THE INTERVIEW PROJECT

by

DORIS DANIEL SANTARONE

(Under the Direction of Denise Spangler)

ABSTRACT

Twenty-nine preservice teachers in a mathematics content course for early childhood education majors participated in an Interview Project. The preservice teachers worked in pairs (and one group of 3) to conduct an interview where they asked a child to solve additive structured story problems. Then they were asked to describe and analyze the child's mathematics using frameworks that were presented to them in class and to discuss any instructional decisions that they made prior to the interview, during the interview, and in a hypothetical second meeting with the child. The goals of the Interview Project were for the preservice teachers to develop knowledge of the frameworks around additive structured story problems and to apply these frameworks in real time and retrospectively while teaching children and for the preservice teachers to develop the ability to listen to and learn from children's mathematical thinking. The purpose of the study was to evaluate whether or not the project met these goals. Pre-Interview Project and post-Interview Project data were collected in order to observe any changes. Data were collected in the form of class products, interviews, and observations. I found that the preservice teachers engaged in a Mathematics Teaching Cycle similar to that described by Simon (1997). The preservice teachers showed a significant improvement in their ability to describe a child's mathematics, to analyze a child's mathematics, and to use their listening to make appropriate instructional decisions. In addition, I found that the preservice teachers were rethinking their definitions of teaching and learning mathematics. They moved away from a view of teaching as telling and toward a view of teaching as posing appropriate tasks. They saw the benefit of incorporating theory into their practice. They also rethought their views of learning mathematics. They began to value reasoning strategically and using intuition to solve problems, rather than relying on traditional algorithms.

INDEX WORDS: Preservice Teachers, Mathematics, Math 2008, Elementary, Children's Mathematics, Constructivism, Field Experience, Hypothetical Learning Trajectory, Mathematics Teaching Cycle

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DEDICATION

This dissertation is dedicated to my husband, Jeff, for his unwavering love and support during this time. I also dedicate this to my children, Izzy and Jack, who have given me the inspiration to continue this journey and to be the best role model that I can be for them.

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CHAPTER 1

INTRODUCTION

For the past six years, I have been teaching in the mathematics department at Georgia College. Initially, my interests were in teaching college level mathematics courses and courses for preservice secondary mathematics teachers. I had no interest in elementary school mathematics. But, through my experiences of teaching courses for the early childhood program and having two children of my own, my interest in the development of a child's mathematics grew. I have especially grown to enjoy teaching the Math 2008 course on numbers and operations, where we discuss the early development of a child's counting strategies and other strategies used to solve additive structured story problems. In this course, I had always shown videos of children solving additive structured story problems and asked the preservice teachers to describe the story's structure (using the Cognitively Guided Instruction framework (1999)) and the strategy that the child used to solve it (using the frameworks of Gelman (1986), CGI (1999) and Van de Walle (2010)). I always felt that this activity was good, but I felt that it could be better if the preservice teachers had an authentic experience working with a child. Thus, I began to incorporate an interview project where the preservice teachers interviewed a child and then described the structure of the story problem and the strategy that the child used to solve the problem. At that time, I felt that this project was successful in accomplishing the goals that I had for it.

In the spring semester of 2012, when I was enrolled in a doctoral course called Advanced Studies of Mathematics Learning, we discussed the levels of whole number development that

Steffe and colleagues had developed (Steffe et al., 1982). For my final project in that class, I attempted to make connections between the frameworks of CGI, Gelman, Van de Walle, and Steffe. I suggested that Steffe's framework could be used to analyze the child's mathematical actions and that a teacher's instructional decisions should be based on this analysis. I realized at that time that I could use Steffe's levels of whole number development in the Math 2008 Interview Project as a way for the preservice teachers to analyze the child's mathematics and make appropriate instructional decisions in addition to the prior stated requirement of describing the child's mathematics. I thought that it would be a great addition to the project, and I began to incorporate it in the fall semester of 2012. After several semesters of incorporating Steffe's levels into the interview project, I thought that there had been a major improvement in the project's impact on the preservice teachers' knowledge and beliefs. Thus, I wanted to formally evaluate the effectiveness of the Interview Project.

Statement of the Problem and Background

Since the 1980s, reformers have sought to improve what students get out of school by advocating for changes in the standards, in the assessment, and in the curriculum (Ball & Cohen, 1999). "Students are to learn mathematics with understanding, engage in and be able to solve real-world and meaningful problems, and develop the confidence and power to think mathematically" (Ball, 1995, p. 3). In response to this reform, teacher education programs face the challenge of improving preservice teachers' conceptual understanding of the subject matter and their abilities to examine students' mathematical thinking in a deeper and more complex way. "Changing teacher preparation to more fully engage core practices and pedagogies of enactment requires a significant shift in the practice of teacher education" (Grossman & McDonald, 2008, p.191). Building teachers' knowledge of how students learn and develop is recommended widely, sometimes in the form of teaching principles or professional standards, as requisite to planning effective instruction (Even & Tirosh, 2008; NCTM, 1991). The 1999 Report of the Consortium for Policy Research in Education (CPRE) called for more opportunities for teachers to "learn not only about the subject matter, but also about how students think about the content" (p. 1). In addition, the National Council of Teachers of Mathematics (2000) called for teachers to have not just a deep understanding of the content that they teach, but also knowledge of their students.

Crockett (2002) found that analyzing student thinking was the most powerful activity to lead teachers to reconsider the teaching and learning of mathematics. Fennema et al. (1993, 1996) claimed that learning about research on children's mathematical thinking and applying it while interacting with students were associated with changes in both teachers' beliefs and the type of instruction that they provided. Given this evidence, I was curious to learn if these results could occur in preservice teacher education. If teachers are expected to analyze students' mathematical thinking, then it is important to engage future teachers in experiences that awaken their curiosity and challenge their thinking about teaching and learning. This literature base lends further support to the experience I created within a mathematics content course for preservice elementary school teachers (Math 2008) that I call the Interview Project.

Math 2008

Math 2008 is a course that is part of the mathematics requirement for early childhood education majors in Georgia.[1] At Georgia College, this course is a requirement for admission into the early childhood cohort. The content of the course includes early number concepts, meaning for the operations, place value, whole number computation, meaning for fractions, and

[1] The early childhood education program in Georgia is for preservice teachers of prekindergarten through fifth grade. In other states, this is commonly called elementary.

operations with fractions. One of the process goals for the course is that preservice teachers understand and use the major concepts of number and operations in mathematics for grades P-5 as defined by the National Council of Teachers of Mathematics (NCTM, 2000). This course should also encourage them to solve problems using multiple strategies including manipulatives and technological tools. They are expected to interpret students' solutions and determine the reasonableness of answers and efficiency of solution methods. All of these components of the class are building blocks in being able to describe and analyze student thinking.

In the early childhood education program at Georgia College, the preservice teachers are required to take four mathematics content courses; Math 2008 is the first of those four. However, unlike many other programs around the country, they never take a mathematics specific methods course. As the instructor of the Math 2008 course, I think that pedagogical experiences needed to be integrated in their content courses.

The Interview Project

The Interview Project is a course assignment where preservice teachers are required to describe a child's mathematical thinking, analyze the child's mathematical thinking using the frameworks introduced in the course, apply their analysis to inform their instructional decisions (if they were to work with this child again), and discuss any on-the-spot instructional decisions they made while working with the child. The content focus of the interview is additive structured story problems. From the Interview Project, I want the preservice teachers to see how capable children are of learning mathematics and solving problems so that they will learn to respect children's mathematical thinking even when they do not understand it. Ultimately, I want preservice teachers to allow what they learn from children to influence how they think about their own mathematical thinking and allow it to inform their teaching. For my research, I was interested in determining what opportunities were afforded by including such a project in a mathematics content course for preservice teachers. In particular, I was interested in determining whether the Interview Project gives the preservice teachers the experience needed for them to reexamine the teaching and learning of mathematics.

Goals of the Interview Project. The Interview Project was created with several goals in mind that all center around the big idea that

> children's knowledge and the teacher's understanding of that knowledge are central to instructional decision making…thus, the processes of learning about research on children's mathematical thinking and using that knowledge while interacting with students are associated with changes in both teacher's beliefs and the type of instruction they provide their students. (Vacc & Bright, 1999, pp. 90-91).

There are a few big overarching goals of the Interview Project and some smaller goals that fall under one of the umbrellas of the main goals. One of the main goals of the Interview Project is for the preservice teachers to develop knowledge of the frameworks around additive structured story problems and for them to apply these frameworks in real time and retrospectively while teaching children. Another main goal is for the preservice teachers to develop the ability to listen to and learn from children's mathematical thinking. "Clearly, the act of unpacking learners' mathematics requires listening to students" (D'Ambrosio, 2004, p. 139). These goals are keystones in developing the ability to describe and analyze a child's mathematical thinking.

The smaller goals, which I call sub goals, are intended to give an assessment of the main goals under whose umbrella they fall. A more detailed description of these sub goals is given below, and the rubric for each is provided in the data analysis section.

Sub goal #1. The first sub goal of the Interview Project falls under the main goal of using the frameworks from class in a real life teaching scenario. One way that the preservice teachers will use the frameworks (in Appendix B) is in describing a child's mathematics. The preservice teachers use the Cognitively Guided Instruction (CGI) framework (Carpenter, 1999) to identify the structure of the story problem. Then, using the CGI and the Van de Walle (Van de Walle, 2010) frameworks, the preservice teachers describe the method that the child used to find the solution to the task. To meet this goal, the preservice teachers need to have knowledge of the problem structures; they need to be able to write story problems within these structures; and they need to be able to identify the many different possible solution methods discussed in the frameworks.

Sub goal #2. The second sub goal of the Interview Project also falls under the main goal of using the frameworks from class in a real life teaching scenario. Another way to use the frameworks (in Appendix B) is to analyze a child's mathematical thinking. What do the child's actions imply about the level at which s/he is operating? Using the Steffe and colleagues (1982) framework and the Gelman (1986) framework, the preservice teacher needs to use the child's responses to the tasks to identify his/her current level of operating as a perceptual counter, motor item counter, Initial Number Sequence, Strategic Additive Reasoning, etc. In addition, if the child uses an invented strategy, the preservice teacher should write a series of equations that justifies whether this strategy will always work. When writing the series of equations, it is important to use the equal sign appropriately and state the properties used at each step.

Sub goal #3. The third sub goal falls under the main goal of learning to listen to and learn from a child. Davis (1996) suggested that while you cannot observe listening occur, you can infer how a teacher is listening through how s/he responds to students. You can also infer how a teacher is listening by the instructional decisions that s/he makes. Some of these decisions can be planned for and some of them cannot. To meet this goal, the preservice teachers need to have knowledge of the possible solution methods and know what task is appropriate for the child based on his/her solution method. In addition, the preservice teacher needs to be able to do all of this on the spot.

Sub goal #4. The fourth sub goal of the Interview Project also falls under the main goal of learning to listen to and learn from a child. This goal is for the preservice teachers to be able to use this experience of interviewing a child to inform future instructional decisions.

Background

In 1986, Shulman (1986) pointed out that a major limitation of the cognitive research on teacher training was the absence of a focus on the subject matter to be taught. Similarly, Romberg and Carpenter (1986) noted that research on the learning of mathematics and on the teaching of mathematics were conducted as two separate disciplines. They suggested that the research on teaching should incorporate an analysis of mathematics content. To show how the subject-matter could be integrated into the study of teachers' thought processes, Shulman provided the example of a teacher's "pedagogical content knowledge," (PCK) which he defined as:

> The ways of representing and formulating the subject that make it comprehensible to others…alternate forms of representation, some of which derive from research where others originate in the wisdom of practice…an understanding of what makes the learning of specific topics easier of difficult: The conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (Shulman, 1986, p. 9)

Shulman's introduction of the term PCK into the lexicon of teacher education launched many research efforts around the idea of teachers' pedagogical content knowledge. In 1989, Peterson, Fennema, Carpenter, and Loef (1989) studied the relationships among first-grade teachers' pedagogical content beliefs, teachers' pedagogical content knowledge, and students' achievement in mathematics. They found that

> compared to teachers with a less cognitively based perspective, teachers with a more cognitively based perspective made extensive use of word problems in introducing addition and subtraction…they spent time developing children's counting strategies before teaching number facts…they had a greater knowledge of word problem types and children's problem-solving strategies." (Peterson, 1989, p. 1)

Fosnot, Shifter, and Simon (Simon & Shifter, 1991; Schifter & Fosnot, 1993; Simon, 1995) studied the effects of a professional development experience where teachers were encouraged to develop mathematical knowledge and a constructivist pedagogy. Results showed that "as teachers learned mathematics, they changed their beliefs about the importance of making instructional decisions based on children's understanding and concurrently changed their instructional practices" (Fennema et al., 1996, p. 404). Cobb, Yackel, and Wood (Cobb, Wood, & Yackel, 1990; Cobb et al., 1991; Cobb, Yackel, & Wood, 1992; Cobb, Wood, Yackel, & McNeal, 1993) had teachers engage in workshop activities where they reflected on their instruction and students' thinking. The teachers were encouraged to ask their students to describe their thought processes as they engaged in the instructional activities provided during the workshop. They reported that the teachers from their workshops made changes in their instruction to consider how students learned and their students had a higher level of conceptual understanding.

One of the largest contributions to the research on enabling teachers to understand their students' thinking is the Cognitively Guided Instruction (CGI) group.

> CGI is based on an integrated program of research focused on the development of students' mathematical thinking, on instruction that influences that development, on teachers' knowledge and beliefs that influence their instructional practices, and on the way teachers' knowledge, beliefs and practices are influenced by their understanding of students' mathematical thinking. (Carpenter, Fennema, Franke, Levi, Empson, 1999, p. 105)

The CGI group has conducted many research efforts (Carpenter, Fennema, Peterson, & Carey, 1988; Peterson, Fennema, Carpenter & Loef, 1989; Fennema, Carpenter, Franke, & Carey, 1992; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996) and found that using a "research based knowledge about children's mathematical thinking and well-defined taxonomies of problem types and children's solution strategies for arithmetic operations results in the ability to make instructional decisions that are appropriate to the mathematical needs of their students" (Vacc & Bright, 1999, p. 90).

Vacc and Bright (1999) sought to determine whether similar findings would accrue to the integration of CGI within preservice teacher education programs. The preservice teachers were introduced to CGI during their methods course, and the researchers wanted to see if this would influence their instruction during the student teaching process. They found that there were significant changes in the preservice teachers' beliefs and perceptions about mathematics instruction. But, some questions were raised. "Preservice teachers may acknowledge the tenets of CGI and yet be unable to use them in their teaching, perhaps in part because of their lack of teaching experience" (Vacc & Bright, 1999, p. 107). For my study, I was also dealing with preservice teachers who lacked in teaching experience. Because of this, I knew that they would not be able to apply all of their knowledge of the frameworks in the moment of the interview. This is why I felt it was important for the preservice teachers to reflect on their interview experience, discussing anything that they could have done differently or anything that they would do differently in a future meeting. I do not believe that it is realistic to expect that the preservice teachers would be able to use the frameworks with complete accuracy during their first experience with a child. Thus, the Interview Project serves as a starting point that will inform their future interactions.

Rationale

Franke, Carpenter, Levi, and Fennema (2001) suggested that "learning to understand the development of children's mathematical thinking could lead to fundamental changes in teachers' beliefs and practices" (p. 105). Further, they claimed that teachers' knowledge of problem types and the strategies that children often use allows them to better analyze children's mathematical thinking. The Interview Project was developed to give preservice early childhood teachers the opportunity to do just that–to use their knowledge of several frameworks to describe and analyze children's mathematical thinking. This project is one that can contribute to the mathematics teacher education community in the effort of reforming mathematics education.

Research Questions

For my dissertation study, I wanted to evaluate the extent to which preservice teachers were able to apply the frameworks they learned in class to the Interview Project. To do this, I was guided by the following research questions:

1) To what extent does the Interview Project increase preservice teachers' knowledge of the research (frameworks) and their ability to apply them in their interactions with a child?

2) To what extent does the Interview Project help preservice teachers learn to listen to and learn from children?

CHAPTER 2

LITERATURE REVIEW

In my literature review, I summarize and critique research that has investigated preservice and inservice teachers field experiences, specifically conducting interviews with children, learning to listen to children and act upon their listening to make instructional decisions. I also review literature on constructivism and how this perspective helped me to determine the theoretical frameworks that I used to situate my findings. In addition, I summarize literature on children's mathematics and the knowledge needed for teachers to describe and analyze a child's mathematics.

Field Experience

Field experiences have been repeatedly identified as the most significant part of teacher preparation programs (Carnegie Forum on Education and the Economy, 1986; Holmes Group, 1986; Knowles & Cole, 1996; National Commission on Excellence in Education, 1983; Zeichner, 1992). Although there are many different experiences that may be considered "field experience," common field experiences are done through classroom observations or through the more intense student-teaching experience. Though highly regarded in preservice teacher education, field experience has quite a controversial history, with respect to its impact on teacher learning. In the 1970s, research on field experience exposed a disconnection between teacher preparation and the practice of teaching. Studies reported negative outcomes of field experience, including changes in preservice teachers' attitudes toward education to reflect the attitudes of their mentor teachers (Mahan & Lacefield, 1978) and the development of bureaucratic orientations, where teachers are expected to be loyal to the school organization, behave consistently according to the rules and regulations, and defer to the authority figures (Hoy, 1977). In response to these reports, in the 1980s, the Carnegie Forum on Education and the Economy (1986), the Holmes Group (1986), and the National Commission on Excellence in Education (1983) recommended that preservice teachers have more authentic experiences to prepare them to handle the challenges that exist in schools. They called for a restructuring of the field experience to consider teacher thinking and teacher learning. This was a major paradigmatic shift that recognized teacher development and field-based experience as complex processes (Zeichner, 1992). Field experiences now focus on reflection and inquiry into teaching with an emphasis on the integration of the study and practice of teaching through collaborations between universities and schools (Knowles & Cole, 1996). Today, field experiences continue to play a major role in preservice teacher education courses.

The rationale for field experience is that this firsthand exposure to the complexity of the classroom provides the preservice teachers with the opportunity to practice instructional strategies and decision making, to reflect on the practice, and to meld theory to practice. This type of experience also gives preservice teachers the opportunity to work hands-on with students and use the students' thinking to tailor their instruction. The National Forum to Accelerate Middle-Grades Reform (2002) suggested that for preservice teachers to have the necessary knowledge, skills, and dispositions to meet the needs of learners, preservice teacher education programs need to focus their efforts on helping preservice teachers to enact reform oriented pedagogy. This enactment can only be done through field experience. Preservice teachers need the opportunity to construct an understanding of the diversity of the social, emotional, physical, and intellectual development of their students. Hororwitz, Darling-Hammond, and Bransford (2005) contended that,

> Understanding a child's thinking is one of the most important keys to shaping appropriate learning tasks that are engaging for students. Helping [preservice teachers] conceptualize developmentally responsive teaching practices through field experiences and continued reflection prepares them to enact developmentally responsive pedagogy. (p. 89)

Research has clearly shown that field experiences are important occasions for preservice teacher learning rather than merely times for preservice teachers to demonstrate or apply things previously learned (Zeichner, 1996).

Although field experiences continue to be viewed as highly important in preservice teacher education, they do not always lead to productive growth for preservice teachers (McIntyre, Byrd & Fox, 1996). Experiences in the field can expose preservice teachers to a limited repertoire of teaching strategies, socializing them into traditional ways of teaching mathematics. Although educational theory, like constructivism, is typically presented within the context of the preservice teacher education courses, the transfer of these theories to practice is challenging when field experiences expose only limited and narrow strategies (Korthagen & Kessels, 1999). Particularly in mathematics education, the reform-minded practices are rarely observed in field experiences. The preservice teachers' mentors may be experts in the teaching of mathematics and have years of successful teaching experience, but they often are not aware of what is known from research about how to support teacher learning and its transfer to the early years of teaching in the context of a university-based teacher education program (Cochran-Smith & Zeichner, 2005; Darling-Hammond, 2006), and they do not necessarily think of themselves as teacher educators. Darling-Hammond (2009) referred to this lack of connection between campus courses and field experiences as the Achilles heel of teacher education.

Another challenge of field experiences is that they typically take place in a classroom full of students. As novices, preservice teachers often are not equipped to sufficiently handle the complexities of an actual classroom in order to focus their attention, in productive ways, on the teaching and learning that is taking place. Researchers have found that for novice teachers in field experiences, attention to procedural and management issues were easy distractions from opportunities to inform their own mathematics and make sense of students' mathematics (Mewborn, 1999; Moore, 2003; Zeichner & Tabacknick, 1981). Field experiences are also typically done alone or with a few other preservice teachers, which limits their opportunities to discuss and reflect critically about their experiences.

The Apprenticeship Approach versus the Laboratory Approach

Dewey (1964b) argued that both practical and theoretical work are required for the professional development of teachers. Philipp et al. have particularized this idea to mathematics education, saying

> Teacher educators' responsibilities are, on one hand, to prepare teachers to manage the practical aspects of teaching that arise on a daily basis and, on the other hand, to prepare teachers to grapple with the deeper questions of the relationship between subject-matter knowledge and educational principles and theory. (Philipp, Ambrose, R., Lamb, L., Sowder, J., Schappelle, B., Sowder, L., Thanheiser, E., Chauvot, J. , 2007, p. 443)

Dewey called the focus on preparation for the practical aspects of a job the "apprenticeship" approach, and he called the focus on the more theoretical aspects of a job the "laboratory" approach.

In the apprenticeship approach teacher education programs help prospective teachers learn how to do that which is currently being done. Student teaching is one example of such an apprenticeship. The laboratory approach is “local, particular, and situated” (Shulman, 1998, p. 512). Examples of the laboratory approach are prospective teachers analyzing students’ understandings of mathematics. In this environment, prospective teachers are learning “to attend to how children perceive their mathematical worlds, so that when they later take the role of teacher, they can connect what they are learning about teaching with what they already know about students’ mathematical understanding” (Philipp et al., 2007, p. 443).

Philipp et al. (2007) claimed that introducing the laboratory approach while preservice teachers are engaged in student teaching would take a radical restructuring of teacher education in the United States. Thus, they suggested instead that the laboratory approach be introduced earlier into teacher education, perhaps in one of the first mathematics content courses taken by the preservice teachers.

The Interview Project is one such example of the laboratory approach, where preservice teachers are analyzing a child’s mathematics and using their analysis to inform their future as teachers. The Interview Project also took place in Math 2008, which is the very first mathematics content course for the early childhood majors at Georgia College. Math 2008 is a requirement for entry into the early childhood program; thus, it is taken very early in their program.

Interviews

Preservice teachers interviewing students is a practice that has grown in popularity as a result of the reform movement in mathematics education (NCTM, 1991, 1995, 2000). One-on-one interviews help the teacher to get past the typical teacher-student interactions in American classrooms, characterized by the Third International Mathematics and Science Study (TIMSS) as a place where students “acquire isolated skills through repeated practice…and give one-word responses to rapid fire questions” (Stigler & Heibert, 1999, p. 45). Designed to foster understandings of students as learners of mathematics, structured interviews are “grounded in clinical interview procedures and, fittingly, create situations in which student reasoning through open-ended mathematics tasks can be observed first-hand” (Jenkins, 2009, p. 149). In an interview setting, the teachers have the opportunity to become more aware of the student’s current understandings; they can ask the student intentional probing questions; they can make sense of the student’s responses and analyze their responses; they can inform their own instruction based on the student’s responses.

Fennema et al. (1996) studied 21 primary grade teachers over a 4-year period, where the teachers interviewed children after being exposed to a specific research-based model of children’s mathematical thinking. Results indicated that when mathematics teachers conduct interviews with students, they shift their beliefs about how mathematics should be taught and modify their teaching practices as a result. These findings suggested that the practice of interviewing in order to develop an understanding of children’s mathematical thinking can be a productive basis for helping teachers to make fundamental changes called for in the reform recommendations.

Research has shown that teachers interviewing students has been a positive tool in preservice teacher education. Bright and Vacc (1994) showed that interviewing children was an instrumental factor in shifting preservice teachers’ beliefs to a more constructivist orientation to teaching mathematics. The preservice teachers developed views of instruction different than that of “telling,” and they placed more emphasis on having students figure out mathematics concepts in meaningful ways. Moyer and Milewitz (2002) studied 48 preservice teachers who conducted one-on-one interviews with a child. Results of the study indicated that the use of one-on-one interviews allowed the preservice teachers to recognize and reflect on effective questioning techniques and shifted their beliefs about mathematics assessment in general. Abney (2007) studied four preservice elementary teachers interviewing children during The School Buddy Experience, where the tasks revolved around arithmetic story problems. She found that, as a result of interviewing children, preservice teachers were “redefining what teaching mathematics meant to them. They were moving toward a definition of teaching mathematics that suggested that teaching was more about listening and posing appropriate tasks for students” (Abney, p. 150). In Jenkin’s study (2010), six middle grades preservice teachers participated in a structured interview as part of a mathematics methods course. The interview was centered around problem solving tasks requiring the application of standards-based middle grades mathematics content and processes (NCTM, 2000). Jenkins found that the preservice middle school teachers developed listening skills for accessing students’ mathematical thinking and awareness of the variety of ways students make sense of mathematics.

While the advantages of interviews are clearly present, there are also some drawbacks. Interviews are, by definition, a form of verbal communication between an interviewer and an interviewee and typically involve questioning by the interviewer. Because questioning is such an integral part of interviewing, it is essential that the preservice teachers have developed the necessary questioning skills (Moyer & Milewicz, 2002, Wassermann, 1991). An interview protocol can be provided, but this does not always guarantee competence in questioning. The preservice teachers cannot plan what the student will say and need to be prepared for these types of instances. One way to develop adequate questioning skills is practice. By having the preservice teachers conduct an interview, they are given the opportunity to intentionally select questions that will probe the student's mathematical thinking. They can then reflect on their questioning, determining what types of questions were beneficial in probing the student's mathematics and what types of questions were not beneficial, and they can develop better questioning strategies.

Moyer and Milewicz (2002) suggested that developing questioning skills should be an integral focus of preservice mathematics education coursework because it is such an important tool in understanding a student's mathematics and how they learn mathematics. Wassermann (1991) claimed that a teacher's questioning strategies are pivotal to the instruction process because questioning is the most frequently used instructional tool. Teachers who can question effectively are "better able to discern the range and depth of children's thinking" (Moyer & Milewicz, 2002, p. 293).

The previously mentioned research was highly valuable for my dissertation study. When creating a project for the Math 2008 course, I felt that it was important to incorporate a field experience where the preservice teachers would have the opportunity to work with a child and to describe and analyze the child's mathematical thinking. I felt that the best way to have the preservice teachers interact with a child was to conduct an interview. I believe that the Interview Project highlights the positive aspects of interviewing, as shown in previous research. For example, the one-on-one (or possibly two-on-one) aspect of the Interview Project allowed the opportunity for the preservice teachers to have a more personal interaction with the student, getting to know him or her better, and not having to deal with the sometimes chaotic classroom scenario. This environment was more inclined to give the preservice teachers the opportunity to make sense of the student's responses, to analyze the student's responses, and to reflect on his or her responses, helping to inform their own instruction.

The aforementioned studies influenced the way that I conducted my research on the Interview Project. The Fennema (1996) study had the biggest impact on my study, as it influenced me to add an additional goal to my project that I did not originally have. Like my study, the Fennema (1996) study used the Cognitively Guided Instruction (1999) framework as a way for the teachers to understand children's thinking. But, as I read this study, I noticed one big difference in my study's original goals and their study's goals. They stated that their goal was

> to help teachers organize and expand their understanding of children's thinking and to explore how to use this knowledge to make instructional decisions such as choice of problems for children to solve, questions to ask children to elicit their understanding, and ways to assist children to solve problems and report their thinking. (Fennema, 1996, p. 407)

When I initially read this, I had not considered studying the instructional decisions made by the preservice teachers. My only goal at that point was to study whether or not preservice teachers could describe and analyze a child's mathematical thinking, but after reading the Fennema (1996) research, I realized that making appropriate instructional decisions was instrumental in determining whether or not the preservice teachers were able to apply the frameworks discussed in class to their interview with the child. Thus, I added this as a goal for my study.

Although the Fennema (1996) study was a great influence for my study, the participants were in-service teachers, whereas my participants were pre-service teachers. In addition, Fennema worked with the teachers in his study for 4 years, whereas I only had one semester to work with mine. The Bright and Vacc (1994) study was one where preservice teachers used the CGI frameworks to inform their interview with a child. This study showed that it was possible for preservice teachers to use research to inform their knowledge of children's mathematics and to change their own beliefs about mathematics in only a two-year teacher education program.

I also used the research to potentially avoid any negative effects of interviewing. For example, the research (Cochran-Smith & Zeichner, 2005; Darling-Hammond, 2006) suggested that while student teaching, preservice teachers rarely observe reform-oriented teaching practices, and they do not attempt to transfer these practices that were learned in their teacher education courses to the classroom because they simply follow along with their mentor teacher's teaching practices. Because the Interview Project is done before the preservice teachers have had any other field experiences in the early childhood program, it is likely the very first time they have worked with a child to understand his/her mathematical thinking. Thus, the preservice teachers can apply the research that is presented to them in class to their interaction with the child without the influences from a past or current mentor teacher.

Another suggestion made by the Moyer and Milewitz (2002) study, the Abney (2007) study, and the Jenkins (2010) was that teachers need adequate questioning skills in order for the interview to be effective. To deal with this, I gave the preservice teachers some literature on questioning and had them write a reflection on the literature as a class assignment. In addition, I asked the preservice teachers to do an exercise where they practiced conducting an interview with a peer. After this exercise, they reflected on their questioning, determined what types of questions were and were not beneficial, and then developed better questions.

Constructivism and the Mathematics Teaching Cycle

Constructivism

Ball (1997) asserted that while knowing what students know is not possible, it is still a worthwhile pursuit, claiming that "issues like these challenge core epistemological and psychological assumptions about what it means to 'know' something" (p. 771). Constructivism is an epistemology that differs from other theories of learning in that it rejects the notion of being able to determine validity of knowledge by matching it to an absolute reality. Instead, knowledge is seen as functional as long as it is useful or viable within the framework of a subject's experience (Steffe & Thompson, 2000). From this perspective, it is impossible to 'know' what students know in mathematics. Because of this impossibility, Steffe and Thompson (2000) suggested that teachers construct a model of a student's mathematics in order to determine what action to take next. According to constructivists, this model is a hypothetical model because the teachers would be interpreting their students' mathematical actions in terms of their own experience. Thus, these hypothetical models are constructed to order, comprehend, and explain their experience (Steffe, von Glasersfeld, Richards, & Cobb, 1983).

Teaching based on constructivist ideas gives preeminent value to the development of students' personal mathematical ideas. Steffe et al. stated that "conceptual knowledge cannot be transferred ready-made from one person to another but must be built up by every knower on the basis of his or her own experience" (Steffe, von Glasersfeld, Richards, & Cobb, 1983, p. 12). With constructivist instruction, students are encouraged to use their own methods for solving problems. They are not asked to adopt the teacher's thinking but encouraged to refine their own. Through specifically designed mathematical tasks and interaction with other students, the student's own intuitive mathematical thinking gradually becomes more abstract.

The Mathematics Teaching Cycle

Simon (1995) offered a framework for teaching, referred to as the mathematics teaching cycle that supports the constructivists' view of learning. Because constructivists believe that teachers cannot simply transfer knowledge to their students, activities where students practice problems and then discuss them are not adequate for promoting learning mathematics with understanding (Simon, 1995). Instead, planning for mathematical activities or tasks should be informed by how students think about mathematics. It is not enough for teachers to attend solely to objectives, content, activities, or assessment; student thinking is an equally important component.

The mathematics teaching cycle is a conceptual framework that describes the "cyclical interrelationships of aspects of teacher knowledge, thinking, decision making, and activity" (Simon, 1995, p. 135). Simon (1995) suggested that the mathematics teaching cycle describes the teacher's decision making process, where teacher's knowledge of learning goals and tasks, as well as their predictions about how students learn should all be a part of the lesson planning process. The diagram below (Figure 1) shows Simon's mathematics teaching cycle.

Because the students' mathematical knowledge can never be known precisely, the teacher must "hypothesize" the nature of the students' understandings based on his interpretations of the students' behaviors using "his own schemata with respect to mathematics, learning, students, and so on" (Simon, 1995, p. 135). The teacher makes a prediction as to the path that learning might take, which Simon calls the "hypothetical learning trajectory." The hypothetical learning trajectory provides the teacher with a rationale for choosing a particular instructional design based on his best guess about how learning might proceed. The teacher uses information about students' current knowledge, the specific content, and pedagogical content knowledge about how students best learn the content in designing a lesson (Simon & Tzur, 2004).

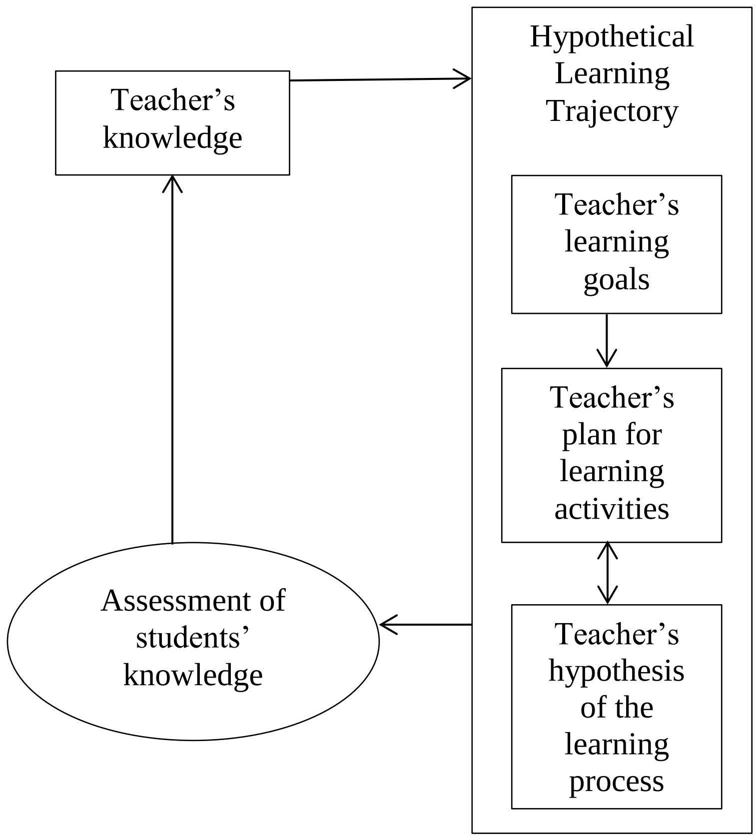

Figure 1. Simon's Mathematics Teaching Cycle

The hypothetical learning trajectory is made up of three parts: the learning goal, the learning activities, and the hypothetical learning process. The hypothetical learning process is "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (Simon, 1995, p. 136). Simon noted that the development of the hypothetical learning process and the development of the learning activities are symbiotic and depend on one another. The generation of the hypothetical learning trajectory happens prior to classroom instruction when the teacher develops a plan for classroom activity. Once the lesson is enacted, the interactions that the teacher has with the students constitute an experience. This experience "by the nature of its social constitution is different from the one anticipated by the teacher" (Simon, 1995, p. 137). Thus, the teacher's knowledge is modified in several ways. First, during the mathematics teaching cycle, student knowledge is continually assessed. Therefore, a completed lesson will influence a teacher's knowledge of students' mathematical understanding. In addition, the classroom experience adds to the teacher's own knowledge of mathematics. These domains of knowledge then, in turn, allow the teacher to create new learning goals and trajectories for learning in future lessons. As the teacher's knowledge is enhanced, new or modified hypothetical learning trajectories are formed.

For my study, I was interested in how preservice teachers describe and analyze students' mathematical thinking and use this knowledge to inform their instruction. Simon's mathematics teaching cycle provided a way for me to frame my study. The preservice teachers began the cycle in the box labelled "Teacher's knowledge." They had knowledge of mathematics, through their own personal experiences as students of mathematics, and they had knowledge of children's mathematics through the lessons and activities enacted in our Math 2008 class. In particular, the preservice teachers gained essential knowledge of additive structured story problems and some typical strategies children use to solve them through watching videos. The videos served as examples of what might happen during their interviews. The preservice teachers used this knowledge in their planning for the interview, but because they could not know what exactly would happen in their interview, their planning was hypothetical. While the preservice teachers planned for their interview with the child, during the interview, the preservice teachers likely had a different experience than what they predicted. After the interview, they described and analyzed the child's mathematics; this, along with the experience of the interview, informed their knowledge of students' mathematics and their own knowledge of mathematics. The preservice teachers were then asked to discuss their hypothetical future instruction, which was likely influenced by the new knowledge that they gained.

Figure 2 is Simon's elaborated version of the mathematics teaching cycle. This diagram describes "the relationships among various domains of teacher knowledge, the hypothetical learning trajectory, and the interactions with students" (Simon, 1995, p. 137). These new pieces of the diagram describe different domains of teacher knowledge and different types of interactions with students. I situated each of my research questions into Simon's mathematics teaching cycle diagram.

1) To what extent does The Interview Project increase preservice teachers' knowledge of the research (frameworks) and their ability to apply them in their interactions with a child?

2) To what extent does The Interview Project help preservice teachers learn to listen to and learn from children?

The first research question involves the preservice teachers' abilities to use the frameworks discussed in class to describe and analyze a child's mathematical thinking. I would place this question in the box labeled "Teacher's models of students' knowledge" and in "Teacher's knowledge of student learning of particular content." The frameworks discussed in the Math 2008 course should have given the preservice teachers knowledge in these domains. The arrow from these boxes to the hypothetical learning trajectory indicates that they directly affect the hypothetical learning trajectory, which, in turn, affects the teacher's interaction with students. In the Interview Project, the preservice teachers were asked to use the knowledge gained from their description and analysis to predict the future learning trajectory.

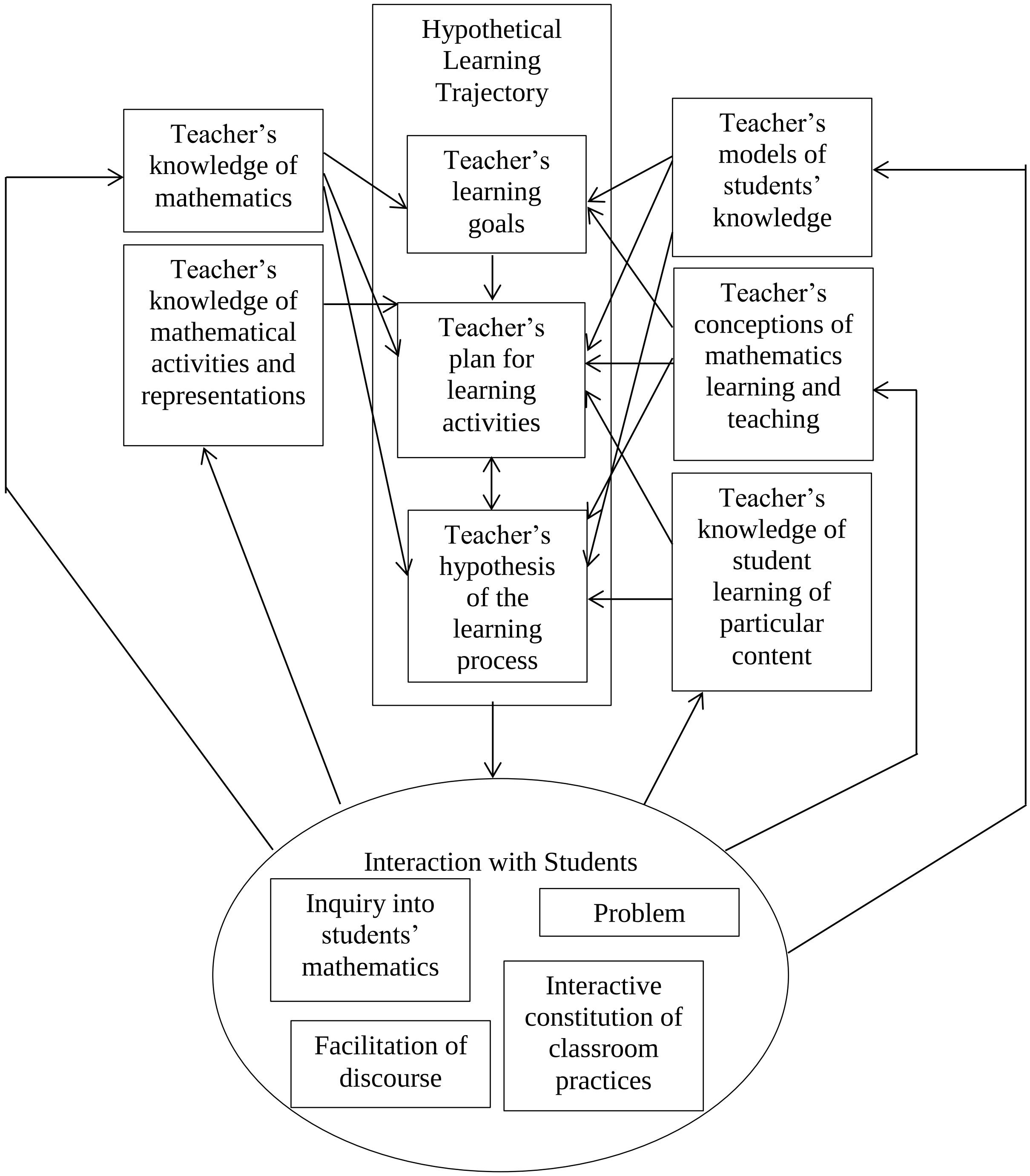

Figure 2. Simon's mathematics teaching cycle (with elaborated domains of teacher knowledge).

The second research question deals with the preservice teachers' abilities to listen to the child. Because I could not directly assess the way one listens, I used their instructional decisions as a way to determine whether or not the preservice teachers were listening to the child. Because the hypothetical learning trajectory is co-constructed between the teacher and the student, it involves a teacher's planned instructional decisions as well as the decisions that are made spontaneously during the interview. Because of this, the second research question can be situated in the box containing the Hypothetical Learning Trajectory as well as in each of the components of the box called "Interaction With Students". The teacher's interaction with the student also directly informs the teacher's knowledge.

Learning Trajectories

Clements and Sarama (2010) claimed that children follow a natural developmental progression in learning mathematics called a learning trajectory. "When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective" (Clements & Sarama, 2010, p. 1). Learning trajectories are made up of three parts: a mathematical goal, a developmental path along which children develop to reach that goal, and a set of instructional activities that are matched to each of the levels of thinking in that path that help children develop higher levels of thinking. The first part, the mathematical goal, is the cluster of mathematical concepts and skills. The second part, the developmental progression, consists of the levels of thinking, where each is more sophisticated than the last. This progression is a typical path that children follow in developing understanding and skills about that particular math topic. The third part of the learning trajectory consists of the instructional tasks. The tasks are specifically designed to help children learn the ideas and skills needed to achieve that level of thinking.

Clements and Sarama (2014) have developed learning trajectories for many different content topics in mathematics, but the one specific to the interest of my research is their learning trajectory for adding and subtracting. They claimed that “most children follow an observable developmental progression in learning to add and subtract numbers with recognizable stages or levels” (Clements & Sarama, 2014, p. B6). They gave a name to each developmental level, an approximate age range, and a description of each level. For example, at around age 5, a child is likely to be on the “find result” level, where a child can find sums by joining two sets or differences by separating a set in a take-away problem. By the time the child is 6 years old, s/he has likely advanced from the “find result” level through the “find change” and “make it” levels and to the “counting strategies” level, where s/he can find a sum using finger patterns or by counting on. By age 7, the child has advanced to the “deriver” level, where his/her strategies are more flexible and s/he can use derived combinations of numbers; then at age 8, the child is likely at the “problem solver” level where s/he can use many known combinations of numbers, becoming even more flexible than when at the “deriver” level.

Although Clements and Sarama have done extensive work in the area of learning trajectories, other researchers have also done similar work. For my study, I chose to focus on the work of the Cognitively Guided Instruction group, Steffe et al., Gelman, and Van de Walle. I chose to use these because the others have been around longer and are therefore embedded in teacher education materials to which I had access.)

Learning trajectories have proven to be effective for the development of early mathematics curricula and for teacher professional development (Clements & Sarama, 2008;

Sarama, Clements, Starkey, Klein, & Wakeley, 2008). Teachers who used learning trajectories saw themselves helping students move through levels of understanding rather than just moving through the curriculum (Fuson, Carroll, & Drueck, 2000). However, there has been little research done with learning trajectories in preservice teacher education. Like Clements and Sarama's (2008, 2010, 2014) research, my study focused on determining a level of development for a child based off of the method that the child used to solve an additively structured problem.

Skills Needed to Enhance the Interview Project

There are many skills that are needed in order to interview a child and describe and analyze his/her mathematical thinking. The skills that I have chosen to focus on with respect to the Interview Project are listening, choosing appropriate tasks, and reflecting on their experience. These skills are included in Simon's diagram of the mathematics teaching cycle in the box labeled "Interaction with Students". These skills are applied and developed during the interaction with students and in planning for the interaction with students.

Listening

A necessary part of describing and analyzing a child's mathematical thinking through an interview is listening. Some research focuses exclusively on the act of listening in order to investigate teachers' constructions of models of students' mathematics (D'Ambrosio, 2004; Davis, 1996). Davis suggested that listening is a fully human endeavor in which all senses are engaged. However, it is also important to note that we are always listening through the filter of our own perception. Davis (1996) believes that "we need to acknowledge that our listening always and inevitably occurs against the backdrop of personal histories that are set in and shaped by cultural, historical, social, and environmental factors" (p. 47).

While Davis identified three types of listening in which teachers and all people could be engaged in, he also acknowledged that listening cannot be reduced to a set of skills and guidelines. The three types of listening identified by Davis are evaluative listening, interpretive listening, and hermeneutic listening. Evaluative listening is characterized by "listening for something in particular rather than listening to the speaker" (Davis, 1997, p. 359). The purpose for listening is to assess the correctness of a response, based on the listener's understanding of the discipline. Interpretive listening is more information seeking. A teacher listening interpretively is working to understand how her students are making sense. However, the purpose of understanding how her students are thinking is to help them get to the "right understanding," according to the teacher's interpretation of the discipline. When a teacher is listening hermeneutically, she is more readily able to learn mathematics from her students. Students' responses and ideas tend to direct the enacted lesson. Opportunities for mathematical exploration come from the ideas of the students.

Evaluative listening is the type of listening that is typical in most mathematics classrooms (D'Ambrosio, 2004). Neither interpretive nor hermeneutic listening is an easy endeavor. Simply engaging in practicum experiences certainly does not guarantee that preservice teachers will learn to listen in more open and productive ways. With all of the constraints and distractions in learning to teach in a whole class setting, I believe that engaging teachers in an interview with one child provides the opportunity to learn to listen to children interpretively or hermeneutically.

Davis (1996) suggested that while you cannot observe listening occur, you can infer how a teacher is listening through how s/he responds to students. You can also infer how a teacher is listening by what s/he is listening for and what s/he chooses to ignore. Questioning is one type of response that teachers use to respond to students. Questions are instructional decisions that often occur on the spot although some are planned in advance. Questions can be categorized in three different ways: probing, prodding, and prompting (Abney, 2007). Probing involves questions to determine what or how a student is thinking; prodding questions are intended to keep a student acting mathematically; and prompting questions attempt to elicit a specific response or strategy to a task (Abney, 2007).

Choosing Appropriate Tasks

Based on their current interpretations of the child's mathematical knowledge, the preservice teacher had to "make decisions concerning situations to create, critical questions to ask, and the types of learning to encourage" (Steffe, 2002, p. 177). When choosing tasks, a teacher has to decide whether her goal is assimilation or accommodation. Steffe and Wiegel (1996) refer to assimilation as concerning "the integration of any sort of reality into a structure, where any sort of reality is interpreted to mean reality from an observer's point of view" (p. 491). For Piaget (1980b), assimilation is the fundamental relation involved in learning. It involves perturbations, but the restoration of equilibrium that occurs as a result of the perturbation does not always result in learning. Without accommodation, one's current ways of operating would never be stretched, allowing for cognitive development. Steffe and Wiegel (1996) refer to accommodation as "a modification of conceptual structure in response to perturbation" (p. 491). If the teacher's goal is to bring an idea to a child's existing structure, or assimilate, then s/he would choose a task that would extend strategies or concepts that the child already knew. However, if the teacher's goal is for the child to make an accommodation, s/he would choose a task that would provoke a perturbation.

Possible tasks can be determined by a teacher in part with respect to a children's zone of proximal development (ZPD) (Vygotsky, 1956).

> The zone of proximal development for a child is the distance between her actual development level as determined by independent problem solving and her level of potential development as determined through problem solving under the guidance or in collaboration with more capable peers (Vygotsky, 1978, p. 86)

From a constructivist's view, a ZPD relative to a child's specific scheme is determined by the modifications of the scheme the child might make during or as a result of his interaction with the teacher. Thus, the ZPD is the teacher's construct. Steffe (1991) calls this the zone of potential construction (ZPC). He defined the zone of potential construction as "the range determined by the modifications of a concept a student might make in, or as a result of, interactive communication in a mathematical environment" (p. 193). This perspective obliges the teacher to consider differences among students' conceptions.

Essentially, for teachers who operate with a constructivist framework, the tasks chosen for a child are based on hypotheses about their interpretations of a child's mathematics. The teacher must decenter and assume the mathematical viewpoint of the child (Steffe, 1991). As the child engages in the task, the teacher's model can be modified as a response to the new observations.

In my study, the participants were preservice teachers with no experience of teaching and likely very little experience working with children. With this lack of experience, preservice teachers had to rely on researchers' experiential models of children's mathematics that were presented to them in class. These particular models of children's mathematics should have been used to help the preservice teachers design or choose appropriate tasks that were in the child's ZPC.

Reflecting on their Experience

Because their participation in the Interview Project was probably the first time that the preservice teachers were learning to listen to children and describe and analyze their mathematical thinking, I felt that it was important that they reflect on their experience. Dewey (Zeichner & Liston, 1996) suggested that the process of reflection for teachers "begins when they experience a difficulty, troublesome event, or experience that cannot be immediately resolved" (p. 8). This sense of uncertainty or unease prompts teachers to step back and analyze their experiences. Munby and Russell (1990) refer to this as *puzzles of practice.* The action of reflection involves many sources of understanding; it involves using both our heads and our hearts, our reasoning capacities and our emotional insights. Dewey (Zeichner & Liston, 1996) listed three attitudes that are essential to reflective teaching. These are openmindedness, responsibility, and wholeheartedness.

The first attitude, openmindedness, is "an active desire to listen to more sides than one, to give full attention to alternative possibilities, and to recognize the possibility of error even in beliefs that are dearest to us" (Zeichner & Liston, 1996, p. 10). An individual who is openminded recognizes that there is not only one perspective and is willing to listen without argument to other perspectives. When looking for evidence of openmindedness by the preservice teachers in my study, I searched for indications that the preservice teachers willingly listened to the child's side and accepted it even though it was different from their own perceptions.

The second attitude of reflective teaching suggested by Dewey was responsibility. Responsibility involves careful consideration of the consequences of one's actions. The consequences could be personal consequences, academic consequences, or social and political consequences. The attitude of responsibility involves reflection about outcomes that are unexpected in teaching because teaching involves both intended outcomes and unintended outcomes. In my study, I looked for the preservice teachers to consider "why they are doing what they are doing in ways that are beyond question of immediate utility to consider the ways in which it is working, why it is working, and for whom" (Zeichner & Liston, 1996, pp. 10-11).

The third attitude necessary for reflection, according to Dewey, was wholeheartedness. Teachers who are wholehearted examine the results of their actions and are always open to learning something new. As teachers, "they continually strive to understand their own teaching and the way in which it impacts their students and they make deliberate efforts to see situations from different perspectives" (Zeichner & Liston, 1996, p. 11). In this project, I was particularly looking for evidence that the preservice teachers strived to understand their own teaching, their child's mathematics, and the way in which it can inform their practice.

Frameworks Needed to Describe and Analyze a Child's Mathematics

In addition to having the skills previously mentioned, the preservice teachers who participated in the Interview Project also needed to have knowledge of children's mathematics. There are several frameworks that I used in order to give the preservice teachers this necessary knowledge. The Cognitively Guided Instruction (1999) framework provided a way of classifying additive structured story problems; it also provided strategies that children use when solving these story problems. The Van de Walle (2010) book, which was the textbook used for the Math 2008 course, provided more strategies that children use when solving additive structured story problems. Gelman (1986) provided a framework for the counting principles, and Steffe et al. (1983, 1988, 2003) gave a framework that describes levels of children's whole number development. The knowledge gained from these frameworks is included in Simon's diagram of the mathematics teaching cycle in the boxes that indicate a Teacher's Knowledge. The knowledge on which I chose to focus includes knowledge of student learning of particular content and mathematical knowledge, which are both explicit components of the Teacher's Knowledge in Simon's mathematics teaching cycle.

Additive Structured Story Problems

Much research has been done on the different ways of classifying additively structured story problems. Fuson (1992) gave one way of categorizing story problems based on their conceptual structure. More recently, the National Research Council (2009) as well as the Common Core State Standards (2010) have given their own ways of classifying story problems based on their semantics. For the Math 2008 course, I chose to use the Cognitively Guided Instruction (CGI) framework in order to provide a way of classifying story problems because this framework is described in the textbook that was used for the Math 2008 course, which was written by Van de Walle.

> CGI is based on an integrated program of research focused on the development of students' mathematical thinking, on instruction that influences that development, on teachers' knowledge and beliefs that influence their instructional practices, and on the way teachers' knowledge, beliefs and practices are influenced by their understanding of students' mathematical thinking. (Carpenter, Fennema, Franke, Levi, Empson, 1999, p. 105)

As early as age 2 or 3, children begin to see addition and subtraction situations in the real world, even though they probably do not recognize the mathematics quite yet. Using story problems that the child can relate to gives him the opportunity to interpret, model, and solve addition and subtraction problems.

The CGI framework defines four structures of additive story problems, join, separate, part-part-whole, and compare. The context of the story can vary, as can the number size and the placement of the unknown, but the basic structure remains the same. The join and separate problems involve action.

Join. A join problem involves a "direct or implied action in which a set is increased by a particular amount" (Carpenter, 1999, p. 7). An example of a join problem is:

> Sam has 4 marbles. His friend, Tommy, gives him 3 more marbles. How many marbles does Sam have altogether?

In a join problem, there is an initial quantity and a change quantity, which is then joined to the starting quantity to produce the final quantity, which is the result. In this example, the initial quantity is four marbles, the change quantity is three marbles, and the result in unknown. Join problems can also be written where the initial quantity is unknown or where the change quantity is unknown. Children view these as different problems although they have the same structure. The CGI researchers suggest that join problems where the initial quantity or change quantity is unknown are more difficult for children to solve than those where the result quantity is unknown (1999).

Separate. A separate problem is similar to a join problem because there is an action that takes place, but with a separate problem, the initial quantity is decreased rather than increased.

> There were 8 dolphins playing. 3 of the dolphins swam away. How many dolphins were still playing?

A separate problem has an initial quantity, then a quantity that is removed, called the change, and the resulting quantity. The example above is a separate problem where the result is unknown, but there are two other distinct types of separate problems, one where the initial quantity is unknown and one where the change quantity is unknown. Like with join problems, CGI researchers suggest separate problems where the initial quantity of the change quantity is unknown are more difficult for children to solve than separate problems where the result is unknown (1999).

Part-Part-Whole. The CGI researchers also identified another type of additive problem, called the part-part-whole problem. Unlike the join problem, the part-part-whole problem does not involve a direct or implied action. This type of problem involves two mutually exclusive subsets of a whole set. An example of a part-part-whole story is:

> Andy has a basket of fruit. There are 5 bananas and 7 apples in the basket. How many pieces of fruit are there?

In a part-part-whole problem, there are two parts, which together make up the whole. In this example, the two parts are the bananas and the apples, and the whole is unknown. Because the two parts assume equivalent roles, there are only two types of part-part-whole problems, one where a part is unknown and one where the whole is unknown.

Compare. The CGI researchers also identify another problem type as a compare problem. A compare problem is one where two distinct, disjoint sets are compared to one another. In a compare problem, one set is compared to the other, which is why we call one a referent set and the other the compared set. The difference, or the amount that one set exceeds the other, is the third quantity in the compare problem. An example of a compare problem is:

> Farrar has 9 stickers in her sticker book, and Sally has 14 stickers in her sticker book. How many more stickers does Sally have than Farrar?

In this example, Farrar's 9 stickers are the referent set, Sally's 14 stickers are the compared set, and the difference is unknown. Compare problems can also be written where the referent set is unknown or the compared set is unknown.

Solution Methods

Methods for solving addition. In addition to providing a way to classify story problems, the CGI group (1999) also named several solution strategies that are common for children to use when solving these types of stories. If the solution to the problem is computed through addition, then the common solution methods are to use direct modelling, counting all, counting on, or using a number fact. *Direct modelling* is the most basic strategy, where children "use physical objects or fingers to directly model the action or relationships described in each problem" (Carpenter, 1999, p. 15). Over time, children's strategies become more abstract and efficient, and they replace direct modelling with counting strategies. The *count all* strategy is one where the child counts out each number that he is adding and also counts out the result. Let us use the following problem as an example:

> Bill has 4 rocks in his pocket. After putting 7 more rocks in his pocket, how many rocks does Bill have now have in his pocket?

To solve this problem using a *direct model* and *count all* strategy, a child would count the first 4 rocks, "1, 2, 3, 4, ", then count the 7 additional rocks, "1, 2, 3, 4, 5, 6, 7"; then the child would put them together and count the total number of rocks, "1, 2, 3, 4, 5, 6, 7, 8, 9, 11…11 rocks."

A more advanced counting strategy is called the *count on* strategy. Using the *count on* strategy, a child would begin counting from one of the addends and then stop counting when the number of steps that represents the other addend has been completed. For example, to solve the problem above, a child might say, "4 [pause], 5, 6, 7, 8, 9, 10, 11…11 rocks." In this example, the child *counted on from the first* addend in the story. If a child is presented with a problem where the first addend is smaller than the second addend, he may *count on from the larger* addend. For example, the child might say, "7 [pause], 8, 9, 10, 11…11 rocks." *Counting on from the larger* addend is an indication that the child grasps the commutative property. In other words, the child realizes that starting with 7 and counting up by 4 will give him the same result as starting with 4 and counting up by 7 and that counting on from larger is more efficient.

Children's solutions to story problems are not limited to direct modelling and counting. As children learn number facts and number combinations, they can use this knowledge to solve story problems as well. Using known number facts and decomposing numbers into their different combinations in order to help solve an additive problem is what Steffe (1982) calls strategic additive learning. For example, to solve 4 + 7, a child might say, "Well, 4 plus 4 is 8 and 3 more is 11." In this example, the child decomposed 7 into 4 + 3 because combining 4 and 4 first was easier than adding 7. Doubles are one common strategy used for strategic additive reasoning. Van de Walle (2010) lists doubles, one more than, two more than, combinations of 10, making a 10, using 5 as an anchor, and near doubles as popular strategies that children use when strategically reasoning.

Methods for solving subtraction. If the solution to the problem is computed through subtraction, then the common solution methods are direct modelling, counting all, counting down, counting down to, counting off, counting on to, or using a number fact. The *direct modelling* strategy, the *count all* strategy, and the *number facts* strategy have all been described in the previous section, and the same descriptions apply to subtraction.

The *counting down* strategy is a backward counting sequence that starts with minuend and then stops counting when the number of steps that represents the subtrahend has been completed. Let us use the following example to illustrate:

> Tommy has 9 pieces of candy. If he eats 6 pieces of candy, how many pieces of candy does Tommy have left?

Using the *count down* strategy, the child would say, "9 [pause], 8, 7, 6, 5, 4, 3…3 candies." The *count down to* strategy is different in that the backward counting sequence stops when it reaches the number indicated by the subtrahend. Thus, the solution would be found by counting the number of steps taken to get from the minuend to the subtrahend. To solve the story problem above, a child might say, "9 [pause], 8, 7, 6…so, 3 candies." Typically, the child monitors the number of steps with his fingers, with an action, or mentally. This strategy can be used to solve the separate problem above where the result is unknown but is more commonly used to solve a separate problem where the change is unknown. Conceptually, this strategy matches the semantics of 9 – ? = 6, rather than 9 – 6 = ?.

Another strategy that children use to compute subtraction is the *count off* strategy. This strategy is slightly different than the *count down* strategy in that the child does not count the number of steps; rather, the child counts the number of number words he is striking off. Thus, the solution is the number word that comes after the last stricken number word. For example, a child might say, "9, 8, 7, 6, 5, 4…3 candies." The numbers 9, 8, 7, 6, 5, 4 were stricken off and the child counted 6 stricken numbers words.

The *count on to* strategy is another strategy that children use when computing subtraction. The *count on to* strategy is a forward counting sequence that begins with the subtrahend and continues until the minuend is reached. The answer is the number of steps taken to get from the subtrahend to the minuend. If a child used the *count on to* strategy to solve the story above, he would say, "6, 7, 8, 9…so, 3 candies." Although the *count on to* strategy can be used to solve the separate problem above with the result unknown, it is more commonly used to solve a join problem where the change is unknown. This strategy matches the semantic structure of 6 + ? = 9 better than 9 – 6 = ?.

Levels of Whole Number Development

Counting principles. The construction of a number sequence is preceded by the basic activity of counting. Gelman and Gallistel (1986) identified five principles that govern and define counting. The first is the stable order principle, stating that you need to know the counting words and be able to recite them in the correct order each time; it is impossible to count up to seven if you know only the first six counting words. The second principle is the one-to-one principle. One, and only one, number word has to be matched to each and every object; lack of co-ordination is a source of potential error. The third principle is the cardinality principle. When correctly following the first two principles, the number name allocated to the last object tells how many objects you have counted. The fourth principle is the principle of abstraction. You can count anything – visible objects, objects of different shapes and sizes, things that are too far away to touch, objects that cannot be moved, moving objects, hidden objects, imaginary objects, sounds, etc. The last of Gellman and Gallistel's principles is the principle of order irrelevance. Objects may be counted in any order provided no other counting principle is violated. Children in the pre-numerical stage are sorting through and learning these counting principles based on their experiences with counting. When children learn the principles of stable order, one-to-one, and cardinality, the result of counting is an "extensive meaning for their number words: counting a collection “one, two, three, four, five” results in the child having “five” things “out there”” (Olive, p. 5).

Pre-numerical stages. There are two counting stages identified by Steffe, von Glasersfeld, Richards, and Cobb (1983) as pre-numerical. The first is the perceptual counting stage. Children in this stage require the collection of countable items to be in their perceptual field. Steffe and Cobb (1988) claim that the need for countable items (i.e. tiles, marbles, fingers, etc.) is due to a lack of awareness of plurality. “An awareness of plurality requires the production of a visualized image of a perceptual item along with its actual repetitions” (Steffe, 1988, p. 136). When children have developed the awareness of plurality and internalized their countable items, they are in the second pre-numerical stage, the figurative stage of counting. In this stage, children can count items that are not in their immediate perceptual field. This development means that the child has constituted the collection of countable items as permanent. The child can re-present an image of the countable items and count these images. Many children in the figurative stage will use sensory-motor items, such as fingers or taps, to stand in for the imagined objects.

In both the perceptual and figurative stages of counting, children are capable of limited additive tasks. These children do not recognize the task as “addition,” but the activity that they engage in produces an additive result. The following task is one that is appropriate for a pre-numerical child:

> Sam has 4 marbles. His friend, Tommy, gives him 3 more marbles. How many marbles does Sam have altogether?

In order to do a join problem like this one, children in the pre-numerical stage would have to use *direct modelling* and/or the *count all* strategy. The perceptual counter would count a collection of four objects, then count a collection of three objects to join with the previous collection, and finally *count all* of the items in the resulting collection. Steffe calls this additive scheme a "perceptual join," combining two separate collections into one single collection of perceptual items (Steffe, 2003, p. 239). The figurative counter is capable of a slightly more advanced *count all* method. This child may be able to count the initial four objects, using his fingers in place of the objects and saying "1, 2, 3, 4" in synchrony; then continue counting his fingers, "5, 6, 7" and stop when he recognizes his finger pattern for three. This child has a finger pattern for three and is simply coordinating this pattern with his number words. Steffe calls this additive scheme "counting all with intuitive extension" (Steffe, 2003, p. 239). Both the perceptual and figurative counters use the *count all* strategy. The necessity to use the *count all* strategy is a major distinction between the pre-numerical child and the numerical child because pre-numerical children have not yet constructed a number sequence with permanence.

Initial number sequence. The figurative counter will begin to develop the ability to unitize. Unitizing is the ability to re-present the countable objects, focusing attention on each individual item, making you explicitly aware of the number of counted items. As the pre-numerical child develops the ability to visualize countable objects, he begins "to internalize (make mental representation of) their counting acts, and eventually interiorize the results of those counting acts" (Olive, 2001, p. 5-6). Now, the child has constructed a numerical concept for his number words. For example, the number word "four" represents the counting sequence "1, 2, 3, 4." Steffe calls this a numerical composite. When a child has established numerical composites, he is in the numerical stage that Steffe calls the Initial Number Sequence.

In solving a join problem where the result is unknown or a part-part-whole problem where the whole is unknown, a child with the Initial Number Sequence would use the *count on* method. For example, a child may be asked the following join question:

> Farrar had 6 dolls. Her parents gave her 3 more dolls for her birthday. How many dolls did Farrar have then?

Because a child with the Initial Number Sequence has a numerical composite for “six”, he knows that the word “six” refers to the counting activity of “1, 2, 3, 4, 5, 6” without actually carrying out the counting activity. Thus, he does not need to count the initial six dolls. So, the child may say, “6…7, 8, 9 – nine dolls.” Using the *count on* strategy actually includes two numerical composites, one for “six” and one for “three”. The numerical composite of “three” is important because the child has to *count on* three more times (from six). Steffe calls this addition scheme “counting on with numerical extension.”

A child with the Initial Number Sequence may also solve a separate problem with the result unknown by using the *count down* strategy. For example, a child might respond to the problem of 8 - 3 by saying, “8 [pause], 7, 6, 5…5.” The child must monitor each counted item and consider each counting act as a unit item. To do this, the child may keep track of his counts by putting up a finger with each count after the initial quantity. The use of the *count down* strategy indicates that the child “took the results of counting as one thing – as material of his unitizing operation” (Steffe, 1988, p. 153). He could see his verbal number sequence as two distinct parts, {8, 7, 6} and the other part which is symbolized by “5”.

In both cases (*count on* and *count down*), the children interpreted the problem as a counting problem, where counting was symbolized by counting words.

> If the children counted the visible items, they could apply the uniting operation to the counted items and then continue to count to the "result" number, whereupon they could once again apply the uniting operation to their records of continuing to count, creating a numerical whole whose numerosity could be established. These are what I call *sequential uniting operations*. (Steffe, 1988, p. 154)

Having the sequential uniting operation is an indication that the child has "reinteriorized his counting acts," which is crucial for the development of the next number sequence.

Tacitly nested number sequence. The Tacitly Nested Number Sequence is a transitional sequence that occurs between the Initial Number Sequence and the Explicitly Nested Number Sequence. Children are typically in this stage for only three to five months before making the vertical move into the Explicitly Nested Number Sequence. When a child with the Initial Number Sequence has shown the ability to "reinteriorize counting acts", it is possible that he will unite the records of counting into a composite unit. This essentially means that the child is aware that a collection of items can be considered one thing, a composite whole. With this new development, the child's monitoring ability has progressed. Putting up fingers has changed from the Initial Number Sequence where fingers were the countable items to now putting up fingers serving as a record of a counting act as well as a countable item. This ability places the child in the Tacitly Nested Number Sequence. Take the following join problem as an example:

> Sally puts 6 coins in a jar. Then she puts some more coins in the jar. If there are 14 coins altogether, how many more did Sally put in the jar?

This problem is a join problem where the change is unknown. A common strategy that children use to solve this problem is the *counting on to* strategy. To answer the question above, a child might say, "6…7, 8, 9, 10, 11, 12, 13, 14 – that is 8 more balls." While uttering the words 7, 8, 9, …14, the child must monitor his counts. One common way to monitor is using fingers; so the child would put up a finger every time a number word is said, starting with 7. The answer, which indicates the "change" in the problem, is found by the number of fingers that are up when 14 is reached. This monitoring activity requires two levels of units, which is a new development for the Tacitly Nested Number Sequence child. While using the *count on to* method, the child makes two composite units, the first 6 counting acts and those that have not yet been carried out.

A child with the Tacitly Nested Number Sequence may also solve a separate problem with an unknown change quantity by using the *count down to* method. A child, using the *counting down to* strategy might solve the previously mentioned problem by saying, "14…13, 12, 11, 10, 9, 8, 7, 6 – that's 6 coins that Sally put in the jar", simultaneously putting up a finger every time he utters a number word. The child is monitoring his counts by putting up fingers, and he stops at 6 and recognizes that there are 8 fingers up and that 8 is the answer.

Explicitly nested number sequence. All of the solution strategies that have been listed in the Tacitly Nested Number Sequence are strategies that are also commonly used by children in the Explicitly Nested Number Sequence. In those cases, we would need to know more about how the child operates in other situations to determine which number sequence he has. There are several main elements that are crucial to make the leap from the Tacitly Nested Number Sequence to the Explicitly Nested Number Sequence. One is the shift from the *sequential uniting operation* to the *progressive uniting operation*. The progressive uniting operation is when a child can

> take the results of using his uniting operation as material of its application. As a consequence, a number word was now a symbol for a unit containing a number sequence from one up to and including that number word (or any other segment of a number sequence of specific numerosity indicated by the number word) (Steffe, 1988, p. 158)

Another element of the Explicitly Nested Number Sequence is the ability to disembed. Disembedding is “a conceptual act that takes elements out of a given composite unit and uses them to make a new composite unit, but the elements that are taken out of the composite unit are left in the composite unit” (Steffe, 2003, p. 243). Essentially, the child can take the composite unit from the whole-number sequence without destroying the sequence. Another element of the Explicitly Nested Number Sequence is the recognition that a number can be constructed from iterable units of “1”. That is, a child sees the number 5 as 1 five times as well as the counting act of “1, 2, 3, 4, 5.” The ability to see iterable units of 1 gives the child the capability of disembedding any number of 1s from the composite unit without destroying the unit. Thus, a child can now see a number as two unitary items. For example, the number 15 can be seen as one unit containing the first ten items, and another unit containing the remaining items of the sequence. Then, each of these unitary items can be disembedded and seen as components apart from 15 and components in 15. This advancement gives the child the ability to construct subtraction as the inversion of addition; it also allows a child to see subtraction as the difference of two numbers. These operations give the child the ability use strategic reasoning.

To identify that a child has advanced from the Tacitly Nested Number Sequence to the Explicitly Nested Number Sequence, it is a good idea to ask them problem types that may elicit these advancements. One problem type might be to a join problem with the change unknown, like the one identified for the Tacitly Nested Number Sequence child. This time, the problem will be stated with larger numbers. Using larger numbers will make it difficult for the child to use his fingers as a monitoring system. Below is an example of a join problem where the change is unknown (using large numbers):

> Tom puts 24 coins in his piggy bank. Then he puts more coins in the piggy bank, and had 57 coins altogether. How many more coins did Tom put in the piggy bank?

The child with the Explicitly Nested Number Sequence may solve this problem using the *count on to* strategy, which is the same counting strategy used by the child with the Tacitly Nested Number Sequence, but the monitoring act is different. For example, a child might say, "25 is one, 26 is two, 27 is three, 28 is four, ….56 is thirty-three, 57 is thirty-four—he added 34 coins to the piggy bank." This monitoring technique is "double counting." Double counting is much more difficult than keeping track of how many times you count by simply putting up fingers. This "double counting" technique is an indication of the recursive property of the child's number sequence--that is, to take the elements of the number sequence as countable items. The strategy of "double counting" is a novelty and indicates that a child may have the Explicitly Nested Number Sequence.

Children with the Explicitly Nested Number Sequence may also begin to use strategic reasoning. Strategic reasoning is the testing ground to find how generative the child is. The join problem with the change unknown that was stated above may be solved using strategic reasoning. For example, a child might say, "24…and 30 more is 54. Then, 3 more would give me 57. So, that's 33 more coins."

The complexity of a child's mathematics is astounding. I have described the five main stages of a child's counting schemes, from perceptual, to figurative, then to the Initial Number Sequence, the Tacitly Nested Number Sequence, and finally the Explicitly Nested Number

Sequence. Each stage is an interiorization of its preceding stage. The stages provide a framework for teachers to better understand their students' construction of number. With this, teachers may be able to listen to their students, and shape their instruction around the students' mathematics. They can give students opportunities to make vertical progress in the construction of their number sequence.

CHAPTER 3

METHODOLOGY

I conducted an evaluative study documenting the overall effects of the Interview Project, a project where preservice teachers engaged in an interview with a child during their first mathematics content course for the early childhood teacher education program. Patton (2002) suggested that a summative evaluation serves as an "overall judgment about the effectiveness of a program, policy, or product" (p. 218). My goal was to determine whether or not the Interview Project was an effective way of helping preservice teachers learn to describe a child's mathematics, analyze a child's mathematics, and make appropriate instructional decisions based on the actions of the child.

Participants

The participants in my dissertation study were 29 preservice early childhood teachers at Georgia College enrolled in my section of the course Math 2008 in the spring semester of 2014. The pre and post Interview Project data were collected from all 29 preservice teachers individually, while the observation and interview data were collected from a subset of seven pairs of preservice teachers.

Data Collection

In order to determine the effectiveness of the Interview Project, I employed several different methods of data collection. Data collection consisted of class products, observations, and interviews with a subset of the preservice teachers. These artifacts provided me with data that gave me insight into the preservice teachers' mathematical knowledge for teaching additive structures aligned with the frameworks from class. More specifically, the data gave insight into the preservice teachers' abilities to describe and analyze a child's mathematics and use that analysis to make instructional decisions. Some data were collected at the beginning of the semester, when no classroom discussions had influenced the preservice teachers' knowledge, and then more data were collected after the Interview Project had been completed to determine any changes that occurred. All discussions and activities in the Math 2008 course that took place between the first day of class and the assignment of the Interview Project were focused on the frameworks needed to successfully participate in the Interview Project. Thus, the changes that occurred could be contributed to both the preservice teachers' knowledge of the frameworks and their participation in the Interview Project.

Class Products

Because I was the instructor of the Math 2008 course, I had the unique opportunity to collect data through class products. I implemented some tasks that were specifically designed to help assess the preservice teachers' knowledge of the frameworks used in the class during the class sessions. To assess the preservice teachers' ability to describe the tasks and the child's mathematics, I used the tasks described here:

Task 1: Additive structured story problem types. To assess whether or not the preservice teachers could recognize different types of additive structured story problems, I gave them a collection of story problems and asked them to organize them into groups. All of the story problems could be solved with the expression 8 + 5 or 13 - 5, but they were different problem types such as join change unknown, separate result unknown, compare difference unknown, part-part-whole part unknown.

Task 2: Knowledge of solution methods. I assessed the preservice teachers' knowledge of the different solution methods used by children to solve additive structured problems using videotapes. The preservice teachers watched videotapes of children solving additive problems and then they were asked to show how these children would solve a related problem. They were also asked to describe how the child solved the problem.

To assess whether or not the preservice teachers' could analyze a child's mathematics and make instructional decisions based on that analysis, I used the following task:

Task 3: Levels of development. The preservice teachers were asked to watch some videos of children solving additive structured story problems. Some of the children solved the problems correctly and some did not. After watching the videos, I asked the preservice teachers to respond in writing to the prompt "Based on what you just saw, what can you tell me about this child's level of development?"

Task 4: Instructional decisions. After the preservice teachers responded to the question above about the child's development for Task 3, I then asked them, "What would you do next? What task would you give this child now that you've seen this clip?" I asked this question to assess the preservice teachers' abilities to make instructional decisions based on a child's current ways of thinking. Again, the preservice teachers responded to this question in writing.

In addition to the task above, I used another task to assess whether or not the preservice teachers could identify the relative difficulty of a pair of story problems.

Task 5: Relative problem difficulty. To assess the preservice teachers' ability to distinguish the relative difficulty of different story problem types, I gave the preservice teachers five pairs of word problems and asked them to identify which was more difficult. They could also respond that the problems were of the same difficulty level. The pairs were different in terms of their problem structure, for example, a join result unknown versus a join change unknown, or a separate result unknown versus a compare result unknown. In addition, I asked them to read a join result unknown story problem that could be solved by a simple addition computation, then I asked them to write a different story problem that was more difficult to solve but could still be solved by the same addition computation. They repeated this task but with a separate result unknown story problem that could be solved by a simple subtraction computation.

The Interview Project was assigned around midterm of the semester, allowing for enough class time to be dedicated to discussions of the frameworks of Cognitively Guided Instruction, Gelman, Steffe et al., Van de Walle, and others. Prior to that point I spent time in class having the preservice teachers classify problems and write their own problems for particular classifications, determining which type of problems are most difficult for children, and what types of strategies children use to solve the problems. These strategies include *direct modeling*, where students use manipulatives to model the story in the problem, a counting scheme, such as *counting all*, *counting on*, or *counting on from largest*, or *strategic reasoning*, where a child uses a closely related fact along with properties involved in the operation to solve the problem. In class, we also spent time determining what the use of these strategies might tell us about the child's level of whole number development and ways to make a vertical leap to the next level of development.

The preservice teachers were given a description of the project and a rubric (see Appendix A), indicating the expectations for the assignment. They were also given a sample interview protocol that they could use as a guide for their interview. They could choose to use the sample interview questions, or they could design their own interview questions. It was suggested that they work in pairs, allowing one preservice teacher to interview and question, freeing the other preservice teacher to take field notes on the child's responses, quoting when possible. I suggested that they interview a child between the ages of 5 and 10 if possible. The preservice teachers could interview a child that they knew personally, or if they could not find a child on their own to interview, I found one for them in one of the local elementary schools. The pairs then wrote a summary of their interview, describing the types of story problems that they asked, the solution methods that the child used, their analysis of the child's actions, and a reflection on their experience and how it affected their current and future instructional decisions.

The main sources of my post-Interview Project data were the preservice teachers' written reports of the Interview Project. As I investigated the data, I was looking for how the preservice teachers were able to describe the child's mathematics and instances where they analyzed their descriptions according to the frameworks from class. I also wanted to determine whether or not they were able to apply their analysis by making instructional decisions, and whether they were able to recognize and make use of mathematical properties in the child's strategies.

Observations

While I believed that the preservice teachers' written reports of the Interview Project would be a rich source of data, I also conducted some observations to help solidify some of the findings from their reports. Because I was interested in the instructional decisions that the preservice teachers constructed and how these models informed their instructional decisions, I needed to study their interactions with the child. Hays (2004) claimed that "the interaction of individuals cannot be understood without observations" (p. 229). Thus, I observed a subset of the preservice teachers' interviews with the child and took field notes, which included the preservice teachers' progression of tasks, the strategies used by the child, and instructional decisions that the preservice teachers made. I also audiotaped the observed preservice teachers' interview sessions for retrospective analysis. The subset consisted of seven pairs of preservice teachers. Seven pairs gave me enough data to get an overall feel of the types of instructional decisions that the preservice teachers were making. I chose the subset of preservice teachers by asking for volunteers, and the observations took place during a mutually agreed upon time after the project assignment had been given.

Interviews

To triangulate the data, I conducted interviews with the preservice teachers that I observed. The purpose of the research interview is "to explore the views, experiences, beliefs and/or motivations of individuals on specific matters. Interviews are believed to provide a 'deeper' understanding of social phenomena than would be obtained from purely quantitative methods, such as questionnaires or surveys" (Gill et al., 2008, p. 292). Thus, interviews served as a way to validate the data collected by the class products and observations.

Session interviews. I interviewed the seven pairs of preservice teachers that I observed immediately after their session with the child. Guided by my research questions, I asked them questions such as:

1) Why did you ask that particular question?
2) Did you deviate from your planned progression of questions? How? Why?
3) How were you thinking about the child's mathematics when you planned your tasks?

These questions were intended to help me understand any instructional decisions that were made, whether they were planned or not planned. The answers to these questions also helped me to confirm or disconfirm the data from my observations and field notes. These interviews took about 15 minutes and occurred immediately after their interview with the child.

Final interviews. In addition, I also conducted interviews with the subset of seven pairs of preservice teachers after the Interview Project was completed and the written report was submitted. This subset of preservice teachers was the same as the subset chosen for the observations. By this point in time, the preservice teachers had had time to describe and analyze the child's mathematics, predict any future instructional decisions, and reflect on the entire experience. For their final interview, I asked the preservice teachers the following questions:

1) One of the goals of the Interview Project was to describe and analyze a child's mathematics. Why or why not do you think describing and analyzing a child's mathematics is a helpful tool for teachers?
2) Another goal of the Interview Project was to learn to listen to children. Why or why not do you think listening is a helpful tool for teachers? Why or why not? Did you use your listening skills during your interview to make instructional decisions? Did you use your listening skills after your interview to make future instructional decisions?
3) What did you get out of the Interview Project?

The answers that the preservice teachers gave during their final interview helped me to confirm or disconfirm the data from the written reports and observations. The first two questions were directly related to my two research questions. The third question was an overall question about whether or not the preservice teacher thought that this project was worthwhile.

The table below summarizes the groupings of the 29 participants, the grade level of the child that each group interviewed, and whether or not the group was observed and interviewed by me.

Table 1

Summary of Participants

Group	Members	Grade Level of Child	Observed and Interviewed?
Group 1	A B	1	no
Group 2	C D	2	yes
Group 3	E F	4	no
Group 4	G H	3	yes
Group 5	I J	2	yes
Group 6	K L	1	yes
Group 7	M N O	2	no
Group 8	P Q	2	no
Group 9	R S	2	yes
Group 10	T U	4	no
Group 11	V W	3	yes
Group 12	X Y	3	no
Group 13	Z AA	5	no
Group 14	BB CC	K	yes

Data Analysis

For the data analysis, I started by looking at the pre-Interview Project data. This included class products. During the first week of class, the preservice teachers completed the five tasks that were mentioned previously. When analyzing these tasks, I was looking for instances where the preservice teachers described the child's mathematics, analyzed the child's mathematics, and made instructional decisions.

Pre-Interview Project: Description of the Child's Mathematics

When analyzing the first task, "Additive Structured Story Problem Types," I looked for whether or not the preservice teachers could categorize the story problems based on their story structure rather than their computational structure. This told me whether or not the preservice teachers could describe the problem types accurately. The second task, "Knowledge of Solution Methods," determined whether or not the preservice teachers could describe children's solution strategies. To analyze this task, I looked for the preservice teachers' ability to identify particular solution strategies used by children, not necessarily by the name of the strategy, but by their ability to reproduce the strategy.

Pre-Interview Project: Analysis of the Child's Mathematics

To assess whether or not the preservice teachers could analyze a child's mathematics, I used Tasks 3 and 4 mentioned previously. When analyzing the Task 3, "Levels of Development," I looked for instances where preservice teacher was able to give some sort of reasoning for the child's thinking process. I looked for the preservice teacher to give insight into the child's solution strategy more than simply saying, "He got it wrong/right." The preservice teacher may have given a reason the child used that particular solution strategy, or s/he may have attempted to describe what the solution strategy suggests about the child's level of understanding. I did not expect that the preservice teachers to use any formal language from the frameworks.

Pre-Interview Project: Instructional Decisions

I used Tasks 4 and 5 to assess whether or not the preservice teachers could make instructional decisions based on the child's responses to whole number additive story problems. When analyzing Task 4, "Instructional Decisions," I was looking for whether or not the preservice teachers could describe a specific task that would logically follow the previous task, based on the child's response. Simply giving a task was not enough; the task needed to be in line with the child's understandings on the previous task. For example, if the child solved a join-result unknown problem correctly using the count on (from the first number) method, the preservice teacher might suggest that the next task be a join-result unknown where the smaller number is written first in the story to see if the child will use the same count on strategy as before (from the first number) or if the child will count on from the larger number. This counting method is slightly more complex and would give more information about the child's level of development.

In order to make these types of instructional decisions, teachers need to be aware that some types of additive story problems are more complex than others. I used Task 5, "Relative Problem Difficulty," to assess whether or not the preservice teachers could identify the relative difficulty of a story problem. When analyzing the first part of this task, I simply looked for whether the preservice teacher could identify the relative difficulty of the problems in the pair. No rationale was needed. For the second part of this task, I determined whether or not the preservice teacher could write a story problem that could be solved by a particular computation but was more difficult than the one given. In order to assess these tasks, I used the rubric in Appendix D.

Once the Interview Project was assigned, I began my post-Interview Project data analysis. This data consisted of class products (written report), observations, and the preservice teacher session interviews and final interviews. When analyzing the written report, I looked for instances where the preservice teachers described the child's mathematics, analyzed the child's mathematics, and made instructional decisions. I also used my observations and field notes to analyze the preservice teachers' instructional decisions during their interview, along with the session interviews that I conducted with the preservice teachers directly after their interview with the child. Below is a more detailed description and rubric of how I analyzed the data.

Post-Interview Project: Description of the Child's Mathematics

In the Interview Project description and rubric, the preservice teachers were given instructions to describe the child's mathematics. More specifically, I was looking for the preservice teachers to accurately describe a child's mathematics using the frameworks discussed in class. The preservice teachers needed to use the Cognitively Guided Instruction (1999) framework to identify the structure of the story problem, such as Join-change unknown or Compare-difference unknown. Then, using the CGI and the Van de Walle (2010) frameworks, the preservice teachers should have described the method that the child used to find the solution to the task. For example, the child might have used a direct model, the counting all method, or strategic additive reasoning. Correctly using language from these frameworks was key.

Post-Interview Project: Analysis of the Child's Mathematics

The preservice teachers were also required to give an analysis of the child's mathematics in the Interview Project write up. I was looking specifically for cases where the preservice teachers analyzed the child's mathematics using the frameworks from the class. What did the child's actions imply about the level at which he/she was operating? Using the Steffe and colleagues (1982) framework and the Gelman (1986) framework, the preservice teacher needed to use the child's responses to the tasks to identify his/her current level of operating as perceptual counter, motor item counter, Initial Number Sequence, Strategic Additive Reasoning, etc. Again, correct use of the language from these frameworks was key. In addition, if the child used an invented strategy, the preservice teacher should have written a series of equations that justified why this strategy will always work. When writing the series of equations, it was important to use the equal sign appropriately and state the mathematical properties used at each step.

In addition to the preservice teachers' written reports, I also used the first question from the final interview with seven pairs of preservice teachers to analyze their ability to describe and analyze a child's mathematics. I looked for instances where the preservice teachers gave particular examples of how they described or how they analyzed the child's mathematics, using language from the frameworks.

Post-Interview Project: Instructional Decisions

To analyze the preservice teachers' instructional decisions, I used three sources of data: observations from seven pairs of preservice teachers, session and final interviews from the same seven pairs of preservice teachers, and the written reports from all 29 preservice teachers. During my observations, I took field notes and audiotaped the interview. I then conducted session interviews with the preservice teachers directly after their interview with the child to help me better understand my observation. The preservice teachers were also asked, in their written report, to reflect on their instructional decisions. From these, I looked for cases of any instructional decisions that the preservice teachers made. Some of these decisions could be planned for and some of them could not. The preservice teachers were expected to have prepared the tasks for the interview and an ideal order in which the tasks were to be completed. Because a child's response cannot be predicted, the preservice teachers were to have considered different scenarios that might occur and be prepared to handle them if they arose, which may have required changing the order of the tasks, the size of the numbers, skipping a task, or doing more of a particular task. But, because not all situations can be planned for, the preservice teachers also needed to be able to spontaneously handle unforeseen situations. They may have needed to answer a question that the child asked, or they may have needed to skip a task if it deemed it too difficult for the child, or they may have needed to change the task to accomplish a particular goal. To make these kinds of instructional decisions, the preservice teachers needed to listen to the child; they needed to have knowledge of the possible solution methods and know what task was appropriate for the child based on his/her solution method. In addition, the preservice teacher needed to be able to do all of this on the spot.

As an additional way of analyzing the preservice teachers' abilities to make instructional decisions, I used the second question from the final interview with seven pairs of preservice teachers. I looked for instances where the preservice teachers gave specific examples of when they used their listening to make instructional decisions during their interview.

I analyzed the preservice teachers' written reports using the rubric in Appendix A. Any data collected from the observations and interviews was used to add depth and detail to the analysis. Figure 1 below shows the overview of the Interview Project's goals and the data that I analyzed in order to evaluate the project.

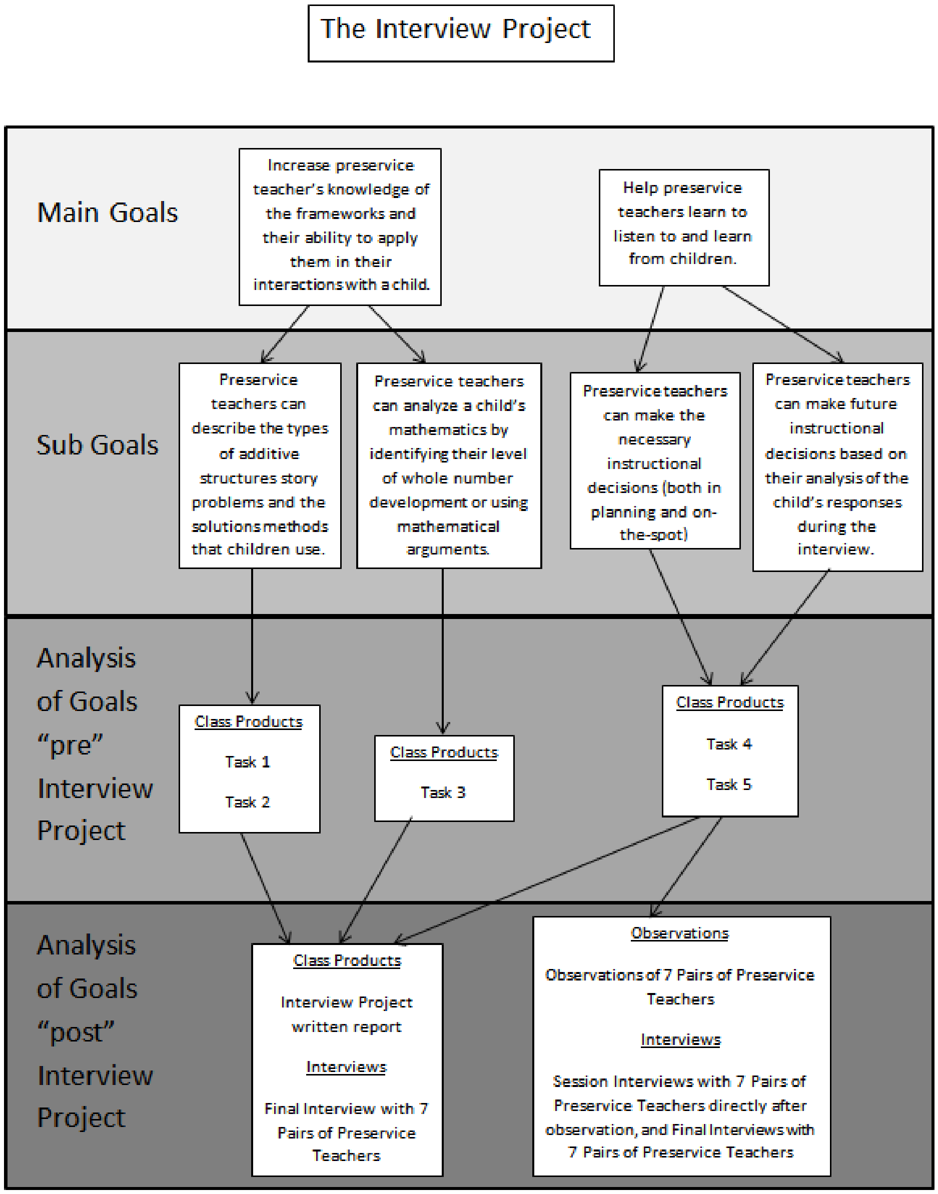

Figure 3. The Interview Project's goals and how they were analyzed.

Limitations

Because the Math 2008 course was taught by me, there were some concerns of subjectivity and validity. Patton (2002) defined qualitative research as attempting to understand the unique interactions in a particular situation. The purpose of understanding is not necessarily to predict what might occur, but rather to understand in depth the characteristics of the situation and the meaning brought by participants and what is happening to them at the moment. Thus, qualitative research begins by accepting that there are many different ways of understanding and of making sense of the world. As the researcher, my goal was to understand what preservice teachers got out of the Interview Project, and because there is no empirical way to understand what is going on inside someone's head, I had to interpret their words, their writings, and their actions. Interpretation, according to Prasad (2005), is an attempt to make sense of an object of study. "The interpretation aims to bring to light an underlying coherence or sense" (Taylor, 1976, p. 153). Our interpretations are affected by our personal life experiences and beliefs and the personal prejudices, or pre-judgments that might exist.

Constructivists embrace subjectivity as "a pathway deeper into understanding the human dimensions of the world in general as well as whatever specific phenomena they are examining" (Patton, 2002, p. 546). As a constructivist, my goal was not to find a singular truth or to make a generalization about the effects of the Interview Project but to develop a deep understanding of the specific cases that I studied. Because I was the instructor of this course and have had experience with this project, I definitely had some feelings, beliefs, and expectations of what the preservice teachers would learn from the Interview Project. I believed that the Interview Project would give the preservice teachers an opportunity to see how capable children are of learning mathematics and solving problems. I believed that they would be learning to respect children's mathematical thinking even when they do not understand it. I believed the preservice teachers would use their experience from the Interview Project to influence how they thought about their own mathematical thinking and to inform their teaching. These beliefs may have affected data analysis and interpretations. Because of this, it was important that I was open and honest about my beliefs and that I reported how these beliefs may have affected the data analysis. It was also important that, when analyzing the data, I reported direct quotations from the preservice teachers' written reports or interviews. Using quotations gives the reader the opportunity to agree or disagree with my interpretations. Constructivists' research is explicitly informed by attention to praxis and reflexivity, that is, "understanding how one's own experiences and background affect what one understands and how one acts in the world" (Patton, 2002, p. 546). Thus, it was important to incorporate procedures for validating and verifying the analysis. I used several strategies suggested by Maxwell (2012).

First, I used triangulation. Triangulation is a technique that assists in the validation of data through cross verification from multiple sources. Through the use of class products, observations, and interviews, I minimized any potential biases that could occur from a single method.

I also used respondent validation during my research to help improve the accuracy and validity of the data. I used respondent validation in two ways. During the session interview and final interview, I restated or summarized their responses and my interpretation of their responses and then questioned the participant to determine the accuracy of my interpretation. I also used respondent validation after the study was completed. I shared all of my findings with the participants via email. This allowed the participants to critically analyze the findings and comment on them. If the participants affirmed that the summaries reflected their views, feelings, and experiences, then I assumed that the study had credibility. In the one case where the participant did not affirm the summaries, then I asked her to clarify her statements. Then, I used respondent validation once again in order to make sure that my interpretation of her responses was valid.

Lastly, to ensure validity, I used numbers when reporting the results of my study. After I scored the assessments with the given rubrics, I reported some simple descriptive statistics that showed the frequency and relative frequency of the possible scores that the preservice teachers received. I reported these numbers to give the reader a sense of the "big picture."

It is also important to note that the Interview Project was a graded assignment for the preservice teachers. I believe that this could have had both positive and negative effects on the research. Hopefully, the preservice teachers thoroughly prepared for the interview and took their analysis seriously because it did affect their grades in the course. On the other hand, holding this sort of leverage against them may have been slightly intimidating and caused the preservice teachers to say or write things because it was what they believed I wanted them to say. This was why participation in the observation and interview portions of the research was optional, even though the project was mandatory. Their grade for the project was based on their written analysis and reflection on the interview with the child, which was another reason I thought it was important to use other sources of data collection. The observations and interviews were not part of their grade for the project. During class sessions, I developed a classroom discourse that encouraged open discussions about their thoughts without fear of being judged or graded. My hope was that this would allow the participants to be candid and honest about their answers in their session and final interview questions and in their reflection writings.

In qualitative research, subjectivity is both a tremendous strength and a potential weakness. The research methods that I chose helped to alleviate any potential weaknesses in order to reap the enormous benefits that resulted in the research.

CHAPTER 4

FINDINGS

Carpenter et. al (1999) stated that

> if teachers are to be expected to plan instruction based on their knowledge of students' thinking, they need some coherent basis for making instructional decisions. To address this problem, we need to help teachers to construct conceptual maps of the development of children's mathematical thinking in specific content domains. (p. 105)

I set out to determine whether or not preservice teachers would be able to use the frameworks presented to them in class to describe and analyze a child's mathematical thinking and apply this in an authentic scenario via the Interview Project. In addition, I wanted to determine whether or not preservice teachers would be able to listen to and learn from a child during their interview. Through my investigation, I determined that my participants showed a significant increase in their abilities to both describe and analyze a child's mathematics and to make appropriate instructional decisions as a result of listening to the child. I also discuss other findings that came out of my research, including the preservice teachers' evolving definitions of teaching and learning mathematics**.**

Preservice Teachers' Knowledge and Application of the Research

The first main goal of the Interview Project was to increase the preservice teachers' knowledge of the research and to apply this research in a real life teaching scenario. To determine whether or not this goal was met, I looked for instances where the preservice teachers used the research frameworks to *describe* the child's mathematics and for instances where they used the frameworks to *analyze* the child's mathematics. I used Tasks 1, 2, and 3 from the pre-Interview Project assessments and I used the written reports from the post-Interview Project assessments to assess this goal and compare pre and post results.

In the Math 2008 class, the preservice teachers were presented the language of Gelman (1986) which describes the counting principles, Cognitively Guided Instruction (1999) to help classify story problems and childrens' solution methods, Van de Walle (2010) which describes more solution methods that children use, and Steffe et. al. (1982) to assist in determining a level of whole number development, throughout the first four weeks of the semester (Appendix F). Therefore, it is not surprising that the preservice teachers' use of this language increased. The results show a significant improvement in the preservice teachers' abilities to both describe and analyze a child's mathematics using the language from the frameworks presented in class. The preservice teachers' abilities to describe and analyze a child's mathematics, influenced by the frameworks from class, informed their knowledge of children's mathematics from a more global perspective. In one pair's written report, they wrote,

> This was a very helpful project because it allowed us to step into the child's mind and analyze what she was thinking. As future teachers, this skill will be very helpful and beneficial to understanding a child's level. This project also helped us to understand that children can use their own logical thinking to solve problems that they haven't been taught previously. Their imagination is often larger than ours when it comes to creating ways to solve a problem.

Description of the Child's Mathematics

In order to assess the first goal of the Interview Project, I broke it down into two sub goals. These sub goals are to describe the child's mathematics and to analyze the child's mathematics. Both of these sub goals require the preservice teachers to use language from the frameworks presented in class. The description process is the first step that is essential in later analyzing the child's mathematics.

Tasks 1 and 2 were used as the pre-Interview Project assessment to determine the preservice teachers' abilities to describe a child's mathematics. Task 1 assessed each individual preservice teacher's ability to recognize different structures of additive story problems. For this task, I gave the preservice teachers a collection of story problems and asked them to organize them into groups. All of the story problems could be solved by the simple computation of 8 + 5 or 13 - 5, but they were different problem types, such as join change unknown, separate result unknown, compare difference unknown, part-part-whole part unknown. I also asked the preservice teachers to describe why they chose their groupings. The task was scored using the rubric in Appendix D with a score of 0, 1, or 2. The overall average on Task 1 was 0.17. Table 2 shows the scores for the 29 preservice teachers.

Table 2

Results of Task 1: Additive Structured Story Problem Types

Score on Task 1	Frequency	Relative Frequency
0	25	86.2%
1	3	10.3%
2	1	3.4%
Total	29	100%

After further analysis of these results, I found that 19 of the preservice teachers were given a score of 0 because they grouped the story problems by their solution's computation. In other words, they grouped all of the story problems that could be solved by addition together and all of the story problems that could be solved by subtraction together. Within their category of addition, they included join result unknown problems, separate initial unknown and vhange unknown problems, compare larger unknown, and part-part-whole whole unknown problems. In their category of subtraction, they included join initial unknown and change unknown problems, separate result unknown problems, compare difference unknown or smaller unknown problems, and part-part-whole part unknown problems. In this categorization, the structure of the story problem was not taken into account; instead, only the computation that is needed to find the answer was taken into consideration. The other 6 preservice teachers who were given a score of 0 on Task 1 received this score due to superficial groupings. For example, some said that they grouped the stories by the verb tense that was used (i.e. present tense or past tense); others said that they grouped the stories by the names that were used (i.e. Juan or Connie). The 3 preservice teachers who received a score of 1 all used the key word approach, grouping stories together that used the word "altogether", "gave" or "more." Using these groupings resulted in a somewhat accurate categorization of the different problem structures because two of the join problems used the word "altogether," all of the separate problems used the word "gave," and all of the compare problems used the word "more". Thus, using the key words did result in a categorization by problem structure for some of the stories but not all of them.

Task 2 assessed the preservice teachers' abilities to recognize different methods of solving an additive structured story problem. For this task, the preservice teachers watched four videotapes of a child solving an additive problem, and they were asked to describe how each child solved the problem and to show how the child would solve a related problem. Each of the four problems was scored using the rubric in Appendix D with a score of 0, 1, or 2; then the average of those was recorded as the score for Task 2. See Table 3 for the results.

Table 3

Results of Task 2: Knowledge of Solution Methods

Score on Task 2	Frequency	Relative Frequency
0	2	6.9%
0.25	5	17.2%
0.5	6	20.7%
0.75	6	20.7%
1	4	13.8%
1.25	2	6.9%
1.5	3	10.3%
1.75	0	0%
2	1	3.4%
Total	29	100%

The overall average of scores on Task 2 was 0.75. With further analysis of these results, I found that the most common reason for receiving a score of 0 on each of the four problems was the lack of detail in the description. Twenty-three of the preservice teachers received a 0 on at least one problem because of the lack of detail, sayings things such as "he used his fingers" or "she used blocks" or "he counted". All of these descriptions are accurate, but a more detailed description is needed in order to determine exactly how the child used his fingers to solve the problem or which counting method was used (i.e. count all, count on, count up to, etc.).

For the post-Interview Project assessment, the written reports were used. For their written reports, the preservice teachers were asked to discuss the classification of the story problems that they asked the child to solve, and they were also asked to discuss what strategies the child used to solve the problem. The written reports were completed in groups of 2 and one group of 3 and were scored with the rubric in Appendix A, receiving a score from 0 to 2. Because the preservice teachers asked the child 6-10 questions, it was possible that some of the descriptions were accurate and some were not within one report. As a result, I awarded half points to those scenarios. Table 4 shows the results. The overall average of these scores was 1.69.

Table 4

Results of Describing a Child's Mathematics in the Written Report (Post-Interview Project)

Score for Describing a Child's Mathematics in Written Report	Frequency	Relative Frequency
0	0	0%
0.5	0	0%
1	4	13.8%
1.5	10	34.5%
2	15	51.7%
Total	29	100%

The scores on Tasks 1 and 2 were averaged together to give an overall pre-Interview Project score for the preservice teachers' abilities to describe a child's mathematical thinking. The results of the pre-Interview Project assessment and the post-Interview Project assessment for the preservice teachers' abilities to describe a child's mathematical thinking are presented in Table 5.

Table 5

Overall Pre and Post Assessment Scores for Describing a Child's Mathematical Thinking

	Average of Pre-Assessment	Average of Post-Assessment	Percentage Increase	t-score	Significance Level
Description of Child's Mathematical Thinking	0.4267241	1.6896552	295.9596%	20.407	$\alpha<0.0005$

Table 6

Individual Pre and Post Assessment Scores for Describing a Child's Mathematical Thinking

Group	Members	Pre	Post
Group 1	A	0.5	2
	B	0.125	
Group 2	C	0.75	2
	D	0.25	
Group 3	E	0.25	2
	F	0.625	
Group 4	G	0.375	2
	H	0.5	
Group 5	I	0.375	1.5
	J	0.375	
Group 6	K	0.375	1
	L	0	
Group 7	M	0.625	2
	N	0.375	
	O	0.125	
Group 8	P	0.375	1
	Q	0.25	
Group 9	R	1	1.5
	S	0	
Group 10	T	0.75	2
	U	0.25	
Group 11	V	0.375	1.5
	W	0	
Group 12	X	1	1.5
	Y	1.5	
Group 13	Z	0.375	1.5
	AA	0.5	
Group 14	BB	0.25	2
	CC	0.125	
AVERAGE		**0.4267241**	**1.6896552**

Overall, the preservice teachers showed a significant improvement in their abilities to use the frameworks from class to describe a child's mathematics. The individual results in Table 6 show that every preservice teacher increased his/her knowledge and use of the language from the frameworks to describe a child's mathematics. They used language such as join result unknown, separate start unknown, direct modeling, counting on, counting all, etc. In a typical excerpt from the written reports, one pair of preservice teachers wrote,

> The first story problem we asked Josh was a join-result unknown problem with small numbers because we wanted to start with a simple problem. We asked him, "Emily has 7 Skittles. Ali gave her 5 more Skittles. How many Skittles does Emily have in all?" Josh first sat quietly and appeared to be thinking about it. After a few minutes of silence, I asked him if he wanted to use the blocks that were sitting on his table. He nodded and then counted out 7 blocks and placed them in front of him. He then counted 5 more blocks and slid them across the table so that they were in the same place as the previous 7 blocks. Then, Josh counted all of the blocks, and said, "12." Josh used the blocks to directly model the problem and he used the count all method because he counted out each number in the problem and he counted out the result. Based on his response to this story problem, Josh would be considered a perceptual counter, meaning that he had to use physical objects in his perceptual field to count 7+5.

This pair of preservice teachers used the language of CGI to describe the structure of the story problem as a join result unknown and to describe the method that the child used to solve the problem as direct modelling and counting all. The preservice teachers also used their description to analyze Josh's mathematical actions. Through their description, they deduced that Josh was a perceptual counter.

Another typical response when describing a child's mathematical actions was

> We asked Ryan this question, "Caleb has 6 apples and 7 pears. How much fruit does Caleb have altogether?" This problem is a part-part-whole problem with the whole unknown. Ryan used strategic additive reasoning with the "using a double" strategy. He said, "6 and 6 is 12, then plus one more is 13." Using strategic reasoning demonstrates that he is on the Explicitly Nested Number Sequence level.

These preservice teachers used the language of CGI to describe the structure of the story problem, and they used the language from Van de Walle to describe his strategy as the "using a double" strategy. In addition, the preservice teachers recognized that the "using a double" strategy is one form of what Steffe et.al. call strategic additive reasoning, and they used this description to further analyze the child's mathematics, placing him on the Explicitly Nested Number Sequence level.

Using the language from the CGI, Van de Walle, and Steffe frameworks was a necessary first step in the preservice teachers' constructions of the students' mathematical thinking. Subsequently, they used their descriptions of their children's mathematics in order to provide evidence for their analyses.

Analysis of the Child's Mathematics

All of the preservice teachers were able to describe the mathematical actions of the child that they interviewed, but they were also able to use their descriptions to delve even deeper. They were able to use their descriptions to assess the level at which the child was operating according to the frameworks presented in class and make inferences about the child's mathematical development. This assessing and inferring is what I call analyzing the child's mathematics.

Task 3 was used as the pre-Interview Project assessment to determine the preservice teachers' abilities to analyze a child's mathematics. In this task, the preservice teachers watched four videos of children solving an additive structured story problem. They were then asked to respond to the following question, "Based on what you just saw, what can you tell me about this child's level of development?" Their response to each video was scored with the rubric in Appendix D, receiving a score of 0, 1, or 2; these scores were averaged to give a final score for Task 3. Table 7 shows the results.

Table 7

Results of Task 3: Levels of Development

Score on Task 3	Frequency	Relative Frequency
0	15	51.7%
0.25	9	31.0%
0.5	5	17.2%
0.75	0	0%
1	0	0%
1.25	0	0%
1.5	0	0%
1.75	0	0%
2	0	0%
Total	29	100%

This task proved to be very difficult for the preservice teachers, with an overall average of only 0.16. Ten of the preservice teachers did not even attempt to determine a level of development for one or more of the children in the videos, providing no response at all. For those who did respond, the most common reason for receiving a score of 0 was simply estimating the child's age or grade level. For example, one preservice teacher wrote, "He looks like he is about 7 years old, like my nephew." Fifteen of the preservice teachers received a score of 0 for this reason. Another common response was for the preservice teacher to simply tell whether or not the child answered the question correctly or not or how quickly the child was able to produce the answer. This type of response was often accompanied by the preservice teacher describing the child as "smart" when the question was answered correctly or "slow" when the question was answered incorrectly. 7 of the preservice teachers were given a score of 0 for these types of responses.

For the post-Interview Project assessment to determine whether or not the preservice teachers could analyze a child's mathematics, I used their written reports. For their reports, the preservice teachers were asked to use Steffe and colleagues' or Gelman's model or language from the Van de Walle text as a framework to analyze the student's mathematical thinking displayed on the tasks. They were also asked to discuss what the child's actions implied about the level at which the child was operating. Each group of preservice received a score of 0 to 2, as described on the rubric in Appendix A. Because the preservice teachers asked the child 6-10 questions, it was possible that some of the analyses were accurate and some were not within one report. As a result, I awarded half points to those scenarios. Table 8 shows the results. The overall average of these scores was 1.83.

Table 8

Results of Analyzing a Child's Mathematics in the Written Report (Post-Interview Project)

Score for Analyzing a Child's Mathematics in Written Report	Frequency	Relative Frequency
0	0	0%
0.5	0	0%
1	0	0%
1.5	10	34.5%
2	19	65.5%
Total	29	100%

The results of the pre-Interview Project assessment and the post-Interview Project assessment to determine the preservice teachers' abilities to analyze a child's mathematics are presented in Table 9.

Table 9

Overall Pre and Post Assessment Scores for Analyzing a Child's Mathematical Thinking

	Average of Pre-Assessment	Average of Post-Assessment	Percentage Increase	t-score	Significance Level
Analysis of Child's Mathematical Thinking	0.16379310	1.8275862	1015.7895%	46.614	α<0.0005

Table 10

Individual Pre and Post Assessment Scores for Describing a Child's Mathematical Thinking

Group	Members	Pre	Post
Group 1	A	0	2
	B	0	
Group 2	C	0.25	2
	D	0	
Group 3	E	0.25	2
	F	0	
Group 4	G	0	2
	H	0.5	
Group 5	I	0	1.5
	J	0	
Group 6	K	0.5	1.5
	L	0	
Group 7	M	0.5	2
	N	0.25	
	O	0	
Group 8	P	0.25	2
	Q	0.25	
Group 9	R	0.5	1.5
	S	0	

Group 10	T	0.25	2
	U	0	
Group 11	V	0.25	1.5
	W	0.5	
Group 12	X	0	1.5
	Y	0	
Group 13	Z	0	2
	AA	0	
Group 14	BB	0.25	2
	CC	0.25	
AVERAGE		**0.16379310**	**1.8275862**

These overall scores show a statistically significant improvement in the preservice teachers' abilities to analyze a child's mathematics using the frameworks from class. The individual results in Table 10 show that every preservice teacher increased his or her knowledge and use of the language from the frameworks to analyze a child's mathematics. They used language such as one-to-one, cardinality, disembedding, Initial Number Sequence (INS), Strategic Additive Reasoning (SAR), and other terms from the literature. In a typical response, one pair of preservice teachers wrote in their written report,

> The next problem was, "Meredith has 9 fish. Joey gives her 3 more fish. How many fish does Meredith have altogether?" This was a join problem with the result unknown. She wrote down 9+3 and got the answer 12. We asked her how she solved the problem and she told us that she counted up on her fingers and showed us how she used her fingers to count. She said, "9…10 [put up finger], 11 [put up another finger], 12 [put up another finger]." Because she used the count on strategy, we found that Sara was on the level of Initial Number Sequence. This indicated that she was definitely numerical, which was a new revelation. On all of the previous questions, she used the blocks. This was the first time she counted on. Being on this level indicates that she knew how to unitize. Sara

> understood that the number 9 represents 9 items. She didn't have to count to 9; instead, she could just start at 9.

After describing the story problem type and the method that the child used to solve the problem, this pair of preservice teachers identified this child to have her Initial Number Sequence, which is language from Steffe et. al. In addition to identifying the level, the preservice teachers described what this indicates about the child's development. They used language such as numerical and unitize, which is language Steffe et. al. use to describe the levels of whole number development.

Another pair of preservice teachers wrote,

> The question we asked was a join-initial unknown, "Bob had some toy cars. He gave 3 toy cars to his sister Katie, and then he had 4 toy cars left. How many toy cars did Bob have before he gave 3 to Katie?" Taylor used the counters and counted out 3, then separately counted out 4 more counters. Then she put both groups of counters together and counted the total, and said "8". Although Taylor miscounted, she had the right idea of portraying Gelman's model of partitioning, in which she counted all. After discovering that Meg had the right idea of how to solve this advanced problem, it helped us realize that she is at least perceptual and could possibly be figurative.

While the preservice teachers incorrectly identified the story problem as a join initial unknown it was clear that they were focusing on the child's method of solving the problem more than the context of the problem. The child's modeling strategy was to join two sets even though this problem was a separate-initial unknown. These preservice teachers were able to recognize the child's counting all strategy, which is language from CGI. They were able to connect this to Steffe's counting levels and identify the child as perceptual because she used items in her perceptual field. They recognized that the child was able to keep track of two different parts of a set, those items that have been counted and those that have not. This ability is what Gelman called partitioning. Not all preservice teachers were able to connect all frameworks, but all used language such as this to describe and analyze the child's mathematical actions. This type of language use is a powerful aspect of the project that allows the preservice teachers to connect the theories from class to practice. This quotation is also evidence of a pair of preservice teachers using the hypothetical learning trajectory because they were actively trying to assess whether a child was capable of a higher level of thinking.

Another typical example of a pair of preservice teachers analyzing a child's mathematical thinking is

> We asked Lucy, "Hannah has 4 more marbles than Max. Max has 18 marbles. How many marbles does Hannah have?" In this problem, Lucy used strategic additive reasoning. She solved by saying that 18 and 2 is 20 and two more is 22. This shows that Lucy is on the Explicitly Nested Number Sequence level because she used derived facts. We made sense of her line of reasoning here:
>
> 18 + 4
>
> = 18 + (2 + 2) decomposed 4
>
> = (18 + 2) + 2 associative property
>
> = 20 + 2 addition
>
> = 22 addition

The preservice teachers were able to precisely describe what the child did to solve the problem; they were able to name this strategy as strategic reasoning and connect this strategy to Steffe's

Explicitly Nested Number Sequence. They were also able to write a series of equations and name the mathematical properties in order to analyze the validity of the child's strategic reasoning.

Another example where mathematical properties were used in the analysis was

> We told her this story problem, "Kennedy has 3 blocks and you give her 6 blocks. How many blocks does Kennedy have altogether?" This is a join problem with the result unknown. She did not use the blocks in front of her but instead used her fingers to count, saying, "6….6, 7, 8, 9…9" Because she put one finger up with every number that she said, we know that she has the one-to-one principle. Also, since she knew that the last number name allocated to the last object gave her the answer, she had cardinality. Even though we stated the smaller number first, she chose to count on from the larger number. Because she counted on, we know that she is at least in the INS level. Because she counted on from the larger number even though it wasn't first in the story, we know that she understands the commutative property. In other words, she knows that if she started with 3 and counted up by 6, it would be the same thing as if she started with 6 and counted up by 3. This makes her on the INS+ level. .

Here, the preservice teachers discussed the child's use of the commutative property to make the problem quicker to solve. The child's use of counting on from the larger number, as opposed to counting on from the first number, is a detail that these preservice teachers brought to light and described as something significant in the child's development.

The frameworks of Steffe et. al. and Gelman provided the necessary language for the preservice teachers to analyze the child's mathematical thinking, using their previously written descriptions. Knowledge of these frameworks, along with the frameworks of CGI and Van de

Walle, gave the preservice teachers the ability to map a child's mathematical thinking, which was not possible for them before the Math 2008 course or the Interview Project.

Simon (1997) stated that "as the teacher, my perception of students' mathematical understandings is structured by my understandings of the mathematics in question. Conversely, what I observe in the students' mathematical thinking affects my understanding of the mathematical ideas involved and their interconnections" (p. 135). This understanding should be used to inform the teachers' instructional decisions as part of their hypothetical learning trajectory. The preservice teachers were able to use language from the frameworks to describe and analyze a child's mathematical actions and use this as their guide to thinking about children's development of whole numbers. The preservice teachers were clearly using these frameworks to directly inform the description and analysis that they wrote following the interview.

Preservice Teachers Listening to and Learning from Children

The second main goal of the Interview Project was to help preservice teachers learn to listen to and learn from children. Davis (1996) suggested that while you cannot observe listening occur, you can infer how a teacher is listening through how he or she responds to students. You can also infer how a teacher is listening by what he or she is listening for and what he or she chooses to ignore. Thus, I assessed this goal by looking for instances where the preservice teachers made any type of instructional decisions, whether they were planned prior to the interview, made on the spot during the interview, or discussed for use when working with the child again in the future. More specifically, I looked for instances where these instructional decisions were informed by their knowledge of the frameworks from class.

Simon (1995) suggests that the hypothetical learning trajectory should “provide the teacher with a rationale for choosing a particular instructional design” (p. 135). Because the teacher cannot know the students’ actual understanding, she must infer the students’ understanding based on his or her interpretations of students’ learning and understanding and use these inferences when choosing the type of instruction and task. This can be seen in the thinking and planning that precedes instruction as well as the spontaneous decisions that are made in response to students’ thinking.

As the teacher interacts with the students, “the teacher and students collectively constitute an experience” (Simon, 1997, p. 78). Consequently, the experience would be different than the one that the teacher anticipated. As a result of these interactions, a teacher’s ideas and knowledge are modified as he or she makes sense of what is happening or what has happened in the classroom. For my research, there was a pair of preservice teachers and one group of three working with just one child, rather than a classroom of children. However, the spirit of Simon’s mathematics teaching cycle was maintained because the preservice teachers were using their knowledge from personal experiences and the experiences of our class to anticipate the child’s hypothetical learning trajectory, modifying their knowledge through their interaction with the child and through their analyses, and then reflecting on the hypothetical repetition of the cycle.

Simon’s hypothetical learning trajectory, as mentioned in the literature review, has three main components: the learning goal that defines the direction, the learning activities, and the hypothetical learning process. The hypothetical learning trajectory is intended to “underscore the importance of having a goal and rationale for teaching decisions and the hypothetical nature of such thinking” (Simon, 1995, p. 136). Two of the essential elements of teachers’ knowledge that inform their hypothetical learning trajectory are a teacher’s model of students’ knowledge and a teacher’s knowledge of student learning of particular content. I wanted to see if the knowledge gained through the Math 2008 course informed the preservice teachers’ instructional decisions.

In planning for their interview with a child, the preservice teachers set goals and created activities for assessing a child’s whole number development, based on their hypothesis about how the child might respond to the different story problem structures. The preservice teachers used the knowledge gained in class (through lectures, activities, and watching videos of children) as a guide for their interview with the child. I can only assume that the knowledge gained in class was the basis of the preservice teachers’ knowledge on children’s whole number development because I did not document other experiences they may have had with children’s mathematics outside of my class. Inevitably, the interview did not go as planned and spontaneous, on the spot, instructional decisions were made. Through their interactions with just one child, the preservice teachers began to modify their knowledge of children’s whole number development. Using their experience of interviewing a child, the preservice teachers were asked to plan appropriate tasks for working with the child again. These tasks were to be informed by their interactions with the child and their analysis of the child’s mathematical thinking. Unfortunately, these future interactions were only hypothetical, meaning that the preservice teachers only completed one full cycle of Simon’s mathematics teaching cycle.

In assessing the second main goal of the Interview Project, to learn to listen to and learn from children, I used Tasks 4 and 5 from the pre-Interview Project assessments. For Task 4, the preservice teachers watched 4 videos and were asked to respond to the question, “What would you do next? What task would you give this child now that you’ve seen this clip?” Their responses were scored according to the rubric in Appendix D. Each of the four responses were given a score of 0, 1, or 2, and these scores were averaged together to give a final score for Task 4. The results are presented in Table 11.

Table 11

Results of Task 4: Instructional Decisions

Score on Task 4	Frequency	Relative Frequency
0	4	13.8%
0.25	6	20.7%
0.5	9	31.0%
0.75	8	27.6%
1	2	6.9%
1.25	0	0%
1.5	0	0%
1.75	0	0%
2	0	0%
Total	29	100%

The overall average of these scores was a 0.48. The most common reason for a score of 0 on Task 4 was the lack of detail. None of the preservice teachers wrote an example of a question that s/he would ask. Instead, they wrote very generalized responses. For example, nearly all of the preservice teachers suggested that when the child correctly answered a story that involved addition computation they would give the child a question that involved subtraction next. When the child correctly answered a story that involved subtraction computation, the preservice teachers suggested giving him a multiplication problem next. Most likely, the preservice teachers learned the operations of whole numbers in this particular order, addition, subtraction, multiplication, then division, which may suggest that subtraction is more difficult than addition and multiplication is more difficult than subtraction. These experiences may justify why the preservice teachers responded in this way.

Task 5 was also used to assess the preservice teachers' abilities to make appropriate instructional decisions. For this task, there were two parts. The first part asked the preservice teachers to read five pairs of additive structured story problems and choose which one is more difficult for a child to answer. They could also respond by saying that they were equally difficult. For the second part of Task 5, the preservice teachers were given a Join Result Unknown story problem that could be solved by the simple addition computation of 8+5, and they were asked to write another story problem that could also be solved by computing 8+5 but was more difficult than the Join Result Unknown story that was given. This same process was repeated but with a Separate Result Unknown story problem given. I scored their responses according to the rubric in Appendix D with a score of 0, 1, or 2 on part 1 and on part 2. These scores were averaged together to give a final score for the pre-assessment score of the preservice teacher's ability to learn to listen to children and make appropriate instructional decisions. The results are in Table 12.

Table 12

Results of Task 5: Relative Problem Difficulty

Score on Task 5	Frequency	Relative Frequency
0	1	3.4%
0.5	14	48.3%
1	10	34.5%
1.5	4	13.8%
2	0	0%
Total	29	100%

The overall average on Task 5 was 0.79. When I further analyzed the results of part 1, I found that two of the pairs of story problems were the most common reason for lower scores. The first pair of story problems was: a) Connie had some marbles. She gave 5 to Juan. Now Connie only has 8 marbles. How many marbles did Connie start with? b) Connie has 13 marbles. She gave 5 to Juan. How many marbles does Connie have now? Eight out of the 29 preservice teachers responded that b) was more difficult. Although I did not ask them to give reasoning to their responses, I speculate that the preservice teachers recognized that b) could be solved by subtraction, while a) could be solved by addition and ranked subtraction as more difficult than addition rather than attending to the fact that a) is a start unknown situation, whereas b) is a result unknown situation. The other pair of story problems that was most commonly missed was: a) Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with? b) Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left? For this pair, eight of the preservice teachers ranked these as equal in difficulty. Again, I did not ask them to justify their responses, but I speculate that they ranked them this way because both of the problems could be solved by 13-5. For both of these pairs of story problems, the preservice teachers did not take the structure of the story problem into consideration; they only considered the computation that is involved in finding the solution. The authors of CGI (1999) suggest that Join and Separate story problems with a result unknown are easier to solve than ones with an initial unknown or a change unknown, no matter what computation is involved in finding the solution. For the second part of Task 5, there were two main problems that arose that resulted in low scores. One of those was that the preservice teachers wrote a question of the same difficulty as the one given instead of writing a story that was more difficult than the one given. For example, when the Join Result Unknown story problem was given, the preservice teacher wrote a story with a different context but it was still a Join Result Unknown story problem. The other main problem was their lack of following directions or possibly misinterpreting the directions. Fifteen of the preservice teachers wrote a story that could not be solved by the given computation. In response to the Join Result Unknown problem that could be solved by 8+5, one preservice teacher wrote the story problem, "John has 8 marbles. Connie gave him some more marbles, and now he has 13. How many marbles did Connie give him?" This is a Join Change Unknown problem, which, according to CGI (1999), is typically more difficult to solve than a Join Result Unknown problem, but this problem is solved by the computation 13-5, not 8+5. It is possible that the preservice teacher simply did not follow the directions given or s/he may have believed that 8+5 and 13-5 are the same computation, given that they are form the same fact family.

For the post-Interview Project assessment of learning to listen to children and make appropriate instructional decisions, I used the written reports, the observations, and the interviews with the preservice teachers directly following their interview with the child. For their written report, the preservice teachers were asked to discuss any instructional decisions that were made, whether they were planned decisions, on the spot decisions, or decisions for future work with the child. In addition, I took field notes while observing seven of the pairs of preservice teachers interview a child and asked them questions during their session interviews about the instructional decisions that I observed. I scored their responses from 0 to 2 using the rubric in Appendix A; these results are presented in Table 13.

Table 13

Results of Listening and Making Instructional Decisions (Post-Interview Project)

Score for Listening	Frequency	Relative Frequency
0	0	0%
0.5	0	0%
1	0	0%
1.5	19	65.5%
2	10	34.5%
Total	29	100%

The scores on Tasks 4 and 5 were averaged together to give an overall pre-Interview Project score for the preservice teachers' abilities to listen to a child and make appropriate instructional decisions. The results of the pre-Interview Project assessment and the post-Interview Project assessment for the preservice teachers' abilities to listen to a child and make appropriate instructional decisions are presented in Table 14.

Table 14

Overall Pre and Post Assessment Scores for Making Instructional Decisions

	Average of Pre-Assessment	Average of Post-Assessment	Percentage Increase	t-score	Significance Level
Making Instructional Decisions	0.67672414	1.6724138	147.134%	24.909	$\alpha<0.0005$

The preservice teachers showed a significant improvement in their ability to make instructional decisions based on their knowledge of the research. The results in Table 15 show that each preservice teacher showed that he or she had advanced in his or her ability to make appropriate instructional decisions.

Table 15

Individual Pre and Post Assessment Scores for Making Instructional Decisions

Group	Members	Pre	Post
Group 1	A	0.375	2
	B	0.625	
Group 2	C	0.625	2
	D	0.75	
Group 3	E	0.875	1.5
	F	0.875	
Group 4	G	0.75	2
	H	0.375	
Group 5	I	0.75	1.5
	J	0.875	
Group 6	K	1	1.5
	L	0.625	
Group 7	M	0.875	1.5
	N	0.5	
	O	0.625	
Group 8	P	0.625	1.5
	Q	0.5	
Group 9	R	0.875	1.5
	S	0.75	
Group 10	T	0.875	1.5
	U	0	
Group 11	V	0.75	1.5
	W	0.75	
Group 12	X	0.625	1.5
	Y	0.875	
Group 13	Z	0.5	2
	AA	0.375	
Group 14	BB	0.875	2
	CC	0.75	
AVERAGE		**0.67672414**	**1.6724138**

Planned Instructional Decisions

For most of these preservice teachers, this was the first time that they worked with a child to learn to listen to and be responsive to the child's mathematics. Thus, their planning could not be based on prior experiences. They relied on the knowledge that they had gained in class to inform their planning. In class, the preservice teachers learned about early number concepts, the structures and methods of solving additive story problems, and how to analyze a child's level of whole number development. This information was presented to them through lectures, class activities, and watching videos of children. The videos were intended to give the preservice teachers a way of "interacting" with a child that was easily accessible to us in class and to show them some examples of how children respond to additive structured story problems during an interview. Collectively, the lectures, activities, and videos gave the preservice teachers knowledge to prepare for their interview with a child. When interviewing a pair of preservice teachers immediately following their interview, I asked them, "Can you tell me a little bit about how you prepared for the interview?" One of the preservice teachers responded by saying,

> Well, we wanted to ask him some simple questions first…you know, to build his confidence. We wanted him to feel comfortable and not be scared or intimidated. So, we asked him a join problem. I don't really remember exactly what it was. Let's see…[looks at paper]. Oh! It was a join problem-result unknown…6 pieces of bubble gum then got 4 more. We gave a join-result unknown problem because, like we talked about in class, the result unknowns are easier than the start or change unknowns, you know? We did do some of those [start and change unknown problems] later, but…We also started with small numbers to see if she could do those, then we went to bigger numbers.

This preservice teacher thought about the potential awkwardness of an interview and chose to start the interview with some easier questions to help build the child's self-confidence and hopefully dissolve any discomfort that the child may have felt. The preservice teacher also chose a particular order for her questions, using her knowledge of the additive structured story problems and each type's difficulty level, relative to the others. In addition, she discussed her decision to begin the interview with smaller numbers. Because she said, "to see if she could do those", it seems that the preservice teacher was also anticipating two possible paths of the interview as well. If the child could easily work with small numbers, then she would begin to introduce larger numbers. If the child could not easily work with small numbers, then she would continue working with small numbers. The preservice teacher was planning for spontaneous changes that might need to be made during the interview.

Only two pairs of preservice teachers wrote about their planning process in their written reports. I interviewed seven pairs of preservice teachers immediately following their interviews, asking them to describe how they planned for the interviews, and all of them showed evidence of preparing for their interviews using the frameworks presented in class as a guide. Although it is likely that that all of the groups of preservice teachers did prepare for their interview using the frameworks from class, there is no evidence that the remaining five groups did so.

Spontaneous Instructional Decisions

As Simon (1995) suggested, the experience between the teacher and student, by the nature of its social constitution, is different from the one predicted by the teacher. Consequently, the preservice teachers had to make spontaneous instructional decisions in response to the child's mathematical actions. Some of the preservice teachers changed the numbers in their problem; some changed the order of the problems; some skipped problems, but all of the preservice teachers showed evidence in their written reports of making at least one spontaneous instructional decision. One pair of preservice teachers told me during their session interview,

> When we asked him the join-change unknown problem, he didn't seem to understand how to do it. We had to help him using the blocks to explain. Then, the same thing

> happened with the join-start unknown problem. He really struggled with those…so we went back to result unknown problems. We did more join and separate with the result unknown….I used bigger numbers though, when I went back to the result unknowns.

This pair of preservice teachers showed that they were listening to the child, noticing his struggle, and they made the spontaneous decision to change the order of their prepared questions. They had prepared their questions in an order where they got progressively more difficult, but when they realized that the child had a difficult time with the change unknown and start unknown problems, they quickly abandoned their plan and made the decision to go back to the questions that were in his zone of potential development.

Another pair of preservice teachers made a spontaneous decision during their interview to change the numbers to larger numbers in order to establish the appropriate level of the child's whole number development. They wrote,

> Since John used the count on method many times, we knew he was on the INS level. So, in order to see if he was on the ENS level, we asked him the question, "Carl has 25 jellybeans. Tyler gives him 16 more. Now how many jellybeans does Carl have?" Originally, this problem had smaller numbers in it, but we changed them to higher numbers because the higher numbers may be difficult for him to count on using his fingers. The larger numbers would encourage him to find other strategies to solve the problem.

This statement suggests that this pair of preservice teachers was making spontaneous instructional decisions based on their in-the-moment analysis of the child's mathematics, and this analysis was informed by the frameworks from class. However, in order for this analysis to occur they had to use questions to probe the child's mathematics. It was clear that these preservice teachers were intentional with their choice of the size of the numbers. These preservice teachers were listening for a particular strategy, specifically counting on or strategic additive reasoning. This is an example of listening evaluatively (Davis, 1996) because they had a hypothesis and used a specific task to test that hypothesis. While Davis (1996) contends that there is no value in asking questions when we already anticipate a response, I feel that this excerpt shows the value in evaluative listening. The structure of the problem and the size of the numbers were specifically chosen in anticipation of a particular response. Listening for this response could help to determine whether or not the child could use strategic reasoning, which would help to determine the child's level of whole number development. Listening for a particular response could also help the preservice teachers in making further instructional decisions, such as which question to ask next. However, I agree with Davis that teachers should not limit themselves to listening only evaluatively.

Besides probing questions, which are information seeking, the preservice teachers also asked questions intending to elicit a particular response. The case below illustrates the preservice teachers' use of prompting questions. Rather than taking the child's word that a particular task was too difficult or just giving the child the correct answer, they used prompting questions to help the child successfully arrive at the answer.

> We posed the question 10 + 20 = ? on a piece of paper. She looked at it for a minute and said that it was too hard. Instead of moving onto another problem, we strategized a little bit and came up with an alternative route. We put one set of ten unifix cubes on the table and asked her how many were there. She replied instantly with, "10." Then, we said, "This is a little bit of a different question so think hard about this one. How many groups of 10 are on the table?" She thought for a moment, and then responded with, "there is 1

> group of 10!" We went on to put 3 rows of 10 unifix cubes on the table and posed the question to her again, "How many set of 10 are on the table?" Again, she understood exactly what we were asking her. When we asked how she knew all of this, she responded with an extremely intelligent answer that surprised us both! She help up her fingers and explained that 10 is like 1 just with a 0 on the end. So, 20 is like 2, 30 is like 3, and so on. After hearing her explain her strategy, we posed the 10+20=? again. She thought for a minute, and came up with 30. When asked how she got that answer, she held up her fingers and said, "Cause 20 is 2 fingers and 10 is 1 finger and 2+1=3, so the answer is 30!"

The preservice teachers' moment of surprise indicates that they were not listening for something in particular. Rather they were listening to the child and interpreting her response based on their own knowledge of the discipline. By using the Unifix cubes, they prompted the child to be able to think in groups of ten but did not anticipate her connection to the symbols. Thus, these preservice teachers were listening interpretively. By their questioning and willingness to be surprised, these preservice teachers were displaying what Dewey (Zeichner, 2013) called openmindedness. They were able to listen to the child's mathematics and make sense of it even though it was different than how they thought about the task.

Instructional Decisions for Future Work with the Child

After their interactions with the child, Simon (1995) suggested that "the teacher's assessment of student thinking can bring about adaptations in the teacher's knowledge that, in turn, leads to a new or modified hypothetical learning trajectory" (p. 137). To assess whether or not the preservice teachers used their interviews to inform their modified hypothetical learning trajectories, they were asked to plan for a future interview with the same child. On the project description the preservice teachers were asked to respond to the question, "if you could continue to work with this child, what concepts or kinds of problems do you think would be productive work for her or him?" All of the pairs were able to thoughtfully respond to this question. In searching for evidence of where the preservice teachers were making instructional decisions for the imagined future session with the child, I found that 6 out of the 14 groups were vague and gave responses that were very explorational. In this excerpt, it seems clear that the preservice teachers were searching for what might be on the cusp of what is possible for the child. However, they did not pinpoint any specific concept or relate their instructional decision back to the framework.

> If we were to continue working with Jeffrey, we would work on multiplication problems. He did not know, after reading the problem, whether or not it was appropriate to multiply. This is his ZPD because he struggled with these problems when he worked on it independently, but was able to solve it when we helped him.

The other 8 groups (out of 14) were able to specifically address the levels in the framework and suggest directions for the child's mathematics related to particular types of word problems. One pair wrote,

> If we were given the chance to work with Bailey again, we would encourage her not to use the blocks as much. She used the blocks to help her answer every problem. We don't know if she used the blocks because they were sitting in front of her and she felt that she had to use them, or maybe she used them because she actually needed them. So, next time, we would give her more join and separate problems with the result unknown, similar to the ones we gave her this time (#1-6). We know she can solve these with the blocks, but we would want to see if she can solve them without the blocks. She may use

> her fingers in place of the blocks. That way, we would be able to tell if she is a figurative counter, rather than just a perceptual counter.

These preservice teachers used their analysis that the child was a perceptual counter to think about pushing the child to the figurative level by getting the child to become less reliant on physical materials. They also named specific problems that they would give to the child in order to accomplish this. This analysis was not done during the interview; it was done after the interview was over, when the preservice teachers reflected on their experience. During my interview with this pair of preservice teachers, one of them stated, "We didn't notice that she used the blocks on every problem until we got home and looked back over our notes and listened to our [audio] tape." This shows that the preservice teachers' abilities to listen to the child's responses and analyze them did not stop at the end of the interview; it was ongoing. This reflective teaching was helpful in informing their future instruction with the child.

Preservice Teachers' Evolving Definitions of Teaching and Learning Mathematics

For my research, I did not set out to study the preservice teachers' beliefs about the teaching and learning of mathematics. But from their written reports and their session interviews with me, I noticed that the preservice teachers seemed to be rethinking their definitions of teaching and learning mathematics. They were troubling the idea of teaching as telling and moving towards the notion of teaching as posing appropriate tasks. The preservice teachers were also becoming more aware of the different ways that children think and the importance of being open to these many ways.

Unfortunately, I did not do a pre-assessment or a post-assessment of the preservice teachers' beliefs of the teaching and learning of mathematics because this was not part of my research goal. Thus, I do not have a way of measuring the possible change in their beliefs. The only evidence that I have are the quotations from the preservice teachers' written reports and their interviews with me. These quotations are evidence of the evolving definitions of teaching and learning mathematics for at least some of the preservice teachers.

Rethinking the Teaching of Mathematics

One of the ways that the preservice teachers were beginning to change their conceptions of teaching mathematics was that they were abandoning their thinking that teaching mathematics was telling a student how to act. There was evidence from 5 of the pairs of preservice teachers that showed this change. One example was from a written report, where one pair of preservice teachers wrote,

> This interview was incredibly informative for our future as a math teacher. It made us realize that skillful questioning is imperative in your instructional decisions. You can't just tell the student how to do the problem because they might not think the same way that you do. You have to let them use their own thinking to do it their own way. But, as the teacher, you have to know what type of questions to pose.

This pair of preservice teachers saw the value of questioning as a tool for teachers. They also highlighted the direct relationship between the teachers' questioning and the students' learning. In my interview with another pair of preservice teachers, one of them stated,

> When I was in elementary school, my teacher just told us how to do problems step-by-step and then we practiced that procedure over and over again. I don't want to teach math like that…I'm scared that I might fall into that because that's all I know, but I think that teaching math should be more about, you know, giving the students opportunities to learn things in their own way. The teacher has to know what kind of tasks to give them to make those opportunities happen.

This preservice teacher was beginning to reconsider her definition of teaching as telling to teaching as giving learning opportunities through appropriate tasks. She also pointed out how difficult it is to teach in a different way than you were taught. Although she does not want to be a teacher who teaches by telling, she admitted that she may be drawn back to this type of teaching. This is consistent with Simon's (1997) claim that "many teachers have developed their models of teaching in the context of thousands of hours as students in traditional classrooms…[which is] difficult to change" (p. 57).

Another preservice teacher showed great enthusiasm over her new view of teaching mathematics during her interview with me. She said,

> I'm so excited to teach with the Common Core if this is what it's going to be like. I never thought about teaching math this way. I always thought it was just the teacher telling the kids the steps and going over examples, and…it's like a robot, you just repeat the same steps every day. But, now I feel like I can teach math so that it's exciting, but also so that the kids learn it better. I mean, if kids are able to use their own thinking, their own logic to solve problems, then it makes so much more sense to them, and math is so much easier if it makes sense to you.

Through other class readings and discussions, this preservice teacher formed an opinion of what teaching through the Common Core would look like. Her vision of teaching through the Common Core was not the traditional one that she was exposed to in school, where the teacher teaches by telling, but one of reform teaching. This comment also shows the preservice teacher's belief that the way in which the mathematics is taught directly corresponds to the mathematical understandings of the students.

The previous two quotations by students emphasized the structure that characterizes most mathematics classrooms throughout the country. NCTM (1989b) described this classroom in the following way:

> Mathematics instruction begins with checking the previous day's assignment. Troublesome problems are worked by the teacher or a student. Then the teacher briefly explains or demonstrates the next piece of material, and the remainder of the time is spent at seat-work on the next assignment. (NCTM, 1989b, p. 2)

This routine is prevalent in all subject areas, but perhaps it is most evident in mathematics instruction. It has been characterized by some educators (Barnes & Shemilt, 1974; NRC, 1989) as the transmission model, where the mathematics is broken down into a body of facts and techniques that can be transmitted from the teacher to the student. Borasi (2005) claims that

> Good mathematics should be conceived not as the clear and efficient transmission of established mathematical results but as the creation of a community of learners engaged collaboratively in the construction of mathematical knowledge…This, in turn, will involve the development of "rich" classroom tasks. (Borasi, 2005, p. 181)

The quotations from several preservice teachers show that they were beginning to change their view of teaching through transmission to teaching through the development of rich tasks.

Incorporating Theory into Practice

It is likely that most, if not all, of the preservice teachers in this study had never considered incorporating theory into their practice of teaching mathematics before this course. After they completed the Interview Project, 8 out of the 14 groups of preservice teachers referred to their interview with the child as a way to see value in using theory in their practice. In their reflection on the project, one pair of preservice teachers wrote in their written report, "We really liked doing this interview because it was really interesting to see what we had learned about in class used in real life." Another preservice teacher had a similar response during her interview with me. She said,

> When we learned the levels, I just kind of memorized them for the test, but then, when I actually had to figure out what level Ta'khia was on, it really made me see how beneficial they are. It doesn't mean as much until you're put in that position yourself.

These preservice teachers emphasized the importance of incorporating the theory that was taught in class into a real life experience with a child. Without this experience, the theory would not have had meaning to them.

Doyle (1990) identifies two types of knowledge. "Propositional knowledge" is the knowledge of research and theory. "Craft knowledge" is the knowledge of the skills of teaching. Doyle claims that these two types of knowledge learned separately are insufficient. He also asserts that preservice teacher education should include opportunities for preservice teachers to develop these two types of knowledge simultaneously. The preservice teachers in my study seemed to be doing just that. One preservice teacher said during her interview with me,

> It was helpful to have all of the information from class, like the different ways they might solve the problem and the Steffe levels. We were able to use it to watch him and understand him and the process of what he's doing while he's solving the problem. And since we knew the different strategies, we recognized them right away and I felt like I knew what level he was on before we even left the interview….It was cool to see all the stuff we talked about in class actually happening. I'm not sure what we would've done without knowing all that stuff.

This preservice teacher used the knowledge of the frameworks from class to inform her interview with the child. She clearly appreciated having the knowledge of the frameworks, implying that the interview would not have been as successful without it. The unique opportunity to interview a child gave her the chance to personalize the theory from class through the Interview Project. Thus, she was incorporating her "propositional knowledge" into her "craft knowledge".

Rethinking the Learning of Mathematics

In the book *Making Sense* (Hiebert et al., 1997), the authors describe four features of a productive mathematics classroom. One of those features is:

> Students have autonomy with respect to the methods used to solve problems. Students must respect the need for everyone to understand their own methods and must recognize that there are often a variety of methods that will lead to a solution.

In their written reports and their interviews with me, the preservice teachers showed a newfound respect for the variety of methods that students use to solve a problem. One pair of preservice teachers wrote,

> We recognize that our students will all have different ways of coming to a solution to a problem, and we think it is important to let them come to that conclusion on their own instead of always making them use the standard algorithm.

When I interviewed this same pair of preservice teachers, one of them stated,

> Before this class, I think I would've just expected the kid to use the standard algorithm to do everything, and if they didn't, I would've thought, 'Oh, they don't know what to do. They should've learned this in school…how to use the algorithm.'

This pair of preservice teachers believed that there was one correct way to solve the problems, which was to use the standard algorithm. But, after their interview with the child, this belief was challenged. They saw the importance of letting students use their own personal strategies, even if it is not the standard procedure taught in most schools. Another pair of preservice teachers wrote,

> We saw that there are many ways to get an answer to a certain problem. Children are more capable than they are given credit for. Teachers need to allow children to use their own mathematical thinking and intuition to solve problems instead of forcing them to use one particular method.

Among the 14 written reports, 12 of them contained comments similar to the ones above. These comments show that the preservice teachers were reconsidering the way that children learn mathematics. Instead of seeing the learner as the passive receiver of the teacher's knowledge, the preservice teachers were beginning to see perceive the learner as "already possessing systematic and relevant knowledge to build off of" (Barnes, 1995, p. 147). Carpenter et al. claim that

> Young children are naturally curious and have a desire to make sense of their world…Until recently, we have not clearly recognized how much young children understand about basic number ideas, and instruction in early mathematics too often has not capitalized on their rich store of informal knowledge…Children may actually understand the concepts we are trying to teach but be unable to make send of the specific procedures we are asking them to use. Children do not always think about mathematics in the same way that adults do (Carpenter, 1999, p. xiii-xiv).

Through their comments in their written reports and their interviews with me, it seemed that the preservice teachers experienced powerful changes in their conceptions of what it means to teach and to learn mathematics. Through constructing children's mathematics, they changed their own constructions of mathematics. They moved away from a view of teaching as telling and toward a view of teaching as posing appropriate tasks. They saw the benefit of incorporating theory into their practice. They also rethought their views of learning mathematics. They began to value reasoning strategically and using intuition to solve problems, rather than relying on traditional algorithms. One pair of preservice teachers wrote in their written report, “Overall, the main thing we got out of this project is a rejuvenated mindset on teaching math and a new appreciation for how children learn math.”

CHAPTER 5

SUMMARY AND CONCLUSIONS

Steffe and D'Ambrosio (1995) claimed that for a mathematics teacher to operate under a contructivist epistemology, s/he must be a teacher "who studies the mathematical constructions of students and who interacts with students in a learning space whose design is based, at least in part, on a working knowledge of students mathematics" (p. 148). Steffe and Wiegel (1996) emphasized that one of the most basic aspects of a contructivist teacher includes close listening to students. However, they warn that close listening is not enough and that teachers need to learn to act on their listening in order to reveal and extend the mathematics of students.

Summary

The purpose of my study was to investigate preservice teachers' abilities to describe and analyze a child's mathematics using the frameworks presented to them in the Math 2008 course. I was also interested in whether or not the preservice teachers could learn to listen to and learn from the child. The 29 preservice teachers that I studied were enrolled in my Math 2008 course, a course on Numbers and Operations for Early Childhood majors, in the spring semester of 2014. In this course, I presented the frameworks of Gelman (1986) (describes the counting principles), Cognitively Guided Instruction (1999) (classifies story problems and childrens' solution methods), Van de Walle (2010) (describes more solution methods that children use), and Steffe et. al. (1982) (describes levels of whole number development). After these frameworks were presented, the preservice teachers participated in a project where they worked in pairs to interview a child. I call this project the Interview Project. For their interview, the preservice teachers ask the child to solve additive structured story problems. The preservice teachers had to plan their interview, carry it out, and then write a report that described and analyzed the child's mathematical actions during their interview. They also wrote a reflection of how this interview has informed their mathematical thinking and their teaching. They are asked to think about the instructional decisions that they made during the interview and what instructional decisions they would make if they worked with the child again in the future. My data included pre-Interview Project assessments and post-Interview Project assessments. In the first few days of the Math 2008 course, I gave the students several tasks (Appendix C) to pre-assess their abilities to describe a child's mathematics, to analyze a child's mathematics, and to make appropriate instructional decisions through listening to the child. The post-Interview Project assessments included the preservice teachers' written reports, my field notes from the observations, and transcripts of the session and final interviews. To analyze the data, I created rubrics for each assessment and compared the pre-Interview Project data to the post-Interview project data in order to see the changes

I found that, overall, there was a statistically significant improvement in the preservice teachers' abilities to describe a child's mathematics, to analyze a child's mathematics, and to listen to a child in order to make appropriate instructional decisions. In their written reports, all of the preservice teachers used language from the frameworks to describe a child's mathematics. They used language from Cognitively Guided Instruction (CGI) such as join-result unknown, direct modelling, and count on. They also used language such as making a double from Van de Walle to describe the child's mathematics during their interview. The preservice teachers also used their descriptions of their children's mathematics in order to provide evidence for their analyses. All of the preservice teachers also showed evidence in their written reports of being able to use the frameworks to analyze the child's mathematics, using language from Gelman, such as partitioning and cardinality, or language from Steffe, such as perceptual, Initial Number Sequence, and Strategic Additive Reasoning.

I also found that the preservice teachers were learning to listen to children and use their listening to inform their instructional decisions, including on-the-spot decisions that were made during their interview with the child and future decisions, such as the tasks and problems they designed to give to the child in a hypothetical future meeting. In their written reports and during their session interviews with me, all of the preservice teachers discussed at least one spontaneous change that they made during their interview. For example, they mentioned skipping a task that they had planned, reordering their tasks, or using larger or smaller numbers than the ones that they had planned. These instructional decisions are evidence that the preservice teachers were listening to the child and using what they heard to inform their instruction. In addition, all of the preservice teachers were able to use their description and analysis to plan for a hypothetical future meeting with the child. Although all of the preservice teachers addressed this future meeting and gave it a lot of thought, only some groups were able to name a specific task and relate it to the frameworks. Overall, the instructional decisions made by the preservice teachers resembled the Mathematics Teaching Cycle by Simon (1995), even though Simon's Mathematics Teaching Cycle was concerned with with inservice teachers in a classroom environment. They used their knowledge from personal experiences and the experiences of our class to anticipate and plan for the child's hypothetical learning trajectory, modifying their knowledge through their interaction with the child and through their analyses, and then reflecting on the hypothetical repetition of the cycle.

These instructional decisions were evidence of the preservice teachers' listening to the child. It was not the questions that were asked, but the way in which the preservice teacher used the questions to attend to the child's mathematics, as D'Ambrosio (2004) suggested. In her study where a preservice teacher responded to a child, D'Ambrosio contended that it is likely that a preservice teacher can only listen to a child evaluatively. She claimed that listening interpretively about less than one full course is unlikely and listening hermeneutically is even more unlikely, if possible at all. My data support the claim that hermeneutic listening is unlikely, but my data do not support the claim that preservice teachers cannot listen interpretively about less than one full course. I found evidence that preservice teachers listened to children both evaluatively and interpretively, but I found no evidence of hermeneutic listening. However, the preservice teachers in my study were able to use their listening to describe and analyze the child's mathematics and use this to inform their instructional decisions.

There were other significant findings that were not directly related to my research questions. From their written reports and their session interviews with me, I noticed that the preservice teachers were rethinking their definitions of teaching and learning mathematics. They were moving away from defining teaching as telling. Instead, they seemed to be defining teaching as listening to children and giving them learning opportunities by questioning and posing appropriate tasks. The preservice teachers were also beginning to find value in using research and theory to inform their actions as teachers. In addition, the preservice teachers were redefining what it means to learn mathematics. They had a newfound appreciation for the many different strategies that children use to solve mathematical problems and began to find value in a child's intuition rather than a reliance on the traditional algorithms.

Conclusions

I set out to study the extent to which the Interview Project increased preservice teachers' knowledge of the research and frameworks and their ability to apply them in their interactions with a child. The preservice teachers in this study showed a statistically significant improvement in their abilities to both describe and analyze a child's mathematics after they had been exposed to the frameworks in class and completed the Interview Project. I do not claim that these changes were due solely to the Interview Project. The Interview Project was a culminating project that, to be effective, required the preservice teachers to learn the necessary language from the frameworks before conducting their interview with a child. Thus, the several weeks spent in class going over these frameworks were an essential factor in the changes that occurred. The Interview Project, though, gave the preservice teachers the authentic experience of working with a child that was necessary for them to apply the frameworks that they learned in class. I claim that the observed changes were due to the experiences and lessons that the preservice teachers had in the Math 2008 class leading up to and including the Interview Project.

I also set out to determine the extent to which the Interview Project helps preservice teachers learn to listen to and learn from children. The preservice teachers showed a statistically significant improvement in their abilities to make instructional decisions based on their ability to listen. I concluded that the preservice teachers were listening both evaluatively and interpretively, but I saw no evidence of hermeneutic listening (Davis, 1996). These findings support D'Ambrosio's (2004) claims that preservice teachers are not likely to engage in hermeneutic listening. However, I found that there is value in evaluative listening, unlike D'Ambrosio's (2004) suggestion that evaluative listening is "not sufficient to help the teacher build a model of the child's mathematics" (p. 139). While interviewing a child, one pair of preservice teachers hypothesized that the child may be able to use strategic reasoning even though there was no evidence of it in the child's previous responses. To test their hypothesis, they asked the child to solve a problem with larger numbers and listened for the response that they had anticipated, strategic reasoning. Listening for this particular strategy helped them to determine the child's level of whole number development and to make further instructional decisions, such as which question to ask next. Although I believe there is value in evaluative listening, I support Davis's (1996) claim that listening should not be limited to evaluative listening.

Perhaps the most significant finding in my study is that through the experience of the Interview Project, the preservice teachers not only learned the frameworks and how to apply them, but they were able to redefine their notion of what it means to teach and learn mathematics. In their research, the Cognitively Guided Instruction group also found that "learning to understand the development of children's mathematical thinking leads to fundamental changes in teachers' beliefs" (Carpenter et al., 1999, p. 105). At the end of the project, the preservice teachers in my study were viewing the teaching and learning of mathematics in a way that is more consistent with what the National Council of Teachers of Mathematics (NCTM, 2000) endorses as reform oriented. This development is an example of a teacher in the beginning of transition. Simon et al. define teachers in transition as "teachers whose practices have changed and are changing as a result of participation in current mathematics education reforms" (Simon et al., 2000, p. 579). They suggest that for this transition to happen in preservice teacher education involves a qualitative reorganization of the preservice teachers' understandings. Goldsmith and Shifter (1997) describe a qualitative reorganization of understanding as the "learner's construction of increasingly complex cognitive structures resulting in the capacity for more complex thought and action" (p. 23). This would allow the preservice teachers to reorganize their thoughts in ways that make them a better mathematician and allow them to redefine teaching mathematics from one of transferring knowledge to one of giving students opportunities to build off of their current understandings.

Implications for Teacher Education

The Interview Project as a Way of Moving Toward Reform

NCTM has, since the early 1980s, been advocating for reform in mathematics education. One of their suggestions on being a successful teacher in this era of reform is discussed in the assessment principle. The assessment principle asserted that

> assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done *to* students; rather, it should also be done *for* students, to guide and enhance their learning (NCTM, 2000, p. 22).

Additionally, NCTM advised that one's beliefs about the teaching and learning of mathematics can be changed during preservice teacher education. Several researchers (Crockett, 2002; Fennema, 1993, 1996; Vacc, 1999) found that giving teachers the opportunity to analyze students' thinking and make instructional decisions is one of the most powerful ways to change a teacher's beliefs to ones that are more consistent with the reform movement suggested by NCTM's *Principles and Standards* (2000) and *Principles to Actions* (2014). The Interview Project is one such opportunity for preservice teachers. In her final interview with me, one preservice teacher said,

> Now, I feel like I know more about what it means to do constructivist teaching. We have talked about it in this class and in other classes, but I struggled to understand what it would look like in a real classroom. This is the first time I understood it…I would have never thought about sitting down with my students and doing something like this, trying to analyze and understand their thinking. I probably would've just taught like I was taught, writing on the board and giving worksheets and homework.

For this preservice teacher, the Interview Project was an essential component that began her transition of beliefs from traditional teaching to reform teaching. She pointed out her struggle to grasp the constructivist orientation of teaching, as did several other preservice teachers in their written reports. This example suggests that mathematics education reform is not going to happen naturally or easily. Thus, teachers and preservice teachers need opportunities to listen to children and make sense of their mathematics.

Early Field Experience

Preservice teacher education programs have implemented a variety of models to help with the task of analyzing student thinking, including field experiences. Field experiences have repeatedly been identified as the most significant part of teacher preparation programs (Carnegie Forum on Education and the Economy, 1986; Holmes Group, 1986; Knowles & Cole, 1996; National Commission on Excellence in Education, 1983; Zeichner, 1992). In order to see the big picture of teaching and learning, preservice teachers need hands-on experiences with children and to be allowed to interpret that experience with guidance from an expert (Wilson et al., 2001). Typically, student teaching occurs in the last semester of a teacher education program, but "early field experiences" have become increasingly popular. Wasburn-Moses et al. (2012) define an "early field experience" as "a field experience that occurs within the first two years of traditional preparation programs" (p. 8). The early field experience was introduced in the 1980s as a response to the need to strengthen partnerships between K-12 and higher education (Huling, 1998). In more recent years, the push toward reform education has emphasized the importance of field experiences, including early field experiences, in teacher education programs. Philipp et al. (2007) conducted a study of 159 preservice teachers who were engaged in an early field experience where they had opportunities to analyze children's mathematics. The feedback that they received from the preservice teachers "conveyed the sense that early field experiences are magical when preservice teachers and children are brought together" (Philipp et al., 2007, p. 470).

Although there has been some recent research on early field experiences that reported positive perceptions among preservice teachers (Gomez et al., 2009), there is "little systematic research available to guide decisions about the implementation of early field experiences" (Gomez et al., p. 120). My research with the Interview Project can add to this very limited area of research. It can serve as an example of a project that could be implemented into a teacher preparation program as an early field experience. My research has given a clear and detailed description of the Interview Project, making it easily accessible to all teacher education programs, and my research has also described its impact on preservice teachers' beliefs toward the teaching and learning of mathematics.

Pedagogical Experience in a Content Course for Early Childhood Majors

"Teaching is about weaving together knowledge about subject matter with knowledge about children and how they learn, about the teachers' role, and about classroom life" (Ball, 1990, p. 12). A mathematics content course can be about numbers and operations, geometry, algebra, or data analysis, and an educational psychology course could focus on theories of learning. But a methods course is typically where preservice teachers have the opportunity to weave everything together. Some early childhood teacher education programs, such as the one in which my participants were enrolled, do not include a methods course that specifically focuses on the content of mathematics. In my participants' program, they were required to take four mathematics content courses, including Math 2008. However, unlike many other programs, they never took a mathematics specific methods course. As an instructor of the Math 2008 course, I felt that pedagogical experiences needed to be integrated in their content courses.

The Interview Project is an example of an experience that weaves together the preservice teachers' mathematics content knowledge and pedagogical knowledge. Their interview with the child gave them the opportunity to enact their content knowledge of additive structured story problems and the solution methods that children use as well as the pedagogical knowledge of how to analyze the child's mathematical actions and make instructional decisions based on these actions. This project can serve other teacher education programs that are in the same situation as my participants' program, whose preservice teachers would not have experienced the intertwining of mathematics content and pedagogy otherwise without the Interview Project.

Future Research

As I reflected on my findings, I saw several things that I could have done differently. These possible adaptations to the research include giving pre and post assessments to measure a change in the preservice teachers' beliefs about the teaching and learning of mathematics and including more interactions or interviews between the preservice teachers and the child. In addition, I question the preservice teachers' abilities to adapt what they learned through the Interview Project to a whole class setting.

Changes in Preservice Teachers' Beliefs

Other studies (Carpenter, 1999; Crocket, 2002; Fennema, 1993, 1996) found that as teachers learn to understand the development of children's mathematical thinking, their beliefs about the teaching and learning of mathematics change. In my study, I found evidence of the preservice teachers shifting their beliefs to a more constructivist way of teaching and learning mathematics. Because I did not set out to measure a change in beliefs, I did not administer an assessment of the preservice teachers' beliefs. Next time, I would like to give a pre-assessment and post-assessment of the preservice teachers' beliefs about the teaching and learning of mathematics to see if there are any changes. For instance, I could pose questions about the role of school mathematics, have them choose and explain a simile that best describes the learning and teaching of mathematics as well as their view of mathematics as a discipline. Because I would want to see if they see their own views as fallible and whether they are able to learn from the children they interview, I would ask similar questions in a post-assessment perhaps as part of the assignment.

More Interactions Between Preservice Teachers and Child

A similar project was conducted in a mathematics methods course on children's construction of numbers and operations at a nearby Southeastern research university. Preservice teachers in this course were expected to conduct an initial interview with a child in a particular local school. After this interview, the preservice teachers planned seven successive sessions with the same child. Four preservice teachers were carefully selected as participants in a study of this project. Each of these four participants was able to construct a working model of her child's mathematics. All four participants were able to describe and analyze the mathematics of their children. They were also able to use their descriptions and analyses to inform their instructional decisions. Specifically, the participants were able to determine where their child was, according to the framework from class, and were able to choose and create tasks that would push their child to the next level.

The preservice teachers in my interview project were able to describe and analyze the children's mathematics using the framework from class, and they were also able to make some instructional decisions. The major difference in this project and mine is the number of sessions that the preservice teacher worked with the child. Instead of seven sessions, my participants only had one face to face interaction with the child and then planned for a hypothetical future meeting. Because any future instructional decisions were hypothetical, the preservice teachers in my study lacked focus and were mostly explorational. For instance, instead of planning tasks that would evoke strategic additive reasoning, those that said anything about future instructional decisions were only able to say things such as, "If I could work with this child again I would like to see what she would do with division or fractions." This leads me to ask whether it would be worth the time and additional efforts from my students' perspective to have them conduct at least a second interview with the same child. I feel that this would give them the opportunity to go beyond the interview and force them to make the instructional decisions. I do not believe that a course focused on teaching mathematical content should require as much time as for the previously described pedagogy project in the methods course. However, I do feel that there are many benefits to including this interview project in the Math 2008 content course. I also believe, based on my results, that a second interview would be worth the logistical challenges.

Preservice Teachers' Abilities to Adapt to Whole Class Setting

In their written reports and the interviews with me, most of the preservice teachers mentioned that they found value in their newfound ability to describe and analyze a child's mathematical thinking and to use this to inform their instruction. One pair of preservice teachers wrote,

> We loved doing this interview, and we hope to incorporate individual interviews in our future classrooms. We think it is important for a teacher to be able to assess each child's level of whole number development and to design their instruction accordingly.

However, there were also many preservice teachers who were reluctant about trying to incorporate this into a whole class setting. In one preservice teacher's final interview with me, she said,

> Something that I'm not sure about is how I will be able to teach like this to a whole class. I mean, I think it's great if you have time to sit down with each child and do interviews like this, but I don't see how that's possible. Think of how much time and energy it took for us to do this one interview.

Many of the studies where teachers or preservice teachers were constructing models of children's mathematical thinking were done in a teaching experiment, where the teacher worked with just one student. In his research with teaching experiments, Steffe (1991) said that "choosing to work in these laboratory conditions should not be construed to mean that I view constructivism as being restricted in its implications to teaching individual children" (p.179). Thus, one possible focus for future research would be to explore how these findings could extend to a whole class setting.

Overall, I am incredibly pleased with the outcome of the Interview Project. The preservice teachers were able to show evidence that they can analyze and describe a child's mathematics as well as inform their teaching through this process. They also showed that they were allowing the interview with a child to inform their own mathematical thinking in ways that they probably did not expect. Although I saw some changes that could be made to this assignment, I strongly believe that this Interview Project has a great impact on the preservice teachers and will change the way they previously felt about teaching mathematics.

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APPENDIX A

DESCRIPTION AND RUBRIC FOR THE INTERVIEW PROJECT

Title: Interview a Child
Project Goals: With this project you are beginning to learn to listen to and learn from children. I want you to see how capable children are of learning mathematics and solving problems. You are learning to respect children's mathematical thinking even when you do not understand it. Allow what you learn from children to influence how you think about your own mathematical thinking and allow it to inform your teaching. In this sense you will be assessing a child's developmental level with respect to whole number. You will be using this interview to describe a child's mathematics, analyze their mathematics using the framework from the course, and apply your analysis to inform your instructional decisions (if you were to work with this child again). In this part you may also discuss any on-the-spot instructional decisions you made while working with this student.
Description: For this project, you will interview a student (elementary age) to learn about her or his strengths and areas of potential development in mathematics. The purpose is to reflect on what you learn from the interview. Write a summary of the interview you conducted. The review should contain the following information:

1. General information such as your name, the name (use a pseudo-name), age, and grade of the student you interviewed, any pertinent information about the child you would like to mention.
2. Your analysis, including all of the mathematical problems you posed and a brief summary of the child's response. Say more than "The child solved the problem correctly." Explain how the child solved the problem or what the child said to indicate that she or he could not solve the problem. Some children will not be able to explain how they solved a problem. If this happens, simply indicate this in your summary. Note any behaviors you see the child exhibiting such as counting on fingers or moving lips. Discuss what you learned from this experience. Did anything surprise you? If you could continue work with this child, what concepts or kinds of problems do you think would be productive work for her or him? What kinds of problems would you think would be in their ZPD or right on the edge or it? Why do you believe this? What, if any, implications does interviewing have for you as a teacher?
3. Note: Avoid evaluative statements about the child, such as, "she was really smart" or "he seemed slow." You do not know enough about the child to make such statements, and besides, those statements provide no useful information. Instead, provide details such as, "When I asked her how many marbles she has if she started with 8 and her friend gave her 9 more, she solved it by saying '8 and 8 is 16, and one more is 17.' I thought that was

neat because I would not have expected a child to do that, "I asked him this question and he just looked at me. I asked him if I should repeat the question and he said 'no.' I did not know how else to reach him."

Rubric:

Components of the Project	**Description**	**Points**
Instrument	Selects or designs an instrument or task that will help assess a student's level of whole number development. Your task(s) need to be open enough to allow for multiple entry points. If the student uses a traditional algorithm, then you may want to ask them to explain it or ask them to try the problem in a different way. You will have a difficult time assessing the SMT if all they do is follow an approach that didn't come out of their own logical necessity.	**4**
Description of SMT	Description of the Mathematical Actions of the student on the task(s) This part needs to be as detailed as possible. You'll need to discuss the classification of the story problems (Join-Change Unknown, etc.) that the children solved and what strategies they took based on the language used from class (Direct Modeling, Counting on from largest, etc.). Be sure to describe a child's strategy in the child's language and use the language from class as well.	**4**
Analysis of SMT	Uses Steffe & colleagues' or Gelman's Model or language from the text as a framework to analyze the student's mathematical thinking displayed on the tasks. You will need to discuss what their actions imply about the level at which the child is operating (Perceptual Counters, Motor Item Counters,…,INS, INS+, SAR). If a child uses an invented strategy, write a series of equations that justifies why this strategy will always work. Be sure to use the equal sign appropriately and state the properties used at each step.	**4**
Application of SMT	What on the spot decisions were made: Questions asked or problems skipped, changed,	**4**

	or enhanced? What next? How will your analysis of the student's mathematical thinking inform your instructional decisions? Based on what you saw the student do, what problems do you believe would be on the on the edge of her/his ZPD? Since this is difficult for even the most veteran teachers, who get to rely on their experiences with children. You will have to rely on the existing literature. You will need to site at least three resources for your project that show you were looking for how to respond to this student. What did you learn from this particular student that you could apply to your future teaching? What, if any, implications does interviewing have for you as a teacher?	
Total		**16**

APPENDIX B

BRIEF DESCRIPTION OF THE FRAMEWORKS PRESERNTED IN MATH 2008 CLASS

	Framework	**Ways of using the Framework for The Interview Project**
Description of Child's Mathematical Thinking	Cognitively Guided Instruction (CGI)	Classifying Story Problems (Join-Result Unknown, Separate-Change Unknown, etc.) Identifying Children's Strategies (Direct Modelling, Counting on from the larger number, etc.)
	Van de Walle	Identifying Children's Strategies (Near doubles, Using Tens, etc.)
Analysis of Child's Mathematical Thinking	Steffe et al.	Identifying Child's Level of Whole Number Development (Perceptual Counter, Initial Number Sequence, Strategic Additive Reasoning, etc.)
	Gelman	Identifying Counting Principles (Cardinality, One-to-one, etc.)

APPENDIX C

5 PRE-INTERVIEW PROJECT TASKS

Task 1

Read the story problems below. Organize them into 2 or more groups. Then, describe why you organized them into these groups.

Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?	Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?	Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?		Connie has 13 marbles. 5 are red and the rest are blue. How many blue marbles does Connie have?
Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Carpenter, T. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Task 2

Watch each videotape of a child solving a story problem. How did the child solve the problem? Can you describe their method? Then, show how the child would solve the following related problem.
(The videos being used are from *Children's Mathematics: Cognitively Guided Instruction* by Carpenter et al.)

1) a) How did the child solve the problem? Describe their method as best you can.

b) How would the child solve the following problem?
<u>Related Problem</u>: To make lemonade, Calvin put 3 lemons in a pitcher. Then, he decided it needed more lemons and added 4 more lemons to the pitcher. How many lemons are in the pitcher now?

2) a) How did the child solve the problem? Describe their method as best you can.

b) How would the child solve the following problem?
<u>Related Problem</u>: Julio has 5 stickers in his sticker book. His friend, Jason, gave him some more stickers for his birthday, and now he has 11 stickers. How many stickers did Jason give him?

3) a) How did the child solve the problem? Describe their method as best you can.

b) How would the child solve the following problem?
Related Problem: Johnny has 8 stickers in his sticker book. His sister has 3 stickers in her sticker book. How many more stickers does Johnny have than his sister?

4) a) How did the child solve the problem? Describe their method as best you can.

b) How would the child solve the following problem?
Related Problem: Debbie has 7 books on her shelf. If she puts 8 more books on her shelf, how many books will she have altogether?

Tasks 3 and 4

After watching each video of a child solving a story problem, respond to the following questions:
a) Based on what you just saw, what can you tell me about this child's level of development?
b) What would you do next? What task would you give this child now that you've seen this clip?

(The videos being used are from *Integrating Mathematics and Pedagogy to Illustrate Children's Reasoning* by Phillip et. al.)

1) a) Based on what you just saw, what can you tell me about this child's level of development?

b) What would you do next? What task would you give this child now that you've seen this clip?

2) a) Based on what you just saw, what can you tell me about this child's level of development?

b) What would you do next? What task would you give this child now that you've seen this clip?

3) a) Based on what you just saw, what can you tell me about this child's level of development?

b) What would you do next? What task would you give this child now that you've seen this clip?

4) a) Based on what you just saw, what can you tell me about this child's level of development?

b) What would you do next? What task would you give this child now that you've seen this clip?

Task 5

Determine which of the two story problems is more difficult. Circle the story problem that you believe is the more difficult one. If you believe that they have the same difficulty level, circle neither.

1)
A) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?
B) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?

2)
A) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether?
B) Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?

3)
A) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?
B) Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?

4)
A) Connie had some marbles. She gave 5 to Juan. Now Connie only has 8 marbles. How many marbles did Connie start with?
B) Connie has 13 marbles. She gave 5 to Juan. How many marbles does Connie have now?

5)
A) Connie had some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with?
B) Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?

6) Below is a story problem that can be solved by the computation 8+5.

Connie has 8 marbles. Juan gave her 5 more marbles. How many marbles does Connie have altogether?

Your task is to write a different story problem that is MORE DIFFICULT than the one above to solve, but can still be solved by the computation 8+5.

7) Below is a story problem that can be solved by the computation 8-5.

Connie has 8 marbles. She gave 5 to Juan. How many marbles does Connie have left?

Your task is to write a different story problem that is MORE DIFFICULT than the one above to solve, but can still be solved by the computation 8-5.

APPENDIX D

RUBRIC FOR PRE-INTERVIEW PROJECT TASKS

Goal		**Does Not Meet Goal (0)**	**Partially Meets Goal (1)**	**Meets Goal (2)**
Description of Child's Mathematical Thinking	Task 1	Organized the story problems into groups that did not have any structural connection (i.e. put all of the "joins" together and "separates", or put all the "initial unknowns" together and the "difference unknowns" together.	Organized the story problems into groups with structural connections but was not able to describe why they organized them into those groups. PST may have also organized the story problems into groups, where some groups had structural connections but others did not.	Organized the story problems into logical groups and gave an accurate description of why these groups were appropriate.
	Task 2	Gave an inaccurate description of how the child solved the story problem or simply repeated verbatim the child's process. Was not able to solve a similar problem using the same method as the child.	Could solve a similar problem using the same method as the child, but was not able to name the method or accurately describe it.	Was able to name the child's solution method (or accurately describe the method) and solve a similar problem using the same method as the child.
Analysis of the Child's Mathematical Thinking	Task 3	PST gives no response or a response that does not accurately describe the child's level of development. The PST may estimate	PST attempts to describe a level of development but cannot justify it.	PST correctly describes a level of development based on the child's response to the story problem and justifies it.

		the child's age or grade level, but offers no developmental level. The PST may describe the child as "slow" or "smart" but does not give a reason for this description. The PST may attempt to describe a level, but their reasoning is based on how quickly the child gives the answer or whether or not the child gives the correct answer. The PST may also simply state what the child did (for example, "the child counted the blocks to find the answer") but does not offer any insight into how this helps to describe the child's developmental level.		
Instructional Decisions	Task 4	PST gives no response or gives or simply suggests that they would give a "harder problem".	PST's response is vague or explorational. The PST may give a suggestion such as "work on subtraction" or "give a problem with larger numbers", but does not identify a specific task.	PST gives a specific task that is on the cusp of the child's developmental ability, using the child's response to the given story problem as guidance.
	Task 5	Part 1 – PST correctly identifies 0-2 of the relative difficulty problems	Part 1 – PST correctly identifies 3-4 of the relative difficulty problems	Part 1 – PST correctly identifies all 5 of the relative difficulty problems

		(correctness is determined by CGI suggestions) Part 2 – PST does not respond to either question or responds to both questions incorrectly. A response is marked incorrect if the story is not more difficult than the one given. The PST may respond with a story problem that can be solved with the same computation but the story has the same structure as the one given. The PST may also respond with a story problem that is more difficult than the one given but it cannot be solved with the required computation.	(correctness is determined by CGI suggestions) Part 2 – PST responds to only one question correctly. A response is marked incorrect if the story is not more difficult than the one given. The PST may respond with a story problem that can be solved with the same computation but the story has the same structure as the one given. The PST may also respond with a story problem that is more difficult than the one given but it cannot be solved with the required computation	(correctness is determined by CGI suggestions) Part 2 – PST responds to both questions with a story problem that is more difficult than the one given, but it can be solved using the same computation.

APPENDIX E

FINAL INTERVIEW QUESTIONS

1) One of the goals of The Interview Project is to describe and analyze a child's mathematics. Why or why not do you think describing and analyzing a child's mathematics is a helpful tool for teachers?

2) Another goal of The Interview Project is to learn to listen to children. Why or why not do you think listening is a helpful tool for teachers? Why or why not? Did you use your listening skills during your interview to make instructional decisions? Did you use your listening skills after your interview to make future instructional decisions?

3) What did you get out of The Interview Project?

APPENDIX F

OUTLINE OF MATH 2008 COURSE

Week:	Content Covered	Activities
1	Syllabus, Introductions, Research Discussion, Pre-assessment for research	Pre-assessment activities
2	Gelman's Counting Principles Counting Methods (count all, count on, etc.) Subitizing	Alphabet Counting Activity (intended to bring out Gelman's Counting Principles and some of the counting methods) Watch videos (IMAP and CGI) to identify Gelman's principles and counting methods.
3	Steffe's Levels of Whole Number Development	Watched videos (IMAP and CGI) to identify child's solution method and to speculate which of Steffe's levels of whole number development.
4	Additive Structured Story Problems (join, separate, compare, and part-part-whole) Models for Solving Additive Structured Problems (set model, length model)	Watched videos (IMAP and CGI) to identify the story problem structure, the child's solution method, and to speculate which of Steffe's levels of whole number development. After identifying all of these, I also asked, "What would you do next? What type of question do you think would be appropriate and why?"
5	Test 1	The Interview Project description and rubric was handed out in class and discussed.
6	Multiplicative Structured Story Problems (multiplication, measurement division, and partitive division) Models for Solving Multiplicative Structured	The Doorbell Rang (read book and handout) What To Do With Those Remainders?

	Problems (set model, length model, area model, array model, and combinations model) Remainders Multiplication and Division by Zero	
7	Computation (Addition and Subtraction) Making Sense of Children's Invented Methods The Traditional Algorithms for Addition and Subtraction	Watched videos (DMI) and wrote a series of equations that proved whether or not the child's strategy was mathematically correct. Using base ten blocks or bundles of toothpicks, we acted out the steps of the traditional algorithm to better understand the reasoning behind each step, bringing special attention to the regrouping step.
8	Computation (Multiplication and Division) Making Sense of Children's Invented Methods The Traditional Algorithms for Multiplication and Division. The Advantages of Invented Strategies over the Traditional Algorithm	Watched videos (DMI) and wrote a series of equations that proved whether or not the child's strategy was mathematically correct. Using base ten blocks or bundles of toothpicks, we acted out the steps of the traditional algorithm to better understand the reasoning behind each step.
9	Test 2	
10	Spring Break	
11	Meaning for Fractions	Observations of Interviews
12	Meaning for Fractions	Observations of Interviews
13	Operations with Fractions	
14	Operations with Fractions	
15	Test 3	

16	The Interview Project Due and Presentations	