" (Moyer & Milewicz, 2002, p. 295-296), and could be "cognitively demanding," as it "requires considerable pedagogical content knowledge[,] and necessitates that teachers know their students well" (Boaler & Brodie, 2004, p. 773).

Prior studies have revealed the heterogeneity of questioning performance that exists between novice and experienced teachers (Hyman, 1979; Sahin & Kulm, 2008; Tienken, Goldberg, & DiRocco, 2009). Generally, novice teachers may experience more anxiety related to posing questions (Brown & Edmondson, 1984; Crespo, 2003). The weaknesses in their questioning could include difficulties in assessing students' understanding (Nicol, 1999), failure to ask probing questions to develop deeper thinking in students (Sahin & Kulm, 2008), and the tendency to ask more leading questions and to overlook opportunities for probing student thinking (Weiland, Hudson, & Amador, 2014). However, field experiences can provide opportunities for preservice teachers to investigate questioning strategies to gain knowledge of students' mathematical thinking (Chamberlin & Chamberlin, 2010; Mewborn & Stinson, 2007).

For example, Chamberlin and Chamberlin (2010) stated that, “[m]any of the teachers mentioned questioning the students to stimulate their thinking, to refocus them on the problem at hand, to understand the students’ thinking, or to challenge the students in their thinking” (p. 402) in preservice teachers’ gifted education field experiences. This finding exemplified what Mayor and Milewicz (2002) concluded: “[H]aving preservice teachers focus on the skill of questioning in a one-on-one diagnostic interview may be an effective starting point for developing the mathematics questioning skills they will use as future classroom teachers” (p. 297). Furthermore, I argue that, after obtaining a comprehensive understanding of preservice teachers’ practices, teacher educators should cultivate and develop preservice teachers’ abilities to enact questioning through constant learning and practice, beginning at the early stages of the teacher education program.

The Importance of this Study

Compared to the studies on experienced teachers’ questioning skills (Boaler & Brodie, 2004; Di Teodoro, Donders, Kemp-Davidson, Robertson, & Schuyler, 2011; Sahin & Kulm, 2008; van Zee & Minstrll, 1997), relatively little research has examined preservice teachers’ questioning performance in the teacher education stage (Moyer & Milewicz, 2002; Nicol, 1999; Weiland et al., 2014). Preservice teachers’ field experiences serve as their first official teaching praxis to apply what they have learned in teacher education courses. Raymond (1997) stressed that “Mathematics educators cannot ignore the fact that teachers are exposed to many factors that may influence practice” (p. 574). Therefore, I conducted this study to investigate how preservice teachers enact and refine questioning strategies to support students’ problem solving in the mathematical field experience.

To provide valuable information to extend our understanding of how preservice teachers enact questioning in a mathematical context, I analyzed, in particular, teacher questioning in the stages of problem solving (Polya, 1957) and considered students' responses during those interactions. By studying both verbal and nonverbal interactional moves, I gained insight into how preservice teachers enact questioning and students' reactions caused by their questioning. The study reveals the features, functions, and constructions of interactional turns, as well as the successes and difficulties in preservice teachers' questioning. Understanding preservice teachers' questioning will not only help teacher educators and researchers facilitate the development of questioning techniques in teacher education programs but will also benefit both teachers and students in their interactions.

The Purpose and Research Questions

The purpose of this study was to investigate elementary preservice teachers' questioning practices when they conducted mathematics interviews with only one student in the context of solving mathematical tasks. During the interviews, the preservice teachers were instructed to focus completely on how to employ questions to learn about their students' mathematical thinking in a field-based activity – a Single Student Mathematics Field Experience (SSMFE), in which preservice teachers conduct eight interviews with the same child over the course of a semester (Sawyer & Lee, 2014). The following questions guided my investigation during this study:

1. What is the nature of enacted questions that elementary preservice teachers employ in their SSMFE interviews?
2. How are the teacher-student interactional turns constructed and functioning in their SSMFE interviews?
3. What are the successes and difficulties in elementary preservice teachers' questioning practices in their SSMFE interviews?

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Questioning is generally accepted as a move in which people employ questions to seek desired information from others, normally in verbal form. According to Green (1971), questioning is a strategic move that is evaluated by the consequences it causes. Questioning has been considered one of the most important methods of instruction since Ross (1860) wrote about it, approximately 150 years ago. In this chapter, I begin by tracing the origin of questioning in history. Then, I synthesize various question-classification systems and the criteria behind them and the literature on student learning and mathematics teaching in relation to teacher questioning. I conclude this chapter by describing literature that informed integrated theoretical framework used in the study.

The Origin of Questioning

The history of questioning can be traced back to Socrates, whose Socratic method has long been recognized as the practice of disciplined, rigorously thoughtful dialogue. For example, in Plato's dialog *Meno* (Long, 2002), Socrates used a series of questions to help a slave boy discover the relationship between two squares. In the vignette, this geometry problem had a determined solution, and the purpose of the Socratic questions was to help the student arrive at a particular conclusion without telling him the answer directly (Graesser, Person, & Huber, 1992). Another example occurred in *The Republic* (Grube, trans. 1992), in which Socrates helped

Glaucon and Adeimantus reflect on a new understanding of justice by purposefully posing strategic questions. This process is similar to the concept of divergent thinking, which "represents intellectual operations wherein the individual was free to generate independently his own data within a data-poor situation or to take a new direction or perspective on a given topic" (Gallagher & Aschner, 1963, p. 187). Although the Socratic method was generally accepted by teachers and educators in conversations about classroom pedagogy, it was actually "deduced from the study of works by Plato, Aristotle, Aristophanes, and Xenophon" (Schneider, 2013, p. 625). In other words, there was no one, definitive "Socratic method," such that it became "something of a pedagogical free-for-all," in which "educators were free to borrow what they liked from others, take their own liberties, and make of the methods what they wished" (p. 632).

The application of the Socratic method has spread widely in several subdomains in education over time. Beginning in the late 19th century, theoretical essays that addressed the art of questioning, particularly with regard to the feasibility and misuse of questioning techniques, emerged. As the earliest record in literature, an essay by Ross (1860) defined the catechetical method as the method of teaching by questioning and distinguished examinatory questioning, which was "used to prove whether what has been previously learned has been rightly understood and is remembered by the pupil" (p. 367), from catechetical questioning. He then listed detailed descriptions, comparisons, and warnings in terms of applying questioning techniques in teaching. In particular, he suggested that teachers anatomize each question beginning with the individual words used, and then move on to syntactical relations, collateral facts, and eventually to the questions' implications. Moreover, he warned that teaching by this catechetical method should avoid preachments and monotony of voice, so as not to descend to incorrect language or manners, not stray too far from the principal point, and not to be beyond respondents' comprehension.

De Garmo (1902) asserted that "[t]o question well is to teach well" (p. 179) and accentuated that the fine art of teaching lies in skillful questioning more than in anything else. He grouped questions into four classes: (1) Analytical (to analyze knowledge into its elements), (2) Development (to aid the pupil in arriving at a clear comprehension of anticipated themes), (3) Review (to reconsider the formed conceptual construction), and (4) Examination (to inspect accomplished procedures). He then further emphasized that "all questions should be definite, comprehensible, and thought-provoking" (p. 181). He insisted that questions should be logical and reasonable and identified the special characteristics of questions that should be avoided, such as obscure and technical expressions, as well as helpful techniques for the formation and implementation of questioning. His essays advanced the general principles by which questions should be framed, delivered, and examined and simultaneously signified the merits and pitfalls of enacting questioning in practice. Although these theoretical, descriptive articles did not provide research-based evidence about questioning, they inspired scholars to conduct empirical studies in classrooms.

Classification of Teacher Questioning

Almost no research-based articles describing teacher questioning in classrooms existed prior to the work by Stevens (1912). A pioneer in examining the practice of teacher questioning, Stevens (1912) conducted 100 observations across multiple subjects in high schools and investigated the number of teachers' questions and students' responses. She found that the average number of questions asked by teachers per day (200 total minutes of class time, on average) was 385 and further discussed the efficiency of instruction reflected in teacher questioning. Stevens (1912) claimed that "[e]fficiency of instruction involves good questioning; good questioning is synonymous with the use of good questions" (p. 71), and she was particularly concerned with two phenomena related to the rapid-fire method of teacher questioning: the fact that (1) the implied "high-pressure atmosphere" created by asking a large number of questions "produced an inconsistency between the pace of teaching and 'nature's own processes of mental activity'" (p. 17); and (2) "the largest educational assets that can be reckoned are verbal memory and superficial judgment" (p. 23). In sum, Stevens (1912) discussed the number of questions and the usage of the question-and-answer exchanges in her study but did not develop categories for the questions.

To properly examine questions enacted in different subject matters and at different grade levels, researchers devised unique criterion measures to categorize the observed questions. The question-classification system developed after Stevens' (1912) study presented, essentially, dichotomies, such as traditional versus unusual questioning procedure (Briggs, 1935) or memory versus thought-provoking questions (Corey & Fahey, 1940; Haynes, 1935). Specifically, analyses of teacher questioning often employed the dichotomy between eliciting factual knowledge and provoking thought. For example, Haynes (1935) found that 77% of teachers' questions observed in elementary school called for a factual answer, with only 17% eliciting student thinking, and other studies demonstrated similar proportions of these two types of questions (Guszak, 1967; Moyer, 1967; Schreiber, 1967).

The question classification system was eventually expanded to more than two categories; the most famous of these expansions was Bloom's Taxonomy (1956), or variations based on it.

To illustrate how various question classification systems have employed Bloom's Taxonomy, it is first necessary to outline the types of knowledge and cognitive processes in Bloom's Taxonomy (see Figure 2-1) and recapitulate the definitions of both dimensions (see Table 2-1) before providing representative systems. In my review of relevant literature, I noticed that the question classification systems related to both the knowledge dimension and cognitive process, so I also adopted the revised Taxonomy and included definitions and examples to expose readers to a comprehensive understanding.

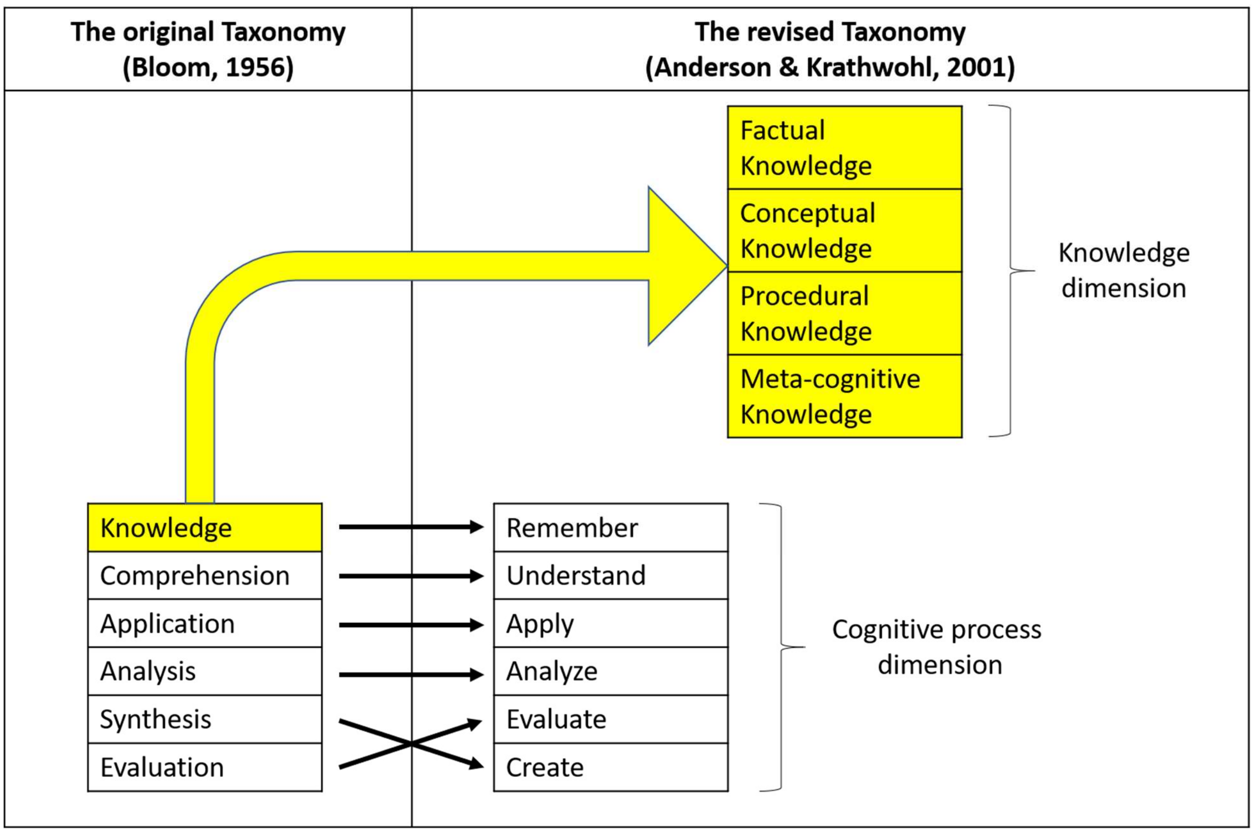

Figure 2-1. The mapping of the original Bloom's Taxonomy and revised Taxonomy.

Table 2-1

Types in the Knowledge Dimension and Categories in the Cognitive Process Dimension in the Revised Taxonomy (Anderson et al., 2001)

The knowledge dimension	
Types	**Examples**
Factual knowledge	The basic elements students must know to be acquainted with a discipline or solve problems in it
Conceptual knowledge	The interrelationships among the basic elements within a larger structure that enable them to function together
Procedural knowledge	How to do something, methods of inquiry, and criteria for using skills, algorithms, techniques, and methods
Metacognitive knowledge	Knowledge of cognition in general as well as awareness and knowledge of one's own cognition
The cognitive process dimension	
Categories	**Definitions**
Remember	Retrieve knowledge from long-term memory
Understand	Construct meaning from instructional messages, including oral, written, and graphic communication
Apply	Applying a procedure to a familiar task
Analyze	Break material into its constituent parts and determine how the parts relate to one another and to an overall structure or purpose
Evaluate	Make judgments based on criteria and standards
Create	Put elements together to form a coherent or functional whole; to reorganize elements into a new pattern or structure

Although Bloom's (1956) Taxonomy was proposed in the 1950s, Hunkins (1968) recognized in the 1960s that it had "seldom been employed as a guide for teachers' questions and as a means for their study" (p. 31). Because Bloom intended his Taxonomy to provide a classification of educational goals instead of teacher questions, researchers seemed to favor Sanders' (1966) classification system, which included an additional category, *interpretation*, over Bloom's Taxonomy, and researchers widely adopted Sanders' work for studying teachers' questions.

Whether the original Taxonomy was publicly mentioned or not, the idea of Bloom's categories formed the central idea in a number of question-classification systems found in a review of the literature on teacher questioning, as shown in Table 2-2. For example, Aschner (1961) found that it was easy for teachers to ask questions that call for remembering, and thus, the most common thinking activity that occurred in a classroom was remembering. When reviewing the outcomes from their studies, I recognized a very similar trend of employing various questions in the cognitive process dimension in that era. Sloan and Pate (1966) concluded that recall questions were used more than any other type of question in both traditional (573 out of 1,517) or new-math curriculum groups (380 out of 1,536). Moyer (1967) found the disappointing outcome in his study that only 29% of the 2,500 questions asked by teachers provoked students' thinking and noted that questioning practices in instruction in the 1960s were not used primarily to "stimulate thinking as opposed to the somewhat discredited question-answer method employed prior to 1920" (p. 214). Likewise, Schreiber (1967) concluded that the most prevalent type of questions asked by 14 teachers in social studies classrooms was the recall of facts (1,076 out of 2,704), and Davis and Tinsley (1967) analyzed questions asked by 44 high school social studies teachers and their students and found 1,313 memory questions out of a total of 2,520 questions. Guszak's (1967) study revealed, among 1,587 questions observed from 12 elementary classrooms, that recall questions occurred in the highest proportion (56.9%) and that most evaluation questions called for a simple yes or no response.

In the early 1970s, the field produced additional comparison studies. Godbold (1970) conducted a comparison study to examine questions asked by four groups of elementary and secondary teachers and found that the proportion of memory questions in each group ranged from 54.84% to 67.78% and occurred in the highest proportion among the eight categories of questions across all groups. Rogers (1970) focused her study on two groups of elementary student teachers (experimental and control groups) with questioning interventions. In the cognitive categories, the mean percentages of memory oral questions in both groups were approximately 51% and 57%, respectively (no statistically significant difference at .05 level), but the mean percentage of memory test questions composed by student teachers was statistically different (36% in the experimental group and 91% in the control group).

Lastly, I concluded that the application of Bloom's Taxonomy to classifying questioning systems seemed to be the foundation of the question-classification systems that considered the cognitive process. Moreover, it could be incorporated into a more complex system that might include more than one parameters. For example, Enokson's (1973) theoretical model, *the simplified teacher question classification*, included one parameter to analyze questioning quality and another on the cognitive level to examine cognition based on Bloom's Taxonomy. In this simplified model, Enokson (1973) only divided the cognitive operations into two categories—low and high mental operations. The low cognitive questions required memory or the simple retrieval of data, so this type of question was also called data recall questions and corresponded to the knowledge category in Bloom's Taxonomy. The high cognitive questions asked students to perform higher-order mental operations and were also defined as data processing questions (see Table 2-2 for the classification).

Table 2-2

Representative Question-Classification Systems

Author (year)	Knowledge (remember)	Comprehension (understand)	Application (apply)	Analysis (analyze)	Synthesis (create)	Evaluation (evaluate)	Other
Aschner (1961)	Memory			Reasoning	Creative thinking	Judgment	
Sloan & Pate (1966)	Recall Recognition	Demonstration Comprehension		Analysis	Synthesis		Opinion Attitude
Moyer (1967)	Name State Describe	Explain Define Illustrate		Compare-contrast	Suggest	Affirm	Opinion Alternate choice Yes-no Action
Sanders (1966)	Memory	Translation Interpretation	Application	Analysis	Synthesis	Evaluation	
Schreiber (1967)	Recall	Making comparisons		Identifying	Speculating		Describing
Davis & Tinsley (1967)	Memory	Translation Interpretation	Application		Synthesis	Evaluation	Affectivity Procedure
Guszak (1967)	Recall Recognition	Translation		Explanation	Conjecture	Evaluation	
Godbold (1970)	Memory	Translation Interpretation	Application	Analysis	Synthesis	Evaluation	Routine
Rogers (1970)	Memory	Translation Interpretation	Application	Analysis	Synthesis	Evaluation	Affective Procedural Textbook Pupil-initiated
Enokson (1973)	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation	Guilford's *Structure-of-Intellect (SI) Model*
	Low mental operations	High mental operations					

Guilford's (1956) *Structure-Of-Intellect* (SOI) Model was another popular question-classification system that incorporated the cognitive process as an individual dimension—operations. A typical and direct application of this model occurred in Gallagher and Aschner's (1963) five-category system (see Table 2-3), which they used to analyze classroom verbal interactions in a variety of subject matter to investigate gifted junior-high school students' productive thought processes. They found that the teacher's thought productions were very similar to those of the students (in a social studies class), and the basis of classroom discourse consisted of cognitive-memory-level teacher questions and student responses.

Table 2-3

Gallagher and Aschner's (1963) Classification System

Categories	**Definition**
Cognitive-memory operations (CM)	Represents the simple reproduction of facts, formulae, or other items of remembered content through the use of such processes as recognition, rote memory, and selective recall.
Convergent thinking (CT)	Represents the analysis and integration of given or remembered data. CT leads to one expected end-remit or answer because of the tightly structured framework through which the individual must respond.
Divergent thinking (DT)	Represents intellectual operations wherein the individual is free to independently generate his own data within a data-poor situation or to take a new direction or perspective on a given topic.
Evaluative thinking (ET)	Deals with matters of judgment, value, and choice, and is characterized by its judgmental quality.
Routine (R)	Contains miscellaneous classroom activities, including the attitudinal dimensions of praise and censure, the dimensions of structuring, a kind of prefatory remark, telling in advance what the speaker intends to say or do, or what s/he expects someone else to say or do.

Unlike previous studies that applied Bloom's (1956) and Guilford's (1956) systems separately, Enokson (1973) considered them "as being interrelated systems which function simultaneously" (p. 28) and incorporated Guilford's (1956) *Structure-of-Intellect (SI) Model* into his classification system as another parameter, the nature of questions, that contained convergent and divergent questions. The convergent category consisted of closed questions that required only a single possible answer and the divergent category included open questions that allowed several possible answers. Although Blosser (1991) criticized this closed and open dichotomy as over-simplified, this classification seemed properly reflected by several research results on the number of questions teachers asked in the classroom—the low cognitive and convergent questions generally outnumbered other types of questions.

Later, in the 1990s, Wilen (1991) proposed the *Gallagher-Aschner and Bloom hybrid system* that subdivided the convergent and divergent operations into four levels as shown in Table 2-4. To highlight the similarities this system shared with others, I have italicized the operations adopted from Bloom and underlined the concepts corresponding to Gallagher-Aschner's system.

Table 2-4

Wilen's (1991) Classification System

Categories	**Definition**
Level I: Low-Order Convergent	Questions requiring students to engage in reproductive thinking (e.g., *recall or recognize* information). Responses can easily be anticipated.
Level II: High-Order Convergent	Questions requiring students to engage in the first levels of productive thinking (e.g., demonstrate an *understanding* of information by organizing material mentally). Responses can be anticipated.
Level III: Low-Order Divergent	Questions requiring students to *think critically* about information (e.g., discover reasons or causes, *draw conclusions or generalizations* to support opinions). Responses may not be anticipated.
Level IV: High-Order Divergent	Higher-order questions requiring students to perform original and evaluative thinking (e.g., make predictions, solve realistic problems, produce original communications, and *judge* ideas, information, actions, and aesthetic expressions). Responses cannot be anticipated.

In addition, it is worth noting the differences between the two systems. First, Gallagher and Aschner's (1963) five-category system described teacher-student interaction with a focus on student's productive thought processes, while Wilen's (1991) classification system categorized only teacher questions. Second, these two systems seemed merely to re-organize the topics in a different way based on the research objectives. In sum, these question-classification systems were primarily based on memory, cognition, and productive thinking. The inclusion of the cognitive process in the analysis of teacher questioning, on the one hand, accentuated students' role in teacher questioning. Hogg and Wilen (1976) advocated that "[s]tudents can be a practical, reliable source of feedback on teachers' performances, for they observe the teacher in action many hours each week" (p. 281). More than simply observing their teachers, students had a high possibility to enhance their learning through retrieving prior knowledge and producing new thoughts through teachers' questions. On the other hand, the approach of categorizing teachers' questions by referring to students' cognitive behavior also decreased the effectiveness of the classification because the construction was considered an inferential result. For instance, the question, "How do you know 8 plus 8 equals 16?" could stimulate high-order cognitive thinking if students showed evidence of analyzing or reasoning by using the given information, but they might merely recall a memorized fact. Although Gall (1970) suggested a solution by appropriately controlling the instrument in research, this difficulty has continued to challenge researchers.

In the comparison of various question classification systems, some researchers incorporated additional or alternate categories into their analytic frameworks (Davis & Tinsley, 1967; Godbold, 1970; Moyer, 1967; Schreiber, 1967; Sloan & Pate, 1966; Rogers, 1970). Most of these categories aimed to elicit students' (a) opinions (e.g., agreement), (b) action (e.g., to pick up things), or (c) attitude (e.g., affectivity) – questions that did not involve any cognitive operations and for which it would be possible to elicit nonverbal responses from students. For example, a teacher might ask "Would you like me to read the problem again?" when she detected her student's hesitation in action, and the student could simply nod to continue this interaction. This exact question could also call for a "pause" instead of "action" when used to clear up confusion in the ongoing interaction.

In terms of affective questions, Davis and Tinsley (1967) defined them as those where "the one questioned responds with a statement of feeling, emotion, or opinion without a standard of appraisal" (p. 23). Hunkins (1976) described five functions of affective questions: (a) to sensitize students to the existence of certain phenomena and stimuli (e.g., "Would you be willing to read the problem for me?"); (b) to sufficiently involve students in their work (e.g., "Are you done with your work?"); (c) to encourage students to value their commitment to a phenomenon (e.g., "Do you think you can try one more time?"); (d) to help students organize the values into a system (e.g., "Do you want to write down all relevant numbers when I read the problem to you?"); and (e) to have students utilize the existing values to regulate their behavior, such as "What would you do to help others to solve this problem?" (p. 62-65). After Hunkins (1976), some researchers produced similar models to classify affective questions, with different foci such as student engagement (Morgan & Saxton, 1994; Wilen, 1987) and attitudes (Sloan & Pate, 1966).

In brief, researchers classified questions through applying alternative systems, such as interrogative versus rhetorical (Hyman, 1979, Moyer, 1967), tempo (Houston, 1938), open or closed questions (Enokson, 1973; Smith, Hardman, Wall, & Mroz, 2004), and hierarchy versus context involvement (Frager, 1979), in addition to incorporating cognitive operations. All these studies diversified question-classification systems and broadened our horizon when evaluating teacher questioning. However, most research on teacher questioning insisted on considering student responses, and this incorporation attracted researchers' attention to the sequential patterns of teacher-student interactions.

The Sequential Patterns in Teacher Questioning

To understand the interaction between teachers and students, it is imperative to examine the elements and patterns of teacher-student exchanges. My review of the sequential pattern of teacher questioning in the literature began with the initial formation of the question-response pattern.

Guszak (1967) analyzed the anatomy of individual teacher-student exchanges and, as a result, developed the concept of the Question-Response Unit (QRU). This QRU pattern may contain a subset of the following elements: (a[1]) the teacher's initiating question; (b) student response [congruent (+) or incongruent (−)]; (c) clarifying, extending, or cueing remarks from the teacher; (d) the teacher's management of time allowed [did not allow time (0) or response allowed but only silence heard (x)]; (e) the referent in the materials for the question; (f) the way the student subsequently dealt with the question; and (g) the phase in which attention was shifted away from the initiating question. The combination of QRU patterns varied: the most frequent was "ab+" (86%), followed by other patterns such as "ab−" (4%) and "a0cb+"(4%). Several trends are notable. First, most QRU patterns consisted of "the teacher initiated a question and the student provided a congruent response" in which a satisfying response seemed always to be

[1] The item letter also served as the code for pattern elements discussed in QRU.

given by students and functioned as a closure in the teacher-student exchanges. Second, regarding the type of questions initiated by the teacher, 65% of the “ab+” pattern belonged to recall questions, and the study also revealed that the dominant “ab+” type (recall questions) was most prominent in lower grade levels (second grade in this study).

Moreover, Guszak (1967) expanded the analysis to a combination of two or more QRU patterns and constructed another concept called the Question-Response Episode (QRE) that contained four types:

1. The setting-purpose follow-up episode occurred when a teacher followed up the initial question with a parallel question (e.g., a second attempt to get a response).
2. The verification episode involved questions in which a congruent response can be verified by referring to the sources or text.
3. The justification episode appeared when a teacher called upon a student to justify a previous response by providing an explanation that often followed a conjectural response.
4. The judgmental episode occurred when a teacher asked for an evaluation of responses to the preceding question.

Although these QUE types were originally used to describe teachers’ questions, I viewed the concept of QRE as a precursor of the Initiation-Response-Feedback (IRF) structure (Sinclair & Coulthard, 1975) because of certain shared features: (1) the QRE was built on the basic unit of “teacher-initiated question followed by student response” that corresponds to the I-R sequence, and (2) the follow-up episodes had correspondent occurrences similar to the feedback in IRF. Another famous interactional sequence was the Initiation-Response-Evaluation (IRE) pattern (Mehan, 1979a), in which the evaluation could be considered an evaluative response that was a simplified follow-up move compared with the structure of repeat, verification, justification, and judgmental follow-up question in QRE. The IRF structure is so dominant that Wells (1993) claimed that it “accounts for some 70% of all the discourse that takes place between teacher and students” (p. 2) in both secondary and elementary schools, and it has been the dominant model even in one-on-one teacher-student interactions. In fact, the analytic model of the application of IRF patterns could be more complicated than merely interpreting IRF as a triadic sequence. For example, Wells and Arauz (2006) analyzed dialogic modes of interaction used in science classrooms by level, as shown in Figure 2-2. In the levels of analysis, the IRF was centered in the nuclear exchanges from any sequence within different episodes collected across seven years. This coding scheme sufficiently represented the layers of elements involved in an interactional episode and outlined the flow of analyzing classroom discourse.

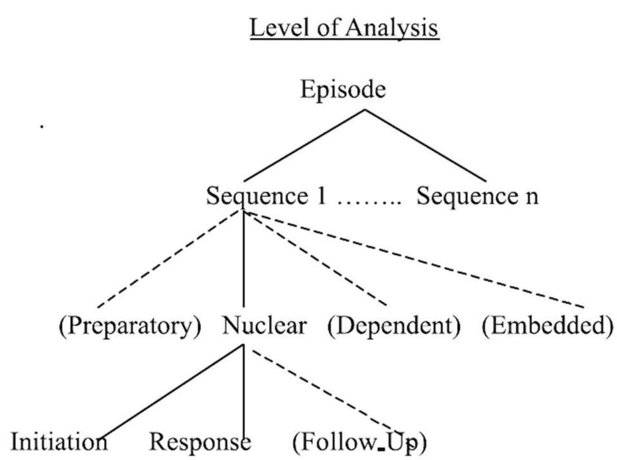

Figure 2-2. An overview of the coding scheme in the Developing Inquiring Communities in Education Project (DICEP) by Wells and Arauz (2006, p. 390).

Similarly, Mehan (1979a) identified the IRE as "the most recurrent pattern" (p. 72) but simultaneously warned that while employing the discourse patterns in instruction, "teachers are sometimes not aware that the child's display of knowledge is constrained by the structure of the task, the organization of discourse, and the physical parameters of the teaching-learning situation" (Mehan, 1979b, p. 294). This lack of awareness could be due to the complexity of the components comprising the IRE patterns, as shown in Table 2-5.

Table 2-5

Potential Components in IRE Patterns

Type of act	Potential components
I: Teacher initiation	Directive, informative, choice elicitation, product elicitation, process elicitation, and metaprocess elicitation
R: Student reply	Non-verbal reaction, acknowledgement, choice response, product response, process response, and metaprocess response
E: Teacher evaluation	Prompt, accept, and praise

While the structure and organization of question-response patterns, such as the speech act by Sinclair and Coulthard (1975) and the initiation act by Mehan (1979a), were exhaustively studied and discussed in the 1970s, the next challenge would be how to leverage established scholarship to improve the classroom discourse and teacher-student interactions. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) advocated changes to enhance the environment of the mathematics classroom and reinforced the role of teacher questioning in teachers' orchestration of classroom discourse. Accordingly, Mewborn and Huberty (1999) urged teachers to listen carefully to students so that they can ask good follow-up questions and encouraged teachers to utilize a triadic sequence—Question-Listen-Question—to "provoke thoughtful responses from students…[and] help students clarify and extend their thinking" (p. 226). Although the structure of classroom discourse has been a topic of continuous interest to researchers, the focus in the analysis of discourse shifted from overemphasizing teachers' performance to increasing students' contributions, as well as the interactive nature of teacher-student exchanges.

van Zee and Minstrell (1997) examined how an experienced science teacher used questioning to guide student thinking and defined a reflective toss, a particular kind of question that enabled the teacher to encourage students to elaborate their thinking. In this structure, the role of the teacher included catching the meaning of the student's statement and then throwing responsibility for thinking back to the student(s). This reflective toss not only exemplified the concept of "listening carefully and asking good follow-up questions," but also fixed the purposes of the teacher's follow-up questions on (a) engaging students in a proposed method, (b) beginning the refinement process by clarifying a discussed method, and (c) evaluating an alternative method that might arise as a byproduct of the discussion. That is, the move repeatedly redirected the focal point of the whole discussion back to the idea students proposed at the moment or their on-going thinking. In particular, the reflective toss also successfully invited other students to help elaborate their peer's idea when needed. Notably, the reflective toss excluded questioning moves like directly accepting the proposed idea or requesting an external evaluation of it.

It could be beneficial to understand how teachers use follow-up moves to value and incorporate students' ideas in their interactions. In her study examining teacher follow-up moves and student learning, Bishop (2008) used *responsiveness* to reflect the extent of how the teacher responds to their students, identifying four categories: (1) Low-level moves mainly evaluate or rebroadcast a student's idea; (2) Medium-level moves focus on the teacher's thinking, with minimal response to the student; (3) High-level (I) moves respond to the student's ideas but display the teacher's thinking; and (4) High-level (II) moves explore student thinking and encourage his or her independent reasoning. This study added a dimension to the aforementioned QRE categories and clarified the goals in the reflective toss. For example, when a student proposes a method, the teacher could follow it up to different extents, from passively repeating what the student said to actively inviting high-level mental operations.

Keeping the levels of responsiveness in mind, I expanded my focus on responsiveness to a mutual relationship between teacher and student due to my purpose of analyzing one-on-one interviews in this study. A more general model emerged in a review of the literature. Hogan, Nastasi, and Pressley (1999) identified three interaction patterns—consensual, responsive, and elaborative—while scrutinizing 32 eighth-grade students' reasoning complexity in science classrooms. Consensual interaction occurred when one of the participants contributed substantive responses to the interaction and the other served as a "minimally verbally active audience" (p. 393). When enacting responsive interaction sequences, two parties of participants equally contributed substantive responses to the interaction and could freely express their ideas on the topic discussed. The elaborative pattern occurred when the participants not only contributed substantive responses but also co-constructed additions, made corrections, or offered a counterargument based on any prior statement (Hogan et al., 1999). In sum, an initial question, generally from the teacher, could always guarantee a sequence of interactional moves that consisted of a response from students, feedback, evaluation, or follow-up questions from the teacher, or multiple recurrent utterances of the response and follow-up subset. When various types of teacher-student exchanges exist, it would be helpful to examine or evaluate them through the lens of the function (e.g., verification, justification, or judgmental purposes in QRE), the structure (e.g., IRF or IRE), the extent (e.g., low, medium, or high level), or the quality (e.g., consensual, responsive, or elaborative pattern).

Teacher Questioning in Mathematics Teaching

In a review of the literature on questioning, I noticed that most research was conducted in social studies classrooms. In this section, I selected studies that focused on teacher questioning in mathematics classrooms.

Comparison of Groups of Teachers in Questioning

Some scholars attempted to compare teachers' questioning practices while establishing different conditions in their study. For example, Sloan and Pate (1966) conducted a quantitative study to compare two groups of elementary mathematics teachers' questioning practices based on what they required of the pupils. The eight categories used to analyze teachers' questions were 1) recognition, 2) recall, 3) demonstration of skill, 4) comprehension, 5) analysis, 6) synthesis, 7) opinion, and 8) attitude. They reported a statistically significant difference between the questioning practices of teachers who participated in the "new math" program, which emphasized the objectives of inquiry and discovery, and teachers in traditional mathematics programs. They found that the new math teachers asked significantly fewer recall and more comprehension and analysis questions.

Perry, VanderStoep, and Yu (1993) conducted a cross-country study in which they employed six types of questions to examine the questioning practices of teachers in Japan, Taiwan, and the U.S., focusing on addition and subtraction in first-grade mathematics classes.

The predetermined categories included 1) computation or rote recall, 2) rule recall, 3) computing in context, 4) making up a problem, 5) problem-solving strategies, and 6) conceptual knowledge, and all categories were completely related to students' problem-solving behavior. They found that Asian teachers asked significantly more questions about conceptual knowledge and problem-solving strategies than did U.S. teachers. Taiwanese teachers asked significantly more questions that were embedded in a concrete context than did U.S. teachers. The kinds of questions typically asked in Japanese and Taiwanese classrooms may contribute to the construction of more sophisticated mathematical knowledge for children in those classrooms.

Sahin and Kulm (2008) compared two sixth-grade teachers' uses of probing, guiding, and factual questions to study the types of questions asked in classrooms and the intentions in relation to their uses of these questions. The novice teacher was a first-year male teacher, and the experienced teacher was a female teacher with seven years of teaching experience on the same topic. The results in this study showed that both teachers asked factual questions more often than other types of questions. Although the teachers used mostly probing questions to ask several students similar questions to focus on the specific exercise instead of extending or generalizing ideas, the novice teacher asked a far higher percentage of probing questions than the experienced teacher, who only enacted more probing questions during the use of manipulatives. In particular, the authors suggested that including students' answers when examining the teachers' questions could be a useful approach to determine the intention of teachers' questions.

Boaler and Brodie (2004) investigated the nature of high-school mathematics teachers' questions and developed nine categories of questions from an analysis of teaching practice as shown in Table 2-6. The data analysis showed that the teachers using a traditional curriculum asked more Type 1 questions (> 95%) than the teachers who used reformed curriculum (range from 60% to 75%). Moreover, this result was consistent with the study by Sloan and Pate (1966), in that they also compared the relationship between curriculum change and teacher questioning in two groups of teachers: those using “new math” materials (School Mathematics Study Group, SMSG) and those who used traditional materials. However, their classification of questions seems debatable because of the high percentage of the Type 1 questions and extremely low percentages of other question types, such as Type 2, 6, 7, and 8. Moreover, although the question types ascertained the practical purpose of teacher questioning, this classification system seems too behavior-oriented to highlight the thematic concepts that might derive from these questioning moves.

Table 2-6

Boaler and Brodie' (2004) Classification System

Question type	Description	Examples
1. Gathering information, leading student through a method	Requires immediate answer Rehearses known facts/procedures Enables students to state facts/procedures	What is the value of x in this equation? How would you plot that point?
2. Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them	What is this called? How would we write this correctly?
3. Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations	Where is this x on the diagram? What does probability mean?
4. Probing, getting students to explain their thinking	Asks student to articulate, elaborate or clarify ideas	How did you get 10? Can you explain your idea?
5. Generating discussion	Solicits contributions from other members of class	Is there another opinion about this? What did you say, Justin?
6. Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life	In what other situations could you apply this? Where else have we used this?
7. Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used	Would this work with other numbers?
8. Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving	What is the problem asking you? What is important about this?
9. Establishing context	Talks about issues outside of mathematics in order to enable links to be made with mathematics	What is the lottery? How old do you have to be to play the lottery?

As shown above, knowing which group of teachers outperformed on questioning under particular conditions would inform researchers about relative advantages and disadvantages when establishing a research environment in the future. In the following, I review two studies that identified the effectiveness of a particular type of question in mathematics classes.

Effectiveness of Particular Types of Questioning

Franke et al. (2009) investigated the questioning practices elementary teachers employed to elicit students' thinking and stimulate mathematical discussion in three elementary classrooms. These teachers were participating in a professional development program that highlighted relational thinking.[2] The authors analyzed the questioning practices these teachers used to follow up on students' initial responses and identified four types of questioning practices: (1) General questions that were not related to anything specific that a student said, (2) specific questions that addressed something specific in a student's explanation, (3) probing sequences of specific questions that consisted of a series of more than two related questions about something specific that a student said and included multiple teacher questions and multiple student responses, and (4) leading questions in which the teacher guided students toward particular answers or explanations and provided opportunities for students to respond (p. 383).

In particular, Franke et al. (2009) discussed the relationship between teachers' questioning practices and students' explanations of their problem-solving strategies and concluded that (a) teacher follow-up questions were not a guarantee of students' further elaboration of their thinking; (b) when teachers asked sequences of specific questions (alone or

[2] For the detail of program content, please refer to *Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School* by Carpenter, Franke, and Levi (2003).

in conjunction with other questioning types), 100% of the target students provided elaboration of their explanations; and (c) the target students, whose initial explanations were incorrect, ultimately produced correct and complete explanations primarily in segments with teachers' probing sequences of specific questions. In short, probing sequences of specific questions always elicited further elaboration from the student and usually allowed the student the opportunity to articulate a correct and complete explanation regardless of whether the initial student explanation was ambiguous, incomplete, or incorrect.

To explore the effect of different types of questions in an elementary teacher's mathematics classroom, Parks (2010) identified four types of questions across two quality dimensions: "Reform versus Traditional" and "Implicit versus Explicit," providing descriptions to illustrate her use of these concepts (see Table 2-7). Parks concluded that implicit questions particularly benefited students who shared a common cultural background and language practices with the teacher and that explicit questions seemed to productively support students' mathematical thinking.

Table 2-7

Parks' (2010) Classification System

	Implicit (questions did not provide hints or clues about the expected answer)	**Explicit** (questions alluded to the kind of answer the teacher desired)
Reform (teaching that includes process skills as well as content)	Examples: Why? What do you notice about this? Why does this make sense? What's a prediction you could make? What can you tell me about this? What do you think?	Examples: Caitlin, can you say why you disagree with Sienna's answer? Tell me why you're adding 32 and 33. Why would 26 not make any sense as an answer?
Traditional (teaching that is more narrowly focused on content)	Examples: What do you do to add two-digit numbers with regrouping? If you haven't memorized your facts, what can you do to get the answer?	Examples: What is four groups of two? What digit is in the one's place, everybody? What do we call the name of this coin? Okay, in Celsius, what temperature does water freeze at?

In some studies, the categories of questions in practice were derived directly from the data and impossible to anticipate in advance, such as Boaler and Brodie's (2004) classification system. However, the following studies tended to describe the outcome teacher questioning might determine with regard to students' mathematical performance.

Teacher Questioning that Elicited Student Work

Martino and Maher (1999) analyzed an elementary teacher's timely questioning in mathematics classrooms and highlighted four functions in teacher questions: 1) facilitating justification, 2) offering an opportunity for generalization, 3) inviting learners to make connections, and 4) facilitating awareness of solutions presented by other students. They then concluded that there was a strong relationship between monitoring students' constructions and posing a timely question, which can challenge learners to advance their understanding. The authors suggested that "the type of question asked by the teacher must be connected to the student's present thinking about a solution" (p. 56).

Adopting this idea of "connecting to the student's thinking," Webb et al. (2006) attempted to analyze the nature of teacher questions by taking a different approach: including the nature of the student's response and of the cognitive processes required to formulate the response (the content). They found that most teacher questions required only low-level information, like a single-number answer, and they claimed that teachers infrequently prompted an elaborated response from students, even though they did detect a slight increase in the questions asking students to describe a computational procedure in general terms. In addition, teachers sparsely requested an explanation with regard to the rationale behind students' procedural understanding. Instead, they required students to engage in only low- or medium-level cognitive processes to formulate a response (the operations). Particularly in problem-solving, calling on students to assign them a more active role only resulted in questions inquiring about numerical procedures, and in this process, teachers' questions "seemed intended to uncover errors that could be corrected rather than to uncover misconceptions that could be rectified" (p. 109). Overall, the findings suggested that the role of teachers was modeled as an active help provider, and the role of students was modeled as a help-seeker or a passive recipient of the teacher's instruction. Moreover, these roles became a consensual model existing lying in teacher-student interactions, particularly while working in a small group.

Researchers have paid substantial attention to in-service and experienced teachers' questioning practices. However, how to examine preservice teachers' questioning practices and evaluate the development of their questioning techniques was in fact noted by scholars approximately a half-century ago (Blosser, 1979; Grager, 1979; Rogers, 1970). A further discussion of this trend is presented below.

Preservice Teachers' Questioning Practices

In her study of learning to teach mathematics by questioning, listening, and responding, Nicol (1999) pointed out that preservice teachers experienced tensions related to posing questions to 1) learn what students are thinking, 2) guide students to the answer, and 3) test students' thinking. She found that prospective teachers asked questions to direct students toward the correct answer in the beginning sessions of her study, although they had been trained not to lead students through the problem. As a result, students only had limited opportunities to demonstrate their thinking and advance their cognitive performance because of a lack of spaces for inquiry in teachers' questions.

Moyer and Milewicz (2002) examined 48 preservice teachers' questioning strategies in their one-on-one diagnostic interviews with students working on rational numbers. This approach was designed to exclude the uncertainty of classroom distraction so that participating teachers could focus on the child's thinking. In the study, the preservice teachers utilized question categories provided by the instructor to analyze their own audiotaped interviews. Three dominant categories were revealed: 1) check-listing, 2) instructing rather than assessing, and 3) probing and follow-up questions. This diagnostic interview structure helped preservice teachers develop questioning techniques that incorporated children's thinking "in the process of interpreting and responding to unexpected answers" (p. 296). Notably, however, the emerging questioning patterns left out the stages of students' problem solving, and the effects of mathematical tasks were underrated. For example, this study did not distinguish the teachers' questions pertinent to eliciting students' existing strategies from those used to confirm the mathematical terminology students produced.

Based on an adaptation of question categories created by Moyor and Milewicz (2002), Weiland et al., (2014) examined the development of elementary preservice teachers' questioning practices and videotaped one pair of teachers working with two students through formative assessment interviews on mathematics for six weeks. They found that, on the one hand, preservice teachers' use of competent follow-up questions, those "attempting to draw students' attention to conceptual meaning" (p. 344), increased over the course of the semester. On the other hand, both teachers missed many opportunities to further explore student thinking while directly instructing and asking leading questions.

Teacher Questioning in Problem Solving

Jacobs and Ambrose (2008) studied 65 K-3 teachers conducting one-on-one interviews of 231 students solving 1,018 story problems. Their study proposed four categories of teaching moves to support a child's thinking before a correct answer is given and four categories to extend a child's thinking after the child gives a correct answer. Later, in 2016, Jacobs and Empson (2016) constructed an amalgamated framework based on Jacobs and Ambrose's (2008) study and their research data—as shown in Figure 2-3—to analyze an experienced elementary teacher's teaching moves.

Jacobs and Ambrose (2008) Analysis Original Categories of Teaching Moves	
Before a correct answer is given	*After a correct answer is given*
Ensuring the child understands the problem	
Exploring what the child has already done	Promoting reflection on the strategy the child just completed
Reminding the child to use other strategies	Encouraging the child to explore multiple strategies and their connections
	Connecting the child's thinking to symbolic notation
Changing the mathematics in the problem to match the child's level of understanding	Generating follow-up problems linked to the problem the child just completed

→

Current Analysis Emerging Framework of Teaching Moves
Ensuring the child is making sense of the story problem
Exploring details of the child's existing strategies
Encouraging the child to consider other strategies
Connecting the child's thinking to symbolic notation
Posing a related problem linked to what the child understands*

Figure 2-3. An amalgamated framework of teaching moves (Jacobs & Empson, 2016, p. 189).

They found that the teacher was responsive to students' thinking and identified four teaching moves that were commonly used, including

1) Regularly checking the student's understanding of the problem or highlighting the specific, challenging parts of the problem in the stage of ensuring the student is making sense of the story problem.

2) Posing general starter questions, inviting explanation, linking representation and the story context, and making a connection in the stage of exploring details of the student' existing strategy.

3) Soliciting alternative strategies or comparing multiple strategies in the stage of encouraging the student to consider other strategies.

4) Requiring formal mathematics notation for the student's solutions in the stage of connecting the student's thinking to symbolic notation.

Jacobs and Empson's (2016) revised framework could be practically mapped onto Polya's (1957) four stages of problem solving (see Figure 2-4) and would be very useful to examine teaching moves in support of students' problem-solving activities in mathematics.

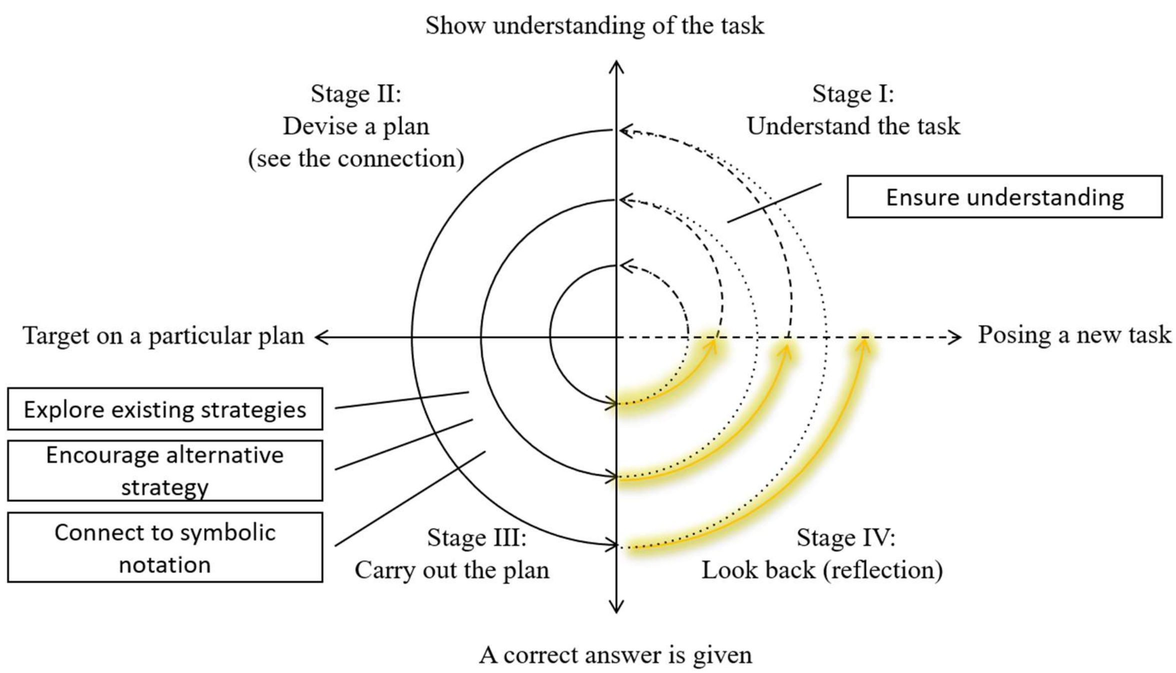

Figure 2-4. Teaching moves in the four-stage problem solving. The tasks become more difficult to the right on the x-axis. In Quadrant IV, the dashed line maintains the difficulty level in the next task, and the yellow line means the cognitive demand of next task will be increased.

An Integrated Framework of Teacher Questioning

Based on my review of the literature, I developed an integrated framework informed by 1) Jacobs and Ambrose's (2008) original work on teacher moves, 2) Jacobs and Empson's (2016) revised framework, and 3) Polya's (1957) four problem-solving stages: (I) understanding the task, (II) devising a plan, (III) carrying out the plan, and (IV) looking back. Because Stage IV, with its extending moves used after the correct answer is given, is beyond the scope of this work, I only examined questioning moves in the first three stages, as shown in Figure 2-5. The original categories proposed by Jacobs and Ambrose (2008) included four types of supporting and four types of extending moves. I included only supporting moves because the focus of teacher questioning in this study was exclusively on the questioning moves teachers used to support students' thinking before the posed mathematical problem was correctly solved. The four categories of teaching moves from Jacobs and Empson's (2016) framework (ensuring understanding, exploring strategies, connecting, and considering other strategies) served as the categories for this framework.

	Framework of Teaching Moves (Jacobs & Empson, 2016)	**Four-stage guideline (Polya, 1957)**	**Current Analytical Framework of Questioning Moves**
Supportive Questioning Moves	• Ensuring the child is making sense of the story problem	Stage (I) For understanding the problem	[SQM1] Ensuring the child is making sense of the task
		Stage (II) For devising a plan	*Adding* *[SQM2] Inquiring the child's plan to solve the task*
	• Exploring details of the child's existing strategies • Connecting the child's thinking to symbolic notation • Encouraging the child to consider other strategies	Stage (III) For carrying out the plan	[SQM3] Exploring details of the child's existing strategies [SQM4] Connecting the child's thinking to symbolic notation [SQM5] Encouraging the child to consider other strategies

Figure 2-5. The integrated framework that was developed based on Jacob and Empson's (2016) teaching moves and three stages in Polya's (1957) problem-solving guidelines.

Given that the teacher-student interactions in this study were situated in only three of Polya's problem-solving stages, a category of questioning moves used for inquiring about the child's problem-solving plan was needed. Therefore, I added a new category, *Supportive Questioning Move 2*, to analyze questioning moves observed in problem-solving Stage II. Hereafter, teachers' supportive questioning moves will be denoted as SQM, corresponding to Jacobs and Empson's teaching moves (see Figure 2-5). Table 2-8 presents the five categories of teachers' questioning moves with the description of their function. Once I determined the category of the questioning move, I examined the openness of the questioning move based on the extent of the student's responses elicited by that questioning move.

Table 2-8

The Five Categories of the Supportive Questioning Move (SQM)

Category of Supportive Questioning Moves	**Description of The Function**
SQM1: Ensuring the child is making sense of the task	To ensure that students understand • the contextual features of the task scenarios • the involved mathematical ideas and relationships within the tasks
SQM2: Inquiring about the child's plan to solve the task	To learn about the child's problem-solving strategies derived from • the given information • the child's prior experience
SQM3: Exploring details of the child's existing strategies	To facilitate preservice teachers' understanding of • the child's procedural understanding • the child's conceptual understanding • the rationale behind the mathematical representations employed
SQM4: Connecting the child's thinking to symbolic notation	To enhance the connection between • the child's thinking and mathematical representations • informal expression and formal mathematical terminology • the child's presenting idea and its corresponding mathematical principle
SQM5: Encouraging the child to consider other strategies	To elicit additional strategies by • providing a hint when the child is struggling to solve the task • leveraging the child's successful strategy

CHAPTER 3

METHODOLOGY

The data for this study were collected in two semesters—Fall 2014 and Spring 2015—to address the following research questions:

1. What is the nature of enacted questions that elementary preservice teachers employ in their SSMFE interviews?
2. How are the teacher-student interactional turns constructed and functioning in their SSMFE interviews?
3. What are the successes and difficulties in elementary preservice teachers' questioning practices in their SSMFE interviews?

Participants

The participants who participated in this study consisted of 13 preservice teachers from two mathematics methods courses at the University of Georgia. All teacher participants were in their junior year and had completed at least two mathematics content courses and other mandatory education courses (e.g., investigating critical and contemporary issues in education, exploring socio-cultural perspectives on diversity, and exploring learning and teaching) for Early Childhood Education majors (certification Pre-K–5).

The participants were selected by convenience sampling (Patton, 2002) from 28 preservice teachers in each methods course. I initially recruited volunteers at the beginning of each EMAT 3400 courses: One was in Fall 2014 and the other is in Spring 2015. The Spring 2015 section was taught by an instructor who was the teaching assistant working in the Fall 2014 section, so both courses were structured the same way on the campus classes (e.g., same topics were addressed in the same order) and tasks designed for SSMFE interviews (e.g., same weekly interview protocols were provided to preservice teachers). In each methods course, approximately ten preservice teachers showed interest in participating in this study. I determined the teacher participants based on my observation of their performance in the methods course. The candidates were those who displayed relatively positive attitudes by asking more questions about the course content and actively discussing their interview plans with the instructor. The student participants consisted of two grade levels of students at a public elementary school: fourth-graders in the Fall 2014 study and first-graders in the Spring 2015 study. At the beginning of the semester in Fall 2014 and Spring 2015, I recruited student participants in the elementary school that cooperated with these two mathematics methods courses, and approximately eight students in each grade were consented to participate in this study.

Based on this convenience sampling (Patton, 2002), I grouped 13 pairs of teacher-student participants were follows: One cohort ($n = 6$) participated in the Fall 2014 study; the other cohort ($n = 7$) participated in the Spring 2015 study. All teacher participants then completed a consent form to indicate their agreement to participate in this study. The student participants' consent forms were completed by their parents or guardian.

Settings

Mathematics Methods Courses (EMAT 3400)

This study was conducted in two mathematics methods courses—the first of two mathematics pedagogical courses in the teacher education program— at the University of Georgia. The mathematics methods courses consisted of field experience at Barrow Elementary School one day per week for eight weeks and class on campus the remainder of the semester. The Spring 2015 section was taught by an instructor who was the teaching assistant working in the Fall 2014 section, so both courses were structured the same way on the campus classes (e.g., same topics were addressed in the same order) and tasks designed for SSMFE interviews (e.g., same weekly interview protocols were provided to preservice teachers).

Barrow Elementary School is located in Athens, Georgia, and is home to a total of 43 teachers and 536 students (Pre-K to Grade 5). The school included 44% White students, 40% African-American students, 7% Asian students, 5% Hispanic students, and 4% multi-racial students. Barrow Elementary School has two pre-K classes, five kindergarten classes, five first-grade classes, four second-grade classes, four third-grade classes, three fourth-grade classes, and three fifth-grade classes. At Barrow Elementary School, preservice elementary school teachers participated in a field-based activity, the *Single Student Mathematical Field Experience* (SSMFE, see Sawyer & Lee, 2014), and concentrated on learning about children's mathematical thinking.

Single Student Mathematical Field Experience (SSMFE)

Moyer and Milewicz's (2002) research result indicated that "[h]aving preservice teachers focus on the skill of questioning in a one-on-one diagnostic interview may be an effective starting point for developing the mathematics questioning skills they will use as future classroom teachers" (p. 297). The field experience at Barrow Elementary School is addressed as the *Single Student Mathematical Field Experience* (SSMFE), in which each preservice teacher worked one-on-one with an elementary student on mathematical topics once a week for eight weeks. During the eight-week period, the teacher participants worked with student participants to develop an understanding of the student's mathematical thinking, explanations, and interpretation in the problem-solving process.

In general, the instructors of EMAT 3400 designed interview protocols for preservice teachers to use in the SSMFE (see Appendix A). However, when preservice teachers decided to incorporate special tasks such as reading children's literature, the course instructors would not prepare tasks for preservice teachers in advance. For each SSMFE interview, preservice teachers selected a subset of interview tasks on the protocol in advance as the intended tasks with potential talk moves (Ginsburg, 1997) they planned to use while working with students. In the actual interview, preservice teachers inevitably had to employ unanticipated questioning moves to accomplish the problem-solving tasks with students. These impromptu questioning moves were termed "enacted questioning moves." During each SSMFE interview, the instructor and one teaching assistant circulated in the hallway and were ready to assist the preservice teachers at any time they needed help. The researcher provided no additional interventions and materials in the SSMFE. Each SSMFE session lasted for 35 to 45 minutes, depending on the type and level of difficulty of the enacted tasks. The structure of SSMFE interviews and intended and enacted questions is shown in Figure 3-1.

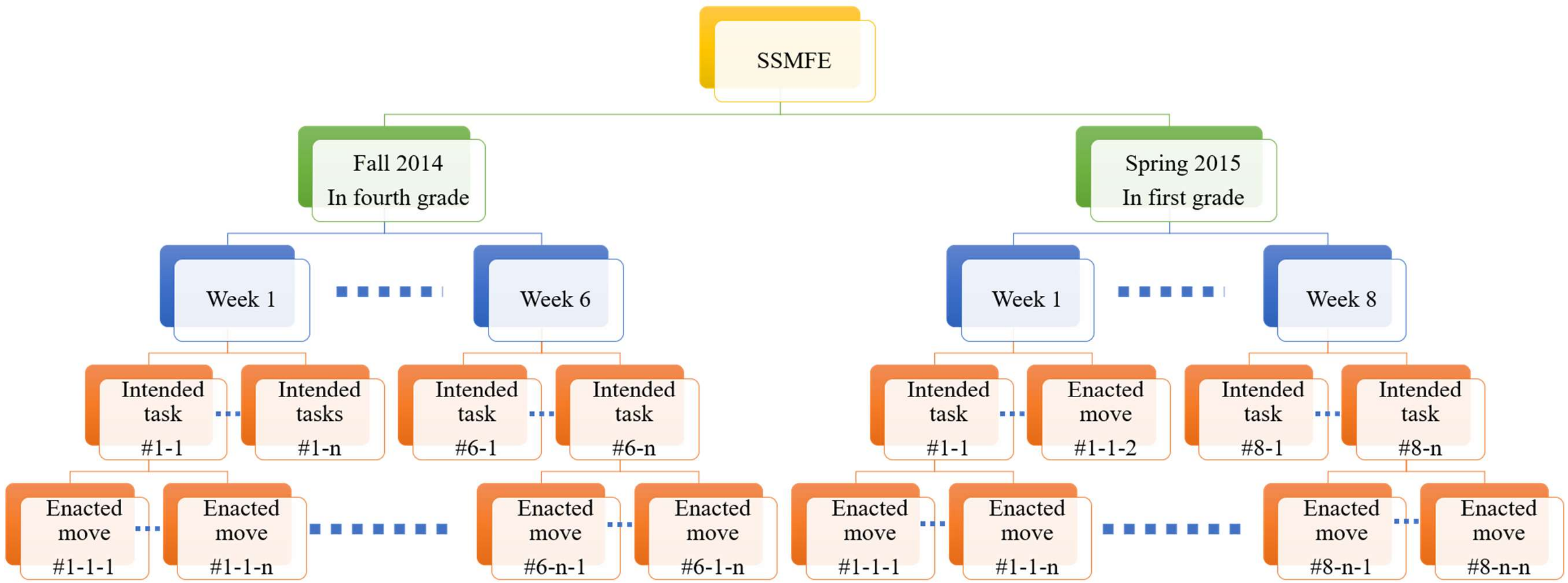

Figure 3-1. The structure of the Single Student Mathematical Field Experience (SSMFE).

After each SSMFE interviews, the preservice teachers optionally wrote follow-up debriefing (see Appendix B). At the end of the course, they were required to construct a final portfolio documenting their reflection and growth over the course of the semester. The debriefing form was an auxiliary assignment to help preservice teachers introspect their work in each SSMFE; and the final portfolio was a formal report that preservice teachers had to describe the interactions by issues, including describing the student's mathematical understanding, analysis of mathematics content, and personal reflection.

Intended Mathematical Tasks

The preservice teachers' work with children on arithmetic tasks, including addition, subtraction, multiplication, and division of whole numbers, was addressed in the Cognitively Guided Instruction (CGI) program (Carpenter, Fennema, Franke, Levi, & Empson, 1999) as the primary basis for the field experience. In addition, preservice teachers could adopt children's mathematical literature books and adapt any mathematics problems that fit elementary students' knowledge and ability. The intended mathematical tasks were categorized into one of the following five mathematical topics: (a) Base-N; (b) Base Ten/Place Value; (c) Number Facts; (d) Fraction or Equal-sharing Problems; and (e) Arithmetic problems including addition, subtraction, multiplication and division adapted from *Children's Mathematics: Cognitively Guided Instruction* (CGI) (Carpenter et al., 1999; Empson & Levi, 2011). Example tasks are listed in Appendix C.

Data Collection

For every SSMFE interview, one pair of participants was selected. Before each SSMFE interview, the selected pair of participants was provided with oral instructions regarding participation in the study, and the participants were seated in front of videotaping equipment including microphones, voice recorders, and camcorders. After both participants agreed to take part in the videotaped activity, they were then audio-recorded and videotaped for 45 to 60 minutes, normally longer than the actual interview. Across the two semesters of the study, I collected 15 videos from 13 pairs of participants, as one pair in Spring 2015 was videotaped twice. To precisely record the fine points of participants' moment-to-moment interactions, one camera filmed the teacher, while the other filmed the student. I collected other relevant data sources as supplementary material for analyzing data, including students' written work as well as preservice teachers' debriefing forms, course assignments, SSMFE final portfolios, and all fieldnotes preservice teachers took during the session. The SSMFE interview data were collected over 6 weeks from September to November in Fall 2014 and over 8 weeks from January to April in 2015.

Data Analysis

Staller (2010) stated that "[q]ualitative research …cover[s] a wide variety of research methods and methodologies that provide holistic, in-depth accounts and attempt to reflect the complicated, contextual, interactive, and interpretive nature of our social world" (p. 1159). Examining teacher questioning occurring in the teacher-student interactions and its accompanying sequence of behavior and conversation exchanges can reveal the complicated, contextual, interactive, and interpretive nature of teaching practice. To investigate teacher questioning in teacher-student interactions in the SSMFE, I transcribed the audio and videotaped recordings verbatim and applied the qualitative method to analyze the collected data.

I conceptualized a *questioning move* as a unit of inquiry activity (Allender, 1969) that the questioner employed to seek information from the listener with respect to an intended purpose and that may initiate a series of conversation exchanges. A questioning move was formulated as an expression that had different syntactic forms and pragmatic categories of speech acts (Graesser et al., 1992). The most conventional format for a questioning move was an interrogative expression that always ends syntactically with a question mark (?) when captured in print, while other formats included directive and informative initiations (Mehan, 1979b).

I analyzed the questioning moves in teacher-student interactions based on interpretive traditions. Prasad (2005) introduced five prominent interpretive traditions and explained that "[a]lthough interpretive traditions uniformly subscribe to the belief that our worlds are socially created, they also assert that these constructions are possible only because of our ability to attach meanings to objects, events, and interactions" (p. 14). The teacher-student interaction in the SSMFE was a co-constructed social activity comprising collaborative work. In this study, I examined naturally occurring interactions to "demonstrate the connection between language use and immediate context (indexicality) that gives it specific local meaning and relevance" (Prasad, 2005, p. 68). Accordingly, when analyzing data in this study, I initially focused on the analysis of the transcripts from video recordings and scrutinized the transcripts along with the videos and relevant materials collected in the field and from the courses.

To properly answer all my three research questions, I first employed framework analysis (Richie & Spencer, 1994) to analyze and categorize preservice teachers' questioning moves (see

Figure 3-2, called as Layer I analysis) based on the proposed integrated framework (see Table 2-8). Figure 3-3 shows the process of conducting framework analysis. During the Layer I analysis, I also identified the level of openness of each move using the criteria listed in Table 3-1, that provides the description of three subcategories of openness (open-ended, intermediate, closed) with examples of students' responses to demonstrate the characteristics of teachers' questioning moves.

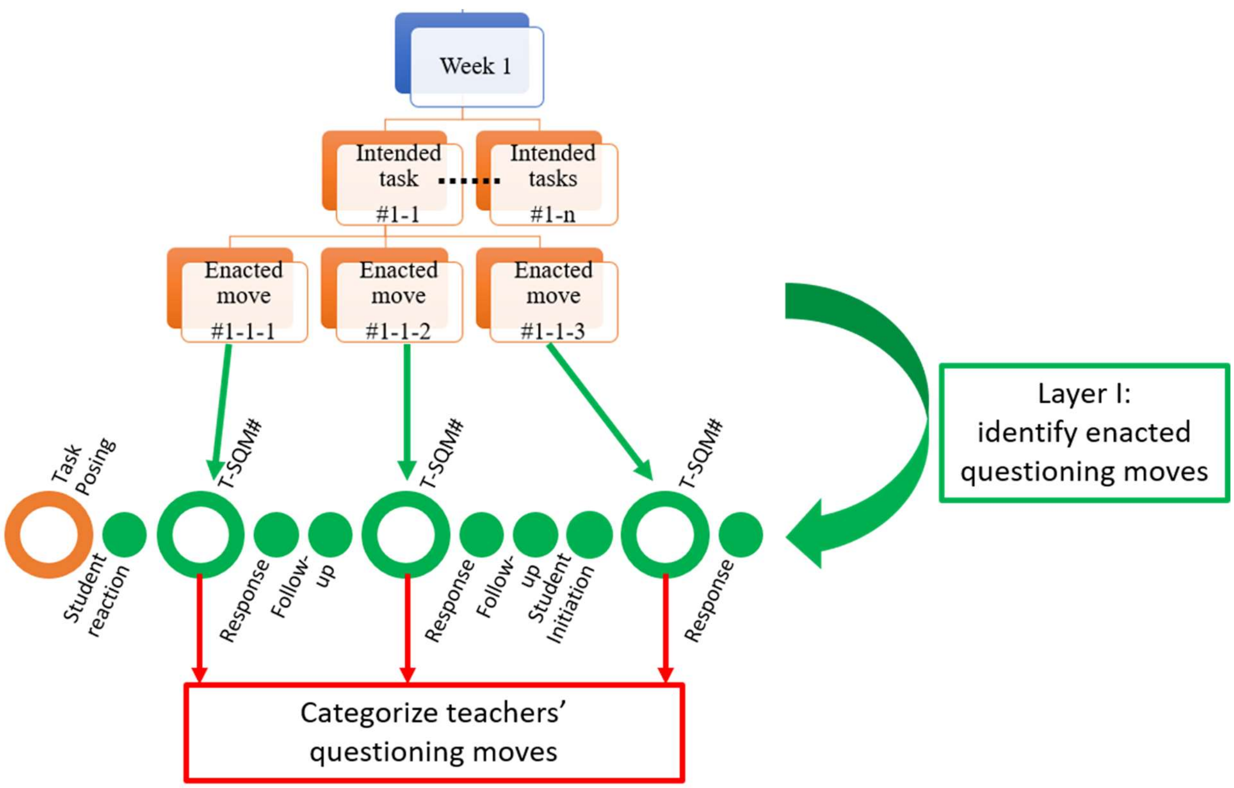

Figure 3-2. Coding flowchart for Layer I analysis: Categorizing teachers' questioning moves by the integrated framework (see Table 2-8).

Table 3-1

The Levels of Openness of the Teacher's Questioning Moves

Levels of openness	Descriptions	Examples of student responses
Open-ended	Allow the students to express their own opinions or interpretations	T: Do you remember what you are trying to find in the problem? S: How many cookies she has in all.
	Allow the students to reflect on the process of making connections or comparisons, such as expressing the grounds of their reasoning	T: Why did you start with the biggest number when adding them? S: Because it is the highest number, and it is easier for me to put it on top.
Intermediate	Allow the students to express (dis)agreement or a response from a list provided in the interaction	T: Is the answer to $18 + 18$ bigger or less than 30? S: Bigger.
	Allow the students to provide a factual response retrieved from their memory	T: What are those two blocks representing? S: Two pieces of candy [in the task].
Closed	Make the students obey teachers' directive initiations	S: [Reaction like nodding] S: Uh-huh!
	Make the students accept teachers' informative initiations	S: [Acknowledgement] S: Okay.

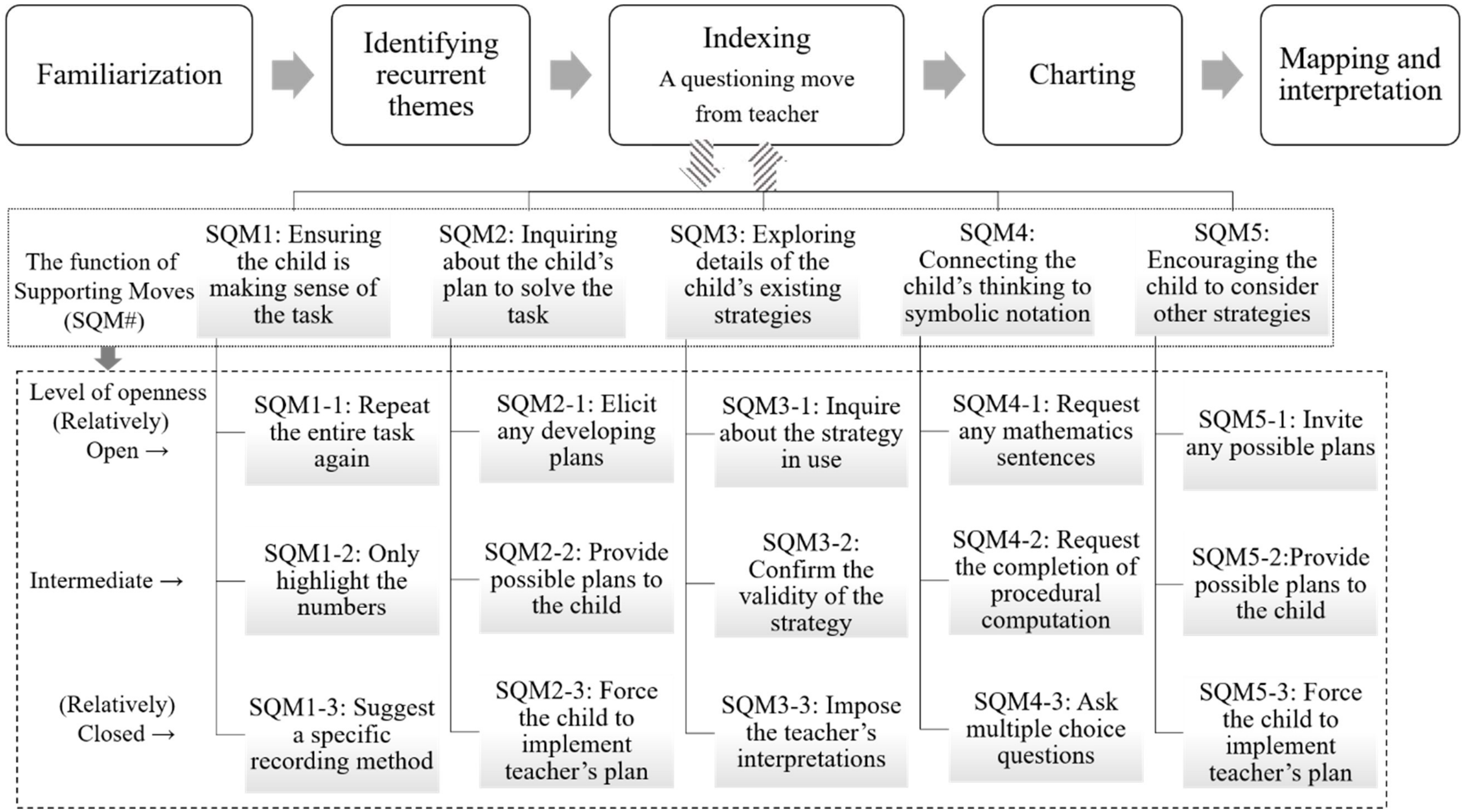

Figure 3-3. Coding flowchart for supportive questioning moves (SQMs) and the level of openness.

Organizing the categorized questions with its relevant interactional turns resulted in segments of interactional moves; I then began to analyze the function of all segments based on the functional categories (see Table 3-2) that stemmed from the questioning moves.

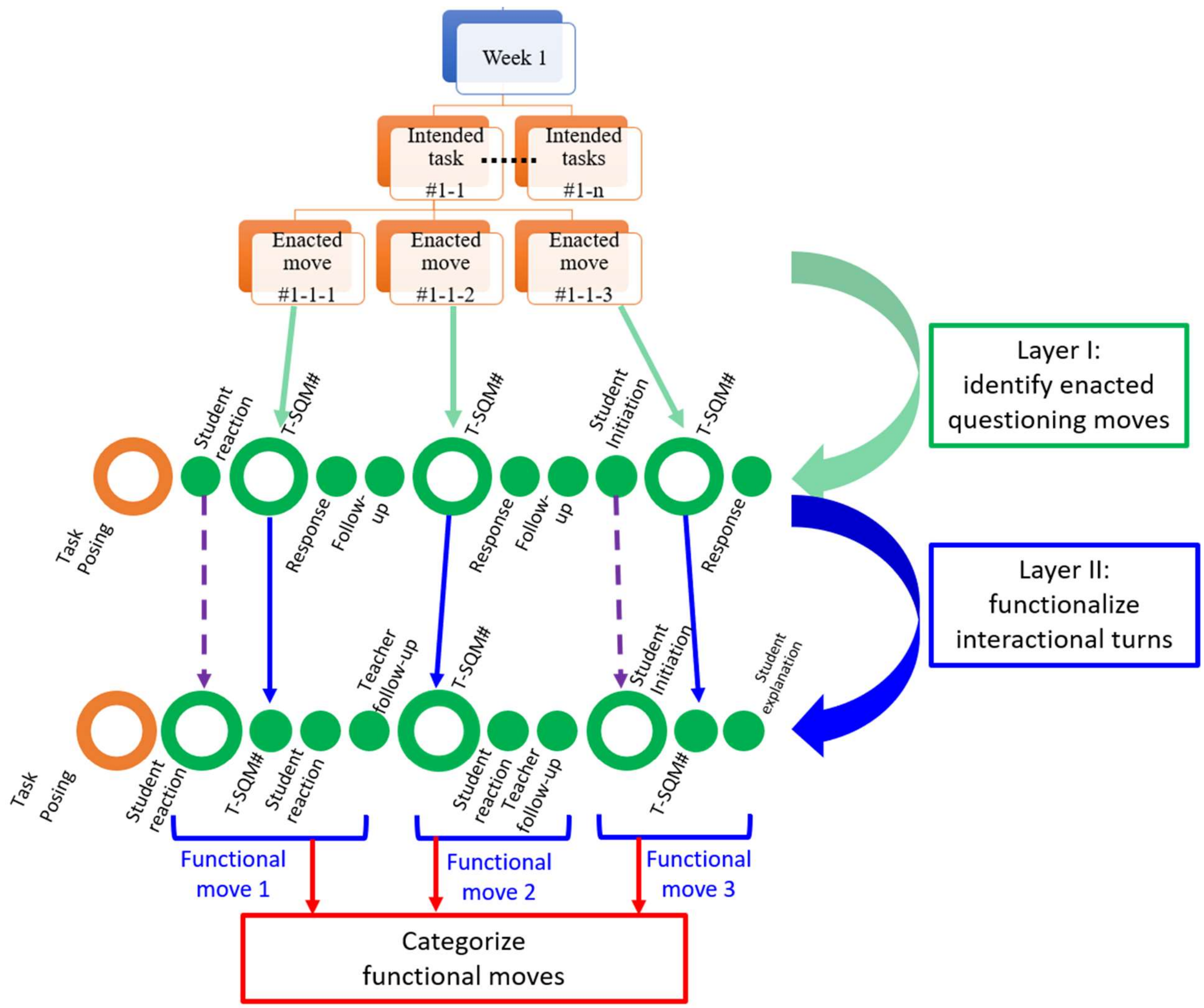

Figure 3-4. Coding flowchart for Layer II analysis: Categorizing teachers' functional moves based on predetermined types of questioning moves.

Table 3-2

The Functional Categories

Functional Category	Description	Sample Initial Turns
Task Clarification (TC)	Clarify or seek the given information in a task	T: Do you want me to read the task again? or S: Can you read the task again?
Plan Elicitation (PE)	Elicit or produce the initial plan	T: What will you do [to solve this task]? or S: This should be a multiplication [instead of a division].
Procedural Understanding (PU)	Explore or explain the procedure involved	T: What did you just do? or S: 2 plus 5 is 7 and 10 plus 10 is 20, so 12 plus 15 is 27.
Making Connections (MC)	Make a connection between the answer and the original task	T: What does 12 mean in the task? or S: The answer 120 means the total number of teeth two dinosaurs have.
Rationale Behind a Strategy (RA)	Inquire about or elaborate on the rationale behind the proposed strategy	T: Why did you do multiplication? or S: I know it is multiplication because I need to find more not less.
Math Terminology (MT)	Elicit or give correct math terminology	T: What do you call that piece? or S: If a cookie was cut into 4 pieces, one piece is a quarter.
Alternative Strategy (AS)	Elicit or propose an alternative strategy	T: What is another way you can solve this task? or S: I want to solve it by using cubes this time.

To expand the analyses of teacher-student interactions, Hogan et al. (1999) identified three interaction patterns—consensual, responsive, and elaborative—that emerged in peer and teacher-guided discussion. To take a one-on-one interaction as an example, the first speaker initiated the conversation, and this initiation could bring up three potential types of interaction patterns. The consensual type occurred when one of the participants contributed substantive responses to the interaction, and the other served as a "minimally verbally active audience" (Hogan et al., 1999, p. 393). While enacting responsive interaction sequences, both participants equally contributed substantive responses to the interaction and could freely express their ideas on the topic discussed. The elaborative pattern occurred when both participants not only contributed substantive responses but also co-constructed additions, made corrections, or offered a counterargument based on any prior statement (Hogan et al., 1999). In each segment of functional moves, I determined who initiated the move and identified the interactional pattern (see Table 3-3). Lastly, with the coded functional moves at hand, I applied thematic analysis (Braun & Clarle, 2006) to appropriately highlight emerging features by theme (see the bottom of Figure 3-5).

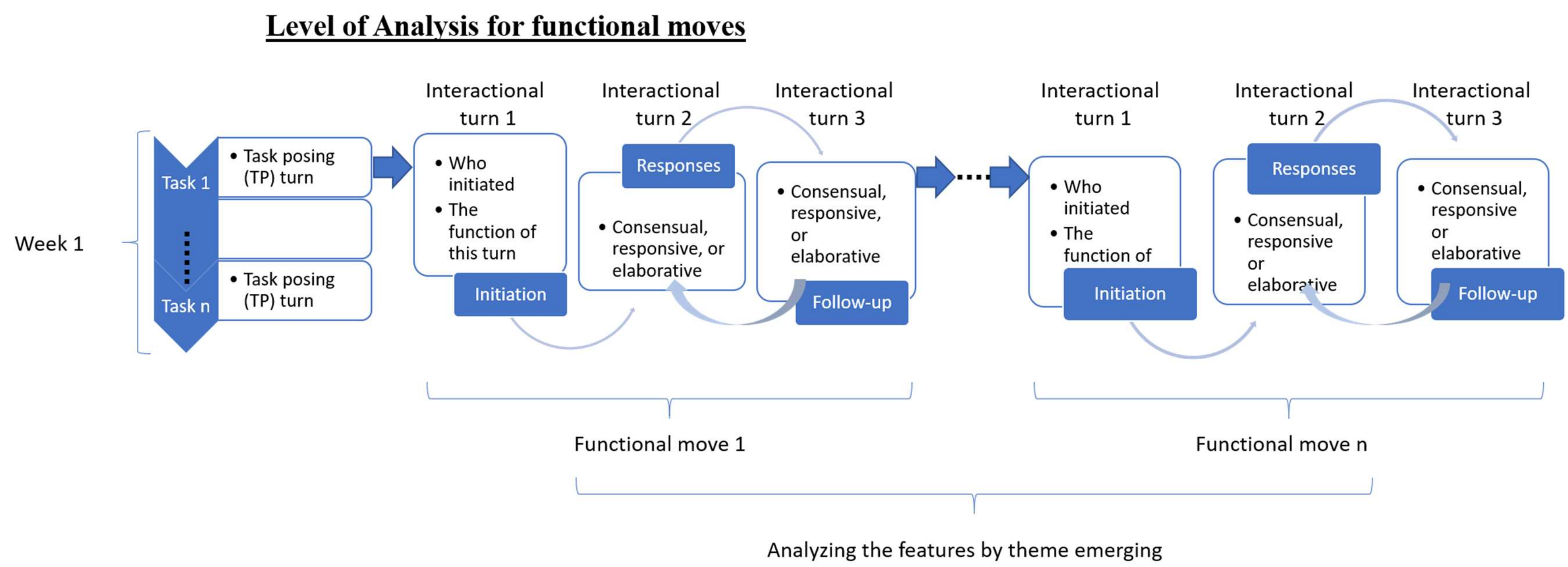

Figure 3-5. Coding flowchart for interactional patterns and analyzing the features of functional moves.

Table 3-3

The Framework of Interactional Patterns Adapted from Hogan et al. (1999)

Category of Interactional Patterns	Description
Teacher-initiated Nonresponse or Consensual reactions (TXC)	The initial interactional turn is initiated by the teacher, and the student replies with nonresponse or only consensual responses.
Teacher-initiated Responsive or Elaborative reactions (TRE)	The initial interactional turn is initiated by the teacher, and the student replies with responsive or elaborative responses.
Student-initiated Nonresponse or Consensual reactions (SXC)	The initial interactional turn is initiated by the student, and the teacher replies with nonresponse or only consensual responses.
Student-initiated Responsive or Elaborative reactions (SRE)	The initial interactional turn is initiated by the student, and the teacher replies with responsive or elaborative responses.

To check for reliability, I repeated the coding process every two weeks, eventually coding all questions at least twice and achieving 86% coding agreement. I was also aware of my subjectivity in my conducting of these qualitative methods, and I took practical steps to deal with it as described in the following section.

Subjectivity

Subjectivity is defined as "the quality of an investigator that affects the results of the observational investigation" (Gove, 2002, Webster's dictionary, Third) stemming from "the circumstance of one's class, statuses, and values" (Peshkin, 1988, p. 17). Of particular relevance to this study, people are not necessarily aware of subjectivity in either the research and non-research environment, which is why researchers should pay more attention to their subjectivity in qualitative research. To seek out possible subjectivity, Peshkin (1988) identified six aspects of subjectivities: (a) the Ethnic-Maintenance I, (b) the Community-Maintenance I, (c) the E-Pluribus-Unum I, (d) the Justice-Seeking I, (e) the Pedagogical-Meliorist I, and (f) the Nonresearch Human I. In the following section, I address three possible subjectivities in my study.

First, the Ethnic-Maintenance I was obvious in my study because I am a “Taiwanese I.” In fact, my ethnicity shaped my being in this country. Moreover, when expanding this Ethnic-Maintenance I in the SSMFE, every preservice teacher encountered this Ethnic-Maintenance I when she had to interact with a student from a different race group. Second, the Community-Maintenance I occurred when people stepped into a new community. I was not as familiar with the U.S. elementary schools as I was with Taiwanese schools. Something natural for the members of the U.S. community might be very strange to me. Fortunately, the observation of the SSMFE drew my attention to the one-on-one interaction in the pair of participants and distracted me from the concerns of these two aspects of subjectivities. Third, I had several years of experience directing preservice teachers in the area of elementary mathematics teaching in my home country, and this prior experience might result in “the Pedagogical-Meliorist I” subjectivity. Peshkin (1988) described this experience in his article: “[A]s I sat in the back of the classroom, I felt that I wanted to remedy the poor teaching I observed. This surprised me because among the first things I explain to any of my study's school personnel is that I am neither evaluator nor reformer” (p. 20). Therefore, I strived to switch my role from a teacher to a researcher, as I believe this is the proper solution to combat this subjectivity.

As an experienced teacher educator and researcher, I maintained strong student-centered and reform-based perspectives about mathematics teaching and learning. This point of view contradicts that of a high proportion of traditional teachers in the field because most teachers experienced their apprenticeship years observing traditional instruction in school math (Ball, 1990). In this research, although I was sometimes positioned as a researcher who tended to make evaluative judgments, implicitly or explicitly, in the process of observation, I strived to try to "consciously attend to the orientations that will shape what I see and what I make of what I see" (Peshkin, 1988, p. 21) in order to tame my subjectivity.

Limitations

The study was limited by the kinds of data collected on the participants. When I selected the preservice teachers in their first mathematics methods course as participant candidates, I recognized the limitation resulting from the lack of interview data because knowing the teachers' intention when asking questions could be as important as knowing the number and categories of questions they asked (Sahin & Kulm, 2008). Teacher questioning has been considered as a professional but personal skill. I decided to exclude interview data from this study for two reasons: 1) several teacher participants expressed their concerns on limited experience working with elementary students on mathematics tasks and worried about their "inexperienced" questioning technique before they signed the consent form; and 2) in the pilot study, the informal post-session interview revealed that some preservice teachers did not really spend their time "contemplating" their questions before enacting them. Accordingly, I reluctantly abandoned interviews while designing this study.

Although I classified the questions preservice teachers enacted in the interview as impromptu moves, the intention behind enacting questioning in a particular way might exist for preservice teachers. Teaching practices were affected by teacher belief (Scott, 2005), knowledge (Grossman & McDonald, 2008), and prior experiences of learning (Ball, 1990). I believe that the motivation or hesitation to ask a particular type of question could have been gained through conducting interviews with the teacher participants. With the interview data remaining a missing piece, teachers' considerations behind selecting tasks and enacting questioning remained implicit.

CHAPTER 4

PRESERVICE TEACHERS' SUPPORTIVE QUESTIONING IN ELEMENTARY STUDENTS' MATHEMATICAL PROBLEM SOLVING

Lee, Y.-J. Submitted to the *Journal for Research in Mathematics Education*

Abstract

The quality of teacher questioning affects the extent of its utilization, particularly in a dynamic teacher-student interaction. I employed an integrated framework to analyze 13 preservice teachers' questioning moves while elementary school students were engaged in mathematical problem solving. The findings revealed four influential features in preservice teachers' questioning: (a) flexibility in the setup, (b) limited extent of inquiry, (c) non-specific probing questions, and (d) neglect of the child's unexamined but valuable strategies. Based on the findings, teacher questioning is a complicated but trainable technique. Suggestions for developing a practice-based training curriculum to enhance the cultivation of teacher questioning are presented.

KEYWORDS: Teacher questioning, problem solving, field experience, methods courses

Introduction

While we often pay very little attention to how we use questions in our daily lives, questions play a considerably important role in the field of education, particularly in inquiry-based teaching and learning environments (Menezes, Guerreiro, Martinho, & Ferreira, 2013; Oliveira, 2010). More than a century ago, Stevens (1912) asserted that when used correctly, question-and-answer exchanges are better at engaging students and facilitating student learning compared to repeating facts, testing facts, and lecturing. She warned that teachers might "foster in [their] pupils negative habits of work, poor associations, and careless impression" (p. 4) if they do not know how to appropriately use questions to assist their students in developing knowledge. Many scholars extoled effective questioning and considered it as an asset in education based on the assumption that teachers may develop new insights into students' thinking through enacting questioning to ascertain students' prior knowledge, to assess students' performance, and to arouse student curiosity (Fitch, 1879; Groisser, 1964; Houston, 1938; Hyman, 1979). The National Council of Teachers of Mathematics (NCTM, 2000) emphasized that, through posing thoughtful questions, teachers can not only motivate students to reexamine their reasoning but also challenge students "with varied levels of expertise…without taking over the process of thinking for them" (p. 19). In the National Research Council's *Adding it up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001), researchers emphasized that "questioning and discussion should elicit students' thinking and solution strategies and should build on them, leading to greater clarity and precision" (p. 426).

The heterogeneity of questioning performance that exists between novice and experienced teachers has been revealed in prior studies (Hyman, 1979; Sahin & Kulm, 2008; Tienken, Goldberg, & DiRocco, 2009). Generally, novice teachers experience more struggles related to posing questions (Brown & Edmondson, 1984; Crespo, 2003). The weaknesses in their questioning could include difficulties in assessing students' understanding (Nicol, 1999), failure to ask probing questions to develop deeper thinking in students (Sahin & Kulm, 2008), and the tendency to ask more leading questions and to overlook opportunities for probing student thinking (Weiland, Hudson, & Amador, 2014). These findings imply that effectively enacting questioning requires multi-faceted knowledge, such as subject-matter knowledge, knowledge of learners and their characteristics, and pedagogical content knowledge (Lappan & Theule-Lubienski, 1994; Shulman, 1987) as well as the skills of classroom management, discussion leading, and communication (Baroody & Coslick, 1998; Ralph, 1999).

Teachers' experience, knowledge of disciplinary content, and instructional objectives exert a substantial influence on the questions they ask (Gall & Rhody, 1987; Haynes, 1935), but teacher questioning cannot be considered in isolation from student learning. Hunkins (1989) asserted that questioning should be viewed as a complicated linguistic device that possesses divergent levels of function, difficulty, interest, and feasibility based on different analytical frameworks. In terms of student learning, teacher questioning has the potential to initiate a series of cognitive processes in students such as remembering, understanding, applying, analyzing, evaluating, and creating (Bloom, 1956; Hyman, 1979; Strayer, 1911). The application of the knowledge and skills mentioned above increases the complexity of questioning in the classroom, and one should consider questioning as both a practical matter and professional technique that teachers master through empirical practice, not merely theoretical discussion (Fitch, 1879).

Martino and Maher (1999) claimed that "[t]he art of questioning may take years to develop for it requires an in-depth knowledge of both mathematics and children's learning of mathematics" (p. 54). Many teacher education and professional development programs are aimed at fostering teachers' questioning strategies for the purpose of enhancing students' learning (Franke, Webb, Chan, Ing, Freund, & Battey, 2009; Kiemer, Groschner, Pehmer, & Seidel, 2015; Ralph, 1999; Weiland et al., 2014). To achieve this goal, teacher educators need to be acquainted with preservice teachers' present questioning performance as well as the function of their enacted questions.

Teacher Questioning

Questioning as a Strategic Act

Teacher questioning is the most commonly used teaching technique (Floyd, 1960; Stevens, 1912) and plays an essential role in effective teaching (Floyd, 1960; Moyer, 1967; Reynolds & Muijs, 1999). Green (1971) considered questioning as a strategic act that should be distinguished from other logical strategies, such as explaining, that can "be evaluated independently of their consequences" (p. 7). While addressing the quality of teacher questioning, one should include a thoughtful examination not only of the number of questions but also the extent to which questioning affects student learning (Houston, 1938). This interdependence between teacher and student suggests that teachers' questioning performance should be evaluated simultaneously based on the quantity and quality of selected questions, the means and timing of asking a question, and the reactions a question elicits in the student. Wassermann (1991) considered these components as the building blocks of effective teacher questioning through which "teachers ascertain what students know, how much they understand, and how well they are able to articulate their ideas" (p. 257).

In mathematical settings, prematurely employing questioning might not only decrease the positive effects of instructional scaffolds but also impede students' development of reasoning and meta-cognitive knowledge. Nicol (1999) pointed out that prospective teachers experienced tensions related to "posing questions to learn what students are thinking versus posing questions to get students to the answer versus posing questions to test students' thinking" (p. 53). She found that prospective teachers asked questions to direct students toward the correct answer in the beginning sessions of her study even though they had been trained not to lead students through the problem. As a result, students only had limited opportunities to demonstrate their thinking and advance their cognitive performance because of a lack of spaces for inquiry in teachers' questions. Moyer and Milewicz (2002) also reported that preservice teachers commonly enacted questioning by checking the listed items in their interview protocol rather than by allowing students to expand on their strategies. They claimed that approximately one-fourth of preservice teachers in their study attempted to instruct students instead of assessing their knowledge during the process of mathematical problem solving. Furthermore, the preservice teachers conceded that they noticed a discrepancy between the intended questions and the enacted questions they had to improvise in the field due to "children's unpredictable responses in a mathematics interaction" (Moyer & Milewicz, 2002, p. 311). As such, the effectiveness of teacher questioning should be strategically determined based on the student responses those questions elicited and what instructional goals they achieved.

Questioning as a Social Act

While analyzing the structure of classroom lessons, Mehan (1979) observed and described Initiation-Reply-Evaluation (IRE) as a common pattern occurring in teacher-student interactions. This structure consists of several iterated or embedded adjacency pairs (Schegloff & Sacks, 1973), in which the first act in a pair is conditionally relevant to the second act, which is meant to occur as expected in the dialogue. Mehan (1979) further identified these acts by pointing out that "an elicitation does not seek just any information, it seeks particular information" (p. 44). That is, every interactional sequence contains a co-occurrence relationship (Garfinkel, 1967; Garfinkel & Sacks, 1970) that was reflexively constructed by teacher and student, and the first part of a sequence ultimately triggers the second part of the sequence, which retrospectively categorizes the initiation. Accordingly, the IRE sequences seem more like "social acts" (Mead, 1934) than "speech acts" (Searle, 1969, 1976) due to the feature of seeking the completion of the adjacency pair and giving meaning in interaction.

When interpreting teacher questioning as a social act, van Zee and Minstrell (1997) defined a particular style of questioning, *a reflective toss*, in which the sequence typically starts with a student statement, proceeds with a teacher question, and continues with additional student statements. They highlighted that the responsibility for thinking in a reflective toss should return to the students and suggested that teachers should "shift toward more reflective discourse by asking questions" (p. 227) that help students clarify meanings, neutrally consider diverse viewpoints, and monitor their own ideas and actions. After successfully inviting students to reflect on their own thinking, teacher questioning would further facilitate collaboration through comparing and contrasting various perspectives among participants. When communication and reasoning flourish, the teacher should allow students to "play more active roles in their own and each other's learning, and thus build a classroom community that invites active participation, confidence, and further learning" (Martina & Maher, 1999, p. 75).

In a mathematical learning community, teachers and students possess different knowledge, beliefs, dispositions, and experiences related to mathematics. Normally, it is the teacher who takes responsibility for facilitating the interaction by applying pedagogical techniques and efficiently orchestrating the shared discourse. To cultivate these professional skills, pedagogy courses in a training program may serve as the first official environment in which preservice teachers can practice and polish questioning techniques before they begin formal classroom teaching. In the following section, I introduce an integrated framework that was used to examine preservice teachers' questioning performance during their early mathematics field experiences.

Conceptual Framework

I view students' problem solving as an internalized process to solve a given mathematical task, so I am interested in positioning the study of teacher questioning in students' problem-solving within distinct stages proposed by Polya (1957). I employed an integrated framework informed by Jacobs and Ambrose's (2008) original work on teacher moves, Jacobs and Empson's (2016) revised framework, and Polya's (1957) four problem-solving stages: (I) understanding the task, (II) devising a plan, (III) carrying out the plan, and (IV) looking back. Because Stage IV, with its extending moves used after the correct answer was given, is beyond the scope of this work, I only examined questioning moves in the first three stages, as shown in Figure 4-1.

	Framework of Teaching Moves (Jacobs & Empson, 2016)	Four-stage guideline (Polya, 1957)	Current Analytical Framework of Questioning Moves
Supportive Questioning Moves	• Ensuring the child is making sense of the story problem →	Stage (I) For understanding the problem	← [SQM1] Ensuring the child is making sense of the task
		Stage (II) For devising a plan	← *Adding* *[SQM2] Inquiring the child's plan to solve the task*
	• Exploring details of the child's existing strategies • Connecting the child's thinking to symbolic notation • Encouraging the child to consider other strategies →	Stage (III) For carrying out the plan	← [SQM3] Exploring details of the child's existing strategies [SQM4] Connecting the child's thinking to symbolic notation [SQM5] Encouraging the child to consider other strategies

Figure 4-1. The integrated framework developed based on Jacob and Empson's (2016) teaching moves and three stages in Polya's (1957) problem-solving guidelines.

I conceptualized a *questioning move* as a unit of inquiry activity (Allender, 1969) that the questioner employed to seek information from the listener with respect to an intended purpose and that may initiate a series of conversation exchanges. All questioning moves presented in this study were teacher-initiated and formulated as an expression that had different syntactic forms and pragmatic categories of speech acts (Graesser et al., 1992). The most conventional format for a questioning move was an interrogative expression that always ends syntactically with a question mark (?) when captured in print. The categories of questioning moves were originally derived from Jacobs and Ambrose's (2008) research, in which they proposed teachers' supporting and extending moves. I only included supporting moves because the focus of this paper was on the questioning moves teachers used exclusively to support students' thinking before the posed mathematical problem was correctly solved.

The four categories of teaching moves from Jacobs and Empson's (2016) framework (ensuring understanding, exploring strategies, connecting, and considering other strategies) served as the categories for this framework. Given that the teacher-student interactions in this study were situated in only three of Polya's problem-solving stages, a category of questioning moves used for inquiring about the child's problem-solving plan was needed. Therefore, I added a new category, *Supportive Questioning Move 2*, to analyze questioning moves observed in problem-solving Stage II. Hereafter, teachers' supportive questioning moves will be denoted as SQM, corresponding to Jacobs and Empson's teaching moves, such as SQM1 (see Figure 4-1). Table 4-1 presents the five categories of teachers' questioning moves with the description of their function.

Table 4-1

The Five Categories of Supportive Questioning Move (SQM)

Category of Supportive Questioning Moves	Description of the Function
SQM1: Ensuring the child is making sense of the task	To ensure that students understand • the contextual features of the task scenarios • the involved mathematical ideas and relationships within the tasks
SQM2: Inquiring about the child's plan to solve the task	To learn about the child's problem-solving strategies derived from • the given information • the child's prior experience
SQM3: Exploring details of the child's existing strategies	To facilitate preservice teachers' understanding of • the child's procedural understanding • the child's conceptual understanding • the rationale behind the mathematical representations employed
SQM4: Connecting the child's thinking to symbolic notation	To enhance the connection between • the child's thinking and mathematical representations • informal expression and formal mathematical terminology • the child's presenting idea and its corresponding mathematical principle
SQM5: Encouraging the child to consider other strategies	To elicit additional strategies by • providing a hint when the child is struggling to solve the task • leveraging the child's successful strategy

Additionally, it is important to analyze teacher questioning on the aspect of openness, by which I mean the degree to which the question allowed the student to make flexible responses. For example, known information questions are closed whereas questions that attempt to elicit students' interpretations and explanations based on their strategies are open. Once I determined the category of the questioning move, I examined the openness of the questioning move based on the extent of the student's responses elicited by that questioning move. Table 4-2 provides the description of three subcategories of openness (open-ended, intermediate, closed) with examples of students' responses to demonstrate the characteristics of teachers' questioning moves.

Table 4-2

The Levels of Openness of the Teacher's Questioning Moves

Levels of Openness	Descriptions	Examples of Students' Responses
Open-ended	Allow the students to express their own opinions or interpretations	T: Do you remember what you are trying to find in the problem? S: How many cookies she has in all.
	Allow the students to reflect on the process of making connections or comparisons, such as expressing the grounds of their reasoning	T: Why did you start with the biggest number when adding them? S: Because it is the highest number, and it is easier for me to put it on top.
Intermediate	Allow the students to express (dis)agreement or a response from a list provided in the interaction	T: Is the answer to 18+18 bigger or less than 30? S: Bigger.
	Allow the students to provide a factual response retrieved from their memory	T: What are those two blocks representing? S: Two pieces of candy [in the task].
Closed	Make the students obey teachers' directive initiations	S: [Reaction like nodding] S: Uh-huh!
	Make the students accept teachers' informative initiations	S: [Acknowledgement] S: Okay.

Due to the inappropriateness of categorizing SQM4 and SQM5 questions by their openness, this open-intermediate-closed scale only applied to the first three thematic categories (SQM1 to SQM3) of questioning moves. The open-ended moves allowed students to express their thoughts to a posed problem. The most significant characteristic of open-ended moves was that students' verbal responses consist of long explanations, rationale of the strategy, and personal opinions. The intermediate moves normally elicited short answers or a selection from a multiple-choice question. The closed moves included informative and directive initiations;

informative initiations generally resulted in acknowledgment, and directives initiated students' non-verbal reactions, such as nodding (Mehan, 1979). Although most of the closed moves were not in the format of a question that could elicit students' verbal elaboration, they set the teacher-student interaction in motion and thus were worth analysis and discussion based on the rationale of this study. To extend the understanding of preservice teachers' questioning in a dynamic problem-solving process, all employed questioning moves were examined in the moment-to-moment interactions. Two research questions guided this study:

1. How do preservice teachers enact questioning in each stage of mathematical problem solving?
2. What features emerge in preservice teachers' utilization of supportive questioning moves while working with elementary students in mathematics?

Methods

Setting and Tasks

This research was conducted in the teacher education program for early childhood majors (certification Pre-K–5) at a Northeast Georgia University. The setting is a school-based activity, the *Single Student Mathematics Field Experience* (SSMFE), which was designed to facilitate preservice teachers' knowledge of students' mathematical thinking and to help them reflect on how student thinking can inform mathematics teaching. The SSMFE required preservice teachers to conduct a one-on-one interview with a single student in a local elementary school once a week for eight weeks during the first mathematics methods course.

During the methods course the preservice teachers were exposed to all problem types with different locations of unknown (i.e., start unknown, change unknown, and result unknown) while being introduced to Cognitively Guided Instruction (CGI, Carpenter, Fennema, Franke, Levi, & Empson, 1999). They practiced how to listen to their classmates' mathematical thinking in the methods course before interviewing elementary students. In addition, preservice teachers worked on writing different types of problems and specifying the problem structure and location of the unknown as course assignments. Mandatory course work and homework assignments included distinguishing problem types, writing problems with particular sets of numbers, listening to mathematical thinking, and sharing ideas and comments. The instructors emphasized teachers' use of their knowledge of problem types and the typical strategies used by children according to the literature to inform their instruction. Due to the adoption of the CGI frame in the course, preservice teachers were encouraged to practice writing problems with different numbers and structures and to understand the mathematical thinking that children unused in their solution strategies.

Preservice teachers were also told not to discourage children's ideas and to pay close attention to how children's strategies developed over time. In particular, preservice teachers were encouraged to point out incongruities in children's strategies and to ask why questions to understand the big mathematical ideas behind children's thinking. The instructor also introduced the possible talk moves that preservice teacher could implement in the SSMFE, such as encouraging the child's way of solving problems (rather than forcing the child to use the teacher's preferred method), asking for justifications, and revoicing the child's explanations. Before the SSMFE began, the instructor walked preservice teachers through the interview protocol and elaborated on the goal of the SSMFE: to understand how your child is thinking.

The focus of this investigation centered around understanding the nature of preservice teachers' questioning performance in the eight sessions of interview-based interactions. Although the SSMFE was quite different from teaching and learning in a classroom setting, prior research has contended that preservice teachers benefit from conducting structured interviews with students, such as one-on-one interactions, in early field experiences (Jacobs & Ambrose, 2008; Weiland et al., 2014). Given that teacher questioning could be considered as a strategic and social act, working with a single student in an inquiry-based setting might positively increase preservice teachers' awareness of their questioning abilities and help them recognize the suitability and effectiveness of questioning strategies without being distracted by other aspects of classroom management. Therefore, preservice teachers were required to focus particularly on inquiring about the students' thinking, explanations, and problem-solving strategies through asking questions.

While working with students on diverse mathematical tasks, preservice teachers had opportunities to intensely practice their questioning skills with regard to different mathematical topics. In every session, the preservice teachers predominantly employed mathematical tasks that were provided by the course instructor in the format of interview protocols, although they were allowed to design their own intended tasks for children who might need extra help or challenge. All tasks were related to one of the following five mathematical topics: (a) Base-N; (b) Base Ten/Place Value; (c) Number Facts; (d) Fraction or equal-sharing problems; and (e) Arithmetic problems including addition, subtraction, multiplication and division adapted from "Children's Mathematics Cognitively Guided Instruction (CGI)" (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Empson & Levi, 2011). Each SSMFE session lasted for 35 to 45 minutes, depending on the type and level of difficulty of the executed tasks. Example tasks are listed in the Appendix D.

Participants

The 13 preservice teachers who participated in this study (hereafter referred to as teacher participants) were eleven Caucasian female and two Asian female undergraduate students who were enrolled in their first mathematics methods course: One cohort ($n = 6$) participated in the Fall 2014 study; the other cohort ($n = 7$) participated in the Spring 2015 study. The participants were selected by convenience sampling (Patton, 2002) from 28 preservice teachers in each methods course. The Spring 2015 section was taught by an instructor who was the teaching assistant working in the Fall 2014 section, so both courses were structured the same way on the campus classes (e.g., same topics were addressed in the same order) and tasks designed for SSMFE interviews (e.g., same weekly interview protocols were provided to preservice teachers). In the recruitment, there were about 10 volunteers from each course, and I selected the final participants according to their participation in the course. All of the teacher participants were in their junior year at the University and had completed at least two mathematics content courses for Early Childhood Education majors.

The student participants consisted of two grade levels of students at a public elementary school: fourth-graders in the Fall 2014 study and first-graders in the Spring 2015 study. At the beginning of the semester in Fall 2014 and Spring 2015, I recruited student participants in the elementary school that cooperated with these two mathematics methods courses, and about right students in each grade were consented to participate in this study. After having the final list of both teacher and student participants, the instructor and I then randomly paired up participants and formed 13 participating pairs for this study.

Data Collection

Every SSMFE week, one pair of participants (a preservice teacher and an elementary school student) was videotaped during their mathematics lesson. Across the two semesters of the study, 13 pairs of participants were audio recorded and videotaped. However, in order to conduct a comparison of the same teacher's questioning performance at two different time points, two pairs of participants from the Spring 2015 class were recorded twice in the SSMFE—the first time in the middle of the field experience and the second time in the last week of the field experience. Therefore, 15 videos were collected from 13 pairs of participants in this study. To effectively collect the moment-to-moment interactions between the teacher and the student, the audio recorder was focused on the teacher's verbal expression, and the video camera was focused on written work produced by both teacher and student. Other data sources were collected as supplementary documentation, including the students' written work as well as the notes preservice teachers took during the session, teachers' debriefing forms, course assignments, and the teachers' SSMFE final portfolios.

Data Analysis

I employed framework analysis to analyze the data sources collected in this study (Richie & Spencer, 1994). This process of analysis contains five key phases: (1) familiarization, (2) identifying recurrent themes, (3) indexing, (4) charting, and (5) mapping and interpretation (see Figure 4-2). I started by watching videos and reading transcripts repeatedly in order to highlight the most frequent patterns in teacher questioning moves based on the framework I developed. The key themes among the examined data were then identified, and the related moves were sifted and sorted into thematic categories (SQM1 to SQM5). In particular, I categorized teachers' questioning moves based on stage of the interaction, not on the teachers' purpose. For example, if a questioning move occurred when the student had not yet started to devise a solution strategy (Stage II), the move was assigned to Stage I because the interaction initiated by this move happened before Stage II.

In addition to categorizing the moves based on the thematic framework, the students' reactions to each questioning move were considered so that each move could be annotated by its level of openness. While viewing the video recording, I took analytical notes to supplement the data analysis with the foreground of an enacted move, the intonation in the questioning moves, and notable reaction a move caused in segments of teacher-student interactions. The assigned code of a questioning move was in the format of "SQM thematic number (1 to 5) - openness number (1 to 3)." For example, SQM1-1 stands for an open-ended question within the first thematic category of making sure the student understands the problem (see Figure 4-2). When a move was classified inconsistently during different coding sessions, the entire dialogue was scrutinized to position the undetermined move within a suitable category. In order to address issues of reliability, I repeated the coding process every two weeks, and all questioning moves were coded more than twice, with 86% agreement of coding achieved. All moves in relation to students' reactions were ultimately mapped and interpreted within each task as a complete scenario.

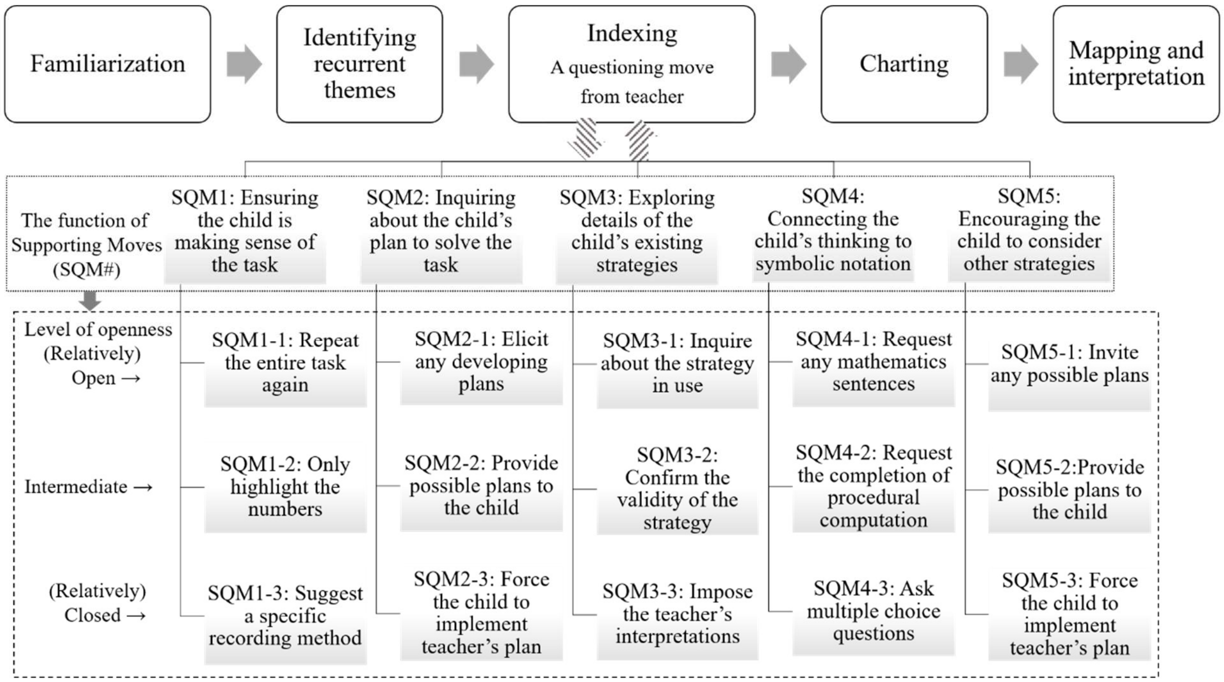

Figure 4-2. Coding flowchart for supportive questioning moves (SQMs) and the level of openness.

As a result of data analysis, I compiled frequency counts of how often each type of move appeared and also identified some qualitative features of teacher questioning. I initially examined the distribution of preservice teachers' questions within the five thematic categories along with their degrees of openness to delineate how preservice teachers utilized questioning moves. With the quantitative data in mind, the qualitative features of preservice teachers' questioning moves will be characterized in isolated representative vignettes with illustrative excerpts from transcripts of the video recordings. Teacher participants' reflective written assignments were adopted as supplementary evidence while interpreting their questioning practices.

Findings: The Distribution of Preservice Teachers' Questions

Table 4-3 presents the frequency distribution of preservice teachers' enacted questioning moves within and across the five thematic categories during 15 SSMFE sessions. In total, 1,027 supportive questioning moves were employed by 13 preservice teachers during 115 implemented mathematical tasks.

Table 4-3

Frequency Table of Preservice Teachers' Questioning Moves in SSMFE

Subtype / Thematic category	Subtype 1 (open-ended)	Subtype 2 (intermediate)	Subtype 3 (closed)	Subtotal
SQM1	163	103	3	269
SQM2	59	48	29	136
SQM3	201	217	96	514
SQM4	40	18	13	71
SQM5	22	10	5	37
Subtotal	485	396	146	1027 (100%)

There could be more than one way to measure the percentage of each type of question. For example, one way to evaluate the data presented above is to compute the percentage that each type of supportive questioning moves occupied among the total number of questions used in SSMFE. The outcome from this perspective would then show that 50% of the teacher questioning moves were categorized as SQM3; 26% of moves were SQM1; 13% were SQM2; 7% were SQM4; and 4% were SQM5. In addition to this breakdown, Figure 4-3 provides a quantitative analysis of the degree of openness of each question within each thematic category.

The results of this analysis showed that teacher participants asked more open-ended questions in almost all categories except SQM3. However, employing open-ended questions within different categories may yield different results in the teacher-student interactions, especially with regard to students' problem-solving abilities, which will be discussed in following sections.

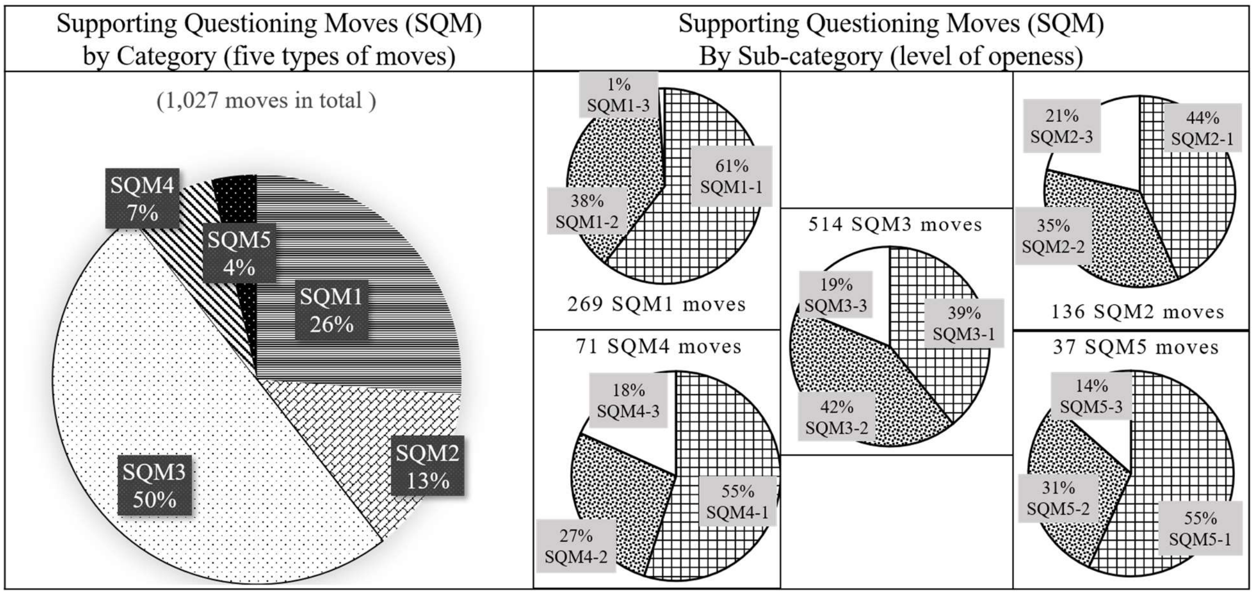

Figure 4-3. Supportive questioning moves by category in the SSMFE.

Findings: The Qualitative Features of Preservice Teachers' Questioning

In following sections, I use representative vignettes taken from SSMFE sessions to illustrate how teacher participants employed questioning moves to support students' problem solving and what influential features emerged in their questioning. The qualitative findings are presented in parallel with the order of students' problem-solving activities from Stage I to Stage III.

Stage I: Questioning Moves Used to Build Children's Understanding of the Task

Polya (1957) determined understanding the task to be solved as the most important stage in problem solving. As can be seen in Table 4-3, 269 of the 1027 questions asked by the preservice teachers were asked to assess students' understanding of the tasks (SMQ1). However, the intent and outcome of enacting SQM1 questions under different circumstances were quite heterogeneous across participants. On one hand, the timing at which SQM1 questions occurred during the session varied: they could occur at the beginning of students' problem solving or in the midst of the interactions. On the other hand, the formats of SQM1 questions were nearly identical even at different time points. Furthermore, a large group of teacher participants did not automatically (re)confirm students' understanding of the tasks until they realized that students were struggling with the problem-solving process due to a lack of knowledge about the mathematical relationships among the numbers involved. The significance of the timing and format of enacting SQM1 questions will be further discussed later in this paper.

Among the 269 SQM1 questions, 61% were coded as open-ended ones. The application of open-ended questions in SQM1 indicated that that the teacher participants devoted sufficient space to allowing children to identify the necessary contextual features and essential mathematical relationships among the given numbers in the setup stage. Jackson, Garrison, Wilson, Gibbons, and Shahan (2013) asserted that "the setup, or introduction, of cognitively demanding tasks is a crucial phase of mathematics instruction" (p. 646). When exploring the relationships between teachers' arrangement of the setup and students' opportunities to learn, they found that it may be helpful for students in making connections between their strategies and correct solutions. This arrangement relied on paying attention to establishing a "taken-as-shared understanding" of contextual features and mathematical relationships, one kind of understanding that was "taken as, or reasonably assumed to be, compatible enough to enable students to communicate in consistent ways about the relevant ideas" (p. 654). In the current study, SQM1 questions normally occurred following the posing of the task in the Stage I. While enacting this type of question to confirm students' understanding, teacher participants might (a) change the numbers but maintain the original mathematical structure, (b) modify the mathematical structure of the task, (c) implement the original task when the child was able to accomplish it, (d) strictly implement preplanned tasks without considering the child's struggles, (e) instruct instead of assessing the child's thinking, or (f) invite students to interpret the task by themselves.

Changing the numbers but maintaining the original mathematical structure. In some cases, preservice teachers only changed the assigned numbers and declared that the change would decrease the cognitive demand for the student. Indeed, there was higher possibility that, while making this change in the interactions, teacher participants did make a problem easier from the aspect of the numbers while maintaining the preplanned mathematical structure. Vignette 1 exemplifies this phenomenon.

> ***Vignette 1***
> At the beginning of one task, Amy (preservice teacher) posed a problem with the mathematical structure of $23 + 7 \times 10$ (23 donuts plus 7 boxes of 10 donuts). However, Bonny (child) seemed challenged by adding 7 groups of ten. Therefore, Amy revised the task by changing the number of boxes from 7 to 4 by saying, "That might be a lot. Here, we can change it to 4 boxes [of donuts]. How about that?" In this case, Amy maintained the mathematical structure in the problem and was still able to assess Bonny's ability to add multiple tens to a given number (Grade 1, SSMFE 8, Task 6).

However, in some special circumstances, changing numbers also changed the mathematical structure of a task.

Modifying the mathematical structure of the task. A few preservice teachers tried to modify the planned tasks when they detected that the students were struggling with solving the original tasks. However, this approach sometimes altered the mathematical structure of the tasks and it turned out students solved the problems by applying completely different mathematical knowledge. For example, one preservice teacher, Mariah, worked with a first-grader on the base-ten topic, in which the warm-up question asked the child to decompose 10 into two numbers and then to list all pair combinations such that one number is greater than the other. However, Mariah changed the number 10 to a relatively easy number, 3, without considering how this change might completely alter the mathematical structure of the make-a-ten method the instructor originally designed for this SSMFE session.

In vignette 2, another preservice teacher, Mary, planned to assess her first-grader, Abby's knowledge of fractions in an equal-sharing structure $3 \div 10 = \_\_$, but she ended this task with a different mathematical structure of whole-number division $20 \div 10 = \_\_$ after modifying the task.

> ***Vignette 2***
> Mary posed the task, "If I am having a party with 10 friends, and we are splitting 3 cookie cakes, how much of the cookie cakes would each friend get?" Abby was struggling with partitioning all three cakes into pieces of the same size; she initially divided all three cakes into 5 unequally sized pieces. After she distributed each piece of cake to ten people, she realized that she would need to cut the last cake into ten pieces if everyone wanted to get an equal portion of cake. In this process, Mary tried to draw from Abby the mathematical terminology for the divided pieces she had created in her drawing and was satisfied that Abby could correctly name them as "one-fifth" and "tenths." However, when Mary asked, "Do you know how much one person would get?" Abby replied "3 pieces." Mary did not correct Abby; instead, she summarized the total pieces of cake in the drawing for Abby by changing the mathematical structure from $3 \div 10 = \_\_$ to $20 \div 10 = \_\_$. The task was then directed to a new scenario. Mary's change of the task elicited a new strategy from Abby, who started to draw 20 tally marks, distributed them to 10 people, and concluded with the answer "2 pieces per person." Mary praised Abby for her answer and wrapped up this task (Grade 1, SSMFE 8, Task 1).

Working with students on operations involving fractions could be more challenging than on whole-number division tasks for teachers. Ball (1993) asserted that "[i]n teaching fractions, the teacher must weigh the relative advantages in providing students with structured representational materials versus having students refine existing models and develop their own representational media" (p. 163). Due to the complexity of teaching fractions, researchers have suggested that teachers provide visual representations, use multiple references to exemplify a fraction, and adopt language familiar to students based on what they already know and how they have learned (Chick, 2007; Dufour-Janvier, Bednarz, & Belanger, 1987; Nesher, 1989). In addition, it is essential to provide a "fruitful thinking space" in which students may explore the relationship between the meaning of fractions and the representation embedded in them (Ball, 1993). Vignette 2 showed that Mary seemed not to provide advanced "thinking space" for Abby in her questioning to help her explore the assigned fraction concepts (10 people are splitting 3 cakes). Instead, after receiving the answer "3 different-size pieces" from Abby, Mary revised the task in a new mathematical structure $20 \div 10 = \_\_$. Although this improvisatory decision to adapt the original structure to an easier one (10 people are sharing 20 pieces of cake) decreased the cognitive demand of the task, it also missed a valuable opportunity to explore the concept of fractions with the child.

Implementing the original task when the child was able to accomplish it. Most preservice teachers chose to implement the preplanned tasks as they were described on the interview protocol without making any change, regardless of what the numbers and the structure were. Although this might encourage the development of students' own interpretation and understanding of the tasks, in most cases, students were more likely exposed to the process of "sinking or swimming." When the child was able to solve the task, implementing the original task reinforced teachers' beliefs about the appropriateness of intended tasks designed by the course instructor and may result in their obedience to the authority in the methods courses. For instance, Amy and Bonny were working on a multiplication problem with the mathematical structure of $3 \times 5 = \_\_$, and Amy posed an "unnatural" problem, that was an intended task provided on the interview protocol, to Bonny in vignette 3.

> ***Vignette 3***
> Amy posed the task to Bonny as it appeared on the protocol: "There are 3 bees. A bee has 5 legs. How many legs are there in all?" Initially, Bonny misunderstood the problem as there being only one bee who has 5 legs, so she solved this problem by drawing three bees, and thought the other two bees were supposed to have 6 legs with a final answer $6 + 6 + 5 = 17$. Amy then asked, "Why does just the last one have 5?" and Bonny explained, "Maybe the bee was born without a leg." Amy agreed but insisted on delivering the preplanned task by saying, "But all bees lost a leg." Later, Bonny successfully solved the problem but felt sad about the fact that every bee lost a leg. Even though Amy admitted that it wasn't a very good math problem, she decided to stick to the problem she got from the instructor (Grade 1, SSMFE 3, Task 2)

In the above vignette, Bonny was still able to solve the problem even though the story struck her as unnatural, so Amy implemented this task without encountering any difficulties in her interaction with Bonny. However, preservice teachers' implementation of inappropriate tasks might hinder student success in problem solving. The task of the Lucky 5 Candy Factory demonstrated this phenomenon.

Strictly implementing preplanned tasks without considering the child's struggles. Inevitably, students sometimes struggle to understand the task when its mathematical relationship is complicated, such as problems including multi-digit numbers, several levels of units, and multiple operations. The Lucky 5 Candy Factory task (see Appendix D) is an example of such a task. While working with her first-grader on this task, Mary employed up to 50 SQM1 questions to inquire about Abby's understanding of the mathematical relationship in the setup throughout this interview session. There are four levels of units involved in this task, which is related to the base-five concept, and it was obvious that dealing with four distinct units (the individual candy, the roll of candy, the box of candy, and the crate of candy) at the same time was a demanding task for Abby. Among those units, five individual candies should be repackaged as one roll of candy, five rolls of candy will become one box of candy, and five boxes of candy must be converted into one crate of candy. To be able to simultaneously manipulate these four units, Abby needed the scheme of coordinating "levels of units" in this activity (Hackenberg, 2010). Those levels of units evolve from the "composite unit," a unit that itself is composed of other units. Students are likely to need more time to comprehend the inclusion relations of this composite unit (Steffe, 1994a, 1994b) and to intentionally dis-embed the subset of units from the whole. In vignette 4, Abby demonstrated her lack of comprehension about the rule of repackaging, even though she had been working on six sub-tasks that complied with the same rule.

> ***Vignette 4***
> The task on the protocol stated: "Another shipment comes into the factory with 3 rolls, 4 boxes, and 1 crate. You place the shipment on the conveyer belts so it can be repackaged. When the machines get done repackaging, how many of each package should we expect to see?" Abby started with having all base-five blocks, and meanwhile, Mary tried to drop a hint by saying, "If repackaging, sometimes they don't have to change the packaging." However, Abby decided to first replace one roll with five individual cubes with a long pause. When Mary inquired about the reason why she made this change, Abby answered, "I am not sure." Since this is the seventh sub-task in the Lucky 5 Candy Factory task, Mary did not repeat how the machines repackaged the candies; instead, she probed "Do you think the machines can change that [pointing to the used base-five blocks] more?" and "How would they change it?" After pausing for approximately 20 seconds, Abby admitted "Not sure" with a frustrated look on her face. Mary immediately offered, "We can come back to it," wrapped up this sub-task, and moved on to next one (Grade 1, SSMFE 4, Task 7).

When realizing that Abby did not really comprehend what the repackaging meant in this task, Mary did not try alternative moves to decrease the cognitive demand of the task for Abby. For example, she could have simplified the task by using only two units, the individual candy and the roll of candy, instead of four units at the same time. Because she did not take this approach, Mary witnessed Abby's failure, which was caused by her lack of comprehension about repackaging among the four units and her inability to transition between them on almost half of the sub-tasks in the Lucky 5 Candy Factory task.

Instructing instead of assessing the child's thinking. It is not uncommon for preservice teachers to enter the field with the intention to lead students to the correct answer (Nicol, 1999). In some cases, teachers explicitly highlighted the mathematical relationship or provided direction to their students to help them solve the problem. In vignette 5, Mary directly showed how to solve the task correctly in a way that Abby could only passively accept and mimic.

> ***Vignette 5***
> The posed task involved finding a way to divide 50 people into teams of 4. Abby initially drew 50 dots to represent 50 people in the pool. Mary then tried to highlight the mathematical relationship for her by saying, "And we have to split them into teams of 4," but Abby did not understand, so she hesitated for a while. Mary then elaborated by emphasizing the number of people needed in each team. Abby proposed splitting the 50 people into 4 teams. Mary clarified by emphasizing that each team should have 4 people. Immediately, Abby understood and asked for guidance with the next step. Mary stepped in and demonstrated how to solve this task: "Why don't we go through and circle…you have four dots right here; this would be a team…Circle your 4s." After this demonstration, Abby began to mimic what Mary had demonstrated until she completed this task (Grade 1, SSMFE 8, Task 2).

Moyer and Milewicz (2002) also observed this phenomenon in their study on preservice teachers' questioning and concluded that one general questioning category preservice teachers commonly utilize is "instructing rather than assessing," in which teachers directly demonstrate problem-solving strategies and expect their students to follow the instruction. In the setting of vignette 5, Mary implemented 10 tasks in 50 minutes, which suggests she had an ambition to accomplish all planned tasks before the session was over. It is possible that, for Mary, to have the student solve all tasks in every session was the main goal in her SSMFE, but without interviewing Mary, I cannot draw an inference about the reason behind this ambition. In line with Moyer and Milewicz's (2002) research, this study also revealed that preservice teachers had the tendency to instruct their students on how to correctly solve the posed task rather than striving to assess student thinking.

Inviting students to interpret the task. Polya (1957) suggested that, to effectively enhance students' understanding of a task in Stage I, teachers can check students' understanding by asking them to repeat the problem they need to solve. Moreover, letting students point out the principal parts of the problem is also a strategic way to ensure they do not miss the essential conditions of the task. For instance, one preservice teacher, Sami, not only asked her student, Vicky, to repeat the task but also required her to interpret the mathematical relationship from her perspective. Therefore, she frequently assessed Vicky's understanding of the task by asking questions like "I forget the problem, can you tell me the problem again? Do you remember the problem?", "Can you repeat that question for me?", "Do you remember what you are trying to find in the problem?" and "Can you tell me what happened in the problem?" in several tasks throughout the session on that day (*Grade 1, SSMFE 2*). Although Vicky spent plenty of time retrieving information and forming the sentence to express what she knew about the task, the wait time Sami applied was fairly appropriate and successful to invite Vicky to interpret the tasks. Because Sami took this approach, Vicky had opportunities to revisit the problems, select essential information, and demonstrate her understanding of the tasks to Sami before she started to solve them. Sami's SSMFE set up a positive questioning example in problem-solving Stage I.

Stage II: Questioning Moves Used to Inquire about the Devised Plan

Devising a plan in problem solving could be described as selecting suitable heuristics based on the solver's understanding of the problem in the first stage. To achieve this, students must monitor their comprehension, examine the connection between the received information and the unknown, and establish potential solution paths (Carlson & Bloom, 2005; Peker, 2009; Printrich, 2002). Polya (1957) pointed out that "[t]he way from understanding the problem to conceiving a plan may be long and tortuous" (p. 8). In SSMFE, there were a total of 136 moves in SQM2, and 44% of these moves were open-ended, which meant that students had an opportunity to elaborate on the reasons behind conceiving a feasible plan. However, most enacted tasks were taught in school mathematics class, so the student participants generally began to solve the posed task right after it was posed by the preservice teachers. In addition, students habitually conceived plans in their head, so the rationale behind the devised plan became relatively implicit in the process of problem solving. It is important to mention that questioning moves positioned in Stage II occurred before students began to carry out their solution strategies. Two phenomena of teachers' questioning behavior are worth discussing: (a) easily accepting an oversimplified reason for the devised strategy and (b) directly suggesting a possible direction to develop a strategy.

Teacher accepted an oversimplified reason for the devised strategy. In many cases, teacher participants chose not to prompt students further to articulate the rationale for their devised plans, so they missed the opportunity to acquire students' insights into the proposed plan. The observed vignettes affiliated with Stage II illustrated the tendency of preservice teachers to complacently accept an oversimplified reason about how the plan was devised or selected, rather than conduct a thorough inquiry about the students' rationale behind their strategies. Vignette 6 demonstrates how the teacher enacted open-ended SQM2 questions to inquire about the child's plan but stopped exploring when receiving a response stemming from an oversimplified reason for the decision on employing subtraction.

> ***Vignette 6***
> The original task was posed as, "Danny has 2001 points on his favorite video game. He forgot to save the game before turning it off, and he loses 956 points. How many points does he have now? Solve the problem in two ways." Sam automatically articulated his plan of solving this task by saying, "Well, you don't need this information that he forgets, but you need that [pointing to 'he loses 965 points'], then I am gonna subtract. So, if I subtract 2001 by 965, I will get my answer." Teresa inquired, "How'd you know that?" and Sam answered, "Because the key word is 'loses', and then he loses 965 points, and he has 2001 points" (Grade 4, SSMFE 4, Task 1).

In vignette 6, Sam demonstrated his ability to sieve the information and devise a strategy, so Teresa tried to inquire about the rationale behind his plan. Sam then indicated that the key word was "loses" so he conducted subtraction. However, Teresa accepted this reply without endeavoring to elicit further explanation on this statement. Beckmann (2016) emphasized that the construction of mathematical relationships among numbers in a math word problem should not be determined by merely focusing on a key word. Due to Teresa's acceptance of Sam's oversimplified reason, she might have unintentionally reinforced the use of key words. Conversely, when Sam asserted that the key word "loses" told him to subtract, one possible move for Teresa was to invite Sam to explain the mathematical reasoning behind his decision by asking questions such as "Why does the word 'loses' mean 'to subtract' to you?", which was a typical question suggested by the instructor of the methods course.

Teacher suggested a possible direction to develop a strategy. In the remaining moves in SQM2, preservice teachers normally directly suggested a possible strategy that was either derived from students' existing work or from the teachers themselves. Vignette 7 shows how the teacher initially tried to discover the child's intended plan via a SQM2 question, "What are you thinking about doing?" but then followed up by suggesting a new strategy without extending the child's proposed idea.

> ***Vignette 7***
> Bonny has been struggling to figure out how to equally distribute 30 basketballs among 10 bins. After some trial-and-error methods, Amy asked, "What are you thinking about doing?" Bonny described her plan and the difficulty she encountered: "I am thinking about how to put all of them [30 basketballs], the same number in here [10 bins], but she has 30 basketballs, and I have nine more bins." Amy then suggested a new strategy by saying, "Can you write a number sentence for it? Remember how last week we wrote a number sentence for that one problem?" based on their prior experience (Grade 1, SSMFE 3, Task 1).

As a result, Bonny wrote the sentence, $18 + 18 = \_\_$, in which she assumed that 30 would be the answer to the unknown number and began solving her equation by arranging 18 cubes plus 18 cubes on the table. Compared to the questioning move of "instructing instead of assessing" shown in vignette 5, Amy's suggestion left relatively more learning space for Bonny in vignette 7. That is, Amy's suggestion of solving the problem using a number sentence without regulating the format of the sentence and the numbers in it did not limit Bonny's creation. According to Polya (1957), there are several functional questions that would help students devise a plan, such as "Did you use all the data? Did you use the whole condition?" and "Could you use any (previous) problem with a similar unknown?" (p. 10). It is evident that knowing how to solve a problem relies on how well the problem solvers understand the mathematical relationships in the task and can make connections between the current situation and their past experiences. Without this firm fundamental understanding, it is difficult for students to solve the tasks successfully.

After watching Bonny struggle for ten minutes, Amy realized that Bonny did not understand the mathematical relationship between the two given numbers. During this ten-minute span, Amy's questions superficially focused on Bonny's trial actions, such as "Why did you try 8?" and "Is 18+18 bigger or less than 30?" However, Bonny still could not move on without devising a proper plan for this task, even though she admitted that this process did challenge her in a positive way. Ultimately, one questioning move enacted by Amy, "Can you think of another way of doing it besides putting them all in one bin at the same time?" stopped Bonny from trying to put several objects in the same container and inspired Bonny by only putting one object in each of the ten bins at one time. This nudge, *besides putting them all in one bin at the same time*, might not suggest a possible plan to solve the task, but it excluded an impractical strategy and finally helped Bonny successfully solve this task. In other cases, however, when students spent more than 3 minutes to figure out how to solve a task, preservice teachers were inclined to terminate the task and move on to the next.

Stage III: Questioning Moves Used to Investigate the Existing Strategy

Half of preservice teachers' questions in SSMFE fell into the category of SQM3, and this use of questioning was consistent with the course goal of understanding children's mathematical thinking. Most preservice teachers were eager to dive into the mission of exploring student thinking while observing the students solving math problems in SSMFE. Although the preservice teachers in this study were provided with exemplary talk moves, such as asking questions like "How did you do that?" "How did you figure that out?" or "Why" (Ginsburg, Jacobs, & Lopez, 1993). The relative nature of enacting SQM3 questioning is in line with the "responsive moves" described by Jacobs and Empson (2016), who proposed that there is no way that preservice teachers can predict the best time to enact SQM3 questioning because those moves are subject to what students do and come in response to the previous question under a specific set of circumstances.

Open-ended SQM3 questions required students to demonstrate their existing strategies in detail. Teachers typically used direct probing questions like “What did you do?” or “How did you do that?” in a problem-solving interview. However, in most cases, these types of questions exclusively elicited students’ procedural knowledge rather than their conceptual knowledge. One could ascribe this outcome to teachers’ deficient questioning techniques, such as (a) using non-specific probing questions and (b) neglecting unexamined but valuable strategy devised by students.

Teachers used non-specific probing questions. Similar to what Moyer and Milewicz (2002) found in their study, preservice teachers in this study used many “non-specific” probing questions. This type of question did not “acknowledge the child’s specific response, resorting instead to general follow-up questions such as ‘What were you thinking?’” (p. 307). In this current study, some teachers risked obscuring their questions by not specifically indicating the parts of the students’ strategies that they intended to probe using SQM3 questions. As a result, teachers failed to tailor their questions to the appropriate responses that really interested them in the interactions. Linked to this issue, even though the first-grade student, Alia, clearly described how she converted a Start Unknown Join problem into a Change Unknown problem adapted from the CGI problems, Tina’s question consisted of non-specific indicator such as the pronoun “that” in this task:

> ***Vignette 8***
> The posed task was, “Julia had some markers. She gave 2 markers to her brother. Now she only has 4 markers. How many markers did Julia start with?” After Alia gave her solution, “She started with 6 markers,” Tina asked her, “How’d you know that?” Alia replied, “Because 2 plus 4 is 6, and…you add those up, and I did an inverse [computing $6 - 2$ on paper], so I got 4…I also knew 4 plus 2 was 6, so if you added 2 plus 4 [it’d be] 6 markers.” Tina did not inquire further about the strategy (Grade 4, SSMFE 1, Task 1).

The pronoun "that" in Tina's question could have meant the answer 6 and been intended to elicit how the 6 was produced in the computational procedure, or it could have meant how the strategy was devised by the student. Preservice teachers might have considered these non-specific questions as "routine questions" that they had to enact in the interview, without contemplating how to use them strategically to elicit pertinent information from the students. Therefore, preservice teachers sometimes wrapped up the task after receiving any response from their students. In this study, most SQM3 questions exclusively ascertained students' computational explanations, and this outcome seemed sufficient to teacher participants because they did not pursue further questioning.

Teachers neglected to examine strategy devised by students. In the next vignette, the preservice teacher Alice missed an opportunity to inquire about her student's initial strategy when the student was trying to solve the task involving the mathematical structure $54 = \_\_ \times 10 + \_\_$.

> ***Vignette 9***
> After posing the task to Sandy, Alice was taking notes to record what Sandy did in her first strategy, and Sandy said: "It was 10 and the other one number [sic] was 54. Let's try…five… [computing 5 times 10 in the long division]…50 equals…so it's 5, remainder of 4 (see Figure 4-4)" Alice immediately asked, "How did you know?" Rather than elaborating on the original strategy, Sandy responded to this probing question by demonstrating another strategy, "I try 2, and do 2 again, and then that's 20, so I can do one to 14. It's gonna be 5" (Grade 4, SSMFE 2, Task 1).

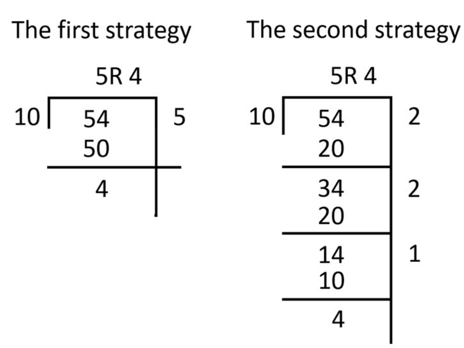

Figure 4-4. Sandy's first strategy and second strategy.

In vignette 9, Sandy's first strategy is unexamined but valuable. However, Alice did not probe the original strategy further, even though she recorded it in her notebook. She accepted that Sandy responded to the question, which was intentionally used to explore how she figured out the first strategy, with her second strategy. In accepting this response without further probing, Alice missed an opportunity to inquire about the reason why Sandy tried the number 5 in her first strategy. During the session, Sandy had explained that she usually employed a different method to check her original solution due to her self-doubt regarding the initial solution. However, Alice could have leveraged Sandy's "old habit" of producing at least two strategies while solving a problem through effectively enacting questioning for both strategies.

Another example was from a fourth-grade session in which the student, Natasha, was working on 60 × 10 by using the partial-products method with a rectangle drawing, called the Box Problem, as shown in Figure 4-5 (SSMFE 5, Task 3). Natasha did not solve this problem correctly, and her explanation of the procedure was flawed. However, the only question from the teacher, Tiffany, was "so is it 6,000 or 600?" and Natasha's strategy remained unexamined. After receiving the confirmation of the incorrect solution, 6,000, from Natasha, Tiffany moved to the next problem without inspecting the pieces of information Natasha provided.

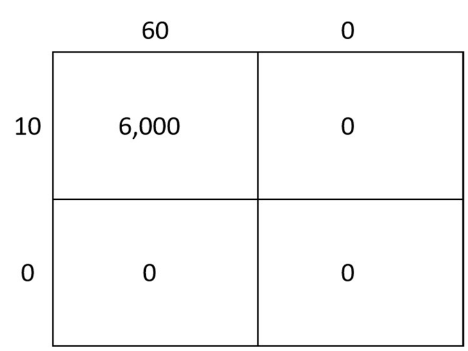

Figure 4-5. Natasha's solution to 60 × 10 by conducting the partial-products method.

In conclusion, when preservice teachers were satisfied with "listening, not to the child's thinking, but for a response which then allows the interview to continue" (Moyer & Milewicz, 2002, p. 301), they did not actively clarify the undefined indicators in their questions and occasionally neglected to probe some underdeveloped responses. Because carefully following up on students' thinking was suppressed by the ambition to complete all tasks in the SSMFE interviews in the time allotted, more than half of teacher participants failed to discover the rationale behind the implemented procedure. Although using the "What," "How," or confirmative questions successfully elicited the steps of a computation procedure, preservice teachers lost valuable opportunities to invite further elaboration and reflection on the students' decision-making processes in problem solving.

Stage III: Questioning Moves Used to Elicit Mathematical Terminology

In terms of using mathematical terminology, NCTM (2000) suggests that teachers should help K-2 students relate everyday language to mathematical language and symbols in a meaningful way. In SSMFE, 55% of SQM4 questions allowed the students to express mathematical representations in the ways they preferred. In these relatively open circumstances, most preservice teachers primarily requested that students write "a mathematical sentence" that comprises two given numbers, one unknown number, and the equal sign. However, students were allowed to decide the type of operations and the location of the unknown number in the sentence. In contrast to this open approach, some preservice teachers insisted on students incorporating the correct mathematical terminology (e.g., the names of unit fractions and the structure of a mathematical problem), which is a relatively closed approach because this questioning behavior confined students' responses to a particular answer.

NCTM (2000) warns that "[i]t is important to avoid a premature rush to impose formal mathematical language" (p. 63) on students and proposed that teachers must provide sufficient time and experiences to help students make a connection to formal mathematical language and to develop their communicative power with conventional mathematical terms. In the next vignette, Mary was delighted that Abby successfully named the correct mathematical terminology in the interaction while they were working on an equal-sharing task.

> ***Vignette 10***
> After Abby partitioned two cakes into five pieces and one into ten pieces with different-size pieces in all three cakes, Mary elaborated on the question in a more specific way by saying, "Let's just take one of the friends. How much of the cookie cake does one friend have? Remember how we talk about, when it's split into five pieces, what's that called?" and simultaneously pointed to the first cake that was partitioned into five pieces. Abby responded, "A fifth?" with an uncertain intonation. Then, Mary pointed to the cake that was divided into 10 pieces and tried to direct Abby to another mathematical term by saying, "So, remember how we have…these are broken up into what, if you have 10 pieces?" Abby answered, "Tenths." Mary was delighted at this response and replied "Right!" (Grade 1, SSMFE 8-4, Task 1).

In this case, Abby was able to recall the terms of unit fractions such as "fifth" after Mary reminded her by stating "Remember how we talk about, when it's split into five pieces, what's that called?" and did the same to address the term of "tenth." Although Abby correctly enunciated the mathematical terminologies, she did not demonstrate her understanding of fractions in the context as she solved this equal-sharing task by concluding that each friend would get three pieces even though the pieces were not properly divided into the same size.

According to Empson and Levi (2011), working with students on equal-sharing problems is a beneficial way to introduce fractions because those problems can help students "understand that a countable set of objects can also include fractions of an object" (p. 6). In this vignette, Mary was satisfied merely with Abby's ability to name the unit fractions, even though the goal of this task was to expose Abby to a setting in which she was supposed to create a fractional quantity that allowed her to relate it to a whole-number quantity. Mary's use of questions like "What would you call this?" and "Do you know its name?" while working on the topics of fractions, division, and other operations evinced her intent of eliciting correct mathematical terms. In Mary's final portfolio, she asserted that focusing on the use of correct language as a follow-up activity on the topic of fractions was the goal in her SSMFE. This goal affected Mary's entire SSMFE, and vignette 11 exhibits how Mary required Abby to identify the type of problem in her questions.

> ***Vignette 11***
> Abby solved the task, "There are 20 petals on a flower. If we wanted to split the petals into groups of 4, how many groups would there be?" by drawing 20 petals on paper and circling groups of 4. She then obtained the correct answer, "5 groups." Mary posed a follow-up question by asking, "Can you tell me what type of problem this is? Is this addition, subtraction, multiplication, or division? Do you know?" Abby was uncertain and replied, "Multiplication?" Mary provided more information related to her question by stating, "If we're divi-, if we're splitting 'em up into groups?" and led Abby to say the word with her by pronouncing, "Di-vi-sion." Once Mary heard what she expected, she wrapped up the task (Grade 1, SSMFE 8-4, Task 7).

For Mary, it seemed very important to maintain the goals of introducing mathematical language and helping Abby use correct mathematical terms, so she insisted on spending time waiting for a correct response to the question, "What is the mathematical word for how much each person would get?" (Mary, final portfolio, p. 12) throughout her SSMFE. However, the instructor's goal of the SSMFE was aimed at providing preservice teachers with an opportunity to understand their students' mathematical thinking, so it is not necessary for preservice teachers to overemphasize the correct use of mathematical terms during the SSMFE. In other words, it was acceptable to allow students to use their own language in their explanations to better express their thinking. Rushing students into using formal mathematical language prematurely could thwart the exploration of their existing knowledge and on-going thinking processes.

Stage III: Questioning Moves Used to Elicit Alternative Strategies

The SQM5 questions (37 out of 1027) were the least prevalent. For most preservice teachers, it was sufficient if their students were able to solve the tasks by employing just one strategy. However, because students' first choice of strategy might fail or the original task might require two different strategies, teachers need to understand the importance and timing of inviting alternative strategies from students. Normally, the preservice teachers in this study did not correct students' strategies and answers, even when those were flawed. Instead, they tried to ask questions to encourage students to share other strategies. In the following vignette, Tiffany successfully elicited a second strategy from her fourth-grader, Natasha, by employing a typical SQM5 question.

> ***Vignette 12***
> After posing the task, "If each T-Rex has 60 teeth, how many would 10 of them have in all?" Natasha solved the task by adding a 0 to her previous answer (120) but obtained a wrong answer of "A thousand two-hundred." Tiffany then asked, "Could you show me another way to solve this problem [60×10]? What's another way you could solve it? Other than using mental math and annexing the zeros?" Natasha immediately proposed her second strategy by saying "Box problem" (Grade 4, SSMFE 5, Task 3).

Even though the term "another way" generally elicited new strategies from the fourth-graders, proposing a different strategy might be difficult for some students, especially for first-graders. In some cases, the first-grade students merely rearranged numbers in the sentence, restated existing strategies with an inverse operation, or accepted what the teacher suggested passively. If students assume that finding another way implies conducting a different rearrangement of their original expression, one way teachers can respond is to clarify their intention and indicate that finding another way means creating a new strategy to solve the task. This phenomenon corresponds to what Lampert (1990) pointed out, "Doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question, mathematical truth is determined when the answer is ratified by the teacher" (p. 32). Finally, only five questions of the 37 SMQ5 posed by the preservice teachers focused on the comparison of strategies rather than encouraging a new strategy. Those comparison questions were mostly conducted by the teacher through initiating the analysis of similarities and differences among distinct strategies, so they are not discussed further in this paper.

Discussion

In this study, I sought not only to categorize preservice teachers' supportive questioning moves but also to characterize those moves in relation to students' reactions in Polya's problem-solving stages. On the basis of the data presented above, I suggest examining teachers' questioning moves in a teacher-student interaction context that encourages students to demonstrate their mathematical thinking with regard to how to interpret a mathematical task, devise a problem-solving plan, justify a strategy, and perform a computational procedure. In particular, the aspects of analysis should include the timing, format, and features of the questioning moves teachers enacted while working with different student populations.

Teachers' Flexibility in the Setup

The task posed in Stage I is the origin point of the entire sequence of a SSMFE session, and it should be understandable and solvable for students in order to accomplish the problem-solving mission successfully. If the task requires spontaneous revision in the SSMFE, to what extent the student participants can understand the task will depend on how capable teachers are of modifying the intended task and posing SQM1 questions appropriately. During the eight-week period, only 4 out of 11 preservice teachers tried to modify pre-planned tasks in setup to decrease the task's cognitive demand. However, the outcomes of such modifications were not positive experiences for them due to the resulting defects in the altered mathematical structure. This phenomenon could stem from preservice teachers' insufficient mathematical content knowledge (Ball, 1990) and might reinforce their beliefs about authority that they should follow the interview protocol provided by the methods course instructor (Cooney, Shealy, & Arvold, 1998; Mewborn, 1999). In addition, preservice teachers' open-ended questions did not always guarantee a fruitful opportunity for students to reason and communicate in Stage I. For example, most teacher participants generally repeated the original syntactic formulation as it was provided on the interview protocol and reenacted it the moment their students showed hesitation in the problem-solving activity. These inflexible SQM1 questions constituted an obstacle for first-grade students who have not sufficiently developed the taken-as-shared understanding of context (Cobb, Wood, Yackel, & McNeal, 1992), use of mathematizing language (Cobb, Gravemeijer, Yachel, McClain, & Whitenack, 1997), and mathematical fluency (Parks, 2010). Hence, teachers' flexibility in the set-up phase deserves attention.

Teachers' Limited Extent of Inquiry

Examining preservice teachers' questioning in relation to students' problem-solving performance also revealed that most teacher participants stopped their inquiry after receiving a plausible response from the students. For example, when the child replied to the question "How did you conceive that plan?" with "my teacher taught me in school," or responded to "Why did you know that strategy is correct?" with "because the keyword more means plus," preservice teachers' inquiries ceased. Given that the process of devising a plan at Stage II and the rationale behind it could reflect the problem solver's mathematical knowledge and logical reasoning, which are fundamental for implementing the strategy in Stage III, the limited extent of inquiry in

Stage II not only shortened the string of teacher-student interactions but failed to inform the teacher with a thorough explanation of the student's plan. As a result, it centered the conversation on computational steps exclusively.

Preservice teachers terminated the task for different reasons. In some cases, when student participants encountered unclear terminology in the question, they responded by saying "I don't know," and preservice teachers stopped questioning, presumably because they had been instructed not to compel the students to continue if they seemed uncomfortable. In other cases, preservice teachers simply accepted student responses and terminated the task without follow up, regardless of students' problem-solving performance. This phenomenon was detected in both Stage II and Stage III, and these instances did not allow the preservice teachers to develop or improve their questioning.

Teachers' Neglect of Unexamined Strategies

Some teacher participants neglected to examine students' valuable strategies in their questioning, regardless of whether the solution was correct or incorrect. This phenomenon was illustrated by vignette 9, in which Sandy's first strategy was completely ignored, even though Alice recorded that strategy in her notebook. The eight-week SSMFE was designed to enhance teacher participants' understanding of students' thinking in mathematical problem-solving activities. Questioning was one of the few assessment techniques preservice teachers could use to inquire into students' conceptions in different types of mathematical tasks. When teachers miss the opportunity to probe into students' responses and the strategies students develop, the learning opportunity preservice teachers might only gain once in their education programs will be in vain.

Conclusions

The teacher participants demonstrated their questioning in an incremental rather than an accumulated progression. That is, every time preservice teachers started a new task, they, except two of the 13 teacher participants, showed improvement in the selection of questions and the extent to which they had to discontinue a particular type of questioning. Through gauging their students' responses and attitudes in previous tasks, they strived to avoid the mistakes that were noticeably harmful. However, since questioning is an art, preservice teachers must experience unpredictable variables in related to their questioning (Dillon, 1983; Fitch, 1879; Wassermann, 1991), and accordingly, the accumulated progression of questioning was hindered. In other words, the student may decide to approach next task by applying another strategy that required different knowledge and skills, and based on what the student performed, the preservice teacher had to discard the questioning techniques that were inappropriate and modified former questions for a specific context.

This study provides insight into how preservice teachers' questioning performance was influenced by interacting with single student on designated mathematical tasks. I concluded that teacher questioning was a contextual, situated behavior and when being promised with an assisted-learning opportunity, they may successfully develop the knowledge and abilities to (a) effectively modify tasks, (b) efficiently extend inquiries, (c) precisely define probing questions, and (d) promptly detect and react to students' underdeveloped responses in their questioning. After learning the nature and features of teacher questioning, questioning should be identified as a teachable technique, in which teacher educators can assist preservice teachers' learning to question through providing a practice-based analytic framework in a field-based activity starting in the early stage of teacher education programs.

Limitations

In this observational study I documented and analyzed teacher questioning that occurred during an 8-week SSMFE in mathematics methods courses. My analysis was based on a short interaction for each pair of participants, which limited its comprehensiveness and applicability. Teacher questioning is a dynamic, moment-to-moment behavior, and teachers might improvise their questions in a flash of intuition or deliberately employ questions with a solid conceptual guideline, but I cannot determine which was the basis for teacher questions in this study. Because I did not conduct interviews with preservice teachers, there are likely many relevant issues that have not been addressed in the findings, such as the teachers' intention and self-reflection process with regard to asking particular questions. Furthermore, the perspective of the student participants in the interaction was not documented other than verbal responses audible on the tape and nonverbal reactions in the video. I did not analyze how students processed the posed questions, which I consider an important aspect of problem solving.

Implications

In closing, I will address two categories of implications are arising from this study: 1) implications for future research on teacher questioning and 2) implications for the design of field experiences and clinical practice. A further discussion of this trend is presented below.

Implications for future research

This study was based on thirteen pairs of preservice teachers and elementary school students' interactions on mathematical tasks. Through analyzing teachers' questioning moves in different stages of problem-solving activities, I have identified the influences of teachers' weakness on modifying the intended tasks, extending the inquiry, utilizing indicators, and following up on students' strategies in their questioning. All influences have close interactions with one another and play a critical role in each stage of mathematical problem solving. Thus, one approach to research about teacher questioning should continue to connect the enacted questioning to the problem-solving context. Future research can draw from the conceptual framework and coding schemes presented in this study, and then further consider employing (a) well-developed measures (Hill, Schilling, & Ball, 2004) to assess teacher participants' content knowledge for teaching elementary mathematics and (b) well-structured interviews to inquire about teachers' intentions behind questioning moves observed in their teaching practice.

The design of field experience

The fact that teacher participants enacted questioning less effectively while working with students on multi-topic mathematical tasks has implications for the design of field experience. NCATE (2008) emphasizes the importance of field experiences and clinical practice in teacher education programs. Well-designed field experiences can expose teacher candidates to an environment where they can "develop and demonstrate the knowledge, skills, and professional dispositions necessary to help all students learn" (p. 12). In general, scholars who focus on teacher questioning have employed a variety of tasks across multiple mathematical topics and paid close attention to how teachers demonstrate their mastery of questioning skills while inquiring about students' thinking. However, to develop advantageous mathematical field experiences, an important procedure is to scrutinize the components needed in teachers' clinical practice including the selection of student populations and the adoption of appropriate mathematical tasks. For example, when Mary adapted the multilevel-unit Lucky 5 Candy Factory task, which had unfamiliar terms and a complicated structure, to her first-grade student, the problem-solving failure frustrated not only the student but also the teacher. This finding not only reinforces DeCorte and Verschaffel's (1987) assertion that the semantic structure impacts first-graders' strategies for solving addition and subtraction word problems but also echoes Linville's (1976) conclusion that syntax and vocabulary levels impact the difficulty of verbal arithmetic problems. In other words, the relationships between the selected student population and the appropriateness of enacted tasks should be thoroughly examined in order to ascertain the appropriateness. Once the working environment and tasks are appropriately set and the student population is properly selected, preservice teachers can concentrate on practicing their questioning technique in the field experience.

The cultivation of teacher questioning

In this study, preservice teachers demonstrated they were good at asking questions that elicited specific information from students (i.e., the procedures used by students). For example, they asked what students understood about the task or what steps they took. However, they tended not to use their questioning to support the problem-solving process from a broader perspective. In particular, they did not effectively help students devise a plan for solving the problem, an essential bridge between understanding the task and carrying out solution strategies. They did not leverage the CGI frame they had being given to elicit students' ideas or extend them.

Clegg (1987) contended that "[t]eachers use questions more than any other activity. They are central to such strategies as recitation, review, discussion, inquiry, and problem solving" (p. 11). While accepting the definition of teaching as "[e]verything that teachers must do to support the learning of their students" (Ball, Thames, & Phelps, 2008, p. 395), I further value effective questioning moves as a professional technique with the characteristics of strategical and social acts that will not only support preservice teachers' learning to teach in the teacher education programs but also empower their learning in the post-training journey.

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APPENDIX D

Mathematical Tasks Within Five Topics in the SSMFE

1) Base-N
In the fourth-grade class: A Base-4 math task In the first-grade class: A Base-5 math task
Example task: The Lucky 5 Candy Factory task, used in the first-grade class At the Lucky 5 Candy factory, machines do the packaging. At this factory, there are the following types of packaging: individual candies, rolls of candies, boxes of candies, and crates of candies. The machines look for groups of 5. This means that every time a machine sees a 5, they put the candies into a container of the next size. So, every time they see 5 individual candies, the machines put them into a roll. Rolls can only hold 5 candies. Every time the machines see 5 rolls, they put them into a box. Boxes can only hold 5 rolls. And every time the machines see 5 boxes, they put them into a crate. Crates can only hold 5 boxes. Question #1: 9 candies were dumped onto a conveyer belt below. How many of each type of package should we expect to see after the machines finish packaging the candies?
2) Base-Ten/Place Value
In the fourth-grade class: Multi-digits operation tasks In the first-grade class: Two-digit adding the multiples of 10 tasks, and place value tasks
Example task: Base-ten subtraction problem, used in the fourth-grade class Nicky has 2001 points on his favorite video game. He forgets to save the game before turning it off, and he loses 956 points. How many points does he have now? Solve the problem in two ways.
3) Number Facts
Example tasks: ____ + ____ = 11 45=25+20=___ + 15 8+9=___ (naked number fact problem) 17+25=___ (naked number fact problem)
4) Fraction or equal-sharing problems
Example task: Equal-sharing problem, used in the first-grade class If I am having a party with 10 friends. We are splitting 3 cookie cakes. How much of the cookie cakes would each friend get?
5) CGI problems types

Join	*(Result Unknown)* Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have all together?	*(Change Unknown)* Connie has 5 marbles. How many more marbles does she need to have 13 marbles all together?	*(Start Unknown)* Connie had some marbles. Juan gave her 8 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	*(Result Unknown)* Connie had 13 marbles. She gave 8 to Juan. How many marbles does Connie have left?	*(Change Unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	*(Start Unknown)* Connie had some marbles. She gave 8 to Juan. Now she has 5 marbles left. How many marbles did Connie have to start with?
Part-Part-Whole	*(Whole Unknown)* Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?		*(Part Unknown)* Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?
Compare	*(Difference Unknown)* Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	*(Referent Unknown)* Connie has 13 marbles. She has 8 more marbles than Juan. How many marbles does Juan have?
Grouping	*(Multiplication)* Bart has 4 boxes of pencils. There are 6 pencils in each box. How many pencils does Bart have all together?	*(Measurement Division)* Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have?	*(Partitive Division)* Bart has 6 boxes of pencils with the same number of pencils in each box. All together he has 24 pencils. How many pencils are in each box?

Sources: https://elemath.hallco.org/web/wp-content/uploads/2015/06/CGI-Problem-Types.pdf

CHAPTER 5

THE FUNCTIONS AND CONSTRUCTIONS OF INTERACTIONAL TURNS BASED ON TEACHER QUESTIONING IN MATHEMATICAL PROBLEM SOLVING

Lee, Y.-J. To be submitted to the *Educational Studies in Mathematics*.

Abstract

This study illustrates the construction and functioning of teacher-student interactional turns in support of elementary students' mathematical problem solving. Data were collected in the form of observations, video recordings, and course assignments. Two frameworks were employed to describe the functions and patterns of interactional turns. In particular, the constructions of sequencing patterns of the functional moves are presented, and the features emerging from them are discussed. The findings indicate that the well-performing functional moves have potential to elicit multidimensional facets of students' mathematical thinking and yet may not be enacted competently in mathematical problem solving. How to effectively utilize student-produced discourse to inform teacher questioning strategies in early field experiences has important implications in learning to teach mathematics.

KEYWORDS: Teacher questioning, problem solving, mathematics methods courses, field experience

Introduction

Teacher-student interactions are ubiquitous in every educational setting, and the modes of interaction vary depending on the teacher's goals or the student's purpose for study. Among all interactional modes, the question-and-answer interaction is a prominent discourse pattern enacted in classrooms, and most teacher-student conversation is led by teacher questioning. To enhance the environment of teaching and learning mathematics, the National Council of Teachers of Mathematics (NCTM) (1991) suggests that teachers should orchestrate classroom discourse by: 1) "posing questions and tasks that elicit, engage, and challenge each student's thinking;" 2) "listening carefully to students' ideas;" and 3) "asking students to clarify and justify their ideas orally and in writing" (p. 35). Accordingly, teacher questioning plays a substantial role in teacher-student interactions, and effective mathematics teaching relies heavily on teachers' questioning techniques. I begin with a close look at the historical development of and prior research on questioning.

More than a century ago, Stevens (1912) asserted that "[t]he question and answer type of recitation, when rightly used, is more fruitful for the teaching process than … the topical recitation, the written lesson, or the lecture"[3] (p. 2). As enacting questioning in the classroom has become a prevalent pedagogical move, the affiliated condition in Stevens' assertion, "when rightly used," has gained relatively less attention than the enacting of questioning itself. Ross (1860) was the earliest educator to overtly discuss the method of teaching by questioning and to

[3] Stevens (1912) defined "the topical recitation as a method employed for repeating facts that are presented and systematized by someone else; the written lesson as a test of the facts a student possesses and, at best, his method of classifying them; and the lecture as a pouring in process" (p. 2).

distinguish the difference between catechetical questioning, the method of teaching by questioning, and examinatory questions, which were used for testing or proving. In historical records, questioning did not become a topic studied in education until Stevens (1912) systematically observed high school teachers' questioning across different subject matters including English, History, Science, Modern Language, and Mathematics. Her research represented a significant step toward the study of teacher questioning in many respects. For example, her research shone a light on how teachers' questions could be tallied and scrutinized in the context of teaching. Research on analyzing the frequency and categories of teachers' questions in classrooms has flourished since.

For much of the 20th century, studies focused on expert teachers and the quantitative and practical characteristics of questions employed in the classroom (Floyd, 1960; Gallagher & Aschner, 1963; Haynes, 1935; Houston, 1938; Hunkins, 1976; Morgan & Saxton, 1994; Moyer, 1967; Wilen, 1987), although researchers have increasingly seemed dissatisfied with merely categorizing teachers' questions as the importance of student participation has become more of a research trend. Hogg and Wilen (1976) suggested that "[s]tudents can be a practical, reliable source of feedback on teachers' performances" (p. 281). Researchers have urged that including students as sources in teacher questioning could reduce the view of teachers as the single authority in classrooms and increase student engagement during the interaction (Di Teodoro, Donders, Kemp-Davidson, Robertson, & Schuyler, 2011; Franke et al., 2009; Ralph, 1999; Sahin & Kulm, 2008; van Zee & Minstrel, 1997). Based on the historical evolution of teacher questioning as well as previous research on questioning in classrooms, I take the analytical perspective that factors the context of mathematical problems in the foreground and examines the outcomes of questions in teacher-student interactions. Accordingly, the primary purpose of this study was to explore the functions and constructions of teacher questioning in a mathematical problem-solving setting.

Conceptual Frameworks

Effective Mathematics Teaching and Questioning in Interactions

Teaching efficacy is a pursuit in the field of education. With regard to mathematics teaching, NCTM (2000) emphasized that "[e]ffective mathematics teaching requires understanding what students know and need to learn and then challenging and support them to learn it well" (p. 11). In practical applications, teaching includes a series of decisions regarding "what to teach, how to teach, who to call on, how fast the lesson should move, how to respond to a child, and so on" (Carpenter, Fennema, Franke, Levi, & Empson, 1999, p. 95). Therefore, effective mathematics teaching is affected by teachers' knowledge, beliefs and attitudes that are "stored as schemas in the mind of the teacher" (Ernest, 1989, p. 13) and should be considered as an evolutionary process.

To further illustrate the components that affect teachers' learning to teach, Ernest (1989) distinguished between teachers' thought processes (e.g., planning, interactive decision-making, and reflection) and thought structures (e.g., knowledge, beliefs, and attitudes) and then offered conceptual models to elucidate mathematics instruction. Ball and McDiarmid (1989) also pointed out that "teachers' conceptions of knowledge shape their practice – the kinds of questions they ask, the ideas they reinforce, the sorts of tasks they assign" (p. 2). That is, teachers' thought structures dominate their instructional practice, and teachers' performance could partially reflect their thought structures. Therefore, examining the modes of teachers' practical work, including verbal expression, the use of multiple representations, and nonverbal behavior, might provide insight into the knowledge, beliefs, and attitude teachers possess.

Questioning, as a strategic act in teaching (Green 1971), normally comprises a sequence of questions that aims to support a predetermined goal the initiator expects to achieve. Mewborn and Huberty (1999) suggested that, after initiating the first question in the sequence, teachers should listen carefully to students so that they can ask good follow-up questions. This observed three-part sequence of the “Question-Listen-Question” technique could be viewed as a structure analyzed from the teacher’s standpoint relative to the well-known “Initiation-Response-Evaluation” (IRE) pattern (Mehan, 1979) and the “Initiation-Response-Feedback” (IRF) sequence (Sinclair & Coulthard, 1975). In addition, Mewborn and Huberty (1999) identified two challenges observed in the classroom including (1) “dealing with incorrect or incomplete solutions” and (2) “finding time to use this type [effective initial and follow-up] of questions” (p. 243).

Many researchers have noted that teachers’ experiences working with students can play a role in teacher questioning (Hyman, 1979; Sahin & Kulm, 2008; Tienken, Goldberg, & DiRocco, 2009). For example, Fitch (1879) emphasized that questioning is a practical matter and usually occurs in an environment where teacher-student dialogue plays a more critical role than teacher monologues do. Furthermore, enacting questioning is considered a complicated undertaking because it demands effort to masterfully accomplish the series of asking an initial question, listening to students, and providing proper feedback (or asking follow-up questions). Accordingly, Martino and Maher (1999) asserted that “[t]he art of questioning may take years to develop for it requires an in-depth knowledge of both mathematics and children’s learning of mathematics” (p. 54). As a result, this mission becomes especially difficult for novice teachers, who have relatively limited experience working with students. Hence, I argue that without knowing the strengths and weakness in novice teachers' questioning, providing any interventions for the purpose of cultivating questioning techniques could be impractical.

Although questioning ability could develop along with teachers' experience of enacting it, every teacher will gain different experiences in learning how to ask questions that depend largely on their respective teaching environment and disposition toward mathematics. In addition, complex psychological elements also play a role in this learning process, such as the questioner's expectations and prediction of the responses and the respondent's cognitive behavior as initiated by the questions. Therefore, one should not expect that teachers will ultimately become experts on questioning at the same satisfactory level while allowing this technique to self-evolve. This complexity is also the reason why helping teachers systematically develop effective questioning techniques—including what question to ask, how to ask it, how the respondent receives it and replies, and how the answer contributes to the interaction—has become indispensable in teacher preparation programs.

Cognitively Guided Instruction (CGI) Context

Cognitively Guided Instruction (CGI) professional development program originally proposed by three researchers—Thomas Carpenter, Elizabeth Fennema, and Penelope Peterson—at the Wisconsin Research and Development Center in 1985 for the purpose of helping teachers utilize research-based knowledge on children's mathematical thinking to make informed instructional decisions in the classroom (Carpenter & Fennema, 1992). The CGI program was grounded on a series of studies on young children's addition and subtraction concept and skills (Carpenter, 1985; Carpenter & Moser, 1984; Riley, Greeno, & Heller, 1983). Along with the analysis of children's solution strategies, CGI researchers identified 11 types of addition and subtraction word problems distinguished by the involvement of action or relationships with different location of unknown quantity in each type of problems (Carpenter et al., 1999).

Based on the problem types and solution strategies, various research stemmed from the CGI program including: (1) teachers' knowledge of students' solution strategies, problem-solving ability, and students' beliefs about learning (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Peterson, Carpenter, & Fennema, 1989); (2) teachers' pedagogical content knowledge (Carpenter, Fennema, Peterson, & Carey, 1988); and (3) teachers' content beliefs in mathematics (Peterson, Fennema, Carpenter, & Loef, 1989). CGI researchers believed that the understanding of the development of children's mathematical thinking would result in changes in teachers' beliefs and practices and then these changes would be ultimately reflected in students' mathematical learning. In short, the main CGI tenet emphasizes that "instruction should be based on what each child knows, [and] it is necessary to continually assess not only whether a learner can solve a particular problem but also how the learner solves the problem" (Carpenter et al., 1989, p. 505). In the past three decades, the CGI program strived to assist teachers in developing the relevant knowledge to evaluate students' knowledge via the guidance of evidence-based research, and eventually help teachers improve their classroom instruction.

The tasks analyzed in this study centered on addition, subtraction, multiplication, and division word problems due to the participating students' grade level. For addition and subtraction problems, four basic classes of problems can be identified: (a) join problems, (b) separate problems, (c) part-part-whole problems, and (d) comparison problems. Join problems are similar to separate problems, and both of them involve a direct or implied action over time with the increase or decrease of the initial quantity in the problems. Part-part-whole problems involve a static relationship that contains a particular set (e.g., marbles) and its disjoint subsets (e.g., red and blue), whereas compare problems involve the comparison of two distinct, disjoint sets (e.g., Jessica's marbles and Carol's marbles).

Although the basic structure involving actions and relations within each class of problems remains the same, the unknown quantity among problems within a class could vary. There are regularly three distinct quantities in each problem, and any one of which can be the unknown. For joining and separating action problems, there are three distinct types of problems that can be generated by varying the unknown: result unknown, change unknown, and start unknown; for part-part-whole problems, there are whole unknown and part unknown problems; and for compare problems, the three distinct types of problems are difference unknown, compare quantity unknown, and referent unknown problems.

The analysis of addition and subtraction in CGI studies provides a framework that can be extended to multiplication and division (Carpenter et al., 1999). The initial discussion of multiplication and division problems in CGI considers the problems in "which collections can be grouped or partitioned into equivalent groups with no remainders" (Carpenter et al., 1999, p. 33). Three basic classes of problems can be identified as multiplication (product unknown), measurement division (number of sets unknown), and partitive division (number of elements in each set unknown), depending on which quantity in the word problem serves as the unknown in the problem (Kouba, 1989). In detail, multiplication problems provide the number of groups and the number of objects in each group, and the unknown is the total number of objects. Measurement Division problems merely have the unknown number of groups, and Partitive Division problems possess an unknown number of objects in each group.

The distinctions among different types of word problems were reflected in students' development of solution strategies in the process of their problem solving. When solving join and separate problems, children may start with modeling the action and relations in the word problems and the location of the unknow quantity played a role in students' selecting of strategies. For example, the joining all strategy was conducted to deal with the result or whole unknown problems and the joining to strategy was for join change unknown problems. When students perceived the needlessness of physically modeling, they began to develop more abstract strategies—the counting strategies, such as counting on from the first, counting on from larger, counting on to, counting down, and counting down to. Lastly, the number facts strategies might be performed to replace the strategies that focus on manipulating counting sequence (Carpenter et al., 1999).

While solving multiplication and measurement division problems, students' solution strategies could be advanced from using the counting-on and counting-down strategies experienced in join and separate problems to counting by multiples (e.g., counting by doubles, by fives, and by tens). Furthermore, multiplication and measurement division problems could also foster the development of base-ten system understanding when they involve the amount of ten in each unit (e.g., the number of cookies in each bag is ten) and multidigit algorithms. To distinguish the two types of division, it is worthwhile to mention that measurement division problems involve the capacity of measuring quantity from the original set, and partitive division problems benefit the development of fractions learning due to their property of equal distribution.

The knowledge derived from CGI-related studies provides teachers a foundation to (1) better understand their students' problem-solving strategies that related distinctions between types of word problems, (2) adapt their own instructional methodologies to match students' learning capacity, (3) reinforce their beliefs about instructing upon students' existing knowledge and (4) experience the role of knowledge facilitator instead of knowledge transmitter in students' learning (Carpenter et al., 1988; Carpenter & Fennema, 1992; Peterson et al., 1989). Carpenter (1988) considered the process of teaching in the CGI context as problem solving. The CGI approach valued the opportunity for teachers to implement their knowledge and guaranteed teachers the time they need to reflect on their practices. As a result, the CGI teachers spent more time on enhancing students' problem-solving ability, allowed students to flexibly use different strategies, and listened to students' solution processes attentively.

While engaging in the CGI approach, teachers had a better understanding of students' mathematical thinking that allowed them "to interpret students' responses and modify questioning or instruction accordingly" (Carpenter & Fennema, 1992, p. 462). Furthermore, Lindquist (2015) concluded that "central to instruction in CGI is the art of questioning" (p. xvi). Accordingly, I situated the investigation of preservice teachers' questioning in the CGI context based on the aforementioned features observed in CGI classrooms.

Categorizing Interactional Patterns

Carpenter, Fennema, Franke, Levi, & Empson (2015) advocated that "interacting with children is essential to learn about children's mathematical thinking" (p. 6). From the perspective of engaging students in mathematics, Battey, Neal, and Hunsdon (2018) emphasized that how teachers handle classroom interactions "plays a role in how all students experience mathematics (p. 433). Accordingly, this section outlines some interactional patterns and elaborates their functions and characteristics in teacher-student interactions.

In his analysis of the anatomy of individual teacher-student exchanges, Guszak (1967) developed the concept of the Question-Response Unit (QRU). This pattern of questioner-respondent exchanges generally develops in a sequence of dialogues and consists of several dynamic sub-chains in discourse. In the mathematics classroom, teacher-student interactions could build on verbal exchanges, along with nonverbal interactions such as gesturing and written responses. Within verbal exchanges, the unit of dialogue is regularly circumscribed as a complete statement made by a speaker, and this is particularly obvious in the analysis of questioning. For example, Mehan (1979a) identified the "Initiation-Response-Evaluation" (IRE) as "the most recurrent pattern" (p. 72) observed in classrooms, in which the questioner, normally the teacher, initiates a question or inquiry followed by the respondent's (normally the student) reply, and then the whole sequence ends with the teacher's evaluation or feedback. Another similar triad is known as the "Initiation-Response-Feedback" (IRF) sequence (Sinclair & Coulthard, 1975), which describes teachers' questions, whether questions were answered and by whom, and the types of feedback given in response to student responses (Smith, Hardman, Wall, & Mroz, 2004). However, this Question-Response type of interactional patterns merely identifies the turn taking in the dialogues and it could be imperative to understand how teachers could follow up students' ideas in their interactions.

van Zee and Minstrell (1997) examined how an experienced science teacher used questioning to guide student thinking and defined *a reflective toss*, a particular kind of question that enabled the teacher to encourage students to elaborate their thinking. In this structure, the role of the teacher included catching the meaning of the student's statement and then throwing responsibility for thinking back to the student(s). This reflective toss not only exemplified the concept of "listening carefully and asking good follow-up questions," but also fixed the purposes of the teacher's follow-up questions on (a) engaging students in a proposed method, (b) beginning the refinement process by clarifying a discussed method, and (c) evaluating an alternative method that might arise as a byproduct of the discussion. That is, the move repeatedly redirected the focal point of the whole discussion back to the idea students proposed at the moment or their on-going thinking. In particular, the reflective toss also successfully invited other students to help elaborate their peer's idea when needed. The reflective toss was situated in one experienced teacher's expertise to interact with his students, and it is necessary to investigate other interactional patterns.

To further investigate the contribution of the single statement in the exchanges, Hogan, Nastasi, and Pressley (1999) identified three interactional patterns—consensual, responsive, and elaborative—that emerged in peer and teacher-guided discussion while scrutinizing 32 eighth-grade students' reasoning complexity in science classrooms. These patterns were different from the IRE and IRF because the contribution of the follow-up statements was factored in the interactions. To take a one-on-one interaction as an example, the first speaker initiated the conversation, and this initiation could bring up three potential types of interaction patterns. The first type was considered as consensual when one of the participants contributed substantive responses to the interaction and the other served as a "minimally verbally active audience" (Hogan, Nastasi, & Pressley, 1999, p. 393). While enacting responsive interaction sequences, both participants equally contributed substantive responses to the interaction and could freely express their ideas on the topic discussed. The elaborative pattern occurred when both participants not only contributed substantive responses but also co-constructed additions, made corrections, or offered a counterargument based on any prior statement (Hogan, Nastasi, & Pressley, 1999).

In this paper, I particularly analyzed the interactional patterns constructed by teacher and student in CGI problem-solving settings through an integrated framework. By describing teacher-student interactions I hope to illuminate ways that teacher preparation and professional development programs can provide appropriate assistance to cultivate preservice and inservice teachers' questioning techniques.

Methods

Study Background and Participants

The setting for the study was a field-based activity named *the Single Student Mathematics Field Experience* (SSMFE), which was embedded in the first mathematics methods course in the teacher education program for early childhood majors (certification Pre-K–5) at a Northeast Georgia university. Although conducting a single student interview is quite different from teaching a class of students, prior research has contended that conducting one-on-one interviews with students in early field experiences could benefit preservice teachers' learning to teach (Jacobs & Ambrose, 2008; Weiland, Hudson, & Amador, 2014).

The one-on-one settings in SSMFE were designed to 1) offer participating teachers a practical, structured opportunity to develop their questioning strategies and 2) allow them to actively listen for and reflect on students' responses to their questions. In the SSMFE interviews, preservice teachers concentrated on understanding what their students know and are able to do in solving arithmetic problems. During the activity, students might fail to respond to teachers' questions or present unexpected solutions, and this could be considered as a valuable opportunity for preservice teacher to learn how to employ follow-up questions to probe student thinking. This setting appropriately reflects Moyer and Milewicz's (2002) assertion that learning to ask a good question requires "shifting the practices and beliefs of the individuals engaged in those interactions" (pp. 295-296). Therefore, the SSMFE is a well-designed setting in which to investigate teacher questioning because preservice teachers could enact and reflect on questioning to learn student thinking in problem solving (Chamberlin & Chamberlin, 2010; Nicol, 1998; Ralph, 1999).

The teacher participants in this study were from two mathematics methods courses: One cohort ($n = 3$) participated in the Fall 2014 study; the other cohort ($n = 3$) participated in the Spring 2015 study. The 6 teacher participants were a subset of 13 preservice teachers who participated in a dissertation study and they were the only teacher participants who conducted whole-number arithmetic word problems including addition, subtraction, multiplication, and division in the CGI context. In the method class, preservice teachers were exposed to children's strategies for all problem types including addition, subtraction, multiplication, and division. In their SSMFE interview, preservice teachers were able to annotate students' strategies—modeling, counting, or number facts—during the session, and almost all teacher participants provided detailed description about what students did while they solved the tasks posed in the interview.

The teacher participants were in their junior year at the University and had completed at least two mathematics content courses and other mandatory education courses (e.g., investigating critical and contemporary issues in education, exploring socio-cultural perspectives on diversity, and exploring learning and teaching) for early childhood education majors. Among them, 4 participants were White females and 2 participants were Asian females. The student participants were selected by convenience sampling (Patton, 2002) and consisted of 3 fourth-grade and 3 first-grade students at a public elementary school.

Data Collection

The participating teachers conducted a one-on-one interview with a single student once a week for eight weeks during a semester, and each SSMFE session lasted for 30 to 45 minutes, depending on the students' problem-solving performance. In order to investigate how preservice teachers adapted intended tasks from the interview protocols and enacted spontaneous questions in the CGI settings, I videotaped only one pair of teacher and student participants per week. To increase the diversity of data collected for this study, my data set consisted of only one interview from per pair of participants. Therefore, I totally collected 6 SSMFE interview sessions from 6 different pairs of participants.

The focus of each interview was to elicit the student's mathematical thinking, explanations, and problem-solving strategies through enacting questioning around tasks conducted in the CGI settings. The majority of the enacted tasks were part of the interview protocol compiled by the course instructors and comprised types of whole-number arithmetic tasks from the CGI problem types as shown in Section 2.2. The rest of the tasks included multi-step arithmetic problems and problems that focused on the properties of operation and number sense.

All the interviews were videotaped by two cameras—one filming the preservice teacher and the other the student—in order to catch the moment-to-moment dynamics in the interactions. Other data sources included the children's written work; preservice teachers' field notes, debriefing form, course assignments, and the SSMFE final portfolio; and the researcher's analytic notes taken while viewing the video recordings. All video recordings were transcribed to enhance the accuracy of analyses.

Analyses

The detailed transcriptions of spoken discourse along with the descriptions of nonverbal moves from the six videotaped interviews were the primary data. During the analytic procedure, I primarily conducted framework analysis (Richie & Spencer, 1994) to examine the trends and details along with quantitative analysis to depict a synoptic view of the relative distribution of different types of moves. An interactional turn was the unit of analysis for the codes in which "a turn began when a person took the floor in a conversation and ended when another person took the floor" (Hogan, Nastasi, & Pressley, 1999, p. 387). It is important to note that first, an interactional turn could be initiated by either teacher or student. For example, in some conversations, the student was the one who initiated a move with a specific function, and the teacher might merely provide encouragement or acknowledgement without any substantial contribution to the content of the conversation. Second, an interactional turn might not have any function. Taking an opening conversation in the interview for an instance, the teacher might greet the student with "how are you today?" or "I like your dress." These turns were considered interactional turns in the conversation, but they did not play a role in my analysis of the function of the turns used in the SSMFE.

In the analysis, the framework employed in this study stemmed from three resources: 1) Jacobs and Empson's (2016) framework of teaching moves; 2) Hogan, Nastasi, and Pressley's (1999) interaction patterns; and 3) the pilot analysis of the data. In a study of teaching moves in one-on-one problem-solving interviews, Jacobs and Ambrose (2008) identified four categories of teachers' supporting moves and four categories of extending moves. After further research, Jacobs and Empson (2016) proposed a revised framework comprising five categories: (1) ensuring the child is making sense of the story problem, (2) exploring details of the child's existing strategies, (3) encouraging the child to consider other strategies, (4) connecting the child's thinking to symbolic notation, and (5) posing a related problem linked to what the child understands. These categories of teaching moves served as a basis to classify the types of functional moves in this study. Below, the coding process for the function of moves is first presented and the categorization of interactional patterns follows.

The functions of moves. Transcripts of each interviews were broken into tasks, and each task started with a Task Posing (TP) turn[4]. All TP turns were enacted by the teachers and were immediately followed by one or several interactional turns, that could be verbally or nonverbally, from the student or the teacher. To further analyzing the functions of interactional turns, I conceptualized a functional move as a sequence of interactional turns containing participants' verbal or visual action that has a particular function, and every functional move was initiated and categorized based on the function of the first observed interactional turn. That is, a functional move consisted of a sequence of interactional turns and was categorized into one of the seven functional categories depending on its initial turn in the sequence. The term "functional moves" emphasized the functions of the moves.

Through repeated readings of transcripts, seven main categories of functional moves emerged (see Table 5-1). A functional move could be initiated by either the teacher or the student with a verbal turn (e.g., asking a question) or visual action (e.g., directly solving the task

[4] Task Posing was a turn because it was normally initiated by the teacher and executed by only the teacher.

on paper). To illustrate the functions of the moves from the teacher and the student respectively, two sample initial turns were provided in each category.

Table 5-1

The Categories of Functional Categories

Category of Functional Move	Description	Sample Initial Turns
Task Clarification (TC)	Clarify or seek the given information in a task	T: Do you want me to read the task again? or S: Can you read the task again?
Plan Elicitation (PE)	Elicit or produce the initial plan	T: What will you do [to solve this task]? or S: This should be a multiplication [instead of a division].
Procedural Understanding (PU)	Explore or explain the procedure involved	T: What did you just do? or S: 2 plus 5 is 7 and 10 plus 10 is 20, so 12 plus 15 is 27.
Making Connections (MC)	Make connections between the answer and the original task	T: What does 12 mean in the task? or S: The answer 120 means the total number of teeth two dinosaurs have.
Rationale Behind a Strategy (RA)	Inquire about or elaborate on the rationale behind the proposed strategy	T: Why did you do multiplication? or S: I know it is multiplication because I need to find more not less.
Math Terminology (MT)	Elicit or give correct math terminology	T: What do you call that piece? or S: If a cookie was cut into 4 pieces, one piece is a quarter.
Alternative Strategy (AS)	Elicit or propose an alternative strategy	T: What is another way you can solve this task? or S: I want to solve it by using cubes this time.

The patterns of functional moves. The pattern of functional moves was categorized based on the types of interaction patterns proposed by Hogan, Nastasi, and Pressley's (1999) (see section 2.3). I coded the initial turn in a functional move by answering two questions: 1) who (the teacher or the student) initiated a move? and 2) what function did the first interactional turn demonstrate? After the initiation of a move was identified, the remainder of the interactional moves consisted of either a single response or a subset of recurring pairs of the response and follow-up turns. The responses and follow-up turns were then identified as a) consensual, b) responsive, or c) elaborative patterns. Because a functional move might comprise several pairs of the response and follow-up turns, it is possible that more than one type of pattern occurred in one move. Under this circumstance, the criterion to determine the type was to prioritize the pattern from elaborative, responsive, to consensual. As a result, there were six categories of interactional patterns emerged in this coding process as shown in Table 5-2, and Figure 5-1 shows the level of analysis in one SSMFE interview in this study.

The analysis of interactional patterns helped me identify the features and contributions of the turns in the conversation between the initiator and respondent at three analytic levels. First, it is imperative to investigate how a single functional move was co-constructed by both participating teacher and student within mathematical problem-solving activities. To reveal this, I explored who took the first move after a task was posed, which function came first and occurred predominantly in the stream of conversation, and under what circumstance the functional move was terminated. Second, an SSMFE interview consisted of several tasks, so it was useful to analyze and compare different functional moves the teacher tended to employ in one interview session. In so doing, the features situated in similar or different tasks from the same teacher could be detected. Last, when the analysis was expanded to all six pairs of participants, a general description with regard to the functional moves with interactional patterns enriched the understanding of how each interactional turns was functioning and constructed in the SSMFE.

Table 5-2

The Categories of Interactional Patterns

Category of Interactional Patterns	Description
Teacher-initiated Nonresponse	The initial interactional turn is initiated by the teacher, and the student replies with nonresponse.
Teacher-initiated Consensual reaction	The initial interactional turn in a functional move is initiated by the teacher, and the student replies with only consensual responses, such as "yeah" or "uh-huh."
Teacher-initiated Responsive reaction	The initial interactional turn in a functional move is initiated by the teacher, and the student replies with responsive responses, such as an answer to a question and an expression of personal ideas
Teacher-initiated Elaborative reaction	The initial interactional turn in a functional move is initiated by the teacher, and the student replies with elaborative responses, such as a co-constructed additions or a counterarguments based on a prior statement.
Student-initiated Nonresponse	The initial interactional turn is initiated by the student, and the teacher replies with nonresponse.
Student-initiated Consensual reaction	The initial interactional turn in a functional move is initiated by the student, and the teacher replies with nonresponse or only consensual responses, such as "yeah" or "uh-huh."
Student-initiated Responsive reaction	The initial interactional turn in a functional move is initiated by the student, and the teacher replies with responsive responses, such as an answer to a question and an expression of personal ideas.
Student-initiated Elaborative reaction	The initial interactional turn in a functional move is initiated by the student, and the teacher replies with elaborative responses, such as a co-constructed additions or a counterarguments based on a prior statement.

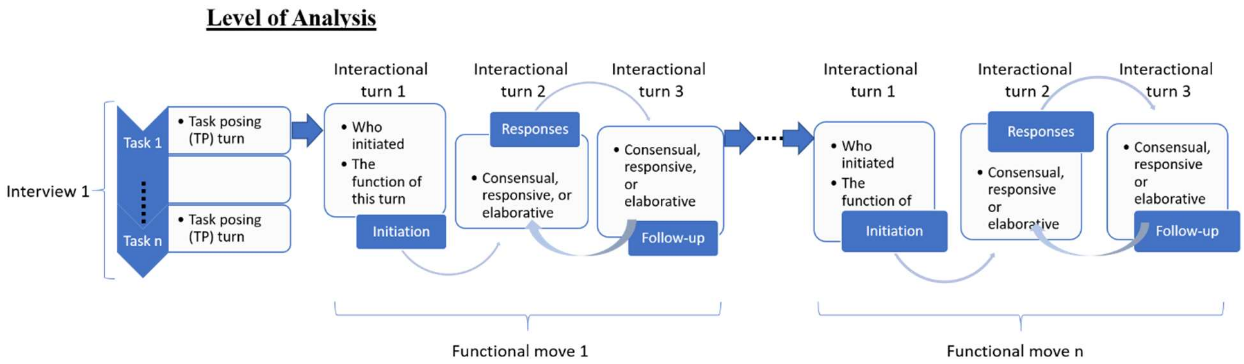

Figure 5-1. Level of analysis of one example SSMFE interview session.

To describe features of functional moves, I employed two types of ratios (in the format of a decimal) to present the descriptive statistics: 1) the "move-count ratio" was defined as the ratio of the number of a particular type of moves to the total number of the employed functional moves in a single task, and 2) the "word-count ratio" was defined as the ratio of the word counts of a particular type of moves to the word counts of all the moves in one task. The move-count ratio reflected how often a particular type of move was employed, whereas the word-count ratio represented the extent to which the move was expanded in a task. For example, the measurement division task "Each dinosaur is given three Hawaiian lei necklaces as they walk in the door. If 783 leis are given out, how many dinosaurs were in attendance?" with the structure of $783 \div 3$ was conducted in week 6 with a fourth grader (see Figure 5-2), and the procedural understanding (PU) move occurred once in task 5, so both the number of times it was employed, and the word counts it occupied across the whole task were counted. Specifically, the move-count ratio of the PU moves to all functional moves in this task was calculated by the formula of "the number of PU moves (= 1) divided by the number of total functional moves (= 4)" to gain 0.25 for this task. Although the move-count ratio of the PU moves was not high, the word-count ratio of the

PU moves was computed as $549 \div 611$, i.e., approximately 0.9 in this task. Every task conducted in the SSMFE was considered as an individual scenario. The number of a particular type of moves (e.g., PU moves) used in all tasks across the SSMFE sessions and the extent to which they were enacted were taken into consideration in the analysis.

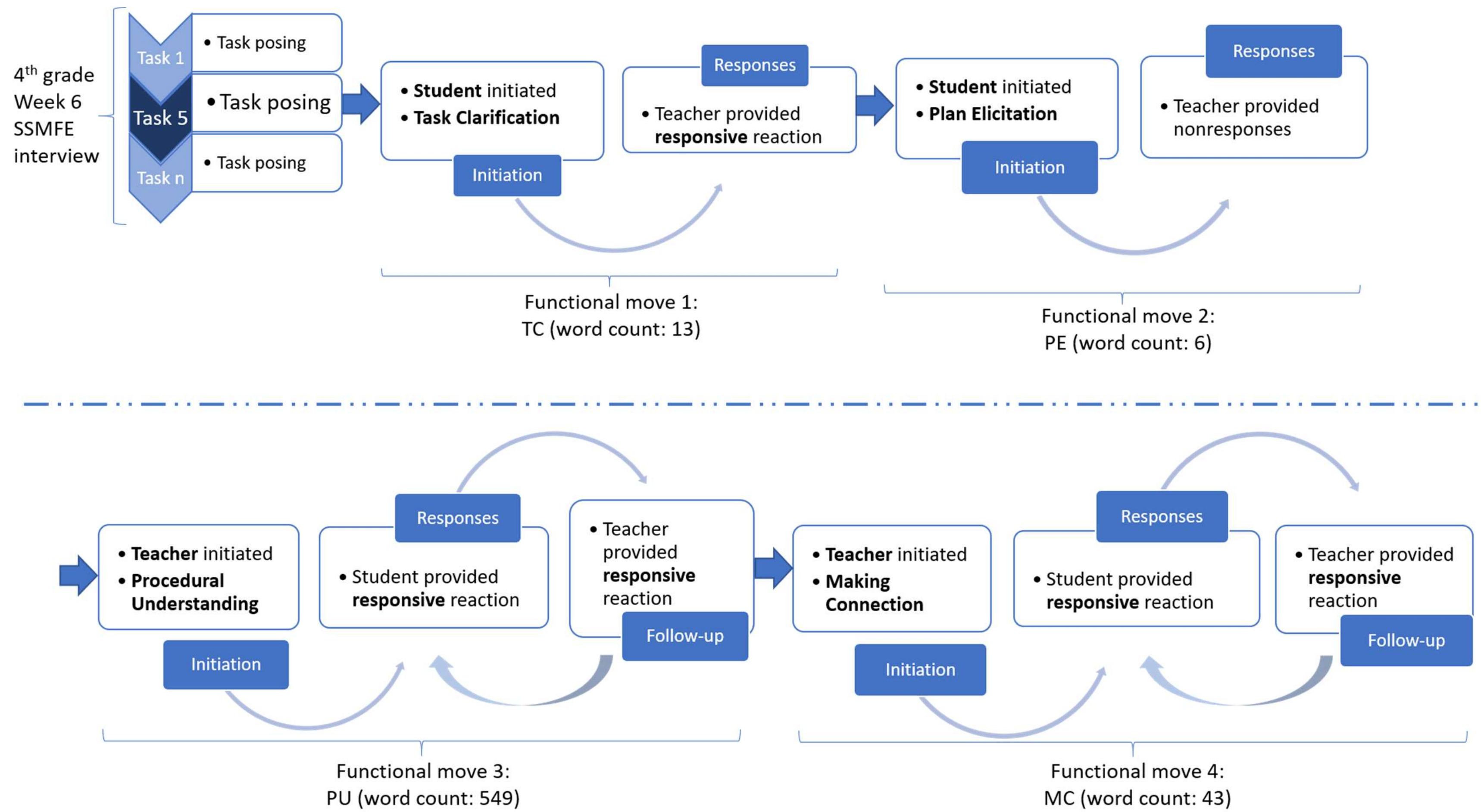

Figure 5-2. The functional moves employed in Task 5 with the structure of 783 ÷ 3 (week 6 in the fourth grade).

Findings and Discussion

I first present an overview of the number of functional moves by category and then discuss the enactment of Procedural Understanding (PU) moves and compare them with Task Clarification (TC) and Alternative Strategy (AS) moves in terms of move-count ratio, word-count ratio, and interactional pattern. Next, I display representative function-switch sequences used in the SSMFE and relate the functional moves to the CGI context.

The Predominance of the Procedural Understanding (PU) Move

Throughout six SSMFE sessions, 281 functional moves were identified in 44 main tasks conducted in the CGI settings. The frequency distribution of the 281 functional moves is shown in Figure 5-3, and three functional categories—Procedural Understanding (PU), Task Clarification (TC), and Alternative Strategy (AS)—comprised more than 10% of the moves, individually, used in SSMFE. Given the focus on eliciting students' mathematical thinking emphasized in the mathematics methods courses, it is legitimate that 41% of the 281 functional moves were used to explore students' procedural understanding.

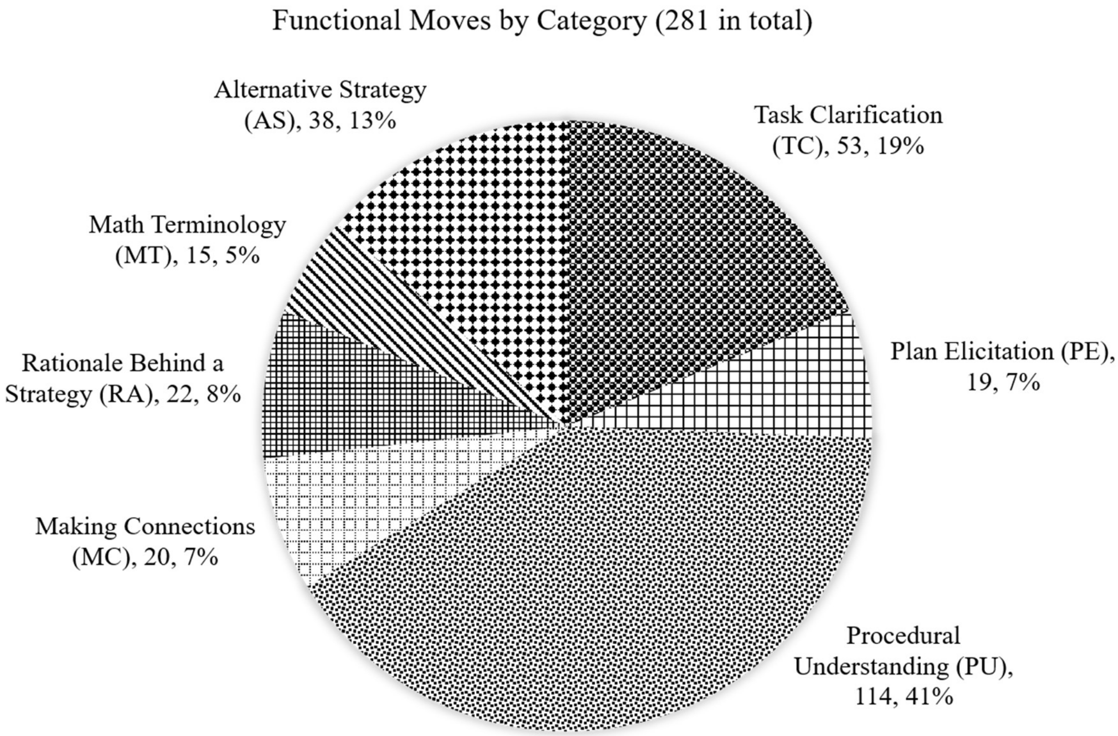

Figure 5-3. Frequency and percentage of functional moves by category.

In addition to the predominance of the PU moves, the significance of the employment of the PU moves emerged under scrutiny in every task by comparison with TC moves and AS moves. To paint an overall portrait of how these moves were functioning in each task, the results of "move-count ratio" and "the word-count ratio" of the three types of functional moves are first presented (see Figure 5-4).

The move-count ratio of PU moves was in the range of 0.22 to 0.71 with an outlier 1. The low boundary 0.22 revealed that participating teachers employed at least one PU move in every task in order to inquire about students' mathematical thinking on the computational procedures. The outlier 1 reported that the PU move was the only type of functional moves used in one particular task. Compared with the PU moves, TC moves and AS moves had lower boundary 0 in both ratios, so they were not employed in some tasks, indicating that the clarification of the given information and alternative strategies were not employed to accomplish the task. In addition, the word-count ratio of PU moves ranged similarly to its move-count ratio (0.19 to 1), and the central 50% of measured word-count ratios of PU moves ranged from 0.4 to 0.77, that were higher than the word-count ratios in TC moves and AS moves. That is, the participating teachers and students had longer conversations while addressing procedural understanding.

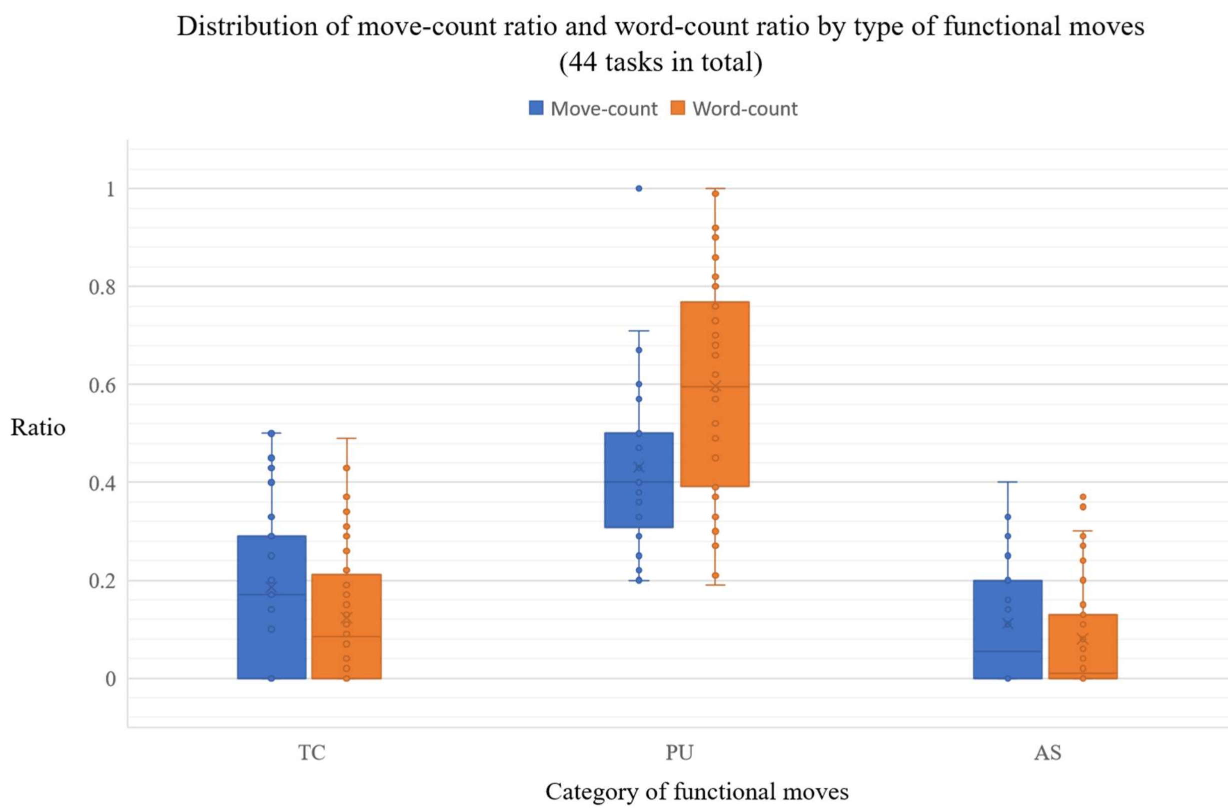

Figure 5-4. Distribution of word-count ratio and move-count ratio of TC, PU, and AS moves.

In terms of the interaction patterns (Hogan, Nastasi, & Pressley, 1999), 85% of the PU moves were initiated by students instead of by teachers, likely because the participating students were used to solving the problem immediately after the teachers posed it even though they might not completely understand the task. In a further pattern analysis, 25% of the student-initiated PU moves were categorized as "Consensual pattern," in which the responders either simply agreed with the statement or actively accepted what was said to keep the conversation continuing, and 19% of the student-initiated PU moves received no responses. Regardless of whether the moves were initiated by teachers or by students, more than 61% of the PU moves were identified as Responsive pattern, in which the responders contributed to the conversations with questions, comments, or concerns regarding the initiated content.

With regard to the length of the PU moves, the longest interaction consisted of more than 500 words[5], and it occurred in a session with a fourth-grade student at the sixth week in the SSMFE. A measurement division task with the structure of $783 \div 3$ was posed (Week 6, Task 5, see Figure 5-2). This task began with a student-initiated TC move after the teacher posed the task, and then the student immediately proposed his plan to solve the problem by saying "partial quotient," which was categorized as a PE move. The teacher then initiated the PU move by asking the question "What you are doing?" and the student was patient in introducing the partial quotient when he realized that his teacher had not learned this method. During the time the student demonstrated how to find a partial quotient correctly, the teacher simply replied "okay" several times until the student had accomplished two sets of 3×100 in the partial quotient procedure. The first half part of this loquacious PU move was interrupted by an off-track conversation that was excluded from the word counts of this PU move. To refocus the student's concentration on the task, the teacher then initiated the second part of the PU conversation by asking a typical question, "What are you thinking now?" and the student then explained his

[5] These are the word counts on the transcript that includes some explanations of nonverbal action, so the real verbal expression might be less than this number.

computational procedures that led to the answer "261." This task was concluded by a MC move by the question "Can you remember what the problem was?" in which the teacher tried to help the student make a connection between what he accomplished and what his answer represented within the scenario, but the student merely summarized the procedural steps by using a mathematical equation $783 \div 3 = 261$ to wind up this task.

The Function-switch Sequences

In all six SSMFE interviews, 48 tasks were posed, including four sub-tasks that were embedded in the original problem theme and context of three tasks, and the middle 50% of the number of functional types ranged from 2 to 4 types in the conversations (out of 7 types). In this section, the task that comprised most functional moves will be presented first.

The more different types of functional moves preservice teachers employed in practices, the more opportunities they elicited different cognitive processes from students. For example, the PU moves evoked students' explanations of computational procedure while the RA moves educed students' reasoning. Therefore, the number of functions in one task could be considered as an indicator of the variety of functions. The task that employed most distinct functions was conducted in Week 1 with a fourth grader. A partitive division task "Seven friends want to share 63 candies equally. How many candies should each friend get?" with the structure of $63 \div 7$ was posed and all seven types of functions were performed during this interview (see Figure 5-5).

The horizontal axis represents the order of functional moves in use throughout the task. After the teacher posed the task, the first move was a student-initiated PU move in which the student wrote down 63×7 on the paper and explicated the procedure of how he obtained the answer "441" after the teacher posed the task. In this initial PU move, the student did not realize that the strategy he used to solve this problem was not correct, and during this move, the teacher asked the question “Can you explain to me what you were doing over here?” that only elicited more explanation of his procedural understanding in solving 63×7. Then the teacher employed an MC move to help the student reexamine the executed procedure within the context by asking “Can you explain what 63 is in the problem?” and the student failed. Hence, the teacher offered to repeat the task, which was the turning point at which the student proposed a correct strategy (PE move) by saying “you have to divide, instead of multiply” as the fourth move in the interview. Instead of letting the student perform the correct strategy directly, the teacher prompted for the rationale behind this proposed strategy (RA move) by asking “Why do you say that?” and then the student elaborated on his strategy with the given numbers and concluded that 7 times 9 would be equal to 63 but struggled to show this process in writing. The teacher then immediately suggested “using the cubes,” and in the rest of the interview, she initiated two more AS moves by asking the student to try “counting the cubes” and “writing the numbers out;” one MK move to discuss the property of even and odd numbers; two RA moves to inquire about the rationale behind the strategy; four MC moves to invite the meaning of the produced numbers in the context; and offered to read the problem again twice during the task. In terms of student-initiated moves, 12 of them were PU moves that were used throughout the interview with the move-count ratio 0.39 and the word-count ratio 0.62. In other words, most of the conversation focused on the student’s procedural understanding with sparse functional moves from other categories.

Although the student ended up with the answer of “each friend would get 8 pieces and then one friend would get 7” based on the final arrangement of the cubes on the table, which was mistakenly set up by the student, the preservice teachers elicited multidimensional facets of the student's mathematical thinking. Figure 5-5 provides the distribution of types of the 31 functional moved employed by the teacher and the word counts in the conversation of each move.

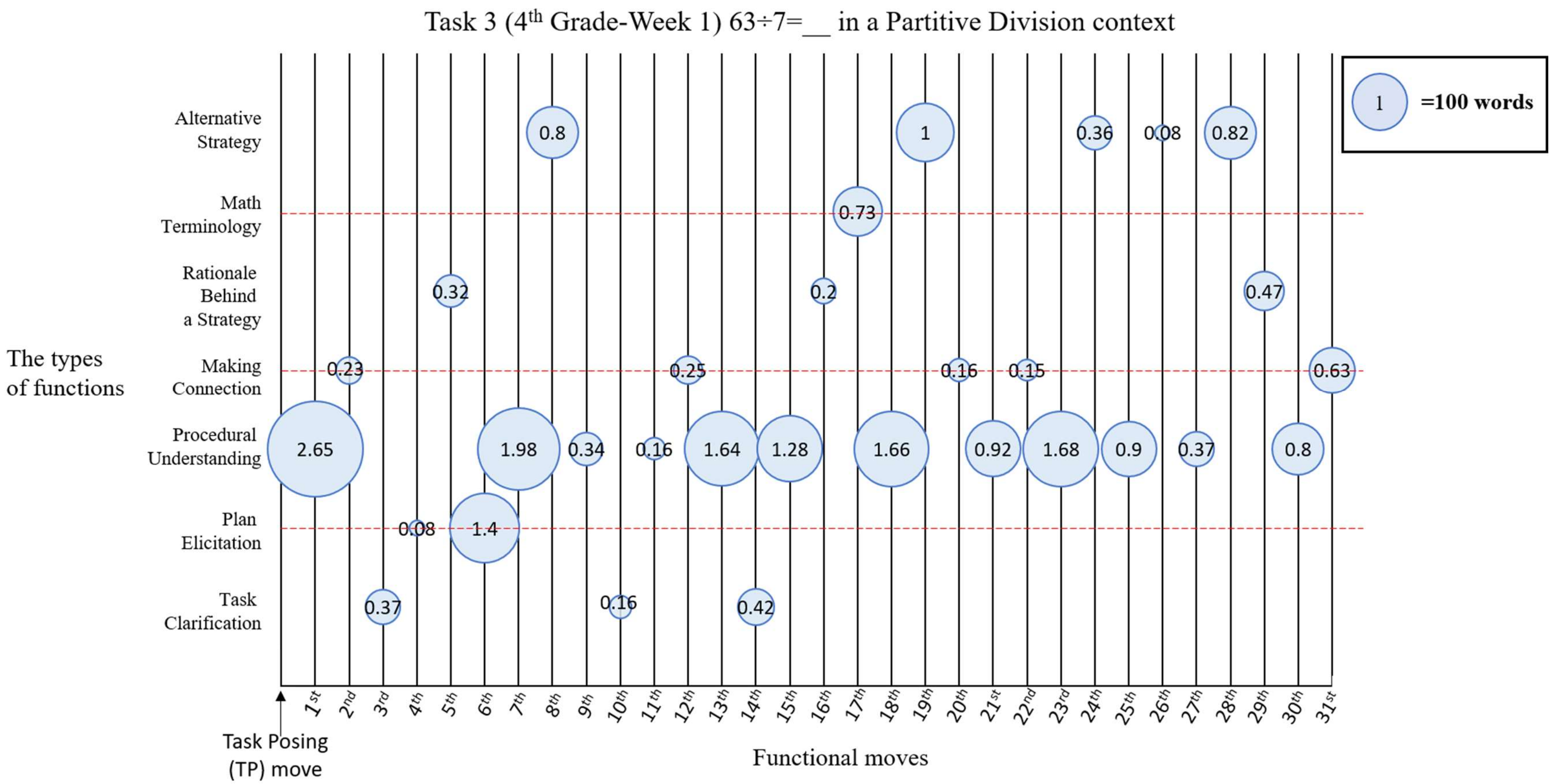

Figure 5-5. The functional moves employed in Task 3 with the structure of 63 ÷ 7 (week 1 in the fourth grade). The horizontal axis represents the order of functional moves in use throughout the task and the first functional moved was a PU move containing 265 words in the teacher-student conversation after the teacher posed the task.

In the following sections, I will illustrate the function-switch sequences in the SSMFE interactions with relevant discussions. All tasks began with the TP turn, and TC moves came after the posed task in 21 out of the 48 tasks, and another 21 tasks had a PU move following the TP move. In the rest of 6 tasks, 5 tasks were followed by a PE move and one task by an AS move. In the 21 "TP-TC" sequential tasks, 20 tasks had student-initiated TC moves. Most students required task clarification right after the task was posed by teachers, and this outcome gives rise to a number of critical questions, such as "Were the tasks too hard/complicated to understand for the students the first time they received them?" "Was the way the teacher posed a task inappropriate?" When expanding the sequence with one more functional move, 16 tasks out of the 48 tasks shared the same sequence of "TP-TC-PU," and 14 out of the 16 tasks were student-initiated PU moves. That is, once the task was clarified, students would begin to solve the problem in their own way. In addition, 9 tasks out of the 48 tasks had the sequence of "TP-PU-AS," and among them, 8 AS moves were initiated by teachers, mostly from the first-grade cohort. That is, when students started solving a problem after task posing without a request of TC move, the teachers had the tendency to suggest alternative strategies in the interactions.

While scrutinizing the 48 initial PU moves, I found that 13 of them had the sequence of "PU-AS," even though the PU moves did not occur immediately after the TP moves. Overall, 13% of functional moves were used to suggest an alternative strategy. Although switching to another strategy did benefit students in some cases, particularly when they were struggling with the prior strategy, employing AS moves too early could cause some problematic issues because 1) it might cease students' spontaneous strategies and 2) it could impose teachers' strategies on students. Furthermore, among the 48 initial PU moves, 10 of them were also the last functional move in the interview, and 5 of these closing moves were from the same interview—the first week interview from the first-grade cohort. In that interview, 3 tasks were not successfully solved by the student, and the teacher did not address the mistakes. Those tasks were relatively short and consisted of only 3 functional moves. The anticipated question arising from this phenomenon would be "Why did the teacher wrap up the task so quickly with acceptance of the student's incorrect procedural understanding?"

It is worth mentioning that there was a total of 15 "PU-MC" and "PU-RA" sequences of moves in the 48 tasks when the PU moves were initially used, 90% of which were teacher-initiated. By initiating MC moves in the interactions, preservice teachers were not only learning what the students can do but also practicing assisting them to discover connections among the numbers, mathematical relationships, and the meaning of the solutions. However, these MC moves were initiated by questions that involved both the given numbers and the unknown number from the tasks, and it is possible that the students' responses did not reflect their ability to make connections. Instead, they merely demonstrated the ability to repeat the exact description about the known numbers. That is, not all MC moves were successfully enacted to make connections in the SSMFE interviews. For example, a teacher began a MC move by asking "What does 6 represent and what does 4 mean, then?" after the student solved the task "There are some kids on the playground. After 6 kids went home, 10 kids were still left on the playground. How many kids did they start off with on the playground?" by performing $10 - 6 = 4$. The student then replied with "6 is the kids that went home and the 4 is the kids that were still on the playground" and the teacher terminated the task by saying "Okay, good." In this explanation, although the student repeated the description about 6, which represents "the kids that went home" from the original task, the function of making connections failed to relate the answer 4 and the given number 10 back to the original mathematical relationship and context of this task.

By enacting the RA moves, preservice teachers actually elevated the cognitive level of their questions to the "know why" stage, which might involve more complicated cognitive processing (Bloom, 1956). On the students' side, they were given the opportunity to reflect on the logic behind their performance; on the teachers' side, learning how to evaluate the logical thoughts in students was challenging but important. However, most preservice teachers failed to follow up students' responses after the RA moves were carried out and were content with students' procedural reasoning. For example, a teacher continuously posed two tasks with the following structures: 12×3 and 3×12. The student immediately answered "The [answer is] same" in the second task 3×12 so the teacher asked, "Why are they the same?" that elicited the response "since multiplication doesn't matter out of order" from the student. However, the conversation ended at this point. This situation should have served as a great opportunity to inquire about the student's thoughts regarding the commutative property of multiplication, but the teacher did not follow up.

The Functional Moves in the CGI Problem-solving Settings

In the analysis of functional moves in tasks in the CGI settings, I purposefully excluded the nontypical task involving mixed operations in this study in order to center the discussion on the problem types (see Section 2.2). The constructions and the features of the functional moves used in four CGI problem types were presented regarding the aspects of the word counts, the number of moves, and the number of functions.

Sixteen addition, seven subtraction, eight multiplication, and 12 division tasks were enacted in six interviews. Most addition tasks were enacted in the first grade, and 50% were the result unknown tasks. The number of moves ranged from one to nine, and four out of the 16 addition tasks were not successfully solved (27%). Two addition tasks that was conducted with first-grade students used the most functional moves (9 moves): A part-part-whole context with the structure of $8 = 3 + \_\_$ contained three TC moves, and the student continually showed uncertainty about the mathematical relationship necessary for solving this task; and a task with the structure of $23 + 4 \times 10$ had the greatest number of different types of functional moves (6 types) because of its two-step procedure. Similarly, most subtraction tasks were enacted in the first grade. Among the seven tasks, only one task was with result unknown structure, and four tasks were not successfully solved (57%) due to the student's misunderstanding of the mathematical relationship in tasks. Overall, the participants employed relatively fewer moves in the subtraction tasks (ranging from two to seven), and the $14 - \_\_ = 3$ task used the greatest number of functions (5 types) in six moves. It is worth mentioning that the same teacher used the greatest variety of the types of functions in both addition and subtraction tasks. Therefore, I argue that the diversity of functions in moves could depend on the participants, which might deserve further investigation.

Most multiplication and division tasks were conducted in the fourth grade, and the number of moves employed was higher compared with addition and subtraction tasks. The multiplication tasks ranged from 4 to 12 moves, and the division tasks ranged from 3 to 12, excluding the two extreme cases with more than 20 moves. Conversations in the two division tasks with more than 20 moves had an extremely large number of words: the week 1 task $63 \div 7$ used approximately 2,300 words, and the week 3 task $49 \div 8$ used 1,117 words. Moreover, the word counts in multiplication and division tasks were more than twice the word counts in addition and subtraction tasks. This difference could be attributed to the fact that the fourth-grade students had more mature language development to express their thoughts as compared with the first-grade participants, and most multiplication and division tasks were enacted in the fourth grade.

Lastly, the findings revealed some features of functional moves while tracing the trends of moves in four CGI problem types. Most addition and subtraction tasks involved 2 to 4 functions while most multiplication tasks used 3 or 4 types of functions, and most division tasks had 3 to 5 types. The division tasks provided a rich environment for teachers and students to clarify tasks, develop strategies, and make connections. If the special cases that involved large numbers of moves or words were excluded, the addition tasks engaged the participants in making connections and exploring new strategies more frequently. Overall, the division tasks were loaded with the most PE moves, and almost all of them were initiated by students. The addition tasks had more moves that were used to evaluate students' knowledge of using correct mathematical representations and terminologies, although some did provoke more abstract mathematical discussions, such as the base-ten system and number theory.

Conclusions

The functions and constructions of interactional turns examined in this paper show 1) the pervasive use of PU moves, 2) the inevitable use of TC moves, and 3) the ineffectiveness of MC moves. Moreover, no matter who initiated a functional move, teacher questioning was the center of the enacted moves in SSMFE interactions.

It is not surprising that there were more student-initiated PU moves in the SSMFE because students were the problem solvers and were encouraged to explain what they were doing in their problem solving. Accordingly, it is important for the teacher to notice what the student did and listen to what the student said in order to effectively enact follow up questions. However, preservice teachers seemed not to leverage the student-initiated PU moves because 50% of student-initiated PU moves in the fourth grade ended up with the teacher either (a) repeating what students said with an upward inflection at the last word in her sentence, or (b) commenting with a short acknowledgment like "okay," "uh-huh [with nodding]," or "gotcha."

No matter who initiated the PU moves, these moves were normally used to elicit students' procedural knowledge during mathematical problem solving. In addition, the analysis also revealed that a portion of preservice teachers nonetheless barely contributed to the student-initiated PU moves: they either merely watched students solving the tasks, or they passively agreed with what the students said in order to reach a solution. This acceptance of students' procedural knowledge with no questions is not an effective teaching move because teachers failed to provide students with an opportunity to clarify and justify their ideas orally and in writing (NCTM, 1991). Based on the questioning performance in Task 3 (see Figure 5-5), the praxis of teacher questioning has the potential to elicit multidimensional facets of students' mathematical thinking even when the construction of PU moves was predominant and inevitable in teacher-student interactions in the setting of mathematical problem solving.

The inevitable use for TC moves was associated with the setup in mathematical task posing and reflected the importance of establishing "taken-as-shared understanding" of contextual features and mathematical relationships to help students make connections between their strategies and correct solutions (Cobb, Wood, Yackel, & McNeal, 1992; Cobb, Yackel, & Wood, 1992). Because most students requested task clarification right after the task was posed, one can assume that they were not sufficiently informed when the teachers merely verbalized the tasks. In most cases, the teachers resolved this issue by repeating the task verbatim, either partially or as a whole, and this approach did not work, particularly in atypical tasks (e.g., the start and change unknown problems). This finding revealed the essential gap between teachers' assumptions and students' abilities about the comprehension of mathematics word problems. It was frustrating but thought-provoking to observe participating students rejecting their teachers' offer to repeat the task and remaining stuck with their own misunderstanding, producing incorrect answers to several tasks. It is imperative to ask why our preservice teachers failed to pose the tasks in a more comprehensible way for students in the interactions. The success of supporting students' problem solving begins with ensuring that students completely understand the task. "Variations in the wording of the problems … can make a problem more or less difficult for children to solve" (Carpenter et al., 2015, p. 12). Therefore, it is paramount to better equip teachers to explain the task in multiple ways based on a structured interview protocol designed to guide their practices.

Not all MC moves were effectively enacted in the SSMFE interviews although MC moves were enacted to invite the meaning of the solution and associate it with the task context. Most preservice teachers were satisfied with eliciting the unit of each number and ignored to accurately engage students in the contextual feature and mathematical relationship. For example, a fourth grader was able to repeat the task "63 means the candies and 7 means there are 7 friends who want to share candies," but after working with cubes, the student concluded that "each friend would have 8 [having 7 groups of 8], and one would get 7. Yes, the total is 63" to the task $63 \div 7$. Throughout the process, the teacher attempted to help the student make connections by asking "what do 63, 7, and the numbers of cubes on the table mean?" four times, but none of them successfully helped the student associate all the numbers with the correct problem context. I conclude that MC moves should be expanded further to connect with prior steps, proposed plans, operations employed, and the manipulatives used or alternative mathematical representations in a broader horizon, rather than being enacted merely to repeat the original task stem and satisfy with the unit of referent numbers.

In their course assignments and final SSMFE portfolio, preservice teachers in this study, to a lesser extent, asserted that they did realize that some of their questioning did not work in the construction of effective functional moves. Although they were encouraged to note both positive and negative experiences of their interactions with elementary students for future reference in their teaching, it was difficult for novice teachers to efficiently analyze and effectively modify their questioning along with students' reactions without a systematic and analytical framework. Teacher education programs should provide an environment that supports preservice teachers in systematically developing inquiry-based questioning techniques to enhance teacher-student interactions in mathematics classroom.

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CHAPTER 6

PRESERVICE TEACHERS' QUESTIONING PRACTICE IN MATHEMATICAL FIELD EXPERIENCE: SUCCESSES AND DIFFICULTIES

Lee, Y.-J. To be submitted to the *Journal of Teacher Education.*

Abstract

I examined six preservice teachers' questioning practices in their single-student mathematical field experiences (SSMFE) on whole-number arithmetic tasks. Each pair of participants was videotaped during one session, and the written work related to the session was collected. Data were analyzed using theme-based coding based on two frameworks: the categories of functional moves and interactional patterns. The analyses show that 1) Task Clarification (TC) moves elicited contextualization when teachers provided flexible support in questioning, and 2) Procedural Understanding (PU), Making Connections (MC), Rationale Behind a Strategy (RA), and Alternative Strategy (AS) moves resulted in deviated from the contextual features and mathematical relationship when students' focus of attention on the superficial feature in a task was not redirected. These findings not only detail the conditions for enacting functional moves but also contribute to better document the successes and struggles a functional move prompted. Findings also suggest implications for curriculum designers, teacher education programs, and for teacher educators and researchers.

KEYWORDS*:* Teacher questioning, mathematics methods courses, field experience

Introduction

Wassermann (1991) pointed out that "[q]uestions are the building blocks of the instructional process" (p. 257), and Aschner (1961) called teachers "professional question maker[s]" because they "probably [devote] more time and thought to asking questions than anybody since Socrates" (p. 44). Studies investigating teachers' use of questions in grade schools support this contention that classroom teachers often use question-and-answer recitations in their instruction (Floyd, 1960; Moyer, 1967; Stevens, 1912). Since the mid-20th century, researchers have increasingly focused on the quality of teacher questioning, including the types (e.g., open-ended or closed, see Hargreaves, 1984), purposes (e.g., eliciting or evaluating thoughts, see Mehan, 1979), patterns (e.g., the IRF structure, see Sinclair & Coulthard, 1975) and the resulting outcomes (e.g., the resultant cognitive process, see Smart & Marshall, 2013). In addition to adopting these determinate divisions to analyze classroom questions, scholars have also developed categories derived from the analysis of teachers' questioning practices in the field (e.g., Boaler & Brodie, 2004; Hufferd-Ackles, Fuson, & Sherin, 2004; Martino & Maher, 1999; Franke et al., 2009; van Zee & Minstrll, 1997). These studies reflect the consensus that teacher questioning is a multi-faceted, pervasive, and "influential teaching act because it is the most basic way teachers use to stimulate participation, thinking and learning in the classroom" (Wilen & Clegg, 1986, p. 53). Moreover, teacher questioning should be regarded as incorporating other teaching practices that could be affected by many factors, such as teachers' beliefs, social teaching norms, and classroom situations (Raymond 1997).

Some researchers have explored the difference between experienced and novice teachers' questioning performance, and their findings reflect convergent views on the phenomenon: novice, including preservice, teachers normally lack efficiency and fluency in their questioning

(Jacobs, Ambrose, Philipp, & Martin, 2011; Tienken, Goldberg, & DiRocco, 2009). Examining novice teachers' questioning performance has illuminated the dilemmas and challenges inexperienced teachers encounter in their praxis (Moyer & Milewicz, 2002; Nicol, 1998; Ralph, 1999; Weiland, Hudson, & Amador, 2014) and has resulted in agreement among scholars that effectively enacting questioning can pose a challenge, particularly for inexperienced teachers. This finding aligns with the observation by Christensen (1991) that "[questioning] requires asking the right questions of the right student at the right time" (p. 154), yet the constraints of the classroom make questioning complicated and difficult, particularly for preservice teachers. Accordingly, it is imperative to scrutinize preservice teachers' questioning, including how their questions function and what effects those questions have in interactions with students. Only once those questions are answered can teacher educators provide appropriate assistance to preservice teachers in teacher education and professional development programs to help them learn to enact effective questioning.

Learning to Enact Questioning in Mathematical Field Experience

Since the initiation of official teacher education programs, there has been substantial discussion and debate regarding the role that field experiences play in teacher preparation (Borrowman, 1956; Lortie, 1975; Zeichner, 1980). Although preservice teachers may enter their teacher education programs with established learning experiences, Ball (1990) found that "many of them lack alternative images of mathematics teaching" (p. 11) and believed that prospective teachers' future experiences are still "affected, redirected, by such changes in ideas, ways of seeing, or ways of doing things" (p. 12). Studies have found that field experiences provide preservice teachers with an opportunity to apply questioning strategies to gain knowledge of students' mathematical thinking (Martino & Maher, 1999; Mewborn & Stinson, 2007) and, further, to analyze or reflect on their questioning strategies (Moyer & Milewicz, 2002; Nicol, 1998). For example, preservice teachers in Chamberlin and Chamberlin's (2010) study mentioned that they had learned "questioning the students to stimulate their thinking, to refocus them on the problem at hand, to understand the students' thinking, or to challenge the students in their thinking" (p. 402) in their field experiences.

Generally, preservice teachers have relatively little experience working with elementary students, particularly on mathematical tasks. Therefore, field-based activities in practicum settings can serve as great resources and opportunities to expose preservice teachers to an environment in which they can gain knowledge of children's thinking in mathematical problem solving. Although Zeichner (2010) warned that there exists "the disconnection between what students are taught in campus courses and their opportunities for learning to enact these practices in their school placements" (p. 91), Feiman-Nemser (2001) advocated "[o]bservation, apprenticeship, guided practice, knowledge application, and inquiry all have a place in field-based learning" (p.1024). Moreover, Mewborn and Stinson (2007) contended that "[f]ield experiences provide a rich ground for questioning why we do the things we do and how we might do them differently if we are serving the goal of creating opportunities for preservice teachers to engage in assisted performance" (p. 1482-1483). Such findings support the idea that developing questioning proficiency relies on using carefully designed tasks in an assisted learning environment instead of on preservice teachers' self-evolving development over time.

There are few studies examining preservice teachers' questioning performance exclusively in mathematical field experiences. Nicol (1998) studied 14 prospective elementary teachers while they were working with small groups of students for 10 weeks and noted that, at the end of their experiences, "[t]hey were posing questions of students for the purposes of learning what students were thinking rather than with the intended emphasis on leading students to the correct answer" (p. 62). Weiland, Hudson, and Amador (2014) examined the development of preservice teachers' questioning practice in weekly formative assessment interviews and concluded that the field-experience approach provided rich opportunities for preservice teachers not only to develop the core practice of questioning but also to practice "adapt[ing] their questioning practice to offer more competent questions in their interactions with students (p. 349).

To diminish the distraction of classroom management and instead focus on the child's thinking, Mayor and Milewicz (2002) investigated 48 preservice teachers in their one-on-one mathematics interviews with elementary students and asserted that "using the diagnostic interview format allowed them [preservice teachers] to recognize and reflect on effective questioning techniques" (p. 293). They further concluded that "[h]aving preservice teachers focus on the skill of questioning in a one-on-one diagnostic interview may be an effective starting point for developing the mathematics questioning skills they will use as future classroom teachers" (p. 297). In summary, these studies revealed the following: 1) preservice teachers' questioning techniques can be challenged and can develop within the context of face-to-face interaction with small numbers of students, 2) their questioning functioned in relation to the students' responses and the milieu in which they were working, and 3) there existed difficulties and dilemmas in their questioning practices that could serve as starting points for teacher educators to intervene.

Accordingly, I addressed the following questions in this study:

1. How do preservice teachers' questioning moves function while working with elementary students on arithmetic word problems?
2. What were the successes and difficulties of enacting functional moves in their SSMFE sessions?

Conceptual Frameworks

To better examine teacher questioning with functions, I employed two frameworks in this study. The first framework consisted of seven categories of functional moves. The types of functional moves stemmed from Jacobs and Empson's (2016) framework of teaching moves, and Figure 6-1 shows the correspondences between the four categories of teaching moves and the seven categories of functional moves. The Task Clarification (TC) was used to ensure the student is making sense of the posed problem, so the occurrence of this move could be either the teacher clarified, or the student sought the given information in the original task. In Jacobs and Empson's (2016) framework, the category of Plan Elicitation (PE) moves was not existing, so I added this category for the purpose of analyzing teachers' questioning moves in students' mathematical problem solving. To explore details of the student's existing strategies, teachers centered their questions on three aspects: the student's procedural understanding (PU), student's ability to make connections (MC), and reasoning about their strategies (RA). The Math Terminology (MT) moves were corresponding to connecting the student's thinking to symbolic notation and the Alternative Strategy (AS) moves accorded with the teaching move "encouraging the child to consider other strategies."

Framework of Teaching Moves (Jacobs & Empson, 2016)	**Functional Moves**	**Description**
Ensuring the child is making sense of the story problem	Task Clarification (TC)	Clarify or seek the given information in a task
	Plan Elicitation (PE)	Elicit or produce the initial plan
Exploring details of the child's existing strategies	Procedural Understanding (PU)	Explore or explain the procedure involved
	Making Connections (MC)	Make connections between the answer and the original task
	Rationale Behind a Strategy (RA)	Inquire about or elaborate on the rationale behind the proposed strategy
Connecting the child's thinking to symbolic notation	Math Terminology (MT)	Elicit or give correct math terminology
Encouraging the child to consider other strategies	Alternative Strategy (AS)	Elicit or propose an alternative strategy (including mathematical representations)

Figure 6-1. The correspondences between the four categories of teaching moves and the seven categories of functional moves.

The second framework was adapted from Hogan, Nastasi, and Pressley's (1999) interaction patterns as shown in Table 6-1 and I combined the nonresponse and consensual reactions as the NC pattern, and the responsive reactions and elaborative reactions merged into the RE pattern. Additionally, I distinguished the NC and RE patters by considering who initiated the unit of functional move, so there were four categories of interactional patterns in total: (1) Teacher-initiated Nonresponse or Consensual reactions (TNC), (2) Teacher-initiated Responsive or Elaborative reactions (TRE), (3) Student-initiated Nonresponse or Consensual reactions (SNC), and (4) Student-initiated Responsive or Elaborative reactions (SRE).

Table 6-1

The Framework of Interactional Patterns Adapted from Hogan, et al. (1999)

Category of Interactional Patterns	**Description**
Teacher-initiated Nonresponse or Consensual reactions (TNC)	The initial interactional turn is initiated by the teacher, and the student replies with nonresponse or only consensual responses.
Teacher-initiated Responsive or Elaborative reactions (TRE)	The initial interactional turn is initiated by the teacher, and the student replies with responsive or elaborative responses.
Student-initiated Nonresponse or Consensual reactions (SNC)	The initial interactional turn is initiated by the student, and the teacher replies with nonresponse or only consensual responses.
Student-initiated Responsive or Elaborative reactions (SRE)	The initial interactional turn is initiated by the student, and the teacher replies with responsive or elaborative responses.

Methodology

Participants and Settings

The participants were six pairs of preservice teachers and elementary school students who were selected by convenience sampling (Patton, 2002). The participating teachers were all female, but the students were both male ($n = 2$) and female ($n = 4$). Three students were from the fourth grade and three were from the first grade in a local public elementary school.

During their first mathematics methods course, all participating teachers conducted single-student mathematical interviews weekly in a public elementary school for eight weeks. The instructor provided tasks that were whole-number word problems involving four operations—addition, subtraction, multiplication, and division and the problems were adapted from the Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). The course instructor provided weekly interview protocol composed of less than ten tasks to preservice teachers before every session but allowed teacher participants to adapt or redesign the given tasks to fit students' problem-solving ability. Additionally, preservice teachers were required to audiotape their interviews as references for accomplishing course assignments.

Data Collection and Analysis

Each interview session lasted approximately 35 to 45 minutes, and the participants were both videotaped and audio recorded. Following the interviews, the protocol, preservice teachers' field notes, and students' written work were collected. In addition, all debriefing forms, course assignments, and the final portfolio from participating teachers served as supplementary materials in the analysis. All video clips were transcribed verbatim, and nonverbal actions were included to enrich the record of the interviews.

Every turn that occurred in teacher-student interactions was defined as an *interactional turn*, which referred to either the teacher's questioning move or the student's response. Most teacher-initiated interactional turns consisted of a questioning move that was conceptualized as a verbal expression that prompted a student to provide a verbal response or nonverbal action. Additionally, I grouped several interactional turns together and assigned a function based on the role the initial turn played. These grouped interactional turns constituted a *functional move*, which was the unit of analysis in this study. In short, a functional move contained several interactional moves that could be initiated by either the teacher or the student.

I analyzed the data by applying thematic analysis (Braun & Clarle, 2006), defined as a method "for identifying, analyzing and reporting patterns (themes) within data" (p. 79). It contained five steps: (1) Familiarization: I delved deeper into the data by repeatedly reading the transcripts, along with watching the corresponding videos; (2) Coding and indexing: I coded the data by highlighting the characteristic of interactional turn based on two frameworks. I first determine whether an interactional turn was a questioning move to precisely distinguish preservice teachers' questioning moves and students' responses. I then grouped interactional turns with the same function into units of functional moves using the functional framework (see Figure 6-1) and detached irrelevant turns. After determining the function of every functional move, I began analyzing "interactional pattern" using the framework of interactional patterns (see Table 6-1); (3) Thematizing: I discerned potential themes and grouped coded data under broad themes. I further read through the coded interactional turns several times to determine and discern potential themes; (4) Reexamining I checked the themes thoroughly to confirm the patterns and reassessed the classification of functional moves if necessary; and (5) Interpreting the themes: I assigned meaning and defined the themes by producing thick description to interpret observed phenomena. To check for reliability, I repeated this coding process on partial data every two weeks, and all questions were coded at least twice, achieving 86% agreement of coding.

Findings

In this section, I first present the frequency and percentage of functional moves by category to provide an overview of the quantitative outcomes in the data and follow it with the analysis of the interactional patterns including the identification of the initiator and the respondent's contributions to the conversation. Figure 6-2 presents a breakdown of the functional moves employed by preservice teachers, indicating the frequency at different grade levels (fourth and first grades).

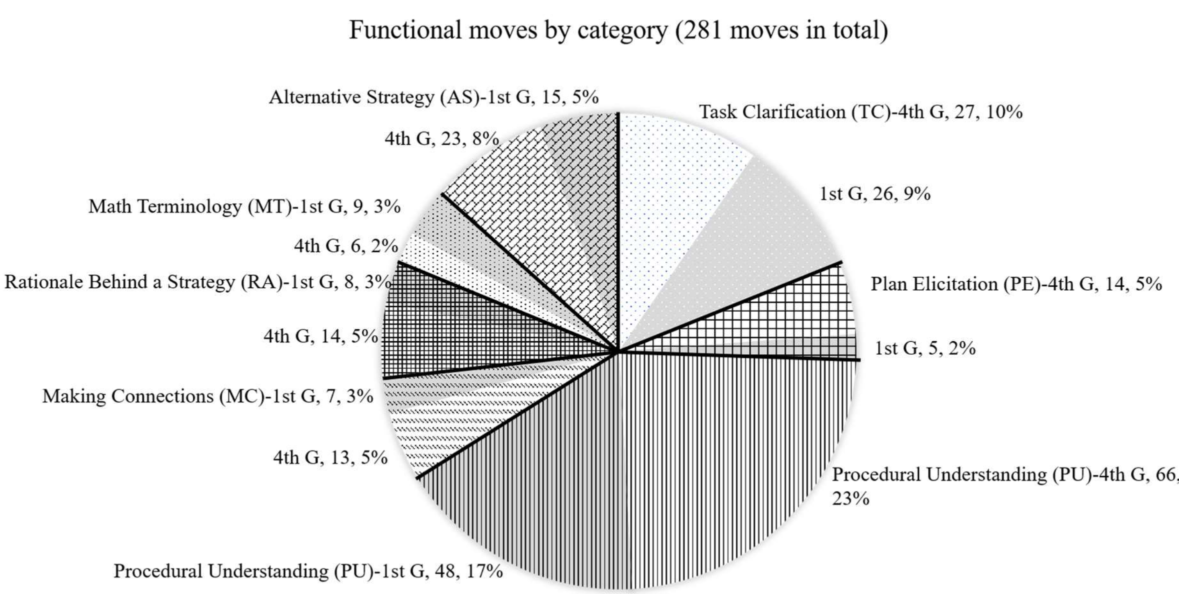

Figure 6-2. Frequency and percentage of functional moves by category and grade (4th G: the fourth grade; 1st G: the first grade).

During the six SSMFE interview sessions, the most frequently used moves were: (1) TC moves (19% of total moves), (2) PU moves (40%), and (3) AS moves (13%). Therefore, this paper focuses specifically on discussing how preservice teachers orchestrated these functional moves to (a) promote task understanding, (b) disclose existing strategy, and (c) incorporate alternative strategy. Furthermore, comparisons between the first-grade and fourth-grade cohorts are discussed in particular.

Promoting Task Understanding-TC Moves

In Figure 6-3, I provided a quantitative comparison of the number of TC moves by grade. In total, 53 TC moves were used, 26 in the first grade and 27 in the fourth grade, demonstrating a balanced usage at different grade levels.

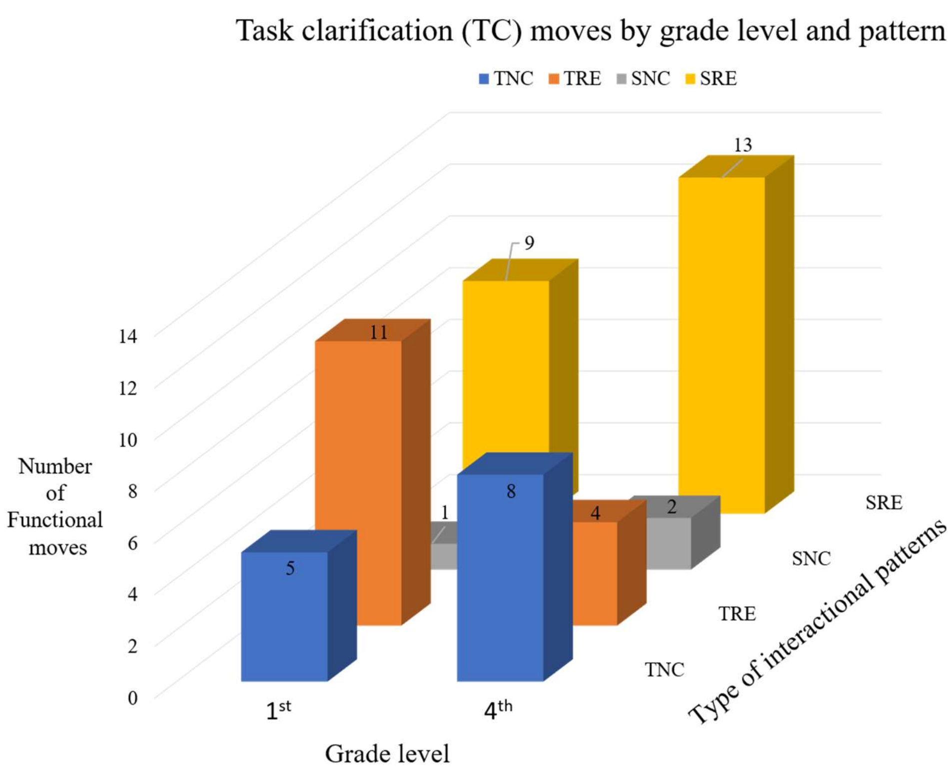

Figure 6-3. Frequency of TC moves by grade level and pattern.

For the TC moves that were initiated by teachers (TNC and TRE), fourth-graders acquiesced to their teachers' offerings more often than first-graders did (8:5), and first-graders demonstrated higher verbal engagement with teacher-initiated TC moves (11:4). When the TC moves were initiated by students (SNC and SRE), teachers tended to be responsive[6] to the student-initiated TC moves in all but three tasks. In terms of the format of TC moves, teachers either repeated the entire task verbatim or merely highlighted a particular part of the task for students. Among the 53 TC moves, most moves successfully facilitated students' understanding of the task, and only six TC moves malfunctioned or were ignored. Below, I present four selective scenarios to illustrate certain outcomes of enacting TC moves.

Scenario 1

Task: Our class made some clay animals. The following day, our class made five more clay animals. Now there are nine clay animals. How many clay animals were there before our class made any more?

S: [Drawing five circles, and then freezing]
T: *Do you want me to read the question again*?
S: [Shaking head]
T: No? Okay.
S: [Drawing nine circles after the 5 circles. Numbering circles from 1 to 5. Writing down $5 + 9 = 14$.] (see Figure 6-4).
(W1-1st G-Task#3)

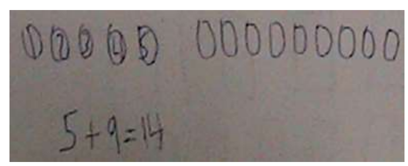

Figure 6-4. The student's solution to $\_ + 5 = 9$

The teacher in this interview spontaneously enacted a TC move (highlighted) after noticing an incorrect written representation and a long pause from the student, but the student rejected this offer and continued to solve this task based on her perceived understanding of the task. Accordingly, this task was concluded with an incorrect answer.

[6] Being responsive meant that the teachers normally contributed essential content to the conversation.

Scenario 2

Task: Eric made eight cookies. Three were chocolate chip and the rest were oatmeal raisin. How many oatmeal raisin cookies did Eric make?

S: [Drawing three more circles, and another set of three, and another set of three, writing down $8 + 3$, then pausing] (see Figure 6-5).

…

T: So, I said that he made three chocolate chip, right? And the rest were oatmeal raisin, so how many oatmeal raisin did he make?

S: Four?

T: So *eight…there are eight total cookies, right?*

S: [Nodding]

T: *And he made three chocolate chip cookies, so how many oatmeal raisin cookies did he make?* You wanna do it with cubes? Maybe it'd be easier to look at?

S: [Pulling out eight cubes, grabbing three more in his hand] …8, and, made 3…

T: Okay, so what do you think the answer is? It's 11, right?

S: [Nodding]

T: Yeah? Okay, alright.

(W1-1st G-Task#10)

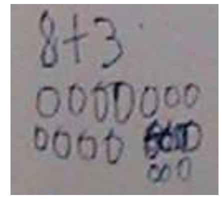

Figure 6-5. The student's solution to $8 = 3 + \_$

In Scenario 2, the teacher, like most other teacher participants, enacted the TC move by restating the given information eight total cookies and three chocolate chip cookies. Without a correct understanding of the mathematical relationship among the given numbers, the student was still not able to solve this task successfully. In other words, no matter what representations (drawn circles, cubes, numerals) he adopted to solve this task, the student was obviously stuck in his misinterpretation of the mathematical relationship between the whole eight cookies and the subset entity three chocolate chip cookies. Such a situation highlights that when the problem solver does not correctly understand the mathematical relationship, enacting the TC move by restating the given information might not be an optimal choice for the purpose of clarifying a mathematical task.

Scenario 3

Task: Seven friends had 63 candies, they want to share equally. How many candies should each friend get?

In this scenario, the student had already correctly solved the task by claiming that she knew the fact that 7 times 9 equals 63. The teacher asked the student to verify the solution using another method, and the student chose to use cubes. Before the student began to employ the second strategy, the teacher employed a TC move as shown in the following exchange. However, the student ignored this offer due to her concentration on a new strategy when the teacher offered to read the problem again.

I: *If you need me to read the problem again, I can totally do that too, okay*?
P: [Pulling out cubes] Three! So you wanna split this into seven groups, so I have 1, 2, 3, 4, …[continuing working on the cubes]
(W1-4th G-Task#3)

Scenario 4

Task: 49 dinosaurs are coming, and they have 8 caves. If they spread the family out as equally as possible, how many dinosaurs will need to sleep in each cave?

In this scenario, the teacher attempted to use a TC move to help the student clarify the task after she noticed that the student was using an incorrect number in his computation.

T: Okay, well, fifty-se--…so what does this number mean? This number down here? We just added up all these [numbers] right?
S: [Adding all numbers in his drawing up by conducting $7 + 7 + 7 + 7 + 8 + 7 + 7 + 7 = 28 + 29$ but resulting in only 47] It's supposed to be 57, but I am checking my answer 'cause we did it so fast.
T: Well, *is that how many dinosaurs were coming, in the beginning*?
S: I am checking my answer to see if we did something. Um, 7, that would be…8…38; I'm off by 10…But where?
T: Wait so, okay, *so repeat the problem to me, what were we trying to do in the beginning*.
(W6-4th G-Task#4)

The analyses show that preservice teachers encountered both successes and difficulties in their questioning practices while working with elementary students on whole-number word problems adapted from CGI. I defined difficulty as a situation in which a relatively negative outcome resulted from the functional move that was enacted in the interaction. For example, when the teacher initiated a MC move to help the student make connections among the given information and produced solution, the outcome was considered a difficulty if a) the move was completely ignored, b) the student only provided superficial connections as responses (i.e., merely addressed the referent unit without mentioning the mathematical relationships), or c) the student failed to make correct connections. If the interaction did not result in the negative outcomes mentioned above, it was considered a successful enactment of functional moves. For example, when the first TC move the teacher employed in Scenario 4 was ignored because of the student's focus on his computation, she attempted to enact another TC move by asking the student to repeat the problem from the beginning, which successfully redirected the student to the original context.

While comparing teachers' questioning practice at different grade levels, I noted that teachers of first-grade students enacted TC moves to (a) answer students uncertainty, (b) ensure students were informed, (c) help students correctly represent the task in their solution (e.g., complete a number sentence or grab correct number of cubes), or (d) correct students' misunderstanding while noticing a mistake in their students' problem solving. For example, one preservice teacher required her first-grader to interpret the mathematical relationship in her own words based on the given information, and only when the student failed, the teacher stepped in by supplying needed information. Teachers of fourth-grade students often enacted TC moves to complement their students' incomplete statement about the task, which mainly occurred at the early stage of the student's problem solving. This is, the first-grade students often initiated TC moves by asking a real question, such as "How many did he have?" and the fourth-grade students were inclined to initiate TC moves by restating the task with the information they perceived, which might be incomplete and incorrect. In addition, I found that first-grade students were more likely given opportunities to make responsive and elaborative responses than fourth-grade students when TC moves were initiated by teachers. It is also worth noting that the teachers working with first-grade students were more adept at helping students "construct" the meaning of the task (e.g., tried to elicit students' interpretation about the task), rather than merely keeping their students "informed" (e.g., provided the answer or explanation students were looking for). As a result, the difference was reflected in the quantitative data (the TRE ratio is 11:4)

Although a move might be ignored with a nonresponse reaction in the interaction in sparse cases, most moves in this study were identified as valid moves because they successfully elicited one of the three reactions: consensual, responsive, and elaborative. Most tasks with the fourth graders contained up to three valid TC moves, except for one task that consisted of five valid TC moves. This special case occurred when solving a multistep problem with the mathematical structure "$5 \times 6 + 4 - 3 = \_ \times 6 + \_$." While delivering this task to a fourth-grade student, the teacher attempted to clarify one condition at a time alongside the student's step-by-step computational procedure. The sequence of interactional turns included the following actions: (1) The teacher posed the task: "Five bags of peanuts and six peanuts in each bag," and the student wrote a vertical form for $5 \times 6 = 30$; (2) the teacher then provided the information that there were "Four extra peanuts not in a bag and three peanuts were given away. How many left?" and the student added 30 and 4 to obtain 34, and then stated that minus 3 equals 31 as the answer; (3) the teacher then continued the second part of the task by asking "how many complete bags of six are there after giving those three away?" after which the student performed the division by using the long-division method to divide 31 by 6, concluding that the answer is 5 with a remainder of 1. In this case, the whole task was broken down into small pieces of information, so all five TC moves were effectively enacted. These breakdown questioning moves created greater leverage for students' problem-solving performance by reducing their working memory loads. In addition, these TC moves were precisely enacted in alignment with their corresponding actions, so they functioned as step-by-step directions in the problem-solving task. From the result, the student successfully solved this task, but it was built on the process in which the teacher had to provide fragmentary information as directions and worse, it narrowed the student's creativity on solving this task.

In sum, the ways preservice teachers enacted TC moves were very similar, and the interactional situations included: (a) students verbally requested repetition or partial clarification of the tasks in order to embark on the problem solving, normally immediately after the task posing; (b) teachers asked students to rephrase the task; (c) teachers actively offered repetition when students showed uncertainty in their problem solving; and (d) teachers highlighted particular pieces of information for students to reinforce partial clarification. TC moves were employed in different format and the way teachers enacted them affected students' performance in their problem solving, so it is important to well-equip teachers with diverse tactics of enacting TC moves.

Disclosing Existing Strategy-PU, MC, RA Moves

Preservice teachers employed three different types of functional moves—Procedural Understanding (PU), Making Connections (MC), and Rationale Behind a Strategy (RA)—to explore students' mathematical thinking related to their work in progress. The quantitative data show that teachers working with the fourth-grade students, compared with teachers working with first-grade students, used more PU, MC, and RA moves (93:63).

Regarding PU moves, it is not surprising that there were more student-initiated moves than teacher-initiated moves (97:17) because students were the problem solver in the activity and they automatically began to solve the task after the teacher posed it. When the PU moves were initiated by teachers, students in both grades attained a 100% rate of providing responsive or elaborative reactions, which means that the students were able and willing to enrich the conversation. Teachers working with first-grade students more actively initiated PU moves in the interactions.

Most teachers in both grades contributed responsive reactions (26 moves in the first grade and 29 moves in the fourth grade) to the student-initiated PU moves with inquiries such as "How did you know that?" or "Show me more about your work." However, it is worth discussing the successes and difficulties that teachers experienced when they encountered student-initiated PU moves in both grades. The imbalanced number of overall student-initiated PU moves (60:37) revealed that the fourth-grade students were more likely to elaborate on their computational procedures than the first graders were, likely due to their language development and mathematical knowledge. Therefore, preservice teachers who were assigned to work with fourth-grade students should have an opportunity to learn more about students' thinking in their problem solving. However, preservice teachers seemed unable to gain this advantage by leveraging this opportunity because 50% of student-initiated PU moves in the fourth grade ended up with the teacher either (a) repeating what students said with an upward inflection at the last word in her sentence, or (b) commenting with a short acknowledgment like "okay," "uh-huh [with nodding]," or "gotcha" to continue the conversation.

Most MC and RA moves were teacher-initiated (see Figure 6-6). Compared by grade, the MC and RA moves seemed to share a similar distribution of the number of moves used in the interactions. For example, when it was the teachers who initiated MC and RA moves, they were more likely to receive responsive reactions from students (the TRE moves). This finding implies that, when employing MC and RA moves, preservice teachers were inclined to ask open-ended questions to invite students' elaboration of the mathematical relationship among the numbers and the rationale behind their strategies.

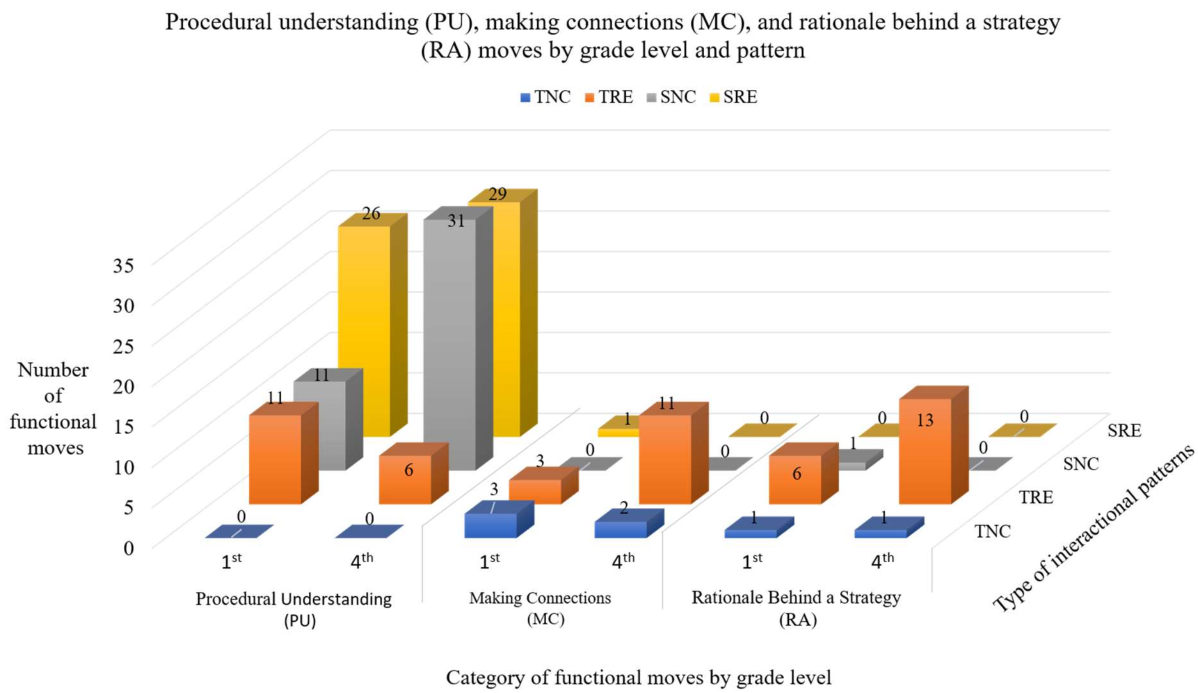

Figure 6-6. Frequency of PU, MC, and RA moves by grade level and pattern.

In the following sections, I present three results of preservice teachers' MC moves: (a) a contextualized connection was successfully produced, (b) the contextual features were maintained but the key mathematical relationship was diminished, and (c) the mathematical relationship was ignored.

The contextualized connection. The preservice teachers working with fourth-grade students more frequently elicited precise description while enacting questioning moves to help students make connections between the solutions and contextual features.

Scenario 5

Task: Julie had some markers. She gave four markers to her brother. Now she only has two markers. How many markers did Julie start with?

After the teacher posed the task, the student came up with her solution strategy (see Figure 6-7) and explained “because 2 plus 4 is 6, and you add those [numbers] up. I did an inverse [$6 - 2$] so I got 4. I also knew 4 plus 2 was 6, so if you added 2 plus 4, it’d be 6 markers. So it’s six markers.” The teacher then initiated the MC move as shown in the following exchange:

T: So the *2 is what in the problem?*
S: The 2 is how many markers she gave to her brother, and the 4 is how many markers she had left.
T: And *what’s the 6?*
S: The 6 is how many she had before she gave 2 to her brother, and before she had four left.
(W1-4^{th} G-Task#1)

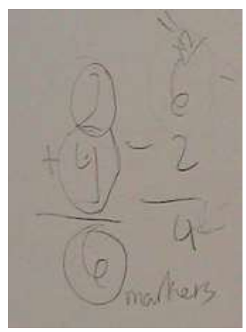

Figure 6-7. The student’s solution to $\_\_ - 4 = 2$

Scenario 6

Task: Rosa had 69 diamond rings, and then she bought 316 more. How many diamond rings does Rosa have now?

After the teacher posed the task, the student then fluently performed a computational procedure by explaining "So, if she had 69 rings and she bought 316. Six plus 9 is 15, so you add the one [in the tens place] up here. Six plus 1 is 7, plus 1 more is 8, so you have 385" with a vertical form of the addition $69 + 316 = 385$. The teacher then prompted with an MC move as shown in following exchange:

T: *So the 69 is what?*
S: Sixty-nine is how many rings she has, and 316 is how many she bought.
T: *And then 385 is what?*
S: How many she has in all.
(W1-4th G-Task#2)

Fourth-grade students seemed more able to make contextualized connections on their own even when the teachers' questions lacked specific indicators to relate to the original task. In Scenario 6, the student not only knew the unit of the numbers 69 and 385, but she also understood the problem structure that involved an action related to the given numbers. However, first-grade students might maintain the contextual features but diminish the key mathematical relationship in a task.

The diminished mathematical relationship. In some first-grade cases, the students simply preserved the contextual feature (e.g., a superficial impression such as the unit of the number) and the mathematical relationship was diminished. Thus, the correct mathematical structure was not completely carried over to the stage of devising strategies, not to mention the stage of carrying out strategies.

Scenario 7

Task: There are some kids on the playground. After six kids went home, 10 kids were still left on the playground. How many kids did they start off with on the playground?

After the teacher posed the task, the student incorrectly represented and solved this problem by drawing 10 circles in total, crossing out 6, then writing down $10 - 6 = 4$ (see Figure 6-8). The student then explained as follows: "First, I did 10. Second, I x-ed out 6. Third, I counted, and then the number was 4…when I counted them. Only 4 were left." The teacher then initiated a MC move as shown in following excerpt:

T: Okay, *what does the 10 and the 6 and the 4 stand for?* What would be the word problem you give them?
S: [long pausing]
T: So if they ask you what does 10 minus 6 equal 4 mean? What would you say then?
S: Equals…
T: So what is… *what does the number 10 mean?* Is it the number of kids on the playground? Or is it the number that left?
S: The number that left… think… okay, the playground?
T: Okay, so it's the kids on the playground, so then *what does 6 represent*?
S: The kids that went home?
T: The kids that went home. And so *what does 4 mean*, then?
S: Kids that were still on the playground.
T: Okay, good!
(W1-1st G-Task#9)

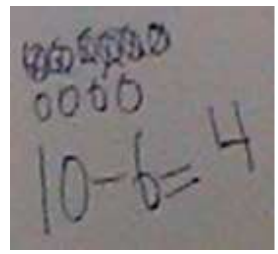

Figure 6-8. The student's solution to $\_ - 6 = 10$

In Scenario 7, the student mentioned contextual features from the task stem, such as "in the playground," "went home," and "left." However, the student did not comprehend the key mathematical relationship in this separate, start unknown problem, and the student's misunderstanding of this task was reflected in her solution strategy: "Ten on the playground, six went home, and four were still on the playground." The student clearly failed to solve this task, but the preservice teachers did not leverage the contextual features the student retained to help her reconstruct the correct mathematical relationship and succeed in his or her problem solving.

Ignoring the contextual features. When students demonstrated their computational procedures, they sometimes completely ignore contextual features of the task.

Scenario 8

Task: Eric brought 11 boxes of cookies to school. There are 12 cookies in each box. How many cookies did Eric bring to school?

While solving this task, the student drew 11 circles with 12 dots in each circle and explained "So 12 plus 12 is 24 [repeating five times] and there's gonna be one extra of 12. So this is 48, this is 48, and this [48 plus 48] would be 96, and you add 96 and 24" (see Figure 6-9). The teacher then initiated an MC move, as shown in the following exchange:

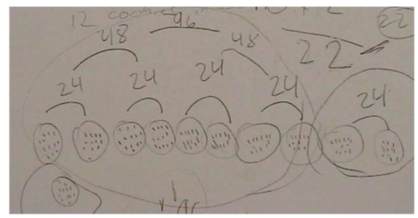

Figure 6-9. The student's solution to $11 \times 12 = \_\_$.

T: Here you had 24…you did 96, I am just making sure I am following, so 96 is this [pointing two sets of 48] number, right?
S: Yes.
T: And *that represents*…
S: Eight plus 8 is 16, but that, you can't do, so you have to add 1 to, um, each of…well, that is 8. So you have to add 1 to 8, which is 9,
T: Okay
S: And you put the 6, is 96
T: And *the 96 represents what*?
S: Um, how many I added up, for…so like, I added those up, which is 24,
T: Okay
S: And I add it 4 up, which is 48,
T: Okay
S: And then, I added those up, and I got 96.
T: So *96 represents*…
S: How many this [circling 8 groups of 12 in her written work] is.
T: Okay! gotcha!
(W1-4th G-Task#4)

In Scenario 8, the teacher enacted MC moves three times, but the student continued to reply with her computational steps and result without associating them to the context of this task.

Scenario 9

Task: Tyrannosaurus Math goes to the World's First Dinosaur prom. Tons of dinosaurs attend. Each dinosaur is given three Hawaiian lei necklaces as they walk in the door. If 783 leis are given out, how many dinosaurs were in attendance?

To solve this problem, the student employed the partial quotient method of division and accomplished all steps by mumbling "One hundred…[writing 100 on the side]…3 times 50…150…and, that would be 3…33…[tried]10 [getting 30] minus 30 [doing $33 - 30$] 3 [figuring out $3 \times \_ = 3$] is 1…plus all these [numbers on the side]…261" (see Figure 6-10). After he concluded that the answer was 261, the teacher tried to help him make connections back to the task context, as shown in the following exchange:

T: So tell me, *what did we just do--all that work?*
S: Partial quotient.
T: No, *but what was the problem?* Can you remember?
S: Seven hundred and eighty-three divided by 3, right here [pointing to her written work].
T: Equals?
S: Two-hundred and sixty-one.
T: Perfect. Okay.
(W5-4th G-Task#5)

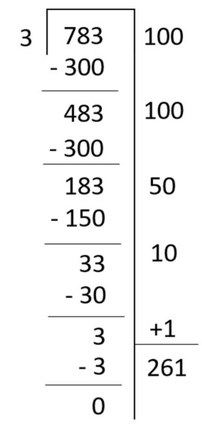

Figure 6-10. The student's solution to $783 \div 3 = \_$.

In these two scenarios, both fourth-grade students were fluent in articulating or demonstrating all computational steps in verbal expressions or written representations and capable of devising a solution strategy for the task, carrying out their plans, and attaining a satisfactory result in their interactions with the teachers. Although these students were able to perform complicated computations and attain the correct answer, the teachers' use of MC moves elicited procedural explanations of the computation rather than conceptual explanations of the way in which the numbers and operations corresponded to the story in the task. Although the teachers each made multiple attempts to elicit contextual explanations using MC moves, each time the students interpreted them as requests for procedural explanations, and the teachers seemed unable to phrase the MC move to elicit a contextual explanation.

In sum, enacting MC moves provided preservice teachers with an opportunity to test students' understanding of the contextual features and mathematical relationships in the task, discover students' capacity to make connections among given and unknown information, and experience diverse outcomes stemming from their moment-to-moment praxis of questioning.

Incorporate Alternative Strategy-AS Moves

The quantitative results (see Figure 6-11) show that most AS moves (82%) were initiated by teachers in both grades, and preservice teachers teaching fourth graders were more likely to employ AS moves compared to teachers working with first graders (23:15). For the teacher-initiated AS moves, students in both grades demonstrated a responsive tendency in the interactional patterns; for the student-initiated moves, teachers displayed more consensual behavior and easily accepted what students proposed.

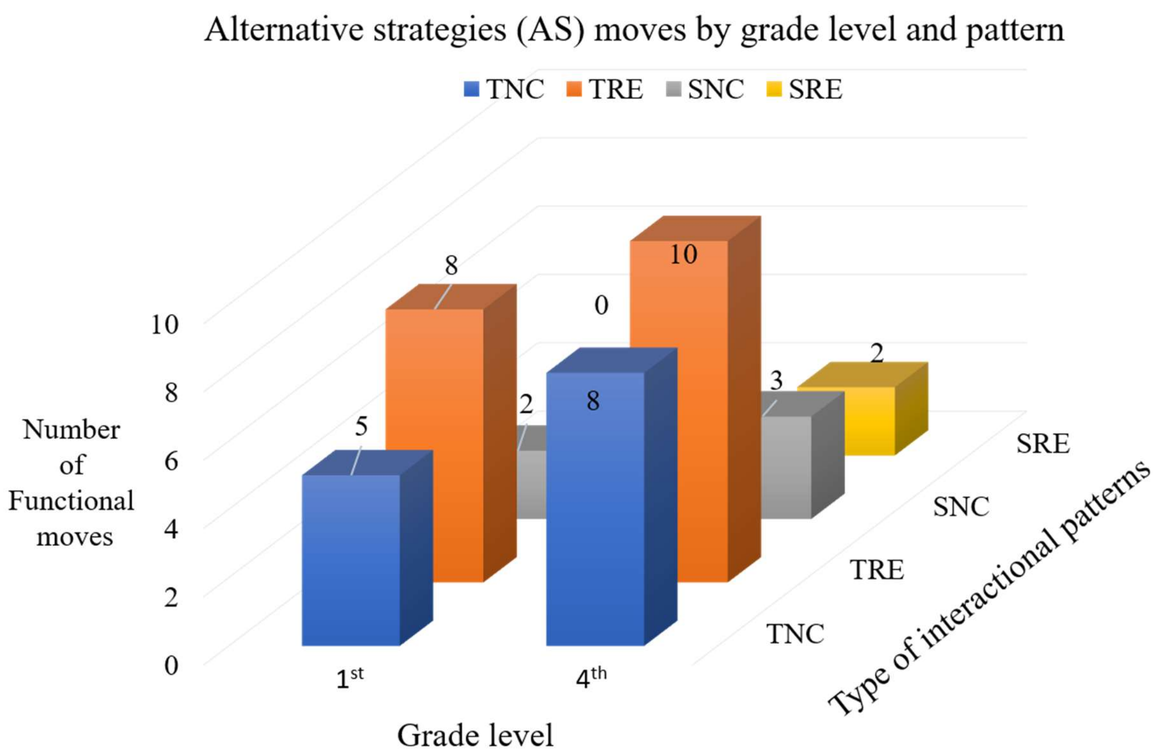

Figure 6-11. Frequency of AS moves by grade level and pattern.

The strategies students used included: (a) a drawing, (b) an equation, (c) an equation corresponding to the mathematical relationship (e.g., horizontal or vertical form), or (d) cubes. Students used an alternative strategy or representation when: (a) the initial strategy had been used to solve the problem and the teacher required another strategy (an open-ended move), (b) students were struggling with the first strategy, so they wanted to try another one, or (c) teachers directly reminded students about other possible strategies regardless of the implementation of the initial strategy (a closed move). In addition, the questions "How can you check your solution by using another way?" or "Is there another way you can write it?" also elicited another strategy (including representation) in students' solutions. The open-ended AS move usually left more freedom for students to employ their desired strategies to solve or check their existing solutions.

However, asking open-ended AS moves also resulted in some consequences that are worth discussing. In the findings addressing the MC move, I discussed how the fourth graders tended to ignore the contextual features of the task and concentrate on computational procedures. Below, I present representative scenarios from the first-grade cohort to illustrate a similar phenomenon, but one caused, in this instance, by AS moves.

Scenario 10

Task: Our class made some clay animals. The following day, our class made five more clay animals. Now there are nine clay animals. How many clay animals were there before our class made any more?

The student initially solved the task by drawing circles and writing an equation corresponding to his misunderstanding. After the student concluded the task with the wrong solution 14, the teacher employed an AS move to suggest using cubes (the highlighted part).

S: [Drawing nine circles after the five circles. Numbering circles from 1 to 5. Writing down $5 + 9 = 14$.] (see Figure 6-4).
T: So what's the answer?
S: 14.
T: *Do you think, um…using the cubes or anything would help you with the problem?* Do you wanna try that?
S: [Nodding]
T: Yeah? You can do that.
S: [Grabbing the bag of cubes, taking some blocks out]
(W1-1st G-Task#3)

The student then put five cubes on the table and held one cube in his hand with a long pause. The student admitted that this task was too hard for him, so the teacher terminated this task. Although the student accepted the suggestion of incorporating cubes, the student's misunderstanding of the mathematical relationship of this task had been constructed. Hence, the teacher's suggestion to use the second strategy resulted in a false duplication of his initial solution strategy. In the next two scenarios, the AS moves were more likely used to emphasize computational understanding instead of strategic reasoning.

Scenario 11

Task: Ms. Park had 13 cookies, and Vail gave her some more. Now Ms. Park had 20 cookies. How many cookies did Vail give Ms. Park?

The student successfully solved this task using the "adding up to 20" strategy and was required to show a different way. She then replied "you could have 7, then you could add 13. It could also equal 20." (W2-1st G-Task#2).

Scenario 12

Task: A teacher brings 24 cookies for her class. Amy's mom brings 12 more cookies. How many cookies does the class have in all?

The student initially tried to solve this task by applying several known facts including $12 + 15 = 27$ and $10 + 15 = 25$, but was interrupted by the teacher's first AS move suggesting her to break down the numbers.

T: *Can you make it break down somehow?*
S: Yeah! Like that other problem that we did.
T: Mm-hmm.
S: 24 [Writes 24 and 12 vertically]
…
S: 36! Cause I know that 4+2=6. So it's gonna be 36.
T: *Is that another way you can write that maybe?*
S: [Writes down $12 + 24 = 36$] 36. There!
T: *Is there any way you can break it down and write it? Do you know?*
S: …If I wanted to do a minus one, then it would be 36 minus 12, or 24.
T: 36 minus what?
S: 36 minus 12 equals 24.
T: How do you know that all equals each other? How do you know that you can flip it around, and that it can do that?
S: Because, um, I know that 24 plus 12 equals 36, so if I did 36 minus 12, it equals 24.
T: Good job.
(W8-1st G-Task#5)

In Scenario 12, the teacher employed AS moves three times, and the student produced (1) a vertical form of the equation $24 + 12 = 36$, (2) another equation $12 + 24 = 36$, and (3) another equation $36 - 12 = 24$ respectively to the three moves. The teacher then inquired about the reasoning by asking "How do you know that all equals each other?" to which the student replied, "Because I know $24 + 12$ equals 36, so if I did $36 - 12$, it equals 24." The teacher praised the student and moved to next task.

The application of arithmetic properties became the focus in Scenario 11 and 12. In Scenario 11, the student essentially applied the commutative property of addition to switch the numbers 7 and 13 as a different way to solve this task. In Scenario 12, the student started with some facts she knew about the given numbers, and the teacher then suggested the "break down" method. The wording in the next question was about "another way you can write it," which elicited the student's knowledge about the addition and subtraction facts that deviated from the contextual features and mathematical relationship of the task. That is, once the number sentence was constructed, the conversation was redirected to the focus of the manipulation of the written equation or the implicit numerical relationship. Hence, the teachers' use of AS moves consistently led to students focusing on the format of written equation rather than linking the numbers and operations to the context of the task.

Discussion and Implications

This study revealed the successes and difficulties of preservice teachers' questioning enacted in their single-student mathematical field experience and provides evidence about the mutual relations between teachers' questions and students' reactions. The literature is clear that, through interacting with students, preservice teachers can not only practice their questioning strategies but also have opportunities to reflect on them (Moyer & Milewicz, 2002; Nicol, 1998; Weiland, Hudson, & Amador, 2014) However, I argue that for preservice teachers for whom this experience is the first time enacting questioning with students, it is imperative that they learn about the practicality and versatility of questioning because "[t]eachers use questions more than any other instructional tool, and good questioning is perceived as a hallmark of good teaching" (Wassermann, 1991, p. 257). Despite a variety of functional moves employed in the mathematical interviews, the scenarios examined in this study shed light on the tendency of teachers to pose questions that lead student to consider numerical expressions and equations decontextualized from the tasks they represent.

While promoting task understanding, preservice teachers habitually repeated tasks verbatim, a move that did not help students rectify misunderstanding, particularly when working with first-grade students. Due to a lack of flexibility in the delivery of intended tasks, preservice teachers were not able to ask effective questions when students experienced difficulty working on the tasks. However, the teachers' use of TC moves did sometimes help students clarify the tasks, particularly when preservice teachers required students to rephrase the mathematical relationship. These successes and difficulties have curriculum implications in relation to field-based activity design and implementation, and teacher educators should consider whether we provide sufficient assistance to help preservice teachers with selecting tasks and enacting them.

In this study teachers were provided with prepared interview protocols and were allowed to modify the tasks in a way that better fit their interviewee's ability and knowledge. In addition to or instead of such protocols, teacher educators should consider providing a frame-based questioning protocol and intervening in teacher-student interactions in interview-based field experiences. This idea echoes the notion Stein, Smith, Henningsen, and Silver (2009) advocated: teacher educators should provide a scaffold for teachers to "allow them to do something that they would otherwise not be able to do" (p. 135). Although preservice teachers in this study had tasks ready for their students, they did not seem well-prepared to implement the tasks they brought into the field. I observed preservice teachers' hesitation about how to better re-pose questions after noticing students' confusion and inability to solve the problem; some preservice teachers showed uncertainty regarding the content of the tasks, such as the correct problem structure and solution in the delivery; moreover, two teachers even admitted in front of their students that the task they posed was difficult for them. If the teachers' understanding about the tasks is uncertain, they will not be able to solidify students' comprehension in their understanding of the task. Enacting TC moves needs to be handled delicately because it could result in deviated from the contextual features and mathematical relationship of the task.

While trying to get students to explain their strategies, preservice teachers successfully employed diverse functional moves, which resulted in diverse responses from students. However, preservice teachers seemed to provide unproductive responses, in which they either repeated what students said or commented with a short acknowledgment. Although the format of follow-up questions could vary under different circumstances (Franke et al., 2009), I found a lack of follow-up questions in preservice teachers' PU moves in this study, particularly in the fourth grade. That is, most preservice teachers granted the students the agency of authority and accepted what the students said. However, I argue that it is beneficial for the teacher to nudge the student further to request more clarification, especially when working with elementary students. At times, students fluently explicated their mathematical thinking with detailed computational procedures. At other times, they were unclear in their explanations and even showed flaws in their computations, but preservice teachers did not push for further explanation. It is critical for preservice teachers to learn how to elicit multiple types of knowledge from students without discouraging their thoughts.

With regard to the MC moves, most successful contextualized connections occurred with fourth-grade students, and a plausible interpretation could be that the fourth-grade students more likely shared a common basis of mathematics language and knowledge with peers and teachers (Cobb, Wood, Yackel, & McNeal, 1992), so they were more capable of making connections between the mathematical elements in use and the solutions they produced. The difficulties normally occurred when the mathematical relationship was diminished (e.g., Scenario 7) or ignored in students' explanations (e.g., Scenario 8 and 9), and preservice teachers were unable to resolve this difficulty through questioning. Because preservice teachers were not encouraged to correct students' mistakes, they used a conservative strategy in their questioning even when receiving irrelevant or wrong connections from students. In some cases, dealing with Start or Change Unknown tasks perplexed younger students (Peterson, Fennema, & Carpenter, 1989), and learning how to support students in solving these types of tasks by enacting questioning is a necessary challenge for preservice teachers.

In this study somepreservice teachers encouraged students to incorporate alternative strategies (including representations) into the existing process either through directly assigning a strategy or overtly inviting "other ways" of solving the problem. Most preservice teachers did not attempt to dictate students' initial problem-solving strategies. Furthermore, for preservice teachers, initiating AS moves also provided students with an opportunity to demonstrate their mathematical thinking through using multiple representations. Promisingly, the AS moves served as an incentive for students to create more ways to solve or represent the posed problem.

However, these moves sometimes resulted in the use of decontextualized number sentences (correct and incorrect) that were not explicitly connected to the context of the task or the process the student used to solve the problem. For example, the teacher employed three AS moves in Scenario 12 and the last one elicited an equation $36 - 12 = 24$, which neither corresponded to the problem structure "join, result unknown" nor matched the student's solution strategy "adding 20 and 10 first and then adding the ones." Although this case has methodological implications for future research, and researchers should investigate the intention behind the enacted moves, a more practical question arises based on this phenomenon: in what way and to what extent can teacher educators provide the assistance preservice teachers need in their early field experiences to cultivate their questioning techniques and optimize their learning to teach?

To better cultivate teachers' ability to enact effective functional moves, educators should encourage teachers to develop and refine their practices with the considerations of key aspects of a task, including contextual features, mathematical relationships, the cognitive demand (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), students' verbal communication skills, the range of methods a student might use to solve a task, and the misconceptions student might have and errors they might make (Stein, Smith, Henningsen, & Silver, 2009). In conclusion, I cited three excerpts from preservice teachers' final portfolios in which they described their questioning experiences in SSMFE:

> Sandy: I asked a lot of fundamental questions such as "How did you so that?" Or "Can you show me how you did that?" to identify which strategies my student used to solve the problem. It was really helpful especially when my student used only her brain to find a solution to the problem (Final portfolio, 2015).

Abby: If I had not asked these fundamental questions, I do not believe she [the student] would have just shown me that she could exhibit commutativity. Because of these questions, I was able to see my student's growth and more of her abilities and understand my student's mathematical understanding at a deeper level (Final portfolio, 2015).

Alisha: It seems that my student knew how to solve problems but only in one specific strategy. It would be great to be able to see if I could probe him to try another way of solving the problem. I want to see if using this move helps my student learn of new ways to solve a problem (Final portfolio, 2015).

I view the SSMFE as a successful activity not only because the teachers recognized questioning as a powerful technique by which they could help students solve most tasks but because they also experienced struggles in their learning to question and then determined to develop their knowledge of learners through questioning. Experiencing these struggles in early field experience and then learning to systematically analyze questions enacted in the teacher-student interactions could help future teachers develop mathematics teaching practices that are responsive to student's thinking and focus on the important mathematical ideas in tasks.

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CHAPTER 7

DISCUSSION AND CONCLUSION

In this chapter, I describe how I examined preservice teachers' questioning practices by classifying their supporting questioning moves within the problem-solving context and further analyze the reorganized interactional turns to scrutinize functional moves between teacher and student in SSMFE interactions. Supported by the findings derived from the data, I discuss the features that emerged from the analyses as an overall phenomenon in terms of the praxis of teacher questioning.

Using an Integrated Framework in a Problem-solving Context

As noted in Chapter 2, I developed an integrated framework based on prior work through a particular procedure: I embedded Jacobs and Empson's (2016) teaching moves into Polya's (1957) four-stage problem-solving process, and then I added a new category, SQM2 inquiring about the child's plan to solve the task, to finalize the question inventory in accordance with the problem-solving stages (see Figure 2-4).

The four original categories of teaching moves—ensuring the child is making sense of the story problem [SQM1], exploring details of the child's existing strategies [SQM3], connecting the child's thinking to symbolic notation [SQM4], and encouraging the child to consider other strategies [SQM5] (see Figure 2-5) in Jacobs and Empson's (2016) framework—were originally proposed by Jacobs and Ambrose (2008) as a result of analyzing 65 teachers interviewing 231 elementary students solving 1,018 story problems. I used this framework to analyze the collected data.

Questioning Moves in Stage I

There is no doubt that the problem solvers must understand the problem and desire its solution in Stage I. Because the SSMFE served as an interactive activity in mathematical problem solving, the factors hampering its accomplishment remained: (1) the problem might not be well selected, such that the level of difficulty is not consistent with the problem solver's capability, or (2) regarding flaws of representation and description, the problem might not be appropriately introduced to the problem solver. When noting students' difficulties in Stage I, teacher participants implemented one or more of five strategies in their questioning practices: (a) changing the numbers but maintaining the original mathematical structure, (b) modifying the mathematical structure of the task, (c) strictly implementing preplanned tasks without considering the child's struggles, (d) instructing instead of assessing the child's thinking, or (f) inviting students to interpret the task by themselves.

In the cases of changing numbers but keeping the structure of the task, preservice teachers took advantage of reducing the students' working load, resulting in successful problem solving. However, it is worth noting that the move of modifying the original mathematical structure exposed students to completely different experiences. The move of modifying the structure of the task deprived students of the opportunity to solve the task designed based on a particular mathematical structure (e.g., an equally sharing task) and more importantly, when the teacher changed the structure of the problem, it was no longer possible to assess the student's understanding of that type of problem. That is, when the structure of the problem was changed, neither the teacher nor the student learns anything about what the student knows about the original type of problem.

In some cases, preservice teachers strictly implemented preplanned tasks without considering the child's struggles and instructing instead of assessing the child's thinking. However, whether students successfully solved the posed task or not, this type of move either overestimated students' cognitive operational abilities or ignored students' role in their problem solving. To effectively but unobtrusively help students in their problem solving, Polya (1957) emphasized that "the teacher should put himself in the student's place…and ask a question or indicate a step that could have occurred to the student himself" (p. 1).

Last, when teachers decided to invite students to interpret the task by themselves, it elicited diverse reactions from students, including (a) maintaining the structure and successfully solving the task, (b) maintaining the structure but failing to solve the task, or (c) changing the structure so that students failed to solve the task. However, I value this questioning move because it was able not only to produce alternative outcomes in students' problem solving but also to enrich preservice teachers' learning regarding the aspects of understanding the learners and enacting questioning. This idea echoes Polya's (1957) suggestion of asking students to repeat the task and "point out the principal parts of the problem" (p. 6).

Questioning Moves in Stage II

Previous question classification systems have lacked corresponding questioning moves in Stage II. Polya (1957) commented that "[i]n fact, the main achievement in the solution of a problem is to conceive the idea of a plan…[that] may emerge…after apparently unsuccessful trials and a period of hesitation" (p. 8). In this study, I found that preservice teachers were: (a) easily accepting an oversimplified reason for the devised strategy, (b) directly suggesting a possible direction to develop a strategy, and (c) asking a “how” instead of a “why” question while trying to discover the rationale behind the proposed strategy. All these phenomena stemmed from the common mathematics knowledge and language that teacher participants shared with the student with whom they were working. For example, when a student claimed that the word “loses” means “subtraction,” the preservice teacher accepted this answer with no questions because this reasoning was logical to the teacher as well. This unquestioning acceptance operated in the cases when teachers directly suggested strategies rather than allowing more time to students and were content with students’ procedural explanations instead of inquiring about the rationale behind students’ explanations. Polya (1957) noted that “the way from understanding the problem to conceiving a plan may be long and tortuous” (p. 8), and I believe that including the move SQM2 in the framework enables researchers to pay special attention to the questioning moves used to inquire about the child’s devising plan in problem solving.

Questioning Moves in Stage III

It is a common conclusion in the literature that teacher questioning has focused on factual knowledge and procedural understanding since the first empirical research conducted by Stevens (1912) in her classroom observations. In this study preservice teachers’ questioning also exclusively elicited students’ procedural knowledge rather than their conceptual knowledge in Stage III, but I would like to discuss two additional features that emerged in the analyses: (1) preservice teachers unwittingly used pronouns or terms that could have different meanings in daily use in their SSMFE sessions; and (2) they neglected unexamined but valuable strategies devised by students while exploring student mathematical thinking. I relate both phenomena in teacher questioning to teachers' immature professional development in noticing students' mathematical thinking (Sherin, Jacobs, & Philipp, 2011). Sherin and van Es (2005) summarized that noticing involves (1) identifying what is important in a situation, (2) making connections between specific interactions and the broader principles, and (3) using what they [teachers] know about the interactional context. For preservice teachers who were working with students on mathematics for the first time, it is not surprising that they had difficulty in refining their use of questions and identifying the important ideas worth pursuing. However, analyzing questioning moves by applying this integrated framework enables not only researchers but also educators and teachers to identify successes and difficulties in teacher questioning and to improve this important teaching technique in practice.

The Features of Functional Moves in Problem Solving

Teacher questioning has pedagogical importance because it elicits "many different kinds of logical operations that might be required to answer a question" (Riegle, 1976). To determine the function of interactional episodes, it is necessary to appraise not only the question asked by teachers but also the answer from students.

Through categorizing all questioning moves (see Figure 3-2, Layer I analysis) and then organizing segments of functional moves (see Figure 3-4, Layer II analysis), I noticed patterns in SSMFE sessions. First, the predominance of the procedural understanding (PU) moves, unsurprisingly, was borne out in teacher-student interactions, and this feature was consistent with the course goal of understanding children's mathematical thinking. However, it is disconcerting to see that some preservice teachers barely responded to student-initiated PU moves. In those cases, they either merely watched students solving the tasks without any reaction, or they passively agreed with what the students performed and said to achieve a solution. While working on arithmetic tasks, students' relational reasoning often remained unarticulated. For example, students may have demonstrated implicit awareness of the commutative property in their arithmetic calculations, but when the teacher did not inquire about the reasoning behind the calculation, the teacher was unable to conclude what the student knew or understood about the commutative property. In this study, I found that teachers' unconditional acceptance of students' PU statements limited the opportunity to help students develop a deeper understanding of particular relationships in arithmetic. Jacobs, Franke, Carpenter, Levi, and Battey (2007) noted that "developing an understanding of this relation in learning arithmetic may go a long way toward preventing common algebraic errors" (p. 261), so it is important to explore ways that teachers can learn to probe students' thinking in similar situations. Therefore, I concluded that learning to elicit legitimate mathematical reasoning that supports students' strategies and solutions through enacting effective questioning is essential in preservice teachers' early field experiences.

Preservice teachers' assumptions about students' learning appeared in the inevitable use of TC moves and the ineffectiveness of MC moves. Both features revealed the gap between what the teacher assumed her student should know and be able to do and what the students really knew and were capable of doing while solving word problems. It was interesting to witness that the preservice teachers were good at clarifying meanings in their everyday conversations with students, but they experienced frustration when trying to explain mathematics tasks when students did not understand them right away. When the routine strategy of restating the task did not resolve students' confusion, preservice teachers were generally unable to use alternative task-posing moves in practice, which could suggest deficiencies in their mathematics content knowledge, pedagogical content knowledge, and the knowledge of learners (Shulman, 1987). With respect to the MC moves, most preservice teachers who worked with fourth graders successfully elicited the connections. To address the struggles participating teachers experienced with the first graders in the SSMFE, I suggest that preservice teachers should expand their MC questioning moves further to connect with prior steps, proposed plans, operations employed, and the manipulatives used or alternative mathematical representations in a broader horizon, rather than being enacted merely to repeat the original task stem and satisfy with the unit of referent numbers. Based on the complexity of this mission, preservice teachers need guidance and opportunities to practice how to realize broader instructional objectives and assist in developing students' power to organize their thoughts in their questioning (Houston, 1938).

Lastly, another salient feature was that preservice teachers seemed to diminish the contextual features or ignore the mathematical relationship of the task while enacting MC and AS moves. In this study, decontextualization refers to the situation in which the students extracted the numbers from the context of the task and focused only on computational operation. The participating students sometimes decontextualized their solutions in problem-solving activities while concentrating on the computational procedures. However, it is worth discussing why teachers were not able to help their students reconnect the solution to the context of the task. First, preservice teachers were not encouraged to correct students' mistakes, so they were generally conservative while enacting questioning. As a result, they accepted whatever their students concluded once the students' answers were correct. Second, some teachers encountered situations in which their students completely misunderstood the mathematical relationship of the task they posed, and they did not do anything to help students make sense of the structure using the context of the problem and eventually posed tasks with a different structure, which their students were able to solve. For example, a teacher who worked with a first grader posed an addition change unknown problem, but her student used the start and result numbers to create a result unknown problem (i.e., instead of solving 7 + ___ = 10, the child solved 7 + 10 = 17). She then tried another set of numbers within the same problem structure (i.e., posing the $12 + \_\_ = 21$ problem), but again the student changed the problem structure to result unknown using the given numbers. The teacher then decided to pose a result unknown problem with the structure of $8 + 7 = \_\_$, which the student was able to solve successfully. In this case, the student solved the tasks in a way he was able to handle, and the teacher initially tried to keep the problem structure but changed the numbers, but the student repeatedly changed the structure (i.e., computing $12 + 21$). As a result, posing the task that the student was able to solve became the teacher's last option. I concluded that teachers need to develop the ability to help students re-contextualize a mathematics task to make it more reasonable and comprehensible. Working one-on-one with students is a particularly valuable opportunity for preservice teachers to learn about and practice recontextualization.

Overall, preservice teachers' development of their questioning skills was an incremental progression. Eleven of the 13 preservice teachers showed improvement in their selection of questions and the extent to which they had to discontinue a particular type of questioning across the enactment of a task. Through better grasping their students' responses and attitude in previous tasks, preservice teachers strived to avoid the mistakes that were noticeably harmful. However, preservice teachers' questioning might not be an accumulated progression in the SSMFE because the student might approach the next task by applying a strategy that required different knowledge and skills from the student, and based on what the student did, preservice teachers had to discard the questioning techniques that were inappropriate and modify previous questions for the specific context.

SSMFE evoked preservice teachers' awareness that some of their questioning moves did not work in the construction of effective functional moves. It was especially difficult for novice teachers to efficiently analyze and effectively modify their questioning along with students' reactions without a systematic and analytical framework. Hence, SSMFE was considered a successful activity to cultivate teacher questioning not only because the teachers recognized questioning as a useful technique they could employ to understand students' mathematical thinking and reasoning behind computational procedures but because they also experienced struggles in their learning to question and then determined to develop their knowledge of learners through effective questioning in their future teaching.

Implications

This dissertation provided evidence of the praxis of preservice teachers' questioning in an early mathematics field experience. The literature showed that relatively few studies focused on preservice teachers' questioning performance (Moyer & Milewicz, 2002; Weiland et al., 2014), limiting our capacity to cultivate preservice teachers' questioning skills, which are "pivotal to the instructional process, [but] teachers do not learn this art from any serious study of questioning strategies during their teacher education programs" (Wassermann, 1991, p. 257).

Implications for Teacher Education

The participants in this study not only needed time to enact questioning but also lacked systematical analysis and an iterative process of reflection on their enacted questions. While they had an opportunity to inquire about students' mathematical thinking in the context of one-on-one interactions, they did not spend much time analyzing students' thinking and their own questioning.

The cultivation of preservice teachers' learning of questioning can be carried out on three aspects:

The contextual aspect. The analysis of participating teachers' questioning performance leads to the first implication: the importance of developing effective questioning ability with careful attention to student responses in the context of mathematical problem solving. CGI tasks provide a promising environment for development these skills because teaching moves, including questioning, are dependent on how well teachers know their students. Both the types of questioning moves and functional moves stem from the stages of students' problem solving, and they are categories for which to examine both preplanned and enacted questioning moves in the context in which students solve mathematical tasks.

The tactical aspect. There are implications related to the implementation of questioning moves to learn children's mathematical thinking. The situated nature of this analysis indicates that how the tasks are selected from the interview protocols and how the questioning moves are enacted in the SSMFE (i.e., change number or structure) can significantly influence PSTs' practices. In addition, this study shows that PSTs do not always follow up their students' thinking with refined, effective questioning moves. Because it is essential for PSTs to develop their questioning moves and reflect on them as they work with students, establishing a mechanism for selecting, enacting, reflecting, and reapplying questioning moves in a dynamic interaction becomes imperative. In short, careful attention should be given to teachers' skills in posing tasks and the ways in which the questioning moves are (re)enacted in a timely and appropriate manner in learning children's mathematical thinking. Simply asking PSTs to reflect is not sufficient; teacher educators need to structure opportunities for PSTs to link their questioning with students' responses and to consider possible alternatives moves.

The instructional aspect. Lastly, this study also has implications related to the role of the methods course instructors in the design of the SSMFE interviews. Through this study, I noticed that the interview protocols and grade levels selected by the instructors played a role in PSTs' questioning. I suggest that instructors explicitly discuss appropriate principles of enacting questioning moves in different conditions, including the mathematics problem structures and students' development in math and language by grade. Furthermore, the methods course instructors can also model appropriate techniques of questioning in a specific context in the SSMFE. In so doing, preservice teachers have an opportunity to expand their questioning inventory and tool kit for their future use.

Overall, teacher questioning is a contextual, situated behavior that relies on the tactic of questioning and when being promised with an assisted-learning opportunity, they may successfully develop the knowledge and abilities to (a) effectively select tasks, (b) precisely enact questions, (c) efficiently extend inquiries, and (d) promptly detect and react to students responses in their questioning.

Implications for Researchers

There exist a variety of question-classification systems that can be employed to investigate how preservice teachers perform questioning in the field. Some researchers adopted predetermined categories for the aspects of the cognitive operations (Sloan & Pate, 1966), action-oriented (Perry et al., 1993), instruction-oriented (Sahin & Kulm, 2008), and formats (Park, 2010). Other scholars reported categories derived from the data (Jacobs & Ambrose, 2008; Moyer & Milewicz, 2002; Weiland et al., 2014). No single best system exists that will be a good fit for all teacher questioning scenarios. One can imagine the result could have been very different from prior studies if I had employed the cognitive process dimension to analyze my data.

My integrated framework allowed me to analyze preservice teachers' questioning practices in the context of students' problem-solving behavior, which allowed me to interweave teacher questions and student responses as a complete interactional scenario and connect teachers' questioning moves to the stages of problem solving: understanding the task, devising a plan, and carrying out the plan. Researchers investigating preservice teachers' questioning practices should also consider the relevant influences, including the interactional context, in addition to the tasks and grade levels, when making sense of teachers' questioning practices.

Future Research

This study cannot necessarily be generalized to different populations because it lacked a large number of preservice teachers from different teacher preparation programs. However, from this investigation, there are multiple approaches researchers can take to develop a better understanding of teacher questioning and seek an appropriate framework that can be used to help preservice teachers analyze and reflect on their questioning moves in practice. Future studies could explore the relationship between preservice teachers' mathematics content knowledge and their praxis of questioning, particularly in the task posing and delivering stage. Ball (1990) found that almost all prospective teachers (including mathematics majors) have difficulty articulating and connecting underlying mathematical concepts, principles, and meanings. She further claimed that "teachers must understand mathematics deeply themselves" (p. 458) to facilitate their pupils' understanding of mathematics concepts and procedures. Future studies could consider teachers' mathematical knowledge for teaching as they try to uncover the reasons preservice teachers hesitate to react to students' mistakes and uncertainty.

Future studies could also explore what specific conditions (e.g., controlling the mathematical tasks) have an influence on the praxis of teacher questioning. Having one teacher one-on-one work with different students at the same grade level on the same tasks could obviate the discrepancy stemming from the tasks. In addition, longitudinal studies that study the same participants could be particularly useful for examining the development of teacher questioning.

Lastly, future research could be conducted in a broader setting than mathematics instruction. For example, mathematics problem solving is a critical component of STEM programs, which reflect NCTM's (1989) recommendation that teachers focus on tasks that encourage students "to explore, to guess, and even to make and correct errors so they gain confidence in their ability to solve complex problems" (p. 5). Solving word problems, working on contextualized tasks, and dealing with real-world problems might require different questioning techniques, and researchers could broaden the horizon of current understanding and application of questioning in education.

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APPENDIX A

<u>Barrow Buddy Interview #2</u>
<u>EMAT3400</u>

Talk moves I want to use:

_______________________________ _________________________________

Some of my planned follow-up questions:

____

____

1. **Partitive Division**

   ____ friends have ____ candies that they want to share equally. How many candies should each friend get?

   Easier (5, 10) Medium (3, 18) Harder (6, 24)

2. **Measurement Division**

   Maria has ____ cookies. She wants to put the cookies into bags with ____ cookies in each bag. How many bags of cookies can she make?

   Easier (12, 3) Medium (20, 5) Harder (27, 3)

3. **Measurement Division with a Remainder**

   Joe has ____ little toys. If he puts ____ toys into each bag, how many bags of toys will he be able to make?

   Easier (13, 4), Medium (17, 5), Harder (25, 7)

4. **Multiplication:**

   Eric brought ____ boxes of cookies to school. There are _____ cookies in each box. How many cookies did Eric bring to school?

   Easier (3, 5) Medium (3, 9) Harder (3, 12)

5. **Partitive Division with a Remainder**

   Jaime was helping put away basketballs in the gym. She had ____ bins to put _____ basketballs in. If she placed an equal amount of basketballs in each bin, how many basketballs should go in each bin?

   Easier (3, 16) Medium (6, 20) Harder (8, 30)

6. **Multistep Problem:**
Tyrell has ____ packages of gum. Each package has ____ pieces of gum. If Tyrell gives ____ pieces of gum to his sister, how many pieces of gum will he have left?

Easier (5, 3, 6)   Medium (6, 4, 14)   Harder (4, 12, 20)

7. **Multistep Problem:**
19 children are taking a mini-bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 7 seats. How many children will have to sit three to a seat, and how many get to sit two to a seat?

8. **Multistep Problem:**
Farmer Brown sent her children to the field to count the number of horses and ducks. Her son returned and said "There are 4 animals!" Her daughter came back and said, "There were 14 legs!" How many ducks and how many horses did Farmer Brown have?

9. **______________________________:**

____

____

____

Easier (____, ____)   Medium (____, ____)   Harder (____, ____)

10. **______________________________:**

____

____

____

Easier (____, ____)   Medium (____, ____)   Harder (____, ____)

11. **______________________________:**

____

____

____

Easier (____, ____)   Medium (____, ____)   Harder (____, ____)

APPENDIX B

Debrief: Multiplication/Division: 2/3/2015

1. Which problems appeared *less* difficult for the child? Why do you think that was?

2. Which problems appeared *more* difficult for the child? Why do you think that was?

3. Were you surprised by anything? If so, what? If not, why not?

4. What did you learn (about the student, about yourself)?

5. What did you struggle with?

6. If you could re-do this interview, would you do anything differently? If so, what would you do differently and why?

7. What additional problems would you like to pose or re-ask your Barrow Buddy during our next session? Why? For example, would you change the context in some way to make it more accessible to your Buddy? Use different number choices?

APPENDIX C

SSMFE Mathematical topics with exemplary tasks

1) Base-N
In the fourth-grade class: A Base-4 math task; in the first-grade class: A Base-5 math task
Exemplary task: The Luck 5 Candy Factory task (1st Grade) At the lucky 5 candy factory, machines do the packaging. At this factory, there are the following types of packaging: individual candies, rolls of candies, boxes of candies, and crates of candies. The machines look for groups of 5. This means that every time a machine sees a 5, they put the candies into a container of the next size. So, every time they see 5 individual candies, the machines put them into a roll. Rolls can only hold 5 candies. Every time the machines see 5 rolls, they put them into a box. Boxes can only hold 5 rolls. And every time the machines see 5 boxes, they put them into a crate. Crates can only hold 5 boxes. Question #1: 9 candies were dumped onto a conveyer belt below. How many of each type of package should we expect to see after the machines finish packaging the candies?

2) Base Ten/Place Value
In the fourth-grade class: Multi-digits operation tasks In the first-grade class: Two-digit adding the multiples of 10 tasks, and place value tasks
Exemplary task: Base ten subtraction (4th Grade) Nicky has 2001 points on his favorite video game. He forgets to save the game before turning it off, and he loses 956 points. How many points does he have now? Solve the problem in two ways.

3) Number Facts
Exemplary task: ____ + ____ = 11 45=25+20=___ + 15 8+9=___ (naked number fact problem)
4) Fractions or equally sharing
Exemplary task (1st Grade): If I am having a party with 10 friends. We are splitting 3 cookie cakes. How much of the cookie cakes would each friend get?

5) CGI problems types

Join	*(Result Unknown)* Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have all together?	*(Change Unknown)* Connie has 5 marbles. How many more marbles does she need to have 13 marbles all together?	*(Start Unknown)* Connie had some marbles. Juan gave her 8 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate	*(Result Unknown)* Connie had 13 marbles. She gave 8 to Juan. How many marbles does Connie have left?	*(Change Unknown)* Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	*(Start Unknown)* Connie had some marbles. She gave 8 to Juan. Now she has 5 marbles left. How many marbles did Connie have to start with?
Part-Part-Whole	*(Whole Unknown)* Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?		*(Part Unknown)* Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?
Compare	*(Difference Unknown)* Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	*(Compare Quantity Unknown)* Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	*(Referent Unknown)* Connie has 13 marbles. She has 8 more marbles than Juan. How many marbles does Juan have?
Grouping	*(Multiplication)* Bart has 4 boxes of pencils. There are 6 pencils in each box. How many pencils does Bart have all together?	*(Measurement Division)* Bart has 24 pencils. They are packed 6 pencils to a box. How many boxes of pencils does he have?	*(Partitive Division)* Bart has 6 boxes of pencils with the same number of pencils in each box. All together he has 24 pencils. How many pencils are in each box?

Sources: https://elemath.hallco.org/web/wp-content/uploads/2015/06/CGI-Problem-Types.pdf