

**MODELING THE LIMITING SIZE-DENSITY RELATIONSHIP OF LOBLOLLY PINE  
(*PINUS TAEDA* L.) PLANTATIONS**

by

**ROBERTO VOLFOVICZ LEON**

(Under the Direction of Bruce E. Borders)

**ABSTRACT**

The effect of stand factors on the limiting size-density relationship was studied. Management intensity, planting density, and site index significantly impacted the parameters of the limiting size-density relationship. Different fitting methods and mathematical forms were proposed to model the size-density trajectories. The mixed-effects analysis technique proved to be a valuable statistical approach for fitting the limiting size-density line and providing the opportunity to test the significance and impact of additional factors. A semiparametric interaction mixed model was able to completely describe the density-size trajectory of even-aged loblolly pine stands in the southeastern U.S.

**INDEX WORDS:** Self-thinning, Size-density trajectories, Mixed models, Semiparametric regression, Stochastic frontier regression, Stand density index, Loblolly pine

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## DEDICATION

I dedicate this dissertation to my wonderful family. Particularly to my understanding and patient wife, Alejandra, who has been proud and supportive of my work and has shared the many uncertainties, challenges and sacrifices for completing this dissertation, and to our precious daughter Martina , who is the joy of our lives. Above all, the sacrifices, patience, and love of wife and daughter have been a thing of beauty. I am humbled by such grace; I love you with all my heart.

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## **CHAPTER 1**

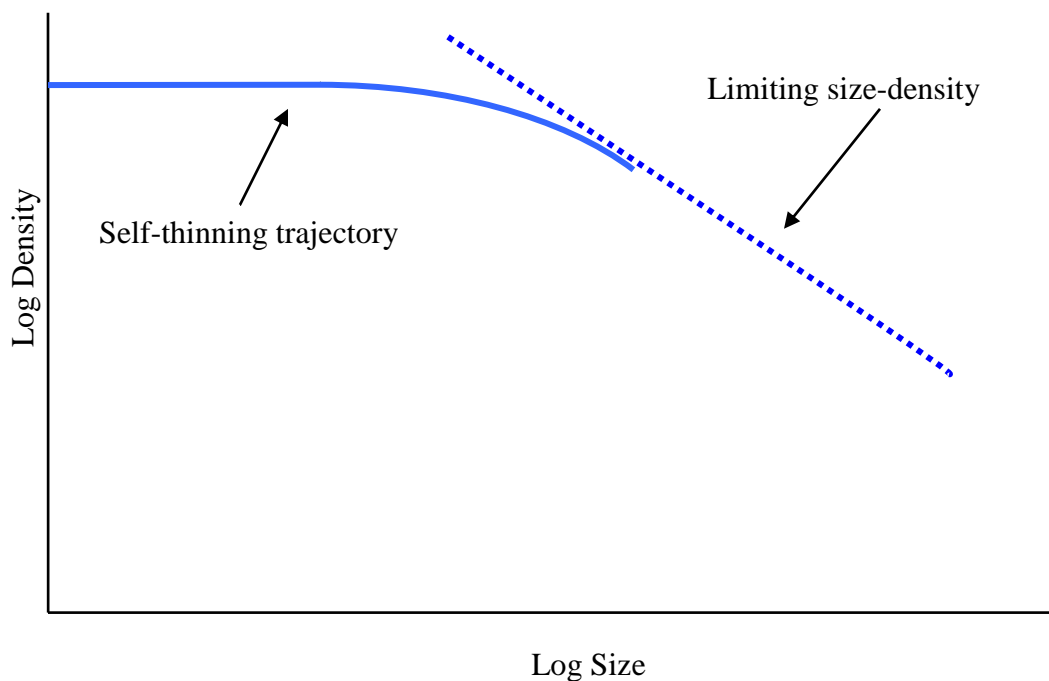
### **Introduction and Review of Literature**

#### **1.1. Introduction**

The relationship between average tree size and tree density has been a topic of discussion and research for more than seventy years (Reineke 1933; Yoda et al. 1963; Pretzsch 2002; Bi 2004; Pretzsch and Biber 2005; Reynolds and Ford 2005; Vanclay and Sands 2009; Comeau et al. 2010; VanderSchaaf 2010). Several investigators have claimed that the relationship between the number of plants per unit area and their size constitutes the foundation of ecology (Drew and Flewelling 1977; Westoby 1981; Zeide 1987; Jack and Long 1996). Size-density relationships have been used to investigate stand development and self-thinning relationships (Rio et al. 2001; Long et al. 2004), to develop relative density indices (Reineke 1933; Curtis 1970) and stand management diagrams (Drew and Flewelling 1977, 1979; Vacchiano et al. 2008), and to serve as a predictor of stand growth (Biging and Dobbertin 1995; Pretzsch 2005). An understanding of the size-density relationships allows silviculturists to design management prescriptions that alter available growing space and influence tree size, growth, and mortality (Lhotka and Loewenstein 2008).

Higher stand densities (e.g. trees per hectare) are associated with smaller average size (e.g. diameter at breast height, quadratic mean diameter, crown width, or leaf area per tree) due

to increased competition for physical space or available resources such as light, nutrients and water (Dean and Long 1992; Jack and Long 1996; Long et al. 2004). Stands growing at high densities are subject to density-dependent mortality or self-thinning (Westoby 1984). For a given average tree size, there is a limit to the number of trees per unit area that may co-exist in an even-aged stand. The relationship linking the average tree size and the maximum number of surviving trees per unit area may be described by means of a self-thinning boundary line (Reineke 1933). The self-thinning line defines a straight upper boundary on log-log scales for all possible combinations of mean individual size and density in plant populations (Figure 1.1).



**Figure 1.1** Number of plants per unit area in relation to average size. The self-thinning trajectory represents a typical development pattern for an individual even-aged stand that undergoes density dependent mortality through time (after Jack and Long 1996).

The existence of a boundary for combinations of average size and density implies that average size increases only to some limit, beyond which there is a reduction in the number of individuals (Jack and Long 1996). In the present chapter important issues of the self-thinning boundary line that relate to the studies reported in this dissertation will be reviewed.

## **1.2. Modeling the limiting size-density relationship**

Tree size and stand density have been traditionally related in two major ways: Reineke's equation (Reineke 1933), and the so-called 3/2 power law of self-thinning (Yoda et al. 1963). However, several researchers have questioned the validity of these two theoretical relations for the purpose of describing the self-thinning process. In the last 30 years numerous attempts to model the limiting size-density relationship have been made, involving the use of diverse mathematical model forms, the inclusion of different explanatory variables into the mathematical models, and the application of diverse fitting methods to estimate the parameters in the model (Drew and Flewelling 1977; Hynynen 1993; Zeide 1995; Robinson 1998; Yang and Titus 2002; Bi 2004; Zhang et al. 2005; Weiskittel et al. 2009).

### **1.2.1. Reineke's size-density boundary line and stand density index**

Based on empirical evidence Reineke (1933) found that the maximum quadratic mean diameter at breast height,  $D_q$ , for a given number of trees per unit area,  $N$ , in even-aged stands of full density could be represented on a log-log scale as

$$[1] \quad \ln(N) = \alpha + \beta \cdot \ln(D_q)$$

Reineke obtained this relationship by graphing  $Dq$  and  $N$  for untreated forest inventory plots on a double-logarithmic grid. He claimed that the slope was constant among species, while the intercept varied over a range of species, but not within species, regardless of site quality and stand age. When the intercept is shifted such that all size-density points fall under the curve, the curve represents the maximum size-density relationship for the species. Four assumptions are associated to Equation [1]: (i) a predictable species-specific relationship exists between maximum size and stand density; (ii) the relationship is linear in log-log scale; (iii) the slope of the limiting size-density boundary line is universal; and (iv) the intercept is constant for a given species and region (Jack and Long 1996). These assumptions are commonly considered as reasonable approximations but have been difficult to assess rigorously (Shaw 2006; Weiskittel et al. 2009). Reineke (1933) found a slope of -1.605 for 12 out of 14 species examined. Steeper slopes were noted for slash pine (*Pinus elliottii* Englem.) and longleaf pine (*Pinus palustris* Mill.). For loblolly pine (*Pinus taeda* L.), the slope was estimated to be -1.707 by MacKinney and Chaiken (1935), -1.696 by Harms (1981), -1.505 by Williams (1996), and -1.926 by Lhotka and Loewenstein (2008). For radiata pine (*Pinus radiata* D. Don), Bailey (1972) reported a slope of -1.58. Drew and Flewelling (1977) gave a slope -1.54 for Douglas-fir (*Pseudotsuga menziesii* [Mirb.]).

From Equation [1], the maximum number of trees per unit area at a given quadratic mean diameter is

$$[2] \quad N = C \cdot Dq^\beta$$

where  $C = e^\alpha$ . This limiting relationship establishes that a given increase in average diameter,

$\frac{dDq}{Dq}$ , eliminates a fixed proportion of trees equal to  $\beta \cdot \frac{dDq}{Dq}$  (Zeide and Stephens 2010).

Reineke defined the expected maximum number of trees at a reference quadratic mean diameter of 25.4 cm as stand density index (SDI)

$$[3] \quad SDI = C \cdot (25.4)^\beta$$

For a stand of known N and Dq, SDI can be written as

$$[4] \quad SDI = N \cdot \left( \frac{25.4}{Dq} \right)^\beta$$

implying that all stands with the same proportion of the limiting number of trees per unit area have the same SDI regardless of the average stand diameter (Clutter et al. 1983).

### 1.2.2. Yoda's $-3/2$ power law of self-thinning

In the late 1950s and early 1960s, Japanese scientists investigated the size-density relationship experimentally, postulating the so-called ' $-3/2$  self-thinning law'. The law was first proposed by Tadaki and Shidei (1959), but is best known from the work of Yoda et al. (1963) and its re-evaluation by White & Harper (1970). Using a diversity of agricultural crops and weed species, Yoda et al. (1963) analyzed mean plant mass across a range of densities and established what they referred to as the  $-3/2$  power law of self-thinning. The authors plotted mean plant mass ( $b$ ) versus density ( $N$ , number of plants per unit area) on the log-log scale, and observed that the relationship followed a straight line with universal slope close to  $-1.5$  for all species in their study:

$$[5] \quad \ln(b) = \alpha' - \frac{3}{2} \cdot \ln(N)$$

Yoda et al. (1963) considered plants as simple Euclidian objects, where all their parts scaled isometrically to each other. Yoda's coefficient  $-3/2$  is based on the cubic relation between

plant diameter and biomass,  $b \propto Dq^3$ , and the quadratic relationship between plant diameter and occupied growing area  $s$ ,  $s \propto Dq^2$ . Since  $s$  is defined as the inverse of number of plants ( $s=1/N$ ), the relationship between  $s$  and  $Dq$  can be written as  $N \propto Dq^{-2}$  or  $Dq \propto N^{-1/2}$ . It follows  $b \propto (N^{-1/2})^3 \propto N^{-3/2}$  (i.e. Equation [5]).

Using total plant weight per unit area ( $B$ ) as a response variable, the self-thinning line can be written as

$$[6] \quad \ln(B) = \alpha' - 0.5 \cdot \ln(N)$$

where the intercept  $\alpha'$  is assumed to be species-specific and environment-specific. Since  $B = b \cdot N$ , then  $B \propto N^{-3/2} \cdot N \propto N^{-1/2}$  (i.e. Equation [6]). Drew and Flewelling (1977) applied this equation to forest populations and noted the relationship to Reineke's maximum size-density model.

Yoda's power law of self-thinning defines a straight upper boundary line on log-log scales for all possible combinations of mean individual biomass and density in plant populations. West et al. (1997) and Enquist et al. (1998) challenged the traditional  $-3/2$  slope of the Yoda's upper boundary line by  $-4/3$ , which was deduced from mechanical theory. They presented a model that considered plants as fractal objects and postulated the generality of quarter-power scaling:  $b \propto N^{-\frac{4}{3}}$  where  $b$  is mean biomass per tree and  $N$  is number of trees per unit area, for unmanaged, fully stocked stands.

### **1.2.3. Reineke's SDI and $-3/2$ power law of self-thinning in application**

In the words of Zeide (2005), Reineke's density assessment with the stand density index "may be the most significant American contribution to forest science". Yoda's self-thinning line and Reineke's stand density index are practical and broadly used in empirical plantation growth and yield models to predict natural mortality (Monserud et al. 2005; Poage et al. 2007), as well as in process-based models (Landsberg and Waring 1997). Biging and Dobbertin (1995) and Pretsch (2005) included stand density indices based on the self-thinning rule as predictors in growth models. Limiting size-density relationships have been used as constraints in growth and yield models for both the  $\log(\text{Volume})$ – $\log(\text{Density})$  relationship (Smith and Hann 1984, 1986; Turnblom and Burk 2000) and the  $\log(\text{Density})$ – $\log(Dq)$  relationship (Hynynen 1993). Models that predict diameter at breast height (DBH), height, volume or weight have been used to forecast stand development until size-density trajectories reach the boundary line and stands are then assumed to self-thin along this constraint (Maguire et al. 1990; Yang and Titus 2002; Monserud et al. 2005).

Reineke's rule has achieved substantial importance for the quantification and control of stand density and modeling of stand development in pure (Long 1985; Newton 1997; Ducey and Larson 1999; Puettmann et al. 1993; Rio et al. 2001; Bi 2004) and mixed (Puettmann et al. 1992; Sterba and Monserud 1993; Weiskittel et al. 2009) stands.

Foresters have also incorporated the maximum size-density relationship into stand management diagrams (Drew and Flewelling 1977, 1979; Farnden 1996; Vacchiano et al. 2008), which graphically depict density and stand dynamics. They describe the relationship between stand density, average tree size, stand yield and dominant height, based on relevant ecological

and allometric relationships such as the self-thinning rule, providing silviculturists with a suitable tool for assessing stand density and planning interventions (Shaw 2006).

Reineke's stand density index and the  $-3/2$  power law are basically the same in that both are used to define a maximum stand density at a given stand average size (Yang and Titus 2002). To relate the number of trees/area with an expression of the size of the average tree, Zeide (1987) recommends using the square mean diameter (Reineke's equation) instead of the average tree biomass (Yoda's expression), since diameter is more correlated with crown width. Additionally, diameter can be measured more accurately and easily than biomass and it is the most practical size variable for management (Rio et al. 2001). The difference in the slopes from Reineke and Yoda arises from the different allometry between quadratic mean diameter and mean plant biomass (i.e. volume or weight). The  $-3/2$  power law exponent relating volume to stem number is equivalent to Reineke's - 1.605 exponent relating quadratic mean diameter  $D_q$  to stem number ( $N$ ) only when volume is proportional to  $D_q^{2.4075}$ , and Reineke's rule becomes a special case of Yoda's (Pretzsch 2009).

#### **1.2.4. Biological interpretation of the self-thinning slope line: self-tolerance**

The parameters and mathematical form of models that reproduce mechanisms of growth can be analyzed in terms of biology, in addition to statistics (Zeide 2005). Reineke's slope is a measure of intraspecific competitive ability of trees, called self-tolerance by analogy with tree tolerance (Zeide 1985). Tolerance indicates the interspecific competitive ability of trees, while self-tolerance refers to the capability of trees to compete with or tolerate conspecifics (Zeide

2005). Self-tolerance is hence important for determining optimal planting density, thinning intensity, and other silvicultural activities in pure stands.

A measure of self-tolerance is the ratio of trees,  $-\frac{dN}{N}$ , eliminated by a given increase in average quadratic diameter,  $\frac{dDq}{Dq}$  :

$$[7] \quad \text{Self - tolerance} = -\frac{dN/N}{dDq/Dq}$$

The smaller the ratio, the fewer trees die with the same increase in quadratic mean diameter, and therefore, the more self-tolerant is the species.

Differentiating Reineke's Equation [1] yields the following expression for the slope  $\beta$  :

$$[8] \quad \beta = -\frac{dN/N}{dDq/Dq}$$

The identity of Equations [7] and [8] results in  $\beta$ , the slope of the self-thinning line, being a measure of self-tolerance in pure stands (Zeide 2005). Species that have high self-tolerance, by this definition, have larger  $\beta$  than those with less self-tolerance (i.e. for a given reduction in density, they are able to make a greater increase in size).

### **1.2.5. Criticism of Reineke's SDI and the $-3/2$ power law of self-thinning. Alternative mathematical forms to model the size-density relationship.**

Reineke's stand density index and the  $-3/2$  power law of self-thinning have received both vast approval (White and Harper 1970; Harper 1977; Hutchings and Budd 1981; Long and Smith 1984; Westoby 1984), and enormous scrutiny. The suitability of these two theoretical relations for the purpose of describing the limiting size-density relationship has been called into question by various authors in the last 30 years (Drew and Flewelling 1977; Weller 1987, 1991; Zeide 1987; Bredenkamp and Burkhardt 1990; Lonsdale 1990; Hynynen 1993; Schutz and Zingg 2010). Long and Smith (1984) referred to the  $-3/2$  power law of self-thinning as “a true law instead of the mere rule”, Robinson (1998) claimed it to be "a rigid manifestation" of the self-thinning property, while Lonsdale (1990) stated that it was too simple to be biologically believable. Biologists have been traditionally reluctant to accepting theories that some call ‘laws’, as the review articles by Weller (1987), Zeide (1987), and Monserud et al. (2005) illustrate.

One of the main criticisms to Reineke's and Yoda self-thinning law has been the assumption of a universally fixed slope coefficient across species, age, and site quality (Gadow 1986; Pretzsch and Biber 2005). Instead of being a constant, the slope parameter of the self-thinning line has been found to be variable for different species (Zeide 1987; Jack and Long 1996; Bi and Turvey 1997; Weiskittel et al. 2009). Several studies have shown that stands self-thin at differing levels of maximum SDI, and there is not agreement in the processes driving this variation, which has led various investigators to conclude that each stand has its own dynamic self-thinning trajectory (Weller 1987, 1991; Pittman and Turnblom 2003; VanderSchaaf 2004; Reynolds and Ford 2005; Weiskittel et al. 2009). Weller (1987, 1991) separated the law into two

concepts, the “dynamic self-thinning line” and the “species boundary line” by defining two types of limiting size-density relationships: (i) individual stand maximum size-density relationship boundaries referred to as dynamic thinning line boundaries, and (ii) species maximum size-density relationship boundary line, defined as a static upper limit of maximum tree density-average tree size relationship that applies to all stands of a certain species within a geographical area. The dynamic thinning line is assumed to be affected by stand conditions, genetics, and silvicultural treatments, while the species maximum size-density relationship boundary line cannot be impacted by them (Weller 1991; VanderSchaaf and Burkhardt 2007). The general acceptance that the maximum size-density boundary line is constant for a given species and therefore independent of site quality and stand age can be attributed then to the lack of rigorous testing of such a relationship rather than to strong evidence in support of it (Jack and Long 1996). Identification of the key factors driving the variation of the self-thinning relationship continues (Weiskittel et al. 2009). Some studies have illustrated that individual stands do not always approach the maximum size-density boundary because of genetics or environmental limitations (Weller 1987; Zeide 1987; Hynynen 1993). Studying the effects of stand conditions on the self-thinning line, Zeide (1987) suggested that steeper slopes are associated with stands of optimal conditions, while flatter slopes are related to those of suboptimal conditions. Hence, variations in the self-thinning slope are not necessarily species-specific, but they may be site-specific. Site index has been identified as a key factor influencing the dynamic and species self-thinning line in Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco) (Pittman and Turnblom 2003) and radiata pine (*Pinus radiata* [D. Don]) (Bi 2001), respectively. Hann et al. (2003) concluded that fertilization had no influence on the limiting size-density trajectory in Douglas-fir. Puettmann et al. (1993) found that planting density had no effect on the

size-density relationship in red alder (*Alnus rubra* [Bong]), while Turnblom and Burk (2000), Reynolds and Ford (2005), and VanderSchaaf and Burkhart (2007) found that initial stand density is an important factor influencing the species self-thinning line. These differing results can be explained by the quality of the available data as well as the variety of statistical methods that have been used to examine the species self-thinning limiting line (Weiskittel et al. 2009).

Several methods have been applied to estimate the parameters of the limiting size-density line (Zhang et al. 2005), among others: arbitrarily hand placing a line above an upper boundary of points (Reineke 1933; Yoda et al. 1963), fitting an ordinary least square regression line (Mackinney and Chaiken 1935; Rio et al. 2001), estimating coefficients through quantile regression (Cade et al. 1999), principal component analysis and reduced major axis regression (Bi and Turvey 1997; Wilson et al. 1999; Lhotka and Loewenstein 2008), adopting stochastic frontier regression to model the limiting line (Bi 2001 and 2004; Weiskittel et al. 2009), or using a linear mixed model approach (Hynynen 1993; VanderSchaaf and Burkhart 2007). Some of these methods have multiple limitations such as using subjective techniques (hand fitting), making statistical inference very difficult (quantile regression), or ignoring autocorrelations between observations from the same plot when determining the boundary line (ordinary least squares and stochastic frontier regression), which make testing the influence of stand and site factors on the limiting size-density relationship problematic. In addition, ordinary least square regression is documented as unsuitable to fit limiting size-density relationships due to the unsatisfied assumption of zero error variance in the explanatory variables (Weller 1987). Likewise, the assumptions of equal error variances for the response and explanatory variables for principal components analysis regression, and error variance proportional to intrinsic variance for reduced major axis regression might not be valid.

Size-density boundary lines are developed from repeated measurement data, which are usually correlated due to multiple measurements made on each experimental plot, and the resulting residual variances are often not homogeneous (Gregoire 1987). Repeated measurement data present two aspects of the data that are interrelated and require modeling: the mean response and the covariance among recurring measurements on the same individuals. This interdependence occurs because the vector of residuals (observed responses minus fitted responses) depends upon the specification of the model for the mean. Failure to take account of the covariance among repeated measures will generally result in incorrect estimates of the sampling variability (Fitzmaurice et al. 2004). With incorrect standard errors, tests statistics and p-values will also be incorrect and hence can result in misleading inferences. Linear mixed-effects analysis can use random plot effects to account for correlation among observations from the same cluster-plot (Lappi and Bailey 1988; Schabenberger and Pierce 2001; VanderSchaaf and Burkhart 2007).

To perform the fitting and to interpret the result as a line of maximum size-density, it is necessary to be certain that the stands analyzed are fully stocked or at maximum density (Weller 1987; Bi and Turvey 1997). Improperly including observations from stands of mean density that have not yet started to self-thin will flatten the estimated slope of the limiting line, while inclusion of observations coming from stands of high density that have not yet started to self-thin will steepen the slope of the line (Zhang et al. 2005). A regularly addressed problem in the fitting of the size-density limiting relationship is the identification of those observations that are occurring in the self-thinning phase of stand development and, more explicitly, what observations occur along the thinning line. Numerous selection methods have been postulated, varying from choosing data points that lie close to an arbitrarily visualized upper boundary

(Westoby 1984; Weller 1987, 1991; Cao et al. 2000; VanderSchaaf and Burkhart 2007) to using some statistically based principle. Blackburn et al. (1992), Bi and Turvey (1997), Yang and Titus (2002) and Comeau et al. (2010) selected data points to fit the limiting relationship by grouping observations into a number of equal size classes and choosing the uppermost points within each size class. Rio et al. 2001 selected only data points from plots where the annual mortality rate was above 1% and did not suffer mortality due to exceptional causes. There is still not agreement among foresters and biologists about the selection criteria to use when choosing what observations occur along the self-thinning line (Smith and Hann 1984; Rio et al. 2001; Zhang et al. 2005).

To avoid subjectivity on the selection of the observations that are used to fit the upper boundary of the tree size and tree density relationship some authors have developed models that describe the whole size-density trajectories of individual stands and hence eliminate the need to censor data. All data points, whether from fully stocked or under stocked stands, are subsequently used in the fitting process. The limiting size-density relationship materializes then from overlapping trajectories of individual stands (Smith and Hann 1986; Puettmann et al. 1993; Tang et al. 1994; Zeide 1995; Cao et al. 2000). Zeide (1987) described the self-thinning process as a nonlinear concave down rather than a straight line ( $\ln(N)-\ln(Dq)$ ). Three phases can be distinguished in the curve: (1) from young stage until crown closure, when the self-thinning is less than the mortality at maximum competition, it is concave and positioned under the line of maximum density; (2) in the intermediate phase of the stand's development, it follows the straight line of maximum density; and (3) in aged stands, when the capability to fill gaps left by dead trees has been reduced, the line tends to curve below and diverge from the line projected by

Reineke's equation (Zeide 1995). Recently, Schutz and Zingg (2010) assessed the curvilinearity of the size-density relationship by introducing  $D_q$  and its square into the Reineke's model.

Smith and Hann (1986), Puettmann et al. (1993), and Tang et al. (1994) proposed a different trajectory for the size-density relationship, in which once the line of limiting density has been reached, the curve then follows it as an asymptote. To describe the size-density trajectory, Zeide (1995) proposed a variation to Reineke's equation considering that the slope would not be constant but varying with age, and including total height as an additional covariate in the model. Yang and Titus (2002) derived a nonlinear regression model to describe the maximum size-density relationship and used it for constraining individual tree mortality functions. Cao and Dean (2008) and VanderSchaaf and Burkhardt (2008) applied segmented regression techniques to modeling the trajectory of stand density and quadratic mean diameter of individual stands through time for slash and loblolly pine stands, respectively.

Bi (2001) introduced a generalized expression of the self-thinning rule by including site index as an explanatory variable in the model. His equation became a two-dimensional self-thinning surface that defined a density-dependent upper frontier of stand biomass over a gradient of site productivity for a given species. Weiskittel et al. (2009) considered the species self-thinning boundary as a multidimensional surface as site index, stand origin, and stand purity showed a significant effect when modeling the limiting size-density relationship in mixed stands of Douglas-fir, western hemlock, and red alder in the Pacific Northwest.

### 1.3. Stand basal area as a measure of stand density

Basal area per hectare is a measure of the cumulative cross-sectional area at breast height (1.37 m above ground) for a stand of trees. Stand basal area is an important indicator of stand density, which simultaneously takes into account the average tree size and the number of trees per unit area. Recently, some authors have introduced the maximum basal area attained during self-thinning as a covariate in the limiting size-density relationship (Skovsgaard and Vanclay 2008; Vanclay and Sands 2009; Tewari 2010).

Goulding (1972) presented a hyperbolic relationship between quadratic mean diameter ( $D_q$ ), dominant height ( $H$ ) and stand density ( $N$ ):

$$[9] \quad D_q = \frac{1}{a \cdot H^b N + c \cdot H^d}$$

where  $a$ ,  $b$ ,  $c$ , and  $d$  are parameters to be estimated. Sterba (1987) extended Goulding's work and showed how to calculate the theoretical maximum basal area along the limiting size-density curve. Substituting Equation [9] into the per unit area basal area ( $BA$ ) gives:

$$[10] \quad BA = \frac{\pi}{4} D_q^2 \cdot N = \frac{\pi}{4} \left( \frac{1}{a \cdot H^b N + c \cdot H^d} \right)^2 \cdot N$$

The number of trees per unit area at maximum basal area ( $N_{BA \max}$ ) may be obtained by setting the first derivative of Equation [10] with respect to  $N$  equal to zero (Sterba 1987):

$$[11] \quad N_{BA \max} = \frac{c}{a} \cdot H^{(d-b)}$$

Replacing  $N$  in Equation [9], we get the quadratic mean diameter at maximum basal area ( $D_{q \max}$ ):

$$[12] \quad Dq_{BA \max} = \frac{1}{2 \cdot c \cdot H^d}$$

Solving Equation [12] for H and replacing the expression in Equation [11] we obtain the number of trees per unit area at maximum basal area (Sterba 1987):

$$[13] \quad N_{BA \max} = \frac{c}{a} \cdot \left( \frac{1}{2} \cdot c \cdot Dq_{BA \max} \right)^{\frac{b}{d}-1}$$

Equation [13] represents a limiting size-density relationship along maximum basal area.

Maximum basal area carrying capacity can be defined as the maximum amount of basal area of a given species that can be sustained on a long-term basis within a stand (Helms 1998). VanderSchaaf and Burkhardt (2007b) analyzed the connection between the limiting size-density relationship and the maximum basal area carrying capacity and concluded that for stands with a limiting size-density relationship (LSDR) slope larger than  $-2$ , maximum basal area carrying capacity will occur after a stand has reached its LSDR stage of stand development (i.e. after Reineke's SDI has been maximized), while if a LSDR's slope is smaller than  $-2$ , a stand will reach its maximum basal area carrying capacity prior to reaching its LSDR stage of stand development. Maximum basal area carrying capacity and the LSDR stage of stand development will occur simultaneously in stands that have a LSDR slope equal to  $-2$  (VanderSchaaf and Burkhardt 2007).

Many silvicultural and forest management considerations, such as thinning intensity, are based on basal area ground measurements. The average annual increment curve of basal area is a valuable tool for management of forest stands and contributes to estimate the timing of intermediate and final cuts (Sun et al. 2007). In addition, basal area is important for forest inventories because it is highly correlated with volume and growth of forest stands.

#### **1.4. Stand development and size-density relationship**

Even-aged stands develop from a set of individual, free to grow trees through the start of competition, to full site occupancy, self-thinning, and finally the development of multicohort stand structures (Long and Smith 1984; Oliver and Larson 1990; Long et al. 2004). The relationship between tree size-density in developing stands has an essential place in population dynamics and quantitative silviculture (Jack and Long 1996). Different combinations of mean size and density can represent the same relative density (i.e. many small trees can have the same relative density as fewer, but larger trees).

The different forms in which stand-level leaf area might be distributed among individual trees can be considered in the context of changes in tree size-density during stand development. At lower relative densities individual trees bear large quantities of leaf area compared to trees of the same age in denser stands. At those low relative densities, stand-level leaf area is below the upper limit, and stand growth, hence, is below potential for the species, site quality and stand age (Long et al. 2004). At higher relative densities, individual tree leaf area, and therefore tree growth, is much lower than that of open grown trees of the same species and age on the same site. Stand-level leaf area approaches its upper limit and stand growth is at its potential (Smith and Long 2001). At full site occupancy of stand development, the specific combination of tree size and stand density determines how a fixed amount of stand foliage is distributed among trees in the stand. As the stand develops, the stronger competitors continue to accumulate more foliage at the expense of weaker trees. The process of self-thinning can be considered as the natural redistribution of a fixed amount of total leaf area onto gradually fewer and larger trees (Assmann 1970; Osawa 1995; Long et al. 2004).

Measures of relative density, as the Reineke's SDI, are important in the design and application of silvicultural treatments. One reason is that site quality does not generally have a substantial effect on the nature of the size–density trajectory during stand development (Harms et al. 2000). Site quality influences the rate at which a stand moves along its trajectory. On high quality sites, individual tree and stand leaf area accumulates faster, resulting in greater potential tree and stand growth. Given the same initial density, stands on more productive sites will reach the beginning of competition, canopy closure, full site occupancy, and self-thinning more rapidly than stands on poorer sites. Hence, while tree size–density relations are basically independent of site quality, the rate of stand development is greatly dependent on site quality (Long et al. 2004).

Silvicultural practices can have long lasting influences on site quality (i.e. drainage of excessively wet sites; fertilization on phosphorus deficient sites). In these situations, stand development and the rate at which stands move along the size–density trajectory may be permanently altered. In other situations, however, silvicultural treatments result in just temporary increases in site growth potential (i.e. a single application of nitrogen generally results in a growth response of relatively few years duration), and the effects of the treatments can be thought as a temporary boost in the rate of stand development. The result is an acceleration of the stand along its size–density trajectory for some years, after which the rate of stand development returns to normal. The main result, however, is that the leap ahead is usually permanent. Stand dynamics and structure are such that the stand appears older and more mature than its actual age (i.e. further along the size–density trajectory) (Jack and Long 1996; Long et al. 2004). While the stand may be further ahead in growth and leaf area accumulation, it is also further ahead in self-thinning. The most important practical result of such temporary accelerations in stand development is a shortened rotation (Miller, 1981).

## **1.5. Silvicultural effects on the size-density relationship**

Responses to silvicultural practices result from their influence on the amount of resources potentially available for growth, the ability of trees to acquire those resources, and the distribution of resources among population components. An outline of the influence of different silvicultural practices on resource availability and growth follows.

### **1.5.1 Site preparation**

Site preparation practices can be classified into two categories: those that manipulate the physical properties of the soil, and those that control competition (Morris and Lowery 1988). Practices that manipulate the soil include plowing, harrowing, mounding, bedding, and subsoiling and they are applied to increase the volume of soil available for root exploration and the pace in which growing space is captured by the trees. Practices to control competition include applying herbicides, scalping, root raking, shearing, chopping, harrowing, burning, dragging, and mulching and they are aimed to reallocate growing space to the desired species. Practices of either group have been found to increase growth and survival when compared to no site preparation. (Haywood and Tiarks 1990; Sutton 1993; Varelides and Kritikos 1995; Haywood et al. 1997; Long et al. 2004, Wagner et al. 2006).

Borders and Bailey (2001) analyzed intensive silviculture treatments for loblolly pine plantation management at six sites in Georgia (including locations with high shrub density). After intensive mechanical site preparation, continuous vegetation control increased

merchantable volume through ages 10–12 years from 37–122%. Adding repeated fertilization further improved yields.

One of the most comprehensive studies examining growth and yield improvements from woody and herbaceous weed control in the southern United States was conducted by Miller et al. (1995, 2003). The experimental design was replicated in 13 plantations across seven southern states and four physiographic provinces of the region. Loblolly pine plantations were monitored for 15 years with near-complete control of woody, herbaceous, and woody plus herbaceous components during the first 3–5 yr. A factorial combination of two woody control treatments (no woody control vs. complete woody plant control) and two herbaceous control treatments (no herbaceous control vs. complete herbaceous plant control) was established. Herbicides were used before planting and annually through crown closure (3–5 years after planting) to establish and maintain the treatments. Pine yields at year 15 were strongly influenced by herbicide treatments applied during the first 3–5 years after planting. Controlling both woody and herbaceous vegetation increased merchantable wood volumes 67% above that on plots that were only site prepared. Control of only woody vegetation increased merchantable pine volume on 11 sites by 14–118% and gains on treated plots increased as hardwood and shrub abundance increased on the check plots. Gains in volume from early control of only herbaceous vegetation (leaving woody vegetation) increased 17–50% on 10 sites.

### **1.5.2. Thinning**

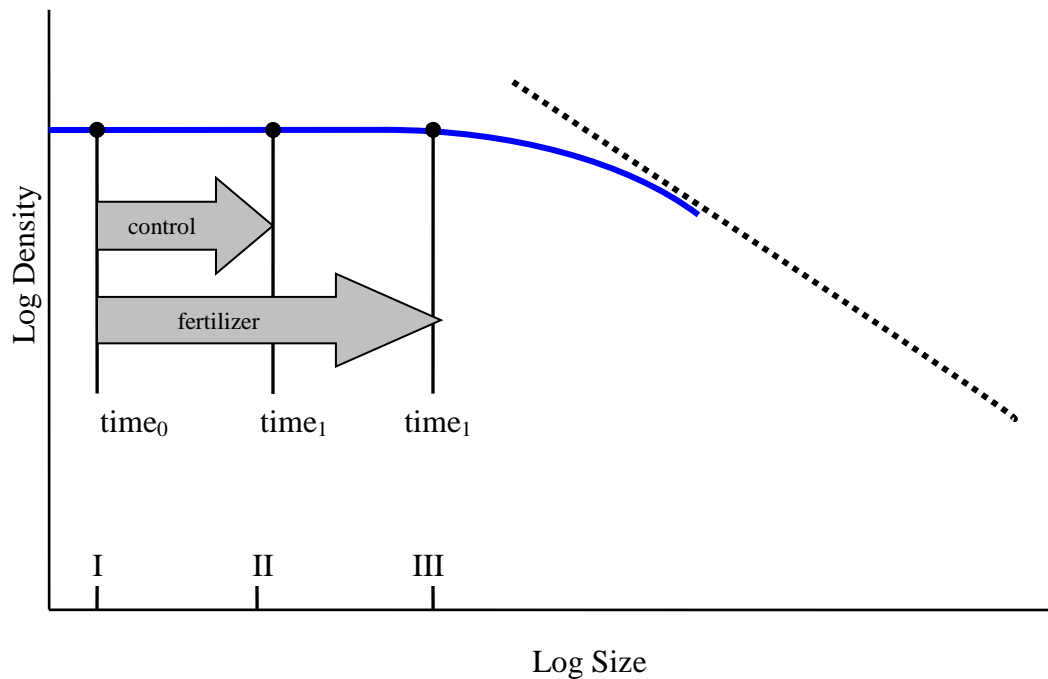
Thinning denotes the treatment of removing selected trees from a stand to enhance the growth of residual trees. Average tree size–density relations are an essential part of thinning

theory and are the basis of many practical thinning tools (Jack and Long, 1996). This practice implies the manipulation of the canopy and its leaf area. Two important components are associated with this treatment: an instant reduction in stand-level leaf area, followed ultimately by an increment in the leaf area of residual trees. The quantity of reduction in stand level leaf area, and the time necessary to recover that leaf area, represent foregone potential stand growth (Zeide 2001, Long et al. 2004). Thinning, over time, proceeds to reallocate leaf area onto residual trees —similar to but more severe than self-thinning (Long et al. 2004). Post-thinning recuperation of stand-level leaf area results from increments in individual tree leaf area that arises faster than it would in dense stands. The increase in individual tree leaf area is associated with greater individual tree growth. Therefore, temporary declines in stand-level leaf area result in the loss of potential stand growth, but increases in individual tree growth. This is the tradeoff associated with allocating leaf area onto fewer and larger trees.

### **1.5.3. Fertilization**

Fertilization is commonly implemented in forest management to enhance productivity. This practice accelerates leaf area growth, increases the growth rate of trees, and reduces the number of years required to achieve maximum current and mean annual increments (Smith and Long 2001). Increments in productivity associated with fertilization result from increases in the total quantity of leaf area, the rate of leaf area accumulation, and the growth efficiency of the leaf area (Albaugh et al. 1998; Fisher and Binkley 2000). The growth response to increased leaf area can be illustrated using average tree size–density relationships (Figure 1.2): the stand follows the same size-density trajectory, but with an augmented rate of development (Long et al. 2004). The

result of fertilization is a reduction in the time required to attain, or regain, maximum leaf area (Mitchell et al. 1996; Will et al. 2002).



**Figure 1.2** Normal progression of a stand from mean size I to mean size II (control arrow). Accelerated progression of the stand to mean size III (fertilizer arrow). The time taken to progress to either II (control) or III (fertilizer) is the same. Stand at mean size III is the same age as at mean size II, but is more developed and appears to be older (after Long et al. 2004).

Fertilizing stands already carrying maximum leaf area (i.e. fully stocked) is not expected to have a significant effect on stand growth as the ability of individual trees to add leaf area in such situations is limited. Fertilization in fully stocked stands of Douglas-fir has even been reported to accelerate the self-thinning process, since site resources are not capable of supporting the increases in stand-level leaf area (Mitchell et al. 1996).

Hann et al. (2003) found that fertilization had no significant effect on the Douglas-fir or western hemlock self-thinning boundary line intercept or slope. Dean and Jokela (1992) found

that fertilization did not affect the maximum values of stand density (measured as number of trees per unit area) obtained for slash pine plantations, suggesting that accelerated stand development does not modify the stand densities associated with the various stages of stand development.

Stand response to fertilization can be long-term or short-term. Long-term effects take place when a slowly soluble nutrient (i.e. phosphorus) is added to the soil. Turner et al. (2002) reported responses to last over 50 years following phosphorous fertilization in a *Pinus radiata* stand. Short-term responses to fertilization are typical of a single application of nitrogen fertilizer and generally last 4 to 8 years. Whereas the response to phosphorous fertilization is equivalent to an increase in site quality, the short-term response typical of nitrogen fertilization is basically a temporary increase in stand development, benefiting trees, but not the site (Miller 1981). Fox et al. (2007) reported growth increases in volume that averaged 25% after nitrogen plus phosphorous fertilization in loblolly pine stand in the southern United States. These responses have normally continued for at least 6–10 years, depending on soil type, fertilizer rates, and stand conditions (Fox et al. 2007).

## **1.6. Objectives**

Based on the above discussion, it is clear that much has been studied about limiting size-density relations and their importance in forestry. However, there is still disagreement regarding adequate mathematical forms and the associated fitting methods that model the limiting-size density relationship in forest stands, and the effects of site quality and silvicultural factors on the relationship. Thus, there is the need to develop and validate new models that describe this

relationship and that can detect and quantify the effects of site quality and silvicultural treatments on the limiting size-density trajectories, providing a better understanding of this important component of forest stand dynamics.

The broad objectives of research documented in this dissertation were:

1. To model the species maximum size-density relationship by integrating a selection of other stand factors beyond the quadratic mean diameter such as silvicultural intensity, planting density, site index, and soil type into the model,
2. To compare and illustrate the application of alternative mathematical forms that have been or can be used to modeling the trajectory of stand density and quadratic mean diameter of individual stands through time, and
3. To compare and illustrate the application of different fitting methods that have been or can be used to estimate the parameters of the limiting size-density trajectory.

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## CHAPTER 2

### **Impact of Silvicultural Treatments and Site Quality on the Limiting Size-Density Relationship in Loblolly Pine Stands (*Pinus taeda* L.): a Mixed-Effect Modeling Approach**

#### **2.1. Abstract**

The effect of stand factors on the limiting size-density relationship in even-aged loblolly pine (*Pinus taeda* L.) stands was studied. The mixed-effects analysis technique proved to be a valuable statistical approach for fitting the limiting size-density line and providing the opportunity to test the significance and impact of additional factors. The assumption that the intercept and slope of the species limiting size-density relationship are invariant to stand and site factors was analyzed using data from a culture/density study in the Lower Coastal Plain region of the southern United States. Management intensity, planting density, and site index significantly impacted the parameters of the species limiting size-density relationship. CRIFF soil types did not significantly affect the slope or intercept of the line.

**Keywords:** Self-thinning, Diameter-density relationship, Stand density index, Loblolly pine, *Pinus taeda* L.,

## 2.2. Introduction

The mortality related to competition among trees within a stand is called self-thinning or density-dependent mortality (Yoda et al. 1963; Hynynen 1993; Monserud et al. 2005). The process of self-thinning occurs when individuals grow at high population density, resulting in a negative relationship between individuals per area (N) and average individual size (Westoby 1984). For any given average tree size, there is a limit to the number of trees per unit area that may co-exist in an even-aged stand. The relationship between the average tree size and the number of trees that occupy an area can be described by means of a limiting contour. Weller (1987) defined two types of limiting size-density relationships: (i) individual stand maximum size-density relationship boundaries referred to as dynamic thinning line boundaries, and (ii) species maximum size-density relationship boundary line, defined as a static upper limit of average tree size-maximum tree density relationship that applies to all stands of a certain species within a geographical area. The dynamic thinning lines can be affected by stand conditions, genetics, or silvicultural treatments, while the species maximum size-density relationship boundary line cannot be impacted by them (Weller 1991; VanderSchaaf and Burkhardt 2007).

Reineke (1933) postulated a linear relationship (on the log-log scale) between the number of trees per unit area (N) and quadratic mean diameter (Dq) in even-aged stands of full density:

$$[1] \quad \ln(N) = \alpha + \beta \cdot \ln(Dq)$$

Reineke (1933) claimed that the slope was universal (-1.605) across species, while the intercept varied over a range of species, but not within species, regardless of site quality and stand age. From Equation [1], the maximum number of trees per unit area at a given quadratic mean diameter is given by

$$[2] \quad N = \alpha^* \cdot Dq^\beta$$

where  $\alpha^* = e^\alpha$ . This limiting relationship establishes that a given increase in average diameter,  $\frac{dDq}{Dq}$ , eliminates a fixed proportion of trees equal to  $\beta \cdot \frac{dDq}{Dq}$  (Zeide and Stephens 2010).

Reneike defined this expected maximum number of trees at a reference quadratic mean diameter of 25.4 cm as stand density index (SDI)

$$[3] \quad SDI = \alpha^* \cdot (25.4)^\beta$$

For a stand of known N and Dq, SDI can be written as

$$[4] \quad SDI = N \cdot \left( \frac{25.4}{Dq} \right)^\beta$$

implying that all stands with the same proportion of the limiting number of trees per unit area have the same SDI regardless of the average stand diameter (Clutter et al. 1983).

Yoda et al. (1963) analyzed the variation in mean plant mass across a range of densities and postulated the so-called “-3/2 power law of self-thinning”. This rule establishes that, on the log-log scale, the relationship between mean plant biomass and plant density (number/area) for a stand undergoing self-thinning is a straight line with a constant slope of -1.5 regardless of species, age, and site quality (Jack and Long 1996).

Yoda’s self-thinning line and Reineke’s stand density index are useful and extensively used in plantation growth and yield models to predict natural mortality (Monserud et al. 2005). Reineke’s stand density index and the -3/2 power law are basically the same in that both are used to define a maximum stand density at a given stand average size (Yang and Titus 2002). To relate the number of trees/area with an expression of the size of the average tree, Zeide (1987) recommends using the square mean diameter (Reineke’s equation) instead of the average tree biomass (Yoda’s expression), since diameter is more correlated with crown width. Additionally,

diameter can be measured more accurately and easily than biomass and it is the most practical size variable for management (Rio et al. 2001).

In the words of Zeide (2005), Reineke's density assessment with the stand density index "may be the most significant American contribution to forest science". However, like Hynynen (1993), Pittman and Turnblom (2003), and Pretzsch and Biber (2005), he calls into question the validity of a universally fixed slope (Pretzsch, 2009). Analyzing the effects of stand conditions on the slope of the self-thinning line, Zeide (1987) suggested that steeper slopes are associated with stands of optimal conditions, while flatter slopes are related to those of sub-optimal conditions, indicating that variation in the self-thinning slope is not necessarily species-specific, but it may be site-specific (Monserud et al. 2005). Reineke's equation assumes that trees die as a result of a single factor: competition due to an increase in the size of adjacent trees. It disregards other stand conditions such as initial density, nutrition, and site preparation, as well as internal physiological and morphological processes that detract from vitality as trees age such as diminishing tolerance to shading, and increase in tree size, slowing the growth by redirecting resources to supporting structure and respiration (Zeide and Stephens 2010).

Several statistical methods have been used to estimate the parameters of the species limiting size-density line (Zhang et al. 2005), among others: arbitrarily hand placing a line above an upper boundary of points (Yoda et al. 1963), fitting an ordinary least square regression (Rio et al. 2001), estimating coefficients through quantile regression (Cade et al. 1999), or adopting stochastic frontier regression to model the limiting line (Bi 2001 and 2004; Weiskittel 2009). Some of these methods have multiple limitations such as using subjective techniques (hand fitting), making statistical inference very difficult (quantile regression), or ignoring autocorrelations between observations from the same plot when determining the boundary line

(ordinary least squares and stochastic frontier regression), which make testing the influence of stand and site factors on the limiting size-density relationship problematic. Theoretically, the limiting size–density line should be an upper boundary line of all the data points (Weller 1987). Numerous studies have failed to account for the asymptotic nature of the maximum size–density line when estimating coefficients. To correct this problem, some authors have shifted the fitted limiting size–density line by increasing the estimated intercept while preserving the estimated slope value (Solomon and Zhang 2002; Yang and Titus 2002).

Few attempts have been made to use mixed-effects procedures to model the species limiting size-density line. Hynynen (1993) used a mixed linear model to estimate the boundary relationship in stands of *Pinus sylvestris*, *Picea abies*, and *Betula pendula*. VanderSchaaf and Burkhart (2007) applied mixed-effects models to estimate the species maximum size-density boundary line in stands of loblolly pine, yet the slopes varied broadly in their analysis (from -1.2 to -2.5) and the factors that contributed to this variation were not addressed.

This chapter reports on the way in which varying silvicultural intensity, planting density, site quality, and soil types affect the species limiting size-density line. The objectives of the present study were: (i) to illustrate the use of mixed-effects linear models to analyze the species limiting size-density relations in loblolly pine stands; (ii) to test the null hypothesis of no effect of silvicultural intensity on the species maximum size-density line followed by populations grown with two silvicultural levels; and (iii) to examine the stability of the parameter estimates of the limiting line across a gradient of planting densities, site qualities, and soils types. Our study differs from the two previous studies that have used mixed-effects linear models to fit the species limiting size-density line (Hynynen 1993; VanderSchaaf and Burkhart 2007) by integrating a

selection of other stand factors beyond quadratic mean diameter such as silvicultural intensity, planting density, and site index into the model.

## **2.3. Materials and Methods**

### **2.3.1. Field Experiment, Design, and Data Collection**

The data for this work were obtained from permanent plots of loblolly pine of the Plantation Management Research Cooperative's (PMRC) Coastal Plain culture/density study. The study was established in 1995/1996 to quantify and contrast the effects of intensive and current operational practices on the growth and yield of loblolly pine plantations across a wide range of planting densities and to investigate potential interactions between cultural intensity and stand density across a range of soil categories. Seventeen installations were established in the Lower Coastal Plain of Georgia, Florida, and South Carolina on five CRIFF soil groups A, B1, B2, C and D. CRIFF soil groups are defined using soil drainage, texture and depth of the subsurface soil layers (Jokela and Long 2000). Characteristics of the CRIFF soil groups used in the study are presented in Table 2.1.

Site preparation and subsequent silvicultural treatments represent two levels of management intensity: operational and intensive culture (Table 2.2). At each installation there was a random allocation of the management regimes to main plots. Within each treatment regime, six loblolly pine subplots with densities of 741, 1483, 2224, 2965, 3706, and 4448 trees per hectare were randomly assigned. To ensure that the initial density was achieved, each planting spot was double-planted and reduced to a single surviving seedling after the first

growing season. A single first generation improved family, planted as 1-0 stock, was used at all installations. Measurement area of subplots ranged from 0.04 to 0.1 ha depending on density treatment. The measurement area plus a surrounding buffer area at least 8 m in width received the prescribed culture and density treatments. Since the installations can be considered as a random sample of all possible locations, they were treated as random. The arrangement of soil groups, management intensity treatments and planting densities resulted in a mixed-model split-split plot design. See Zhao et al. (2011) for a more detailed description of the study.

### 2.3.2. Data

The dataset under analysis consisted of 1134 measurements collected on 204 permanent plots of loblolly pine. Measures of quadratic mean diameter and number of trees per hectare were conducted every two years, from age two to twelve. Summary statistics of stand variables for the whole dataset are presented in Table 2.3. A summary of stand attributes by silvicultural treatment regime is shown in Table 2.4. Base age 25 years site index values were estimated using the equation of Borders et al. (2004) for loblolly pine plantations:

$$[5] \quad S = a \left( 1 - \left( 1 - \left( \frac{HD}{a} \right)^{1/b} \right)^{25/A} \right)^b$$

where  $a=117.6$  and  $b=1.336527$  using English units,  $S$  is site index estimated at the age ( $A$ ) of the oldest measurement,  $HD$  is dominant height defined as the average height of trees with diameter at breast height (DBH) larger than the median DBH of the stand.

The trajectories of stand density (natural logarithm of trees per ha) and quadratic mean diameter (natural logarithm of quadratic mean diameter at breast height in cm) by plot are shown in Figure 2.1.

Due to the repeated measures of the study, separate ANOVAs were performed at each measurement age to test the effects of silvicultural treatment, initial planting density, soil groups, and their interactions, on the quadratic mean diameter and, Reineke's stand density index (SDI) (Table 2.5). The separate ANOVAs indicate that planting density and the silvicultural treatments affected loblolly pine quadratic mean diameter and SDI at all measured ages.

### **2.3.3. Model Development**

Limiting size-density boundary lines are developed from repeated measurement data, which are usually correlated due to multiple measurements made on each experimental plot, and the resulting residual variances are often not homogeneous (Gregoire 1987; Littell et al. 2006). Repeated measurement data present two aspects of the data that are interrelated and require modeling: the mean response and the covariance among recurring measurements on the same individuals. This interdependence occurs because the vector of residuals (observed responses minus fitted responses) depends upon the specification of the model for the mean. Failure to take account of the covariance among repeated measures will generally result in incorrect estimates of the sampling variability (Fitzmaurice et al. 2004). With incorrect standard errors, tests statistics and p-values will also be incorrect and hence can result in misleading inferences. Linear mixed-effects analysis can use random plot effects to account for correlation among

observations from the same cluster-plot (Lappi and Bailey 1988; Schabenberger and Pierce 2001; VanderSchaaf and Burkhart 2007).

Using vector and matrix notation, the general specification of a linear mixed-effects model can be expressed as:

$$\begin{aligned}
 \mathbf{Y} &= \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon} \\
 \mathbf{u} &\sim \mathbf{N}(\mathbf{0}, \mathbf{D}) \\
 \boldsymbol{\varepsilon} &\sim \mathbf{N}(\mathbf{0}, \mathbf{R}) \\
 \text{Cov}(\mathbf{u}, \boldsymbol{\varepsilon}) &= \mathbf{0}
 \end{aligned}
 \tag{6}$$

In Equation [6],  $\mathbf{Y}$  represents a  $n \times 1$  vector of continuous responses,  $\mathbf{X}$  is a  $n \times p$  design matrix of covariates,  $\boldsymbol{\beta}$  is a  $p \times 1$  vector of unknown fixed effects parameters,  $\mathbf{u}$  is a  $q \times 1$  vector of random effects, and  $\mathbf{Z}$  is a  $n \times q$  matrix of covariates, with  $q \leq p$ . We assume that the  $q$  random effects in the  $\mathbf{u}$  vector follow a multivariate normal distribution, with mean vector  $\mathbf{0}$  and a variance-covariance matrix denoted by  $\mathbf{D}$ :  $\mathbf{u} \sim \mathbf{N}(\mathbf{0}, \mathbf{D})$ .  $\mathbf{D} = \text{VAR}(\mathbf{u})$  is a  $q \times q$  matrix that is symmetric and positive definite. The  $\boldsymbol{\varepsilon}$  vector in Equation [6] is a vector of  $n$  residuals, and it is assumed that they are random variables that follow a multivariate normal distribution with mean vector  $\mathbf{0}$  and a positive definite symmetric covariance matrix  $\mathbf{R}$ :  $\boldsymbol{\varepsilon} \sim \mathbf{N}(\mathbf{0}, \mathbf{R})$ . Finally, we assume that the vector of residuals  $\boldsymbol{\varepsilon}$  and random effects  $\mathbf{u}$  are independent of each other.

Fitting mixed models implies that an appropriate mean structure as well as a covariance structure needs to be specified. A key part of fitting mixed models is to determine which parameters should be modeled as fixed and which should be modeled as mixed (fixed plus a random component). The process of building a linear mixed model for a given dataset of repeated measurements is an iterative one that requires a series of model-fitting steps and explorations, and selection of suitable mean and covariance structures for the observed data. Following the steps suggested by Verbeke and Molenberghs (2000) and West et al. (2007) for

building a linear mixed model, a full model with random effects associated with all the parameters in the equation was fitted first by assuming a diagonal variance-covariance matrix structure for random effects and an independent structure for within plot error. The next step involved the selection of a random effects structure. Several reduced (nested) models were fitted by dropping the random effect terms associated with the parameters in the full model. The need for including the selected random effects is tested by performing restricted ML (REML)-based likelihood ratio tests for the associated covariance parameters. Sixteen different mean and random effects structures were fitted in this analysis. Next, we selected a covariance structure for the residuals in the model. Once the fixed and random effects were added to the model, the remaining variation in the observed responses is due to the residual error, and an appropriate covariance structure for the residuals was examined. Different variance-covariance structures such as a first-order autoregressive model (AR(1)), variance components model (VC), unstructured covariance model (UN), exponential, and Gaussian serial correlation functions were used with the data to account for dependence across repeated measurements within each plot. The best of these models is selected by comparing the fitted equations using Akaike's information criterion (AIC, Akaike, 1973) and Schwarz's Bayesian information criterion (BIC, Schwarz, 1978). Among plausible models, the model that minimizes AIC, and BIC is preferred. When AIC, or BIC, is close, the simpler equation is generally considered preferable in the interest of using a parsimonious model. The last step in the model building process involved model reduction by using Type-III F- tests to determine if the fixed effects parameters were significant. Model diagnostics were carried out in the final model to check whether the distributional assumptions for the residuals were satisfied.

To perform this fitting and to interpret the result as a line of maximum size-density, it is necessary to be certain that the plots analyzed are fully stocked or at maximum density (Weller 1987; Bi and Turvey 1997). A visual inspection of graphs of  $\ln(N)$  vs.  $\ln(Dq)$  was conducted for each experimental plot to determine when trajectories have reached the boundary line. After data examination, we disregarded data where the annual mortality rate of the plot was under 1%, as well as data from the first measurement of each plot, since we do not know if there was previous density-dependent mortality (Rio et al. 2001; Hynynen 1993). Applying these principles, the data from the 204 plots were reduced to 142 plots for the study of the limiting size-density line fitted using linear mixed-effects models (Table 2.6). Various authors consider that the limiting relationship between the quadratic mean diameter and the planting density is curvilinear instead of linear on a log-log scale over the whole range of self-thinning (Zeide 1987; Cao et al. 2000; VanderSchaaf and Burkart 2007). In this study we deal only with the linear section of the trajectory.

In Equation [7] below, we fit a linear regression model for each plot, which describes their natural logarithm of trees/hectare ( $\ln(N)$ ) as a function of the natural logarithm of their quadratic mean diameter in cm ( $\ln(Dq)$ ). This initial model includes the fixed effects of  $\ln(Dq)$ , silvicultural management group, and their interaction. We also include two random effects associated with each plot: a random intercept, and a random slope effects. This allows each plot to have a unique linear trajectory, with coefficients that vary randomly around the fixed effects defining the mean limiting size-density line for each silvicultural management group. We use restricted maximum likelihood (REML) to estimate the variances and covariance of the two random effects. Equation [7] also includes residuals associated with the  $\ln(N)$  observations, which conditionally on a given plot are assumed to be independent and identically distributed.

The model specification for an individual plot response,  $\ln N_{ti}$ , on plot  $i$  at the  $t$ -th age is given by:

$$[7] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{\text{Management}} + u_{0i}) + (\beta_{11} + \beta_{12} \times I_{\text{Management}} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti}$$

One dummy variable ( $I_{\text{Management}}$ ) is introduced to identify the two levels of silvicultural treatment:

$$I_{\text{Management}} = \begin{cases} 1, & \text{if Intensive silvicultural regime} \\ 0, & \text{if Operational silvicultural regime} \end{cases}$$

The parameters  $\beta_{01}$  and  $\beta_{11}$  represent the fixed effect associated with the intercept and slope, respectively, for a plot under the operational silvicultural regime treatment (reference group). The fixed effects  $\beta_{02}$  and  $\beta_{12}$  represent the difference in the intercept and slope for the intensive vs. the operational regimes. The terms  $u_{0i}$ ,  $u_{1i}$  are cluster-specific random effects to be predicted and assumed to be  $N(0, \sigma_0^2)$ , and  $N(0, \sigma_1^2)$ , respectively. A cluster is defined as an individual plot (indexed by  $i$ ). The distribution of the vector of the two random effects associated with the plot  $i$  is assumed to be multivariate normal:

$$\mathbf{u} = \begin{pmatrix} u_{0i} \\ u_{1i} \end{pmatrix} \sim N(\mathbf{0}, \mathbf{D})$$

Each of the two random effects has a mean of 0, and a positive-definite variance-covariance matrix  $\mathbf{D}$ . The term  $\varepsilon_{ti}$  in Equation [7] represents the residual associated with the observation at time  $t$  on plot  $i$ . The distribution of the residuals can be written as:

$$\varepsilon_{ti} \sim N(\mathbf{0}, \mathbf{R})$$

We assume that the residuals are independent of the random effects.

To specify the appropriate mean structure of the model, all 15 nested sub models of Equation [7] (full model) were fitted by sequentially dropping fixed and random effect terms associated with the parameters. Allowing both the slope and intercept to differ for the two levels of silvicultural management regime, and with random effects associated only with the slope results in Equation [8]:

$$[8] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{Management}) + (\beta_{11} + \beta_{12} \times I_{Management} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting both parameters differ across silvicultural regimes, and removing the random effects from the slope yields Equation [9]:

$$[9] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{Management}) + (\beta_{11} + \beta_{12} \times I_{Management}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting only the intercept to differ across silvicultural groups, and including random effects associated only with the slope results in Equation [10]:

$$[10] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{Management}) + (\beta_{11} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

allowing only the slope to differ across silvicultural groups, and including random effects associated with this parameter yields Equation [11]:

$$[11] \quad \ln N_{ti} = \beta_{01} + (\beta_{11} + \beta_{12} \times I_{Management} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

permitting only the intercept to differ across silvicultural groups, and including random effects only on it results in Equation [12]:

$$[12] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{Management} + u_{0i}) + \beta_{11} \ln Dq_{ti} + \varepsilon_{ti} ;$$

allowing only the intercept to differ across silvicultural groups, and with random effects associated with all the parameters in the model yields Equation [13]:

$$[13] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{Management} + u_{0i}) + (\beta_{11} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting only the slope to differ across silvicultural regimes, and with random effects associated with all parameters yields Equation [14]:

$$[14] \quad \ln N_{ti} = (\beta_{0i} + u_{0i}) + (\beta_{11} + \beta_{12} \times I_{Management} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting only the slope to differ across silvicultural regimes, and with random effects associated only with the intercept results in Equation [15]:

$$[15] \quad \ln N_{ti} = (\beta_{0i} + u_{0i}) + (\beta_{11} + \beta_{12} \times I_{Management}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

allowing a unique intercept and slope, and with random effects associated with all parameters yields Equation [16]:

$$[16] \quad \ln N_{ti} = (\beta_{0i} + u_{0i}) + (\beta_{11} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti}$$

allowing unique intercept and slope, and with random effects associated only with the intercept results in Equation [17]:

$$[17] \quad \ln N_{ti} = (\beta_{0i} + u_{0i}) + \beta_{11} \ln Dq_{ti} + \varepsilon_{ti} ;$$

allowing unique intercept and slope, and with random effects associated only with the slope results in Equation [18]:

$$[18] \quad \ln N_{ti} = \beta_{0i} + (\beta_{11} + u_{1i}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting the intercept and slope to differ across silvicultural regimes, and considering random effects associated with the intercept results in Equation [19]:

$$[19] \quad \ln N_{ti} = (\beta_{0i} + \beta_{02} \times I_{Management} + u_{0i}) + (\beta_{11} + \beta_{12} \times I_{Management}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting only the slope to differ across silvicultural regimes, and considering all parameters as fixed results in Equation [20]:

$$[20] \quad \ln N_{ti} = \beta_{0i} + (\beta_{11} + \beta_{12} \times I_{Management}) \ln Dq_{ti} + \varepsilon_{ti} ;$$

letting only the intercept to differ across silvicultural regimes, and considering all parameters as fixed results in Equation [21]:

$$[21] \quad \ln N_{ti} = (\beta_{01} + \beta_{02} \times I_{Management}) + \beta_{11} \ln Dq_{ti} + \varepsilon_{ti} \quad ;$$

finally, allowing a unique intercept and slope, and considering all parameters as fixed yields Equation [22]:

$$[22] \quad \ln N_{ti} = \beta_{01} + \beta_{11} \ln Dq_{ti} + \varepsilon_{ti} \quad .$$

To analyze the invariance of the slope and intercept of the species limiting size-density line for the two management regimes across different stand conditions, the final model was refitted using subsets of data that reflected different planting densities, soil types, and site qualities. The data set was split into three subsets according to the initial planting densities: one for low densities (741 and 1483 trees/ha), one for medium densities (2224 and 2965 trees/ha), and one for high densities (3706 and 4448 trees/ha). Regarding soil type, the dataset was divided into four subsets corresponding to the CRIFF soil groups A, B1, B2 and C. The CRIFF soil group D was not considered in this analysis due to lack of enough data to carry out the fitting procedures. Finally, the data set was split into two subsets according to the estimated site index values: lower site qualities (where site index is less than or equal to the median value for the whole dataset), and higher site qualities (where site index is strictly greater than the median value). The final species limiting size-density model was fitted to each of the subsets. Parameter estimates of the final model were obtained from each subset, and stability of the slope and intercept for the two management intensities was analyzed across each of the subsets. A graphical comparison of the differences among these curves was done by plotting them on top of each other. The Kolmogorov- Smirnov test (Conover 1999) was carried out to identify any significant difference between each of the resulting curves fitted with the subsets and the overall

curve for all the densities, soils and site qualities. The principle of the test is to rank all the observations from two populations, say  $X_1$  and  $X_2$ ; establish the empirical cumulative distribution functions  $F_1(X_1)$  and  $F_2(X_2)$ ; and compute the maximum vertical distance between the two empirical distribution functions:  $T = \max |F_1(X_1) - F_2(X_2)|$ . The maximum difference is then compared with a critical value from a Kolmogorov-Smirnov table at a given level of significance  $\alpha$  and sample sizes of both populations. If the Kolmogorov-Smirnov statistic  $T$  is smaller than the critical value, then the null hypothesis is accepted, indicating that no significant difference between the two populations exists (Conover 1999). Each of the fitted lines for the different initial planting densities, soil groups, and site index classes were used to predict the natural log of stand densities in trees/ha ( $Ln(TPH)$ ) for varying natural log of quadratic mean diameters and silvicultural management regimes. The results were then compared with the overall model to detect any significant difference using the Kolmogorov Smirnov test.

The combined effect of silvicultural regimes, site index, planting density, and CRIFF soil groups on the species limiting size-density line intercept and slope was examined. The general model form used in this analysis was:

$$[23] \quad Ln(N) = \beta_0 + \beta_1 Ln(Dq) + \beta_2 Ln(SI) + \beta_3 Ln(PTPH) + \beta_4 Management + \beta_{5i} Soil_i$$

where  $N$  represents stand density (trees/ha),  $Dq$  is quadratic mean diameter (cm),  $SI$  is site index (m),  $PTPH$  is the initial planting stand density (trees/ha),  $Management$  is an indicator variable for silvicultural management (1 if intensive, 0 if operational), and  $Soil_i$  is a set of three indicator variables that represents the four CRIFF soil groups included in the analysis. Additionally, all interactions, as well as random effects associated with all parameters, were tested. The final equation was developed by stepwise procedure where a series of likelihood ratio tests were conducted between the full model and the reduced model by dropping the non-significant

parameters from the full model. Information criterion (AIC, BIC) was used for selecting the best variance-covariance structure. The final mixed-effects linear model was built using the SAS procedure PROC MIXED implemented with a SAS macro application.

## 2.4. Results

We begin by fitting Equation [7], the full mean model structure with different intercept and slope for the two management groups, and that allows the intercept and the slope to vary randomly from one plot to another. Several reduced models were fitted by dropping the random effect terms associated with the parameters as well as fixed effects. Goodness of fit statistics for all competing linear mean mixed-effects model structures are presented in Table 2.7.

We encountered estimation problems with the variances and covariance parameters for the random effects when we fitted Equation [7]. We noted that the estimate for the variance of the random intercepts ( $\sigma_0^2$ ) was set to zero, indicating that the maximum of the REML log-likelihood function was really on the boundary of the parameter space. We refitted the model without any restriction on the parameter space (i.e. by removing the restriction that the variance components have to be positive), resulting in a negative estimate for the variance of the random intercepts. We formally tested the need for random plot-specific intercepts by using a REML-based likelihood ratio test (LRT). To perform this test, we fitted a nested model (Equation [8]) that omitted the random intercept effects. The resulting LRT statistics was not significant (p-value= 0.0952), so we removed the random effects associated with the plot-specific intercepts in Equation [7] and in all subsequent models. We decided that Equation [8], with random effects only associated with the slopes, was preferred at this stage of the analysis.

To select the final structure for the random effects we tested whether the random effects associated with the slopes can be omitted from the model. We fitted Equation [9] by removing the random plot-specific slopes from Equation [8], while retaining the same fixed effects. A likelihood ratio test statistic was then calculated by subtracting the -2 REML log-likelihood value for Equation [8] (the reference model) from the value for Equation [9] (the nested model). The asymptotic distribution of the likelihood ratio test statistics is a  $\chi^2_1$  distribution. The resulting test statistic (118.3) was significant (p-value <.0001), therefore we retained the random effects associated with the slope in Equation [8].

We tried to reduce the model by removing nonsignificant fixed effects. First we tested whether the fixed effect associated with the Management\*Ln(Dq) interaction ( $\beta_{12}$ ) could be omitted from Equation [8] to form the nested Equation [10]. This is equivalent to testing if there is a significant difference in the slopes of the limiting size-density line across the two management regimes. This was tested using Type III F-tests since likelihood ratio tests give incorrect p-values for fixed effects (Pinheiro and Bates 2000). Based on the significant results of this test ( $F = 6.05$ , p-value=0.0148), we concluded that the fixed effect associated with the management\*Ln(Dq) interaction was significant and should be retained in the model. The data support the hypothesis that the slope of the species limiting size-density line does differ for different levels of silvicultural management group, and we kept the Management\*Ln(Dq) interaction term in Equation [8]. Next we tested whether the fixed effect associated with the silvicultural management regimes term ( $\beta_{02}$ ) can be omitted from the model (Equation [8] vs. [11]). This is equivalent to testing if there is a significant difference in the intercepts of the species limiting size-density line across the silvicultural management regimes. The corresponding Type III F-test was significant ( $F = 7.23$ , p-value = 0.0078) so we concluded that

the intercept of the species limiting size-density line does differ for different levels of silvicultural management group, and we kept the management term in Equation [8]. For comparison purposes, the performance of all possible sub models of the full mean equation structure is shown in Table 2.7. Observe that those models that omitted the silvicultural management effect on the species limiting size-density line showed poorer fitting performance (higher AICs and BICs values).

Subsequently we tested if the variance of the residuals was the same (homogenous) for the two silvicultural management regimes. We refitted Equation [8] allowing the residuals variances for observations at different levels of silvicultural management to differ, and performed a REML-based likelihood ratio. The test statistic has a  $\chi^2$  distribution with 1 degree of freedom because of the additional residual variance component. The test result was not significant (p-value = 0.5271), therefore we decide that Equation [8] with homogeneous residual variance for the intensive and operational silvicultural treatment regimes is our preferred model at this phase of the analysis.

The last step in our model building was to select a residual covariance structure for Equation [8] through information criteria. The first-order autoregressive structure, (AR-1, with autoregressive parameter estimate  $\hat{\phi}=0.8417$ ) provided the best fit (lowest AIC and BIC) and was chosen as the best representation for the covariance structure of the residuals.

Summarizing the model building process, Equation [8] with intercept and slope that vary across silvicultural management regimes and that allows random effects associated only with the slope was chosen as the best mixed effect model form to describe the mean species limiting size-density line in our study (AIC=-216.1, BIC= -210.2). A first-order autoregressive (AR-1) covariance matrix for the residuals provided the best fit (AIC= -221.6, BIC = -212.7). The

parameter estimates from the final fitted model and the goodness of fit statistics are presented in Table 2.8.

All the parameters in the final model were significant at the 0.05 level. The self-thinning lines followed by populations from the two silvicultural regimes levels differed from each other (Figure 2.2). Silvicultural management level affected the slope of the thinning line significantly, with more mortality per unit increment of quadratic mean diameter at the intensive silvicultural level (steeper slope). The intensive silvicultural management regime yielded a larger intercept (12.70 vs. 11.69), the difference being significant at 0.01 level ( $p\text{-value} = 0.0078$ ), and a steeper slope (-1.8754 vs. -1.5265), the difference being significant at 5% level ( $p\text{-value} = 0.0148$ ), than the operational silvicultural regime. The slope obtained for the intensive regime (-1.8754) is significantly different from the one given by Reineke (-1.605) at the 95% confidence level, while the slope estimated for the operational regime (-1.5265) was not significantly different from the stated slope of the self-thinning rule at 95% confidence level.

Figure 2.2 shows the reduced data from the plots assumed to be at maximum density and that have been used to fit the models, and the resulting species maximum size-density lines for the two silvicultural groups obtained with Equation [8]. Since about half of the points were above the resulting line, and the remaining were below the line, it was required to shift the line up vertically in order to be placed above all points to represent the limiting boundary line (Solomon and Zhang 2002; Yang and Titus 2002). This was obtained by increasing the parameter estimates corresponding to the intercept ( $\beta_{01}$  and  $\beta_{02}$ ) while keeping the parameter estimates for the slopes ( $\beta_{11}$  and  $\beta_{12}$ ). The mean structure of the final model after shifting the line vertically is

$$[24] \quad \ln N = 12.18 + 0.97 \cdot I_{\text{Management}} + (-1.5265 - 0.3489 \cdot I_{\text{Management}}) \ln Dq$$

or explicitly by silvicultural regime

$$[25] \quad \begin{aligned} \text{Intensive:} \quad & \ln(N) = 13.15 - 1.8754 \cdot \ln(Dq) \\ \text{Operational:} \quad & \ln(N) = 12.18 - 1.5265 \cdot \ln(Dq) \end{aligned}$$

The implied maximum SDI values (predicted trees/ha when  $Dq = 25.4$  cm) for the intensive and operational silvicultural regimes were 1192, and 1397 respectively.

Residual diagnostics were performed for the final model [8]. To check the assumption of normality for the residuals, normal Q-Q plots were obtained for the intensive and operational silvicultural regimes (Figure 2.3). The plots show that the distribution of the conditional residuals does not depart substantially from a normal distribution. The Shapiro-Wilk test of normality revealed that the assumption of normality for the conditional residuals is met in both the operational regime (Shapiro-Wilk  $W = 0.987$ ,  $p\text{-value} = 0.1544$ ) and the intensive group ( $W = 0.979$ ,  $p\text{-value} = 0.0947$ ).

Plots of the standardized residuals vs. predicted values for each level of silvicultural management (Figure 2.4) show that the residuals were randomly distributed and had no systematic trend. We do not see strong evidence of nonconstant variance for the groups, and the variance of the residuals appears to be fairly constant across the fitted values. Scatterplots of the observed  $\ln(N)$  vs. the predicted values show agreement between the observed and predicted values for each silvicultural group ( $R^2 = 0.9982$  for the intensive, and  $0.9934$  for the operational regime) (Figure 2.5). The distribution of slopes are presented in Figure 2.6 for the intensive and operational silvicultural treatments based on Equation [8]. The random effects associated with the slope seem to be normally distributed for both regimes, and the corresponding values appear

to be centered on their means. Lag-residual plots for Equation [8] do not show evidence of correlation among residuals for any of the silvicultural management groups (Figure 2.7).

The coefficients of the three species limiting size-density lines (based on Equation [8]) fitted separately for the low (741 and 1,483 trees/ha), medium (2,224 and 2,965 trees/ha), and high (3,766 and 4,448 trees/ha) subsets of planting density classes are presented in Table 2.9. For all planting density classes, the intensive silvicultural regime consistently yielded steeper slopes and larger intercepts than the operational regime, the differences being significant for all but the lower planting density class. The slopes varied from -1.3633 to -1.8754 for the intensive regime, and from -1.0764 to -1.5265 for the operational one. For the intensive regime, the slopes tended to increase as planting density increased. Considering the stability of the intercepts, the intensively managed plots yielded parameter estimates that ranged from 10.9590 to 12.7010, while the operational managed plots resulted in intercept estimates that ranged from 9.6834 to 11.6879 when including different planting densities. An overlaid graph of these lines showed that the curves deviated from each other (Figure 2.8). This was corroborated by the Kolmogorov-Smirnov test (p-values < .0001 across all planting densities). Both graphical comparison and the Kolmogorov-Smirnov test indicate that planting density affects the species limiting size-density relationship.

Parameter estimates based on Equation [8] fitted separately for the CRIFF soils groups are shown in Table 2.10. For all soil groups, the intensive silvicultural regime consistently yielded steeper slopes and larger intercepts than the operational regime, but the differences were not significant at 5% level. Differences between regimes in the slopes and intercepts, respectively, were significant only for the whole dataset. The slopes varied from -1.2662 to -3.0348 for the intensive regime, and from -1.0603 to -2.6035 for the operational treatment. The

intensively managed plots yielded intercepts parameter estimates that ranged from 11.0414 to 15.4001, while the operational managed plots resulted in intercept estimates that ranged from 10.5156 to 14.6828 for the different soil groups. The Kolmogorov-Smirnov test showed that for the two regimes the lines did not significantly deviate from each other (the corresponding p-values for CRIFF soil groups A, B1, B2, and C were respectively 0.6728, 0.4015, 0.5811, and 0.7159) (Figure 2.9). The test indicates that the species limiting size-density relationship for a given silvicultural treatment was not affected by the CRIFF soil groups.

The parameter estimates of the species limiting size-density line based on Equation [8] fitted separately for the two site index classes are provided in Table 2.11. The low site index class included values that were smaller than or equal to the median site index for all plots (26.6 m), while the high class included values that were greater than the median. An overlaid graph between these limiting size-density relationships did not show large deviations (Figure 2.10). This was corroborated by the Kolmogorov-Smirnov test (the corresponding p-values for the low and high site index classes were respectively 0.7329 and 0.6927). A visual comparison of the plots in Figure 2.10 and the Kolmogorov-Smirnov test indicate that site quality did not affect the species limiting size-density line when considering these two site index classes and controlling for silvicultural management. Only the high site index class showed significant differences in the slope and intercept between silvicultural regimes (Table 2.11). The slopes varied from -1.9713 to -2.1363 for the intensive regime, and from -1.7429 to -1.7166 for the operational one. Considering the stability of the intercepts, the intensively managed plots yielded parameter estimates that ranged from 12.9119 to 13.5011, while the operational managed plots resulted in intercept estimates that ranged from 12.1541 to 12.2966 by site index classes. The influence of

site index in the limiting size-density relationship was considered later by incorporating it as a covariate in the species limiting size-density model.

Likelihood ratio tests and substantial reductions in the information criterion (AIC, BIC) indicated that other stand-level variables significantly influenced the slope and intercept of the species limiting line. Equation [26] below was developed by a stepwise procedure where a series of likelihood ratios tests were conducted between the full model and the reduced model by dropping the non-significant parameters from the full model. Random effects associated only with the  $\ln(Dq)$  term, and an autoregressive structure for the residual errors provided the best fit. All the parameters in the final model were significant at the 0.01 level. The parameter estimates from the final fitted model and the information criterion are presented below (standard errors are shown in parenthesis):

$$\begin{aligned}
 \ln(N) = & 8.6870 - 1.6268 \cdot \ln(Dq) + 0.6988 \cdot \ln(SI) \\
 & (0.5239) \quad (0.05412) \quad (0.1665) \\
 [26] \quad & - 0.001745 \cdot \ln(Dq) \cdot \ln(PTPH) - 0.03015 \cdot \ln(Dq) \cdot \text{Management} \\
 & (0.0008307) \quad (0.009924)
 \end{aligned}$$

Information criterion : AIC = -422.4 , BIC = -416.5

The slope of the species limiting size-density relationship was significantly influenced by planting density and silvicultural management regime (interaction terms in Equation [26]). The slope gets steeper as planting density increases, and is significantly flatter in plots under the operational management regime. The slope was not significantly influenced by site index. The intercept of the species limiting size-density relationship was significantly influenced by site index, increasing its value as site index increased (Figure 2.11). The CRIFF soil group factor did

not have a significant influence on the slope or intercept of the species limiting size-density relationship. Equation [26] shows a substantial reduction in the information criterion statistics, indicating that including site index, silvicultural management and planting density in the model improves the fitting of the species limiting size-density relationship. Comparing Equations [8] and [26], AIC was reduced from -221.6 to -422.4 and BIC from -212.7 to -416.5.

## **2.5. Discussion**

Earlier studies on the species limiting size-density line regarded it to be a one-dimensional, or at most a two-dimensional surface (Zeide 1995; Bi 2001 and 2004). More recently Weiskittel et al. (2009) introduced the effects of site index, stand origin, and stand purity on the boundary line of mixed species stands. In the study reported here results from a mixed-effect model approach indicate that the limiting boundary line is a multidimensional surface as planting density, management intensity, and site index significantly influenced the parameters of the species limiting size-density relationship in even-aged stands of loblolly pine (Equation [26]). These results support the findings of Reynolds and Ford (2005), and Turnblom and Burk (2000) who concluded that differences in initial stands conditions, such as site quality and planting density, affect the self-thinning trajectories.

The hypothesis of no effect of silvicultural management level on the slope of self-thinning lines was not supported in this study. The parameter estimates of the species limiting size-density line were significantly different among management regimes, which suggest that their values are not constant and can vary among silvicultural treatments. Different thinning lines for each silvicultural level were observed, with a steeper slope and a larger intercept at the more

intensive silvicultural level. Self-thinning is the final expression of competition in dense stands of plants (Yoda et al. 1963) and as a result, the stand density versus size line that populations under self-thinning traverse is the result of competitive effects operating in the stand (Morris 2002). The different line patterns can be explained by competition developing differently in stands grown at each silvicultural level. The intensive management regime, which included complete vegetation control as well as heavy and repeated refertilization, caused trees to grow larger (Harrison and Kane 2008) resulting in more severe competition with each other for limited growing space and resources such as light, water, and nutrients. This might explain the higher mortality per unit increment of quadratic mean diameter (steeper slope) observed in the intensive silvicultural level. Silvicultural practices like site preparation, competition control, and fertilization can have major influence on the rate at which individual trees accumulate leaf area and how fast stands reach canopy closure, full site occupancy and self-thinning (Long et al. 2004). Considering the same planting density, stands on better sites may achieve canopy closure and self thinning faster than on poorer sites. Our results above suggest that the intensive silvicultural treatment accelerated the rate of stand development, stands appear more mature than indicated by their age alone, that is, further ahead in their size-density trajectories and in self-thinning. This can explain the fact that the majority of the data used to fit the models, and hence assumed to be in the self-thinning phase, came from plots that were in the intensive silvicultural regime (Table 2.6).

Our study shows that the intercept of the self-thinning limiting-size density line did not remain constant across management intensities, and in fact it increased with silvicultural level. The results of this work support the argument of Jack and Long (1996) and Bi (2001, 2004) for careful and more rigorous testing of claims in the literature that the species limiting size-density

line is site-independent (Yoda et al. 1963; White and Harper 1970; Drew and Flewelling 1979; Begin et al. 2001). It is suggested that the observed or assumed site-independence of the self-thinning line in the literature could be due to the lack of rigorous statistical testing rather than evidence proving site-independence. This claim is supported by several studies (Morris and Myerscough 1991; Hynynen 1993; Bi 2001, 2004; Morris 2002).

The implied species maximum SDI values across silvicultural regimes based on the final model [8] were higher than the maximum SDI reported by Reineke (1933) for loblolly pine (1,140 trees/ha). The difference may be attributed to differing statistical techniques to estimate the parameters of the self-thinning line (Weiskittel et al. 2009), and the relatively young age of the stands in our study (Zeide and Stephens 2010). This finding is consistent with results of Zeide and Stephens (2010), who found that the predicted maximum stand density index for stands of loblolly pine of 9 and 10 years old was higher than Reineke's (1,140 trees/ha). Diminishing tolerance to shading, senescence, and obstacles related with increasing tree size (diversion of resources to support structures and respiration) as trees get older contribute to tree mortality. These arguments suggest that the younger the stand, the higher the maximum stand density index (Zeide and Stephens 2010). Regarding silvicultural intensity, the operationally managed stands yielded a higher maximum SDI (1,397 trees/ha) than the intensively managed ones (1,192 trees/ha), hence allowing more trees/ha for a given value of quadratic mean diameter. The same degree of site occupancy was therefore reached by a smaller number of larger trees (intensively managed stands) or a larger number of smaller trees (operationally managed stands). This suggests that by increasing fertilization and entirely removing competing vegetation, stands can change the allometric relationship between the size of the tree (e.g. stem diameter) and the area occupied by the tree (e.g. crown size), or particularly, between stem

diameter and the area needed to satisfy photosynthetic and respiratory demands, resulting in changes in competition and in the self-thinning trajectories of the stands.

Our findings support the idea that the species limiting size-density line can vary significantly with site index (Zeide 1987; Bi 2001; Pittman and Turnblom 2003; Weiskittel et al. 2009). The results of this study suggest that, similar to silvicultural management, site index can modify the allometric relationship between the size of the tree and the area occupied by the tree, resulting in changes in competition and in the self-thinning trajectories of the stands. Superior quality sites would be able to support a larger number of trees and be likely to evolve through stand development at faster rates than poor sites. This conclusion differs from those of others who have claimed that species limiting relationship is invariant with site index (Yang and Titus 2002, Tang et al. 2005; Schutz and Zingg 2010).

The self-thinning relationship was not affected by the CRIFF soil factor when controlling by site index, management regime, and planting density. The soil factor was left out of multiple regression Equation [26] since it was not significant at 5% level. This is consistent with results of Smith and Hann (1984) and Weskittel et al. (2009) who found that soil type did not affect either the intercept or slope parameters of Douglas fir, western hemlock, and red alder.

Our findings disagree from those of others who have concluded that the species limiting size-density boundary line was invariant to planting density (Puettmann et al., 1993; Tang et al., 1995). For loblolly pine in this study, the magnitude of variation in the estimated slope over the gradient of planting densities ranged in value from -1.0764 to -1.8754. These values are consistent with those reported in other studies. Yang and Titus (2002) reported slopes varying from - 0.5926 to -1.6869 in mixed stands dominated by trembling aspen (*Populus tremuloides* Michx.), white spruce (*Picea glauca* (Moench) Voss), and lodgepole pine (*Pinus contortia* var.

*latifoliada* Engelm). Pretzsch and Biber (2005) estimated slopes changing from -1.204 to -2.027 in mixed stands of common beech (*Fagus sylvatica* L.), Norway spruce (*Picea abies* [L.] Karst.), Scots pine (*Pinus sylvestris* L.) and common oak (*Quercus petraea* [Mattuschka] Liebl.). For loblolly pine, different studies estimated the slope to be -1.707 (MacKinney and Chaiken 1935), -1.696 (Harms 1981), and -1.505 (Williams 1996). VanderSchaaf and Burkhart (2008) reported slopes varying across planting densities from - 0.9850 to -2.5987 in loblolly pine plantations in the southeastern US. The magnitude of variation in the estimated intercept over the gradient of planting densities ranged in value in our study from 9.6834 to 12.7010, also consistent with values reported in the literature (Yang and Titus 2002; Reynolds and Ford 2005; VanderSchaaf and Burkhart 2008; Weiskittel et al. 2009; VanderSchaaf 2010). The parameters of the self-thinning line in the present study appeared to be a function of planting density as there was a direct relationship between stand density and values of the slope and intercept in the intensively managed plots. As stand density increased, so did the slopes and intercepts of the self-thinning line, indicating higher mortality per unit increment of  $D_q$  (on a log-log scale). At low planting densities individual tree size would be at its maximum as determined by species, age and site quality, but site occupancy is below its upper limit resulting in light mortality due to competition. As planting densities increase, site occupancy would be near or at its upper limit, resulting in an increase in competition related mortality.

It is important to mention the impact that the structure of our data can have on the parameter estimates of the self-thinning line. Differing findings can partly be attributed to the quality and size of the available data as well as the variety of statistical methods used to examine the limiting size-density relationship. The data used in this study was from young stands, ages 2-12, which were probably in the first phase of self-thinning. The development pattern associated

with age may have a confounding effect with the factors that were analyzed: site quality, initial density, soil type, and silvicultural management regimes. Refitting our models with new measurements from the same permanent plots might show the effect of stand age on the self-thinning line. Extreme values of initial stand density can also have a confounding effect on the boundary line. Erroneously including data points from stands of mean density (v.g. trees/ha) that have not yet begun to thin will flatten the estimated slope of the limiting line, while inclusion of data points coming from stands of high density that have not yet begun to thin will steepen the slope (Westoby 1984, Zhang et al. 2005).

## **2.6 Conclusions**

Mixed effects analysis techniques proved to be an effective method for modeling and recognizing the source of variation in the species limiting size-density line of stands of loblolly pine. The intercept and slope of the boundary line were significantly affected by silvicultural management, planting density, and site index. Different thinning lines for each silvicultural level were observed, with a steeper slope and a larger intercept at the more intensive silvicultural level. The slope corresponding to the intensive regime was significantly larger than the value proposed by Reineke (1933), while the slope of the operational regime was not. This is important as estimates of the slope are required to use Reineke's model as a stand density index (SDI). The estimated values of the maximum SDI for both silvicultural levels were larger than the previously published value of 1,140 trees/ha for loblolly pine. When site index, initial density, and silvicultural level were simultaneously included as input in the species size-density boundary relationship it was found that the intercept significantly increased with site index. The

slope was also significantly affected by planting density and silvicultural regime, becoming steeper as the initial density increased, and as the management changed from operational to intensive level. These results show that the boundary self-thinning line is driven by several stand factors that affect the relationship between tree density and average tree size.

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**Table 2.1.** CRIFF soil groups used in the Coastal Plain culture/density study.

<b>CRIFF Soil Group</b>	<b>Drainage</b>	<b>Features</b>
A	Very poor to somewhat poor	Sand to loamy sand surface layer, with a finer textured argillic soil horizon < 20" below.
B1	Very poor to somewhat poor	Sand to loamy sand surface layer, with a finer textured argillic soil horizon 20 – 40" below.
B2	Very to somewhat poor	Sand to loamy sand surface layer, with a finer textured argillic soil horizon > 40" below or absent.
C	Poor to somewhat poor	Spodic with argillic horizons below the surface layer.
D	Poor to moderately well	Spodic without argillic horizon below the surface layer.

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**Table 2.2.** Summary of the silvicultural treatments used in the study<sup>a</sup>.

Operational Treatment	Intensive Treatment
Bedding	Bedding
Fall banded chemical site preparation	Fall broadcast chemical site preparation
	Tip moth control
Herbaceous weed control: first year banded	Repeated herbicide application to achieve complete and sustained vegetation control through year 12
Fertilization: at planting, 561 kg/ ha of 10–10–10; before 8 <sup>th</sup> and 12 <sup>th</sup> growth seasons, 224 kg/ha N+ 28 kg/ ha P	Fertilization: at planting, 561 kg/ha of 10–10–10 ; Spring 3rd grow season, 673 kg/ ha 10–10–10 + micronutrients + 131 kg/ha NH <sub>4</sub> NO <sub>3</sub> ; Spring 4 <sup>th</sup> grow season 131 kg/ha NH <sub>4</sub> NO <sub>3</sub> ; Spring 6th grow season 336 kg/ha NH <sub>4</sub> NO <sub>3</sub> ; Spring 8 <sup>th</sup> , 10 <sup>th</sup> , and 12 <sup>th</sup> grow season 224 kg/ha N+ 28 kg/ha P

<sup>a</sup>Adapted from Zhao et al. 2010.

**Table 2.3.** Plot-level description of data (n=1,134 observations)<sup>a</sup>.

<b>Variable</b>	<b>Mean</b>	<b>S.D.</b>	<b>Min</b>	<b>Max</b>
Trees/ ha	2,310	1,154	484	4,448
Dq (cm)	10.67	5.41	1.16	27.77
H (m)	9.4	4.8	1.5	18.7
SI (m)	26.6	1.5	22.4	31.3

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<sup>a</sup> S.D.: standard deviation , Dq: quadratic mean diameter (cm), H: average total height (m), SI: site index base age 25 years (m)

**Table 2.4.** Plot-level description of data by silvicultural regime<sup>a</sup>.

	Operational Management					Intensive Management				
Variable	n	Mean	S.D.	Min	Max	n	Mean	S.D.	Min	Max
Trees/ ha	570	2,340	1,159	598	4,448	564	2,281	1,146	484	4,448
Dq (cm)	570	9.58	5.08	1.16	23.31	564	11.76	5.51	1.32	27.77
H (m)	570	8.6	4.5	1.5	18.6	564	10.1	4.9	1.9	18.7

<sup>a</sup> n: number of observations, Dq: quadratic mean diameter (cm), H: average total height (m)

**Table 2.5.** ANOVA p-values for treatments effects (soil group, silvicultural management, initial planting density) and their interactions on quadratic mean diameter and Reineke's stand density index<sup>a</sup>.

Source of Variation	Age 2	Age 4	Age 6	Age 8	Age 10	Age 12
<b>Variable: Quadratic mean diameter</b>						
Soil	0.081	0.563	0.353	0.050	<b>0.048</b>	<b>0.038</b>
Management	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>
Soil x Management	<b>0.014</b>	0.248	0.118	<b>0.0146</b>	<b>0.048</b>	0.058
Initial Density	<b>0.012</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>
Soil x Initial Density	0.139	0.072	0.084	0.131	0.448	0.168
Management x Initial Density	<b>0.0004</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>
<b>Variable: Stand Density Index</b>						
Soil	<b>0.039</b>	0.517	0.476	0.233	0.493	0.274
Management	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>0.0005</b>
Soil x Management	<b>0.005</b>	0.224	0.349	0.149	<b>0.028</b>	<b>0.012</b>
Initial Density	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>&lt;.0001</b>
Soil x Initial Density	<b>&lt;.0001</b>	0.311	0.171	0.318	0.059	0.522
Management x Initial Density	<b>&lt;.0001</b>	<b>&lt;.0001</b>	<b>0.0012</b>	<b>0.008</b>	0.065	0.200

<sup>a</sup> Significant effects at 5% level are noted in bold.

**Table 2.6.** Data reduction results by silvicultural management.

<b>Management Regime</b>	<b>Total Number Plots</b>	<b>Total Number Data Points</b>	<b>Reduced Number Plots</b>	<b>Reduced Number Data Points</b>
All reduced data	204	1134	142	248
Intensive	102	564	85	152
Operational	102	570	57	96

**Table 2.7.** Comparisons of linear mixed-effects model performance (REML estimation method with independent diagonal covariance structure for the residuals).

MODEL	-2Log-likelihood	AIC*	BIC*
3	-220.1	-216.1	-210.2
4	-220.1	-216.1	-210.2
5	-101.8	-99.8	-96.3
6	-216.6	-212.6	-206.7
7	-207.6	-203.6	-197.7
8	-216.6	-212.6	-206.7
9	-213.5	-209.5	-203.6
10	-213.5	-209.5	-203.6
11	-204.6	-200.6	-194.7
12	-215.8	-211.8	-205.9
13	-207.8	-203.8	-197.9
14	-215.8	-211.8	-205.9
15	-211.9	-207.9	-202
16	-101.1	-99.1	-95.6
17	-103.7	-101.7	-98.2
18	-81.1	-79.1	-75.6

**Note:** AIC, Akaike information criterion (smaller is better); BIC, Bayesian information criterion (smaller is better).

**Table 2.8.** Linear mixed-effects model parameter estimates for the species maximum size-density relationship in loblolly pine (Model [8])

Effect	Parameter	Estimate	Std Error	t-value	p-value
<i>Fixed-Effect Parameter</i>					
Intercept (Operational Management)	$\beta_{01}$	11.6879	0.2906	40.22	<.0001
Intercept (Intensive vs. Operational)	$\beta_{02}$	1.0130	0.3767	2.69	0.0078
Slope (Operational)	$\beta_{11}$	-1.5265	0.1116	-13.67	<.0001
Slope (Intensive vs Operational)	$\beta_{12}$	-0.3489	0.1419	-2.46	0.0148
<i>Covariance Parameter</i>					
Variance of Random Slopes	$\sigma_1^2$	0.007439			
Residual Variance	$\sigma^2$	0.003495			
Autoregressive parameter AR(1)	$\phi$	0.8417			
Observations		248			
AIC		-221.6			
BIC		-212.7			

**Table 2.9.** Parameter estimates of the species limiting size-density line based on Equation [8] using mixed-effect methods for the full data set and for subsets of the data consisting of varying planting densities.

	n	Intercept (Std.Error)		Slope (Std.Error)	
		Intensive	Operational	Intensive	Operational
<b>Planting Density (trees/ha)</b>					
Low ( < 2,224)	57	10.9590 <sup>a</sup> (0.6397)	10.1967 <sup>a</sup> (1.3447)	-1.3633 <sup>a</sup> (0.2124)	-1.1872 <sup>a</sup> (0.4635)
Medium (2,224 - 2,965)	93	11.7399 <sup>a</sup> (0.3514)	9.6834 <sup>b</sup> (0.3579)	-1.5058 <sup>a</sup> (0.1293)	-1.0764 <sup>b</sup> (0.1369)
High ( > 2,965)	98	12.4384 <sup>a</sup> (0.3031)	10.8232 <sup>b</sup> (0.3106)	-1.7322 <sup>a</sup> (0.1179)	-1.0990 <sup>b</sup> (0.1247)
All densities	248	12.7010 <sup>a</sup> (0.2398)	11.6879 <sup>b</sup> (0.2906)	-1.8754 <sup>a</sup> (0.08754)	-1.5265 <sup>b</sup> (0.1116)

**Notes:** for a given planting density class, different letters indicate significant differences at 5% level between the silvicultural management regimes in terms of the slopes and intercepts, respectively.

n = number of observations using in the parameter estimation, Std.Error = standard error of the estimate.

**Table 2.10.** Parameter estimates of the species limiting size-density line based on Equation [8] using mixed-effect methods for the full data set and for subsets of the data consisting of varying CRIFF soil groups.

		Intercept (Std.Error)		Slope (Std.Error)	
		Intensive	Operational	Intensive	Operational
CRIFF Soil	n				
<b>A</b>	61	11.0414 <sup>a</sup> (0.4037)	10.5156 <sup>a</sup> (0.4271)	-1.2662 <sup>a</sup> (0.1449)	-1.0603 <sup>a</sup> (0.1617)
<b>B1</b>	41	14.1005 <sup>a</sup> (0.3348)	13.8004 <sup>a</sup> (0.3849)	-2.3979 <sup>a</sup> (0.1207)	-2.3064 <sup>a</sup> (0.1444)
<b>B2</b>	45	15.4001 <sup>a</sup> (0.6451)	14.6828 <sup>a</sup> (0.5342)	-3.0348 <sup>a</sup> (0.2560)	-2.6035 <sup>a</sup> (0.1954)
<b>C</b>	88	13.1961 <sup>a</sup> (0.3911)	12.9510 <sup>a</sup> (0.5143)	-2.0618 <sup>a</sup> (0.1438)	-2.0058 <sup>a</sup> (0.1983)
<b>All soils</b>	248	12.7010 <sup>a</sup> (0.2398)	11.6879 <sup>b</sup> (0.2906)	-1.8754 <sup>a</sup> (0.08754)	-1.5265 <sup>b</sup> (0.1116)

**Notes:** for a given planting density class, different letters indicate significant differences at 5% level between the silvicultural management regimes in terms of the slopes and intercepts, respectively.

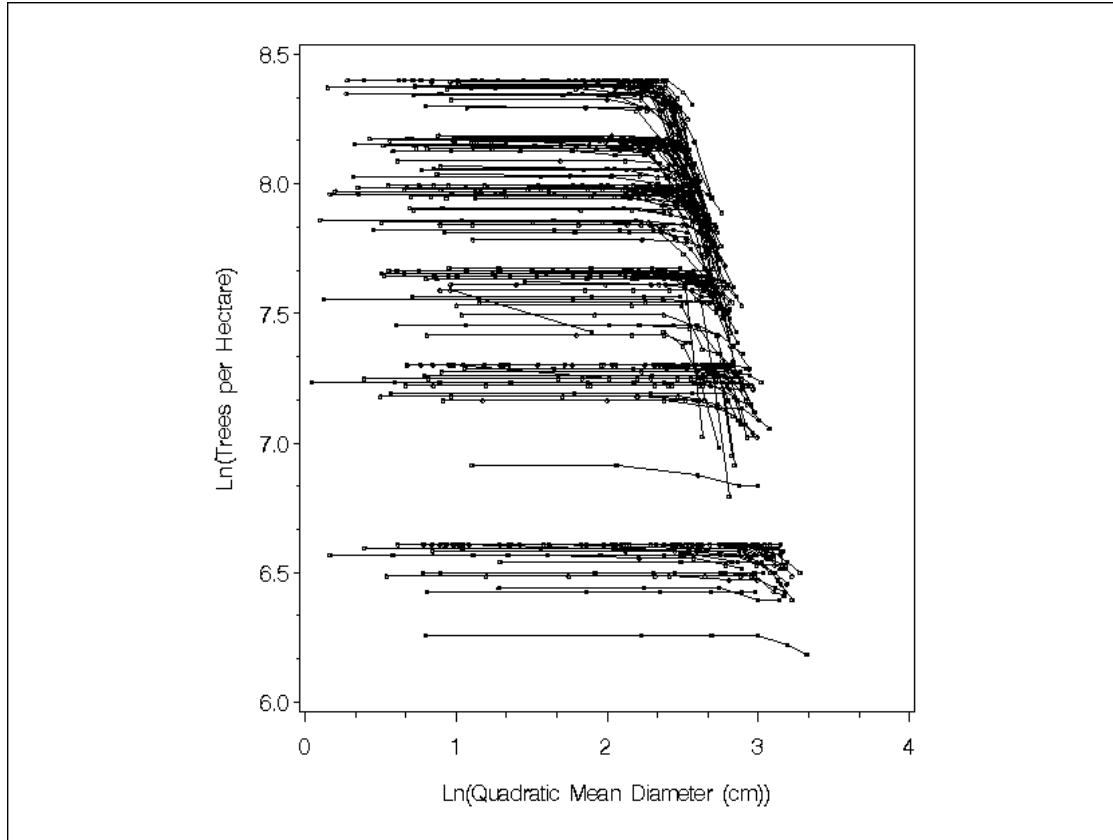
n = number of observations using in the parameter estimation, Std.Error = standard error of the estimate.

**Table 2.11.** Parameter estimates of the species limiting size-density line based on Equation [8] using mixed-effect methods for the full data set and for subsets of the data consisting of varying site index values.

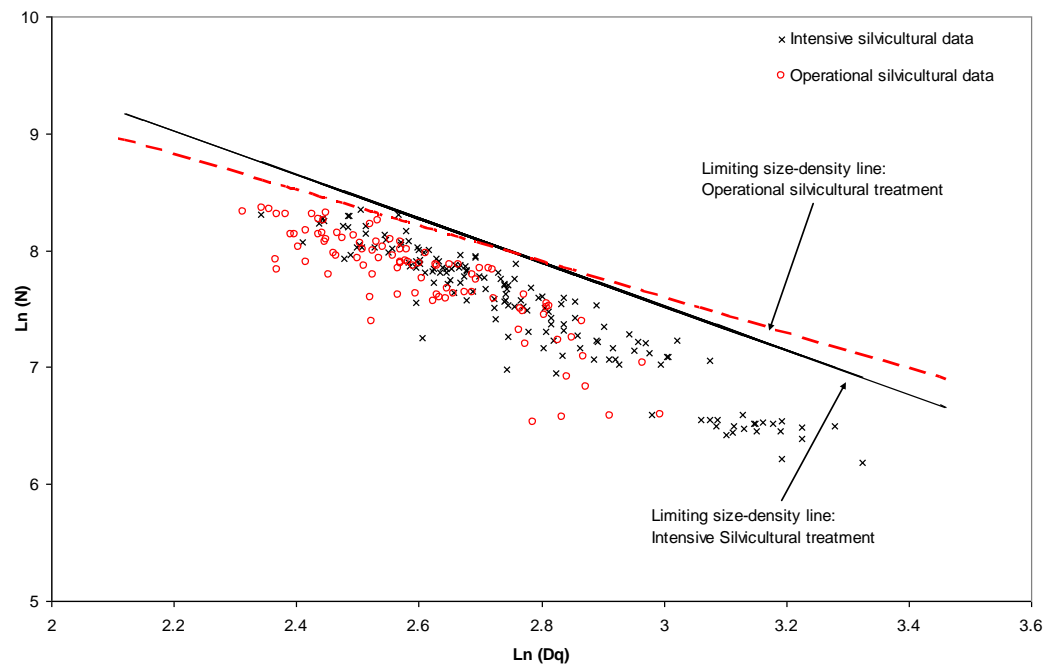
Site Index class	n	Intercept (Std.Error)		Slope (Std.Error)	
		Intensive	Operational	Intensive	Operational
Low	133	12.9119 <sup>a</sup> (0.3284)	12.1541 <sup>a</sup> (0.1210)	-1.9713 <sup>a</sup> (0.4051)	-1.7429 <sup>a</sup> 0.1549
High	115	13.5011 <sup>a</sup> (0.3155)	12.2966 <sup>b</sup> (0.4075)	-2.1363 <sup>a</sup> (0.1133)	-1.7166 <sup>b</sup> (0.1569)
All Site indexes	248	12.7010 <sup>a</sup> (0.2398)	11.6879 <sup>b</sup> (0.2906)	-1.8754 <sup>a</sup> (0.08754)	-1.5265 <sup>b</sup> (0.1116)

**Notes:** for a given planting density class, different letters indicate significant differences at 5% level between the silvicultural management regimes in terms of the slopes and intercepts, respectively.

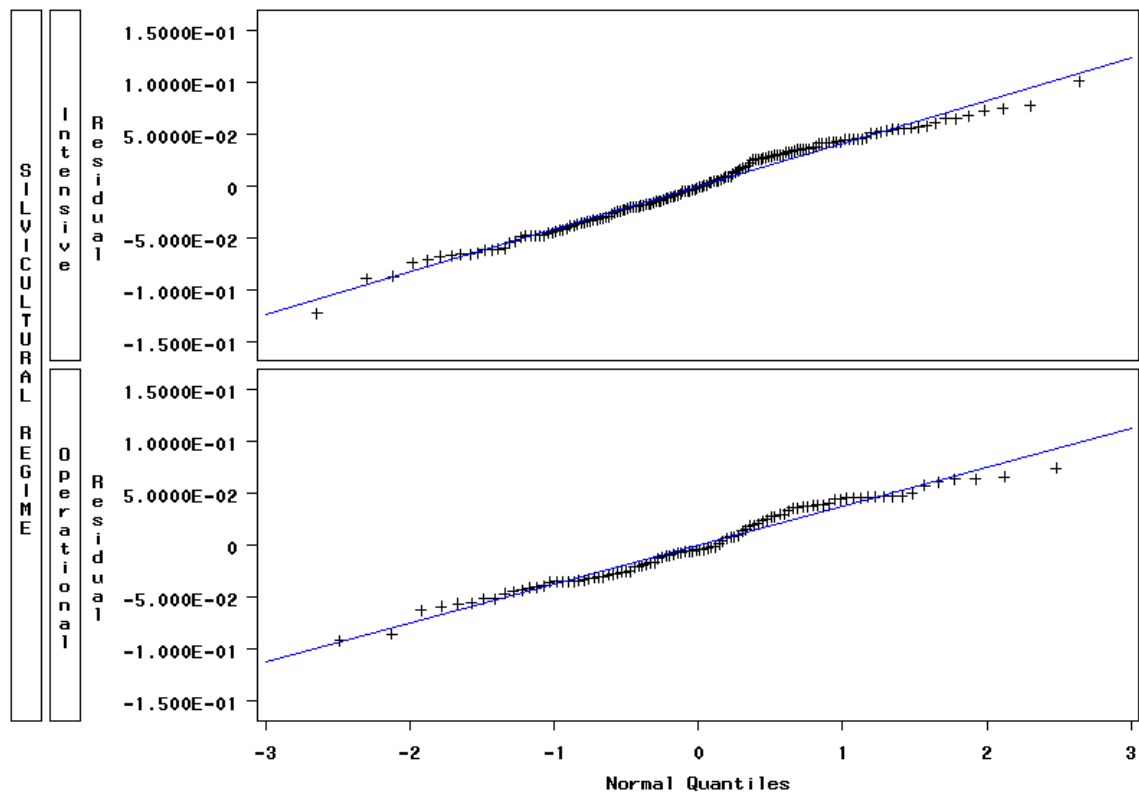
n = number of observations using in the parameter estimation, Std.Error = standard error of the estimate.



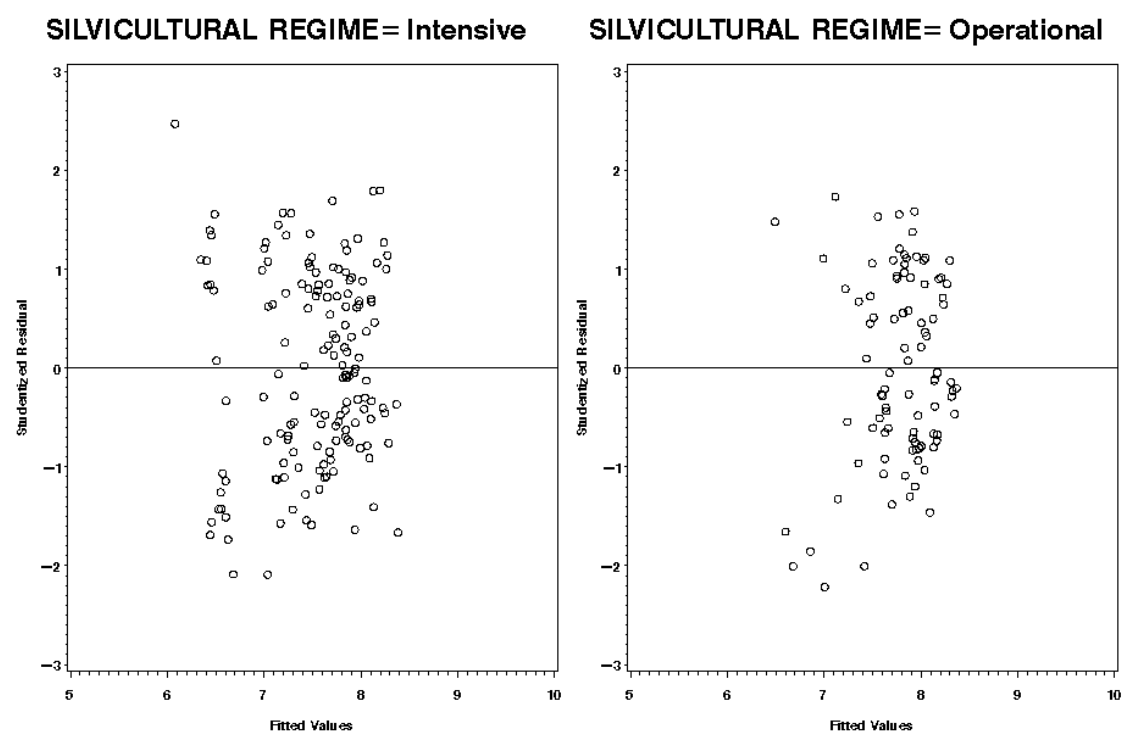
**Figure 2.1** Observed density-size trajectories from 204 permanent plots in loblolly pine stands.



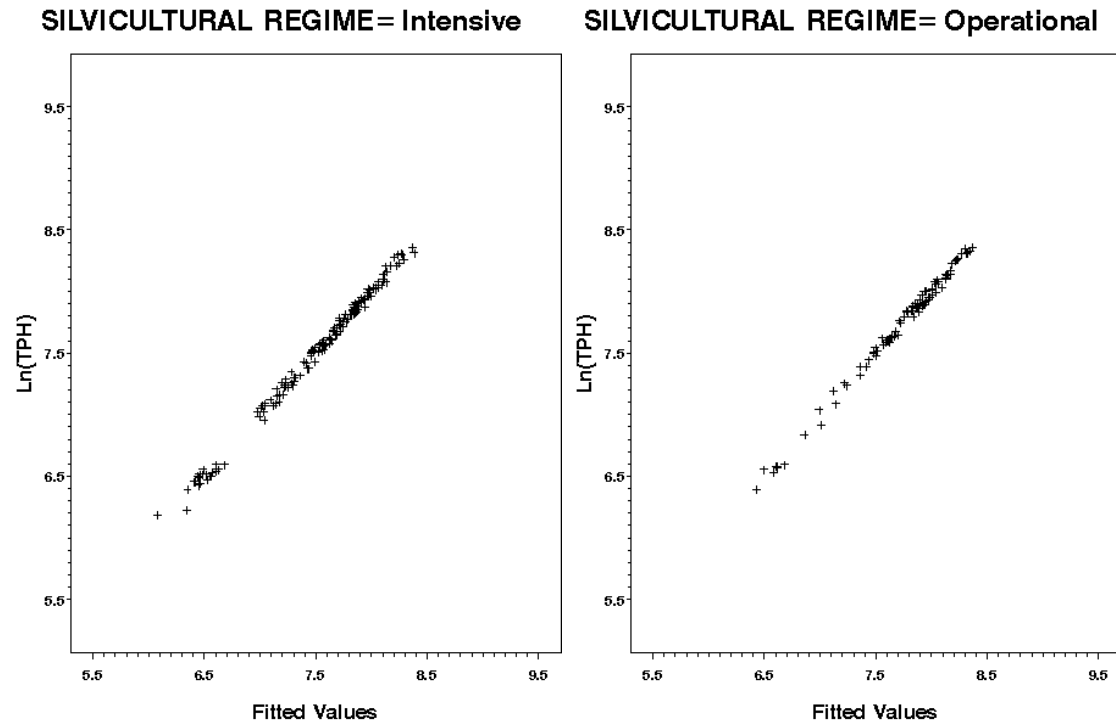
**Figure 2.2** Species limiting size-density line for the intensive and operational treatment groups, based on Equation [8]. The boundary lines were vertically shifted above all points.



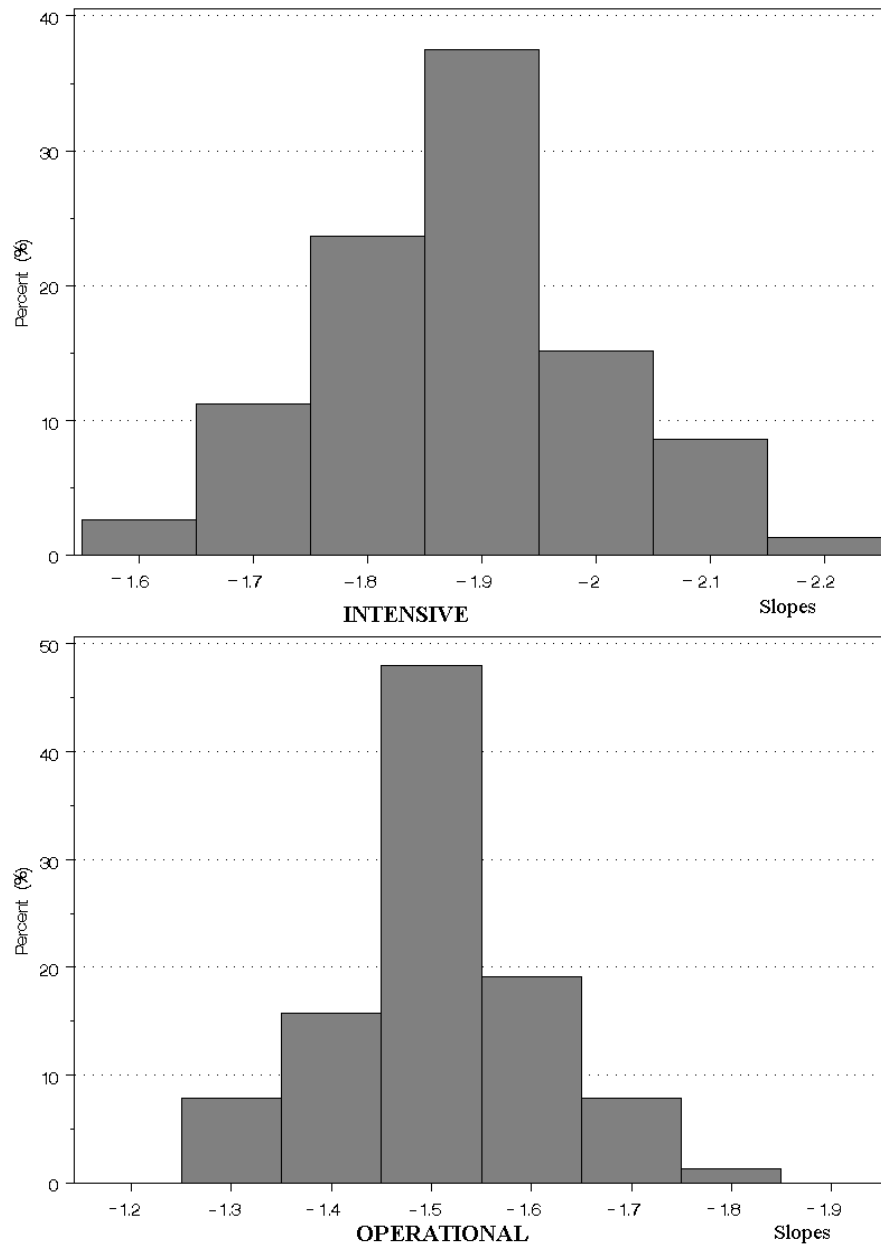
**Figure 2.3** Normal Q-Q plots for the conditional residuals in the intensive and operational silvicultural treatment groups, based on the fit of Equation [8].



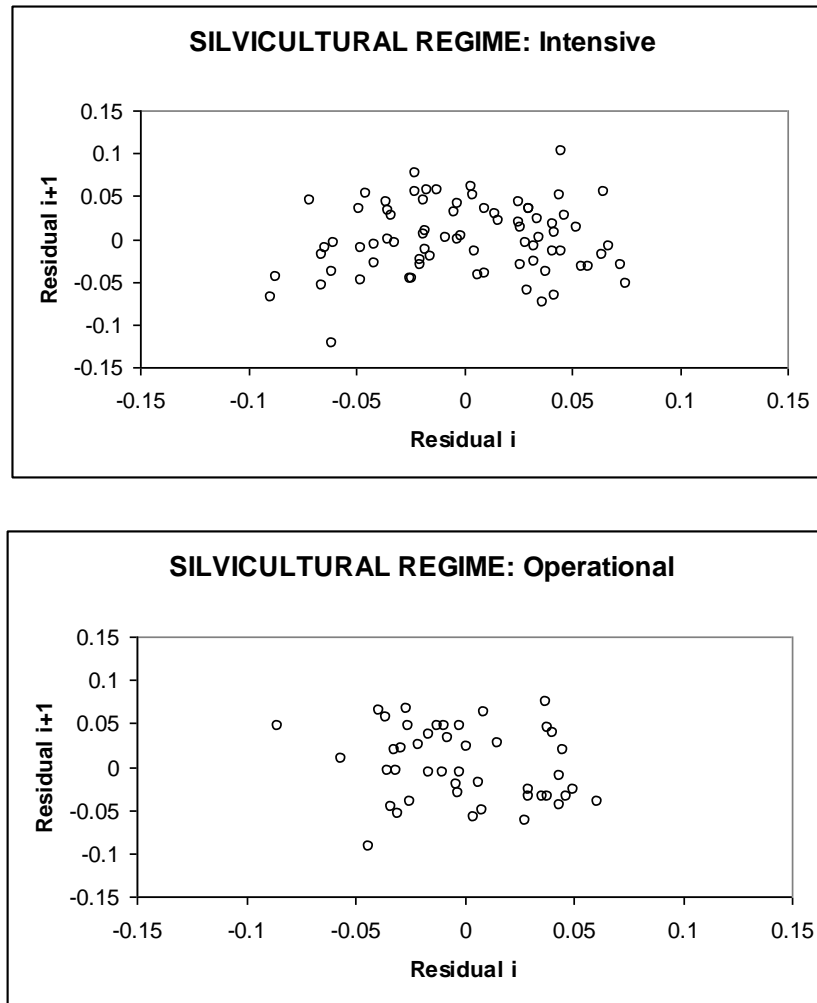
**Figure 2.4** Scatterplots of conditional residuals vs. predicted values in the intensive and operational silvicultural treatment groups, based on Equation [8].



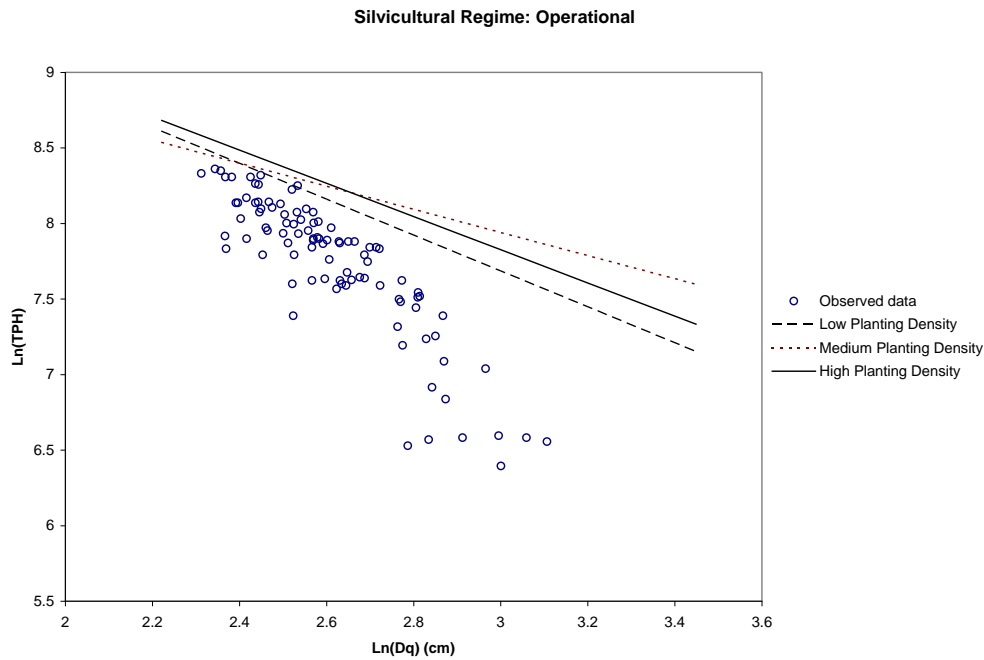
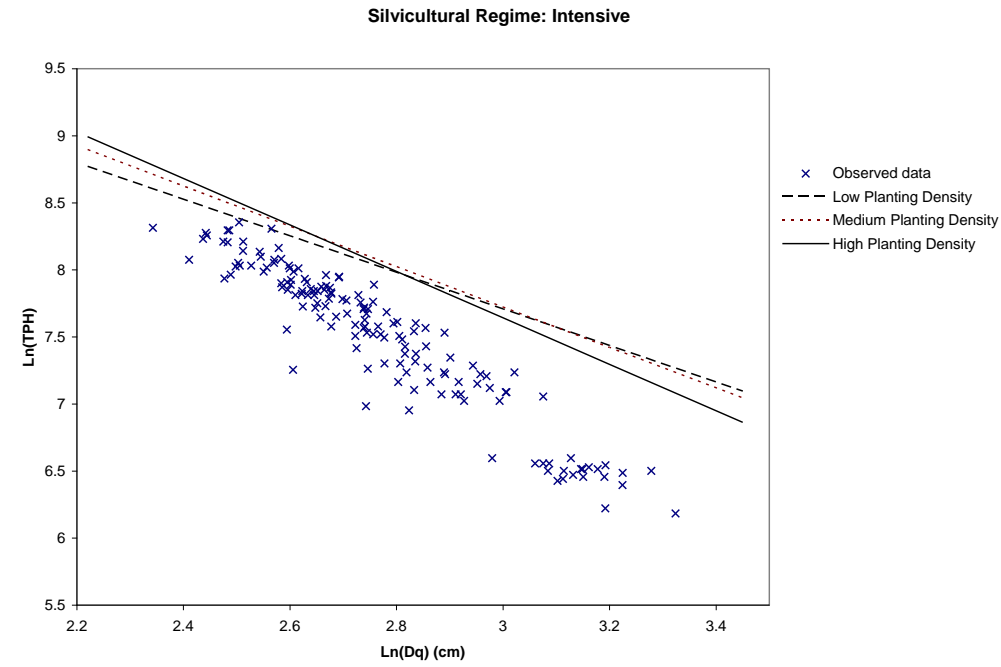
**Figure 2.5** Agreement of observed  $\text{Ln}(N)$  with predicted  $\text{Ln}(N)$  for each level of silvicultural treatment, based on Equation [8].



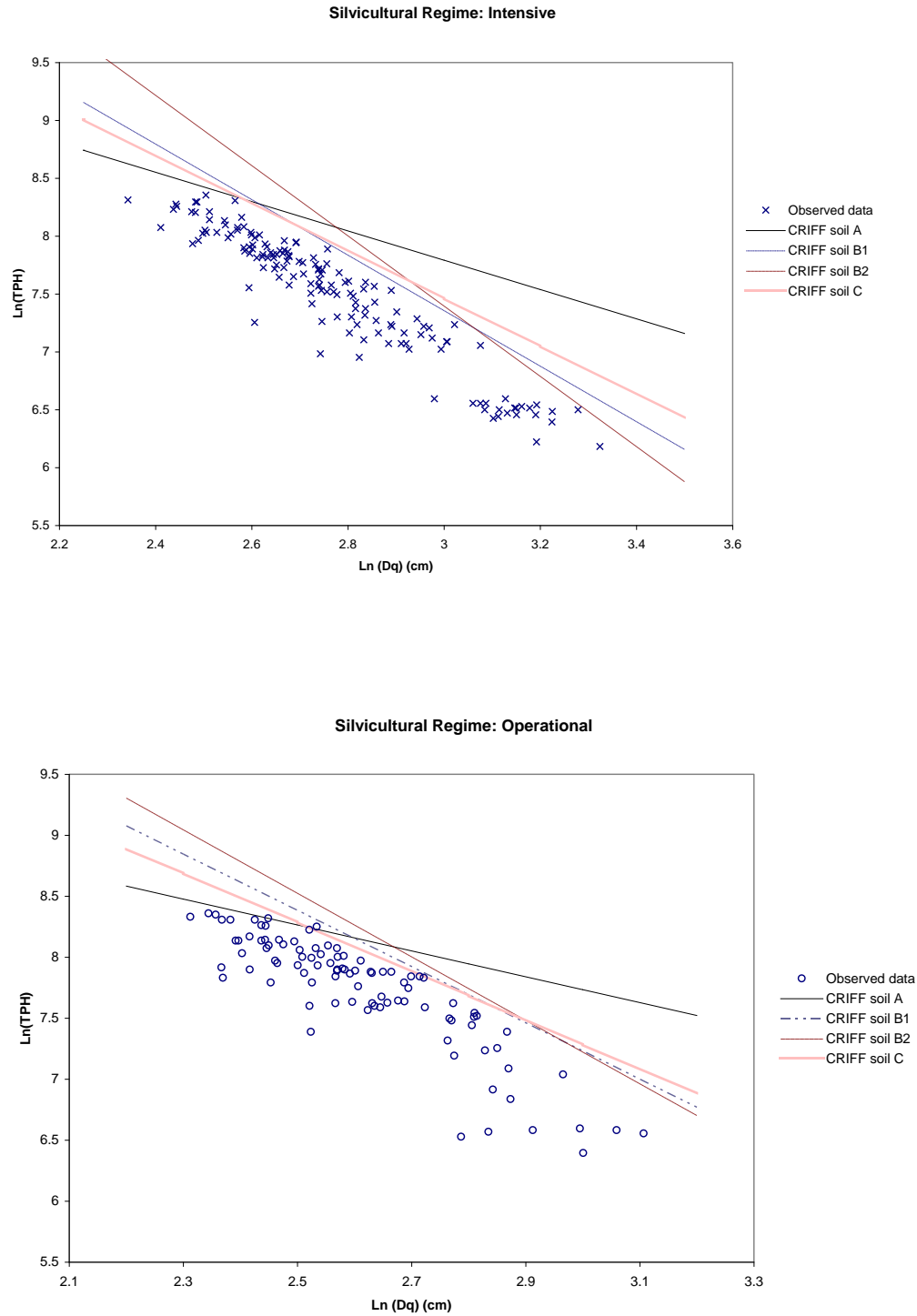
**Figure 2.6** Distribution of slopes for each level of the silvicultural treatment based on Equation [8].



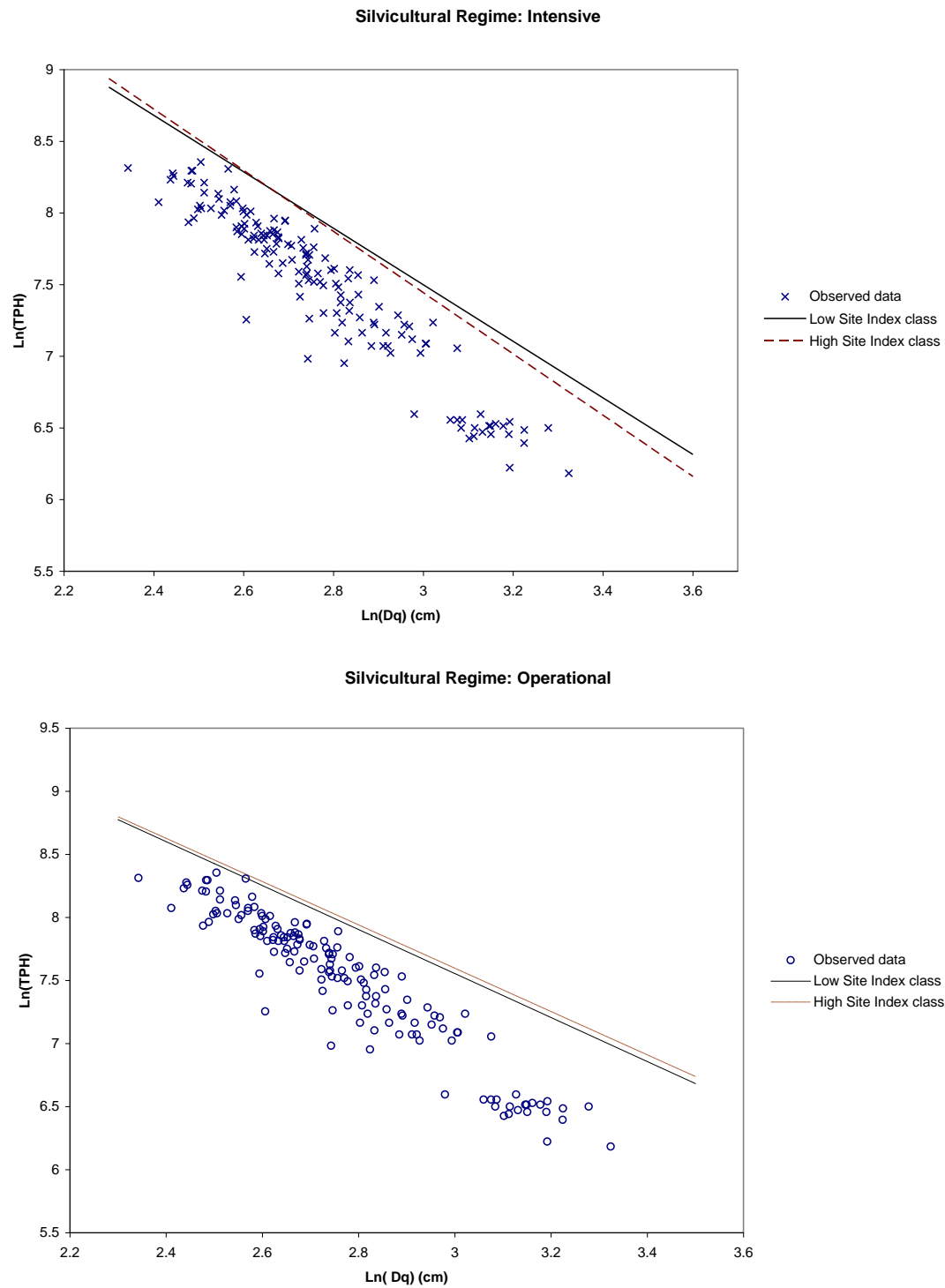
**Figure 2.7** Lag-residual plots in the intensive and operational silvicultural treatment groups based on Equation [8].



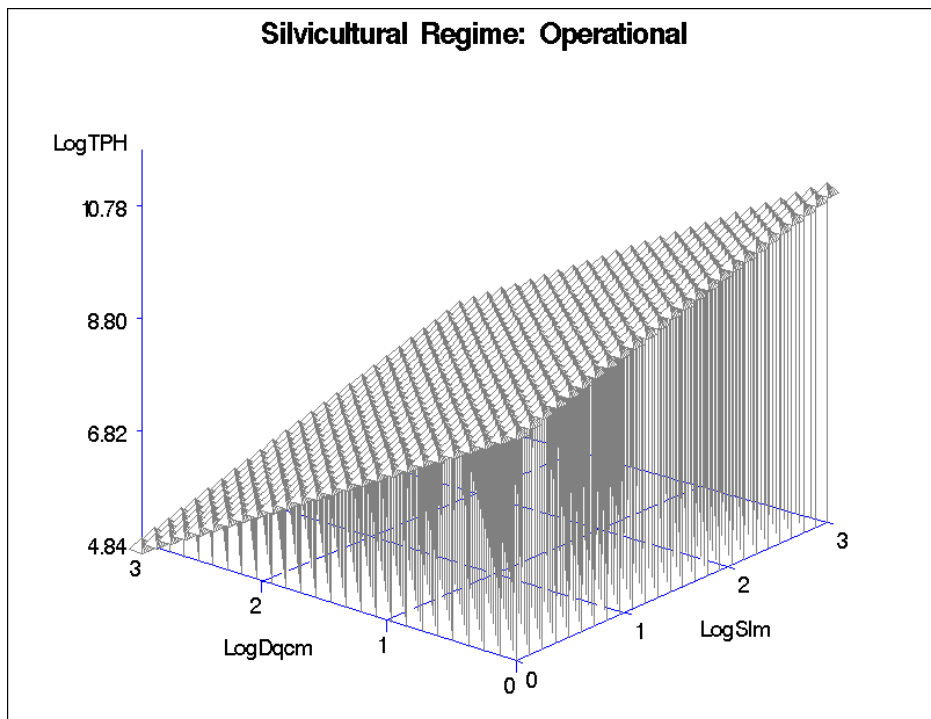
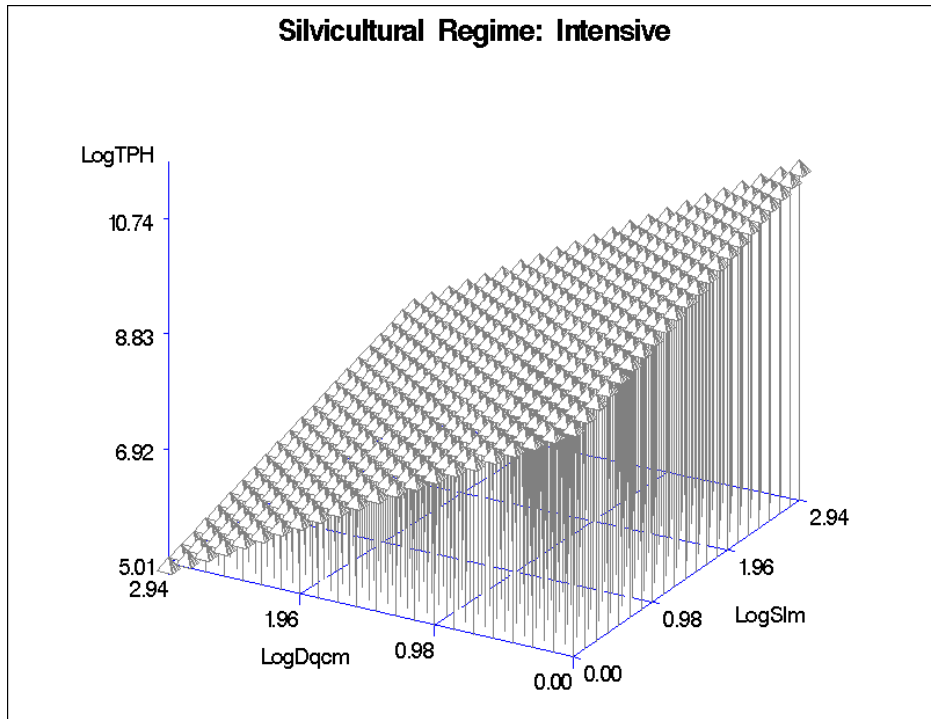
**Figure 2.8** Species limiting size-density lines by silvicultural regime (based on Equation [8]) by planting densities classes. The boundary lines were vertically shifted above all points.



**Figure 2.9** Species limiting size-density lines by silvicultural regime (based on Equation [8]) by CRIFF soil groups. The boundary lines were vertically shifted above all points.



**Figure 2.10** Species limiting size-density lines by silvicultural regime (based on Equation [8]) by site index class. The boundary lines were vertically shifted above all points.



**Figure 2.11** Species limiting size-density surface (based on Equation [26]) of stand density ( $\text{Ln}(\text{TPH})$ ) over quadratic mean diameter ( $\text{Ln}(\text{Dq})$ , in cm) and site index (m) by silvicultural regime for a given planting density (2,224 trees/ha).

## CHAPTER 3

### Semiparametric Regression Analysis of the Size-Density Relationship in Loblolly Pine (*Pinus taeda* L.) Stands

#### 3.1. Abstract

This chapter investigated the application of semiparametric regression analysis to modeling the trajectory of tree density and quadratic mean diameter through time for individual stands. A semiparametric interaction mixed model was able to completely describe the density-size trajectory of even-aged loblolly pine stands in the southeastern U.S. The model addressed the non-linear trend in  $\ln(\text{trees/ha})$  with  $\ln(\text{quadratic mean diameter})$ , and the effect of initial density on the trajectory. An independent upper boundary line fitted using stochastic frontier regression was introduced to study how the semiparametric model approaches the limiting size-density relationship. The semiparametric regression approach is attractive for modeling complex functional forms such as the size-density relationship presented in this chapter.

**Keywords:** Self-thinning, Diameter-density relationship, Semiparametric regression, Stochastic frontier regression, Loblolly pine, *Pinus taeda* L.

### 3.2. Introduction

When even-aged monocultures of plants are subject to severe intraspecific competition, death of suppressed plants occurs in the process known as self-thinning or density-dependent mortality (Yoda et al. 1963, Westoby 1984). The most common density indexes for the analysis of the impact of stand density on self-thinning combine an expression of the size of the average tree (diameter at breast height, biomass or volume) with the number of trees per unit area (Curtis 1970, Long 1985, Long et al. 2004). Reineke's equation (Reineke, 1933) and the “-3/2 power law” of self-thinning (Yoda et al. 1963) are the most commonly used. The rules state that, in log-log scales, the relationship between average plant size and stand density is a straight line (self-thinning line or maximum size–density relationship) for a stand undergoing density-related mortality. Reineke's equation is written as

$$[1] \quad \ln(TPH) = \alpha + \beta \cdot \ln(Dq)$$

where TPH is trees per hectare, and Dq is quadratic mean diameter.

There is a large, and at times controversial, body of literature concerning the self-thinning law covering the past 40 years (Westoby 1984; Weller 1987, 1991; Lonsdale 1990; Pretzsch 2002; Pretzsch and Biber 2005). The controversy has primarily focused on whether the slope of the maximum size-density relationship line is constant (Zeide 1987, Lonsdale 1990, Hynynen, 1993).

Some authors considered that the self-thinning relationships for given data sets were curvilinear instead of linear on a log–log scale (Zeide 1987, Cao et al. 2000, Bi 2001, Yang and Titus 2002, Weiskittel et al. 2009). Zeide (1987) claimed the self-thinning trajectory as a concave down curve rather than a straight line ( $\ln(TPH)$ - $\ln(Dq)$ ). The curve has been described

as having three phases: (I) from young stage until crown closure, when the self-thinning is less than the mortality at maximum competition, it is concave and lies under the straight line of maximum density; (II) in the intermediate phase of the stand's development, it follows the straight line of maximum density; and (III) in older-aged stands, when the capability to fill gaps left by dead trees has been lost, the line inclines to curve below the straight line. Smith and Hann (1984), Puettmann et al. (1993), and Tang et al. (1994) have proposed a different kind of trajectory for the related number of trees per unit area-quadratic mean diameter, in which once the line of maximum density has been reached, the curve follows it as an asymptote.

Another important concern centers on the most suitable methods for data selection and parameter estimation in the self-thinning equations (Bi and Turvey 1997, Bi et al. 2000, Zhang et al. 2005). To perform the fitting and to interpret the result as a line of maximum size-density it is essential that the stands analyzed are fully-stocked or at maximum density (Weller 1987, Rio 2001). Data used to model the maximum size–density relationship should be carefully selected from those stands that are in the self-thinning stage of stand development. Numerous selection methods have been proposed, from visually selecting observations (Harms 1981, Weller 1987, VanderSchaaf and Burkhardt 2007) to more statistically based criteria (Smith and Hann 1984, Bredenkamp and Burkhardt 1990, del Rio et al. 2001, Zhang et al. 2005, Poage et al. 2007). There is still no agreement on the selection criteria to apply when determining what observations occur along the self-thinning lines (Zhang et al. 2005, VanderSchaaf and Burkhardt 2008).

Zhang et al. (2005) compared several statistical techniques for estimating the limiting size-density relationship and concluded that stochastic frontier regression (SFR) had advantages over other techniques such as quantile regression, ordinary least squares (OLS), and principal components analysis (PCA). Stochastic frontier regression models seek to explain boundary,

frontier or optimal behavior rather than average behavior as in ordinary regression models (Aigner et al. 1977). Stochastic frontier regression is an econometric method that has been adopted to estimate the self-thinning boundary line as a density-dependent stochastic frontier (Bi et al. 2000, Zhang et al. 2005, Weiskittel et al. 2009). This method uses all the data points to fit the limiting line. Since no subjective data selection is involved and no information contained in the data is lost, the boundary line can be estimated without subjectivity and more efficiently than the common methods of subjective data selection. Consequently, statistical inferences about the estimated self-thinning boundary line can be made objectively and more precisely.

Modeling the complete size–density trajectories of individual stands removes the need to censor data to study the properties of the upper boundary of the tree size and tree density relationship. The boundary develops from overlapping trajectories of individual stands (Smith and Hann 1986, Cao and Dean 2008). Individual trajectories can also be used as constraints in stand growth and yield models (Hynynen 1993, Tang et al. 1994, Monserud et al. 2005, Poage et al. 2007). Growth and yield systems can combine mortality functions with height, diameter, volume, or weight equations to estimate an approach to a limiting size-density constraint. To incorporate this technique the models need to appropriately describe the full developmental trajectories of the stand. Modelers generally assume a monotonic increase in mortality with increasing proximity to the size–density boundary after canopy closure (Cao and Dean 2008). Long and Smith (1984) suggested a developmental stage between canopy closure and self-thinning called full-site occupancy characterized by accelerated size differentiation among trees in the stand and skewness in size distribution. Pre- and post-canopy closure mortality rates indicate that a complete description of a stand’s developmental trajectory requires numerous curve segments. This chapter reports on the application of semiparametric regression, a

technique that allows the flexible incorporation of nonlinear functional relationships in regression models without having parametric restrictions, to describe the complete size-density relationship of loblolly pine stands.

The objectives of this study were to (1) apply semiparametric regression analysis to modeling the trajectory of stand density and quadratic mean diameter of individual stands through time; and (2) analyze the approach of the semiparametric model to a boundary size-density relationship fitted using stochastic frontier regression methods.

### 3.3. Data

The data for this work were obtained from permanent plots of loblolly pine from a spacing study established on six sites in southeastern Georgia (Chatham, Long, Tattnall, and Effingham counties) (Figure 3.1). Three replicates of a randomized complete block design were established at each location in 1985, 1986, 1989 or 1991. Ten initial planting configurations were established, ranging from 749 to 2990 trees per hectare (749, 897, 1078, 1122, 1347, 1495, 1683, 1794, 2244, and 2990 trees/ha). A total of 180 experimental units (plots) were established when all six sites were combined (6 sites  $\times$  3 replications  $\times$  10 planting configurations). Plot size ranged from 0.024 to 0.085 ha. Each plot was measured from three to four times, at 3-5 year intervals, resulting in a total of 648 observations.

Site index was calculated using the site index equations developed by Borders et al. (2004) for second rotation loblolly pine plantations:

$$[2] \quad S = a \left( 1 - \left( 1 - \left( \frac{HD}{a} \right)^{1/b} \right)^{25/A} \right)^b$$

where  $a=117.6$  and  $b=1.336527$  using English units,  $S$  is site index estimated at the age ( $A$ ) of the oldest measurement,  $HD$  is dominant height defined as the average height of trees with diameter at breast height (DBH) larger than the median DBH of the stand. Summary statistics of plot-level characteristics for the whole dataset are presented in Table 3.1. The trajectories of stand density (trees per hectare) and quadratic mean diameter (cm) in log-log scale for these measurements are shown in Figure 3.2.

Preliminary Analysis of Variance (ANOVA) was performed to test the effect of location on the quadratic mean diameter, Reineke's stand density index (SDI), stand basal area/ha, and trees/ha. All analyses showed that there were no significant differences among locations for the above variables. Since location was not shown to affect the variables related with the size-density trajectories, data were pooled across all planting configurations, replications and locations to fit the semiparametric regression model.

### **3.4. Methods**

#### **3.4.1. Model development: semiparametric interactive mixed model**

Semiparametric regression is a combination between traditional parametric regression analysis and nonparametric regression methods. Nonparametric regression (i.e. scatterplot smoothing) refers to a set of statistical techniques used to summarize bivariate relationships in scatterplots. Through more commonly used "parametrical" statistical techniques, the relationship between two variables is summarized with a parameter. With nonparametric regression, there is no single parameter and the relationship between the variables is summarized with a line drawing

(Keele 2008). Semiparametric analysis combines research across several branches of statistics: parametric and nonparametric regression, longitudinal and spatial data analysis, mixed and hierarchical Bayesian models, Expectation-Maximization (EM) and Markov Chain Monte Carlo (MCMC) algorithms (Rupert et al. 2003, 2009). Semiparametric regression is concerned with the flexible incorporation of nonlinear functional relationships in regression models without having parametric restrictions. Engle et al. (1986) were among the first to develop semiparametric regression models to analyze the relationship between temperature and electricity usage. Since then, many disciplines have embraced semiparametric models. Examples include economics (Linton and Hardle 1996), political science (Beck and Jackman 1998), ecology (Roland et al. 2000), and wood quality (Jordan et al. 2008, Finto 2010).

Semiparametric regression models based on penalized splines can be embedded in the mixed model framework, allowing for mixed model estimation and for inferential and computational tools to be used (Ngo and Wand 2004). The mixed model representation of penalized splines allows for a fusion between parametric mixed models and smoothing, which is called *semiparametric* mixed models (Rupert et al. 2003).

In our study, semiparametric techniques were applied to model trajectories of stand density and quadratic mean diameter through time. When describing the complete size-density trajectory of individual stands (Figure 3.2), the linearity assumption underlying Reineke's model (Equation [1]) is no longer reasonable. The non-linear trend for the complete trajectories of stand density ( $N$ , trees/ha) and quadratic mean diameter ( $D_q$ , cm) in log-log scale can be described by a mixed model representation. The model specification for an individual plot response is given by

$$y_{ij} = U_i + f(x_{ij}) + \varepsilon_{ij},$$

[3]

$$1 \leq j \leq n_i, 1 \leq i \leq m, \quad \varepsilon_{ij} \sim i.i.d. N(0, \sigma_\varepsilon^2), \quad U_i \sim i.i.d. N(0, \sigma_U^2)$$

where  $y_{ij} = \text{Ln}N_{ij}$  represents the number of trees per hectare (Ln) on the  $i$ -th plot at the  $j$ -th measurement,  $U_i$  indicates the set of random effects in the mixed model associated with the plots,  $f$  is some smooth function (i.e. differentiable over its complete domain) to be defined that describes the trajectory of stand density ( $\text{Ln}N$ ) and size ( $\text{Ln}Dq$ ) for individual plots, and  $x_{ij} = \text{Ln}Dq_{ij}$  represents the quadratic mean diameter (Ln) on the  $i$ -th plot at  $j$ -th measurement. For our study,  $m = 180$  plots, and  $n_i \in \{3, 4\}$ , indicating that each plot was measured either 3 or 4 times.

Penalized smoothing splines, i.e. curves formed by splicing low-order polynomials at known knot locations, were used to define the smooth function. Using a quadratic spline basis for the smooth function  $f(x_{ij})$  we see that Equation [3] can be written as follows

[4]

$$y_{ij} = U_i + \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \sum_{k=1}^K u_k (x_{ij} - \kappa_k)_+^2 + \varepsilon_{ij}$$

$$1 \leq j \leq n_i, 1 \leq i \leq m, \quad U_i \sim i.i.d. N(0, \sigma_U^2), \quad u_k \sim i.i.d. N(0, \sigma_u^2), \quad \varepsilon_{ij} \sim i.i.d. N(0, \sigma_\varepsilon^2)$$

In this equation,  $\kappa_1, \kappa_2, \dots, \kappa_K$  represent  $K$  knot locations within the range of the  $x_{ij}$ 's, and  $(x_{ij} - \kappa_k)_+$  is the positive function where “+” sets it to zero for those values of  $x_{ij}$  such that

$$x_{ij} - \kappa_k \text{ is negative (i.e. } x_{ij} < \kappa_k), \text{ i.e. } (x_{ij} - \kappa_k)_+ = \begin{cases} x_{ij} - \kappa_k, & \text{if } x_{ij} > \kappa_k \\ 0, & \text{else} \end{cases}$$

Note that Equation

[4] is a special case of the Gaussian linear mixed model. If we define

$$\mathbf{Y} = \begin{bmatrix} y_{11} \\ \cdot \\ \cdot \\ y_{1m_1} \\ \cdot \\ y_{nm_n} \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{11}^2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & x_{1m_1} & x_{1m_1}^2 \\ \cdot & \cdot & \cdot \\ 1 & x_{nm_n} & x_{nm_n}^2 \end{bmatrix},$$

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & x_{11} - \kappa_{1+}^2 & \cdot & \cdot & \cdot & \cdot & x_{11} - \kappa_{\kappa+}^2 \\ & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & x_{1m_n} - \kappa_{1+}^2 & \cdot & \cdot & \cdot & \cdot & x_{1m_n} - \kappa_{\kappa+}^2 \\ & & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & x_{nm_n} - \kappa_{1+}^2 & \cdot & \cdot & \cdot & \cdot & x_{nm_n} - \kappa_{\kappa+}^2 \end{bmatrix},$$

and

$$\mathbf{u} = \begin{bmatrix} \mathbf{U}_1 \\ \vdots \\ \mathbf{U}_m \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_K \end{bmatrix}$$

then we can simultaneously estimate variance components for the random intercept and the amount of smoothing for  $f$  by using the mixed model:

$$[5] \quad Y = X\beta + Zu + \varepsilon, \quad \text{Cov} \begin{bmatrix} \mathbf{u} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} \sigma_U^2 \mathbf{I} & 0 & 0 \\ 0 & \sigma_u^2 \mathbf{I} & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \mathbf{I} \end{bmatrix}$$

Here  $\sigma_U^2$  measures the between-plot variation,  $\sigma_\varepsilon^2$  measures within-plot variation, and  $\sigma_u^2$  controls the amount of smoothing done to estimate  $f$  through the parameter  $\lambda = \sigma_\varepsilon^2 / \sigma_u^2$ . The maximum likelihood estimate for  $\hat{\beta}$  (**MLE**), and an empirical best linear unbiased predictor for  $\hat{\mathbf{u}}$  (**BLUP**) were obtained using PROC MIXED in SAS (SAS Institute Inc. 2008).

This study is concerned with the effects of planting density on self-thinning trajectories. As described in section 3.3, the data can be categorized according to ten planting densities: 749, 897, 1078, 1122, 1347, 1495, 1683, 1794, 2244, and 2990 trees/ha. To address this question of interest, we define indicator variables  $Z_{it}$  for nine of ten planting densities and consider the model

$$[6] \quad y_{ij} = U_i + \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \sum_{k=1}^K u_k x_{ij} - \kappa_k^2 + \beta_3 I_{749} + \beta_4 I_{897} \\ + \beta_5 I_{1078} + \beta_6 I_{1122} + \beta_7 I_{1347} + \beta_8 I_{1495} + \beta_9 I_{1683} + \beta_{10} I_{1794} + \beta_{11} I_{2244} + \varepsilon_{ij}$$

$$1 \leq j \leq n_i, 1 \leq i \leq m, U_i \sim i.i.d. N(0, \sigma_U^2), u_k \sim i.i.d. N(0, \sigma_u^2), \varepsilon_{ij} \sim i.i.d. N(0, \sigma_\varepsilon^2)$$

With this formulation, the planting density of 2290 trees/ha comprises the reference group and  $\beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}, \beta_{11}$  represent mean differences in LnN between the other planting density groups.

Equation [6] corresponds to an additive model, i.e. the joint effect of all predictor variables upon the response is expressed as a sum of individual effects. These individual effects show how the expected response varies as any single predictor varies with the others held fixed at arbitrary values. However, there are no guarantees that an additive model will provide a satisfactory fit in any given situation. Non-additive or interaction models are those such that as one predictor is varied, the effect on the expected response depends on the fixed values of the other predictors.

Let  $z_i \in \{1, 2, 3, \dots, L=10\}$  represent the planting density coded factor corresponding to the ten initial densities, and let  $k_1, k_2, \dots, k_K$  be a set of knots inside the range of the  $x_i$ . Define

$$z_{it} = \begin{cases} 1 & \text{if } z_i = t \\ 0 & \text{otherwise} \end{cases}, \text{ for } t=1,2, \dots, L=10.$$

The inclusion of interaction terms yields a *semiparametric interactive mixed model* (in Equation [7]).

$$\begin{aligned} y_{ij} = & U_i + \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \sum_{k=1}^K u_k \left( x_{ij} - \kappa_k \right)^2 + \\ [7] \quad & + \sum_{t=2}^L z_{it} \left( \gamma_{0t} + \gamma_{1t} x_{ij} + \gamma_{2t} x_{ij}^2 \right) \\ & + \sum_{t=2}^L z_{it} \left( \sum_{k=1}^K v_k^t \left( x_{ij} - \kappa_k \right)^2 \right) + \varepsilon_{ij} \end{aligned}$$

$$1 \leq j \leq n_i, \quad 1 \leq i \leq m,$$

$$U_i \sim i.i.d. N(0, \sigma_U^2), \quad u_k \sim i.i.d. N(0, \sigma_u^2),$$

$$\varepsilon_{ij} \sim i.i.d. N(0, \sigma_\varepsilon^2), \quad v_k^t \sim i.i.d. N(0, \sigma_v^2)$$

In model [7],  $\sum_{k=1}^K u_k' (x_{ij} - \kappa_k)^2$  represents deviations from the overall smooth term  $\sum_{k=1}^K u_k' (x_{ij} - \kappa_k)^2$ . The vector  $[\beta_0 \ \beta_1 \ \beta_2 \ u_1 \ u_2 \ \dots \ u_K]$  denotes the initial density of 2290 trees/ha (t=1, reference group) and additional terms represent mean differences in LnN between the other planting density groups.

### 3.4.2. Testing for additivity

A test for additivity can be conducted by comparing the additive model [6] to the interaction model [7] and checking whether the interaction model offers a significant improvement in fit (Hastie and Tibshirani 1990). In particular, we can compare the log-likelihoods for the additive and interaction model through the likelihood ratio test statistic. In terms of the parameters in [7], the null hypothesis of additivity is

$$H_0: \gamma_{jt} = 0, \quad t = 1, 2, \dots, L = 10, \quad j = 1, 2, 3, \quad \text{and } \sigma_v^2 = 0$$

After accounting for the constraints imposed on the fixed effects to ensure identifiability, the null hypothesis restricts (L-1)\*3 fixed effects parameters and one variance component.

### 3.4.3. Semiparametric first order derivative estimation

The derivatives of the regression function  $f$  can be of importance as well as  $f$  itself. Derivative estimation plays an important role in the exploration of structures in curves, inference of significant features in data, trend analysis, comparison of regression scatterplots, and analysis of growth data (Park and Kang 2008). All the previous analysis techniques are based on the

inference about slopes (and hence the derivative) of the regression estimates. Therefore, the study of estimating derivatives (first and higher orders) semiparametrically is as equally important as regression estimation.

The derivative of the semiparametric interactive mixed model [7] can be used to explore the rate of change of density (LnN) with respect to size (LnDq). Differentiating Equation [7] with respect to  $x_{ij}$  we have

$$[8] \quad y' = \frac{\partial y_{ij}}{\partial x_{ij}} = \beta_1 + 2\beta_2 x_{ij} + \sum_{k=1}^K 2u_k (x_{ij} - \kappa_k) + \sum_{t=2}^L z_{it} \gamma_{1t} + 2\gamma_{2t} x_{ij} + \sum_{t=2}^L z_{ij} \left( \sum_{k=1}^K 2v_k^t (x_{ij} - \kappa_k) \right)$$

Operationally, a derivative estimate at  $x$  can be obtained from the quadratic fit coefficients

$$\boldsymbol{\beta} = [\beta_0 \quad \beta_1 \quad \beta_2]^T \quad \text{and} \quad \mathbf{u} = [u_1, \dots, u_K]^T \quad \text{by setting}$$

$$\mathbf{X}'_x = [0 \quad 1 \quad 2x] \quad \text{and} \quad \mathbf{Z}'_x = \begin{bmatrix} 2(x - \kappa_1) \\ \vdots \\ 2(x - \kappa_K) \end{bmatrix}.$$

It follows

$$y' = \mathbf{X}'_x \boldsymbol{\beta} + \mathbf{Z}'_x \mathbf{u}$$

#### 3.4.4. Knot specification

The set  $\kappa_1, \kappa_2, \dots, \kappa_K$  represent  $K$  knot locations to be determined within the range of the  $x_{ij}$ . The goal is to choose enough knots to resolve the structure of the underlying regression function. A practical default is to choose the knots to guarantee that there are a fixed number of

unique observations (4-5) between each knot (Ruppert et al. 2003). For large data sets this can lead to an unnecessary number of knots, so a maximum number of acceptable knots is recommended, usually 20-40 (Ruppert et al. 2003). Wand (2003) proposed the following rule for knot location:

$$\kappa_k = \left( \frac{k+1}{K+2} \right) \text{th sample quantile of the unique } x_{ij}, \quad 1 \leq k \leq K$$

with  $k = \min(4/n, 35)$ .

As an alternative, the user may choose  $K$  based on visual inspection of the scatterplot to determine the complexity of the function  $f$  relative to the structure of the data (Wand 2003)

French et al. (2001), Ruppert (2002), and Wand (2003) claimed that knot specification has a minor impact on the fitting of semiparametric regression models. Rupert et al. (2003), Jordan et al. (2008), and Finto (2010) used evenly spaced knots in fitting the semiparametric regression.

For our study, we compared the fitting result of using evenly spaced knots between the minimum and maximum of LnDq from the available data, and that resulting from choosing the knots based on visual inspection of scatterplots and first derivative plots.

### 3.4.5. Using stochastic frontier regression to model the boundary size-density relationship

Stochastic frontier models of production systems were originally proposed by Aigner et al. (1977), and by Meeusen and Van den Broeck (1977). The model can be written in the form,

$$\begin{aligned} y_i &= f(X_i, \beta) + v_i - u_i \\ [9] \quad &= \beta_0 + \beta_1 x_i + \varepsilon_i \\ &= \beta_0 + \beta_1 x_i + v_i - u_i \end{aligned}$$

where  $\beta$  is a known parameter vector to be estimated,  $\varepsilon_i = v_i - u_i$  is a compound error term with  $u_i \geq 0$  and  $v_i$  unrestricted. The terms  $v_i$  correspond to a two-sided random variable assumed to be iid  $N(0, \sigma_v^2)$ , and  $u_i$  is one-sided random variable which can be assumed to have any distribution restricted to only positive or negative values, such as half normal or truncated-normal. The terms  $u_i$  and  $v_i$  are assumed to be independent. When  $u_i$  takes on only negative values, then  $y_i$  would be less than the frontier  $[f(X_i, \beta) + v_i]$ , describing a stochastic maximum relationship. A number of specifications have been considered for the  $u_i$  terms: a half-normal distribution  $u_i \sim N(0, \sigma_u^2)$  (Aigner et al. 1977), an exponential distribution, and a truncated normal (Stevenson 1980). Preliminary analysis showed no significant different log-likelihood values between specifications of the  $u_i$  terms, and hence a half-normal model was applied. The vector parameter  $\beta$  was estimated together with the variance components:  $\sigma^2$  and  $\gamma$ , where  $\sigma^2 = \sigma_u^2 + \sigma_v^2$ ,  $\gamma = \sigma_u^2 / \sigma^2$ . The parameters  $\sigma^2$  and  $\gamma$  are diagnostic statistics that indicate the relevance of the use of the stochastic frontier function and the accuracy of the assumptions made on the distribution form of the error term. The parameters in the model were estimated by maximum likelihood methods with FRONTIER v4.1 (Coelli 1996).

### **3.4.6. Relating the stochastic frontier boundary model with the semiparametric regression trajectories**

The stochastic frontier regression method has the potential to estimate an upper limiting boundary line above all plots for the maximum size-density relationship. The slope of the stochastic frontier model can be integrated into the semiparametric regression trajectories as follows. Let  $G(x)$  be a nonlinear segmented model that has the following form:

$$\begin{aligned}
[10] \quad & \mathbf{G}(\mathbf{x}) = Y_1(x) \quad \text{if } x < K \\
& \mathbf{G}(\mathbf{x}) = Y_2(x) \quad \text{if } x \geq K
\end{aligned}$$

where  $Y_1(x)$  is the semiparametric interactive mixed model presented in Equation [7],  $Y_2(x)$  is the stochastic frontier regression model showed in Equation [9], and  $x$  represents the quadratic mean diameter expressed as  $\text{Ln}(\text{Dq})$ . That is, for values of  $x$  less than  $K$ , the equation relating  $y$  ( $\text{Ln}(\text{trees/ha})$ ) and  $x$  ( $\text{Ln}(\text{Dq})$ ) is the semiparametric model, and, for values of  $x$  greater than  $K$ , the equation is a line with the slope given by the stochastic regression model. PROC NLMIXED in SAS can fit such a segmented model even when the joint point,  $K$ , is unknown. To impose continuity (the two sections meet at  $x=K$ ), and smoothness (the first derivatives with respect to  $x$  are the same at  $K$ ) we have:

$$\begin{aligned}
[11] \quad y_{ij} = & U_i + \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \sum_{k=1}^K u_k (x_{ij} - \kappa_k)^2 + \\
& + \sum_{t=2}^L z_{it} (\gamma_{0t} + \gamma_{1t} x_{ij} + \gamma_{2t} x_{ij}^2) \\
& + \sum_{t=2}^L z_{it} \left( \sum_{k=1}^K v_k^t (x_{ij} - \kappa_k)^2 \right) + \varepsilon_{ij} \\
= & \beta_{0i}^* + \beta_{1\_SFR} x_{ij} + v_i - u_i,
\end{aligned}$$

at  $x_i = K$ , where  $\beta_{1\_SFR}$  is the slope of the stochastic frontier model, and  $\beta_{0i}^*$  correspond to a set of intercepts varying by planting density. Solving Equation [11] for  $\beta_{0i}^*$ , we obtain a parameter that allows continuity between the two sections. Considering the smoothness of the new equation we require:

$$\begin{aligned}
y' = \frac{\partial y_{ij}}{\partial x_{ij}} = & \beta_1 + 2\beta_2 x_{ij} + \sum_{k=1}^K 2u_k (x_{ij} - \kappa_k) + \\
& + \sum_{t=2}^L z_{it} (\gamma_{1t} + 2\gamma_{2t} x_{ij}) \\
& + \sum_{t=2}^L z_{it} \left( \sum_{k=1}^K 2v_k^t (x_{ij} - \kappa_k) \right)
\end{aligned}$$

$$= \frac{\partial y}{\partial x} (\beta_{0\_SFR} + \beta_{1\_SFR} x_i) =$$

$$= \beta_{1\_SFR}$$

Following Cao and Dean (2008), the joint point  $x=K$  is estimated for each stand density by fitting first a two segment quadratic linear-regression model with an unknown join point. The validity of this joint point can be assessed by a visual inspection of the scatterplot and first derivative plots.

The final model will then have the form:

$$\begin{aligned}
 [12] \quad G(x) \quad y_{ij} = & U_i + \beta_0 + \beta_1 x_{ij} + \beta_2 x_{ij}^2 + \sum_{k=1}^K u_k (x_{ij} - \kappa_k)^2 + \\
 & + \sum_{t=2}^L z_{it} (\gamma_{0t} + \gamma_{1t} x_{ij} + \gamma_{2t} x_{ij}^2) \\
 & + \sum_{t=2}^L z_{it} \left( \sum_{k=1}^K v_k^t (x_{ij} - \kappa_k)^2 \right) + \varepsilon_{ij}, \text{ if } x < K \\
 & y_{ij} = \beta_{0_i}^* + \beta_{1\_SFR} x_i, \text{ if } x \geq K
 \end{aligned}$$

### 3.5. Results and discussion

#### 3.5.1. Semiparametric regression model

Models [6] (semiparametric additive mixed model) and [7] (semiparametric interaction mixed model) were fitted using PROC MIXED, a SAS mixed model procedure (SAS Institute

Inc. 2008), applying the method of restricted maximum likelihood (*REML*) and a Toeplitz covariance structure of the errors to account for the longitudinal aspect of the data.

Based on visual inspection of the scatterplot (Figure 3.2), six evenly spaced knots located in the last phase of the size-density trajectories ( $2.5 \leq \text{LnDq} \leq 3.5$ ) were chosen to capture the nonlinear trend and the monotonic decreasing trend of the curves. Using evenly spaced knots through the whole range of  $\text{LnDq}$  resulted in illogic results (i.e. increasing number of trees/ha as the quadratic mean diameter increases; trees/ha was predicted to be greater than the planting densities) and hence poor fit of the semiparametric curve.

The semiparametric interaction mixed model was able to completely describe the density-size trajectories of our data, addressing both the non-linear trend in  $\text{Ln}(\text{trees/ha})$  with  $\text{Ln}(\text{quadratic mean diameter})$ , and the effect of initial planting density on the trajectories. A test for additivity was conducted by using a likelihood ratio statistics by fitting the full model with interaction (Equation [7]) and the nested reduced model (Equation [6]). The test statistics,  $2 \times \log\text{-Likelihood} = -1159.9 - (-1307.8) = 147.9$  has an asymptotic  $\chi^2$  distribution with 28 ( $9 \times 3 + 1$ ) degrees of freedom. Based on the significant result of this test ( $p\text{-value} < 0.0001$ ) we concluded that the effects associated with initial-density\* $\text{LnDq}$  interaction were significant and should be retained in the model. Model [7] therefore provided a better fit to our data.

A plot of the predicted trajectories from the full model [7] for four planting densities (2990, 2244, 1794, and 1683 trees/ha) is presented in Figure 3.3. The plot shows that four of the predicted trajectories intersect at an approximate value of  $\text{LnDq} = 3.0$ , or  $\text{Dq} = 20$  cm. Crossing trajectories violates the assumption that individual stands asymptote to a common upper size-density frontier. Our results, however, support the findings of Drew and Flewelling (1979), Hibbs (1987), Dean and Baldwin (1993), and Cao and Dean (2008) that individual trajectories

can cross each other. Although that assumption essentially simplifies curve fitting, curves fit to individual stands should reproduce their individual patterns, permitting crossovers if they exist in the data set (Cao and Dean 2008).

Close examination of the curves in Figure 3.3 reveals that the size-density trajectories can be divided into at least two zones, in agreement with the stages of stand development that have been described by Drew and Flewelling (1979), Williams (1994), and VanderSchaaf and Burkhart (2008): (I) an initial stage where the impact of competition is minor in which mortality is independent of stand density (density independent mortality stage), and (II) a stage with competition-induced mortality, also called the self-thinning stage. Within the density induced mortality, three phases of stand development can be found: the first phase is described by a nonlinear zone of the trajectory, followed by the second phase, a linear portion of the trajectory, and the third phase is characterized by a divergence of the size-density trajectory from the linear portion. We observe that the trajectories seemed to follow stage I (independent mortality stage), and the first two phases of stage II (nonlinear trend followed by a linear portion) (Figure 3.3).

Plots of predicted  $\text{Ln}(\text{trees/ha})$  with 95% individual prediction bands are presented in Figure 3.4 to Figure 3.7 for initial densities 1683, 1794, 2244, and 2990 trees/ha, respectively. The estimated 95% confidence bands indicate that the predictions are fairly precise, i.e. the variability around the predictions was quite narrow.

Plots of the first derivative for the semiparametric interaction mixed model [7] for selected initial stand densities are presented in Figure 3.8. Derivative plots allow for a richer interpretation of the semiparametric regression model. Based on the plot, we can observe up to three segments in the trajectories, corresponding to three phases of stand development. In the first phase  $\text{Ln}(\text{trees/ha})$  remains mostly constant until approximately  $\text{Lndq} = 2.0$  (7.39 cm). This

first segment represents the density-independent mortality stage. After that, as tree size increases ( $\ln D_q$ ), stand density ( $\ln(\text{trees/ha})$ ) decreases with a certain mortality rate until approximately  $\ln D_q = 2.5$  (12.2 cm). This suggests the start of the density-induced mortality phase. Then, there is an increase in mortality rate,  $\ln(\text{trees/ha})$  decreases at an increasing rate over the range of  $\ln D_q$ . With increases in tree size and the death of trees, the size-density trajectory is assumed to become linear.

### 3.5.2. Stochastic frontier regression model

Preliminary analysis showed no significant different log-likelihood values between specifications of the  $u_i$  terms, and hence a half-normal model was applied. Parameter estimates for the half-normal stochastic frontier are given in Table 3.2. The  $\sigma^2$  and  $\gamma$  coefficients are the diagnostic statistics that indicate the appropriateness of the use of the stochastic frontier function and the accuracy of the assumptions made on the distribution form of the error term. The value of  $\gamma = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$  was highly significant ( $\gamma = 0.9828$ , S.E.=0.0029), indicating that SFR is appropriate to estimate the boundary. The parameter estimate  $\sigma_v^2$  was close to zero ( $\sigma_v^2 = 0.002174$ ), indicating that the stochastic frontier model can yield a true upper limiting boundary line (Zhang et al. 2005, Weiskittel et al. 2009). The estimated limiting size-density relationship as a stochastic frontier for loblolly pine is (Figure 3.9):

$$[13] \quad \ln(\text{trees/ha}) = 12.443 - 1.820 \ln(D_q)$$

The standard errors for the estimates of  $\beta_0$  and  $\beta_1$  were 0.1964 and 0.0673 respectively. The estimated slope (-1.820) is significantly different from the one given by Reineke (-1.605), at

the 95% confidence level). For loblolly pine, the slope was estimated to be -1.707 by MacKinney and Chaiken (1935), -1.696 by Harms (1981), -1.505 by Williams (1996), and -1.926 by Lhotka and Loewenstein (2008).

### **3.5.3 Relating the stochastic frontier boundary model with the semiparametric regression trajectories**

The complete predicted size-density trajectories obtained by semiparametric regression (Equation [7]) and the upper limiting size-density relationship obtained by stochastic frontier regression (Equation [9]) for loblolly pine corresponding to four planting densities (1683, 1794, 2244, and 2990 trees/ha) are shown in Figure 3.10 to Figure 3.13. Stands with higher values of initial density show trajectories that are approaching, or traveling along, the upper boundary described by the stochastic frontier model. For comparative purposes, the trajectories of the four initial densities are shown in Figure 3.14. The curves suggest that the stands under study are in the second stage of self-thinning, i.e. the trajectories approach and trail the straight line of maximum density, without departing from the linear trend. In aged stands (the data used for this study correspond to relatively young stands, ranging from 5 to 20 years old) it is expected that the trajectories incline below the boundary as a consequence of losing the capability to fill the gaps left by dead trees (Bredenkamp and Burkhart 1990, Cao et al. 2000, Vanderschaaf and Burkhart 2008).

To incorporate the slope of the stochastic frontier regression into the size-density trajectories a two-segment model was fitted to the data (Equation [12]). The first segment corresponds to the semiparametric model, while the last part corresponds to a linear model with

the slope given by the stochastic frontier regression function ( $\beta_{1\_SFR} = -1.820$ ), and intercepts varying by planting density.

For each planting density, a join point was estimated by fitting a two-segment polynomial regression model to the data (Cao and Dean 2008). The estimated point was corroborated for accuracy by a visual inspection of scatterplots and first derivative curves (Figure 3.8). Most join points were in the range of  $\ln(Dq) = 2.7-3.1$ . As an example, the parameters corresponding to the initial stand density of 2244 trees/ha were  $\beta_{0i}^* = 12.667$ , and  $K=2.7$ . Larger initial stand densities yielded smaller join point values, indicating that self-thinning started earlier in crowded stands. Figure 3.15 shows the trajectories of the 2-segment semiparametric-stochastic regression model.

### 3.6. Conclusions

The main purpose of this study was to demonstrate the process of using semiparametric regression to modeling the trajectory of tree density and quadratic mean diameter through time for individual stands. The full equation, a semiparametric interaction mixed model, was able to completely describe the density-size trajectory of loblolly pine stands. It addressed the non-linear trend in  $\ln(\text{trees/ha})$  with  $\ln(\text{quadratic mean diameter})$ , and the effect of initial density on the trajectory. Including the slope of the stochastic frontier regression into the trajectories appears to be appropriate for this dataset. Data used in this study came from relatively young stands (age ranged from 5-20 years), where most of the trajectories were already in, or approaching, the linear phase of self-thinning. To our knowledge, this has been the first time that semiparametric regression is used to model self-thinning trajectories.

The semiparametric regression approach is attractive for modeling complex functional forms as the size-density relationship presented in this chapter.

### **3.7 Acknowledgements**

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**Table 3.1.** Plot-level description of data (n=648)<sup>a</sup>

<b>Variable</b>	<b>Mean</b>	<b>S.D.</b>	<b>Min</b>	<b>Max</b>
Trees/ ha	1393	571.9	585	2990
Dq (cm)	15.7	5.2	3.9	29.9
BA (m <sup>2</sup> /ha)	26.7	13.0	1.2	55.0
H (m)	12.6	5.2	3.2	25.4
SI(m)	25.2	1.2	23.6	27.8
Age (years)	11	3.9	5	20

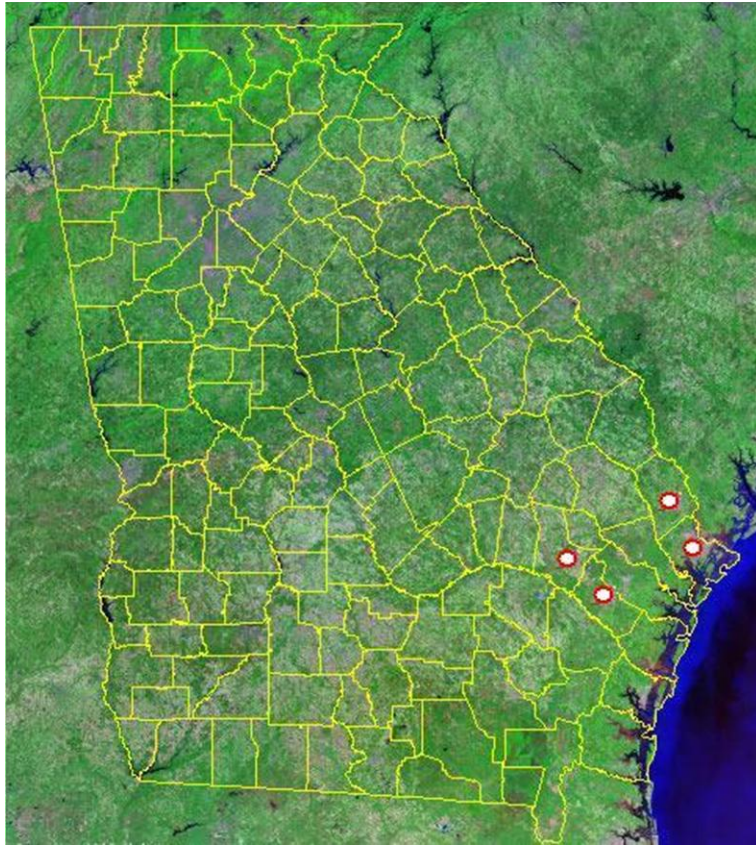
<sup>a</sup> S.D.: standard deviation , Dq: quadratic mean diameter, BA: basal area, H: average total height, SI: site index base age 25 years

**Table 3.2.** Stochastic frontier regression model parameter estimates of the limiting size-density relationship in loblolly pine (Equation [9])

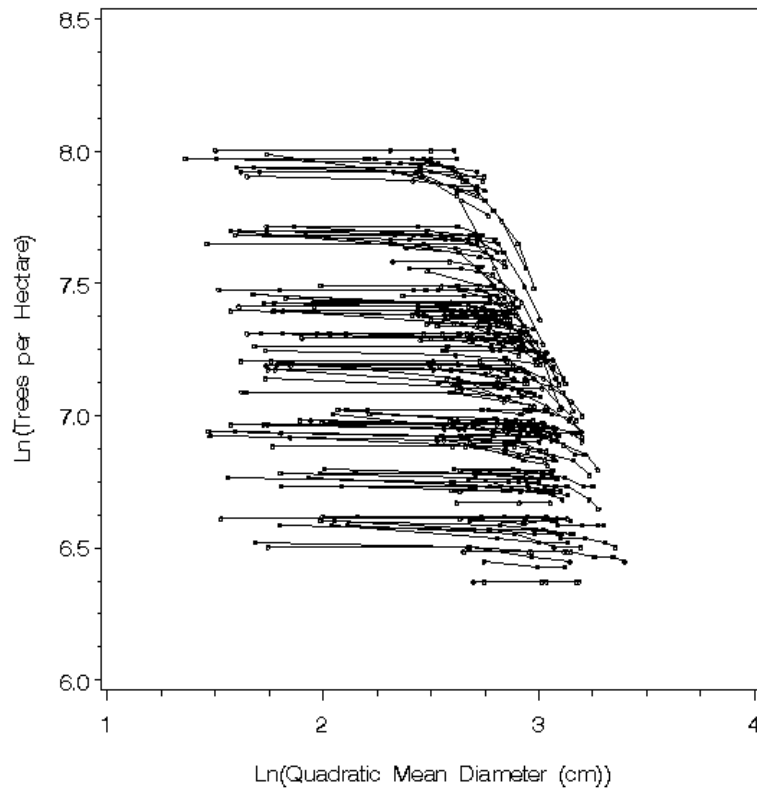
Parameter	Estimate	S.E
$\beta_0$	12.443	0.1964
$\beta_1$	-1.820	0.0673
$\sigma^2$	0.1264	0.0167
$\gamma$	0.9828	0.0029
Log-likelihood	258.7	

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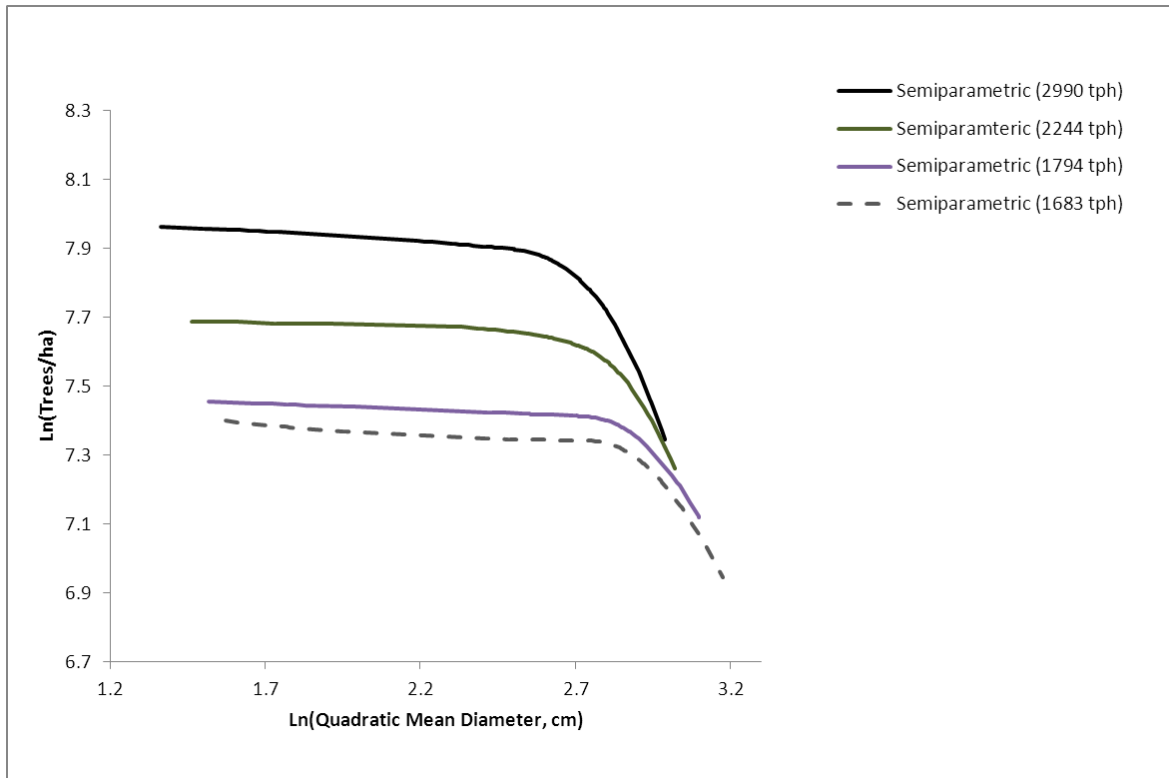
**Note:**  $\sigma^2 = \sigma_u^2 + \sigma_v^2$  ,  $\gamma = \sigma_u^2 / \sigma^2$



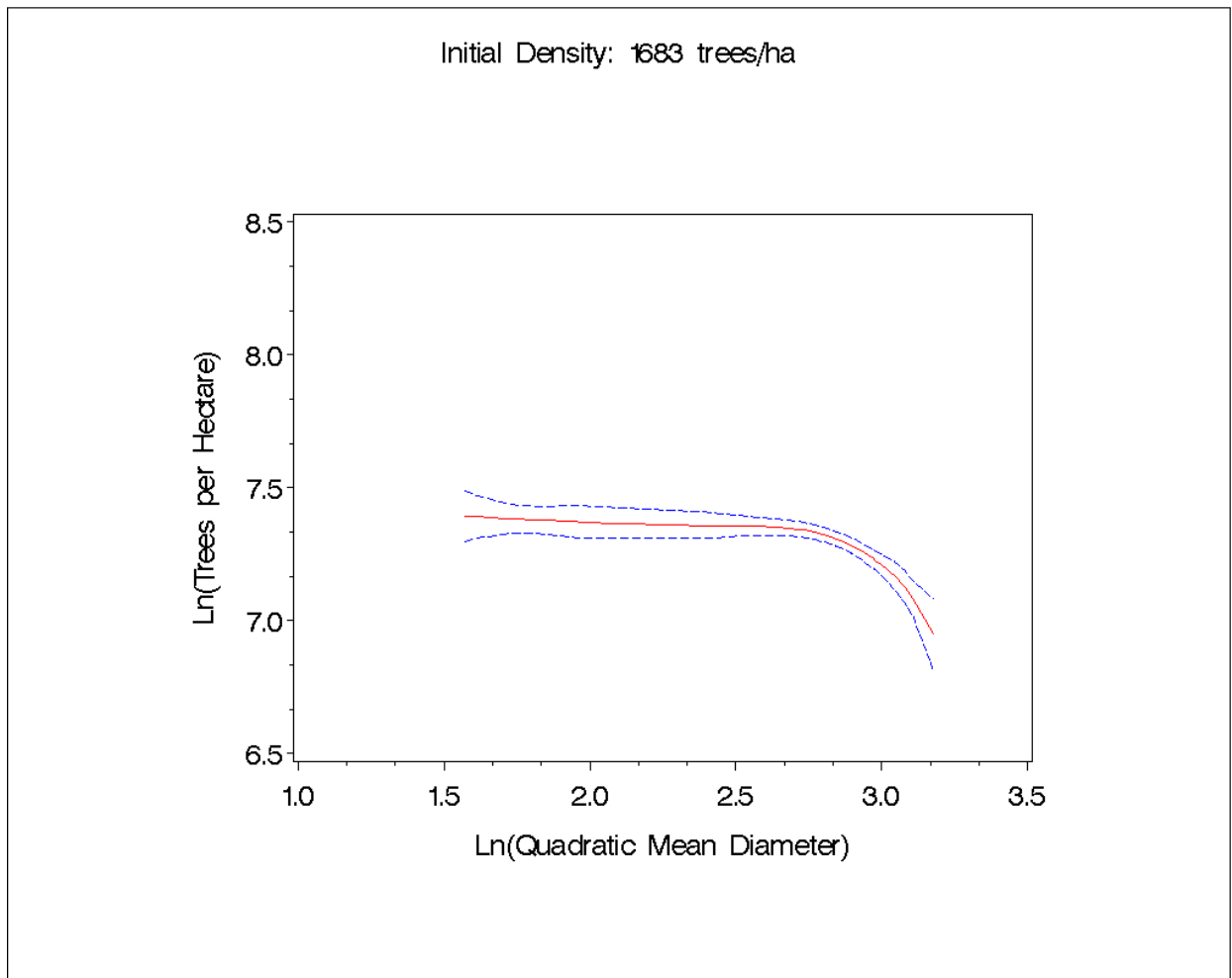
**Figure 3.1:** Location of the loblolly pine research installations used in this analysis (○).



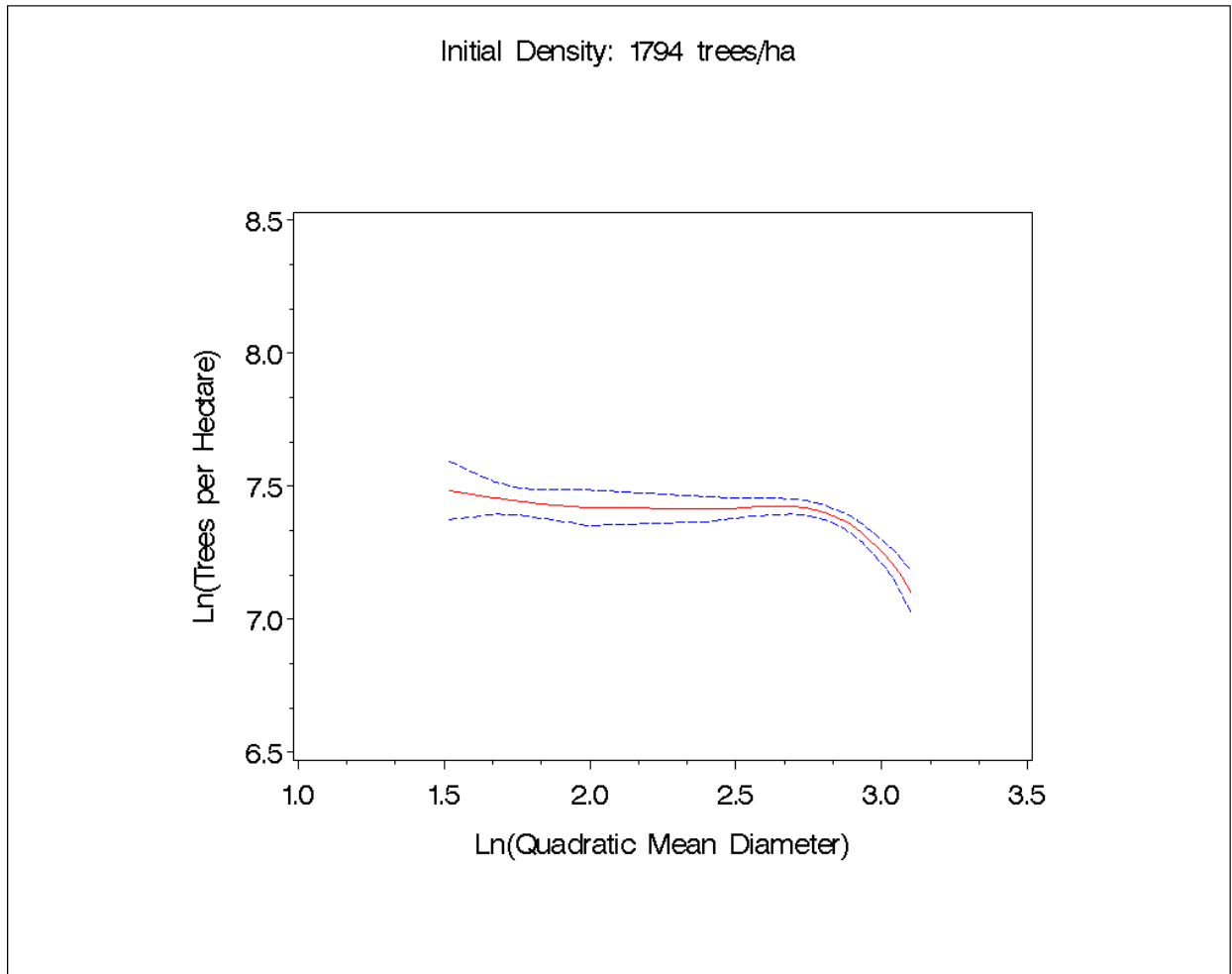
**Figure 3.2** Density-size trajectories observed from 180 plots from the loblolly pine spacing study. There are between three to four measurements per plot, 648 in total. The repeated measures for each experimental plot are connected in the graph.



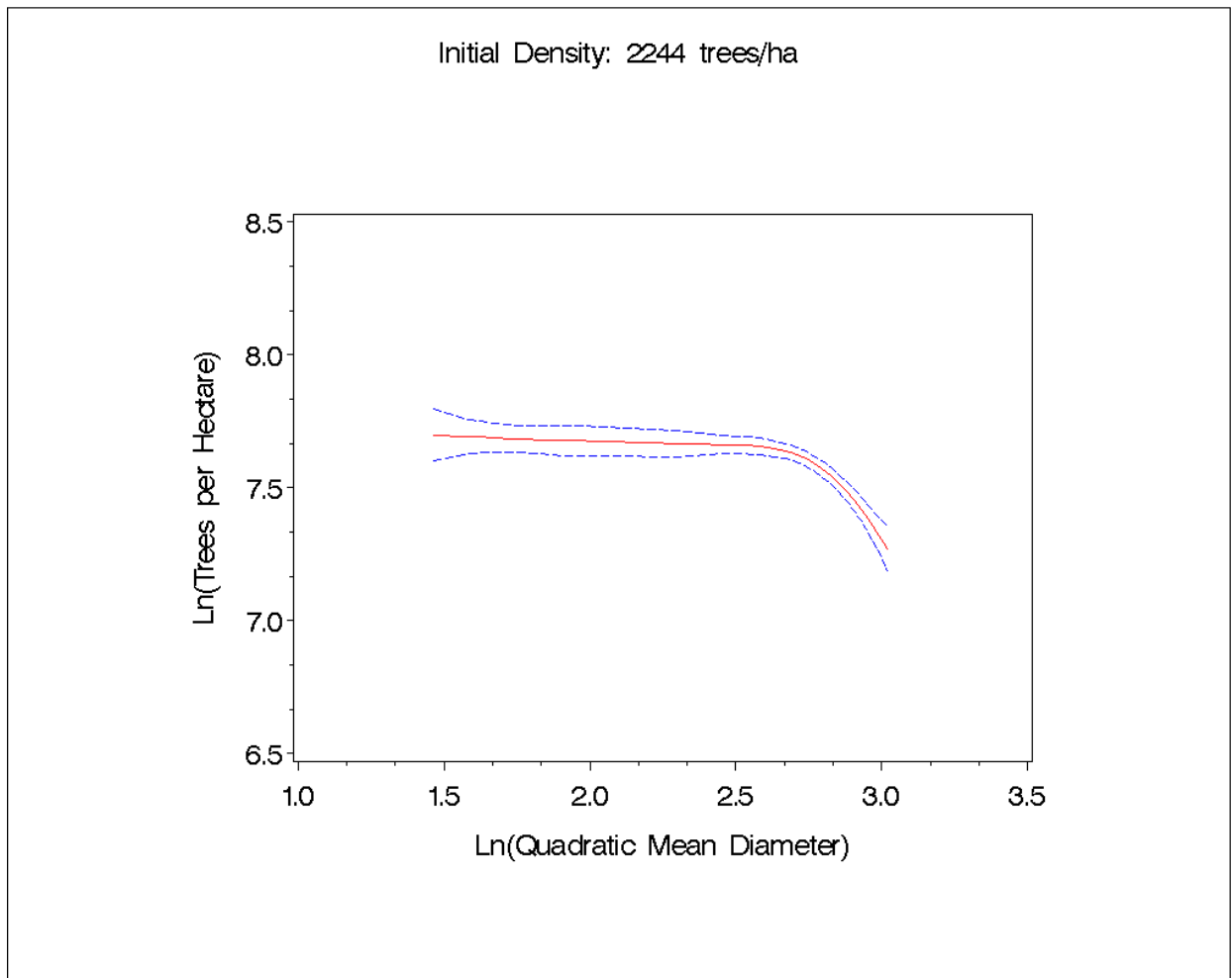
**Figure 3.3** Predicted Ln(trees/ha) by four planting densities from the semiparametric interaction mixed model [7].



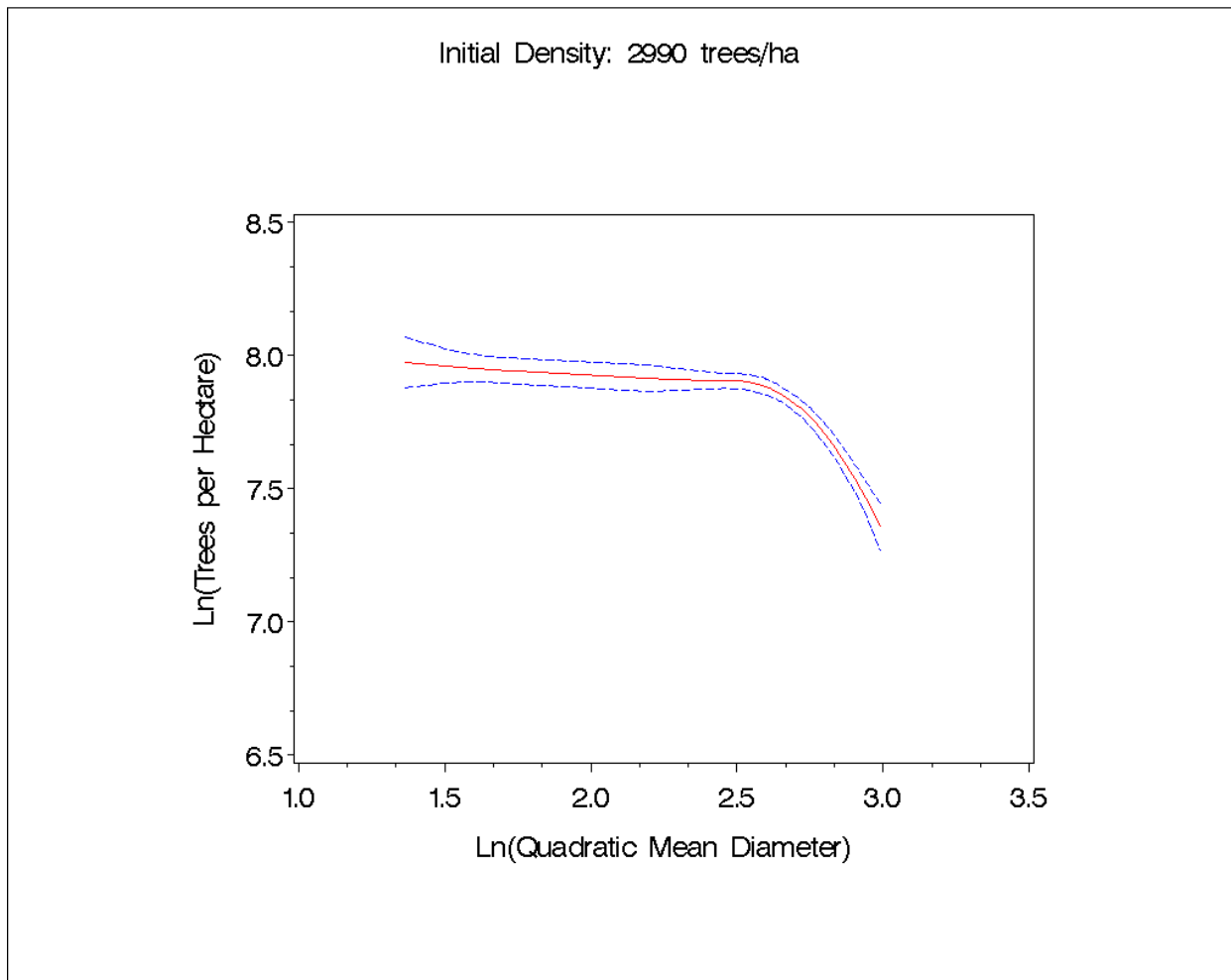
**Figure 3.4** Predicted Ln(trees/ha) with 95% individual confidence band from semiparametric interaction mixed model [7] at an initial density of 1683 trees/ha.



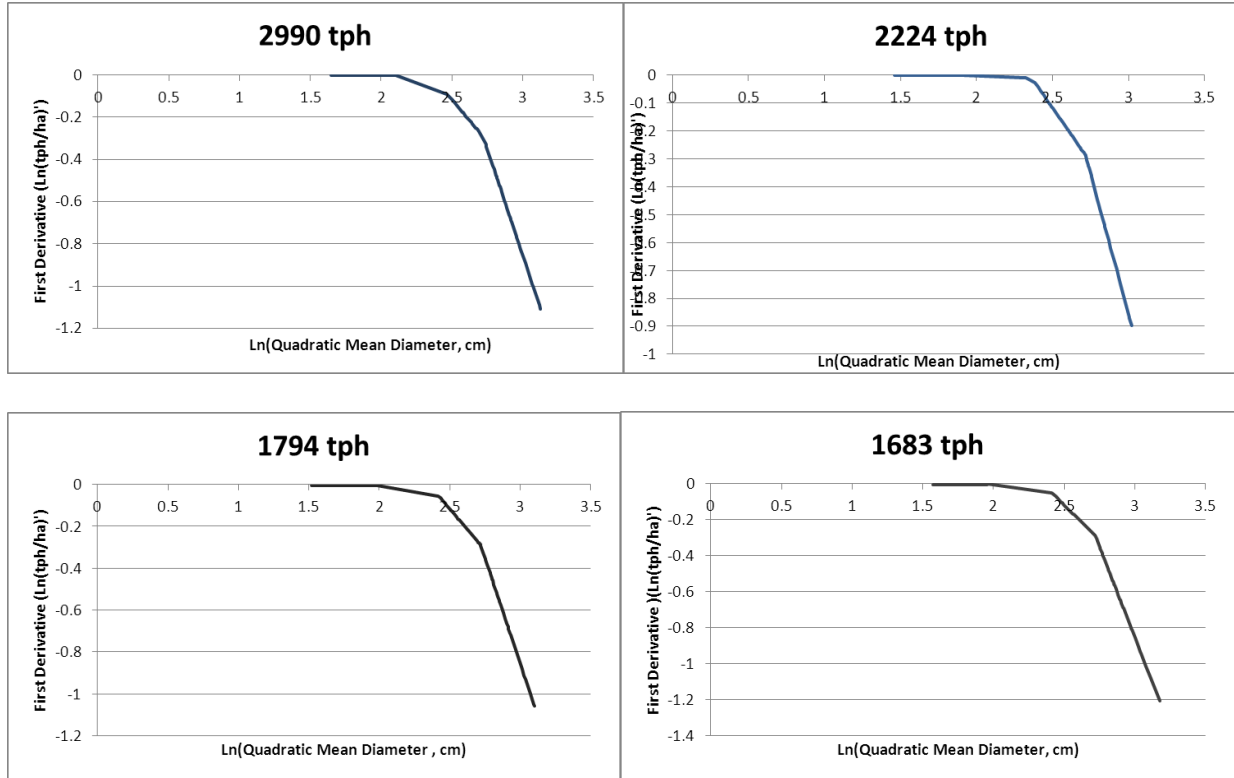
**Figure 3.5.** Predicted Ln(trees/ha) with 95% individual confidence band from semiparametric interaction mixed model [7] at an initial density of 1794 trees/ha.



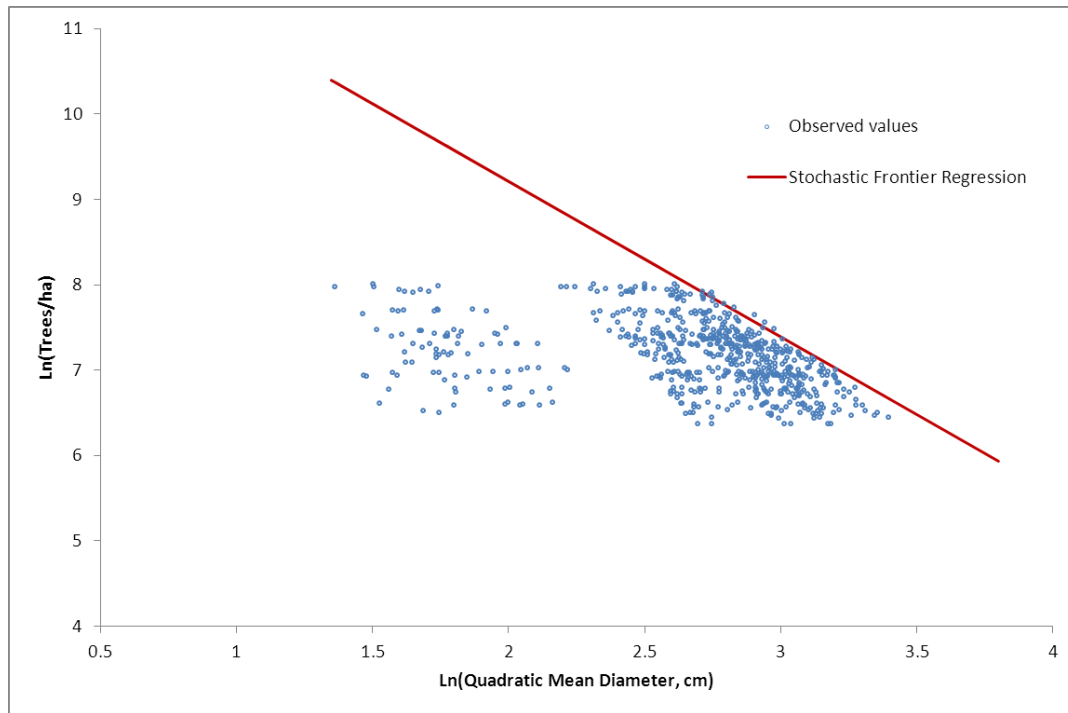
**Figure 3.6** Predicted Ln(trees/ha) with 95% individual confidence band from semiparametric interaction mixed model [7] at an initial density of 2244 trees/ha.



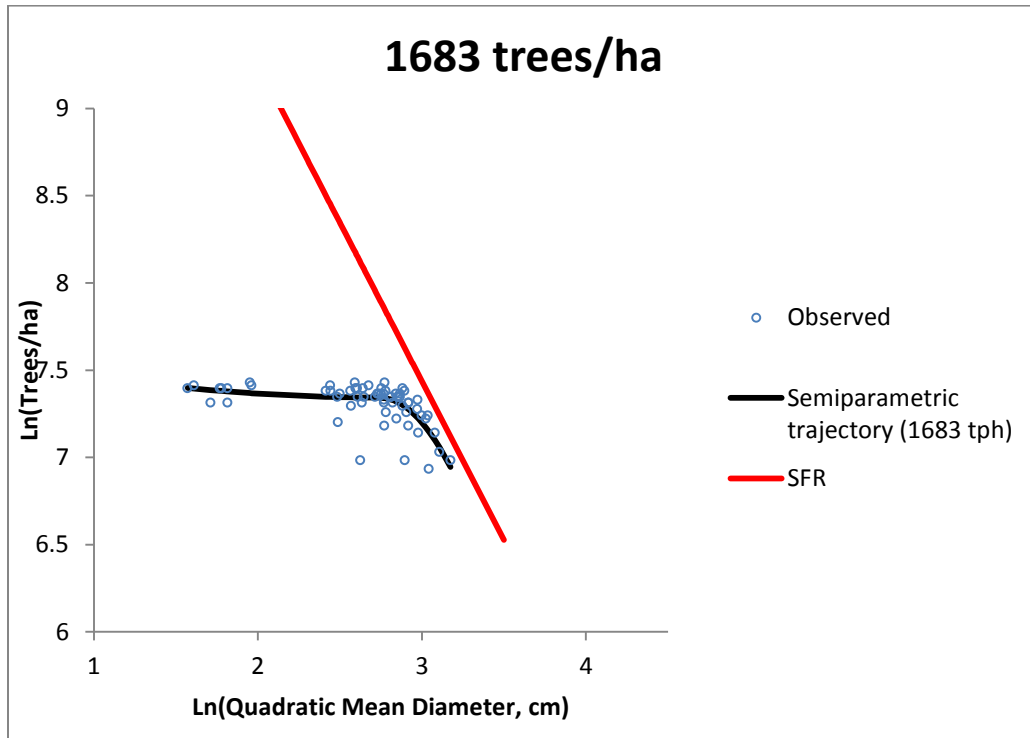
**Figure 3.7** Predicted Ln(trees/ha) with 95% individual confidence band from semiparametric interaction mixed model [7] at an initial density of 2990 trees/ha.



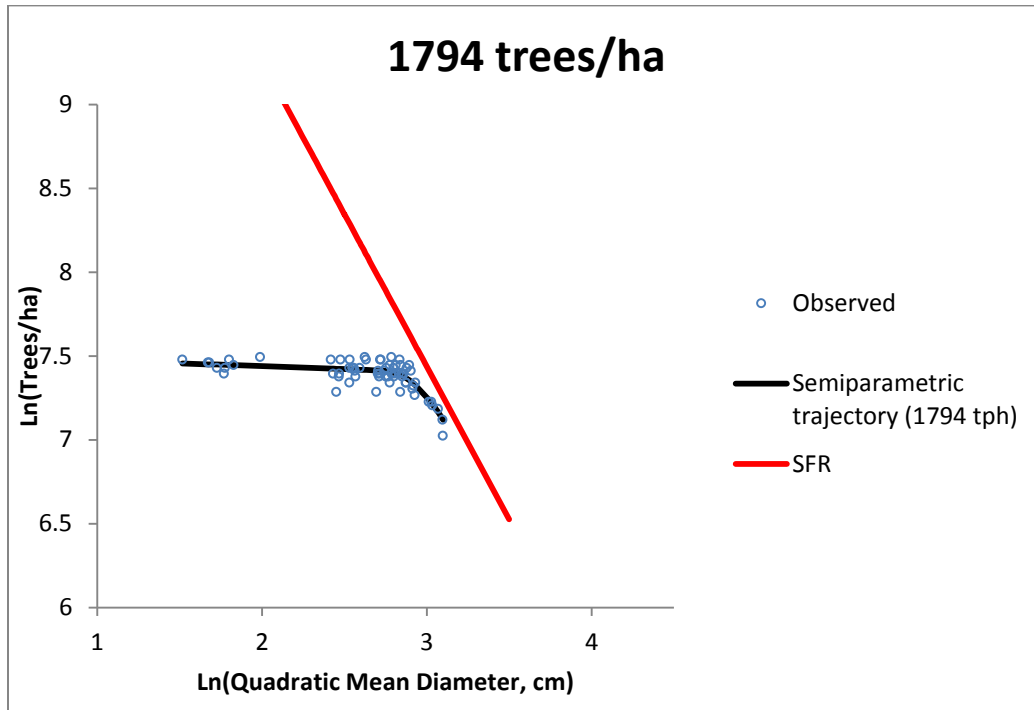
**Figure 3.8** First derivative plot from semiparametric interaction mixed model [8] by four planting densities



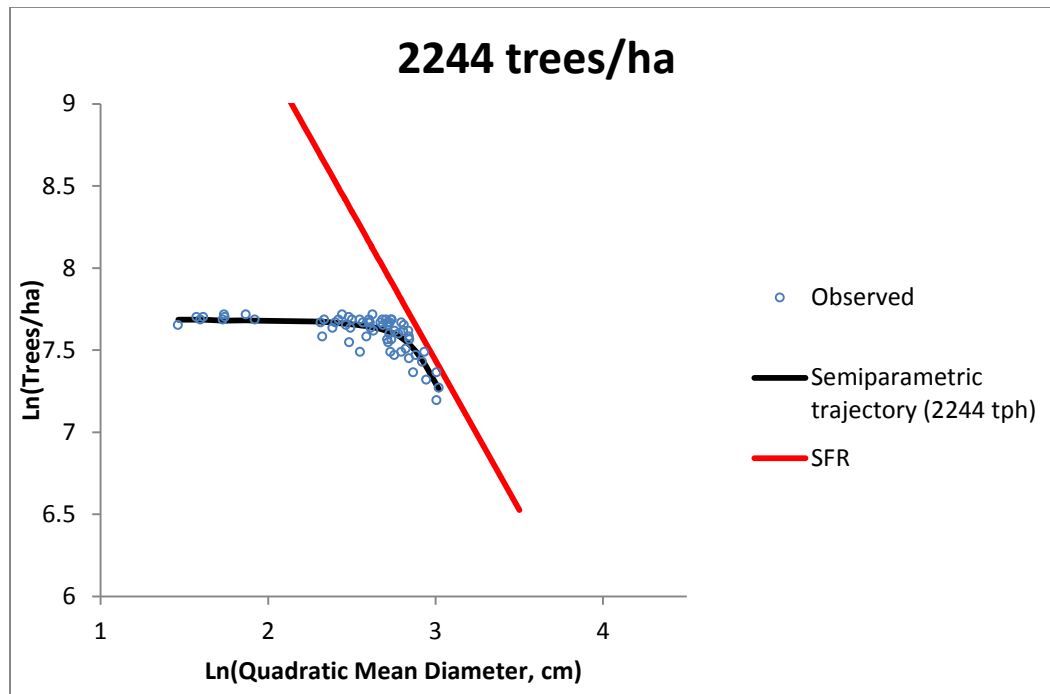
**Figure 3.9** The limiting size-density relationship for loblolly pine obtained by stochastic frontier regression (Equation [13])



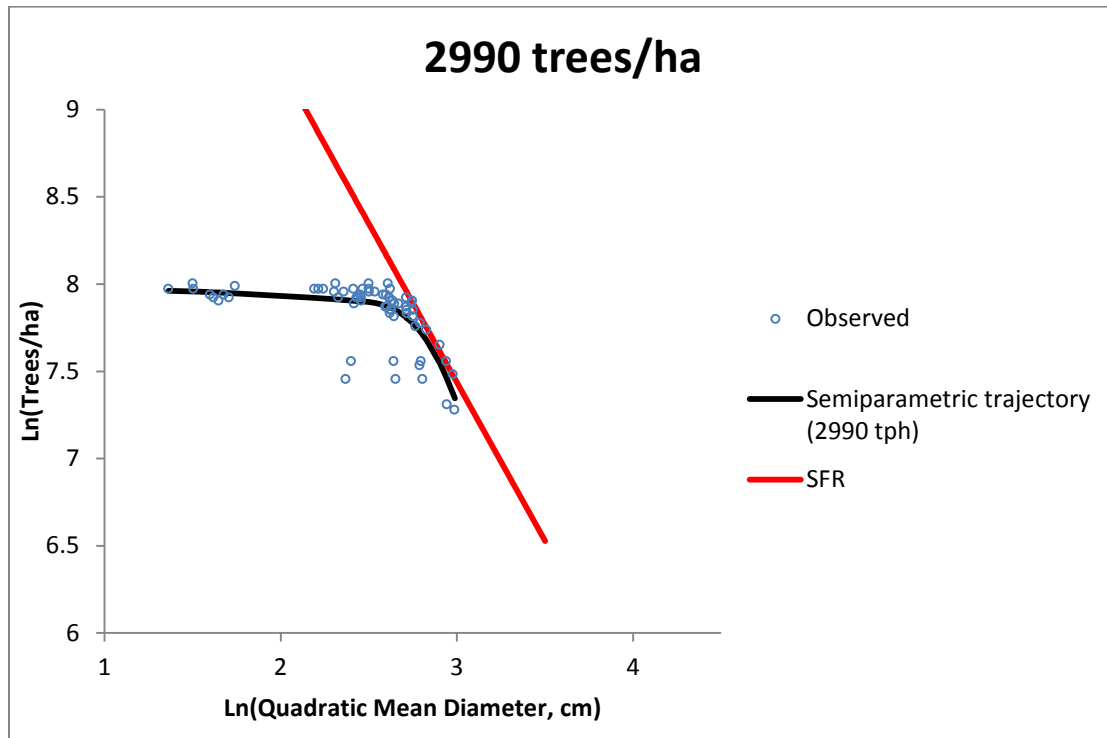
**Figure 3.10** Predicted size-density trajectory obtained by semiparametric regression (Equation [7]) and the upper limiting size-density relationship obtained by stochastic frontier regression (Equation [13]) for loblolly pine corresponding to a planting density of 1683 trees/ha.



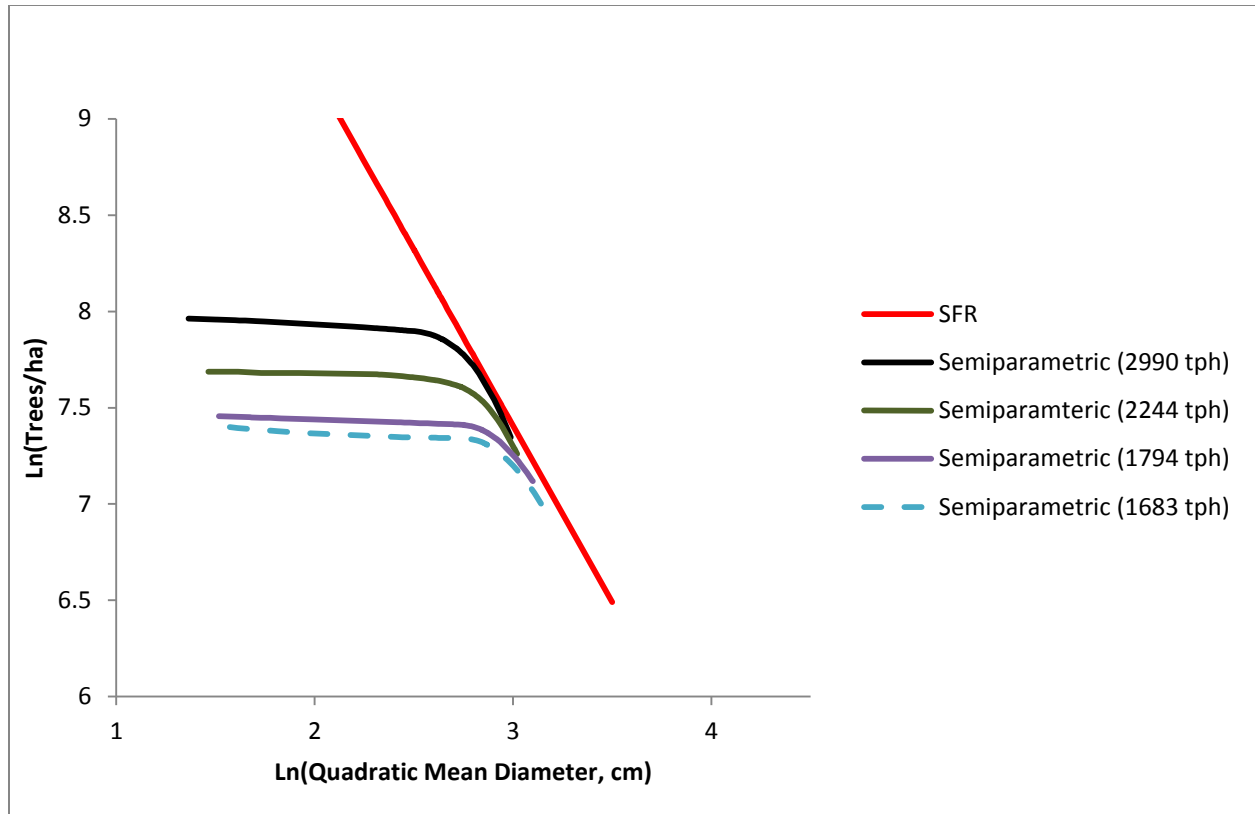
**Figure 3.11** Predicted size-density trajectory obtained by semiparametric regression (Equation [7]) and the upper limiting size-density relationship obtained by stochastic frontier regression (Equation [13]) for loblolly pine corresponding to a planting density of 1794 trees/ha.



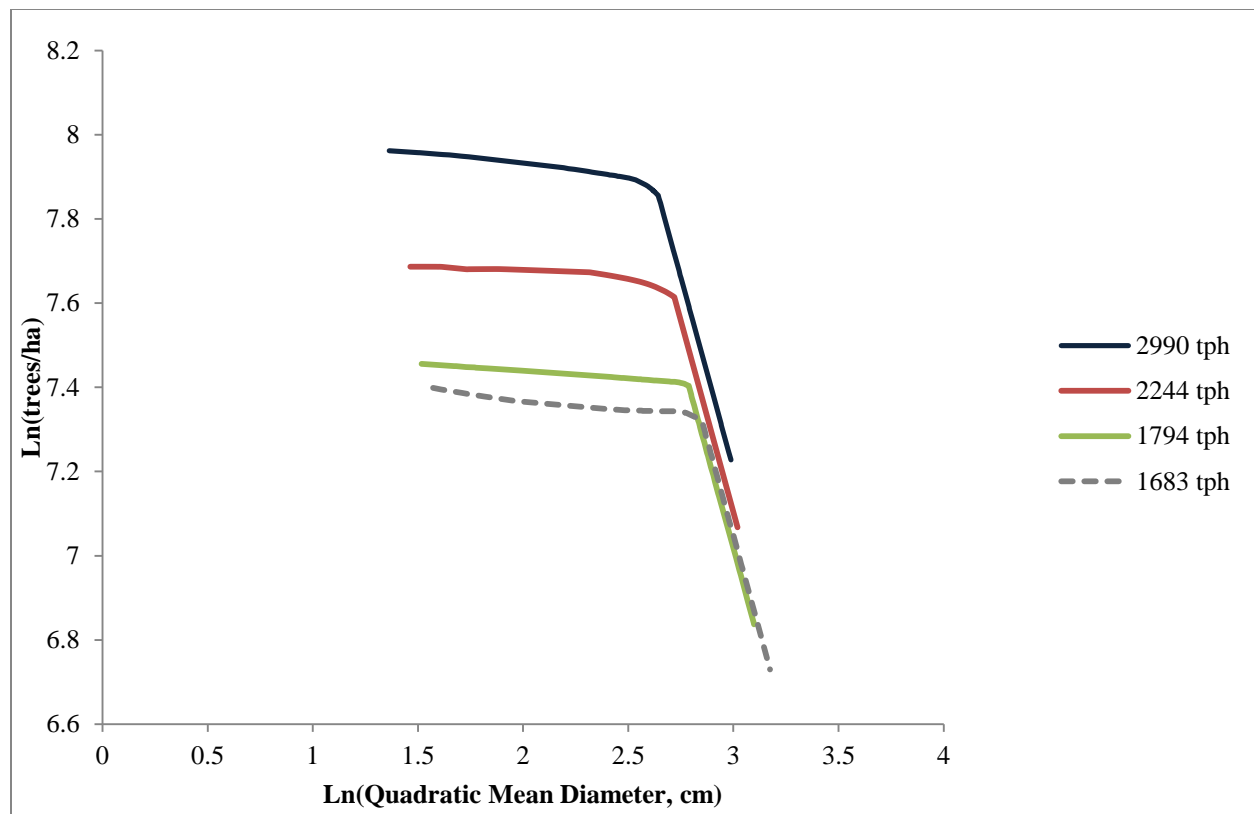
**Figure 3.12** Predicted size-density trajectory obtained by semiparametric regression (Equation [7]) and the upper limiting size-density relationship obtained by stochastic frontier regression (Equation [13]) for loblolly pine corresponding to a planting density of 2244 trees/ha.



**Figure 3.13** Predicted size-density trajectory obtained by semiparametric regression (Equation [7]) and the upper limiting size-density relationship obtained by stochastic frontier regression (Equation [13]) for loblolly pine corresponding to a planting density of 2990 trees/ha.



**Figure 3.14** Predicted size-density trajectory obtained by semiparametric regression (Equation [7]) and the upper limiting size-density relationship obtained by stochastic frontier regression (Equation [13]) for loblolly pine corresponding to the higher four planting densities (1683, 1794, 2244, 2990 trees/ha).



**Figure 3.15** Predicted size-density trajectories obtained by the 2-segment model (semiparametric regression + stochastic frontier regression slope) (Equation [12]) by four planting densities (1683, 1794, 2244, and 2990 trees/ha).

## **Chapter 4**

### **Summary and Conclusions**

Self-thinning is a critical process during stand development. As single trees in a stand grow in size, they begin to compete for limited resources such as light, water, mineral nutrients, and space. Competition arises when the density of a stand surpasses the level at which each individual can obtain its maximum growing space. Natural mortality results when the minimum growing space requirements of all trees exceed the resources of the site. The dynamic equilibrium between tree growth and death induced by competition has been historically governed by the Reineke's equation and the so-called  $-3/2$  power law of self-thinning. The law states that in log-log scale the relationship between average tree size and stand density is a straight line (called the self-thinning or limiting size-density line). Size-density relationships have been used to investigate stand development and self-thinning relationships, to develop relative density indices and stand management diagrams, and to serve as a predictor of stand growth. An understanding of the size-density relationships allows silviculturists to design management prescriptions that alter available growing space and influence tree size, growth, and mortality. This dissertation was an effort to model the maximum size-density relationship by integrating a selection of other stand factors such as silvicultural intensity, planting density, site index, and soil type into the model, and to compare and illustrate the application of alternative

mathematical forms and fitting methods that have been or can be used to modeling the trajectory of stand density and size of individual stands through time.

The effect of stand factors on the limiting size-density relationship in even-aged loblolly pine (*Pinus taeda* L.) stands in the U.S. Southeast was studied. The mixed-effects analysis technique proved to be a valuable statistical approach for fitting the limiting size-density line and providing the opportunity to test the significance and impact of additional factors. The assumption that the intercept and slope of the species limiting size-density relationship are invariant to stand and site factors was analyzed using data from a culture/density study in the Lower Coastal Plain region of the southern United States. Management intensity, planting density, and site index significantly impacted the parameters of the species limiting size-density relationship. CRIFF soil types did not significantly affect the slope or intercept of the line.

The application of semiparametric regression analysis to modeling the trajectory of tree density and quadratic mean diameter through time for individual stands was investigated. A semiparametric interaction mixed model was able to completely describe the density-size trajectory of even-aged loblolly pine stands in the southeastern U.S. The model addressed the non-linear trend in  $\text{Ln}(\text{trees/ha})$  with  $\text{Ln}(\text{quadratic mean diameter})$ , and the effect of initial density on the trajectory. To account for the linear phase of self-thinning, and to achieve a better representation of this important factor in stand dynamics, the slope of the stochastic frontier regression was incorporated into the semiparametric model. To our knowledge, this is the first time that semiparametric regression has been used to model size-density trajectories.