Small-Sample Prediction of Estimated Loss Potentials

by

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(Under the direction of Lynne Seymour)

Abstract

This thesis constructs predictions for the 2003 General Liability premises and operations estimated loss potentials (ELPs) for Manufacturers and Contractors (MC) and Owners, Landlords, and Tenants (OLT). The dataset contains yearly ELPs from 1990-2002 for 23 MC class codes and 57 OLT class codes, which came from three Insurance Services Office (ISO) circulars. Bootstrapping was performed on the MC and OLT 2003 predicted ELPs to be able to construct 95% confidence intervals. In spite of the small series, the results appear to be good.

Index words: Actuarial, Bootstrapping, Time Series Analysis

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Dedication

I would like to thank God first of all for the ability to embark on such a difficult task of writing a thesis. Secondly, I would like to thank my wife Julia for all of her patience and support. Lastly, I would like to thank my family for always encouraging me to dream big and make the most of every opportunity the Lord brings into my life.

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Chapter 1

Introduction

1.1 Actuarial Background

Suppose that we were an underwriter for an insurance company that is established in a particular line of business within insurance; e.g., commercial auto, general liability, or excess and surplus. There are many factors that are used to quote a premium, but for simplicity, suppose that the company that is interested in being insured does not have any insurance losses (i.e., claims). They only request general liability premises and operations coverage with limits of $100,000, and they are requesting coverage from an a-rated class code with a payroll exposure base. The first step would be to decide which a-rated class code would best capture their risk. The chosen class code would have a basic limit pure premium, which is equal to the average loss per exposure unit on a basic limit basis (a loss that is limited to $100,000). The basic limit pure premium would include all of the expenses that can be tied to a specific claim in the chosen class code, which is known as the allocated loss adjustment expense (ALAE). Once we have the basic limit pure premium, we would need to use the number of exposures for the risk that was being priced. The number of exposures would be given to the underwriter by the company requesting insurance. We would then divide the total number of exposures by 1,000 since each loss cost is per 1,000 units of exposure (payroll in this example). Finally, we would multiply the basic limit pure premium and (exposures/1,000) to get the total expected losses. However, the insurance company that insures the risk also has expenses that cannot be tied to a specific claim, which are known unallocated loss adjustment expenses (ULAE). In addition, to account for the commission, overhead costs, and taxes, a loss cost multiplier (LCM) is used. Therefore, in order to come up with a premium for our example, we take the (basic limit loss cost $\times$ (payroll exposures/1,000) $\times$ LCM) + ULAE.

An Expected Loss Potential (ELP), also known as a basic pure premium, is equal to the average loss per exposure unit on a basic limit basis (a loss that is limited to $100,000 and includes all ALAE). Each ELP represents a risk classification, which is a group of individual policies that: "(1) are relatively homogeneous (all group members have similar costs), (2) are sufficiently large to estimate relative cost differences (credibility), and (3) maintain stable means over time (reliability)". [1]

Most risk classifications, which are each assigned a separate class code, have a multitude of loss data. Classes that have little loss data are considered a-rated classes. Thus, coming up with a statistically sound future ELP for a-rated classes is quite challenging. This thesis looks to use appropriate time series analysis and bootstrapping techniques to predict future ELPs for class codes with small sample sizes. Thus, we are not coming up with premiums, just the basic pure premiums for class codes with small sample sizes for 2003. In order to come up with a premium for a policy in 2008, we would need to apply a trend factor for each year, to get the basic limit pure premium to 2008 dollars. Finally, we would need the LCM, ULAE, and number of exposures to get the final premium.

1.2 ELP Data

It is difficult for the average-sized insurance company to be able to make credible inferences on the class codes that they insure; therefore, most companies have to purchase loss data that comes from many different insurance companies. Most insurance companies use the Insurance Services Office (ISO), which collects data for many different lines of insurance. This thesis will focus on the General Liability line of insurance. General Liability insurance is primarily concerned with bodily injury or property damage that occurs to third parties either from the day to day operations on the premises of the business or from the products that the business produces.

We will use three ISO reports published in 1996, 2000, and 2004, which give basic limit pure premiums (ELPs) for multi-state risks for the a-rated General Liability premises and operations as well as products and completed operations class codes for each of the years 1990-2002. [3] [4] [5]

For our analysis, we have decided to focus on the premises and operations class codes because they have fewer missing and zeroed-out values, and are made up of two distinct groups, manufacturers and contractors (MC), and owners, landlords, and tenants (OLT). Missing values only occur when a class code was not in existence, as some class codes were created after 1990 to incorporate new risk classifications, and zeroed-out values occurred when there was not a single loss reported during that particular year.

1.3 The General Statistical Challenge

Because of the short length of the time series for each class code, any inference made using traditional time series techniques [8] must include at least one assumption. We assume that class codes can be grouped into sets that have a similar behavior in their autocorrelation structures. Under this assumption, we devise a bootstrapping [2] strategy for inference. Thus, class codes for the premises and operations risk were combined to form larger groups based on two groups: manufacturers and contractors (MC), and owners, landlords, and tenants (OLT). The MC group is made up of 61 class codes, and the OLT group is made up of 98 class codes. However, all of the class codes that had missing or zeroed out values were removed. We were then left with 23 class codes in the MC group and 57 in the OLT group, each with 13 observations per class code for the years 1990-2002. [1]

The ELPs for the different class codes within the MC and OLT groups varied significantly from year to year. This was expected in order for the requirements of risk classification to be met. For example, risks with lower average claims should not have a premium which is equal

[1]The 1996 and 2000 circulars had a 1994 ELP, but we used the 1994 ELP from the 2000 circular since it was more recent. The 1998 ELP was also duplicated in the 2000 and 2004 circulars, and we took the 1998 ELP from the 2004 circular.

to that for risks with much higher average claims. However, the ELPs should be in some way comparable. So, all of the class code values for each year were divided by the maximum ELP for the 13 years within each class code. Thus, the predicted ELPs for each of the class codes will be multiplied by the maximum ELP during the 13 years to get the "true" 2003 predicted ELP for that particular class code.

Many of the class codes display a large trend, while some displayed only slight trends. In order to incorporate the trend into predicting the 2003 ELP, we must build a model that can capture as much of the trend as possible. Also, because we did not know anything about the distributions for the MC and OLT groups, we use bootstrapping to determine OLT and MC prediction distributions. These distributions allow us to determine if the OLT and MC predictions are significant or not. Each of these concerns will be addressed in the next chapter.

Chapter 2

Statistical Analysis

Even within the two groupings, the behavior of the ELP trends can vary greatly. Figures 2.1 and 2.2 each show a sample of three class code ELP plots per year for MC and OLT, respectively. Figure 2.3 shows the class code descriptions for the MC and OLT class codes.

Although the trends are short, a nonlinear trend captures more of the apparent variability, so a quadratic regression model was fit to each ELP series.

The trend model that was used for predicting the 2003 ELP for each of the class codes was [6]

$$\vec{y} = X\beta + \vec{\epsilon}\ \text{, with}$$

$$\vec{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_{13} \end{pmatrix},\ X = \begin{pmatrix} 1 & 1 & 1^2 \\ 1 & 2 & 2^2 \\ 1 & 3 & 3^2 \\ 1 & 4 & 4^2 \\ \vdots & \vdots & \vdots \\ 1 & 13 & 13^2 \end{pmatrix},\ \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta 2 \end{pmatrix} \text{ and } \vec{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{13} \end{pmatrix}$$

where $\vec{y}$ is the vector of ELP values; X and $\vec{\beta}$ are the design matrix and parameter vector, respectively, for a quadratic regression; and $\vec{\varepsilon}$ is a vector of correlated errors. Because the series is short, we assume that errors are auto-regressive time series of order one: $\vec{\epsilon} = \vec{y} - X\vec{\beta} \sim AR(1)$. Therefore, the model for the error term may be written $\epsilon_i = \phi\epsilon_{i-1} + Z_i$, where Z_i is white noise with mean zero and variance σ^2, and ϕ is the autoregressive parameter, with $|\phi| < 1$. [8]

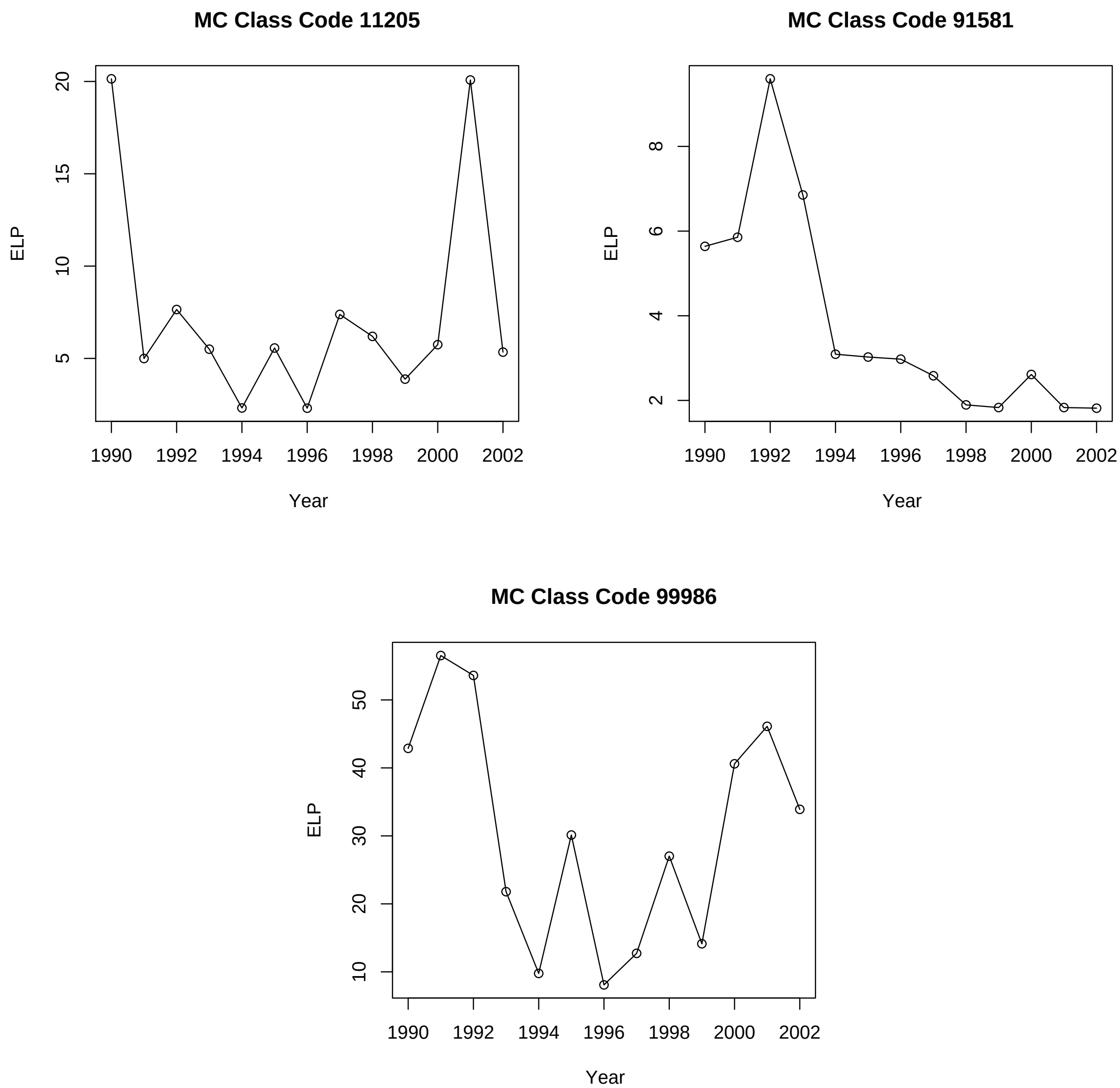

Figure 2.1: Sample of MC Class Code Plots

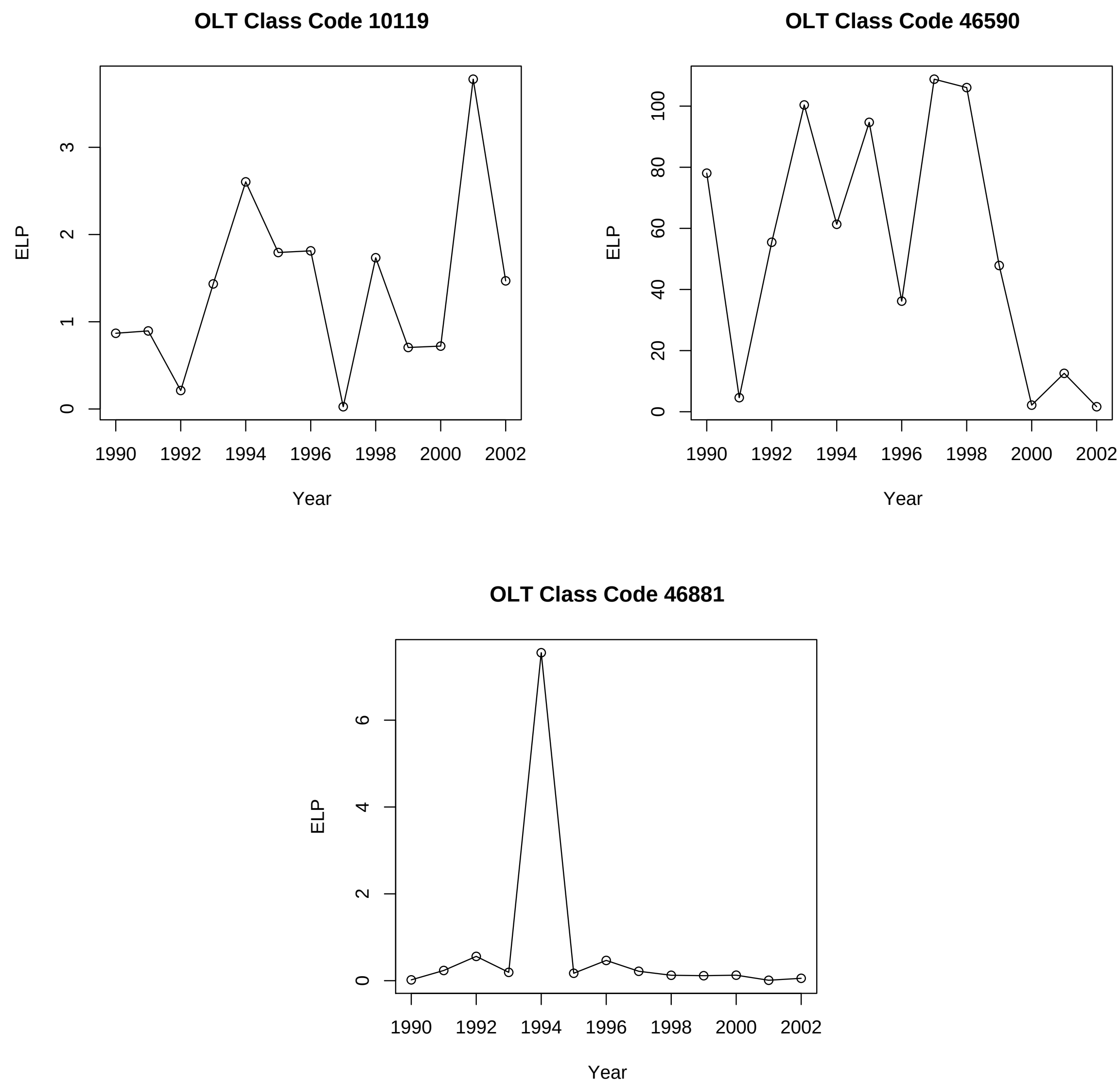

Figure 2.2: Sample of OLT Class Code Plots

Group	Class Code	Description
MC	11205	Contractors Equipment - Earth Moving - Rented with Operator
MC	91581	Contractors - Subcontracted Work - Construction, not Buildings
MC	99986	Wrecking - Buildings or Structures
OLT	10119	Boats - Rented to Others
OLT	46590	Parades
OLT	46881	Professional and Trade Associations - Office Building - For Profit

Figure 2.3: Class Code Descriptions for Sample MC and OLT Class Codes

Figures 2.4 and 2.5 each show the sampled standardized class code series plotted along with the fit for MC and OLT, respectively. Clearly, the fits are not perfect, but a linear fit would be much worse.

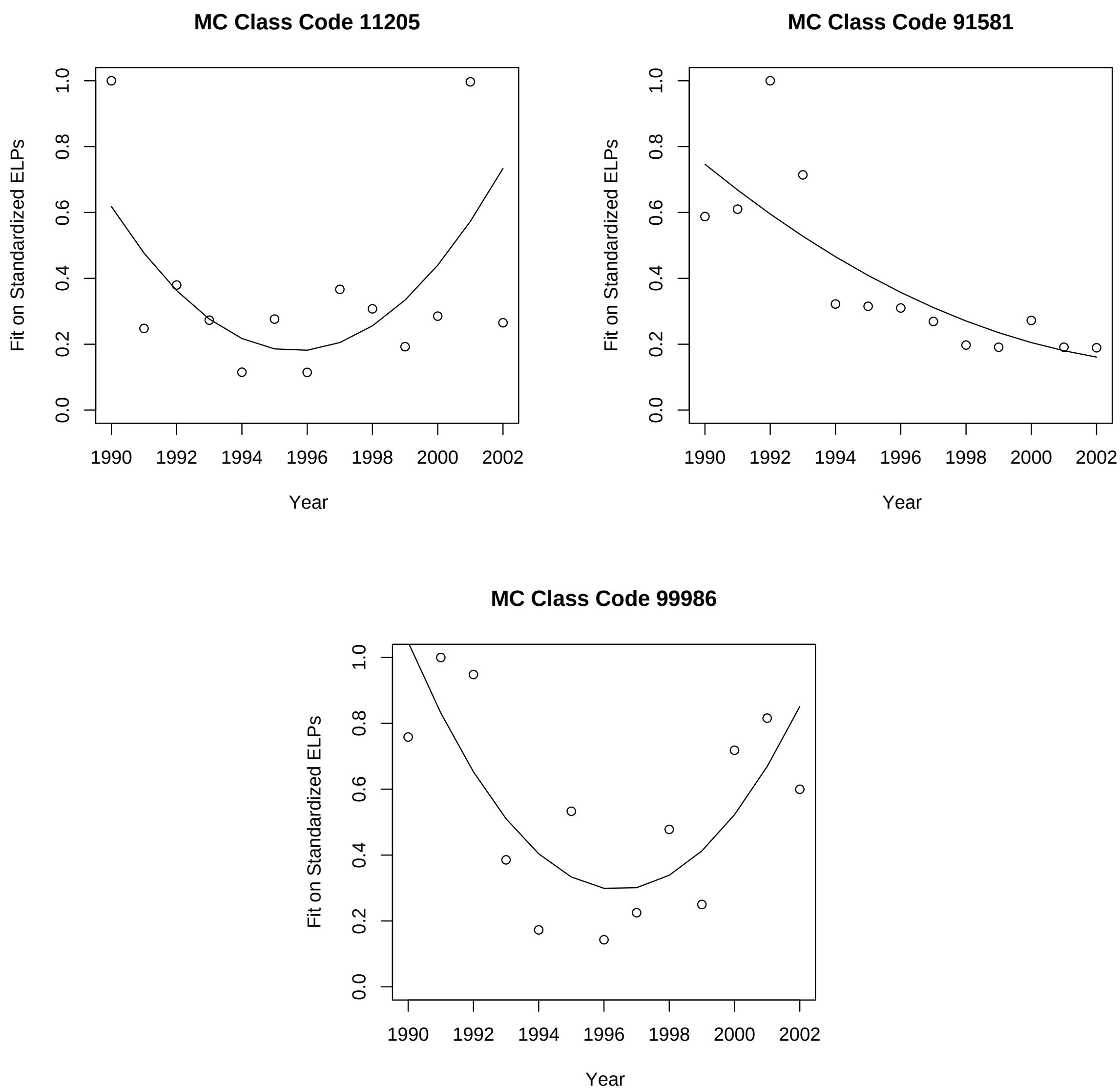

Figure 2.4: Sample of MC Class Code Fitted Plots

All of the estimated coefficients for $\hat{y}_{14}$ came from the `arima` procedure in R. [7] This procedure in R calculated the coefficients by minimizing a conditional sum of squares to obtain the starting values for a maximum likelihood procedure, based on a Gaussian likelihood. We decided to fit an AR(1) because the length of the time series is so short. In addition, we

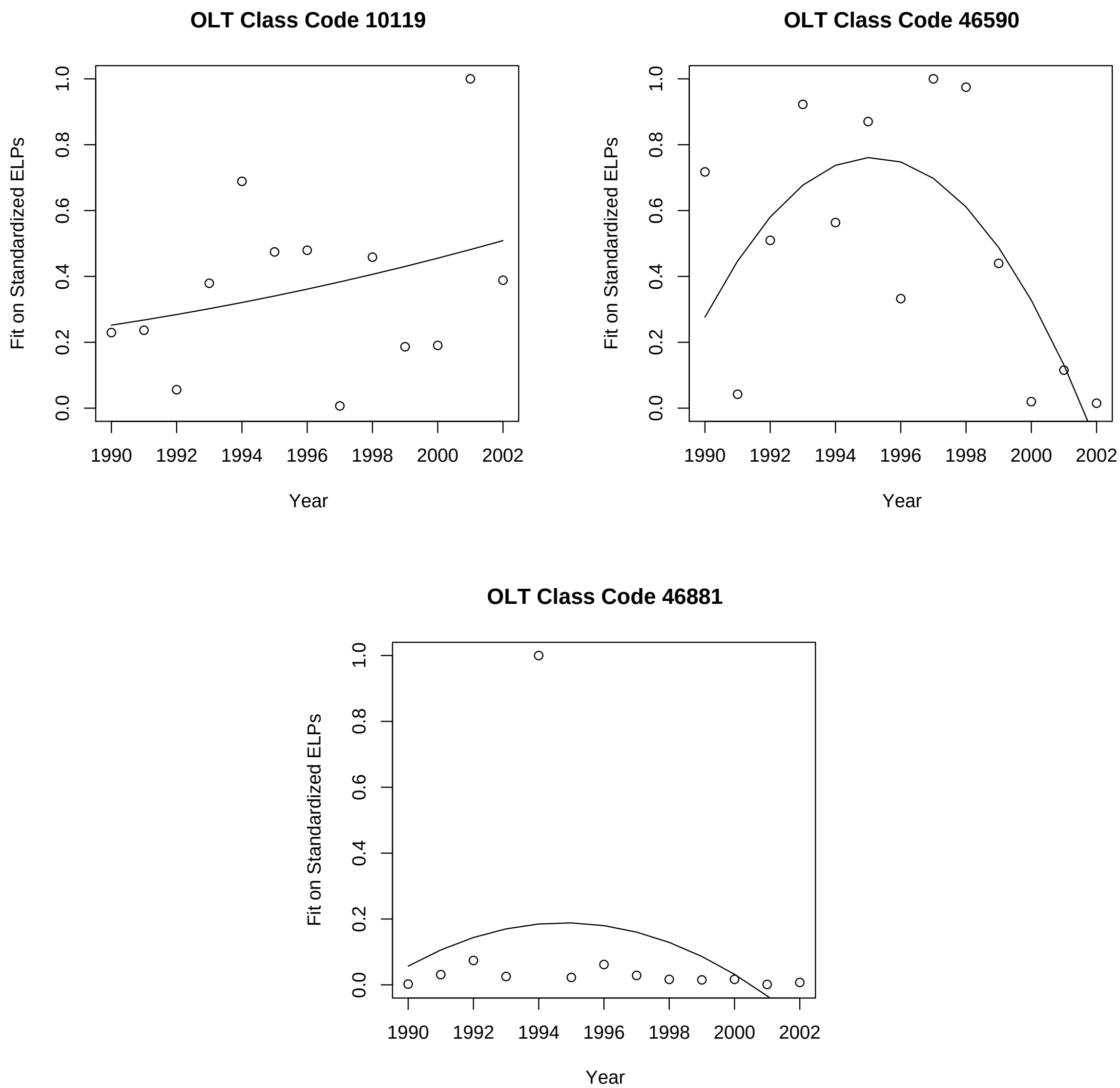

Figure 2.5: Sample of OLT Class Code Fitted Plots

added a design matrix in the arima procedure, which allowed us to add external regressors so that we would have a polynomial model. Then by substituting in the coefficients into

$$\hat{y}_{14} = \hat{\beta}_0 + \hat{\beta}_1 * 14 + \hat{\beta}_2 * 14^2 + \hat{\epsilon}_{14}$$

$$\hat{\epsilon}_{14} = \hat{\phi}\varepsilon_{13}$$

we compute estimates for the 2003 ELPs for each of the class codes in the MC and OLT groups. [7] [8]

We should expect the ELP predictions to fall close to the value of the fitted line at 2002. However, some of the class codes had decreasing trends, so that future ELP predictions were negative, which makes no sense because insurance companies do not *pay* premiums. Therefore, the predicted ELP for such cases is taken to be the mean ELP for the 13 years. There were two OLT class codes that had a negative prediction. Also, none of the MC class codes had a negative prediction.

A table with the necessary coefficients to calculate $\hat{y}_{14}$, along with the calculated $\hat{y}_{14}$ is given in the appendix for each of the MC and OLT groups.

We then bootstrapped the $\hat{y}_{14}$ values from these to construct the MC and OLT prediction distributions. To build bootstrap distributions for inference on $\hat{y}_{14}$, we resample the 23 MC predictions and 57 OLT predictions separately, and with replacement. We then compute the percentiles for the 95% confidence limits for each of the MC and OLT bootstrap distributions of $\hat{y}_{14}$. We repeat this 1000 times. The confidence bounds used for prediction are the averages of the 1000 bootstrapped confidence bounds. [2]

Chapter 3

Results

Figure 3.1 shows sample histograms of the bootstrapped $\hat{y}_{14}$ values, with the 2.5th and 97.5th percentiles highlighted. Note that we took 1000 such samples and that the histograms below are only from the first bootstrap sample.

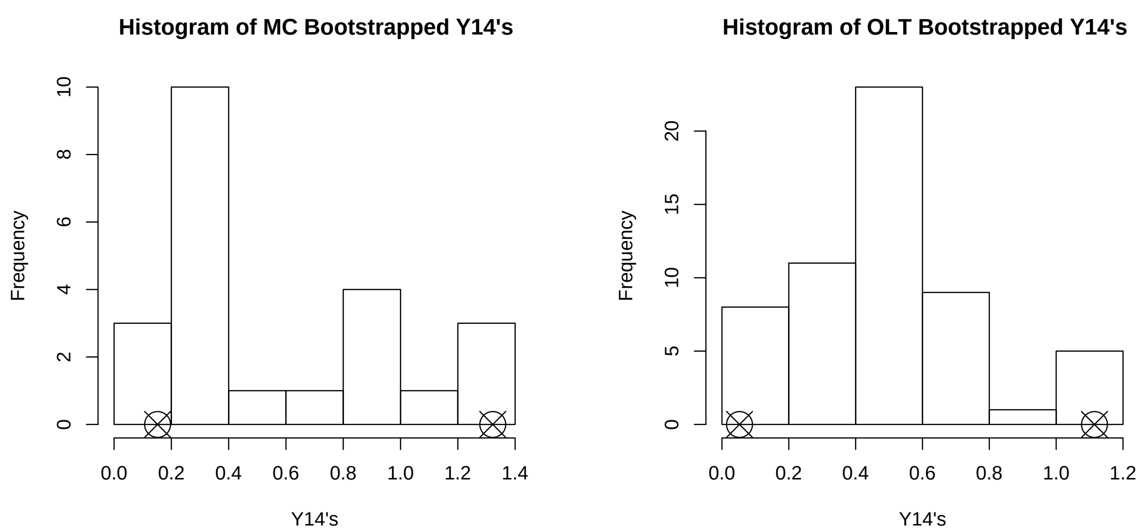

Figure 3.1: MC and OLT Histograms for Bootstrapped $\hat{y}_{14}$'s

Figures 3.3 and 3.4 show histograms of the 1000 MC and OLT $\hat{y}_{14}$ 2.5th and 97.5th percentiles. We can see that the histogram of the 2.5th percentiles is not as compact as the histogram of the 97.5th percentiles. This could occur because the class codes with higher ELPs tended to be less volatile than those with lower ELPs. Figure 3.2 shows the mean of the 2.5th and 97.5th percentiles for MC and OLT.

By having a 95% confidence interval for the 2003 predicted ELPs, we can say that approximately 95% of the confidence intervals will contain the 2003 predicted ELPs within the upper

Group	2.5th Percentile	97.5th Percentile
MC	0.11976	1.30389
OLT	0.10656	1.08600

Figure 3.2: Mean of 1000 MC and OLT bootstrapped $\hat{y}_{14}$ 2.5th and 97.5th Percentiles

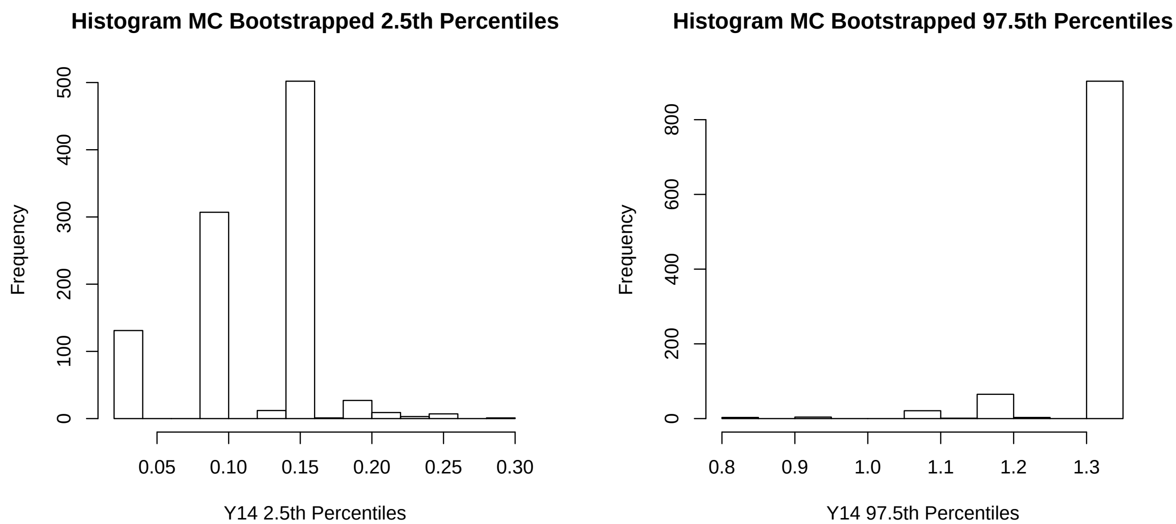

Figure 3.3: MC Histograms for 2.5th and 97.5th Percentiles

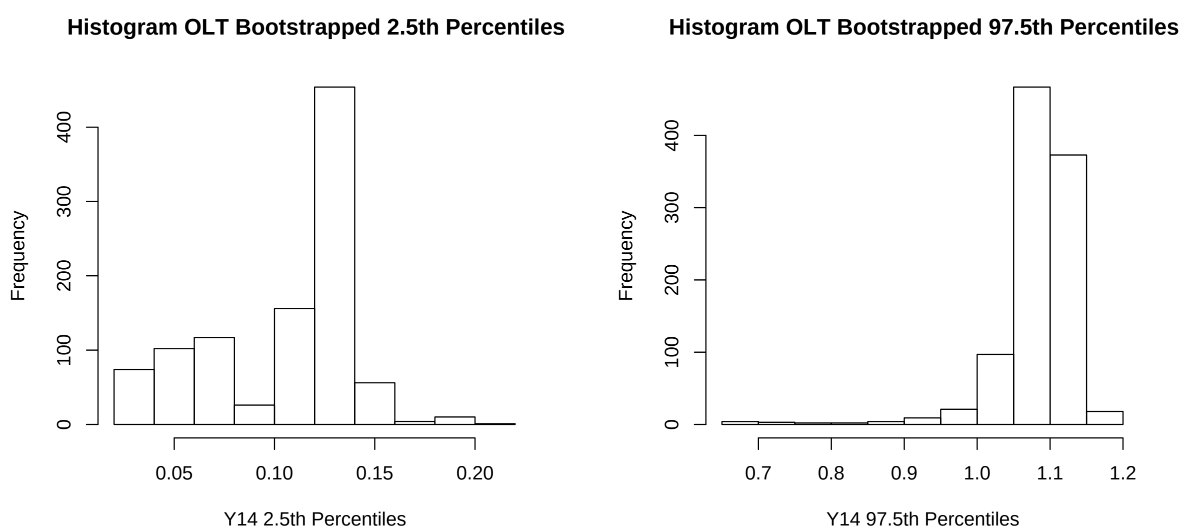

Figure 3.4: OLT Histograms for 2.5th and 97.5th Percentiles

and lower bounds and that approximately 5% of the confidence intervals will not contain the 2003 predicted ELPs.

Figures 3.5 and 3.6 show earlier sample class code plots with the 2003 predicted value and 95% confidence bounds for MC and OLT, respectively.

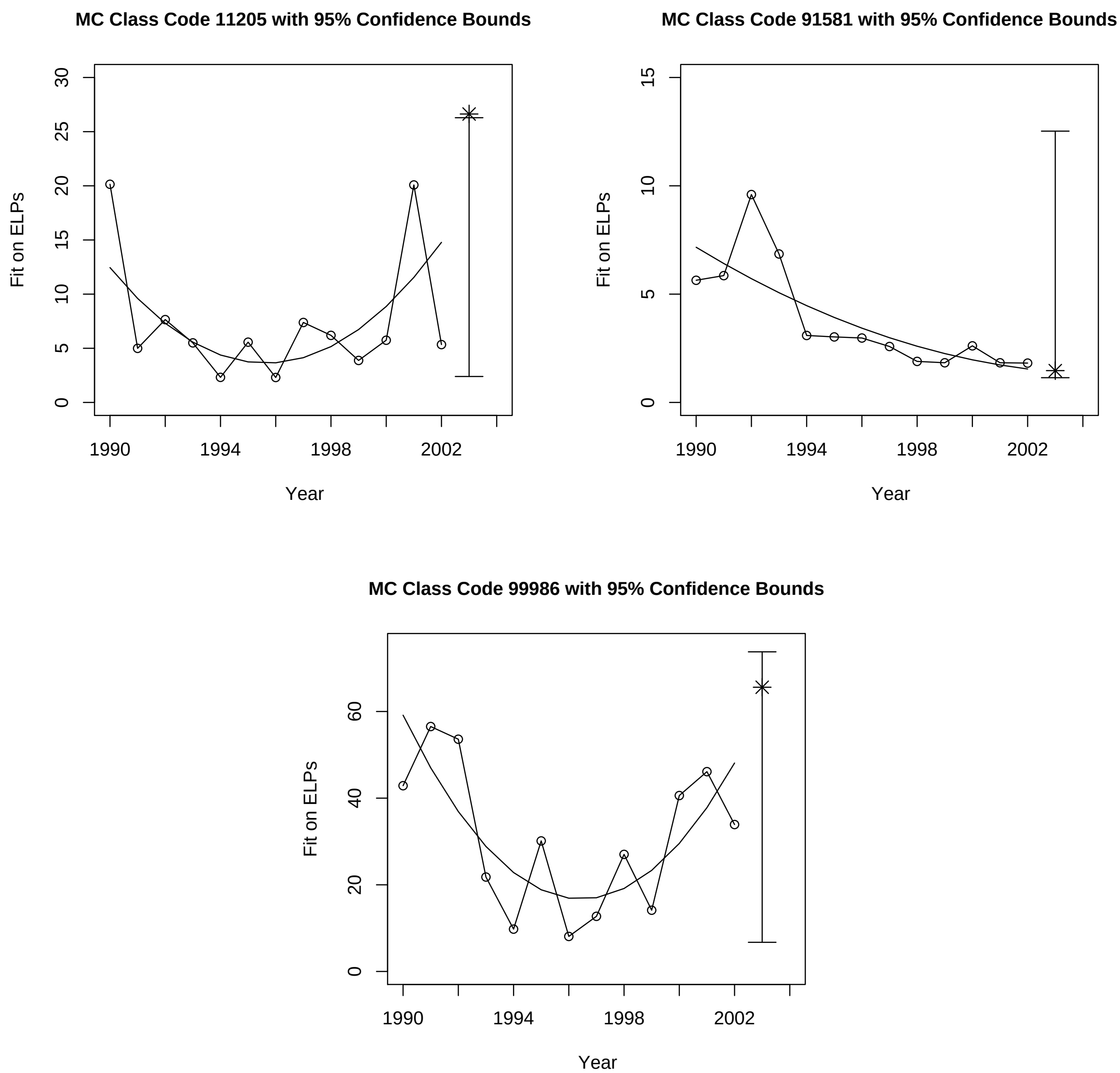

Figure 3.5: Sample of MC Class Code Fitted Plots

Clearly, the confidence bounds are very large.

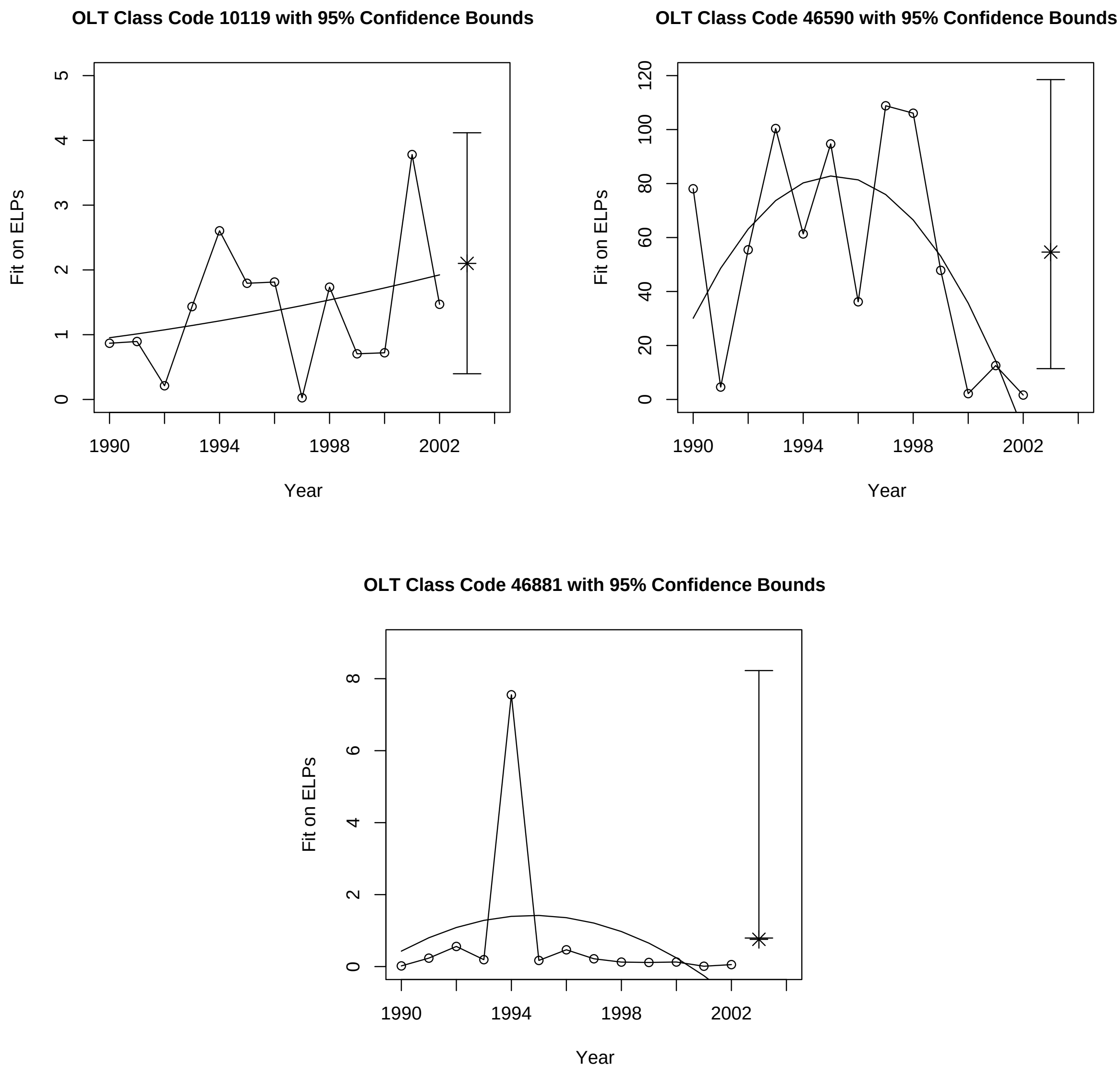

Figure 3.6: Sample of MC Class Code Fitted Plots

Table 3.1: Sample MC and OLT Class Codes with Coefficients and Confidence Bounds

Group	Code	$\hat{y}_{14}$	Max	Max $\times$ $\hat{y}_{14}$	AR1	Intercept	Beta1	Beta2	e13	L. bound	U. bound
MC	11205	1.322	20.143	26.629	-0.854	0.787	-0.183	0.014	-0.468	2.397	26.287
MC	91581	0.153	9.597	1.469	0.219	0.830	-0.086	0.003	0.028	1.142	12.524
MC	99986	1.160	56.530	65.598	-0.363	1.297	-0.269	0.018	-0.251	6.727	73.772
OLT	10119	0.556	3.781	2.100	-0.155	0.238	0.014	0.001	-0.120	0.397	4.118
OLT	46590	0.502	108.820	54.604	-0.345	0.069	0.225	-0.018	0.117	11.426	118.505
OLT	46881	0.100	7.553	0.757	-0.205	-0.004	0.066	-0.006	0.118	0.793	8.225

Chapter 4

Conclusion

4.1 Directions for Further Development

Overall, it will be very interesting to see how many of the 2003 ELPs actually fall into the confidence bounds we constructed. Further data may be incorporated into this analysis for improved estimates of the confidence bounds, likely giving smaller confidence intervals. As more data becomes available, inferences may be made on the class codes that were recently created and unfortunately had to be left out of the analysis in this thesis, but may be included in future analyses. In addition, a method to handle the zero observations may be developed, so that these class codes may be included.

The General Liability 2003 products and completed operations class code ELPs could also be predicted. In addition, more sophisticated models could be made to capture more of the trends in the different class codes, possibly through Bayesian analysis and pulse functions for outliers. This thesis also only looked at the multi-state a-rated premises and operations class codes. Finding the a-rated premises and operations as well as the a-rated products and completed operations class code ELPs by state would be more ideal. However, finding the data would be challenging and very expensive, but larger insurance carriers have such valuable data.

Appendix A

Data

Table A.1: MC Original Data

Code	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
11205	20.143	4.996	7.649	5.501	2.318	5.566	2.306	7.384	6.195	3.879	5.746	20.080	5.343
13411	0.350	0.089	0.056	0.298	0.091	0.292	0.195	0.450	0.688	0.443	0.568	0.365	0.484
15060	0.505	1.072	0.586	0.725	0.682	0.366	0.632	0.314	0.363	0.169	0.373	0.228	0.132
15061	0.368	0.474	0.470	0.316	0.162	0.176	0.139	0.111	0.150	0.150	0.180	0.080	0.107
54444	0.416	0.561	0.491	0.361	0.130	0.070	0.146	0.084	0.140	0.076	0.097	0.322	0.196
91210	72.429	35.861	13.358	94.995	108.400	123.170	53.974	72.625	42.167	112.090	112.980	128.780	134.670
91280	25.486	23.742	77.903	117.190	22.627	12.333	13.822	48.295	40.828	16.426	14.105	3.470	20.274
91581	5.640	5.855	9.597	6.854	3.092	3.025	2.975	2.583	1.895	1.832	2.613	1.832	1.817
91582	4.454	5.750	3.634	3.256	1.750	1.291	1.551	1.332	2.101	1.264	1.539	1.052	0.697
91583	2.084	2.012	2.426	2.843	1.035	0.696	0.707	0.587	0.815	0.658	1.148	0.905	0.496
91584	2.133	2.637	2.860	3.795	1.357	1.335	1.968	1.393	1.564	1.331	1.498	1.365	1.162
91585	3.173	3.015	3.085	3.537	1.800	1.425	1.370	1.044	1.293	1.336	1.254	1.149	1.287
91586	1.689	1.149	0.966	2.048	0.584	0.874	3.500	2.447	1.200	0.833	0.778	0.532	0.729
91587	1.922	4.616	9.073	8.841	3.968	3.636	9.492	5.768	2.185	0.893	1.858	2.386	2.255
91588	1.558	6.231	3.939	3.628	3.535	1.058	0.853	0.533	2.161	1.606	1.442	1.684	2.680
91589	5.354	3.639	3.802	3.877	3.516	2.636	3.511	2.426	2.974	1.922	2.608	2.687	2.894
94444	2.144	0.930	0.674	2.248	0.992	0.462	1.332	2.656	0.273	0.085	0.098	0.307	0.363
95648	4.510	6.005	4.539	6.285	3.789	4.286	4.854	3.984	8.489	9.771	7.800	5.249	5.743
96703	2.908	3.687	3.122	4.546	1.049	1.123	2.625	20.556	3.158	6.840	10.444	15.047	11.811
97002	0.541	1.169	1.066	2.623	0.926	0.619	0.859	4.174	0.436	0.373	0.868	0.419	0.529
97221	5.378	5.103	10.953	0.857	1.218	0.905	1.590	1.981	0.418	0.753	0.736	0.532	0.857
98150	10.486	25.286	11.586	16.687	4.841	15.023	6.506	3.855	5.265	4.610	4.516	9.739	9.102
99986	42.875	56.530	53.603	21.796	9.772	30.131	8.086	12.726	27.021	14.136	40.603	46.117	33.908

Table A.2: OLT Original Data

Code	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
10020	11.878	18.328	20.538	36.9	17.099	23.976	8.108	23.772	25.669	8.087	6.873	4.785	3.979
10119	0.868	0.895	0.212	1.434	2.604	1.794	1.813	0.026	1.734	0.705	0.721	3.781	1.469
10135	0.007	3.781	0.062	14.543	0.097	0.67	3.275	0.267	40.629	68.435	74.809	2.688	38.792
10375	18.437	55.326	38.591	43.143	38.757	84.219	92.869	95.159	77.263	49.186	46.696	46.483	31.047
11101	0.204	0.221	0.589	0.446	0.243	0.197	0.111	0.315	0.144	0.639	0.955	0.661	0.853
15119	4.609	78.703	10.672	11.063	6.766	8.094	1.745	3.39	4.619	3.005	1.743	5.265	4.316
15300	1.964	2.03	0.718	1.442	0.988	0.858	1.28	0.248	0.252	0.796	1.614	0.793	0.67
16722	4.523	2.912	3.821	3.437	2.274	2.458	2.613	2.655	2.61	2.355	2.691	2.48	2.748
16723	0.469	0.309	0.733	0.518	1.594	1.165	1.088	0.148	1.47	0.969	1.416	0.904	0.459
18991	4.074	2.281	1.866	1.417	4.077	2.197	3.539	2.389	5.106	2.008	2.459	2.049	1.526
40015	709.35	64.277	181.02	405.66	1505.4	29.76	6.756	355.35	893.69	15.79	19.312	757.4	165.4
40031	408.98	209.84	168.74	121.78	132.6	82.977	73.279	80.622	103.18	102.2	154.2	27.795	55.014
40040	3.739	1.284	3.953	3.616	2.379	0.772	0.336	0.875	1.723	6.076	7.507	3.654	1.482
40066	1.302	7.138	3.422	2.299	0.726	0.179	0.595	1.763	0.418	1.632	1.099	1.765	0.053
40067	0.221	0.765	9.265	0.882	0.966	1.078	0.777	0.497	0.714	0.485	0.436	0.542	0.35
40069	311.7	343.84	100.91	201.47	489.3	260.89	134.51	77.569	60.233	106.99	37.978	10.666	46.837
40072	280.82	254.87	335.01	34	416.08	375.27	205.23	7.785	34.567	348.3	158.17	54.003	62.094
40115	154.89	184.65	189.75	160.08	55.315	87.907	276.06	44.007	110.36	146.9	108.75	125.33	57.292
40117	222.65	100.4	116.28	102.62	127.79	93.275	50.964	32.529	87.753	118.15	79.951	203.32	261.31
40140	1293.7	156.68	162.03	815.23	386	155.98	146.14	323.39	332.88	634.17	764.49	428.44	39.292
41210	197.17	136.18	142.26	166.1	96.317	156.31	188.51	30.171	67.137	26.451	110.94	7.197	77.457
41666	6.915	5.498	10.05	7.938	8.915	6.385	3.444	5.08	4.60	2.63	10.201	2.757	1.853
41700	517.97	110.42	642.18	879.51	377.71	510.25	471.89	947.42	351.3	232.6	384.99	69.886	113.97
43424	2.259	1.29	5.392	4.532	12.352	6.184	32.326	12.164	2.808	7.467	3.385	5.502	3.032
43517	941.23	344.22	1234.1	123.18	551.9	1094.3	734.58	555.67	804.66	577.9	1383	944.25	1396.9
43754	23.816	570.89	104.85	119.44	1.922	42.931	38.876	70.609	94.581	4.855	25.903	36.097	57.447
43945	20.484	15.651	37.996	11.439	17.974	7.079	12.097	5.769	55.12	17.432	15.33	64.78	14.816
43946	35.535	35.077	129.89	79.784	15.516	820.88	500.02	34.933	210.92	82.959	98.225	66.029	123.73
44105	0.473	4.223	3.149	2.816	0.409	0.238	0.306	0.102	0.645	3.517	2.467	5.526	11.227

Table A.3: OLT Original Data Continued

Code	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
44106	10.086	13.672	6.247	2.914	0.037	0.059	0.164	0.877	4.024	7.519	14.29	6.505	0.406
44113	1.704	1.132	1.482	0.767	0.525	0.938	0.244	0.743	0.521	3.115	0.799	1.069	1.139
44193	255.04	634.12	378.7	333.56	113.82	221.43	37.051	115.64	229.38	199.97	131.87	463.65	116.18
44194	2108.8	0.603	15.085	233.46	806.67	475.04	270.83	173.59	209.49	480.22	261.57	328.24	371.44
45539	0.809	0.663	0.622	0.679	0.609	0.359	0.339	0.779	1.635	2.35	0.987	0.829	0.874
45993	0.104	0.199	0.105	0.07	0.096	0.088	0.09	0.066	0.105	0.11	0.088	0.119	0.03
46510	9.529	81.47	68.954	0.006	14.721	6.575	2.899	1.364	6.551	6.162	62.521	0.569	0.197
46590	78.071	4.6	55.463	100.39	61.331	94.7	36.194	108.82	106.07	47.849	2.165	12.564	1.64
46671	56.454	80.85	90.482	77.08	56.225	57.089	55.299	30.876	41.572	37.822	52.12	44.854	78.181
46773	196.56	1665.2	498.31	191.96	168.86	42.694	4.502	183.32	241.89	108.45	13.55	90.486	95.038
46822	5.041	0.207	109.74	34.426	2.729	0	0.384	0.514	97.229	2.67	0.064	2.002	1.422
46881	0.019	0.235	0.56	0.192	7.553	0.171	0.467	0.217	0.125	0.115	0.127	0.01	0.055
47051	8.773	7.888	6.617	6.45	7.368	7.198	4.652	5.836	10.984	2.662	2.544	2.451	0.857
47052	4.011	5.79	4.941	3.593	3.092	2.403	2.299	1.772	3.223	3.767	2.996	5.388	2.115
47103	11.958	16.081	11.163	12.958	12.087	1.584	2.735	3.367	23.501	6.424	10.316	8.477	2.461
47146	0.746	2.727	0.609	2.624	1.258	0.788	2.786	1.153	0.851	2.796	1.742	0.902	1.61
48177	0.081	0.039	0.646	0.593	1.499	0.362	0.579	2.076	12.665	11.641	14.4	9.986	5.578
48178	0.749	11.789	1.734	8.339	68.24	65.104	11.373	0.558	15.257	22.807	19.89	26.151	70.518
48727	37.751	34.993	42.441	33.382	29.994	25.306	29.626	30.404	31.955	27.622	30.24	34.525	35.55
48924	16.791	32.951	107.69	21.418	8.941	4.429	1.822	2.873	4.864	10.545	6.826	12.062	5.607
49451	1.341	1.317	0.928	0.7	0.731	0.716	0.507	0.518	0.716	0.633	0.8	0.738	0.508
49452	0.516	0.76	0.949	0.73	0.758	0.919	0.645	0.866	1.316	1.386	1.882	0.619	0.835
49800	3.549	59.626	20.522	39.485	49.517	13.339	11.207	71.008	88.567	15.679	55.417	34.076	96.171
49890	0.077	34.275	23.455	10.572	1.974	0.856	0.34	2.257	3.625	2.776	3.486	2.844	2.735
49891	0.654	0.165	1.106	3.604	1.079	2.8	2.768	0.891	1.22	0.822	1.5	1.074	1.61
63219	5.455	8.183	62.921	56.112	12.34	8.04	23.128	50.124	34.255	17.18	21.635	28.236	15.685
63220	6.987	4.627	83.008	3.918	21.311	21.121	24.045	4.484	15.736	0.371	1.086	5.625	6.843
64500	24.804	30.642	23.945	17.59	16.274	18.688	45.374	11.341	10.7	22.868	14.662	42.639	33.351

Appendix B

Coefficients

Table B.1: MC Coefficients

Code	$\hat{y}_{14}$	Max	Max $\times$ $\hat{y}_{14}$	AR1	Intercept	Beta1	Beta2	e13
11205	1.32198	20.14300	26.62859	-0.85429	0.78690	-0.18254	0.01373	-0.46845
13411	0.84585	0.68800	0.58194	-0.06410	0.15485	0.04489	0.00029	-0.08420
15060	0.15047	1.07200	0.16131	-0.88535	0.88718	-0.07441	0.00140	-0.03387
15061	0.25231	0.47400	0.11960	0.24442	1.08359	-0.14162	0.00589	-0.01214
54444	0.60265	0.56100	0.33809	-0.12060	1.27497	-0.25117	0.01445	-0.10205
91210	1.06438	134.67000	143.34047	0.14604	0.43626	-0.00151	0.00329	0.02694
91280	0.28652	117.19000	33.57700	0.15875	0.35230	0.02192	-0.00349	0.12494
91581	0.15311	9.59700	1.46938	0.21850	0.83008	-0.08626	0.00268	0.02842
91582	0.26254	5.75000	1.50958	-0.07266	1.05463	-0.15987	0.00734	-0.09597
91583	0.22609	2.84300	0.64277	0.25009	0.96639	-0.11740	0.00468	-0.05685
91584	0.28229	3.79500	1.07130	-0.04146	0.78378	-0.05102	0.00109	0.00212
91585	0.37235	3.53700	1.31699	0.19010	1.13460	-0.14047	0.00613	0.02028
91586	0.02902	3.50000	0.10157	0.03968	0.25707	0.07952	-0.00686	0.07664
91587	0.46106	9.49200	4.37638	0.17871	0.35961	0.09942	-0.00945	0.18308
91588	0.37344	6.23100	2.32689	-0.45609	0.95254	-0.15447	0.00828	0.08637
91589	0.50315	5.35400	2.69389	-0.61865	0.96161	-0.09012	0.00422	0.03776
94444	0.36388	2.65600	0.96646	-0.12744	0.56227	-0.00034	-0.00316	0.11307
95648	0.66127	9.77100	6.46124	0.28448	0.42525	0.03217	-0.00096	-0.09390
96703	0.81840	20.55600	16.82309	-0.29691	0.13321	-0.00618	0.00376	-0.11441
97002	0.26910	4.17400	1.12323	-0.26886	0.11130	0.08589	-0.00705	0.08967
97221	0.13789	10.95300	1.51029	-0.26866	0.79521	-0.14406	0.00692	-0.01334
98150	0.39410	25.28600	9.96511	-0.86349	1.00689	-0.16510	0.00875	0.01992
99986	1.16041	56.53000	65.59822	-0.36345	1.29706	-0.26885	0.01804	-0.25108

Table B.2: OLT Coefficients

Code	$\hat{y}_{14}$	Max	Max × $\hat{y}_{14}$	AR1	Intercept	Beta1	Beta2	e13
10020	0.43776	36.90000	16.15323	-0.27605	0.35710	0.09992	-0.00982	0.11104
10119	0.55554	3.78100	2.10049	-0.15468	0.23754	0.01401	0.00053	-0.12019
10135	0.66626	74.80900	49.84256	0.16561	-0.09267	0.03929	0.00114	-0.09292
10375	0.13079	95.15900	12.44565	0.26929	-0.01327	0.21940	-0.01496	0.01509
11101	1.08974	0.95500	1.04070	0.14589	0.43534	-0.08022	0.00910	-0.03650
15119	0.27461	78.70300	21.61298	-0.85356	0.74242	-0.16359	0.00897	-0.07622
15300	0.63772	2.03000	1.29457	-0.21783	1.12644	-0.17483	0.00980	-0.17903
16722	0.66356	4.52300	3.00126	-0.43188	0.97652	-0.09529	0.00519	-0.00774
16723	0.40545	1.59400	0.64629	-0.30378	0.04317	0.16045	-0.00988	-0.17162
18991	0.33189	5.10600	1.69464	-0.40674	0.38997	0.06375	-0.00500	-0.07420
40015	0.18526	1505.40000	278.89201	-0.22230	0.26973	0.00905	-0.00117	-0.08008
40031	0.29218	408.98000	119.49554	0.17351	0.95198	-0.16825	0.00875	-0.10857
40040	0.39347	7.50700	2.95381	0.39226	0.43698	-0.03238	0.00261	-0.25988
40066	0.44186	7.13800	3.15399	-0.58143	0.84602	-0.16486	0.00905	-0.22448
40067	0.14096	9.26500	1.30600	-0.22836	0.32558	-0.02811	0.00025	0.03589
40069	0.34317	489.30000	167.91485	0.04218	0.67987	-0.03722	-0.00120	0.10316
40072	0.53118	416.08000	221.01531	0.22322	0.73543	0.01395	-0.00480	0.04386
40115	0.39611	276.06000	109.34950	-0.34598	0.66202	-0.03258	0.00071	-0.15015
40117	1.13116	261.31000	295.58382	-0.00213	0.98214	-0.22401	0.01676	0.09720
40140	0.44631	1293.70000	577.38765	-0.07861	0.64197	-0.09442	0.00561	-0.33266
41210	0.03152	197.17000	6.21508	-0.39432	0.94761	-0.05612	-0.00023	0.21436
41666	0.25266	10.20100	2.57742	-0.18961	0.74851	-0.00178	-0.00252	-0.11833
41700	0.45550	947.42000	431.54585	-0.37402	0.23387	0.13924	-0.01205	0.11319
43424	0.23485	32.32600	7.59177	-0.18381	-0.17587	0.15924	-0.01121	0.09411
43517	0.98478	1396.90000	1375.64296	-0.56743	0.60347	-0.06362	0.00674	0.08515
43754	0.24430	570.89000	139.46772	-0.76965	0.71228	-0.14866	0.00807	-0.04214
43945	1.02686	64.78000	66.51991	-0.59429	0.45145	-0.07552	0.00703	-0.42895
43946	0.20930	820.88000	171.80754	-0.01475	-0.14409	0.13929	-0.00987	0.15217
44105	0.98605	11.22700	11.07033	-0.00864	0.50584	-0.16814	0.01447	0.23449
44106	0.27336	14.29000	3.90631	0.45483	0.80264	-0.13504	0.00768	-0.31665
44113	0.60739	3.11500	1.89203	-0.24597	0.54392	-0.08126	0.00596	-0.12998
44193	1.03475	634.12000	656.15618	-0.85309	1.09538	-0.21992	0.01376	-0.37778
44194	0.31031	2108.80000	654.37919	-0.20142	0.55778	-0.10864	0.00644	-0.05741
45539	0.44935	2.35000	1.05597	0.39715	0.23388	0.02393	-0.00039	-0.10641

Table B.3: OLT Coefficients Continued

Code	$\hat{y}_{14}$	Max	Max × $\hat{y}_{14}$	AR1	Intercept	Beta1	Beta2	e13
45993	0.64644	0.19900	0.12864	-0.45122	0.79010	-0.07778	0.00409	-0.31910
46510	0.34326	81.47000	27.96530	-0.20399	0.78671	-0.14909	0.00815	-0.22407
46590	0.50179	108.82000	54.60438	-0.34468	0.06956	0.22530	-0.01835	0.11731
46671	0.77702	90.48200	70.30589	0.23188	0.97620	-0.10166	0.00602	0.19247
46773	0.24253	1665.20000	403.85522	-0.62350	0.76090	-0.16079	0.00862	-0.06995
46822	0.17975	109.74000	19.72523	-0.11028	0.19073	0.02955	-0.00344	0.01932
46881	0.10028	7.55300	0.75738	-0.20493	-0.00386	0.06646	-0.00574	0.11799
47051	0.52020	10.98400	5.71385	-0.17759	0.65279	0.03186	-0.00566	-0.03226
47052	1.16290	5.79000	6.73317	-0.65115	1.15826	-0.18045	0.01157	-0.40309
47103	0.35178	23.50100	8.26714	-0.10789	0.62622	-0.05013	0.00206	-0.21803
47146	0.46072	2.79600	1.28817	-0.58848	0.54263	0.01842	-0.00156	0.05753
48177	0.54564	14.40000	7.85715	0.47385	-0.19143	0.09199	-0.00223	-0.24037
48178	0.86275	70.51800	60.83960	0.34676	0.13316	0.01119	0.00235	0.32434
48727	0.89177	42.44100	37.84779	-0.03618	1.02478	-0.08555	0.00543	0.00676
48924	0.08420	107.69000	9.06711	-0.04218	0.53068	-0.08675	0.00392	-0.01262
49451	0.57397	1.34100	0.76969	0.14897	1.12003	-0.14846	0.00793	-0.15204
49452	0.52093	1.88200	0.98039	0.08190	0.20941	0.07387	-0.00364	-0.11075
49800	0.62581	96.17100	60.18462	-0.34356	0.31056	-0.00081	0.00223	0.32276
49890	0.19162	34.27500	6.56782	-0.31491	0.79174	-0.15267	0.00778	-0.04147
49891	0.06586	3.60400	0.23737	-0.14545	0.07547	0.13064	-0.00921	0.22946
63219	0.14159	62.92100	8.90877	0.17110	0.16143	0.10501	-0.00761	0.00874
63220	0.18456	83.00800	15.32015	-0.40915	0.29458	0.00634	-0.00242	0.1135
64500	0.86028	45.37400	39.03414	-0.22612	0.74481	-0.09950	0.00768	-0.01418

Appendix C

R Code

```
mcanalysis <- function(code)
   {
    mc = read.table("MC.txt",header=T,sep=",")
    classcode = ts(c(mc[code,2],mc[code,3],mc[code,4],mc[code,5],mc[code,6],
      mc[code,7],mc[code,8],mc[code,9],mc[code,10],mc[code,11],mc[code,12],
      mc[code,13],mc[code,14]))
    year = c(1990,1991,1992,1993,1994,1995,1996,1997,1998,1999,2000,
      2001,2002,2003)
    max = max(classcode)
    for (i in 1:13){
     classcode[i] = classcode[i]/max
     }
    n = length(classcode)
    arimaxreg = t(matrix(c(1:n, (1:n)^2), ncol=n, byrow=T))
    mcout = arima(classcode,c(1,0,0),xreg=arimaxreg)
    arimacoef = mcout$coef
    arimares = mcout$res
    names(arimacoef) = c("","","","")
    xmatrix2 = t(matrix(c(1,1,1,1,1,1,1,1,1,1,1,1,1, 1:n, (1:n)^2), ncol=n,byrow=T))
    Beta = matrix(c(arimacoef[2],arimacoef[3],arimacoef[4]),ncol=1,byrow=T)
    XtBeta = xmatrix2%*%Beta
    YminusXtBeta = classcode - XtBeta
    res13 = YminusXtBeta[13]
    y14 = arimacoef[2] + (arimacoef[3]*14) + (arimacoef[4]*14^2) +
      arimacoef[1]*res13)
    codesum = 0
    for (i in 1:13){
      codesum = codesum + classcode[i]
    }
    if (y14 < 0) y14 = codesum/13
    analysisout = t(matrix(c(mc[code,1],y14,max,max*y14,arimacoef[1],arimacoef[2],
     arimacoef[3],arimacoef[4],res13)))
    analysisout
  }

mcy14 = mcoutput[,2]
samplemc = matrix(1,23,1)
mctwohalf = matrix(1,1000,1)
mcntysvnhalf = matrix(1,1000,1)
for (i in 1:1000){
  for (j in 1:23){
   randommc = as.integer((runif(1)*sample(23,1,replace=T))+1)
   samplemc[j,] = mcy14[randommc]
  }
 mctwohalf[i,] = quantile(samplemc,probs=.025)
 mcntysvnhalf[i,] = quantile(samplemc,probs=.975)
}
```

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