THE EFFECTS ON PARAMETER ESTIMATION OF SAMPLE SIZE RATIO, TEST LENGTH AND TRAIT CORRELATION IN A TWO-DIMENSIONAL, TWO-PARAMETER, COMPENSATORY ITEM RESPONSE MODEL WITH DICHOTOMOUS SCORING

by

ERIC C. POPP

(Under the Direction of Gary Lautenschlager)

ABSTRACT

This study used a monte carlo simulation to determine the influence of sample size ratio, test length and trait correlation on obtaining quality parameter estimates for a two-dimensional, two- parameter, compensatory item response model with dichotomous scoring. The study found that the quality of parameter estimates dropped sharply at low levels of sample size ratio and test length. The drop was greater when the traits were correlated. Generally, sample size ratio was the biggest influence on the quality of parameter estimates with it uniquely accounting for up to 87% of the variance observed. Test length was the second largest influence with the percentage variance accounted for being in the mid teens. However, for bias of discrimination parameters trait correlation was the largest influence accounting for up to 60% of the variance. The influence of sample size ratio, test length and trait correlation varied across item difficulty and discrimination levels. To aid the test developer in determining where the drop in parameter estimate quality occurs, tables were compiled providing the ratio of variance, root mean square error, and bias to average parameter level for various combinations of sample size ratio, test length and trait correlation. Similar tables listing the correlation of parameter estimates with true parameters were also compiled.

INDEX WORDS: Multidimensional Item Response Theory, Parameter Recovery, Monte Carlo

THE EFFECTS ON PARAMETER ESTIMATION OF SAMPLE SIZE RATIO, TEST LENGTH AND TRAIT CORRELATION IN A TWO-DIMENSIONAL, TWO-PARAMETER, COMPENSATORY ITEM RESPONSE MODEL WITH DICHOTOMOUS SCORING

by

ERIC C. POPP

B.S., The University of Tennessee at Chattanooga, 1982

M.Div., Reformed Theological Seminary, 1988

M.S., The University of Georgia, 2002

A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2004

THE EFFECTS ON PARAMETER ESTIMATION OF SAMPLE SIZE RATIO, TEST LENGTH AND TRAIT CORRELATION IN A TWO-DIMENSIONAL, TWO-PARAMETER, COMPENSATORY ITEM RESPONSE MODEL WITH DICHOTOMOUS SCORING

by

ERIC C. POPP

Major Professor: Gary Lautenschlager

Committee: Garnett Stokes
Robert Mahan

Electronic Version Approved:

Maureen Grasso
Dean of the Graduate School
The University of Georgia
May 2004

DEDICATION

To Joy, my bride of sixteen years. May her support of and dedication to me in this endeavor be richly rewarded. To my dear children, Anakela, Jared, Nathan, and Maile, whose patience, understanding and support have eased the weight of this task.

ACKNOWLEDGEMENTS

I would like to thank Dr. Gary Lautenschlager for his efforts in making this project a success. Gary's willingness to act as a sounding board in both scheduled meetings and in my many "drop by" meetings is greatly appreciated. I also thank Drs. Garnett Stokes and Rob Mahan for their willingness to serve on my committee and to plow through the details of this paper. I would like to thank Mr. Colin Fraser for his help with the NOHARM program. His work at adapting the program to allow for the running of large batch files made my task much easier. I appreciate the willingness of ETS to grant permission to use the RESGEN program in creating the initial data sets for this study. Family and friends have played an important role in encouraging and supporting me in this process. While it would be impossible to mention everyone, there are a few I would like to particularly thank. My parents' encouraging words and generosity were a great boost. Through many great meals and good fellowship on Friday nights, Lewis and Sandy Stowe have helped me maintain a balance in life during this process. Their friendship will always be cherished. Although many friends put up with listening to me drone on about this study, David Hunt had a particular patience and good humor about listening to me. I would also like to thank my fellow students, Lawrence Michaels, David Finch, Nancy Yanchus and Harriet Landau. Many times Lawrence willingly stopped his work in the computer lab for me to bounce ideas of him. His input and enthusiasm about my study were an encouragement. David as a fellow traveler in this process provided good friendship and encouragement along the way. As my office mate, Nancy listened to many mumblings as I plowed through this work. Harriet's friendship and contribution of a SAS book helped me along in this process. Thanks to all.

TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

Item response theory is an approach to testing and measurement that uses a mathematical function to relate the probability of a correct response with the level of the underlying trait of interest (Hambleton, Swaminathan, & Rogers 1991). Although the original emergence of item response theory can be traced to the 1960's, it was not widely used in the field of psychology through the 1970's and 1980's (Embretson & Reise, 2000). However, item response theory has now become mainstream and is being applied to a variety of measurement situations.

To be able to apply item response theory three primary assumptions must be met (Hambleton, Swaminathan, & Rogers, 1991). The first assumption is that the item response theory model used fits the data. This means that the mathematical function that relates the probability of a correct response to an item with the level of the underlying trait of interest adequately captures the nature of that relationship. The second assumption is that there must local independence among the items on the test. This assumption means that if the trait of interest is held constant there is statistical independence among the examinees' responses to any pair of items. In other words, the trait of interest is the only factor influencing the examinees' responses. The third assumption is that the items are unidimensional. This means that the items are either measuring only the trait of interest or if other traits are systematically influencing examinees' responses these traits have equal influence on all items.

If the unidimensionality assumption is violated this can lead to items functioning differently for different groups (Robie, Zickar, & Schmit, 2001). It may also result in poor model fit or in inaccurate item parameter estimates (Kirisci, Hsu, & Yu, 2001). These factors can bring into question the validity of the test scores obtained. Because in practical applications the assumption of unidimensionality can be tenuous, researchers have sought to develop multidimensional item response models that can handle more than one trait systematically influencing how examinees' respond to items.

Generally there are two types of multidimensional item response models for dichotomous scoring: compensatory and noncompensatory (Ackerman, 1994, Reckase, 1997). In the noncompensatory models, the probability of answering an item correctly is a multiplicative function of the unidimensional models that describe the relationship between the item and each of the dimensions. Therefore, in terms of answering an item correctly, having a high level on one of the traits will not compensate for a lower level on another one of the traits. In this model there are discrimination and difficulty parameters for each dimension being measured by the item.

Compensatory models assume that the probability of a correct response increases monotonically as each trait level increases (Reckase, 1985). In these models, in regard to responding correctly to an item, being high on one trait can compensate to a degree for being low on another trait. In the compensatory model there are discrimination parameters for each of the dimensions, but there is only one overall difficulty parameter for the item (Ackerman, 1994).

Some Potential Applications of Multidimensional Item Response Theory

Multidimensional item response theory (MIRT) models have the potential to be applied to a number of situations to help in the measurement of complex psychological processes (Reckase, 1997). Reckase notes that MIRT has been shown to have usefulness in investigating the structural details of skills needed to respond to test items. Luecht (1996) provides an example of this by applying MIRT to computerized adaptive testing for medical licensing/certification. By design, items on this type of licensure test integrate content covering numerous domains causing the items to be multidimensional in nature. However, the outcome measure traditionally has been univariate. Luecht demonstrated that the application of MIRT to this type of testing could allow for reporting of subscore profiles that help examinees diagnose their strengths and weaknesses in core areas. In a similar type application, a polytomous componential MIRT model was used to measure the task of synonym generation in which the two components of the task required separate abilities (Hoskens & De Boeck, 2001). This approach allowed for examining combination effects of the abilities needed for each component.

In addition to possible applications to ability measurement, MIRT holds potential in regard to attitude measures that may be driven by several separate traits, but where not all item responses may be influenced to the same degree by these traits. For example, Steinberg and Thissen (1995) note that MIRT has great potential in personality measurement. One possible application in this area would be to separate out the influence the personality traits of interest have on the probability of endorsing an item from the influence of social desirability factors.

Due to the ability to model multiple traits measured in one item, MIRT has the potential to be useful in describing and understanding why items may be functioning differently for various subgroups when they are analyzed unidimensionally (Reckase, 1997). McDonald (2000) notes that MIRT may be able to offer improvements over classical test theory in the several areas including dimensionality and interpretation; convergent/discriminant validity; and eliminating measurement error correlations.

Traditionally Accepted Sample Size Requirements for Multidimensional Item Response Models

Despite the potential usefulness of MIRT models in addressing various measurement problems, the traditional assumption that large sample sizes are needed to accurately estimate the parameters of MIRT models has generally limited their applications to large scale testing projects

(Ackerman, 1994). Historically, the accepted sample size needed to estimate the parameters in a two-dimensional model has been 2,000. This figure seems to have emerged in the late 1980's. In conducting a monte carlo study designed to test a MIRT scale equating procedure, Hirsch (1989) chose to use a sample size of 2,000 for a 40-item test. However, he did not offer a reference or a rationale for this choice. In 1993 Batley and Boss used a simulated data study to examine the impact of trait correlation and trait distribution restriction on MIRT trait and parameter estimations. They also used a sample size of 2,000 for a 104-item test without providing a reference or a rationale. In an article describing what multidimensional items are actually measuring, Ackerman (1994) notes that a sample size of a least 2,000 is needed to obtain satisfactory parameter estimates for a two-dimensional model. However, again no reference or rationale is provided. In a monte carlo study examining a MIRT scale equating model, Li and Lissitz (2000) used a sample size of 2000 as the base line in part one of their study and as the accepted sample size in part two of the study. They referenced both Ackerman and Batley & Boss as the basis for this choice.

An indication that smaller sample sizes might be viable was given by McDonald (1997) in describing the multidimensional parameter estimation procedure used in the computer program NOHARM (Fraser, 2003). McDonald stated that some unpublished studies suggest that the unweighted least squares approach used allows for accurate parameter estimation with sample sizes as small as 100 and that the procedure is fairly robust in face of violations of the assumption of normality of the latent trait distributions. In testing a procedure to determine the standard error of multidimensional parameter estimates generated by NOHARM, Maydeu-Olivares (2001) found that adequate estimates could be obtained with a sample as small as 100 for a eight-item, two-dimensional test. Maydeu-Olivares noted that further research with tests of more realistic lengths is needed. Besides the short test length Maydeu-Olivares's study had several other limitations. The items used were simulated to measure only one of the two dimensions. Four items measured dimension one and four measured dimension two. Therefore, the compensatory nature of the model was not really tested. The only other sample size tested was 1,000. Two potentially important factors were not varied in the study. There was no variation in the distribution of the latent traits or in the correlation of the latent traits. The study was also limited to two-dimensions.

Purpose of Current Study

Reckase (1997) noted that little is known about data requirements for parameter estimation in MIRT and that this is a rich area for future research. If MIRT models are going to be able to be applied to situations where large samples are not available, research is needed to determine realistic sample size requirements for various test lengths to estimate the parameters. Also studies are needed to determine how robust the estimations are in the face of various conditions such as correlated traits.

This current study seeks to address some of these gaps in our understanding by using a monte carlo approach to determine adequate sample sizes for parameter recovery for a dichotomously-scored, two-dimensional, two-parameter compensatory MIRT model with various factors being manipulated. Factors addressed in this study are test length (TL), sample size/ test length ratio (SSR) and trait correlation (TC). This type of parameter recovery research is typically approached using monte carlo designs (Harwell, Stone, Hsu, & Kirisci, 1996).

Summary of Two-Dimensional Item Response Theory

In two-dimensional item response models a surface in the two-dimensional trait space describes the probability of a correct response given a trait combination. This surface is referred to as the item characteristic surface (ICS).

A two-parameter logistic version of a two-dimensional compensatory model (Reckase, 1985) expresses the probability of a correct response on item i by person j as

$$P\left(x_{ij} = 1 \mid \underline{a}_i, d_{i,} \underline{q}_j\right) = \frac{e^{\underline{a}_i' \underline{q}_j + d_i}}{1.0 + e^{\underline{a}_i' \underline{q}_j + d_i}}, \qquad (3)$$

where $\underline{a}_i$ is a vector of discrimination parameters, a_1 and a_2 for item i;

$d_{i,}$ is an overall difficulty level for item i;

and $\underline{q}_j$ is a vector of ability parameters, q_1 and q_2 for person j.

Figure 1 illustrates the ICS generated by this model with parameter values of a_{i1} = 1.25, a_{i2} = .75 and $d_{i,}$ = 1. It is important to note that in this model larger values of d_i represent easier items.

In MIRT, the meanings of the model parameters are not as clear-cut as in unidimensional IRT. In the unidimensional case a higher level of b_i indicates that to have a .5 probability of responding correctly to an item examinees need to have a higher level of the trait being measured. In the multidimensional case different items may be measuring the traits of interest to different degrees. For example, math "word problems" measure both math ability and reading comprehension. One item may require a high level of reading ability but a low level of math ability. Another may require a low level of reading ability but a high level of math ability. Thus, the ICS's of the two items could be quite different. However, both items could have the same d_i value. This makes it impossible to compare the difficulty of multidimensional items just using the d_i parameter (Reckase, 1985).

A similar issue arises for the discrimination parameters. In unidimensional item response theory, a larger discrimination value indicates that the item is more discriminating. However, in the multidimensional case there is a discrimination parameter for each dimension. Building on the example above, one "word problem" item may have a discrimination parameter of .5 for reading comprehension and a discrimination of 1.5 for math ability. Another item may have a value of 1.0 for both discrimination parameters. This makes comparing the overall discrimination of multidimensional items difficult.

In the two-parameter, unidimensional case, the difficulty of the item is determined at the point of steepest slope of the ICC. This is also where the probability of a correct response is .5. The discrimination of an item is related to the slope of the ICC at this point. This concept does not directly generalize into MIRT because the multidimensional case involves an item response surface rather than a single line. In some cases the point on the ICS with the steepest slope occurs when one of the traits approaches infinity. In other cases there may be an infinite number of points of steepest slope (Reckase & McKinley, 1991). Also, at any point the slope of the surface depends upon in which direction the slope is measured (Ackerman, 1994; Reckase, 1985).

To address this problem both an overall multidimensional measure of difficultly, MID, (Reckase, 1985) and an overall multidimensional measure of discrimination, MDISC, (Reckase & McKinley, 1991) have been developed. Each of these measures is based on the assumption that the trait axes are orthogonal. The MID consists of statistics for a distance and a direction in the trait space. To develop the MID, Reckase proposed that the slope of the ICS at any point in the multidimensional space be measured in the direction from the origin of the trait space to the point of interest. For two-parameter models, Reckase demonstrated that in any particular direction the slope of the surface is steepest when the surface crosses the .5 plane. Reckase also demonstrated that the direction that would result in the steepest slope at the .5 plane could be determined by a set of equations. For the two-dimensional case these equations are:

$$\cos \boldsymbol{a}_{ik} = \frac{a_{ik}}{\left(\sum_{k=1}^{2} a_{ik}^{2}\right)^{1/2}} \qquad (4)$$

where α_{ik} is the angle from the k-dimension axis to the point in trait space where the maximum slope occurs. These two angles define the direction from the origin to the point of steepest slope.

The signed distance from the origin in the trait space to the point of maximum slope when the slope is measured in the direction from the origin is designated as item parameter, D_i. For the two-dimensional case it is defined as:

$$D_i = \frac{-d_i}{\left(\sum_{k=1}^{2} a_{ik}^2\right)^{1/2}}. \tag{5}$$

This is analogous to the b_i in unidimensional item response theory.

The MDISC is the discrimination ability of the item in the direction of the MID (Reckase & McKinley, 1991). For the two-dimensional case it is given by:

$$\left(\sum_{k=1}^{2} a_{ik}^2\right)^{1/2}. \tag{6}$$

These definitions of multidimensional item difficulty and multidimensional discrimination create the same relationship between MID and MDISC as exists between a_i and b_i in the unidimensional case. That is, MID is located at the point of maximum slope of the ICS and MDISC is proportional to the slope at that point. Figure 2 illustrates these statistics for a two-dimensional case with parameters $a_{i1} = 1.5$, $a_{i2} = .75$, $d_i = -1.0$, $MDISC = 1.677$, $D_i = .596$, $\alpha_{i1} = 26.56^o$ and $\alpha_{i2} = 63.44^o$.

Ackerman (1994) highlights the point that the directional nature of the MID and MDISC indicates that multidimensional items discriminate optimally in one direction in the trait space. This means that while an item can measure any composite of trait scores, it has optimal discrimination for only one composite. This is analogous to a unidimensional item having maximum discrimination at only one trait level. For the two-dimensional case Ackerman notes that this optimal composite is given by:

$$q_{BMC_i} = (\cos a_{i1})(q_1) + (\sin a_{i1})(q_2)\,. \qquad (7)$$

Factors Effecting Parameter Estimation in Item Response Models

In the unidimensional item response theory arena, a number of monte carlo studies have been conducted to examine the impact of various factors on the recovery of parameters. The results of some of these studies can provide clues as to which factors might be important to examine in regard to their impact on parameter recovery in multidimensional models. Hulin, Lissak, and Drasgow (1982) examined the influence of sample size and test length on parameter recovery in a two- and three-parameter logistic model. While they found that were trade offs between sample size and test length, they noted that for a 30-item test a sample size of 500 produced adequate estimates for a two-parameter model and a sample size of 1000 produced adequate estimates for a three-parameter model. In general shorter tests needed larger sample sizes to get accurate estimates.

Stone (1992) examined a marginal maximum likelihood estimation of a two-parameter logistic model. He varied test length, sample size and trait distributions. He found that difficulty estimates were fairly stable even in face of short tests, small sample sizes and non-normal (skewed and platykurtic) trait distributions though difficultly estimates were more subject to bias in shorter tests when the normality assumptions were not met. Estimations for item discrimination parameters were precise and stable under normal trait distributions. Skewed and platykurtic trait distributions did tend to positively bias discrimination estimations.

De Ayala and Sava-Bolesta (1999) investigated the impact of sample size/test length ratio, trait distribution and item information on parameter estimation in a nominal response model. They found that trait distribution accounted for 42.5 % of the variability in the accuracy of estimations for discrimination followed by sample size/test length ratio at 29.5% and item information at 3.5%. As the trait distribution departed from normal, larger sample sizes were needed. Under normal trait distributions a sample size/test length ratio of 10:1 produced accurate parameter estimates. Other researchers compared the parameter estimation of the nominal response model using the marginal maximum likelihood estimation and the Markov chain monte carlo estimation (Wollack, Bolt, Cohen, and Lee, 2002).

Kirisci, et al. (2001) compared the robustness of the parameter estimation of several different computer programs in the face of violations of the unidimensionality and normality assumptions. They found that the estimation program used influenced the robustness of the unidimensional assumption. However, in general across the programs when using a sample size of 1,000 and a test length of 40, the estimations were insensitive to the underlying trait distribution. This finding is surprising in light of De Ayala and Sava-Bolesta's (1999) finding that trait distribution accounted for a bulk of the variability in parameter estimation.

Less work has been done in parameter recovery in MIRT models. Of the few studies done most have focused on either detecting multidimensionality or on the effect of multidimensional data on parameter estimation in unidimensional models (Harwell, Stone, Hsu, & Kirisci, 1996). Bately and Boss (1993) examined the impact of trait correlation and trait range restriction on parameter recovery in a two-dimensional, two-parameter model. They found that correlation of the two traits had only a minor adverse influence on the recovery of the difficulty parameters. Range restriction in one trait did adversely impact the recovery of the difficulty parameters. The recovery of discrimination parameters was adversely impacted by both correlation of the traits and range restriction in one of the traits.

Factors Chosen for Examination in this Current Study

As Maydeu-Olivares (2001) highlighted the need for estimation studies to be conducted on more realistic test lengths, test length was chosen as one of the factors to manipulate in this current study. Because there is no extensive published study of the impact of sample size on multidimensional parameter recovery, sample size was also chosen as a factor to manipulate. The manipulation of sample size was done via manipulating the sample size/test length (SSR). The correlation of the traits was also chosen as a manipulation factor. This choice as prompted by two considerations. First, Bately and Boss's (1993) study indicated that correlation had a large adverse impact on estimating discrimination parameters in multidimensional models. Second, the MID and MDISC are based on the assumption of orthogonal trait dimensions, but in actual testing situations this assumption is likely to be violated. To make the study manageable in size the analysis was limited to a two-dimensional case with normally distributed latent traits.

The Need for Scale Equating in Multidimensional Item Response Monte Carlo Studies

One characteristic of item response theory is that the scaling of parameters and trait estimates is invariant with respect to linear transformations. This produces indeterminacy in the scale used for the estimations (Hambleton, et al., 1991). Therefore, when parameter estimates are derived from different samples, they must be placed on a common scale before they can be compared. The implication for this in monte carlo studies of parameter estimation is that to assess the quality of the parameter estimates, the estimates must be placed on the same scale as the true parameters used to generate the data (Baker & Al-Karni, 1991; Yen, 1987). In this monte carlo study, the parameter estimates made for each replication of a condition were compared to the true parameters used to generate the data for that condition. For these comparisons to be made the parameter estimates for each replication must be on the same scale as the true parameters for that condition. This process of placing parameters on the same scale is known as scale equating. How this need for scale equating was addressed in this current study is outlined below.

In unidimensional item response models both the slope and the difficulty scale must be transformed to place the parameter estimates on the same scale. When placing parameters from sample X on the same scale as parameters from sample Y, the transformations can accomplished using

$$a_Y = \frac{a_X}{a} \tag{9}$$

and

$$b_Y = ab_X + b \tag{10}$$

where a and b are scaling constants which can be determined via several possible methods (Hambleton, et al., 1991).

In MIRT a third transformation enters into the equating procedure. This additional transformation is the orientation of the trait axes in trait space. Li and Lissitz (2000) provided a graphical representation of the transformations necessary in equating two-dimensional item response models. (See Figure 3.) The three transformations that must be addressed are the orientation of the trait axes (rotation), the point of origin in the trait space (translation) and unit of measure used (dilation). Rotation involves making X_{q_1} and X_{q_2} parallel with Y_{q_1} and Y_{q_2} respectively. Translation involves shifting O_X to O_Y. Dilation involves equating the distance from O_X to U_X with the distance from O_Y to U_Y.

While several methods have been proposed for multidimensional scale equating (Hirsch, 1989; Li & Lissitz, 2000), the procedures are complex and there are still many unresolved issues related to them. As Li and Lissitz noted the impact on MIRT scale equating of sample size, of the number of anchoring items, and of the item discrimination and parameter estimation procedures are still unexplored areas. This lack of understanding of the dynamics of multidimensional scale equating could be problematic in a monte carlo study designed to examine the effects of sample characteristics on parameter estimation. The problem faced is determining if inaccuracies in the parameter estimates are due to sample characteristics or due to equating procedures or due to a combination of both.

Fortunately, for the purpose of the current study there are ways to work around both the rotational and scale indeterminacy aspects (translation and dilation) of multidimensional scale equating that do not require applying untested equating procedures to the parameter estimates. Scale indeterminacy can be addressed if the estimation program constrains the trait distributions to be a multivariate normal distribution with means of zero and variances of one. As Li & Lissitz (2000) note, this constraint causes the variances and covariances of the discrimination parameter estimates to capture the original (unstandardized) variances and covariances of the traits. The impact of this constraint on resolving the scale indeterminacy can be seen in the transformation formulas for the item discrimination parameters and the difficulty parameter (Li & Lissitz). For the two-dimensional case the formulas are:

$$a_{Yi1} = s_{q_1} a_{Xi1}, \tag{11}$$

$$a_{Yi2} = s_{q2} a_{Xi2}, \tag{12}$$

and

$$d_{Yi} = d_{Xi} + a_{Xi1} m_{q_1} + a_{Xi2} m_{q_2}, \tag{13}$$

where s_{q_1} and s_{q_2} are the variances of the trait estimates,

and m_{q_1} and m_{q_2} are the means of the trait estimates.

When the estimated trait variances are constraint to be one and the means are constrained to be zero the formulas collapse to:

$$a_{Yi1} = a_{Xi1}, \qquad (14)$$

$$a_{Yi2} = a_{Xi2}, \qquad (15)$$

and

$$d_{Yi} = d_{Xi}. \qquad (16)$$

NOHARM (Fraser, 2003), the estimation program chosen for this study, uses the necessary constraint of a multivariate normal distribution of trait estimates with means of zero and variances of one (McDonald, 1997) to address these scale indeterminacy issues.

The rotational indeterminacy problem can be addressed by creating at least one item per trait that measures just that one latent trait. McDonald (1997) refers to these types of items as basis items. By constraining the discrimination parameters of the dimensions not measured by the item to be zero in the parameter estimation process, these items serve to orient the axes in the trait space. McDonald strongly encourages their use in multidimensional modeling, as without them it is impossible to understand the structure of the data.

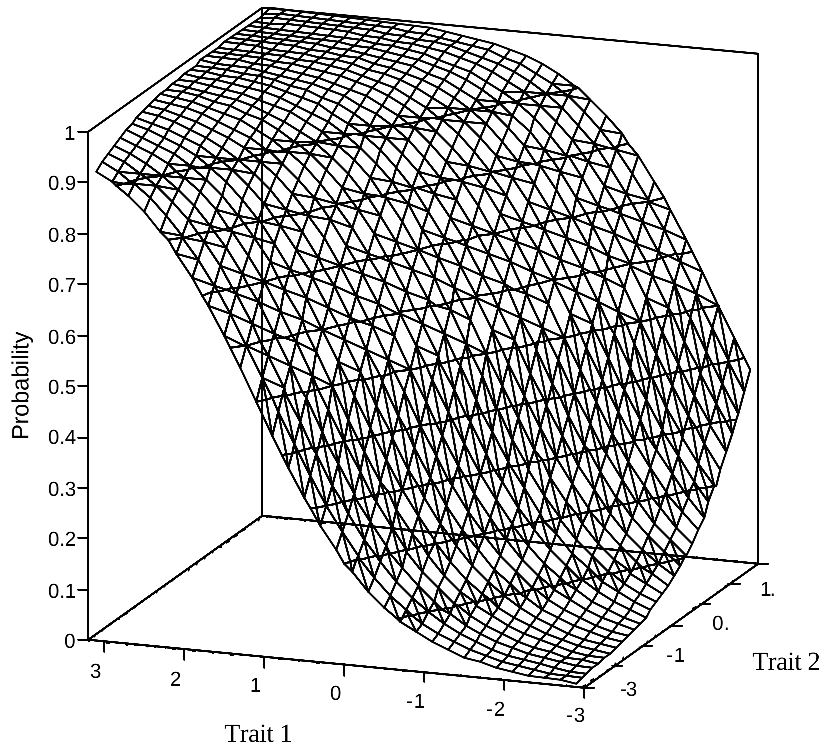


Figure 1. Item characteristic surface (ICS) for a two-dimensional, two-parameter compensatory model with parameters $a_{i1} = 1.25$, $a_{i2} = .75$ and $d_i = 1$.

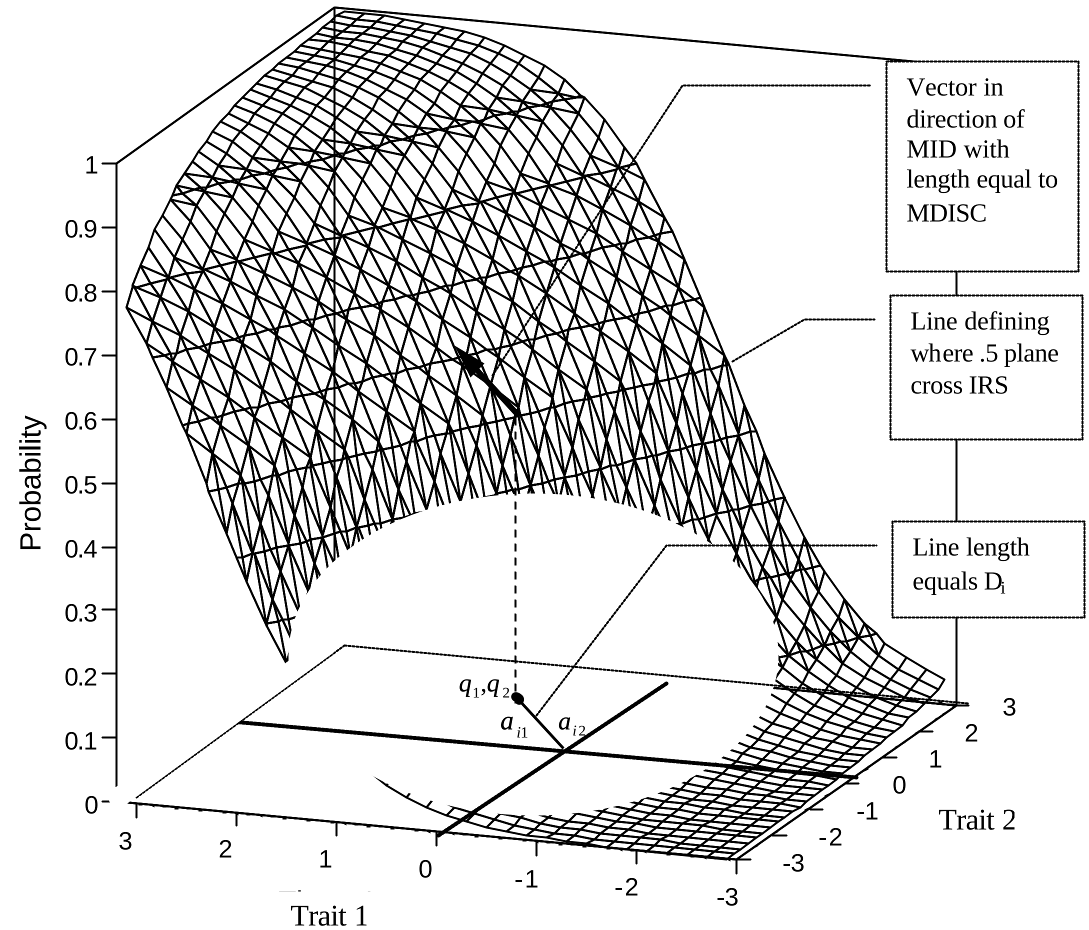


Figure 2. Graphical representation of MID and MDISC with $a_{i1} = 1.5$, $a_{i2} = .75$, $d_i = -1.0$, $MDISC = 1.677$, $D_i = .596$, $a_{i1} = 26.56^o$ and $a_{i2} = 63.44^o$.

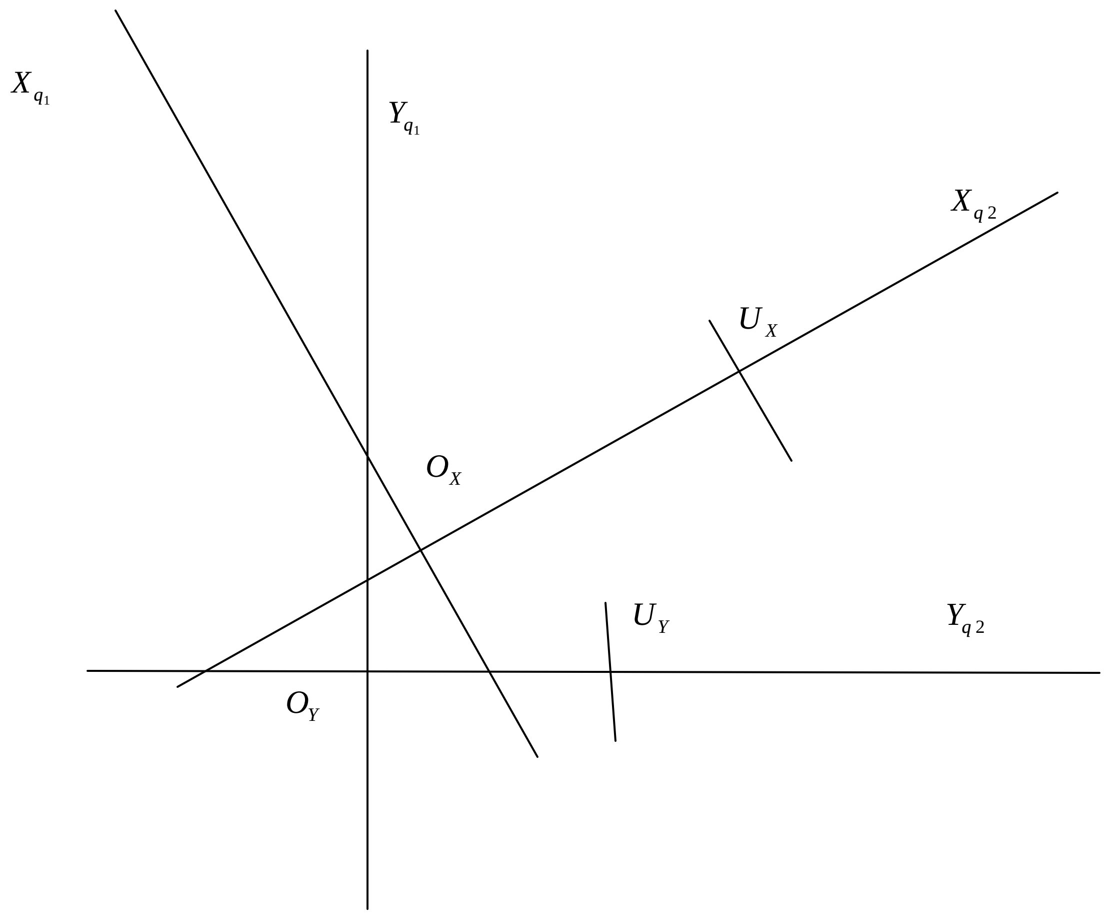


Figure 3. Graphical representation of needed transformations in multidimensional item response scale equating: rotation, translation and dialation. Modified from Li & Lissitz (2000).

CHAPTER 2

METHODS

The current study tested a two-dimensional, two-parameter compensatory model as described in equation 3.

Conditions Tested

Three factors potentially impacting parameter estimation were examined in this study: test length (TL), sample size/test length ratio (SSR) and trait correlation (TC). Four test lengths were used: 23, 44, 65, and 86. These test lengths were chosen to allow for two basis items per test and for a uniform distribution of items over seven difficulty levels while also reflecting a range of reasonable test lengths. Six sample size/test length ratios were used: 5:1, 10:1, 20:1, 30:1, 50:1, 100:1. These figures were chosen based on De Ayala and Sava-Bolesta's (1999) finding that in a unidimensional nominal model SSR's from 5:1 to greater than 10:1 were needed depending on the underlying trait distribution. They used a SSR of 20:1 as their base line. Because one of the goals of this research was to determine the minimum sample sized needed for parameter recovery a 5:1 SSR was included. However, for a multidimensional model larger SSR's may be needed because more parameters are being estimated than in the unidimensional case. SSR's up to 100:1 were included so that the shortest test length (23 items) would have one condition with a sample size over 2,000. This was done to allow comparing the results of various sample sizes with the traditionally accepted 2,000 figure.

Two levels were included for the trait correlation: 0.0 and 0.3. Crossing all of these conditions produced a 4 x 6 x 2 design resulting in a total of 48 conditions.

To allow for testing of the impact of these factors at various difficulty levels a uniform distribution of difficulty levels, d_i, was chosen. Seven levels of difficulty were chosen: -1.0, -0.66, -0.33, 0.0, 0.33, 0.66, and 1.0. To allow for testing of the impact of the conditions on different item discrimination levels various combinations of discrimination parameters were used. For the 23-item test three combinations were used: $a_1 = 1.0$, $a_2 = 1.0$; $a_1 = 0.5$, $a_2 = 1.0$; $a_1 = 1.0$, $a_2 = 1.5$. Items where a1 and a2 were both 1.0 had a mid discrimination level. Items with of one discrimination parameter being 0.5 and the other being 1.0 had a low discrimination leve l. Items with one discrimination parameter being 1.0 and the other being in 1.5 had a high discrimination level. For the 44-item test three additional combinations were included which reversed the discrimination values for the dimensions: $a_1 = 1.0$, $a_2 = 1.0$; $a_1 = 1.0$, $a_2 = 0.5$; $a_1 = 1.5$, $a_2 = 1.0$. For the 65-item test the first set of combinations were repeated and for the 86-item test the second set of combinations were added again. This approach provided a degree of balance on the discrimination ability of the tests on each of the dimensions.

The two basis items in each test were simulated to have d_i values of 0.0 and discrimination parameter combinations of $a_1 = 1.0$, $a_2 = 0.0$, and $a_1 = 0.0$, $a_2 = 1.0$ respectively.

Data Generation

The computer program RESGEN 4.0 (Muraki, 2000) was used to generate the simulated response patterns based on the chosen item parameters and trait correlations. The program generates a simulee’s response pattern based on the item parameters and the simulee’s trait levels by comparing the simulee’s probability of passing an item with a random number between zero and one. If the random number is less than the probability then the simulee is credited as having passed the item. The normal-ogive item response model option was chosen for the data generation.

Dependent Variables

In a two-dimensional, two-parameter model a number of parameters are available for examination: a_{i1}, a_{i2}, d_i, MDISC and MID which includes D_i and α_{ik}. As each of these parameters provides insight into how an item behaves, each was examined in this study.

A commonly used measure of the quality of parameter recovery is the root mean square error (RMSE) (Harwell, et al., 1996). This is given by:

$$RMSE_{\hat{l}} = \left[\frac{\sum_{i=1}^{n} \left(\hat{l}_i - l_i \right)^2}{n} \right]^{1/2}, \qquad (17)$$

where $\hat{l}_i$ is the estimated parameter of interest;

l_i is the actual parameter used to generate the data;

and n is the number of items across which the parameter is averaged.

The RMSE was then averaged across all the replications within a condition to provide a mean RMSE (MRMSE) for that condition. This MRMSE serves as a global measure of parameter recovery. However, by itself the MRMSE does not separate out the bias of the estimation and the precision of the estimation. To measure the bias the of parameter estimations the average signed deviation was used. This is given by:

$$BIAS_{\hat{l}} = \frac{\sum_{i=1}^{n} \left(\hat{l}_i - l_i \right)}{n}, \qquad (18)$$

where the terms are defined as above. As with the RMSE, the BIAS was averaged across replications within each condition to provide a measure of mean BIAS (MBIAS) for that condition.

The variance of the parameter estimations across simulation replications was used as a measure of the precision of the estimations. This is given by:

$$VAR_{\hat{l}} = \frac{\sum_{r=1}^{R}\left(\hat{l}_{ir} - \overline{\hat{l}_i}\right)^2}{R}, \qquad (19)$$

where $\hat{l}_{ir}$ is the parameter estimation for replication r;

$\overline{\hat{l}_i}$ is the average parameter estimation across all replications;

and R is the number of replications.

VAR was then averaged across items to provide a mean VAR (MVAR).

Another commonly used statistic in monte carlo studies is the correlation between the estimated parameters and the true parameters (Harwell, et al., 1996). This measure was also included in the study.

Number of Replications

Harwell, et al., (1996) recommend a minimum of 25 replications of each condition in monte carlo item response theory studies. However, Cohen, Kane, & Kim (2001) note that several factors can influence the number of replications that are needed to accurately determine the variability of the precision of recovery of item parameters. They found that although testing the effects of manipulating sample size does not seem to require a large number replications, manipulating test length does require a large number of replications. Cohen, et al. noted that the magnitude of the differences between conditions to be measured, Δ, determines how small the standard error of the statistic of interest needs to be for the difference to be detectable. They suggested that a stringent tolerance criterion would be of the magnitude of:

$$\frac{SE}{\Delta} \le .1. \qquad (20)$$

In the examples they provide, one case required 1,122 replications to meet this standard and another case required over 28,000. These values well exceed the minimum 25 replications that Harwell, et al. recommend.

From a practical point of view using 28,000 replications would severely limit the number of conditions that could be tested. Therefore, to balance the need for power in the study and the need for manageable sized data sets, a minimum of 2,000 replications was used for each condition in the study.

Parameter Estimation

The Windows version of the program NOHARM (Normal-Ogive Harmonic Analysis Robust Method) (Fraser, 2003) was used in the parameter estimation. This is a commonly used program in MIRT (Maydeu-Olivares, 2001) for modeling dichotomously scored items. The model is based on nonlinear factor analysis and is an approximation of the multidimensional normal-ogive model (Maydeu-Olivares; McDonald, 1997). The program uses a polynomial series to approximate the multidimensional model based on a harmonic analysis. Parameter estimations are made in reference to the observed proportion of examinees passing each item. The estimations for d_i values are found in closed form from the appropriate sample analogue. The estimations for the a_i values are found through minimizing the appropriate equation using an unweighted least squares approach. (For details of the underlying theory and the model for parameter estimation see McDonald, (1997), and the NOHARM manual (Fraser, 2003).)

In the current study the solutions provided by NOHARM were constrained to have trait correlations equal to those used in the data generation. For the anchor items used to orient the trait axes, the discrimination value for one trait was constrained to be zero while the other discrimination value was free to be estimated. One anchor item allowed the discrimination value for trait one to be free to be estimated and the other anchor item allowed the discrimination value of trait two to be estimated. The discrimination values fixed to zero were not used in the calculations of MRMSE, MBIAS and MVAR.

CHAPTER 3

RESULTS FOR ENTIRE TEST

In monte carlo studies where a number of factors are examined, it can be difficult to interpret the findings based only on descriptive statistics. Therefore, researchers have been encouraged to include inferential analysis (Harwell, 1997; Harwell, et al., 1996). In light of this recommendation, regression analyses were conducted on the data. Regression analysis is particularly attractive in item response theory monte carlo studies as the independent variables tend to be metric (Harwell, et al.).

The data were analyzed in several ways to help shed light on the dynamics of MIRT parameter recovery. The three dependent measures, $MRMSE_{\hat{i}}$, $MBIAS_{\hat{i}}$, and $MVAR_{\hat{i}}$, were calculated for each of the parameters of interest, a_{i1}, a_{i2}, d_i, MDISC and MID which includes D_i and a_1. Because only two-dimensional cases are being investigated a_2 is not needed to define the direction as MID assumes orthogonal dimensions. The orthogonal assumption constrains a_2 to equal $90^o - a_1$ in the two-dimensional case. In addition to being calculated across the entire test, the dependent measures were also calculated within each item difficulty level and within each item discrimination level.

To determine the influence of the independent variables (SSR, TL and TC) on the dependent variables (MVAR, MRMSE and MBIAS) for each of the parameter estimates (a1, a2, d, MDISC, D of MID and a_1 of MID), hierarchical regression was used (Pedhazer, 1997). SSR, TL and TC were entered in the first step. In the second step the product terms SRR x TL, SSR x TC, and TL x TC were added. The product term SSR x TL x TC was entered in the third step. If the data plots or the plots of the residuals with the independent variables suggested that additional terms (i.e. squared or reciprocal terms of the independent variables) were needed to better specify the model, these terms were added in subsequent steps. The type II squared semi-partial correlations were used to examine the unique variance in the dependent variables accounted for by the terms in the step in which they were first entered.

For some replications, solutions for the parameter estimates created by NOHARM included negative values for either a1 or a2. Because negative discrimination values are not valid in MIRT, solutions with negative estimates for either a1 or a2 were considered inadmissible. Table 1 lists the number of replications with admissible solutions for each condition. Because of these inadmissible solutions, the data points for all conditions did not contribute equal amounts of information to the regression analyses. Therefore, the data points were weighted by the number of valid replications in the condition.

Before the regression analyses were conducted, a log transformation was applied to MVAR, MRMSE and MBIAS. This was done for two reasons. First, there was evidence of unequal variance in each of these dependent variables across the levels of each of the independent variables (Appendix A, Tables A1 to A5) for each of the parameter estimates. A log transformation helped in equalizing the variances. Second, in some cases the large range of values (Tables 2, 6, 10, 14 and 18) made it difficult to discern patterns in the data. By expanding values in the low range and compacting values in the high range, the log transformation helped make patterns in the data more visible.

The use of the log transformation with MBIAS could be problematic because MBIAS can assume negative values. For situations where there were negative values of MBIAS two types of regression analyses were conducted. First, to examine the influence of SSR, TL and TC on the magnitude of MBIAS, a hierarchical regression analysis was conducted on the log of the absolute values of MBIAS. This allowed for the examination of the unique variance in the magnitude of

MBIAS accounted for by each of the independent variables. Second, to examine the influence of the independent variables on the direction of MBIAS, logistic regression was conducted using a coding system where all negative values of MBIAS were assigned a value of minus one and all positive values were assigned a value of one. This coding allowed for the examination of the influence the independent variables had on the probability of negative MBIAS occurring.

For each hierarchical regression analysis conduced, the assumption of normality of residuals was tested using Shapiro-Wilk's W (Shapiro & Wilk, 1965). Because the regression analyses had samples of 48 or less, this test was chosen due to it having been demonstrated to have adequate power with smaller sample sizes (Shapiro, Wilk & Chen, 1968). To assess the assumption of the homoscedasticity of residuals Glejser's (1969) approach was used. In this approach the absolute values of the residuals are regressed on the independent variables. If the regression coefficient for a variable is significant, then there is evidence of heteroscedasticity of residuals along that variable. Glejser recommends using a significance level of p = .11 to have a 5% probability of type I errors (detecting heteroscedasticity when it is not present).

Analyses for Discrimination Estimates – a1 and a2

The mean, maximum value, minimum value, and SD for MVAR, MRMSE and MBIAS and their log transformation are presented in Table 2. The MVAR, MRMSE and MBIAS for a1 and a2 for each condition are presented in Appendix B, Tables B1 to B6.

MVAR of a1 and a2

The plots of the log of MVAR with SSR, TL and TC for a1 (Figure 4) revealed two distinct groups of data points. There was a clear break in the data where the log of MVAR for a1 dropped suddenly. A similar pattern existed for a2 (Appendix C, Figure C1). All of the conditions in the high log MVAR groups for a1 and a2 (Appendix D, Table D1) had either a SSR of 5 or a TL of 23. The plots of the log of MVAR of a1 with SSR and TL for TC equal to 0.0 and 0.3 (Figure 5) revealed a "mountain" region where the log of MVAR values increased sharply for low values of SSR and TL. The ridges of these "mountains" ran along the edges of the plot where SSR equaled 5 and TL equaled 23. For SSR equal to five, the height of the ridge decreased as TL increased. For TL equal to 23, the height of the ridge decreased as SSR increased. For conditions with TC equal to 0.3 the slopes of the "mountain" were steeper and the ridges were higher. There was also a "plain" region where the log of MVAR values were lower and more stable. Generally, this "plain" region began when SSR equaled 10 and TL equaled 44. A similar pattern was seen for a2 (Appendix C, Figure C2)

This "mountain" and "plain" pattern suggested two perspectives from which the data could be modeled. From the first perspective, the variations in the data could be viewed as being continuous with the extreme values of MVAR resulting from an interaction between SSR and TL. From this perspective small values of SSR along with small values of TL produced exceptionally large values of MVAR. From the second perspective, variations in the data could be viewed as discontinuous with a distinct jump occurring in the values of MVAR when a minimum threshold in the value of SSR and/or TL was crossed.

To examine which perspective best fits the data two hierarchical regression analyses were conducted. To model the continuous data perspective, a regression model was fit to the full data set which included all the conditions. To model the discontinuous perspective, a regression model was fit to a reduced data set consisting of conditions in the "plain" region. To create the reduced data set conditions with a SSR of 5 or a TL of 23 were dropped. This resulted in 18 conditions being removed. Although five of the conditions eliminated were not the in high MVAR group for a1 or a2, their elimination maintained a balanced experimental design. The mean, maximum value, minimum value, and SD for MVAR, MRMSE, MBIAS and their log transformations for the reduced data set are presented in Table 3.

An examination of Figure 5 suggested that a hyperbolic function may be involved in the relationship between the log of MVAR and SSR and TL (Berry & Feldman, 1985). Therefore the reciprocal terms of SSR and TL were entered into the equation after the product terms. Detailed results of the regression analysis using the full data set (from the continuous data perspective) are presented in Appendix E, Tables E1 and E2. A summary of the results is presented in Table 4. The significant terms at each step in the regression analyses were the same for a1 and a2.

The R^2 values of .8772, $F(7, 40)=30.17$, $p <.0001$, for the final regression step for a1 and of .8797, $F(7, 40) = 30.86$, $p < .0001$ for a2 suggested a good model fit. Glejser's test did not identify any heteroscedasticity for the residuals of a1 or a2. However, for both a1 and a2 the residuals did not meet the normality assumption with Shapiro-Wilk W's of 0.955786, $p = .0682$ and 0.944455, $p = .0242$. An examination of the plots of the residuals with the predicted values (Figure 6) indicated that the fit of the model might not be appropriate. These plots showed a distinct clustering of the residuals for the conditions in the high and low MVAR groupings. For low predicted values, the residuals were positive indicating under prediction. As the predicted values increased, the residuals slid into negative values indicating over prediction. However, the residuals jumped back into positive values as the data entered the high MVAR group of conditions. This indicated a systematic bias in the predictions of the model.

The analysis from the discontinuous data perspective used the reduced data set. The detailed results of the second analysis are presented in Appendix E, Tables E3 and E4. A summary of the results is presented in Table 4. The R^2 values of .9973, $F(9, 20) = 826.13$, $p <.0001$ for a1 and of .9972, $F(9, 20) = 792.40$, $p < .0001$ for a2 suggested a good model fit after the fourth step. However, the residual plot with SSR suggested the need for a higher order SSR term in the model. Therefore, a fifth step was added to the hierarchical regression analysis in which a SSR^2 term was entered. The results of this additional step are included in Tables E3 and E4. After the addition of the SSR^2 term the final R^2 values were .9997, F(10, 19) = 6064.77, p < .0001 for a1 and .9996, F(10, 19) = 5365.08, p < .0001. The residuals meet the normality assumption with Shapiro-Wilk's W's of 0.976623, p = 0.7304 for a1 and of 0.95016, p = 0.1707 for a2. Based on Glejser's test there was no evidence of heteroscedasticity. The plots of the residuals with the predicted values (Figure 7) did not indicate a prediction bias.

MRMSE of a1 and a2

As with MVAR, the values of the log of MRMSE for a1 and a2 (Appendix C, Figures C3 and C4) clustered into low and high log of MRMSE groups. However, the break point for the conditions was not as distinct as it was for MVAR. The conditions which provided the data points for the high log MRMSE group for a1 and a2 are presented in Appendix D, Table D2. All of the high log MRMSE conditions either had SSR equal to 5 or TL equal to 23. The plots of the log of MRMSE of a1 with SSR and TL for TC equal to 0.0 and 0.3 (Figure 8) had the "mountain/plain" characteristics. The same pattern existed for a2 (Appendix C, Figure C5). This suggested the possibility of modeling the data from both the continuous and discontinuous perspectives. Because of the similarity of the plots for MVAR and MRMSE, the same variables were used in the regression analyses.

The detailed results of the regression analyses for the log of MRMSE for a1 and a2 from the continuous perspective are presented in Appendix E, Tables E5 and E6. A summary of the findings is presented in Table 4. The R^2 values of .8962, F(9,38) = 36.44, p <.0001, for the final regression step for a1 and of .8558, F(9,38) = 25.06, p < .0001 for a2 suggested a good model fit. The residuals for a1 met the normality assumption with a Shapiro-Wilk W's of 0.97856, p = .5202. However, the residuals of a2 did not meet the normality assumption with a Shapiro-Wilk W's of 0.94751, $p = .0319$. Glejser's test indicated heteroscedasticity for the residuals of a1 along SSR, $t(1) = 1.70$, $p = .0968$. Tests for a2 indicated heteroscedasticity of residuals along SSR, $t(1) = -2.42$, $p = .0196$, and TL, $t(1) = -2.41$, $p = .0204$. The residual plots for a1 and a2 with the predicted values (Appendix C, Figure C6) showed the same prediction bias pattern seen for MVAR.

Based on the results for MVAR, the term SSR^2 was included in the analysis from the discontinuous perspective. The results of this analysis are presented in Appendix E, Tables E7 and E8. A summary of the findings are presented in Table 4. The R^2 values of .9991, $F(10, 19) =2078.17$, $p <.0001$, for the final regression step for a1 and of .9993, $F(10, 19) = 2720.81$ $p < .0001$ for a2 suggested a good model fit. Neither the residuals for a1 or a2 violated the normality assumptions with Shapiro-Wilk W's of 0.978316, $p <.9994$ and 0.969187, $p = .5172$ respectively. Glejser's test did not reveal any heteroscedasticity for a1 or a2. The plots of the residuals with the predicted value (Appendix C, Figure C7) did not give any indication of the systematic bias in prediction evidenced in the plots in the analysis using all conditions.

<u>*MBIAS of a1 and a2*</u>

Because no values for MBIAS for a1 and a2 were negative, the log transformation could be made without any other transformations to the data. The values of the log of MBIAS with for a1 and a2 (Appendix C, Figures 20 and 21) grouped into conditions with high and low MBIAS. The conditions which provided the data points for the high log MBIAS group (Appendix D, Table D3) all had either a SSR of 5 or a TL of 23. The plot of the log of MBIAS with SSR and TL for TC equal to 0.0 and 0.3 for a1 (Figure 9) had the "mountain/plain" characteristics. The data for a2 showed the same characteristics (Appendix C, Figure C10) thus analyses were conducted from both the continuous and discontinuous data perspectives.

The detailed results of the analysis on the full data set are presented in Appendix E, Tables E9 and E10. A summary of the findings is presented in Table 4. The final R^2 values of 0.8519, F(9, 38) = 24.29, p < .0001 and 0.8256, F(9, 38) = 19.99, p < .0001 for a2 suggested a good model fit. The residual values for a1 conformed to the normality assumption with Shapiro-Wilk W's of .973808, p = .3537, however the residuals for a2 did not, W = .927092, p = .0053. Glejser's test showed no evidence of heteroscedasticity for the residuals for a1. However, the test indicated heteroscedasticity for the residuals of a2 along SSR, t(1) = -2.07, p = 0.0446, and along TL, t(1) = -2.35, p = 0.0234. The plots of the residuals with the predicted values (Appendix C, Figure C11) indicate the systematic prediction bias pattern seen for MVAR.

The plots for the log of MBIAS with SSR and TL for TC equal to 0.0 and 0.3 for a1 (Figure 10) for the reduced data set showed that although the data in the "plain" region was smoother than for the full set, there are irregularities in the behavior of the log of MBIAS. The "plain" region for a2 (Appendix C, Figure C12) also had irregularities. The detailed results for the analysis from the discontinuous perspective on the reduced data set are given in Appendix E, Tables E11 and E12. A summary of the findings is presented in Table 4. The final R^2 values of 0.9844, F(10, 19) = 120.07, p < .0001 for a1 and .9835, F(10, 19) = 113.50, p < .0001 for a2 indicated a good model fit. The residuals for both a1 and a2 meet the normality assumption with Shapiro-Wilk W's 0.966844, p = 0.4568 and 0.946717, p = 0.1380. Glejser's tests indicated some heteroscedasticity in the residuals for a2 along SSR, t(1) = -1.69, p = 0.1029. No heteroscedasticity was seen for the residuals of a1. The plots of the residuals with the predicted values (Appendix C, Figure C13) did not indicate prediction bias.

Correlations between estimates and parameters for a1 and a2

The average within condition correlation between the estimated values of a1 and a2 and the parameter values of a1 and a2 across the all the conditions and across the conditions in the reduced data set are given in Table 5. The average correlation was greater for the reduced data set.

Analyses for Difficulty Estimates - d

The mean, maximum value, minimum value, and SD for MVAR, MRMSE and MBIAS for d and their log transformations are presented in Table 6. The MVAR, MRMSE and MBIAS for d across for each condition are presented in Appendix B, Tables B7 to B9.

MVAR for d

The plots of the log of MVAR of d with SSR, TL and TC (Figure 11) revealed high and low MVAR groups. The conditions which provided the data points for the high log MVAR group (Appendix D, Table D4) all had a SSR of 5 or a TL of 23. Plots of the log of MVAR of d with SSR and TL for TC equal to 0.0 and 0.3 (Figures 12) had the "mountain/plain" characteristics. Therefore, regression models from both the continuous data perspective and from the discontinuous data perspective were fit to the data. The results of the regression analyses on the full data set from the continuous data perspective are presented in Appendix E, Table E13. A summary of the findings is presented in Table 7. The final R^2 value of 0.8550, $F(9, 38) = 24.99$, $p = .0001$ suggested a good model fit. The residuals met the normality assumption with a Shapiro-Wilk W of 0.975697, $p = 0.4145$. Glejser's test indicated a heteroscedasticity of variance for the residuals along SSR, $t(1) = -2.72$, $p = .0094$. The plot of the residuals with the predicted values (Figure 13) revealed prediction bias.

The mean, maximum value, minimum value, and SD for MVAR, MRMSE and MBIAS for d and their log transformations for the reduced data set are presented in Table 8. The detailed results of the regression analysis on the reduced data set are provided in Appendix E, Table E14. A summary of the findings is presented in Table 7. The final R^2 of 0.9995, $F(10, 19) = 3663.82$, $p < .0001$ indicated a good model fit. The residuals met the normality assumption with a Sharpiro-Wilk W of 0.98051, p =0.8390. Glejser's test did not indicate any heteroscedasticity in the residuals. The plot of the residuals with the predicted values (Figure 14) did not indicate prediction bias.

MRMSE for d

The values of the log of MRMSE for d (Appendix C, Figure C14) grouped into high and low MRMSE groups although the break in the data was not as distinct as with MVAR. All conditions which provided the data points for the high log of MRMSE group (Appendix D, Table D5) either had a SSR of 5 or a TL of 23. Again analyses were conducted from the continuous and discontinuous perspectives because the plots of the log of MRMSE with SSR and TL with TC = 0.0 and TC = 0.3 (Figure 15) had the "mountain/plain" characteristics.

The details of the results of the analysis for the full data set are given in Appendix E, Table E15. A summary is provided in Table 7. The final R^2 was 0.8510, $F(9, 38) = 24.12$, $p < .0001$. The residuals did not meet the normality assumption with a Shapiro-Wilk's W of .909668, p = 0.0013. Glejser's test indicated heteroscedasticity of the residuals along SSR, $t(1) = -2.61$, $p = 0.0124$ and TL, $t(1) = -1.95$, $p = 0.0575$. The plot of the residuals with the predicted values (Appendix C, C15) showed a systematic prediction bias similar to that found for MVAR of d.

The detail of the results of the analysis for the reduced data set are given in Appendix E, Table E16. A summary is provided in Table 7. The final R^2 was 0.9995, $F(10, 19) = 3684.14$, $p < .0001$. The residuals met the normality assumption, W = 0.968176, p = .4906. Glejser's test did not indicate heteroscedasticity of the residuals. The plot of the residuals with the predicted values (Appendix C, Figure C16) did not indicate prediction bias.

MBIAS of d

Because some of the values of MBIAS for d assumed negative values a direct log transformation was not performed. A hierarchical regression analysis was conducted on the log of the absolute values of MBIAS to examine the influence of the independent variables on the magnitude of MBIAS of d. To examine the influence of the independent variables on the direction of the MBIAS, logistic regression was conducted using a coding system where all negative values of MBIAS were assigned a value of minus one and all positive MBIAS values were assigned a value of one.

Examination of the plots of the log of absolute value of the d with SSR, TL and TC (Figure 16) did not reveal a natural break of the MBIAS values into high and low groups. However, the plots with SSR and TL for TC equal to 0.0 and 0.3 (Figure 17) did reveal the familiar “mountain/plain” pattern although the “plain” region had some irregularities as for the MVAR and MRMSE. Therefore, analyses were conducted on both the full and reduced data sets to examine the model fit from both the continuous and discontinuous data perspectives.

Details of the analysis on the full data set are given in Appendix E, Table E17. A summary is given in Table 7. The final R^2 was 0.6836, $F(9, 38) = 9.12$, $p = 0.0001$. The residuals did not meet the normality assumption with a Shapiro-Wilk’s W of .914834, $p = 0.0020$. Glejser’s test did not indicate heteroscedasticity of residuals. The plot of the residuals with the predicted values (Figure 18) suggested a cyclical pattern.

The details of the analysis for the reduced data set are given in Appendix E, Table E18. A summary is given in Table 7. The R^2 value was 0.7341, $F(9, 20) = 6.14$, $p < .0004$. The residuals met the normality assumption with a Shapiro-Wilk’s W of 0.976191, $p = .7178$. Glejser’s test did not indicate heteroscedasticity. The plot of the residuals with the predicted values (Figure 19) hinted at a cyclical pattern but it was not as distinct as for the full data set.

For the logistic regression analysis of the sign of MBIAS for d, all of the independent variables and their product terms were enter simultaneously. However, the solution would not converge with the product term, SSR x TL x TC in the model. Once this term was removed, the solution converged and the Hosmer and Lemeshow goodness-of-fit test, $?^2(8) = 13.1728$, p = 0.1060, indicated an acceptable fit. The results for the logistic regression indicated that SSR was the only significant term in predicting the probability of MBIAS being negative. SSR's coefficient estimate was -0.1987, Wald $?^2(1) = 5.1503$, p = 0.0232. As SSR increased the probability of MBIAS being negative decreased.

Correlations between estimates and parameters for d

The average within condition correlation between the estimated value of d and the parameter value of d across the all the conditions and across the conditions in the reduced data set are given in Table 9. The average correlation was slightly greater for the reduced data set.

Analyses for Multidimensional Discrimination Estimates - MDISC

The mean, maximum value, minimum value, and SD for MVAR, MRMSE and MBIAS for MDISC and for their log transformations are presented in Table 10. The MVAR, MRMSE and MBIAS for MDISC for each condition are presented in Appendix B, Tables B10 to B12.

MVAR for MDISC

Plots of the log of MVAR of MDISC with SSR, TL and TC for MDISC (Figure 20) reveal high and low MVAR groups. The conditions which provided the data points for the high log MVAR group (Appendix D, Table D6) all had either a SSR value of 5 or a TL value of 23. The plots of the log of MVAR of MDISC with SSR and TL for TC equal to 0.0 and 0.3 (Figure 21) had the "mountain/plain" characteristics. In this case the ridge along TL equal to 23 dropped more quickly as

SSR increased. The data was again examined from the continuous and discontinuous data set perspectives.

The results of the regression analyses for log of MVAR of MDISC for the full data set are presented in Appendix E, Table E19. A summary of the findings is presented in Table 11. The final R^2 of .7877, $F(9,38) = 15.67$, $p < .0001$, suggested a moderate model fit. The residuals met the normality assumption with a Shapiro-Wilk' W of 0.971466, $p = 0.2886$. Glejser's test indicated residual heteroscedasticity along SSR, $t(1) = -3.54$, $p = 0.001$. The plot of the residual with the predicted values (Figure 22) indicated prediction bias.

The descriptive statistics for the reduced data set used in the discontinuous model perspective are presented in Table 12. The results of the regression analyses for the reduced data set are presented in Appendix E Table E20. A summary of the findings is presented in Table 11. The final R^2 of 0.9973, $F(9, 20) = 816.31$, $p < .0001$ indicated a good model fit. The residuals met the normality assumption with a Shapiro-Wilk's W of 0.946603, $p = 0.1370$. Glejser's test did not indicate any heteroscedasticity of the residuals. The plot of the residuals with the predicted values (Appendix C, Figure C17) did not indicate prediction bias.

MRMSE for MDISC

The values of the log of MRMSE for MDISC (Appendix C, Figure C18) clustered into high and low MRMSE groupings. The conditions which provided the data points for the high log MRMSE group for MDISC (Appendix D, Table D7) all had a SSR value of 5 or a TL value of 23. The plots of the log of MRMSE of MDISC with SSR and TL for TC equal to 0.0 and 0.3 (Figure 423) have the "mountain/plain" characteristics. As with MVAR, the ridge along TL equal to 23 dropped quickly as SSR increased. Again the data was considered from the continuous and discontinuous model perspectives.

The results of the regression analyses on the full data set from the continuous perspective are presented in Appendix E, Table E21. A summary of the findings is presented in Table 11. The final R^2 of .7975, $F(9,38) = 16.63$, $p < .0001$, suggested a moderate model fit. The residuals did not meet the normality assumption with a Shapiro-Wilk' W of 0.901369, $p = 0.0007$. Glejser's test indicated residual heteroscedasticity along SSR, $t(1) = -3.09$, $p = 0.0035$. The plot of the residual with the predicted values (Appendix C, Figure C19) showed a prediction bias similar to that seen for MVAR of MDISC.

The results of the regression analyses for the reduced data set are presented in Appendix E, Table E22. A summary of the findings is presented in Table 11. The final R^2 was 0.9986, $F(10, 19) = 1357.51$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk's W of 0.974479, $p = 0.6674$. Glejser's test indicated heteroscedasticity along TC, $t(1) = 2.29$, $p = 0.0303$. The plot of the residual with the predicted values (Appendix C, Figure C20) did not indicate prediction bias.

MBIAS of MDISC

None of the values of the MBIAS for MDISC assumed negative values so the direct log transformation was performed. The values of the log of MBIAS for MDISC (Appendix C, Figure C21) grouped into high and low MBIAS conditions although the high MBIAS group was small. The conditions which provided the data points for the high log of MBIAS group for MDISC (Appendix D, Table D8) all had a SSR value of 5 or a TL value of 23. Plots of the log of MRMSE of MDISC with SSR and TL for TC equal to 0.0 and 0.3 (Figure 24) had the "mountain/plains" characteristics with the quick drop along the TL equal to 23 ridge. Two analyses were conducted to fit models from the continuous and discontinuous perspectives.

The results of the regression analyses for the log of MRMSE of MDISC from the continuous perspective are presented in Appendix E, Table E23. A summary of the findings is presented in Table 11. The final R^2 was .7885, $F(9, 38) = 15.74$, $p < .0001$. The residuals did not meet the normality assumption with a Shapiro-Wilk's W of 0.860175, p = 0.0001. Glejser's test indicated heteroscedasticity of the residual along SSR, $t(1) = 2.58$, $p < .0001$. The plot of the residuals with the predicted values (Appendix C, Figure C22) showed a systematic prediction bias similar to that for MVAR and MRMSE for MDISC.

The log of MBIAS of MDISC plotted with SSR and TL for TC equal to 0.0 and 0.3 for the reduced data set (Figure 25) showed some irregularities in the "plain" region of the data. The results of the regression analyses for the log of MBIAS for the reduced data set are presented in Appendix E Table E24. A summary of the findings is presented in Table 11. The final R^2 was 0.9993, $F(10, 19) = 176.09$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk's W of 0.957179, p = 0.2619. Glejser's test did not indicate any heteroscedasticity of the residuals. The plot of the residuals with the predicted values (Appendix C, Figure C23) did not indicate prediction bias.

<u>Correlations between estimates and parameters for MDISC</u>

The average within condition correlation between the estimated value of MDISC and the parameter value of MDISC across the all the conditions and across the conditions in the reduced data set are given in Table 13. The average correlation was greater for the reduced data set.

Analyses for Multidimensional Item Difficulty Estimates – D (distance)

The mean, maximum value, minimum value, and SD for MVAR, MRMSE and MBIAS for D of MID and for their log transformations are presented in Table 14. The MVAR, MRMSE and MBIAS for D of MID for each condition are presented in Appendix B, Tables B13 to B15.

MVAR for D of MID

The plots of the log of MVAR of D with SSR, TL and TC (Figure 26) did not show high and low MVAR groups. The surfaces of the plots of MVAR with SSR and TL for TC equal to 0.0 and 0.3 (Figure 27) presented smooth surfaces rather that the “mountain/plain” pattern seen for the other parameter estimates. These plots did not suggest the need to consider the data from the discontinuous perspective. Therefore, the regression analysis was only conducted on the full data set.

The details of the regression analysis are presented in Appendix E, Table E25. A summary is given in Table 15. The final R^2 was 0.9991, $F(10, 37) = 4180.47$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk W of 0.958472, $p = 0.0876$. Glejser’s test did indicate some heteroscedasticity of the residuals along SSR, $t(1) = -2.59$, $p = 0.0129$. The plot of the residual with the predicted values (Appendix C, Figure C24) did not indicate prediction bias.

MRMSE for D of MID

The values of the log of MRMSE of D (Appendix C, Figure C25) did not cluster into clear show high and low MRMSE groups. Also the surfaces of the plots of MRMSE with SSR and TL for TC equal to 0.0 and 0.3 (Figure 28) presented smoother surfaces than seen for the full data set of the other parameter estimates. These plots did not suggest the need to consider the data from the discontinuous perspective. Therefore, the regression analysis was only conducted on the full data set. The details of the regression analysis are presented in Appendix E, Table E26. A summary is given in Table 15. The final R^2 was 0.9990, $F(10, 37) = 3727.07$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk W of 0.963363, $p = 0.1380$. Glejser’s test did indicate some heteroscedasticity of the residuals along SSR, $t(1) = -2.50$, $p = 0.0163$. The plot of the residuals with the predicted values (Appendix C, Figure C26) did not indicate prediction bias.

MBIAS of D of MID

For D of MID, MBIAS assumed negative values in some cases. Therefore, as with the MBIAS of d, both a hierarchical regression on the absolute values of MBIAS of D and a logistic regression on the sign of the MBIAS were conducted.

The values of the log of the absolute value of MBIAS of D (Appendix C, Figure C27) did not cluster into two distinct groups of values into high and low MBIAS. However, the plot of the log of the absolute values of MBIAS of D with SSR and TL for TC equal to 0.0 and 0.3 (Figure 29) hinted at the "mountain" and "plain" pattern. Although in this case the "plain" was somewhat erratic. Because the "mountain" and "plain" pattern was suggested, analyses were conducted from both the continuous and discontinuous model perspectives.

The detailed results of the regression for the full data set are presented in Appendix E, Table E27. A summary is provided in Table 15. The final R^2 was 0.5815, $F(9, 38) = 5.87$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk's W of 0.952849, $p = 0.0520$. Glejser's test indicated heteroscedasticity with the residuals along TL, $t(1) = -2.24$, $p = 0.0302$. The plot of the residuals with the predicted values (Figure 30) suggested a systematic prediction bias within high and low predicted values.

The mean, maximum value, minimum value, and SD for absolute value of MBIAS for D of MID and for the log transformation for the reduced data set are presented in Table 16. The detailed results of the regression for the reduced data set are presented in Appendix E, Table E28. A summary is provided in Table 15. The final R^2 was 0.5928. $F(9,20) = 3.23$, $p = .0138$. The residuals met the normality assumption with a Shapiro-Wilk W of 0.95728, $p = .2635$. Glejser's test did not indicate heteroscedasticity. The plot of the residuals with the predicted values (Appendix C, Figure C28) did not indicate a selection bias although there are two groupings of predicted values.

For the logistic regression analysis of the sign of MBIAS for D of MID, all of the independent variables and their product terms were entered simultaneously. The Hosmer and Lemeshow goodness-of-fit test, $?^2(8) = 10.3308$, $p = 0.2426$, indicated an acceptable fit. The results for the logistic regression indicated that SSR was the only significant term in predicting the probability of MBIAS being negative. SSR's coefficient estimate was 0.1776, Wald $?^2(1) = 3.9311$, $p = 0.0474$. As SSR increased the probability of MBIAS for D being negative also increased.

Correlations between estimates and parameters for D of MID

The average within condition correlation between the estimated value of D of MID and the parameter value of D of MID across the all the conditions and across the conditions in the reduced data set are given in Table 17. The average correlation was only slightly greater for the reduced data set.

Analyses for Multidimensional Item Difficulty Estimates – a_1 *(direction)*

The mean, maximum value, minimum value, and SD for MVAR, MRMSE and MBIAS for a_1 (in radians) of MID and for their log transformations are presented in Table 18. The MVAR, MRMSE and MBIAS for a_1 of MID for each condition are presented in Appendix B, Tables B16 to B18.

MVAR of a_1 *of MID*

The plots of the log of MVAR of a_1 with SSR, TL and TC (Figure 31) did not show high and low MVAR groups. Also the surfaces of the plots of MVAR with SSR and TL for TC equal to 0.0 and 0.3 (Figures 32) presented smoother surfaces similar to those for D of MID. These plots did not suggest the need to examine the data from the discontinuous perspective. Therefore, an analysis was only conducted on the full data set.

The details of the regression analysis are presented in Appendix E, Table E29. A summary is given in Table 19. The final R^2 was .9895, $F(9, 38) = 397.87$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk's W of 0.962322, $p = .1253$. Glejser's tests showed no evidence of heteroscedasticity. The plot of the residuals with the predicted values (Appendix C, FigureC29) did not indicate a prediction bias.

MRMSE for a_1 *of MID*

The values of the log of MRMSE of a_1 (Appendix C, C30) did not group into high and low MVAR conditions. Also the surfaces of the plots of MRMSE with SSR and TL for TC equal to 0.0 and 0.3 (Figures 33) presented smoother surfaces similar to those for D of MID. The details of the regression analysis are presented in Appendix E, Table E30. A summary is given in Table 19. The final R^2 was 0.9912, $F(9, 38) = 474.16$, $p < .0001$. The residuals met the normality assumption with a Shapiro-Wilk W of 0.979877, $p = 0.5736$. Glejser's test did not indicate any heteroscedasticity of the residuals. The plot of the residuals with the predicted values (Appendix C, C31) did not indicate any prediction bias.

MBIAS for a_1 *of MID*

For a_1 of MID, MBIAS assumed negative values in some cases. Therefore, both a hierarchical regression on the absolute values of MBIAS of a_1 and a logistic regression on the sign of the MBIAS were conducted.

The values of the log of absolute value of MBIAS of a_1 of MID (Appendix C, Figure C32) did not suggest two distinct groups. Nor do the plots with SSR and TL for TC = 0.0 and TC = 0.3 (Figure 34) show the clear “mountain/plain” pattern. Instead the plots are erratic. Therefore the hierarchical analysis was conducted only on the full data set. The detailed results for the regression for the full data set are in Appendix E, Table E31. A summary is provided in Table 19. The final R^2 value was 0.6099, $F(9,38) = 6.60$, $p = 0.0001$. The residuals did not meet the normality assumption with a Shapiro-Wilks W of 0.892374, $p < .0004$. Glejser's test indicated heteroscedasticity of residuals long SSR, $t(1) = -$

2.55, p = .0142. The plot of the residuals with the predicted values (Figure 35) did not indicate prediction bias although the predicted values cluster into two groups.

For the logistic regression analysis of the sign of MBIAS for a_1 of MID, all of the independent variables and their product terms were enter simultaneously. However, the solution would not converge with the product term, SSR x TL x TC in the model. Once this term was removed, the solution converged and the Hosmer and Lemeshow goodness-of-fit test, $?^2(7) = 3.1662$, p = 0.8692, indicated an acceptable fit. The results for the logistic regression did not indicate any terms being significant in predicting the probability of MBIAS being negative.

Correlations between estimates and parameters for a_1 *of MID*

The average within condition correlation between the estimated value of a_1 of MID and the parameter value of a_1 of MID across the all the conditions and across the conditions in the reduced data set are given in Table 20. The average correlation was greater for the reduced data set.

Table 1. Number of admissible replications by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	1692	1967	1997	2000
10	1933	2000	2000	2000
20	1996	2000	2000	2000
30	2000	2000	2000	2000
50	2000	2000	2000	2000
100	2000	2000	2000	2000
TC = 0.3				
5	1368	1840	1962	1992
10	1792	1993	2000	2000
20	1958	2000	2000	2000
30	1994	2000	2000	2000
50	2000	2000	2000	2000
100	2000	2000	2000	2000

Table 2. Descriptive statistics for MVAR, MRMSE and MBIAS of a1 and a2

	N	Mean	Max	Min	SD
MVAR a1	48	7953.70	356108.71	0.0012740	51386.88
Log MVAR a1	48	-2.0003242	12.7829913	-6.6655882	1.5781183
MVAR a2	48	3711.70	159043.03	0.0012374	22975.63
Log MVAR a2	48	-2.3734792	11.9769301	-6.6947819	1.4878419
MRMSE a1	48	1.9956309	48.6500638	0.0452405	7.6162561
Log MRMSE a1	48	-1.4933858	3.8846531	-3.0957631	1.5781183
MRMSE a2	48	1.7858783	44.7472320	0.0442344	7.0354046
Log MRMSE a2	48	-1.6030143	3.8010296	-3.1182514	1.4878419
BIAS a1	48	0.5356659	14.9638633	0.0168888	2.2552221
Log MBIAS a1	48	-2.7213772	2.7056382	-4.0811062	1.4479838
BIAS a2	48	0.4636720	13.1287639	0.0159252	1.9817513
Log MBIAS a2	48	-2.8429809	2.5748055	-4.1398536	1.3889524

Table 3. Descriptive statistics for MVAR, MRMSE and MBIAS of a1 and a2 for reduced data set

	N	Mean	Max	Min	SD
MVAR a1	30	0.011654	0.054013	0.001274	0.011713
Log MVAR a1	30	-4.87166	-2.91853	-6.66559	0.945634
MVAR a2	30	0.011866	0.057771	0.001237	0.012257
Log MVAR a2	30	-4.86608	-2.85127	-6.69478	0.9574
MRMSE a1	30	0.107003	0.234502	0.045241	0.0445165
Log MRMSE a1	30	-2.31401	-1.45029	-3.09576	0.4037908
MRMSE a2	30	0.108262	0.242203	0.044234	0.0467814
Log MRMSE a2	30	-2.3079	-1.41798	-3.11825	0.4169859
BIAS a1	30	0.035227	0.061564	0.016889	0.011438
Log MBIAS a1	30	-3.40077	-2.78767	-4.08111	0.345096
BIAS a2	30	0.03575	0.070707	0.015925	0.015219
Log MBIAS a2	30	-3.42236	-2.64921	-4.13985	0.441912

Table 4. Summary of hierarchical regression results for a1 and a2

	N	Step 1: SSR	Step 1: TL	Step 1: TC	Step 2: SSR x TL	Step 2: SSR x TC	Step 2: TL x TC	Step 3: SSR x TL x TC	Step 4: 1/SSR	Step 4: 1/TL	Step 5: SSR^2	Normality of residuals	Homoscedasticity of residuals	Apparent prediction bias	Final R^2
Log MVAR															
a1	48	22.269	37.008	-	6.245	-	-	-	12.173	8.988	N/A	No	Yes	Yes	.8772
a2	48	26.581	31.111	-	8.440	-	-	-	14.488	6.724	N/A	No	Yes	Yes	.8797
a1	30	72.048	13.020	5.052	-	-	-	-	9.455	0.142	0.237	Yes	Yes	No	.9997
a2	30	71.149	13.109	6.063	-	-	-	-	9.298	0.0638	0.244	Yes	Yes	No	.9996
Log MRMSE															
a1	48	26.148	31.846	-	7.411	-	-	-	13.829	8.336	N/A	Yes	No	Yes	.8962
a2	48	27.610	24.847	-	7.061	-	-	-	18.294	5.245	N/A	No	No	Yes	.8558
a1	30	64.103	10.641	12.753	-	-	-	-	11.131	0.404	0.249	Yes	Yes	No	.9991
a2	30	61.371	10.998	15.143	-	-	-	-	11.627	-	0.286	Yes	Yes	No	.9993
Log MBIAS															
a1	48	20.151	26.863	6.755	10.629	-	-	-	12.573	9.367	N/A	Yes	Yes	Yes	.8519
a2	48	23.527	18.341	8.712	10.067	-	-	-	18.245	4.821	N/A	No	No	Yes	.8256
a1	30	29.075	2.128	59.517	-	1.743	-	-	4.635	0.785	-	Yes	Yes	No	.9844
a2	30	25.366	3.053	59.829	-	-	-	-	8.688	-	-	Yes	No	No	.9835

Note. A – indicates that the term was not statistically significant when entered into the regression equation. N/A indicates that the term was not used in the regression equation.

Table 5. Average correlations between estimates and parameters for a1 and a2 across conditions for full and reduced data sets

	N	Mean	Max	Min	SD
Full Set					
a1	48	0.88836	0.9918982	0.0166158	0.3924676
a2	48	0.89677	0.9919856	0.0030454	0.3754829
Reduced Set					
a1	30	0.95989	0.9918982	0.8111556	0.0434169
a2	30	0.95984	0.9919856	0.8079091	0.0431432

Note. Mean based on r to z transformation (Corey, Dunlap, & Burke, 1998) and weighted by number of admissible replications.

Table 6. Descriptive statistics for MVAR, MRMSE and MBIAS of d

	N	Mean	Max	Min	SD
MVAR d	48	870.62717	37286.768	0.00060	5390.1329
Log MVAR d	48	-3.72932	10.52639	-7.42647	4.11539
MRMSE d	48	0.71931	19.43081	0.02569	2.96136
Log MRMSE d	48	-2.24705	2.96686	-3.66161	1.33879
BIAS d	48	-0.03223	0.21892	-1.50590	0.22067
Log absolute value of MBIAS d	48	-5.35484	0.40939	-9.05893	1.6266

Table 7. Summary of hierarchical regression results for d

			Percentage variance in dependent variable accounted by term in the regression step in which it was entered.													
			Step 1			Step 2			Step 3	Step 4		Step 5				
		N	SSR	TL	TC	SSR x TL	SSR x TC	TL x TC	SSR x TL x TC	1/SSR	1/TL	SSR^2	Normality of residuals	Homoscedasticity of residuals	Apparent prediction bias	Final R^2
Log MVAR																
	d	48	27.966	22.906	-	6.919	-	-	-	21.325	4.008	N/A	Yes	No	Yes	0.8550
	d	30	76.636	10.976	-	-	-	-	-	10.375	0.135	0.232	Yes	Yes	No	0.9995
Log MRMSE																
	d	48	33.240	20.120	-	5.349	-	-	-	21.783	3.367	N/A	No	No	Yes	.8510
	d	30	74.598	10.244	3.128	-	-	-	-	11.450	0.144	0.246	Yes	Yes	No	.9995
Log absolute value of MBIAS																
	d	48	39.565	9.435	-	-	-	-	-	18.623	-	N/A	No	Yes	Yes	0.6836
	d	30	47.375	7.378	-	11.563	-	-	-	-	-	N/A	Yes	Yes	No	.7341

Note. A – indicates that the term was not statistically significant when entered into the regression equation. N/A indicates that the term was not used in the regression equation.

Table 8. Descriptive statistics for MVAR, MRMSE and MBIAS of d for reduced data set

	N	Mean	Max	Min	SD
MVAR d	30	0.0046070	0.0192623	0.000595285	0.0043565
Log MVAR d	30	-5.7661852	-3.9496046	-7.4264696	0.9071450
MRMSE d	30	0.0627753	0.1372314	0.0256911	0.0276430
Log MRMSE d	30	-2.8572681	-1.9860866	-3.6616089	0.4283839
MBIAS d	30	-0.00024461	0.0042321	-0.0147611	0.0048553
Log of absolute value of MBIAS d	30	-5.95977	-4.21576	-8.42368	0.95895

Table 9. Average correlations between estimates and parameters for d across conditions for full and reduced data sets

	N	Mean	Max	Min	SD
Full Set					
d	48	0.98731	0.9992618	0.0296081	0.3027852
Reduced Set					
d	30	0.99629	0.9992618	0.9791318	0.0047344

Note. Mean based on r to z transformation (Corey, Dunlap, & Burke, 1998) and weighted by number of admissible replications.

Table 10. Descriptive statistics for MVAR, MRMSE and MBIAS of MDISC

	N	Mean	Max	Min	SD
MVAR MDISC	48	11577.794	522445.79	0.00157	75383.655
Log MVAR MDISC	48	-2.80209	13.16628	-6.45545	4.38662
MRMSE MDISC	48	1.98270	61.30859	0.05579	9.09218
Log MRMSE of MDISC	48	-1.56685	4.11592	-2.88610	1.37500
BIAS MDISC	48	0.58287	19.42632	0.02519	2.82973
Log MBIAS of MDISC	48	-2.51932	2.96663	-3.68111	1.23862

Table 11. Summary of hierarchical regression results for MDISC

	N	Percentage variance in dependent variable accounted by term in the regression step in which it was entered. Step 1: SSR	Step 1: TL	Step 1: TC	Step 2: SSR x TL	Step 2: SSR x TC	Step 2: TL x TC	Step 3: SSR x TL x TC	Step 4: 1/SSR	Step 4: 1/TL	Step 5: SSR^2	Normality of residuals	Homoscedasticity of residuals	Apparent prediction bias	Final R^2
Log MVAR															
MDISC	48	25.728	13.967	-	-	-	-	-	31.098	-	N/A	Yes	No	Yes	.7877
MDISC	30	75.433	12.238	2.035	-	-	-	-	9.845	0.131	N/A	Yes	Yes	No	.9973
Log MRMSE															
MDISC	48	27.098	14.977	-	5.336	-	-	-	27.266	-	N/A	No	No	Yes	.7975
MDISC	30	61.324	9.051	15.234	-	-	-	-	12.703	0.162	0.257	Yes	No	No	.9986
Log BIAS															
MDISC	48	22.659	13.282	10.389	6.613	-	-	-	24.026	2.287	N/A	No	No	Yes	.7885
MDISC	30	28.850	2.991	57.617	-	-	-	-	8.086	-	-	Yes	Yes	No	.9993

Note. A – indicates that the term was not statistically significant when entered into the regression equation. N/A indicates that the term was not used in the regression equation.

Table 12. Descriptive statistics for MVAR, MRMSE and MBIAS of MDISC for reduced data set

	N	Mean	Max	Min	SD
MVAR MDISC	30	0.0139503	0.0658335	0.0015719	0.0142965
Log MVAR MDISC	30	-4.7040085	-2.7206268	-6.4554505	0.9582814
MRMSE MDISC	30	0.1264392	0.2676545	0.0557932	0.0500774
Log MRMSE MDISC	30	-2.1399231	0.3849483	-1.3180582	-2.8861037
BIAS MDISC	30	0.0539990	0.1082727	0.0251949	0.0212940
Log MBIAS MDISC	30	-2.9955798	-2.2231024	-3.6811130	0.4041458

Table 13. Average correlations between estimates and parameters for MDISC across conditions for full and reduced data sets

	N	Mean	Max	Min	SD
Full Set					
MDISC	48	0.91668	0.9904059	0.0207168	0.3018099
Reduced Set					
MDISC	30	0.95732	0.9904059	0.8097515	0.0446260

Note. Mean based on r to z transformation (Corey, Dunlap, & Burke, 1998) and weighted by number of admissible replications.

Table 14. Descriptive statistics for MVAR, MRMSE and MBIAS of D of MID

	N	Mean	Max	Min	SD
MVAR D of MID	48	0.00457	0.02614	0.00028	0.00572
Log MVAR D of MID	48	-6.02036	-3.64429	-8.18204	1.15657
MRMSE D of MID	48	0.05630	0.15375	0.01660	0.03339
Log MRMSE D of MID	48	-3.03692	-1.87244	-4.09850	0.57106
MBIAS D of MID	48	0.00373	0.01441	-0.00267	0.00520
Log absolute value of D of MID	48	-6.18578	-4.23997	-9.32653	1.46365

Table 15. Summary of hierarchical regression result for D of MID

		Percentage variance in dependent variable accounted by term in the regression step in which it was entered.													
		Step 1			Step 2			Step 3	Step 4		Step 5				
	N	SSR	TL	TC	SSR x TL	SSR x TC	TL x TC	SSR x TL x TC	1/SSR	1/TL	SSR^2	Normality of residuals	Homoscedasticity of residuals	Apparent prediction bias	Final R^2
Log MVAR															
D of MID	48	67.439	18.863	-	-	-	-	-	13.303	0.689	0.733	Yes	No	No	.9991
Log MRMSE															
D of MID	48	68.354	17.684	-	-	-	-	-	13.578	0.603	0.770	Yes	No	No	.9990
Log abs MBIAS															
D of MID	48	33.472	-	-	-	-	-	-	18.851	-	N/A	Yes	No	Yes	0.5815
D of MID	30	15.306	-	-	-	-	-	-	37.265	-	N/A	Yes	Yes	No	0.5928

Note. A – indicates that the term was not statistically significant when entered into the regression equation. N/A indicates that the term was not used in the regression equation.

Table 16. Descriptive statistics for MBIAS for D of MID for reduced data set

	N	Mean	Max	Min	SD
MBIAS D of MID	30	0.0012174	0.0116826	-0.0026691	0.0036041
Log absolute value of MBIAS D of MID	30	-6.7161810	-4.4496589	-8.9189843	1.2224443

Table 17. Average correlations between estimates and parameters for D of MID across conditions for full and reduced data sets

	N	Mean	Max	Min	SD
Full Set					
D of MID	48	0.99504	0.9994049	0.9463469	0.0116531
Reduced Set					
D of MID	30	0.99710	.9994049	0.9854412	0.0035159

Note. Mean based on r to z transformation (Corey, Dunlap, & Burke, 1998) and weighted by number of admissible replications.

Table 18. Descriptives for MVAR, MRMSE and MBIAS of a_1 of MID

	N	Mean	Max	Min	SD
MVAR a_1 of MID	48	0.01322	0.09133	0.00044	0.01877
Log MVAR a_1 of MID	48	-5.13318	-2.39331	-7.72583	1.32199
MRMSE a_1 of MID	48	0.09052	0.30777	0.02043	0.06322
Log MRMSE a_1 of MID	48	-2.61423	-1.17839	-3.89096	0.65595
MBIAS a_1 of MID	48	-0.00700	0.00486	-0.11404	0.02089
Log absolute value of MBIAS a_1 of MID	48	-6.18308	-2.17121	-10.18053	1.56227

Table 19. Summary of hierarchical regression results for a_1 of MID

		Percent variance in dependent variable accounted by term in the regression step in which it was entered.													
		Step 1			Step 2			Step 3	Step 4		Step 5				
	N	SSR	TL	TC	SSR x TL	SSR x TC	TL x TC	SSR x TL x TC	1/SSR	1/TL	SSR^2	Normality of residuals	Homoscedasticity of residuals	Apparent prediction bias	Final R^2
Log MVAR															
a_1 of MID	48	47.273	34.766	6.930	-	-	-	-	8.152	3.657	N/A	Yes	Yes	No	.9895
Log MRMSE															
a_1 of MID	48	48.444	33.312	6.932	-	-	-	-	8.848	3.456	N/A	Yes	Yes	No	.9912
Log absolute value of MBIAS															
a_1 of MID	48	-	33.342	-	-	-	-	-	-	21.478	N/A	No	No	No	0.6099

Note. A – indicates that the term was not statistically significant when entered into the regression equation. N/A indicates that the term was not used in the regression equation.

Table 20. Average correlations between estimates and parameters for a_1 of MID across conditions for full and reduced data sets

	N	Mean	Max	Min	SD
Full Set					
a_1 of MID	48	0.93522	0.9954196	0.2969974	0.1762735
Reduced Set					
a_1 of MID	30	0.97137	0.9954196	0.8373739	0.0366022

Note. Mean based on r to z transformation (Corey, Dunlap, & Burke, 1998) and weighted by number of admissible replications.

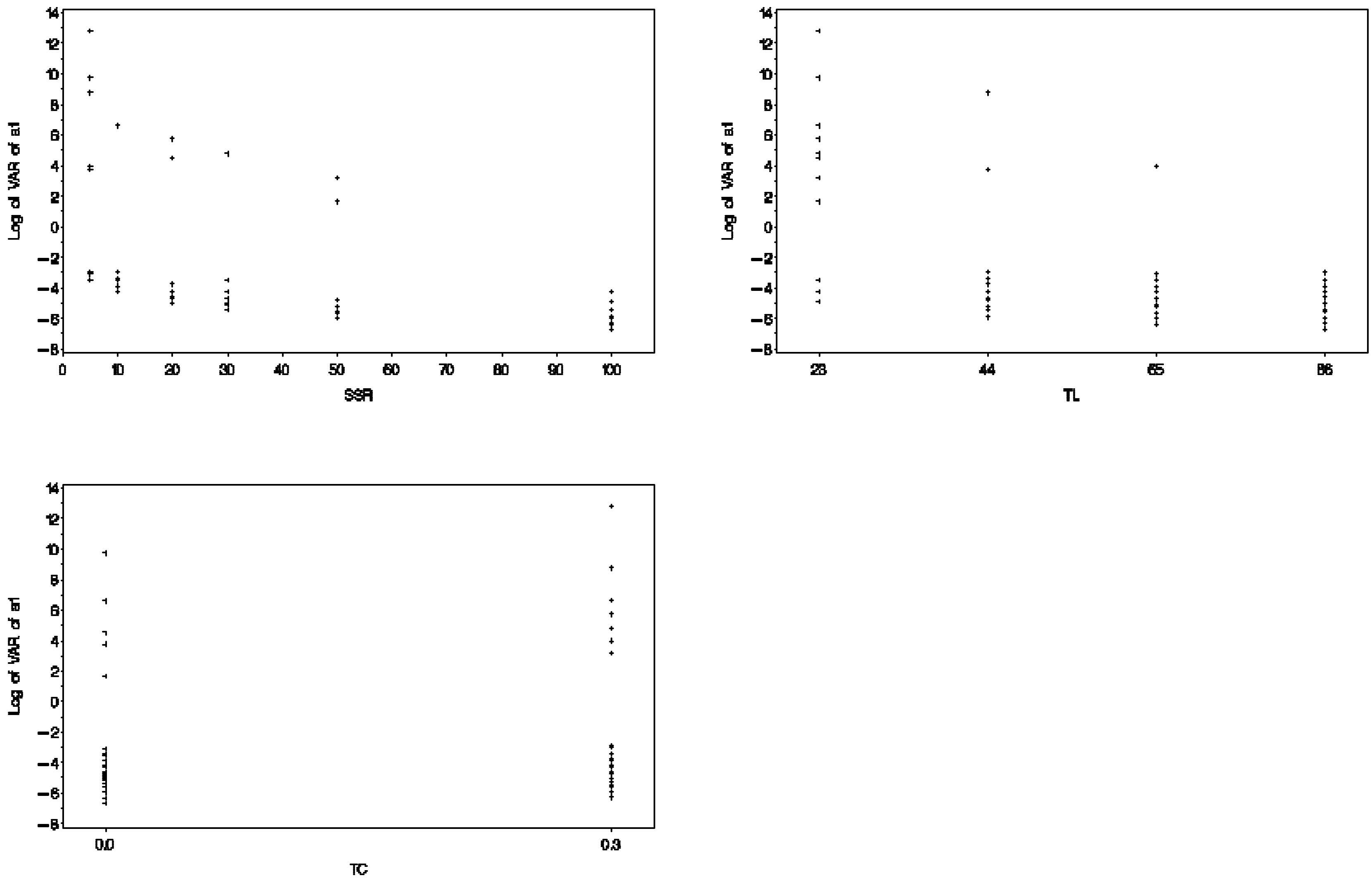
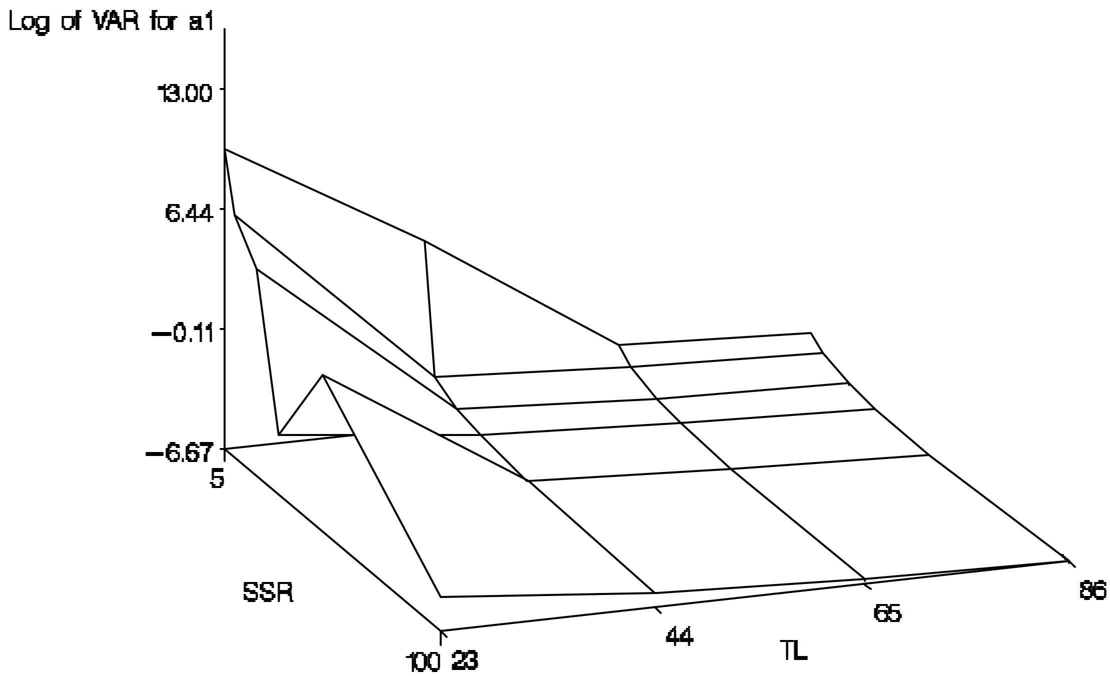


Figure 4. Log of MVAR of a1 plotted with SSR, TL and TC.

TC = 0.0



TC = 0.3

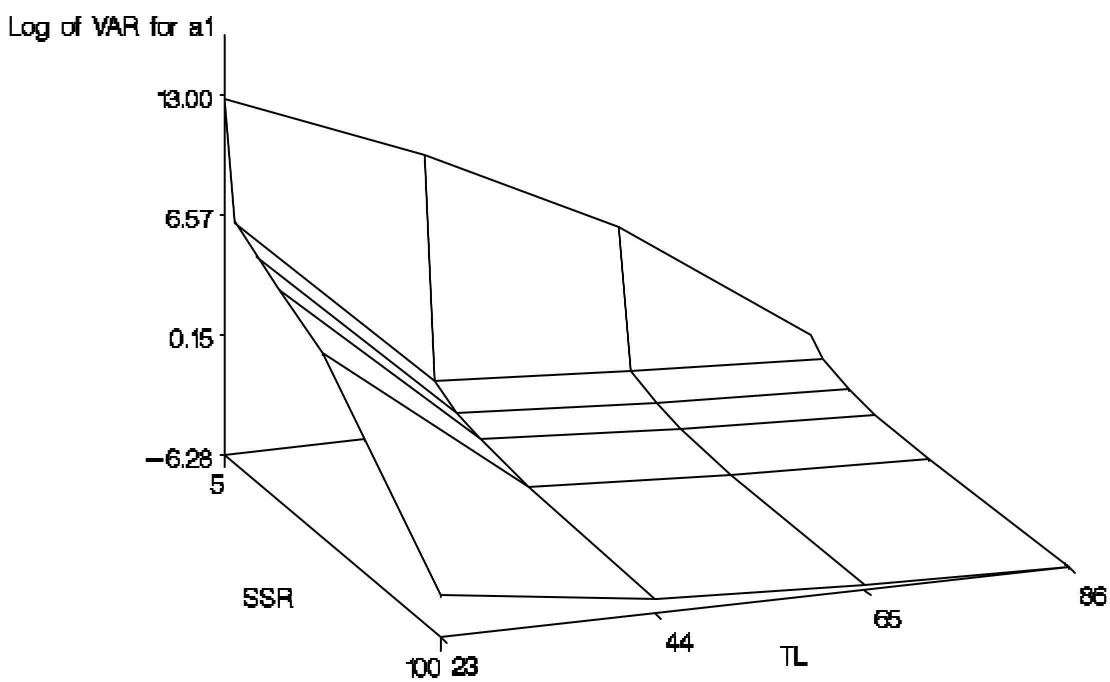
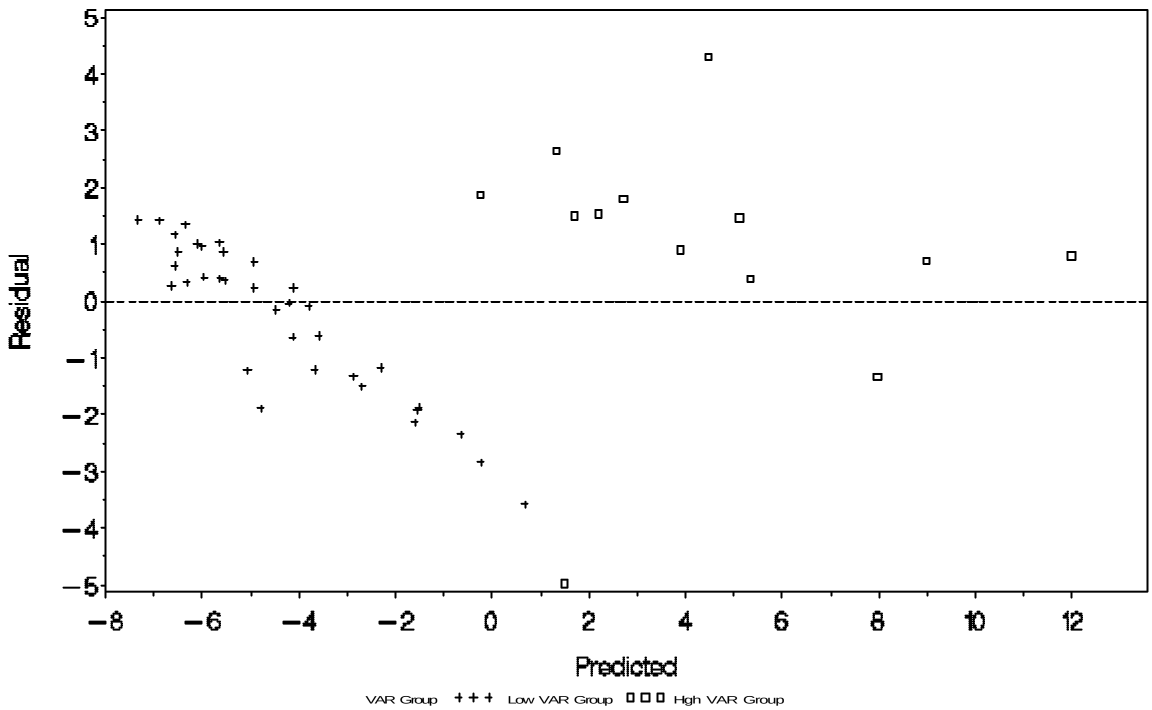


Figure 5. Log of MVAR of a1 plotted with SSR and TL.

a1



a2

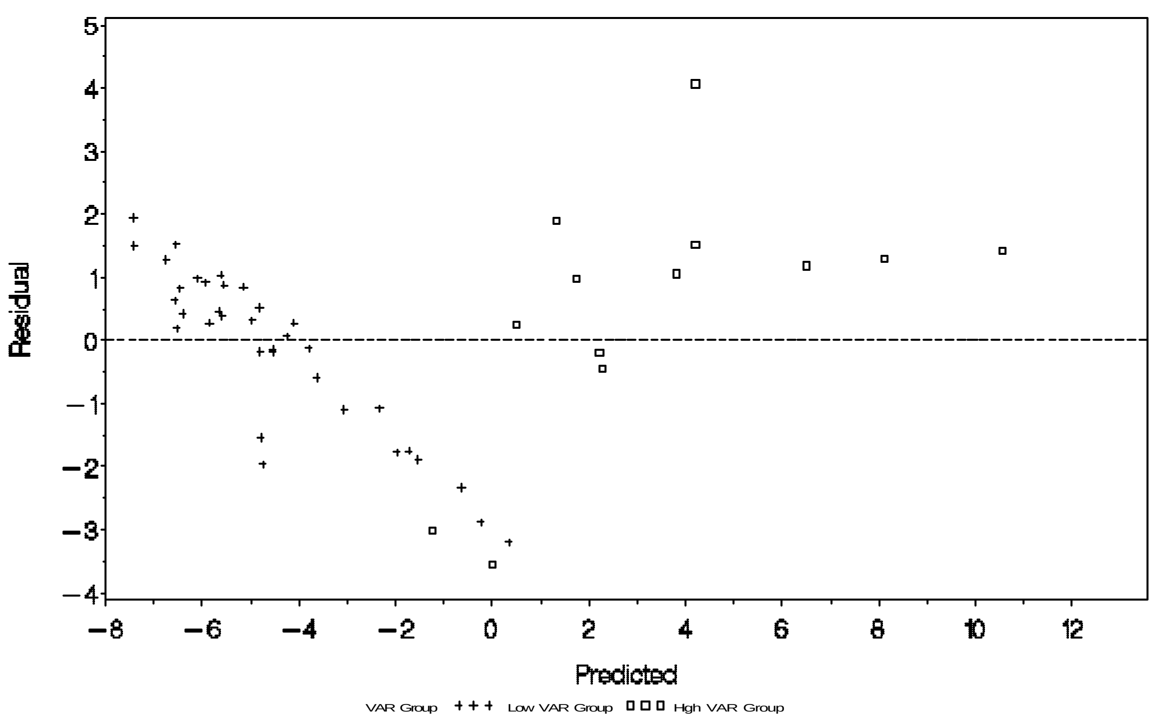


Figure 6. Residual plots of log of MVAR of a1 and a2 with predicted values.

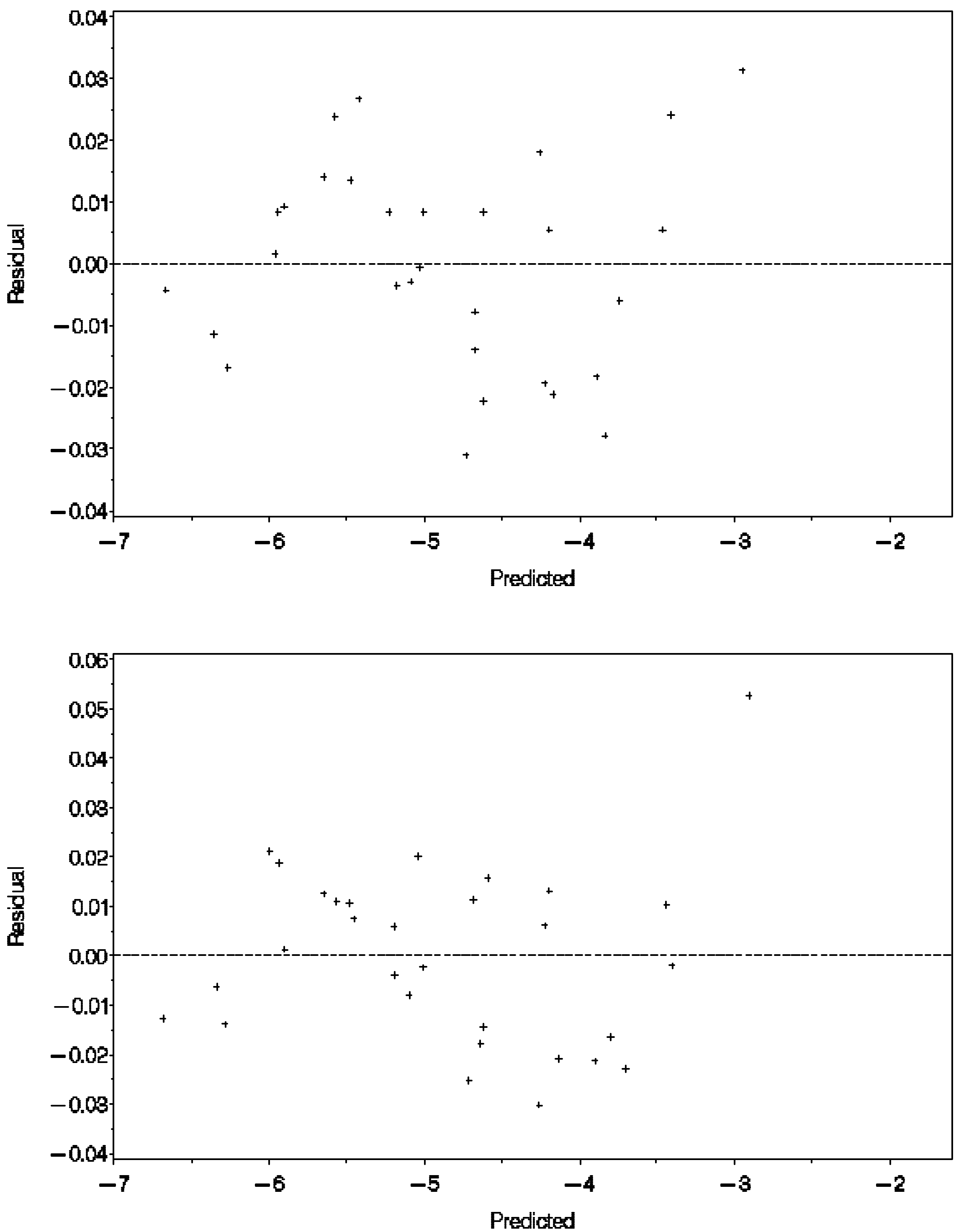
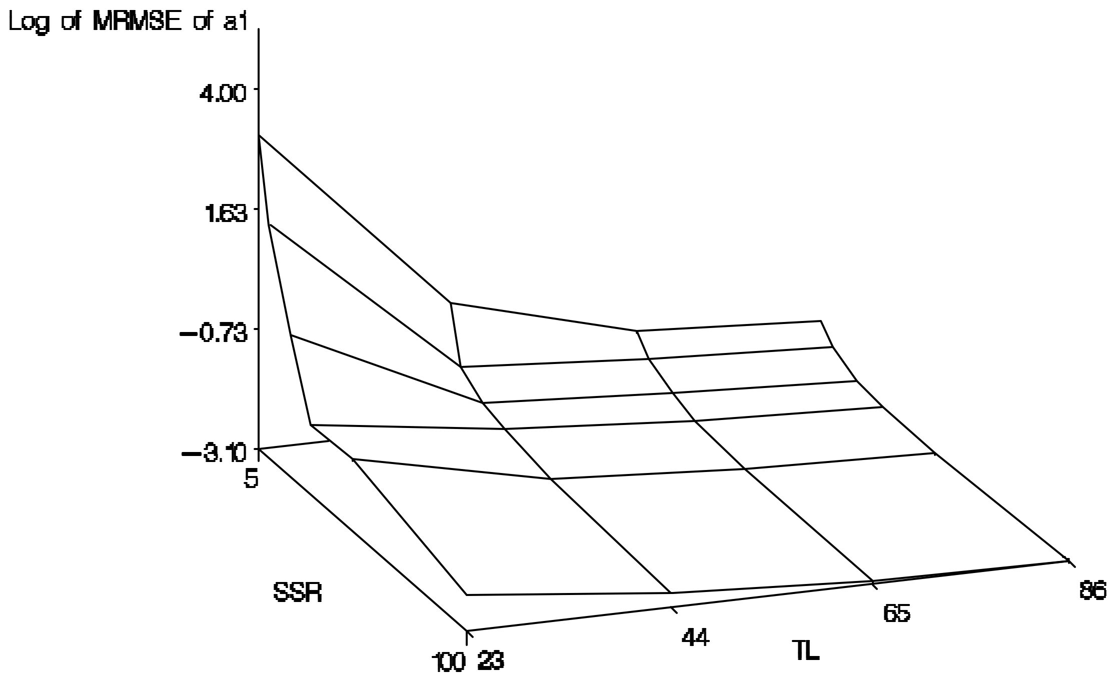


Figure 7. Residual plots of log of MVAR with predicted values for a1 and a2 for reduced data set after the addition of the SSR^2 term.

TC = 0.0



TC = 0.3

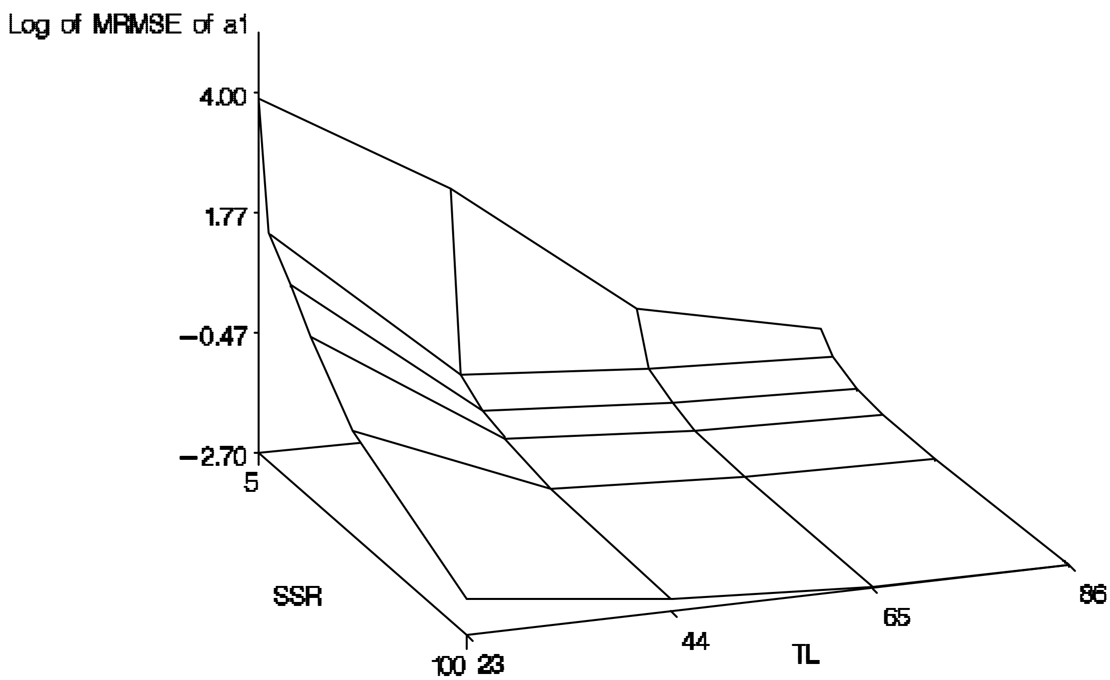
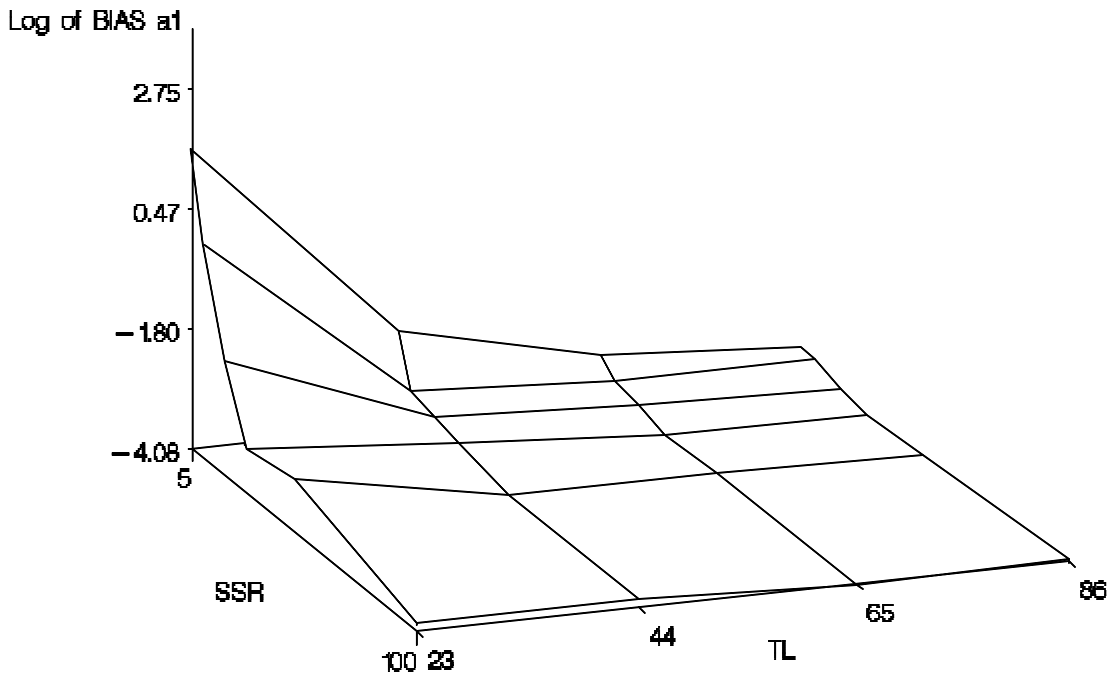


Figure 8. Log of MRMSE of a1 plotted with SSR and TL.

TC = 0.0



TC = 0.3

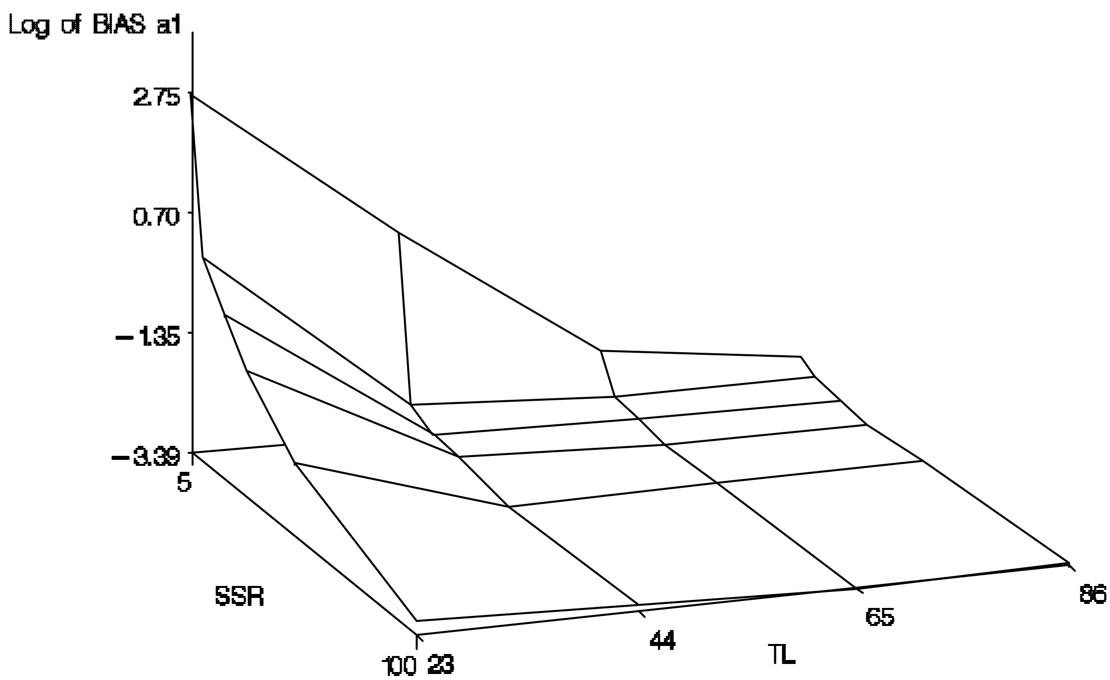
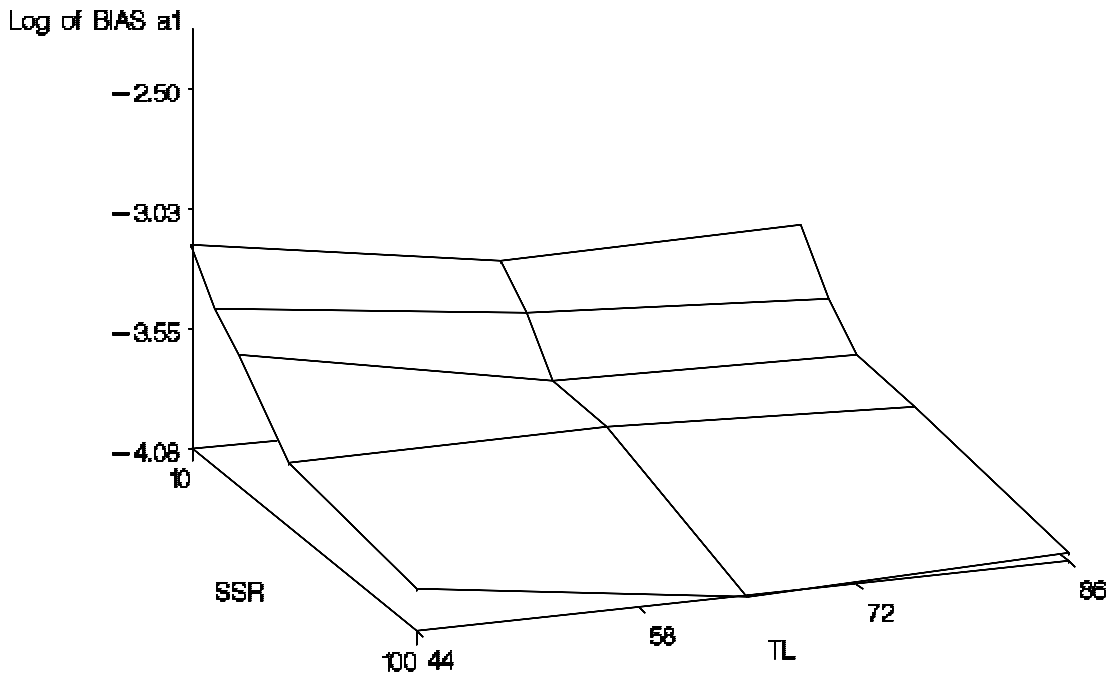


Figure 9. Log of MBIAS of a1 plotted with SSR and TL.

TC = 0.0



TC = 0.3

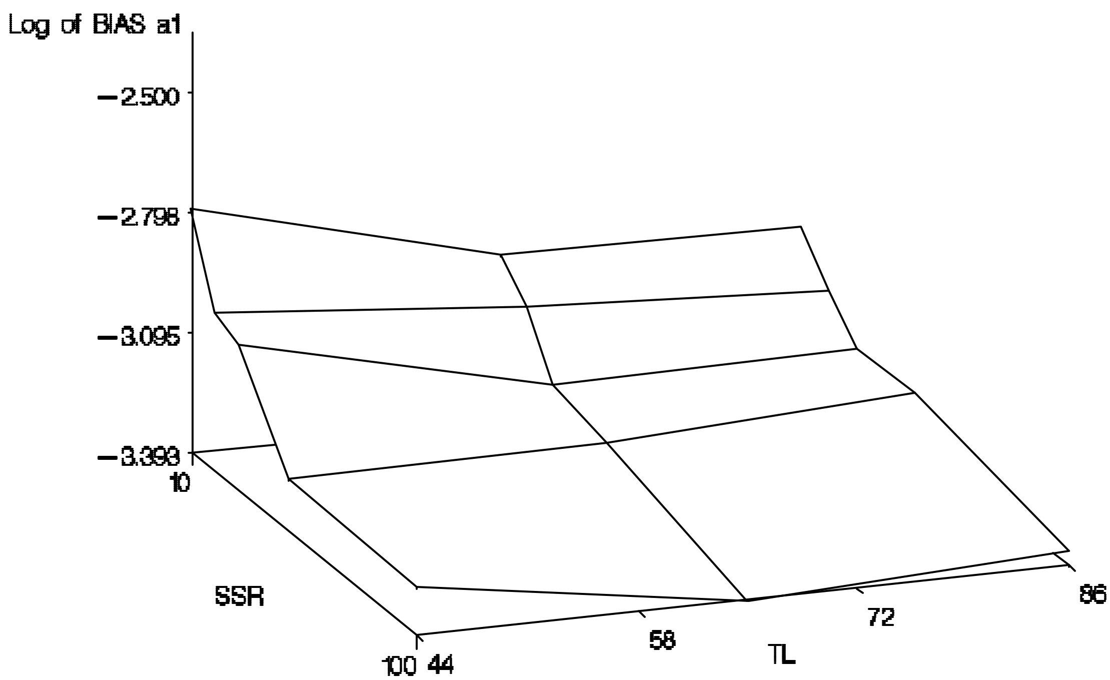


Figure 10. Log of MBIAS of a1 plotted with SSR and TL for reduced data set.

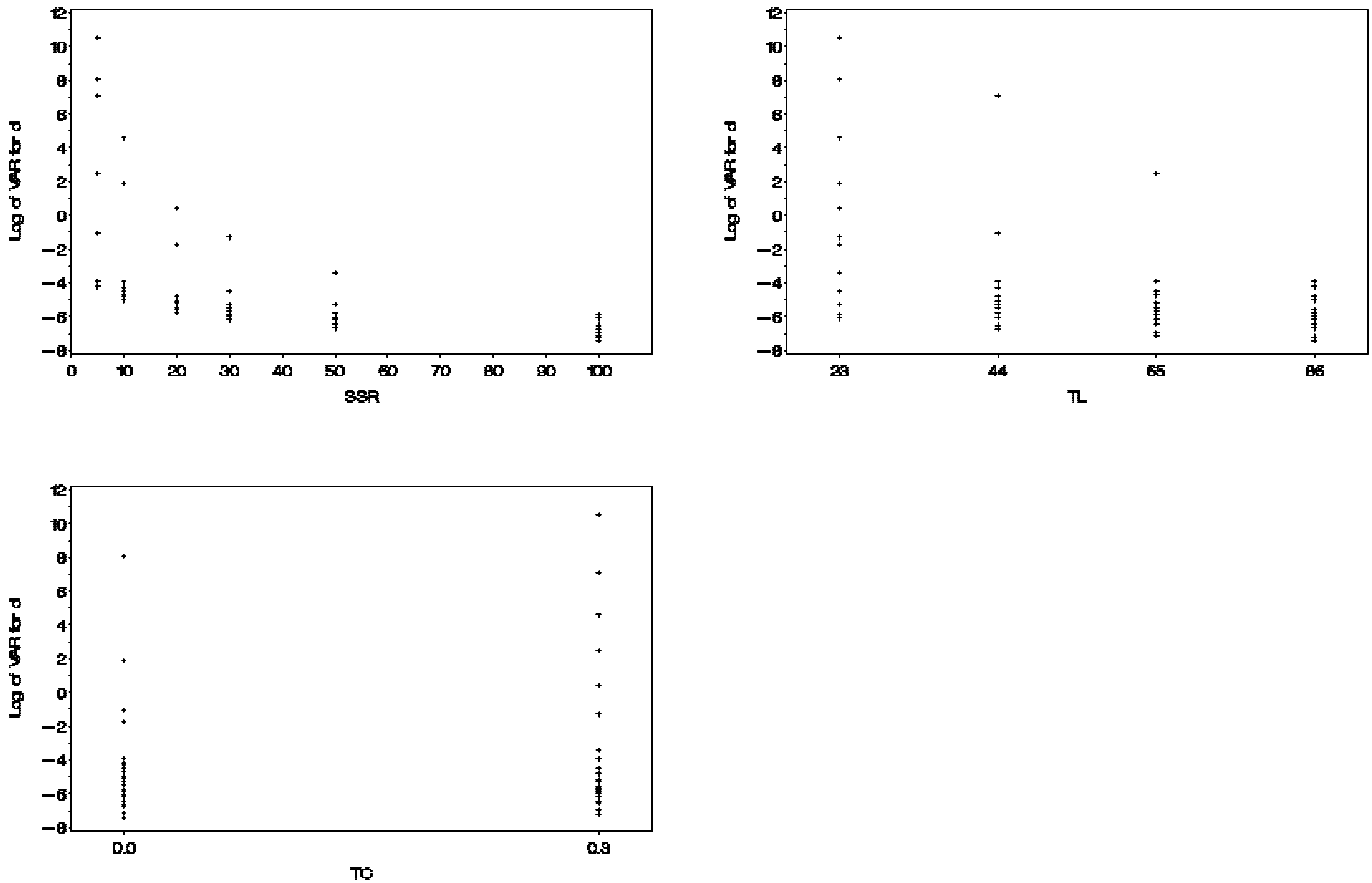
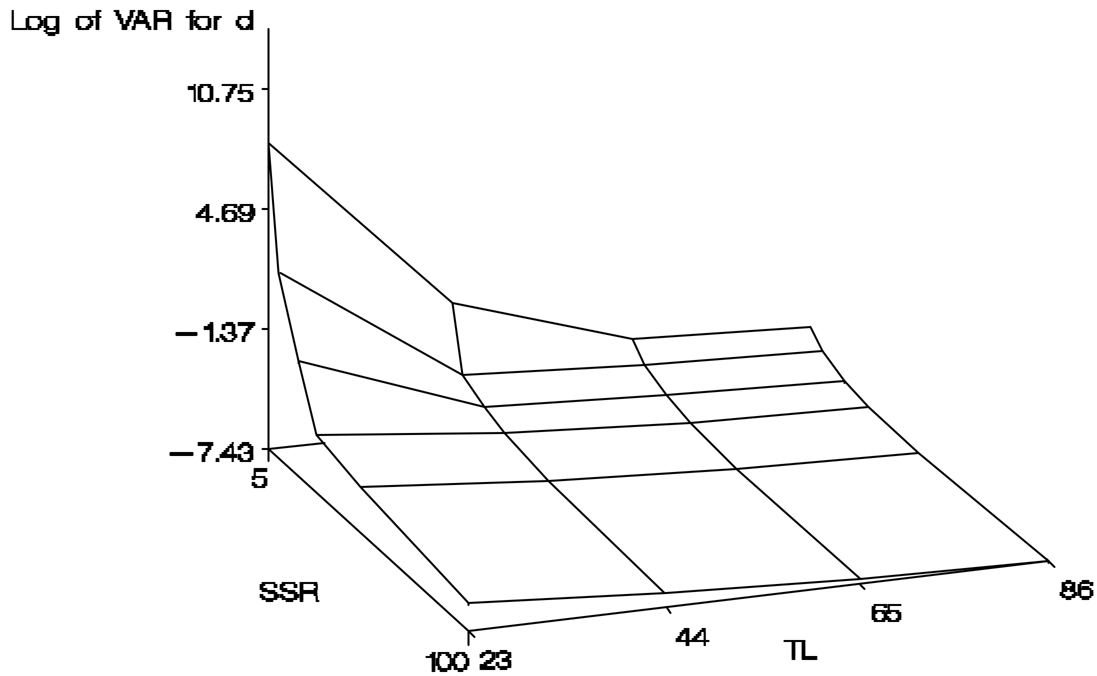


Figure 11. Log of MVAR of d plotted with SSR, TL and TC.

TC = 0.0



TC = 0.3

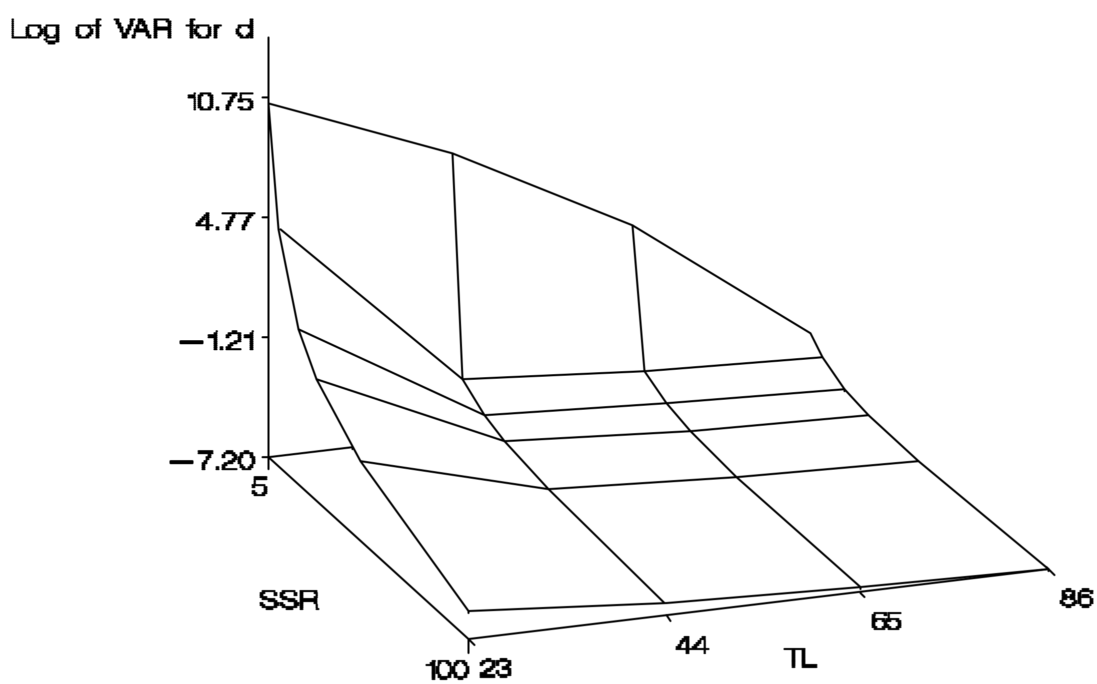


Figure 12. Log of MVAR of d plotted with SSR and TL.

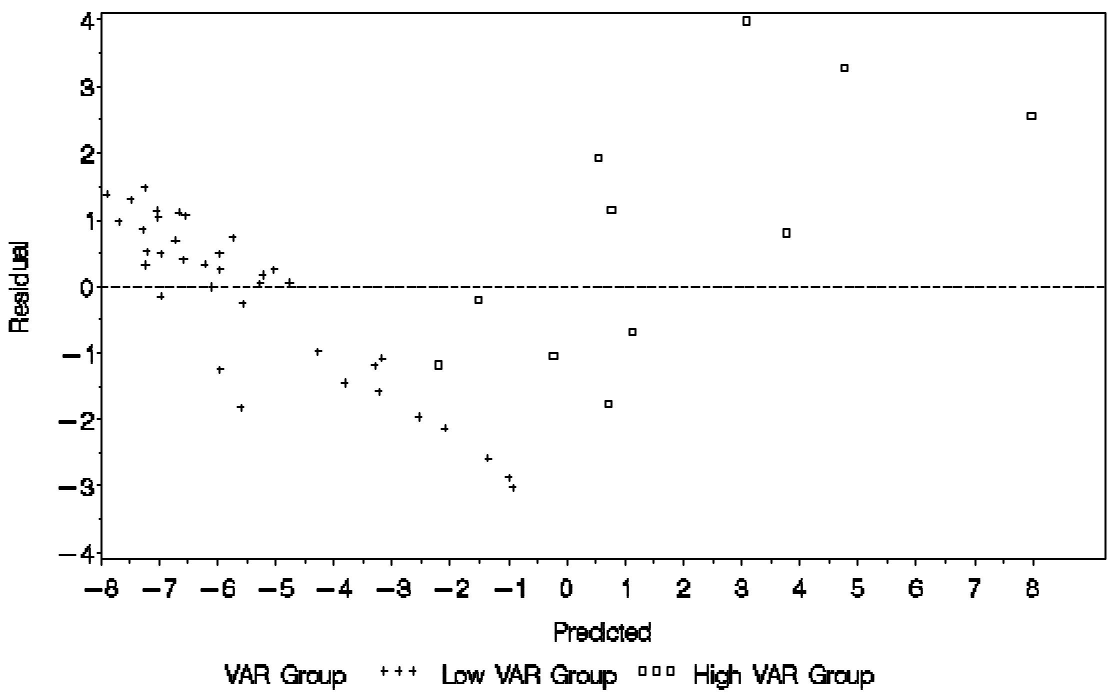


Figure 13. Residual plots of log of MVAR of d with predicted values.

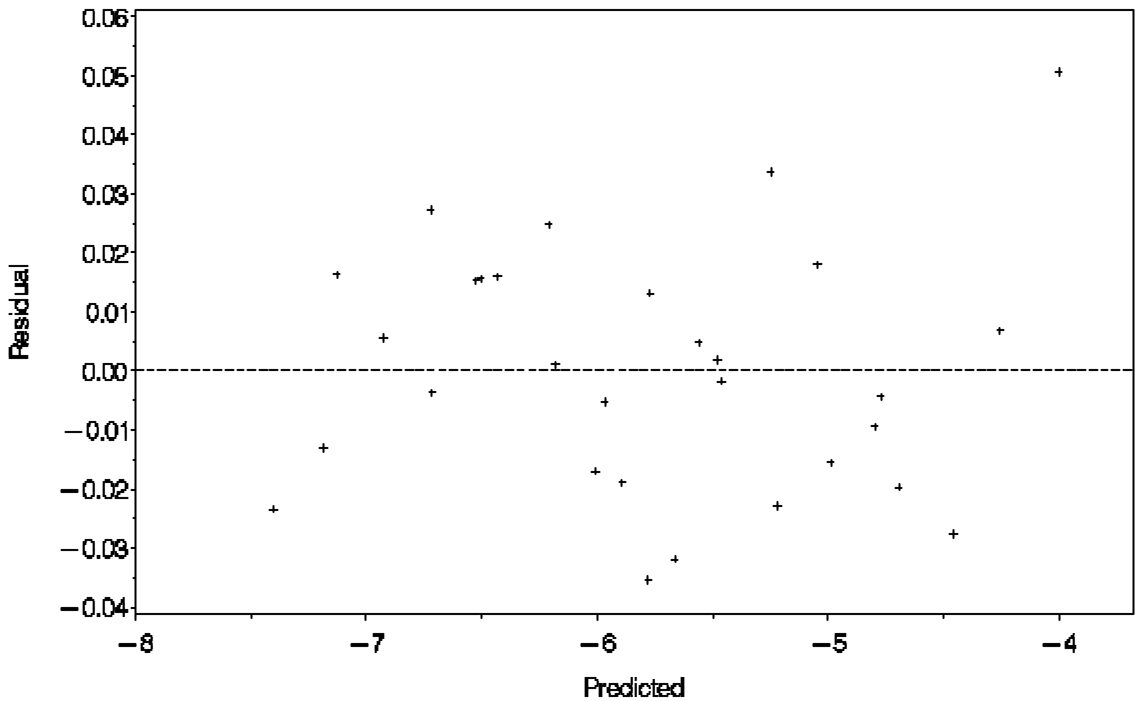
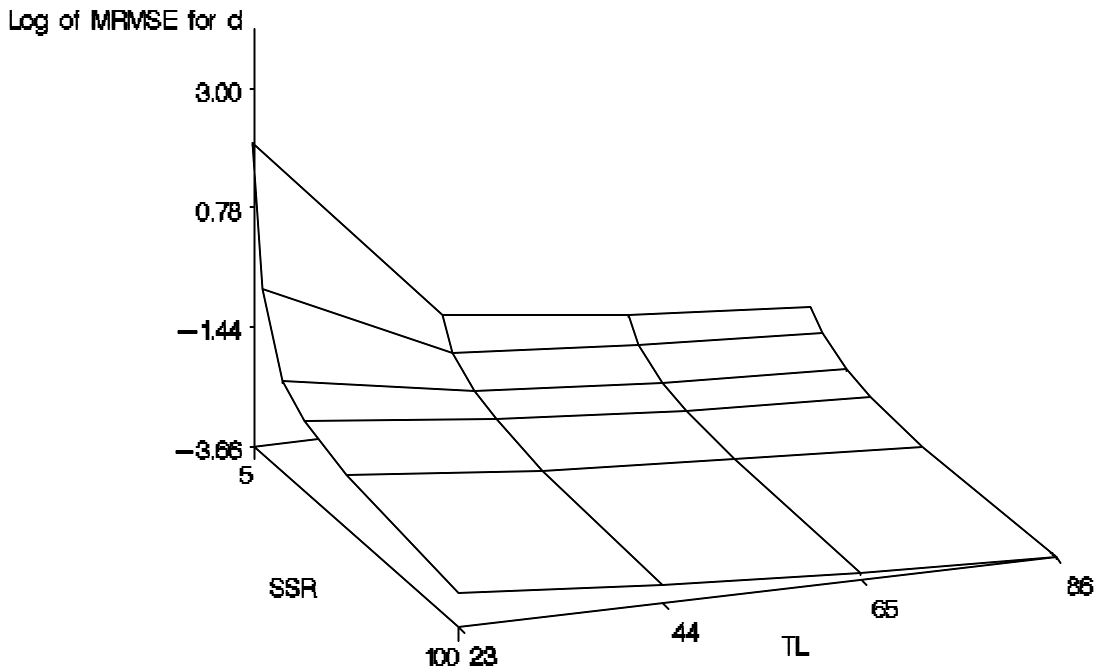


Figure 14. Residual plots of log of MVAR of d with predicted values for reduced data set.

TC = 0.0



TC = 0.3

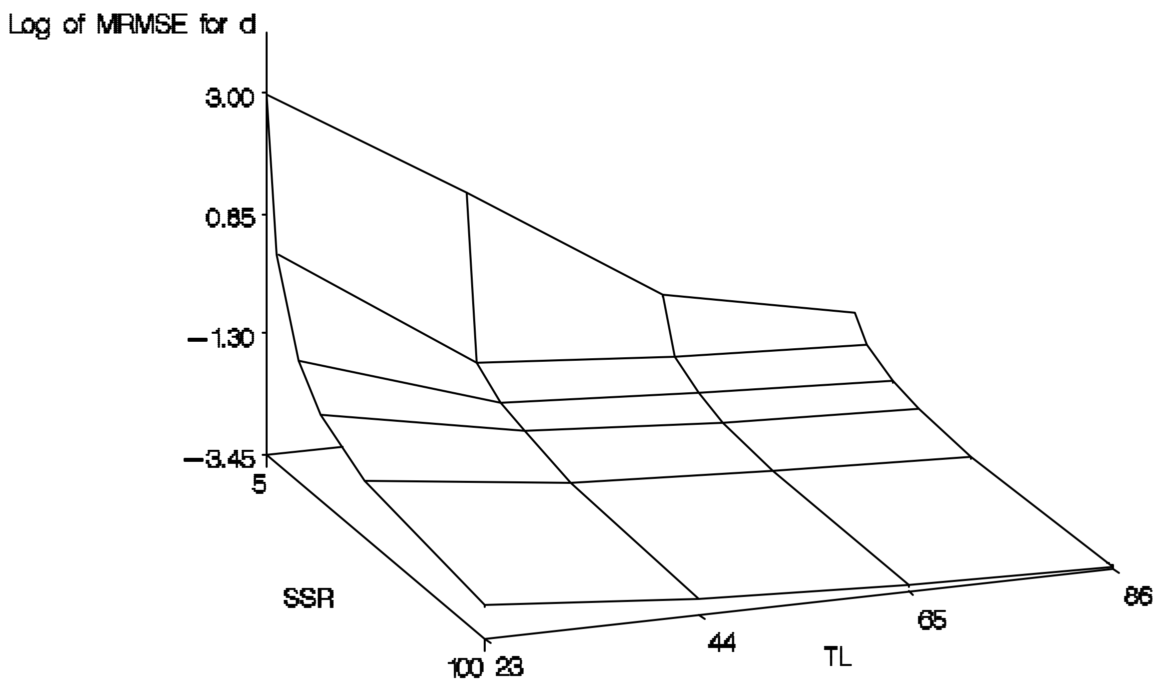


Figure 15. Log of MRMSE of d plotted with SSR and TL.

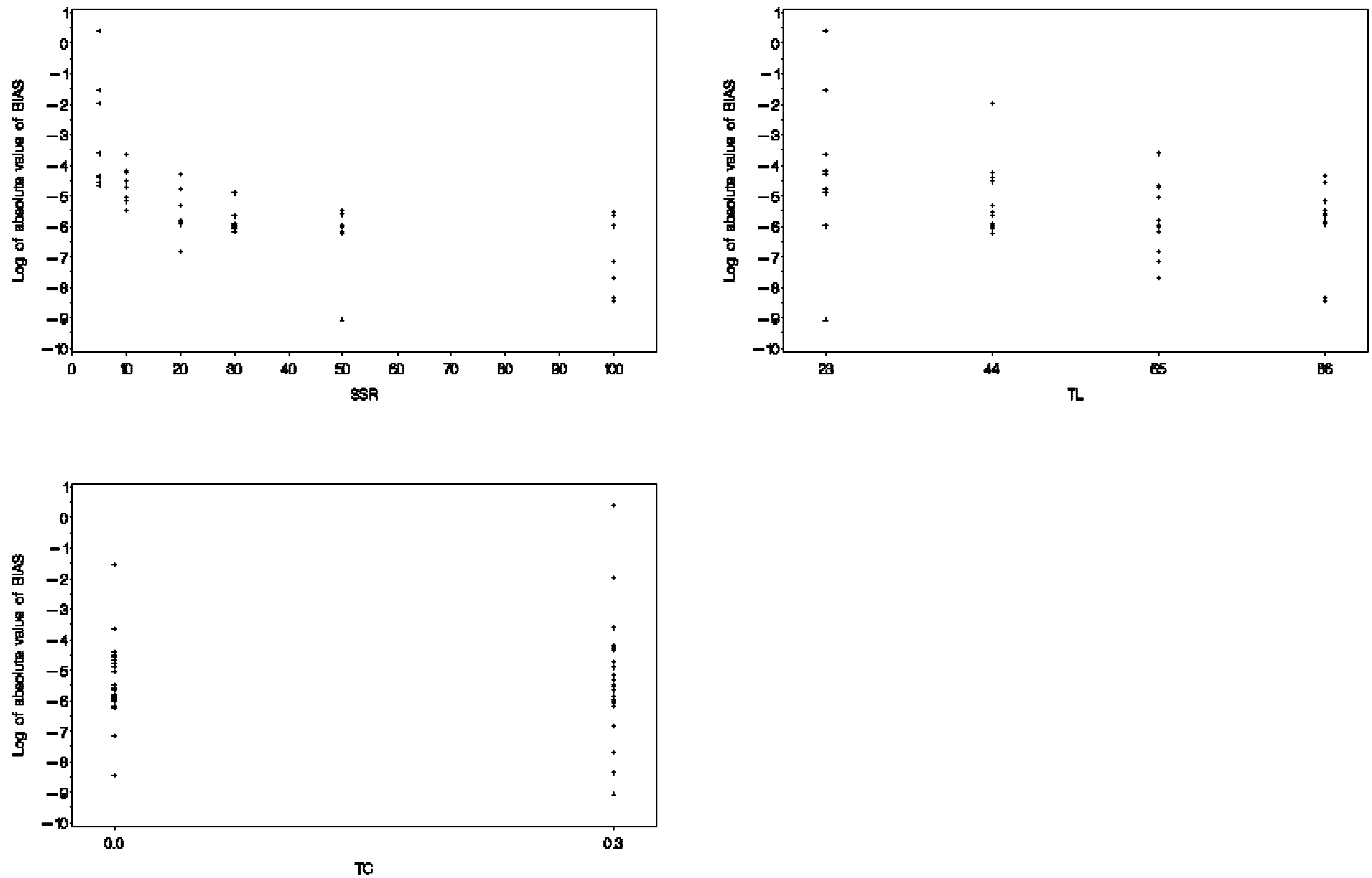
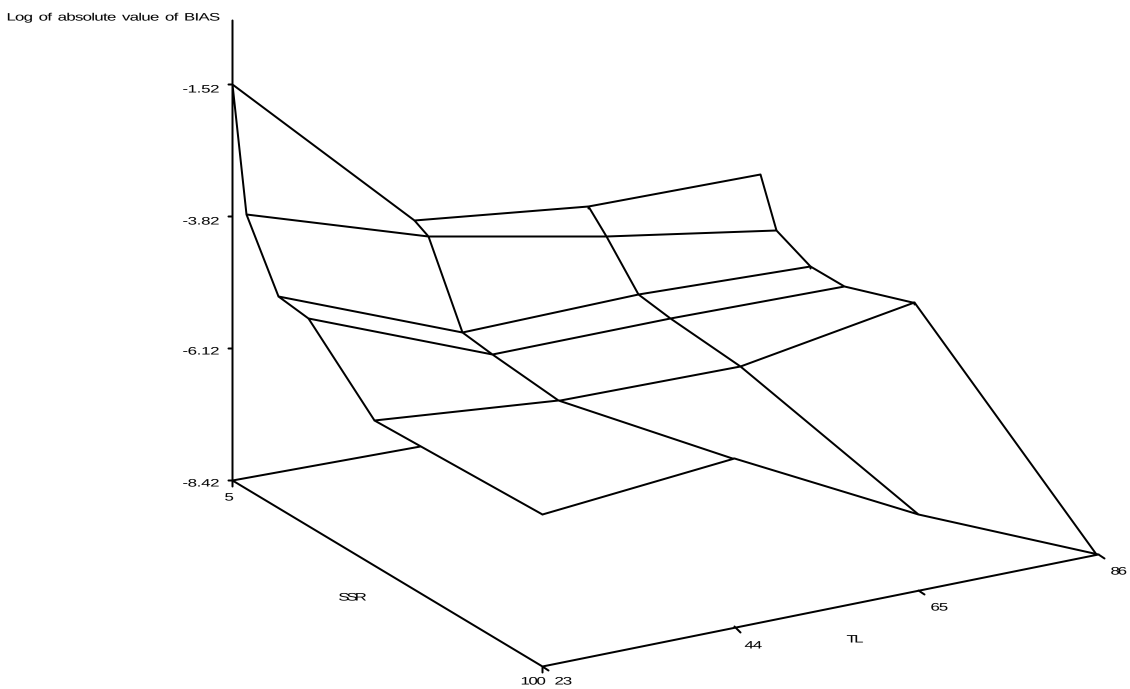


Figure 16. Log of absolute value of MBIAS of d plotted with SSR, TL and TC.

TC = 0.0



TC = 0.3

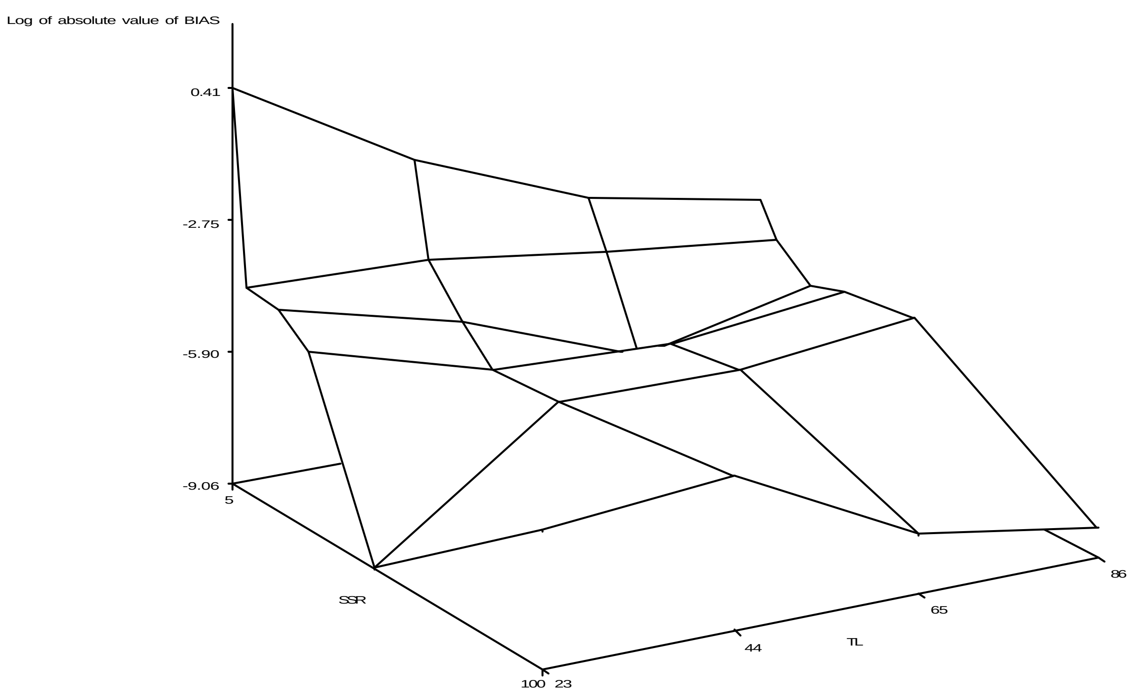


Figure 17. Log of absolute value of MBIAS of d plotted with SSR and TL.

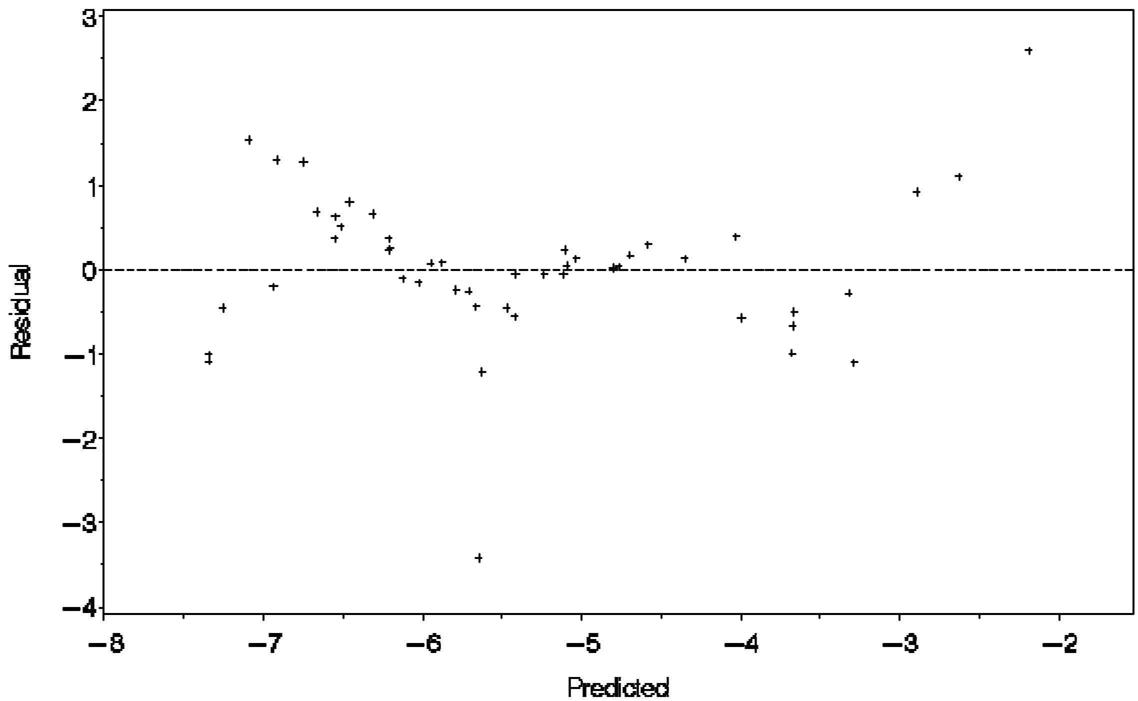


Figure 18. Residual plots of log of absolute value of MBIAS of d with predicted values.

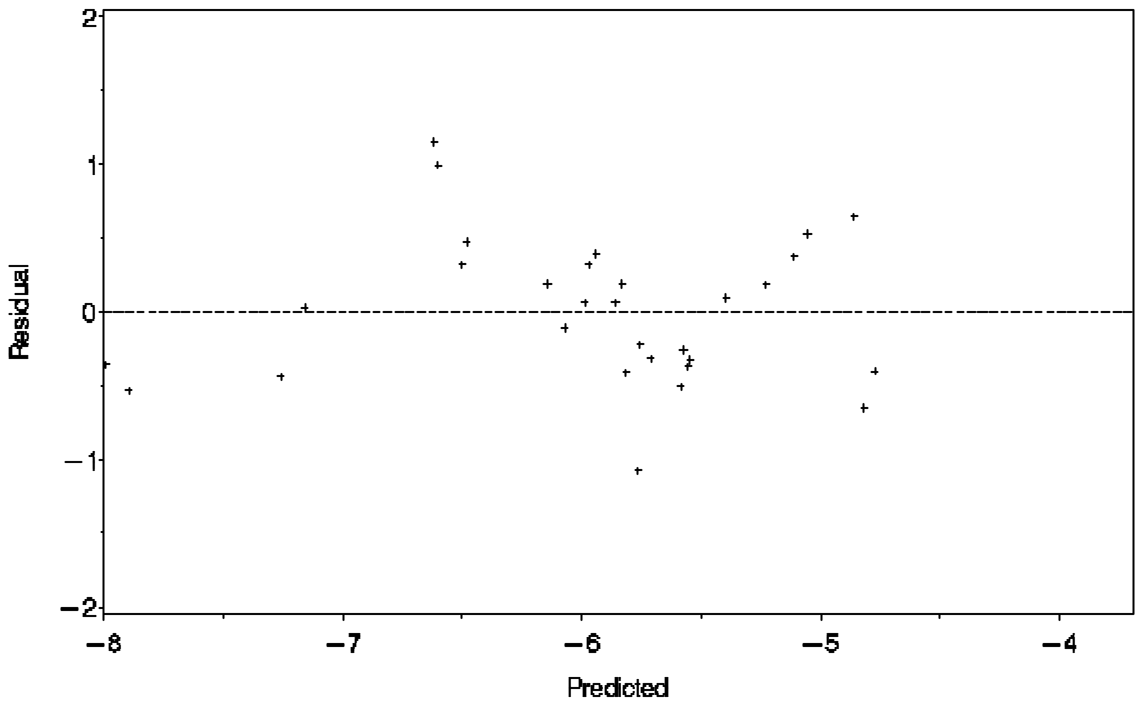


Figure 19. Residual plots of log of absolute value of MBIAS of d with predicted values.

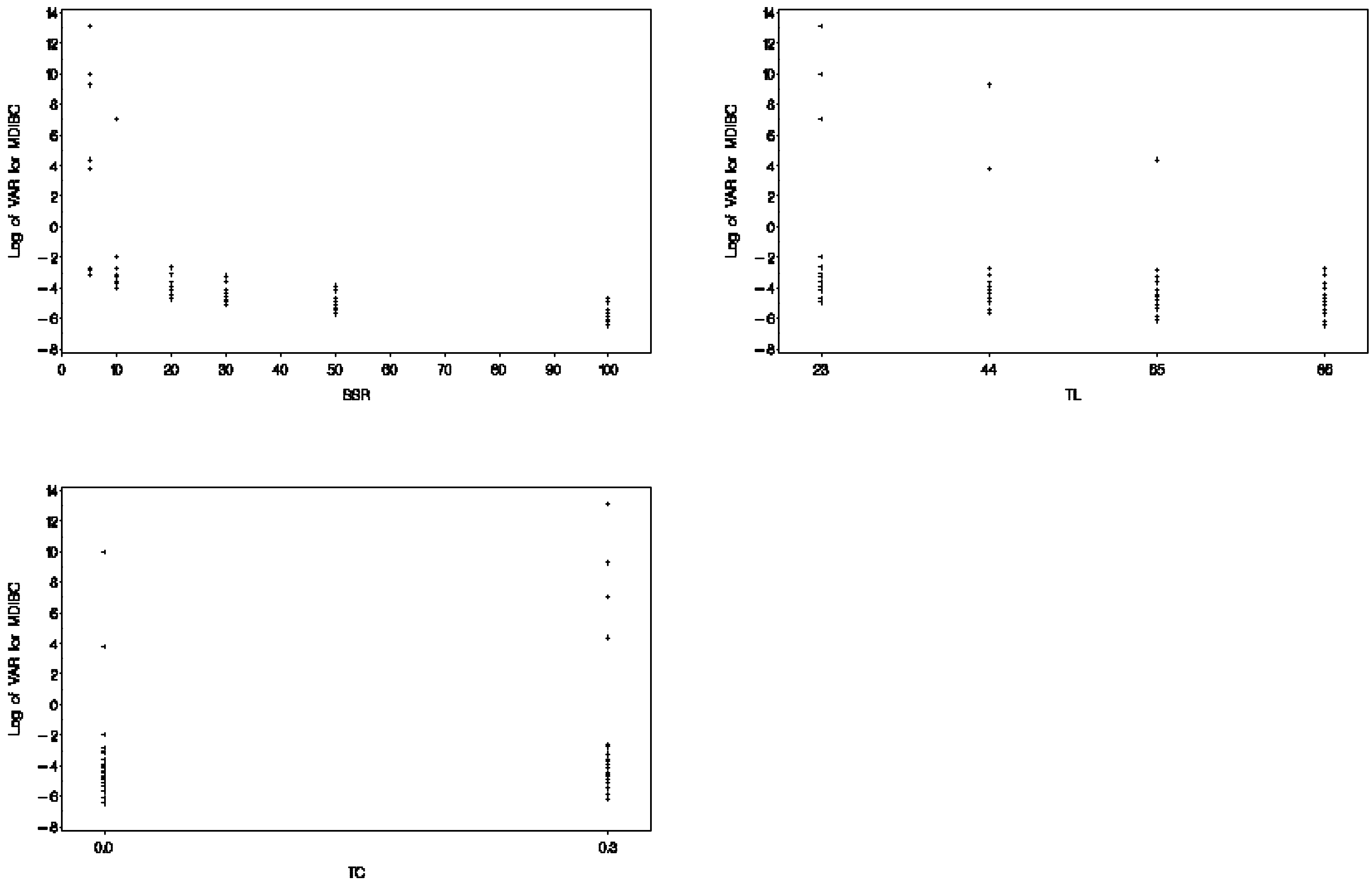
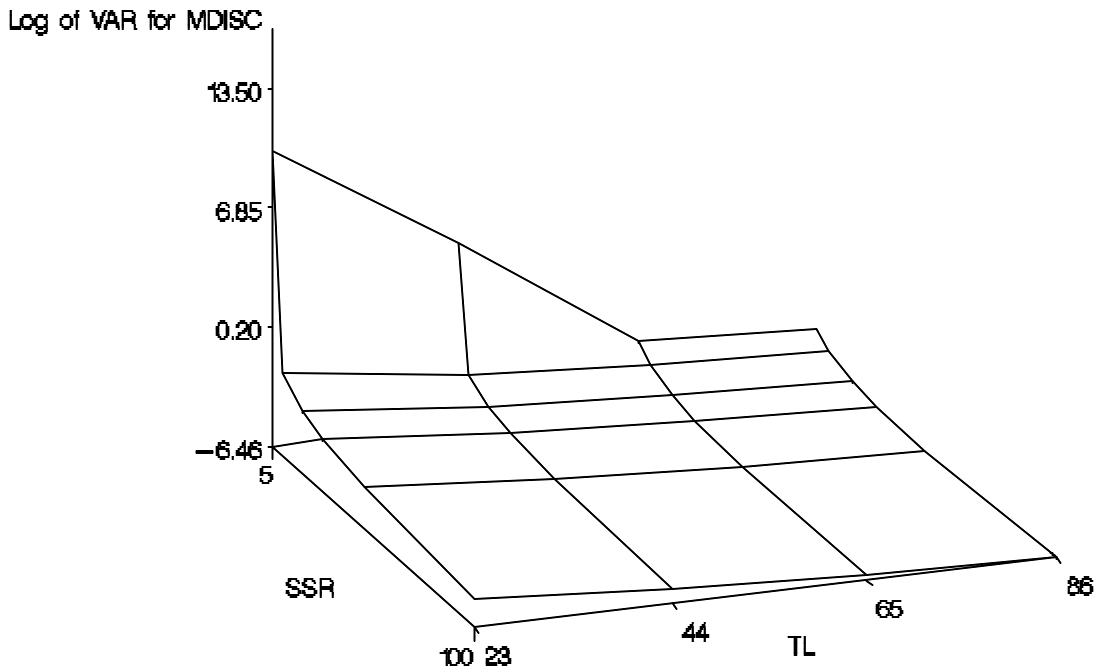


Figure 20. Log of MVAR of MDISC plotted with SSR, TL and TC.

TC = 0.0



TC = 0.3

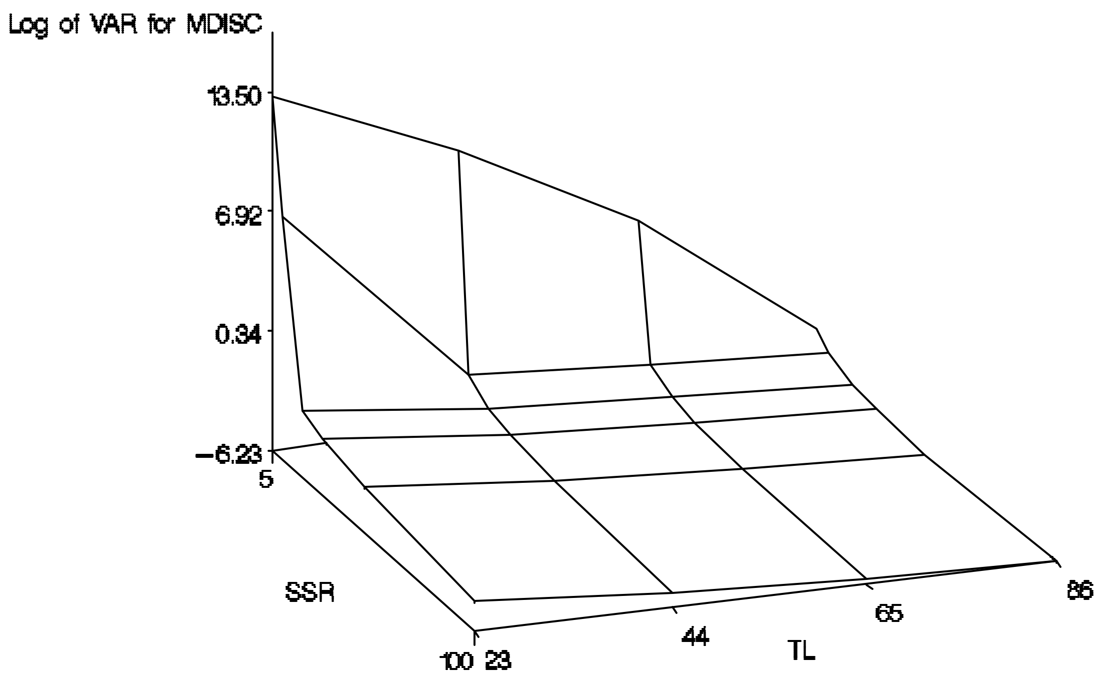


Figure 21. Log of MVAR of MDISC plotted with SSR and TL.

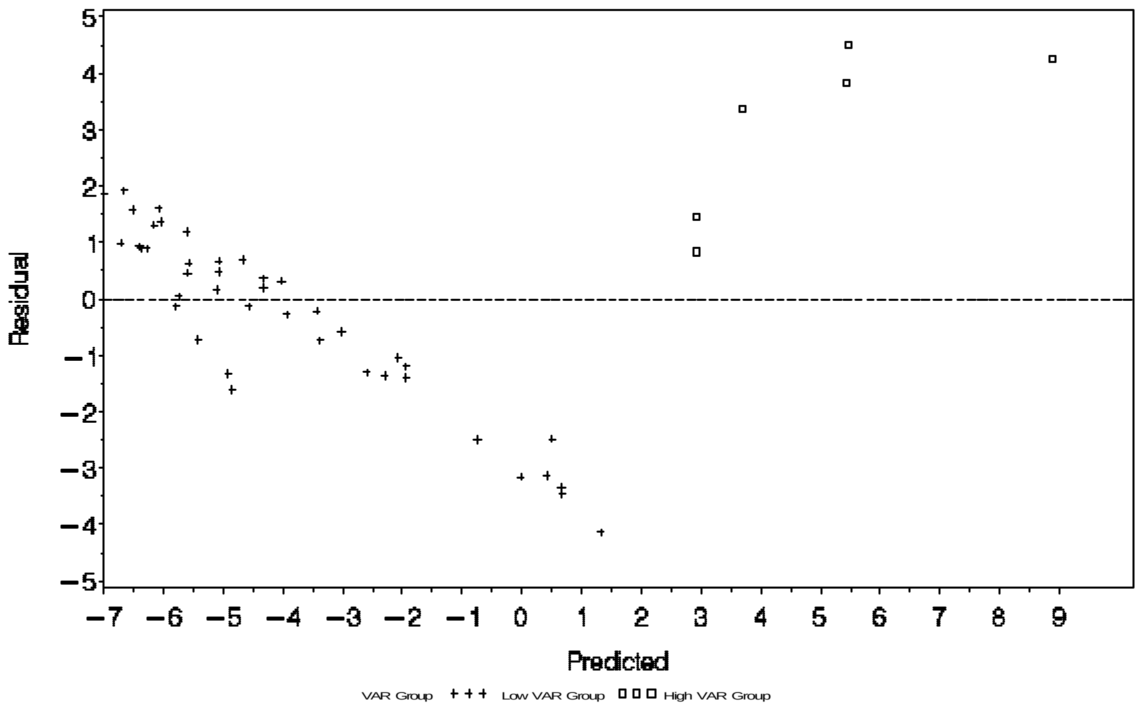
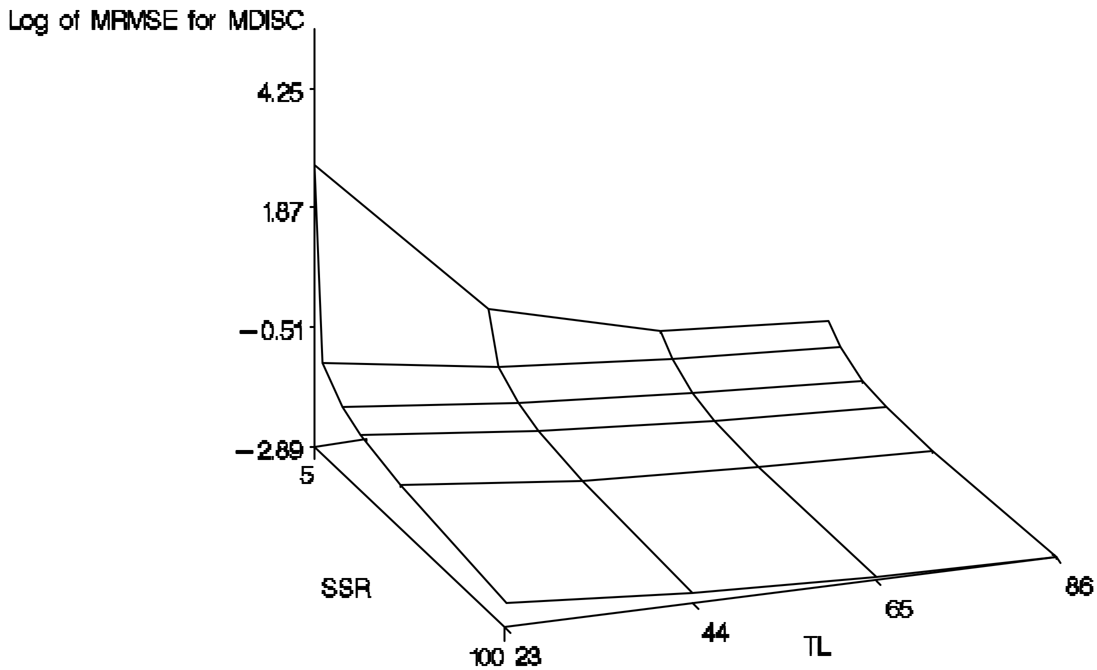


Figure 22. Residual plots of log of MVAR of MDISC with predicted values.

TC = 0.0



TC = 0.3

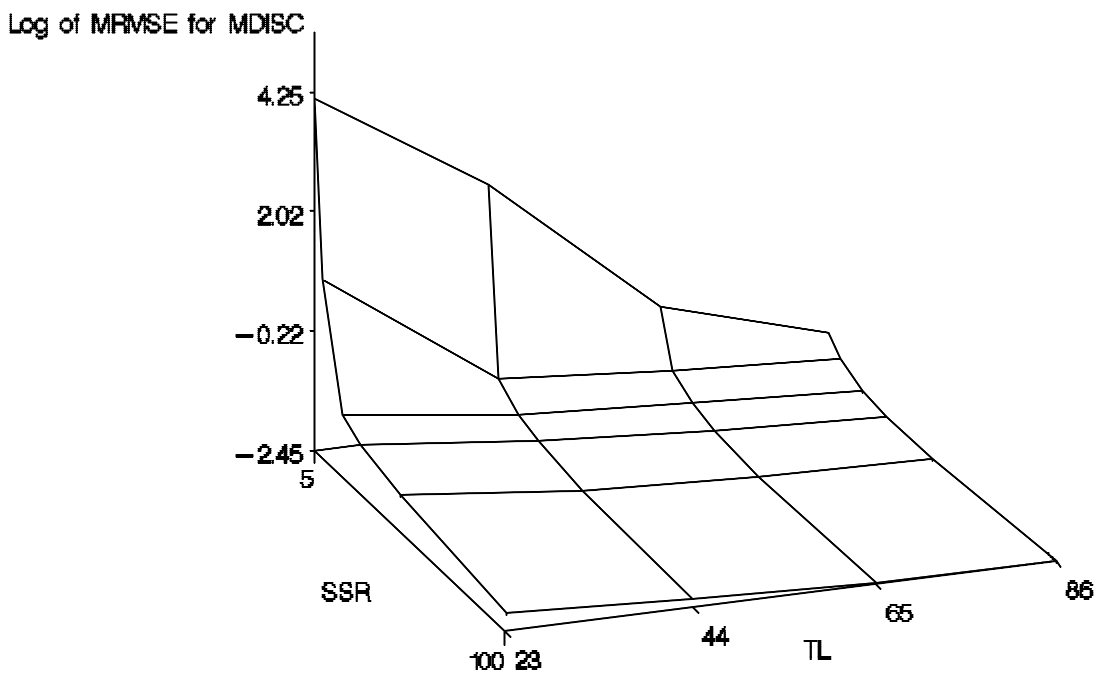
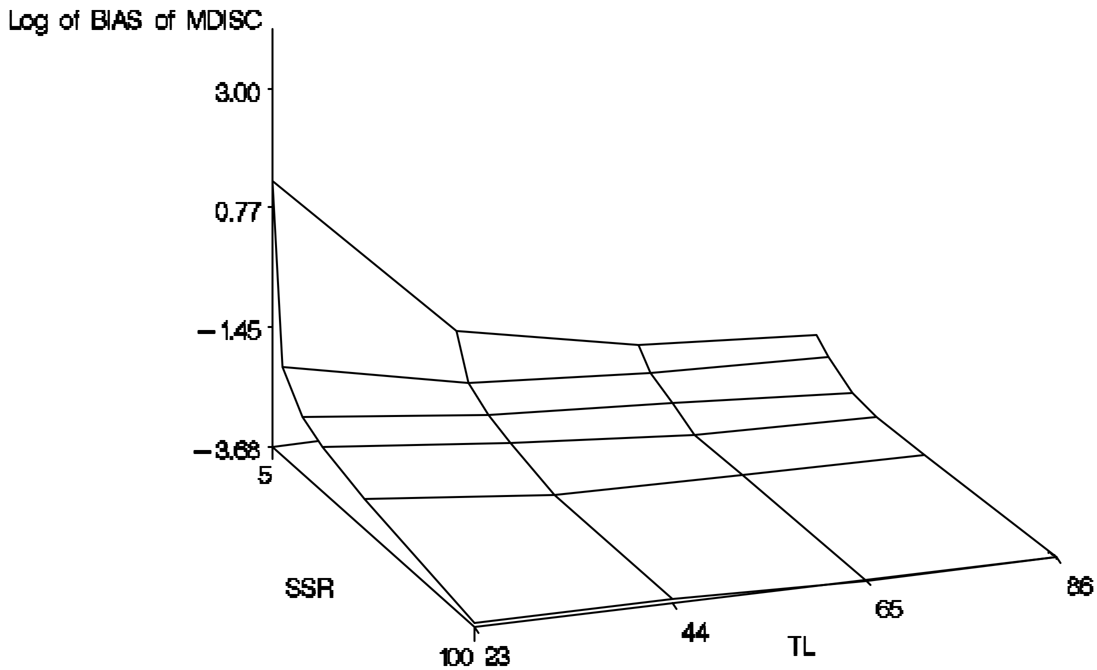


Figure 23. Log of MRMSE of MDISC plotted with SSR and TL.

TC = 0.0



TC = 0.3

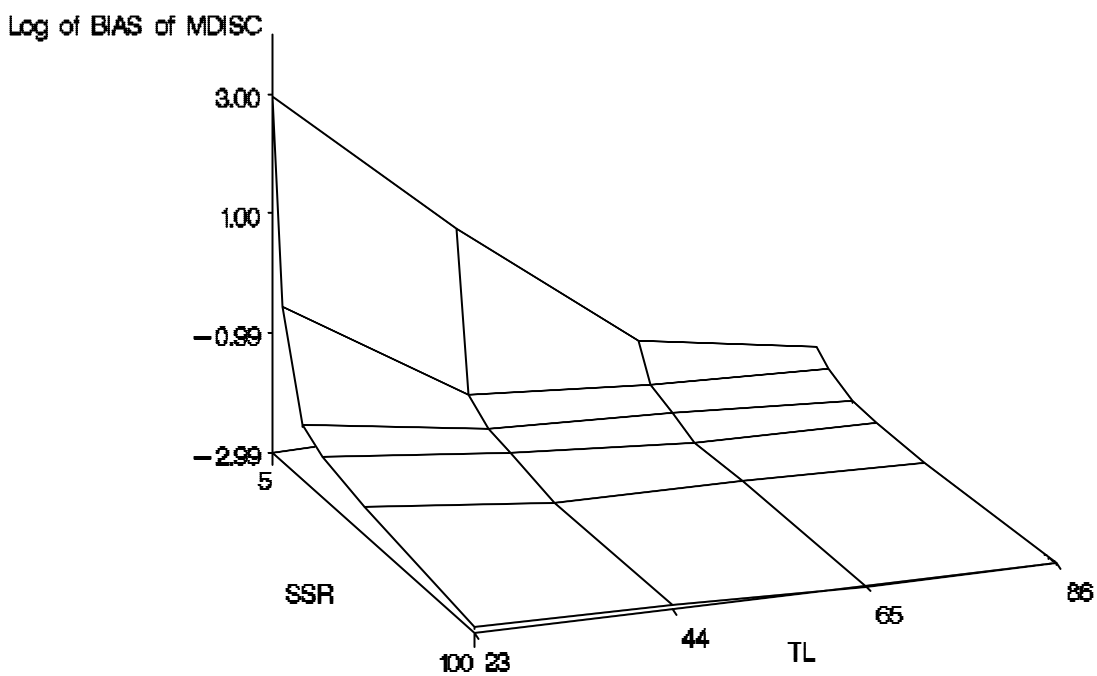
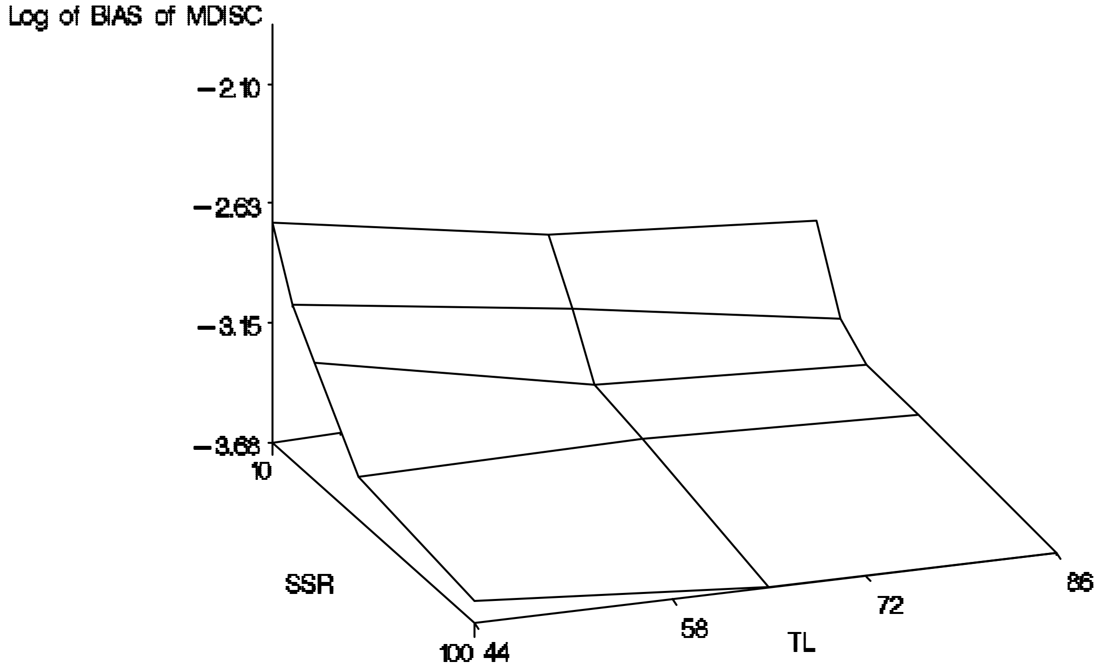


Figure 24. Log of MBIAS of MDISC plotted with SSR and TL.

TC = 0.0



TC = 0.3

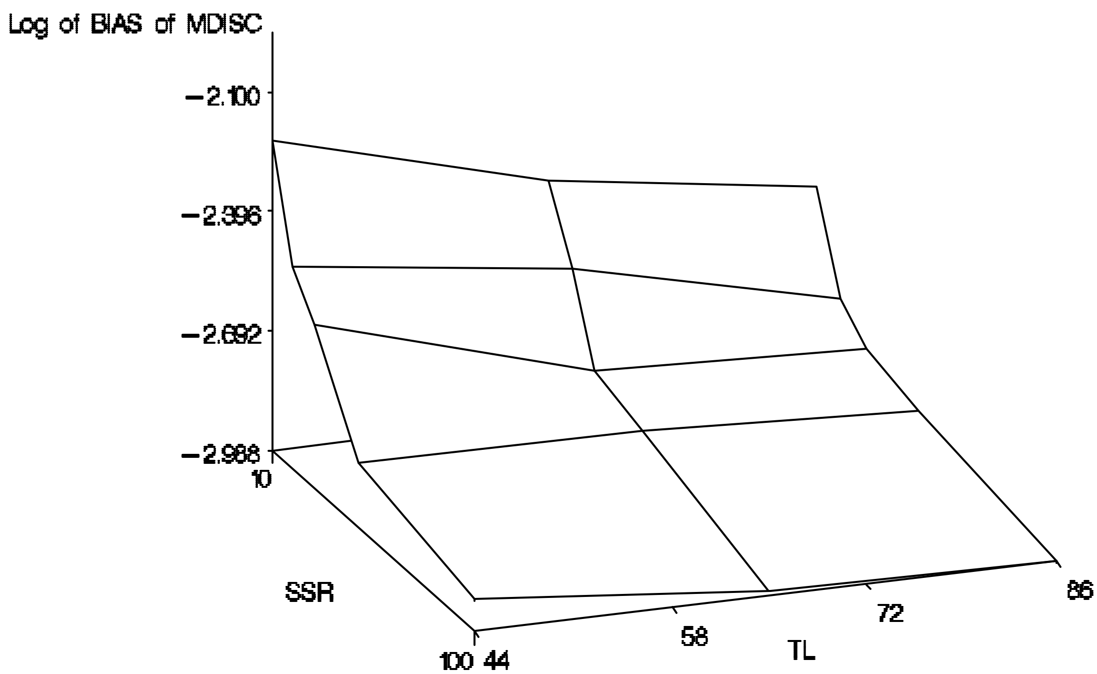


Figure 25. Log of MBIAS of MDISC plotted with SSR and TL for reduced data set.

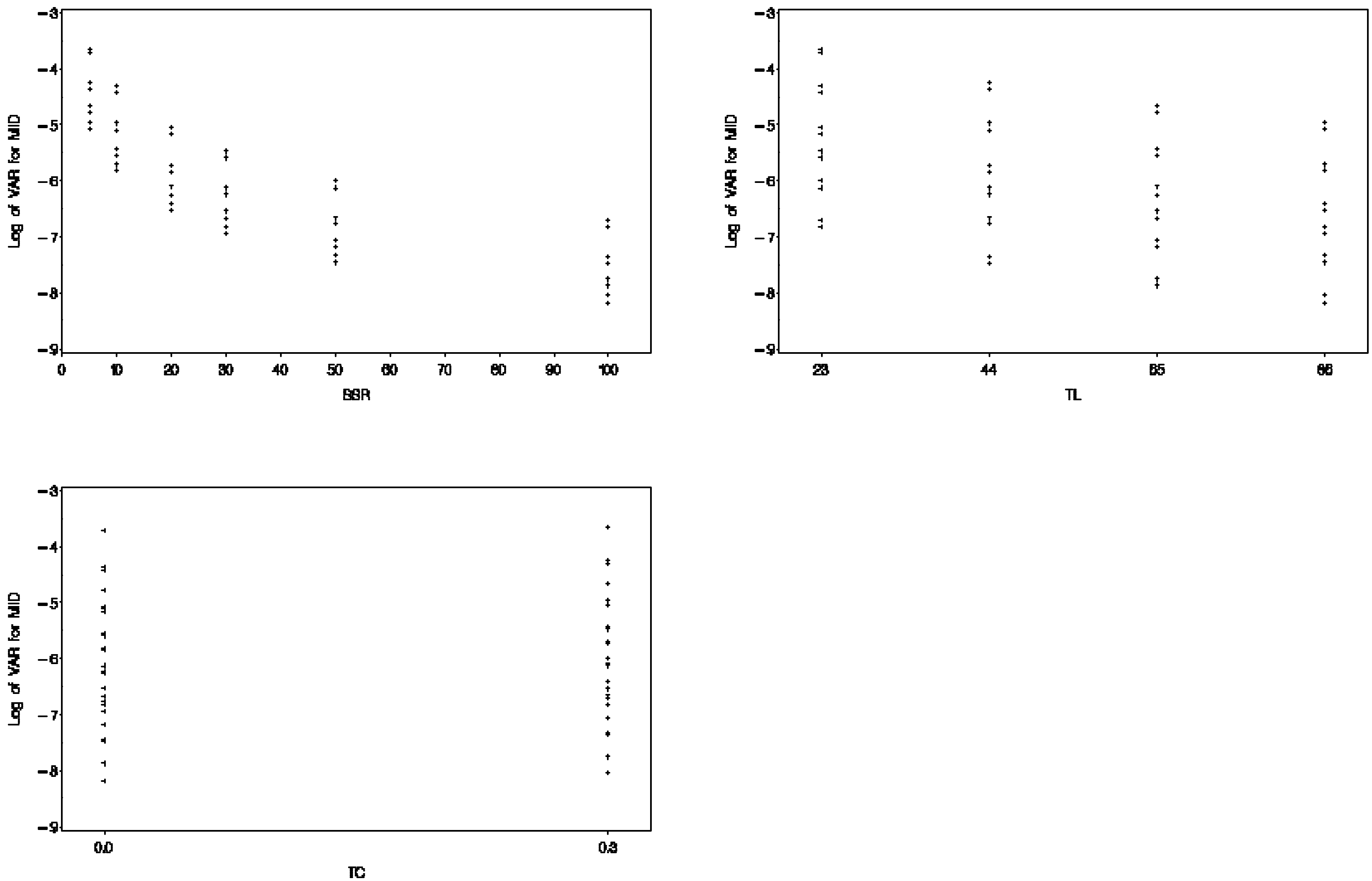
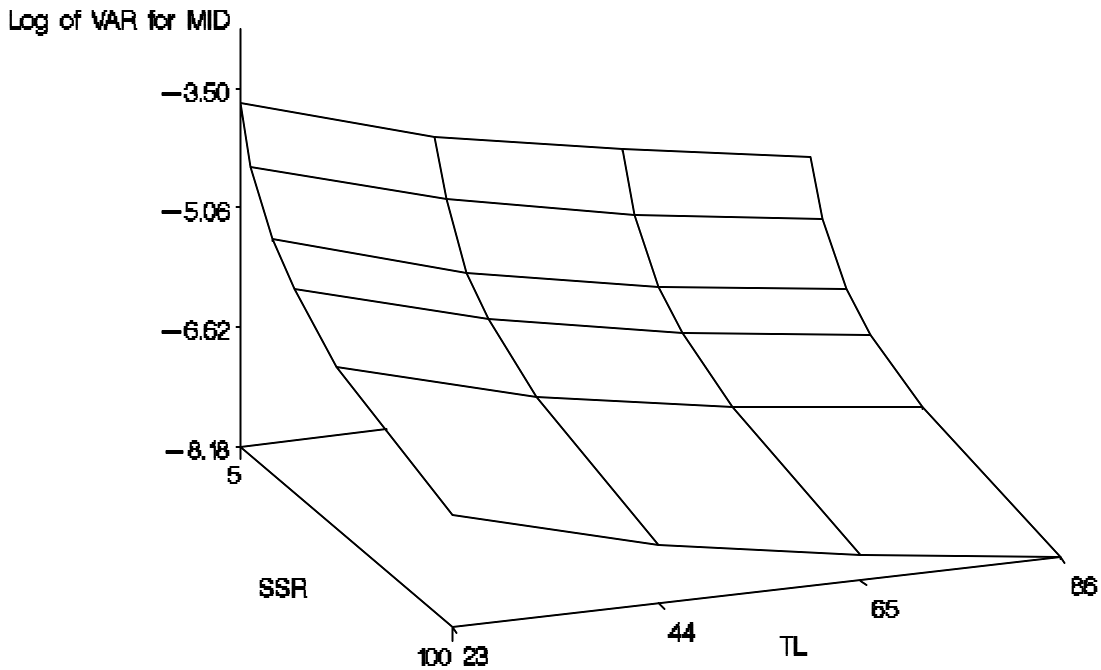


Figure 26. Log of MVAR of D of MID plotted with SSR, TL and TC.

TC = 0.0



TC = 0.3

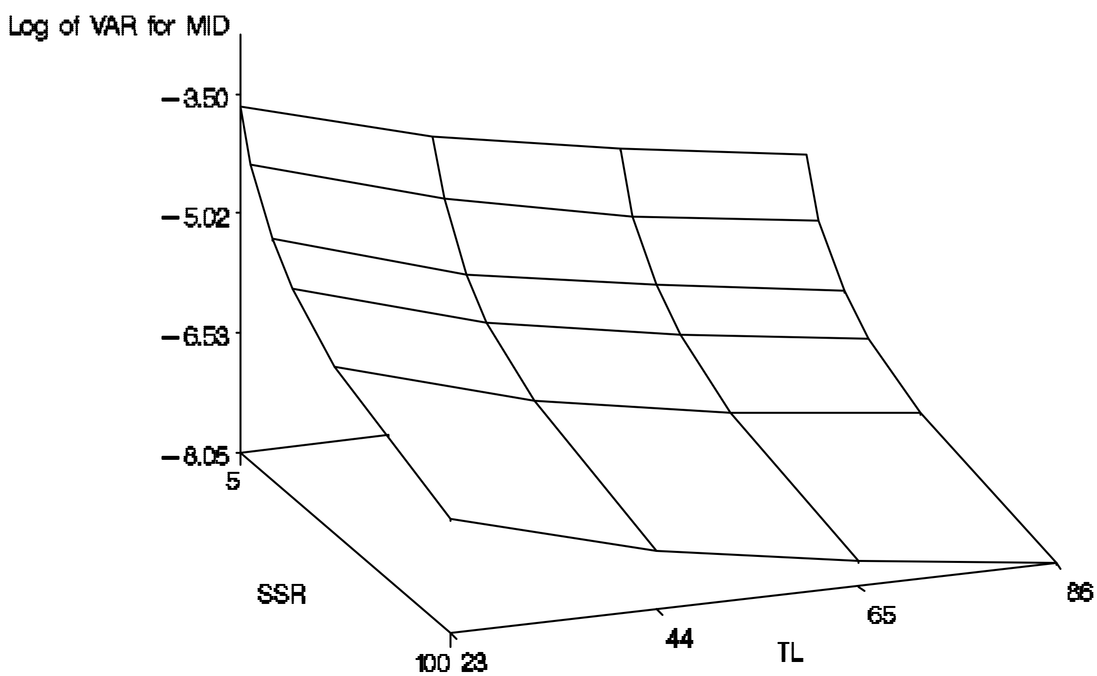
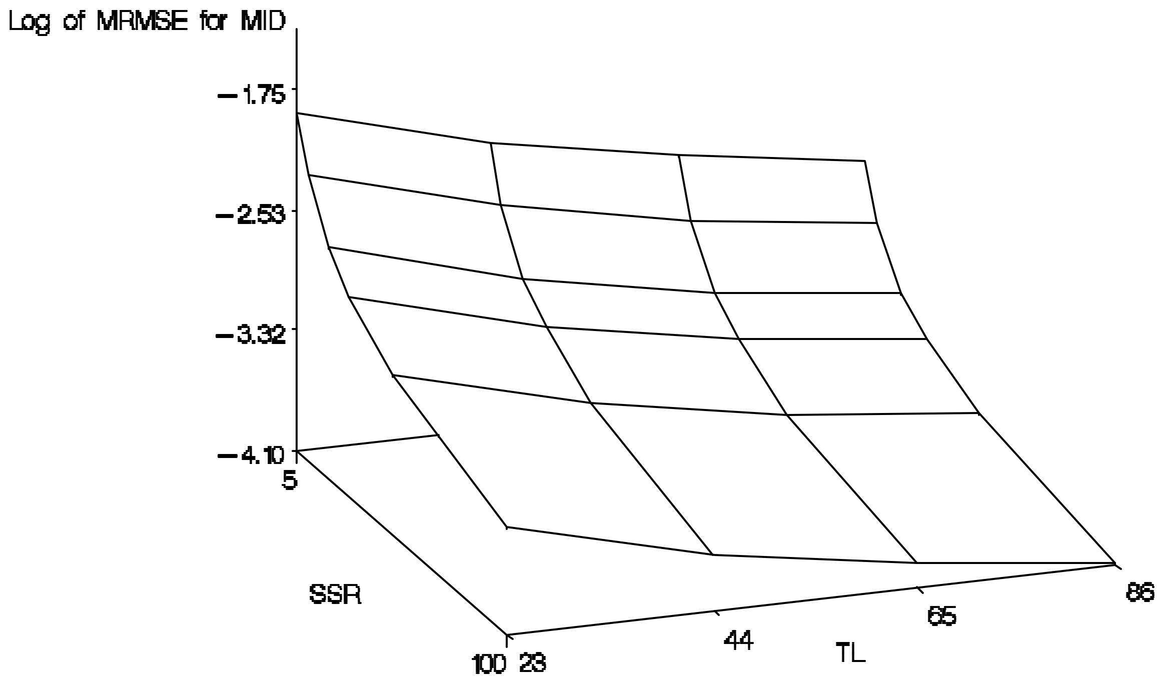


Figure 27. Log of MVAR of D of MID plotted with SSR and TL.

TC = 0.0



TC = 0.3

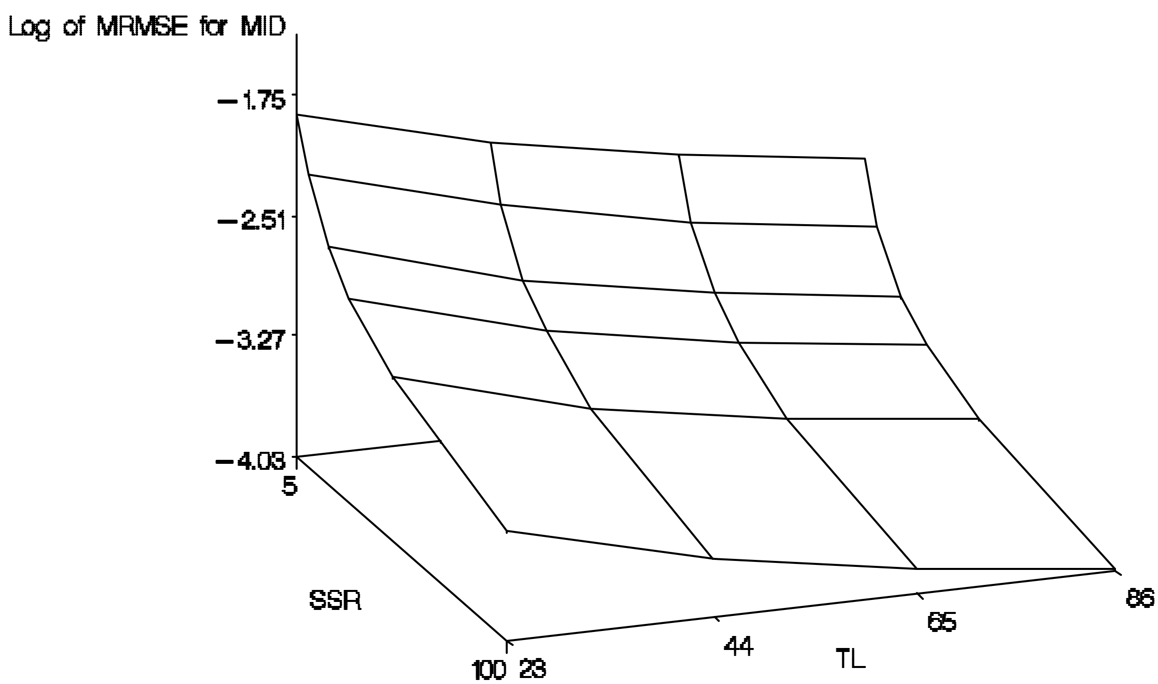
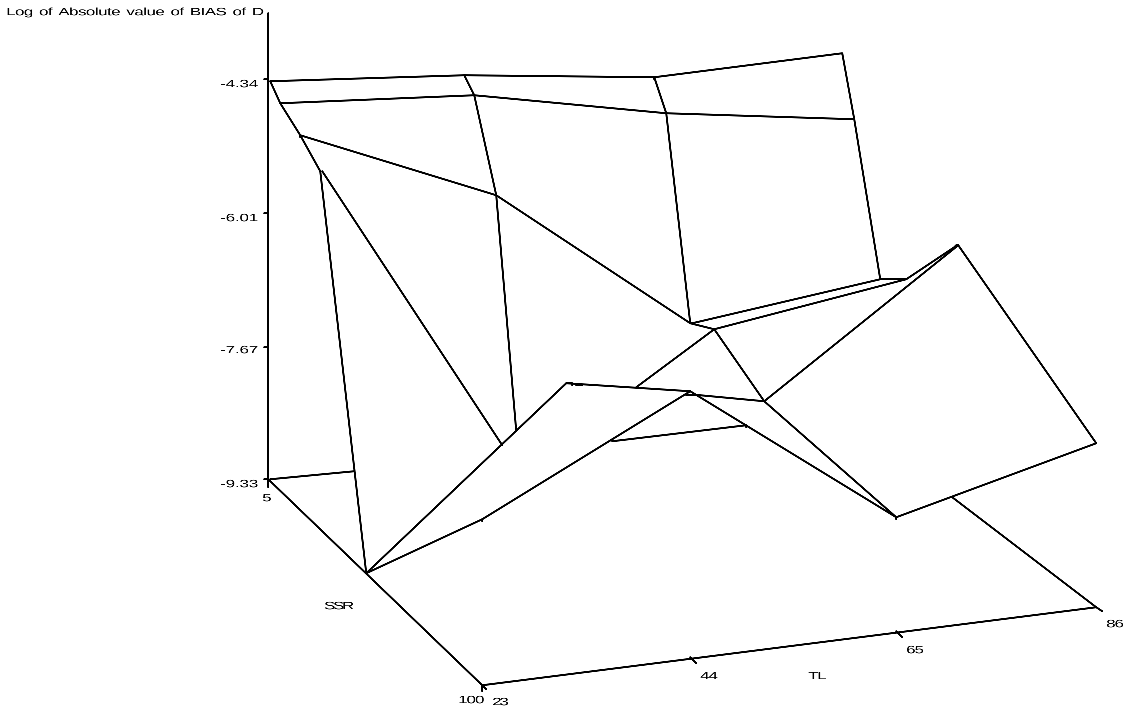


Figure 28. Log of MRMSE of D of MID plotted with SSR and TL.

TC = 0.0



TC = 0.3

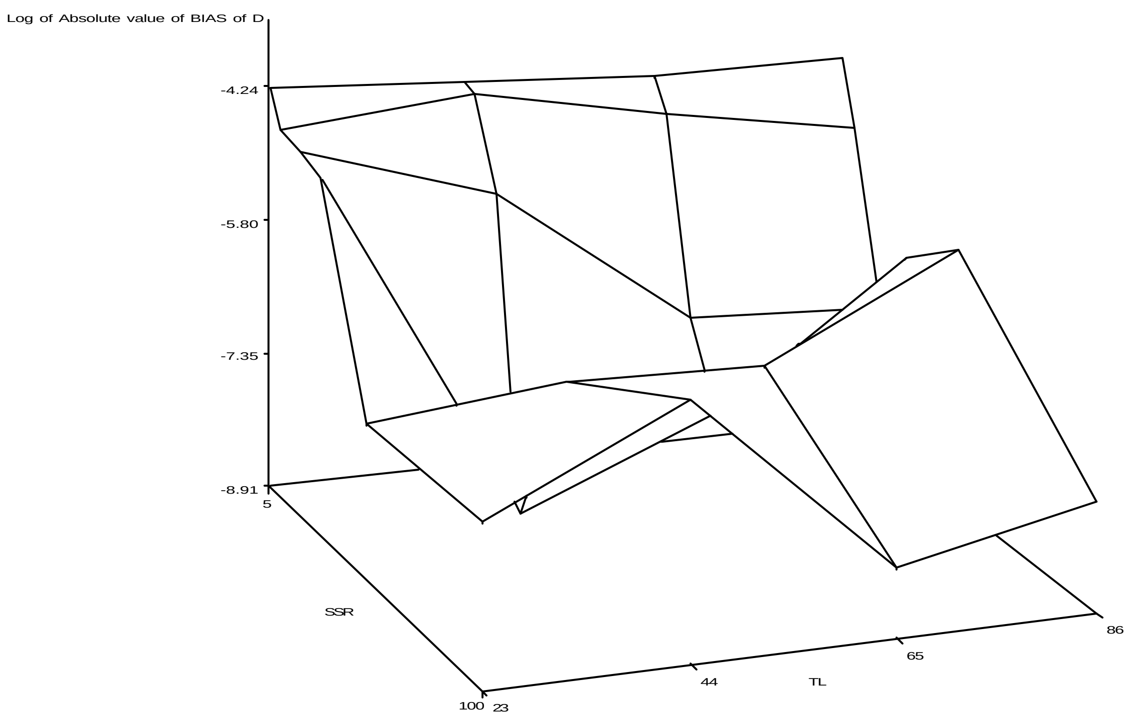


Figure 29. Log of absolute value of MBIAS of D of MID plotted with SSR and TL.

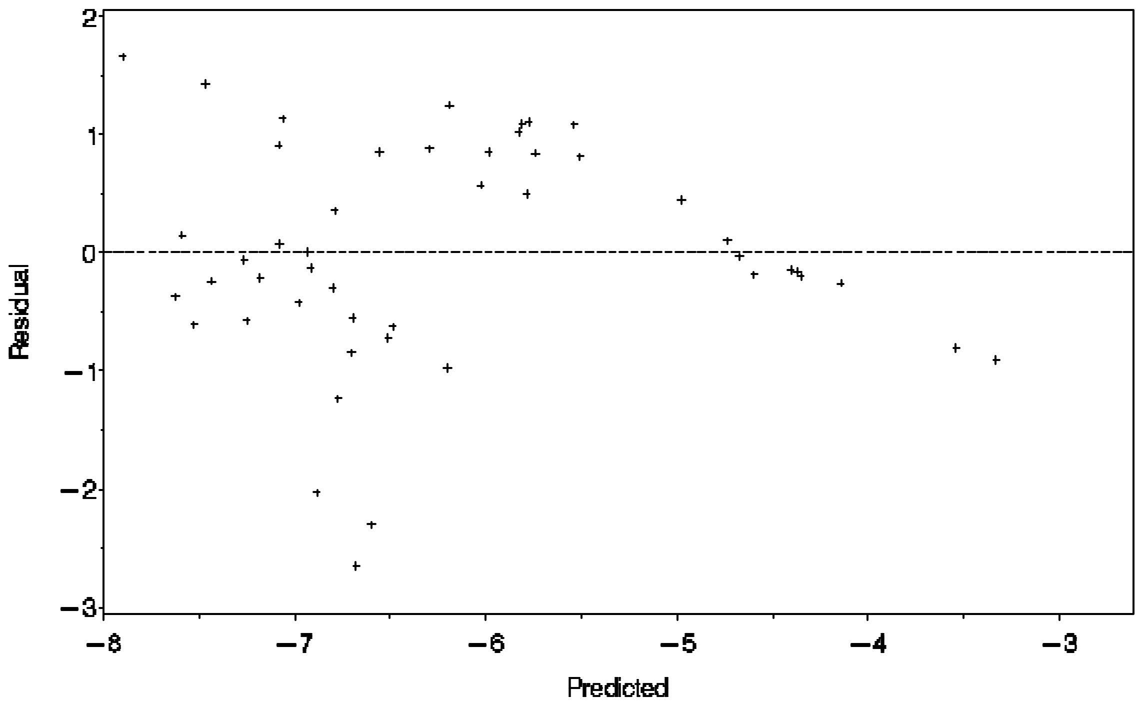


Figure 30. Residual plots of log of absolute value of MBIAS of D of MID with predicted values.

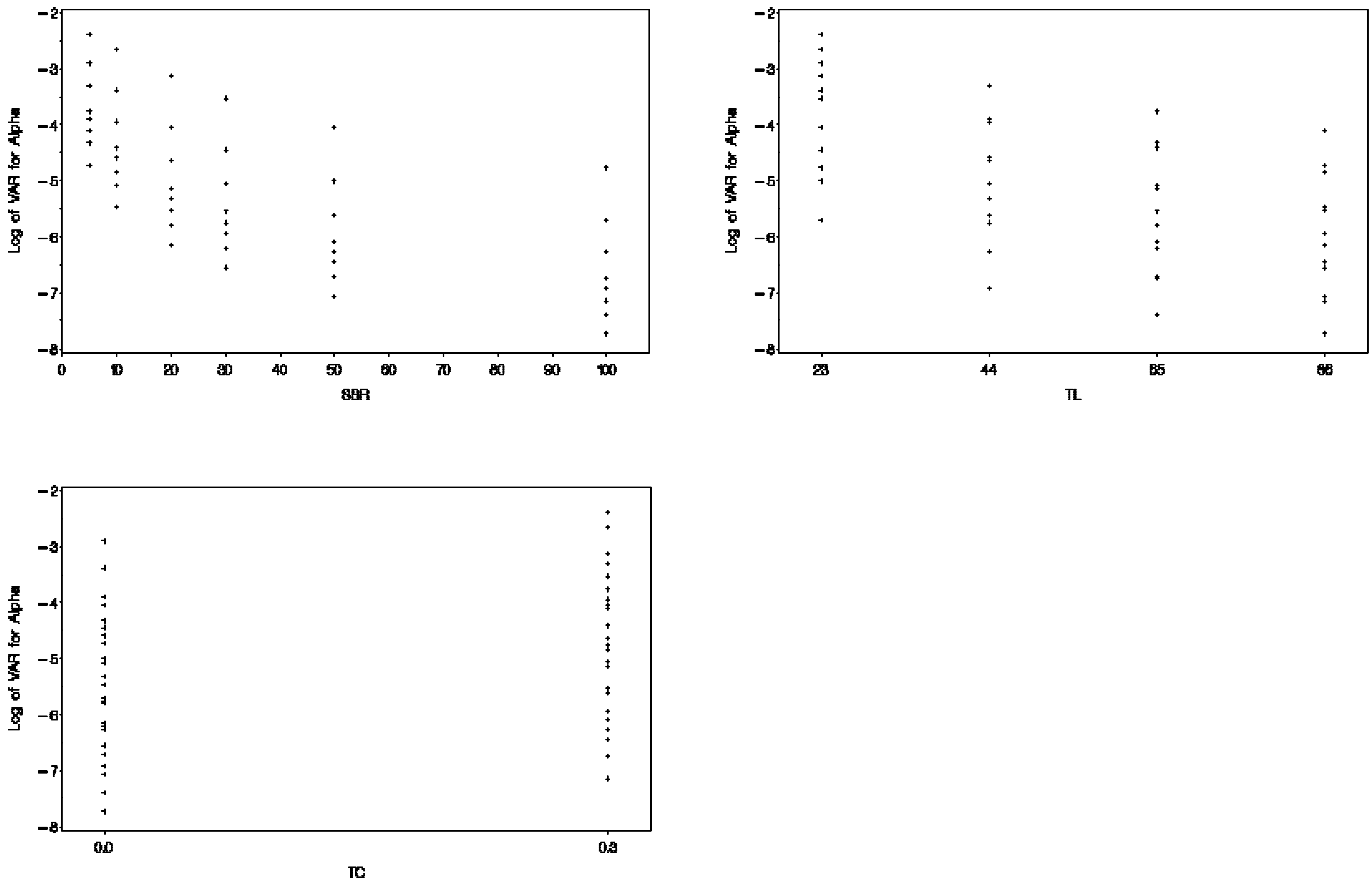
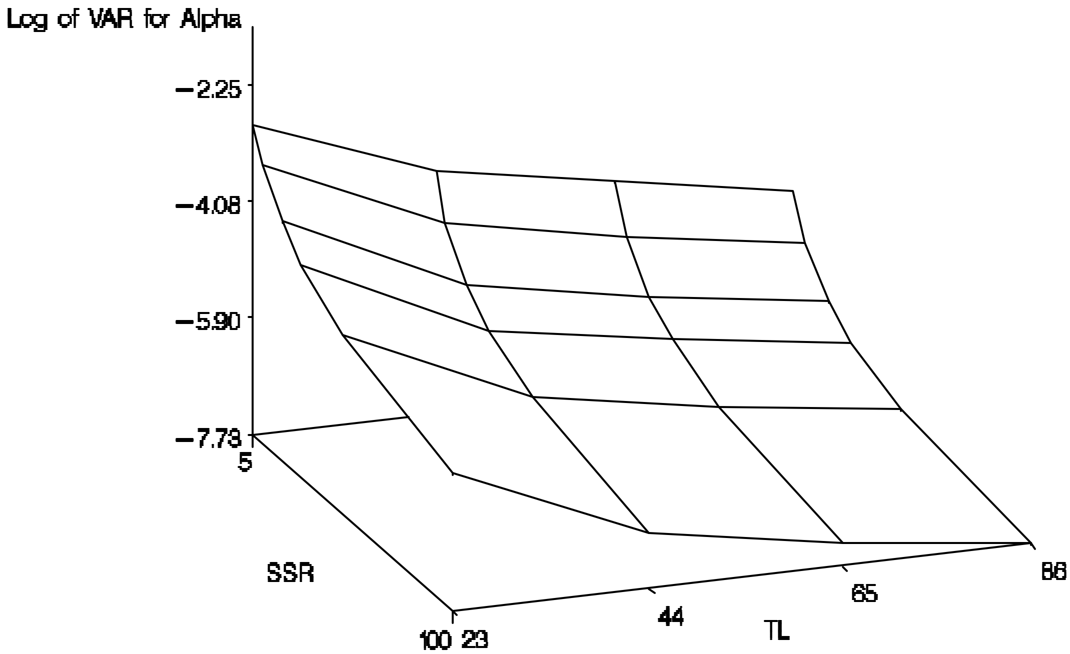


Figure 31. Log of MVAR of a_1 of MID plotted with SSR, TL and TC.

TC = 0.0



TC = 0.3

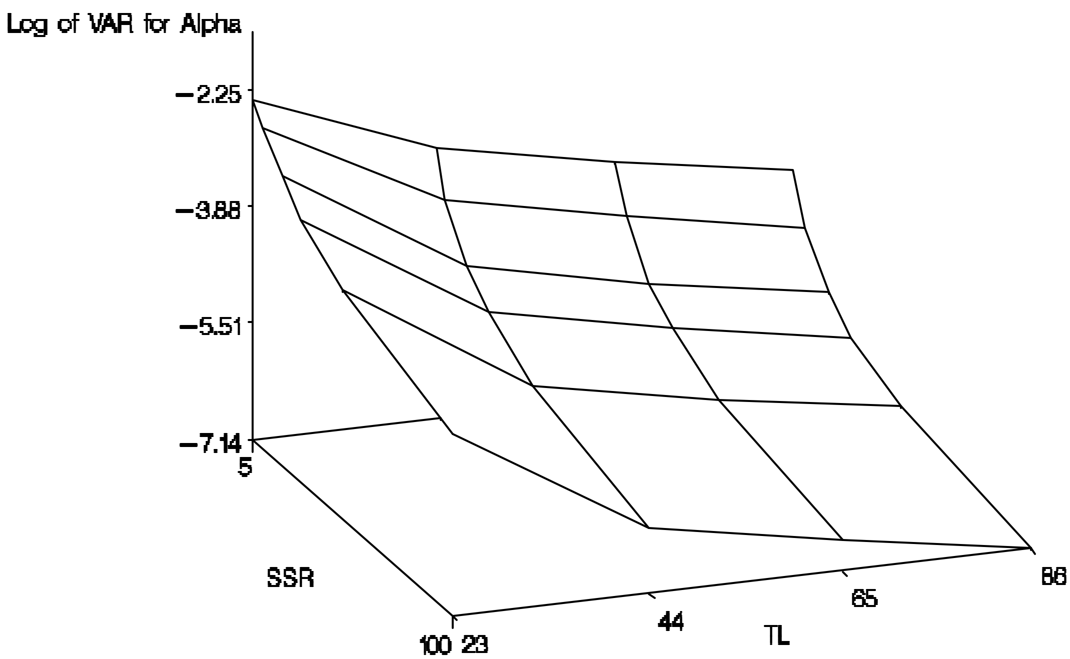
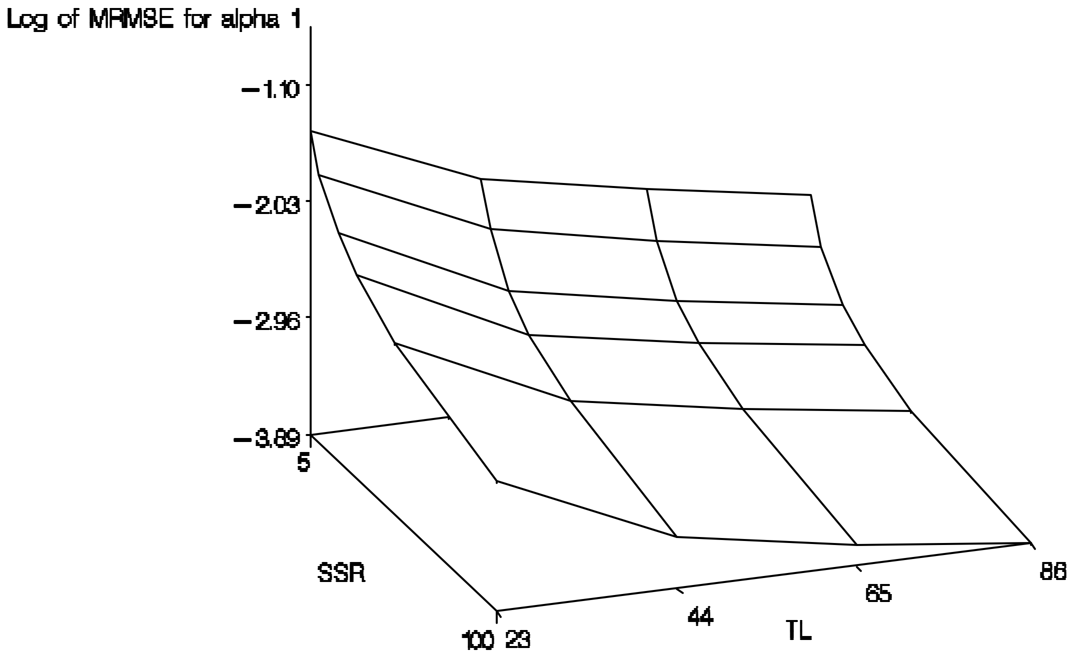


Figure 32. Log of MVAR of a_1 of MID plotted with SSR and TL.

TC = 0.0



TC = 0.3

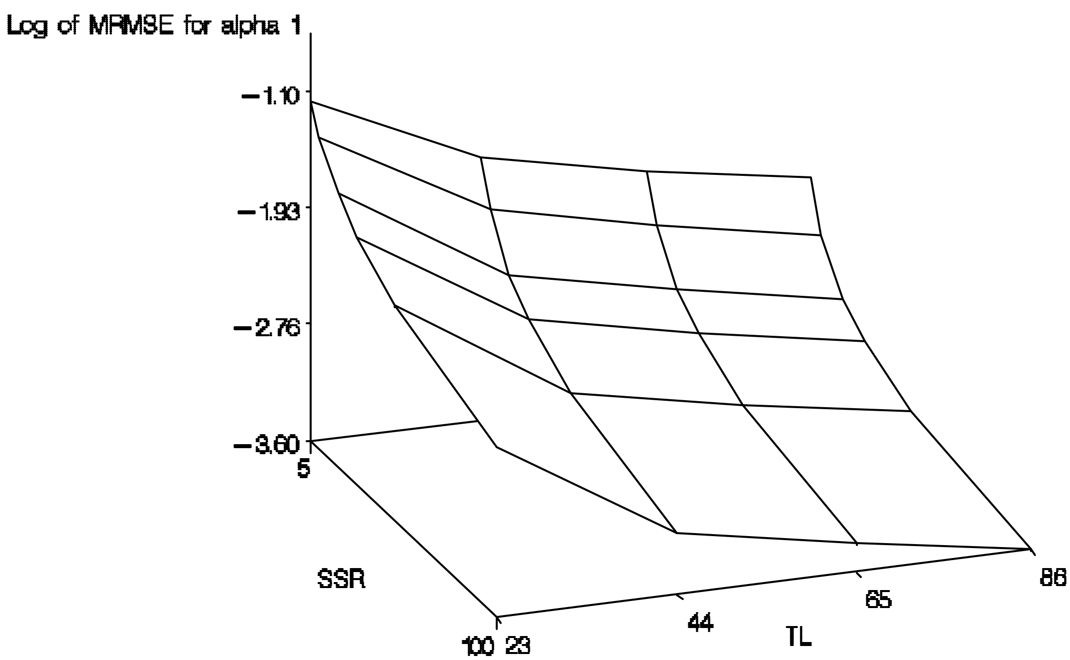
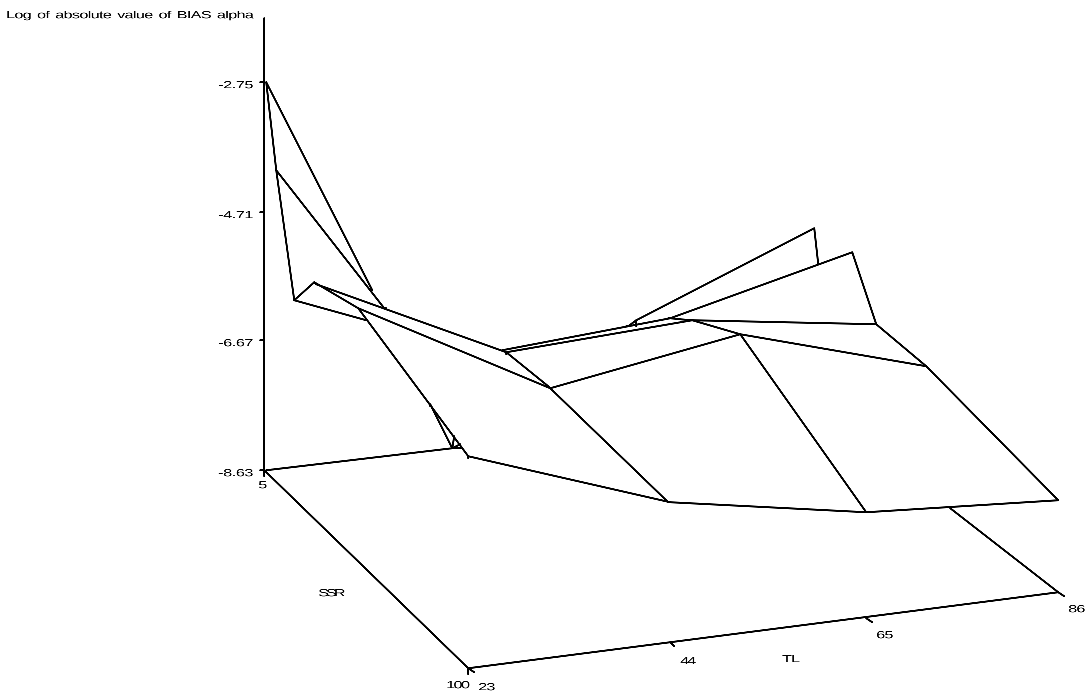


Figure 33. Log of MRMSE of a_1 of MID plotted with SSR and TL.

TC = 0.0



TC = 0.3

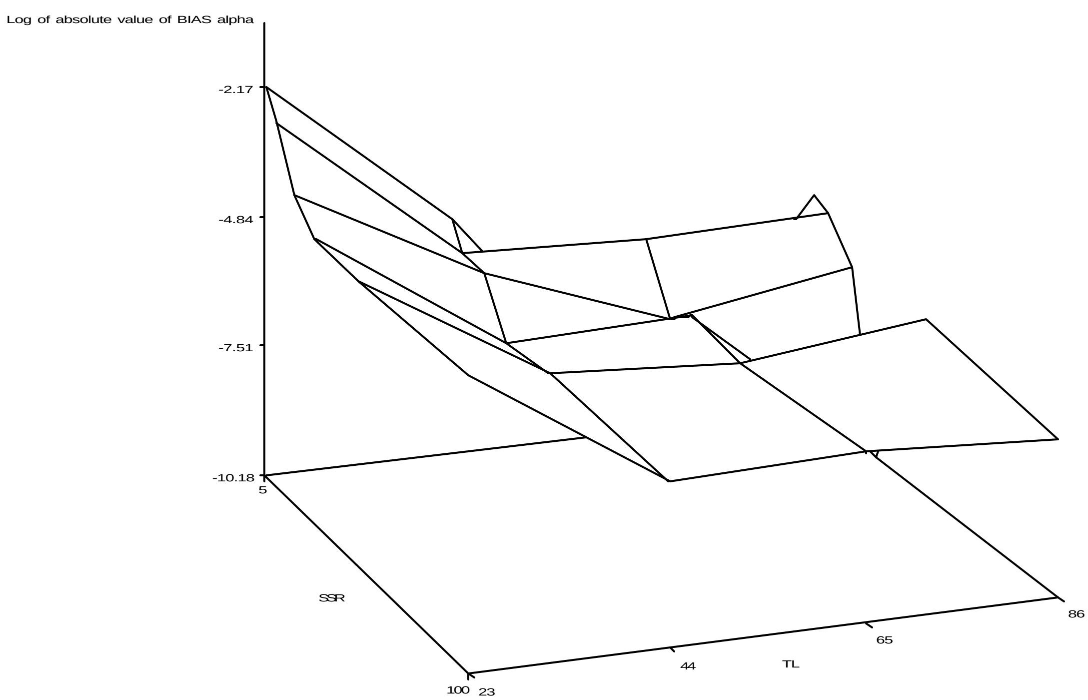


Figure 34. Log of absolute value of MBIAS of a_1 of MID plotted with SSR and TL.

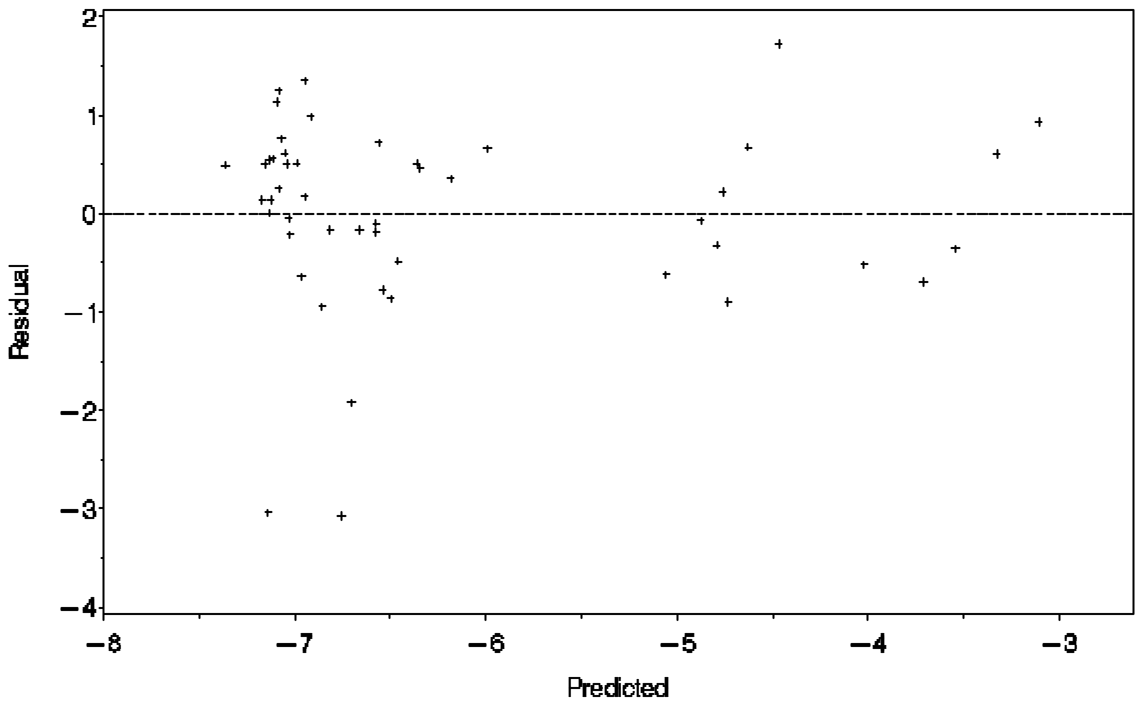


Figure 35. Residual plots of log of the absolute value of MBIAS of a_1 of MID with predicted values.

CHAPTER 4

DISCUSSION FOR ENTIRE TEST

Assessment model perspectives

The plots of MVAR, MRMSE and MBIAS with SSR and TL for TC equal to 0.0 and 0.3 for a1, a2, d, and MDISC all have a "mountain/plain" pattern where the dependent variable values rise sharply from a level "plain" region to "mountain" region for low levels of SSR and TL. This pattern is also seen for MBIAS for D of MID. This "mountain/plain" pattern is not found for MVAR or MRMSE for D and a_1 nor for the MBIAS of a_1.

This "mountain/plain" pattern suggests two possible perspectives from which the data sets can be modeled. From the first perspective, the pattern in the data can be viewed as being continuous with the extreme values of MVAR, MRMSE and MBIAS, resulting from an interaction between SSR and TL. From this perspective small values of SSR along with small values of TL produce exceptionally large values of MVAR, MRMSE and MBIAS. From the second perspective the pattern in the data can be viewed as discontinuous with a distinct jump occurring in the values of MVAR, MRMSE and MBIAS, once a minimum threshold in the value of SSR and/or TL is crossed.

To examine which of these two perspectives provides the most useful model for understanding the data, two hierarchical regression analyses were conducted for the combinations of dependent variables and parameter estimates with the "mountain/plain" plots. To fit the continuous perspective model to the data, the full data set with all conditions included was used. To fit the discontinuous perspective model a reduced data set, in which only the conditions that were in the "plain" region of the plot were included, was used. To maintain a balanced experimental design for the discontinuous models, all conditions with the either SSR equal to 5 or TL equal to 23 were eliminated even though this included a few data points at the edge of the "plain." The fit of the two analyses were then compared to see which model best described the observed data.

Four criteria from the regression analyses were used to assess the goodness of the fit of the models. First, the amount of variance in MVAR, MRMSE, or MBIAS that the model accounted for was considered. The final R^2 value was used to measure this criterion. Second, how well the residuals met the normality assumption was considered. Shapiro-Wilk's W (Shapiro & Wilk, 1965) was used to measure this criterion. The third criterion considered was how well the residuals met the homoscedasticity assumption along SSR, TL and TC. To measure this, Glejser's (1969) approach of regressing the absolute values of the residuals on the terms in the model regression equation was used. The fourth criterion used was the existence of prediction bias. This was assessed by examining the plots of the residuals with the predicted values of MVAR, MRMSE and MBIAS to see if prediction bias was apparent.

The correlations of parameter estimates with the true parameters were used as a fifth criterion in determining which of the two perspectives were most useful in understanding the data. Average correlations were calculated across all the conditions and across only those conditions in the reduced data set. Higher average correlations indicate a better fit.

The summaries of these criteria (Tables 4, 7, 11, and 15) indicate that the models from the discontinuous perspective fit the data better than do the models from the continuous perspective. The R^2's for the reduced data sets are consistently larger than the R^2's for the full data sets. The models for the reduced data sets meet more of the residual assumptions and demonstrate less prediction bias. Also the average correlations between the parameter estimates and the true parameters are larger for the reduced data sets.

This consistency of better fits for the models from the discontinuous perspective indicates that for the combinations of dependent variables and parameter estimates with the “mountain/plain” pattern (i.e. MVAR, MRMSE and MBIAS for a1, a2, d, and MDISC; and MBIAS for D) the quality of parameter estimates is best viewed as being discontinuous with thresholds where large jumps in stability, precision and accuracy occur. This discontinuity in the quality of the parameter estimates raises two important questions. The first question is, where does the jump in MVAR, MRMSE and MBIAS occur? That is, what is the minimum threshold of SSR and TL needed to obtain acceptable parameter estimates? This question is important for those seeking to obtain useable parameter estimates with the smallest sample size ratio or shortest test possible.

The second question is, what is the influence of SSR, TL and TC on the levels of MVAR, MRMSE and MBIAS once one is working in the stable “plain” region? This question is important for those seeking to increase the stability, precision or accuracy of parameter estimates. Knowing the influence of SSR and TL would aid in making decisions on what factors to adjust to achieve better estimates. While the correlation of the traits of interested cannot be adjusted, knowing the influence of TC on the estimation of parameters helps the test developer plan the needed SSR and TL depending on the correlation of the traits being measured by the test. This second question will be addressed first.

Influence of SSR, TL and TC on MVAR, MRMSE and MBIAS

The results of the hierarchical regression analyses need to be considered in assessing the influence of independent variables (SSR, TL and TC) on dependent variables (MVAR, MRMSE and MBIAS). In these analyses, the unique variance explained by each term in the step in which it was entered was determined using the type II squared semi-partial correlation. The influence of the independent variables on the dependent variables is manifested both in the terms SSR, TL and TC as well as in terms involving a transformation of the independent variables such as SSR^2, 1/SSR and

1/TL. For example, to understand the full impact of SSR on MVAR of a1 the unique variance accounted for by SSR, 1/SSR and SSR^2 all need to be considered. Table 21 provides a summary of the percentage variance in the dependent variables associated with each independent variable either directly or through a transformation of the variable.

The direction of influence of terms in the regression equation is reflected in the sign of the regression coefficient of that term. Because hierarchical regression was used to determine the unique contribution of each term in the step in which it was entered, the coefficients of interest are those associated with a term in the step in which it was entered. A negative coefficient indicates that as the term increases the dependent variable decreases. However, special consideration needs to be given to how this is to be interpreted for the reciprocal terms, 1/SSR and 1/TL. In this case, a positive coefficient indicates that as SSR or TL increases the dependent variable decreases. This is because the reciprocal term decreases as the independent variable increases.

For the combinations of dependent variables and parameters estimates with the "mountain/plain" data patterns, the results of the hierarchical regressions from the discontinuous models are reported in Table 21. For the combinations with the smoother patterns the results from the continuous models are reported. Generally, for the models from the continuous perspective product terms are not significant in uniquely accounting for variance in the dependent variables. As noted in the discussion below, in only two situations were product terms significant.

Influence of independent variables on MVAR

SSR is the dominating influence on of MVAR across all parameter estimates. As SSR increases the log of MVAR decreases. For all parameter estimates except a_1, the variance in the log of MVAR associated with SSR is in the 80%'s. For a_1, SSR's influence is less with 55.4% of the variance being associated with it.

TL is the second largest influence on MVAR for all parameter estimates. As TL increases, the log of MVAR decreases. Except for a_1, percentage variance in the log of MVAR associated with TL ranges from the low to high teens. For a_1, the percentage associated with TL is 38.4.

TC has the least influence on MVAR for all parameter estimates with values for the percentage variance in the log of MVAR accounted for being in the single digits. As TC increases MVAR also increases. TC has no influence on the MVAR for parameters reflecting item difficulty, d and D of MID. This finding is consistent with Bately and Boss's (1993) finding that trait correlations do not impact difficulty estimates.

Influence of independent variables on MRMSE

SSR is also the dominant influence on MRMSE across all the parameter estimates. The percentage variance in the log of MRMSE accounted for by SSR ranges from the mid 70's to the mid 80's except for a_1. For a_1, SSR accounts for 57.3% of the variance. As SSR increases, MRMSE decreases.

For d and D of MID, TL is the second largest influence on MRMSE accounting for 10.4% and 18.3%, respectively, of the variance in the log of MRMSE. For the discrimination parameters (a1, a2 and MDISC) TL accounted for between 10% and 11% of the variance which is less than that accounted for by TC. For a_1 TL accounts for 36.8% of the variance in the log of MRMSE. As TL increases MRMSE decreases.

TC is the second largest influence on MRMSE for a1, a2 and MDISC accounting for 12.8%, 15.1% and 15.2% of the variance in the log of MRMSE. For d, TC accounts for just 3.1% of the variance and TC has no influence on the variance for D of MID. For a_1 of MID, TC accounted for 6.9% of the variance in the log of MRMSE. As TC increases, MRMSE also increases.

Influence of independent variable on MBIAS

The MBIAS of the discrimination parameter estimates (a1, a2 and MDISC) is positive for all conditions indicating that these discrimination estimates were consistently larger than the true parameters. This consistent positive MBIAS could be the result of eliminating replications with inadmissible solutions from the analyses. Because solutions with negative discrimination values are considered inadmissible, a lower limit of zero is placed on discrimination estimates used in the analyses. Therefore, the minimum values for MBIAS for discrimination estimates ranges from -1.5 to -.5 depending on value of the true parameter. At the same time, there is no limit on how large MBIAS for discrimination estimates can be. However, this pattern of positive MBIAS for the discrimination parameters is also seen within conditions that have no inadmissible replications. This suggests that the positive MBIAS found for discrimination estimates is not just a result of eliminating inadmissible solutions but is reflective of true positive bias in discrimination estimates.

The magnitude of the MBIAS for the item difficulty parameters, d and D of MID, and for a_1 of MID are less predictable than for the discrimination parameters. This is seen in the lower R^2's found as well as in the erratic nature of the data even in the "plain" regions. Also the direction of the MBIAS for a_1 of MID appears to be random with no terms influencing the probability of the bias being positive or negative.

SSR is the dominate influence in the magnitude of the MBIAS for the difficulty parameters, d and D of MID, accounting for 47.4% and 52.6% of the variance in the log of the absolute values of MBIAS respectively. As SSR increases MBIAS decreases. SSR is the only term which influences the probability of the direction of the MBIAS for d and D of MID. As SSR increases the probability of MBIAS of d being negative decreases. For D of MID as SSR increases the probability of MBIAS being negative increases. It can be seen that these findings are not contradictory when the nature of d and D of MID is considered. D of MID is a negative function of d. Negative values of d indicate more difficult items than do positive values of d, while positive values of D of MID indicate more difficult items than do negative values of D. So for both d and D of MID, as SSR increases the absolute value of MBIAS decreases and the probability that the item's difficulty will be underestimated increases.

SSR's influence on the MBIAS for the discrimination estimates is less with the percentage variance in the log of MBIAS being in the mid 30%'s. SSR has no influence on the MBIAS of a_1 of MID.

TL has minimal influence on the MBIAS of the discrimination estimates with the percentage variance accounted for in the log of MBIAS being in the low single digits. As TL increases MBIAS decreases. Its influence on the magnitude of MBIAS of d is 7.4% of the variance. The product term between SSR and TL accounts for 11.6 % of the variance for d. As the term increases, MBIAS decreases. TL has no influence on the MBIAS of D of MID. However, TL was the only term to uniquely influence the magnitude of the log of absolute value of MBIAS of a_1 accounting for 54.8% of the variance. However, TL does not help in predicting the direction of the MBIAS of a_1.

TC had the greatest influence on the MBIAS of a1, a2 and MDISC with the percentage variance in the log of MBIAS accounted for being in the high 50's. The product term between SSR and TC accounts for an additional 1.7% of the variance for a1. The regression coefficient for this term is positive making it difficult to interpret. As SSR and TC increase, MBIAS increases. This would indicate that large values of SSR would lead to greater MBIAS. However, this does not fit with the other findings for SSR in this study. Considering the small value of variance accounted for by this term and considering that it is not significant for a2, it is possible that this finding is a type II error. TC has no influence on the MBIAS of d, D or a_1 of MID.

Locating minimum acceptable thresholds for SSR and TL

The “mountain/plain” nature of the relationship of SSR and TL with MVAR, MRMSE, and MBIAS for many of the parameter estimates raises the question of when does one cross from the “mountain” into the “plain.” As mentioned earlier this is an important consideration for the test developer seeking to get quality parameter estimates with the smallest sample size ratio and shortest test length possible.

The answer to this question is complicated by two factors. The first factor is that SSR, TL and TC all influence the quality of the estimations. An SSR that produces an acceptable level of MVAR at one TL may not produce acceptable levels at a lower TL. This interaction dynamic is seen in the SSR x TL product term being significant in many of the models fitted to the data from the continuous perspective. This interactive nature prevents the creation of a “rule of thumb” of a minimum acceptable SSR, TL or sample size. For example, a TL of 44 and a SSR of 20 gives a sample size of 880. At TC = 0.0 this sample yielded a MVAR for a1 of 0.015136 (Table Appendix B, Table B1). A TL of 23 and SSR of 50 yields a sample size of 1,150 yet yields a much higher MVAR for a1 at 5.23.

The second factor that complicates the answer to this question is that there is no standard of what are acceptable levels of MVAR, MRMSE or MBIAS in MIRT parameter estimates. For example, for a TL of 44 and a TC of 0.0, is a MVAR of 0.034 for an average a1 value of 1.0 acceptable or is it more appropriate to achieve a MVAR of 0.001. Based on the results of this study, this improvement would require going from a sample size of 440 to one of 1,320. Another issue is whether or not the acceptable level of MVAR for an estimate is different dependent on the average level of that estimate. For example, is a MVAR of 0.03 acceptable for both an average a1 of 0.5 and for an average a1 of 1.5?

To provide guidelines to the test developer as to when parameter estimates are in the "plain" region, two sets of tables were composed. (These tables are also provided for the combinations of parameter estimates and dependent variables not having the "mountain/plain" characteristics as they may still prove to be helpful tools.) The first set of tables (Appendix F) divides the mean of MVAR, MRMSE and MBIAS found in this study by the average parameter value used to generate the data for that condition. This provides a measure of the relative size of MVAR, MRMSE and MBIAS, when compared to the true parameter value. Smaller values of these ratios indicate higher quality parameter estimates. For d and D of MID the averages used are of the absolute values of the parameters. In these cases the absolute values are used because the simulated tests were designed with a balanced number of negative and positive values for d and D. Therefore the averages would be zero.

The second set of tables, found in Appendix G, has the correlation between the estimates and the parameters used to create the data for each condition. This gives an additional measure of the quality of the estimates obtained with higher correlations indicating more higher quality parameter estimates.

By considering the information on these tables, the test developer can make judgments on what combinations of SSR and TL are needed to provide estimates with appropriate levels of MVAR, MRMSE and MBIAS, for their particular applications. Measures subject to close scrutiny, such as selection tools, may require lower levels of MVAR, MRMSE and MBIAS than measures that are used for less "high stakes" applications.

Table 21. Percentage variance in the dependent variables associated with each independent variable (rounded to nearest tenth)

		Data Set	SSR	TL	TC
MVAR					
	a1	Reduced	81.7	13.1	5.1
	a2	Reduced	80.7	13.2	6.1
	d	Reduced	87.2	11.0	0.0
	MDISC	Reduced	85.3	12.4	2.0
	D of MID	Full	81.4	19.6	0.0
	a_1 of MID	Full	55.4	38.4	6.9
MRMSE					
	a1	Reduced	75.5	11.0	12.8
	a2	Reduced	73.3	11.0	15.1
	d	Reduced	86.3	10.4	3.1
	MDISC	Reduced	74.3	9.2	15.2
	D of MID	Full	82.7	18.3	0.0
	a_1 of MID	Full	57.3	36.8	6.9
MBIAS					
	a1	Reduced	33.71	2.9	59.5
	a2	Reduced	34.1	3.1	59.8
	d[1]	Reduced	47.4	7.4	0.0
	MDISC	Reduced	36.9	3.0	57.6
	D of MID	Reduced	52.6	0.0	0.0
	a_1 of MID	Full	0.0	54.8	0.0

[1]Note: The SSR x TL accounted for 11.6% for the variance.

CHAPTER 5

RESULTS AND DISCUSSION FOR DIFFICULTY AND DISCRIMINATION LEVELS

It is possible that the influence of SSR, TL and TC on MVAR, MRMSE and MBIAS are different for items with different difficulty levels or for items with different discrimination levels. To consider this, hierarchical regressions were run for items at the seven item difficulty levels and the three item discrimination levels used in this study. For these analyses, the same model/data set combinations that yielded the best fit for the full test were used. For example, for a1 the reduced data set was used and the terms SSR, TL, TC, SSR x TL, SSR x TC, TL x TC, SSR x TL x TC, 1/SSR, 1/TL and SSR^2 were included in the regression model. The terms were entered in the same steps as they were for the regressions on the entire test. The unique variances in MVAR, MRMSE and MBIAS accounted for by each term in the step in which it was entered for the various difficulty levels are recorded in Appendix H, Tables H1 – H18. The results for the various discrimination levels are in Appendix I, Tables I1 – I18.

Alf and Graf (1999) presented a method of establishing confidence limits for the difference between two squared correlation coefficients. However, they commented that it should be used cautiously with sample sizes between 60 and 200. With larger samples it can be readily used. In this study the sample sizes for the actual regression analyses from which the type II squared semi-partial correlations were obtained were either 30 (for the reduced data sets) or 48 (for the full data sets). These small sample sizes make applying Alf and Graf's approach inappropriate here. Therefore, visual observation was used to look for possible differences in the variance accounted for by terms across the difficulty and discrimination levels.

Discussion of results of regressions at difficulty levels

MVAR

The influence of SSR, TL and TC on MVAR across difficult levels appears to be fairly stable. There is perhaps a slight drop in the influence in SSR on the discrimination estimates and on D of MID for smaller absolute values of item difficulty and a corresponding increase in the influence of TC. However, the changes are so small as to have no practical implications.

MRMSE

There appears to be a significant drop in the influence of SSR on the MRMSE of a1, a2, MDISC and D of MID as the absolute values of item difficulty decrease. For example, for a2 the SSR accounts for 71.7% of the variance in the log of MRMSE for items at d equal to minus one. However, at d equals zero the variance accounted for is 54.1%. There is a corresponding increase in the variance accounted for by TC which goes from 7.1% at d equals minus one to 23% at d equals 0. These changes could be large enough to warrant consideration in test development. If a test was designed to have a large number of items in the middle difficulty range, a larger SSR might be needed to achieve quality discrimination estimates. The fluctuation in the influence of the terms for the MRMSE of d and a_1 appears small and of little practical concern.

MBIAS

For the discrimination estimates the pattern of influence on MBIAS is similar to that of MRMSE. The influence of SSR appears to drop with smaller absolute values of item difficulty while the influence of TC increases. For the difficulty estimates, d and D of MID, and for a_1 of MID, the change in term influence across the difficulty levels is sporadic. This highlights the difficulty in modeling the MBIAS of difficulty parameters. The sporadic behavior across difficulty levels calls into question the validity of the findings concerning the influence of the independent variables on MBIAS for the whole test. Therefore, these findings should be viewed with caution.

Discussion of results of regressions at discrimination levels

MVAR

For a1, a2, and d, the influence of SSR on MVAR appears to drop slightly as item discrimination increases. There is a corresponding increase in the influence of TC. However, these fluctuations are too small to be of practical importance. No change in influence pattern was noted for MDISC, D or a_1 of MID.

MRMSE

For a1, a2, and MDISC, the influence of SSR and of TL drops as item discrimination increases. Again, there is a corresponding increase in the influence of TC. The changes appear large enough to merit consideration in test development. This decreased influence of SSR may indicate that if highly discriminating items are used, larger sample sizes will be needed for quality parameter estimates particularly if the traits are correlated. This pattern was not apparent for difficulty estimates nor for a_1.

MBIAS

For a1, a2, and MDISC, the influence of SSR on MBIAS appears to drop sharply as item discrimination increases. The influence of TL drops slightly. The influence of TC increases sharply. These changes are large enough to merit consideration in test development. If highly discriminating items are used, larger sample sizes may be needed to reduce MBIAS to an acceptable level particularly if the traits are correlated. No clear fluctuation in influence patterns were noted for d, D or a_1 of MID.

CHAPTER 6

LIMITATIONS AND FUTURE RESEARCH

Limitations

There are several limitations of this study that should be noted. The relatively large changes in the test lengths used limited the study's ability to pinpoint at what test length the quality of parameter estimates dropped. This study puts it somewhere between a length of 23 and 44 items which from a practical test development perspective is a fairly large range.

A second limitation is that the study only used two levels of trait correlation. This makes it difficult to see trends in the data that may be caused by changes in trait correlation. The addition of one or two more trait correlation levels both extending the range (perhaps to 0.5) and filling in between the tested values would help clarify possible patterns.

A third limitation is that the study did not include non-normal distributions of traits. Thus the impact of skewed or truncated trait distributions could not be examined.

A fourth limitation is that the difference in influence of SSR, TL and TC on MVAR, MRMSE and MBIAS at different difficulty and discrimination levels could not be tested statistically. This leaves conclusions in this area up to subjective judgments. Although some patterns appeared to emerge they could not be objectively tested.

Future Research

Additional simulations are needed to help pinpoint the edge of the "plain" region where the quality of parameter estimates drops. Studies with several test lengths between 23 and 44 would be helpful in clarifying the edge of the plain. Also the "ridges" of the "mountains" need to be explored to understand how quickly parameter estimates improve as sample size ratio or test length increases while the other factor remains at a low level.

The smoothing effect of D and alpha on the "mountain/plain" pattern warrants further investigation. An important question to answer is whether D and alpha still accurately describe an item if the a1, a2 or d estimates used to calculate them came from the "mountain" region.

Work also needs to be done to understand the shifts in term influence at different item difficulty and discrimination levels. Studies designed to allow for the statistical testing of trends across item difficulty and discrimination levels need to be conducted.

Studies also need to be designed to examine the quality of estimated trait levels of individual subjects under various data conditions.

Research on the sample size ratios and test lengths needed to obtain quality parameter estimates needs to be extended to studies with actual subjects. Comparing parameter estimates obtained from smaller samples with those obtained from larger samples could provide insight into where the borders of the "plain" region are in actual test development situations. Comparing parameter estimates of long test with estimates obtained from a subset of questions from the long test could also help clarify the "plain" region.

An important issue that needs to be explored in applying MIRT to actual testing situations is the number and type of basis items required to orient the trait axes. Consideration needs to be given to designing items that measure only one of the traits of interest at a time. Studies need to be conducted to determine how many basis items are needed to orient the axes in an actual scale development situation.

.

REFERENCES

Ackerman, T. A. (1994). Using multidimensional item response theory to understand what items and tests are measuring. *Applied Measurement in Education, 7,* 255-278.

Alf, E. F. Jr., & Graf, R. G. (1999). Asymptotic confidence limits for the difference between two squared multiple correlations: A simplified approach. *Psychological Methods, 4, 70 – 75.*

Baker, F. B., & Al-Karni, A. (1991). A comparison of two procedures of computing irt equating coefficients. *Journal of Educational Measurement, 28,* 147-162.

Batley, R., & Boss, M. W. (1993). The effects of parameter estimation of correlated dimensions and a distribution-restricted trait in a multidimensional item response model. *Applied Psychological Measurement, 17,* 131-141.

Berry, W. D. & Feldman, S. (1985). Multiple regression in practice. Sage University Paper series on Quantitative Applications in the Social Sciences, 50 (Series No. 07-050). Newbury Park and London: Sage Publications.

Cohen, A. S., Kane, M. T., & Kim Seock-Ho (2001). The precision of simulation study results. *Applied Psychological Measurement, 25,* 136 - 145.

Corey, D.M.., Dunlap, W. P., & Burke, M. J. (1998). Averaging correlations: Expected values and bias in combined Pearson rs and Fisher's z transformations. *The Journal of General Psychology, 125,* 245-261.

De Ayala, R. J., & Sava-Bolesta, M. (1999). Item parameter recovery for the nominal response model. *Applied Psychological Measurement, 23,* 3-19.

Embretson, S. E., & Reise, S. P. (2000). *Item response theory for psychologists.* Mahwah, NJ: Erlbaum.

Fraser, C. (2003). *NOHARM: A Computer Program for Fitting Both Unidimensional and Multidimensional Normal Ogive Models of Latent Trait Theory.*

Glejser, H. (1969). A new test for heteroskedasticity. *Journal of the American Statistical Association, 64,* 316 – 323.

Hambleton, R. K., Swaminathan, H., & Rogers, H. J. (1991). *Fundamentals of Item Response Theory.* Newbury Park, NY: SAGE.

Harwell, M. R. (1997). Analyzing the results of monte carlo studies in item response theory. *Educational and Psychological Measurement, 57,* 266-279.

Harwell, M., Stone, C. A., Hsu, T., & Kirisci, L. (1996). Monte carlo studies in item response theory. *Applied Psychological Measurement, 20,* 101-125.

Hirsch, T. M. (1989). Multidimensional equating. *Journal of Educational Measurement, 26,* 337-349.

Hoskens, M., & De Boeck, P. (2001). Multidimensional componential item response theory models for polytomous items. *Applied Psychological Measurement, 25,* 19-37.

Hulin C. L., Lissak R. I., & Drasgow, F (1982). Recovery of two- and three-parameter logistic item characteristic curves: A monte carlo study. *Applied Psychological Measurement, 6,* 249-260.

Kirisci, L., Hsu, T., & Yu, L. (2001). Robustness of item parameter estimation programs to assumptions of unidimensionality and normality. *Applied Psychological Measurement, 25,* 146-162.

Li, Y. H., & Lissitz, R. W. (2000). An evaluation of the accuracy of multidimensional irt linking. *Applied Psychological Measurement, 24,* 115-138.

Luecht, R. M. (1996). Multidimensional computerized adaptive testing in a certification or licensure context. *Applied Psychological Measurement, 20,* 389- 404.

Maydeu-Olivares, A. (2001). Multidimensional item response theory modeling of binary data: Large sample properties of NOHARM estimates. *Journal of Educational and Behavioral Statistics, 26,* 51-71.

McDonald, R. P. (1997). Normal-ogive multidimensional model. In W. J. van der Linden & R. K. Hambleton (Eds.), *Handbook of modern item response theory.* New York: Springer.

McDonald, R. P. (2000). A basis for multidimensional item response theory. *Applied Psychological Measurement, 24,* 99- 114.

Muraki, E. (2000). RESGEN: Item Response Generation (Version 4.0) [Computer software and manual]. Princeton, NJ: ETS.

Pedhazer, E. J. (1997). *Multiple Regression in Behavioral Research.* 3rd ed. Fort Worth, TX: Harcourt Brace.

Reckase, M. D. (1985). The difficulty of test items that measure more than one ability. *Applied Psychological Measurement, 9,* 401-412.

Reckase, M. D. (1997). The past and future of multidimensional item response theory. *Applied Psychological Measurement, 21,* 25-36.

Reckase, M. D., & McKinley, R. L. (1991). The discriminating power of items that measure more than one dimension. *Applied Psychological Measurement, 15,* 361-373.

Robie, C., Zickar M. J., & Schmit, M. J. (2001). Measurement equivalence between applicant and incumbent groups: A IRT analysis of personality scales. *Human Performance, 14,* 187-207.

Shapiro, S. S., & Wilk, M. B., (1965). An analysis of variance test for normality (Complete Samples). *Biometrika, 52,* p. 591 – 611.

Shapiro, S. S., Wilk, M. B., & Chen, H. J. (1968). A comparative study of various tests for normality. *Journal of the American Statistical Association, 63,* p.1343-1372.

Steinberg, L., & Thissen, D. (1995). Item response theory in personality research. In P. E. Shrout & S. T. Fiske (Eds.), *Personality research, methods, and theory: A festschrift honoring Donald W. Fiske* (pp. 161-181). Hillsdale, NJ: Erlbaum.

Stone, C. A. (1992). Recovery of marginal maximum likelihood estimates in the two-parameter logistic response model: An evaluation of MULTILOG. *Applied Psychological Measurement, 16,* 1-16.

Wollack, J. A., Bolt, D. M., Cohen, A. S., & Lee, Y. (2002). Recovery of item parameters in the nominal response model: A comparison of marginal maximum likelihood estimation and markov chain monte carlo estimation. *Applied Psychological Measurement, 26,* 339-352.

Yen, W. M. (1987). A comparison of the efficiency and accuracy of bilog and logist. *Psychometrika, 52,* 275 – 291.

APPENDICES

APPENDIX A

VARIANCE IN DEPENDENT VARIABLES

AT VARIOUS LEVELS OF INDEPENDENT VARIABLES

Table A1. Variance in MVAR, MRMSE, and MBIAS of a1 and a2 across SSR, TL and TC

	MVAR		MRMSE		MBIAS	
	a1	a2	a1	a2	a1	a2
SSR						
5	15588639416	3088046606	300.6334980	255.4243112	27.5798533	21.1681498
10	120676.44	586003.70	4.0189315	4.1455577	0.2043472	0.1875776
20	12127.84	2085.48	0.6549785	0.1346154	0.0300550	0.0056164
30	1858.34	5.2395845	0.1723245	0.0064079	0.0078969	0.000324601
50	77.3137755	0.000081435	0.0166251	0.0014242	0.000832452	0.000140792
100	0.000021502	0.000017662	0.000687975	0.000626734	0.000107544	0.000085379
TL						
23	10488706777	2084278657	207.2872983	179.4155314	18.7831268	14.6474075
44	3578184.31	1276102.38	3.2900114	2.7832651	0.0843439	0.0792597
65	242.3250722	52.1712395	0.0139769	0.0118400	0.000473533	0.000615829
86	0.000215074	0.000228531	0.0029056	0.0032162	0.000174904	0.000301605
TC						
0.0	11952655.26	6518388.60	19.3114863	16.4959943	1.0554445	0.8034789
0.3	5275570851	1051139548	97.9741059	83.4681835	9.2062170	7.1091767

Table A2. Variance in MVAR, MRMSE, and MBIAS of d across SSR, TL and TC

	MVAR	MRMSE	MBIAS
SSR			
5	169244595	46.1475433	0.2936286
10	1199.16	0.1844294	0.000136719
20	0.3103165	0.0052456	0.000038727
30	0.0097882	0.0010213	0.000022309
50	0.000128732	0.000269263	1.499415E-6
100	6.1067047E-7	0.000090433	2.5686278E-6
TL			
23	114839352	32.6469173	0.1972878
44	119270.83	0.4589662	0.0016108
65	12.1883113	0.0047970	0.000079313
86	0.000036502	0.0012860	0.000034702
TC			
0.0	423052.10	2.0834353	0.0021049
0.3	57811986.07	15.6122767	0.0943013

Table A3. Variance in MVAR, MRMSE, and MBIAS of MDISC across SSR, TL and TC

	MVAR	MRMSE	MBIAS
SSR			
5	33582639007	445.1641236	44.8242570
10	174415.81	0.6155426	0.0409016
20	0.000446640	0.0030259	0.000720620
30	0.000137581	0.0017636	0.000491412
50	0.000036712	0.000914245	0.000322558
100	7.6987273E-6	0.000448466	0.000202567
TL			
23	22604964502	313.8870426	31.0100674
44	9333330.89	6.3034930	0.1765008
65	535.2533027	0.0229025	0.0014947
86	0.000352220	0.0039494	0.000606809
TC			
0.0	19070201.18	9.3536421	0.4986485
0.3	11355151502	156.5609010	15.5522484

Table A4. Variance in MVAR, MRMSE, and MBIAS of D of MID across SSR, TL and TC

	MVAR	MRMSE	MBIAS
SSR			
5	0.000059907	0.000839772	3.9534845E-6
10	0.000016115	0.000464070	7.5796337E-6
20	3.6115826E-6	0.000221252	0.000017231
30	1.5135929E-6	0.000142373	0.000014973
50	5.1450803E-7	0.000077827	1.1876199E-6
100	1.2744082E-7	0.000039346	1.3267675E-6
TL			
23	0.000076726	0.0017604	0.000029958
44	0.000021503	0.000969852	0.000032754
65	9.582647E-6	0.000650561	0.000017786
86	5.2594503E-6	0.000486932	0.000019733
TC			
0.0	0.000030781	0.0010901	0.000024454
0.3	0.000035941	0.0011825	0.000030678

Table A5. Variance in MVAR, MRMSE, and MBIAS of a_1 of MID across SSR, TL and TC

	MVAR	MRMSE	MBIAS
SSR			
5	0.000773103	0.0052051	0.0019631
10	0.000487846	0.0039210	0.000571981
20	0.000194110	0.0023000	0.000050850
30	0.000088704	0.0015487	0.000018380
50	0.000030982	0.000911082	0.000014967
100	7.3022608E-6	0.000443060	0.000011882
TL			
23	0.000770418	0.0060434	0.0012044
44	0.000114291	0.0021714	4.3109878E-6
65	0.000046176	0.0013724	2.0513735E-6
86	0.000021969	0.000933110	4.3322194E-6
TC			
0.0	0.000160385	0.0025854	0.000176840
0.3	0.000521414	0.0050863	0.000705261

APPENDIX B

MEAN VALUES OF DEPENDENT VARIABLES BY CONDITION

Table B1. MVAR of a1 by condition

		TL			
		23	44	65	86
	SSR				
TC = 0.0					
	5	16959.18	42.91985	0.045676	0.031189
	10	733.7877	0.033895	0.020096	0.014413
	20	92.28709	0.015136	0.009266	0.006781
	30	0.030699	0.00968	0.00614	0.004567
	50	5.230606	0.005639	0.003592	0.002576
	100	0.007699	0.002761	0.001717	0.001274
TC = 0.3					
	5	356108.7	6556.504	53.93854	0.04982
	10	766.6658	0.054013	0.031384	0.02101
	20	310.9905	0.023584	0.014485	0.009981
	30	121.9407	0.015214	0.009238	0.00655
	50	25.07758	0.008549	0.005419	0.003892
	100	0.014932	0.004239	0.002641	0.001871

Table B2. MRMSE of a1 by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	21.4348	0.551334	0.213546	0.178466
10	4.407804	0.18471	0.143922	0.125194
20	0.695721	0.127146	0.101223	0.088618
30	0.171319	0.103693	0.082863	0.07418
50	0.179508	0.08011	0.06633	0.059415
100	0.088625	0.059882	0.048549	0.04524
TC = 0.3				
5	48.65006	6.432877	0.485747	0.22711
10	4.592031	0.234502	0.182082	0.156117
20	2.419285	0.161451	0.13104	0.115193
30	1.281332	0.135003	0.108056	0.097761
50	0.44383	0.105188	0.08897	0.083437
100	0.126799	0.082929	0.070028	0.067256

Table B3. MBIAS of a1 by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	5.023968	0.110103	0.047482	0.039509
10	0.962635	0.041189	0.033997	0.035433
20	0.146724	0.033824	0.029261	0.027544
30	0.03852	0.029935	0.023331	0.023051
50	0.044121	0.02204	0.022611	0.02153
100	0.019767	0.020035	0.016889	0.017497
TC = 0.3				
5	14.96386	1.047417	0.100592	0.063448
10	1.077138	0.061564	0.051224	0.051329
20	0.532238	0.049716	0.046933	0.04561
30	0.285002	0.047969	0.040428	0.04117
50	0.109808	0.038024	0.03841	0.040328
100	0.042814	0.037516	0.033606	0.03482

Table B4. MVAR of a2 by condition

	TL			
SSR	23	44	65	86
TC = 0.0				
5	12518.44	7.628871	0.044953	0.030565
10	310.7623	0.032481	0.01986	0.013741
20	15.69045	0.014768	0.009387	0.006623
30	2.195375	0.0095	0.006078	0.004325
50	0.014245	0.005591	0.003579	0.002539
100	0.006921	0.002753	0.001754	0.001237
TC = 0.3				
5	159043	3913.914	25.03487	0.051798
10	2187.653	0.057771	0.033215	0.021993
20	130.4835	0.024134	0.015277	0.010342
30	6.422898	0.01572	0.009713	0.006677
50	0.029539	0.008759	0.005553	0.003869
100	0.013786	0.004202	0.002701	0.001844

Table B5. MRMSE of a2 by condition

	TL 23	TL 44	TL 65	TL 86
SSR				
TC = 0.0				
5	20.01951	0.379174	0.21584	0.179201
10	1.933968	0.182831	0.146644	0.122801
20	0.293459	0.125678	0.102435	0.085509
30	0.195556	0.101534	0.082787	0.070952
50	0.122194	0.078303	0.066339	0.0575
100	0.086072	0.058354	0.050619	0.044234
TC = 0.3				
5	44.74723	5.92214	0.45055	0.234963
10	5.914895	0.242203	0.194021	0.162022
20	1.168369	0.166497	0.139475	0.114565
30	0.312341	0.138286	0.112579	0.097559
50	0.175424	0.106146	0.093261	0.081513
100	0.123418	0.082137	0.074791	0.066279

Table B6. MBIAS of a2 by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	4.42251	0.085214	0.054567	0.046062
10	0.417903	0.042964	0.038635	0.033877
20	0.066692	0.031698	0.028509	0.020498
30	0.042886	0.025977	0.021161	0.019474
50	0.026726	0.018039	0.01919	0.018459
100	0.018088	0.016886	0.017132	0.015925
TC = 0.3				
5	13.12876	1.015857	0.10646	0.073512
10	1.273523	0.070707	0.064963	0.058634
20	0.250284	0.056985	0.054525	0.042382
30	0.074191	0.050766	0.042505	0.040678
50	0.047967	0.040154	0.040733	0.037011
100	0.032555	0.035551	0.035535	0.03294

Table B7. MVAR of d by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	3186.732	0.349225	0.020821	0.01451
10	6.766852	0.01426	0.008989	0.006716
20	0.179231	0.006543	0.004179	0.003151
30	0.011147	0.004241	0.002712	0.002068
50	0.005185	0.002421	0.001635	0.001242
100	0.002265	0.001207	0.000815	0.000595
TC = 0.3				
5	37286.77	1196.386	12.09944	0.019685
10	98.69335	0.019262	0.011287	0.008413
20	1.596242	0.008169	0.00544	0.003873
30	0.284192	0.00529	0.003364	0.002549
50	0.034332	0.002982	0.002059	0.001524
100	0.002782	0.001487	0.000989	0.000749

Table B8. MRMSE of d by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	7.14956	0.205142	0.142385	0.119598
10	0.548495	0.118211	0.094205	0.081516
20	0.136951	0.080049	0.064501	0.056367
30	0.091555	0.064553	0.052301	0.045871
50	0.068714	0.049216	0.040943	0.036122
100	0.047368	0.035327	0.029466	0.025691
TC = 0.3				
5	19.43081	2.427483	0.279459	0.139589
10	1.300112	0.137231	0.107018	0.092749
20	0.273217	0.090816	0.074819	0.064236
30	0.143728	0.073748	0.060042	0.053041
50	0.085362	0.056271	0.047832	0.042522
100	0.054076	0.041448	0.035273	0.031872

Table B9. MBIAS of d by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.21892	-0.01248	-0.00943	-0.01044
10	-0.02615	-0.01084	-0.00645	-0.00423
20	-0.00847	-0.00267	0.003045	0.002924
30	-0.00771	0.002411	0.0026	0.002677
50	0.002531	0.001979	0.002065	0.003671
100	0.002556	0.003546	0.0008	0.00022
TC = 0.3				
5	-1.5059	-0.13972	-0.02737	-0.01305
10	0.015498	-0.01476	-0.00877	-0.00567
20	-0.0138	-0.005	0.001074	0.002841
30	-0.00743	0.002262	0.002078	0.003514
50	0.000116	0.002527	0.002469	0.004232
100	0.002541	0.003897	0.000453	-0.00024

Table B10. MVAR of MDISC by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	21395.44	43.57796	0.060495	0.042
10	0.141032	0.043503	0.026233	0.018776
20	0.044458	0.019157	0.011975	0.00879
30	0.027457	0.012288	0.007904	0.005844
50	0.014874	0.00717	0.004583	0.003278
100	0.007182	0.0034	0.00214	0.001572
TC = 0.3				
5	522445.8	10586.9	80.16062	0.062431
10	1181.29	0.065833	0.036246	0.024715
20	0.070003	0.02563	0.016241	0.011355
30	0.039238	0.016311	0.010136	0.007334
50	0.020585	0.009146	0.005849	0.004161
100	0.009309	0.004272	0.002703	0.001964

Table B11. MRMSE of MDISC by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	15.12986	0.564509	0.252102	0.212859
10	0.345602	0.212692	0.170687	0.1483
20	0.210121	0.146763	0.119943	0.103807
30	0.168724	0.119882	0.097842	0.08726
50	0.127199	0.092467	0.079354	0.071443
100	0.089518	0.070403	0.060347	0.055793
TC = 0.3				
5	61.30859	8.870383	0.621586	0.267461
10	2.43257	0.267655	0.212641	0.185419
20	0.27346	0.184221	0.156953	0.135747
30	0.215985	0.155858	0.128338	0.117722
50	0.166409	0.121972	0.109884	0.1026
100	0.119475	0.100086	0.090817	0.08628

Table B12. MBIAS of MDISC by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	3.508371	0.14709	0.081646	0.067262
10	0.126976	0.066054	0.055861	0.052503
20	0.070693	0.049569	0.043251	0.036156
30	0.056077	0.041839	0.033308	0.032015
50	0.043292	0.030166	0.030887	0.029983
100	0.028029	0.027734	0.02537	0.025195
TC = 0.3				
5	19.42632	1.51807	0.168775	0.110277
10	0.651168	0.108273	0.091903	0.08476
20	0.12066	0.082829	0.076696	0.066333
30	0.097703	0.074936	0.062343	0.061258
50	0.07931	0.058889	0.058772	0.05738
100	0.056055	0.054264	0.051071	0.050374

Table B13. MVAR of D of MID by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.0246	0.012585	0.008483	0.006215
10	0.012071	0.006129	0.003929	0.002967
20	0.005711	0.002902	0.001903	0.001458
30	0.003738	0.001945	0.001273	0.000974
50	0.002176	0.001144	0.000756	0.000581
100	0.001091	0.000567	0.000384	0.00028
TC = 0.3				
5	0.02614	0.014223	0.009505	0.007106
10	0.013421	0.006959	0.004399	0.003308
20	0.006487	0.003265	0.002241	0.001638
30	0.004215	0.002185	0.00145	0.001082
50	0.002488	0.001281	0.000861	0.000652
100	0.001228	0.000642	0.00043	0.00032

Table 14. MRMSE of D of MID by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.149077	0.109053	0.089988	0.077639
10	0.10478	0.076507	0.06161	0.053533
20	0.072634	0.052354	0.042719	0.037503
30	0.058829	0.042831	0.034942	0.030678
50	0.044593	0.032954	0.027004	0.023815
100	0.031644	0.023331	0.01933	0.016597
TC = 0.3				
5	0.153748	0.115272	0.095016	0.082731
10	0.11035	0.081376	0.06513	0.056468
20	0.077191	0.05549	0.04611	0.039654
30	0.06237	0.045417	0.037221	0.032282
50	0.047477	0.034795	0.028796	0.025277
100	0.033479	0.024851	0.020449	0.017725

Table B15. MBIAS of D of MID by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.012974	0.010851	0.008397	0.009126
10	0.010838	0.009361	0.005986	0.004308
20	0.008981	0.003358	-0.00053	-0.00071
30	0.007154	-0.00013	-0.00061	-0.00087
50	-8.9E-05	-0.00066	-0.0004	-0.00209
100	-0.00059	-0.00195	0.000338	0.000617
TC = 0.3				
5	0.014408	0.012276	0.010552	0.010562
10	0.009716	0.011683	0.007473	0.00505
20	0.009149	0.004473	0.000821	-0.00072
30	0.008163	-0.00014	-0.00033	-0.00161
50	0.000771	-0.00098	-0.00091	-0.00267
100	-0.00082	-0.00239	0.00029	0.000462

Table B16. MVAR of a_1 of MID by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.055227	0.020459	0.01322	0.008834
10	0.034073	0.010199	0.006228	0.004266
20	0.017603	0.004907	0.003075	0.002143
30	0.011694	0.003181	0.00202	0.001428
50	0.006703	0.001905	0.001211	0.000859
100	0.003312	0.000987	0.000623	0.000441
TC = 0.3				
5	0.091326	0.036855	0.023373	0.016387
10	0.069759	0.019414	0.012094	0.007886
20	0.043558	0.009603	0.005886	0.003956
30	0.02941	0.006426	0.003914	0.002622
50	0.017432	0.003679	0.002285	0.001603
100	0.008502	0.001885	0.001186	0.000789

Table B17. MRMSE of a_1 by condition

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.229132	0.136594	0.109853	0.090208
10	0.172506	0.096246	0.075708	0.062829
20	0.122822	0.066765	0.053287	0.044622
30	0.100033	0.053986	0.043172	0.036487
50	0.076294	0.041795	0.03357	0.028326
100	0.053523	0.030092	0.024087	0.020426
TC = 0.3				
5	0.307774	0.184432	0.146609	0.122722
10	0.251844	0.132521	0.105023	0.08546
20	0.189751	0.093135	0.073472	0.060448
30	0.155435	0.076108	0.059956	0.049295
50	0.119927	0.057799	0.045812	0.038517
100	0.084497	0.041432	0.033157	0.027399

Table B18. MBIAS of a_1 of MID

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	-0.06417	0.00018	0.000924	0.002878
10	-0.01929	0.000406	0.000495	-0.00063
20	-0.00358	-0.00114	-0.00145	-0.00291
30	-0.00602	-0.00154	-0.00182	-0.00127
50	-0.00708	-0.0016	-0.00257	-0.00116
100	-0.00339	-0.00129	-0.0008	-0.00067
TC = 0.3				
5	-0.11404	0.004856	5.37E-05	0.003735
10	-0.06583	0.002957	0.002666	0.002935
20	-0.02035	0.002802	0.000723	-0.00137
30	-0.0122	0.00096	-0.00109	3.79E-05
50	-0.01068	0.001086	-0.00088	-0.00141
100	-0.01071	-0.00093	-0.00104	-0.00085

APPENDIX C

SUPPLEMENTAL PLOTS

Figure C1. Log of MVAR of a2 plotted with SSR, TL and TC.

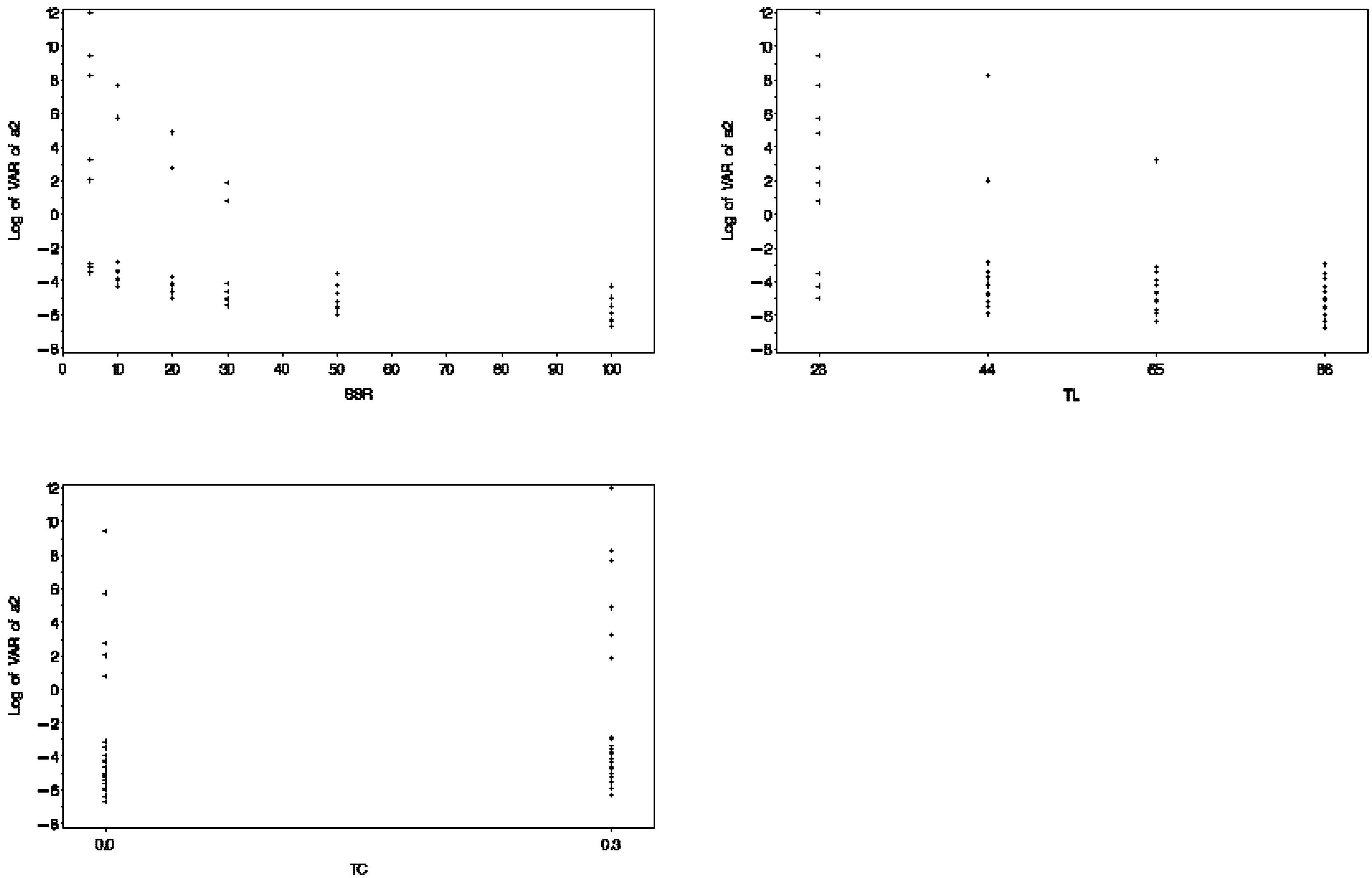
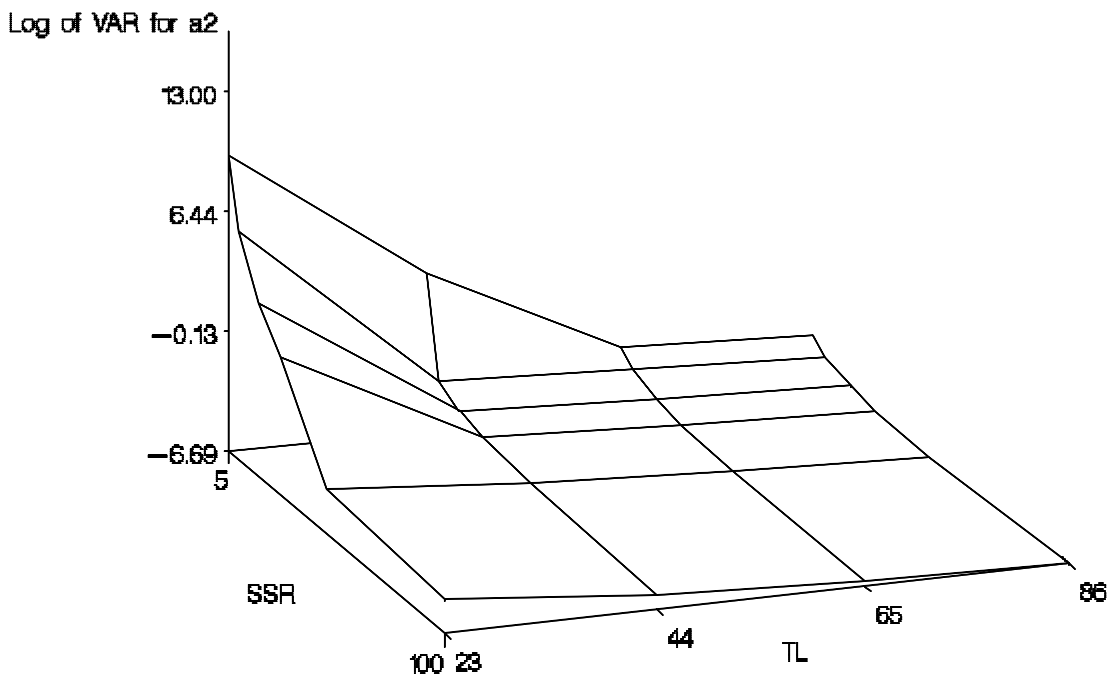


Figure C2. Log of MVAR a2 plotted with SSR and TL.

TC = 0.0



TC = 0.3

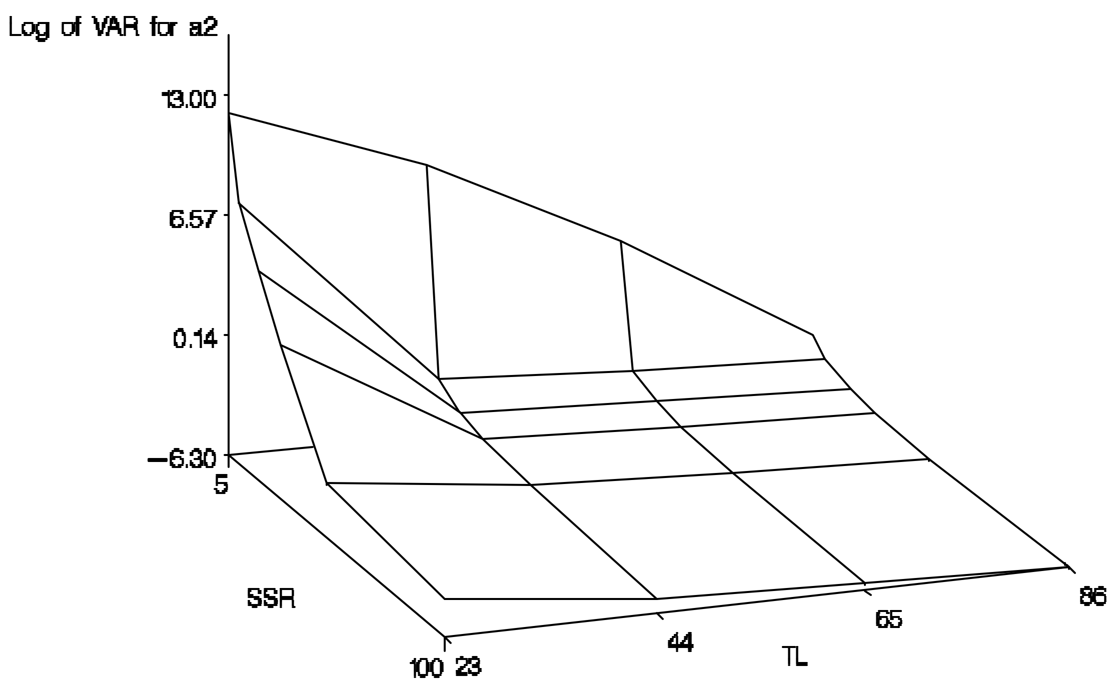


Figure C3. Log of MRMSE of a1 plotted with SSR, TL and TC.

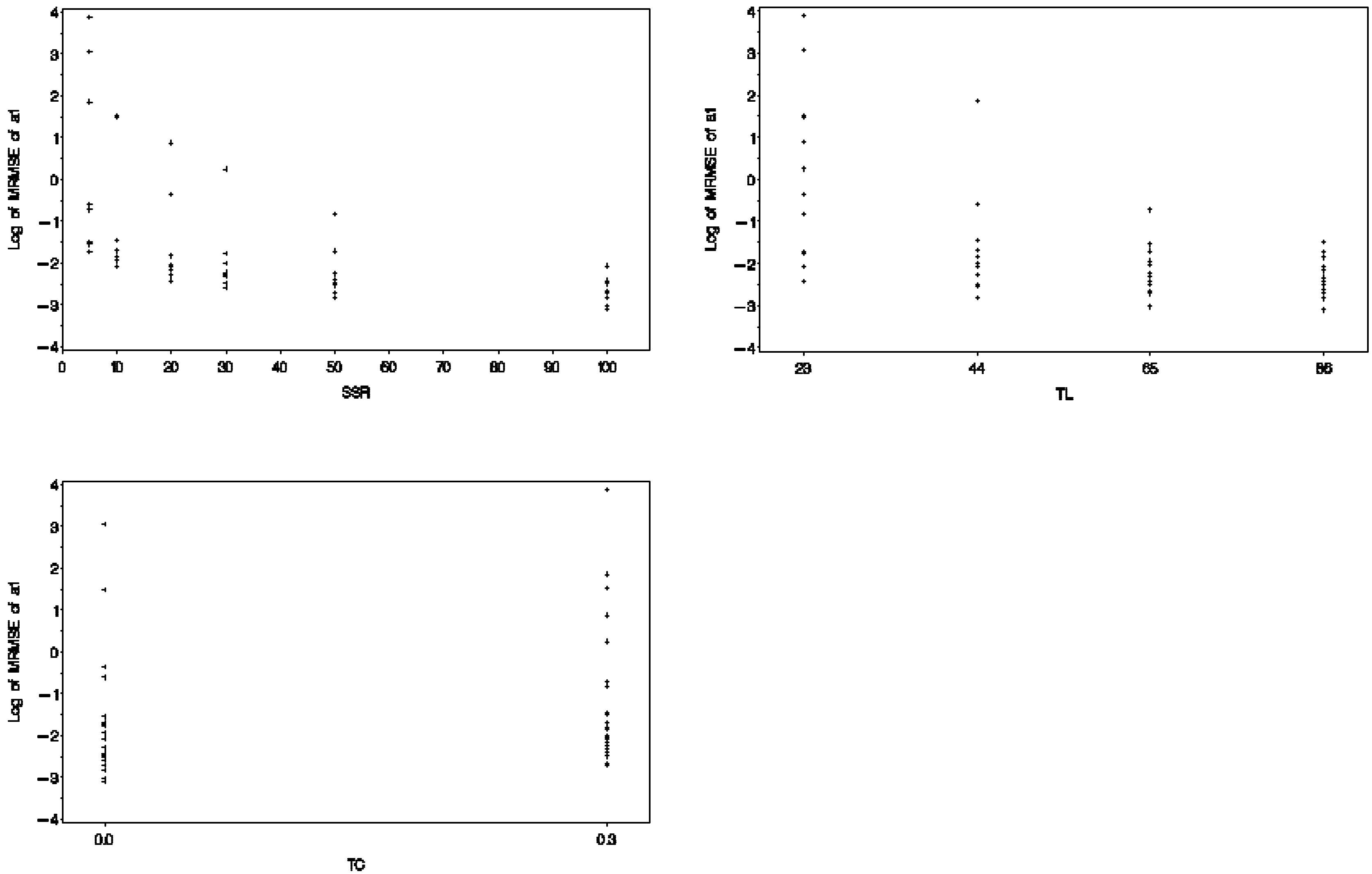


Figure C4. Log of MRMSE of a2 plotted with SSR, TL and TC.

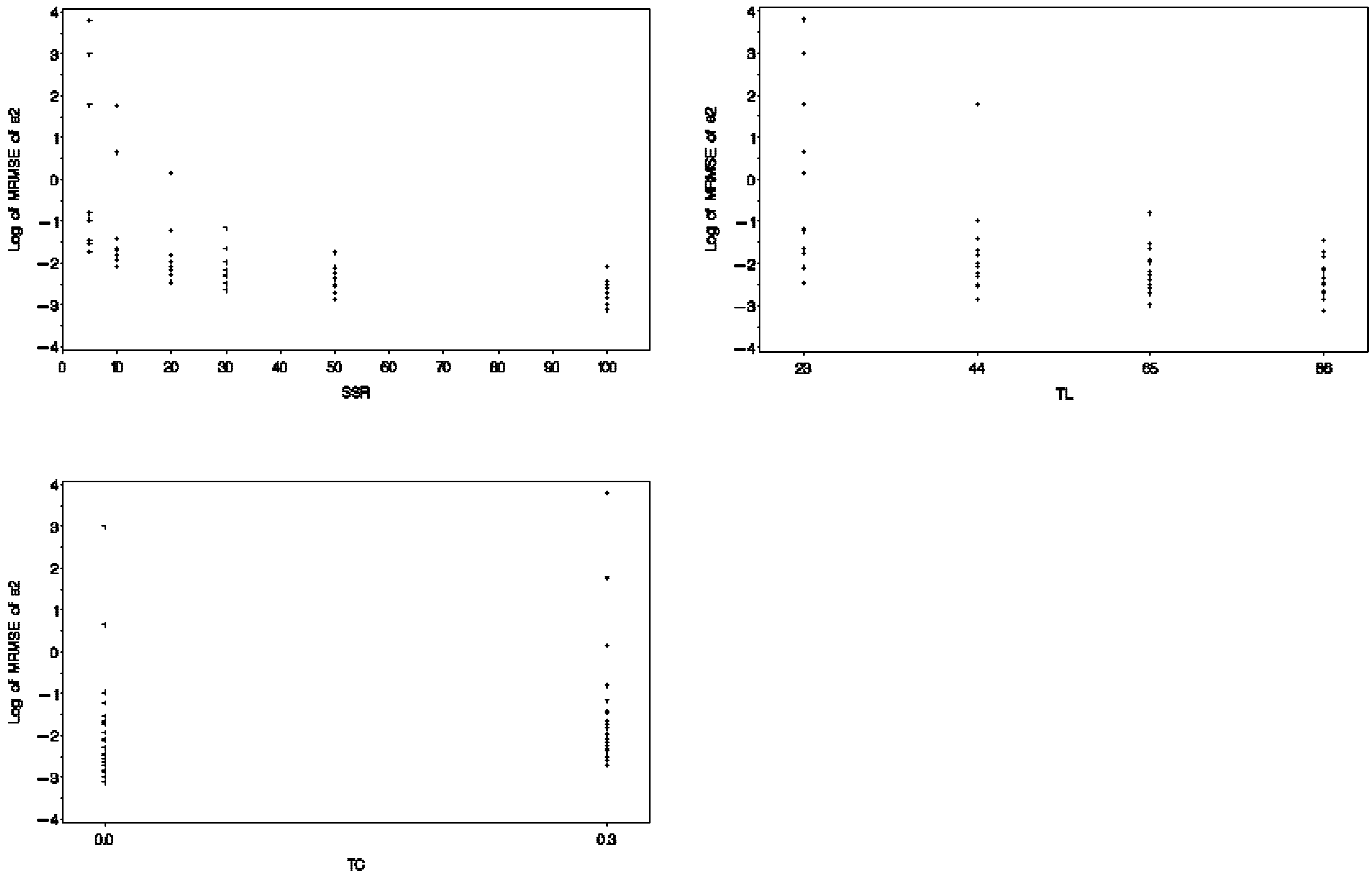
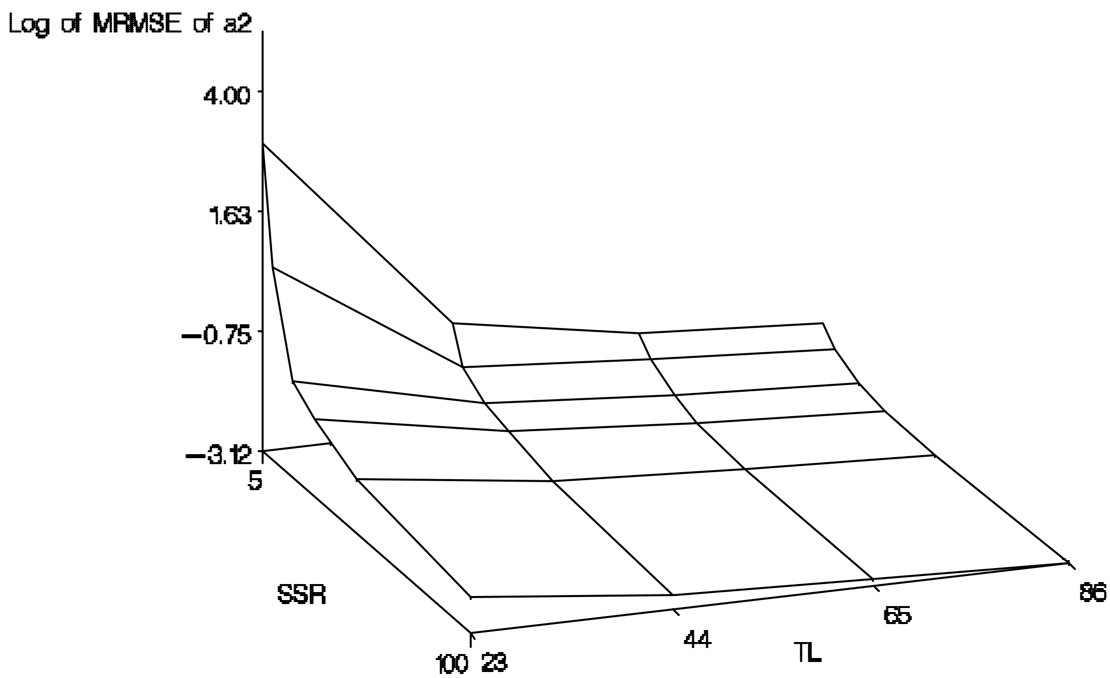


Figure C5 Log of MRMSE a2 plotted with SSR and TL.

TC = 0.0



TC = 0.3

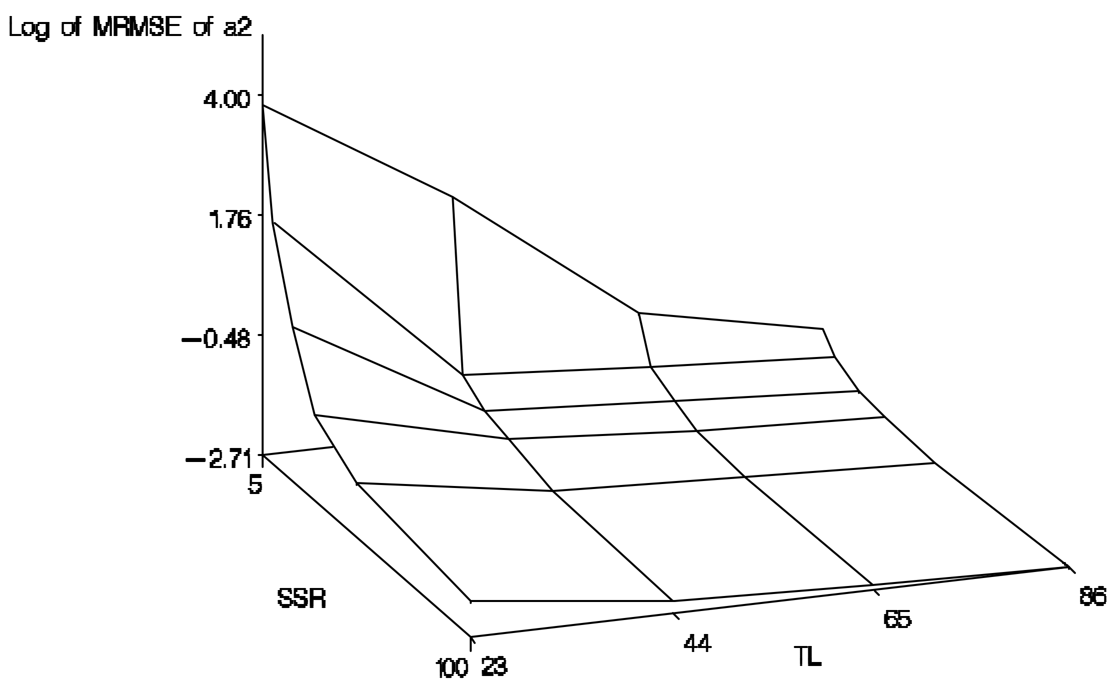
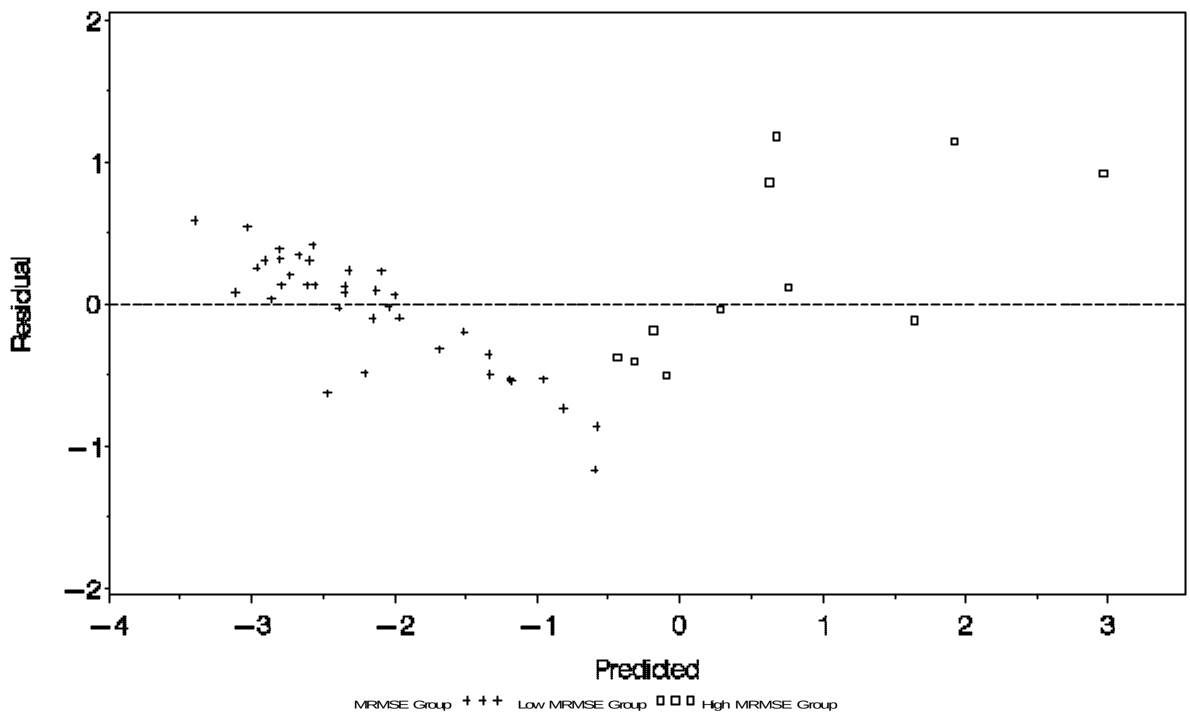


Figure C6. Residual plots of log of MRMSE with predicted values for a1 and a2.

a1



a2

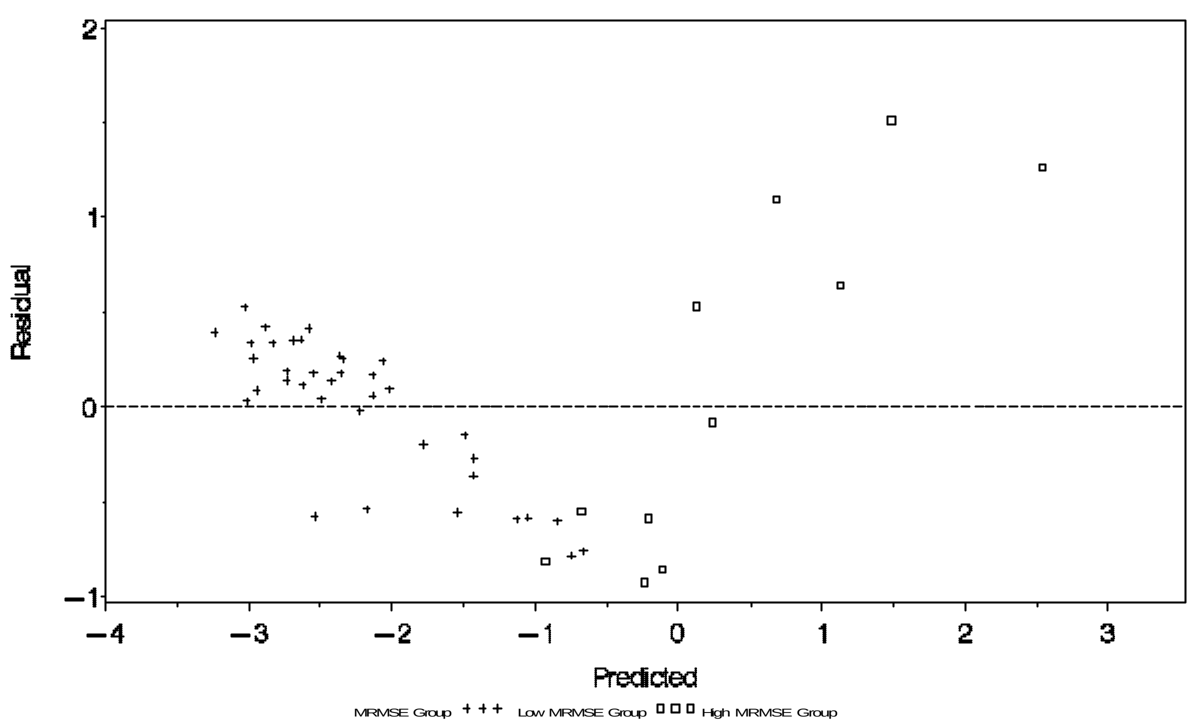
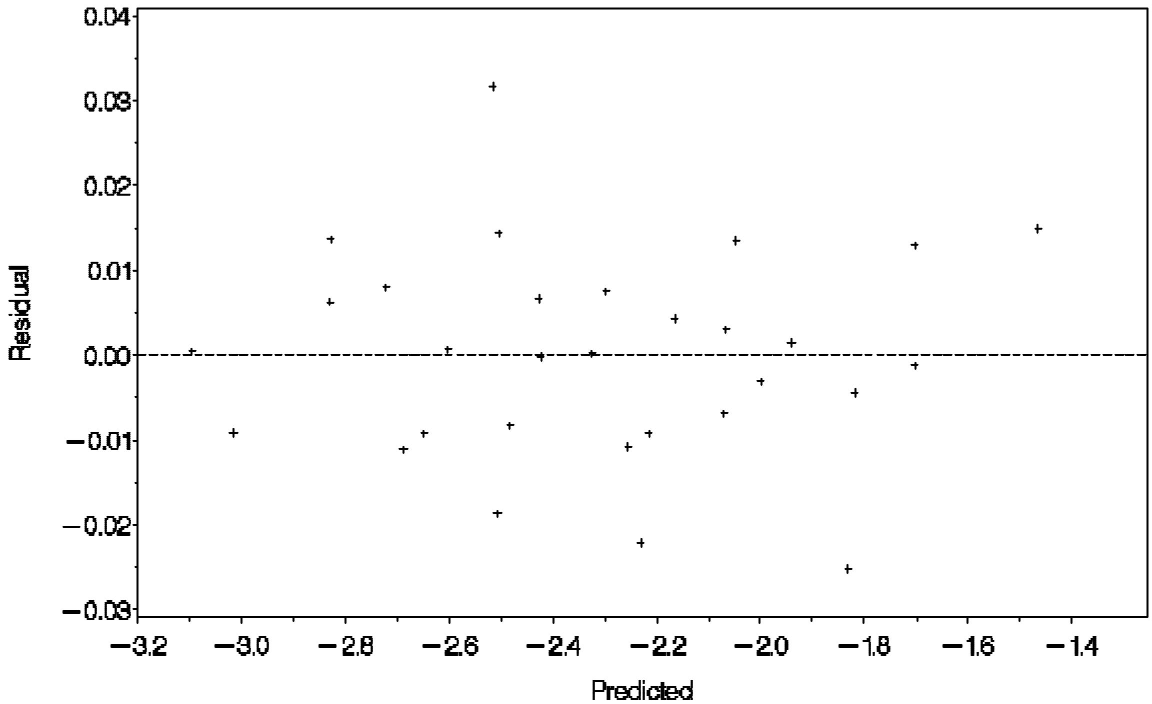


Figure C7. Residual plots of log of MRMSE with predicted values for a1 and a2 for reduced data set.
a1



a2

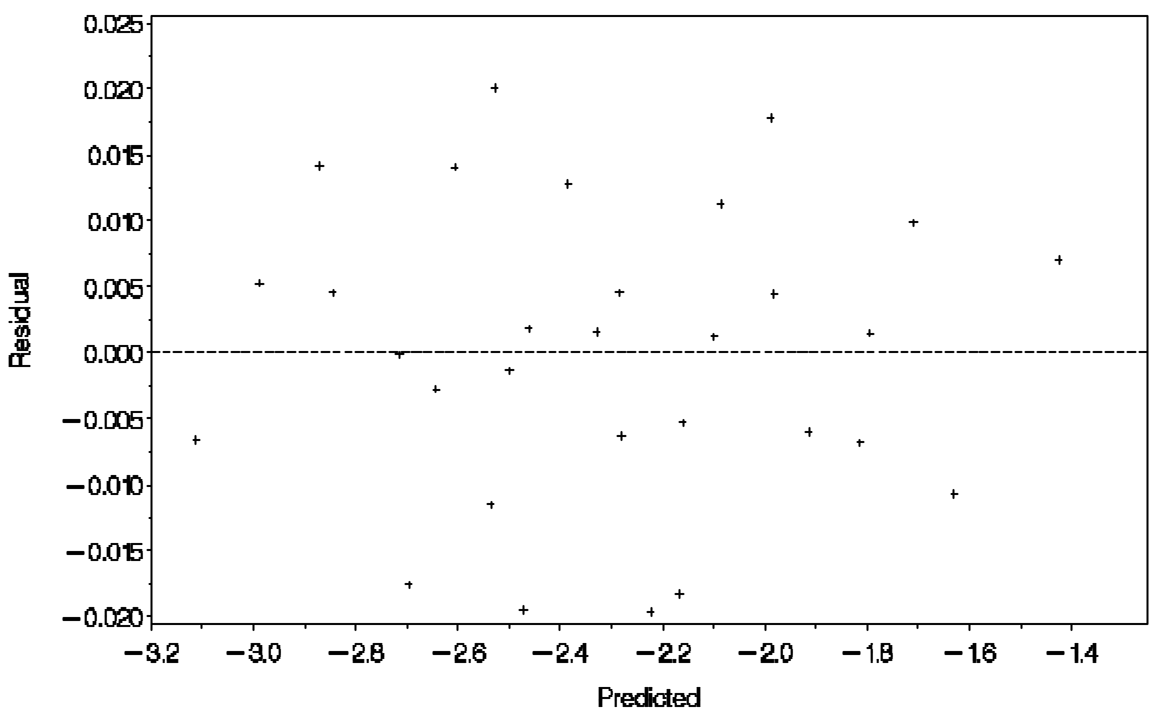


Figure C8. Log of MBIAS of a1 plotted with SSR, TL and TC.

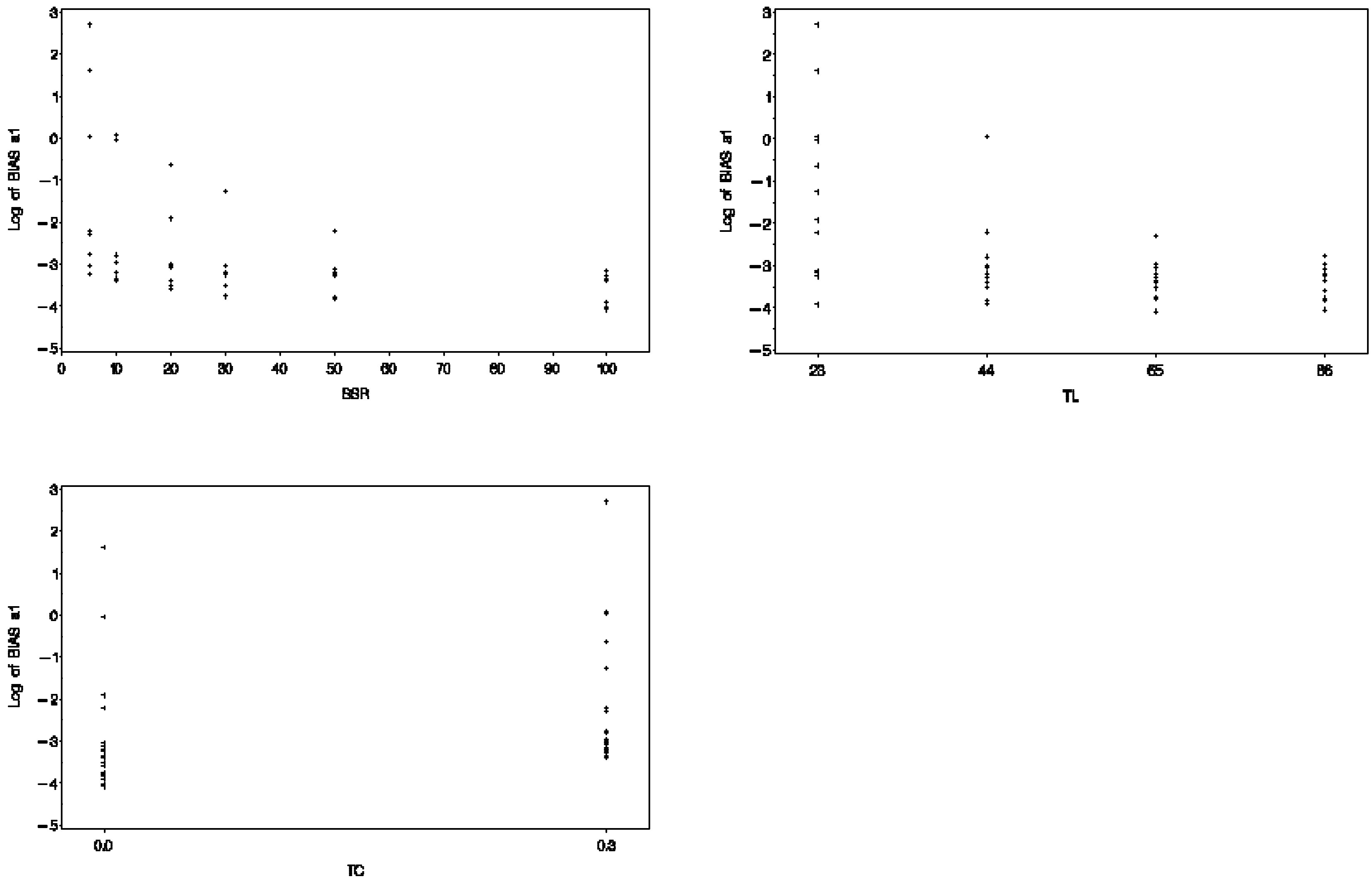
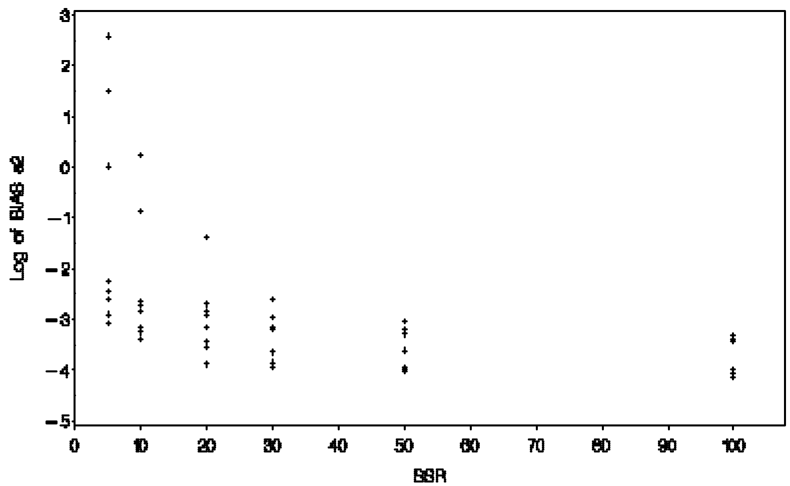
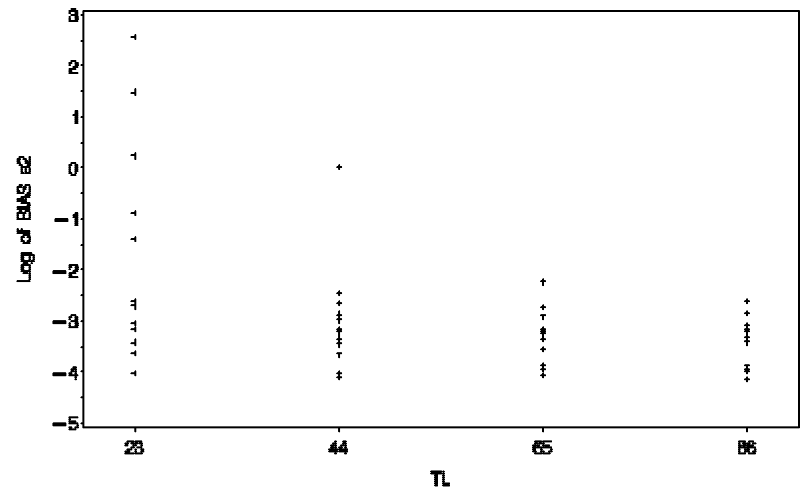


Figure C9. Log MBIAS of a2 plotted with SSR, TL and TC.





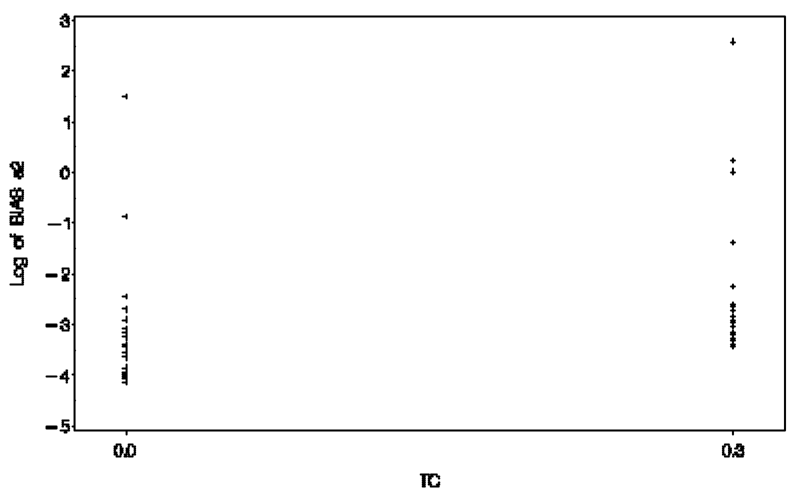
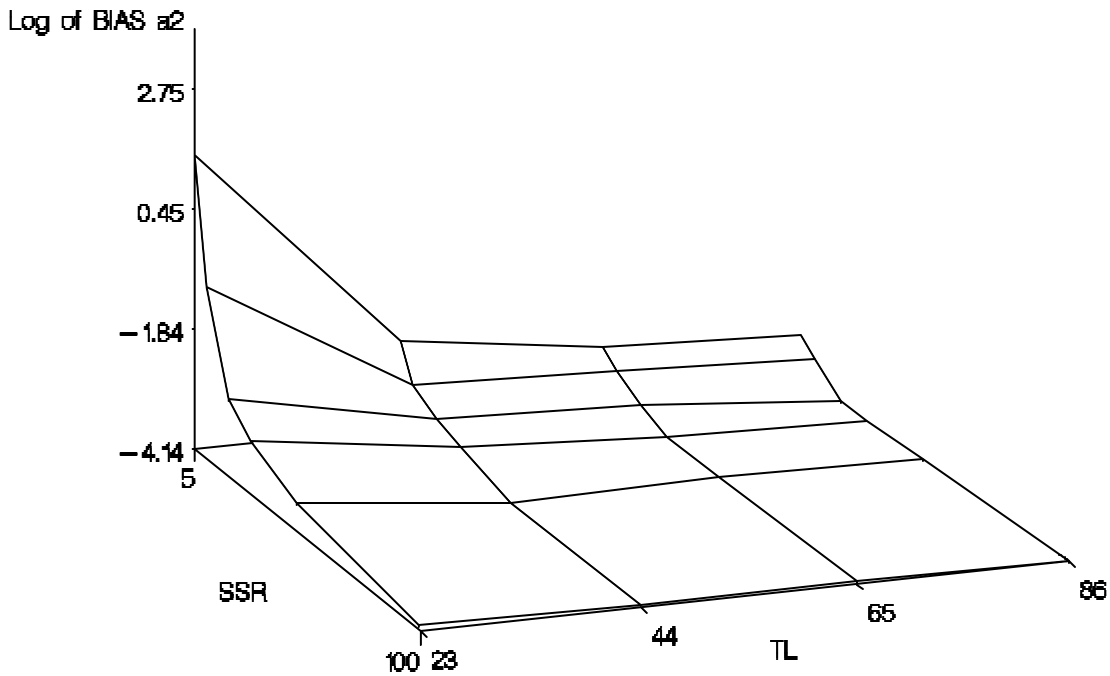


Figure C10. Log of MBIAS a2 plotted with SSR and TL.
TC = 0.0



TC = 0.3

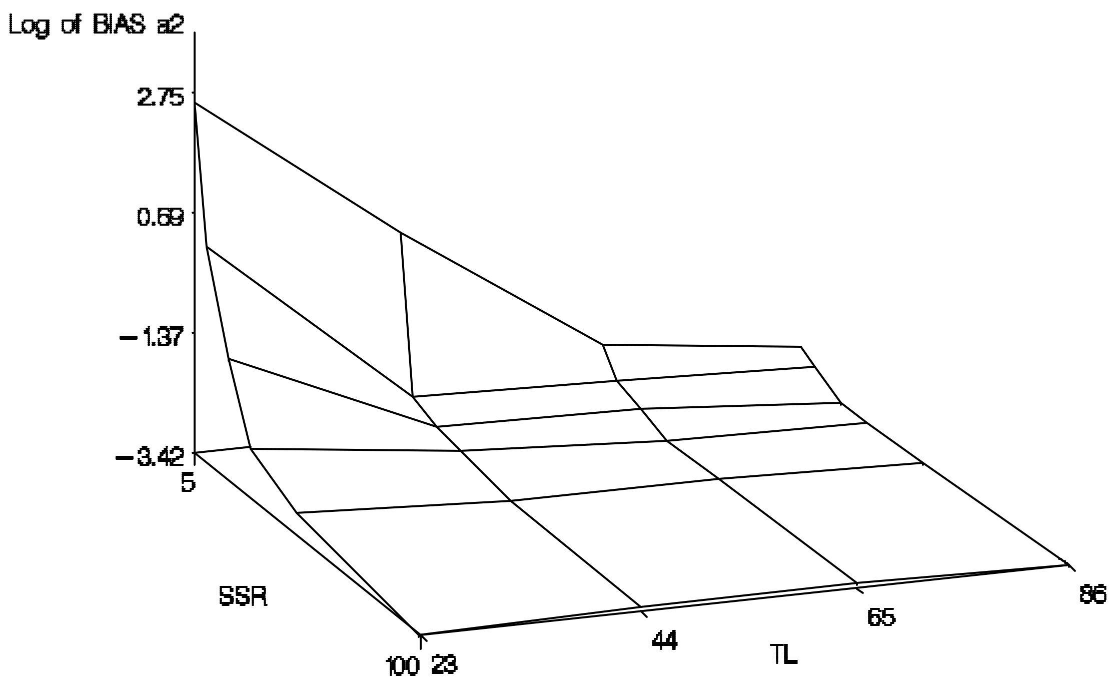
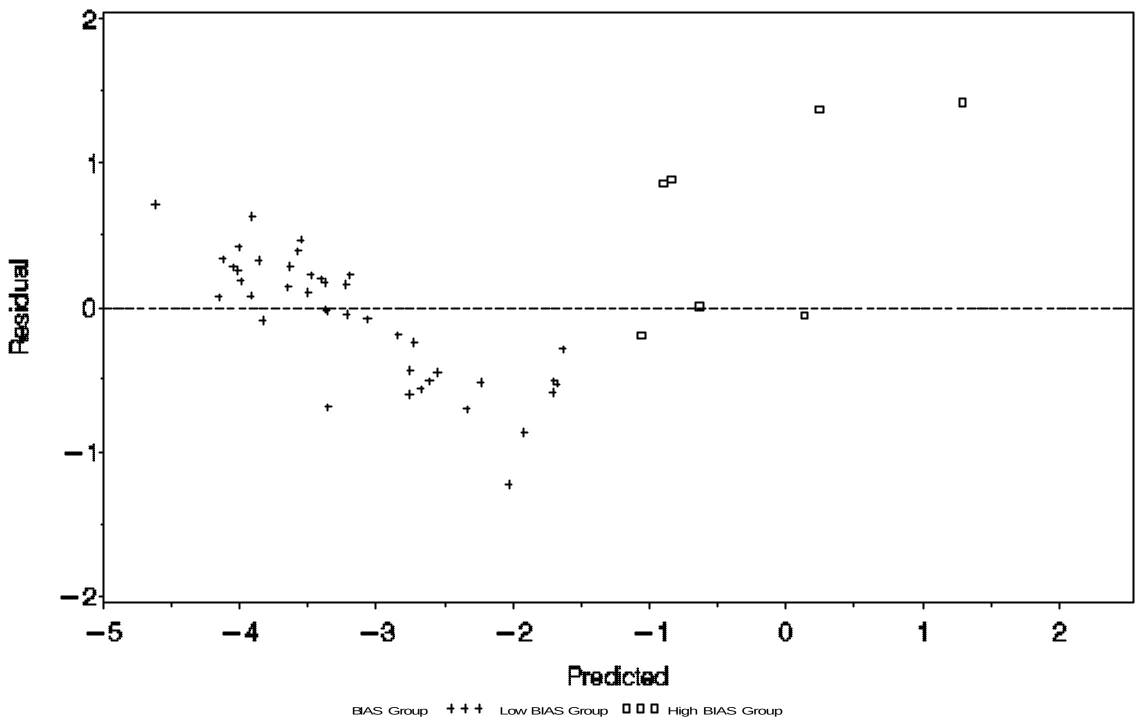


Figure C11. Residual plots of log of MBIAS with predicted values for a1 and a2.
a1



a2

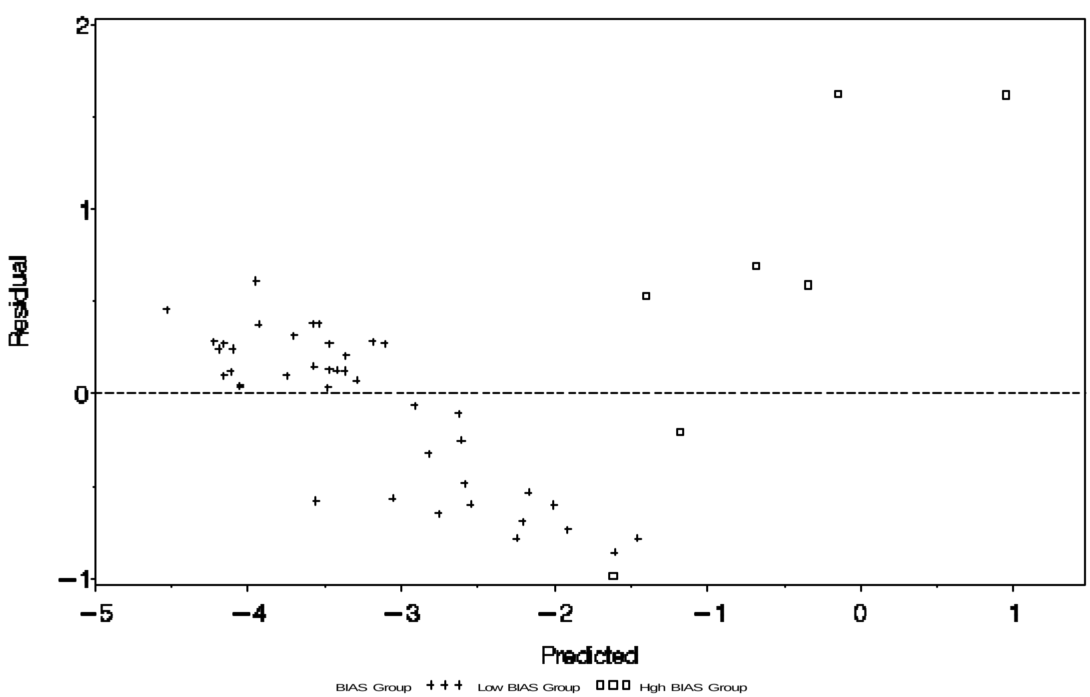
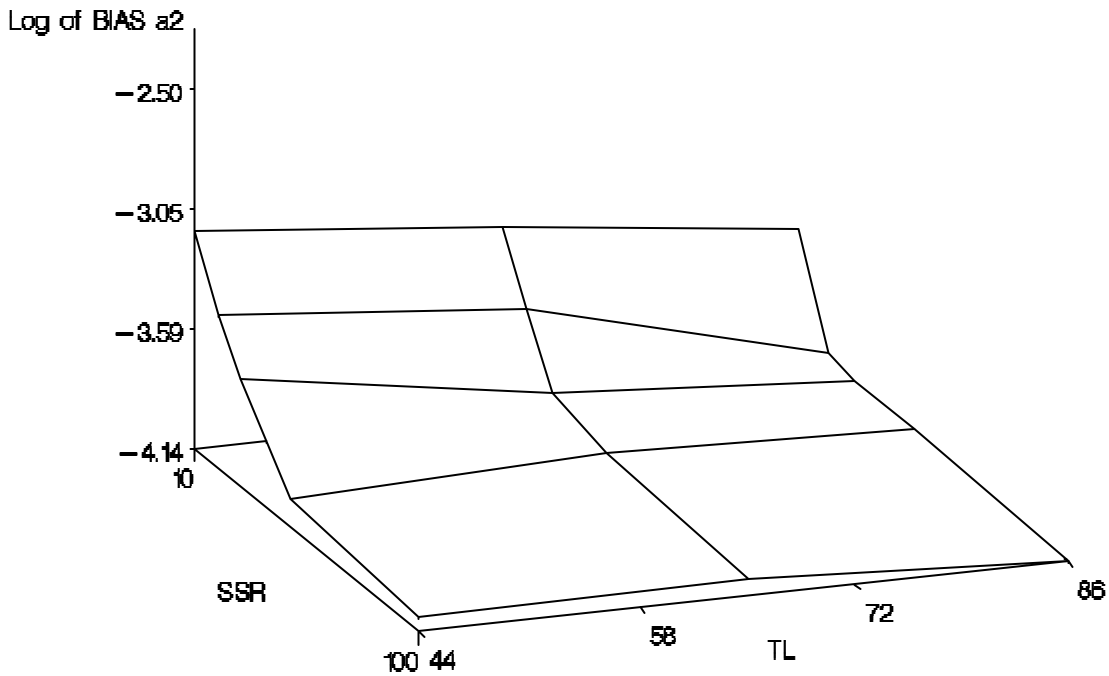


Figure C12. Log of MBIAS a2 plotted with SSR and TL.
TC = 0.0



TC = 0.3

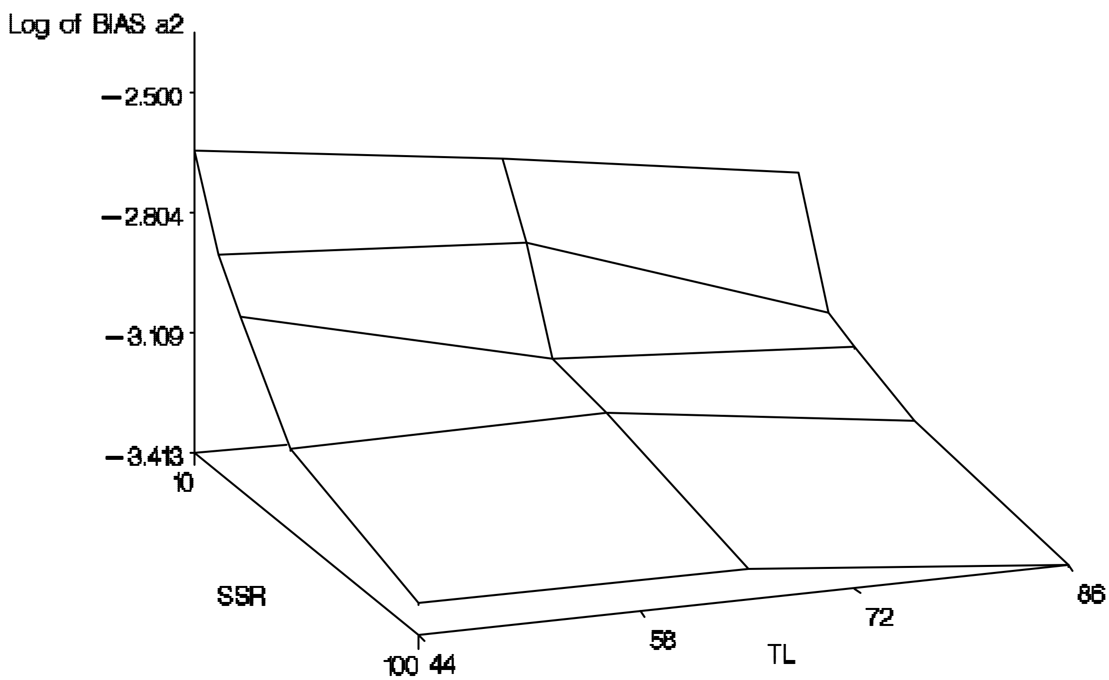
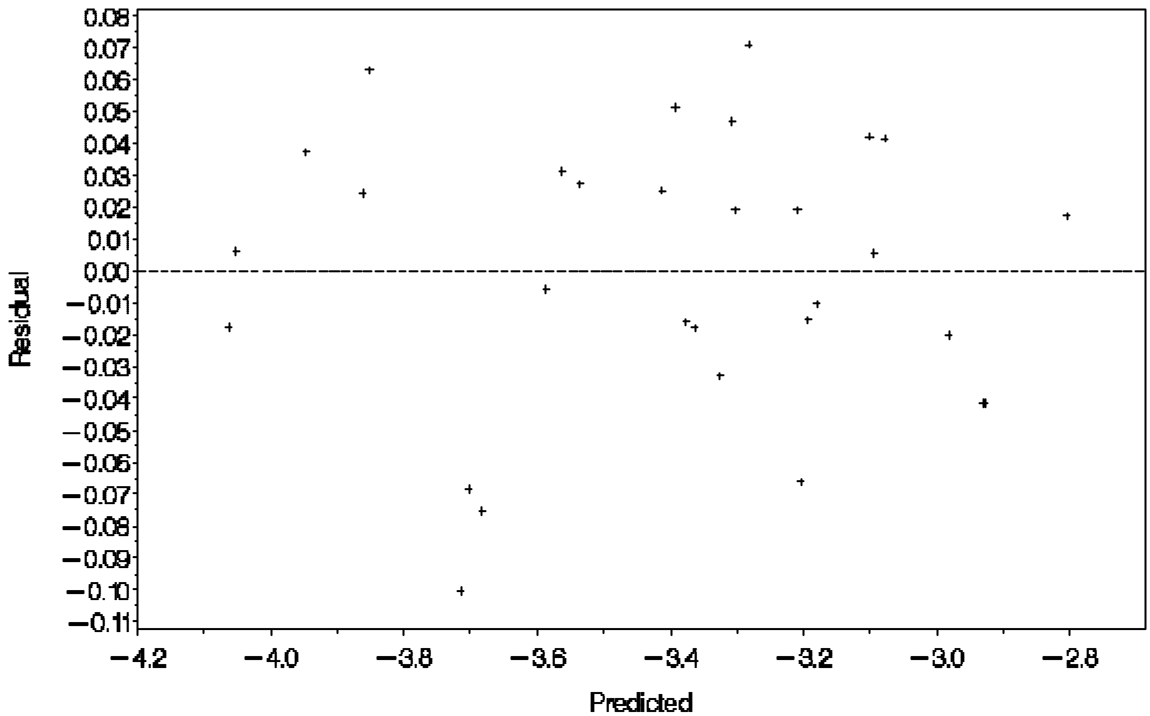


Figure C13. Residual plots of log of MBIAS with predicted values for a1 and a2 for reduced data set.
a1



a2

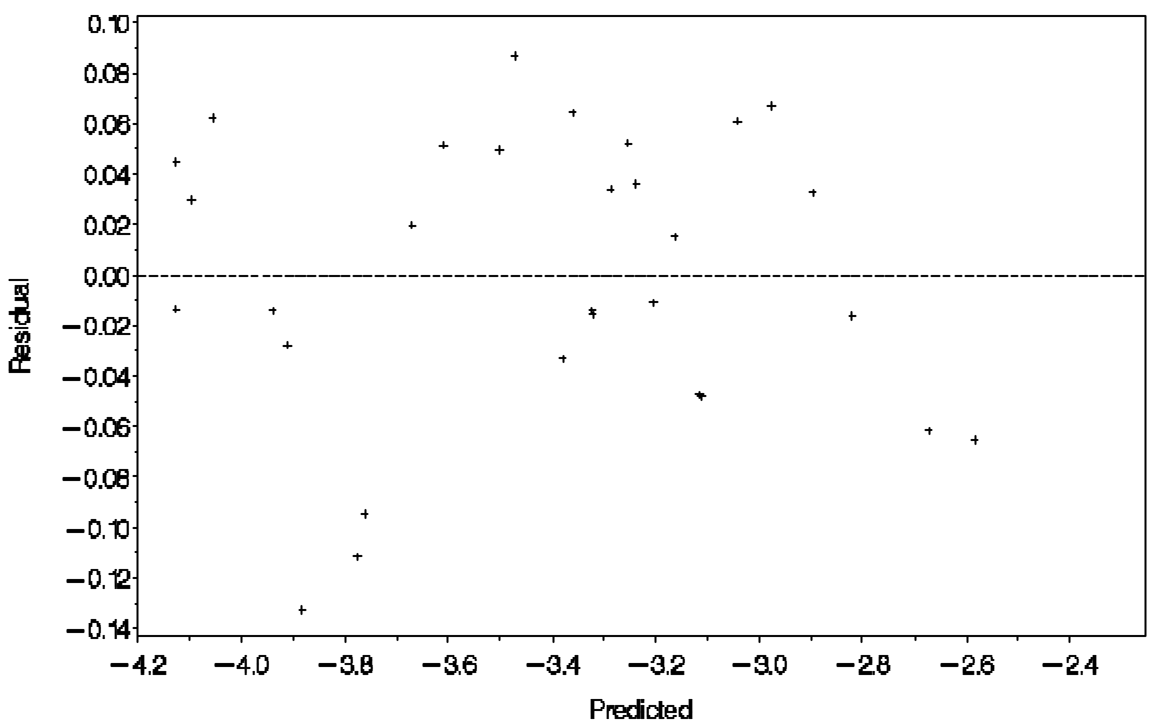


Figure C14. Log of MRMSE of d plotted with SSR, TL and TC.

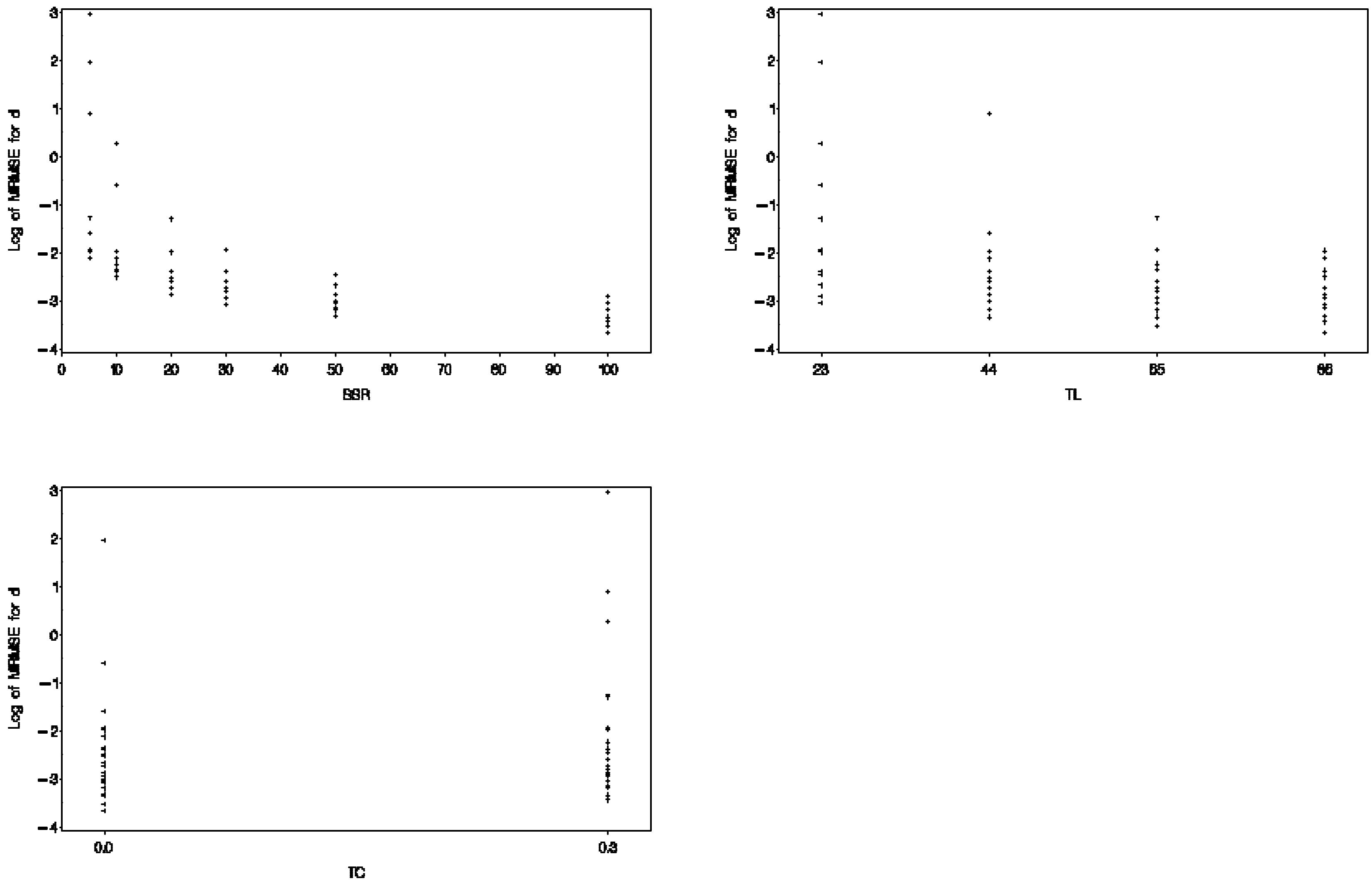


Figure C15. Residual plots of log of MRMSE of d with predicted values.

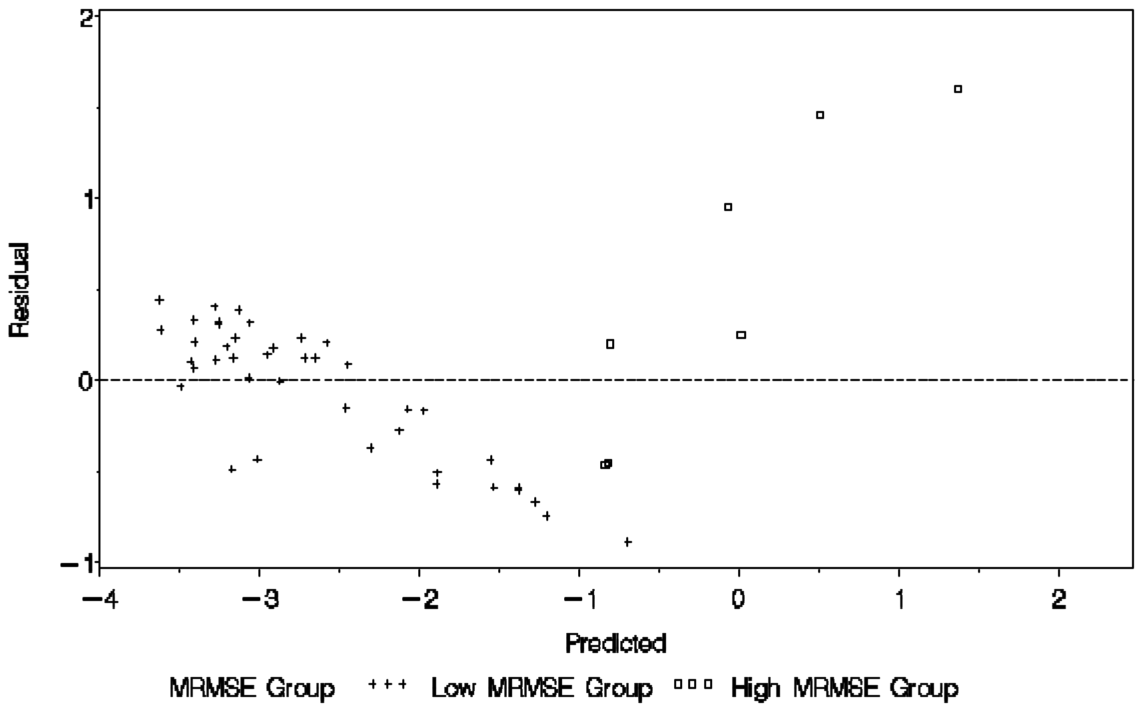


Figure C16. Residual plots of log of MRMSE of d with predicted values for reduced data set.

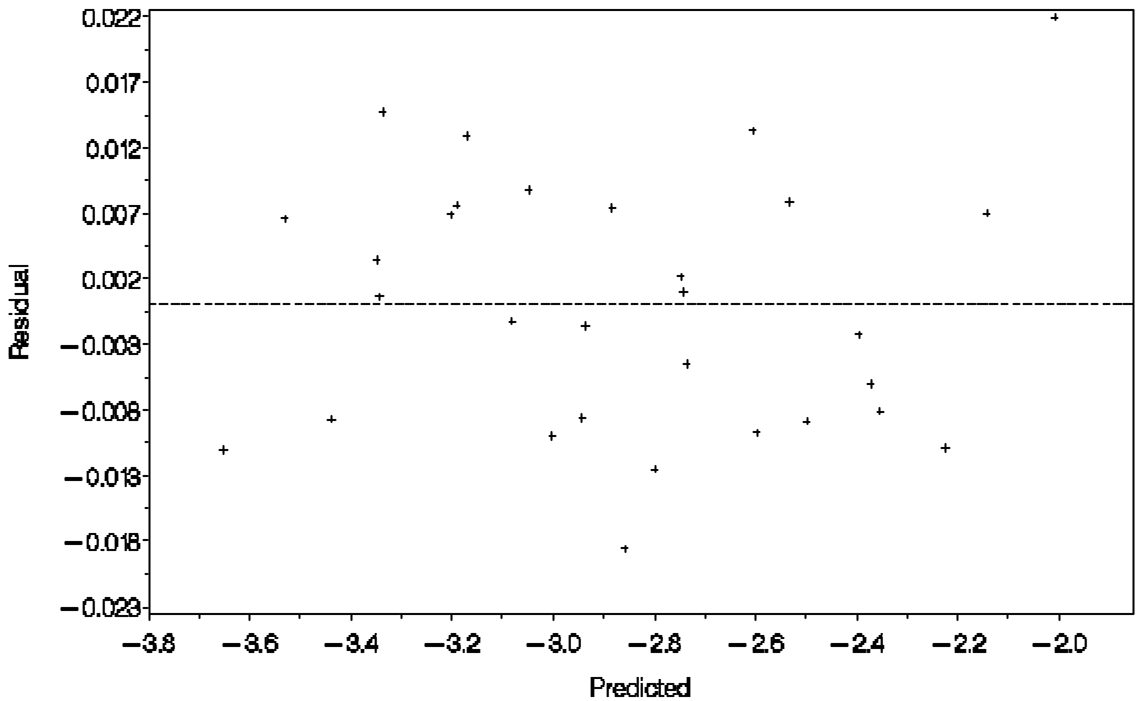


Figure C17. Residual plots of log of MVAR of MDISC with predicted values for reduced data set.

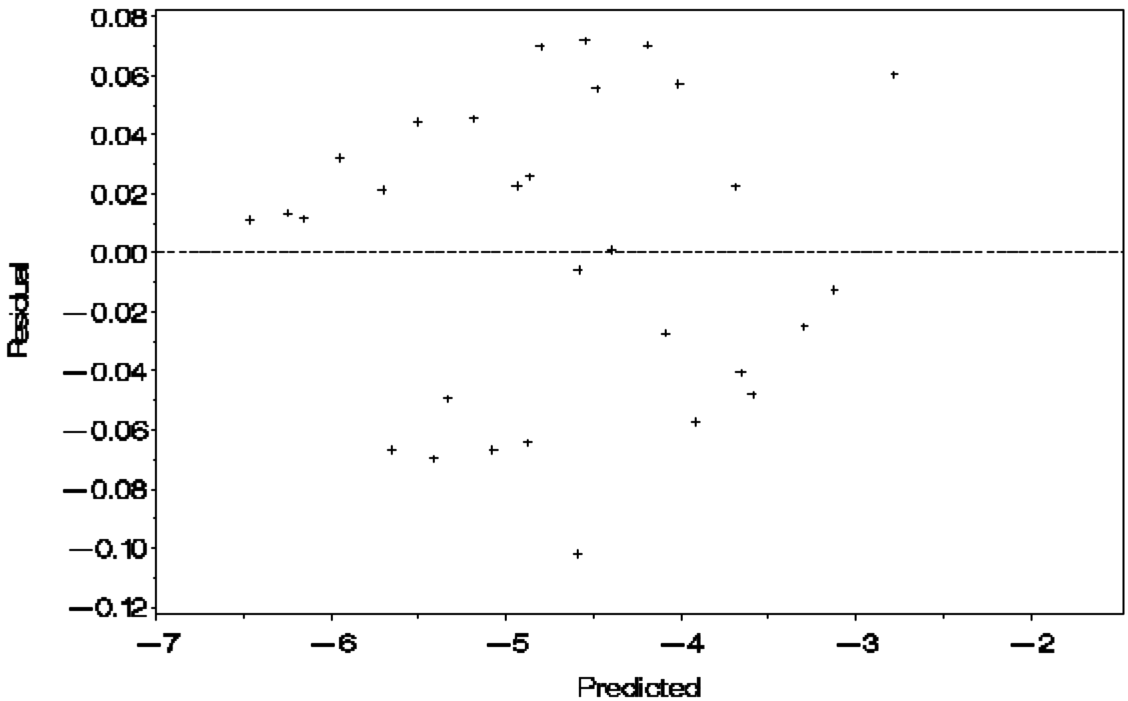


Figure C18. Log of MRMSE of MDISC plotted with SSR, TL and TC.

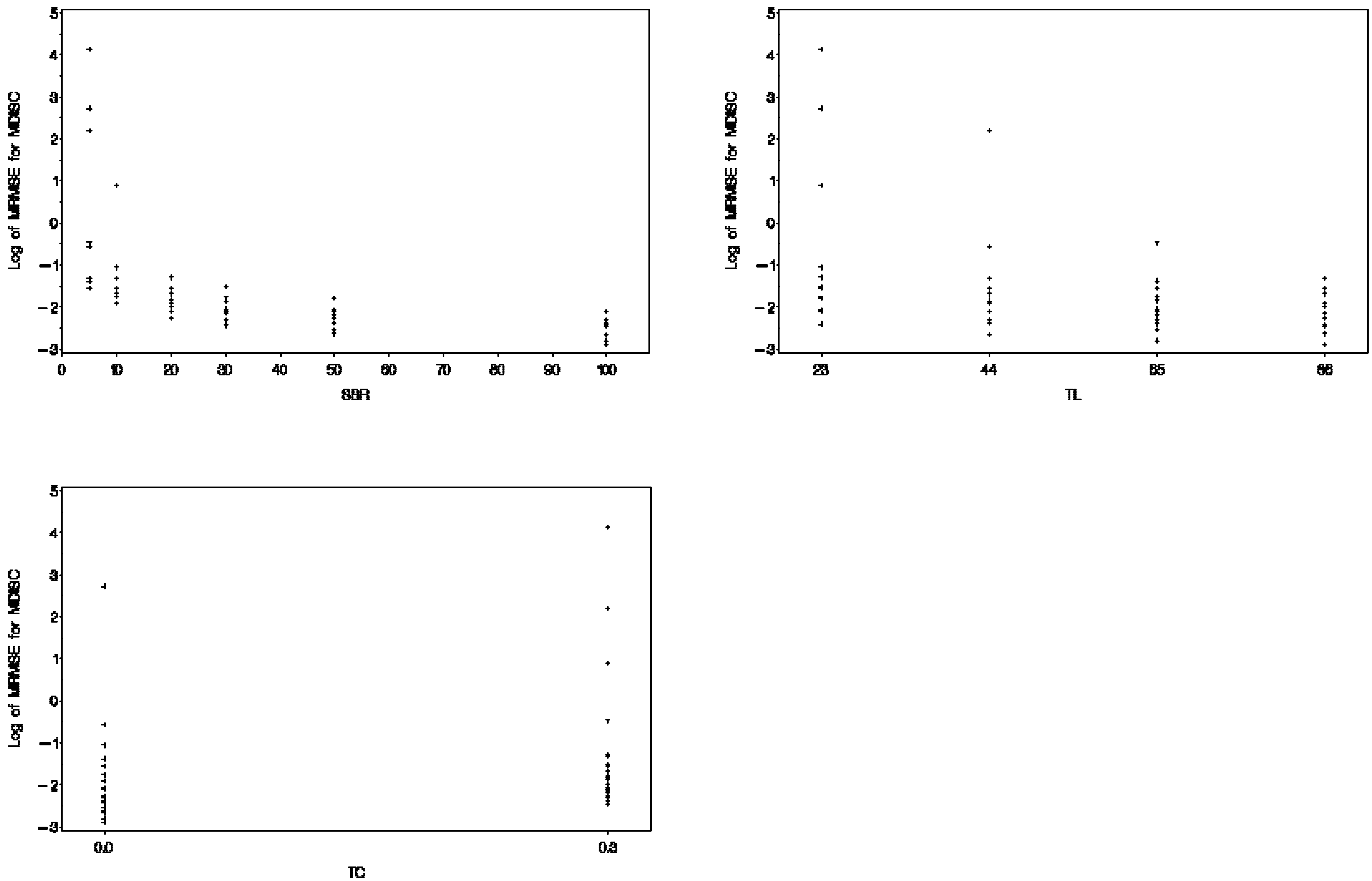


Figure C19. Residual plots of log of MRMSE of MDISC with predicted values.

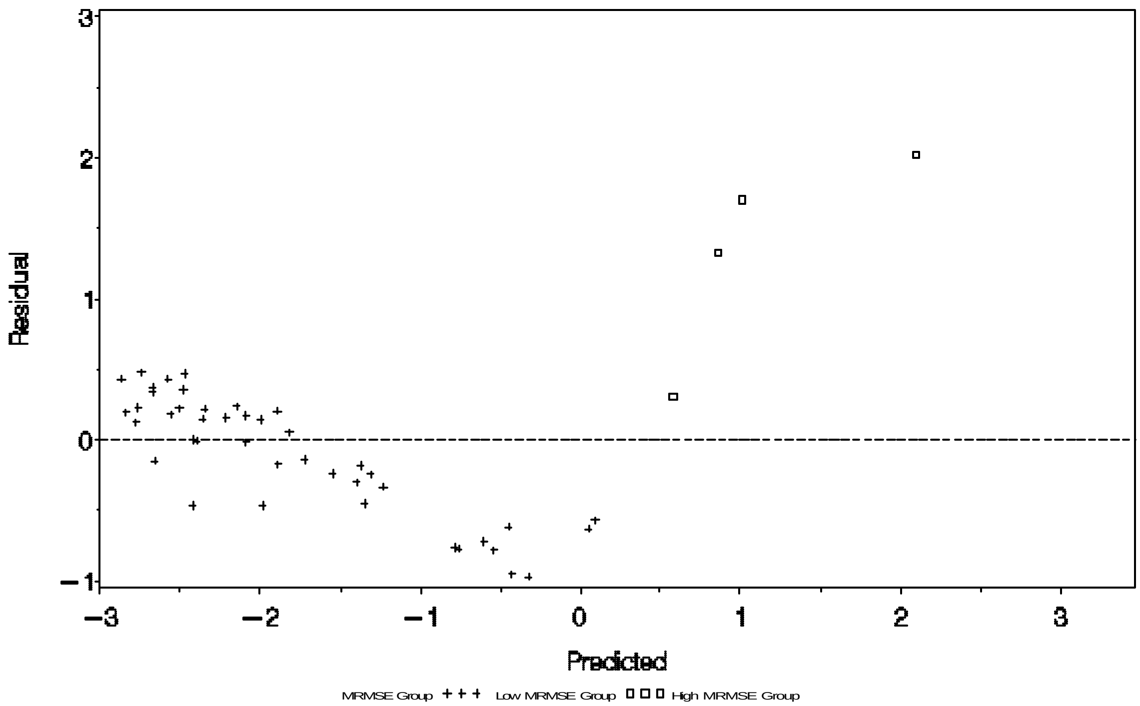


Figure C20. Residual plots of log of MRMSE of MDISC with predicted values for reduced data set.

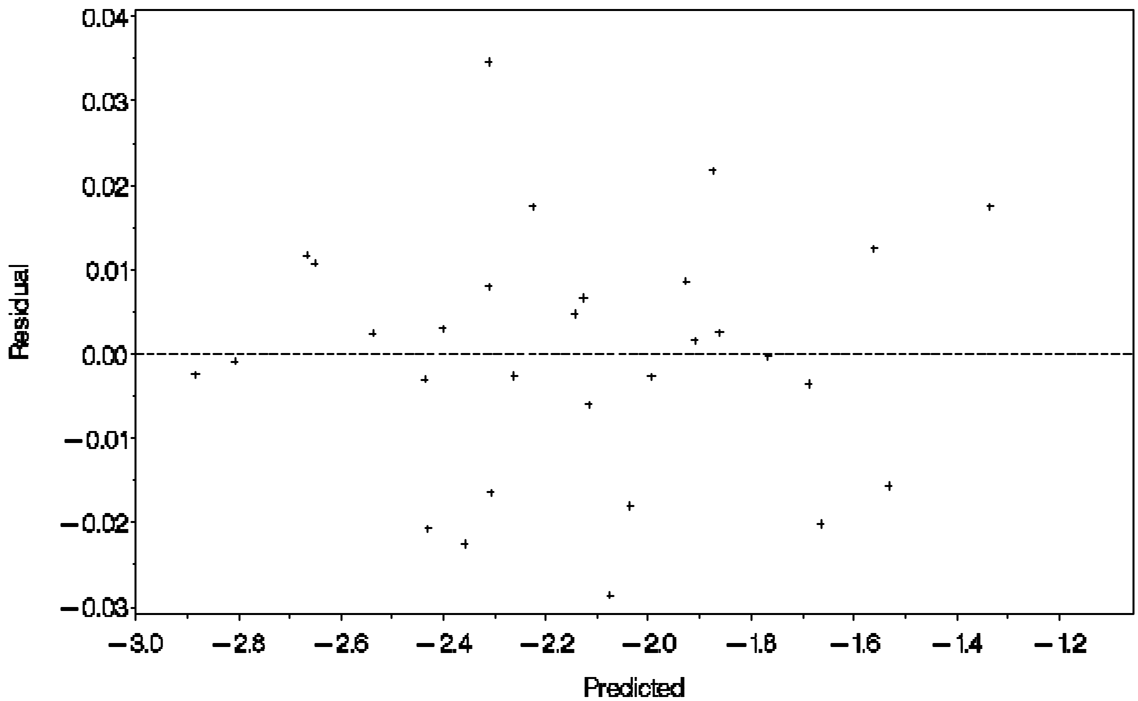


Figure C21. Log of MBIAS of MDISC plotted with SSR, TL and TC.

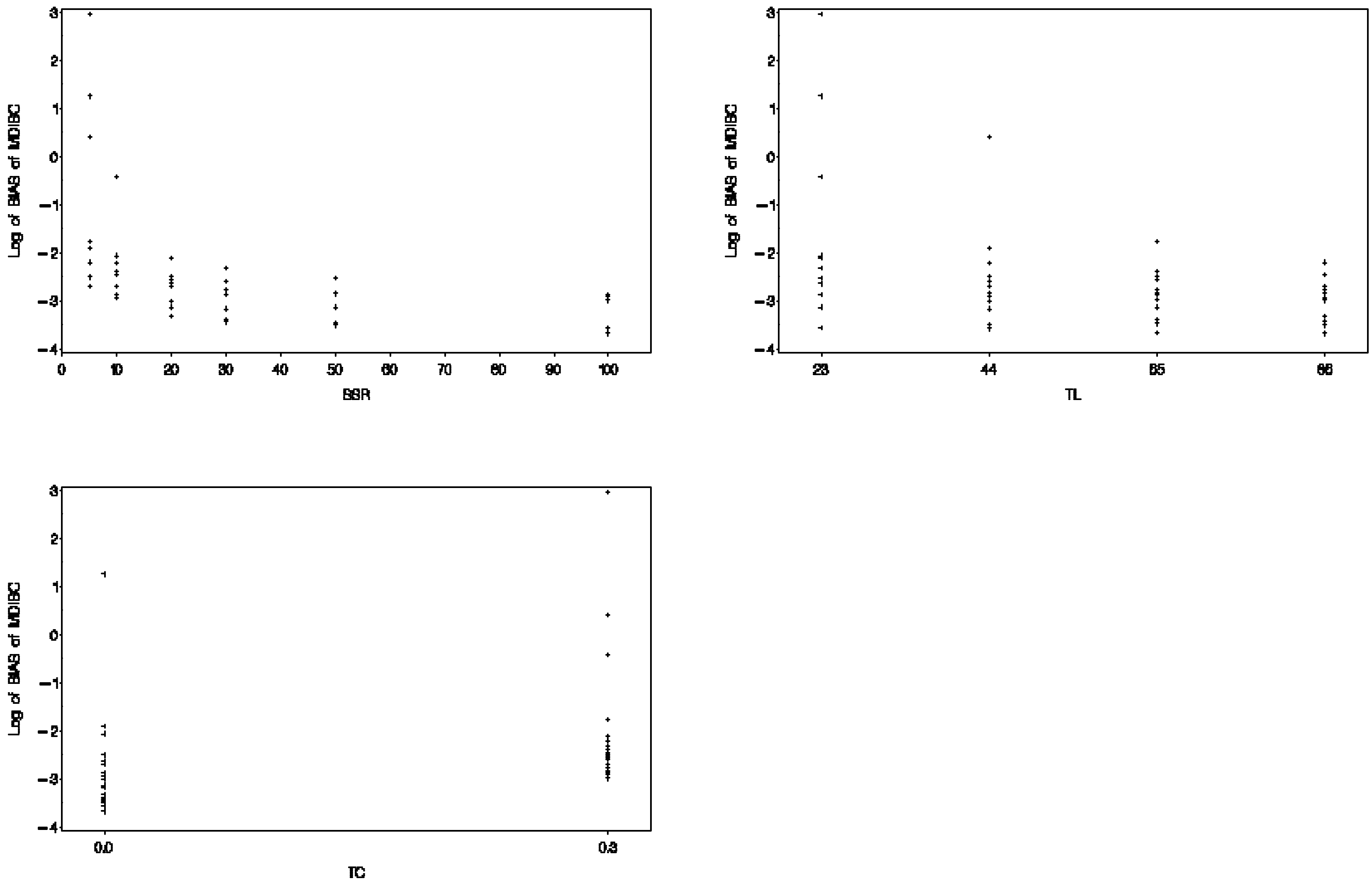


Figure C22. Residual plots of log of MBIAS of MDISC with predicted values.

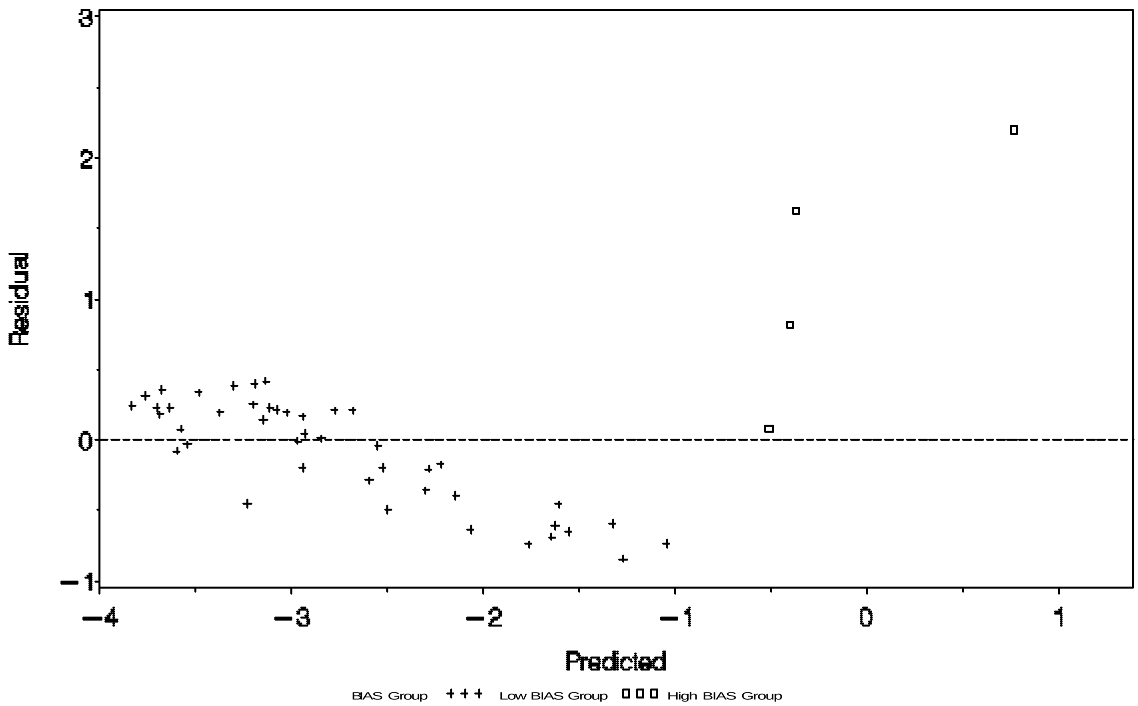


Figure C23. Residual plots of log of MBIAS of MDISC with predicted values for reduced data set.

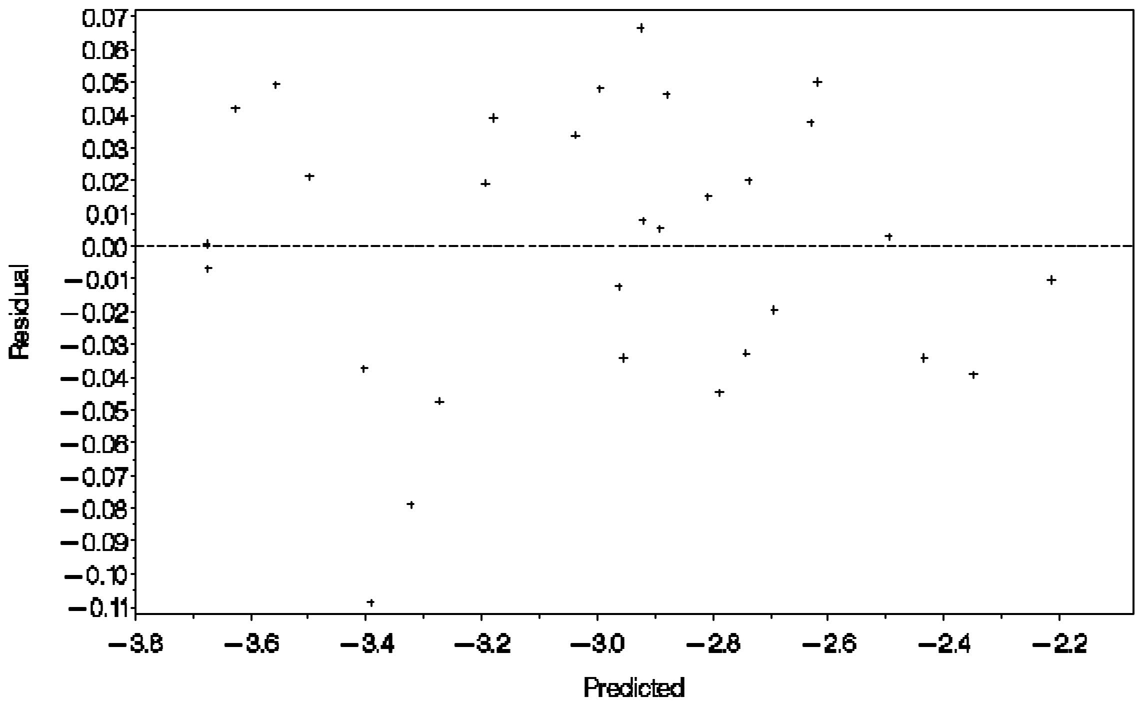


Figure C24. Residual plots of log of MVAR of D of MID with predicted values.

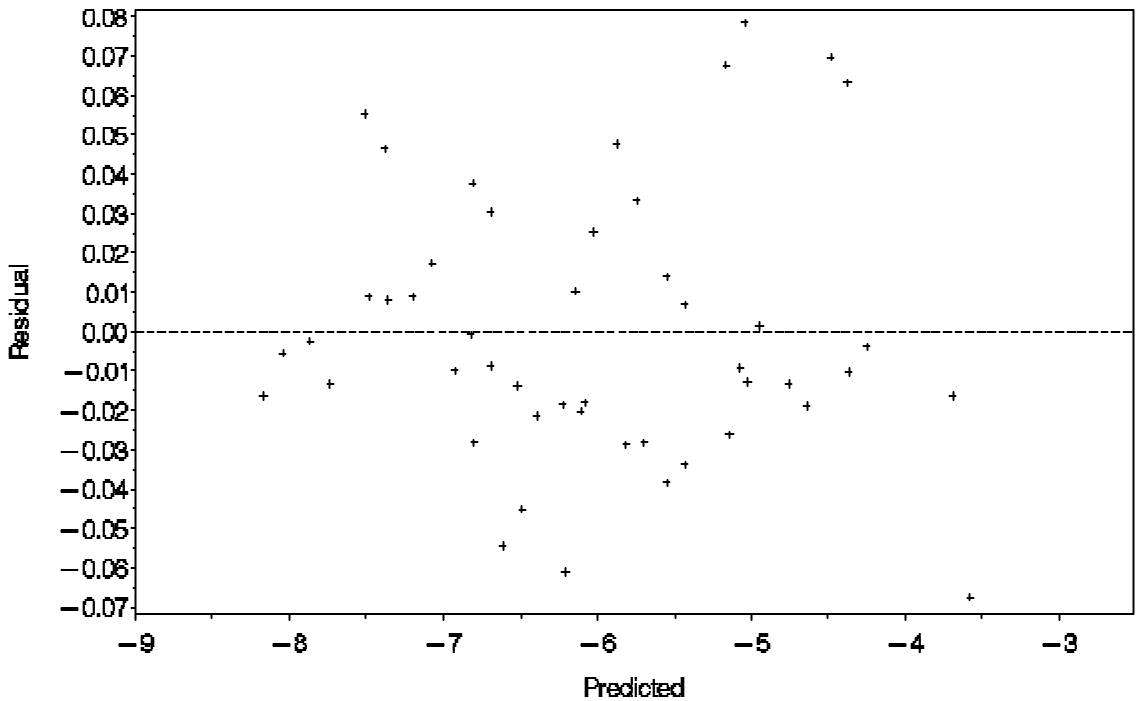


Figure C25. Log of MRMSE of D of MID plotted with SSR, TL and TC.

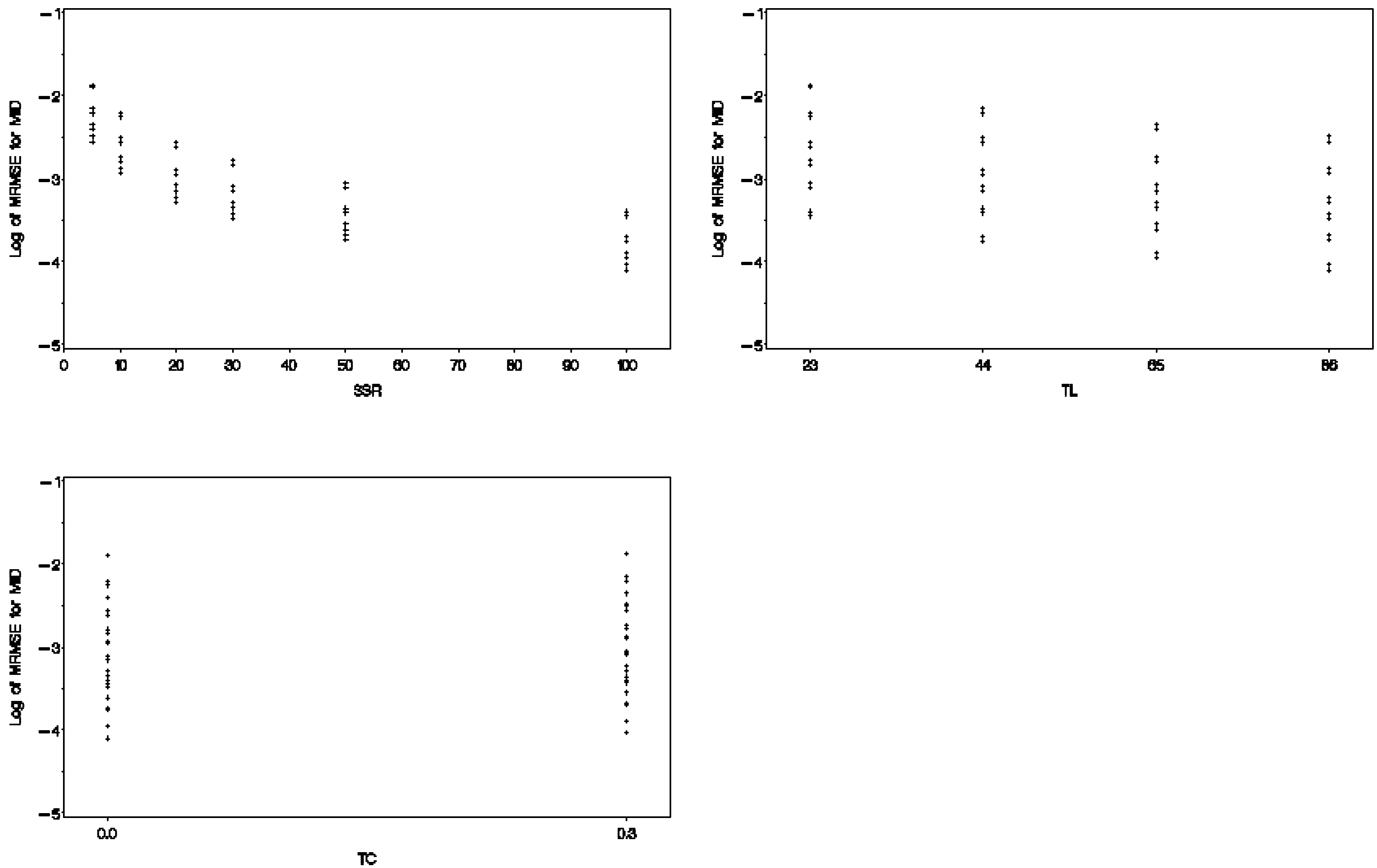


Figure C26. Residual plots of log of MRMSE of D of MID with predicted values.

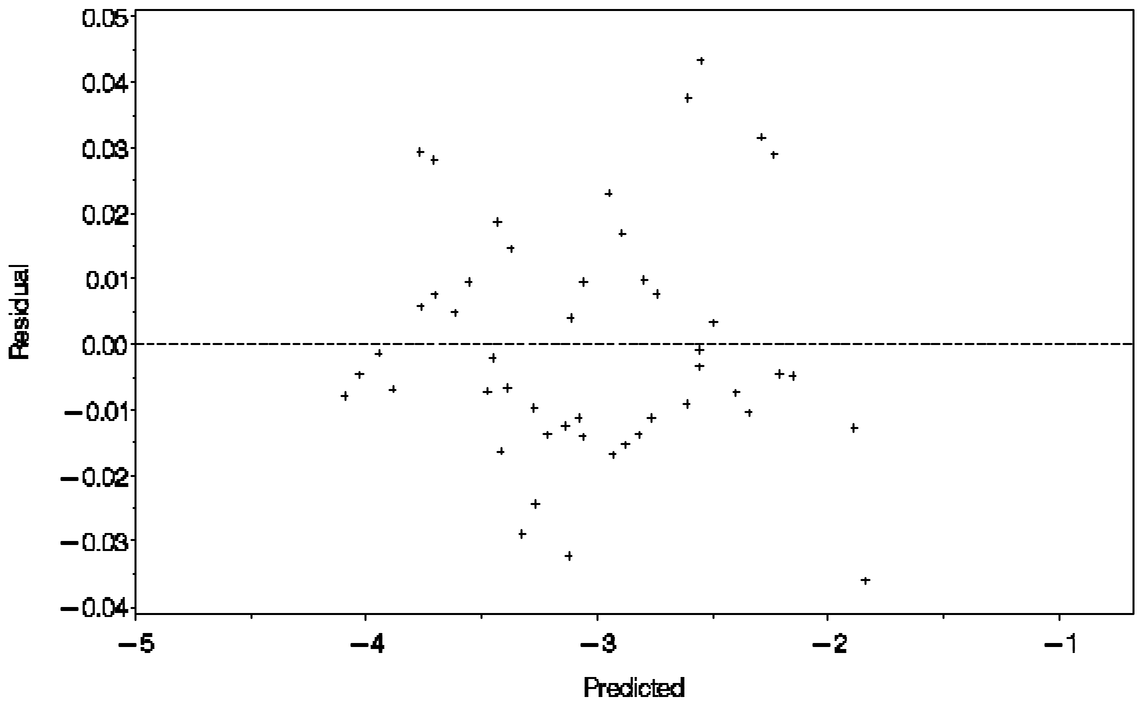


Figure C27. Log of absolute value of MBIAS D of MID plotted with SSR, TL and TC.

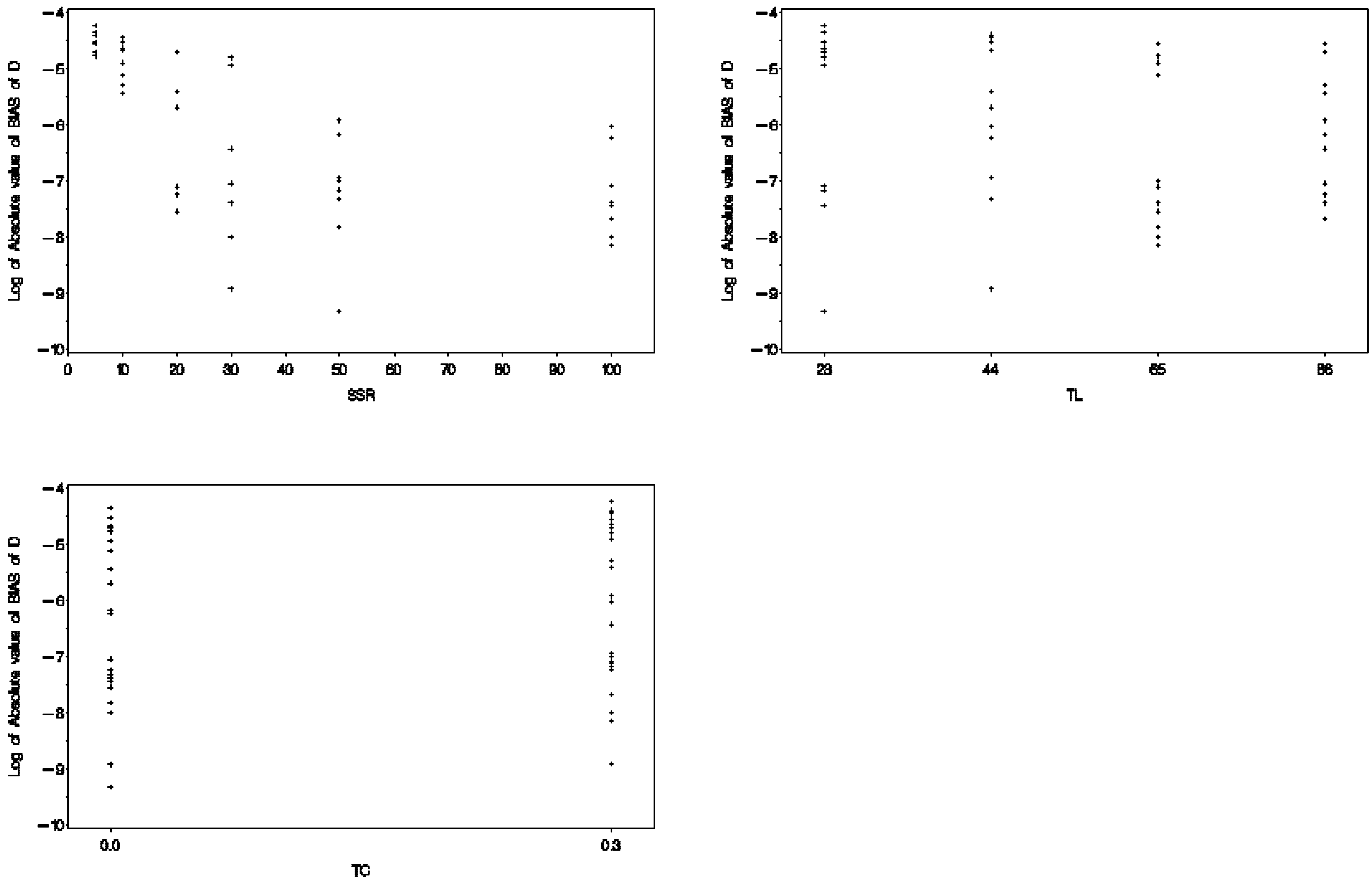


Figure C28. Residual plots of log of MBIAS of D of MID with predicted values for reduced data set.

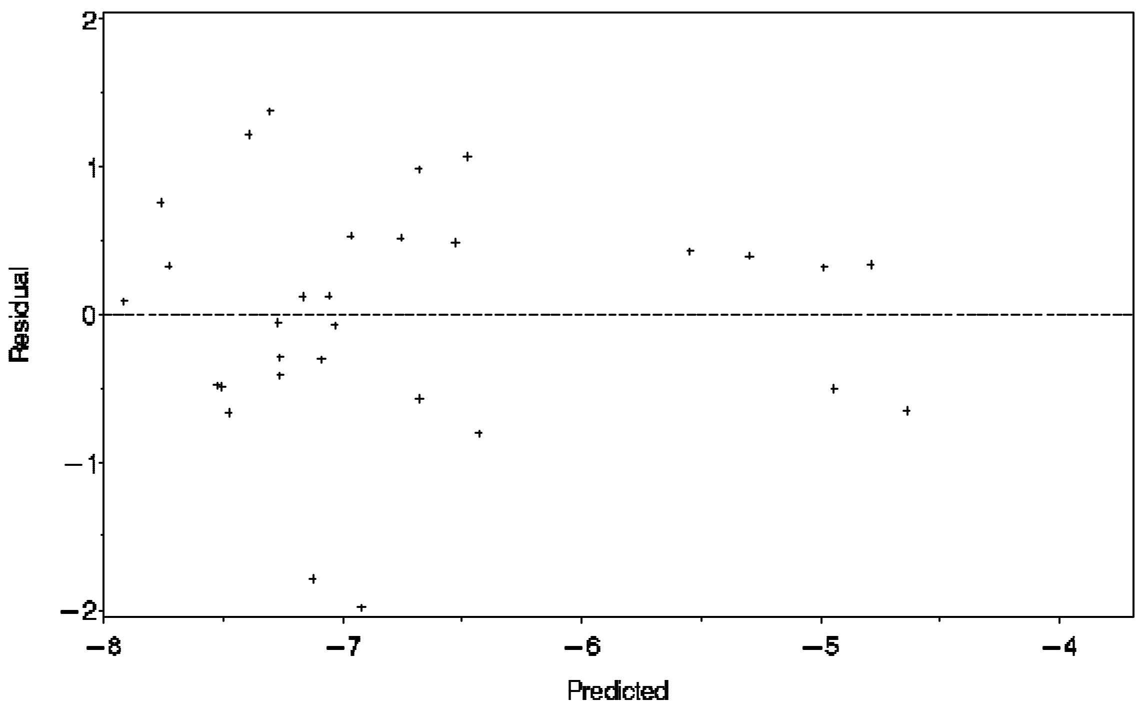


Figure C29. Residual plots of log of MVAR of a_1 of MID with predicted values.

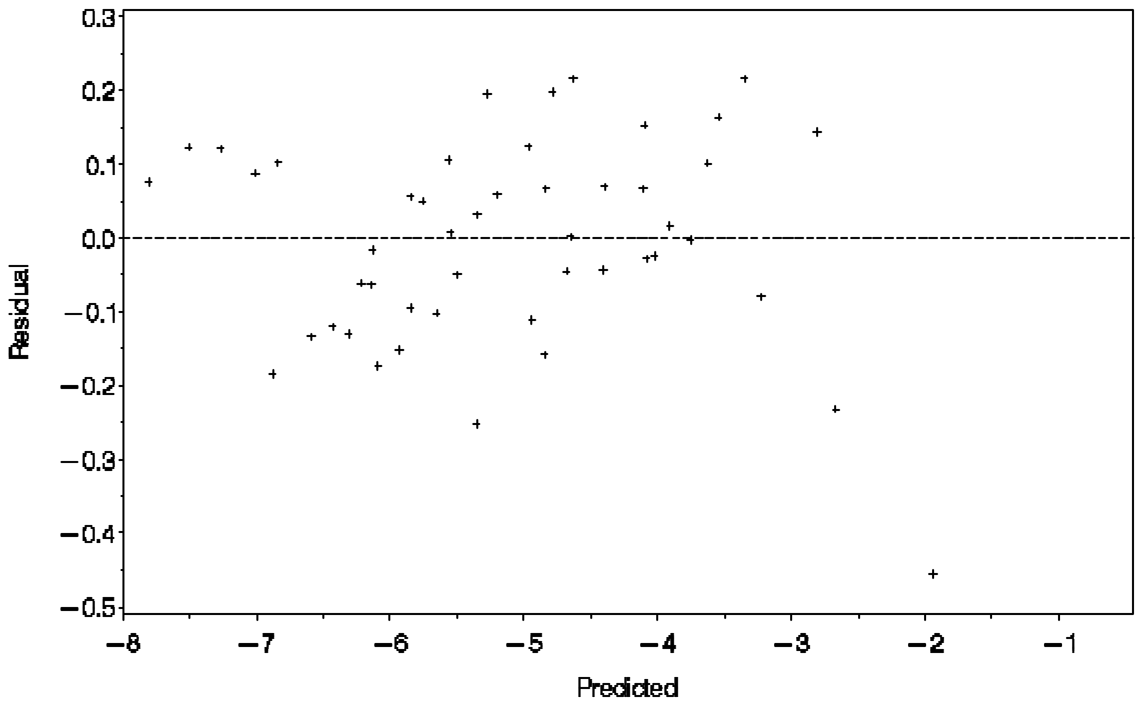


Figure C30. Log of MRMSE of a_1 of MID plotted with SSR, TL and TC.

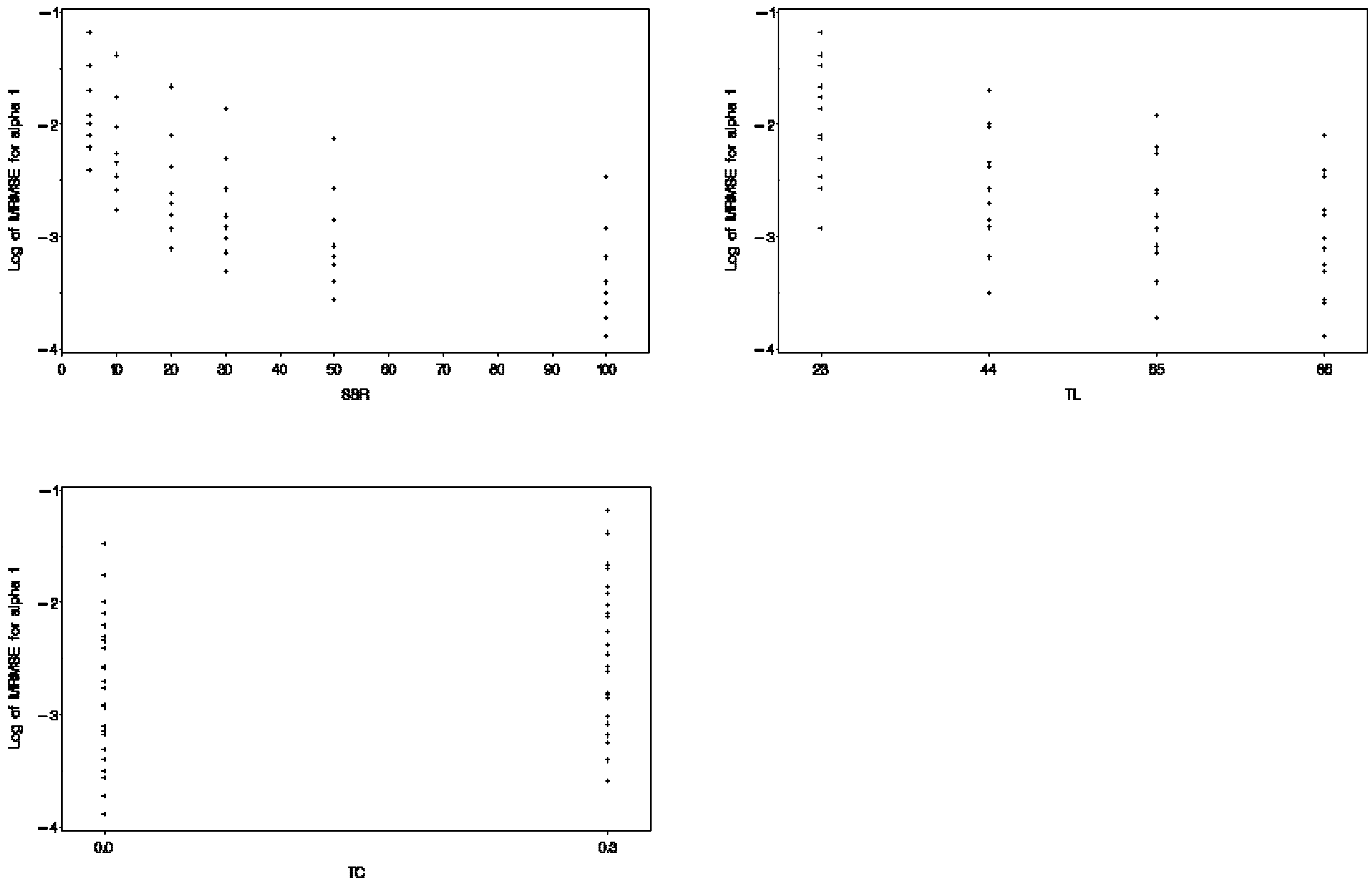


Figure C31. Residual plots of log of MRMSE of a_1 of MID with predicted values.

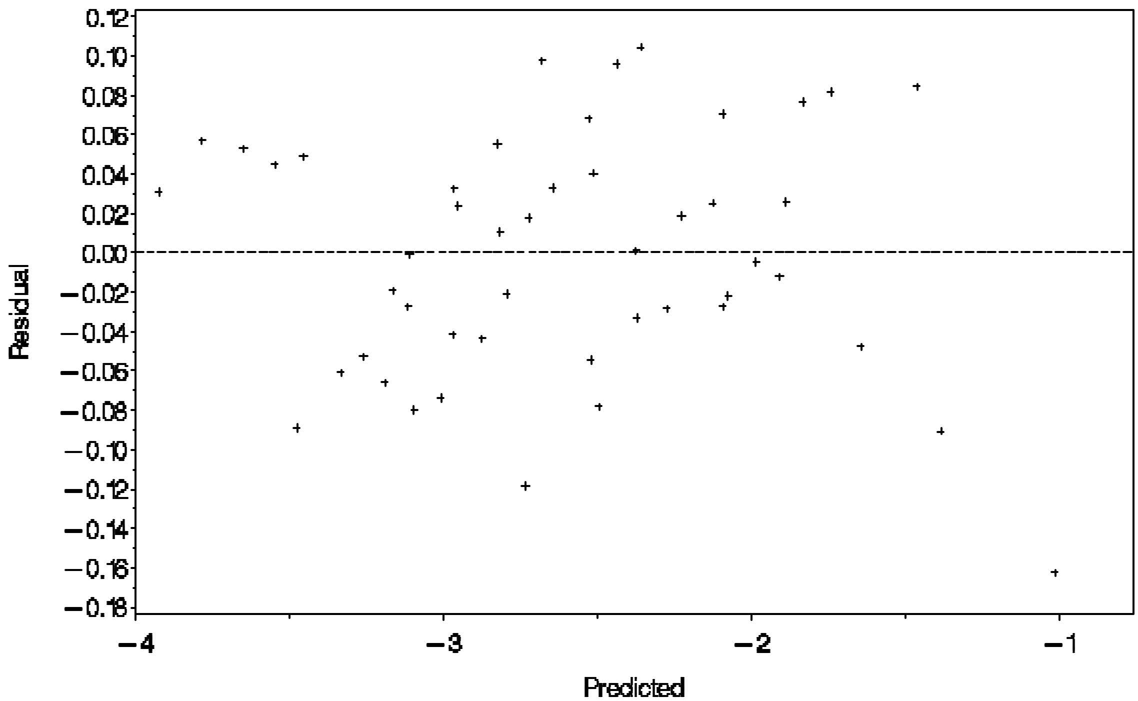
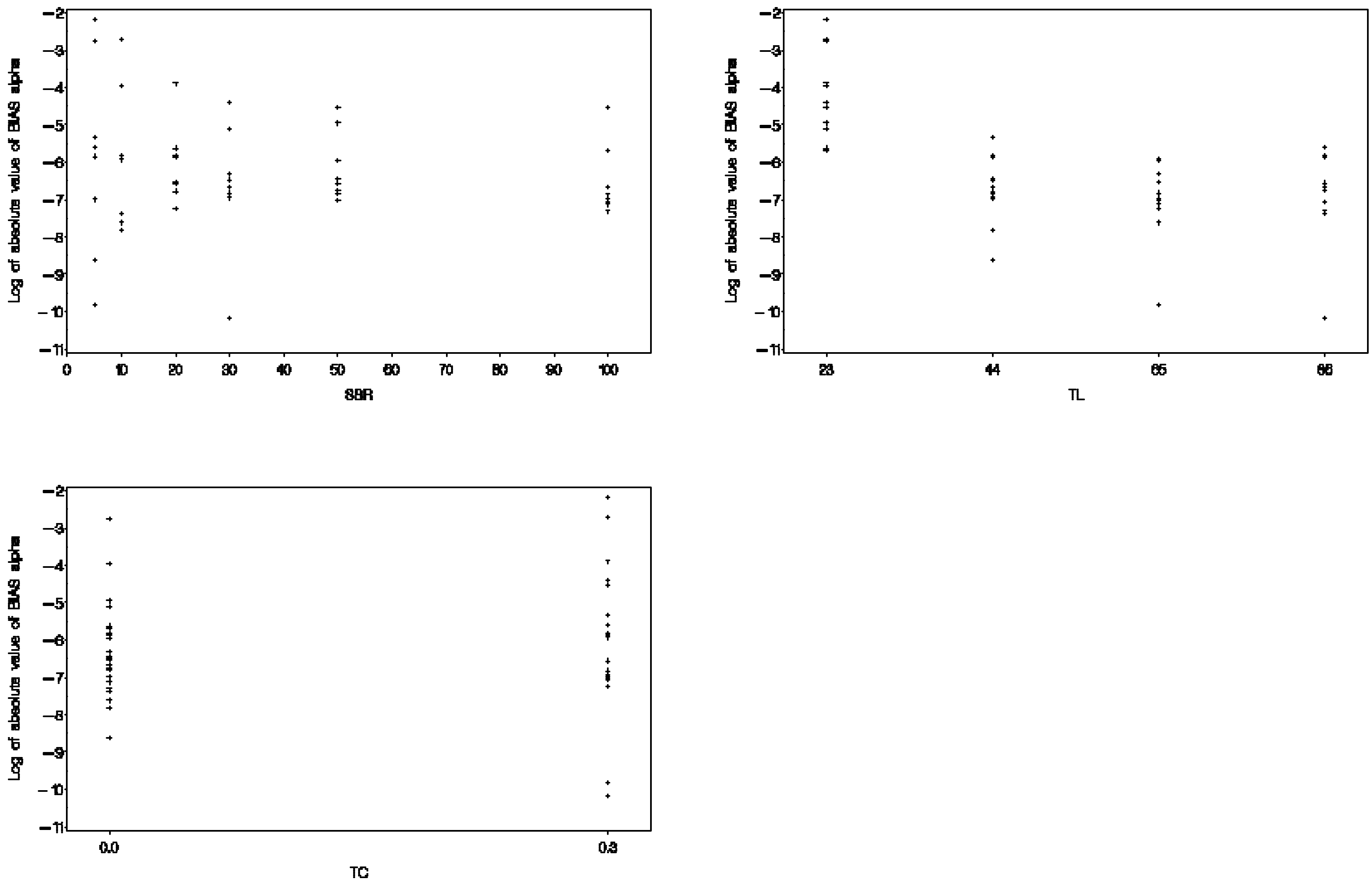


Figure C32. Log of absolute value of MBIAS of alpha of MID plotted with SSR, TL and TC.



APPENDIX D

CONDITIONS IN HIGH DEPENDENT VARIABLE GROUPS

Table D1. Log of MVAR of a1 and a2, number of admissible replications, SSR, TL, TC and for conditions in high MVAR grouping

Condition	Log MVAR a1	Log MVAR a2	SSR	TL	TC	Num of admissible replications
1	9.738565	9.434958	5	23	0.0	1692
2	6.59822	5.739028	10	23	0.0	1933
3	4.524904	2.753052	20	23	0.0	1996
4	*	0.786353	30	23	0.0	2000
5	1.654527	*	50	23	0.0	2000
7	12.78299	11.97693	5	23	0.3	1368
8	6.642051	7.690585	10	23	0.3	1792
9	5.739762	4.871247	20	23	0.3	1958
10	4.803535	1.859869	30	23	0.3	1994
11	3.221974	*	50	23	0.3	2000
13	3.759334	2.03194	5	44	0.0	1967
19	8.788213	8.272293	5	44	0.3	1840
31	3.987845	3.22027	5	65	0.3	1962

Note. * indicates that this condition for this variable was not in the high MVAR group.

Table D2. Log of MRMSE of a1 and a2, SSR, TL, TC and number of admissible replications for conditions in high MRMSE grouping

Condition	Log MRMSE a1	Log MRMSE a2	SSR	TL	TC	Num of admissible replications
1	3.065016	2.996707	5	23	0.0	1692
2	1.483377	0.659574	10	23	0.0	1933
3	-0.36281	*	20	23	0.0	1996
7	-2.42334	3.80103	5	23	0.3	1368
8	3.884653	1.777474	10	23	0.3	1792
9	1.524322	0.155609	20	23	0.3	1958
10	0.883472	*	30	23	0.3	1994
11	0.2479	*	50	23	0.3	2000
13	-2.06515	*	5	44	0.0	1967
19	-2.81538	1.778698	5	44	0.3	1840
31	-0.72207	*	5	65	0.3	1962

Note. * indicates that this condition for this variable was not in the high MRMSE group.

Table D3. Log of MBIAS of a1 and a2, SSR, TL, TC and number of admissible replications for conditions in high MBIAS grouping

Condition	Log MBIAS a1	Log MBIAS a2	SSR	TL	TC	Num of admissible replications
1	1.61422	1.486707	5	23	0.0	1692
2	-0.03808	-0.87251	10	23	0.0	1933
7	2.705638	2.574805	5	23	0.3	1368
8	0.074308	0.241787	10	23	0.3	1792
9	-0.63066	-1.38516	20	23	0.3	1958
10	-1.25526	*	30	23	0.3	1994
19	0.046327	0.015733	5	44	0.3	1840

Note. * indicates that this condition for this variable was not in the high MBIAS group.

TableD4. Log of MVAR of d, number of admissible replications, SSR, TL, TC and for conditions in high MVAR grouping

Condition	Log MVAR d	SSR	TL	TC	Num of admissible replications
1	9.738565	5	23	0.0	1692
2	6.59822	10	23	0.0	1933
3	4.524904	20	23	0.0	1996
7	12.78299	5	23	0.3	1368
8	6.642051	10	23	0.3	1792
9	5.739762	20	23	0.3	1958
10	4.803535	30	23	0.3	1994
11	3.221974	50	23	0.3	2000
13	3.759334	5	44	0.0	1967
19	8.788213	5	44	0.3	1840
31	3.987845	5	65	0.3	1962

Table D5. Log of MRMSE of d, number of admissible replications, SSR, TL, TC and for conditions in high MRMSE grouping

Condition	Log MRMSE d	SSR	TL	TC	Num of admissible replications
1	1.967051	5	23	0.0	1692
2	-0.60058	10	23	0.0	1933
7	2.96686	5	23	0.3	1368
8	0.262451	10	23	0.3	1792
9	-1.29749	20	23	0.3	1958
19	0.886855	5	44	0.3	1840
31	-1.2749	5	65	0.3	1962

Table D6. Log of MVAR of MDISC, number of admissible replications, SSR, TL, TC and for conditions in high MVAR grouping

Condition	Log MVAR MDISC	SSR	TL	TC	Num of admissible replications
1	9.970933	5	23	0.0	1692
7	13.16628	5	23	0.3	1368
8	7.074363	10	23	0.3	1792
13	3.774551	5	44	0.0	1967
19	9.267372	5	44	0.3	1840
31	4.384032	5	65	0.3	1962

Table D7. Log of MRMSE of MDISC, number of admissible replications, SSR, TL, TC and for conditions in high MRMSE grouping

Condition	Log MRMSE MDISC	SSR	TL	TC	Num of admissible replications
1	2.71667	5	23	0.0	1692
7	4.11592	5	23	0.3	1368
8	0.888948	10	23	0.3	1792
19	2.182718	5	44	0.3	1840

Table D8. Log of MBIAS of MDISC, number of admissible replications, SSR, TL, TC and for conditions in high MBIAS grouping

Condition	Log MBIAS MDISC	SSR	TL	TC	Num of admissible replications
1	1.255152	5	23	0.0	1692
7	2.966629	5	23	0.3	1368
8	-0.42899	10	23	0.3	1792
19	0.41744	5	44	0.3	1840

APPENDIX E

DETAILED RESULTS OF HIERARCHICAL REGRESSION ANALYSES

Table E1. Hierarchical Regression Analysis for log of MVAR of a1 with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.06930	0.01400	<.0001	-0.47198	22.269
TL	-0.12323	0.01931	<.0001	-0.60845	37.008
TC	4.35008	3.00912	0.1554	0.13785	1.900
Step 2					
SSR	-0.14744	0.03606	0.0002	-1.00415	-
TL	-0.16812	0.03279	<.0001	-0.83012	-
TC	11.02736	7.96734	.1738	0.34945	-
SSR x TL	0.00157	0.0005644	.0082	0.69360	6.245
SSR x TC	-0.05397	0.08800	.5431	-0.09971	0.304
TL x TC	-0.08486	0.12139	.4885	-0.17258	0.395
Step 3					
SSR	-0.14477	0.04788	0.0043	-0.98597	-
TL	-0.16635	0.03903	0.0001	-0.82139	-
TC	11.69065	11.15048	0.3007	0.37047	-
SSR x TL	0.00152	0.000804	0.066	0.67204	-
SSR x TC	-0.07198	0.22724	0.7531	-0.13298	-
TL x TC	-0.09686	0.18581	0.605	-0.197	-
SSR x TL x TC	0.000328	0.00381	0.9318	0.03704	0.0001
Step 4					
SSR	-0.09811	0.03092	0.003	-0.66818	-
TL	-0.01146	0.03853	0.7678	-0.05659	-
TC	12.89411	6.96737	0.072	0.4086	-
SSR x TL	0.00158	0.000503	0.0033	0.69694	-
SSR x TC	-0.0891	0.14197	0.534	-0.16462	-
TL x TC	-0.11298	0.11609	0.3366	-0.22978	-
SSR x TL x TC	0.000557	0.00238	0.8161	0.06291	-
1/SSR	35.82548	5.83639	<.0001	0.48583	12.173
1/TL	324.15073	61.45386	<.0001	0.83442	8.988

Note. R^2 = .6000 for Step 1. ?R^2 = .0688 for Step 2. ?R^2 = 0.0001 for Step 3. ? R^2 = .2083 for Step 4.

Table E2. Hierarchical Regression Analysis for log of MVAR of a2 with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.06987	0.01322	<.0001	-0.51565	26.581
TL	-0.10426	0.01823	<.0001	-0.55787	31.111
TC	3.69001	2.84079	0.2007	0.12671	1.605
Step 2					
SSR	-0.15361	0.03319	<.0001	-1.13367	-
TL	-0.16004	0.03018	<.0001	-0.85632	-
TC	8.0457	7.33444	0.2791	0.27628	-
SSR x TL	0.00168	0.00052	0.0024	0.80633	8.440
SSR x TC	-0.05822	0.08101	0.4764	-0.11657	0.415
TL x TC	-0.03972	0.11174	0.7241	-0.08753	0.102
Step 3					
SSR	-0.14945	0.04407	0.0016	-1.10299	-
TL	-0.15729	0.03593	<.0001	-0.84158	-
TC	9.07863	10.26296	0.3817	0.31175	-
SSR x TL	0.00161	0.00074	0.0359	0.76994	-
SSR x TC	-0.08627	0.20915	0.6822	-0.17272	-
TL x TC	-0.05841	0.17102	0.7345	-0.12874	-
SSR x TL x TC	0.000511	0.00351	0.8848	0.0625	0.0002
Step 4					
SSR	-0.10218	0.02825	0.0009	-0.75411	-
TL	-0.03418	0.03521	0.3377	-0.1829	-
TC	10.20172	6.3658	0.1173	0.35032	-
SSR x TL	0.00166	0.000459	0.0009	0.79511	-
SSR x TC	-0.1023	0.12971	0.4352	-0.20481	-
TL x TC	-0.07334	0.10607	0.4935	-0.16163	-
SSR x TL x TC	0.000724	0.00217	0.741	0.08854	-
1/SSR	36.06764	5.33247	<.0001	0.53002	14.488
1/TL	258.72651	56.14783	<.0001	0.72170	6.724

Note. R^2 = .5813 for Step 1. ?R^2 = .0891 for Step 2. ?R^2 = 0.0002 for Step 3. ? R^2 = .2091 for Step 4.

Table E3. Hierarchical Regression Analysis for log of MVAR of a1 reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.02475	0.0018	<.0001	-0.84881	72.0480
TL	-0.01956	0.00334	<.0001	-0.36083	13.020
TC	1.39294	0.38231	0.0012	0.22477	5.0520
Step 2					
SSR	-0.02597	0.00773	0.0027	-0.89037	-
TL	-0.01971	0.00686	0.0086	-0.36357	-
TC	1.70266	1.67892	0.3211	0.27475	-
SSR x TL	1.96E-05	0.000111	0.8619	0.04766	0.0133
SSR x TC	-0.00041	0.01273	0.9745	-0.0041	0.0005
TL x TC	-0.0045	0.02367	0.851	-0.05036	0.0155
Step 3					
SSR	-0.02558	0.01083	0.0274	-0.87709	-
TL	-0.01946	0.00849	0.0319	-0.35896	-
TC	1.81119	2.69167	0.508	0.29227	-
SSR x TL	1.36E-05	0.000161	0.9333	0.03316	-
SSR x TC	-0.003	0.05104	0.9537	-0.02979	-
TL x TC	-0.00617	0.04004	0.879	-0.06905	-
SSR x TL x TC	3.98E-05	0.000759	0.9587	0.02696	0.0000
Step 4					
SSR	-0.01421	0.00192	<.0001	-0.48737	-
TL	-0.00849	0.00367	0.0316	-0.1566	-
TC	1.81626	0.46554	0.0009	0.29309	-
SSR x TL	1.36E-05	2.79E-05	0.6298	0.03316	-
SSR x TC	-0.00306	0.00883	0.7321	-0.03047	-
TL x TC	-0.00623	0.00692	0.3788	-0.06977	-
SSR x TL x TC	4.06E-05	0.000131	0.7603	0.02755	-
1/SSR	14.57984	0.54916	<.0001	0.49637	9.4550
1/TL	41.51228	12.73695	0.0039	0.20584	0.1420

Note. R^2 = 0.9010 for Step 1. ?R^2 = 0.0003.for Step 2. ?R^2 = 0.0000 for Step 3. ? R^2 = 0.096 for Step 4. ?R^2 = Step 5.

Table E3 (continued) Hierarchical Regression Analysis for log of MVAR of a1 reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 5					
SSR	-0.0315	0.00159	<.0001	-1.08025	-
TL	-0.00849	0.00129	<.0001	-0.15661	-
TC	1.81605	0.1632	<.0001	0.29305	-
SSR x TL	1.36E-05	9.76E-06	0.1787	0.03316	-
SSR x TC	-0.00306	0.00309	0.335	-0.03044	-
TL x TC	-0.00623	0.00243	0.0189	-0.06974	-
SSR x TL x TC	4.06E-05	4.6E-05	0.3891	0.02753	-
1/SSR	10.30513	0.40519	<.0001	0.35084	-
1/TL	41.51022	4.46498	<.0001	0.20583	-
SSR^2	0.000123	1.03E-05	<.0001	0.48948	0.237

Note. R^2 = 0.9010 for Step 1. ?R^2 = 0.0003.for Step 2. ?R^2 = 0.0000 for Step 3. ? R^2 = 0.096 for Step 4. ?R^2 = 0.0024 Step 5.

Table E4. Hierarchical Regression Analysis for log of MVAR of a2 reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.0249	0.0018	<.0001	-0.8435	71.1490
TL	-0.01987	0.00335	<.0001	-0.36206	13.1090
TC	1.54488	0.38312	0.0004	0.24623	06.0630
Step 2					
SSR	-0.02554	0.00774	0.0031	-0.8649	-
TL	-0.01995	0.00687	0.008	-0.36354	-
TC	1.96033	1.68051	0.2554	0.31245	-
SSR x TL	1.72E-05	0.000112	0.8785	0.0414	0.0100
SSR x TC	-0.00326	0.01275	0.8006	-0.03199	0.0274
TL x TC	-0.00429	0.02369	0.858	-0.0474	0.0137
Step 3					
SSR	-0.025	0.01084	0.0309	-0.84665	-
TL	-0.01961	0.0085	0.0309	-0.3572	-
TC	2.11123	2.69407	0.4416	0.3365	-
SSR x TL	8.95E-06	0.000161	0.9562	0.02149	-
SSR x TC	-0.00685	0.05109	0.8946	-0.06727	-
TL x TC	-0.00661	0.04007	0.8705	-0.07307	-
SSR x TL x TC	5.53E-05	0.00076	0.9427	0.03703	0.0000
Step 4					
SSR	-0.01359	0.00199	<.0001	-0.46017	-
TL	-0.01217	0.0038	0.0044	-0.22175	-
TC	2.11624	0.48123	0.0003	0.3373	-
SSR x TL	8.95E-06	2.88E-05	0.7592	0.02149	-
SSR x TC	-0.00692	0.00913	0.4573	-0.06793	-
TL x TC	-0.00667	0.00716	0.3625	-0.07377	-
SSR x TL x TC	5.61E-05	0.000136	0.6837	0.03761	-
1/SSR	14.63843	0.56767	<.0001	0.49225	9.2980
1/TL	28.13295	13.16613	0.0452	0.13779	0.0638

Note. R^2 = 0.9031 for Step 1. ?R^2 = 0.0005 for Step 2. ?R^2 = 0.0000 for Step 3. ? R^2 = 0.0936 for Step 4. ?R^2 = for Step 5.

Table E4 (continued). Hierarchical Regression Analysis for log of MVAR of a2 reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 5					
SSR	-0.03136	0.00171	<.0001	-1.06212	-
TL	-0.01217	0.00139	<.0001	-0.22176	-
TC	2.11602	0.17567	<.0001	0.33726	-
SSR x TL	8.95E-06	1.05E-05	0.4053	0.02149	-
SSR x TC	-0.00691	0.00333	0.0517	-0.06791	-
TL x TC	-0.00667	0.00261	0.0195	-0.07374	-
SSR x TL x TC	5.61E-05	4.96E-05	0.2718	0.03758	-
1/SSR	10.24438	0.43615	<.0001	0.34449	-
1/TL	28.13084	4.80613	<.0001	0.13778	-
SSR^2	0.000126	1.1E-05	<.0001	0.49696	0.244

Note. R^2 = 0.9031 for Step 1. ?R^2 = 0.0005 for Step 2. ?R^2 = 0.0000 for Step 3. ? R^2 = 0.0936 for Step 4. ?R^2 = 0.0024 for Step 5.

Table E5. Hierarchical Regression Analysis for log of MRMSE of a1 with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.02333	0.00436	<.0001	-0.51144	26.148
TL	-0.03552	0.00602	<.0001	-0.56442	31.846
TC	1.71283	0.9373	0.0744	0.17468	3.051
Step 2					
SSR	-0.05121	0.01106	<.0001	-1.12231	-
TL	-0.0506	0.01006	<.0001	-0.80403	-
TC	3.66518	2.44362	0.1413	0.37378	-
SSR x TL	0.000531	0.000173	0.0038	0.75556	7.411
SSR x TC	-0.00873	0.02699	0.7479	-0.05192	0.082
TL x TC	-0.02942	0.03723	0.4339	-0.19257	0.491
Step 3					
SSR	-0.04885	0.01468	0.0019	-1.07067	-
TL	-0.04904	0.01196	0.0002	-0.77921	-
TC	4.25061	3.41759	0.2208	0.43348	-
SSR x TL	0.000488	0.000247	0.0546	0.69432	-
SSR x TC	-0.02463	0.06965	0.7255	-0.14643	-
TL x TC	-0.04002	0.05695	0.4863	-0.26193	-
SSR x TL x TC	0.00029	0.00117	0.8053	0.1052	0.0495
Step 4					
SSR	-0.03335	0.00884	0.0005	-0.73096	
TL	-0.00277	0.01101	0.8029	-0.04399	
TC	4.63564	1.99119	0.0253	0.47275	
SSR x TL	0.000506	0.000144	0.0011	0.71996	
SSR x TC	-0.03011	0.04057	0.4625	-0.17906	
TL x TC	-0.04516	0.03318	0.1815	-0.29556	
SSR x TL x TC	0.000363	0.00068	0.5968	0.13178	
1/SSR	11.86537	1.66797	<.0001	0.51782	13.829
1/TL	96.99905	17.5628	<.0001	0.80355	8.336

Note. R^2 = .5980 for Step 1. ?R^2 = .093 for Step 2. ?R^2 = 0.0005 for Step 3.
? R^2 = .2184 for Step 4.

Table E6. Hierarchical Regression Analysis for log of MRMSE of a2 with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.02237	0.00432	<.0001	-0.52554	27.610
TL	-0.02928	0.00596	<.0001	-0.49855	24.847
TC	1.67905	0.92871	0.0775	0.18351	3.367
Step 2					
SSR	-0.04698	0.01117	0.0001	-1.10346	-
TL	-0.04391	0.01016	<.0001	-0.74777	-
TC	3.31592	2.46859	0.1866	0.36241	-
SSR x TL	0.000484	0.000175	0.0085	0.7375	7.061
SSR x TC	-0.01317	0.02727	0.6318	-0.08389	0.215
TL x TC	-0.02078	0.03761	0.5836	-0.14576	0.282
Step 3					
SSR	-0.04299	0.01481	0.006	-1.00988	-
TL	-0.04127	0.01207	0.0015	-0.7028	-
TC	4.30581	3.44773	0.219	0.4706	-
SSR x TL	0.000411	0.000249	0.1063	0.62652	-
SSR x TC	-0.04004	0.07026	0.5719	-0.25516	-
TL x TC	-0.0387	0.05745	0.5045	-0.27145	-
SSR x TL x TC	0.00049	0.00118	0.6798	0.19063	0.163
Step 4					
SSR	-0.02622	0.00972	0.0103	-0.61583	-
TL	-0.0073	0.01211	0.5502	-0.12433	-
TC	4.67674	2.1895	0.0392	0.51114	-
SSR x TL	0.000428	0.000158	0.01	0.65296	-
SSR x TC	-0.04535	0.04461	0.3158	-0.289	-
TL x TC	-0.04359	0.03648	0.2396	-0.30576	-
SSR x TL x TC	0.00056	0.000748	0.4588	0.21789	-
1/SSR	12.7342	1.83409	<.0001	0.59559	18.294
1/TL	71.79468	19.31187	0.0006	0.6374	5.245

Note. R^2 = .5467 for Step 1. ?R^2 = .0751 for Step 2. ?R^2 = 0.0016 for Step 3. ? R^2 = .2324 for Step 4.

Table E7. Hierarchical Regression Analysis for log of MRMSE of a1 reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.00997	0.000864	<.0001	-0.80065	64.1030
TL	-0.00755	0.00161	<.0001	-0.3262	10.6410
TC	0.94498	0.18362	<.0001	0.35712	12.753
Step 2					
SSR	-0.01313	0.00362	0.0014	-1.05433	-
TL	-0.0095	0.00322	0.0071	-0.41052	-
TC	0.58306	0.78718	0.4664	0.22034	-
SSR x TL	3.76E-05	5.22E-05	0.4789	0.2141	2.6800
SSR x TC	0.00477	0.00597	0.4325	0.11105	0.330
TL x TC	0.00249	0.0111	0.8247	0.06519	0.0260
Step 3					
SSR	-0.01244	0.00507	0.0226	-0.99884	-
TL	-0.00906	0.00398	0.0329	-0.39123	-
TC	0.77659	1.26096	0.5443	0.29348	-
SSR x TL	2.7E-05	7.55E-05	0.7242	0.15356	-
SSR x TC	0.000162	0.02391	0.9947	0.00377	-
TL x TC	-0.00049	0.01876	0.9793	-0.01287	-
SSR x TL x TC	7.09E-05	0.000356	0.8439	0.11261	0.0214
Step 4					
SSR	-0.00717	0.000924	<.0001	-0.57598	-
TL	-0.00117	0.00177	0.5146	-0.05061	-
TC	0.77901	0.22387	0.0024	0.2944	-
SSR x TL	2.7E-05	1.34E-05	0.0578	0.15356	-
SSR x TC	0.000129	0.00425	0.976	0.00301	-
TL x TC	-0.00052	0.00333	0.8771	-0.01368	-
SSR x TL x TC	7.13E-05	6.32E-05	0.2723	0.11327	-
1/SSR	6.75494	0.26409	<.0001	0.53858	11.131
1/TL	29.83745	6.12512	<.0001	0.34649	0.404

Note. R^2 = 0.8748 for Step 1. ?R^2 = 0.0062 for Step 2. ?R^2 = 0.0003 for Step 3.
? R^2 = 0.1153 for Step 4. ?R^2 = .0025 Step 5.

Table E7. (continued). Hierarchical Regression Analysis for log of MRMSE of a1 for the reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 5					
SSR	-0.01474	0.00116	<.0001	-1.18366	-
TL	-0.00117	0.000939	0.2271	-0.05062	-
TC	0.77891	0.11901	<.0001	0.29436	-
SSR x TL	2.7E-05	7.12E-06	0.0012	0.15356	-
SSR x TC	0.000131	0.00226	0.9544	0.00304	-
TL x TC	-0.00052	0.00177	0.772	-0.01365	-
SSR x TL x TC	7.13E-05	3.36E-05	0.0471	0.11324	-
1/SSR	4.8841	0.29547	<.0001	0.38941	-
1/TL	29.83655	3.25598	<.0001	0.34648	-
SSR^2	5.38E-05	7.47E-06	<.0001	0.50169	0.249

Note. R^2 = 0.8748 for Step 1. ?R^2 = 0.0062 for Step 2. ?R^2 = 0.0003 for Step 3. ? R^2 = 0.1153 for Step 4. ?R^2 = .0025 Step 5.

Table E8. Hierarchical Regression Analysis for log of MRMSE of a2 for reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.01007	0.000892	<.0001	-0.7834	61.371
TL	-0.00793	0.00166	<.0001	-0.33164	10.998
TC	1.06336	0.18952	<.0001	0.38914	15.143
Step 2					
SSR	-0.0133	0.00376	0.0018	-1.03461	-
TL	-0.00996	0.00334	0.0066	-0.4168	-
TC	0.78352	0.81679	0.3474	0.28673	-
SSR x TL	4.14E-05	5.42E-05	0.4527	0.22831	0.305
SSR x TC	0.0036	0.00619	0.5672	0.08109	0.176
TL x TC	0.00198	0.01152	0.8649	0.05031	0.0155
Step 3					
SSR	-0.01274	0.00526	0.0243	-0.99049	-
TL	-0.0096	0.00413	0.0297	-0.40147	-
TC	0.94241	1.30884	0.4791	0.34488	-
SSR x TL	3.27E-05	7.83E-05	0.6806	0.18018	-
SSR x TC	-0.00019	0.02482	0.9941	-0.0042	-
TL x TC	-0.00046	0.01947	0.9812	-0.01175	-
SSR x TL x TC	5.82E-05	0.000369	0.8762	0.08953	0.0135
Step 4					
SSR	-0.00718	0.000976	<.0001	-0.55831	-
TL	-0.00813	0.00187	0.0003	-0.3401	-
TC	0.9448	0.23645	0.0007	0.34575	-
SSR x TL	3.27E-05	1.42E-05	0.0317	0.18018	-
SSR x TC	-0.00022	0.00448	0.9616	-0.00493	-
TL x TC	-0.00049	0.00352	0.8899	-0.01252	-
SSR x TL x TC	5.86E-05	6.67E-05	0.3901	0.09016	-
1/SSR	7.12952	0.27892	<.0001	0.55046	11.627
1/TL	5.55175	6.46918	0.401	0.06243	0.0131

Note. R^2 = 0.8749 for Step 1. ?R^2 = 0.0050 for Step 2. ?R^2 = 0.0001 for Step 3.
? R^2 = 0.1164 for Step 4. ?R^2 = 0.0029 Step 5.

Table E8 (continued). Hierarchical Regression Analysis for log of MRMSE of a2 for reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 5					
SSR	-0.01556	0.00105	<.0001	-1.20983	-
TL	-0.00813	0.000847	<.0001	-0.34011	-
TC	0.9447	0.10742	<.0001	0.34572	-
SSR x TL	3.27E-05	6.43E-06	<.0001	0.18018	-
SSR x TC	-0.00022	0.00204	0.9162	-0.0049	-
TL x TC	-0.00049	0.0016	0.7616	-0.01249	-
SSR x TL x TC	5.86E-05	3.03E-05	0.0682	0.09013	-
1/SSR	5.05815	0.2667	<.0001	0.39053	-
1/TL	5.55076	2.9389	0.0743	0.06242	-
SSR^2	5.95E-05	6.74E-06	<.0001	0.53789	0.286

Note. R^2 = 0.8749 for Step 1. ?R^2 = 0.0050 for Step 2. ?R^2 = 0.0001 for Step 3. ? R^2 = 0.1164 for Step 4. ?R^2 = 0.0029 Step 5.

Table E9. Summary of Hierarchical Regression Analysis for log of MBIAS of a1 with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.01861	0.0043	<.0001	-0.44898	20.151
TL	-0.02964	0.00593	<.0001	-0.51839	26.863
TC	2.31577	0.92453	0.016	0.25992	6.755
Step 2					
SSR	-0.05029	0.01068	<.0001	-1.213	-
TL	-0.04746	0.00971	<.0001	-0.83003	-
TC	3.58348	2.3594	0.1365	0.40221	-
SSR x TL	0.000578	0.000167	0.0013	0.90486	10.629
SSR x TC	-0.00043	0.02606	0.9868	-0.00283	0.00025
TL x TC	-0.02241	0.03595	0.5364	-0.16147	0.346
Step 3					
SSR	-0.04882	0.01418	0.0014	-1.17755	-
TL	-0.04649	0.01156	0.0002	-0.81299	-
TC	3.94866	3.30129	0.2387	0.4432	-
SSR x TL	0.000551	0.000238	0.0259	0.86282	-
SSR x TC	-0.01035	0.06728	0.8785	-0.06772	-
TL x TC	-0.02903	0.05501	0.6007	-0.20909	-
SSR x TL x TC	0.000181	0.00113	0.8735	0.07222	0.023364
Step 4					
SSR	-0.03543	0.00959	0.0007	-0.85462	-
TL	-0.00184	0.01195	0.8783	-0.03221	-
TC	4.29452	2.16057	0.0541	0.48202	-
SSR x TL	0.000567	0.000156	0.0008	0.88817	-
SSR x TC	-0.01527	0.04403	0.7306	-0.09992	-
TL x TC	-0.03366	0.036	0.3557	-0.24246	-
SSR x TL x TC	0.000247	0.000738	0.7401	0.09856	-
1/SSR	10.2797	1.80986	<.0001	0.49376	12.573
1/TL	93.42462	19.05675	<.0001	0.8518	9.367

Note. R^2 = .5262 for Step 1. ?R^2 = .1094 for Step 2. ?R^2 = 0.0003 for Step 3. ? R^2 = .2160 for Step 4.

Table E10. Summary of Hierarchical Regression Analysis for log of MBIAS of a2 with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.01915	0.00423	<.0001	-0.48513	23.527
TL	-0.02332	0.00583	0.0002	-0.42834	18.341
TC	2.50402	0.90886	0.0085	0.29518	8.712
Step 2					
SSR	-0.04774	0.0107	<.0001	-1.20947	-
TL	-0.0414	0.00973	0.0001	-0.76045	-
TC	3.28283	2.36372	0.1724	0.38699	-
SSR x TL	0.000536	0.000167	0.0027	0.88059	10.067
SSR x TC	-0.00543	0.02611	0.8363	-0.03731	0.0425
TL x TC	-0.01024	0.03601	0.7775	-0.07749	0.0796
Step 3					
SSR	-0.04452	0.01419	0.0032	-1.12795	-
TL	-0.03927	0.01156	0.0016	-0.72127	-
TC	4.08229	3.30333	0.2237	0.48124	-
SSR x TL	0.000477	0.000238	0.0522	0.78392	-
SSR x TC	-0.02714	0.06732	0.689	-0.1865	-
TL x TC	-0.02471	0.05505	0.6559	-0.18698	-
SSR x TL x TC	0.000396	0.00113	0.7278	0.16606	0.124
Step 4					
SSR	-0.02898	0.00991	0.0058	-0.73413	-
TL	-0.00911	0.01235	0.4653	-0.16727	-
TC	4.42226	2.23233	0.0549	0.52131	-
SSR x TL	0.000493	0.000161	0.004	0.81005	-
SSR x TC	-0.032	0.04549	0.486	-0.21997	-
TL x TC	-0.02919	0.0372	0.4374	-0.22086	-
SSR x TL x TC	0.00046	0.000762	0.5502	0.19298	-
1/SSR	11.79057	1.86997	<.0001	0.5948	18.245
1/TL	63.81632	19.68971	0.0025	0.6111	4.821

Note. $R^2 = 0.4950$ for Step 1. $?R^2 = 0.1016$ for Step 2. $?R^2 = 0.0012$ for Step 3. $? R^2 = 0.2278$ for Step 4.

Table E11. Hierarchical Regression Analysis for log of MBIAS of a1 for the reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.00574	0.000636	<.0001	-0.53921	29.075
TL	-0.00289	0.00118	0.0218	-0.14588	2.128
TC	1.74482	0.13523	<.0001	0.77147	59.517
Step 2					
SSR	-0.00868	0.00242	0.0015	-0.81586	-
TL	-0.00469	0.00215	0.0393	-0.23704	-
TC	1.00064	0.52487	0.0692	0.44243	-
SSR x TL	2.37E-05	3.48E-05	0.5032	0.15781	0.146
SSR x TC	0.00937	0.00398	0.0275	0.25519	1.743
TL x TC	0.0054	0.0074	0.4733	0.16552	0.167
Step 3					
SSR	-0.00849	0.00338	0.0199	-0.79801	-
TL	-0.00457	0.00265	0.0994	-0.23084	-
TC	1.05383	0.8414	0.2235	0.46595	-
SSR x TL	2.08E-05	5.04E-05	0.684	0.13835	-
SSR x TC	0.0081	0.01596	0.6167	0.2207	-
TL x TC	0.00458	0.01252	0.718	0.14042	-
SSR x TL x TC	1.95E-05	0.000237	0.9353	0.03621	0.0022
Step 4					
SSR	-0.00559	0.00182	0.0061	-0.52513	-
TL	0.00483	0.00349	0.1817	0.24398	-
TC	1.05528	0.44227	0.027	0.46659	-
SSR x TL	2.08E-05	2.65E-05	0.4419	0.13835	-
SSR x TC	0.00808	0.00839	0.3468	0.22016	-
TL x TC	0.00456	0.00658	0.4963	0.13986	-
SSR x TL x TC	1.97E-05	0.000125	0.876	0.03667	-
1/SSR	3.72577	0.52172	<.0001	0.34755	4.635
1/TL	35.55003	12.10045	0.0081	0.483	0.785

Note. R^2 = 0.9070 for Step 1. ?R^2 =0.0206 for Step 2. ?R^2 =0.0000 for Step 3. ? R^2 = 0.0542 for Step 4. ?R^2 = 0.0026 Step 5.

Table E11 (continued). Hierarchical Regression Analysis for log of MBIAS of a1 for reduced data set (N = 30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 5					
SSR	-0.0122	0.0041	0.0077	-1.14625	-
TL	0.00483	0.00331	0.1615	0.24397	-
TC	1.05519	0.42005	0.0212	0.46655	-
SSR x TL	2.08E-05	2.51E-05	0.419	0.13835	-
SSR x TC	0.00808	0.00797	0.323	0.22019	-
TL x TC	0.00456	0.00625	0.4744	0.13989	-
SSR x TL x TC	1.97E-05	0.000118	0.8696	0.03664	-
1/SSR	2.09138	1.04292	0.0594	0.19509	-
1/TL	35.54925	11.49252	0.006	0.48299	-
SSR^2	4.7E-05	2.64E-05	0.0909	0.51278	0.260

Note. R^2 = 0.9070 for Step 1. ?R^2 =0.0206 for Step 2. ?R^2 =0.0000 for Step 3. ? R^2 = 0.0542 for Step 4. ?R^2 = 0.0026 Step 5.

Table E12. Hierarchical Regression Analysis for log of MBIAS of a2 for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.00686	0.000917	<.0001	-0.50364	25.366
TL	-0.00443	0.0017	0.0153	-0.17473	3.053
TC	2.24018	0.19485	<.0001	0.77349	59.829
Step 2					
SSR	-0.01195	0.00376	0.0042	-0.87644	-
TL	-0.00746	0.00334	0.0354	-0.2943	-
TC	1.85169	0.81589	0.0329	0.63935	-
SSR x TL	6.45E-05	5.41E-05	0.2458	0.33549	0.658
SSR x TC	0.00593	0.00619	0.3476	0.12623	0.426
TL x TC	0.00214	0.0115	0.8539	0.05133	0.016
Step 3					
SSR	-0.01307	0.00525	0.0209	-0.95875	-
TL	-0.00818	0.00412	0.0595	-0.32291	-
TC	1.53746	1.30524	0.2514	0.53085	-
SSR x TL	8.17E-05	7.81E-05	0.3067	0.42529	-
SSR x TC	0.01341	0.02475	0.5933	0.28538	-
TL x TC	0.00698	0.01942	0.7228	0.16713	-
SSR x TL x TC	-0.00012	0.000368	0.7576	-0.16705	0.047
Step 4					
SSR	-0.00798	0.00234	0.0028	-0.58516	-
TL	-0.01255	0.00448	0.0111	-0.4952	-
TC	1.53951	0.56805	0.0135	0.53156	-
SSR x TL	8.17E-05	3.4E-05	0.026	0.42529	-
SSR x TC	0.01339	0.01077	0.2284	0.28478	-
TL x TC	0.00695	0.00845	0.4204	0.16651	-
SSR x TL x TC	-0.00011	0.00016	0.4823	-0.16654	-
1/SSR	6.53191	0.67009	<.0001	0.47583	8.688
1/TL	-16.5185	15.54164	0.3005	-0.17526	0.103

Note. R^2 = 0.8823 for Step 1. ?R^2 =0.0110 for Step 2. ?R^2 =0.0005 for Step 3. ? R^2 = 0.0879 for Step 4. ?R^2 = 0.0018 Step 5.

Table E12. (continued). Summary of Hierarchical Regression Analysis for log of MBIAS of a2 for reduced data set (N=30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 5					
SSR	-0.01506	0.00539	0.0116	-1.10508	-
TL	-0.01255	0.00436	0.0097	-0.49521	-
TC	1.53943	0.55301	0.0118	0.53153	-
SSR x TL	8.17E-05	3.31E-05	0.0232	0.42529	-
SSR x TC	0.01339	0.01049	0.2171	0.28481	-
TL x TC	0.00695	0.00823	0.4086	0.16654	-
SSR x TL x TC	-0.00011	0.000156	0.471	-0.16656	-
1/SSR	4.77996	1.37303	0.0025	0.3482	-
1/TL	-16.5193	15.13011	0.2886	-0.17527	-
SSR^2	5.04E-05	3.47E-05	0.1633	0.42924	0.182

Note. R^2 = 0.8823 for Step 1. ?R^2 =0.0110 for Step 2. ?R^2 =0.0005 for Step 3.
? R^2 = 0.0879 for Step 4. ?R^2 = 0.0018 Step 5.

Table E13. Hierarchical Regression Analysis for log of MVAR of d with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.0627	0.01236	<.0001	-0.52891	27.966
TL	-0.07827	0.01706	<.0001	-0.47869	22.906
TC	3.90598	2.65711	0.1487	0.15331	2.350
Step 2					
SSR	-0.12511	0.03183	0.0003	-1.05539	-
TL	-0.11481	0.02894	0.0003	-0.70212	-
TC	11.16796	7.03357	0.12	0.43834	-
SSR x TL	0.00133	0.000498	0.0106	0.73006	6.919
SSR x TC	-0.07269	0.07769	0.3549	-0.16635	0.846
TL x TC	-0.08327	0.10716	0.4416	-0.20977	0.583
Step 3					
SSR	-0.11669	0.04222	0.0086	-0.98435	-
TL	-0.10922	0.03442	0.0029	-0.66798	-
TC	13.26038	9.8329	0.1851	0.52046	-
SSR x TL	0.00118	0.000709	0.1041	0.64581	-
SSR x TC	-0.12951	0.20039	0.5218	-0.29636	-
TL x TC	-0.12115	0.16385	0.464	-0.30518	-
SSR x TL x TC	0.00104	0.00336	0.7595	0.14471	0.094
Step 4					
SSR	-0.06608	0.02709	0.0195	-0.5574	-
TL	-0.02705	0.03376	0.4279	-0.16545	-
TC	14.32178	6.10392	0.0243	0.56212	-
SSR x TL	0.00123	0.00044	0.0082	0.67297	-
SSR x TC	-0.14474	0.12438	0.2518	-0.33122	-
TL x TC	-0.13507	0.10171	0.1921	-0.34023	-
SSR x TL x TC	0.00123	0.00208	0.5572	0.17262	-
1/SSR	38.28468	5.1131	<.0001	0.64304	21.325
1/TL	174.7551	53.83803	0.0024	0.55717	4.008

Note. R^2 = 0.5215 for Step 1. ?R^2 = 0.0825 for Step 2. ?R^2 = 0.0010 for Step 3. ? R^2 = 0.2505 for Step 4.

Table E14. Hierarchical Regression Analysis for log of MVAR of d for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.02449	0.0018	<.0001	-0.87542	76.636
TL	-0.01723	0.00335	<.0001	-0.33131	10.976
TC	0.74757	0.38348	0.0621	0.12575	1.581
Step 2					
SSR	-0.02567	0.00776	0.0031	-0.91759	-
TL	-0.0179	0.00689	0.016	-0.34422	-
TC	0.88334	1.68485	0.6051	0.14859	-
SSR x TL	2.08E-05	0.000112	0.8541	0.05272	0.016
SSR x TC	-0.00115	0.01278	0.9292	-0.0119	0.004
TL x TC	-0.00135	0.02376	0.9553	-0.01572	0.002
Step 3					
SSR	-0.02506	0.01086	0.0309	-0.89583	-
TL	-0.01751	0.00852	0.052	-0.33666	-
TC	1.05382	2.70095	0.7002	0.17726	-
SSR x TL	1.14E-05	0.000162	0.9442	0.02898	-
SSR x TC	-0.00521	0.05122	0.92	-0.05396	-
TL x TC	-0.00397	0.04018	0.9222	-0.04633	-
SSR x TL x TC	6.24E-05	0.000762	0.9354	0.04415	0.003
Step 4					
SSR	-0.01364	0.0019	<.0001	-0.48758	-
TL	-0.00726	0.00362	0.059	-0.13953	-
TC	1.0589	0.45937	0.032	0.17812	-
SSR x TL	1.14E-05	2.75E-05	0.6819	0.02898	-
SSR x TC	-0.00527	0.00871	0.5516	-0.05467	-
TL x TC	-0.00403	0.00683	0.5616	-0.04708	-
SSR x TL x TC	6.33E-05	0.00013	0.6305	0.04477	-
1/SSR	14.65152	0.54188	<.0001	0.51997	10.375
1/TL	38.79602	12.56809	0.0058	0.20053	0.135

Note. R^2 = 0.8918 for Step 1. ?R^2 =0.0002 for Step 2. ?R^2 =0.0001 for Step 3. ? R^2 = 0.1051 for Step 4. ?R^2 = 0.0023 for Step 5.

Table E14 (continued). Summary of Hierarchical Regression Analysis for log of MVAR of d for reduced data set (N=30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 5					
SSR	-0.03005	0.00196	<.0001	-1.07424	-
TL	-0.00726	0.00159	0.0002	-0.13954	-
TC	1.05869	0.2014	<.0001	0.17808	-
SSR x TL	1.14E-05	1.21E-05	0.3547	0.02898	-
SSR x TC	-0.00527	0.00382	0.1835	-0.05464	-
TL x TC	-0.00403	0.003	0.1943	-0.04705	-
SSR x TL x TC	6.33E-05	5.68E-05	0.2793	0.04474	-
1/SSR	10.59382	0.50005	<.0001	0.37596	-
1/TL	38.79407	5.5103	<.0001	0.20052	-
SSR^2	0.000117	1.26E-05	<.0001	0.48433	0.232

Note. R^2 = 0.8918 for Step 1. ?R^2 =0.0002 for Step 2. ?R^2 =0.0001 for Step 3. ? R^2 = 0.1051 for Step 4. ?R^2 = 0.0023 for Step 5.

Table E15. Hierarchical Regression Analysis for log of MRMSE of d with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.02195	0.00389	<.0001	-0.57664	33.240
TL	-0.02356	0.00536	<.0001	-0.44863	20.120
TC	1.10142	0.83569	0.1943	0.13461	1.812
Step 2					
SSR	-0.04042	0.01027	0.0003	-1.06162	-
TL	-0.03458	0.00934	0.0006	-0.65843	-
TC	2.68378	2.26938	0.2438	0.328	-
SSR x TL	0.000377	0.000161	0.0241	0.6419	5.349
SSR x TC	-0.01486	0.02507	0.5564	-0.10591	0.343
TL x TC	-0.01874	0.03458	0.5908	-0.14695	0.286
Step 3					
SSR	-0.03662	0.01361	0.0103	-0.96192	-
TL	-0.03206	0.01109	0.0062	-0.61052	-
TC	3.62687	3.169	0.2592	0.44326	-
SSR x TL	0.000307	0.000229	0.1866	0.52367	-
SSR x TC	-0.04047	0.06458	0.5345	-0.28837	-
TL x TC	-0.03581	0.05281	0.5016	-0.28086	-
SSR x TL x TC	0.000467	0.00108	0.6688	0.2031	0.185
Step 4					
SSR	-0.02017	0.00883	0.0281	-0.5298	-
TL	-0.00795	0.01101	0.4743	-0.15146	-
TC	3.96447	1.99001	0.0536	0.48452	-
SSR x TL	0.000323	0.000144	0.0303	0.55055	-
SSR x TC	-0.04532	0.04055	0.2708	-0.32293	-
TL x TC	-0.04022	0.03316	0.2326	-0.31549	-
SSR x TL x TC	0.00053	0.00068	0.4404	0.23069	-
1/SSR	12.42623	1.66698	<.0001	0.64991	21.783
1/TL	51.44063	17.55238	0.0057	0.5107	3.367

Note. R^2 =0.5411 for Step 1. ?R^2 = 0.0592 for Step 2. ?R^2 = 0.0019 for Step 3. ? R^2 = 0.2488 for Step 4.

Table E16. Hierarchical Regression Analysis for log of MRMSE of d for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.01141	0.000899	<.0001	-0.8637	74.598
TL	-0.00786	0.00167	<.0001	-0.32006	10.244
TC	0.49655	0.19106	0.0152	0.17687	3.128
Step 2					
SSR	-0.01313	0.00385	0.0024	-0.99391	-
TL	-0.00898	0.00342	0.015	-0.36557	-
TC	0.32931	0.83551	0.6971	0.1173	-
SSR x TL	2.19E-05	5.54E-05	0.6959	0.1178	0.080
SSR x TC	0.00196	0.00634	0.76	0.043	0.049
TL x TC	0.00131	0.01178	0.9126	0.0323	0.006
Step 3					
SSR	-0.01255	0.00539	0.0294	-0.94965	-
TL	-0.0086	0.00422	0.0539	-0.35018	-
TC	0.49313	1.33883	0.7161	0.17566	-
SSR x TL	1.3E-05	8.01E-05	0.8731	0.0695	-
SSR x TC	-0.00194	0.02539	0.9398	-0.04259	-
TL x TC	-0.00121	0.01991	0.952	-0.02998	-
SSR x TL x TC	6E-05	0.000378	0.8752	0.08984	0.014
Step 4					
SSR	-0.00688	0.000917	<.0001	-0.52076	-
TL	-0.0036	0.00175	0.0536	-0.14642	-
TC	0.49565	0.22225	0.0374	0.17655	-
SSR x TL	1.3E-05	1.33E-05	0.3419	0.0695	-
SSR x TC	-0.00197	0.00421	0.6445	-0.04334	-
TL x TC	-0.00125	0.00331	0.7104	-0.03077	-
SSR x TL x TC	6.04E-05	6.27E-05	0.3467	0.09049	-
1/SSR	7.2687	0.26217	<.0001	0.54626	11.450
1/TL	18.93625	6.08062	0.0055	0.20727	0.144

Note. R^2 = 0.8796 for Step 1. ?R^2 =0.0013 for Step 2. ?R^2 =0.0002 for Step 3.
? R^2 = 0.1150 for Step 4. ?R^2 = 0.0025 for Step 5.

Table E16 (continued). Summary of Hierarchical Regression Analysis for log of MRMSE of d for reduced data set (N=30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 5					
SSR	-0.01487	0.000925	<.0001	-1.12532	-
TL	-0.0036	0.000748	0.0001	-0.14643	-
TC	0.49555	0.09485	<.0001	0.17652	-
SSR x TL	1.3E-05	5.68E-06	0.0342	0.0695	-
SSR x TC	-0.00197	0.0018	0.2863	-0.04331	-
TL x TC	-0.00124	0.00141	0.389	-0.03074	-
SSR x TL x TC	6.04E-05	2.68E-05	0.0359	0.09046	-
1/SSR	5.29403	0.23549	<.0001	0.39786	-
1/TL	18.93531	2.59494	<.0001	0.20726	-
SSR^2	5.68E-05	5.95E-06	<.0001	0.49912	0.246

Note. R^2 = 0.8796 for Step 1. ?R^2 =0.0013 for Step 2. ?R^2 =0.0002 for Step 3. ? R^2 = 0.1150 for Step 4. ?R^2 = 0.0025 for Step 5.

Table E17. Hierarchical Regression Analysis for log of absolute value of MBIAS of d with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.0298	0.00513	<.0001	-0.62911	39.565
TL	-0.02007	0.00708	0.0069	-0.30723	9.435
TC	0.27162	1.10258	0.8066	0.02668	0.071
Step 2					
SSR	-0.02666	0.01439	0.0711	-0.5628	-
TL	-0.02302	0.01308	0.0859	-0.35233	-
TC	0.50168	3.17903	0.8754	0.04928	-
SSR x TL	2.02E-05	0.000225	0.9291	0.02764	0.01
SSR x TC	-0.02842	0.03511	0.423	-0.16277	0.810
TL x TC	0.01461	0.04843	0.7645	0.09209	0.112
Step 3					
SSR	-0.02233	0.01908	0.2488	-0.47138	-
TL	-0.02015	0.01555	0.2025	-0.3084	-
TC	1.57754	4.44272	0.7244	0.15497	-
SSR x TL	-5.9E-05	0.00032	0.855	-0.08077	-
SSR x TC	-0.05763	0.09054	0.5281	-0.33007	-
TL x TC	-0.00487	0.07403	0.9479	-0.03069	-
SSR x TL x TC	0.000532	0.00152	0.7276	0.18622	0.155
SSR	-0.00323	0.01601	0.8411	-0.06822	-
TL	-0.01145	0.01995	0.5695	-0.17523	-
TC	1.9146	3.60804	0.5988	0.18808	-
SSR x TL	-4.3E-05	0.00026	0.8689	-0.05923	-
SSR x TC	-0.06251	0.07352	0.4005	-0.35801	-
TL x TC	-0.00919	0.06012	0.8793	-0.05794	-
SSR x TL x TC	0.000595	0.00123	0.6321	0.20812	-
1/SSR	14.29483	3.02236	<.0001	0.60092	18.623
1/TL	19.98436	31.82373	0.5338	0.15947	0.328

Note. R^2 =0.4839 for Step 1. ?R^2 = 0.0094 for Step 2. ?R^2 = 0.0016 for Step 3. ?R^2 = 0.1887 for Step 4.

Table E18. Hierarchical Regression Analysis for log of absolute value MBIAS of d for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.02036	0.0039	<.0001	-0.6883	47.375
TL	-0.01493	0.00725	0.0495	-0.27162	7.378
TC	0.13604	0.82869	0.8709	0.02165	0.047
Step 2					
SSR	0.01901	0.01443	0.2007	0.64261	-
TL	0.01078	0.01281	0.4086	0.19614	-
TC	0.94977	3.13413	0.7646	0.15113	-
SSR x TL	-0.00059	0.000208	0.0097	-1.40648	11.563
SSR x TC	-0.00823	0.02377	0.7322	-0.08072	0.174
TL x TC	-0.0072	0.04419	0.8719	-0.07951	0.039
Step 3					
SSR	0.02035	0.02021	0.3249	0.68805	-
TL	0.01165	0.01585	0.47	0.21194	-
TC	1.32619	5.02394	0.7943	0.21102	-
SSR x TL	-0.00061	0.000301	0.0557	-1.45606	-
SSR x TC	-0.01719	0.09527	0.8584	-0.16858	-
TL x TC	-0.01299	0.07473	0.8636	-0.14344	-
SSR x TL x TC	0.000138	0.00142	0.9234	0.09222	0.0144
Step 4					
SSR	0.02845	0.01939	0.158	0.96191	-
TL	0.05494	0.03708	0.154	0.99931	-
TC	1.33063	4.70033	0.78	0.21173	-
SSR x TL	-0.00061	0.000281	0.0432	-1.45606	-
SSR x TC	-0.01725	0.08914	0.8485	-0.16917	-
TL x TC	-0.01305	0.06992	0.8538	-0.14406	-
SSR x TL x TC	0.000139	0.00133	0.9178	0.09273	-
1/SSR	10.39038	5.54466	0.0756	0.34881	4.669
1/TL	163.812	128.5989	0.2173	0.80094	2.157

Note. R^2 = 0.5479 for Step 1. ?R^2 =0.1178 for Step 2. ?R^2 =0.0001 for Step 3. ? R^2 = 0.0683 for Step 4.

Table E19. Hierarchical Regression Analysis for log of MVAR of MDISC with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.06367	0.01455	<.0001	-0.50732	25.728
TL	-0.06471	0.02008	0.0024	-0.37379	13.967
TC	3.78012	3.12774	0.2333	0.14014	1.964
Step 2					
SSR	-0.1154	0.03901	0.0051	-0.91943	-
TL	-0.09672	0.03547	0.0094	-0.55871	-
TC	10.94283	8.61872	0.2114	0.40568	-
SSR x TL	0.00117	0.000611	0.0621	0.60549	4.759
SSR x TC	-0.08449	0.09519	0.38	-0.18262	1.019
TL x TC	-0.07378	0.13131	0.5773	-0.17555	0.408
Step 3					
SSR	-0.10076	0.05168	0.0582	-0.80283	-
TL	-0.08702	0.04213	0.0454	-0.50268	-
TC	14.57873	12.03446	0.2328	0.54047	-
SSR x TL	0.000904	0.000868	0.3042	0.46722	-
SSR x TC	-0.18321	0.24525	0.4594	-0.396	-
TL x TC	-0.1396	0.20054	0.4904	-0.33215	-
SSR x TL x TC	0.0018	0.00411	0.6641	0.23751	0.253
Step 4					
SSR	-0.03539	0.03476	0.315	-0.28195	-
TL	-0.0559	0.04331	0.2046	-0.3229	-
TC	15.73644	7.83133	0.0516	0.58339	-
SSR x TL	0.000958	0.000565	0.0982	0.49514	-
SSR x TC	-0.19996	0.15958	0.2178	-0.43221	-
TL x TC	-0.15445	0.13049	0.2439	-0.36749	-
SSR x TL x TC	0.00201	0.00267	0.4562	0.2659	-
1/SSR	48.94634	6.56011	<.0001	0.77653	31.098
1/TL	71.17097	69.07418	0.3094	0.21433	0.593

Note. R^2 =0.4085 for Step 1. ?R^2 = 0.0611 for Step 2. ?R^2 = 0.0015 for Step 3. ? R^2 = 0.3156 for Step 4.

Table E20. Hierarchical Regression Analysis for log of MVAR of MDISC for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.02567	0.00186	<.0001	-0.86852	75.433
TL	-0.01922	0.00346	<.0001	-0.34983	12.238
TC	0.89592	0.39538	0.032	0.14266	2.035
Step 2					
SSR	-0.02661	0.00798	0.0029	-0.90044	-
TL	-0.01957	0.00709	0.0111	-0.35625	-
TC	1.27717	1.73437	0.4689	0.20337	-
SSR x TL	2.19E-05	0.000115	0.8506	0.0526	0.016
SSR x TC	-0.00321	0.01315	0.8093	-0.0315	0.027
TL x TC	-0.00379	0.02445	0.8782	-0.04187	0.011
Step 3					
SSR	-0.02578	0.01118	0.031	-0.87218	-
TL	-0.01903	0.00877	0.0411	-0.34643	-
TC	1.51111	2.78001	0.5922	0.24063	-
SSR x TL	9.07E-06	0.000166	0.957	0.02176	-
SSR x TC	-0.00878	0.05272	0.8693	-0.08614	-
TL x TC	-0.00739	0.04135	0.8598	-0.08163	-
SSR x TL x TC	8.57E-05	0.000784	0.914	0.05736	0.006
Step 4					
SSR	-0.01402	0.00196	<.0001	-0.47449	-
TL	-0.00837	0.00374	0.037	-0.15229	-
TC	1.51634	0.47459	0.0045	0.24146	-
SSR x TL	9.07E-06	2.84E-05	0.7528	0.02176	-
SSR x TC	-0.00885	0.009	0.3371	-0.08684	-
TL x TC	-0.00746	0.00706	0.3035	-0.08236	-
SSR x TL x TC	8.66E-05	0.000134	0.5252	0.05796	-
1/SSR	15.0767	0.55984	<.0001	0.50651	9.845
1/TL	40.35905	12.98445	0.0055	0.19748	0.131

Note. R^2 = 0.8969 for Step 1. ?R^2 = 0.0006 for Step 2. ?R^2 = .0000 for Step 3. ? R^2 = 0.0998 for Step 4.

Table E21. Hierarchical Regression Analysis for log of MRSME of MDISC with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.02019	0.00434	<.0001	-0.52065	27.098
TL	-0.0207	0.00599	0.0012	-0.38707	14.977
TC	1.60967	0.93287	0.0915	0.19314	3.730
Step 2					
SSR	-0.039	0.01161	0.0017	-1.00568	-
TL	-0.03194	0.01056	0.0043	-0.59717	-
TC	3.20212	2.56524	0.219	0.38421	-
SSR x TL	0.000383	0.000182	0.0412	0.64111	5.336
SSR x TC	-0.01495	0.02833	0.6006	-0.10459	0.334
TL x TC	-0.01886	0.03908	0.632	-0.14521	0.279
Step 3					
SSR	-0.03347	0.01536	0.0353	-0.86298	-
TL	-0.02827	0.01252	0.0295	-0.5286	-
TC	4.57698	3.57661	0.208	0.54918	-
SSR x TL	0.000282	0.000258	0.281	0.47189	-
SSR x TC	-0.05228	0.07289	0.4774	-0.36574	-
TL x TC	-0.04374	0.0596	0.4673	-0.33687	-
SSR x TL x TC	0.00068	0.00122	0.5808	0.29068	0.378
Step 4					
SSR	-0.01462	0.01049	0.1716	-0.3769	-
TL	-0.01215	0.01307	0.3585	-0.22711	-
TC	4.93112	2.36327	0.0437	0.59167	-
SSR x TL	0.000299	0.00017	0.088	0.49956	-
SSR x TC	-0.05739	0.04816	0.2408	-0.40148	-
TL x TC	-0.04832	0.03938	0.2273	-0.37214	-
SSR x TL x TC	0.000746	0.000807	0.361	0.31892	-
1/SSR	14.16071	1.97965	<.0001	0.72711	27.266
1/TL	35.25693	20.84458	0.0989	0.34364	1.524

Note. R^2 =0.4488 for Step 1. ?R^2 = 0.0590 for Step 2. ?R^2 = 0.0038 for Step 3. ? R^2 = 0.2859 for Step 4.

Table E22. Hierarchical Regression Analysis for log of MRMSE of MDISC for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.0093	0.000884	<.0001	-0.7831	61.324
TL	-0.00664	0.00164	0.0004	-0.30086	9.051
TC	0.98461	0.18779	<.0001	0.39031	15.234
Step 2					
SSR	-0.01317	0.00365	0.0015	-1.10934	-
TL	-0.00911	0.00324	0.01	-0.41265	-
TC	0.471	0.79301	0.5584	0.18671	-
SSR x TL	4.51E-05	5.26E-05	0.4007	0.26916	0.423
SSR x TC	0.0063	0.00601	0.3061	0.15377	0.633
TL x TC	0.00383	0.01118	0.7348	0.10543	0.068
Step 3					
SSR	-0.0125	0.00511	0.0229	-1.05295	-
TL	-0.00867	0.00401	0.0416	-0.39305	-
TC	0.65853	1.27039	0.6094	0.26105	-
SSR x TL	3.48E-05	7.6E-05	0.652	0.20763	-
SSR x TC	0.00183	0.02409	0.9401	0.04473	-
TL x TC	0.000949	0.0189	0.9604	0.02609	-
SSR x TL x TC	6.87E-05	0.000358	0.8498	0.11446	0.022
Step 4					
SSR	-0.00714	0.000951	<.0001	-0.60121	-
TL	-0.00391	0.00182	0.044	-0.17708	-
TC	0.66091	0.23045	0.0095	0.26199	-
SSR x TL	3.48E-05	1.38E-05	0.0203	0.20763	-
SSR x TC	0.0018	0.00437	0.6849	0.04395	-
TL x TC	0.000918	0.00343	0.7915	0.02526	-
SSR x TL x TC	6.91E-05	6.5E-05	0.3005	0.11514	
1/SSR	6.87939	0.27184	<.0001	0.57535	12.703
1/TL	18.03565	6.30489	0.0097	0.21969	0.162

Note. R^2 = 0.8559 for Step 1. ?R^2 =0.0113 for Step 2. ?R^2 =0.0002 for Step 3. ? R^2 = 0.1286 for Step 4. ?R^2 = 0.0026 for Step 5.

Table E22. (continued). Summary of Hierarchical Regression Analysis for log of MRMSE of MDISC for reduced data set (N=30)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 5					
SSR	-0.01447	0.00137	<.0001	-1.21858	-
TL	-0.00391	0.00111	0.0022	-0.17709	-
TC	0.66082	0.14034	0.0002	0.26196	-
SSR x TL	3.48E-05	8.4E-06	0.0006	0.20763	-
SSR x TC	0.0018	0.00266	0.5069	0.04398	-
TL x TC	0.000919	0.00209	0.6646	0.02529	-
SSR x TL x TC	6.91E-05	3.96E-05	0.0972	0.11511	-
1/SSR	5.06742	0.34844	<.0001	0.42381	-
1/TL	18.03478	3.83964	0.0002	0.21968	-
SSR^2	5.21E-05	8.81E-06	<.0001	0.50969	0.257

Note. R^2 = 0.8559 for Step 1. ?R^2 =0.0113 for Step 2. ?R^2 =0.0002 for Step 3. ? R^2 = 0.1286 for Step 4. ?R^2 = 0.0026 for Step 5.

Table E23. Hierarchical Regression Analysis for log of MBIAS of MDISC with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.01651	0.00387	0.0001	-0.4761	22.659
TL	-0.01744	0.00533	0.0021	-0.36451	13.282
TC	2.40279	0.83071	0.0059	0.32233	10.389
Step 2					
SSR	-0.03689	0.0102	0.0008	-1.06363	-
TL	-0.02949	0.00927	0.0028	-0.61634	-
TC	3.49432	2.25276	0.1286	0.46876	-
SSR x TL	0.00039	0.00016	0.0189	0.72972	6.613
SR x TC	-0.00695	0.02488	0.7815	-0.05434	0.0902
TL x TC	-0.01502	0.03432	0.6639	-0.12935	0.222
Step 3					
SSR	-0.03314	0.01351	0.0186	-0.95534	-
TL	-0.027	0.01101	0.0187	-0.5643	-
TC	4.42749	3.14583	0.167	0.59395	-
SSR x TL	0.000321	0.000227	0.1645	0.60131	-
SSR x TC	-0.03228	0.06411	0.6173	-0.25251	-
TL x TC	-0.03192	0.05242	0.5461	-0.27479	-
SSR x TL x TC	0.000462	0.00107	0.6698	0.22058	0.218
Step 4					
SSR	-0.01735	0.00959	0.0783	-0.50019	-
TL	-0.00908	0.01195	0.4522	-0.18969	-
TC	4.73664	2.1601	0.0345	0.63542	-
SSR x TL	0.000336	0.000156	0.0375	0.62832	-
SSR x TC	-0.03674	0.04402	0.4092	-0.28732	-
TL x TC	-0.03594	0.03599	0.3244	-0.3094	-
SSR x TL x TC	0.00052	0.000738	0.4856	0.24823	-
1/SSR	11.88945	1.80946	<.0001	0.68255	24.026
1/TL	38.62648	19.05259	0.0497	0.42092	2.287

Note. R^2 =0.4536 for Step 1. ?R^2 = 0.0819 for Step 2. ?R^2 = 0.0022 for Step 3. ? R^2 = 0.2608 for Step 4.

Table E24. Hierarchical Regression Analysis for log of MBIAS of MDISC for reduced data set (N= 30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.00669	0.000794	<.0001	-0.53712	28.850
TL	-0.00401	0.00148	0.0116	-0.17295	2.991
TC	2.01047	0.16877	<.0001	0.75907	57.619
Step 2					
SSR	-0.01106	0.00323	0.0023	-0.8876	-
TL	-0.00657	0.00287	0.0315	-0.28351	-
TC	1.59472	0.70169	0.0327	0.6021	-
SSR x TL	5.27E-05	4.66E-05	0.2695	0.29974	0.525
SSR x TC	0.0063	0.00532	0.2489	0.14645	0.574
TL x TC	0.00233	0.00989	0.816	0.06099	0.023
Step 3					
SSR	-0.01126	0.00452	0.0209	-0.9033	-
TL	-0.0067	0.00355	0.0725	-0.28896	-
TC	1.53991	1.12494	0.1849	0.58141	-
SSR x TL	5.57E-05	6.73E-05	0.4169	0.31687	-
SSR x TC	0.0076	0.02133	0.725	0.1768	-
TL x TC	0.00317	0.01673	0.8514	0.08308	-
SSR x TL x TC	-2E-05	0.000317	0.9501	-0.03186	0.002
Step 4					
SSR	-0.00677	0.00178	0.0011	-0.54288	-
TL	-0.00311	0.00341	0.3719	-0.13427	-
TC	1.5419	0.43181	0.0019	0.58216	-
SSR x TL	5.57E-05	2.58E-05	0.0435	0.31687	-
SSR x TC	0.00757	0.00819	0.3661	0.17617	-
TL x TC	0.00315	0.00642	0.6296	0.08242	-
SSR x TL x TC	-2E-05	0.000122	0.8729	-0.03132	-
1/SSR	5.76288	0.50937	<.0001	0.45905	8.086
1/TL	13.56319	11.81405	0.2645	0.15736	0.083

Note. R^2 = 0.8944 for Step 1. ?R^2 =0.0113 for Step 2. ?R^2 =0.0000 for Step 3. ? R^2 = 0.0817 for Step 4. ?R^2 = 0.0019 for Step 5.

Table E24. (continued). Summary of Hierarchical Regression Analysis for log of MBIAS of MDISC for reduced data set (N=30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 5					
SSR	-0.01349	0.00397	0.003	-1.08216	-
TL	-0.00311	0.00321	0.3449	-0.13428	-
TC	1.54181	0.40721	0.0012	0.58213	-
SSR x TL	5.57E-05	2.44E-05	0.0339	0.31687	-
SSR x TC	0.00757	0.00772	0.339	0.1762	-
TL x TC	0.00315	0.00606	0.6093	0.08245	-
SSR x TL x TC	-2E-05	0.000115	0.8653	-0.03134	-
1/SSR	4.10106	1.01103	0.0007	0.32668	-
1/TL	13.56239	11.14101	0.2384	0.15735	-
SSR^2	4.78E-05	2.56E-05	0.0773	0.44522	0.196

Note. R^2 = 0.8944 for Step 1. ?R^2 =0.0113 for Step 2. ?R^2 =0.0000 for Step 3. ? R^2 = 0.0817 for Step 4. ?R^2 = 0.0019 for Step 5.

Table E25. Hierarchical Regression Analysis for log of MVAR of D of MID (N = 48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.02858	0.00201	<.0001	-0.82136	67.439
TL	-0.02085	0.00278	<.0001	-0.4344	18.863
TC	0.37146	0.43283	0.3954	0.04967	0.247
Step 2					
SSR	-0.02816	0.0057	<.0001	-0.80923	-
TL	-0.02071	0.00518	0.0003	-0.43138	-
TC	0.23653	1.25907	0.8519	0.03162	-
SSR x TL	-1.1E-05	8.92E-05	0.9044	-0.02009	0.005
SSR x TC	0.00115	0.01391	0.9348	0.00893	0.002
TL x TC	0.00169	0.01918	0.9301	0.01452	0.003
Step 3					
SSR	-0.02856	0.00757	0.0005	-0.82058	-
TL	-0.02097	0.00617	0.0015	-0.43684	-
TC	0.1384	1.76211	0.9378	0.0185	-
SSR x TL	-3.6E-06	0.000127	0.9778	-0.00663	-
SSR x TC	0.00381	0.03591	0.916	0.0297	-
TL x TC	0.00347	0.02936	0.9066	0.02976	-
SSR x TL x TC	-4.9E-05	0.000602	0.9361	-0.02312	0.002
Step 4					
SSR	-0.01674	0.0019	<.0001	-0.48095	-
TL	-0.01127	0.00236	<.0001	-0.23469	-
TC	0.35929	0.42716	0.4055	0.04804	-
SSR x TL	6.75E-06	3.08E-05	0.8277	0.01259	-
SSR x TC	0.000622	0.0087	0.9434	0.00485	-
TL x TC	0.000614	0.00712	0.9317	0.00527	-
SSR x TL x TC	-7.4E-06	0.000146	0.9601	-0.0035	-
1/SSR	8.87654	0.35782	<.0001	0.5079	13.303
1/TL	21.2659	3.76765	<.0001	0.23097	0.689

Note. R^2 = .8527 for Step 1. ?R^2 = 0.0001 for Step 2. ?R^2 = 0.0000 for Step 3.
? R^2 = 0.1390 for Step 4. ?R^2 = 0.0073 for Step 5.

Table E25. (continued) Hierarchical Regression Analysis for log of MVAR of D of MID (N = 48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 5					
SSR	-0.04044	0.00149	<.0001	-1.16211	-
TL	-0.01125	0.000786	<.0001	-0.23432	-
TC	0.35516	0.14203	0.017	0.04749	-
SSR x TL	6.3E-06	1.02E-05	0.5421	0.01176	-
SSR x TC	0.000697	0.00289	0.811	0.00543	-
TL x TC	0.000654	0.00237	0.7837	0.00562	-
SSR x TL x TC	-8.2E-06	4.85E-05	0.8675	-0.00388	-
1/SSR	6.10654	0.19792	<.0001	0.3494	-
1/TL	21.2419	1.25274	<.0001	0.23071	-
SSR^2	0.000186	1.06E-05	<.0001	0.58907	0.733

Note. R^2 = .8527 for Step 1. ?R^2 = 0.0001 for Step 2. ?R^2 = 0.0000 for Step 3. ? R^2 = 0.1390 for Step 4. ?R^2 = 0.0073 for Step 5.

Table E26. Hierarchical Regression Analysis for log of MRMSE of D of MID (N = 48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.01421	0.001	<.0001	-0.82691	68.354
TL	-0.00997	0.00138	<.0001	-0.4206	17.684
TC	0.1802	0.21553	0.4076	0.04878	0.238
Step 2					
SSR	-0.01409	0.00284	<.0001	-0.81951	-
TL	-0.00995	0.00258	0.0004	-0.41986	-
TC	0.11005	0.62701	0.8615	0.02979	-
SSR x TL	-4E-06	4.44E-05	0.9291	-0.01501	0.003
SSR x TC	0.000615	0.00693	0.9297	0.00971	0.003
TL x TC	0.000868	0.00955	0.9281	0.01508	0.003
Step 3					
SSR	-0.01427	0.00377	0.0005	-0.82993	-
TL	-0.01007	0.00307	0.0022	-0.42486	-
TC	0.06555	0.87754	0.9408	0.01774	-
SSR x TL	-7.02E-07	6.33E-05	0.9912	-0.00265	-
SSR x TC	0.00182	0.01788	0.9193	0.02878	-
TL x TC	0.00167	0.01462	0.9095	0.02907	-
SSR x TL x TC	-2.2E-05	0.0003	0.9418	-0.02122	0.002
Step 4					
SSR	-0.00836	0.000963	<.0001	-0.4866	-
TL	-0.00561	0.0012	<.0001	-0.23665	-
TC	0.17476	0.21709	0.4258	0.0473	-
SSR x TL	4.39E-06	1.57E-05	0.7805	0.01659	-
SSR x TC	0.000247	0.00442	0.9558	0.00389	-
TL x TC	0.000264	0.00362	0.9423	0.00458	-
SSR x TL x TC	-1.7E-06	7.42E-05	0.9823	-0.0016	-
1/SSR	4.4296	0.18185	<.0001	0.51312	13.578
1/TL	9.83054	1.91482	<.0001	0.21616	0.603

Note. R^2 = 0.8502 for Step 1. ?R^2 = 0.0001 for Step 2. ?R^2 = 0.0001for Step 3.
? R^2 = .1409 for Step 4. ?R^2 = 0.0077 for Step 5.

Table E26. (continued) Hierarchical Regression Analysis for log of MRMSE of D of MID (N = 48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 5					
SSR	-0.02037	0.000781	<.0001	-1.18495	-
TL	-0.0056	0.000411	<.0001	-0.23627	-
TC	0.17266	0.0743	0.0257	0.04674	-
SSR x TL	4.17E-06	5.36E-06	0.4417	0.01573	-
SSR x TC	0.000285	0.00151	0.8519	0.00449	-
TL x TC	0.000284	0.00124	0.8195	0.00494	-
SSR x TL x TC	-2.1E-06	2.54E-05	0.9356	-0.00199	-
1/SSR	3.02686	0.10353	<.0001	0.35063	-
1/TL	9.81839	0.65531	<.0001	0.21589	-
SSR^2	9.43E-05	5.56E-06	<.0001	0.60393	0.770

Note. R^2 = 0.8502 for Step 1. ?R^2 = 0.0001 for Step 2. ?R^2 = 0.0001for Step 3. ? R^2 = .1409 for Step 4. ?R^2 = 0.0077 for Step 5.

Table E27. Hierarchical Regression Analysis for log absolute value of MBIAS of D of MID with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.02587	0.00536	<.0001	-0.57865	33.472
TL	-0.01072	0.0074	0.1544	-0.17382	3.020
TC	0.76781	1.15254	0.5088	0.0799	0.638
Step 2					
SSR	-0.03388	0.0151	0.0303	-0.75778	-
TL	-0.01373	0.01373	0.3233	-0.22257	-
TC	1.46902	3.33645	0.662	0.15287	-
SSR x TL	0.00014	0.000236	0.5578	0.20273	0.534
SR x TC	0.00224	0.03685	0.9519	0.01358	0.006
TL x TC	-0.01415	0.05083	0.7821	-0.09451	0.118
Step 3					
SSR	-0.03938	0.02001	0.056	-0.88062	-
TL	-0.01737	0.01631	0.2933	-0.2816	-
TC	0.10437	4.6594	0.9822	0.01086	-
SSR x TL	0.00024	0.000336	0.4793	0.34839	-
SSR x TC	0.03929	0.09496	0.6812	0.23838	-
TL x TC	0.01055	0.07764	0.8926	0.07047	-
SSR x TL x TC	-0.00068	0.00159	0.6737	-0.25023	0.280
Step 4					
SSR	-0.02135	0.01739	0.227	-0.47748	-
TL	0.00327	0.02167	0.8807	0.05308	-
TC	0.45786	3.9176	0.9076	0.04765	-
SSR x TL	0.000257	0.000283	0.3697	0.37235	-
SSR x TC	0.0342	0.07983	0.6708	0.20751	-
TL x TC	0.00595	0.06528	0.9278	0.03977	-
SSR x TL x TC	-0.00061	0.00134	0.6516	-0.2257	-
1/SSR	13.57666	3.28168	0.0002	0.6046	18.851
1/TL	44.46977	34.55418	0.2059	0.37591	1.824

Note. R^2 =0.3672 for Step 1. ?R^2 = 0.0065 for Step 2. ?R^2 = 0.0028 for Step 3. ? R^2 = 0.2050 for Step 4.

Table E28. Hierarchical Regression Analysis for log absolute value of MBIAS of D of MID for reduced data set (N=30)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.01475	0.00678	0.0388	-0.39123	15.306
TL	-0.00301	0.0126	0.8128	-0.043	0.185
TC	0.57377	1.43998	0.6935	0.07162	5.13
Step 2					
SSR	-0.00265	0.02903	0.928	-0.07036	-
TL	0.00487	0.02578	0.8518	0.06952	-
TC	1.30234	6.30656	0.8382	0.16256	-
SSR x TL	-0.00017	0.000418	0.6937	-0.31397	5.76
SR x TC	-0.00832	0.04783	0.8635	-0.06397	0.110
TL x TC	-0.00583	0.08892	0.9482	-0.05053	0.016
Step 3					
SSR	-0.0071	0.04065	0.863	-0.1883	-
TL	0.002	0.03188	0.9506	0.02852	-
TC	0.0569	10.10557	0.9956	0.0071	-
SSR x TL	-9.9E-05	0.000605	0.8721	-0.1853	-
SSR x TC	0.02133	0.19164	0.9124	0.16405	-
TL x TC	0.01332	0.15032	0.9302	0.11539	-
SSR x TL x TC	-0.00046	0.00285	0.8743	-0.23935	0.097
Step 4					
SSR	0.02207	0.0306	0.479	0.58543	-
TL	0.08781	0.05849	0.1489	1.25286	-
TC	0.07125	7.41498	0.9924	0.00889	-
SSR x TL	-9.9E-05	0.000444	0.8265	-0.1853	-
SSR x TC	0.02114	0.14062	0.882	0.16255	-
TL x TC	0.01314	0.1103	0.9063	0.11382	-
SSR x TL x TC	-0.00045	0.00209	0.8305	-0.23806	-
1/SSR	37.42075	8.74695	0.0004	0.98546	37.265
1/TL	324.707	202.8707	0.1252	1.24543	5.216

Note. R^2 =0.1600 Step 1. ?R^2 = 0.0070 Step 2. ?R^2 = 0.0010 for step 3.
? R^2 = 0.4248 for Step 4.

Table E29. Hierarchical Regression Analysis for log of MVAR of a_1 of MID with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MVAR Accounted for
Step 1					
SSR	-0.02737	0.00215	<.0001	-0.68767	47.273
TL	-0.03237	0.00297	<.0001	-0.58974	34.766
TC	2.25177	0.46268	<.0001	0.26327	6.930
Step 2					
SSR	-0.02545	0.00606	0.0001	-0.63957	-
TL	-0.02931	0.00551	<.0001	-0.53396	-
TC	2.66693	1.33825	0.053	0.31181	-
SSR x TL	-4.4E-05	9.48E-05	0.6452	-0.0717	0.067
SR x TC	0.0033	0.01478	0.8247	0.02246	0.015
TL x TC	-0.00976	0.02039	0.6346	-0.07326	0.071
Step 3					
SSR	-0.02703	0.00803	0.0017	-0.67912	-
TL	-0.03035	0.00655	<.0001	-0.55296	-
TC	2.27591	1.87094	0.2309	0.26609	-
SSR x TL	-1.5E-05	0.000135	0.9108	-0.0248	-
SSR x TC	0.01391	0.03813	0.7171	0.09484	-
TL x TC	-0.00268	0.03118	0.9318	-0.02014	-
SSR x TL x TC	-0.00019	0.000639	0.7637	-0.08056	0.029
Step 4					
SSR	-0.01661	0.00245	<.0001	-0.41732	-
TL	-0.0037	0.00305	0.2336	-0.06736	-
TC	2.52209	0.55231	<.0001	0.29487	-
SSR x TL	-3.7E-06	3.98E-05	0.9266	-0.00602	-
SSR x TC	0.0104	0.01125	0.3614	0.07088	-
TL x TC	-0.00595	0.0092	0.5216	-0.04467	-
SSR x TL x TC	-0.00015	0.000189	0.4413	-0.06113	-
1/SSR	7.94621	0.46265	<.0001	0.39757	8.152
1/TL	56.04275	4.87149	<.0001	0.53225	3.657

Note. R^2 =0.8713 for Step 1. ?R^2 = 0.0015 for Step 2. ?R^2 = 0.0003 for Step 3. ? R^2 = 0.1164 for Step 4.

Table E30. Hierarchical Regression Analysis for log of MRMSE of a_1 of MID with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MRMSE Accounted for
Step 1					
SSR	-0.01372	0.00108	<.0001	-0.69614	48.444
TL	-0.01569	0.00149	<.0001	-0.57727	33.312
TC	1.11502	0.23145	<.0001	0.2633	6.932
Step 2					
SSR	-0.01314	0.00304	<.0001	-0.66677	-
TL	-0.01448	0.00276	<.0001	-0.53261	-
TC	1.32325	0.67077	0.0553	0.31247	-
SSR x TL	-1.4E-05	4.75E-05	0.7667	-0.04673	0.028
SR x TC	0.00132	0.00741	0.8598	0.01814	0.010
TL x TC	-0.00467	0.01022	0.6501	-0.07077	0.066
Step 3					
SSR	-0.01377	0.00403	0.0015	-0.69893	-
TL	-0.0149	0.00328	<.0001	-0.54806	-
TC	1.16581	0.93815	0.2212	0.27529	-
SSR x TL	-2.6E-06	6.77E-05	0.9694	-0.0086	-
SSR x TC	0.00559	0.01912	0.7714	0.07699	-
TL x TC	-0.00182	0.01563	0.9079	-0.02758	-
SSR x TL x TC	-7.8E-05	0.00032	0.8092	-0.06551	0.019
Step 4					
SSR	-0.00839	0.00111	<.0001	-0.42575	-
TL	-0.00209	0.00139	0.1409	-0.07671	-
TC	1.29027	0.25071	<.0001	0.30468	-
SSR x TL	3.21E-06	1.81E-05	0.86	0.01058	-
SSR x TC	0.00381	0.00511	0.46	0.0525	-
TL x TC	-0.00347	0.00418	0.4115	-0.05257	-
SSR x TL x TC	-5.4E-05	8.56E-05	0.5295	-0.0457	-
1/SSR	4.09892	0.21001	<.0001	0.4142	8.848
1/TL	26.97406	2.2113	<.0001	0.51741	3.456

Note. R^2 = 0.8686 for Step 1. ?R^2 = .0011 for Step 2. ?R^2 = 0.0001 for Step 3. ? R^2 = .1114 for Step 4.

Table E31. Hierarchical Regression Analysis for log of the absolute value of MBIAS of a_1 of MID with all conditions included (N=48)

Variable	B	SE B	p	ß	Percent Variance in MBIAS Accounted for
Step 1					
SSR	-0.00532	0.00565	0.3518	-0.11455	1.312
TL	-0.03697	0.00779	<.0001	-0.57753	33.342
TC	0.68111	1.21399	0.5776	0.06828	0.466
Step 2					
SSR	-0.01105	0.01545	0.4784	-0.23811	-
TL	-0.03039	0.01405	0.0364	-0.47465	-
TC	5.70991	3.41308	0.102	0.57243	-
SSR x TL	0.000147	0.000242	0.5463	0.20567	0.549
SR x TC	-0.01609	0.0377	0.6717	-0.09407	0.270
TL x TC	-0.0808	0.052	0.1279	-0.51988	3.582
Step 3					
SSR	-0.00619	0.02048	0.7642	-0.13329	-
TL	-0.02716	0.01669	0.1116	-0.42428	-
TC	6.91875	4.7691	0.1546	0.69362	-
SSR x TL	5.82E-05	0.000344	0.8665	0.08136	-
SSR x TC	-0.04892	0.09719	0.6175	-0.28592	-
TL x TC	-0.10268	0.07947	0.2038	-0.66067	-
SSR x TL x TC	0.000598	0.00163	0.7155	0.21354	0.204
Step 4					
SSR	-0.00495	0.01742	0.7778	-0.10671	-
TL	0.04965	0.02171	0.0279	0.77558	-
TC	7.15726	3.92597	0.0762	0.71753	-
SSR x TL	6.94E-05	0.000283	0.8076	0.09709	-
SSR x TC	-0.0522	0.08	0.518	-0.30512	-
TL x TC	-0.10613	0.06542	0.113	-0.68287	-
SSR x TL x TC	0.000646	0.00134	0.6329	0.23053	-
1/SSR	1.43464	3.28868	0.6651	0.06155	0.195
1/TL	158.3854	34.62796	<.0001	1.28984	21.478

Note. R^2 =0.3483 for Step 1. ?R^2 = 0.0434 for Step 2. ?R^2 = 0.0020 for Step 3. ? R^2 = 0.2162 for Step 4.

APPENDIX F

RATIO OF MVAR, MRMSE AND MBIAS TO AVERAGE PARAMETER VALUE BY CONDITION

Table F1. Ratio of MVAR of a1 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	20351.02	42.91985	0.048363	0.031189
10	880.5452	0.033895	0.021278	0.014413
20	110.7445	0.015136	0.009812	0.006781
30	0.036839	0.00968	0.006502	0.004567
50	6.276727	0.005639	0.003803	0.002576
100	0.009239	0.002761	0.001818	0.001274
TC = 0.3				
5	427330.4	6556.504	57.1114	0.04982
10	919.9989	0.054013	0.033231	0.02101
20	373.1886	0.023584	0.015337	0.009981
30	146.3289	0.015214	0.009781	0.00655
50	30.0931	0.008549	0.005738	0.003892
100	0.017919	0.004239	0.002796	0.001871

Table F2. Ratio of MRMSE of a1 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	25.72176	0.551334	0.226108	0.178466
10	5.289365	0.18471	0.152388	0.125194
20	0.834865	0.127146	0.107177	0.088618
30	0.205583	0.103693	0.087738	0.07418
50	0.21541	0.08011	0.070232	0.059415
100	0.106351	0.059882	0.051405	0.04524
TC = 0.3				
5	58.38008	6.432877	0.51432	0.22711
10	5.510438	0.234502	0.192793	0.156117
20	2.903142	0.161451	0.138748	0.115193
30	1.537598	0.135003	0.114412	0.097761
50	0.532597	0.105188	0.094203	0.083437
100	0.152158	0.082929	0.074148	0.067256

Table F3. Ratio of MBIAS of a1 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	6.028761	0.110103	0.050275	0.039509
10	1.155162	0.041189	0.035997	0.035433
20	0.176069	0.033824	0.030983	0.027544
30	0.046225	0.029935	0.024703	0.023051
50	0.052945	0.02204	0.023941	0.02153
100	0.023721	0.020035	0.017882	0.017497
TC = 0.3				
5	17.95664	1.047417	0.106509	0.063448
10	1.292566	0.061564	0.054237	0.051329
20	0.638685	0.049716	0.049694	0.04561
30	0.342003	0.047969	0.042806	0.04117
50	0.13177	0.038024	0.04067	0.040328
100	0.051377	0.037516	0.035583	0.03482

Table F4. Ratio of MVAR of a2 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	10730.1	7.628871	0.042587	0.030565
10	266.3677	0.032481	0.018815	0.013741
20	13.44896	0.014768	0.008893	0.006623
30	1.88175	0.0095	0.005758	0.004325
50	0.01221	0.005591	0.003391	0.002539
100	0.005933	0.002753	0.001662	0.001237
TC = 0.3				
5	136322.6	3913.914	23.71725	0.051798
10	1875.131	0.057771	0.031467	0.021993
20	111.843	0.024134	0.014473	0.010342
30	5.505341	0.01572	0.009202	0.006677
50	0.025319	0.008759	0.00526	0.003869
100	0.011817	0.004202	0.002559	0.001844

Table F5. Ratio of MRMSE of a2 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	17.15958	0.379174	0.20448	0.179201
10	1.657687	0.182831	0.138926	0.122801
20	0.251536	0.125678	0.097044	0.085509
30	0.167619	0.101534	0.07843	0.070952
50	0.104738	0.078303	0.062847	0.0575
100	0.073776	0.058354	0.047955	0.044234
TC = 0.3				
5	38.35477	5.92214	0.426836	0.234963
10	5.06991	0.242203	0.18381	0.162022
20	1.001459	0.166497	0.132134	0.114565
30	0.26772	0.138286	0.106654	0.097559
50	0.150363	0.106146	0.088353	0.081513
100	0.105787	0.082137	0.070854	0.066279

Table F6. Ratio of MBIAS of a2 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	3.790723	0.085214	0.051695	0.046062
10	0.358202	0.042964	0.036601	0.033877
20	0.057165	0.031698	0.027009	0.020498
30	0.036759	0.025977	0.020047	0.019474
50	0.022908	0.018039	0.01818	0.018459
100	0.015504	0.016886	0.01623	0.015925
TC = 0.3				
5	11.25323	1.015857	0.100857	0.073512
10	1.091591	0.070707	0.061544	0.058634
20	0.214529	0.056985	0.051655	0.042382
30	0.063593	0.050766	0.040268	0.040678
50	0.041114	0.040154	0.038589	0.037011
100	0.027905	0.035551	0.033665	0.03294

Table F7. Ratio of MVAR of d to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	5604.804	0.614215	0.036619	0.025521
10	11.9015	0.02508	0.015809	0.011811
20	0.315231	0.011508	0.007349	0.005542
30	0.019605	0.007459	0.00477	0.003637
50	0.009119	0.004257	0.002876	0.002185
100	0.003984	0.002123	0.001433	0.001047
TC = 0.3				
5	65579.74	2104.197	21.28041	0.034622
10	173.5813	0.033878	0.019851	0.014796
20	2.807461	0.014368	0.009567	0.006812
30	0.499835	0.009303	0.005917	0.004483
50	0.060383	0.005245	0.003621	0.00268
100	0.004893	0.002616	0.001739	0.001317

Table F8. Ratio of MRMSE of d to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	17.49523	0.50199	0.348422	0.292661
10	1.342188	0.289266	0.230524	0.199473
20	0.335124	0.195883	0.157836	0.137933
30	0.224037	0.157964	0.127982	0.112247
50	0.168145	0.120435	0.100188	0.088392
100	0.11591	0.086446	0.072104	0.062867
TC = 0.3				
5	47.54788	5.940137	0.683846	0.341579
10	3.181421	0.33581	0.261877	0.226961
20	0.668573	0.222231	0.183085	0.157187
30	0.351708	0.180464	0.146926	0.129794
50	0.208883	0.137697	0.117047	0.104052
100	0.132325	0.101425	0.086314	0.077993

Table F9. Ratio of MBIAS of d to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.385035	-0.02194	-0.01659	-0.01836
10	-0.04599	-0.01907	-0.01135	-0.00744
20	-0.0149	-0.00469	0.005355	0.005143
30	-0.01355	0.004241	0.004573	0.004708
50	0.004451	0.003481	0.003633	0.006457
100	0.004495	0.006237	0.001407	0.000386
TC = 0.3				
5	-2.64856	-0.24574	-0.04814	-0.02296
10	0.027259	-0.02596	-0.01542	-0.00997
20	-0.02427	-0.00879	0.001888	0.004996
30	-0.01307	0.003978	0.003655	0.00618
50	0.000205	0.004445	0.004343	0.007443
100	0.004469	0.006854	0.000797	-0.00042

Table F10. Ratio of MVAR of MDISC to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	14806.46	30.15759	0.041865	0.029065
10	0.097599	0.030106	0.018154	0.012994
20	0.030766	0.013257	0.008287	0.006083
30	0.019001	0.008504	0.00547	0.004044
50	0.010293	0.004962	0.003171	0.002268
100	0.00497	0.002353	0.001481	0.001088
TC = 0.3				
5	361552.3	7326.532	55.47418	0.043204
10	817.4976	0.045559	0.025084	0.017104
20	0.048445	0.017737	0.011239	0.007858
30	0.027154	0.011288	0.007014	0.005076
50	0.014246	0.006329	0.004048	0.00288
100	0.006442	0.002956	0.001871	0.001359

Table F11. Ratio of MRMSE of MDISC to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	10.47043	0.390662	0.174464	0.147307
10	0.23917	0.147191	0.118122	0.102629
20	0.145411	0.101566	0.083005	0.071838
30	0.116764	0.082963	0.06771	0.060387
50	0.088027	0.063991	0.054916	0.049441
100	0.06195	0.048722	0.041762	0.038611
TC = 0.3				
5	42.42786	6.13864	0.430161	0.185093
10	1.683431	0.185227	0.147156	0.128317
20	0.189244	0.127488	0.108618	0.093942
30	0.14947	0.107859	0.088815	0.081468
50	0.115161	0.084409	0.076044	0.071003
100	0.082681	0.069263	0.062849	0.059709

Table F12. Ratio of MBIAS of MDISC to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	2.427925	0.101792	0.056502	0.046548
10	0.087872	0.045712	0.038658	0.036334
20	0.048922	0.034304	0.029931	0.025021
30	0.038808	0.028954	0.02305	0.022156
50	0.02996	0.020876	0.021375	0.020749
100	0.019397	0.019193	0.017557	0.017436
TC = 0.3				
5	13.44375	1.050562	0.116799	0.076316
10	0.450633	0.074929	0.0636	0.058657
20	0.083502	0.057321	0.053077	0.045905
30	0.067614	0.051859	0.043144	0.042393
50	0.054886	0.040753	0.040673	0.039709
100	0.038792	0.037553	0.035343	0.034861

Table F13. Ratio of MVAR of D of MID to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.060197	0.030796	0.020758	0.015207
10	0.029538	0.014998	0.009614	0.00726
20	0.013976	0.007102	0.004658	0.003569
30	0.009146	0.00476	0.003114	0.002384
50	0.005325	0.0028	0.00185	0.001422
100	0.002669	0.001388	0.000939	0.000684
TC = 0.3				
5	0.063966	0.034803	0.023259	0.017388
10	0.032841	0.01703	0.010763	0.008095
20	0.015875	0.007989	0.005484	0.004007
30	0.010313	0.005348	0.003548	0.002648
50	0.006087	0.003135	0.002107	0.001595
100	0.003005	0.001572	0.001052	0.000783

Table F14. Ratio of MRMSE of D of MID to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.364796	0.266856	0.220205	0.189985
10	0.256401	0.187216	0.150763	0.130997
20	0.177739	0.128113	0.104535	0.091771
30	0.143956	0.10481	0.085504	0.07507
50	0.109121	0.080639	0.06608	0.058277
100	0.077434	0.057092	0.047301	0.040615
TC = 0.3				
5	0.376227	0.282075	0.232508	0.202445
10	0.270031	0.199129	0.159375	0.138179
20	0.18889	0.135787	0.112833	0.097035
30	0.152621	0.111137	0.09108	0.078995
50	0.116179	0.085144	0.070464	0.061853
100	0.081925	0.060811	0.050038	0.043373

Table F15. Ratio of MBIAS of D of MID to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.031749	0.026553	0.020549	0.022333
10	0.026521	0.022906	0.014648	0.010543
20	0.021976	0.008217	-0.00129	-0.00174
30	0.017507	-0.00033	-0.0015	-0.00213
50	-0.00022	-0.00161	-0.00098	-0.00512
100	-0.00144	-0.00478	0.000828	0.00151
TC = 0.3				
5	0.035257	0.030039	0.025822	0.025847
10	0.023776	0.028588	0.018286	0.012357
20	0.022387	0.010946	0.00201	-0.00177
30	0.019976	-0.00033	-0.00081	-0.00394
50	0.001887	-0.00239	-0.00222	-0.00653
100	-0.00202	-0.00584	0.000711	0.001131

Table F16. Ratio of MVAR of a_1 of MID to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.057622	0.02605	0.01568	0.011248
10	0.035551	0.012986	0.007388	0.005432
20	0.018366	0.006248	0.003647	0.002729
30	0.012201	0.00405	0.002396	0.001818
50	0.006994	0.002425	0.001436	0.001093
100	0.003456	0.001257	0.000738	0.000562
TC = 0.3				
5	0.095286	0.046926	0.027724	0.020864
10	0.072783	0.024719	0.014345	0.010041
20	0.045446	0.012227	0.006981	0.005037
30	0.030685	0.008182	0.004643	0.003339
50	0.018188	0.004685	0.00271	0.002041
100	0.00887	0.002401	0.001407	0.001005

Table F17. Ratio of MRMSE of a_1 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.239066	0.173917	0.1303	0.114856
10	0.179985	0.122544	0.089799	0.079997
20	0.128147	0.085008	0.063205	0.056815
30	0.10437	0.068736	0.051207	0.046457
50	0.079602	0.053215	0.039819	0.036065
100	0.055843	0.038315	0.02857	0.026007
TC = 0.3				
5	0.321117	0.234826	0.173897	0.156254
10	0.262762	0.168731	0.124571	0.108811
20	0.197977	0.118583	0.087147	0.076965
30	0.162173	0.096904	0.071116	0.062764
50	0.125127	0.073591	0.054339	0.049041
100	0.08816	0.052753	0.039328	0.034885

Table F18. Ratio of MBIAS of a_1 to average parameter value

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	-0.06695	0.000229	0.001096	0.003664
10	-0.02013	0.000517	0.000588	-0.0008
20	-0.00373	-0.00145	-0.00172	-0.0037
30	-0.00628	-0.00196	-0.00216	-0.00161
50	-0.00739	-0.00204	-0.00305	-0.00148
100	-0.00353	-0.00164	-0.00095	-0.00085
TC = 0.3				
5	-0.11898	0.006183	6.37E-05	0.004756
10	-0.06869	0.003765	0.003162	0.003737
20	-0.02123	0.003567	0.000857	-0.00175
30	-0.01273	0.001222	-0.00129	4.83E-05
50	-0.01114	0.001383	-0.00105	-0.0018
100	-0.01117	-0.00118	-0.00123	-0.00108

APPENDIX G

CORRELATION OF PARAMETER ESTIMATES WITH TRUE PARAMETERS BY CONDITION

Table G1. Correlation of estimates of a1 with parameters by conditions

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.027311	0.057884	0.819659	0.870027
10	0.031415	0.858667	0.903683	0.931066
20	0.033667	0.92635	0.951008	0.964717
30	0.81751	0.950325	0.965981	0.975271
50	0.11089	0.969381	0.978898	0.985167
100	0.939305	0.983793	0.989013	0.991898
TC = 0.3				
5	0.016784	0.016616	0.049852	0.825812
10	0.032428	0.811156	0.868533	0.909573
20	0.032285	0.897737	0.929622	0.951361
30	0.03728	0.92892	0.951768	0.966181
50	0.058866	0.956285	0.969289	0.977694
100	0.8954	0.975323	0.982718	0.987386

Table G2. Correlation of estimates of a2 with parameters by conditions

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.020324	0.117328	0.829734	0.873905
10	0.003045	0.865546	0.907668	0.933762
20	0.064336	0.928038	0.950557	0.964652
30	0.171519	0.9501	0.966423	0.976005
50	0.906054	0.96909	0.979256	0.985096
100	0.948863	0.983583	0.98892	0.991986
TC = 0.3				
5	0.034418	0.020746	0.072881	0.825384
10	0.007625	0.807909	0.870361	0.906726
20	0.014534	0.897017	0.929144	0.949345
30	0.10788	0.926544	0.950931	0.964935
50	0.847623	0.954649	0.969354	0.97757
100	0.914716	0.975335	0.982771	0.987351

Table G3. Correlation of estimates of d with parameters by conditions

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.038796	0.755219	0.977788	0.984259
10	0.249827	0.98416	0.989934	0.992461
20	0.838602	0.992474	0.9952	0.996377
30	0.986835	0.995047	0.996846	0.997595
50	0.993707	0.997135	0.998066	0.998525
100	0.99719	0.998536	0.999007	0.999262
TC = 0.3				
5	0.029608	0.03147	0.197863	0.979196
10	0.083786	0.979132	0.987506	0.99059
20	0.464466	0.990659	0.993735	0.995489
30	0.775987	0.993799	0.996022	0.996942
50	0.96144	0.996401	0.997475	0.998087
100	0.996468	0.9981	0.998685	0.998962

Table G4. Correlation of estimates of MDISC with parameters by conditions for TC = 0.0

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.028736	0.059368	0.80175	0.846336
10	0.689897	0.840673	0.891288	0.917602
20	0.839851	0.915022	0.942983	0.956492
30	0.886727	0.940879	0.960007	0.969654
50	0.930722	0.963052	0.975502	0.98199
100	0.962556	0.980948	0.987424	0.990406
TC = 0.3				
5	0.031003	0.020717	0.046648	0.818537
10	0.031991	0.809751	0.875087	0.907602
20	0.808048	0.90271	0.933647	0.950612
30	0.869775	0.932656	0.954635	0.965815
50	0.918711	0.957837	0.97134	0.978577
100	0.956715	0.977412	0.984437	0.987931

Table G5. Correlation of estimates of D of MID with parameters by conditions

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.950745	0.974361	0.982482	0.987097
10	0.975184	0.98727	0.991752	0.993727
20	0.988048	0.993873	0.995963	0.996921
30	0.992144	0.995883	0.997316	0.997941
50	0.995397	0.997591	0.998399	0.998766
100	0.997705	0.998797	0.999186	0.999405
TC = 0.3				
5	0.946347	0.970723	0.980204	0.985114
10	0.971885	0.985441	0.990697	0.992973
20	0.986246	0.993062	0.99521	0.996516
30	0.991025	0.995335	0.996922	0.997692
50	0.994679	0.997279	0.998158	0.998604
100	0.997395	0.998626	0.999077	0.99931

Table G6. Correlation of estimates of a_1 of MID with parameters by conditions

	TL			
	23	44	65	86
SSR				
TC = 0.0				
5	0.426751	0.831091	0.876001	0.918004
10	0.550187	0.906381	0.935713	0.957785
20	0.69425	0.951568	0.966872	0.978274
30	0.767302	0.967887	0.978029	0.985442
50	0.846685	0.980639	0.986668	0.991163
100	0.916749	0.989829	0.993099	0.99542
TC = 0.3				
5	0.296997	0.728968	0.802175	0.859945
10	0.383954	0.837374	0.884267	0.925346
20	0.499326	0.91023	0.938743	0.960811
30	0.586189	0.937499	0.958411	0.973681
50	0.69211	0.963304	0.975268	0.983647
100	0.815287	0.980726	0.986941	0.991842

APPENDIX H

HIERARCHICAL REGRESSION RESULTS AT ITEM DIFFICULTY LEVELS

Table H1. Summary of hierarchical regression for log of MVAR of a1 at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	72.049	71.96	71.601	71.739	71.212	71.912	72.431
TL	13.034	12.885	13.189	13.119	13.084	13.091	12.953
TC	5.15	5.35	5.431	5.33	5.564	5.093	4.631
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	9.264	9.311	9.341	9.381	9.648	9.528	9.531
1/TL	0.112	0.131	0.143	0.154	0.192	0.135	0.154
SSR^2	0.334	0.324	0.244	0.194	0.19	0.169	0.224
R^2	0.9996	0.9997	0.9997	0.9992	0.9994	0.9996	0.9994

Table H2. Summary of hierarchical regression for log of MVAR of a2 at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	71.646	71.48	70.992	70.447	70.397	70.17	71.351
TL	13.167	13.052	12.858	13.507	12.95	13.324	13.318
TC	5.459	6.034	6.654	6.692	6.645	6.292	5.82
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	9.34	9.122	9.11	8.924	9.621	9.71	9.019
1/TL	0.093735	0.059191	0.0814	-	-	-	-
SSR^2	0.173	0.209	0.227	0.298	0.213	0.258	0.349
R^2	0.9990	0.9996	0.9996	0.9996	0.9994	0.9991	0.9997

Table H3. Summary of hierarchical regression for log of MRMSE of a1 at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	71.265	64.898	59.3	57.985	60.766	67.576	72.865
TL	10.358	9.323	9.058	9.164	9.043	9.446	10.622
TC	7.543	13.241	18.127	19.22	16.836	11.277	6.084
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	10.058	11.066	11.583	11.748	11.597	10.594	9.837
1/TL	0.194	0.337	0.49	0.514	0.476	0.274	0.203
SSR^2	0.337	0.313	0.232	0.218	0.229	0.206	0.249
R^2	0.9996	0.9992	0.9987	0.998	0.998	0.9987	0.999

Table H4. Summary of hierarchical regression for log of MRMSE of a2 at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	71.704	64.894	57.037	54.107	55.861	62.44	69.921
TL	11.263	10.181	9.463	9.566	9.103	9.59	10.815
TC	7.115	13.604	20.385	23.006	20.876	15.107	8.608
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	9.603	10.703	12.008	12.019	12.9	11.79	10.02
1/TL	0.055444	-	-	-	-	-	-
SSR^2	0.196	0.214	0.254	0.315	0.257	0.382	0.421
R^2	0.9994	0.9986	0.9982	0.9974	0.9988	0.9988	0.9991

Table H5. Summary of hierarchical regression for log of MBIAS of a1 at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	32.025	17.329	16.635	18.312	25.23	39.824	70.701
TL	-	-	2.258	3.304	3.33	2.852	3.647
TC	55.861	74.264	74.131	71.477	62.464	46.715	14.909
SSR x TL	-	-	-	-	-	-	-
SSR x TC	4.171	1.186	1.338	-	-	2.065	3.167
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	3.379	3.506	3.127	3.307	4.529	5.405	4.485
1/TL	-	0.676	0.974	1.149	0.986	0.584	-
SSR^2	-	-	-	-	-	0.562	0.525
R^2	0.9695	0.9703	0.9803	0.9795	0.9738	0.9754	0.9686

Table H6. Summary of hierarchical regression for log of MBIAS of a2 at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	41.6	20.407	16.694	17.072	21.06	29.056	45.378
TL	1.971	2.717	2.97	4.263	3.836	3.127	3.036
TC	44.539	67.194	70.377	68.97	62.941	54.186	35.575
SSR x TL	-	-	-	-	-	-	-
SSR x TC	4.76	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	4.273	6.736	7.088	7.234	9.191	9.801	9.862
1/TL	-	-	-	-	-	-	-
SSR^2	-	-	-	-	-	0.393	1.103
R^2	0.9749	0.978	0.981	0.9843	0.984	0.985	0.9779

Table H7. Summary of hierarchical regression for log of MVAR of d at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	75.609	76.152	76.592	76.675	76.578	75.923	76.56
TL	11.476	11.399	11.192	10.964	11.048	11.38	11.48
TC	-	-	-	-	-	1.793	-
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	10.878	10.263	10.21	10.258	10.265	10.476	9.961
1/TL	0.165	0.175	0.112	0.17	0.133	0.112	0.146
SSR^2	0.161	0.208	0.23	0.29	0.24	0.245	0.266
R^2	0.9991	0.9994	0.9994	0.9993	0.9994	0.9996	0.9994

Table H8. Summary of hierarchical regression for log of MRMSE of d at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	74.953	72.828	75.071	77.551	76.212	74.444	76.571
TL	9.605	9.595	9.776	10.055	9.987	9.65	9.691
TC	2.681	3.932	2.536	-	2.802	4.359	2.825
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	12.178	12.742	12.048	10.453	10.506	10.892	10.361
1/TL	0.157	0.144	0.081232	0.151	0.147	0.103	0.12
SSR^2	0.204	0.279	0.292	0.267	0.197	0.248	0.302
R^2	0.9995	0.9994	0.9995	0.9993	0.9989	0.9991	0.9993

Table H9. Summary of hierarchical regression for log of absolute value of MBIAS of d at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	34.246	23.262	20.735	43.322	-	-	30.596
TL	-	-	-	-	-	-	-
TC	24.652	38.192	31.392	-	20.919	77.423	58.568
SSR x TL	12.184	6.022	7.113	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	14.889	26.486	30.718	9.426	40.428	12.64	-
1/TL	-	-	-	8.515	-	-	-
R^2	0.9312	0.9739	0.9439	0.6923	0.8585	0.9482	0.9241

Table H10. Summary of hierarchical regression for log of MVAR of MDISC at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	75.732	75.679	75.13	75.024	74.728	75.051	76.503
TL	12.329	12.262	12.162	12.478	11.938	12.312	12.101
TC	-	1.919	2.646	2.697	2.695	1.978	-
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	10.072	9.737	9.64	9.35	10.157	10.155	9.752
1/TL	0.156	0.117	0.136	0.113	0.13	0.136	0.132
R^2	0.9963	0.9972	0.9976	0.997	0.9972	0.9972	0.9969

Table H11. Summary of hierarchical regression for log of RMSE of MDISC at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	74.985	63.985	54.482	51.871	54.831	64.316	74.843
TL	9.154	8.165	7.713	7.989	7.542	7.843	8.967
TC	4.156	13.959	23.007	25.541	22.283	13.768	4.448
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	10.899	12.101	12.575	12.502	13.246	12.371	10.97
1/TL	0.179	0.171	0.184	0.118	0.121	-	-
SSR^2	0.222	0.239	0.209	0.254	0.255	0.312	0.379
R^2	0.9991	0.9985	0.998	0.9972	0.9974	0.9991	0.9995

Table H12. Summary of hierarchical regression for log of MBIAS of MDISC at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	43.039	22.688	19.34	19.876	24.748	35.496	58.034
TL	-	2.33	3.322	4.417	4.154	3.518	3.801
TC	43.162	65.818	68.683	67.258	60.556	49.014	25.933
SSR x TL	-	-	-	-	-	-	-
SSR x TC	5.57	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	4.145	6.856	6.576	6.576	8.154	9.000	8.343
1/TL	0.483	-	-	-	-	-	-
SSR^2	-	-	-	-	-	0.423	0.836
R^2	0.9809	0.9864	0.9902	0.9902	0.9882	0.9905	0.9898

Table H13. Summary of hierarchical regression for log of MVAR of D of MID at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	79.212	78.652	78.273	77.944	77.84	77.824	78.001
TL	11.258	11.251	11.25	10.889	11.155	11.276	11.393
TC	-	-	-	-	-	-	-
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	8.645	8.881	9.254	9.924	10.01	10.115	9.917
1/TL	0.168	0.18	0.109	0.152	0.114	0.072399	0.099506
SSR^2	0.28	0.338	0.284	0.312	0.234	0.201	0.25
R^2	0.9992	0.9994	0.9996	0.9995	0.9996	0.9994	0.9993

Table H14. Summary of hierarchical regression for log of MRMSE of D of MID at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	74.985	63.985	54.482	51.871	54.831	64.316	74.843
TL	9.154	8.165	7.713	7.989	7.542	7.843	8.967
TC	4.156	13.959	23.007	25.541	22.283	13.768	4.448
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	10.899	12.101	12.575	12.502	13.246	12.371	10.97
1/TL	0.179	0.171	0.184	0.118	0.121	-	-
SSR^2	0.222	0.239	0.209	0.254	0.255	0.312	0.379
R^2	0.9991	0.9985	0.998	0.9972	0.9974	0.9991	0.9995

Table H15. Summary of hierarchical regression for log of absolute value of MBIAS of D of MID at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	-	-	15.473	38.783	-	26.405	-
TL	-	-	-	-	-	-	-
TC	-	-	-	-	-	-	-
SSR x TL	16.332	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	18.231	-	-	-	36.184	26.391	29.885
1/TL	-	-	-	11.062	-	-	-
R^2	0.5684	0.2264	0.4372	0.6229	0.6984	0.6926	0.461

Table H16. Summary of hierarchical regression for log of MVAR of a_1 of MID at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	47.498	47.401	47.351	47.193	47.093	47.129	47.185
TL	34.408	34.558	34.878	35.156	35.119	34.725	34.546
TC	7.237	7.023	6.65	6.428	6.629	7.035	7.421
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	7.925	8.032	8.169	8.203	8.322	8.312	8.127
1/TL	3.626	3.75	3.722	3.798	3.657	3.569	3.494
SSR^2	0.642	0.617	0.622	0.629	0.639	0.637	0.638
R^2	0.9953	0.9955	0.9958	0.9957	0.9961	0.9959	0.9958

Table H17. Summary of hierarchical regression for log of MRMSE of a_1 of MID at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	49.403	49.285	49.114	48.914	48.983	49.198	49.465
TL	32.243	32.334	32.58	32.811	32.667	32.287	32.224
TC	7.207	7.001	6.724	6.543	6.774	7.051	7.304
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	8.705	8.816	9.042	9.14	9.152	9.114	8.804
1/TL	3.333	3.493	3.471	3.531	3.385	3.274	3.142
SSR^2	0.646	0.617	0.632	0.657	0.659	0.655	0.647
R^2	0.9971	0.997	0.9972	0.9973	0.9975	0.9974	0.9974

Table H18. Summary of hierarchical regression for log of absolute value of MBIAS of a_1 of MID at difficulty levels. Percentage variance explained by term in step in which it was entered.

	Item Difficulty (d)						
Term	-1.0	0.66	0.33	0.0	-.33	0.66	1.0
SSR	-	-	-	-	11.636	6.857	6.819
TL	20.05	12.716	26.724	33.171	34.659	26.075	21.456
TC	-	-	-	-	-	-	22.917
SSR x TL	-	-	-	-	-	-	-
SSR x TC	-	-	-	-	-	-	-
TL x TC	-	-	-	-	-	-	-
SSR x TC x TL	-	-	-	-	-	-	-
1/SSR	-	-	-	-	-	-	-
1/TL	32.713	32.946	35.608	13.982	17.811	12.417	17.577
R^2	0.6424	0.5331	0.6714	0.5849	0.7139	0.5414	0.7227

APPENDIX I

HIERARCHICAL REGRESSION RESULTS AT ITEM DISCRIMINATION LEVELS

Table I1. Summary of hierarchical regression for log of MVAR of a1 at item discrimination levels. Percentage variance accounted for by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	73.032	72.705	70.997
TL	13.97	12.909	12.835
TC	3.406	4.813	6.065
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.102	9.24	9.591
1/TL	0.187	-	0.237
SSR^2	0.266	0.275	0.21
R^2	0.9998	0.9997	0.9996

Table I2. Summary of hierarchical regression for log of MRMSE of a1 at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	73.221	70.432	58.163
TL	13.422	11.937	9.077
TC	3.55	7.398	18.953
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.282	9.763	11.634
1/TL	0.22	-	0.793
SSR^2	0.273	0.286	0.213
R^2	0.9999	0.9995	0.9963

Table I3. Summary of hierarchical regression for log of MBIAS of a1 at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	69.188	33.949	19.213
TL	6.974	3.676	1.364
TC	5.606	53.043	71.158
SSR x TL	3.056	2.617	
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	-	3.871	4.45
1/TL	-	-	1.755
SSR^2	-	-	-
R^2	0.9435	0.9788	0.9907

Table I4. Summary of hierarchical regression for log of MVAR of a2 at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	72.795	71.865	69.883
TL	14.107	12.986	12.964
TC	3.801	5.688	7.317
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	8.968	8.948	9.447
1/TL	0.057819	0.195	-
SSR^2	0.253	0.284	0.227
R^2	0.9999	0.9999	0.9993

Table I5. Summary of hierarchical regression for log of MRMSE of a2 at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	72.651	69.467	54.635
TL	13.607	11.94	9.558
TC	4.035	8.392	22.165
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.379	9.612	12.391
1/TL	-	0.231	-
SSR^2	0.272	0.304	0.266
R^2	0.9999	0.9997	0.9986

Table I6. Summary of hierarchical regression for log of MBIAS of a2 at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	59.898	30.49	17.394
TL	9.175	4.351	2.224
TC	16.053	52.312	70.262
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	10.353	8.496	7.724
1/TL	-	-	0.418
SSR^2	-	-	-
R^2	0.9675	0.9751	0.9891

Table I7. Summary of hierarchical regression for log of MVAR of d at item item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	77.549	76.722	75.281
TL	11.381	11.228	11.293
TC	-	-	2.482
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	10.086	10.258	10.453
1/TL	0.133	0.113	0.171
SSR^2	0.299	0.236	0.198
R^2	0.9998	0.9995	0.9991

Table I8. Summary of hierarchical regression for log of MRMSE of d at item item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	77.749	76.447	71.012
TL	10.946	10.685	9.697
TC	-	-	6.331
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	10.339	10.674	12.114
1/TL	0.126	0.116	0.174
SSR^2	0.286	0.233	0.23
R^2	0.9998	0.9995	0.9991

Table I9. Summary of hierarchical regression for log of absolute value of MBIAS of d at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	42.86	37.26	43.813
TL	-	-	-
TC	-	-	-
SSR x TL	-	-	14.333
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	7.767	-	-
1/TL	-	9.304	-
R^2	0.6804	0.607	0.7373

Table I10. Summary of hierarchical regression for log of MVAR of MDISC at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	76.808	77.833	73.907
TL	13.008	11.68	12.216
TC	-	-	3.408
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.455	9.427	9.981
1/TL	0.157	0.099306	0.136
R^2	0.9975	0.9973	0.9971

Table I11. Summary of hierarchical regression for log of MRMSE of MDISC at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	76.51	73.348	53.66
TL	12.538	10.654	8.205
TC	-	4.611	23.303
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.949	10.63	12.812
1/TL	0.146	0.092416	0.172
SSR^2	0.267	0.309	0.224
R^2	0.9998	0.9995	0.9977

Table I12. Summary of hierarchical regression for log of MBIAS of MDISC at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	66.525	35.912	19.964
TL	13.778	5.21	2.019
TC	17.111	48.379	69.074
SSR x TL	1.554	-	-
SSR x TC	10.36	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	-	-	-
1/TL	-	-	-
SSR^2	-	-	-
R^2	0.9728	0.9873	0.992

Table I13. Summary of hierarchical regression for log of MVAR of D of MID at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	78.462	78.286	78.084
TL	11.249	11.261	11.158
TC	-	-	-
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.588	9.473	9.420
1/TL	0.135	0.101	0.142
SSR^2	.267	0.275	0.260
R^2	.9998	0.9997	.9996

Table I14. Summary of hierarchical regression for log of MRMSE of D of MID at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	78.522	78.522	78.522
TL	10.774	10.774	10.774
TC	-	-	-
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	9.767	9.767	9.767
1/TL	0.092215	0.092215	0.092215
SSR^2	0.247	0.247	0.247
R^2	0.9997	0.9997	0.9997

Table I15. Summary of hierarchical regression for log of absolute value of MBIAS of D of MID at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	-	-	28.05
TL	-	-	-
TC	-	-	-
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	40.079	27.635	22.835
1/TL	9.154	-	-
R^2	0.601	0.4299	0.6118

Table I16. Summary of hierarchical regression for log of MVAR of a_1 of MID at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	67.202	63.517	63.719
TL	14.548	13.833	14.152
TC	9.622	14.101	13.561
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	8.271	8.162	8.198
1/TL	0.069635	-	-
SSR^2	0.272	0.294	0.281
R^2	0.9999	0.9997	0.9997

Table I17. Summary of hierarchical regression for log of MRMSE of a_1 of MID at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	67.435	63.945	63.677
TL	14.149	13.451	13.496
TC	9.676	14.028	13.943
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	8.386	8.204	8.484
1/TL	0.065113	-	-
SSR^2	0.276	0.291	0.3
R^2	0.9999	0.9998	0.9998

Table I18. Summary of hierarchical regression for log of absolute value of MBIAS of alpha of MID at item discrimination levels. Percentage variance explained by term in step in which it was entered.

	Item Discrimination Level		
Term	Low	Mid	High
SSR	-	-	7.769
TL	35.152	20.57	42.877
TC	-	-	-
SSR x TL	-	-	-
SSR x TC	-	-	-
TL x TC	-	-	-
SSR x TC x TL	-	-	-
1/SSR	-	8.853	-
1/TL	30.323	-	26.785
R^2	0.7411	0.4344	0.8137

APPENDIX J

TERMS AND DEFINITIONS

a1 Item discrimination for trait 1

a2 Item discrimination for trait 2

a_1 of MID Alpha 1 of multidimensional item difficulty: the angle from the trait one axis to the point in trait space of maximum slope on the ICS (Direction of MID)

$$\cos a_{ik} = \frac{a_{ik}}{\left(\sum_{k=1}^{2} a_{ik}^{2} \right)^{1/2}}$$

d Item difficulty

D of MID The signed distance from the trait space origin to the point of maximum slope on the ICS

$$D_i = \frac{-d_i}{\left(\sum_{k=1}^{2} a_{ik}^{2} \right)^{1/2}}$$

ICS Item characteristic curve

MDISC Multidimensional discrimination: discrimination ability of an item in the direction of MID

$$\text{MDISC} = \left(\sum_{k=1}^{2} a_{ik}^{2} \right)^{1/2}$$

MID Multidimensional item difficulty: item difficulty measure consisting of a distance and direction from the trait origin

MIRT Multidimensional item response theory