SOLUTION TO THE MULTIPLE REGION DYNAMIC RATIONAL EXPECTATIONS

COMMODITY MARKET MODEL: APPLICATION TO A COTTON MARKET

by

OLEKSIY TOKOVENKO

(Under the Direction of Lewell F. Gunter)

ABSTRACT

Modeling international commodity markets is a complicated issue in economics. Contemporary models suggest a variety of strategies to solve for optimal policies. Although they have proved to be efficient in many aspects of theoretical analysis, certain limitations always exist in applications to the real world problems. In this study we developed a multiple region dynamic rational expectation commodity model that is in general more flexible than conventional ones. The essential proposition on separability of solutions for given policy functions was made. A successive approximation algorithm was used to obtain an approximate solution to a model designed. The results support main assumptions of the model. However, the algorithm is characterized by a slow convergence, which limits the results obtained so far.

INDEX WORDS: commodity market model, numerical methods, rational expectations, successive approximation

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Присвячується моїм батькам

Dedicated to My Parents

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CHAPTER 1

INTRODUCTION

1.1. Background

Commodity markets are complex systems. They are characterized by highly volatile commodity prices behavior, seasonal demand and supply schemes, complicated government market intervention programs, and policy regulations. The contemporary theory of commodity markets tries to model the behavior of commodity prices to explain the factors that generates the price fluctuations and thus to be able to make predictions of future prices. The commodity prices are the conventional indicator of the state of a market, both current and future. They are also the most useful tool for the market regulation.

There are a variety of approaches that are currently used to find empirical support for the theoretical foundations of commodity market processes. Most of them utilize complicated econometric techniques and generate results that with a certain confidence could provide explanations of specific characteristics of real markets. However, these approaches are based solely on past information and thus generally the results obtained are backward looking. This means that the success of forecasts or policies designed from econometric analysis will be conditional on the confined relevance of conditions observed in the past. At this point it seems to be really important to employ a forward looking methodology that can explain the behavior of the market itself, and reveal the most important linkages between the commodity market elements. In other words one needs to understand the underlying process of the market activity to make a consistent analysis of the real world problem.

Recent literature on commodity market models introduces a variety of methodologies for solving the problems by numerical methods. Although many problems associated with the solution of large scale commodity models can be solved relatively easily now, most studies are still oriented on one region policy optimization that requires solving for the quite limited number of state variables and correspondent strategies. Complicated market structures that include public and speculative storage, seasonality and non-stationarity are brought in for analysis, but, they are often inapplicable to international markets. Any economic process clearly specified and modeled locally generates a net of interactions in multiple region environments. Providing that the model should be solved for all regions simultaneously, the optimization task then is formulated as a huge system of nonlinear equations, the solution to which transforms the optimal planner's problem into an unstable and burdensome procedure.

Multiple region models currently developed have proved to be efficient in general. However, often they assume certain trade offs that make them limited to specific cases. Hence it is vitally important to find a way to develop a more general framework that can use conventional optimality principles, but at the same time can be flexible enough to utilize only a limited number of model restrictions.

1.2. Objectives

The current study has three main objectives:

- develop a generalized theoretical framework for the multiple region dynamic rational expectations model and its solutions, that allows the relaxation of conventional limiting assumptions;
- discuss the main issues that arise in the process of application of the solution procedure to models of real world markets;
- obtain an approximate solution to the selected market model, check the assumptions that were utilized in theoretical developments, and analyze the implications of results.

1.3. Organization

The organization of this thesis is as follows. In the first chapter a short statement on the background of a problem that is the subject of our study is provided. Chapter 1 also defines the general idea and the specific objectives of this research. The literature review in Chapter 2 summarizes the main developments made in modeling the problem of commodity market policy optimization, and recent advances in the field of stochastic dynamic modeling. The general theoretical model framework and numerical solution strategy are presented in Chapter 3. This is the most essential part of the study that reveals the opportunity to improve contemporary international commodity models. The next Chapter discusses the step by step approach in design and implementation of the solution algorithm proposed in Chapter 3. Chapter 5 is oriented to the analysis of the results found from the empirical application of the model. The last part of the thesis also includes the possible implications and limitations of the developed model framework.

CHAPTER 2

LITERATURE REVIEW

2.1. One region models

A variety of commodity models that studied the price storage relationship can be found in economic literature before 1950s. Most utilize a simplified market framework and thus return quite a generalized result.

Gustafson (1958) first introduced dynamic programming methods in the field of modeling commodity markets. Gustafson suggests deriving the optimal policy rules as the instrument of optimal control, rather then finding equilibrium quantities. This work proves an equivalence of social value function maximization as an optimal planner's strategy to profit maximization as a competitive market approach in an undistorted environment. This is the first research that provides the fundamental mathematical model for the optimal decision process in commodity markets. The type of model introduced emphasizes nonlinearity in the storage – expected price relationship as the result of the condition that aggregate storage cannot be negative. Although Gustafson (1958) obtain the results for a certain environment and stationary consumption, which makes the empirical part produce quite an approximate solution, this work is classical in a sense of the approach introduced. Many later researchers benefited from it by utilizing the main principles and optimality conditions derived almost fifty years ago.

The subject of Gardner (1979) is a strategy of optimal stockpiling of grains. Gardner studies an uncertain environment, with extensions to the Gustafson model such as non-stationarity of model parameters, stochastic demand, deviations in production and elastic supply response. This work provides a good idea of solution strategy, however it utilizes a linear quadratic specification and thus the results are not accurate.

Newbery and Stiglitz (1981, 1982) provided a comprehensive theoretical work on commodity price stabilization. This research employs algebraic methods to study all possible aspects of different price stabilization schemes under uncertainty. The authors use both partial and general equilibrium analysis. They also represent a dynamic analysis of buffer stock schemes and application of theory to the theoretical framework for the problems of stabilizing agricultural commodity markets.

Williams and Wright (1991) presents the synthesis of the modern theory of competitive storage and the classical model by Gustafson. They combine spatial market clearing conditions with the intertemporal arbitrage equation of the classic model and rational expectations hypothesis.

The recent literature on commodity markets theory often utilize numerical solution methods based on a functional approximation method to replace the original functional equation problem with a finite dimensional problem in the form of a system of nonlinear equations.

Williams and Wright (1991) introduce a successive approximation algorithm based on a curve fitting technique. Miranda and Glauber (1995) presented an advanced version of it based on a Chebychev orthogonal collocation method.

Deaton and Laroque (1992, 1995, 1996) use maximum likelihood estimation of a nonlinear dynamic rational expectations commodity model.

Some of the latest applications of the rational expectations storage models to commodity-specific markets have focused on simulating policy scenarios (Miranda and Helmberger (1988), Miranda and Glauber (1993), Ng (1996), Ng and Ruge-Murcia (2000), Michaelides and Ng (2000)).

Although most of studies utilize annual models, other models also exist (Peterson and Tomek (2000), Lowry *et al* (1987)).

Carter and Revoredo (2000) study interactions between working and speculative stocks. Many other researches introduce public storage (Gardner (1979), Williams and Wright (1991), Miranda and Glauber (1993)).

2.2. Multiple region models.

Gustafson (1958) derives modifications of his approach in application to the several regions international grain market. Although results of his works are considered to be classical, they mainly elaborate on the deterministic type model.

Gardner (1979) provides an application of his model to the study of the world wheat market. An important advance made in his work is introduction of international market policy optimization from the point of view of a single country, taken the policy environment in the rest of the world as given. Gardner points out that the regions (countries) should not necessary adopt a worldwide consistent set of policies.

Williams and Wright (1991) presents a comprehensive section on trade storage relations, where the two region model framework is discussed. Their version of extension of the one country algorithm utilizes the conventional optimality conditions. The authors also introduced an additional spatial condition for future prices.

The latest versions of multiple region models are basically presented by Miranda, who efficiently combined space, time and uncertainty in one model. The approach presented in Miranda and Glauber (1995) is demonstrated to be extremely effective, however their empirical example is based on artificial data.

Makki, Tweeten and Miranda (1996) present probably the only study that with enough confidence is successful in explanation of the behavior of a real market. Still, it also has certain limitations, as the authors utilize an assumption on specified export or import orientation of countries, which is not always the case.

CHAPTER 3

GENERAL FRAMEWORK

3.1. Theoretical framework

This section presents a three-region dynamic world cotton market model. In each period any region can be either a net exporter or net importer. The trade flows are assumed to occur from the exporters to the importers, with no trade between homogenous types of agents. In any period *t*, the supply q_t^i initially available in any region *i* is composed of a carryover from the preceding period s_{t-1}^i and new production, which is determined by an exogenous random yield $\tilde{y}_t^i$ on the acreage a_{t-1}^i, planted the preceding period:

$$q_t^i = a_{t-1}^i \tilde{y}_t^i + s_{t-1}^i, \qquad \forall i \qquad (1)$$

The region must allocate total supply available q_t^i among consumption c_t^i, future storage s_t^i and amount traded $\sum_j x_t^{ij}$:

$$q_t^i = c_t^i + s_t^i + \sum_j x_t^{ij}, \qquad \forall i, \quad \forall j \neq i, \qquad (2)$$

where *ij* denotes a trade flow from region *i* to region *j*.

If $q_t^i > c_t^i + s_t^i$, one observes an excess supply in the region *i*, which is then a net exporter ($\sum_j x_t^{ij} > 0$). Otherwise, in case of $q_t^i < c_t^i + s_t^i$, region *i* is a net importer ($\sum_j x_t^{ij} < 0$). It is also possible for $q_t^i = c_t^i + s_t^i$, which corresponds to a closed economy, but it has a low probability to occur in the model.

The resulting spatial equilibrium is summarized in the following material balance equation:

$$\sum_i q_t^i = \sum_i c_t^i + \sum_i s_t^i + \sum_i \sum_j x_t^{ij} \,, \qquad \forall i \,, \quad \forall j \neq i \tag{3}$$

Clearly, at least one net exporting and one net importing region should exist at any given period *t* (neglecting the situation when all three agents are closed economies), for the world market to be balanced. Hence, the model has to satisfy the following condition:

$$\sum_i \sum_j x_t^{ij} = 0 \,, \qquad \forall i \,, \quad \forall j \neq i \tag{4}$$

Then, necessary conditions for the spatial equilibrium to hold are those described by equations (2) and (5):

$$\sum_i q_t^i = \sum_i c_t^i + \sum_i s_t^i \,, \qquad \forall i \,, \quad \forall j \neq i \tag{5}$$

Following Makki, Tweeten and Miranda (2001), we define this specification as one that assumes no losses in storage and no qualitative differences between the stored commodity and the freshly harvested commodity.

Current consumption (use) level c_t^i of commodity in region *i* is a strictly decreasing function of the market clearing price p_t^i:

$$c_t^i = \alpha^i (p_t^i)^{\beta^i} \,, \qquad \forall i \,, \tag{6}$$

where $\alpha^i > 0$ is a constant term of region *i* demand equation, $\beta^i < 0$ is the price elasticity of demand in region *i*.

Consumption in any region *i* is assumed to be nonstochastic and consumer's income is assumed to be constant.

The acreage a_t^i planted by rational producers in region *i* is a strictly increasing function of the price expected at harvest time ($E_t[p_{t+1}^i]$):

$$a_t^i = \gamma^i (E_t[p_{t+1}^i])^{\eta^i}, \qquad \forall i, \tag{7}$$

where $\gamma^i > 0$ is a constant term of region *i* acreage response equation, $\eta^i > 0$ is the price elasticity of acreage response in region *i*.

We generalize the perfect foresight assumption of the model by assuming that the expectations are formed in the sense of Muth (1961). The rationality assumption implies that the price expectations formed by storers and producers in the model are consistent with the stochastic price distributions implied by the model.

To solve the optimization problem, we construct the welfare measure which will reflect the corresponding social value of the commodity available to the different types of agents — consumers and suppliers. The current social welfare is a sum of consumer and producer surpluses induced by their decisions on the action variables in period *t*:

$$V_t^i = VC_t^i + VSUP_t^i, \qquad \forall i \tag{8}$$

The current consumer surplus VC_t^i in any region *i* is measured by the area under the demand curve minus the revenue, generated by the commodity consumed at the current equilibrium price $p_t^i(c_t^i)$:

$$VC_t^i(q_t^i, s_t^i, x_t^i) = \int_0^{c_t^i = q_t^i - s_t^i - x_t^i} p_t^i(c_t^i)dc - p_t^i(c_t^i)c_t^i, \qquad \forall i \tag{9}$$

Alternatively, the current consumer surplus may be interpreted as the present social value of the commodity consumed to the consumer type of agents.

The model assumes that the consumers do not differentiate commodity on the basis of quality, political considerations, historical trading patterns and the country or origin. We use homogeneity of commodity purchased to imply the homogeneity of commodity consumed. Then we can treat the consumer surplus as unique, rather then sum of consumer value of commodity produced in the region *i* and imported.

Under the assumption of zero income elasticity the Marshallian surplus is equivalent to the Hicksian compensating and equivalent variation each period *t*. The model is thus not affected by the intertemporal change in consumer income and its structure does not need to elaborate on the various income effects. The assumption of zero income elasticity is made solely for the simplicity purposes and may be relaxed relatively easily.

The general model structure implies that supply side of the market is represented by the three types of agents: producers, storers and traders (importers/exporters). Each group of agents gains a specific surplus corresponding to the activity it conducts.

The current producer surplus VP_t^i in any region *i* is measured by the profit from the production activity of farmers. Producer receives revenue generated by the commodity produced at the current equilibrium price $p_t^i(c_t^i)$, discounted for the production cost:

$$VP_t^i(q_t^i, s_t^i, x_t^i) = p_t^i(c_t^i) a_{t-1}^i (E_{t-1}[p_t]) y_t^i - a_{t-1}^i (E_{t-1}[p_t]) \omega_t^i, \qquad \forall i, \qquad (10)$$

where ω_t^i is a fixed per acre production cost.

Each period *t* storer pays the total cost of commodity purchased to carry into the next period plus the total cost of storage of the amount to store. These costs compose the negative side of the storer balance. Each period the storer's gain is equal to the revenue generated from selling the commodity stored the preceding period at the current equilibrium price $p_t^i(c_t^i)$. Then, the present storer surplus VS_t^i in any region *i* is measured by the profit from the competitive storage:

$$VS_t^i(q_t^i, s_t^i, x_t^i, s_{t-1}^i) = p_t^i(c_t^i) s_{t-1}^i - p_t^i(c_t^i) s_t^i - K_t^i(s_t^i), \qquad \forall i, \qquad (11)$$

where $K_t^i(s_t^i)$ is the region *i* storage cost function or the current value of commodity stored.

According to the theory of price of storage, the total cost of storage consists of three components (Brennan 1958): the total physical cost of storage $O_t^i(s_t^i)$, the total risk-aversion factor $R_t^i(s_t^i)$ and the total convenience yield on stocks $CY_t^i(s_t^i)$, so that the storage cost function has the following form:

$$K_t^i(s_t^i) = O_t^i(s_t^i) + R_t^i(s_t^i) - CY_t^i(s_t^i)\,, \qquad \forall i \tag{12}$$

The total physical cost of storage includes rent for storage space, handling charges, interest, insurance, etc. The planner can meet an unexpected raise in consumption. This generates the convenience yield which can be considered to be the opportunity cost of holding some working stocks of commodity and, alternatively, mobilizing the resources to meet the unexpected demand shock. Once the stock rises above the certain high level convenience yield goes to zero. This level of commodity storage amount may be naturally interpreted as the maximum level of working stocks to have. Brennan (1958) suggests expecting total risk aversion to be an increasing function of stocks. If planner holds a comparatively small amount of stocks than the risk involved in storage decisions, namely investment in stocks, is also small. This implies that the third component of storage cost function — marginal risk aversion factor — is expected to small as well. Respectively, holding high level of storage increases the possible economic losses if unexpected fall in future prices take place. In this case, the marginal risk aversion factor increases as the stock goes over the certain level.

Both marginal physical costs $o_t^i(s_t^i)$ and marginal risk-aversion factor $r_t^i(s_t^i)$ are either constant or increasing functions of s_t^i, $o_t^i(s_t^i) > 0$, $(o_t^i(s_t^i))' \geq 0$, $r_t^i(s_t^i) > 0$, $(r_t^i(s_t^i))' \geq 0$. The marginal convenience yield is decreasing function of s_t^i, $cy_t^i(s_t^i) < 0$, $(cy_t^i(s_t^i))' \geq 0$. The net marginal cost of storage may be written as:

$$k_t^i(s_t^i) = o_t^i(s_t^i) + r_t^i(s_t^i) - cy_t^i(s_t^i), \qquad \forall i \tag{13}$$

The net carrying charge of storage $k_t^i(s_t^i)$ is specified as the difference between current and expected prices.

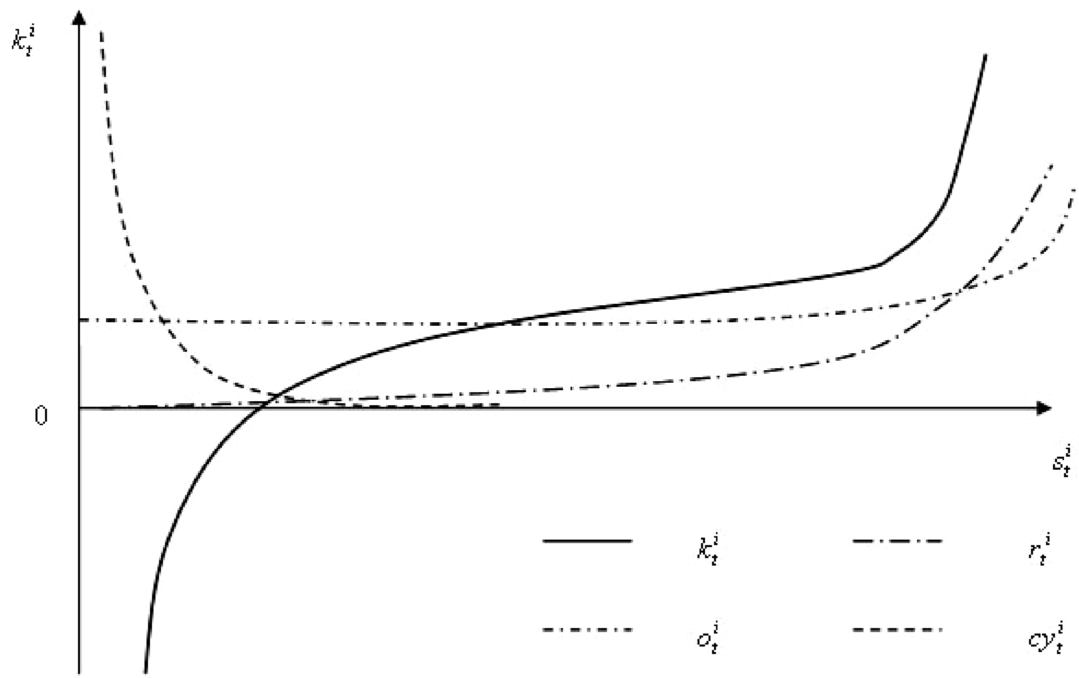

Figure 3.1. General form of storage cost function.

Source: Brennan (1958).

Finally, the present trader (exporter/importer) surplus VXM_t^i in any region *i* is measured by the profit from the competitive trade activity:

$$VXM_t^i(q_t^i, s_t^i, x_t^i) = \sum_j (-p_t^i(c_t^i)x_t^{ij} + p_t^j(c_t^i)x_t^{ij} - \tau_t^{ij}x_t^{ij}), \tag{14}$$

$\forall i$, $\forall j \neq i$

where τ_t^{ij} is per unit of commodity traded fixed cost of transportation from region *i* to region *j*.

We use the assumption of symmetry of the transportation cost and export flows which implies that $\tau_t^{ij} = \tau_t^{ji}$ and $x_t^{ij} = x_t^{ji}$.

The equation of a trader surplus assumes two situations. The first, when the agent is a pure exporter and x_t^{ij} is positive, defines the current surplus as the difference between the total cost of commodity purchased at the home equilibrium price p_t^i from the home producer for export plus the total cost of transportation and the revenue generated by the amount of commodity traded at the foreign equilibrium price p_t^j . The second case, when the agent is a pure importer and x_t^{ij} is negative, defines the current surplus as the difference between the total cost of commodity purchased at the price $p_t^j + \tau_t^{ji}$ from the foreign importer and the revenue generated by the amount of commodity purchased at the home equilibrium price p_t^i , when is sold to the home consumer.

Now assume, for simplicity, that we have only one representative supplier, who simultaneously conducts production, storing and trade. Then, the current supplier surplus $VSUP_t^i$ is simply the sum of surpluses generated by the activities stated above:

$$VSUP_t^i = VP_t^i + VS_t^i + VXM_t^i \qquad (15.1)$$

$$VSUP_t^i(q_t^i, s_t^i, x_t^{ij}) = p_t^i(c_t^i)c_t^i - a_{t-1}^i(E_{t-1}[p_t])\omega_t^i - K_t^i(s_t^i) -$$

$$-\sum_j \tau_t^{ij} x_t^{ij} + \sum_j p_t^j(c_t^i) x_t^{ij} , \qquad \forall i , \quad \forall j \neq i \qquad (15.2)$$

Combining (9) and (15.2) we obtain the equation for the current social surplus (welfare):

$$V_t^i = VC_t^i + VSUP_t^i \tag{16.1}$$

$$V_t^i(q_t^i, s_t^i, x_t^{ij}) = \int_0^{c_t^i = q_t^i - s_t^i - x_t^i} p_t^i(c_t^i)dc - a_{t-1}^i(E_{t-1}[p_t])\omega_t^i - K_t^i(s_t^i) -$$

$$-\sum\nolimits_j \tau_t^{ij} x_t^{ij} + \sum\nolimits_j p_t^j(c_t^i) x_t^{ij}, \qquad \forall i, \qquad \forall j \neq i \tag{16.2}$$

Planner observes the state of market q_t^i, takes actions $s_{t=0}^i$ and $\sum_j x_t^{ij}$ from which society earns a reward $V_t^i(q_t^i, s_t^i, x_t^{ij})$. The planner optimization problem is to select the storage $s_{t=0}^i$ and amount traded $\sum_j x_t^{ij}$ (decision variables), such that they will maximize the discounted stream of expected future surplus (welfare) W^i in the infinite time horizon, given the initial supply $q_{t=0}^i$.

$$W^i(q^i) = \max_{s^i \in S^i \geq 0, x^{ij} \in X^{ij}} E\sum_{t=0}^{\infty} \delta_t^i \left[V_t^i(q_t^i, s_t^i, x_t^{ij})\right], \tag{17}$$

$$\forall i, \qquad \forall s^i \geq 0, \qquad \forall j \neq i,$$

where δ_t^i is the region *i* discount factor.

We use the stationarity assumption, specified by Gustafson (1958), as the situation when the value function, storage cost function, discount rate and frequency distribution of yield are expected to remain unchanged in future. The model is constructed to have no growth or seasonality.

In its general form, the equilibrium payoff function (W^i) for each region *i* should satisfy Bellman's principle of optimality (Bellman 1959):

$$W^i(q^i) = \max_{s^i \in S^i \geq 0, x^{ij} \in X^{ij}} [V^i(q^i, s^i, x^{ij}) + \delta^i E_{\tilde{y}} W^i(g^i(s^i, \tilde{y}^i)], \tag{18}$$

$$\forall i, \qquad \forall s^i \geq 0, \qquad \forall j \neq i$$

where $g^i(s^i, \tilde{y}^i)$ is the state variable transition function. The state of market is a controlled Markov process. Our model implies that the state of market — initial supply of commodity q^i_{t+1} available in the next period $t+1$ — depends on the storage decision s^i_t made in period *t* and an exogenous random yield $\tilde{y}^i_{t+1}$ that is unknown in period *t*:

$$g^i = a^i_{t-1}\tilde{y}^i_t + s^i_{t-1}, \qquad \forall i \tag{19}$$

Here, as the planner's decision problem has an infinite horizon, the payoff functions do not depend on time *t* and the Bellman equations has the form of functional fixed-point equation.

The equilibrium conditions for discrete time, continuous state, continuous choice Markov decision problems are derived by applying the Karush-Kuhn-Tucker and Envelope Theorem to the optimization problem embedded in the Bellman equation; for more details on continuous state dynamic programming see Miranda (2002, Chapter 8).

We assume that actions are unconstrained. More specifically, decisions on the amount of commodity traded may be positive, negative, or equal to zero, which correspond to export, import and no-trade situation; storage decisions may be non-negative only, by definition. However, by including a convenience yield and risk payment into the storage cost function, we design the model to perform no-stock out and no-overstock scenarios, which implies optimal storage decisions to be bounded indirectly.

The Karush-Kuhn-Tucker condition for the presented unconstrained optimization problem imply that the optimal actions s^i and x^{ij}, given state of market q^i, satisfy the following equimarginality conditions:

$$\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial s^i} + \delta^i E_{\tilde{y}}[\lambda(g^i(s^i, \tilde{y}^i))\frac{\partial g^i(s^i, \tilde{y}^i)}{\partial s^i}] = 0, \tag{20.1}$$

$\forall i$, $\forall j \neq i$,

and

$$\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial x^{ij}} + \delta^i E_{\tilde{y}}[\lambda(g^i(s^i, \tilde{y}^i))\frac{\partial g^i(s^i, \tilde{y}^i)}{\partial x^{ij}}] = 0 , \tag{20.2}$$

$\forall i$, $\forall j \neq i$,

where λ is the marginal value of initial supply available to the planner or, using the terminology of optimal control, "shadow" price of initial supply q^i :

$$\lambda(q^i) \equiv \frac{\partial W^i}{\partial q^i} \tag{21}$$

Applying the Envelope Theorem to the same optimization problem results in:

$$\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial q^i} + \delta^i E_{\tilde{y}}[\lambda(g^i(s^i, \tilde{y}^i))\frac{\partial g^i(s^i, \tilde{y}^i)}{\partial q^i}] = \lambda(q^i) , \qquad \forall i \tag{22}$$

In our application, the state transition depends only on the action taking by the planner, so that:

$$\frac{\partial g^i(s^i, \tilde{y}^i)}{\partial q^i} = 0 , \qquad \forall i \tag{23}$$

In this case we may substitute the expression derived using the Envelope Theorem (22) into the expressions derived using the Karush-Kuhn-Tucker conditions (20). This procedure eliminates the shadow price function as an unknown and simplifies the Euler conditions into two functional equations in two unknowns, the optimal trade x^{ij} and storage s^i policies:

$$\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial s^i} + \delta^i E_{\tilde{y}}[\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial q^i}\frac{\partial g^i(s^i, \tilde{y}^i)}{\partial s^i}] = 0 , \tag{24.1}$$

$\forall i$, $\forall j \neq i$

and

$$\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial x^{ij}} + \delta^i E_{\tilde{y}}[\frac{\partial V^i(q^i, s^i, x^{ij})}{\partial q^i} \frac{\partial g^i(s^i, \tilde{y}^i)}{\partial x^{ij}}] = 0, \qquad (24.2)$$

$\forall i, \qquad \forall j \neq i$

Applying the Euler equations (24.1) and (24.2) to the planner optimization problem (18) results in:

$$-p^i(q^i - s^i - x^i) - k^i(s^i) + \delta^i E_{\tilde{y}}[p^i] = 0, \qquad \forall i, \qquad \forall s^i \geq 0 \qquad (25.1)$$

and

$$-p^i(q^i - s^i - x^i) - \tau^{ij} + p^j(q^j - s^j - x^j) = 0, \qquad (25.2)$$

$\forall i, \qquad \forall s^i \geq 0, \qquad \forall j \neq i$

The first equation is the intertemporal arbitrage condition of the social planner *i* that may be represented as:

$$\mu(s^i) = -p^i(c^i) - k^i(s^i) + \delta^i E[p^i], \qquad \forall i, \quad \forall s^i \geq 0 \qquad (26)$$

where $\mu(s^i)$ is the marginal profit from storage.

The optimal choice for the amount to store corresponds to the zero marginal profit, which satisfies Pareto optimality criterion. Discounted future price cannot exceed the current price by more than marginal cost of commodity stored. Otherwise, the expected profit opportunities from storing the commodity will exist in the economy, lowering the expected future price and raising the current price. The negative expected marginal profit will prevent agents from storing the commodity above the certain optimal level as it will cause economic losses. This results in the following optimality condition:

$$\delta^i E[p^i] = p^i(c^i) + k^i(s^i), \qquad \forall i, \qquad \forall s^i \geq 0 \qquad (27)$$

The second equation is the spatial arbitrage condition of the social planner *i* that may be represented as:

$$\chi(x^{ij}) = -p^i(c^i) - \tau^{ij} + p^j(c^j), \qquad \forall i, \qquad \forall j \neq i \qquad (28)$$

For the decision on the amount of commodity traded to be optimal the marginal profit from trade activity should be equal to zero, i.e. the higher price cannot exceed the lower price by more than marginal cost of transportation. Otherwise, the spread between the low price of export $p^i(c^i) + \tau^{ij}$ and the high import price $p^j(c^j)$ will create additional profit opportunities and generate extra commodity flows from low-price region to a high-price region, until the difference between prices evolves. The negative marginal profit will prevent traders from negotiating on the amount over the equilibrium quantity, as it will cause economic loses for them. This results in the following spatial arbitrage complementarity slackness conditions for exporter (29.1) and importer (29.2):

$$p^i(c^i) + \tau^{ij} \leq p^j(c^j) \perp \left|x^{ij}\right| \geq 0, \qquad \forall i, \quad \forall j \neq i \qquad (29.1)$$

$$p^j(c^j) + \tau^{ji} \leq p^i(c^i) \perp \left|x^{ji}\right| \geq 0, \qquad \forall i, \quad \forall j \neq i \qquad (29.2)$$

The formulation as a planner's problem defines the storage and trade amount as control variables. In a stochastic environment, the planner cannot foresee the yield shock and thus cannot predict the exact size of future harvests. He must instead anticipate his strategy for future storage and trade decisions conditional on future supply. Employing the terminology of dynamic programming, the planner needs an optimal decision rule — an optimal relationship between the current storage, amount traded and availability of commodity at the beginning of period. Those relationships are defined in Williams and Wright (1991) as storage and trade rule.

Define the functions f_s^i and f_x^i that return equilibrium storage level s^i and amount traded x^{ij} as a function of supply q^i currently available in region *i*:

$$s^i = f_s^i(q^i = a^i \tilde{y}^i + s_{t-1}^i), \qquad \forall i \tag{30.1}$$

$$x^{ij} = f_x^i(q^i = a^i \tilde{y}^i + s_{t-1}^i), \qquad \forall i \tag{30.2}$$

For the infinite planning horizon, the storage rule is stationary; for more details on stationarity of optimal rules in infinite horizon dynamic problems see Bertsekas (1987, Chapter 5). As a result, the relation between the average price the next period and future availability $E_t p_{t+1}^i(a_t^i \tilde{y}_{t+1}^i + s_t^i, a_t^{j1} \tilde{y}_{t+1}^{j1} + s_t^{j1}, a_t^{j2} \tilde{y}_{t+1}^{j2} + s_t^{j2})$ is stationary.

If private storers believe this relationship to be a particular function, through their collective actions $E_t p_{t+1}^i(a_t^i \tilde{y}_{t+1}^i + s_t^i, a_t^{j1} \tilde{y}_{t+1}^{j1} + s_t^{j1}, a_t^{j2} \tilde{y}_{t+1}^{j2} + s_t^{j2})$ should in fact be that function. In other words, their expectations should be self-fulfilling, that is, "rational" by definition.

To capture the rationality assumption about the acreage allocation decision process algebraically let us define the λ^i function that gives the expected equilibrium price in any region *i* in terms of the initial supplies q^i, q^{j1} and q^{j2} available in region *i*, *j1* and *j2*, at the beginning of period $t+1$ respectively. Knowing the equilibrium price functions for all three regions, λ^i, λ^{j1} and λ^{j2}, the expected prices implied by the model could be computed by integrating over the yield distributions (Miranda and Glauber):

$$E_t p_{t+1}^i = E_t \lambda^i (a_t^i \tilde{y}_{t+1}^i + s_t^i, a_t^{j1} \tilde{y}_{t+1}^{j1} + s_t^{j1}, a_t^{j2} \tilde{y}_{t+1}^{j2} + s_t^{j2}) \tag{31}$$

$$\forall i \qquad \forall j \neq i$$

Combining the necessary conditions derived above, namely equations (2), (6), (7), (27), (29), (31), results in the system of nonlinear equations that describes nonlinear rational expectation commodity model designed in this study. Now we have six unknown variables c_t^i,

s_t^i, a_t^i, p_t^i, x_t^{ij} and $E_t[p_{t+1}^i]$, one predetermined endogenous variable q_t^i for each region *i* and six corresponding conditions that determine the values of those unknowns. In dynamic environment, four of all unknowns — c_t^i, x_t^{ij}, s_t^i, a_t^i — are formally the subject to planner's decision, in other words, they should be considered as the planner's "control" variables. Two other unknowns — the current p_t^i and expected $E_t[p_{t+1}^i]$ prices — characterize the state of market and are the response of a system to the planner's actions. This fact implies that the values of control variables chosen optimally will result in optimal response of a market through the establishment of the equilibrium prices best possible. In mathematical sense it means the sufficiency of finding solutions for the values of c_t^i, x_t^{ij}, s_t^i and a_t^i to solve the nonlinear rational expectation commodity model in our case. Applying this logic by a simple direct substitution technique results in reduced system of nonlinear equations of the following form:

$$a_t^i = \gamma^i (E_t \lambda^i (a_t^i \tilde{y}_{t+1}^i + s_t^i, a_t^{j1} \tilde{y}_{t+1}^{j1} + s_t^{j1}, a_t^{j2} \tilde{y}_{t+1}^{j2} + s_t^{j2}))^{\eta_i} \tag{32.1}$$

$\forall i \qquad \forall j \neq i$

$$q_t^i = c_t^i + s_t^i + \sum_j x_t^{ij}, \qquad \forall i, \quad \forall j \neq i \tag{32.2}$$

$$\delta^i E_t \lambda^i (a_t^i \tilde{y}_{t+1}^i + s_t^i, a_t^{j1} \tilde{y}_{t+1}^{j1} + s_t^{j1}, a_t^{j2} \tilde{y}_{t+1}^{j2} + s_t^{j2}) = \left(\frac{c^i}{\alpha^i} \right)^{\frac{1}{\beta_i}} + k^i (s^i), \tag{32.3}$$

$\forall i, \qquad \forall s^i \geq 0$

$$\left(\frac{c^i}{\alpha^i} \right)^{\frac{1}{\beta_i}} + \tau^{ij} \leq \left(\frac{c^j}{\alpha^j} \right)^{\frac{1}{\beta_j}} \perp \left| x^{ij} \right| \geq 0, \qquad \forall i, \quad \forall j \neq i \tag{32.4}$$

$$\left(\frac{c^j}{\alpha^j} \right)^{\frac{1}{\beta_j}} + \tau^{ji} \leq \left(\frac{c^i}{\alpha^i} \right)^{\frac{1}{\beta_i}} \perp \left| x^{ji} \right| \geq 0, \qquad \forall i, \quad \forall j \neq i \tag{32.5}$$

Williams and Wright (1991) suggest using $E_t[p_{t+1}^i]$ as a function of current storage decisions s_t^i rather then function of the future supply q_t^i available in all regions *i* at the beginning of each period *t*. One can justify this suggestion by considering a_t^i being, by assumption proposed, the function of $E_t[p_{t+1}^i(s_t^i)]$:

$$a_t^i = E_t[p_{t+1}^i(s_t^i)], \qquad \forall i \tag{33}$$

This implies further that:

$$E_t[p_{t+1}^i] = E_t \lambda^i (a_t^i(s_t^i)\tilde{y}_{t+1}^i + s_t^i, a_t^{j1}(s_t^{j1})\tilde{y}_{t+1}^{j1} + s_t^{j1}, a_t^{j2}(s_t^{j2})\tilde{y}_{t+1}^{j2} + s_t^{j2}),$$

$$\forall i, \qquad \forall j \neq i, \tag{34.1}$$

which in turn, can be represented as

$$E_t[p_{t+1}^i] = \lambda^i(s_t^i, s_t^{j1}, s_t^{j2}), \qquad \forall i, \quad \forall j \neq i \tag{34.2}$$

Equation (33) has its own application to the problem discussed. Under assumption (34.2) a_t^i as unknown appears only in one equation of the system, namely (32.1). Moreover, it is a function of only one control variable — amount to store s_t^i. Hence, optimal solution for the value of s_t^i directly determines the optimal solution for the acreage to plant a_t^i. By employing this simple idea we may reduce the model equations to the system of three conditions and three unknowns per region to solve for.

As q_t^i is predetermined each period, i.e. known for us, the conditions (2) and (3) imply the same logic as before — optimal solution for consumption c_t^i and storage s_t^i gives the optimal value of the amount traded x_t^{ij}. This can be interpreted as the direct application of Walras law, i.e. equilibrium in consumption and storage market requires equilibrium in market of commodity traded to hold.

The balance equation (4) makes possible the application of Walras law to the market of commodity consumed. Now, the resulting conditions may be defined as equations (5) and (32.3).

Direct substitution for consumption variable c_t^i generates the final equation of form:

$$\sum_i q_t^i = \sum_i \alpha^i (\delta^i \lambda^i (s_t^i, s_t^{j1}, s_t^{j2}) - k^i (s^i))^{\beta^i} + \sum_i s_t^i \,, \qquad \forall i \,, \quad \forall j \neq i \qquad (35)$$

Initial formulation of a problem required simultaneous solution of eighteen equations for eighteen unknowns. Provided derivations allows the planner to solve only one equation, namely (35), for three unknowns s^i, s^{j1} and s^{j2} initially. To obtain the equilibrium values for fifteen other variables the optimizer need to apply the conditions to the found optimal values of storage variables. This process is expected to require less effort as one will solve only a few if not one equation per unknown at a time.

3.2 Numerical solution strategy

Still, the nonlinear rational expectations commodity model cannot be solved using conventional algebraic techniques. The reason for that, as described in Miranda (1998) and Miranda and Glauber (1995), is that the expected price functions λ^i, λ^{j1} and λ^{j2} are not known initially. To solve the dynamic commodity market model one need first to derive them.

At this point, solution of equation (35) becomes a functional equation problem. The equilibrium expected price functions λ^i, λ^{j1} and λ^{j2} must simultaneously satisfy an infinite number of conditions — for every realizable combination of current storage levels s^i, s^{j1} and s^{j2}, relations $E_t[p_{t+1}^i] = \lambda^i (s_t^i, s_t^{j1}, s_t^{j2})$ should solve the equation (35) and then the system of equations (32.1) – (32.5) for equilibrium values of unknown model variables.

Although we cannot employ conventional algebra or mathematical programming, the commodity market model can be solved numerically. To do this, one must approximate the infinite-dimensional functional equation problem posed by the equation (35) with a finite-dimensional problem. The process of forming such an approximation is called discretization. A variety of discretization techniques can be used. We will benefit from the comprehensive comparative survey on numerical strategies for solving the nonlinear rational expectations commodity market model presented in Miranda (1998) by using the polynomial collocation method as the most effective discretization approach suggested for such type of models.

Technically, collocation method replaces the infinite-dimensional functional equation problem with a finite-dimensional nonlinear equation problem (Judd 1998, Miranda 2003). By collocation method the unknown function $\hat{f}$ is approximated using a linear combination of known functions $\phi_0, \phi_1, \phi_2, ..., \phi_n$, called the basis functions:

$$\hat{f} = \sum_{j=0}^{n} b_j \phi_j(s) \ , \tag{36}$$

n coefficients $b_0, b_1, b_2, ..., b_n$ are fixed by requiring the approximant to satisfy the functional equation, not at all possible points of the domain, but rather at *n* specially chosen in $[a, b]$ interval points $s_0, s_1, s_2, ..., s_n$, called collocation nodes.

Application of polynomial collocation to a specific functional equation problem of this study calls for each expected price function λ^i to be approximated using 3-dimensional n_i^{th} degree polynomial. The approximating polynomials are expressed as linear combinations of the tensor product of 1-dimensional basis polynomial functions ϕ_j^i of order *j*:

$$\lambda^i = \sum_{j1=0}^{n1} \sum_{j2=0}^{n2} \sum_{j3=0}^{n3} b_{ij1j2j3} \phi_{j1}(s) \phi_{j2}(s) \phi_{j3}(s) \,, \quad i = 1,2,3 \tag{37}$$

The $3(n_1+1)(n_2+1)(n_3+1)$ unknown coefficients $b_{ij1j2j3}$ are fixed by imposing $3(n_1+1)(n_2+1)(n_3+1)$ conditions that the polynomial approximants λ^1, λ^2 and λ^3 exactly fit expected prices implied by model equations (2), (6), (7), (27), (29) and (31) at a specified grid of $(n_1+1)(n_2+1)(n_3+1)$ collocation nodes ($s^1_{k1}, s^2_{k2}, s^3_{k3}$), where $k^i = 1,2,3,...,n_i$. For more details on using polynomial approximation in multiregional nonlinear rational expectation commodity market model see Miranda and Glauber (1995) and Rui and Miranda (1996).

Collocation schemes differ in how the collocation nodes and basis functions are selected. In this particular study we are going to use Chebychev collocation method. This method is proven to be highly accurate and efficient technique for solving the functional equation problems in economic applications (see e.g. Judd 1988, Chapter 11). Miranda and Glauber (1995) and Makki, Tweeten and Miranda (1996, 2001) presents more specific example of employing of Chebychev collocation for solving the nonlinear rational expectations commodity market model.

In collocation scheme named above the collocation nodes are the Chebychev collocation nodes selected so as to minimize the maximum approximation error which, in this case, is guaranteed to go to zero as the number of nodes increases by the property of Chebychev nodes (for more details on Chebychev nodes see Atkinson 1978, Chapter 4):

$$s^i_{ki} = 0.5(s^i_{\min} + s^i_{\max}) + 0.5(s^i_{\max} - s^i_{\min})\cos\left(\frac{k_i + 0.5}{n_i + 1}\right), \qquad \forall i \qquad (38)$$

where $s^i_{\min}$ and $s^i_{\max}$ are the lower and upper bounds on the storage decisions in region *i*.

The second step of polynomial collocation method requires the basis functions to be selected so as to minimize the rounding error and computational cost associated with computing the coefficients $b_{ij1j2j3}$ of the polynomial approximants. If Chebychev nodes are chosen at the previous step of collocation scheme, then the Chebychev basis functions are the best choice for the second step. Othogonality of Chebychev polynomials as basis functions in combinations with useful properties of Chebychev nodes produces stable interpolation matrix, the fact that is very important in case of using multidimensional approximation schemes (see Miranda 2003, Chapter 6 and Atkinson 1978, Chapter 4 for more details). The Chebychev polynomials are defined by employing the triple recursive relation:

$$\phi_{j1+1}(s^i) = 2s^i\phi_{j1}(s^i) - \phi_{j1-1}(s^i), \qquad \forall i \tag{39}$$

To solve for the expected price function approximants one can use various approaches. Our choice is for the successive approximation algorithm suggested in Williams and Wright (1991) and Miranda and Glauber (1995). Both versions of its application to solution of rational expectations commodity market models are equivalent in general. But the design of the model this particular study is focused on requires the combination of certain features of those approaches. More specifically, we need to combine the three-period solution step described in Williams and Wright (1991) with the Chebychev orthogonal collocation strategy introduced in Miranda and Glauber (1995).

The current form of equation (35) suggest the solution based formally on simultaneous computing values of parameters in two periods *t* and *t*+1: the current equilibrium storage levels s^i, s^{j1} and s^{j2}, and the current expectations on equilibrium price of future period $E_t[p^i_{t+1}] = \lambda^i(s^i_t, s^{j1}_t, s^{j2}_t)$. Williams and Wright (1991) suggest to use collocation nodes chosen for period *t*-1 such that the supply available initially in period *t* is no longer predetermined itself, but rather a function of predetermined variables – collocation nodes. Applying the state transition function (19) to the equation (35) we receive new three-period equilibrium condition to solve:

$$\sum_i a^i_{t-1}\tilde{y}^i_t + s^i_{t-1} = \sum_i \alpha^i(\delta^i\lambda^i(s^i_t, s^{j1}_t, s^{j2}_t) - k^i(s^i))^{\beta^i} + \sum_i s^i_t, \tag{40}$$

$\forall i$, $\forall j \neq i$

We use Gaussian quadrature rules to replace the continuous yield distribution with an approximating *m*-point discrete distribution. The values $e_1^i, e_2^i, e_3^i, ..., e_{mi}^i$ are assumed to the discrete yield variables with the associated probability weights $w_1^i, w_2^i, w_3^i, ..., w_{mi}^i$, that are fixed by requiring the discrete yield distribution to possess the same first $2m^i - 1$ moments as the original yield distribution (see Atkinson (1978, Chapter 5) and Miranda (2003, Chapter 5) for more details on Gaussian quadrature).

Below we present a pseudo code for the successive approximation algorithm (premier code written for Matlab presented in Appendices A and B):

0. Initial Step: Select the degrees of approximation in each dimension n^i; for $i = 1,2,3$, select the storage bounds $s_{\min}^i$ and $s_{\max}^i$ and compute the Chebychev collocation nodes s_{ki}^i for $k^i = 0,1,2,...,n^i$; for $i = 1,2,3$ and $j^i = 0,1,2,...,n^i$ make initial guess for the coefficients of the approximating polynomial $b_{ij1j2j3}$; for $i = 1,2,3$, compute the values of discrete yield distribution $e_1^i, e_2^i, e_3^i, ..., e_{mi}^i$ and corresponding probability weights $w_1^i, w_2^i, w_3^i, ..., w_{mi}^i$.

1. Solution Step: For $i = 1,2,3$ and $s_{ki}^i \in [s_{\min}^i, s_{\max}^i]$, let

$$\lambda^i = \sum_{j1=0}^{n1} \sum_{j2=0}^{n2} \sum_{j3=0}^{n3} b_{ij1j2j3} \phi_{j1}(s) \phi_{j2}(s) \phi_{j3}(s)$$

For all $k = 1,2,3,...,k^1 \times k^2 \times k^3$ repeat Procedure A.

Procedure A: For $i = 1,2,3$ compute q_{li}^i where $l^i = 1,2,3,...,m^i$.

For $i = 1,2,3$ and $l^i = 1,2,3,...,m^i$, solve the nonlinear equation

$$\sum_i q_{li}^i = \sum_i \alpha^i (\delta^i \lambda^i (s^1, s^2, s^3) - k^i (s^i))^{\beta^i} + \sum_i s^i$$

For $i = 1,2,3$ and $l^i = 1,2,3,...,m^i$, solve the problem

$$p_k^i = \sum_{li} (\delta^i \lambda^i (s^1, s^2, s^3) - k^i (s^i)) \times w_{li}^i$$

2. Update Step: Find the coefficient $b'_{ij1j2j3}$, $i = 1,2,3$, $j^i = 0,1,2,...,n^i$, that solve the linear equation system

$$\sum_{j1=0}^{n1} \sum_{j2=0}^{n2} \sum_{j3=0}^{n3} b_{ij1j2j3} \phi_{j1}(s) \phi_{j2}(s) \phi_{j3}(s) = p_k^i$$

3. Convergence Step: Convergence Check: If $\left\| b'_{ij1j2j3} - b_{ij1j2j3} \right\| < \varepsilon$ for $i = 1,2,3$, $j^i = 0,1,2,...,n^i$ and some convergence tolerance ε, update the coefficients by setting $b'_{ij1j2j3} \leftarrow b_{ij1j2j3}$ and stop; otherwise update the coefficients and return to Step 1.

The successive approximation algorithm suggested above should return the function that optimally relates equilibrium expected prices and amounts of commodity to store. Calculation of storage rule is only one step ahead because the system of model equations has now a close form solution. To obtain this relationship, the optimizer needs to solve the equation (41) for q_t^i given the function λ^i and predetermined levels of storage s^i, s^{j1} and s^{j2}, defined as Chebychev nodes.

$$q^i = a^i (\arg \lambda^i) y^i + \arg \lambda^i , \qquad \forall i , \qquad (41)$$

where $\lambda^i = \delta^i \lambda^i (s^1, s^{j1}, s^{j2}) - k^i (s^i)$.

Computation of equilibrium trade decisions requires substituting the equilibrium levels of consumption, storage and initial supply into equation (32.2).

CHAPTER 4

ISSUES ON SUCCESSIVE APPROXIMATION ALGORITHM

The Step 1 of the algorithm is oriented on the solution to the system of model equations. In case of one region model the optimization problem at this point is characterized by one equation based on the intertemporal arbitrage condition. Providing that the functional relation between the current storage decision and the expected price is known the optimal storage decision can be found by solving the fixed point equation problem:

$$s_t^i = q_t^i - \left(\frac{\delta E_t^i p_{t+1}(s_t^i) - k^i(s_t^i)}{\alpha^i} \right)^{\frac{1}{\beta^i}}, \qquad \forall i, \qquad (42)$$

using fixed point function iteration or by solving the root-finding problem:

$$\alpha^i (q_t^i - s_t^i)^{\beta^i} - k^i(s_t^i) + \delta E_t^i p_{t+1}(s_t^i) = 0, \qquad \forall i, \qquad (43)$$

using the Newton or quasi-Newton methods. Both approaches are guaranteed to converge under quite mild assumptions on values of equation parameters. The problem solution is relatively easy in a technical sense.

A multiregional case makes the problem more complicated as the planner solution has to satisfy the set of $6 \times n$, where n is the number of regions, nonlinear equations rather then one intertemporal arbitrary condition. The optimization task can be solved by the Newton method as it suggested in Miranda and Glauber (1995).

Although this approach is proved to be efficient for a two-country model, increasing the number of regions will inevitably cause problems in computations.

Applying the Newton type methods in this case requires the calculation of the inverse of the matrix of first derivatives to get the value of iteration step. The Jacobian for this kind of problem should have a size of $(6\times n)\times(6\times n)$ elements. Assuming that the equilibrium conditions for the given region *i* typically have zero derivatives with respect to most decision variables of the planner from any region *j*, the matrix we are interested in tends to be sparse. In numerical sense, it means that the constructed Jacobian has zero determinant and its inverse results in Newton iteration step to be equal to infinity for each unknown. Computational methods suggest using the alternative methods with the iteration step adjusted to be bound, such as damped Newton iterations (Miranda and Glauber 1995). Applying those methods should solve the problem in most cases; however, increasing number of regions for $n>2$ will cause the growth in total number of zero elements of Jacobian matrix, and this may result in slowing the speed of convergence to the solution.

The computational methods are very sensitive to the parameters and data they are working with. Inappropriate scaling of inputs will make the Jacobian singular to the working precision of machine computation. Even if the number of zero elements is small enough, the derivatives based on the value of the specified parameters of the system may happen to be too low due to the choice of units of data. In this case the difference between their values and zero are often not feasible to the machine. Although this problem has the different nature than the one stated above, the result is basically the same. As a possible remedy, the theory of numerical computations provides singular value decomposition technique that allows finding one of the possible solutions to the singular value problem.

Taking into account the possible difficulties discussed above, we have chosen to utilize the approach described in the Chapter 3 of this study, as the most efficient combination of time and efforts spent on the solution to the Step 1 available given the structure of the model of our particular interest.

According to the strategy chosen, the optimization task formulated in Step 1 of the current version of the algorithm requires finding the solution to the following root-finding problem:

$$\sum_i q_t^i - \sum_i s_t^i - \sum_i \alpha^i (\delta^i E_t^i p_{t+1}^i (s_t^1, s_t^2, s_t^3) - k^i (s_t^i))^{\beta_i} = 0, \qquad \forall i \qquad (44)$$

Apparently, the problem stated cannot be solved by Newton type methods: the equilibrium condition of one equation has to be solved for three unknowns. We suggest using derivative-free methods, such as Nelder-Mead upward simplex method (Miranda 2002), or direct search methods, based on the evolutionary, genetic or reinforced learning principle (Spall 2003). The former strategy option is preferred for its simplicity in case of a low number of regions included in the model and a simple form of demand function which guarantees the left-hand side of the equation to be a smooth and continuous function of unknown variables in each dimension. The latter approach is expected to be more efficient in case of highly dimensional optimization problem, in other words when number of regions is large. It also makes it theoretically possible to introduce the demand function of a flexible form.

According to the criteria discussed above, the structure of the studied model implies higher comparative efficiency of following the first strategy — Nelder-Mead method. The version of the algorithm used in our research was designed for univariate and multivariate maximization problem. Since the current representation of optimization task does not guarantee the global maximum of the left-hand side function to be equal zero, the root-finding and maximization problem formulation are not equivalent in this case. Hence, to make the procedure utilized by Nelder-Mead algorithm appropriate for optimization problem embedded in (44), we need to construct a merit function that will evaluate the model trade balance associated with the values of storage decisions chosen. The transformation applied to equilibrium condition of a form (44) results in the following maximization problem formulation:

$$\max_{s^i \in S^i}\left(-\left|\sum_i q_t^i - \sum_i s_t^i - \sum_i \alpha^i (\delta^i E_t^i p_{t+1}^i (s_t^1, s_t^2, s_t^3) - k^i (s_t^i))^{\beta_i}\right|\right), \qquad \forall i \tag{45}$$

It is clear, that the absolute value of the expression for material balance cannot be negative. Thus, the expression in brackets defined on the interval $(s_{\min}^i, s_{\max}^i)$ in each dimension can have values in range $(-\infty;0]$. Apparently, the maximum of the whole function is zero, providing the realistic values of model parameters that guarantee the sustainability of material balance. In other words, some combinations of demand and supply elasticities may cause the system to perform an extreme behavior. In economic sense, it means that the model was designed not to support an assumption of closed market and thus the planner might observe a predetermined material misbalance. In mathematical sense, the function in brackets of (45) simply has a negative maximum by its specification.

The convergence tolerance for Nelder-Mead algorithm is by default equal to maximum machine precision possible. Since the algorithm computes the merit function at each step, it may be reasonable to use its value to set up our own convergence tolerance appropriate for the problem. Recall that by chosen scale of data input one unit of physical production, consumption or trade is equal to one billion metric tons in real terms. Hence it may be enough to have the total trade balance discrepancy for less than 100 tons. This amount is arbitrary, of course, and chosen only for example purpose. For computer accuracy it means that the algorithm should be stopped once the change in value of a merit function is less than 10^{-7}. The default precision criterion is somewhere close to 10^{-12}—10^{-14}, which corresponds to the level of misbalance measured in kilos or even grams. In many cases the optimizer does not need to be that precise. In the current study we use a tolerance level set at 10^{-8}. It has lowered the iteration time required by Nelder-Mead by 1/3 according to our observations. Following this logic, one may consider that this level of accuracy is too high for practical purposes. Indeed, it might seem to be rational to use ε equal to 10^{-6} (hundreds of tons) or even 10^{-4} (thousands). The achieved time decrease will be $\approx 1/2$. In a large dimensional or many nodes problems that require thousands of iterations any time cost reduction is extremely important. But one needs to be careful with these methods. Decrease in accuracy of results to the desired level, obtained in this manner, may turn in false convergence, when the optimal solution is not really a maximum we are interested in. In mathematical sense, the higher is the residual of (45) the wider is the interval for choice variables that in combination may result in the same value of merit function. In other words, the less is the discrepancy in material balance the more confidence we have on optimal choice.

Step 3 of the algorithm is oriented solely on update of collocation coefficients. It requires finding the solution to the system of linear equations. The seminal literature discussed in Chapter 2 suggests using of L-U factorization (Miranda 2002) as the most efficient method.

Combination of all steps of the algorithm in practice reveals the main problem of multiple stage methods like successive approximation associated with the low speed of convergence and, thus, the long time required for the whole optimization problem to evolve. Let us denote the time cost of one iteration of the Nelder-Mead algorithm by *t* (time required for Step 0 and 2 is neglected for the purpose of exposition), the number of Chebychev nodes in one dimension by n_i, where *i* is the index of dimension, the number of Gaussian quadrature nodes m in each dimension and the number of iterations to solve for collocation coefficients by *it*.

In full information models the planner need to solve only one step for the unique equilibrium to obtain optimal rules.

Found optimal levels of initial supply q^i and the optimal levels of storage decisions associated with them are then used to obtain the optimal storage rule of form:

$$s^i = f_s^i(q^i)\,, \qquad \forall i \tag{46}$$

As one can see time cost *TFI* (time required for solution for full information specification) for this problem is mainly induced by the procedure based on the one used in Step 1 of the algorithm, that is:

$$TFI = t(n_1 n_2 n_3) \tag{47}$$

In case of incomplete information, when the true functional relationships between the storage decisions and the expected prices defined by (37) are known *a priori*, the planner needs to employ multiple stage algorithms, that utilize simple idea of sequential update of equilibrium solution obtained by running Step 1 and collocation coefficients, that specify λ^i, found in Step 2. This procedure assumes time cost *TII* (time required for solution for incomplete information specification) of total:

$$TII = (t(n_1 n_2 n_3))it + t(n_1 n_2 n_3) \tag{48}$$

In case of stochastic problem this also requires computed expectations of prices. This raises time cost by the number of nodes defined in Gaussian quadrature. Then the approximate time *TII* required by the successive approximation is:

$$T = (mt(n_1 n_2 n_3))it + t(n_1 n_2 n_3) \tag{49}$$

One can see that under the incomplete information specification the computation will take approximately $it+1$ times longer than if the complete information is available when the problem has deterministic nature. Respectively, it will take $(m)it+1$ times longer than in case of stochastic optimization.

Apparently, unlike *m*, *n* and *t* that are directly defined by the optimizer it is a product of model specification and thus is relative. Williams and Wright (1991) discuss the algorithm versions for two kinds of models. The first one assumes inelastic supply response. It basically utilizes fixed acreage levels that do not depend on storage decisions. In this case equilibrium obtained in Step 1 should be updated with respect to the new values of collocation coefficients that define new corrected function for expected prices. The algorithm is expected to perform steady convergence to the equilibrium form of λ^i in the direction specified by the initial guess (in other words in the direction specified by whether the initial guess on collocation coefficients was higher or lover than their resulting values). In this case the total number of iterations *it* on equilibrium function of expected prices may be reasonably low. Our model is of the second kind, which assumes the reaction of supply based on the expectation of future prices. Now, equilibrium obtained in Step 1 should be updated to take into account not only the new collocation coefficients, but also the corresponding new production levels calculated in accordance with new prices expectation generated by the corrected λ^i. In this case, as we observed, the algorithm may often perform an oscillating type of convergence that in general requires relatively higher number of iterations *it* to solve the problem. As it was discussed above, time costs of computation grow by factor *it* as the problem formulation becomes more complicated. This means that reduction in total number of iterations is very important policy to optimizer. Moreover, besides the clear fact that outer cycle of the successive approximation algorithm indirectly generates additional massive computations by multiplication time required by the inner cycle and thus lowering *it* will lead to the extreme reduction in time consumed by the solution process, one may notice another valuable opportunity in this policy. The strategy of decreasing the number of Gaussian-Chebychev nodes, regardless of its attractiveness, always has a payoff that appears in loss of accuracy. Accelerating convergence of collocation coefficients is not associated with accuracy of computations and can be naturally obtained by an accurate initial guess. In case of more complicated type of problem the theory suggests to apply the series convergence accelerating methods, e.g. Aitken acceleration (Small and Wang 2003).

Solution to the successive approximation algorithm returns *i* expected price functions λ^i of storage decision made in *i* regions. At this point the planner has the full information set of model parameters, thus the further optimization procedures concerning the solution for the optimal storage rule is similar to one described above in this chapter under the assumption of full information available *a priori* (recall the corresponding equations 45 – 46). The proposition on the separability of the policy solutions is also useful if one is only interesting in determining the optimal storage policy. In this case the solution to the equation 45 is sufficient and an optimizer does not need to conduct additional computations on optimal consumption, acreage and trade decision, which is an inevitable procedure in case of simultaneous solution of the whole set of optimal spatial and intertemporal conditions. Although for the small scale models that include one, two or three regions the total time required for the last step of solution that follows the successive approximation algorithm is not high, increasing dimensions of an optimization problem or some specific assumption may be associated with the far more complex computations.

Given the simple Cobb-Douglas type consumer demand function (6), one can find the corresponding equilibrium consumption level by rearranging terms in (27):

$$c^i = \alpha^i (\lambda^i (s^1, s^2, s^3) - k^i (s^i))^{\beta^i}, \qquad \forall i \qquad (50)$$

An optimal amount of commodity traded x^i can be obtained in a straight way by substituting found optimal levels for s^i, q^i and c^i into the material balance equation for region *i*

(2). Many applications in international commodity markets require to know the specified flows of commodity that is to define x^{ij}, which will be optimal given the structure of a model and constraints set to keep the system balanced. One may notice that the procedure designed to obtained optimal values for trade flows and thus to solve for an optimal trade policy is related to the dimensionality of a model in a more complex way, rather then the other steps in solution to the optimization problem. Apparently, if any region *i* is allowed to export or to import in or from any number $n-1$ of regions *j*, the total number of trade flows that may possible simultaneously exist in the market in any period *t* growth explosively as the number of regions included in the model increases. This is the matter of counting for all possible different combinations of trade schemes that theoretically can be established in the particular model as the equilibrium ones given the optimal set of other choice variables. Solution to the spatial optimization problem in a discussed type of models is basically a solution to the system of linear. The whole system must satisfy a general market trade balance (4). This is the necessary condition for the equilibrium to exist. Of course, if the model refers to an open regional market and does not include all the regions of a world market than the condition specified in (4) can be relaxed in variety of ways, e.g. introducing an external trade shocks in an "islands model" style. Each equation specifies the region *i* trade balance and is described by the following simple condition:

$$x^i = \sum_j x^{ij} \quad , \qquad \forall i , \qquad \forall j \neq i \tag{51}$$

It means that the total amount traded by the region *i* must be equal exactly to the sum of the commodity trade flows exported in or imported from $n-1$ possible regions *j*. In a mathematical sense regional trade balance equations create a set of n necessary conditions that should be solved for $n(n-1)$ unknowns that are the values of each possible trade flow. One can easily see that the system can be solved in theory for only the cases where $n \leq 2$.

Applying the assumption of the symmetry of trade flows decreases the number of unknowns in two times so that the optimizer seems to be able to solve the model, which includes $n \le 3$ regions. This specification of spatial conditions assumes that there is no trade between the agents of homogenous types. Our version of a model does not count for transit trade. We may see that in case of n regions the system of spatial conditions contains $2n$ equations for $n(n-1)$ unknowns. If the solution procedure utilizes only the symmetry assumption, then the actual system is a set of n equations for $n(n-1)/2$ unknowns. At this point the problem seems to be specified for $n \le 3$ regions, so that in the matrix representation it has a full rank and thus can theoretically be solved.

The model of international commodity model presented here is designed such that no assumptions about trade flows rather then conventional were made. It means that the optimization problem can be solved in theory for any number of regions. However, it is sill not possible to solve for the optimal trade flows x^{ij} in most cases. By the same logic as employed above for any number of regions $n > 3$ one will have more unknowns then the corresponding equations that describe the model. Apparently, because the trade between two exporters or two importers is prohibited, the only situation that assumes an existence of possible solution to this kind of problem is when there is either only a single exporter or a single importer. Then the optimal amount traded x^i obtained for each given region *i* should be counted directly as a single optimal trade flow from this specific region to the only one region of an opposite trading type acting in the market. In any other case when the model includes more then one region of each type, representative exporter or importer of any region *i* is actually indifferent in his choice of a counterpart, because the only criterion, which should be met is a resulting difference in prices. For this type of model to have a specific solution one will need to employ a certain definition of consumer preferences such as Armington assumptions. Then the optimal choice of planner's decisions on trade will be based on additional parameters specifying the system of model equations.

CHAPTER 5

RESULTS, SUMMARY AND CONCLUSIONS

In accordance with the third objective and as the logical step to complete the research we have performed an experiment to obtain an approximate solution to a multiple region model using the framework developed in Chapters 3 and 4 of this study. Since the current thesis is not concentrated on a study of the specific effects of economic policies, our main objective was to test the whole concept discussed above in general, that is to find a possible solution given real world data and check if the results can provide us with information in favor or against the assumptions made in the theoretical developments for this research.

5.1. Data

As an application for the experiment the world cotton market was chosen. We have used annual data from 1972 to 2003. All data were obtained from the USDA Economic Research Service and Foreign Agricultural Service Database.

The main source was the statistical data from Cotton and Wool Yearbook <http://www.ers.usda.gov/publications/so/view.asp?f=field/cws-bby/>.

We also used miscellaneous issues of Cotton and Wool outlook published by USDA ERS. The latest report is available at USDA <http://www.ers.usda.gov/publications/so/view.asp?f=field/cws-bb/2004>.

Additional information on missing quantities and prices were found in 1996 to 2004 Cotton: World Markets and Trade reports <http://www.fas.usda.gov/cotton_arc.html>.

5.2. Market outlook and model calibration

We have chosen three regions to be United States (US), Republic of China (CH), and all other countries are aggregated in the Rest of the World (ROW).

The data were scaled so that one unit of physical values corresponds to one billion metric tons of cotton and one unit of monetary value is equivalent to one dollar per metric ton of cotton.

Since we require only an approximate solution, the parameters of supply and demand equations were calibrated to fit the historically observed data and are represented in Table (5.1).

Table 5.1. Calibrated model parameters of supply and demand equations

Parameter	US	CH	ROW
Constant term of demand function (α)	62.16271	113.73892	1004.69964
Price elasticity of demand (β)	-0.48	-0.53	-0.59
Constant term of supply function (γ)	1.17924	0.46485	1.62920
Price elasticity of supply (η)	0.20	0.30	0.35

For simplicity the discount rate is specified to be conventional 0.95 for all three regions. To create more precise measure for the agents' time preferences one may discount the conventional rate for the taxation level as suggested in Makki, Tweeten and Miranda (1996).

We set the following minimum and maximum values for storage decisions based on the historically observed levels of carryover (Table 5.2).

Table 5.2. The minimum and maximum levels of carryover

Bounds	US	CH	ROW
	--------------Billion metric tons--------------		
$s_{\min}$	0.5	1.0	2.9
$s_{\max}$	2.1	5.5	6.2

Providing the established bounds we have chosen six Chebychev nodes in each dimension as a reasonable combination of time required for computation and solution accuracy.

The stochastic component of the problem is modeled by selecting four Gaussian quadrature nodes in each dimension. We have assumed that the cotton yields follow the lognormal distributions that are characterized by the first two moments shown below:

Table 5.3. Mean and variance of cotton yield distributions

Parameter	US	CH	ROW
Mean	6.5947	6.8332	6.3704
Variance	0.00025	0.00025	0.00025

The initial guess for the collocation coefficients were found in the following manner. First, we utilized the simple curve fitting technique to estimate the relationship between the current price level and the beginning stocks observed. Obtained coefficients for the 4-th degree polynomial regression were used to fit the specified Chebychev nodes and generate approximate corresponding prices. Pre-multiplying the vector of estimated prices by the inverse of collocation matrix of Chebychev basis functions evaluated at the specified levels of carryover resulted in a vector of coefficients that we are interested in. This vector was basically used as an initial guess. In other words, we perform Step 3 of the algorithm, using an estimated price level. A similar approach is suggested in Williams and Wright (1991). Although this approach is not best in terms of the quality of approximation due to the simplicity of the regression technique chosen, the experiment does not require an optimal performance of the algorithm.

5.3. Results

Using the successive approximation algorithm presented in the Chapter 3, the three region model designed in this study was solved on 2.80 GHz Pentium® 4 Dell Personal computer using the Mathworks Matlab 6.5 programming environment. To implement the Nelder-Mead algorithm and Gaussian and Chebychev nodes, and Chebychev basis functions we utilize routines written by Miranda and Fackler (1999). Solving the model took approximately 5 hours.

The successive approximation algorithm was actually stopped after performing twelve iterations. Although the convergence criteria was not meet in general the difference between the results of iterations decreased to a reasonably low level. The analysis of collocation coefficients obtained for the last three iterations has shown that most of them stabilized. However, we observed several extreme values that may be explained by the algorithm hitting the lower bound set as a constraint on the storage decisions set in the experiment:

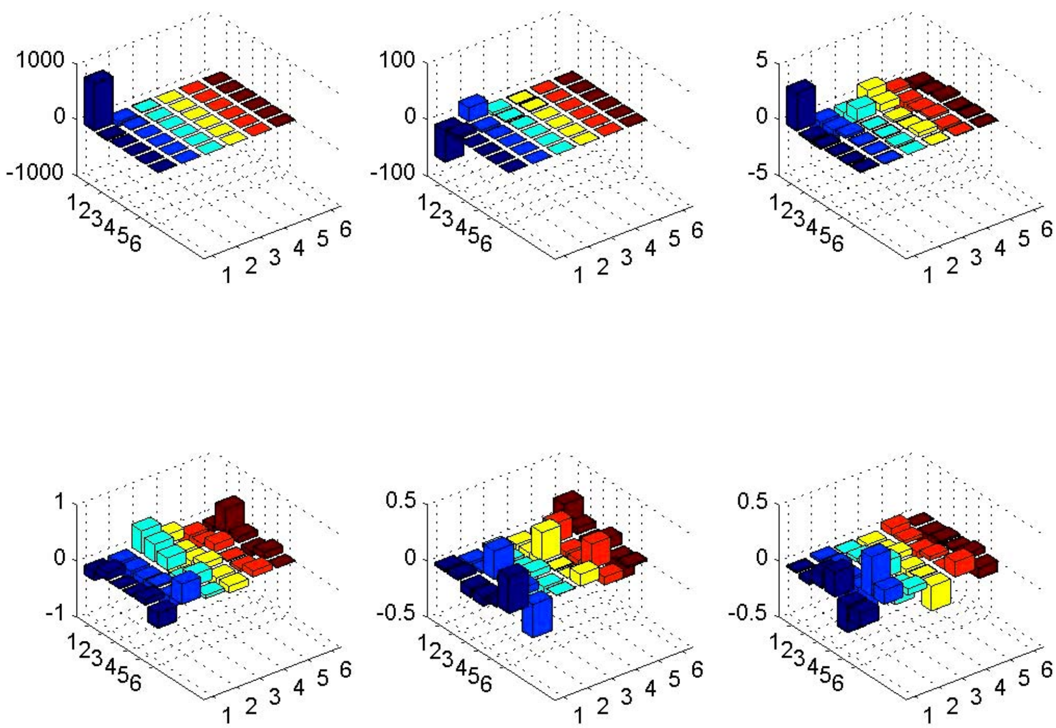

Figure 5.1. Computed collocation coefficients for the equilibrium expected price function (US) for six ROW stock scenarios*

*Horizontal axes represent US and CH storage levels. ROW stocks are fixed at the six possible levels starting from lower bound to the upper bound level.

The theory of numerical analysis relates this effect to the special property of Chebychev polynomials to produce relatively higher errors in the ends of the interval of interpolation, as discussed before. The simplified version of the algorithm used to conduct the experiment utilizes linear cost functions, hence the certain disturbances in the computed vector of coefficients are predetermined. One can observe that with the increase in ROW stock level the variation of coefficients decreases starting from about 100 in the first and second cases (coefficient with a 1000 level in the first case corresponds to the intercept and thus not taken into account) to 0.5 in the latter case. As approaching the upper bound of storage decisions the collocation coefficients stabilize.

The model assumes elastic supply response for all three regions. As it was expected the algorithm performs a kind of oscillating convergence — at each iteration, the resulting function of expected prices updates both general level of future price response on carryover and the curvature of the function itself to count for the acreage decisions in all three regions.

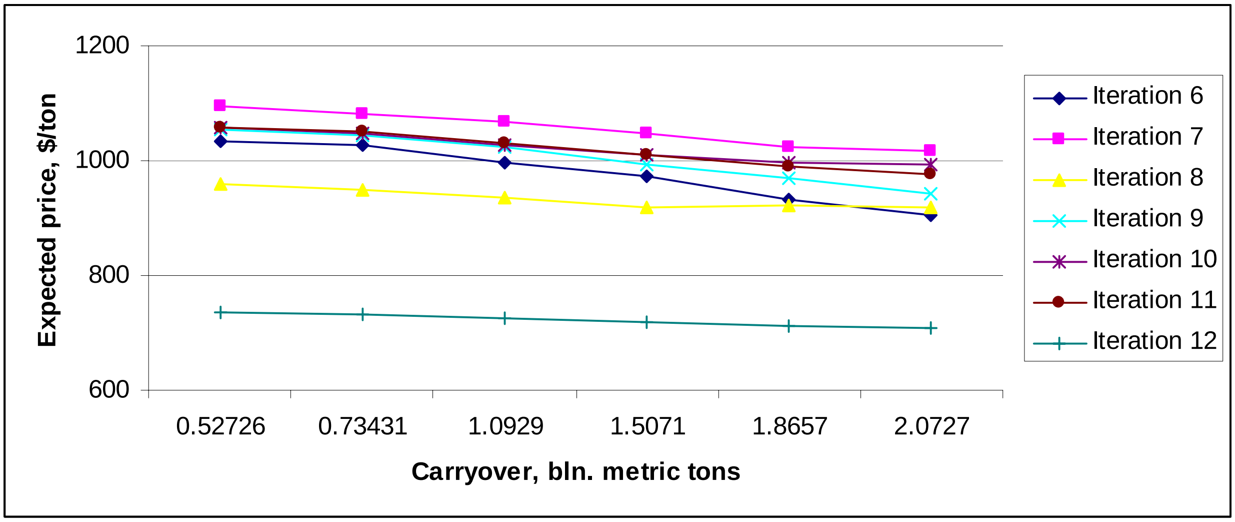

Figure 5.2. Iterated equilibrium expected price function for US*

*Carryover level is fixed at 2.6677 billion metric tons for China and 6.1438 billion metric tons for the Rest of the World

Nevertheless, the last seven iterations of the expected price function for US described by Figure 5.2 tend to stabilize the search for the equilibrium in an interval from $900 to $1100 per metric ton. Hence, both effects of disturbances in values of collocation coefficients and oscillating convergence may theoretically evolve if the constraints are set properly and the number of iterations run is relatively high.

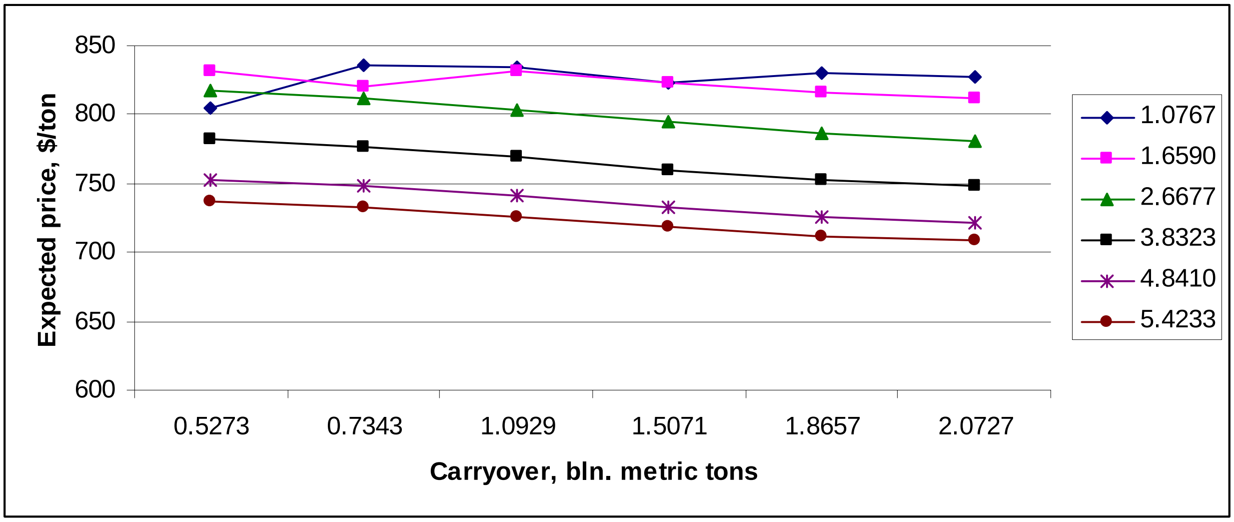

Figure 5.3. Equilibrium expected price function for US*

*Carryover of cotton for China is fixed at the levels specified in legend to the diagram (in billion metric tons).

The resulting function of expected prices and stock levels describes the stable negative relationship predicted by the theory. However, it can be observed that the low levels of stocks correspond to a higher nonlinear relationship between the carryover of cotton and future prices. Despite this fact, Figure 5.3 represents the monotonic negative response of the US price expectations to the increase in storage levels in the third region. This important observation justifies the general statement the regional models should count for international effects to produce unbiased results.

One of the most essential developments of our approach was decreasing number of conditions that should be solved for simultaneously. According to the theory discussed in Chapter 3, the solution for the equilibrium storage, trade and consumption decisions may be obtained separately. The results presented above do not prove this to be fact by themselves. To check whether our assumptions are supported by the experiment we have computed the difference between the vectors of equilibrium expected prices obtained from the last twelve iterations. It was found that this difference is never more than a few cents per unit traded (except for some extreme case associated with the disturbances in collocation coefficients values discussed before). The steady difference was equal to 67.55 – 67.58 dollars per one billion metric tons of cotton that is traded between the US and China, and 72.42 – 72.43 dollars per one billion metric tons of cotton that is traded between the China and the Rest of the world. The marginal transportation costs between US and the Rest of the World were 139 – 140 dollars. Taking into account that the computation of each specific combination of expected prices is independent of any other, obtained results on differences in prices may be concluded to support the main assumption on separability of optimal conditions.

5.4. Summary and Conclusions

The theory underlying the model developed in this research suggests that the agents acting in the market are rational in their decisions and thus the future price behavior may be explained as a reaction to the aggregate current activity of the consumers and suppliers. Economics of commodity markets argues that the optimal planner’s decision is equivalent to the one that is an equilibrium result of competitive market activity of the agents. Based on the planner optimization problem as a maximizing of the social welfare of agents active in the international market, the theoretical model generates the set of optimal conditions required to be satisfied. A combination of these conditions and the material balances arising in the international environment allowed the author to modify the solution strategy suggested earlier in the works of Williams and Wright (1991) and Miranda and Glauber (1995). The resulting algorithm does not require the direct specification of the trade orientation of regions and thus may be theoretically applied for solution of a model designed for any number of regions.

An empirical part of the study was oriented to a three region international cotton market model. An approximate solution obtained in the experiment supports in general the main statements made in theoretical developments. The resulting expected price functions describe a negative nonlinear relationship between the price expectations and the storage decisions made. As expected, the model assumes the monotonic negative response of home price expectations to the current storage levels available in foreign regions. This finding makes clear that it is important for models that study local markets to account for international effects. In context of the current research this suggests more emphasis be placed on the commodity traded. In a broader sense there may be the need to introduce exchange rates, international trade regulations, etc. This model also supports the suggested separability of the solution for optimal policies, as the differences in international prices were observed to be stable for all the possible combinations of storage decisions. The theory of international commodity markets predicts that price stabilization may be obtained for less cost if the international trade is allowed. The results of experiment also support the idea of an existence of a direct constraint for international price variation — generated spatial conditions assume that the difference between the prices observed in different regions should not exceed the unit transportation cost of commodity.

Although the current study does not include advanced policy analysis, the developed approach to a solution of this type of model is very flexible and leaves a lot of space for extensions such as government support to farmers, export subsidies, endogenous market protections, multilateral trade agreements, etc. However the nature of the algorithm makes it a subject of the “curse of dimensionality” and the linear convergence that is the typical feature of the multiple stage procedures. Both factors increase time required for computation. As a result, despite that the whole strategy is expected to generate growth in efficiency in general, this computational procedure will fail to be of benefit in large scale problems. This seems to be the main limitation of this approach. The other problem that arises in application is an extreme sensitivity to the changes of parameters. It makes the algorithm extremely difficult to implement when dealing with incomplete information. Our study has achieved the current objective, but the experience of empirical experiment requires for cardinal improvement in solution procedures to make them work at the maximum potential in a complicated model environment. That is an important goal for the future research.

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APPENDICES

A. Main module (themodel6)

```
% The model

clc;
clear;
close all;

load Data;

n = 6;
fspace = fundefn('cheb',[n n n],[0 0 0],[3000 6000 7000]);
G = funnode(fspace);
S = gridmake(G);
F = funbas(fspace,S);

[e1,w1] = qnwlogn(4,6.5947,0.00025);
[e2,w2] = qnwlogn(4,6.8332,0.00025);
[e3,w3] = qnwlogn(4,6.3704,0.00025);

load prices;

P = Xp;

c = F\Xp;

delta  = [0.95   0.95  0.95];     % time preference parameter (discount rate)

input = [1000,1300,3000]';
for it = 1:7;

    a(:,1) = 0.08*P(:,1).^0.55;
    a(:,2) = 0.15*P(:,2).^0.50;
    a(:,3) = 2.55*P(:,3).^0.30;

    in = [1;2;3];

    for i = 1:16;

        optset('syseq6','c',c);

        Y1 = a(i,1)*e1;
        Y2 = a(i,2)*e2;
        Y3 = a(i,3)*e3;
```

```
        A1 = Y1 + S(i,1);
        A2 = Y2 + S(i,2);
        A3 = Y3 + S(i,3);

        optset('syseq6','fspace',fspace);
        optset('syseq6','n',n);

            P1 = zeros(1,4);
            P2 = zeros(1,4);
            P3 = zeros(1,4);

        for j = 1:4;
            T1 = A1(j)/1000;
            T2 = A2(j)/1000;
            T3 = A3(j)/1000;
            optset('syseq6','c',c);
            optset('syseq6','fspace',fspace);
            optset('syseq6','T1',T1);
            optset('syseq6','T2',T2);
            optset('syseq6','T3',T3);
            optset('syseq6','i',i);
            optset('syseq6','j',j);
            optset('neldmead','tol',0.0001);
            es = neldmead('syseq6',in);
            in = es;
            E(i,1:3)=es';
            S1(j) = es(1);
            S2(j) = es(2);
            S3(j) = es(3);
            p = funeval(c,fspace,[S1(j)*1000 S2(j)*1000 S3(j)*1000]);
            P1(j) = p(1);
            P2(j) = p(2);
            P3(j) = p(3);
            K1(j) = 50;
            K2(j) = 50;
            K3(j) = 50;
        end;
        P1 = delta(1)*P1'-K1';
        P2 = delta(2)*P2'-K2';
        P3 = delta(3)*P3'-K3';
        EP(i,1) = P1'*w1;
        EP(i,2) = P2'*w2;
        EP(i,3) = P3'*w3;
    end;
    cold = c;
    P = EP;
    c = F\P;
    if norm(c-cold)<0.001,break,end;
end;
disp('Final iteration');
disp(it);
```

B. Sub-routine (syseq6)

```
function [y] = syseq6(in);

delta  = [0.95   0.95  0.95];     % time preference parameter (discount rate)
tau    = [40   100  140];         % transportation cost per unit
beta   = [-2.1 -1.9 -1.7];
alpha  = [6000 8000 90000];
k      = [100    95   93];        % cost of storage per unit

clc;

           c = optget('syseq6','c',1);
           fspace = optget('syseq6','fspace',1);
           T1= optget('syseq6','T1',1);
           T2= optget('syseq6','T2',1);
           T3= optget('syseq6','T3',1);
           i = optget('syseq6','i',1);
           j = optget('syseq6','j',1);

k(1) = 100;
k(2) = 100;
k(3) = 100;

p = funeval(c,fspace,[in(1)*1000 in(2)*1000 in(3)*1000]);
d1 = ((delta(1)*p(1)-k(1))/alpha(1)).^(1/beta(1));
d2 = ((delta(2)*p(2)-k(2))/alpha(2)).^(1/beta(2));
d3 = ((delta(3)*p(3)-k(3))/alpha(3)).^(1/beta(3));

fprintf(1,'Iteration %2.0f.%2.0f\n',i,j);

fprintf(1,'US consumption is          %6.4f mln tonns\n',d1);
fprintf(1,'China consumption is       %6.4f mln tonns\n',d2);
fprintf(1,'ROW consumption is         %6.4f mln tonns\n',d3);

fprintf(1,'US    exports - imports is %6.4f mln tonns\n',T1-d1-in(1));
fprintf(1,'China exports - imports is %6.4f mln tonns\n',T2-d2-in(2));
fprintf(1,'ROW   exports - imports is %6.4f mln tonns\n',T3-d3-in(3));

y = -abs(T1+T2+T3-in(1)-in(2)-in(3)-d1-d2-d3);
if abs(y)<0.000000001
    y=0;
end;

fprintf(1,'The merit function is %16.14f \n',y);

disp('Equilibrium storage');
disp(in);


return;
```