WENDY MICHELLE BURLESON SANCHEZ
Conceptualizing Mathematics Teachers' Use of Open-Ended Assessment Items
(Under the Direction of THOMAS J. COONEY)

Research indicates a gap between the assessment practices recommended in mathematics education reform literature and teachers' actual assessment practices. One way that has been suggested to help move toward the reform vision is the use of open-ended assessment items. The purpose of this study was to understand the factors that influence teachers' use of open-ended assessment items

Theories of intellectual development formed a framework for investigating teachers' beliefs and relation to authority. These theories, along with theories of reflection, were helpful in conceptualizing interpretations of teachers' actions.

Case studies focused on three secondary mathematics teachers who had participated in projects designed to enable them to create and use open-ended assessment items in their teaching. Seven face-to-face interviews, approximately 24 hours of classroom observation, and the collection of artifacts comprised the data for each participant. Inductive analysis was used to analyze the data.

Findings indicated four salient factors that affected the teachers' use of open-ended assessment items: beliefs and authority, reflectivity, knowledge, and system constraints. Some of the factors interacted. A description of how each factor influenced teachers' use of open-ended items is provided.

The findings suggest that preservice and inservice teacher education programs go beyond an attempt to get teachers to use open-ended assessment items to focus specifically on how to use student responses to such items to inform teaching. The study adds to the robustness of the claim that a relationship exists between an individual's relation to authority, reflectivity, and the extent to which he or she is able to implement reform. It is suggested that staff development, if it is to help teachers move in the direction of reform, should focus on more than teaching or assessment techniques per se, and begin to challenge the fundamental assumptions that underlie teaching and assessment in the first place. Staff development, like preservice teacher education, should provide contexts for reflection and challenges of beliefs that hinder teachers' movement toward reform.

INDEX WORDS: Secondary Mathematics Teaching, Assessment, Open-ended Items, Reform, Teacher Beliefs, Authority, Reflective Thinking

CONCEPTUALIZING MATHEMATICS TEACHERS' USE OF OPEN-ENDED ASSESSMENT ITEMS

by

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B.S.Ed. The University of Georgia, 1992

M.Ed. The University of Georgia, 1997

A Dissertation Submitted to the Graduate Faculty of

The University of Georgia in Partial Fulfillment

Of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2001

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August 2001

DEDICATION

To my husband, who enlightens me daily, and to the children that we hope to have. I hope that one day they develop the appreciation and love of learning that their parents have.

ACKNOWLEDGEMENTS

People who know me might be amazed to know that I, Wendy Sanchez, am at a loss for words. There are so many people to whom I owe a debt of gratitude that can never be repaid. Dr. Tom Cooney has walked with me on a path that began with me searching for "the right way to teach" in 1991 and he is still with me as I begin a whole new journey. The words *teacher* and *mentor* only begin to describe the role he has played in my development as a teacher, researcher, and thinker. I hope that we continue to work together for years to come. I will keep those emails rolling!

Dr. Pat Wilson taught my undergraduate methods' course and will sit beside me at my graduation. I will never forget the day she took me on a picnic to discuss my mathematical struggles. Her insights in research have awed and inspired me. I wish to thank her sincerely. Dr. Jeremy Kilpatrick is as amazing as his reputation. Because of him, readers of this dissertation will perhaps not cringe at my yet to be developed writing style. His time and patience with me is appreciated. Dr. Laurie Hart has always challenged me to think about alternative explanations and brought my attention to context and issues of power. I hope our conversations will continue. Dr. Sybilla Beckmann-Kazez helped me feel mathematically capable in my weakest moments. The University of Georgia is lucky to have her in its mathematics department. Mathematics professors like Dr. Beckmann can help narrow the gap between mathematicians and mathematics educators. I hope that I am able to develop the kind of feel for my students' mathematical knowledge that Dr. Beckmann has for hers.

Dr. Denise Mewborn has been an intellectual and emotional support for me, and I thank her from the bottom of my heart. She has a way of making students feel capable when they feel they are at the end of their rope. Dr. Lou Ann Lovin, who finished a year before me, has given me advice and support from the “been-there-done-that” perspective. I see many PME papers in our future, and we’ll always be roommates!

To my graduate school colleagues who have read countless versions of this document and have listened to me complain, cry, and rejoice—how can I thank you? The members of my writing seminar (Keith, Lisa, Signe, Nancy, John, Robyn) convinced me that I had something good even when I thought I had nothing to offer. Special thanks to Keith, whom I think I bothered the most. You are a dear friend whom I will miss every day. I thank Lisa for her “what do you want to say here…just write that” advice. I appreciate the comic relief of Andy and Chris. Nickey Ice has been a wonderful colleague and friend. I know we have many years ahead of working together to improve education in Georgia.

My parents, Larry and Helen Burleson, have been such a source of inspiration in my life. I think my Dad truly believes that I could figure anything out if I just thought about it long enough and worked hard enough. Just as I always thought he “could move Stone Mountain,” he always made me feel that I, too, could move mountains. My mother has inspired me to be a confident, assertive woman. She has encouraged me and comforted me as only a mother can. Thank you, Mom and Dad, for my first degree and for all your help getting me through the second two (this is the last one, though—I promise!). My sister, Robin, has listened to me complain about “gradual school” for a long time. She can finally tell Alex that I do not keep failing out of school, and that I am finally done!

I appreciate Robie, Stephanie, and Kelly for helping take my mind off of my dissertation by simply having fun with me. Now that this is over, I will either have more time for fun or more stress to alleviate from my new job, so don’t go anywhere!

And finally, my husband, Gregory. I do not know another woman who has as much support, love, help, empathy, loyalty, and friendship in a husband as I do. Thank you for paying for me and for loving me. It is your turn, baby! I am proud of everything you do.

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CHAPTER 1

INTRODUCTION AND BACKGROUND

Reform efforts in mathematics education emphasize the processes of communicating, reasoning, connecting, representing, and problem solving (National Council of Teachers of Mathematics [NCTM], 2000). If these processes are to be central to mathematics learning, then strategies must be developed to assess them. Traditional mathematics questions that require students to generate specific numerical or algebraic answers are limited in their ability to provide information about students' reasoning processes and their ability to connect and communicate about mathematical topics. One strategy for assessing such processes is the use of open-ended assessment items. In this study, I am defining open-ended items as items that have more than one correct answer or more than one way of arriving at a specific answer. Since these questions move beyond requiring students to mimic skills and procedures to asking them to explain why procedures work and how concepts are connected and represented, open-ended items can be a powerful tool for eliciting information about students' reasoning and conceptual understanding. Research on teachers' assessment practices indicates that teachers are reluctant to use open-ended items (Cooney, 1992 ; Senk, Beckmann, & Thompson, 1997). The need for strategies such as open-ended items that assess the kinds of outcomes advocated in reform initiatives and the reluctance of teachers to use these items has provided a context for investigating factors that influence teachers to use (or not use) open-ended items. Thus, the purpose of this study was to understand the factors that influence teachers' use of open-ended assessment items.

Setting: The Projects

The participants in this study were chosen from a group of teachers who were engaged in the third of three projects[1] aimed at enabling them to create and use open-ended assessment items in their teaching. The project was a joint effort between a large, metropolitan school district in Georgia and the University of Georgia (UGA). The

project leaders were the mathematics coordinator for the school system and a mathematics education professor at UGA. I was one of two doctoral students assigned to the project. The first had begun in the spring of 1997 and involved approximately 30 teachers of Algebra I and Geometry in the school system. Since Algebra I was taught at both the middle school and high school, there were some middle school teachers who participated in the project; the majority of the participants were high school teachers.

Initial training had occurred in the spring of 1997 and focused on informing teachers about reform efforts in mathematics education in the areas of curriculum, teaching, and assessment. The training also provided a rationale for using open-ended assessment items. Strategies for scoring responses to open-ended items were shared with the project teachers, and they practiced using a rubric to score responses. Discussions about how individual groups of teachers scored responses helped the teachers feel more comfortable with their concerns about grading. Teachers from a neighboring county who had participated in previous assessment projects with UGA shared examples, anecdotes, and student work to help the project participants recognize the benefits of using open-ended assessment items. Characteristics of quality assessment were shared with the participants along with heuristics for creating open-ended items such as “Who is correct and why?” and “What is wrong with this?,” as illustrated in the examples in Figure 1.

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[1] These projects were supported under the Eisenhower Higher Education Act under the direction of Drs. Thomas J. Cooney and Laura Grounsell.

Heuristic	Example
Who is correct and why?	Austin says he multiplied two complex numbers of the form $a + bi$ and obtained a real number. Jenna claimed that could only happen if either a or b is zero. Do you agree with Jenna? Why or why not?
What is wrong with this?	Alex divided 202 by 7 on his calculator and got 28.8571429. He claimed that the number was irrational since it appeared to neither terminate nor repeat. What is wrong with Alex's reasoning?
Create an example or situation.	Draw and label a parallelogram and a trapezoid that have equal areas. Show how you know that the two figures have the same area.

Figure 1. Examples of open-ended items generated with heuristics

After the initial training, the writing phase of the first project had begun in the summer of 1997. Project participants met in small groups for two weeks to create open-ended items. They used the system's curriculum objectives as a guide. Several items were created for each curricular objective. During this phase of the project, the participants also came back together in a large group to discuss issues that arose during the writing. Certain teacher-generated items were showcased as exemplars, and others were used to discuss potential problems. For example at times the teachers generated an item with one of the heuristics that they thought was non-traditional. For example, they wrote items like "Susanna solved $2x + 4 = 8$ and got 2, and Franco got 4. Who is correct and why?" This question is not much different from asking students to simply solve $2x + 4 = 8$. Issues such as this were discussed in the larger group to help the teachers recognize and create items that were both open-ended and non-traditional.

After the items were generated, the UGA team (the professor and two doctoral students) reviewed and edited them. Some items were deleted, and some were rewritten, attempting to stay as true to the teacher-generated originals as possible. The resulting items were sent to the project teachers for field testing during the school year 1997-1998. A subset of the project teachers (the advisory board) met periodically during the year to review students' responses. Based on these responses, the advisory board recommended that each item be accepted, revised, or deleted. The UGA team reviewed the responses along with the advisory board's recommendation and made the next round of decisions about accepting, revising, or deleting each item. Finally, the items were given to a research mathematician, who reviewed them for mathematical significance and clarity. The UGA team reviewed the mathematician's comments and made final decisions about the items, which were placed in a searchable database for use across the county in the Algebra I and Geometry courses. Once the database was in place, the county required that 20% of each teacher's tests include open-ended assessment items.

Two more projects followed, each following a similar procedure. The second project, in 1998-1999 focused on the Pre-Algebra course, and the third, in 1999-2000 focused on the Algebra II course. The participants in the present study all had been involved in the Pre-Algebra project, and one of them had been involved in the Algebra I and Geometry project.

Rationale

Mathematics education reform efforts in the late 1980s and throughout the 1990s included an emphasis on assessment. The authors of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) argued,

> In an instructional environment that demands a deeper understanding of mathematics, testing instruments that call for only the identification of single correct responses no longer suffice. Instead, our instruments must reflect the scope of our instructional program to have students solve problems, reason, and communicate. (p. 192)

NCTM's publication of the *Assessment Standards for School Mathematics* in 1995 further outlined their position on assessment. These standards asserted that assessment should,

1. Reflect the mathematics that all students need to know and be able to do,
2. Enhance mathematics learning,
3. Promote equity,
4. Be an open process,

5. Promote valid inferences about mathematics learning, and
6. Be a coherent process. (NCTM, 1995, pp. 11-21)

Other publications have offered practical suggestions for moving toward the vision of mathematics assessment set forth in the *Standards* documents. For example, Stenmark (1991) produced a guide for teachers who were attempting to use forms of assessment beyond traditional single-answer questions. NCTM's 1993 yearbook (Webb & Coxford, 1993) was dedicated to assessment. Other publications (e.g., Hancock, 1995; Kulm, 1994: Lambdin, Kehle, & Preston, 1996; Moon & Schulman, 1995) provided rationales for different assessment strategies and suggestions for implementing them.

One assessment alternative that was suggested as a way to move beyond traditional assessment methods was to ask open-ended questions. Hancock (1995) noted that "because of the wider range of solution methods they allow students, open-ended questions are thought to be better at revealing students' thinking" (p. 496). Moon and Schulman (1995) said,

> Open-ended problems often require students to explain their thinking and thus allow teachers to gain insights into their learning styles, the "holes" in their understanding, the language they use to describe mathematical ideas, and their interpretations of mathematical situations. When no specific techniques are identified in the problem statement,…teachers learn which techniques the students choose as useful and get a better picture of their students' mathematical power. (p. 30)

Cooney, Badger, and Wilson (1993) helped characterize the nature of open-ended assessment items with criteria for good assessment tasks. These criteria asserted that a good assessment task should involve significant mathematics, be solvable in a variety of ways, elicit a range of responses, require students to communicate, and stimulate the best possible performance on the part of the student. Clarke (1996) offered four principles often implicit in mathematics assessment: a) Assessment involves the exchange of information, b) assessment must optimize students' expression of their learning, c) assessment should have instructional value, and d) assessment should anticipate action. Clarke suggested that one strategy that embodies these principles is the use of open-ended tasks.

The literature in assessment reform not only proposes asking different kinds of questions, but, more importantly, it also suggests asking different questions for different purposes. The nature of open-ended items allows teachers to glean more information about student understanding, as Hancock (1995) and Moon and Schulman (1995) pointed out. But what teachers do with that information is another issue. The authors of the *Assessment Standards* (1995) said that although one purpose for assessment is to evaluate students' achievement, there are other purposes as well. They argued that assessment should help educators monitor student progress, make instructional decisions, and evaluate programs. Hence, if teachers are to implement reform, they must do more than merely ask different questions. They must use the information gained from asking those questions for different purposes. If open-ended items are viewed as a vehicle for reform, the view for investigating teachers' use of the items must be expanded beyond whether teachers use them to include how they use them and for what purpose.

This study focused on a particular assessment strategy, the use of open-ended items, on how teachers used that strategy in their teaching, and on the factors influencing that use. Wilson (1993) suggested that many factors may influence teachers' choices of assessment strategies; for example,

> their own beliefs about mathematics, about teaching and learning, and about authority. They may also include the size of their classes and their teaching load, the support of the administration, the expectations of students and parents, or the influence of mandated testing. (p. 14)

Senk et al. (1997) found that teachers' beliefs and knowledge as well as the curriculum and textbook they used influenced teachers' choices of assessment instruments. Because the teachers in the present study were involved in extensive training supported by the school system on using open-ended items, the effects of some of the potentially constraining factors were reduced. These factors included lack of support, lack of understanding of reform, and concerns about grading.

Several researchers have found relationships between teachers' beliefs, their relation to authority, and their practice. For example, Cooney et al. (1998) argued that "a teacher's movement from conceptualizing knowledge as something emanating from external beings toward conceptualizing knowledge as something emanating from interrelationships between self and others is an important consideration in conceptualizing teachers' professional development" (p. 329). When teachers locate authority outside of themselves, they do not participate in teaching as problem solving. Mewborn (1999) explained, "The issue is who has the authority to generate, reason about, and test hypotheses about mathematics teaching and learning" (p. 335). She found that when preservice elementary school teachers had an external locus of authority, they did not engage in these processes—they left them to the authorities. What would it mean for teachers' use of open-ended items if they did not generate, reason about, and test hypotheses about teaching and learning? Indeed, what is meaningful assessment if it is not testing hypotheses about teaching and learning? One justification for using open-ended items is that responses to those items enable teachers to get a better glimpse into student understanding for purposes beyond merely assigning grades. One of those purposes is to inform instruction. Teachers can use the information they get from student responses to open-ended items to make decisions about how to teach their particular students, who have their own particular misconceptions. If teachers rely on an external authority to inform their instruction as opposed to making decisions based on observations of their particular students, they are less likely to situate instruction within their own particular context.

The purpose of this study was to investigate some factors that affect teachers' use of open-ended items. I did not go into the study with no ideas about the nature of those factors. I was aware of what the literature had to say about influences on teachers' practices. In particular, it was natural to question how beliefs, orientation to authority, and the system constraints affected teachers' use of open-ended items. I also expected that other factors might emerge that would help explain teachers' use of open-ended assessment items.

Overview of the Study

Three secondary mathematics teachers, who had been involved in the projects previously described, participated in the study. The primary research question that guided the inquiry was: What factors influence teachers' use of open-ended assessment items? Two additional questions expanded upon the primary question: How do teachers' beliefs facilitate or hinder their use of open-ended assessment items? and How do teachers' orientations to authority facilitate or hinder their use of open-ended assessment items?

A review of the literature on teachers' assessment practices and a theoretical framework for understanding beliefs and relation to authority are provided in chapter 2. Chapter 2 also contains a theoretical framework for understanding reflective thinking. During the data collection, I began to notice qualitative differences in how the participants were using open-ended items in their teaching. As I struggled to understand the nature of those differences, some literature I had read on reflective thinking (Dewey, 1933; Van Manen, 1977) provided language to describe them. Thus, I included literature on reflective thinking in my theoretical perspectives.

Data collection began on January 19, 2000 and ended on May 2, 2000. From January 19 to March 15, I observed one teacher each day and interviewed each teacher weekly. Seven face-to-face interviews were conducted with each participant. The nature of the interviews is described in chapter 3. Each teacher was observed for approximately 12 hours in each of two different classes for a total of approximately 24 hours of classroom observation. The nature and purpose of the observations are also described in chapter 3.

A first phase of analysis, done through inductive analysis (Patton, 1990) generated themes that are described in chapter 4. These themes related to the teachers' beliefs and actions with respect to mathematics, teaching, learning, and assessment. A second phase of analysis resulted in a description of the teachers in terms of the theoretical framework. Each teacher was described in terms of her or his relation to authority and the extent of her or his reflectivity. This description is provided in chapter 5 as a cross-case analysis. Chapter 6 provides a summary of the study along with implications for teacher education and research.

CHAPTER 2

RELEVANT LITERATURE: A FRAMEWORK

Research on Teachers' Assessment Practices

In a 1994 dissertation, Hancock observed that the "research literature on classroom assessment is not extensive" (p. 25). It is still the case that this research base is small, although important. The development of an understanding of teachers' assessment practices is essential to understanding how they might attempt change in those practices. Information on how teachers currently assess, and perhaps more importantly why they assess the way they do, is essential for conceptualizing staff development or pre-service teacher education programs aimed at changing assessment practices. Several fundamental studies have investigated teachers' assessment practices.

Teachers' Conceptions of Reform

A gap exists between teachers' assessment practices and practices advocated by the reform. Research can shed some light on that gap. Cooney (1992) reported that teachers have little understanding of the NCTM's *Curriculum and Evaluation Standards*. Since the *Standard*s were relatively new in 1992, this finding is not surprising. Perhaps more surprising is Senk et al.'s (1997) conclusion that the shift toward the recommendations of the *Assessment Standards* had just begun to occur in about two-thirds of the 19 classrooms they studied.

One specific area of mismatch between the reform vision and teachers' practices reported in the literature is a lack of alignment between instruction and assessment. NCTM (1989) asserted that "methods and tasks for assessing students' learning should be aligned with the curriculum's instructional approaches and activities, including the use of calculators, computers, and manipulatives" (p. 193). Stiggins and Conklin (1992) found that although teachers' mathematics tests contain mostly items at the inference level, oral questions asked during instruction tend to be at the recall level (of Quellmalz's Taxonomy of Thinking[2]). They noted that this discrepancy results "in a thinking skill level mismatch between oral questions and expectations for written work" (p. 165).

The 1989 Evaluation Standards recommended that assessment be an integral part of instruction. Stiggins and Conklin (1992) found that "teachers view instruction and assessment as entirely distinct functions and do not know how to integrate instruction and assessment in planning class time" (p. 152). Similarly, Hancock (1994) found that teachers' tests and quizzes are clearly distinguishable from their instructional activities. The empirical literature on teachers' assessment practices suggests that the majority of the assessment that occurs in teachers' classrooms is summative, not formative, in nature. If assessment is summative, it occurs at the end of instruction and therefore cannot inform that instruction, which is one of the purposes assessment should serve, according to the reform literature.

Teachers' Use of Test Items

Both Cooney (1992) and Senk et al. (1997) reported that mathematics tests contain mostly low-level items. Cooney (1992), however, noted that the "non-geometry teachers were more likely to create low level items to assess a deep and thorough understanding of mathematics, whereas the geometry teachers tended to create more application, multistep, or open-ended items to test a deep and thorough understanding" (p. 9). Stiggins and Conklin (1992), however, found that in mathematics, "only 19 percent of the items [on participant's assessment instruments] assess recall [again, using of Quellmalz's Taxonomy of Thinking], whereas 72 percent of all items tap inference" (p. 158). According to their study, mathematics teachers' assessment instruments assess high-level thinking skills. They did not report the domain of mathematics they studied but merely that they studied mathematics classes. If the classes in their sample were

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[2] Quellmalz's Taxonomy of Thinking characterizes thinking from low-level to high-level using the following levels: recall, analysis, comparison, inference, and evaluation.

geometry classes, then their results do not necessarily conflict with Cooney's (1992) findings.

Cooney (1992) pointed out that teachers, when asked to create an item that assessed a deep and thorough understanding of a mathematical topic, often create items that are more difficult but not more conceptually challenging. He explained that those teachers have the perception that a deep and thorough understanding means students have an ability to perform multiple steps. He also found that low-level students are more likely to be given skill oriented items than higher-level students and that less experienced teachers are less likely to create items that assess higher order thinking skills than more experienced teachers.

Teachers, according to Cooney's (1992) findings, "feel more comfortable with items that require the generation of a specific number rather than an item that requires an open-ended response" (p. 11). Hancock (1994) found that most of the items on teachers' tests and quizzes are completion items answered by a single numerical or polynomial expression. Cooney (1992) reported that teachers claim students lack certain prerequisites for answering open-ended questions (e.g., reading and thinking skills).

Mathematics teachers tend to be comfortable with the perceived certainness of their field. Brown, Cooney, and Jones (1990) explained that "many believe that one of the unique characteristics of mathematics is that it is a well-defined body of knowledge" (p. 646). They further stated that "such a view of mathematics entices researchers to believe that testing for mathematical ability, competence, and achievement is relatively easy, as compared with other subjects" (p. 646). Mathematics teachers often share this view and see their subject as inherently objective. Introducing subjectivity into the mathematics classroom via asking open-ended items and using a scoring rubric to assess students' ability to reason and communicate about mathematics is problematic for teachers who take comfort in the objective decision to mark a response with a checkmark or an ×. Because of the comfort teachers feel in objectively scoring answers to mathematics questions, it is not surprising that Stiggins and Conklin (1992) found that teachers are not familiar with methods of assessing performance or that Cooney, Bell, Fisher-Cauble, and Sanchez (1996) found that they are not comfortable with subjectivity in grading. The kinds of items teachers choose for their mathematics tests tend not to be the kinds of items that can adequately assess a reform-based curriculum. If instruction is based on the processes of reasoning, problem solving, communication, representation, and connections, then assessment items must reflect those processes. Understanding the factors that influence teachers' choices of assessment items is a first step in enabling teachers to choose tasks for their students that go beyond mimicking procedures to explaining processes.

Factors That Influence Teachers' Assessment Practices

The literature speaks to factors that can influence teachers' assessment practices. Hancock (1994) found that changes in state policy do not substantially affect teachers' practices. Nash (1993) found that teachers need support when implementing innovative assessment strategies and that they only feel comfortable implementing one innovative strategy at a time. Senk et al. (1997) reported that teachers' knowledge and beliefs about assessment can influence their use of new assessment techniques, as can the instructional materials available. According to the literature, perceived lack of time is a significant constraint on teachers' use of innovative assessment strategies (Cooney et al., 1996; Nash, 1993). Wilson (1993) found that the expectations of others, an institutionalized curriculum, the structure of the school, and working conditions can all serve as limiting factors with regard to assessment.

Many of the factors described above are external to the teacher. For example, policy changes, support, available materials, time, curriculum, and social conditions are all part of the system in which teachers live. The only internal factor mentioned in the literature as affecting teachers' assessment practices is their knowledge and beliefs. Missing in the literature is an exploration of whether knowledge and beliefs affect the way teachers cope with the external factors they encounter in their teaching. The external factors are there, but how a teacher copes with them is likely affected by her or his beliefs and how he or she comes to believe.

Teachers' Grading Practices

Wilson's (1993) finding that what gets graded reflects what is valued is an important notion for this study. One important way teachers inform students about the importance of a topic is through assessment. The tasks teachers choose to grade are the ones they place the most value on, and students know that. If the only items teachers choose are algorithmic procedural items, students believe that these skills are the most important ones. Moreover, they will begin to associate mathematics with algorithmic processes only. Written tests are the primary determinant of student grades (Cooney, 1992; Hancock, 1994; Senk et al., 1997). It stands to reason that if test items are mainly items calling for a single number response, students will develop a limited view of mathematics.

Stiggins and Conklin (1992) reported that affective measures such as effort are often included in grading for motivational purposes. Measurement experts recommend that only achievement information be included in grading (Stiggins, Frisbie, & Griswold, 1989). How concerned a teacher is with the affective dimension of teaching and learning is likely to affect the way he or she grades.

Summary

Though the literature base on mathematics teachers' classroom assessment practices is not extensive, there have been some fundamental studies that give insight into how and why mathematics teachers assess the way that they do. Change in assessment practice can happen, but as the studies described above show, teachers need support and training in order to align their assessment practices with reform efforts. Even with support, teachers struggle with certain issues, such as subjectivity in grading, time constraints, a limited view of mathematics, and perceptions of their students' ability. Teachers' knowledge and beliefs also likely influence their assessment practices, including their use of open-ended assessment items. This study was designed to facilitate a better understanding of the factors that affect teachers' assessment practices so that ways can be found to change them in a direction more aligned with reform initiatives in mathematics education.

Theoretical Perspectives

Two bodies of literature provided theoretical perspectives for the study. As previously described, beliefs and relation to authority have been shown to affect teachers' instructional practice and it is reasonable to anticipate that these factors affect their assessment practice as well. Given how the extent to which teachers reflected seemed to affect their use of open-ended assessment items, Idecided to develop a theoretical model for understanding reflective thinking. The literature on beliefs and reflective thinking enabled me to operationalize important constructs for studying teachers' assessment practices.

Understanding Beliefs

The concept of belief has been defined differently across the literature and has been compared and contrasted to the concept of knowledge. In this study, *belief* was taken to mean "all the matters of which we have no sure knowledge and yet which we are sufficiently confident of to act upon and also the matters that we now accept as certainly true, as knowledge, but which nevertheless may be questioned in the future" (Dewey, 1933, p. 6). Thompson (1992) pointed out the difficulty in teasing out the difference between knowledge and beliefs. She also pointed out that perhaps we did not need to waste time agonizing over the distinction since the two constructs are so intertwined. In the present study, knowledge was taken to be a subset of belief.

The way that beliefs are established, evaluated, or judged varies significantly among different groups of people. A large literature exists on theories of ways people come to know (Baxter Magolda, 1992; Belenky, Clinchy, Goldberger, & Tarule, 1986; King & Kitchener, 1994; Perry, 1999). These theories all posit distinct stages or categories in which people can be positioned along a continuum from knowledge given from authority to knowledge developed in context. Many researchers (e.g. Cooney, Shealy, & Arvold, 1993; Shealy, 1994) have used the literature on ways of knowing to investigate teachers' beliefs. How people come to believe is important to understanding what it is that they believe.

Many questions can be asked about how a person comes to hold her or his beliefs. Green (1971) made the distinction between evidentially held beliefs and nonevidentially held beliefs. As the name implies, non-evidentially held beliefs are held without regard to evidence. Green explained that "beliefs held nonevidentially cannot be modified by introducing evidence or reasons" (p. 48). Evidentially held beliefs, on the other hand, "are held on the basis of evidence or reasons, [so] they can be modified in the light of further evidence or better reasons" (p. 48).

Although a given individual ultimately makes her or his own decisions, differences occur in what those decisions are based. On one extreme of the ways of knowing continuum lies a position in which individuals rely on authority for knowledge. Copes (1982) described the early stages of Perry's (1999) scheme as relying on authority to "know and deliver [the] answer and deliver truths" (p. 38). People who rely exclusively on external sources for their beliefs make their decisions based solely on what authorities tell them is true. Belenky, et al. (1986) described women who rely on authority as their source of knowledge as equating "receiving, retaining, and returning the words of authorities with learning" (p. 39). In their scheme, King and Kitchener (1994) explained that, in the early stages of intellectual development "what the individual believes to be true is not differentiated from what authorities say is true" (p. 49). When individuals who have an external source for authority make decisions, they base those decisions on the words of authority as if those words were absolute truth.

In the early stages of intellectual development, beliefs are not evidentially held. King and Kitchener (1994) explained that "beliefs need no justification since there is assumed to be an absolute correspondence between what is believed to be true and what is true" (p. 49). At these stages, what is directly perceived or what is given by authority is the only evidence needed for belief. Belenky et al. (1986), in their description of received knowers, said, “They do not evaluate the idea. They collect facts but do not develop opinions. Facts are true; opinions don’t count” (p. 42). Evidence is not part of the equation for those whose beliefs are derived from what authorities tell them is true.

Individuals who view knowledge as absolute believe “that every question has an answer, [and] that there is a solution to every problem” (Copes, 1982, p. 38). They have a dualistic view of knowledge—a proposition is either right or wrong. King and Kitchener (1994) described these individuals’ view of knowledge as “absolute and predetermined, [with] the existence of legitimate alternatives [being] denied” (p. 48). Individuals with this view of knowledge believe that everything is knowable; even if they do not know something, someone else must. Toward the end of the early phases of intellectual development, individuals begin to recognize that some knowledge is not known. At this point, however, those individuals believe that knowledge is eventually certain. If something is not known, it is just because the authorities have not figured it out yet. But everything is still knowable.

As people move along in their development, they begin to recognize that they may play a part in their own knowledge or belief formation. Baxter Magolda (1992) described this shift by saying, “Authorities are no longer seen as the only source of knowledge; instead [people] view themselves as equals and hold their own opinions as valid” (pp. 137-138). Similarly, Belenky et al. (1986) described a shift to subjectivism in which the locale of truth “now resides within the person and can negate answers that the outside world supplies” (p. 54). When people’s source for knowledge begins to move away from authority, they turn to authorities when they do not know the answers themselves. They see all opinions as valid and “do not acknowledge qualitative differences between the opinions of experts and their own opinions or between different expert’s opinions” (King & Kitchener, 1994, p.61). This “everything goes” attitude is aligned with Perry’s (1999) multiplism.

At this point, individuals begin to seek reasons for their beliefs. As Perry (1999) explained, “The bridge to the new world of comparative thought lies in the distinction between *an* opinion (however well-'expressed') and a *supported* opinion" (p. 110, emphasis in original). King and Kitchener (1994) described a position during which "beliefs are justified by giving reasons and using evidence, but the arguments and choice of evidence are idiosyncratic—for example, choosing evidence that fits an established belief" (p. 61).

In the center of the ways of knowing continuum, knowledge is often viewed as partially certain. Baxter Magolda (1992) described this shift, explaining that "absolute knowledge exists in some areas, [but]…uncertainty exists in others. Discrepancies among authorities in these uncertain areas are viewed as a result of the answers being unknown" (p. 47). Individuals who view knowledge as partially certain recognize that there are some truly problematic situations in which we just do not know the answers.

Toward the other extreme of the ways of knowing continuum, the source for knowledge becomes internal and individuals judge propositions for themselves. They may base those judgments on personal experience or on evidence. Some individuals with an internal source for knowledge make their decisions based on what feels right and what is aligned with their personal experiences. Others base judgments on external evidence. They will likely use evidence from a variety of sources (some external, some internal), but the acceptance or denial of the proposition is their decision to make. People who have an internal source for their knowledge recognize that "ill-structured problems require solutions that must be constructed and that even experts are involved in a similar process" (King & Kitchener, 1994, p. 66). Belenky et al. (1986) explained that during the time people are shifting from an external to an internal source for knowledge, there may be an outright rejection of authority, with persons believing only what feels right for them. Later, though, these same persons may seek to integrate the opinions of authority with their own thoughts. During the later stages of intellectual development, people recognize that "all knowledge is a construction and truth is a matter of the context in which it is embedded" (Belenky et al., 1986, p. 138). People with an internal source for knowledge make their own decisions in context; however, they integrate the voices of others with their own in the decision-making process. That is, they listen to the opinions of others and consider those opinions in their decision making, but they make the decisions based on what they believe is true. In these later stages, judgments are based on evidence, but the evidence itself is judged. More than just requiring evidence for justification, individuals on the other extreme of the ways of knowing continuum require that the evidence was generated in a way that they believe is valid.

In the later stages of intellectual development, knowledge is viewed as relative. Persons who view knowledge as relative recognize that different views or opinions can be compared and that one can be judged as better than the other. But this judgment is made in context. King and Kitchener (1994) offered that "interpretations that are based on evaluations of evidence across contexts and on the evaluated opinions of reputable others can be known" (p. 68). Knowledge is relative because it is based on assumptions that must continually be reevaluated. King and Kitchener (1994) explained,

> Knowledge is the outcome of a process of reasonable inquiry in which solutions to ill-structured problems are constructed. The adequacy of those solutions is evaluated in terms of what is most reasonable or probable on the basis of the current evidence and is reevaluated when relevant new evidence, perspectives, or tools or inquiry become available. (p. 71)

Individuals who view knowledge as relative recognize that knowledge is developed under current paradigms and that when those paradigms shift, our understanding may have to shift along with them.

Understanding Reflection

Reflective Thinking Defined

Dewey (1933) defined reflective thinking as the "active persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" (p. 9). He further explained that reflective thinking "includes a conscious and voluntary effort to establish belief upon a firm basis of evidence and rationality" (p. 9). He described reflective thinking as constructing a metaphorical chain with each link built from the previous one. Ideas tied together consecutively to form a consequence make up reflective thinking. Random ideas that pass through our heads do not constitute reflective thinking. Dewey explained that "reflection…implies that something is believed in (or disbelieved in), not on its own direct account, but through something else which stands as witness, evidence, proof, voucher, warrant; that is, as *ground of belief*" (p. 11, emphasis in original).

Dewey (1933) identified two phases of reflective thinking. First, an individual recognizes something as problematic. This phase is similar to the notion of a perturbation. It is at this phase that thinking originates. After recognizing a situation as problematic, the next phase is an act of search in order to solve the problem. During this search, ideas come to mind (and are linked together as a chain, as described above) in order to generate hypotheses. These hypotheses are tested in an effort to create evidence. Beliefs can be established as a result of this reflective process.

<u>Relating Beliefs and Reflective Thinking</u>

Reflective thinking can be viewed as a way to establish, confirm, or refute a belief. In other words, people can come to believe through reflective thinking. There are other ways, too, that people come to believe. In their reflective judgment model, King and Kitchener (1994) posited seven stages of reflective judgment. They grouped the seven stages into three categories: pre-reflective, quasi-reflective, and reflective. It is in the reflective category (the last two stages) that King and Kitchener (1994) claim that true reflective thinking (in the Deweyian sense) occurs. Pre-reflective thinkers view knowledge as certain. Viewing knowledge as certain does not leave room for recognizing situations as problematic. Thus, reflective thinking is cut off at the first phase. Quasi-reflective thinkers recognize that some situations are problematic, and they recognize the need for evidence, but they use evidence in an idiosyncratic way. These thinkers get cut off in the second phase of reflective thinking. Although ideas might come to mind, they might not be linked together in a logical way. These quasi-reflective thinkers may grab onto any evidence they come across without testing it or judging its viability. In the reflective category, true reflective thinking guides the formation of beliefs.

Not all thinking is reflective thinking, but thinking that is not reflective should not necessarily be considered low level. King and Kitchener (1994) explained that

> Individuals who do not recognize uncertainty may be able to solve other kinds of problems such as figuring out the circumference of a circle, translating a set of instructions into a computer language, or playing a game of chess, quite adequately. These activities do not require reflective thinking in the way Dewey used the term. Mathematical formulas, logic, or rules of play may be adequate to derive successful solutions to these problems. While such activities require intelligence and astute thinking skills,…they are not truly problematic and…are characteristic of a different type of problem. (p. 7)

Thinking may be high level and require a good deal of knowledge or intelligence, but not constitute reflective thinking.

Levels of Reflection

Thinking can be classified as reflective or not. But reflective thinking can be classified further according to the depth or level of reflection. Van Manen (1977) described three levels of reflectivity. The first level of reflection is concerned with the means needed to obtain given ends. A person reflecting at the first level would take for granted that the ends are desirable and would not question them. The criteria for reflection at the first level are limited to economy, efficiency, and effectiveness. Individuals reflecting at the first level are mostly concerned with what works. They determine their practical actions by deciding on the quickest path to a given goal. A teacher reflecting at the first level would be looking for methods of teaching given topics that enable students to answer mathematics questions correctly. The teacher's reflection on her or his teaching of the topic would be limited to the time it took to teach the topic and the extent to which students were able to get correct answers after instruction.

At the second level of reflection, the person engages in a debate over principles and goals. At this level, attention is paid to “analyzing and clarifying individual and cultural experiences, meanings, perceptions, assumptions, prejudgments, and presuppositions, for the purpose of orienting practical actions” (Van Manen, p. 226). At this second level, one assesses the implications and consequences of actions and beliefs. The ends are still not questioned at this second level. Individuals reflecting at the second level follow their actions to their logical conclusions and assess whether or not the results are aligned with their principles and goals. At this second level of reflection, one is interested in more than what works. One attempts to orientone’s practical action within a larger system of principles and goals. Teachers reflecting at the second level would be interested in more than whether students are able to get correct answers. They would want to know that their instructional techniques were aligned with their larger goals for teaching. If teachers believed that teaching should promote understanding or that students should be able to apply mathematics, they would concern themselves with whether their chosen strategy facilitated understanding or application. They would move beyond what works to whether it worked for their goals.

At the third level of reflection, the ends or goals themselves are questioned. At this level, one questions the “worth of knowledge and…the nature of the social conditions necessary for raising the question of worthwhileness in the first place” (Van Manen, 1977, p. 227). Embedded in this level is a “constant critique of domination, of institutions, and of repressive forms of authority” (Van Manen, 1977, p. 227). Individuals question the morality or value of propositions given the conditions of society at large. It is only at this third level of reflection that a teacher would critically examine the ends of education. The ends as they are currently defined would be up for debate and not accepted as given.

Goodman (1984) used Van Manen’s levels of reflection in his analysis of a teacher education seminar. He found that the issues the preservice teachers focused on were practical and utilitarian and that “the curriculum (what’s worth teaching and why), the nature and purpose of instruction, the complexity of interpersonal relationships, the power structure of schools/classrooms, and the role of school in society were rarely discussed" (p. 16). In other words, he found that preservice teachers' reflection is limited to the first of Van Manen's levels. Goodman (1984) explained that "reflection, for these individuals, meant thinking about which techniques seemed best in any given situation. The techniques were seen as ends in themselves rather than means to implement some broader educational purpose" (p. 16). If open-ended items are viewed as a technique, teachers may have the same view of using those items as Goodman's preservice teachers did, that is, as ends in themselves. The purpose of asking open-ended questions is to elicit different information about student understanding than is elicited by traditional assessment items and then to do something with that information. If teachers do not recognize this larger educational purpose, then their use of open-ended items is unlikely to be a step toward the vision for assessment described in the reform literature.

Relating the Theoretical Perspectives to the Research Question

The empirical literature on teachers' assessment practices says that knowledge and beliefs influence assessment practice. It indicates that textbooks, curriculum, time pressure, expectations of others, and similar factors affect assessment practice. These factors outside of knowledge and beliefs seem to have a strong impact on what teachers are willing to do with assessment in their classrooms, and there is a common thread woven throughout them. All are external, and are related to authority. All seem to collapse into one: beliefs (and the relation to authority). Hence, I clarified my primary research question by adding two secondary questions. The literature on beliefs and authority was essential for addressing my research questions. The reflective thinking literature helped me develop a theoretical orientation toward the observations I made while collecting data. The more I read the literature on reflective thinking, the more I recognized the relationship between reflective thinking and orientation to authority (Cooney et al., 1998; Mewborn, 1999). All of these constructs seem to weave together as factors that might affect teachers' use of open-ended assessment items.

CHAPTER 3

METHODOLOGY

Previous research on teachers' assessment practice has focused on the kinds of assessment strategies that teachers use rather than on a particular strategy and how teachers use it. The present study focused on one particular assessment strategy, the use of open-ended items, and sought to understand the factors that influence teachers' use of this strategy. Because existing theories have not identified the factors that influence teachers' use of open-ended items, a qualitative design was appropriate (Merriam, 1998). The study involved three interpretive case studies. Merriam (1998) explained that interpretive case studies are useful when there is a lack of theory. The "researcher gathers as much information about the problem as possible with the intent of analyzing, interpreting, or theorizing about the phenomenon" (p. 38). Interpretive case studies move beyond description of phenomena and attempt to explain them,. In this chapter, I describe the selection of participants, data collection, data analysis, and other methodological issues.

Participants

The participants were chosen from a group of 11 high school mathematics teachers who were involved the Algebra II project described in chapter 1. Because I was assigned to this project as a doctoral student, I had access to these teachers and worked with them for a year. All teachers in the project seemed to have "bought into" using open-ended items in their teaching. I used purposeful selection of participants. Merriam (1998) explained that "purposeful sampling is based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (p. 61). Because the research question in this study concerned teachers' use of open-ended items, choosing teachers who were adept at writing open-ended items helped ensure that such items would be used. Additionally, prior research indicated that few teachers use open-ended items. Selecting teachers who seemed committed to open-ended items helped me understand how those teachers who *want* to use open-ended items cope with the demands that prevent other teachers from using them. The selection of participants from this project generated a sample from which much could be learned.

I explained the focus and intent of the research to all the project participants and asked them to fill out a form indicating whether they were willing to participate, would need more information, or were not willing to participate. I used three selection criteria. The first was willingness to participate. I wanted to observe each teacher in two different classes, one lower ability level and one upper ability level, because the literature indicated differences in teachers' assessment practices when they taught different courses. The second criterion was that the teachers taught two classes that differed in ability level. The final criterion was convenience. I wanted to observe two different classes each visit and to conduct an interview every other visit. I needed to have the two classes and a free period (or the beginning or end of school) scheduled, if possible, consecutively so that I would not be in the school for long periods of time without collecting data.

The choice of participants was greatly reduced when all three criteria were applied. In fact, one of the participants that I chose had indicated on his form that he would need more information before agreeing to participate. I went to his school to better explain the research procedures, and he agreed to participate. I chose three participants—Robin, Todd, and Keith—who met all of the criteria.

Data Collection

The data sources, for each participant, included seven face-to-face interviews, an average of 24 hours of classroom observation, and the collection of artifacts. A rationale and description are provided below for each data source.

Face-to-face interviews were important for determining the nature of teachers' beliefs. McCracken (1988) explained,

> The long interview is one of the most powerful methods in the qualitative armory…. The method can take us into the mental world of the individual, to glimpse the categories and logic by which he or she sees the world. It can also take us into the lifeworld of the individual, to see the content and pattern of daily experience. The long interview gives us the opportunity to step into the mind of another person, to see and experience the world as they do themselves. (p. 9)

In attempting to understand the teachers' beliefs, the face-to-face interview was a natural choice for data collection. Each interview was audiotaped and transcribed.

Although face-to-face interviews are an excellent way of investigating teachers' beliefs, Wilson's (1993) study showed that a teacher's stated beliefs may be very different from the beliefs that can be inferred from her or his action. In an effort to fully investigate the participants' beliefs, classroom observations were conducted. And because teachers' behaviors or attitudes can be different depending upon the subject (recall Cooney's 1992 finding that geometry teachers write higher level items than other teachers), each teacher was observed in two different classes. Classroom observations done not only in an attempt to understand teachers' beliefs, but also to gather data about the open-ended items that teachers gave during class. Fieldnotes were taken during each observation.

The artifacts included any handout given during observation, all tests and quizzes given during the data collection period, graded responses to open-ended items, and so on. These artifacts provided information about the kinds of questions teachers asked both in class and on tests and quizzes.

I observed two classes each day I visited a participant. I observed Robin teaching Algebra I and Calculus. I observed Todd teaching Algebra Concepts (a traditional Algebra I course taught over two years) and Algebra II. I observed Keith teaching Algebra II and Pre-Calculus. Keith was on block scheduling, so his classes were approximately twice as long as Robin and Todd's classes. Hence, I observed Keith once a week and the other two teachers twice a week for seven full weeks and two partial weeks. Figure 2 summarizes the data collection schedule. The seventh interview was completed after the observation period ended. The seventh interview occurred on May 1, April 24, and May2, for Robin, Todd, and Keith, respectively. Observations were not conducted on those days.

Following NCTM's *Professional Standards for Teaching Mathematics* (1991), I observed three aspects of these teachers' mathematics classes: (a) tasks, (b) discourse, and (c) environment. Tasks included examples and problems worked by the teacher or posed by the teacher for the students to work. Tasks also included homework problems, quiz or test items, and other assignments. Observations of the kinds of tasks that a teacher gave helped me understand what mathematics the teacher chose to emphasize in her or his teaching. When discourse was observed, I considered interactions between the teacher and the students, including the questions the teacher asked and how the teacher responded to student questions. I considered both one-on-one and group interactions with students, from which I hoped to glean information about the teacher's beliefs about mathematics, mathematics teaching, and mathematics learning. The environment encompassed both the physical environment and the general atmosphere of the classroom. Such features as classroom decor, tones of voice, and gestures were included as part of the environment.

The first two interviews were the same for each teacher (see Appendix for interview protocols). The first interview focused on the teachers' beliefs about mathematics and mathematics teaching and learning. In this interview, I asked questions such as, "If mathematics were an animal, what would it be?" and "Would you say that mathematics is discovered or invented?"

Monday	Tuesday	Wednesday	Thursday	Friday
1/17	1/18	1/19 Robin Observation	1/20 Keith Observation	1/21 Todd Observation
1/24 Snow day	1/25 Snow day	1/26 Robin Observation and Interview 1	1/27 Robin Observation	1/28 Keith Observation
1/31 Robin Observation and Interview 2	2/1 Robin Observation	2/2 Todd Observation	2/3 Keith Observation and Interview 1	2/4 Todd Observation
2/7	2/8 Todd Observation and Interview 1	2/9 Keith Observation and Interview 2	2/10 Todd Observation and Interview 2	2/11 Robin Observation and Interview 3
2/14 Todd Observation	2/15 Todd Observation and Interview 3	2/16 Robin Observation	2/17 Keith Observation and Interview 3	2/18 Holiday
2/21 Holiday	2/22 Robin Observation	2/23 Robin Observation	2/24 Todd Observation and Interview 4	2/25 Todd Observation
2/28 Robin Observation	2/29 Robin Observation and Interview 4	3/1 Todd Observation and Interview 5	3/2 Keith Observation and Interview 4	3/3 Todd Observation
3/6 Keith Observation and Interview 5	3/7 Todd Observation	3/8 Todd Observation and Interview 6	3/9 Robin Observation	3/10 Robin Observation and Interview 5
3/13	3/14 Keith Observation and Interview 6	3/15 Robin Interview 6	3/16	3/17

Figure 2. Data collection schedule

I also asked about learning mathematics. For example, I asked teachers to choose from a list or to create their own metaphor for learning mathematics. Some of the choices were building a house, cooking with a recipe, conducting an experiment, and creating a clay sculpture. I also asked teachers to describe the best environment for learning mathematics. With respect to teaching, I again asked the participants to choose a metaphor from a list (or create their own) for teaching mathematics. Some of the choices were news broadcaster, gardener, orchestra conductor, and coach. I also asked the participants to describe the characteristics of a good mathematics teacher and a poor mathematics teacher.

The second interview focused on mathematics assessment. In this interview, I asked teachers about their assessment practices, including what constituted good assessment items, when and where they felt most comfortable using open-ended assessment, and how they chose test items. I asked them to analyze one of their tests and determine the level (from Cooney et al., 1993) of each item. We discussed how they felt about the kinds of questions they asked on their tests.

The third interview focused on issues that came from the classroom observations, so they were different for each teacher. For example, Todd's third interview focused on some particular open-ended items he asked in the two classes I observed. We talked about how the items were graded, how the students had responded to the items, and possible reasons that students had performed poorly on an open-ended test item that had been posed in class the day before. Robin's third interview focused on an open-ended warm-up she had assigned in Algebra I. We discussed why she had assigned the item, what information she had obtained about student understanding from assigning the item, and how she had felt when she graded the item. Keith's third interview was, for the most part, role playing. I asked Keith to pretend that I was a new teacher in his school (Keith was the mathematics department chair) and that we were having a conversation in the teachers' lounge. I asked him questions about open-ended assessment as though he was my department head. I focused my questions on the factors reported in the assessment literature as constraining teachers' use of alternative assessment. For example, I told Keith that asking open-ended questions was taking too much time and that I was afraid I would not finish the curriculum. I expressed concern about grading and unpredictability. Through the role playing, I was able to elicit information about how Keith coped with these constraints in his own teaching.

Interview 4 contained teacher-specific questions. For example, I asked Todd why he was so willing to use open-ended items in his teaching. I asked Robin about a decision she had made to retest a certain chapter in Algebra I when the students performed poorly on the test, and I asked Keith how he resolved a problem he had when teaching synthetic division in Pre-Calculus. Overall, however, the fourth interview was similar for the three teachers. In each case, we discussed what they felt was their responsibility as a mathematics teacher, to whom they felt responsible, and who they perceived was their boss. We discussed whose responsibility it was to decide what was taught in their classrooms. Further, I posed two arguments about finishing the curriculum and asked the teachers who they agreed most with and why. Teacher A claimed that it is a teacher's responsibility to finish the curriculum so that there would be consistency across classes no matter who taught the course. Teacher B claimed that developing mathematical understanding takes time and would not sacrifice understanding in order to finish the curriculum. The participants were asked to align themselves with one of the positions and to discuss their own position.

Interview 5 also contained some teacher-specific questions but was similar in focus for each participant. In this interview, we discussed how the teachers carried out their teaching roles. I asked about lesson planning and whether future topics affected the way they taught current topics. We discussed what they perceived to be their strengths and weaknesses as mathematics teachers. I asked them whether they attended more to the practical or the theoretical in their teaching and about their biggest dilemma in teaching mathematics. I asked about grading open-ended items and what advice they would give to a new teacher about using such items.

For Interview 6, I looked back at each interview protocol for each teacher and found questions that had elicited particularly informative responses. If I had not asked a teacher those questions, I did so in Interview 6. I also asked some political questions; for example, about what they thought of the new governor's education plan.

Interview 7 was the same for each participant. I had not seen the participants for at least a month, so I asked them whether anything had happened since I last saw them that they wanted to discuss. We discussed the extent to which they reflected on their teaching. Before the interview, I had reviewed the data. I asked the teachers in Interview 7 to clarify anything about which I was confused. For example, each teacher clarified what he or she meant by certain words like *application*. I also gave each teacher the topic of logarithms, and we talked about how they would teach it.

Data Analysis

Data analysis occurred in two phases. The first phase involved open coding of interview transcripts and highlighting any pieces of data that seemed significant in terms of the research question or the theoretical framework. Each of these highlighted pieces was placed on a colored card (a different color for each participant). I also coded the field notes taken during observations. These pieces of data were also put on the colored cards. The cards were sorted into piles using inductive analysis. Patton (1990) explained that "inductive analysis means that the patterns, themes, and categories of analysis come from the data; they emerge out of the data rather than being imposed on them prior to data collection an analysis" (p. 390).

The first sorting resulted in piles for each participant in the domains of mathematics, teaching, learning, and assessment. Within each pile, I further sorted the cards into piles that centered around the same theme. For example, Robin spoke often about connections and emphasized connections in her teaching. One pile of cards that focused on connections was created for Robin as a subset of the teaching pile. Several of these piles were created and were later collapsed into themes. For example, the piles labeled "connections", "going backwards", and "applications" were collapsed into the theme “teaching should promote understanding.” Robin used connections, applications, and the technique of presenting mathematical procedures as the reverse of previously learned procedures to promote understanding. Using this part of the analysis, I wrote the case studies.

During the second phase of analysis, I returned to the data for a more theoretical look at the themes that had emerged during the first phase. I looked at the first phase of analysis as finding out and reporting what was happening. What was happening in these teachers’ classrooms? What did they believe about mathematics, teaching and learning mathematics, and assessment? In the second phase, I wanted to explain what was happening in theoretical terms. At this point I returned to the data to look specifically for incidences in which I could glean information about authority, reflective thinking, and levels of reflection. The codes I used in the second phase of analysis were tied to my theoretical framework. For example, I coded AVM for an absolute view of mathematics and L1R, L2R, and L3R for Levels 1, 2, and 3 of reflectivity, respectively. I noted when the participants used evidence or authority in their decision making. I was also able to use the results of the first phase of analysis to glean information about beliefs, authority and reflection. Searches for disconfirming evidence helped solidify the themes.

Role of the Researcher

In Robin’s class, I was able to remain an onlooker, not participating in the class. I had more trouble with that role in Keith and Todd’s classes. It was not so much an issue with Keith, but it became an important issue with Todd. In Keith’s classes, his students often worked in groups. In Keith’s pre-calculus class, the students quickly realized that I knew mathematics and asked me to help them with their group work. Keith felt comfortable with me doing this. So on occasion I would interact with the students, asking them questions and helping them. Similarly, in Todd’s class, I sometimes helped students when they were working in groups. Often when Todd’s students were in groups, he was sitting at his desk grading, planning, or doing other activities. Therefore, the students turned to me. What became an issue with Todd was when he started interacting with me during instruction. He watched me while he was teaching, and if my facial expression changed, he would ask me what was wrong. A few times, he made mistakes on the board and he could tell from my facial expression that he was doing something wrong. One time, he even stopped class to ask me what was wrong. Another time, he asked me to explain to the students how to use a certain feature on the calculator with which he was not familiar. I became an authority figure in Todd's classroom because he asked me questions in front of his students. I was not comfortable with this role and tried to minimize the changes in my facial expressions in an attempt to address the problem. Apart from these instances, I sat in the back of the room and did not interact with the students or the teacher during class.

My relationship with the participants varied as far as the extent to which I knew them and interacted with them prior to the study. I had worked with Robin for 3 years on the assessment projects. I did some work with the school system outside of my duties as the doctoral student assigned to the project. As a contractor, I taught "train the trainer" sessions to prepare project teachers to inform teachers across the county how to use and access the item-bank created during the project. Robin was one of the trainers. I had many conversations with Robin, and I would say we became friends. Robin most likely viewed me as a colleague—an equal. Similarly, I had observed some of Keith's classes the year before the present study and interviewed him four times as part of a small study I conducted for a qualitative research class. We talked about educational issues and discovered that we thought similarly about many issues. Keith and I also became friends and I sensed that he viewed me as a colleague as well. I barely knew Todd when the study began. I had not interacted with him outside of the assessment project. Todd did not know me and the only role he had seen me in prior to data collection was as one of the project leaders. Todd and I were roughly the same age, but I had more teaching experience and more education than he had.

Limitations of the Study

With respect to generalization in the traditional sense, which is maximized by controlling sample size, using random sampling, and so forth, the findings of this study are not generalizable. However, two different conceptions of generalizability can be used to address the issue of generalizability. Merriam (1998) explained, “Drawing on tacit knowledge, intuition, and personal experience, people look for patterns that explain their own experience as well as events in the world around them” (p. 211). A deep understanding of the particular can allow an individual to recognize similarities across different contexts. Suppose I lifted a rock and saw a snake. I could not generalize from this one particular situation; that is, I could not say that there is a snake under every rock. But I am now attuned to the presence of snakes under rocks. I might be more careful the next time I pick up a rock. Now I realize that snakes go under rocks. If, in this study, I found that Factor A affected a given teacher’s use of open-ended assessment, then when I think about other teachers, I would at least be attuned to looking for Factor A as a possible influence on their use of open-ended assessment items. This ability to apply knowledge of the particular to different situations is referred to as naturalistic generalization. Even across very different contexts, certain ways of thinking can be useful in understanding specific situations. The knowledge that a certain factor influenced the way one teacher used open-ended items can be useful in thinking about the way another teacher may use them, even if the particular situations are different.

Another conception of generalizability is the notion of user generalizability. This type of generalizability “involves leaving the extent to which a study’s findings apply to other situations up to the people in those situations” (Merriam, 1998, p. 211). The researcher cannot anticipate all the situations in which her or his research will be used. To a great extent, it is up to the reader to decide how much the research applies to her or his specific situation. To this end, I have attempted to provide a thick description so that the reader can assess the relevancy of my research to her or his own situation.

Another limitation of the study was that reflective thinking became an important piece of my theoretical framework after most of the data were collected. It was during data collection that I became aware of the importance of reflective thinking in teachers' use of open-ended assessment items. I asked a few questions about reflection in later interviews, but I had not yet developed a theoretical way of thinking about reflection when I asked those questions. Had I a better theoretical grasp on reflection, I probably would have asked different questions and probed in different ways. Although I was able to make inferences about the teachers' reflection from the data I gathered, it would have been better to have gone into the study with a theoretical orientation toward reflective thinking.

CHAPTER 4

CASE STUDIES

The case studies inform the reader of the themes that emerged from the initial phase of data analysis. In this chapter, I describe each teacher's beliefs in the domains of mathematics, teaching, learning, and assessment. In the domain of mathematics teaching, I present each teacher's view of the purpose for teaching and the role of the teacher. Under that topic, I also describe how each teacher approaches making decisions about her or his teaching. In the domain of mathematics assessment, I present each teacher's view of the purpose of assessment and what he or she views as important characteristics of assessment.

The Case of Robin

Robin was a white female who had been teaching for 26 years. The school she taught in had approximately 2250 students from a suburban population. Eighty five percent of the 180 faculty held postgraduate degrees. The school was a Georgia School of Excellence for the school year 1995-1996. It was a U.S. Department of Education Blue Ribbon School for the school year 1997-1998. Robin was the mathematics department chair at the school. Robin was a leader in her county, serving on several committees and project teams and as the lead teacher in county staff development courses. She had taught every level of secondary school mathematics from pre-algebra through calculus, and she had also taught middle school mathematics. As an undergraduate at a large university, she took 60 quarter hours of mathematics courses, including courses in computer programming, number theory, and topology. She graduated magna cum laude. She completed a master of education degree in mathematics education, during which she took courses in modern algebra, geometry, and analysis.

The year following data collection, Robin was voted teacher of the year at her school and also for the entire county.

Themes Related to Mathematics

Robin felt that mathematics was like a puzzle. She enjoyed the process of fitting the pieces together for herself and for her students. She liked the logical structure of mathematics. Asked whether mathematics was discovered or invented, Robin said she believed it was discovered. She explained,

> Like Euclid, when he put all the theorems in together into one unique whole of all these other pieces. And he put them together and put them sequentially—of how you prove one, then you go to the next, and then you go to the next. I think it was a discovery that was already there; he just kind of pulled it together. You can't say he invented geometry; he just kind of discovered how the pieces fit. (Interview 1, 1/26)

When asked whether mathematics was creative, Robin said, "Maybe in the sense that you are using your brain to kind of create a solution. Creative in coming up with new mathematics—no" (Interview 1, 1/26). She did not view mathematics as a human invention. She viewed mathematics as something that is present in nature for us to discover. Robin told a story of attending the state governor's honors summer program in English as a high school student. She said people were sitting around discussing opinions about English, and that was not exciting to her. This experience influenced her decision to be a mathematics teacher—she did not like discussing opinions. She wanted something more exact. She liked mathematics because it is logical and structured. Robin defined mathematics as "a collection of useful knowledge that humans have tried to develop into a structured, connected set of theorems, rules, and truths" (Interview 7, 5/1).

Robin had a well-connected view of mathematics. When she talked about mathematics, she often spoke about how topics were related to other topic s. When asked how she would teach logarithms, she said she would help students understand the concept of logarithmic growth. She explained,

> How many numbers are going to have a logarithm base five between three and four? Well, a whole bunch, right? Because you are going to go from one hundred twenty-five to six hundred and twenty-five. So, you have got a whole bunch of numbers now whose logarithms are between those two digits, those two units. So, well what happens on the next one [between four and five]? So, what are you doing? You are showing them logarithmic growth because they don't grow very fast. I mean, look at the mapping if you want to think of it that way. Look into the mapping of all of these numbers into this space, and there is even a lot more numbers into the same amount of space. (Interview 7, 5/1)

She pointed out that she wanted students to understand the logarithm as a function and how that function behaved and not merely to know the definition of a logarithm and how to translate expressions and equations from logarithmic form to exponential form. I could easily tell from our conversations that Robin's mathematical knowledge was solid and well-connected.

Themes Related to Teaching Mathematics

For Robin, teaching was all about understanding. She believed that teachers should pay attention to topic development , making sure that students saw the connections between what they were learning and what they already knew. Robin took a reflective approach to her teaching, constantly evaluating her effectiveness as a teacher. She thought deeply about her responsibilities as a teacher as well as about her beliefs about teaching. At times, Robin felt a certain tension between her responsibilities and her beliefs.

Purpose of Teaching

Robin believed that teaching should promote understanding. She did not want her students to merely be able to compute correct answers to mathematics problems. She wanted her students to understand why procedures work, how they are connected to other procedures and how the procedures could be applied in the world. She believed that it is important for students to see how certain skills are developed from other skills and how many mathematical skills are the reverse processes of previously learned skills. During instruction, Robin asked questions such as, “Why is 40 the common denominator?” (Algebra I, 2/11), “Tell me why $x^2 + 4 \neq (x + 2)(x + 2)$” (Algebra I, 1/27), “What does it mean that I have two answers?” (Algebra I, 1/19), and “Do you notice a pattern?” (Calculus, 1/27). Her focus was clearly on more than computing answers.

Recall that Robin believed that mathematics fits together. Like a puzzle, the pieces of mathematics can be linked together in some way. This view of mathematics played out strongly in Robin’s beliefs about mathematics teaching. Part of a teacher’s job is to help students link the pieces of mathematics together in a meaningful way. The idea of connections was central to Robin’s beliefs about teaching. She explained, “I like to always give my students a reasoning or a kind of a developmental sequence of how this came about or how you can fit this in with this” (Interview 1, 1/26). Later she said,

> If there is anything that I think I do better than other teachers, it’s showing the connection from one topic to the next….My whole emphasis in mathematics is, Don’t learn math as a bunch of rules, but develop the why and the understanding of it, or the connections between. (Interview 5, 3/10)

Perhaps more important than Robin’s emphasis on connections is that she carried it out in her teaching. *Connections* was not just an NCTM buzzword to Robin; it was more than rhetoric, it was an essential aspect of mathematics and mathematics teaching. Consider the following teaching episode from her Algebra I class on January 31. Robin asked, “Is $(m - 3)$ the same thing as $(3 - m)$?” A student said no, because if $m = 1$, then $m - 3 = -2$, but $3 - m = 2$. Robin chose many different values for m. The students concluded that $(3 - m)$ is the *opposite* of $(m - 3)$. She went on to create the chart in Figure 3, writing the expression and asking students for the opposites. Then Robin asked, “Why am I stressing all this opposite business?” What if I had $\frac{-2}{2}$ or $\frac{8}{-8}$ or $\frac{-10}{10}$? Could I reduce? You can cancel, but you have to leave a negative 1 somewhere. So, back to number 4, $\frac{m-3}{3-m}$ is negative 1." Robin took a substantial portion of class time to connect simplifying rational expressions to what the students already knew about simplifying rational numbers.

Expression	Opposite
2*x* – 5	-2*x* + 5 or 5 – 2*x*
x – *y*	*y* – *x* or -*x* + *y*
-3 + *m*	-*m* + 3 or 3 – *m*
-5 – 7*x*	7*x* +5 or 5 + 7*x*

Figure 3. Robin's opposite expression chart.

Similarly, on February 16 in her Calculus class, Robin had the students compute the area of the first quadrant of the unit circle using methods from integral calculus. After they found their solution, they used the formula for the area of a circle from geometry and saw that the two methods yielded the same answer. On January 31, Robin put the chart in Figure 4 on the overhead projector for students to copy as part of the their notes for the day. Robin emphasized connections when speaking about teaching, and she also emphasized connections during her teaching.

Arithmetic	______________________________	Algebra
Fractions	______________________________	Rational Expressions
Multiply	______________________________	Multiply
$\frac{3}{5} ? \frac{5}{8} = \frac{3}{8}$	______________________________	$\frac{3(x+3)}{15} ? \frac{5(x+2)}{(x+3)} = (x+2)$

Figure 4. Robin's connection chart.

Another way Robin showed her commitment to teaching for understanding was by emphasizing applications in mathematics. For example, when introducing the process for computing the area under a curve in the Calculus class, Robin asked the students why they might need to know the area under a curve. The students discussed finding the area of a plot of land. Whenever possible, Robin would tell students about or ask them for a real-world situation in which a certain process could be used. Before studying systems of linear equations, she posed a problem regarding items bought at a concession stand at a ball game. The problem could have been solved by setting up and solving a system of linear equations. The students used a guess-and-check approach. After teaching the unit, Robin posed the problem again. She described what happened. She had said,

> "Okay, now we have just spent the last two weeks solving systems of equations. I want you to try this method on this problem that you have seen before." And so I pulled out the transparency, and I gave it to them. And they didn't remember the answer. I was kind of surprised about that, and I said, "Now, how would you work that now?" And the quickest ones off the bat, you know, "Well, call this *x*, call that *y*," did it in systems of equations, and they were done. I said, "Do you realize what you did was that you just showed me that you knew how to apply the mathematics we have been learning?" And when we started this chapter, it was just guess and check." And they went "Oh, yeah, okay, I guess we are learning something after all". So, I was real tickled on how that turned out. (Interview 1, 1/26)

It was exciting to Robin that students saw a reason for learning the mathematics. She believed using applications of mathematics helps students understand the mathematics better. For example, she used a motion detector along with a CBL (Calculator-Based Laboratory) for an activity in which students walked in front of the motion detector, and a graph of time versus distance was displayed. She said

> To me, [meaningful mathematics is] showing just the motion of walking away from the wall and getting into a line of positive slope; walking toward the wall and getting a line of negative slope. The relationship between what is the graph, you know? Here is the graph. It is distance and time. What does slope mean? You know, walk faster, oh, that was steeper Why? Why is it steeper? You know, because you are changing the distance quicker and in less time. You know, that to me is some meaningful mathematics. (Interview 6, 3/15)

Robin thought it was important for students to be able to use the mathematics they learned in class to describe real-world phenomena. She believed that such use would promote understanding.

Robin had developed a belief that one way to help students learn with understanding is to help them recognize that many procedures in mathematics are the reverse of other procedures; for example, factoring reverses multiplication, and anti-differentiation reverses differentiation. Last year, Robin had a particular Calculus class that had struggled with topics from differential calculus. She was worried that the students would really struggle with topics from integral calculus. She thought about how to approach anti-derivatives for this particular group. She explained,

> Well, [I thought] maybe I should just emphasize this going backwards, and talk about it in terms of inverse operations, you know. If you follow a road map to get somewhere, how do you get back again? And if you add, how do you get back? You subtract, you know? And I started talking about that with them, and we started talking about derivatives and the power rule. And it just worked, it just worked great. And, in fact, they caught on to integrals so much better than I thought they would. (Interview 1, 1/26)

Once Robin had found success with this method, she used it often in her teaching. In the Algebra I class, she asked her students what the relationship was between multiplying and factoring. The students said they were backwards of one another. Robin noted that one operation "undoes" what the other one does. She was fond of the kind of questions that asked students to do the opposite of what traditional questions ask. For example, she asked the Algebra I students to create a rational expression that reduced to a given rational expression. She liked the fact that students were having to go backward to answer this question. Robin believed that going backward deepens a student's mathematical understanding—that it takes their understanding to a higher level.

Role of the Teacher

Robin was a reflective person who thought deeply about teaching. Several times during data collection, she would tell me when we sat down for an interview that she had been thinking about what we had talked about in the last interview. She wanted to revisit topics from previous interviews once she had time to think about things more. For example, in Interview 4 (2/29), I asked Robin, "Is it possible that you have a student in your class who you don't physically see their solution to any problem other than on a quiz or test?" Robin was quick to tell me that this was not possible because she walked around the room looking at homework, called students to the board, and collected warm-up activities. But at the next interview (3/10), Robin said,

> Okay, well you asked a question, "Was it possible that the only work I ever saw from a student was only a quiz or a test?" And of course my initial reaction was no, that's not possible. But, the more I think about it, it is possible. It is possible to have overlooked a student or walk by that student's desk when they hadn't really done anything on their paper, and so … you didn't really look closely at their work. (Interview 5 , 3/10)

Robin did not just answer the question and move on. The question had stayed with her, and she re-evaluated her initial response. That was typical for Robin. Another time we discussed the fact that students have a hard time understanding factoring. I asked her what it was that the students did not understand. She said,

> I guess, Wendy, they don't know what factors are. I mean, they don't realize that maybe it boils down to the symbolism, the variables. Maybe we haven't built up the whole concept of the expression with *x*'s in it. (Interview 3, 2/11)

Later, when I was sitting in Robin's class to observe, she explained to me that she had been thinking about what we had talked about in Interview 3. She said she had been thinking about how teachers emphasize the procedural, especially with factoring. Students do not understand that $x^2 + 8x + 15$ is a *number*, and that its factors are two numbers that multiply to yield that number. Perhaps, she said, if we emphasize plugging in some values for *x*, it would help. For example, $x^2 + 8x + 15 = (x + 3)(x + 5)$. Let $x = 2$. The factors would be 2 + 3 and 2 + 5 (5 and 7) so the product $x^2 + 8x + 15$ should be 35, and it is (because $2^2 + 8(2) + 15$ is 35). After our interview, Robin kept thinking about what it was that students do not understand about factoring, and she devised an alternative way of presenting material that she thought might enhance understanding.

Robin's reflective nature kept her thinking about her responsibilities and her beliefs, and she often found herself in conflict between the two. Robin believed that it was her responsibility to cover the entire curriculum, but that belief sometimes conflicted with her belief that teaching should promote understanding. She saw teaching for understanding as difficult given the curriculum and time constraints under which she worked.

I asked Robin if she was able to be the best teacher she could be. She responded by saying,

> You are just so pushed for covering this curriculum objective, and this curriculum objective, that you can't really slow down and do it as thoroughly as you want to do it. And so, in that sense, no, I think I could be a much better teacher if I had more time to develop topics. (Interview 2, 1/31)

There were times when Robin chose to teach a topic differently than she wanted to because of lack of time. For example, I observed her the day she taught the slope-intercept form of a line. The last 15 minutes of class, she explained that in the equation $y = mx + b$, the "*m*" was the slope and the "*b*" was the *y*-intercept, or where the line crossed the *y*-axis. She explained how to put a point on the *y*-intercept, count using the slope to determine a second point, and then connect the points to graph the line (Observation, 3/10). Later that day, during an interview, Robin discussed a different method of teaching the slope-intercept form of a line. She described a discovery, or inductive, approach to teaching the topic in which students would graph lines using a table of values (a skill they had already mastered) and then notice patterns, eventually discovering that the *y*-intercept is the "*b*" in the slope-intercept form of a line and that the slope of the line is the "*m*" in the equation. She explained her thinking before teaching this topic:

> Well, you know you've got 15 minutes. You can't do your guided discovery in 15 minutes. You certainly can't do any inductive learning in 15 minutes. So you come down to this, okay, guess what? This is a shortcut, you know. If you notice this equation, that equation is going to tell you the slope, it's going to tell you the

> *y* intercept. You know… I didn't feel good about that lesson at all! And uh… and I don't think the students were ready to really assimilate. I don't think they had assimilated what, you know, what I was trying to get across to them either. So it's kind of an unsuccessful lesson. And part of it was, that you just feel pushed to do it; uh, today is the day that we've got to do it. So… you know, it's just a matter of, how are you going to balance everything that you need to do? (Interview 5, 3/10).

Robin described how she could have taught the lesson in a way that would better promote student understanding, but enacting the lesson she had made a decision to just tell. She had made a conscious decision not to use an inductive approach but instead to tell students "the rule." She was not comfortable with that decision. Reflecting on it, she said the lesson was unsuccessful. But finishing the curriculum was important enough to Robin to sacrifice understanding for it.

Robin felt obligated to finish the curriculum for two reasons. First, she was involved in writing curricula for her county. She felt that if the writers could not finish it, how could they expect others to? Secondly, as department head at her school, she felt that she should be a model for the other teachers in her school. Robin was to serve on a curriculum committee the following year, and she said, "I'm hoping that on this curriculum committee that we're on, that we do start narrowing down the topics" (Interview 5, 3/10).

Robin did not merely go through the motions of teaching. She was dedicated, reflective, confident, experienced, and guided by her beliefs. But she really struggled because of her commitment to the curriculum. It was a daily struggle for Robin to mediate between her beliefs about teaching for understanding and her commitment to finishing the curriculum. During data collection, I asked Robin questions about her beliefs, and we discussed her teaching. We discussed that her beliefs about teaching were not always aligned with her actual teaching practice. That discrepancy really bothered Robin; in fact, one day she got quite emotional, saying she was a hypocrite. Robin professed to believe that using open-ended items was an important way of gathering information about student understanding. Yet she rarely used open-ended items during class. Open-ended items did appear on her tests but were seldom used as part of instruction or as homework questions. Robin began to question why she was not doing what she believed she should do. She said,

> First you go through cycles of "I should be doing more, why am I not doing more?" And then you go into cycles, "Well, I sure am doing a lot more than most people, and I'm doing this as much as I can work in or as much as I have time for or as much as my energy allows me to do." And you know, you just go around in circles like that because you have so many parameters, you know? On one hand, you've got the curriculum guide that you are juggling. You got a class of thirty individuals that you are juggling, and to be honest with you, they would rather have a ditto of twenty problems that are just alike. And then you have, "Well, what does my own philosophy say I ought to be doing?" And you are just juggling all of those things. And another thing that I have to juggle is being the department chair and feeling like, well, we have to go along with this curriculum. I have to help my other teachers come with me. (Interview 7, 5/1)

Robin concluded that she used much more open-ended assessment than before, and that everything could not change overnight. She recognized using alternative assessment as a process—one with which she was not finished.

To a great extent, Robin was a traditional teacher. Although she did have a focus on understanding and making connections, and she used technology as an integral part of her teaching, on any given day if you walked into Robin's classroom, you would see her lecturing. Students were seated in rows and usually copied notes off the overhead projector or worked problems as Robin walked around the room. She focused on procedures and on the structure and steps that must be followed to understand mathematics. She wanted students to understand why the procedures worked, but nonetheless, the focus was still on the procedures. Even though Robin emphasized understanding in her teaching, she was attempting to transmit the understanding. She pointed out connections. She explained the reasons that procedures worked. When I asked her about the ideal environment for learning mathematics, she discussed resources such as chalkboards, books, measurement tools, and calculators—tools that were consistent with a transmission model of teaching. When I asked her what her ultimate responsibility as a teacher was, Robin responded by saying, "Well, I think my ultimate responsibility is to kind of turn the light on to my students so that they can learn mathematics" (Interview 4, 2/29).

Decision-Making

Robin valued her many years of teaching experience. She believed that because of her experience, she was justified in making decisions about teaching. I asked her what she would do if she did not like the curriculum she was asked to teach. She said,

> I don't know, Wendy, what I would do. Uh, I'm stubborn enough to say, well, I'm going to do it my way. But I've had 20 some years of experience, and uh, you know a first-year teacher who comes in and says, "Well, I want my students to understand everything that they're doing," and only covers the first five chapters of Algebra I is maybe saying the same thing that I'm saying. But I've been teaching 25 years. And I think that if I have made a decision that, uh, the pacing was wrong or that these objectives were not necessary, …I think that would be more reasonable. (Interview 4, 2/29)

Robin had faith in her experience and felt justified making decisions based on that experience. She felt that one's experiences with mathematics helps form one's beliefs about teaching. She said, "I do think there are some teachers who believe that they are doing the right thing just by teaching Skill 1, Skill 2, Skill 3. And I just think they believe that because that is their own experience with mathematics" (Interview 7, 5/1).

Certainly the county curriculum guided Robin's teaching but also, her beliefs guided her teaching. I asked her to whom she felt responsible in her teaching. She responded,

> Well, in a sense I’m responsible to the county, following the county curriculum and having a county instructional folder. And in a sense I’m responsible to the administration of our local school. And in a sense I’m responsible to the parents. And I’m responsible to the students. But uh, but what it really boils down to, the bottom line is, I feel a responsibility to myself more than I do to any of those other outside groups or people or anything. Because I’m…gonna do what I need to do in the classroom because of why I think I’m here. Not because [the] county tells me to cover XYZ or to have an instructional folder that has all my lessons in it, blah, blah, blah. But because, you know, I’m a serious committed person to teaching, and I’m going to do it the way that I feel it needs to be done. (Interview 4, 2/29)

Robin’s dedication to teaching, her reflective nature, and her commitment to her beliefs about teaching made her a confident decision maker in her classroom.

Themes Related to Learning Mathematics

When students did not do as well as she thought they should, Robin did not blame the students. She was willing to consider that the way she taught the material may have been a cause for the students’ lack of success. A group of algebra students did not grasp a topic as well as her previous classes did, which led Robin to comment,

> I don’t know if it was the group I had the year before that was just a little bit sharper as far as understanding the real meaning of *variable*, or whether I just did a better job teaching it, or whether—I don’t know. (Interview 2, 1/31)

She looked for reasons students did not understand and considered that she might be one of those reasons, instead of assuming that the students did not try hard or study. When she developed an alternative approach for teaching anti-derivatives to her weak Calculus class, she said,

> But I just, you know, had pulled the lesson together so much better [teaching antiderivatives], I guess. You know, I related it to a lot of other things that they were familiar with, I guess. And I think, too, I was a little bit clearer with what I

> wanted to do and where I wanted to go with the idea, too. So, you know, you just have to sit down and reflect about it, "Okay, how am I going to teach this lesson?" Sometimes you do more of that than others, and I guess because I was dreading teaching it, I spent more time thinking about it. (Interview 1, 1/26)

When students did better than she expected, Robin thought about what she had done that better facilitated their understanding. When they did worse than she expected, she thought about what she might have done that hindered their understanding and how she could do it better in the future.

Themes Related to Assessment

Robin believed that teachers and students could and should learn from assessment. She used assessment in a meaningful way to inform her teaching. Robin believed that assessment should be fair and should include opportunities for students to show a conceptual understanding of mathematics.

Purpose of Assessment

Robin felt that one purpose of assessment was to provide feedback about her teaching. She believed that both students and teachers could learn from assessment. For example, Robin gave the open-ended item in Figure 5. She was surprised that there were students who could not give any answers. She commented,

> So, of course, that tells me that, well, next year I need to go back and think about that again. And, you know, maybe build up the whole idea, the fact that I can change the middle term, because I didn't really do that this year. I didn't say, "Okay, look at all of these three. They have the first coefficient the same and the constant term the same, but here are two different items with the same middle term. What makes them different? And are they both factorable?" So, you know, next year when I start talking about factoring, I am going to at least include that idea. Because I felt like I overlooked it, or I would have had at least one answer from everybody. So that made it a good item, too, that fact that not only

did I get information about what they could do, I also got information about what I could do. (Interview 2, 1/31)

Find all linear terms so that the trinomial will factor with integer coefficients. How do you know that these are all the possible terms?

$x^2 + \_\_\_\_\_ - 6$

Figure 5. Robin's factoring item.

There were instances in which Robin saw a student's method for working a problem and decided that the student's method made more conceptual sense than the method she had taught. When speaking about percent problems, Robin said, "[The students] can do those things if you set them up into, 'Okay, *is* over *of* equals percent over a hundred.' You know they can do that thing, but it doesn't mean anything to them" (Interview 7, 5/1). One of Robin's students had a method for answering percent questions in which she related everything back to one percent. Robin said,

> To me, that girl knew what she was doing. I mean, she related it to, and she could do all three of those types of problems by relating it to, "Okay, if I have got five percent and it is this, what is one percent?" And to me, that's so much more meaningful than to set up this proportion, cross multiply, and divide, you know? Number 1, it shows a little bit of sense with numbers. And I think that other method is just pushing numbers around on the page.…They don't understand the relationship between numbers. And so, I am going to use that [method the next time I teach this topic]. (Interview 7, 5/1)

Robin did not merely grade her students' responses to assessment items, she thought about how the students were thinking and how her instruction impacted or should impact the way they were thinking. She was not willing to immediately place blame on the students when they missed an item. She did not assume that students were not prepared or had not practiced enough. She was willing to consider that her teaching may not have facilitated the student's understanding the way she had anticipated it would. . Robin sought to inform her teaching through her assessment practice.

Robin used the information in a student's response not only to ascertain that the student had not mastered a skill or concept but also to understand exactly what misconception the student had. For example, Robin asked her students to write a rational expression that reduced to a given rational expression. From one response, she made the following observations:

> What they thought they were doing was squaring both the pieces. Well, they weren't really squaring the pieces, and they weren't really multiplying them both by the same number. But they still said just because they squared every term in there that it would reduce to the one we wanted. Now that student has major problems. I mean, that shows a real lack of even understanding fractions, because a fraction 4/9 does not reduce to 2/3. And yet that is what the student was showing me algebraically. (Interview 3, 2/11)

Again, Robin did more than just mark the student's paper wrong. She determined that this student believed that one can square the numerator and denominator of a fraction and have an equivalent fraction. This was powerful information that Robin could use in her teaching of this student.

Not only did Robin believe that she should learn from assessment, but she also believed that students should learn from assessment as well. She encouraged students to think about their own understanding. For example, at the beginning of the second semester, Robin asked her Calculus students to work in groups to solve problems from the previous semester's final exam. The groups were to solve the problem, give an alternate solution, and provide hints, common mistakes, and pitfalls. Several times, I heard Robin ask for volunteers to read a good response to a particular item. She encouraged the students to think about their own responses and what was lacking from them. Robin became discouraged when students were not willing to correct themselves. She said,

> The ones who self-correct, or the ones who really look at their homework and re-evaluate what they did or make corrections or do some reflecting on, "What was it I was doing wrong?" are the ones who are going to come out with better skills in the long run. (Interview 6, 3/15)

Several times, Robin told students to correct their work, to write down what they did wrong, or to write down the steps for doing a procedure correctly. She wanted the students to reflect on their own thinking.

Robin would never claim that mathematical skills are not important. In fact, she often worried that she emphasized skills too much. But she believed that skills and procedures are not the only thing that should be emphasized in teaching or in assessment. She believed that assessment should go beyond the purpose of merely assessing skills to include an emphasis on concepts. She said,

> My fundamental belief is that students should know why mathematics works. They should see the reason for what they are doing. And their assessment should be able to reflect that they see the reason for what they are doing—that they can explain what they have done. And that they could take something that is a little bit different from a homework problem and still come up with the answer to it, because they understand mathematics. (Interview 5, 3/10)

<u>Characteristics of Assessment</u>

Anyone who has spent any time talking to teachers knows that they will usually go out of their way to be fair. Not surprisingly, Robin said she believed that assessment should be fair. What is interesting is what she meant by *fair*. Fairness, in Robin's eyes, was multi-dimensional. First of all, in order for assessment to be fair, it must be aligned with instruction. Secondly, fairness meant including opportunities for all students to show their mathematical understanding. Robin explained, "In *fair*, I mean that you are actually putting questions on the test that match the kind of lessons that you have taught—not only topics but the amount of time, the amount of energy, the amount of depth" (Interview 2, 1/31). It angered Robin to think that some teachers use publishergenerated tests without ever considering whether those tests emphasize the same topics they emphasize in their teaching. In order to be fair, teachers must ask themselves, “Was this topic taught so that the students could master it?” (Interview 2, 1/31) before assessing the topic. It is important to note that Robin did not feel obligated to give only test items that were like ones to which the students had previously been exposed. She felt comfortable giving students higher-level items they had not seen before but for which they had the foundation to answer.

Robin believed that fair assessment should help her sort her students along a continuum of understanding. Assessment should be discriminating, distinguishing the As from the Bs, and so on. But beyond just helping her determine grades, Robin expected more from her assessment. Through asking open-ended items, Robin came to believe that there are students with a conceptual understanding of mathematics but who make mistakes with skills and procedures. These students, according to Robin, are unfairly assessed by traditional assessment questions. Consider the case of Alex, one of Robin’s students. On an Algebra I test, Robin asked 17 traditional questions and 3 open-ended questions. Alex scored a 65% on the test. He got full credit on the open-ended items and all of the points he missed came from the other items. Figure 6 shows the responses Alex gave to the open-ended items and the mistakes he made in the traditional items.

Open-ended question: Mark claims that $\frac{x+3}{x+6}=\frac{1}{2}$ because the *X*’s cancel and $\frac{3}{6}$ reduces to $\frac{1}{2}$. Is his reasoning correct? Justify your response.	Traditional question: Multiply. Simplify your answer. $\frac{x^2-4}{2x-8} ? \frac{x^2-16}{4x-8}$
Alex’s response: He is incorrect because the *x* + 6 and the *x* + 3 must stay together.	Alex’s response: $\frac{(x+2)(x-2)(x-4)(x+4)}{2(x-4) ? 4(x-2)}$ $\frac{x^2+8}{8}=x^2$

Figure 6. Alex’s responses to a traditional and an open-ended item.

Besides the fact that Alex multiplied (x + 2) and (x + 4) and got x^2 + 8, in his last step, he canceled the 8s. This cancellation was contrary to what he wrote in the open-ended response. Alex troubled Robin. She said, "I did Alex an injustice by giving all of those skill problems and only two or three alternative assessment problems" (Interview 7, 5/1). Another time she said, "Some students just seem to make those careless mistakes all through when they are really able to do more of the mathematics than they are getting credit for" (Interview 6, 3/15). Many teachers, when trying alternative assessment, worry about whether they can be fair when using open-ended items. Robin asserted that it can be unfair not to use open-ended items. She explained,

> I've seen several of my students in Algebra I who will miss all of these little details in working out the problem, but yet they know what they are doing. They can explain the alternative item and just get it right and then just miss all of the details in the other. So in a sense, it is not fair to them not to put [alternative items on the test]. (Interview 6, 3/15)

In Robin's view, providing opportunities for all students to show their mathematical understanding meant giving students who have a conceptual understanding of mathematics but lack a facility with mathematical skills a chance to demonstrate the kind of understanding they have. She noted that it is not always those students with a mastery of the skills who give the most eloquent answers to open-ended items. She said,

> And little Cindy Carmichael—who is a good thinker but she doesn't follow through with the mechanics very well, very careless, makes a lot of errors and things like that—she had a great answer, too. And so I really enjoyed the next day. (Interview 3, 2/11)

Robin recognized that open-ended items gave different students the opportunity to shine. Not using these kind of items would, in Robin's eyes, be unfair.

Robin wanted assessment to be a fair way of gathering information about student understanding, and she recognized that creating fair assessment was problematic. For example, Robin talked about when students do poorly on a test. She told me,

> There is not a good way to get around how you are going to mark them, because I have had classes where if I threw out a test, they didn't take the next one seriously….So you don't get anywhere by throwing one [test] out completely. And then again, you throw out an item, then you are really penalizing the kids that spent more time on that item. (Interview 2, 1/31)

Robin recognized the complexity of assessment and viewed it as problematic.

Robin thought it was important that mathematics assessment include more than just algorithmic, procedural thinking. When speaking about the possibility of standardized end-of-course testing, Robin said, "If it was a test that was simply skills, no high-level thinking skills, no thought questions, if it was just strictly 'factor this divide this,' that would scare me" (Interview 6, 3/15). When asked why it would scare her, Robin explained that such a test would communicate to students that mathematics is just a set of skills, and she believed that there is more to mathematics than just skills.

Some teachers worry that decreasing their use of traditional items could put their students at a disadvantage on standardized tests. Robin said,

> I guarantee you that the students that came out of a course like that where 80 percent [of the test items] were open-ended, I think they could answer any SAT item that has ever been published. I really do, because they would know their mathematics. They would be able to communicate it. They would be able to think on their feet. (Interview 7, 5/1)

Robin liked assessment items that require students to rethink something or to show their creativity. She liked that open-ended items provide students the opportunity to look at mathematics from a variety of perspectives (e.g., graphical, algebraic).

Although Robin believed in the use of open-ended items as an important piece of her assessment program, such items were not common in her teaching. Open-ended items appeared on every test, but only occasionally did Robin ask open-ended questions during class. Robin explained that using open-ended items takes extra preparation. Further, the county curriculum guide contained a set of behavioral objectives that focused on skills, and Robin felt obligated to teach and assess those skills. Robin felt strongly about finishing the curriculum, and because open-ended items take more time for students to complete and more time to discuss, their use in class was limited. Open-ended items are also more difficult to generate than traditional items. Despite all these constraints, Robin still felt that open-ended items are important. She said, “You know, you do what you can, and every year you are going to build up more and more and more. But you know, it [using open-ended items as part of instruction and assessment] is not just going to happen overnight” (Interview 5, 3/15). Robin realized that implementing open-ended assessment in her class was a process. She explained that she used many more open-ended items that year than she did the previous year. And the process got easier. Robin explained “Once you start writing [open-ended items], and once you start using them, then it is easier to write them and come up with them and discuss this question or things like that. But again, it is just going to take time and doing it more” (Interview 5, 3/15).

The Case of Todd

Todd was a white male in his third year of teaching. The school he taught in had a student population that was over 95% African-American. The school was a mathematics and science magnet school. Prior to becoming a mathematics teacher, Todd had worked as a buyer for a whole food and beverage company. His first degree was in finance. Todd was dual-certified in science and mathematics and taught science classes as well as mathematics classes. The highest level mathematics course he had taught was Algebra II which he was teaching for the first time at the time of the study.

Themes Related to Mathematics

Todd saw mathematics as an unchanging universal constant. He liked the concreteness of mathematics. He said that with mathematics,

> There is always an answer. While there may be trouble finding the answer, there is always an answer of some sort, so I am good with those things. I like dealing with concrete issues where there is an answer of some sort. (Interview 1, 2/8)

When asked whether mathematics was discovered or invented, Todd said that it was both. He explained that while the mathematics was there all along, people provided ways for us to think about it or see it. For example, Descartes provided us with a way of visualizing geometric ideas with the coordinate plane. Todd recognized that people can see mathematics from different perspectives. Some people might view mathematics from a business perspective, others from an engineering perspective, and still others from a philosophical perspective.

Todd saw mathematics as a process, much like conducting a science experiment. He explained,

> If you conduct an experiment, you have to ask yourself a question. You have to then go through the steps of solving your question and then you have to see if your answer is actually going to work. So it kind of takes you through the steps that a normal mathematical problem would take you through, and, uh, observation, thinking about it, doing the math, and then checking what you got. (Interview 1, 2/8)

He saw mathematics as a set of useful rules and truths. He contrasted mathematics to science by explaining that one although could disbelieve a scientific theory such as evolution, “It’s hard to say I don’t believe in the quadratic formula, you know, I don’t believe in scientific notation—you either do or you’re not teaching math. It’s kind of the way that it is” (Interview 5, 3/1).

Todd saw two distinctive sides of mathematics—the philosophical and the useful. The philosophical side of mathematics was concerned with proving theorems and understanding why procedures worked. The useful side of mathematics was concerned with applying mathematics and doing the procedures of mathematics. Todd found applying mathematics and doing the procedures of mathematics more useful in his business career than the philosophical side of mathematics. Therefore, he was more interested in the useful aspects of mathematics than in the philosophical.

Todd's mathematical knowledge did not seem solid or well connected. His mathematical language was often imprecise. For example, several times he used the word *equation* when he should have used *inequality*. He explained, "I'm a little better at using the everyday language than I am about using the technical language, which is probably one of my faults" (Interview 5, 3/1). Todd made several mathematical errors on the board during the time I observed his teaching. For example, on March 1, he put Figure 7 on the board.

$$\frac{(-a)^8 c}{a^2 c^5} = -a^{8-2} c^{1-5} = -a^6 c^{-4} = \frac{-a^6}{c^4}$$

Figure 7. Todd's error in simplifying a rational expression.

When asked how he would teach logarithms, Todd said he would relate them to exponentials and try to show real-world applications of them. His discussion of logarithms lacked specificity, and when pushed to give a definition of a logarithm, he said, "I really would probably have trouble giving a good definition of logarithm off the top of my head" (Interview 7, 4/24). Todd was uncomfortable with this discussion. Todd knew a lot of mathematical facts and procedures but was not very interested in the mathematics behind those facts and procedures. He was more interested in the applications of mathematics than he was in the, as he called it, philosophical aspect of mathematics.

Todd's view of mathematics did not seem well connected. For example, when I asked him if he planned to teach the fundamental theorem of algebra and its consequence (an n^{th} degree polynomial has exactly n roots over the complex numbers) he replied, "That might be a little far for my students. I may bring that up. If it brings it up in the book, I'll definitely talk about it" (Interview 3, 2/15). Then he commented, "The thing that I think I'll get a lot of trouble with is that the kids will go, 'Well, what about no solution?'" When I reminded him that all polynomials with real coefficients had solutions over the complex numbers, he replied, "They are having a little trouble with understanding the complex. So, they might not be able to see the difference. They will look at the real number situation with the integers" (Interview 3, 2/15). Todd hypothesized that students would learn the fundamental theorem of algebra when they took calculus. He did not appear to have the number systems and the fundamental theorem of algebra straight in his own mind. He relied on the textbook, not his own knowledge of mathematics, to decide what mathematics was important for his students.

Themes Related to Teaching Mathematics

For Todd, teaching was all about preparing students to be productive members of society. He wanted his students to know that mathematics could be applied in the real world. His focus was on skills and procedures. He strove to be a supportive yet demanding force in his students' lives. It was important to Todd that his students view him as a mathematical authority.

Purpose of Teaching

Todd believed that teaching should provide students with a facility with basic mathematical procedures so that they could become productive members of the workforce. He said, "My whole idea behind teaching is to set my kids up for being solid workers in the workplace" (Interview 4, 2/24). He wanted students to realize the power of mathematics. He thought that students should understand that mathematics was useful and relevant for their future. He explained,

> Knowing mathematics is to understand the importance of it. Because if you don't understand the importance of it, chances are you're not going to take it seriously when it's taught to you, and you're never going to truly understand it. So I think understanding the importance of what math can do for you personally, knowing it can free you from the bounds of certain problems a lot of people have. So to know mathematics, I would say you understand its power, you understand the concepts, and [are] able to apply what you know in mathematics to your world. (Interview 1, 2/8)

It was important to Todd that students see the usefulness in mathematics. For example, when he introduced scientific notation, he explained that this notation had applications in astronomy, (e.g., if one wanted to report the distance between two planets), chemistry, (e.g., if one wanted to report the size of an atom) and biology (e.g., if one wanted to discuss the size of a cell) (Algebra Concepts, 3/1). When he introduced quadratic functions, he said there were applications of quadratic functions in business, science, and war machines (Algebra II, 2/15). Todd's focus was not as much on students being able to actually apply mathematics in the real world as it was on their ability to recognize where mathematics could be applied. For example, when he taught scientific notation, he explained why the notation was useful but when he explained how to convert between scientific and standard notation, he focused on the procedure. He showed students the rules for converting between notations but he never explained that the numbers, no matter how they were represented, had the same value. Todd was much more concerned with students understanding why they needed to know certain mathematical procedures rather than why mathematical procedures worked. He said,

> If I start going into the whole reason of why it works this way, in some instances,… it's going to be like, whoosh, you know, lost. I wouldn't try to blow the whole class away with a beautiful proof on the board, because they're going to get that in other classes here that teach that. And so, while I believe it's unbelievably important to understand why it works, if I can get them to understand why they need it, that's the first hurdle to overcome. And that's usually, with kids, the first thing that they want to know. They don't necessarily want to know why it works. They want to know "Why do I need to know this?" (Interview 5, 3/1)

Todd's lack of concern with why mathematics works was demonstrated when a student asked him why the leading coefficient of a quadratic equation had to be 1 in order to use the completing the square method. Todd replied, "Because it works in the formula. Trust me—there are some things in math that I won't be able to explain. It just works." (Algebra II, 2/14).

When asked if the NCTM *Standards* had affected his teaching, Todd explained that because he taught mostly low-level classes, he tended to

> put the standards down a level and look at more what I think [the students] need to know to get by. Are they going to be able to balance a checkbook? Are they going to be able to order if they are a buyer? Are they going to be able to calculate miles per gallon in their car? Those are they types of things that they are going to need to know every day. Can they calculate a percentage if they got a sale price? Are they going to be taken by someone? That's what I'm worried about because they are not going to be going to college, and they are not going to be applying the Pythagorean Theorem unless they are going into construction, you know? And in that case, I would want them to know that. (Interview 2, 2/10)

Todd believed that most of the students he taught would not go to college and would instead go immediately into the business world. He said, "If you can't readily use it, why teach it to them, they are going to forget it anyway. Focus on what they need, and consider everything else sort of fluffy stuff" (Interview 2, 2/10).

When Todd assigned homework, he usually assigned half of the problems in each textbook section (the odd numbered ones). He did not leave out the application problems. When he went over homework, however, he rarely spent time going over the application problems. He explained,

> I like to have [the students] at least attempt the application problems even though I don't spend a lot of time on them in class. I always give them two or three application problems—just part of homework—and then all of the others. I always try to include the critical thinking questions, which is like the alternative questions in a way, just because I want them to think about it. So I try to assign them a wide range of the problems. (Interview 2, 2/10)

Todd felt it was important for students to recognize that mathematics was applicable in the real world. However, he did not use class time to discuss application problems. It appeared that his purpose for assigning application problems was to merely expose students to them. First and foremost, Todd was concerned that students be able to "do the math." He felt that his students did not have the prerequisite knowledge that they should. Consequently, he explained that he put fewer applications on his tests than he would like to. "If I had kids that I really felt like were on level, you know, I would probably give many more application problems, many alternative assessment questions and only a few of the typical mathematics questions" (Interview 2, 2/10). When Todd was short on time and needed to delete a section in the textbook, he chose an application section to delete. It was more important to him that he cover the skills in the chapter because he believed that students were more likely to see a problem requiring them to graph a quadratic inequality on the graduation test than they were to see an application problem.

Todd's concern for the students' preparation for the workforce led his emphasis on skills and procedures in. He wanted students to have "a basic understanding of your four operations…basic algebra…how to solve for an unknown variable" (Interview 1, 2/8). Todd advocated a common-sense approach to teaching, in which students learn to think logically and be proficient in adding and subtracting positive and negative numbers and dealing with percents. Todd explained that he was "definitely more skill-oriented because my kids aren't. So to be conceptual, you need the skill level first" (Interview 4, 2/24).

When Todd was teaching compound inequalities in Algebra Concepts on February 2, he told the students that when there is an *and* problem, there will be one letter in the middle and two numbers on the outside—always. In his teaching, he was quick to provide students with a rule. There were times when Todd tried an inductive approach. For example, in Algebra Concepts on February 25, he was teaching how to divide monomials and gave the examples in Figure 8. He pointed out that you could subtract the exponents to get the answer. Immediately, he wrote the rule in Figure 9 on the board. Although he gave examples to illustrate the rule, the students did not come up with it themselves. Instead, Todd told them the rule.

$\frac{8}{4} = 2$ $\frac{2^3}{2^2} = 2^1 = 2$	$\frac{16}{2} = 8$ $\frac{2^4}{2^1} = 2^3 = 8$	$\frac{32}{8} = 4$ $\frac{2^5}{2^3} = 2^2 = 4$

Figure 8. Todd's examples of simplifying fractions with exponents.

Any number a where a cannot = 0 and integer m and n, $\frac{a^m}{a^n} = a^{m-n}$

Figure 9. Todd's rule for simplifying fractions with exponents.

Similarly, in Algebra II on 2/10, Todd asked students to graph $y_1 = x^2$ and $y_2 = -x^2$ on their graphing calculators. He pointed out that the two graphs were mirror images of one another and asked the students which one was going down. They answered $-x^2$. Todd asked them what they could conclude. Before anyone could answer, he said that if there is a negative in front of the first term the graph will probably open down. He gave only one example before providing the students with a rule. Several times Todd asked students to draw conclusions, but when they failed to make a quick conjecture, he told them a rule.

Role of the Teacher

Todd saw the teacher's role as a coach—one who is supportive and encouraging, but also demanding. It was important to Todd that his students feel comfortable enough in class to ask questions. He said, "I try to let my students know that they should always feel free to ask a question in class, to not be embarrassed about not understanding something" (Interview 5, 3/1). He wanted his students to know "that I'm behind you either way, good or bad" (Interview 1, 2/8). Todd compared being a teacher to being a gardener, where

> you can consider maybe the student coming into your class as the seed. And you plant your seed, and you have to nurture it and water it with the information, and give it sun and room to grow and things like that. And then it blossoms. You know, you could probably tie that into education by, you know, giving them food

by the education you give and space by letting people grow at their own pace. And you know, the blossoming would be passing the class. (Interview 1, 2/8)

Todd found success in nurturing and encouraging students. He told the story of Darnell, who had not been successful in mathematics prior to being in Todd's class. Todd encouraged and nurtured Darnell who ended up with the highest grade in the class. At the end of the semester, Todd gave him an award. Todd described Darnell's happiness, his smile, and his excitement as he told Todd that he was going to put the award on his refrigerator.

Todd was clear that, although it is important for teachers to be encouraging and nurturing, at the same time they must be objective and demanding. When Darnell was suspended, and Todd had to go to a parent conference, he let Darnell know that he was disappointed in him. Todd said, "[I'm] not giving him [Darnell] any special privileges just because he is my top student" (Interview 5, 3/1). He explained that being encouraging and objective go hand in hand:

> Being objective is probably the most important thing that you can do as a teacher. As, you know, just because, if you start playing the game of "Well, he's a nice student, so I'll give him these breaks," and other things like that, you start setting a dangerous precedent. (Interview 5, 3/1)

Todd believed in the idea of "tough love." It was not necessary that his students liked him. He wanted them to respect him and feel supported by him, but he explained, "It's not written anywhere that I have to be nice to anybody" (Interview 5, 3/1). Todd explained that the teachers the students perceived as nice were usually those who allowed students to take advantage of them. Todd admired his mentor [a more experienced teacher assigned as Todd's mentor] because "his demands are so high for his kids" (Interview 6, 3/8). Like a coach who pushed his players to their limit, Todd had high expectations for his students.

In addition to being encouraging and demanding, Todd thought that teachers should be adaptive. They should change with the times, learning to use modern equipment. They should not become stagnant, using the same lesson plans year after year. Todd believed that teachers should adapt to the particular students they are teaching. He explained,

> You have to be able to learn from what [the students] are doing. If they are, you know, if they're struggling in your class, and you're not picking up on it, then you're not doing well for the students. Where I teach, I would probably do a lot, I would probably go about the whole class differently in another school, because the students will be different. So you have to be able to read and learn from your students to know what you're going to teach. Some classes are going to be real quick in picking up on things. Others might be much slower and if you try to teach the same thing the same way to different students, you're not going to be successful in that. (Interview 6, 3/8)

Todd demonstrated his own adaptability on February 8 in Algebra Concepts. He assigned a problem for students to work in groups. The students were supposed to write an open sentence involving absolute value for the situation in Figure 10. Todd asked whether any students had never been bowling. One student raised his hand. Todd explained the sport of bowling and then it to basketball. He said, "Suppose he's within 10 baskets of this 30-point average. What does that mean? It means his points could range from 20 to 40." Todd related a situation that he thought his students might not find familiar to a situation with which he was certain they would be familiar.

Mary's bowling score was within 10 pins of her average score of 105

Figure 10. Todd's bowling question.

Beyond being encouraging, demanding, and adaptive Todd wanted his students to perceive him as a mathematical authority. He explained, "Let's say that someone asked me a question that I don't know the answer to, and then I try to put it on the board, and it doesn't work. I get a little uncomfortable about that" (Interview 3, 2/15). Todd believed that it is important for a teacher to know her or his material, and he was not comfortable being confused in front of his class. I asked him how long he would be willing to try to figure out a problem with the class. He replied,

> Well, if we are all trying to figure it out together, I might let it go on until I actually can't do it….But I try to work [it] through with the class. But I will stop if I am working with the class, and I see that we are going in a bad direction, and I am, maybe, putting information out there that isn't true, you know, like trying to make quick generalizations, which is an easy thing to do when you are making mistakes. So, if I catch myself doing that, I'll just stop it and move on. I usually try not to waste more than a couple minutes on a problem. So if it exceeded that time,…then I just move on and say I'll come back to it later on. (Interview 3, 2/15)

Several times during my observation of Todd's teaching, he told students he would have to think more about a particular question and get back to them. He explained, "You can't really have a lot of dead air in class, because then they start slipping. So I usually say I'll get back to it the next day, and I either do or don't, because they forget" (Interview 2, 2/10).

Todd saw a teacher as "the person who enlightens [the students]" (Interview 4, 2/24). He said he would not be comfortable teaching Calculus because

> I'm really bad in the philosophical areas of mathematics, like proofs and things…. [Calculus] requires a lot of proof and other things like that, which I am not a hundred percent awesome with.… And so I think as a teacher you need to know your limitations…what you can teach, what you don't think you can teach. (Interview 1, 2/8)

Todd told of a time he taught a lesson wrong and had to reteach the topic. He said that, "having to go back and explain to everybody that I taught them wrong was pretty painful in my book" (Interview 7, 4/24). It was so important to Todd to be perceived as a mathematical authority in his classroom that he would stop a discussion and move on, making a sort of "sweeping under the rug" motion to alleviate his discomfort. Todd found it painful to face students with his own mathematical uncertainty.

Todd's teaching was, for the most part, "by the book." His lessons were centered around the textbook. When I asked him how he would introduce the concept of logarithms, he said,

> Since I don't know necessarily what the book or the curriculum wants me to teach, if someone just asks me about logarithms, I probably would try to talk about from maybe a graphical standpoint or a scientific standpoint, which tends to be the way I look at things. (Interview 7, 4/24)

Todd was willing to adjust the curriculum for his particular situation, but the curriculum and textbook had a tremendous impact on the activities of his classroom. Despite Todd's espoused emphasis on applications and preparing students for the workforce, he taught many topics that were not applicable to the business world. For example, he taught graphing compound inequalities on the number line. I asked him why he was willing to spend time on a topic that was not something students would likely need in the real world. I asked him how he coped with the difference between what he was supposed to teach and what he thought was important. He said that the reason he taught graphing on a number line was that it was on the graduation test but that he did not emphasize graphing as much as solving inequalities in his class. He would emphasize circle graphs and trend lines because they would be useful in business, but graphing on the number line would not be useful in business. Even though Todd did not perceive graphing on a number line to be an important skill for his students, he taught the skill because it was part of the curriculum and part of the graduation test. Todd said that "the fact that we are on such a defined curriculum really hinders exceptional teaching" (Interview 1, 2/8). Todd wished he had more time. He said, "I love to cook, and I'd love to bring in ideas…into my class, but I don't have time to do that in a lot of instances" (Interview 1, 2/8). He said, "I know that I get paid to teach the curriculum, and so I am going to follow the curriculum" (Interview 2, 2/10).

Todd seemed to follow the textbook without really thinking about how topics connected to each other. For example, on February 14 in Algebra II, Todd went over a homework problem that asked students to solve $d^2 + 6d + 8 = 0$ by graphing. First, he factored the quadratic equation and set both factors equal to zero to find the zeros of the function (at this point he had already solved the equation). He went on to create a table of values containing the points (-2, 0), (-4, 0), (0, 8), (-6, 8), and (-3, -1). He plotted the points and commented that "most likely (-3, -1) is the lowest point." Later, in an interview I asked him whether he would be addressing how to find the lowest point (the vertex). He said that they did not do vertex and axis of symmetry in Algebra II; those topics came in Trigonometry. In a later interview, Todd added the following.

> The other day we were talking about, "Are we going to get into the vertex and axis of symmetry?" And I said, "I don't think so in this level." And then bam—two weeks later—axis of symmetry and vertex. And so, that's a good example of the fact that I know what we were doing with the quadratics, but I didn't know…how deep we were going to get in with them. (Interview 5, 3/1)

It was Todd's first time through the Algebra II curriculum, which surprised him more than once. He described a lesson in which he taught a topic a certain way and the later thought, "I should have done it this way, because it would have helped out" (Interview 5, 3/1). When Todd taught solving radical equations and worked homework problems on the board, he neglected to consider both roots. For example, he worked the problem in Figure 11 on February 2. A student asked, "What's with the ± in the back of the book?" Todd did not hear the student's question and moved on. Later, I asked him why he did not address both the positive and negative roots, and he explained that the book focused on the principal root. Later, when he taught solving quadratics by completing the square, he got to the equation $(x + 3)^2 = 25$. He said took the square root of both sides and obtained $(x + 3) = 5$ and $(x + 3) = -5$. This was the first time he had addressed both roots. He was not consistent in how he approached this topic.

$$y = \sqrt{r^2 + s^2} \quad \text{Solve for } r.$$
$$y^2 = r^2 + s^2$$
$$r^2 = y^2 - s^2$$
$$r = \sqrt{y^2 - s}$$

Figure 11. Todd's solution of a radical equation.

I asked Todd whether topics coming later in the curriculum affected the way he taught particular topics, and he said that he believed that they should but that he did not know the Algebra II curriculum enough for that to be the case this first time through. He said, "I don't necessarily look through those [later textbook] sections to see connections, and so I think, probably in the future,…I definitely will" (Interview 5, 3/1). When I asked him how far in advance he planned, he said that he tried to stay ahead by three to five days but that he usually ended up planning a day or two before he taught the lesson.

Todd did not see technology as playing an important role in teaching mathematics. He recognized that graphing calculators were a powerful tool to help students understand graphs of functions. But to a large extent, Todd thought "about [teaching] the old fashioned way, and you don't need tools to learn math" (Interview 1, 2/8). He spoke about the development of mathematics and that the greatest mathematicians did not have electronic tools. He was worried that students were becoming too dependent on technology and were not thinking enough for themselves. He described a study that was reported on the television program "20/20" in which one class had complete access to technology and another class had no access to technology. Todd elaborated,

> They found out that the kids with the technology were very lazy. They didn't have a full grasp or understanding of the situation.…The whole [learning] process was slowed by [the use of technology]. And I thought, that's totally different than what I've been told, and so it seemed so odd and ludicrous. And then I started thinking about it, and it really made sense to me. Because if you think about

> it—and I see it in my students, and I think that's when it really started hitting me, was when I asked kids what 8 times 5 is, and it would take them five minutes. And then they would end up getting a calculator to do it. I think I'm one of the rare teachers out there who maybe has even heard of this study and also who believes in it. And I think that puts me aside a little bit. But I would defend my beliefs to anybody. Because I really think that…when I really explain to them how I feel about it, I think they'd see it too. And so I really think that the best learning environment does not necessarily need to have any kind of electronic component whatsoever. (Interview 1, 2/8)

During the time I observed Todd's teaching, I saw him use graphing calculators with his students only once. On February 10 in Algebra II, he used calculators to establish that parabolas whose equations have a negative leading coefficient go down and those with a positive leading coefficient go up. On the same day, he discussed solving quadratic equations by graphing and showed students the Trace and Table functions. He was unclear about how to set the parameters for the Table and asked me to explain it to the class, which I did.

Todd's teaching was, for the most part, traditional. A typical day in Todd's classroom saw him going over the previous night's homework, working examples from the textbook for the section he was covering that day, and then assigning new homework. Todd did ask a variety of questions, including open-ended questions and application problems (which will be discussed in themes related to assessment), but he did not spend much time discussing these types of questions. His focus was on "doing the math." Mathematical topics in Todd's class were very discrete. For the most part, he did not make connections from one topic to another or from one day to the next.

<u>Decision-Making</u>

Todd recognized that he was an inexperienced teacher, so he often turned to more experienced teachers to help him make decisions about his teaching. For example, when he chose to omit a chapter from the course, I asked him how he made that decision. He said, “I chose it but with a lot of—. I take into consideration a lot of what other teachers’ advices [sic] are because I am new to the state, and I don’t really know everything about what is going to be on the graduation test” (Interview 4, 2/24). Todd often took his tests to his mentor teacher before giving them, asking whether the test was too hard, too easy, too long, or too short. I asked Todd whose responsibility it was to determine what happens in his classroom. His immediate response was this it was his responsibility, but then he said the politically correct response would be the board of education. I asked him for the truth and he said,

> The truth is the board of education, but I decide how long and how much of it I am really doing. They tell me what they want me to teach and when they want me to be done with it. They say, “Take chapter 6 and spend 14 days on it.” Now, I can take chapter 6 and spend 14 days, or I can spend 10, or I can spend 20. Technically I am going against what the curriculum states, but I do control what we do in our class, so ultimately I am responsible for that. (Interview 4,2/2)

When I asked Todd what he would do if he did not like the curriculum he was given to teach, he said he would check with other teachers to see how they felt. He explained,

> If I’m around a group of like-minded people that agreed with what I thought, I might stay and try to fight to have it changed. If I’m on the out—meaning I might be one of the only people thinking that way—while I’m always one that likes to affect change, it’s often a losing battle. So I think I’d probably look for a different school. (Interview 5, 3/1)

Todd was willing to make some decisions on his own—for example, when he left out the application section in Algebra II—but most of the time he consulted other teachers before making decisions about his teaching.

Themes Related to Learning Mathematics

Todd did not view the classes he taught as something that should be very problematic for students. He said,

> Basic algebra is not rocket science. So, you know, what I do with homework and everything else you should be able to catch what is going on. And by the way I test, you know, I don't feel like I am throwing anything like curveballs at you. So you should be able to get it. (Interview 2, 2/10)

Todd felt that if students tried in his class, then they should be successful. He placed the burden of learning on the student. He asked,

> Is it my fault that someone didn't learn? Am I supposed to be able to get someone to pass my class who doesn't study, who doesn't put effort in or doesn't do anything else? Am I to be held responsible? This is the parent's fault and the child's fault, not mine. (Interview 2, 2/10)

Todd was quick to conclude that students were not trying hard enough. I observed him when he graded the fourth quiz in Algebra Concepts. One question on the quiz was, "Is $(a + b)^2 = a^2 + b^2$?" None of the students got the question right, and most of them left it blank. I asked Todd why he thought students left it blank. Why didn't they at least plug in numbers? He replied, "They have a block." When I asked what causes that, he said, "Poor teaching before and not studying" (Algebra Concepts, 3/3). Todd believed that other teachers were too easy on students and passed the buck. He had an attitude of "the buck stops here." Other teachers let students by without understanding the material, but Todd said he was not willing for that to happen in his class. If the students did not know how to answer this quiz question, it was because their previous teachers passed them on or they were not trying hard enough. In general, Todd did not consider his own teaching to be a source of student misunderstanding.

One exception is the following: On February 14 in Algebra Concepts, a beginning activity was "Create an inequality with no solution or as a solution." A student came to the board and wrote $0 > s < 0$. Todd told the student, "The way you wrote it is kind of weird, I'd like to see it written a little differently, but I see what you were trying to do. A number can't be both less than zero and greater than 0." The student's inequality did have a solution—any number less than zero. Later in the lesson, Todd gave the example of $\frac{1}{2}x\ \frac{1}{2} = -4$ as a correct response to the question (an equation, not an inequality) because it had no solution. The next day, this same item appeared on a test. Immediately following the test, I looked over the responses to this item. Todd asked me how the students had done, and I said none of them got the item correct. He said, "Can you believe that? We did that in class just the day before" (Algebra Concepts, 2/15). We talked about this item further in an interview. We discussed the student's response to the item in class. I asked Todd what he thought contributed to the students' poor responses on the item. He said,

> I didn't spend a whole lot of time focusing on problems that didn't have a solution, so I don't think that they were accustomed to seeing them….And thinking back on it, I didn't focus on it as much as I should have. So, that would be one reason, I think. Another reason is, as you see in my class, I have some students that are paying attention, some students that are talking, and some students that don't care. And so the few students that pay attention most of the time will get it right.…I also sort of said one of the responses was maybe correct, but it wasn't, so I also kind of shot myself in the foot on that one. So, I would say that there is definitely a fifty-fifty fault rate on my own and theirs. And that is rare. I wouldn't usually give myself that high of a fault rate, but I would say that I probably dropped the ball a little bit on the teaching. (Interview 4, 2/24)

Todd was willing to look at his teaching after we discussed what went on in class. But even after we discussed the particulars of his teaching, he still assumed that most of the students missed the question because they had not tried hard enough or listened well enough. He admitted that he had been partially responsible for the students missing the item, but he seemed to look at this as an isolated event. When reflecting on this incident, Todd focused more on the fact that he had not spent much time on questions with no solution than on how he had handled the student's solution. He did not seem to make the connection between the way he dealt with the student's solution and the way his students answered the test question. Todd did not use the results of assessment to inform or reflect on his teaching in this situation.

Todd felt that the burden of learning lies with students and their parents. He felt that students have been empowered to a point where a lack of effort is acceptable. He did not feel that the discipline policies were strong enough in his school. He believed that if schools removed trouble makers from classrooms then there would be fewer distractions and students would learn more.

Themes Related to Assessment

Purpose of Assessment

Assessment served several purposes in Todd's classroom. He used assessment for the purpose of assigning grades. Also, Todd believed that assessment can be a motivating factor when students' efforts are rewarded, and heused assessment to support his instruction.

Todd used assessment for determining grades, but, his approach to grading seemed haphazard. He graded very quickly. He did much of his grading while students were working independently or in groups. On February 25 in Algebra Concepts, Todd gave a quiz. One of the items is shown in Figure 12. During the time that the students were taking the quiz, I worked the quiz as well. My answer to this question was 27 different ways. After class, I asked Todd for his answer, and he said 26. I told him that my answer was 27. He graded the quizzes and accepted both 26 and 27 (he wrote "OK" by the responses of 27). After he finished the grading, we compared answers and decided that 27 was the correct answer. He did not alter his grading.

> In the last Los Angeles Lakers game Kobe Bryant scored 15 points. How many different ways are there for him to do this? (include 3 point shots, 2 point shots, and free throws)

Figure 12. Todd's basketball question.

Todd's grading of open-ended items did not seem to be based on a rubric. He explained that when he used open-ended items on tests, he "more or less" used the rubric developed in the assessment project in which he was participating. On quizzes, however, his questions were worth only 2 points each and so, he said, "I sort of apply that rubric, but I…condense it" (Interview 5, 3/1). I observed Todd while he graded the fourth quiz in Algebra II. He did not use a rubric when he graded the open-ended items. He graded the quizzes quickly and did not appear to be reflecting on the students' responses. Figure 13 shows an open-ended item and Todd's scoring and comments on selected student responses. Todd assumed the student who gave Response 1 meant *k* rather than *h* and gave full credit. Response 2 said nothing about the graphs of the functions, merely that the equations were different in that one had a *k* and one did not. Responses 3 and 4 indicated that these students believed that the parameter *k* shifted the graph left or right (rather than up or down) and received full credit. Response 5 indicated that the student believed that the parameter *k* affected the direction of the parabola and got half credit. The student who gave Response 6 did not take into consideration that *k* could be negative and thus the graph could be shifted down rather than up. A look at the differences between Responses 4 and 6, each of which received full credit, indicates the lack of reflection that went into the grading.

Item: How does the graph of $y = (x - h)^2 + k$ compare to the graph of $y = (x - h)^2$?		
Student Response	Credit given	Todd's comment
1. They are the same. *h* translates either up or down	Full credit	Todd changed the *h* to a *k* in the student response
2. Difference in *k*	Half credit	No comment
3. The *k*'s differ which determine wheter [sic] the parabola is translated on the left or right side (of the *y*-axis)	Full credit	No comment
4. It is going to either go left or right	Full credit	No comment
5. One parabola goes up and the other goes down	Half credit	No comment
6. It has a *k*, so that means it moved up *k* units	Full credit	No comment

Figure 13. Todd's grading of an open-ended item.

On the Algebra Concepts test on February 15 (the one previously discussed), recall the item “Create an inequality with no solution or the empty set as a solution.” Recall also that in class the day before Todd had given ½*x*½= -4 as being a correct response. In grading the test Todd gave the response -½*n*½= 6 full credit and wrote “Good” beside it. Even though this grading was consistent with what he had taught, Todd wrote “Good” by a response that was an equation to an item asking for an inequality. He also gave the response ½0½= 4 full credit and wrote “OK” by it. Again, this is an equation, and not even a true statement (there is no variable, so what is the solution?). Todd was in a hurry to grade, and he graded whenever he had a spare moment. His grading process was not a reflective one.

Beyond grading, Todd also used assessment for the purpose of motivation. He explained,

> I think if you don’t put a stipulation on even the fact that [the students] may be graded, you tend to lose the non-motivated students right off the bat. And that is sad, but it is a realistic look at life. So, I definitely at least threaten to grade. That is a harsh word, *threaten*, but I at least use that there could be a grade to it as a motivational factor to at least try to work it out. (Interview 2, 2/10)

Although Todd found grades to be motivating, he claimed that he did not completely believe in using assessment as punishment. He said, “I think using a test as fear and punishment is bad in some aspects. In some aspects, it can be a motivational tool” (Interview 2, 2/10). When I asked him how he chose his test questions, he said,

> I choose test items a number of different ways. One, if I want to be mean, and I see that the kids aren’t paying attention in class, and they are not helping out, and they are not doing what they need to do, and I notice that, I may choose the exact problem that is giving them the hardest time and assign it. (Interview 2, 2/10)

As was previously discussed, Todd believed that the responsibility for learning lies with the student. When he felt that students were not trying hard enough, it angered him.

Todd used the phrase “it’s all in the attempt” several times both with students and with me during interviews. He explained,

> So, if you can get them to see that it is all in the attempt—. ...I think too many people worry about the finished product when you really should be focusing on what is going on during the making of the finished product. Yes, you want the finished product to be great, but if the kid passes, or if the finished product is great, but the middle stuff is all crazy, then you really don’t have a great finished product, do you? (Interview 2, 2/10)

Many days, when students entered the room, Todd had a beginning activity on the board. The students were encouraged to come to the board and work the problems. Todd gave credit to students who attempted the problems (only one student per problem), regardless of whether their answer was correct. The primary purpose of the beginning activity was to motivate students to begin thinking.

Todd assigned open-ended items as group work for students several times. While the students were working on the items, Todd did not circulate about the room. When the students were finished with their problems, they were to bring their group’s solution to Todd to check off. He explained how he scored this assignment:

> I don’t like to give anybody failing work for group work if they answer all the questions—because they try. So what I did was, if they got them all right, they got a perfect score; and if they got one wrong, I give them an eight out of ten; and if they missed any more than one, they got a seven. Then when I went around and had them explain the answers, and that was when they earned extra points….On group work I am trying focus more on the thought process than I am with the outcome, and that is really what my whole class is about. (Interview 3, 2/15)

On tests, Todd said he was interested in the outcome of processes but in class he was more interested in the process itself. He provided students with several opportunities to earn points by attempting problems. He said, “To me it’s all in the attempt. You can’t learn if you don't want to make the attempt. And if you don't give people credit for making the attempt, then they're not going to do it again" (Interview 5, 3/1).

Todd asked open-ended questions during instruction to assess students' understanding of terms or concepts. For example, on March 7 in Algebra Concepts, Todd taught ascending and descending order of the terms in polynomials. He had already taught the concepts of degree and naming polynomials by their number of terms (monomial, binomial, etc.). He asked students to create examples of polynomials that fit certain criteria. For example, he asked them to create a 4^{th}- degree polynomial, a 5^{th}-degree trinomial, and a 10^{th}- degree binomial. They discussed how to arrange the terms in each of these in ascending and descending order. The students were eager to participate. Later, Todd commented about the use of open-ended items during instruction:

> Today…I made everybody have to participate, as opposed to making them raise their hands. And so, when they didn't know, I kept going back and making them answer whether they were right or wrong. And that helps out because you've just involved the student who normally wouldn't be involved and also, um,… maybe they learned. Hopefully they learned. (Interview 6, 3/8)

<u>Characteristics of Assessment</u>

Todd believed that assessment should be aligned with instruction. He believed that it is important "that you're testing what you have covered" (Interview 2, 2/10). He clarified what he meant by saying, "Now, I always try to extend a little bit because I want to try to make them do the extension to something maybe that we haven't done" (Interview 2, 2/10). Todd wanted his assessment to be challenging. He said he would worry if all of his students made A's on his tests.

Todd used a variety of assessment strategies in his teaching. He used open-ended questions, projects, application problems, multiple-choice items, and traditional problems. He said,

> A lot of problems that I see is that teachers fall into a rut of giving the book made test all of the time. And the book-made test doesn't test any concepts fairly at all. I mean, there are some application problems in there, which is fine, but really they are very rare in the alternative assessment. Even though we do have alternative assessment books that come along with each of these [textbooks], they don't tend to be that great. (Interview 4, 2/24)

Todd was committed to using open-ended assessment and used it in all areas of his teaching. He did not want to rely on any one method of assessment exclusively. He said,

> Different students learn with different things, and so if you can include enough different things you can really have a better understanding [of whether] the student knows what is going on or not. Because if they fail and you have given them a number of different ways to test, then you know that they are really lost or they are not understanding it. And so by just testing one way, I think you hurt the kids because they are only used to that way, you know. They are only used to multiple-choice, or they are only used to short answer. (Interview 2, 2/10)

This view went beyond Todd's classroom. He believed that tests are only one way to assess a student's capability. For example, he did not believe that the graduation test should be weighted as much as it was. He would rather have seen a more cumulative measure of students' high school experiences such as the extra curricular activities in which they were involved and their course grades.

Todd also liked open-ended assessment items because he believed that they better assessed conceptual understanding than traditional mathematics questions. He said, "Alternative assessing is another way of making sure that [students] understand things on a more verbal level from a different approach. You're definitely approaching it on a higher level, so you get more understanding out of it" (Interview 6, 3/8). Even though he had not felt successful when he first started using open-ended assessment, he stuck with it because he believed that open-ended items were getting at things that he wanted his students to know. He said, "The more you use it, you get more correct answers later on" (Interview 6, 3/8).

The Case of Keith

Keith was a white male in his sixth year of teaching at a suburban school with approximately 1109 students. He served as department head at his school. His current position was his first teaching position. The year following data collection, he was selected as teacher of the year at his school. Prior to becoming a mathematics teacher, Keith worked as an industrial engineer. Keith had taken 4 courses in calculus, 2 courses in abstract algebra. He also took real analysis at both the undergraduate and graduate levels, statics, time and motion studies, combinatorics, math modeling, linear algebra, linear optimization, and statistics at both the undergraduate and graduate levels. He had taken complex analysis, topology, graph theory, Pascal programming, and discrete mathematics. Keith's background in mathematics was strong and diverse. He held a master of education degree in mathematics.

Themes Related to Mathematics

Keith believed that mathematics is absolute. He believed that mathematics exists in some pure sense in nature and that humans find or discover it. He said,

> I think that there is a natural order around us, and I think that mathematics is out there and you just find it. I don't think you make it up. You might make up names for certain properties or certain applications, but I think the mathematics is there. (Interview 1, 2/3)

Although he saw mathematics as absolute, he also believed that people had different visions of mathematics. Like Plato's parable of the cave, Keith believed that we only see shadows of the real mathematics, and we all see it differently. He explained,

> I am kind of an engineering, skilled-type person, and that is the way that I see [mathematics]. That is my perspective on a lot of things. But I think that there are other people that—.I go down to the art department every now and then, and

> you know, they see mathematics in a whole different way than I do. (Interview 1, 2/3)

He compared learning mathematics to building a clay sculpture—people all have the same raw materials, but they build a different sculpture. Although mathematics exists in a pure sense, people have different views or visions of it.

Themes Related to Teaching Mathematics

For Keith, teaching and learning were integrally connected. He believed that if the students did not learn then the teacher had not taught. It was important to Keith that his students develop an understanding of mathematics rather than merely an ability to carry out mathematical procedures. He believed that students learn and perceive mathematics differently, and the teacher's job is to use everything available to make decisions that would facilitate all students' learning. He approached teaching in a reflective way in an attempt to help his students understand and apply mathematical concepts.

Purpose of Teaching

The whole purpose of teaching, according to Keith, was student learning. Keith believed that teaching was more than just "[standing] up there and [saying], 'okay, here it is'" (Interview 3, 2/17). Rather, he explained, "my job is a teacher, and teaching does not happen until learning happens" (Interview 4, 3/2). More than just learning facts and procedures, Keith wanted his students to understand mathematics. He wanted them to know more than just why they needed to know mathematics—he wanted them to understand why and how mathematics worked. He explained,

> The majority of [the students] just want to know, "Why would we ever want to do this? Why would we use it?" And, if we're trying to raise and train learners, life-long learners, then I think it's more important to kind of think about why you can do this and … why these things can do that.…[And] that's how all these scientists and mathematicians have made new discoveries that make our lives easier; because they asked that kind of why question, rather than "Why do I need

> to learn this?"…I think that you stifle a lot of creativity and you stifle other perspectives and other approaches if students don't see those kinds of things. (Interview 5, 3/6)

Keith recognized that students were perhaps more interested in knowing why they needed to learn mathematics, but he believed it was important for them to understand how mathematics worked as well. He wanted them to be able to do more than memorize and imitate; he wanted them to understand.

Keith wanted his students to develop a sense of the big picture in mathematics, so that they understood its importance and how it was relevant in the world. He saw this "big picture" view as more important than merely the development of mathematical skills: "If you get or grasp the whole picture, I think then it will make the skills come" (Interview 4, 3/2). Keith saw beauty in mathematics, and he explained, "[I like] the applications, the way it applies to our everyday lives, and I like to look for the mathematics in everything I see around me" (Interview 1, 2/3). In class, Keith liked to use applications to motivate mathematical topics. He said, "I like to start abstract and go down to concrete" (Interview 4, 3/2). For example, Keith described an activity he obtained from an NCTM journal and used in his pre-calculus class:

> It was a way for students to measure the distance from the ceiling to the minute hand on the clock. And they were able to, like, mark the number of hours that had passed and then the distance from the ceiling. And we did this before we started anything with graphs of sine functions and cosine functions. And so this activity gave them a chance to…connect the two and see that they are things that are periodic. And they were able to come up with the ideas of periodicity and frequency on their own through that activity, and that was a good activity. It was much easier for them to do it that way and internalize it than it would have been for me to stand up and say, "Here are these sine functions that are periodic." (Interview 2, 2/9)

Keith liked the fact that the students were able to develop the ideas of periodicity and frequency from an application rather than from him telling them about the concepts.

Keith believed that teaching should prepare students to be successful in society. He told his students, "Your boss is not going to come up to you and say, 'Here, graph $y = -6 + \frac{3}{4}x$ for me'.…Instead, he's going to come to you with a problem,[and] you'll have to come up with a model" (Algebra II,2/9). He wanted his students to be able to interpret situations and model them mathematically. For example, on February 9 in Algebra II, Keith did an activity using a motion detector and a CBL (Calculator-Based Laboratory) in which he asked students to work in groups to generate the graphs in Figure 14 by walking in front of the motion detector. For each graph, he asked them to sketch the graph they generated using the CBL and to interpret each graph in terms of what happened to their position as time passed and the rate at which they were walking. Later, Keith described using the CBL and motion detector to investigate parabolic curves using the motion of a ball (Interview 7, 5/2).

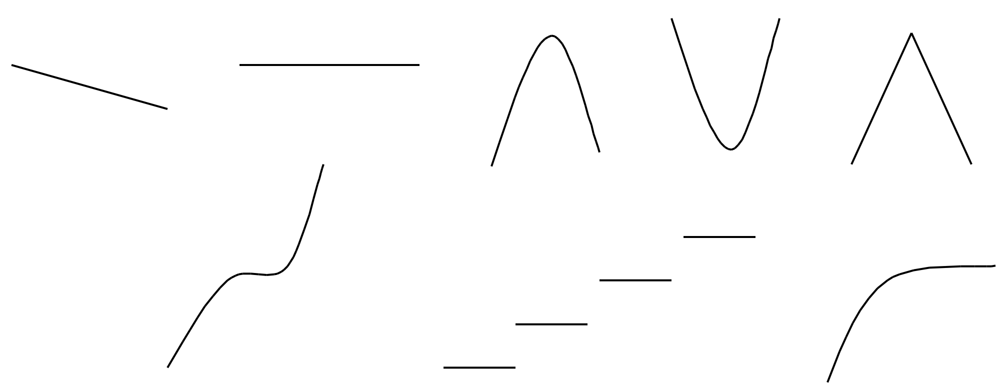

Figure 14. Graphs for Keith's students to create on the CBL.

Applications played a dual role in Keith's teaching. He used them to help motivate students (Why do we need to know this?) and also as a way to help students investigate mathematics on their own in order to see mathematical relationships rather than just telling them the relationships. He wanted students to understand how to model real-world situations mathematically (a skill he believed would be useful in society), to recognize the usefulness of mathematics, and to learn mathematics in the context of applications.

Role of the Teacher

Keith believed that teachers should be knowledgeable, motivating, and enthusiastic. He believed that students were capable and that teachers should value individual student approaches. At the same time, he believed that students should be held accountable for their own learning.

Keith's teaching was conversational. The students were involved in his teaching, even when he was giving notes or going over homework. He valued the way students approached problems; he did not force them to work them his way. For example, on February 9 in Algebra II, Keith solved a problem in which he graphed $y = -\frac{2}{3}x + 2$. A student said, "Last year the teacher taught us to graph $y = -\frac{2}{3}x + 2$. We put a dot on 2 and then go down 2 and over 3." Keith said, "That's exactly right. You can do that method too." The student said, "But when I did it that way, I got a different answer than when I did it with a table." Keith said, "Let's do it both ways." He graphed the line using a table and then graphed it using the *y*-intercept and the slope to help the student identify his mistake. He took the time in class to value the student's different approach to the problem. On February 17 in Algebra II, when working an example on the board, Keith said, "Francisco, how do you graph? Do you like to use slope-intercept form or graph using *x* and *y* intercepts?" He graphed it the way Francisco preferred. This practice was common in Keith's class. When he assigned an application problem in pre-calculus, the directions he handed out described an approach to solving the problem that would utilize radical equations, a topic they were currently studying. One student approached the problem in a different way than was suggested by the directions. Keith explained,

> And see, to me, she didn't follow Steps 1, 2, 3, 4, 5 that on that sheet. But she probably learned as much [as], if not more, than anybody else did following Steps 1 through 10. And so I think that's one thing about being objective and not saying, "Okay, that's wrong because you didn't do this." (Interview 5, 3/6)

Keith thought it was important to value how students approached problems rather than insisting that they approach them in a uniform way. He did not agree with a newscaster metaphor for teaching, because a newscaster "is just saying, 'Okay, here it is,' and doesn't even follow up or doesn't even help or guide students along" (Interview 1, 2/3). He believed that teaching should involve interaction between the teacher and the students.

Even when he gave students rules or procedures for solving problems, Keith encouraged the students to justify the rules for themselves. For example, he described how to graph a linear inequality by giving steps: The line divides the coordinate plane into two half-planes, and one of the half-planes will contain all the solutions and the other one will not. To determine which half plane is the solution, test a point. Then he said "Now, everybody just believed me, but I only tested one point. You should take a moment to choose another point in that half-plane to see if it's a solution" (Algebra II, 2/17). After he gave the students time to choose and test a point, he asked each student (it was a small class) what her or his point was, and he tested each point on the overhead projector. He used the students' points to help them see that all the points in that half-plane satisfied the inequality. Similarly, in Algebra II on March 14, Keith conjectured that if he multiplied a matrix containing the coordinates of the vertices of a triangle by 2, then he would double the perimeter of the triangle. After he multiplied the matrix by 2, he asked students to graph the original triangle and the new one on graph paper in order to see if the perimeter was doubled. Even though he suggested the method, Keith expected his students to justify the method for themselves. He did not want to be the only mathematical authority in the classroom. He wanted students to be their own mathematical authority. When he made mistakes on the board, Keith felt comfortable having students work with him to find the error. He explained,

> I try to kind of put us on the same playing field and not—.I don't want them to think, "Okay here's this big expert up here and he's supposed to be able to tell me everything." I would rather them say, "You know, he can go through it, and we can work through it and find out if we did everything right or wrong." (Interview 5, 3/6)

Keith viewed students as capable and valued their mathematical thinking.

Although Keith believed that teaching did not happen unless learning occurred, he did not believe that teachers were solely responsible for student learning. He believed that students had to do their part as well. But rather than just blaming students for not working hard enough, Keith had strategies to help ensure that students were doing their part. From observing teachers, I have noticed that many teachers, when going over homework with students, end up working students' homework for them. They work many, if not most, of the homework problems in class, and students are able to copy down correct solutions. Too, these teachers use much of their instructional time in working homework problems for students. Keith's way of dealing with homework avoided this problem. When students asked Keith to work a homework problem, he asked them questions about their approach. He asked questions such as the following: What did you try first? What was your first step? What did you do next? And what did you get? And then what? What would you suggest I do? What would you suggest I do first, Bethany ? What do I do next? Okay, tell me what the result is. Okay, Melanie, tell me how you set it up. During my observations, I never saw students ask questions about homework problems that they had not attempted. Keith used his questioning as a way to make sure the students were doing their homework, and to learn about the kinds of mistakes the students were making. He still walked around the room to look at students' homework papers, but he also learned about how they were doing on their homework when he worked homework problems on the board and interacted with a student about her or his solution process. In Pre-Calculus on February 9, Keith noticed, when he walked around the room looking at the papers, that many students had problems with the previous night's homework. He told the students,

> There are a lot of questions on the homework. I will tell you several people to work with. Compare your homework and try to figure out where you made your mistakes. Then we'll narrow down the number of problems we have to work together. I'll work no more than 3 problems from each section. We will do this activity for about 10 minutes.

Keith allowed students to work on this activity for about 25 minutes, but the students were on task. Some groups of students worked problems on the board that other groups could not solve. Keith gave them time to work together to solve the problems they found difficult. But he did not "let them off the hook" as far as homework went. It is important to note that Keith made the decision to do this activity because of what he learned by walking around the room looking at the students' homework. He altered his teaching activities on the spot using information he gained by interacting with his students.

<u>Decision-Making</u>

Keith believed that the decisions teachers make should be based on several available resources. For example, Keith worked with the College Board on "vertical teaming" to identify the five or six most important topics in each course in his county's curriculum. When making decisions about teaching, Keith kept in mind what he believed to be the most important topics for the course. He considered the recommendations of NCTM. He used the county curriculum and the textbook in his decision-making process. He considered each of these resources as a guide rather than a prescription for teaching. He explained, "We do have a curriculum that's given to us, [but I use] the curriculum as just kind of an outline….So, it's my responsibility to decide what it is that the students learn" (Interview 4, 3/2). Similarly, Keith used the textbook as a resource in his decision making. I asked him if he looked at how the textbook treated particular topics when he decided how to teach them and he replied, "Yes, because it's just another point of view from mine. If I know of better ways to present the topic, I usually present them my own way" (Interview 4, 3/2). He was willing to look at how the textbook presented the topic, but he might or might not choose to present it the same way, depending on what he thought made sense for his students.

When making decisions about teaching, Keith tried to think about how he could best prepare his students for society. His focus was never on finishing the curriculum or on doing "what he was supposed to do." In Keith's view, what he was supposed to do was facilitate student understanding of mathematics so that his students would be prepared for what came after high school. It was his job to determine how to best achieve that goal. He used resources to help him make decisions, but the decisions were his to make.

Although Keith used the county curriculum in his decision making, he did not follow it exactly. He was more concerned that students understood what he taught rather than finishing all the objectives. He explained,

> It's kind of a balancing act, and it's…your philosophy of education kind of against the curriculum. And if you would rather students have read every single page in the book and not know much, or if you would rather them know something well and maybe not have covered as many topics, then I think [the latter] might be the better choice. (Interview 3, 2/17)

Keith seldom finished the curriculum for the courses he taught, and he was not particularly bothered by that. Still, as with most teachers, time was an issue for Keith. It was important to him that he taught what he considered to be the five or six most important ideas or concepts for each of his courses. He wanted to make sure that he allocated his limited time to the most important topics for each course. But a bigger issue for Keith than how much time he had with his students was how much time he had for planning and preparation. When Keith spoke of time limitations, he was usually talking about how he never felt that he had enough time to prepare. As department head at his school, Keith was often involved in school and county meetings. He also helped many of the faculty members at his school with their computer problems. He told me,

> Time is a huge issue. I don't know what else to say. You know, if I had the time to sit down and create items on my own, good items, then I think that my instruction would be much more in line with my assessment. (Interview 2, 2/9)

Keith wanted to use activities that would help his students understand mathematics better. Finding or creating such activities takes time, and Keith wished he had more of it. He said,

> In a lot of the topic areas—especially, like, pre-calculus,—I think there are a lot of good applications out there and a lot of good models that we could do here in the school environment. But it just takes some research, and it takes some time to come up with good applications that have something to do with verifying trig identities, you know? There has got to be something out there that we could do with it, I just don't have the time always to sit down and think about it and find them. (Interview 1, 2/3)

Keith wanted to spend time thinking about, planning, and preparing for teaching for understanding. He believed that applications and hands-on activities better facilitated understanding than lecture, but he did not feel that he had enough time to find such activities and applications.

Themes Related to Learning Mathematics

Keith believed that everyone can learn mathematics. He thought that some people are more interested in mathematics than others, and an interest and appreciation for mathematics might cause students to perform better in mathematics. But he believed that all students are capable of learning mathematics, and if students are not learning then the teacher is not teaching. Someone once asked Keith what he did with unteachable students. He said he had thought about that question many times.

> I just don't know that there is such a thing. I think it is the way you approach it, and if that student sees it differently than I do then maybe I need to learn a little

> bit about the way that student sees the mathematics to get the student interested or motivated to look at it. (Interview 1, 2/3)

Keith believed that if a student was not understanding mathematics, it was because he was not understanding the student's thinking well enough to effectively communicate about the mathematics with the student.

Although he believed that everyone could learn mathematics, he also believed that people learned mathematics differently. Some people just "see" mathematics, whereas others need to investigate mathematics in order to learn it. Working with other people and hearing their perspectives can help some people learn mathematics. Tools and manipulatives can help some people learn mathematics. Learning mathematics, to Keith, means understanding how mathematics works, how it is connected, and how to apply it to situations in order to solve problems. Models and hands-on materials can help students understand mathematical situations. Experimenting and playing around with mathematics can help learning. People learn mathematics in different ways, and Keith believed that it is a teacher's job to figure out how her or his students learn mathematics so he or she can teach it effectively.

Themes Related to Assessment

Keith struggled with assessment and recognized it as problematic. He wanted his assessment to accurately reflect what students know and to be aligned with his instruction. He thought assessment should reflect his values with respect to teaching. He wanted to learn about his students and their understanding through his assessment.

Purpose of Assessment

Asked to define assessment, Keith replied, "assessment to me is just the method of finding out what students know" (Interview 2, 2/9). What concerned him about assessment was that teachers' assessment instruments often provide more information about what students do not knew than what they do know. Keith wanted to provide his students an opportunity to demonstrate what they know, and how they knew it. He understood that if he gave a student a particular question that he or she could not answer, it did not necessarily mean that the student had no understanding of the concept. Keith explained that a good assessment item "is just something that allows the student to kind of do their own thinking and show you what they know in their own way" (Interview 2, 2/9). He believed that students understand and approach mathematics differently and that fair assessment should allow them to demonstrate their own kind of knowing:

> You want [students] to have to make choices and to be able to use their own reasoning skills and their own logic when they approach a problem. And I think it's important in two ways: One, it kind of lets you, it does let you know what the student knows and how they understood what you taught them. But it also gives them the chance to present what they know—maybe not the same way that you expected them to or the same way you presented it to them. (Interview 3, 2/17)

Keith valued the way students approach mathematics and recognized that they may approach it differently than he does. He still used traditional skill problems on his assessment instruments. He believed it was important for students to be able to do the mathematical procedures they discussed in class, and he assessed those skills. But he also thought it was important to allow students to demonstrate a different kind of thinking. He said, "I try to choose things that will let students demonstrate to me what they know that I might not have asked" (Interview 2, 2/9). Keith recognized that assessment gives only a snapshot of student understanding and that what he chose to put on a test had an effect on how students did on the test. For that reason, he tried to include items that allowed students to demonstrate their own kind of knowing. Keith believed that the purpose of assessment is to understand what students know, but he did not see that as a simple process. He recognized complexities that make assessment problematic. He understood that understanding student thinking is a complex endeavor:

> When you have a pencil-and-paper test, and you just say, "Okay here's a problem, solve it," there's one answer, they know how to do it, and they're done. But you don't really know whether they're just mimicking what you did in class, or

> whether they really understand the concept or understand the problem. (Interview 3, 2/17)

Keith recognized that being able to do mathematics problems did not necessarily imply that the students had a conceptual understanding of the mathematical concepts he was assessing. When asked what he liked best about using open-ended items, he elaborated,

> It gives that student that maybe I haven't matched their learning style, or maybe I haven't been able to get to them, or maybe they can't produce all the kinds of things that I want to see over here on these skills, but I can get some sort of idea of what they did understand and how they did perceive what they were taught. (Interview 2, 2/9)

For, Keith, assessment was problematic in the sense that it can give an inaccurate view of students' knowledge. Keith felt that open-ended items helped him obtain a better view of what his students did know.

Keith used the information he gained from assessment for more than just determining grades. His assessment also informed his teaching. For example, he described a situation in which he posed an open-ended question about systems of linear equations during class. His students had not offered any satisfactory answers. He said,

> They didn't come up with any good responses for [the question] at first until we kind of led a discussion on it, but I hope that is the way that people will use those questions too. If they do get something where everybody says, "I don't know," I hope then that they can say, "Okay, well, then this is a good teachable moment for us and maybe we can go back and at least, kind of share a thought process and see what people can come up with." (Interview 6, 3/14)

Indeed, Keith did use this instance as a teachable moment. On March 6, in Algebra II, he posed the question in Figure 15. He gave students several minutes to work on the problem and then asked them to pass their papers around so everyone could read everyone else's response (there were only six students in the class).

One strategy used to solve a system of linear equations is to add them together. Explain why this is permissible.

Figure 15. Keith's solving systems item.

Keith then asked for a volunteer to share her or his answer. Melanie said, "Well, if you add them, you have only one equation instead of two, and that's easier to work with." He replied, "You're telling me why we would want to do that, not why we can. Let's look at $3x + 5 = 7$. Let's say I want to add 3 to the left side and –2 to the right side. Is that okay?" The students said that it was not okay, Keith asked them to think about the original question again for one minute, and he timed them. Another student offered a suggestion, and Keith told him that, like Melanie, he was focusing on why one would want to use the method rather than why the method worked. Then Keith asked, "Why was it not okay to add 3 to one side and –2 to the other? Suppose I have

$$\begin{array}{r} 2x + 3y = 5 \\ \underline{+ -2x + 5y = 7} \end{array}$$

I'm adding $2x + 3y$ to one side and 5 to the other. Is that okay?" The students said that it was okay because $2x + 3y$ was equal to 5. Keith then asked Catherine to explain it again or to restate what they had just discussed. In this situation, Keith posed a question and when the students did not give him satisfactory answers, he led a discussion to help students understand. Had he not posed the question, he might not have known that the students did not understand why the method worked. He used the results of an assessment activity to direct his teaching.

I asked Keith if any of the responses to an open-ended item he had given on a test surprised him. He explained that he was surprised that one particular student seemed to have no idea how to approach the problem. Based on what the student had done in class, Keith believed that he understood the concept and therefore he was surprised that the student did not successfully answer the item. Later, during a quiz, the student asked Keith if he could talk to him in the hallway. The student started crying. Keith concluded that the student probably had test anxiety. He used the information he gained during class and compared it to the information he was gathering during formal assessment and concluded that, although the student understood concepts, he was unable to demonstrate that understanding on a test.

Characteristics of Assessment

Keith had some overall goals for teaching and learning, and the way he viewed assessment was consistent with those goals. Keith believed that teaching was about facilitating learning so students would be prepared for their life in society. He approached teaching by identifying five or six specific concepts that students should understand for each course. He valued conceptual understanding over an ability to execute mathematical procedures. He believed that his assessment should reflect the overall values he had concerning teaching, and he believed that his assessment should be aligned with his instruction.

Like most teachers, Keith found that students resisted his use of open-ended items at first. He admitted that sometimes it was easier not to give open-ended items, because he would meet less resistance, but he persisted:

> I think it better just to keep doing it and keep doing it and keep making them think and keep giving them those kinds of problems so that then they realize, hey, this guy is serious about it, we are going to have to think. (Interview 3, 2/17)

Keith recognized that if students believed that he valued thinking and understanding, then eventually they would think and perhaps understand. Like his teaching, Keith approached his assessment in a "big picture" kind of way. I asked him what kind of advice he would give a new teacher regarding assessment, and he said,

> To kind of step back and look at the entire curriculum or, you know, what the mathematics is and think about several things. One, how are [the students] going to use it when they leave high school? You know, when you go out and get a job in private industry somewhere, your boss doesn't come to you and say, "Okay, here $4x = 18$, tell me what x is." You know they don't do that. They want you to

> see the whole picture also and to know how to use all of these things and put these things together. So I think when you develop assessment, you kind of have to keep all of that in your mind, too, as you go through and think about how important Number 9 is to the whole picture.

Keith believed that the way one assesses should reflect one's overall values for teaching: "Those open-ended assessments kind of give you an idea about what my attitudes are towards mathematics and what I think the big picture is. And I hope that it gives the students that same idea" (Interview 2, 2/9). He explained that students know "what it takes to get by and get the grade" (Interview 2, 2/9) and that in his class he wanted students to realize that they had to think to get the grade. He wanted students to realize that the mathematics in his class was more than "just [moving] everything to the left side and [setting] it equal to zero" (Interview 2, 2/9). Those values did not differ between lower-ability level classes and higher-ability level classes. Keith valued conceptual understanding in all levels of mathematics. He said, "I don't think there is anything different about asking a high-level question in a low-level math class than asking a high-level question in a high-level class" (Interview 2, 2/9). He recognized that the concepts are different in different levels, but he believed that all students should be encouraged and expected to think at a high level.

Although Keith believed that his assessment should reflect his values, his tests still contained mostly skill-oriented items. Every test that Keith gave during the observation period contained some open-ended items, but gave some quizzes that did not contain open-ended items. I asked Keith to place an $\times$ on a continuum between skills and concepts to indicate where he believed that the emphasis should be placed in high school mathematics. He put the $\times$ about $\frac{2}{3}$ of the way towards concepts. Later, he said,

> I've been thinking too about this continuum. You know I put my $\times$ up here; most of my questions are over here [towards skills]. You know we struggle, we

> struggle to get 20% of our tests to be alternative. And, yet I placed the importance up here, and most of my questions are over here. (Interview 4, 3/2)

Keith was quick to recognize, when I asked him where his values lie, that his assessment instruments were not consistent with what he believed he valued. Keith used open-ended assessment during class, and every test contained open-ended items, and some quizzes also contained open-ended items. But Keith was still concerned that more of his assessment was not open-ended. He explained that open-ended items were more difficult to create than skill-oriented items. Perhaps most interesting about Keith's struggle with this issue is that I did not ask him (I planned to, but he brought it up before I did) where his assessment instruments lay on the continuum. I only asked him about his beliefs about where emphasis should be placed in high school mathematics. It was Keith that began to question how his instruments compared to his beliefs. It was clear that he wanted his instruments to reflect his beliefs and that it was problematic for him that they might not.

Keith wanted his assessment to be aligned with his beliefs and values, and he also wanted his assessment to be aligned with his instruction. When I asked Keith if he used open-ended items on tests, he said, "I do, but I try to practice that same kind of questioning in class as we're discussing different concepts. I also try to use [such items] as little group activities" (Interview 3, 2/17). It was important to Keith that his students be asked to do the same kind of thinking during class that he was asking them to do on tests: However, Keith did not believe that students should only see on tests the kinds of items they saw during class. "It doesn't always have to be exactly the same kind of problem as they did in class" (Interview 2, 2/9). He liked problems that extended what the students had done in class.

After he had done the activity using the CBL and motion detector to generate graphs of different functions, Keith included on his quiz two questions that presented graphs and asked for a motion that would yield each graph using the CBL and motion detector. Keith's questions were directly tied to the activity the students had done in class. This assessment was aligned with his instruction and with his belief that students should be able to model real-world phenomena.

CHAPTER 5

A CROSS-CASE ANALYSIS

Across the three cases, there were four salient factors that affected the teachers' use of open-ended assessment items: beliefs and authority, reflectivity, knowledge, and system constraints. In the following sections, each of these factors will be discussed in terms of how it affected the teachers' use of open-ended assessment items. At the end of the chapter, the findings from the study are situated in the broader literature on teachers' assessment practices.

Beliefs and Authority

Beliefs

Chapter 4 described the participants' beliefs about mathematics, mathematics teaching and learning, and assessment. In summary, while Keith had an absolute view of mathematics, he recognized differences in how individuals come to know and understand mathematics. He believed that the purpose of teaching mathematics was to facilitate mathematical learning. He wanted the learning of mathematics to focus beyond skills and procedures to conceptual understanding, a "big-picture" view of mathematics, an appreciation of mathematics, and an ability to use mathematics to model real-world phenomena. Keith's view of teaching was similar to problem solving; he strove to figure out what and how students knew what they knew and then to devise ways of communicating with them based on that knowledge. He believed that the burden of learning lay, for the most part, with the teacher. Although students were held accountable for doing their part, Keith believed that if students were not learning then he was not teaching. Keith defined assessment as finding out what students know. He considered this process problematic because individuals can know mathematics in different ways. He felt a responsibility to ask questions that would provide students the opportunity to show him not only what they knew but also how they knew it.

Like Keith, Robin had an absolute view of mathematics. However, Robin saw inherent structure in mathematics that was governed by logic. Like a puzzle, mathematics fit together in a logical way in Robin's mind. She recognized different approaches to mathematics, but these different approaches were like putting the puzzle pieces together in a different order. Everything still fit together the same way, even if put together in a different order. It was important to Robin that her students develop a sense of the structure of mathematics. She emphasized connections in her teaching; she wanted students to understand how the pieces of mathematics fit together. Robin's view of teaching was explaining or transmitting the structure of mathematics to students through mostly lecture. She saw value in guided discovery but often sacrificed the use of this technique in the interest of time. Robin believed that she was obligated to teach the entire county curriculum. She felt responsible for her students' learning and when they did not succeed she reflected on her teaching as a source for the students' lack of understanding. Robin believed that assessment should be a fair process of determining students' ability to perform mathematical procedures and to explain mathematical connections. She believed that both students and teachers should learn from assessment.

Todd also had an absolute view of mathematics, but he recognized that different people had different perspectives on mathematics. He saw two distinct sides of mathematics, the practical and the philosophical. Todd was interested in the practical aspects of mathematics and was not interested in the philosophical aspects. For Todd, teaching was all about preparing his students for life in society. For his particular students, he perceived that his role was to prepare them for the business world because he believed that they were not college-bound. In Todd's own background in business, he found basic mathematical skills and procedures to be the essential aspects of mathematical knowledge. Therefore, he wanted to focus his teaching on helping students become proficient in carrying out basic mathematical procedures like adding, subtracting, multiplying, dividing, and solving algebraic equations. Todd used a lecture format to show students how to perform the mathematical skills and procedures that were in the textbook. Todd believed that the burden of learning lay with the students. He believed that if students worked hard then they would understand mathematics. When Todd's students demonstrated a lack of mathematical understanding, he usually did not look to his teaching as an explanation. Todd used assessment mostly for the purpose of determining grades. He believed that assessment should be fair in the sense that it should involve questions similar in focus to those asked during class. Todd also used assessment for motivational purposes.

The way the teachers used open-ended assessment items was affected by their beliefs about mathematics, teaching, learning, and assessment. All three of the teachers used open-ended assessment items, but they did so for different purposes and in different ways. These purposes and ways were aligned with their beliefs. For example, Keith believed that students had different ways of thinking about mathematics. Hence, he used open-ended items so that students "have to make choices and to be able to use their own reasoning skills and their own logic when they approach a problem" (Interview 3, 2/17). Robin believed that open-ended items allow students who have a conceptual understanding of a topic but "miss all of these little details" (Interview 6, 3/15) to demonstrate their understanding. She asserted that not including open-ended items in assessment would be unfair for such students. She valued conceptual thinking and used open-ended items as a way to elicit that kind of understanding. Todd believed that assessment should be aligned with instruction. He felt that teachers should test what they taught. Todd used the same open-ended items on tests as he had discussed in class. His students were used to seeing open-ended items. Todd recognized the importance of asking open-ended items during instruction if he planned to ask them on tests and quizzes.

Todd used considerably more open-ended items than the other two participants. Over the course of the observation period, he used 25 open-ended items whereas Keith used 16 and Robin used 10. By nature, open-ended items focus more on conceptual understanding than on procedural understanding. If Todd's interests lay with the practical and procedural aspects of mathematics, why did he use so many open-ended items? Todd explained,

> Because…that is the concept part. I really think that if you know the skill part and you know the concept part, you are dangerous…And my whole idea behind teaching is to set my kids up for being solid workers in the work place. I don't even look at the college aspect. I want them to be able come to class and apply what they learn in the real world. And you are not going to be able to do just the skill part in the business world. You are going to need to know the concept part and…doing the concept part, I think also helps to cement the skills into their head…It shows that you really mastered something and at the same time it also helps cement it into your head…Plus, it also breaks the monotony of regurgitation of knowledge and it makes things a little easier, makes things a little bit more fun or interesting. (Interview 4, 2/24)

Todd related his use of open-ended items to his belief that teaching should prepare students for life in society. He indicated that both conceptual understanding and procedural understanding were necessary for students' success in the business world. He also said, "I think the skills [as opposed to concepts] are probably the most important as far as what you would use in the real world sense….But I often tell my students that the highest paying jobs to deal with math will be the conceptual jobs" (Interview 4, 2/24).

Todd viewed conceptual understanding much like icing on a cake. It would be nice if his students had a conceptual understanding but in order to obtain a conceptual understanding, first students must understand the procedures. It was important to Todd that his students knew the skills, and open-ended items might help "cement" those skills into the students' heads. Too, open-ended items were a motivational tool because they were "fun and interesting" and the kind of thinking required by open-ended items might lead students into higher paying jobs. In his Algebra Concepts class, Todd asked the students how they liked these kinds of questions [open-ended items]. They said they liked them. Todd told me,

> When I asked them in class today and they said they enjoyed and glad we did it and so that really took me by surprise. And so I think if they like it that means that they are having fun doing it, which means they probably understand what they are doing for most part. And for the most part they got all of them right, you know, with a little help here and there. So, normally I would say the alternative questions, I don't think tend to favor the kids in the lowest levels because most of the alternative questions are written in a way that you need to understand the math to be able to apply it to answer the question…I think they know more than they want to let themselves believe they know so in a way it is kind of tricking them into actually doing the math that they probably didn't think they could do. (Interview 2, 2/10)

Todd's emphasis was on "doing the math" and open-ended items motivated or tricked students into doing the math.

The teachers' beliefs about how people know mathematics affected their use of open-ended items. Even though Keith saw mathematics as absolute in the sense that it exists in some ideal way, he believed that how students perceived mathematics varied. Therefore, he spent a considerable amount of time trying to understand his students' thinking. Open-ended items facilitated Keith's desire to know more about his students' thinking. Keith's belief that different students learned and perceived mathematics differently (a relativistic view of knowledge) made open-ended assessment items a particularly useful tool for his teaching.

Robin spent a lot of time thinking about logical ways to present mathematical topics. Although she would not argue that she had completely figured it out, Robin believed that there was a right way to teach and she felt that she was moving towards it. She valued her experience and was committed to her beliefs about teaching. Therefore, Robin spent most of her time using what she considered tried and true methods of explaining mathematics and its connections to students. The methods were still open to scrutiny and she constantly searched for new ways of presenting the material. Robin was more concerned about getting students to see mathematics as she saw it (a connected whole) than in trying to understand how they saw it. Robin saw mathematics as a logical system. Mathematical topics are connected in a logical way. This logic is the same for everyone. Consequently, Robin did not feel as strong of a need as Keith did to understand her students' thinking in order to inform her instruction. She used open-ended items to identify whether or not students had made the connections that she transmitted. She reflected on student responses to open-ended items to identify misconceptions and used that information to inform her teaching. But her use of the items was more about checking for understanding than about trying to make sense of the students' understanding. When Robin used open-ended items, she often asked questions that required students to identify and explain procedural errors, or questions that asked who is right and why. These questions helped Robin assess whether students were getting the connections she was emphasizing in her teaching. In class, Robin often asked students why certain procedures did or did not work. The open-ended items she chose assessed the underlying concepts.

Todd's belief that if students try then they will understand prevented him from considering evidence that indicated his teaching could be a source of students' misconceptions. When evidence was pointed out to him that his teaching affected the way students responded (incorrectly) to an open-ended test item, he was reluctant to consider himself responsible. Even though students gave answers to the item that indicated that they paid attention to Todd's instruction and still did not understand, he largely ignored this evidence because it contradicted his belief that if students tried hard and paid attention then they would understand.

Authority

The teachers' orientations to authority also affected the way they used open-ended assessment items. Keith and Robin had an internal locus for authority with respect to their beliefs about teaching, while Todd had an external locus for authority. Both Robin and Keith made their own decisions about their teaching. Although they used external sources such as the textbook, the curriculum, and the NCTM recommendations to aid them in decision-making, they decided how to teach based on their own system of beliefs.

Keith and Robin had an internal source for authority, which led them to integrate the use of open-ended items into their larger scheme for teaching. Keith was, for the most part, unaffected by the authority of his school system and the textbook in his teaching. He used open-ended items because they helped him better understand his students' thinking. Robin made decisions about teaching based on her experiences and her beliefs about teaching. One of those beliefs was that she should finish the curriculum. This belief may be related to authority, but more likely it is related to the fact that Robin helped create the curriculum and felt ownership of it. Her commitment to the curriculum sometimes overrode her commitment to open-ended assessment, but when she did use open-ended items, the way she used them was consistent with her beliefs about teaching. The fact that Robin did not always choose to include 20% open-ended items on her tests illustrated her willingness to deviate from the authority of her school system. She did not use open-ended items because she was supposed to, she used them because they helped her reach her broader educational goals.

Todd, on the other hand, relied more on authority in developing his beliefs about teaching. Although he expressed reasons for using open-ended items, such as encouraging higher-level thinking, the way he used open-ended items minimized the potential for fostering higher-level thinking in his students. Recall that Todd was relatively new to the teaching profession. He did not consider himself to be an authority on teaching and consequently sought advice from other teachers about certain aspects of his practice. He used the textbook as a prescription for teaching. Todd used a lot of "they" language when he talked about how and what he taught. When asked if he planned to address a particular topic, he would respond with something such as "They don't get into that," they meaning the authors of the textbook. When asked how he would teach logarithms (a topic he had not yet taught), he responded, "I don't know necessarily what the book or the curriculum wants me to teach" (Interview 7, 4/24).

Todd, who also taught science classes, pointed out differences between the science and mathematics curricula:

> That's always been my problem with math teaching that I never really realized until I got into student teaching, and that is the fact that it's so defined, the curriculum. It's like the most defined curriculum of any of the curriculums that I've ever seen. In my science class, it tells the objective that I want to, that they want me to teach, but it doesn't tell me what chapter I have to use if I have to use that book. They kind of leave it up to me how I want to teach it, whereas math, they gave me the book. Every student has to have the book, 'cause that's what it is, that's the book we're using, and we need to do these chapters. (Interview 1, 2/8)

Todd was willing to teach the way he wanted to teach in his science courses because the curriculum was less specific about what he should do. In mathematics, however, Todd saw the curriculum as more specific, more defined. The "they" language was present in the above excerpt; it is clear that Todd intended to do what "they" wanted him to do.

Robin and Keith each paid attention to context when making judgments. For example, Robin said, "I do think there are some teachers who believe that they are doing the right thing just by doing Skill 1, Skill 2, Skill 3. And I just think they believe that because that is their own experience with mathematics" (Interview 7). She recognized that experience affects belief. Because some teachers had only experienced mathematics in a procedural way, Robin saw it as natural that those teachers would believe in teaching in a procedural way.

When I asked Keith if there was a right way to teach mathematics, he said, There are many different approaches. I just think that you have to think about everything that is available to you, and pick out the best, and do the best you can with it" (Interview 5). At the same time, Keith also believed that there was a wrong way to teach mathematics. He believed that teachers who did not know their content, focused only on procedural understanding, or taught to standardized tests were not teaching well. He thought these teachers were not affecting learning. Even though he recognized that there may be many “right” ways to teach, he believed that good teaching resulted in certain outcomes; if those outcomes were not realized, then good teaching had not occurred. He did not have an “anything goes” approach. He recognized that judgments had to be made in context but that some judgments were better than others.

Because Keith and Robin used evidence in the formation of their beliefs and judged that evidence in context, they were able to use student responses to open-ended questions to empower their teaching. When their students’ responses to open-ended questions were incorrect, Keith and Robin took this as evidence of a misconception and considered their teaching as a possible source for the misconception. When they identified a misconception, they used that information in their teaching. Keith claimed that with traditional assessment “You don’t really know whether they’re just mimicking what you did in class or whether they really understand the concept” (Interview 3, 2/17). He wanted his students to have a conceptual understanding of mathematics, and he was able to use responses to open-ended items as evidence for that kind of understanding or lack thereof. Robin, too, looked for evidence of understanding in student responses to open-ended items. She said,

> Next year when I start talking about factoring, I am going to at least include that idea because I felt like I overlooked it or I would have had at least one answer from everybody. So that made it a good item, too: the fact that not only did I get information about what they could do, I also got information about what I could do. (Interview 2, 1/31)

When Robin’s students performed poorly on an open-ended assessment item, she used their responses as evidence that: a) they did not understand, and b) she needed to adjust her teaching.

Todd did not pay as much attention to context as did Keith and Robin. Todd used evidence in an idiosyncratic way, aligning himself with different authorities according to how the authorities fit with his experiences. Todd looked for evidence to justify his beliefs but did not evaluate the way in which the evidence was generated. For example, Todd believed that technology usually hindered students' ability to think. He said, "Once in a while I'll let them use a calculator just to prove to them that having a calculator in hand is not going to increase their scores, and in fact, I've often found decreasing scores" (Interview 1). Decreased scores when students used calculators supported Todd's belief that technology hindered students' ability to think. But Todd did not change his instruction or the construction of the test when he allowed students to use calculators on the test and found decreasing scores. He did not indicate whether additional time was allowed for using technology. These and other factors may have influenced the reduction in scores, but Todd noticed the reduction and took that as evidence to support his already-formed belief. He did not judge the evidence in the context in which it was generated. Similarly, recall Todd's description of the study reported on the program "20/20" concerning technology in schools. At first, he thought that the results of the study were "ludicrous" because they were "totally different than what [he'd] been told." Once he started thinking about it, he decided that students were calculator-dependent in his class. He did not question how the study was conducted. He did not question how technology was used in the two classrooms that were compared. But once a perceived authority had made a claim that technology was bad, Todd had evidence for what he perceived in his own classroom. He did not change his belief based on his experience. He changed his belief when an authority confirmed what he experienced.

Todd was able to recognize the importance of context and the fallibility of authorities, at times. To a certain extent, Todd appeared to want to "buck" authority, but at the same time, he did what he was supposed to do. He said,

> I mean, I know what I get paid for and I don't believe in, I mean, I haven't put myself at the level where I am in charge in the curriculum planning but I do feel

> that who makes the curriculum,…some of these people haven't been ever in the business world. Some of these people have no idea how hard it is to teach certain subjects so I often think that the people who design the curriculum aren't often, unfortunately, out for the best interest of the child. I think it is more of we have adopted this book, let's get done with the book, let's teach the book or whatever. They look at how to adopt the book with the curriculum that they chose. I think that happens sometimes and I don't think it should. I really think that our math would be a hundred percent different if we looked at the type of kids we are teaching, where we think they are going and how we should focus our math to them because I really think that we are teaching kids things, in some instances, that they don't absolutely need to know. (Interview 2, 2/10)

Although Todd recognized the importance of doing what is best for his particular group of students, and at the same time pointed out that the authorities may not be in the best position to make curricular decisions, his teaching was still dictated by the textbook. Todd acted in accordance with authority even when he did not agree with authority.

Todd's intellectual development seemed to be in transition. Although he recognized the need for evidence and was beginning to form some of his own beliefs about teaching, he was still relying heavily on authority to inform his teaching. He professed a belief that applications were important, yet glossed over them in class. He did not use the evidence he gathered from student responses to open-ended items to inform his teaching. He used open-ended items much the same way he used other items. He did not attend to the evidence his students provided him about their lack of understanding in order to make decisions about his teaching. Rather, his teaching was dictated by authority (the textbook) and he asked open-ended questions, for the most part, because authority (the county) directed him to.

Reflectivity

Cooney, Shealy and Arvold (1998) found a relationship between an individual's beliefs and relation to authority and the extent to which he or she reflected. They maintained that an individual's voice needed to be a partner in her or his knowledge construction in order for the individual to practice their teaching reflectively. Similarly, Mewborn (1999) found that "When the locus of authority was internal to the preservice teachers, they felt empowered to think reflectively in the manner described by Dewey" (p. 335). The findings of the present study also point to a relationship between an individual's relation to authority and reflective thinking. Both Keith and Robin integrated other voices with their own in their knowledge construction while Todd was more dependent on authority. Keith and Robin approached assessment more reflectively than Todd did.

Van Manen's (1977) levels of reflection provide a means for conceptualizing the extent of the teachers' reflectivity. Each of the three teachers exhibited reflective thinking at some level. They each recognized certain aspects of teaching as problematic. Each of them also searched for ways to address aspects of teaching that they viewed as problematic. Although the three teachers all exhibited reflective thinking, the levels at which they reflected varied. All three teachers reflected at the first of Van Manen's levels. Each of them was concerned with how to reach her or his goals in the most efficient way possible, but some of the teachers reflected more deeply.

Keith's Reflective Nature

Keith recognized aspects of his teaching as problematic, and he searched for ways of addressing those aspects. He explained,

> Before I got involved with alternative assessment, it was like, I don't know what to do—something is missing. And now I have something I can grab onto. I've always been somebody who likes to ask questions [while teaching]. It is just that I didn't always have, I don't think, the framework that I needed to ask a good question, thinking about possible responses and thinking about how you would determine if those were good responses. Now I'm thinking about that all of the time. Not only do I just reflect about what I have taught, but I think more about it before I teach. (Interview 7)

Keith considered learning, assessment, and teaching all problematic. Keith felt that different people experienced mathematics differently, and he was constantly trying to understand how his students thought about mathematics. If students misunderstood, he assumed that he needed to understand them better and devise a way of communicating with them about mathematics the way they saw it.

Keith's reflection moved beyond the first level. He thought it was important to look beyond what the textbook or the curriculum said and to decide what was most important for students. Like he said, "I think when you develop assessment, you kind of have to…think about how important Number 9 is to the whole picture" (Interview 2, 2/9). Keith believed it was important to look at broader educational goals. He wanted students to be prepared for success in life, and he wanted to be sure that the questions he asked facilitated that broader purpose. Keith's focus on the big picture and the fact that he reflected about how his teaching fit in with the big picture illustrated that he reflected at the second of Van Manen's levels. Keith also questioned the ends, not just the means, of teaching. He did not accept the curriculum or the textbook as recipes for teaching. He decided on what was best for his students and taught the way he believed was best. Keith planned to serve on a curriculum committee the summer following data collection. He expressed a desire to change the curriculum to include fewer objectives and to be more focused on what he perceived to be the big ideas for each course. He recognized a need to change the curriculum (the "ends"), and was excited about participating in the process of change. Moreover, he was willing to not follow the curriculum because he believed that it contained too many objectives. It is conceivable that a teacher could not agree with a curriculum but still teach it because he or she was required to do so. Keith felt so strongly opposed to the pacing in the curriculum that he did not follow it. Not only did Keith recognize a need for change, but he also refused to use the curriculum as it was stated because he did not agree with its fast pace. Keith demonstrated Level 3 reflection in that he was willing to question authority and to examine the educational ends, not accept them as given, and to decide his own course of action.

Keith constantly tried to step outside of his situation in order to view the "big picture." He challenged educational ends in his own mind and took steps to challenge them systemically by volunteering to serve on a curriculum committee. Keith was able to inform his teaching through asking open-ended assessment items by reflecting on student responses to the items. Keith's reflective nature facilitated his use of open-ended assessment in a way that was aligned with reform initiatives in mathematics education.

Robin's Reflective Nature

Like Keith, Robin looked to her teaching when students did not understand. She recognized teaching as problematic. Recall her thoughts about teaching factoring. She recognized a problematic situation (students did not understand factoring), and then initiated a search in order to address the issue (she came up with a new approach to teaching factoring). She intended to test her method the next time she taught factoring. Reflective thinking was evident throughout Robin's data.

Robin's level of reflectivity also went beyond Van Manen's (1977) first level. For example, Robin reflected on students' problems with simplifying rational expressions. She said, "That may be where we're missing the boat from the very beginning. Are they using the concept of variable, I mean are they using the symbols for variables way before they understand what it means" (Interview 3)? Robin wanted students to understand mathematics. She wanted to know more than whether the students could do the problems. She wanted to know that they understood the concepts behind the problems. When students did not understand, she questioned why they did not. She was concerned with more than what worked. Robin consistently reflected at the second of Van Manen's levels, but she also reflected at the third level. For example, when I asked her if American education was broken, she said,

> You get all of the reports about well, our best students don't do as good as the best students in Japan, and our middle students don't do as good as the middle students in Japan, but sometimes that is more a reflection of the diversity of our culture than anything else. And to me, who cares about the average? What is the

> average? Who is average? You know, we are still being successful. Our whole system is still successful, our democracy, our free enterprise, I mean, I don't see major problems there. (Interview 5, 3/15)

Robin looked beyond what the test results said and whether or not students could answer questions correctly. She reflected on the system as a whole. She looked at the social conditions in which education is situated in order to evaluate it.

Reflectivity empowered Robin's use of open-ended items. The way Robin used open-ended items (to inform her teaching practice, to identify misconceptions) was aligned with the assessment reform initiatives in mathematics education. Using these items gave Robin insights into her students' thinking, and she was able to use those insights to inform her teaching. Because Robin reflected on her students' responses to open-ended items, she obtained more information about their conceptual understanding and misconceptions than she would have been able to get from traditional questions. Because she reflected deeply (at Levels 2 and 3), Robin was better able to align her assessment practices with her principles and goals for teaching.

<u>Todd's Reflective Nature</u>

Todd did not think that the mathematics he taught should be problematic for students. Nor did Todd view assessment as problematic. He explained,

> I've had many times where I have had one or two people only pass a test out of twenty, and I'll let the class have it.…A lot of that has to do with the fact that they don't put effort into it. I mean, you know, I know you can attribute some of that to probably my teaching, but at the same point, this stuff is pretty self-explanatory. If you try, you should be able to get it. Basic algebra is not rocket science. So you know, what I do with homework and everything else you should be able to catch what is going on. And by the way I test, you know, I don't feel like I am throwing anything like curveballs at you. So you should be able to get it. (Interview 2, 2/10)

Todd believed that if students did their part and if the teacher put questions on the test similar to those he or she asked in class, then students should pass the test. Because Todd found neither high school mathematics nor assessment problematic, it is not surprising that his thinking was not particularly reflective in these domains.

Todd's reflection was, for the most part, limited to the first of Van Manen's levels. He professed a belief that teaching should prepare students to be productive members of the workforce and that students should be able to apply mathematics. But his teaching did not focus on mathematical applications. Although he assigned application problems, he did not spend class time discussing those applications. Todd did not reflect on how his teaching was related to his overall goals. Todd wanted students to be able to apply mathematics, but he did not concern himself with whether his teaching facilitated an ability to apply mathematical concepts to real-world situations. He did not consider how his teaching might affect students' understanding of future topics. Although Todd questioned whether the writers of the curriculum were out for the best interests of the students, he accepted the curriculum as given. It did not bother Todd enough that the curriculum was not aligned with his particular principles and goals for teaching mathematics for him to act in opposition to the curriculum.

Todd's lack of reflectivity in certain domains was due, in part, to his reliance on the authority of the textbook and the graduation test. But beyond that, it was also due to his desire to be perceived as an authority. Todd was not willing to appear to his students as unknowledgeable in mathematics, and consequently, was uncomfortable when situations arose in which he was unsure of the mathematics. He would often sweep under the rug those situations that caused discomfort. Therefore, he cut off his reflective thinking in the first phase because he refused to recognize a situation as mathematically problematic in front of his students. Too, when a student offered a response to an open-ended item that Todd did not understand, he made little attempt to understand the student's approach because, again, spending too much time thinking in front of the students indicated a weakness in his own mathematical understanding. The school system expected Todd to use open-ended items, and so he did. However, Todd was uncomfortable when students came up with answers that he did not immediately recognize as correct or incorrect.

Although Todd used more open-ended items than either of the other two participants, he did not use them in a way that informed his teaching. His haphazard approach to grading provided evidence of his lack of reflection on the student responses to open-ended items. Todd did not learn about student understanding from reading responses to open-ended items. Even when students mimicked what he had done in class the day before, Todd considered himself only partially responsible for their poor performance on an open-ended test item. He did not reflect on his teaching as a source of student misunderstanding; therefore his teaching was not informed by his students' responses to open-ended items. Although open-ended items were assigned often during class and also appeared on formal assessments, not much class time was dedicated to discussing different approaches or even correct answers. Most of the time, credit was given for attempting open-ended items. He minimized the potential effect of open-ended items on his students' learning. The fact that Todd did not recognize assessment as problematic limited his ability to reflect on assessment.

Summary

Reflectivity was a powerful force that affected how teachers used open-ended assessment items. When, as Goodman (1984) described, "reflection…meant thinking about which techniques seemed best in any given situation [and when] the techniques were seen as ends themselves rather than means to implement some broader educational purpose" (p. 16), the use of open-ended items was not as powerful, as in the case of Todd. However, when teachers reflected more deeply and focused on open-ended items as a way to achieve a broader purpose, the use was more meaningful as in the cases of Keith and Robin.

Knowledge

Content Knowledge

Shulman (1986) explained that a teacher's content knowledge should facilitate an understanding not only

> *that* something is so; the teacher must further understand *why* it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in justification can be weakened and even denied. Moreover, we expect the teacher to understand why a given topic is particularly central to a discipline whereas another may be somewhat peripheral. (p. 9, emphasis in original)

Robin's focus on the "whys" and the "connections between" demonstrated the richness of her content knowledge. It was important to Robin that her students appreciate the development of mathematical topics and the structure of the body of mathematics. Her focus went beyond a discrete set of skills to include an emphasis on structure and understanding. Similarly, Keith was interested in educating life-long learners who thought about "Why you can do this?" Keith saw beauty in mathematics and wanted his students to develop a "big-picture" view of mathematics. He felt comfortable letting go of some of the authority in his classroom in order to encourage students to justify mathematical truths for themselves. Although Todd recognized the importance of understanding why mathematics works, he did not focus on why it worked in his own teaching. He claimed that students were more interested in knowing why they needed to know mathematics rather than in why it worked. Recall that when a student asked a question about how mathematics worked, Todd replied, "Because it works in the formula. Trust me—there are some things in math that I won't be able to explain, it just works" (Algebra II, 2/14). Todd wanted his students to view him as a mathematical authority and he brushed aside questions about content with which he was uncomfortable. Todd was often surprised by topics as they arose in the curriculum. Consequently, his teaching did little to set the stage for the learning of subsequent material.

The teachers' content knowledge affected their use of open-ended items. Because Keith and Robin had a solid knowledge of content, and focused their teaching on why mathematics works, open-ended items were useful for them. Open-ended items encourage students to investigate the whys of mathematics, hence they were aligned with Keith and Robin's goals for teaching mathematics. Todd was uncomfortable when he was not able to immediately determine the correctness of a student's response to an open-ended item. This discomfort caused Todd to abandon discussions of open-ended items, thus limiting their impact on student understanding.

Pedagogical Content Knowledge

In his definition of pedagogical content knowledge, Shulman (1986) included,

> The most useful forms of representation of…ideas, the most powerful analogies, illustrations, examples, explanation, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no singe most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. (p. 9).

Pedagogical content knowledge can be viewed as having a reciprocal relationship with a meaningful use of open-ended assessment items. If a teacher's pedagogical content knowledge is well developed, then students should be better able to develop a conceptual understanding of the subject and open-ended items can help teachers assess that understanding. On the other hand, if teachers use open-ended items meaningfully, then they would use the student responses to the items to inform their instruction, thus informing their pedagogical content knowledge.

Robin reflected on her teaching and on her students' responses to assessment items to inform her teaching. She identified particular misconceptions and thought about ways to address them in her teaching. As an experienced, reflective teacher, Robin's pedagogical content knowledge was well developed. Keith searched for different ways to present mathematical topics that would address different ways of student knowing. He valued students' approaches to mathematics and incorporated their approaches into his teaching. Keith, too, had a well-developed knowledge of pedagogical content.

Todd, being relatively new to the teaching profession, relied on the textbook to inform his teaching. He had not accumulated or developed a variety of approaches to mathematical topics and relied on the single approach provided by the textbook. An important question is whether Todd's pedagogical content knowledge will increase as he gains experience. Surely, more experience will provide more context for developing pedagogical content knowledge, but the question remains, will Todd attend to the necessary aspects of his practice and reflect on his practice enough to have a real impact on his pedagogical content knowledge? If Todd does not view assessment or the content of his courses as problematic for students, to what extent will he reflect on those aspects of his practice?

System Constraints

Keith, Robin, and Todd each felt constrained by the school system, a factor that inhibited their use of open-ended items. But perhaps as interesting as what did constrain them was what did not constrain them. Grading was not a hindering factor in these teachers' use of open-ended items. In fact, when I asked Keith about the hardest part of using open-ended assessment, he replied, "It's not the grading" (Interview 2, 2/9). None of the three teachers was concerned with grading responses to open-ended assessment items. They felt comfortable with the grading rubric used in the project. Also, these teachers had a source for open-ended items (the item bank), so they had access to a large number of items. Keith did comment that he was teaching classes for which an item bank did not exist and that it was difficult to find time to search for or create items for those classes. The teachers felt supported in their efforts to use open-ended assessment items (the county required that tests contain such items).

All three teachers felt that they could be better teachers if they had more time. Robin and Todd wished they either had more time with the students or fewer objectives in the curriculum. Keith wished that he had more time to think about and prepare for his teaching. Both Robin and Todd felt an obligation to finish (or, in Todd's case, nearly finish) the county curriculum. This constraint did affect Robin's use of open-ended assessment, but did not affect Todd's use. Because of the way Todd used open-ended items, their use did not require any more time than traditional items. Therefore, Todd felt free to use open-ended items often. For Robin, however, using open-ended items took valuable class time that she would not easily forfeit. Although she valued using open-ended assessment, her dedication to finishing the curriculum would sometimes override her commitment to using the assessment items.

Reflection and time seemed to interplay in how they affected the teachers' use of open-ended assessment. The more reflective the teacher was, the more time he or she spent when using open-ended items; consequently, time became a salient issue. Keith did not feel obligated to finish the curriculum; so even though open-ended assessment took more time, he was willing to use it in the interest of promoting conceptual understanding. Time did affect Keith's use of open-ended items in the sense that they took more time to create or find. Keith's issue with time was in planning and preparation. He felt like he would use more open-ended assessment if he had more time to dedicate to planning.

Situating the Study

The purpose of this section is to situate the present study within the broader empirical literature on teachers' assessment practices. Comparisons and contrasts will be drawn between the results of prior literature and the results of the present study, and explanations will be offered for differences.

Teachers' Conceptions of Reform

Prior research indicates a gap between teachers' practices and reform initiatives. Specifically, Cooney (1992) and Senk et al. (1997) reported that teachers had little understanding of the NCTM *Standards*. Todd exhibited a limited understanding of the *Standards*, but Keith and Robin demonstrated a knowledge and appreciation for the spirit of the *Standards*. A question is begged by this finding: Do teachers need to be reform-minded in order to use open-ended assessment in ways that empower their teaching? Guskey (1986) posited a claim that a change in practice could facilitate a change in beliefs. Todd's case provides an argument against Guskey's claim. Reform initiatives advocate a use of open-ended items as one vehicle for implementing reform. Todd used open-ended items (i.e., he changed his practice), yet he still found the NCTM *Standards* irrelevant for his students (i.e., he did not change his beliefs). Using open-ended assessment items did not facilitate a shift in Todd's thinking about reform. Perhaps being reform-minded to begin with is what enabled Keith and Robin to use open-ended items in meaningful ways. Todd's limited understanding of the reform hindered his meaningful use of open-ended items.

For the most part, the teachers in the present study had assessment practices that were aligned with their teaching practice. Each of the three teachers used open-ended items on tests, and they also used them in class. Although Robin used open-ended items a great deal less than the other two teachers did, her instruction focused on understanding and connections, and her tests contained items that assessed understanding and connections. Stiggins and Conklin (1992) reported a lack of alignment between teachers' instruction and their assessment. The teachers in the present study felt comfortable including open-ended items on their tests and in their teaching, and that was likely due to their participation in the assessment projects. In general, their instruction and assessment were aligned, a result contrary to that of Stiggins and Conklin (1992).

Stiggings and Conklin (1992) and Hancock (1994) each indicated that assessment was not an integral part of instruction for the teachers they studied. This, too, was the case for Todd. Todd did not use assessment to inform instruction. Both Keith and Robin, however, were able to use student responses to open-ended items to inform their teaching. What enabled Keith and Robin to use assessment as such an integral part of their instruction? The fact that Keith and Robin each viewed assessment as problematic and reflected deeply on responses to open-ended items enabled them to better understand student thinking which in turn informed their teaching. Todd's lack of reflection on student responses to open-ended items prevented him from informing his instruction with his assessment. Too, each teacher's view of certainty of mathematical knowledge affected the way they used open-ended assessment and the extent to which they used assessment as an integral part of instruction. Keith recognized that students view mathematics differently and consequently recognized a need to understand student thinking. He used assessment as a way to gain insight into that thinking. He used that insight in his instruction. Todd did not recognize the need to consider different approaches or to understand student thinking. After all, basic algebra is not "rocket science."

Teachers' Use of Test Items

Previous research indicated that teachers' tests contained mostly low-level items (Cooney, 1992; Senk et al, 1997) and that teachers were not comfortable using items that required an open-ended response (Cooney, 1992; Hancock, 1994). That was not the case for the teachers in the present study. The teachers in this study each used open-ended items that assessed higher-level thinking skills on every test. That was likely due to the teachers' participation in the assessment projects and the support they received from their school system for using open-ended items. Too, the system actually required that they use open-ended items on tests, so the teachers' orientation to authority also affected their use of open-ended items on tests. Keith and Robin, having an internal source for authority, used open-ended items because they fit their broader educational purposes. But Todd did not use open-ended items in an effort to achieve an educational goal; he used them because he was supposed to.

Cooney et al. (1996) reported that mathematics teachers were not comfortable with subjectivity in grading. The teachers in the present study, however, were comfortable using open-ended items on tests and never expressed a concern with subjectivity in grading. Keith and Robin were comfortable with subjectivity in grading partly because they had an internal source for knowledge. That is, they were comfortable making decisions about the value of student responses based on evidence they saw in the response. Keith and Robin recognized assessment as problematic, but they felt comfortable examining evidence and making decisions on the value of student responses. BecauseTodd did not view assessment as problematic, the issue of subjectivity never surfaced for him.

Factors that Influence Teachers' Assessment Practices

Hancock (1994) found that changes in state policy did not substantially affect teachers' practices. The school system in which the participants in the present study were employed implemented a policy requiring that 20% of test items be open-ended. The policy probably affected teachers' use of the items in the sense that they all actually asked open-ended items on tests. What the policy did not affect was the way in which the teachers used the items. A policy change may cause teachers to use certain strategies, but other factors are more salient for understanding how they use those strategies.

Previous research has indicated that knowledge and beliefs, instructional materials available (Senk et al., 1997), time (Cooney, et al., 1996, Nash, 1993), and an institutionalized curriculum (Wilson, 1993) influenced teachers' assessment practices. The present study corroborates these findings. In Chapter 2, I pointed to a gap in the literature about whether internal factors (knowledge and beliefs) affected how teachers cope with the external factors such as time, curriculum, and materials. The results of this study indicated that they do. For example, Keith's belief that teaching should promote understanding and his emphasis on the big picture affected his view of the curriculum and time constraints of his system. His internal source for his beliefs enabled him to choose the extent to which he followed the rules. Keith did not need permission to deviate from the curriculum. He did so because he believed that it was best for his students. Thus, the curriculum constraint was not an issue for Keith. Similarly, Todd's lack of reflectivity on student responses to open-ended items meant that using the items did not take him any more time than using traditional items. Thus, Todd's lack of reflectivity on student responses (an internal factor) caused time (an external factor) to be a less salient issue.

Teachers' Grading Practices

Cooney (1992), Hancock (1994), and Senk et al. (1997) each reported that written tests were the primary source for student grades. This was also the case in the present study. Wilson (1993) found that "what gets graded is what gets valued." Data from students were not collected in the present study, so it is not clear whether the participants' students valued the kind of thinking required for answering open-ended items. But Keith did express a concern that he wanted his students to know that they had to think on a conceptual level and thus he had to ask and evaluate items that assessed such understanding. The teachers were comfortable having open-ended assessment items on their tests and did not express concern about grading them.

Summary

Previous research reported factors that affect teachers' assessment practices. The results of the present study provide a deeper understanding of how those factors affected teachers' use of a particular assessment strategy. Too, this study added reflective thinking into the mix of factors that influence teachers' assessment practices. The study also indicated that some factors influence others; for example, internal factors such as beliefs and orientation to authority influence the extent to which external factors such as time and curriculum impact teachers' assessment practices.

CHAPTER 6

SUMMARY AND IMPLICATIONS

Purpose

Whereas previous studies focused on which assessment strategies teachers use (e.g., see Cooney, 1992; Senk, Beckmann & Thompson, 1997), this study focused on a particular assessment strategy, viz., using open-ended items, and the factors that influenced teachers' use of that strategy. Because the participants were drawn from a group of teachers who had participated in staff development projects aimed at enabling them to use open-ended assessment in their teaching, some of the constraining factors described in previous studies were removed. For example, Cooney (1992) found that teachers were uncomfortable with questions that did not require the generation of a specific number and that they were reluctant to assess students' performance subjectively. The teachers in the present study felt comfortable using open-ended items and expressed no concerns about using a rubric in their grading of open-ended responses. Also because of their involvement in the assessment projects, the teachers felt comfortable using open-ended items on tests. Unlike Senk et al.'s (1997) finding in their study of 19 high school mathematics teachers that "test items generally were low level,…involved very little reasoning, and were almost never open-ended" (p. 187), the teachers in this study included open-ended items on every test. This practice was due, in part, to the system's directive that 20% of test items should be open-ended. But the teachers felt comfortable using the items and used them in class as well as on tests. This group of teachers was an appropriate group of informants for research because of the fact that some of the constraining factors described in other studies were removed.

At the outset of the study, I believed that *if* teachers used open-ended items, then their teaching practice would be affected in a way that moved them closer to the reform initiatives in mathematics education. I believed that the goal was getting teachers to use open-ended items. If they used them, then they would understand their students' thinking better and use that information to inform their teaching. I assumed that the very act of asking open-ended items was a vehicle for implementing reform. Hence, I set out to understand the factors that influenced teachers' use of open-ended items so that I could better understand how to get teachers to use them. However, the term *use* in my research question originally referred to *whether* they used open-ended items rather than *how* they used them. If I could understand what made some teachers use them (or not), then I could develop better ways to convince other teachers to use them. Through the course of the study, the meaning of the research question changed. I was studying a group of teachers that did use open-ended items, but I quickly realized that the way they used them varied. My question became more than what factors influence teachers to use open-ended items. I turned my attention to understanding the factors that enable or hinder teachers use of open-ended items in ways that empower their teaching. My primary research question remained: What factors influence teachers' use of open-ended assessment items? But I turned my focus from what factors affect where, when, and how often teachers use such items to what factors facilitate or hinder teachers' meaningful use of them. Two questions expanded upon the primary question: (1) How do teachers' beliefs facilitate or hinder their use of open-ended assessment items? and (2) How do teachers' orientations to authority facilitate or hinder their use of open-ended assessment items?

Theoretical Perspectives

Many researchers have found connections between teachers' beliefs and their practical actions. Specifically, Senk et al. (1997) suggested that teachers' knowledge and beliefs affected their assessment strategies. It was natural to look at beliefs as a possible influence on teachers' use of open-ended items. Theories about intellectual development (Baxter Magolda, 1992; Belenky et al., 1986; King & Kitchener, 1994; Perry, 1999) have been used to investigate teachers' beliefs. These theories all suggest that people's knowledge and beliefs fall along a continuum from knowledge being given by authority to knowledge being developed and evaluated in context. The theories of intellectual development (Baxter Magolda, 1992; Belenky et al., 1986; King & Kitchener, 1994; Perry, 1999) were used as a framework for understanding teachers' beliefs and orientations to authority.

At the outset of the study, I intended to limit my theoretical perspectives to those involving teachers' beliefs and relation to authority. During data collection, and concurrent with redefining the research question, I began noticing a difference in how the teachers used open-ended items and not just whether they used them. I began to question the cause of this difference. As I watched the teachers use and grade open-ended items, I noticed a difference in how much they were thinking about the student responses to the items. The literature I had read on reflective thinking (Dewey, 1933; Van Manen, 1977) came to mind as a way of making sense of the differences I observed during data collection. Dewey (1933) provided a definition and description of reflective thinking and Van Manen (1977) provided a way to conceptualize depth of reflection, each of which was helpful in conceptualizing my interpretations of the teachers' actions.

Researchers (Cooney et al., 1998; Mewborn, 1999) have posited a relationship between an individual's relation to authority and the extent of his or her reflectivity. Mewborn (1999) found that "the decisive issue that determined whether the preservice teachers were inclined to think reflectively was the locus of authority for pedagogical ideas" (p. 335). The relationship between relationship to authority and reflectivity was investigated in the present study.

Methodology

Three teachers who had been involved in staff development aimed at enabling them to create and use open-ended assessment in their teaching participated in the study. Each teacher was interviewed seven times and observed for approximately 24 hours in two different classes (approximately 12 hours of classroom observation in each class). Artifacts including tests, quizzes, handouts, and graded open-ended assessment items were collected.

The first part of the analysis was done through inductive analysis (Patton, 1990) and was aided by the creation of colored cards that were sorted in order to identify themes in the data. The second part of the analysis involved a return to the data and coding in terms of the theoretical framework. In this phase of analysis, the data were coded for source, evidence, certainty, acts of reflective thinking, and levels of reflection.

Findings

Figure 16 summarizes the participants' beliefs in the areas of mathematics, teaching, learning, and assessment. All three teachers believed that mathematics is "out there" to be discovered and that it is not a human invention. Keith believed that although mathematics was absolute, people had different views or perspectives on mathematics. Robin saw mathematics as logical, with pieces of mathematics fitting together like pieces of a puzzle. Todd saw two distinctive sides of mathematics: the philosophical (including theory and proof) and the useful. He was more interested in the useful side of mathematics.

Keith and Robin focused their teaching on conceptual understanding, whereas Todd focused more on procedural understanding. The central focus in Keith's teaching was learning; he believed that teaching did not occur if learning did not occur. He focused more on the big picture than on isolated skills. Robin's central focus in her teaching was on making connections between mathematical topics for her students. Todd's main focus was on demonstrating relevance and preparing his students for society by helping them become proficient in basic skills. Keith and Robin both believed that they were largely responsible for their students' learning. Each of them looked at their teaching as a possible source if students did not understand. Todd, on the other hand, placed more of the burden of learning on the students. He believed that students were responsible for their own learning, and if they did not learn then they did not try hard enough. Keith believed that everyone could learn mathematics, but that people learn it differently.

	Keith	Robin	Todd
Mathematics	· Mathematics is "out there" · People can have different views of mathematics	· Mathematics is logically structured · Mathematics fits together · Mathematics is "out there"	· Mathematics is an unchanging, universal constant · Mathematics is "out there" but people invent the language to describe it · Mathematics is two-sided—philosophical and useful
Teaching	· Teaching should promote learning · Teaching should promote understanding · The "big picture" is more important than isolated skills · Teaching should prepare students for success in society · Teachers should value student thinking	· Teaching should promote understanding (connections, applications, going backwards) · It is a teacher's responsibility to finish the curriculum	· Teaching should prepare students to be successful members of society · Teachers should demonstrate relevance · The basics should be the primary focus for teaching · Teachers should be encouraging but demanding · Teachers should be adaptive · Teachers should be an authority in their subject area
Learning	· Everyone can learn mathematics · Different people learn mathematics differently · Teachers are responsible for student learning	· Teachers and students share the responsibility for learning	· If students do their work, learning should occur.
Assessment	· Assessment should provide students and opportunity to show what they know and how they know it · Assessment should inform instruction · Assessment should reflect the teacher's values and should be aligned with instruction	· Teachers and students should learn from assessment · Assessment should be fair · Assessment should give all students the opportunity to demonstrate their understanding	· Assessment can be used for motivation (it's all in the attempt) · Assessment can be used during instruction · Assessment should be aligned with instruction · Assessment methods should be varied

Figure 16. Summary of the Participants Beliefs about Mathematics, Teaching, Learning, and Assessment

Both Keith and Robin used assessment to inform their instruction. All three of the teachers wanted assessment to be a fair measure of what students know. All three of the teachers used open-ended items both during class and during instruction. Todd used open-ended items during class more than the other two teachers did, but Keith used them often during class. Robin seldom used open-ended items during class.

The teachers' beliefs affected the way they used open-ended assessment items. For example, Keith used open-ended items as a way to understand students' thinking that might be different from his own way of thinking. He believed that people understood mathematics differently and open-ended items helped him identify those different ways of understanding. Robin wanted her students to understand mathematical connections, and open-ended items helped her assess whether students had made those connections. Todd believed that assessment could be a motivating factor for students, one of the reasons he used open-ended items.

Keith and Robin had an internal source for authority, which led them to use open-ended items to support their broader educational goals. Todd's reliance on the authority of the school system led him to use open-ended items, but he did not use them in ways that informed his teaching.

All three of the teachers thought reflectively on some level about particular aspects of their teaching. Robin and Keith both found teaching and assessment problematic, and thus were able to think reflectively in these domains. Todd did not find teaching or assessing the mathematics he taught problematic, and only reflected at a surface level in these domains. Keith and Robin each reflected consistently at the second of Van Manen's levels of reflection, and each of them showed evidence of Level 3 reflection. Todd's reflection was mostly limited to the first level. Keith and Robin's reflective approach to assessment enabled them to use open-ended assessment in ways that informed their teaching. Todd did not reflect on student responses to open-ended items (as was evidenced by his haphazard approach to grading) and thus he was not able to use student responses to open-ended assessment items to inform his teaching. Todd was using open-ended items because he was required to do so, but, for the most part, he did not use them to inform his teaching.

Content knowledge and pedagogical content knowledge facilitated Keith and Robin's meaningful use of open-ended items and hindered Todd's meaningful use of such items. Keith and Robin were comfortable enough with their own mathematical knowledge to investigate student approaches to open-ended items whereas Todd felt uncomfortable when he could not immediately recognize a response as correct or incorrect.

Time was the only constraint with which these teachers really struggled. Robin's reflective use of open-ended items took up time that she would not easily give up. She was committed to finishing the curriculum, and would sacrifice using open-ended items in order to move more quickly through the curriculum. Although Todd claimed that time was a constraint for him, it did not affect his use of open-ended items. When Todd used open-ended items, they took no more time than traditional items, therefore he felt free to use them often. Keith was not constrained by the curriculum. He was not committed to finishing the curriculum so he used open-ended items often in an effort to promote students' understanding. Keith did express concern about time in that he wished he had more time to find or create open-ended items.

Implications for Teacher Education

Current publications for teachers (Brahier, 2001; Danielson & Marquez, 1998; Dyer & Moynihan, 2000) on alternative assessment strategies focus on providing teachers with a rationale for using open-ended items, giving strategies for creating and grading them, and sources for finding them. They do not go beyond the issues of finding, creating, and grading open-ended assessment items to what may be the more important issue of how to use these strategies to empower teaching and learning. Even the training that I helped design ignored this aspect of the process. Previous research indicates that using open-ended items is difficult for teachers and that there are factors that constrain teachers that are difficult to overcome. If using open-ended items does not move teachers in the direction of reform, then why do educators want to support that use?

This study has shown that it is possible for a teacher to use open-ended items often and yet that usage can have little impact on teaching. The problem goes beyond getting teachers to use open-ended items to getting teachers to use them in meaningful ways. Beliefs, orientation to authority, and reflectivity are factors that can both facilitate and hinder teachers' meaningful use of open-ended assessment items. Time also played a constraining role in how teachers used open-ended assessment, although it interacted with other issues. Open-ended assessment did take longer than traditional assessment for those teachers who used it meaningfully. The extent to which the teacher was committed to finishing the curriculum affected the amount of time he or she was willing to spend on asking open-ended items.

The findings from this study suggest several implications for teacher education. First, preservice and inservice teacher education programs should focus specifically on how to use student responses to open-ended items to inform teaching. For example, a teacher educator could hand out a set of student responses to an open-ended item and ask teachers to create an activity based on the responses to that item. The case of Todd implied that it cannot be assumed that teachers will know how to use open-ended responses to inform their teaching just because teacher educators convince them that including open-ended items is an important part of an assessment program. This study suggests that teacher educators need to be more explicit about how to learn from student responses to these items and how to adjust teaching based on those responses. In order to use open-ended assessment items meaningfully, teachers need to value the kind of thinking that is elicited by those items. Providing contexts for preservice teachers to engage in this kind of thinking and to discuss the nature of different kinds of thinking would be a step towards helping them to develop an appreciation for it. It would be helpful if preservice teachers experienced open-ended assessment not only in their mathematics education classes but in their mathematics classes as well. Experiencing open-ended items as a student may help preservice teachers develop an appreciation for the cognitive demands of this kind of thinking.

Second, teacher education programs should focus on developing teachers who have the characteristics that enable them to implement reform. How can teacher educators educate teachers to be reflective? How can they encourage teachers to integrate other voices with their own and evaluate evidence in context? How can they get teachers to rely less on textbooks and authority and more on their own system of beliefs that they developed reflectively? Obviously, these are not new questions, but this study adds to the robustness of the claim that a relationship exists between an individual's relation to authority, reflectivity, and the extent to which he or she is able to implement reform. Efforts are being made to address these issues. For example, Mewborn and Wilson (1999) posited a relationship between what preservice teachers *see* in their experiences with their ability to reflect on those experiences:,

> It is important to help preservice teachers see teaching and learning as complex activities that involve multiple, interacting variables. So…to "see better" means to see complexity, to see phenomena that need to be discussed and reconsidered, and to see multiple courses of action. In other words,…to see teaching as problem solving. (p. 5)

A recognition of teaching as problematic is a first step toward reflecting on teaching practice. Mewborn and Wilson contended that "what is seen serves as a foundation and helps to focus reflection, and reflection enhances what we see" (p. 6). They offered teaching strategies that helped preservice teachers "see better." They argued that "we can help preservice teachers see teaching and learning as problematic by employing the following strategies: 1) revisit situations; 2) contrast examples; 3) limit role of preservice teacher; 4) provide opportunities for both individual and shared processing of what was seen" (Mewborn & Wilson, 1999, p. 29). These strategies can be used in teacher education as a basis for discussions about assessment in order to encourage reflection.

Mewborn and Wilson (1999) described a situation in which preservice teachers observed a master teacher and then observed a student teacher. These contrasting situations enabled the preservice teachers to see and articulate better the specific actions of the master teacher that facilitated learning. The provision of contrasting examples of meaningful uses of open-ended assessment items and trivial uses of those items may help teachers to see better what constitutes a meaningful use of the items. Mewborn and Wilson's third suggestion (limiting the role of the preservice teacher) could be used to focus preservice teachers on assessment in their field experiences. When preservice teachers observe teachers in the field, they could be encouraged to focus only on the kinds of questions that the teacher asked and the kinds of responses that students gave. Back in their university setting, the preservice teachers could discuss the nature of questions the teacher asked and what they learned about student understanding from the responses to those questions. They could also discuss how the teacher used (or did not use) the students' responses to inform her or his instruction. Mewborn and Wilson advocated a move away from viewing preservice teachers as unreflective and toward directing instruction in ways that will help them become reflective and to recognize teaching as problematic.

Finally, Robin showed that there are teachers who are capable of and willing to teach for understanding but who sometimes make a conscious decision to teach by telling because of a strong commitment to their system's curriculum. The literature has pointed to an institutionalized curriculum as a constraining factor in teachers' use of alternative assessment (Senk et al., 1997; Wilson, 1993). I believe that this concern is trivialized in teacher education. Teacher educators seem to have a hidden desire for teachers to ignore their system's curriculum in favor of teaching for understanding. Other professionals cannot ignore what their company wants them to do and do what they believe is best. Can anyone blame teachers for doing what they are contracted to do? In general, curriculum guides contain sets of behavioral objectives that are focused on procedural knowledge. There are so many of these objectives that teaching for understanding and covering all the objectives is a difficult task. Many teacher educators want to encourage teachers to reflect on their situation, make decisions based on evidence in context, and to aware of their options within their given situation. But one cannot ignore the system as it is. I do not argue that teacher educators must accept the system as it is and work within it, but I question whose job is it to reform curriculum framework on a state level. Whose responsibility is it to reduce the number of curricular objectives to a reasonable level? An individual school system must follow state guidelines. Who reforms those guidelines? The mathematics education research community has some insights into the conditions under which teachers can implement reform. How can these insights be used to persuade state and local systems to create environments that will facilitate, not hinder, reform? Researchers should continue the line of research that points to constraints of the system as a factor that hinders teaching for understanding and should focus on making this research known to policy makers.

Implications for Research

The findings of this study suggest several avenues for future inquiry. First, ways of educating teachers who have beliefs, orientations to authority, and reflectivity that are conducive to implementing reform need to be conceptualized as described in the previous section. I suggest that staff development, if it is to help teachers move in the direction of reform, should focus on more than teaching or assessment techniques per se and begin to challenge the fundamental assumptions that underlie teaching and assessment in the first place. Staff development, like preservice teacher education, should provide contexts for reflection and challenges of beliefs that hinder teachers' movement toward reform. After such programs are implemented, investigations could be made into the extent to which they are effective in changing teachers' beliefs, orientations to authority, and reflectivity in directions that facilitate reform teaching. A next step would be to investigate whether any changes in these structures affect changes in teachers' practices. Participation in staff development helped the teachers in this study cope with some of the external constraints with which other teachers struggled, such as subjectivity and grading. One of the effects of the staff development may have been that teachers actually used open-ended assessment items, but the project did not, in and of itself, precipitate a meaningful use of the items.

In the same way that teachers have students in their classes who are at a variety of levels of learning, staff development courses have teachers at a variety of places in the progression towards reform. Some teachers are ready for the assessment reform message. A gentle nudge and they may profess a realization that they are not asking the right questions. They realize that valuing understanding necessitates asking questions that elicit information about understanding. Other teachers hold tightly to their perception that the purpose for assessment is to see where students stand and that traditional mathematics questions are a viable (and perhaps the best) way of achieving that goal. Staff developers have to work with both of these groups of teachers. Future research could investigate the salient features of successful staff development projects to inform future staff development initiatives.

The case of Todd illustrated the importance of looking beyond the surface when investigating teachers' practices. On a surface level, Todd used open-ended assessment items often and put them on his tests. It could be argued that Todd was implementing assessment reform initiatives in his teaching. A closer look at Todd's teaching, however, revealed that open-ended assessment items had little impact on Todd's teaching. Studies that merely report the surface-level attributes of teachers' practice are not as useful as studies that strive for more depth.

Concluding Remarks

Although the focus of this study was on teachers' use of open-ended assessment items, I view it as a piece of a larger research agenda in mathematics teacher education. Many researchers are asking questions about how teacher educators can help teachers focus their instruction on conceptual rather than procedural understanding. One way of understanding teachers' commitment to focusing on conceptual understanding is to look at the kinds of questions they feel compelled to ask their students. More importantly, researchers can look at why they feel compelled to ask (and not ask) the questions they do and how they use the information they get from their students when they ask those questions. Time and again, the research literature points to teachers' orientations to authority, their beliefs, and the extent of their reflection as being important factors for understanding teachers' practical actions. Across different domains such as problem solving, elementary preservice teacher education, secondary preservice teacher education, and teachers' assessment practices, these same factors seem to be salient in understanding the extent to which teachers are able to implement reform. The reform-minded teacher, it seems, must have an internal locus of authority, view teaching as problematic, and reflect on her or his teaching practice. This study suggests a continued attention to developing ways to educate teachers with these attributes.

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APPENDIX

INTERVIEW PROTOCOLS

Interview 1 (Keith, Robin, and Todd)

1. What do you like best about mathematics[3]?

2. What do you like least about mathematics?

3. If mathematics were an animal, what would it be? Why?

4. Would you say mathematics is discovered or invented? Elaborate.

5. Is there more than one way to think about mathematics? *[If so, follow up with: Are some ways of thinking about mathematics better than others?]*

6. What made you want to teach mathematics rather than another subject?

7. Do you differentiate between pure and applied mathematics? Why or why not? *[And how?—try to get the teacher to say whether or not they think one is better than the other]*.

8. What do high school students need to know about the Pythagorean Theorem? *[The goal with this question and the next one is to see whether they gravitate towards procedural, utilitarian aspects of mathematics or towards connections and understanding]*

9. Is mathematics a creative subject*? [Follow up with: What does it mean to be creative in mathematics?]*

10. What does it mean to know mathematics? *[Follow up with: To what extent is that culture dependent?]*

11. When it comes to learning mathematics, would you argue more for nature or for nurture?

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[3] Questions 1-3, 12-14, and 18-21 in this interview were taken or adapted from RADIATE—a National Science Foundation project directed by Drs. Thomas J. Cooney and Patricia S. Wilson at the University of Georgia.

12. Consider the following similes: Learning mathematics is like:

Working on an assembly line	Watching a movie
Cooking with a recipe	Picking fruit from a tree
Working a jigsaw puzzle	Conducting an experiment
Building a house	Creating a clay sculpture
Working as an apprentice	Working on a corporate project team

13. Choose the simile that you believe best describes learning mathematics and explain your choice.

14. Describe the characteristics of a good mathematics student.

15. Describe the characteristics of a poor mathematics student.

16. How do you feel about cooperative learning in mathematics? Why? *[I want to know here if the teacher believes that learning mathematics is an individual endeavor or if it is learned in interaction.]*

17. Describe the best environment for mathematical learning. *[Follow up with questions like: What would students be doing? What would the teacher be doing? What would the room look like? What materials would be present in the room?]*

18. If money were not a factor, what materials would you want your students to have at home to assist in their mathematical learning? *[ask about computers, calculators, manipulatives, tools]*

19. Describe the characteristics of a good mathematics teacher.

20. Describe the characteristics of a poor mathematics teacher.

21. Consider the following similes. A mathematics teacher is like:

News broadcaster	Entertainer
Doctor	Orchestra conductor
Gardener	Coach
Missionary	Social worker

22. Choose the simile that you believe best describes a mathematics teacher and explain your choice.

23. Who or what has influenced you to pursue mathematics teaching? What about them influenced you?

24. Describe a particularly successful teaching episode that you remember. What made it successful?

25. What makes a mathematics lesson unsuccessful?

26. If you could create the perfect teaching environment, what characteristics would it have?

27. Do you feel that you are able to be the best teacher you can be?

Interview 2 (Keith, Robin, and Todd)

I will ask teachers to write an item that would assess a minimal/deep and thorough understanding of a given topic and bring it to this interview. I will choose the topic based on classroom observations so that the topic is one they are currently teaching.

1. How do you define assessment in mathematics?

2. How are assessment and grading related?

3. Fill in the blanks: Good assessment should ____________________.

   Bad assessment ____________________________.

1. How do you choose test items?

2. How do you choose homework problems?

3. What are the characteristics of a good assessment item?

4. Consider the two items you brought in. What about the first item makes it assess a minimal understanding of the topic? What about the second item makes it assess a deep and thorough understanding of the topic?

5. Suppose you grade a set of tests and the grades were much lower than you expected. How would you react?

6. Would you say that your assessment is aligned with your instruction? In what ways is it aligned/misaligned?

7. Do the NCTM Standards affect your assessment practices? In what way?

8. Think of a mathematical task that you assigned recently that you particularly liked. What did you like about it?

9. Do you expect students to learn during a test? Elaborate.

10. When/where do you mostly use open-ended items?

11. Show teachers Cooney's levels:

Level 1	Simple computation or recognition
Level 2	Comprehension. Student must make some decision but once made the solution process is straightforward, e.g., a simple one-step problem.
Level 3	An application or multistep problem in which student must make several decisions about solution process or what operations to use
Level 4	Nonroutine or open-ended problem

Ask: About what percent of your test items are at each level? (They can look at a recent test if they need to). Follow up with: How do you feel about that distribution?

12. Is there a particular type of open-ended item that you like to use? Why?

13. What is the hardest part about using open-ended items?

14. What do you like best about using open-ended items?

Interview 3

<u>Keith</u>

Today we are going to do a role play. I hope you like drama! Let's pretend I am a new teacher that you hired. We have the same planning period. During pre-planning, you gave me a hard copy of the alternative assessment item banks for algebra I and geometry, and you also showed me how to access them on the computer. I am teaching both of these courses. One day during our planning period, I bring up the item banks. I'll begin the conversation, and then we'll just play the conversation out.

Wendy: You know, Keith, you gave me these item banks at the beginning of the year and I thought there were some really good questions in there. But I gave a few to my students and they just can't do them. They just leave them blank. So I stopped giving them—they just made my grades lower.

At this point, I will respond to things Keith says. At some point, I will make the following points:

1. Those items are also really hard to grade. It's hard to be objective when grading them. It's much more objective when there is one right answer.

2. They take a long time for the students to answer. I have to get through this curriculum. I just don't have the time to spend on this stuff. I have to make sure they can work the normal problems, and that's hard enough.

3. When I give some of those questions, I can't always anticipate what the students will say. What if they come up with an answer that I don't understand? Leaving things that open makes things less predictable. I don't know how to prepare for what the students might say.

4. What about in my classes that don't have an item bank? These items are really hard to write. If I don't have a big store of them, I just don't have the time to create them from scratch.

5. Well, I could see using these items as a warm-up, or maybe as a group activity, but I don't know about using them on tests. I mean, I only like to put things on tests that I have covered in class. It's not really fair to throw them something they've never seen before on a test.

6. You told me at the beginning of the year that 20% of my test items have to assess higher-order thinking skills. What exactly does that mean? Will anyone ever check?

7. Sometimes I feel like my students won't even try to answer a problem written in words. That's why sometimes I will give them credit just for trying a problem. What do you think of that?

After the role play, I will ask Keith about the responses and grades of the open-ended items he will bring to the interview.

Some questions:

1. Were there some misconceptions that the students had of which you were unaware?

2. Did any of the responses surprise you?

3. Do you feel that your students were prepared to answer these items? In what way?

4. What did you learn about student understanding from the responses to these items?

I would also like to ask Keith the following questions:

1. You said you thought it was harder to come up with open-ended items in pre-calculus than for some of the lower-level classes. Why do you think that is?
2. Is it okay for students to feel uncomfortable during your mathematics class?
3. Is it okay for *you* to feel uncomfortable during your mathematics class? Does that ever happen? When? Can you think of an example? How do you cope with feeling uncomfortable?
4. What is your job as a mathematics teacher? What do you get paid to do? What earns your paycheck?
5. To whom are you responsible when it comes to your mathematics teaching?

<u>Robin</u>

1. Let's talk about the warm-up you assigned and took up the last time I was here. I asked you to write a paragraph after grading that item explaining what you learned from the student responses to the item and whether or not anything surprised you. Tell me about what you wrote. [*Lots of follow-ups here*!]
2. Did that item count for a grade? In what way?
3. Do you give items like this one often in class, or was this new for you? [Follow up with what holds you back from using more items like these or what makes you choose these kinds of items to use during class—depending on the answer.]
4. Do you use items like this one more often in any certain classes than in other classes? Are there any classes you would feel uncomfortable using such items?
5. What made you decide to assign that item?
6. What, if anything, is different in the mathematics of the item you assigned for the warm-up and the items you had previously assigned concerning simplifying rational expressions?
7. Do you believe that your students were prepared to answer that item? [Follow up with should they have been based on what you taught? or, in what way were they prepared?]
8. What kinds of things were going through your mind as you were grading/marking the students' papers?
9. As far as assessment, what did you get out of assigning this item? Do you think the students got anything out of answering the item?

10. When you return[ed] the papers, what happens/happened next? Will [Did] a discussion follow? Tell me about that.

11. Did it ever happen that two students both got the highest score on the item but you really liked one of the student's answers better than the other? Can you show me an example? What made you like the one more than the other?

12. How do you feel about the students that gave you a multiplication problem involving two rational expressions for which the product simplified to the correct result (as opposed to one rational expression)?

13. What does the main misconception seem to be regarding simplifying rational expressions in this class? Will you address that misconception any differently now or in the future?

14. Show me the two items you brought in (minimal/deep and thorough). What about the first item makes it assess a minimal understanding of the topic? What about the second item makes it assess a deep and thorough understanding of the topic?

15. When would you most likely use the first item? The second item?

16. Would you learn anything different from the responses to the second item than the first? Explain.

17. What instruction would you use to prepare students for answering the first item? How about the second item?

<u>Todd</u>

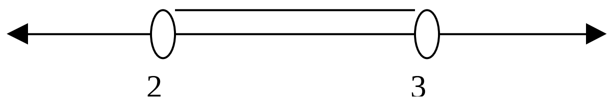
1. Is it okay for students to feel uncomfortable during your mathematics class? When/under what circumstances? Elaborate.

2. Is it okay for you to feel uncomfortable during your mathematics class? When/under what circumstances? Do you avoid feeling uncomfortable? How?

3. Let's talk about Terry's answer to the beginning activity:

   Create an inequality with no solution or as a solution. Terry wrote: $0 \geq s < 0$. You said, "if you take off the equal to part, it might be no solution—I'll have to think about that some more. I see what you're trying to do—a number can't be greater than zero and less than zero at the same time. But the way you wrote it is kind of weird." But $0 \geq s < 0$ just says $s < 0$, which does have solutions. You didn't seem to really look at what his solution said, but you were quick to look at what he might be trying to say. Tell me what was going on in your mind then.

4. Consider the following problem: $x > 2$ and $x > 3$. The solution set for the inequality is $x > 3$. Do you think your students understand this? How likely do you think it would be for them to graph the solution set the following way:



5. How much do you think your students think about what the problems say as opposed to implementing a procedure (specifically the Algebra Concepts students when they are trying to solve and graph compound inequalities)?

6. After you assigned the four alternative assessment items in Algebra Concepts, you asked your students how they liked these problems. They said they liked them and wanted more like them. What do you think about that?

7. Were the alternative assessment items graded? How? Did you look at each student’s response to each item? Did you look at each group’s response to each item? Were the application problems graded? How?

8. Did anyone in Algebra II get the extra credit problem in which you asked them to create a quadratic function that opens to the left or right?

9. In Algebra II, do you plan to discuss the fundamental theorem of Algebra and its consequences? (i.e., an nth degree polynomial has exactly n roots).

10. Do you plan to discuss the fact that a quadratic function will always have exactly two solutions (counting multiplicities)?

11. You went over the following Algebra II homework problem during class:

    Solve by graphing: $d^2 + 6d + 8 = 0$. When you worked it, you factored the quadratic as $(d + 2)(d+4)$, set each factor equal to zero and found the solutions of $d = -2$ and -4. Then you created the following table of values and plotted the graph:

d	*y*
-2	0
-4	0
0	8
-6	8
-3	-1

You said, “(-3,-1) is most likely the lowest point.” How can you be sure? How can you determine the vertex? How can you convince students that (-3,-1) is the vertex? Do you care at this point whether they can locate the correct vertex?

Interview 4

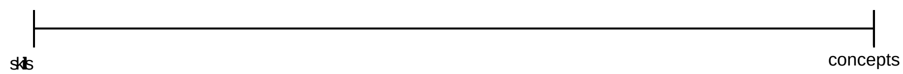
Keith

1. Tell me how you resolved the synthetic division problem.

2. What is your ultimate responsibility as a mathematics teacher?

3. Who’s responsibility is it to decide what is taught in your classroom?

4. Consider the following continuum. As far as mathematical importance, put an X on the continuum where you believe the importance or emphasis should be in mathematics teaching.



5. What is the best way to assess an understanding of skills?

6. What is the best way to assess an understanding of concepts?

7. Do your assessment instruments reflect where you fell on the continuum?

8. Have you ever or would you ever consider writing curriculum? Why or why not?

9. Two teachers were having a discussion in the teachers’ lounge. With whom do you most agree and why?

   Teacher A: We have the curriculum for a reason. It shouldn’t matter which teacher a student has. Algebra I is Algebra I. Everyone should cover the same material. If a teacher doesn’t finish the curriculum, she has really done her students a disservice. They won’t be prepared for the next class. Even if you can’t do everything in detail, you should at least expose the students to all the material in the curriculum. Our job is to teach the curriculum.

   Teacher B: Nobody can predict how long it will take a certain group of students to understand a given topic. It’s okay to use the curriculum as a guide, but we have to honor the pace of our students. Developing mathematical understanding takes time. Getting students to make and test conjectures and draw conclusions takes time. I’m not willing to sacrifice understanding in order to finish the curriculum. Our job is to facilitate students’ mathematical understanding.

10. Describe your mathematical experiences for me. When did you first realize you were talented in mathematics? Was mathematics ever a struggle for you, or did it always come easy?

11. Are you happy with the number and frequency of open-ended questions that you ask your students?

12. The purpose of high school mathematics is to enable students to be able to:

    function (or excel) in college mathematics courses
    function in society
    reason mathematically
    see mathematics as a connected whole
    see the beauty in mathematics
    do well on standardized tests
    solve mathematical problems

    Rank the above purposes in order from most important to least important. Explain why you ranked them the way you did.

13. How do you decide how you will teach a particular topic?

<u>Robin</u>

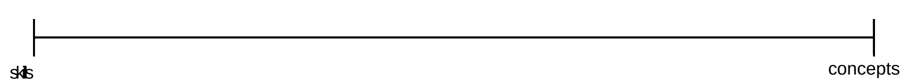
1. I'm going to ask you three questions to help you understand what kind of response I'm looking for. So, I'll ask the three of them, and then you respond. What is your job as a teacher? What do you get paid to do? What earns your paycheck? [Follow up with how do you facilitate that and how do you assess your success at your job].

2. To whom are you responsible in your mathematics teaching? Who is your boss?

3. What is your ultimate responsibility as a mathematics teacher?

4. Who's responsibility is it to decide what is taught in your classroom?

5. Consider the following continuum. As far as mathematical importance, put an X on the continuum where you believe the importance or emphasis should be in mathematics teaching.



6. What is the best way to assess an understanding of skills?

7. What is the best way to assess an understanding of concepts?

8. Do your assessment instruments reflect where you fell on the continuum?

9. Have you ever be involved in writing curriculum? Tell me about the process.

10. Two teachers were having a discussion in the teachers' lounge. With whom do you most agree and why?

    Teacher A: We have the curriculum for a reason. It shouldn't matter which teacher a student has. Algebra I is Algebra I. Everyone should cover the same material. If a teacher doesn't finish the curriculum, she has really done her students a disservice. They won't be prepared for the next class. Even if you can't do everything in detail, you should at least expose the students to all the material in the curriculum. Our job is to teach the curriculum.

    Teacher B: Nobody can predict how long it will take a certain group of students to understand a given topic. It's okay to use the curriculum as a guide, but we have to honor the pace of our students. Developing mathematical understanding takes time. Getting students to make and test conjectures and draw conclusions takes time. I'm not willing to sacrifice understanding in order to finish the curriculum. Our job is to facilitate students' mathematical understanding.

11. When you graded the poorly done tests in Algebra I, how did you feel?

12. What do you think contributed to the students' poor performance on the test?

13. How do you think the students felt when they got their papers back? How do you feel about that?

14. What made you decide to retest?

15. Won't this put you behind in the curriculum? How do you feel about that?

16. How did you choose the items for "reviewing basic skills" in Algebra I (the ones you put on the overhead after you passed back the poor tests)?

17. Let's look back to the mid-chapter Algebra I test for chapter 8. You said two other people wrote the test. Would you have included as many open-ended items if you had written it yourself? Why or why not?

18. Look at Student A's response to No. 20. Tell me why you gave full credit to the response. Would you revise the question in any way? Does it bother you that they wrote a multiplication problem rather than a single rational expression?

19. Now look at Student C's response to No. 20. Again, they gave a multiplication problem. Compare the answers of Students A and C. Do you like one better than the other? How do you feel about awarding both answers the same number of points?

20. Did it surprise you that students thought that (x^2 + 8) = (x + 4) (x – 4)?

21. Compare Student B's response to numbers 18, 19, and 20 with his response to numbers 5-14. What do you think is going on here?

22. Is it possible that you have a student in your class for which you do not physically see their solution to any problems other than quiz or test pro

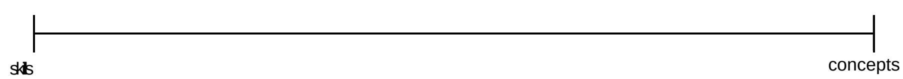
<u>Todd</u>

1. To whom are you responsible in your mathematics teaching? Who is your boss?

2. What is your ultimate responsibility as a mathematics teacher?

3. Who's responsibility is it to decide what is taught in your classroom?

4. Consider the following continuum. As far as mathematical importance, put an X on the continuum where you believe the importance or emphasis should be in mathematics teaching.



5. What is the best way to assess an understanding of skills?

6. What is the best way to assess an understanding of concepts?

7. Do your assessment instruments reflect where you fell on the continuum?

8. Would you ever be interested in writing curriculum? Why or why not?

9. Two teachers were having a discussion in the teachers' lounge. With whom do you most agree and why?

   Teacher A: We have the curriculum for a reason. It shouldn't matter which teacher a student has. Algebra I is Algebra I. Everyone should cover the same material. If a teacher doesn't finish the curriculum, she has really done her students a disservice. They won't be prepared for the next class. Even if you can't do everything in detail, you should at least expose the students to all the material in the curriculum. Our job is to teach the curriculum.

   Teacher B: Nobody can predict how long it will take a certain group of students to understand a given topic. It's okay to use the curriculum as a guide, but we have to honor the pace of our students. Developing mathematical understanding takes time. Getting students to make and test conjectures and draw conclusions takes time. I'm not willing to sacrifice understanding in order to finish the curriculum. Our job is to facilitate students' mathematical understanding.

10. Recall the item on your Algebra Concepts test in which you asked students to create an inequality that had no solution. What do you think contributed to the students' poor performance on that item?

11. Describe your mathematical experiences for me. When did you first realize you were talented in mathematics? Was mathematics ever a struggle for you, or did it always come easy?

12. In Algebra II, in one class, you went over homework problem No. 29 (the derivation of the quadratic formula) and in the other class you didn't. Could you tell a difference the next day during instruction between the two classes? Describe.

13. You use a lot of open-ended items. You use them during class, on quizzes, and on tests. What makes you so willing to use them?

Interview 5

Keith

1. Do you write down your lesson plans? Do you have one I can look at? If not, how do you plan your lessons? How far in advance do you plan?

2. Do topics that will come later in the curriculum affect how you teach current topics? In what way?

3. Before you give open-ended items in class, do you think about possible solutions, ideal solutions, or possible mistakes students might make, or do you deal with those more as they arise? Why?

4. Do you use the Fulton County rubric when you grade open-ended items on tests and quizzes? If not, how do you grade them?

5. What are your strengths as a mathematics teacher?

6. What are your weaknesses as a mathematics teacher?

7. What would you do if you <u>really</u> didn't like the county curriculum?

8. Out of the following list, which characteristic is the most important for a mathematics teacher to have? Least important?

Fair
Knowledgeable about content
Understanding
Kind
Objective
Nurturing
Protective

9. Put an X on the continuum where you think the majority of high school mathematics should focus. Would it change for any particular courses? Why or why not?

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practical theoretical

10. Is there a right way to teach mathematics? Is there a wrong way to teach mathematics?

11. How important is it for mathematics teachers to be precise in their language? Can you give me an example?

12. Teachers often talk about wanting their students to know "why." There are different ways to look at the "why" issue. A teacher can want her or his students to know why they need to know a particular topic, for example, scientific notation. If one needs to report a very small measurement, such as the size of an atom, scientific notation is useful. Alternatively, a teacher may want her or his students to understand why things work. For example, rather than just understanding that in order to put 52300 in scientific notation, you put the decimal after the first number and multiply by 10 to the number of spaces you moved, the teacher may want students to know why $5.23 \times 10^4 = 52300$? Which of these why questions is more important to you?

13. What is the most important thing you could tell me about your beliefs about mathematics teaching and mathematics assessment?

14. What is your biggest dilemma as a mathematics teacher?

15. Tell me about your experience as an engineer. How did you relate with your boss? Did you work in a group or on your own most of the time? Were you able to "do your own thing" or were you closely monitored? Did you have strict deadlines? Did you finish things on time?

Robin

1. Do you write down your lesson plans? Do you have one I can look at? If not, how do you plan your lessons? How far in advance do you plan?

2. Do topics that will come later in the curriculum affect how you teach current topics? In what way?

3. Before you give open-ended items in class, do you think about possible solutions, ideal solutions, or possible mistakes students might make, or do you deal with those more as they arise? Why?

4. Do you use the Fulton County rubric when you grade open-ended items on tests and quizzes? If not, how do you grade them?

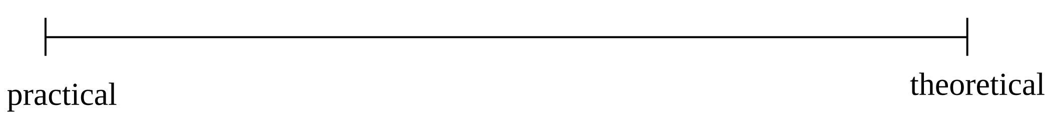
5. What are your strengths as a mathematics teacher?

6. What are your weaknesses as a mathematics teacher?

7. Out of the following list, which characteristic is the most important for a mathematics teacher to have? Least important?

   Fair
   Knowledgeable about content
   Understanding
   Kind
   Objective
   Nurturing
   Protective

8. Put an X on the continuum where you think the majority of high school mathematics should focus. Would it change for any particular courses? Why or why not?



9. Is there a right way to teach mathematics? Is there a wrong way to teach mathematics?

10. How important is it for mathematics teachers to be precise in their language? Can you give me an example?

11. Teachers often talk about wanting their students to know why. There are different ways to look at the "why" issue. A teacher can want her or his students to know why they need to know a particular topic, for example, scientific notation. If one needs to report a very small measurement, such as the size of an atom, scientific notation is useful. Alternatively, a teacher may want her or his students to understand why things work. For example, rather than just understanding that in order to put 52300 in scientific notation, you put the decimal after the first number and multiply by 10 to the number of spaces you moved, the teacher may want students to know why $5.23 \times 10^4 = 52300$? Which of these why questions is more important to you?

12. What is the most important thing you could tell me about your beliefs about mathematics teaching and mathematics assessment?

13. What is your biggest dilemma as a mathematics teacher? Do you believe it's important to use open-ended items at every level of mathematics?

14. If students never answer any open-ended items in your class, could they still be successful? What does it mean to be successful in your class?

15. If you were giving advice to a new teacher regarding open-ended items, what would you say?

<u>Todd</u>

1. Do you write down your lesson plans? Do you have one I can look at? IF not, how do you plan your lessons? How far in advance do you plan?

2. Do topics that will come later in the curriculum affect how you teach current topics? In what way?

3. Before you give open-ended items in class, do you think about possible solutions, ideal solutions, or possible mistakes students might make, or do you deal with those more as they arise? Why?

4. When do you create the answer key to tests or quizzes? (Before or after you give the test or quiz)

5. Do you use the County rubric when you grade open-ended items on tests and quizzes? If not, how do you grade them?

6. What are your strengths as a mathematics teacher? (Possibly ask about weaknesses)

7. What would you do if you <u>really</u> didn't like the county curriculum?

8. Out of the following list, which characteristic is the most important for a mathematics teacher to have? Least important?

   Fair
   Knowledgeable about content
   Understanding
   Kind
   Objective
   Nurturing
   Protective

9. Put an X on the continuum where you think the majority of high school mathematics should focus. Would it change for any particular courses? Why or why not?

   |———————————————————————————|

   practical   theoretical

10. Is there a right way to teach mathematics? Is there a wrong way to teach mathematics?

11. How important is it for mathematics teachers to be precise in their language? Can you give me an example?

12. After their group work in Algebra Concepts last Thursday, you put two problems on the board. One was the four by four checkerboard problem and the other one was "Is $(a + b)^m = a^m + b^m$?" Where did you get that question? Why did you ask it?

13. Why did you decide to skip section 8.5 in Algebra II? Is this section included in the curriculum?

14. Tell me about your career prior to your mathematics teaching. What were your responsibilities? Did you work in teams or alone? How did you relate to your boss?

Interview 6

<u>Keith</u>

1. Do you believe it's important to use open-ended items at every level of mathematics?

2. If students never answer any open-ended items in your class, could they still be successful? What does it mean to be successful in your class?
3. Do you feel that your teacher education program prepared you well to teach secondary mathematics? If not, what was lacking?

4. Is American education “broken”? What would it take to “fix” it?

5. What do you know about the governor’s education plan? What do you think about it?

6. Would you ever be interested in an administrative position? Why or why not?

7. You’ve talked often about the importance of applications in mathematics. How much attention in class work, homework, quizzes, and tests do applications get in your class? Explain.

8. You told me in interview 1 that you like to focus on “how it works and where it works.” How does that play out in your classes? In your assessment?

9. If you could design and attend a workshop, what would happen there?

10. Tell me about participating in this research.

11. Is it possible that, for a particular student in your class, for a particular unit, the only worked out problems you review for that student were quiz or test problems? How do you feel about that?

12. Let’s discuss your grading.

Robin

1. What is your biggest dilemma as a mathematics teacher?

2. Do you believe it’s important to use open-ended items at every level of mathematics?

3. If students never answer any open-ended items in your class, could they still be successful? What does it mean to be successful in your class?

4. If you were giving advice to a new teacher regarding open-ended items, what would you say?

5. Do you believe it’s important to use open-ended items at every level of mathematics?

6. Is American education “broken”? What would it take to “fix” it?

7. What do you know about the governor’s education plan? What do you think about it?

8. Would you ever be interested in an administrative position? Why or why not?

9. If you could design and attend a workshop, what would happen there?

10. Do you write down your lesson plans? If so, describe one for me.

11. When you interview a teacher for a potential teaching position, what are you looking for? What could a potential teacher say that would prevent you from hiring her or him?

12. Tell me about participating in this research.

<u>Todd</u>

1. What is the most important thing you could tell me about your beliefs about mathematics teaching and assessment?

2. Do you believe it's important to use open-ended items at every level of mathematics?

3. If students never answer any open-ended items in your class, could they still be successful? What does it mean to be successful in your class?

4. If you were giving advice to a new teacher regarding open-ended items, what would you say?

5. Do you feel that your teacher education program prepared you well to teach secondary mathematics? If not, what was lacking?

6. What would it take to "fix" American education?

7. What politics are acting on education?

8. What do you know about the governor's education plan? What do you think about it?

9. What educational issues would you like to see brought to the forefront in the presidential election? Why? What is your position?

10. You talked about eventually moving into an administrative position like principal, mathematics or science coordinator, or superintendent. What would be your main agenda in such a position.

11. You have talked before about having "Doug" down the hall as your mentor and how you really respect him as a teacher. What about him makes him a good teacher, in your mind?

12. Yesterday you asked a series of questions in class: Create a 4^{th} degree polynomial, create a 5^{th} degree trinomial, and create a 10^{th} degree binomial. Tell

me your rationale for asking those questions. How do you think that went? Will you ask those kinds of questions again?

13. Perhaps discuss finding the axis of symmetry and vertex in a parabola in $y = ax^2 + bx + c$ form.

Interview 7 (Keith, Robin, and Todd)

1. Is there anything that has happened since the last time I was here that you wish I had seen or that you wish you could have told me?

2. To what extent do you reflect on your teaching? Do you feel that you reflect the same amount, or in the same way, before, after, and while participating in this research?

3. To what extent do you reflect on your assessment practices? Compare before, after and during the research.

4. John Dewey said, "reflective thinking, in short, means judgment suspended during further inquiry; and suspense is likely to be somewhat painful" (p. 13 of *How We Think*). What do you think he meant by that? Have you ever experienced this? Describe.

5. You have talked a lot about real-world applications. In one of our interviews, we discussed a continuum between skills and concepts. Are application problems, by nature, more towards the skill side of the continuum or more towards the concepts side? Why?

6. Which of the following best describes your view of mathematics:
   a) a set of rules and truths
   b) an unquestioned body of useful knowledge
   c) a body of structured, pure knowledge
   d) a personal experience
   e) a changing body of knowledge that is constructed socially

7. Consider the topic of logarithms.
   a) How would you introduce the topic?
   b) What aspects of the topic would you focus on?
   c) Would you bring in real-world applications? Can you think of an example?
   d) What problems would you expect students to have with the topic? What
   e) makes you think that? How would you address those problems?

8. Is there anything else you would like to tell me?