THREE ESSAYS ON INSURANCE DEMAND FOR AMBIGUOUS ATEMPORAL AND TEMPORAL RISKS

by

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(Under the Direction of Arthur A. Snow)

ABSTRACT

Drawing on the models of Klibanoff et al.(2005, 2009), Chapter 1 shows that, in the presence of ambiguity, fair pricing remains a necessary and sufficient condition for full insurance coverage to be optimal. This result holds in both atemporal and temporal contexts. With unfairly priced insurance for a temporal risk, a *small* amount of ambiguity aversion leads an ambiguity-averse insurance applicant to demand more coverage and save less than an applicant who is ambiguity-neutral when ambiguity preferences exhibit constant ambiguity aversion. This result holds for an *arbitrary* amount of ambiguity only when ambiguity preferences exhibit a critical degree of increasing absolute ambiguity aversion.

Chapter 2 and 3 attempt to quantitatively confirm the results in Chapter 1 by assuming regular isoelastic specifications for utility functions. What is found is that for the highly regular isoelastic specification, coverage goes up as individual becomes more risk averse or ambiguity averse or both, though ambiguity aversion is not equivalent to an increase in risk aversion in terms of its effect on the strength of demand. In atemporal case and coverage goes up and saving goes down as either risk aversion or ambiguity aversion increases, the latter being consistent with the prediction for small introductions of ambiguity aversion reported in Chapter 1. Since the isoelastic functions may be amenable to empirical estimation, the way may be open to testing this conclusion empirically.

INDEX WORDS: Risk Aversion Ambiguity Aversion Insurance Demand Comparative Statics

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DEDICATION

I dedicate my dissertation work to my parents Qingli Liang and Jie Ren, and also to my aunt Qingbo Liang, who offered me unconditional love and support throughout the completion of this dissertation.

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CHAPTER 1

INSURANCE DEMAND FOR ATEMPORAL AND TEMPORAL RISKS IN THE PRESENCE OF AMBIGUTY

1.1 Introduction

It has been known at least since Knight (1926) that there are two types of risky situations; those in which the probability of the alternative states is known, which he termed one of risk, and those in which the probabilities are not known with precision, which he termed one of uncertainty. Since Ellsberg (1961), the latter has been called a situation of ambiguity. Knight recognized that probabilities are not the elements of a decision that may be uncertain. Langewisch and Choobineh(1996) describe several other ambiguities decision makers may face, such as uncertainty about what can be done. Ellsberg reported on his casual experiments that showed people typically prefer risk to ambiguity, that is, they are ambiguity-averse. The difference between risk and ambiguity is that ambiguous events have a much higher degree of uncertainty, not only the uncertainty about the outcome of an action conditional on the state of nature, but also uncertainty about the probability of a given state of nature occurring. Risk preferences reflect an individual's attitude toward bearing risk about wealth while ambiguity preferences reflect an individual's attitude toward bearing uncertainty about expected utility.

Empirical documentation of the prevalence of ambiguity aversion has accumulated in the intervening years,[1] while casual reflection suggests that situations of risk are relatively rare

[1] See, for example, Camerer and Weber(1992), Einhorn and Hogarth (1986) Sarin and Weber(1993), Chow and Sarin (2001), Chesson and Viscusi (2003), and Halevy (2007)

compared to situations of ambiguity. Given the prevalence of ambiguity and ambiguity aversion, their implications for economic behavior are worthy of study.

However, the early models of ambiguity (discussed below) relied on distortion of probabilities to capture both ambiguity and ambiguity preferences or reflecting extreme ambiguity aversion. The recent literature develops smooth Recursive Expected Utility models [e.g Ergin and Gul (2004), Klibanoff et al. (2005), Nau(2005), Seo (2007), Ahn (2008)]. These models separate risk preferences from ambiguity preferences and distinguish between ambiguity preferences and ambiguous beliefs. Using the model of Klibanoff et al. (2005), several papers have examined the comparative-statics effects of ambiguity aversion on the value of information [Snow (2010)], optimal self-protection [Snow (2011), Huang (2012)], and portfolio choice [Gollier (2011)]. This paper is concerned with the implication of ambiguity aversion for insurance demand.

It is well known that, in an atemporal context, an expected-utility maximizer demands full insurance coverage if and only if the insurance is fairly priced. We extend this conclusion to an environment where an individual is uncertain about the risk of loss and is ambiguity-averse. We then turn to the implications of ambiguity aversion for insurance demand when insurance pricing is unfair. In an atemporal context with ambiguity, ambiguity aversion increases the decision maker's aversion to bearing risk about wealth. Hence, ambiguity aversion increases the demand for insurance when pricing is unfair. In some contents, such as the market for long-term care insurance, there is an important temporal aspect to insurance demand. We first show that, in a temporal context, an ambiguity-averse individual demands full insurance coverage if and only if insurance is fairly priced. Turning to unfair pricing, we show that, in a temporal context with ambiguity, a small increase in the degree of ambiguity aversion increases insurance demand if ambiguity preferences exhibit constant absolute ambiguity aversion, but no sign-definite predictions are possible with either decreasing or increasing ambiguity aversion. However, limited predictions are possible for an arbitrary increase in the degree of ambiguity aversion. Insurance demand increases if for a given degree of ambiguity aversion, ambiguity preference exhibits a critical degree of increasing absolute ambiguity aversion (IAAA).

There are several models that incorporate ambiguity when describing individuals' economic behavior that is inconsistent with expected-utility model. The rank-dependent model developed by Quiggin (1982) and Yaari (1987) and Choquet expected-utility model developed by Schmeidler (1989) characterize ambiguity as distortion of probability but make no distinction between ambiguity and attitude toward bearing ambiguity. Hence, in these models, it is not possible to derive comparative statics predictions for ambiguity aversion. The max/min model developed by Gilboa and Schmeidler (1989) does not allow for changes in the degree of ambiguity aversion. The model of Kahn and Sarin (1988) combines a probability-weighting function to account for ambiguity preferences with a second-order probability distribution representing ambiguous beliefs. Since the probability-weighting function depends on the objective probability to satisfy the expected-utility criterion when ambiguity resolves, risk beliefs and ambiguity preference are not separated.

In this paper, we adopt the smooth recursive models developed by Klibanoff et al. (2005) and temporal (2009) decision environments to carry out our analysis as they separate ambiguous beliefs from ambiguity preferences and risk preferences. These models are well-suited to our purposes as they represent uncertainty about the risk of loss by a second-order probability distribution defined on possible values of risk, and capture attitudes towards bearing this uncertainty in a preference functional defined on expected utility. This separation of risk preferences and ambiguity preferences allows us to determine the effect of ambiguity aversion on insurance demand in the presence of a given ambiguity. In addition, the smooth recursive structure of the model allows us to adapt well-known techniques for analyzing risk preferences to an investigation of ambiguity preferences. Throughout the paper, we assume time separability for preferences in a temporal context so that we can apply in the temporal context insights gleaned in the atemporal analysis.

1.2 Insurance Demand for Atemporal Risk

In an atemporal context, a risk-averse, expected-utility maximizer facing a known risk of loss demands full coverage if and only if insurance pricing is fair, with partial coverage demanded when insurance pricing is unfair[2]. We first provide a formal proof of this result as a benchmark for contrasting it with subsequent proofs of the same result in the presence of ambiguity.

We assume there are two states of nature, an accident state *A* in which a loss *L* is incurred and a state *N* when no loss is incurred, so that state-contingent wealth is given by

$$W_A = M - pc - L + c \tag{1a}$$

$$W_N = M - pc \tag{1b}$$

where *M* is initial wealth, *p* is the insurance premium rate, and *c* is the quantity of coverage. Combining equations (1a) and (1b) by eliminating *c* yields the budget equation

$$pW_A + (1-p)W_N = M - pL\ . \tag{1c}$$

1.2.1 Insurance Demand without Ambiguity

The individual's problem in the absence of ambiguity is to maximize expected utility

$$EU(W_A, W_N, \bar{\theta}) \equiv \bar{\theta}u(W_A) + (1-\bar{\theta})u(W_N) \tag{2}$$

[2] Georges Dionne, Handbook of Insurance (Schlesinger, 2000),134

where $\bar{\theta}$ is the objective probability of state *A* and *u* is a von Neumann-Morgenstern utility function defined over final wealth, assumed to be everywhere differentiable with $u' > 0$ and $u'' < 0$[3]. The concavity of *u* indicates risk aversion.

Introducing the Lagrange multiplier λ for the budget constraint (1c), the first-order conditions for maximizing expected utility (2) with respect to W_A and W_N are

$$\bar{\theta}u'_A - \lambda p = 0 \tag{3a}$$

$$(1-\bar{\theta})u'_N - \lambda(1-p) = 0\,. \tag{3b}$$

Together, these equations yield the tangency condition

$$MRS^{EU} \equiv \frac{(1-\bar{\theta})u'_N}{\bar{\theta}u'_A} = \frac{1-p}{p} \tag{3c}$$

equating the marginal rate of substitution with the price ratio.

Proposition 1: In an atemporal context without ambiguity, a risk-averse insurance applicant maximizing expected utility (2) demands full insurance coverage if and only if insurance pricing is fair.

Proof. Necessity: With full insurance coverage, $c = L$ and therefore $W_A = W_N$ and $u'_A = u'_N$.

The tangency condition (3c) becomes

$$\frac{1-\bar{\theta}}{\bar{\theta}} = \frac{1-p}{p}\,. \tag{4}$$

For this equality to hold, *p* must equal $\bar{\theta}$.

Sufficiency: With fair insurance pricing, $p = \bar{\theta}$ and marginal condition (3c) becomes

[3] Throughout, primes are used to denote derivatives of univariate functions

$$\frac{u'_N}{u'_A} = 1. \tag{5}$$

For this equality to hold, W_A must equal to W_N, which implies $c = L$. Therefore, under fair pricing, full coverage insurance is optimal. ■

1.2.2 Insurance Demand with Ambiguity

When the individual is uncertain about the risk of loss, the demand for insurance coverage depends on the individual's attitude toward this uncertainty as well as the individual's willingness to bear risk. To introduce ambiguity, we adopt the atemporal smooth recursive model developed by Klibanoff et al. (2005) and assume that the individual is uncertain about the probability of experiencing the accident. We represent this uncertainty by a (second-order) probability distribution $F(\theta)$ defined on the possible values for the probability of loss, with support contained in the interval [0,1]. An expected-utility maximizer is ambiguity-neutral, since expected utility is linear in probabilities. We are interested in the behavior of decision makers who are ambiguity-averse.

The axiom system developed by Klibanoff et al. (2005) captures the willingness to bear ambiguity in a preference functional defined on expected utility. The resulting decision criterion is

$$AEU(W_A, W_N) \equiv \varphi^{-1}\left(\int \varphi(EU(W_A, W_N, \theta)\, dF(\theta)\right) \tag{6}$$

where $F(\theta)$ represents the individual's beliefs about the probability of loss and φ is an increasing function defined on expected utility. With ambiguity neutrality, φ is linear; with ambiguity aversion, φ is concave.

We assume that the insurer knows that the objective probability of loss is $\bar{\theta}$, while the insurance applicant is uncertain about the value of θ but has beliefs that are unbiased in the sense that $\int \theta \, dF(\theta) = \bar{\theta}$. This assumption, which follows from the observation that ambiguity cannot affect the behavior of an ambiguity-neutral decision maker, ensures concordance between the beliefs of the insurer and those of the applicant.

Introducing the Lagrange multiplier λ^a for the budget constraint (1c), the first-order conditions for maximizing ambiguous expected utility (6) with respect to W_A and W_N are

$$[1/\varphi'(AEU)]\int \varphi' \cdot \theta \, dF(\theta) u'_A - \lambda^a p = 0 \tag{7a}$$

$$[1/\varphi'(AEU)]\int \varphi' \cdot (1-\theta) \, dF(\theta) u'_N - \lambda^a (1-p) = 0 \, . \tag{7b}$$

Together, these equations yield the tangency condition

$$MRS^{AEU} = \frac{\int \varphi' \cdot (1-\theta) \, dF(\theta) u'_N}{\int \varphi' \cdot \theta \, dF(\theta) u'_A} = \frac{1-p}{p} \tag{7c}$$

equating the ambiguous marginal rate of substitution with the price ratio. Our next result shows that the conclusion of Proposition 1 carries over from contracting without ambiguity to contracting with ambiguity and ambiguity aversion.

Proposition 2: In an atemporal context with ambiguity, a risk-averse and ambiguity-averse insurance applicant with decision criterion (6) demands full insurance coverage if and only if insurance pricing is fair.

Proof. Necessity: With full insurance coverage, $c = L$ and therefore $W_A = W_N$ and $u'_A = u'_N$, while expected utility $EU(W_A, W_N, \theta)$ and, hence, φ' are independent of θ. The tangency condition (7c) then becomes

$$\frac{1-\bar{\theta}}{\bar{\theta}} = \frac{1-p}{p}. \tag{8}$$

For this equality to hold, p must equal $\bar{\theta}$.

Sufficiency: With fair insurance pricing, $p = \bar{\theta}$ and $c = L$ satisfies the tangency condition (7c). Suppose that $c < L$ is optimal. In that event, $W_A < W_N$ and condition (7c) then implies

$$\int \varphi' \cdot (1-\theta)\, dFu'_N \bar{\theta} = \int \varphi' \cdot \theta\, dFu'_A (1-\bar{\theta}) > \int \varphi' \cdot \theta\, dFu'_N (1-\bar{\theta}) \tag{9}$$

which yields $\int \varphi' \cdot (\bar{\theta}-\theta)\, dFu'_N > 0$. However, *EU* declines as θ increases, while φ' is decreasing in *EU* given ambiguity aversion. It follows that φ' increases as θ increases, which implies the contradiction

$$0 > \text{cov}(\varphi', \bar{\theta}-\theta) = \int \varphi'(\bar{\theta}-\theta) dF(\theta). \tag{10}$$

Hence, the supposition must be false. A similar argument shows $W_A > W_N$ cannot be optimal. Therefore, under fair pricing full insurance coverage is optimal. ■

Note that, while the proof of necessity follows the same course here as the proof of Proposition 1, the proof of sufficiency involves a different line of reasoning. In the absence of ambiguity, fair pricing implies equality between the marginal utilities of the two states, as the tangency condition (3c) for optimal coverage implies equation (5) equating the two marginal utilities. By contrast, in the presence of ambiguity, equation (7c) is the tangency condition for optimal coverage, and fair pricing does not immediately imply equation (5), but rather proof by contradiction establishes the equality.

1.2.3 Insurance Demand with Ambiguity and Unfair Pricing

To analyze the effect of ambiguity aversion insurance demand with unfair pricing, we first show that, in the atemporal context, ambiguity aversion increases aversion to bearing risk about wealth in the presence of ambiguity.

Proposition 3: In the presence of ambiguity, ambiguity aversion increases aversion to bearing risk about wealth for the ambiguous expected utility criterion (6).

Proof: Proposition 2 implies that the individual's marginal rate of substitution is equal to the fair-odds ratio when $W_A = W_N$. To verify this, observe that

$$MRS^{AEU}\Big|_{W_A=W_N} = \frac{\int \varphi' \cdot (1-\theta)\, dF u_N'}{\int \varphi' \cdot \theta\, dF u_A'} = \frac{1-\bar{\theta}}{\bar{\theta}}, \tag{11}$$

where the final equality follows from our assumption that ambiguous beliefs are unbiased. For contingent wealth pairs with $W_A < W_N$, introducing ambiguity aversion in the presence of ambiguity reduces the marginal rate of substitution if

$$MRS^{AEU}\Big|_{W_A<W_N} = \frac{\int \varphi' \cdot (1-\theta)\, dF u_N'}{\int \varphi' \cdot \theta\, dF u_A'} < \frac{(1-\bar{\theta})u_N'}{\bar{\theta} u_A'}. \tag{12}$$

The inequality requires a negative value for $\int \varphi' \cdot (\bar{\theta} - \theta)\, dF$, which we have established at inequality (10) given ambiguity aversion. Inequality (12) is reversed for wealth pairs with $W_A > W_N$. It follows that the Yaari (1969) acceptance sets for ambiguous expected utility (6) are nested within those for expected utility (2), indicating that AEU is more averse to risk about wealth than is $EU(\bar{\theta})$. See Figure 1. ■

Since partial coverage is demanded when insurance pricing is unfair, we conclude from Proposition 3 that insurance demand for ambiguity-averse decision makers is greater in the presence of ambiguity than in its absence. Thus, in the atemporal context with ambiguity and unfair insurance pricing, an ambiguity-averse decision maker has a greater demand for insurance than an ambiguity-neutral decision maker with the same risk preferences.

1.2.4 Wealth Effects with Ambiguity

To examine the effect of changes in wealth on insurance demand with unfair pricing, we begin with an ambiguity neutral insurance applicant who maximizes *EU* defined at equation (2). The effect of greater wealth on *MRS* defined at equation *(3c)* is given by

$$\frac{\partial MRS^{EU}}{\partial M} = MRS^{EU}\left[R_u(W_A) - R_u(W_N)\right], \tag{13}$$

where $R_u(w) \equiv -u''(w)/u'(w)$ is the index of absolute risk aversion for *u*. With decreasing absolute risk aversion (DARA), equation (13) is positive and the income expansion path in wealth space is flatter than the 45 degree line. Hence, with DARA, insurance coverage is inferior.

For ambiguous expected utility, *AEU* defined at equation (6), the marginal rate of substitution is given at equation (7c). The effect of greater wealth on *MRS* is given by

$$\begin{aligned}\frac{dMRS^{AEU}}{dM} &= MRS^{AEU}\left(\frac{\int \varphi'' u' \cdot (1-\theta)\, dF(\theta) u_N' + \int \varphi' \cdot (1-\theta)\, dF(\theta) u_N''}{\int \varphi' \cdot (1-\theta)\, dF(\theta) u_N'}\right. \\ &\quad \left. - \frac{\int \varphi'' u' \cdot \theta\, dF(\theta) u_A' + \int \varphi' \cdot \theta\, dF(\theta) u_A''}{\int \varphi' \cdot \theta\, dF(\theta) u_A'}\right) \\ &= MRS^{AEU}\left[R_u(W_A) - R_u(W_N) + \Omega\right]\end{aligned} \tag{14}$$

where

$$\Omega \equiv \frac{\int \varphi'' \cdot (1-\theta)\, dF(\theta) u'_N}{\int \varphi' \cdot (1-\theta)\, dF(\theta)} - \frac{\int \varphi'' \cdot \theta\, dF(\theta) u'_A}{\int \varphi' \cdot \theta\, dF(\theta)}. \tag{14a}$$

With constant absolute ambiguity aversion (CARA) or DARA, the difference in risk aversion measure is nonnegative. We next show that Ω is positive.

Let $u' = \theta u'_A + (1-\theta) u'_N = \hat{u}' + \varepsilon$ where $\hat{u}' = \hat{\theta} u'_A + (1-\hat{\theta}) u'_N$ and ε is the residual of u' from $\hat{u}'$. The critical value $\hat{\theta}$ is identified below. In both numerators on the right hand side of (14a), multiply the integrand by φ' / φ' With CAAA, the index of absolute ambiguity aversion, $R_\varphi(EU) = -\varphi''(EU) / \varphi'(EU)$, is constant and therefore can be factored out of each integral, resulting in

$$\begin{aligned}
\Omega &= \frac{\int \varphi'' \cdot (\varphi'/\varphi') \cdot (1-\theta)\, dF(\theta) u'_N}{\int \varphi' \cdot (1-\theta)\, dF(\theta)} - \frac{\int \varphi'' \cdot (\varphi'/\varphi') \cdot \theta\, dF(\theta) u'_A}{\int \varphi' \cdot \theta\, dF(\theta)} \\
&= -R_\varphi [\frac{\int \varphi' \cdot (1-\theta)\, dF(\theta)(u'_N)}{\int \varphi' \cdot (1-\theta)\, dF(\theta)} - \frac{\int \varphi' \cdot \theta\, dF(\theta)(u'_A)}{\int \varphi' \cdot \theta\, dF(\theta)}] \\
&= -R_\varphi \cdot \frac{1}{\int (1-\theta)\varphi' dF} [\int \varphi' \varepsilon dF - \int \theta \varphi' \varepsilon \frac{\int \varphi' dF}{\int \theta \varphi' dF}] \\
&== -R_\varphi \cdot \frac{1}{\int (1-\theta)\varphi' dF} [\int \varphi' \varepsilon (1 - \theta \frac{\int \varphi' dF}{\int \theta \varphi' dF}) dF]
\end{aligned} \tag{15}$$

Let $\hat{\theta}$ be the critical value such that $1 - \hat{\theta} \frac{\int \varphi' dF}{\int \theta \varphi' dF} = 0$. It then follows that the term

$1 - \theta \frac{\int \varphi' dF}{\int \theta \varphi' dF}$ changes signs from positive to negative at $\hat{\theta}$ as θ increases, while

$\varepsilon\big|_{\theta=\hat{\theta}} = u'\big|_{\theta=\hat{\theta}} - \hat{u}' = 0$ and ε then changes signs from negative to positive at $\hat{\theta}$ as θ increases given ambiguity aversion, Therefore, the product of the two is always negative, leading Ω to be positive. Thus with nonincreasing absolute risk aversion (NIARA) and CAAA, $\frac{\partial MRS^{AEU}}{\partial M} > 0$,

For *AEU,* insurance is inferior. See Figure 3.

Proposition 4: If u exhibits nonincreasing absolute risk aversion NIARA and φ exhibits CAAA, then *AEU* behaves as if it were a risk-averse *EU* with DARA.

This proposition shows that a decision maker with CARA behaves with DARA in the presence of ambiguity when ambiguity preferences exhibit CAAA.

1.3 Insurance Demand for Temporal Risk

In a two-period context of temporal risk, the insurance applicant faces a risk of loss in the second period, and can save and buy insurance coverage in the first period. In the second period, the individual consumes the return to saving in the first period and receives coverage in the accident state. Final wealth in the first period is denoted W_1, while W_A and W_N denote the state contingent, second-period wealth. These wealth levels are given by

$$W_1 = M - pc - s \tag{16a}$$

$$W_A = M + s - L + c \tag{16b}$$

$$W_N = M + s \tag{16c}$$

where the initial wealth endowment *M* is the same in both periods, s is the amount saved, and the interest rate has been normalized to equal zero. Combining equation (16a)-(16c) by eliminating *s* and *c* yields the budget equation

$$W_1 + pW_A + (1-p)W_N = 2M - pL\,. \tag{16d}$$

1.3.1 Insurance Demand without Ambiguity

The individual's decision criterion in the absence of ambiguity is intertemporal expected utility with time-separable preferences

$$u(W_1) + EU(W_A, W_N, \bar{\theta}) \equiv u(W_1) + \bar{\theta}u(W_A) + (1-\bar{\theta})u(W_N) \qquad (17)$$

where, for simplicity, the pure rate of time preference has been normalized to equal zero. Using λ to denote the Lagrange multiplier for the budget constraint (1dd), the first-order conditions for optimizing the Lagrangian with respect to W_1, W_A, and W_N to maximize criterion (17) are

$$u_1' - \lambda = 0 \qquad (18a)$$

$$\bar{\theta}u_A' - \lambda p = 0 \qquad (18b)$$

$$(1-\bar{\theta})u_N' - \lambda(1-p) = 0 \qquad (18c)$$

Observe that the optimal rate of substitution between wealth in states *A* and *N* is

$$\frac{(1-\bar{\theta})u_N'}{\bar{\theta}u_A'} = \frac{1-p}{p} \qquad (18d)$$

just as in the atemporal context as shown in equation (3c).

Proposition 4: In a temporal context without ambiguity, a risk-averse expected utility maximizer demands full insurance coverage if and only if insurance pricing is fair.

Proof. Since marginal condition (18d) is equivalent to condition (7c), the proof is the same as the proof of Proposition 1. ■

1.3.2 Insurance Demand with Ambiguity

To analyze insurance demand in a temporal context with ambiguity, we adopt the temporal decision criterion developed by Klibanoff et al. (2009), imposing the assumption of time-separable preferences.

$$u(W_1) + AEU(W_A, W_N) = u(W_1) + \varphi^{-1}\left(\int \varphi(EU(W_A, W_N, \theta)\, dF(\theta)\right) \qquad (19)$$

Using λ^a to denote the Lagrange multiplier for budget constraint (16d), the first-order conditions for maximizing criterion (19) with respect to W_1, W_A, and W_N are

$$u_1' - \lambda^a = 0 \qquad (20a)$$

$$[1/\varphi'(AEU)]\int \varphi' \cdot \theta\, dF(\theta) u_A' - \lambda^a p = 0 \qquad (20b)$$

$$[1/\varphi'(AEU)]\int \varphi' \cdot (1-\theta)\, dF(\theta) u_N' - \lambda^a (1-p) = 0\,. \qquad (20c)$$

The marginal rate of substitution between wealth in states *A* and *N* is

$$\frac{\int \varphi' \cdot (1-\theta)\, dF(\theta) u_N'}{\int \varphi' \cdot \theta\, dF(\theta) u_A'} = \frac{1-p}{p} \qquad (20d)$$

just as in the atemporal context as shown in equation (7c).

Proposition 5: In a temporal context with ambiguity, a risk-averse and ambiguity-averse insurance applicant with decision criterion (19) demands full insurance coverage if and only if insurance pricing is fair.

Proof: Since the tangency condition (20d) is equivalent to condition (3c), the proof is the same as the proof of Proposition 1. See Figure 2. ■

1.3.3 Insurance Demand with Ambiguity and Unfair Pricing

In this section, the implications of ambiguity aversion for insurance demand are examined in a temporal context where ambiguity is present and insurance pricing is unfair, so that *p* exceeds the objective risk of loss $\bar{\theta}$. In the atemporal context considered in section 2.3, ambiguity aversion is shown to increase insurance demand in the presence of ambiguity with unfair pricing since it increases aversion to bearing risk about wealth. This result does not carry over to the temporal context without qualification since the saving decision transfers wealth across time periods, introducing an additional response to ambiguity that bears on the insurance decision.

1.3.3.1 Ambiguity Aversion in the Small

We first consider the implications of introducing a small degree of ambiguity aversion in an environment with ambiguity present.

> *Proposition 6*: In the presence of ambiguity and unfair insurance pricing, introducing a small degree of ambiguity aversion results in an increase in insurance demand and lower saving for insurance applicants with decision criterion (19) whose ambiguity preferences exhibit constant absolute ambiguity aversion.

Proof: Using equations (16a)-(16d), define $V(c,s,\bar{\theta})$ to be the intertemporal expected utility criterion without ambiguity (17), and define $\hat{V}(c,s,\alpha)$ to be criterion (19) with ambiguity, introducing the shift parameter α for the ambiguity preference functional φ such that φ is ambiguity neutral when α equals α^o, so that $\hat{V}(c,s,\alpha^o) \equiv V(c,s,\bar{\theta})$, and φ becomes ambiguity averse with an incremental increase α in above α^o.

The first-order conditions for maximizing $\hat{V}_c(c,s,a)$are

$$\hat{V}_c(c^o,s^o,\alpha) = -pu'(W_1) + \partial AEU / \partial c = 0 \tag{21a}$$

$$\hat{V}_s(c^o,s^o,\alpha) = -u'(W_1) + \partial AEU / \partial s = 0 \tag{21b}$$

where φ is linear and c^o and s^o are the optimal saving and insurance coverage choices when $\alpha = \alpha^o$.

The comparative statics effect of increasing α above α^o are obtained by totally differentiating the first-order conditions (21a) and (21b) to arrive at

$$\begin{bmatrix} V_{cc} V_{cs} \\ V_{sc} V_{ss} \end{bmatrix} \begin{bmatrix} dc^o / d\alpha \\ ds^o / d\alpha \end{bmatrix} = \begin{bmatrix} -\hat{V}_{c\alpha} \\ -\hat{V}_{s\alpha} \end{bmatrix} \tag{22}$$

Solving the equation system (22) for the comparative statics effects on *c* yields

$$\frac{dc}{d\alpha}|_{\alpha=\alpha^o} = [-\hat{V}_{c\alpha} V_{ss} + \hat{V}_{s\alpha} V_{cs}]/H \ , \tag{23}$$

where *H* is the determinant of the Hessian matrix for $\hat{V}(c,s,\alpha)$. In the appendix, the following inequalities are established: $V_{ss} < V_{cs} < 0$ and $V_{cc} < V_{cs}$. It is also shown that *V* is a strictly concave function of *c* and *s* since *u* is risk averse, so that *H* is positive. Therefore, the comparative statics effect on *c* of introducing ambiguity aversion has the same sign as

$$-V_{ss}\hat{V}_{c\alpha} + V_{cs}\hat{V}_{s\alpha} . \tag{24}$$

To determine the sign of $\hat{V}_{c\alpha}$, observe that $\hat{V}_{c\alpha}$ is positive if

$$\hat{V}_c(c,s,\alpha) > \hat{V}_c(c,s,\alpha^o) \equiv V_c(c,s,\bar{\theta})$$

This inequality requires

$$-pu'_1 + [1/\varphi'(AEU)]\int \varphi'\theta dF(\theta) u'_A > -pu'_1 + \bar{\theta} u'_A \tag{25a}$$

$$\Leftrightarrow [1/\varphi'(AEU)]\int \varphi'\theta dF(\theta) > \bar{\theta} \tag{25b}$$

$$\Leftrightarrow \int \varphi'\theta dF(\theta) > \bar{\theta} \cdot \varphi'(AEU) . \tag{25c}$$

Since φ' is increasing in θ, we have

$$0 < \mathrm{cov}(\varphi', \theta) = \int \varphi'\theta dF(\theta) - \int \varphi' dF(\theta) \cdot \bar{\theta} . \tag{26}$$

Therefore inequality (25c) holds, and $\hat{V}_{c\alpha}$ is positive if

$$\int \varphi' dF \geq \varphi'(AEU) . \tag{27}$$

For $AEU(W_A, W_N)$, introduce π_φ to denote the ambiguity premium and ψ_φ to denote ambiguity prudence premium, so that

$$AEU(W_A, W_N, \alpha) = \varphi^{-1}(\int \varphi(EU(\theta))dF(\theta)) = EU(\bar{\theta}) - \pi_\varphi \tag{28a}$$

$$\int \varphi'(EU(\bar{\theta}))dF(\theta) = \varphi'(EU(\bar{\theta}) - \psi_\varphi)\,. \tag{28b}$$

Thus, inequality (27) is equivalent to

$$\varphi'(EU(\bar{\theta}) - \psi_\varphi) \geq \varphi'(EU(\bar{\theta}) - \pi_\varphi) \tag{29}$$

By analogy with expected utility theory, this inequality holds if ambiguity preferences exhibit nonincreasing absolute ambiguity aversion, for then the degree of ambiguity aversion is less than or equal to the degree of ambiguity prudence and $\psi_\varphi \geq \pi_\varphi$. Under this condition $\hat{V}_{c\alpha}$ is positive.

Similarly, the marginal utility of saving with ambiguity aversion is

$$\hat{V}_s(c, s, \alpha) = -u_1 + [1/\varphi'(AEU)]\int \varphi'[\theta u'_A + (1-\theta)u'_N]dF(\theta) \tag{30}$$

while with ambiguity neutral, we have

$$\hat{V}_s(c, s, \alpha^o) \equiv V_s(c, s, \bar{\theta}) = -u_1 + \bar{\theta}u'_A + (1-\bar{\theta})u'_N\,. \tag{31}$$

Comparing (30) and (31), we see that the term involving u'_A in (30) is greater than the corresponding term in (31) if inequality (25c) holds, for which $\psi_\varphi \geq \pi_\varphi$ is sufficient. As for the term involving u'_N, we have

$$1 - \bar{\theta} > [1/\varphi'(AEU)]\int \varphi'(1-\theta)dF(\theta) \tag{32a}$$

$$\Leftrightarrow (1-\bar{\theta}) \cdot \varphi'(AEU) > \int \varphi'(1-\theta)dF(\theta)\,. \tag{32b}$$

Since φ' is increasing in θ, we have

$$0 > \text{cov}(\varphi', 1-\theta) = \int \varphi'(1-\theta) dF(\theta) - (1-\bar{\theta}) \int \varphi' dF(\theta) \tag{33}$$

Therefore, inequality (32b) holds if

$$\int \varphi' \, dF(\theta) \leq \varphi'(AEU), \tag{34}$$

which requires nondecreasing absolute ambiguity aversion or $\psi_\varphi \leq \pi_\varphi$.

We conclude that, with constant absolute ambiguity aversion, we have $\hat{V}_{c\alpha} > 0$ from inequality (25c), while inequality (32b) implies $\hat{V}_{c\alpha} > \hat{V}_{s\alpha}$.

With $\hat{V}_{ss} < \hat{V}_{cs} < 0$, (24) is then positive. Therefore, the introduction of a small degree of ambiguity aversion results in an increase in demand of insurance coverage given constant absolute ambiguity aversion.

Similarly, the effect of introducing a small degree of ambiguity aversion on saving behavior is given by

$$\frac{ds}{d\alpha}\Big|_{\alpha=\alpha^o} = [-\hat{V}_{s\alpha} V_{cc} + \hat{V}_{c\alpha} V_{sc}]/H . \tag{35}$$

With $\hat{V}_{c\alpha} > 0$, $\hat{V}_{c\alpha} > \hat{V}_{s\alpha}$ and $\hat{V}_{sc} < \hat{V}_{cc} < 0$, it is clear that (35) is negative.■

In sum, behavioral changes associated with ambiguity aversion in the small do not depend on risk preferences. When a small degree of ambiguity-aversion is introduced, an individual with constant absolute ambiguity aversion saves less and demands more insurance coverage. Finally, it is worth mentioning that, aside from the second derivatives of *V* the only sign-definite information is provided by the two covariance expressions (26) and (33) which have opposite signs. Thus, no predictions are forthcoming with either decreasing or increasing absolute ambiguity aversion; only constant ambiguity aversion yields sign-definite predictions.

1.3.3.2 Ambiguity Aversion in the Large

A more limited result is obtained for large increases in ambiguity aversion. The first-order condition for saving by an insurance applicant who is ambiguity neutral is

$$-u'(W_1) + \bar{\theta} u'_A + (1-\bar{\theta}) u'_N = 0 \qquad (36)$$

With the introduction of ambiguity aversion, the net marginal benefit of saving becomes positive if

$$-u'(W_1) + \frac{1}{\varphi'(AEU)} \int \varphi' \cdot [\theta u'_A + (1-\theta) u'_N] dF > -u'(W_1) + \bar{\theta} u'_A + (1-\bar{\theta}) u'_N \qquad (37)$$

$$\Leftrightarrow \int \varphi' \cdot [\theta u'_A + (1-\theta) u'_N] dF > \varphi'(AEU) \cdot [\bar{\theta} u'_A + (1-\bar{\theta}) u'_N]$$

Define $u' = \theta u'_A + (1-\theta) u'_N$ and $\bar{u}' = \bar{\theta} u'_A + (1-\bar{\theta}) u'_N$. Since φ' and u' are both decreasing functions in θ,

$$0 < \operatorname{cov}(\varphi', u') = \int \varphi' \cdot u' dF - \int \varphi' dF \cdot \bar{u}'$$

Therefore, the sufficient condition for (35) to hold is

$$\int \varphi' dF \cdot \bar{u}' \geq \varphi'(AEU) \cdot \bar{u}'$$

$$\Leftrightarrow \int \varphi' dF \geq \varphi'(AEU),$$

which, as we have seen, requires $\psi_\varphi \geq \pi_\varphi$.

Therefore, with NIAAA,the marginal benefit of saving increases after introducing an arbitrary degree of ambiguity aversion. In order to reach the new optimal allocation of wealth, W_1 must decline and *s+pc* must increase. The effect on 2nd-period wealth is then

$$p dW_A + (1-p) dW_N = p(ds + dc) + (1-p) ds = ds + p \cdot dc > 0$$

That is, with NIAAA, the 2nd period budget line shifts out. As shown in Proposition 4, with NIARA and CAAA, AEU exhibits DARA and the income expansion path is flatter than 45 degree line. As a consequence, no prediction is forthcoming for the effect of ambiguity aversion on optimal saving and insurance coverage when the decision maker exhibits NIAAA. See Figure 5. It follows that sign-definite predictions are possible only if ambiguity aversion has no effect on the marginal benefit of saving, which requires IAAA. See Figure 4.

Proposition 7: In the presence of ambiguity and unfair insurance pricing, ambiguity aversion results in greater insurance demand and lower saving for insurance applicants with decision criterion (19) if for a given degree of ambiguity aversion, ambiguity preferences exhibit a critical degree of IAAA.

Proof: With the introduction of ambiguity aversion, the net marginal benefit of saving does not change if

$$-u'(W_1)+\frac{1}{\varphi'(AEU)}\int\varphi'\cdot[\theta u'_A+(1-\theta)u'_N]dF=-u'(W_1)+\bar{\theta}u'_A+(1-\bar{\theta})u'_N \qquad \Leftrightarrow$$

$$\int\varphi'\cdot[\theta u'_A+(1-\theta)u'_N]dF=\varphi'(AEU)\cdot[\bar{\theta}u'_A+(1-\bar{\theta})u'_N] \tag{38}$$

$$\Leftrightarrow \int\varphi'\cdot u'dF=\varphi'(AEU)\cdot\bar{u}' \tag{38a}$$

Subtract $\int\varphi' dF\bar{u}'$ on both sides to arrive at,

$$\int\varphi'\cdot(u'-\bar{u}')dF=[\varphi'(AEU)-\int\varphi' dF]\cdot\bar{u}' \text{ ,}$$

which can be rewritten as

$$\varphi'(\bar{u}-\pi_\varphi)-\varphi'(\bar{u}-\psi_\varphi)=\frac{\int\varphi'\cdot(u'-\bar{u}')dF}{\bar{u}'}. \tag{39}$$

As the right-hand side is positive ψ_φ must exceed π_φ by a critical amount. For equation (39) to hold, ambiguity preference must exhibit a critical degree of IAAA. ■

1.4 Conclusion

We have shown that in the presence of ambiguity, fair pricing is still a necessary condition for full-coverage insurance to be optimal as is the case in expected utility theory in both atemporal and temporal contexts and the sufficiency side of this result is established through proof by contradiction. In the atemporal context, ambiguity aversion increases the aversion to bearing risk about wealth and ambiguous expected utility is more averse to risk than expected utility, implying that for an ambiguity-averse decision makers demand more insurance in the presence of ambiguity than in its absence. This result does not carry over to the temporal context without further qualification since the saving decision transfers wealth across time periods and introduce an additional response to ambiguity that bears on the insurance decision.

With unfairly priced insurance, introducing a small amount of ambiguity results in an increase in insurance coverage and a decrease in savings for an ambiguity-averse insurance applicant if the ambiguity preference exhibits constant ambiguity aversion (CAAA). The monotonicity property of the marginal utility of coverage and savings with respect to ambiguity aversion parameter is guaranteed only if the degree of ambiguity aversion equal to the degree of ambiguity prudence, which requires constant absolute ambiguity aversion.

We also discuss that if an arbitrary amount of ambiguity is introduced, no prediction is forthcoming for the effect of ambiguity aversion on insurance demand and saving if the decision maker exhibits nonincreasing absolute ambiguity aversion (NIAAA). This is because with

NIAAA. , This is because with NIAAA, AEU exhibits DARA and the strength of DARA results in opposing effects of ambiguity aversion on optimal insurance coverage and savings. Therefore, the sign-definite predictions are possible only if ambiguity aversion has no effect on the marginal benefit of saving, which requires ambiguity preference exhibit a critical degree of IAAA.

A APPENDIX

A.1 Proofs for Ambiguity Aversion in Small

The second derivative of $V(c,s,\bar{\theta})$ are given by

$$V_{ss} = u_1'' + \bar{\theta}u_A'' + (1-\bar{\theta})u_N'' \quad \text{(A.1a)}$$

$$V_{sc} = V_{cs} = pu_1'' + \bar{\theta}u_A'' \quad \text{(A.1b)}$$

$$V_{cc} = p^2u_1'' + \bar{\theta}u_A'' \quad \text{(A.1c)}$$

The we have the following

$$V_{cs} - V_{ss} = -(1-p)u_1'' - (1-\bar{\theta})u_N'' \quad \text{(A.2a)}$$

$$V_{cc} - V_{sc} = -p(1-p)u_1'' \quad \text{(A.2b)}$$

Since *u* is concave, the second order derivatives are negative and (A.2a) and (A.2b) are both positive. Therefore,

$$V_{ss} < V_{cs} < 0 \quad \text{(A.3a)}$$

$$V_{sc} - V_{cc} < 0 \quad \text{(A.3a)}$$

Finally, the determinant of the Hessian matrix for *V* is

$$\begin{aligned} H &= V_{cc}V_{ss} - V_{cs}^2 \\ &= (p^2u_1'' + \bar{\theta}u_A'')(u_1'' + \bar{\theta}u_A'' + (1-\bar{\theta})u_N'' - (pu_1'' + \bar{\theta}u_A'')^2 \\ &= \bar{\theta}(1-p)^2u_1''u_A'' + p^2(1-\bar{\theta})u_1''u_N'' + \bar{\theta}(1-\bar{\theta})u_1''u_N'' > 0 \end{aligned} \quad \text{(A.4)}$$

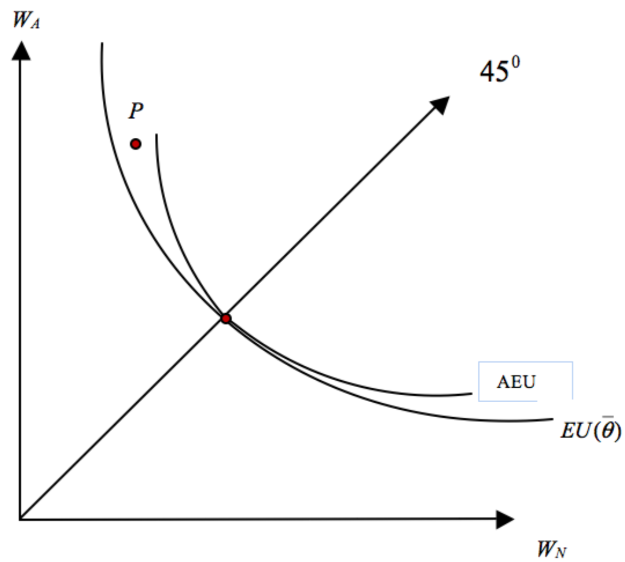

Figure 1-1

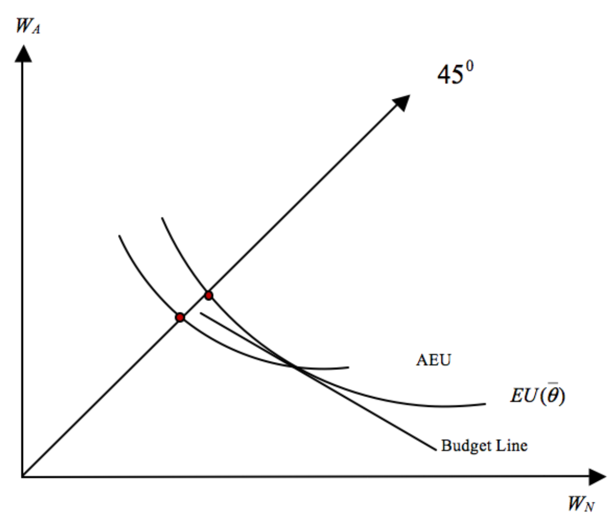

Figure 1-2

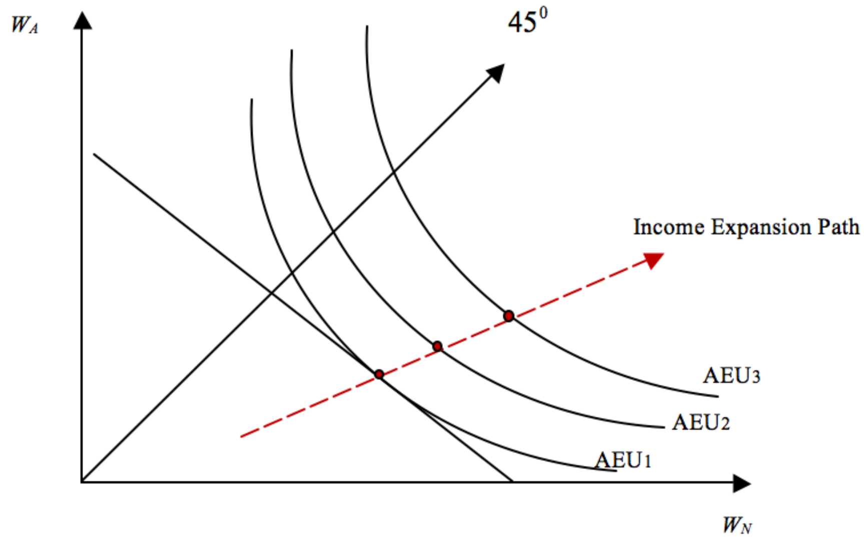

Figure 1-3

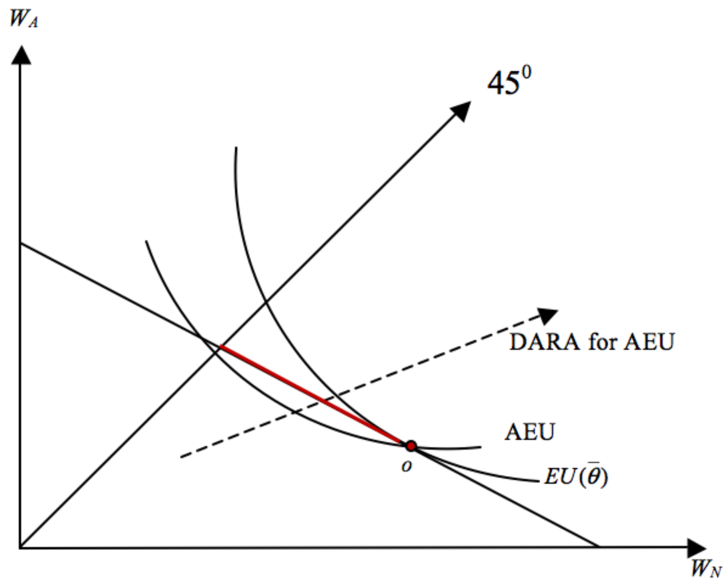

Figure 1-4

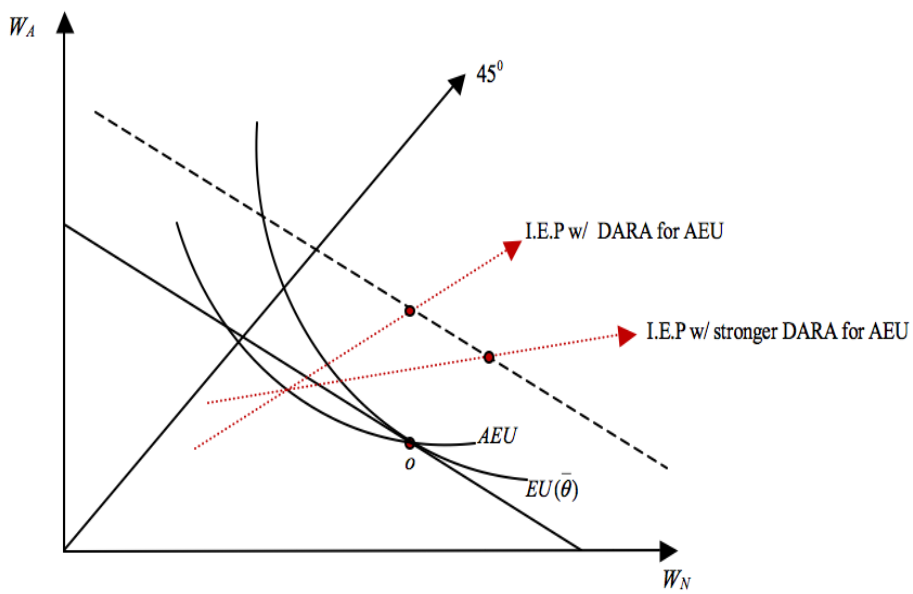

Figure 1-5

CHAPTER 2

A NUMERICAL STUDY OF AMBIGUITY AVERSION ON INSURANCE DEMAND FOR ATEMPORAL RISK

2.1 Introduction

In chapter 1, we demonstrated that, in an atemporal context, ambiguity aversion increases the demand for insurance coverage. Specifically, in the presence of ambiguity, if an ambiguity-neutral insurance applicant demands positive, but less-than-full coverage, then the same applicant demands more coverage when ambiguity averse, while still demanding less than the full coverage. In this chapter, we complement this qualitative result with a quantitative analysis designed to address the question: How much does ambiguity aversion matter for insurance demand?

It is well-established that an increase in risk aversion reduces the demand for risky assets (Arrow(1965) and Pratt(1964)) and increases the demand for insurance. The comparative statics of ambiguity aversion with respect to insurance demand is parallel to that for risk aversion. Consistent with the intuition that ambiguity aversion reinforces risk aversion, ambiguity aversion should raise the demand for insurance.

We show that although both risk aversion and ambiguity aversion have positive effects on insurance demand, ambiguity aversion is not equivalent to an increase in risk aversion in terms of its effect on the strength of demand. We also find a diminishing marginal effect of an increase in risk aversion on the insurance demand for ambiguity-neutral individuals.

Gollier (2011), using a standard static portfolio model with one safe asset and one uncertain asset , shows that the introduction of ambiguity aversion reduces investor's demand for the uncertain asset. However, it is not necessary the case that greater ambiguity aversion reduces demand without specific conditions being met. Corgnet et al. (2012) develop an experiment to evaluate a trader's reaction under ambiguity and find that ambiguity plays a limited role in explaining financial anomalies and the change in market valuation is line with fundamentals. Specifically, they examined how traders respond to dividend realizations and how asset prices, volatility, and trading volumes change in the presence of ambiguity. In their paper, the dividends are designed to follow two unknown regimes, each with three possibilities. While no public news is available in the first period of trading, information about the regime is released in the second period and information about dividend is released in the third period. They are, therefore, able to control for the degree of ambiguity to partial out the effect of ambiguity aversion. Our results are consistent with Corgnet et al. in that we find insurance demand is sensitive to individuals' risk preference while it does not change significantly in response to greater ambiguity aversion given the level of risk aversion.

2.2 Insurance Demand with Ambiguity

We are mostly interested in analyzing a setting with two states of nature: an accident state A with a loss L and a state N with no loss incurred. This simplified structure of uncertainty, adopted in Chapter 1, allows us to get clear-cut results concerning the effects of ambiguity aversion on insurance demand. State-contingent wealth is given by

$$W_A = M - pc - L + c \tag{1a}$$

$$W_N = M - pc \tag{1b}$$

where M is endowed wealth, c is the amount of coverage chosen, and p is the per-unit price of coverage. The expected utility when θ is the probability of an accident is given by

$$EU(W_A, W_N, \theta) \equiv \theta u(W_A) + (1-\theta)u(W_N) \tag{2}$$

where u is a von Neumann-Morgenstern utility function defined over final wealth, assumed to be everywhere differentiable with $u' > 0$ and $u'' < 0$.

As in Chapter 1, when the accident probability q is not known with certainty, we assume that the decision criterion for ambiguous expected utility is given by the Klibanoff, Marinacci and Mukerji (KMM)(2005) criterion

$$AEU(W_A, W_N) \equiv \varphi^{-1}\left(\int \varphi(EU(W_A, W_N, \theta)\, dF(\theta)\right), \tag{3}$$

in which $F(\theta)$ represents the individual's subjective beliefs about the probability of an accident, and φ captures the individual's attitude toward bearing the subjective uncertainty about expected utility. With ambiguity aversion the functional φ is concave, and with ambiguity neutrality φ is linear. We assume that the expected value $\bar{\theta}$ of the accident probability is equal to the objective probability to ensure that ambiguity has no effect on the behavior of an ambiguity-neutral decision maker.

In our quantitative analysis, we specify two functional forms to represent preferences regarding risk and ambiguity, isoelastic and exponential.

2.2.1 Isoelastic Preferences

For the isoelastic specifications, the utility function is assumed to be

$$u(W) = \frac{(W)^{1-\tau}}{1-\tau}, \tau > 0 \text{ and } \tau \neq 1, \tag{4a}$$

where τ is the relative risk aversion parameter, and the ambiguity preference functional is assumed to be

$$\varphi(EU) = \frac{(EU)^{1-\eta}}{1-\eta}, \eta > 0 \text{ and } \eta \neq 1, \tag{4b}$$

where η is the ambiguity aversion parameter. The concavity of u in (2a) indicates risk aversion while concavity of φ in (2b) indicates ambiguity aversion. In the knife-edge case when $\tau = 1$ and $\eta = 1$, (2a) and (2b) reduce to

$$u(W) = \ln(W), \tau = 1, \tag{5a}$$

$$\varphi(EU) = \ln(EU), \eta = 1, \tag{6b}$$

respectively. As Ju and Miao (2007) point out, the KMM criterion with an isoelasitc utility function is not necessarily well-defined in the quantitative analysis. To see this, suppose η equals ½. Then φ in (2b) requires that the expected utility inside the exponential function be nonnegative. However, empirical estimates of the risk aversion parameter (τ) are typically greater than 2, which results in a negative value for expected utility. In other words, if $\eta = 1/2$ and $\tau > 2$, then φ equals a complex number.

The axiom system developed by Hayashi and Miao (2011) (HM thereafter) addresses this issue by assuming a preference functional defined on expected utility that is, itself, a composition mapping with u^{-1}, and they use this mapping to capture the willingness to bear ambiguity. We will also examine the HM criterion in an intertemporal model of insurance demand and saving in Chapter 3. For the static setting in this Chapter, we define *AEU* in the HM model as

$$AEU(W_A, W_N) \equiv \varphi^{-1}\left(\int \varphi \circ u^{-1}(EU(W_A, W_N, \theta) dF(\theta)\right) \tag{6}$$

In this model, $G \equiv \varphi \circ u^{-1}$ captures ambiguity preferences as φ alone does in the KMM model. In particular, the HM criterion is ambiguity averse if and only if G is concave, which further requires that φ is more concave than u, and in the isoelastic specification η exceeds τ. To verify this, observe that equation (2a) representing risk preferences yields the inverse mapping

$$\mathrm{u}^{-1}(\mathrm{EU}) = (\mathrm{EU}(1-\tau))^{\frac{1}{1-\tau}} = E[W^{1/(1-\tau)}]^{1/(1-\tau)} \tag{7}$$

which is always positive. From equation (2b) we obtain

$$G(c) = \varphi \circ u^{-1}(EU) = \frac{[(\mathrm{EU}(1-\tau))^{\frac{1}{1-\tau}}]^{1-\eta}}{1-\eta} = \frac{\{E[W^{1/(1-\tau)}]^{1/(1-\tau)}\}^{1-\eta}}{1-\eta} \tag{8}$$

which implies

$$G'(EU) = \frac{[(\mathrm{EU}(1-\tau))^{\frac{1}{1-\tau}}]^{1-\eta}}{EU(1-\tau)} \text{ and} \tag{9}$$

$$G''(EU) = \frac{(\tau-\eta)[(\mathrm{EU}(1-\tau))^{\frac{1}{1-\tau}}]^{1-\eta}}{EU^2(1-\tau)^2} \tag{10}$$

Thus, G is concave and the HM criterion is ambiguity averse if and only if $\eta > \tau$.

The functional form of the HM model in (5) is, thus, slightly different from the KMM model (4), but is well-suited to our computational purposes. The model allows us to examine whether the effect of ambiguity aversion on insurance demand differs from that of risk aversion for any positive values for risk aversion. We have shown in Chapter 1 that in an atemporal context, insurance demand increases with the degree of ambiguity aversion. To quantatively measure the increase in insurance demand under different degrees of risk aversion and ambiguity aversion, we first consider the special cases when $\eta = \tau$ and the individual is ambiguity neutral, in which event ambiguous expected utility reduces to the expected utility model as if there were no ambiguity. Then we move to ambiguity-averse cases where $\eta > \tau$. The higher the value η, the lower the value of ambiguous expected utility as the higher degree of ambiguity aversion discounts utility more through the curvature of the preference functional φ.

We have shown in Proposition 2 in Chapter 1 that, in an atemporal context, fair pricing is a necessary and sufficient condition for full insurance coverage to be optimal for the KMM criterion with ambiguity aversion. This is also true for the HM decision criterion (5) if and only if φ is a concave transformation of *u*. To see this, the marginal condition for optimal insurance demand for the HM model is

$$\frac{\int G' \cdot (1-\theta) dF(\theta) u'_N}{\int G' \cdot \theta dF(\theta) u'_A} = \frac{1-p}{p}. \tag{11}$$

The proof of necessity is similar to that for Proposition 2. The difference is that *G* now is in a compositional form and its concavity depends on the concavity of φ relative to the concavity of *u*.

For sufficiency, observe that, with fair insurance pricing $p = \overline{\theta}$, and $c = L$ satisfies the tangency condition (10). Suppose that $c < L$ is optimal; then $W_A < W_N$ and condition (10) then implies

$$\int G' \cdot (1-\theta)\, dFu'_N \bar{\theta} = \int G' \cdot \theta\, dFu'_A (1-\bar{\theta}) > \int G' \cdot \theta\, dFu'_N (1-\bar{\theta}), \tag{12}$$

which yields $\int G' \cdot (\bar{\theta}-\theta)\, dFu'_N > 0$. However, *EU* declines as θ increases, while G' is decreasing in *EU*, given ambiguity aversion. ($G''(EU)$ is negative if $\eta > \tau$.) It follows that G' increases as θ increases, which implies the contradiction

$$0 > \text{cov}(G', \bar{\theta}-\theta) = \int \varphi'(\bar{\theta}-\theta) dF(\theta). \tag{13}$$

Hence, the supposition must be false. A similar argument shows $W_A > W_N$ cannot be optimal. Therefore, under fair pricing full insurance coverage is optimal. It follows that partial coverage is optimal if insurance pricing is favorable but unfair.

2.2.2 Exponential Preferences

For the exponential specifications, risk preferences exhibit constant absolute risk aversion (CARA) and ambiguity preferences exhibit constant absolute ambiguity aversion (CAAA). For CARA utility

$$u(W) = -e^{-\tau(W)}, \tau > 0, \tag{14}$$

where τ is the parameter of CARA and for CAAA utility

$$v(u^{-1}) = -e^{-\eta(u^{-1})}, \eta > 0, \tag{15}$$

where η is the parameter of CAAA.

The HM criterion is ambiguity averse if and only if *G* is concave, which further requires that φ is more concave than *u*, and in the exponential specification η exceeds τ. To verify this, observe that equation (13a) representing risk preferences yields the inverse mapping

$$\text{u}^{-1}(\text{EU}) = \text{-log(-EU)}/\tau\ , \tag{16}$$

From equation (13b) we obtain

$$G(EU) = \varphi \circ u^{-1}(EU) = -(-EU)^{\frac{\eta}{\tau}} \tag{17}$$

which implies

$$G'(EU) = \frac{\eta}{\tau}(-EU)^{\frac{\eta}{\tau}-1} \text{ and} \tag{18}$$

$$G''(EU) = -\frac{\eta}{\tau}\cdot(\frac{\eta}{\tau}-1)\cdot(-EU)^{\frac{\eta}{\tau}-2}\ . \tag{19}$$

From the definition of *u* and *EU*, it is expected that -*EU* is always positive without any constraints on parameter η or τ. Therefore, $G'(EU) > 0$ and $G''(EU) < 0$ if and only if $\eta > \tau$, i.e. *G* is concave if the individual is ambiguity averse.

Similarly, we next show fair pricing is a necessary and sufficient condition for full insurance coverage to be optimal for the HM decision criterion (5) if and only if φ is a concave transformation of *u* in this exponential case. To see this, the marginal condition for optimal insurance demand for the HM model is

$$\frac{\int G'(1-\theta)dF(\theta)u'_N}{\int G'\theta dF(\theta)u'_A} = \frac{1-p}{p}\ . \tag{20}$$

The proof of necessity is similar to that for Proposition 2 in Chapter One. The difference is that *G* now is in a compositional form and its concavity depends on the concavity of φ relative to the concavity of *u*.

For sufficiency, observe that, with fair insurance pricing $p = \bar{\theta}$, and $c = L$ satisfies the tangency condition (10). Suppose that $c < L$ is optimal; then $W_A < W_N$ and condition (10) then implies

$$\int G' \cdot (1-\theta)\, dFu'_N \bar{\theta} = \int G' \cdot \theta\, dFu'_A (1-\bar{\theta}) > \int G' \cdot \theta\, dFu'_N (1-\bar{\theta}), \tag{21}$$

which yields $\int G' \cdot (\bar{\theta} - \theta)\, dFu'_N > 0$. However, *EU* declines as θ increases, while G' is decreasing in *EU*, given ambiguity aversion. ($G''(EU)$ is negative if $\eta > \tau$.) It follows that G' increases as θ increases, which implies the contradiction

$$0 > \text{cov}(G', \bar{\theta} - \theta) = \int \varphi'(\bar{\theta} - \theta) dF(\theta). \tag{22}$$

Hence, the supposition must be false. A similar argument shows $W_A > W_N$ cannot be optimal. Therefore, under fair pricing full insurance coverage is optimal. It follows that partial coverage is optimal if insurance pricing is favorable but unfair.

2.3 Parameter Selection

In this section, I discuss the choice of the parameters in the model specification and narrow down parameter selection without loss of generality. I then implement the optimization of insurance coverage for different levels of risk aversion and ambiguity aversion. Recognizing that state-contingent wealth depends on the choice of coverage, we are interested in an optimal choice of coverage solving

$$\max_c \{AEU(W_A, W_N) \mid 0 < c < M / p\}.$$

The insurance premium per dollar of coverage *p* has to be greater than the objective risk of accident to guarantee that the insurer earns a return sufficient to cover operating expenses proportional to the amount of coverage. If the premium rate *p* exceeds $\bar{\theta}$ by a critical amount, insurance demand falls to zero for a given insurance applicant. This is not surprising because if the insurance is too expensive, only those who are extremely risk-averse and/or ambiguity-averse would be willing to pay for insurance and many potential insurance applicants would be driven out of the market. To better capture this pricing restriction, define ς to be the percentage that the premium deviates from the expected probability, and allow this ratio to be between 0 and 50%:

$$\varsigma = \frac{p - E(\theta)}{E(\theta)} \in (0, 50\%)$$

The amount that any insurance applicant should have to pay an insurer equals the expected loss plus an amount sufficient to cover the insurer's expenses for selling and providing services.

For the choice of initial wealth amount *M* and loss amount *L*, intuitively, when individuals are very rich, they become less risk averse because of decreasing absolute risk aversion and are then indifferent between taking the risk and purchasing the insurance for certainty, which results in a very small insurance coverage or zero coverage. Mathematically, in this model specification, if *M* is very large relative to the loss L and τ (the relative risk aversion parameter) approaches 1 (e.g. τ = 0.9), then the exponential power (1-τ) for the wealth in the isoelastic utility function is very small, which severely compresses the resulting value of risk averse utility. Any relatively small change in wealth (an insurance purchase or loss occurrence) would not affect utility much or the corresponding optimal insurance demand. Therefore, to better monitor this effect, define the loss-to-wealth ratio $r = L / M$. We allow *r* to vary within a range of 0 and 20%: i.e, the possible loss is restricted to be no more than 20% of wealth. For illustrative purposes, M is set equal to 2 and L is set equal to 1. In addition, we require that wealth in both states is non-negative meaning, in the worst case, an individual may lose everything when an accident happens but will not be in debt, which further requires $0 <= c <= M/p$. We include this constraint in the optimization process.

2.4 Distributions for Ambiguity

We assume three different regimes for the distribution of the accident state probability θ: uniform distribution, beta distribution and triangular distribution. In all three cases, the expected value of the accident probability is set at 0.25, for both practical reasons and illustrative purposes. We start with the uniform distribution case, where we allow θ~U [0, 0.5], so the accident probability could be any value between 0 and 50% with equal chance, and the expected value is $\bar{\theta} = (1/2)^2 = 0.25$.

In the second scenario, the probability of an accident is assumed to follow a beta distribution $Beta(\alpha,\beta)$ with two shape-control parameters α and β. In Bayesian updating inference, the beta distribution is a conjugate prior, where the posterior distribution is in the same family as the prior distribution and the updated information is added in the parameters. The uniform distribution is a special case of the beta distribution with two parameters α = 1 and β = 1 and θ defined on [0,1]. The probability density function (p.d.f) for the beta distribution is

$$f(\theta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{Beta(\alpha,\beta)}.$$

where α and β control the shape of the curve. Also see figure 2.2.1 and figure 2.2.2. In particular, when α and β are both greater than 1 then the p.d.f is a unimodal curve while if both

α and β are less than one, the curve is U-shaped. We also assume the mean-preserving property such that $E(\theta) = \dfrac{\alpha}{(\alpha+\beta)} =$ 0.25, which requires that β is threefold of α. We let α = 1 and β = 3, which is right-skewed and differentiated from the symmetric uniform and triangular distributions.

In a third scenario, I assume that θ~triangular [0, 0.5], with parameters a = 0, b = 0.5, c = 0.25, so that E(θ) = 0.25. The probability density function is (see Figure 2.1)

$$f(\theta) = \begin{cases} 16\theta, \\ 8 - 16\theta \end{cases}$$

Note that the p.d.f. is nondifferentiable at the peak of the curve.

2.5 Numerical Results

The optimization process is implemented in Mathematica 8. For details, note that, first, for optimization to be achieved numerically, ranges for parameters have to be specified before maximizing ambiguous expected utility in equation (5) symbolically. Otherwise, the integrated symbolic function is a very large input and Mathematica reports errors due the complexity of maximization process. Additionally, the target optimal coverage has also to be assumed before integration. I use a ?numericQ command in Mathematica to change the order of evaluation. It holds the symbolic expression within the integration so the function cannot be evaluated symbolically until the value of *c* is plugged in.

2.5.1 Isoelastic Preferences

The effect of risk aversion on insurance demand is achieved by analyzing an ambiguity-neutral but risk-averse individual who behaves in the same manner as a risk-averse individual with no ambiguity. The risk parameter (τ) is set equal to the ambiguity parameter (η). To be consistent with current literatures, the relative risk parameter τ is defined over the range 0 to 4. Figure 2.4 illustrates the result with $M = 2$, the accident loss L=1 and insurance premium rate p=0.3. We also require $M > L$ and $0 < c < M/p$, which allows wealth in the accident-free state to be nonnegative. There is an increasing concave curve of optimal coverage as a function of risk aversion. For a small degree of ambiguity aversion, individuals opt out of the insurance market because the insurance is too costly and the marginal benefit of insurance is less than the marginal cost. This result can be seen in the Figure 2-4 when the relative risk aversion parameter is very small, insurance demand is close to zero. As the individual becomes more ambiguity averse, the optimal insurance demand gradually approaches full coverage at a decreasing rate, indicating that insurance demand is sensitive to changes in risk aversion of low level. Note that since the accident loss is assumed to be one, with unfair pricing, the optimal coverage will not exceed 1. In addition, the result is robust to the choice of probability distribution for θ due to the mean-preserving assumption. All the three cases exhibit the same increasing insurance demand at a decreasing rate as risk aversion increases. The optimal coverage is robust to different distribution assumptions. See Figure 2-4 to Figure 2-6

Secondly, the effect of ambiguity aversion on insurance demand can be analyzed by changing the parameter η for a given level of risk aversion. To better interpret the results, risk preferences are predetermined at two separate levels, 0.7 and 2. Figure 2-6 and Figure 2-6 shows the ambiguity effect with τ equal 0.7 and 1. 2, respectively. The values of τ are chosen because utility function u flips signs at τ =1where the behavior of optimal demand might change. See Figure (2.7).

Figure 2-6 and Figure 2-6 shows the effects of ambiguity aversion on demand for the risk aversion when θ is uniformly distributed. These trends are approximately linear, rather than concave, as in the ambiguity-neutral case. In general, insurance demand increases when individual becomes more ambiguity averse and the insurance demand of high-risk aversion is lying above the demand with low risk aversion, indicating risk aversion is still a main driver for the coverage decision.

As the individual become more risk-averse, additional uncertainty does not affect the choice of insurance much and effect of ambiguity aversion is diminishing. Table2-1 shows the increase in insurance demand for every unit change of ambiguity aversion. In terms of the magnitude, optimal coverage increases 47% from 0.42 to 0.62 for increases in ambiguity aversion from 0.7 to 4 while it increases10% from 0.66 to 0.73 for ambiguity aversion from 1.2 to 4. Finally, both τ and η are allowed to change at the same time. Figure 2-8 is the three dimensional graph for optimal insurance versus the relative risk aversion and ambiguity aversion parameters. We see that when at least one parameter (either for risk aversion or for ambiguity aversion) is small, insurance demand is almost zero. The main increase in demand comes from the change risk aversion and only a moderate increase along the ambiguity aversion axis.

Similar results obtain when θ is triangular distributed and beta distributed as shown in Figure 2-10 and Figure 2-12, respectively. Figure 2-6 and Figure 2-6 show the corresponding maximized ambiguous expected utility as a function of ambiguity aversion when θ is triangular distributed and Beta distributed. The changes in optimal ambiguous expected utility are very unnoticeable for a given change in ambiguity aversion. In all, insurance demand is not sensitive to changes in ambiguity preference change and ambiguity aversion plays a limited role in the choice of insurance coverage. Figure 2-14 and Figure 2-15 show that ambiguity aversion has the largest effect on demand when θ is beta distributed.

2.5.2 Exponential Preferences

The analytical results for exponential preferences are similar to isoelastic case and can be found through Figure 2-18 to Figure 2-23. Starting from ambiguity neutral case, the effect of risk aversion on insurance demand is achieved by analyzing an ambiguity-neutral but risk-averse individual who behaves in the same manner as a risk-averse individual with no ambiguity. The risk parameter (τ) is set equal to the ambiguity parameter (η).The relative risk parameter τ is also defined over the range 0 to 4 as in isoelastic utility. Figure 2-18 illustrates the result with M = 2, the accident loss L=1 and insurance premium rate p=0.3. There is an increasing concave curve of optimal coverage as a function of risk aversion. The optimal coverage increases as individual becomes more risk averse, since the loss is set to 1, the optimal coverage infinitely approaches 1 at a decreasing rate without achieving it under unfair pricing setting.

To examine the effects of ambiguity aversion, the risk aversion is fixed at 0.7 and 1.2, respectively for each θ distribution from Figure 2-18 and Figure 2-19. If an individual is less risk averse, then his/her optimal choice of coverage is more sensitive to ambiguity aversion, which is reflected by the curvature of optimal coverage line. In addition, the overall coverage level is higher when an individual is more ambiguity averse.

2.6 Conclusions

This chapter implements a quantitative analysis designed to measure the effect of ambiguity aversion on insurance demand. It discusses why HM model is more suitable for the quantitative analysis than the KMM model. Two model specifications are assumed to represent CRAA and CAAA and each of the parameters is chosen carefully to maintain the generality of the analysis as well as to be economically meaningful. The probability of an accident follows a mean-preserving uniform, triangular and beta distribution, respectively to capture ambiguity. For both CRAA and CAAA scenarios, the finding is consistent with Corgnet et. al (2012) that ambiguity aversion plays a limited role in determining the insurance coverage as opposed to the larger effects of risk aversion and this result is robust to probability distributions and levels of risk aversion.

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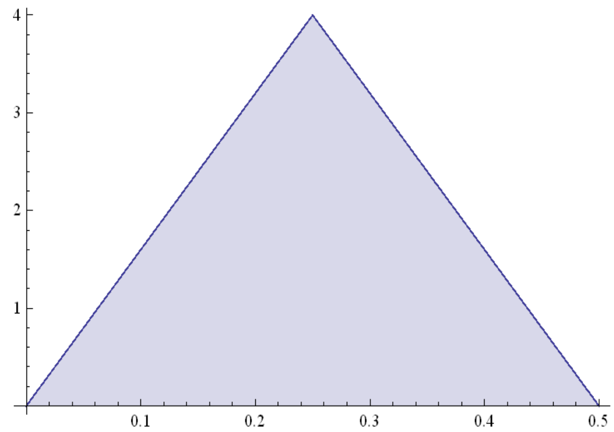

Figure 2-1 Triangular Distribution [0, 0.5]

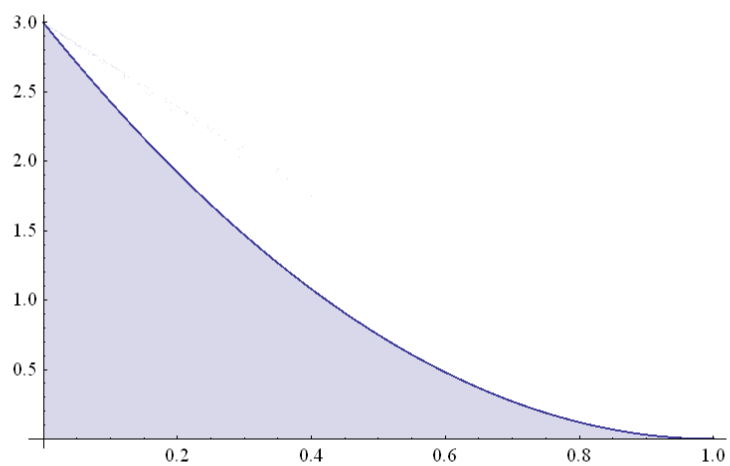

Figure 2-2 Beta[1,3]

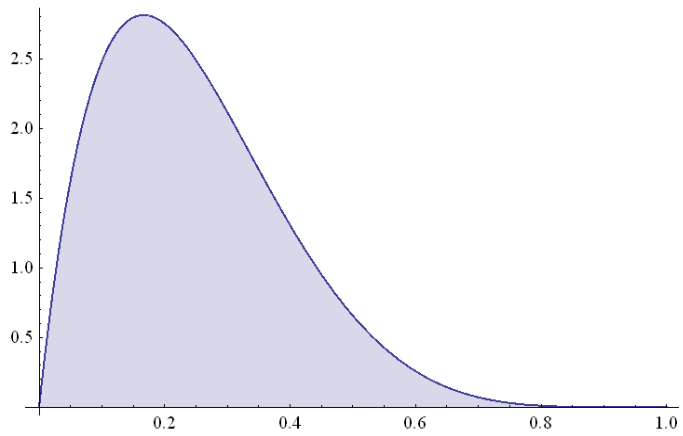

Figure 2-3 Beta[2,6]

	Uniform	Triangular	Beta
Ambiguity Neutral	0.229	0.229	0.229
Ambiguity Averse (tau=0.7)	0.056	0.032	0.078
Ambiguity Averse (tau=1.2)	0.024	0.013	0.037

Table 2-1: Effects on Insurance Demand for Unit Change in Ambiguity for Atemporal Risk

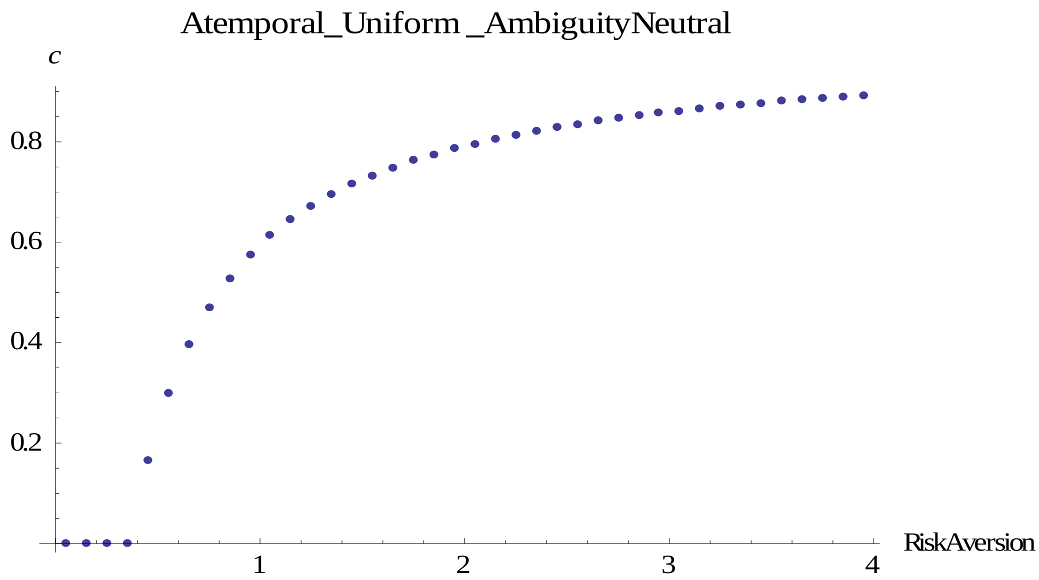

Figure 2-4: Insurance Demand vs Risk Aversion for Atemporal Risk with Uniformly Distributed Ambiguity

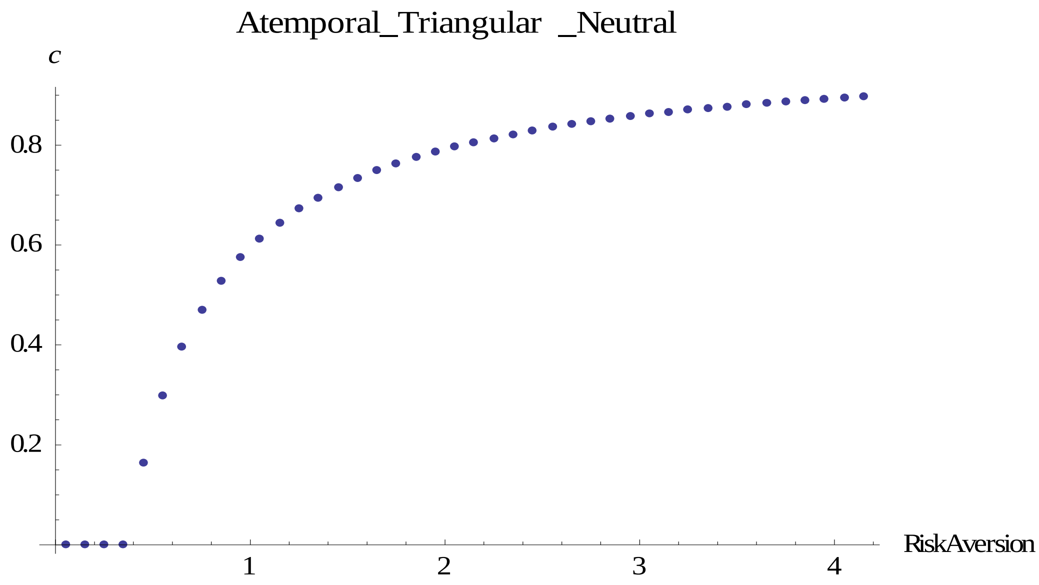

Figure 2-5: Insurance Demand vs Risk Aversion for Atemporal Risk with Triangular Distributed Ambiguity

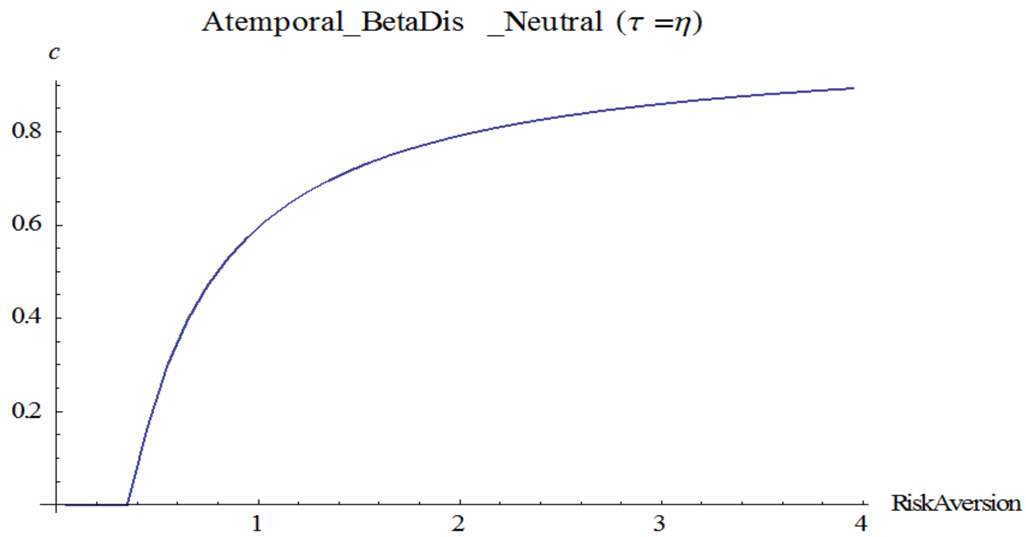

Figure 2-6: Insurance Demand vs Risk Aversion for Atemporal Risk with Beta Distributed Ambiguity

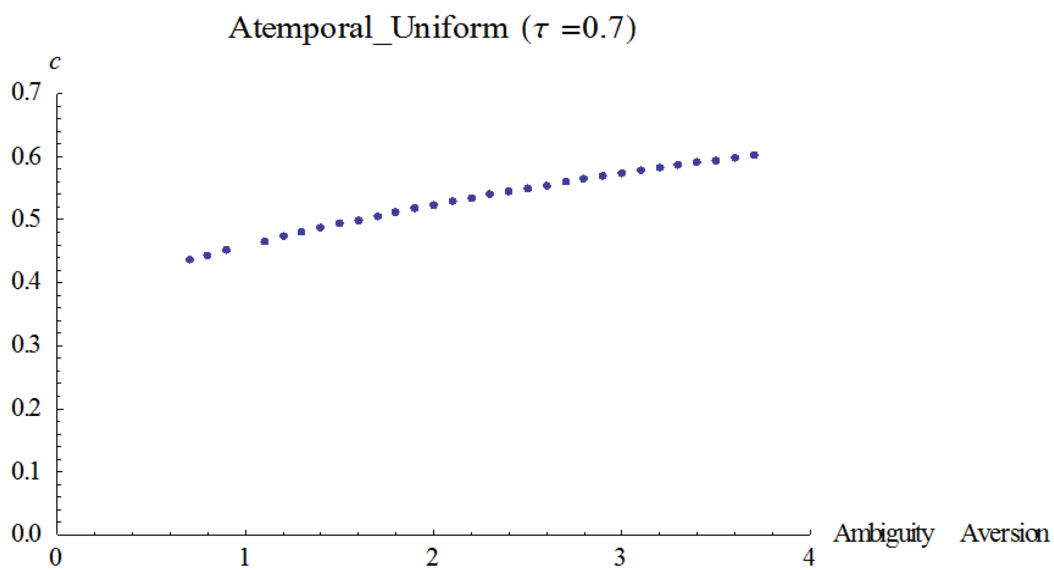

Figure 2-7: Insurance Demand vs Ambiguity Aversion with Uniform Distributiuon and Risk Aversion=0.7

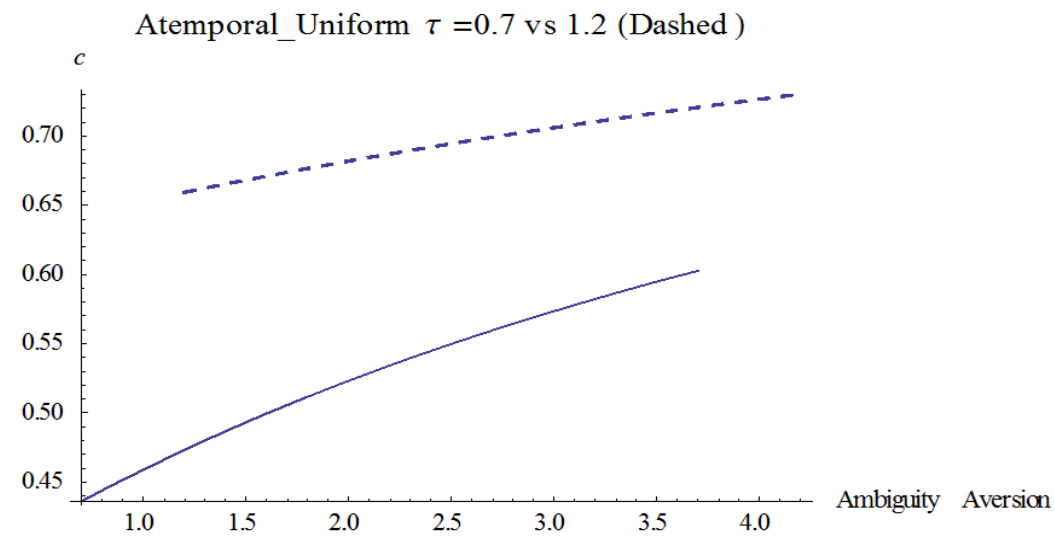

Figure 2-8: Insurance Demand vs Ambiguity Aversion at Risk Aversion 1.2 and 0.7 for Uniform Distributiion

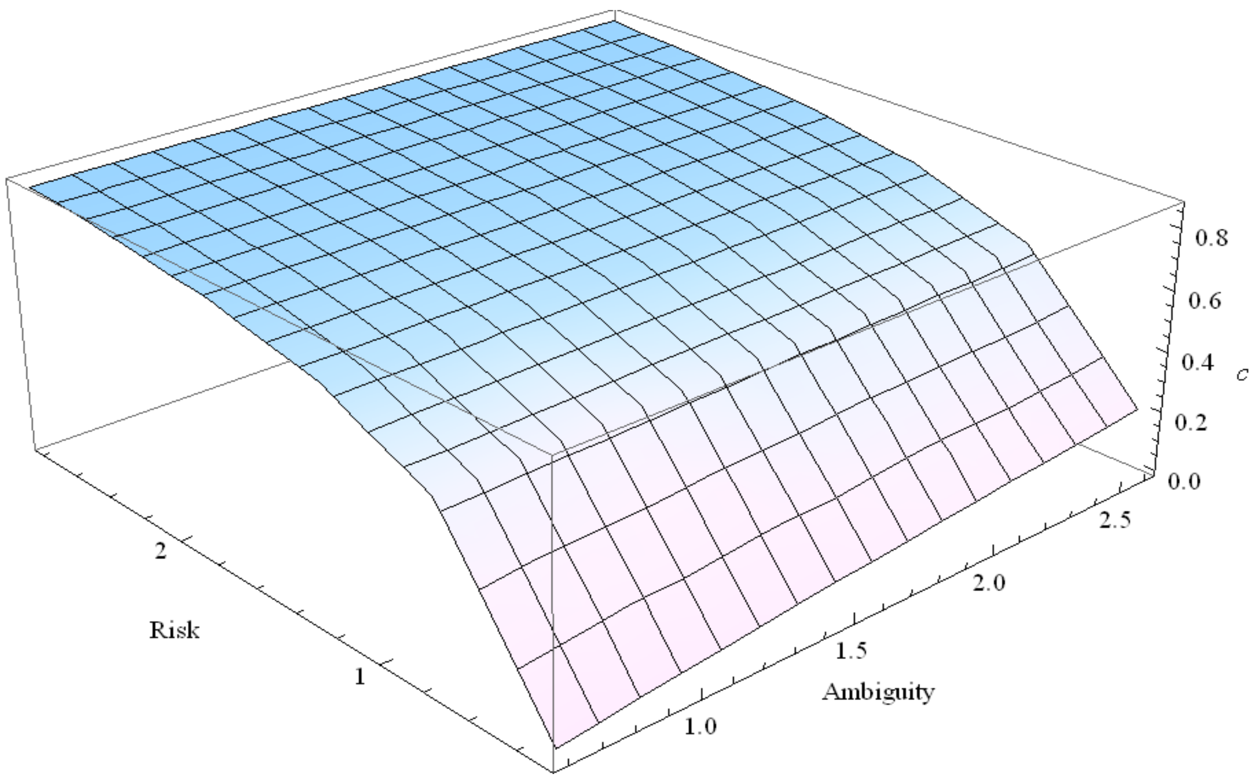

Figure 2-9 : Insurance Demand vs Ambiguity Aversion and Risk Aversion for Uniform Distributiion

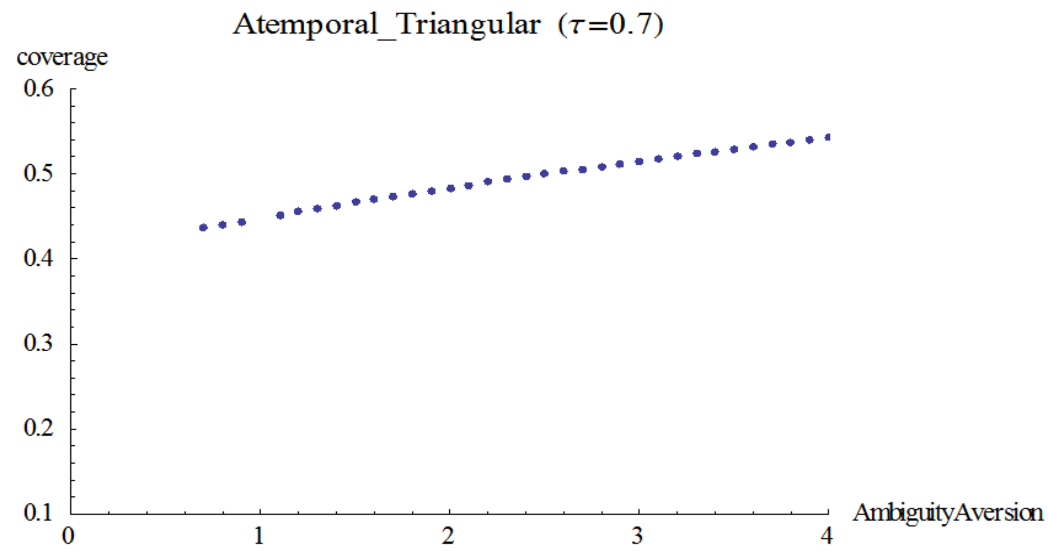

Figure 2-10: Insurance Demand vs Ambiguity Aversion with Triangular Distributiuon and Risk Aversion=0.7

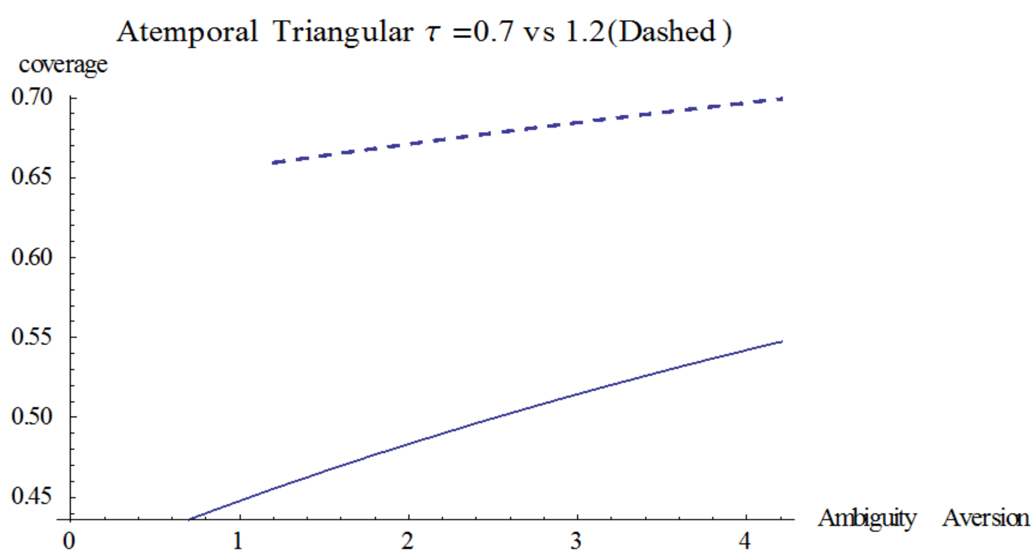

Figure 2-11: Insurance Demand vs Ambiguity Aversion at Risk Aversion 1.2 and 0.7 for Triangular Distributiion

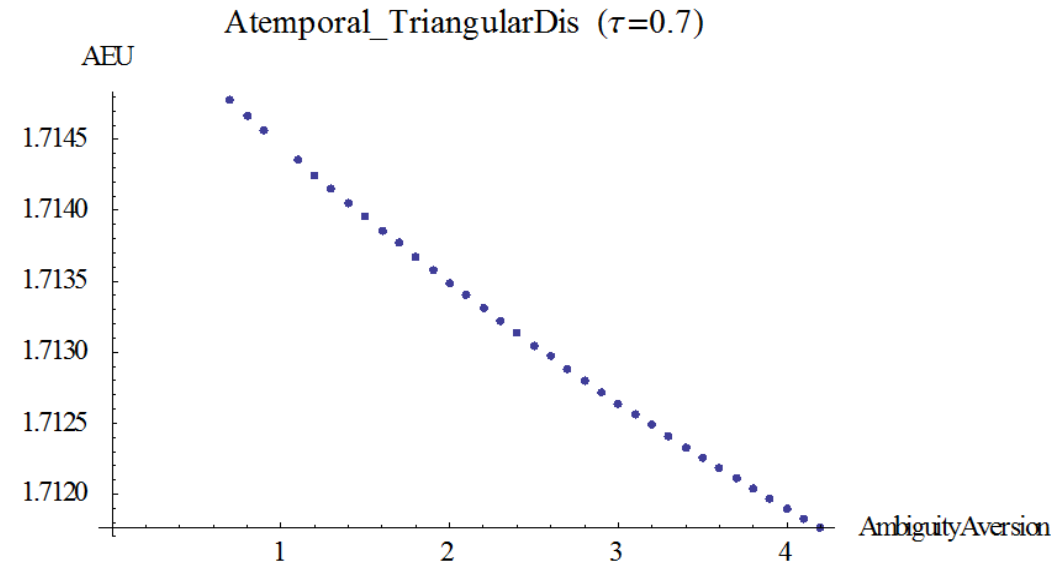

Figure 2-12: Ambiguious Expected Utility vs Ambiguity Aversion with Triangular Distributiuon and Risk Aversion=0.7

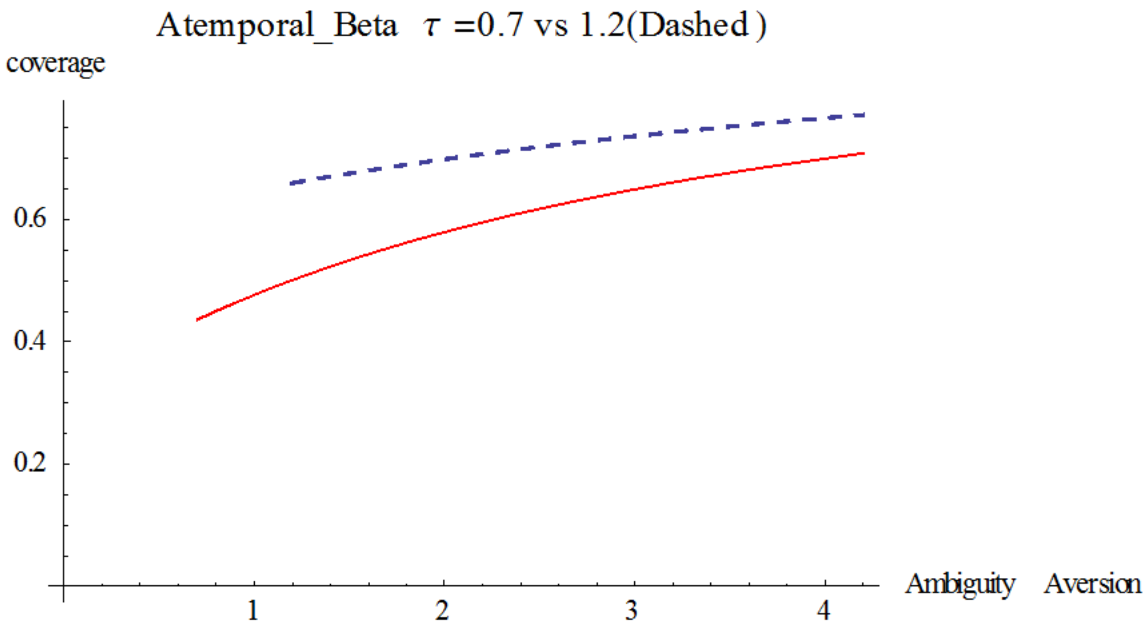

Figure 2-13: Insurance Demand vs Ambiguity Aversion at Risk Aversion 1.2 and 0.7 for Beta Distributiion

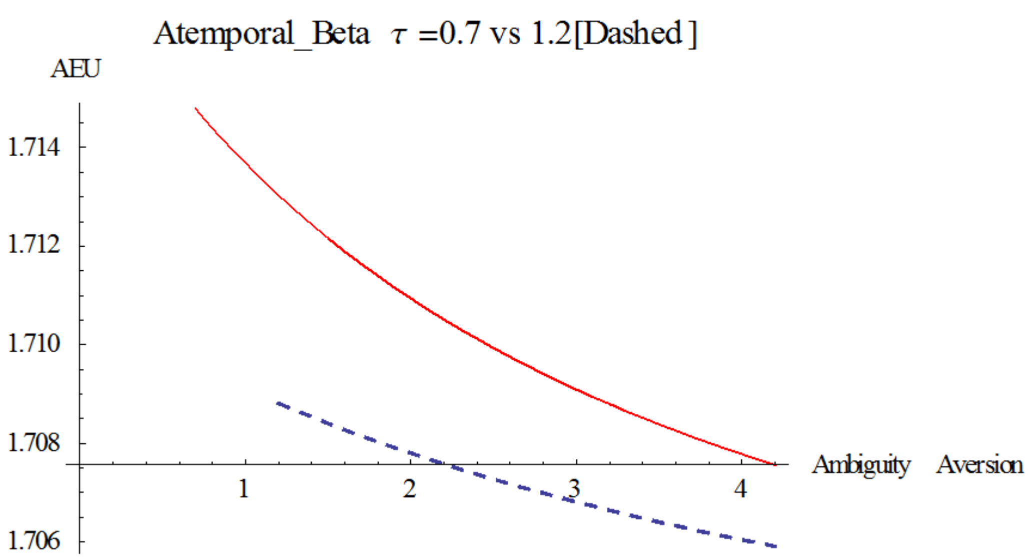

Figure 2-14: Ambiguious Expected Utility vs Ambiguity Aversion at Risk Aversion 1.2 and 0.7 for Beta Distributiion

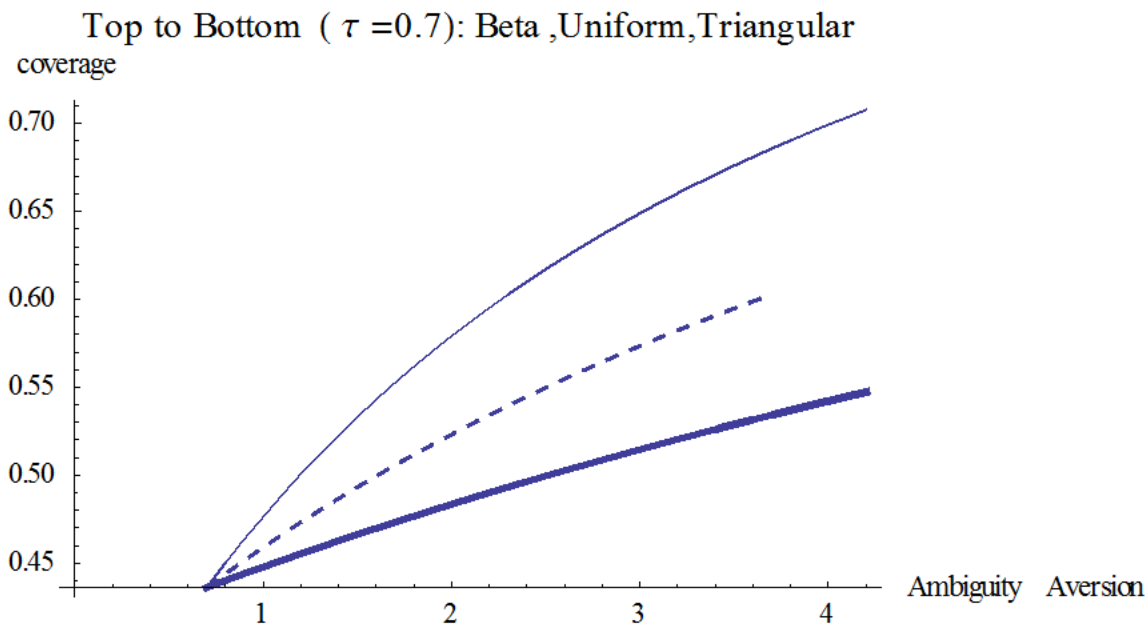

Figure 2-15: Comparisons of Insurance Demand with Uniform, Beta and Triangular Distributed Ambiguity with Risk Averseion =0.7

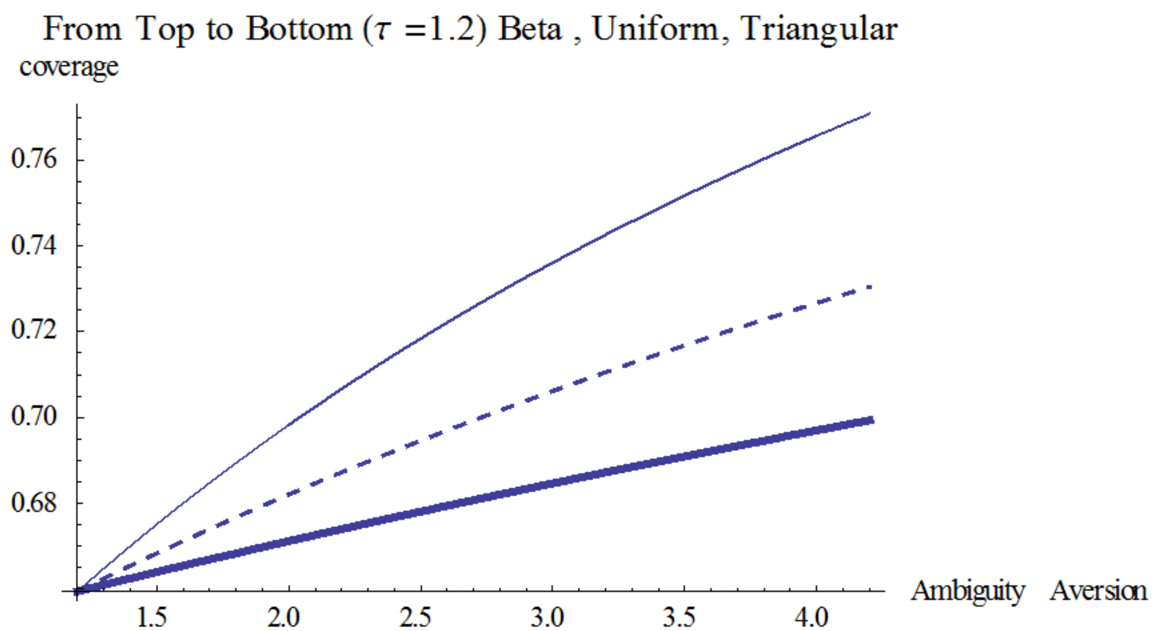

Figure 2-16: Comparisons of Insurance Demand with Uniform, Beta and Triangular Distributed Ambiguity with Risk Averseion =1.2

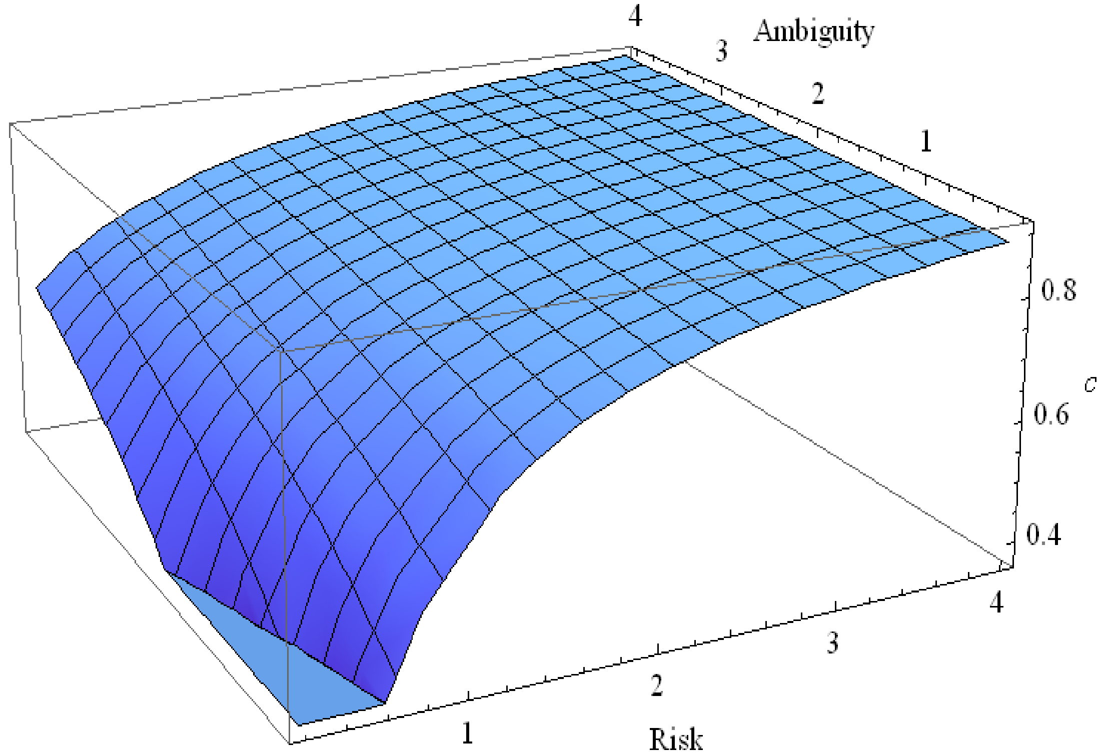

Figure 2-17 Insurance Demand vs Ambiguity Aversion and Risk Aversion for Beta Distributiion

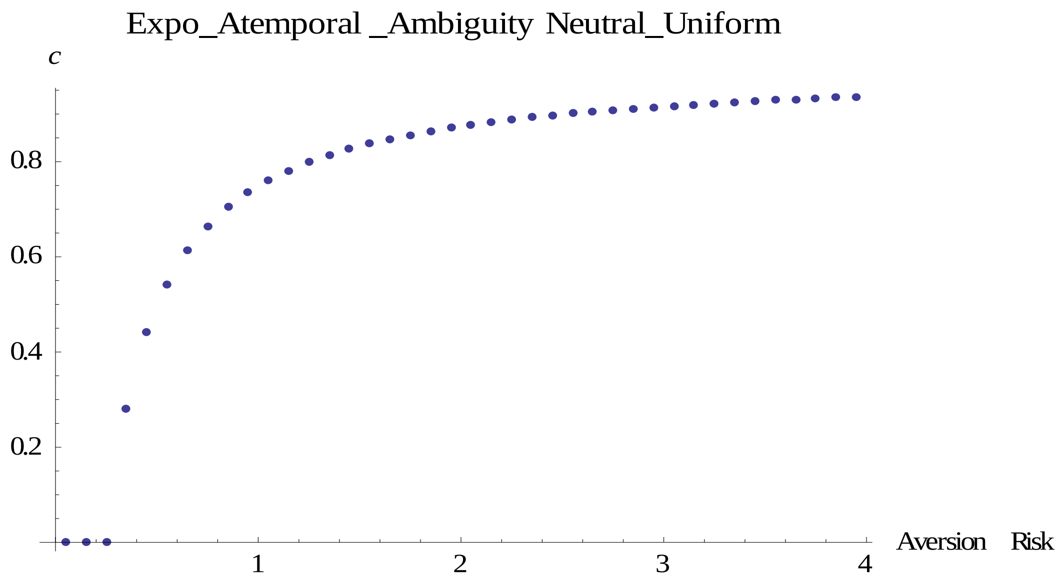

Figure 2-18: Exponential Preferences Insurance Demand vs Risk Aversion with Ambiguity Neutrality for Uniform Distributiion

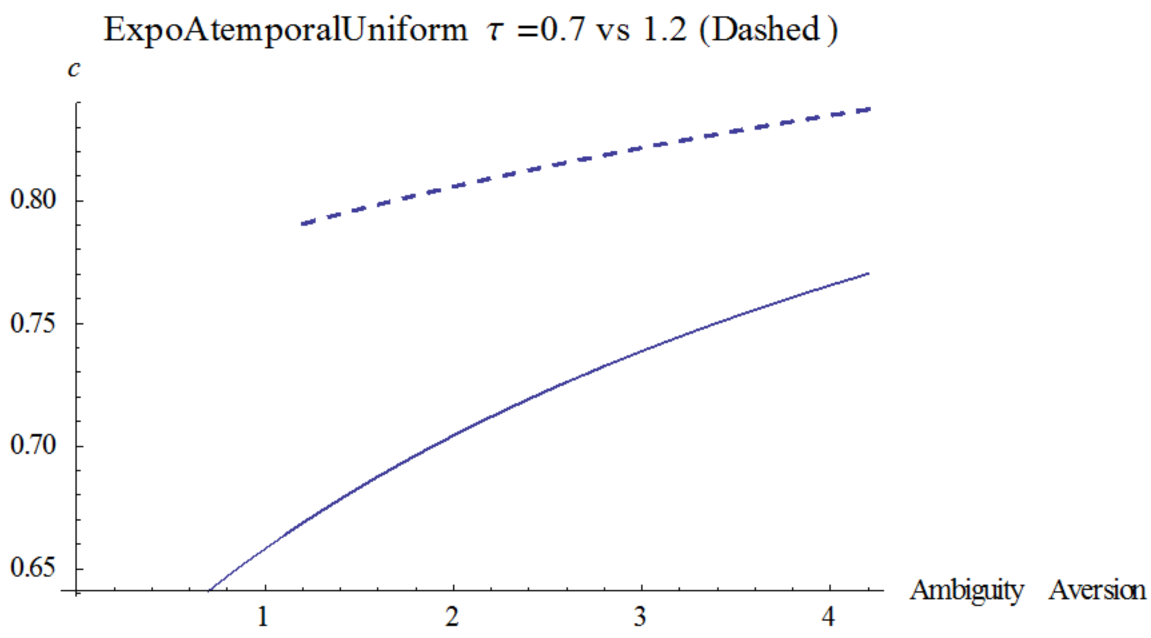

Figure 2-19: Exponential Preferences Insurance Demand vs Ambiguity Aversion with Risk Aversion=0.7 and 1.2 for Uniform Distributiion

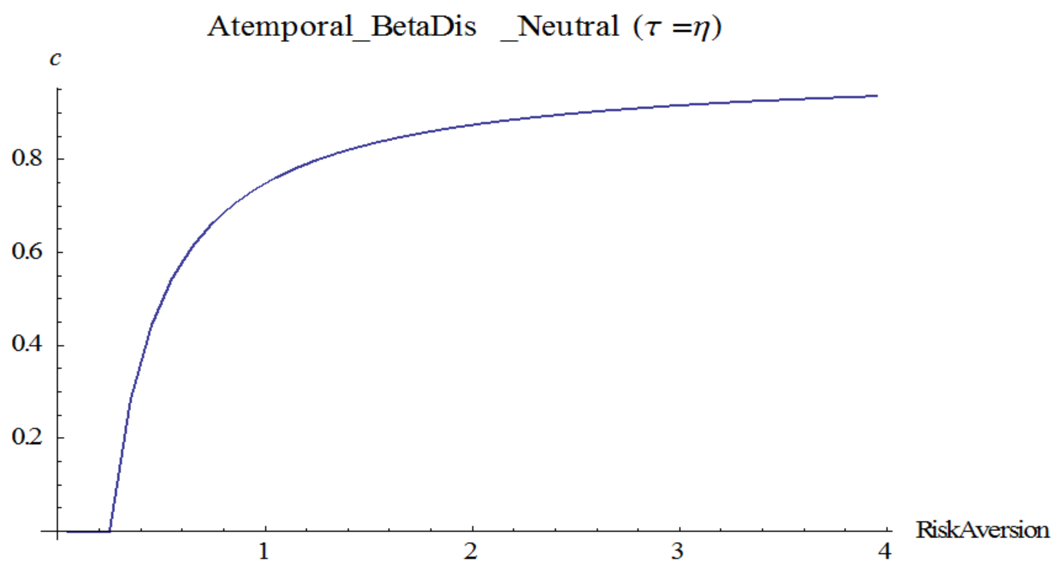

Figure 2-20: Exponential Preferences Insurance Demand vs Risk Aversion with Ambiguity Neutrality for Beta Distributiion

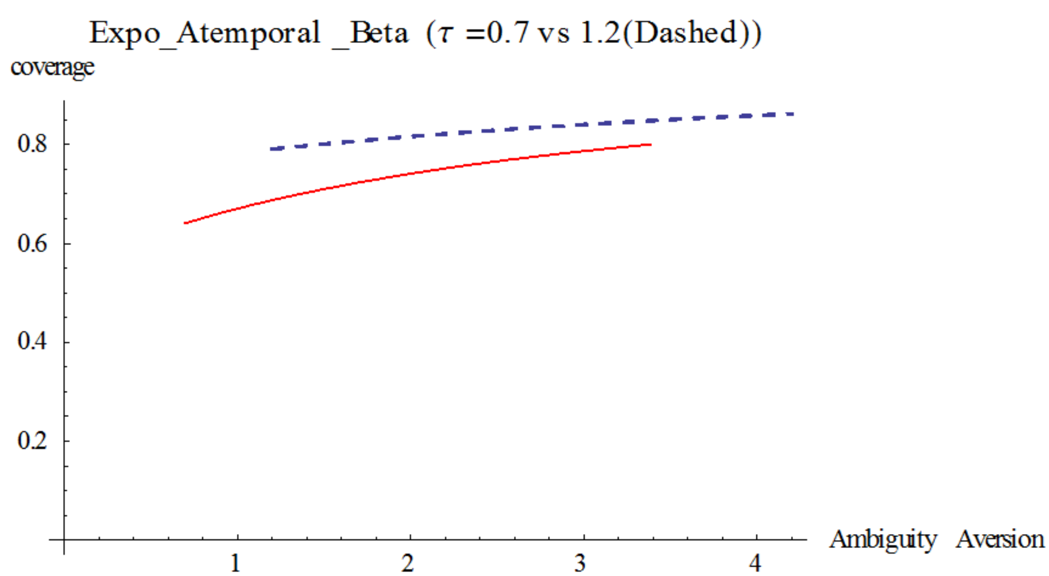

Figure 2-21: : Exponential Preferences Insurance Demand vs Ambiguity Aversion with Risk Aversion=0.7 and 1.2 for Beta Distributiion

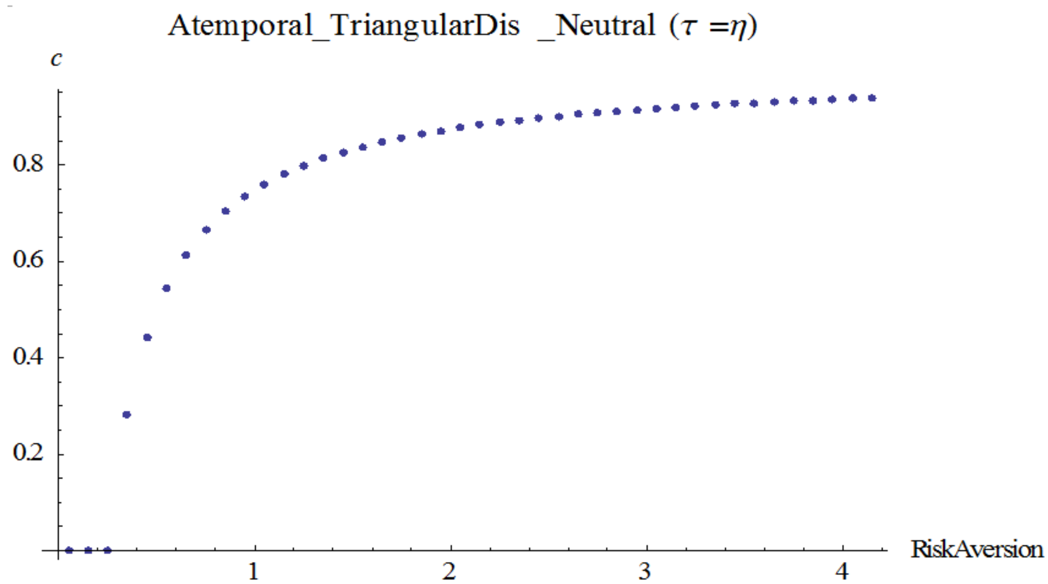

Figure 2-22: Exponential Preferences Insurance Demand vs Risk Aversion with Ambiguity Neutrality for Triangular Distributiion

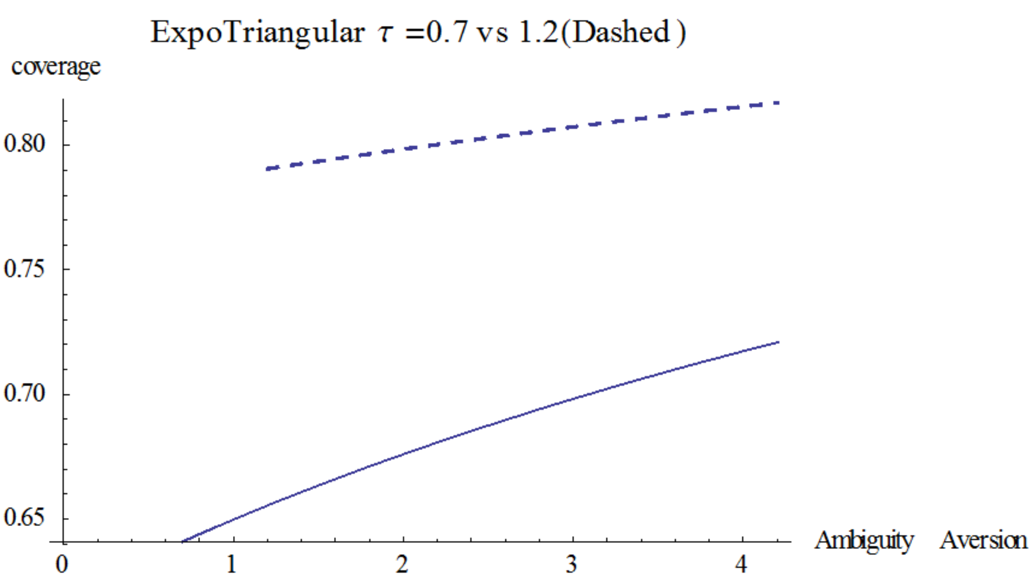

Figure 2-23: : Exponential Preferences Insurance Demand vs Ambiguity Aversion with Risk Aversion=0.7 and 1.2 for Triangular Distributiion

CHAPTER 3

A NUMERICAL STUDY OF AMBIGUITY AVERSION AND INSURANCE DEMAND FOR TEMPORAL RISK

3.1 Introduction

In Chapter 1, we showed that no sign-definite predictions are forthcoming concerning the qualitative effect of ambiguity aversion on insurance demand in a temporal context, except for the special case of the introduction of a marginal degree of ambiguity aversion. Even in that case constant absolute ambiguity aversion is needed to predict lower saving and higher coverage. In this chapter, the issue is addressed numerically. Extending the analytical approach of Chapter 2 from the atemporal to the temporal context, we investigate quantitatively the effect of ambiguity preference on insurance demand and savings.

3.2 Insurance demand for temporal risk

We adopt the temporal model setting in Chapter 1 and assume both exponential and isoelastic utility functions to explore the effects of ambiguity aversion on insurance demand. In a two-period context of temporal risk, the insurance applicant faces a risk of loss in the second period, and can save and/or buy insurance coverage in the first period. In the second period, the individual consumes the return to saving in the first period and receives coverage in the accident state. Final wealth in the first period is denoted by W_1, while W_A and W_N denote the state contingent, second-period wealth. These wealth levels are given by

$$W_1 = M - pc - s \tag{1a}$$

$$W_A = M + s - L + c \tag{1b}$$

$$W_N = M + s \tag{1c}$$

where the initial wealth endowment M is the same in both periods, s is the amount saved, and the interest rate has been normalized to equal zero. The price of insurance coverage c is denoted by p. Combining equations (1a)-(1c) by eliminating s and c yields the budget equation

$$W_1 + pW_A + (1-p)W_N = 2M - pL\,. \tag{1d}$$

In the absence of ambiguity, the individual's decision criterion is intertemporal expected utility with time-separable preferences

$$u(W_1) + EU(W_A, W_N, \bar{\theta}) \equiv u(W_1) + \bar{\theta}u(W_A) + (1-\bar{\theta})u(W_N)\,. \tag{2}$$

When the accident probability θ is not known with certainty, we assume that the decision criterion for ambiguous expected utility is given by the Klibanoff, Marinacci and Mukerji (KMM 2005) criterion

$$AEU(W_A, W_N) \equiv u(W_1) + \varphi^{-1}\left(\int \varphi(EU(W_A, W_N, \theta)dF(\theta)\right) \tag{3}$$

in which $F(\theta)$ represents the individual's subjective beliefs about the probability of an accident, and φ captures the individual's attitude toward bearing the subjective uncertainty about expected utility. With ambiguity aversion the functional φ is concave, and with ambiguity neutrality φ is linear. We assume that the expected value $\bar{\theta}$ of the accident probability is equal to the objective probability to ensure that ambiguity has no effect on the behavior of an ambiguity-neutral decision maker.

As discussed in Chapter 2, we assume two utility specifications to quantitatively measure the effect of ambiguity aversion, one is exponential utility for constant absolute risk aversion (CARA) and constant absolute ambiguity aversion (CAAA) and the other is isoelastic utility for constant relative risk aversion (CRRA) and constant relative ambiguity aversion (CRAA).

For the isoelastic setting, the risk-averse utility function is assumed to be

$$u(W) = \frac{W^{1-\tau}}{1-\tau}, \tau > 0 \neq 1, \tag{4a}$$

where τ is the parameter of CARA and the larger the value of τ, the more risk averse the decision maker. Similarly, for the ambiguity-averse preference function, assume

$$\varphi(u^{-1}) = \frac{(u^{-1})^{1-\eta}}{1-\eta}, \eta > 0 \neq 1, \tag{4b}$$

where η is the parameter of CAAA and the larger the value of η, the more ambiguity averse the decision maker. In the knife-edge case when $\tau = 1$ and $\eta = 1$, (2a) and (2b) reduce to

$$u(W) = \ln(W), \tau = 1, \tag{5a}$$

$$\varphi(x) = \ln(x), \eta = 1\,. \tag{5b}$$

As the second scenario, where the insurance applicant is assumed to have constant absolute risk aversion (CARA), utility is given by

$$u(W) = -e^{-\tau(W)}, \tau > 0 \tag{6a}$$

where τ is the parameter of CARA, and for constant absolute ambiguity aversion (CAAA), utility is given by

$$v(u^{-1}) = -e^{-\eta(u^{-1})}, \eta > 0\,, \tag{6b}$$

η is the parameter of CAAA.

As discussed in Chapter2, the functional form in KMM model in equation (3) essentially requires EU to be positive for *AEU* to be well defined for any positive value of η. However, empirical literature show the estimated risk aversion τ is greater than one, which yields the possibility of a complex number for *AEU*. The axiom system developed by Hayashi and Miao (2011) (HM thereafter) addresses this issue by assuming a preference functional defined on expected utility that is, itself, a composition mapping with u^{-1}, and they use this mapping to capture the willingness to bear ambiguity. The temporal decision criterion based on Hayashi and Miao (HM 2011), imposing the assumption of time-separable preferences,is

$$u(W_1) + AEU(W_A, W_N) \equiv u(W_1) + \varphi^{-1}\left(\int \varphi \circ u^{-1}(EU(W_A, W_N, \theta)dF(\theta)\right). \tag{7}$$

In this model, $G \equiv \varphi \circ u^{-1}$ is a composition function that captures ambiguity preferences as φ alone does in the KMM model. In particular, the HM criterion is ambiguity averse if and only if G is concave, so that φ is more concave than u which requires that η exceeds τ.

We have shown in Proposition 2 in Chapter 1 that, in an atemporal context, fair pricing is a necessary and sufficient condition for full insurance coverage to be optimal for the KMM criterion with ambiguity aversion. This is also true for the HM decision criterion (7) if and only if φ is a concave transformation of u. To see this, using λ^a to denote the Lagrange multiplier for budget constraint (1d), the first-order conditions for maximizing criterion (7) with respect to W_1, W_A, and W_N are

$$u_1' - \lambda^a = 0 \tag{8a}$$

$$[1/\varphi'(AEU)]\int G' \cdot \theta \, dF(\theta) u_A' - \lambda^a p = 0 \tag{8b}$$

$$[1/\varphi'(AEU)]]\int G' \cdot (1-\theta) \, dF(\theta) u_N' - \lambda^a (1-p) = 0\,. \tag{8c}$$

Then the marginal rate of substitution between wealth in state *A* and *N* for the HM model is

$$\frac{\int G'(1-\theta)dF(\theta)u_N'}{\int G'\cdot\theta dF(\theta)u_A'}=\frac{1-p}{p} \quad , \tag{10}$$

which is the same as the marginal condition in the atemporal context. The proof of necessity is similar to that for Proposition 2. The difference is that *G* now is in a compositional form and its concavity depends on the concavity of φ relative to the concavity of *u*.

For sufficiency, observe that, with fair insurance pricing $p=\bar{\theta}$, and $c=L$ satisfies the tangency condition (10). Suppose that $c<L$ is optimal; then $W_A<W_N$ and condition (10) then implies

$$\int G'(1-\theta)dFu_N'\bar{\theta}=\int G'\theta dFu_A'(1-\bar{\theta})>\int G'\theta dFu_N'(1-\bar{\theta}) \tag{11}$$

which yields $\int G'(\bar{\theta}-\theta)dFu_N'>0$. However, *EU* declines as θ increases, while G' is decreasing in *EU*, given ambiguity aversion. ($G''(EU)$ is negative if $\eta>\tau$.) It follows that G' increases as θ increases, which implies the contradiction

$$0>\text{cov}(G',\bar{\theta}-\theta)=\int G'(\bar{\theta}-\theta)dF(\theta). \tag{12}$$

Hence, the supposition must be false. A similar argument shows $W_A>W_N$ cannot be optimal. Therefore, under fair pricing full insurance coverage is optimal. It follows that partial coverage is optimal if insurance pricing is favorable but unfair.

We have shown that in a temporal context, insurance demand increases with the degree of ambiguity aversion, and saving decreases, when the introduced ambiguity aversion is small. Here we quantitatively measure the effect of ambiguity aversion by assuming isoelastic utility functions and exponential utility functions (CARA and CAAA, respectively). In application, as in Chapter2, we consider the special case when $\eta = \tau$, and the individual is ambiguity-neutral, in which event-ambiguous expected utility reduces to the expected utility model as if there were no ambiguity. If $\eta > \tau$, then the individual is ambiguity-averse.

3.3 Parameter Selection

There are two choice variables, insurance demand and saving. The parameter values are chosen without loss of generality. I implement optimization in Matlab with different levels of risk aversion and ambiguity aversion. Recognizing that state-contingent wealth depends on the choice of coverage and saving, we are interested in an optimal choice solving

$$\max_{c,s}\{AEU(c,s) \mid W_A > 0, W_N > 0\}.$$

With respect to the price of coverage, define ς to be the percentage that premium deviates from the expected probability and allow this ratio to be between 0 and 50%. We also assume a moderate range for the ratio of loss to wealth $r = \frac{L}{M}$, which is allowed to vary over the range 0 and 20%. For illustrative purposes, M is set equal to 2 and L is set equal to 1. In addition, the wealth for each state is restricted to be positive so that in the worst case, an individual loses everything but is not in debt.

3.4 Distributions for Ambiguity

Since the probability of the accident state θ is unknown, we assume three different regimes: the uniform distribution U[0, 0.5], the beta distribution Beta[1,3] and the symmetric triangular distribution defined over 0 to 0.5. In all three cases, the expected value of probability is set to be 0.25, which is chosen for both practical reasons and illustrative purposes. To capture the iteration accuracy, the step value of saving and coverage is set to be 0.005. The optimization process is implemented in Matlab.

3.4.1 Numerical Results: Exponential Functions

Figure 3.1 to Figure 3.7 show the effect of ambiguity aversion on insurance coverage and savings when utility functions are in exponential form while Figure 3.8 to Figure 3.14 shows the same results when utility functions are in isoelastic form. In Chapter 2, it has been shown that in an atemporal context, coverage increases as the decision maker become more risk averse for atemporal risks. This result carries over to the case of temporal risks. See Figure 3-1. The optimal coverage gradually approaches one but never equals one. However, for optimal *saving*, we see a non-monotonic trend: as the decision maker becomes more risk averse, saving decreases first to a critical value and then climbs up moderately thereafter. This suggests that when risk-aversion is at a moderate level, an individual purchases more insurance coverage and saves less. The effects of coverage dominate the effect of saving. When risk aversion exceeds a critical value, the marginal benefit brought by additional insurance coverage alone does not compensate for the risk the individual is taking, therefore, the individual purchases more insurance but also saves more.

Figure 3-2 to Figure 3-7 illustrate the effect of ambiguity aversion on insurance demand and savings when θ is uniformly distributed, triangular distributed and beta distributed, respectively. We find consistent results with theoretical prediction from Chapter 1: when ambiguity aversion changes by a small amount, an individual with constant absolute ambiguity aversion saves less and demands more coverage, and this result does not depend on the level of risk aversion. We select two risk aversion levels, one is greater than one and the other is less than one. For the uniform distribution, saving decreases and coverage increases as an individual becomes more ambiguity averse. See Figure 3-2and Figure 3-2. The monotonic trend of coverage and saving does not depend on the choice of risk aversion. However, an overall higher coverage and an overall lower saving are observed for a more risk adverse individual.

The patterns of optimal coverage and saving are less clear when θ follows a triangular distribution. When absolute risk aversion is fixed at 0.7, there are a few jumps for both saving and coverage as an individual becomes more ambiguity averse and the demands for coverage and saving change in opposite directions for ambiguity aversion between 1 and 3. For example, when coverage reaches its peak value, almost hitting 0.78, saving drops to its lowest value 1.56. When ambiguity aversion is about 2.2, coverage reaches its lowest value around 0.43, while saving reaches peak around 1.65. However, in general it is still the case that when demand for coverage goes up, saving goes down. See Figure 3-2. When risk aversion is 1.2, coverage and saving change in the same direction instead of opposite direction when ambiguity aversion is less than 2.5, for example, both coverage and saving reach local maximum values when ambiguity aversion equals 1.9 and 2.2. However, when ambiguity aversion is 2.6, coverage reaches its global minimum while saving reaches global maximium. When ambiguity aversion goes beyond 3, saving goes down and coverage goes up. See Figure 3-2 . One reason for this non-stable trend for both coverage and saving may be the non-differentiable peak of the triangular distribution.

When θ follows a beta distribution, the effects of ambiguity aversion are more expected. From Figure 3-6 and Figure 3-7, we see a monotonic trend for both savings and coverage. As an individual becomes more ambiguity aversion, coverage increases while saving decreases.

3.4.2 Numerical Results: Isoelastic Functions

Isoelastic cases with ambiguity also yields interesting results that reinforce the results obtained for exponential functions. The changes for both saving and coverage are monotonic as an individual becomes more ambiguity averse: the individual purchases more insurance and saves less. This result holds for both the uniform distribution and the beta distribution. In terms of magnitude, the effect of ambiguity aversion on coverage is greater in the isoelastic case than in the exponential case. Moreover, coverage is more sensitive to ambiguity aversion than is saving when θ is uniformly distributed and beta distributed. However, when θ has a triangular distribution, it is not guaranteed that coverage increases and saving decreases as an individual becomes more ambiguity averse. As in Figure 3-13 and Figure 3-14, coverage drops at a level of ambiguity aversion of 3.7 and saving jumps at the same time. Therefore , when one of saving and coverage changes, the other changes in the opposite way simultaneously.

3.5 Conclusion

At least for the highly regular exponential and isoelastic specifications, coverage goes up and saving goes down as either risk aversion or ambiguity aversion increases when distribution of θ is differentiable. This result is consistent with the prediction for small introductions of ambiguity aversion reported in Chapter 1. Since the functional forms are amenable to empirical estimation, the way may be open to testing this conclusion empirically.

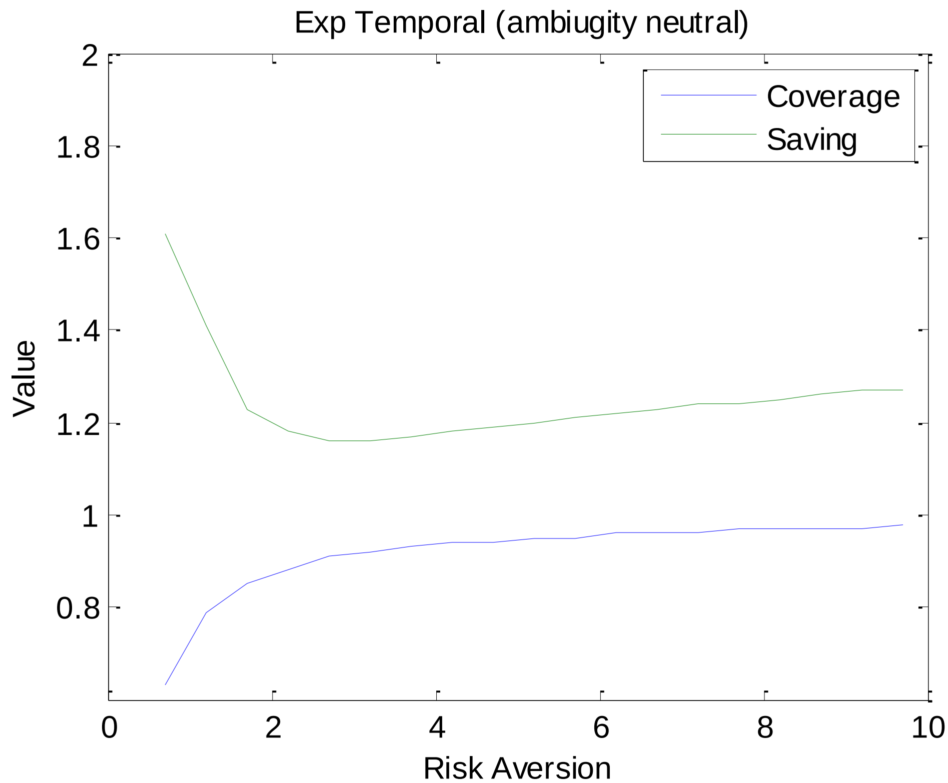

Figure 3-1 Exponential Preferences Insurance Demand and Savings vs Risk Aversion with Ambiguity Neutrality for Temporal Risks

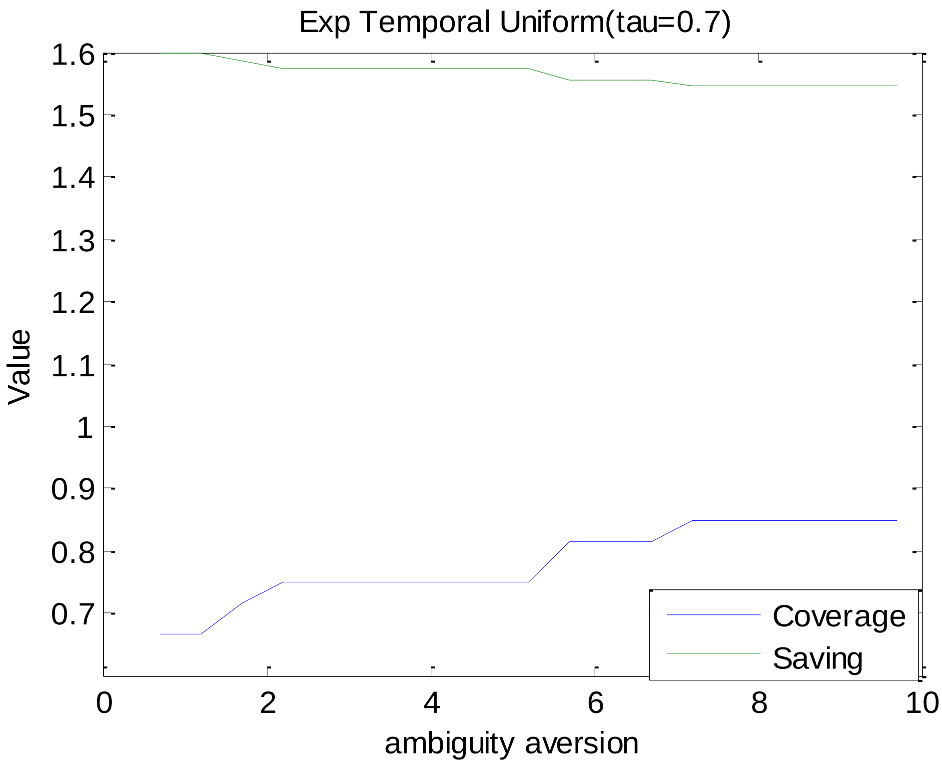

Figure 3-2: Exponential Preferences Insurance Demand and Savings vs Ambiguity Aversion with Unform Distributuion and Risk Aversion=0.7 for Temporal Risks

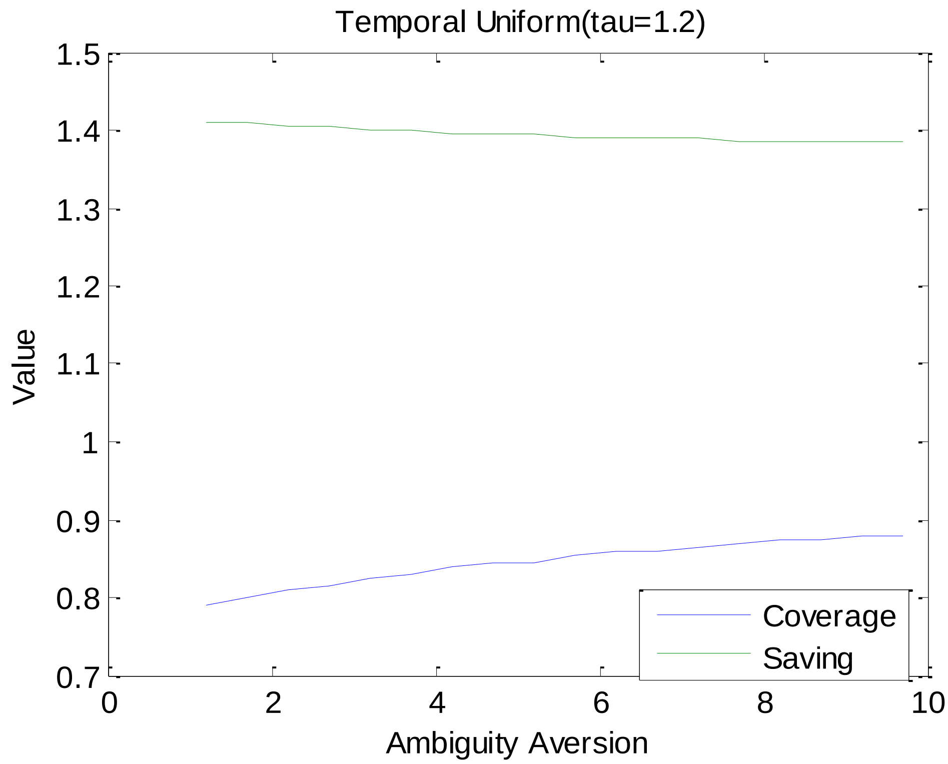

Figure 3-3: Exponential Preferences Insurance Demand and Savings vs Ambiguity Aversion with Unform Distributuion and Risk Aversion=1.2 for Temporal Risks

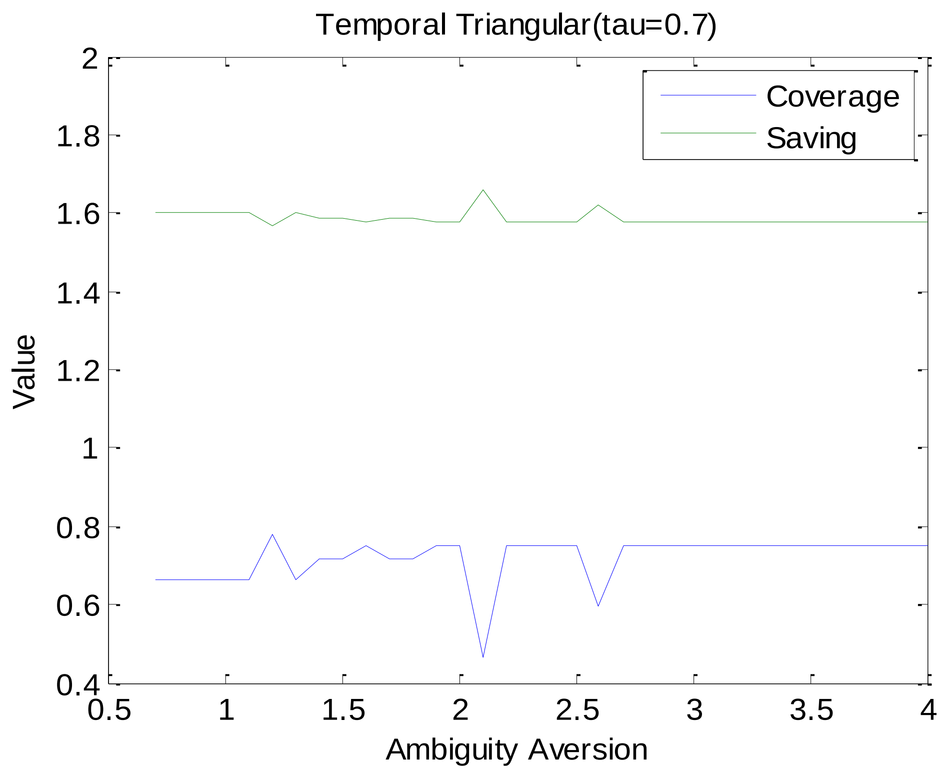

Figure 3-4: Exponential Preferences Insurance Demand and Savings vs Ambiguity Aversion with Triangular Distributuion and Risk Aversion=0.7 for Temporal Risks

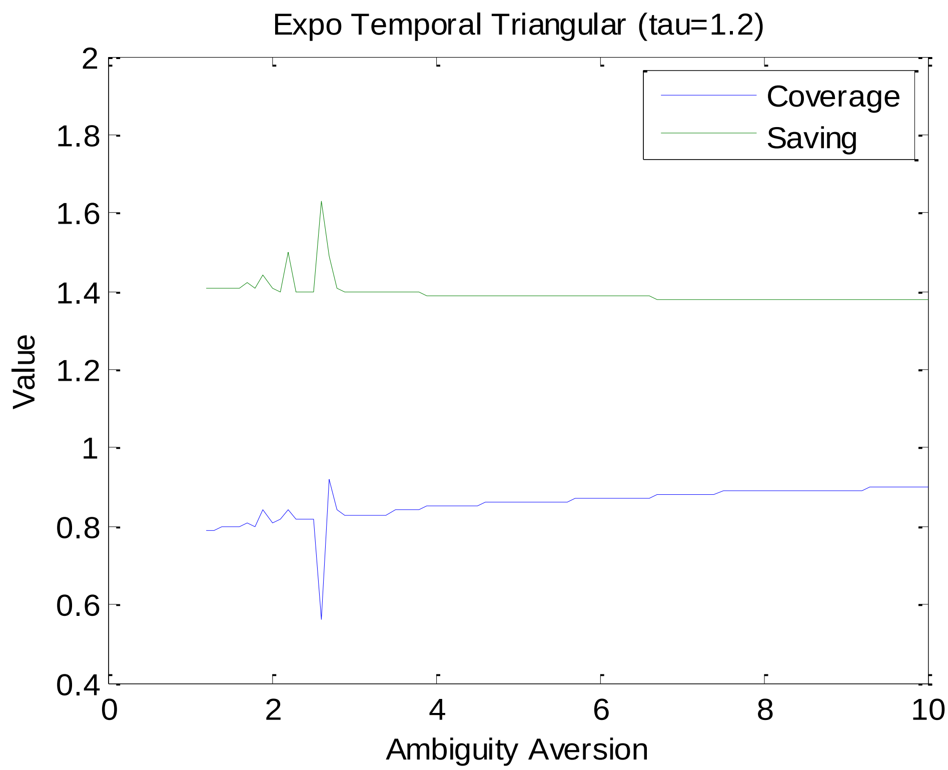

Figure 3-5: Exponential Preferences Insurance Demand and Savings vs Ambiguity Aversion with Triangular Distributuion and Risk Aversion=1.2 for Temporal Risks

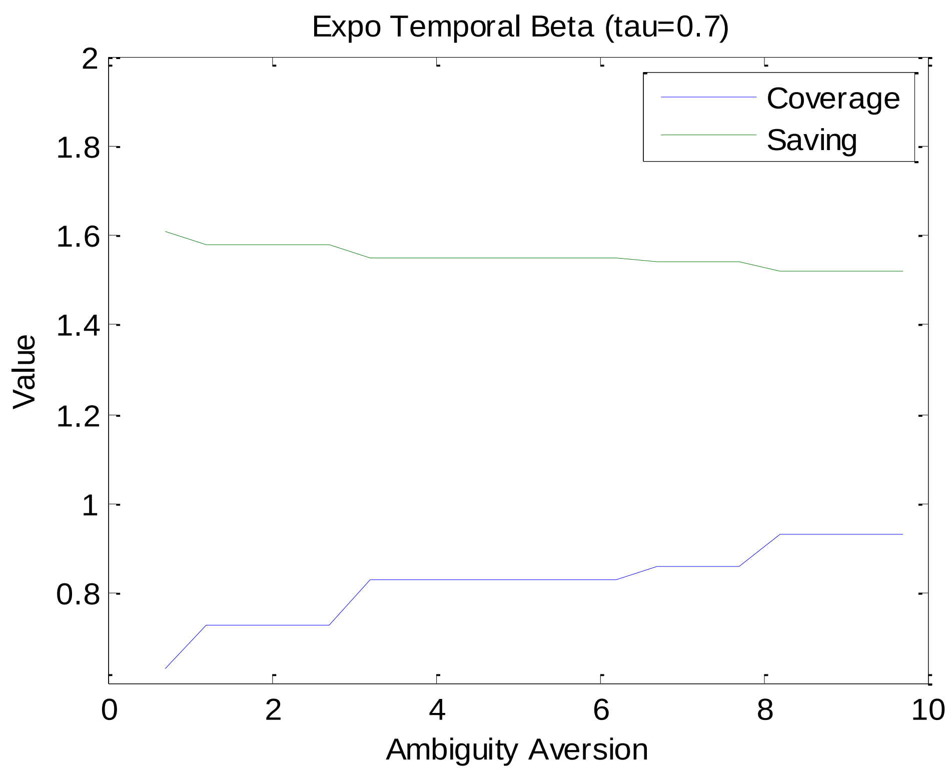

Figure 3-6: Exponential Preferences Insurance Demand and Savings vs Ambiguity Aversion with Beta Distributuion and Risk Aversion=0.7 for Temporal Risks

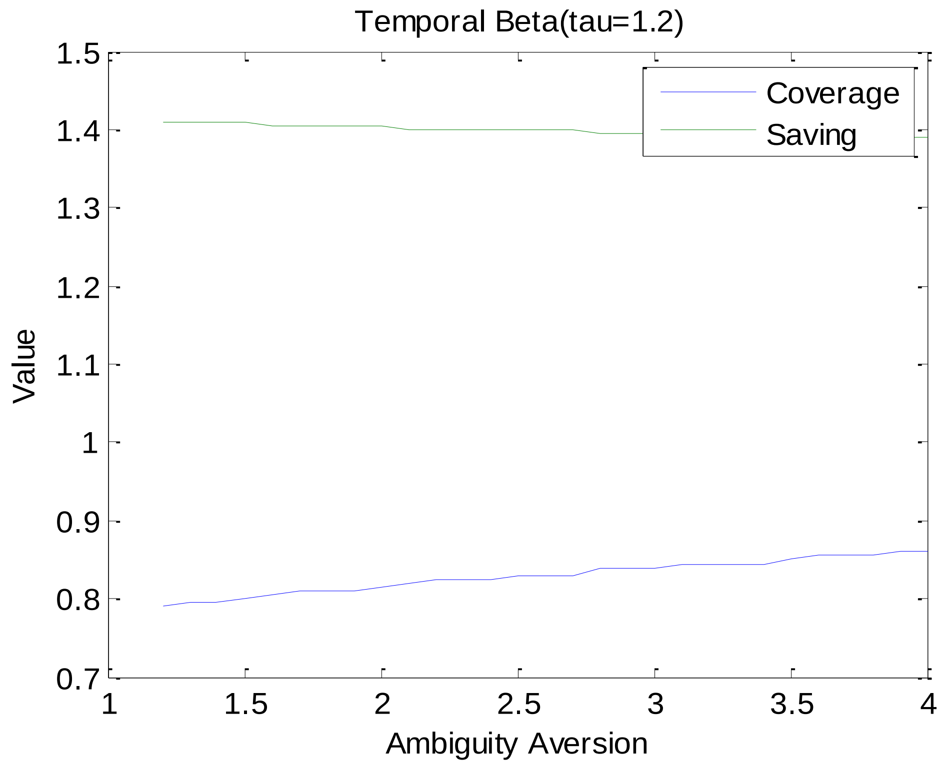

Figure 3-7: Exponential Preferences Insurance Demand and Savings vs Ambiguity Aversion with Beta Distributuion and Risk Aversion=1.2 for Temporal Risks

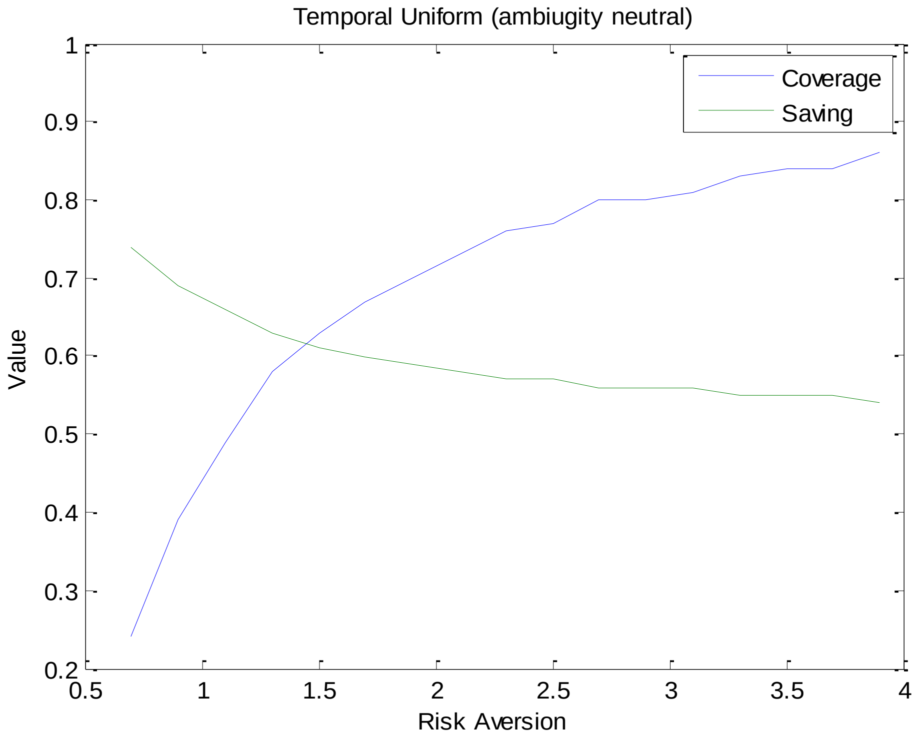

Figure 3-8: Isoelastic Preference Insurance Demand and Savings vs Risk Aversion with Unform Distributuion Temporal Risks

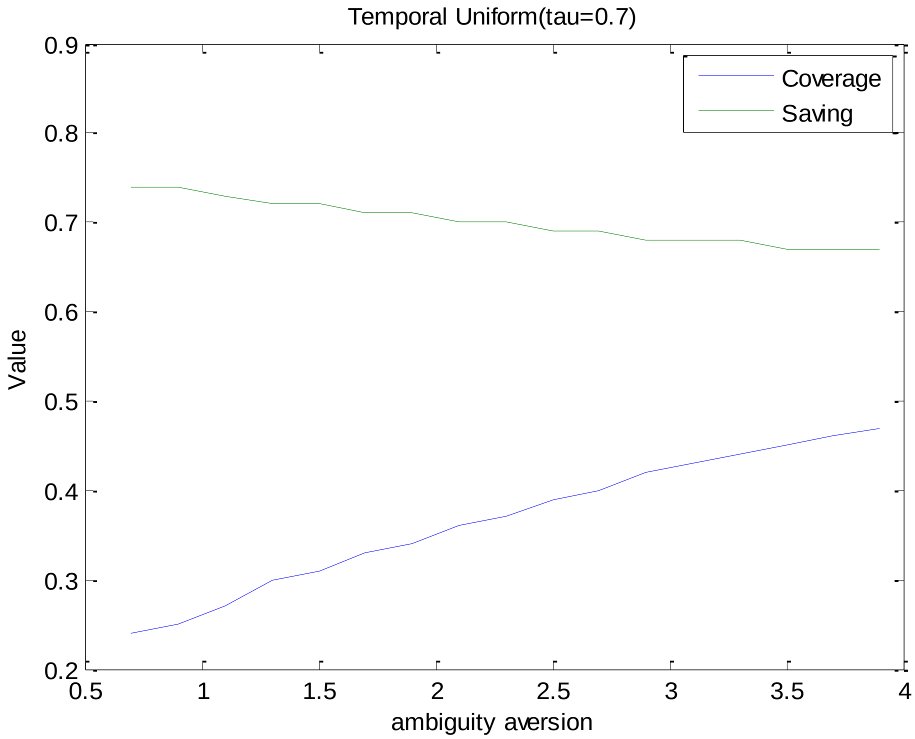

Figure 3-9: Isoelastic Preferences Insurance Demand and Savings vs Ambiguity Aversion with Unform Distributuion and Risk Aversion=0.7 for Temporal Risks

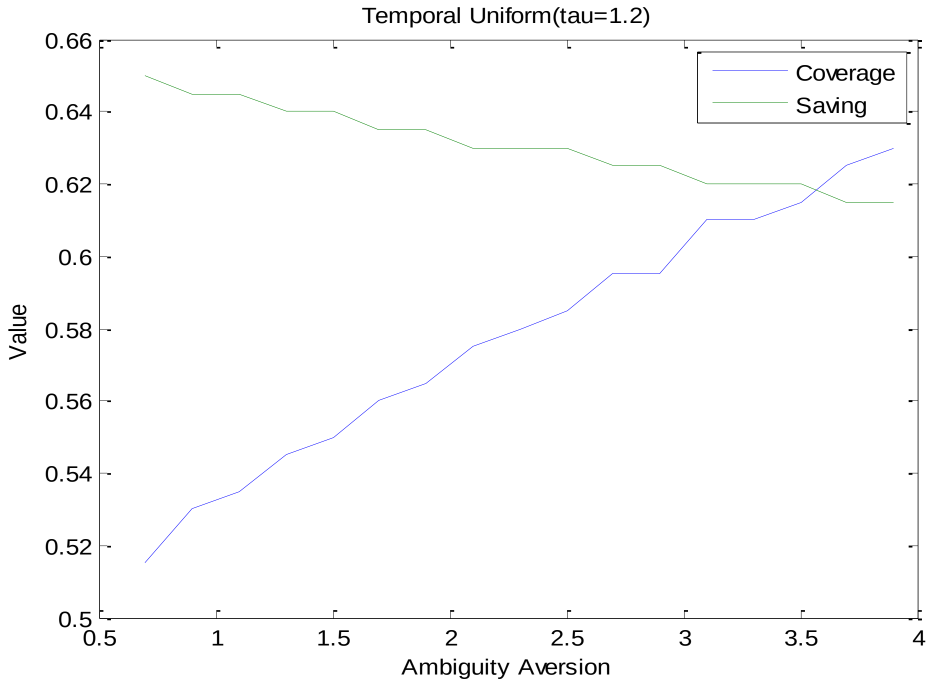

Figure 3-10: Isoelastic Preferences Insurance Demand and Savings vs Ambiguity Aversion with Unform Distributuion and Risk Aversion=1.2 for Temporal Risks

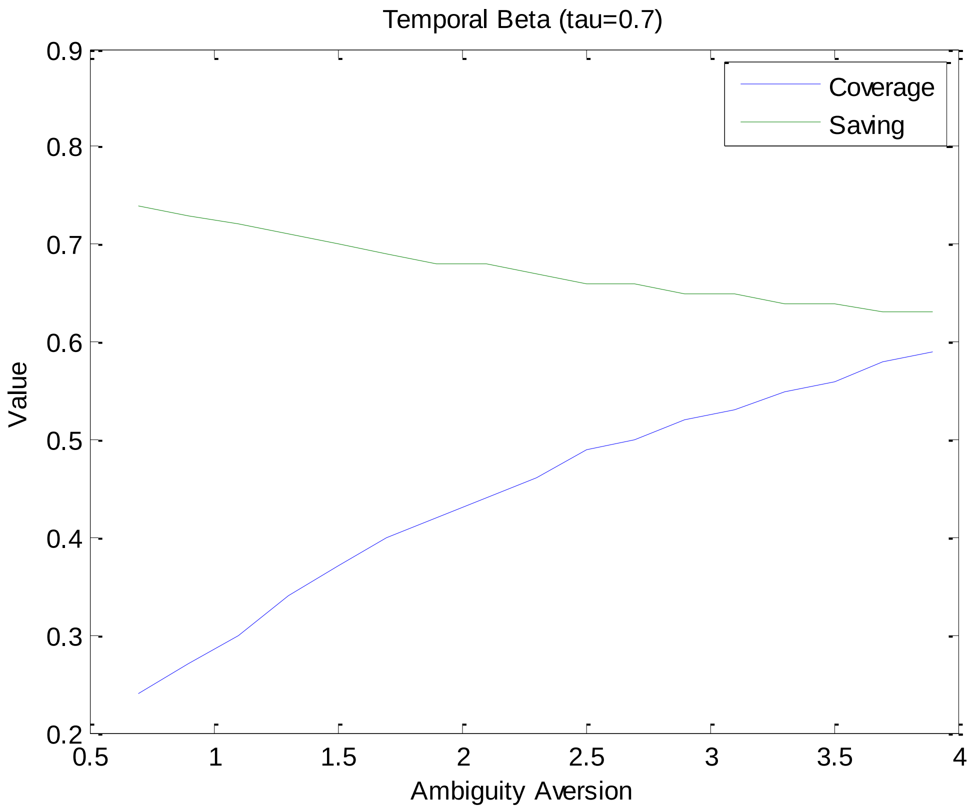

Figure 3-11: Isoelastic Preferences Insurance Demand and Savings vs Ambiguity Aversion with Beta Distributuion and Risk Aversion=0.7 for Temporal Risks

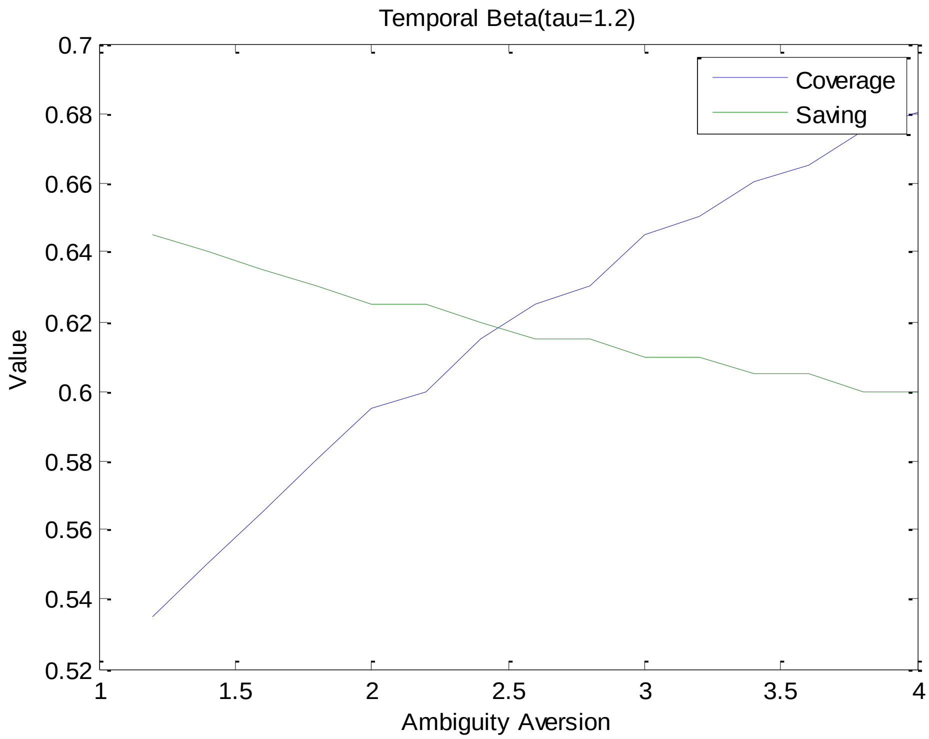

Figure 3-12: Isoelastic Preferences Insurance Demand and Savings vs Ambiguity Aversion with Beta Distributuion and Risk Aversion=1.2 for Temporal Risks

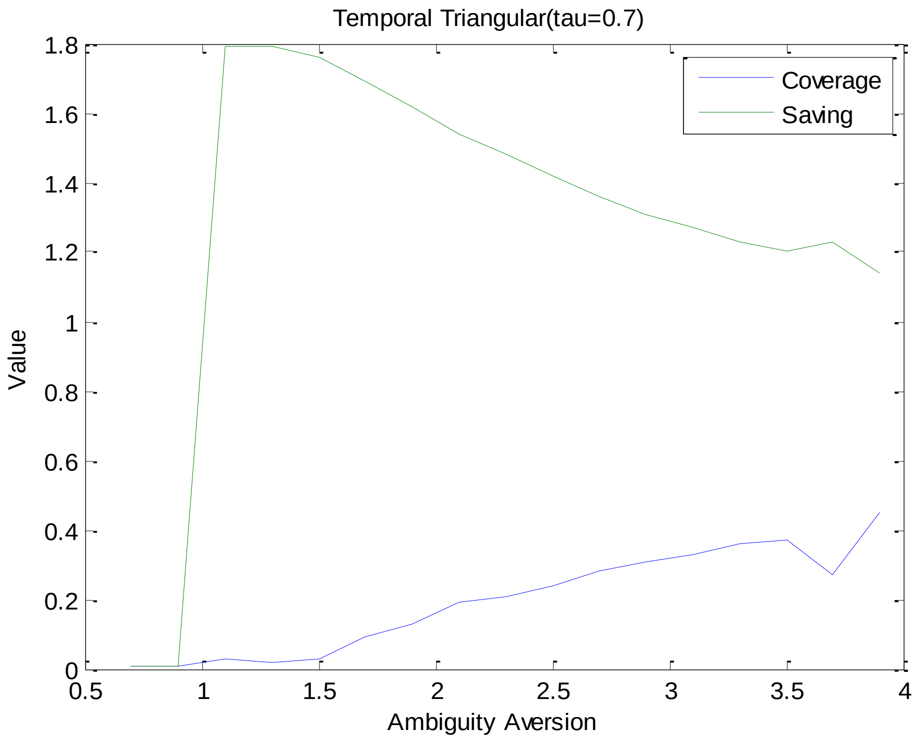

Figure 3-13 Isoelastic Preferences Insurance Demand and Savings vs Ambiguity Aversion with Triangular Distributuion and Risk Aversion=0.7 for Temporal Risks

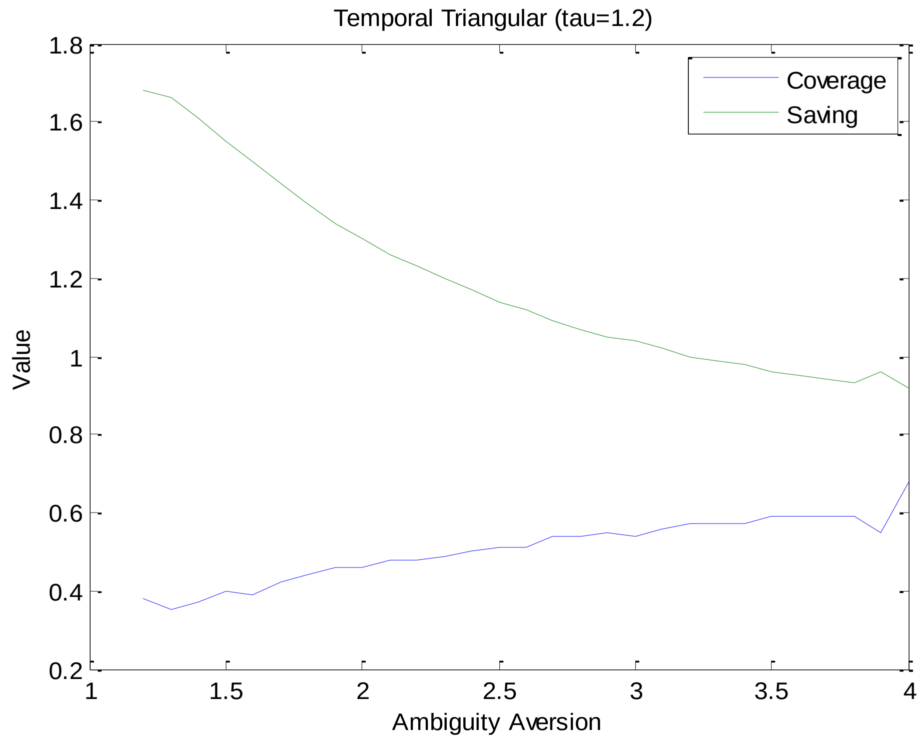

Figure 3-14: Isoelastic Preferences Insurance Demand and Savings vs Ambiguity Aversion with Triangular Distributuion and Risk Aversion=1.2 for Temporal Risks

CONCLUSIONS

Drawing on the models of Klibanoff et al.(2005, 2009), the first result shown in Chapter 1 is that, in the presence of ambiguity, fair pricing is still a necessary condition for full-coverage insurance to be optimal as is the case in expected utility theory in both atemporal and temporal contexts and the sufficiency side of this result is established through proof by contradiction.

In the atemporal context, ambiguity aversion increases the aversion to bearing risk about wealth and ambiguous expected utility is more averse to risk than expected utility, implying that for an ambiguity-averse decision maker demands more insurance in the presence of ambiguity than in its absence. This result does not carry over to the temporal context without further qualification since the saving decision transfers wealth across time periods, introducing an additional response to ambiguity that bears on the insurance decision. In the temporal context with unfair insurance pricing, introducing a small amount of ambiguity aversion results in an increase in insurance coverage and a decrease in savings for an ambiguity-averse insurance applicant if the ambiguity preferences exhibit constant absolute ambiguity aversion (CAAA). The monotonicity property of the marginal utility of coverage and savings with respect to ambiguity aversion is guaranteed only if the degree of ambiguity aversion is equal to the degree of ambiguity prudence, which requires constant absolute ambiguity aversion.

It is also discussed that if an arbitrary amount of ambiguity aversion is introduced, no prediction is forthcoming for the effect of ambiguity aversion on insurance demand and saving if the decision maker exhibits nonincreasing absolute ambiguity aversion (NIAAA). This is because with NIAAA, AEU exhibits DARA, resulting in opposing effects of ambiguity aversion on optimal insurance coverage and saving. Therefore, sign-definite predictions are possible only if ambiguity aversion has no effect on the marginal benefit of saving, which requires ambiguity preferences exhibit a critical degree of IAAA.

Chapter 2 and Chapter 3 attempt to quantitatively confirm the results in Chapter 1 by assuming regular isoelastic and exponential specifications for utility functions.
Chapter 2 implements a quantitative analysis designed to measure the effect of ambiguity aversion on insurance demand. It discusses why HM model is more suitable for the quantitative analysis than the KMM model. Two model specifications are assumed to represent CRAA and CAAA and each of the parameters is chosen carefully to maintain the generality of the analysis as well as to be economically meaningful. The probability of an accident follows a mean-preserving uniform, triangular and beta distribution, respectively to capture ambiguity. For both CRAA and CAAA scenarios, the findings are consistent with Corgnet et. al (2012) that coverage increases as individual becomes more risk averse or ambiguity averse or both, though ambiguity aversion is not equivalent to an increase in risk aversion in terms of its effect on the strength of demand.

Chapter 3 shows that at least for the highly regular exponential and isoelastic specifications, coverage goes up and saving goes down as either risk aversion or ambiguity aversion increases when distribution of the loss probability θ is differentiable. This result is consistent with the prediction for small introductions of ambiguity aversion reported in Chapter 1. However, when ambiguity is triangular distributed with non-differential property at some point, the optimal saving and coverage show a non-monotonic trend as the individual becomes more ambiguity averse, which supports the theoretical result in chapter one when an arbitrary ambiguity aversion is introduced. Since the functional forms are amenable to empirical estimation, the way may be open to testing this conclusion empirically.

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