THE MATH WARS: COMMUNITIES OF MEANING WITHIN THE CRITICS OF SCHOOL MATHEMATICS REFORMS

by

PATRICIA ANNE WAGNER

(Under the Direction of Jeremy Kilpatrick and AnnaMarie Conner)

ABSTRACT

Over the course of United States history, there have been numerous attempts to reform school mathematics in order to increase student achievement. Although the methods of reform have varied, a common theme has emerged: The reform encounters a political backlash that forces a retreat into traditional instructional materials and methods. This research study examined the beliefs and motivations of those on one side of the "math wars," a struggle over the goals and methods for school mathematics that originated in the 1990s. In her policy work, Yannow (2000) described individuals reacting to policies as inhabiting *communities of meaning*: groups in which "cognitive, linguistic, and cultural practices reinforce each other, to the point at which shared sense is more common than not, and policy-relevant groups become 'interpretive communities' sharing thought, speech, practice, and their meanings" (p. 10). I drew upon this interpretation to describe the communities of meaning of those who took a reactive position in the math wars; that is, critics of school mathematics reforms. Using this framework in conjunction with Green's (1971) and Rokeach's (1968) interpretations of belief systems, I identified three communities of meaning and described their primary lenses for viewing school

mathematics and reforms. These descriptions enabled me to infer each group's motivation for

political activism against the reforms. The findings from this study have implications for the

political advocates of reforms, educational researchers, and those charged with implementing

school mathematics reforms.

INDEX WORDS:

Math Wars, Mathematics Education, Mathematics Education Reform,

Education Policy, Beliefs

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DEDICATION

I dedicate this work to my husband, John Wagner, whom I love, admire, and respect more with each passing day. You have been my inspiration and source of support not only during my time in graduate school, but throughout our entire journey together. Your quiet confidence in me has gradually transformed me from an insecure girl into a confident and determined woman. I love our life together and I love you more than I can say.

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CHAPTER 1

SCHOOL MATHEMATICS REFORM CONTROVERSIES

I have sometimes thought that theology had the deepest and strongest hold upon the human mind...; but that is not true; educational ideas are by far the slowest to change. Noah Webster is mightier than Jonathan Edwards, technical grammar than predestination. It is useless for anyone who attempts to improve education to complain; the right way is to recognize the situation and make the best of it. (Parker, 1902)

Grade 8 Math Night was an event held in a Fairfield, Connecticut, Board of Education office on Wednesday, October 24, 2012. The goal of the meeting was to introduce parents to College Preparatory Mathematics (CPM), a curriculum that was being piloted in Fairfield middle schools that year. The Fairfield Public School Department of Education (DOE) had been looking ahead to the adoption of a mathematics curriculum in spring 2013 and had entered an agreement with the publisher to pilot CPM in the district's pre-algebra classes. During Grade 8 Math Night, DOE official Dr. Paul Rasmussen explained to the crowd that Connecticut's adoption of the Common Core State Standards in Mathematics (CCSSM) (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) necessitated new instructional practices, such as a focus on problem solving, critical thinking, and communication, and that CPM was designed to support those practices. His presentation attempted to familiarize parents with the new curriculum, offer research evidence that supported the new instructional practices, and engage families in actual tasks from the curriculum. At least one teacher was present to speak in support of the curriculum (Grosso, 2012; Rasmussen, 2012a; Sayed, 2012).

Six days later, the Minuteman News Center (Grosso, 2012) published *Parents Question a New Method for Teaching Math*, which highlighted parents' continued apprehensions. The DOE issued a follow-up document addressing frequently asked questions from the meeting in an attempt to ease concerns (Rasmussen, 2012b). Many parents were not appeased, however, and voiced their strong objections at the following school board meeting. Board members, unaware of the piloting program, demanded an accounting from district personnel at a meeting to be held two weeks later (Fairfield Public Schools, 2012).

On December 11, 2012, dozens of parents listened quietly as Fairfield Public School's Deputy Superintendent, the Director of Secondary Education, the Curriculum Leader of Grades 6-12 Mathematics, and a practicing teacher addressed the BOE and the parents, speaking passionately about the benefits of the new curriculum and instructional practices. The speakers acknowledged that although mistakes had been made in communication, they had operated within their authority and in the best interest of Fairfield's children. Following the 40-minute presentation, school board members' reactions ranged from confusion ("When does the instruction occur?" (Sayed, 2012, 02:25:00)) to skepticism ("I'm hearing overwhelmingly opposing points of view to what you're saying" (Sayed, 03:05:59)). One school board member stated, "I have gotten numerous comments back from teachers within the district who love this model and are telling me it's a successful model and to please support it. On the other side I'm getting parents who are absolutely frustrated with it" (Sayed, 03:08:30). Another school board member claimed that the issue had pitted teachers against parents. After a lengthy discussion, a motion was made and seconded that a discussion to mandate the discontinuance of CPM by all teachers be added to that night's agenda. The room erupted in applause (Sayed, 2012).

During the resulting discussion, board members decided to table the vote mandating the discontinuance of CPM in favor of creating a district website forum to collect the public's feedback about the middle school mathematics curriculum. In the days that followed, Fairfield parents organized and created a website titled *Fairfield Math Advocates* (www.fairfieldmathadvocates.com) and urged the public to sign a petition for the immediate removal of CPM materials from classrooms. On May 21, 2013, after gathering public feedback, district leaders voted to discontinue the use of CPM materials and adopted a more traditional curriculum (Gerber, 2013).

The Struggle to Implement Reforms in School Mathematics

To those attempting to enact school mathematics reforms in United States classrooms, stories like this are troubling. Researchers in mathematics education have generally supported curricula and instructional practices typified by CPM. They point to research that demonstrates higher achievement of underprivileged children and better critical thinking and problem-solving abilities in students who are taught with a reform curriculum (e.g., Boaler, 1998; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Riordan & Noyce, 2001; Senk & Thompson, 2003).

Additionally, researchers have promoted school mathematics reforms as crucial elements of social justice and equity (e.g., Boaler, 2002; Gutstein, 2003; Schoenfeld, 2002). In this sense, the outcome of the events in Fairfield, Connecticut, is tremendously important to its children. The Fairfield Public Schools, however, are not unique in this controversy over school mathematics reforms. Similar controversies have occurred in school districts in California, Michigan, New York, and other states around the country.

School Mathematics Reform

What *is* school mathematics reform? To what aspects of it do critics object? Although the phenomenon of parents and other stakeholders organizing in opposition to mathematics curricular or instructional changes has become a familiar one, the answers to these questions are unclear. The current controversies, labeled the *math wars* by commentators and the media, are generally viewed as involving advocates for reforms and a reactionary group opposing those reforms. Commentators have characterized the opposition in different ways, describing them as professional mathematicians, the right wing, or parents. The advocates, in contrast, are usually identified as members of the National Council of Teachers of Mathematics (NCTM). In particular, NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics* and (2000) *Principles and Standards for School Mathematics* have been characterized as the main drivers behind and descriptors of mathematics education reforms (Klein, 2007; Schoenfeld, 2004; Talbert, 2002).

Developed in response to *A Nation at Risk* (National Commission on Excellence in Education, 1983), NCTM's *Curriculum and Evaluation Standards for School Mathematics* was published with the goal of creating "a coherent vision of what it means to be mathematically literate" (NCTM, 1989, p. 1) in order "(1) to ensure quality, (2) to indicate goals, and (3) to promote change" (p. 2) in school mathematics instruction. The social shift in the United States from the industrial age to the information age was viewed as necessitating changes in the goals and purposes of school mathematics education. New goals included producing "mathematically literate workers" (p. 3) capable of lifelong learning, increasing diversity among those taking higher level mathematics and science courses, and instilling the technological understanding necessary to ensure an informed electorate. NCTM's (2000) *Principles and Standards for School*

Mathematics built upon this earlier work, and was "grounded in the belief that all students should learn important mathematical concepts and processes with understanding" ("Preface", para. 1).

The ambiguity surrounding those who oppose school mathematics reforms raises three questions. First, who are they? Second, what conceptions of school mathematics and reforms, if any, do these individuals share? And third, are there conceptions about the appropriate goals and methods of school mathematics or objections to school mathematics reforms that separate these individuals into distinct groups with common understandings? Answers to these questions are complicated by the fact that school mathematics reforms can describe changes affecting Grades K–12 mathematics at the local level; for example, a new grading scale in mathematics classes or a revised algebra end-of-course test. In such cases, local opposition may develop over these changes, and one may characterize these changes as school mathematics reforms. In general, however, these are not the kind of controversies to which the infamous math wars have referred. In particular, the math wars have transcended any specific local controversy and have played out at the state and national levels. In considering the opposition, therefore, it is important to distinguish between local reforms and broader controversies surrounding change in school mathematics classrooms. With this in mind, I answer the first question generally: The opposition to school mathematics reforms is any individual or group who enters a public forum to make known his, her, or their dissatisfaction with nonlocal school mathematics reforms. My second and third questions raise the prospect that the critics of school mathematics reforms have common understandings that unite them in oppositional activities, but also have significant differences in the ways they conceptualize the reforms. Yannow (2000) described individuals

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¹ I use *conceptions* to mean "general mental structures, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (Thompson, 1992, p. 130).

reacting to policies as inhabiting *communities of meaning*: groups in which "cognitive, linguistic, and cultural practices reinforce each other, to the point at which shared sense is more common than not, and policy-relevant groups become 'interpretive communities' sharing thought, speech, practice, and their meanings" (p. 10). The second and third questions, therefore, might be answerable by determining what communities of meaning exist within the critics of school mathematics reforms.

Mathematics Education Advocates and the Math Wars

Since the 1990s, when the term *math wars* was reintroduced to describe the controversies surrounding school mathematics, a pattern of reform policy implementation followed by a retreat from those policies has developed on both the state and local levels. The reaction of reform advocates to this pattern of reform-retreat in schools and states around the country may be characterized as threefold. First, some have speculated about the purposes and concerns of the opposition, taking positions that range from ascribing nefarious purposes (Kohn, 1998) to contemplating more reasoned motivations (although less-than-generous interpretations are usually offered) (Becker & Jacob, 2000; Schoenfeld, 2004). These speculations are usually springboards for arguments against the supposed concerns of the opposition. Second, some reform advocates have attempted to enact reforms through prospective teacher education and practicing teachers' professional development, in hopes of circumventing the treacherous policy process. In this vein, prospective and practicing teachers are initiated into the vision of mathematics education proposed by NCTM and taught to modify curriculum and instructional practices to conform to that vision. Third, some reform advocates have taken an optimistic waitit-out position. They have simply chosen to ignore the opposition, focusing on other areas of research or advocacy, perhaps assuming that over time incremental changes will take place in

policies and classroom practice in the direction of reform. The relative quiet of the media about mathematics curriculum controversies, in comparison to the extensive coverage in the 1990s, serves to support the view that the debates and controversies have subsided.

The optimism of those preferring the wait-it-out approach may have been fed by the media's shifted focus to the controversy surrounding the CCSSM. This dispute concerns mathematics education, yet the battle does not appear to be forming along the familiar pro and anti school mathematics reform lines. For example, Hung-Hsi Wu (1996), formerly a vocal critic of mathematics education reforms, has indicated his support for the Common Core State Standards in Mathematics (Wu, 2011a). This change may be interpreted as a positive sign; however, the apparent truce may also be attributed to unrelated factors, particularly given that other opponents of school mathematics reforms have expressed dissatisfaction with the CCSSM (e.g., Milgram, 2011; Stotsky & Wurman, 2010). Additionally, the events occurring in Fairfield, Connecticut, demonstrate that the controversy is alive and well in pockets of communities, and at least in Fairfield, the strategy of appealing to the supposed demands of the CCSSM did not serve to dissuade parents determined to eliminate a reform-style curriculum.

The view that teachers dedicated to reforms may be able to implement them in their classrooms and schools is also questionable. Talbert (2002) detailed the story of teachers in a California school who had collaboratively embraced and enacted reform pedagogies and curriculum. The reforms, which the teachers implemented without controversy, had been in place for 5 years when a group of parents lobbied district leaders for traditional courses. District leaders acquiesced to their demands, eventually returning all mathematics classes to the traditional model, against the teachers' wishes. Talbert summarized, "The course and consequence of anti-reform mobilization in California and elsewhere in the U.S. makes clear that

parents and public have ultimate authority to set the terms of American education, over-riding that of the teaching profession and bureaucracies" (p. 341).

The Fairfield Public Schools controversy similarly reveals the weakness of an overreliance on teachers to implement reforms, even when supportive policies are in place. In Fairfield, teachers found themselves in opposition to the parents of their students, which affected the morale of some of the teachers. In the public comments period at the end of the December 11th board meeting, a Fairfield teacher addressed the parents in the room:

It's upsetting to see the papers, the blogs, the opinion pieces, where there are numerous questions asked that imply that we've done something detrimental to our students. With no basis for it. And to me, that's where the outrage is coming from. When you read these. I have to tell you, as a teacher who lives in this town, to wake up in the morning and read the paper and see the vile things said about me, a mathematics teacher, and the vicious things about me, a mathematics teacher who has dedicated my life to taking care of and teaching these children, and given up hours upon hours from my own children; to know that the parents think of me that way isn't a great feeling. (Sayed, 2012, 03:47:15)

Given that placing the burden for reform on teachers may be tenuous or costly to teacher morale and that the controversy surrounding school mathematics reforms is likely not going away, the most productive reaction to the controversies may be speculation about the motivations and purposes of those who oppose the reforms. I would argue, however, that speculation by itself does little to fix the problem. Any form of resolution is unlikely unless reform advocates' speculations about the positions of the opposition are accurate.

Analysis of the politics of the curriculum wars requires that we try to characterize what is being contested. Loveless [2001] underscores why clarifying underlying issues is so important. In accounting for how so many policies and programs fail either in adoption or in implementation, he says we must understand that policies can and often do run into political trouble because they are "perceived to be—plain and simple—bad ideas. The content of policies affects the politics they engender." Thus, we may find that once issues are clearly delineated, we will be able to see both the truth that inspires partisanship on each side and the errors that inspire resistance. (Mitchell & Boyd, 2001, p. 68)

Martin (2003), reflecting on the math wars, advised,

We need to learn from the critics, rather than treating them only as adversaries to be defeated.... Our critics can help us see where we have gone too far, to help keep us in check. Some of their concerns are valid. When they are, we should acknowledge them, learn from them, and use them to move forward. (para. 29)

Heeding Martin's advice, I propose that in regards to mathematics education reform, a genuine willingness to understand contrary points of view may reveal areas of weakness in our own understandings or gaps within the mathematics education literature. It may also reveal areas of commonality where the beginning of fruitful dialogue may be possible. For this reason, it is necessary to make the individuals who inhabit communities of meaning, specifically those opposed to school mathematics reforms, the object of research.

In the story that began this chapter, Fairfield Department of Education official Paul

Rasmussen prepared his presentation for the parents of Grade 8 mathematics students using the knowledge that the mathematics education research community made available to him. He had the citations of numerous studies that demonstrated the effectiveness of the instructional techniques; he had examples of tasks and demonstrated their use; and he had compelling arguments for the necessity of developing students' critical thinking and problem-solving skills.

But what Rasmussen did not have was an understanding of the probable interpretations, concerns, beliefs, or experiences that define the communities of meaning of those who would not be appeased by his presentation. Malcolm Gladwell (2012) said, "The key to good decision making is not knowledge. It is understanding. We are swimming in the former. We are lacking in the latter" (p. 188). The present study was envisioned to offer an understanding of those opposed to school mathematics reforms.

Purpose of the Study

The purpose of this study was to explore the motivations and conceptions of individuals who publicly oppose mathematics education reform in order to describe their communities of meaning. The following research questions guided the study:

- 1. What experiences or affiliations do those who oppose school mathematics reforms have in common?
- 2. What beliefs about mathematics as a domain are shared by individuals in each community of meaning?
- 3. What beliefs about the goals of school mathematics and how those goals should be achieved are shared by individuals in each community of meaning?
- 4. How does each community of meaning characterize school mathematics reforms and to what elements of their perceptions of the reforms do they object?
- 5. How does each community of meaning interpret and react to the evidence supporting school mathematics reforms?

In the chapters that follow, I review the literature relevant to these questions, detail the methods and procedures I used in my study, present my findings, and discuss the implications of those findings.

CHAPTER 2

LITERATURE REVIEW

In this chapter I examine the literature relevant to my study. I have divided this into two main sections. First, I present the literature related to the math wars, beginning with an overview, including where and how the math wars originated and the major players involved. Other accounts of the math wars have been written, often reflecting the biases and perspectives of the author. For the purpose of this chapter, I have attempted to present only those details that remain uncontested. Then I summarize the existing literature that speculated upon the identities and motives of those opposing school mathematics reforms. In the second main section, I detail the theoretical perspectives that guided my study.

The Math Wars

Although pockets of controversy involving school mathematics may have occurred prior to the mid 1990s, the current iteration of the math wars hit the national stage following the California Department of Education's revision of its state mathematics frameworks in 1992. Motivated by state law requiring revision of the state frameworks every seven years, the 1992 mathematics *Frameworks* drew some of its recommendations from the NCTM's (1989) *Curriculum and Evaluation Standards for Teaching Mathematics*. The *Frameworks* made recommendations concerning both the content and pedagogy of Grades K–12 mathematics and offered a new approach to the teaching of mathematics.

Shortly following the adoption of the 1992 *Frameworks*, California entered its regular adoption process of instructional materials. State funds for curriculum materials for Grades K–8

could be used only for state-approved instructional materials, which by law had to align with the frameworks. (The purchase of high school materials was left to each school district's discretion.) "At the national level, it was considered financial suicide for any major text series to fail to meet California, Texas, and New York's adoption criteria (Schoenfeld, 2004, p. 261); therefore, publishers eagerly produced curricula that met the new content and pedagogical recommendations in California's 1992 *Frameworks*, and these curricula were adopted by California's State Board of Education in 1994.

Prior to introduction of the newly adopted curricula in California schools, the state had suffered two events that heightened political tensions surrounding education. First, there were ongoing contentious debates about the substance of California's state assessments. Second, in both 1992 and 1994, California's students performed uncharacteristically poorly on the National Assessment of Educational Progress (NAEP), ranking near the bottom of state outcomes. This low performance led to a critical focus on the mathematics frameworks (Wilson, 2003). The Frameworks enjoyed a sort of abstract ambiguity as long as it was a written document of recommendations; however, "the curricula developed to bring about the reforms [in the Frameworks] provided more concrete targets for criticism" (Schoen, Fey, Hirsch, & Coxford, 1999, p. 449). In particular, some parents were shocked by the unfamiliar approach of their children's textbooks. Other parents, in STEM fields or academic mathematicians themselves, voiced critical concerns (Wilson, 2003, p. 152). Politically conservative columnists derided the content of some of the curriculums as liberal fluff. Gradually, concerned individuals connected with each other and began to organize. Websites, such as Mathematically Correct, disseminated information opposing the new curricula and served to connect individuals with like concerns, advancing political pressure against the curricula.

Amid mounting criticism, the California state board pushed forward its regular revision of the mathematics framework. Although the 7-year schedule set the next revision for 1999, "the state appointed a committee to produce the next mathematics curriculum framework a year ahead of schedule" (Jackson, 1997, p. 822). The appointment of members to the committee was contentious, with claims leveled that the "deck was stacked" against one side or the other. At the same time, a group appointed by the governor, legislature, and state superintendent was charged with drafting California's first Grades K–12 mathematics content standards. According to Jacob (2001), "the standards were to describe the topics all students should study, and the frameworks were to articulate a vision of how to get there" (p. 264).

Authors' accounts of the development and adoption of the 1999 Frameworks are highly variable and tend to depend on the author's sympathies in the debate. What appears to be the least contested account is that the group charged with writing the content standards finished their work behind that of the framework committee. The mathematics frameworks, however, were supposed to reflect the work of the content committee. In late October, it became apparent that the new mathematics framework reflected a traditional view of school mathematics, whereas the content standards were more in line with the 1992 Frameworks. At the request of the state board, four Stanford mathematicians rewrote the content standards to align with the more traditional frameworks.

The 1999 *Frameworks* recommended comparatively traditional pedagogy, and subsequent curriculum adoptions reflected this focus. Many of the critics of earlier curricula were appeared by the reintroduction of more familiar instructional materials. In October 1999, however, the

U.S. Department of Education released a report designating ten mathematics programmes as 'exemplary' or 'promising'. Several of the programmes on the list, including

MathLand, had been sharply criticized by mathematicians and parents for much of the decade. The imprimatur of the US government carried by these controversial programmes threatened not only to undermine California's new direction in mathematics education, it could marginalize criticisms of the NCTM aligned textbooks nationwide. Within a month of the release of the Education Department's report, more than 200 university mathematicians added their names to an open letter to [U.S.] Secretary [of Education Richard] Riley calling upon him to withdraw those recommendations. (Klein, 2007, p. 30)

The organizers of the "Riley letter" additionally published a copy of the letter in a paid full-page ad in the *Washington Post*, where it generated national attention to the math wars. Prior to this event, Secretary Riley (1998) had addressed the most vocal critics on both sides of the California math wars at a meeting of the American Mathematical Society:

This leads me back to the need to bring an end to the shortsighted, politicized, and harmful bickering over the teaching and learning of mathematics. I will tell you that if we continue down this road of infighting, we will only negate the gains we have already made, and the real losers will be the students of America. (p. 489)

Two years later, the math wars were a national phenomenon.

Speculations About the Critics of Reforms

As discussed in the previous chapter, advocates of mathematics education reforms have speculated about the identities and motives of those opposed to school mathematics reforms, as well as about their specific objections to those reforms. Klein (2007), an applied mathematician, provided a broad description of the critics to reforms from the perspective of someone opposed to the reforms. In the following, I summarize how Klein and other authors have characterized school mathematics reform critics, including their motivations for and objections to reforms.

Perspectives of Advocates for Reform

Authors who advocated for reforms differed from each other in their characterizations of those opposed to reforms. Becker and Jacob (2000) claimed the opposition was composed mostly of professional mathematicians and argued that those who were opposed to reforms believed that "learning mathematics consists mainly of learning procedures by rote" (p. 534). Becker and

Jacobs used multiple examples to indicate these mathematicians had unrealistic expectations about the level of mathematical formality that is appropriate for young children, believing that "if students do not use formal mathematical language and reason to support answers, their learning may be in jeopardy" (p. 532) Later, Jacob (2001) wrote that James Milgram and Hung-Hsi Wu, both mathematicians opposed to the reforms, expressed concern that calculators would interfere with children learning basic skills, that long division was no longer emphasized, and that there was inadequate attention to proof.

Other advocates of school mathematics reforms took a broader view of the critics, characterizing them as "dissenting mathematicians, teachers, and other citizens" (Schoen, Fey, Hirsch, & Coxford, 1999, p. 445), some of whom "have no apparent expertise in mathematics and no experience teaching mathematics at any level" (O'Brien, 1999, p. 434). According to O'Brien (1999) and Battista (1999), critics of reforms desired a return to teaching the basics of mathematics, and Schoen, Fey, Hirsch, and Coxford (1999) claimed that the critics particularly objected to the de-emphasis of arithmetic and algebraic skills. Schoen et al. additionally speculated that opponents of the reforms worried that there was inadequate attention to formal reasoning and proof and that the reformed school mathematics did not sufficiently prepare students for college. In addition, Schoen et al. hypothesized that opponents to reforms could not conceive of a mathematics curriculum that is able to challenge high-ability students as well as below-average and average students in the same classroom, and therefore assumed that such a curriculum was necessarily watered down. O'Brien added over-controlling parents to those opposed to reforms, suggesting that the traditional way of teaching mathematics

reflects a deep-seated longing to control children through external rewards and punishments, rather than to harness children's urge to make sense of things. We've all seen controlling parents at poolside: "Jonathan, get out of the pool. You're grounded for

five minutes." "Gee, Dad, what did I do?" "Out! Now you're grounded for 10 minutes!" (p. 435)

Reys (2001), Battista, and Schoen et al. also speculated that opponents to reforms were concerned that they were being implemented without adequate prior testing.

Other advocates for reform claimed that critics were primarily political conservatives, usually dismissed as the "far right" or individuals who would never consciously align themselves with the far right, but were nevertheless unconsciously advancing its cause. Schoenfeld (2004) noted that "neither the extreme reform camp nor the extreme traditionalist camp is monolithic; each can be considered a confederation of strange bedfellows" (p. 281); yet Kohn (1998) and Schoenfeld and Pearson (2009) claimed that all the critics shared, consciously or not, the belief that

the primary purpose of education [is] to train workers just skilled enough to fill the jobs needed but not skilled enough to question the inequities of power and wealth in the society, perpetuating the status quo by reproducing the social and economic order in each generation. (Schoenfeld & Pearson, p. 561)

Kohn, in particular, painted a harsh picture of reform critics, claiming some were "your upperclass, high-achieving parents who feel that education is competitive, that there shouldn't be anyone else in the same class as my child, and we shouldn't spend a whole lot of time with the have-nots" (pp. 569–570). Kohn characterized these opponents as middle or upper-middle class Whites who did not want students of color in their children's classes and who were not satisfied with their child's success unless it came at the price of other students' failures. Schoenfeld and Pearson similarly, but more tactfully, implied that those opposed to reforms were racist or classist. To Kohn, such attitudes were expected of the far right, but were shamefully exposed in the opposition to reforms by affluent, mainstream, or liberal Whites.

Klein (2007) offered a perspective uniquely different from that of the advocates of reform, primarily because he counted himself as opposed to school mathematics reforms. He

classical traditionalist and progressivist education. To Klein, the opposition to reforms included those holding to the classical traditionalist view, which "traces its origins to Plato, who argued that education for a just society requires the reinforcement of the rational over the instinctive and emotional aspect of human nature" (p. 22). Klein, however, noted that political events had generated the appearance that the opposition was composed mainly of political conservatives, a narrative, Klein suggested, seized upon and promoted by the advocates for reforms. He went on to argue that although the prevalence of political conservatives among those opposed to the reforms was understandable because the "roots of progressive education are intertwined with anti-authoritarian ideals" (p. 31), the issue was not a left-right dispute and should not be characterized as such. In sum, then, Klein framed the opposition as composed of two fairly distinct groups: classical traditionalists, of whom some were mathematicians, and political conservatives.

Education and Politics

The phenomenon of parents and other local stakeholders organizing and participating in the political realm to reverse local educational policies is evidence that educating children in mathematics (or any subject) is a political activity. Researchers in education have long focused on the intersection of education and politics, particularly with respect to matters related to the implementation of new policies (e.g., Cohen & Spillane, 1992; McLaughlin, 1987; Spillane, Reiser, & Reimer, 2002). Most noncritical research addressing mathematics education and policy focuses on what I call *insiders*, i.e., students and those with careers involving Grades K–12 education. This policy research may study the interpretations of the policy on those tasked with its implementation or explore the effects of the policy on students or teachers. However, the

effects or interpretations of the policy on *outsiders*, (e.g., parents or other stakeholders outside the Grades K-12 educational system) are seldom studied. McDonnell (2004) noted that in educational research, parents are usually studied with respect to their activities aligning with those sanctioned by the schools, such as their assistance with their child's homework or volunteering in the schools. She noted, "Rarely do researchers study parents who oppose established policies or who actively question educators' decisions" (p. 108). Yet, outsiders hold a lot of power in our educational system, which privileges local control and parental involvement (Cohen & Spillane, 1992).

The power that outsiders possess regarding education reform efforts is evidenced by the many times that attempts to reform mathematics education have been "undone" by organized groups of opposition. Pierson (1993) noted that policies may create alliances between individuals and groups that heretofore had no motivation to see each other as allies. For example, new educational policies may disrupt established structures privileging some groups over others, which may motivate opposition activity from those seeking to sustain their advantages.

Additionally, given that all policies, including those in education, contain implicit values that those with an interest in the policy may or may not hold (McDonnell, 2004), new policies may serve to generate interest groups that oppose one or more of those values. Thus, although a common purpose underlies the alliances, the motivations for political engagement of the individuals may vary. In the case of the controversies surrounding school mathematics reforms, this means we must allow for the possibility that different interests or interpretations may motivate those opposing them.

Interpretive Policy Analysis

Yannow (1996) recognized the need to consider individual interpretations in her work as a policy analyst. She questioned the positivist approach to policy analysis that was prevalent in the field, arguing that policies are not objectively or neutrally interpreted. Further, she argued, people perceive policies in unique ways, their perceptions being dependent on previous experiences and beliefs. She saw different interpretations of the same policy as underlying many conflicts surrounding policies and suggested that when parties find themselves at odds,

a policy analyst would be far more productive in helping the parties understand the differences underlying one another's positions—that they are situated knowers arguing from different standpoints (rather than attributing stupidity or "blindness" to reality to the opposing side)—than by providing econometric data. (2000, p. 9)

Yannow (2000) assumed that policy conflicts are a result of contending interpretations of the policy by different communities of meaning.

From the literature speculating on the characteristics and motivations of those opposed to school mathematics reforms, we can surmise that political controversies might generate unusual alliances, for individuals with different interpretations and motivations may find themselves political allies. Using a wide lens, these individuals may appear similar because they are in agreement concerning a political goal; however, upon closer examination it might be possible to identify multiple communities of meaning residing within the same side of a political debate. In trying to understand the positions of those on a single side of a political debate, therefore, it may be helpful to identify these communities of meaning. In fact, it may be necessary to do so.

Yannow (2000) described communities of meaning as groups that share common ways of framing a policy that depend on deeper commonalities of experiences, beliefs, or values. As an example of how one's experience can influence his or her interpretation, consider how a man who had recently traveled to Athens, Greece, might interpret an individual's comment about the

beautiful weather in Athens. His interpretation of the statement, being influenced by his recent travels, causes him to consider the weather from where he had traveled, whereas the speaker was discussing the weather in Athens, Georgia. Yannow, therefore, characterized communities of meaning as groups that similarly interpret the various dimensions of the policy and therefore, "speak the same language" in the way they react to, talk about, and characterize the policy and its effects.

What constitutes a community of meaning? What factors might unite individuals and thus represent areas of commonality underlying a community of meaning? Yannow (1996) suggested "prior experience, education, training, and so forth constitute the 'frame' or 'lens' through which one sees the world and makes sense of what is seen" (p. 6); therefore, communities of meaning may be determined by identifying common experiences and affiliations. Alternatively, in his review of research on teacher beliefs, Philipp (2007) described beliefs as "a lens through which one looks when interpreting the world" (pp. 257–258), which suggests that communities of meaning may be determined by identifying commonly held beliefs. Either way, a community of meaning in regards to a policy is defined by the lens through which its members interpret the policy.

It may be argued that my framework is weakened by the fact that the math wars are not necessarily a reaction to a specific policy. How am I to identify how individuals interpret a policy when I do not even know what the policy is? In response, I point out that even in the case where the policy in question has been identified, individuals' perceptions of the policy may differ to such an extent that one may question whether they are discussing the same policy. Therefore, regardless of whether communities of meaning form around a written document or

whether they form around an idea, the researcher's job is the same: He or she must determine how individuals describe and interpret it.

Structures of Beliefs

Philipp's (2007) observation that beliefs shape individuals' interpretations of experiences is a convincing argument for the need to attend to individual beliefs as a potential unifying element in communities of meaning. To guide my study, I adopted Pajares's (1992) adaptation of Harvey's (1986, cited by Philipp) definition of beliefs as "a conceptual representation which signifies to its holder a reality or given state of affairs of sufficient validity, truth and/or trustworthiness to warrant reliance upon it as a guide to personal thought and action" (p. 660). McDonnell (2004) observed that values are embedded in all policies, which raises the question of how values are related to beliefs. Rokeach (1973, cited by Philipp, 2007) conceptualized values as a *subset* of beliefs, an interpretation echoed by Philipp, who reasoned that "a belief *that* is about beliefs, but a belief *in* is about values" (p. 265). Therefore, for this study, I have taken the position that the term *value* is subsumed by the term *belief*.

To conduct a study in which the identification of beliefs is a primary component, it is necessary to go beyond a mere definition of belief in order to conceptualize its relevant characteristics. Green (1971) and Rokeach (1968) presented complementary and sometimes overlapping descriptions of the structure of beliefs in relation to each other. Green attributed three dimensions to *belief systems*. First, he asserted that beliefs exist in a quasi-logical structure, meaning that some beliefs are derived from other beliefs. According to Green, *primary beliefs* are the beliefs that an individual assumes to be true and are, therefore, the beliefs from which other beliefs are derived. Primary beliefs are analogous to the axioms in mathematics. *Derivative*

beliefs are the beliefs that arise from logical conclusions resulting from primary beliefs, and as such, are analogous to mathematical propositions that arise from the axioms.

Second, Green (1971) described beliefs as varying in terms of their importance to the individual. Rokeach (1968) focused primarily on how strongly beliefs are held and how difficult certain beliefs may be to change as compared with others. Both Green and Rokeach used metaphors to describe their conception of the strengths of beliefs. Rokeach envisioned beliefs as analogous to the atom, in which the most strongly held and resistant-to-change beliefs were represented by the nucleus. In Rokeach's analogy, these beliefs are as difficult to change as it is to unsettle the nucleus of an atom. As one moves outwards from the nucleus, one identifies beliefs that are held less strongly, as are electrons in subsequent shells of an atom. Similarly, Green envisioned the psychological strength of beliefs as a series of concentric circles, stating, "Within the core circle will be found those beliefs held with greatest psychological strength, those we are most prone to accept without question, those we hold most dearly, and which therefore we are least able to debate openly and least able to change" (p. 46). Green called the most strongly held beliefs psychologically central as compared to those beliefs most open to change being psychologically peripheral. Significantly, Green did not equate psychologically central beliefs with primary beliefs, noting that a psychologically central belief could be either a primary or derivative belief.

Rokeach (1968) suggested that the strength of one's belief is dependent on its connectedness to other beliefs; that is, "the more a given belief is functionally connected or in communication with other beliefs, the more implications and consequences it has for other beliefs and, therefore, the more central the belief' (p. 5). Rokeach concluded that beliefs

connected to individual identity and that are shared with others are more connected, and therefore, psychologically central.

Green's (1971) third dimension claimed that "beliefs are held in clusters, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs" (p. 48). This allows the possibility that psychologically central beliefs may be inconsistent with one another.

Green (1971) distinguished between beliefs based on evidence and those held nonevidentially, claiming, "It follows immediately that beliefs held nonevidentially cannot be modified by introducing evidence or reasons" (p. 48). In an analysis of public reaction to research in education, Kilpatrick (2001) similarly noted that "those calling the loudest for research can be counted on to pay the least attention to it" (p. 426). These observations highlight the difficulty in determining whether a belief is based on evidence or whether the evidence is accepted simply because it supports a previously held belief. Pajares (1992) claimed, "Theorists generally agree that beliefs are created through a process of enculturation and social construction" (p. 316) and that individual experiences are, in turn, filtered through individually held beliefs, where interpretations of the experience may be shaped to conform to previously established beliefs. This suggests a reflexive connection between beliefs and evidence, where beliefs may be based on evidence that has been internally modified to fit the belief. In this case, attention to individual interpretations of evidence that supports or contradicts a belief may provide insight into the strength of the belief.

Beliefs About Mathematics Teaching and Learning

In attempting to determine the conceptions of individuals opposing school mathematics reforms, I was faced with the question of which particular conceptions should be included. Was

it sufficient to analyze how individuals characterized the reforms and the reasons for their objections? Jacobsen (2009) suggested that fundamental assumptions about the goals and purposes of reform should also be considered. In calling for research into public beliefs about education, she argued that educational reforms are too often undertaken without acknowledging their embedded assumptions about the goals and purposes of education, assumptions that might conflict with those of the public. As an example, she noted that an implicit goal of education embedded in No Child Left Behind was mastery of academic skills. Jacobsen noted, "The public may agree with this; however, we have limited—and often flawed—data to support this conclusion" (p. 308). She argued that by misjudging public assumptions, "the risks to the institution may be quite high as people become increasingly dissatisfied" (p. 308). This suggests that opponents to school mathematics reforms may primarily object to perceived assumptions regarding the goals of school mathematics embedded in the reforms. In this case, it might be valuable to attend to how individuals interpret the goals of reforms, as well as determining what they believe should be the goals of school mathematics. Therefore, for my study I framed individuals' reactions to school mathematics reforms within the context of their beliefs about the goals of Grades K-12 mathematics and how those goals can and should be achieved.

Ernest (1991) provided a useful direction for considering individuals' beliefs about the goals and purposes of education, as well their beliefs about teaching and learning. He proposed five social groups based upon adherents' educational philosophies, which he determined by considering two levels of ideology. First, Ernest described primary elements as "comprising the deeper elements of the ideology" (p. 131), which were often abstract and tacit beliefs not directly related to school mathematics. Primary elements included the individual's political ideology, moral views, and theory of society. An additional component was the individual's philosophy of

mathematics. Ernest provided complex and detailed discussions about the different philosophies of mathematics; however, each could be characterized as one of two fundamental views. *Absolutists* "view mathematics as an objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic" (Ernest, 2004, p. 8). The absolutist philosophy of mathematics holds that mathematics exists outside the experience of the individual and that it is discovered rather than created. On the other hand, *fallibilists* view "mathematics as human, corrigible, historical and changing" (Ernest, 2004, p. 9). The fallibilist philosophy of mathematics holds that mathematics is socially constructed and therefore open to error and revision.

Second, Ernest (1991) described secondary elements, as "derived elements pertaining to education" (p. 131). These elements were theoretically largely determined by the primary elements. Secondary elements included the goals of school mathematics and theories about the teaching and learning of mathematics. In particular, Ernest considered views of the role of ability and effort in mathematics achievement, what constitutes mathematics success and how it is assessed, what resources (i.e., technology and manipulatives) are appropriate for mathematics instruction, and theories related to social diversity in mathematics education.

Ernest's (1991) five identified social groups were specific to the social and political conditions that existed in the United Kingdom at the time of his writing; however, Klein's (2007) two distinct groups composing the critics of reforms are mirrored in two of Ernest's social groups. Specifically, Ernest's *industrial trainers* were individuals holding a "radical right" political viewpoint, an absolutist view of mathematics, and a "Protestant work ethic, which places a premium on work, industry, thrift, discipline, duty, self-denial and self-help" (p. 141). This could be descriptive of Klein's political conservatives. Similarly, Ernest's *old humanists*

were individuals holding conservative views of society, but perhaps politically liberal positions, who held beliefs about knowledge that could be traced back to Plato, "who saw mathematical knowledge in absolute, transcendental terms as pure, true and good" (p. 169). According to Ernest, old humanists held an absolutist view of mathematics and were interested in preserving and transmitting cultural values and knowledge to the next generation. This could be descriptive of Klein's classical traditionalists. Regardless, the elements to which Ernest attended in order to distinguish his groups provided direction for determining communities of meaning that I found useful for my study. In my analysis, I similarly attended to these components in order to build a conception of the ideologies that informed the reactions of those who opposed school mathematics reforms.

In the preceding discussion, I have presented the literature that informed my initial perspectives about the impetus for the math wars and the characterizations of those opposed to school mathematics reforms. Additionally, I outlined the theoretical perspectives that guided my study and my analysis decisions. In the following chapter, I describe my methodology in detail.

CHAPTER 3

DESCRIPTION OF THE STUDY

My methodology was framed within the theoretical framework of symbolic interactionism. This research tradition focuses on human meaning and operates under three fundamental assumptions: People act towards things according to the meaning they have ascribed to them, these meanings are a result of individual interpretations derived from interaction with a larger society, and, individual interpretations are in a constant state of flux as a result of continuing personal and social interaction (Prasad, 2005). In symbolic interactionism, human meaning, or perception, becomes the focus of interest, for the assumption is that individual perception is reality. The research process is necessarily reflexive, because the perceptions of the researcher determine his or her reality and this fluidic reality is caught up in determining the constructed reality of others.

To answer my research questions, I chose a grounded theory qualitative approach as outlined by Corbin and Strauss (2008) because the approach accounts for the assumptions of symbolic interactionism and "the procedures of grounded theory are designed to develop a well integrated set of concepts that provide a thorough theoretical explanation of social phenomena under study (Corbin & Strauss, 1990, p. 5). Given that my goal was to develop a theoretical description of the different beliefs and interpretations that describe the communities of meaning of opponents to school mathematics reforms, this methodology was appropriate for both my assumptions and

goals of the study. In the following, I describe my personal subjectivities as a researcher and my methods of data collection and analysis.

Researcher Subjectivity Statement

Preissle (2008) described a subjectivity statement as "a summary of who researchers are in relation to what and whom they are studying" (p. 844). The purpose of the subjectivity statement is twofold. First, the researcher is motivated to consider his or her own biases that could influence his or her interpretations of the data; and second, readers are provided information that allows them to consider the results and interpretations with knowledge of the interpretative biases of the researcher. In this spirit, I provide the following information about myself.

I count myself as an advocate for school mathematics reforms. I recognize, however, that my interpretation of reforms is my own and does not necessarily align with the interpretations of other advocates (or opponents) of the reforms. My personal interpretation of school mathematics reforms has been shaped by the NCTM's (2000) *Principles and Standards for School Mathematics*. I agree with the six principles for school mathematics described therein and interpret the intent of the process standards to be that those standards are meant to complement the content standards. In my opinion, attention to both sets of standards in the classroom results in synergistic effects in student learning.

I believe that school mathematics reforms do not call for strict instructional practices, but instead encourage a flexible approach that accommodates the students' personal interests and motivations, cultures, and prior mathematical knowledge. An important characteristic of reforms, in my view, is the intentional engagement of students

in instructional activities, for it is only through their cognitive engagement that learning occurs. Consequently, the reforms call for instructional practices that enable the teacher to focus on student thinking so that he or she may assess students' understandings and facilitate their learning.

Given that some of the opponents of reforms are political conservatives, it is important to acknowledge that I myself am politically conservative. In my opinion, the school mathematics reforms do not reflect liberal ideology, although I did not always hold that opinion. When I was first introduced to the theories and practices of the reforms, some of the articles supporting them were noticeably (to me) framed from a leftist perspective. This framing caused me to push back initially, but as I separated out the less politically biased research and examined it carefully; as I watched examples of the instructional techniques that facilitated student engagement with mathematics while making their understandings more explicit to the teacher; and as I interacted with a few politically like-minded faculty and graduate students, I began to see the value of the reforms with respect to my personal and political beliefs.

I believe my personal experiences with the school mathematics reforms have uniquely positioned me to take a noncritical look at the opponents of reforms, for in them I see myself. That is not to say that I believe the opponents would embrace the reforms if they simply understood them better. Rather, I believe I am able to consider their concerns from a sympathetic point of view, overlooking what might otherwise be hurtful or offensive accusations in order to try to understand the beliefs that drive them. I believe that through this empathetic process, I can attempt to see Grades K–12 mathematics and the reforms through the eyes of the critics in order to present their conceptions of the

reforms here. By doing so, I hope to ease the implementation of educational ideas that I believe are crucial to the mathematical achievement of all students.

Data Collection and Analysis

In grounded theory, data are collected using a *theoretical sampling* method, in which the analytic needs of the study determine the scope and depth of data collection (Corbin & Strauss, 2008). This method offered a realistic way of collecting and analyzing data because, rather than predetermining the data that would guide my study, the focus of the analysis became the development of *concepts* that emerged from analyses of data sets. That focus allowed me to collect the appropriate types of data to fully develop a conception of the communities of meaning that were within the larger group of individuals opposing school mathematics reforms.

Grounded theory methodology involves an inductive process of data collection and analysis. "Analysis begins after the first day of data gathering. Data collection leads to analysis. Analysis leads to concepts. Concepts generate questions. Questions lead to more data collection so that the researcher might learn more about those concepts" (Corbin & Strauss, 2008, pp. 144–145). Therefore, after the initial collection of data, subsequent collections take the form of theoretical sampling. In determining what specific data to begin with, I noted that the opponents of school mathematics reforms often connected with each other virtually, especially concerning the reforms on a national scale. According to Yannow (2000), "Interpretive policy analysis often begins with document analysis" (p. 31); therefore, I began with the written documents of opponents to school mathematics reforms that were readily available on the Internet.

Most of the written documents of my initial data set were collected from the websites

Mathematically Correct and New York City HOLD National (NYC HOLD). These two national

advocacy group websites are "devoted to the concerns raised by parents and scientists about the invasion of our schools by the *New-New Math* and the need to restore basic skills to math education" (Mathematically Correct, n.d.). I chose these two sites because they present themselves as representative of those who object to mathematics education reform policies and they offered a large number of articles by multiple authors who expressed their objections to the general ideas of school mathematics reforms. I initially collected articles indiscriminately, attempting to find articles reflecting a variety of authors. Because I was interested in identifying communities of meaning, it was important to look for areas of commonality between multiple authors as opposed to capturing the views of a particular individual.

I used *QSR International's NVivo 10* (NVivo) qualitative data analysis software to aid my analysis of the documents. I initially read each document and then coded during a second reading. The software allowed me to capture specific elements within each piece of data (such as text or picture) and assign codes or categories to it. My initial categories were broad and corresponded to each of my research questions. For example, to address the question of experiences and affiliations, I developed codes related to individual careers. During the course of my analysis I generated hundreds of codes under these broad categories. I also introduced new categories for prevalent comments, such *as who counts as expert in school mathematics*. Corbin and Strauss (1990) cautioned, "Every concept brought into the study or discovered in the research process is at first provisional. Each concept earns its way into the theory by *repeatedly* being present" (p. 7). Consequently, some of the codes I created were subsequently eliminated because they were not commonly expressed. Occasionally, I collapsed two or more codes or subcategories into a single code or subcategory on the basis that keeping them separate did not offer any substantial benefit and that the presence of both was complicating the analysis. For

example, for Research Question 4, *How does each community of meaning characterize school mathematics reforms and to what elements of their perceptions of the reforms do they object*, I initially had two subcategories: how the reforms were characterized and objections to reforms. As analysis of my data progressed, it became clear that the opponents' characterizations of the reforms and their objections to them were mostly the same. By keeping the subcategories separate, the codes under each were usually repeats of each other, making the process of coding unwieldy. I therefore chose to collapse the two subcategories into one.

After my initial round of data collection and analysis, I sought out more data to answer questions I had. These questions included whether codes that had weak support were so because they represented comments that were not common or whether there were others who expressed the same sentiment in other writings. Additionally, I sought out more data to expand the number of authors included in my analysis. I collected new data on multiple occasions, once after noticing that none of the documents I had analyzed reflected right-wing political ideology, in contrast to Klein's (2007) suggestion that some opponents were political conservatives. Using references provided by Klein, I searched specifically for documents written by political conservatives concerning school mathematics. I searched websites that provide news and commentary from politically conservative perspectives and think tanks composed of mostly conservative scholars, such as Town Hall and the American Enterprise Institute, respectively. Finally, I did general Internet searches using keywords such as math wars, math curriculum, fuzzy math, and the names of specific curricula that had been associated with the school mathematics reforms in order to find others who were speaking about the reforms on a nonlocal level.

Although I was analyzing through coding, I refrained from making final determinations about what communities of meaning were represented in my data until I reached *theoretical saturation*, that is, the sense that all the categories had been completely captured and described based on the absence of any new concepts in subsequently collected data (Corbin & Strauss, 2008). At this point, my data consisted of web pages, written documents posted to websites, commentaries, PowerPoint documents that some opponents had used to persuade the general public about school mathematics reforms, books, journal articles, and a video of an interview of opponents. Most of my data sources were obtained from the Internet.

Because I was not interested in the opinions of a single individual, but instead a collective understanding, I decided interviews of opponents to the reforms would not be beneficial to this study. In a sense, websites were the "gathering places" of the opponents to the reforms and the articles and commentaries on those pages were the way they virtually communicated with each other. Through analysis of these products, I was participating in their activities. By viewing the PowerPoint presentations they prepared, I was in a sense attending their meetings. A potential criticism of relying heavily on virtual materials for my data is that because the documents were public, the authors may have been guarded in the views they expressed. Although that is a legitimate concern, conducting interviews with opponents would carry the same limitation because it is unlikely they would be less guarded in discussions with a stranger. In fact, because the websites were the "meeting places" of the opponents and the documents were their communications with each other, it is possible that these documents represented their most unguarded expressions.

Upon reaching theoretical saturation, I had developed a large number of codes. During my analysis, axial coding allowed me to collapse codes under a common category; however, I

still kept and used the individual codes within these categories. I did that because it was impossible to know, prior to determining the individual communities of meaning, whether the codes I had collapsed into a single category would regain independent significance after I separated the data by communities of meaning. Consequently, I sometimes had codes that went 3 or 4 levels deep. The use of NVivo allowed me to efficiently deal with such a large number of codes and to choose when and if any code would aggregate the subcodes falling within it.

Although my analysis had already given me an initial impression of likely communities of meaning, I used NVivo to perform numerous queries along different codes and categories (called *matrix coding*) to see if they differentiated groups in a meaningful way (see Figure 1 for an example of matrix coding). As an example, in characterizing the reforms, many opponents made comments concerning basic skills. These comments took two forms: asserting that basic skills had been de-emphasized and complaints related to algorithms. Matrix coding enabled me to compare codes in other categories or subcategories with respect to which types of comments the opponents made.

		A: RE basic skills ▼	B: de-emphasized ♥	C : RE algorithms ▼	D : de-emphasized ▼	E: invent their own ♥	F : non-standard algorithms 7
1 : Education-Traditional	V	44	17	24	16	5	9
2 : Budd et al	V	1	1	0	0	0	0
3 : Carson	V	23	9	12	9	3	3
4 : Garelick	V	13	4	9	5	1	6
5 : Hirsch	V	1	1	0	0	0	0
6 : Loveless	V	0	0	0	0	0	0
7 : Mass	V	2	0	1	1	0	0
8 : misc	V	1	1	0	0	0	0
9 : Pappas	V	0	0	0	0	0	0
10 : Stotsky	V	2	1	1	0	1	0
11 : USWCMath	V	1	0	1	1	0	0

Figure 1. Picture of a matrix coding query using NVivo.

As another example of matrix coding, under the code, *algorithm*, I had an additional three subcodes: complaints that the reforms asked students to invent their own algorithms, complaints that no algorithms were developed, and complaints that nonstandard algorithms were being

taught. By performing matrix coding along these three codes, I could determine whether these concerns were the basis of distinctions between communities of meaning. Queries such as this one produced little variation among the other codes, suggesting that these codes did not represent the basis for distinctions between communities of meaning, so I returned to considering my initial impressions further. This analysis (described more fully in the next chapter) resulted in my identifying three communities of meaning.

In order to describe each community of meaning, I used the codes to help me determine which community of meaning each of my authors most fully occupied. In this process I faced a dilemma about how to determine authorship of general web pages. Specifically, two websites, Mathematically Correct and the United States Coalition for World Class Mathematics had produced documents for which no particular authorship was credited. After discussing this with my major professors, we decided to credit authorship to the website itself and treat it as any other author. A second question was how to determine the authorship of documents with multiple named authors. I determined that each document should be attributed to one author only, in order to avoid double counting the codes within it. I therefore attributed each document to the first author. In all but one case, documents with multiple authors were easily placed into a community of meaning because each author could be independently placed into the same community of meaning on the basis of other documents. The single exception was a document written by multiple authors who occupied different communities of meaning. The contents of the document did not place it firmly into a particular community of meaning; therefore, a characteristic of the first author was used as the determining factor.

After determining the community of meaning to which each author belonged, I created categories of each community of meaning and coded the data by author. This way I was able to

perform matrix coding along the authors in each community of meaning. For example, I determined that Milgram, an author of some of my documents, resided in a community of meaning that I called *Math-Traditionalist*. I coded every piece of data for which Milgram was an author as Math-Traditionalist with the subcode, Milgram. By doing this, I was able to attribute every coded reference that I had made in Milgram's documents to him. To describe each community of meaning, I performed matrix coding for each author in the community of meaning, as well as for their combined responses. Figure 1 provides an example of a matrix coding query along authors in the Education-Traditionalist community of meaning with respect to their comments concerning basic skills, as described earlier. The number 44 in the top left depicts the total number of coded references concerning basic skills in that community of meaning. Seventeen comments were coded as complaints that the reforms de-emphasized basic skills, and 24 represented some complaint about algorithms. The code basic skills aggregated the codes de-emphasized and algorithms, whose sum was not 44. Therefore there were 3 coded references concerning basic skills that pertained neither to their de-emphasis nor to algorithms. Similarly, one can see that the code *RE algorithms* aggregated the subcodes in the columns to its right. In this case, the sum of the number of subcodes exceeds the code that aggregated them, which means that some comments were coded with more than one of these subcodes; that is, a single comment may have both complained that students were asked to invent their own algorithms and that nonstandard algorithms were taught.

By looking at the distribution of codes according to author, I was able to determine occasions when a single author was producing a code, which would call into question whether it represented a common understanding. This phenomenon presented the dilemma of how many authors needed to express the same idea in order for it to be considered as a common

understanding. I determined that, in the spirit of triangulation, I would consider an idea represented by a code as shared within the community of meaning if at least three authors shared the code. That meant, for example, that in Figure 1, the complaint that nonstandard algorithms were being taught did not apply to this community of meaning because only Carson and Garelick were coded as expressing this concern.

The final step of my analysis was a process through which I "became" a member of each community of meaning in order to describe it. In this step, I wrote out a full description of each community of meaning from a sympathetic position. By looking at the issues through their eyes, I attempted to represent their conceptions as accurately as possible. This step preceded the final write-up of my findings, presented with respect to each of my five research questions, which follows in the next chapter.

CHAPTER 4

COMMUNITIES OF MEANING

My findings are the product of my analysis of 99 documents written by 41 authors, producing 1,800 coded references. I easily identified the two communities of meaning suggested by Klein (2007), specifically *classical traditionalists* and *political conservatives*. About one-fifth of the authors that fell in the classical traditionalist division were self-proclaimed liberals or Democrats (or both) or were identified as such by Klein. But more importantly, the classical traditionalists' comments did not imply a conservative political position. Additionally, Abigail Thompson broadly characterized mathematicians as non-conservative:

Following a presentation to the California Board of Education, Abigail Thompson, a mathematics professor at the University of California at Davis . . . was invited to speak at a local Republican convention. A liberal Democrat, Thompson was stunned. Mathematicians tend to jump into such issues with both feet, she says, "and then they find themselves labeled as right-wing conservatives. And it's pretty hilarious. I don't know any mathematicians who are right-wing conservatives." (Klein, 2007, p. 27)

Therefore, with one exception, none of the mathematicians were categorized as politically conservative. The single mathematician categorized as a political conservative identified himself as such.

Within the classical traditionalists, a comparison of codes according to whether the author was a mathematician highlighted important differences. Specifically, the mathematicians were the only authors who expressed concern that the reforms *redefined mathematics*. I further noted that mathematicians' comments produced more codes in the area of mathematics as a domain than the nonmathematician traditionalists, and their descriptions of the reforms and their objections to them were more explicit. In fact, the nonmathematician traditionalists expressed

conceptions—of mathematics as a domain, the goals of mathematics education and how those goals should be achieved—that were more akin to those of political conservatives than to those of the traditionalists who were mathematicians. The views of political conservatives in other areas, however, distinguished them from the nonmathematician traditionalists. These observations led me to conclude that the authors occupied one of three communities of meaning, which was determined by the primary lens through which each author viewed Grades K–12 mathematics and the reforms.

The communities of meaning that I describe emerged from commonalities in the data. All of the data that I analyzed were placed into one of the three communities of meaning; however, the distribution of data between the three communities was not equal and the number of coded references each collection of data produced ranged widely (see Table 1). As stated, my inference about the primary lens through which each individual viewed school mathematics reforms distinguished the three communities. Although the authors in one community of meaning might have shared some of the characteristics of other communities of meaning, I inferred each author's community based on what appeared to be the primary lens for his or her perceptions of Grades K–12 mathematics and the reforms. It was possible for an individual to reside within more than one community and employ multiple lenses to form his or her understanding. For clarity, however, I treated each of these communities as distinct.

The community of meaning that I called *Math Traditionalists* (MTs) accounted for the largest number of easily accessible online documents, many of them located in the popular websites *MathematicallyCorrect*.com and *NYCHold*.com. The documents represented the views of 12 authors, 11 of whom identified with the role of an academic mathematician, having currently or formerly served in such a role. (The one exception was the author previously

described as *Mathematically Correct*, whose documents, I determined, fit the MT community of meaning.) The authors in this community of meaning frequently cited each other's work, collaborated in producing a single document, or otherwise indicated that they had interacted with each other in producing their writings, such as thanking others for their helpful suggestions. The documents varied considerably in their length and purpose, each yielding nuanced perspectives in at least one of my categories of interest.

Table 1

Distribution of Documents and Coded References among Communities of Meaning

Community of Meaning	Number of Authors	Number of Documents	Number of Coded References
Math Traditionalists	12	53	1,457
Education Traditionalists	11	21	165
Political Conservatives	18	25	174

The community of meaning that I called *Education Traditionalists* (ETs) encompassed 11 authors of various occupations; although none of the first authors was an academic mathematician, some worked in STEM fields, whereas others' careers were far removed from mathematics. One document was the product of collaboration between ETs and MTs. This document was difficult to place because it had characteristics of both communities of meaning; however, because the first author of the document was not a mathematician, I ultimately grouped it with the ET data set. The number of documents that fell into the ET community of meaning was about half that in the MTs' but produced only about one-tenth of the number of coded references of the MT documents.

The community of meaning that I called *Political Conservatives* (PCs) emerged from documents written by 18 authors, but the number of documents and coded references in the data set mirrored that of the ETs. The smaller number of coded references in the ETs' and PCs' documents as compared to the MTs' was partly the product of fewer documents to code, but was more significantly due to ETs' and PCs' less nuanced and complex conceptions of Grades K–12 mathematics and the reforms. Below, I describe the primary lens through which each community of meaning filtered the issue of mathematics education reforms and characterize their views in the categories for which they shared understanding and language.

The Primary Lenses for Viewing School Mathematics Reforms

The first research question sought to determine the experiences or affiliations that those who oppose mathematics education reforms have in common. The data suggested that the MTs shared experiences inherent to becoming and working as academic mathematicians and the PCs shared their political affiliation. The ETs, on the other hand, shared with each other and the PCs a commonality in their experiences with school mathematics and an affiliation with the MTs in political activity opposing the reforms. Their affiliation with MTs was particularly evident in the website, *NYCHold*.com where they collaborated in opposing school mathematics reforms. The ETs' commonalities with both the PCs and the MTs were evident in the ways the ETs' conceptions of school mathematics and the reforms overlapped with those in the other groups. As such, the views of the ETs represented a sort of crossing of the MTs and the PCs, yet the ETs also expressed unique emphases along certain codes.

By considering the common beliefs of the MTs and the ETs that may have facilitated their collaboration, I realized that each group was defined by the primary lens through which they viewed school mathematics and the reforms and that these lenses were reflected in and

shaped by each group's experiences and affiliations. In short, the MTs filtered the reforms primarily through their view of mathematics; the ETs viewed educational reforms through the lens of their perspective on education; and the PCs viewed mathematics education reforms primarily through the lens of their conservative worldview. In the following I describe each lens in detail.

The Math-Traditionalist Lens: A View of Mathematics

The MTs found commonality in their view of mathematics as a discipline, and this view acted as the primary lens through which they viewed mathematics education reforms. The MTs spent more time characterizing mathematics than those in any other community of meaning, such comments comprising their third most prevalent type of comment as compared with being the least prevalent type of comment for ETs and PCs.

The MTs valued the coherent nature of mathematics as evidenced by their discussions of its internal structure and hierarchical nature. They claimed that what one knows in mathematics is dependent on previously proven facts and skills, most of which have been committed to memory. For example, Milgram (2010) wrote, "Math is hierarchical: entirely supported by the material learned in the lowest grades" (Slide 3) and Klein et al. (2005) asserted, "The essence of mathematics is its coherent quality. Knowledge of one part of a logical structure entails consequences that are inescapable and can be found out by reason alone" (p. 33). Many MTs claimed that definitions formed the foundation of mathematical structure and that those definitions must be precise. Ironically, Milgram (2004), wrestling with the difficulty of defining mathematics itself, wrote,

A few years back a more profound attempt at a definition [of mathematics] was given privately by Prof. N. Gottlieb at Purdue. He suggested "Mathematics is the study of precisely defined objects." A number of people participating in this discussion said, in effect, "Yes, that's very close, but let's not publicize it since it would not sound very

exciting to the current MTV generation, and would tend to confirm the widely held belief that mathematics is boring and useless." (p. 2)

The MTs repeatedly used the terms *precision* and *precise* or some synonym of these, such as *exact*, in their descriptions of mathematics. Wu (1999) wrote, "There are at least two special features about mathematics...: It is cumulative and it is precise" (p. 10). They asserted that definitions of "terms, operations, and the properties of these operations" (Milgram, 2005, p. 9) must be precise, in addition to mathematical discourse itself. Milgram (n.d.c), in particular, emphasized the importance of precision in posing problems, arguing that posed problems must remove all ambiguity:

In a classroom where there are no definitions there can be no well-posed questions, and hence no mathematics. While many educators have embraced the notion of problem solving and often quote the four steps of problem solving outlined by G. Polya, his model assumed that problems were necessarily well-posed before a student would see them, which is often not the case. Thus, it is necessary to extend Polya's model for problem solving to include these issues, and the failure to do this previously has resulted in gross misconceptions about what mathematics is and how it can be successfully taught. (p. 1)

In an interview with Leong (2013), Wu reinforced Milgram's sentiment;

Mathematics should not leave people in doubt as to what is true or what is not true. You either say it is true or false, or you can just say you don't know. You should not say something and leave it hanging, and then let students decide whether it is true or not true. That's not acceptable, and in any case that is not the kind of mathematics I know. (p. 32)

Precision in definitions, mathematical discourse, and problem posing was characterized as an important property of mathematics, but it was also framed as critical because of the abstract nature of mathematics. Wu (1999) wrote,

The precision of mathematics stems from its abstract nature. Whereas even in a rigorous discipline such as physics, a photograph or a measurement by a laboratory equipment can render verbal explanations superfluous, the basic concepts of mathematics reside only in the realm of ideas and therefore must be meticulously described. (p. 10)

The MTs viewed mathematics itself as abstract, yet applicable to the concrete, physical realm. According to Allen (1997), "the abstract quality of mathematics produces power" (p. 3) and enables the "spectacular triumph of logic over intuition" (p. 11).

An important aspect of mathematics to the MTs was the role of proof, which was often referred to as deduction, logical reasoning, or mathematical reasoning. In the view of MTs, proof was closely related to conceptual understanding, for "students demonstrate genuine conceptual understanding when they can supply the mathematical reason for each step in a method" (Quirk, 2002, "How the NCEE Redefines Conceptual Understanding," para. 6). In fact, at least three MTs viewed proof as a vehicle for mathematical understanding. For example, Wu (1999) described an episode in which he helped an individual learn the polynomial division algorithm. In his account, Wu realized that his student was unable to prove the standard long division algorithm, so he carefully guided him through a proof by induction, providing additional information about complete induction as a method of proof when it became clear that the student lacked such knowledge.

Finally he got it done. The whole session took something like two hours. I had no doubt that he really learned the algorithm through this tortuous process, and it is likely that for most students this is the only way to learn it. (p. 3)

Additionally, the MTs characterized mathematics as a human construction that had developed over centuries. The tone of their descriptions was that of respect, almost reverence. For example, Klein (2005) wrote, "Mathematics is the oldest and most universal part of our culture. In fact, we share it with all the world, and it has its roots in the most ancient of times and the most distant of lands" (pp. 32–33). Notably, although the MTs characterized mathematics as a human construction, their comments suggested they conceived the content of school

mathematics as a fixed body of knowledge, largely unaffected by mathematics' ever-increasing boundaries.

The MTs was the sole group for which I identified a secondary lens through which they filtered school mathematics. Although their view of mathematics was their primary lens, their view of education agreed with that of an Education Traditionalist (ET). The impact of this lens presented itself in some of their views of teaching and learning mathematics, as well as their reactions to some of the characteristics of the school mathematics reforms. Therefore, the MTs' secondary lens is that of the ETs.

The Education-Traditionalist Lens: A View of Education

The ETs' educational views corresponded to those of *academic-traditionalists* (hereafter referred to simply as *traditionalists*) as opposed to *progressive-experimentalists* (hereafter *progressivists*), and they equated modern-day education reforms to educational philosophies promoted by progressivists. The controversy over these two views of education reaches back to the early 1900s in the United States; therefore, to properly describe the ET lens, it is necessary to frame their views within the history of the controversy between traditionalists and progressivists.

Traditionalists versus progressivists: An age-old controversy. In the first half of the 20th century, a philosophical war raged over Grades K–12 education in the United States. On one side were progressivists, represented most notably by John Dewey, and on the other were traditionalists, represented by those from two schools of thought: essentialism and perennialism.

Perennialists, most visibly represented by Robert Maynard Hutchins (1899–1977) and Mortimer J. Adler (1902–2001), emphasized the "generic similarity" (Hutchins, 1936, p. 67) of the human race and the need to capitalize on common bonds in the development and preservation of intellectual truths that remained "the same in any time or place" (p. 66). They believed that the

failure to attend to these truths would lead to cultural corruption and economic disorder (Adler, 1940), and they pointed to the current economic challenges facing America and the war in Europe as evidence for their claims (Hutchins, 1936). They believed that society would be best served by the cultivation of intellectual virtues, arguing that these virtues transcended circumstance and experience (Hutchins, 1936). The essentialists, most visibly represented by Isaac L. Kandel (1881–1965) and William C. Bagley (1874–1946), held a similar reverence for "cultural heritage" (Kandel, 1938, p. 22) and focused on the preservation of the democracy. They saw the present state of education, which they characterized as "weak and ineffective" (Bagley, 1938, p. 241), as a source of much of the current economic and political unrest.

On the other side of the philosophical war, the progressivists believed that to properly consider the collective of society it was necessary to recognize the individual. Dewey (1902/2010) claimed that "only by being true to the full growth of all the individuals who make it up, can society by any chance be true to itself" (p. 6). Whereas the traditionalists viewed society as a collective governed by the same immutable truths that had existed for ages, the progressivists saw the imposition of supposed "truths" as a danger, for "there is implicit in every assertion of fixed and eternal first truths the necessity for some *human* authority to decide ... just what these truths are and how they shall be taught" (Dewey, 1937a, p. 400). To the progressivists, knowledge was not the "truth" proposed by the perennialists; instead, knowledge was "no longer an immobile solid; it [had] been liquefied. It [was] actively moving in all the currents of society itself" (Dewey, 1902/2010, p. 13). The progressivists believed that society was constantly changing, which required the active updating of methods and ideas (Dewey, 1937a, 1937b, 1988). In fact, the progressivists believed that the failure to employ modern

resources in the battle against social evils was "one factor that [had] given totalitarian philosophies their present power" (Dewey, 1941/1988, p. 319).

The modern-day controversy. Modern traditionalists' and progressivists' visions of education are not diametrically opposed. There is a wide chasm, however, between the two philosophies that underlie them. In a nutshell, the traditionalist philosophy holds that humanity's basic instincts are untrustworthy and that the qualities necessary for a civilized society are contrary to the human condition (Bagley, 1938; Hirsch, 2001; Hutchins, 1936; Kandel, 1938; Loveless, 2001). Consequently, traditionalists view education as a means by which these qualities are cultivated via the disciplined and systematic acquisition of a reasonably fixed body of knowledge (Bagley, 1938; Hirsch, 2001; Kandel, 1938). On the other hand, progressivist philosophy holds that humanity's basic instincts include curiosity, the desire to solve problems, and the need for peer approval (Dewey, 1902/2010; Kilpatrick, 1918). Consequently, progressivists assert that effective education occurs through attention to each individual's experiences and interests in order to induce an active personal construction of knowledge that will serve him or her into adulthood. According to progressivists, this personal knowledge mimics the nature of collective knowledge, which is viewed as fluid and ever-changing (Dewey, 1902/2010, 1941/1988; Kilpatrick, 1918; Loveless, 2001).

Progressivists desire an education system characterized by social interaction among peers, active student involvement in problem-solving situations, integrated and interdisciplinary subject matter, and student-centered instruction (Counts, 1932; Dewey, 1902/2010; Hirsch, 2001; Kilpatrick, 1918; Klein, 2007). They frame their conception of subject matter in light of the needs, interests, and experiences of individual students. The progressivist classroom is characterized by project-based learning that capitalizes on student interests to create learning

opportunities; therefore, subject matter is integrated and fluid. Additionally, value judgments or preferences are often considerations in these projects; and therefore assessments are generally formative and informal, acknowledging that ambiguity precludes a "right" answer. Because progressivists believe that each individual's understanding of subject matter is unique and personal, they argue that the teacher's role is to facilitate students' constructions of subject matter with attention to ensuring that the students' understanding does not directly conflict with established collective understanding.

In contrast, traditionalists believe education should be characterized by the development of self-control and intellectual virtues (Hirsch, 2001; Hutchins, 1936), teacher-centered instruction (Loveless, 2001), and the dissection of subject-matter, both between and within subjects (Bagley, 1938; Hirsch, 2001). Therefore, although traditionalists acknowledge advancements and changes to fields of study, they view most Grades K–12 subjects, such as mathematics, as consisting of material that is fixed and not subject to change. This material is what traditionalists believe needs to be passed along to students via direct instruction. The traditional classroom is characterized by a separation of subject matter into individual subjects, which are further separated into topics, all designed for the disciplined and systematic acquisition of its content. Traditionalists hold subject matter in the highest regard, valuing it far more than social goals or learning processes, and they believe that students' knowledge of this subject matter can and should be systematically assessed.

The primary lens through which the ETs viewed school mathematics education reforms was that of a modern traditionalist. In particular, some ETs wrote that the current education reforms were rooted in ideas promoted by "John Dewey and his followers" (Stotsky, 1999, p. 249), equating the reforms with progressivist ideology. Those ETs who did not explicitly align

themselves with modern traditionalists expressed views similar to those who did, and I therefore categorized them as ETs. This similarity suggests that the lens through which the ETs viewed Grades K–12 mathematics education reforms was not dependent on knowledge of the traditionalist-progressivist controversy; rather, the authors in this community of meaning shared the philosophical and educational understanding of modern traditionalists.

The Political-Conservative Lens: A Conservative Worldview

The term *conservative* applies to a large number of people with diverse views; therefore no single summary can capture the complexities involved nor accurately describe the views of those who identify as conservative. This description of the PCs' lens has been partially informed by my own understanding and the values and ideologies implied by the PC authors in their comments on school mathematics reforms. In addition to detailing the conservatives' views, I include some of their conceptions of liberal ideology, which the PCs associated with Grades K—12 mathematics education reforms, and their conceptions of academia, which they saw as partially responsible for those reforms.

Conservatives' view of humanity. Conservatives do not believe that humankind is naturally good. They believe individuals are born with certain proclivities and areas of potential and that those qualities are fixed but not deterministic. Human's natural tendencies, however, are "selfish and hedonistic by design. Given his nature ... it is a wonder he ever chooses conformity" (Adams, 2009, para. 3). In the view of conservatives, individuals require guidance and moral education to suppress their natural inclinations. Thus, children are socialized by family, religion, and community to conform to social standards. Government also plays a role by enforcing the rule of law that society has determined (Adams, 2009; intellectual takeout, n.d; Weinberger, 2011).

Liberals, according to conservatives, believe that individuals begin as blank slates and that their natures are a product of social conditioning. Therefore, the liberals believe, mankind is generally good and "to the extent that people become 'bad' it is because 'society' corrupted them" (Adams, 2009, para. 6). Because individuals are a product of their conditioning, those who behave badly can be reconditioned to behave well; it is simply a matter of understanding where things went wrong.

Conservative values. Fundamentally, conservative values revolve around the belief that evil exists in the world; that it can be recognized, and it can be fought (Conservapedia, 2013a; Downing, 2001). Because conservatives believe that human nature is not inherently good, they recognize a standard outside of the individual that represents absolute moral truth. For many conservatives, that truth is God. Additionally, to conservatives the existence of moral truth implies the existence of physical truth; that is, the physical world *is* and human's attempts to describe it are either *right* or *wrong*. To conservatives, these moral and physical truths are more fully known by some people than others—for example, in most cases a parent knows more about truth than his or her child—and the authority of these people should be respected, "but not to the point of accepting orders or assertions that are contrary to logic or morality" (*Conservapedia*.com, 2013a, para. 3).

In the view of conservatives, individuals are in a constant state of choice between good and evil, and maturity marks an individual who chooses self-control, responsibility, and duty to others (good) over his or her selfish desires (evil) (Conservapedia, 2013a; Downing, 2001; Laser, 2010). Conservatives assume that in most cases, the "self-indulgent search for instant gratification of desires" (Conservapedia, 2013a, para. 3) is the reason people find themselves in unhappy social or financial conditions. Additionally, conservatives view human emotions as

unpredictable and unreliable and likely to lead astray those who yield to them. Therefore, emotions are suspect and meant to be controlled.

Because conservatives view the human condition as a constant battle between right and wrong—producing self-control and irresponsibility, respectively—they believe that individuals should be judged according to their personal choices and allowed to reap or suffer the consequences of those choices (Downing, 2001). Thus, conservatives value personal responsibility because that characteristic evidences an individual who is aware of the connection between his or her condition and choices. Conservatives respect those who do not "complain and instead take practical action to improve one's situation" (Conservapedia, 2013a, para. 3), because they believe that in the United States, individuals' conditions are (for the most part) a product of the choices they have made. Conservatives view competition as a motivator for people to make choices that will improve their condition and, by extension, improve the community in which they live. Therefore, they accept unequal outcomes, insisting instead on equal rights and opportunity (Laser, 2010).

Because conservatives believe in good and evil—right and wrong—and that these can be recognized, they believe that societies and cultures can be judged as conforming more or less to each. Therefore, conservatives are less likely to value unknown cultures or celebrate diversity for diversity's sake. Rather, they embrace American culture and values, which they know and believe to be good (Downing, 2001).

Finally, conservatives also believe that "families know best how to raise their children. They need to be strengthened and have the right to raise their children the way they want" (Downing, 2001, para. 2). To conservatives, parents should have the ultimate say in what and

how their children should be educated and, having the most intimate knowledge of their child, are in the best position to make those decisions.

Conservatives' perspectives on liberals. Conservatives believe that liberals are driven by their sense of compassion (Laser, 2010), which, they admit, is a noble quality. They would argue, however, that liberals' compassion fails to account for human nature; therefore, liberals' acts of compassion often have the unintended consequence of exacerbating problems by reducing or eliminating the consequences individuals would normally experience when they submit to their selfish tendencies. To conservatives, liberals have distanced individuals from the consequences of their poor choices by promoting policies that minimize or eliminate those consequences. In the view of conservatives, liberals desire equal benefits for all, regardless of effort or product. Conservatives believe that taken to its conclusion, this philosophy would lead to the downfall of society, for if consequences of decisions are eliminated, individuals will have no motivation to act in ways contrary to their natural tendencies.

Conservatives assume that liberals have further distanced individuals from the consequences of their decisions by promoting the narrative of victimization. In conservatives' opinion, liberals "see society as composed of groups: black vs. white, old vs. young, rich vs. poor, male vs. female" (Downing, 2001, para. 3). Conservatives argue that by framing the individuals in some groups as victims, liberals reduce the power individuals in those groups have to change their circumstances, simply because those individuals will no longer believe they would benefit by making socially desirable decisions.

Conservatives claim that liberals do not believe in absolute truth, instead existing in a state of moral relativism in which individuals determine their own sense of right and wrong and are slaves to their emotional states (Downing, 2001). Within this context, conservatives believe

liberals view all cultures as equal and worthy of celebration, regardless of the culture's positions regarding "freedom, rights, and life" (para. 25). Significantly, conservatives believe that the term *social justice* encompasses liberals' ideologies (Downing, 2001).

Conservatives' perspectives on academia. "Education in America, especially at its higher level, is typically seen by conservatives as promoting liberalism and helping to promote moral degeneration" (Conservapedia, 2013b, para. 8). As evidence, conservatives point to studies that show most professors' ideology is left of center (Hurtado, Eagan, Pryor, Whang, & Tran, 2012; Klein, 2009; Inbar & Lammers, 2012). Based on anecdotes of liberal professors berating conservative students (Hannity, 2013; Limbaugh, 2013, 2013), conservatives have concluded that a goal of higher education is to indoctrinate students with liberal ideology. Because of this conclusion, conservatives approach ideas that they perceive as having originated in higher education with the suspicion that those ideas are pushing a liberal agenda.

Conservatives have particular reservations about professors connected to education. In particular, they suspect that these educators believe that they know what is good for children better than the children's parents (e.g. Beck, 2013; Limbaugh, 2012), to which conservatives strongly object. Additionally, they believe that education professors have concentrated on children's self-esteem to the extent that children are shielded from hard truths about the outcomes of their effort, which serves to distance children from the consequences of their actions and decisions (Limbaugh, 2007). Conservatives view this practice as ultimately harmful to children.

The Views of Mathematics

My second research question sought to determine the beliefs about mathematics that were shared by individuals in each community of meaning. The MTs' views of mathematics defined

their particular community of meaning and have been described in a previous section. The ETs and the PCs expressed very little in their writings about their conceptions of mathematics as a domain (see Table 2). Of the ETs who did, all referred to it as hierarchical: "Mathematics is cumulative. The prerequisites accumulate as you move to the next topic of study" (Pappas, 2003).

Table 2

Number of Coded References for Each Community of Meaning in Categories of Mathematics and the Teaching and Learning of Mathematics

Community of Meaning	View of Mathematics	Who Counts As Expert	Student Ability and Effort	Goals of School Mathematics	How Goals Should Be Achieved
Math Traditionalists $(n = 53)$	71	76	37	66	58
Education Traditionalists $(n = 21)$	10	44	10	29	49
Political Conservatives $(n = 25)$	13	27	13	20	52

Note. n = number of documents in community of meaning.

This conception of mathematics arguably corresponds to traditionalists' penchant for dissecting subject matter. That is, given this conception, mathematics can be viewed as content to be served in small "bites," where subsequent bites build on what has already been consumed. In this view, all of mathematics hinges on previous learning, much like construction scaffolding depends on a secure foundation. Loveless (2004), evidenced this view by framing mathematics as building upon basic skills:

Basic skills are necessary to advance in math. Insisting that students master computation skills is not to advocate that they stop at the basics. Basic skills are a floor, not a ceiling.

Students must learn arithmetic so that they can move on to more demanding mathematics— algebra, geometry, calculus. An emphasis on the basics should never be used as an excuse to straightjacket students or to slow their progress in the math curriculum. ("Why Important," para. 2)

Although the PCs' view of mathematics corresponded to the ETs' conception of mathematics as hierarchical, only half of the PCs commenting about mathematics as a domain expressed this view. Instead, the PCs also expressed a conception similar to MTs' assertion that mathematics had been developed over hundreds or thousands of years. PCs' less focused conceptions of mathematics as a domain as compared to ETs' highlights the centrality of the ETs' belief that subject matter can be dissected, as compared to the PCs' more peripheral beliefs about mathematics. Notably, ETs, PCs and MTs all shared the opinion that mathematics is hierarchical.

The Views of Teaching and Learning Mathematics

My third research question sought to determine each community of meaning's beliefs as to what should be the goals of school mathematics and how those goals can or should be achieved. In these categories, the MTs' number of comments per document was similar to or less than those of the ETs' and the PC's (see Table 2); however, the MTs' comments were distributed more evenly between larger numbers of codes. In comparison, the ETs' and the PCs' views of teaching and learning mathematics were simple and homogenous. Additionally, both the ETs and the PCs rarely stated their vision of teaching and learning mathematics explicitly; rather, I had to infer their views from their objections to Grades K–12 mathematics reforms. These observations underscore the primary lenses through which each community viewed school mathematics. The MTs' rich descriptions reflected their attention to the various elements of mathematics, whereas the ETs' and PCs' lack of explicit discussion reflected their preoccupation with matters unrelated

to mathematics, namely interpretations of reforms based on a general view of education and a conservative worldview, respectively.

Student Ability and Effort

In my analysis, I captured author comments that either explicitly or implicitly revealed the author's beliefs about the roles of ability and effort in learning mathematics. Neither the ETs nor the PCs expressed commonality within their groups regarding their views of how, or if, natural ability or effort affects student achievement in mathematics. As shown in Table 2, authors in both communities made comments that I coded as indicating some opinion; however, none of these opinions were shared by at least three members in their community of meaning; consequently, I was unable to determine whether these opinions represented a common understanding. In contrast, the MTs shared an understanding that, in essence, dodged the question. According to Raimi (2002), "We must have the courage to recognize that not everyone will succeed, even if everyone can. We need not decide the difference is genetic, or caused by bad companions, or whatever 'cause' is popular at the moment' (para. 10), but "it is important for us to ... admit that some will—for whatever reason—learn substantially less than others, no matter what we do" (para. 7). The MTs assumed stratified student achievement levels, but demurred on identifying the causes for different levels of achievement. MTs expressed the belief, however, that students who fell on the low end of achievement had the potential to learn more mathematics than had traditionally been expected of them. Speaking of "at-risk" students, Milgram (n.d.a) stated, "They can learn much more material than people here believe if it is just presented appropriately" (Slide 37).

Who Counts As Expert

In my analysis, I captured comments that revealed who the authors considered to be an authority for mathematics education. In doing so, I also identified comments that revealed a belief about who should *not* be considered an authority. I separated these comments according to whether it assigned (or denied) authority in regards to determining the content, pedagogy, or the training of teachers for school mathematics.

Teacher training. Of the three communities of meaning, only the MTs made comments concerning teacher training. They believed that mathematicians should determine or provide the content training of prospective teachers; however, as discussed later, they were less convinced of their ability to handle issues related to pedagogical practices and suggested the training of teachers in this area should be a combined effort of mathematicians and educators.

School mathematics content. The MTs, ETs, and PCs all viewed mathematicians as expert in determining the content for school mathematics. Birkett (2012), a PC, wrote,

Math is unusual in the degree to which it builds on itself, so that deficiencies in a child's math training often show up many years later. There is reason to think, for example, that the single most important determinant of a student's success in calculus and other college-level math is the quality of the instruction he received in grades 4 through 6. So when choosing elementary curricula, it's essential to get input from those teaching math at the college level. There's an important sense in which they are the only people who really know whether any of our math teaching is working. ("Some History," para. 5)

The ETs also looked to the mathematics content in high-achieving countries as an example of what ought to be included in U.S. Grades K–12 mathematics. Loveless (2004) explained, "If we are serious about providing all students with a challenging mathematics curriculum it must be coherent, focused and demanding not by our own sense of what this might mean, but by international standards" ("Why Important," para. 14).

School mathematics pedagogy. The ETs expressed no consensus about who counted as expert in determining pedagogy in the classroom. Most ETs were silent on the matter, perhaps reflecting the traditionalists' preoccupation with content. The MTs were also not convinced of their ability to handle issues related to pedagogy. The MTs believed that mathematicians and educators must work closely together to improve Grades K–12 mathematics education. Schmid (2000) said:

Mathematicians do not want to invade the educators' turf. We are not qualified to do their work. We are qualified as critics of reforms in math education. We should call attention to reforms we see as well meaning, but hectic and harmful. Most music critics would not do well as orchestra musicians. They do have acute hearing for shrill sounds from the orchestra! (para. 14)

The MTs believed, however, that mathematics educators did not have strong enough knowledge of mathematics as a domain to be trusted to make pedagogical decisions without the input of mathematicians. Milgram (n.d.c) suggested that "the isolation between mathematicians and math educators over the last 70 to 80 years has led to a significant loss of understanding of mathematics on the part of math educators" (p. 8). Therefore, they suggested that mathematicians and educators should share authority concerning pedagogical practices.

The PCs did not differentiate between those who were in positions of expertise regarding content versus pedagogy; therefore, they considered mathematicians expert in pedagogy by default. The PCs frequently mentioned the controversial 1999 decision by the U.S. Education Department of Education to label some reform-style curricula as *promising* or *exemplary*, after which "more than 200 prestigious mathematicians and scholars, including four Nobel laureates and two winners of the Fields Medal, the highest math honor, published a full-page ad in the Washington Post criticizing the 'exemplary' curricula' (Schlafly, 2006, para. 7) and sent a similar letter to then-Secretary of Education, Richard Riley. Of the eight PC authors who made comments recognizing the expertise of some group or individual in mathematical content or

pedagogy, half specifically mentioned this historical incident. Therefore, it can be argued that the mathematicians involved in this incident, many of whom might have been MTs, had considerable influence on the way Grades K–12 mathematics reforms were perceived at the time, as well as many years later.

The PCs heaped scorn upon *educrats*, a term some PCs used to differentiate between traditional educators and, ostensibly, the developers and advocates of reforms. According to Saunders (1998), the difference between educators and educrats is "simply put, [that] educrats believe in process—as opposed to educators, who believe in results" (para. 3). The PCs made it clear that they did not view educrats as experts. Cheney (1998), granting expertise to a university physics professor named Alan Cromer, wrote:

Mr. Cromer speculates that the reason constructivism has taken hold is that it confers status. Math and science educators are stuck in education departments, he observes, completely cut off from math and science departments. "What is their expertise?" he asks. "Constructivism gives them something to be expert on. It helps the professional lives of a marginalized group of people." (para. 6)

But, according to Williams (2007), these educators—educrats—lacked the intellectual wherewithal to contribute to education in a meaningful way:

American education will never be improved until we address one of the problems seen as too delicate to discuss. ... Schools of education, either graduate or undergraduate, represent the academic slums of most any university. As such, they are home to the least able students and professors with the lowest academic respect. Were we serious about efforts to improve public education, one of the first things we would do is eliminate schools of education. (para. 4)

The Goals of Grades K-12 Mathematics

In my analysis, I captured comments that addressed writers' conceptions about what ought to be considered a goal of Grades K–12 mathematics. The ETs and the PCs identified just one goal, that is, the mastery of fundamental skills, of which standard algorithms are an important subset. In fact, every PC who revealed an opinion about the goals of school

mathematics—9 of the 13 authors—alluded to this view. Representing the ETs, the *United States Coalition for World Class Math* (2009) website (http://usworldclassmath.webs.com) posted, "The ability to use [standard algorithms] fluently provides the foundation for mathematical competence" (para. 1). Notably, the ETs' insistence that students master fundamental skills highlights the primacy of content knowledge in the traditionalists' vision of education. Budd et al. (2005), a group of ETs, explicitly supported this interpretation:

What is taught in math is the most critical component of teaching math. How math is taught is important as well, but is dictated by the 'what'. Much of understanding comes from mastery of basic skills—an approach backed by most professors of mathematics (row 4, column 2).

The ETs argued, however, that their vision was not one that replicates what has been done in the past. They wanted this content to be taught *well*. Loveless (2004) stated it as follows:

I am mystified when some analysts refer to a concern for arithmetic and computation skills as advocating "back to basics." Well, as an old elementary teacher, I am very concerned about American fourth graders learning arithmetic. A 50% proficiency rate is unacceptable. But I don't want to go back to anything. I want to go forward on the basics. Back to basics implies there was a golden age when everyone learned essential skills. That age has never existed. To ensure that every fourth grader is proficient at whole number arithmetic means that we must go forward, not backward. We must go forward on basic skills if a more equitable school system is a national goal; we must go forward if American students are to be prepared for higher level mathematics; we must go forward if young people are to master the skills correlated with middle class employment as adults. Back to basics is a bad idea. There is nothing to go back to but mediocrity and failure. It is time to go forward as a nation on basic skills. ("Why Important," para. 4)

In comparison to the ETs and the PCs, the MTs expressed a richer conception of the goals of school mathematics. The MTs' goals for Grades K–12 mathematics were framed by fundamental assumptions about learning. According to Quirk (n.d.), the MTs' assumptions were as follows:

- 1. Math is a man-made abstraction that only exists in the human mind or in written form.
- 2. There is an established body of math knowledge that different people can understand in the same correct way.

- 3. There is a stable foundational "K-12 math subset" that can be understood by different K-12 students in the same correct way.
- 4. K-12 math teachers can lead K-12 students to a correct understanding of K-12 math. (para. 1)

Similar to the ETs and the PCs, the MTs agreed that mastering fundamental skills and algorithms should be a goal of school mathematics, but for the MTs this goal was embedded in a larger vision of teaching and learning mathematics. In sum, the MTs believed the goal of Grades K-12 mathematics should be to move the student forward in his or her mathematical understandings. In their view, this is an iterative process: Learning involves an introduction to new topics, which may shed light on previous learning as well as form the basis for future learning. The MTs assumed students meet the introduction of new topics with varying levels of understanding. For example, at times the student may merely memorize an algorithm or definition, but at other times his or her initial exposure to a new concept will be accompanied by deeper understanding. In general, the MTs would say, the level of conceptual understanding the student attains when learning a new topic is determined by the amount of mathematical knowledge he or she currently holds when learning the new material and the connections he or she makes between the new topic and his or her currently held knowledge. Regardless, according to the MTs, mathematical understandings will develop as students progress in their learning, enabling them to fill gaps in their previous understanding. Quirk (n.d.) described the MTs' view this way:

- Progress is slow at the beginning:
 - o Each of us begins with no remembered math knowledge.
 - o Initially, the mind has no orientation information.
- Progress is faster and faster as the knowledge base grows:
 - As knowledge grows, the mind has an increasingly richer frame of reference.
 - o It becomes increasingly easier to build new knowledge on the already existing and ever expanding remembered math knowledge base.

- "Understanding" grows as the knowledge base grows:
 - o Newly acquired knowledge helps to clarify old knowledge.
 - O Example: A first grader needs to memorize 2 + 2 = 4, but a mathematically correct proof of this fact ... must be delayed until the child has acquired a richer math knowledge base.
 - Frequently we just need to memorize, to get the knowledge in our brain. Then the brain can do its magic, leading to what we call "understanding". Newly remembered knowledge is integrated with previously remembered knowledge and "understanding" evolves. It may happen instantly, or it may take years. ("Understanding the Process," para. 1)

This conception of learning mathematics sheds light on why the MTs may have been untroubled by a student's short term use of "parroting." In the view of the MTs, this response may be a necessity until the student has developed the knowledge to establish a better understanding of the concept. Quirk (2002) wrote, "There's another important fact about conceptual understanding: it deepens as newly acquired knowledge provides an increasingly richer frame of reference" (p. 4). Although the MTs recognized that parroting may be necessary at times because of the limitations posed by children's developmental capabilities, they preferred that children hold the mathematical tools necessary to form connections with, and see the reasoning for, new mathematical material. Therefore, the MTs would argue that for students to develop mathematically, they must commit previously learned material to memory. Absent memorization, they would argue, the iterative process fails.

The MTs' view of learning reflects their beliefs about the coherent and hierarchical nature of mathematics. To MTs, all the components of mathematics, from computational skills to mathematical reasoning, are vital to a solid understanding of mathematics. The website *Mathematically Correct* (n.d.) posted, "The truth is that in mathematics, skills and understanding are completely intertwined" (para. 4). In this view, the knowledge one gains through procedural fluency is as important as understanding the conceptual basis for the procedure. Given their belief that learning mathematics is an iterative process, the MTs wanted students to commit facts

to memory and master procedures to the point of fluency so that employing them in learning new material would not pose an undue burden on students' working memories. This fluency, said MTs, frees students cognitively so that they may focus solely on integrating the new material with what they already know. For example, Ocken (2007) wrote, "It's crucial to perform most procedures automatically, in order to free the mind for the study of higher level questions" (para. 6).

Like the ETs and the PCs, the MTs believed that students ought to learn standard algorithms in school. The MTs viewed algorithms as serving a double purpose: providing efficient methods that allow a student's attention to remain on the essential ideas at hand, and providing a foundation for future skills, some of which may not be seen until their college years. For instance, Quirk (2013) wrote:

The standard algorithms are efficient, general methods. For example, multi-digit multiplication only requires knowing single digit multiplication facts, single digit addition facts, and the idea of carrying. And carrying works the same for both addition and multiplication. Students can learn how to carry out this procedure automatically, freeing the conscious mind for higher level thought. ("Why are the Standard Algorithms," para. 1)

Raimi (2006) also argued:

Long division is a pre-skill that all students must master to automaticity for algebra (polynomial long division), pre-calculus (finding roots and asymptotes), and calculus (e.g., integration of rational functions and Laplace transforms.) Its demand for estimation and computation skills during the procedure develops number sense and facility with the decimal system of notation as no other single arithmetic operation affords. (para. 18)

The MTs' emphasis on procedural fluency and learning algorithms may have been a product of their view of how students learn mathematics; but it may have equally been motivated by their view of precision as an inherent property of mathematics. Algorithms and fluent computational skills privilege precision by streamlining student work and maximizing efficiency. Precision itself, however, was also explicitly stated as a goal of Grades K–12 mathematics education. In

particular, MTs argued for the development of precision in discourse, both written and verbal, and in computation. Milgram (2004) wrote, "Students must learn precision because if they do not, they will fail to develop mathematical competency. There is simply no middle ground here" (p. 24).

The MTs suggested that students should be exposed to mathematical proof during their school years but appeared to be conflicted about the depth and level of formality that should be expected. For example, in his interview with Leong (2013), Wu expressed modest expectations when he stated, "Even in school mathematics, people should be able to say, 'Here is the formula. You can use it to derive certain facts. I can give you the reason why it is true'" (p. 32). Yet, in a different writing, Wu (2010) suggested a fairly formal proof as an acceptable alternative for high school students. To properly represent the formality, the full proof is replicated here:

In America, the Most Frequently Asked question in school mathematics is "Why is negative × negative equal to positive?"

Mathematicians consider this to be obvious. They prove something more general: For any numbers x, y, (-x)(-y) = xy.

Proof: We first prove that (-x)z = -(xz) for any x and z. Observe that if a number A satisfies xy + A = 0, then A = -(xy). But by the distributive law, $xy + \{(-x)y\} = (x + (-x))y = 0$. Now let z = (-y), then we have (-x)(-y) = -(x(-y)), which by the commutative law is equal to -((-y)x) = -(-(yx)) = yx = xy. So (-x)(-y) = xy.

University mathematicians usually do not recognize how sophisticated this simple argument really is. It is not suitable for the consumption of school students.

The basic resistance to accepting negative \times negative = positive is a psychological one. If we can explain this phenomenon for integers (rather than fractions), most of the battle is already won.

We will give a relative simple explanation of why $(-2)(-3) = 2 \times 3$. The key step lies in the proof of

$$(-1)(-1) = 1$$

If we want to show a number is equal to 1, the most desirable way is to get it through a computation, e.g., if

$$A = (12 \times 13) - (6 \times 25) - 5$$
, then $A = 1$ because $A = 156 - 150 - 5 = 1$

But sometimes, such a direct computation is not available. Then we have to settle for an indirect method of verification. (Think of dipping a pH strip into a solution to test for acidity.)

So to test if a number A is equal to 1, we ask: is it true that A+(-1)=0? If so, then we are done.

```
Now let A = (-1)(-1). We have A+(-1) = (-1)(-1)+(-1) = (-1)(-1)+1 \cdot (-1)
By the distributive law, (-1)(-1)+1 \cdot (-1) = ((-1)+1)(-1) = 0 \cdot (-1) = 0
So A+(-1) = 0, and we conclude that A = 1, i.e., (-1)(-1) = 1
Now we can prove (-2)(-3) = 2 \times 3.
We first show (-1)(-3) = 3. We have (-1)(-3) = (-1)((-1)+(-1)+(-1))
which, by the distributive law, is equal to (-1)(-1)+(-1)(-1)+(-1)(-1) = 1+1+1 = 3
Thus (-1)(-3) = 3.
Then, (-2)(-3) = ((-1)+(-1))(-3) = (-1)(-3)+(-1)(-3). By what we just proved, the latter is 3+3=2\times 3. So (-2)(-3)=2\times 3.
Proof of (-m)(-n) = mn for whole numbers m, n is similar. (Slides 15–17)
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This argument, which Wu provided as a simpler alternative for high school students, suggests a more sophisticated level of proof than "I can give you the reason that it is true." A possible explanation for this apparent incongruence is that the MTs expected teachers to present somewhat formal proofs for their students, but the students themselves were to be held to lower expectations in producing their own proofs. Regardless, the MTs clearly viewed proof as an important element of Grades K–12 mathematics education. Klein (2005) argued that reasoning (i.e., proof) is the lifeblood of mathematics and that its absence in school mathematics is the cause of students' failure to form a cohesive understanding of mathematical concepts. Similarly, Allen (1997) wrote, "Proof, properly introduced, DOES NOT make mathematics more austere, forbidding and difficult. On the contrary, it can be an exciting game which provides the only path to understanding" (p. 3).

How Grades K-12 Mathematics Goals Should Be Achieved

In my analysis, I coded comments that indicated how the author envisioned the goals of school mathematics could or should be achieved. The three communities of meaning agreed with each other in the belief that student practice and direct instruction are effective instructional practices although their support for, and defense of, these instructional practices varied. The ET and the MT communities suggested additional effective practices that the PCs did not; however, the PCs did not make comments that explicitly or implicitly disagreed with the other

communities' suggestions. The PCs' relative inattention to how mathematics should be taught in school is a reflection of their primary lens, for the PCs were less preoccupied with conceptions of school mathematics than they were with how changes to traditional practices might conflict with their beliefs or values. In the following, I summarize the three communities' conceptions of student practice and direct instruction, as well as the additional suggestions made by the MTs and the ETs.

Practice. The MTs' most repeated pedagogical recommendation was that students be provided ample opportunity to practice skills and algorithms, particularly computation and algebraic manipulations. Allen (1997) claimed teachers must "recognize the indispensable role of well-planned practice, repetition, review and drill in the development of the work and study habits that are necessary for the acquisition of mathematical knowledge" (p. 5). Milgram (2005) pointed out that students must also practice moving from the concrete to abstract, as that is where the power of mathematics resides. The MTs believed students' early years should be devoted to practicing and memorizing the skills that would enable them to engage in procedures automatically and to recall facts and definitions easily.

The PCs similarly and unabashedly recommended extensive student practice. As an example, Birkett (2012), a PC, simply stated, "One thing that has become clear through the Math Wars is that the drill-and-practice method works extremely well for virtually all students" ("Assessing Curricula," para. 8). In contrast, although the ETs suggested that practice was an important element of learning mathematics, they were careful not to promote drill. Carson bristled under the suggestion that her "emphasis on math algorithms might result in the drill and kill approach" (Lehman College Multimedia Center, 2011, 03:51), emphatically stating, "First of all, there's no one advocating for a program that is nothing but drill. That's not even close. ...

[But] the fact is children do need to practice" (03:52). Instead, the ETs' descriptions of practice focused on their conception of an incremental approach to learning. Pappas (2003) exhibited such a view when he wrote:

Accurate recall will remind you that learning to read did not happen overnight (how many years was it?). In the beginning you had to learn the alphabet symbols, the sound for each symbol, how to pronounce combinations of symbols (words), and on and on. It was not easy. However, you learned by doing. The more you read the better you could read. In this sense learning mathematics is not different nor more difficult than learning to read. (p. 5)

Notably, Pappas's analogy highlights his traditionalist lens, in which the process of learning to read was a matter of dissecting the task into small chunks and working bit by bit, via practice and memorization, toward the ability to read.

Direct instruction. Both the ETs and the PCs were unreserved in their recommendation of direct instruction as an effective instructional practice and both the ETs and MTs explicitly called for teacher-centered classrooms. The MTs, however, expressed defensiveness about their support for direct instruction. Wu (1999) posted a 15-page unpublished manuscript to his university webpage supporting the traditional lecturing format. Quirk (2002) wrote, "Today's educationists deplore 'teaching by telling', but eliminating telling changes the very definition of 'to teach'. How and when to tell may be debatable, but not telling is malpractice" (para. 7). In describing their view of direct instruction, the MTs countered the usual picture of a disengaged teacher lecturing at the front of rows of nonparticipating students. Instead, the MTs' vision was one in which teachers actively engaged their students via questions and solicitations for ideas. For example, Allen (1997) suggested, "Proofs should be constructed in class with teacher and students cooperating in the construction" (p. 10), and Quirk (n.d.) advised teachers to provide "examples, examples, examples. Continually ask questions to test understanding. Give

immediate and constant feedback" (para. 7). In an interview with Sriskandarajah (n.d.), Askey couched his advice within a story about his own experience as a learner:

The teacher started by saying that we would be learning how to solve some equations we did not know how to solve, and gave an example something like $x^2 + 4x + 5 = 0$. I raised my hand and said the solutions were $x = -2 \pm i$. He looked surprised and asked me where I had learned this. My answer was a good illustration of how to teach, tell the truth but not the whole truth [emphasis added]. I said I had learned it in a different school system. I did not say that it was an eighth grade class, which it was. (p. 1)

The MTs' defensiveness about their support for direct instruction was most likely a response to their perception that it was disparaged in education; yet their adamant support of it reflects their traditionalist educational views. On the other hand, it is possible that their support of direct instruction was partly attributable to their conflicted views of the role of proof in Grades K–12 mathematics. In particular, recognizing that elementary school students (and perhaps even older students) are likely not developmentally ready for formal proof, the MTs may have embraced direct instruction as the method by which students could still engage in it.

Technology. The PCs did not express a common opinion about technology in the classroom. The ETs and the MTs, however, both expressed reservations about the use of calculators in school classrooms. They believed that the use of calculators should be limited or nonexistent so that students would develop competence in solving problems without them. In particular, they worried that by encouraging calculator use too early, arithmetic competency could be compromised. The MTs guardedly supported the use of other technology in the classroom, recognizing the affordances of technology in terms of mathematical exploration and student motivation. Yet they cautioned against students viewing technology as a mathematical authority and worried that technology such as Geometer's Sketchpad® "tend to blur the distinction between illustration and proof" (Allen, 1997, p. 15).

Progression through content. To avoid the misconception that they were promoting pedagogy in which students practice and memorize a series of skills and facts in a disconnected fashion, individual MTs wrote extensively about how certain mathematical topics should unfold within school mathematics. In doing so, they modeled their vision of how students could form the connections and understanding that a devotion to learning basic skills would enable. For example, Raimi (2004) provided this example of how students could develop an appropriate understanding of why we "invert and multiply" when dividing fractions:

Two fractions are equivalent if they represent the same number. Thus 3/4 is equivalent to 15/20, for example, or to 6/8. Anyone who uses fractions at all must understand these equivalences, and the very idea of "percentage" is of course a matter of finding, for a fraction, an equivalent fraction with denominator 100. The way this is done is by multiplying numerator and denominator by the same thing. Now, well before introducing "invert and multiply", the problem 5 divided by 3/4 can be solved with the use of equivalent fractions: 5, or 5/1 if you like, is equivalent to 20/4, and the result of dividing 20/4 by 3/4 may be construed as a fraction with numerator 20/4 and denominator 3/4, which is equivalent to 20/3, or 6 and 2/3. A middle school student should, in any good program, be given a good work-out in finding and recognizing equivalent fractions, in multiplying fractions (which is easily reduced to multiplication of integers, with a resulting single fraction), and then in the use of these techniques to change a fraction with fractional numerator and denominator to a fraction with integral numerator and denominator. When the time comes to speak of fractions more generally, "a/b" instead of some particular numerical example, it will be discovered without effort that this procedure, already understood from earlier numerical examples, produces the computation

(a/b)/(c/d) = (ad/bd)/(cb/db) = ad/cb, which is the result of "invert and multiply", that is, $(a/b)/(c/d) = (a/b) \times (d/c)$, or ad/cb. This result is a theorem, something to prove once and for all and then put away for future mindless use, the way we end up memorizing and using $6 \times 9 = 54$ without having each time to draw a 6-by-9 rectangle and count squares. ("Is Today's Doctrine," para. 8)

According to Raimi, each concept follows naturally from what came before, and therefore students would build upon what they had previously committed to memory. In their own way, the ETs echoed the MTs' suggestion that an appropriate progression through the content is essential to student learning. In particular, the ETs believed that teachers and students should begin with a focus on basic skills and procedures, particularly standard algorithms, and move

"step by step" (Stotsky, 2009, para. 9) to higher levels of formality and applications of those procedures. In the ETs' view, procedures were a necessary prerequisite to problem solving.

Carson said,

In basic arithmetic the purpose of these procedures is that you get to the point where you don't have to think about it anymore; so you can go on to levels of mathematics where it becomes a lot more interesting and it becomes more creative and wonderful in the applications. (Lehman College Multimedia Center, 2011, 01:50)

The MTs' unique views. The MTs believed that the content knowledge of the teacher was a major determining factor in whether school mathematics goals would be achieved. In their view, teacher content knowledge may be the most important element of Grades K–12 mathematics education. Quirk (n.d.) stated, "Know math yourself. You can't teach math if you don't know math. You should know the underlying 'whys' and how to build math knowledge" (para. 7). This assertion is perhaps unsurprising given that the MTs' primary lens was their view of mathematics.

According to the MTs, teachers in the United States have traditionally been underprepared in content knowledge, and they suggested that this might be the primary reason for poor student achievement in mathematics. In fact, the MTs suspected that a preoccupation with pedagogy in education had contributed to the problem. Schmid (2000) exclaimed, "Teacher training in America has traditionally and grossly stressed pedagogy over content. The implicit message to the teachers: if you know how to teach, you can teach anything!" (para. 13).

Additionally, Wu (2011b) wrote, "What must not be left unsaid is the obvious fact that, without a solid mathematical knowledge base, it is futile to talk about pedagogical content knowledge" (p. 381). The MTs recommended that prospective teachers receive focused instruction in the mathematical content they would teach, as well as content their students would encounter a few years later. This training, they argued, should be provided through mathematics departments

rather than mathematics education or education departments in order for teachers to develop an accurate sense of what mathematics is. Wu stated, "The cumulative gap between what (research) mathematicians take for granted as mathematics and what teachers and educators perceive to be mathematics has caused enormous damage in mathematics education" (p. 382).

The MTs disdained alternative assessments, group assessments, and assessments that measure what they viewed as nonmathematical outcomes, such as mathematical disposition or the ability to cooperate. Instead, they asserted that student success should be measured by "objective tests" (Quirk, n.d., "The NCTM Calls Them Standards," para. 2) that are "fair and accurate" (Raimi, 2002, para. 10). These tests, said the MTs, should be designed to diagnose students who are falling behind as well as identify exceptional students. Raimi (2002) described it as follows:

Galton once said that in comparison of intellect, Isaac Newton was to the man in the street as the man in the street was to his dog. We must identify the "village Milton" if we can, while advancing the sturdy yeoman at the same time. Pretending there is no difference is hurtful to them both. We need examinations much more demanding at the high end than those we are seeing now, and we need minimal scores that are sufficient but not attainable by luck. (para. 9)

The Conceptions of School Mathematics Reforms

My fourth research question sought to determine how each community of meaning characterized school mathematics reforms and to what elements of these perceptions they objected. Because the documents in my data set were specifically written to address the school mathematics reforms, it is unsurprising that the bulk of authors' comments addressed this question. A more interesting observation, however, is on what aspects of the reforms each community of meaning focused. As can be seen in Table 3, the majority of the MTs' comments concerned the content and pedagogy of the reforms, which evidences that their view of

mathematics served as their primary lens. To the MTs, the mathematics content and how mathematics was portrayed to students were important considerations.

Table 3

Number of Coded References for Each Community of Meaning in Categories of Conceptions of School Mathematics Reforms

Community of Meaning	Impetus for and Goals of Reform	Content of Reform	Pedagogy of Reform	Assessment in Reform	Costs of Reform
Math Traditionalists $(n = 53)$	93	122	90	16	30
Education Traditionalists $(n = 21)$	41	66	67	11	43
Political Conservatives (n = 25)	87	50	68	15	37

Note. n = number of documents in community of meaning.

Table 3 shows the ETs' broader approach to the reforms, reflecting their primary lens, which was a traditional view of education. Their attention was on content and pedagogy; however, compared with the MTs and, to a lesser extent, the PCs, they dwelt heavily on the potential costs of the school mathematics reforms. This emphasis might have been expected given that traditionalists believe that their vision of education is necessary for the individual and society.

The PCs focused primarily on the impetus for reforms and the reform goals. We can see that the PCs' number of comments in this category was twice that of the ETs' and nearly equaled those made by the MTs in twice the number of documents. As I discuss below, the PCs' primary

lens is evidenced in their attention to this category. In the following, I will more fully describe each of these categories and each community of meaning's conceptions within each category.

The Impetus for and Goals of School Mathematics Reforms

In my analysis, I coded comments that revealed the authors' conceptions of the motivations for school mathematics reforms and to whom they attributed their origins. These comments were often closely connected to their perception of the goals of the school reforms, which I also captured and coded. All three communities attributed school mathematics reforms at least partially to NCTM and its documents, and they frequently cited those documents as evidence for their assertions regarding the characteristics of reforms. According to Klein (2007), an MT,

In a long series of documents published by the NCTM, three have been especially influential: An Agenda for Action (1980), Curriculum and Evaluation Standards for School Mathematics (1989), and Principles and Standards for School Mathematics (2000). The latter two are referred to respectively as the 1989 NCTM Standards and the 2000 NCTM Standards, or just Standards when the context is clear. (p. 23)

When speaking about Grades K–12 mathematics education reforms, the communities often blurred the line between mathematics education researchers, education specialists, and curriculum developers, lumping them together as advocates for reforms, often under the banner of the NCTM. The PCs, however, appeared to be more aware than the ETs and the MTs that the NCTM was an organization of mathematics stakeholders.

All three communities also expressed the belief that the constructivist learning theory motivated school mathematics reforms. According to Vukmir (2001), a PC, constructivism holds that "children understand and learn only those concepts that they 'construct' or discover on their own" (para. 5) and Wu (1998), an MT, described constructivism as follows:

Roughly, [constructivist learning theory] is the education philosophy which holds that the acquisition of knowledge takes place only when the external input has been internalized

and integrated into one's own mind. Thus learning requires the construction of a mental image in response to the external input. So far so good, except that current proponents of constructivism go further and stipulate that classroom time should be used for the students to re-discover or re-invent the concepts or the methods of solution in order to help along this mental construction. (p. 8)

Additionally, all the communities characterized the school mathematics reforms as following a progressivist philosophy of education. Garelick (2005), an ET, may have expressed the view of all three communities of meaning by attributing the reforms to NCTM, progressivism, and constructivism in this way:

The NCTM standards were a brew of progressivism—a nod to the 1920s when math was supposed to be practical—and constructivism, which was progressivism that adapted research from cognitive psychology to the task of teaching and called it discovery learning. ("Some Secrets," para. 1)

Both the MTs and the ETs suggested that the reforms were based on the assumption that mathematical processes are more important than mathematical content. For example, Garelick (2007), an ET, complained:

[NCTM-advocated] standards de-emphasize learning basic skills, relying instead on "strategies for learning". They are informed by a dubious educational theory and philosophy that holds that understanding the big picture concepts of math builds the foundation upon which students learn math facts and develop skills. (para. 5)

The ETs, in particular, believed that the primary goal of reformed Grades K–12 mathematics was to develop these processes, which they claimed was hidden beneath "a brand-new objective—conveniently indefinable and immeasurable—called 'deep conceptual understanding'" (Stotsky, 2009, para. 3). The ETs expressed frustration with the perception that reformers were laying claim to conceptual understanding, a perception for which they had some evidence. For example, in an interview, Carson was asked, "What's so terrible with children having a conceptual understanding?" Carson's exasperated response suggested that that was not the first time she had encountered the assumption that traditional mathematics education ignored conceptual

understanding. She emphatically stated, "Of course we want conceptual understanding! There's no one who doesn't want conceptual understanding" (Lehman College Multimedia Center, 2011, 01:50). Garelick (2013) similarly expressed defensiveness at what he perceived was an accusation that traditional methods preclude conceptual understanding:

Among the many arguments about "balance", in the end it all comes down to "understanding". The reform camp argues that they do in fact teach procedures—they just teach them with meaning, so that students can understand what they are doing. The implication is that those on the traditional/classical side of teaching math do not teach procedures with meaning—i.e., "understanding." (para. 5)

The ETs' perception of reforms' claim to conceptual understanding, as well as their accusation that reformers had used the term to obscure the real goals of school mathematics reforms, deviated from that of the MTs, who spent little time discussing conceptual understanding apart from their assertions of what it means for a student to understand mathematics. Although the MTs acknowledged the contention that the reforms were meant to foster students' conceptual understanding, their comments concerning the goals of reforms were focused more on other areas. Similarly, the PCs acknowledged that a claimed goal of Grades K–12 mathematics reform was the conceptual understanding of mathematics, but their comments focused more on other perceived goals. Whereas the ETs expressed irritation at the reformers' apparent monopoly of the claim to conceptual understanding, the PCs merely dismissed the claim as untenable. For example, Schlafly (2006) wrote, "This silliness is based on the false notion that children can develop a deeper understanding of mathematics when they invent their own methods for performing basic calculations" (para. 4)

The MTs and the PCs agreed that a motivation for Grades K–12 mathematics reforms was a desire to increase success in mathematics by traditionally disadvantaged students; therefore, they believed, a goal of mathematics education reforms was to close what had

commonly been called the achievement gap, that is, to eliminate the discrepancy in achievement between students based on socioeconomic status, gender, and race or ethnicity. However, the MTs and the PCs perceived that in order to increase the success of disadvantaged groups, reformers simply reduced the expectations for all students. Klein (2007), an MT, stated,

The NCTM reform was an attempt to redefine mathematics in order to correct social inequities. To make mathematics more accessible to minority groups and women, progressive educators argued for programmes that eliminated basic skills and the intellectual content that depends on those skills. (p. 32)

The MTs, but not the PCs, responded to their interpretation that reforms reduced expectations by accusing reform advocates of sexism and racism. In the MTs' view, the reformers were operating on the assumption that certain subgroups are incapable of real mathematical achievement, and therefore the reformers sought to redefine what mathematical achievement is. Although the MTs readily admitted that some groups had not achieved well in the past, they attributed this phenomenon to other reasons, such as the poor content knowledge of teachers or improper sequencing of ideas.

The MTs identified a closely related, yet different, motivation for school mathematics reforms. They believed a goal of reform was for *all* students to be successful, not just the disadvantaged. Therefore, the MTs asserted that in order to ensure that goal, the reformers had devised nonmathematical objectives that students could be said to achieve. An example of this view is a comment posted to a mathematics discussion board in reference to a mathematics education research study that was challenged by a few MTs as being inaccurate and biased (the comment, and the entire discussion thread, was subsequently removed by the website moderator). Hansen (2013), although not an MT, followed the resulting controversy and captured the views of many MTs with his tongue-in-cheek post:

I was hoping for something like the following from [the researcher].... "I realize that the reader might infer from my conclusion and comments in my original study that the students ... were passing, or even doing very well, in these advanced classes. Heaven's no! Nothing could be further from the truth. Almost none of these students did well enough to even pass. Indeed, they were failing. But that wasn't what my study was about. My study was about showing that if we adopt an open and encouraging environment in mathematics education then we can achieve a semblance of success in mathematics. I realize that most readers are puzzled and will ask why we would want a semblance of success and not actual success, and more importantly, why I didn't point this detail out in my original study? This is because my study was not about mathematics, not in the common sense, it was about equality. But in order to achieve a semblance of equality we must establish a semblance of success in mathematics. Now, I don't have an answer to the question "Why does equality, or even a semblance of equality, have anything to do with mathematics?" But if equality can be reduced to a semblance of equality based on a semblance of success in mathematics then my study shows a possible way of achieving that semblance." (para. 2)

In addition to contriving "phantom" areas of mathematics success, the MTs believed reformers strove towards a goal of fostering positive student dispositions toward mathematics. Raimi (2004) observed, "There is actually an easy way to succeed: The way to guarantee an education available to everyone is ... to make it easy, and to make it fun" (para. 6). Wu (1998), however, described why that goal troubled the MTs:

A battle cry of the reform is "Mathematics for all!" In an attempt to make this come true, there is presently a conscientious effort to spread the news that "Math is fun!" While applauding the good intention, we nevertheless must ask whether the constant repetition of this slogan like a mantra helps students learn mathematics. Have students been told that this kind of "fun" includes the fun of working hard to solve difficult problems? Nothing good comes cheaply, and learning mathematics is no exception. We owe it to the students to tell them honestly about the hard work needed to learn mathematics. (p. 10)

MTs' concerns arose out of their belief that the reformers lacked the content knowledge necessary to have an accurate picture of what mathematics is and therefore what constitutes success in it. As evidence, the MTs highlighted mathematical errors in what they characterized as reform documents, curriculum materials, and mathematics education research studies, and they expressed general skepticism about relying on reformers to make decisions about Grade K–12 mathematics:

[Reformers'] "vision" is limited to the needs of everyday life. The concept of prerequisite knowledge is never mentioned. They're not concerned with setting the stage for learning more advanced math. Many of their high school math examples belong at the elementary school level. They claim to emphasize conceptual understanding, but give no evidence that they understand how math ideas are connected. They appear blind to the vertically-structured nature of the math knowledge domain. (Quirk, 2002, "How the NCEE Redefines Conceptual," para. 14)

The examples of mathematical errors offered by the MTs were most often instances in which the posed problem or expected solution suffered from ambiguity or a lack of precision. Pattern problems were often cited as examples in which a lack of precision was problematic. Milgram (n.d.c) complained:

The fault is in the whole idea of expecting an answer to a problem of the form "continue this pattern". There are no incorrect answers to such a problem, for a pattern can be continued any way one likes with equal justice—unless the RULE for the pattern is made explicit and unambiguous. (p. 6)

In these cases, the MTs insisted that the reformers' expectation for particular answers to these pattern problems was mathematically erroneous. Other examples included problems in which students were to make mathematical inferences based on a graph or table. Milgram (n.d.d) argued, "What actually happens is that students are aware of the popular—but mathematically ridiculous—assumption that if it appears the graph has an interesting point that seems to be near an integer... then it is, in fact, integral" (p. 8). The MTs found these types of mathematical errors to be so common that Wu coined it the *wishful thinking syndrome*: "Give out partial information and students will automatically fill in the missing information to achieve a complete conceptual understanding on their own" (Milgram, n.d.d, p. 20). The MTs complained that the ramification of implying that such problems had definite answers would be that students might develop misconceptions about what mathematics is and can do.

The PCs expressed some of their own unique interpretations of the impetus for and goals of school mathematics reforms. The PCs believed that a primary driver of reforms was the

developers' liberal ideology. In fact, of all the motivations for reforms mentioned by the PCs, this claim was repeated the most often. As an example, Korol (2013) predicted, "Direct teacher instruction will be replaced by self-directed learning, group-think, with emphasis on subjectivity, feelings, emotions, beliefs, multiculturalism, political correctness, social engineering, globalism, sexual freedom, environmental extremism, victimization, moral relativism, and redistribution of wealth" (para. 3). The PCs reacted negatively to the idea of infusing social justice into mathematics instruction, based on their conception that the term *social justice* was equivalent to liberal ideology. Olson (2011) wrote:

Those of us who attended public schools before "social justice" spread through the curriculum like a bad infection probably remember sitting in math class and working through problems such as this one:

Leroy has one quarter, one dime, one nickel, and one penny. Two of the coins are in his left pocket and the other two coins are in his right pocket. The coins have been randomly placed in the two pockets.

What is the probability that Leroy will be able to purchase a 30-cent candy bar with the two coins in his left pocket? Using the coins, explain your reasoning.

We didn't know it at the time, but while we busily charted all of Leroy's different coin combinations, we were actually being taught to tacitly approve of America's exploitative capitalistic system. (para. 1-4)

Olson then claimed that in the K–12 mathematics reforms, the goal would not simply be the correct mathematical answer to the problem:

The average American "oppressor" would say that the correct answer to sample problem is, "Leroy has a one-in-three chance of having the right combination of coins in his pocket to buy the candy bar."

But according to the social justice crowd, the correct answer should be, "Leroy is contributing to the oppression of the cocoa bean pickers of the world by purchasing a non-fair trade candy bar." (Students who suggest charging Leroy with a hate crime would be given extra credit.) (para. 8-9)

The PCs suggested that states, districts, schools, and even teachers had been pressured or tricked into adopting the Grades K–12 mathematics reforms. For example, Norton (2005) warned

that "teachers have been fired for refusing to teach the [reform math] program" ("A Few Facts," para. 11). Cheney (1998), writing during the Clinton presidency, reported:

A dozen members of Congress have sent a letter to President Clinton expressing their disapproval of the NSF's [National Science Foundation's] attempt to influence the California State Board of Education. "To use the hammer of possible withdrawal of federal funds to force a state into compliance with unproven practices is unconscionable," they write. (para. 11)

Therefore, to the PCs, the motivation for Grades K–12 mathematics education reforms was perhaps sinister. Already wary of educational changes, the PCs found evidence that supported their suspicions that the reforms were designed around an agenda with which they did not agree.

Based on their assumptions of the motivation for the reforms, it is perhaps unsurprising that some PCs believed that a goal of the Grades K–12 mathematics reforms was to "indoctrinate students with a pre-planned social agenda" (Chapman, 2013, para. 15). In particular, some PCs posited that the reforms were designed to create a wedge between parents and their children. Norton (2011) warned:

It is a very real possibility that ... all these other educators have embraced constructivist math not only for the social engineering aspects, but because it's another barrier between parent and child. Parents don't know how to do this method of math, so they may figure that it will serve to separate the parents a little further from their children and get children to believe that their teacher at school is the source of knowledge they should turn to. (para. 9)

The Content of School Mathematics Reforms

In my analysis, I coded comments that characterized or expressed an objection related to the mathematics content in Grades K–12 mathematics reforms. All three communities expressed the common interpretation that the reforms de-emphasized basic skills, including algorithms, and they all characterized the reforms as "dumbed down" school mathematics. Raimi (2004), an MT, wrote:

The [reform] is briefly described as a galloping anti-intellectualism, a "dumbing-down" of the curriculum for all students—but in the name of improved "understanding". The

pretentious phrase "higher-order thinking skills", so prevalent in the education community today, is about as good a symptom as one can find: It permits, indeed insists on, the avoidance of mathematics itself. It pursues "real education", stuff that goes with words like "today's world" and "the new century", replacing or adding to the progressive education cant of sixty years ago, where "vibrant" and "rich" were the adjectives describing an empty education program. (para. 1)

The ETs, in particular, were troubled by their perception that the reforms "neglected the teaching of standard algorithms ... insisting instead on the value of student-developed algorithms" (Stotsky, 2009, para. 9). Garelick (2013) wrote,

I wouldn't be quite so much against the strategies that are taught in lieu of the standard algorithms, if they were used to help explain how the standard algorithms work, thus effecting the "understanding" the reformers claim to be so concerned about. In actual practice, however, the alternatives are left by themselves. Students are left to work with partial sums and partial products (and other methods such as the lattice method for multiplication) in the early grades, and do not learn the standard algorithms until 4th and 5th grades. ("Balance," para. 7)

In the view of both the ETs and the MTs, acquiring a foundation of basic skills was critical to the students' future learning of mathematics. Raimi (2004), an MT, argued,

Mathematics is intended to make hard things easy by accumulating the discoveries of our ancestors and passing them on to the young in organized form. Today's doctrine in the schools denies this, by requiring a constant return to first principles as evidence of "understanding." What turns out to be true is that such teaching makes mathematics difficult in the long run, not easy; but school is not the long run. This particular NCTM ... prescription has made things look easy by omitting the truly important knowledge that makes them truly easy. ("Is Today's Doctrine," para. 11)

In the opinion of the ETs and the MTs, de-emphasizing basic skills and algorithms meant student achievement would suffer.

Whereas the ETs focused exclusively on a de-emphasis of basic skills when considering the content of reforms, the MTs included many other aspects of content. The ETs' singular focus on basic skills is perhaps attributable to their less sophisticated view of mathematics as a domain. Given that traditionalists prioritize content, the ETs objected to the content changes they could identify, which was naturally less than what an academic mathematician could notice. When

perceiving these changes through their traditionalist lenses, however, both the ETs and the MTs concluded that what they believed to be a progressivist handling of basic skills and algorithms would be harmful to student learning.

In contrast, the PCs simply complained about what they perceived as a de-emphasis on basic skills without providing a reason for why they found this problematic. For example, Malkin (2007) wrote, "Are you worried that your third-grader hasn't learned simple multiplication yet? ... Join the club" (para. 1). Instead, the PCs derided the content changes, implying that they were the products of liberals acting on their unrealistic views of human nature, their desire for equal outcomes, or their belief that all cultures are worthy of celebration. For example, McNew and McNew (2010) wrote, "Human beings have been performing simple math since hunter-gatherers realized they had digits and things that needed to be counted. Only a starry-eyed progressive fool would attempt improvement upon methods of simple addition and subtraction" (para. 21). The PCs' strong reactions reflected their assumption that the reforms advanced an ideology contrary to their most fundamental beliefs. As another example, Malkin (2010) wrote:

Longtime readers of this blog are familiar with my critiques of Fuzzy Math, New Math, New New Math ... and every other social justice-tainted effort by educrats to corrupt and undermine rigorous math education in this country. Today's stupid education fad of the day?

"Mayan Math." I kid you not. (para. 1)

As discussed earlier, to conservatives, non-American cultures may or may not be good. Therefore, they are hesitant to celebrate unknown cultures, preferring instead to direct attention to the "goodness" of American culture. The PCs objected to what they perceived to be a preoccupation with celebrating diverse cultures at the expense of learning mathematics. In a review of *Glencoe Pre-Algebra*, Van Court (1999) wrote:

Like most of today's schoolbooks, Glencoe Pre-Algebra is full of ostentatious items that reflect the fad for promoting awareness of "cultures." The Glencoe writers have contrived a host of these things, which we can describe by borrowing a term from the art critic

Robert Hughes: ethnokitsch. Most of Glencoe's ethnokitsch items are trivial and superficial, and they are significant only because they take up space, clutter pages, and create distractions. ("Ethnokitsch," para. 1)

Although the PCs rarely offered specific reasons for their objections to the content in reforms, they might have believed that more specific objections were unnecessary because they had usually already made the case that the motivation for and goals of reforms were suspect.

The bulk of the MTs' writing about school mathematics reforms concerned content. In addition to their general agreement with the ETs and the PCs that the reforms de-emphasized basic skills, they asserted that the reforms focused instead on problem solving or "understanding," allowing students to use calculators or other technology to perform computations when necessary. Schmid (2000), an MT, characterizing reforms and what he believed were reformers' arguments, wrote:

Mathematicians are perplexed, and the proverbial man on the street ... appears to be perplexed as well: improve mathematical literacy by downgrading computational skills? Yes, precisely, say the reformers. The old ways of teaching mathematics have failed. Too many children are scared of mathematics for life. Let's teach them mathematical thinking, not routine skills. Understanding is the key, not computations. (para. 4)

But the MTs were unmoved by their interpretation of reformers' arguments. Schmid continued:

Mathematicians are not convinced. By all means liven up the textbooks, make the subject engaging, include interesting problems, but don't give up on basic skills! Conceptual understanding can and must coexist with computational facility—we do not need to choose between them! (para. 5)

The MTs also lamented a de-emphasis on algorithms for less pragmatic reasons. Quirk (2013) wrote, "They refuse to acknowledge the elegant design of the standard algorithms. And they're blind to what is lost" (para. 12).

Whereas the ETs' and the PCs' observations concerning the content of school mathematics reforms were confined to basic skills and algorithms, the MTs offered more sophisticated and complex discussions concerning content. Specifically, the MTs made

comments concerning precision and proof in school mathematics reforms. Below, I describe their conceptions of each.

Precision. The MTs complained about a lack of precision in school mathematics reforms. Their views fell into two qualitatively different interpretations of precision. First, the MTs expressed concern about a lack of precision as related to correctness. As discussed earlier, the MTs expressed reservations about reformers' mathematical content knowledge and suggested that this deficit led to documents, curriculum materials, and research studies rife with mathematical errors. Klein (2006) held up this example, which he claimed was taken from a seventh-grade quiz in materials from the *Connected Mathematics Program* (CMP), a program the MTs associated with reform:

Problem: Find the slope and y-intercept of the equation 10 = x - 2.5.

Solution: The equation 10 = x - 2.5 is a specific case of the equation y = x - 2.5, which has a slope of 1 and a y- intercept of -2.5. (para. 1)

Klein complained that "students instructed and graded in this way learn incorrect mathematics, and teachers who know better may be undermined by their less informed peers, armed with the 'solution'" (para. 2).

Second, the MTs expressed concern about a lack of precision as it relates to formality. Wu (1998) wrote:

A second area of concern in the current reform is the fuzzification of mathematics. Precision is a defining characteristic of our discipline, but the present tendency is to move mathematics completely back into the arena of everyday life where ambiguity and allusiveness thrive. (p. 6)

This interpretation of Grades K–12 mathematics reforms was responsible for the MTs' tendency to refer to those reforms as "fuzzy math." As discussed earlier, the MTs viewed precision as an

important characteristic of mathematics; therefore, in their eyes, the lack of attention to this element would undermine a students' understanding of what mathematics is.

Proof. As discussed earlier, the MTs viewed proof as an important aspect of mathematics, but they were ambivalent about the role it should play in Grades K–12 mathematics. When considering school mathematics reforms, however, the MTs were decidedly dissatisfied. Specifically, the MTs complained that the reforms had minimized the role of proof in Grades K–12 mathematics classrooms to an unacceptable level. Allen (1997) wrote:

During roughly the first half of this century, [an] emphasis on proof-directed exposition would have been considered redundant by experienced teachers. In presenting proof they were seeking CLARITY IN EXPOSITION rather than pretentious rigor. Today, teachers trained in the last ten years in our Schools of Education may reject it as too rigorous, too austere, too "mathematical." (p. 15, emphasis in original)

The MTs warned that the reforms blurred the line between empirical arguments and mathematical proof and would therefore fail to instill in students any sense of what constitutes mathematical proof. For example, Wu (1998) wrote:

The cavalier manner in which the reform texts treat logical argument is nothing short of breath-taking. Heuristic arguments are randomly offered or withheld and, in case of the former, whether these are correct proofs or not is never made clear. Rather than being the underpinning of mathematics, logical deduction is now regarded as at best irrelevant. (p. 3)

Wu's sentiment underscores the MTs' concern that a lack of precision regarding proof could undermine students' understanding of what mathematical proof is. Additionally, the MTs' warnings about students developing an empirical view of proof were related to their claims that reform materials were full of mathematical errors; in particular, the MTs worried that the reformers might hold an empirical view of mathematical proof. Therefore, the MTs' concerns about proof were linked to their concerns about precision in regards to both formality and correctness.

The Pedagogy of School Mathematics Reforms

In my analysis, I coded comments either characterizing or objecting to the pedagogy of school mathematics reforms. Looking across the communities of meaning, all three characterized the reforms as emphasizing group work and the use of calculators and manipulatives, while deemphasizing memorization and practice. They also shared the perspective that the school mathematics reforms emphasized a discovery, or inquiry, approach and learning through problem solving. Clopton (2000), an ET, captured the concerns of all three groups when he claimed, "Time for mathematics, both in class and at home, is seriously limited and must be used as efficiently as possible. These activities are inefficient learning methods" (para. 10). Pride (2008), a PC, was blunter: "To develop math to its present form took thousands of adult mathematicians a millennium and a half. One kid can't be expected to work it out in six years of elementary school" ("Too Young to Derive," para. 1). Notably, the MTs and the ETs characterized these pedagogical practices as student-centered, reflecting their association of the reforms with progressivism.

The most pronounced difference between the PCs and both the ETs and MTs was the PCs' focus on the role of correct answers in the reforms. The PCs likened the reforms' deemphasis of correct answers to moral relativism. To conservatives, denying the existence of truth (e.g., *correct* answers) in the physical or abstract world is the same as denying the existence of moral truth. Therefore, the PCs viewed the de-emphasis of correct answers as part of a liberal agenda to cast doubt on any claim to absolute truth. Chapman (2013) complained, "Teachers are no longer supposed to teach a body of knowledge (i.e. 'facts' and 'truth'), but are now facilitators charged with helping students construct their own realities" (para. 6). The PCs suspected that in constructing those realities students would be directed to depend on their

emotions—which, as previously discussed, conservatives believe to be untrustworthy—to develop their conceptions of right and wrong. The PCs further believed the de-emphasis of correct answers was a product of liberals' tendencies to distance individuals from the consequences of their actions. Malkin (2008) wrote, "Too many teachers are too busy bloviating about the self-esteem benefits of Everyday Math to bother with the basics. 1 + 1 = I feel good about math, so who cares?" (para. 3).

The MTs' unique views. The MTs differed from the ETs and the PCs by painting a detailed and connected picture of the pedagogical practices of reforms. In their descriptions they outlined the arguments they believed the advocates of reforms were offering in support of those practices followed by their rebuttal to those arguments. Additionally, the MTs acknowledged criticisms of traditional mathematics education by voicing what they believed were reformers' concerns about them. In general, the MTs stated that reformers were concerned that students were memorizing a series of disconnected facts and procedures without understanding and that they were being subjected to monotonous and tedious drills. Klein (2007) acknowledged that, ironically, in the view of reformers, traditional school mathematics was a "dumbed down" (p. 28) version of reformed school mathematics. Mathematically Correct (1996) acknowledged, "Perhaps the most viable criticism of traditional programs offered by the proponents of the new programs is that traditional students do not do as well on problem solving (meaning word problems) as they do on straight computation" (para. 11).

In describing the pedagogical practices of reform, the MTs painted a picture of students who have been given control of their learning in the interest of making mathematics fun. This was the MTs' general interpretation of student-centered instruction. The MTs believed that in the classroom, groups of students worked "with peers in cooperative learning groups to 'construct'

strategies for solving math problems" (Quirk, 1997/2005, para. 4). In these environments, the teacher presumably acted as a facilitator rather than an instructor, confined to being a resource for students as opposed to the director of learning. According to the MTs, the use of manipulatives was either a part of making mathematics fun for students or was meant to provide students a way to compensate for their lack of knowledge of basic skills. Additionally, they believed classroom activities involved a large amount of written and verbal communication, which was conducted in students' vernacular.

The MTs reacted to their interpretations of this pedagogical approach by stating their reservations about some of its components. For example, they expressed a number of concerns related to the discovery approach via a post on the Mathematically Correct (1996) website:

[Discovery learning] holds that students will learn math better if they are left to discover the rules and methods of mathematics for themselves, rather than being taught by teachers or textbooks. This is not unlike the Socratic method, minus Socrates. One of the problems with this approach is that teachers must be extremely skilled in these methods. Another is that "discovery" takes so long that considerably less material can be covered. A third problem is that the children sometimes "discover" the wrong "rules" and teachers don't always catch the error. (para. 2)

The MTs allowed that a discovery strategy is an appropriate instructional method when used sparingly but reiterated their concerns about the length of time required for exploration. Wu (1999) worried, "On the debit side, guided discovery and cooperative learning slow down the pace of a course, at least by half" (p. 12).

Closely related to their concern about the length of time that a discovery approach would require was the MTs' caution about what they perceived to be an extensive use of written and verbal communication. In addition to the time spent away from mathematics to focus on "writing essays" (Mathematically Correct, n.d., para. 28), they lamented the informal nature of ordinary

language and how its lack of precision could foster misconceptions of the language of mathematics. Mathematically Correct (1996) posted:

Proponents ... like to talk about "communicating mathematically." However, the students do not learn the language of mathematical exposition—the terminology, symbols, and syntax needed for communicating mathematically. Instead, their products are better characterized as "communicating about math,"—written and spoken words and pictures that have something to do with math but are a far cry from "communicating mathematically." (para. 4)

Additionally, the MTs wondered whether a focus on communication would disadvantage students who struggled with verbal or written communication. According to Mathematically Correct, in reform mathematics classrooms

students end up spending hours working on essays, again detracting from their chance to practice basic skills. This has the substantial risk that potentially controversial moral lessons make their way into assignments. Another problem with the emphasis on language skills, which many feel is misplaced in math class, is the disadvantage to students with speech and hearing problems or non-native English speakers. (para. 3)

The MTs also questioned teachers' mathematical capabilities to navigate an exploratory instructional environment. Wu (2005) wrote, "The demand put on teachers' knowledge of mathematics by the discovery method is at times so enormous as to defeat even professional mathematicians" (p. 4). As discussed earlier, the MTs suspected that teachers lacked the necessary mathematical content knowledge to appropriately teach using traditional methods. Therefore, they argued, teachers will certainly not do well using a method that makes greater demands upon them.

Similarly, the MTs expressed concerns about the use of group work in the classroom. Their understanding was that reformers advocate group work under the assumption that such cooperation mimics, and therefore prepares them for, future roles in the workforce. The MTs rejected this perceived argument, averring that "real world experience shows that those who do math in organizations rarely do so in a group setting" (Mathematically Correct, 1996, para. 8).

However, the MTs saw this argument as moot; that is, even if the reformers' assumption was correct, the MTs argued that group work presents challenges that supersede the need to prepare students for the workforce in this way. Specifically, the MTs expressed deep concerns about the distribution of work and learning in a group setting. Mathematically Correct (1996) posted,

When children work in groups in school, the distribution of work, and of learning, is not equal.... Problems often occur from unequal ability levels within a group. In such cases, the most advanced students do the bulk of the work, with the others copying from them. In groups of equal ability levels, students have been known to split up the work and then copy answers. (para. 8)

Assessment in School Mathematics Reforms

In my analysis, I coded comments that shed light on the way the authors conceptualized assessment in school mathematics reforms. Although some ETs made comments about assessments, they did not express any commonalities in their understandings. The PCs, however, were concerned that what they identified as a refusal to acknowledge the existence of correctness, and hence, truth, in the pedagogical practices of reform extended to the assessments in reform. In particular, they believed that assessments were subjective and too focused on preserving students' self-esteem. In VanCourt's (1999) review of *Glencoe Pre-Algebra*, he described an end-of-chapter assessment and compared it with an alternative assessment that was also offered at the end of the chapter. After implying that the alternative assessment was easy, he wrote:

Here Glencoe is clearly declaring two expectations. First, some students will be completely defeated by the material in chapter 10 and will be unable to perform the exercises in the "Study Guide and Assessment" section. Second, teachers will want some pretext for awarding good grades to such students and for pushing the students ahead to be defeated by chapter 11. The "Alternative Assessment" section provides the desired pretext (para. 25).

The PCs suspected that the liberal proponents of reforms desired equal outcomes and therefore rigged the assessments so there would be no losers.

The MTs shared the PCs' concern that the assessments were subjective and focused on students' dispositions. Although the concerns expressed by the MTs and the PCs may have been superficially similar, the MTs' concerns originated from a different place than did the PCs'. In particular, the MTs' suspicions regarding assessments were based on their belief that a goal of the reforms was to develop positive student dispositions about mathematics and to ensure success for all students. Some MTs specifically mentioned "math appreciation" as an assessed goal, whereas others suggested calculator skills or social goals (such as cooperation) as items that might be assessed in the reforms. To MTs, the inclusion of these nonmathematical outcomes in assessments amounted to unnecessary coddling of students and, by association, as a way for reformers to hide from the failures of their own reforms. According to Klein (1999), using the assessments of reform "in place of examinations with consequences, and little if any importance placed on student discipline and responsibility in the reform literature, help to make reform math an object of ridicule among vocal parents' groups" (para. 34).

The Potential Costs of School Mathematics Reforms

My analysis captured authors' expressions of concern about who the reforms might harm and warnings in the form of predications about the costs to national society. In regards to the latter, both the ETs and the MTs predicted a shortage of competent workers in STEM fields. For example, Wu (1998) warned, "There are valid reasons to fear that the reform will throttle the normal process of producing a competent corps of scientists, engineers and mathematicians" (p. 2), and Loveless (2004), an ET, worried that the reforms would "have implications for our nation as a whole" for "it is not clear how long we can make up for deficiencies in our own educational system by importing the needed talent from other countries" ("Why Important," para. 8).

Each community of meaning focused primarily on different groups of students when expressing the potential costs of the reforms. The MTs opined that the costs of Grades K–12 mathematics reforms would be heavy for high-ability or "bright" students. For example, Milgram (2005) wrote,

It is a grim thing to watch otherwise very bright students struggle with more advanced courses because they have to figure everything out at a basic verbal level. What happens with such students, since they do not have total fluency with basic concepts, is that—though they can often do the work—they simply take far too long working through the most basic material, and soon find themselves too far behind to catch up. (p. 12)

Additionally, Bachelis (n.d.) stated, "My main concern about these programs is that, in an effort to keep everyone entertained, they tend to turn off the good students who would do well if given the traditional math" (para.4). Perhaps it is unsurprising that the MTs expressed such concern for high-ability students given that they may identify an earlier version of themselves with these students. Raimi's (2006) angst is evident in his question:

But is it not unconscionable that NCTM should outline a program for the middle schools that absolutely prevents everyone's attaining the skill in arithmetic that some of them would ultimately need, in college or in later life? If not everyone can be an engineer, shall we therefore arrange our public educational system so that nobody can? (para. 37)

The PCs worried about the costs to the college bound and to all students in a general sense. The PCs' concern for the college bound was based on their perception that students who had been subjected to the Grades K–12 mathematics reforms were increasingly finding themselves ill-prepared for college and that the reforms were responsible for increased enrollment in college remedial mathematics courses. Cheney (1997) claimed that parents "have complained about ... high school graduates who get A's and B's in [reform] math classes and have to do remedial work in college" (para. 9). But the PCs' concerns about the potential costs of reforms were also expressed towards students in general. Cheney (1998) claimed that "the Department of Defense has found that ... [reform] math hurts everyone" (para. 10).

The PCs also expressed a concern for the costs of the Grades K–12 mathematics reforms to parents, which was unique to this community. As discussed earlier, conservatives strongly believe that parents are in the position to know what is best for their children and should have the right to make that determination. The PCs noted that parents were frequently at a loss as to how to help their children with homework because the mathematics content was strange to them. To some PCs, this phenomenon was by design. Price (2013) wrote, "New and Reform Math like to throw around complex terms, probably to bamboozle parents" (para. 13). McNew and McNew (2010) recounted a story in which a teacher undermined a set of parents who had tutored their child how to perform a standard algorithm by telling the child "that her parents had taught her math 'the old way' and that it was 'confusing and a step behind'" (para. 6). To the PCs, stories like these reinforced their conception that educators believed that they knew best what was good for children and that they would undermine parents' authority in the eyes of their children.

The ETs registered a higher number of comments in this category than either the MTs or the PCs. Their concern was almost exclusively reserved for the disadvantaged. In particular, the ETs believed that although students from families with intellectual or financial capital had the resources to overcome the failures of schools, the "students who [would] pay the biggest price were those with the least to lose, those for whom the educational system has never worked very well" (Loveless, 2004, "Why Important," para. 1). Therefore, the ETs predicted, mathematics education reforms would increase social injustice and aggravate the achievement gap. Hirsch (1997) wrote:

The oppressed class should be taught to master the tools of power and authority—the ability to read, write, and communicate—and should gain enough traditional knowledge to understand the worlds of nature and culture surrounding them. Children, particularly the children of the poor, should not be encouraged to follow "natural" inclinations, which would only keep them ignorant and make them slaves of emotion. They should learn the

value of hard work, gain the knowledge that leads to understanding, and master the traditional culture in order to command its rhetoric (p. 42).

Hirsch's words underscore the ETs' traditionalist lens, in which the natural instincts of the oppressed, as in all of humanity, are seen as untrustworthy; making it necessary for society to instill in its children the qualities needed for individual success. According to Hirsch, children from families lacking resources would fail to "secure the knowledge and skills that will enable them to improve their condition" (p. 42). Therefore, he reasoned, "political liberals really ought to oppose progressive educational ideas because they have led to practical failure and greater social inequity. The only practical way to achieve liberalism's aim of greater social justice is to pursue conservative educational policies" (p. 42).

The Conceptions of the Evidence for School Mathematics Reforms

My last research question sought to determine how each community of meaning interpreted and reacted to the evidence supporting school mathematics reforms. In this category, I coded 57 comments from the MTs, 49 comments from the ETs, and 52 comments from the PCs. All three communities of meaning agreed that the school mathematics reforms were not supported by evidence. The MTs and the ETs argued that research supporting reforms was inconclusive or in error, whereas the PCs were either not aware or did not acknowledge that research supporting reforms existed. Regardless, the conclusion was the same in all three communities: Grades K–12 mathematics reforms were experimental. Mathematically Correct (1996) posted, "In fact, the lack of research support is striking. Perhaps the most unifying feature of these new programs is that they are all experimental" (para. 14) and Malkin (2007), a PC, claimed the reforms treated students as "guinea pigs" (para. 9).

The MTs approached the prospect of instructional changes from a *first do no harm* perspective; that is, they suggested that before instructional changes are implemented, there

should be unequivocal evidence supporting those changes. For example, Mathematically Correct (1996) posted, "One might reasonably expect that such radical departures from traditional methods would be based on clear, well-documented, overwhelmingly compelling, quantitative evidence of their superiority. Sadly, this is not the case at all" (para. 14). The MTs positioned themselves as willing to accept evidence that supported reform methods; but, they insisted, no such evidence existed.

All three communities of meaning apparently assumed that school reforms were implemented in schools immediately following the publication of NCTM's (1989) *Curriculum and Evaluation Standards*, as evidenced by their assertions that the reforms had caused subsequent drops in standardized test scores and increases in college remediation rates. For example, when asked why he spent so much of this time on mathematics education in Grades K–12 schools, Askey, an MT, responded, "Over 20 years ago I was finding it harder to teach calculus than before, and I wanted to find out why and see what could be done to reverse this trend" (Sriskandarajah, n.d., p. 3). Garelick (2013), an ET, rhetorically asked, "Reform math in various forms has pervaded math teaching for over twenty years. Could it be that the failures of understanding are due to the beliefs and techniques of the reformers rather than the other way around?" ("What are the Two," para. 2).

The MTs and the PCs each offered evidence to support the traditional methods of teaching and learning mathematics. Vukmir (2001), a PC, offered an anecdote of parents who, when encountering reform methods in their children's school, decided to homeschool their children in mathematics. Vukmir claimed, "The children thrived under their parent's tutelage, scoring at the 'advanced proficiency' level on the Wisconsin Student Assessment System (WSAS) math test" (para. 3). Other PCs asserted that countries with high student achievement in

mathematics were teaching using traditional methods or that research existed to support traditional methods. For example, Carnine (2000) described what he stated was "probably the largest education experiment in the United States" (p. 4), which evaluated reform methods against direct instruction in mathematics and other subject areas. Carnine claimed that the study, dubbed *Project Follow Through*, found direct instruction to be the most effective method. He wrote,

Longitudinal studies were undertaken using the high school records of students who had received Direct Instruction through the end of third grade as well as the records of a comparison group of students who did not receive Direct Instruction. Researchers looked at test scores, attendance, college acceptances, and retention. When academic performance was the measure, the Direct Instruction students outperformed the control group in the five comparisons whose results were statistically significant. The comparisons favored Direct Instruction students on the other measures as well (attendance, college acceptances, and retention) in all studies with statistically significant results. (p. 7)

Klein (1999), an MT, referred to Jamie Escalante, a teacher made famous by the Hollywood movie *Stand and Deliver*, as an example of an individual whose use of traditional teaching methods resulted in minority students' success, and Milgram (2010) referenced his own work in an inner city school in San Jose as evidence that traditional methods along with increased teacher content knowledge leads to gains in student achievement. Interestingly, some MTs cited their own or each other's unpublished research, some of which appeared to be fairly extensive, as evidence in support of traditional practices. For example, Milgram (n.d.b) reported a study conducted by Bachelis, who described his methods in greater detail in an appendix. Bachelis's quasi-experimental study tested a cohort of students using a reform-based curriculum (the treatment) against students using a nonreform curriculum (the control). According to Milgram,

The data represents the first glimpse-to the best of our knowledge-of how students trained in this new way perform in a university environment, and, frankly, the results are not encouraging. First, almost all the [treatment] students were severely critical of the program, many bluntly blaming it for their difficulties in university level courses.

Moreover, there was no measure represented in the survey, such as ACT scores, SAT Math scores, grades in college math courses, level of college math courses attempted, where the [treatment] students even met, let alone surpassed the comparison group of [control] students. (para. 3)

The MTs did not provide any reasons why their studies remained unpublished; however, they revealed their conceptions of published mathematics education research that supported mathematics education reforms. The MTs were highly skeptical of mathematics education research, calling into question researchers' methods and motives and equating mathematics education researchers to proponents of school mathematics reforms. Specifically, the MTs suggested that those involved in mathematics education research were affected by their own biases and an inability or unwillingness to question their own assumptions. As evidence, the MTs noted what they deemed as a lack of objectivity and restraint in mathematics education research reports. For example, Raimi (2004) noted that "NCTM hired a publicity organization to make sure the country took their Standards to heart, and its regional and national meetings featured only papers pressing and praising their progressivist doctrine" ("Rise," para. 1). Raimi then accused mathematics education research journals of shutting out nonconforming points of view, warning, "An iron conformity has descended upon the world of mathematics education in the United States" ("Rise," para. 2). Additionally, Wu (1999) claimed that mathematics education researchers' bias was evident in the one-sided nature of their results:

A more balanced approach to the subject of lecturing might begin by listing its strengths, its weaknesses, and the range within which it would be effective. ... The overriding fact remains that the current discussion of pedagogy fails to meet the most basic requirements of scholarship: any advocacy should state clearly its goal, its benefits, and its disadvantages. In this light, the advocacy of the guide-on-the-side pedagogy has been presented more like an infomercial than a scholarly recommendation. It is all good and nothing bad could possibly come of it (p. 11).

The ETs shared this general skepticism of educational research. For example, Garelick (2005) wrote, "In the various arguments about how best to teach math, educationists make the

point that research shows that their approaches work best. I tend to be suspicious of that research, and apparently others are as well" ("An Evidentiary," para. 1). He continued with a quote from a 2004 press release by the National Academy Press. Interestingly, Carson (2006), another ET, shared the same quote in one of her documents:

"On Evaluating Curricular Effectiveness: Judging the Quality of K–12 Mathematics Evaluations," National Academies' Mathematical Sciences Education Board, National Research Council (2004)

Findings: "Evaluations of mathematics curricula provide important information for educators, parents, students and curriculum developers, but those conducted to date on 19 curricula (which included all 13 NSF funded K–12 math programs) fall short of the scientific standards necessary to gauge overall effectiveness." (press release National Academy Press) (slide 30)

The PCs² conceptions of the evidence for school mathematics reforms were actually less a conception about the evidence supporting reforms than a conception of the evidence in support of traditional methods. Given the PCs² distrust of academia, it is perhaps unsurprising that most of them spent little time considering its products. For the PC community of meaning, one might predict that research evidence supporting reforms would have little impact in convincing them (Kilpatrick, 2001). Seen through the lens of their conservative worldview, the reforms represented an ideology that the PCs rejected on the most fundamental level. Similarly, the ETs viewed the school mathematics reforms within the context of their beliefs about the role of education in society and how children are best prepared for the demands of adulthood. The MTs viewed the reforms through their beliefs about what mathematics is and how it should be taught. As noted by Hiebert (1999), research cannot determine goals or values. This observation is perhaps particularly relevant in considering the way these communities of meaning responded to the evidence for reforms.

CHAPTER 5

DISCUSSION AND IMPLICATIONS

In this study, I sought to explore the motivations and conceptions of individuals who had publicly opposed school mathematics reforms in order to determine the commonalities that united them in their political activism against the reforms. As discussed in the previous chapter, those opposed to reforms shared many commonalities in their conceptions of the goals and best practices for school mathematics, the way they characterized the school mathematics reforms, and their conceptions of the evidence for the reforms. Although the critics may have recognized each other as allies in their activism against the reforms, under these somewhat superficial commonalities lay fundamental differences in the way they viewed the reforms, the importance they assigned to the various characteristics of the reforms, and their motivations for rejecting them. Below, I discuss my findings with respect to my theoretical perspectives and infer the primary motivation for each community of meaning's political activism in opposing the reforms. Then I compare my findings with the speculations I found in the literature. Finally, I discuss the implications of this study.

Discussion

The three communities of meaning agreed about certain elements of Grades K–12 mathematics and the reforms. In particular, they "spoke each other's language" in terms of their conceptions of mathematics as hierarchical, their view that mathematicians should be considered experts of the content of school mathematics, their belief that basic skills and algorithms are important elements in school mathematics, and their view that students should be taught these

skills though the use of direct instruction and student practice. Additionally, all three communities of meaning shared the view that the school mathematics reforms were a product of the NCTM advocating a progressive teaching philosophy on the basis of the constructivist learning theory. They agreed that some of the characteristics of the reforms were a de-emphasis of basic skills, including algorithms, memorization, and practice, and an emphasis on the use of calculators, group work, manipulatives, and discovery learning through problem solving. Each community of meaning determined that the reforms were "dumbed down" from traditional school mathematics and that they had not been adequately tested before being implemented in schools. Philosophically, all three communities of meaning agreed that human instincts cannot be trusted, a view MTs shared as a part of their secondary traditionalist lens, and I argue that this belief affected all three communities' view of what education can and should look like. These shared common understandings of school mathematics and the reforms enabled them to see each other as allies in the math wars; however, despite these similarities, there were fundamental differences between the communities of meaning in the way they perceived school mathematics reforms and why they objected to them.

The primary lens in each community of meaning identifies what Green (1971) called the primary beliefs of the individuals in the community. These lenses, or these primary beliefs, shaped the opponents' subsequent logical conclusions resulting from their primary beliefs, that is, their derivative beliefs, which were their interpretations of and objections to the reforms. That the lenses represent primary beliefs, however, does not indicate that they are psychologically central. To better understand the psychological strength of their beliefs, it may be helpful to consider why each community of meaning was passionate enough about the controversy to insert itself into the political sphere.

Motivations for Political Activism

The primary lens through which the MTs viewed school mathematics and the reforms was their conception of mathematics as a domain. The MTs were troubled that reformers were redefining mathematics as a domain and, by extension, mathematics success. They worried that left unchecked school mathematics reforms would fundamentally change the subject with which they strongly identified. The MTs frequently accused mathematics education researchers (nearly all of whom MTs believed to be advocates of reform) of designing assessments that would ensure the outcome of their studies, which revealed their belief that they could not trust the expertise of researchers in mathematics education.

In a recent contentious and public dispute, a few MTs challenged the research findings of a highly regarded mathematics education researcher, implying—if not outright accusing—the researcher of academic fraud (Milgram, 2012). Aside from issues related to the appropriateness of how the dispute was handled by either party, a contributing factor to the controversy was each side's view of what mathematics is and what constitutes success in it. For the MTs, it can be argued that this fundamental question is highly personal, for the answer is closely linked to their identity as mathematicians. In this case, a lack of understanding of this fundamental difference in beliefs about mathematics learning may have generated fertile ground for a painful and divisive disagreement. By extension, this perceived challenge to their professional identities might explain the MTs' decisions to enter the political fray in opposition to reforms. Rokeach (1968) argued that beliefs connected to individual identity are strongly connected to other beliefs, and therefore are psychologically central. We can surmise, then, that MTs' primary lens—their view of mathematics—represents both primary and psychologically central beliefs.

Although the MTs' perception—that the reforms redefined mathematics—explains their strong reaction to the reforms, it is more difficult to ascertain why the ETs invested effort in publicly opposing the Grades K–12 mathematics reforms. Perhaps some ETs were parents motivated by what they believed were practices that would be costly to their child, as suggested by Kohn (1998). That inference, however, is not supported by the data. Although the data does not refute Kohn's claim, the ETs' public arguments focused on children in general, and particularly those who were disadvantaged. Given the primary lens of the ETs, it is reasonable to assume that they were at least partially motivated by their concern for disadvantaged children. I make this argument based on the ETs' frequent comments concerning the costs of reform to disadvantaged students as well as the ETs' assumption that society is best served through the systematic acquisition of content and practices that have been implicitly determined through culture. From the ETs' perspective, failure to acquire this content and these practices is the primary cause of individuals' inability to better their economic positions. The ETs surmised that disadvantaged families lacked the economic and social resources that would ensure their children received this cultural conditioning, as opposed to the advantaged, whose induction into these practices and understandings was ensured by their more affluent parents. Therefore, having framed the controversy within their traditionalist ideology, the ETs may have chosen to engage politically to stop what they believed were educational practices that would be harmful to disadvantaged children and society as a whole.

This inference about the ETs' motivation for political activism sheds little light as to the psychological strength of the beliefs represented by their lens. Some ETs' identities may be connected to their traditionalist view of education. For example, E. D. Hirsch, an ET, has developed an entire curriculum series based upon his views of education, which evidences that

his beliefs are psychologically central. Other ETs, however, have less cause to incorporate their views of education as a part of their personal identities. Rokeach (1968) suggested that psychologically central beliefs are those that are shared with others because interactions with those who share the same beliefs increase the connections of those beliefs with individuals' other beliefs. In this vein, the very act of organizing to oppose school mathematics reforms served to reinforce the ETs' lens and increase its psychological strength.

The primary lens through which PCs viewed school mathematics and the reforms was their conservative worldview. As such, the psychological centrality of their lens is immediately obvious. To the PCs, the reforms represented a rejection of their fundamental beliefs and value systems and a subtle, yet perhaps effective, attempt to indoctrinate children into liberal ideology. The PCs appeared to be conflicted in their view as to whether the liberal ideology they saw as prevalent in the reforms was intentional or, alternatively, was the product of liberal advocates who were blind to their own assumptions. To the PCs, if the reforms were the latter, then they were a ridiculous product of naïve advocates who had an unrealistic view of humanity, if the former, then they were a dangerous attack. Thus, the tone of the PCs' commentaries about the Grades K–12 mathematics reforms fluctuated between derision and alarm. The motivation for the PCs' engagement in the politics of the school mathematics reforms was that they perceived them to be counter to their deeply held belief system.

Connections to the Literature

In Chapter 1, I argued that speculation of critics' concerns about school mathematics reforms might offer little possibility to affect the math wars unless we verify reform advocates' speculations about the positions of the opposition. I now compare my findings with those presented in the literature by the advocates of reforms.

With respect to the identities of those opposing reforms, all of the authors' speculations at least partially matched my findings. The one exception was that a few authors identified parents acting on behalf of their children; although some of the critics in my study may have been parents, they did not refer to their position as parents in the arguments about the reforms. It is likely, however, that my focus on the reforms on the national scale prevented me from identifying this group, which might be more visible in local anti-reform efforts.

In the literature, different authors focused on different "types" of people opposed to reforms. For example, Becker and Jacob (2000) described the opposition as mathematicians, and Kohn (1998) and Schoenfeld and Pearson (2009) described them as the "far right." Each of these descriptions is at least partially true with respect to my findings. As suggested by Becker and Jacob, one community of meaning, the MTs, was mathematicians. Although one could argue about the difference between political conservatives and the *far* right, it is likely that Kohn and Shoenfeld and Pearson were referencing the PC community of meaning. With respect to the identities of the critics of reform, Schoenfeld (2004) offered the most accurate description as it relates to my findings: that they were a conglomeration of individuals with different motivations and understandings.

Klein's (2007) description of the opponents of reforms not only matched my findings, but also helped me identify two communities of meaning, the ETs and the PCs. Ironically, Klein did not identify the community of meaning in which I determined he belonged: the MTs. Ernest (1991) described social groups that closely matched Klein's descriptions of the two communities that comprised the opponents to school mathematics reforms. Ernest's industrial trainers—which matched Klein's political conservatives—match the PC community of meaning in my findings in some respects. Ernest described industrial trainers as politically conservative (although he

implied they were radically so) and as holding an absolutist view of mathematics as well as a Protestant work ethic, which was true of the PCs. The PCs, however, deviated from Ernest's description in that they did not express the view that intelligence and mathematical ability are necessarily inherited or fixed, nor can one reasonably describe their positions as "extreme Jingoistic values of monoculturalism, crypto-racism and xenophobia" (p. 151).

Ernest's (1991) description of old humanists mostly matched the ETs, and to a lesser extent, the MTs. The old humanists held conservative views of education but were often politically liberal and valued the transmission of cultural values and knowledge to the next generation, which matched the ETs and MTs. Ernest, however, characterized old humanists as assuming a fixed view of intelligence and mathematical ability, which does not apply to the ETs or the MTs, and of holding an absolutist view of mathematics, which does not apply to the MTs.

My findings either do not support or do not address speculations offered by some advocates of the school mathematics reforms. Specifically, Kohn (1998) and Schoenfeld and Pearson (2009) claimed that those opposed to reforms held beliefs that framed education as a vehicle for social efficiency and, subtly or not so subtly, implied opponents were racist and classist. As to the latter, the data did not specifically address the accusation; for although the data did not reveal any overt racism or classism, it was not sufficient to rule out subtle or ingrained forms of racism and classism. In the case of the former, however, the data did not support the claim that critics of reforms held social-efficiency views of education. Specifically, both the MTs and the ETs exhibited educational beliefs that aligned with classical traditionalism as opposed to the view of education attributed to them by Schoenfeld and Pearson. Similarly, the PCs did not stray far from the traditionalist views of education held by both the MTs and the ETs.

My findings support some speculations offered by advocates for reforms; specifically, all of my communities of meaning objected to the overuse of calculators and the de-emphasis of algorithms, as was suggested by Jacob (2001). Additionally, they objected to what they perceived as a de-emphasis of computational and algebraic skills as suggested by Schoen et al. (1999). My findings also agree with Schoen et al.'s contention that those opposed to the school mathematics reforms could not conceive of a mathematics curriculum that could simultaneously challenge the lowest and highest ability students, leading them to characterize the reforms as "watered down." Reys's (2001), Battista's (1999), and Schoen et al's contention that those opposed to reforms believed that they had been implemented without adequate testing was also supported by my findings. O'Brien (1999) and Battista (1999) speculated that those opposed to reforms desired a return to teaching the basics of mathematics, which is at least partially supported by my findings for only the PC community of meaning.

Although advocates for reforms correctly identified some of the critics' understandings and objections to reforms, they overlooked the fundamental bases for critics' objections. In a sense, the advocates were distracted by the minutia of the controversy, which acted as noise that masked the primary objections of the critics, which are best understood from the perspective of each community of meaning's primary lens for viewing the reforms and their motivations for engaging politically to overturn them. By engaging the critics in the minutia of the controversy, rather than in the fundamental bases for critics' concerns, advocates could have been perceived as dismissive of opponents' centrally held beliefs. This perception might have not only driven a deeper wedge between the two sides, but also motivated the critics to seek others who understood and agreed with their concerns.

Implications

The findings from this study have implications for three groups. First, those who are charged with implementing Grades K–12 mathematics reforms can gain insight into common objections to reforms and the primary beliefs that may underlie those objections. Second, there are lessons for the advocates of reforms, as well as questions to consider. Third, this study contributes to the literature by increasing the call for educational research that speaks to policymakers and laypeople and offers additional insights related to Ernest's (1989) theoretical framework.

Implications for the Implementers of Reforms

Classroom teachers, principals, district leaders and others charged with carrying out school mathematics reforms may find themselves the initial point of contact for individuals concerned about the reforms. By cutting through the noise of specific objections to reforms and concentrating on the primary beliefs that drive their objections, implementers may be able to alleviate concerns, or at a minimum, refrain from exacerbating them. This would involve careful questioning to determine whether the critic expresses beliefs that place him or her in one of the three communities of meaning identified in this study. Table 4 summarizes the primary lens and the motivations for political engagement of each of the communities of meaning. The third column specifies what I call *flash points*, that is, the terminology or ideas that may appear harmless to implementers but are particularly problematic to communities of meaning and could therefore derail a productive discussion.

In my findings, the MTs expressed frustration with what they believed were incorrect views of mathematics. They were particularly displeased with pattern problems because of their inherent ambiguity. Although it is unlikely the MTs would object to the assertion that noticing

and describing patterns is an important process in mathematics, they noted that *mathematics* occurred when *verifying* these conjectures. In a conversation, an implementer needs to be aware that discussions involving pattern problems (or any other problem-solving situation with imprecise parameters) could be tenuous. The MTs are sensitive to impressions that those charged with presenting mathematics to school children have an incorrect view of what mathematics is; therefore, implementers need to be clear about their views of mathematics and how they might conflict with an MT. The MTs additionally expressed frustration with the message that "math is fun," which they believed reduced mathematics to games and minimized the expectation that students must work hard to learn it.

Table 4

Description of Interpretations of Reforms

Community of Meaning	Primary Lens	Motivation for Political Activism	Flash Points
Math Traditionalists	View of mathematics	Reforms redefine mathematics	 Math is fun Redefines mathematics Math is pattern finding
Education Traditionalists	Traditionalist view of education	Concern for disadvantaged and society	AlgorithmsConceptual understandingDrill and kill
Political Conservatives	Conservative worldview	Reforms are liberal indoctrination	 Social justice Multicultural education Answer not as important as process

The ETs were particularly concerned about a de-emphasis of standard algorithms. It is important to them that these algorithms be mastered to the point of fluency. Implementers should

predetermine how they will handle this assertion. Implementers, however, must be careful about appealing to the need to develop students' conceptual understanding, as the ETs were particularly frustrated by the implied assumption that traditional mathematics instruction neglects to do this. The ETs were also sensitive to the contention that traditional mathematics instruction involved a "drill and kill" approach. In discussing school mathematics reforms with a member of the ET community of meaning, care must be taken not to belittle traditional methods of instruction, as these represent ETs' primary lens, which may also be centrally held beliefs.

The PCs were sensitive to language and practices that they believed promoted liberal ideology. Because the PCs equate the term *social justice* with liberalism, their perception of what an implementer means when using this term is likely far removed from what he or she actually intends. Likewise, the PCs generally consider multicultural education to mean practices that they view as unproductive or even harmful. PCs are particularly troubled by the suggestion that correct answers are unimportant or are less important than mathematical processes, because not only does this appear counterintuitive to them, they associate this belief with relativism, that is, the rejection of absolute truth.

By attending to these cautions when dealing with a critic who resides in one of this study's communities of meaning, implementers may avoid some of the particular pitfalls into which they may have easily fallen. Engaging critics at this level, however, is only helpful to the extent that the implementers understand their own commitment to the reforms and have developed answers to the questions that the findings of this study suggest are important for advocates of reforms, which are posed in the following section.

Implications for Advocates of Reforms

All three communities of meaning agreed that mathematicians are experts for determining school mathematics content. From this, it is apparent that the *visible* support of mathematicians is crucial to the success of reforms. The NCTM has included mathematicians in its work and some advocates of school mathematics reforms are professional mathematicians (e.g., Jacobs, 2001). The letter to Secretary Riley, signed by hundreds of mathematicians, however, likely did serious damage to the cause of school mathematics reforms by way of convincing some of the public that most mathematicians opposed them. Advocates of reforms must be politically savvy in recruiting and highlighting more than just a few mathematicians as spokespeople for school mathematics reforms in order to persuade both the public and other mathematicians of their viability.

The MTs contended that the reforms attempt to redefine mathematics. This raises a question for advocates of reform; namely, is their accusation true? Some advocates for reforms have made such a claim (e.g., Meyer, D., 2010; Romberg & Kaput, 1999; Skovsomose & Valero, 2001), but it is unclear in what way they are saying mathematics is being redefined or even if their interpretations match. It is possible that advocates' claims about redefinition refer to *school* mathematics as opposed to mathematics as a discipline, basing their claim on the belief that the reforms offer students a more authentic experience in doing mathematics. The MTs, however, claimed that reforms attempt to redefine mathematics as a discipline. Moreover, they rejected the assumption that the reforms offer a more authentic experience, arguing, for example, that reforms' inattention to precision shifts the activities away from the field of mathematics.

It is possible that while attending carefully to the question of the nature of mathematics—that is, whether mathematics is discovered or created—and the implications to school

mathematics the answer to this question poses—advocates have overlooked the question of the fundamental characteristics of mathematics. Advocates must not only consider this question, but determine how educators can ensure that students do not develop improper understandings when mathematics is "bent" in order to accommodate the developmental limitations of children. In addition, if advocates determine that, indeed, the reforms constitute a redefinition of mathematics as a domain, we must ask, in what ways do the reforms redefine mathematics and what possible repercussions might there be to this redefinition, both in term of mathematics as a discipline and in the scientific advances mathematics has traditionally enabled? These questions must be answered by advocates of reforms if they want to legitimately engage MTs in discussions about school mathematics reforms; but more importantly, it may be important for advocates to consider these questions because by doing so, they enter the political fray with full knowledge of what it is they are advocating.

The literature and my findings supported some advocates' contention that those opposing school mathematics reforms were politically conservative. In most cases, advocates causally dismissed the concerns of these individuals with terminology implying their views were extreme (e.g. *far right, radical right, religious right*). The message was clear: *Those* people and their concerns do not count and are not worth debate. For example, Ernest (1991) critiqued the social group that correlated to the PC community of meaning by stating,

The industrial trainer aims are based on an extreme and largely rejected set of principles and moral values.... The 'protestant work ethic' and other moral values, such as 'original sin', are an extremist and inappropriate foundation for educational policy. Their basis is one that is rejected by the weight of Western intellectual thought. (p. 151)

In this, Ernest appears to support the PCs' suspicions that the school mathematics reforms are incompatible with their conservative worldview. In reply to Kohn's (1998) description of the characteristics and motivation of the opponents to school reforms, Rochester (1998), a political

science professor, suggested that the advocates themselves were perpetuating the perception that the reforms are based on liberal ideology:

Actually, Kohn does us a real service in unwittingly revealing just how neatly the assorted jargon of contemporary progressive rhetoric fits into a single left-wing ideological critique of traditional education. The leftist roots of the "school reform" movement, fully bared, are as plain as day. (p. 176)

This raises important questions for the advocates of school mathematics reforms. *Are* the reforms based on liberal ideology? That is, are they incompatible with conservative beliefs? If so, what does that mean for the viability of reforms given the political make up of our country? A Gallup poll from January 2014 measuring political ideology of the U.S. population reported 38% as conservative, 34% as moderate, and 23% as liberal (Jones, 2014). In some "red states," nearly 50% of the population is politically conservative (Gallup, 2014). As noted earlier, the PCs' conservative worldview encompassed their psychologically central beliefs; therefore, the PCs cannot be argued out of their positions. The only possibility for easing the concerns of PCs is to determine that the school mathematics reforms can fit into their belief systems. In this respect, we have some reassurance from, ironically, Klein (2007), who wrote, "In the course of the math wars, parents of school children and mathematicians who objected to the dearth of content were dismissed as right wing, but there is nothing inherently left wing about the NCTM aligned mathematics programmes" (p. 32).

If Klein's (2007) assertion is correct, we can consider important cautions for mathematics teacher educators (MTEs) who promote the instructional practices called for in school mathematics reforms. Specifically, these MTEs need to be aware of the perspectives of their students. Given the distribution of political conservatives in our country, it is likely that some practicing and prospective teachers will hold these views. It is important, therefore, to avoid framing the instructional practices of reform in ways that these teachers will perceive as

politically liberal. By initially avoiding terminology that PCs associate with liberal ideology (e.g. *social justice, multicultural education*), and replacing it instead with actual descriptions of what is meant prior to the introduction of such terms, MTEs can reduce the possibility that teachers will misinterpret what they are saying. Additionally, just as MTEs ought to attend to racial and cultural aspects of the prospective or practicing teachers in their classes, they should adjust their practices to accommodate political ideologies, for these ideologies may have a greater impact on teachers' receptivity to the methods of reform than previously acknowledged.

Contributions to the Literature

The MTs in this study suggest that it may be necessary to consider an additional philosophical view of mathematics that affects the instructional practices of teachers. In *The Impact of Beliefs on the Teaching of Mathematics*, Ernest (1989) distinguished between two forms of the absolutist view of mathematics, the *instrumentalist view* and the *Platonist view*. Teachers holding an instrumentalist view characterize mathematics as "a set of unrelated utilitarian rules and facts" (p. 250), while those holding a Platonist view characterize mathematics as a "static but unified body of certain knowledge" (p. 250) that exists outside the experience of the individual and is therefore discovered rather than created. Ernest proposed a single view that corresponded with the fallibilist philosophy of mathematics. He described the *problem-solving view* of mathematics as a "dynamic, continually expanding field of human creation and invention" (p. 250), and is therefore "perpetually open to revision" (Ernest, 1991, p. 18). Ernest (1989) argued that teachers' personal philosophies of mathematics play a role in their instructional practices and suggested that the instructional practices of school mathematics reforms rely on a problem-solving view of mathematics.

The MTs in this study exhibited a fallibilist view of mathematics as evidenced by their assertions that it resides in the individual or collective mind. Because they believe mathematics is created rather than discovered, it follows they view it as subject to change and error. On the other hand, the MTs did not fit Ernest's (1989) description of a problem-solving view of mathematics, the only view corresponding to a fallibilist philosophy of mathematics.

Specifically, the MTs assumed a fixed subset of mathematics appropriate for school mathematics. The MTs might argue that this subset of mathematics is fixed, not because it is proven to be without error, but because for all practical purposes, it works. Ernest (1991), after arguing against the validity of an absolutist philosophy of mathematics, reassured the reader that embracing a fallibilist philosophy of mathematics "does not represent a loss of knowledge" (p. 20). As an analogy, he noted that

in modern physics, General Relativity Theory requires relinquishing absolute, universal frames of reference in favour of a relativistic perspective.... But what we see here [is] not the loss of knowledge of absolute frames and certainty. Rather we see the growth of knowledge, bringing with it a realization of the limits of what can be known. (p. 20)

In Ernest's analogy, the absolute view was unsettled by the realization that "absolute, universal frames of reference" were a mirage; yet even so, this realization represented an advance in the field. Ernest was arguing that mathematics as a domain is not minimized by the realization that its tenets are uncertain. The MTs, however, might expand on Ernest's analogy, noting that despite the advance of Einstein's Theory, Newton's laws of motion and universal gravitation are still taught in schools, and appropriately so. That is, expansions in the field, even when such advancements uncover limitations or errors in what had been known, do not delegitimize the necessity for and value of earlier theories. For just as we can still use Newton's laws to predict when a ball thrown into the air with a specific initial velocity will land, we can be reasonably certain that the standard algorithm for computing 36×54 will yield useful information that will

apply to other areas of mathematics regardless of future advances in the field. In this way, the MTs' philosophical view of mathematics differs from that described by Ernest (1989, 1991) in that it assumes a relatively fixed subset. Although this study does not provide the empirical evidence necessary to link such a view with corresponding views of teaching and learning and the resulting instructional practices of teachers, future research may uncover practices unique to this view of mathematics.

All three communities of meaning expressed similar sentiments about research in education that suggests that the field suffers a perception problem. Orland (2009) explored the reasons that educational research is often ignored or marginalized by policymakers and noted "throughout its some 100-year history, educational research has not enjoyed a reputation for scientific rigor within either academic or policy circles" (p. 116). Kilpatrick (2001) cautioned that the gold-standard for quality research— experimental and quasi-experimental methods—cannot serve all the needs of educational research because education poses challenges that do not exist in areas such as medicine. But, Kilpatrick argued,

the avoidance of experimentation—and of quantitative methods in general—by researchers in mathematics education over the past several decades has rendered the field unable to contribute much in the way of evidence to the public discourse. Practices being recommended by reformers and challenged by others have not been submitted to empirical tests, and they should be. (p. 424)

Orland echoed Kilpatrick's argument, noting that policymakers do not perceive educational research as offering information any more credible than other information sources. These observations point to the need for educational research—including mathematics education research in the context of school mathematics reforms—that policymakers and others will find credible and useful. Certainly, attending to Kilpatrick's call will help; however, the PCs offer insights that may also apply to educational research.

The PCs perceived higher education as a bastion of liberal ideology. Although it is unlikely that university environments are as left wing as the PCs suggest, the liberal leanings of professors has been well documented (Jaschik, 2012; Klein, 2005; Maranto & Woessner, 2012; Wood, 2011). In qualitative research in particular, which is prevalent in educational research, political ideologies often shape the questions that are asked, the theoretical assumptions underlying the research, and the interpretation of data. Given the prevalence of leftist political positions in academia, research that can be identified as favoring a political position will almost certainly lean left. Therefore, it is reasonable to assume that policymakers and the general public will, just as PCs, view educational research as essentially liberal. Given the current political make up of U.S. society, which is more conservative than liberal, policymakers and others might conclude that educational research is out-of-touch with their values and beliefs.

A solution to this perception problem is not easy. The lack of political diversity in the field of education research may be viewed simplistically (and perhaps offensively) as analogous to the ongoing lack of gender and racial diversity in many areas of U.S. society; therefore, the way forward may be to employ similar concern and attention to fostering welcoming environments for political diversity. Therefore, to Kilpatrick's (2001) call for more experimental and quasi-experimental studies in mathematics education research, I add a call for increased political diversity within the field of education research generally and mathematics education research specifically.

The events in Fairfield Public Schools described in my opening chapter have continued over the time that I have conducted this dissertation research. One of the strongest opponents of the district's use of the CPM curriculum materials was elected to the school board in November 2013. The website that critics originally developed to mobilize opponents of the CPM curriculum

continues to be updated. In April 2014, three documents were uploaded to the site. One was the data and summary of a district-sponsored study of the one-year pilot study of the CPM materials. The results were that students' marks remained constant as compared to prior years and that midterm exams did not show significant differences, despite an increase in rigor in the test. The other two documents posted to the site were unsolicited reports to the Fairfield Public School Board of Education criticizing both the methods and results of the district's study. The authors were James Milgram and Wayne Bishop, both members of the MT community of meaning.

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APPENDIX

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