CONCEPTUAL, PROCEDURAL, AND METASTRATEGIC KNOWLEDGE IN MATHEMATICS

by

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(Under the Direction of Martha Carr)

ABSTRACT

This study examines a modification of the iterative model (Rittle-Johnson & Siegler, 1998), in which metastrategic knowledge was hypothesized to be a mediator underlying the relationship between conceptual and procedural knowledge. Fifty-six fifth-graders participated in this study. The children solved fraction problems, half of which were computation problems and half of which were word problems. Two approaches were used to assess conceptual and procedural knowledge: the same-problem approach and the different-problem approach. Both approaches yielded similar findings. Conceptual, procedural, and metastrategic knowledge correlated positively with each other. The correlation between conceptual and procedural knowledge was reduced after controlling for metastrategic knowledge, suggesting that metastrategic knowledge played a role in the relationship between conceptual and procedural knowledge. The relationships among the three knowledge types grew stronger over trials, indicating a developing link among them.

INDEX WORDS: Metastrategic Knowledge, Conceptual Knowledge, Procedural Knowledge, Mathematics Problem Solving

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CHAPTER 1

INTRODUCTION

Understanding the key factors in successful mathematical problem solving is one of the central goals of educational psychologists. Conceptual knowledge and procedural knowledge have been widely accepted as key factors affecting the development of mathematical ability (Hiebert & Lefevre, 1986; Rittle-Johnson & Siegler, 1998). Other researchers assert that a child's explicit metacognitive knowledge leads to flexible use of mathematics strategies, which in turn improves mathematics ability (Carr & Jessup, 1995; Lucangeli, Cornoldi, & Tellarini, 1998). Despite extensive research examining each factor independently, there is an absence of research that explores the possible links and relationships between metacognitive knowledge, conceptual knowledge, and procedural knowledge in the development of mathematics problem solving. The purpose of this study was to examine the roles of metacognitive, conceptual, and procedural knowledge on children's ability to solve computation and word problems in the fraction domain. It is argued that the three types of knowledge work together to improve performance in mathematics.

In the following sections, an argument will be presented for the role of metacognition in the development of conceptual and procedural knowledge. First, the literature on metacognitive knowledge and its impact on mathematics will be discussed. Following that, conceptual and procedural knowledge as predictors of mathematics ability will be discussed. Three developmental theories of conceptual and procedural knowledge in mathematics (concept first, procedure first, iterative) will be presented and critiqued. Finally, an alternative version of the iterative model that incorporates conceptual, procedural, and metacognitive knowledge will be discussed.

Metacognitive Knowledge

Metacognition, as coined by John Flavell in 1971, is "cognition about cognition". Metacognition has two separate but interrelated facets, namely the knowledge and understanding of cognitive phenomena, and the regulation and control of cognitive action (Flavell, 1976). The notion of the importance of examining one's own cognitive processes is not a new one. For example, Piaget's (1971) concept of 'reflective abstraction' as a mechanism for extracting, reorganizing, and consolidating knowledge is similar to the construct of metacognition.

Earlier studies in the area of metacognition focused primarily on metamemory, or knowledge about memory processes and contents. Specifically, metamemory refers to children's knowledge about memory strategies such as rehearsal and elaboration, how they work, and what factors influence memory functioning (Flavell, 1971). Researchers have been interested on metamemory because theoretically metamemory plays a critical role in memory performance. Empirical findings indeed suggest that children who are aware of the relationship between strategy use and recall often perform better in memory tasks than children who do not possess such awareness (e.g., Justice, Baker-Ward, Gupta, & Janning, 1997; Schneider & Pressley, 1997). Based on a meta-analysis of 60 studies (7097 subjects), Schneider and Pressley (1997) found that the overall metamemory-memory correlation coefficient was .41, suggesting that metamemory makes a reliable contribution to memory performance. The relationship was also found to be stronger among older children than among younger children. For example, adolescents are more likely than younger children to transfer elaboration strategies. Although older grade-school children can be taught many elaboration strategies, they do not transfer those strategies as readily as adolescents. This evidence suggests that metacognitive knowledge is not necessarily qualitatively different in development from other kinds of knowledge. Rather, it is acquired gradually over time and develops through experience with appropriate strategies.

More contemporary models of metacognition encompass multidimensional constructs including awareness, understanding, monitoring, and management of one's strategic performance of many kinds of cognitive tasks (Carr & Biddlecomb, 1998; Kuhn & Pearsall, 1998). Metacognition is believed to play an important role in cognitive activities that are related to problem solving and learning. Good information processors, as described by Pressley, Borkowski, and Schneider (1987) are reflective, planful, and resourceful. They deploy attention appropriately, possess sufficient short-term memory space, and continually monitor their performance. In a learning context, metacognition essentially involves knowledge of when to use, how to coordinate, and how to monitor various skills in problem solving (Mayer, 1998).

In line with this, metacognitive theory assumes that at least with cognitive tasks that entail effortful cognitive processing, strategy selection and performance are influenced by children reflecting upon their understanding of task, available cognitive resources, and their experience with similar problems (Kuhn, Garcia-Mila, Zohar, & Anderson, 1995). Kuhn and Pearsall (1998) proposed two components associated with metastrategic understanding in their reasoning tasks. These two components are the awareness of the nature and requirements of the task, and the awareness of the strategies that exist in one's repository that may be applicable to the task. To test this hypothesis, Kuhn and Pearsall (1998) measured the two components of metastrategic understanding separately and examined their relation to strategic performance in a task that involved inductive causal reasoning in a multivariable context. They used a microgenetic method in which fifth-grade students were presented the task repeatedly over a seven-week period. The two portions of the metastrategic knowledge were assessable as distinct entities and they were related to the mastery of inductive strategy. Children were unlikely to attain strategic mastery until they had reached a minimum level of metastrategic understanding. The authors suggested that the two components of metastrategic understanding are necessary but not sufficient for mastery in strategic performance. Also, they recognized that multiple possibilities exist in the direction of causality between strategic performance and metastrategic understanding. They proposed a bidirectional model, in which the successful use of strategies may "feed up" to the metastrategic level and result in strengthened awareness of the strategy and understanding of its value. Similarly, both components of metastrategic knowledge may "feed down" to the strategic level, guiding the application of strategies. In sum, the authors advocate for a model of multiple and bidirectional paths of influence between strategic performance and metastrategic understanding.

Impact of Metacognitive Knowledge in Mathematics

The development of metacognitive knowledge is receiving increasing attention among educators in response to students' rote learning and their resulting inability to extend their understanding to new contexts (Brown, 1997; Kuhn, 1999). Lucangeli and Cornoldi (1997) believed that one of the reasons why mathematical learning is difficult for students is the level of specificity of context. For example, being able to carry out arithmetic operations does not necessarily guarantee a student's ability to perform correctly in word problems that resemble a real world context. This is because performing arithmetic operations is only a part of mathematics problem solving. Mathematics problem solving also requires the student to extract relevant information from the problem, decide what needs to be done and when, thus the importance of metacognition.

Metacognitive theory predicts that correct mathematics strategy use would partly result from children's understanding of why and when to use mathematics strategies and the monitoring of problem solving activity (Schoenfeld, 1987). Having multiple strategies provides flexibility for children to handle complex information more efficiently and respond to novel problems more adaptively. Lucangeli and Cornoldi (1997) hypothesized that metacognitive knowledge contributes to the flexibility and attentiveness of one's conscious use of cognitive abilities. Specifically, metacognitive knowledge is more important for some aspects of mathematics in which more complex and flexible thought processes are necessary. Aspects of mathematics that require mostly automatic processes and overlearned knowledge will not benefit from metacognitive reflection. Research findings support this hypothesis. Carr, Alexander, and Folds-Bennett (1994) examined how metacognitive knowledge about mathematics strategies affects second grade children's correct use of strategies over a five-month period. They found that even second graders possess metacognitive knowledge about mathematics strategies. Metacognitive knowledge was related to students' immediate correct use of mathematics strategy and also latent correct use of mathematics strategy five months later. In a follow-up study, Carr and Jessup (1995) found that metacognition comes into play in second graders' strategy use for a relatively new and effortful strategy (decomposition) that is in the process of being acquired, but is not influential for highly automated, less effortful strategies (retrieval or min strategy). Similarly, Lucangeli and Cornoldi (1997) found that metacognitive components are related to third and forth graders' mathematical performance in a standardized test of arithmetic, geometry, and problem solving. Specifically, this relationship is stronger for tasks that are less automatized: problem solving and geometrical tasks. For arithmetic calculation, the relationship is present only for third graders but not for fourth graders. They proposed that this is due to the higher degree of automatization in arithmetic skills for fourth graders in comparison to third graders.

Garofalo and Lester (1985) presented a cognitive-metacognitive framework in an attempt to specify key points at which metacognitive decisions are likely to influence cognitive actions. The four stages are namely, orientation (strategic behavior to assess and understand a problem), organization (planning of behavior and choice of actions), execution (regulation of behavior to conform to plans), and verification (evaluation of decisions made and of outcomes of executed plans). Similarly, Mayer (1985) proposed four cognitive processes involved in mathematical problem solving. The four processes are, namely, translation (the conversion of problem to mental representation), integration (the combination of inferences in a consistent representation of the whole problem context), planning (the preparation of an action plan), and execution (the action of solving the problem). These theoretical frameworks identify aspects of metacognitive skills and have been applied in instructional programs to promote students' mathematics performance.

Lucangeli, Cornoldi, and Tellarini (1998) conducted a series of instructional research projects to examine the importance of metacognitive knowledge in learning mathematics. The theoretical framework used to create the instructional program was similar to the framework put forward by Mayer (1985) and Garofalo and Lester (1985). In the first investigation, fifth grade high and low achievers in arithmetic reasoning and problem solving were found to differ in their metacognitive skills, including prediction of success or failure in a task, planning for correct steps leading to a goal, monitoring strategy use and other cognitive activities, and evaluation of performance. In the second investigation, an instructional program designed to improve the above aspects of metacognitive skills was developed for children between 8 and 12 years of age.

The experimental condition participants received the instructional program whereas the control group received standard mathematical instruction. After completion of the program, experimental condition students were found to be significantly better than the control condition students in problem solving and logical reasoning. Hohn and Frey (2002) also developed a heuristic strategy instructional program (SOLVED) for solving word problems based on Mayer's (1985) framework. Children were trained over several periods to use the strategy to solve different types of word problems. Results of two experiments involving 223 third-, fourth-, and fifth-grade students indicated that SOLVED was more effective in aiding both short-term (after each lesson) and delayed problem solving (two weeks after instruction) than traditional problem-solving instruction. Accuracy in problem solving was significantly correlated with metacognitive processing. These research findings suggest that metacognition is likely to influence performance at various points of problem solving activity. In addition, metacognition is a skill that can be improved through appropriate intervention.

Conceptual and Procedural Knowledge

The distinction between conceptual and procedural knowledge has been the focus of studies in various areas of cognition, including memory (Anderson, 1993), propositional reasoning (Byrnes, 1988), and mathematics (Hiebert & Lefevre, 1986). Conceptual knowledge consists of the core concepts for a domain and their interrelations (i.e., "knowing that"). Procedural knowledge is the ability to execute the steps required to attain various goals (i.e., "knowing how"). In the research literature, these two types of knowledge are not always considered as distinct entities but may be considered as knowledge developing on a continuum. In the following sections, developmental theories of conceptual and procedural knowledge in mathematics will be presented.

Impact of Conceptual and Procedural Knowledge in Mathematics

There are disagreements among researchers regarding the developmental relation between conceptual knowledge and procedural knowledge (Rittle-Johnson & Siegler, 1998). Attempts to resolve this issue are of both theoretical and practical importance. The widespread observation that many American children perform poorly in school mathematics in comparison to their counterparts in other countries (Towse & Saxton, 1998) has resulted in calls for improved instruction. As a result of differing views of the developmental sequencing and the relationship between conceptual and procedural knowledge, instructional reforms have bounced back and forth from emphasizing procedures to emphasizing concepts. In response to this, the National Council of Teachers of Mathematics (2000) recently recommended that both the conceptual understanding and the practice of routine skills be taught. In order to develop effective instruction that will incorporate both types of knowledge, it is necessary to first understand how conceptual and procedural knowledge develop and work together to play a crucial role in mathematical proficiency. There are three theories about the developmental relationship between conceptual and procedural knowledge: (1) Conceptual knowledge develops before procedural knowledge; (2) Conceptual knowledge develops after procedural knowledge; (3) Conceptual and procedural knowledge develop iteratively. Each of these approaches will be presented below.

Concept First

Hiebert and Lefevre (1986) proposed that conceptual knowledge is important for procedure selection, procedure monitoring, and the transfer of procedural knowledge to new situations. Research findings suggest that in some areas of mathematics, conceptual understanding plays an important and leading role in procedure adoption and generation (e.g., Byrnes & Wasik, 1991; Canobi, Reeve, & Pattison, 1998). Canobi, Reeve, and Pattison (1998) used a mathematics task that consisted of consecutive problems in which their relational properties were manipulated to reflect aspects of additive composition (natural numbers are composed by addition), commutativity ($a + b = b + a$), and associativity principles [$(a + b) + c = a + (b + c)$]. They found that children who were conceptually competent in recognizing and explaining properties of addition tended to solve problems more quickly and accurately compared to children who were less conceptually competent. Also, children who spontaneously used conceptually based procedures were also able to explain and justify these procedures, but the reverse was not necessarily true. In the Hiebert and Wearne study (1996), 70 children were followed over the first three years of school while they were learning about place value, multidigit addition, and subtraction. It was found that students who demonstrated conceptual understanding (called the understanders) were more likely than their peers (the nonunderstanders) to invent new procedures and modify old ones to solve new problems when facing challenging problem solving tasks. Nonunderstanders, however, appeared to be incapable of developing appropriate procedures and performing correctly without instruction. These findings suggest that early conceptual understanding plays a crucial role in stimulating and guiding the development of procedural skills.

Procedure First

On the other hand, some theories suggest that procedural knowledge precedes conceptual knowledge. For example, Karmiloff-Smith (1992) suggests that knowledge begins at a basic, procedural level that becomes increasingly complex and conceptual over time. Several studies have shown that young children can count correctly before they understand certain counting principles, such as order irrelevance of addition (e.g., Fuson, 1988; Wynn, 1990). Siegler and

Stern (1998) used inversion problems (a + b – b, the principle that adding and subtracting the same number leaves the result unchanged) to examine the process in which second graders' discover the shortcut strategy based on this principle (simply ignore the number that is both added and subtracted). They found that most children generated fast solution times that were indicative of use of the shortcut strategy before they could explicitly report using it. In other words, children first executed the shortcut strategy at an implicit, unreportable level before they were able to report their use of the strategy explicitly. This indicates that procedural knowledge develops before the emergence of conceptual knowledge.

The Iterative Model

Due to the use of different assessment tools and examination of different content areas in mathematics, the findings on concept-first and procedure-first present a paradox. In view of this, Rittle-Johnson and Alibali (1999) argued that such a debate is misguided. As an attempt to reconcile the two bodies of research literature, they hypothesized a gradual, bidirectional iterative model for the development of conceptual and procedural knowledge. The model suggests that conceptual and procedural knowledge develop in a hand-over-hand, mutually supportive manner. In other words, none of the knowledge types develops in a global way. Instead, increases in one type of knowledge (for example, conceptual knowledge) lead to gains in the other type of knowledge (procedural knowledge), which in turn will lead to further increases in the first type of knowledge (conceptual knowledge).

In two intervention studies, Rittle-Johnson, Siegler, and Alibali, (2001) examined the iterative development of conceptual and procedural knowledge in children's learning about decimal fractions using multifaceted and continuous measures of knowledge. In both studies, conceptual and procedural knowledge appeared to develop in a gradual, hand-over-hand process.

Pretest conceptual knowledge was positively related to the amount of procedural knowledge acquired during the intervention phase. In turn, procedural knowledge assessed during the intervention phase predicted the improvement of conceptual knowledge from pretest to posttest. In addition, the relation between pretest conceptual knowledge and subsequent procedural knowledge was substantially reduced when frequency of correct problem representation was included in the analyses, suggesting a mediating role for problem representation. In other words, forming a correct internal representation of a problem in working memory during problem solving is one of the mechanisms underlying the iterative development of procedural and conceptual knowledge.

Purpose of the Present Study

It is evident that multiple factors interact in mathematics problem solving. However, most of the studies examine these factors independently; consequently, we do not have insight into how they develop and work together. The dearth of studies linking metacognitive, conceptual, and procedural knowledge calls for further research in this area. The present study examined an alternative version of the iterative model, in which metacognition is hypothesized to be a mediator underlying the relations between conceptual and procedural knowledge. In other words, the link from initial conceptual knowledge to improved procedural knowledge will be explained, at least in part, by improvement in metacognitive knowledge (refer to Figure 1). As metacognition is a multifaceted construct, this study focused on metastrategic understanding (Kuhn & Pearsall, 1998) as the mediator of the relationship between procedural and conceptual knowledge. Metastrategic understanding includes the awareness of the nature and requirements of the task in hand, and also the awareness of the strategy that may be applicable to the task. These two components of metastrategic understanding are considered important for mathematical performance and support the connection between conceptual and procedural knowledge to be formed.

Fraction problems were used as the problem task in this study because many children encounter significant difficulties in this domain of mathematics (Smith, 1995). There is empirical evidence for a relationship between conceptual and procedural knowledge in fraction decimals (Byrnes & Wasik, 1991; Hecht, 1998). Byrnes and Wasik (1991) examined conceptual knowledge of fraction using tasks of picture-symbols (selecting pictures depicting a fraction), simple morphism (identifying drawings of the same fraction), and order problems. To measure procedural knowledge, they measured participants' accuracy on multiplication and addition problems. Performance on each of the conceptual knowledge tasks significantly correlated with fraction computation accuracy (overall $r = .55$). Therefore, children who have both conceptual and procedural knowledge of fractions tend to perform better in mathematical tasks.

The microgenetic approach will be used to detect the gradual, bidirectional relations between conceptual and procedural knowledge. Unlike a prototypic microgenetic study, the present study involved only a single session assessment. However, the spirit of microgenesis was followed by using dense sampling of behavior and trial-by-trial analysis. The problem solving tasks are designed to assess conceptual knowledge (understanding of why strategy is appropriate), procedural knowledge (the ability to execute the algorithmic steps), and metastrategic knowledge (the ability to perceive the relationship between a consecutive pair of problems based on the requirements of the task, and applicable strategies for the problems).

CHAPTER 2

METHOD

Participants

Fifty-six fifth-grade students, 29 boys and 27 girls, from five classrooms in three suburban elementary schools participated in this study. Students participated near the end of their school year in May with permission from parents and schools. Fifth graders were selected for this study because fraction comparison and fraction addition were part of the mathematics instruction for fifth grade. However, as fifth graders had been exposed to fraction knowledge only for one year, it was expected that the participants would show lower level of performance. This was desirable for the study as it allowed the observation of the growth of knowledge in the fraction domain. In the beginning of the data collection, the researcher entered classrooms to invite the students to participate and gave out consent forms. Only those children who returned completed consent forms were interviewed.

Procedures

The children were interviewed individually, outside of the classroom. All of the interviews were audiotaped for later coding. The children were told explicitly that they were not being tested and that the researcher was only interested in learning about the different ways children solve problems. The researcher told the participants that they would be asked questions about how they solved the problems and about why they solved the problems the way they did. The researcher presented the mathematics task, which consisted of twenty fraction problems, to each participant. The participants were instructed to solve the problems using paper and pencil.

They had unlimited time to solve each problem and were encouraged to write down their work in as much detail as possible.

Mathematics Task

The mathematics task was designed to assess participants' conceptual and procedural knowledge of fractions involved in solving computational problems and word problems. Participants' ability to monitor and connect their knowledge between computational problems and word problems was assessed as a part of this task and served as an index for metastrategic knowledge.

The mathematics problems were generated from several mathematics textbooks recommended by the Georgia Department of Education. The complete mathematics task can be found in Appendix A. The task involved twenty fraction problems of two domains, namely fraction comparison and fraction addition. Within each domain there were ten problems, half were the computation problem type and half were the word problem type. As can be seen from Appendix A, each computation problem was paired up with a word problem. Each pair of problems required the same conceptual and procedural knowledge necessary to solve the particular pair of problems accurately. Paired computation and word problems were always presented consecutively with the order of presentation being random (computation first or word first). Problem pairs of the two domains (comparison and addition) were presented in alternating order. A pilot study with ten participants was conducted before the actual study was conducted to ensure that the mathematics task was appropriate in level of difficulty and length.

Assessment and Coding of Conceptual and Procedural Knowledge

Studies in the research literature that assess conceptual and procedural knowledge use different assessment tasks (e.g., Byrnes & Wasik, 1991; Rittle-Johnson & Alibali, 1999; Rittle-

Johnson, Siegler, Alibali, 2001). For example, Byrnes and Wasik (1991) measured conceptual knowledge using picture-symbols (selecting pictures depicting a fraction), simple morphism (identifying drawings of the same fraction), and order problems. For procedural knowledge, they used multiplication and addition problems of fractions. An argument against this approach is that it assumes the assessment tasks only measure one particular type of knowledge. This assumption can be problematic because conceptual and procedural knowledge are intertwined and it may not be clear what type of knowledge the assessment task is really measuring. An alternative approach is to assess both conceptual and procedural knowledge using the same task. For example, in the Canobi, Reeve, and Pattison (1998) study described above, conceptual understanding was assessed by the ability to spontaneously apply the relational properties of the problems in the procedure of problem solving. In this way, both the conceptual and the procedural understanding that are required to solve the same problem are assessed.

In this study, both approaches to assessing conceptual and procedural knowledge were used (described below). After solving each problem, the children were prompted with questions and asked to explain their problem solving procedure: "What did you do just now to solve the problem?" Children were also asked to explain the rationale of their procedure: "Why did you solve the problem this way?" The coding of conceptual and procedural knowledge was approached in two ways:

(a) Same-Problem Approach

Conceptual knowledge. For each problem, children received two points for a verbal response that showed complete conceptual understanding (e.g., the fraction with bigger numerator is bigger because it has more parts; you need to change the fractions into the same unit in order to add them). One point was awarded for a response that showed partially correct conceptual understanding (e.g., ¼ and 2/5 are the same because their numerators are equally away from denominators; to add fractions you need to change the smaller denominator to the bigger one). Zero points were given for a response that indicated no conceptual understanding. The total possible conceptual score across all problems was 40.

Procedural knowledge. For each problem, procedural knowledge was coded based on both verbal reports and procedures written on paper. Two points were awarded when the child used the correct procedures and got the right answer. One point was awarded when the correct procedure was used but the answer was wrong due to a computational error. Zero points were given when the wrong answer was shown and the procedures were incomplete or incorrect. The total possible procedural score across all problems was 40.

(b) Different-Problem Approach

Conceptual knowledge. Conceptual knowledge about fractions is theoretically linked to fraction word problem solving skills (Hecht, 1998). This is because conceptual understanding may be used during the process of constructing mental models for fraction word problems. Conceptual knowledge was coded based on participants' ability to translate the word problems into appropriate computation equations. Two points were given for a completely correct computation equation. One point was given for a partially correct computation equation and zero points when the equation was totally incorrect or when the child was unable to construct an equation. The total possible conceptual score was 20.

Procedural knowledge. Procedural knowledge was coded based on participants' ability to solve the computation problems. Two points were awarded when the child used the correct procedures and got the right answer. One point was awarded when the correct procedure was used but the answer was wrong due to a minor computational error. Zero points were given when the wrong answer was shown and the procedures were incomplete or incorrect. The total possible procedural score was 20.

Assessment and Coding of Metastrategic Knowledge

For both approaches, participants' ability to detect the connection between the computational and word problems served as an index for metastrategic knowledge. This is because in order to detect the connection between the computational and word problem, participants had to be aware of the requirement and nature of the problems, and the strategies that were applicable to the problems. After solving each pair of questions, participants were provided a probe to elicit their understanding of the connection between the computational problem and the word problem within the same set: "You have just solved these two problems. Do you see anything special about them?" The participants' verbal response was coded according to two criteria, that is, understanding that the two problems had the same objective or requirement, and understanding how the same strategy was applicable to both problems. Zero points were given for a response that indicated no understanding of the similarities between the problems. One point was awarded for a response that indicated that the child understood only one of the above criteria, but not both. Two points were awarded for a response that indicated understanding of both criteria. The total possible metastrategic score was 20.

CHAPTER 3

RESULTS

The results are presented in two sections. First, an overview of children's performance across trials is presented. Next, correlation analyses among the main variables (conceptual, procedural, and metastrategic knowledge) are presented. Within each section, results are presented both from the same problem approach and from the different problem approach. No gender difference was found in the analyses.

Overview of Performance Across Trials

(a) Same Problem Approach

Overall means and standard deviations for conceptual, procedural, and metastrategic scores are presented in Table 1. As expected, participants performed poorly in the measures of conceptual, procedural, and metastrategic knowledge because the mean scores of each knowledge types were only 50% or less out of the total possible score. Conceptual and procedural scores of comparison problems were similar in magnitude compared to the conceptual and procedural scores of addition problems.

On the basis of the iterative model, changes of conceptual and procedural knowledge were expected to be related to each other. To examine the pattern of change across trials, composite conceptual and procedural scores were created by summing scores for paired computation and word problems. Then, percentages of this composite sum across all participants (N = 56) were computed to create two variables: percentage conceptual score and percentage procedural score. Similarly, metastrategic scores for each pair of problems across all participants were computed and converted to a percentage metastrategic score. A line graph depicting the changes of percentage of conceptual, procedural, and metastrategic scores can be found in Figure 2 (for comparison problems) and Figure 3 (for addition problems).

Visual inspection of the figures indicates that conceptual and procedural scores showed similar patterns for both comparison and addition problems. Despite the relatively poor performance of children on these tasks, there was relatively little change in conceptual and procedural score across trials. However, there was a gradual growth of metastrategic knowledge across trials for both types of problems.

(b) Different Problem Approach

Overall means and standard deviations for conceptual, procedural, and metastrategic scores are presented in Table 2. As can be seen from the table, the levels of conceptual, procedural, and metastrategic knowledge were generally low (the mean scores of each knowledge types were only 50% or less out of the total possible scores). Similar to the same problem approach, participants performed similarly on the measures conceptual and procedural knowledge for comparison problems but showed a gap in their conceptual and procedural scores for addition problems.

To examine the pattern of change across trials, composite conceptual and procedural scores were created within problem domains (comparison or addition problem) by summing across all participants and converting to percentages (percentage conceptual score and percentage procedural score). As with the same problem approach, composite metastrategic scores for each pair of problems across all participants were computed and converted to a percentage metastrategic score. A line graph depicting the changes of percentage of conceptual, procedural, and metastrategic scores can be found in Figure 4 (for comparison problems) and Figure 5 (for addition problems).

For comparison problems, visual inspection of the figures indicates that conceptual and procedural scores showed some slight fluctuations. Conceptual and procedural scores were similar in magnitude except at trial two. The reason for this fluctuation is not clear and may be just due to the problems' different level of difficulty. For addition problems, conceptual and procedural scores showed similar patterns of change. Overall, similar to the results yielded from the same problem approach, there was relatively little change in levels of conceptual and procedural knowledge for both comparison and addition problems.

Notice that the levels of conceptual and procedural knowledge are closer to each other in the comparison problem than in the addition problem. This is found in both the same problem approach and the different problem approach. These may be due to the effect of classroom instruction. When the study was conducted, students had received some amount of instruction on fraction comparison. They possessed sufficient levels of conceptual knowledge and procedural knowledge about fraction comparison problems to carry out the calculations. On the other hand, students were just learning fraction addition. They may have known that they needed to have an equal denominator to add two fractions, but did not yet possess the procedural skills to carry out the computation. Metastrategic scores, on the other hand, showed a gradual growth across trials from both the same problem approach and the different problem approach. As children solved more problems, an understanding of the nature of the problems started to emerge. In other words, they became better at perceiving the connections between the computation and word problems. Also, they become aware that the same potential strategies could be applied to the problems.

Correlation Analyses

(a) Same Problem Approach

If conceptual, procedural, and metastrategic knowledge interact iteratively during learning, conceptual, procedural, and metastrategic score should correlate with each other. To test this, the total number of correct answers, and conceptual, procedural, and metastrategic scores across all problems were correlated. The correlation matrix of these variables is presented in Table 3. Children who were more likely to correctly solve the problems were more likely to have high scores on the conceptual, procedural, and metastrategic measures. In line with Figure 2 and Figure 3, conceptual and procedural scores showed the highest positive correlation, reflecting the stability of both types of knowledge across problems. Metastrategic scores also showed lower but significant correlations with conceptual and procedural scores.

On the basis of the iterative model, the relationships among the three types of knowledge were expected to grow stronger across trials. In order to examine the development of the relationships among conceptual, procedural, and metastrategic knowledge, participants' performance on these measures was split into halves, the first half (questions one to ten) and the second half (questions 11 to 20). Correlation analyses among the variables were conducted with each half (see Table 4). As can be seen from the table, the overall correlations among conceptual, procedural, and metastrategic knowledge were found to be higher in the second half of the task. The correlation between conceptual and procedural score, which was already very high in the first half, showed a small increase in the second half. Whereas the correlation between metastrategic score and the other two scores showed higher increases.

If metastrategic knowledge mediated the relation between conceptual and procedural knowledge, the correlation between conceptual and procedural scores found previously should be reduced when metastrategic score is controlled. Partial correlation analyses were conducted to examine the metastrategic score as a mediator between conceptual and procedural scores. For the first half of the task, the correlation between conceptual and procedural scores dropped from 0.834 to 0.788 after the metastrategic score was entered as a control variable. For the second half of the task, the correlation between conceptual and procedural score dropped more from 0.900 to 0.735 after the metastrategic score was entered as a control variable. The correlations between conceptual and procedural score were still significant in both analyses even after the metastrategic score was controlled. However, the more drastic drop at the second half of the task suggested that metastrategic knowledge played a higher contribution in the relation between conceptual and procedural knowledge at the later phase.

These results are consistent with the hypothesis that metastrategic knowledge is important in the iterative relationship between conceptual and procedural knowledge. Children's conceptual and procedural knowledge were found to be correlated with each other. These components of knowledge were found to be correlated with the total number of correct answers in the mathematics task, suggesting that they are important for success in mathematics problem solving. In addition, metastrategic knowledge was found to be related to conceptual and procedural knowledge. The relationships among the three variables grew stronger over trials, indicating a developing link among the three knowledge types. In addition, the reduced correlation between conceptual and procedural knowledge after metastrategic knowledge was controlled suggested the role of metastrategic knowledge as an underlying factor linking conceptual and procedural knowledge.

(b) Different Problem Approach

Similar analyses based on the same rationales were conducted using the data from the different problem approach. First, the correlation matrix of the total number of correct answers, conceptual, procedural, and metastrategic scores is presented in Table 5. Similar to the results yielded from the same problem approach, children who were more likely to correctly solve the problems were also more likely to have high scores on the conceptual, procedural, and metastrategic measures. Conceptual and procedural scores showed a lower but still significant positive correlation (r = 0.57 compared to r = 0.92 from the same problem approach). This correlation is more in line with the magnitude of correlation that is usually found in the research literature. Metastrategic scores showed similar correlations with conceptual and procedural scores, and total number of correct answers as with the same problem approach.

Correlation analyses among conceptual, procedural, and metastrategic knowledge were conducted with the first and second half of the task (see Table 6). As can be seen from the table, the correlations among conceptual, procedural, and metastrategic knowledge were once again found to be higher at the second half of the task. In line with the gradual growth of metastrategic score depicted in the line graphs, the correlation between metastrategic score and the other two scores (conceptual and procedural score) were higher at the second half of the task.

In the next step, partial correlation analyses were conducted to examine metastrategic knowledge as a mediator between conceptual and procedural knowledge. For the first half of the task, the correlation between conceptual and procedural scores dropped from 0.428 (p=.001) to a nonsignificant 0.222 (p=.10). For the second half of the task, the correlation between conceptual and procedural scores dropped from 0.487 (p=.000) to a nonsignificant –0.167 (p= .22). It is apparent that by using the different problem approach to assess conceptual and procedural knowledge, metastrategic knowledge showed a stronger role in the relationship between the two types of knowledge. This suggests that metastrategic knowledge was more relevant to the kinds of conceptual and procedural knowledge that were assessed in the different problem approach. In this approach, conceptual knowledge was assessed by the participants' ability to transform word problems into working equations and theoretically this ability should be related to the awareness of task requirement component of metastrategic knowledge. In line with this, procedural knowledge was assessed by the participants' ability to carry out computational steps correctly in the computation problems. This ability is more in line with the awareness of potential strategy component of metastrategic knowledge. This is the possible reason why metastrategic knowledge contributed more variance to conceptual and procedural knowledge in the different problem approach.

Table 1

Means and Standard Deviations (in parentheses) for Conceptual, Procedural, and Metastrategic Scores Using the Same Problem Approach

Knowledge	Problem Domain	
	Comparison	Addition
Conceptual	9.04 (3.79)	9.93 (5.79)
Procedural	8.18 (4.15)	6.25 (7.46)
Metastrategic	3.16 (2.67)	4.59 (2.88)

Note: Possible scores for conceptual and procedural knowledge range from zero to 20. Possible scores for metastrategic knowledge range from zero to 10.

Table 2

Means and Standard Deviations (in parentheses) for Conceptual, Procedural, and Metastrategic Scores Using the Different Problem Approach

Knowledge	Problem Domain	
	Comparison	Addition
Conceptual	4.18 (1.90)	5.79 (2.56)
Procedural	3.98 (2.28)	3.43 (4.09)
Metastrategic	3.16 (2.67)	4.59 (2.88)

Note: Possible scores for conceptual, procedural, and metastrategic knowledge range from zero to 10.

Table 3

Intercorrelations Between Conceptual, Procedural, Metastrategic Scores and Number of Correct Answers across All Problems Using the Same Problem Approach

Variable	1	2	3	4
1. Conceptual	-	0.923**	0.771**	0.838**
2. Procedural		-	0.691**	0.864**
3. Metastrategic			-	0.620**
4. Number of correct				-

** $p < .01$

Table 4

Intercorrelations Between Variables in First Half and Second Half of Task Using the Same Problem Approach

Variables	Conceptual	Procedural	Metastrategic
First Half			
Conceptual	-	0.834**	0.590**
Procedural	-	-	0.450**
Metastrategic	-	-	-
Second Half			
Conceptual	-	0.900**	0.832**
Procedural	-	-	0.769**
Metastrategic	-	-	-

** $p < .01$

Table 5

Intercorrelations Between Conceptual, Procedural, Metastrategic Scores and Number of Correct Answers across All Problems Using the Different Problem Approach

Variable	1	2	3	4
1. Conceptual	-	0.570**	0.726**	0.640**
2. Procedural		-	0.698**	0.855**
3. Metastrategic			-	0.620**
4. Number of correct				-

** $p < .01$

Table 6

Intercorrelations Between Variables in First Half and Second Half of Task Using the Different Problem Approach

Variables	Conceptual	Procedural	Metastrategic
First Half			
Conceptual	-	0.428**	0.564**
Procedural	-	-	0.472**
Metastrategic	-	-	-
Second Half			
Conceptual	-	0.487**	0.739**
Procedural	-	-	0.759**
Metastrategic	-	-	-

** $p < .01$

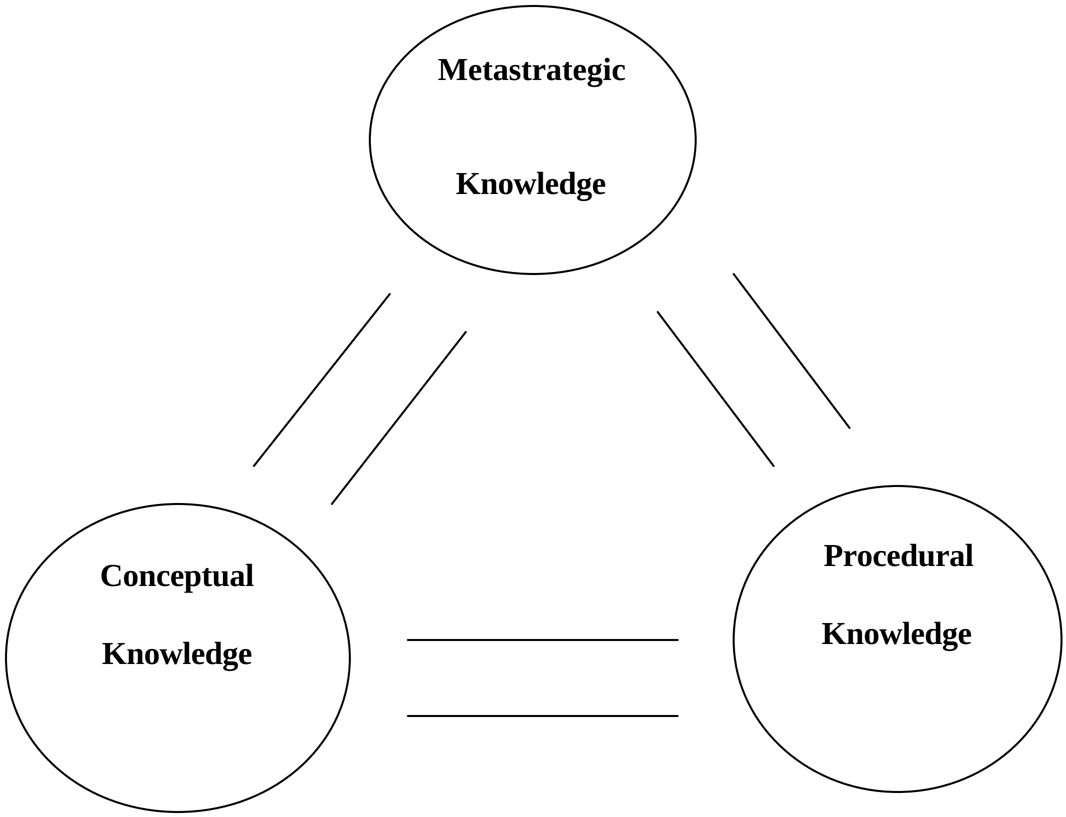

Figure 1. Iterative model for the development of conceptual and procedural knowledge

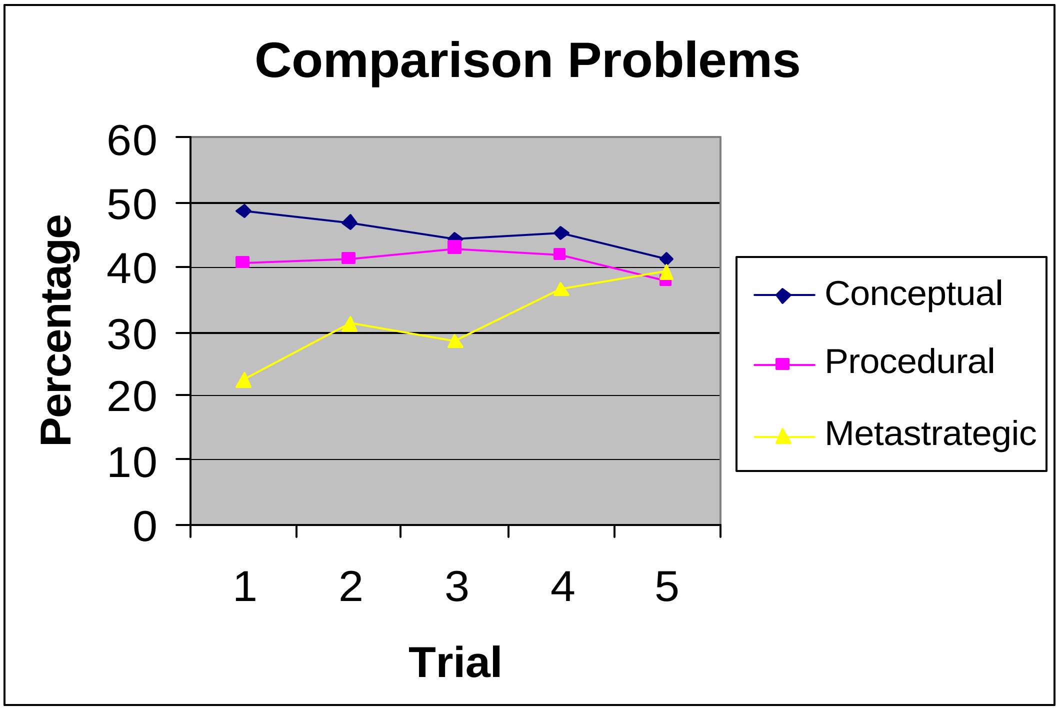

Figure 2. Percentage of conceptual, procedural, and metastrategic scores for comparison problems using the same problem approach.

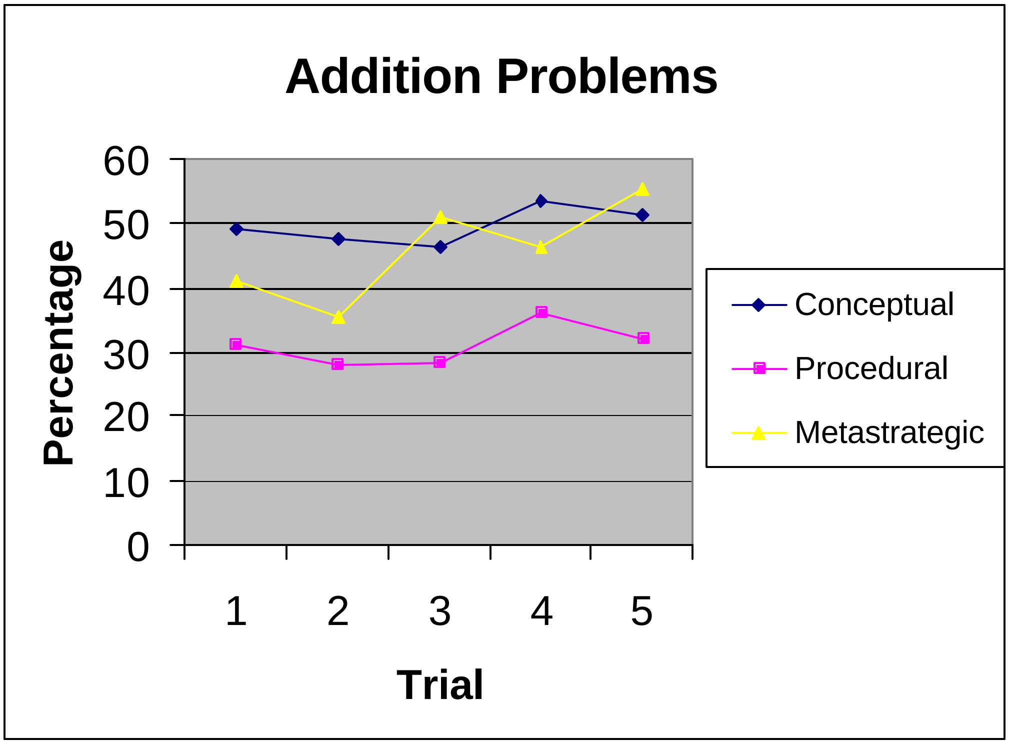

Figure 3. Percentage of conceptual, procedural, and metastrategic scores for addition problems using the same problem approach.

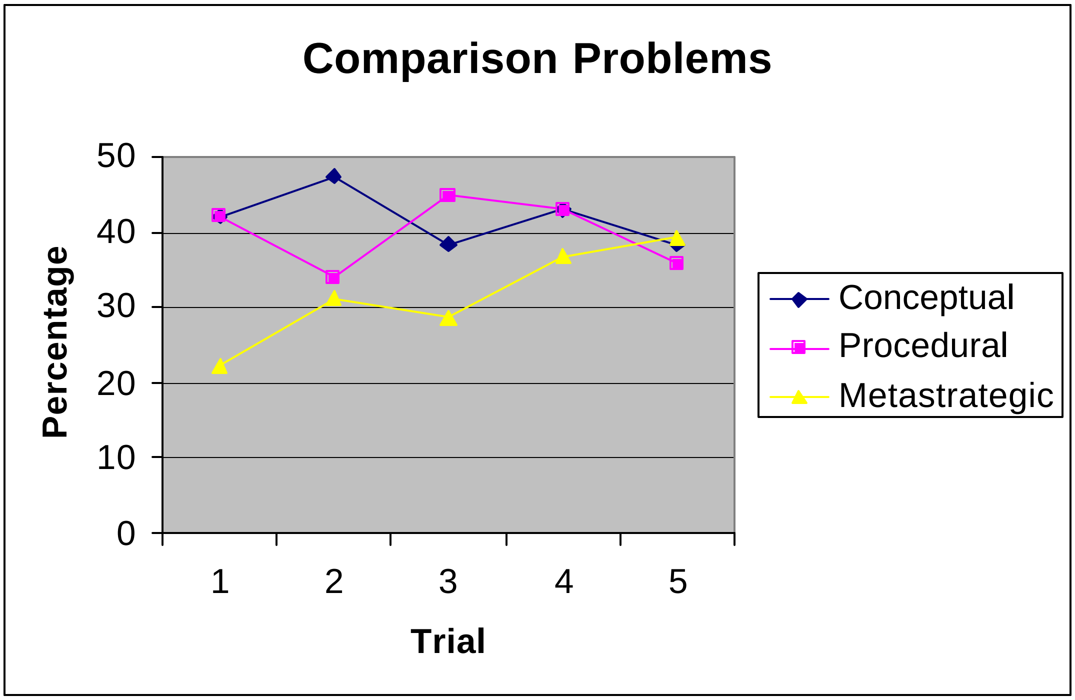

Figure 4. Percentage of conceptual, procedural, and metastrategic scores for comparison problems using the different problem approach.

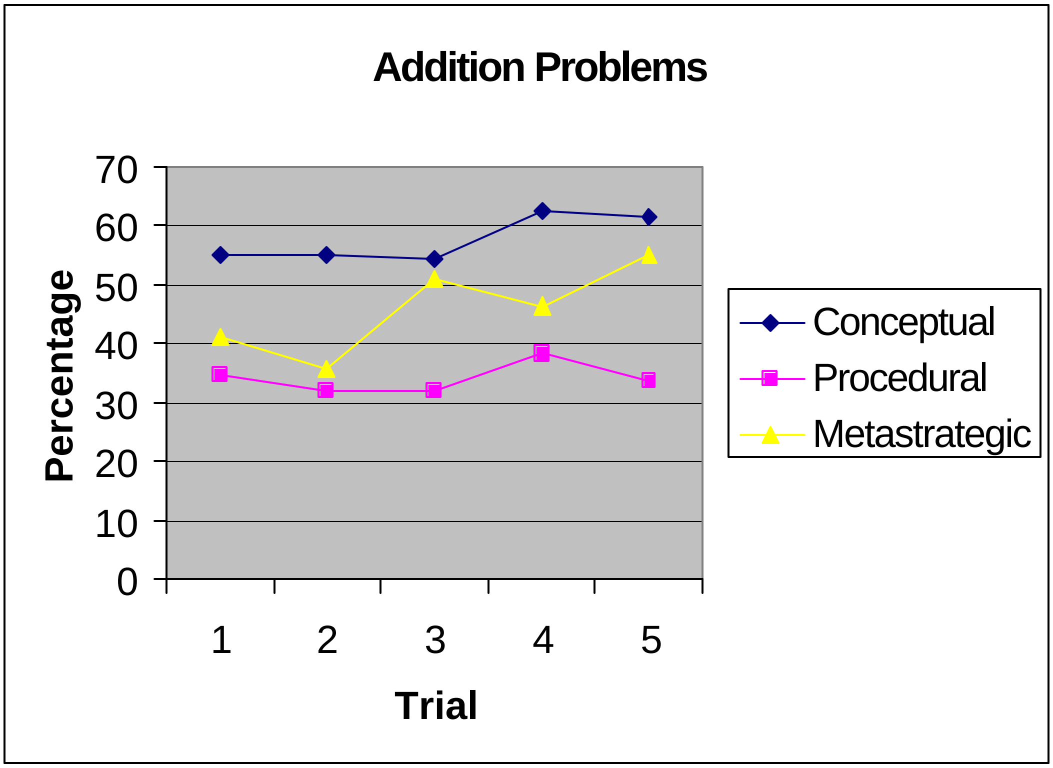

Figure 5. Percentage of conceptual, procedural, and metastrategic scores for addition problems using the different problem approach.

CHAPTER 4

DISCUSSION

This study examined the relationship of conceptual and procedural knowledge in the domain of fraction knowledge. Based on the iterative model, children's conceptual and procedural knowledge were expected to be correlated and develop hand-in-hand. Despite differences in the magnitude of correlation, both the same problem approach and the different problem approach used to assess the relationship between conceptual and procedural knowledge indicated that conceptual and procedural knowledge were related to each other. Furthermore, the relationship between conceptual and procedural knowledge strengthened with practice. The correlation was stronger during the second half of the mathematics task, suggesting a growing relationship between the two knowledge types as the children worked on the problems. These findings are in line with other research indicating that conceptual and procedural knowledge influence one another and develop in tandem (Rittle-Johnson & Alibali, 1999).

In addition, the correlation between conceptual and procedural knowledge was reduced after controlling for metastrategic knowledge, suggesting that metastrategic knowledge played a role in the relationship between conceptual and procedural knowledge. The correlation between conceptual and procedural knowledge was reduced more during the second half of the mathematics task. This shows that the relationships among the three knowledge types grew stronger over trials, indicating a developing link among them. The findings of the present study add to the iterative model suggesting that metastrategic knowledge is a mechanism underlying the relationship between conceptual and procedural knowledge. Metastrategic understanding includes the awareness of the nature and requirements of the task, and the awareness of the strategies available in one's repertory that are potentially applicable to the task (Kuhn & Pearsall, 1998). Theoretically, both types of metastrategic understanding are relevant to the development of conceptual and procedural knowledge. For mathematical competence, one needs to have adequate cognitive resources (such as working memory, attentional resources, conceptual knowledge, and procedural knowledge) and use them well by having metastrategic knowledge to control over what is done and how it is done. By having an understanding of the nature of a task, it is easier for children to learn and develop the conceptual knowledge that is necessary to solve the task accurately. Similarly, by having an awareness of the strategies potentially applicable to a task, the process of selecting the appropriate procedures and strategies is facilitated and made easier. In the present study, the children who were more likely to perceive the connections between the word and computation problems possessed higher levels of conceptual and procedural knowledge, and hence showed better performance in the mathematics task.

Even though not directly examined in this study, it is important to note that the above processes do not function independently, but mutually influence each other in a bidirectional way (Kuhn & Pearsall, 1998). Over time, with increasing conceptual and procedural knowledge, the growth of metastrategic knowledge will be supported. In turn, the emergence of metastrategic knowledge will support the connection between conceptual and procedural knowledge to promote mathematical competence. Therefore, within the iterative model, conceptual and procedural knowledge supports the emergence of metastrategic knowledge, which in turn allows conceptual and procedural knowledge to develop faster and more fluently.

Prior research found that teaching children to use elaborative strategies and related metacognitive knowledge resulted in improved mathematics achievement (e.g., Lucangeli, Cornoldi, & Tellarini, 1998). Children who know why, when, where, and how to use different strategies are more successful in mathematics than students who do not have this knowledge. Metacognition is most related to strategies that are both effortful and in the process of being acquired, but not to highly automated, less effortful strategies (Carr & Jessup, 1995). In general, there seems not to be a linear relationship between metacognition and mathematical competence. Rather, metacognition plays an important role in the process of learning and that metacognition is a skill that facilitates learning, but is not required for learning. Also, metacognition is not needed as much when strategies are already well acquired and are being used in a familiar context.

Although the present study provides some information about the role of metastrategic knowledge in the relationship between conceptual and procedural knowledge, many questions remain to be answered. This study is limited by a number of methodological constraints. Specifically, conceptual, procedural and metastrategic knowledge were assessed over only a short period of time. As a result, conceptual and procedural knowledge were found to be relatively stable in this study. This may be due to the nature of the domain in that fraction knowledge is an area that many students find difficult to master despite extensive experience (Smith, 1995). It is likely that additional instruction is necessary to allow for significant growth in conceptual and procedural knowledge of fraction knowledge. In addition, most of the findings in this study was based on correlational analyses. Therefore, the next step will be to use multiple sessions to assess the causal links between conceptual knowledge and procedural knowledge.

In order to examine closely the causal relationship among the knowledge types, experiments should be conducted such that children are randomly assigned to conditions in which one type of knowledge is instructed over multiple sessions, and the other type of knowledge is examined. This will allow an examination of the initial state of conceptual and procedural knowledge and how the two knowledge types develop gradually in tandem over time. Experimental manipulation on metastrategic knowledge will allow direct examination of metastrategic knowledge as a mediator between the development of conceptual and procedural knowledge. For example, participants can be assigned to experimental condition in which they have to generate self-explanation regarding the nature and applicable strategies of the mathematics task in hand. Findings from Crowley and Siegler (1999) suggest that generating verbal explanation of a newly acquired strategy facilitates generalization of the strategy to other contexts. This is probably because explanation promotes more effective management of the new strategy's goal structure by making it easier to keep track of subgoal execution within the strategy. Therefore, manipulating whether or not children make verbal explanation will allow us to examine the effects of metastrategic knowledge on the development of conceptual and procedural knowledge.

The converging evidence produced by the same problem approach and the different problem approach suggests that using both approaches may be worthwhile and useful to assess conceptual and procedural knowledge. Instead of using assessment tasks that assess types of knowledge dichotomously on the basis of arbitrary criteria, using both approaches to analyze the data may yield more meaningful and convincing evidence. Nevertheless, both same problem approach and different problem approach used in this study were dependent on verbal ability and may have underestimated the actual level of participants' knowledge. Nonverbal methods or methods that rely less on verbal ability need to be developed for future studies.

Instructional research suggests that high and low performers differ in terms of their metacognitive understanding (Hohn & Frey, 2002; Lucangeli, Cornoldi, & Tellarini, 1998). The present study did not have sufficient sample size to allow the comparison of high and low performers on the measures of knowledge types. However, the correlation between total number of correct and metastrategic score indicated that students who performed better in the math task had higher metastrategic knowledge. This is because students who performed better in the math task were also more capable of seeing the connections between the computation and the word problems. This is somewhat in line with the findings from Chi, Feltovich, and Glaser (1981) that showed that experts and novices in physics differ in their categorization of physics problems. Whereas experts focused on the deep structure of the problems (underlying physical principles) for categorization, novices tended to rely on surface features (words and diagrams) of the problems for categorization. Therefore, future research should address the possible qualitative and quantitative differences between low and high performers in terms of their conceptual, procedural and metastrategic knowledge, and the relationships among these knowledge types Theoretically, high performers should make more explicit connections among the three types of knowledge, which in turn should help them to perform better in mathematics tasks.

The connection between performance and understanding has been a topic that has received much attention from educators (Salomon & Perkins, 1989). It has been suggested that students in the United States acquire facts that they cannot access and use appropriately and that it is the passive nature of learning that results in poor performance and understanding (Brown, 1997). Strategy instruction that is passive and does not require students to actively process what is being learned does not guarantee that students will continue to use the strategies (maintenance) or flexibly deploy these strategies in a new context (transfer). Therefore, the question of how metastrategic knowledge as a form of explicit awareness relates to performance in mathematics is one worthy of investigation. Theoretically, effective learners operate best when they possess insight into their own strengths and weaknesses and have access to their knowledge of problem solving strategies. Instead of making children learn facts and information via passive, rote memorization, the ultimate goal of educators should be to see that children develop into adaptive problem solvers who are capable of solving problems flexibly in different contexts. Within the mathematics domain, it has been recommended that both conceptual and procedural knowledge be taught in the classrooms (NCTM, 2000). The results of the current study suggest that metastrategic knowledge should also be included in the instructions to support the development of conceptual and procedural knowledge.

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APPENDIX

MATH TASK

Comparison Problems (in consecutive pairs of computation and word problem):

1) Compare the fractions and list them in order from the least to the greatest:

1/4, 2/5

2) John's party has 1 pizza for 4 people and Gary's party has 2 pizzas for 5 people. At which party would you eat the most pizza if all the pizza was eaten, and everyone had the same amount?

3) Is 84/100 or 17/25 larger?
4) Jenny correctly answered 84 out of the 100 questions on a science test. John answered 17 out of 25 correctly. Who received the higher score?

5) List the following fractions from the greatest to the least.

4, 3 1/3, 16/4

6) To make a cake, Elaine uses a total of 11 1/3 cups of flour. She uses 16/4 cups of whole wheat flour, 4 cups of white flour, and 3 1/3 cups of regular flour. Which kind of flour does Elaine use least in the cake?

7) Give one number that is right in the middle between 3/5 and 6/5.
8) Mary is making rice for dinner. To make a cup of rice, 3/5 cup of water is needed. To make 2 cups of rice, 1 1/5 cup of water is needed. Mary wants to make 1 ½ cups of rice. How much water does she need?

9) List the following fractions from the greatest to the least:

5/12, 1/4, 2/3

10) Mary has 18 ounces of juice. She divides it evenly among 3 glasses so that each glass holds the same amount of juice. Glass A is 2/3 full, Glass B is 5/12 full, and Glass C is 1/4 full. Which glass has the biggest size?

Addition Problems (in consecutive pairs of computation and word problem):

11) ¼ + ¼ + 1/6 + 1/6 = ?
12) There are 30 days in April. Last week Gary went to school for 2 days. Each day it took him 1/4 hour to get to the school and 1/6 hour to get back home. How many hours did he spend on traveling back and forth the school last week?

13) 6/8 + ¾ = ?
14) Every week, Mabel watched 3/4 hour of science video and 1/2 hour of cartoon. Tom watched 1/5 hour of cartoon and 6/8 hour of science video. How many hours of science video do they watch in 1 week?

15) 5/8 + 1/4 = ?
16) On the editorial staff of a local newspaper, 5/8 of them are reporters, 1/8 of them are writers, and 1/4 of them are photographers. How many reporters and photographers are there on the editorial staff?

17) 1/3 + 2/5 = ?
18) A mosaic design has red, yellow, and green tiles. 4/15 are yellow, 1/3 are red, and 2/5 are green. How many tiles are not yellow?

19) 2/3 + 1/6 = ?
20) 5th graders at Green Elementary voted on what symbol to use on their school T-shirts. How many students did not vote for the bear and for the eagle?