A COGNITIVE MODEL OF INDIVIDUAL VARIATION IN MATHEMATICS ACHIEVEMENT IN COLLEGE STUDENTS REFERRED FOR LEARNING DIFFICULTIES

By

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(Under the Direction of K. Noël Gregg and Randy W. Kamphaus)

ABSTRACT

This study analyzed, via Structural Equation Modeling (SEM) the contribution of auditory long-term memory, auditory working memory, and reasoning to mathematics calculation and mathematics reasoning in a sample of 401 college-age students referred for assessment of possible learning difficulties. Three nested models were examined in an attempt to determine the differential effects of cognitive processes on mathematics calculation and mathematics reasoning. Measures were chosen from a comprehensive neuropsychological battery administered to individuals seeking evaluations at The University of Georgia – Regents' Center for Learning Disorders (UGA-RCLD). Auditory long-term memory, auditory working memory, and reasoning contributed 14% and 44 % of the variance to mathematics calculation and mathematics reasoning, respectively. Nested model analyses failed to demonstrate differential contributions of variance to mathematics calculation and mathematics reasoning.

INDEX WORDS: Math, Mathematics Achievement, Calculation, Cognitive Ability, Cognitive Processing, Long-term Memory, Reasoning, Working Memory

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A Dissertation Submitted to the Graduate Faculty of The University of Georgia in Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2005

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December 2005

DEDICATION

This dissertation is dedicated to the memory of my parents.

ACKNOWLEDGEMENTS

I am indebted to my dissertation committee for their generous time-commitment and support, and in particular, to my chairs, Drs. Gregg and Kamphaus, and to Dr. Bandalos, without whose help I could not have undertaken this project.

I would also like to express my appreciation to Drs. Davis and Miller and to Chris Coleman at the Learning Disabilities Center (LDC). It was because of their passion for the study of learning disabilities that I became interested in the topic of this dissertation, and I am grateful for the knowledge that they shared with me over the three years that I worked at the LDC.

I would also like to thank my friends at UGA who helped me through many professional and personal obstacles, particularly Jamilia Blake, Lauren Jones, and Kadi Moon.

TABLE OF CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Background

Due to the importance of mathematical skills for college and for adequate adult functioning, mathematics courses are required and are a major component of both the primary and secondary grades of education (Patton, Cronin, & Bassett, 1998). However, according to the US Department of Education, only 20 percent of twelfth-graders, nationally, were proficient in mathematics, based on testing conducted in 2000 (http://www.ed.gov/nclb/methods/math/math.html/).

The increasing demands of a technologically advanced society create a greater need for mathematics knowledge and skills, and individuals without these skills may be incapable of finding lucrative and personally rewarding positions (Fourqurean, Meisgeier, Swank, & Williams, 1991). Furthermore, many of today's technology jobs require advanced mathematics skills and training beyond high school. Well-paying jobs in general, and not only those in science and technology fields, often require some college training (Fourqurean et al., 1991), and the general lack of mathematics skills in the adult population puts this country at a disadvantage when competing globally (Battista, 1999).

The mathematics achievement of United States citizens has been a concern of the public and policy makers for some time, resulting in the recent legislation aimed at addressing this problem. This concern has manifested in a number of trends and legislative remedies including increasing "high stakes testing" particularly in recent years with the passing of the No Child Left Behind Act (NCLB) in 2001 (Miksch, 2003). In an effort to ensure that all students are academically proficient, NCLB mandates that each state administer annual assessments in language arts and mathematics in grades three through eight and at least once in grades ten through twelve by the year 2005 (Coleman, Palmer, & Garrett, 2003). A specific concern is that these tests are not based on sound scientific research regarding an understanding of these subject areas or appropriate expectations for mathematical understanding at certain grade levels (Battista, 1999). Penalties for failure of high stakes tests have severe implications for older students. The cumulative nature of mathematics knowledge makes passing from one grade to the next unlikely when high stakes testing in middle or high school relies on information not learned in previous years (Ackerman & Dykman, 1995). Older students at risk for not passing are at an increased risk of dropping out of school either after failing the exam or to avoid taking it (Miksch, 2003), and students who either drop out or are unable to pass the graduation exit exam leave school with even poorer employment prospects having been rendered unable to pursue postsecondary educational avenues, as a high school diploma is often mandatory for employment, on-the-job training, college, and the military (Albrecht & Joles, 2003; Miksch, 2003).

These substantial societal implications emphasize the need to better understand the cognitive abilities that contribute to mathematics achievement, especially since such an understanding could contribute information necessary to design better formative and summative achievement measures. Relatively little is known about the cognitive processes that impact the higher-order mathematics skills of adults (Geary, 2005). Yet, between five and eight percent of elementary school children demonstrate cognitive deficits that interfere with mathematics achievement (Geary, 2005). Identifying a model of cognitive variables contributing to variation in mathematics skills in adults will contribute to a better understanding of cognitive and linguistic processes impacting mathematics in a college population and help in developing accommodations for those students with learning disabilities impacting mathematics.

By comparison, while extensive studies have been conducted on the cognitive and linguistic processes underlying reading decoding, relatively little has been accomplished in regard to uncovering the cognitive processes that contribute to variability in mathematics performance (Robinson, Menchetti, & Torgesen, 2002). Working memory, linguistic abilities, and other individual cognitive abilities are known contributors to reading achievement. Similar progress toward defining the cognitive abilities underlying mathematics performance and their interactive role in promoting or retarding mathematics achievement, however, has been far less rapid.

Identification of the relationship between cognitive or linguistic processes and mathematics performance could be identified in the same way that studies focusing on reading decoding have supported a relationship between the cognitive and linguistic processes and reading achievement (Padget, 1998). The same cognitive and linguistic processes implicated in reading achievement may also explain individual variation in mathematics achievement as well (Fletcher, 2005; Geary, 1993). While research has been conducted with children identifying deficits in cognitive processes related to poor mathematics achievement, studies regarding the mathematics skills of adults have generally focused on mathematics failure rather than identifying the cognitive processes associated with individual variation in mathematics performance across skill levels (Cirino, Morris, & Morris, 2002). Currently, models of cognitive processing variables accounting for the heterogeneity of mathematics performance in adults are lacking (Cirino et al., 2002). Further research investigating a cognitive model accounting for individual variation in mathematics performance in adults is needed to provide differential intervention and/or accommodations. The testing of models of cognitive processes that undergird mathematics achievement is a needed first step, one that may be studied in a population at risk for achievement problems, because this population provides a range of proficiency levels that may be ideal for examining individual differences in achievement (Cirino et al., 2002).

Previous research studies investigating mathematics skills have been largely applied to child populations, and studies regarding adult learners of mathematics and science have been largely absent (Croucher & Houssart, 1997; Ashcraft, 1992). Thus, much of the current insight into these relationships is necessarily gleaned from the child literature. Geary, for example, proposed a cognitive model describing a profile of a subset of children demonstrating specific learning disabilities interfering with mathematics performance (Geary, 2005). A review of existing studies, such as those by Geary and his colleagues, in addition to research conducted with patients with focal brain lesions, is provided to identify commonalities among the studies regarding cognitive processes that likely contribute to individual differences in mathematics performance in older adolescents and adults. Despite the difficulty with mathematics that many adolescents and adults experience, the majority of educational research conducted with college-level populations has focused on the humanities and social science fields (Croucher & Houssart, 1997).

Research points to developmental growth and decline of cognitive and linguistic abilities across the lifespan that are related to mathematics skills (McGrew & Woodcock, 2001). Several notable differences between the literature regarding children and adults exist (Cirino et al., 2002). For example, whereas typical elementary school children use counting strategies in order to solve simple mathematics problems (e.g., 5 + 4), typical adults rely almost exclusively on retrieval strategies and only rely on counting when retrieval fails to produce a correct answer (Ashcraft, 1992). Despite this reliance on retrieval, some evidence suggests that adults may utilize other cognitive processes to an equal or greater degree, such as working memory and reasoning (Floyd, Evans, & McGrew, in press). Additionally, the support for a visual-spatial factor underlying mathematical skills in adults has varied widely across studies (Cirino et al., 2002). Thus, different cognitive abilities are employed to differing degrees based on age. This study will attempt to examine whether children and adults are cognitively and linguistically similar with regard to mathematics achievement.

The majority of research regarding the relationship between cognitive processes and mathematics achievement has focused on five cognitive abilities: long-term memory (Geary, 1993; Sokol, McCloskey, Cohen, & Aliminosa, D. 1991; Zentall, 1990), working memory (Gathercole & Pickering, 2000; Keeler & Swanson, 2001; McLean & Hitch, 1999; Swanson, 1994), reasoning (Ablard & Tissot, 1998; Cifarelli, 1998; Rourke & Conway, 1997), visual-spatial ability (Assel, Landry, Swank, Smith, & Steelman, 2003; Harnadek & Rourke, 1994; Hermelin & O'Connor, 1986; McLeod & Crump, 1978), and verbal ability (Abedi & Lord, 2001; Pennington, 1991). The research regarding the relationships between these variables and mathematics achievement provides a basis to form an exploratory, theoretically-based model of cognitive processing variables related to mathematics achievement in older adolescents and adults that may be tested via structural equation modeling (SEM).

The purpose of the current study was to develop and test an exploratory model of cognitive and linguistic processes contributing to individual variability in mathematics achievement in a post-secondary sample of clinic-referred college students. This research systematically examined the cognitive processes that explain or predict achievement and provided external validity for specific cognitive variables predicting mathematics achievement in young adults (McGrew & Flanagan, 1997). It was also designed to contribute to an emerging body of literature regarding the relationships between cognitive and linguistic processes and mathematics achievement by exploring the relationships between cognitive processing variables related to two specific areas of mathematics achievement, mathematics calculation and mathematics reasoning, in a college population. In addition, the study will contribute to understanding effective accommodations and interventions for populations with different learning profiles.

Exploratory Model of Cognitive Processes

The cognitive variables chosen for the model studied include long-term memory, working memory, reasoning, visual-spatial ability, and vocabulary knowledge, as defined by subtests on various cognitive ability measures. Mathematics achievement was defined by the Math Calculation Skills (Calculation and Math Fluency subtests) and Math Reasoning (Applied Problems and Quantitative Concepts subtests) clusters of the Woodcock-Johnson III Tests of Achievement (WJ III: Woodcock, McGrew, & Mather, 2001). Each of the cognitive domains was selected based on theoretical support provided in the literature and represent the endogenous latent variables of the model tested in this study. The hypothesis tested is that the cognitive processes and language abilities identified predict mathematics calculation and problem-solving as measured by subtests of the WJ III in a best-fitting model identified via structural equation modeling (SEM). Mathematics achievement and each of the cognitive and language abilities included have been described in various ways, but in this study, were chosen based on support from previous studies conducted (Geary & Widaman,1992; Gersten, Jordan, & Flojo, 2005; Keeler & Swanson, 2001; McGrew & Hessler, 1995; Swanson, 1994) with adolescents and adults and on practical consideration of available measures. Specific terms are provided in Table 1 to assist the reader in understanding the latent variables included in the model.

Table 1. Terms and Definitions

Latent Variable	Definition
Long-term Memory	Ability to store information and retrieve it later through association (McGrew & Flanagan, 1997).
Auditory Working Memory	The conscious process of mentally replaying experiences, actions, or mental processes and considering their results or how they are composed (Battista, 1999).
Reasoning	Thinking in a logical manner, formulating and testing conjectures, making sense of things, and forming and justifying judgments, inferences, and conclusions (Battista, 1999).
Visual-spatial Ability	The ability to perceive the visual world and recreate aspects of one's visual experience (Gardner, 1983).
Vocabulary	Measure of language development using knowledge of definitions of words
Mathematics Calculation	Measure of computational skills and automaticity with basic mathematical facts (Mather & Woodcock, 2001)
Mathematics Reasoning	Measure of problem solving, analysis, reasoning, and vocabulary relating to mathematical knowledge (Mather & Woodcock, 2001)

Cognitive and Linguistic Variables

It is difficult to differentiate the impact of verbal processes from nonverbal processes on mathematics achievement measures. Mathematics problems rarely contain strictly verbal, or strictly visual, information. Isolation of the verbal and nonverbal cognitive processes that potentially influence mathematical skills is equally challenging. Thus, many of the cognitive variables chosen in the current model include both verbal and visual components. Some researchers propose that individuals experiencing mathematical difficulties do so because of deficits in verbal skills that globally impact achievement across reading and mathematical domains (Fletcher, 2005; Pennington, 1991). Therefore, in the current model, a measure of word knowledge, namely vocabulary skills, was chosen to measure the impact of this variable on mathematics achievement. The use of vocabulary as defined by the Peabody Picture Vocabulary Test – Third Edition (PPVT-III; Dunn & Dunn, 1997) and Wechsler Adult Intelligence Scale – Third Edition (WAIS-III; Wechsler, 1997b) as a measure of verbal skills has been previously supported in the literature (Cirino et al., 2002; Fletcher, 2005) and is used to determine if significant contributions from the other four cognitive processing variables are apparent over and above the contribution of vocabulary skills.

The influence of long-term memory on mathematics achievement has been studied in both children and adults. It is most frequently described in terms of a network retrieval model in which information, including basic mathematics facts (e.g., 4 + 2 = 6), are stored in and retrieved from a memory network (Ashcraft & Battaglia, 1978; Geary, Widaman, Cormier, & Little, 1987; Ashcraft, 1992). Although the source of long-term memory deficits continue to be debated, deficits associated with mathematics performance are generally described as either impairments in the ability to retrieve mathematics facts from long-term memory or difficulties inhibiting the retrieval of irrelevant associations in memory (Geary, 2005). Researchers often examine the role of long-term memory retrieval in mathematics achievement by testing an individual’s ability to retrieve mathematical facts from memory. The current study defines long-term memory in terms of subtests taken from the California Verbal Learning Test – Second Edition (CVLT-II: Delis, Kramer, Kaplan, & Ober, 2000) and Wechsler Memory Scales – III (WMS-III: Wechsler, 1997c) in order to measure auditory-verbal and visual-nonverbal tasks unrelated to mathematics.

While some studies have included both visual and auditory working memory in investigating the relationship between working memory and mathematics achievement (Wilson & Swanson, 2001), this study includes auditory working memory and visual-spatial skills as two distinct variables. Research suggests that visual working memory may be related to mathematical achievement; however, auditory working memory measures, such as the reverse digit span task, have been more commonly used in studies investigating mathematics achievement and have gained more support in the empirical literature, (Gersten et al., 2005). Thus, the current study includes measures of auditory working memory as defined by selected subtests of the WAIS-III and WJ III. In this study, working memory is distinguished from short-term memory, as only measures requiring the individual to briefly store and mentally manipulate auditory-verbal information were included in the model. Memory span measures that simply require subjects to repeat information after holding it in memory for a brief period of time were excluded, as research appears to support a stronger link between measures of auditory working memory and mathematics achievement than measures of short-term memory and mathematics achievement (Wilson & Swanson, 2001).

Definitions of reasoning ability and terminology vary widely across studies, but most point to an ability to "conceptualize" or to create abstract representations of information presented within mathematics problems (Cifarelli, 1998). Future studies may seek to narrow a definition of reasoning, isolate its constructs, and determine if verbal or visual reasoning tasks better predict mathematics achievement in specific populations. Measures of reasoning were defined using both auditory-verbal and visual-nonverbal measures taken from the WAIS-III and the WJ III.

Visual-spatial ability, as it relates to mathematics achievement, has been one of the most frequently examined cognitive processes in the literature (Assel, et al., 2003). While many early studies suggested a clear relationship between visual-spatial ability and mathematical skills (Rourke & Strang, 1978), more recent studies have failed to support this relationship (Floyd et al., in press). In the current study, visual-spatial skills are represented by measures on the WAIS-III that assess the ability to spatially represent and/or rotate visual information.

While the vast majority of studies have defined mathematics achievement by success in calculation of basic mathematics facts, the current study examines two aspects of mathematics achievement as defined by the WJ III, namely that of mathematics calculation and mathematics reasoning. Support for these separate skills can be found in the standardization studies of the WJ III and in research identifying different types of mathematics achievement in older adolescents and adults (Floyd et al., in press). The separation of these skills seemed particularly relevant in this study, which attempts to examine variation in achievement across a range of skills in a college sample, rather than simply examining the impact of cognitive processing on basic calculation skills.

A Model of Mathematics Achievement

The model tested is based on the hypothesis that all independent variables have a direct influence on mathematics achievement. The hypothesized relationships among latent variables are depicted in Figure 1.

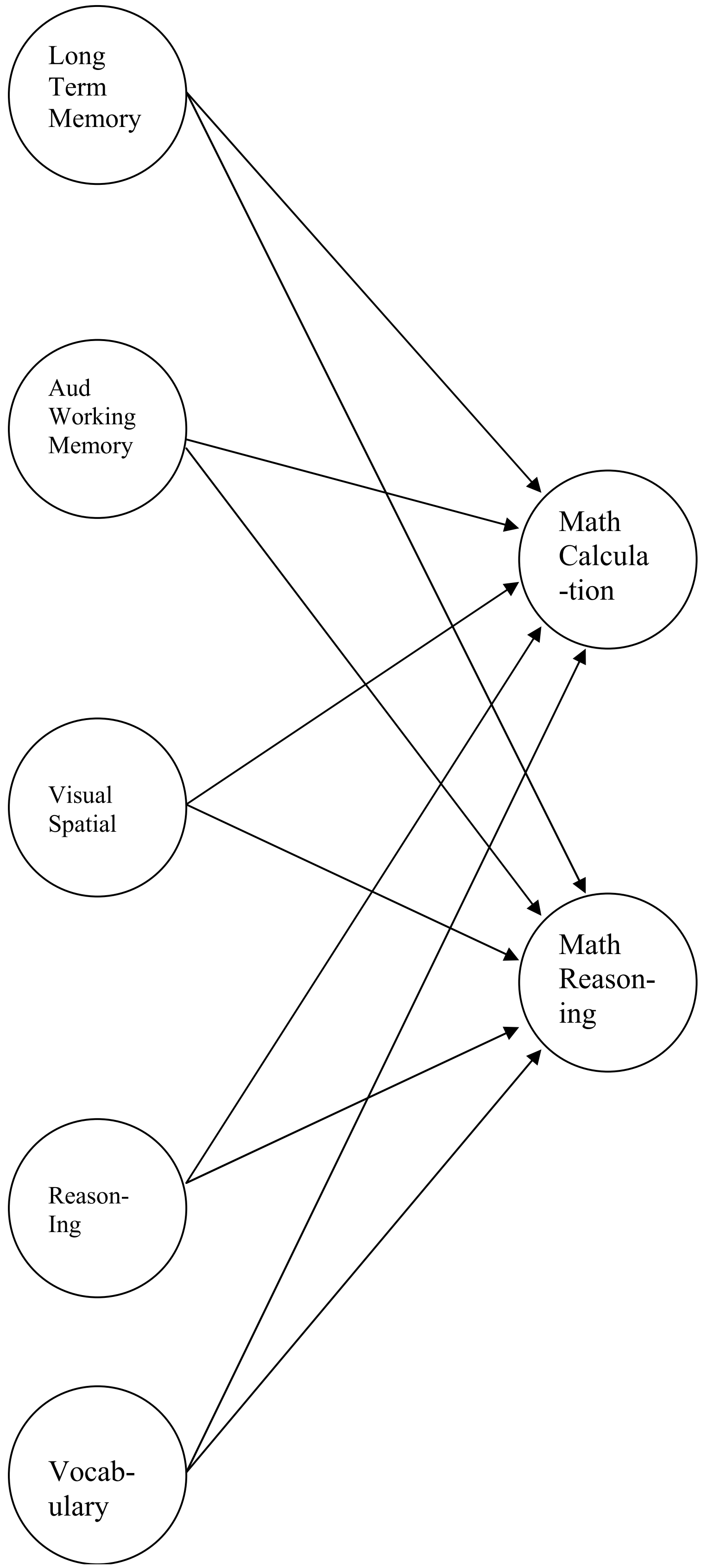

Figure 1. Relationships among Latent Variables

In addition, three nested models, identical to the full model but with paths from each of the exogenous latent variables to the endogenous latent variables set to be equal, were estimated. In this way, it may be determined that the exogenous latent variables have a differential impact on mathematics reasoning and mathematics calculation.

Structural Equation Modeling (SEM) was used to investigate the multivariate relationships among the variables in the current study. As the latent variables (i.e., long-term memory, auditory working memory, reasoning, visual-spatial ability, vocabulary, mathematics calculation, and mathematics reasoning) are not directly observable, SEM includes a measurement model, in addition to the structural model, that depicts the relationships among observed variables, or measurements, that constitute the latent variables. The full model (measurement model + structural model) is presented in Figure 2.

Research Questions

The following research questions were addressed by comparing a series of three nested models to the full structural model. Nested models contained equal paths between one exogenous latent variable at a time and the endogenous latent variables. The overall fit of the models and the influence of each exogenous latent variable were assessed in a sample of college students at risk for learning problems.

1. Do the cognitive and linguistic abilities of long-term memory, auditory working memory, reasoning, visual-spatial ability, and vocabulary contribute significant variance to mathematics calculation and mathematics reasoning?

2. What are the strengths of the relationships between the exogenous latent variables (i.e., long-term memory, auditory working memory, reasoning, visual-spatial ability, and vocabulary) and endogenous latent variables (i.e., mathematics calculation and mathematics reasoning)?

3. Do the exogenous latent variables explain differential variance in the endogenous latent variables of mathematics calculation and mathematics reasoning?

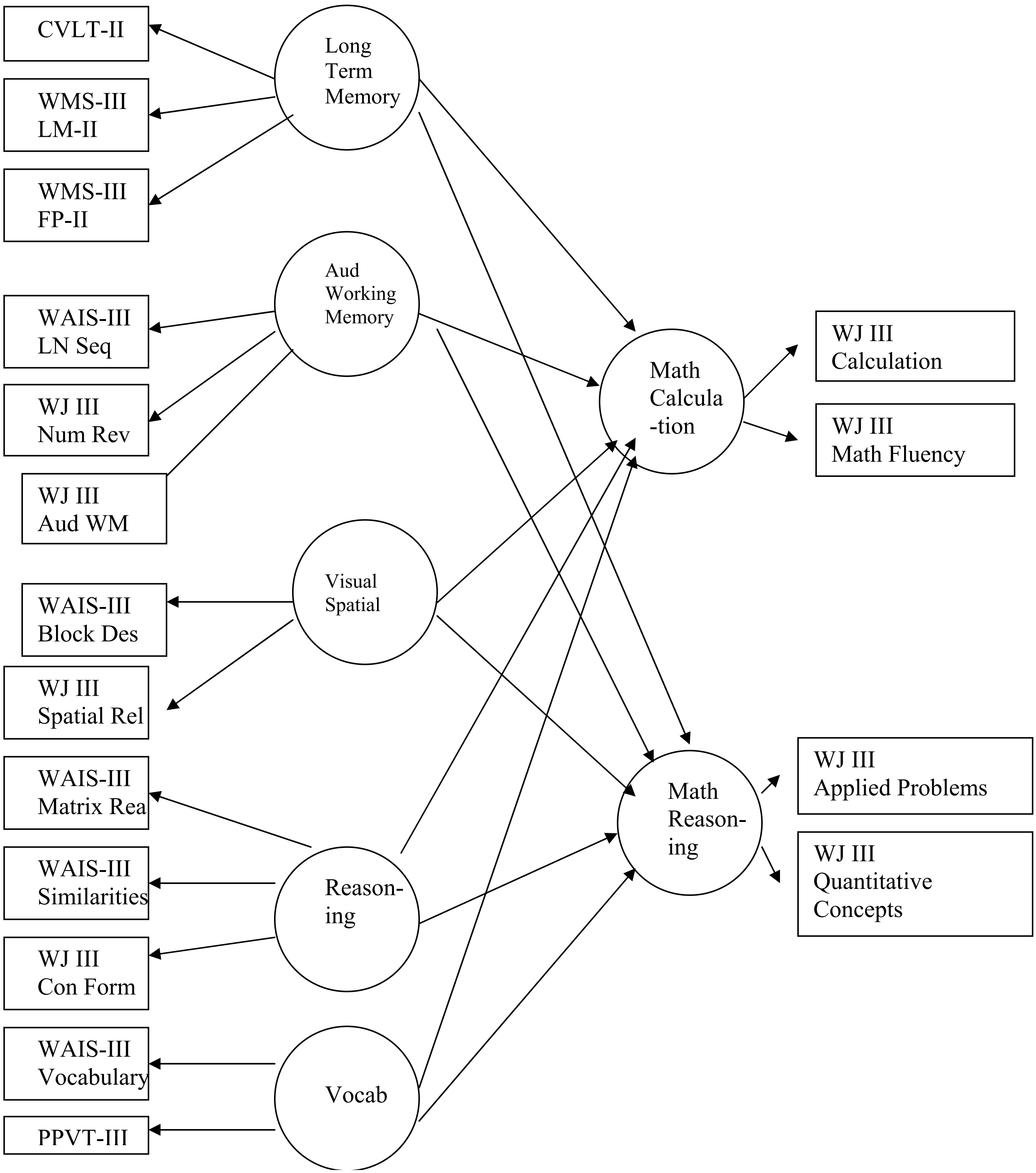

Figure 2. Structural and Measurement Model

CHAPTER 2

LITERATURE REVIEW

Mathematics underachievement can limit an adult's academic and vocational success. Studies investigating the etiologies of mathematics difficulties in older adolescents and adults are needed for the purpose of identifying better interventions and accommodations. One method of investigating the possible causes of these difficulties is to study the relationships of cognitive and linguistic processes to mathematics performance in the areas of calculation and problem solving (Cirino et al., 2002). Cognitive and linguistic processes may be defined as underlying achievement constructs that indicate a person's level of cognitive functioning in specific areas (McGrew & Flanagan, 1997). In this study, these constructs are represented by long-term memory, reasoning, auditory working memory, visual-spatial skills, and vocabulary. These abilities can be used as a guide to interpret how specific types of information are processed and can be used to predict achievement. While the adult neuropsychological literature has suggested memory and visual-spatial deficits in patients with focal brain lesions, and other investigations into the relationship between cognitive processing and mathematics achievement have implicated reasoning, working memory, and long-term memory, little empirical evidence exists that supports a model of cognitive variables linked to typical mathematics achievement in adults. The current study proposes a model of cognitive and linguistic processes for young adults at risk for poor achievement.

The Development of Mathematical Skills

Counting is the basis from which individuals learn mathematics skills, both procedurally and conceptually (Geary, 2005; Gersten, et al., 2005). Learning to count usually begins at approximately age four and is guided by five implicit principles (Gallistel & Gelman, 1992). These principles are referred to as the essential features of counting, and implicit adherence to them is necessary for accurate counting. The first principle is one-one correspondence, which is defined as assigning one, and only one, word (e.g., one, two, three, etc.) to each counted item. The second principle is stable order, which means that the order of the assigned words must not vary across counted sets. The third principle, cardinality, means that the value of the final word represents the quantity of the total set of items. Fourth is abstraction, which indicates that any object can be counted. The fifth, and final, essential counting principle is order-irrelevance, which states that items in a set can be counted in any sequence (Gelman & Gallistel, 1978). Several researchers have assessed the degree to which children attend to the essential features of counting, and also, make errors by attending to unessential features of counting. Children who make frequent errors by attending to unessential features of counting (e.g., believing that it is necessary to begin counting from the left end of a series of objects) understand counting as a rote activity but may not understand the conceptual nature of counting (Geary, Hoard, & Hamson, 1999).

The seminal work by Gelman and Gallistel (1978) outlines the evolutionary theory on which the essential features of counting are based. According to Gelman and Gallistel's principles first theory, evidence that humans not only count in a rote fashion but use cognitive processes, such as reasoning and memory to conceptually represent counting, is inherent in animal studies. Gallistel and Gelman (1992) cite a study by Meck and Church (1983) describing a system in which animals store numerical information in memory and compare new numerical information to the stored information, in a manner that might be analogous to working memory, in order to make decisions about the environment. “If the magnitudes that represent numerosities enter into processes equivalent to ordering, addition, subtraction, multiplication, and division, then, by our definition, the animals reason arithmetically” (Gallistel and Gelman, 1992, p. 53). According to Gallistel and Gelman (1992), these preverbal conceptualizations of number are then incorporated into a “bidirectional mapping system” when humans learn to count. Other preverbal mechanisms that lay the foundation for later mathematical skills may include infants’ manipulation of objects of varying distances, which aids in the development of spatial conceptualization (Sutaria, 1985).

Human beings not only rely on preverbal, or primary abilities, however, but must learn to make analogies between preverbal concepts of numerosity and learned number words and then incorporate these number words into knowledge of digit intervals and other number concepts. As evidence for the principles first theory, Gallistel and Gelman (1992) suggest that both children and adults make more errors and are slower in identifying which of two quantities is larger or smaller when the difference between the quantities is small, a procedure that relies on linking preverbal concepts of number to number words and mathematics processes, such as addition.

One competing hypothesis to Gelman and Gallistel’s theory suggests that children simply imitate counting, so initially, counting is nothing more than a rote task. In this, principles after theory, the essential and nonessential features are inferred after repeated imitation (Briars & Siegler, 1984; Fuson, 1988; Fuson & Hall, 1983; Dehaene, 1992). However, in both the principles first and principles after theories, the development of mathematics computation skills stems from the ability to discern the essential and nonessential features of counting.

Once counting is mastered, the development of basic arithmetic skills typically begins when children add using one of several counting procedures. The min, otherwise known as the counting-on procedure, is the most efficient of the counting methods and is executed by counting up from the largest addend to the value of the smallest addend (e.g., 7 +3 is calculated by stating, “seven...eight, nine, ten”). The max procedure is similar, but rather than beginning with the largest addend, children begin with the smallest and count up through the quantity of the larger (e.g., “three...four, five, six, seven, eight, nine, ten”). In the sum, or counting-all, procedure, children count the values of both integers. Decomposition occurs when children use either a base of 10 or “ties” as a starting point from which to add integers. For example, using a base of 10 strategy, a child might calculate, 8 + 5, by conceptualizing the problem as 8 + 2 = 10, and 10 + 3 = 13. Like solutions to problems using the base of 10 strategy, problems with ties (e.g., 6 + 6) appear to be more easily stored and/or recalled from memory. Thus, using ties, a child might solve the problem, 6 + 7, by conceptualizing 6 + 6 = 12, and 12 + 1 = 13. The fingers and counting fingers strategies are also commonly used by children to solve simple addition problems. Children use the fingers strategy when they use their fingers to look at them, without actually counting them, which seems to prompt memory retrieval, while the counting fingers procedure involves the child counting his or her fingers (Geary & Brown, 1991). In a study by Geary and Brown (1991), third and fourth grade gifted, academically normal, and children with learning disabilities impacting mathematics were compared to determine the strategies they used for solving simple addition problems. As gifted children used memory retrieval to an equal extent as adults, and children with learning disabilities used retrieval the least frequently, instead using finger counting strategies, Geary and Brown (1991) concluded that typical children gradually progress through stages, whereby finger-counting is replaced with memory retrieval. Geary and Brown (1991) also noted that older children used verbal counting more often than finger-counting, suggesting that finger-counting strategies are not instantaneously replaced by retrieval, but rather, during typical development, children gradually develop new strategies, eventually preferring retrieval to solve basic arithmetic facts, a strategy that continues into adulthood.

By sixth grade counting procedures are nearly entirely replaced by retrieval in typically developing children to solve most simple addition problems, as children are able to quickly and automatically recall answers from memory without having to calculate. However, the transition to retrieval does not occur instantaneously, but rather, old procedural strategies, such as the max procedure, are gradually replaced with more efficient strategies (Ashcraft, 1992).

Ashcraft (1992) also recognized the influence of memory processes in mathematics development. Ashcraft proposed three network retrieval models for explaining how mathematical information is accessed and used. These models answer the questions, "How is a person's knowledge of number and mathematics organized in memory and what are the processes by which this knowledge is accessed and applied in various settings?" (Ashcraft, 1992, p.77). In other words, Ashcraft assumed the accuracy of the retrieval model and only debated how the retrieval model is organized. In the network retrieval model proposed by Ashcraft, mathematical information is accessed and retrieved via spreading activation, where the strength of memory nodes are influenced by practice and the frequency with which certain mathematical facts are utilized. Similar models that build on Ashcraft also support the network system, such as Siegler and Shrager's (1984) distribution of associations model and Campbell's (1987) network interference model. While these models differ slightly in the type of information that is held in memory (e.g., correct and incorrect answers versus associations between whole problems and answers), all three of these models stipulate that mathematical information is retrieved from a memory network (Ashcraft, 1992). Although these models tout the importance of retrieval in mathematics processing for individuals of all ages, only basic mathematics facts, such as in simple addition and multiplication, are accounted for, and as these network nodes cannot be seen or specifically measured, the models represent best guesses about the manner in which memory influences mathematics processing and knowledge.

Ashcraft's theory focused on a method for how memory and mathematics are linked, but memory retrieval is not the only cognitive process underlying mathematics skills development that Ashcraft noted. For example, in Ashcraft's network retrieval model, working memory also plays a role, as information that undergoes a delay in storage or activation is subject to decay, indicating that working memory deficits may impact storage and/or retrieval of mathematical information. Furthermore, as these network retrieval models only address basic arithmetic problems, other, more complex problem-solving procedures may rely on working memory to carry out the multiple steps involved in arriving at a solution. Even processes like carrying and borrowing in multiple-digit addition and subtraction require individuals to mentally hold on to information while performing operations with the digits of the problem at hand (Ashcraft, 1992). Additionally, Siegler and Shrager (1984) focused on choice of solution strategies in the network retrieval model, in which problem difficulty determines the use of reasoning in mathematical problem-solving, as the errors noted in children's attempts to solve problems suggest misunderstanding in conceptualization of the problem. The transition from counting to utilizing other procedural strategies and the eventual graduation to retrieval represents only one major shift in development related to the acquisition of mathematics skills. Another shift is the transition children make once they have achieved mastery of basic arithmetic and must learn more advanced mathematical concepts such as algebra. In a rare study examining the mathematical difficulties of adolescents, Greenstein & Strain (1977) measured the Key Math Diagnostic Arithmetic Test's (Connolly, Nachtman, & Pritchett, 1976) ability to discriminate students with mathematics disabilities from those without mathematics disabilities. The mean performance of the students identified with mathematics disabilities fell at approximately the fourth grade level despite actual grade level ranging between eighth and eleventh grades. The results of this study may suggest that fourth grade represents a new shift in mathematics skills, one in which a transition from concrete to abstract thinking is necessary for mathematics skill acquisition to progress.

Historical Studies

Longitudinal studies may be the most effective way to study the progression of mathematics development from childhood to adulthood (Geary et al., 1999; Gersten, et al., 2005). However, such longitudinal studies are costly, time-consuming, and relatively rare. In lieu of longitudinal studies, neuropsychological studies investigating the relationship between focal brain lesions, their corresponding cognitive deficits, and their resulting impact on mathematics skills may provide information about how cognitive processing is related to mathematics skills in adults. While these studies are frequently based on mathematical deficits rather than analyses of normal mathematical functioning, such studies can provide insight into the cognitive processes underlying variability in mathematics performance (McCloskey, Harley, & Sokol, 1991).

Mathematical difficulties likely stem from a variety of factors. Neuropsychologists have examined the brain in an effort to uncover anomalies that might account for mathematical deficits. Therefore, most of what is known about the association between neurology and mathematics skills has been revealed in studies of brain-injured adults (Sutaria, 1985). This information is helpful in understanding the mathematical abilities of adults, as it is often difficult to draw parallels from the child literature to adult functioning. In addition, these neurological studies may help identify differences between mathematical skills in children and adults.

In the early 1920's Henschen (1925) conducted the first large-scale statistical analysis of mathematics ability in adults and coined the term, "Akalkulia" to describe a disorder in which mathematics ability was affected in people with acquired brain damage (Rourke & Conway, 1997). Since then, neuropsychological studies have sought to uncover a wide range of physical characteristics and anomalies associated with mathematics disabilities. For example, some studies have suggested a link between birth complications, such as low birth weight and toxemia during delivery, to poor mathematics performance later in life (Badian, 1999). Other studies have examined the relationship between low mathematics scores and right versus left hemisphere deficits (Rourke, 1993). The revised term, "acalculia" was designated to define difficulties with mathematics calculation that resulted from neurological damage and to distinguish mathematics disorders from reading and writing disorders (Sutaria, 1984).

Between 1924 and 1930, Gerstmann conducted research with individuals who appeared to exhibit four behavioral symptoms together in a developmental syndrome, one of which included an inability to perform mathematics operations. These operations led to the development of a new term, "dyscalculia." Dyscalculia is similar to acalculia in that it is defined by impairments in the ability to perform mathematics calculations, but its origin stems from congenital, rather than acquired, brain anomalies (Kosc, 1974).

Gerstmann believed that the syndrome resulted from a lesion in the left-parietal occipital area of the brain (Gerstmann, 1957). However, others have pointed to lesions in the superior temporal and supramarginal gyri and the angular gyrus. Contradictory research results, in addition to variation in the degree and nature of symptoms, make Gerstmann's syndrome a controversial disorder (Rourke & Conway, 1997). Although the existence of the syndrome has never been validated, it represents one of the first examples of developmental impairments in mathematics skills, and many of the symptoms affecting mathematics performance continue to mimic those characterized by individuals with learning disabilities impacting mathematics (Rourke & Conway, 1997).

The evidence of involvement of such cognitive abilities as visual-spatial ability, linguistic processes, and concept formation (i.e., reasoning) has influenced several researchers' conceptualizations of the underlying cognitive processes impacting mathematics achievement (McLeod & Crump, 1978; Kosc, 1993; Rourke & Conway, 1997; Badian, 1999; Hécaen, Angelergues, & Houillier, 1961). For example, Rourke and Conway (1997) and Badian (1999) summarized Hécaen's et al. (1961) theory of mathematics disability by describing three subtypes. The first of the three subtypes, alexia or agraphia for numbers, encompasses difficulties in mathematics that result from verbal deficits and is characterized by difficulty in reading and writing numbers (Rourke & Conway, 1997). The second subtype, spatial acalculia, is influenced by visual-spatial deficits, and the third subtype, anarithmetria, describes individuals who have difficulty in carrying out mathematics operations despite intact visual-spatial skills (Rourke & Conway, 1997). Badian (1999) cited evidence for a visual-spatial subtype of mathematics disabilities that described students' confusion in working with digits aligned horizontally and vertically, errors made by subtracting top numbers from bottom numbers, reversing two-digit numbers when carrying, ignoring place values, and omitting zeros in three digit numbers when the zero is located in the tens place. Badian (1999) also cited Hécaen's anarithmetria subtype, suggesting that individuals with this subtype of mathematics disability know mathematics facts and exhibit no spatial deficits, but confuse procedures. Badian suggested that "disturbances in memory" may be involved in the anarithmetria subtype, but stated that these memory disturbances differ from memory impairments that impede the learning of mathematics facts, procedures, and multiplication tables. Badian also expanded on Hécaen's model of mathematics disability by adding a subtype that she called attentional-sequential dyscalculia in which children forget to add all the digits in a column, have difficulty remembering multiplication tables, omit decimal points and dollar signs, and make frequent errors in adding and subtracting (Badian, 1983).

Kosc's (1974) six subtypes deviated slightly from Hécaen's model by including two verbal subtypes, the first of which he called verbal dyscalculia, characterized by impairments in the ability to name mathematics terms, quantities, and numbers. The second verbal type, lexical dyscalculia, is defined by deficits in the ability to read numbers and operational symbols. The third subtype, Kosc named graphical dyscalculia, which characterizes deficits in the manipulation of mathematical symbols in writing. Next, Kosc identified operational dyscalculia, which is equivalent to Hécaen's anarithmetria. Then, Kosc named a visual-spatial type of mathematics disability that he referred to as practognostic dyscalculia, meaning dysfunction in the ability to mentally, or otherwise, manipulate objects affecting such skills as estimating and comparing quantities and objects. Lastly, Kosc recognized a subtype that he called ideognostic dyscalculia that explained difficulties in performing mathematics calculations that is not caused by linguistic or visual-spatial deficits but is characterized by deficits in the ability to understand mathematics ideas and relationships (Rourke & Conway, 1997). In more recent years, neurological evidence has continued to reveal mathematical deficits. However, specific brain sites have not been reliably linked to specific processing deficits due to the small number of studies and the fact that brain injuries rarely result in localized damage that impacts a single cognitive skill (Geary, 1993).

Relationship between Cognitive and Linguistic Processes and Mathematics

Support for the role of memory retrieval can be found in recent studies conducted by Geary on the impact of mathematical learning disabilities in children (Geary, Hamson & Hoard, 2000; Geary, Hoard & Hamson, 1999; Geary, 1993; Geary, Brown & Smaranayake, 1991, Gersten, et al., 2005). A model of three primary cognitive variables impacting mathematics skills was posited by Geary from his work with children and is supported by much of the research conducted with children and adults. In Geary's studies of calculation skills, he identified deficits in the "representation and retrieval of arithmetic facts from long-term semantic memory" (Geary, 1993, p. 346). The second cognitive processing domain that Geary proposed is "manifested by the use of developmentally immature arithmetical procedures and a high frequency of procedural errors" (Geary, 1993, p. 346). Procedural errors may include skills deficits in counting and using computational strategies (Geary, Bow-Thomas & Yao, 1992). The third cognitive processing variable described by Geary's model includes visuospatial skills, which according to Geary, lead to a "disruption of the ability to spatially represent numerical information" (Geary, 1993, p.346).

Geary proposes that the poor mathematics skills of children with learning disabilities impacting mathematics result from both memory and procedural deficits. "Mathematically disabled children are also less skilled than normal children in the use of counting, or computational, procedures to solve arithmetic problems…" (Geary et al., 1992, p. 372) and that memory retrieval deficits demonstrate a different developmental trajectory and result from a different etiology than procedural deficits (Geary et al., 1992). Thus, not only are children who demonstrate mathematics difficulties less likely to use retrieval strategies in solving basic arithmetic problems, but they make more errors in retrieval and computational procedures. However, Geary contends that these computational errors represent a developmental delay rather than a true deficit. As evidence for a dual developmental trajectory, one in which computational skills catch up with other developmentally appropriate skills, Geary cites his studies showing that computational errors disappear by second grade (Geary et al., 1992). However, it seems likely that errors made in the procedural algorithms for solving mathematical problems may be related to a lack of conceptual understanding of mathematics. As these errors do not appear to be related to memory deficits, they may be related to other underlying cognitive processes, such as reasoning, which Geary has not examined. Again, Geary's studies in regard to computational and procedural errors deal only with basic mathematics problems in young children and do not address other cognitive processes, aside from memory retrieval, that may impact students' difficulties with mathematics later in life.

Despite the strong evidence that memory retrieval is the primary strategy in solving basic mathematics problems once children reach the sixth grade, Geary's conclusion that poor arithmetic performance is primarily explained by disturbances in long-term memory networks may not account for the wide range of difficulties in mathematics that students experience. As

Geary's studies examine only very basic mathematics problems, one cannot generalize to other types of mathematics problem-solving that is more typical of students in high school and college.

Although few investigations into the cognitive processes underlying mathematics skills in adults have been conducted, one such study by Geary et al. (1987) supported the role of retrieval in the mathematics achievement of undergraduate college students. In this study several cognitive processes, such as counting, memory retrieval, and rule-based procedural processes were investigated by predicting reaction times when performing simple and complex (e.g., involving carrying) addition problems. As in many of the studies involving child populations, this study was limited in the scope of mathematical skills examined, and all cognitive processes investigated were assumed to correlate with reaction times. However, this study represents one early attempt to test the importance of retrieval in the mathematics skills of young adults.

Floyd et al. (in press) investigated the cognitive and linguistic processes underlying mathematics skills (calculation and problem solving) of adults by using a large nationally represented sample used to norm the WJ III. The study included subjects ranging in age from six years to over 80 years to predict the WJ III Clusters, such as Comprehension-Knowledge (Gc), Fluid Reasoning (Gf), Short-term Memory (Gsm), Processing Speed (Gs), Long-term Storage and Retrieval (Glr), and Visual-Spatial Thinking (Gv), that were most predictive of mathematics achievement across the lifespan. Unlike previous studies identifying mathematics achievement in terms of basic arithmetic skills alone, Floyd's study utilized all the mathematics achievement subtests of the WJ III including Calculation, Math Fluency, Applied Problems, and Quantitative Concepts to develop two criterion variables, Math Calculation Skills, made up of the Calculation and Math Fluency subtests, and Math Reasoning, consisting of the Applied Problems and Quantitative Concepts subtests.

Results of Floyd's study led to interesting implications for adults. Both Comprehension-Knowledge and Fluid Reasoning demonstrated significant relationships to mathematics achievement across the lifespan. Whereas Fluid Reasoning demonstrated moderate correlations with Math Calculation Skills that increased from approximately age 6 to age 70, Fluid Reasoning demonstrated an even stronger relationship with Math Reasoning that increased after age 25.

Another interesting finding of the study involved the clusters related to memory. The Long-term Storage and Retrieval Cluster demonstrated a significant relationship with mathematics achievement only during the elementary school years and demonstrated the most important contribution before age 7. However, the Short-term Memory and Working Memory Clusters demonstrated significant and moderate correlations with mathematics achievement across age groups into middle adulthood. The relationship between Working Memory demonstrated a stronger relationship with both the Math Calculation Skills and Math Reasoning Clusters than did the Short-term Memory Cluster, which includes both Short-term Memory subtests and Working Memory Subtests. The effects were apparent across age groups into adulthood, but were most evident between the ages of 6 and 10.

In contrast to several neuropsychological studies investigating the cognitive processes underlying mathematics skills in adults, the Visual-Spatial Thinking Cluster of the WJ III failed to yield significant correlations between the cluster and mathematics achievement across the lifespan.

The results of this study indicate several notable variations between the cognitive processes important for the development of mathematics skills in childhood and those that underlie mathematics skills in adulthood. For example, the implications for reasoning skills in the mathematics skills of adults is emphasized by this study, where as long-term memory may be less important. However, the role of auditory working memory is supported, but the role of visual-spatial skills is questioned.

Cognitive Processing Variables

Variation in mathematics achievement may be explained by variations in the cognitive processes underlying achievement (Geary et al., 1999). While many studies identifying cognitive processes that contribute to reading achievement variability have been conducted, studies investigating the cognitive processes contributing to variability in mathematics achievement are needed, particularly for older adolescents and adults (Silver, Pennett, Black, Fair, & Balise, 1999). As no cognitive processes have been identified that contribute to mathematics achievement with equal empirical support as that of the literature on reading achievement (Robinson et al., 2002), the current study aims to add to the mathematics achievement literature for this age group.

Long-Term Memory

In regard to mathematics, Gelman and Gallistel's (1978) theory of preverbal concepts of numerosity suggest that cognitive processing domains, such as long-term memory, must function adequately to ensure the normal development of mathematics skills, such as adding and multiplying basic facts. For example, studies investigating preverbal processes, otherwise known as primary abilities, indicate that stimuli is stored in memory from which information regarding number may be inferred (Geary, 1995). Meck and Church (1983) proposed a model suggesting that a series of pulses or noises are stored in memory after the last noise or pulse is perceived. The number of pulses stored in memory may then be compared against another group of successive pulses differing in quantity. When animals and very young children base decisions on such information a process analogous to subtraction occurs because the first set of stimuli must be compared to the second set in order to make a decision.

The relationship between long-term memory retrieval and mathematics has been studied in a variety of ways. One common method is to directly assess an individual's facility at retrieving the answers to basic mathematical facts from memory (Geary, 1993). For example, Geary et al. (1987) administered a battery of tests measuring Numerical Facility, Perceptual Speed, and Spatial Relations to undergraduate students and asked them to verify whether simple and complex addition and multiplication problems were correct or incorrect. The results from reaction time measurements indicated that fact retrieval, carrying, and encoding contributed to the Numerical Facility and Perceptual Speed factors but not to the Spatial Relations factor. Measuring retrieval in this way assumes that frequent solving of basic calculations eventually leads to the storage of the correct answers to basic mathematics facts (e.g., 4 x 2 = 8) in memory (Geary, 1993). However, this method is domain-specific in that it measures retrieval related specifically to mathematical information. The method says little about the relationship between the more general cognitive processing domain of long-term memory and mathematics. Additionally, such a domain-specific method considers only a very narrow range of mathematical information, which in most cases includes elementary knowledge of mathematics facts (e.g., simple addition and multiplication facts). A model assessing a broader range of mathematical information may provide a means of studying the relationship between long-term memory and mathematics achievement across a range of skills, including advanced mathematics, that is typically required of older adolescents and university students. Thus, we can determine if variability in the ability to store and retrieve information in long-term memory results in variability in the accuracy of performing mathematical calculations and solving mathematics problems (Cirino et al., 2002).

Typical studies linking long-term memory to mathematical skills stems from the child literature and often includes empirical studies supporting the role of long-term memory retrieval in learning disabilities impacting mathematics (Fleishner, Garnett, & Shepherd, 1982; Garnett & Fleishner, 1983; Geary et al., 1987; Gersten, et al., 2005; Goldman, Pellegrino, & Mertz, 1988). In a study by Geary (1990), first and second graders with and without learning disabilities were separated into three groups. The first group consisted of children demonstrating underachievement in mathematics but who improved over the course of an academic year. The second group was comprised of children exhibiting low mathematics achievement who showed no improvement over the school year, and the third group of children demonstrated grade-level mathematics achievement. The second (no-improvement) group was presumed most likely to demonstrate true mathematics disabilities, as their failure to improve over time indicated neurological deficits leading to underachievement in mathematics. All three of Geary's groups utilized verbal counting and retrieval more than any other strategy. However, the no-improvement, underachieving group utilized retrieval strategies significantly less often than counting strategies. The relationship between counting and retrieval was -.45 for the no-improvement group compared to -.93 for the normally achieving group and -.91 for the mathematics improved group. Therefore, the results of this study indicate that children with mathematics disabilities rely more heavily on strategies other than retrieval to solve simple basic arithmetic problems. In addition to relying less on retrieval to solve mathematics problems, the no-improvement group made significantly more errors when using the retrieval strategy, suggesting abnormalities in long-term memory storage and/or retrieval (Geary, 1990).

The implications of such studies are that young children and individuals with learning disabilities may use counting strategies to solve basic mathematics facts, but once in adolescence mastery of basic addition facts is achieved when all basic facts can be retrieved accurately and automatically from long-term memory (Geary, 1994). Robinson, et al. (2002) argued that difficulty in retrieving number facts from long-term memory is caused by semantic or phonological deficits because both represent cognitive errors in the storage mechanism. In other words, semantic features of numbers are misrepresented in long-term memory, which leads to deficits in number sense. The more likely it is that new information makes sense to the individual (i.e., stored in a semantic context), the more likely it is that the information will be easily recalled.

Although research on the relationship between memory retrieval and mathematics achievement has been conducted primarily with children, Svenson (1985) demonstrated that adults also use memory retrieval as their primary strategy to solve arithmetic problems. This finding is consistent with Geary's (1993) theory regarding long-term memory and its pervasive and permanent effect on mathematics throughout the lifespan. For example, frequent counting eventually leads to the long-term memory associations between addends and their sums (Geary et al., 1999). As children develop they rely more heavily on retrieval and increasingly less on other types of strategies, such as counting (Geary, 1993). With increasing maturity and skill development, individuals gradually shift from using learned procedures and algorithms to direct retrieval of answers from memory, at least for solving high-frequency arithmetic facts. Thus, adults rely heavily on long-term memory to solve basic mathematics problems. "Basic arithmetic facts constitute a well-defined and circumscribed set of facts that is learned by virtually every adult." (McCloskey et al., 1991, p. 377). In typically developing individuals, retrieval of declarative knowledge is presumed to underlie addition performance (Geary et al., 1987). Therefore, it is the ability to store and retrieve semantic information in memory that allows adults to quickly and automatically provide answers to simple mathematical problems.

In a study of adults with acquired brain damage, Sokol, et al. (1991) examined patterns of calculation errors to make inferences about the cognitive processes that aid typical adults in performing calculations. While the authors suggest that cognitive processes, such as long-term memory retrieval, problem comprehension, and production of retrieved answers, contribute to mathematics calculation skill, several limitations of the study appear to attenuate these results. For example, calculation skills of typical adults were examined by presenting a limited scope of problems, primarily single-digit multiplication problems, to two patients with brain damage incurred in potentially non-identical areas of the brain. Sokol et al. (1991) tested an unconfirmed model of mathematics calculation that assumed the existence of two essential components, including number processing, defined by the ability to comprehend and produce numbers, and calculation (e.g., operations signs/words and mathematics facts). Additionally, these processes were inferred from the patterns of errors made in performing the calculation tasks. According to Sokol et al., (1991), calculation requires individuals to first translate mathematics problems into abstract, internal representations, suggesting abstract reasoning involvement. The next step requires the individual to retrieve answers to these representations from memory and then translate and produce the answer into the appropriate form. Despite the implication of mediating cognitive processes, such as reasoning, the authors state that the errors of the patients with brain damage indicate deficits in long-term memory retrieval, rather than in the processes required to carry out calculation processes (e.g., carrying).

Reasoning

Because human beings develop verbal abilities, preverbal mathematics concepts are eventually linked with words that represent numbers (Ashcraft, 1992). Therefore, deficits may appear in the processes that link preverbal concepts and the words learned to define and work with them. This shift from preverbal understanding of numerosity to analogous verbal conceptualization allows human beings to build on their mathematics knowledge by learning, through a verbal medium, lessons taught in school. While studies that examine mathematics abilities in young children often refer to children's "conceptualizations," the shift from concrete thinking to more abstract thinking impels the inclusion of "reasoning" skills when the mathematics skills of adolescents and adults are studied. Conceptual knowledge is defined as an understanding of the principles that govern the domain (of mathematics) and the interrelations between pieces of knowledge in that domain, although it is not necessary for that knowledge to be explicit (Rittle-Johnson & Siegler, 1998). In order to be successful in advanced mathematics courses, such as algebra, complex reasoning skills are required in addition to knowledge of operations (Ablard & Tissot, 1998). Students who do not possess these reasoning skills and attempt to perform algebra by simply adhering to procedural rules will be unlikely to master algebra and advance to more complex mathematics (Ablard & Tissot, 1998). While it is recognized that the "conceptualization" of mathematics that occurs in young children does not correspond perfectly to "reasoning" skills in adolescence and adulthood, presumably the latter is built upon the former, as the abstraction and verbal skills of children develop over time.

Geary (1993) has conducted extensive research regarding the relationship between children's understanding of mathematical concepts and achievement. According to Geary's theory, children who exhibit developmental delays in the acquisition of conceptual knowledge use immature strategies and display frequent procedural errors. Conceptual knowledge is related to counting in children aged five and younger. Counting provides the first conceptual basis for the development of basic mathematics skills because understanding the counting process and the underlying assumptions involved in number sense is a prerequisite for learning basic addition. Later, knowledge of complex mathematical procedures is built upon these early skills. Thus, counting errors made in childhood suggest deficits in conceptualizing numbers and their symbolic representations, and this lack of conceptual knowledge may persist into adolescence and adulthood. Geary also contends that children with learning disabilities impacting mathematics often demonstrate developmental delays in their acquisition of conceptual knowledge (Geary, 2005). This conceptual knowledge is needed to implement strategies for accurately solving mathematical problems. When children lack conceptual understanding of mathematical principles, they employ ineffective strategies and make procedural errors (Geary, 1993). As Geary's research has been conducted primarily with children, it is unclear if deficits in mathematics achievement seen in adolescence and adulthood result from deficits in reasoning or because basic skills were not sufficiently acquired at a young age for more complex skills to be built upon. For this reason, studying how reasoning impacts adult mathematics performance is important.

Further support for the relationship between reasoning and mathematics has emerged from research regarding strategy choice in children with learning disabilities. Geary and Brown (1991) found that when 41 third and fourth grade students were separated into gifted, average, and learning disabilities groups, and the distributions of strategy choices for solving mathematics problems were plotted, the learning disabled group's distribution showed overuse of immature strategies, in contrast to use of a retrieval strategy. Therefore, children diagnosed with mathematics disabilities used less efficient strategies for retrieving information from long-term memory than their peers (Geary & Brown, 1991; Geary et al., 1991). While these results might suggest deficits in storage or retrieval of basic facts from long-term memory, choosing less efficient strategies to solve mathematics problems may also represent a lack of understanding of the conceptual nature of mathematics problems.

Studies investigating cognitive processes underlying mathematics performance have seldom focused on reasoning skills (Citarelli, 1998). In addition, terms describing reasoning skills have varied widely. For example, reasoning ability is often implied through other terms, as in Geary's contention that children develop an understanding of numbers with little direct instruction (Geary, 1995). In a study by Geary and Widaman (1992), numerical facility is defined by the measures used in the study that "require the execution of arithmetic operation to solve the presented problems." In other words, the mere inclusion of numbers does not necessarily indicate numerical facility. In attempting to replicate an earlier study (Geary, et al. 1987), Geary and Widaman (1992) measured several cognitive processes underlying air force recruits' mathematical skills by measuring reaction times to verify answers to addition and multiplication problems. The cognitive processes measured included Numerical Facility, Perceptual Speed, Spatial Relations, General Reasoning, and Memory Span. Rather than directly measuring the relationship of these cognitive domains to mathematical ability, Geary and Widaman (1992) attempted to show that arithmetic fact retrieval contributes to reasoning measures. Although the tasks were called reasoning measures, each involved arithmetic skills to perform the task, so rather than assessing the direct relationship of mathematical skills to reasoning, reasoning was compounded with mathematics skills to assess the relationship to retrieval. Therefore, an assumption was made about the relationship of retrieval, not only to mathematics achievement, but to other cognitive processes, as well.

Other studies have investigated the processes of production and comprehension of numerical information separately and have examined the individual processes involved in mathematical problems written in numeric versus lexical formats (McCloskey et al., 1991). The study of these separate components implies that this information is processed, or reasoned, differently by individuals in order to make sense of the information. The reasoning abilities required in performing mathematical problems was summed up by Cifarelli (1998, p. 239) "The success of capable problem solvers [in mathematics] may be due in large part to their ability to construct appropriate problem representation in problem-solving situations, and to use these representations as aids for understanding the information and relationships of the situation."

Some studies of more advanced mathematics refer to how individuals form internal representations of the information provided in the problem. For example, one study attempted to discern how algebra students were able to recognize and construct mathematical relationships involving rate, motion, proportions, and probabilities. When students were able to recognize similarities across tasks, it was assumed that they used reasoning skills to form accurate internal representations of the problems (Cifarelli, 1998). Therefore, reasoning skills appear to play a strong role in the advanced mathematics skills required of older adolescents and adults, as they are no longer simply required to know mathematical procedures but also know how to use these procedures (Ablard & Tissot, 1998).

Auditory Working Memory

The relationship between working memory capacity and mathematics ability of older children and adults has been examined in several studies. However, many of these studies have merely compared the performance of children and adults and concluded that adults perform better due to greater working memory capacity. For example, Holzman, Pellegrino, and Glaser (1982) determined that college undergraduates performed better than fourth and fifth graders on complex mathematical tasks involving inductive skills due to the greater working memory capacities of the college students. Even in simple addition tasks, greater working memory capacity has been suggested to contribute to the increased performance of junior high school and college students (Little and Widaman, 1995). While these studies suggest that working memory ability increases throughout development from childhood to adulthood, the individual variation in working memory skills contributing to different types of mathematical tasks has not been well studied in adults. Whether the relationship between working memory and mathematics skills is as strong in older adolescents and adults as it is in children is a question that continues to require clarification (Geary, et al., 1991). However, some studies investigating the relationship between working memory and procedural errors have suggested that working memory functions optimally in young adults in their twenties versus older adults between the ages of 40 and 60 (Campbell & Charness, 1990).

Some studies examining the impact of working memory and mathematics have focused on narrowing the definition of working memory and determining if visual or auditory working memory contributes greater variability to mathematics achievement. A study by Swanson (1994) examined short-term memory and working memory in 75 children and adults with learning disabilities in order to determine if these two constructs are independently related to achievement. Whereas short-term memory was considered to be utilized when verbal information is simply maintained in memory through rehearsal or other mneumonic strategies, working memory was defined as the “simultaneous storage and processing” of the information so that information is not only held, but manipulated, in memory (Swanson, 1994). While working memory is assumed to access long-term memory either through the encoding process or by drawing on information that has been previously learned, short-term memory is thought to occur at only a "surface level" having no access to long-term memory (Swanson, 1994). Results of this study indicated that, overall, visual-spatial measures did not distinguish between the learning disabled and non-learning disabled groups, nor did short-term memory tasks. Furthermore, confirmatory factor analyses indicated a two-factor model consisting of separate working memory and short-term memory factors for both ability groups, with some evidence that a three factor model, including visual-spatial working memory, may fit better for the learning disabled group. Verbal working memory measures contributed significantly more variance than short-term memory to achievement for both ability groups (Swanson, 1994).

Despite the implications for the relationship between working memory and mathematics achievement, this study did not directly examine the relationship among the constructs and mathematics skills, as individuals with learning disabilities were identified according to scores on reading achievement scores. Nor did the study address possible variation in results between children and adults. Thus, further investigation into the impact of working memory on mathematics achievement in adults is needed.

Working memory is an important skill in performing calculations because learning mathematics facts involves the simultaneous activation of procedural knowledge as well as mentally holding onto and manipulating numbers (Geary, 1993). In regard to learning mathematics skills, working memory deficits negatively impact the retention of mathematics information by resulting in working memory overload. If working memory span is atypically short or brief, the amount of information passing into long-term memory and/or the time required for retention to take place is insufficient for long-term storage (Ashcraft, 1982). According to Ashcraft (1982), in addition to the influence working memory has on retrieval, holding partial answers in memory while employing procedural algorithms (e.g., for carrying or borrowing) also uses working memory and affects more complex problem-solving such as multi-digit addition and subtraction.

The relationship between working memory and mathematics has been studied in a number of ways. Some studies have investigated how working memory deficits impact mathematics by looking at children's ability to develop strategies and to perform mental addition. For example, Keeler and Swanson (2001) studied the relationship between strategy choice and children's working memory, as children with working memory deficits have been noted to lack automaticity in implementing efficient strategies. The stability of strategy choice, rather than utilizing any one particular strategy, was hypothesized to mediate working memory in children with mathematics disabilities, based on studies by Coyle, Read, Gaultney, and Bjorklund (1998) who found that gifted and non-gifted children demonstrated differences in strategic knowledge, which related to recall and use of consistent strategies. Results showed that stable strategy choices were related to verbal and visual-spatial working memory performance, while no specific strategy choice emerged that was significantly related to working memory. In the same study, mathematics performance was significantly correlated with verbal working memory, visual-spatial working memory, and reading. Furthermore, working memory composite scores contributed an additional 34% of variance to a regression equation after reading. These results indicate that working memory and strategy choice are not independent constructs, and both are related to mathematics achievement.

Adams and Hitch (1998) investigated the role of working memory deficits in mental addition by disrupting children's subvocalization strategies for holding information in working memory to complete calculations. The results indicated that articulatory suppression significantly impaired children's ability to perform mental arithmetic, lending support for the role of working memory and the phonological loop in performing mental arithmetic.

Visual-spatial Ability

Early studies suggested that mathematics difficulty was primarily explained by deficits in visual-spatial processing, and visual-spatial deficits were often presumed to account for errors in alignment in writing mathematics problems and in incorrectly representing place values (Geary, 1993). For example, Badian (1999) suggested that vertical and horizontal alignment errors in written addition and subtraction, calculating two or three-digit problems in random places other than the right-hand column, subtracting the top digit from the bottom digit, particularly when the top digit is smaller than the digit underneath it, difficulty telling time, and reversing two-digit numbers and omitting zero and making other place value errors are indicative of spatial deficits. Greenstein and Strain (1977) analyzed error patterns of individuals demonstrating mathematics difficulties and determined that errors in misaligning numbers occurred more often in those diagnosed with learning disabilities than in a control group. Badian (1983), following Hécaen's original model described a subtype of learning disabilities whose deficits focused on visual-spatial deficits, called spatial acalculia, defined by impaired calculation caused by spatial processing deficits. According to Badian, the spatial acalculia subtype included the second largest number of individuals with mathematics disabilities after attentional-sequential dyscalculia.

Visual-spatial deficits have been frequently cited in early studies investigating mathematics performance, as problems with counting were thought to contribute to errors children make in drawing pictures that depict an incorrect number of fingers and facial features. For example, children as old as nine who demonstrate mathematics difficulties have been noted to omit noses in their drawings of human figures (Badian, 1983). In a study by Badian (1983), 21% of 669 preschool children omitted the nose. Seventeen five-year-old boys who left out the nose in drawings were compared to 17 age- and kindergarten-screening score matched controls that included noses in their drawings. The groups were compared across achievement measures seven years later, and the no-nose group was one-year behind the control group in mathematics. Badian explained this finding by suggesting that the no-nose group lacked attention to visual detail, which tends to predict later mathematics ability.

Much of the literature linking visual-spatial skills to mathematics achievement stems from the field of neuropsychology. A study by Rourke (1989) revealed that students with mathematics disabilities exhibited poor visual-spatial ability, psychomotor coordination, and tactile-discrimination, and students diagnosed with comorbid reading and mathematics disabilities demonstrated poor performance on verbal and auditory-perceptual tasks. In another study, Rourke (1993) demonstrated that children diagnosed with learning disabilities performed more poorly on the Mazes subtest of the WISC-III, pointing to spatial deficits, and White, Moffitt, and Silva (1992) also interpreted visual-spatial deficits from children's relatively poor performance on Making Trails (Reitan, 1958), Grooved Pegboard (Klove, 1963), and WISC-R Coding (Wechsler, 1974). However, other studies have shown no group differences on the Mazes subtest (Geary et al., 2000). Morris et al. (1998) also failed to identify deficits in spatial skills in children with mathematics and reading disabilities.

Neuropsychological evidence has also suggested a relationship between visual-spatial deficits and poor mathematics achievement. For example, while the majority of males and approximately half of females with Fragile X syndrome are mentally retarded, approximately 50% of females demonstrate average overall intelligence and significantly weak mathematics achievement relative to reading and spelling. These individuals also tend to perform poorly on tasks measuring visual rotation, but not on all visual-spatial tasks (Mazzocco, 2001).

Despite these early theories based on children's errors in alignment and the neuropsychological evidence supporting the contribution of visual-spatial ability in mathematics achievement, several studies have failed to support this relationship. A study comparing the roles of verbal ability and visual-spatial ability in the mathematics skills of first through fifth graders demonstrating mathematics difficulties determined that verbal ability more strongly predicted mathematics achievement on the Key Math Diagnostic Arithmetic Test (Connolly, Nachtman, & Pritchett, 1971) than visual-spatial ability (McLeod & Crump, 1978). In a more recent study of college students using a similar model as in this study, researchers examined the relationship of visual-spatial ability, executive functioning, and semantic retrieval to calculation skills via SEM analysis. Visual-spatial ability did not contribute significantly to variation in calculation skills in this population (Cirino, et al., 2002).

While it is possible that the strength of the relationship between visual-spatial skills and mathematics achievement weakens with age, accounting for the lack of findings, it is also possible that different types of visual-spatial tasks contribute to variability in mathematics achievement in adults (Gersten, et al., 2005). In addition, the severity of deficits in visual-spatial ability caused by brain anomalies documented in some early studies may have a more profound impact on mathematical skills. However, it is important to determine if tasks commonly employed to measure visual-spatial ability may be used to account for individual variability on different types of mathematics tasks as the model in this study attempts to show.

Linguistic Processes

There is no question that a relationship exists between verbal ability and mathematics achievement, as individuals learn and carry out mathematics operations, in large part, through verbal mediums. For example, verbal steps in solving many mathematical problems are often memorized and one must rely heavily on verbal skills in order to successfully solve word problems. The question is not whether a relationship between verbal ability and mathematics exists, but how strong is the relationship, particularly given the difficulty in separating the impact of reading skills on mathematical performance. For example, the fact that so many students with documented reading disabilities demonstrate difficulties with mathematics have led some to believe that all learning disabilities are essentially reading disabilities and that often mathematics skills are impacted secondarily, as at least average reading skills are necessary to read, carry out, and learn mathematics computation (Padget, 1998).

Pennington (1991) defined "core" symptoms of a disorder as those that are most directly caused by the underlying neurological deficit. For learning disabilities impacting reading, the core symptom is difficulty reading and spelling, and the underlying cognitive deficit is typically identified as phonological processing. Because individuals with poor reading skills often exhibit poor mathematics skills, children diagnosed with learning disabilities in reading may also be diagnosed with comorbid mathematics disabilities (Geary, 1993; Cirino et al., 2002). Pennington (1991) also defined secondary symptoms as those that are consequences of core symptoms.

Thus, the question of whether similar underlying cognitive processing domains may be present in both reading and mathematics variability is valid. As discussed previously, the transition between preverbal mathematics concepts and learned mathematics skills occurs concomitantly with verbal ability, and continuing knowledge of mathematics facts is mediated by verbal ability. Geary et al., (2000), who has postulated his own three-subtype model of learning disabilities impacting mathematics recognizes that mathematics and reading disabilities may be associated with the same cognitive deficits. He suggests that deficits in retrieval of mathematical facts from long-term memory represent a general deficit in the ability to represent and retrieve information stored in phonetic and/or semantic codes. The comorbidity of reading and mathematics disabilities, then, can be explained by a single deficit in retrieval of arithmetic facts and terms from long-term memory. In fact, individual differences in phonological processing skills may explain variance related to mathematics calculation ability, and the correlation between mathematics calculation and word-level reading skills has been found to be as high as .59 (Hecht, Torgesen, Wagner, & Rashotte, 2001). In order to write and comprehend numbers, an individual must continuously convert Arabic symbols to number words and vice versa, or in other words translate representations of numbers into semantic codes (Geary et al., 1999). However, an inability to translate these codes may imply quantitative and/or reading deficits, and children with comorbid mathematics and reading difficulties demonstrated the highest rate of errors for these tasks (Geary et al., 1999).

Rourke (1993) has conducted a number of studies related to the cognitive processes underlying reading and mathematics. In a comparison of two groups of children, the first exhibiting low arithmetic scores and average reading and spelling scores, and the second exhibiting low arithmetic and even lower reading and spelling scores, a difference was noted in cognitive processes associated with right and left hemispheres of the brain. The first group demonstrated poor performance on right hemispheric measures, such as visual-spatial, psychomotor, and nonverbal reasoning tasks, while the second group demonstrated poor performance on left hemispheric measures, such as auditory-perceptual tasks and printed word problems.

In another study, Jordan (1995) found that young children with no spatial or language deficits and children with spatial deficits but no language deficits demonstrated significantly better performance on calculation measures than children with language deficits and no spatial deficits (Cirino et al., 2002). Evidence of a similar pattern was found with adolescents when improved performance was noted on mathematics word problems when linguistic modifications were made to simplify wording in tests given to eighth grade students (Abedi & Lord, 2001).

Other Cognitive Processes

The focus of much of the research conducted to date that attempts to explain the relationship between mathematics achievement and cognitive processing domains investigates lower order mathematics skills of children, such as simple addition and multiplication (Geary, 2005). However, the cognitive variables chosen for this study are those that have gained the most empirical support for their relationship to mathematics achievement, particularly for older adolescents and adults. Cognitive processes that have been studied less frequently, such as attention and processing speed, may contribute to variation in mathematics achievement. Additionally, the impact of higher order cognitive processes, such as metacognition and executive functioning, on mathematics achievement across the lifespan is still unclear (Geary, 1993). The following is a brief discussion of several studies that address these domains.

Support for the relationship between attention and mathematics achievement is found in research involving individuals diagnosed with Attention Deficit Hyperactivity Disorder (ADHD), as a large proportion of individuals with ADHD experience learning and academic problems (Marshall, Hynd, Handwerk, & Hall, 1997). For example, in a study by Carlson, Lahey, and Neeper (1986), mathematics achievement, in addition to reading and spelling achievement, were significantly lower for an ADHD group without hyperactivity when children diagnosed with ADHD with and without hyperactivity were compared in an analysis of covariance, controlling for IQ. Zentall & Ferkis (1993) suggested that certain symptoms of ADHD, such as disorganization and poor attention are related to poor mathematics computation skills, whereas deficits in memory are correlated with decreased comprehension and problem-solving.

A study by Douglas, Barr, O'Neill, & Briton (1986) showed that children with ADHD who were administered psychostimulant medication improved in mathematics performance accuracy, mathematics speed, and self-correcting behaviors over children administered a placebo. However, different ADHD subtypes may impact mathematics performance to varying degrees. According to the Diagnostic and Statistical Manual of Mental Disorders-Forth Edition (DSM-IV) (American Psychiatric Association, 1994), ADHD, Predominantly Hyperactive-Impulsive type is characterized by excessive movement, difficulty staying quiet, interrupting, and being unable to take turns in conversation or play. In contrast, ADHD, Predominantly Inattentive type includes symptoms of inattention while hyperactive and impulsive symptoms are largely absent. A third diagnostic category, ADHD, Combined Type, includes both inattentive and hyperactive symptoms.

Hynd (1991) compared those with and without hyperactivity on measures of reading, mathematics, and spelling. Although individuals without hyperactivity scored lower in all academic areas, score differences were significant only for mathematics. These results were confirmed by several studies that suggest that students diagnosed with ADHD without hyperactivity have poorer achievement in mathematics than their peers (Nussbaum, Grant, Roman, Poole, & Bigler, 1990; Semrud-Clikeman et al., 1992; Zentall & Ferkis, 1993.) Furthermore, Barkley, Dupaul, and McMurray (1990) found that children diagnosed with ADHD with hyperactivity were more likely to be placed in behavior disorder classrooms, while children diagnosed with ADHD without hyperactivity were more likely to be placed in classes for learning disabled children. Marshall et al. (1997) conducted a study with 182 school-age children diagnosed with ADHD with and without hyperactivity. The students were compared on several achievement measures. Results indicated that students without hyperactivity scored significantly lower than students with hyperactivity on the Basic Achievement Skills Individual Screener: Math (BASIS; Floden & Schutz, 1983) and Wide Range Achievement Test – Revised: Arithmetic (WRAT-R; Jastak & Wilkinson, 1984).

However, individuals diagnosed with ADHD have a myriad of deficits, which vary in scope and degree, not all of which involve attention. Some researchers attribute mathematics disabilities to deficits in automaticity of retrieval of number facts, which may directly result from attention deficits or from diminished learning of mathematics facts due to mathematics avoidance (Marshall et al., 1997). Other researchers have hypothesized that poor mathematics skills result from secondary deficits to attention problems because attention problems interfere with the acquisition of new skills (Ackerman, P.T., Anhalt, J.M., Dykman, R.A., & Holcomb, P.J., 1986). Badian (1983) supported a three-subtype model of mathematics disabilities that was originally developed by Hécaen et al. (1961). This model included alexia and agraphia for numbers, spatial acalculia, and anarithmetia, and Badian added a fourth subtype that she termed attentional-sequential dyscalculia based on the frequency with which a subgroup of individuals made careless errors in executing mathematical procedures and recalling mathematics facts.

Although children with mathematics disabilities are not consistently slower than typical children in performing all mathematics operations, the heterogeneity of mathematics deficits may account for processing speed differences in some subtypes (Geary, 1993). Processing speed deficits can affect mathematics performance directly or by acting on working memory because slower processing allows more time for information held in working memory to decay (Towse & Hitch, 1995). A study by Adams and Hitch (1998) indicated that speed has the same relationship with addition span as working memory does in eight to eleven year olds. Speed of processing is especially important for working memory because working memory is directly affected by the time information is held in memory (Adams & Hitch, 1998). However, speed of processing in performing basic arithmetic will increase with development as slower arithmetic algorithms and strategies are replaced by quicker retrieval methods (Adams & Hitch, 1998). Larson and Saccusso (1989) also suggested that variable rates of information processing may be related to working memory deficits.

In addition to contributing to working memory, processing speed may contribute to higher order mathematics skills, such as problem-solving (Kaye, deWinstanley, Chen, & Bonnefil, 1989; Kaye, Post, Hall, & Dineen, 1986). Geary et al. (1992) tested four latent factors, including Processing Speed, Numerical Facility, General Reasoning, and Memory Span contributing to mathematics ability in Air Force recruits. Results indicated that Memory Span was correlated with Perceptual Speed, while the rate of fact retrieval and carrying processes contributed to General Reasoning ability.

CHAPTER 3

METHODS

Model Justification

Mathematics success may be measured by one's score on a standardized achievement test (Geary & Brown, 1991). However, if that score is low, indicating poor mathematics success, the achievement test does not provide an explanation for the poor performance. Models of cognitive processes contributing to variation in mathematics performance such as the one proposed by Geary (1993) attempt to provide such an explanation. While Geary's theory seeks to explain the cognitive processes underlying the basic mathematical skills of young children, the literature regarding cognitive processes contributing to the more complex mathematical skills of older adolescents and adults suggests differences between how children and adults process mathematical information. However, to date, no theoretically sound model of the cognitive processes contributing to mathematics achievement variability in adults has been proposed, and within the literature regarding mathematics achievement in adults, there is a lack of theories explaining the mathematics difficulties of adults (Robinson et al., 2002). It is the purpose of this study to propose a theoretical explanation for the variation in mathematics achievement in a sample of adults.

One method of identifying the cognitive processes that are related to mathematics achievement is to analyze, via structural equation modeling, the relatedness of specified cognitive processes to mathematics achievement. The method chosen for this study is to test the fit of a full model (see Figure 1) to the data and compare a series of three nested models, which depict hypothesized relationships between cognitive processes in certain areas (e.g., long-term memory, auditory working memory, visual-spatial skills, reasoning, and vocabulary) and mathematics performance (i.e., problems answered correctly on tests of mathematics achievement).

Benefits of SEM

SEM is a multivariate form of statistical analysis that requires the development of a model, typically represented in drawings, that hypothesizes causal variables and their relationships to other variable(s) on which they are presumed to have an effect (Keith, 1999). Despite the investigation of causal hypothesis through SEM, the technique used is correlational, rather than experimental. Thus, random assignment of subjects, manipulation of independent variables, and experimental control is not typically applied (Keith, 1999). Cause and effect is not proved by the confirmation of a model, but rather, the models are hypothesized, without manipulating the independent variables, and tested by determining if the direction and strength of the hypothesized relationships are sufficient to conclude that the a priori models “fit” the data. The fit is assessed by determining how well the model accounts for the data (Kline, 1998, p. 50). In other words the correlations themselves do not determine whether variables are causal, but the model chosen and drawn a priori to the analysis implies cause and effect (Keith, 1999).

In addition to being a powerful method of analysis because, simultaneously, the measurement model tests the degree to which the variables measure the latent constructs, while the structural model explains the relationships among the latent constructs, SEM has several advantages over other statistical techniques (Keith, 1999). While analysis of variance (ANOVA) designs are most appropriate when testing categorical rather than continuous independent variables and assumes non-correlated independent variables, multiple regression (MR) allows the analysis of continuous predictor variables, and simultaneously measures the direct effects of a number of variables at once. However, results may vary widely depending on the MR technique used. For example, in hierarchical regression, variables are entered one at a time with order of entry influencing results, whereas stepwise regression measures predictive power by successively assessing the contribution of each added variable, rather than analyzing the influence of all variables equally (Keith, 1999).

Structural equation modeling, on the other hand, not only simultaneously measures the direct effects of a number of variables, but can measure the direct, indirect, and total effects (i.e., sum of the direct and indirect effects) of variables on a criterion variable (Floyd et al., in press). In addition, errors of measurement are analyzed separately from the structural model, increasing the stability of analysis of the effects of one variable on another, as SEM controls for measurement error by considering only the common variance of several observed measures in order to measure the latent construct. Thus, path coefficients among the latent variables in the structural model are not determined by unique variance (i.e., error variance specific to the measures in addition to random error). SEM may also help determine the relationships of indirect effects, or intervening variables (Keith, 1999). For example, if it is hypothesized that working memory has a causal effect on mathematics achievement, it is also possible to determine if working memory is indirectly related to mathematics achievement by having an effect on long-term memory. In other words, a subsequent test might be to determine if long-term memory helps explain the effect of working memory on mathematics achievement (Keith, 1999). SEM allows the researcher to calculate direct, indirect, and total effects of the hypothesized independent, or exogenous, latent variables on the criterion, or endogenous, latent variables (Keith, 1999).

SEM is well-suited to the current study because it is a multivariate technique for analyzing a complex model including relationships among mathematics achievement and several cognitive variables. Furthermore, in predictive procedures, such as in regression methods, no overall test of model fit is provided as it is in SEM methods. Thus, SEM is an excellent way to develop, modify, and test competing theories (Keith, 1999). SEM is ideal for testing theory because the analysis allows the examination of relationships among latent constructs while excluding random error variance and specific measurement error variance, thus providing parameter estimates that more precisely explain relationships (Anderson & Gerbing, 1988).

Furthermore, SEM can be used to test competing hypotheses about causal relationships and thus help to strengthen the arguments made in favor of theories made and tested via SEM models (Keith, 1999). Despite these advantages, SEM cannot prove the existence of the model but only provide support for the model by disconfirming other models (Anderson & Gerbing, 1988).

SEM in the Current Study

A two-step procedure was used in the current study where a measurement model was hypothesized first, and then tested via confirmatory factor analysis. The measurement model depicts the hypothesized relationships among the observed variables and the constructs, or latent variables, that they are hypothesized to measure (Keith, 1999). This measurement model was then analyzed via confirmatory factor analysis utilizing the following five steps: Model specification, identification, parameter estimation, testing fit by determining how well the model replicates the covariance matrix, and examining how the model might be modified to provide for better fit. This last step makes the term, "confirmatory" a misnomer, as initial lack of fit may require re-specification (Anderson & Gerbing, 1988). However, re-specification is based on theoretical considerations rather than on statistical benefits so as to avoid capitalizing on random sampling error, but any model modification done post hoc will still capitalize on chance to some degree (Anderson & Gerbing, 1988).

The second step in SEM consists of hypothesizing the causal relationships of the latent variables to one another in the structural model. The structural model defines the causal relationships in accordance with a theory (Anderson & Gerbing, 1988). The model's latent and observed variable structure represents the proposed theory and indicates the direction of causality (Kavale & Nye, 1991). Together, the measurement and structural models represent a test of validity for the theory. Testing the overall model assesses convergent and discriminant validity because when data support a priori hypotheses predicting that certain variables are correlated while predicting that other variables will not be related, then fit of the model provides a measure of construct validity (Wechsler, 1997a). As long as analysis of the measurement model yields good statistical fit to the data, the analysis of the structural model provides a measure of theoretical validity (Bentler, 1990).

In order for the measurement model to be identified, the model must have as many covariances as path values to be estimated, and it is preferable for the model to be overidentified so that there are more covariances than unknown path values. According to Bollen (1989), the measurement model will be identified if the following criteria are met: At least three indicators are correlated with each factor, each indicator loads on only one factor, and measurement errors are not correlated. Models may still be identified when there are only two indicators per factor if measurement errors are not correlated, each indicator loads on only one factor, and all factors are correlated with at least one other factor. By choosing multiple indicators for a factor, measurement error can be estimated, and the validity of the construct is strengthened because the construct is measured in different ways (Bollen, 1989).

After the hypothesized models are specified, parameters are estimated. In the CFA, factor loadings, measurement error variances, factor variances, factor correlations, and measurement error correlations, if any, are estimated. In addition, paths among latent variables and disturbance terms (i.e., unexplained variance of each criterion variable) are estimated via SEM. Although several estimation methods are available, Maximum Likelihood (ML) estimation was chosen for the current analysis due to its advantages over other methods with sample sizes less than 1000. In addition, the Sattorra-Bentler technique was implemented to adjust to the level of multivariate kurtosis. The choice of estimation method is important only when models are misspecified and data are not multivariate normally distributed. As these two assumptions are rarely perfectly met, it is important to choose an estimation method that most accurately estimates empirical fit (Olsson, Foss, Troye, & Howell, 2000). Maximum Likelihood (ML) estimation techniques have demonstrated better overall performance than Generalized Least Squares (GLS) when models are misspecified, providing less biased, and more consistent, parameter estimates, and demonstrating higher accuracy in empirical and theoretical fit (Olsson et al., 2000). Both ML and GLS are based on an assumption of multivariate normality. Therefore, the only variation in fit occurs due to misspecification of the models, and results from difference in the weight matrixes used in the discrepancy functions for each estimation method (Olsson et al., 2000). The discrepancy function assesses the weighted difference between the elements of the original sample matrix and the reproduced covariance matrix based on the proposed model. The weight matrix in ML changes with each iteration. The weight matrix for GLS is equal to the inverse of the original sample covariance matrix, whereas the weight matrix of ML is the inverse of the reproduced covariance matrix. The weight matrix in ML is updated at each iteration and produces the most unbiased (i.e., the sample mean is an adequate estimate of the population mean), consistent (i.e., the mean of the sample gets closer to the population mean as sample size increases), and efficient (i.e., the estimator results in minimal variability) estimates when the model is misspecified. The more statistically significant the parameter estimates, the greater the confidence one may have in the theoretical fit of the model tested (Olsson et al., 2000). Therefore, it is important that the estimation method chosen yields the most accurate parameter estimates, not only the estimates that produce the best statistical fit, because statistical fit at the expense of theoretical fit is not better when parameter estimates are biased (Olsson et al., 2000).

When data are not multivariate normally distributed, model fit and tests of significance may be affected (Jöreskog & Sörbom, 1984). Therefore, data must be screened for skewness (i.e., symmetry) and kurtosis (i.e., peakedness) prior to analysis. As kurtosis is more likely to threaten tests of variances and covariances than skewness, it is particularly important to ensure that values do not exceed /3.0/, which represents normally distributed data (Browne, 1984). When values exceed /3.0/, standard errors of parameter estimates may be underestimated, thus affecting significance tests. Jöreskog and Sörbom (1996) recommend values of less than /2.0/ for multivariate normality. In sum, empirical fit is rarely perfect, and the models tested are less than perfect representations of the true model. However, the estimation method chosen should yield parameter estimates that most closely reflect the true parameter estimates (Olsson et al., 2000). Because ML yields standard errors of the parameter estimates, it is possible to test individual parameters for significance as well as testing significance of overall model fit (Anderson & Gerbing, 1988). However, in ML estimation nonnormality may result in the underestimation of standard errors, which affects tests of significance by increasing t-values, which makes t-values for parameter estimates higher (Hu & Bentler, 1995). Yet, ML produces weighted residuals rendering scale of measurement irrelevant, so that variables can utilize different scales of measurement.

Overall fit is assessed via fit indexes, which compare the observed variance-covariance matrix and the estimated matrix implied by the theoretical model. The degree of congruence between the matrixes represents the degree to which the theoretical model represents the data (Keith, 1999). Chi-square is generally reported, as it is the only fit index that provides a significance test that suggests the probability that the theoretical model accurately represents the data. However, chi-square is sensitive to sample size, so that one may obtain a statistically significant chi-square, suggesting poor fit, when only a small difference exists between the actual and implied matrixes (Keith, 1999). In addition to examining fit indexes, fit of the model may be assessed by making sure that the standardized residuals are minimal (i.e., values less than /2.0/), indicating only small discrepancies between the sample matrix and reproduced matrix, and by checking for significant t-values for each path (i.e., t-values greater than /2.0/), which test if the path value significantly deviates from zero. In addition, improper solutions and unexpected signs or values for paths should be noted. Furthermore, in ML estimation nonnormality may cause the chi-square statistic to be inflated, increasing the likelihood of Type I errors (West, Finch, & Curran, 1995). Therefore, chi-square is always accompanied by other fit indexes. In the current analysis the Minimum Fit Function Chi-square test was supplemented with the Comparative Fit Index (CFI; Bentler, 1990), the Non-Normed Fit Index (NNFI or TLI; Tucker & Lewis, 1973) and the Root Mean Square Error of Approximation (RMSEA; Steiger, 1990). The CFI and NNFI are incremental fit indexes, indicating they compare the hypothesized model to a null or baseline model in which the measured variables are assumed to be unrelated (Keith, 1999). The CFI is an incremental fit index that measures the proportion of improvement of the proposed model over the null model. The NNFI is similar to the CFI but includes a correction for model complexity (Kline, 1998). Both the CFI and NNFI are less affected by sample size than chi-square (Keith, 1999). A cutoff of .95 or above is used to identify good fitting models with the CFI and NNFI (Hu & Bentler, 1995). The RMSEA is a stand-alone index, less susceptible to model complexity than the CFI or NNFI, which will always reflect better model fit in more complex models (i.e., models containing more paths) (Keith, 1999). The RMSEA assesses fit by determining how close the population data is to the covariance matrix implied by the model. According to Hu and Bentler (1999) values less than .08, in conjunction with CFI and NNFI indexes greater than .95, are recommended for adequate fit for the RMSEA.

Measurement Models

Subtests selected to measure the long-term memory (LTM) factor included the Long Delay Free Recall subtest of the California Verbal Learning Test – Second Edition (CVLT-II: Delis, Kramer, Kaplan, & Ober, 2000) and the Logical Memory II and Family Pictures II subtests of the Wechsler Memory Scales – III (WMS-III: Wechsler, 1997c). Subtests hypothesized to measure the auditory working memory (AWM) factor included the Letter-Number Sequencing subtest of the Wechsler Adult Intelligence Scale – Third Edition (WAIS-III: Wechsler, 1997b) and the Auditory Working Memory and Numbers Reversed Subtests of the WJ III. Observed variables included in the visual-spatial (VS) factor are the WAIS-III Block Design and WJ III Spatial Relations subtests. The reasoning (REA) factor consisted of the Matrix Reasoning and Similarities subtests of the WAIS-III and the Concept Formation subtest of the WJ III. Subtests chosen to measure the vocabulary (VOC) factor included two vocabulary subtests, the Vocabulary subtest of the WAIS-III and the Vocabulary subtest of the PPVT-III. The two endogenous latent variables representing mathematics achievement were made up of mathematics calculation (MCALC), which included the Calculation and Math Fluency subtests of the WJ III, and mathematics reasoning (MREA), which included the Applied Problems and Quantitative Concepts subtests of the WJ III (See Figure 2).

Structural Models

These studies, which describe the current literature trends regarding mathematics achievement across the lifespan, were used to develop a full structural model of select cognitive processes associated with variability in mathematics achievement, where mathematics achievement was defined as mathematics calculation and mathematics reasoning (See Figure 1). Next, nested models were tested with paths from each exogenous latent variable to the endogenous latent variables successively set to be equal. Successively setting these paths to be equal allows examination of the differential effects of each exogenous variable on the endogenous variables in order to determine which model most accurately represents the cognitive variables underlying mathematics achievement in a sample of adults who were referred to the Regents' Center for Learning Disorders at the University of Georgia due to academic difficulties.

All variables in the model were hypothesized to have a direct and positive relationship with mathematics achievement, which was defined by the two mathematical clusters of the Woodcock Johnson III Tests of Achievement (WJ III: Woodcock, McGrew, & Mather, 2001), Math Calculation Skills and Math Reasoning. Each exogenous latent variable was hypothesized to correlate with each of the two endogenous latent variables (See Figure 1). In the first nested model, the same relationships are hypothesized among the latent variables. However, the paths from long-term memory to the endogenous latent variables are set to be equal to each other. Setting these paths to be equal allows a comparison to be made between the full model and the nested model. This comparison will show the differential effects of long-term memory on mathematics calculation and mathematics reasoning. Vocabulary is included as an exogenous latent variable in both models due to its high correlation with general verbal ability.

Measures

The measures used to examine the relationship between cognitive processing and mathematics achievement in the adult population in this study were chosen from among the standard battery given to participants evaluated at the University of Georgia – Regents' Center for Learning Disorders (UGA-RCLD). All of the measures represent individual subtests included on widely recognized tests of ability and achievement. The WAIS-III (Wechsler, 1997) was developed as a test of intelligence, standardized on 2,450 individuals ranging in age from 16 to 89. Individual subtests measure aspects of intelligence. The Letter-Number Sequencing, Block Design, Matrix Reasoning, Similarities, and Vocabulary subtests of the WAIS-III were included in the proposed models. See Table 2 and 3 for reliability and validity information for the WAIS-III subtests used in the current study. Letter-number sequencing, a subtest included in the WAIS-III working memory factor was selected for the current study to measure working memory. On this subtest, examinees must repeat strings of items, consisting of both letters and numbers, of increasing length. The examinee is required to mentally manipulate the items by putting them in chronological and alphabetical orders. The test contains 21 items when an examinee completes all test items. The block design subtest, which asks examinees to manipulate blocks in order to match a target design presented on a stimulus card contains 14 items of increasing difficulty. It was included in the current study to measure visual-spatial ability. Matrix reasoning contains 26 items that require the examinee to choose the item that completes a geometric pattern or sequence from one of five choices. In the current study, the subtest was selected as a measure of reasoning. The Similarities subtest contains 19 items that ask the examinee to explain how two objects or ideas are conceptually related. The vocabulary subtest contains 33 items that ask the examinee to provide definitions for words that are presented verbally by the examiner and visually on a stimulus card.

Table 2. WAIS-III Reliability by Subtest

Subtests	Test-Retest	Avg. Internal Consistency
Vocabulary	.89-.94	.93
Similarities	.74-.88	.86
Matrix Reasoning	.75-.81	.90
Block Design	.80-.88	.86
Letter-Number Seq.	.70-.80	.82

Note: The range of test-retest stability coefficients is reported for the four age groups (16-29, 30-54, 55-74, 75-89), and were tested within an interval ranging from 2 to 12 weeks. The mean retest interval was 34.6 days (WAIS-III; Wechsler, 1997). Internal consistency coefficients were determined using a split-half method and were averaged across the four age groups.

Table 3. Criterion-related Validity for the WAIS-III

Subtest of WAIS-III	WAIS-R	WISC-III	SPM	SB-IV
Full Scale IQ	.93	.88	.64	.88
Vocabulary	.90	.83		
Similarities	.79	.68		
Block Design	.77	.80		

Note: Criterion-related validity coefficients for the matrix reasoning and letter-number sequencing subtests were unavailable, as these subtests were not included on either the WAIS-R (Wechsler Adult Intelligence Scale – Revised) (WAIS-R; Wechsler, 1981) or the WISC-III (Wechsler Intelligence Scale for Children-Third Edition) (WISC-III; Wechsler, 1991). In addition to the WAIS-R and WISC-III, the full scale score of the WAIS-III was compared to the Standard Progressive Matrices (SPM; Raven, 1976) and the global composite of the Stanford-Binet Intelligence Scale-Fourth Edition (SB-IV; R.L. Thorndike et al., 1986).

Each subtest of the WJ III measures a single narrow ability according to Cattell-Horn-Carroll theory of intelligence on which the WJ III is based. As each subtest measures only one narrow ability, any variance that is irrelevant to the ability is decreased. In addition, growth curves, which depict the relationships between ability levels and age, show the expected patterns of growth and decline among the narrow abilities, providing evidence for their validity (McGrew & Woodcock, 2001). Subtests of the WJ III chosen in the current study include the following: Numbers Reversed, Auditory Working Memory, Spatial Relations, and Concept Formation. Numbers Reversed and Auditory Working Memory measure auditory working memory in the current study. Numbers reversed contains 30 items for which the examiner or a recorded message presents strings of numbers and asks the examinee to repeat the number strings in reverse order. Auditory Working Memory, which includes 42 items, requires examinees to repeat strings that contain numbers and objects. Examinees must first repeat the objects, then repeat the numbers, in the same order in which they were presented. Spatial Relations was chosen in order to measure visual-spatial processing and asks the examinee to choose the shapes that complete a geometric design and consists of 81 possible points. Concept Formation was selected as a measure of nonverbal reasoning. Concept formation consists of 40 items and requires examinees to state the rule that explains why varying colors and numbers of shapes are included in a series. See Table 4 for median split-half internal consistency estimates reported across ages. Median stability coefficients, collapsed across years, taken at intervals of less than one year, between one and two years, and between three and ten years are reported as .77, .75, and .62, respectively for the Concept Formation subtest. Stability coefficients for the other subtests included in the proposed model were not reported in the manual.

Table 4. WJ III Reliability by Subtest

Subtest	Internal Consistency
Spatial Relations	.81
Concept Formation	.94
Numbers Reversed	.87
Auditory WM	.87

The Wechsler Memory Scale – Third Edition (WMS-III) was standardized on 1250 individuals between the ages of 16 and 89 and included an equal number of males and females. Subtests used for the current study included tests of long-term verbal and visual memory. The Logical Memory II subtest measures verbal long-term memory by asking examinees to recall details of two stories that were read to them 20-30 minutes previously and consists of a possible total 50 points. Family Pictures II is a measure of visual long-term memory and contains 64 items that asks examinees to recall which family members were in four scenes, where they were, and what they were doing when the scenes were shown to them 20-30 minutes earlier. Reliability for the WMS-III was established using a split-half method for internal consistency. Stability coefficients were estimated with a test-retest interval between 2 and 12 weeks with an average of 35.6 days and were calculated with two age groups, 16 to 55 year olds and 55-89 year olds. See Table 5 for reliability data reported in the manual. The WMS-III was correlated with the Wechsler Memory Scales – Revised (WMS-R; Wechsler, 1987) and the Children's Memory Scale (CMS; M. Cohen, 1997), an individually administered test of memory functioning in children and adolescents aged 5 to 16 years, in order to establish criterion-related validity.

The correlation coefficients estimating the relationship between the WMS-III Auditory and Visual Indexes and the corresponding Indexes of the CMS ranged from .26 (Visual-Delayed Indexes) to .74 (Auditory/Verbal Immediate Indexes) (Wechsler, 1997).

Table 5. WMS-III Reliability by Subtest

Subtest	Internal Consistency	Stability Coefficients	
	Age	16-55	55-89
Logical Memory II	.79	.79	.79
Family Pictures II	.84	.68	.71

The CVLT-II is a test that measures learning by listing 16 words that the examinee must remember and repeat. The test, as a whole, measures verbal learning and memory in individuals between the ages of 16 and 89 years. The Long Delay Free Recall (LDFR) subtest of the CVLT-II measures long-term verbal memory of the list of 16 words presented five times approximately 20 minutes earlier. Although the manual reports internal consistency and stability reliability estimates, only the stability coefficient is reported for the LDFR subtest. Internal consistency coefficients are reported for the five immediate recall trials in which the 16 words are presented immediately before asking the examinee the repeat the words that he or she remembers. Internal consistency estimates are reported in Table 6 for the immediate recall trials using a split-half technique with the total sample, with three subsamples using age- and gender-corrected scores, and using the number of times each of the 16 words were recalled across the immediate recall trials. Table 6 also reports the stability coefficient for the LDFR trial in which the test was administered twice to 78 subjects with a median test-retest interval of 21 days.

Table 6. CVLT-II Internal Consistency and Stability Coefficients

Internal Consistency (Immediate Recall)			Stability Coefficient (LDFR)
Total Sample	Subsamples	# Words	
.94	.87-.89	.79	.88

In order to investigate validity, a comparison study of the CVLT-II and original CVLT was conducted with a sample of 62 nonclinical adults who ranged in age from 19 to 71 years. The correlation coefficient for the LDFR subtest was estimated at .78. Construct validity of the test as a whole is discussed in the manual in the context of numerous prior studies conducted with the original CVLT and are not specifically cited. While the authors state that studies have failed to demonstrate the contribution of the delayed recall test over and above the immediate recall tests, they assert the importance of delayed memory tests in general due to certain clinical populations' impairments in these measures.

The Vocabulary test of the PPVT-III is an achievement test of hearing vocabulary in English that may be given to individuals from 2 years, 6 months to 57 years, 11 months of age.

Like the Vocabulary subtest of the WAIS-III, the PPVT-III measures vocabulary word knowledge, but rather than asking examinees to provide definitions for words, the words are given, to which the examinee must respond by pointing to corresponding pictures. The test is made up of 204 items. To obtain estimates of internal consistency, alpha reliability coefficients and split-half reliability coefficients were calculated. The median value for internal consistency was .95 across age groups. The median split-half reliability coefficient for internal consistency was .94, and test-retest reliability ranged from .91 to .94 across age groups after a one-month time interval. Alternate forms reliability was also calculated on the entire standardization sample, and stability was established through a test-retest method with a month interval between testing. Alternate forms reliability estimates ranged from .89 to .99 with a median of .95 across age groups.

The following WJ III achievement tests make up the observed variables that are hypothesized to correlate with the endogenous latent variables measuring achievement. Quantitative Concepts A consists of 34 items that ask the examinee to provide the meaning of mathematics terms and symbols and thus represent mathematics fact knowledge, while Quantitative Concepts B has 23 items and asks examinees to complete sequences of number patters and was used in the this study as a measure of quantitative reasoning. The Mathematics Fluency subtest is a timed achievement task that requires examinees to solve as many simple arithmetic problems as quickly as they can in three minutes. The Applied Problems subtest measures mathematics achievement by requiring examinees to solve 63 word problems. They are allowed to use scrap-paper, and the test is not timed. The calculation subtest is another test of mathematics achievement that provides examinees with 45 addition, subtraction, multiplication, division, fraction, decimal, and algebra problems and asks them to solve as many items as they can at their own pace. See Table 7 to view median internal consistency estimates collapsed across age groups for all achievement tests.

Table 7. WJ III Achievement Tests - Reliability by Subtest

Subtest	Internal Consistency
Quantitative Concepts	.91
Applied Problems	.93
Calculation	.86
Math Fluency	.90

Participants

The sample for this study was taken from archival data collected from the Regents' Center for Learning Disorders at the University of Georgia (UGA-RCLD). The UGA-RCLD provides evaluations for individuals experiencing academic difficulties in order to determine eligibility for accommodations in college classes and during exams. Four-hundred-one predominantly white undergraduate and graduate college students participated in the study. Table 8 provides descriptive information for the sample.

Table 8. Descriptive Information for Sample

	Mean	Standard Deviation
Age at Testing	22.50	6.27
Verbal IQ	107.47	11.65
Performance IQ	105.21	14.44
Full Scale IQ	106.87	12.50

As part of the evaluation process, all individuals were administered an extensive test battery including measures of intellectual ability, attention, working memory, learning, long-term memory, executive functions, reasoning, visual-spatial skills, phonological/orthographic processing, verbal fluency, vocabulary, listening comprehension, receptive syntax, social-emotional status, reading decoding, reading rate, reading comprehension, spelling, grammar, punctuation, mathematics calculation, mathematics reasoning, and mathematics fluency.

The sample was collected between 2001 and 2003 and consisted of older adolescents and adults between the ages of 18 and 57, and had a mean age of 22.50. One hundred thirty-eight individuals were diagnosed by the LDC with learning disabilities (LD), ninety-six individuals were diagnosed with Attention Deficit Hyperactivity Disorder (ADHD), and thirty-six were diagnosed with both LD and ADHD as defined according to Diagnostic and Statistical Manual of Mental Disorders – Fourth Edition (DSM-IV) criteria (American Psychiatric Association, 1994). The remaining one hundred thirty-one individuals in the sample were not diagnosed with LD or ADHD.

A team of psychologists, a linguistic expert, and Masters level clinicians relied on clinical judgment to interpret tests and make diagnoses. All students met the criteria for the diagnosis of learning disabled prescribed by the Georgia Board of Regents of the University System of Georgia, which include an average score on a measure of cognitive ability, underachievement in an academic area, and a processing deficit that can be used to explain the underachievement. No diagnoses are made on the basis of a single test score or a discrepancy formula. Rather, a careful analysis by the team of each individual's cognitive, achievement, and social-emotional profile is completed, using quantitative data, qualitative data, and clinical judgment.

Diagnoses of ADHD were based on clinical judgment by examiners after reviewing information from test results, observations, rating scales, and responses to interview questions. All diagnoses were made according to Diagnostic and Statistical Manual for Mental Disorders – Fourth Edition (DSM-IV, American Psychiatric Association, 1994) criteria.

Procedures

A full structural model and a series of three nested models were tested through a sequence of steps. First, the data were screened for outliers, nonnormality, and multicollinearity. Second, the measurement models were evaluated for identification and examined for model fit. Third, the full structural model was tested for model fit and subsequently compared to nested models (Keith, 1999).

Limitations of SEM in the Current Study

While it is possible to hypothesize and test the fit of a causal model of variables via SEM, the method does not prove causality. Instead, SEM provides a statistical method of estimating the magnitude of the effects of independent variables on criterion variables within the constraints of a causal model. The accuracy of the causal interpretations of analyses depends on how accurately the hypothesized model reflects the reality of relationships among the variables (Keith, 1999). In order to determine causality, use of an experimental design is more appropriate. However, in many instances, such as with the current study, the random assignment of subjects to groups who receive different treatments, or as in this study, different cognitive processing skills before measurement, was not possible.

In addition to the limited use of experimental design in many areas of social science research, it is also unlikely that the measures used in the social sciences, such as the tests used in the current study have perfect reliability and validity. Imperfect reliability and validity may lead to errors in the estimation of the magnitudes of path values (Keith, 1999). Validity may be helped by using multiple measures to represent latent variables. However, the accessibility of collecting multiple measures precluded doing so in the current study.

Three conditions must be met to make inferences about causality in SEM. First, the variables represented in the hypothesized model must be related to one another. Second, the assumed cause must occur prior to the assumed effect. In this study, all measures are collected at the same point in time, so this second condition cannot be fulfilled. Third, the relationship that is interpreted to be causal must not be spurious (Keith, 1999). In other words, the causal relationship between an independent variable and criterion variable depicted in the model must not be caused by a different, unspecified variable. Omitting such variables may result in inflation of the hypothesized cause on the hypothesized effect (Keith, 1999). This limitation is particularly pertinent to the population under study in the current model, as the only way to decrease the likelihood of omitting causal variables in the hypothesized model is to depict the model based on a thorough understanding of the theories and previous research relevant to the current study (Keith, 1999). However, as little prior research has been conducted regarding mathematics achievement in older adolescents and adults, the hypothesized models may be vulnerable to variable omission.

CHAPTER 4

RESULTS

Prior to running analyses, the full data set, which included 401 cases, was screened for outliers and nonnormality using DeCarlo's Macro (DeCarlo, 1997). Univariate tests of skew noted a value of 5.6335 on the WJ III Calculation variable, indicating positive skew (i.e., most values fall below the mean). All other variables contained values less than /3.0/. Univariate tests of kurtosis revealed a kurtosis index of 52.92 for WJ III Calculation, indicating a leptokurtic distribution. Kline (1998, p.82) suggested that values greater than /3.0/ represent extreme skew, and values greater than /10.0/ indicate problematic kurtosis, while kurtosis values greater than /20.0/ indicate seriously problematic kurtosis. Nonnormality of data can result in the underestimate of standard errors for parameter values, thus impacting tests of significance (Browne, 1984). To address this problem, Satorra-Bentler Scaled Chi-square was implemented to adjust chi-square and standard errors to the degree of multivariate kurtosis.

Three cases with Mahalanobis distances exceeding critical values were noted as outliers. The data set was scanned to detect obvious data-entry errors; however, none were noted. Next, SPSS, version 12.0, was used to determine upper and lower bounds for each measured variable in outlying cases in order to further scan for discrepancies in entered data. As scores in the outlying cases did not appear to deviate from other cases included in the data set, outliers were treated as extreme cases within the target population and included in the analysis. The percentage of missing values in the total sample equaled 27.64 percent with a total effective sample of 153 after listwise deletion of incomplete cases.

Initial runs of the original data resulted in a nonpositive definite input covariance matrix. Calculation of the weight matrix involves taking the inverse of the covariance matrix during the initial step of the iterative process in ML estimation. Thus, correlations greater than /1.0/ among latent variables indicate that the matrix does not have a positive determinant, which leads to an undefined value when the weight matrix is divided by the determinant. Examination of the intermediate solution revealed correlations among latent variables greater than /1.0/, indicating multicollinearity among latent variables. In addition, several path values for measured variables contained low R^2 values, indicating poor measurement of the latent factor. The issue of multicollinearity was addressed by successively deleting latent variables from the analysis. Measured variables that resulted in paths with R^2 values less than .10 were also deleted, which created one exogenous latent factor and two endogenous latent factors with single indicators. Measurement error variances of single indicators used to measure latent factors were set to the variance of the measured variables multiplied by one minus its reliability $[(1\text{-}r)(\sigma^2)]$ to account for lack of perfect reliability of the measures (Keith, 1999). After the exogenous latent factors of visual-spatial ability and vocabulary were eliminated, the analysis converged; however, negative parameter estimates for the structural equations emerged. Although these path values were nonsignificant, all covariances among variables were positive. Thus, negative parameter estimates suggest continued problems with multicollinearity among the exogenous latent factors. The resulting model included three, rather than five, exogenous latent variables, namely, long-term auditory memory, auditory working memory, and reasoning (See Figure 3). Data from the sample are presented in Table 9, including means, standard deviations, and univariate skew and kurtosis values for each measured variable included in the analysis.

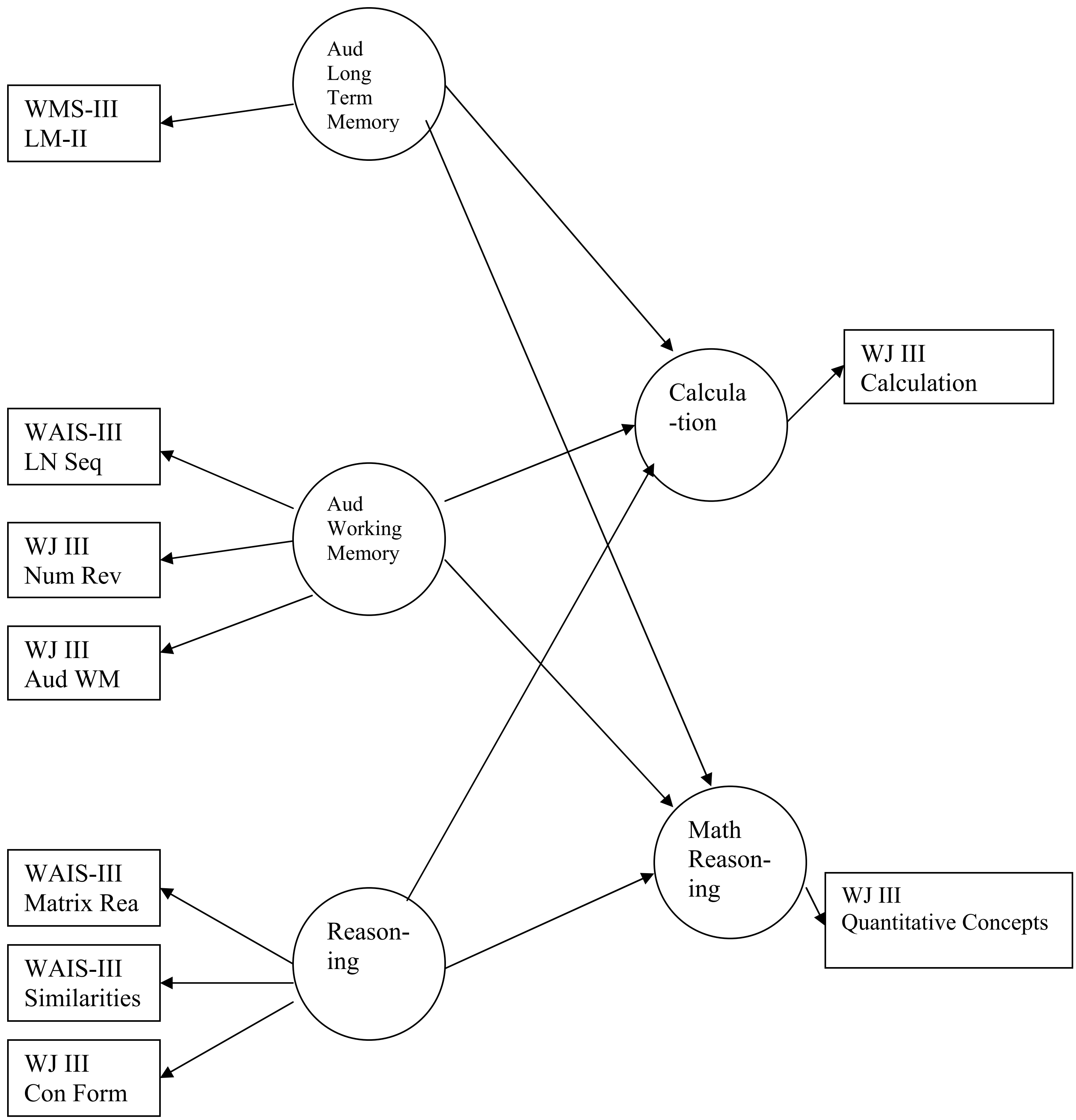

Figure 3. Revised Structural and Measurement Model

Table 9. Descriptive Information for Measured Variables

	Sim.	LNSeq	MR	CF	NR	AuWm	Calc	QC	LM
Mean	24.33	11.67	19.84	33.51	16.59	28.82	29.55	43.55	28.71
S.D.	4.01	2.46	3.55	5.77	3.95	5.55	7.32	4.53	7.13
Skew	-0.403	0.133	-1.627	-1.381	0.079	-1.196	5.634	-1.184	-0.083
Kurtosis	-0.550	-0.161	4.427	1.750	0.010	2.292	52.917	3.035	-0.672

Prelis, version 2.72, provided tests of univariate and multivariate normality for variables included in the adjusted model, in addition to constructing correlation and covariance matrixes that were later used in SEM analyses. Multivariate kurtosis values, as opposed to skewness, are typically considered to evaluate normality of data used in SEM analyses given the impact that kurtosis has on variances and covariances (DeCarlo, 1997; Olsson et al., 2000). Relative Multivariate Kurtosis was 2.029, slightly above the /2.0/ cutoff suggested by Jöreskog and Sörbom (1996). The correlation matrix of observed variables (See Table 10) was constructed to screen for bivariate multicollinearity among variables. Correlations among observed variables were relatively low. None of the correlations among measured variables were greater than .50, including those hypothesized to measure the same latent factor, which suggests that some factors may not be represented well by the measured variables. For example, while both the auditory working memory and reasoning factors were represented by three indicators, the highest correlation among indicators for auditory working memory was .463, and for reasoning, .427.

Table 10. Correlation Matrix of Observed Variables

	Sim.	LNSeq	MR	CF	NR	AuWm	Calc	QC	LM
Sim	1.000								
LNSeq	0.326	1.000							
MR	0.359	0.234	1.000						
CF	0.421	0.360	0.427	1.000					
NR	0.326	0.463	0.250	0.376	1.000				
AuWm	0.201	0.456	0.149	0.380	0.413	1.000			
Calc	0.135	0.128	0.198	0.303	0.131	0.050	1.000		
QC	0.326	0.282	0.458	0.482	0.313	0.268	0.137	1.000	
LM	0.278	0.151	0.051	0.291	0.148	0.198	0.033	0.190	1.000

Satorra-Bentler Scaled Chi Square indicated good fit of the structural model ($\chi^2 = 13.86$ (21), $p = .88$). As the RMSEA was less than the suggested .08 cutoff, and the NNFI and CFI were greater than .95, the model was considered to be a good fit (See Table 11). Values of greater than 1.0, such as the NNFI, indicate an overidentified model with almost perfect fit to the data (Kline, 1998, p. 129). In other words, there are more paths in the model than are needed for a good fit.

Table 11. Fit Indexes for Full Structural Model

	DF	SB X^2	RMSEA	NNFI	CFI
Full Model	21	13.86 ($p = 0.88$)	0.0	1.03	1.00

Model fit was further assessed by reviewing individual path t-values (see Table 12) and standardized residuals (see Table 13). All path values for measurement equations were significant; however, R^2 values for WAIS-III Similarities and WAIS-III Matrix Reasoning were relatively low (< .40), indicating poor measurement of these variables on their respective factors. In addition, path values for the long-term memory and auditory working memory factors were not significant on either of the structural equations. R^2 values were low to moderate, .14 for mathematics calculation and .44 for mathematics reasoning. Two standardized residuals (approximately 6 % of the total elements in the residual matrix) had values greater than /2.0/, indicating adequate model fit. As expected, all measurement path values were positively correlated with latent variables; however, unexpected negative path values were detected between exogenous and endogenous latent variables (See Table 14).

Table 12. Parameter Estimates for Measurement Equations

Path From:	Path To:	Path Value	T-Value	R^2
MCAL	Calc	----------	--------	0.86
MREA	WJQC	----------	--------	0.91
REA	Sim	2.27	8.58	0.32
AWM	LNSeq	1.71	8.70	0.48
REA	MR	2.00	4.54	0.32
REA	Con	4.51	7.84	0.61
AWM	NR	2.66	7.70	0.45
AWM	AuWm	3.50	6.89	0.40
LTM	LM	6.89	20.24	0.93

Table 13. Standardized Residuals

	Calc	WJQC	Sim	LNSEQ	MR	Con	NR	AuWm	LM
Calc									
WJQC	-0.25								
Sim	-3.38	-1.15							
LNSeq	0.51	-0.28	1.21						
MR	1.62	1.22	0.78	-0.51					
Con	-----	-0.34	-1.37	-0.24	-0.14				
NR	1.69	0.49	1.16	-0.13	-0.15	0.16			
AuWm	-0.41	-0.10	-1.11	0.33	-1.49	0.37	-0.54		
LM	-1.42	-0.01	1.71	-0.34	-2.68	0.71	-0.33	0.65	

Table 14. T-Values for Structural Equations

	LTM	AWM	REA	R^2
MCAL	-1.49	-1.07	3.23	0.14
MREA	-0.29	-0.0053	2.54	0.44

As can be seen in Table 14, the long-term memory and auditory working memory factors were not correlated with the endogenous latent factors, mathematics calculation and mathematics reasoning.

Modification Indexes (MI) suggested that allowing WJ III Calculation to load on mathematics reasoning, and conversely, allowing WJ III Quantitative Concepts to load on mathematics calculation would result in significant path values, suggesting that the endogenous latent factors may be highly related, or measuring the same construct. MIs also suggested significant path values resulting from allowing WAIS-III Similarities and WAIS-III Matrix Reasoning to load onto the long-term memory factor, which again points to poor representation of the long-term memory factor. While several MIs suggested allowing measurement errors to covary, none were considered theoretically sound.

Nested Model Analyses

To assess the hypothesis that exogenous latent variables (i.e., auditory long-term memory, auditory working memory, and reasoning) differentially impact mathematics calculation and mathematics reasoning, a set of procedures using nested models was employed. These procedures created nested models by successively setting the paths from one exogenous latent variable at a time to each endogenous latent variable (i.e., mathematics calculation and mathematics reasoning) equal to each other. Thus, the two models are hierarchically related, as Model 1 (See Figure 4) is nested under the Full Structural Model, as are Models 2 and 3 (See Figures 5 and 6). Satorra-Bentler chi-square differences were computed. The value of chi-square generally increases as paths are eliminated, so the goal is to find the most parsimonious model that fits the data as well as the full structural model (Kline, p. 132). Thus, in this instance chi-square difference tests provide a test of whether imposition of the equality constraints in the more constrained models (i.e., Models 1, 2, and 3) result in a significant decrement in the fit of the nested model. A significant chi-square difference implies that values of the parameters actually differ significantly from one model to the next, and therefore, do not predict equally well. It should be noted that while fit indexes and examination of residuals for the full model indicated good fit, problems detected in the measurement of exogenous latent variables and low correlations among exogenous latent variables and endogenous latent variables attenuate the results of nested model analyses. For this reason, only the Satorra-Bentler chi-square difference test conducted between the Full Structural Model and Model 3 is discussed in detail, as among the three exogenous latent variables, reasoning appears to have the strongest relationship to the mathematics calculation and mathematics reasoning variables. However, the cursory results of all nested model analyses are reported below.

In addition to the Satorra-Bentler Scaled Chi-Square, the Comparative Fit Index (CFI; Benter, 1990), the Non-Normed Fit Index (NNFI or TLI; Tucker and Lewis, 1973), and the Root Mean Square Error of Approximation (RMSEA; Steiger, 1990) were employed to evaluate fit of nested models. Thus, primary statistics used to assess change in model fit were the Satorra-Bentler chi-square difference tests as well as inspection of other fit indexes. Significance of path values and R^2 values were used to evaluate models for misspecification.

Results from model comparisons (See Table 15), revealed non-significant Satorra-Bentler chi-square difference tests ($\rho > .05$) for each of the nested models (i.e., Models1 -3) when each model was compared with the Full Structural Model, indicating that the overall fits of the models are comparable. It should be noted that neither the path from long-term memory, nor from auditory working memory to endogenous latent variables was significant. When paths that are not significant are deleted from the model, or constrained, as in this study, it is not surprising that the chi-square difference test is also non-significant (Kline, p. 133). Therefore, the most interesting finding from the nested model analyses is that Model 3 did not demonstrate a significant decrement in fit when the paths between reasoning and the endogenous latent variables were constrained to be equal. Examination of the parameter estimates for the structural equations in Model 3 indicated that the path between reasoning and each of the endogenous latent variables

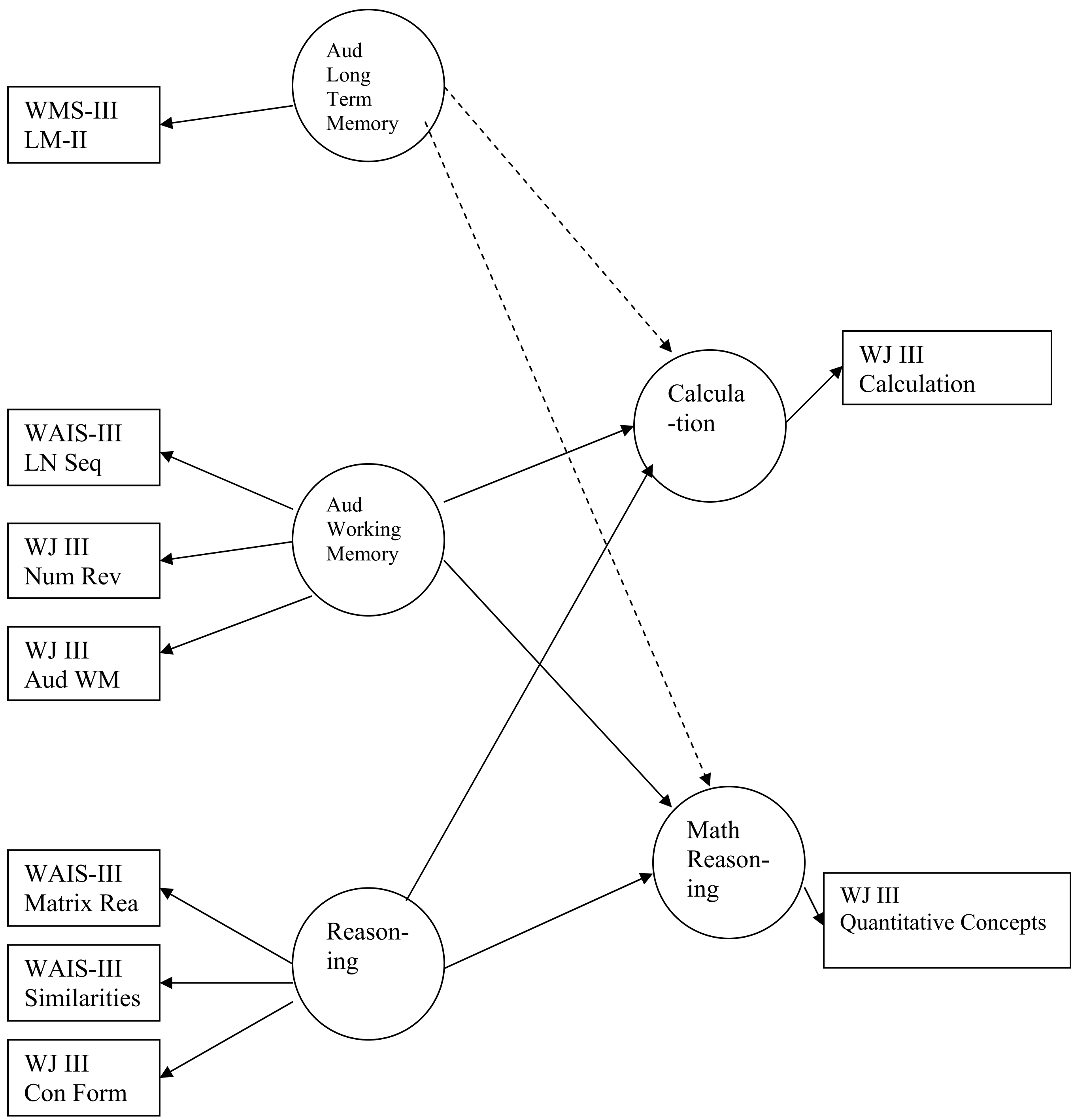

Figure 4. Model 1

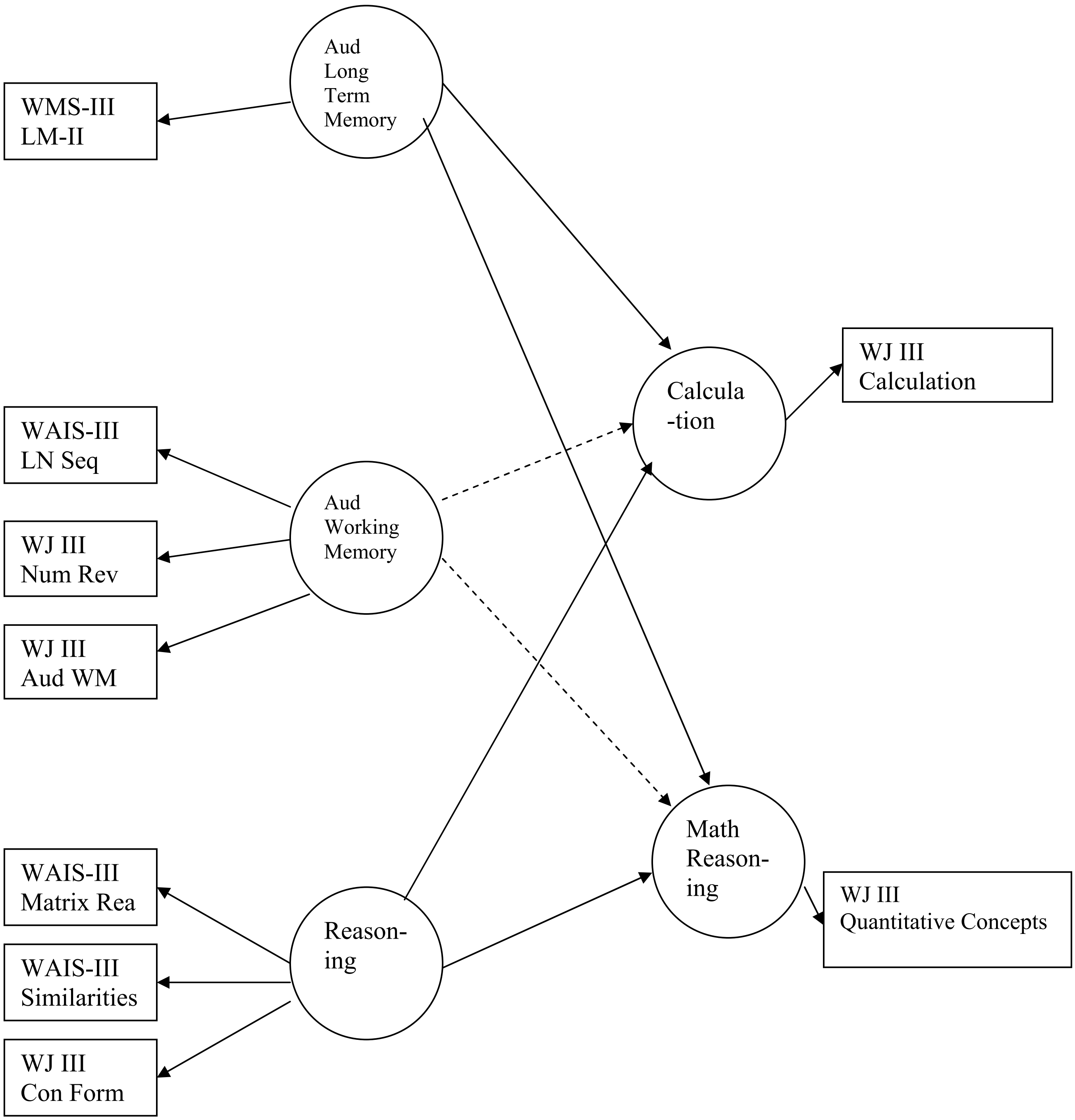

Figure 5. Model 2

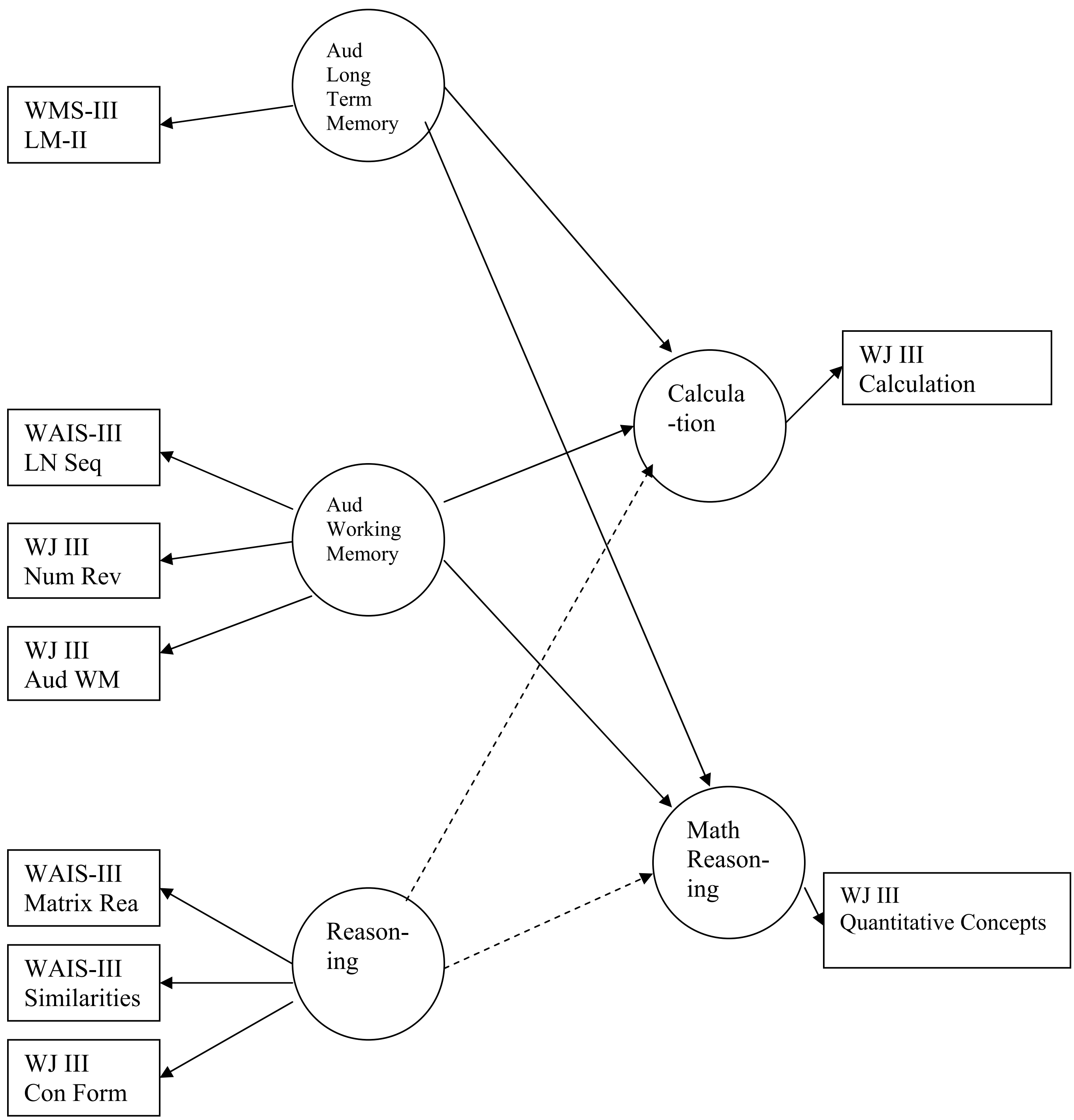

Figure 6. Model 3

was significant, and R^2 values for mathematics calculation and mathematics reasoning were .13 and .45, respectively, indicating that very little of the variance in mathematics calculation was accounted for by exogenous latent variables. As chi-square did not demonstrate a significant change from the Full Structural Model to Model 3, it is not surprising that Model 3 provided good fit to the data, given that the Full Structural Model also demonstrated good fit. Satorra-Bentler Scaled Chi-Square was non-significant, and other fit indexes demonstrated good fit (CFI = 1.00, NNFI = 1.03, and RMSEA = 0.0). However, small R^2 values for the structural equation for mathematics reasoning and generally low and negative path values of long-term memory and auditory working memory on mathematics calculation and mathematics reasoning indicate model misspecification.

Table 15. Nested Model Analyses

	SB χ^2	df	CFI	NNFI	RMSEA	Δdf	Δ SB χ^2
Full Structural Model	13.86	21	1.00	1.03	0.0	---	---
Model 1	14.32	22	1.00	1.03	0.0	1	0.40
Model 2	14.12	22	1.00	1.03	0.0	1	0.24
Model 3	14.31	22	1.00	1.03	0.0	1	0.07

* Each Model comparison was made with the Full Structural Model

CHAPTER 5

DISCUSSION

The purpose of this study was to contribute to the current research regarding the relationships between cognitive and linguistic processes and mathematics achievement. A series of models were tested to examine the strengths of relationships between auditory long-term memory, auditory working memory, and reasoning and mathematics achievement and to compare the differential effects of auditory long-term memory, auditory working memory, and reasoning on mathematics calculation and mathematics reasoning in adults. This study demonstrated that the cognitive processing domains of auditory long-term memory, auditory working memory, and reasoning contribute to variation in the performance of adults on measures of mathematics calculation and mathematics reasoning. The first goal of the study was to assess overall fit of the cognitive model. While fit indexes suggested adequate fit of the structural model, R^2 values for the structural equations suggested that a large proportion of variance was unaccounted for by the latent variables, particularly in regard to variation in the mathematics calculation factor. Low correlations between the auditory long-term memory and auditory working memory factors and the endogenous latent factors (mathematics calculation and mathematics reasoning) and nonsignificant parameter estimates for these factors were detected. In contrast, the parameter estimate for reasoning was significant for both the mathematics calculation and mathematics reasoning factors, indicating that reasoning demonstrated the strongest relationship with both mathematics calculation and mathematics reasoning. However, several indications of mulitcollinearity were detected. While removing the visual-spatial and vocabulary factors from the original model helped address this problem and allowed the model to converge, the presence of negative path values in the structural equations for the auditory long-term memory and auditory working memory factors continued to suggest linear dependency of the factors. This issue points to overlap in skills measured by the factors. Choosing different indicators and refining the factors to more specific areas of measurement (e.g., verbal reasoning) to improve the model for future studies is suggested. While the paths chosen for this model appear to fit the data, other models may fit equally well. Future studies may determine that more specific measures of the cognitive processes examined in this study provide good fit to the data while eliminating multicollinearity problems due to overlap in cognitive abilities. A further improvement may be made by including visual-spatial abilities and linguistic processing measures, as these processes are supported by research, and omission of them may lead to model misspecification.

The second goal of this study was to conduct nested model analyses to determine the differential impact that auditory long-term memory, auditory working memory, and reasoning have on mathematics calculation and mathematics reasoning. However, as the auditory long-term memory and auditory working memory factors were not correlated with mathematics calculation and mathematics reasoning, it was not theoretically justifiable to compare the differential impact of these factors on the endogenous latent factors, and any results of such analyses would not be meaningful. While reasoning did demonstrate a significant relationship to both mathematics calculation and mathematics reasoning, constraining the paths between reasoning and the endogenous latent factors did not demonstrate a significant decrement in fit, indicating that the reasoning factor did not demonstrate a differential impact on mathematics calculation and mathematics reasoning. However, refining the reasoning factor by choosing different indicators and/or improving the measurement of the mathematics calculation and mathematics reasoning factors may result in different findings.

The results of this study are consistent with the literature regarding the developmental trajectory of mathematics achievement, as young children appear to transition from counting as a primary strategy in solving mathematics calculation problems to memory retrieval when they reach approximately sixth grade. Previous research conducted with child populations has supported the theory that once children are able to retrieve mathematics facts from memory rapidly and automatically, their attentional resources may be allocated to other higher-order processes (Gersten et al., 2005). Thus, long-term memory skills are equally important for both children and adults, to proficiently learn and memorize basic mathematics facts and to allow allocation of attentional resources to higher order skills. However, very little is known about how older adolescents and adults use cognitive processing abilities to solve higher-order mathematics problems such as algebra and geometry (Geary, 2005). Several investigations, such as one conducted by Floyd et al. (in press) suggest that reasoning skills are more strongly related to mathematics achievement in adults than long-term memory skills. The results of the current study provide support to the theory that cognitive processes other than long-term memory, such as reasoning and auditory working memory make significant contributions of variance to adult achievement in mathematics. This study hypothesized that specific cognitive abilities demonstrate a differential impact on mathematics calculation and mathematics reasoning. A frequent problem that occurs when using broad-band measures of achievement, as was employed in this study, is the lack of specificity in measuring a vast array of mathematical skills across a number of skill domains (Geary, 2005). This lack of specificity in skill measurement may have resulted in the failure of the nested model comparisons in this study to identify differential effects of cognitive processes on mathematics calculation and mathematics reasoning. Future studies should continue to investigate the strength of these relationships and consider whether reliance on long-term memory in childhood diminishes in favor of reasoning skills in adulthood.

Limitations

While results of SEM analyses indicated a good fit of the Full Structural Model to the data according to a combination of commonly employed fit indexes, examination of standardized residuals, and significant path values for measurement equations, several measured variables, represented by subtests on nationally standardized and widely used measures of cognition and achievement, did not appear to adequately measure the latent factors. The most severe measurement problems were apparent with the long-term memory and auditory working memory factors, as evidenced by low correlations among observed variables representing the auditory working memory factor and modification indexes that suggested improved fit with the addition of paths to the long-term memory factor. While measurement issues related to the long-term memory and auditory working memory factors appeared most problematic, low factor loadings on the endogenous latent factors made it necessary to reduce the representation of each factor to single indicators, compromising the integrity of the mathematics achievement factors. While deletion of the Math Fluency and Applied Problems subtests improved the measurement of the endogenous latent factors from a statistical standpoint, allowing single indicators to represent factors often leads to psychometric inadequacy of measured variables and increased measurement error (Keith, 1999). One manifestation of this problem in the current study can be seen with the Quantitative Concepts subtest chosen to represent the mathematics reasoning factor. In addition to items requiring an examinee to complete number sequences, the subtest includes items that tap knowledge of mathematics terminology, compromising the purity of the subtest as a measure of mathematics reasoning. The mathematics calculation factor was also reduced to include only a single indicator when the Math Fluency and Calculation subtests making up the factor appeared unrelated. However, Calculation, the indicator chosen to represent the mathematics calculation factor was severely skewed and leptokurtic.

The combination of measurement problems and mulitcollinearity among latent factors, resulted in nonpositive definite matrixes upon initial runs of the data. While eliminating the visual-spatial ability and vocabulary factors from the analysis helped solve some of the problems caused by multicollinearity, the presence of negative parameter estimates for structural equations after the removal of these latent factors, suggested continued problems with multicollinearity among exogenous latent factors, particularly in regard to long-term memory and auditory working memory.

While comparisons of nested models indicated that none of the exogenous latent factors contributed to mathematics calculation and mathematics reasoning differentially, measurement problems related to latent factors precluded definitive results. Due to low correlations among the measured variables chosen to represent the factors and nonsignificant parameter estimates for paths between the long-term memory and auditory working memory factors and endogenous latent variables, nonsignificant differences between nested models and the Full Structural Model were anticipated. In addition, the questionable measurement of the endogenous latent factors rendered the nested model analyses less meaningful.

While reasoning demonstrated the strongest relationship to endogenous latent variables, correlations among the measured variables representing the reasoning factor were moderate at best. Furthermore, R^2 values for structural equations were low to moderate, .14 and .44, for mathematics calculation and mathematics reasoning, respectively. While improving measurement of the latent factors may increase R^2 values, it is likely that misspecification of the model resulted from omission of causal variables. Omission of these variables can lead to biased parameter estimates of paths included in the model (Keith, 1998, p. 99).

Because the sample utilized in this study consisted of individuals referred for evaluation due to academic difficulties, the seemingly poor measurement of cognitive factors calls into question whether these measures assess individuals with learning difficulties in the same way that they assess normally-achieving adults. The utilization of an at-risk population may also restrict generalizability of the results to other populations. The size of the sample presented another limitation to the study, and also impacts generalizability. Although the original sample consisted of 401 participants, the sample decreased to 153 participants after listwise deletion. While medium-size samples (i.e., between 100 and 200 participants) are not uncommon in SEM studies, larger samples are recommended as models become more complex. Insufficient sample sizes can lead to sampling error, instability of estimates, and less power (i.e., the likelihood that null hypotheses are rejected correctly) (Kline, 1998).

Future Studies

In addition to the tests included in this study, many other neuropsychological and educational measures exist that may more accurately measure these cognitive processes for future research. While it is probably not prudent to define latent constructs too narrowly, some of the cognitive processing measures included in this study may be replaced with measures that refine the scope of the cognitive processes investigated, such as verbal versus visual working memory and long-term memory, and verbal versus nonverbal reasoning abilities. In addition, mathematics reasoning and mathematics calculation require sound measurement to determine if they are indeed separate constructs, and if so, if they are differentially impacted by specific cognitive processes. It should be noted that the measures included in this study were comprised of broad measures of mathematics achievement, rather than component processes of mathematics calculation and reasoning.

Future investigations into mathematics achievement in adults should also address the contributions of other cognitive processes that were not included in this study but are clearly supported by research, such as visual-spatial skills, executive functioning, processing speed, and linguistic processes. Unfortunately, the deletion of the vocabulary factor precluded the possibility of assessing the contribution of the other factors over and above the contribution of vocabulary as a language measure. Future studies should further investigate the contribution of language to mathematics achievement.

While it was beyond the scope of this study to investigate the impact of cognitive processes on mathematics achievement associated with learning and emotional disorders, studies investigating these processes in individuals with learning disabilities, ADHD, and other disorders will make important contributions to the mathematics achievement literature.

Conclusion

This study supported the relationship of auditory long-term memory, auditory working memory, and reasoning with mathematics achievement in adults. While nested model analyses did not appear to support differential effects of the cognitive processes on mathematics calculation and mathematics achievement, measurement problems attenuated results of nested model analyses, making further investigation of this hypothesis an important consideration for future research. Improving measurement conditions may result in higher R^2 values, indicating higher proportions of variance accounted for by the cognitive processes included in this study. However, re-specification of the model by including omitted cognitive processes may also contribute to variance in mathematics achievement. Studies such as these, which investigate the relationships among cognitive processes and mathematics achievement in adults may lead to a better understanding of the differences between the manner and method that children and adults learn and process mathematics information. Creation of such knowledge will, hopefully, lead to better instructional techniques for college students and accommodations and interventions for older students who demonstrate difficulties in mathematics.

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