DISTRIBUTED CONTROL IN THE ENVIRON NETWORKS OF A SEVEN COMPARTMENT MODEL OF NITROGEN FLOW IN THE NEUSE RIVER ESTUARY, NORTH CAROLINA, USA

by

JOHN SCHRAMSKI

(Under the Direction of Bernard C. Patten)

ABSTRACT

Minimizing subtle variations in network environ analysis (NEA), the two mathematical building blocks of NEA, the law of conservation and the definition of throughflow, are separately developed and then carefully combined to construct a consistent NEA derivation. The conservation equations are derived using the Reynolds transport theorem with an Eulerian control volume to differentiate the concepts of flow and storage. Terms and concepts related to NEA, such as self-flow, turnover rate, storage, accumulation, and the necessary inclusion of a discrete time step in environ storage analysis, are clarified. A comparative NEA methodology is then derived to holistically explore controlling relationships in ecosystems where the term *distributed control* is adopted to describe a diffuse and decentralized concept of control residing in the complexity of network organization. Starting with "open-loop" control theory, three ecological control metrics (*control ratio,* CR; *control difference,* CD; and *system control,* sc_j) are defined in an environ-theoretic framework by considering pair-wise and system-wide distributed control relationships. These control relationships are then explored using a sixteen-season seven compartment steady-state model of nitrogen flow in the Neuse River estuary, North Carolina, USA (Christian, R.R, Thomas, C.R., 2003. Network analysis of nitrogen inputs and cycling in the Neuse River Estuary, N.C., USA. Estuaries 26 (3):815-828). Model compartments of particulate nitrogen in sediment (Sediment) and Nitrates-Nitrites (NO_x) are shown to participate in opposing roles. If a greater nitrogen-exchange magnitude denotes proportional dominance and therefore control, the control metrics reveal Sediment is overwhelmingly controlled by all components, whereas NO_x controls all components. However, if a limiting factor perspective is used (e.g., Sediment sequestering N with a controlled release to NO_x), the conclusions are opposite; Sediment controls all other components and all other components control NO_x. The meaning of "control" in connection with resource stocks and flows in ecosystems still needs resolution. Low ratios of component throughflow and their respective boundary flows are shown to be possible indicators of a component's control dominance.

INDEX WORDS: Network Environ Analysis, Network analysis, Distributed control, Ecological modeling, Open-loop control, Throughflow, Conservation equations.

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DEDICATION

To Richard Leroy Schramski

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GLOSSARY OF MATHEMATICAL SYMBOLS

Symbol	Definition	First Use
$\mathbf{A}$	Adjacency matrix	53
A	Surface area	33
$\mathbf{A}, \mathbf{B}, \mathbf{C}$	State-space equations' coefficient mapping matrices. Definitions used one time only in this section.	8
$\mathbf{C}$	Composite turnover rate matrix created by definition	57
CD	Control difference matrix	125
cd_{ij}	Pair-wise control difference of j oriented toward I	125
CR	Control ratio matrix	125
cr_{ij}	Pair-wise control ratio of j oriented toward i	125
E	Energy	36
$\mathbf{F}$	Flow matrix	53
f_{ij}	Transactional flow from component j to component i	43
$\mathbf{G}$	Direct throughflow intensity matrix	49
g	Gravitational constant ($9.8\ m/s^2$)	38
g_{ij}	Direct throughflow intensity	49
H_i	Theoretical representation of a hierarchically embedded system	113
$\mathbf{I}$	Identity matrix	49
m	Mass	34
$\mathbf{N}$	Integral flow matrix (nondimensional), $\mathbf{N} = (\mathbf{I} - \mathbf{G})^{-1}$	49
n	Integral flow	50
n	Vector size or matrix rank,	8
n	Vector size or matrix rank,	114
$\mathbf{P}$	Composite fraction of storage matrix created by definition, $\mathbf{P} = \mathbf{I} + \mathbf{C}\Delta t$	58
p	Pressure	38
Q	Heat	36
$\mathbf{Q}$	Integral storage matrix (nondimensional), $\mathbf{Q} = (\mathbf{I} - \mathbf{P})^{-1}$	60
S	Extensive (mass dependent) flow quantity (e.g., mass, momentum, energy, etc.)	32
$\mathbf{S}$	Integral turnover time matrix, $\mathbf{S} = \mathbf{Q}\Delta t$	60
s	Intensive flow quantity per unit mass (mass independent)	32
sc_j	System control vector	126
$\mathbf{T}$	Throughflow vector	45

t	Time	8
TC	Total system control value	157
TST	Total system throughflow	46
TSx	Total system storage	46
$\mathbf{u}$	Input vector for common automatic control theory	8
u	Internal energy	38
V_n	Velocity component normal to the surface	33
V_t	Velocity component tangential to the surface	33
W	Work	36
$\mathbf{x}$	State-space vector	8
$\mathbf{x}$	Storage vector	43
$\mathbf{y}$	Output vector	8, 43
$\mathbf{z}$	Input vector	43
z	Vertical height	38
Δt	Change in time	58
η_{ij}	Fractional transfer coefficient	123
ρ	Density or mass per unit volume	32
τ_{ij}	Component stock turnover time	55
$\forall$	Control volume	32
$\dot{m}$	Mass flow	35

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CHAPTER 1

INTRODUCTION

1.1 PREFACE

> In an ecosystem the organisms and the inorganic factors alike are components which are in relatively stable dynamic equilibrium. Succession and development are instances of the universal processes tending towards the creation of such equilibrated systems.
>
> — Arthur G. Tansley, July , 1935

While species independently interact with their environment in subtle and virtually imperceptible ways, the world's behavior remains reasonably predictable. As an example, consider that an ecologist and an engineer shared their thoughts on ecosystem functionality while taking two walks. Their first stroll was through the maritime forest of Cumberland Island, a barrier island off the Coastal Plain region of southern Georgia, USA. The ecologist identified some of the common, possibly salt-spray climax, flora species including cabbage palm (*Sabal palmetto*), saw palmetto (*Serenoa repens*), salt meadow cord-grass (*Spartina patens*), juniper (*Juniperus virginiana*), Spanish moss (*Tillandsia usneoides*), resurrection fern (*Polypodium polypodioides*), and live oak (*Quercus virginiana*), and summarized from previous studies that the plants were uniquely adapted to both the wind and the salt spray. The engineer observed the salt and wind-sculpted system articulated by the ecologist but then considered the physics of the cantilevered live oak branch. Its cantilevering posture was certainly assured and predictable. A quick calculation (reference Appendix G) showed the weight of a commonly found 0.25 meter diameter, 12 meters long, branch generated a twisting torque of ~56,000 Joules at its base, an enormously large number given the stature and calm stability evident before their eyes. He interrupted the pause and highlighted the magnitude by proceeding to show this was equivalent to the torque output of ~150 large 4.6L truck engines. This magnitude of force can produce a lot of damage. They paused, looked around, and realized yet longer and larger diameter branches were everywhere. The ecologist climbed on the branch and, at equilibrium, despite the internal stresses, it supported him nicely.

On their second hike, the engineer and ecologist stood together on the Porter Creek trail in the Smoky Mountains National Park, USA, and considered the downhill flow of the nearby river to be a fairly constant and predictable event. The ecologist described evapotranspiration while also considering the watershed catch basin. The engineer considered the energy of the water descending down the mountain. While they both stood on the bank and watched, the air they inhaled contained but a few drops of the water that was simultaneously being transported back upstream. Only together, with their mutual observations, did they both reach the epiphany. The imposing majesty of the river flows in two directions! Downhill we clearly witness as it splits its watershed, and the other, uphill, decentralized in its mechanics, such that we are not privileged to watch. Indeed, considering the processes involved, the uphill rise is more awe inspiring. The energy input to push the heavy water back uphill uniquely equals the energy output extracted by the system during the water's descent. On another scale, consider the energy expenditure to push the Amazon river back upstream. Yet, this is happening in a way that we cannot watch.

Through their collaborative observations, together they concluded that the subtle and diffuse events in an ecosystem combine into choreographed and predictable, albeit sometimes unseen, behavior. To the ecologist, the system is comprised of multiple species acting in concert. To the engineer, the system produces raw demonstrations of energy, with water moving up and down hills, and wood subjected to, but withstanding, massive internal stresses. Their separate perspectives combined to better articulate basic ecosystem functionality. Given this functionality, what manages, controls, or balances the decentralized fabric of the innumerable and almost imperceptibly small relationships that lead to these macroscopic and predictable actions? Through such personal experiences as just described lies the impetus for this dissertation.

The specific focus of this research is to further develop the foundations of a formal mathematical approach to ecosystem control while specifically accounting for both the networks under which ecosystems operate and the indirect effects generated within them which dominate (Patten, 1983). Advances in this area will augment our understanding of ecosystem function, and then importantly for us, ecosystem sustainability. The process of achieving this goal traverses many developing concepts in systems ecology such as environ analysis, distributed control, biogeochemical cycling, indirect effects, and basic first principles equation development. This Introduction discusses each of these, and then concludes with a conceptual preface to the chapters that follow.

1.2 CONTROL THEORY

> In our search for the direct road to truth, we should busy ourselves with no object about which we cannot attain a certitude equal to that of the demonstration of arithmetic and geometry.
>
> — René Descartes, 1644

The concept of control, by definition, is predicated on a dynamic system where the purpose of control is to alter the dynamic behavior. The word *control*, noun or verb, has hundreds of synonyms or definitions including regulate, restrain, oppress, manage, rule, check, monitor, manipulate, influence, or limit, to name only a few. Unfortunately, the diversity of our natural ecosystem functionality affords the possibility of using all these definitions to describe the organizational multitudes of biotic and abiotic interactions of materials and energy. In fact, it is the diversity of our ecosystem's interactions which requires, as of yet, the likewise diversity in our vernacular to effectively model or describe the controlling or dynamic system. However, without a commonly accepted framework or model, the various perspectives of ecosystem control are lost or diluted in synonymous but slightly nuanced definitions. We lose our ability to understand ecosystem functionality on a greater scale because the piecewise descriptions of our model interactions are as wide and varied as the vocabulary we use.

As such, in the spirit of Descartes' first-principle pursuits, we are primarily driven to begin laying a common mathematical foundation for the distributed constraints under which our ecosystems function. The generally accepted open-loop closed-loop automatic control logic derivations and their supporting mathematics for today's engineered systems do not approach the ecosystem as a holistic unit. Patten (1978b), Patten and Auble (1981), and Dame and Patten (1981) introduce a distributed and holistic control measure based on input and output flow environs which specifically accounts for the dominance of indirect effects (Patten, 1983). Fath (1998, 2004) extended Patten's distributed control logic into an alternative and contrasting representation of top-down (Hairston et al., 1960; Paine, 1966, 1974) and bottom-up (White, 1978; Power, 1992) control actions in ecosystems. Further considering Patten's original distributed control ratio concept, the work herein methodically reviews the original distributed control development, redefines its objectives from an automatic control theory perspective, and expands its applicability.

1.2-1 CONTROL THEORY BACKGROUND

The history of control theory primarily originates from the development of the generally recognized lexicon of automatic control theory in engineered systems. James Watt's eighteenth century centrifugal governor linking output to input (currently known as a servomechanism) for the speed control of a steam engine is often listed as the first significant work of systems control. In the 1920's, Minorsky, while working with ship controllers, demonstrated the first aspects of stability from the differential equations describing his systems. In the 1930's, Nyquist simplified the confirmation of closed-loop systems' stability by considering the open-loop response to steady-state sinusoidal inputs. Around the same time, Hazen worked with the design of relays (*servomechanisms*, a term he derived) to help follow changing input signals. Frequency response, followed by root-locus methods (developed through the 1940's and early 1950's) are the core of linear time-invariant classical control theory, and their utilization has led to systems which both satisfy a wide range of performance indices and are stable. The term *frequency response* refers to the steady-state response of a system to a sinusoidal input. Typically the frequency of the input signal is varied over a certain range while simultaneously observing the resulting system's response characteristics. Judicious adjustments are then made to stabilize the system. W. R. Evan's *root-locus method* plots the closed-loop poles (roots) of the system's characteristic equation for varying gains of the open-loop transfer functions. The basic transient characteristic of a closed-loop response can be ascertained from the closed-loop poles where the open-loop gains are then adjusted, as appropriate, to stabilize the system near a requisite performance point. The frequency response and root-locus methods can lead to stable, albeit simple (minimal-input and -output) systems operating at predetermined, but not necessarily optimum, performance levels. As such, since the 1950's, the emphasis in control design has been towards optimization in some meaningful way and towards handling the increased complexity of multiple-input and multiple-output systems. Modern control theory now includes the optimal control of deterministic and stochastic systems as well as the adaptive and learning control of complex systems where applications of modern control theory exist and continue to develop in non-engineering fields including biology, economics, and sociology.

A fundamental concept within automatic control theory directly applicable to ecological network analysis is the concept of stability in the sense of Liapunov. For a given control system, its stability is usually the most important or pursued characteristic. Stability in the sense of

Liapunov provides a uniquely applicable perspective for ecosystems. Considering a typical mechanics problem by comparison, the objective of the designer is to control the motion, flows, vibrations, etc. when these are objectionable and to enhance these activities when useful. The Nyquist or Routh's stability criteria and others are available to illuminate the behavior of linear and time-invariant systems. However, for nonlinear or linear but time-varying systems, the second method of Liapunov is the most general method for determining the stability of systems of any order. Classical mechanics theory shows that a vibratory system is stable if its total energy (a positive definite function) is continually decreasing (time derivative of the total energy function must be negative definite) until an equilibrium state is reached. Liapunov's second method, otherwise called the *direct method* for stability assessment, utilizes the physics of this observation. The total energy of an unforced, dissipative mechanical system decreases as the state of the system evolves in time. The correlation of a dissipative mechanical system to the solar driven ecosystem is sufficiently obvious. Absent an energy influx, sunlight for example, an ecosystem's stored energy would dissipate to equilibrium. Therefore, according to Liapunov's direct method, if the system has an asymptotically stable equilibrium state, then the displaced (within the domain of the attractor) but stored energy of the system decays with increasing time until it eventually assumes its minimum value at the equilibrium state. The state vector approaches a constant value corresponding to zero energy as time increases. It is often difficult or impossible to define the system's state vector or, in this case, the energy function for nonlinear or time-variant systems. Liapunov circumvented this difficulty by introducing the scalar Liapunov function, a fictitious energy function, as a substitute. Although derived from and for energy considerations, his idea is widely applicable. Absent the formal definition, if a suitable Liapunov function can be found, a system when disturbed can be shown to be, in the sense of

Liapunov, either stable (returning to a nearby equilibrium state), asymptotically stable (returning to the same equilibrium state from which displaced), or unstable (leaving all nearby equilibrium states). Given the nonlinear or linear time-variant characteristics of ecosystems, stability in the sense of Liapunov is a useful characterization for ecosystem behavior.

Another foundational concept in ecosystem control analysis involves Kalman's (1963) *controllability* and *observability* requirements for control design. An example serves to illustrate the concept. Consider the system S defined by the linear differential equations:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)\,, \tag{1-1}$$

$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t)\,; \tag{1-2}$$

A, **B**, and **C** are respectively, n × n, n × r, and p × n matrices. The n × 1 vector **x** is the state of the system, the r × 1 vector **u** the input, and the p × 1 vector **y** the output. The vector **u**(t) is common in control theory representation of inputs whereas the alternative notation **z**(t) is often used to represent inputs in ecological network or environ theory. The coefficient **A** matrix in equation (1-1), also a common variable choice in control theory, represents the weighted mapping of the state vector **x**(t) into the state transition, $\dot{\mathbf{x}}(t)$, and is not related to the adjacency matrix, introduced later in Chapter 2 from graph theory and used throughout ecological network theory. Equation (1-1) is the state transition function and Equation (1-2) is the state response function. The matrices, **A**, **B**, and **C** determine the relationships between **u**(t), **x**(t), $\dot{\mathbf{x}}(t)$, and **y**(t). Controllability involves the influence of the input signal on the state vector, equation (1-1), and therefore does not involve the output equation (1-2). Formally defined, a system is controllable if, over a given time interval [t_0, t_f], any initial state of $x(t_0) = x_0$, can be guided to a zero state of $x(t_f) = 0$, by a continuous input signal of **u**(t). Algebraically, such control exists if,

$$\operatorname{rank}[\mathbf{B}\;\mathbf{AB}\ldots\mathbf{A}^{n-1}\mathbf{B}] \;=\; n\,. \tag{1-3}$$

This is because the n independent variables used to relate the input of the system to the state of the system require n independent equations for the combined relationship to have one solution. Without a single solution, the system in uncontrollable. Stated in another way, if the column vectors $\mathbf{B}$, $\mathbf{AB}$, …, $\mathbf{A}^{n-1}\mathbf{B}$ etc. are linearly independent, then the n × n matrix of equation (1-3) is of rank n and the column vectors span the state space of the system. Observability, on the other hand, involves the effect of the state vector on the output of the linear state equation. Formally defined, a system is observable on the time interval $[t_0, t_f]$ if any initial state, $x(t_0) = x_0$, can be uniquely determined from the output $\mathbf{y}(t)$ for any time in the interval $[t_0, t_f]$. Algebraically, a system is observable if,

$$\text{rank}[\mathbf{C}' \, \mathbf{A}'\mathbf{C}' \ldots \mathbf{A}'^{(n-1)}\mathbf{C}'] = n. \tag{1-4}$$

Similar to controllability, to assure one solution, n independent equations are needed for the n independent variables which are used to relate the state of the system to output from the system. Without a single solution to this relationship, the system in not observable. In large dimensional systems, determining the rank conditions of equations (1-3) and (1-4) is difficult at best. Fixing rank deficiencies can be harder. Controllability and observability are the two basic requirements in the control design of simple and complex systems yet their computational requirements require excessive efforts for even the simplest models.

Fortunately, Śiljak (1991) defined the more accessible concepts of *input* and *output reachability* which explicitly determine controllability and observability, without knowledge of the system parameters (e.g., matrices **A**, **B**, and **C** in the example above). Input and output reachability, and therefore controllability and observability, respectively, can be determined from the reachability matrix (Harary, 1969). This is calculated from the interconnection (adjacency) matrix, which is simply determined from the digraph structure of the model. The ease of this method is particularly helpful with large-scale models. Śiljak's input and output reachability concepts as developed within the framework of Lin's (1974) graph-theoretic concept of structural controllability is an indispensable perspective for ecosystem control analysis. For example, the structural perspective allows a simple discussion of system vulnerability. A control system is vulnerable when removal of a component-to-component interconnection, severing a connection in the corresponding digraph, destroys input reachability. When abiotic or biotic changes occur in an ecosystem, such as man-made or natural reductions in biodiversity that result in structural changes in corresponding interactions, input reachability of system inputs can be affected. Pichai et al. (1981) have developed a theory to identify the minimal set of connections between components essential for preserving system input reachability.

Stability in the sense of Liapunov provides a means to evaluate the stability of complex nonlinear or linear time-variant characteristics of ecosystems without knowing the actual system equations. Input and output reachability provides a perspective to ascertain controllability and observability, by inspection of the model's digraphs, in the control design of large complex systems. Together, these concepts are key fundamentals to the control design of ecosystem models.

1.2-2 ECOSYSTEM CONTROL

Holistic knowledge of controlling relationships in ecosystems is one of the primary reasons for understanding system structure and function through modeling. Knowledge of proximate (direct), or especially distal (indirect), control points affords one the ability to exert control, either close-in or at a network distance — in other words, one should have a capability to implement the elusive concept of "ecosystem management."

Ecological systems are complex and, more importantly, distributed constructs of growth (positive feedback) and subsequent restrictions (negative feedback) maneuvering in concert towards various behaviors (Rosenblueth, 1943) or homeostatic equilibria (performance indices). When described as such, modern control theory concepts are intrinsically woven into this model of biological systems operation at virtually all hierarchical levels of resolution. Verhulst's (1838) *Carrying Capacity*, Liebig's (1840) *Law of the Minimum*, Blackman's (1905) *Law of the Maximum*, Shelford's (1911) *Law of Tolerance*, Chapman's (1928) *Environmental Resistance*, and Tilman's (1982) *Resource Competition Theory* represent but a few examples of various forms of feedback (or possibly feed-forward) restrictions to growth. Similarly, but with considerably less volume of activity, the concept of positive feedback can also be found in the historical literature with Malthus' *exponential growth* (1798) and Chapman's *Biotic Potential* (1928), or the modern reviews by DeAngelis and Post (1991) and Ulanowicz (1991). And finally, performance indices (i.e., teleology, goal functions, eco-targets, orientors, etc.) have been debated in various forms (Clements, 1936; Gleason, 1926; Whittaker, 1953) and again are garnering focus (Müller et al., 1998; Jørgensen, 2000; Fath et al., 2001).

However, the umbrella of ecology quickly conjures a complexity (e.g., behavioral, evolutionary, temporal, spatial) that stretches the basic modern control theory's ability to successfully or usefully map the expected system behavior. Subsequently, notions of controlling relationships in ecology are dispersed from the formal theory to more project-specific terms, e.g., dominating/subordinating, augmenting/diminishing, releasing/constraining, liberating/regulating, bottom-up/top-down, etc. Accordingly, herein, ecosystems are considered to have no explicit controllers that parallel the thermostats and autopilots of man-made systems. The concept of control somehow resides in the checks-and-balances complexity of organization inherent in the interactive networks that join things together, and in the environs into which these networks can be decomposed. The concept is thus one of diffuse, decentralized, even (at a network distance) remote control; for which the term *distributed control* is used. Within the framework of *network environ analysis* (NEA) (Patten, 1978b; Patten and Auble, 1981; Fath and Patten, 1999; Fath, 2004) this kind of control can, with definition of appropriate measures derived from the primary environ analyses of pathways and throughflows, be fruitfully investigated.

1.3 NITROGEN CYCLING

> Taken overall, any reaction in the nitrogen cycle may act as a rate-limiting step and hence control the overall process.
>
> — Janet I. Sprent, 1987

An ecological neologism for the expression *stuff happens* could simply be, *nitrogen cycles*. It is a fact of life, literally. Patten's original distributed control methodology was built from an energetics perspective where all of the initial presentations (Patten, 1978b; Patten and Auble, 1981; Dame and Patten, 1981) involved energy models. While individual and ecosystem energetics have been widely studied (Pandian and Vernberg, 1987; Wiegert, 1988; Wright et al., 1994; Brown, 1995), alternatives to energy being the primary currency have developed. Manson and McGlade (1993) scrutinized energy-based approaches to ecosystem dynamics and evolutionary biology. Redfield et al. (1963) hypothesized from his ocean studies that deep water ratios of C:N:P at 100:16:1 represented the relative requirements of living matter. Recently, others present elementary stoichiometry (typically focused on the varying combinations of the C:N:P ratios) as a causal mechanism linking cellular, ecosystemic, and evolutionary processes (Reiners, 1986; Sterner et al., 1992; Elser and Dobberfuhl, 1996; Elser et al., 2000; Sterner and Elser, 2002). Schlesinger (1997) suggests theoretically that since N-fixing organisms have a high demand for P (linking the global cycles of N and P) that P could possibly be the ultimate limit on nitrogen availability and net primary production. Levin (1989), however, demonstrates that net primary production in most terrestrial and marine ecosystems usually shows an immediate response to additions of N. Additionally, White (1993) articulates that, due to disparate nitrogen compositions between consumers and their foods, energy availability [less than 10% of the sun's energy is captured (Radmer and Kok, 1977) by the ecosystems of the world] is less important than nitrogen in the reproductive success of animals and their subsequent population dynamics. White posits that nitrogen plays one of the pivotal roles in ecosystem functionality. Boyer et al. (1994) identify the literature confirming N (and in particular, not P) is the critical limiting factor in both coastal marine waters in general and in the Neuse River estuary in particular.

Assuming the environment for all organisms is inadequate at some point, populations continue to grow until the limit of a minimum resource is reached. *Survival of the Fittest* is a contrapositive expression inherently describing those individuals or species that cannot cope with the specific limiting resource and subsequently change requirements, move, or die. Following an abbreviated form of White's (1993) argument, assume that the supply of life's basic chemicals is finite. Although carbon, oxygen, hydrogen, and nitrogen are all in great abundance, nitrogen is calculated as the only chemical not readily available. Ninety nine and ninety five hundredths percent (99.95%) of the total nitrogen in the biosphere is in the form of the extremely inert gas, N_2, comprising over 80% of the earth's atmosphere. Only half of a percent (0.5%) of the world's supply of nitrogen is ever fixed, however, and combined with other chemicals. Only half of this small quantity is organic ($0.5 \times 0.005 = 0.0025 = 0.25\%$ organic), and 95% of this is trapped in abiotic litter, soil, or particulate and dissolved matter in the oceans ($0.95 \times 0.0025 = 0.00238 = 0.238\%$ abiotic material). Hence, of the essential chemicals necessary for biotic processes, nitrogen is considered the least available and most limiting

(Delwiche, 1970; Rosswall, 1983; Stewart et al., 1983), yet it is second, only to carbon, with regard to quantities required to sustain life's processes. As such, the Neuse River estuary nitrogen model (Christian and Thomas, 2000, 2003) presents an ideal opportunity to not only augment further development of a distributed control theory for ecology, but also to provide a contrasting view to energy being the primary regulatory currency.

Nitrogen fixation, assimilation, nitrification, and denitrification are simultaneously combined in a choreographed complex sequence of events whose controlling characteristics involve a distributed fabric of continuous interactions. The complexity of every single nitrogen compound interaction has yet to be understood. For example, assume the life cycle begins with the availability of N_2 in atmospheric air or water. Considering the endothermic energetics associated with the initial reaction to break the N_2 bond during nitrogen fixation, nitrogenase enzymes, using energy derived from photosynthesis and chemosynthesis, provide the necessary boosting catalyst to their host autotrophic organisms. The amount of energy used varies widely with both the organisms involved and the prevailing environmental conditions (Sprent, 1979; Harris, 1982). As such, varying energy sources subsequently drive nitrogenase reaction rates in a variable manner which then results in fluctuating reaction efficiencies (energy used per nitrogen molecule reduced). Despite the clarity of this generalized recipe, the endothermic reaction of breaking the N_2 bond will probably never be completely understood (see Postgate, 1987 for a reasonable summary). This single-step complexity is played out over the entire nitrogen cycle in an incomprehensibly broad span of abiotic and biotic interactions. Absent a thermostat-like controller regulating these nitrogen related interactions as the multitude of independent variables (e.g., temperature, pressure, pH, salinity, humidity, space, time, etc.) continue to impact the system's time-forward progress, a mosaic of seemingly invisible distributed controllers manages the outcome. This perspective on the nitrogen cycle in the Neuse River estuary provides a novel and unique look into the controlling actions of nitrogen in an ecosystem.

1.4 NEUSE RIVER ESTUARY MODEL

Network environ analysis (NEA) may proceed from stocks or throughflows. Using NEA throughflow analysis, the framework of distributed control will be developed and explored for steady-state models of nitrogen cycling in the Neuse River estuary, North Carolina, for 16 consecutive seasons from spring 1985 through winter 1989. The nitrogen models (Christian and Thomas, 2000, 2003) have seven components or state variables: Phytoplankton Particulate Nitrogen (PN-Phyto), Heterotroph Particulate Nitrogen (PN-Hetero), Sediment Nitrogen (Sediment), Dissolved Organic Nitrogen (DON), Nitrate and Nitrites (NO_x), Ammonium (NH_4), and Abiotic Particulate Nitrogen (PN-Abiotic). The Neuse River receives water from a 16,000 km^2 watershed which together with the smaller Trent River (9% of Neuse and Trent combined) empties into the 400 km^2 Neuse River estuary, thence ultimately into Pamlico Sound. Many studies focusing on nutrient cycling and cyanobacterial blooms in this area have been conducted over the last 30 years (Boyer et al., 1988; Christian et al., 1984, 1989, 1991; Hobbie and Smith, 1975; Paerl, 1987; Stanley, 1983, 1988).

1.5 EQUATION DEVELOPMENT

> The first step to be taken toward the operational realization….is that of developing an appropriately general class of models by which to describe complex systems mathematically and an associated mathematical theory to deal with the manipulation of such models.
>
> — A. Wayne Wymore, 1967

The premise behind a distributed control logic is that a system is comprised of connected components all of which participate simultaneously in a managed time-forward trajectory. The theory owes its general formulation to the concept of state, which, by definition, is primarily tied to the mathematical model of the system under study. Basically, only a set of numbers collectively containing sufficient past information relevant to the determination of the system's future behavior is all that is needed to define the state of a specific system. State-space theory (Zadeh and Desoer, 1963; Zadeh, 1964; Kalman et al., 1969; Patten et al., 1976) provides both a logical foundation and a mathematical representation for the state of systems. Defining a generalized notion of state applicable to all systems is a formidable task and remains one of the primary pursuits of systems analysis (Wymore, 1967; Bertalanffy, 1968; Klir, 1969; Hanson, 1995). Skipping the formal definition, the equations of state for a dynamical system encompass the current state of the system, the input, the output, and two relationships that map the input to both the future state on one hand, and the future output on the other. It is generally assumed that, past states, inputs, and outputs precede or are contemporary with future states, where the designation "dynamical" signifies causal, meaning determinate (unique input time sequence → unique output sequence) and nonanticipatory (Zadeh and Desoer, 1963; Zadeh, 1964). Basically, the successful mathematical notion of a dynamical system depicts the unidirectional temporal flow of causation (Patten et al., 1976). Equations of state in this formulation are uniquely suited for control and stability analyses. In fact, most mathematical system theories or the more specific linear system theories introduce the state-space concept and corresponding equations, then quickly divert to the issues of control and stability.

Network environ analysis (NEA) (Patten, 1978a) is a state-space-based network analysis (e.g., Ponstein, 1966; Patten et al., 1976; Higashi and Burns, 1991; Margalef, 1991; Patten and Jørgensen, 1996; Newman, 2003; Borrett, 2005). It is predicated on a two-environment version of state-space theory in which objects taken as open systems simultaneously participate, by conservative material and energy exchanges, in outgoing and incoming environments. The duality of environment, later attributed to von Uexküll (1926), was first developed as an ecological systems theory by Patten et al. (1976). Subsequently, Patten (1978a) provided the name *environs* and presented three propositions (Patten, 1978a) which solidified *environ analysis* as an environmental system theory available for further development [e.g., by Barber et al., 1979; Matis and Patten, 1981; Patten, 1981; Patten and Matis, 1982; Hippe, 1983; Fath, 1998; Fath and Patten, 1999; Borrett, 2006; Gattie et al., 2006a, 2006b (submitted); and Schramski et al., 2006] as an extension of Leontief's (1936, 1965, 1966) economic input–output analysis. Environ theory unfolds mathematically from the definition of throughflow and the premise that transactional elements (mass – energy) are conserved.

1.6 DISSERTATION PREFACE

The specific focus of this dissertation is development of a formal mathematical approach to ecosystem control within the framework of network environ analysis (NEA). The core results are in Chapters 2-4, each written as an in independent publishable paper. The present Chapter 1 articulates automated control theory, ecosystem control theory, nitrogen cycling, and network environ analysis as necessary preludes to the core chapters. Definitions and equation derivations in NEA are crucial to the interpretations of distributed control theory. Chapter 2 addresses this by meticulously deriving, from the Reynolds transport theorem, the fundamental throughflow and storage environ equations using a consistent methodology. Throughflow equations are derived identically in parallel with the storage equations leading to a unified foundation for the subsequent distributed control mathematics. This chapter is the result of discussions among the coauthors, others, and of many mathematical false starts searching for a novel way to tie throughflow and storage environ analysis together in a common methodological framework. Chapter 3 develops the mathematics of environ distributed control by formally considering individual pair-wise relationships in a network. Relative and absolute control perspectives are identified. Distributed control within the 16-model average model of nitrogen cycling in the Neuse River estuary is evaluated; in this, NO_x and Sediment are found to play leading roles. Chapter 4 pushes the mathematics and interpretations further in evaluating distributed control through the 4-year, 16-season temporal sequence represented by the 16 season Christian and Thomas (2000) models. Low component throughflow-to-boundary flow ratios (T_i/z_i or T_i/y_i) are shown to be possible indicators of components with high levels of control in the network suggesting that components dominated by their boundary flows may act as gatekeepers for the system's relationship to its environment across the system boundary, and thus dominate in the network. Chapter 5 provides a summary of the dissertation's key objectives and a discussion of the larger significances Finally, the appendix provides supporting calculations and the Neuse River estuary raw data such that calculations can be independently repeated or expanded.

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CHAPTER 2

DISTRIBUTED CONTROL IN THE ENVIRON NETWORKS OF A SEVEN-COMPARTMENT MODEL OF NITROGEN FLOW IN THE NEUSE RIVER ESTUARY, USA: EQUATION DEVELOPMENT[1]

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[1] Schramski, J.R., Gattie, D.K., and Patten, B.C. to be submitted to *Ecological Engineering*

ABSTRACT

Recognized as the two mathematical building blocks of network environ analysis (Patten, 1978a), the law of conservation and the definition of throughflow are both separately derived and combined into the directional two-environment, efferent and afferent, ecological systems theory. The conservation equations are derived using the Reynolds transport theorem and an Eulerian control volume model which provide a framework defining and separating the concepts of flow and storage. The definition of throughflow and the component conservation equations are systematically combined in a consistent methodology for both throughflow and storage environ analysis. The energy conservation equation, in particular, is expanded for a more thorough understanding of its individual terms as derived for an ecological system. Terms and concepts common to environ analysis, such as self-flow, turnover rate, storage, accumulation, and the necessary inclusion of a discrete time step in environ storage analysis, are clarified. An averaged model of the Neuse River estuary nitrogen cycle (Christian and Thomas, 2000) is evaluated, where connectivity, environ throughflow, and environ storage are collectively utilized to provide a picture of each component's individual participation in the Neuse ecosystem without having to remove it from the network to which it belongs. Accounting for all indirect and direct pathways, the Sediment compartment is shown to have a relatively weak role in the throughflow cycling networks and a contrastingly dominant role in the steady-state exchange of component storage contents. In comparison, PN-Heterotrophs compartment are discovered to dominate the throughflow cycling of nitrogen quantities and contrastingly play a minimal role in the steady-state storage exchange of component storages.

2.1 INTRODUCTION

The development of Patten's (1978a) NEA over the last 28 years, by a variety of authors (e.g., Patten, 1978b; Barber et al., 1979; Matis and Patten, 1979; Dame and Patten, 1981; Patten and Auble, 1981; Patten, 1981, 1982, 1985; Fath and Patten, 1999; Gattie et al., 2006a; Schramski et al., 2006, etc.) has significantly expanded the original presentation. As to be expected, minor variations have occurred in the derivations and their subsequent interpretations. For example, the original derivation (Patten, 1978a) and immediate successor (Barber et al., 1979) specifically included the prospect of a self-flow, f_{ii}, in their flow conservation equations without elaborating on their essence. Flow equations used by Matis and Patten (1979) to formulate the concept of environ-storage analysis, by definition, did not include self-flows and therefore were not discussed. Later, the descriptive essence of a self-flow varied including references to storage (Patten 1981, 1982), time delay, and node queues (Patten, 1985). Fath and Patten (1999) subtly imply self-flows are related to storage but include them in their flow-equation derivations and then specifically state that they will not be algebraically considered in subsequent development. Recently, through personal discussions, technical paper preparations, and classroom material development in the University of Georgia's Institute of Ecology and the Systems and Engineering Ecology program, environ-throughflow equation development has expressed self-flows similar to the Matis and Patten (1979) method where they are not defined in the flow equations. Self-flow inclusion, or not, while using the adjacency matrix in network path-length proliferation analysis or the simple decision as to when self-flows should be pictorially included on a network digraph can be unclear and then perpetuate further confusion. These minor inconsistencies and others are the specific focus of Chapter 2. The specific intent is to organize the work of others while including some novel additions to create an NEA foundation which is concise, consistent, and clear both algebraically and interpretatively for the subsequent expansion of NEA into the realm of control theory.

Although other methods of coordination may exist, Chapter 2 provides a single presentation which organizes the history of NEA development into one ordered stream of uninterrupted construction. Specifically, we start by reaching back to the equation development of Barber et al. (1979) which separately formulates the conservation equations and then incorporates the definition of throughflow to generate the efferent- and afferent-oriented NEA equations. We augment this approach by borrowing from the mature field of mass transport dynamics and use the Reynolds transport theorem to formulate the conservation equations which explicitly provide a framework defining and separating the concepts of flow and storage. Although a large portion of the chapter is relegated to repeating previously published material, the consistent and continuous stream of development is a necessary strategy to both improve algebraic clarity to aid unforeseen future NEA development and to generate additional novel improvements to the overall NEA theory. As a direct result of the rigidity afforded this meticulous derivation and the consistent methodology used between the throughflow- and storage-environ equation development, such concepts including but not limited to self-flow, turnover rate, storage, accumulation, and the necessary inclusion of a discrete time step in environ storage analysis, are easier to define and articulate.

An efferent and afferent environ review of an averaged model of nitrogen cycling in the Neuse River estuary (Christian and Thomas, 2000) is provided for two specific reasons. First, the results serve to complete the continuous stream of NEA development, a core objective of this chapter. It is hoped that this chapter can be used as a single introductory guide to new users of NEA to which very little additional reference to other materials will be required. An empirical example is appropriate to accompany and augment the theoretical development. Second, the NEA Neuse River estuary results in Chapter 2 form a preliminary basis of empirical understanding of the Neuse nitrogen cycle prior to delving into the control theory of Chapters 3 and 4. Pathway proliferation analysis, and nitrogen throughflow- and storage-environ analysis are coupled into a dynamic introspection of the participation of each of the seven model components (PN-Phytoplankton, PN-Heterotrophs, Sediment Nitrogen, Dissolved Organic Nitrogen, Nitrates and Nitrites, Ammonium, and PN-Abiotic) which together comprise the Neuse River estuary nitrogen model.

2.2 EQUATIONS-OF-CONSERVATION DEVELOPMENT

Network environ analysis (NEA) is predicated on a two-environment view of state-space system theory. The dual environment concept, first developed as an ecological systems theory (Patten et al., 1976), was then reorganized into three propositions (Patten, 1978a) that solidified the input–output oriented environ theory. NEA is mathematically developed both from the definition of throughflow and the premise that a transactional entity (e.g., mass or energy) is conserved. By definition, the concept of throughflow is bi-directional. There exists a total throughflow in, T^{in}, and a total throughflow out, T^{out}, where at steady-state conditions, $T^{in} = T^{out}$. Similarly, the conservation equations are also directional wherein each term represents flow arriving, leaving, or staying in a particular control volume. At steady-state, mass or energy do not accumulate in the control volume and the flows leaving the control volume equal the flows arriving.

The conservation equations, particularly with conservation of energy, have remained largely assumed in previous network analysis or more specifically in NEA (Finn, 1976, 1977; Hannon, 1973; Patten, 1978a, 1978b, 1981, 1982, 1985; Patten et al., 1976, 1981, 1982). Using the Reynolds transport theorem, the conservation of energy and the continuity equations for a typical ecological network model are derived below. The formulation of these equations to a rigid control volume with its control surface provides a framework to better separate and interpret NEA flows and storages. The definition of throughflow, one of the two foundations of the mathematics of directional environ analysis, is separately developed later.

The model development, clearly representing both the derivation and final representation of the basic conservation of mass and energy equations fundamental to network environ analysis (NEA) is borrowed, in concept, from the fields of thermo and fluid dynamics. Virtually all fluid and thermodynamic textbooks carry the control volume (CV) Eulerian derivation of the continuity and energy conservation equations. These equations serve as first-principle foundations throughout the study of transport dynamics. The conservation equation development below, using Reynolds transport theorem, is a hybrid of several methods (White, 1994; Wark, 1983), with most of the presentation emanating from Jenna's (1993) derivation. The latter is particularly easy to understand and well suited for the perspective it presents which is subsequently helpful to NEA equation development. The presentation of equations (2-14) through (2-28) or equations that are very similar is common in transport dynamics. Although abbreviated, the time and space allowed herein for Reynolds transport development is intended to provide the discerning reader, without additional references, a sense of how the mathematics of conserved systems remains formal and therefore helpful to long term model development and interpretation in any field. The understanding and on-going application of these common equations represent one of the core pursuits of mass transport dynamics.

2.2-1 CONTROL VOLUME PERSPECTIVE

Lagrangian and Eulerian perspectives on models are applicable to the study of mass or energy transfer. Lagrangian methodology addresses particle motion considered individually by position as a function of time. This is impractical for most reasonable analyses and attainable supporting data. The Eulerian or *control volume* (CV) method is more applicable to the study of entire regions (systems of particles) of mass and energy flow. A CV analysis of a specific region can be established through the *Reynolds transport theorem* conversion relating the time derivative of a system property to the rate of change of that property within a specified region. A fixed CV, uniquely chosen to represent the respective region of study, is usually bounded by an artificial *control surface* (CS) separating the region of study from the surrounding environment. Pictorially speaking, the CS is a dashed line surrounding the CV as demonstrated in Figure 2.1. [Figure 2.1 here] The object is to relate the area outside the control surface (environment) to the region within the control volume (system) and, accounting for all of the mass or energy both within the CV or crossing the CS, generate the conservation equations. The procedure is to start with the system,

$$S = \iiint s\,\rho\, d\cancel{V}, \tag{2-1}$$

where S is an extensive (mass dependent) flow quantity (e.g., mass, momentum, energy, etc.) which we desire to monitor, where s is the intensive flow quantity per unit mass (mass independent), ρ is the density or mass per unit volume, and $\cancel{V}$ is the system volume of interest. In Figure 2.2, let volume 1, $\cancel{V}_1$, at t_1 be represented by $\cancel{V}_A$ plus $\cancel{V}_B$ and volume 2, $\cancel{V}_2$, at t_2 by $\cancel{V}_B$ plus $\cancel{V}_C$. The control volume is bounded by the dashed line. The change in the extensive property S of the system $\cancel{V}$ over the time interval t_1 to t_2 can be written as,

$$\Delta S = (S_{B2} + S_{C2}) - (S_{A1} + S_{B1}). \tag{2-2}$$

[Figure 2.2 here] Rearranging the terms to organize the extensive property inside and outside the prescribed control volume and dividing by the time interval ($\Delta t = t_2 - t_1$), the change in the extensive property S on a per unit time basis is,

$$\frac{\Delta S}{\Delta t} = \frac{S_{B2} - S_{B1}}{\Delta t} + \frac{S_{C2} - S_{A1}}{\Delta t} . \tag{2-3}$$

To determine the instantaneous time rate of change of flow quantity S, take the limit as Δt approaches zero:

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \lim_{\Delta t \to 0} \frac{S_{B2} - S_{B1}}{\Delta t} + \lim_{\Delta t \to 0} \frac{S_{C2} - S_{A1}}{\Delta t} . \tag{2-4}$$

The left-hand-side can be rewritten as the instantaneous time rate of change of the total system S:

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \left. \frac{dS}{dt} \right|_{\text{system}} . \tag{2-5}$$

The first right-hand-side term can be rewritten as the partial time rate of change of the flow quantity S within the control volume:

$$\lim_{\Delta t \to 0} \frac{S_{B2} - S_{B1}}{\Delta t} = \left. \frac{\partial S}{\partial t} \right|_{CV} . \tag{2-6}$$

The second right-hand-side term in equation (2-4) represents the net rate flow quantity S out of the control volume:

$$\lim_{\Delta t \to 0} \frac{S_{C2} - S_{A1}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\delta S}{\Delta t} , \tag{2-7}$$

(where δ signifies the net rate out, a.k.a., out minus in) and requires additional manipulation to obtain the specific limiting expression.

Now consider Figure 2.3 showing a differential area, dA, on the control surface of the control volume. [Figure 2.3 here] The tangential velocity, V_t, carries no matter out of the CV by crossing the CS. As such, all matter leaving dA can be assumed in the normal direction represented by the normal component of the velocity, V_n. Over the time interval Δt, the mass of the matter crossing dA can be written as:

$$dm = \rho \, d\forall \, , \tag{2-8}$$

where the differential volume, $d\forall$, is the three dimensional product of the cross-sectional area and height,

$$d\forall = (V_n \times \Delta t) \, dA \, . \tag{2-9}$$

Assuming that the amount of quantity S moving through the area dA is,

$$\delta S = s \, dm \, , \tag{2-10}$$

substitute equations (2-8) and (2-9) and divide through by Δt to obtain the equation,

$$\frac{\delta S}{\Delta t} = s \, \rho \, V_n \, dA \, . \tag{2-11}$$

The limit as Δt approaches zero allows equation (2-11) to be written as,

$$\lim_{\Delta t \to 0} \frac{\delta S}{\Delta t} = \iint_{CS} s \, \rho \, V_n \, dA \, . \tag{2-12}$$

Substituting equations (2-5), (2-6), and (2-12) into equation (2-4) generates the general conservation equation as derived from the Reynolds transport theorem:

$$\left. \frac{dS}{dt} \right|_{system} = \left. \frac{\partial S}{\partial t} \right|_{CV} + \iint_{CS} s \, \rho \, V_n \, dA \, . \tag{2-13}$$

In practical terms, equation (2-13) states,

instantaneous time rate of change in S for a system of particles (total particles)	=	instantaneous time rate of accumulation of S within the control volume (amount stored)	+	amount of S leaving the control volume minus the amount of S entering (net rate out)

2.2-2 CONTINUITY EQUATION

For conservation of mass, let the system S = m and s = m/m = 1: then, the Reynolds transport theorem represented by equation (2-13) becomes,

$$\left.\frac{dm}{dt}\right|_{\text{system}} = \left.\frac{\partial m}{\partial t}\right|_{CV} + \iint_{CS} \rho\, V_n\, dA\,. \tag{2-14}$$

Considering the term on the left-hand-side of equation (2-14), the total mass in a system must remain constant (mass can neither be created nor destroyed in all practical circumstances), as such,

$$\left.\frac{dm}{dt}\right|_{\text{system}} = 0\,. \tag{2-15}$$

Considering the second term on the right-hand-side of equation (2-14), assuming the inlets and outlets through the CS are one dimensional, the integral reduces to,

$$\iint_{CS} s\, \rho\, V_n\, dA = \sum_i (\rho_i A_i V_i)_{\substack{in\\CS}} - \sum_i (\rho_i A_i V_i)_{\substack{out\\CS}}\,. \tag{2-16}$$

The quantity ρAV is the mass flow, $\dot{m}$, passing through the one-dimensional cross section with units of mass per time. Substitute equations (2-15) and (2-16) into equation (2-14) to get the useful form,

$$\left.\frac{\partial m}{\partial t}\right|_{CV} = \sum_{\substack{in\\CS}} \dot{m} - \sum_{\substack{out\\CS}} \dot{m}\,, \tag{2-17}$$

which states that the change of mass (rate of accumulation) in the CV is equal to the net difference between the mass flow in and the mass flow out, both across the CS. At steady state, there is no change of mass in the CV,

$$\left.\frac{\partial m}{\partial t}\right|_{CV} = 0\,, \tag{2-18}$$

and as such equation (2-17) is usually simplified to

$$\sum_{\substack{\text{out}\\ \text{CS}}} \dot{m} = \sum_{\substack{\text{in}\\ \text{CS}}} \dot{m} . \tag{2-19}$$

Per derivation of the equation from the Reynolds transport theorem, the in-bound and out-bound mass flows are defined as only those that specifically cross the control surface. This observation remains important to flow associated with self-flows of path length one in NEA models, which will be discussed later.

2.2-3 ENERGY EQUATION

First consider, for an adiabatic system, the conservation of energy statement for a closed system (e.g., control volume enclosed by a control surface) is the definition,

$$dW \equiv (E_2 - E_1) = dE . \tag{2-20}$$

The work done on the system, dW, is defined as the change in energy ($E_2 - E_1$) of the system, where E_1 and E_2 indicate the initial and final states. The concept of work, W, quantifies the interaction of a system with it surroundings (in effect, work crossing the system boundary) and includes, for example: shaft, electric and magnetic, viscous shear, or flow work. Numerous energy forms can constitute the total energy, E, of a macroscopic system including, for example: internal, kinetic, gravitational potential, electrostatic, chemical, nuclear, magnetic, or strain energy.

For a nonadiabatic system the conservation of energy equation is defined as,

$$Q \equiv (E_2 - E_1) - W, \tag{2-21}$$

where the difference between the change of energy of the system and the work done on the system is the heat interaction that transpired during the process (positive Q for heat added to the system). Rearranging (2-21) and writing in a differential form provides the equation,

$$dE = dQ + dW, \tag{2-22}$$

where dQ can be generally thought of as an energy transfer process driven by the temperature difference between two systems. The mechanisms commonly describing dQ are radiation, convection, and conduction. The energy in a system (control volume) changes by an amount equal to that which crosses the system boundary (control surface). Energy can cross a system boundary in the form of heat transfer (Q), work (W), or by mass entering or leaving. For conservation of energy, let the system S = E and s = E/m = e. Then the Reynolds transport theorem represented by equation (2-13) becomes,

$$\left.\frac{dE}{dt}\right|_{system} = \left.\frac{\partial E}{\partial t}\right|_{CV} + \iint_{CS} e\,\rho\,V_n\,dA\,. \qquad (2\text{-}23)$$

Substituting equation (2-22) into equation (2-23) yields,

$$\left[\frac{dQ}{dt}+\frac{dW}{dt}\right]_{system} = \left.\frac{\partial E}{\partial t}\right|_{CV} + \iint_{CS} e\,\rho\,V_n\,dA\,, \qquad (2\text{-}24)$$

where the heat transfer to the system, $\frac{dQ}{dt}$, is through conduction, convection, or radiation. The work done on the system, $\frac{dW}{dt}$, (e.g., shaft, viscous, pressure, electric, etc.) plus the heat transfer into the system, $\frac{dQ}{dt}$, equals the accumulation of energy in the control volume, $\left.\frac{\partial E}{\partial t}\right|_{CV}$ (for a non-steady-state system), plus all the energy associated with mass entering or leaving the system across the control surface, $\iint_{CS} e\,\rho\,V_n\,dA$ (e.g., internal, kinetic, potential, chemical, nuclear, etc.). Equation (2-24) is generally referenced as the mathematical representation of the first law of thermodynamics. Thermodynamic analysis of a given system involves independently evaluating the heat transfer, the work quantities, and the changes in specific forms of energy associated with the mass flows across the system's boundaries.

For example, it is a moderately difficult but a common derivation from equation (2-24), when performing an engineering analysis of the thermal and mechanical energies of a system (where e includes internal, u; kinetic, KE; and potential energies, PE), to obtain the equation,

$$-\frac{dQ}{dt}_{\text{losses}} + \frac{dW}{dt}_{\text{shaft}} = \dot{m}\left[\sum_{\text{out}}\left(\frac{p}{\rho}+\frac{V^2}{2}+gz\right)-\sum_{\text{in}}\left(\frac{p}{\rho}+\frac{V^2}{2}+gz\right)\right]. \qquad (2\text{-}25)$$

The system is assumed to be at steady state, such that the accumulation of energy $\left.\frac{\partial E}{\partial t}\right|_{CV}$, is approximately zero, and mass flows into the system approximately equal mass flows out such that $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$. For convenience, the loss term, $-\frac{dQ}{dt}_{\text{losses}}$, has combined, by definition, the total system heat transfer with the internal energy change, where $-\frac{dQ}{dt}_{\text{losses}} = \frac{dQ}{dt} + \dot{m}(u_{in} - u_{out})$. For incompressible flows, a typical situation, changes in internal energy Δu are usually insignificant. The shaft work, designated $\frac{dW}{dt}_{\text{shaft}}$, is a simplification of the total work crossing the control surface where all other forms of work (except work due to mass flow) are assumed negligible (e.g., electric, magnetic, shear, viscous, etc. are approximately zero). The work due to mass flow, $\frac{dW}{dt}_{\text{flow}}$, is the result of mass movement across the system control surface and is therefore, through an auxiliary derivation, rewritten in the form $\dot{m}\left[\sum_{\text{in}}\frac{p}{\rho}-\sum_{\text{out}}\frac{p}{\rho}\right]$, where p and ρ are fluid pressures and densities, respectively. These new terms are then algebraically transferred to the right-hand-side of equation (2-25). The change in kinetic and potential energies of the mass flows are captured by $\dot{m}\left[\sum_{\text{out}}\frac{V^2}{2}-\sum_{\text{in}}\frac{V^2}{2}\right]$ and $\dot{m}\left[\sum_{\text{out}} gz-\sum_{\text{in}} gz\right]$, respectively, where V, g, and z are the fluid velocity, the gravitational constant (9.8 m/s^2), and the fluid vertical height. Equation (2-25), indispensable in modern society, is a practical representation of the energies associated with a thermal and mechanical system.

Representing a living ecosystem with a practical formulation of equation (2-24) is the next logical step. Solar energy is electromagnetic radiation. When incident on a perfectly absorbing black surface, it is completely converted to heat, Q. Natural abiotic and biotic processes are constantly transforming solar energy to work, W, for example: 1) Solar energy is converted to gravitational potential energy through the heating of water, subsequently producing evaporation, which eventually causes precipitation at higher altitudes. 2) Kinetic energy is produced from solar heating of the atmosphere and the earth, ultimately creating atmospheric winds. 3) Primary producers use and convert solar energy to move raw materials of lower chemical energy (e.g. H_2O and CO_2) to higher energy materials (e.g., carbohydrates) by photosynthesis. 4) Secondary consumers consume these materials, increase their biomass, and produce useful work. Inefficiencies in these processes are lost to the greater environment as heat eventually radiated back to space.

Certainly, ecosystem types vary widely and, correspondingly, are orchestrated tradeoffs between heat transfer Q, work W, energy stored $\left.\frac{\partial E}{\partial t}\right|_{CV}$, and the energy associated with mass flow across the control surface system boundaries $\iint_{CS} e\, \rho\, V_n\, dA$. Although in-depth energy balances of specific ecosystems or components of ecosystems exist, for example, green leaves (Aber and Melillo, 2001) or bodies of water (the psychometric chart in all elementary thermodynamics texts), as of yet, the practical technology associated with data acquisition in network ecosystem analysis of energy flow has been grossly simplified to food-web style contact graphs (Odum H., 1956, 1957; Tilly, 1968; Williams and Crouthamel, 1972; Dame and Patten,

1981) somewhat expanded to include abiotic as well as biotic aggregate components. The vast majority of heat transfer, work, changes in internal energy, etc., are loosely combined into respiration. The energy associated with component across-boundary mass flows is primarily the chemical energy associated with food. As such, with the exception of primary producers, all input flows to specific model components either from outside the system or from intercompartmental flows are assumed to be in the form of chemical energy (e.g., carbohydrates). Primary producers receive their input from outside the system in the form of electromagnetic radiation. Output flows, defined as leaving the system boundary (no consumer available within the defined system), include the energies associated with component metabolism (basal and non-basal respiration), net component growth (Hannon, 1973), and that which is contained in exported biomass (e.g., mortality, ingested but not assimilated, etc.). Variations exist on the groupings of these system-level output energy flows (Hannon, 1985; Odum, 1957; Dame and Patten; 1981) but these variations, as of yet, reveal no additional information regarding the fundamental differences between the work, the heat transfer, or the energy associated with mass transfer. Intercompartmental output flows are consumer driven and, coupled with their corresponding intercompartmental input flows (consumption), simply involve the transfer of chemical energy. Subsequently, for ecological network analysis of energy flow in a network, equation (2-24) will be taken in the following form,

$$\left.\frac{dQ}{dt}\right|_{\text{system}} = \left.\frac{\partial E}{\partial t}\right|_{CV} + \sum_{\substack{\text{out}\\ CS}} \dot{m}_{\text{out}}\, e_{\text{out}} - \sum_{\substack{\text{in}\\ CS}} \dot{m}_{\text{in}}\, e_{\text{in}}\ , \tag{2-26}$$

where e is the energy transfer associated with mass flow, in this case usually biomass with stored chemical energy. The net heat transfer, $\frac{dQ}{dt}$, is solar electromagnetic radiation and respiration when applicable. The work, $\frac{dW}{dt}$, performed by or on a component (e.g., a biotic organism physically moving materials or an abiotic material moving vertically lower to earth along with a stream dropping downhill) is combined with the heat transfer term, $\frac{dQ}{dt}$, to generally represent energy incoming or outgoing to or from the system which is not specifically associated with mass flow across the system boundary. At steady-state, with no accumulation of energy in the CV, $\left.\frac{\partial E}{\partial t}\right|_{CV} = 0$, then equation (2-26) is often simplified to,

$$\left.\frac{dQ}{dt}\right|_{system} = \sum_{\substack{out\\CS}} \dot{m}_{out}\, e_{out} - \sum_{\substack{in\\CS}} \dot{m}_{in} e_{in} \tag{2-27}$$

where the heat flow from and to the system, $\pm\left.\frac{dQ}{dt}\right|_{system}$, could be confused with the absent accumulation term, $\left.\frac{\partial E}{\partial t}\right|_{CV}$. If the system is assumed adiabatic, $\left.\frac{dQ}{dt}\right|_{system} = 0$, then equation (2-26) becomes,

$$\left.\frac{\partial E}{\partial t}\right|_{CV} = \sum_{\substack{in\\CS}} \dot{m}_{in}\, e_{in} - \sum_{\substack{out\\CS}} \dot{m}_{out} e_{out} \tag{2-28}$$

which is a subtly but distinctly different equation when compared to equation (2-27). Considering equations (2-24), (2-26), (2-27) or (2-28), a conglomeration of energy inputs, $+\frac{dQ}{dt}$ (e.g., radiation), outputs, $-\frac{dQ}{dt}$ (e.g., respiration), and storage accumulations, $\pm\left.\frac{\partial E}{\partial t}\right|_{CV}$ (e.g., biomass), of the system, are often brazenly written interchangeably with the incoming and outgoing mass flow terms on the right-hand-side. To begin clarifying these issues, for each energy conservation equation in an ecological network model (generally n equations for an n-component model), equation (2-24) should be the starting point with assumption-labeled steps clearly leading to the final equation form. One area of interest with possible applications in ecological network analysis is the study of exergy (Rant, 1956) and its sub-discipline eco-exergy (Jørgensen, in press), which continues to help articulate and improve the understanding of the various forms of energy in ecosystems (Susani et al., in press; Sciubba, 2004).

2.3 FOUNDATION OF NETWORK ENVIRON THROUGHFLOW ANALYSIS

Developed from general systems theory (Zadeh and Desoer, 1963; Zadeh, 1964; Kalman et al., 1969), network analysis (Hannon, 1973; Platt et al., 1981; Ulanowicz and Platt, 1985; Wulff et al., 1989; Higashi and Burns, 1991; Patten and Jørgensen, 1996) is an environmental application of economic input–output analysis (Leontief 1936, 1951, 1966) modeled as interconnected compartments with a common currency (usually energy or mass but can also include information). Network environ analysis (NEA) (Patten, 1978a, 1981, 1982; Fath and Patten, 1999) represents every object as existing within a system of two environments defined as input-oriented and output-oriented environs. The object is a partition of two mutually exclusive halves, one comprising the inflow and the other outflow (von Uexküll, 1926), of which, each direction includes an independent intersystem flow network originating or terminating at the system boundary. As a direct result, distinct input and output environments can be defined and investigated for every component in a system. The reader is encouraged to consult Fath and Patten (1999) for a concise review of NEA. Two fundamental precepts of network environ analysis are the law of conservation and the definition of throughflow.

2.3-1 NETWORK CONSERVATION EQUATIONS

Figure 2.4 shows a typical model component within a multi-component model where the material storage, either mass or energy, is usually denoted x, and inter-component mass or energy flow quantities are denoted f. [Figure 2.4 here] Given the model convention shown in Figure 2.4, equations (2-17) or (2-28) can be written for one component in the model as a difference or differential equation; for example, the latter case,

$$\left.\frac{dx_i}{dt}\right|_{CV} = \sum_{\substack{j=1\\ \text{in}\\ CS}}^{n} f_{ij}(t) + z_i(t) - \sum_{\substack{h=1\\ \text{out}\\ CS}}^{n} f_{hi}(t) + y_i(t), \qquad t_0 \le t < \infty, \tag{2-29}$$

where f_{ii} represents flow from a component to itself (e.g., cannibalism). The formalities of the Reynolds transport theorem require that the storage accumulation on the left-hand-side of equation (2-29) occur in the control volume and the flows on the right-hand-side of the equation cross the control surface. For an n-component system, equation (2-29) is further defined for each of the components,

$$\left.\frac{dx_i}{dt}\right|_{CV} = \sum_{\substack{j=1\\ \text{in}\\ CS}}^{n} f_{ij}(t) + z_i(t) - \sum_{\substack{h=1\\ \text{out}\\ CS}}^{n} f_{hi}(t) + y_i(t), \quad i = 1, 2, \ldots, n, \quad t_0 \le t < \infty. \tag{2-30}$$

Equation (2-30) written for a steady-state system, $\left.\frac{dx_i}{dt}\right|_{CV} = 0$, where accumulation of mass or energy is zero, becomes,

$$\sum_{\substack{h=1\\ \text{out}\\ CS}}^{n} f_{hi} + y_i = \sum_{\substack{j=1\\ \text{in}\\ CS}}^{n} f_{ij} + z_i, \quad i = 1, 2, \ldots, n, \qquad t_0 \le t < \infty. \tag{2-31}$$

Equation (2-30) or (2-31) is the typical network conservation equation for an n-component network, reference Figure 2.4, where the flow f represents a non-negative intercompartmental flow, z an input boundary flow, and y an output boundary flow. Strictly interpreting equation (2-30), all intercompartmental flows, f_{ij} in, or f_{hi} out, including self-flows f_{ii}, participate in the component storage x_i, rate of change by helping to add $\left.\frac{dx_i}{dt}\right|_{CV} > 0$, subtract $\left.\frac{dx_i}{dt}\right|_{CV} < 0$, or hold steady $\left.\frac{dx_i}{dt}\right|_{CV} = 0$, each component's storage. Combined together in total, the intercompartmental flows can be evaluated to elicit this component storage information. However, independently removed from the system to which they belong, each f (including f_{ii}) does not provide sufficient information to determine either the component's total rate of storage accumulation (for a non steady-state system) or its turnover time (for a steady-state system).

2.3-2 COMPONENT SELF FLOWS, f_{ii}

Considering the f_{ii} terms from the perspective of the Reynolds transport theorem [second term on the right-hand-side of equation (2-13)], all observed flows f, including f_{ii}, shall cross the defined control surface. This is a subtle but important attribute of the control volume model for the corresponding empirical data-gathering. Therefore, by definition from the Reynolds transport theorem, self flows out and back into a component, f_{ii}, shall be an observed flow quantity. However, in all models to date, the observed field-acquired data corresponding to the f_{ii} terms have been zero or inconsequential and therefore, assumed zero. Secondly, if the observed flow values, f_{ii}, were something other than zero, considering equation (2-29) for example, the f_{ii} term in the out-flow summation $\sum_{out}$ and the f_{ii} term in the in-flow summation $\sum_{in}$ would actually cancel. Despite these two somewhat nullifying issues, the f_{ii} term should always remain in the standard network equation development as carried forward from equation (2-29) for at least three reasons: (1) Although flow from a component to itself is almost never observed over a path length of one (having one time step duration for discrete time analysis), subsequent mathematical analysis will show that a cycle from a component back to itself is common over path lengths greater than one (indirect flows routed through other components when considering discrete time steps greater than one). Not acknowledging and including the f_{ii} term for the first time step (path length equal to one), makes the subsequent longer path length calculations mathematically impossible with the corresponding matrix algebra. (2) The concept of throughflow, introduced later, will be substituted into equation (2-29) and will effectively enfold one of the f_{ii} terms (by incorporating one of the two summations $\sum_{out}$ or $\sum_{in}$) into the throughflow definition while the other f_{ii} term (in the remaining of the two summations $\sum_{out}$ or $\sum_{in}$) remains visible and available to the remaining matrix algebra calculations of indirect flows at path lengths greater than one. Per the rationale listed for (1) above, the f_{ii} that is still available (not enfolded in the throughflow substitution) for algebraic manipulation is mathematically necessary for subsequent calculations used to elicit indirect effects at path lengths greater than one. As such, though the two f_{ii} terms could cancel in equation (2-29), they are each mathematically necessary and they do not cancel in subsequent algebraic manipulation. (3) Although, to date, no f_{ii} flows of consequential magnitude have been found, they certainly will be, and will need to be incorporated in future NEA analyses.

2.3-3 THE CONCEPT OF THROUGHFLOW

With regard to energy or mass, a component's storage and the various flows to and from its neighboring compartments, in a multi-compartment system, represent quantifiable activities or performance indices. One such index, *component throughflow*, T_i, is a key network property and serves as an indicator of a component's activity within a system. Referring to Figure 2.4, throughflow is defined as the sum of all flows in or out of component i in an n-component system,

$$T_i^{in}(t) \equiv z_i(t) + \sum_{j=1}^{n} f_{ij}(t), \quad i = 1, 2, \ldots, n, \tag{2-32}$$

$$T_i^{out}(t) \equiv y_i(t) + \sum_{h=1}^{n} f_{hi}(t), \quad i = 1, 2, \ldots, n. \tag{2-33}$$

The values T_i^{in} or T_i^{out} are directional and unequal in an unsteady conservative system [reference equations (2-17) and (2-28)]. The throughflows are related to the component storage accumulation where the unsteady equation balancing throughflows in and out is:

$$\left.\frac{dx_i}{dt}\right|_{CV} = T_i^{in}(t)_{CS} - T_i^{out}(t)_{CS}, \quad i = 1, 2, \ldots, n, \qquad t_0 \leq t < \infty. \tag{2-34}$$

Throughflow is both a property of the individual system components as well as the whole system where *total system throughflow* is the sum of all component throughflows,

$$TST^{in}(t) = \sum_{i=1}^{n} T_i^{in}(t), \tag{2-35}$$

$$TST^{out}(t) = \sum_{i=1}^{n} T_i^{out}(t), \tag{2-36}$$

whose values TST^{in} and TST^{out} are also directional and different in a conservative unsteady system. Therefore, following from equation (2-34), *total system storage*, TSx, and total system throughflows are related by the following dynamical equation:

$$\left.\frac{dTSx}{dt}(t)\right|_{CV} = TST^{in}(t)_{CS} - TST^{out}(t)_{CS}, \qquad t_0 \leq t < \infty. \tag{2-37}$$

For a steady-state system where all component and total system storages remain constant, $\frac{dx_i}{dt} = 0$ and $\frac{dTSx}{dt} = 0$, equations (2-34) and (2-37) reduce to

$$T_i = T_i^{out} = T_i^{in}, \qquad i = 1, 2, \ldots, n, \tag{2-38}$$

$$TST = TST^{out} = TST^{in}, \tag{2-39}$$

respectively. Computationally, at steady-state the directionality of throughflows becomes moot by the equalities in equations (2-38) and (2-39). Algebraically however, for steady and unsteady systems, a throughflow's directional orientation in equations (2-32) and (2-33) is one of the fundamental constructs of input–output analysis (Leontief 1936, 1966; Hannon, 1973; Patten, 1978a) and is fundamental to NEA's mutually distinct and directional input and output oriented environs. Throughflows are by definition [equations (2-32), (2-33), (2-35), and (2-36)] enfolded composite flows with input and output boundary flows and directional intercompartmental flows as constituents. As such, although throughflows can be a key system representation of network activity, they can also be further parsed into their respective constituents of directional boundary (z_i or y_i) and intercompartmental partitioned microdynamic flows revealing additional component and network characteristics (Gattie et al., 2006a, 2006b).

2.3-4 EFFERENT FLOW EQUATIONS

The efferent or time-forward environ flow equations are developed by combining the outward oriented throughflow equation (2-33) for T_i^{out} with the conservation equation (2-30) to obtain:

$$\frac{dx_i}{dt} = \sum_{\substack{j=1 \\ in}}^{n} f_{ij}(t) + z_i(t) - T_i^{out}(t), \quad i = 1, 2, \ldots, n, \quad t_0 \leq t < \infty, \qquad (2\text{-}40)$$

where the control volume and control surface designations are dropped to minimize repetition. At steady-state, $\frac{dx_i}{dt} = 0$, equation (2-40) becomes,

$$T_i^{out} = \sum_{\substack{j=1 \\ in}}^{n} f_{ij} + z_i, \quad i = 1, 2, \ldots, n, \qquad (2\text{-}41)$$

where the right-hand-side is T_i^{in}, defined in equation (2-32). Equations (2-40) and (2-41) show that the incoming throughflow T_i^{in}, expressed as the sum of incoming intercompartmental flows f_{ij}, and the incoming boundary flow z_i, drive the future ($t_0 \le t < \infty$) behavior of the system. Further, the input–output dual perspective of environ analysis interprets intercompartmental flows f_{ij}, simultaneously as an outflow from j or an inflow to i. Taking f_{ij} in its output from j orientation, a corresponding flow intensity g can be defined as a function of the output throughflow of j:

$$g_{ij}(t) \equiv \frac{f_{ij}(t)}{T_j^{out}(t)}. \qquad (2\text{-}42)$$

Here, g_{ij} is the fraction (therefore dimensionless) of output throughflow at donor component j contributed to the focal component i. Rearranging (2-42) and substituting into (2-40) yields:

$$\frac{dx_i}{dt} = \sum_{\substack{j=1 \\ in}}^{n} g_{ij}(t)T_j^{out}(t) + z_i(t) - T_i^{out}(t), \quad i = 1, 2, \ldots, n, \quad t_0 \le t < \infty, \qquad (2\text{-}43)$$

and at steady-state,

$$T_i^{out} = \sum_{\substack{j=1 \\ in}}^{n} g_{ij}T_j^{out} + z_i, \qquad i = 1,2,\ldots,n. \qquad (2\text{-}44)$$

Development of input–output mathematics for the unsteady case depicted in equation (2-43) still remains to be done. Hippe’s (1983) successful time-dependent environ analysis for even a simple case illustrates how difficult this will be. His approach sidestepped equation (2-43) and developed a set of state equations from equation (2-44) (therefore, in the neighborhood of steady-state) which allowed time-dependent inflows, $z_i(t)$, to generate time-dependent environ dynamics.

For continuity and completeness, I'll complete this section with a review of NEA theory. Considering the steady-state case to aid subsequent algebra, equation (2-44) is written in matrix form for an n-component system as,

$$\mathbf{T}^{\text{out}}_{\text{n x 1}} = \mathbf{G}_{\text{n x n}}\mathbf{T}^{\text{out}}_{\text{n x 1}} + \mathbf{z}_{\text{n x 1}}. \tag{2-45}$$

Solve for $\mathbf{T}^{\text{out}}_{\text{n x 1}}$:

$$\mathbf{T}^{\text{out}}_{\text{n x 1}} = \left[\mathbf{I}_{\text{n x n}} - \mathbf{G}_{\text{n x n}}\right]^{-1}\mathbf{z}_{\text{n x 1}}, \tag{2-46}$$

where **I**, the identity matrix, is dimensionless. For convenience, the term $\left[\mathbf{I}_{\text{n x n}} - \mathbf{G}_{\text{n x n}}\right]^{-1}$ is redefined as,

$$\mathbf{N}_{\text{n x n}} \equiv \left[\mathbf{I}_{\text{n x n}} - \mathbf{G}_{\text{n x n}}\right]^{-1}, \tag{2-47}$$

such that equation (2-46) is generally written in the form,

$$\mathbf{T}^{\text{out}}_{\text{n x 1}} = \mathbf{N}_{\text{n x n}}\mathbf{z}_{\text{n x 1}}. \tag{2-48}$$

Hannon (1973), using existence conditions given by Ortega (1972), described necessary physical conditions assuring that **N** exists. Each irreducible block diagonal submatrix of **G** (presence of at least one non-zero element in all rows provides the condition of irreducibility) defined as $\hat{\mathbf{G}}$ with elements $\hat{g}_{ij}$ must meet the criterion, $\sum_{j}\hat{g}_{ij} \le 1$, for at least one row, i. Hannon demonstrated, using an early ecological energy model for Silver Springs, Florida (Odum, 1957), that inclusion of the sun as an energy input guaranteed this criterion in his analysis. Patten (1978a) later noted that at least one component in each submatrix of **N** generated by the block diagonalization procedure must receive input from the system's input environment from across the system boundary. This solidified the significance of the directional mapping in equation (2-48) of the boundary input vector **z**, from outside the environment, into system throughflows (e.g., Figure 2.4).

Hannon referred to the individual n_{ij} coefficients as structural elements of the ecosystem since **z** can be varied to produce changes in **T** without changing **N**. In this case **N** acts as a steady-state transfer function, taking boundary inputs to the system into their interior throughflows. However, it should be noted from the definitions of **N** (2-47), **G** (2-42), and **T** (2-41) that change in **z** also entails change in **N**. The mapping matrix **N** is known as a transitive closure or integral flow matrix because it accounts for all direct and indirect flows between each interior component (Patten, 1978a; Fath and Patten, 1999; Gattie et al., 2006a) over all paths of all lengths. **N** may be decomposed by path length m into an infinite series (Ore, 1962),

$$\mathbf{N} = [\mathbf{I} - \mathbf{G}]^{-1} = \mathbf{G}^0 + \mathbf{G}^1 + \mathbf{G}^2 + \mathbf{G}^3 +, \ldots, \mathbf{G}^m = \sum_{m=0}^{\infty} \mathbf{G}^m , \qquad (2\text{-}49)$$

which converges if and only if $0 \le g_{ij} \le 1$. This series partitions input-to-throughflow mapping coefficients into contributions from paths of each length m carrying substance from the system boundary to interior compartments. The ecological significance of equation (2-49) cannot be over stated. The convergent power series confers and confirms potentially infinite connectivity of indirect relationships between all compartments of a system with well-connected graphs of transactions. This is central to the premise that indirect effects dominate the activities of a system (Patten, 1983; Higashi and Patten, 1986; 1989). Moreover, output-oriented environs are implicit in the **N** matrix. Each component's efferent connectivity, in a multi-component system, subsequently maps this boundary flow time-forward through its output environ-oriented flows entailing all direct and indirect pathways, m, of all lengths (m → ∞).

2.3-5 AFFERENT FLOW EQUATIONS

The development of environ afferent or time-backward flow equations is similar in scope to the efferent flow equations and will be presented, in some cases, in an abbreviated format. The afferent flow equations are developed by combining the inbound oriented throughflow equation (2-32) for T_i^{in} with the conservation equation (2-30) to obtain:

$$\frac{dx_i}{dt} = T_i^{in}(t) \; - \sum_{\substack{h=1 \\ out}}^{n} f_{hi}(t) + y_i(t) \qquad i = 1, 2, \ldots, n\,, \quad -\infty < t \leq t_0, \qquad (2\text{-}50)$$

or for the steady-state case,

$$T_i^{in} = \sum_{\substack{h=1 \\ out}}^{n} f_{hi} + y_i\,, \qquad i = 1, 2, \ldots, n\,. \qquad (2\text{-}51)$$

The right hand side of equation (2-51) includes all outgoing flows from component i [definition of outgoing throughflow, T_i^{out}, reference equation (2-33)]. This includes outgoing boundary flow, y_i, and all intercompartmental flows, f_{hi}, to other components h (h = 1, 2,…, n). Equations (2-50) and (2-51) show that the outgoing throughflow, T_i^{out}, expressed as the sum of outgoing intercompartmental flows, f_{hi}, and boundary flow, y_i, are driven by the past ($-\infty < t \leq t_0$) behavior of the system. Further, each empirical flow, f_{hi}, can be considered an outflow from i and inflow to h. The nondimensional function g' is defined by orienting the perspective of f_{hi} as an inflow to h allowing f_{hi} to be expressed as a fraction of the input throughflows, T_h^{in} :

$$g'_{hi}(t) \equiv \frac{f_{hi}(t)}{T_h^{in}(t)}\;. \qquad (2\text{-}52)$$

Rearranging equation (2-52) and substituting into equation (2-50) yields:

$$\frac{dx_i}{dt} = T_i^{in}(t) - \sum_{\substack{h=1 \\ out}}^{n} g'_{hi}(t) T_h^{in}(t) + y_i(t), \qquad i = 1, 2, \ldots, n\,, \qquad -\infty < t \leq t_0, \qquad (2\text{-}53)$$

and for the steady-state case,

$$T_i^{in} = \sum_{\substack{h=1 \\ out}}^{n} g'_{hi} T_h^{in} + y_i, \quad i = 1, 2, ..., n. \tag{2-54}$$

To maintain continuity, I'll finish this section with a brief review of the remaining NEA theory development. Considering only the steady-state case, to aid subsequent algebra, equation (2-54) is written in matrix form as:

$$\mathbf{T}_{1\,x\,n}^{in} = \mathbf{T}_{1\,x\,n}^{in} \mathbf{G}'_{n\,x\,n} + \mathbf{y}_{1\,x\,n}. \tag{2-55}$$

Rearranging to solve for $\mathbf{T}_{1\,x\,n}^{in}$, yields,

$$\mathbf{T}_{1\,x\,n}^{in} = \mathbf{y}_{1\,x\,n} \left[\mathbf{I}_{n\,x\,n} - \mathbf{G}'_{n\,x\,n} \right]^{-1}, \tag{2-56}$$

where $\mathbf{I}$ is a nondimensional identity matrix. For convenience, the term $\left[\mathbf{I}_{n\,x\,n} - \mathbf{G}'_{n\,x\,n} \right]^{-1}$ is redefined as

$$\mathbf{N}'_{n\,x\,n} \equiv \left[\mathbf{I}_{n\,x\,n} - \mathbf{G}'_{n\,x\,n} \right]^{-1}, \tag{2-57}$$

such that equation (2-56) becomes:

$$\mathbf{T}_{1\,x\,n}^{in} = \mathbf{y}_{1\,x\,n} \mathbf{N}'_{n\,x\,n}. \tag{2-58}$$

Equation (2-56) corresponds to Leontief's (1936) original Input-Output Analysis relationship wherein economic activity upstream necessary to produce an industrial output, y, could be determined and evaluated. Patten's afferent input-oriented environs are implicit in the $\mathbf{N}'$ matrix of equation (2-58). Component outputs, $\mathbf{y}$, can be mapped backwards through these input environs containing all the direct and indirect pathways of all lengths [as $m \to \infty$; equation (2-59)], to incoming component throughflows, $\mathbf{T}^{in}$. Similar to equation (2-49), $\mathbf{N}'$ can also be written:

$$\mathbf{N}' = \left[\mathbf{I} - \mathbf{G}'\right]^{-1} = \mathbf{G}'^{\,0} + \mathbf{G}'^{\,1} + \mathbf{G}'^{\,2} + \mathbf{G}'^{\,3} +, \ldots, \mathbf{G}'^{\,m} = \sum_{m=0}^{\infty} \mathbf{G}'^{\,m} . \qquad (2\text{-}59)$$

The transitive closure matrix, $\mathbf{N}'$, is the summation of the infinite power series of $\mathbf{G}'$ which represents the partitioning coefficients for mapping boundary output, y_i, at any compartment, i, into flows to i from the remaining components in the system along all pathways of all lengths, m. Each component's afferent connectivity establishes first a reference to the outside system through the output boundary flow, **y**, and then subsequently maps this boundary flow backwards in time through its input-oriented environ flows over all direct and indirect pathways, m, of all lengths ($m \rightarrow \infty$).

2.3-6 PATHWAY ANALYSIS WITH THE ADJACENCY MATRIX

A simple binary matrix of ones and zeros is used to facilitate a discussion of a network model’s pathways (structure) and the corresponding concept of pathway proliferation. The appropriately titled adjacency matrix from mathematical graph theory, $\mathbf{A} = (a_{ij})$, sometimes called an interconnection matrix (Śiljak, 1991), represents a system’s direct component connectivity. Values of $a_{ij} = 1$ and $a_{ij} = 0$ signify flow and no flow connections, respectively, from component j to i. The adjacency matrix is therefore isomorphic to the corresponding flow matrix, $\mathbf{F} = (f_{ij})$. The adjacency matrix raised to the m'th power, $A^m = (a_{ij})^m$ (matrix multiplication not scalar powers), enumerates quantities of pathways of length m directed from each j to each i in the system. Specifically, the whole number in an i, j interstitial location of the matrix $(a_{ij})^m$ is the number of indirect pathways between components j and i of path length m where the i, j entry of $(a_{ij})^m$ is denoted $a_{ij}^{(m)}$ by standard notation. Values of $a_{ij}^{(m)}$ increase without bound as $m \rightarrow \infty$, signifying that the quantity of indirect pathways between j and i increase without bound as path length m increases. Pathway proliferation analysis systematically evaluates increasing powers of the adjacency matrix for trends with indirect connectivity between various components in a multi-component system.

The physical interpretation of the **A** matrix diagonal terms can be further considered given the clarity and rigidity of the conservation equations (2-30) as developed from the control volume model. Self-flows clearly exist as flows among other flows in the right-hand-side summation. However, in all models thus far in the NEA literature, the f_{ii} diagonal terms are zero or not specifically addressed since there are no observed self-flows in the empirical data. Correspondingly there is no self-connectivity ($a_{ii} = 0$). As such, pathway proliferation analysis of this type of system as $m \rightarrow \infty$ starts with zeros on the diagonal of the **A** matrix. Lacking the self-flow inclusion provided by equation (2-30), similar mathematical exercises introduced self-flow connections ($a_{ii} = 1$) to path length proliferation analysis by calling them node queues, storages, or time delays (for example, see Patten 1985). This methodology showed that quantities of paths, at higher path lengths, of these latter hypothetical exercises ($a_{ii} = 1$) proliferate faster as $m \rightarrow \infty$ than the corresponding path quantities observed in the empirically supported models ($a_{ii} = 0$). As it would be inappropriate to randomly place a one in any off diagonal location of the **A** matrix without a corresponding empirical flow f_{ij}, it is now felt that placing ones on the diagonal of the **A** matrix a_{ii}, without supporting empirical self-flows f_{ii}, should be avoided. For example, the common self-flow arcs (arrows) on the components of a multi-component network digraph is not supported by the existence of the respective corresponding empirical flows when $f_{ii} = 0$. If no supporting empirical data exist, from this point forward, leaving self-flow arcs off the typical digraph pictorial diagram is an appropriate practice to improve clarity and minimize confusion. However, they can certainly be added for discussion purposes. Additionally, it is now clear that the concept of storage or time delay, from the differential equation (2-30) perspective, has no unique relationship to the diagonal self-flows of the **F** and **A** matrices. The connectivity and flow from a component to itself, a_{ii} and f_{ii}, are only one flow relationship, among many, that a component has within the intercompartmental flow network. All of a component's flows (f_{ij} for i and j = 1, 2, …, n) with their isomorphic connectivities (a_{ij} for i and j = 1, 2, …, n) simply contribute to a component's accumulation as per equation (2-30) or storage turnovers as per the steady-state equations (2-62) and (2-84) discussed later.

2.4 EFFERENT STORAGE EQUATIONS

Similar to network throughflow analysis, storage analysis maps the boundary flows, **z** and **y**, forward or backward, respectively, through an environ network of component storages (Matis and Patten, 1981). The efferent or time-forward steady-state storage equations, as generated by boundary input **z**, are developed by first defining two terms. Each i'th component's steady-state donor specific total *turnover rate* coefficient (with dimensions of reciprocal time, T^{-1}) is defined as,

$$\tau_i^{-1} \equiv \frac{T_i^{out}}{x_i}, \quad i = 1, 2, ..., n, \tag{2-60}$$

where T_i^{out} is the total steady-state throughflow out of a donor component i and x_i is the total steady-state storage (dimensioned ML^{-2}) at donor component i. The inverse of τ_i^{-1} is the turnover time τ_i (with dimensions T) of the component's storage. Turnover rates can be partitioned by those intercompartmental flows, f_{ij}, which contribute to the total component turnover rate. As such, for a component j, the donor specific steady-state *partial turnover rate* is:

$$\tau_{ij}^{-1} \equiv \frac{f_{ij}}{x_j}, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, n. \qquad (2\text{-}61)$$

where f_{ij} is the intercompartmental flow from the donor component j to the receiver component i and x_j is the storage at donor component j. τ_{ij}^{-1} represents the partial turnover rate the intercompartmental flow from j specifically oriented towards i, f_{ij}, contributes to the overall storage turnover of component j. τ_{ij}^{-1} is often called the "rate coefficient" of component j oriented towards component i whereas τ_i^{-1} is simply called the turnover rate of component i. The individual partitioned turnover rates combine to equal the total component turnover rate,

$$\tau_j^{-1} = \sum_{i=1}^{n} \tau_{ij}^{-1} + \frac{y_j}{x_j}, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, n. \qquad (2\text{-}62)$$

The total turnover rate of equation (2-60) is expressed in terms of its donor specific partitioned turnover rates from both intercompartmental and boundary flows for a specified component j. The reciprocal turnover time, τ_{ij}, represents component j's storage turnover time attributed specifically to the partitioned intercompartmental flow directed towards component i.

Since flow from a component back to the same component, f_{ii}, is usually zero or assumed zero, τ_{ii}^{-1} is also usually zero. Recall, this also requires that zeros are placed on the diagonal in the adjacency matrix, $a_{ii} = 0$ for i = 1, 2, 3, …, n. If there is no intercompartmental flow across the component control surface and back to itself during one time step, $f_{ii} = 0$, there can be no partial contribution towards the component storage turnover rate, τ_{ii}^{-1}. Additionally, the partial storage turnover time, τ_{ii}, attributed to a zero flow from a component to itself, justifiably, remains undefined (i.e., $x_j / f_{ii} = x_j / 0$ = undefined). If no self-flow exists, $f_{ii} = 0$, the partial turnover rate specifically attributed to this flow is zero, $\tau_{ii}^{-1} = 0$, and the time this flow would require to turn over the component storage, x_i, cannot be calculated and τ_{ii} is undefined.

Substituting the defined total [equation (2-60)] and partial [equation (2-61)] donor-storage turnover rates into the efferent flow equation (2-41) reformulates the flows of the linear throughflow model into fractions of the donor compartments. First, rearranging equation (2-60) to express the throughflow out of component i, T_i^{out}, as a product of the component's turnover rate, τ_i^{-1}, and storage, x_i,

$$T_i^{out} = \tau_i^{-1} x_i, \quad i = 1, 2, \ldots, n, \tag{2-63}$$

and similarly rearranging equation (2-61),

$$f_{ij} = \tau_{ij}^{-1} x_j, \quad j = 1, 2, \ldots, n, \;\; i = 1, 2, \ldots, n, \tag{2-64}$$

equations (2-63) and (2-64) are then substituted into equation (2-41) to yield,

$$\tau_i^{-1} x_i = \sum_{j=1}^{n} \tau_{ij}^{-1} x_j + z_i, \quad i = 1, 2, \ldots, n\,. \tag{2-65}$$

This equation in matrix form for an n-component system is,

$$\begin{bmatrix} \tau_1^{-1} & 0 & \cdots & 0 \\ 0 & \tau_2^{-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tau_n^{-1} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \tau_{11}^{-1} & \tau_{12}^{-1} & \cdots & \tau_{1n}^{-1} \\ \tau_{21}^{-1} & \tau_{22}^{-1} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \tau_{n1}^{-1} & \cdots & \cdots & \tau_{nn}^{-1} \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}. \tag{2-66}$$

Here the partial turnover matrix results and subsequent interpretations follow directly from the empirically observed data in the observed flows matrix, **F**. The matrices $\boldsymbol{\tau}^{-1}$, **F**, and **A** are all isomorphic. Combine similar terms,

$$\begin{bmatrix} \left(\tau_1^{-1} - \tau_{11}^{-1}\right) & -\tau_{12}^{-1} & \cdots & -\tau_{1n}^{-1} \\ -\tau_{21}^{-1} & \left(\tau_2^{-1} - \tau_{22}^{-1}\right) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ -\tau_{n1}^{-1} & \cdots & \cdots & \left(\tau_n^{-1} - \tau_{nn}^{-1}\right) \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}, \tag{2-67}$$

and then finally define the composite matrix **C** which is comprised of the donor controlled, output-oriented, partial turnover rates (rate coefficients) combined with the total turnover rates on the diagonals as,

$$[C] \equiv \begin{bmatrix} -\left(\tau_1^{-1} - \tau_{11}^{-1}\right) & \tau_{12}^{-1} & \cdots & \tau_{1n}^{-1} \\ \tau_{21}^{-1} & -\left(\tau_2^{-1} - \tau_{22}^{-1}\right) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \tau_{n1}^{-1} & \cdots & \cdots & -\left(\tau_n^{-1} - \tau_{nn}^{-1}\right) \end{bmatrix}. \tag{2-68}$$

Recall that the f_{ii} terms are usually zero, as such, the corresponding rate coefficients τ_{ii}^{-1} are also usually zero. The diagonals would then solely be defined by the component throughflow as shown in equation (2-60). However, in some future model if f_{ii} is not zero, the definition of **C** in equation (2-68) is still adequate to carry this empirical data. Substitute the definition of the donor controlled turnover rate matrix **C** into equation (2-67) and multiply by a ratio of a delta time step equal to one ($\Delta t/\Delta t$) to yield,

$$-\mathbf{C}_{n \times n}\left(\frac{\Delta t}{\Delta t}\right)\mathbf{x}_{n \times 1} = \mathbf{z}_{n \times 1}. \tag{2-69}$$

Pursuing a convergent series format similar to equations (2-46) and (2-49), first a non-dimensional **P** is defined as,

$$\mathbf{P}_{n\times n} \equiv \mathbf{I}_{n\times n} + \mathbf{C}_{n\times n}\Delta t, \tag{2-70}$$

where **I** is the identity matrix. By definition, **P** is constructed of both **C** and **I**. The **P** matrix is a uniquely revealing construct of information regarding the storage turnover relationships between components. The off diagonal elements of the **P** matrix, p_{ij}, represent fractions of storage x_j in component j that will be transferred to storage at i, x_i, during the interval t to t + Δt. The diagonal terms, p_{ii}, are different, due to the algebraic construction that generates the composite **P** matrix. The diagonal terms identify the fraction of the original quantity of mass or energy remaining in each component after the discrete time step Δt. Recall that the system is assumed to be at steady-state. As such, as mass or energy leave a component's storage, an equal amount replaces it. Total storage content, x_i, remains constant.

The time interval introduced in equations (2-69) and (2-70) has one constraint,

$$\frac{\Delta t\, T_i^{out}}{x_i} \le 1 \qquad i = 1, 2, \ldots, n\,, \tag{2-71}$$

assuring that the selected discrete time step does not allow more than the total stock stored in each component to be moved during one time step. As such, the necessary constraint for an appropriate time step is established as,

$$\Delta t \le \frac{x_i}{T_i^{out}} \qquad i = 1, 2, \ldots, n\,. \tag{2-72}$$

As a method of consistent practice, the Δt, for a given multi-component system's storage environ analysis, is sized such that the highest component turnover rate, τ_{max}^{-1}, multiplied by one time step, Δt, equals one:

$$\Delta t\, \tau_{max}^{-1} = \Delta t \left(\frac{T_i^{out}}{x_i} \right)_{max} = 1 \qquad i = 1, 2, \ldots, n\,, \tag{2-73}$$

where the time interval for the entire storage analysis is then,

$$\Delta t = \left(\frac{x_i}{T_i^{out}} \right)_{min} \qquad i = 1, 2, \ldots, n\,. \tag{2-74}$$

Considering the pursuit of a converging series format and with the appropriate constraints applied to Δt, first rearrange and then rewrite equation (2-69) by adding and subtracting the identity matrix:

$$\left[-\mathbf{C}_{n\times n} \Delta t + \mathbf{I}_{n\times n} - \mathbf{I}_{n\times n} \right] \underset{n\times 1}{\mathbf{x}} = \Delta t \underset{n\times 1}{\mathbf{z}}\,, \tag{2-75}$$

and then substitute equation (2-70) for **P**;

$$\left[\mathbf{I}_{n\times n}-\mathbf{P}_{n\times n}\right]\underset{n\times 1}{\mathbf{x}}=\Delta t\,\underset{n\times 1}{\mathbf{z}}\,. \tag{2-76}$$

Solve for **x**,

$$\underset{n\times 1}{\mathbf{x}}=\Delta t\left[\mathbf{I}_{n\times n}-\mathbf{P}_{n\times n}\right]^{-1}\underset{n\times 1}{\mathbf{z}}\,; \tag{2-77}$$

and for convenience define **Q**;

$$\mathbf{Q}\equiv[\mathbf{I}-\mathbf{P}]^{-1}\,, \tag{2-78}$$

allowing equation (2-73) to be written as,

$$\underset{n\times 1}{\mathbf{x}}=\Delta t\,\underset{n\times n}{\mathbf{Q}}\;\underset{n\times 1}{\mathbf{z}}\,, \tag{2-79}$$

for all $\Delta t \leq T_i/x_i$ per equation (2-72). **Q** represents the transitive closure or integral storage specific flow matrix [similar to N in equations (2-47) through (2-49)] for discrete time inputs, $\mathbf{z}\Delta t$. **Q** accounts for all direct and indirect storages as it can alternately be written as:

$$\mathbf{Q}=[\mathbf{I}-\mathbf{P}]^{-1}=\mathbf{P}^0+\mathbf{P}^1+\mathbf{P}^2+\mathbf{P}^3+,\ldots,\mathbf{P}^m=\sum_{m=0}^{\infty}\mathbf{P}^m, \tag{2-80}$$

where the term $[\mathbf{I}-\mathbf{P}]^{-1}$ is expanded to the converging series $\mathbf{P}^0 + \mathbf{P}^1 + \mathbf{P}^2 + \mathbf{P}^3 + ,\ldots, \mathbf{P}^m$ for $m \to \infty$ if the condition $0 \leq p_{ij} \leq 1$ exists (for i, j = 1, 2, …, n), which is assured if the time interval criterion of equation (2-72) is maintained. The storage-specific series (2-80) confers and confirms an infinite connectivity (in the limit) of indirect relationships between the storages of all system components. Patten's output-oriented storage environs are therefore implicit in the **Q** matrix. Each component's efferent connectivity first establishes a reference to the outside system through the input boundary flow, **z**, and then maps this boundary flow time-forward through the output-oriented environ network of all direct and indirect connections as $m \to \infty$. For continuous-time inputs **z**, the transitive closure matrix **Q** and the discrete time step Δt are combined to form the **S** matrix ($\mathbf{S} = \mathbf{Q}\Delta t$) such that equation (2-79) becomes,

$$\underset{n\times 1}{\mathbf{x}}=\underset{n\times n}{\mathbf{S}}\;\underset{n\times 1}{\mathbf{z}}\,. \tag{2-81}$$

S is the transitive closure matrix, comprised of integral turnover times, τ_{ij}, that map time-continuous inputs **z** into component storages, **x**. Comparing equations (2-81) and (2-69) shows that $\mathbf{S} = -\mathbf{C}^{-1}$.

2.5 AFFERENT STORAGE EQUATIONS

Afferent steady-state storage equations tracing flows to storage, **x**, backward from output, **y** are developed in the same way as the efferent case. A component's recipient-specific turnover rate at steady-state is defined as,

$$\tau_i'^{-1} \equiv \frac{T_i^{in}}{x_i}, \quad i = 1, 2, ..., n, \tag{2-82}$$

where T_i^{in} is the total steady-state throughflow into a recipient component i and x_i is storage (units ML^{-2}) at i. The recipient-specific partial turnover rate is defined as,

$$\tau_{hi}'^{-1} \equiv \frac{f_{hi}}{x_h}, \quad h = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, n, \tag{2-83}$$

where f_{hi} is the intercompartmental flow to the receiver component h from the donor component i and x_h is the storage at receiver component h. The reciprocal partitioned turnover time, τ'_{hi}, represents component h's storage turnover time attributed specifically to the partitioned intercompartmental flow oriented from i. The individual partitioned turnover rates combine to equal the total component-specific turnover rate,

$$\tau_h^{-1} = \sum_{i=1}^{n} \tau_{hi}'^{-1} + \frac{z_h}{x_h}, \quad h = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, n, \tag{2-84}$$

where, for a specified component h, equation (2-81) is expressed in terms of its receiver-specific partitioned turnover rates from both intercompartmental and boundary flows.

Substituting the defined total [equation (2-81)] and partial [equation (2-83)] receiver-storage turnover rates into the afferent flow equation (2-51) reformulates the flows of the linear throughflow model into fractions of the recipient component's storage. First, rearrange equation (2-82) to express the throughflow into component i, T_i^{in}, as a product of the component's turnover rate, τ_i^{-1}, and storage, x_i:

$$T_i^{in} = \tau_i'^{-1} x_i, \quad i = 1, 2, \ldots, n, \tag{2-85}$$

where $\tau_i' = \tau_i$ at steady state conditions. Similarly rearranging equation (2-83),

$$f_{hi} = \tau_{hi}'^{-1} x_h, \quad h = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, n, \tag{2-86}$$

equations (2-85) and (2-86) are then substituted into equation (2-51) to yield:

$$\tau_i^{-1} x_i = \sum_{h=1}^{n} \tau_{hi}'^{-1} x_h + y_i, \quad i = 1, 2, \ldots, n. \tag{2-87}$$

Equation (2-87) can be expanded to the following n-component matrix form:

$$[x_1 \ \cdots \ x_n] * \begin{bmatrix} \tau_1^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \tau_n^{-1} \end{bmatrix} = [x_1 \ \cdots \ x_n] * \begin{bmatrix} \tau_{11}'^{-1} & \cdots & \tau_{1n}'^{-1} \\ \vdots & \ddots & \vdots \\ \tau_{n1}'^{-1} & \cdots & \tau_{nn}'^{-1} \end{bmatrix} + [y_1 \ \cdots \ y_n], \tag{2-88}$$

where similar terms can be combined to achieve,

$$[x_1 \ x_2 \ \cdots \ x_n] * \begin{bmatrix} \left(\tau_1^{-1} - \tau_{11}'^{-1}\right) & -\tau_{12}'^{-1} & \cdots & -\tau_{1n}'^{-1} \\ -\tau_{21}'^{-1} & \left(\tau_2^{-1} - \tau_{22}'^{-1}\right) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ -\tau_{n1}'^{-1} & \cdots & \cdots & \left(\tau_n^{-1} - \tau_{nn}'^{-1}\right) \end{bmatrix} = [y_1 \ y_2 \ \cdots \ y_n]. \tag{2-89}$$

Comprised of the recipient controlled, input-oriented, partial turnover rates (rate coefficients) combined with total turnover rates on the diagonals, the **C'** matrix is defined as:

$$[C'] \equiv \begin{bmatrix} -\left(\tau_1^{-1} - \tau_{11}'^{-1}\right) & \tau_{12}'^{-1} & \cdots & \tau_{1n}'^{-1} \\ \tau_{21}'^{-1} & -\left(\tau_2^{-1} - \tau_{22}'^{-1}\right) & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ \tau_{n1}'^{-1} & \cdots & \cdots & -\left(\tau_n^{-1} - \tau_{nn}'^{-1}\right) \end{bmatrix}. \tag{2-90}$$

Substitute the definition of the receiver controlled turnover rate matrix **C**' into equation (2-89) and multiply by a ratio of a delta time step equal to one (Δt/Δt) to yield,

$$\underset{1\times n}{\mathbf{x}}\left[-\mathbf{C}'\right]_{n\times n}\left(\frac{\Delta t}{\Delta t}\right)=\underset{1\times n}{\mathbf{y}}\,. \tag{2-91}$$

Again, pursuing a converging series format similar to equations (2-57) and (2-59), first a non-dimensional **P**' is defined:

$$\mathbf{P}'_{n\times n}\equiv\mathbf{I}_{n\times n}+\mathbf{C}'_{n\times n}\Delta t\,, \tag{2-92}$$

where **I** is the identity matrix. The time interval constraint is

$$\frac{\Delta t\,T_i^{in}}{x_i}\leq 1 \qquad i=1,2,\ldots,n\,, \tag{2-93}$$

assuring that the turnover rates, τ_i^{-1}, over one time step are not larger than the available stocks x_i. The necessary constraint becomes:

$$\Delta t\leq\frac{x_i}{T_i^{in}} \qquad i=1,2,\ldots,n\,. \tag{2-94}$$

Δt is sized similar to the development of equations (2-73) and (2-74) where:

$$\Delta t\,\tau_{max}^{-1}=\Delta t\left(\frac{T_i^{in}}{x_i}\right)_{max}=1 \qquad i=1,2,\ldots,n\,. \tag{2-95}$$

The time interval is then

$$\Delta t=\left(\frac{x_i}{T_i^{in}}\right)_{min}, \qquad i=1,2,\ldots,n\,. \tag{2-96}$$

In a steady-state system, $T_i^{in}=T_i^{out}$, whereupon equations (2-74) and (2-96) calculate the same Δt. Pursuing a converging series format, rearrange and rewrite equation (2-91) by adding and subtracting the identity matrix,

$$\underset{1\times n}{\mathbf{x}}\left[-\mathbf{C}'_{n\times n}\Delta t+\mathbf{I}_{n\times n}-\mathbf{I}_{n\times n}\right]=\Delta t\,\underset{1\times n}{\mathbf{y}}\,, \tag{2-97}$$

then substitute **P** [equation (2-92)],

$$\underset{1\times n}{\mathbf{x}}\left[\mathbf{I}_{n\times n}-\mathbf{P}'_{n\times n}\right]=\Delta t\,\underset{1\times n}{\mathbf{y}}, \tag{2-98}$$

and solve for **x**,

$$\underset{1\times n}{\mathbf{x}}=\underset{1\times n}{\mathbf{y}}\left[\mathbf{I}_{n\times n}-\mathbf{P}'_{n\times n}\right]^{-1}\Delta t. \tag{2-99}$$

For convenience, **Q'** is defined as,

$$\mathbf{Q}' \equiv [\mathbf{I}-\mathbf{P}']^{-1}, \tag{2-100}$$

allowing equation (2-73) to now be written as,

$$\underset{1\times n}{\mathbf{x}}=\underset{1\times n}{\mathbf{y}}\mathbf{Q}'\Delta t, \tag{2-101}$$

for all $\Delta t \leq T_i/x_i$ per equation (2-94). **Q'** represents the transitive closure or integral storage specific matrix [similar to **N'** in equations (2-57) through (2-59)] for discrete time inputs, $\mathbf{y}\Delta t$, accounting for all direct and indirect storages. **Q'** can alternatively be written as,

$$\mathbf{Q}'=\left[\mathbf{I}-\mathbf{P}'\right]^{-1}=\mathbf{P}'^{0}+\mathbf{P}'^{1}+\mathbf{P}'^{2}+\mathbf{P}'^{3}+,\ldots,\mathbf{P}'^{m}=\sum_{m=0}^{\infty}\mathbf{P}'^{m}, \tag{2-102}$$

where the term $[\mathbf{I}-\mathbf{P}']^{-1}$ is expanded to the convergent series $\mathbf{P}'^{0}+\mathbf{P}'^{1}+\mathbf{P}'^{2}+\mathbf{P}'^{3}+,\ldots,\mathbf{P}'^{m}$ for $m \to \infty$ if the condition $0 \leq p'_{ij} \leq 1$ exists, which is assured if the time interval criterion per equation (2-94) is maintained. The input-oriented storage environs are implicit in the **Q'** matrix. Each component's afferent connectivity, in a multi-component system, is first referenced to the outside system through the output boundary flow, **y**, and then subsequently mapped time-backwards from this boundary flow through its input-environ network of component storages through all direct and indirect connections as $m \to \infty$. For continuous-time outputs, **y**, the transitive closure matrix, **Q'**, and the discrete time step, Δt, are combined to form the **S'** matrix, where equation (2-101) becomes,

$$\underset{1\times n}{\mathbf{x}}=\underset{1\times n}{\mathbf{y}}\underset{n\times n}{\mathbf{S}'}. \tag{2-103}$$

S' is the transitive closure matrix for mapping time-continuous outputs, **y**, from component storages, **x**. Comparing equations (2-103) and (2-91) shows that $\mathbf{S'} = -\mathbf{C'}^{-1}$.

2.6 THE STEADY-STATE RELATIONSHIP BETWEEN STORAGE AND THROUGHFLOW

In a steady-state system, the turnover rates, τ_i^{-1}, and their reciprocal times, τ_i, establish the relationship between the integral storage, **Q** and **Q'**, and throughflow, **N** and **N'**, matrices. Equation (2-79) written in scalar form,

$$x_i = \Delta t \, q_{ij} \, z_j, \qquad (2\text{-}104)$$

and then multiplied by the turnover rate ratio $\frac{T_i}{x_i}$, yields:

$$x_i \left(\frac{T_i}{x_i} \right) = \Delta t \, q_{ij} \, z_j \left(\frac{T_i}{x_i} \right). \qquad (2\text{-}105)$$

Rearranged and substituting $\tau_i^{-1} = \frac{T_i}{x_i}$,

$$T_i = \tau_i^{-1} \, q_{ij} \, \Delta t \, z_j. \qquad (2\text{-}106)$$

Comparing this to the scalar form of equation (2-48) shows that for the efferent orientation that,

$$n_{ij} = \tau_i^{-1} q_{ij} \Delta t, \qquad (2\text{-}107)$$

or reciprocally,

$$n_{ij} \, \tau_i = q_{ij} \, \Delta t. \qquad (2\text{-}108)$$

Similarly, for the afferent orientation,

$$n'_{ij} = \tau_j^{-1} q'_{ij} \Delta t, \qquad (2\text{-}109)$$

and,

$$n'_{ij} \, \tau_j = q'_{ij} \, \Delta t. \qquad (2\text{-}110)$$

As evident in the original substitutions of equations (2-65) and (2-87), component turnover times or rates establish the direct relationship between the throughflow, **N** and **N'**, and storage, **Q** and **Q'**, integral matrices and their corresponding maps (convergent series) of direct and indirect connectivity at all path lengths.

2.7 EXAMPLE: STEADY-STATE NITROGEN CYCLING IN THE NEUSE RIVER ESTUARY

An average steady-state nitrogen cycle of the Neuse River estuary, North Carolina, (Christian and Thomas, 2000) was used for the NEA throughflow and storage analyses. This dynamic steady-state model represents an average of 16 consecutive seasons from spring 1985 through winter 1989 (Christian and Thomas, 2003) monitored as a part of a Neuse River Modeling and Monitoring program (ModMon). This program (a consortium of academia, government, and industry) was developed to improve water quality by reducing nitrogen loading to a eutrophic ecosystem. Christian and Thomas (2000, 2003) employed ecological network analysis (ENA) to specifically understand the relationship between varying nitrogen inputs to the system and the respective nitrogen recycling patterns and rates. They concluded: (1) Recycling of nitrogen as measured by the Finn cycling index (Finn, 1976) was high, averaging 89%; (2) Freshwater residence time was long, averaging 51 days; (3) The rate of nitrogen loading was a small fraction of the total microbial processing of nitrogen; and (4) The controls on primary production (an area of focus in their study) tended to be associated with conditions in the estuary rather than import. Processes within the Sediment component, including denitrification, benthic filter feeding, and burial, were considered a weakness in model construction.

The Neuse River receives water from a 16,000 km^2 watershed which together with the smaller Trent River (9% of Neuse and Trent combined) empties into the 400 km^2 Neuse River estuary, ultimately emptying into Pamlico Sound. Many studies focusing on nutrient cycling and cyanobacterial blooms in this area have been conducted over the last 30 years (Boyer et al., 1988; Christian et al., 1984, 1989, 1991; Hobbie and Smith, 1975; Paerl, 1987; Stanley, 1983, 1988). Due to the size of Pamlico Sound which separates river flow from the Atlantic Ocean, residence times of freshwater in the sound approach one year and salinities are one-half to two-thirds that of seawater salinities (Giese et al., 1979). Also, as such, astronomical tides are insignificant and wind tides coupled with river discharge fluctuations dominate water level fluctuations. The model made no distinction between molecular N or gaseous end products of denitrification. All inputs are dominated by fluctuating riverine loading with DON, NO_x, and NH_4 receiving input from precipitation and PN-Heterotrophs, including some immigration. All outputs except from Sediment were discharges to the downstream Pamlico Sound or N_2 from denitrification, reflecting respiratory loss. The output from Sediment (top one centimeter) was considered burial beyond the modeled sediment thickness. PN-Heterotrophs experienced some emigration.

The model includes seven components: phytoplankton particulate nitrogen, PN-Phyto-1; heterotroph particulate nitrogen, PN-Hetero-2; sediment particulate nitrogen, Sediment-3; dissolved organic nitrogen, DON-4; nitrate and nitrites, NO_x-5; ammonium, NH_4-6; and abiotic particulate nitrogen, PN-Abiotic-7. Table 2.1 lists the boundary flows, standing stocks, and inter-component flows for the averaged data set. [Table 2.1 here] Some portions of the model were directly sampled and measured while others were interpolated both from literature (e.g., data from similar estuaries) and subsequent mass balance. A digraph of model connectivity is shown in Figure 2.5. [Figure 2.5 here]

2.7-1 NEUSE RIVER ESTUARY NITROGEN CONSERVATION EQUATIONS

Establish the control surfaces around the control volumes of each model component as illustrated by the PN-Phyto-1 component in Figure 2.5. The nitrogen mass balance of the steady-state Neuse River estuary model can be represented by equation (2-19) modified slightly by dividing through by A^2, to allow for the dimensional units of the Neuse River estuary data:

$$\sum_{\substack{\text{in} \\ \text{CS}}} \frac{\dot{m}}{A^2} - \sum_{\substack{\text{out} \\ \text{CS}}} \frac{\dot{m}}{A^2} = 0 . \tag{2-19a}$$

The dimensions of $\dot{m}$ are $M \cdot T^{-1}$. As such, equation (2-19a) with units of $M\,A^{-2}\,T^{-1}$, represents the Neuse River estuary model's dimensional units of (mmol - N)/season collected on a m^2 basis. Considering the mass flows which cross the CS, the mass balance for PN-Phyto-1 can be written as,

$$(f_{21} + f_{31} + f_{41} + f_{71} + y_1) - (f_{14} + f_{15} + f_{16} + z_1) = 0. \tag{2-19b}$$

Assuming similar CV's and CS's for the remaining components in the model, the mass balances are,

$$(f_{32} + f_{42} + f_{62} + f_{72} + y_2) - (f_{21} + f_{23} + f_{24} + f_{25} + f_{26} + f_{27} + z_2) = 0, \tag{2-19c}$$

$$(f_{23} + f_{53} + f_{63} + y_3) - (f_{31} + f_{32} + f_{35} + f_{36} + f_{37} + z_3) = 0, \tag{2-19d}$$

$$(f_{14} + f_{24} + y_4) - (f_{41} + f_{42} + z_4) = 0, \tag{2-19e}$$

$$(f_{15} + f_{25} + f_{35} + y_5) - (f_{53} + f_{56} + z_5) = 0, \tag{2-19f}$$

$$(f_{16} + f_{26} + f_{36} + f_{56} + y_6) - (f_{62} + f_{63} + z_6) = 0, \tag{2-19g}$$

$$(f_{27} + f_{37} + y_7) - (f_{71} + f_{72} + z_7) = 0. \tag{2-19h}$$

2.7-2 NEUSE RIVER ESTUARY NITROGEN THROUGHFLOW EQUATIONS

Its fairly straight forward to calculate the throughflows, as shown in Table 2.1, associated with the seven components in the Neuse River estuary model. Using equations (2-32) and (2-33) for component (1), PN-Phyto, the calculations are:

$$\begin{aligned} T_1^{in} &= z_1 + f_{11} + f_{12} + f_{13} + f_{14} + f_{15} + f_{16} + f_{17} = \\ &16 + 0 + 0 + 0 + 1{,}176 + 918 + 4{,}816 + 0 = 6{,}926 \frac{\text{mmol} - \text{N}}{\text{m}^2 \cdot \text{season}} \end{aligned} \tag{2-32a}$$

$$\begin{aligned} T_1^{out} &= y_1 + f_{11} + f_{21} + f_{31} + f_{41} + f_{51} + f_{61} + f_{71} = \\ &37 + 0 + 4{,}859 + 611 + 1{,}363 + 0 + 0 + 57 = 6{,}927 \frac{\text{mmol} - \text{N}}{\text{m}^2 \cdot \text{season}} \end{aligned} \tag{2-33a}$$

As these equations illustrate, empirical data almost never balance identically; the imbalance in this model is, however, within the 5% limitation suggested by Kay et al. (1976). Although the sensitivity of results mass-balance variation remains an area for further study, most tests of output sensitivity indicate small output variation associated with throughflow variation (Gattie et al., 2006a). The input and output throughflows [mmol–N/(m^2 · season)] at each node are:

$$T^{in} = \begin{bmatrix} 6{,}926 \\ 15{,}013 \\ 1{,}255 \\ 2{,}688 \\ 1{,}579 \\ 11{,}955 \\ 2{,}101 \end{bmatrix}, \quad (2\text{-}32b) \qquad T^{out} = \begin{bmatrix} 6{,}927 \\ 15{,}005 \\ 1{,}255 \\ 2{,}688 \\ 1{,}578 \\ 11{,}955 \\ 2{,}099 \end{bmatrix}. \quad (2\text{-}33b)$$

2.7-3 NEUSE RIVER ESTUARY NITROGEN EFFERENT FLOW EQUATIONS

Moving towards a matrix algebra expression of equation (2-45), inter-compartmental flows are organized in compact form as the **F** matrix,

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 & 1{,}176 & 918 & 4{,}816 & 0 \\ 4{,}859 & 0 & 555 & 1{,}388 & 583 & 5{,}999 & 1{,}605 \\ 611 & 7 & 0 & 0 & 66 & 80 & 463 \\ 1{,}363 & 1{,}138 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 123 & 0 & 0 & 1{,}037 & 0 \\ 0 & 11{,}714 & 159 & 0 & 0 & 0 & 0 \\ 57 & 2{,}005 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{\mathrm{mm-N}}{\mathrm{m}^2 \cdot \mathrm{season}}, \tag{2-111}$$

oriented (as in Table 2.1) from columns to rows. The corresponding nondimensional **G** matrix, equation (2-42), is calculated by dividing each column by its respective throughflow,

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & .438 & .581 & .403 & 0 \\ .702 & 0 & .442 & .516 & .369 & .502 & .764 \\ .088 & .000 & 0 & 0 & .042 & .007 & .220 \\ .197 & .076 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .098 & 0 & 0 & .087 & 0 \\ 0 & .780 & .127 & 0 & 0 & 0 & 0 \\ .008 & .134 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2-112}$$

When the input and output throughflows are not equal as shown in equations (2-32a) and (2-32b), by convention, the output throughflows [equation (2-32b)] are used to formulate the **G** matrix. The **G** matrix simply enables the inter-component empirical flows of equation (2-111) to be represented as fractions of the donor component throughflows. For example, considering g_{21}, 70.2% of the nitrogen throughflow output from PN-Phyto-1, flows to PN-Hetero-2. Notice the diagonals are zero. Zero percent of the nitrogen throughflow output, over a given time interval, flows from a component out across its control surface back into the same component across the same control surface. The **G** matrix, often referenced as the donor throughflow intensity matrix, provides a snapshot, over one time step, of the inter-component throughflow intensities. For example, considering the second row of the **G** matrix, every component allocates a significant portion of output throughflow to PN-Hetero-2 (from 36.9% to 76.4%).

The dimensionless integral **N** matrix is calculated with either equation (2-47) or (2-49):

$$\mathbf{N} = \begin{bmatrix} 9.40 & 8.73 & 5.87 & 8.62 & 8.93 & 8.98 & 7.96 \\ 19.16 & 20.15 & 13.27 & 18.79 & 19.13 & 19.58 & 18.32 \\ 1.58 & 1.56 & 2.04 & 1.50 & 1.58 & 1.57 & 1.64 \\ 3.30 & 3.24 & 2.16 & 4.12 & 3.21 & 3.25 & 2.95 \\ 1.47 & 1.53 & 1.12 & 1.43 & 2.47 & 1.58 & 1.42 \\ 15.15 & 15.92 & 10.61 & 14.85 & 15.13 & 16.48 & 14.50 \\ 2.64 & 2.76 & 1.82 & 2.58 & 2.63 & 2.69 & 3.51 \end{bmatrix}. \qquad (2\text{-}113)$$

Barber et al., (1979) provide three separate interpretations of equations (2-48) and (2-113). Consider interstitial element $n_{25} = 19.13$. This is the throughflow-out of component 2 generated by one unit of boundary input to component 5. It is also the throughflow from component 5 to component 2, over all direct and indirect paths, due to one unit of input into component 5. Thirdly, it is also the number of times a unit of throughflow introduced at component 5 will pass through component 2 before leaving the system. The transitive closure matrix **N** relates the system-level boundary input vector, **z**, over all direct and indirect pathways to the compartmental throughflow vector $\mathbf{T}^{\mathbf{out}}$.

PN-Hetero-2 and NH_4-6 are consistently the largest purveyors of nitrogen previously input across the system boundary at all components. For example, a unit of nitrogen input at NO_x-5, will pass through PN-Hetero-2, 19.13 times before it leaves the system. In total, considering the second row of the **N** matrix, a unit of nitrogen input at components 1 through 7, will pass through PN-Hetero-2, an average of 18.34 times ranging from 18.32 at PN-Abiotic-7 to 20.15 at PN-Hetero-2. Similarly, considering the sixth row of the **N** matrix, a unit of nitrogen input at components 1 through 7, will pass through NH_4-6 an average of 14.66 times ranging from 10.61 at Sediment-3 to 16.48 at NH_4-6. PN-Hetero and NH_4 both play large rolls in the cycling of nitrogen in the output environs of all seven components.

Sediment-3 and NO_x-5 are consistently the smallest purveyors of nitrogen previously input across the system boundary at all components. Considering the third row of the **N** matrix, a unit of nitrogen input at components 1 through 7, will only pass through Sediment-3 an average of 1.64 times ranging from 1.50 at DON-4 to 2.04 at Sediment-3. Similarly, considering the fifth row of the **N** matrix, a unit of nitrogen input at components 1 through 7 will only pass through NO_x-5 an average of 1.57 times ranging from 1.12 at Sediment-3 to 2.47 at NO_x-5. Sediment and NO_x both play small rolls in the cycling of nitrogen within the output environs of all seven components.

2.7-4 NEUSE RIVER ESTUARY NITROGEN AFFERENT FLOW EQUATIONS

The nondimensional **G'** matrix, equation (2-52), is calculated by dividing each row by its respective throughflow,

$$\mathbf{G}' = \begin{bmatrix} 0 & 0 & 0 & .170 & .132 & .695 & 0 \\ .324 & .000 & .037 & .092 & .039 & .400 & .107 \\ .487 & .006 & 0 & 0 & .053 & .064 & .369 \\ .507 & .423 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .078 & 0 & 0 & .657 & 0 \\ 0 & .980 & .013 & 0 & 0 & 0 & 0 \\ .027 & .954 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2-114}$$

This matrix represents the inter-component flows of equation (2-111) expressed as fractions of recipient component throughflows. For example, considering g'_{21}, 32.4% of the nitrogen throughflow input to PN-Hetero-2, comes from PN-Phyto-1. The zero diagonal elements again indicate that none of the recipient nitrogen throughflow of each component, over a given time interval, came to the component from across its control surface having emanated originally from the same component across the same control surface. The **G'** matrix provides a snapshot, over one time step, of the inter-component recipient throughflow intensities. For example, considering the fourth column of the **G'** matrix, only two components have a portion of their input throughflows generated from DON-4, 17.0% of PN-Phyto-1 and 9.2% of PN-Hetero-2. The remaining components, 3 through 7, have no portions of their input throughflows (g'_{34} through g'_{74}) emanating from DON-4.

The dimensionless integral **N'** matrix is calculated by either equation (2-57) or (2-59):

$$\mathbf{N}' = \begin{bmatrix} 9.40 & 18.91 & 1.06 & 3.34 & 2.04 & 15.50 & 2.41 \\ 8.83 & 20.15 & 1.11 & 3.36 & 2.01 & 15.59 & 2.56 \\ 8.73 & 18.65 & 2.04 & 3.21 & 1.99 & 14.96 & 2.75 \\ 8.51 & 18.12 & 1.01 & 4.12 & 1.88 & 14.46 & 2.31 \\ 6.44 & 14.58 & 0.89 & 2.44 & 2.47 & 11.99 & 1.89 \\ 8.78 & 20.00 & 1.11 & 3.34 & 2.00 & 16.48 & 2.55 \\ 2.69 & 19.75 & 1.09 & 3.30 & 1.98 & 15.30 & 3.51 \end{bmatrix}. \qquad (2\text{-}115)$$

Considering element $n'_{25} = 15.59$, Barber et. al's (1979) three interpretations of equations (2-58) and (2-115) include: (1) The throughflow into component 5 required to generate one unit of output from 2, (2) The throughflow from 5 contributing one unit of output from 2 over all direct and indirect paths connecting the two compartments, (3) The number of times a unit of output from component 2 passed through component 5 since entering the system. The transitive closure matrix **N'** relates the system level boundary output vector, **y**, over all direct and indirect pathways back to the compartmental throughflow vector $\mathbf{T^{in}}$.

Similar to the output environ efferent case, component's PN-Hetero-2, and NH_4-6, respectively, are consistently the largest purveyors of nitrogen that ultimately exits the system control surface from all components. Considering the second column of the **N'** matrix, a unit of nitrogen output from components 1 through 7 has passed through PN-Hetero-2 an average of 18.59 times ranging from 14.58 at NO_x-5 to 20.15 at PN-Hetero-2. Similarly, considering the sixth column of **N'**, a unit of nitrogen output from components 1 through 7 has passed through

NH_4-6 an average of 14.90 times ranging from 11.99 at NO_x-5 to 16.48 at NH_4-6. PN-Heterotrophs and NH_4 both play large rolls in the nitrogen cycling within the input environs of all seven components.

Sediment-3 and NO_x-5 are consistently the smallest purveyors of nitrogen ultimately discharging as boundary output at all components. Considering the third column of **N',** a unit of nitrogen output from components 1 through 7 has only passed through Sediment-3 an average of 1.19 times ranging from 1.50 times at DON-4 to 2.04 times at Sediment-3. Similarly, considering column 5 of **N'**, nitrogen output from components 1 through 7 has only passed through NO_x-5 an average of 2.05 times ranging from 1.12 at Sediment-3 to 2.47 at NO_x-5. Sediment and NO_x both play small rolls in the nitrogen cycling within the output environs of all seven components.

2.7-5 NEUSE RIVER ESTUARY NITROGEN PATHWAY ANALYSIS WITH THE ADJACENCY MATRIX

The adjacency matrix for the Neuse River estuary model (Figure 2.5), isomorphic to the **F** matrix in equation (2-111), is:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \qquad (2\text{-}116)$$

The seven component model has 22 of 49 possible connections for an overall system connectivity of 45%. The rows and columns represent direct (one path length) incoming and outgoing flows, respectively. The largest single component connectivity is depicted by the second row where all components, except PN-Hetero-2 itself, flow nitrogen to PN-Hetero-2. The second largest is shown in the third row, where all components, except Sediment-3 and DON-4, flow nitrogen to Sediment-3. The next highest connectivities are all represented by outgoing flows where PN-Phyto-1, PN-Hetero-2, and NH_4-6 each have four components to which they flow nitrogen over direct links of path length one. Considering incoming and outgoing flows in combined, PN-Hetero-2 followed by Sediment-3, and then PN-Phyto-1, in decreasing order, are the most connected components. PN-Abiotic-7 and DON-4 are the least connected.

The adjacency matrix raised to the second power enumerates paths of length two:

$$\mathbf{A}^2 = \begin{bmatrix} 1 & 2 & 2 & 0 & 0 & 1 & 0 \\ 3 & 4 & 2 & 1 & 2 & 3 & 1 \\ 2 & 2 & 3 & 2 & 2 & 3 & 1 \\ 1 & 0 & 1 & 2 & 2 & 2 & 1 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 & 2 & 1 \end{bmatrix}. \qquad (2\text{-}117)$$

Considering both incoming and outgoing flows combined, similar to the single path length conclusions, PN-Hetero-2 and Sediment-3 exhibit the greatest, and PN-Abiotic-7 and DON-4 the least, levels of connectance. However, NH_4-6 with relatively low direct connectivity (path length of one) has significant second-order connectivity (path length of two). The connectivities from DON-4 to NO_x-5, $a_{17} = a_{17}^{(2)} = 0$, and from PN-Abiotic-7 to PN-Phyto-1, $a_{54} = a_{54}^{(2)} = 0$, are the only two directional connectances which show no connectivity at path lengths of one and two.

Equation (2-116), populated by the field-acquired empirical data, shows no self-flows for all components as reflected by zeros along the diagonal. However, equation (2-117) shows there is self-connectivity at the level of two path-lengths. In particular, both PN-Hetero-2, $a_{22}^{(2)} = 4$, and Sediment-3, $a_{33}^{(2)} = 3$, already highly-directly-connected components, equation (2-116), experience high second-order self cycling of nitrogen, equation (2-117). In fact, the highest number of connections of path length two, considering the entire $\mathbf{A}^2$ matrix, occurs from PN-Hetero-2 to itself. The diagonal elements, nonexistent to the empirical data gatherer at the level of direct connections, quickly become viable routes of nitrogen flow at path lengths of two. A general review of indirect connectivity at higher path lengths, $\mathbf{A}^m$ as m increases, shows that PN-Hetero-2 and Sediment-3 both consistently maintain high numbers of incoming connections. The number of connections from NH_4-6 to PN-Hetero-2 is usually the highest at all path lengths, followed closely by the paths from NH_4-6 to Sediment-3 and PN-Hetero-2 back to itself. These three unidirectional flows generate a lot of activity. Interestingly, one of them, the self-flow of PN-Hetero-2 is not evident in the direct connection digraph of Figure 2.5, or its supporting empirical data.

2.7-6 NEUSE RIVER ESTUARY EFFERENT STORAGE

Using equation (2-60), component steady-state turnover rates for the standing stocks of nitrogen are calculated as:

$$\tau_i^{-1} = \begin{bmatrix} 81.49 & 234.45 & 0.24 & 9.96 & 26.75 & 332.08 & 34.41 \end{bmatrix} \text{ season}^{-1}, \qquad (2\text{-}118)$$

where the reciprocal τ_i is the turnover time:

$$\tau_i = \begin{bmatrix} 0.0123 & 0.0043 & 4.1667 & 0.1004 & 0.0374 & 0.0030 & 0.0291 \end{bmatrix} \text{ season}. \qquad (2\text{-}119)$$

The turnover rates range from highest at NH_4-6 to lowest for Sediment-3, respectively, 332.08 and 0.24 per season. Reciprocally, NH_4-6 and Sediment-3 require 0.003 and 4.1667 seasons to turn over their respective storages. These donor-specific steady-state partial turnover rates are further partitioned per equation (2-61) as follows,

$$\tau_{ij}^{-1} = \begin{bmatrix} 0 & 0 & 0 & 4.36 & 15.56 & 133.78 & 0 \\ 57.16 & 0 & 0.11 & 5.14 & 9.88 & 166.64 & 26.31 \\ 7.19 & 0.11 & 0 & 0 & 1.12 & 2.22 & 7.59 \\ 16.04 & 17.78 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 & 0 & 28.81 & 0 \\ 0 & 183.03 & 0.03 & 0 & 0 & 0 & 0 \\ 0.67 & 31.33 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{season}^{-1}. \qquad (2\text{-}120)$$

Each τ_{ij}^{-1} represents the partial turnover rate the intercompartmental flow from donor component j, oriented toward component i, contributes to the total turnover rate of donor component j. For example, consider $\tau_{62}^{-1} = 183.03$; 78.1% (183.03/234.45 * 100) of the turnover rate of PN-Hetero-2 is directed towards NH_4-6. The partitioned turnover matrix (2-120) can be written as percentages of the donor components' total turnover rates:

$$\frac{\tau_{ij}^{-1}}{\tau'_j} * 100 = \begin{bmatrix} 0 & 0 & 0 & 43.8 & 58.2 & 40.3 & 0 \\ 70.1 & 0 & 45.8 & 51.6 & 36.9 & 50.2 & 76.5 \\ 8.8 & 0.0 & 0 & 0 & 4.2 & 0.7 & 22.1 \\ 19.7 & 7.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.3 & 0 & 0 & 8.7 & 0 \\ 0 & 78.1 & 12.5 & 0 & 0 & 0 & 0 \\ 0.8 & 13.4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \%, \text{ and} \qquad (2\text{-}121)$$

$$\frac{\left(y_j / x_j\right)}{\tau'_j} \times 100 = \begin{bmatrix} 0.5 & 1.0 & 33.5 & 4.6 & 0.7 & 0.2 & 1.5 \end{bmatrix} \%, \qquad (2\text{-}122)$$

where equation (2-122) represents the output boundary flows', y_j, percentage contributions from the total turnover rates of each j. The partitioned turnover rates in the columns of matrix (2-121) plus the respective boundary flow contributions (2-122) total to 100 percent of each component's turnover rate per equation (2-62). Most notably, as indicated by row 2 of (2-121), PN-Hetero-2 is the recipient of a large percentage of the remaining components' outflow turnover rates, varying from 36.9% for NO_x-5 to 76.5% for PN-Abiotic-7. PN-Phyto-1 and PN-Hetero-2 combined receive significant portions of the outflow turnover rates directed from components DON-4, 43.8 + 51.6 = 95.4%; NO_x-5, 58.2 + 36.9 = 95.1%; and NH_4-6, 40.3 + 50.2 = 90.5%. Matrix (2-122) shows one-third, 33.5%, of the Sediment's (3) outflow turnover rate to be associated with boundary outflow, y_3. These turnover rate results and subsequent interpretations are derived directly from the empirical data in the flow matrix, **F**. As previously noted, the turnover matrix, $\boldsymbol{\tau}^{-1}$, observed flow matrix, **F**, and adjacency matrix, **A**, are all isomorphic.

The donor-specific turnover rate matrix **C**, equation (2-67), is:

$$\mathbf{C} = \begin{bmatrix} -81.49 & 0 & 0 & 4.36 & 15.56 & 133.78 & 0 \\ 57.16 & -234.45 & 0.11 & 5.14 & 9.88 & 166.64 & 26.31 \\ 7.19 & 0.11 & -0.24 & 0 & 1.12 & 2.22 & 7.59 \\ 16.04 & 17.78 & 0 & -9.96 & 0 & 0 & 0 \\ 0 & 0 & 0.02 & 0 & -26.75 & 28.81 & 0 \\ 0 & 183.03 & 0.03 & 0 & 0 & -332.08 & 0 \\ 0.67 & 31.33 & 0 & 0 & 0 & 0 & -34.41 \end{bmatrix} \text{season}^{-1}. \quad (2\text{-}123)$$

This is a composite matrix used for equation development concisely showing the partitioned donor specific partitioned turnover rates, τ_{ij}^{-1}, on the off-diagonals with the negative of the total component turnover rates on the diagonals. Recall, by definition of the composite **C** matrix, equation (2-68), that the diagonals theoretically contain two entities: (1) The partitioned donor-specific turnover rate attributed to a flow from a component back to itself, τ_{ii}^{-1}, (these are typically zero because f_{ii} are typically zero in the empirical data), and (2) the negative of the total turnover rate, $-\tau_i^{-1}$.

The maximum turnover rate in the Neuse River estuary occurs at the NH_4-6 component where $\tau_{\max}^{-1} = 332.08\ \text{season}^{-1}$. As such, as previously explained, the time step, Δt, is selected to assure that the stock of this component will not turn over in a period of time less than the derived discrete time interval, Δt. Therefore, using equation (2-74), Δt is set identical to the NH_4-6 turnover time, τ_6:

$$\Delta t = \left(\frac{x_i}{T_i^{out}} \right)_{min} = \left(\tau_i \right)_{min} = \left(\frac{x_6}{T_6^{out}} \right) = \left(\frac{36}{11955} \right) = 0.00301\,\text{seasons}. \tag{2-124}$$

The dimensionless storage probability matrix, **P**, calculated with equation (2-70),

$$\mathbf{P} = \begin{bmatrix} 0.75 & 0 & 0 & 0.01 & 0.05 & 0.4 & 0 \\ 0.17 & 0.29 & 0 & 0.02 & 0.03 & 0.5 & 0.08 \\ 0.02 & 0 & 1 & 0 & 0 & 0.01 & 0.02 \\ 0.05 & 0.05 & 0 & 0.97 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.92 & 0.09 & 0 \\ 0 & 0.55 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.09 & 0 & 0 & 0 & 0 & 0.9 \end{bmatrix}, \tag{2-125}$$

is relative to the discrete time interval, Δt = 0.00301 season, which by definition, sets p_{66} equal to zero. The remaining diagonal terms, by the same definition in equation (2-74), are then between or equal to zero and one. The off-diagonal terms, p_{ij}, represent the fraction of nitrogen storage in component j, x_j, which will be transferred to the storage at i, x_i, during the interval from t to t + Δt. Since **P** is an algebraic construct of both **C** and the identity matrix **I**, the diagonal terms, p_{ii}, represent the fraction of original nitrogen that remains after one Δt. For example, since Δt was calculated based on the turnover time, τ_6, of NH_4-6, there remains exactly a 0.0 fraction of x_6 remaining ($p_{66} = 0.0$) of the original nitrogen in NH_4 storage after a time passage of Δt = 0.00301 seasons. Of course, under the assumed steady-state conditions, additional nitrogen has moved into these components to replace the nitrogen that has left which maintains the storage contents, x_i, constant. The diagonal elements of **P** denote the fractions of nitrogen remaining compared to the original amounts at the start of the time step. Considering additional diagonal terms, 75% of the original nitrogen remains in PN-Phyto-1, $p_{11} = 0.75$, after the same Δt. The turnover time of Sediment-3 is so large, $\tau_3 = 4.14$ seasons, compared to NH_4-6, $\tau_6 = 0.00301$ seasons, that 99.93%

[rounded to 1.00 in p_{33} of equation (2-125)] of Sediment-3 remains after Δt. NH_4-6 followed by PN-Hetero-2, $\tau_2 = 0.00427$ seasons, and then PN-Phyto-1, $\tau_1 = 0.01227$ seasons, have the shortest turnover times as further illuminated by considering the fractions of nitrogen remaining after one discrete time step (p_{66} = 0%, p_{22} = 29%, p_{11} = 75%). The remaining four components all maintain over 90% of their original nitrogen after each Δt.

The efferent integral discrete- and continuous-time integral storage matrices, **Q** (dimensionless) and **S**, as calculated from equation (2-78) and **S** = **Q**Δt are:

$$\mathbf{Q} = \begin{bmatrix} 38.75 & 36.02 & 24.24 & 35.55 & 36.87 & 37.05 & 32.89 \\ 27.47 & 28.90 & 19.03 & 26.94 & 27.46 & 28.08 & 26.30 \\ 2{,}203.81 & 2{,}173.7 & 2{,}830.49 & 2{,}086.59 & 2{,}203.53 & 2{,}188.63 & 2{,}286.47 \\ 111.48 & 109.65 & 73.03 & 138.75 & 108.42 & 109.82 & 99.95 \\ 18.48 & 19.30 & 14.08 & 18.05 & 30.88 & 19.90 & 17.86 \\ 15.34 & 16.13 & 10.75 & 15.04 & 15.34 & 16.68 & 14.71 \\ 25.77 & 27.02 & 17.80 & 25.22 & 25.72 & 26.29 & 34.24 \end{bmatrix}, \qquad (2\text{-}126)$$

$$\mathbf{S} = \begin{bmatrix} 0.117 & 0.108 & 0.073 & 0.107 & 0.111 & 0.112 & 0.099 \\ 0.083 & 0.087 & 0.057 & 0.081 & 0.083 & 0.085 & 0.079 \\ 6.636 & 6.546 & 8.523 & 6.283 & 6.635 & 6.591 & 6.885 \\ 0.336 & 0.330 & 0.220 & 0.418 & 0.326 & 0.331 & 0.301 \\ 0.056 & 0.058 & 0.042 & 0.054 & 0.093 & 0.060 & 0.054 \\ 0.046 & 0.049 & 0.032 & 0.045 & 0.046 & 0.050 & 0.044 \\ 0.078 & 0.081 & 0.054 & 0.076 & 0.077 & 0.079 & 0.103 \end{bmatrix} \text{ seasons.} \qquad (2\text{-}127)$$

Similar to the **N** matrix, the **S** matrix (integral turnover times) of equation (2-81) maps all boundary inputs, **z**, through all direct and indirect pathways to the respective component storages, **x**. Following Barber et al.'s (1979) similar interpretive logic, consider interstitial element $s_{35} = 6.635$. This is the amount of storage forced out of Sediment-3 to contribute one unit of input to NO_x-5. It is also the amount of storage passing from component 5 to component 3, over all direct and indirect paths of all lengths, to provide one unit of input into component 5.

Clearly, Sediment-3 dominates the relative magnitudes in the matrix. Row three's values are over one hundred times greater than the smallest value in the matrix, $s_{63} = 0.032$. A single unit of nitrogen input z_4, into DON-4, produces 6.283 mmol-N/m^2 of storage routed from DON's storage to Sediment's storage. The highest integral transfer in the matrix occurs at s_{33}. For a unit of boundary flow, z_3, into Sediment-3, 8.523 mmol-N/m^2 of storage is transferred from Sediment's storage throughout the system, across all direct and indirect pathways, and finally back to Sediment's storage. The magnitudes associated with the third row of **S** identify the significant contributions each of the system's seven components make to the large Sediment storage reservoir. Row four's values are, on average, approximately ten times the lowest magnitude at s_{63}. Relatively compared, all components, except Sediment at $s_{43} = 0.220$, route a major amount of their boundary inputs to the storage at DON-4.

2.7-7 NEUSE RIVER ESTUARY AFFERENT STORAGE

The steady-state transfer rate coefficients, τ_i^{-1}, equation (2-118), and their reciprocal time coefficients, τ_i, equation (2-119), for afferent storage analysis are identical to those calculated for the efferent analysis. The recipient-specific steady-state partial turnover rates are partitioned per equation (2-83) as follows:

$$\tau_{ij}'^{-1} = \begin{bmatrix} 0 & 0 & 0 & 13.84 & 10.8 & 56.66 & 0 \\ 75.92 & 0 & 8.67 & 21.69 & 9.11 & 93.73 & 25.08 \\ 0.12 & 0.00 & 0 & 0 & 0.01 & 0.02 & 0.09 \\ 5.05 & 4.21 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.08 & 0 & 0 & 17.58 & 0 \\ 0 & 325.39 & 4.42 & 0 & 0 & 0 & 0 \\ 0.93 & 32.87 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{season}^{-1}. \qquad (2\text{-}128)$$

Each $\tau_{ij}'^{-1}$ represents the observed partial turnover rate of recipient component i due to the intercompartmental flow to recipient component i coming from j. For example, consider

$\tau'^{-1}_{26} = 93.73$; this means that 40.0% (93.73/234.45 * 100) of the turnover rate of PN-Hetero-2 is derived from NH4-6. The partitioned turnover rates can be written as percentages of the recipient component's total turnover rates:

$$\frac{\tau'^{-1}_{ij}}{\tau'_j} * 100 = \begin{bmatrix} 0 & 0 & 0 & 17.0 & 13.2 & 69.5 & 0 \\ 32.4 & 0 & 3.7 & 9.2 & 3.9 & 40.0 & 10.7 \\ 48.7 & 0 & 0 & 0 & 5.4 & 6.2 & 37.1 \\ 50.7 & 42.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.8 & 0 & 0 & 65.7 & 0 \\ 0 & 98.0 & 1.3 & 0 & 0 & 0 & 0 \\ 2.7 & 95.5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \%, \text{ and} \tag{2-129}$$

$$\frac{\left(z_i / x_j \right)}{\tau_j} \times 100 = \begin{bmatrix} 0.2 & 0.2 & 2.2 & 6.9 & 26.5 & 0.7 & 1.9 \end{bmatrix} \%, \tag{2-130}$$

where equation (2-130) represents the input boundary flows, z_i, percentage contributions to the total turnover rates of each j. The rows of the partitioned turnover rates in matrix (2-129) and the respective boundary flow contribution (2-130) combine to total 100 percent of each component's turnover rate per equation (2-84). Components NH_4-6 and PN-Abiotic-7, receive 98.0% and 95.5%, respectively, of their inflow-generated turnover rates from PN-Hetero-2. Considering the second row of equation (2-129), PN-Hetero is the only component receiving an inflow turnover rate contribution from all other components. The third column of equation (2-129) shows that a very small or zero percentage of any component's inflow-generated turnover rate is derived from Sediment-3, whereas Sediment's inflow contributions to its turnover rate come primarily from PN-Phyto-1 and PN-Abiotic-7, 48.7% and 37.1%, respectively. Finally, 26.5% of the inflow-generated turnover rate of NO_x-5 is due to input z_5, per equation (2-130).

The recipient-specific turnover rate matrix **C'** from equation (2-90) is:

$$\mathbf{C}' = \begin{bmatrix} -81.49 & 0 & 0 & 13.84 & 10.80 & 56.66 & 0 \\ 75.922 & -234.45 & 8.67 & 21.69 & 9.11 & 93.73 & 25.08 \\ 0.12 & 0 & -0.24 & 0 & 0.01 & 0.02 & 0.09 \\ 5.05 & 4.21 & 0 & -9.96 & 0 & 0 & 0 \\ 0 & 0 & 2.08 & 0 & -26.75 & 17.58 & 0 \\ 0 & 325.39 & 4.42 & 0 & 0 & -332.08 & 0 \\ 0.93 & 32.87 & 0 & 0 & 0 & 0 & -34.41 \end{bmatrix} \text{season}^{-1}. \quad (2\text{-}131)$$

This composite matrix used for equation development shows the partial turnover rates, $\tau_{ij}'^{-1}$, on the off-diagonals and the negative total turnover rates entered along the principal diagonal. At steady-state where $T_i^{in} = T_i^{out}$, equations (2-74) and (2-96) calculate the same Δt = τ_6 = 0.00301 seasons, which assures that the stock of the system's maximum-turnover-rate component, NH_4-6 ($\tau_{max}'^{-1} = 332.08\ \text{season}^{-1}$), does not turn over in one time step. The dimensionless storage probability matrix, equation (2-92) expressed for the discrete time interval Δt = 0.00301 season (which sets p_{66} equal to zero), is:

$$\mathbf{P}' = \begin{bmatrix} 0.75 & 0 & 0 & 0.04 & 0.03 & 0.17 & 0 \\ 0.23 & 0.29 & 0.03 & 0.07 & 0.03 & 0.28 & 0.08 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.02 & 0.01 & 0 & 0.97 & 0 & 0 & 0 \\ 0 & 0 & 0.01 & 0 & 0.92 & 0.05 & 0 \\ 0 & 0.98 & 0.01 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 & 0 & 0.9 \end{bmatrix}. \quad (2\text{-}132)$$

The remaining diagonal terms, by the same definition in equation (2-96), lie between or are equal to zero and one. Since **P'** is an algebraic construct of both **C'** and **I**, the diagonal terms, p'_{ii}, represent the fraction of original nitrogen that remains after one Δt time step. As such, the diagonal terms numerical magnitudes are identical and interpreted the same as the diagonal terms of p_{ii} expressed in equation (2-125). The off-diagonal terms, p'_{ij}, represent fractions of nitrogen storage in component i, x_i, transferred from storage at j, x_j, during each Δt.

The afferent discrete- and continuous-time integral storage matrices, **Q'** (dimensionless) and **S'**, as calculated from equation (2-102) and **S'** = **Q'**Δt are:

$$\mathbf{Q}' = \begin{bmatrix} 38.75 & 27.12 & 1{,}482.78 & 112.94 & 25.59 & 15.69 & 23.61 \\ 36.49 & 28.90 & 1{,}546.25 & 113.67 & 25.31 & 15.79 & 25.07 \\ 36.02 & 26.75 & 2{,}830.49 & 108.34 & 25.00 & 15.15 & 26.82 \\ 35.10 & 25.99 & 1{,}406.50 & 138.75 & 23.69 & 14.64 & 22.58 \\ 26.62 & 20.93 & 1{,}241.02 & 82.59 & 30.88 & 12.14 & 18.47 \\ 36.23 & 28.68 & 1{,}552.73 & 112.82 & 25.13 & 16.68 & 24.92 \\ 35.9 & 28.35 & 1{,}517.27 & 111.65 & 24.87 & 15.51 & 34.24 \end{bmatrix}, \qquad (2\text{-}133)$$

$$\mathbf{S}' = \begin{bmatrix} 0.117 & 0.082 & 4.465 & 0.340 & 0.077 & 0.047 & 0.071 \\ 0.110 & 0.087 & 4.656 & 0.342 & 0.076 & 0.048 & 0.075 \\ 0.108 & 0.081 & 8.523 & 0.326 & 0.075 & 0.046 & 0.081 \\ 0.106 & 0.078 & 4.235 & 0.418 & 0.071 & 0.044 & 0.068 \\ 0.080 & 0.063 & 3.737 & 0.249 & 0.093 & 0.037 & 0.056 \\ 0.109 & 0.086 & 4.676 & 0.034 & 0.076 & 0.050 & 0.075 \\ 0.108 & 0.085 & 4.569 & 0.336 & 0.075 & 0.047 & 0.103 \end{bmatrix} \text{seasons.} \qquad (2\text{-}134)$$

Similar to the **N'** matrix, the integral **S'** matrix (comprised of integral turnover times) of equation (2-103) maps all outputs back from across the system boundary, **y**, through all direct and indirect pathways to respective component storages, **x**. As an example, consider element s'_{53} = 3.737. This represents the amount of storage in Sediment-3 that contributed to one unit of output from NO_x-5. Similar to the efferent direction conclusions from equation (2-127), Sediment-3 also dominates the relative magnitudes in matrix (2-134). Column 3's values are over one-hundred times the smallest magnitude in the matrix, s'_{56} = 0.037 seasons. As another example, s'_{43} = 4.235 indicates that one unit of nitrogen flowing out across the boundary as y_4 from DON-4 requires a production of 4.235 mmol-N/m^2 of storage routed from Sediment storage to DON storage. The highest integral transfer in the matrix occurs at s'_{33}. For a unit of boundary flow out of Sediment, y_3, 8.523 mmol-N/m^2 of storage is transferred from Sediment storage throughout the system, across all direct and indirect pathways, and finally back to Sediment storage. The magnitudes associated with the third column of **S'** identify the significant contribution Sediment storage makes to the system's seven component storages. The values in column 4, representing DON, are, on average, approximately ten times the lowest magnitude, s'_{56}. Relatively compared, all components (with the minimized exception of NH_4-6, $s'_{64} = 0.034$) route a large amount of their boundary input to the storage of DON-4.

2.8 DISCUSSION

The law of conservation and the efferent and afferent definitions of throughflow are the analytical foundations of network environ analysis (NEA) (Barber et al., 1979). Generally assuming the past precedes the future, where the term "dynamical" has roughly the same meaning as causal or nonanticipatory, state-space theory (Zadeh, 1964; Kalman et al., 1969; Rugh, 1996**)** provides a mathematical representation for the state of systems. Patten et al. (1976) employed this mathematical representation in an ecological rendering of the physical aspects of causal chains by conservative energy or matter flows transferred within or between ecological components. Environ theory (Patten 1978a) is designed to reconnect ecology to the object of its interest — the external environment that lies outside ecological control surfaces at all levels of organization which impinge on these to become drivers of organism-environment coupling and dynamics. The rigidity of form provided by the transport theory's control volume approach helps to establish a rigorous inside/outside distinction in the context of real (CV's) instead of abstract (compartmental) spaces — a connection helpful for NEA because intercompartmental- and self-flows and component storages then maintain consistent definitions at model conception and subsequent analytical interpretations. Ecology can be defined as the biological science of environment, but in its history it has often lost sight of what is system and what is environment.

The classical population equations, devoid as they are of explicit connection to environment, are a prime example of this lapse. The boundary of environ theory, however, is explicit, and its rendering as a control surface makes it more concrete and physical than it typically is in zero-dimensional compartmental modeling.

2.8-1 CONSERVATION EQUATIONS

The conservation laws underlying NEA, although accurate, remain largely assumed in ecological applications. That is, ecologists take such laws of physics and chemistry for granted in their empirical and theoretical research. Basically, a successful accounting can be made of all conserved quantities where a statement similar to equation (2-17) can be written for mass and some primitive energy models by inspection with very little understanding of the derivation or limitations of the conservation laws. However, equation (2-17), having been derived from an Eulerian control volume, correctly identifies that the rate of accumulation term on the left is within the CV and the mass flows on the right side of the equations must pass through the defined CS to satisfy the integral calculus operations evident in equation (2-14). This rigidity of form is helpful to subsequent interpretation wherein its not always appropriate to simplify (particularly with energy models) where interpretive subtleties can be lost.

For instance, several points are easily clarified with the subsequent NEA interpretations given the derivations and formalities of the Reynolds transport theorem equations (2-1) through (2-13), the continuity equations (2-14) through (2-19), and the energy equations of (2-20) through (2-28):

(1) The rate of accumulation term [left side of equation (2-17)] is otherwise loosely referenced as the storage term in conservation-law mathematics. With present (but not permanent) NEA methods, however, the system is usually assumed to be at steady-state,

and the accumulation term, as a direct result of this assumption, is zero. The NEA study of storage is largely a study of turnover times, τ, and rates, τ^{-1}, facilitated by dividing the flows [right side of equations (2-17), (2-26) and (2-30)] by the donor or recipient steady-state storages, $\mathbf{x}$. NEA storage analysis remains a study of the right-hand-side of the conservation equation comprised of terms which, by definition, cross the CS. Turnover times and rates illuminate the understanding of component steady-state storage relations, however, current NEA acquires this information through the balanced flows across the CS, not by a nonzero accumulation term in the CV on the left-hand-side. Analysis of nonsteady-state systems, when that becomes available, will advance environ storage analysis beyond where steady-state turnover rates and times can take it.

(2) Augmenting the observation and brief discussion in section 2.32, flow from a component to itself over one path length (self-flow) represents only one flow of many [all of which exist on the right side of equations (2-17), (2-26) and (2-30)] which contribute to component storage turnover rates and their reciprocal turnover times. A component self-flow out and back to itself over one path length is sometimes incorrectly called a storage or a quasi-storage related term. Technically derived from the Reynolds transport integral calculus operations, however, a flow only exists in the conservation equation if it crosses the CS. If it does cross the CS (and then eventually come back across the CS into the component), it is only one flow of many on the right side of the conservation equation contributing to the component's storage turnover rate or turnover time and therefore does not inherit, over the other intercompartmental flows, any special recognition regarding the storage of a component. Self-flows are simply one flow of many in a multi-component system. If a flow does not cross the defined CS of the component, then by

definition, the flow does not exist on the right side of the conservation equation. The flow remains within the CV and the conservation equation has no capacity to include or review the flow's essence or activity. However, the matrix mathematics associated with NEA requires a self-flow for each component, zero or otherwise, because the diagonals of **F**, **G**, **G'**, etc. matrices must be implemented to permit subsequent matrix operations. If self-flows equal to zero ($f_{jj} \equiv f_{jj}^{out} \equiv f_{jj}^{in} = 0$) are included, as diagonal entries in **F** to meet the requirements of matrix mathematics, then by definition in the Reynolds transport theorem, these zero flows must conceptually cross the CS when exiting the component (i.e., as f_{jj}^{out}) and then re-cross this same CS again when returning (as f_{jj}^{in}). Per the Reynolds transport theorem derivation, self-flows are flows, not storages.

(3) The development of the energy conservation equations (2-20) through (2-28) identifies the abundance of assumptions necessary to arrive at a suitable ecological network analysis equation. Energy within ecosystems has a variety of manifestations only coarsely represented by the quantity of terms in a comprehensive energy conservation equation. Furthermore, energy continuously transforms itself through a variety of forms and thereby cycles (Patten, 1985) through a combinatorially vast array of microscopic fluxes as it degrades ultimately to the predictable diurnally cycled heat radiating back to space. These assumptions and the corresponding lumping of terms are rarely discussed in depth beyond the pertinent energy-flow description leading directly to the simplified equations (2-27) or (2-28). For example, the output flow respiration term, r_i, in Hannon's (1973) model combines the three energy terms: (1) heat flow out due to respiration, (2) net component growth of energy content, and (3) energy content of exported biomass. These three terms involve three distinctly different energy manifestations in the

conservation equation, respectively: (1) the heat flow out due to respiration is Q, (2) the net component growth of energy content represents the nonsteady-state accumulation term, $\left.\frac{\partial E}{\partial t}\right|_{CV}$, and (3) the energy content of the exported biomass is an energy flow associated with mass flow which is one of the terms in the summation $\sum_{\substack{out \\ CS}} \dot{m}_{out} e_{out}$. For the most part, the differentiation of the forms of energy adds nothing to the current NEA methodology. However, it is always scientifically appropriate to concisely and clearly state all assumptions in any equation development and consolidation. As of yet, these assumptions have not been articulated with regard to ecological network equation development. The Reynolds transport equations (2-13), (2-14), and (2-23) provide a framework to adequately describe all assumptions leading to the final NEA conservation equation. These assumptions, like all assumptions should, can now be used and then subsequently evaluated for their impact on the final results. They can also be isolated and independently explored in later studies.

The laws of conservation, written as equations to model a system, are simply reformulations of the common system description of equation (2-1). Their mathematical derivation, however, establishes a model whose rigid framework must be respected with subsequent assumptions, mathematical manipulations, and interpretations of results. The same rigidity within the framework of derivation also helps to simplify conceptual interpretations. For example, in summary: (1) The accumulation term, $\left.\frac{dS}{dt}\right|_{CV}$, which represents nonsteady-state storage, increases or decreases within a component and is not the focus of NEA storage analysis. Rather, NEA steady-state storage analysis represents a study of component turnover times and rates as determined from the flows across the control surfaces. The component steady-state storages, x_i, are algebraically introduced to the conservation equation to facilitate this analysis. (2) When the control volume and its control surface are established, flows from a component to itself are easy to interpret. (3) The assumptions associated with algebraic representations of complex energy flows in an ecosystem are clearly articulated within the rigidity of the Reynolds transport theorem development process. The Reynolds transport theorem conservation equations for both continuity and energy are useful and necessary models to frame ecological network analysis.

2.8-2 NETWORK ENVIRON ANALYSIS

The network environ analysis derivation reestablished herein, although developed in parts in many publications, is subtly different than recent trends as it reaches back to the original development scheme shown in Barber et al. (1979). This derivation independently and thoroughly derives the equations of conservation and the definition of throughflow which then combine to form the efferent or afferent oriented perspectives. This subtlety more clearly elicits the essence of an oriented network analysis within the two environment perspective. The law of conservation accounts for all flows, coming, going, or staying. With no accumulation ("staying") for steady-state systems, this coming or going vernacular description begins to identify one way of orienting the flows. Similarly, for steady-state systems, the definition of throughflow also has an orienting aspect by considering either the total flows coming into a component, T^{in}, or the total flows leaving a component, T^{out}. By substitution, the two concepts of conservation and throughflow, each with algebraic terms explicitly oriented across the modeled control surface, combine to provide the oriented perspective to ecological network analysis inherent to network environ analysis. As a direct result, the efferent flow equations map the inputs, **z**, through the output environs implicit in **N**, to the output throughflow, T^{out} and the afferent flow equations map the outputs, **y**, back through the input environs implicit in **N'** to the component input throughflow, T^{in}. The orientating capability of network environ analysis is a fundamental construct of the oriented nature of the conservation and throughflow equation's terms. An important aspect to NEA, the origination of this equation orientation can be lost in the, often too coarse, algebraic substitutions leading to the final environ implicit equations (2-48), (2-58), (2-81), and (2-103).

2.8-3 STORAGE EQUATIONS

Network environ storage analysis, originally conceived by Matis and Patten (1981), reparameterized the conservation equations by dividing the component flows with their respective donor or recipient steady-state storages, **x**, (for efferent or afferent analysis, respectively) and subsequently created and then defined the new algebraically constructed transfer coefficient terms. Equations (2-61) and (2-83) subtly refine this approach by clarifying and redefining the transfer coefficient terms as partial turnover rates, τ_{ij}^{-1}, which when summed together, including the partial turnovers due to boundary flows as in equations (2-62) and (2-84), generate the total component turnover rates, τ^{-1}. This partial-turnover-of-storage presentation illuminates the contribution of inter-component flows to the Matis and Patten methodology. Each intercompartmental flow contributes to its donor component's total turnover rate. Also, the partial turnover matrix, equations (2-61) and (2-83), a construct of the flow matrix, is isomorphic to both the flow, **F**, and adjacency, **A**, matrices and, as such, is a natural progression in the development of an introspective directional environ flow analysis. The algebraically composite **C** and **C'** matrices, constructed from the combination of the conservation equations and the definition of throughflow (both of which are converted to their partial and total turnover rate format by dividing through by their respective component storages), identify the foundation of the storage environ analysis. The efferent **C** and afferent **C'** matrices concisely include and present, respectively, the system donor and receiver storage turnover rate dynamics. Although they are algebraic composites, they are reasonably easy to interpret and, as such, aid in the interpretation of NEA. The defined efferent **P** and afferent **P'** matrices, through a uniquely chosen time step, Δt, harness the component turnover rate information of the **C** and **C'** matrices and display their information in a uniquely revealing fraction-of-storage-transferred format. When the time step, Δt, is chosen, by consistent practice, such that the highest component turnover rate, $\tau_{\max}^{-1}$, empties that component completely, $\Delta t\ \tau_{\max}^{-1} = 1$, then the **P** and **P'** turnover rates are normalized on a zero to one scale. Together, the efferent **C** and **P** and afferent **C'** and **P'** matrices transform the observed flows of an ecosystem network into an illuminating exchange of component storages through varying rates of transfer.

Importance of the turnover rate, τ^{-1}, is further shown by the mathematical relationship between environ throughflow and storage analyses where the environ-implicit throughflow matrices **N** and **N**' and the environ-implicit storage matrices **S** and **S**' are simply numerically equated through the turnover rate, τ^{-1}, as shown in equations (2-107) and (2-109). Environ throughflow analysis is algebraically augmented early in the equation development by converting the flows of the conservation equations to partial and total turnover rates (dividing flows by respective component storages). The composite turnover matrices, **C**, **P**, **C'**, and **P'** are determined and the final environ-implicit **Q**, **S**, **Q'**, and **S'** environ storage mapping matrices are the direct result of the subsequent algebra. However, equations (2-107) and (2-109) mathematically show that the substitutions made early in the environ storage equation development are also evident, as expected, in the final results. Although environ storage analysis (primarily due to the insights offered from the review of storage turnover rates and times) is revealing as a separate environ perspective within NEA, it should always be carefully viewed as an extension of environ throughflow analysis since it deals with the flows on the right-hand-side of the conservation equation (2-30).

2.8-4 NEUSE RIVER ESTUARY THROUGHFLOW AND STORAGE NETWORK ENVIRON ANALYSIS

NEA affords the network analyst both a practical means to review an ecosystem's activity and the mathematical prowess to uncover the vastly underestimated indirect pathways and effects within that activity. In some cases, the results when indirect effects are considered reinforce the practical-direct-relationship values, while in other cases the indirect effects indicate completely different or somewhat unexpected results. PN-Hetero-2 appear to play a strong role both from the directly observed model flow activity and also from the mathematically determined indirect effects. Practically stated, the highest throughflow in the model exists in the PN-Hetero-2 component at 15,005 mmol-N m^{-2} $season^{-1}$ and PN-Hetero-2 is also the most directly connected component in the model (10 connections of 22 total, reference Figure (2.5) and equation (2-116); there are 49 possible connections). The adjacency matrices continued to show that PN-Heterotrophs were consistently one of the most connected at virtually all path lengths. Furthermore, when considering the efferent and afferent throughflow integral matrices **N** and **N'**, PN-Hetero-2 is the highest purveyor, over all path lengths, of nitrogen either originating at the boundary of the system, **z**, or leaving the boundary, **y** [in **N**, equation (2-113) the second row average of 18.34 is highest and in **N'**, equation (2-115), the second column average of 18.59 is also highest; note, in addition, that the highest matrix value is the PN-Hetero-2 self-flow, $n_{22} = n'_{22} = 20.15$]. Interestingly, although no PN-Hetero-2 self-flow exists at a path length of one [reference equations (2-111) and (2-116) where $f_{22} = 0$ and $a_{22} = 0$, respectively], the connectivity at higher path lengths (as $m \to \infty$) is almost always greatest between PN-

Hetero-2 and itself [e.g., at path length 2, the highest number of pathways occurs at the PN-Hetero-2 self-flow location; reference equation (2-117) where $a_{22}^{(2)} = 4$]. PN-Hetero-2 demonstrates that self-flows, mistakenly innocuous at path-lengths of one due in large part to their consistent zero flows, (a.k.a, $f_{ii} = 0$ or otherwise known as zeros on the diagonals), become increasing important in higher path length analysis. PN-Hetero-2 has the second highest turnover rate, 234 seasons^{-1}. NH_4-6 is the highest at 332 season^{-1}. Looking at the direct storage turnover analysis, equation (2-121), all components distribute a significant portion of their storage turnover to PN-Hetero-2 ranging from a low of 37% of NO_x-5 storage to a high of 76% of PN-Abiotic-7 storage being specifically directed to PN-Hetero-2. PN-Hetero-2 is the only component, equation (2-129), which receives some of its storage from all of the other six components. Contrastingly, consider either the efferent **Q** and **S**, or the afferent **Q'** and **S'** integral storage transfer matrices [equations (2-126), (2-127), (2-133), and (2-134)]. Over all path lengths, PN-Hetero-2 receives somewhat less storage turnover from other components, when relatively compared to other integral storage transfers in the system. For example, consider equation (2-126) where the average of row 2 is 26.31 which is lower than the averages from row 1 (34.48), row 3 (2,281.89), and row 4 (107.30). This observation is markedly different than what is suggested by the directly observed turnover matrices of equations (2-121) and (2-129) where PN-Hetero-2 appears to dominate.

The role of Sediment-3 is more difficult to ascertain. Its component throughflow and storage turnover rate are the system's lowest at 1,255 mmol-N m^{-2} season^{-1} and 0.24 season^{-1}, respectively. For example, 99.9% of the original Sediment remains in storage after one time step (Δt established by convention to be the time for NH_4-6 storage to completely empty). Additionally, considering both the efferent and afferent integral throughflows implicit in **N** and

N', Sediment-3 is the smallest purveyor of nitrogen, over all path lengths, either originating at the boundaries of the system, **z**, or leaving the boundaries of the system, **y**. For example, reference the matrix **N**, equation (2-113), where the third row average of 1.64 is the matrix's second lowest row average. Also consider the matrix **N'**, equation (2-115), where the third column average of 1.19 is the matrix's lowest column average. The lowest matrix value is Sediment's (3) self-flow where $n_{33} = n'_{33} = 2.04$. Yet, contrastingly Sediment-3 has the highest component steady-state storage at 5,200 mmol-N m^{-2} and is the second most connected component in the system when considering a pathway analysis using the third row and column of all powers of the adjacency matrix, $\mathbf{A}^{(m)}$ as $m \rightarrow \infty$. The turnover rate matrices show very little direct contribution of each component's turnover rate of their storage to Sediment's (3) storage. For example, reference the third row of equation (2-121) where contributed storage turnover dedicated to Sediment-3 ranges from 0% of DON-4 storage up to 22.1% of PN-Abiotic-7 storage. Also reference the third column of equation (2-129) where contributed storage turnover dedicated to Sediment-3 ranges from 0% of PN-Phyto (2) and DON-4 storage to 7.8% of NO_x-5. However, the environ storage analysis results clearly show that Sediment-3 dominates the storage-component transfers when all pathways are considered in the integral matrices of **Q**, **S**, **Q'**, and **S'**. This is shown by the relative magnitudes of the third rows in equations (2-126) and (2-127) and the third columns of equations (2-133) and (2-134). In fact, the highest integral transfer in the matrix occurs at $s'_{33} = s_{33}$ with a magnitude of 8.523. A single unit of nitrogen crossing the system boundaries into or out of Sediment-3 will require 8.523 mmol-N/m^2 of storage routed from Sediment's storage, across all direct and indirect pathways, and ultimately back to Sediment's storage before the unit of nitrogen then leaves the system. This magnifying effect is true despite the fact Sediment-3 contributes 33.5% of its storage turnover to boundary output flow [reference equation (2-122)] which leaves the system completely. Although not readily apparent, Sediment-3 is the grand internal augmenter of the circulation of nitrogen storage in the Neuse River estuary nitrogen cycle.

This work provides a foundation of meticulous environ equation development coupled with a comprehensive NEA review of an ecological model which simultaneously includes analyses of system connectivity, system throughflows, and system storage turnovers. The conservation of energy equation development, although unused thereafter its presentation, will hopefully provide some insight to the future empirical data gathering activities and subsequent lumping assumptions necessary to get workable energy equations. The conservation of mass equations provide a consistent methodology to derive both the environ throughflow and environ storage bi-directional equations. The process is consistent within itself and consistent to standard practices by presenting first principles (conservation equations) and definitions (definition of throughflow) and then algebraically modifying the equations from that point forward. The Neuse River estuary NEA results show that PN-Heterotrophs (2) play a smaller role in the storage transfer process over all indirect pathways of all lengths as compared to its dominant role in intercompartmental throughflow propagation. Conversely, Sediment-3 plays a dominant role in the storage transfer process over all indirect storage pathways of all lengths as compared to its minimized role in intercompartmental throughflow propagation. The incorporation of a well developed environ storage analysis into the commonly used environ throughflow analysis will continue to provide yet more insights into the power of indirect effects.

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2.11 TABLES

Table 2.1: Average data set (as calculated by Christian and Thomas, 2000) representing 16 consecutive seasons from Spring 1985 through Winter 1989 for the Neuse River estuary nitrogen model. Intercompartmental flows, f_{ij}; boundary flows, z and y; and throughflows, T, have dimensional units of mmol-N/(m^2 × season). Storage, **x**, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow, T	Storage, x
PN-Phyto	0	0	0	1,176	918	4,816	0	16	6,927	85
PN-Hetero	4,859	0	555	1,388	583	5,999	1,605	24	15,005	64
Sediment	611	7	0	0	66	80	463	28	1,255	5,200
DON	1,363	1,138	0	0	0	0	0	187	2,688	270
NO_x	0	0	123	0	0	1,037	0	419	1,578	59
NH_4	0	11,714	159	0	0	0	0	82	11,955	36
PN-Abiotic	57	2,005	0	0	0	0	0	39	2,099	61
Output, y	37	148	418	124	11	23	31		TST = 41,517	

2.12 FIGURE LEGENDS

Figure 2.1: Fixed control volume showing its corresponding control surface of a liquid draining from a holding tank.

Figure 2.2: The volume of an extensive property, S, at two sequential times, t_1 and t_2. The control volume of interest, $\cancel{V}_B$, is bounded by the control surface represented by a dashed line.

Figure 2.3: Flow through a differential area, dA, of a control surface surrounding a control volume. V is the velocity of some entity through the differential area, dA, and is together comprised of its tangential, V_t, and normal, V_n, velocity components.

Figure 2.4: Typical model component within an n-component system showing its mass or energy storage, x_i; control surface, CS; control volume, CV; intercompartmental flows into i, f_{ij}; intercompartmental flows out of i, f_{hi}; input boundary flow, z_i; and output boundary flow, y_i. j and h represent the rest of the components in the system, where $j = h = 1, 2, \ldots, n$; $j \neq i$; $h \neq i$. By convention, flow f_{ij} is from j to i and f_{hi} is from i to h.

Figure 2.5**:** Seven component Neuse River estuary nitrogen model (Christian and Thomas 2000, 2003). Figure includes the system control volume (CV) and the system control surface (CS) and an example (PN-Phyto) of a typical individual component's CV and CS. Defining the appropriate CS's and CV's is required per the Reynolds transport theorem. Figure shows the intercompartmental flows f_{ij}, input boundary flows z_i, and output boundary flows y_j,

Figure 2.1

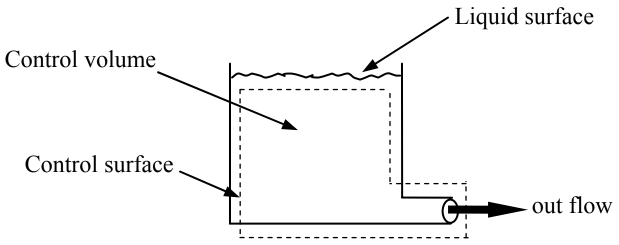


Figure 2.2

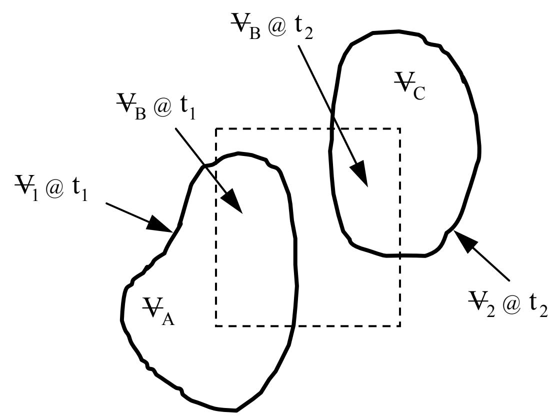


Figure 2.3

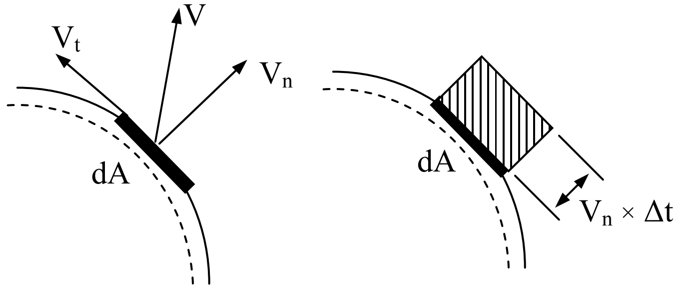


Figure 2.4

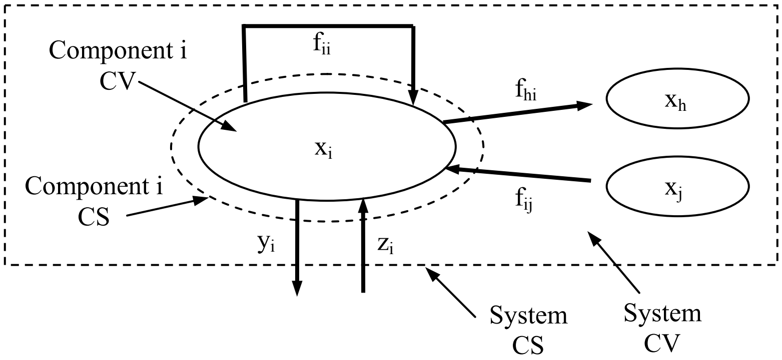
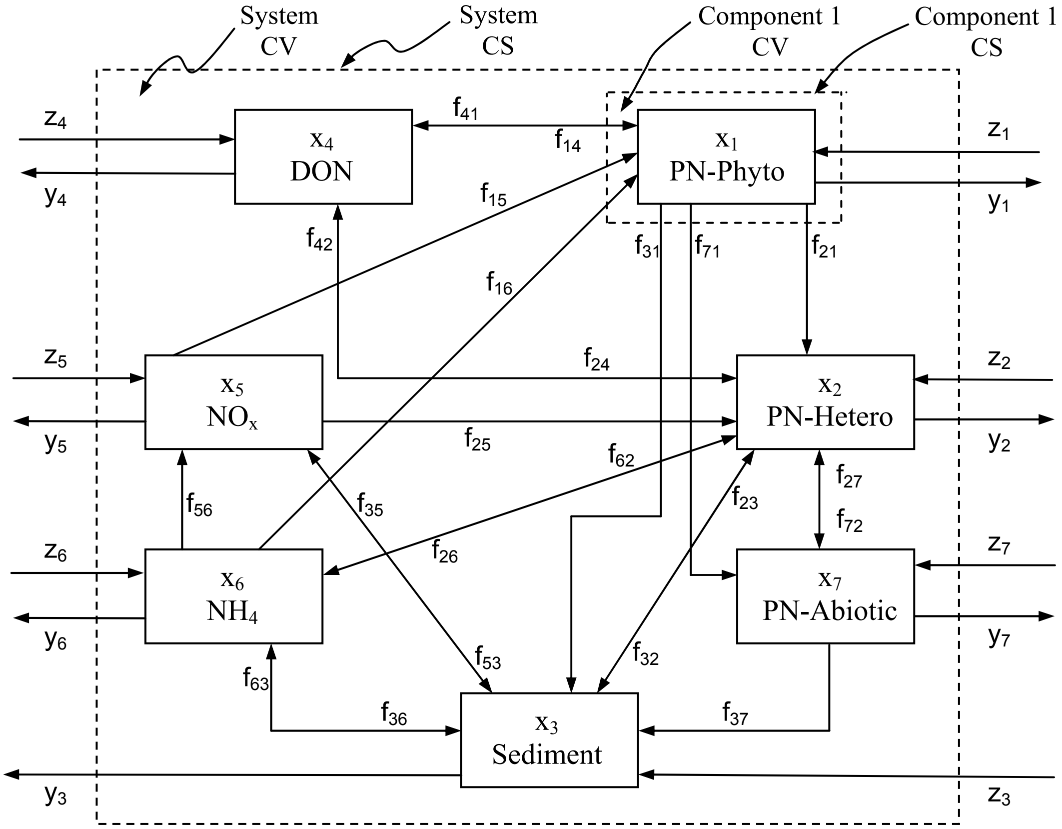

Component i CV
Component i CS
f_{ii}
x_i
f_{hi}
x_h
f_{ij}
x_j
y_i
z_i
System CS
System CV

Figure 2.5


System CV
System CS
Component 1 CV
Component 1 CS
z4
y4
x4 DON
f41
f14
x1 PN-Phyto
z1
y1
f15
f42
f31
f71
f21
f16
z5
y5
x5 NOx
f24
f25
x2 PN-Hetero
z2
y2
f62
f27
f72
f56
f35
f23
f26
z6
y6
x6 NH4
x7 PN-Abiotic
z7
y7
f53
f32
f63
f36
f37
x3 Sediment
y3
z3

CHAPTER 3

DISTRIBUTED CONTROL IN THE ENVIRON NETWORKS OF A SEVEN-COMPARTMENT MODEL OF NITROGEN FLOW IN THE NEUSE RIVER ESTUARY, USA—STEADY-STATE ANALYSIS[1]

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[1] This chapter advances the concepts and formulations of a previously published paper: Schramski, J.R., Gattie, D.K, Patten, B.C., Borrett, S.R., Fath, B.D., Thomas, C.R., and Whipple, S.J., 2006. Ecological Modelling, **194**:189-201, reprinted here with permission of publisher

ABSTRACT

Comparative network environ analysis (NEA) is used to holistically explore controlling relationships in ecosystems. The term *distributed control* for the present series of papers is adopted to describe a diffuse and decentralized concept of control residing in the complexity of organization. Starting with "open-loop" control theory parlance, three ecological control metrics (*control ratio, control difference, and system control*) are derived in an environ-theoretic framework by considering pair-wise and system-wide distributed control relationships. These control relationships are then explored using the seven compartment steady-state model of average nitrogen flow in the Neuse River estuary, North Carolina, USA (Christian and Thomas 2003). The model compartments of Sediment and Nitrates-Nitrites (NO_x) are shown to participate in opposing roles. If nitrogen-exchange magnitude denotes proportional dominance and therefore control, the control metrics reveal that Sediment is overwhelmingly controlled by all components, whereas NO_x exhibits control over all components. However, if a limiting factor perspective is used (e.g., Sediment sequestering N with a controlled release to NO_x), the conclusions are opposite; Sediment controls all other components and all other components control NO_x. As such, the interpretation of the ecological control relationship is still under review. The control metrics developed should aid in this assessment.

3.1 INTRODUCTON

Ecologists often speak loosely about “control” and “regulation.” Holistic knowledge of controlling relationships in ecosystems is one of the primary reasons for understanding system structure and function through modeling. Knowledge of proximate (direct), or especially distal (indirect), control points affords managers the ability to exert control, either close-in or at a network distance — in other words, one should have a capability to implement the elusive concept of "ecosystem management."

Ecological systems are complex constructs of growth (positive feedback) and subsequent restrictions (negative feedback) maneuvering in concert towards various behaviors (Rosenblueth et al., 1943) or homeostatic equilibria (performance indices). When described as such, modern control theory lexicon is intrinsically woven into this model of biological systems operation at virtually all hierarchical resolution levels. Chapman’s (1928) *Environmental Resistance*, von Liebig’s (1841) *Law of the Minimum*, Blackman’s (1905) *Law of the Maximum*, Shelford’s (1911) *Law of Toleration*, Verhulst’s (1838) *Carrying Capacity*, and Tilman’s (1982) *Resource Competition* theory represent but a few examples of various forms of feedback (or possibly feedforward) restrictions to growth. Similarly, but with considerably less volume of activity, the concept of positive feedback can also be found in the historical literature with Malthus’ exponential growth (1798) and Chapman’s *Biotic Potential* (1928) or the modern review by DeAngelis and Post (1991). And finally, performance indices (i.e., teleology, goal functions, eco-targets, orientors, etc.) have been debated in various forms (Clements, 1936; Gleason, 1926; Whittaker, 1953) and again are garnering attention (Müller et al., 1998; Jørgensen, 2000; Fath et al., 2001).

However, the umbrella of ecology quickly conjures a complexity (e.g., behavioral, evolutionary, temporal, spatial, etc.) which challenges the basic modern control theory's ability to successfully or usefully map the expected system behavior. Subsequently, notions of controlling relationships in ecology are dispersed from the formal theory to more project-specific terms, e.g., dominating/subordinating, augmenting/limiting, releasing/constraining, liberating/regulating, bottom-up/top-down, etc.

Accordingly, herein we consider that in ecosystems there are no explicit controllers that parallel the thermostats and autopilots and other discrete regulators in man-made systems. Instead, the concept of control somehow resides in the complexity of organization — in the interactive networks that join things together, and specifically in the environs into which these networks can be decomposed. The concept is thus one of diffuse, decentralized, and even (at a network distance) remote control; for present purposes in this set of papers we have adopted the term *distributed control*. Within the framework of *network environ analysis* (NEA) (Patten, 1978a, b; Patten and Auble, 1981; Fath and Patten, 1999; Gattie et al., 2006a) this kind of control can, with definition of appropriate measures derived from the primary environ analyses of pathways and throughflows, be fruitfully investigated.

NEA assumes a system is both a component of a greater system and also itself comprised of components. A three-level hierarchy exists to model this interpretation, with Koestler's (1967) term *holon* H used to denote any of these hierarchical levels (Patten and Auble, 1981). As such, a system H, is by definition a connected set of holons H_i, i = 1,2,…, *n*. Each component H_i has an incoming interactive network that brings in energy and matter, and an outgoing network that takes energy and matter away. These are, respectively, input and output environments of that holon, and when they are bounded by systems to which the holons belong, they are *environs*. These environs are partition elements of the entire interior flow–storage network, in two orientations: afferent (input environs) and efferent (output environs). The system of interacting components has an external reference environment with which it, as an open system, exchanges energy, matter, and information. As measurable units, environs stop at the system boundary where formal system description ends.

Equations (3-1) and (3-2) represent the throughflow equations for an *n*-component system model at steady-state, as developed from the component conservation equations (Leontief, 1936; Barber et al., 1979):

$$[\mathrm{T}]^{\mathrm{out}}_{n\times 1} = [\mathrm{N}]_{n\times n}\times[\mathrm{z}]_{n\times 1} \tag{3-1}$$

$$[\mathrm{T}]^{\mathrm{in}}_{1\times n} = [\mathrm{y}]_{1\times n}\times[\mathrm{N'}]_{n\times n} \tag{3-2}$$

Where the italic *n* is the number of system components. The transitive closure matrices N and N' relate the system level boundary input z and output y vectors, respectively, over all direct and indirect pathways, to the compartmental throughflow vector T. By definition (Patten, 1978a), the input and output environs are implicit in the transitive closure matrices N' and N (aptly named as they account for all causal transmissions of introduced energy or matter while it remains in the system). Component inputs, z, are mapped forward through the output environs implicit in N to the respective component output throughflows, $\mathrm{T}^{\mathrm{out}}$. Similarly, component outputs, y, can be mapped backwards through the input environs implicit in N' to the component input throughflows, T^{in}. Gattie et al. (2006a) review the environ networks of the average steady-state nitrogen flows of the Neuse River estuary. Considering the enfolded indirect effects of the transitive closure matrices, N and N', Borrett et al. (2006) explore the temporal variation of these indirect effects within the nitrogen flows over 16 seasons in the same estuary.

3.2 PAIR-WISE ENVIRON RELATIONS IN A NETWORK

A perspective on a decentralized or diffuse relationship begins by considering a pair-wise view of two components H_i and H_j in a multi-component ($i, j = 1, \ldots, n$) system. Assuming the two-environment partitioning property of environ theory (Patten, 1978a, b), and an internally connected system, any two components (H_i and H_j) are directly and indirectly related to each other to the extent that their respective input and output environs overlap. Accounting for the throughflow at both H_i and H_j, equations (3-1) and (3-2) are rewritten in scalar form as summations in Table 3.1. [Table 3.1 here] The pair-wise dual environ relationship specifically between H_j and H_i is found from the respective terms (partition elements) in the summations of the T_i equations (3-3) and (3-4) where $k = j$. Then, $T_{ij}^{in} = n_{ij} \times z_j$ and $T_{ji}^{out} = y_j \times n'_{ji}$, respectively. Similarly, the specific reciprocal relationship between H_i and H_j is found from the respective partitions of the T_j in equations (3-5) and (3-6) where $k = i$. Then, $T_{ji}^{in} = n_{ji} \times z_i$ and $T_{ij}^{out} = y_i \times n'_{ij}$, respectively. Figure 3.1 displays these pair-wise dual environ relationships, with originating and terminal components reversed in Figures 3.1a and 3.1b [Figures 3.1 here]. The environs depicted in Figures 3.1a and 3.1b, derived from the respective terms from equations (3-3) through (3-6), represent a pair-wise controlling relationship differential. That is, considering Figure 3.1a, H_j's relationship with H_i, extending forward, n_{ij}, is different than H_j's relationship with H_i reaching backward via n'_{ji}. This relationship between two compartments H_i and H_j is comprised of at least two qualities: (1) The existence of a distinct topological connectance comprised of both direct and indirect pathways, and (2) A magnitude of throughflow between the two components as ultimately generated by flow across the system boundary. Magnitudes of throughflow are almost certainly coupled to the respective pair-wise connectance between two components. To the contrary, however, connectance can be evaluated independently of the associated flows.

The adjacency matrix, A = (a_{ij}), represents a system's direct component connectance where $a_{ij} = 1$ and $a_{ij} = 0$ are interpreted as a direct connection or no connection (no direct flow), respectively, from H_j to H_i (note the columns-to-rows orientation established in these definitions). Further, the adjacency matrix raised to the m power $(a_{ij})^m$ (matrix multiplication not scalar powers) enumerates the indirect pathways of length m between two components wherein the numerical whole number in an i, j interstitial location of the matrix $(a_{ij})^m$ is the number of indirect pathways between components H_j and H_i of length m. The i, j entry of $(a_{ij})^m$ is denoted $a_{ij}^{(m)}$ by standard notation. A comparison of the same interstitial i, j location, in both the efferent and afferent adjacency matrices, $[(a_{ij})]^m$ and $[(a_{ij})^T]^m$ (transpose), respectively, shows that pathway proliferation between two components is increasingly different depending on direction (H_j extending forward to H_i versus H_j reaching back to H_i) as the path length, m, increases without bound:

$$a_{ij}^{(m)} \neq a_{ij}^{T(m)}\text{, as path length, m, increases.} \tag{3-7}$$

Therefore, connectance between two components, H_i and H_j, is distinctly directional and ultimately different between input and output environs.

3.3 SYSTEM-LEVEL CONTROL RELATIONSHIPS

Differences in afferent versus efferent connectance reflect the inherent differences between respective input and output environs. Therefore, considering Figure 3.1a, the relationship of H_i in the output environ of H_j will be different from H_i in the input environ of H_j. Two observations can be made concerning this type of network model of a system at steady-state: (1) The transitive closure matrices can be considered simple open-loop controller models of the components in the network, where input or output across the system boundary is modified (augmented or diminished) before being routed to or from another component in the system; and (2) Although feedback certainly exists in the system, the boundary-component-boundary relationships as represented by each term in the summation equations (3-3) through (3-6) in the steady-state model are transactional open-loop controllers with no modeled feedback. The feedback is inherent in the distributed interactions of the open-loop controllers acting in parallel. Considered in total, the sum of the pair-wise component relationships throughout a system can be considered a distributed control network.

In an open-loop control system, as shown in Figure 3.2, the output is neither measured nor fed back for comparison with the input. Nor does the output have an effect on the controlling action. [Figure 3.2 here] Certainly feedback exists in numerous dimensions, at multiple resolutions, in any ecosystem. However, an open-loop control model of a steady-state system is an opportunity to inspect the resultant system performance after the feedbacks have brought the system to a reasonable homeostatic equilibrium. That is, in ecosystems after a disturbance, a transient positive/negative feedback control scheme runs its course (non steady-state), and the resultant steady-state construct of components and flows is a balanced cyclic mapping of inputs, outputs, and controllers (amplifiers and attenuators). Such distributed networks of open-loop controllers, constructed of the integral matrices, constitutes a system of checks and balances within the direct and indirect flow networks. Considering Figure 3.1a, left side, H_j modifies (augments or attenuates) the input from H_i and then returns a different magnitude y_j out across the boundary of the system. Similarly, Figure 3.1a, right side, if z_j is retrieved across the boundary, H_j modifies the input from the boundary and then returns a modified value to H_i. The additional magnitude which has been added or subtracted during its passage through H_j in either direction is captured, or distributed, respectively, elsewhere in the system. This difference between mutual input and output environ networks represents a pair-wise control or dominance relationship (Patten, 1978b; Patten and Auble, 1981). For present purposes, if a component H_i in a system contributes more energy-matter to H_j than H_j to H_i, then H_i dominates H_j and to that extent can be said, in an augmentative sense, to "control" it. That is, assuming magnitude denotes dominance and if one holon is dominant over another in a system, the former will be said to control the latter. The distribution of these pair-wise transfer functions provides for a system level distributed control model.

Two mathematical perspectives of open-loop transfer functions exist: 1) a model comprised of partitions of throughflows can be constructed of functional controllers that operate in a component-relative distributed control model, and 2) a model comprised of fractions of throughflows can be constructed of fractional controllers which operate in a system-absolute distributed control model. Although the next two sections review both to differentiate the two, this chapter will focus on system-absolute control measures. Component-relative control measures will follow in a later study.

3.4 RELATIVE ENVIRON CONTROL

The partitioned component throughflows to which each summation in equations (3-3) through (3-6) is comprised represent a set of functional open-loop controllers, n_{ik}, coupled with their boundary flows, z_k or y_k, all ultimately contributing to the total throughflow at a component.

Example, Equation (3-3) expanded for a focal component H_3 in a multi-component system takes the form:

$$T_3^{out} = [n_{31} \times z_1]^{in} + [n_{32} \times z_2]^{in} + \ldots + [n_{3n} \times z_n]^{in}, \qquad n = \text{number of system components}. \qquad (3\text{-}3a)$$

From equation (3-3a) and Figure 3.1a right side, a partition of the total input throughflow to component H_3, T_3^{in}, contributed from component H_1 could be written:

$$T_{31}^{in} = n_{31} \times z_1 \tag{3-3b}$$

Considering equation (3-3b), z_1 is the input to H_1, n_{31} is the functional controller or functional transfer function and T_{31}^{in} is the directed input to H_3 from H_1 generated by z_1. Similarly, equation (3-4) expanded for a focal component H_3 in a multi-component system takes the form:

$$T_3^{in} = \left[y_1 \times n'_{13}\right]^{out} + \left[y_2 \times n'_{23}\right]^{out} + \ldots + \left[y_n \times n'_{n3}\right]^{out}, \quad n = \text{number of system components} . \tag{3-4a}$$

From equation (3-4a) and Figure 3.1a left side, a partition of the total output throughflow from component H_3, T_3^{out}, contributed to component H_1 could be written:

$$T_{13}^{out} = y_1 \times n'_{13} \tag{3-4b}$$

Considering equation (3-4b), y_1 is the output of H_1, n'_{13} is the functional controller or functional transfer function, and T_{13}^{out} is the directed output from H_3 to H_1 required to generate y_1.

Considering the terms of the summation equation (3-3), z_k are the inputs and n_{ik} are the functional controllers where T_{ik}^{in} then represents the respective partitioned throughflow inputs to H_i from each of the components H_k for each z_k. In NEA methodology, n_{ik} implicitly represents the output environs that map each input vector z_k forward in time from each component H_k to the resultant T_{ik}^{in} partitioned input throughflow at component H_i. Likewise, considering the terms of the summation equation (3-4), y_k are the outputs and n'_{ki} are the functional controllers where T_{ki}^{out} then represents the respective partitioned throughflow outputs from H_i to each of the other components H_k for each y_k. In NEA methodology, n'_{ki} implicitly represents the input environ flows that map the outputs of y_k backward from component H_k to the resultant historical T_{ki}^{out} partitioned output throughflow at component H_i.

Numbers of pathways (H_j forward to H_i versus H_i backward to H_j) between any two components are equal as shown in powers of the system adjacency matrices,

$$a_{ij}^{(m)} = a_{ji}^{T(m)} \qquad m = \text{pathway length} = 1, 2, \ldots, \infty. \tag{3-8}$$

Yet, the corresponding input and output environ elements (n'_{ij} and n_{ij}) as developed in equations (3-3) and (3-6) are not equal. The conservation of mass in these equations (Hannon, 1973; Barber et al., 1979) is component specific in that each equation represents a mass balance of throughflow at each component. The partition transfer functions detailed in equations (3-3) and (3-6) are a function of their component throughflows. For example, consider the environ perspective depicted in Figure 3.1 Since n_{ij} (Figure 3.1a, right side) is derived from the perspective of H_i, it is a function referenced to T_i [equation (3-3)]. In comparison, n'_{ij} (Figure 3.1b, left side) relates to the perspective of H_j and is therefore referenced to T_j, [equation (3-6)]. As such, since n_{ij} and n'_{ij} are calculated with respect to different component throughflows, they are directionally specific and therefore $n_{ij} \neq n'_{ij}$. H_j controls H_i through the output environ, implicit in n_{ij}, of H_j (Figure 3.1a, right side). However, considered from H_i's perspective, H_i is controlled by H_j through H_i's input environ contained in n'_{ij} (Figure 3.1b, left side). Therefore, in relative environ controlling relationships, since $n_{ij} \neq n'_{ij}$, the control relationship propagated from H_j through its output environ to H_i (Figure 3.1a, right) can be different in magnitude than the reciprocal control relationship received at H_i through its input environ from H_j (Figure 3.1b, left). The control measure calculations are relative to a specific component's throughflow, and as such, relative environ control measure calculations are component-relative computations. Relative environ controlling relationships and their component-relative consequences will be considered further in future studies.

3.5 ABSOLUTE ENVIRON CONTROL

The transfer weightings are normalized on the basis of their component throughflows, where equations (3-3) through (3-6), shown in Table 3.1, are rewritten from a partition throughflow derivation to a fraction of throughflow derivation as shown in Table 3.2. [Table 3.2 here] Unlike each of the parenthetical terms in the summations of equations (3-3) through (3-6) which were partition elements of throughflow, the parenthetical nondimensional term in each new summation represents a directional specific (the interstitial subscripts on the respective elements of the transitive closure matrices, N and N', denote donor and recipient components) fraction of throughflow. Considering two components H_i and H_j in an *n*-component system, the pair-wise dual environ relationship between H_j and H_i is found from the respective fractional terms of T_i in equations (3-9) and (3-10), where k = j. Then, $T_{ij}^{in}/T_i^{out} = (n_{ij} \times z_j)/T_i^{out}$ and $T_{ji}^{out}/T_i^{in} = (y_j \times n'_{ji})/T_i^{in}$, respectively. Similarly, the reciprocal relationship specifically between H_i and H_j is found from the respective fractional terms of T_j in equations (3-11) and (3-12), where k = i. Then, $T_{ji}^{in}/T_j^{out} = (n_{ji} \times z_i)/T_j^{out}$ and $T_{ij}^{out}/T_j^{in} = (y_i \times n'_{ij})/T_j^{in}$, respectively. Figure 3.3 graphically displays these pair-wise dual relationships. [Figure 3.3 here] The environs depicted in Figures 3.3a and 3.3b, as derived from the respective fractional controllers from equations (3-9) through (3-12), are the foundation of a pair-wise controlling relationship differential. That is, considering Figure 3.3a, H_j's relationship, n_{ij}/T_i^{out}, with H_i as extended forward through H_j's output environ, is different than H_j's relationship, n'_{ji}/T_i^{in}, with H_i reaching backward through H_j's input environ. That is, $n_{ij}/T_i^{out} \neq n'_{ji}/T_i^{in}$. The difference in magnitude of these weightings establishes a pair-wise control measure differential formulated below.

The fractional throughflows of which each summation in equations (3-9) through (3-12) are comprised can also be interpreted as a set of open-loop controllers, e.g., the product of n_{ik}/T_i^{out} and the boundary flow z_k contribute a fraction of throughflow, T_{ik}^{in}/T_i^{out}, to the total normalized throughflow at a component.

Example: Equation (3-9) expanded for a focal component H_3 in a multi-component system takes the form:

$$1=\frac{[n_{31}\times z_1]}{T_3^{out}}+\frac{[n_{32}\times z_2]}{T_3^{out}}+\ldots+\frac{[n_{3n}\times z_n]}{T_3^{out}}, \qquad n = \text{number of system components}\,. \tag{3-9a}$$

From equation (3-9a) and Figure 3.3a right side, the fraction of the total output throughflow at component H_3, T_3^{out}, contributed from component H_1 could be written:

$$\frac{T_{31}^{in}}{T_3^{out}}=\frac{n_{31}}{T_3^{out}}\times z_1 \tag{3-9b}$$

Considering equation (3-9b), z_1 is the input of H_1, n_{31}/T_3^{out}, expressed in reciprocal throughflow units, is the modeled fractional controller or fractional transfer function, and T_{31}^{in}/T_3^{out} is the fraction of throughflow from H_1 oriented to H_3 generated by z_1. Similarly, equation (3-10) expanded for a focal component H_3 in a multi-component system takes the form:

$$1=\frac{[y_1\times n'_{13}]}{T_3^{in}}+\frac{[y_2\times n'_{23}]}{T_3^{in}}+\ldots+\frac{[y_n\times n'_{n3}]}{T_3^{in}}, \qquad n = \text{number of system components}\,. \tag{3-10a}$$

From equation (3-10a) and Figure 3.3a left side, the fraction of the total input throughflow at component H_3, T_3^{in}, contributed to component H_1 could be written:

$$\frac{T_{13}^{out}}{T_3^{in}}=y_1\times\frac{n'_{13}}{T_3^{in}}\,. \tag{3-10b}$$

> Here, y_1 is the output of H_1, n'_{13}/T_3^{in}, again expressed in reciprocal throughflow units, is the modeled fractional controller or fractional transfer function, and T_{13}^{out}/T_3^{in} is the fraction of throughflow from H_3 oriented to H_1 required to generate y_1.

Considering the fractions in the summation equation (3-9), z_k are the inputs and n_{ik}/T_i^{out} are the fractional controllers, where T_{ik}^{in}/T_i^{out} can then represent the respective fractions of throughflow outputs from each of the other components H_k to the focal component H_i for each input to H_k, z_k. In NEA methodology, the magnitude of n_{ik}/T_i^{out}, in reciprocal throughflow units, implicitly represents the output environs which map inputs z_k forward in time from each component H_k to each corresponding fraction of throughflow at H_i, T_{ik}^{in}/T_i^{out}. Likewise, considering the fractions in the summation equation (3-10), y_k are the outputs and n'_{ki}/T_i^{in}, in reciprocal throughflow units, are the fractional controllers where T_{ki}^{out}/T_i^{in} can then represent the respective fractions of throughflow inputs to each of the other components H_k from H_i for each output at H_k, y_k. In NEA methodology, the magnitude of n'_{ki}/T_i^{in} implicitly represents the input environs which map the outputs in vector y_k back from each component H_k to their corresponding fraction of throughflow T_{ki}^{out}/T_i^{in} at H_i.

The respective fractional transfer coefficients η_{ij} and η_{ji} are by definition:

$$\eta_{ij} \equiv \left[\frac{n_{ij}}{T_i^{out}}\right] = \left[\frac{n'_{ij}}{T_j^{in}}\right] \qquad i, j = 1, 2, \ldots, n, \qquad (3\text{-}13)$$

$$\eta_{ji} \equiv \left[\frac{n'_{ji}}{T_i^{in}}\right] = \left[\frac{n_{ji}}{T_j^{out}}\right] \qquad i, j = 1, 2, \ldots, n, \qquad (3\text{-}14)$$

where typical dimensional units are reciprocal mass $[M^{-1}L^2T]$ or energy $[M^{-1}T^3]$ throughflow per area in the MLT system of units. The lower case italic *n* is the number of compartments in the system. Note, on the left-hand sides, the use of the Greek letter eta, η, for the fractional transfer coefficients, specifically differentiating from the Arabic n used throughout the text for the transitive closure matrix values. Referring to equations (3-9) with (3-13) and (3-10) with (3-14), it is evident that the η_{ij} coefficients reference driving inputs, z_k, and concluding outputs, y_k, to unit throughflows, $T_i^{out} = T_i^{in} = 1$ at every component, i = 1, 2, …, *n*. This partitioning of unit throughflows is evident in rewriting (3-9), and (3-10) as:

$$1 = \eta_{i1}z_1 + \eta_{i2}z_2 + \ldots + \eta_{in}z_n, \tag{3-9b}$$

and,

$$1 = y_1\eta_{j1} + y_2\eta_{j2} + \ldots + y_n\eta_{jn}. \tag{3-10b}$$

These partition elements (each term) show at a glance the involvement of each boundary input and output with the unitized steady-state throughflows (see later, equation (3-21) for the Neuse River estuary nitrogen system). The existence of the two equations (3-13) and (3-14) for η_{ij} demonstrates the directional reciprocity of the fractional control measure. Unlike the relative environ control measure, for a given control relationship between two components, the absolute environ control measure calculations are not specific to the component under consideration. For example, consider the two components H_i and H_j as represented in equation (3-14). The focal component H_i [Figure 3.3a, left side and equation (3-10)], oriented forward through the input environ of H_j, controls H_j as represented in magnitude by n'_{ji}/T_i^{in}. From a different component perspective, the focal component H_j is controlled by H_i, as represented in magnitude by n_{ji}/T_j^{out}, oriented backwards through H_i's output environ [Figure 3.3b, right side and equation (3-11)]. However, since $n'_{ji}/T_i^{in} = n_{ji}/T_j^{out}$, equation (3-14), the transfer function relationship is not component specific. Once the direction of control is established, in this example H_i controlling H_j, the same quantitative control relationship exists for H_i over H_j regardless of whether it is oriented forward through H_j's input environ or backward through H_i's output environ. Fractional environ control transfer function values are not relative to the focal component under consideration and are otherwise stated to be system absolute. This observation is the core premise and foundation behind throughflow-based *absolute environ control* terminology and methodology.

Mindful of the equivalences expressed in equations (3-13) and (3-14), several absolute control metrics can be defined. Pair-wise component comparisons are enabled by a dimensionless *control ratio*:

$$cr_{ij} \equiv \frac{\eta_{ij} - \eta_{ji}}{\max(\eta_{ij}, \eta_{ji})}, \qquad i, j = 1, 2, \ldots, n, \qquad 0 \leq |cr_{ij}| \leq 1. \qquad (3\text{-}15)$$

System-based comparisons of the fractional transfer values are can be made by the *control difference*:

$$cd_{ij} \equiv \eta_{ij} - \eta_{ji}, \qquad i, j = 1, 2, \ldots, n, \qquad 0 \leq |cd_{ij}| < \infty. \qquad (3\text{-}16)$$

Both cr_{ij} and cd_{ij} are interpreted from j to i using “controls” for cr_{ij}, $cd_{ij} > 0$ and “is controlled by” for cr_{ij}, $cd_{ij} < 0$. For example, cr_{13} means, H_3 controls H_1 with a magnitude of $|cr_{13}|$ when $cr_{13} > 0$ and H_1 controls H_3 with a magnitude of $|cr_{13}|$ when $cr_{13} < 0$. The magnitudes $|cr_{ij}|$ and $|cd_{ij}|$ denote the level of control, and the signs, + or −, denote direction of control. By definition, the control ratio matrix, CR, is a collection of values each calculated with different denominators. As such, adding individual cr_{ij} values across rows or down columns is not mathematically appropriate or ecologically revealing. The benefit of the cr_{ij} calculation is that it represents the strength of the difference between the two fractional open-loop control magnitudes referenced to the maximum open-loop control magnitude of only those two components. Each cr_{ij} is a unique pair-wise comparison of the input and output fractional transfer weights between H_i and H_j. The cd_{ij} values in the control difference matrix CD are differences between the fractional transfer values ($\eta_{ij} - \eta_{ji}$) acting from H_j and H_i. Since these are not scaled to the local pairs involved as lacking the division operation of the largest fractional transfer value [max (η_{ij} – η_{ji})], does for the cr_{ij} values, the cd_{ij} values remain comparable across all (H_i, H_j) pairs and thus can be considered system-based metrics whose magnitudes are referenced to the whole system. The magnitudes of the cd_{ij} values throughout the CD matrix can be freely compared and contrasted among each one an other. Further, absolute environ control methodology is independent of component pair orientation. Opposite elements across the main diagonals of the CD and CR asymmetric matrices (e.g., cr_{13} versus cr_{31}) are the same magnitudes ($|cr_{13}| = |cr_{31}|$) with opposite signs (directions of control) and are independent of the opposing component pairs (e.g., H_3 and H_1) differing throughflows.

Since magnitudes of the control differences, cd_{ij}, are all referenced to the same system, cd_{ij} values are additive. This additive quality affords the computation of a system versus component magnitude of control weighting by summing the rows of the CD matrix to calculate the *system control* vector where by definition:

$$sc_j \equiv \sum_{k=1}^{n} cd_{kj} \qquad j = 1, 2, \ldots, n, \qquad 0 \leq |sc_j| < \infty \qquad (3\text{-}17)$$

where the italic *n* is the number of system components, the "$\sum cd_k$" portion of $\sum cd_{kj}$ represents the system comprised of all recipient components, and the subscript j represents the specific donor component, H_j. Therefore a positive sc_j value denotes a specific focal component's, H_j, control of the system (remaining components combined, k = 1, 2, …, *n*; k ≠ j). Likewise a negative sc_j value denotes that a specific focal component H_j is controlled by the system (remaining components combined, k = 1, 2, …, *n*; k ≠ j). The system's components are effectively ranked on a continuous scale from the most negative to the most positive sc_j where a level of hierarchy for each ecosystem (model) component can be attributed to the sc_j magnitude. All sc_j added together will equal zero denoting total system balance.

3.6 RESULTS: STEADY-STATE NEUSE RIVER ESTUARY

The afferent and efferent dimensionless transitive closure matrices, N and N', and the dimensional throughflow matrix, T, for the 16-sample average of the Neuse River nitrogen cycling data (Christian and Thomas, 2000; Gattie et al., 2006a) are as follows:

$$N = \begin{bmatrix} 9.40 & 8.73 & 5.87 & 8.62 & 8.93 & 8.98 & 7.96 \\ 19.16 & 20.16 & 13.27 & 18.79 & 19.14 & 19.58 & 18.32 \\ 1.58 & 1.56 & 2.04 & 1.50 & 1.58 & 1.57 & 1.64 \\ 3.30 & 3.25 & 2.16 & 4.12 & 3.21 & 3.25 & 2.96 \\ 1.47 & 1.53 & 1.12 & 1.43 & 2.47 & 1.58 & 1.42 \\ 15.15 & 15.92 & 10.61 & 14.85 & 15.13 & 16.48 & 14.50 \\ 2.64 & 2.76 & 1.82 & 2.58 & 2.63 & 2.69 & 3.51 \end{bmatrix} \tag{3-18}$$

$$N' = \begin{bmatrix} 9.40 & 18.92 & 1.06 & 3.35 & 2.04 & 15.50 & 2.41 \\ 8.84 & 20.16 & 1.11 & 3.36 & 2.01 & 15.59 & 2.56 \\ 8.73 & 18.65 & 2.04 & 3.21 & 1.99 & 14.96 & 2.75 \\ 8.51 & 18.12 & 1.01 & 4.12 & 1.88 & 14.46 & 2.31 \\ 6.44 & 14.59 & 0.89 & 2.44 & 2.47 & 11.99 & 1.89 \\ 8.78 & 20.00 & 1.11 & 3.34 & 2.00 & 16.48 & 2.55 \\ 8.69 & 19.75 & 1.09 & 3.30 & 1.98 & 15.30 & 3.51 \end{bmatrix} \tag{3-19}$$

$$T = \begin{bmatrix} 6{,}927 & 15{,}005 & 1{,}255 & 2{,}688 & 1{,}578 & 11{,}955 & 2{,}099 \end{bmatrix}^{T} \frac{\text{mmol}-\text{N}}{\text{m}^2 \times \text{season}} \tag{3-20}$$

The model has seven components, expressed as nitrogen standing stocks (mmol-N/m^2): H_1 = phytoplankton, H_2 = heterotrophs, H_3 = sediments, H_4 = dissolved organic nitrogen, H_5 = NO_x, nitrates-nitrites, H_6 = NH_4, ammonia, and H_7 = abiotic matter. Mnemonic names and numbers for these were established in Chapter 2: PN-Phyto-1, PN-Hetero-2, Sediment-3, DON-4, NO_x-5, NH_4-6, and PN-Abiotic-7.

3.6-1 RELATIVE ENVIRON CONTROL

Consider the relationship between PN-Phyto-1 and Sediment-3 as generalized in Figure 3.1a and shown in Figure 3.4a. [Figure 3.4a here] The input and output environs implicit in n'_{13} =

1.06 and n_{31} = 1.58 (i.e., transfer functions, controllers, etc.), respectively, are not the same which illuminates the differences as to how PN-Phyto-1 receive nitrogen from Sediment-3 and, conversely how PN-Phyto-1 route nitrogen to Sediment-3. This difference is inherent in the transactional (both magnitude and connectance) relationship between the compartments. The transitive closure matrix values [equations (3-18) and (3-19)] which serve as the multipliers for the open-loop controllers of throughflow in the steady-state system also, through their magnitudes, identify the importance of one component in the environ of another. That is, Phyto (1) has a direct and indirect transactional relationship with Sediment-3 expressed through its output environ as specified by the value n_{31} = 1.58. Conversely, PN-Phyto-1 also has a relationship with Sediment-3 looking backward through its input environ expressed in the value n'_{13} = 1.06. Since this specific transactional pair-wise relationship can be represented by $n_{31} > n'_{13}$, on a per unit of boundary flow basis, then PN-Phyto-1 contributes more to Sediment-3 than Sediment-3 to PN-Phyto-1 and, as such, PN-Phyto-1 controls Sediment-3 by the dominance, not limiting-factor, interpretation of "control". Similarly, an example depicting Figure 3.1b is shown in Figure 3.4b where the same conclusion can be derived; $n'_{31} > n_{13}$, therefore PN-Phyto-1 controls Sediment-3. [Figure 3.4b here] However, recall the relative environ control measure is specific to each component. Thus, Figure 3.4a, whose values represent partitions of T_3, shows a magnitude differential of ($n_{31} - n'_{13} = 0.52$). Figure 3.4b, which represents partitions of T_1, shows a differential of ($n'_{31} - n_{13} = 2.86$). The result is the same, PN-Phyto-1 controls Sediment-3. However, the magnitude of the controlling relationship is specific to the component from which it is considered. The relative environ control measure derived with functional transfer values is a component-relative computation, i.e., relative environ control magnitude is in the eyes of the beholder.

3.6-2 ABSOLUTE ENVRION CONTROL

Given equations (3-18), (3-19), and (3-20) for the Neuse River estuary nitrogen cycling model and using equation (3-13) or (3-14) the fractional transfer matrix η is:

$$\begin{array}{rl} & {}_{\text{to } i}\diagdown^{\text{from } j} \\ \eta\times10^{3} = & \begin{bmatrix} 1.36 & 1.26 & 0.85 & 1.24 & 1.29 & 1.30 & 1.15 \\ 1.28 & 1.34 & 0.88 & 1.25 & 1.28 & 1.30 & 1.22 \\ 1.26 & 1.24 & 1.63 & 1.19 & 1.26 & 1.25 & 1.31 \\ 1.23 & 1.21 & 0.80 & 1.53 & 1.19 & 1.21 & 1.10 \\ 0.93 & 0.97 & 0.71 & 0.91 & 1.56 & 1.00 & 0.90 \\ 1.27 & 1.33 & 0.89 & 1.24 & 1.27 & 1.38 & 1.21 \\ 1.26 & 1.32 & 0.87 & 1.23 & 1.25 & 1.28 & 1.67 \end{bmatrix} \dfrac{\text{m}^{2}\times\text{season}}{\text{mmol}-\text{N}}. \end{array} \tag{3-21}$$

Figures 3.5a and 3.5b demonstrate the premise of equations (3-13) and (3-14). The output environ of component H_1, PN-Phyto, oriented towards component H_3, Sediment, is identical to the input environ of component H_3, Sediment, oriented from component H_1, PN-Phyto. That is, n_{31} and n'_{31} map identically to η_{31} through their relationships to T_3 and T_1, respectively. Although η_{31} is one number, it possesses information pertaining to both perspectives. The fractional transfer matrix relationship between a pair-wise relationship of two components is equal and independent of perspective and, as such, represents system-absolute values. This orientation independence identity is isomorphic to the pathway relationship in equation (3-8). Similarly, the output environ relationship of H_3 oriented to component H_1 as depicted by η_{13} in Figure 3.5b, left side, is identical to the input environ relationship of H_1 oriented from component H_3 in Figure 3.5a, right side. [Figure 3.5a and 3.5b here] In Figure 3.5a, the output environ relationship of H_1 oriented toward H_3, is quantified by $\eta_{31} = 1.26 \times 10^{-3}$ ($\text{m}^2 \times$ season)/mmol-N. This is greater than the input environ relationship of H_1 oriented from H_3, quantified by $\eta_{13} = 0.85 \times 10^{-3}$ ($\text{m}^2 \times$ season)/mmol-N. As such, H_1 is shown again to control H_3.

Equations (3-15) and (3-16) provide two methodologies to quantify or relate the magnitude of controls. The *control ratio* matrix, computed in accordance with equation (3-15) is as follows:

$$\begin{array}{rl} & \diagdown \text{ from } j \\ \text{to } i & \\ CR = & \begin{bmatrix} 0 & -0.01 & -0.33 & 0.01 & 0.28 & 0.02 & -0.08 \\ 0.01 & 0 & -0.29 & 0.04 & 0.24 & -0.02 & -0.07 \\ 0.33 & 0.29 & 0 & 0.33 & 0.44 & 0.29 & 0.34 \\ -0.01 & -0.04 & -0.33 & 0 & 0.24 & -0.03 & -.11 \\ -0.28 & -0.24 & -0.44 & -0.24 & 0 & -0.21 & -0.28 \\ -0.02 & 0.02 & -0.29 & 0.03 & 0.21 & 0 & -0.05 \\ 0.08 & 0.07 & -0.34 & 0.11 & 0.28 & 0.05 & 0 \end{bmatrix} . \end{array} \qquad (3\text{-}22)$$

The presentation of the CR matrix of values is identical in concept to the control matrix (c_{ij}) in Patten's (1978b) and Patten and Auble's (1981) study of the Cone Spring energy model with a few minor distinctions. The cr_{ij} relationship maintains column to rows progression, i.e., component H_1 controls component H_3 with a weighted value of $cr_{31} = 0.33$. This j to i format is consistent with the N and N' matrices that construct the η and cr_{ij} values. Also, the reciprocity of the fractional controlling relationship, as equated in equations (3-13) and (3-14), i.e., $cr_{ij} = -cr_{ji}$, is recognized and all elements including the equal in magnitude but opposite in sign negative control ratios, $-cr_{ij}$, remain in the matrix. This is somewhat redundant information but, left in place, demonstrates the overall controlling relationship balance necessary for whole system balance. The principal diagonal elements of CR, cr_{ii}, are 0, which can be interpreted identically to Patten's (1978b) and Patten and Auble's (1981) diagonal elements of $c_{ii} = 1$, meaning that a component H_i is equally important in its own output environ implicit in N per unit input z, as in its input environ implicit in N' per unit output y. H_i is related to itself equally through either its output or input environs and, as such, maintains no controlling advantage over itself as the result of an afferent or efferent perspective.

Each element of the CR matrix has a range of $0 \leq |cr_{ij}| \leq 1$. A value (magnitude) of 1 represents maximum control of a component by another (as explained in Section 3.5, positive or negative signs represent direction of control), and is indicative of an observed direct flow of substance from the former to the latter, with no direct or indirect return flow. An example of such a pair-wise maximal control can be found in the Cone Spring Energy Model (Patten, 1978b; Patten and Auble, 1981). In their modeled aquatic energy cycle, plants have complete control ($c_{ij} = +1$) over every other component. Reciprocally, the other components (detritus, bacteria, detritivores, and carnivores) have no control over plants as there is no appreciable flow of energy from the remaining components in the Cone Spring Model back to the plants. Patten and Auble's control measures, at the maximal control extremes, make intuitive sense in this circumstance.

Considering individual component-to-component comparisons, NO_x-5 and Sediment-3 show enhanced but opposite roles in the seven component model. Sediment is controlled ($cr_{i3} < 0$ for $i = 1, 2, \ldots, 7, i \neq 3$) by all other components whereas NO_x controls all other components ($cr_{5j} > 0$ for $j = 1, 2, \ldots, 7, j \neq 5$). Further, the magnitudes of the control ratios associated with Sediment and NO_x are the highest in the CR matrix, the maximum being $cr_{35} = 0.44$ representing the NO_x controlling Sediment. The remaining five components appear to have no discernible pattern of involvement as a group to the remaining components, and all appear diminished compared to the controlling relationships of NO_x and Sediment to the other components.

Using equation (3-16), equation (3-23) documents the fractional *control difference* measure between pairs of components. Similarly, equation (3-24) displays the results of equation (3-17), the *system control* vector, by calculating each component's aggregate (columns) fractional transfer functions:

$$\mathrm{CD} \times 10^4 = \begin{array}{r} \diagdown\ \text{from j} \\ \text{to i} \end{array} \begin{bmatrix} 0 & -0.2 & -4.1 & 0.2 & 3.6 & 0.3 & -1.0 \\ 0.2 & 0 & -3.6 & 0.4 & 3.0 & -0.3 & -0.9 \\ 4.1 & 3.6 & 0 & 3.9 & 5.5 & 3.6 & 4.4 \\ -0.2 & -0.4 & -3.9 & 0 & 2.9 & -0.3 & -1.3 \\ -3.6 & -3.0 & -5.5 & -2.9 & 0 & -2.6 & -3.5 \\ -0.3 & 0.3 & -3.6 & 0.3 & 2.6 & 0 & -0.7 \\ 1.0 & 0.9 & -4.4 & 1.3 & 3.5 & 0.7 & 0 \end{bmatrix} \frac{\mathrm{m}^2 \times \mathrm{season}}{\mathrm{mmol-N}} \qquad (3\text{-}23)$$

$$\mathrm{sc}_{\mathrm{j}} \times 10^4 = \begin{bmatrix} 1.3 \\ 1.2 \\ -25.2 \\ 3.2 \\ 21.2 \\ 1.3 \\ -3.0 \end{bmatrix} \frac{\mathrm{m}^2 \times \mathrm{season}}{\mathrm{mmol-N}} \qquad (3\text{-}24)$$

The CD matrix, interpreted as before from columns to rows (j → i), represents scaled control measures from 0 to the maximum magnitude in the matrix. For example, $cd_{31} = 4.1 \times 10^{-4}$ ($m^2 \times$ season)/mmol-N means that H_1, PN-Phyto, controls H_3, Sediment, with a magnitude of 75% ($4.1/5.5 \times 100$) of the maximum controlling relationship, which is $cd_{35} = 5.5 \times 10^{-4}$ ($m^2 \times$ season)/mmol-N [NO_x-5 controlling Sediment-3]. The cd_{ij} control measures are absolute values which allow comparison of two cd_{ij} magnitudes by either a direct comparison or relative percentages. Recall the cr_{ij} values are calculated with different denominators where individual comparisons need to be qualified appropriately. Considering the CD matrix, the highest magnitude of control exists over Sediment-3. Not only are there no constituents on which Sediment-3 exerts control (the third column of CD has all negative numbers) but the magnitudes of control on Sediment-3 (third row) represent the seven highest values in the matrix (cd_{31} through $cd_{37} \geq$ remaining cd_{ij}). Although by smaller magnitudes, the remaining system components except Sediment (H_1 through H_6, except H_3) also control PN-Abiotic, H_7, i.e., cd_{71},

cd_{72}, cd_{74}, cd_{75}, and cd_{76} are all positive numbers. To the contrary, NO_x-5 controls each component in the system with relatively high magnitudes (fifth column elements of CD all positive). While Sediment-3 is overwhelmingly controlled by the system, NO_x-5 overwhelmingly controls the system. Although by small magnitudes, PN-Abiotic-7 is also controlled by the remaining system components except Sediment-3. The system control vector, sc_j, concisely shows that the system overwhelmingly controls Sediment-3 [$sc_3 = -25.2 \times 10^{-4}$ $(m^2 \times \text{season})/\text{mmol-N}$] while, on the other hand, NO_x-5 overwhelmingly controls the system [$sc_5 = 21.2 \times 10^{-4}$ $(m^2 \times \text{season})/\text{mmol-N}$].

3.7 DISCUSSION

In steady-state network environ analysis, the transitive closure matrices, N and N', represent an augmentation or attenuation of the boundary flows (z and y) of a system through all direct and indirect pathways to the respective component throughflows. The individual interstitial transitive closure values, n_{ij} and n'_{ij}, or their magnitude of throughflow fractional equivalents, η_{ij}, can be treated as distributed open-loop controllers or transfer functions in the classical control theory sense. In this, the closed-loop feedback is inherent in the distributed interactions of the open-loop controllers acting in parallel. The strength or magnitude of incoming versus outgoing (environ theory perspective) controllers can then be compared to identify augmentative or suppressive relationships. The controller magnitudes, per se, offer a weighted means to facilitate this comparison. Two physical aspects which contribute to the transfer weights between two components in a multi-component system are system connectance (structure) and throughflow (function) magnitudes. Using the adjacency matrix in equation (3-7), efferent and afferent connectance over increasing path lengths between two components are shown to be different. This identifies, in part, at least one of the reasons for directionally dependent transfer function weightings. This directional dependency is fundamental to the comparative controller differentials which environ distributed control theory captures.

The control ratio, cr_{ij}, inspired by Patten's (1978b) and Patten and Auble's (1981) original environ control measure c_{ij}, offers a preliminary pair-wise comparison of input and output environ transfer functions, and helps introduce the interpretive characteristics of the environ control metrics. In a broad qualitative sense, environ elements implicit in the transitive closure matrices quantify the level of importance of one component in another component's respective environ. "Importance" in the Neuse River estuary model is defined on a unit boundary flow basis as the amount of nitrogen committed as a partition or fraction of a component's throughflow. Considering that each component has a specific amount of nitrogen (total component throughflow) to divide appropriately among six other components, how much does it allocate to each specific component? The varying fractions of nitrogen throughflow committed from one component to another, in a qualitative sense, indicate varying hierarchies of efficiency or importance. Basically, environ elements on a partition-of-throughflow, n_{ij} and n'_{ij}, or fraction-of-throughflow, η_{ij}, basis offer quantitative values to compare a qualitative level of importance of one component in a network to another. If one component is more important or efficient than another, for purposes herein, it can be regarded as dominant. Consider PN-Phyto-1 and Sediment-3. PN-Phyto-1 dominates Sediment-3 as indicated by a relative control ratio weighting of $cr_{31} = 0.33$. As such, at least the following statements are quantifiably true for a unit boundary fractional-environ-control comparison of PN-Phyto-1 and Sediment-3: (1) Sediment, through its output environ, generates a fraction of throughflow [$\eta_{13} = 0.00085$ ($m^2 \times$ season)/mmol-N] that is 33% less than the fraction of throughflow [$\eta_{31} = 0.00126$ ($m^2 \times$ season)/mmol-N] available to Sediment through its input environ from PN-Phyto. (2)

Conversely, PN-Phyto, through its output environ, generates a fraction of throughflow [η_{31} = 0.00126 (m^2 × season)/mmol-N] to Sediment that exceeds the fraction of throughflow [η_{13} = 0.00085 (m^2 × season)/mmol-N] available to PN-Phyto through its input environ from Sediment. (3) PN-Phyto are more efficient at moving nitrogen to Sediment on a per throughflow basis than conversely. Thus, PN-Phyto dominates Sediment. Further, on a unit boundary-flow basis, all components are more efficient at moving nitrogen to Sediment (cr_{31} = 0.33, cr_{32} = 0.29, cr_{34} = 0.33, cr_{35} = 0.44, cr_{36} = 0.29, cr_{37} = 0.34) than conversely. Alternatively, using the Total Dependency and Total Contribution matrices of Ecological Network Analysis (ENA), Christian and Thomas (2003) showed that phytoplankton depended on sediment for 50% of its combined direct and indirect throughflow. Additional analysis by Christian and Thomas (2003, and personal communication), using this technique, revealed that phytoplankton is least dependent on sediment nitrogen and, conversely, sediment is most dependent on phytoplankton for its respective contributions to throughflows. Also, Sediment was shown to be least dependent on itself (cycling) for throughflow. Although the correlation of the environ control magnitudes to throughflow magnitudes is inconclusive thus far, Christian and Thomas's throughflow magnitude-based conclusions support the present control ration results.

While Sediment-3 is decisively controlled by the system, NO_x-5 dominates all other components (cr_{15} = 0.28, cr_{25} = 0.24, cr_{35} = 0.44, cr_{45} = 0.24, cr_{65} = 0.21, cr_{75} = 0.28). This finding has ecological management ramifications as NO_x-5 is the greatest source of readily available nitrogen [z_5 = 417 mmol-N/(m^2 × season)] loaded into the Neuse River estuary. The most dominant of the pair-wise comparative environ controls is between NO_x and Sediment, where cr_{35} = 0.44. Input–output analysis by Christian and Thomas (2003) showed the majority of imported NO_x-N was buried in the Sediment in all summers, falls and half the springs during the four-year study. Sediment and NO_x dominate from opposite perspectives of the absolute environ control spectrum in the seven-component model. All other control ratios are significantly less, ranging from a virtually inconsequential $cr_{21} = cr_{14} = 0.01$ to a small $cr_{74} = 0.11$ (DON controlling PN-Abiotic). The controlling pair-wise relationships among the seven components remain distributed, with Sediment and NO_x maintaining significant roles.

Although, these significant roles are interesting and seemingly intuitive, absent future review of this methodology and application to additional models, these broad results remain hypotheses. For example, contrary to the present results, the Total Dependency matrix results of Christian and Thomas (2003) could be interpreted such that PN-Hetero-2 is the controlling element in the system since, on average, the system elements are generally most dependent on PN-Hetero for throughflow. PN-Hetero, a major transformer of nitrogen within the system, is also the major participant in the cycle with the largest flow [PN-Hetero-2 to NH_4-6 back to PN-Hetero-2]. Most of the NO_x boundary input becomes involved in this cycle while very little goes to Sediment. Yet, burial in Sediment is the dominant export route for boundary input to NO_x, while export as NO_x to Pamlico Sound is the smallest.

Assessing the variation of the distributed controlling relationships (comparison of the different magnitudes) within a CD or CR matrix or between the CD and CR matrices requires specific consideration and interpretation of each interstitial value. In all cases, for the Neuse River estuary model, Sediment is controlled by all other components, NO_x controls all other components, and the largest differential of control magnitude is NO_x over Sediment [$cd_{35} = 5.5 \times 10^{-4}$ ($m^2 \times$ season)/mmol-N]. The controlling relationships among the other components are significantly less, ranging from $cd_{21} = cd_{14} = 0.20 \times 10^{-4}$ to $cd_{74} = 1.3 \times 10^{-4}$ ($m^2 \times$ season)/mmol-N. Utilizing the additive quality of the cd_{ij} metrics, the sc_j vector captures the system (remaining six components) to individual component (focal component) controlling relationships. Similar to the interpretive results garnered by a review of the CD matrix, the sc_j vector identifies a significant control of the system on Sediment, $sc_3 = -25.2 \times 10^{-4}$ (m^2 × season)/mmol-N, and of NO_x on the system, $sc_5 = 21.2 \times 10^{-4}$ (m^2 × season)/mmol-N. Since both the system/component and component/system controlling relationships in the sc_j metric are balanced, for interpretive clarification, the vector can be normalized by the total control differential magnitude ($\sum_{j=1}^{n} \frac{|sc_j|}{2}$) and rewritten on a percent basis. For example, $\frac{sc_3}{\sum_{j=1}^{n} \frac{|sc_j|}{2}} \times 100 = \frac{-25.2}{2.82} \times 100 = -89.4\%$. Thus, for the Neuse River estuary average data set, the sc_j vector written as percentages are 4.6%, 4.3%, −89.4%, 11.3%, 75.2%, 4.6%, and −10.6%. Of the controlling relationship differential between the system and each of the components, 89.4% is dedicated to controlling Sediment alone. The remaining 10.6% is reserved for system-level control of PN-Abiotic. Considering component control of the system, NO_x dominates the individual controlling relationships at 75.2 %, with remaining control distributed among the four different components: DON-4 11.3%, PN-Phyto-1 4.6%, NH_4-6 4.6%, and PN-Hetero-2 4.3%.

Component throughflows, T_j, are not necessarily correlated with the corresponding level of distributed control magnitudes. Sediment-3 and NO_x-5 have the two lowest throughflows [$T_3 = 1,255$ and $T_5 = 1,578$ mmol-N/(m^2 × season)] but exert the two highest magnitudes of absolute control on the system [$sc_3 = -25.2 \times 10^{-4}$ versus $sc_5 = 21.2 \times 10^{-4}$ (m^2 × season)/mmol-N]. To the contrary, PN-Hetero-2 and NH_4-6 each have relatively high throughflows [$T_2 = 15,005$ and $T_6 = 11,955$ mmol-N/(m^2 × season)] but maintain low controlling relationships [$sc_2 = 1.2 \times 10^{-4}$ versus $sc_6 = 1.3 \times 10^{-4}$ (m^2 × season)/mmol-N]. The remaining three components have no consistent throughflow or system control relational patterns [PN-Phyto-1, DON-4, and PN-Abiotic-7, $T_1 = 6{,}927$, $T_4 = 2{,}688$, and $T_7 = 2{,}099$ mmol-N/(m^2 × season) versus $sc_1 = 1.3 \times 10^{-4}$, $sc_4 = 3.2 \times 10^{-4}$, and $sc_7 = -3.0 \times 10^{-4}$ (m^2 × season)/mmol-N, respectively].

An assumption was made that magnitude denotes dominance and therefore control. Considering the example in Figure 3.5a, the output environ of H_1 oriented to H_3, implicit in $\eta_{31} = 1.26 \times 10^{-3}$ (m^2 × season)/mmol-N, is greater than the input environ of H_1 as oriented from H_3, implicit in $\eta_{13} = 0.85 \times 10^{-3}$ (m^2 × season)/mmol-N. As such, component H_1 controls component H_3 [$cd_{31} = 4.1 \times 10^{-4}$ (m^2 × season)/mmol-N]. It remains the hypothesis of this research, an extension of Patten's (1978b) and Patten and Auble's (1981) original idea, that this overall weighted environ comparison offers a quantitative methodology to compare pair-wise dominance in a network. The term "control" is not used in the established control-theory perspective of a transient feedback closed-loop system. Rather, given a dynamic steady-state model, control resides in the component's actions in an open-loop control sense, with fixed, augmenting or diminishing transfer functions. The closed-loop feedback is inherent in the distributed interactions of the open-loop controllers acting in series and in parallel diffusely distributed in a system of checks and balances.

The orientation of the controlling relationship (greater direct plus indirect flow denotes dominance, therefore control) was based on an energy flow derivation in the Cone Spring Energy Model (Patten, 1978b; Patten and Auble, 1981). In the aquatic energy cycle, plants have complete control ($c_{ij} = 1$) over every other component. This established the direction of control (greater magnitude dominates) that remained plausible with the that energy model's remaining component's pair-wise relations. The direction of control herein, carried forward from the Cone Spring control model, although intuitively plausible (e.g., Sediment is controlled by all other components while NO_x controls all other components), remains inconclusive absent additional research. For instance, if a limiting factor perspective is used (e.g., Sediment sequesters N with a controlled release to NO_x), the conclusions could be opposite. Then Sediment would control all other components and all other components would control NO_x. The direction of control remains open for further study where storage, turnover rates, and a transient analysis are currently being considered. For example, Christian et al. (1996) discussed the difference between food-web energy models and biogeochemical models. Energy models are not suppose to have feedbacks to primary producers whereas biogeochemical models do (but see Patten, 1985 for a contrary view). Such system differences will affect the respective system indices. Baird and Ulanowicz (1993) found that food web models with high Finn Cycling Indices are experiencing stress, but Christian et al. (1996) found that biogeochemical cycles experiencing stress have low Finn cycling indices. Although direction of control remains open for further study, with the Neuse River estuary average nitrogen cycle, comparative environ analysis quantitatively shows that NO_x and Sediment decisively remain on opposite ends of a dominance spectrum.

Environ control methodology raises interesting questions based on both the results themselves and their ramifications. For instance, within the results themselves, it is not immediately clear why NH_4-6 when compared to NO_x-5, also a dissolved inorganic nitrogen, with roughly similar storage ($X_5 = 59$ versus $X_6 = 36$ mmol N/m^2), does not have a similar controlling environ influence [$sc_5 = -21.2 \times 10^{-4}$ versus $sc_6 = -1.3 \times 10^{-4}$ (m^2 × season)/mmol-N]. NH_4 throughflow is markedly higher than NO_x [$T_5 = 11{,}955$ versus $T_6 = 1{,}578$ mmol-N/(m^2 × season)]. Likewise, regarding the ramifications of the results, Sediment (dredging activities, low turnover rates therefore long repair cycles, etc.) and NO_x (industrial agriculture, non-point sources, etc.) are directly involved in ecosystem management. As such, their decisively controlling relationships in a distributed control scheme lend credence to the need for further review.

For these reasons, additional research is planned in at least the following areas: (1) Throughflow, by definition, is a component-specific quantity. It represents the total in-flow or out-flow of focal components. Since the present environ control methodology is throughflow based, the component-relative nature of the relative control metrics remains an important part of the distributed control concept in environ theory. What additional qualitative value is afforded a component-specific (relative) versus system-wide (absolute) distributed control metric quantification? (2) Component storage and turnover rates certainly play a role in the inertia of material movement through a dynamic system. The environ control metrics derived are throughflow based and do not relate to the component storages in the system. Storage (e.g., Sediment very high at $X_3 = 5{,}200$, NO_x relatively low at $X_5 = 59$ mmol-N/m^2), turnover rates, and throughflows [e.g., Sediment and NO_x low at 1,255 and 1,578 mmol-N/(m^2 × season), respectively] play interactive roles and are subjects for further review. Storage-based environ networks (Matis and Patten, 1981) and their potential controlling relationships will be further investigated. (3) A component can participate both with internal cycling and with flows across the system boundary. To what extent does the magnitude of this participation determine a component's controlling hierarchy? For example, NO_x and Sediment have high input [$z_5 = 419$ mmol-N/(m^2 × season)] and output [$y_3 = 418$ mmol-N/(m^2 × season)] boundary flows, respectively. In highly cyclic systems (Neuse River estuary Finn Cycling Indices are generally greater than 89%), are the components with high boundary flows valued for their pipeline to the outside? The correlated relationship of cycling and boundary flows to overall control magnitudes may begin to reveal practical ecosystem management results and applications. (4) Present environ analysis assumes a system is at a dynamic steady-state wherein the internal and boundary flows are constant and balanced. Although steady-state analyses may begin to capture the underlying principles in a complex network, the extent these principles adjust under transient or discrete-time step conditions needs to be pursued.

3.8 ACKNOWLEDGEMENTS

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3.10 TABLES

Table 3.1: Throughflow summations for two components, H_i and H_j, in an *n*-component system, as the product of the boundary flows, z and y, and their corresponding transitive closure matrices, N and N'. Equations (3-3) through (3-6) are conservation equations for a steady-state system where the total throughflow out, T^{out}, of a component equals the total throughflow in, T^{in}, to the same component.

$T_i^{out} = \sum_{k=1}^{n} (n_{ik} \times z_k)^{in}$ (3-3)	Each term in the summation represents a partition, T_{ik}^{in}, of the total throughflow, T_i, specifically from H_k into H_i as generated by input z_k into H_k.
$T_i^{in} = \sum_{k=1}^{n} (y_k \times n'_{ki})^{out}$ (3-4)	Each term in the summation represents a partition, T_{ki}^{out}, of the total throughflow, T_i, specifically from H_i to H_k as generated by an output y_k from H_k.
$T_j^{out} = \sum_{k=1}^{n} (n_{jk} \times z_k)^{in}$ (3-5)	Each term in the summation represents a partition, T_{jk}^{in}, of the total throughflow, T_j, specifically from H_k into H_j as generated by input z_k into H_k.
$T_j^{in} = \sum_{k=1}^{n} (y_k \times n'_{kj})^{out}$ (3-6)	Each term in the summation represents a partition, T_{kj}^{out}, of the total throughflow, T_j, specifically from H_j to H_k as generated by an output y_k from H_k.

Table 3.2: Fractions of throughflow summations for paired components, H_i and H_j, in an *n*-component system, as a function of the boundary flows, z and y, and their corresponding transitive closure matrices, N and N'.

$1 = \sum_{k=1}^{n} \left[\frac{n_{ik}}{T_i^{out}} \times z_k \right]$	(3-9)	Each term in the summation represents the fraction T_{ik}^{in}/T_i^{out} of throughflow partitioned to H_i specifically from H_k as generated by input z_k.
$1 = \sum_{k=1}^{n} \left[y_k \times \frac{n'_{ki}}{T_i^{in}} \right]$	(3-10)	Each term in the summation represents the fraction T_{ki}^{out}/T_i^{in} of throughflow partitioned from H_i specifically oriented to H_k to produce by output y_k.
$1 = \sum_{k=1}^{n} \left[\frac{n_{jk}}{T_j^{out}} \times z_k \right]$	(3-11)	Each term in the summation represents the fraction T_{jk}^{in}/T_j^{out} of throughflow partitioned to H_j specifically from H_k as generated by input z_k.
$1 = \sum_{k=1}^{n} \left[y_k \times \frac{n'_{kj}}{T_j^{in}} \right]$	(3-12)	Each term in the summation represents the fraction T_{kj}^{out}/T_j^{in} of throughflow partitioned from H_j specifically oriented to H_k to produce output y_k.

3.11 FIGURE LEGENDS

Figure 3.1: Component environ pairs in a multi-component system.

Figure 3.2: Open-loop control system

Figure 3.3: Component environ pairs in a multi-component system. η_{ji}, as defined by equation (3-14), is represented in both Figure 3.3a left and Figure 3.3b right. Similarly, η_{ij}, as defined by equation (3.13), is represented in both Figure 3.3a right and Figure 3.3b left.

Figure 3.4a: Input and output environ analysis of a pair-wise transactive relationship between component's H_1 and H_3 using equations (3-4) and (3-3), respectively, where y and z are boundary flows, mmol-N/(m^2 · season). The partitioned output throughflow from H_3 oriented towards H_1, T_{13}^{out}, is represented in magnitude by the combination of the output boundary flow and the transitive closure term, $y_1 \times n'_{13}$. The partitioned input throughflow at H_3 from H_1, T_{31}^{in}, is represented in magnitude by the combination of the input boundary flow and transitive closure term, $n_{31} \times z_1$.

Figure 3.4b, Input and output environ analysis of a pair-wise transactive relationship between component's H_3 and H_1 using equations (3-5) and (3-6), respectively, where y and z are boundary flows, mmol-N/(m^2 · season). The partitioned output throughflow from H_1 oriented towards H_3, T_{31}^{out}, is represented in magnitude by the combination of the boundary flow and the transitive closure term, $y_3 \times n'_{31}$. The partitioned input throughflow at H_1 from H_3, T_{13}^{in}, is represented in magnitude by the combination of the input boundary flow and the transitive closure term, $n_{13} \times z_3$.

Figure 3.5a, Pair-wise transactive relationship between components H_1 and H_3 using equations (3-9) and (3-10) showing the fractions of throughflow (dimensionless) represented in magnitude by the fractional environ transfer coefficients, η_{13} and η_{31}, where y and z boundary flows, mmol-N/($m^2 \cdot$ season).

Figure 3.5b, Pair-wise transactive relationship between components H_3 and H_1 using equations (3-11) and (3-12) showing the fractions of throughflow (dimensionless) represented in magnitude by the fractional environ transfer coefficients, η_{31} and η_{13}, where y and z are boundary flows, mmol-N/($m^2 \cdot$ season).

Figure 3.1

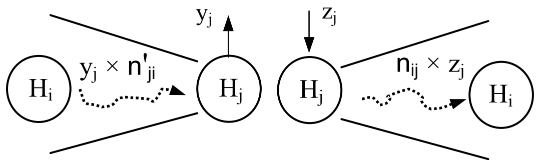


Figure 3.1a, Right: throughflow output environ of H_j oriented forward towards H_i represented in magnitude by $n_{ij} \times z_j$ [equation (3-3) when $k = j$]. Left: throughflow input environ of H_j oriented backwards toward H_i represented in magnitude by $y_j \times n'_{ji}$ [equation (3-4) when $k = j$].

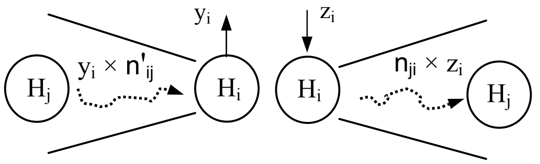


Figure 3.1b, Right: throughflow output environ of H_i oriented forward towards H_j represented in magnitude by $n_{ji} \times z_i$ [equation (3-5) when $k = i$]. Left: throughflow input environ of H_i as oriented backwards toward H_j represented in magnitude by $y_i \times n'_{ij}$, [equation (3-6) when $k = i$].

Figure 3.2

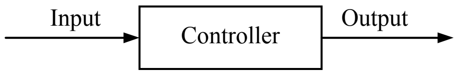


Figure 3.3

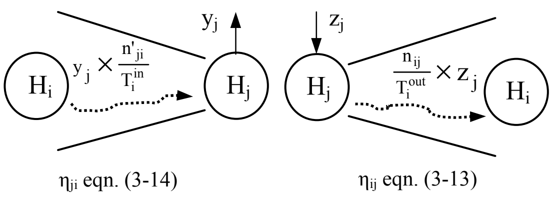


Figure 3.3a, Right: fraction of throughflow output environ represented in magnitude by $n_{ij} / T_i^{out} \times z_j$ of H_j oriented towards H_i, reference equation (3-9) when k = j. Left: fraction of throughflow input environ represented in magnitude by $y_j \times n'_{ji} / T_i^{in}$ of H_j as oriented from H_i, reference equation (3-10) when k = j.

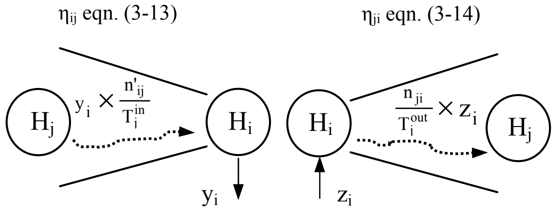


Figure 3.3b, Right: fraction of throughflow output environ represented in magnitude by $n_{ji} / T_j^{out} \times z_i$ of H_i oriented towards H_j, reference equation (3-11) when k = i. Left: fraction of throughflow input environ represented in magnitude by $y_i \times n'_{ij} / T_j^{in}$ of H_i as oriented from H_j, reference equation (3-12) when k = i.

Figure 3.4a

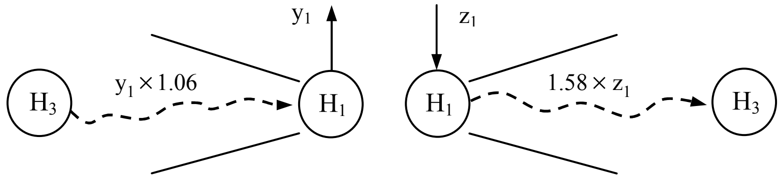


Figure 3.4b

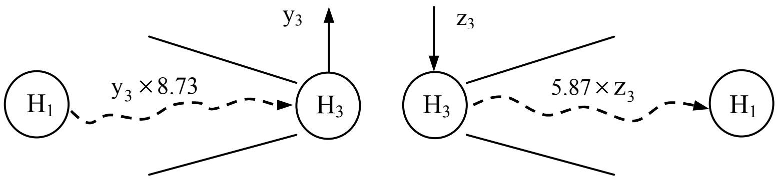


Figure 3.5a

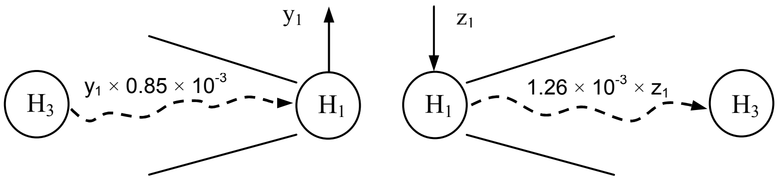
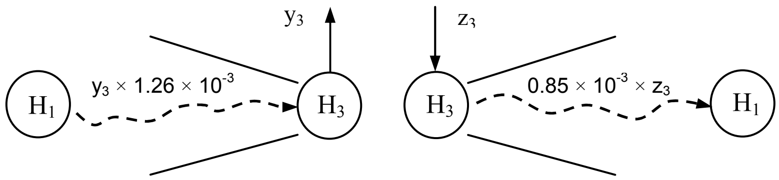


Figure 3.5b



CHAPTER 4

DISTRIBUTED CONTROL IN THE ENVIRON NETWORKS OF A SEVEN-COMPARTMENT MODEL OF NITROGEN FLOW IN THE NEUSE RIVER ESTUARY, USA: TIME SERIES ANALYSIS[1]

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[1] Schramski, J.R., Gattie, D.K., Patten, B.C., Borrett, S.R., Bata, S.A., Fath, B.D., and Whipple, S.J., Submitted to *Ecological Modelling*.

ABSTRACT

The methods of network environ analysis (NEA) (e.g., Patten 1978a, b; Barber et al., 1979; Fath and Patten, 1999) apply to dynamical steady-state models. Networks of real ecosystems are near steady-state in long-term mean characteristics, but are dynamic in short-term responses. A formal mathematical approach to dynamic NEA analysis has never been fully developed, though see Hippe (1983) for one approach. Another potential approach to addressing this limitation is to analyze a discrete-time series of steady-state models, each a snapshot for the time period it represents. Using concepts from "open-loop" control theory, four throughflow-based ecological control terms (*control ratio,* CR; *control difference,* CD; *system control,* sc_j; and *total system control,* TC), as developed using an environ framework in Schramski et al. (2005), are evaluated for 16 consecutive seasons of nitrogen cycling in the Neuse River estuary, North Carolina, USA (Christian and Thomas, 2003). Results of this assessment offer a quantitative measure of the quasi-dynamic distributed control in this network. Nitrate-nitrites (NO_x) and Sediment assume opposing but dominant roles (high sc_j magnitudes) in all 16 seasons. Low ratios of total component throughflow (T_i) to respective boundary inflow (z_i) or outflow (y_j) are shown to be indicators of component control dominance, suggesting a role for boundary flows in determination of a system component's dominance. This conclusion may also derive from the high cycling in this nitrogen model (average Finn cycling index was 89% for all 16 seasons). TC appears to be correlated with total system throughflow (TST), suggesting that this property may indicate a system's distributed control patterns. This correlation may also be fruitful if TST proves to be an indicator of system stability, an important consideration in the notion of control. Future efforts will focus on the effects of throughflow magnitude, component storage, and residence times on distributed control.

4.1 INTRODUCTION

One insight attributed to network environ analysis (NEA) is its ability to quantify the integral (direct, indirect, and boundary) relationships between compartments. NEA, although offering a holistic perspective, is currently a steady-state analysis methodology. The foundational conservation equations assume that the rate of mass or energy accumulation (dx/dt) is zero, significantly simplifying the subsequent equation development (Barber et al., 1979). Networks of real ecosystems are not steady-state and change over time. Although advanced dynamic simulation software exists, the mathematics required for true theoretical dynamic NEA are daunting, and have yet to be fully developed. The non-steady-state case is discussed by Patten et al., (1976). Hippe (1983) explored the very specific time-dependent input function which does yield time dependent results. Hannon (1986) and Levine (1988) pursued dynamic-linear, but nevertheless time invariant, studies of input–output network analysis.

One approach to overcoming this limitation is to analyze a discrete-time series of steady-state models, each a fixed snapshot for the period it represents (Leontief, 1970). The NEA control metrics generated for each steady-state network can then be contrasted to determine how they change over time to generate a quasi-dynamic perspective. Some network measures (e.g., component throughflow, T_j, and total system throughflow, TST) are sensitive to the number of components in the model. In this work, we circumvent this issue by comparing NEA control measures in a time sequence of models with identical structure. This paper is the last of a series exploring indirect effects and distributed control in the nitrogen cycles of the Neuse River estuary, North Carolina, USA. Gattie et al., (2006a) and Borrett et al., (2006) examined indirect effects while two other papers, Chapter 3 of this dissertation [Schramski et al., (2006)] and this chapter [Schramski et al., (submitted to Ecological Modelling)], are devoted to distributed control.

Ecosystems, when studied in total, do not contain obvious singular sources of control. This property must somehow reside in the complexity of organization, a complexity captured in the interactive networks of ecosystem models. We speak of such control as diffuse, decentralized, or remote, for which we adopt the general term *distributed control* (Patten, 1978b). Considering an "open-loop" control theory foundation (e.g., Ogata, 1970) within the NEA framework, distributed control can be investigated by defining appropriate concepts and measures from those originated by Patten (1978b), Patten and Auble (1981), and Dame and Patten (1981) and further explored by Fath (2004). In Schramski et al. (2006, Chapter 3 here) we distinguish three concepts of what we represent as absolute control: the *control ratio*, the *control difference*, and the *system control* metrics, within the environ control methodology. By *absolute control* we mean magnitudes of control which are independent of component-specific throughflows. This is in contradistinction to *relative control* which is specifically calculated as a function of a specific component's throughflow embedded in a multi-component system. The first two expressions (control ratio, cr_{ij}, and control difference, cd_{ij}) denote a pair-wise dominance relationship between two components [theoretically considered hierarchically embedded *holons*, H (Koestler, 1976; Patten and Auble, 1981; and Schramski et al., 2006)], H_i and H_j, proximately (directly) or distally (indirectly) connected in a network. When one component H_i transfers a greater fraction of energy or matter throughflow to another H_j, than H_j does to H_i, then H_i will be considered to control H_j in a dominance sense (Patten, 1978b; Patten and Auble, 1981; Dame and Patten, 1981; Schramski et al., 2006). The reverse of this logic is the limiting-factor-perspective in which the component withholding a resource is the one controlling the other. The control ratio, cr_{ij}, offers a quantitative value representing the level of dominance between the (H_i, H_j) component pair on a scale specific to the magnitude of the relationship between those two components. Unlike the control ratio, dimensionless and derived with uncommon denominators, control difference magnitudes, dimensioned measures, use a system-wide scale and can be compared and contrasted among each other for varying control strengths between component pairs (Schramski et al., 2006). As such, control difference magnitudes are system-level measures. Utilizing this property, the control difference magnitudes can also be column-summed to determine a focal component H_j's relative combined strength as compared to the remaining system in total. The third index, the s*ystem control* vector, sc_j, captures this additive quality of the control difference metrics such that $sc_j < 0$ denotes system control of the specific component H_j and $sc_j > 0$ denotes a focal component, H_j's, control of the entire system (all remaining components combined). The fourth control measure, *total system control* (TC), is developed and defined to quantify the system-wide input–output total environ differential. These four throughflow-based ecological control measures (control ratio, CR; control difference, CD; system control, sc_j; and total system control, TC), developed in the environ framework both by Schramski et al. (2006) and herein below, are evaluated for 16 consecutive seasons of measured and modelled nitrogen cycling in the Neuse River estuary, North Carolina, USA by Christian and Thomas (2003). The results augment the steady-state methodology of the other papers in this series by producing a quantitative discrete-time series measure of the seasonal dynamics of distributed control.

4.2 ABSOLUTE ENVIRON CONTROL, PAIR-WISE DIFFERENTIAL

The control ratio and control difference parameters are comprised of oriented fractional transfer coefficients (FTC's), η_{ij} and η_{ji}, where the FTC's (with dimensional units of reciprocal throughflow) are modified parameters of the transitive closure values, n_{ij} and n'_{ij}. Specifically, the fractions of throughflow summations are [see Schramski et al. (2006) for derivation]:

$$1=\sum_{k=1}^{n}\left[\frac{n_{ik}}{T_i^{out}}\times z_k\right] \tag{4-1}$$

$$1=\sum_{k=1}^{n}\left[y_k\times\frac{n'_{ki}}{T_i^{in}}\right] \tag{4-2}$$

$$1=\sum_{k=1}^{n}\left[\frac{n_{jk}}{T_j^{out}}\times z_k\right] \tag{4-3}$$

$$1=\sum_{k=1}^{n}\left[y_k\times\frac{n'_{kj}}{T_j^{in}}\right]. \tag{4-4}$$

These equations (4-1) through (4-4) represent all input and output-environ combinations between any two components, H_i and H_j, in an *n*-component system, as a function of the boundary flows, z and y, and their corresponding transitive closure matrices, N and N'. Expressions (4-1) and (4-2), or (4-3) and (4-4) are duals. The bracketed expressions in the summations are the nondimensional transitive closure fractions of throughflow comprised of the total combined direct and indirect fractions of throughflow between all components in a multi-component system. The ratios $\frac{n_{ij}}{T_i^{out}}$ and $\frac{n'_{ji}}{T_i^{in}}$ or $\frac{n_{ji}}{T_j^{out}}$ and $\frac{n'_{ij}}{T_j^{in}}$ represent the open-loop control transfer functions, or FTC's, which dualistically transform the boundary flows z and y, respectively, into combined direct and indirect fractions of throughflow:

$$\eta_{ij}\equiv\left[\frac{n_{ij}}{T_i^{out}}\right]=\left[\frac{n'_{ij}}{T_j^{in}}\right],\qquad i, j = 1, 2, \ldots, n, \tag{4-5}$$

$$\eta_{ji} \equiv \left[\frac{n'_{ji}}{T_i^{in}}\right] = \left[\frac{n_{ji}}{T_j^{out}}\right], \qquad i, j = 1, 2, \ldots, n, \qquad (4\text{-}6)$$

where typical dimensional units are reciprocal mass $[(L^2 \cdot T)/M]$ or energy $[M^{-1}T^3]$ throughflow per area as denoted in the MLT system of units. Considering modern control theory, the FTC's, even though they contain network cycles, can be interpreted to represent open-loop gains. Each component in the system acts individually as a transfer function specifying the input–output relationship of a linear time-invariant system. Transfer functions include dimensions necessary to relate input and output but do not provide information concerning the physical structure of the system, which remains a "black box." Assuming a system with cycling, the transfer functions together are embedded in a network of transfer functions each acting as an infinite (closed cycles are infinitely long) array of simultaneous feedback loops. While equations (4-5) and (4-6) are distinct, either one will successfully populate the entire FTC, η_{nxn}, matrix for the *n*-component system even though each equation is created from the perspective that different components, with their respective throughflows, are oriented in opposing directions (throughflow-in or throughflow-out). This reciprocity represents the directional similarity introduced to the system analysis by normalizing the transitive closure values n_{ij} and n'_{ij} by their respective throughflows.

Graphically the FTC's are shown in Figures 4.1a and 4.1b. [Figure 4.1 here] The directional difference between H_i oriented to H_j and H_j oriented to H_i quantified in η_{ij} and η_{ji}, respectively, can be attributed, in part, to the afferent and efferent connectance differences within the respective input and output environs between two components (Schramski et al., 2006). With the magnitude of the FTC's, η_{ji} and η_{ij}, capturing the distinct directional relationship between each H_i, H_j pair an *n*-component system, their relationship quantitatively captures the afferent or efferent controlling relationship between these components. Pair-wise comparisons of individual fractional transfer values are accomplished by the dimensionless control ratio:

$$cr_{ij} \equiv \frac{\eta_{ij} - \eta_{ji}}{\max\,(\eta_{ij}, \eta_{ji})} \qquad i, j = 1, 2, \ldots, n \qquad 0 \le |cr_{ij}| \le 1, \qquad (4\text{-}7)$$

while a system-based comparison of the pair-wise fractional transfer values is realized by the control difference:

$$cd_{ij} \equiv \eta_{ij} - \eta_{ji} \qquad i, j = 1, 2, \ldots, n \qquad 0 \le |cd_{ij}| < \infty, \qquad (4\text{-}8)$$

where cr_{ij} and cd_{ij} are interpreted directionally from j to i. Pair-wise individual comparisons, in this case, signify that the control ratios are fractions of the difference between two FTC's and, as such, their magnitudes cannot be compared to each other. The cd_{ij} values of the CD matrix, however, are differences in the fractional transfer values ($\eta_{ij} - \eta_{ji}$) acting between two components, H_i and H_j. Lacking the division operation of the largest fractional transfer value [max ($\eta_{ij} - \eta_{ji}$)], as used to calculate cr_{ij} values, the cd_{ij} values remain referenced to a system wide-scale. The magnitudes of the cd_{ij} values, throughout the CD matrix, can be compared and contrasted among each other for their varied strengths. Matrices of these coefficients, CR and CD, quantify the difference in magnitudes of the component's specific directional open-loop controllers. Combined, the individual open-loop controllers are embedded in a network of open-loop controllers each acting as an infinite array of simultaneous feedback loops to each other. Many closed-loop feedback pathways are implicit in the CR and CD measures. Figure 4.2 pictorially demonstrates the pair-wise control difference relationship using the FTC environ diagrams. [Figure 4.2 here]

4.3 ABSOLUTE ENVIRON CONTROL, AGGREGATE DIFFERENTIAL

Considering system-level control relations, the system control vector, sc_j, represents the control difference relationship between a single component and the remaining aggregate components of the *n*-component system:

$$\mathrm{sc}_{\mathrm{j}} \equiv \sum_{\mathrm{k}=1}^{n} \mathrm{cd}_{\mathrm{kj}} \qquad \mathrm{j} = 1, 2, \ldots, n \qquad 0 \le |\mathrm{sc}_{\mathrm{j}}| < \infty \qquad (4\text{-}9)$$

The dual equations (4-1), (4-2) or (4-3), (4-4) can be graphically displayed to demonstrate the relationship of a selected component, H_j, to all system components as quantified in equation (4-9). [Figure 4.3 here] Expand equation (4-9) for components k through *n* to generate equation (4-9a):

$$\mathrm{sc}_{\mathrm{j}} = (\eta_{\mathrm{kj}} - \eta_{\mathrm{jk}}) + (\eta_{(\mathrm{k}+1)\mathrm{j}} - \eta_{\mathrm{j}(\mathrm{k}+1)}) + (\eta_{(\mathrm{k}+2)\mathrm{j}} - \eta_{\mathrm{j}(\mathrm{k}+2)}) + (\eta_{(\mathrm{k}+3)\mathrm{j}} - \eta_{\mathrm{j}(\mathrm{k}+3)}) \ldots + \ldots (\eta_{n\mathrm{j}} - \eta_{\mathrm{j}n}), \qquad (4\text{-}9\mathrm{a})$$

and consider Figure 4.3 to illuminate the system control calculation which effectively adds the columns of the cd_{ij} matrix to quantitatively capture component H_j's relationship with the remaining *n*-component system. Further, the combined differentials of FTC's focused on each component, i.e., sc_1, sc_2, sc_3, etc., represent a total-system accounting balance such that:

$$\sum_{\mathrm{j}=1}^{n} \mathrm{sc}_{\mathrm{j}} = 0. \qquad (4\text{-}10)$$

For every throughflow fraction of open-loop augmentation between a component and its system, an equal and opposite level of open-loop attenuation must exist between other components and their respective remaining components in the system as a whole. Whole-system balance is always maintained. Also, the total amount of environ differentiation which exists in a given system is a measure of the total-control magnitude distributed among the controlling relationships between components. This is *total system control* (TC) and is calculated as

$$\mathrm{TC} \equiv \pm \sum_{\mathrm{j}=1}^{n} \frac{|\mathrm{sc}_{\mathrm{j}}|}{2}, \qquad \mathrm{j} = 1, 2, \ldots, n, \qquad 0 \le |\,\mathrm{TC}\,| < \infty. \qquad (4\text{-}11)$$

TC maintains the same units as the *fractional transfer coefficients* and, as such, represents the accumulated fraction of throughflow gain available to be augmented (positive values, +TC, called *donor* TC) or attenuated (negative values, −TC, called *receiver* TC) as related to the corresponding incoming or outgoing boundary flows. *Donor* and *receiver* TC's cancel (−TC + TC = 0) per equation (4-10) to assure whole-system balance.

4.4 NEUSE RIVER ESTUARY MODEL DESCRIPTION

Using NEA throughflow analysis, the framework of distributed control was explored using steady-state models of nitrogen cycling in the Neuse River estuary, North Carolina, for 16 consecutive seasons from Spring 1985 through Winter 1989 (Christian and Thomas, 2003). The nitrogen models include seven components: Phytoplankton Particulate Nitrogen, PN-Phyto-1; Heterotroph Particulate Nitrogen, PN-Hetero-2; Sediment Nitrogen, Sediment-3; Dissolved Organic Nitrogen, DON-4; Nitrate and Nitrites, NO_x-5; Ammonium, NH_4-6; and Abiotic Particulate Nitrogen, PN-Abiotic-7. A diagraph of connectivity is shown in Figure 4.4. [Figure 4.4 here]

The Neuse River receives water from a 16,000 km^2 watershed which together with the smaller Trent River (9% of Neuse and Trent combined) empties into the 400 km^2 Neuse River estuary, ultimately emptying into Pamlico Sound. Many studies focusing on nutrient cycling and cyanobacterial blooms in this area have been conducted over the last 30 years (Boyer et al., 1988; Christian et al., 1984, 1989, 1991; Hobbie and Smith, 1975; Paerl, 1987; Stanley, 1983, 1988). Due to the size of Pamlico Sound, which separates river flow from the Atlantic Ocean, residence times of freshwater in the sound approach one year and salinities are one-half to two-thirds that of seawater (Giese et al., 1979). Astronomical tides are insignificant and wind tides coupled with river-discharge fluctuations dominate water-level fluctuations. The model made no distinction between molecular N and gaseous end products of denitrification. All inputs are dominated by fluctuating riverine loading with DON-4, NO_x-5, and NH_4-6 receiving input from precipitation and PN-Hetero-2, including some immigration. All outputs except Sediment-3 were discharges downstream to Pamlico Sound or N_2 from denitrification, reflecting respiratory loss. The output from Sediment-3 (top one centimeter) was considered burial beyond the modeled sediment thickness. PN-Hetero-2 experienced some emigration.

Table 4.1 lists the standing stocks, boundary flows, component throughflows, and total system throughflows for the 16-season Neuse River estuary nitrogen cycle model as determined by Christian and Thomas (2000, 2003) as a part of a Neuse River Modeling and Monitoring program (ModMon). Some portions of the model were directly sampled and measured while others were interpolated both from literature (e.g., data from similar estuaries) and subsequent mass balance. [Table 4.1 here]

The ModMon program (a consortium of academia, government, and industry) was developed to improve water quality by reducing nitrogen loading to a eutrophic ecosystem. Christian and Thomas (2003) employed ecological network analysis (ENA) to understand the relationship between nitrogen inputs to the system and nitrogen cycling within the system. They concluded: 1) Cycling of nitrogen was high [average Finn (1976) cycling index = 0.89]; 2) Freshwater residence time averaged a long 51 days; 3) The rate of new nitrogen loading was a small fraction of the total microbial processing of nitrogen; and 4) The controls on primary production (an area of focus in their study) tended to be associated with conditions in the estuary rather than import. Processes within the sediments, including denitrification, benthic filter feeding, and burial, were considered a weakness in model construction.

4.5 EXAMPLE, STEADY-STATE NEUSE RIVER ESTUARY, SUMMER 1986

Tables 4.2 through 4.7 and equation (4-11a) provide an example of the network environ analysis (NEA) distributed control calculations. Starting with the nitrogen flow characteristics captured in the respective summer of 1986 steady-state model, NEA begins by organizing the inter-component flows into the f_{ij} matrix where flows are oriented from columns j to rows i. Table 4.2 shows the f_{ij} matrix, along with the system's corresponding boundary flows and calculated throughflows. [Table 4.2 here] Considering the conservation of mass equations for each component in the system and the definition of throughflow, the transitive closure (direct plus indirect) matrices N and N' are calculated from f_{ij} and the throughflow vectors. Table 4.3 shows both the output- and input-oriented, N and N' matrices, respectively (Fath and Borrett, 2005). [Table 4.3 here] The fractional transfer coefficients (FTC), η_{ij}, equation (4-5), are shown in Table 4.4. [Table 4.4 here] The control ratios, cr_{ij}, equation (4-7), are given in Table 4.5, [Table 4.5 here] and control differences, cd_{ij}, equation (4-7), in Table 4.6. [Table 4.6 here] The system control vector, sc_j, equation (4-9), is shown in Table 4.7. [Table 4.7 here] Total system control (TC) is the aggregate of the respective directional (positive or negative) system control values using equation (4-11):

$$TC = 10.51 \times 10^{-3} \text{ (m}^2\text{ season)/mmol-N} \tag{4-11a}$$

4.6 DISCRETE TIME-SERIES ANALYSIS OF 16 SEASONS

A pattern of distributed control among the seven components for the 16 season time series is demonstrated by the system control vectors, sc_j (Table 4.8). [Table 4.8 here] Positive values indicate the magnitude of control exerted by each row component on the remaining aggregate system (six components). Negative values denote the magnitude of control by the latter on the corresponding individual focal (column) component. The individual system control values thus quantify the differential environ relationships between each column component and the remaining components comprising the total system. As an example, in the summer of 1986, the PN-Phyto component, PN-Phyto-1 controls the remaining aggregate system with a differential magnitude of 0.64×10^{-3} (m^2 season)/mmol-N. In general over a four year period, PN-Phyto-1, PN-Hetero-2, DON-4, and NH_4-6 control the system with relatively small magnitudes (averages: $sc_1 = 0.77 \times 10^{-3}$, $sc_2 = 1.23 \times 10^{-3}$, $sc_4 = 1.75 \times 10^{-3}$, and $sc_6 = 1.27 \times 10^{-3}$ (m^2 season)/mmol-N). On a few occasions these four components are controlled by the system (negative values). Spring 1985 is one unique period in which all four of these components are controlled by the system, contrary to their 16-season averages. When considering boundary flows, component storage levels, nitrogen throughflows, surface water salinity, and water flushing times, no apparent correlations can be found to explain the control parameters calculated for the spring of 1985. In fact, the only unusual environmental event was a large winter-spring discharge beginning in 1987 (Christian et al., 1991; Boyer et al. 1994). Additionally, the four identified components perform very different functions within the ecosystem. Two are highly interconnected biotic components [PN-Phyto-1 and PN-Hetero-2], and two are relatively less connected abiotic components [DON-4 and NH_4-6]. It appears that an exterior event (storm water loading, etc.) was not driving the rare simultaneous switching of control in these four components. Since the nitrogen data for each steady-state model remain averaged, both temporally due to sampling frequencies, and spatially given the size of the estuary, the reasons for these control point switches are obscure. The moderately connected PN-Abiotic-7 was controlled by the system during 14 of the 16 seasons, with an average magnitude of $sc_7 = -1.28 \times 10^3$ (m^2 season)/mmol-N). PN-Abiotic-7 and Sediment-3 (average $sc_3 = -12.68 \times 10^3$) together, on average, comprise the receiver transfer functions of the system. In fact, Sediment-3 remained negative for all 16 seasons. Recall equation (4-10) wherein the system must maintain balance. The system control trend lines from Table 4.8, shown in Figure 4.5, depict the spread between the donor and receiver TC relationships in the decentralized system. [Figure 4.5 here]

4.6-1 TOTAL SYSTEM CONTROL

Seasonally, each of the seven components participates in either a controlled ($sc_j < 0$) or controlling ($sc_j > 0$) relationship with the remaining total system. The aggregate of the components which attenuate, otherwise called the *receiver* (controlled) components, represent the negative TC. The aggregate of the components which augment, *donor* (controlling) components, are represented by the positive TC entries. These values, from the second to last row in Table 4.8, are the uppermost line (diamond data marker) in Figure 4.5. In comparison, Sediment-3 and PN-Abiotic-7 together comprise the vast majority of the receiver TC [negative of donor TC per equation (4-11)] across all 16 seasons. The receiver TC values are lowermost (square data marker) in Figure 4.5. Sediment system control magnitude, sc_3, drives the TC receiver magnitude. The donor and receiver TC lines necessarily mirror each other in Figure 4.5, reflecting the magnitude of controls one way (system on components) or the other (components on system). That is, the vertical spread between the positive (donor) and negative (receiver) TC lines graphically demonstrates the total system control magnitudes the system then individually distributes among its components. The average TC_{avg} magnitude is $\pm 14.30 \times 10^{-3}$ (m^2 season)/mmol-N. Fall and winter TC magnitudes exceed the average while spring and summer magnitudes fall below the average in all cases except winter of 1987 and spring of 1988. Looking for an explanation, winter–spring water discharge flow in the estuary was highest in early months of 1987, but a similar high flow did not occur during the spring of 1988. Further, surface water salinities and flushing times show no readily apparent correlation to these discrepancies (Christian et al., 1991). Generally, nitrogen throughflow is highest in the spring and summer and lowest in the fall and winter, presumably reflecting trends in biological activity.

Figure 4.6 explores the correlation between total system throughflow (TST) and TC. There is some logarithmic correlation between the two, particularly at higher throughflows. [Figure 4.6 here] The average nitrogen TST for the complete 16 seasons is 10,373 mmol-N/(m^2 season). Two of the springs and all of the summers exceeded the average. As such, recognizing some correlation between TST and TC, as TST increases in the springs and summers TC magnitude is low. Where TST decreases in the falls and winters, TC magnitude is usually higher. The dynamic give and take of the control differential is evident in Figure 4.5. In an attempt to explain the TC variation, besides the mild seasonal correlation to TST, no other decisive or reasonable correlation to TC variation was found.

Sediment-3 and NO_x-5 remain decisively opposed in their receiver/donor controlled/controlling relationship across all 16 seasons. Three observations are noteworthy regarding their apparently unique position in the system:

(1) Although not a particularly strong correlation, the system controls, sc_3 and sc_5, correlated logarithmically with TST ($R^2 = 0.41$ in both cases). These values are larger than the remaining five components' ($R^2 = 0.01$ to 0.30) correlations with TST;

(2) The system controls, sc_3 and sc_5, logarithmically correlated with their own throughflows T_3 and T_5, ($R^2 = 0.45$ and $R^2 = 0.67$, respectively) with larger R^2 correlations than the remaining five component system controls correlated with their throughflows (remaining R^2s varied from 0.00 to 0.17); and

(3) With two exceptions, the flows y_3 and z_5 were the highest boundary flows.

4.6-2 BOUNDARY FLOWS

The consistently high magnitudes of sc_3 and sc_5 values show that Sediment-3 and NO_x-5 play decisive roles in the seven compartment system over the entire 16 seasons. The first two observations above indicate there is a possible relationship between the differential environ controls, sc_3 and sc_5, and both total system throughflow, TST, and also their respective component throughflows, T_3 and T_5. Also, boundary flows, through a basic inspection of magnitudes, appear to play a role. Table 4.9 combines the last two observations by calculating the ratio of component throughflows, T_i, to their respective boundary flows, z_i and y_i. [Table 4.9 here] In all cases, the input boundary flow z_5 to NO_x-5 represents the highest percentage of input z_i to a respective component's throughflow, T_i. Similarly, in all cases, the output boundary flow y_3 from Sediment-3 represents the highest percentage of output boundary flow y_i with respect to a component's throughflow, T_i. In general, in virtually all 16 seasons, the highest boundary flows were z_5 and y_3. Additionally, Table 4.9 shows that z_5 and y_3 play dominant roles in their respective components' internal throughflows.

4.6-3 UNIT ENVIRON ANALYSIS

The NEA methodology of partitioning a system into a set of mutually disjoint and exhaustive object/environment elements (environs; Patten, 1978a) is used to consider the NO_x-5 and Sediment-3 participation in each steady-state system. If H_i represents any component in the *n*-component system, it is simultaneously comprised of a within-system output and input environment, represented by E_i and E'_i, respectively. The output-oriented transitive closure matrix, n_{ij} (e.g., Table 4.3), implicitly defines the output environs E_i of the steady-state condition normalized to one unit of input, z_i, from each component, H_i. Similarly, the input-oriented transitive closure matrix, n'_{ij}, implicitly defines the input environs E'_i of the steady-state condition normalized to one unit of output, y_i, for each component, H_i. The unit environs map all of the subsequent direct throughflows associated with a unit boundary flow. See Patten (1978a) and Dame and Patten (1981) for theory and methods and also reference Fath and Borrett (2005) for supporting software to facilitate calculations. Each environ is considered one partition element and each such element is mutually disjoint from the others created by their respective unit boundary flows. As a result, this methodology can be used (Christian and Thomas, 2003) to evaluate the magnitude and distribution of impact of a particular boundary flow.

Figure 4.7 is one conclusion from the output environ analysis of NO_x-5 Using the transitive closure matrix, n_{ij}, and a unit input ($z_5 = 1$) to NO_x-5, the fractions of boundary outputs, y_i, from each component are determined. [Figure 4.7 here] Through the 16-season distribution of partitioned fractions of output, the highest fractions of output, y_j, are consistently from Sediment-3; spring 1985 and winter 1987 are the only exceptions. Spring 1985 was the one unique period in which the four components, PN-Phyto-1, PN-Hetero-2, DON-4, and NH_4-6, were uniquely controlled by the system contrary to their 16-season averages' generally controlling the system. Winter 1987 was the period of highest water flow, and one of only two seasons (the second was spring 1988) which did not follow the seasonal trend of fall and winter TC remaining above average and spring and summers falling below average.

Figure 4.8 shows the Figure 4.7 data averaged across the 16 seasons. [Figure 4.8 here] The average output is highest from Sediment-3, representing and average of 61% of the input to NO_x-5. This comprises a greater magnitude of nitrogen flow than the other components combined. Figure 4.9 considers an input-environ analysis of the Sediment-3 component. Using the transitive closure matrix, n'_{ij}, and a unit boundary output ($y_3 = 1$) from Sediments (3), the fractions of boundary inputs, z_i, to each component embodied in this output are determined.

[Figure 4.9 here] The fraction of input to NO_x-5 as generated by a unit output from Sediment-3 ranged from 0.41 in the summer of 1986 to a high of 0.71 in the winter of 1988. In all cases, through the entire 16-season distribution of partitioned fractions of input, the highest fractions of input, z_i, are to NO_x-5 Figure 4.10 shows the 16-season average fraction of boundary input to each component given a unit boundary output from Sediment-3. [Figure 4.10 here] Over 16 seasons, an average of 51% of the output from Sediments (3), y_3, entered the system from NO_x-5, comprising slightly over half the total boundary output from Sediments (3).

4.7 DISCUSSION

Nutrient cycles are by definition a biogeochemical circular casual system. Patten et al. (1976) demonstrated that all components in an ecosystem can be affected by and affect every other, directly or indirectly, through circular causal networks. These biogeochemical circular networks, or feedback "loops" ("cycles" in graph theory terminology), represent a closed and simultaneously self-controlling dynamic. Each component simultaneously functions as both: 1) an open-loop transfer function augmenting or diminishing boundary flows passing across the system boundary into the circular casual system, and 2) a potentially infinite quantity (when all path lengths of connectance are included) of closed-loop feedback cycles within the multiple component system. Closed-loop control is implicit in the open-loop model and its associated indices.

The notion of control implies the coordination of change which is, by definition, a dynamic concept. Although the comparison of open-loop controller strengths in a steady-state network model successfully identifies dominance relationships, a perspective of the same controller relationships adjusting over time begins to elicit the dynamic coordination of components in an ecosystem. The present NEA distributed control methodology captures the pair-wise controlling relationships in a complex network organization. The subsequent discrete-time-series analysis of a sequence of steady-state models captures the interactive and dynamic networks of the respective ecosystem. This study elucidates the dynamic controlling interactions of the seven-component Neuse River estuary nitrogen model over 16 seasons. Novel insight into the distributed control of a multi-component system is offered by first quantifying the controlling relationships and then reviewing the results for potential pragmatic correlations useful to the ecosystem manager or observer.

Considering the system control vectors, PN-Phyto-1, PN-Hetero-2, DON-4, and NH_4-6 generally control the system with relatively small magnitudes (Table 4.8). In rare instances, these four components are controlled by the system. Most notably, in the spring of 1985, all four are controlled by the system simultaneously. Additionally, these four components represent very different components of the ecosystem where two are highly inter-connected biotic components [PN-Phyto-1 and PN-Hetero-2] and two are somewhat less, relatively, connected abiotic components [DON-4 and NH_4-6]. No obvious influence (e.g., water flow, salinity, nitrogen throughflows) could be identified to explain their sporadic and rare behaviour of switching from controlling to controlled by the system. Additionally, the tipping point from controlled to controlling is $sc_j = 0.0$ (m^2 season)/mmol-N. Considering the average sc_j values for these four components are relatively small, it remains an interesting observation that these components do not switch from controlling to controlled more often. There remains a consistency to PN-Phyto-1, PN-Hetero-2, DON-4, and NH_4's (6) control of the system with only rare interruptions. A similar observation is evident for the moderately connected PN-Abiotic-7. Its average system control magnitude is relatively small (close to the tipping point) yet the component is remarkably consistent (14 of the 16 seasons) in its position of being controlled by the system. Similar to the averaged single-model steady-state findings of Schramski et al. (2006), both NO_x-5 and Sediment-3 remain decisively consistent (all 16 seasons) with regard to their controlling relationships within the seven component system over the entire 16-season review. NO_x-5 controls the system while Sediment-3 is controlled. Although variation in magnitude exists, in 11 of 16 seasons Sediment-3 and PN-Abiotic-7 are controlled by the system and the remaining five components control the system. While allowing for some variation in overall magnitudes, there is arguably some consistency with each component's controlled/controlling role in the decentralized network throughout 16 seasons which represent a wide range of throughflows and biological activity.

Total system control (TC) is the total open-loop magnitude (gain) differential available to be partitioned for controlled and controlling magnitudes throughout a multi-component system. The positive or *donor* TC balances the negative or *receiver* −TC as mirror images (Figure 4.5) of each other as compared to the horizontal axis or control tipping point [$\pm TC = 0.0$ (m^2 season)/mmol-N]. Their symmetry about the horizontal axis helps to identify the total system balance evident between the open-loop magnitude augmentation and its counteracting attenuation throughout the decentralized system. The system remains balanced at all magnitudes of TC. The TC variation over the 16-season analysis is further illuminated in the variation of the overall vertical spread between the *donor* and *receiver* TC lines shown in Figure 4.5. The average magnitude is $TC_{avg} = 14.30 \times 10^{-3}$ (m^2 season)/mmol-N. Fall and winter TC magnitudes exceed the average while spring and summer magnitudes fall below in all cases except winter 1987 and spring 1988. As a direct result of biological activity, nitrogen throughflow is generally highest in the spring and summer and lowest in the fall and winter, providing some parallel to the TC magnitude fluctuation. The logarithmic correlation between total system throughflow (TST) and TC was significant enough for consideration. As nitrogen throughflow increases in spring and summer, the TC of the system appears to logarithmically decrease. Control of the system over one component or vice-versa, the control of any one component over the total system, is reduced at times of high biological and throughflow activity.

Stability, a term closely coupled to the concept of control, develops as a consideration during this balancing throughflow magnitude with any particular component's level of dominance. If stability is the measure of a system's ability to return to its nominal state, then it could be hypothesized that steady-state systems with high TC values are less stable and more sensitive to outside perturbation (i.e., take longer to reach equilibrium, bifurcate to a new equilibrium, etc.). TST appears to play a predictive role for TC and, to a lesser extent, both TST and TC could be related to the elusive concept of system stability.

Boundary flows appear to play a role in determining dominance. NO_x-5 and Sediment-3 generally had the highest boundary flows (z_5 and y_3, respectively), but more importantly, these boundary flows were always the highest percentage of component throughflow, T_i, for their respective component as compared to the other component boundary flow to throughflow rations. Several hypotheses can be considered from these results for subsequent research. These two components appear to be the system's link to the outside environment beyond the boundaries of the model. NO_x-5 could be construed as the predominant or, at least, the controlling source of nitrogen for the estuary. Using the same argument, Sediment-3 is then viewed as the predominant or, at least, the controlled sink of nitrogen leaving the estuary. The internal relationship through the direct and indirect pathways between NO_x-5 and Sediment-3 is also evident. Environ analysis showed that the 61% majority of nitrogen that entered the NO_x-5 component from across the system boundary, z_5, subsequently left across the system boundary from Sediment-3, y_3. Further, similar environ analysis showed that a 51% majority of nitrogen leaving Sediment-3 across the system boundary, y_3, originated from system boundary input to NO_x-5, z_5. Not only are the inputs, z_5, to NO_x-5 and the outputs, y_3, from Sediment-3 relatively dominant, but the internal relationship between NO_x-5 and Sediment-3 appears to be strong as well. NO_x-5 and Sediment-3 may be gatekeepers with special roles or significance in the Neuse River estuary. Gatekeepers, in this sense, could determine how much recycling or indirect flow is evident in a system. Recycling and the corresponding indirect flows which result, play a large role in whole system emergent properties.

The remaining five components [PN-Phyto-1, PN-Hetero-2, DON-4, NH_4-6, and PN-Abiotic-7] do not have the same correlation levels with activities across the system boundary. Additionally, the overall seven-component system demonstrates extremely high internal cycling with a Finn (1976) cycling index averaging 89% for all 16 models. These final observations support a general hypothesis wherein, for high internal cycling systems, those components which maintain strong connectivity across the system boundaries tend to be at a strong point in the control scheme spectrum. One inference from Christian and Thomas's (2003) findings was that short-term control of primary production associated with PN-Phyto-1 tends to be associated with conditions in the estuary, not external boundary flows. The authors noted that phytoplankton uptake correlated with TST and that, on average, 89% of TST was involved with recycling. Also, total nitrogen (TN) loading accounted for only 11%, on average, of phytoplankton uptake. It is surmised that the high recycling nature of this low-flushing-rate biogeochemical system drives components with interdependence based on biological activity. The description of control relations and their rationale for these components may very well lie within the system boundaries. These general observations and this subsequent hypothesis remain specific to the model until additional models are studied. In particular, energy models, typically much less cyclical than biogeochemical models with little to no feedback to primary producers, could produce decidedly different results. Nevertheless, a means to quantify distributed control affords the ability to continue to ask these questions.

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4.10 TABLES

Table 4.1: Summary of data for 16 steady-state seasonal nitrogen networks of the Neuse River estuary, North Carolina (Christian and Thomas, 2003). The model components are: PN-Phyto-1, PN-Hetero-2, Sediment-3, DON-4, NO_x-5, NH_4-6, and PN-Abiotic-7. X_i = component storages (mmol-N)/m^2, z_i = boundary inputs (mmol-N)/(m^2 × season), y_i = boundary outputs (mmol-N)/(m^2 × season), T_i = component throughflows (mmol-N)/(m^2 × season), and TST = total system throughflow (mmol-N)/(m^2 × season).

	Spr 85	Sum 85	Fall 85	Win 86	Spr 86	Sum 86	Fall 86	Win 87	Spr 87	Sum 87	Fall 87	Win 88	Spr 88	Sum 88	Fall 88	Win 89
X_1	18	29	23	14	16	29	17	21	21	33	21	28	11	24	13	20
X_2	14	22	18	12	18	18	15	18	16	18	18	8	12	22	12	13
X_3	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300	1,300
X_4	81	68	35	48	89	99	73	51	40	94	108	15	84	78	71	47
X_5	5	6	4	17	5	14	16	84	18	7	4	17	10	7	9	12
X_6	15	5	9	17	13	8	17	14	7	11	5	3	3	5	5	6
X_7	13	25	17	9	17	20	15	13	15	20	17	7	12	24	12	9
z_1	3	4	4	1	3	3	1	12	15	2	1	4	3	1	1	6
z_2	5	5	7	7	5	5	4	15	7	4	4	7	5	4	4	8
z_3	5	8	9	6	5	8	9	6	5	8	9	6	5	8	9	6
z_4	41	26	51	55	36	55	19	146	93	19	16	4	36	27	30	94
z_5	59	50	80	92	57	67	49	464	179	38	39	123	96	64	65	152
z_6	15	20	23	20	17	21	14	37	28	15	12	21	26	23	15	19
z_7	5	6	7	6	5	6	4	11	7	4	4	6	5	5	4	6
y_1	11	7	4	2	4	7	4	31	45	5	4	9	2	6	4	2
y_2	9	3	8	10	8	3	4	371	0	0	0	0	32	0	0	131
y_3	27	83	154	106	74	127	61	156	162	67	56	121	119	90	94	132
y_4	61	18	8	61	28	20	18	81	66	12	19	18	15	26	22	21
y_5	1	0	0	0	2	2	4	7	16	0.1	1	7	1	1	1	1
y_6	15	1	2	4	6	2	5	15	17	2	2	12	3	2	3	2
y_7	9	7	5	4	6	4	4	30	28	3	3	4	4	7	4	2
T_1	1,111	2,499	1,778	665	1,557	2,177	1,985	1,300	2,736	3,092	942	627	1,511	3,331	1,326	1,065
T_2	3,545	7,984	2,946	2,764	5,174	4,064	3,411	2,813	3,681	5,369	2,792	3,683	2,243	5,819	1,783	1,962
T_3	283	558	320	143	298	512	252	194	320	548	296	197	224	502	200	170
T_4	711	1,747	422	535	972	782	645	670	861	589	202	477	583	844	325	385
T_5	333	469	342	318	320	514	355	681	413	512	283	314	312	483	309	354
T_6	2,737	5,959	2,498	2,178	4,001	3,318	2,649	1,843	2,786	4,703	2,559	3,130	1,595	4,914	1,456	1,491
T_7	400	966	474	277	593	613	566	406	736	808	251	252	430	921	333	312
TST	9,120	20,182	8,780	6,880	12,915	11,980	9,863	7,907	11,533	15,621	7,325	8,680	6,898	16,814	5,732	5,789

Table 4.2: Summer 1986 data set for the Neuse River estuary nitrogen model. Inter-component flows, f_{ij}; boundary flows, z and y; and throughflows, T, have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	379	379	1,416	0	3	2,177	29
PN-Hetero	1,453	0	233	383	131	1,432	427	5	4,064	18
Sediment	264	2	0	0	2	54	182	8	512	1300
DON	435	292	0	0	0	0	0	55	782	99
NO_x	0	0	33	0	0	414	0	67	514	14
NH_4	0	3,178	119	0	0	0	0	21	3,318	8
PN-Abiotic	18	589	0	0	0	0	0	6	613	20
									TST = 11,980	
Output, y	7	3	127	20	2	2	4			

Table 4.3: Transitive closure matrices, N = (n_{ij}) and N' = (n'_{ij}), and component throughflows, T_i, for summer 1986 for the Neuse River estuary nitrogen model. Throughflow dimensional units are mmol-N/(m^2-season).

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	13.7826	13.2383	10.0261	13.1635	13.5756	13.4524	12.1982
PN-Hetero	24.7974	25.9522	19.2800	24.7288	24.9737	25.2130	23.8019
Sediment	3.1224	3.1204	3.3461	3.0416	3.1106	3.1219	3.1671
DON	4.5357	4.5099	3.3886	5.4071	4.5070	4.4996	4.1476
NO_x	2.7113	2.8238	2.1939	2.6971	3.7274	2.8766	2.6184
NH_4	20.1170	21.0196	15.8544	20.0445	20.2521	21.4419	19.3489
PN-Abiotic	3.7079	3.8707	2.8772	3.6928	3.7317	3.7654	4.5505
n'_{ij}							
PN-Phyto	13.7826	24.7131	2.3580	4.7285	3.2053	20.5030	3.4348
PN-Hetero	13.2834	25.9522	2.4290	4.7583	3.1586	20.5849	3.5902
Sediment	13.2763	24.7686	3.3461	4.6456	3.1228	20.2311	3.7918
DON	12.6269	23.4376	2.2187	5.4071	2.9624	19.0916	3.2512
NO_x	11.4836	22.3268	2.1854	4.1033	3.7274	18.5692	3.1227
NH_4	13.1991	25.7455	2.4465	4.7242	3.1373	21.4419	3.5747
PN-Abiotic	13.1681	25.6618	2.4031	4.7109	3.1290	20.3810	4.5505
T_i	2,177	4,064	512	782	514	3,318	613

Table 4.4: Fractional transfer coefficients (FTC's), η_{ij}, for summer 1986 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	6.33	6.08	4.61	6.05	6.24	6.18	5.60
PN-Hetero	6.10	6.39	4.74	6.08	6.15	6.20	5.86
Sediment	6.10	6.09	6.54	5.94	6.08	6.10	6.19
DON	5.80	5.77	4.33	6.91	5.76	5.75	5.30
NO_x	5.27	5.49	4.27	5.25	7.25	5.60	5.09
NH_4	6.06	6.34	4.78	6.04	6.10	6.46	5.83
PN-Abiotic	6.05	6.31	4.69	6.02	6.09	6.14	7.42

Table 4.5: Control ratios, cr_{ij}, for summer 1986 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.00	−0.24	0.04	0.15	0.02	−0.07
PN-Hetero	0.00	0.00	−0.22	0.05	0.11	−0.02	−0.07
Sediment	0.24	0.22	0.00	0.27	0.30	0.22	0.24
DON	−0.04	−0.05	−0.27	0.00	0.09	−0.05	−0.12
NO_x	−0.15	−0.11	−0.30	−0.09	0.00	−0.08	−0.16
NH_4	−0.02	0.02	−0.22	0.05	0.08	0.00	−0.05
PN-Abiotic	0.07	0.07	−0.24	0.12	0.16	0.05	0.00

Table 4.6: Control difference parameters, cd_{ij}, for summer 1986 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N. Numbers in the table have been multiplied by 10^3.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	−0.02	−1.49	0.25	0.96	0.12	−0.45
PN-Hetero	0.02	0.00	−1.35	0.32	0.65	−0.13	−0.46
Sediment	1.49	1.35	0.00	1.61	1.81	1.32	1.49
DON	−0.25	−0.32	−1.61	0.00	0.52	−0.29	−0.72
NO_x	−0.96	−0.65	−1.81	−0.52	0.00	−0.51	−0.99
NH_4	−0.12	0.13	−1.32	0.29	0.51	0.00	−0.31
PN-Abiotic	0.45	0.46	−1.49	0.72	0.99	0.31	0.00

Table 4.7: System control vector, sc_j, for the summer of 1986 Neuse River estuary nitrogen model. Dimensional units for sc_j are (m^2 season)/mmol-N. Numbers in the table have been multiplied by 10^3.

	$sc_j \times 10^3$
PN-Phyto	0.64
PN-Hetero	0.95
Sediment	−9.07
DON	2.66
NO_x	5.44
NH_4	0.82
PN-Abiotic	−1.44

Table 4.8: System control vectors, sc_j, for 16 consecutive seasons from spring 1985 through winter 1989 for the seven component Neuse River estuary nitrogen model. TC = total system control. Dimensional units for sc_j and TC are (m^2 season)/mmol-N. TST = total system throughflow (mmol-N/(m^2 × season)).

$sc_j \times 10^3$	Spr 85	Sum 85	Fall 85	Win 86	Spr 86	Sum 86	Fall 86	Win 87	Spr 87	Sum 87	Fall 87	Win 88	Spr 88	Sum 88	Fall 88	Win 89
PN-Phyto	−0.32	0.37	1.26	2.06	1.11	0.64	1.19	0.74	0.61	0.31	−0.26	−0.54	1.95	0.62	1.32	1.22
PN-Hetero	−0.48	0.53	1.43	3.52	0.82	0.95	1.30	−0.24	0.64	0.65	2.06	2.87	1.55	0.56	3.14	0.40
Sediment	−3.79	−7.33	−15.99	−23.09	−11.16	−9.07	−13.40	−6.72	−9.07	−7.82	−12.23	−20.81	−17.57	−8.10	−21.25	−15.46
DON	−1.86	0.74	5.19	2.42	1.35	2.66	1.33	1.11	1.19	1.46	−0.25	1.06	3.06	0.76	2.74	5.03
NO_x	8.10	6.22	9.21	10.99	8.36	5.44	9.44	5.81	7.35	5.78	12.26	14.51	10.73	6.89	13.30	8.68
NH_4	−0.39	0.60	1.55	3.53	0.89	0.82	1.34	0.19	0.33	0.58	1.99	2.42	1.95	0.58	2.99	0.92
PN-Abiotic	−1.26	−1.13	−2.64	0.56	−1.37	−1.44	−1.19	−0.89	−1.06	−0.97	−3.60	0.49	−1.67	−1.30	−2.25	−0.79
TC × 10^3	8.10	8.46	18.64	23.09	12.53	10.51	14.59	7.85	10.13	8.79	16.34	21.36	19.24	9.40	23.50	16.25
TST	9,120	20,182	8,780	6,880	12,915	11,980	9,863	7,907	11,533	15,621	7,325	8,680	6,898	16,814	5,732	5,789

Table 4.9: Nondimensional ratios of individual component throughflows, T_i, to boundary flows, z_i and y_j for nitrogen flow in the Neuse River estuary, USA: 16 seasons. The symbol ∞ denotes division by an output boundary flow, y, very close to zero.

T/z	Spr 85	Sum 85	Fall 85	Win 86	Spr 86	Sum 86	Fall 86	Win 87	Spr 87	Sum 87	Fall 87	Win 88	Spr 88	Sum 88	Fall 88	Win 89
PN-Phyto	370	625	445	665	519	726	1,985	108	182	1,546	942	157	504	3,331	1,326	178
PN-Hetero	709	1,597	421	395	1,035	813	853	188	526	1,342	698	526	449	1,455	446	245
Sediment	57	70	36	24	60	64	28	32	64	69	33	33	45	63	22	28
DON	17	67	8	10	27	14	34	5	9	31	13	119	16	31	11	4
NO_x	6	9	4	3	6	8	7	1	2	13	7	3	3	8	5	2
NH_4	182	298	109	109	235	158	189	50	100	314	213	149	61	214	97	78
PN-Abiotic	80	161	68	46	119	102	142	37	105	202	63	42	86	184	83	52
T/y																
PN-Phyto	101	357	445	333	389	311	496	42	61	618	236	70	756	555	332	533
PN-Hetero	394	2,661	368	276	647	1,355	853	8	∞	∞	∞	∞	70	∞	∞	15
Sediment	10	7	2	1	4	4	4	1	2	8	5	2	2	6	2	1
DON	12	97	53	9	35	39	36	8	13	49	11	27	39	32	15	18
NO_x	333	∞	∞	∞	160	257	89	97	26	5,120	283	45	312	483	309	354
NH_4	182	5,959	1,249	545	667	1,659	530	123	164	2,352	1,280	261	532	2,457	485	746
PN-Abiotic	44	138	95	69	99	153	142	14	26	269	84	63	108	132	83	156

4.11 FIGURE LEGENDS

Figure 4.1: Fractional transfer coefficients (FTC's), η_{ji} and η_{ij}, with their respective boundary flows z and y, between component pairs, H_i and H_j, in an *n*-component system. η_{ji}, defined by equation (4-6), is represented in Figure 4.1a. Similarly, η_{ij}, defined by equation (4-5), is represented in Figure 4.1b.

Figure 4.2: Control difference, cd_{ij}, relationship defined by equation (4-8).

Figure 4.3: System diagrams for equations (4-3) and (4-4) of an *n*-component model showing, in particular, the *fractional transfer coefficients* (FTC's), η_{jk} and η_{kj}, between components H_k and H_j, where $j \neq k$.

Figure 4.4: Digraph of nitrogen flux in the Neuse River estuary, North Carolina, USA (Christian and Thomas, 2003).

Figure 4.5: System control vectors, sc_j, for nitrogen flow in the Neuse River estuary, 16 consecutive seasons.

Figure 4.6: Nondimensional total system control, TC, as a function of total system throughflow, TST, for nitrogen flow in the Neuse River estuary, discrete time analysis over 16 consecutive seasons.

Figure 4.7: Boundary outputs from all model components, y_j, for the Neuse River estuary nitrogen model given a unit boundary input, $z_5 = 1$, to NO_x-5, discrete time analysis over 16 consecutive seasons.

Figure 4.8: Sixteen season average of boundary outputs, y_j, for the Neuse River estuary nitrogen model as a result of a unit boundary input, $z_5 = 1$, to NO_x-5.

Figure 4.9: Boundary inputs from all model components, z_i, for the Neuse River estuary nitrogen model given a unit boundary output, $y_3 = 1$, from Sediment-3, discrete time analysis over 16 consecutive seasons.

Figure 4.10: Sixteen season average of boundary inputs, z_i, for the Neuse River estuary nitrogen model as a result of a unit boundary output, $y_3 = 1$, from Sediment-3.

Figure 4.1

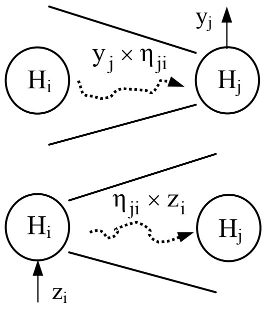


Figure 4.1a, Top: fraction of throughflow input environ represented by $y_j \times \eta_{ji}$ of H_j as oriented from H_i, reference equations (4-2) and (4-6) when k = j. Bottom: fraction of throughflow output environ represented by $\eta_{ji} \times z_i$ of H_i oriented towards H_j, reference equations (4-3) and (4-6) when k = i. Top and bottom figures show the FTC, η_{ji}, is identical for H_i oriented towards H_j either through H_i's output environ or through H_j's input environ.

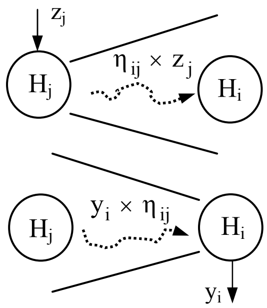


Figure 4.1b, Top: fraction of throughflow output environ represented by $\eta_{ij} \times z_j$ of H_j as oriented towards H_i, reference equations (4-1) and (4-5) when k = j. Bottom: fraction of throughflow input environ represented by $y_i \times \eta_{ij}$ of H_i as oriented from H_j, reference equations (4-4) and (4-5) when k = i. Top and bottom figures show the FTC, η_{ij}, is identical for H_j oriented towards H_i either through H_j's output environ or through H_i's input environ.

Figure 4.2

The diagram to the right shows one control difference combination out of four possible from the FTC environ diagrams of Figures 4.1a and 4.1b. The FTC's model the relationship between H_i and H_j as open-loop controllers which augment boundary inputs, z, to the total combined direct and indirect fractions of throughflow between two components in an *n*-component system. The strongest open-loop controller dominates and establishes direction of control. $cd_{ij} = \eta_{ij} - \eta_{ji}$ quantifies both magnitude and orientation.

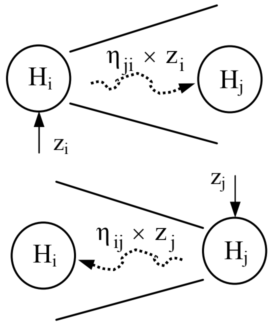


Figure 4.3

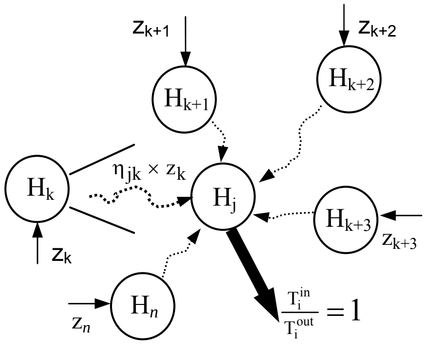


Figure 4.3a, Top: fraction of throughflow output environ represented by $\eta_{jk} \times z_k$ of H_k oriented forward towards H_j, reference equation (4-3) coupled with equation (4-6), as embedded in an *n*-component model. The remaining FTC's for components k + 1 through *n* are not shown for clarity.

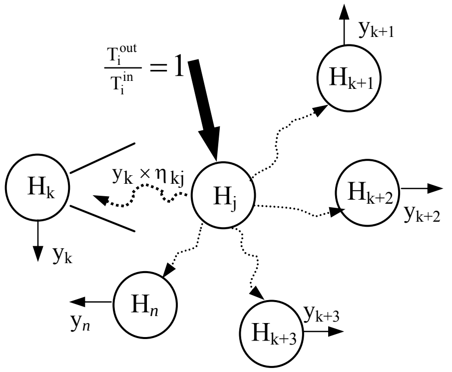


Figure 4.3b, Bottom: fraction of throughflow input environ represented by $y_k \times \eta_{kj}$ of H_k oriented back towards H_j, reference equation (4-4) coupled with equation (4-5), as embedded in an *n*-component model. The remaining FTC's for components k + 1 through *n* are not shown for clarity.

Figure 4.4

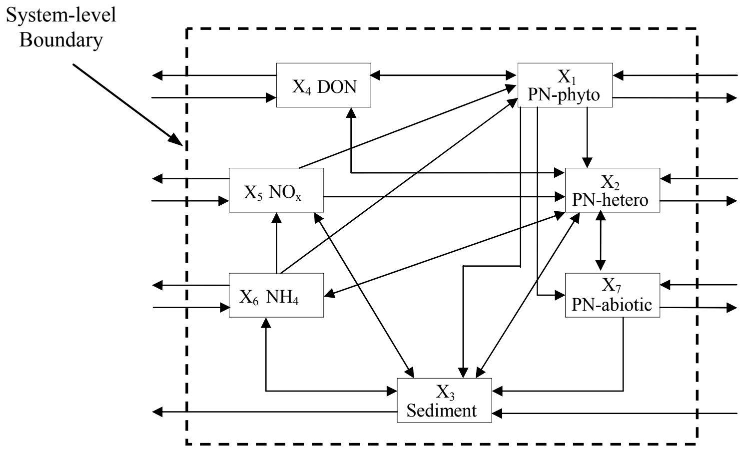

System-level
Boundary
X_4 DON
X_1
PN-phyto
X_5 NO_x
X_2
PN-hetero
X_6 NH_4
X_7
PN-abiotic
X_3
Sediment

Figure 4.5

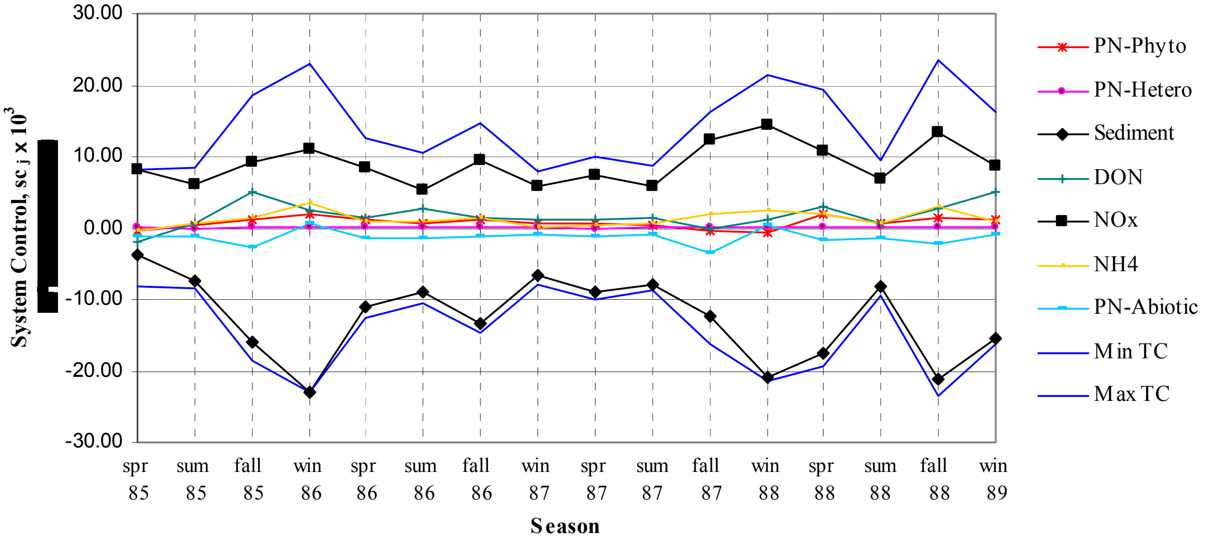

30.00
20.00
10.00
0.00
-10.00
-20.00
-30.00
System Control, sc j x 10^3
spr 85
sum 85
fall 85
win 86
spr 86
sum 86
fall 86
win 87
spr 87
sum 87
fall 87
win 88
spr 88
sum 88
fall 88
win 89
Season
PN-Phyto
PN-Hetero
Sediment
DON
NOx
NH4
PN-Abiotic
Min TC
Max TC

Figure 4.6

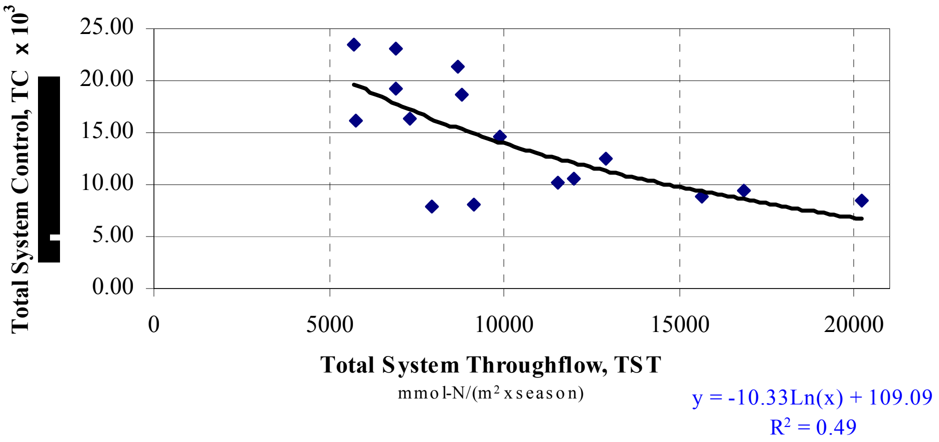

Total System Control, TC x 10^3
25.00
20.00
15.00
10.00
5.00
0.00
0
5000
10000
15000
20000
Total System Throughflow, TST
mmol-N/(m^2 x season)
y = -10.33Ln(x) + 109.09
R^2 = 0.49

Figure 4.7

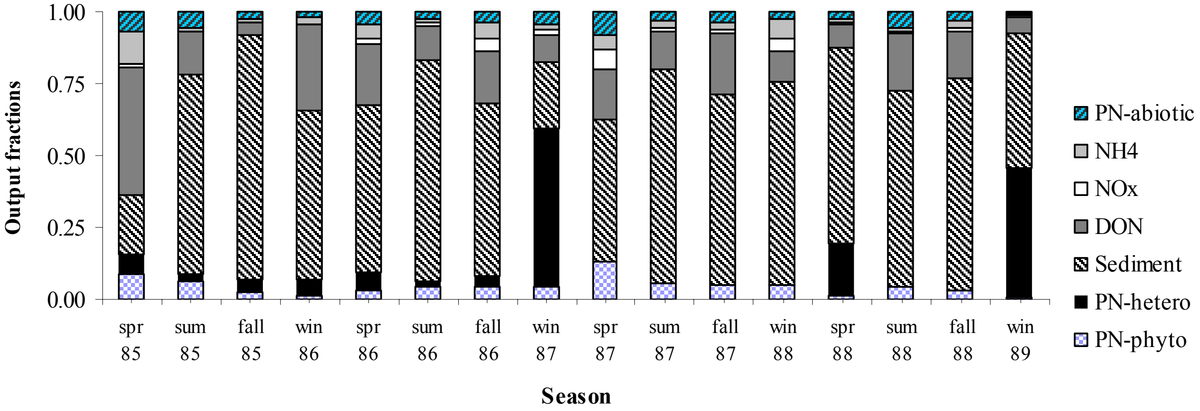

Output fractions
1.00
0.75
0.50
0.25
0.00
spr 85
sum 85
fall 85
win 86
spr 86
sum 86
fall 86
win 87
spr 87
sum 87
fall 87
win 88
spr 88
sum 88
fall 88
win 89
Season
PN-abiotic
NH4
NOx
DON
Sediment
PN-hetero
PN-phyto

Figure 4.8

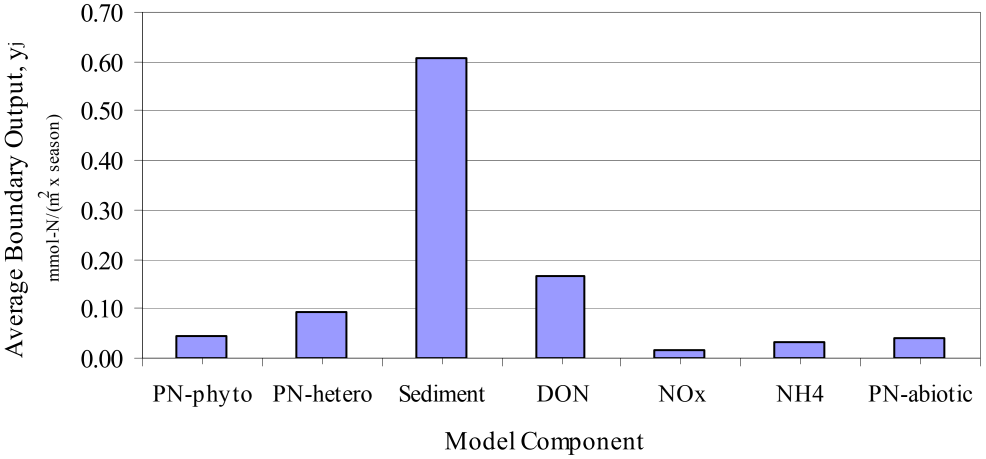

Average Boundary Output, yj
mmol-N/(m^2 x season)
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
PN-phyto
PN-hetero
Sediment
DON
NOx
NH4
PN-abiotic
Model Component

Figure 4.9

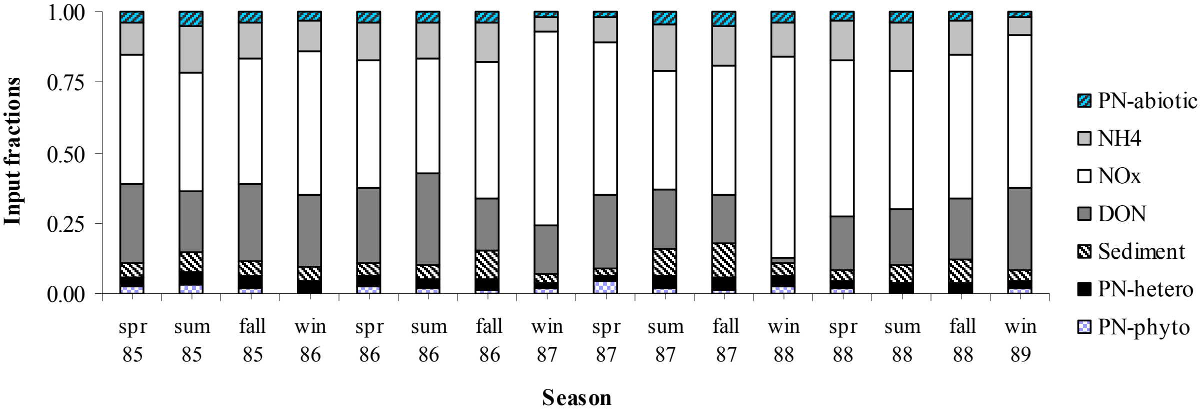
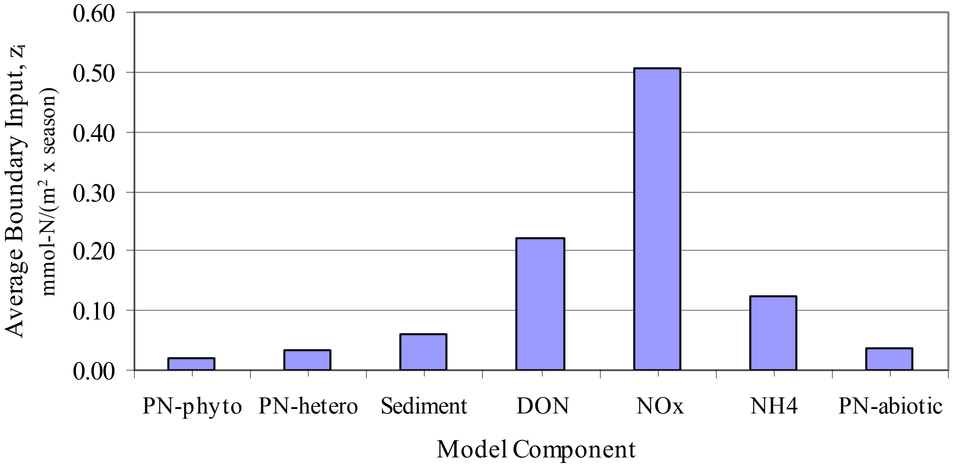

1.00
0.75
0.50
0.25
0.00
Input fractions
spr 85
sum 85
fall 85
win 86
spr 86
sum 86
fall 86
win 87
spr 87
sum 87
fall 87
win 88
spr 88
sum 88
fall 88
win 89
Season
PN-abiotic
NH4
NOx
DON
Sediment
PN-hetero
PN-phyto

Figure 4.10


0.60
0.50
0.40
0.30
0.20
0.10
0.00
Average Boundary Input, z_i
mmol-N/(m^2 x season)
PN-phyto
PN-hetero
Sediment
DON
NOx
NH4
PN-abiotic
Model Component

CHAPTER 5

CONCLUSIONS

5.1 INTRODUCTION

The specific focus of this dissertation has been development of a formal mathematical approach to ecosystem control within the framework of network environ analysis (NEA). While codifying the dissertation's key concepts and results, this chapter elicits or addresses the significance of the more salient points and begins to develop their broader implications.

5.2 PREFACE

> … by long and often cruel experience and by collecting and analyzing the historical material, we are gradually learning to get a clear view of the indirect, more remote, social effects of our productive activity, and so the possibility is afforded us of mastering and controlling these effects as well.
>
> To carry out this control requires something more than mere knowledge. It requires a complete revolution in our hitherto existing mode of production, and with it our whole contemporary social order.
>
> — Frederick Engels, Dialectics of Nature, (written 1874-1882, published posthumously 1940)

Engels was specifically addressing how the reigning human social order affected the earth's natural systems. Assuming, what are now called ecosystems serve as natural resource suppliers necessary for human survival, both he and Marx continually posited that a decentralized societal power structure would synchronize the human experience with the earth's ecosystems. Engels' ecological systems observations, in particular, the indirect effects, were way ahead of the systems science to which he inadvertently referred. Further, his dialog uniquely mirrors the current state of *ecosystem control* in several ways. First, we are only now, after persistent labor, beginning to appreciate the indirect and remote effects fundamental to consideration of the concept. Second, something more than the existing mainstream knowledge will be required to move us forward. Although somewhat allegorical in its true sociological underpinnings, Engels' reference to a "…long and often cruel experience…" uniquely describes the current states of *control* and *ecosystem control* as they pertain to our studies of the natural systems which blanket and sustain us.

5.3 CONTROL

What is control? *Control* is an ambiguous concept when using the hundreds of related synonyms available in the English language vernacular. Scientifically, however, the *control* sequence, in its simplest interpretation, is about the propagation of cause (Patten et al., 1976). Specifically, control is rooted in determinism's small but significant sub-category of causation (the coupling of cause-and-effect through state). If the cause has produced an effect, a control action has occurred by the state. Although the philosophical principles of determinism and causality are profound in their broadest scope, the mechanistic or somewhat practical view of causation is that the *state* of the system generally links the past with the future. For each unique input, a system generates a unique output. The unique antecedent is always followed by the same consequence. All of the previous events constitute the complete cause. This is the main pillar of inductive science (Mill, 1843).

The system's continuous link or linkages (causal bond) between the cause-and-effect are necessary in the controlling action. Kalman's (1963) controllability and observability and later Šiljak's (1991) input and output reachability are concepts which acknowledge the necessity of the transactive connectivity in the system necessary for the propagation-of-cause chain (a.k.a., control) to sequence properly. To *cause* is to initiate control. An *effect* is having been controlled. According to Kalman and Šiljak, the cause-and-effect need to be connected. Simple enough. Using modern parlance (Dorf, 1980), the *cause* is the input. The system state is the open–loop controller. The effect is the output. Aristotle's (1947) belief in the non anticipatory nature of *efficient* cause-and-effect are relayed properly in the open–loop model. Von Uexkūll's (1926) function–circles, which identify that an organism's output (world-of-action) over a time-forward march can then reach the same organism's input (world-as-sensed), acknowledge the classical closed-loop control configuration's existence in natural systems. Patten (1978a) grouped the world-of-action and the world-as-sensed together in a comprehensive whole he simply called *the surrounding world.* Cause-and-effect through state are inherently, by definition, an open-loop, non-anticipatory control sequence. The ecosystem, a "surrounding world", is inherently, almost by definition, a web of closed-loop control sequences that allow for non-anticipatory causal action.

5.4 ECOSYSTEM CONTROL

What is ecosystem control? The broadest perspective on the concept suggests that it is about tracing and quantifying cause-and-effect sequences in natural systems. The analytical process of considering or observing these natural actions and reactions is pervasive throughout the typical ecological contemplation. We qualify, quantify, or sometimes just hypothesize why events unfold as witnessed. Considering common macro-descriptions in mainstream cataloging of phenomena (in population, community, ecosystem, empirical, and theoretical ecology, etc.), the typical subtopics (such as growth, demographics, density dependence, limiting factors, life history traits, predation, resources, competitors, metapopulations, climax theory, etc.) can be aligned with either positive growth, negative feedback, equilibria, or factors that affect these three areas. For example, evolution is about speciation (positive pressure to expand) and extinction (feedback to check expansion) occurring near some spatially and temporally heterogeneous equilibrium. Evolution, the contrapositive of *survival of the fittest*, meaning death to the unfit, is fundamentally a negative feedback process. In living systems, speciation and biotic growth always represent positive pressure counterpoised by biotic (e.g., competition) or abiotic (e.g., resource deficiencies) negative pressures moving towards local or global equilibria.

So long as a biotic component exists in our ecosystems, the classical c*ontrol* model is applicable within or across any spatial or temporal hierarchy.

Consider, bottom-up and top-down control in food webs. Comfortably ensconced in the Lindeman (1942) trophic level paradigm and coaxed by Hairston et al., 1960, we generally refer to control sequences as top-down (via. consumption) (Shears and Babcock, 2002; Worm and Myers, 2003), bottom-up (via. resource limitation) (Power, 1992; Brose, 2003), or combinations of both (Slobodkin et al., 1967; Hunter and Price, 1992; Rosemond et al., 2001; Worm et al., 2002). With a controlled perturbation, starting at some trophic level or species, we trace the direct cause-and-effect sequence through several trophic levels or species in a food web, carefully noting the ecosystem subtleties. Subsequently, and fortunately, our understanding of ecosystem functionality increases with each confirmed cause-and-effect observation. However, as the basic model descriptions we use to observe these action-reaction sequences are challenged over time [e.g., trophic levels (Polis, 1991); ecosystems (O'Neill, 2001)], our periscopic interpretation of direct causal linkages in a highly connected multi-dimensional system are also ripe for critique based on the obvious complexity. After all, in a very simple input and output reachable system, matrix multiplication of the adjacency matrix in graph theory shows that pathway connectivity between two components is a divergent function. When all path lengths are considered, there are in the limit an infinite number of routes (passing through the remaining system's components) between two components. To move beyond the one-dimensional top-down, bottom-up, alternating subtleties, the deep understanding of these innumerable and multi-dimensional indirect pathways and their effects becomes central to our continued pursuit of first-principle rules within ecosystem functionality.

5.5 CHAPTER 1, OBJECTIVES AND CONCLUSIONS

Fortunately Engel's first assertion (in the quotation at the head of the chapter) was also optimistic. By virtue of the current circumstances, we are afforded the possibility to control or at least master our understanding of these indirect effects. Network analysis, coupled with environ theory's two-environment approach (Patten, 1978a; Patten and Auble, 1981; Fath and Patten, 1999; Fath, 2004), has helped broach the mathematics necessary to quantify these indirect manifestations. Specifically, this dissertation has begun to address, while accounting for the embedded indirect effects, the mathematics associated with the distributed controlling pair-wise relationships that occur between components in a multi-component system.

Chapter 1's main objective was to presuppose the distributed control methodology using the nitrogen cycle of the Neuse River estuary by providing a literature based foundation of key concepts necessary for the dissertation's premise. The ecologically surmised control theory of growth and feedback leading to some homeostatic equilibria remains the essential model where the most basic concept of control is initially embedded in Zadeh's premise mechanistic (Zadeh and Desoer, 1963; Zadeh, 1964) that cause drives effect through state. Linear systems theory specifically models this cause-and-effect sequence as an open-loop controller (see, for example, Dorf, 1980). Open-loop control sequences, connected in temporally varying parallel arrangements, create closed-loop non-anticipatory but error-correcting arrangements with the power to regulate outputs. Clearly, this model can be applied to ecosystems growing (positive feedback) against resistance (negative feedback) towards some homeostatic equilibrium (performance indices) at any temporal or spatial hierarchy. Fortunately, the mathematical mapping of an ecosystem as a set of distributed controllers allows for the quantitative accounting of cause-and-effect controlling relationships in a network. As such, NEA coupled with automatic control theory, ecosystem control theory, and nitrogen as a controlling transactional element were together articulated as important dimensions in the foundation of NEA distributed control.

5.6 CHAPTER 2, OBJECTIVES AND CONCLUSIONS

Chapter 2's purpose — to tie NEA mathematics back to first principles — specifically laid the analytical foundation for the distributed control methodology. This was accomplished by achieving three objectives: (1) incorporating a strict Eulerian conservation-law methodology into NEA, (2) synthesizing the throughflow- and storage-based environ-equation development into a seamless, consistent derivation, and (3) clarifying various vague concepts and terminologies such as self-flow, storage, turnover rate, and the necessary inclusion of a discrete time step in environ storage analysis. The 16-model average of the Neuse River estuary nitrogen cycle was then investigated with an enhanced NEA perspective. Three avenues emerged for further consideration.

(1) Due to its theoretical and mathematical nature, the current natural science audience for NEA is limited. Significant systems and mathematical skills are necessary to elicit the essence of the methodology, its results, or corresponding conclusions. This chapter, intended for the journal *Ecological Engineering*, attempts to extend NEA accessibility in the literature. Mass and energy transport dynamics are a mature science outside ecology whose primary laws and equations are fundamental to the typical engineering curriculum. Tying NEA back to this field opens its methodology to additional prospective uses and, therefore, to a much wider audience. Those individuals well versed in transport dynamics will find this chapter a seamless derivation of the environ equations using well established vernacular and mathematical techniques. The allure of capturing or quantifying the power of indirect effects in networks currently resides only

in the mathematics. Developing a wider audience beyond the mathematics will create more opportunities for practical applications.

(2) The Eulerian control surface's existence and analytical rigidity, embedded in the mathematical model, significantly helps streamline NEA equation development and clarify subsequent interpretation. The control surface separates the focal unit (compartment or system) from its environment, both conceptually and pictorially (ref. Figure 2.1), and analytically [conservation equation's development where equations (2-1)–(2-19) are for mass and equations (2-20)–(2-28) are for energy]. The Eulerian mathematical model respects the control surface's three-dimensional location and orientation whereas the generalized focal unit or "compartment" common in ecological network analysis or NEA does not posses this information. Integral calculus is then able to elegantly evaluate and, if necessary, compensate for location and orientation, particularly when three-dimensional constructs are simplified to two- or one-dimensional systems, as in NEA. The accounting of all directions, orientations, and qualities of flow is driven by the analytical description of the control surface. For example, the previously vague, if not multi-defined, *self-flow* was clarified in a simple interpretation of the unyielding analytical model. Although a mass conservation development was used with the nitrogen cycle example, the larger impact of the Eulerian control surface contribution to ecological network analysis may eventually reside with the conservation of energy. The types and qualities of energy crossing the control surface due to mass flow, heat transfer, or work are specifically organized into their respective analytical terms. Simplifying assumptions which combine these terms, common in transport dynamics, are not clearly or correctly defined in the ecological literature. The Eulerian control surface construct will help ecological network analysis in general become better aligned with the more mature field of transport dynamics.

(3) Lastly, some commentary regarding network analysis is appropriate given both Christian and Thomas's (2000, 2003) original research on the Neuse River estuary and the NEA methodology as articulated herein. Specifically, how have (a) NEA methodology and (b) Christian and Thomas's work benefited from this dissertation?

(a) The NEA derivation herein, is subtly different. The conservation equations are derived using a Reynolds transport Eulerian control volume approach. This perspective provides a rigid analytical model of flows and storages necessary for model interpretation and helpful to subsequent NEA development. Most importantly, similar to the mature field of transport dynamics, the control volume (CV)/control surface (CS) derivation clarifies the importance of the latter to the model's analytical development and interpretation. For example, self flows can be defined nicely within the original conservation equation format. Then reaching back and embracing the original methodology (Patten, 1978a; Barber et al., 1979), the concepts of conservation and throughflow, each with algebraic terms directionally oriented across the modeled control surface, were introduced separately at first [conservation equations (2-29)–(2-31); throughflow equations (2-32)–(2-39)] and then combined [equations (2-40) and (2-50)] to provide a clear expression of the oriented perspective to ecological network analysis inherent to NEA. The origination of this explicit equation orientation is a critical aspect to the understanding of NEA and can be lost in the, often too coarse, algebraic substitutions leading to the final environ implicit equations. The new, parallel and consistent, algebraic development of the throughflow and storage NEA equations also helps to minimize the same algebraic difficulties and improve the subsequent system interpretations. Also, while this discussion is addressing Chapter 2, it is still important to briefly acknowledge by stretching into Chapter 3 that the NEA distributed control methodology successfully quantifies the virtually imperceptible, minute magnitudes of pair-wise flow relationships in a network that are distributed over a diverging set of indirect pathways. Establishing and quantifying magnitude and direction (+/−) of basic cause-and-effect relationships between compartment pairs in a multi-component system, which includes these unobservable and divergent pathways, significantly enhances the power of NEA to both identify but also capture the importance of indirect effects.

(b) Christian and Thomas (2000) eloquently summarized the various network theories and supporting software available at the time of their analysis. Their summary demonstrated that ecological network analysis has grown considerably in its logical breadth and pragmatic offerings. Therein lays the significance of this dissertation to Christian and Thomas's Neuse River estuary study. Our distributed control model offers some inviting considerations regarding the potential makeup of the Neuse nitrogen cycle including most prominently the roles of both nitrates and sediment as donors and receivers in the extended nitrogen pathways. As such, the model stimulates new questions as a result of a new perspective. However, for the moment, the primary advancement is not the new questions asked or the potential novel findings. Simply, it's the continued work in ecological network analysis that eventually produces yet more practical tools for the ecosystem manager. Whereas the work herein relies on Christian and Thomas's empirical model, their model relied on the theoretical and empirical models before them. Their deft use of established ecological network theory and the conclusions they achieved demonstrate the need to continually push for additional theoretical investigations of the network perspective. Practical and pragmatic uses emerge from this process. Chapter 2's objectives fundamentally streamlined the development of NEA while making its use more practical and accessible to a wider audience. The necessary analytical foundation for the distributed control methodology was clearly formulated.

5.7 CHAPTER 3, OBJECTIVES AND CONCLUSIONS

Chapter 3's purpose — to articulate and demonstrate a distributed control in the environ structure of a multi-component network model — established a comprehensive methodology for both current consideration and future growth. Chapters one and two, and the literature they reference, gradually research, streamline, but then also subtly force the confrontation of the epistemological underpinnings and shortcomings of the current state of control theory in ecological multi-component systems — dominated by indirect interactions. The fortuitous nature of the current situation lends credence to Engel's optimistic assertion elucidating the rationale for taking the next step. By virtue of the current circumstances, "…the possibility is afforded us of mastering and controlling these [indirect] effects as well." Chapter 3's purpose — to develop the distributed control methodology within environ theory — was accomplished by achieving these three objectives: (1) providing a historical & mathematical foundation for a distributed control perspective, (2) formulating the analytical pair-wise transactive relationships in a network, and (3) expanding the pair-wise relationships in a network to establish a distributed control methodology. A 16-model-average representation of nitrogen flow in the Neuse River estuary was then investigated. Achieving the objectives opened three areas for further discussion.

(1) This research initially assumed that magnitude differentials in transport flows represented dominating/subordinating associations between donors and receiver components. These pairings were deemed controlling/controlled relationships. It is worth reviewing this logic based on the study's overall assertions. *Control* is about causing action. The mechanistic and practical view of cause-and-effect, fundamental to inductive science, is that cause drives effect through state. The potential transactional or relational materials present in the causal actions and

reactions is as diverse as the imagination. To quantify causal sequences, we chose to focus on conservative transactions. If the state (temperature, pressure, amount, etc.) of a material changes, the initiating cause has sequenced to a subsequent effect. To track and capture these sequences, we focused on material quantities. Evidence of a control action is now available and with historical precedence, this sequence is modeled as an open-loop controller.

In steady-state NEA, the transitive closure matrices, N and N', once computed augment or attenuate the boundary flows (y and z) of a system through all direct and indirect pathways to the respective component throughflows. The scalar transitive closure values, n_{ij} and n'_{ij}, or their fractional equivalents, η_{ij}, are regarded as distributed open-loop controllers or transfer functions in the classical control theory sense wherein the closed-loop feedback is inherent to the distributed interactions of the open-loop controllers acting in parallel. The strength or magnitude of the incoming versus outgoing environ contributions to control can then be compared to identify augmentative and suppressive relationships. The controller magnitudes, per se, offer a weighted means to facilitate this comparison. As such, the distributed control methodology successfully tracks and accounts for magnitude differentials of the transactive element, subsequently allowing attribution of quantitative significances to long and indirect causal chains in systems.

(2) Inevitably the expressions *unit input* and *unit output* are mentioned when NEA is discussed. The proclivity towards these expressions is most likely owed to some of the earlier (and more powerful and recognizable) environ analyses (Patten 1978a) which consist of tracing a system’s transactions of a steady-state unitary input-into or output-from one compartment in the environ network of a model. For example, a steady-state unit input can be supplied into the output environ of one component in a multi-component model and traced forward using the

information in the N matrix through the system's internal cycling until its fractions completely exit from multiple components across the system boundary. Christian and Thomas (2003) used this technique (a unidirectional environs analysis) in their study of NO_x-N entering the Neuse River estuary. They demonstrated that the majority of imported NO_x-N was deposited in the Sediment compartment in all summers and falls, half of the springs, and never in the winter before it exited the system. They also showed that Phytoplankton uptake of an atom of imported N ranged from about 2 to 35 times during its stay in the estuary with strong seasonality over their 16-season data set. Although the nitrogen entered as NO_x, most of the Phytoplankton uptake was in the form of NH_4-N as a result of recycling. Unit boundary values allow for easy accounting of flows as they simply split into fractions of the original unit. By superposition, if the inputs and outputs are scaled-up to their original values, then added into the model one-by-one, the original model steady-state flows are eventually obtained. In short, unit environ analysis scales the model flows to values that allow convenient readout of magnitudes and routings.

However, a *per unit of boundary flow* statement is germane to other areas of environs analysis unrelated to the above example, and subsequently, can be misinterpreted. Consider, that the original environ *control* matrix is a construct of the transitive closure matrices N and N' (Patten 1978b). Discussions of the ratios calculated from the respective interstitial matrix values, n_{ij} and n'_{ij}, often referenced the material or energy transferred as *per unit of input*, z_i, or *per unit of output*, y_j (e.g., Patten and Auble, 1981; Patten, 1982). Similar statements have been made in this dissertation. These statements, although technically true and appropriate in their context, can also be misleading if not completely understood, and have led to the incorrect labeling of this distributed control evaluation as a unit environ analysis. In fact, this is not a unit boundary analysis. The statements merely acknowledge that the boundary flow vector, z_i or y_j, when multiplied together with the respective interstitial transitive closure matrix value, n_{ij} or n'_{ij}, will scale-up the final combination to the total throughflow magnitude, T^{out} or T^{in}. Considered another way, without the boundary flow vector magnitude, the interstitial matrix value, n_{ij} or n'_{ij}, when removed from its algebraic equation is technically on a *per unit* of boundary flow basis. We are acknowledging the dimensional units which carry with the numerical magnitude of n_{ij} or n'_{ij}. This is not suggesting unit environ analysis. The analytical open-loop control model developed and used herein helps to clarify this point further. The interstitial transitive closure values, n_{ij} and n'_{ij}, are considered transfer functions in an open-loop control configuration which augment or attenuate the flows they receive. Traditional control theory measures dominant and suppressive pairings by comparing the strengths (magnitudes) of the transfer functions. Control theory does not compare the transfer functions coupled with their inputs which, in the end, would just be a comparison of the two systems' outputs. We are measuring the *ability to change*, not the result of the change. Given an open-loop controller, what is its propensity to enact change? It is the weight of the transfer function on a per unit basis. Between two transfer functions, which is the stronger? The one with a greater capacity to enact change when the two are compared, on a per unit basis.

As a final thought, consider that a steady-state system is really dynamic over short-term micro adjustments but approaches an assumed steady-state operation over the long term. During these micro transient periods with the inputs and outputs varying ever so slightly, the strength of the transfer functions, also varying ever so slightly, represent the weighted capability for each component to enact change in the system. A comparison of these transfer functions determines which is the one that will institute the greatest impact on the micro change of delta input that it receives as each input magnitude varies ever so slightly. As such, both the original and the present environ *control* analyses are not unit environ analyses in the traditional sense, they are comparisons of individual component's abilities to enact change to the input they receive, on a per unit input basis. Although one component may receive a larger input than another component in the same system, the first component's overall strength to modify the signal could be far weaker than the second component's. Subtle to tease out in a so-called steady-state model, the clarity of the distinction between a *unit analysis* and a *per unit input* can be more evident when the system's micro transient dynamics are included in the descriptive comparison.

(3) The Neuse River estuary control metric results are intriguing but require a realistic appreciation of their significance. As Patten (1982) expressed with his original formulation, "The dominance expressed in this example is not exactly like that which results from direct competition in the 'struggle for existence.'" The control metrics meet the technical definition of control, as derived herein, but practically applied, the results represent the quantification and comparison of nitrogen flow potentials through all the direct and indirect pathways of the Neuse River estuary. Without mentioning the specific cause-and-effect sequencing, at least the following explicit statements are quantifiably true, from the mathematics, for a per-unit boundary, fractional-environ control comparison of NO_x and Sediment: (1) Sediment, through its output environ oriented towards NO_x, generates a fraction of throughflow [$\eta_{53} = 0.00071$ (m^2 × season)/mmol-N] that is 44% less than the fraction of throughflow [$\eta_{35} = 0.00126$ (m^2 × season)/mmol-N] available to Sediment through its input environ from NO_x; (2) Conversely, NO_x, through its output environ oriented towards Sediment, generates a fraction of throughflow [$\eta_{35} = 0.00126$ (m^2 × season)/mmol-N] to Sediment that exceeds the fraction of throughflow [$\eta_{53} = 0.00071$ (m^2 × season)/mmol-N] that is available to NO_x through its input environ oriented from Sediment by 77%; (3) NO_x is more efficient at moving nitrogen to Sediment on a per-

throughflow basis than conversely. Thus, NO_x supplies a greater fraction of throughflow through all the direct and indirect pathways to Sediment than Sediment returns back to NO_x. Further, all components are more efficient at moving nitrogen to Sediment ($cr_{31} = 0.33$, $cr_{32} = 0.29$, $cr_{34} = 0.33$, $cr_{35} = 0.44$, $cr_{36} = 0.29$, $cr_{37} = 0.34$; Equation 3-22), than conversely, and NO_x is more efficient at moving nitrogen to all other components ($cr_{15} = 0.28$, $cr_{25} = 0.24$, $cr_{35} = 0.44$, $cr_{45} = 0.24$, $cr_{65} = 0.21$, $cr_{75} = 0.28$; Equation 3-22), than conversely.

It remains the hypothesis of this research, an extension of Patten's (1978b) and Patten and Auble's (1981) original idea, that this overall weighted environ comparison offers a quantitative methodology to compare pair-wise dominance or control relations in a network. The term "control" is not used in the common perspective of a transient closed-loop negative feedback system, but rather, given a dynamic steady-state model, control resides in the component's action as an open-loop, augmenting or diminishing, checks-and-balances-type transfer function representing potential for change. The closed-loop feedback is inherent in the distributed interactions of the open-loop controllers acting in parallel. The orientation of the controlling relationship (greater magnitude denotes dominance, therefore control) was derived based on an energy flow derivation in the Cone Spring energy model (Patten, 1978b; Patten and Auble, 1981). In the aquatic energy cycle, plants have complete control ($c_{ij} = 1$) over every other component — "bottom-up control" in contemporary parlance. This established the direction of control (greater magnitude dominates) that remained plausible with the remaining component pair-wise relations when considered independently. The established direction of the controlling relationship was then carried forward to this review of nitrogen cycling. The direction of control herein, although intuitively plausible (e.g., Sediment is controlled by all other components while NO_x controls all other components), remains inconclusive absent additional research. For instance, if a limiting factor perspective is used (e.g., Sediment sequesters N with a controlled release to NO_x), the conclusions could be opposite, i.e., Sediment controls all other components and all other components control NO_x. I close the discussion of this section with the same statement that I used to introduce it, "the Neuse River estuary control metric results are intriguing but require a realistic appreciation of their significance." We have successfully mapped the cause-and-effect distributed controlling magnitudes and directions (+/−) of a conservative material through the direct and indirect pathways of a multi-component system.

5.8 CHAPTER 4, OBJECTIVES AND CONCLUSIONS

Chapter 4's primary purpose was to identify correlations or causations within the indirect environ control patterns exhibited by a sequence of static models approximating a time-continuous dynamic model. The mathematics for a dynamic NEA has yet to be developed, necessitating the discrete-time approximation approach. Chapter 4's purpose — to review temporal control relations — was achieved through the following specific objectives: (1) dynamic distributed control characteristics inherent in Christian and Thomas's (2003) sixteen season nitrogen cycling models were calculated, (2) temporal control patterns were elicited, and (3) external and internal parameter correlations or causations relating to these control patterns were identified. The results are interesting but not conclusive. Those concerning NO_x and Sediment are summarized here.

Considering all seven model components, the only statistically significant sc_j correlations to TST were sc_3 (Sediment) and sc_5 (NO_x) (sc_3 vs. $\log_e$ TST, $r = 0.63$, $n = 16$, $p < 0.01$; sc_5 vs. $\log_e$ TST, $r = 0.64$, $n = 16$, $p < 0.01$). The remaining five component's ($r = 0.10$ to 0.54) did not correlate. The only statistically significant sc_j correlations with respective component throughflows, T_j, were also sc_3 and sc_5 (sc_3 vs. $\log_e T_3$, $r = 0.67$, $n = 16$, $p < 0.01$; sc_5 vs. $\log_e T_5$,

$r = 0.82$, $n = 16$, $p < 0.001$). These r values are larger than the remaining five component's ($r = 0.00$ to 0.41) which did not statistically correlate. Both TST and T_j reflect internal system activity since they include recycling. This is particularly pertinent for the Neuse models because their recycling rates were extremely high with the Finn Cycling Index averaging over 89%.

No system control parameter correlated with TN loading, an external system driver. Christian and Thomas (2003) similarly concluded that primary production tends to be associated with nitrogen cycling inside the estuary (correlated with TST) rather than nitrogen import through TN loading. Nitrogen cycling within the system was very high whereas TN boundary inflow averaged only 11% of the total phytoplankton uptake. The present analysis similarly found no correlations between the distributed control metrics and either TN loading or individual nitrogen inputs, z_i, or outputs, y_j. Direct causation or correlation from outside the system boundaries was not statistically evident. However, with two exceptions, the boundary flows, y_3 and z_5, were the highest boundary flows over all 16 seasons.

There appears to be a relationship between the dominance of boundary flows, within a component's throughflow and the indicated control magnitudes of NO_x and Sediment. The input boundary flow at NO_x, z_5, and the output boundary flow from Sediment, y_3, dominated their respective component throughflows (i.e., $T_3/y_3 < T_j/y_j$ for $j = 1, 2, 4, 5, 6, 7$; $T_5/z_5 < T_i/z_i$ for $j = 1, 2, 3, 4, 6, 7$). Further consideration of this finding leads to conflicting conclusions. On average, most boundary input to NO_x leaves the system as boundary output from Sediment. This suggests a unique connectivity relationship in the indirect pathways of the network interconnecting these components. However, the correlation of T_3/y_3 to sc_3 was statistically significant (sc_3 vs. T_3/y_3, $r = 0.68$, $n = 16$, $p < 0.01$) whereas that of T_5/z_5 to sc_5 was not (sc_5 vs. T_5/z_5, $r = 0.36$, $n = 16$, p at $\alpha_{0.05} = 0.50$).

Thus, control strengths of NO_x and Sediment appear to correlate with internal system, presumably biological, activity reflected in both global (TST) and local (T_5 and T_3) throughflow characteristics. Also, their consistent control direction over all 16 seasons appears related to the consistent dominance of y_3/T_3 and z_5/T_5 ratios. The control parameter correlations to these ratios and the unit environ analysis of the NO_x and Sediment connectivity are conflicting but nevertheless encouraging for future pursuits.

5.9 POSTSCRIPT

Research always involves assumptions and concessions, etc. that are mitigated or resolved along the way or remain for future endeavors. The pair-wise environ control approached developed herein revealed the importance of component throughflows in mathematically formulating the distributed control concept. This was codified with the formulation of the relative vs. absolute control designations. The fraction-of-throughflow concept of absolute control was explored, but the magnitude-of-throughflow concept of relative control remains for further work. The open-loop control strategy of comparing transfer-function strengths for dominance aligns with automatic control theory. This is appropriate for steady-state analysis, but in a dynamic NEA theory with time-varying transfer functions a closed-loop approach might be better. Although the open-loop concept was used here to identify and capture transfer function magnitudes, the generalization is made that network structure inherently involves closed-loop dynamics — hence the system of checks and balances one can envision distributed control to represent. The mathematics associated with combining open-loop transfer functions both in parallel and in series are complex and still remain to be explored within environ methodology.

In closing, this research further develops the formal mathematics of distributed control in ecosystems. Network environ analysis and its dual environment model are strengthened and expanded as a means to trace the indirect effects pervasive and dominant in natural systems. A significant amount of work remains to understand distributed control within the hierarchical fabric of decentralized networks where Engel's dialectic prenotion,

> "To carry out this control requires something more than mere knowledge. It requires a complete revolution in our hitherto existing mode of production, and with it our whole contemporary social order."

foretells an extended era of discovery.

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APPENDIX

Appendix A

Inter-compartment flows, f_{ij}; boundary flows, z and y; throughflows, T; and Storage, X, values as collected and determined by Christian and Thomas for the Neuse River estuary nitrogen model, North Carolina, USA. The 17 tables below include 16 consecutive seasons from Spring 1985 to Winter 1989 and an average data set as calculated by Christian and Thomas which represents all 16 seasons.

Table A1, Average data set (as calculated by Christian and Thomas) representing 16 consecutive seasons from Spring 1985 through Winter 1989 for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T, have dimensional units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	1,176	918	4,816	0	16	6,927	85
PN-Hetero	4,859	0	555	1,388	583	5,999	1605	24	15,005	64
Sediment	611	7	0	0	66	80	463	28	1,255	5,200
DON	1,363	1,138	0	0	0	0	0	187	2,688	270
NO_x	0	0	123	0	0	1,037	0	419	1,578	59
NH_4	0	11,714	159	0	0	0	0	82	11,955	36
PN-Abiotic	57	2,005	0	0	0	0	0	39	2,099	61
Output, y	37	148	418	124	11	23	31		TST = 41,517	

Table A2, Spring 1985 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	191	191	726	0	3	1,111	18
PN-Hetero	735	0	201	459	104	1,745	296	5	3,545	14
Sediment	131	1	0	0	37	14	95	5	283	1,300
DON	222	448	0	0	0	0	0	41	711	81
NO_x	0	0	37	0	0	237	0	59	333	5
NH_4	0	2,704	18	0	0	0	0	15	2,737	15
PN-Abiotic	12	383	0	0	0	0	0	5	400	13
Output, y	11	9	27	61	1	15	9		TST = 9,120	

Table A3, Summer 1985 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	508	366	1,621	0	4	2,499	29
PN-Hetero	1,706	0	323	1,221	101	3,897	731	5	7,984	22
Sediment	264	2	0	0	2	54	228	8	558	1300
DON	499	1,222	0	0	0	0	0	26	1,747	68
NO_x	0	0	33	0	0	386	0	50	469	6
NH_4	0	5,820	119	0	0	0	0	20	5,959	5
PN-Abiotic	23	937	0	0	0	0	0	6	966	25
Output, y	7	3	83	18	0	1	7		TST = 20,182	

Table A4, Fall 1985 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	238	238	1,298	0	4	1,778	23
PN-Hetero	1,237	0	128	176	103	960	335	7	2,946	18
Sediment	167	2	0	0	1	7	134	9	3,20	1,300
DON	355	16	0	0	0	0	0	51	4,22	35
NO_x	0	0	31	0	0	231	0	80	3,42	4
NH_4	0	2,468	7	0	0	0	0	23	2,498	9
PN-Abiotic	15	452	0	0	0	0	0	7	474	17
Output, y	4	8	154	8	0	2	5		TST = 8,780	

Table A5, Winter 1986 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	108	108	448	0	1	665	14
PN-Hetero	458	0	0	366	184	1,517	232	7	2,764	12
Sediment	64	1	0	0	26	5	41	6	143	1,300
DON	133	347	0	0	0	0	0	55	535	48
NO_x	0	0	22	0	0	204	0	92	318	17
NH_4	0	2,143	15	0	0	0	0	20	2,178	17
PN-Abiotic	8	263	0	0	0	0	0	6	277	9
Output, y	2	10	106	61	0	4	4		TST = 6,880	

Table A6, Spring 1986 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	260	260	1,034	0	3	1,557	16
PN-Hetero	1,111	0	169	684	21	2,721	463	5	5,174	18
Sediment	116	2	0	0	37	14	124	5	298	1,300
DON	311	625	0	0	0	0	0	36	972	89
NO_x	0	0	37	0	0	226	0	57	320	5
NH_4	0	3,966	18	0	0	0	0	17	4,001	13
PN-Abiotic	15	573	0	0	0	0	0	5	593	17
Output, y	4	8	74	28	2	6	6		TST = 12,915	

Table A7, Summer 1986 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	379	379	1,416	0	3	2,177	29
PN-Hetero	1,453	0	233	383	131	1,432	427	5	4,064	18
Sediment	264	2	0	0	2	54	182	8	512	1300
DON	435	292	0	0	0	0	0	55	782	99
NO_x	0	0	33	0	0	414	0	67	514	14
NH_4	0	3,178	119	0	0	0	0	21	3,318	8
PN-Abiotic	18	589	0	0	0	0	0	6	613	20
Output, y	7	3	127	20	2	2	4		TST = 11,980	

Table A8, Fall 1986 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	364	250	1,370	0	1	1,985	17
PN-Hetero	1,446	0	153	263	100	992	453	4	3,411	15
Sediment	124	2	0	0	1	7	109	9	252	1,300
DON	397	229	0	0	0	0	0	19	645	73
NO_x	0	0	31	0	0	275	0	49	355	16
NH_4	0	2,628	7	0	0	0	0	14	2,649	17
PN-Abiotic	14	548	0	0	0	0	0	4	566	15
Output, y	4	4	61	18	4	5	4		TST = 9,863	

Table A9, Winter 1987 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	326	326	636	0	12	1,300	21
PN-Hetero	903	0	1	263	322	992	317	15	2,813	18
Sediment	96	2	0	0	26	5	59	6	194	1,300
DON	258	266	0	0	0	0	0	146	670	51
NO_x	0	0	22	0	0	195	0	464	681	84
NH_4	0	1,791	15	0	0	0	0	37	1,843	14
PN-Abiotic	12	383	0	0	0	0	0	11	406	13
Output, y	31	371	156	81	7	15	30		TST = 7,907	

Table A10, Spring 1987 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	580	273	1,868	0	15	2,736	21
PN-Hetero	1,980	0	103	215	87	690	599	7	3,681	16
Sediment	153	2	0	0	37	14	109	5	320	1,300
DON	544	224	0	0	0	0	0	93	861	40
NO_x	0	0	37	0	0	197	0	179	413	18
NH_4	0	2,740	18	0	0	0	0	28	2,786	7
PN-Abiotic	14	715	0	0	0	0	0	7	736	15
Output, y	45	0	162	66	16	17	28		TST = 11,533	

Table A11, Summer 1987 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	333	333	2,424	0	2	3,092	33
PN-Hetero	2,211	0	329	244	176	1782	623	4	5,369	18
Sediment	300	2	0	0	2	54	182	8	548	1,300
DON	558	12	0	0	0	0	0	19	589	94
NO_x	0	0	33	0	0	441	0	38	512	7
NH_4	0	4,569	119	0	0	0	0	15	4,703	11
PN-Abiotic	18	786	0	0	0	0	0	4	808	20
Output, y	5	0	67	12	0.1	2	3		TST = 15,621	

Table A12, Fall 1987 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	61	61	819	0	1	942	21
PN-Hetero	602	0	202	122	220	1,518	124	4	2,792	18
Sediment	153	2	0	0	1	7	124	9	296	1,300
DON	168	18	0	0	0	0	0	16	202	108
NO_x	0	0	31	0	0	213	0	39	283	4
NH_4	0	2,540	7	0	0	0	0	12	2,559	5
PN-Abiotic	15	232	0	0	0	0	0	4	251	17
Output, y	4	0	56	19	1	2	3		TST = 7,325	

Table A13, Winter 1988 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	74	74	475	0	4	627	28
PN-Hetero	360	0	39	385	207	2,469	216	7	3,683	8
Sediment	127	1	0	0	26	5	32	6	197	1,300
DON	125	348	0	0	0	0	0	4	477	15
NO_x	0	0	22	0	0	169	0	123	314	17
NH_4	0	3,094	15	0	0	0	0	21	3,130	3
PN-Abiotic	6	240	0	0	0	0	0	6	252	7
Output, y	9	0	121	18	7	12	4		TST = 8,680	

Table A14, Spring 1988 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	386	172	950	0	3	1,511	11
PN-Hetero	1,116	0	50	182	102	449	339	5	2,243	12
Sediment	80	1	0	0	37	14	87	5	224	1,300
DON	302	245	0	0	0	0	0	36	583	84
NO_x	0	0	37	0	0	179	0	96	312	10
NH_4	0	1,551	18	0	0	0	0	26	1,595	3
PN-Abiotic	11	414	0	0	0	0	0	5	430	12
Output, y	2	32	119	15	1	3	4		TST = 6,898	

Table A15, Summer 1988 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	446	446	2,438	0	1	3,331	24
PN-Hetero	2,419	0	260	372	34	2,034	696	4	5,819	22
Sediment	218	2	0	0	2	54	218	8	502	1,300
DON	666	151	0	0	0	0	0	27	844	78
NO_x	0	0	33	0	0	386	0	64	483	7
NH_4	0	4,772	119	0	0	0	0	23	4,914	5
PN-Abiotic	22	894	0	0	0	0	0	5	921	24
Output, y	6	0	90	26	1	2	7		TST = 16,814	

Table A16, Fall 1988 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	261	3	1,061	0	1	1,326	13
PN-Hetero	951	0	68	42	304	172	242	4	1,783	12
Sediment	95	1	0	0	1	7	87	9	200	1,300
DON	265	30	0	0	0	0	0	30	325	71
NO_x	0	0	31	0	0	213	0	65	309	9
NH_4	0	1,434	7	0	0	0	0	15	1,456	5
PN-Abiotic	11	318	0	0	0	0	0	4	333	12
Output, y	4	0	94	22	1	3	4		TST = 5,732	

Table A17, Winter 1989 data set for the Neuse River estuary nitrogen model. Inter-compartment flows, f_{ij}; boundary flows, z and y; and throughflows, T have units of mmol-N/(m^2 × season). Storage, X, has dimensional units of mmol-N/m^2. TST is total system throughflow.

f_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic	Input, z	Throughflow T	Storage, X
PN-Phyto	0	0	0	190	190	679	0	6	1,065	20
PN-Hetero	748	0	1	174	137	625	269	8	1,962	13
Sediment	91	1	0	0	26	5	41	6	170	1,300
DON	212	79	0	0	0	0	0	94	385	47
NO_x	0	0	22	0	0	180	0	152	354	12
NH_4	0	1,457	15	0	0	0	0	19	1,491	6
PN-Abiotic	12	294	0	0	0	0	0	6	312	9
Output, y	2	131	132	21	1	2	2		TST = 5,739	

Appendix B

Efferent and afferent transitive closure matrices, N and N', for the Neuse River estuary nitrogen model, North Carolina, USA. The 17 tables below include 16 consecutive seasons from Spring 1985 to Winter 1989 and an average data set as calculated by Christian and Thomas which represents all 16 seasons.

Table B1, Transitive closure matrices, n_{ij} and n'_{ij}, for the average data set representing 16 consecutive seasons from Spring 1985 through Winter 1989 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	9.4017	8.7262	5.8723	8.6192	8.9334	8.9804	7.9603
PN-Hetero	19.1590	20.1550	13.2690	18.7890	19.1350	19.5800	18.3210
Sediment	1.5820	1.5589	2.0433	1.4971	1.5807	1.5703	1.6411
DON	3.3025	3.2450	2.1615	4.1205	3.2085	3.2515	2.9553
NO_x	1.4691	1.5340	1.1208	1.4349	2.4674	1.5831	1.4189
NH_4	15.1490	15.9230	10.6120	14.8500	15.1300	16.4770	14.5030
PN-Abiotic	2.6361	2.7635	1.8204	2.5803	2.6290	2.6889	3.5123
n'_{ij}							
PN-Phyto	9.4017	18.9150	1.0641	3.3451	2.0366	15.5010	2.4147
PN-Hetero	8.8386	20.1550	1.1092	3.3641	2.0125	15.5920	2.5639
Sediment	8.7308	18.6480	2.0433	3.2065	1.9888	14.9590	2.7474
DON	8.5093	18.1240	1.0092	4.1205	1.8847	14.4610	2.3099
NO_x	6.4441	14.5850	0.8908	2.4426	2.4674	11.9860	1.8879
NH_4	8.7766	19.9970	1.1140	3.3390	1.9984	16.4770	2.5488
PN-Abiotic	8.6898	19.7470	1.0874	3.3012	1.9758	15.3000	3.5123

Table B2, Transitive closure matrices, n_{ij} and n'_{ij}, for Spring 1985 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	9.1070	8.3876	7.6378	7.8613	8.6917	8.5550	8.0208
PN-Hetero	27.0471	28.2265	25.3557	25.4880	27.1463	27.6508	26.9096
Sediment	2.1359	2.0924	2.9079	1.9246	2.2017	2.1061	2.2390
DON	5.2378	5.2431	4.7305	5.7919	5.1674	5.2038	5.0034
NO_x	2.0774	2.1494	2.0709	1.9457	3.0930	2.1998	2.0824
NH_4	20.7664	21.6633	19.5254	19.5637	20.8463	22.2250	20.6681
PN-Abiotic	3.0205	3.1402	2.8219	2.8386	3.0268	3.0798	3.9939
n'_{ij}							
PN-Phyto	9.1070	26.7633	1.9455	5.0309	2.6052	21.0755	2.8878
PN-Hetero	8.4765	28.2265	2.0242	5.1120	2.5500	21.3484	3.0363
Sediment	8.3850	26.2102	2.9079	4.8352	2.5906	20.3688	3.1647
DON	8.1846	26.1420	1.8829	5.7919	2.4202	20.0321	2.8149
NO_x	6.9310	22.8819	1.7600	4.1543	3.0930	18.0810	2.5014
NH_4	8.4295	28.0586	2.0189	5.0821	2.5363	22.2250	3.0205
PN-Abiotic	8.3895	27.8298	1.9965	5.0456	2.5198	21.0734	3.9939

Table B3, Transitive closure matrices, n_{ij} and n'_{ij}, for Summer 1985 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	21.6213	21.0205	17.9552	20.9786	21.4763	21.1821	20.1447
PN-Hetero	67.3262	68.8826	58.4318	67.7203	67.6234	68.2714	65.9168
Sediment	4.6817	4.6710	4.9779	4.6260	4.6806	4.6765	4.7096
DON	14.6220	14.7403	12.5286	15.5540	14.6386	14.6790	14.1115
NO_x	3.5206	3.5933	3.1222	3.5352	4.5346	3.6296	3.4561
NH_4	50.0764	51.2086	43.6559	50.3518	50.2928	51.7643	49.0549
PN-Abiotic	8.1004	8.2775	7.0228	8.1407	8.1339	8.2073	8.9214
n'_{ij}							
PN-Phyto	21.6213	67.1578	4.0092	14.6657	4.0306	50.5100	7.7870
PN-Hetero	21.0732	68.8826	4.0838	14.8181	3.9724	50.9556	7.9754
Sediment	20.9669	66.8334	4.9779	14.4831	3.9341	49.9415	8.1531
DON	20.9161	67.3648	4.0017	15.5540	3.9299	50.0699	7.8029
NO_x	18.7591	61.1709	3.7147	13.1683	4.5346	46.1174	7.1185
NH_4	21.0003	68.6105	4.0879	14.7616	3.9583	51.7643	7.9522
PN-Abiotic	20.9553	68.4137	4.0566	14.7224	3.9491	50.6285	8.9214

Table B4, Transitive closure matrices, n_{ij} and n'_{ij}, for Fall 1985 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	10.4139	9.8658	5.1600	9.9879	10.2335	10.1635	8.4314
PN-Hetero	16.4525	17.4965	8.9893	16.5760	16.7451	16.8467	14.9070
Sediment	1.7707	1.7666	1.9168	1.7354	1.7699	1.7681	1.7904
DON	2.1686	2.0649	1.0791	3.0842	2.1342	2.1208	1.7644
NO_x	1.4497	1.5302	0.8860	1.4558	2.4723	1.5724	1.3319
NH_4	13.8218	14.6963	7.5727	13.9245	14.0669	15.1519	12.5274
PN-Abiotic	2.6121	2.7677	1.4227	2.6275	2.6555	2.6705	3.3583
n'_{ij}							
PN-Phyto	10.4139	16.3469	0.9287	2.3706	1.9684	14.2793	2.2477
PN-Hetero	9.9296	17.4965	0.9764	2.3744	1.9439	14.2848	2.3985
Sediment	9.8386	16.2639	1.9168	2.2886	1.8916	13.8020	2.6521
DON	9.1370	14.4149	0.8183	3.0842	1.7296	12.5538	1.9818
NO_x	7.5367	13.1809	0.8290	1.7963	2.4723	11.4853	1.8460
NH_4	9.8379	17.3320	0.9701	2.3523	1.9259	15.1519	2.3771
PN-Abiotic	9.7983	17.2018	0.9605	2.3392	1.9160	14.0737	3.3583

Table B5, Transitive closure matrices, n_{ij} and n'_{ij}, for Winter 1986 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	4.3889	3.7288	0.9775	3.4369	3.7280	3.8514	3.2677
PN-Hetero	15.0461	16.8056	4.0353	14.5343	15.1640	16.2298	14.6728
Sediment	0.7742	0.7482	1.2016	0.6681	0.7941	0.7575	0.8045
DON	2.7667	2.8556	0.7021	3.5121	2.6493	2.8078	2.4956
NO_x	1.2194	1.3429	0.4897	1.1648	2.2312	1.3962	1.1972
NH_4	11.7469	13.1083	3.2547	11.3389	11.8403	13.6628	11.4606
PN-Abiotic	1.4845	1.6439	0.3957	1.4243	1.4877	1.5906	2.4355
n'_{ij}							
PN-Phyto	4.3889	15.4984	0.2102	2.7650	1.7827	12.6139	1.3612
PN-Hetero	3.6200	16.8056	0.2088	2.8133	1.7446	12.7889	1.4705
Sediment	3.6004	14.4609	1.2016	2.4996	1.7659	11.5371	1.5583
DON	3.4390	14.7530	0.1877	3.5121	1.5747	11.4306	1.2921
NO_x	2.5499	11.6721	0.2202	1.9597	2.2312	9.5630	1.0428
NH_4	3.5866	16.6352	0.2137	2.7853	1.7287	13.6628	1.4576
PN-Abiotic	3.5638	16.4039	0.2043	2.7509	1.7079	12.5068	2.4355

Table B6, Transitive closure matrices, n_{ij} and n'_{ij}, for Spring 1986 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	12.9582	12.2545	9.2394	12.0897	12.4010	12.4157	11.5000
PN-Hetero	41.3773	42.6411	31.7001	41.0747	40.0827	42.0677	39.9218
Sediment	2.3185	2.3039	2.7380	2.2415	2.3516	2.3085	2.3714
DON	7.5865	7.5986	5.6747	8.3765	7.3189	7.5616	7.1195
NO_x	2.0873	2.1402	1.7218	2.0644	3.0355	2.1724	2.0311
NH_4	31.8568	32.8246	24.4643	31.6202	30.8664	33.3854	30.7443
PN-Abiotic	4.7072	4.8404	3.5997	4.6653	4.5585	4.7784	5.5320
n'_{ij}							
PN-Phyto	12.9582	40.7224	1.7684	7.5473	2.5487	31.9045	4.3799
PN-Hetero	12.4516	42.6411	1.8258	7.7164	2.4790	32.5305	4.5755
Sediment	12.1140	40.0020	2.7380	7.3111	2.5252	30.9939	4.7189
DON	12.1525	40.4479	1.7398	8.3765	2.4095	31.1254	4.3435
NO_x	10.1562	34.6041	1.6035	6.2706	3.0355	27.1621	3.7638
NH_4	12.3972	42.4480	1.8221	7.6818	2.4687	33.3854	4.5567
PN-Abiotic	12.3594	42.2330	1.8089	7.6470	2.4599	32.2404	5.5320

Table B7, Transitive closure matrices, n_{ij} and n'_{ij}, for Summer 1986 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	13.7826	13.2383	10.0261	13.1635	13.5756	13.4524	12.1982
PN-Hetero	24.7974	25.9522	19.2800	24.7288	24.9737	25.2130	23.8019
Sediment	3.1224	3.1204	3.3461	3.0416	3.1106	3.1219	3.1671
DON	4.5357	4.5099	3.3886	5.4071	4.5070	4.4996	4.1476
NO_x	2.7113	2.8238	2.1939	2.6971	3.7274	2.8766	2.6184
NH_4	20.1170	21.0196	15.8544	20.0445	20.2521	21.4419	19.3489
PN-Abiotic	3.7079	3.8707	2.8772	3.6928	3.7317	3.7654	4.5505
n'_{ij}							
PN-Phyto	13.7826	24.7131	2.3580	4.7285	3.2053	20.5030	3.4348
PN-Hetero	13.2834	25.9522	2.4290	4.7583	3.1586	20.5849	3.5902
Sediment	13.2763	24.7686	3.3461	4.6456	3.1228	20.2311	3.7918
DON	12.6269	23.4376	2.2187	5.4071	2.9624	19.0916	3.2512
NO_x	11.4836	22.3268	2.1854	4.1033	3.7274	18.5692	3.1227
NH_4	13.1991	25.7455	2.4465	4.7242	3.1373	21.4419	3.5747
PN-Abiotic	13.1681	25.6618	2.4031	4.7109	3.1290	20.3810	4.5505

Table B8, Transitive closure matrices, n_{ij} and n'_{ij}, for Fall 1986 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	20.8420	20.2396	15.3705	20.0148	20.4221	20.5190	19.1589
PN-Hetero	34.9741	35.9204	27.0713	34.3840	34.8244	35.2260	33.9624
Sediment	2.5130	2.5065	2.8969	2.4402	2.4840	2.5038	2.5640
DON	6.5164	6.4595	4.8916	7.3113	6.4224	6.4687	6.1119
NO_x	3.1137	3.1886	2.5299	3.0573	4.0981	3.2365	3.0392
NH_4	27.0156	27.7445	20.9375	26.5589	26.8994	28.2094	26.2375
PN-Abiotic	5.7658	5.9136	4.4576	5.6652	5.7388	5.8040	6.5914
n'_{ij}							
PN-Phyto	20.8420	34.7795	1.9513	6.5035	3.6523	27.3828	5.4629
PN-Hetero	20.3529	35.9204	2.0000	6.5018	3.6243	27.3567	5.6355
Sediment	19.7950	33.9277	2.8969	6.2459	3.4992	26.3202	5.7588
DON	20.0544	34.1600	1.9111	7.3113	3.5348	26.5669	5.3633
NO_x	17.4104	30.6372	1.7959	5.5549	4.0981	24.1507	4.8456
NH_4	20.2438	35.7253	1.9918	6.4668	3.6049	28.2094	5.6060
PN-Abiotic	20.2211	35.6383	1.9847	6.4559	3.5994	27.1640	6.5914

Table B9, Transitive closure matrices, n_{ij} and n'_{ij}, for Winter 1987 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	2.4750	1.5492	0.3734	1.8124	1.9316	1.8934	1.2639
PN-Hetero	4.0660	4.6309	0.8320	3.7962	4.1678	4.3389	3.7366
Sediment	0.2883	0.2324	1.0538	0.2315	0.2882	0.2579	0.3346
DON	0.8757	0.7454	0.1528	1.7187	0.7775	0.7861	0.6042
NO_x	0.3090	0.3402	0.1842	0.2839	1.3158	0.4295	0.2924
NH_4	2.6111	2.9664	0.6112	2.4349	2.6759	3.7825	2.4049
PN-Abiotic	0.5764	0.6448	0.1167	0.5336	0.5853	0.6082	1.5204
n'_{ij}							
PN-Phyto	2.4750	3.3522	0.0557	0.9341	1.0119	2.6842	0.3947
PN-Hetero	1.8791	4.6309	0.0574	0.9042	1.0090	2.8428	0.5393
Sediment	1.9320	3.3703	1.0538	0.7996	1.0115	2.4505	0.7003
DON	1.6991	3.1294	0.0442	1.7187	0.7902	2.1622	0.3661
NO_x	0.5898	1.4053	0.0525	0.2793	1.3158	1.1623	0.1743
NH_4	1.8418	4.5276	0.0643	0.8852	0.9888	3.7825	0.5298
PN-Abiotic	1.8458	4.4676	0.0558	0.8806	0.9817	2.7611	1.5204

Table B10, Transitive closure matrices, n_{ij} and n'_{ij}, for Spring 1987 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	9.0494	8.4902	4.1676	8.2161	8.1437	8.7671	7.5271
PN-Hetero	11.6975	12.5941	5.9782	11.0247	10.9208	11.7646	11.1352
Sediment	0.9653	0.9675	1.4782	0.8918	0.9743	0.9632	1.0063
DON	2.5111	2.4545	1.1924	3.3045	2.2838	2.4591	2.1742
NO_x	0.7311	0.7786	0.4915	0.6869	1.6913	0.8051	0.7065
NH_4	8.7615	9.4290	4.5331	8.2566	8.1839	9.8113	8.3452
PN-Abiotic	2.3184	2.4897	1.1825	2.1835	2.1629	2.3300	3.2014
n'_{ij}							
PN-Phyto	9.0494	11.4227	0.4874	2.5856	1.2293	8.9274	2.0248
PN-Hetero	8.6945	12.5941	0.5197	2.5787	1.2253	8.9041	2.2264
Sediment	8.2533	11.1291	1.4782	2.3996	1.2575	8.3855	2.3145
DON	7.9796	10.4937	0.4432	3.3045	1.0955	7.9570	1.8586
NO_x	4.8436	6.9395	0.3808	1.4321	1.6913	5.4312	1.2590
NH_4	8.6043	12.4581	0.5207	2.5517	1.2132	9.8113	2.2046
PN-Abiotic	8.6186	12.4521	0.5141	2.5543	1.2137	8.8199	3.2014

Table B11, Transitive closure matrices, n_{ij} and n'_{ij}, for Summer 1987 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	35.1918	34.7431	30.5486	34.2889	34.9507	34.9309	33.6693
PN-Hetero	60.1896	61.4011	53.6578	59.4652	60.4630	60.5737	59.4289
Sediment	6.0918	6.1003	6.3523	5.9712	6.0838	6.0947	6.1344
DON	6.4854	6.4072	5.6329	7.3209	6.4425	6.4392	6.2090
NO_x	5.2939	5.3912	4.7937	5.2264	6.3151	5.4185	5.2366
NH_4	52.5440	53.5768	47.0420	51.9014	52.7749	53.8715	51.9059
PN-Abiotic	9.0164	9.1911	8.0331	8.9051	9.0550	9.0711	9.8962
n'_{ij}							
PN-Phyto	35.1918	60.3285	5.4142	6.5318	5.7874	53.1306	8.7985
PN-Hetero	34.6631	61.4011	5.4767	6.5236	5.7659	53.0598	8.9437
Sediment	34.3720	59.7675	6.3523	6.4180	5.6842	52.3053	9.0449
DON	34.0458	58.4043	5.2408	7.3209	5.6003	51.4153	8.5176
NO_x	31.9701	56.5344	5.1307	6.0124	6.3151	49.7723	8.2640
NH_4	34.5452	61.1639	5.4814	6.5001	5.7454	53.8715	8.9177
PN-Abiotic	34.5033	61.0732	5.4482	6.4915	5.7378	52.7987	9.8962

Table B12, Transitive closure matrices, n_{ij} and n'_{ij}, for Fall 1987 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	11.9181	11.5099	9.3393	10.5506	11.5495	11.6289	10.3000
PN-Hetero	32.8652	34.8469	28.1769	30.9708	34.2730	34.1194	31.1353
Sediment	3.4943	3.5130	3.8466	3.1769	3.4978	3.5039	3.6358
DON	2.3374	2.2774	1.8473	3.0813	2.2808	2.2939	2.0377
NO_x	2.8615	3.0135	2.5441	2.6842	3.9685	3.0407	2.7456
NH_4	29.9815	31.7848	25.7247	28.2506	31.2623	32.1227	28.4111
PN-Abiotic	2.9207	3.0789	2.4901	2.7415	3.0318	3.0203	3.7512
n'_{ij}							
PN-Phyto	11.9181	34.1143	2.9346	2.2624	3.4698	31.5907	2.7445
PN-Hetero	11.0885	34.8469	2.9872	2.2407	3.4740	31.2721	2.7991
Sediment	11.1204	33.1364	3.8466	2.1680	3.3441	30.2925	3.0831
DON	10.9002	31.4774	2.7069	3.0813	3.1953	29.0600	2.5320
NO_x	9.5248	29.7308	2.6609	1.9159	3.9685	27.4954	2.4351
NH_4	11.0366	34.6788	2.9756	2.2300	3.4573	32.1227	2.7867
PN-Abiotic	10.9613	34.2478	2.9365	2.2063	3.4183	30.7927	3.7512

Table B13, Transitive closure matrices, n_{ij} and n'_{ij}, for Winter 1988 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO$_x$	NH$_4$	PN-Abiotic
PN-Phyto	4.3203	3.8816	1.4800	3.8032	3.6996	3.9196	3.5150
PN-Hetero	20.5525	24.3236	9.0188	22.8207	21.6254	23.4879	21.9940
Sediment	1.1721	1.1338	1.4386	1.0970	1.1428	1.1363	1.1545
DON	2.8033	3.0721	1.1472	3.9145	2.7809	3.0008	2.7789
NO$_x$	1.0680	1.2346	0.5757	1.1621	2.1132	1.2509	1.1313
NH$_4$	17.3549	20.5200	7.6860	19.2547	18.2540	20.8182	18.5646
PN-Abiotic	1.3806	1.6222	0.6019	1.5235	1.4446	1.5681	2.4669
n'_{ij}							
PN-Phyto	4.3203	22.8006	0.4650	2.8933	1.8528	19.5669	1.4127
PN-Hetero	3.4989	24.3236	0.4824	2.9556	1.8437	19.9612	1.5049
Sediment	3.7306	21.1974	1.4386	2.6561	1.8215	18.0534	1.4769
DON	3.6848	23.7205	0.4738	3.9145	1.8306	19.6905	1.4681
NO$_x$	2.1325	14.4806	0.3612	1.7654	2.1132	12.4696	0.9079
NH$_4$	3.4765	24.1454	0.4838	2.9343	1.8312	20.8182	1.4947
PN-Abiotic	3.4351	23.7082	0.4705	2.8837	1.8000	19.4766	2.4669

Table B14, Transitive closure matrices, n_{ij} and n'_{ij}, for Spring 1988 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO$_x$	NH$_4$	PN-Abiotic
PN-Phyto	9.4802	8.6817	4.0800	8.9870	8.5484	9.0856	7.6699
PN-Hetero	13.3702	13.9565	6.2839	13.2093	12.6787	13.3703	12.2743
Sediment	1.2521	1.2395	1.5894	1.2159	1.2840	1.2527	1.2988
DON	3.3552	3.2596	1.5018	4.2390	3.0934	3.2763	2.8737
NO$_x$	1.2557	1.2990	0.7645	1.2369	2.2076	1.3680	1.1788
NH$_4$	9.3459	9.7503	4.4730	9.2317	8.8703	10.3460	8.5919
PN-Abiotic	2.5368	2.6392	1.1896	2.5035	2.4024	2.5340	3.3214
n'_{ij}							
PN-Phyto	9.4802	12.8875	0.6048	3.4675	1.7651	9.5907	2.1827
PN-Hetero	9.0069	13.9565	0.6276	3.4333	1.7636	9.5076	2.3531
Sediment	8.4461	12.4114	1.5894	3.1647	1.7884	8.9201	2.4931
DON	8.6959	12.5410	0.5770	4.2390	1.6555	8.9636	2.1195
NO$_x$	6.0812	9.3385	0.5489	2.3112	2.2076	6.9935	1.6246
NH$_4$	8.8537	13.7116	0.6282	3.3743	1.7351	10.3460	2.3163
PN-Abiotic	8.9142	13.7669	0.6197	3.3943	1.7431	9.3992	3.3214

Table B15, Transitive closure matrices, n_{ij} and n'_{ij}, for Summer 1988 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	25.9462	25.3899	20.9266	24.9017	25.8325	25.6413	24.1405
PN-Hetero	44.7044	45.7467	37.3040	43.7866	44.6545	45.0324	43.4006
Sediment	3.8057	3.8164	4.1300	3.6932	3.7999	3.8117	3.8616
DON	6.3477	6.2636	5.1521	7.1151	6.3237	6.2953	5.9529
NO_x	3.2008	3.2688	2.7514	3.1322	4.1971	3.3010	3.1215
NH_4	37.5630	38.4203	31.5710	36.7837	37.5207	38.8334	36.5070
PN-Abiotic	7.0395	7.1960	5.8694	6.8916	7.0311	7.0879	7.8273
n'_{ij}							
PN-Phyto	25.9462	44.3542	3.1538	6.3095	3.7458	37.8269	6.6747
PN-Hetero	25.5903	45.7467	3.2182	6.3509	3.7065	38.0287	6.8692
Sediment	25.2527	44.2383	4.1300	6.2093	3.6561	37.3121	7.0848
DON	25.0525	43.1844	3.0644	7.1151	3.6189	36.6529	6.4960
NO_x	22.0742	39.3817	2.8597	5.4732	4.1971	33.5838	5.9522
NH_4	25.4624	45.4961	3.2252	6.3177	3.6879	38.8334	6.8423
PN-Abiotic	25.4599	45.4651	3.1992	6.3154	3.6873	37.8174	7.8273

Table B16, Transitive closure matrices, n_{ij} and n'_{ij}, for Fall 1988 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	11.3443	10.8140	5.7359	10.5079	10.7678	11.1470	9.3574
PN-Hetero	14.1184	15.0416	7.9333	13.2820	14.9610	14.2919	13.0038
Sediment	1.5642	1.5725	1.8324	1.4594	1.5682	1.5638	1.6215
DON	2.5047	2.4143	1.2798	3.3235	2.4037	2.4682	2.0889
NO_x	1.9116	2.0215	1.2268	1.7964	3.0113	2.0782	1.7896
NH_4	11.4096	12.1524	6.4446	10.7333	12.0874	12.5492	10.5152
PN-Abiotic	2.6121	2.7724	1.4625	2.4560	2.7576	2.6414	3.3969
n'_{ij}							
PN-Phyto	11.3443	14.5411	0.8651	2.5755	2.5092	12.2399	2.3499
PN-Hetero	10.4997	15.0416	0.8899	2.4210	2.5928	11.6708	2.4286
Sediment	10.3704	14.0187	1.8324	2.3715	2.4228	11.3844	2.6998
DON	10.2192	13.2450	0.7876	3.3235	2.2853	11.0575	2.1403
NO_x	8.2031	11.6647	0.7941	1.8894	3.0113	9.7925	1.9286
NH_4	10.3909	14.8817	0.8852	2.3958	2.5653	12.5492	2.4049
PN-Abiotic	10.4015	14.8444	0.8784	2.3970	2.5589	11.5494	3.3969

Table B17, Transitive closure matrices, n_{ij} and n'_{ij}, for Winter 1989 for the Neuse River estuary nitrogen model.

n_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	4.2995	3.4958	0.8443	3.7017	3.7225	3.8756	3.1249
PN-Hetero	6.9180	7.6100	1.5532	6.8534	6.7722	7.1632	6.7653
Sediment	0.5824	0.5323	1.1308	0.5280	0.6017	0.5648	0.6075
DON	1.1344	1.0023	0.2306	2.0128	1.0137	1.0599	0.8945
NO_x	0.7018	0.7568	0.2976	0.6884	1.6914	0.8420	0.6916
NH_4	5.1888	5.6982	1.2532	5.1360	5.0822	6.3693	5.0776
PN-Abiotic	1.0851	1.1797	0.2423	1.0687	1.0567	1.1171	2.0490
n'_{ij}							
PN-Phyto	4.2995	6.4401	0.1348	1.3382	1.2373	5.4258	0.9155
PN-Hetero	3.7552	7.6100	0.1346	1.3448	1.2219	5.4436	1.0758
Sediment	3.6488	6.1434	1.1308	1.1958	1.2529	4.9536	1.1150
DON	3.1380	5.1078	0.1018	2.0128	0.9321	4.1047	0.7249
NO_x	2.1113	4.1945	0.1429	0.7487	1.6914	3.5465	0.6096
NH_4	3.7063	7.4982	0.1429	1.3262	1.2066	6.3693	1.0625
PN-Abiotic	3.7039	7.4186	0.1320	1.3187	1.1990	5.3382	2.0490

Appendix C

Fractional Transfer Coefficients (FTC's) for the Neuse River estuary nitrogen model, North Carolina, USA as calculated by equations (3-5) or (3-6). The 17 tables below include 16 consecutive seasons from Spring 1985 to Winter 1989 and an average data set as calculated by Christian and Thomas which represents all 16 seasons.

Table C1, Fractional Transfer Coefficients (FTC's), η_{ij}, for the average data set representing 16 consecutive seasons from spring 1985 through winter 1989 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	1.36	1.26	0.85	1.24	1.29	1.30	1.15
PN-Hetero	1.28	1.34	0.88	1.25	1.28	1.30	1.22
Sediment	1.26	1.24	1.63	1.19	1.26	1.25	1.31
DON	1.23	1.21	0.80	1.53	1.19	1.21	1.10
NO_x	0.93	0.97	0.71	0.91	1.56	1.00	0.90
NH_4	1.27	1.33	0.89	1.24	1.27	1.38	1.21
PN-Abiotic	1.25	1.32	0.87	1.23	1.25	1.28	1.67

Table C2, Fractional Transfer Coefficients (FTC's), η_{ij}, for Spring 1985 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	8.20	7.55	6.87	7.08	7.82	7.70	7.22
PN-Hetero	7.63	7.96	7.15	7.19	7.66	7.80	7.59
Sediment	7.55	7.39	10.28	6.80	7.78	7.44	7.91
DON	7.37	7.37	6.65	8.15	7.27	7.32	7.04
NO_x	6.24	6.45	6.22	5.84	9.29	6.61	6.25
NH_4	7.59	7.91	7.13	7.15	7.62	8.12	7.55
PN-Abiotic	7.55	7.85	7.05	7.10	7.57	7.70	9.98

Table C3, Fractional Transfer Coefficients (FTC's), η_{ij}, for Summer 1985 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	8.65	8.41	7.18	8.39	8.59	8.48	8.06
PN-Hetero	8.43	8.63	7.32	8.48	8.47	8.55	8.26
Sediment	8.39	8.37	8.92	8.29	8.39	8.38	8.44
DON	8.37	8.44	7.17	8.90	8.38	8.40	8.08
NO_x	7.51	7.66	6.66	7.54	9.67	7.74	7.37
NH_4	8.40	8.59	7.33	8.45	8.44	8.69	8.23
PN-Abiotic	8.39	8.57	7.27	8.43	8.42	8.50	9.24

Table C4, Fractional Transfer Coefficients (FTC's), η_{ij}, for Fall 1985 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	5.86	5.55	2.90	5.62	5.76	5.72	4.74
PN-Hetero	5.58	5.94	3.05	5.63	5.68	5.72	5.06
Sediment	5.53	5.52	5.99	5.42	5.53	5.53	5.60
DON	5.14	4.89	2.56	7.31	5.06	5.03	4.18
NO_x	4.24	4.47	2.59	4.26	7.23	4.60	3.89
NH_4	5.53	5.88	3.03	5.57	5.63	6.07	5.01
PN-Abiotic	5.51	5.84	3.00	5.54	5.60	5.63	7.08

Table C5, Fractional Transfer Coefficients (FTC's), η_{ij}, for Winter 1986 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	6.60	5.61	1.47	5.17	5.61	5.79	4.91
PN-Hetero	5.44	6.08	1.46	5.26	5.49	5.87	5.31
Sediment	5.41	5.23	8.40	4.67	5.55	5.30	5.63
DON	5.17	5.34	1.31	6.56	4.95	5.25	4.66
NO_x	3.83	4.22	1.54	3.66	7.02	4.39	3.76
NH_4	5.39	6.02	1.49	5.21	5.44	6.27	5.26
PN-Abiotic	5.36	5.93	1.43	5.14	5.37	5.74	8.79

Table C6, Fractional Transfer Coefficients (FTC's), η_{ij}, for Spring 1986 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	8.32	7.87	5.93	7.76	7.96	7.97	7.39
PN-Hetero	8.00	8.24	6.13	7.94	7.75	8.13	7.72
Sediment	7.78	7.73	9.19	7.52	7.89	7.75	7.96
DON	7.81	7.82	5.84	8.62	7.53	7.78	7.32
NO_x	6.52	6.69	5.38	6.45	9.49	6.79	6.35
NH_4	7.96	8.20	6.11	7.90	7.71	8.34	7.68
PN-Abiotic	7.94	8.16	6.07	7.87	7.69	8.06	9.33

Table C7, Fractional Transfer Coefficients (FTC's), η_{ij}, for Summer 1986 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	6.33	6.08	4.61	6.05	6.24	6.18	5.60
PN-Hetero	6.10	6.39	4.74	6.08	6.15	6.20	5.86
Sediment	6.10	6.09	6.54	5.94	6.08	6.10	6.19
DON	5.80	5.77	4.33	6.91	5.76	5.75	5.30
NO_x	5.27	5.49	4.27	5.25	7.25	5.60	5.09
NH_4	6.06	6.34	4.78	6.04	6.10	6.46	5.83
PN-Abiotic	6.05	6.31	4.69	6.02	6.09	6.14	7.42

Table C8, Fractional Transfer Coefficients (FTC's), η_{ij}, for Fall 1986 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	10.50	10.20	7.74	10.08	10.29	10.34	9.65
PN-Hetero	10.25	10.53	7.94	10.08	10.21	10.33	9.96
Sediment	9.97	9.95	11.50	9.68	9.86	9.94	10.17
DON	10.10	10.01	7.58	11.34	9.96	10.03	9.48
NO_x	8.77	8.98	7.13	8.61	11.54	9.12	8.56
NH_4	10.20	10.47	7.90	10.03	10.15	10.65	9.90
PN-Abiotic	10.19	10.45	7.88	10.01	10.14	10.25	11.65

Table C9, Fractional Transfer Coefficients (FTC's), η_{ij}, for Winter 1987 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	1.90	1.19	0.29	1.39	1.49	1.46	0.97
PN-Hetero	1.45	1.65	0.30	1.35	1.48	1.54	1.33
Sediment	1.49	1.20	5.43	1.19	1.49	1.33	1.72
DON	1.31	1.11	0.23	2.57	1.16	1.17	0.90
NO_x	0.45	0.50	0.27	0.42	1.93	0.63	0.43
NH_4	1.42	1.61	0.33	1.32	1.45	2.05	1.30
PN-Abiotic	1.42	1.59	0.29	1.31	1.44	1.50	3.74

Table C10, Fractional Transfer Coefficients (FTC's), η_{ij}, for Spring 1987 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	3.31	3.10	1.52	3.00	2.98	3.20	2.75
PN-Hetero	3.18	3.42	1.62	3.00	2.97	3.20	3.03
Sediment	3.02	3.02	4.62	2.79	3.04	3.01	3.14
DON	2.92	2.85	1.38	3.84	2.65	2.86	2.53
NO_x	1.77	1.89	1.19	1.66	4.10	1.95	1.71
NH_4	3.14	3.38	1.63	2.96	2.94	3.52	3.00
PN-Abiotic	3.15	3.38	1.61	2.97	2.94	3.17	4.35

Table C11, Fractional Transfer Coefficients (FTC's), η_{ij}, for Summer 1987 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	11.38	11.24	9.88	11.09	11.30	11.30	10.89
PN-Hetero	11.21	11.44	9.99	11.08	11.26	11.28	11.07
Sediment	11.12	11.13	11.59	10.90	11.10	11.12	11.19
DON	11.01	10.88	9.56	12.43	10.94	10.93	10.54
NO_x	10.34	10.53	9.36	10.21	12.33	10.58	10.23
NH_4	11.17	11.39	10.00	11.04	11.22	11.45	11.04
PN-Abiotic	11.16	11.38	9.94	11.02	11.21	11.23	12.25

Table C12, Fractional Transfer Coefficients (FTC's), η_{ij}, for Fall 1987 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	12.65	12.22	9.91	11.20	12.26	12.34	10.93
PN-Hetero	11.77	12.48	10.09	11.09	12.28	12.22	11.15
Sediment	11.81	11.87	13.00	10.73	11.82	11.84	12.28
DON	11.57	11.27	9.14	15.25	11.29	11.36	10.09
NO_x	10.11	10.65	8.99	9.48	14.02	10.74	9.70
NH_4	11.72	12.42	10.05	11.04	12.22	12.55	11.10
PN-Abiotic	11.64	12.27	9.92	10.92	12.08	12.03	14.94

Table C13, Fractional Transfer Coefficients (FTC's), η_{ij}, for Winter 1988 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	6.89	6.19	2.36	6.07	5.90	6.25	5.61
PN-Hetero	5.58	6.60	2.45	6.20	5.87	6.38	5.97
Sediment	5.95	5.76	7.30	5.57	5.80	5.77	5.86
DON	5.88	6.44	2.41	8.21	5.83	6.29	5.83
NO_x	3.40	3.93	1.83	3.70	6.73	3.98	3.60
NH_4	5.54	6.56	2.46	6.15	5.83	6.65	5.93
PN-Abiotic	5.48	6.44	2.39	6.05	5.73	6.22	9.79

Table C14, Fractional Transfer Coefficients (FTC's), η_{ij}, for Spring 1988 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	6.27	5.75	2.70	5.95	5.66	6.01	5.08
PN-Hetero	5.96	6.22	2.80	5.89	5.65	5.96	5.47
Sediment	5.59	5.53	7.10	5.43	5.73	5.59	5.80
DON	5.76	5.59	2.58	7.27	5.31	5.62	4.93
NO_x	4.02	4.16	2.45	3.96	7.08	4.38	3.78
NH_4	5.86	6.11	2.80	5.79	5.56	6.49	5.39
PN-Abiotic	5.90	6.14	2.77	5.82	5.59	5.89	7.72

Table C15, Fractional Transfer Coefficients (FTC's), η_{ij}, for Summer 1988 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	7.79	7.62	6.28	7.48	7.76	7.70	7.25
PN-Hetero	7.68	7.86	6.41	7.52	7.67	7.74	7.46
Sediment	7.58	7.60	8.23	7.36	7.57	7.59	7.69
DON	7.52	7.42	6.10	8.43	7.49	7.46	7.05
NO_x	6.63	6.77	5.70	6.48	8.69	6.83	6.46
NH_4	7.64	7.82	6.42	7.49	7.64	7.90	7.43
PN-Abiotic	7.64	7.81	6.37	7.48	7.63	7.70	8.50

Table C16, Fractional Transfer Coefficients (FTC's), η_{ij}, for Fall 1988 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	8.56	8.16	4.33	7.92	8.12	8.41	7.06
PN-Hetero	7.92	8.44	4.45	7.45	8.39	8.02	7.29
Sediment	7.82	7.86	9.16	7.30	7.84	7.82	8.11
DON	7.71	7.43	3.94	10.23	7.40	7.59	6.43
NO_x	6.19	6.54	3.97	5.81	9.75	6.73	5.79
NH_4	7.84	8.35	4.43	7.37	8.30	8.62	7.22
PN-Abiotic	7.84	8.33	4.39	7.38	8.28	7.93	10.20

Table C17, Fractional Transfer Coefficients (FTC's), η_{ij}, for Winter 1989 for the Neuse River estuary nitrogen model. Numbers in the table have been multiplied by 10^3 for efficiency of presentation. Dimensional units for η are (m^2 season)/mmol-N.

$\eta_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	4.04	3.28	0.79	3.48	3.50	3.64	2.93
PN-Hetero	3.53	3.88	0.79	3.49	3.45	3.65	3.45
Sediment	3.43	3.13	6.65	3.11	3.54	3.32	3.57
DON	2.95	2.60	0.60	5.23	2.63	2.75	2.32
NO_x	1.98	2.14	0.84	1.94	4.78	2.38	1.95
NH_4	3.48	3.82	0.84	3.44	3.41	4.27	3.41
PN-Abiotic	3.48	3.78	0.78	3.43	3.39	3.58	6.57

Appendix D

Control Ratios, cr_{ij}, for the Neuse River estuary nitrogen model, as calculated by equation (3-7). The 17 tables below include 16 consecutive seasons from Spring 1985 to Winter 1989 and an average data set as calculated by Christian and Thomas which represents all 16 seasons.

Table D1, Control Ratios, cr_{ij}, for the data set representing an average (as calculated by Christian and Thomas) of 16 consecutive seasons from spring 1985 through winter 1989 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.01	-0.33	0.01	0.28	0.02	-0.08
PN-Hetero	0.01	0.00	-0.29	0.04	0.24	-0.02	-0.07
Sediment	0.33	0.29	0.00	0.33	0.44	0.29	0.34
DON	-0.01	-0.04	-0.33	0.00	0.24	-0.03	-0.10
NO_x	-0.28	-0.24	-0.44	-0.24	0.00	-0.21	-0.28
NH_4	-0.02	0.02	-0.29	0.03	0.21	0.00	-0.05
PN-Abiotic	0.08	0.07	-0.34	0.10	0.28	0.05	0.00

Table D2, Control Ratios, cr_{ij}, for Spring 1985 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.01	-0.09	-0.04	0.20	0.01	-0.04
PN-Hetero	0.01	0.00	-0.03	-0.03	0.16	-0.01	-0.03
Sediment	0.09	0.03	0.00	0.02	0.20	0.04	0.11
DON	0.04	0.03	-0.02	0.00	0.20	0.02	-0.01
NO_x	-0.20	-0.16	-0.20	-0.20	0.00	-0.13	-0.17
NH_4	-0.01	0.01	-0.04	-0.02	0.13	0.00	-0.02
PN-Abiotic	0.04	0.03	-0.11	0.01	0.17	0.02	0.00

Table D3, Control Ratios, cr_{ij}, for Summer 1985 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.00	-0.14	0.00	0.13	0.01	-0.04
PN-Hetero	0.00	0.00	-0.13	0.01	0.10	0.00	-0.04
Sediment	0.14	0.13	0.00	0.13	0.21	0.13	0.14
DON	0.00	-0.01	-0.13	0.00	0.10	-0.01	-0.04
NO_x	-0.13	-0.10	-0.21	-0.10	0.00	-0.08	-0.12
NH_4	-0.01	0.00	-0.13	0.01	0.08	0.00	-0.03
PN-Abiotic	0.04	0.04	-0.14	0.04	0.12	0.03	0.00

Table D4, Control Ratios, cr_{ij}, for Fall 1985 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.01	-0.48	0.09	0.26	0.03	-0.14
PN-Hetero	0.01	0.00	-0.45	0.13	0.21	-0.03	-0.13
Sediment	0.48	0.45	0.00	0.53	0.53	0.45	0.46
DON	-0.09	-0.13	-0.53	0.00	0.16	-0.10	-0.25
NO_x	-0.26	-0.21	-0.53	-0.16	0.00	-0.18	-0.30
NH_4	-0.03	0.03	-0.45	0.10	0.18	0.00	-0.11
PN-Abiotic	0.14	0.13	-0.46	0.25	0.30	0.11	0.00

Table D5, Control Ratios, cr_{ij}, for Winter 1986 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.03	-0.73	0.00	0.32	0.07	-0.08
PN-Hetero	-0.03	0.00	-0.72	-0.01	0.23	-0.02	-0.11
Sediment	0.73	0.72	0.00	0.72	0.72	0.72	0.75
DON	0.00	0.01	-0.72	0.00	0.26	0.01	-0.09
NO_x	-0.32	-0.23	-0.72	-0.26	0.00	-0.19	-0.30
NH_4	-0.07	0.02	-0.72	-0.01	0.19	0.00	-0.08
PN-Abiotic	0.08	0.11	-0.75	0.09	0.30	0.08	0.00

Table D6, Control Ratios, cr_{ij}, for Spring 1986 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.02	-0.24	-0.01	0.18	0.00	-0.07
PN-Hetero	0.02	0.00	-0.21	0.02	0.14	-0.01	-0.05
Sediment	0.24	0.21	0.00	0.22	0.32	0.21	0.24
DON	0.01	-0.02	-0.22	0.00	0.14	-0.02	-0.07
NO_x	-0.18	-0.14	-0.32	-0.14	0.00	-0.12	-0.17
NH_4	0.00	0.01	-0.21	0.02	0.12	0.00	-0.05
PN-Abiotic	0.07	0.05	-0.24	0.07	0.17	0.05	0.00

Table D7, Control Ratios, cr_{ij}, for Summer 1986 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.00	-0.24	0.04	0.15	0.02	-0.07
PN-Hetero	0.00	0.00	-0.22	0.05	0.11	-0.02	-0.07
Sediment	0.24	0.22	0.00	0.27	0.30	0.22	0.24
DON	-0.04	-0.05	-0.27	0.00	0.09	-0.05	-0.12
NO_x	-0.15	-0.11	-0.30	-0.09	0.00	-0.08	-0.16
NH_4	-0.02	0.02	-0.22	0.05	0.08	0.00	-0.05
PN-Abiotic	0.07	0.07	-0.24	0.12	0.16	0.05	0.00

Table D8, Control Ratios, cr$_{ij}$, for Fall 1986 for the Neuse River estuary nitrogen model.

cr$_{ij}$	PN-Phyto	PN-Hetero	Sediment	DON	NO$_x$	NH$_4$	PN-Abiotic
PN-Phyto	0.00	-0.01	-0.22	0.00	0.15	0.01	-0.05
PN-Hetero	0.01	0.00	-0.20	0.01	0.12	-0.01	-0.05
Sediment	0.22	0.20	0.00	0.22	0.28	0.20	0.23
DON	0.00	-0.01	-0.22	0.00	0.14	0.00	-0.05
NO$_x$	-0.15	-0.12	-0.28	-0.14	0.00	-0.10	-0.16
NH$_4$	-0.01	0.01	-0.20	0.00	0.10	0.00	-0.03
PN-Abiotic	0.05	0.05	-0.23	0.05	0.16	0.03	0.00

Table D9, Control Ratios, cr$_{ij}$, for Winter 1987 for the Neuse River estuary nitrogen model.

cr$_{ij}$	PN-Phyto	PN-Hetero	Sediment	DON	NO$_x$	NH$_4$	PN-Abiotic
PN-Phyto	0.00	-0.18	-0.81	0.06	0.69	0.03	-0.32
PN-Hetero	0.18	0.00	-0.75	0.18	0.66	-0.04	-0.16
Sediment	0.81	0.75	0.00	0.81	0.82	0.75	0.83
DON	-0.06	-0.18	-0.81	0.00	0.64	-0.11	-0.31
NO$_x$	-0.69	-0.66	-0.82	-0.64	0.00	-0.57	-0.70
NH$_4$	-0.03	0.04	-0.75	0.11	0.57	0.00	-0.13
PN-Abiotic	0.32	0.16	-0.83	0.31	0.70	0.13	0.00

Table D10, Control Ratios, cr$_{ij}$, for Spring 1987 for the Neuse River estuary nitrogen model.

cr$_{ij}$	PN-Phyto	PN-Hetero	Sediment	DON	NO$_x$	NH$_4$	PN-Abiotic
PN-Phyto	0.00	-0.02	-0.50	0.03	0.41	0.02	-0.13
PN-Hetero	0.02	0.00	-0.46	0.05	0.36	-0.06	-0.11
Sediment	0.50	0.46	0.00	0.50	0.61	0.46	0.49
DON	-0.03	-0.05	-0.50	0.00	0.37	-0.04	-0.15
NO$_x$	-0.41	-0.36	-0.61	-0.37	0.00	-0.34	-0.42
NH$_4$	-0.02	0.06	-0.46	0.04	0.34	0.00	-0.05
PN-Abiotic	0.13	0.11	-0.49	0.15	0.42	0.05	0.00

Table D11, Control Ratios, cr$_{ij}$, for Summer 1987 for the Neuse River estuary nitrogen model.

cr$_{ij}$	PN-Phyto	PN-Hetero	Sediment	DON	NO$_x$	NH$_4$	PN-Abiotic
PN-Phyto	0.00	0.00	-0.11	0.01	0.09	0.01	-0.02
PN-Hetero	0.00	0.00	-0.10	0.02	0.06	-0.01	-0.03
Sediment	0.11	0.10	0.00	0.12	0.16	0.10	0.11
DON	-0.01	-0.02	-0.12	0.00	0.07	-0.01	-0.04
NO$_x$	-0.09	-0.06	-0.16	-0.07	0.00	-0.06	-0.09
NH$_4$	-0.01	0.01	-0.10	0.01	0.06	0.00	-0.02
PN-Abiotic	0.02	0.03	-0.11	0.04	0.09	0.02	0.00

Table D12, Control Ratios, cr_{ij}, for Fall 1987 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.04	-0.16	-0.03	0.18	0.05	-0.06
PN-Hetero	-0.04	0.00	-0.15	-0.02	0.13	-0.02	-0.09
Sediment	0.16	0.15	0.00	0.15	0.24	0.15	0.19
DON	0.03	0.02	-0.15	0.00	0.16	0.03	-0.08
NO_x	-0.18	-0.13	-0.24	-0.16	0.00	-0.12	-0.20
NH_4	-0.05	0.02	-0.15	-0.03	0.12	0.00	-0.08
PN-Abiotic	0.06	0.09	-0.19	0.08	0.20	0.08	0.00

Table D13, Control Ratios, cr_{ij}, for Winter 1988 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.10	-0.60	0.03	0.42	0.11	0.02
PN-Hetero	-0.10	0.00	-0.57	-0.04	0.33	-0.03	-0.07
Sediment	0.60	0.57	0.00	0.57	0.68	0.57	0.59
DON	-0.03	0.04	-0.57	0.00	0.37	0.02	-0.04
NO_x	-0.42	-0.33	-0.68	-0.37	0.00	-0.32	-0.37
NH_4	-0.11	0.03	-0.57	-0.02	0.32	0.00	-0.05
PN-Abiotic	-0.02	0.07	-0.59	0.04	0.37	0.05	0.00

Table D14, Control Ratios, cr_{ij}, for Spring 1988 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.04	-0.52	0.03	0.29	0.03	-0.14
PN-Hetero	0.04	0.00	-0.49	0.05	0.26	-0.02	-0.11
Sediment	0.52	0.49	0.00	0.53	0.57	0.50	0.52
DON	-0.03	-0.05	-0.53	0.00	0.25	-0.03	-0.15
NO_x	-0.29	-0.26	-0.57	-0.25	0.00	-0.21	-0.32
NH_4	-0.03	0.02	-0.50	0.03	0.21	0.00	-0.09
PN-Abiotic	0.14	0.11	-0.52	0.15	0.32	0.09	0.00

Table D15, Control Ratios, cr_{ij}, for Summer 1988 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.01	-0.17	-0.01	0.15	0.01	-0.05
PN-Hetero	0.01	0.00	-0.16	0.01	0.12	-0.01	-0.05
Sediment	0.17	0.16	0.00	0.17	0.25	0.15	0.17
DON	0.01	-0.01	-0.17	0.00	0.13	0.00	-0.06
NO_x	-0.15	-0.12	-0.25	-0.13	0.00	-0.10	-0.15
NH_4	-0.01	0.01	-0.15	0.00	0.10	0.00	-0.03
PN-Abiotic	0.05	0.05	-0.17	0.06	0.15	0.03	0.00

Table D16, Control Ratios, cr_{ij}, for Fall 1988 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.03	-0.45	0.03	0.24	0.07	-0.10
PN-Hetero	-0.03	0.00	-0.43	0.00	0.22	-0.04	-0.12
Sediment	0.45	0.43	0.00	0.46	0.49	0.43	0.46
DON	-0.03	0.00	-0.46	0.00	0.21	0.03	-0.13
NO_x	-0.24	-0.22	-0.49	-0.21	0.00	-0.19	-0.30
NH_4	-0.07	0.04	-0.43	-0.03	0.19	0.00	-0.09
PN-Abiotic	0.10	0.12	-0.46	0.13	0.30	0.09	0.00

Table D17, Control Ratios, cr_{ij}, for Winter 1989 for the Neuse River estuary nitrogen model.

cr_{ij}	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.07	-0.77	0.15	0.43	0.04	-0.16
PN-Hetero	0.07	0.00	-0.75	0.25	0.38	-0.04	-0.09
Sediment	0.77	0.75	0.00	0.81	0.76	0.75	0.78
DON	-0.15	-0.25	-0.81	0.00	0.26	-0.20	-0.32
NO_x	-0.43	-0.38	-0.76	-0.26	0.00	-0.30	-0.42
NH_4	-0.04	0.04	-0.75	0.20	0.30	0.00	-0.05
PN-Abiotic	0.16	0.09	-0.78	0.32	0.42	0.05	0.00

Appendix E

Control Difference, cd_{ij}, parameters for the Neuse River estuary nitrogen model, North Carolina, USA as calculated by equation (3-8). The 17 tables below include 16 consecutive seasons from Spring 1985 to Winter 1989 and an average data set as calculated by Christian and Thomas which represents all 16 seasons.

Table E1, Control Difference, cd_{ij}, parameters for the data set representing an average (as calculated by Christian and Thomas) of 16 consecutive seasons from spring 1985 through winter 1989 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.02	-0.41	0.02	0.36	0.03	-0.10
PN-Hetero	0.02	0.00	-0.36	0.04	0.30	-0.03	-0.09
Sediment	0.41	0.36	0.00	0.39	0.55	0.36	0.44
DON	-0.02	-0.04	-0.39	0.00	0.29	-0.03	-0.13
NO_x	-0.36	-0.30	-0.55	-0.29	0.00	-0.26	-0.35
NH_4	-0.03	0.03	-0.36	0.03	0.26	0.00	-0.07
PN-Abiotic	0.10	0.09	-0.44	0.13	0.35	0.07	0.00

Table E2, Control Difference, cd_{ij}, parameters for Spring 1985 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.08	-0.67	-0.29	1.58	0.11	-0.33
PN-Hetero	0.08	0.00	-0.24	-0.18	1.20	-0.12	-0.26
Sediment	0.67	0.24	0.00	0.15	1.56	0.31	0.86
DON	0.29	0.18	-0.15	0.00	1.42	0.17	-0.06
NO_x	-1.58	-1.20	-1.56	-1.42	0.00	-1.01	-1.31
NH_4	-0.11	0.12	-0.31	-0.17	1.01	0.00	-0.15
PN-Abiotic	0.33	0.26	-0.86	0.06	1.31	0.15	0.00

Table E3, Control Difference, cd_{ij}, parameters for Summer 1985 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.02	-1.21	0.03	1.09	0.07	-0.32
PN-Hetero	0.02	0.00	-1.05	0.04	0.81	-0.04	-0.31
Sediment	1.21	1.05	0.00	1.12	1.73	1.05	1.17
DON	-0.03	-0.04	-1.12	0.00	0.84	-0.05	-0.35
NO_x	-1.09	-0.81	-1.73	-0.84	0.00	-0.70	-1.05
NH_4	-0.07	0.04	-1.05	0.05	0.70	0.00	-0.26
PN-Abiotic	0.32	0.31	-1.17	0.35	1.05	0.26	0.00

Table E4, Control Difference, cd_{ij}, parameters for Fall 1985 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.04	-2.63	0.48	1.52	0.18	-0.77
PN-Hetero	0.04	0.00	-2.47	0.73	1.21	-0.16	-0.78
Sediment	2.63	2.47	0.00	2.87	2.94	2.49	2.59
DON	-0.48	-0.73	-2.87	0.00	0.80	-0.55	-1.36
NO_x	-1.52	-1.21	-2.94	-0.80	0.00	-1.03	-1.71
NH_4	-0.18	0.16	-2.49	0.55	1.03	0.00	-0.62
PN-Abiotic	0.77	0.78	-2.59	1.36	1.71	0.62	0.00

Table E5, Control Difference, cd_{ij}, parameters for Winter 1986 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.16	-3.94	0.00	1.77	0.40	-0.45
PN-Hetero	-0.16	0.00	-3.77	-0.08	1.26	-0.15	-0.63
Sediment	3.94	3.77	0.00	3.36	4.01	3.80	4.20
DON	0.00	0.08	-3.36	0.00	1.29	0.04	-0.48
NO_x	-1.77	-1.26	-4.01	-1.29	0.00	-1.05	-1.61
NH_4	-0.40	0.15	-3.80	-0.04	1.05	0.00	-0.48
PN-Abiotic	0.45	0.63	-4.20	0.48	1.61	0.48	0.00

Table E6, Control Difference, cd_{ij}, parameters for Spring 1986 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.13	-1.85	-0.04	1.44	0.01	-0.55
PN-Hetero	0.13	0.00	-1.60	0.12	1.06	-0.07	-0.45
Sediment	1.85	1.60	0.00	1.68	2.51	1.63	1.89
DON	0.04	-0.12	-1.68	0.00	1.08	-0.12	-0.54
NO_x	-1.44	-1.06	-2.51	-1.08	0.00	-0.93	-1.34
NH_4	-0.01	0.07	-1.63	0.12	0.93	0.00	-0.37
PN-Abiotic	0.55	0.45	-1.89	0.54	1.34	0.37	0.00

Table E7, Control Difference, cd_{ij}, parameters for Summer 1986 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.02	-1.49	0.25	0.96	0.12	-0.45
PN-Hetero	0.02	0.00	-1.35	0.32	0.65	-0.13	-0.46
Sediment	1.49	1.35	0.00	1.61	1.81	1.32	1.49
DON	-0.25	-0.32	-1.61	0.00	0.52	-0.29	-0.72
NO_x	-0.96	-0.65	-1.81	-0.52	0.00	-0.51	-0.99
NH_4	-0.12	0.13	-1.32	0.29	0.51	0.00	-0.31
PN-Abiotic	0.45	0.46	-1.49	0.72	0.99	0.31	0.00

Table E8, Control Difference, cd_{ij}, parameters for Fall 1986 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.06	-2.23	-0.02	1.52	0.14	-0.54
PN-Hetero	0.06	0.00	-2.01	0.07	1.23	-0.15	-0.49
Sediment	2.23	2.01	0.00	2.10	2.73	2.03	2.30
DON	0.02	-0.07	-2.10	0.00	1.34	0.00	-0.53
NO_x	-1.52	-1.23	-2.73	-1.34	0.00	-1.04	-1.58
NH_4	-0.14	0.15	-2.03	0.00	1.04	0.00	-0.35
PN-Abiotic	0.54	0.49	-2.30	0.53	1.58	0.35	0.00

Table E9, Control Difference, cd_{ij}, parameters for Winter 1987 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.25	-1.20	0.09	1.03	0.04	-0.45
PN-Hetero	0.25	0.00	-0.90	0.24	0.98	-0.07	-0.26
Sediment	1.20	0.90	0.00	0.97	1.21	1.00	1.44
DON	-0.09	-0.24	-0.97	0.00	0.74	-0.15	-0.41
NO_x	-1.03	-0.98	-1.21	-0.74	0.00	-0.82	-1.01
NH_4	-0.04	0.07	-1.00	0.15	0.82	0.00	-0.19
PN-Abiotic	0.45	0.26	-1.44	0.41	1.01	0.19	0.00

Table E10, Control Difference, cd_{ij}, parameters for Spring 1987 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.07	-1.49	0.09	1.21	0.06	-0.40
PN-Hetero	0.07	0.00	-1.40	0.14	1.08	-0.19	-0.36
Sediment	1.49	1.40	0.00	1.40	1.85	1.38	1.54
DON	-0.09	-0.14	-1.40	0.00	0.99	-0.11	-0.44
NO_x	-1.21	-1.08	-1.85	-0.99	0.00	-0.99	-1.23
NH_4	-0.06	0.19	-1.38	0.11	0.99	0.00	-0.17
PN-Abiotic	0.40	0.36	-1.54	0.44	1.23	0.17	0.00

Table E11, Control Difference, cd_{ij}, parameters for Summer 1987 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.03	-1.24	0.08	0.96	0.12	-0.27
PN-Hetero	-0.03	0.00	-1.14	0.20	0.73	-0.11	-0.31
Sediment	1.24	1.14	0.00	1.33	1.74	1.12	1.25
DON	-0.08	-0.20	-1.33	0.00	0.73	-0.10	-0.48
NO_x	-0.96	-0.73	-1.74	-0.73	0.00	-0.64	-0.98
NH_4	-0.12	0.11	-1.12	0.10	0.64	0.00	-0.19
PN-Abiotic	0.27	0.31	-1.25	0.48	0.98	0.19	0.00

Table E12, Control Difference, cd_{ij}, parameters for Fall 1987 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.45	-1.89	-0.37	2.15	0.63	-0.70
PN-Hetero	-0.45	0.00	-1.78	-0.18	1.63	-0.20	-1.11
Sediment	1.89	1.78	0.00	1.59	2.83	1.78	2.36
DON	0.37	0.18	-1.59	0.00	1.81	0.32	-0.83
NO_x	-2.15	-1.63	-2.83	-1.81	0.00	-1.47	-2.38
NH_4	-0.63	0.20	-1.78	-0.32	1.47	0.00	-0.93
PN-Abiotic	0.70	1.11	-2.36	0.83	2.38	0.93	0.00

Table E13, Control Difference, cd_{ij}, parameters for Winter 1988 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.61	-3.59	0.19	2.50	0.71	0.13
PN-Hetero	-0.61	0.00	-3.31	-0.24	1.94	-0.18	-0.47
Sediment	3.59	3.31	0.00	3.16	3.97	3.31	3.47
DON	-0.19	0.24	-3.16	0.00	2.13	0.14	-0.22
NO_x	-2.50	-1.94	-3.97	-2.13	0.00	-1.85	-2.13
NH_4	-0.71	0.18	-3.31	-0.14	1.85	0.00	-0.29
PN-Abiotic	-0.13	0.47	-3.47	0.22	2.13	0.29	0.00

Table E14, Control Difference, cd_{ij}, parameters for Spring 1988 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.22	-2.89	0.19	1.63	0.15	-0.82
PN-Hetero	0.22	0.00	-2.73	0.30	1.49	-0.15	-0.67
Sediment	2.89	2.73	0.00	2.85	3.28	2.79	3.03
DON	-0.19	-0.30	-2.85	0.00	1.34	-0.17	-0.89
NO_x	-1.63	-1.49	-3.28	-1.34	0.00	-1.18	-1.81
NH_4	-0.15	0.15	-2.79	0.17	1.18	0.00	-0.51
PN-Abiotic	0.82	0.67	-3.03	0.89	1.81	0.51	0.00

Table E15, Control Difference, cd_{ij}, parameters for Summer 1988 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.06	-1.30	-0.05	1.13	0.05	-0.40
PN-Hetero	0.06	0.00	-1.19	0.10	0.91	-0.08	-0.35
Sediment	1.30	1.19	0.00	1.25	1.87	1.17	1.32
DON	0.05	-0.10	-1.25	0.00	1.01	-0.03	-0.43
NO_x	-1.13	-0.91	-1.87	-1.01	0.00	-0.80	-1.17
NH_4	-0.05	0.08	-1.17	0.03	0.80	0.00	-0.27
PN-Abiotic	0.40	0.35	-1.32	0.43	1.17	0.27	0.00

Table E16, Control Difference, cd_{ij}, parameters for Fall 1988 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	0.24	-3.50	0.22	1.93	0.57	-0.79
PN-Hetero	-0.24	0.00	-3.41	0.02	1.85	-0.33	-1.03
Sediment	3.50	3.41	0.00	3.36	3.87	3.39	3.72
DON	-0.22	-0.02	-3.36	0.00	1.58	0.22	-0.95
NO_x	-1.93	-1.85	-3.87	-1.58	0.00	-1.58	-2.49
NH_4	-0.57	0.33	-3.39	-0.22	1.58	0.00	-0.71
PN-Abiotic	0.79	1.03	-3.72	0.95	2.49	0.71	0.00

Table E17, Control Difference, cd_{ij}, parameters for Fall 1989 for the Neuse River estuary nitrogen model. Dimensional units for cd_{ij} are (m^2 season)/mmol-N.

$cd_{ij} \times 10^3$	PN-Phyto	PN-Hetero	Sediment	DON	NO_x	NH_4	PN-Abiotic
PN-Phyto	0.00	-0.24	-2.63	0.53	1.51	0.16	-0.54
PN-Hetero	0.24	0.00	-2.34	0.89	1.31	-0.17	-0.33
Sediment	2.63	2.34	0.00	2.51	2.70	2.48	2.80
DON	-0.53	-0.89	-2.51	0.00	0.69	-0.69	-1.10
NO_x	-1.51	-1.31	-2.70	-0.69	0.00	-1.03	-1.43
NH_4	-0.16	0.17	-2.48	0.69	1.03	0.00	-0.17
PN-Abiotic	0.54	0.33	-2.80	1.10	1.43	0.17	0.00

Appendix F

Total system throughflow, TST; System Control, sc_j; and Total System Control, TC; for the Neuse River estuary nitrogen model, North Carolina, USA. The table below includes 16 consecutive seasons from Spring 1985 to Winter 1989 and an average data set as calculated by Christian and Thomas which represents all 16 seasons.

Table F-1, System control vectors, sc_j, for nitrogen flow in the Neuse River estuary, USA: 16 seasons. TST = total system throughflow (mmol-N/($m^2 \times$ season). TC = total system control. Dimensional units for sc_j are ($m^2 \times$ season)/mmol-N.

$sc_j \times 10^3$	Avg	Spr 85	Sum 85	Fall 85	Win 86	Spr 86	Sum 86	Fall 86	Win 87	Spr 87	Sum 87	Fall 87	Win 88	Spr 88	Sum 88	Fall 88	Win 89
PN-Phyto	0.13	-0.32	0.37	1.26	2.06	1.11	0.64	1.19	0.74	0.61	0.31	-0.26	-0.54	1.95	0.62	1.32	1.22
PN-Hetero	0.12	-0.48	0.53	1.43	3.52	0.82	0.95	1.30	-0.24	0.64	0.65	2.06	2.87	1.55	0.56	3.14	0.40
Sediment	-2.52	-3.79	-7.33	-15.99	-23.09	-11.16	-9.07	-13.40	-6.72	-9.07	-7.82	-12.23	-20.81	-17.57	-8.10	-21.25	-15.46
DON	0.32	-1.86	0.74	5.19	2.42	1.35	2.66	1.33	1.11	1.19	1.46	-0.25	1.06	3.06	0.76	2.74	5.03
NO_x	2.12	8.10	6.22	9.21	10.99	8.36	5.44	9.44	5.81	7.35	5.78	12.26	14.51	10.73	6.89	13.30	8.68
NH_4	0.13	-0.39	0.60	1.55	3.53	0.89	0.82	1.34	0.19	0.33	0.58	1.99	2.42	1.95	0.58	2.99	0.92
PN-Abiotic	0.30	-1.26	-1.13	-2.64	0.56	-1.37	-1.44	-1.19	-0.89	-1.06	-0.97	-3.60	0.49	-1.67	-1.30	-2.25	-0.79
TC	28.2	8.10	8.46	18.64	23.09	12.53	10.51	14.59	7.85	10.13	8.79	16.34	21.36	19.24	9.40	23.50	16.25
TST	41,517	9,120	20,182	8,780	6,880	12,915	11,980	9,863	7,907	11,533	15,621	7,325	8,680	6,898	16,814	5,732	5,789

Appendix G

Forces in a Cantilevered Live Oak Branch

Assume:
L = 12 meter section
D = 0.25 meter diameter
ρ = 14,455 N/m^3 green density for Live Oak
10% estimated multiplier for branches

Weight = Volume x Density = $(\pi R^2 L) \times \rho$

Weight = $(1.1)\pi(0.25\text{m}/2)^2 \times 12\ \text{m} \times 14{,}455\ \text{N/m}^3 = 9{,}364$ newtons

From standard static beam analysis assuming a uniformly distributed load:

Twisting moment at the base, $M = W \times L/2$

$M = 9{,}364\ \text{N} \times 12\ \text{m}/2 = 56{,}182$ joules

Torque output of Ford F150 4.6L engine at 2,500 rpm = ~375 joules

Equivalent quantity of Ford F150 engines = 56,182 joules / 375 joules = ~**150**