ABSTRACT

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Human Random Capacities through Repeated Numeric Sampling
(Under the Direction of DR. NICOLE LAZAR)

Randomness in the macroworld of human actions would contradict the theory of causal determinism, that all events occur as a direct result from antecedent factors, and be a pivotal contribution to philosophical debates on free will. The aim of this research is to understand human capacities and motivations of randomness. Can human beings act truly randomly? Participants completed a short survey and entered 100 “random” digits from 1 to 10 into a grid. The numeric sequences generated were statistically analyzed through tests described by Donald Knuth in *The Art of Computer Programming, Vol. 2* (1981) to determine their degree of randomness. These sequences were compared against sequences generated from different methods of randomization consisting of dice rolls, decimal digits of pi, and deterministic formulas used by Texas Instruments programmable calculators and Java. Hypotheses of uniformity were tested using Chi-Squared analysis in frequency, serial, gap, and poker tests. Half the sequences generated from dice did not adequately pass the frequency test. Java, a Texas Instruments calculator, the decimal digits of pi and a majority of human participants produced sequences that did adequately pass the frequency test. Due to the theoretical foundation of the research question, it is impossible to produce concrete conclusions. However, higher proportions of survey sequences failed the various tests as they progressed in rigor and a similar pattern was present only in sequences produced from deterministic formulas in Java.

INDEX WORDS: Randomness, Determinism, Pseudorandom Number Generator, Frequency Test, Serial Test, Gap Test, Poker Test

HUMAN RANDOM CAPACITIES THROUGH

REPEATED NUMERIC SAMPLING

by

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CHAPTER 1
INTRODUCTION

Philosophy and the Problem of Free Will

Avrum Stroll notes that the age-old philosophical debate of free will centers around one question: "Is all human comportment determined by antecedent factors, including one's genetic makeup, so that no one can choose or act differently from the way that he or she does?" (Stroll, 2004, p.124) If this is the case, then free will in the sense meant by most, the ability to freely choose one's actions, does not exist. This would parallel the notion of fate, that something external to the agent predetermines the future and he is helpless to change it. If free will does not exist, and people cannot choose their own actions, then should they be held responsible for them? And if they are not to be held responsible for their actions, should they be punished for them? It can then be seen that the problem of free will is irrevocably intertwined with broader social issues such as moral responsibility and criminal punishment.

The theory of causal determinism holds that an event occurs because of the condition created from all events preceding it. Given a particular condition there will be only one directly resulting outcome. Causal determinism further stipulates that because every action is the result of causes, given a complete knowledge of all causes and events prior to the action, one could predict the resulting action. Hard determinism, or simply determinism, relates causal determinism to human actions and thus seems to undermine the idea of free will. If this cause-and-effect relationship exists within human actions, then life would progress down a singular path determined by preceding factors much like a recurrence relation. Because this argument alone begs an infinite regress to the agent's origins, hard determinism further maintains that at birth the causes of human actions come solely from heredity. From this point onward, determinism also allows for the external influences of the environment to alter the path of the agent. Another regress naturally arises when searching for origins further back. However, this leads to an argument well outside the scope of this research.

There are three central philosophical stances on the free will problem. They range in their acceptance of determinism. The hard determinist stands on one end, firmly asserting the validity of determinism and drawing the conclusion that free will cannot possibly therefore exist. A libertarian stands solidly opposing this on the other extreme, asserting that free will does exist and because of its incompatibility with determinism, refutes the notion altogether. The compatibilist account of free will lies in between both and tries to reconcile the differences mainly through a semantic argument.

Determinism can coexist alongside free will if what is meant by free will is further studied. Under hard determinism, a person's will is determined by his heredity and environment, but his heredity and environment *are* the person (Blatchford, 2005, p. 471). If the object of desire is already determined due to a person's nature, it only proves that something is being desired – that a will exists. What makes this will free is that it is not controlled or coerced by anyone or anything else. So long as one's intentions are his own, he is acting of his own "free will." The actions of a person should not be thought of as a result of free will in the sense meant by most, but rather as actions willed freely.

Randomness (Philosophical and Statistical Definitions)

Paramount to this free will debate is also the notion of randomness. Under causal determinism, from one set of conditions creating one total cause, there would be only one possible outcome, or effect. What then, about randomness? Many philosophers never answer this problem fully. Random might describe the pattern a leaf takes while falling from a branch to the ground. To a hard determinist, the patter of the leaf is not actually random at all, but rather the necessary pattern from the wind gusts and aerodynamics of the leaf. To a compatibilist, this may constitute a random event, but would be seen as an event in nature and part of environmental influence external to the agent releasing it from the same hard-fast, cause and effect relationship.

Subatomic events at the quantum level seem to present examples of truly random processes. While these random events do occur in nature, and could fall under the realm of the environmental influence, they would also occur at the subatomic level within the agent. Some philosophers have met this opposition to determinism by classifying a *microworld* and a *macroworld*. They place certain quantum processes from quantum theory in a microworld that has no upward translation into a macroworld of human thoughts and actions. Justifications for this compartmentalization of events can be seen in both Henrik Walter (2001, pp. 24-25) and Wilfrid Sellars (1966, p.143). A random event must ultimately be found in the macroworld and within a person's thoughts or actions to pose a threat to determinism.

In this human macroworld, philosophers start off debates clarifying what exactly is meant by 'random.' Many definitions concern issues of causality. Random events by one definition are events that have no cause. If they have no cause and their existence is conceded, then determinism is negated because in determinism everything has a cause. Random events must then be defined as events that have an *unknown* cause. John Thomas (1997, p. 265) concludes, "though we do not *experience* our choices as caused, if in fact they are caused by factors inherent in, and necessitated by, the total physical contest in which we act, then behavior can be meaningfully understood and assessed in terms of causes (as determinists typically have hoped to do).” Simply because a person acts so quickly that he is unaware of the choice he is making, does not mean there are not reasons and causes that the person chose accordingly.

Under causal determinism, every event in the macroworld has a cause. If the cause of an event were known, its outcome would be predictable. By contraposition then, if an event is unpredictable, the cause of the event’s occurrence is unknown. It is then possible for an unpredictable event to have an underlying cause that is simply unknown. This is what is meant by a “random event.” Bennett (1998, p.83) intuitively asks, “Is a random outcome completely determined, and random only by virtue of our ignorance of the most minute contribution factors?” According to the causal determinist, yes.

Statisticians argue similarly fundamental concepts concerning definitions of randomness. Theories include notions of equiprobability and Bayesian epistemic views “that it is a primitive notion rather than something to be defined in terms of probability” (McGrew, 2001, p. 33). One statistical definition of randomness is having equally likely probabilities. This definition is most commonly associated with fairness. If two events are equally likely and randomness is used to choose between the two, it removes the subjectivity in the decision. Flipping a coin before a sporting event ensures that both teams have equally likely chances of winning the advantage because both sides of the coin are assumed to have equally likely chances of landing up. Having equally likely probabilities also limits the ability to predict the outcome with any degree of certainty.

Probability Theory

Probability theory seeks to bring an element of predictability to randomness by analyzing different probability distributions of random variables. The simplest and most intuitive distribution is the uniform distribution. Under this distribution, random variables have equal probabilities, like rolls of a fair die or cards drawn from a shuffled deck. In other distributions, certain outcomes are more likely than others. The normal distribution, or commonly, the bell curve, gives higher probabilities to outcomes around the center, or mean, of the distribution. Test scores are often normally distributed, yielding more B's than A's or C's. Many more probability distributions exist that model items as diverse as allocation of wealth to number of hurricanes in a year. Probabilities assigned to random events seek to provide an amount of certainty to a prediction. An outcome with a probability of 70% is more likely than one with 30%. However, even a probability of 99% does not guarantee the outcome and therefore the event remains random. "Once the accuracy becomes 100 percent, or the outcome certain, the event is purely deterministic and no uncertainty is involved. No uncertainty – no randomness" (Bennett, 1998, p. 87).

While statistics has used probabilities to describe random variables, probability distributions cannot fully encompass the notion of randomness. Fetzer (1997) brings up the problem of single-case probabilities. The frequentist ideal of assigning probabilities is based on frequencies and observed patterns from long-run exposures. How then, can a probability be assigned to something that will only happen once? When random events in fair sampling are defined *a priori* by appealing to equal probabilities, McGrew (2001) concludes that an infinite regression looms. Even for events that are similar, never again will the same exact situation come about so as to necessitate the same probability being assigned to it. If a person is choosing between chocolate and vanilla ice cream, even if it is known that the person likes chocolate more than vanilla, at that precise moment, a probability of choice cannot be assigned since the same conditions that brought about this particular preference could not possibly have been seen previously. This is an extreme example of single-case probabilities, but the problem still remains.

Another paradox arises with the confusion between probability and possibility. McGrew (2001, p. 48) poses the following interesting example: A professor at a university goes to a Coke machine to get a Coke. There are 50 sodas in the machine. The professor assumes he will receive a Coke from the machine with probability 1. However, he knows the soda stocker has a certain malicious bent and that there is a *possibility* that the stocker could have added a Mello Yello to the Coke machine. The teacher should not abandon his assumption of receiving a Coke, but rather should reevaluate the situation after assigning a probability to the stocker adding a Mello Yello. It is key here to recognize that, "possibilities do not eliminate probabilities" (McGrew, 2001, p.49). Another example involves tossing a coin. Normally it would be assumed that the probability of the coin landing heads or tails is .5. But, is it physically *possible* for the coin to land on its side? If the answer is yes, this clearly threatens our assumption for the .5 probability of heads or tails. Since the coin landing on its side constitutes another event in the sample space, it must also be assigned a probability, which combined with the probabilities of heads and tails should sum to 1. Simply reevaluating the situation will show that the probability of this possibility is so small that for any practical application it will not play any role. However, while the possibility of something should not cause us to forget probabilities, so should we not rely too heavily on probabilities that we forget other possibilities.

With parallels to the example above, some argue for pragmatics over probabilities. Monty Hall was the host of the game show "Let's Make a Deal." The correct strategies of the game have been much debated by statisticians, mathematicians, game theorists, teachers, and students. It is known as the "Monty Hall Problem" and has changed a little from the original design of the 1960s game show. In the problem, a contestant is asked to choose one of three doors behind two of which are goats and behind the third a car. After the contestant chooses a door, the host, Monty Hall, opens one of the remaining doors to reveal a goat. He then asks the contestant if he would like to stay with his initial door or switch to the other door. Should the contestant stay or switch? Conditional probabilities as well as empirical studies confirm that switching doors yields a 2/3 probability of winning, whereas staying, only a 1/3 probability. If a contestant chooses the correct door initially and switches, then he loses (Tierney, 1991). So after Monty Hall opens the door to reveal the goat, Thomas (2008) argues that it is no longer a matter of probabilities, but of pragmatics. How confident is the contestant that his door is the door with the car? If his door has the car, then the probability of winning if he switches is 0, not 2/3 (John Thomas, personal communication, April 15, 2008). Also, the contestant is only on the show once. So this argument is an example of single-case probability. The contestant does not have multiple chances on the show to achieve the expected value of the game, a win two-thirds of the time, from switching every time[1]. Probabilities thus only go so far in helping to explain chance and randomness.

Psychology of Probability and Randomness

Games of chance and statistical paradoxes such as the Monty Hall problem show us how our intuitive sense of probabilities and randomness is often incorrect. Most people confronted with the Monty Hall problem for the first time conclude that at the final stage, the probability of winning is .5 because there are two remaining options. Even at a second, closer examination, the correct result is by no means obvious. This problem was posed to Marilyn vos Savant, listed in the Guiness Book of World Records Hall of Fame for "Highest I.Q.", in her weekly "Ask

[1] While this is true, rationally considering the situation will result in the contestant switching. The contestant might have chosen door 1 for example, because he followed his intuition or he "just knows." Unless someone behind stage tipped him off and he *actually* knows extra information about the game, he can at most assign a probability of 1/3 to the car being behind door 1 no matter how much he convinces himself he has chosen the correct door. The 2:1 odds against the contestant are certainly not in his favor and he would do better to switch doors.

Marilyn" column in *Parade* magazine in 1990. Her response to switch doors sparked a nation-wide mathematical debate that was even discussed inside the walls of the Central Intelligence Agency (Tierney, 1991). This example is still used today in elementary statistics and game theory classes to baffle students.

Another simple example often used in classrooms of how randomness and probabilities can be misleading is the likelihood of having two people in one room with the same birthday. One might reason that there are 365 days in a year and assuming a class size of 30, the odds of two students having the same birth date is very low. However, it's actually around 70% ("Birthday problem," n.d., paras. 4-5). The easy mistake is that as a student in the classroom, rather than calculating the probability that someone has the same birthday as them self, must remember that the question asks for the probability that *any* two people have the same birthday. This adds an element of combinatorics to the problem and increases the chance of this seemingly infrequent occurrence.

"The brain, no matter how well schooled, is just plain bad at dealing with randomness and probability. Confronted with situations that require an intuitive grasp of the odds, even the best mathematicians and scientists can find themselves floundering," concludes George Johnson (2008, p.14) in a book review of Leonard Mlodinow's *The Drunkard's Walk: How Randomness Rules Our Lives.* Many like Johnson and Mlodinow have studied how human brains perceive and relate to mathematics and randomness. Stanislas Dehaene (1997) is yet another author and psychologist that has studied this field. He writes about his findings in *The Number Sense: How the Mind Creates Mathematics*. On a simple frequency scale, Dehaene's theories on human perceptions of randomness involve familiarity with the number line. Because the lower counting numbers are more prevalent in our daily lives, we fall back on these numbers more often in attempts to be random.

> […] let us pretend that you are a random number generator and that you have to select numbers at random between 1 and 50. Once this experiment is performed on a large number of subjects, a systematic bias emerges: Instead of responding randomly, we tend to produce smaller numbers more frequently than larger ones— as if smaller numbers were overrepresented in the "mental urn" from which we were drawing. (Dehaene, 1997, p. 76)

Dehaene notes that biases occur in human representations of randomness. One bias that he observes is a trend to over-represent lower numbers due to their higher familiarity. Using similar reasoning, perhaps other biases exist on a larger scale, to numbers like 50 and 100, or to numbers of preference such as 7 as a typical "lucky" number. In any case, Dehaene's research shows the disparity between theoretical definitions and human interpretations of randomness. This conclusion, however, is not a new discovery. Throughout the ages, people have used other sources of randomization to determine unbiased, fair outcomes in life. Sortition, or casting lots, was a method used in Athenian democracy and commonly referred to in the Old Testament as a means of equal-chance selection. The US draft at one time selected applicants through a random selection of birthdays. Using an equal-probability method of selection and randomization removes human bias from the selection process and places it more in line with the notion of fate.

Pseudorandom Number Generators

History has shown a long list of attempts to provide such unbiased sources of randomness. Primitive attempts to generate random numbers came from recording dice rolls, tables of logarithms, and census data (Bennett, 1998). With the modern advent of vast sources of data and information, equally large quantities of random numbers are needed. Thus, researchers have turned to mathematical algorithms to generate random numbers because of their potential unbiased characteristics and limitless supply of numbers. Even the great mathematician and father of modern game theory, John von Neumann (as cited in Bennett, 1997, p. 141), saw their potential. "Conceptually, the idea of an arithmetically determined random digit was both desirable and undesirable." However, the idea of using a concrete algorithm to model randomness is counterintuitive and von Neumann ultimately concluded that "anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin" (as cited in Bennett, 1997, p. 141). He committed this very sin, however, when he proposed the method of middle-squares to generate random digits.

Random number generators are generally called "pseudorandom" number generators because the sequences and numbers they produce are not technically random since they come from a deterministic formula. Pseudorandom number generators often take an initial seed value and manipulate it through a recurrence relation to create the desired sequence. Von Neumann's method of middle-squares takes an n digit number, squares it, and uses the middle n digits as the next number in the sequence. However, his method has a fatal flaw that makes it unsuitable for any rigorous form of randomization; zero continually propagates itself in the same position after its first occurrence.

Von Neumann was correct in being wary of using mathematics as a source of randomization. There are many things to consider when looking for a good pseudorandom number generator. These formulas generally implement recursion and one drawback to recursion is that all formulas that using it will repeat after some time. The length of the sequence until it repeats is called the period so understandably, a good pseudorandom number generator should have a very long period. Certain seed values tend to create long or short periods, so this must also be considered. Most importantly, the pseudorandom number generator must also produce numbers in accordance with the probability distribution required. A generator seeking to replicate a die should, for example, follow a uniform distribution over the range one through six and not display a tendency to produce one number more frequently than others. Because of their varying underlying formulas, seed values, and methods, pseudorandom number generators have different characteristics making them more or less suited for certain applications. A pseudorandom number generator used in cryptology to encode messages should, for obvious reasons, display strong characteristics of unpredictability in both the sequence produced and the ability to discern the underlying formula. A generator used for model simulation involving large quantities of data should have an equally large period and reflect the desired distribution even at a sample size of five million. However, a generator used to produce the table of random digits in the back of an introductory statistics book does not have to be as robust as those previously mentioned because it is adequate enough for its purpose.

One of the most common methods used in pseudorandom number generators is a linear congruential generator (LCG) first proposed by D. H. Lehmer in 1949 (and cited in Knuth, 1981, p. 9). This method uses an underlying formula of the basic form

$$x_{n-1} = (ax_n + c) \pmod{m} \qquad \text{where a,c,m are integers and } n = 1,2,3,\ldots$$

General guidelines for producing a sequence with random characteristics and a longer period are taking $m = 2^w$ with $w \geq 0$. This formula generates a sequence with $0 \leq x_n \leq m$. The simple transformation $u_n = x_n/m$ will achieve a sequence of real numbers in [0,1) (Tezuka, 1995) which is the main focus of many researchers in the field of pseudorandom number generation.

However, for a better comparison to human sampling, the sequences produced from LCGs should be sequences of integers.

Donald Knuth, a well-known name in computer science and programming theory, reviews many methods for checking the randomness of sequences produced by pseudorandom number generators in *The Art of Computer Programming* (1981). His tests range in their rigor from simple frequencies to multidimensional lattice analysis. The binary sequence {0,1,0,1,0,1,…} would pass the less rigorous frequency test with flying colors, since the test only looks at total frequencies observed for each value in the range. However, no one would consider this sequence random. Accordingly, the sequence would fail a more rigorous test like the serial test since there are no occurrences of the same value in consecutive positions.

Like pseudorandom number generators themselves, many of these tests have specific characteristics making them more or less suited to check certain types of sequences. Knuth analyzes four tests first proposed by Maurice Kendall and Bernard Smith in 1938. These original tests were designed mainly to check sequences of integers. The frequency, or equidistribution, test counts the occurrences of each value in the range and uses a chi-square test as a goodness-of-fit check for the uniform distribution. A similar test, the serial (n) test, looks at the grouping of each possible n values within the range and uses frequency analysis to determine if the digits in the sequence occur independently of each other. The gap test checks the length of "gaps" between two occurrences of the same values in the sequence. Finally, the poker test checks the occurrences of variations of the basic types of five-card poker hands: five of a kind, four of a kind, full house, three of a kind, two pairs, one pair, all different. Both the gap test and poker test use the chi-square test statistics and easily computable probabilities to verify the goodness of fit.

Research Question

Can sequences generated from deterministic formulas be more random than those from human beings? How do human beings generate random numbers and are their actions predetermined or can they act truly randomly? Can the compatibilist claims of free will be reconciled with notions of randomness? Under the compartmentalization of compatibilism, randomness at quantum levels can be considered truly random and is left to influences of the environment or the confines of the microworld. Randomness in the macroworld of human actions then becomes the main focus. The aim of this research is to further study and explore the human understanding of randomness and their capacity to act randomly with the wider goal of integrating randomness into the compatibilist account of free will.

Stanislas Dehaene has shown the tendency for humans to favor numbers lower on the number scale when sampling from a wide range. Does this theory hold over a smaller range from one to ten, or do other biases emerge? His research serves as both an inspiration and base line to this study. Donald Knuth provides the means to test the randomness of the sequences produced through various methods in this research. The four tests: frequency test, serial test, gap test, and poker test, first proposed by Kendall and Smith and modified and cited in Knuth (2001) will serve to analyze certain desirable characteristics of randomness present in the sequences under consideration. With the development of newer technology, Knuth also suggests other, even more rigorous tests, however, due to the small sample sizes used in this research and the simple nature of the generators implemented, these four main tests will suffice.

CHAPTER 2
EXPERIMENTAL DESIGN

Five main different methods of randomization were used to create sequences: dice, the decimal digits of pi, a TI-83 Plus calculator, a Java program, and human sampling. Dice represent, in theory, a more pure form of randomization. In society, they are associated with chance and used quite frequently as a means to fairly choose between outcomes. A coin better displays this association and representation of true randomization. However, the range of a die is more closely related to the other ranges in this study. The particular dice used in this study are not claimed to be specially fair or unweighted as they were taken from simple board games. It is impossible to verify this fairness assumption by means of experimentation or observation, unfortunately, since this is the very research question itself. Pi is an interesting case as it represents a sequence coming from a deterministic formula, but not one that was designed to pass tests of randomness. Different formulas exist to generate pi's decimal expansion, but they all arrive at the same sequence. It is well-known and undisputed that pi seems to be a random sequence. However, this startling result has yet to be proved for certain although many mathematicians have tried.

Both Texas Instruments calculators and Java use pseudorandom number generators. However, the method used by Java should be more advanced than that of the TI calculator. This follows from the differing complexities of their generators' respective applications for which they were designed: the calculator as a hand-held computing device and Java as a sophisticated programming language used all over the world. Whether the difference in the methods should be evident from the simple tests conducted is unknown. In line with the nature of this research and their difficulty to obtain access to, none of the more rigorous methods for generating pseudorandom numbers as mentioned previously are represented. Accordingly, more rigorous methods of testing the randomness of the sequences were not used. Sequences from human participants were compared to those produced by these four methods in an attempt to draw comparisons between the underlying methods used to generate the sequences by drawing comparisons in the analysis of sequences produced.

Dice, Pi, Calculator, Java

Four different six-sided dice from various board games were used to generate four separate sequences. Designations given to the dice were as follows: die1- from Trivial Pursuits, die2 – first die from Monopoly, die3 – second die from Monopoly, and die4 – from Scene It. Die1, die2, and die3 are basic white dice with black indents marking the sides, and die4 is a smooth sided die with stickers indicating the numbers. A sample size of 504 was used for die1, die2, die3, and die4. Sample size was chosen on the basis of practicality and divisibility by six.

The number pi has many interesting applications and characteristics, one of which being a seemingly random sequence of decimal digits. For ease in calculations and runtime in JCreator, the decimal digits of pi were used through the 1600^{th} digit and obtained from the internet (Pi, 2008). The exact method used to obtain these values is unknown, but the accuracy of the digits has been verified through other sites.

The exact formula used by Texas Instruments in the TI-83 Plus programmable calculator is unknown, however, it is probably a rather simple pseudorandom number generator and most likely a basic linear congruential generator. According to the TI-83 Plus manual (1999), the command `randInt(`*`lower`*`,`*`upper`*`)` “generates and displays a random integer within a range specified by *lower* and *upper* integer bounds" (Texas Instruments, 1999, p. 2-22). Six total sequences each of sample size 800 were generated using the command `randInt(1,10)` and recorded manually. Since "the TI-83 Plus generates the same random-number sequence for a given seed value," six different seed values were used. Storing zero restores the factory-set seed value. Designation of the sequence denotes the seed value used: calc0, calc1, calc2, calc4, calc10, and calc100.

Java statedly implements a linear congruential formula modeled after those described by Knuth and a 48-bit seed value for its pseudorandom number generator (Sun Microsystems, Inc., 2004). The class `Random` "initializes to a value based on the current time" noting that "objects created within the same millisecond will have the same sequence of random numbers." A manual seed value can be specified using the `setSeed(long seed)` method and taking a 64-bit seed value, but only using 48-bits of the value. The `nextInt(int n)` method "returns a pseudorandom, uniformly distributed `int` [integer] value between 0 (inclusive) and the specified value (exclusive), drawn from this random number generator's sequence." The Java documentation states that "all `n` possible `int` values are produced with (approximately) equal probability" specifically noting that the method "is only approximately an unbiased source of independently chosen bits" (Sun Microsystems, Inc., 2004). The algorithm for the `nextInt(int n)` method uses multiple `if`-statements to ensure that the sequence produced will be adequately random and the period adequately long enough, acknowledging flaws in its own design and checking to ensure they are not exploited. The code used to produce the Java sequences is in Appendix C. Four sequences were created in order using the manual seed values: 1 – java1, 03031987 − java2, 1111111111111111111 − java3, 999999999999999999 – java4 and five were created using time codes as the initial seed value: time1 – java5, time2 – java6, time3 – java7, time4 – java8, and time5 – java9. Each has a sample size of 1000.

Survey

Sequences from human participants were collected through survey from various STAT 2000 lab sessions over two days. Students ages 18-25 were given a survey (see Appendix A) and told to read the directions on the survey. Very little, if any, extra verbal instruction or information about the survey was given so as not to influence any of the students' preconceived notions on randomness. Question 1 asked the student to "pick a random number from 1 to 10." All of the responses from this question were combined to draw inferences from the distribution of the population as a whole. Question 2 asked students to consider carefully *why* they chose the number they did for question 1. Students were given a list of options as well as an 'other' category and asked to explain further. If a reason listed under 'other' was felt to be adequately reflected by another option, the response was changed when recording the data. Responses to question 2 help to analyze whether a cause-and-effect relationship exists in this context, whether this relationship is conscious if it does, or the general perceptions people have in regards to randomness. 180 observations were obtained for each of questions 1 and 2.

For question 3, participants were asked to fill in a ten by ten grid with random numbers from one to ten. The intent was for the students to fill out the grid across and sequentially since the order of the numbers is equally as important as what numbers are filled in, however there was no way to police or verify that this was done. Some students thought through the wording of the question that the numbers '1' though '10' *had* to occur in each row. It was addressed in the classroom after the surveys were handed out, that this was not the case. It is also possible that this trend was so prevalent due to the current popularity of Sudoku puzzles. Upon later analysis of the surveys collected those that used the numbers one though ten in each row, whether they felt they were instructed to or simply did (perhaps noting that it was a ten by ten grid), were discarded. The proportions of surveys that had to be excluded varied by classroom, but always constituted a minority of the surveys and numbered 55 in total. A very few surveys also had to be thrown out because they contained real numbers or letters. 125 total surveys were considered acceptable and given designations survey1,…, survey125.

Because the wording of the third question posed a few problems and the proportion of surveys thrown out for each classroom was a larger than desired, it was reworded and a new survey was distributed to the final two classrooms. A chi-square test of independence confirms that there was a significant (p-value = 0.0022) lessening in the number of surveys that had to be excluded due to the rewording of the question. The wording was changed from,

"Working **left to right** write a random number from 1 to 10 in each individual cell" to

"Repeat Question 1 for each box and fill in your responses working ACROSS."

89 usable surveys were gathered using the first wording, and 36 using the second. Surveys 1-89 come from the first set and surveys 90-125 come from the second. The sequences were also combined to form one large overall sequence with sample size 12500. Two surveys from separate days, survey34 and survey91, listed fours in every cell. These were considered acceptable surveys, but in later analysis it is noted when these sequences are omitted so the results are not skewed. A further combined sequence was also created leaving out the values from samples survey34 and survey91. The methods for all the sequences are summarized in Table 2.1 and notations for all the sequences are summarized in Appendix B.

Table 2.1 Summary of Sequence Comparisons

Method	Range	Sample Size	Repetitions
Dice	1-6	504	4
Pi	0-9	1600	1
Calculator	1-10	800	6
Java	1-10	1000	9
Survey	1-10	100	125

CHAPTER 3
DATA ANALYSIS

Sequences generated from dice and the calculator were generated and recorded manually and then input into Microsoft Excel. Sequences from pi and Java were copied directly from the cited web site and program output file respectively. The original sequences were manipulated through Java programs to produce sequences for the serial test, gap test, and counts of differences for the poker test. Most of the chi-square test statistics and probabilities were calculated in Excel, and the more advanced statistical tests and graphing were conducted in SAS. The rewording of question 3 did not significantly affect the number of sequences that failed each test. The results of the chi-square tests are shown in Table 3.2 at the end of the chapter. Summaries of all the test statistics and probabilities for each test and sequence are also contained in Appendix E.

Frequency Test

For dice with sample size 504, the expected total number of occurrences for each value is 84. The lowest frequency was 68 from die3 and the high was 108 also from die3. The range for die1, die2, and die4 were 77 to 103, 75 to 97, and 70 to 105 respectively. Die1 and die2 passed the frequency test and had probabilities 0.2903 and 0.5223 respectively, while die3 did not with a probability of 0.0066. Using alpha level 0.05, die4 also failed (p-value 0.0460). For pi, the expected frequency was 160 from a sample size of 1600. The range of frequencies was 149 to 175 and the sequence passed the test and had probability 0.8813. All six of the calculator-generated sequences passed the frequency test at alpha level 0.05, while two of nine the Java sequences did not (p-values 0.0484 and 0.0216). A majority of survey sequences passed the frequency test at alpha level 0.05. A chi-square test for independence on a 2x2 contingency table confirms that the different wording of question 3 on the later surveys did not significantly affect the proportion of sequences that failed the various tests (probability 0.5622).

Serial Tests

Only die3 failed both the serial (2) and (3) tests as compared to die3 and die4 that failed the frequency test. The pi sequence was again, not statistically significant, and the proportion of sequences that failed the serial tests for the calculator and Java sequences increased. Only calc100 was significant with probability 0.0308 for the serial (3) test. Java3, java7, and java8 failed both serial (2) and (3) tests. Because the serial tests use the chi-square test statistic to compare observed frequencies with expected frequencies, the assumption of using the chi-square test statistic that all frequencies are greater than or equal to five must be checked. The sample sizes of the various sequences were previously large enough to ensure this assumption was met for the frequency test. Due to the ranges being drastically expanded in accordance with the nature of the serial test, however, this assumption does not hold for a majority of the sequences as shown in Table 3.1.

Table 3.1 Average Proportion of Cells that had Fewer than 5 Observations

	Dice	Pi	Calculator	Java	Survey
Serial (2)	0.0025	0.0000	0.1033	0.0333	0.9810
Serial (3)	0.9075	0.9900	0.9983	0.9960	0.9997

While the assumption was adequately met for the dice, pi, calculator and Java data for serial (2), due to the drastic change in proportion of cells that had fewer than five observations there was not a significant increase or decrease in the number of significant sequence under each method for serial (3). A majority of the survey sequences failed the serial (2) and (3) tests.

However, due to their smaller sample size, most surveys did not meet the assumptions of the chi-square test statistic for either the serial (2) or serial (3) test.

Gap Test

The length of a gap between two occurrences of the same value in a sequence could in theory be quite long. In actuality, this was often the case as some sequences experienced longest gaps of 77 and 105. Because the frequency of these rather long gaps was low, an effort was taken in accordance with Knuth's advice, to ensure there would be five or more observations of each gap length to justify use of the chi-square test statistic. The length of gaps was capped after observations began to continually drop below 5 and a 'greater than or equal to' category was created. Similarly, every three gap lengths were combined for the survey data because of the smaller sample size. Making these adjustments and accordingly calculating probabilities justifies use of the chi-square test statistic.

Die2, die3, and die4 failed the gap test and had probabilities 0.0208, 0.0070, and 0.0102 respectively. All the sequences from pi and the calculator passed the test. Only one sequence, java6, failed this test (p-value 0.0083). A majority (87%) of the survey data failed the gap test.

Poker Test

The original intent of the poker test was to use categories taken straight from poker: four of a kind, two pairs, full house, etc. However, checking the occurrences of these subsequences of length five using a computer program creates difficulties for the programmer as they have no one systematic similarity. Knuth addresses this problem and says,

> It is reasonable to ask for a somewhat simpler version of this test, to facilitate the programming involved. A good compromise would simply be to count the number of *distinct* values in the set of five. We would then have five categories:

5 different = all different;

4 different = one pair;

3 different = two pairs, or three of a kind;

2 different = full house, or four of a kind;

1 different = five of a kind.

This breakdown is easier to determine systematically, and the test is nearly as good (Knuth, 1981, p. 62).

A Java program was used to count the occurrence of these differences. Knuth also gives the formula for determining the probabilities of r differences for each group of k successive numbers from a set with d different values.

$$p_r = \frac{d(d-1)\ldots(d-r+1)}{d^k} \cdot \{^k{}_r\}$$

Where $\{^k{}_r\}$[2] are the Stirling numbers from Appendix D (as cited in "Stirling numbers of the second kind," n.d., Tables of values, para. 1).

Because the probability of 5 consecutive numbers of the same value is so low over these ranges, the '1 different' category almost always had zero observations, which is well below the five required by the chi-square test statistic. All dice and calculator sequences passed the modified poker test, while the sequence from pi failed (p-value 0.0001). Four sequences from Java and 91% of the survey sequences also failed the poker test.

[2] The 'k' and 'r' should not written as offset, but as one directly over the other.

Table 3.2 Chi-Square Contingency Tables

Wording of Question 3 by Significance of Specified Test

Frequency Test	**Original**	**New**		**Expected**			
Significant	12	4	16	11	5	Chi-Square:	**0.13**
Not	77	32	109	78	31	Probability:	**0.7193**
	89	36	125				

Serial (2) Test	**Original**	**New**		**Expected**			
Significant	58	26	84	60	24	Chi-Square:	**0.58**
Not	31	10	41	29	12	Probability:	**0.4469**
	89	36	125				

Serial (3) Test	**Original**	**New**		**Expected**			
Significant	58	25	83	59	24	Chi-Square:	**0.21**
Not	31	11	42	30	12	Probability:	**0.6467**
	89	36	125				

Gap Test	**Original**	**New**		**Expected**			
Significant	80	29	109	78	31	Chi-Square:	**2.00**
Not	9	7	16	11	5	Probability:	**0.1573**
	89	36	125				

Poker Test	**Original**	**New**		**Expected**			
Significant	80	34	114	81	33	Chi-Square:	**0.66**
Not	9	2	11	8	3	Probability:	**0.4154**
	89	36	125				

CHAPTER 4
RESULTS

Question 1 and 2

Question 1 asked students to “pick a random number from 1 to 10.” Looking at the histogram of the 180 total responses from question 1 on the survey as seen in Figure 4.1, two trends are noticeable. ‘7’ was the most frequent response by more than double any of the others at 31% with the next closest at 14% being ‘3’. Also, the values seem to fall into a bimodal distribution centered on ‘3’ and ‘7’. Both ‘3’ and ‘7’ are equidistant from the center and endpoints of the range.

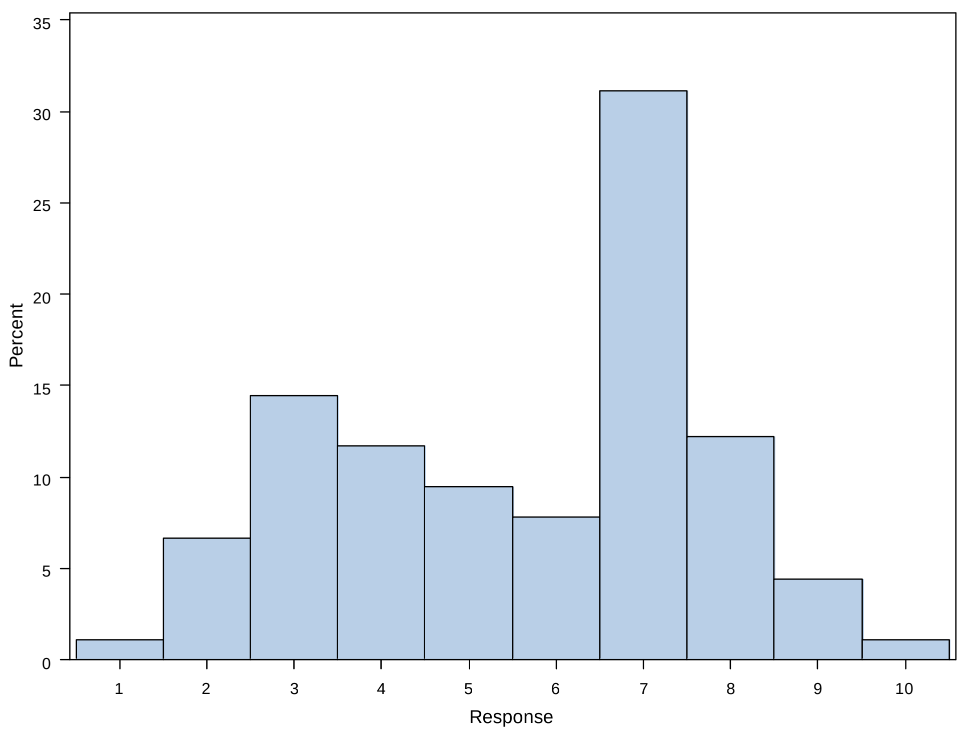

Figure 4.1 Question 1 Responses (n=180)

Of the 56 students who chose ‘7’, 47% gave (a), special number, as the reason and 49% gave reason (b), first number to pop into head. This is comparable to the 52 percent of students that responded (a) overall and the 40 percent that responded (b) overall. Histograms of the responses to question 1 by letter response for question 2 in Figure 4.2 show that the bimodal distribution is more present under response (a). Of those that chose (b) and said their response was the first number they thought of, ‘7’ again was the clear mode. Outside this number, however, there appears to be a uniform distribution between ‘3’ and ‘8’. Only eight percent of students responded with (c), that they thought their number was random. A chi-square test with alpha 0.05 proves that the choice of number in question1 is independent (p-value 0.0746) from the choice given in question 2 although there were several cells that had fewer than 5 observations.

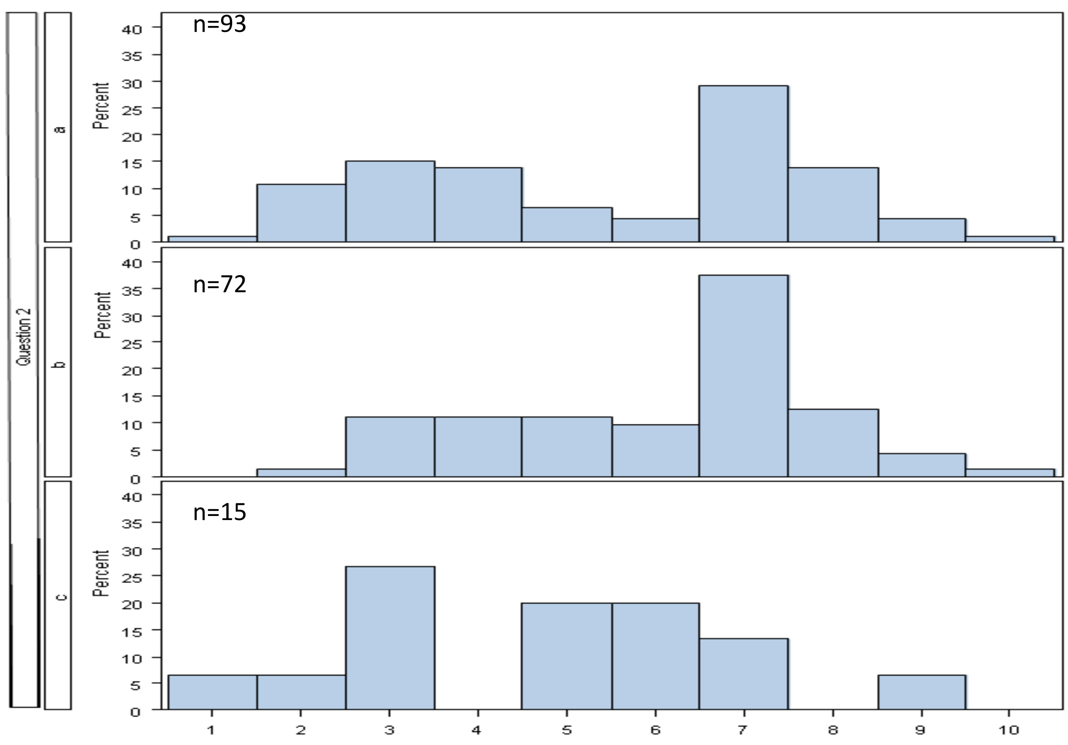

Figure 4.2 Question 1 Responses by Question 2

Dice, Pi, Calculator, Java, Survey

In theory, the four tests should progress in rigorousness from the frequency test to the serial, gap and then poker test. This progression would then also be evident from the increase of proportion of sequences that were significant under the various tests. Table 4.1 shows how this is clearly the case for the survey dataset. The Java data also generally portray this trend. The dice, pi, and calculator datasets have smaller sample sizes and a smaller total number of sequences than the Java and survey data sets, possibly accounting for the lack of trend.

Table 4.1 Proportion of Sequences that Failed the Specified Test

	Frequency	Serial (2)	Serial (3)	Gap	Poker
Dice	0.50	0.25	0.25	0.75	0.00
Pi	0.00	0.00	0.00	0.00	1.00
Calculator	0.00	0.00	0.17	0.00	0.00
Java	0.22	0.33	0.33	0.11	0.44
Survey	0.13	0.68	0.66	0.87	0.91

Combined Survey Data

As mentioned previously, sequences survey34 and survey91 contained values of all fours. Because of this, these observations were dropped so that their values would not skew the results when looking at the combined data sets. The graph of the overall frequencies of the values from the survey sequences is shown in Figure 4.3. The high frequency of value ‘7’ is no longer present as with the responses for question 1. Instead, an overall slightly negative trend is seen with ‘1’ as the most frequent value and ‘10’ as the least frequent.

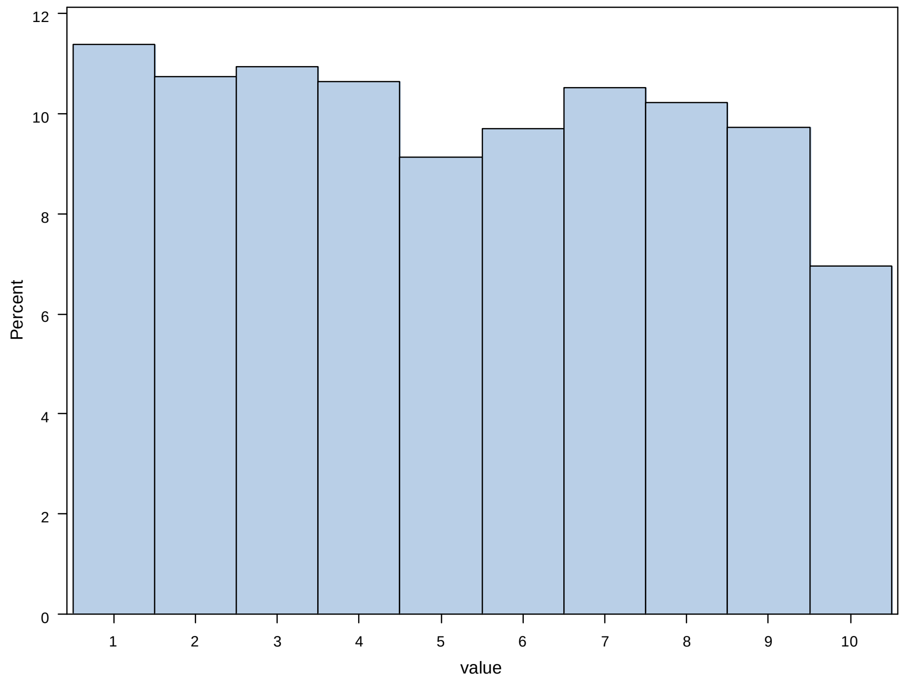

Figure 4.3 Values for Combined Dataset (Survey34, Survey91 dropped)

Side-by-side histograms of the values from all of the 123 sequences by the significance of the frequency test for those sequences in Figure 4.4 also show some noticeable trends. If the probability from the frequency test for a sequence was below 0.05 it was coded 1 and if the sequence was not significant at alpha level 0.05, then it was given a code of 0. 109 surveys were coded '0' (total sample size 10900) and 14 surveys were coded '1' (total sample size 1400). Survey34 and survey91 were not included for this analysis. Looking at the histogram of those sequences that failed the frequency test seems to show the trend to over-represent lower numbers mentioned by Dehaene (1997, p.76). '1' can clearly be seen as the most frequent value and '10' as clearly the lowest. The values '2' through '9' seem to follow a slightly less negative trend in between. To further examine this trend, all observations of '1' and '10' were removed from this subsequence. This smaller subsequence of values '2' through '9' failed a further frequency test (p-value 0.0164). Again, removing all observations of ‘2’ from this subsequence however, achieved a sequence that passed the frequency test (p-value 0.1776). This seemingly decreasing trend can actually better be characterized as a decrease across ‘1’, ‘2’, ‘3’-‘9’, and ‘10’.

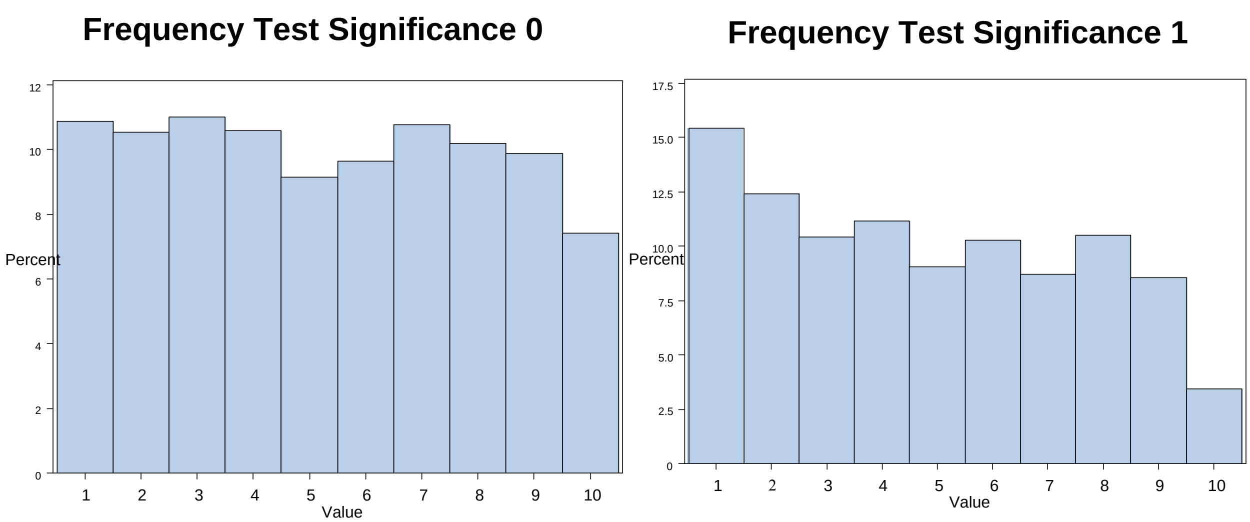

Figure 4.4 Sequence Values by Frequency Test Significance

A vast majority of surveys passed the frequency test. If the sequence passed the frequency test (was not significant), then the frequency of values in the sequence were statistically uniform. Accordingly, looking at the histogram of this data subset verifies that the values have approximately equal frequencies.

CHAPTER 5
CONCLUSION

Increasing proportions of survey sequences failed each test as they progressed in rigor. The worth of the results for the serial tests and to some extent the poker test could be argued because the chi-square tests consistently had fewer than 5 observations. The frequency and gap tests however, still show a vast difference in the proportion of sequences that failed the respective tests in line with the vast difference between the rigors of the tests. In this sense human sampling mimics a poor pseudorandom number generator based on a deterministic formula that has obvious tendencies and biases. For both the humans and the pseudorandom number generator, the requirement of equiprobability is easily met, however the more complex concepts like order of occurrence and independence are harder to grasp. A primitive sequence of ten ones followed by ten twos and so on would certainly pass the frequency test and certainly fail the gap test. Accordingly, this sequence would not be considered random by definition.

Strictly comparing the results in terms of proportion of sequences that failed a given test, those from the survey data most closely resemble those from the Java data. It is not a strict association as the Java data shows a decreased proportion with the gap test. With only nine observations though, there are allowances for random variation as well as the chance choice of manual and time seed values. With 125 observations, there is less leeway and the strictly increasing progression is perfectly modeled. The progress of smallest to largest proportion is the most similar between these two sources and suggests a stronger relationship between these two methods of randomization.

Other factors should be considered. Due to their smaller number of sequences, random variation in the dice, pi, or calculator data would greatly affect the proportions. Because comparable seed values were not used in calculator and Java data sequences, human intervention in this regard could have also upset the natural progress that may be present if the manual seed values for the calculator had been chosen more arbitrarily. Pi is famous also for its known ability to mimic random properties even if the justification for this is unknown. Using pi as a generator of a random sequence then, and especially at a small sample size of 1600 when compared to the over 1,000,000 known digits, is to know the result *a priori* as its fame has been established by passing these very tests. In this case, pi represents a very robust pseudorandom number generator which would have no difficulties in passing the frequency, serial and gap tests, and perhaps only start to show weakness at the poker test. The small sample size of 100 makes it very hard for the survey sequences to perform as well as pi in the various tests because they leave little room for random error at such short sequences. One sequence from the surveys actually did pass all of the tests, and a handful only failed one.

There was no significant correlation between response in question 2 and response in question 1. Regardless of *why* the student believed they had chosen the number in question 1, overall, they drew from the same distribution as everyone else. The distribution of special or lucky numbers is the same as the distribution of numbers that just popped into students' heads. Students giving the justification that their number was special in some way have a specific reason for picking their number. If someone admits to a favorite or special number, a preference for the number has been well established, as a jersey number throughout grade school or a birthday for example. Students who acted on first instinct and claimed no reason for their action preformed exactly as these students that had distinct motivations for their actions. Perhaps there was some overlap in these categories. The first number to occur to someone could easily be their favorite number since it is their favorite, and someone could choose their favorite number first also, since it is their favorite. However, upon the participant's realization of the number chosen, his reason for its choice should more naturally be the answer with a clear association if it had a prior existence- option 'a'. With 165 of 180 students choosing either (a) or (b), the minimal overlap that might have occurred can not fully explain why these two groups of students gave such similar distributions for question 1.

The values from all the survey sequences, naturally excluding observations 34 and 91, demonstrate Dehaene's previous findings adequately enough. Even at the small range of one to ten, the lower numbers have higher frequencies even if the numbers three through nine show no significant difference. His reason is their higher familiarity in our daily lives and this explanation is both reasonable and persuasive. Also, perhaps since the students participating in this study had to physically write the values and '1' consists of a single stroke this could explain its increased frequency, as the two-digit '10' could explain its low frequency. Two and the next category of three through nine also significantly show this trend, however, so the secondary justification is not as likely.

The distribution of values does not itself seem random as there appears to be a clear downward trend in the data. The presence of a trend raises the question of a reason for the trend. While Dehaene gives a possible answer to this question and it has found fair substantiation at least from this study, what is of more interest here is that there exists a *reason* at all. To stubbornly seek the cause for this trend-occurrence presupposes that there is a cause to find. This paradox lies at heart of the debate and due to its fundamentally theoretical foundations it is unlikely a concrete answer will ever be reached. A trend in frequency implies preference. A preference is a predisposed or initial state of mind much akin to a seed value in a recurrence relation. When human participants were asked to generate sequences of random numbers, they preformed much like a pseudorandom number generator using their prior preconceived notions of randomness and familiarity with the number line to sample from the required range. They were able to easily grasp the concept of equiprobability, however many were unfortunately unable to go beyond that. Whether this is due to a lack of knowledge or education on the topic or to a lack of capacity is unknown.

Further study and research in this area would certainly help to solve more of the unknowns uncovered in this experiment. Greater repetitions of each of the non-human methods would certainly allow for a better comparison of proportions of sequences that failed each test. For another survey it would be beneficial to have a follow-up with the participants to understand more about their thought process. It would also be interesting to see how they would perform a second time after a follow-up interview and if their ability to mimic randomness could improve with more knowledge of how their sequences were being analyzed. If this were the case, it might show that a lack of knowledge was the primary reason for their original poor performance. To reduce the physical task of writing taking a part in the production of the sequences it would also be better to have different methods of recording the sequences such as through tape recorder or computer input. Similar written surveys could also be conducted with various ranges such as '0' to '9' or '1' to '100' to provide more insight into how the human mind behaves, although with larger ranges the required sample sizes would also have to be greatly increased and this might cause too much of a strain on participants.

APPENDIX A
RESEARCH SURVEY

1. Pick a random number from 1 to 10: ___________________

2. Why did you choose that particular number? Circle the answer that best explains why:
   - a. special number (favorite number, sport number, lucky number, etc.)
   - b. first number to pop into head
   - c. thought it was 'random'
   - d. other ____

3. Working **left to right** write a random number from 1 to 10 in each individual cell.

APPENDIX B
DESIGNATION CODING FOR SEQUENCES

Code	Source	Seed Value	Sample Size
die1	Die: Trivial Pursuits		504
die2	Die: Monopoly (1)		504
die3	Die: Monopoly (2)		504
die4	Die: Scene It		504
pi	Pi		1600
calc0	TI-83 Plus	0	800
calc1	TI-83 Plus	1	800
calc2	TI-83 Plus	2	800
calc4	TI-83 Plus	4	800
calc10	TI-83 Plus	10	800
calc100	TI-83 Plus	100	800
java1	Java – LCG	0 – factory preset	1000
java2	Java – LCG	03031987	1000
java3	Java – LCG	1111111111111111111	1000
java4	Java – LCG	99999999999999999	1000
java5	Java – LCG	Time (1)	1000
java6	Java – LCG	Time (2)	1000
java7	Java – LCG	Time (3)	1000
java8	Java – LCG	Time (4)	1000
java9	Java – LCG	Time (5)	1000
survey1	Survey: Original Q3		100
… survey89			
survey90	Survey: New Q3		100
… survey125			

APPENDIX C
JAVA PROGRAMS

```
/ *********************************************************
//
// RandomGeneratorTester.java
// Author: Aileen Thomas
// Date: June 19, 2008
//
// Purpose: To generate sequences of random numbers.
//
// ********************************************************

import java.util.Scanner;
import java.io.*;

public class RandomGeneratorTester
{
      public static void main (String[] args) throws Exception
      {
            Scanner scan = new Scanner(System.in);
            RandomGenerator randGen = new RandomGenerator();
            FileOutputStream fout;
            fout = new FileOutputStream("C:\\Documents and
Settings\\Aileen\\My Documents\\RESEARCH\\output.txt");

            System.out.println("Enter seed value (0-none, else seed value)");
            long seed = scan.nextLong();
            if(seed==0)
            {
                  randGen.compute();
            }
            else
            {
                  randGen.setSeed(seed);
                  randGen.compute();
            }

            new PrintStream(fout).println(randGen.toString(seed));
            fout.close();
      }
}
```

```
// ********************************************************
//
// RandomGenerator.java
// Author: Aileen Thomas
// Date: May 12, 2008
//
// Purpose: To operate with RandomGeneratorTester
//
// ********************************************************

import java.util.Random;

public class RandomGenerator
{
      int randnum;
      int[] array = new int[100];
      Random generator = new Random();

      public void setSeed(long value)
      {
            generator.setSeed(value);
      }

      public void compute()
      {
            for (int i=0; i<array.length; i++)
            {
                  randnum = generator.nextInt(10)+1;
                  array[i] = randnum;
            }
      }

      public String toString(long seed)
      {
            String asString = "";
            for (int i=0; i<array.length; i++)
          {
            asString=asString + array[i] + "\r\n";
          }
          return asString;
      }
}
```

APPENDIX D
STIRLING NUMBERS

k \ *r*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	1									
1	0	1								
2	0	1	1							
3	0	1	3	1						
4	0	1	7	6	1					
5	0	1	15	25	10	1				
6	0	1	31	90	65	15	1			
7	0	1	63	301	350	140	21	1		
8	0	1	127	966	1701	1050	266	28	1	
9	0	1	255	3025	7770	6951	2646	462	36	1

APPENDIX E
TEST RESULTS SUMMARY

FREQUENCY TEST

die1

Total:	804		Expected:	84	
Frequencies:					
1	**2**	**3**	**4**	**5**	**6**
103	85	75	77	85	79
Chi-Square:		**6.17**			
Probability:		**0.2903**			

die2

Total:	804		Expected:		84
Frequencies:					
1	**2**	**3**	**4**	**5**	**6**
80	97	85	75	77	90
Chi-Square:		**4.19**			
Probability:		**0.5223**			

die3

Total:	804		Expected:	84	
Frequencies:					
1	**2**	**3**	**4**	**5**	**6**
108	72	101	78	68	77
Chi-Square:		**16.07**			
Probability:		**0.0066**			

die4

Total:	804		Expected:		84
Frequencies:					
1	**2**	**3**	**4**	**5**	**6**
97	79	78	70	75	105
Chi-Square:		**11.29**			
Probability:		**0.0460**			

pi

Total:	1600		Expected		160				
Frequencies:									
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
151	171	163	153	149	167	153	158	160	175
Chi-Square:		**4.43**							
Probability:		**0.8813**							

Calculator

Total: 800 Expected 80

Frequencies:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	Chi-Sq:	Prob:
calc0	80	70	87	98	75	76	82	76	80	76	**6.88**	**0.6501**
calc1	77	91	65	79	77	70	96	71	81	93	**12.15**	**0.2050**
calc2	87	60	91	96	71	65	84	76	78	92	**16.40**	**0.0590**
calc4	74	78	62	82	97	68	86	88	78	87	**11.76**	**0.2270**
calc10	79	90	78	82	82	88	68	68	78	87	**6.48**	**0.6916**
calc100	82	80	73	85	82	83	96	83	77	59	**10.08**	**0.3444**

Java

Total: 1000 Expected 100

Frequencies:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	Chi-Sq:	Prob:
java1	91	87	83	114	97	96	109	89	108	126	**17.02**	**0.0484**
java2	111	109	97	97	108	106	109	94	91	78	**10.02**	**0.3489**
java3	101	101	84	103	92	135	88	90	95	111	**19.46**	**0.0216**
java4	82	88	110	111	89	99	100	117	88	116	**15.00**	**0.0909**
java5	112	100	94	110	99	94	104	103	103	81	**7.12**	**0.6246**
java6	106	101	89	98	95	90	113	99	119	90	**9.19**	**0.4208**
java7	97	112	94	104	105	80	104	122	96	86	**13.42**	**0.1445**
java8	83	93	114	89	98	80	105	113	108	117	**16.06**	**0.0656**
java9	79	96	104	106	91	114	88	112	107	103	**11.32**	**0.2544**

SERIAL (2) TEST

Dice df=35

die1		**die2**	
Chi-Square:	**31.13**	Chi-Square:	**34.57**
Probaability:	**0.6555**	Probability:	**0.4889**
die3		**die4**	
Chi-Square:	**80.80**	Chi-Square:	**43.01**
Probability:	**0.00002**	Probability:	**0.1657**

Pi df=99

Chi-Square:	**92.42**
Probability:	**0.6666**

Calculator df=99

	Chi-Square:	Probability:
calc0	**87.23**	**0.7950**
calc1	**112.01**	**0.1751**
calc2	**94.24**	**0.6164**
calc4	**114.01**	**0.1437**
calc10	**88.73**	**0.7607**
calc100	**97.50**	**0.5239**

Java df=99

	Chi-Square:	Probability:
java1	**110.21**	**0.2074**
java2	**90.59**	**0.7149**
java3	**132.43**	**0.0140**
java4	**98.00**	**0.5096**
java5	**94.19**	**0.6178**
java6	**91.39**	**0.6941**
java7	**132.23**	**0.0144**
java8	**129.83**	**0.0205**
java9	**88.99**	**0.7547**

SERIAL (3) TEST

Dice df=215

die1		**die2**	
Chi-Square:	**213.98**	Chi-Square:	**209.68**
Probaability:	**0.5067**	Probability:	**0.5896**
die3		**die4**	
Chi-Square:	**314.67**	Chi-Square:	**220.87**
Probability:	**0.00001**	Probability:	**0.3773**

Pi df=999

Chi-Square:	**883.48**
Probability:	**0.9963**

Calculator df=999

	Chi-Square:	Probability:
calc0	**941.35**	**0.9033**
calc1	**1034.08**	**0.2146**
calc2	**1021.55**	**0.3031**
calc4	**1018.78**	**0.3248**
calc10	**928.82**	**0.9445**
calc100	**1084.21**	**0.0308**

Java df=999

	Chi-Square:	Probability:
java1	**1002.00**	**0.4673**
java2	**1030.06**	**0.2412**
java3	**1106.21**	**0.0098**
java4	**937.87**	**0.9164**
java5	**985.97**	**0.6094**
java6	**995.99**	**0.5209**
java7	**1076.15**	**0.0447**
java8	**1110.22**	**0.0078**
java9	**955.91**	**0.8323**

GAP TEST

Dice df=15

die1		**die2**	
Chi-Square:	**8.91**	Chi-Square:	**28.12**
Probaability:	**0.8821**	Probability:	**0.0208**
die3		**die4**	
Chi-Square:	**31.72**	Chi-Square:	**30.50**
Probability:	**0.0070**	Probability:	**0.0102**

Pi df=30

Chi-Square:	**20.36**
Probability:	**0.9067**

Calculator df=25

	Chi-Square:	Probability:
calc0	**16.32**	**0.9050**
calc1	**21.18**	**0.6825**
calc2	**15.19**	**0.9368**
calc4	**14.30**	**0.9562**
calc10	**14.44**	**0.9535**
calc100	**18.21**	**0.8335**

Java df=25

	Chi-Square:	Probability:
java1	**21.14**	**0.6845**
java2	**20.13**	**0.7402**
java3	**28.49**	**0.2858**
java4	**13.01**	**0.9763**
java5	**19.45**	**0.7749**
java6	**45.03**	**0.0083**
java7	**21.13**	**0.6851**
java8	**31.76**	**0.1651**
java9	**27.91**	**0.3121**

Frequency Summaries for Gap Lengths:

'Gap' in the charts refers to gap length. Gap Length is the number of digits between two occurences of the same value.

Dice

Expected	Probability	Gap	die1	die2	die3	die4
84	0.1667	**0**	85	80	55	79
70	0.1389	**1**	79	67	94	75
58	0.1157	**2**	58	74	57	60
49	0.0965	**3**	46	33	47	53
41	0.0804	**4**	33	52	47	50
34	0.0670	**5**	37	21	37	32
28	0.0558	**6**	26	29	34	15
23	0.0465	**7**	29	30	19	12
20	0.0388	**8**	19	21	23	11
16	0.0323	**9**	11	15	22	20
14	0.0269	**10**	11	13	10	14
11	0.0224	**11**	10	5	8	15
9	0.0187	**12**	6	9	4	11
8	0.0156	**13**	6	7	10	13
7	0.0130	**14**	8	12	5	12
33	0.0649	**15+**	34	30	26	26

Pi

Expected	Probability	Gap	Pi
160	0.1000	**0**	156
144	0.0900	**1**	147
130	0.0810	**2**	118
117	0.0729	**3**	122
105	0.0656	**4**	107
94	0.0590	**5**	95
85	0.0531	**6**	82
77	0.0478	**7**	76
69	0.0430	**8**	63
62	0.0387	**9**	67
56	0.0349	**10**	58
50	0.0314	**11**	50
45	0.0282	**12**	36
41	0.0254	**13**	37
37	0.0229	**14**	35
33	0.0206	**15**	41
30	0.0185	**16**	30
27	0.0167	**17**	35
24	0.0150	**18**	34
22	0.0135	**19**	24
19	0.0122	**20**	18
18	0.0109	**21**	18
16	0.0098	**22**	17
14	0.0089	**23**	10
13	0.0080	**24**	11
11	0.0072	**25**	11
10	0.0065	**26**	5
9	0.0058	**27**	12
8	0.0052	**28**	9
8	0.0047	**29**	6
68	0.0424	**30+**	60

Calculator

Expected	Probability	Gap	calc0	calc1	calc2	calc4	calc10	calc100
80	0.1000	**0**	79	74	81	79	75	72
72	0.0900	**1**	68	78	66	78	74	72
65	0.0810	**2**	70	66	69	68	71	62
58	0.0729	**3**	53	66	59	57	54	64
52	0.0656	**4**	55	52	53	48	56	62
47	0.0590	**5**	48	38	56	49	42	46
43	0.0531	**6**	46	42	44	43	44	38
38	0.0478	**7**	35	37	32	34	31	45
34	0.0430	**8**	27	43	32	31	36	28
31	0.0387	**9**	32	18	23	31	31	35
28	0.0349	**10**	35	24	27	22	22	18
25	0.0314	**11**	24	36	32	25	22	27

23	0.0282	**12**	20	22	19	23	27	22
20	0.0254	**13**	23	22	14	13	27	21
18	0.0229	**14**	25	15	14	15	19	19
16	0.0206	**15**	13	15	16	20	18	18
15	0.0185	**16**	10	16	17	20	15	19
13	0.0167	**17**	18	10	11	18	13	16
12	0.0150	**18**	12	9	13	11	12	9
11	0.0135	**19**	9	12	11	10	6	9
10	0.0122	**20**	8	7	13	9	13	5
9	0.0109	**21**	6	11	8	9	11	6
8	0.0098	**22**	7	8	6	6	10	11
7	0.0089	**23**	3	8	10	10	5	6
6	0.0080	**24**	6	5	5	3	6	5
57	0.0718	**25+**	58	56	59	57	50	55

Java

Expected	**Probability**	**Gap**	**java1**	**java2**	**java3**	**java4**	**java5**	**java6**	**java7**	**java8**	**java9**
100	0.1000	**0**	106	101	98	95	107	89	93	108	105
90	0.0900	**1**	90	83	75	96	72	96	98	103	80
81	0.0810	**2**	77	79	77	69	69	66	78	87	99
73	0.0729	**3**	74	69	90	70	75	90	72	66	68
66	0.0656	**4**	73	59	71	66	62	61	64	77	58
59	0.0590	**5**	54	47	64	59	49	76	56	67	54
53	0.0531	**6**	44	61	47	54	63	48	55	49	49
48	0.0478	**7**	44	44	34	40	56	43	43	35	48
43	0.0430	**8**	52	45	53	44	43	42	39	46	52
39	0.0387	**9**	42	46	42	36	43	35	42	25	37
35	0.0349	**10**	30	43	39	42	32	35	41	28	32
31	0.0314	**11**	36	39	25	33	37	30	33	30	31
28	0.0282	**12**	28	24	40	30	33	26	30	21	36
25	0.0254	**13**	22	28	29	34	25	24	15	20	28
23	0.0229	**14**	26	21	14	25	22	14	26	19	22
21	0.0206	**15**	20	26	18	27	27	13	21	12	11
19	0.0185	**16**	23	17	16	16	20	18	21	19	15
17	0.0167	**17**	10	16	12	15	15	19	13	15	18
15	0.0150	**18**	13	10	13	13	12	17	26	18	21
14	0.0135	**19**	8	17	15	11	15	27	11	12	8
12	0.0122	**20**	12	14	12	12	10	12	13	10	8
11	0.0109	**21**	9	5	10	11	14	14	10	10	8
10	0.0098	**22**	10	9	8	8	10	18	9	13	14
9	0.0089	**23**	13	11	9	8	8	9	12	12	11
8	0.0080	**24**	2	7	7	6	9	6	7	4	11
72	0.0718	**25+**	72	69	72	70	62	62	62	84	66

POKER TEST

Dice	df=4				
die1			**die2**		
Chi-Square:	**3.71**		Chi-Square:	**0.88**	
Probaability:	**0.4473**		Probability:	**0.9277**	
die3			**die4**		
Chi-Square:	**7.84**		Chi-Square:	**3.00**	
Probability:	**0.0978**		Probability:	**0.5579**	

Pi	df=4
Chi-Square:	**24.36**
Probability:	**0.0001**

Calculator	df=4	
	Chi-Square:	Probability:
calc0	**3.19**	**0.5269**
calc1	**0.64**	**0.9591**
calc2	**0.64**	**0.9591**
calc4	**1.47**	**0.8325**
calc10	**1.77**	**0.7775**
calc100	**3.82**	**0.4303**

Java	df=4	
	Chi-Square:	Probability:
java1	**3.71**	**0.4467**
java2	**11.73**	**0.0195**
java3	**10.19**	**0.0374**
java4	**3.40**	**0.4932**
java5	**7.37**	**0.1176**
java6	**9.98**	**0.0408**
java7	**0.76**	**0.9440**
java8	**12.77**	**0.0125**
java9	**2.97**	**0.5632**

Frequency Summaries for Poker Groupings:

'Poker' in the charts refers to the category of grouping for a subsequence of five consecutive integers. The category name is how many different values there were in the subsequence of length five.

Dice

Expected	**Probability**	**Poker**	**die1**	**die2**	**die3**	**die4**
0	0.0008	**1**	0	0	0	0
29	0.0579	**2**	36	28	23	26
193	0.3858	**3**	200	189	172	196
231	0.4630	**4**	225	239	247	241
46	0.0926	**5**	39	44	58	37

Pi

Expected	**Probability**	**Poker**	**Pi**
0	0.0001	**1**	2
22	0.0135	**2**	21
287	0.1800	**3**	260
804	0.5040	**4**	825
483	0.3024	**5**	488

Calculator

Expected	**Probability**	**Poker**	**calc0**	**calc1**	**calc2**	**calc4**	**calc10**	**calc100**
0	0.0001	**1**	0	0	0	0	0	0
11	0.0135	**2**	13	13	10	14	7	10
143	0.1800	**3**	126	143	138	149	143	123
401	0.5040	**4**	416	403	411	398	411	410
241	0.3024	**5**	241	237	237	235	235	253

Java											
Exp.	**Prob.**	**Poker**	**java1**	**java2**	**java3**	**java4**	**java5**	**java6**	**java7**	**java8**	**java9**
0	0.0001	**1**	0	0	0	0	0	0	0	0	0
13	0.0135	**2**	9	6	20	9	17	8	11	16	14
179	0.1800	**3**	196	196	157	164	151	150	175	215	179
502	0.5040	**4**	503	463	485	517	503	540	509	503	525
301	0.3024	**5**	288	331	334	306	325	298	301	262	278

SURVEY DATA

The first column in the chart refers to the survey number. '1' corresponds to survey1 and so on. Surveys 90-125 were given the survey with the second wording, although the sequences from differently worded surveys did not perform statistically differently over the various tests. Survey34 and survey91 were those that contained entries of all 4s.

	Frequency Test		Serial Test (2)		Seial Test (3)		Gap Test		Poker Test	
	Chi-Sq	Prob	Chi-Sq	Prob	Chi-Sq	Prob	Chi-Sq	Prob	Chi-Sq	Prob
1	4.00	0.9114	111.00	0.1928	1044.00	0.1570	18.80	0.0009	52.30	0.0000
2	2.60	0.9781	157.00	0.0002	1184.00	0.0000	47.34	0.0000	149.32	0.0000
3	21.20	0.0118	181.00	0.0000	1444.00	0.0000	7.47	0.1129	15.67	0.0035
4	3.20	0.9558	237.00	0.0000	1624.00	0.0000	43.56	0.0000	92.66	0.0000
5	4.80	0.8514	117.00	0.1046	1004.00	0.4496	7.00	0.1357	6.79	0.1477
6	14.80	0.0966	117.00	0.1046	1064.00	0.0751	24.41	0.0001	36.08	0.0000
7	8.40	0.4944	247.00	0.0000	1464.00	0.0000	4.70	0.3190	14.29	0.0064
8	6.60	0.6787	199.00	0.0000	1404.00	0.0000	56.44	0.0000	159.90	0.0000
9	7.80	0.5544	133.00	0.0128	1064.00	0.0751	25.58	0.0000	93.20	0.0000
10	10.20	0.3345	163.00	0.0001	1204.00	0.0000	28.23	0.0000	85.42	0.0000
11	9.80	0.3669	287.00	0.0000	2244.00	0.0000	19.45	0.0006	49.07	0.0000
12	3.00	0.9643	127.00	0.0304	1044.00	0.1570	61.68	0.0000	176.57	0.0000
13	4.20	0.8978	139.00	0.0050	1044.00	0.1570	61.68	0.0000	60.64	0.0000
14	3.80	0.9241	119.00	0.0835	964.00	0.7815	18.03	0.0012	4.92	0.2954
15	16.00	0.0669	151.00	0.0006	1044.00	0.1570	7.19	0.1264	6.74	0.1501
16	7.20	0.6163	107.00	0.2739	944.00	0.8923	15.31	0.0041	75.43	0.0000
17	2.40	0.9835	157.00	0.0002	1324.00	0.0000	27.10	0.0000	138.72	0.0000
18	14.20	0.1154	147.00	0.0013	1124.00	0.0034	68.89	0.0000	54.71	0.0000
19	13.20	0.1538	143.00	0.0025	1124.00	0.0034	14.95	0.0048	78.91	0.0000
20	4.60	0.8677	155.00	0.0003	1264.00	0.0000	33.82	0.0000	139.16	0.0000
21	9.20	0.4190	161.00	0.0001	1284.00	0.0000	44.77	0.0000	69.90	0.0000
22	9.40	0.4012	143.00	0.0025	1224.00	0.0000	29.51	0.0000	35.11	0.0000
23	4.80	0.8514	117.00	0.1046	1044.00	0.1570	24.84	0.0001	78.06	0.0000
24	7.40	0.5955	97.00	0.5381	944.00	0.8923	38.74	0.0000	111.23	0.0000
25	0.00	1.0000	721.00	0.0000	6484.00	0.0000	44.01	0.0000	360868	0.0000
26	2.40	0.9835	147.00	0.0013	1244.00	0.0000	218.89	0.0000	70.29	0.0000
27	8.80	0.4559	173.00	0.0000	1364.00	0.0000	37.82	0.0000	68.77	0.0000
28	8.20	0.5141	193.00	0.0000	1364.00	0.0000	60.51	0.0000	87.98	0.0000
29	4.60	0.8677	121.00	0.0659	1004.00	0.4496	29.36	0.0000	96.27	0.0000
30	6.60	0.6787	113.00	0.1590	1084.00	0.0311	48.45	0.0000	41.33	0.0000

31	6.20	0.7197	113.00	0.1590	1004.00	0.4496	16.63	0.0023	124.72	0.0000
32	16.90	0.0503	215.00	0.0000	1500.00	0.0000	33.38	0.0000	67.01	0.0000
33	5.20	0.8165	147.00	0.0013	1124.00	0.0034	46.31	0.0000	27.74	0.0000
34	900.00	0.0000	9703.00	0.0000	95944.00	0.0000	10.85	0.0283	921508	0.0000
35	13.40	0.1453	107.00	0.2739	904.00	0.9854	253.20	0.0000	7.27	0.1223
36	10.00	0.3505	191.00	0.0000	1344.00	0.0000	4.32	0.3645	69.90	0.0000
37	8.20	0.5141	131.00	0.0173	1044.00	0.1570	27.02	0.0000	102.00	0.0000
38	9.20	0.4190	165.00	0.0000	1244.00	0.0000	44.87	0.0000	84.80	0.0000
39	41.80	0.0000	197.00	0.0000	1184.00	0.0000	24.80	0.0001	4.08	0.3952
40	5.00	0.8343	125.00	0.0398	1184.00	0.0000	12.42	0.0145	90.86	0.0000
41	2.20	0.9879	123.00	0.0514	1144.00	0.0009	41.01	0.0000	97.24	0.0000
42	7.80	0.5544	101.00	0.4252	1044.00	0.1570	29.38	0.0000	2.80	0.5920
43	17.20	0.0457	105.00	0.3209	1024.00	0.2845	12.83	0.0121	5.50	0.2395
44	2.20	0.9879	93.00	0.6509	964.00	0.7815	4.62	0.3291	44.15	0.0000
45	4.20	0.8978	175.00	0.0000	1324.00	0.0000	18.23	0.0011	119.07	0.0000
46	200.00	0.0000	883.00	0.0000	9524.00	0.0000	36.96	0.0000	81.71	0.0000
47	3.80	0.9241	145.00	0.0018	1084.00	0.0311	193.47	0.0000	43.35	0.0000
48	14.00	0.1223	163.00	0.0001	1244.00	0.0000	21.32	0.0003	93.20	0.0000
49	7.60	0.5749	109.00	0.2311	1104.00	0.0111	30.41	0.0000	28.07	0.0000
50	4.20	0.8978	153.00	0.0004	1144.00	0.0009	13.04	0.0111	102.76	0.0000
51	1.80	0.9942	131.00	0.0173	1044.00	0.1570	36.86	0.0000	106.22	0.0000
52	57.20	0.0000	421.00	0.0000	3784.00	0.0000	31.49	0.0000	33141.52	0.0000
53	13.60	0.1373	137.00	0.0069	1104.00	0.0111	71.85	0.0000	89.25	0.0000
54	2.20	0.9879	135.00	0.0095	1224.00	0.0000	36.84	0.0000	111.23	0.0000
55	2.60	0.9781	135.00	0.0095	1044.00	0.1570	47.65	0.0000	64.62	0.0000
56	5.40	0.7981	99.00	0.4811	1004.00	0.4496	25.44	0.0000	73.13	0.0000
57	0.60	0.9999	103.00	0.3715	944.00	0.8923	21.50	0.0003	80.53	0.0000
58	1.70	0.9954	181.00	0.0000	1340.00	0.0000	34.31	0.0000	96.75	0.0000
59	7.20	0.6163	117.00	0.1046	1004.00	0.4496	51.96	0.0000	76.83	0.0000
60	7.80	0.5544	155.00	0.0003	1124.00	0.0034	26.44	0.0000	92.66	0.0000
61	5.80	0.7598	119.00	0.0835	1044.00	0.1570	25.38	0.0000	54.68	0.0000
62	5.60	0.7792	91.00	0.7043	1004.00	0.4496	25.20	0.0000	22.83	0.0001
63	16.60	0.0554	159.00	0.0001	1084.00	0.0311	13.51	0.0090	102.00	0.0000
64	3.00	0.9643	189.00	0.0000	1344.00	0.0000	39.04	0.0000	51.87	0.0000
65	10.80	0.2897	185.00	0.0000	1384.00	0.0000	17.09	0.0019	67.83	0.0000
66	7.20	0.6163	87.00	0.8001	924.00	0.9561	27.33	0.0000	6.14	0.1888
67	11.00	0.2757	149.00	0.0009	1104.00	0.0111	3.25	0.5173	111.23	0.0000
68	9.40	0.4012	141.00	0.0036	1144.00	0.0009	49.24	0.0000	69.90	0.0000
69	0.20	1.0000	75.00	0.9655	944.00	0.8923	31.45	0.0000	3.15	0.5323
70	15.80	0.0712	201.00	0.0000	1524.00	0.0000	3.23	0.5197	1036.04	0.0000
71	2.60	0.9781	115.00	0.1297	1144.00	0.0009	11.54	0.0211	123.49	0.0000
72	2.60	0.9781	167.00	0.0000	1264.00	0.0000	38.87	0.0000	81.03	0.0000
73	2.60	0.9781	119.00	0.0835	1024.00	0.2845	48.71	0.0000	78.06	0.0000
74	22.00	0.0089	265.00	0.0000	1564.00	0.0000	33.59	0.0000	48.25	0.0000
75	25.40	0.0026	225.00	0.0000	1404.00	0.0000	25.10	0.0000	56.83	0.0000
76	0.80	0.9998	93.00	0.6509	964.00	0.7815	41.20	0.0000	106.94	0.0000
77	2.00	0.9915	109.00	0.2311	1144.00	0.0009	46.35	0.0000	68.94	0.0000
78	6.20	0.7197	163.00	0.0001	1284.00	0.0000	20.90	0.0003	51.41	0.0000
79	3.40	0.9463	125.00	0.0398	1084.00	0.0311	14.71	0.0053	21.09	0.0003

80	8.80	0.4559	149.00	0.0009	1144.00	0.0009	8.73	0.0683	134.24	0.0000
81	7.40	0.0000	167.00	0.0000	1344.00	0.0000	50.93	0.0000	165.28	0.0000
82	19.40	0.0220	179.00	0.0000	1264.00	0.0000	24.61	0.0001	130.01	0.0000
83	10.40	0.3191	195.00	0.0000	1744.00	0.0000	27.86	0.0000	3689.38	0.0000
84	6.40	0.6993	105.00	0.3209	1024.00	0.2845	13.52	0.0090	68.10	0.0000
85	2.00	0.9915	115.00	0.1297	1104.00	0.0111	15.98	0.0030	139.16	0.0000
86	1.00	0.9994	89.00	0.7544	1004.00	0.4496	65.58	0.0000	33.69	0.0000
87	26.20	0.0019	177.00	0.0000	1164.00	0.0002	33.19	0.0000	17.38	0.0016
88	4.20	0.8978	199.00	0.0000	1384.00	0.0000	11.96	0.0177	128.54	0.0000
89	20.20	0.0167	141.00	0.0036	1144.00	0.0009	37.58	0.0000	14.17	0.0068
90	4.00	0.9114	235.00	0.0000	1904.00	0.0000	15.52	0.0037	46.07	0.0000
91	880.20	0.0000	9507.00	0.0000	94004.00	0.0000	18.72	0.0009	902409	0.0000
92	9.20	0.4190	129.00	0.0230	1184.00	0.0000	248.07	0.0000	92.28	0.0000
93	13.60	0.1373	131.00	0.0173	1064.00	0.0751	29.53	0.0000	62.12	0.0000
94	11.00	0.2757	145.00	0.0018	1164.00	0.0002	27.32	0.0000	37.80	0.0000
95	12.60	0.1816	151.00	0.0006	1164.00	0.0002	19.91	0.0005	155.28	0.0000
96	30.00	0.0004	221.00	0.0000	1464.00	0.0000	12.80	0.0123	19.70	0.0006
97	7.60	0.5749	133.00	0.0128	1104.00	0.0111	22.85	0.0001	2.12	0.7131
98	3.80	0.9241	157.00	0.0002	1144.00	0.0009	3.12	0.5385	23.87	0.0001
99	7.40	0.5955	257.00	0.0000	1884.00	0.0000	3.21	0.5236	110.55	0.0000
100	17.80	0.0376	167.00	0.0000	1204.00	0.0000	57.85	0.0000	110.55	0.0000
101	5.20	0.8165	185.00	0.0000	1544.00	0.0000	45.67	0.0000	1351.93	0.0000
102	12.60	0.1816	149.00	0.0009	1144.00	0.0009	3.65	0.4561	124.72	0.0000
103	2.40	0.9835	127.00	0.0304	1164.00	0.0002	62.71	0.0000	67.01	0.0000
104	4.00	0.9114	111.00	0.1928	1064.00	0.0751	36.86	0.0000	67.01	0.0000
105	15.60	0.0757	163.00	0.0001	1184.00	0.0000	29.05	0.0000	84.34	0.0000
106	7.60	0.5749	161.00	0.0001	1264.00	0.0000	38.78	0.0000	13.87	0.0077
107	16.60	0.0554	169.00	0.0000	1144.00	0.0009	7.61	0.1069	48.25	0.0000
108	2.60	0.9781	149.00	0.0009	1344.00	0.0000	26.39	0.0000	97.24	0.0000
109	16.60	0.0554	151.00	0.0006	1064.00	0.0751	55.51	0.0000	51.41	0.0000
110	16.40	0.0590	139.00	0.0050	1144.00	0.0009	26.12	0.0000	55.38	0.0000
111	4.20	0.8978	133.00	0.0128	1064.00	0.0751	11.77	0.0191	133.43	0.0000
112	7.20	0.6163	123.00	0.0514	1224.00	0.0000	41.63	0.0000	80.74	0.0000
113	24.80	0.0032	225.00	0.0000	1524.00	0.0000	16.65	0.0023	460.52	0.0000
114	7.40	0.5955	143.00	0.0025	1124.00	0.0034	8.52	0.0743	65.11	0.0000
115	16.40	0.0590	195.00	0.0000	1284.00	0.0000	28.30	0.0000	25.38	0.0000
116	10.40	0.3191	123.00	0.0514	1124.00	0.0034	17.32	0.0017	66.35	0.0000
117	7.20	0.6163	111.00	0.1928	1064.00	0.0751	32.34	0.0000	88.25	0.0000
118	4.60	0.8677	103.00	0.3715	1044.00	0.1570	30.69	0.0000	62.52	0.0000
119	6.20	0.7197	111.00	0.1928	1024.00	0.2845	20.82	0.0003	114.98	0.0000
120	3.80	0.9241	125.00	0.0398	1104.00	0.0111	35.84	0.0000	124.72	0.0000
121	5.60	0.7792	141.00	0.0036	984.00	0.6264	31.78	0.0000	82.49	0.0000
122	15.60	0.0757	109.00	0.2311	1064.00	0.0751	29.19	0.0000	4.10	0.3920
123	7.80	0.5544	87.00	0.8001	964.00	0.7815	6.34	0.1748	83.45	0.0000
124	6.60	0.6787	119.00	0.0835	1004.00	0.4496	41.59	0.0000	20.85	0.0003
125	4.40	0.8832	181.00	0.0000	1324.00	0.0000	9.33	0.0534	138.72	0.0000

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