SELECTIVE REDUCTION OF ACTIVITY TIME UNCERTAINTY TO REDUCE RISK OF UNACCEPTABLE SYSTEM PERFORMANCE

by

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(Under the Direction of Richard L. Daniels)

ABSTRACT

One of the major problems in managing a new product development (NPD) project is the difficulty in accurately predicting the duration of project activities. Often only rough design information and preliminary product specifications are available at the early stages of project planning for NPD. Uncertainty concerning how much work must be performed to complete an activity and how productive assigned resources will be complicates the task of accurately estimating the expected time of activity duration. Minimizing the risk of unacceptable project performance (lost customers) is a major concern of decision makers in the NPD process.

This research focuses on resource allocation in project environments with considerable activity time uncertainty, that is, new product development, and explores the implications of considering both the mean and variance of project duration in evaluating alternative approaches. We demonstrate that an objective of increasing the probability that a project completes by a target due date is a reasonable objective that requires the consideration of all activities in the project. In so doing, we demonstrate that resource allocation approaches considering both the mean and variance of activity time provide effective means for improving project performance and improvement approaches need not be confined exclusively to activities on the critical path.

INDEX WORDS: Project Management, Activity Time Uncertainty, New Product Development

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DEDICATION

To God, for I can do all things through Him who strengthens me. And to my wife, Vivian, and sons Jeffrey and Kyle for their continuous encouragement to press-on and for all those years they proceeded with their lives while I was locked away studying.

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CHAPTER 1

INTRODUCTION

Project lateness is a major concern in project management. Minimizing the risk of project lateness is of particular interest to decision makers in the new product development process. When a promised date for product introduction must be met or when there is a lead-time threshold for product delivery beyond which customers are lost, the project management objective is to ensure that actual system performance is acceptable relative to a target date. In these situations a decision maker may prefer a resource allocation policy with low performance variability to an alternative policy with optimal average performance (Daniels and Carrillo, 1997). The risk of achieving unacceptably poor system performance, i.e., project lateness, motivates project management to consider both the average system performance measured as expected project duration and performance variability measured as project duration variance.

The research in this dissertation will result in the analysis of conceptual approaches to the allocation of a resource, available in limited supply, to selectively reduce activity time variance inherent in a project (represented as a network structure). The goal of proposed allocation approaches is the improvement of the probability of achieving desired project duration with the aim of maximizing this likelihood. Subsequently, with the application of the approaches, benefits to project management through a reduction in uncertainty, e.g., managerial time and effort spent interacting with customers and workers to better understand the detailed requirements of a particular activity (Daniels and Carrillo, 1997), and a reduction in risk, e.g., achieving unacceptably poor system performance (Daniels and Carrillo, 1997), are realized.

A network structure representing a project assigned to a single organizational unit embedded in a larger organizational structure is considered. A consequence of activity time uncertainty in this environment is that system performance, as measured by project duration, can vary with the actual duration of the activities. It is assumed that resource-allocation decisions can reduce activity time uncertainty. This study models the selective reduction of activity time uncertainty by assuming that there exists a resource(s), available in limited supply, which can be allocated to project activities to linearly decrease the associated expected activity time or activity time variance. Examples of a resource capable of reducing expected activity time include contracted temporary manpower or equipment employed to reduce activity time. Examples of a resource capable of reducing activity time variance include utilization of a quality function deployment (QFD) process or adoption of the ISO9000 standard aimed at reducing activity time (design process) variance.

Importance of the Research

Delays in projects, particularly new product development, are common and problematic for management. For example, the product development project to launch Apple Computer's first attempt at a portable computer was at least two years late. Microsoft Corporation experienced problems in managing the development of Office 2000 to the point that the product shipped eight months late despite a development process that was geared to shipping on time. Makita Corporation, a leading developer of power tools for construction of commercial and residential buildings, experienced a delay of two years in introducing a 'me-too' product for the construction industry.

A successful new product development process allows firms to evolve in tandem with their ever-changing marketing and technical environments (Brown and Eisenhardt, 1995; Clark and Fujimoto, 1991). Firms have increased operations around the world and broadened the competitive arena to include more unpredictable business foes (Eisenhardt & Martin, 2000). New product development is a key success factor of an innovating organization evolving in step with an ever-changing environment. A firm competing by being innovative must be able to execute the NPD process efficiently and effectively to meet the challenge of a competitor's new product offering, or perhaps more importantly, initiate the challenge. Minimizing the risk of achieving unacceptably poor system performance is an important concern of top-level managers in the NPD process.

It has been noted that network analysis of a project seldom proves to be a useful tool to top-level management (Burt, 1977). This problem has been attributed to the failure of classical network analysis methods in dealing with activity time uncertainty and resource effectiveness. For example, uncertainty in activity time is often heightened by system nervousness (Herroelen and Leus, 2005) stemming from resource allocation decisions made by management as a reaction to the latest information on the status of the project. As decision-making continues over the duration of the project, uncertainty becomes problematic as the project manager allocates or re-allocates resources based on new information (Herroelen and Leus, 2005; Vaziri, et al., 2005).

The resource allocation problem exists because of the realities of project management. Decision making related to resource allocation is not a static process (Burt, 1977), particularly in projects such as NPD that characteristically have high levels of uncertainty and activity time variability (Bajaj, et al., 2004; Brown and Eisenhardt, 1995). The time required to complete an activity is seldom known with certainty in advance, especially for unique non-repetitive projects. Further, the time required to complete activities is seldom independent of the resources allocated to them, and allocating resources to activities is a primary responsibility of management (Burt, 1977). Consequently, developing more effective decision rules to allocate resources is of operational importance to the project manager as it affects the time and cost performance of the project.

For projects such as NPD, activity time uncertainty, i.e. variance, may be more critical than expected activity time, i.e. mean, as a focal point for the project manager interested in reducing the risk of unsatisfactory project performance. One of the most difficult issues facing the project manager is the effect of errors in estimating time and resource requirements. Time estimates are obtained from technical persons associated with the project and usually expressed in probability terms (Malcolm et al., 1959). Different estimators can be expected to have different degrees of bias. If activity times are uncertain, the duration of a project can only be described probabilistically. Subsequently a degree of uncertainty remains in the prospects for completing a project by a specific target date. However, given probabilistic activity time information, the distribution of achievable system performance associated with any resource-allocation decision can be determined, thus providing a means for evaluating alternative allocation approaches.

Purpose of the Research

Research in project management using probabilistic activity networks typically assess performance by focusing on the mean of the project duration distribution and seek to identify the resource-allocation schedule that achieves the optimal system performance, measured as the expected duration of the project (see, e.g., Demeulemeester and Herroelen, 1992; Golenko-Ginzburg and Gonik, 1998; Gutierrez and Paul, 2001; O'Connor, 2006; Pontrandolfo, 2000; Ringer, 1971; Williams, 1998; Yau and Ritchie, 1988). The classical PERT (Program Evaluation and Review Technique) scheduling model focuses exclusively on expected project duration. However, allocation approaches driven by the expected project duration do not necessarily address the concerns of project managers who are more interested in minimizing the risk of achieving unacceptably poor system performance than in optimizing average performance (Daniels and Carrillo, 1997). An alternate approach is to focus on both the average performance, i.e., mean, and the variance of project duration, since a project manager may reasonably prefer a schedule with higher expected duration but little or no variance to a schedule with optimal expected duration but high variance. This approach has been applied to other scheduling environments (see, e.g., Daniels and Carillo, 1997).

Project managers confronted with considerable activity time uncertainty often discover that a budgeted resource allocation plan that is optimal relative to a project schedule performs poorly relative to the actual activity times. Actual activity time values are realized only after resource allocation decisions have been made. The project manager must balance the needs of multiple activities on numerous network paths, and resources must be assigned among the activities to complete the project on time. NPD projects illustrate many of the issues germane to this research. Project management in general largely concentrates on the generation of a schedule of activities that is feasible in terms of precedence relationships among activities, and a resource schedule that optimizes the desired objective, most often project duration (Herroelen and Leus, 2005). In practice, this schedule is called the baseline schedule and serves as the basis for planning internal and external activities.

The project management literature has approached the problem of scheduling activities of uncertain duration by assuming the uncertainty away. Deterministic project network models assume that activity times can be specified precisely in advance of resource allocation.

Stochastic project network models adopt a probabilistic view in which activity time durations are independent random variables with known distributions (Daniels and Kouvelis, 1995).

Stochastic models are usually formulated with the objective of optimizing expected system performance (Daniels and Kouvelis, 1995). According to Daniels and Kouvelis, distributions that capture the uncertainty surrounding important job-specific (or activity/task specific) attributes are often unknown or specified imprecisely. For example, in coordinating a new product through a prototype development facility, uncertainty concerning what or how much work must be performed to complete the development of the one-of-a-kind product complicates the task of accurately estimating activity time distributions. The PERT scheduling model requires that underlying uncertainty be completely ignored to obtain a point estimate of the activity time for use in a deterministic approximation of the problem. The underlying uncertainty must be partially ignored in a stochastic approximation of the problem, and distributional independence of network paths must be assumed to make the problem tractable. However, correlation among associated probability distributions may also be important, as factors such as shared resources and worker skill levels determine the uncertainty of many activities in the network.

Resource allocated to an activity either reduces its activity time or its variance. Either reduction has an impact on that activity's probability distribution, and thus can affect the likelihood of completing the project by the target date. Resource allocation approaches are sensitive to the relationship between the resource allocated to the activity and the probability distribution for the duration of the activity. Consequently, as an objective of this study, the performance of resource allocation rules aimed at achieving the desirable objective of maximizing the probability of project completion to target date, has been computationally tested by applying the allocation approaches to a number of experimental scenarios.

Approaches for the allocation of resource to project activities will be analyzed using three distinct views. First, the allocation of a resource to reduce expected activity time (*et*) is examined. This approach is traditionally used by project management focused on optimizing expected project duration (*eT*). Second, the allocation of a resource to reduce activity time variance *Var*(*t*) is examined. This approach uses information about the uncertainty of activity times, i.e. variance, to influence resource allocation decisions to reduce project duration variance and improve the probability of achieving project completion by a desired target date. Third, uncertainty in project duration is captured in a set of analytical scenarios generated by Risk Calc® (Ferson et al., 1998), a commercially available analytical software based on a proprietary algorithm. In each approach the selective reduction of project duration (*eT*) or project variance *Var*(*T*) is accomplished by assuming that there exists a resource, available in limited supply, that can be applied to a project activity to linearly decrease the associated activity expected time or activity time variance. Resource-allocation approaches are evaluated by determining the allocation that maximizes the likelihood of achieving actual performance no worse than the target date, subsequently minimizing the risk of unacceptable system performance.

In summary, the purpose of this research is to offer project managers a resource allocation approach to maximize the probability of meeting or minimize the risk of exceeding a given project target date. In so doing, we wish to explore a resource that can be selectively allocated to individual activities to (*i*) reduce mean activity time or (*ii*) reduce activity time variance. Allocation approach (*ii*) is warranted since project completion time has variance because activity times have variance. This is presented as an alternative to the traditional approach of managing project completion time based exclusively on expected activity time. As part of the study the effectiveness of several resource allocation rules are tested on an example project network of relatively simple structure. The results of the computational analysis indicate some general resource allocation concepts that should be applicable to the management of larger projects.

Outline of the Dissertation

This study develops allocation approaches and supporting computational analysis for the allocation of resource to activities in a project with considerable uncertainty, e.g. new product development. Chapter 1 describes the importance of the research and reviews the relevant literature.

Chapter 2 provides a problem statement and defines a project, activities, and the resources under consideration. A classic definition of a project, activities, resources, and structure is given. The process for determining expected activity times and project duration is outlined. Sources of uncertainty and variability inherent to a NPD process are also introduced.

Chapter 3 considers the problem of allocating resource to reduce expected time, *(et)*, of an individual activity, including discussion of analytical results. A resource capable of reducing the expected time of an activity is described. Chapter 4 considers the problem of allocating resource to reduce the variance, *Var (t)*, of an individual activity time, including discussion of the analytical results. A resource capable of reducing the variance of an individual activity time is described.

Chapter 5 presents the results of a computational study designed to illustrate the characteristics of resource allocation policies and to test conventional wisdom about how such a resource should be allocated. Generalization to New Product Development projects is discussed in Chapter 6. Examples of real world product development projects with unacceptably poor system performance are described. Chapter 7 summarizes the findings from the study and discusses how the research can be extended.

Literature Review and Prior Related Work

This study draws on literature related to project management network analysis in three major areas: scheduling under uncertainty, probabilistic PERT analysis, and project management in the context of new product development (NPD). Additionally, Chapter 6 offers support from organizational theory and strategic management literature that a high level of uncertainty exists in NPD projects.

- Scheduling under Uncertainty

Effectively allocating resources to activities under uncertainty has been an area of interest for researchers since the 1970s. Early analytical work on the project scheduling problem (Davis and Heidorn, 1971) was aimed at minimizing expected project duration subject to given resource and precedence constraints. The computation of the optimal allocation of limited resources to achieve a given project duration was the focal point. The problem was formulated with a finite set of activities, each with fixed integer duration, and was concerned with the allocation of resources to concurrent activities to minimize project duration beyond the expected time of the critical path. The objective of Davis and Heidorn's study was to minimize the time required to complete *all* activities, i.e. the project network, subject to given constraints. Burt (1977) was one of the first to consider how the allocation of resources to project activities might affect the probability distributions of activity durations (Vaziri, et al., 2005). The problem of interest was defined with respect to a management planning and control objective. In Burt's case, the project manager might want to allocate resources to minimize expected (mean) project duration, or to minimize the longest possible project completion time, or perhaps to minimize the variance of the project duration. The premise underlying the managerial problem is that the expected time to complete the entire project depends on more than the mean time of each path. It is dependent on the probability distributions that govern the time to complete each path. Burt developed heuristic decision rules that provided a simple method for examining the effects of additional resource allocations to activities. In this model, one effect of allocating additional resources is to shift the right-tail of the activity time distribution to the left. Thus, the addition of resources had a specific effect on both the expected time and variance of path duration.

The distributions of activity times and path duration represent uncertainty in project planning. Probabilistic activity durations combined with the effects of resource constraints represent complex uncertainty and are difficult to solve analytically (Williams, 1990). Simulation frameworks have been used (Williams, 1990, 1992) to study uncertainty in its more complex form. Heuristic decision rules have been used to optimally schedule resources with priority ordering taking account of target activity completion times, target path completion times, latest possible activity start time, total slack or float time, or expected (mean) duration. Williams (1992) argued that if there are no resource constraints in a project scheduling problem, the importance of an activity depends on how near-critical it is when considering a particular point in the probability distribution governing the duration of the entire project. This research supported the incorporation of managerial decision models in the planning and control strategies for scheduling resources, and the implementation of contingency plans in project management.

The classical resource-constrained project scheduling problem (RCPSP) has been difficult for researchers to solve because exact methods for scheduling resources to project activities to minimize project duration (Herroelen and Leus, 2005) are generally not practical for anything other than small networks (Vaziri et al., 2005). In recent years, much work has focused on generating solutions for this problem (see, e.g., Li and Willis (1992); Demeulemeester and Herroelen, (1992, 1997); Hartmann (1998); and Golenko-Ginzburg and Gonik (1998)). Substantial research has focused on the use of algorithms and the development of heuristics to solve the RCPSP (Vaziri et al., 2005).

Li and Willis (1992) regarded heuristic resource-constrained scheduling as the only feasible method of handling practical problems in a project scheduling environment. Their paper presented a new procedure for scheduling projects where the availability of resources is constrained. Under this procedure, a project is scheduled forwards and backwards iteratively until there is no further improvement in project completion time. During the iterative process, successive improvements in project completion time are achieved by incorporating a backward schedule into the succeeding forward schedule. The procedure demonstrated that near-optimal resource allocation policies can be generated in a deterministic environment. Demeulemeester and Herroelen (1992) attempted to improve the computational time required to solve the resource-constrained project scheduling problem by fine-tuning the commonly used branch-and-bound algorithm. Their work led to efficiency improvements in average computational time and variance in computational time from problem to problem. However, the method continued the traditional aim of minimizing expected project duration subject to precedence and resource constraints. Additionally, the project network studied was small, with only nine activity nodes. Demeulemeester and Herroelen (1997) enhanced the computational efficiency of the branch-and-bound procedure by incorporating truncating heuristics. Although notable, this research again considered only expected project duration.

Hartmann (1998) developed a heuristic algorithm to generate near-optimal schedules for projects with a large number of activities. To date, no solution procedure can consistently generate optimal solutions for problems with 60 activities or more.

Golenko-Ginzburg and Gonik (1998) further developed the resource constrained project scheduling model by considering projects with random activity durations. A procedure is presented to determine for each activity (i) the starting time, (ii) timing of allocation of resources, and (iii) the assigned resource capacities. The objective was to minimize expected project duration; however, consideration was given to the influence of resource capacity on activity time duration and the corresponding probability density function. Thus, a stochastic optimization problem was substituted for the classical deterministic model.

The resource-constrained project scheduling problem, as well as the common PERT approach to project scheduling, depend on the assumption that the expected time of the longest path, or critical path, determines the expected project duration. Ranasinghe (1994) argued that while this assumption gives the maximum expected duration it does not evaluate the uncertainty of project duration, in that the higher variance of shorter but more uncertain paths is not considered. Ranasinghe proposed that the correlation among the multiple paths of a project network is important in estimating project duration. Classical approaches assume the correlation is zero and the longest path represents all paths. However, when activities are shared between two paths there is correlation in the duration of those paths. As a result, one or more near-critical paths may contribute significantly to actual project duration. Resources allocated to activities on an individual path should be distributed according to how each activity's variance contributes to the variance in path duration.

Bowers (1995) proposed that when resources can be readily transferred between activities, the assumption of an unchanging allocation is unrealistic. When resources are less specialized and easily moved between activities, the demand on the resource may be a more critical factor than the precedence constraint established by the longest path. A conclusion of this study is that the allocation of the scare resource should be based on the importance of lowering uncertainty in the duration of activities that contribute to the uncertainty of project duration. Buss and Rosenblatt (1997) offered similar research supporting the importance of each activity's duration to the duration of a project. This research focused on the effect of activity time uncertainty on the net present value of a project. The results indicate that shifting from an emphasis on minimizing the expected duration of the critical path to a more complex tradeoff between activity durations and cash flow may be a better approach for rescheduling resources and implementing contingency plans in project management.

The research of Daniels and Kouvelis (1995) and Daniels and Carrillo (1997) supported the importance of explicitly considering activity time uncertainty. Although their work centered on production scheduling, the essence of the problem of managing uncertainty in project management was captured. Production schedulers confronted with processing time uncertainty often find that the schedule which is optimal with respect to expected processing times performs poorly when compared to actual processing times. Deterministic scheduling models are based on the assumption that all processing time parameters can be specified with precision prior to scheduling. A process is developed for identifying robust schedules, i.e., schedules whose worst-case performance is optimal (Daniels and Kouvelis, 1995). Daniels and colleagues (1995, 1997) propose that approaches based on point estimates of processing time fail to recognize that decision makers concerned with the management of uncertainty may be more interested in reducing the risk of poor system performance relative to actual processing times than in optimizing expected system performance. Deterministic models do not consider the risk of poor performance of the optimal schedule. In particular, Daniels and Kouvelis found that this characteristic of deterministic models may be particularly problematic in projects from highly competitive environments where efficiency and responsiveness to customer demand is important.

Daniels and Carrillo (1997) extended the research on the effect of processing time uncertainty on system performance by developing scheduling approaches that are evaluated on a measure based on the likelihood of achieving system performance (i.e., total flow time across all jobs) no worse than a given target level. The processing times of individual jobs are considered independent random variables. As noted by Daniels and Carrillo, hedging against unacceptably poor performance is particularly relevant in project environments where there is a clear distinction between good and bad performance. Environments such as production delivery or new product development often include a clear delivery date to a customer or market beyond which customers are lost. The scheduling objective in this type of environment may reasonably be to maximize the likelihood of achieving acceptable system performance instead of optimal average performance.

Gutierrez and Paul (2001) also studied the interaction between activity time variability and system performance. This paper was aimed at analyzing the impact of an increase in activity time variability on the mean remaining completion time of a project at two different stages of project execution. Results of this analysis indicate the importance of investing resources to reduce activity time variability in projects. For projects with a dominant critical path, allocating resources to reduce variability at an early stage of the project was more effective than at a later stage of the project. Wang (2004) considered the uncertainty involved exclusively in product development projects to develop a scheduling methodology for optimizing worst-case schedule performance. Wang's approach provided a way to evaluate the risk of poor schedule performance similarly to the research in production scheduling (Daniels and Kouvelis, 1995; Daniels and Carrillo, 1997). The premise of the study is that, unlike production processes, product development projects are usually unique in nature. The duration of an activity in the project may be extremely difficult to predict accurately because only rough design information is available at the early stage of project planning. This condition makes it difficult to collect enough data to obtain the distribution of the random variable representing the duration of an activity. Uncertain factors in the product development environment are not easily modeled using classical deterministic scheduling methods (Wang, 2004). The scheduling methodology developed can be used by risk-averse product development project managers to evaluate resource allocation decisions according to their impact on minimizing the risk of the project being late.

Leus and Herroelen (2004) studied the effect of resource allocation on protecting a baseline project schedule against variability in activity times. Project activities were assumed to be subject to considerable uncertainty that leads to schedule disruptions. Uncertainty arises from numerous sources such as unavailability of resources or activities taking more time than expected in the deterministic baseline schedule. Resource allocation decisions enable a reactive scheduling policy that allows efficient implementation of managerial contingency plans to compensate for activity time variability.

Research on project scheduling under uncertainty has significantly expanded over the last decade. Herroelen and Leus (2005) conducted a comprehensive survey of the fundamental approaches for scheduling under uncertainty. Among the numerous findings of the study was that most of the major approaches dealing with scheduling risk have been studied in the machine or production scheduling environment. Most of the past research in project scheduling assumed complete information about project parameters and a deterministic environment. As stated earlier, this assumption tends to lead to the generation of schedules that optimize expected project duration. However, for many environments project activities are subject to considerable uncertainty that may lead to schedule disruptions that must be gradually resolved during the execution of the project (Herroelen and Leus, 2005).

Recent research in scheduling under uncertainty (Vaziri, et al., 2005) has provided a means for project managers to optimally allocate resources to individual tasks on several competing projects. Instead of the traditional use of allocation schedules, planned resource allocation to activities is managed by control policies that take into account uncertainty associated with activity durations and the effect of resource allocation on project duration.

In summary, the literature on scheduling under uncertainty has highlighted the importance of considering the entire distribution of processing times in determining how a limited resource should be allocated.

Since projects, especially new product development projects, typically are unique attempts where little past experience can be leveraged, activity time uncertainty is expected. This research considers the relationship between resource allocation and the entire distribution of activity time when the objective is to maximize the likelihood of completing the product by a given target date.

- <u>Probabilistic PERT Analysis</u>

In PERT analysis (MacCrimmon and Ryavec, 1964) activities in projects are assumed to be unique and not routine or repetitive in nature. Consequently, estimates of the duration of activities are uncertain. Particularly in NPD projects where creativity is often important, the ability of resources assigned to activities is inherently hard to measure, and thus estimating activity completion time without error is difficult.

Therefore, in order to capture uncertainty in activity time estimates, a stochastic representation of activity duration must be used. PERT models this uncertainty by assuming that activity duration follows a beta distribution. Although PERT makes an assumption about the form of activity time distributions, the actual shape of the distribution is unknown. When the distributions of times for activities comprising the project are added together, project duration is derived as a distribution whose parameters are a function of the parameters of the activity time distributions. Path duration is treated as a normally-distributed random variable with a mean and standard deviation that can be calculated from the means and variances of the activity times that make-up the project. Given that the distribution of the project duration is normal, the probability of completing a project by a target date can be determined. The compositions and definitions of traditional PERT formulas specifying activity duration (Battersby, 1970) are discussed further in Chapter 2, Problem Statement.

Leading criticisms of the PERT assumptions are centered on the resulting bias of activity time estimates toward optimistic activity durations, i.e., shorter rather than longer activity times (MacCrimmon and Ryavec, 1963; Klingel 1966). Estimated durations are thought to be most often conservative ('most likely' time estimate shifted toward optimistic time estimate), with the level of imprecision varying with the uniqueness of the activity and the experience of the individuals involved in the estimation (Klingel 1966). Further, the expected project duration is obtained by summing the expected times of activities along the longest, i.e., critical path. Hence, given that bias exists in activity time estimates, bias can be expected to exist in expected project duration. Bias in activity time estimates is problematic when seeking to maximize the probability of achieving a given target date.

As stated earlier, for risk assessment purposes the probability of completing a project by a given target date is of interest to management. According to Klingel (1966) and Keefer (1994), relying on the probability of achieving a given project duration to assess risk is hazardous for the project manager. When a manager undertakes planning for a project with numerous activities spanning a substantial length of time, he is usually given or imposes a target date for completion. If the probability of completing the project by the target date is high, the minimum necessary resources may be allocated. If the probability is low, the project manager must evaluate contingency plans to improve performance early in the resource allocation process. Thus, it is imperative for the manager to know as early as possible the likelihood of on-time completion. PERT calculations may mislead the project manager and give him a false sense of security if the expected project duration is optimistically biased (Klingel 1966).

Britney (1976) conducted a study of the problem of substituting deterministic equivalents for randomly distributed activity times. At issue was the PERT methodology for estimating activity means. Britney concluded that the mean is not an optimal point for decision making when distributions of activity durations are skewed and not symmetrically distributed. Grant (1983) also recognized the problem associated with activity durations not being symmetrically distributed and offered a method for reducing variance in PERT networks by using a mathematical transformation to estimate expected project completion time.

More recent research continues to question the validity of the mathematical assumptions used to estimate activity times. Littlefield and Randolph (1991) asserted that the mathematics of estimating PERT activity time appear to be of little practical significance. It is the estimation process encouraging communication between the project manager and those involved in estimating activity times that is important. Cottrell (1999, 2001) offered a modification to the traditional PERT technique for estimating activity durations by proposing that only two durations (most likely and pessimistic) be estimated in order to generate a normal distribution of expected activity times. According to Cottrell, the estimation of three times is unduly complicated and adds little to the accuracy of deterministic equivalents of stochastic activity times in distributions that are not highly skewed.

A second assumption of PERT is that activity times in the project network are independently distributed. However, in actual project conditions there may be dependence among activity durations when scare resources are shared. Project managers may seek to improve actual project duration by switching resources between serial or concurrent activities. Ringer (1971) studied the correlation of activities in series and parallel and introduced statistical dependency into PERT network analysis. How to compensate for dependency among activity durations remains unclear. According to Ringer, even if a manager attempts to manage resource allocation to compensate for dependency relationships it would not be obvious which activity or activities to focus on for contingency resource allocation; thus, little success could be expected.

One of the main purposes of using PERT analysis in project management is to identify the activities and network paths that are critical to the completion of a project on-time. Dodin and Elmaghraby (1985) studied the question of criticality in the PERT model by determining which path is the most likely to be critical and which activities are the most critical. Their work attempted to develop measures of the degree of criticality of an activity. The approach proposed for approximating degree of criticality showed promise. However, it was noted that the structure of a network greatly affects the measurability of criticality. Further, according to the research of

Kulkarni and Adlakha (1986) and Adlakha and Kulkarni (1989), evaluating the distribution of project duration, the probability that a path is the critical path, or the probability that an activity belongs to the critical path is computationally intractable when activity durations are normally-distributed random variables. The difficulty in evaluating any of these measures stems from the statistical dependence created by activities that are common to more than one path in the network (Kulkarni and Adlakha, 1986). Additionally, Yau and Ritchie (1988) presented a method for estimating resource levels in relation to target completion times and also identified network complexity as a factor in project duration.

The allocation of resources to activities based on activity time variance contributing most to the variance of path duration was studied by Williams (1998). Results indicated that an appropriate allocation of resource to an activity should be proportional to the covariance between an activity's duration and the total project duration. Williams' argument was based on two points of reasoning. If an activity has a great amount of variability, the upper end of the probability distribution may have a large effect on the variability of project duration, thus it should be allocated a larger share of resource. Conversely, if an activity has a great amount of slack it will have the lowest covariance with project duration and can be allocated a smaller share of resource since it has little effect on the expected duration of the project. Williams' study sought to determine which activities the project manager should apply resources to in order to reduce the risk of unacceptable project duration. O'Connor (2006) reiterated the key issue of Williams' study by stating that the central difficulty in PERT is the calculation of the start time distribution of each activity when activity start times are related to the distribution of the maximum of a set of dependent random variables.

Pontrandolfo (2000) analyzed the problem of scheduling projects with uncertain activity times and offered the perspective that resources should be allocated based on the ways in which a given project may evolve in relation to due dates and sequences of events occurring from the start of the project to completion. The analytical approach required input on the mean duration, variance, and occurrence probability of every path. Equations were derived to compute mean and variance of the project duration from activity and path data. A conclusion of Pontrandolfo's study was that focusing on path data as a predictor of project duration enhances the project manager's comprehension of the project evolution, allowing better control of the project.

A study conducted by Zhong and Zhang (2003) concerning the use of PERT for the management of construction projects focused on the importance of managing slack on near-critical paths in a project network. Theoretically, there is a non-zero probability that any path in a project network may have an actual duration that exceeds that of the critical path. The actual duration of a near-critical path may exceed that of the critical path if the variance of the near-critical path is high due to activity timing and site condition issues. The authors concluded that the risk of an unacceptable delay of a project can be greatly reduced by effectively controlling the variance of a non-critical path by managing path slack. Kim and Ellis (2005) also conducted a study concerning the management of construction projects that centered on minimizing project duration by the effective allocation of resources. Resources could be split by breaking down an activity into a sequential group of activities whose times can be more accurately predicted. This increased accuracy translates into a benefit for the project manager.

Berleant et al., (working paper) found that the determination of project duration in a network of activities is difficult when completion times are described by probability distributions and extremely challenging when these distributions are neither assumed independent nor to have a known dependency relationship. Berleant's study considered various factors influencing the distribution of activity times, including whether the activity durations are independent random variables, positively correlated (as when activities proceed in the same managerial environment), negatively correlated (as when resources are shared), or dependent in a way that is difficult to characterize, i.e., unknown dependency. As with earlier studies, e.g., Ringer (1971), how to compensate for dependency among activity durations remains unclear.

In summary, notable from the literature on probabilistic PERT analysis is the problem associated with the use of deterministic equivalents of randomly distributed activity times. Relying on expected activity time generated by PERT formulations introduces bias in the risk analysis of project completion to target date. Additionally, focusing exclusively on the expected time of the longest, i.e., critical, path can lead the project manager to ignore a near-critical path that may actually take longer than the critical path. This study aims to extend research and help managers of projects under considerable uncertainty, i.e., those with high activity time variance, by developing and testing a resource allocation approach aimed at reducing project completion time variance to maximize the probability of the project achieving a given completion time.

- Project Management and the Context of New Product Development (NPD)

One application of resource scheduling under uncertainty is the management of new product development projects, typically undertaken by the research and development function within a firm. Often classical PERT analysis is utilized in NPD project management to help in scheduling and resource allocation to activities of projects that usually have lengthy completion times.

Controlling the progress of new product development projects is acknowledged as a complex undertaking (Malcolm et al., 1959). Some projects involve thousands of activities extending many months or years into the future. Often early planning schedules are set up in a backward scheduling approach to conform to time deadlines to meet the market requirements of the product. This forces some activities to be compressed into unrealistic time durations that subsequently cause slippage of the schedule to targeted dates. One of the most difficult issues facing the project manager is the effect of errors in estimating time and resource requirements. Time estimates are obtained from responsible technical persons associated with the project and expressed in probability terms (Malcolm et al., 1959). Different estimators can be expected to have different degrees of bias, subsequently a degree of uncertainty remains in the analysis of the prospects for completing a project by a specific target date. One countermeasure is to deploy a program to compare estimates with actual performance over a period of time to potentially help estimators calibrate their assessments (Malcolm et al., 1959).

Rubenstein and Schroder (1977) identified three sources of variation in the assessment of subjective probabilities of success of a project: personal, organizational, and situational. Subjective probabilities by their nature vary from person to person involved in project duration estimation. Those involved in assessment of scheduling and resource requirements may bias their estimates due to cognitive reliance on knowledge about other projects or because of perceived rewards associated with desirable estimations. Furthermore, the reliability and validity of probability assessment is subject to the degree of association with actual project durations (Rubenstein and Schroder, 1977).

Rubenstein and Schroder report that personal, organizational, and situational influences affect subjective probabilities in numerous ways. When the estimator has participated in project-idea generation and is likely to have responsibility in the project, a relatively optimistic assessment results. Higher ranked estimators are more pessimistic than lower ranked technical experts due to their conservatism and knowledge about organizational constraints. Also, the reward system employed in the project management environment has an effect (although unknown) on the perception of those interested in the project. These findings indicate that estimates of project duration are uncertain due to the dependence of project control decisions on biased estimators. Tushman and Katz (1980) expanded the sources of uncertainty in estimating project duration in their study of the effect of external communication on project performance. The results indicate that organizations, particularly NPD units, are dependent on timely and accurate information from a variety of external sources that require communication linkages to the project manager. Managers must develop and facilitate appropriate mechanisms to meet the external information demands of the project. Project managers unable to develop different information collection and processing methods and evolve them with changing information needs will starve the project duration estimators of bias-reducing data.

In similar research aimed toward understanding personal, organizational, and situational influences that affect subjective probabilities of project duration, a study conducted by Gutierrez and Kouvelis (1991) noted that accurate estimates of project duration obtained from traditional procedures such as PERT are difficult to achieve and are usually biased toward optimism. An underestimation of project duration (i.e. bias toward optimism) occurs due to work force behavioral issues related to Parkinson's Law. The study explores the relationship between the amount of work to be completed in an activity, target completion time, and actual completion time. Parkinson's Law states that work will expand to fill the time available for completion. Gutierrez and Kouvelis' research used a stochastic model to express the expected completion time of an activity as a function of the time allocated to that activity. The authors developed models that provide an organized framework for the project manager to analyze different policies for setting deadlines for activity completion. As part of the study, it was assumed that PERT analysis with activity time estimates having beta distributions represented activity time variability. However, PERT analysis fails to account for time delays from path interactions stemming from dependency of shared activities. The study found that, at times, the more paths sharing an activity, the larger the deviation of actual project duration from expected project duration.

The effect of personal, organizational, and situational influences on project management was also examined by Dougherty and Heller (1994). This study developed the idea that activities of innovative projects, i.e., new product development, do not fit the institutionalized practices of the organization. Either the nature of the activity contradicts the existing system of thought or the content of the activity is new to the existing project management system. Thus, there is no shared understanding for action among those involved in the project. The study found numerous ways that activities in new product development do not fit into the system of thought and action of an organization. Problems include the need for creativity in design (the act of being innovative), existence of departmental barriers that may affect securing human resources and maintenance of team dynamics, and fit of the project into the structure, strategy and climate of the organization. Lack of fit between the NPD project and the organization leads to shortcomings among project participants in linking markets to technologies, working effectively in a creative context, and connecting the new product with the existing set of resources. The lack of knowledge between requirements for development of the new product and resources leads to errors in activity duration estimation and uncertainty in project completion.

Research on project management has expanded to incorporate risk management in recent years. Williams (1995) compiled a bibliography of research related to project risk management to provide an academic framework for integrating uncertainty in project management. Identified in the review as a source of increased risk is the uncertainty of achieving timescale. Most of the studies focused on the analysis of project duration. Notable was the reoccurring focus on critical activities in determining project duration. Common sources of uncertainty in project duration included resource availability, resource requirements that vary over time, effects of operating over a range of activities and resources, and probabilistic or conditional branching of the network structure as the project progresses. The review found that during most analyses certain expected activity times could be estimated, but generating other parameters of the distribution was difficult. Therefore, distributions such as the Beta or the Triangular must be assumed for calculation purposes. This supports the continued use of PERT as a suitable methodology for project analysis.

The management of dependency relationships among activities in product development projects was studied by Smith and Eppinger (1997), who developed a model of the engineering design iteration process. Specific problems associated with building the model centered on the uncertainty inherent in the timing and work flow of product development projects. According to Smith and Eppinger, timing of technology innovations cannot be accurately predicted (due to their novelty), routine engineering functions experience variable timing, and the flow of development activities to be performed cannot be completely specified *a priori*. Dependency among a complex set of activities affects project duration as the work of one design activity affects many other development decisions throughout the organization. Coordinating design decisions is an important responsibility of project management as the increased number of inputs may slow down the development process. The authors concluded that it is better to allocate resources to an activity only after the execution of other activities upon which it is strongly dependent, and to schedule longer activities later in the process so that they are repeated a minimum number of times during design iteration.

Dawson and Dawson (1998) worked with project planning models common to software programs and found multiple types of uncertainty and risk inherent in project planning. In particular they discovered that some activities by their nature have a wide range of completion times, may need to be repeated, may need to be abandoned before completion, or may not be needed at all depending on the outcome of preceding activities. Through simulation the probability distribution for the overall project duration was analyzed and found to be more credible than analysis conducted with activity estimates using a single value.

In a study conducted by Terwiesch and Loch (1999) activity time uncertainty was treated as a moderating variable to the effect of concurrent engineering on NPD project management. The effect of overlapping the activities of a project to accelerate completion was influenced by the resolution of uncertainty during the project. Uncertainty was found to originate from rapidly changing markets and emerging technology that affects the ability of the project team to define project specifications. Furthermore, the resolution of uncertainty may be an organizational capability that must be learned over the course of several projects involving changing project teams. Terwiesch and Loch treated uncertainty resolution as an exogenous variable to their model and left the estimation of uncertainty and how it can be managed to future research. Tatikonda and Montoya-Weiss (2001) also treated market and environmental uncertainty as a moderator. The relationship of project outcomes to market outcomes was affected by the level of external uncertainty. The results indicate that in an environment of market uncertainty, being late to the target date for completion increases the risk of product obsolescence due to changing customer preferences or some other uncontrollable factor from competitor activity. Therefore, understanding which factors and capabilities to emphasize in the product development process is important in reducing the risk of being late to the target completion date.

Sanchez and Perez (2004) conducted an empirical study of the effect of more thoroughly analyzing the termination of a project based on warning signals from the market and the project itself. They proposed that delays have more serious implications in high-technology and research and development environments. New product development is risky as projects may necessarily be discontinued at various stages from idea generation to test marketing. According to Sanchez and Perez, unfruitful projects can consume resources that might be used on projects with higher potential for profitability. Therefore, it is important to identify failing projects as early as possible to allow the resources allocated to them to be reassigned to other projects with less uncertainty and lower risk of failure. Results of the factor analysis indicated that ‘project time deviations’ was a significant contributor to the variability of profitability. Therefore, it can be assumed that monitoring this indicator is important in deciding whether or not to terminate a project. Subsequently, effectively allocating resources to prevent project completion time deviations may be critical to project continuation.

In summary, the resource allocation problem considered here exists because of the realities of project management. Prior research supports the fact that time required to complete an activity is seldom known with certainty in advance, especially for unique non-repetitive projects. In the particular context of NPD, timing of innovation cannot be accurately predicted due to the novelty of the product, what seems to be routine engineering functions experience variable timing, and the flow of development activities to be performed cannot be completely specified prior to undertaking the project. The early work of Burt (1977) established the need for solutions to the resource allocation problem in project management as it was noted that the time required to complete activities is seldom independent of the resources allocated to them, and allocating resources to activities is a primary responsibility of management.

CHAPTER 2

PROBLEM STATEMENT

Definition of a Project

In practice, a project is defined as a series of related jobs directed toward some major output and requiring a significant period of time to perform (Chase et al., 2006). A network diagram, Figure 1, represents the project in the present study used to illustrate how a resource allocation approach can be applied to achieve improvements in project timeliness. The project involves the product marketing and distribution preparation phase, i.e. launch phase, of a NPD project. The subject network is a portion of a much larger network diagram capturing the complex, multi-phase NPD process of a real world company operating as a leading developer of consumer products (see Appendix F).

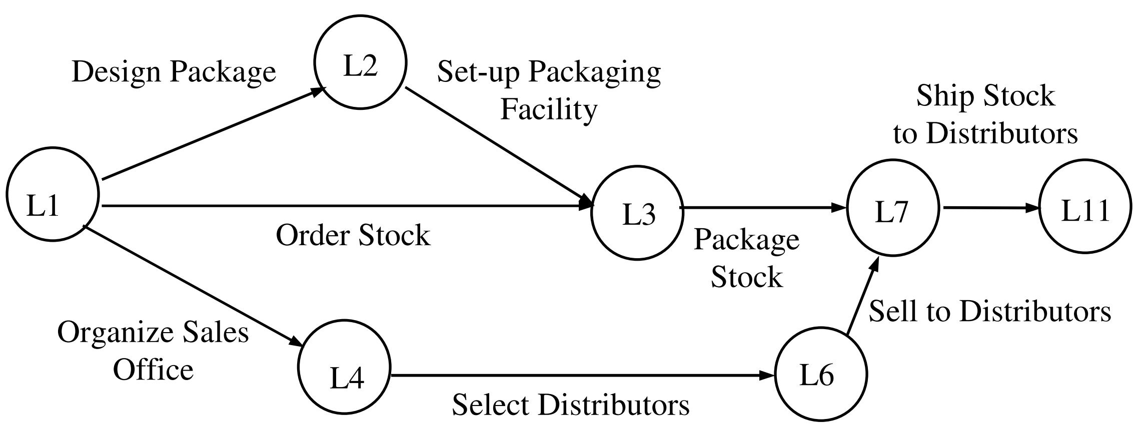


Figure 1. Network Diagram of the Project of Interest

The project of interest is represented as an activity-on-arrow (AOA) network constructed by the precedence diagramming method. Due to technological requirements, there are precedence relationships among the activities. Nodes define the zero-lag finish-start precedence relationships among the activities. Nodes L1 and L11 are source and sink nodes respectively and represent the start and end of the project. Arrows represent activities defining the duration of activities and the project as a whole. The duration of activities can only be specified imprecisely due to uncertainty and incomplete information about resource requirements.

In general, we assume a given network G={V, E}, where V is the set of nodes and E is the set {1,2,…,*n*} of *n* activities. We assume further that the duration of each activity *i*, δ_i, is a normally distributed random variable. The distribution of δ_i is characterized according to three estimates: the optimistic time, t_i^o, the pessimistic time, t_i^p, and the most likely time, t_i^m. From these estimates the mean, variance, and standard deviation of the duration of activity *i* can be computed based on traditional PERT methods (Battersby, 1970; MacCrimmon and Ryavec, 1963; Malcolm et al., 1959; US Navy, 1958):

$$\mu_i = \frac{(t_i^o + 4t_i^m + t_i^p)}{6} \tag{1}$$

$$\sigma_i = \frac{(t_i^p - t_i^o)}{6} \tag{2}$$

$$Var(\delta_i) = \sigma_i^2 = \frac{(t_i^p - t_i^o)^2}{36} \tag{3}$$

See Table 1 for results for a given instance of the problem.

Table 1. Computational results of given network, G={V, E}:

Activity [i]	Description	t_i^o	t_i^m	t_i^p	μ_i	σ_i^2	σ_i
L1-L2	DESIGN PACKAGE	5	10	15	10	2.778	1.667
L2-L3	SET-UP PACKAGING FACILITY	30	45	90	50	100.000	10.000
L1-L3	ORDER STOCK	40	60	110	65	136.111	11.667
L3-L7	PACKAGE STOCK	20	30	40	30	11.111	3.333
L1-L4	ORGANIZE SALES OFFICE	25	30	35	30	2.778	1.667
L4-L6	SELECT DISTRIBUTORS	30	40	80	45	69.444	8.333
L6-L7	SELL TO DISTRIBUTORS	25	30	35	30	2.778	1.667
L7-L11	SHIP STOCK TO DISTRIBUTORS	25	30	35	30	2.778	1.667

- Statement of the Problem

The PERT method assumes that project duration is represented as the duration of the longest path through the project network. This assumption yields the expected project duration but does not necessarily assess the likelihood that actual project duration will meet a given target due date, because shorter but more uncertain paths are ignored (Ranasinghe, 1994). Therefore, to accurately estimate the probability that the actual project duration meets a target due date, the distribution of times associated with all paths through the project network must be explicitly considered. In the problems considered in Chapters 3 and 4, the objective is to determine the allocation of resource that maximizes the likelihood that the project completes at or before target due date T, which might represent a promised date for product introduction or a lead-time threshold for product delivery beyond which customers are lost.

Given network G, we can identify the set L = $\{1,2,\ldots,l\}$ of l paths through the network, where a path is defined as an uninterrupted series of nodes that connect the source and sink nodes of the project network. For each path $j \in L$, we can identify the set A_j = {the activities on path j}. The actual duration of path j, $\Delta_j = \sum_{i \in A_j} \delta_j$, is clearly a normally distributed random variable. The mean and variance of this distribution can be calculated as:

$$D_j = \sum_{i \in A_j} \mu_i \qquad , \; j = 1,2,...,l \tag{4}$$

$$Var(D_j) = \sum_{i \in A_j} \sigma_i^2 \qquad , \; j = 1,2,...,l \tag{5}$$

The associated standard normal statistic, $z_j(T) = \dfrac{(T - D_j)}{\sqrt{Var(D_j)}}$, then yields the likelihood that path j completes at or before target due date T, $P_j(G,T) = \text{Prob}[\Delta_j \le T]$.

The actual duration of the entire project depends on the actual duration, δ_i, of all activities i. Specifically, from any realization of individual activity durations, $\delta = \{\delta_1, \delta_2, ..., \delta_l\}$, the actual duration of all of the paths through the project network $\Delta = \{\Delta_1, \Delta_2, ..., \Delta_l\}$ can be calculated. The actual duration of the project, C, is then determined by the duration of the longest path through the network, the so-called critical path, i.e., $C(G,\delta) = \max_{j \in L}\{\Delta_j\}$.

Since the entire project completes at or before target due date T if and only if the duration of all of the paths is less than or equal to T, we can express the likelihood of meeting the target due date as follows:

$$\text{Prob}\left[C(G,\delta) \le T\right] = \prod_{j \in L} P_j(G,T) \tag{6}$$

This expression shows that the likelihood of achieving a given project completion time is affected by all of the paths through the network. This is in direct contrast with the PERT approach, which focuses exclusively on the critical path.

In this research we assume that a resource exists, available in limited quantity, which can be selectively allocated to individual activities to reduce either the expected duration or the duration variance. Allocating resource to an individual activity clearly affects the distribution of duration of every path that includes that activity.

We also assume that the impact of allocating the first type of resource to activity *i* is to reduce its expected duration by a known amount. More specifically, if x_i units of resource are allocated to activity *i*, then the resulting expected duration is given by:

$$\mu_i = \mu_i' - a_i x_i \qquad , \ i = 1,2,\ldots,n \tag{7}$$

where μ_i' is the expected duration of activity *i* if no resource is allocated, and a_i is the rate at which a unit of resource reduces the expected duration of activity *i*. The objective then is to determine the allocation of resource across activities that maximizes the likelihood that the project completes at or before target due date T.

$$\max_{\{x_1, x_2, \ldots, x_n\}} \prod_{j \in L} P_j(G,T) \tag{8}$$

s.t. (4), (5), and (7),

$$z_j = \frac{(T - D_j)}{\sqrt{Var(D_j)}} \qquad , \ j = 1,2,\ldots,l$$

$$P_j = F^{-1}(z_j) \qquad , \ j = 1,2,\ldots,l$$

$$x_i \geq 0 \qquad , \ i = 1,2,\ldots,n$$

where $F^{-1}(z_j)$ denotes the mass of the standard normal distribution to the left of z_j. Chapter 3 presents and analyzes different strategies for solving this problem.

Alternatively, we assume that the impact of allocating the second type of resource to activity *i* is to reduce the variance in activity time by a known amount. More specifically, if y_i units of resource are allocated to activity *i*, then the resulting activity time variance is given by:

$$Var(\delta_i) = Var'(\delta_i) - b_i y_i \qquad , \ i = 1,2,\ldots,n \tag{9}$$

where $Var'(\delta_i)$ is the activity time variance for activity *i* if no resource is allocated, and b_i is the rate at which a unit of resource reduces the activity time variance of activity *i*. Again, the objective is to determine the allocation of resource across activities that maximizes the likelihood that the project completes at or before target due date T.

$$\max_{\{x_1,x_2,\ldots,x_n\}} \prod_{j \in L} P_j(G,T) \tag{10}$$

s.t. (4), (5), and (9),

$$z_j = \frac{(T - D_j)}{\sqrt{Var(D_j)}} \qquad , \ j = 1,2,\ldots,l$$

$$P_j = F^{-1}(z_j) \qquad , \ j = 1,2,\ldots,l$$

$$y_i \geq 0 \qquad , \ i = 1,2,\ldots,n$$

where again $F^{-1}(z_j)$ denotes the mass of the standard normal distribution to the left of z_j. Chapter 4 presents and analyzes different strategies for solving this problem.

Inherent Uncertainty in NPD Activity Times

- Stochastic Durations of Activity Times

Activity times derived as the weighted average of three time estimates are assumed to follow a beta distribution. Expressions (1), (2), and (3), derived from the classical PERT method, are actually approximations of the mean and variance of a beta distribution. Criticism of this statistical treatment has brought the reliability of expected time estimates derived from the PERT method into question. When compared to the basic formulas, expressions (2) and (3) could lead to absolute errors on the order of 10 percent for the project duration variance and 5 percent for activity duration time variance (Chase et al., 2006). Additionally, given that activity time distributions are continuous, unimodal, and have finite positive end points, other distributions with the same properties, such as the uniform distribution, would yield different means and variances (MacCrimmon and Ryavec, 1964; Chase et al., 2006).

In practice, especially in projects that are unique such as NPD, it is often difficult to arrive at one activity time estimate with confidence, and the subjective definitions of optimistic and pessimistic times are quite ambiguous. The seminal work in the study of the PERT assumptions by MacCrimmon and Ryavec (1964) characterized optimistic time as an arbitrary time somewhat longer than the minimum time in which an activity could possibly be completed and pessimistic time as the longest time an activity could conceivably ever take to complete. Since the parameters of the probability distribution of activity duration rely on human estimates, errors in the estimates are possible, even likely (MacCrimmon and Ryavec, 1964). It was assumed in the study that the human estimates would be incorrect to only a conservative +\-10 or 20 percent of the range. However, estimation error could be greater and will vary with the nature of the activity and resources involved. Conversely, when considering activities as part of a network with serial and parallel relationships, some cancellation of errors could occur. Thus the effect of estimation errors is largely unknown (Dodin and Elmaghraby, 1985). Project management efforts to cope with estimation errors may include transferring resources between activities to offset actual activity durations that are above or below average; this is evidence that correlation may exist among activities in a network structure. Research supports that statistical dependence is derived from activities common to more than one network path (Ringer, 1971; Britney, 1976; Grant, 1983; Kulkarni and Adlakha, 1986; Adlakha and Kulkarni, 1989; O'Connor, 2006).

The PERT model assumes activity times have a beta distribution; however, activity times are subsequently represented as normally distributed for project planning. The suitability of using the mean as a deterministic equivalent of a beta distribution and as an optimal point estimate for project planning when distributions are skewed has been questioned (Britney, 1976; Keefer, 1994). Furthermore, extant research on PERT argues that the true distribution of an activity's duration is not known for certain (MacCrimmon and Ryavec, 1964; Littlefield and Randolph, 1991; Cottrell, 1999) and by assuming a particular distribution some error is introduced into the PERT calculations. Different distributions may accommodate the properties of continuity, unimodality, and finite positive end points while having different means and standard deviations.

- <u>Distribution of Duration Times of Multiple Paths</u>

In network-planning models, expected path duration is the sum of the expected times of activities on that path. The variance of project duration is assumed to equal the sum of the variances of activities along the longest, i.e. critical, path. Central to critical path analysis is that the focus should always remain on activities that make up the critical path. One criticism of the

PERT method is that the longest path does not always determine project duration. In practice, as a project progresses activities not on the critical path often become delayed to such a degree that the project is delayed beyond the expected project duration.

In classical PERT, activity times are assumed to be independent, identically distributed random variables; as such, path duration can be assumed to be normally distributed. Figure 2a presents a stylized representation of the probability distribution associated with the duration of the critical path of network G, including a vertical line indicating the expected project duration *(eT)*. In classical PERT analysis, to determine the probability of completing the critical path activities by a target project duration time T, the position where the target level falls on the probability distribution should be analyzed. Figure 2b shows a representation of T that falls to the right of *eT*. Given that the probability of achieving *eT* is .50, then the probability of T>*eT* is greater than 50%.

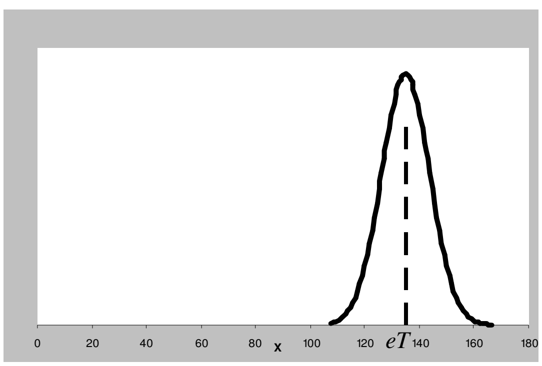
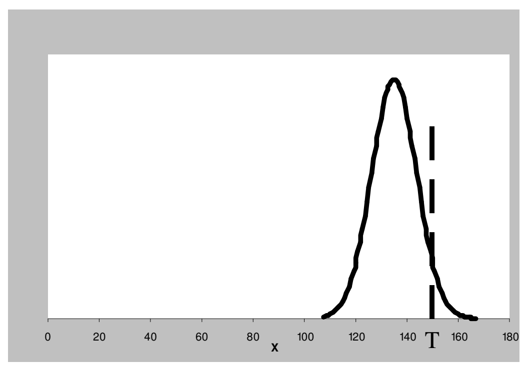


Figure 2a. PDF of Critical Path, *eT*=135



2b. PDF of Critical Path, T=150

As stated earlier, the standard normal statistic, $z_j(T) = \dfrac{(T - D_j)}{\sqrt{Var(D_j)}}$, yields the likelihood of achieving actual project duration no worse than the specified target level T, $P_j(G,T) =$ $\text{Prob}[\Delta_j \leq T]$. The standard-normal statistic z measures the number of standard deviations σ to the right or to the left of *eT* in the distribution. A value associated with a particular *z* statistic represents the area under the distribution to the left of T. As example, for the target duration time T=150 in Figure 2b the area represented by *z* equals 0.956, as generated using the NORMDIST (Z) function of Microsoft Excel.

The existence of other paths through the project network reduces the validity of estimates of project duration based on the means and variances of activities on the critical path (Williams, 1998). If a non-critical path has a mean duration close to that of the critical path, the activities on the non-critical path contribute, perhaps strongly, to the distribution of project duration (MacCrimmon and Ryavec, 1964; Klingel, 1966). Project duration and path durations are related through the probabilities that the project follows each of the individual paths (Pontrandolfo, 2000).

Multiple paths cause the actual project duration to be larger than the expected project duration *eT* (MacCrimmon and Ryavec, 1964; Klingel 1966). The greater the number of paths in a network the greater the error incurred if estimates of project duration are based exclusively on the critical path. However, non-critical paths contribute only weakly to the distribution of project duration if their expected duration is small relative to that of the critical path. Additionally, correlation may exist among activities common to more than one path (Ringer, 1971). The degree to which multiple paths and correlation among activities affect actual project duration depends on network configuration and resource performance.

Research suggests that for stochastic models critical activity analysis may be a valid approach (MacCrimmon and Ryavec, 1964; Chase et al., 2006). As a complementary approach to project planning based on the critical path, critical activity analysis adds robustness in hedging against uncertainty. This is based on the assumption that the critical path may not contain the most critical activities, i.e. those that have large variance but do not lie on the longest path. Theoretically, there is a non-zero probability that the duration of any non-critical path could exceed that of the critical path (Zhong and Zhang, 2003). In the critical activity approach, management attention would focus on activities that have a high activity time variance and lie on the critical path or near-critical path. A near-critical path is one that does not necessarily share activities with the critical path but could become critical if one or a few activities along the path take longer to complete than expected. Typically this is the non-critical path with the least slack time, but a higher slack time path could come into play if it contains activities with high levels of activity time variance. Consequently, the greater the number of paths in the network structure, the more likely one or more near-critical paths exist (Cottrell, 1999; Chase et al., 2006).

- Qualitative Sources of Uncertainty in the NPD Process

The parameters of the probability distribution of activity duration rely on human estimates that can be in error. The activities in a complex new product development project are usually unique to a particular project and are seldom routine or repetitive in nature and thus difficult to estimate with accuracy. In practice, there are many different approaches to structuring the NPD process. Even within the same industry the approach to structuring NPD will vary in some important ways to accommodate organizational, project, and resource differences (Foster, 2003; Ittner and Larcker, 1997).

However, there are similarities in structuring NPD that have emerged across organizations, such as the involvement of a project team rather than a single designer working independently, that represent a framework of a generally accepted NPD process model. It is widely accepted in operations and marketing management that NPD is conducted in multiple steps and is generally understood to occur in two major phases: design and product realization

(Bajaj et al., 2004). Prescriptive organization-oriented methods (Kessler and Chakrabarti, 1996) have emerged from groups such as the International Organization of Standards, e.g., ISO9000, and leading experts such as Robert G. Cooper, e.g., Stage-Gate® (1993, 1998).

In the present study, as a model for the discussion of sources of uncertainty Figure 3 should be viewed as an illustration of a general approach to new product development. Within this model there are nine interrelated phases serving as areas for the introduction of uncertainty in activity times.

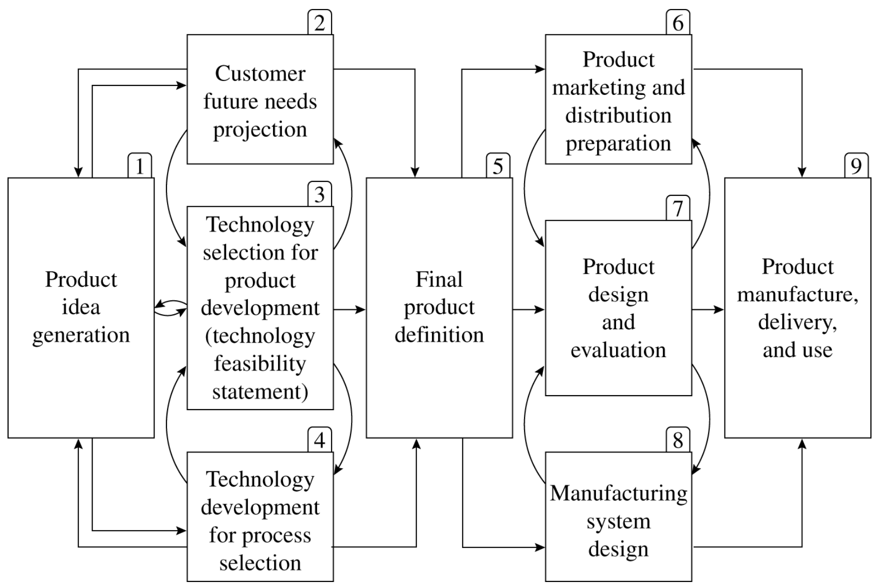


Figure 3. Wilson et al. (1995) *Superior Product Development,* Blackwell

Phases [1] to [4] in the model tend to be characterized as the 'fuzzy front-end' of the NPD project. During this portion of the project, much uncertainty and high levels of activity time variance exist due to the complexity involved in gathering, sorting, and analyzing relevant information and subsequently synthesizing key information into product and process specifications. Phase [1] is product idea generation. During this step external and internal sources brainstorm new concepts. Internal sources include marketing, management, the research and development staff, and other employees. External sources include the customer, suppliers, industry experts, consultants, competitors, and inventors. At this step a preliminary or informal assessment of the marketability of the product is often performed and funding for further development provided if accepted as a viable idea.

Completion times of activities involved in phase [1] are often subject to considerable uncertainty when the project is a unique attempt for which there is little past experience. Flexibility of resources is required to handle changing situations and unpredictable information flow as the phase progresses. Projects change in content over time (Chase et al., 2006), thus a network-planning model made at the beginning of a project may be highly inaccurate later, affecting the ability of management and project resources to bring the product to market. New product ideas resulting from phase [1] take on one of two natures that dramatically affect the uncertainty involved in the project as it progresses. Ideas that are 'leading edge' tend to be groundbreaking (Foster, 2003), technologically innovative (Hill & Rothaermel, 2003), radical in comparison to existing products (Rice et al. 2001), and risky in that they are disruptive to the organization and market (Christensen & Bower, 1996; Christensen & Overdorf, 2000; Christensen, 2001). Errors in estimating activity times and activity time variance are assumed to be high for projects developing products of this nature. Conversely, ideas driven by marketing and customers tend to be incremental (Hill & Rothaermel, 2003) and sustain the organization's capabilities (Christensen & Bower, 1996) building on previous products and better aligned with current customer needs (Foster, 2003). Activity times are known with some accuracy and activity time variance is assumed to be relatively modest due to the experience of the organization with projects for products similar in nature.

From the research of Foster (2003), phases two, three, and four have a relationship to each other and to phase one that is iterative in nature, as information in each step is determined or defined leading to a rippling or reciprocating effect looping forward or backward to reconcile the new information with the other steps. Phase [2] attempts to predict future needs of the target customer in relation to the new idea through the use of market research and data, phase [3] is technology selection for product development in which materials and technologies providing the best performance for the customer needs at an acceptable cost are pre-selected, and phase [4] is technology development for process selection in which the manufacturing processes needed to transform the materials and technologies chosen in phase three are selected. During the completion of the first four phases of the development process, evolution of the product idea, creation of the specifications, planning of materials and processes all occur in a simultaneous nexus of activities. During this rush of activity information asymmetry is present between functional areas, managers, project enablers, and customers. Decision-making for resource allocation based on formal analysis and complete data is difficult due to the project being in states of emergence and progression at the same time. Accurate estimation of activity time duration for these phases of the NPD project is challenging.

Phase [5] is a stage in the NPD process aimed at reducing the uncertainty in the project timeline. Final specifications are defined and drawings are produced for the new product. Variability of activity times is less during this phase since the attributes of the work involved are less uncertain. The creation of drawings and specifications is the repetitive production portion of new product development in which activity times can be predicted more precisely.

Phases six, seven, and eight also have a relationship to each other that is iterative in nature as information in each step is determined or defined by an expanded group of participants working collectively to bring the product to realization. Marketing, sales, manufacturing, and distribution become directly involved in this stage of the project. Analysis and input provided by the expanded team lead to a rippling or reciprocating effect looping forward or backward to reconcile the new information with the other steps. Phase [6], product marketing and distribution preparation, involves the definition of customers, marketing plans, and distribution systems. The design of after-sales processes such as product maintenance and repair may affect the final specifications of the product. Marketing plans may affect the packaging of the product. Phase [7], product design and evaluation, requires the final end-user testing of the product and the system for production. During this phase the product design specification demonstrates the design to be implemented with its major features and conditions for use. From this phase, the expected life of the product, packaging needs, and production infrastructure are verified. Product modifications or special development needs often emerge during this time. Phase [8], manufacturing system design, is the final selection of the processes that will be stable and capable of producing a product that meets the specifications. The result of the process selection is based on projected demand for the product and capital requirements. Design changes may be warranted to improve the manufacturability of the product or to accommodate existing equipment and worker skills.

Phase [9], product manufacture, delivery, and use, represents the traditional operations production function and is not within the scope of this study. At this stage the product design is complete and has been released from the NPD process.

Resulting from the complex NPD process shown in Figure 3 is almost always a product that stretches the organization either incrementally or radically in developing a product to meet the needs of ever-more demanding customers (Christensen, 2001). Issues related to the gathering and processing of technical information, analysis of feedback, the sharing of specialist resources, execution of the design process, as well as numerous product performance difficulties create variability and uncertainty in project duration. Under these conditions, accurate estimation of activity time duration for the various phases of the NPD project is extremely challenging.

CHAPTER 3

SOLUTION APPROAC H:

ALLOCATION OF RESOURCE TO REDUCE EXPECTED ACTIVITY TIMES *(et)*

Consistent with the traditional PERT/CPM methodology, the first solution approach presented here will be exclusively focused on the reduction of the expected activity time of selected activities in the network. There are two parts to the analysis of this solution approach. First, a baseline resource allocation has been developed identifying path durations through the project network. For the baseline schedule, no resource is allocated to reduce expected activity time. We then consider resource allocation and determine the effect on the likelihood of achieving target project duration. The focus of analysis is exclusively on the critical path. Second, a series of analyses are conducted to develop resource allocation approaches that consider expected activity time and variance of each path in the project.

Definition of Resource Inherently Capable of Reducing Activity Time

Given probabilistic activity time information, the distribution of achievable project duration associated with any resource-allocation decision can be determined, thus providing a means for evaluating alternative allocation approaches. This chapter models the selective reduction of activity time uncertainty by supposing that there exists a resource(s), available in limited supply, that can be allocated to project activities to linearly decrease the associated expected activity time (represented as a_i, the rate at which a unit of resource reduces the expected duration of activity *i*).

A resource inherently capable of reducing expected activity time is most often a resource associated with activity direct costs. This type of resource may be related to a worker (i.e. labor) or non-worker (i.e. equipment or facility) resource. Worker-related resources employed to reduce expected activity time include overtime work for existing employees, transferring employees from other jobs, hiring additional employees, contracted temporary manpower, and outsourced man-hours. In the case of new product development this could be knowledge workers such as the engineer, designer, technician or other development team member assigned to project activities for a period of time. The impact of allocating additional units of the worker-related resource is a linear reduction in the duration of the selected activity.

Alternatively, a non-worker related resource capable of reducing expected activity time may be a resource such as equipment or support facilities. Non-worker related resources include additional purchased or leased equipment, more efficient replacement equipment, outsourced resource hours, and the addition of support facilities through vertical or horizontal acquisition. In the case of new product development, this could be development work aids such as a computer-aided design (CAD) workstation, testing apparatus deployed to reduce technician man-hours, or outsourcing of specification development work to a design firm. The impact of allocating additional units of a non-worker related resource is a linear reduction in the duration of the selected activity.

<u>Allocation of Resource</u>

- <u>Goal: Improve Probability of Achieving Desired Project Duration by Reducing Expected Activity Time.</u>

We assume that the impact of allocating the first type of resource to a selected activity is to reduce its expected duration by a known amount. The new expected activity time is derived by subtracting from the expected time of the activity prior to allocation the number of resource units allocated multiplied by the rate at which a unit of the resource reduces the expected duration. The objective then is to determine the allocation of resource across activities that maximizes the likelihood that the project completes at or before target due date T. The allocation approach for this particular phase of the research was to assign all available resource, assumed to be 10 man-days, to a single activity for each scenario of analysis. We assume that this allocation results in a 10-day reduction in the expected duration of the selected activity.

- <u>Allocation Approach to Activities of a Given Project</u>

The allocation approach is applied to a project consisting of $n = 8$ activities represented by the network diagram shown in Figure 1. The given project involves the product marketing and distribution preparation phase, i.e., launch phase of a NPD project. Activities comprising the project are represented by arcs with beginning and end points represented by nodes identified with a label L indicating 'launch phase' and a number indicating a sequential order in the network diagram. The mean activity time and variance for each activity were provided by a test problem formulated for illustrative purposes (Daniels (lecture paper), 2006). See Table 1 for activity time, mean, and variance values.

Given the data in Table 1 and the network diagram in Figure 1, three distinct paths through the network can be identified. Table 2 shows each of these paths along with the expected duration of each. Path 3 is identified as the longest path, or critical path. Figures 2a and 2b show the probability distribution associated with the duration of the critical path, with a vertical line indicating the expected project duration eT (in Figure 2a) as 135 days and the desired project duration of this study, T (in Figure 2b), as 150 days.

Table 2. Baseline Project Schedule

	Paths		
	1	**2**	**3**
	L1-L2	L1-L3	L1-L4
	L2-L3	L3-L7	L4-L6
	L3-L7	L7-L11	L6-L7
	L7-L11		L7-L11
	Paths		
	1	**2**	**3**
	10.0	65.0	30.0
	50.0	30.0	45.0
	30.0	30.0	30.0
	30.0		30.0
Average path time (Days)	120.0	125.0	135.0

Note: Critical path identified with shaded cells

To demonstrate the impact of allocating resource to reduce expected activity times, we first assume that resource allocation has the same effect on the expected duration of any activity, i.e., a_i is constant for all activities *i*. We also assume that activity times are independent random variables. We first allocate 10 man-days of resource to activity L4-L6, the critical activity with the longest expected duration, resulting in a 10-day reduction in the expected duration of this activity. Note that the 10-day reduction in expected activity time is achieved by reducing both optimistic and pessimistic estimates by 10 days.

The results are reported in Table 3 for target completion times of T=135 and T=150. We first focus exclusively on the critical path in calculating the probability that the project completes on-time, consistent with PERT/CPM methodologies that anchor on this path to the exclusion of all others. We see in Table 3 that if no resource is allocated to activity L4-L6, then there is a 50% and 95.6% probability, respectively, that the project will complete by target date T=135 and T=150. If the expected duration of the activity is reduced by 10 days, the likelihoods improve to 87.2% and 99.8%, respectively. This represents an improvement of 74.4% for T=135 and 4.4% for T=150.

Table 3. Summary of computational results of Analysis 1 (for resource reducing *et*)

COMPUTATIONAL RESULTS-Microsoft Excel®				
Desired Project Duration (*T*) Unit = workday	T=135	T=150		
	P (Project Completion Time ≤T), %			
Baseline Case	50.0	95.6		
Scenario [S1]: Allocate resource to critical path activity with greatest mean time, Factor [F2].				
			% Improvement v. Base	
Treatment 1, Critical Path Activity L4-L6	87.2	99.8	74.4	4.4

The results change in important ways if all of the paths through the network are included in the calculations. As shown in Table 4, the likelihood of on-time project completion drops to 36.4% for T=135 and 93.3% for T=150 when no resource is allocated. These probabilities improve to 63.4% and 97.4%, respectively, when 10 man-days of resource are allocated to activity L4-L6. Again, this represents an improvement of 74.2% and 4.4%, respectively, over the baseline case.

Table 4. Summary of computational results of Analysis 2 (for resource reducing *et*)

COMPUTATIONAL RESULTS-Microsoft Excel®				
Desired Project Duration (*T*) Unit = workday	T=135	T=150		
	P (Project Completion Time ≤T), %			
Base Case	36.4	93.3		
			% Improvement v. Base	
Scenario [S3]: Allocate resource to critical path activity with greatest mean time [F2].				
Treatment 1, Critical Path Activity L4-L6	63.4	97.4	74.2	4.4
Scenario [S3]: Allocate resource to non-critical path activity [F2].				
Treatment 1, Non Critical Path Activity L2-L3	39.2	93.6	7.7	0.3

Table 4 also presents the results when the same 10 man-days of resource are allocated to non-critical activity L2-L3. The likelihood of on-time project completion still improves, to 39.2% and 93.6% respectively, but the improvement is far smaller, 7.7% and 0.3%, than that realized when activity L4-L6 is expedited.

While in this example expediting a critical activity improved the probability of on-time project completion more than expediting a non-critical activity by the same amount, the reverse can also be true. At times the variance in the duration of a non-critical path will greatly exceed that of the critical path. To demonstrate this condition, data from Table 1 was changed to reflect a lower path duration variance for the critical path. Optimistic and pessimistic time estimates for critical path activity L4-L6 were adjusted as shown in Table 5 to reduce activity time variance equal to other critical path activities, while expected activity time remained the same. Hence, no change in duration of the critical path occurred.

Table 5. Computational results of given network, G={V, E}**-** Reduced *Var(T)cp*

Activity [i]	Description	t_i^o	t_i^m	t_i^p	μ_i	σ_i^2	σ_i
L1-L2	DESIGN PACKAGE	5	10	15	10	2.778	1.667
L2-L3	SET-UP PACKAGING FACILITY	30	45	90	50	100.000	10.000
L1-L3	ORDER STOCK	40	60	110	65	136.111	11.667
L3-L7	PACKAGE STOCK	20	30	40	30	11.111	3.333
L1-L4	ORGANIZE SALES OFFICE	25	30	35	30	2.778	1.667
L4-L6	SELECT DISTRIBUTORS	35	40	45	45	2.778	1.667
L6-L7	SELL TO DISTRIBUTORS	25	30	35	30	2.778	1.667
L7-L11	SHIP STOCK TO DISTRIBUTORS	25	30	35	30	2.778	1.667

Note: Changed data in shaded cells.

Given the data in Table 5, consider expediting critical path activity L4-L6 and non-critical path activity L1-L3 by the same 10 man-days. Table 6 shows the results when 10 man-days are allocated to reduce the expected duration of each activity. The optimal allocation is easily identified as activity L4-L6 for T=135 and activity L1-L3 for T=150.

Table 6. Summary of computational results of Analysis 2 - Reduced *Var(T)cp*

COMPUTATIONAL RESULTS-Microsoft Excel®				
Desired Project Duration (*T*) Unit = workday	T=135	T=150		
	P (Project Completion Time ≤T), %			
Base Case	36.4	97.7		
			% Improvement v. Base	
Scenario [S3]: Allocate resource to critical path activity with greatest mean time [F2].				
Treatment 1, Critical Path Activity L4-L6	72.7	97.7	99.7	0.0
Scenario [S3]: Allocate resource to non-critical path activity [F2].				
Treatment 1, Non Critical Path Activity L1-L3	43.5	99.5	19.5	1.8%

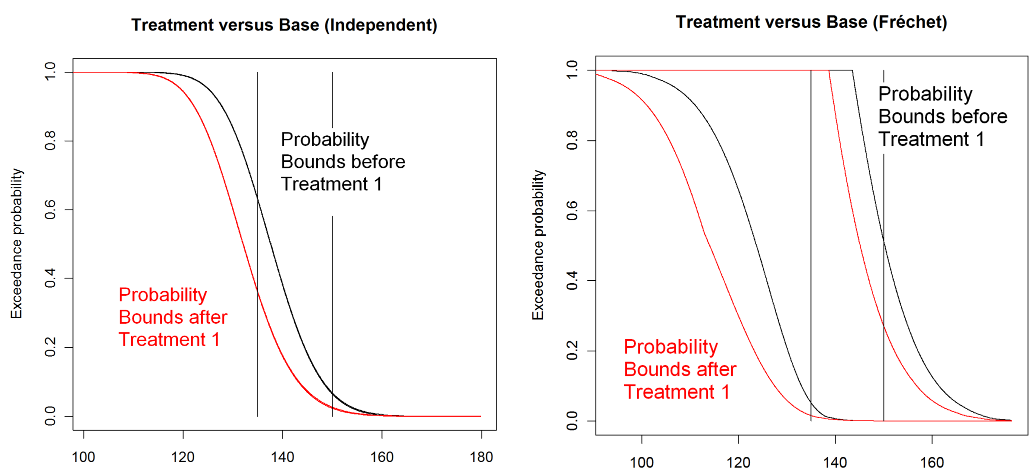
We see that the likelihood of on-time project completion for T=150 increases to 99.5% when resource is allocated to non-critical path activity L1-L3 while no increase is achieved when the same amount of resource is allocated to critical path activity L4-L6. For activity L1-L3 this represents an improvement of 1.8% over the baseline case. This is because the non-critical path that includes activity L1-L3 contributes more to the likelihood that the duration of the project will exceed T=150 than the critical path.

To support the proposition that reduction in activity time to increase the likelihood of on-time project completion is an important focal point in project management, a third analytical approach, Analysis 3, was developed. In Analysis 3, uncertainty in project duration is captured in a set of analytical scenarios generated by Risk Calc® (Ferson, 2002). In the solution approach, activity time can be described as a number, interval, or distribution function. As in Analysis 1 and 2, activity times were first assumed to be independent random variables. In addition, experiments were conducted where activity times were considered to have an unknown level of dependence (the Frechet case of unknown dependence). It should be noted, the determination of an optimal decision strategy for resource allocation to activities with dependency relationships by analytical methods is computationally intractable (Berleant, et al., 2006; Burt, 1977; Daniels and Kouvelis, 1995; Nadas, 1979; O'Connor, 2006). However, under both dependence assumptions, Risk Calc computes probability bounds on the result (usually mathematically the best possible) that circumscribe all possibilities (Ferson et al., 1998). Results are shown as probability bounds consisting of non-crossing cumulative distribution functions (CDF) that enclose the paths of all CDF(s) consistent with the problem (Berleant et al., 2006).

The generation of probability bounds to enclose the CDF(s) is an attractive alternative to calculating the joint probability function. Joint probability refers to a single dimension (e.g., completion time) rather than a multidimensional probability distribution. The determination of a joint probability distribution function is computationally intractable (Berleant, et al., 2006; Nadas, 1979; O'Connor, 2006). Therefore the analysis of probability bounds as presented in Analysis 3 is a parsimonious method for studying the effect of resource allocation decisions on the entire network simultaneously.

In an example analysis using Risk Calc, an experiment was conducted using Scenario [S1], Treatment 1 applied to critical path activity L4-L6. Recall that Treatment 1 is the modification of an activity's expected time by reducing the mean activity time by 10 days without changing activity time variance. As described in Appendix A, this is accomplished by reducing the optimistic, pessimistic, and most likely times by 10 days each. A graphical representation of the resulting solution is shown in Figure 4. Computational results are shown in Table 9.

Figure 4. Graphical representation of Scenario [S1], Treatment 1, from Analysis 3



The results are graphically shown as complementary cumulative distribution functions (a.k.a., exceedance probability) and represented as probability bounds. In the case of the independence assumption (graph on left) the probability bounds appear as a single CDF as a result of post-processing in R to achieve more precision (see Appendix E for discussion). Note that the result is supposed to be a precise distribution. The reason the upper and lower probability bounds are not coincident is due to discretization error from using only 2000 discretization levels. This is analogous to a Monte Carlo result based on a finite number of replications; probability bounds generated by Risk Calc are rigorous in that they are sure to contain the true distribution (Monte Carlo is always only approximate). As stated earlier, Risk

Calc computes probability bounds on the result, usually mathematically the best possible, that circumscribe all possibilities (Ferson et al., 1998). The number of discretization levels could be increased with a sacrifice of computational speed to improve the precision and the probability bounds interval would continue to decrease.

In Figure 4, we see that the probability of project duration exceeding target duration labeled along the x-axis as the corresponding value on the y-axis. At T =135, the probability of project duration exceeding the target duration of 135 days is approximately 63% (actual probability bounds interval width is 62.9% to 63.1%) prior to Treatment 1 being applied and approximately 36% (actual 35.8% to 36.2%) after Treatment 1 is applied. See computational results in Table 9, Scenario S1, Treatment 1, for values.

In the case of the Frechet unknown dependence assumption (graph on right) the probability bounds appear as dual CDF(s) with a wide interval between bounds (Ferson et al., 1998). The wide 'P-box'-type interval results from the additional level of uncertainty added to the calculations when unknown dependence is assumed.

Results from the example analysis above indicate that a more rigorous approach to analyzing resource allocation decisions aimed at maximizing the probability of meeting an acceptable target completion date can be achieved through probability bounds analysis conducted in Risk Calc. First, under the traditional PERT independence assumption a more realistic view of the probability of all paths in the network structure being completed by the target date is derived by the probability bound interval whereby all possibilities are circumscribed by the computation (Ferson et al., 1998). Second, breaking from traditional PERT assumptions, a dependence assumption for activity times can be incorporated into the experimental design, thus allowing the researcher to explore an even more realistic view that there exists some unknown level of dependence among activities in the network. Computational results from Risk Calc for applying Treatment 1 to each of the remaining activities are shown in Table 9.

We will continue to compare the impact of reducing expected activity times with other approaches for improving the probability of on-time project completion. Chapter 4 will consider the companion approach of allocating resource to selectively reduce activity time variance.

CHAPTER 4

SOLUTION APPROACH: ALLOCATION OF RESOURCE TO REDUCE ACTIVITY TIME VARIANCE *Var*(*t*)

Consistent with the traditional PERT/CPM methodology, the first solution approach was exclusively focused on the reduction of expected activity time. We now turn our attention to allocating resource to individual activities in order to reduce activity time variance. We show that effective allocation of this type of resource can also improve the likelihood of achieving on-time project performance.

There are three parts to this analysis. We first consider symmetric reductions in activity time variances, where variance reduction involves an increase in the optimistic estimate of activity time that equals the corresponding decrease in the pessimistic estimate. We next consider asymmetric reductions in activity time variance that are achieved through a decrease in the pessimistic estimate of an activity's duration only. Finally, uncertainty in project duration is captured in a set of analytical scenarios generated by Risk Calc® (Ferson, 2002), a commercially available analytical software based on a proprietary algorithm. Risk Calc supports probability bounds analysis similar to applications conducted in commercially available Monte Carlo software packages.

Definition of Resource Inherently Capable of Reducing Variance

Given probabilistic activity time information, the distribution of achievable project duration associated with any resource-allocation decision can be determined, thus providing a means for evaluating alternative allocation approaches. This chapter models the selective reduction of activity time uncertainty by supposing that there exists a resource, available in limited supply, which can be allocated to project activities to linearly decrease the associated activity time variance.

A resource inherently capable of reducing activity time variance is most often a resource associated with activity indirect costs. This type of resource may be related to an organizational process or organizational system. Process-related resources include process improvement methods that assist in controlling the variability of the process performing to specifications (i.e. standard operating procedures, work instructions, job description, etc.). Examples of resource inherently capable of reducing activity time variance in organizational processes include utilization of quality function deployment (QFD), Quality Circles (i.e. employee problem-solving teams), failure mode and effect analysis (FMEA), Taguchi design of experiments (DOE), mistake-proofing (Poka-yoke), continuous improvement (Kaizen approach), waste reduction (5S approach), and statistical process control (SPC).

In the case of new product development, examples include development process aids such as the House of Quality matrix that involves the use of cross-functional teams of engineering, marketing, and design staff to make design decisions based on group evaluation of customer feedback. The matrix provides the team with a framework for organizing information from the customer to develop valid operating and engineering goals that require little modification as the project proceeds. Failure mode and effect analysis (FMEA) combined with rapid prototyping and design for manufacturing and assembly (DFMA) tools can help reduce NPD schedule variability leading to more robust designs for manufacturing, assembly and customer use. Products developed when these methods are deployed early in the design cycle are more likely to achieve optimal functionality and efficient production with less of a requirement for design iterations (backward and forward loops in the design process to accommodate new information).

Further, technology may be utilized to reduce the variability in the time required to generate and execute innovative ideas by the knowledge worker. The computer-aided design (CAD) workstation can help the designer or engineer evaluate design concepts. Testing devices can be used to control a specified testing regime to prevent unintended analysis and variability caused by subjective evaluation of testing criteria. Subsequently, the impact of allocating additional units of a process-related resource is a linear reduction in the variability in the selected activity's duration.

Organizational system-related resources include system improvement methods that assist in controlling the variability of the organization performing to expectations, i.e. quality standards, delivery promises, and productivity goals. Examples of resource inherently capable of reducing activity time variance in organizational systems include the implementation of the ISO9000 or QS9000 quality standard, Total Quality Management (TQM), the high-involvement-workplace (HIWP), Just-in-time (JIT)/Lean production, and manufacturing resource planning (MRP II).

In new product development, resources include development system methods such as ISO9000. The ISO9000 management system method for product development requires the documentation of how the organization will (*i*) plan and develop the processes needed to provide the product to the market, (*ii*) determine requirements specified by the customer, (*iii*) control the design and development process, and (*iv*) validate with evidence that the product conforms to customer requirements. The certification of an organization to the ISO9000 standard enhances organizational members' understanding and control of processes and methods used in the NPD system. Another example of a variance reduction methodology is the deployment of control processes such as the Stage-Gate® process. The Stage-Gate process is an operational flow-chart for executing NPD from idea to product launch. Stage-Gate divides the product development process into discrete stages separated by management decision points (Cooper, et al, 2002). Cross-functional teams complete a prescribed set of related activities in each discrete stage prior to obtaining management approval to proceed to the next stage of product development (Cooper, et al, 2002).

Ultimately, the impact of allocating additional units of a system-related resource is a linear reduction in the variability in the selected activity's duration. It should be noted that both process-related and system-related resources may not reduce variance symmetrically about the expected activity time. Resources that reduce variance symmetrically are those that both increase optimistic time estimates and decrease pessimistic time estimates (refer to equations *(1)* and *(3)*). This could be any resource that requires additional time to deploy at the beginning of the activity but yields a reduction in time required at the end of the activity. An example would be the utilization of the House of Quality matrix to brainstorm product requirements early in the design activity, which may delay the start of work for the design team but reduce the number of design iterations required prior to activity completion, hence reducing worst-case duration. Alternatively, a process-related or system-related resource could reduce variance asymmetrically about the expected activity time. A resource that reduces variance asymmetrically is one that increases optimistic time estimates and decreases pessimistic time estimates by an unequal amount. An example would be the implementation of a systematic approach to undertaking NPD by requiring design team members to follow a methodical management review process, such as Stage-Gate®, subsequently reducing unwarranted delays in project duration from a phenomenon like Parkinson's Law (Gutierrez and Kouvelis, 1991). Application of such a resource may not increase the optimistic time estimate of an activity but may certainly reduce the pessimistic time estimate if followed successfully.

<u>Allocation of Resource</u>

- <u>Goal: Improve Probability of Achieving Desired Project Duration by Reducing Variance</u>

We assume that the impact of allocating resource to a selected activity is to reduce the activity time variance by a known amount. More specifically, if δ_i is the duration of activity *i*, $Var'(\delta_i)$ the variance in the activity time if no resource is allocated, y_i the amount of resource allocated to activity *i,* and b_i the rate at which a unit of resource decreases the variance in the duration of activity *i,* then the resulting variance in the duration of activity *i* is given by $Var(\delta_i) = Var'(\delta_i) - b_i y_i$.

The objective then is to determine the allocation of resource across activities that maximizes the likelihood that the project completes at or before target due date T. As in Chapter 3, the allocation approach for this particular phase of the research was to assign all available resource, assumed to be 10 man-days, to a single activity for each scenario of analysis.

- <u>Allocation Approach</u>

The allocation approach is applied to a project consisting of n = 8 activities represented by the network diagram shown in Figure 1. See Table 1 for the optimistic, pessimistic, and most likely times, along with the translation of these times into the expected time and variance in duration for each activity. Given the data in Table 1 and the network diagram in Figure 1, a baseline project schedule has been generated with results shown in Table 2.

To demonstrate the impact of allocating resource to reduce the variance in the duration of activities, we first assume that b_i is constant for all activities. We again assume that activity times are independent random variables. Results are reported in Table 7 for target completion times T=135 and T=150. We first focus our calculations of the probability of achieving a project completion time less than or equal to the target date to the activities on the critical path. This is the same starting point used in Chapter 3, as illustrated by the 50% and 95.6% chance that the duration of the critical path is no more than T=135 and T=150, respectively, if no resource is allocated to critical activities.

Alternatively, if resource is allocated to the critical activity with the largest variance in duration, we can assume that variance is reduced symmetrically by both increasing the optimistic time and decreasing the pessimistic time by 10 man-days. The results are shown as Treatment 2 in Table 7. We see from Table 7 that this allocation has no impact on the likelihood that the critical path achieves a duration of no more than T=135. However, the probability that the critical path completes in no more than T=150 improves to 99.5%, a 4.1% improvement.

Table 7. Summary of computational results of Analysis 1 (for resource reducing *Var*(*t*))

COMPUTATIONAL RESULTS-Microsoft Excel®				
Desired Project Duration (*T*) Unit = workday	T=135	T=150		
	P (Project Completion Time ≤T), %			
Baseline Case	50.0	95.6		
Scenario [S1]: Allocate resource to critical path activity with greatest variance [F3, F6].				
			% Improvement v. Base	
Treatment 2, Critical Path Activity L4-L6	50.0	99.5	0	4.1
Treatment 3, Critical Path Activity L4-L6	59.1	98.9	18.2	3.5

To illustrate asymmetric improvement of activity time variance, we assume that allocating resource to an activity only decreases that activity's pessimistic time by 10 man-days. The results are shown as Treatment 3 in Table 7. We see that this allocation improves the likelihood that the critical path completes by time T=135 to 59.1%, an 18.2% improvement over the case where no resource is allocated. If the target completion time is T=150, then resource allocation increases the likelihood of achieving target critical path duration to 98.9%, a 3.5% improvement. Given the assumption that the sum of the variances of the activities along the critical path equals the variance of the time to complete the project (Chase et al., 2006), these results are in line with the expectations of network-planning models where the focus of project management is reduction of expected activity times. Each scenario-factor-treatment combination except [S1], Treatment 2 reduced the expected time of critical path activity L4-L6, which is the aim of traditional PERT/CPM project management. However the reduction of expected activity time was an artifact of reducing variance, the focal point of this chapter.

As in Chapter 3, the results change in important ways when the durations of all the paths are considered. As shown in Table 8, if no resource is allocated to any activity but the duration of all the paths through the network are included, the likelihood that the project completes by time T=135 and T=150 drops to 36.4% and 93.3%, respectively. Table 8 also reproduces the results from Chapter 3 of allocating resource to activity L4-L6 to reduce its expected duration by 10 man-days. This improves the likelihood of achieving a project completion time no worse than T=135 and T=150 to 63.4% and 97.4%, respectively. This represents a 74.2% improvement for T=135 and a 4.4% improvement for T=150.

Table 8 refers to three treatments applied to a total of 4 activities. To summarize, Treatment 1 represents a reduction in the expected duration of the given activity of 10 man-days, with no effect on the variance in the duration of the activity. Treatment 2 addresses activity time variance without affecting expected activity time by simultaneously increasing the optimistic time and decreasing the pessimistic time of the activity by 10 man-days. Treatment 3 addresses both activity time mean and variance by reducing the pessimistic time of the activity by 10 man-days. These three treatments are applied to the critical activities with the largest and second-largest variances, and likewise applied to the non-critical activities with the largest and second-largest variances.

Results are reported in Table 8 for target completion times T=135 and T=150. We first focus our calculation on applying Treatment 2 and 3 to the critical path activity with the largest variance, L4-L6. We see in Table 8 that when 10 man-days of resource are allocated to L4-L6 to symmetrically reduce activity time variance, i.e. Treatment 2, the likelihood of on-time project completion improves to 97.2 % for T=150, a 4.2% improvement over the baseline case. When the same 10 man-days of resource are allocated to L4-L6 asymmetrically, both mean and activity time variance are reduced and the likelihood of on-time project completion improves to 43.0% and 96.6% for T=135 and T=150, respectively. This represents an 18.1% improvement for T=135 and a 3.5% improvement for T=150.

These results show that variance reduction does improve performance, except when resource is applied symmetrically to activities on the critical path. The impact of variance reduction depends on whether resource is applied symmetrically or asymmetrically. The asymmetric allocation of resource, i.e. Treatment 3, to activity L4-L6 yielded an improvement in the likelihood of on-time project completion, where symmetric allocation of resource, i.e. Treatment 2, did not. Treatment 3 performs better than Treatment 2 because the reduction in expected time dominates the additional variance.

Table 8 also presents results when the same 10 man-days of resource are allocated to the non-critical activity with the largest variance, L1-L3. When resource is allocated to L1-L3 to symmetrically reduce activity time variance the likelihood of on-time project completion improves to 39.6% and 95.0 % for T=135 and T=150, respectively, an improvement of 8.8% for T=135 and 1.8% for T=150 over the baseline case. When the same 10 man-days of resource are allocated to L1-L3 asymmetrically, both mean and activity time variance are reduced and the likelihood of on-time project completion improves to 39.6% and 94.7% for T=135 and T=150, respectively. Again, this represents an 8.8% improvement for T=135 and a 1.5% improvement for T=150.

These results show that variance reduction does improve performance when applied to activities on non-critical paths. The impact of variance reduction on non-critical activities may not depend on whether resource is applied symmetrically or asymmetrically. An equal improvement in the likelihood of on-time project completion was yielded for activity L1-L3 with symmetric versus asymmetric allocation of resource. Treatment 3 performs similar to Treatment 2 because variance reduction dominates the reduction in expected time for non-critical activities in this case.

To demonstrate the impact of allocating resource to critical and non-critical activities with different levels of variance, the three treatments were applied to each of the remaining activities; complete results will be discussed in Chapter 5. As an example here, activities L1-L4 and L2-L3 were selected for treatment first because of their second-largest variance among critical activities and non-critical activities, respectively (see Table 1). Previous results have shown that, for Treatment 1, i.e. focusing solely on the reduction of expected activity time, resource allocation to critical path activities always out performs resource allocation to non-critical activities. The aim of this comparison is to determine if an order of preference exists among the four activities for allocating resource to activities on or off the critical path when focusing on activity time variance.

Table 8. Summary of computational results of Analysis 2 (for resource reducing *Var*(*t*))

COMPUTATIONAL RESULTS-Microsoft Excel®				
Desired Project Duration (*T*)	T=135	T=150		
Unit = workday				
	P (Project Completion Time ≤T), %			
Base Case	36.4	93.3		
			% Improvement v. Base	
Scenario [S3]: Allocate resource to critical path activity with greatest mean time [F2] & variance [F3, F6].				
Treatment 1, Critical Path Activity L4-L6	63.4	97.4	74.2	4.4
Treatment 2, Critical Path Activity L4-L6	36.4	97.2	0.0	4.2
Treatment 3, Critical Path Activity L4-L6	43.0	96.6	18.1	3.5
Scenario [S5]: Allocate resource to non-critical path activity with greatest variance [F3, F6].				
Treatment 1, Non Critical Path Activity L1-L3	43.5	95.1	19.5	1.9
Treatment 2, Non Critical Path Activity L1-L3	39.6	95.0	8.8	1.8
Treatment 3, Non Critical Path Activity L1-L3	39.6	94.7	8.8	1.5
Scenario [S3]: Allocate resource to critical path activity with 2nd greatest mean [F2] & variance [F3, F6].				
Treatment 1, Critical Path Activity L1-L4	63.4	97.4	74.2	4.4
Treatment 2, Critical Path Activity L1-L4	36.4	93.6	0.0	0.3
Treatment 3, Critical Path Activity L1-L4	39.2	94.3	7.7	1.1
Scenario [S3]: Allocate resource to non-critical activity with 2nd greatest mean [F2] & variance [F3, F6].				
Treatment 1, Non Critical Path Activity L2-L3	39.2	93.6	7.7	0.3
Treatment 2, Non Critical Path Activity L2-L3	38.6	93.6	6.0	0.3
Treatment 3, Non Critical Path Activity L2-L3	38.2	93.6	4.9	0.3

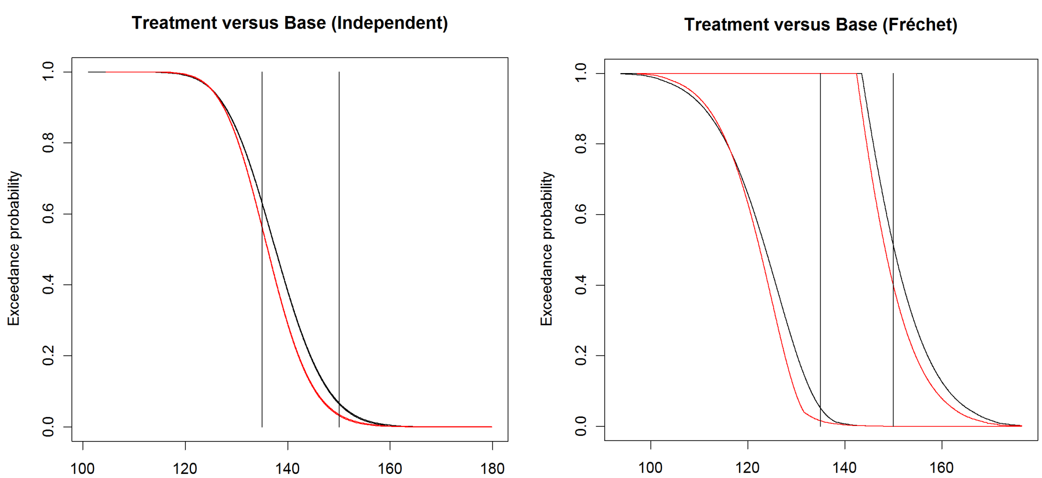
Table 8 presents the results when the same 10 man-days of resource are allocated to activities L1-L4 and L2-L3. When resource is allocated to critical activity L1-L4, the likelihood of on-time project completion improves to 93.6% for T=150 for Treatment 2 and 39.2% and 94.3%, respectively, for Treatment 3. These represent improvements of 0.3% for Treatment 2 and 7.7% and 1.1%, respectively, for Treatment 3. When resource is allocated to non-critical activity L2-L3, the likelihood of on-time project completion improves to 38.6% and 93.6%, respectively, for Treatment 2, and 38.2% and 93.6%, respectively, for Treatment 3. These represent an improvement of 6.0% and 0.3%, respectively, for Treatment 2, and 4.9% and 0.3%, respectively, for Treatment 3.

Given these results, when considering T=135 and Treatment 2, allocating resource to non-critical activities L1-L3 and L2-L3 would be preferred to allocating to either of the two critical activities since a greater improvement in the likelihood of project completion is realized. Also when considering T=135, but Treatment 3, allocation to non-critical activity L1-L3 would be preferred to critical activity L1-L4. When considering T=150, in both Treatments 2 and 3, allocation to non-critical activity L1-L3 would be preferred to critical activity L1-L4.

The results of Analysis 2 show that variance reduction does improve performance, except when resource is applied symmetrically to activities on the critical path, as with activities L4-L6 and L1-L4. Therefore, the impact of variance reduction may depend on whether resource is applied symmetrically or asymmetrically. The asymmetric allocation of resource (Treatment 3) to critical path activities L4-L6 and L1-L4 yielded an improvement in the likelihood of on-time project completion where symmetric allocation of resource (Treatment 2) did not. Treatment 3 performs better than Treatment 2 in this case because the reduction in expected time dominates the additional variance. Analysis 2 also shows that allocating resource to non-critical activities can lead to better performance than allocating the same amount to critical activities. This was demonstrated with the allocation of resource to activity L1-L3 outperforming allocation of resource to activity L1-L4 in both Treatments 2 and 3, and allocation of resource to activity L2-L3 outperforming L1-L4 in Treatment 2.

In an example analysis using Risk Calc, an experiment was conducted using Scenario [S1], Treatment 3 applied to critical path activity L4-L6. Recall that Treatment 3 is the asymmetric reduction of activity time variance. As described in Appendix A, this is accomplished by reducing only the pessimistic time by 10 days. A graphical representation of the resulting solution is shown in Figure 5. Computational results are shown in Table 9.

Figure 5. Graphical representation of Scenario [S1], Treatment 3, from Analysis 3



In Figure 5, we see that the probability of project duration exceeding target duration labeled along the x-axis as the corresponding value on the y-axis. At T =135, the probability of project duration exceeding the target duration of 135 days is approximately 63% (actual probability bounds interval width is 62.9% to 63.1%) prior to Treatment 3 being applied and approximately 56% (actual 56.0% to 56.3%) after Treatment 3 is applied. Computational results from Risk Calc for applying Treatments 2 and 3 to each of the remaining activities are shown in Table 9.

CHAPTER 5

DISCUSSION OF COMPUTATIONAL RESULTS

To consider all activities in the network, a comprehensive computational exercise following the experimental design in Appendix A was conducted. To draw comparisons between the joint probability view (Analysis 2) and the probability bounds view (Analysis 3), additional data expanding Table 8 were generated. Resulting from the exercise are 126 computational results derived from 21 scenario-factor-treatment combinations applied to 2 target project durations and analyzed from 3 views, i.e., (*i*) joint probability, (*ii*) probability bounds with the degree of dependence as none (i.e., independent), and (*iii*) probability bounds with the degree of dependence as unknown (i.e., Frechet). Additionally, calculations were made to derive the proportion change and percentage change of the improved probability of each scenario-factor-treatment combination versus the baseline case. The results are reported in Table 9. See Appendices B, C, and D to review graphical outputs and script of resulting solutions. A copy of a complete set of computations produced by Risk Calc for all problem instances considered may be requested from the author by e-mail at strejeff@isu.edu. An example set, Scenario S2, is provided in Appendix D.

Table 9 reproduces from Table 8 the results of allocating resource to four activities introduced in Chapter 4. Additionally, Table 9 reports the results of allocating resource to three of the four remaining activities found in the *n*=8 network. The results from treating one activity, L1-L2, are omitted from Table 9. Error in generating computational results occurs for L1-L2 because estimates for optimistic and most likely times are too low to produce a non-negative or non-zero number when 10 man-days are allocated under Treatments 1 and 2. Both Microsoft Excel® and Risk Calc® generated errors for this condition.

Results presented in Table 9 support the proposition that a reduction in activity time is important as a focal point of project management for increasing the likelihood of on-time project completion. Analyses 2 and 3 show that a reduction in activity time of either a critical path or non-critical path activity improves the likelihood of on-time project completion. This result is found in all scenarios of analysis as Treatment 1 for both T=135 and T=150.

Results presented in Table 9 also support the proposition that activity time variance reduction is important as a focal point of project management for increasing the likelihood of on-time project completion. Analyses 2 and 3 show that a reduction in activity time variance of either a critical path or non-critical path activity improves the likelihood of on-time project completion.

This result was found when 10 man-days of resource was allocated as Treatments 2 and 3 to each of the four activities considered in Chapter 4 as listed under Scenarios S1, S2, S3, and S4 in Table 9. These results were discussed in Chapter 4. Additionally, when the same 10 man-days of resource was allocated as Treatments 2 and 3 to three additional activities from the network, the same result was found. When resource is allocated to critical activity L6-L7, the likelihood of on-time project completion improves to 93.6% for T=150, Treatment 2, and 39.2% and 94.3%, respectively, for T=135 and T=150, Treatment 3. These represent improvements of 0.3% for Treatment 2 and 7.7% and 1.1%, respectively, for Treatment 3. When the same resource is allocated likewise to critical activity L7-L11, the likelihood of on-time project completion no worse than T=135 and T=150 improves to 36.6 % and 93.7%, respectively, for Treatment 2, and 40.7% and 94.7%, respectively, for Treatment 3. These represent improvements of 0.3% and 0.5%, respectively, for Treatment 2, and 4.3% and 11.8%, respectively, for Treatment 3. Also, when resource is allocated to non-critical activity L3-L7, the likelihood of on-time project completion no worse than T=135 and T=150 improves to 37.2% and 93.8%, respectively, for Treatment 2, and 39.5% and 94.2%, respectively, for Treatment 3. These represent an improvement of 2.2% and 0.5%, respectively, for Treatment 2, and 8.5% and 1.0%, respectively, for Treatment 3. These results along with data reproduced from Table 8 clearly show that a reduction in activity time variance improves the likelihood of on-time project completion and is subsequently an important focal point of project management.

Further, results in Table 9 also indicate that instances occur when a greater improvement in project performance can be achieved by applying a treatment to a non-critical activity versus applying the same treatment to a critical activity. As in the instance of Scenario S2, Treatment 2 versus Scenario S1, Treatment 2, improvement was made in the likelihood of on-time project completion when an allocation of resource was made to reduce the variance of non-critical activity L1-L3 versus no improvement when an equal allocation was made likewise to critical activity L4-L6. For T=135, an improvement of 8.8% from treatment of the non-critical activity was found versus no improvement from treatment of the critical activity.

Similar results occur in Scenario S5, Treatment 2, and Scenario S6A, Treatment 2 versus Scenario S1, Treatment 2. When an allocation of resource was made to reduce the variance of non-critical activities L2-L3 and L3-L7, an improvement in the likelihood of on-time performance no worse than T=135 of 6.0% and 2.2%, respectively, was found versus no improvement from an equal allocation made likewise to critical activity L4-L6.

In these instances, Treatment 2 performs better in improving the likelihood of on-time project completion when allocating resource to non-critical activities versus critical activities.

Recall that Treatment 2 is a symmetrical reduction in activity time variance. A reduction in variance of a non-critical activity reduces the variance of the associated non-critical path. When considering all three paths simultaneously, any reduction in the variance of a non-critical path shifts more of that probability distribution function of project completion time to the left of expected project duration (i.e., mean of the critical path). Hence, for T=135, where expected project duration and target due date are equal, an improvement in the likelihood of achieving on-time project completion results. In this instance, this improvement does not occur when resource is applied symmetrically to reduce variance of activities on the critical path.

Analysis 3 conducted in Risk Calc, where the degree of dependence assumption was defined as unknown (Frechet), generated additional notable results not found elsewhere in the exercise. In Scenarios S3 and S3A, Treatment 2, versus Scenario S1, Treatment 2, it was found that a greater reduction in the upper probability bound of the risk associated with exceeding the target date was achieved when the variance reduction treatment was applied to a critical path activity with lesser variance than an alternate critical path activity. For critical activities L1-L4 and L6-L7, when 10 man-days of resource was allocated to each to reduce activity time variance, the probability bound interval improved to a width of 0.1% to 38.1% in both instances (note, this interval should be viewed as the probability that all three paths will *exceed* the target date, i.e. risk of unacceptable performance). This is an improvement to the baseline case interval calculated as 0.0% to 51.4%. It is also a greater improvement than the result from treating an alternate critical activity having higher variance; that is L4-L6. When the same 10 man-days of resource was allocated to L4-L6 to reduce activity time variance, the probability bound interval only improved to a width of 0.0% to 39.4%. When considering probability bounds as an assessment of risk of unacceptable performance, the upper probability bound reflects worst-case probability of exceeding any given target date (see Figures 4 and 5). Thus, any reduction in the upper probability bound for a given target date, represented as a movement to the left of the upper probability bound, is a reduction in the risk of unacceptable project performance to due date. This result indicates that a useful approach to reducing variance of project duration could be to allocate resource to activities based on their relative amount of activity variance.

In Scenario S6, Treatment 3, versus Scenario S1, Treatment 3, it was found that a greater reduction in the upper probability bound was achieved when the variance reduction treatment was applied to a critical path activity found on the most paths (critical and non-critical). For critical activity L7-L11, when 10 man-days of resource was allocated to reduce activity time variance, the probability bound interval improved to a width of 0.0% to 38.3%. Again, this is an improvement to the baseline case interval calculated as 0.0% to 51.4%. It is also a greater improvement than the result from treating an alternate critical activity having higher variance, but found only on the critical path; that is L4-L6. When the same 10 man-days of resource was allocated to L4-L6 to reduce activity time variance, the probability bound interval only improved to a width of 0.0% to 40.0%. This result indicates that a useful approach to reducing variance of project duration could be to allocate resource based the contribution of an individual activity's variance to project duration variance.

In summary, results presented here support the proposition that allocation approaches to reduce activity time and activity time variance increase the likelihood of on-time project completion. It can be seen from the data presented in Table 9 that the optimal allocation of resource, given that we are allocating all 10 man-days to one activity, is to critical activity L7-L11 for Treatment 1 for both T=135 and T=150, to non-critical activity L1-L3 for Treatment 2 for T=135, critical activity L4-L6 for Treatment 2 for T=150, and critical activity L4-L6 for Treatment 3 for both T=135 and T=150.

These results further show that a reduction in activity time variance of either a critical path or non-critical path activity improves the likelihood of on-time project completion and that allocating resource to non-critical activities can lead to better performance than allocating the same amount to critical activities.

Table 9: COMPUTATIONAL RESULTS-Analyses 2 and 3 conducted in Microsoft Excel® and Risk Calc®

Analytical Software:			Microsoft Excel®						Risk Calc®											
Analytical View of Probability:			Analysis 2: Joint Probability, P(Project Duration<T); *See Note 1*						Analysis 3:Complimentary Cumulative (a.k.a. Exceedance) Distribution Function, P(Project Duration>T); *See Note 2*											
Dependency Assumption:			Factor F7 - Degree of dependence; None (i.e. Independent)						Factor F7 - Degree of dependence; None (i.e. Independent)						Factor F7 - Degree of dependence; Unknown (i.e. Frechet)					
Desired Project Duration (*T*)			T=135	T=150	T=135		T=150		T=135	T=150	T=135		T=150		T=135	T=150	T=135		T=150	
Unit = workday; 5 = I calendar week																				
			P(Project Duration<T), %						P(Project Duration>T),%						P(Project Duration>T),%					
Baseline Case; *See Note 3*			36.4	93.3					62.9, 63.1	6.5, 6.8					5.2, 100.0	0.0, 51.4				
					Improvement v Base		Improvement v. Base				Improvement v. Base		Improvement v. Base				Improvement v. Base		Improvement v. Base	
					Δ	%	Δ	%			Δ	%	Δ	%			Δ	%	Δ	%
Scenario S1: Focus on longest path (critical path); *Apply treatments 1-3 to critical path activity with greatest mean time.*																				
Treatment	Factor	Activity																		
1	F2	L4-L6	63.4	97.4	27.0	74.2	4.1	4.4	35.8, 36.2	2.4, 2.7	26.7, 27.3	42.5, 43.3	3.8, 4.5	58.9, 65.4	1.6, 100.0	0.0, 27.1	(94.9), 98.5	(1841.7), 98.5	(27.1), 51.4	(INF), 100.0
2	F3	L4-L6	36.4	97.2	0.0	0.0	3.9	4.2	62.7, 62.9	2.6, 2.9	0.0, 0.5	0.0, 0.7	3.6, 4.3	55.0, 62.5	1.6, 100.0	0.0, 39.4	(94.9), 98.5	(1841.7), 98.5	(39.4), 51.4	(INF), 100.0
3	F6	L4-L6	43.0	96.6	6.6	18.1	3.3	3.5	56.0, 56.3	3.2, 3.5	6.6, 7.1	10.4, 11.3	3.0, 3.7	45.7, 53.7	1.6, 100.0	0.0, 40.0	(94.9), 98.5	(1841.7), 98.5	(40.0), 51.4	(INF), 100.0
Scenario S2: Focus on path with most variance (not critical path); *Apply treatments 1-3 to non-critical path activity with greatest mean and variance.*																				
Treatment	Factor	Activity																		
1	F2, F4	L1-L3	43.5	95.1	7.1	19.5	1.8	1.9	56.0, 56.3	4.7, 5.0	6.6, 7.1	10.5, 11.3	1.5, 2.1	22.5, 30.9	5.2, 100.0	0.0, 43.9	(94.9), 94.9	(1841.7), 94.9	(43.9), 51.4	(INF), 100.0
2	F3, F4	L1-L3	39.6	95.0	3.2	8.8	1.7	1.8	59.6, 59.9	4.8, 5.1	3.0, 3.5	4.8, 5.5	1.4, 2.0	20.9, 29.4	5.2, 100.0	0.0, 45.6	(94.9), 94.9	(1841.7), 94.9	(45.6), 51.4	(INF), 100.0
3	F6, F4	L1-L3	39.6	94.7	3.2	8.8	1.4	1.5	59.7, 59.9	5.1, 5.4	3.0, 3.5	4.7, 5.5	1.1, 1.7	16.3, 25.0	5.2, 100.0	0.0, 46.7	(94.9), 94.9	(1841.7), 94.9	(46.7), 51.4	(INF), 100.0
Scenario S3: Focus on activities that are on critical path or near-critical path; *Apply treatments 1-3 to critical path activity with 2nd greatest mean and variance.*																				
Treatment	Factor	Activity																		
1	F2	L1-L4	63.4	97.4	27.0	74.2	4.1	4.4	35.8, 36.2	2.4, 2.7	26.7, 27.3	42.5, 43.3	3.8, 4.5	58.9, 65.4	1.6, 100.0	0.0, 27.1	(94.9), 98.5	(1841.7), 98.5	(27.1), 51.4	(INF), 100.0
2	F3	L1-L4	36.4	93.6	0.0	0.0	0.3	0.3	62.9, 63.1	6.2, 6.5	(0.3), 0.3	(0.4), 0.4	0.0, 0.6	(0.8), 8.8	16.7, 100.0	0.1, 38.1	(94.9), 83.4	(1841.7), 83.4	(38.1), 51.3	(INF), 99.8
3	F6	L1-L4	39.2	94.3	2.8	7.7	1.0	1.1	60.2, 60.5	5.5, 5.9	2.4, 3.0	3.8, 4.7	0.6, 1.3	9.3, 19.1	7.6, 100.0	0.0, 42.5	(94.9), 92.4	(1841.7), 92.4	(42.5), 51.4	(INF), 100.0
Scenario S3A: Focus on activities that are on critical path or near-critical path; *Apply treatments 1-3 to critical path activity with 2nd greatest mean and variance.*																				
Treatment	Factor	Activity																		
1	F2	L6-L7	63.4	97.4	63.4	74.2	97.4	4.4	35.8, 36.2	2.4, 2.7	26.7, 27.3	42.5, 43.3	3.8, 4.5	58.9, 65.4	1.6, 100.0	0.0, 27.1	(94.9), 98.5	(1841.7), 98.5	(27.1), 51.4	(INF), 100.0
2	F3	L6-L7	36.4	93.6	36.4	0.0	93.6	0.3	62.9, 63.1	6.2, 6.5	(0.3), 0.3	(0.4), 0.4	0.0, 0.6	(0.8), 8.8	16.7, 100.0	0.1, 38.1	(94.9), 83.4	(1841.7), 83.4	(38.1), 51.3	(INF), 99.8
3	F6	L6-L7	39.2	94.3	39.2	7.7	94.3	1.1	60.2, 60.5	5.5, 5.9	2.4, 3.0	3.8, 4.7	0.6, 1.3	9.3, 19.1	7.6, 100.0	0.0, 42.5	(94.9), 92.4	(1841.7), 92.4	(42.5), 51.4	(INF), 100.0
Scenario S4: Focus on activity most efficiently expedited; *Apply treatment 1 to selected activity with large mean value.*																				
Treatment	Factor	Activity																		
1	F2	L2-L3	39.2	93.6	2.8	7.7	0.3	0.3	60.3, 60.6	6.3, 6.6	2.3, 2.9	3.7, 4.5	(0.1), 0.5	(1.6), 7.4	5.2, 100.0	0.0, 46.5	(94.9), 94.9	(1841.7), 94.9	(46.5), 51.4	(INF), 100.0
Scenario S5: Focus on activity easiest to reduce variance; *Apply treatments 2 <u>and</u> 3 to selected activity with high variance.*																				
Treatment	Factor	Activity																		
2	F3	L2-L3	38.6	93.6	2.2	6.0	0.3	0.3	60.8, 61.1	6.3, 6.6	1.8, 2.3	2.8, 3.6	(0.1), 0.5	(1.6), 7.4	5.2, 100.0	0.0, 47.0	(94.9), 94.9	(1841.7), 94.9	(47.0), 51.4	(INF), 100.0
3	F6	L2-L3	38.2	93.6	1.8	4.9	0.3	0.3	61.2, 61.4	6.3, 6.6	1.5, 2.0	2.3, 3.1	(0.1), 0.5	(1.6), 7.4	5.2, 100.0	0.0, 47.9	(94.9), 94.9	(1841.7), 94.9	(47.9), 51.4	(INF), 100.0
Scenario S6: Focus on activity found on the most paths. *Apply treatments 1-3 to selected activity; include non-critical and critical paths.*																				
Treatment	Factor	Activity																		
1	F2	L7-L11	81.8	99.5	45.4	124.7	6.2	6.6	17.9, 18.2	0.3, 0.5	44.7, 45.3	71.0, 71.7	6.0, 6.5	92.2, 95.6	0.1, 87.3	0.0, 12.7	(82.1) 99.9	(1594.2) 99.9	(12.7), 51.4	(INF), 100.0
2	F3	L7-L11	36.6	93.7	0.2	0.5	0.4	0.4	63.2, 63.4	6.2, 6.4	-0.5, 0.0	(0.8), (0.1)	0.0, 0.7	0.8, 9.6	16.7, 100.0	0.1, 31.9	(94.9), 83.4	(1841.7), 83.4	(31.9), 51.3	(INF), 99.8
3	F6	L7-L11	40.7	94.7	4.3	11.8	1.4	1.5	59.0, 59.3	5.1, 5.4	3.6, 4.1	5.6, 6.5	1.1, 1.7	16.3, 25.0	7.6, 100.0	0.0, 38.3	(94.9), 92.4	(1841.7), 92.4	(38.3), 51.4	(INF), 100.0
Scenario S6A: Focus on activity found on the most (non-critical) paths. *Apply treatments 1-3 to selected activity; exclude critical path.*																				
Treatment	Factor	Activity																		
1	F2	L3-L7	46.9	95.3	10.5	28.8	2.0	2.1	52.8, 53.1	4.5, 4.8	9.8, 10.4	15.6, 16.4	1.7, 2.3	26.4, 33.8	5.2, 100.0	0.0, 37.9	(94.9), 94.9	(1841.7), 94.9	(37.9), 51.4	(INF), 100.0
2	F3	L3-L7	37.2	93.8	0.8	2.2	0.5	0.5	62.3, 62.5	6.1, 6.4	0.4, 0.9	0.6, 1.3	0.1, 0.8	1.6, 11.0	10.7, 100.0	0.5, 38.6	(94.9), 89.3	(1841.7), 89.3	(38.6), 50.9	(INF), 99.1
3	F6	L3-L7	39.5	94.2	3.1	8.5	0.9	1.0	59.9, 60.2	5.6, 5.9	2.7, 3.2	4.2, 5.1	0.6, 1.3	9.3, 18.4	5.2, 100.0	0.0, 42.1	(94.9), 94.9	(1841.7), 94.9	(42.1), 51.3	(INF), 99.9

NOTE: [1] Analysis 2: Joint probability, viewed as the probability that all three paths will complete by the target date (i.e. probability of acceptable performance).
[2] Analysis 3: Complementary cumulative distribution function, or Exceedance, viewed as the probability that all three paths will exceed the target date (i.e. probability, or risk, of unacceptable performance).
[3] Baseline case considers Factor F1 - Number of activities; Low [8]

CHAPTER 6

GENERALIZATION OF PROBLEM TO NPD PROJECTS

The resource allocation approaches presented in this research can be important in controlling uncertainty in new product development projects. PERT analysis is often utilized in NPD project management to allocate resource to activities with lengthy durations. These activities are usually unique to a particular project and are seldom routine or repetitive in nature, thus difficult to estimate with accuracy. Thus, estimating activity times and resource effectiveness can be challenging. However, controlling uncertainty is especially important in new product development, since this function can create significant value for the organization if executed properly.

The allocation approaches presented in this research allow resource to be allocated to activities based on based on the associated impact on the likelihood of completing the project on-time. By distinguishing between the effects of controlling expected activity duration and activity time variance, we provide insight into how different types of resource can be used to improve on-time performance.

Delays in new product development projects are common and clearly problematic for management. As an example, the product development project to launch Apple Computer's first attempt at a portable computer was at least two years late. The product was overweight and oversized and missed the market signal that the portable computer market was shifting to the smaller notebook design. The Design Management Institute in Boston, MA cites numerous reasons for the misfire of the product launch in its case study (March, 1994). Organizational issues included a slow moving “time to perfection” mentality and centralized decision-making power concentrated in a single manager. Also contributing was a company culture in which no product would be shipped until it was “insanely great for the individual”. Design issues included the existence of an inflexible corporate-mandated design language used for all products, the outsourcing of industrial design to outside design firms, and the fact that the user-testing lab reported to another manager outside the design group. The product development process was slow and uncompromising. Design-for-manufacturability was deeply institutionalized and caused conflict between user wants and manufacturing’s expectations. Price wars among competitors in the market led to shifts in Apple’s strategy as the project progressed. All of these issues (organization, company culture, design process, and strategic change) as well as numerous product performance challenges created variability and uncertainty in the process of managing the project’s timeline to market.

Microsoft Corporation experienced problems in managing the development of Office 2000 to the point that the product shipped eight months late despite a development process that was geared to shipping on time. MacCormack (2002) in his Harvard Business School analysis of the Office 2000 project describes the delay as somewhat unexplainable given that Microsoft’s design and development process was flexible and had the ability to respond to new information as it proceeded. The process emphasized keeping to a pre-defined schedule rather than pre-defined specifications. Two main concepts of milestones and daily builds were used to break the development work into smaller stages of design and testing to ensure that new design elements worked well with existing designs.

The Office 2000 project highlights the effect of uncertainty present at the beginning and throughout a development project. Microsoft used several mechanisms to gather information at the start of the project to prevent wasteful iterations of design change and redesign later in the process. Test versions of the software, focus groups of users, off-site planning meetings among technical staff, and an advisory council of representative users were used to gather information to establish the direction of the product and offer feedback at each milestone. Yet even with the sophisticated system of gathering information and ensuring feedback on the evolving design, many problems occurred with project implementation. Timing problems with checking the language code developed by 400 writers working at the same time led to developers violating the daily build concept. Additionally, feedback from verification teams was not received until several days too late. Much information asymmetry and tension existed among the members of the different development teams in designing a product for which there was a diverse set of users. Issues related to gathering and processing of technical information, analysis of feedback, and sharing of specialist resources created variability and uncertainty in the process of managing the Office 2000 project timeline.

Makita Corporation, a leading developer of power tools for construction of commercial and residential buildings, experienced a delay of two years in introducing a “me-too” product for the drywall construction industry. The portable cutout tool was to be a close knock-off of a leading competitor’s established product that had been on the market for many years. Yet, with design specifications that were easy to gather from the competitor’s product, a decentralized design team located close to the customer within the leading market for the product, and a target customer base that was simplistic and uniform in wants and needs, the product was delayed well beyond the original launch date.

Makita operated a disciplined and somewhat rigid product development process that entailed numerous design approval gates and review meetings. The development process was deployed at the Makita corporate headquarters in Japan with great precision. The process was well documented at the subsidiary and met the strict procedural requirements of corporate headquarters. However, the complexity of the process, in terms of the review and approval system, was not well understood by the decentralized (overseas) design team. This caused many delays as the team submitted design and test results to the system approval gates. Organizational complexity, managerial oversight, and information asymmetry created variability and uncertainty in the management of the Makita cutout tool project timeline to market.

The above three examples demonstrate that efficient and effective scheduling of NPD is difficult. A project to develop a competitive product in a complex organization may involve hundreds of activities. For each activity to be successfully executed, the effective allocation of resource is necessary to achieve the project's desired duration.

Variability Inherent in the NPD Process

A new product development project is usually unique in nature and a high level of imprecision exists throughout the project, especially in the early stages of the design. Variability and uncertainty can arise from multiple sources, e.g., (*i*) from the customer in the form of specification uncertainty and changing wants and needs leading to information asymmetry, (*ii*) internally from the design staff in the form of on-time performance, lateness, and design iteration, (*iii*) from the testing specialists through test process performance, test device breakdown, product failure, and (*iv*) from the firm's management through the effects of decision making as the project unfolds. Distinctive characteristics of the NPD process make it a logical context to apply efforts toward the reduction of uncertainty.

- Network Complexity

There are many approaches to designing new products and technologies. Even within the same industry the procedures and processes of firms will vary in some recognizable ways (Foster, 2003; Ittner and Larcker, 1997). Recall that Figure 1 is a network diagram representing the project of interest in this study. The simple network represented in Figure 1 is actually a portion of a much larger network diagram capturing the multi-phase NPD process of a real world company (see Appendix F). In terms of complexity and scope, the NPD process represented in the larger network diagram is typical of the NPD process common to organizations throughout many industries. The project structure is a complex nexus of inter-related activities. There are precedence relationships among the activities due to technological requirements. Perhaps hundreds of activities are linked through the sharing of information, resources, and common managerial oversight.

The effect of project complexity complicates planning using network analysis. As an example, the simple n=8 project represented in Figure 1 has three paths through the network, each having path duration variance ranging from 77.8 days to 150 days. The typical number of activities on a path in this network is four. Therefore, the average variance for activity time in this project is roughly 20 to 40 days. The larger network diagram, representing the typical NPD process, has n=78 activities and an estimated 34 paths through the expanded network, each having an estimated 27 linked activities. Average path duration variance for any of the 34 paths could easily be equivalent to years in project duration. From the examples of real world NPD project delays given earlier, this is a realistic view of the effect of accumulated activity time variance.

- Information Asymmetry

Efficiency in product development is a key success factor because as a competitor launches innovative products more quickly and markets them aggressively, rival firms are compelled to manage their product introduction process for rapid introduction to avoid product obsolescence and decreased competitiveness (Cordero, 1991). During the rush of activity to move the project through process steps, information must be exchanged between functional areas, managers, project enablers, and customers.

Sources of information within the firm and industry structure contribute to the execution of new product development and influence the firm's ability to stay in step with the market in NPD and product launch. Sources internal to the firm include marketing, management, and research and development personnel. Sources in the firm's industry include suppliers, industry experts, consultants, and competitors. Firms may target the development and entry of future products into the market based on information about customer acceptance, competitor reaction, or perceived opportunities with strategy or position among industry rivals.

Under these conditions, decision-making for resource allocation based on formal analysis and complete data is difficult due to the complex flow of information resulting from the project being in states of emergence, progression, and iteration at the same time. For NPD, vast information asymmetry between resources linked through the NPD process adds variability in activity time and subsequently, the execution of the project.

- Knowledge Management

As a tool for innovation, at the center of the NPD process are activities associated with the management of knowledge (Davenport et al., 2002) and creative ability (MacCrimmon and Ryavec, 1964). Knowledge management is effectively combining information with experience and context (Davenport et al., 1998) to make decisions about product specifications and project plans. Making knowledge available to project participants is difficult, consumes resources, and is often untimely causing inefficient iterations within the process (Davenport and Glaser, 2002). Creative ability is hard to measure in individuals and makes the estimation of activity time for activities requiring creative ability imprecise (MacCrimmon and Ryavec, 1963).

- Estimation Error

One of the major problems in managing a NPD project is that the duration of project activities are difficult to predict accurately (Giachetti et al., 1997). This is because often only rough design information and preliminary product specifications are available at the early stages of NPD. Uncertainty concerning how much work must be performed to complete an activity and how productive assigned resources will be complicates the task of accurately estimating the distribution of activity duration (Wang, 2004). One of the most difficult issues facing the project manager is the effect of errors in estimating time and resource requirements. Time estimates are obtained from responsible technical persons associated with the project and expressed in probability terms (Malcolm et al., 1959). Different estimators can be expected to have different degrees of bias, thus, the parameters of the probability distribution of activity duration relying on human estimates can be in error. Subsequently, a degree of uncertainty remains in the analysis of the prospects for completing a project by a specific target date.

Additionally, activity durations are subject to considerable uncertainty (Vaziri, et al, 2005) resulting from the actual allocation of resource. Decision-making about the allocation of resource does not always reduce the uncertainty in activity durations, e.g., constant rescheduling of resources based on the latest information about activity times can create system nervousness from instability in the project management process (Leus and Herroelen, 2004).

- Managerial Decision-Making

As a system, the new product development function creates value for the organization by being reliable and responsive in matching product offering and demand. To maintain market share, firms need to effectively manage their product development projects and bring their products to market as early as possible (Wang, 2004). Planning for activity time uncertainty in new product development is important in making the process as efficient as possible. Decisions must be made about allocating resources to development efforts as information circulating in the industry indicates that an impending new product by a competitor is near (Bayus, 1997).

In practice, top-management oversight adds to the variability and uncertainty of NPD from the effects of resource allocation decisions as a project unfolds. During the execution of the new product development process, top management may choose to intervene in the organization's NPD process by rejecting one project in favor of another or suspending product development due to conflicts with the firm's evolving strategy or changing economic conditions. Schilling (1998) argues that organizations may strategically time the development and entry of future products into the market based on perceptions of customer acceptance. Hill and Rothaermel (2003) assert that firms may choose to withhold a radical technology that may destroy the demand for existing products. Situations such as these disrupt NPD and interfere with the efficient execution of the process.

- Industry Factors

Industry factors exert a strong effect on the progress of NPD and product launch (Bayus et al., 1997; Cordero, 1991, Datar et al., 1997). The rate of technological progress in an industry plays an important role in NPD process cycle length (Cohen et al., 1996; Morgan et al., 2001). Industries that have a relatively high rate of quality improvement, such as automobile and pharmaceutical, may delay launch of a product to continue work toward achieving a superior level of performance (Bayus et al., 1997; Datar et al., 1997; Kessler and Chakrabarti, 1996; Morgan et al., 2001; Souza et al., 2004). Industries that have high project development costs, such as computer processors, tend to apply resources to shorten cycle times. Conversely, industries that have high fixed costs in the form of engineering and testing requirements, such as aircraft and medical devices, tend to experience longer NPD cycles (Ittner and Larcker, 1997; Morgan et al., 2001).

Therefore, depending on the cost implications, project conditions may lead to resource allocation decisions to either expedite the project timeline by adding resource or delay activity start-up to take advantage of slack. Circumstances such as these create variance in NPD that may disrupt the process from being effectively and efficiently executed. Hence, organizations may experience high levels of variability in NPD system performance from project to project (Brown and Eisenhardt, 1997).

In summary, distinctive characteristics of the NPD process make it a logical context to apply efforts toward the reduction of uncertainty. Clearly, estimating activity times and resource effectiveness is challenging in the NPD project environment. However, developing approaches to control uncertainty in new product development projects is important because, as a system, the new product development function creates value for the organization if executed properly.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

This research has focused on resource allocation approaches in project environments with activity time uncertainty, and explored the implications of considering both the mean and variance of project duration in evaluating alternative approaches. NPD projects illustrate many of the issues germane to this research. Project management traditionally concentrates on the generation of a schedule of activities that is feasible in terms of precedence relationships among activities, and a resource schedule that optimizes the desired objective, most often the expected time of the project as determined by the critical path. We defined a measure of allocation effectiveness that represents the likelihood of achieving project duration no worse than a given target level. The probability that a project completes by given time T is an important objective that is (*i*) distinct from that considered in traditional PERT/CPM analysis, and (*ii*) relevant to real project managers, especially in NPD. To implement this new objective, all paths through the network must be considered, because a project completes by time T if and only if all of the paths through the network complete by time T.

For project environments where activity completion times are independent random variables and the performance measure of interest is project duration, we first established baseline project performance. We then considered allocation approaches aimed at controlling uncertainty associated with individual activity times. Results indicated that allocation approaches that consider both the mean and variance of activity time provide effective means for improving the likelihood of on-time project completion. Specifically, through an illustrative example, we showed that resource selectively allocated to reduce the expected duration of activities improves the probability that the project completes by time T. Alternatively, we showed that resource selectively allocated to reduce the variance of activity time also improves the probability that the project completes by time T.

The development of solution approaches to reduce expected activity time and activity time variance is important because as a project manager allocates more or less resource to an activity, the probability distribution of that activity's processing time is affected. By applying more resource to a particular activity, its duration can be probabilistically shortened or its variance reduced (Burt, 1977). The allocation approaches presented here considered the relationship between the resource allocated to the activity and the probability distribution for the duration of the activity.

The first resource allocation approach focused exclusively on the reduction of expected activity time and was presented in Analysis 1. The baseline schedule was developed with no resource allocated. We then considered resource allocation and determined the effect on the likelihood of achieving target project duration. The focus of analysis was exclusively on the critical path, ignoring near-critical paths that could potentially extend project duration. Given the assumption that the expected time to complete the critical path is the sum of the critical path activity times, the results from Analysis 1 were consistent with traditional PERT/CPM network analysis.

As an alternative view to the focus of project management exclusively on the critical path to improve project performance, Analysis 2 was conducted in which the joint probability of all paths through the network being completed by the target date was considered. Given the assumption that the expected time to complete a project is the sum of the activities found on *all* paths in the network, the aim of Analysis 2 was to show that project performance to target duration can be improved by means other than focusing exclusively on the critical path.

In Analysis 2, a series of computational analyses was conducted to evaluate a resource allocation approach that considered duration and variance of each path in the project from a joint probability view. The experimental design shown in Appendix A was followed to select relevant combinations of analytical scenarios and project characteristic. Information about expected activity time and activity time variance was used to maximize the probability that the project duration achieved a target completion date. It was demonstrated that resource can be allocated to (*i*) reduce activity mean time, (*ii*) symmetrically reduce activity time variance, and (*iii*) asymmetrically reduce activity time variance simultaneously reducing activity mean time. Results indicated that allocation approaches considering both the mean and variance of activity time provide effective means for improving the likelihood of on-time project completion.

Analysis 3 was conducted in which uncertainty in project duration was captured in a set of analytical scenarios generated by Risk Calc® (Ferson et al., 1998). Results were shown as probability bounds consisting of non-crossing cumulative distribution functions (CDF) that enclose the paths of all CDF(s) consistent with the problem. The generation of probability bounds to enclose the CDF(s) was considered to be an attractive alternative to calculating the joint probability function. Joint probability refers to a single dimension (e.g., completion time) rather than a multidimensional probability distribution.

Results from Analysis 3 indicate that a more rigorous approach to analyzing resource allocation decisions aimed at maximizing the probability of meeting an acceptable target date can be achieved through probability bounds analysis. A more realistic view of the probability of all paths in the network being completed by the target date was derived by probability bounds in which all possibilities are circumscribed by the computation (Ferson et al., 1998).

Additionally, Risk Calc allowed the incorporation of a dependence assumption for activity times in the experimental design. With this assumption we were allowed to explore a potentially more realistic view that there exists some unknown level of dependence among activities common to a network structure. The incorporation of the Frechet unknown dependence assumption was supposed to add robustness to network analysis by viewing the network as a nexus of activities instead of a collection of independent paths. The computational results were somewhat disappointing in that probability bounds generated under the dependence assumption were at such wide intervals that interpretation was meaningless beyond statements about improvements in the worst-case.

In summary, this research has demonstrated that (*i*) the objective of increasing the probability that a project completes by a target due date is a reasonable objective that requires consideration of duration of all activities in the project, (*ii*) resource allocation approaches considering both the mean and variance of activity time provide effective means for improving performance, and (*iii*) resource allocation aimed at improving project performance need not be confined exclusively to activities on the critical path.

- <u>Limitations</u>

This study is limited in that the problem instances were generated from a single project of simple structure. However, the network model used does embody important characteristics common to larger networks, such as (*i*) multiple paths through the network, (*ii*) the presence of a near-critical path based on expected path duration, (*iii*) activity times of both long and short duration, (*iv*) activity time variance of both high and low levels. A further limitation of this study is the reliance upon computational results to represent real world experiment.

- <u>Future Research</u>

Future research extending this study appears promising. Conclusions from this phase of the research can be tested further on larger networks of more complex structure, e.g., bushy versus sparse. Research could take into consideration the kind of environment in which resource allocation approaches presented will work, e.g., projects with homogenous activity times but heterogeneous activity time variance. Finally, future research could seek to demonstrate which type of resource is preferred by looking at the relative effectiveness of allocation, including a combination of the two resources.

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Appendix A: EXPERIMENTAL DESIGN

Purpose of Study: Resource allocation of limited resources to increase probability of completing a project by a target completion date: Prob $[C(G,\delta) \leq T] = \prod_{j \in L} P_j(G,T)$

Context: Project of interest is network G={V, E}, where V is the set of nodes and E is the set {1,2,…,*n*} of *n* activities.

Types of Resources: Two types exist represented as a_i, the rate at which a unit of resource reduces the expected duration of activity *i* or represented as b_i, the rate at which a unit of resource reduces the activity time variance of activity *i*.

Treatments: Applied to activity selected for its contribution to project duration.

[T1] Reduce expected activity time by 10 man-days

[T2] Reduce activity time variance by symmetrically reducing t_i^o and t_i^p by 10 man-days

[T3] Reduce activity time variance by asymmetrically reducing t_i^p by 10 man-days

Scenarios of Analysis:

[S1] Focus on longest path (critical path)

[S2] Focus on path with most variance (assuming not critical path above)

[S3] Focus on activities that are on critical path or near critical

[S4] Focus on activity most efficiently expedited

[S5] Focus on activity easiest to reduce variance

[S6] Focus on activity found on the most paths

Factors and Levels: Consider what makes a project unique -

[F1] Number of activities (low [8,19]; medium [20,50]; high [>50])

[F2] Amount of activity completion time (*et*)

[F3] Amount of activity time variance (*Var(t)*)

[F4] Variability of path duration across activities ($\Sigma\sigma^2_{\delta p}$; δ= {critical or near-critical})

[F5] Variability in the efficiency to expedite mean time across activities (mean$\sigma^2 i$ - $a_i\, m_i$)

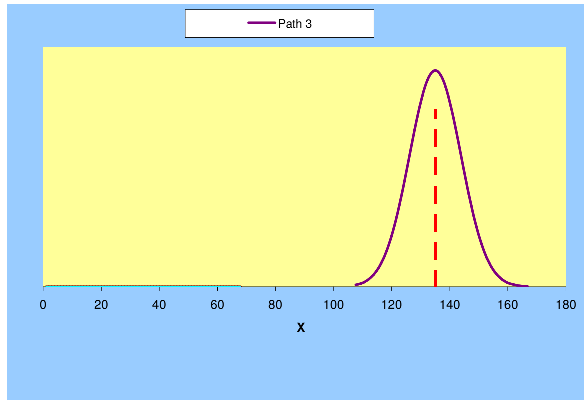
[F6] Variability in the efficiency to reduce variance across activities (mean$\sigma^2 i$ - $a_i\, v_i$)

[F7] Degree of dependence (none, unknown)

APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Baseline Case, Critical Path, (T=135)

Activity	a	m	b	mean	variance
L1-L2	5	10	15	10.0	2.778
L2-L3	30	45	90	50.0	100.000
L1-L3	40	60	110	65.0	136.111
L3-L7	20	30	40	30.0	11.111
L1-L4	25	30	35	30.0	2.778
L4-L6	30	40	80	45.0	69.444
L6-L7	25	30	35	30.0	2.778
L7-L11	25	30	35	30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 50.0 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		30.0

Average path time 135.0

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 0.000

P(T < Target date) = 0.500 50.0%

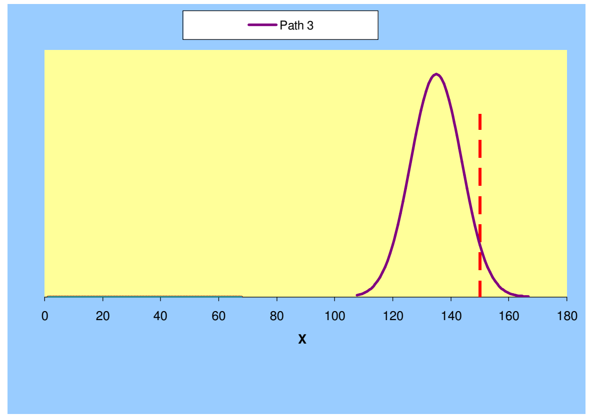
P(T < 135) = 50.0

APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Baseline Case, No Treatment, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 95.6 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		30.0

Average path time 135.0

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 1.701

P(T < Target date) = 0.956 95.6%

P(T < 150) = 95.6

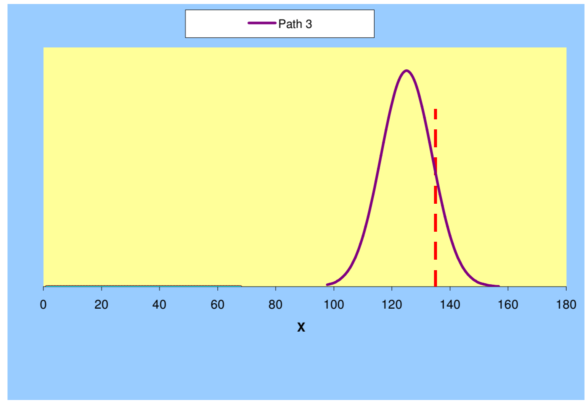
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 1, T=135

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	20	30	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
35.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 87.2 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		35.0
		30.0
		30.0

Average path time 125.0

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 1.134

P(T < Target date) = 0.872 87.2%

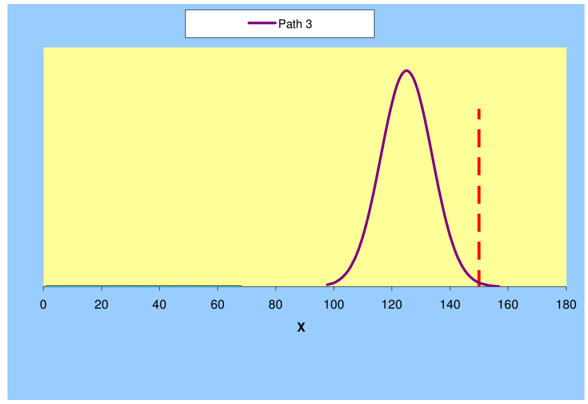
P(T < 135) = 87.2

APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 1, T=150

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	20	30	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
35.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 99.8 %

Paths

1	2	3
0		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		35.0
		30.0
		30.0

Average path time 125.0

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 2.835

P(T < Target date) = 0.998 99.8%

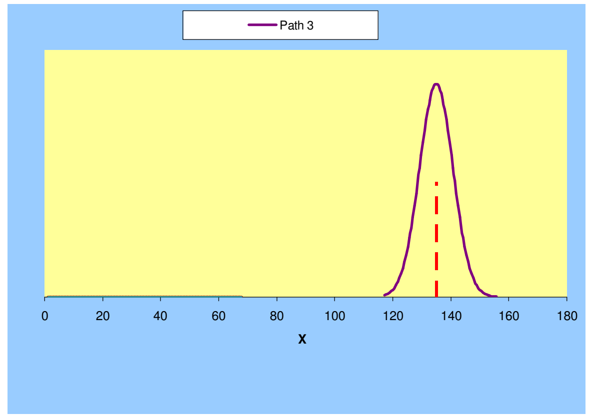
P(T < 150) = 99.8

APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 2, T=135

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	40	40	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	25.000
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 50.0 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		30.0

Average path time 135.0

Paths

1	2	3
		2.778
		25.000
		2.778
		2.778

Variance of path time 33.333

Z 0.000

P(T < Target date) = 0.500 50.0%

P(T < 135) = 50.0

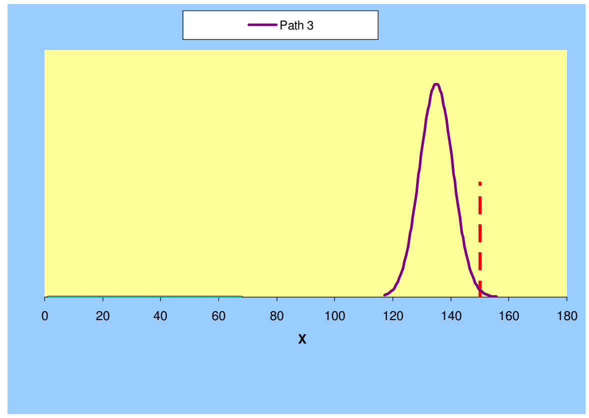
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 2, T=150

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	40	40	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	25.000
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 99.5 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		30.0

Average path time 135.0

Paths

1	2	3
		2.778
		25.000
		2.778
		2.778

Variance of path time 33.333

Z 2.598

P(T < Target date) = 0.995 99.5%

P(T < 150) = 99.5

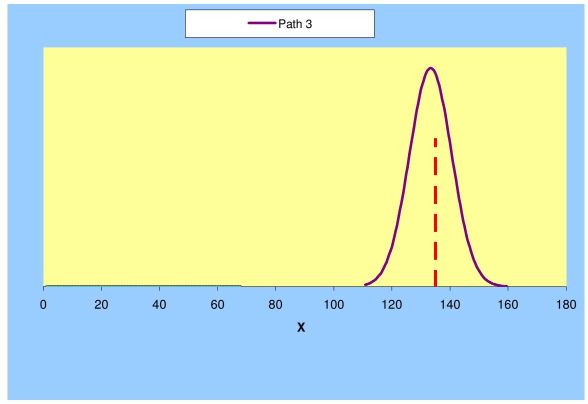
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 3, T=135

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
43.3	44.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 59.1 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		43.3
		30.0
		30.0

Average path time 133.3

Paths

1	2	3
		2.778
		44.444
		2.778
		2.778

Variance of path time 52.778

Z 0.229

P(T < Target date) = 0.591 59.1%

P(T < 135) = 59.1

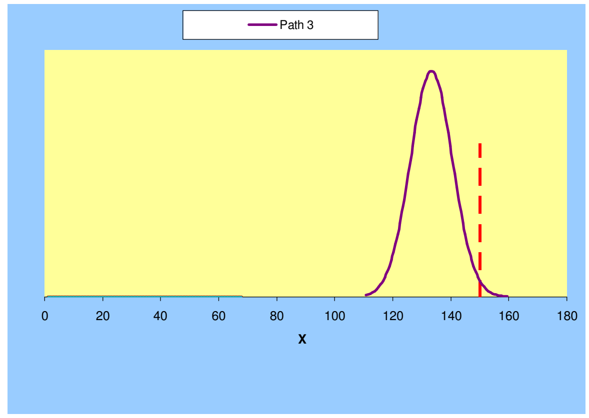
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 3, T=150

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
43.3	44.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 98.9 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		43.3
		30.0
		30.0

Average path time 133.3

Paths

1	2	3
		2.778
		44.444
		2.778
		2.778

Variance of path time 52.778

Z 2.294

P(T < Target date) = 0.989 98.9%

P(T < 150) = 98.9

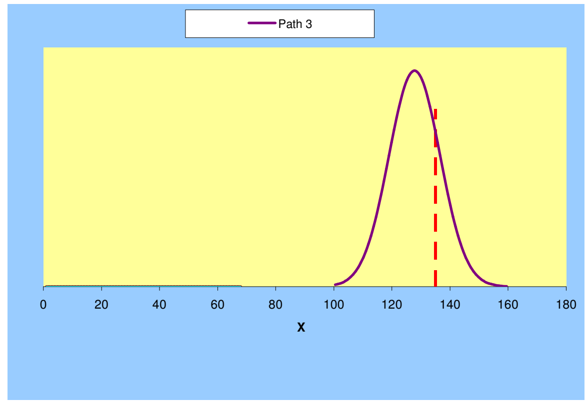
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 4, T=135

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	20	34.2	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
37.8	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 79.3 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		37.8
		30.0
		30.0

Average path time 127.8

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 0.816

P(T < Target date) = 0.793 79.3%

P(T < 135) = 79.3

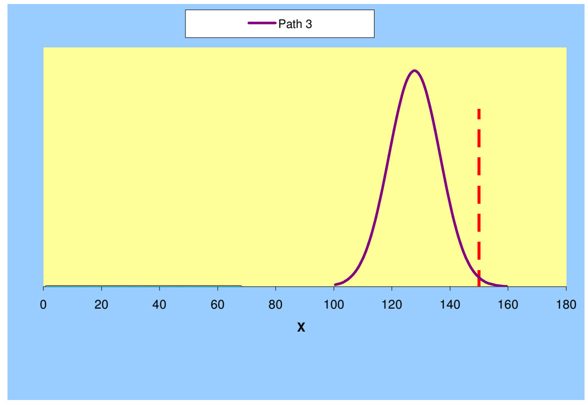
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 4, T=150

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	20	34.2	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
37.8	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 99.4 %

Paths

1	2	3
0		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		37.8
		30.0
		30.0

Average path time 127.8

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 2.517

P(T < Target date) = 0.994 99.4%

P(T < 150) = 99.4

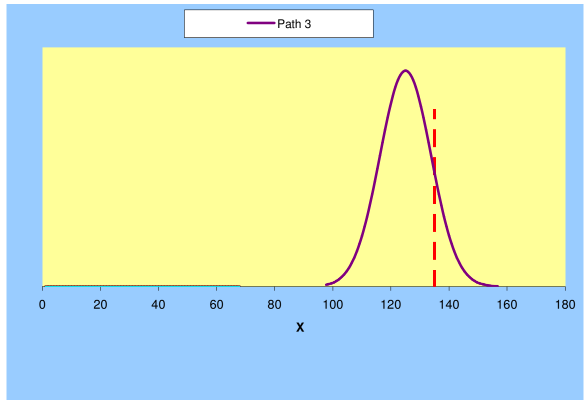
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 1, T=135

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	15	20	25

Treatment cells highlighted in yellow

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
20.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 87.2 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		20.0

Average path time 125.0

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 1.134

P(T < Target date) = 0.872 87.2%

P(T < 135) = 87.2

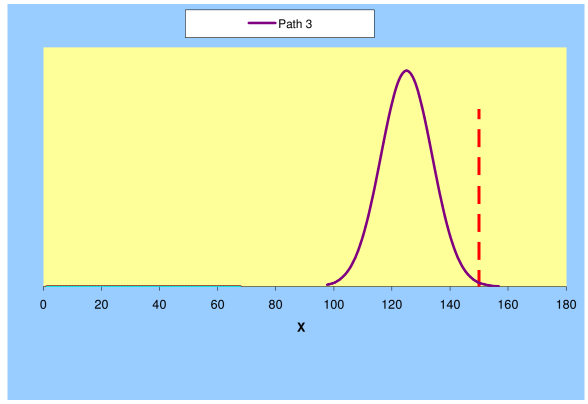
APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 1, T=150

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	15	20	25

Treatment cells highlighted in yellow

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
20.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 99.8 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		20.0

Average path time 125.0

Paths

1	2	3
		2.778
		69.444
		2.778
		2.778

Variance of path time 77.778

Z 2.835

P(T < Target date) = 0.998 99.8%

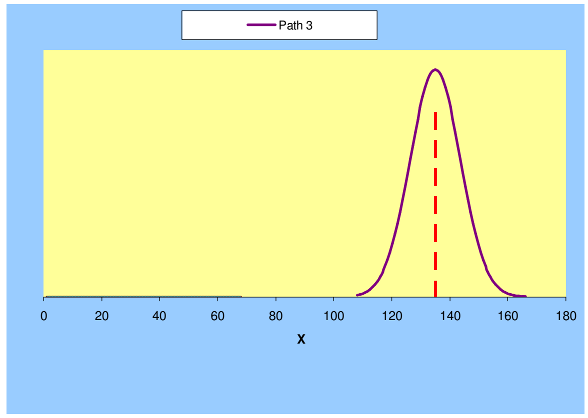
P(T < 150) = 99.8

APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 2, T=135

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	30	30	30

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	0.000



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 50.0 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		30.0

Average path time 135.0

Paths

1	2	3
		2.778
		69.444
		2.778
		0.000

Variance of path time 75.000

Z 0.000

P(T < Target date) = 0.500 50.0%

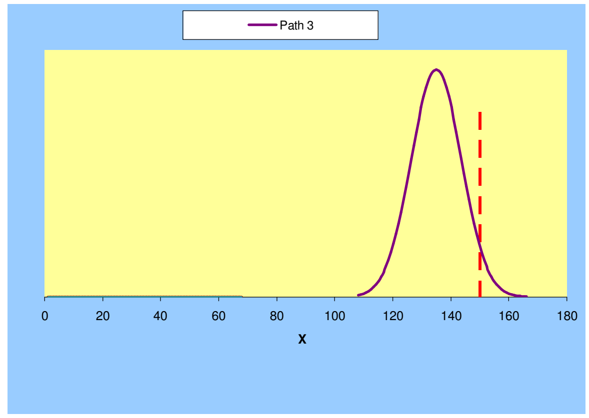
P(T < 135) = 50.0

APPENDIX B: COMPUTATIONAL RESULTS-First Analytical Approach using Microsoft Excel®

Analysis 1, Treatment 2, T=135

Activity	a	m	b	mean	variance
L1-L2	5	10	15	10.0	2.778
L2-L3	30	45	90	50.0	100.000
L1-L3	40	60	110	65.0	136.111
L3-L7	20	30	40	30.0	11.111
L1-L4	25	30	35	30.0	2.778
L4-L6	30	40	80	45.0	69.444
L6-L7	25	30	35	30.0	2.778
L7-L11	30	30	30	30.0	0.000

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 95.8 %

Paths

1	2	3
		L1-L4
		L4-L6
		L6-L7
		L7-L11

Paths

1	2	3
		30.0
		45.0
		30.0
		30.0

Average path time 135.0

Paths

1	2	3
		2.778
		69.444
		2.778
		0.000

Variance of path time 75.000

Z 1.732

P(T < Target date) = 0.958 95.8%

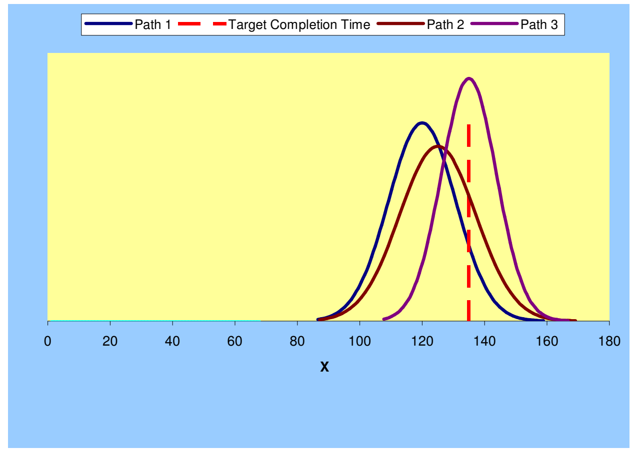
P(T < 150) = 95.8

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Baseline Case, Multiple Paths, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 36.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	1.389	0.8165	0.000	
P(T < Target date) =	0.918	0.793	0.500	36.4%

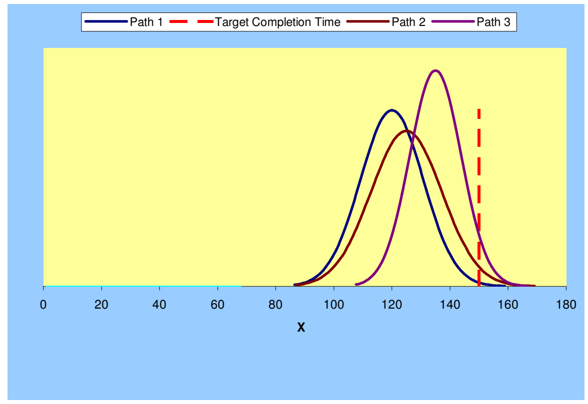
P(T < 135) = 36.4

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Baseline Case, Multiple Paths, No Treatment, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.3 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.777	2.0412	1.701	
P(T < Target date) =	0.997	0.979	0.956	93.3%

P(T < 150) = 93.3

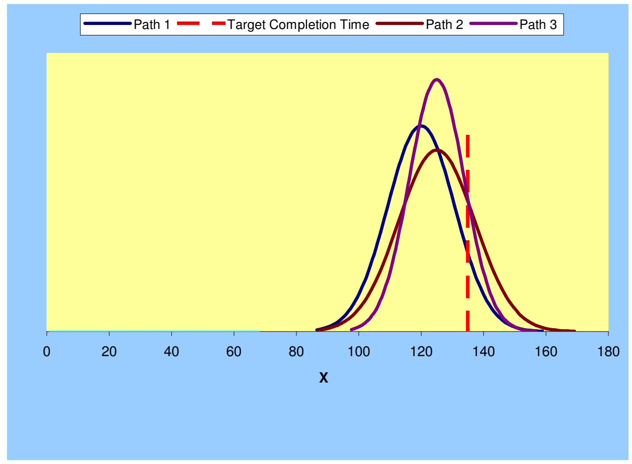
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S3, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	15	20	25
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
20.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 63.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	20.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 125.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	1.389	0.8165	1.134	
P(T < Target date) =	0.918	0.793	0.872	63.4%

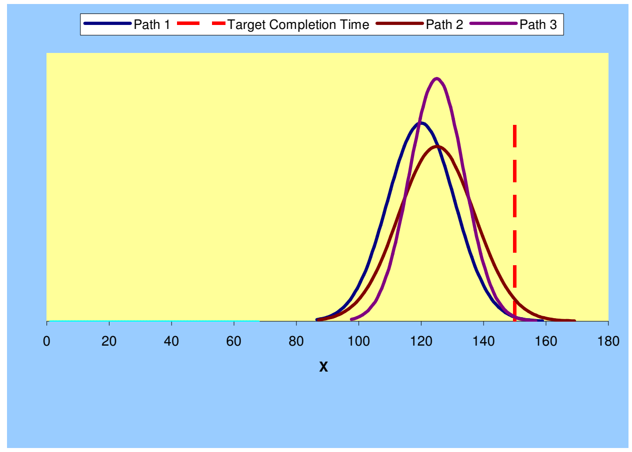
P(T < 135) = 63.4

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S3, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	15	20	25
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
20.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 97.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	20.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 125.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.777	2.0412	2.835	
P(T < Target date) =	0.997	0.979	0.998	97.4%

P(T < 150) = 97.4

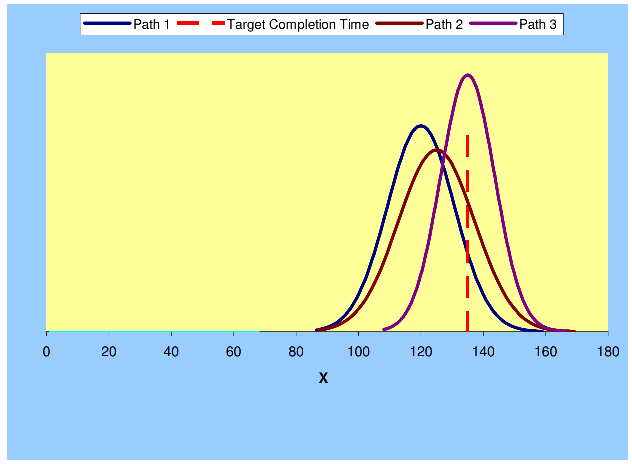
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S3, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	30	30	30
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	0.000
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 36.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	0.000
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	75.000	
Z	1.389	0.8165	0.000	
P(T < Target date) =	0.918	0.793	0.500	36.4%

P(T < 135) = 36.4

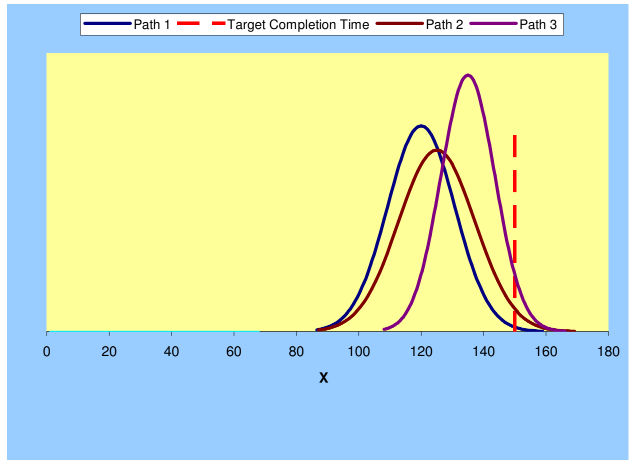
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S3, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	30	30	30
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	0.000
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	0.000
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	75.000	
Z	2.777	2.0412	1.732	
P(T < Target date) =	0.997	0.979	0.958	93.6%

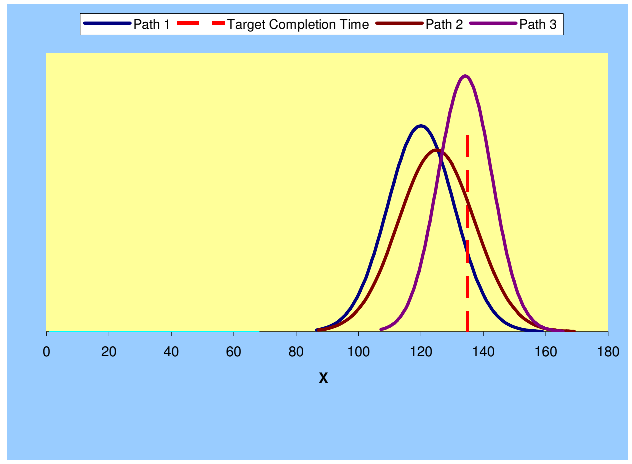
P(T < 150) = 93.6

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S3, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	30
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
29.2	0.694
45.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	29.2
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 134.2

Paths

1	2	3
2.778	136.111	0.694
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	75.694	
Z	1.389	0.8165	0.096	
P(T < Target date) =	0.918	0.793	0.538	39.2%

P(T < 135) = 39.2

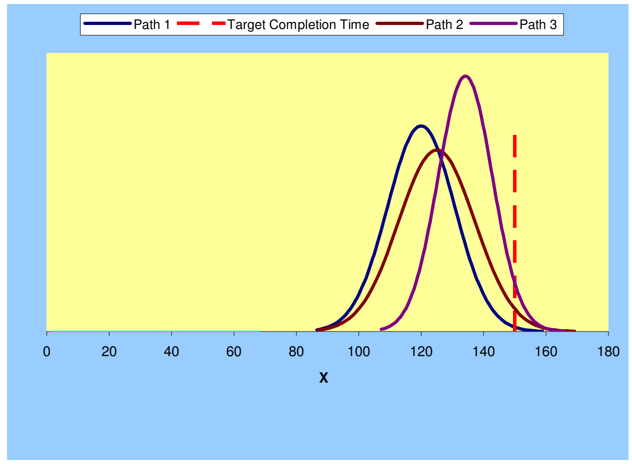
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S3, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	30
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
29.2	0.694
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 94.3 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	29.2
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time	120.0	125.0	134.2

Paths

1	2	3
2.778	136.111	0.694
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	75.694	
Z	2.777	2.0412	1.820	
P(T < Target date) =	0.997	0.979	0.966	94.3%

P(T < 150) = 94.3

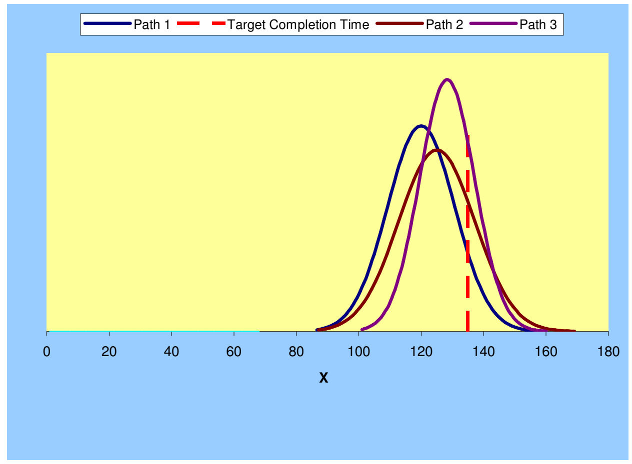
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S3, Treatment 4, [Factor: F5], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	15	25	25
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
23.3	2.778
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 56.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	23.3
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 128.3

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	1.389	0.8165	0.756	
P(T < Target date) =	0.918	0.793	0.775	56.4%

P(T < 135) = 56.4

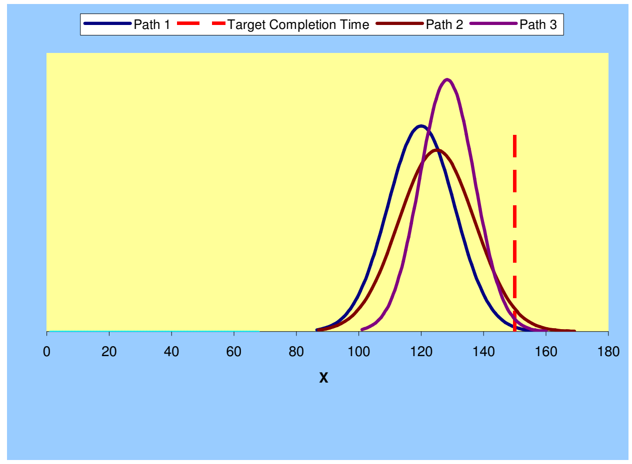
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S3, Treatment 4, [Factor: F5], (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	15	25	25
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
23.3	2.778
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 97.0 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	65.0	23.3
	50.0	30.0	45.0
	30.0	30.0	30.0
	30.0		30.0
Average path time	120.0	125.0	128.3

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	11.111	69.444	
	11.111	2.778	2.778	
	2.778		2.778	
Variance of path time	116.667	150.000	77.778	
Z	2.777	2.0412	2.457	
P(T < Target date) =	0.997	0.979	0.993	97.0%

P(T < 150) = 97.0

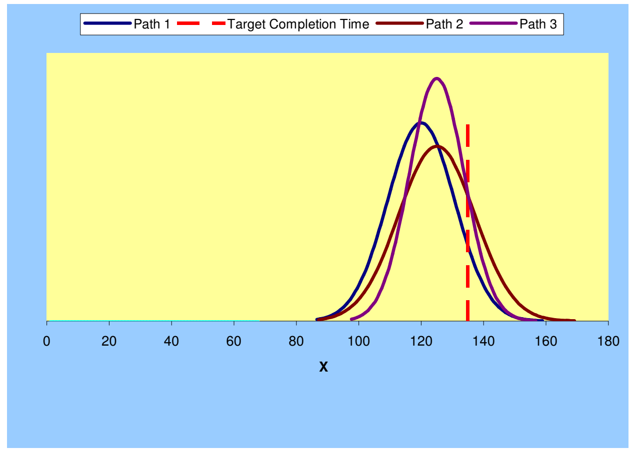
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	20	30	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
35.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 63.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	35.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 125.0

Paths

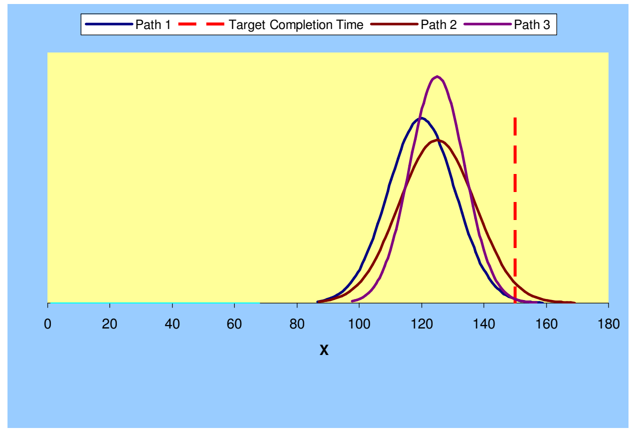
1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	1.389	0.8165	1.134	
P(T < Target date) =	0.918	0.793	0.872	63.4%

P(T < 135) = 63.4

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S1, (T=150)



Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	20	30	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
35.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 97.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	65.0	30.0
	50.0	30.0	35.0
	30.0	30.0	30.0
	30.0		30.0
Average path time	120.0	125.0	125.0

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	11.111	69.444	
	11.111	2.778	2.778	
	2.778		2.778	
Variance of path time	116.667	150.000	77.778	
Z	2.777	2.0412	2.835	
P(T < Target date) =	0.997	0.979	0.998	97.4%

P(T < 150) = 97.4

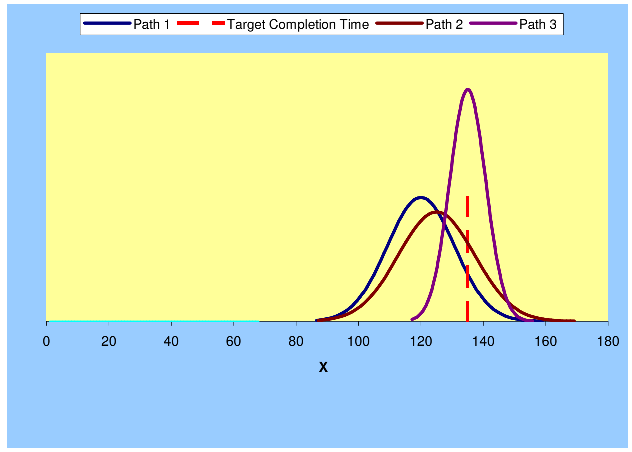
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	40	40	70
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	25.000
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 36.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	25.000
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	33.333	
Z	1.389	0.8165	0.000	
P(T < Target date) =	0.918	0.793	0.500	36.4%

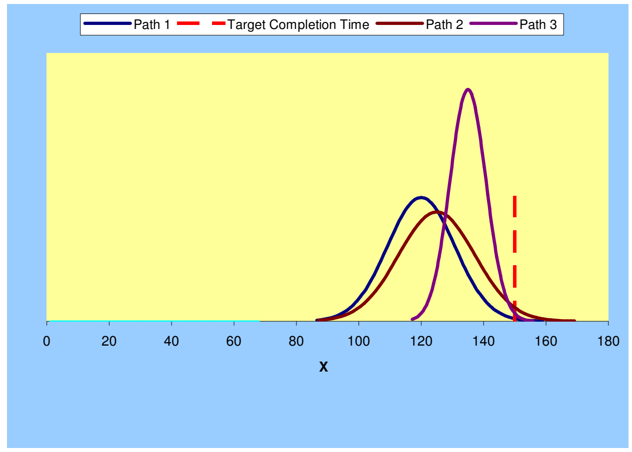
P(T < 135) = 36.4

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S1, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	40	40	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	25.000
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 97.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	25.000
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	33.333	
Z	2.777	2.0412	2.598	
P(T < Target date) =	0.997	0.979	0.995	97.2%

P(T < 150) = 97.2

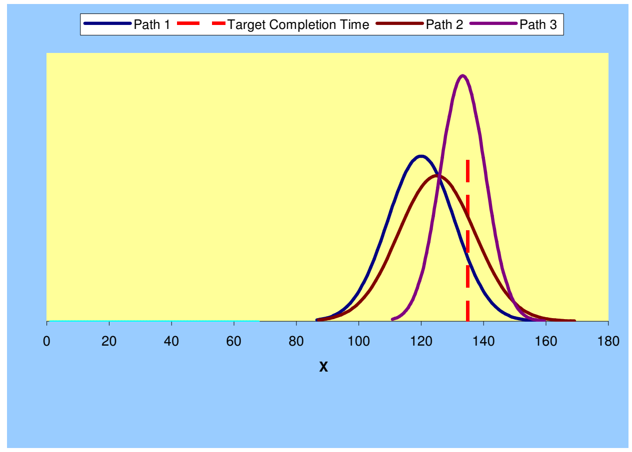
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
43.3	44.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 43.0 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	43.3
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 133.3

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	44.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	52.778	
Z	1.389	0.8165	0.229	
P(T < Target date) =	0.918	0.793	0.591	43.0%

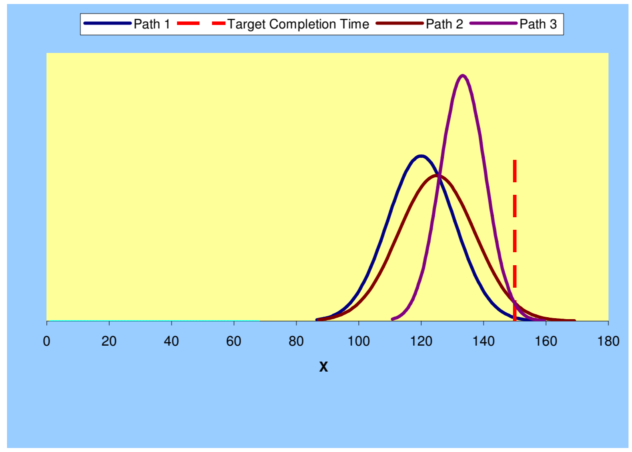
P(T < 135) = 43.0

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S1, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	70
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
43.3	44.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 96.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	65.0	30.0
	50.0	30.0	43.3
	30.0	30.0	30.0
	30.0		30.0
Average path time	120.0	125.0	133.3

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	11.111	44.444	
	11.111	2.778	2.778	
	2.778		2.778	
Variance of path time	116.667	150.000	52.778	
Z	2.777	2.0412	2.294	
P(T < Target date) =	0.997	0.979	0.989	96.6%

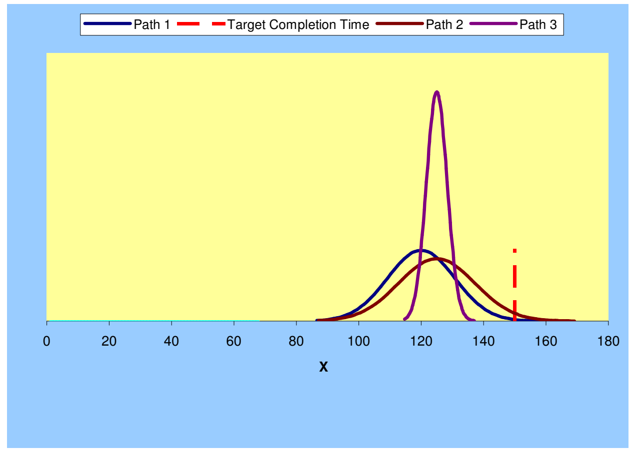
P(T < 150) = 96.6

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Manipulation, VARcp, Treatment 1, T=150

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	35	32.5	45
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
35.0	2.778
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 97.7 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	35.0
30.0	30.0	30.0
30.0		30.0

Average path time	120.0	125.0	125.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	2.778
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	11.111	
Z	2.777	2.0412	7.500	
P(T < Target date) =	0.997	0.979	1.000	97.7%

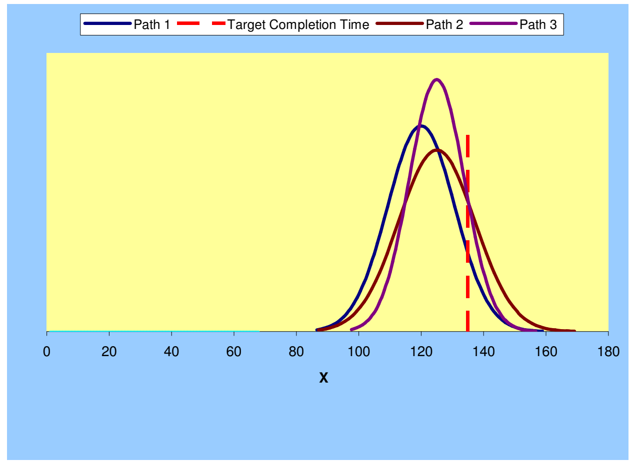
P(T < 150) = 97.7

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S3, (T=135)

Activity	a	m	b	mean	variance
L1-L2	5	10	15	10.0	2.778
L2-L3	30	45	90	50.0	100.000
L1-L3	40	60	110	65.0	136.111
L3-L7	20	30	40	30.0	11.111
L1-L4	25	30	35	30.0	2.778
L4-L6	30	40	80	45.0	69.444
L6-L7	15	20	25	20.0	2.778
L7-L11	25	30	35	30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 63.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	20.0
30.0		30.0

Average path time 120.0 125.0 125.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	1.389	0.8165	1.134	
P(T < Target date) =	0.918	0.793	0.872	63.4%

P(T < 135) = 63.4

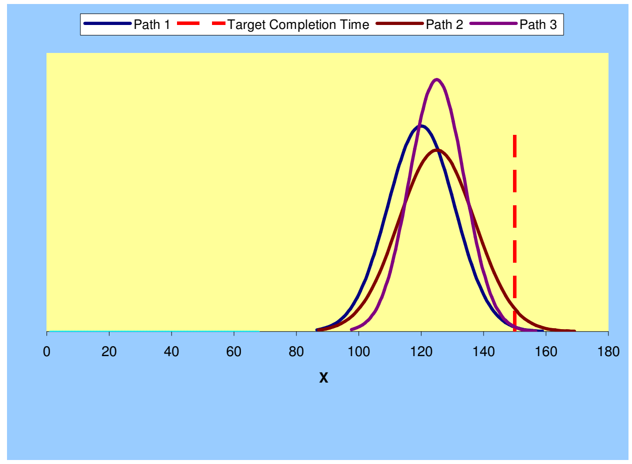
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S3, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	15	20	25
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
20.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 97.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	20.0
30.0		30.0

Average path time 120.0 125.0 125.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.777	2.0412	2.835	
P(T < Target date) =	0.997	0.979	0.998	97.4%

P(T < 150) = 97.4

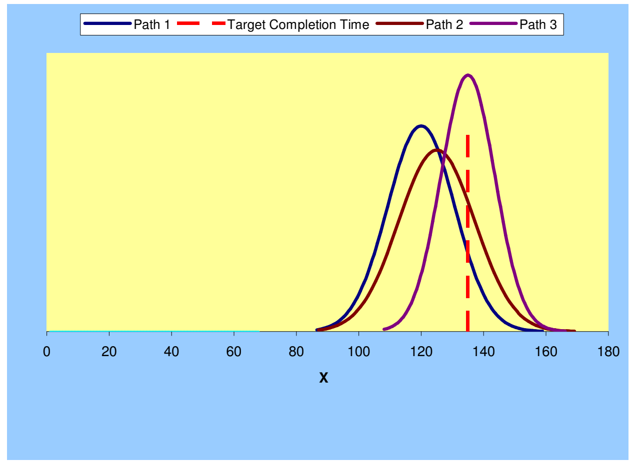
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S3, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	30	30	30
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	0.000
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 36.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	0.000
2.778		2.778

Variance of path time	116.667	150.000	75.000	
Z	1.389	0.8165	0.000	
P(T < Target date) =	0.918	0.793	0.500	36.4%

P(T < 135) = 36.4

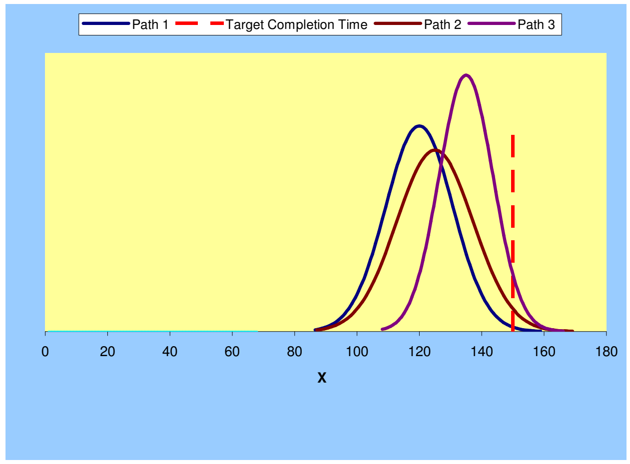
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S3, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	30	30	30
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	0.000
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

	1	2	3
Average path time	120.0	125.0	135.0

Paths

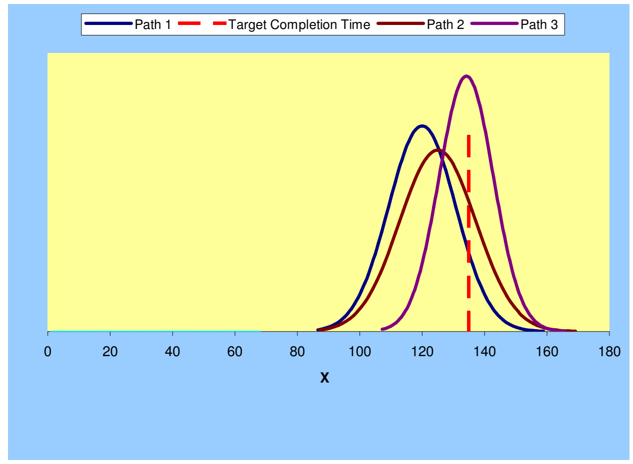
1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	0.000
2.778		2.778

	1	2	3	
Variance of path time	116.667	150.000	75.000	
Z	2.777	2.0412	1.732	
P(T < Target date) =	0.997	0.979	0.958	93.6%

P(T < 150) = 93.6

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S3, (T=135)



Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	30
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
29.2	0.694
30.0	2.778

Treatment cells highlighted in yellow/shaded

What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	29.2
30.0		30.0

Average path time 120.0 125.0 134.2

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	0.694
2.778		2.778

Variance of path time	116.667	150.000	75.694	
Z	1.389	0.8165	0.096	
P(T < Target date) =	0.918	0.793	0.538	39.2%

P(T < 135) = 39.2

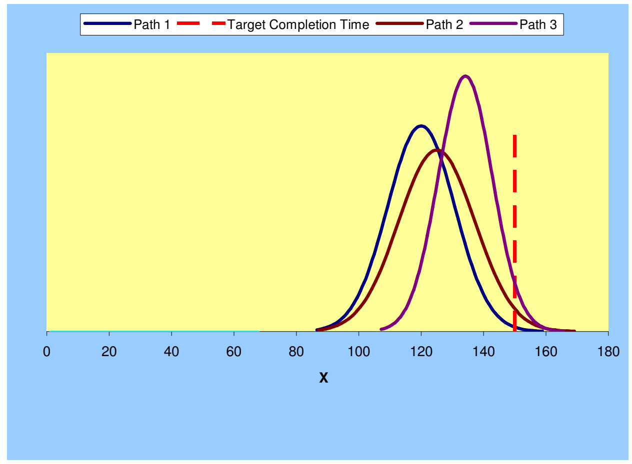
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S3, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	30
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
29.2	0.694
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 94.3 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	29.2
30.0		30.0

Average path time 120.0 125.0 134.2

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	0.694
2.778		2.778

Variance of path time	116.667	150.000	75.694	
Z	2.777	2.0412	1.820	
P(T < Target date) =	0.997	0.979	0.966	94.3%

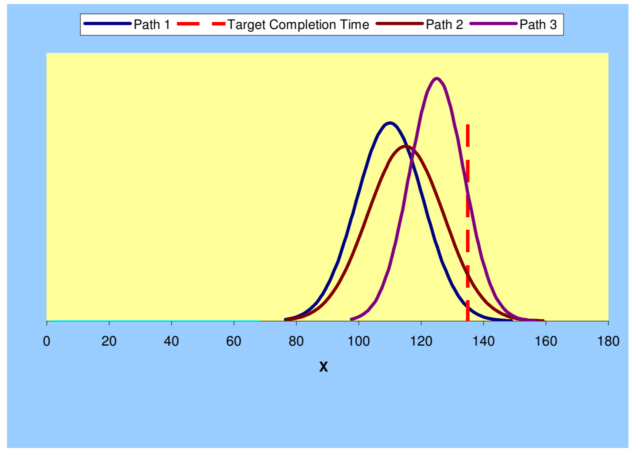
P(T < 150) = 94.3

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6, Treatment 1, [Factor: F2], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	15	20	25

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
20.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 81.8 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	65.0	30.0
	50.0	30.0	45.0
	30.0	20.0	30.0
	20.0		20.0
Average path time	110.0	115.0	125.0

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	11.111	69.444	
	11.111	2.778	2.778	
	2.778		2.778	
Variance of path time	116.667	150.000	77.778	
Z	2.315	1.6330	1.134	
P(T < Target date) =	0.990	0.949	0.872	81.8%

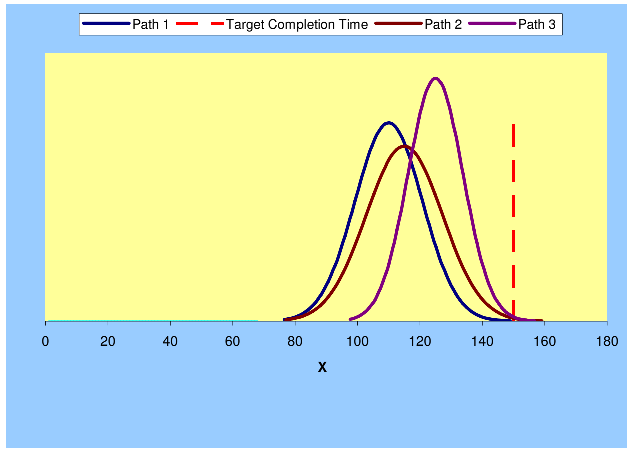
P(T < 135) = 81.8

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6, Treatment 1, [Factor: F2], (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	15	20	25

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
20.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 99.5 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	20.0	30.0
20.0		20.0

Average path time	110.0	115.0	125.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	3.703	2.8577	2.835	
P(T < Target date) =	1.000	0.998	0.998	99.5%

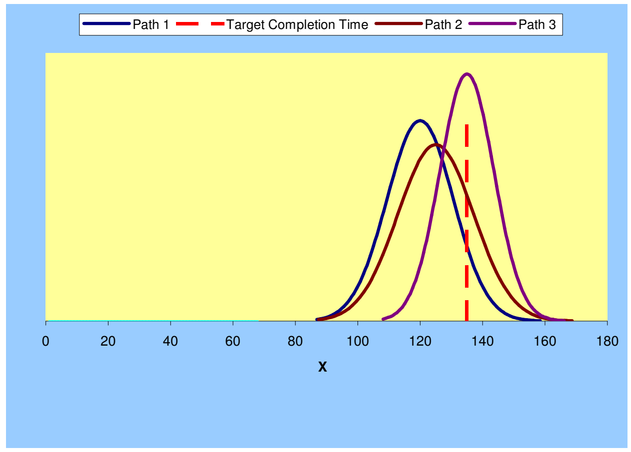
P(T < 150) = 99.5

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6, Treatment 2, [Factor: F2], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	30	30	30

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	0.000



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 36.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	0.000	2.778
0.000		0.000

Variance of path time	113.889	147.222	75.000	
Z	1.406	0.8242	0.000	
P(T < Target date) =	0.920	0.795	0.500	36.6%

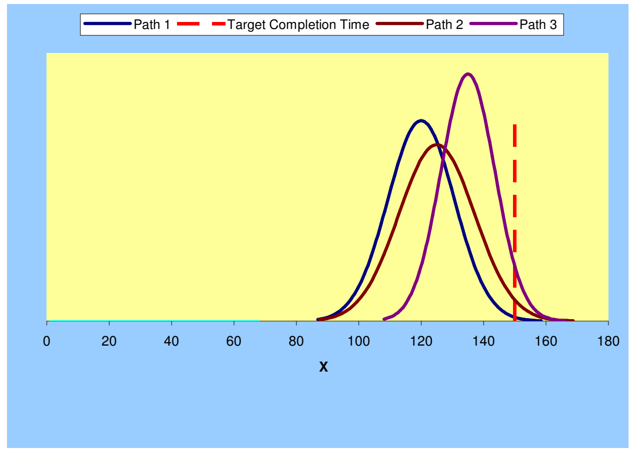
P(T < 135) = 36.6

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6, Treatment 2, [Factor: F2], (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	30	30	30

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	0.000



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.7 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	65.0	30.0
	50.0	30.0	45.0
	30.0	30.0	30.0
	30.0		30.0
Average path time	120.0	125.0	135.0

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	11.111	69.444	
	11.111	0.000	2.778	
	0.000		0.000	
Variance of path time	113.889	147.222	75.000	
Z	2.811	2.0604	1.732	
P(T < Target date) =	0.998	0.980	0.958	93.7%

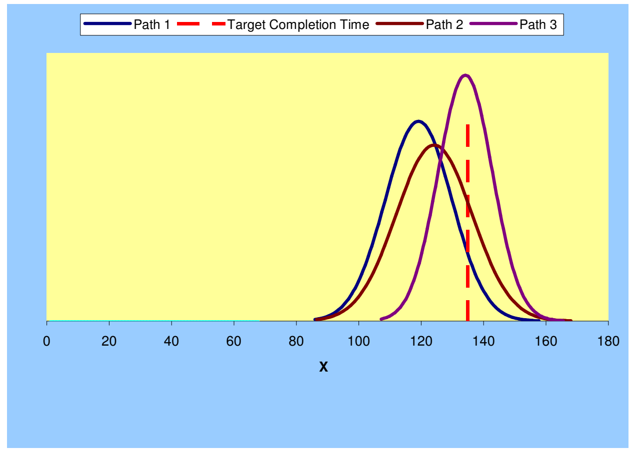
P(T < 150) = 93.7

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6, Treatment 3, [Factor: F2], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	30

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
29.2	0.694



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 40.7 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	29.2	30.0
29.2		29.2

Average path time	119.2	124.2	134.2

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	0.694	2.778
0.694		0.694

Variance of path time	114.583	147.917	75.694	
Z	1.479	0.8907	0.096	
P(T < Target date) =	0.930	0.813	0.538	40.7%

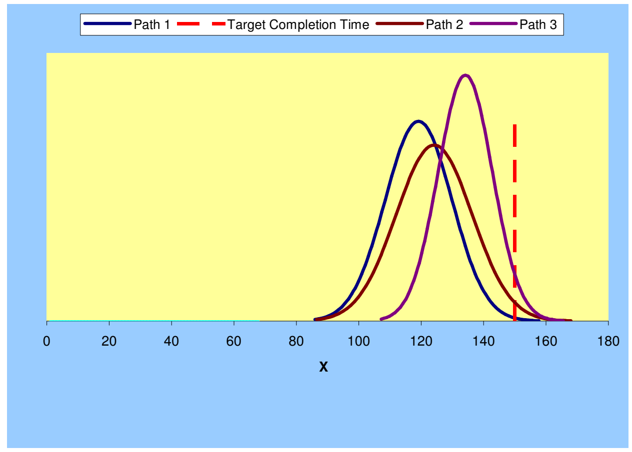
P(T < 135) = 40.7

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6, Treatment 3, [Factor: F2], (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	30

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
29.2	0.694



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 94.7 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	29.2	30.0
29.2		29.2

Average path time	119.2	124.2	134.2

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	0.694	2.778
0.694		0.694

Variance of path time	114.583	147.917	75.694	
Z	2.880	2.1241	1.820	
P(T < Target date) =	0.998	0.983	0.966	94.7%

P(T < 150) = 94.7

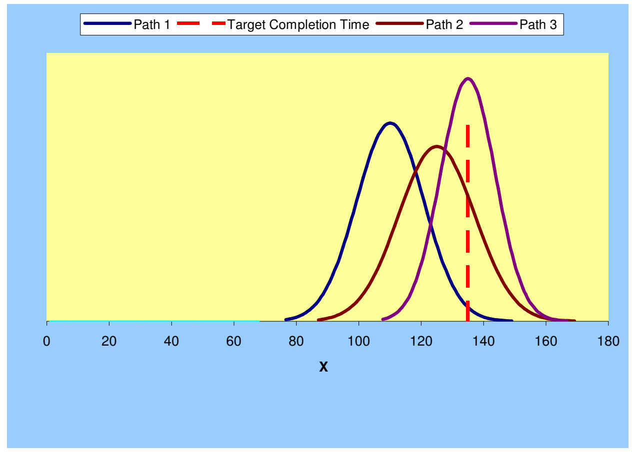
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S4/NC, (T=135)

Activity	a	m	b
L1-L2	0	0	0
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
0.1	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
0.1	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 110.1 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.305	0.8165	0.000	
P(T < Target date) =	0.989	0.793	0.500	39.2%

P(T < 135) = 39.2

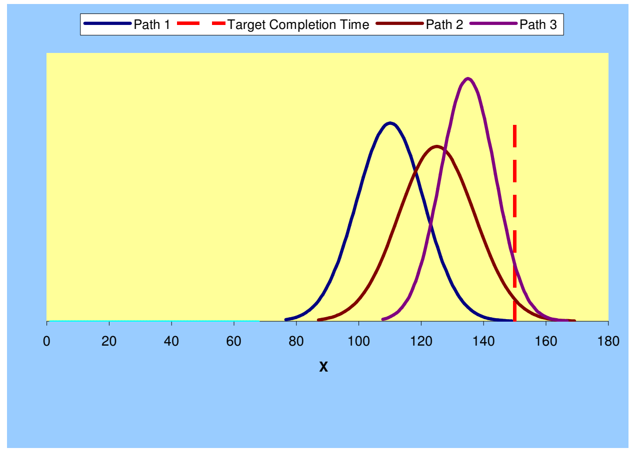
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S4/NC, (T=150)

Activity	a	m	b
L1-L2	0	0	0
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
0.1	2.778
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
0.1	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 110.1 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	3.694	2.0412	1.701	
P(T < Target date) =	1.000	0.979	0.956	93.6%

P(T < 150) = 93.6

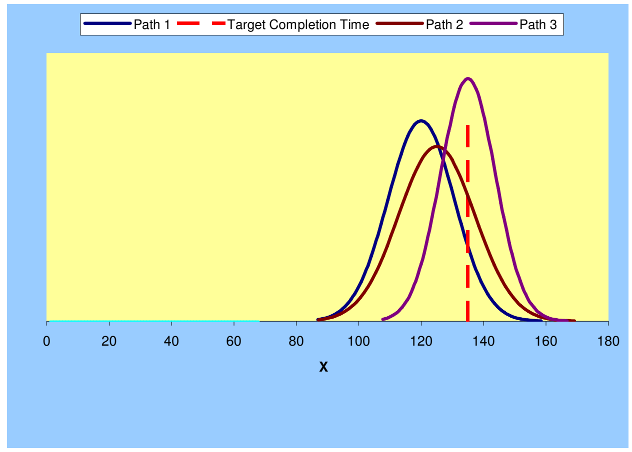
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S4/NC, (T=135)

Activity	a	m	b
L1-L2	0	15	0
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	0.000
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 36.5 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
0.000	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	113.889	150.000	77.778	
Z	1.406	0.8165	0.000	
P(T < Target date) =	0.920	0.793	0.500	36.5%

P(T < 135) = 36.5

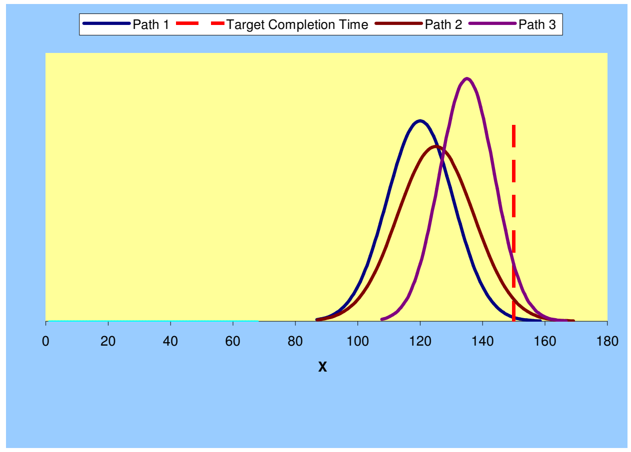
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S4/NC, (T=150)

Activity	a	m	b
L1-L2	0	15	0
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	0.000
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
0.000	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	113.889	150.000	77.778	
Z	2.811	2.0412	1.701	
P(T < Target date) =	0.998	0.979	0.956	93.4%

P(T < 150) = 93.4

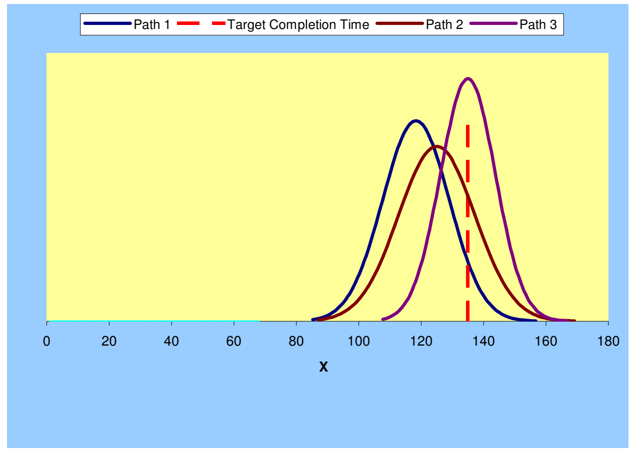
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S4/NC, (T=135)

Activity	a	m	b
L1-L2	5	10	5
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
8.3	0.000
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 37.3 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
8.3	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 118.3 125.0 135.0

Paths

1	2	3
0.000	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	113.889	150.000	77.778	
Z	1.562	0.8165	0.000	
P(T < Target date) =	0.941	0.793	0.500	37.3%

P(T < 135) = 37.3

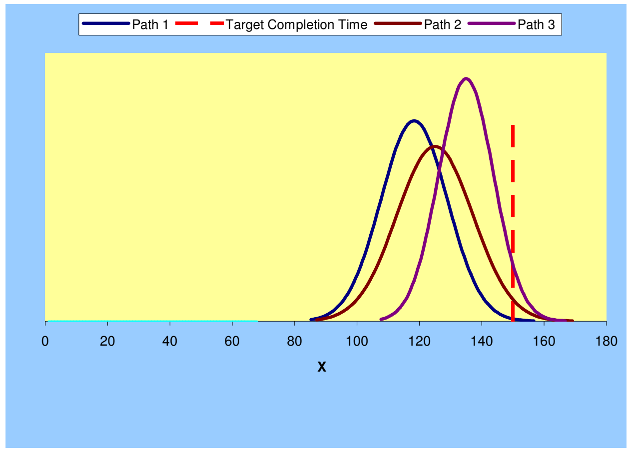
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S4/NC, (T=150)

Activity	a	m	b
L1-L2	5	10	5
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
8.3	0.000
50.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.4 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
8.3	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 118.3 125.0 135.0

Paths

1	2	3
0.000	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

	1	2	3	
Variance of path time	113.889	150.000	77.778	
Z	2.967	2.0412	1.701	
P(T < Target date) =	0.998	0.979	0.956	93.4%

P(T < 150) = 93.4

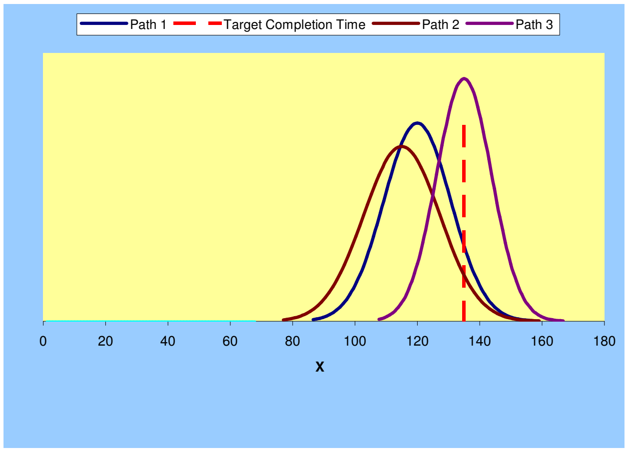
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S2, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	30	50	100
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
50.0	100.000
55.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 43.5 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	55.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time	120.0	115.0	135.0	

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	1.389	1.6330	0.000	
P(T < Target date) =	0.918	0.949	0.500	43.5%

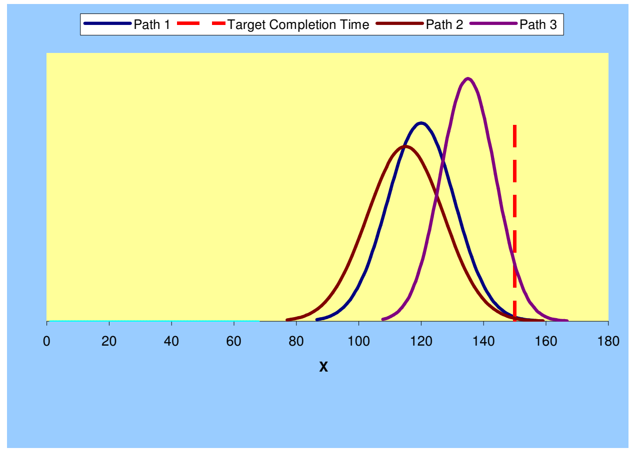
P(T < 135) = 43.5

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S2, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	30	50	100
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
55.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 95.1 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	55.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 115.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.777	2.8577	1.701	
P(T < Target date) =	0.997	0.998	0.956	95.1%

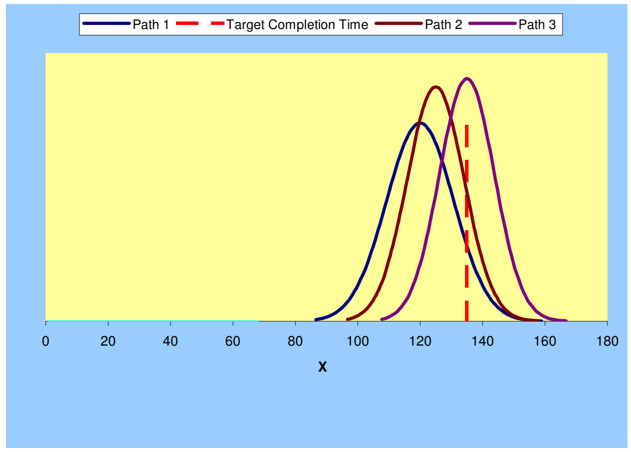
P(T < 150) = 95.1

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S2, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	50	60	100
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	69.444
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

	1	2	3
Average path time	120.0	125.0	135.0

Paths

1	2	3
2.778	69.444	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

	1	2	3	
Variance of path time	116.667	83.333	77.778	
Z	1.389	1.0954	0.000	
P(T < Target date) =	0.918	0.863	0.500	39.6%

P(T < 135) = 39.6

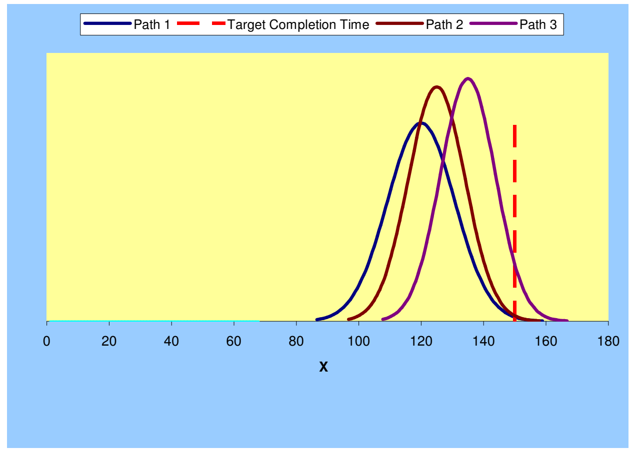
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S2, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	50	60	100
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	69.444
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 95.0 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	69.444	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	83.333	77.778	
Z	2.777	2.7386	1.701	
P(T < Target date) =	0.997	0.997	0.956	95.0%

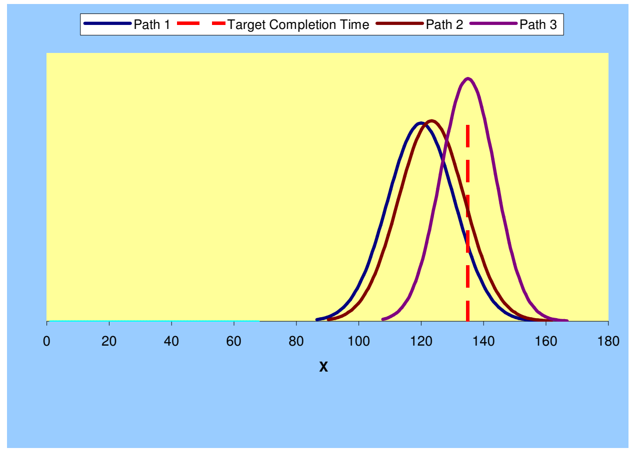
P(T < 150) = 95.0

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S2, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	100
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
63.3	100.000
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	63.3	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 123.3 135.0

Paths

1	2	3
2.778	100.000	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	113.889	77.778	
Z	1.389	1.0932	0.000	
P(T < Target date) =	0.918	0.863	0.500	39.6%

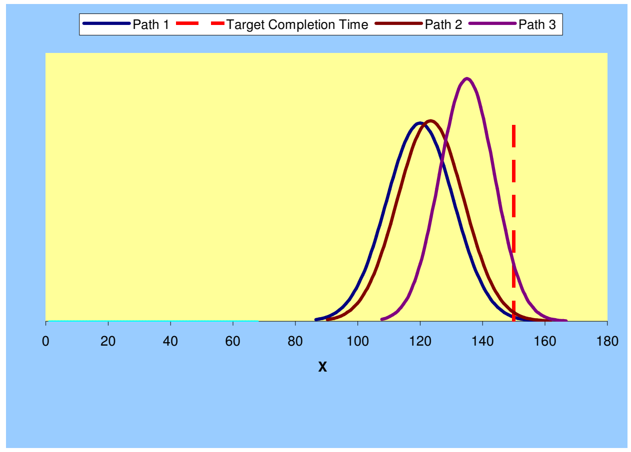
P(T < 135) = 39.6

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S2, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	100
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
63.3	100.000
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



Treatment cells highlighted in yellow/shaded

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 94.7 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	63.3	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time	120.0	123.3	135.0

Paths

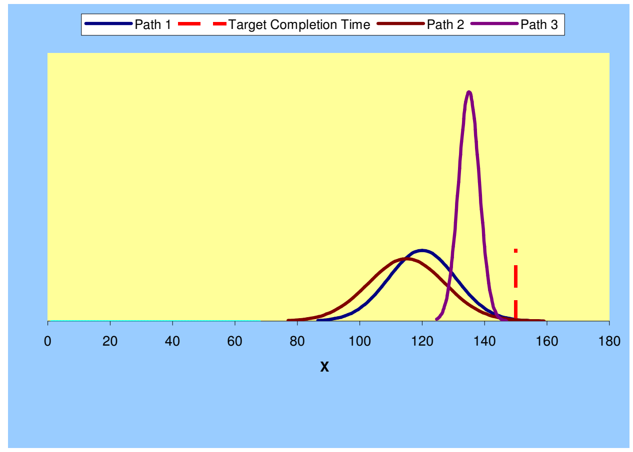
1	2	3
2.778	100.000	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	113.889	77.778	
Z	2.777	2.4988	1.701	
P(T < Target date) =	0.997	0.994	0.956	94.7%

P(T < 150) = 94.7

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Manipulation, VARcp, Treatment 1, T=150



Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	45	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	35	47.5	45
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
55.0	136.111
30.0	11.111
30.0	2.778
45.0	2.778
30.0	2.778
30.0	2.778

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 99.5 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	55.0	30.0
	50.0	30.0	45.0
	30.0	30.0	30.0
	30.0		30.0
Average path time	120.0	115.0	135.0

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	11.111	2.778	
	11.111	2.778	2.778	
	2.778		2.778	
Variance of path time	116.667	150.000	11.111	
Z	2.777	2.8577	4.500	
P(T < Target date) =	0.997	0.998	1.000	99.5%

P(T < 150) = 99.5

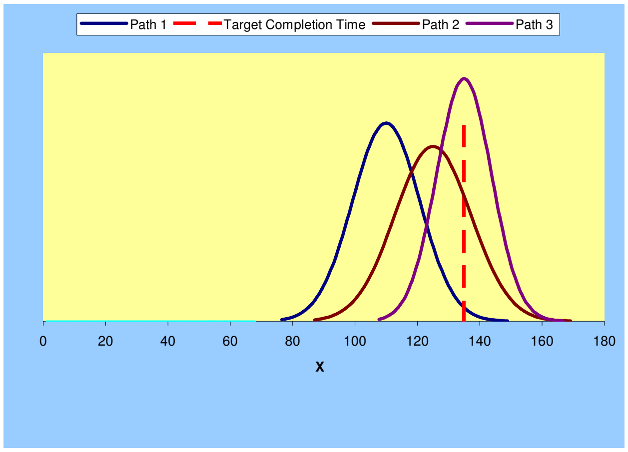
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S3/NC, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	20	35	80
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
40.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
40.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 110.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.315	0.8165	0.000	
P(T < Target date) =	0.990	0.793	0.500	39.2%

P(T < 135) = 39.2

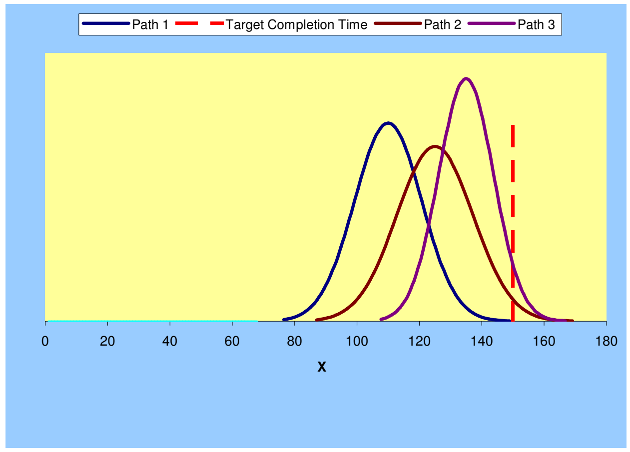
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 1, Scenario S3/NC, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	20	35	80
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
40.0	100.000
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
40.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time	110.0	125.0	135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	3.703	2.0412	1.701	
P(T < Target date) =	1.000	0.979	0.956	93.6%

P(T < 150) = 93.6

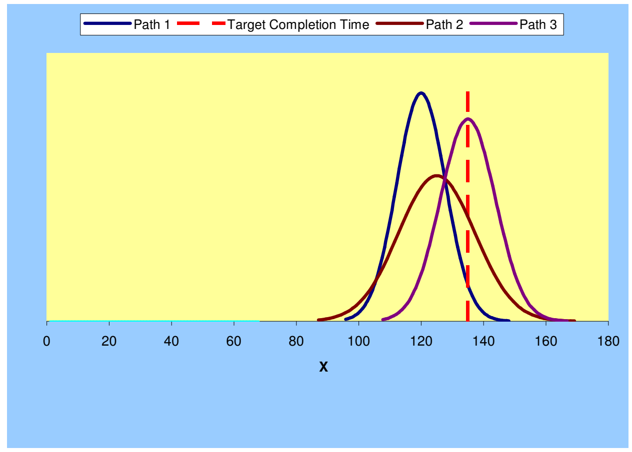
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S3/NC, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	40	45	80
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	44.444
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 38.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 120.0 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
44.444	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	61.111	150.000	77.778	
Z	1.919	0.8165	0.000	
P(T < Target date) =	0.972	0.793	0.500	38.6%

P(T < 135) = 38.6

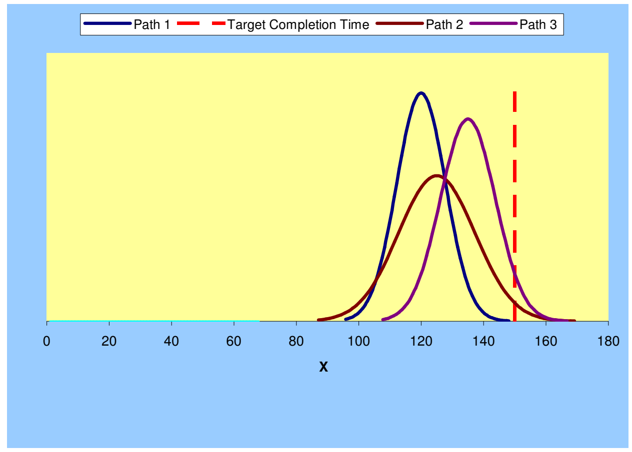
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 2, Scenario S3/NC, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	40	45	80
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	44.444
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778

Treatment cells highlighted in yellow/shaded



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time	120.0	125.0	135.0

Paths

1	2	3
2.778	136.111	2.778
44.444	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	61.111	150.000	77.778	
Z	3.838	2.0412	1.701	
P(T < Target date) =	1.000	0.979	0.956	93.6%

P(T < 150) = 93.6

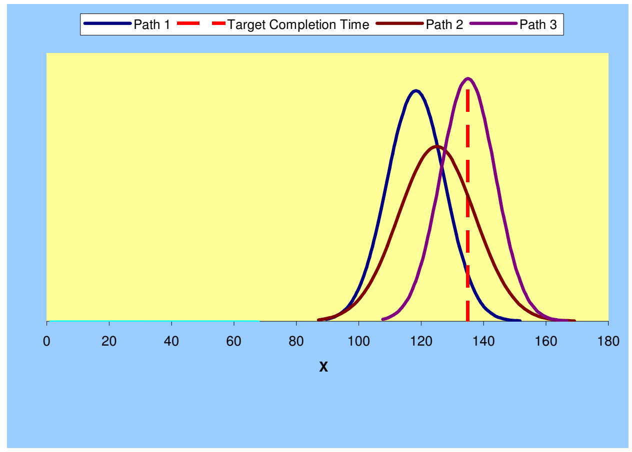
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S3/NC, (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	80
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
48.3	69.444
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 38.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
48.3	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time 118.3 125.0 135.0

Paths

1	2	3
2.778	136.111	2.778
69.444	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	86.111	150.000	77.778	
Z	1.796	0.8165	0.000	
P(T < Target date) =	0.964	0.793	0.500	38.2%

P(T < 135) = 38.2

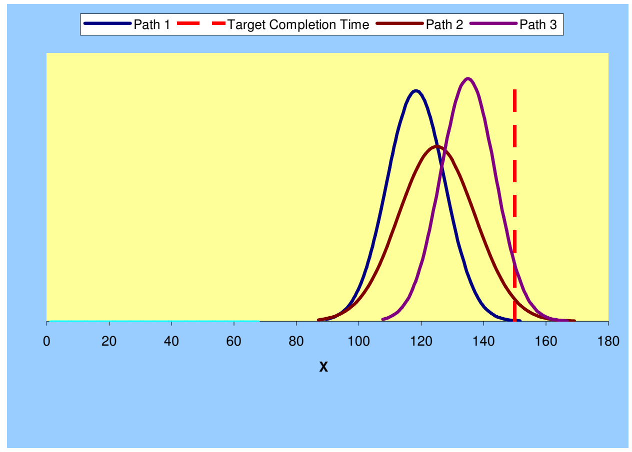
APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Treatment 3, Scenario S3/NC, (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	80
L1-L3	40	60	110
L3-L7	20	30	40
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

Treatment cells highlighted in yellow/shaded

mean	variance
10.0	2.778
48.3	69.444
65.0	136.111
30.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.6 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
48.3	30.0	45.0
30.0	30.0	30.0
30.0		30.0

Average path time	118.3	125.0	135.0

Paths

1	2	3
2.778	136.111	2.778
69.444	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	86.111	150.000	77.778	
Z	3.413	2.0412	1.701	
P(T < Target date) =	1.000	0.979	0.956	93.6%

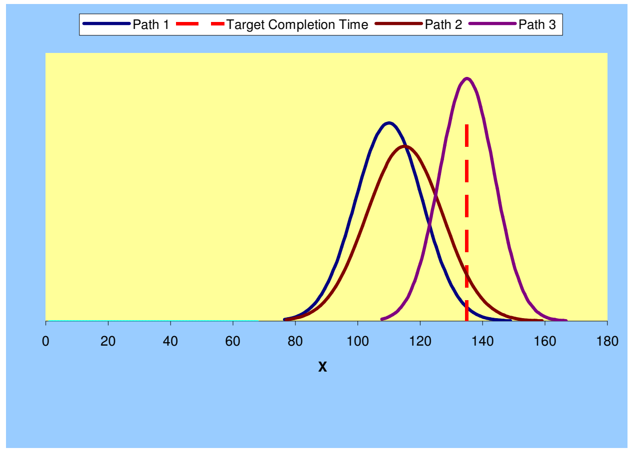
P(T < 150) = 93.6

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6A, Treatment 1, [Factor: F3, F6], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	10	20	30
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
20.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 46.9 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	20.0	45.0
20.0	30.0	30.0
30.0		30.0

Average path time 110.0 115.0 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	2.315	1.6330	0.000	
P(T < Target date) =	0.990	0.949	0.500	46.9%

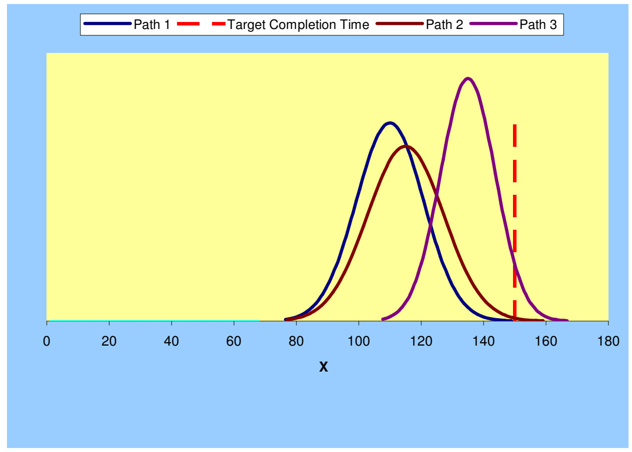
P(T < 135) = 46.9

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6A, Treatment 1, [Factor: F3, F6], (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	10	20	30
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
20.0	11.111
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 95.3 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	20.0	45.0
20.0	30.0	30.0
30.0		30.0

Average path time	110.0	115.0	135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	11.111	69.444
11.111	2.778	2.778
2.778		2.778

Variance of path time	116.667	150.000	77.778	
Z	3.703	2.8577	1.701	
P(T < Target date) =	1.000	0.998	0.956	95.3%

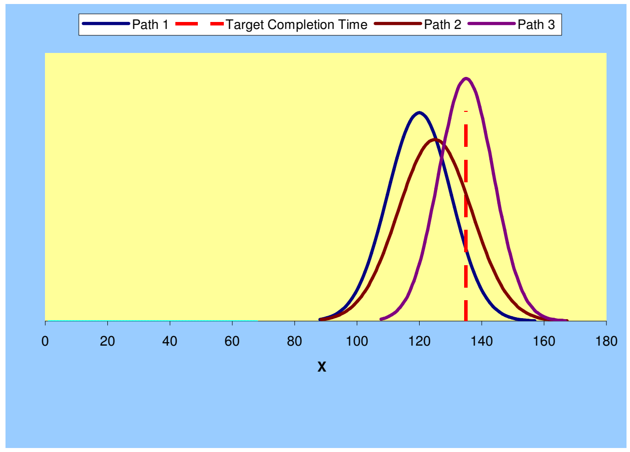
P(T < 150) = 95.3

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6A, Treatment 2, [Factor: F3, F6], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	30	30	30
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	0.000
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 37.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	30.0	45.0
30.0	30.0	30.0
30.0		30.0

	1	2	3
Average path time	120.0	125.0	135.0

Paths

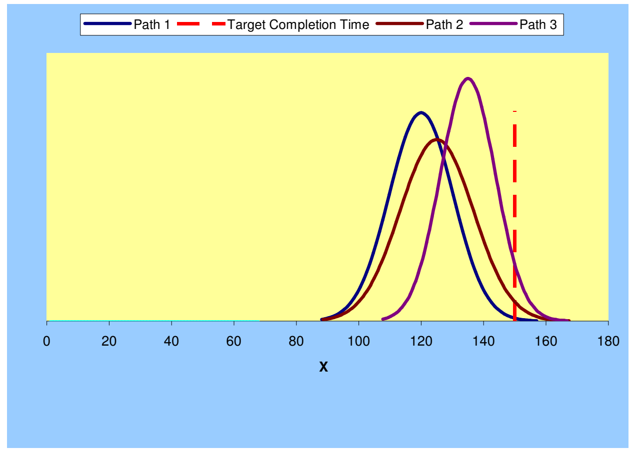
1	2	3
2.778	136.111	2.778
100.000	0.000	69.444
0.000	2.778	2.778
2.778		2.778

	1	2	3	
Variance of path time	105.556	138.889	77.778	
Z	1.460	0.8485	0.000	
P(T < Target date) =	0.928	0.802	0.500	37.2%

P(T < 135) = 37.2

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6A, Treatment 2, [Factor: F3, F6], (T=150)



Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	30	30	30
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
30.0	0.000
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778

What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 93.8 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

	1	2	3
	10.0	65.0	30.0
	50.0	30.0	45.0
	30.0	30.0	30.0
	30.0		30.0
Average path time	120.0	125.0	135.0

Paths

	1	2	3	
	2.778	136.111	2.778	
	100.000	0.000	69.444	
	0.000	2.778	2.778	
	2.778		2.778	
Variance of path time	105.556	138.889	77.778	
Z	2.920	2.1213	1.701	
P(T < Target date) =	0.998	0.983	0.956	93.8%

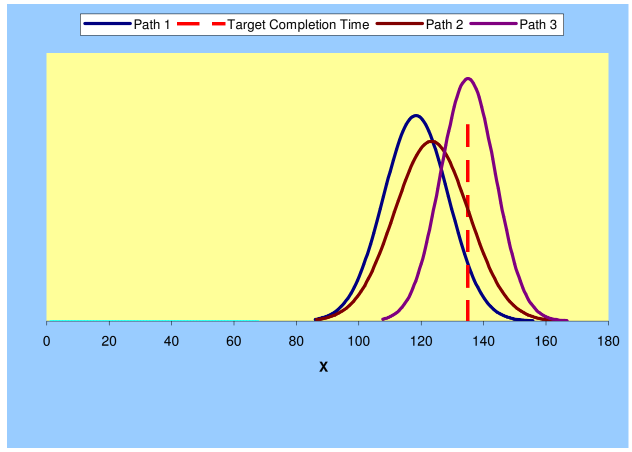
P(T < 150) = 93.8

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6A, Treatment 3, [Factor: F3, F6], (T=135)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	30
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
28.3	2.778
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 135

Likelihood of completing the project in 135 or less time units: 39.5 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	28.3	45.0
28.3	30.0	30.0
30.0		30.0

Average path time 118.3 123.3 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	2.778	69.444
2.778	2.778	2.778
2.778		2.778

Variance of path time	108.333	141.667	77.778	
Z	1.601	0.9802	0.000	
P(T < Target date) =	0.945	0.837	0.500	39.5%

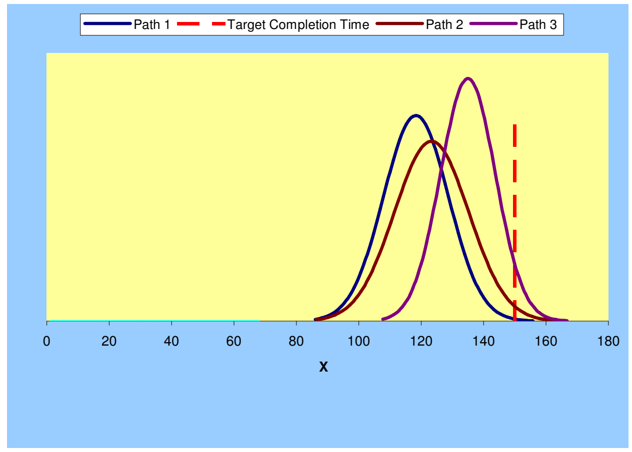
P(T < 135) = 39.5

APPENDIX C: COMPUTATIONAL RESULTS-Second Analytical Approach using Microsoft Excel®

Analysis 2, Scenario S6A, Treatment 3, [Factor: F3, F6], (T=150)

Activity	a	m	b
L1-L2	5	10	15
L2-L3	30	45	90
L1-L3	40	60	110
L3-L7	20	30	30
L1-L4	25	30	35
L4-L6	30	40	80
L6-L7	25	30	35
L7-L11	25	30	35

mean	variance
10.0	2.778
50.0	100.000
65.0	136.111
28.3	2.778
30.0	2.778
45.0	69.444
30.0	2.778
30.0	2.778



What is the projected completion time? 150

Likelihood of completing the project in 150 or less time units: 94.2 %

Paths

1	2	3
L1-L2	L1-L3	L1-L4
L2-L3	L3-L7	L4-L6
L3-L7	L7-L11	L6-L7
L7-L11		L7-L11

Paths

1	2	3
10.0	65.0	30.0
50.0	28.3	45.0
28.3	30.0	30.0
30.0		30.0

Average path time 118.3 123.3 135.0

Paths

1	2	3
2.778	136.111	2.778
100.000	2.778	69.444
2.778	2.778	2.778
2.778		2.778

Variance of path time	108.333	141.667	77.778	
Z	3.042	2.2404	1.701	
P(T < Target date) =	0.999	0.987	0.956	94.2%

P(T < 150) = 94.2

APPENDIX D: COMPUTATIONAL RESULTS-Second Analytical Approach using Risk Calc®

Exemplar Script for Analysis 2, Scenario [S1], Treatment 1

```
source('S4pbox.r')


pbox.steps <- 2000   # makes the calculations more accurate, but a lot slower

is.scalar <- function(x) {
     if (is.pbox(x) && isTRUE(all.equal(left(x),right(x)))) return(TRUE)
  if (is.interval(x) && isTRUE(all.equal(left(x),right(x)))) return(TRUE)
  if (is.numeric(x) && isTRUE(all.equal(1,length(x)))) return(TRUE)
  FALSE
  }

normal0 <- function(normmean, normstd, name=''){
  m <- left(normmean)
  s <- left(normstd)
  pbox(u=qnorm(iii(),m,s), d=qnorm(jjj(),m,s), shape='normal', name=name, ml=m, mh=m,
vl=s^2, vh=s^2)
  }

Pproportion <- function(pN,pN2) if (left(pN) <= 0) env.interval(-Inf,1-pN2/right(pN)) else 1 -
pN2/pN

summaries <- function(N,N2, T1,T2) {
  pN  <- N > T1
  pN2 <- N2 > T1
  cat('(T1) Base:', sayint(pN), ' Improved:', sayint(pN2), ' Difference:', sayint(pN - pN2), '
Proportion:', sayint(Pproportion(pN,pN2)), '\n')

  pN  <- N > T2
  pN2 <- N2 > T2
  cat('(T2) Base:', sayint(pN), ' Improved:', sayint(pN2), ' Difference:', sayint(pN - pN2), '
Proportion:', sayint(Pproportion(pN,pN2)), '\n')
  }

drawthresholds <- function() {
  lines(c(T1,T1),c(0,1))
  lines(c(T2,T2),c(0,1))
  }

a <- normal(10,sqrt(2.778))    # units('workday') // design package
```

```
b <- normal(50,sqrt(100))      # units('workday') // set up packaging facility
c <- normal(65,sqrt(136.111))  # units('workday') // order stock
d <- normal(30,sqrt(11.1111))  # units('workday') // package stock
e <- normal(30,sqrt(2.778))    # units('workday') // organize sales office
f1<- normal(45,sqrt(69.444))   # units('workday') // select distributors
f2<- normal(35,sqrt(69.444))   # units('workday') // select distributors
g <- normal(30,sqrt(2.778))    # units('workday') // sell to distributors
h <- normal(30,sqrt(2.778))    # units('workday') // ship stock to distributors

# all the convolutions
ni <- function() return(pmaxI.pbox(pmaxI.pbox(a %|+|% b, c) %|+|% d, e %|+|% f %|+|% g)
%|+|% h)

# fewer convolutions, and thus more precision
ni <- function() return(pmaxI.pbox(pmaxI.pbox(N(mean(a)+mean(b),sqrt(var(a)+var(b))), c)
%|+|% d, N(mean(e)+mean(f)+mean(g),sqrt(var(e)+var(f)+var(g)))) %|+|% h)


f  <- f1
NI  <-  ni()

f  <- f2
NI2 <-  ni()

T1 <- 135  #workday
T2 <- 150  #workday

summaries(NI,NI2,T1,T2)

rigorous <- 1 - (NI2 > T1)/(NI > T1)
rigorous2 <- 1 - (NI2 > T2)/(NI > T2)

# Frechet ##########################################################


nd <- function() return(pmax(pmax(a %+% b, c) %+% d, e %+% f %+% g) %+% h)

f  <- f1
ND  <-  nd()

f  <- f2
ND2 <-  nd()

summaries(ND,ND2,T1,T2)
```

```
#####################################################################

pbox.cumulative <- FALSE  # use exceedance plots since it's large values that are the problem

plot(NI, col='black')
title("Treatment versus Base (Independent)")
lines(NI2,col='red')
drawthresholds()


one <- recordPlot()


#####################################################################

windows()  # make a new window for Frechet graphics

plot(ND, col='black')
title("Treatment versus Base (Fréchet)")
lines(ND2,col='red')
drawthresholds()

two <- recordPlot()


######################################################################

# Monte Carlo simulations


windows()  # make a new window for graphics
one        # redisplay first graph on second graphics window

many <- 100000


firstMC <- NULL

secondMC <- NULL


goMC <- function() {

  a <- rnorm(many,10,sqrt(2.778))    # units('workday') // design package
  b <- rnorm(many,50,sqrt(100))      # units('workday') // set up packaging facility
```

```
  c <- rnorm(many,65,sqrt(136.111))  # units('workday') // order stock
  d <- rnorm(many,30,sqrt(11.1111))  # units('workday') // package stock
  e <- rnorm(many,30,sqrt(2.778))    # units('workday') // organize sales office
  f1<- rnorm(many,45,sqrt(69.444))   # units('workday') // select distributors
  f2<- rnorm(many,35,sqrt(69.444))   # units('workday') // select distributors
  g <- rnorm(many,30,sqrt(2.778))    # units('workday') // sell to distributors
  h <- rnorm(many,30,sqrt(2.778))    # units('workday') // ship stock to distributors

  ni  <- function() return(pmax(pmax(a + b, c) + d, e + f + g) + h)

  f   <- f1
  NI  <<-  ni()

  f   <- f2
  NI2 <<- ni()

  T1 <<- 135  #workday
  T2 <<- 150  #workday

  pNI <- sum(NI > T1) / many
  pNI2 <- sum(NI2 > T1) / many
  cat('(T1) Base:', pNI, ' Improved:', pNI2, ' Difference:', pNI - pNI2, ' Proportion:',
Pproportion(pNI,pNI2), '\n')

  firstMC <<- c(firstMC, 1 - pNI2/pNI)

  pNI <- sum(NI > T2) / many
  pNI2 <- sum(NI2 > T2) / many
  cat('(T2) Base:', pNI, ' Improved:', pNI2, ' Difference:', pNI - pNI2, ' Proportion:',
Pproportion(pNI,pNI2), '\n')

  secondMC <<- c(secondMC, 1 - pNI2/pNI)
}


for (i in 1:200) goMC()


edf <- function(x, exceedance=FALSE, ...) {
  # the empirical distribution function (Sn)
  n <- length(x)
  s <- sort(x)
  lines(c(s[[1]],s[[1]]),updown(!exceedance,c(0,1/n)), ...); for (i in 2:n) lines(c(s[[i-
1]],s[[i]],s[[i]]), updown(!exceedance,c(i-1,i-1,i)/n), ...)
  }
```

```
edf(NI, TRUE, col='black',lwd=4)
edf(NI2, TRUE, col='red',lwd=4)

three <- recordPlot()


##################################################################

windows()  # make a new window for graphics

plot(c(min(firstMC,left(rigorous)),max(firstMC,right(rigorous))),c(0,1),col='white');
edf(firstMC);
lines(c(left(rigorous),left(rigorous),right(rigorous),right(rigorous)),c(0,1,1,0),lwd=5)

four <- recordPlot()


##################################################################

windows()  # make a new window for graphics

plot(c(min(secondMC,left(rigorous2)),max(secondMC,right(rigorous2))),c(0,1),col='white');
edf(secondMC);
lines(c(left(rigorous2),left(rigorous2),right(rigorous2),right(rigorous2)),c(0,1,1,0),lwd=5)

five <- recordPlot()


# use savePlot to save the graph to a file for inclusion in a document
```

APPENDIX D: COMPUTATIONAL RESULTS-Second Analytical Approach using Risk Calc®

Exemplar Output for Analysis 2, Scenario [S1], Treatment 1

```
R version 2.5.0 (2007-04-23)
Copyright (C) 2007 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> source('S4pbox.r')
:pbox> library loaded
:pbox> demo loaded
>
>
> pbox.steps <- 2000   # makes the calculations more accurate, but a lot slower
>
> is.scalar <- function(x) {
+      if (is.pbox(x) && isTRUE(all.equal(left(x),right(x)))) return(TRUE)
+   if (is.interval(x) && isTRUE(all.equal(left(x),right(x)))) return(TRUE)
+   if (is.numeric(x) && isTRUE(all.equal(1,length(x)))) return(TRUE)
+   FALSE
+   }
>
> normal0 <- function(normmean, normstd, name=''){
+   m <- left(normmean)
+   s <- left(normstd)
+   pbox(u=qnorm(iii(),m,s), d=qnorm(jjj(),m,s), shape='normal', name=name, ml=m, mh=m,
vl=s^2, vh=s^2)
+   }
>
```

```
> Pproportion <- function(pN,pN2) if (left(pN) <= 0) env.interval(-Inf,1-pN2/right(pN)) else 1 -
pN2/pN
>
> summaries <- function(N,N2, T1,T2) {
+   pN  <- N > T1
+   pN2 <- N2 > T1
+   cat('(T1) Base:', sayint(pN), ' Improved:', sayint(pN2), ' Difference:', sayint(pN - pN2), '
Proportion:', sayint(Pproportion(pN,pN2)), '\n')
+
+   pN  <- N > T2
+   pN2 <- N2 > T2
+   cat('(T2) Base:', sayint(pN), ' Improved:', sayint(pN2), ' Difference:', sayint(pN - pN2), '
Proportion:', sayint(Pproportion(pN,pN2)), '\n')
+   }
>
> drawthresholds <- function() {
+   lines(c(T1,T1),c(0,1))
+   lines(c(T2,T2),c(0,1))
+   }
>
> a <- normal(10,sqrt(2.778))    # units('workday') // design package
> b <- normal(50,sqrt(100))      # units('workday') // set up packaging facility
> c <- normal(65,sqrt(136.111))  # units('workday') // order stock
> d <- normal(30,sqrt(11.1111))  # units('workday') // package stock
> e <- normal(30,sqrt(2.778))    # units('workday') // organize sales office
> f1<- normal(45,sqrt(69.444))   # units('workday') // select distributors
> f2<- normal(35,sqrt(69.444))   # units('workday') // select distributors
> g <- normal(30,sqrt(2.778))    # units('workday') // sell to distributors
> h <- normal(30,sqrt(2.778))    # units('workday') // ship stock to distributors
>
> # all the convolutions
> ni <- function() return(pmaxI.pbox(pmaxI.pbox(a %|+|% b, c) %|+|% d, e %|+|% f %|+|% g)
%|+|% h)
>
> # fewer convolutions, and thus more precision
> ni <- function() return(pmaxI.pbox(pmaxI.pbox(N(mean(a)+mean(b),sqrt(var(a)+var(b))), c)
%|+|% d, N(mean(e)+mean(f)+mean(g),sqrt(var(e)+var(f)+var(g)))) %|+|% h)
>
>
> f   <- f1
> NI  <-  ni()
>
> f   <- f2
> NI2 <-  ni()
>
> T1 <- 135  #workday
```

```
> T2 <- 150  #workday
>
> summaries(NI,NI2,T1,T2)
(T1) Base: [0.6285,0.631]  Improved: [0.358,0.3615]  Difference: [0.267,0.273]  Proportion:
[0.424821,0.4326466]
(T2) Base: [0.0645,0.068]  Improved: [0.0235,0.0265]  Difference: [0.038,0.0445]  Proportion:
[0.5891473,0.6544118]
>
> rigorous <- 1 - (NI2 > T1)/(NI > T1)
> rigorous2 <- 1 - (NI2 > T2)/(NI > T2)
>
> # Frechet ###########################################################
>
>
> nd <- function() return(pmax(pmax(a %+% b, c) %+% d, e %+% f %+% g) %+% h)
>
> f   <- f1
> ND  <-  nd()
>
> f   <- f2
> ND2 <-  nd()
>
> summaries(ND,ND2,T1,T2)
(T1) Base: [0.0515,1]  Improved: [0.0155,1]  Difference: [-0.9485,0.9845]  Proportion: [-
18.41748,0.9845]
(T2) Base: [0,0.5135]  Improved: [0,0.271]  Difference: [-0.271,0.5135]  Proportion: [-Inf,1]
>
>
> ######################################################################
>
>
> pbox.cumulative <- FALSE  # use exceedance plots since it's large values that are the problem
>
> plot(NI, col='black')
> title("Treatment versus Base (Independent)")
> lines(NI2,col='red')
> drawthresholds()
>
>
> one <- recordPlot()
>
>
> ######################################################################
>
> windows()  # make a new window for Frechet graphics
>
```

```
> plot(ND, col='black')
> title("Treatment versus Base (Fréchet)")
> lines(ND2,col='red')
> drawthresholds()
>
> two <- recordPlot()
>
>
> ####################################################################
>
> # Monte Carlo simulations
>
>
> windows()  # make a new window for graphics
> one        # redisplay first graph on second graphics window
>
> many <- 100000
>
>
> firstMC <- NULL
>
> secondMC <- NULL
>
>
> goMC <- function() {
+
+   a <- rnorm(many,10,sqrt(2.778))    # units('workday') // design package
+   b <- rnorm(many,50,sqrt(100))      # units('workday') // set up packaging facility
+   c <- rnorm(many,65,sqrt(136.111))  # units('workday') // order stock
+   d <- rnorm(many,30,sqrt(11.1111))  # units('workday') // package stock
+   e <- rnorm(many,30,sqrt(2.778))    # units('workday') // organize sales office
+   f1<- rnorm(many,45,sqrt(69.444))   # units('workday') // select distributors
+   f2<- rnorm(many,35,sqrt(69.444))   # units('workday') // select distributors
+   g <- rnorm(many,30,sqrt(2.778))    # units('workday') // sell to distributors
+   h <- rnorm(many,30,sqrt(2.778))    # units('workday') // ship stock to distributors
+
+   ni  <- function() return(pmax(pmax(a + b, c) + d, e + f + g) + h)
+
+   f   <- f1
+   NI  <<-  ni()
+
+   f   <- f2
+   NI2 <<- ni()
+
+   T1 <<- 135  #workday
+   T2 <<- 150  #workday
```

```
+
+   pNI <- sum(NI > T1) / many
+   pNI2 <- sum(NI2 > T1) / many
+   cat('(T1) Base:', pNI, ' Improved:', pNI2, ' Difference:', pNI - pNI2, ' Proportion:',
Pproportion(pNI,pNI2), '\n')
+
+   firstMC <<- c(firstMC, 1 - pNI2/pNI)
+
+   pNI <- sum(NI > T2) / many
+   pNI2 <- sum(NI2 > T2) / many
+   cat('(T2) Base:', pNI, ' Improved:', pNI2, ' Difference:', pNI - pNI2, ' Proportion:',
Pproportion(pNI,pNI2), '\n')
+
+   secondMC <<- c(secondMC, 1 - pNI2/pNI)
+ }
>
>
> for (i in 1:200) goMC()
(T1) Base: 0.63143  Improved: 0.36253  Difference: 0.2689  Proportion: 0.4258588
(T2) Base: 0.0666  Improved: 0.02559  Difference: 0.04101  Proportion: 0.6157658
(T1) Base: 0.63294  Improved: 0.36074  Difference: 0.2722  Proportion: 0.4300566
(T2) Base: 0.06741  Improved: 0.02648  Difference: 0.04093  Proportion: 0.60718
(T1) Base: 0.62802  Improved: 0.36073  Difference: 0.26729  Proportion: 0.4256075
(T2) Base: 0.06772  Improved: 0.0262  Difference: 0.04152  Proportion: 0.6131128
(T1) Base: 0.62883  Improved: 0.35758  Difference: 0.27125  Proportion: 0.4313566
(T2) Base: 0.06569  Improved: 0.02498  Difference: 0.04071  Proportion: 0.619729
(T1) Base: 0.62808  Improved: 0.35752  Difference: 0.27056  Proportion: 0.4307731
(T2) Base: 0.06553  Improved: 0.02464  Difference: 0.04089  Proportion: 0.623989
(T1) Base: 0.6315  Improved: 0.36102  Difference: 0.27048  Proportion: 0.4283135
(T2) Base: 0.06773  Improved: 0.02546  Difference: 0.04227  Proportion: 0.6240957
(T1) Base: 0.62998  Improved: 0.35928  Difference: 0.2707  Proportion: 0.4296962
(T2) Base: 0.06635  Improved: 0.0254  Difference: 0.04095  Proportion: 0.6171816
(T1) Base: 0.62944  Improved: 0.35809  Difference: 0.27135  Proportion: 0.4310975
(T2) Base: 0.06711  Improved: 0.02536  Difference: 0.04175  Proportion: 0.622113
(T1) Base: 0.62812  Improved: 0.35862  Difference: 0.2695  Proportion: 0.4290581
(T2) Base: 0.06618  Improved: 0.02513  Difference: 0.04105  Proportion: 0.620278
(T1) Base: 0.63058  Improved: 0.35843  Difference: 0.27215  Proportion: 0.4315868
(T2) Base: 0.06642  Improved: 0.02555  Difference: 0.04087  Proportion: 0.6153267
(T1) Base: 0.62901  Improved: 0.35785  Difference: 0.27116  Proportion: 0.4310901
(T2) Base: 0.06609  Improved: 0.02514  Difference: 0.04095  Proportion: 0.6196096
(T1) Base: 0.62794  Improved: 0.35858  Difference: 0.26936  Proportion: 0.4289582
(T2) Base: 0.06734  Improved: 0.02565  Difference: 0.04169  Proportion: 0.6190971
(T1) Base: 0.62947  Improved: 0.35917  Difference: 0.2703  Proportion: 0.4294089
(T2) Base: 0.06647  Improved: 0.02568  Difference: 0.04079  Proportion: 0.6136603
(T1) Base: 0.62987  Improved: 0.35859  Difference: 0.27128  Proportion: 0.4306920
(T2) Base: 0.06656  Improved: 0.02567  Difference: 0.04089  Proportion: 0.6143329
```

(T1) Base: 0.62942 Improved: 0.36112 Difference: 0.2683 Proportion: 0.4262655
(T2) Base: 0.06684 Improved: 0.02592 Difference: 0.04092 Proportion: 0.6122083
(T1) Base: 0.63045 Improved: 0.36067 Difference: 0.26978 Proportion: 0.4279166
(T2) Base: 0.06755 Improved: 0.02567 Difference: 0.04188 Proportion: 0.6199852
(T1) Base: 0.6278 Improved: 0.35882 Difference: 0.26898 Proportion: 0.4284486
(T2) Base: 0.06562 Improved: 0.02512 Difference: 0.0405 Proportion: 0.6171899
(T1) Base: 0.62828 Improved: 0.35829 Difference: 0.26999 Proportion: 0.4297288
(T2) Base: 0.06576 Improved: 0.02481 Difference: 0.04095 Proportion: 0.622719
(T1) Base: 0.62955 Improved: 0.35889 Difference: 0.27066 Proportion: 0.4299261
(T2) Base: 0.06651 Improved: 0.02571 Difference: 0.0408 Proportion: 0.6134416
(T1) Base: 0.63046 Improved: 0.36137 Difference: 0.26909 Proportion: 0.4268153
(T2) Base: 0.06675 Improved: 0.02534 Difference: 0.04141 Proportion: 0.6203745
(T1) Base: 0.62993 Improved: 0.35892 Difference: 0.27101 Proportion: 0.4302224
(T2) Base: 0.06614 Improved: 0.02562 Difference: 0.04052 Proportion: 0.6126399
(T1) Base: 0.63003 Improved: 0.36058 Difference: 0.26945 Proportion: 0.4276780
(T2) Base: 0.06611 Improved: 0.02512 Difference: 0.04099 Proportion: 0.6200272
(T1) Base: 0.63059 Improved: 0.35972 Difference: 0.27087 Proportion: 0.4295501
(T2) Base: 0.0657 Improved: 0.02492 Difference: 0.04078 Proportion: 0.6207002
(T1) Base: 0.62905 Improved: 0.35826 Difference: 0.27079 Proportion: 0.4304745
(T2) Base: 0.06657 Improved: 0.02515 Difference: 0.04142 Proportion: 0.6222022
(T1) Base: 0.63042 Improved: 0.35947 Difference: 0.27095 Proportion: 0.4297928
(T2) Base: 0.06678 Improved: 0.02442 Difference: 0.04236 Proportion: 0.6343217
(T1) Base: 0.63146 Improved: 0.36327 Difference: 0.26819 Proportion: 0.4247142
(T2) Base: 0.0665 Improved: 0.02569 Difference: 0.04081 Proportion: 0.6136842
(T1) Base: 0.6322 Improved: 0.3619 Difference: 0.2703 Proportion: 0.4275546
(T2) Base: 0.06775 Improved: 0.02593 Difference: 0.04182 Proportion: 0.6172694
(T1) Base: 0.62805 Improved: 0.3563 Difference: 0.27175 Proportion: 0.4326885
(T2) Base: 0.06644 Improved: 0.0254 Difference: 0.04104 Proportion: 0.6177002
(T1) Base: 0.62909 Improved: 0.35726 Difference: 0.27183 Proportion: 0.4321003
(T2) Base: 0.06731 Improved: 0.02502 Difference: 0.04229 Proportion: 0.628287
(T1) Base: 0.62934 Improved: 0.35942 Difference: 0.26992 Proportion: 0.4288938
(T2) Base: 0.06787 Improved: 0.02596 Difference: 0.04191 Proportion: 0.617504
(T1) Base: 0.62925 Improved: 0.3577 Difference: 0.27155 Proportion: 0.4315455
(T2) Base: 0.06657 Improved: 0.02551 Difference: 0.04106 Proportion: 0.6167944
(T1) Base: 0.63157 Improved: 0.35906 Difference: 0.27251 Proportion: 0.4314803
(T2) Base: 0.06807 Improved: 0.02501 Difference: 0.04306 Proportion: 0.6325841
(T1) Base: 0.62696 Improved: 0.35625 Difference: 0.27071 Proportion: 0.4317819
(T2) Base: 0.0662 Improved: 0.02551 Difference: 0.04069 Proportion: 0.6146526
(T1) Base: 0.63127 Improved: 0.3593 Difference: 0.27197 Proportion: 0.4308299
(T2) Base: 0.06635 Improved: 0.02553 Difference: 0.04082 Proportion: 0.6152223
(T1) Base: 0.63226 Improved: 0.36084 Difference: 0.27142 Proportion: 0.4292854
(T2) Base: 0.06819 Improved: 0.02563 Difference: 0.04256 Proportion: 0.6241384
(T1) Base: 0.63013 Improved: 0.36029 Difference: 0.26984 Proportion: 0.4282291
(T2) Base: 0.06795 Improved: 0.02636 Difference: 0.04159 Proportion: 0.6120677
(T1) Base: 0.6301 Improved: 0.35806 Difference: 0.27204 Proportion: 0.431741
(T2) Base: 0.06777 Improved: 0.02539 Difference: 0.04238 Proportion: 0.6253505

(T1) Base: 0.63063 Improved: 0.36002 Difference: 0.27061 Proportion: 0.4291106
(T2) Base: 0.06658 Improved: 0.02598 Difference: 0.0406 Proportion: 0.6097927
(T1) Base: 0.6296 Improved: 0.35965 Difference: 0.26995 Proportion: 0.4287643
(T2) Base: 0.06792 Improved: 0.02548 Difference: 0.04244 Proportion: 0.6248528
(T1) Base: 0.62886 Improved: 0.35811 Difference: 0.27075 Proportion: 0.430541
(T2) Base: 0.06676 Improved: 0.02578 Difference: 0.04098 Proportion: 0.6138406
(T1) Base: 0.62662 Improved: 0.35544 Difference: 0.27118 Proportion: 0.4327663
(T2) Base: 0.06597 Improved: 0.0253 Difference: 0.04067 Proportion: 0.6164923
(T1) Base: 0.62967 Improved: 0.35894 Difference: 0.27073 Proportion: 0.4299554
(T2) Base: 0.06631 Improved: 0.0253 Difference: 0.04101 Proportion: 0.6184588
(T1) Base: 0.63031 Improved: 0.3606 Difference: 0.26971 Proportion: 0.4279006
(T2) Base: 0.06636 Improved: 0.02532 Difference: 0.04104 Proportion: 0.6184448
(T1) Base: 0.62835 Improved: 0.35977 Difference: 0.26858 Proportion: 0.4274369
(T2) Base: 0.06565 Improved: 0.02484 Difference: 0.04081 Proportion: 0.6216299
(T1) Base: 0.63025 Improved: 0.35885 Difference: 0.2714 Proportion: 0.4306228
(T2) Base: 0.06774 Improved: 0.02645 Difference: 0.04129 Proportion: 0.6095365
(T1) Base: 0.62741 Improved: 0.3581 Difference: 0.26931 Proportion: 0.4292408
(T2) Base: 0.0659 Improved: 0.02538 Difference: 0.04052 Proportion: 0.614871
(T1) Base: 0.62673 Improved: 0.35772 Difference: 0.26901 Proportion: 0.4292279
(T2) Base: 0.0675 Improved: 0.02531 Difference: 0.04219 Proportion: 0.625037
(T1) Base: 0.63148 Improved: 0.36131 Difference: 0.27017 Proportion: 0.4278362
(T2) Base: 0.06809 Improved: 0.02594 Difference: 0.04215 Proportion: 0.6190336
(T1) Base: 0.63134 Improved: 0.35922 Difference: 0.27212 Proportion: 0.4310197
(T2) Base: 0.06583 Improved: 0.02468 Difference: 0.04115 Proportion: 0.625095
(T1) Base: 0.6317 Improved: 0.35915 Difference: 0.27255 Proportion: 0.4314548
(T2) Base: 0.06746 Improved: 0.02576 Difference: 0.0417 Proportion: 0.6181441
(T1) Base: 0.63261 Improved: 0.3616 Difference: 0.27101 Proportion: 0.4283998
(T2) Base: 0.06639 Improved: 0.02568 Difference: 0.04071 Proportion: 0.6131948
(T1) Base: 0.63044 Improved: 0.36103 Difference: 0.26941 Proportion: 0.4273365
(T2) Base: 0.06871 Improved: 0.02566 Difference: 0.04305 Proportion: 0.6265464
(T1) Base: 0.62925 Improved: 0.35981 Difference: 0.26944 Proportion: 0.4281923
(T2) Base: 0.06549 Improved: 0.02518 Difference: 0.04031 Proportion: 0.6155138
(T1) Base: 0.63248 Improved: 0.36096 Difference: 0.27152 Proportion: 0.4292942
(T2) Base: 0.06563 Improved: 0.0247 Difference: 0.04093 Proportion: 0.6236477
(T1) Base: 0.63201 Improved: 0.36045 Difference: 0.27156 Proportion: 0.4296767
(T2) Base: 0.06783 Improved: 0.02562 Difference: 0.04221 Proportion: 0.622291
(T1) Base: 0.63172 Improved: 0.35927 Difference: 0.27245 Proportion: 0.4312828
(T2) Base: 0.06606 Improved: 0.02456 Difference: 0.0415 Proportion: 0.6282168
(T1) Base: 0.62854 Improved: 0.36004 Difference: 0.2685 Proportion: 0.4271804
(T2) Base: 0.06728 Improved: 0.02632 Difference: 0.04096 Proportion: 0.608799
(T1) Base: 0.62931 Improved: 0.35712 Difference: 0.27219 Proportion: 0.4325213
(T2) Base: 0.06756 Improved: 0.02539 Difference: 0.04217 Proportion: 0.6241859
(T1) Base: 0.63143 Improved: 0.3585 Difference: 0.27293 Proportion: 0.4322411
(T2) Base: 0.06663 Improved: 0.02475 Difference: 0.04188 Proportion: 0.6285457
(T1) Base: 0.63129 Improved: 0.36039 Difference: 0.2709 Proportion: 0.4291213
(T2) Base: 0.06546 Improved: 0.02503 Difference: 0.04043 Proportion: 0.6176291

(T1) Base: 0.62726 Improved: 0.35758 Difference: 0.26968 Proportion: 0.4299334
(T2) Base: 0.06514 Improved: 0.02573 Difference: 0.03941 Proportion: 0.6050046
(T1) Base: 0.63103 Improved: 0.36038 Difference: 0.27065 Proportion: 0.4289020
(T2) Base: 0.06591 Improved: 0.02573 Difference: 0.04018 Proportion: 0.6096192
(T1) Base: 0.62904 Improved: 0.35931 Difference: 0.26973 Proportion: 0.4287963
(T2) Base: 0.06582 Improved: 0.02552 Difference: 0.0403 Proportion: 0.6122759
(T1) Base: 0.63048 Improved: 0.35828 Difference: 0.2722 Proportion: 0.4317346
(T2) Base: 0.06572 Improved: 0.02449 Difference: 0.04123 Proportion: 0.6273585
(T1) Base: 0.62827 Improved: 0.36086 Difference: 0.26741 Proportion: 0.4256291
(T2) Base: 0.06692 Improved: 0.02569 Difference: 0.04123 Proportion: 0.6161088
(T1) Base: 0.63061 Improved: 0.36056 Difference: 0.27005 Proportion: 0.4282362
(T2) Base: 0.06522 Improved: 0.02466 Difference: 0.04056 Proportion: 0.6218951
(T1) Base: 0.62963 Improved: 0.36056 Difference: 0.26907 Proportion: 0.4273462
(T2) Base: 0.06591 Improved: 0.02568 Difference: 0.04023 Proportion: 0.6103778
(T1) Base: 0.63183 Improved: 0.36057 Difference: 0.27126 Proportion: 0.4293243
(T2) Base: 0.06664 Improved: 0.02526 Difference: 0.04138 Proportion: 0.6209484
(T1) Base: 0.63068 Improved: 0.35923 Difference: 0.27145 Proportion: 0.4304084
(T2) Base: 0.0661 Improved: 0.02584 Difference: 0.04026 Proportion: 0.6090772
(T1) Base: 0.63097 Improved: 0.36129 Difference: 0.26968 Proportion: 0.4274054
(T2) Base: 0.06616 Improved: 0.02509 Difference: 0.04107 Proportion: 0.6207678
(T1) Base: 0.62654 Improved: 0.3585 Difference: 0.26804 Proportion: 0.4278099
(T2) Base: 0.06547 Improved: 0.02512 Difference: 0.04035 Proportion: 0.6163128
(T1) Base: 0.63022 Improved: 0.35816 Difference: 0.27206 Proportion: 0.4316905
(T2) Base: 0.0672 Improved: 0.02616 Difference: 0.04104 Proportion: 0.6107143
(T1) Base: 0.6299 Improved: 0.35968 Difference: 0.27022 Proportion: 0.4289887
(T2) Base: 0.06771 Improved: 0.02555 Difference: 0.04216 Proportion: 0.6226554
(T1) Base: 0.63066 Improved: 0.36051 Difference: 0.27015 Proportion: 0.4283608
(T2) Base: 0.06722 Improved: 0.02558 Difference: 0.04164 Proportion: 0.6194585
(T1) Base: 0.63081 Improved: 0.35746 Difference: 0.27335 Proportion: 0.4333317
(T2) Base: 0.06681 Improved: 0.02529 Difference: 0.04152 Proportion: 0.6214639
(T1) Base: 0.6273 Improved: 0.35734 Difference: 0.26996 Proportion: 0.4303523
(T2) Base: 0.0672 Improved: 0.02541 Difference: 0.04179 Proportion: 0.621875
(T1) Base: 0.62811 Improved: 0.35814 Difference: 0.26997 Proportion: 0.4298132
(T2) Base: 0.06734 Improved: 0.02599 Difference: 0.04135 Proportion: 0.6140481
(T1) Base: 0.63327 Improved: 0.36209 Difference: 0.27118 Proportion: 0.4282218
(T2) Base: 0.06655 Improved: 0.02566 Difference: 0.04089 Proportion: 0.6144252
(T1) Base: 0.63171 Improved: 0.36119 Difference: 0.27052 Proportion: 0.4282345
(T2) Base: 0.06672 Improved: 0.0251 Difference: 0.04162 Proportion: 0.623801
(T1) Base: 0.62865 Improved: 0.35662 Difference: 0.27203 Proportion: 0.4327209
(T2) Base: 0.06582 Improved: 0.02484 Difference: 0.04098 Proportion: 0.6226071
(T1) Base: 0.6272 Improved: 0.35786 Difference: 0.26934 Proportion: 0.4294324
(T2) Base: 0.06575 Improved: 0.02501 Difference: 0.04074 Proportion: 0.6196198
(T1) Base: 0.62695 Improved: 0.35968 Difference: 0.26727 Proportion: 0.4263019
(T2) Base: 0.06718 Improved: 0.02605 Difference: 0.04113 Proportion: 0.6122358
(T1) Base: 0.62892 Improved: 0.35829 Difference: 0.27063 Proportion: 0.4303091
(T2) Base: 0.06673 Improved: 0.02547 Difference: 0.04126 Proportion: 0.6183126

(T1) Base: 0.62922 Improved: 0.35847 Difference: 0.27075 Proportion: 0.4302947
(T2) Base: 0.06503 Improved: 0.02555 Difference: 0.03948 Proportion: 0.6071044
(T1) Base: 0.6276 Improved: 0.35939 Difference: 0.26821 Proportion: 0.4273582
(T2) Base: 0.06536 Improved: 0.02512 Difference: 0.04024 Proportion: 0.6156671
(T1) Base: 0.62954 Improved: 0.36157 Difference: 0.26797 Proportion: 0.42566
(T2) Base: 0.06742 Improved: 0.02557 Difference: 0.04185 Proportion: 0.6207357
(T1) Base: 0.62841 Improved: 0.35922 Difference: 0.26919 Proportion: 0.4283668
(T2) Base: 0.06687 Improved: 0.02515 Difference: 0.04172 Proportion: 0.6238971
(T1) Base: 0.6302 Improved: 0.36058 Difference: 0.26962 Proportion: 0.4278324
(T2) Base: 0.06711 Improved: 0.02558 Difference: 0.04153 Proportion: 0.6188347
(T1) Base: 0.62731 Improved: 0.3579 Difference: 0.26941 Proportion: 0.4294687
(T2) Base: 0.06633 Improved: 0.02524 Difference: 0.04109 Proportion: 0.6194784
(T1) Base: 0.63306 Improved: 0.36025 Difference: 0.27281 Proportion: 0.4309386
(T2) Base: 0.06624 Improved: 0.02579 Difference: 0.04045 Proportion: 0.6106582
(T1) Base: 0.63067 Improved: 0.35928 Difference: 0.27139 Proportion: 0.4303201
(T2) Base: 0.06576 Improved: 0.02494 Difference: 0.04082 Proportion: 0.6207421
(T1) Base: 0.62774 Improved: 0.35968 Difference: 0.26806 Proportion: 0.4270239
(T2) Base: 0.06569 Improved: 0.02511 Difference: 0.04058 Proportion: 0.61775
(T1) Base: 0.62746 Improved: 0.3602 Difference: 0.26726 Proportion: 0.4259395
(T2) Base: 0.06581 Improved: 0.02596 Difference: 0.03985 Proportion: 0.6055311
(T1) Base: 0.62799 Improved: 0.36082 Difference: 0.26717 Proportion: 0.4254367
(T2) Base: 0.06672 Improved: 0.02576 Difference: 0.04096 Proportion: 0.6139089
(T1) Base: 0.63112 Improved: 0.36037 Difference: 0.27075 Proportion: 0.4289992
(T2) Base: 0.06704 Improved: 0.0253 Difference: 0.04174 Proportion: 0.6226134
(T1) Base: 0.62872 Improved: 0.35758 Difference: 0.27114 Proportion: 0.4312572
(T2) Base: 0.06714 Improved: 0.02588 Difference: 0.04126 Proportion: 0.6145368
(T1) Base: 0.63018 Improved: 0.36106 Difference: 0.26912 Proportion: 0.4270526
(T2) Base: 0.06462 Improved: 0.02477 Difference: 0.03985 Proportion: 0.6166821
(T1) Base: 0.6304 Improved: 0.35865 Difference: 0.27175 Proportion: 0.4310755
(T2) Base: 0.06679 Improved: 0.02518 Difference: 0.04161 Proportion: 0.6229975
(T1) Base: 0.63001 Improved: 0.36005 Difference: 0.26996 Proportion: 0.4285011
(T2) Base: 0.0667 Improved: 0.02551 Difference: 0.04119 Proportion: 0.6175412
(T1) Base: 0.62936 Improved: 0.35829 Difference: 0.27107 Proportion: 0.4307074
(T2) Base: 0.06681 Improved: 0.02502 Difference: 0.04179 Proportion: 0.6255052
(T1) Base: 0.62898 Improved: 0.36033 Difference: 0.26865 Proportion: 0.4271201
(T2) Base: 0.06572 Improved: 0.02463 Difference: 0.04109 Proportion: 0.6252282
(T1) Base: 0.63256 Improved: 0.36158 Difference: 0.27098 Proportion: 0.4283862
(T2) Base: 0.06663 Improved: 0.02593 Difference: 0.0407 Proportion: 0.610836
(T1) Base: 0.62884 Improved: 0.35565 Difference: 0.27319 Proportion: 0.4344348
(T2) Base: 0.06539 Improved: 0.02479 Difference: 0.0406 Proportion: 0.62089
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(T1) Base: 0.6274 Improved: 0.3593 Difference: 0.2681 Proportion: 0.4273191
(T2) Base: 0.06656 Improved: 0.02572 Difference: 0.04084 Proportion: 0.6135817
(T1) Base: 0.63056 Improved: 0.35917 Difference: 0.27139 Proportion: 0.4303952
(T2) Base: 0.06641 Improved: 0.02534 Difference: 0.04107 Proportion: 0.618431

(T1) Base: 0.63177 Improved: 0.35747 Difference: 0.2743 Proportion: 0.434177
(T2) Base: 0.06696 Improved: 0.02534 Difference: 0.04162 Proportion: 0.6215651
(T1) Base: 0.6309 Improved: 0.36172 Difference: 0.26918 Proportion: 0.4266603
(T2) Base: 0.06673 Improved: 0.02495 Difference: 0.04178 Proportion: 0.6261052
(T1) Base: 0.63067 Improved: 0.35553 Difference: 0.27514 Proportion: 0.4362662
(T2) Base: 0.06658 Improved: 0.02525 Difference: 0.04133 Proportion: 0.620757
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(T2) Base: 0.067 Improved: 0.02563 Difference: 0.04137 Proportion: 0.6174627
(T1) Base: 0.63125 Improved: 0.35961 Difference: 0.27164 Proportion: 0.4303208
(T2) Base: 0.06681 Improved: 0.0255 Difference: 0.04131 Proportion: 0.6183206
(T1) Base: 0.63188 Improved: 0.36046 Difference: 0.27142 Proportion: 0.4295436
(T2) Base: 0.06666 Improved: 0.02553 Difference: 0.04113 Proportion: 0.6170117
(T1) Base: 0.63076 Improved: 0.35938 Difference: 0.27138 Proportion: 0.4302429
(T2) Base: 0.06673 Improved: 0.02597 Difference: 0.04076 Proportion: 0.6108197
(T1) Base: 0.62694 Improved: 0.35988 Difference: 0.26706 Proportion: 0.4259738
(T2) Base: 0.06613 Improved: 0.02517 Difference: 0.04096 Proportion: 0.619386
(T1) Base: 0.62872 Improved: 0.3593 Difference: 0.26942 Proportion: 0.4285214
(T2) Base: 0.06646 Improved: 0.02527 Difference: 0.04119 Proportion: 0.6197713
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(T2) Base: 0.06637 Improved: 0.02585 Difference: 0.04052 Proportion: 0.6105168
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(T1) Base: 0.62874 Improved: 0.36001 Difference: 0.26873 Proportion: 0.4274104
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(T1) Base: 0.62872 Improved: 0.35779 Difference: 0.27093 Proportion: 0.4309231
(T2) Base: 0.0677 Improved: 0.02573 Difference: 0.04197 Proportion: 0.6199409

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(T2) Base: 0.06547 Improved: 0.02568 Difference: 0.03979 Proportion: 0.6077593
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(T2) Base: 0.06737 Improved: 0.02587 Difference: 0.0415 Proportion: 0.6160012
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(T2) Base: 0.06668 Improved: 0.02546 Difference: 0.04122 Proportion: 0.6181764
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(T2) Base: 0.06681 Improved: 0.02595 Difference: 0.04086 Proportion: 0.6115851
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(T2) Base: 0.06733 Improved: 0.02574 Difference: 0.04159 Proportion: 0.6177038
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(T1) Base: 0.62844 Improved: 0.3584 Difference: 0.27004 Proportion: 0.4296989
(T2) Base: 0.06582 Improved: 0.02464 Difference: 0.04118 Proportion: 0.6256457
(T1) Base: 0.62768 Improved: 0.35904 Difference: 0.26864 Proportion: 0.4279888
(T2) Base: 0.06702 Improved: 0.0254 Difference: 0.04162 Proportion: 0.6210087
(T1) Base: 0.63062 Improved: 0.36284 Difference: 0.26778 Proportion: 0.4246297
(T2) Base: 0.06632 Improved: 0.02551 Difference: 0.04081 Proportion: 0.6153498
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(T1) Base: 0.62767 Improved: 0.35902 Difference: 0.26865 Proportion: 0.4280115
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(T1) Base: 0.62569 Improved: 0.35656 Difference: 0.26913 Proportion: 0.4301331
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(T2) Base: 0.06697 Improved: 0.02568 Difference: 0.04129 Proportion: 0.6165447
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(T2) Base: 0.06705 Improved: 0.02582 Difference: 0.04123 Proportion: 0.6149142
(T1) Base: 0.62627 Improved: 0.35719 Difference: 0.26908 Proportion: 0.4296549
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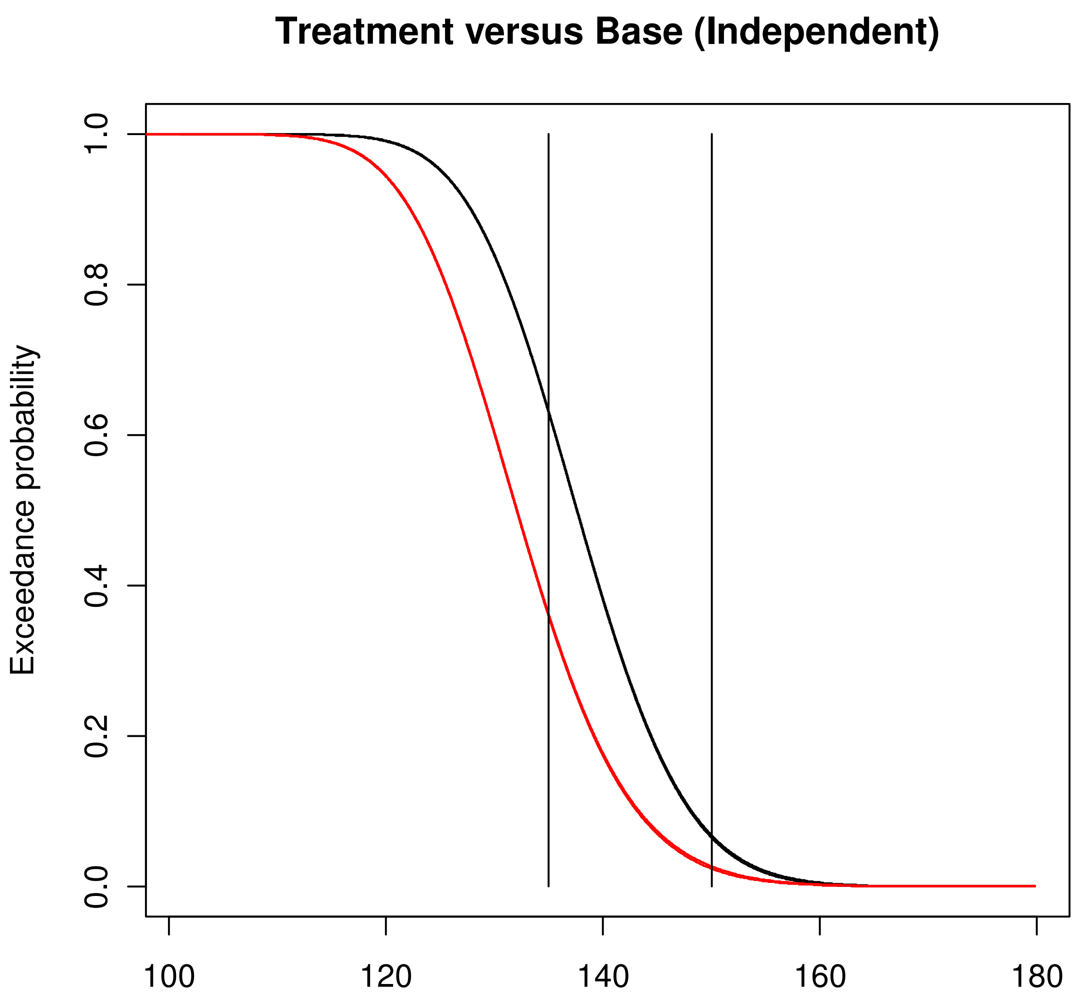
(T1) Base: 0.62881 Improved: 0.35887 Difference: 0.26994 Proportion: 0.4292871
(T2) Base: 0.06703 Improved: 0.02505 Difference: 0.04198 Proportion: 0.6262867
(T1) Base: 0.63084 Improved: 0.35909 Difference: 0.27175 Proportion: 0.4307748
(T2) Base: 0.06781 Improved: 0.02571 Difference: 0.0421 Proportion: 0.6208524
(T1) Base: 0.62975 Improved: 0.35925 Difference: 0.2705 Proportion: 0.4295355
(T2) Base: 0.0665 Improved: 0.02541 Difference: 0.04109 Proportion: 0.6178947

```
(T1) Base: 0.62936  Improved: 0.35885  Difference: 0.27051  Proportion: 0.4298176
(T2) Base: 0.0671  Improved: 0.02545  Difference: 0.04165  Proportion: 0.6207154
(T1) Base: 0.62893  Improved: 0.35819  Difference: 0.27074  Proportion: 0.4304772
(T2) Base: 0.06655  Improved: 0.02635  Difference: 0.0402  Proportion: 0.6040571
>
>
> edf <- function(x, exceedance=FALSE, ...) {
+   # the empirical distribution function (Sn)
+   n <- length(x)
+   s <- sort(x)
+   lines(c(s[[1]],s[[1]]),updown(!exceedance,c(0,1/n)), ...); for (i in 2:n) lines(c(s[[i-
1]],s[[i]],s[[i]]), updown(!exceedance,c(i-1,i-1,i)/n), ...)
+   }
>
>
> edf(NI, TRUE, col='black',lwd=4)
> edf(NI2, TRUE, col='red',lwd=4)
>
> three <- recordPlot()
>
>
> ######################################################################
>
> windows()  # make a new window for graphics
>
> plot(c(min(firstMC,left(rigorous)),max(firstMC,right(rigorous))),c(0,1),col='white');
edf(firstMC);
lines(c(left(rigorous),left(rigorous),right(rigorous),right(rigorous)),c(0,1,1,0),lwd=5)
>
> four <- recordPlot()
>
>
> ######################################################################
>
> windows()  # make a new window for graphics
>
> plot(c(min(secondMC,left(rigorous2)),max(secondMC,right(rigorous2))),c(0,1),col='white');
edf(secondMC);
lines(c(left(rigorous2),left(rigorous2),right(rigorous2),right(rigorous2)),c(0,1,1,0),lwd=5)
>
> five <- recordPlot()
>
>
> # use savePlot to save the graph to a file for inclusion in a document
>
>
```

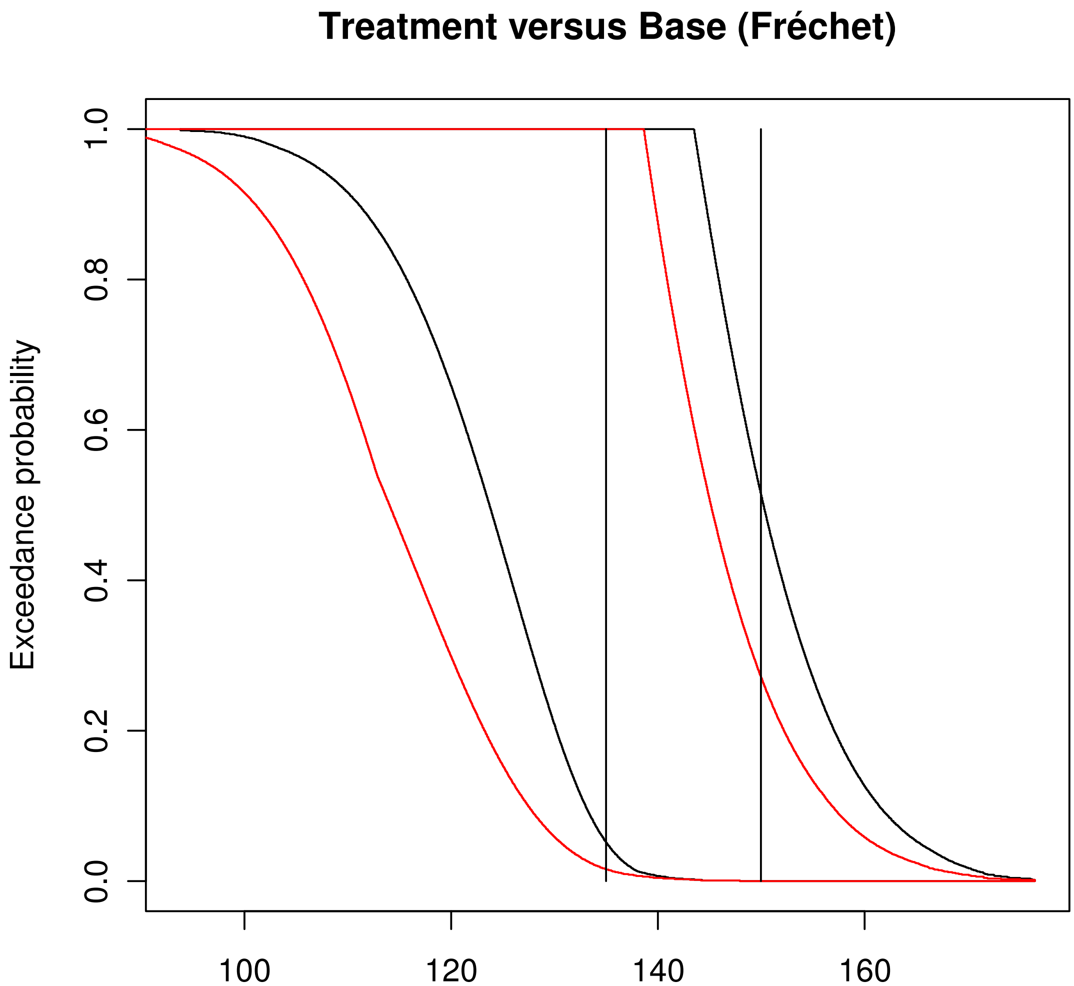
APPENDIX D: COMPUTATIONAL RESULTS-Second Analytical Approach using Risk Calc®

Analysis 2, Scenario [S1], Treatment 1



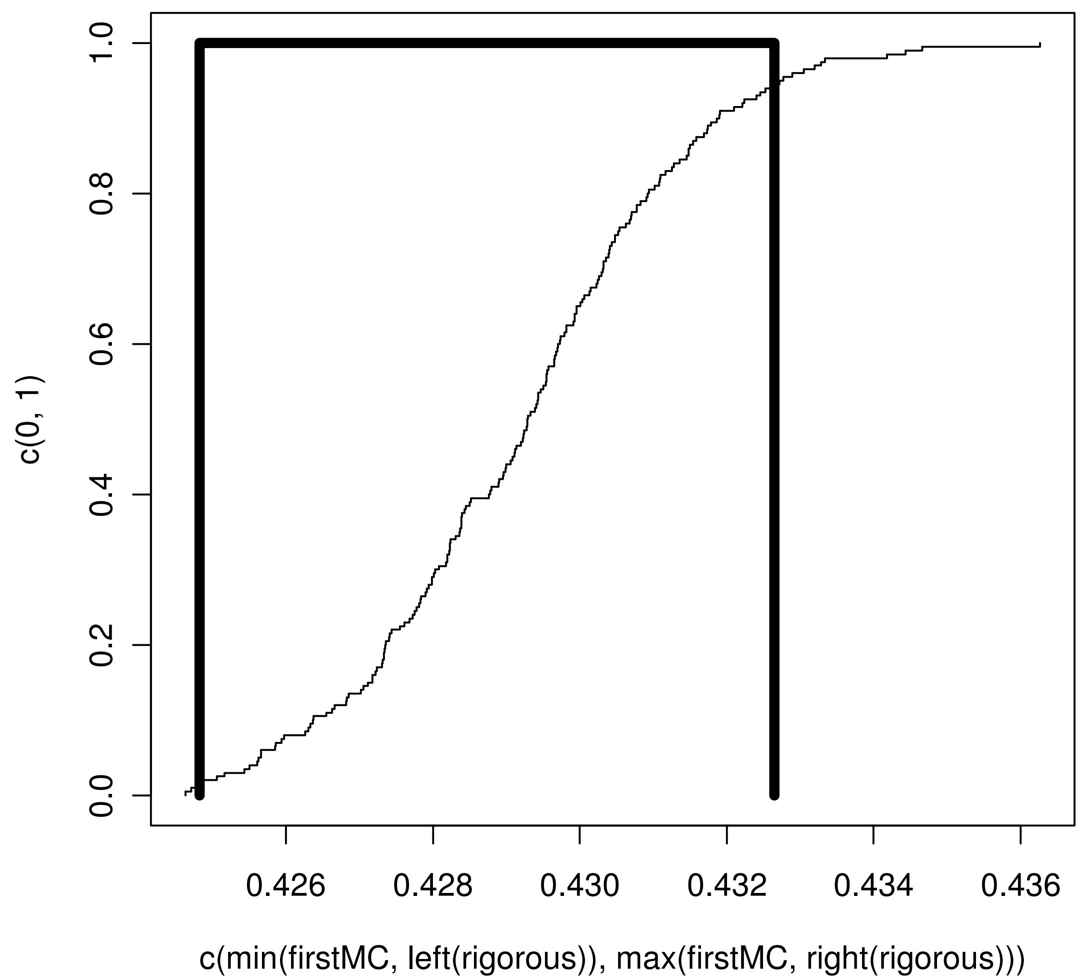
APPENDIX D: COMPUTATIONAL RESULTS-Second Analytical Approach using Risk Calc®

Analysis 2, Scenario [S1], Treatment 1



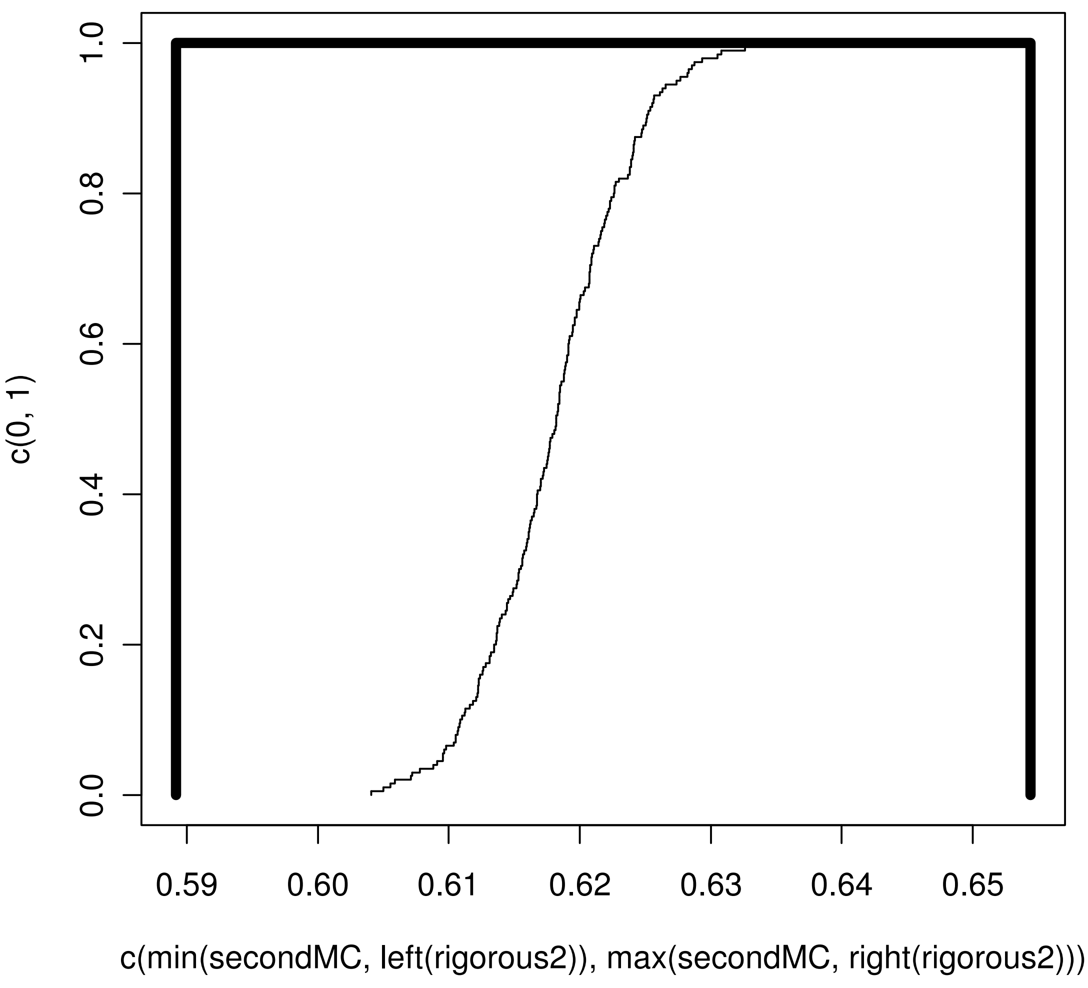
APPENDIX D: COMPUTATIONAL RESULTS-Second Analytical Approach using Risk Calc®

Analysis 2, Scenario [S1], Treatment 1



APPENDIX D: COMPUTATIONAL RESULTS-Second Analytical Approach using Risk Calc®

Analysis 2, Scenario [S1], Treatment 1



Appendix E: Introduction of Risk Calc® and Post-processing in R

The capabilities of Risk Calc have been described in numerous publications by the developer and colleagues (Ferson and Kuhn, 1994; Ferson, 1997, 2001, 2002). There have been numerous studies in the biological sciences utilizing the methods employed in the software to analyze uncertainty and perform risk assessment (see, e.g., Cooper, et al., 1996; Ginzburg, et al., 1996; Kriegler and Held, 2005; Regan, et al., 2002). For further information about the application of Risk Calc, a comprehensive listing of publications may be viewed at the website http://www.ramas.com/riskcalc.htm#refs.

Software Information:
Risk Calc version 4.0

Setauket, New York

Risk Calc does not require the researcher to specify precise details of statistical distributions or dependency relationships when empirical data are lacking. The software uses interval arithmetic and probability theory to cultivate uncertainty through numerical calculations, allowing the researcher to make a risk assessment with confidence (Ferson, 2002). Risk Calc computes with scalars, intervals, fuzzy numbers, probability distributions, and interval bounds on probability distributions. It carries all the uncertainties through calculations automatically.

In analyses conducted using Risk Calc, project activity times may be described as a number, an interval, or a distribution function. The software computes probability bounds on the result (mathematically the best possible) that circumscribe all possibilities (Ferson et al., 1998). The researcher decides what information or assumptions should be used, and the software calculates bounding estimates of risk. Results are shown as probability bounds consisting of non-crossing cumulative distribution functions (CDF) that enclose the paths of all CDF(s) consistent with the problem (Berleant et al., 2006). A limitation of the software is that Risk Calc generates outputs only to a discretization level of 100 (Ferson, 2007). This computational limitation generates probability bounds which are at wide intervals and difficult to interpret with confidence in terms of risk assessment. Ultimately the researcher prefers the generation of perfectly precise distributions with no probability bound width.

To overcome the computational limitation of Risk Calc, the programming language R can be used as a post-processing tool to reduce discretization error. R is a widely used statistical software available under a general public license. According to the website, http://www.r-project.ogr/, R has become a standard among statisticians. The capabilities of R allow the researcher to set an arbitrary number of discretization levels. It was determined from a trial run that 2000 discretization levels was adequate to reduce the error found in the problem of this study. To conduct the post-processing, R requires the addition of an operational library (pbox.r) created as an interface by Risk Calc developers.

Software Information:
R version 2.5.0 (2007-04-23)
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APPENDIX F: New Product Development Process of Company XYZ