MOTION AND THE ACTUALITY OF THE INFINITE IN ARISTOTLE'S *PHYSICS* III

by

JOSEPH P. CARTER

(Under the Direction of Richard D. Winfield)

In this dissertation, I examine Aristotle's *Physics* III exegetically and philosophically to reconstruct how Aristotle discovers the nature of motion as an incomplete actuality, which he uses to argue for an actual infinite. The standard interpretation of Aristotelian infinity is that it exists *only* potentially because, as Aristotle shows in *Phy*. III.5, no actual infinite physical body exists whatsoever. However, I contend that because the infinite must exist in order to preserve time, magnitudes, and number, and because any existing thing has some degree of actuality, then the infinite must in some way be actual. The difficulty is figuring out *how*. I argue that the infinite exists actually as a proximate attribute of motion and primarily as an essential attribute of physical substances. The infinite is an attribute of motion since motion is indefinite. Evidence for this is that in *Phy*. III.6, Aristotle argues that the infinite is actual like the day and the Olympic games because they are inherently incomplete insofar as they are *always* in a state of becoming. The day and the games serve as models for how the constant incompleteness of motion is the very phenomenality of infinity. However, as Aristotle argues in *Phy*. III.1, motion manifests differently across the highest categories of being. This means that not all motions will be strictly the same. Motion is one only by analogy. So, to which motion does the infinite properly belong? Since Aristotle defines the infinite as a sort of quantity, I argue that the infinite belongs to

quantitative change, specifically the activities of division and addition. These activities are inherently incomplete, just like the day and the games, whereas all other motions eventually reach completion. But not only is there a peculiar sort of actuality of infinity with respect to motion. I also argue that the actuality of infinity is an essential attribute of physical substances insofar as physical substances undergo division by nature. Substance is the primary source of the actuality of infinity since when actualized in the act of division, albeit incompletely, infinity is always *within* the actual finite substance.

INDEX WORDS: Aristotle, Plato, physics, motion, infinity, actuality, potentiality, division, addition

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DEDICATION

For my parents and my brother.

Leb wohl gegenwärtiges Leben, das ich führe.

Du kannst so nicht bleiben. Vornehm warst du.

Reiner Geist. Still und einsam.

Leb wohl Ehre beim ersten öffentlichen Schritt.

- Paul Klee, *Tagebücher* #725

…Moon dives beneath indigo loam at the horizon, filling her craters with

hairy golden roots, murmuring earthworms asleep,

and five hundred million hibernating gems

which she surrenders to Plato's Unending.

Each of my fingers becomes a fragrant, violent petal,

and with each step I bruise my new petal-toes.

Here is where I go when I love you…

- Jessie Eisenmann, from *Untitled*

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Dissertations are inconspicuous testaments to friendship and community. As much as one might spend long hours, even days and nights, laboring in isolation parsing conceptual granularities one after another, none of it is ever accomplished apart from a network of people whose company anchors and nourishes one's life. As Friedrich Hölderlin once wrote, "It is good to hold fast unto another. For no one can carry this life alone" (*The Titans*).

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To all of you with love—thank you. This is only the beginning.

ABBREVIATIONS

Abbreviations for Ancient Greek authors follow the standards set forth in the *Oxford Classical Dictionary* except for the following Aristotelian texts:

APo. – Posterior Analytics

DA – De Anima or 'On the Soul'

DC – De Caelo or 'On the Heavens'

GC – On Generation and Corruption

Meta. – Metaphysics

Phy. – Physics

SS – Sense and Sensibilia

Translations of Ancient Greek texts are my own unless explicitly cited otherwise. For Aristotle's Greek, I have used the standard Oxford Classical Texts of the *Corpus Aristotelicum*. The standard Greek texts for Plato are the Burnet editions of the *Platonis Opera.* For the Presocratics Greek texts, I have consulted the Diels-Kranz (DK) edition of *Die Fragmente der Vorsakratiker* and Daniel Graham's recent edition of *The Texts of Early Greek Philosophy.* For Aristotle's ancient commentators, I have also consulted the *Commentaria in Aristotelem Graeca.*

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INTRODUCTION

Ralph Waldo Emerson, championing the Heraclitean impermanence of nature, declares in *Circles* (1841) that "[e]verything looks permanent until its secret is known […] Permanence is but a word of degrees."[1] To be sure, Aristotle is no Emerson. The highest kind of reality for Aristotle is indeed permanence—ἐντελέχεια. An ἐντελέχεια is an 'actuality,' or a 'being-at-work-*staying-itself*.'[2] This is because a thing's form, when fully completed, pertains to an activity that sustains itself. A prime example of this is the unmoved mover, for as thought-thinking-itself, it *always* remains the same activity (*Meta.* Λ.9). Therefore, an ἐντελέχεια is *not* changing. Still, ἐντελέχεια has a secret all of its own, especially in nature. At first glance, so much, if not all, of the natural world looks impermanent. For Aristotle, though, even impermanence has an immutable quality. The scope of actuality is so broad that it applies to things that are by their very nature transient and *incomplete*—especially motion. Motion (κίνησις/μεταβολή) is defined as an ἐντελέχεια, albeit incomplete (*Phy.* III.1, 201a9-10). In many ways, this seems strange because how could change have anything about it that is unchanging? As it turns out, when it is happening, motion in a way remains the same. It's an activity; even though something is undergoing a change, the changing itself retains the same character—incompleteness. What is

[1] Ralph Waldo Emerson, "Circles," *Essays and Lectures* (New York, NY: Library of America, 1983), p. 404.

[2] Joe Sachs, *Aristotle's Physics: A Guided Study* (New Brunswick, NJ: Rutgers Univ. Press, 1998), p. 245 explains 'being-at-work-staying-itself' as a "fusion of the idea of completeness with that of continuity and persistence. Aristotle invents the word by combining *enteles* (complete, full-grown) with *echein* (=*hexis*, to be a certain way by the continuing effort of holding on in that condition), while at the same time punning on *endelecheia* (persistence) by inserting *telos* (completion)."

noteworthy, then, is that motion is intelligible. As an ἐντελέχεια, an entity's motion may be defined and understood as an incomplete actuality.

In this dissertation, I suggest something a bit more audacious: in *Phy.* III, Aristotle uses the account of motion (*Phy.* III.1-3) to give a positive account of the infinite (*Phy.* III.4-8). I argue that in *Phy.* III.6-7, Aristotle ascribes a peculiar sort of actuality to infinity (ἄπειρον) insofar it is *proximately* attributed to an actual motion, namely the activities of division and addition. A 'proximate attribute' is an attribute that belongs to a subject in which it most immediately appears, even though that subject might not be the primary subject of the attribution. Infinity manifests most immediately with respect to motion, since (as I will show) motion is incomplete and incompleteness is a kind of unboundedness (ἄπειρον). Aristotle is clear that there is no actual infinite body (*Phy.* III.5). Nevertheless, in nature, infinity must exist. Aristotle claims that not only does the infinite exist, it is *essential* or '*per se*' to nature (*Phy.* III.1, 200b19-20). For, infinite divisibility is a necessary condition to physicality. Without it, time, extension, and number cease to exist. How then does infinity exist? The typical view is that the infinite exists *only* potentially. As I read Aristotle, however, because it exists even as a potentiality, the infinite must bear some actuality. For, if there is anything axiomatic about Aristotelian ontology it is that existence entails actuality to some degree or another. This is because what exists has a form, and the form indicates the actuality. Since Aristotle *defines* the infinite as 'that outside of which there is always something more,' the infinite too has a form. Thus, the infinite has an actuality. The question is how do we understand this actuality? Not everything is actual in the same way, since there are different forms, and not all forms are fully actual, e.g. motion. Like Emerson said, "permanence is but a word of degrees." But, if the infinite is actual in any way, it too is intelligible, which might be surprising, since intelligibility

is usually with respect to what is finite and definite. The problem then is precisely that the infinite has an actuality, since forms typically pertain to finite entities. So, the difficulty for Aristotle, let alone any reader of the *Physics*, is to figure out *how* the infinite is intelligible as an actuality.

In chapter 1 of this dissertation, I will lay out the problem, the method, and the literature for solving the problem. In order for time, magnitudes, and number to exist, the infinite must exist. For Aristotle, time is infinite because it has neither a beginning nor an end, magnitudes are infinitely divisible because they are continuous quantities, and number has no upper limit since it is always possible to add one more number. However, Aristotle makes clear in *Phy.* III.5 that no actual infinite substance or attribute exists. But, the infinite must exist as attribute of nature. The standard interpretation of Aristotle's infinite suggests that the infinite exists only potentially as an attribute. But, anything that exists, for Aristotle, is actual to some degree. This applies to the infinite too. And this is really the problem: how can the infinite exist actually as an attribute when actuality typically pertains to finite things, including attributes? The solution is that the infinite exists *proximately* as an actual attribute of *motion* and ultimately an essential attribute of moving substances. The method used to arrive at this solution is by first examining what motion is, and thereby attributing the infinite to motion, since motion is inherently indefinite as it is happening. The specific motions to which the infinite belongs are the activities of division and addition since these are *always* incomplete. But since motion itself is attributed to substances, Aristotle may also ultimately attribute the actual infinite to the *moving* substance insofar as substances actively undergo division by nature. Because I interpret *Physics* III differently than many mainstream interpretations, I will also place my reading in the context of the scholarship.

Chapter 2 addresses the account of motion in *Phy*. III.1-3. Motion is defined as the 'actuality of potentiality qua potential.' Essentially, motion is an incomplete actuality or what I will call an *active potentiality*. However, this is an *equivocal* definition since the actuality and potentiality in each category that admits of motion are unique. This precludes Aristotle from providing a scientific definition of motion. That is to say, the definition of motion is not a formula (λόγος) of an essence (τό τι ἦν εἶναι). It does not define an individual thing. A scientific definition indicates individuals univocally since it indicates a single specific nature (*Meta*. Z.4; *APo*. II.2-31). This sort of definition is univocal since it pertains to *one* kind of thing. Motion, however, lacks this strict character because it indicates different motions depending on the *kind* of thing undergoing a change. Substantial change is not the same as alteration, and alteration is generically distinct from increase/decrease, etc. The reason for this is that different kinds of forms and materials cause each type of motion, but there is no common form and material that applies to all motion in the same way. Despite this, Aristotle clearly defines motion at *Phy*. III.1, 201a9-10 as the actuality of a potentiality *qua* potential. How is this possible? Drawing from discussions about likeness (ὁμοίος) in *Topics* I.18 where Aristotle shows how to construct analogical definitions, I will argue that because motion crosses multiple categories Aristotle's solution is to make its definition analogical. Aristotle can define motion only on the grounds of partial similarities between substantial change, qualitative change, quantitative change, and locomotion despite the fact that each shares no common form and material.

More importantly, because motion is defined as an incomplete actuality, it is also inherently indefinite. Aristotle argues for this in *Phy*. III.2. So, I will also show in chapter 2 that motion's incompleteness is the first indication of the connection between motion and infinity. Motion is incomplete since it is *in-between* two determinate actualities: the actuality of the

material and that of the new form. Since motion is neither, it is indefinite. For, as an incomplete activity, there is something more of the final actuality left to obtain. That is to say, there is more of the potential left to be taken in the process. This squares with Aristotle's definition of the infinite in *Phy*. III.6 as 'that outside of which there is always something more.' But while in *Phy.* III.2 it appears that this applies to all motions in the same way, Aristotle will later show in *Phy.* III.6 that the infinite is only properly attributed to quantitative change, which I will show in chapter 4.

Chapter 3 addresses the existence of the infinite in *Phy.* III.4-5. *That* the infinite exists Aristotle does not deny. According to Aristotle's predecessors, in *Phy.* III.4, infinity is a principle of motion. In *Phy.* III.5, however, we see Aristotle argue that there is no actual infinite body whatsoever precisely because it undermines motion. This is because the infinite seems to be neither an actual substance nor an actual attribute, and motion exists only in respect to substance and its attributes. But this leaves us in an aporia. There are only two ways to exist for Aristotle, either substantially or as an attribute. The infinite seems to be neither on this account. But the infinite must exist at least attributively because time, number, and magnitudes exist, each of which assumes the infinite as a necessary condition. But, if the infinite exists, it must do so actually to some degree since any existing thing has some measure of actuality.

How ought the infinite exist? What is infinity, if neither an actual substance nor an attribute of a body? Textually, Aristotle twice calls the infinite actual (*Phy.* III.6, 206a24-25 and 206b13). Aristotle does not deny its actual existence; again, the problem is *how.* Chapter 4 of this dissertation shows how Aristotle argues that the infinite exists 'like the day and the games' as a necessary attribute of motion as *both* actual *and* potential. In this way, I will show that the infinite is something like motion's active potentiality. It is an inherently incomplete actuality.

Nevertheless, because motion is defined analogically, motion in general is not instructive for revealing the actuality of the infinite. Rather, the infinite is an attribute of a specific motion. The specific motion to which it belongs most proximately is *quantitative*: the activities of division and addition. The acts of dividing and adding are active potentialities, as I will argue. Because the act of division happens in physical magnitudes, which are continuous, division is inexhaustible in the direction of reduction whereas addition is infinite in the direction of increase insofar as the divisions are *counted* without reaching an upper limit. As division reduces the magnitude, the number of divisions increases. This means that division never traverses the entire magnitude. So, as I will show, the actuality of the infinite is the *manifestation* of the physical magnitude's inexhaustible capacity to be divided in the very act of dividing and counting. Since magnitudes are primarily attributes of physical substances—whole finite bodies—the infinite as an activity will also be a part of the physical substance.[3] In this way, I will show also that Aristotle's infinite is ultimately an *essential attribute* of the physical substance insofar as substances most properly undergo division.

[3] In this dissertation, I will use 'physical substance' interchangeably with 'whole finite body.' For Aristotle, a physical substance is essentially a whole finite body containing all of its parts functioning as together as *one* body bounded by a surface. This applies to both natural beings like organisms and also artifacts.

CHAPTER 1

THE PROBLEM OF MOTION AND THE INFINITE IN *PHYSICS* III

§1.1. Motion, infinity, and the problem of attribution

Every Aristotelian science investigates a subject matter—a genus and its *per se* attributes (*APo* I.28, 87a38-39; I.10, 76b11-22). Physics (φυσική), since it is indeed claimed to be a science (*Phys.* I.1, 184a10-15; *DC* I.1, 268a1-6; *Meta.* K.7, 1064a10), is no different. The genus is nature (φύσις). Assuming this, there ought to be a set of *per se* attributes. A *per se* (καθ'αὐτό) attribute is what belongs to something universally and necessarily. It is what belongs to something *essentially.* At the beginning of *Phy.* III.1, Aristotle identifies motion and the infinite as some of the *per se* attributes of nature because every physical thing is capable of some sort of motion, and time has neither a beginning nor end and magnitudes are infinitely divisible. Nevertheless, motion and infinity are inherently difficult to know (*Phy.* III.2, 206b33; III.4, 207a30-32). On the one hand, motion is an incomplete actuality since it is neither the latent material/potential out of which it begins nor the form/complete actuality to which it aims; yet, an 'actuality' is complete by definition. Does this mean that motion is *both* incomplete *and* complete? How are we to understand this? On the other, the infinite somehow exists in a determinate cosmos wherein every physical substance and its attributes are by definition finite. How does the infinite exist as an attribute in a world populated by nothing but finite entities?

With these questions in mind, the fact that Aristotle deals with the nature of infinity *immediately* after the account of motion reveals one of the exigent problems in *Physics* III: what

is the connection between motion and infinity, and how does this connection enable Aristotle to secure the infinite's *actual* existence in the natural order? Like motion, the infinite is a strange entity. So much of the being of the infinite must be understood in terms of potentiality because it pertains to the magnitude's *capacity* to be endlessly divided, and yet potentiality is understood *always* with respect to a corresponding actuality. As an existing thing, then, the infinite must be actual to *some* degree. An indication of this is at *Phy.* III.6, 206a18-25 and 206b12-14 where Aristotle twice calls the infinite an actuality *in connection to* motion:

> What remains, then, is the infinite as a potentiality. But, it is not necessary to take 'potentiality' just as if something were potentially a statue, since what is potentially a statue *will be* a statue, and in this way the infinite *will be* in actuality. But since being is said in many ways, just as the day and the Olympic games come to be as always different, in this way the infinite exists. (For with respect to these things, there is something that is <u>*both* potentially *and* actually</u>, for the Olympic games exist in the sense that the games are *both* capable of occurring *and* that they *are* occurring) (*Phy.* 206a18-25).

> In no other way, then, does the infinite exist than potentially and by reduction [of the magnitude] (but additionally, <u>it exists in actuality</u> [ἐντελεχείᾳ] as we say the day and games exist) (*Phy.* III.6, 206b12-14).

I suggest that here Aristotle is arguing for the actuality of the infinite insofar as the infinite is connected to an incomplete activity—motion. But what kind of motion? Most, if not all motions,

do in fact come to an end because they are defined by actualizable goals. Furthermore, how can the infinite be actual in any respect, if Aristotle's universe is finite? The infinite indeed exists for Aristotle, and all existing things are actual to some degree. But, an actuality typically pertains to a finite entity since the actuality is defined according to a *form* with a *specific* set of attributes. So, what kind of thing is Aristotle's infinite, and how can it be actual, when Aristotle's universe seems to be full of nothing but finite entities?

If the infinite has anything to do with motion, especially in a fundamentally finite universe, we must first understand what Aristotle means by motion. The problem Aristotle faces at the beginning of *Phy.* III.1 is that the character of motion precludes it from a scientific definition—a genus and an essential differentia. A scientific definition is *univocal* since it pertains to *one* kind of thing. For example, the definition of 'animal'—a living thing with the capacity for sensation—applies to all species of animals in the exact same way, even though there are many different animals. This is because all animals ultimately fall under the same genus. At *Phy.* III.1, 200b25-201a11, however, Aristotle argues that motion is *equivocal* or what I call *transcatgorial* since, as I will argue in Chapter 2 below, motion is something that crosses categories without it having a common nature that pertains to a single category or genus.[4] For Aristotle, motion lacks a strict generic character because it manifests in fundamentally different categories of being—substantial change, qualitative change, quantitative change, and locomotion:

[4] Something is 'transcategorial' if it manifests in different genera in similar ways without itself having its own genus. Another example of this is 'being' (τὸ εἶναι); everything that is, is said 'to be.' But the being of anything that is, is different because things have fundamentally different forms. In short, there is no common form or actuality by which things are said to be. Nevertheless, everything has some sort of form even though there is no separate genus or category of 'form' or 'being.' Thus, being crosses categories insofar as everything has some sort of form albeit it differently. See Edward Halper, *One and Many in Aristotle's Metaphysics: Books A-Δ* (Las Vegas: Parmenides Publishing, 2009), pp. 53-78.

> While there is indeed something in actuality only, there are those things which are both potential and actual: the individual substance, quantity, quality, and similarly with the other categories of being. As for relation, something can be said according to excess and deficiency, active and affective, and generally as moved and mover. For the mover is a mover in relation to what is movable and the movable is movable by means of a mover. But there is no motion apart from beings, for something changes either substantially, quantitatively, qualitatively, or with respect to place; and there is nothing to take up as common to these which is, as we say, a this, a quantity, a quality, or any of the other categories. Thus, neither motion nor change will be in any way apart from what has been said, for there is no being apart from them. Now, each of these manifests in all things in two ways: the this as either something's form or privation, qualitatively as either white or black, quantitatively as either complete or incomplete, and similarly for place as either up or down, or as light or heavy. Thus, there are as many types [εἴδη τοσαῦτα] of motion and change as there are sorts of being. Since the distinction has been made for each kind of being between actuality and potentiality, motion is [therefore] the actuality of a potentiality so long as there remains [ᾗ] a potential (*Phy.* III.1, 200b25-201a11)

In this passage, Aristotle shows that motion exists in different genera without belonging to a category all of its own (*cf. Cat.* 14). Κίνησις does not manifest in any single category univocally. Substantial change is not the same as alteration and alteration is generically distinct from increase/decrease because each type of motion requires a different set of form and material as their respective causes. Just like 'being,' there is no genus of motion. For, as Aristotle says, there

is no motion apart from the highest categories (*Phy.* III.1, 201a1-13). Therefore, motion seems not to have a scientific definition. Despite this, Aristotle clearly defines it at *Phy.* III.1, 201a9-10 as 'the actuality of a potentiality *qua* potential.' What kind of definition is this? The definition alone is already a challenge to understand; but what is Aristotle's procedure for discovering the definition in the first place? How is Aristotle able to discover such a definition when motion lacks a common genus? Drawing from discussions about likeness (ὁμοίος) in *Topics* I.18 and Simplicius' commentary on *Physics* III, my interpretation will work out the reasoning at *Phy.* III.1, 200b28-201a9 and subsequently show how this sets the framework for the account of motion in *Phy.* III.1-3. I will argue that because motion crosses categories, Aristotle's solution is to make its definition *analogical*. Motion must be defined in such a way as to apply to all types of motion similarly (substantial, qualitative, quantitative, and locomotive) without jeopardizing their real categorial differences.

Now, what does the account of motion in *Phy.* III.1-3 tell us about the infinite? We know from *Phy.* II.9 that all motions are for the sake of some form. So, with respect to the new form, motions are finite precisely because they are categorially determined. The form is the specific τέλος towards which a thing moves. However, in *Phy.* III.2, we learn something different. There, Aristotle is refers to motion with respect to itself. With respect to itself, motion is indefinite insofar as it is an *incomplete activity*. Because motion is an actuality *of* a potential, and since every potential is incomplete (ἀτελές), then the actuality of the motion itself is incomplete. As incomplete, motion is *indefinite*, since incompleteness pertains to a lack of a boundary. To be sure, without much argument on his part, Aristotle seems to assume throughout the *Corpus Aristotelicum* that there is a strong correlation, if not identity, between the infinite (τὸ ἄπειρον)

and the indefinite (τὸ ἀόριστον).[5] It seems fair to say that if what is incomplete is indefinite, and what is indefinite is unbounded or limitless, then it is a good assumption for Aristotle to treat the indefinite and infinite synonymously. Both are understood with respect to incompleteness. In fact, motion seems to be a good example. The indefiniteness of motion is such that while motion is happening, there is still *more* potentiality remaining to actualize. This is consistent with the definition of the infinite in *Phy.* III.6 as *that outside of which there is always something more.* As I will show, in this way, motion *reveals* the infinite as an actual attribute of a thing undergoing change. Infinity arises *within* a process, since a process is by nature indefinite.

On the surface, therefore, it looks as if the infinite is *per se* to motion.[6] This seems to be their *connection.* If motion is indefinite, and what is indefinite is also infinite, then the infinite surely seems to belong essentially to motion. What might help us understand this better? We know from *Phy.* II.1 that motion is *per se* to nature. Because 'nature' (φύσις) may be defined generally as anything with the capacity for motion, motion is a *per se* attribute of everything in

[5] See §2.2 below, especially fn. 100.

[6] Aristotle distinguishes between four senses of *per se* attribution in *APo.* I.4, 73a34-73b17: (1) *per se* with respect to the account of an essence (73a34-37); (2) *per se* with respect to the *per se* substrate—the subject—in the attribute's account (73a37-73b5); (3) *per se* with respect to the substance as a *this* (τόδε τι) since it is said of no other substrate (73b6-10); (4) *per se* with respect to necessity (73b10-16). It is important to point out that when Aristotle speaks of the "account" (λόγος) or "that which is said in the account" (ἐν τῷ λόγῳ τῷ λέγοντι), he seems to refer to the definition—the genus and the *per se* differentiae. Though λόγος may sometimes pertain to the entire demonstration—the syllogism—the most natural reading of it here is the definition. *Per se*$_2$ and *per se*$_4$ are quite interesting in their own right. *Per se*$_2$ suggests that even certain contraries are *per se.* Then, *per se*$_4$ is somewhat curious not only because of the absence of mathematical examples, but also that it seems to pertain to the relationship between multiple natures and necessary/accidental *circumstances*. Aristotle's examples indicate this. It is not because of what walking is that in the event of walking, lightening occurs; lightening happens often without walking. Thus, lightening while walking is accidental. But it is because of what sacrifice is—throat slitting (σφαγή)—that in the event of a sacrifice, death occurs. Therefore, it is necessary that death *happens* in sacrifice. What happens because of sacrifice is inherent *in* the nature of sacrifice. See also Pierre Pellegrin, "Definition in Aristotle's *Posterior Analytics*," in *Being, Nature, and Life in Aristotle,* ed. James Lennox and Robert Bolton (Cambridge: CUP, 2010), pp. 127-129.

the realm of nature.[7] This falls under the first sense of '*per se*' which is delineated at *APo* I.4, 73a34-37: whatever belongs to what something is (τί ἐστι) and is included in its account (λόγος) is *per se*. So, just as motion is necessarily included in the account of nature, it might be the case that the infinite belongs to motion as included in the account of motion. We know from *Phy.* III.2 that motion is *inherently* indefinite, which is another way of attributing infinity *essentially* to motion. So, the infinite seems to be *per se* of motion. However, there is a significant problem. Unlike motion's attribution to nature, infinity cannot be *per se* to motion *simply* (ἁπλώς), since motion is itself an attribute of the physical substance. Properly speaking, only substances have attributes, and it is often by analogy to the substance—treating something substantially—that an

[7] See also *Meta.* K.7, 1064a15-16. *Cf.* Edward Halper, "Aristotle on the Knowledge of Nature," *Review of Metaphysics* 37 (4), p. 825. Although there is no common nature when taken strictly in terms of its initial definition (*Phys.* II.1, 192b13-14), seen more loosely, φύσις does, in fact, exhibit a sense in which it is applicable to all physical things: 'a certain kind of being with the capacity to move.' The strict definition of nature given at the beginning of *Phy.* II.1—nature is an internal principle of motion and rest—may then be subsumed under this general formulation since it pertains to a more specific capacity to move in substances. This definition might in fact serve as a paradigm for all physical things. *Cf.* Edward Halper, "Aristotle's Scientific Method," pp. 81-92. This is an important clarification, since φύσις seems to pertain properly only to substances. But, in the *Physics,* the inquiry stretches to cover not only substances, but also artifacts and the elements, in spite of the fact that technically the latter are not substantial beings. Artifacts and the elements are treated substantially by likening their activities to those of proper substances, usually with respect to the form; the elements, still, are closer to substances than artifacts since while strictly they do not have natures (φύσιν ἔχοντα), they are with respect to/by some nature (κατὰ φύσιν/φύσει, 192b36-193a1). At the end of *Phy.* II.1, Aristotle argues that form is the primary sense of nature (ἡ ἄρα μορφὴ φύσις, *Phys.* II.1, 193b18). The μορφή points to the composite character of physical beings, which accounts for the capacity for movement (*cf. Phy.* II.2, 194a12-27). In this way, I think, Aristotle extends the inquiry to artifacts and the elements since they too have μορφαί—form and matter. This generic sense of nature is consistent with *Meta.* Δ.4, 1015a13, where Aristotle claims that in an extended sense nature is *a certain sort* of οὐσία (ἡ φύσις οὐσία τίς ἐστιν). That is to say, broadly speaking, nature is the *kind* of being with the capacity to move, in contrast to immovable οὐσίαι such as the unmoved movers and, more loosely, mathematical objects. See also *Meta.* Γ.3, 1005a33-36 and I.10, 1058b26-29. Augustin Mansion, *Introduction à la physique aris*to*télicienne* (Louvain: Éditions de l'Institut supérieur de philosophie, 1946), p. 42 argues similarly: "La physique étant l'étude de la nature ou du monde matériel, atteint précisément une donnée de ce *genre*: quelque chose qui est, *une certaine sorte d'êtres, non* to*us les êtres*" (emphasis mine).

attribute can have attributes.[8] Furthermore, according to *Phy.* III.1, motion does not even have a proper, *univocal* definition; motion is not properly a 'thing.' And yet, at least in the *Physics*, it looks as if the infinite is indeed *per se* with respect to motion. Motion is, after all, an incomplete activity which is necessarily indefinite. But is this true of *all* motions, if not all motions are meant in the same way? How, then, are we to think of the *per se* attribution of infinity if that to which it first appears to be *actually* attributed—motion—is not meant in one way?

One strategy could be that it is enough to *treat* motion as a substance in order to delineate any further *per se* attribution of infinity. However, the problem of motion's equivocity remains. To treat something as a substance requires that its definition be univocal, since the definition is for the purpose of using it as a *causal* middle term in a scientific demonstration to account for *per se* attributes. Substances are always univocal; their essences are universal and necessary. That is to say, the definitions of substances are *causes.* Now, this can be extended to non-substantial things if their essences are also univocal. Even though a triangle is not strictly a substance, it can be *treated* as one because it has an essence that applies causally to all types of triangles. Because its definition accounts for the *per se* attributes of all triangles, a triangle can be treated as if it were a substance. A triangle is defined as a plane figure with three sides whose angles equal 180 degrees. This applies to an equilateral triangle in the same way as an isosceles triangle, even though they are differentiated by the proportions of their angles. In this way, the essence of triangles, like that of a substance, is univocal. Just as the definition of 'animal' applies

[8] Admittedly, this is a common method for Aristotle, especially concerning mathematical entities. Mathematical beings are not substances; they too are attributes. But, for the purposes of study—e.g. arithmetic or geometry—they may be treated as substances in order to demonstrate what is *per se* to them. See *Meta.* M.3-4 and Edward Halper, "Some Problems in Aristotle's Mathematical Ontology," *Form and Reason* (Albany, NY: SUNY, 1993), pp. 143-146. Even in *APo.* I.4, 73a34-73b17, where he discusses the different kinds of *per se* attribution, Aristotle appeals to mathematical examples.

to all animals in the same way, since all animals are capable of sensation, the definition of triangle applies to all triangles insofar as they all have three angles that equal 180 degrees.

However, this univocity is not possible for motion. Unlike mathematical objects and animals, motions are known with respect to specific forms and materials *across different* genera. This means that the definition of motion cannot apply to all motions in the same way. Again, substantial change requires substantial forms and materials, whereas qualitative change requires qualitative forms and materials, etc. This makes it difficult, then, to treat motion as a substance for the purpose of demonstrating *its* essential attributes. For, unlike the triangle, it is not clear how the definition of motion—the actuality of potentiality *qua* potential—can be used causally to demonstrate what is universal and necessary for all motions since each motion has different forms and materials. The definition of motion is so ambiguous that it is practically useless in a demonstration. It does not refer to a *specific* form as a cause. So, it cannot tell us scientifically what is essential to all motions—especially the infinite! In short, motion is not the *cause* of the infinite, for its definition cannot account for why the infinite belongs to it in all cases.

How then is the infinite attributed to motion, particularly as an actuality? What is the *cause* of the infinite's actual attribution to motion? To understand this, I suggest that we treat motion *heuristically,* which is to say that we begin with what *first appears* to us—motion's indefiniteness—and we work *backwards* to find the explanation for this. Because the infinite first appears in motion, motion is that from which Aristotle starts the inquiry. In this case, motion functions as what I call a 'threshold phenomenon' to look for what the infinite is in actuality and *why* it belongs essentially to the *moving substance*. What is a 'threshold phenomenon' though? *Metaphysics* Δ.18, 1022a14-19 gives us a clue. There, Aristotle distinguishes between two senses of "that on account of which" (τὸ καθ'ὅ): (1) on account of the form and substance (τὸ εἶδος καὶ

ἡ οὐσία) and (2) on account of the first thing in which something naturally manifests (ἐν ᾧ πρώτω πέφυκε γίγνεσθαι)—i.e. the proximate subject.[9] Aristotle's example for (2) is the color of a surface. Even though color is predicated primarily of the substance, its most immediate subject is the surface of the body, since the surface is the matter most proximate to the color. The surface is proximate because the color's inherence seems first to be with respect to that in which it *appears* to us. Surfaces are *seen* by their colors. So, just as much as the surface is the proximate subject of color, color in turn is the proximate attribute of the surface. In fact, Aristotle's Greek at *Meta.* Δ.18, 1022a16 tells us this much: ἐν ᾧ πρώτω πέφυκε γίγνεσθαι, "that *in* which something naturally *first* comes to be." In this way, the surface acts as a sort of *threshold* for color as primarily an attribute of the substance. For, the proximate subject is often another attribute of a substance, e.g. a surface is part of the physical body as whole. It must also not go unnoticed that the word translated as "surface"—ἐπιφανεία—may be taken as a sort of pun. Ἐπιφανεία, the *visibility* of a body, literally renders as what *shows up* (ἐπι-φανεία). The surface is, therefore, a proximate subject in which the color first *appears*. As proximate, the surface is *an entry point*-—a threshold phenomenon—to understand why and how color belongs to the ultimate subject, which is the physical body bounded by the surface.

With this in mind, in chapter 4 of this dissertation, we will see that motion, like the surface of a body, is a proximate subject acting as a threshold for how the infinite is an actual, essential attribute within the physical substance. Like the surface, motion is that in which infinity first manifests as an actuality insofar as motion is actually indefinite (ἀόριστον) or incomplete (ἀτελές) as it is actually happening. Therefore, the actual infinite is a *proximate* attribute of motion. But, because Aristotle defines motion analogically, not every motion reveals the

[9] There are two other senses, but for our purposes here, they are not relevant. See *Meta.* Δ.18, 1022a19-25.

actuality of the infinite. In which type of motion does the infinite clearly manifest as an actuality? The infinite is a quantity since it is defined as some *amount* outside of which there is *always* more. Aristotle will argue that the infinite is manifest in the quantitative *acts* of division (δίαιρεσις) and addition (προσθέσις). For, in dividing a magnitude, the activity never reaches an indivisible point or smallest part; thus, the activity never comes to an end, since there is always another part to be divided. In turn, each division is being counted indefinitely insofar as there is always another division to be counted. So, the specific motion to which infinity manifests proximately is a quantitative change—the activities of division and addition. Still, the infinite's ultimate subject is not the act of division or addition, but rather the physical substance. The infinite is essential to the physical substance primarily by virtue of the fact that physical substances are extended bodies and every extended body is infinitely divisible *by nature*. Physical bodies undergo division and are counted. The acts of dividing and adding actualize the infinite *incompletely* in the moving substance. In short, the actuality of the infinite is the *manifestation* of the physical body's inexhaustible capacity to be divided in the very acts of dividing and counting.

We will see that the physical substance's nature is the *cause* of the infinite's actual attribution to motion. Because physical substances are inexhaustibly divisible, it is always possible to add things indefinitely. In other words, the infinite is *in* the finite substance as a *per se*, albeit *peculiar* kind of attribute because it is necessarily incomplete. Again, the trick to see this is by first encountering the infinite as it manifests actually in motion insofar as motion is its proximate subject. This is why motion is so important to the account of the infinite. Because motion, like the surface, is also a part of the whole substance, both everything proximately attributed to motion and motion itself are on account of the substance as the proper substrate,

which is τὸ καθ'ὅ in the primary sense. However, it is not any motion—the infinite is most proximate to the acts of division and addition. To be sure, division and addition are not the *causes* of the infinite. But, Aristotle can work backwards from them as the proximate phenomena by which we arrive at what is better known by nature—the *substance* undergoing division.

§1.2. The literature: reception and interpretations of *Physics* III

§1.2.1. The definition of motion (*Phy.* III.1-3)

Since David Ross's 1936 commentary on the *Physics*, much of the literature on the definition of motion has more or less responded to his interpretation of ἐντελέχεια in the definition.[10] Ross argues for what I call the *passage reading*.[11] In order to capture the transitory character of motion, Ross suggests that Aristotle's use of ἐντελέχεια in the definition means 'actualization' since "[motion] is the *passage* from potentiality to actuality."[12] As such, Ross reads the definition as *the actualization of a potentiality as a potential.* Motion is the process of obtaining or losing a form; that is to say, an incomplete activity. But ἐντελέχεια pertains to completeness and stability. So, how do we understand ἐντελέχεια here? Ross argues that it is

[10] W.D. Ross, *Aristotle's* Physics (Oxford: Clarendon Press, 1936), pp. 535-537. *Cf.* Hans Wagner, *Physikvorlesung* (Berlin: Akademie-Verlag, 1972), p. 59 gives a similar reading in German, "Die *Verwirklichung* des Möglichseienden, insofern es von letzterer Artung ist, heißt Proseß."

[11] While this is not the place to tease out the nuances of each treatment, it is enough to say that those I have listed here more or less follow along with Ross. See J.L. Ackrill, 'Aristotle's Distinction between *Energeia* and *Kinesis*,' *Essays on Plato and Aristotle* (Oxford: Clarendon, 1965), pp. 142-178; Robert Heinaman, "Is Aristotle's Definition of Change Circular?", *Apeiron* 27 (1994), pp. 25-37 and "Kosman on Activity and Change," *Oxford Studies in Ancient Philosophy* 12 (1994), pp. 207-218 and "Activity, Change and *De Anima* II.5," *Phronesis* 52 (2007), pp. 139-187; A.L. Peck, 'Aristotle on Κίνησις,' in J. Anton and G. Kustas (eds.), *Essays in Ancient Greek Philosophy* (Albany, NY, 1971), pp. 478-490; J. Kostman, "Aristotle's Definition of Change," *History of Philosophy Quarterly* 4 (1987), pp. 3-16; Daniel Graham, "Aristotle's Definition of Motion," *Ancient Philosophy* 8.2 (1988), pp. 209-215.

[12] Ross, *Aristotle's* Physics, p. 536, his emphasis.

essential to render the definition in the way that most accurately conveys the *process* of motion. The definition should express, Ross thinks, neither the material before being set into motion nor the product. For example, the definition refers neither to the wood and bricks before being shaped into a house nor the finished house. Motion is neither the state of being cold prior to being heated nor the temperature at which the heating ends. Rather it is the *passage* from the former to the latter—this is Ross's understanding of ἐντελέχεια as 'actualization.'

Ross's interpretation is at least *descriptively* instructive. It attempts to draw attention to the state of affairs *in-between* the inactivated materials and the result of the motion. Nevertheless, according to L.A. Kosman, Ross's interpretation fails as a way to read the definition *as a definition*.[13] Kosman maintains that Ross's reading already assumes motion in order to define it, since 'actualization' is itself a change. So, the reading is circular. Furthermore, it does not successfully capture how the in-between state between old and new forms is also itself an *actuality*—something complete—which Kosman describes as "a potentiality *in its full manifestation*."[14] The standard interpretation follows Kosman.[15] Call this the *qualification*

[13] L.A. Kosman, "Aristotle's Definition of Motion," *Phronesis* 4 (1969), 40-62 and *The Activity of Being* (Harvard University Press, 2013), pp. 45-68. See also Kosman, "The Activity of Being in Aristotle's *Metaphysics*," *Unity, Identity, and Explanation in Aristotle's* Metaphysics, ed. T. Scaltsas, D. Charles, and M.L. Gill (Oxford: Clarendon Press, 1994), pp. 202-204 and "Substance, Being, and *Energeia*," *Oxford Studies in Ancient Philosophy* 2 (1984), pp. 128-131.

[14] Kosman, "Aristotle's Definition of Motion," p. 62.

[15] The impact of Kosman's interpretation is far reaching. Those following Kosman focus on the *ontological* consequences of the definition of motion. Mary Louise Gill, "Aristotle's Theory of Causal Action in *Physics* III.3," *Phronesis* 25 (1980), 129-147 and "Aristotle's Distinction Between Change and Activity, *Axiomathes* 14 (2004), pp. 3-22; Ursula Coope, "Change and its Relation to Actuality and Potentiality," in George Anagnostopoulos (ed.), *A Companion* to *Aris*t*otle* (OUP, 2009), 277-291 and *Time for Aris*t*otle* (OUP, 2005), pp. 6-7; Edward Halper, "Aristotle on the Knowledge of Nature," pp. 832-34; Edward Hussey, *Aristotle's* Physics, *Books III and IV* (OUP 1983); Myles Burnyeat, "*De Anima* II 5," *Phronesis* 47 (2002), 28-90 and *'Kinêsis vs. Energeia:* A Much-Read Passage in (but not of) Aristotle's *Metaphysics*, *Oxford Studies in Ancient Philosophy* 24 (2008), pp. 219-292; Sarah Waterlow, *Nature, Change, and Agency in Aristotle's* Physics: *A Philosophical Study*; Andreas Anagnostopoulos, "Change in

reading.[16] While I cover this in greater detail in Chapter 2, the qualification reading is worth a cursory sketch here since so many interpreters who take issue with any form of the passage reading more or less agree with Kosman.

At *Phy.* III.1, 201a29-201b5, Aristotle uses *qua* or 'insofar as' (ᾗ) in the definition so to qualify how the actuality and potentiality pertains to motion but neither the latent material nor the final product.[17] According to Kosman, on one hand the material *qua* material is the first-order potentiality; the bronze *qua* bronze isn't doing anything other than what it is to be bronze, which in one respect is to be the potential for a statue. On the other hand, the statue *qua* statue is the second-order or *full* actuality. It too is not doing anything other than being a statue. Motion, then, is *both* the second-order potentiality *and* the first-order actuality. The second-order potentiality is the material becoming the new form and the first order actuality is the new form itself being actualized, e.g. bronze actively being shaped into the statue and the statue being generated. As such, according to Kosman, motion is qualified as the *constitutive actuality* which picks out "the actuality of a potentiality in the sense of […] an actuality which *is* a potentiality in its *full manifestation*."[18] In contrast to Ross, this interpretation also allows us to read ἐντελέχεια naturally as 'actuality' without smuggling in any notion of passage so as to avoid rendering the

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Aristotle's *Physics* 3," *Oxford Studies in Ancient Philosophy* 39 (2010), pp. 33-79; Helen Lang, *The Order of Nature in Aristotle's* Physics (CUP, 1998), p. 56-57; Joe Sachs, *Aristotle's* Physics*: A Guided Study* (New Brunswick: Rutgers University Press, 1995), pp. 78-80; Richard Rorty, *The Concept of Potentiality,* PhD diss. (Yale University, 1956), pp. 15-19; Rémi Brague, "Aristotle's Definition of Motion and its Ontological Implications," *Graduate Faculty Philosophy Journal* 13.2, pp. 10-11.

[16] A. Anagnostopoulos, *op. cit.*, p. 34-35 calls this the "consensus interpretation, since it has become entrenched in recent years."

[17] Kosman, "Definition of Motion," pp. 50-54. I might even suggest that Ross's reading at least hints at this, even if Ross himself did not intend it.

[18] Kosman, *ibid.,* p. 50, emphasis mine.

definition circular. In this way, the qualification reading bears witness to the *concomitance* of the material's potentiality as a potential and the form's actuality as it is being generated.

Now, I use Ross and Kosman only as figureheads to represent a deeply divided and nuanced scholarly tradition. The commentary that has followed over the past 50 years or so has either taken sides with or against these very readings. My purpose here, however, is quite different. I do not wish to add to this tradition other than a few finer points I contend are warranted. While I more or less agree with the qualification reading (I will give my hesitancies and objections later), I contend there is a more significant problem that the literature as a whole neglects: if *Phy.* 201a9-10 is in fact a *definition*, then what is the *entity* to which it refers? What is the referent? What *specific* actuality is the definition of motion meant to pick out? In order to answer this question, one must have in mind the entity's *genus* and the *essential difference.* The definition of 'human being' refers to this entity writing his dissertation *in the same way as* the reader because both are instantiations of an animal exercising its capacity for reason. Also, because I am defined as an animal, I exercise my capacity for sensation *in the same way* as a dog or horse. But, to what does 'the actuality of a potentiality *qua* potential' refer? What's the genus? The answer is *inherently* ambiguous because neither actuality nor potentiality constitute a genus; they are *transcategorial*. However, the literature largely neglects this as a problem. For example, Kosman believes that Ross's explanation of the definition of motion fails because it is ultimately vacuous. I agree with Kosman; but even he misses the issue. The problem is not merely with Ross's explanation. It's Aristotle own formulation of the definition; it's not really a definition. It specifies no particular genus or species because that's not the function of Aristotle's formulation. Aristotle's definition of motion is not to be read in any precise sense because it's not even an ordinary scientific definition. It does not refer to a particular motion, let alone an entity.

Our expectations that the definition of motion indicates anything in particular grows out of assuming that Aristotle is even able to discover any sort of definition of motion *qua* scientific in the first place.[19] I suggest that the definition is not meant to pick out anything at all—it's a sort of first approximation for a solution to motion's *categorial* ambiguity. Commentators work diligently to disambiguate Aristotle's definition. While this is philosophically and exegetically commendable, I contend such efforts lessen the impact of the definition as picking out something *intrinsically* ambiguous. Aristotle addresses motion's categorial ambiguity at *Phy.* III.1, 200b25-201a9. Modern scholarship often glosses over these initial categorial remarks. Instead it focuses on the definition of motion alone at *Phy.* III.1, 201a9-11. While several ancient and medieval commentators offer readings that take into account Aristotle's opening arguments, this is not the trend amongst modern scholars, except for Edward Halper.[20] Drawing from the account of substantial generation in *Phy.* II, Halper argues that "just as the actual substance comes to be from a potential substance, an actual quantity comes to be from a potential quantity, and analogously for changes in the other categories."[21] Halper's position is that Aristotle uses substantial generation as a paradigm for other types of changes. In order for Aristotle to define

[19] *Cf. Meta.* Z.4, 103018-26; ὁρισμός extends to the categories in the same way as it applies to substances insofar as it pertains to an *essence.* That is to say, just like for substances, the essence of a quantity or quality is also *univocal* because it pertains to one kind of thing. Brague, *op. cit.*, p. 13 recognizes Aristotle's difficulty in *defining* (ὁρίζειν) motion as also intrinsically indefinite, but even Aristotle acknowledges this in *Phy.* III.2. The definitional challenges Aristotle faces are more far reaching than how something indefinite may be defined. For instance, since the definition of motion is analogical, it cannot serve as a middle term in a scientific demonstration. Therefore, it cannot be used to infer anything *per se* about nature. *Cf.* Halper, "Aristotle's Scientific Method," p. 95.

[20] Anagnostopoulos p. 45 fn. 25 recognizes the categorial issues but not the analogical implications for motion. *Cf.* Heinaman, "Circular Change," p. 30.

[21] Halper, "Aristotle's Scientific Method," p. 94; *cf.* Halper, "*Metaphysics* I and the Difference It Makes," *Proceedings of the Boston Area Colloquium in Ancient Philosophy* 22 (2006), p. 77 fn. 11.

motion, there needs to be some sort of genus even though strictly none is available.[22] Halper claims that, in *Phy.* III.1, Aristotle defines motion "[w]ithout explaining how he arrives at it," which seems to be partially why Halper appeals to the account of substantial change in *Physics* II.[23] But, I contend Aristotle's explanation for how he arrives at the definition can be found precisely in the categorical remarks at *Phy.* III.1, 200b28-201a9, to which Halper does not give due diligence. Even though my reading is consistent with Halper's interpretation, I argue that Aristotle draws up the definition of motion directly as a result of his analysis of the relationship motion has to the categories and not directly from *Phy.* II. So, an analysis of Aristotle's categorial remarks is needed. To this end, we to turn to Simplicius.

Simplicius' treatment of Aristotle's categorial remarks is the most thorough. Philoponus also reports that "the commentators" (οἱ ἐξηγηταί) who also support this reading are likely referring at least to Simplicius.[24] Commenting on Aristotle's third categorial remark at *Phy.* III.1, 201a15f, Simplicius argues explicitly that this is evidence for the analogical or 'equivocal' (ὁμώνυμος) character of motion:

> Having said that one can find nothing common to the different sorts of change, [Aristotle] made clear what sort of common element he is denying by shifting the discussion to the categories in which there is change when he said 'which is neither a particular thing nor a quantity nor a quality.' But if change is equivocally named, how does [Aristotle] define it? For there are no definitions of the equivocally named, or else the definition of the

[22] Halper, "Aristotle's Scientific Method," p. 94
[23] *Ibid.*
[24] Philoponus*, In Phys.* 348.15. Philoponus and Simplicius also cite Alexander as a proponent of this reading; see Philoponus, *In Phys.* 349.5-6 and Simplicius, *In Phys.* 403.13. It is noteworthy however that Philoponus, *In Phys.* 349.16-26 leans toward the Theophrastian view that the categories are *interwoven*. I see no evidence of this in Aristotle's categories.

> equivocally named are also equivocal. For since 'principle' [ἀρχή] is equivocally named, the definition that says that a 'principle' is the first in each thing is also equivocal [*Top.* I.18, 108b27]. Moreover, the actuality of the changed *qua* changed will be equivocal. For the product of the equivocals is equivocal. Therefore, he had to add this lemma lest someone who heard the definition of change should think that the definition is given of it as a single genus, and so that it should be clear that the account is given through an equivocal expression as being of an equivocal expression [*APo.* II.17, 99a16-17] (modified Urmson translation).[25]

Simplicius' analysis relies on the account of likeness (ὁμοίος) at *Top.* I.18, 108b7-31 and Aristotle's brief claim at *APo.* II.17, 99a16-17 that analogical demonstrations have similar middle terms.[26] 'Principle' (ἀρχή) is not a genus like 'animal' or 'quality' but a name that

[25] Simplicius, *In Phys.* 403.36-404.15. Though Simplicius does not acknowledge it, Aristotle's discussion in *Topics* I.18, 108b7-31 comes from Archytas (DK47a22 = *Meta.* H.2, 1043a19-25). See also Carl A. Huffman, *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King* (Cambridge: CUP, 2005), pp. 490-505. I discuss this connection in greater detail below.

[26] The scholarly literature on the status of analogical demonstrations, even ἀναλογία in general, in Aristotle is sparse. There are those (mostly the ancient commentators) who argue that analogies are scientifically relevant, especially when it comes to locating the principles of a science. See Alexander of Aphrodisias, *In Aristotelis topicorum libros octo commentaria*, 117.3-31 and 124. 11-33; Thomas Aquinas, *Commentary on Aristotle's* Posterior Analytics, trans., Richard Berquist (Notre Dame, IN: Dumb Ox, 2007), pp. 80-84, 321, and 328-331; Eustratius, *In analyticorum posteriorum librum secundum commentarium*, Commentaria in Aristotelem Graeca XXI.9, ed. Michael Hayduck (Berlin: Georg Reimer, 1907), pp. 230.13-24 and 244.4-35; Wilfried Fiedler, *Analogiemodelle bei Aristoteles: Untersuchungen zu den Vergleichen zwischen den einzelnen Wissenschaften und Künsten* (Amsterdam: B.R. Grüner, 1978); Mary Hesse, "Aristotle's Logic of Analogy," *The Philosophical Quarterly* 15 (61), pp. 328-340; G.L. Muskens, *De vocis ἀναλογίας significatione ac usu apud Aristotelem* (Groningen: J.B. Wolters, 1943); Thomas M. Olshewsky, "Aristotle's Use of 'Analogia,'" *Apeiron: A Journal for Ancient Philosophy and Science* 2 (2), pp. 1-10; Joseph Owens, *The Doctrine of Being in the Aristotelian* Metaphysics*: A Study in the Greek Background of Mediaeval Thought* (Toronto: Pontifical Institute of Mediaeval Studies, 1951), pp. 123-125. Then there are those who doubt the scientific relevance of analogy since it is read heuristically and tentatively: G.E.R. Lloyd, *Aristotelian*

indexes a group of widely divergent things which typically fall in different classes (*Top.* I.18, 108b24). Principles, for example, are understood with respect to something's nature; but there are many different natures. This means there are different principles without a common nature. However, Aristotle thinks it is still possible to collect all principles under the heading 'principle' because they function similarly but not univocally. Whatever way 'principle' is to be defined will, therefore, apply *equivocally* to each sort. Simplicius believes that the definition of motion functions in the same way; motion is defined in such a way as to index in a single formula the different ways a potential is actualized. The causes of motion—form and matter—are fundamentally different in each category that admits motion. Simplicius' discussion falls short, however, by not specifying exactly how this equivocal expression is supposed to function; that requires a careful analysis of To*pics* I.18 with respect to the account of motion in the *Physics*.

§1.2.2. Aristotelian infinity and its connection to motion (*Phy.* III.4-8)

Everyone agrees that infinity exists for Aristotle. The debate, however, centers on *how*. The standard position is that Aristotelian infinity *only* exists potentially.[27] It argues that a

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Explorations (Cambridge: CUP, 1996): pp. 138-159; Lloyd, *Polarity and Analogy: Two Types of Argumentation in Early Greek Thought* (Cambridge: CUP, 1966), pp. 403-420; Terrence Irwin, "Homonymy in Aristotle," *Review of Metaphysics* 34 (3), pp. 523-544; Christof Rapp, "Ähnlichkeit, Analogie, und Homonymie bei Aristoteles," *Zeitschrift für philosophische Forschung* 46 (4), pp. 526-544.

[27] David Bostock, "Aristotle, Zeno, and the Potential Infinite," *Space, Time, Matter, and Form: Essays on Aristotle's* Physics (Oxford: OUP, 2006), pp. 116-126; William Charlton, "Aristotle's Potential Infinities," *Aristotle's* Physics: *A Collection of Essays* (Oxford: Clarendon Press, 1991), pp. 129-49; David J. Furley, "Aristotle and the Atomist on Infinity," *Naturphilosophie bei Aristoteles und Theophrast* (Heidelberg: Lothar Stiehm Verlag, 1969), pp. 87-88; Kurt von Fritz, "Das ἄπειρον bei Aristoteles," *Naturphilosophie bei Aristoteles und Theophrast* (Heidelberg: Lothar Stiehm Verlag, 1969), p. 71; Thomas Heath, *Mathematics in Aristotle* (Bristol: Thoemmes Press), pp. 107-8 and *A History of Greek Mathematics* (New York: Dover, 1981), p. 342; Edward Hussey, *Aristotle's* Physics*: Books III and IV* (Oxford: Clarendon Press), pp. 82-88; Jonathan Lear, "Aristotelian Infinity," *Proceedings of the Aristotelian Society*, 80 (1979-1980),

physical magnitude is *only* able to be divided infinitely many times, while the actual divisions are always numerically and extendedly finite. Still, Aristotle claims several times in *Phy.* III.6 that the infinite seems to bear some degree of actuality especially in connection to motion (206a23-25; 206a29-33; 206b13-14). While most scholars dismiss or ignore the connection, some try to make sense of it in terms of actuality.[28] What do we do with these claims?

William Charlton and Richard Sorabji reject out of hand Aristotle's reference to an actual infinite, and argue that he refers only to a potential infinite.[29] To set the context for Philoponus' rejection of Aristotle's view of the eternity of the universe, Sorabji argues that Aristotle had a "merely potential and never actual" infinite, even though Sorabji cites as a "qualification" at *Phy.* III, 206b13 where Aristotle seems to use ἐντελεχεία in connection to infinity.[30] Sorabji never explains the nature of the qualification; the infinite exists only as potentially infinite divisions. Jonathan Lear agrees with Sorabji, going even further by minimizing the role of motion or "process" in Aristotle's account. Lear argues against Hintikka, who according to Lear fails "to distinguish between an actual process bearing witness to the existence of the potential infinite and an actual process bearing witness to the existence of an actual infinite."[31] Lear denies the latter and supports the former because he assumes that an actual infinite pertains only to an

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pp. 187-210; Friedrich Solmsen, *Aristotle's System of the Physical World: A Comparison with his Predecessors* (New York: Cornell University Press, 1960), p. 173 fn. 57; Richard Sorabji, "Infinity and Creation," *Philoponus and the Rejection of Aristotelian Science*, ed. Richard Sorabji (Ithica, NY: Cornell Univ. Press, 1987), pp. 170-171.

[28] John Bowin, "Aristotelian Infinity," *Oxford Studies in Ancient Philosophy* 32 (2007), pp. 233-250; Antoine Côté, "Aristote admet-il un infini en acte et en puissance en »Physique III, 4-8«?" *Revue Philosophique de Louvain* 88 (1990), pp. 487-503; Jaakko Hintikka, "Aristotelian Infinity," *The Philosophical Review* 75 (2), pp. 197-218; Pascal Massie, "The Actual Infinite as a Day or the Games," *Review of Metaphysics* 60 (Dec. 2006), pp. 573-596; Ross, *Aristotle's* Physics, pp. 556-58; Wolfgang Wieland, *Die aristotelische Physik* (Göttingen: Vandenhoeck & Ruprecht, 1970), pp. 293-307.

[29] William Charlton, *op. cit.,* pp. 140-42; Sorabji, "Infinity and Creation," pp. 171.

[30] Sorabji, "Infinity and Creation," p. 171.

[31] Lear, *op. cit.*, p. 191.

actual infinite magnitude. Nevertheless, Lear seems to hedge; later Lear claims that 206b13-14 is evidence for how a process could indicate an infinite magnitude "actually coming to be" even though such a process reveals only that the magnitude is potentially infinite.[32] Lear however insists later that Aristotle "categorically denies" that any motion attests to an infinite by addition (πρόσθεσις), which is a surprising observation given that *Phy.* III.6, 206b3-33 is all about establishing the convertibility (ἀντεστραμμένως) of διαιρέσις and πρόσθεσις. It remains unclear to me how Lear wishes to deal with infinity's connection to motion in Aristotle's account.[33] What is clear, though, is that in Lear's mind, there is *only* a potential infinite for Aristotle and that appeals to processes only muddle the account.

David Bostock, despite his eventual misgivings, entertains how Aristotle's claims for the connection between infinity and motion might point to an actual infinite.[34] He will in the end reject such a connection on the basis of how Aristotle understands the actualization of a point:

> In fact, [Aristotle] distinguishes between the actual and the merely potential existence of a point in such a way that a point is not said to exist actually until it *has been actualized*, so that there could be an actual infinity of points on a line at one time only if infinitely many points of the line had been actualized by that time. But, assuming that one cannot actualize infinitely many points *all at once,* this must involve the completion of an infinite process of actualizations, and that we have (for a moment) agreed to be impossible.[35]

[32] *Ibid.*
[33] *Ibid.,* pp. 194-195.
[34] Bostock, *op. cit.,* p. 116-118.
[35] *Ibid.,* p. 118.

For this reason, Bostock only sees a potential infinite in Aristotle. Bostock focuses on how the potential infinite shows up in the physical magnitude with respect to points *qua* divisions. To his credit, Bostock's rejection of an actual infinite turns not on minimizing the connection to a *process* of dividing a line—in fact, he thinks "this *is* very close to what [Aristotle] is claiming."[36] Instead, Bostock holds suspect Aristotle's understanding of how points are actualized. For there to be an actual infinite, every point would have to be actualized, which would entail an actual infinite *magnitude*. And Bostock is correct; an actual infinite body is impossible for Aristotle. But this isn't the only way actualities exist, nor is it the only way points are actualized in a magnitude. In fact, Aristotle does not need an infinite magnitude to actualize points, since a finite magnitude works just as well, since a magnitude is infinitely divisible by virtue of its continuity.

The way Bostock rejects an actual infinite in Aristotle is significant. It is an example of the peculiar tendency of some scholars either to skirt or soften Aristotle's (admittedly) terse claims that the infinite is *both* actual *and* potential like the day and the games. Such tendency seems to be rooted in not cashing out how the infinite *manifests* actually as a *phenomenon* that is an attribute of motion and ultimately the moving substance. To be sure, Bostock does point to the *process* of division as a possible locale for an actual infinite; but he slips back into a *hard* distinction between a full actuality and full potentiality with no concomitance of the two, and he assumes that an actual infinite would pertain only to a body. For Bostock, Sorabji, Lear, and others of the same interpretive stripe, when it comes to the infinite, either it exists only potentially in the magnitude or actually with respect to the actualization of *every point* in the magnitude, i.e. a completed infinite magnitude. Such a distinction assumes that actuality is meant in only one respect—completeness of a *body*. But, Aristotle is quite clear that there are

[36] *Ibid.*

incomplete actualities such as motion. Again, Aristotle claims *twice* that the infinite exists *actually* like the day and the games, which are indeed motions. How do we make sense of that?

A contingent of scholars stretching back to Ross believes that there is a sort of actual infinite for Aristotle precisely because of its connection to motion. To clarify Aristotle's claim at *Phy.* III.1, 206b13-14 that the infinite exists as an ἐντελέχεια, Ross argues that Aristotle

> first makes a statement which reproduces what he has said in [206]a16-18, that the infinite exists potentially, and by way of division (i.e. as the infinitely divisible, not as the infinitely extended). He then remarks parenthetically that (while it does not exist at any time as a given entity), it does exist actually in the *special sense* that, when division in the line is going on, a process which is in principle endless is being progressively actualized, as a day or a contest is progressively actualized.[37]

Ross's comment is striking because it takes seriously the prospect of an unusual or "special" type of actuality that might help us understand how the infinite actually exists. While Ross does not spell it out, this special sense seems to pertain to that of a *motion.* Ross believes Aristotle draws a distinction between the actuality of "a given entity"—i.e. a body—and that of the *process* of division—a quantitative motion. Motion, for Aristotle, is not strictly an entity not only because of the categorial problems we've touched on, but also because it is indefinite. Compared to the actuality of a body, the actuality of division is fundamentally incomplete. The progressive realization of the day and the annual repetition of the games are always happening without end

[37] Ross, *Aristotle's* Physics, p. 556, emphasis mine.

(ἀτελός).[38] The infinite, as Ross understands it, exists actually as something like a perpetual *act* of division.

John Bowin also explicitly ties the infinite to the process of division, but stops short of calling it an actuality.[39] Bowin's account locates the potential infinite only in goalless activities. Arguing that the potential infinite exists in the process of division as something incomplete, Bowin claims that

> [p]rocesses are incomplete because they are actualities that fall short of a goal state. What is unusual about processes that go on and on indefinitely is that they have no goal state. In [*Phy.* 207a7-15], Aristotle argues that what is incomplete always has something outside it, and 'nothing is complete [τέλειον] which has no end [τέλος]; and the end is a limit' (*Phys.* 207a14-15). The absence of a goal or end, then, is a sufficient condition for a process always to have something outside of itself, which is, in turn, a sufficient condition for the infinite to be suspended in a perpetual state of potentiality. Thus, as Aristotle says about the process of dividing a continuous magnitude, 'the fact that the indefinitely extendable process of dividing never comes to an end ensures that this activity exists potentially' (*Metaph.* Θ 6, 1048b14-17).[40]

So according to Bowin, the infinite indeed exists and does so with respect to the incompleteness of motion. However, he thinks that motion's incompleteness is what keeps the infinite only in a state of potentiality. For Bowin then, the infinite exists *in potentia* for the specific process of

[38] *Cf.* Bowin, *op. cit.,* pp. 241 and 247-250.
[39] *Ibid.,* pp. 241-247.
[40] Bowin, *op. cit.,* p. 241.

division, which is also why he later argues that the infinite is *per se* to the category of quantity—but only potentially.[41] To say that the potential infinite *exists* in the process of division but not to grant such process any actuality is peculiar. Goalless processes are actualities too, as I will argue in chapter 4, because they still occur in an actual thing. Unlike Ross, Bowin seems unwilling to grant the infinite any actuality even as it appears in the goalless process of division.

Pascal Massie argues for what Bowin seems to miss and what Ross's account implies: Aristotle objects not to the infinite as an actual goalless activity but to an actual infinite *body*:

> What Aristotle rejects here [in *Phy.* III.6] is not at all the concept of actual infinity *simpliciter*, but the idea that infinity could be actual in the sense of something simultaneously given as a whole: that is, that infinity could be something complete, achieved and separate. The so-called Aristotelian rejection of actual infinity in Book 3 of the *Physics* concerns, in fact, the impossibility of an actual infinite body (a body that would infinitely add something to itself). This, argues Aristotle, is never the way a body is. But it does not follow that infinity can only be in potency and never in act.[42]

As Massie's argument suggests, an actuality need not pertain to a *body*. There are activities that admit of both actuality and potentiality that are not themselves extended such as motion. To be sure, *Phy.* III.5 is all about undermining the existence of an actual infinite body. However, it does not follow, as Massie points out, that no infinite exists "in act." There is never a *completely* actual infinite, since this would pertain to an extended magnitude; but, this does not preclude incomplete activities. Additionally, Massie's reading implies a corollary: were the infinite to

[41] *Ibid.* pp. 247-250.
[42] Massie, *op. cit.,* p. 579. *Cf.* Côté, *op. cit.* pp. 400-401.

exist only *in potentia* and "never in act," the infinite would be a *pure* potentiality. If there is no actual infinite *whatsoever,* then the only remaining option is an infinite as a *pure* potentiality. This is a problematic consequence of the standard reading that I explore in chapter 4.

To make sense of Aristotle's claim at *Phy.* III.6, 206b13 that the infinite is an ἐντελέχεια like the day and the games, Wolfgang Wieland distinguishes between an actual infinite that is always coming to be—i.e. motion—and one that is "in sich selbst." For Wieland, the latter is impossible for Aristotle, but the former is plausible. It turns on the *eigentümliche Verschränkung* of actuality and potentiality in motion:

> Die eigentümliche Verschränkung der Modalkategorien bei den Bewegungsphänomenon beruht gerade auf ihrer Zeitstruktur: wirklich ist immer nur das, was gerade gegenwärtig ist; zu dieser Wirklichkeit gehört aber bei Phänomenon wie Tag oder Wettkampf daß sie die Möglichkeit haben, immer wieder ein anderes zu werden. So ruht diese Wirklichkeit niemals in sich selbst. Es ist daher nur in einer ersten Näherung richtig, wenn Simplicius die Schwierigkeit dadurch lösen zu könne glaubt, daß er δύναμις und ἐνέργεια beim ἄπειρον schlechthin dasselbe sein läßt ([*In Arist. Phy.*] 493.19f).[43]

For Wieland, the actuality of the infinite stands or falls not with respect to any antithetical relationship between actuality and potentiality, but whether the actuality exists "immer wieder ein anderes zu werden" or "in sich selbst." Aristotle unequivocally denies the existence of an infinite *in sich selbst* because again this would pertain to an *actual body*—a physical substance. However, the actuality that is "immer wieder ein anderes zu werden" is like the *motion* of the

[43] Wieland, *op. cit.*, p. 299.

day and the games. Wieland comments on the *Zeitstruktur* of Aristotle's examples are interesting, but this is not the place to explore such implications.[44] For now, it is enough to suggest that it is with respect to the actuality of motion that we may grasp the actuality of the infinite.

The ancient commentators also suggest that the infinite is actual with respect to motion. The reason is that the actuality of the infinite, according to Simplicius, parallels that of motion:

> And just as the actuality and the changeable preserving the potential is change, so is the actuality of the unlimited. Just as things having their being in becoming lose their being in losing their becoming, so things whose being is in potentiality exist *just so long as their potentiality exists*. But [Aristotle] says, if someone seeks in these cases that which is potential as such, let him assume the moment when the magnitude is not yet being divided and when the contest can be completed but is not yet being completed. Its actuality is being present with the potentiality in the process of bringing to completion, rather than in its having been completed as in the case of the statue. For what is unlimited is not a whole but part by part.[45]

Here, Simplicius clearly appeals to motion to explain Aristotle's claim to the infinite's ἐντελέχεια. As I read Simplicius's argument, the actuality of the infinite exists similarly to

[44] Wieland's Heideggerianism is tacit, but not inconsequential. He's reading Oskar Becker, Walter Bröcker, and Hans Georg Gadamer, each of which is quite familiar with Heidegger's temporalizing of Aristotle. See Oskar Becker, *Mathematische Existenz: Untersuchungen zur Logik und Ontologie mathematischer Phänomene* (De Gruyter, 1973); Walter Bröcker, *Aristoteles* (Frankfurt am Main, 1935); Hans Georg Gadamer, "Der aristotelische Protreptikos und die entwicklungsgeschitchtliche Betrachtung der aristotelische Ethik," *Hermes* 63 (1928), pp. 138-164.

[45] Simplicius, *In Phys.,* 493.24-32, modified Urmson translation.

motion's. It is what I will call an *active potentiality.* The infinite's potentiality is *entirely mixed* (συμμεμιγμένον) with actuality because it something like becoming which is always both actual and potential.[46] In fact, this type of actuality is not *eigentümlich* at all, if we discard a hard and fast distinction between actuality and potentiality that some commentators seem to commit to Aristotle. The nonexistence of an actual infinite body does not entail a strictly potential infinite. Not all actualities are complete, let alone physical bodies. The infinite has a chance at an actuality if it makes sense to attribute it to something changeable, like a moving body.

Everything falling in the physical realm is *mixed* with both actuality and potentiality in different degrees. This includes the infinite! Overlooking or tempering the concomitance of actuality and potentiality seems to be the common mistake scholars make when wrestling with the nature of Aristotelian infinity. As we can see, the extent to which one is willing to allow an actuality to be incomplete seems to correlate to the willingness to entertain some measure of an actual infinite. Even those commentators who appeal to the connection between infinity and motion in Aristotle in order explain the actuality of the former do not provide a robust account of *what kind of motion* admits of an actual infinite. But to see how Aristotelian motion helps us account for an actual infinite, we need to first understand in detail Aristotle's account of motion.

[46] *Ibid.*, 497.17.

CHAPTER 2

MOTION, CATEGORIAL BEING, AND ANALOGY (*PHY.* III.1-3)

Standard readings of *Phy.* III.1-3 usually focus only on clarifying the definition of motion itself at *Phy.* III.1, 201a9-11.[47] This, however, neglects the overarching categorial issues which frame it. In this chapter, I will show that *Phy.* III.1-3 aims not to give a definition of motion in any strict sense—i.e. a scientific definition with a single genus and specific differentia—but to 'define' it in such a way that both captures what motion is while also resolving categorial issues arising from the fact that the causes of motion—and by that means, motion itself—are in multiple categories. Substance, quality, quantity, and place each have their respective forms and materials. Since, each category has its own forms and materials appropriate to it, there is no common form or material in which each category shares, because there is no genus higher than the categories of being. We know from *Phy.* II.3, 195a26-195b30 that as causes of motion, form and material manifest differently because they are causes which pertain to different *kinds* of things. It follows, then, that respective categorial *motions* will be different and uncommon as well. But, in order to give a proper scientific definition, something has to belong to *one* kind of thing in the same way as other members of the genus. A scientific definition indicates universally and necessarily a thing's genus and essential attribute(s). Both dogs and humans share the same basic features of 'animal'—the capacity for sensation. 'Sensation' is predicated of 'animal' *univocally*. Dogs, cats, and horses have just as much the capacity to see, hear, taste, smell, and

[47] See §1.2.1 above for a full discussion of this literature.

touch as human beings. But when it comes to natural entities in general, motion is predicated *equivocally* because the causes of motion—form and material—fall in fundamentally different categories. That is to say, there is no genus of form and material that is common to all motions. How is a proper scientific definition possible for motion, if its causes are different across categories? What is motion universally and necessarily, if its causes are not always the same?

Also in this chapter, I will show how Aristotle begins to gesture toward his account of the infinite in *Phy.* III.4-8. The analogical character of motion and motion's indefiniteness will converge in the account of the infinite. In *Phy.* III.2, we have a first indication of this. There, Aristotle shows that motion is inherently difficult to understand because it is an incomplete actuality. As incomplete, it is *indefinite*. The indefiniteness of motion is an indication of the existence of the infinite, since what is indefinite is something unbounded or without a limit. I will argue that this is the first gesture toward the account of the infinite. But because motion is analogical, Aristotle will need to determine whether the infinite applies to all motions in the same way or only to particular motions. Does infinity apply to all motions in the same way, or if it applies only to particular motions, are other motions infinite analogously? I will show that for Aristotle in *Phy.* III.6, the infinite will apply specifically to quantitative change of division and addition, but by analogy to motions in different categories.

§2.1. The transcategorial status of motion (*Phy.* III.1)

The definition of motion is notoriously unclear as to what exactly it refers to as 'motion,' and because of this, it has made for considerable debate in the scholarship. Part of the frustration is that interpreters tackle the definition headlong often without due diligence to the categorial issues preceding it. What ought we say about Aristotle's opening discussion at *Phy.* III.1, 200b25

on the categories and motion's relationship to them? How might they influence the way Aristotle constructs the definition and what is Aristotle's method? Without giving a rigorous analysis of the initial categorial issues prior to any engagement with the definition, our understanding of the whole account in *Phy.* III.1-3 begins on the wrong footing.

To fill this gap, I approach the text from a different vantage than most of the scholarship: the categorial remarks at 200b26-201a9 set the stage not only for the definition of motion, but also for the rest of the account in III.1-3, because they raise the very issue with which Aristotle will be wrestling—motion as a category problem. The ontology of motion is indeed a central issue for Aristotle. But, I contend that this arises from a deeper problem of what to do about formulating a definition of motion *qua* definition. Because motion is found in multiple categories, and because each category is a different kind of being, motion manifests differently in each. This means that it cannot be defined with respect to one genus, since the categories are the highest genera; there is nothing else higher than the categories by which they could share something in common. How ought we define motion, then, when properly speaking a scientific definition refers to an individual or at least a single kind of thing?[48] What is motion, if there is no univocal articulation of it?

What seems puzzling at first is precisely *that* Aristotle begins the account of motion by addressing the categories. Edward Hussey, for example, is concerned that the turn to categorial issues is abrupt.[49] This is not altogether an unreasonable worry. There is no immediate context

[48] Owen Goldin, *Explaining the Eclipse: Aristotle's* Posterior Analytics *2.1-10* (Ann Arbor: Univ. of Michigan Press, 1996), p. 94 nicely argues why the conditions for scientific definitions are so strict: "Since a scientific definition presupposes the existence of the definiendum, such a definition presupposes an account by which this existence is posited or justified." The definition of motion, however, does not have a definiendum, at least not straightforwardly. In fact, it has *many different* definienda, but this is the problem! To which does it refer?

[49] Hussey, *op. cit.,* p. 56.

for why Aristotle should even begin the account by commenting on the relationship between actuality/potentiality, the categories, and motion, nor is there is any initial dialectical account so as to give us generally accepted reasons for such a starting point. Normally, the ἔνδοξα—Aristote's predecessors and commonly accepted beliefs—lay down a sort of foundation from which to commence an inquiry. They provide context for the subject matter, identify what counts as acceptable or unacceptable assumptions, and supply Aristotle with problems he believes to be inherent to the subject matter. However, the dialectical treatment concerning motion comes later in III.2, *after* Aristotle has given what he thinks is a proper treatment (καλῶς εἴρηται, 201b16) of motion. Why does Aristotle not begin from III.2, especially since he addresses his predecessors on categorial issues? Aristotle's seemingly precipitous turn to the categories should be cause for pause. Why does Aristotle begin the account in this manner? What is the purpose of his remarks on the categories? In *Categories* 14, Aristotle addresses the many types of motion, where he already argues that alteration and increase are not the same type of motion (*Cat.* 14, 15a20-31). Why redress the issue here?

Let us consider this problem in another way: why is it that Aristotle gives no obvious reason to tackle motion with respect to the categories, when in *Phy.* II he seems previously to have given a fairly thorough and general account of motion with respect to nature (φύσις) as the *substance*? Could book II's analysis of substantial change suffice for an explanation of motion, especially since it identifies nature as motion's cause? Edward Halper recently argued that in book III Aristotle turns to the accidental categories in order to show how the account of substantial change in book II is extended analogically to all categorial changes.[50] *Phy.* II demonstrates universally for all physical entities that nature causally governs substantial

[50] Edward Halper, "Aristotle's Scientific Method," pp. 93-95.

generation. The demonstrative power of a nature lies in the role it serves as the middle term in a scientific syllogism "whose conclusion is that what has the form is necessarily in motion."[51] In other words, nature *qua* form dictates why and how a natural substance comes to be:

> We know that things are in motion, and we know that of the three principles of motion, form is primary because it accounts for the other two. But form exists together with matter in a composite. Because the composite is not eternal, the form must come to be present in the composite, and this requires some pre-existing form [to] act upon a matter so that the matter develops both an organ capable of performing the function that the form is and other organs capable of performing functions that sustain this primary organ. Because the material in these organs must eventually wear out, it will necessarily lose its functions, and the substantial form must cease, sooner or later, to be present in the composite. Hence, form is necessarily the cause of form's coming to be present in the matter and of the course of decay through which the form ceases to be present. These processes—that is, generation, development, and decay—are the characteristic motions that the form necessarily causes. The form is, thus, the middle term of a syllogism whose conclusion is that what has the form is necessarily in motion. As such the form is properly causal in so far as it accounts for the thing's necessarily being in motion and for the characteristic motion it undergoes.[52]

At *Phy.* III.1, 200b14, Aristotle asks explicitly, "What is motion?" (τί ἐστι κίνησις). For Halper, this question resembles the sort of procedurally scientific question set down in *APo.* II.1-2,

[51] *Ibid.,* p. 91
[52] *Ibid.,* pp. 91-92.

which is to look for a causal middle term, just as Aristotle does when defining the lunar eclipse.[53] When Aristotle asks, 'What is the lunar eclipse?' the answer is given through the *nature* of the moon. The lunar eclipse is a *per se* attribute of the moon because it is an essential part of the nature of the moon to be interposed between the Earth and the Sun at specific points of its circuit around the Earth (*APo.* II.2). The nature accounts for a thing's *per se* attributes (*APo.* II.1-2), which are demonstrated by a syllogism whose terms, especially the middle term (which is the nature *qua* cause), apply universally and necessarily. In this way, the eclipse is univocally attributed to the moon. But in *Phy.* III, there's a difficulty—as Halper points out, "the eclipse is a *simple* phenomenon; motion is *many*."[54] Here, Halper points to the difficulty Aristotle faces in the *Physics* when testing the demonstrative power of nature as a *cause* of motion. A scientific syllogism demonstrates the essential and necessary connection between an attribute and its substrate *via* a nature—i.e. a form. Unlike the eclipse's *per se* attribution to the moon, motion is attributed *equivocally* because there is no primary form that explains motion universally, since the form governing motion are different depending on the category in which motion manifests. In other words, multiple motions exist because there are different forms that account for each, but there is no common form that accounts for all motion in the same way.

The solution, Halper argues, is that nature still functions causally for *all* types of motion since "all the genera of motion can be treated together in one science of physics in that they are

[53] Aristotle uses the account of the eclipse's *per se* attribution to the moon in *APo.* II.1-2 as an archetype for scientific demonstrations which require scientific definitions as middle terms. See Halper's analysis of Aristotle's account of the eclipse in "Aristotle's Scientific Method," pp. 60-71. *Cf.* Goldin, *Eclipse*, pp. 17-25. See also L.A. Kosman, "Understanding, Explanation, and Insight in Aristotle's *Posterior Analytics*," *Exegesis and Argument: Studies in Greek Philosophy Presented to Gregory Vlastos,* ed. E.N. Lee *et al* (Netherlands: Koninklijke, 1973), pp. 375-377.
[54] Halper, "Aristotle's Scientific Method," p. 93, my emphasis.

all attributes of the categorial genus of substance."[55] Even though motions differ categorially, Halper argues that Aristotle may cull together each type by "making an analogy between substance and the other categorial genera."[56] This analogy is the source of the definition of motion:

> There is nothing common to beings in distinct categories. Nor, consequently, can the changes in those categories—namely, qualitative change, quantitative change, and change in place—have anything in common except, perhaps, belonging as attributes to the same substance (200b34–201a1). Having worked through the account of nature in book II, we can see that Aristotle is extending to all beings the account of substantial change he developed there by making an analogy between substance and other categorial genera. Just as each substance has a motion in accordance with its nature, namely the motion by which a matter realizes this nature, so too in the other categories there is a motion through which something in that category comes to be actualized in a matter. And just as the actual substance comes to be from a potential substance, an actual quantity comes to be from another quantity that is potentially that quantity, and analogously for changes in the other categories. In each genus, a potentiality for a form is realized when it comes to be an actuality in that genus.[57]

[55] Halper, "Aristotle's Scientific Method," p. 95.
[56] *Ibid.,* p. 94.
[57] *Ibid.*

The problem Aristotle faces, according to Halper, is grasping how these different potentialities in each genus are actualized similarly.[58] This is what the definition of motion should solve. But it's not entirely clear from the definition alone. The specific potential in each genus is rendered into its respective actuality. Just as substances develop from and breakdown into their proximate materials, something transitions from white to black (vice versa), big to small, etc. In each categorial genus admitting motion, there is a pair of contraries which pertain to the material and form requisite for that particular type of motion. Each type of motion is an actuality of a potentiality *qua* potential, modeled most of all by substantial generation. For, according to Halper, accidental motions are like substantial change because they occur *in* the substance as a primary matter:

> […] substance either comes to be itself or serves as the matter for non-substantial motions. To this extent, even the motions in other categories are, in a way, motions of substances, and physics is concerned primarily with the motions of substances.[59]

So, Aristotle may solve the problem of how different potentials are actualized in similar ways because they are ultimately actualized in a substance similarly to the way the substance itself is actualized. Every pair of contraries requires a substrate. That substrate is ultimately the substance. The substance serves as the material substrate for every categorial motion, according to Halper. So, by grasping how the potentiality of the substance is actualized—which is the aim of *Phy.* II—Aristotle may extend that analysis to all other motions analogously.

[58] *Ibid.*
[59] *Ibid.,* p. 95.

Halper believes the account of substantial change in book II is extended analogically to the other categorial changes. And I mostly agree. It will be crucial to the account of the actual infinite that motion pertains to that *of substances*, since division is primarily *in* the substance.[60] Still, Halper claims that Aristotle introduces the definition in *Phy.* III.1 "[w]ithout explaining how he arrived at it."[61] But this is not correct. Aristotle arrives at the definition by working through the relationship between motion and the accidental categories, however briefly it might be. Halper does not give an analysis of Aristotle's categorical remarks in *Phy.* III.1. This is a significant lacuna, since book III investigates motion with respect to the *accidental categories*, which is the focus on Aristotle's initial categorial remarks. In other words, Halper does not consider III.1 as providing a methodological alternative to accounting for motion in non-substantial categories, even if they are analogous to substantial change. The scientific syllogism does not apply straightforwardly in the account of motion precisely because the definition of motion does not contain a causal middle term. The way Aristotle recognizes this is by analyzing how motion operates analogically in the accidental categories. In other words, in *Phy.* III.1, Aristotle formulates the definition of motion as a solution to a category problem.

§2.1.1. Motion as a category problem (*Phy.* III.1, 200b26-201a9)

Aristotle's categorial remarks on motion (*Phy.* III.1, 200b26-201a9) consist of four brief arguments. Simplicius calls them "useful assumptions" (τὰ χρήσιμα λήμματα); but to what extent are they "useful"?[62] It is not exactly clear how each remark follows from the other. But when read as a convergent argument, the categorial remarks altogether reveal that motion is

[60] See §4.1.3 below.
[61] Halper, "Aristotle's Scientific Method," p. 95.
[62] Simplicius, *In Phys.* 413.15.

categorially problematic insofar as it manifests in different ways across the highest categories and yet motion still requires a definition.

Argument one is that the scope of motion is categorial being because motion pertains to those things which are both actual and potential (200b26-28). Argument two pertains to how the mover and what is moved is exclusive to the category of relation, since the relation between mover and moved is predicated *univocally* of particular entities (200b28-32). Argument three shows that there is no motion apart from particular entities (τὰ πράγματα), i.e. no higher genus of motion, because motion is *always* with respect to the categories (200b32-201a3). Finally, argument four shows each category is predicated of entities in two ways—form and privation—since each has a pair of contraries to which and from which there is motion (201a3-201a9). The tacit assumption in each argument is that form and material account for the motions in the categories. In (1), actuality and potentiality correspond to form and material respectively; in (2) form and material function in terms of the mover and what is moved respectively, since the mover acts upon some material; for (3), the assumption that motion is always with respect to the categories is because form and material, as the causes of motion, only belong to them as well; in (4), Aristotle assumes that categories have their own pair of contraries precisely because each contrary is a pair of form and material/privation. When taken altogether, these four arguments reveal that motion is categorially problematic—it is transcategorial. Aristotle must locate a way to define motion that, on one hand, does not undermine its transcategorial status, but on the other, provides another way to insight motion in some unified manner without having to investigate all the particular ways it manifests in the categories.[63] This will be by analogy.

[63] *Cf.* Kosman, "Understanding, Explanation," pp. 383-392 for his discussion on *APo.* II.19 regarding the role of νοῦς and ἐπαγωγή in locating universal principles without investigating every particular.

Argument one (200b26-32) reminds us that the scope of the inquiry is restricted to categorial entities—the natural substance and all of the ways it is predicated:

> While there is indeed something in actuality only, there are those things which are both potential and actual, [for there is] substance, quantity, quality, and similarly with the other categories of being (*Phy.* III.1, 200b26-32).

Motion pertains only to those beings which are both actual and potential because on one hand to exist in any respect is to be actual to some degree since an existing thing has form and the form is the actuality, while on the other hand motion requires some sort of potential since all motions require some specific kind of material that takes on a new form. The relevant spheres of physics are the terrestrial and the heavens, since these are wherein things are both actual and potential—categorial being.[64] Entities bereft of materiality, therefore, never admit of motion, i.e. pure actualities. Even though Aristotle is not explicit here, the primary candidates for pure actualities are those that are eternal—i.e. the unmoved mover. The ultimate conclusion of the *Physics* is that because there is motion, and yet it is impossible for there to be an infinite chain of causes, there must be a mover that is itself incapable of motion (*Phy.* VIII.9-10). Therefore, this primary mover, whose character is left for first philosophy or 'metaphysics' to determine, is a pure actuality (τὸ ἐντελεχείᾳ μόνον). A pure actuality is necessarily motionless, since it is immaterial. As motionless, it orders the entire cosmos by being that for which the outermost heavenly sphere strives, which in turn moves the remaining lower spheres, including the terrestrial (*Meta.* Λ.9).

[64] See Simplicius, *In Phys.* 399.19-400.23 for a similar argument for the extensionality of nature and categorial being.

The concomitance of form and material determines the scope in which there is motion. Actuality and potentiality *together* belong similarly (ὁμοίως) to substance, quantity, quality and the other categories of being (τῶν ἄλλων τῶν τοῦ ὄντος κατηγοριῶν). Because form and material correspond to actuality and potentiality respectively, and because the categories are extensional with entities which are both actual and potential, whatever is purely actual (or purely potential, were this possible) is necessarily excluded from any inquiry into motion. But, Aristotle's argument here is not simply to restrict motion to categorial being, as if a general, universal account can be given *univocally*. The tacit assumption here is that the categories of being are the highest kinds in which both form and material manifest as causes. Since form and material manifest in each category in different respects, the way that we understand how form and material function generally as causes of motion is inextricable from how actuality and potentiality function differently in each category. Actuality and potentiality are therefore transcategorial.

As Aristotle turns to relation (τὸ πρός τι) in the second argument, the discussion shifts from the transcategorial status of actuality and potentiality to a brief but peculiar treatment of the category of relation:

> But in the case of relation [τοῦ πρός τι], in one way it is said [λέγεται] according to excess and deficiency, but in another it is activity and passivity, and generally as the mover and the moveable. For the mover is a mover in relation to what is movable and the movable is movable by means of a mover (*Phy.* III.1, 201a28-32).

Ross suggests that Aristotle restricts the mover (τὸ κινητικόν) and what is moved (τὸ κινητόν) to the category of relation, whereas motion in general manifests across categories, given what we know about the transcategorial status of actuality and potential in the first argument.[65] Hussey, however, takes this argument as an "obscure" parenthetical remark on the first argument.[66] Despite this, what is so significant about relation that deserves its own separate argument? While actuality and potentiality operate similarly in each category, at least according to the first argument, Aristotle believes he needs to address more unambiguously how excess (ὑπεροχή) and deficiency (ἔλλειψις) function in relation. But, for whatever reason, Aristotle does not indicate why or how. The argument appears dialectical (λέγεται, 'it is said,') and its implicit target is Plato's notion of the Great and the Small.[67] Is motion a relation, understood Platonically, as what pervades everything as an overarching category of relation—the Great and the Small—or is it because every motion is *attributed* univocally by a relation between a mover and what is moved? That is to say, is motion itself identical to the genus of relation or is relation its own category? Plato's notion of excess and deficiency pertains to a genus of relation, which is the Great and the Small, out of which everything is generated. While Aristotle returns to this issue in greater detail in *Phy.* III.2, he addresses it dialectically here in order to rebuff a Platonic classification of motion that treats κίνησις *separately* from physical individuals. Aristotle seems to take excess and deficiency categorially for Plato, and one in which all generated entities participate. Both greatness/smallness and excess/deficiency refer to the same class of things for Plato—relation

[65] Ross, Aristotle's *Physics*, p. 535
[66] Hussey, *op. cit.,* p. 58
[67] Simplicius, *In Phys.* 401.30-402.8 recognizes this as well. For further evidence that Aristotle's remark here about 'relation' is a dialectical response to the Platonists, see *Meta.* A.9, 990b17-23 where Aristotle contrast his categorial notion of relation to the Indefinite Dyad. Lang, *Order of Nature*, p. 58 also thinks Aristotle's target might be Plato, but for different reasons—to rebuff Plato's notion of the self-moving soul. I see no evidence for this reading, as it does not make sense of excess and defect in the argument.

itself. Relation, understood in terms of excess and deficiency, is a reference to plurality, and plurality is a source of generation, specifically of mathematical objects and number.[68] But unlike this Platonic category of Relation, for Aristotle there is no such general class—the Great and the Small—upon which a form acts so as to generate something (*Meta.* A.9, 990b17-19).[69] For, form and material always pertain to specific thing—a *particular* substance, quality, quantity, etc. in relation to something of the same kind. Because the relationship between the mover and the moved is in terms of activity and passivity, the mover acts upon a *specific* kind of material/potential, and what is moved is acted upon by a specific form/actuality (*cf. GC* I.7, 323b15-324a9).

This now could help explain why Aristotle places excess and deficiency in the conclusion; it follows dialectically that, for motion, there is no class of relation which is said (λέγεται) in terms of excess and deficiency that is common to all generated entities. Even when Plato argues in the *Timaeus* that there is always a mover for whatever is moved, it is still on the assumption that what is acted upon is the Great and the Small (*Tim.* 57c7-58c4), and not a particular thing with a specific capacity that is to be actualized by a corresponding *particular* form acting upon it. The distinction is subtle, to be sure, but hardly inconsequential to Aristotle's view of motion. What is acted upon is the specific sort of material *in* the thing that is set into

[68] See *Meta.* M.9, 1085b5-13 for Aristotle's discussion of oneness and plurality in the Platonists' account of the generation of number. Apostle, *Aristotle's* Metaphysics, p. 431 remarks that Speusippus makes a finer distinction between the 'specific plurality' that pertains to the Indefinite Dyad or the Great and the Small, and the 'primary plurality' out of which the former are generated. However, Aristotle argues that there is really no difference, which Apostle points out: "For Speusippus, [the Indefinite Dyad] is *Plurality*, in the general sense, not a specific plurality like the *Dyad* or the *Great* and *Small* (which Plato posits). Of course, the *Dyad* is not like Two, which has two units, but has the *Great* and *Small* as its two parts, so to speak. Probably Plato did not elaborate about these parts; so finding it difficult to get at Plato's meaning, Aristotle proceeds to criticize Plato's principles by assigning to terms which Plato used their ordinary meaning."

[69] Simplicius. *In Phys.* 401.8-23.

motion, where the form is the actuality that acts upon this specific material/potential, in contrast to an independent substance like the Great and the Small which is acted upon for the purpose of generation.

In the third argument (200b32-201a3), Aristotle shows more explicitly that there is no genus of motion precisely because motion does not exist apart from particular *concrete* entities (τὰ πράγματα) in any univocally common manner:

> But there is no motion apart from particular beings, for something changes either substantially, quantitatively, qualitatively, or with respect to place; and there is nothing to take up as common [κοινὸν] to these which is, as we say [ὡς φαμέν], a this, a quantity, a quality, or any of the other categories. Thus, neither motion nor change will be in any way apart from what has been said, for there is no being apart from them (*Phy.* III.1, 200b32-201a3).

Just as we see in the first argument concerning the scope of actuality and potentiality, physical being extends only as far as the highest categories; again, physical being *is* categorial being. Because the causes of motion are form and material, and because there is no form or material apart from the categories, motion always manifests within the scope of categorial being. Therefore, motion extends only as far as categorial being. But as we know from the second argument concerning relation, this is also because form and material are *in* the mover and what is moved. Each mover acts upon another always with respect to a specific kind of thing. When we read this argument as framed and informed by the preceding arguments, the inference that there is no motion apart from particular entities follows. Motion and change occur when a certain kind

of thing acts upon another thing of the *same* kind. For, substantially (κατ' οὐσίαν), the human being acts upon another human being so as to generate a child; qualitatively (κατὰ ποιόν), what is hot acts upon something cold whereupon the latter is heated and vice versa; quantitatively (κατὰ ποσόν), what is more or less is added to/subtracted from that which has less or more, the result of which is increase and decrease respectively; and locatively (κατὰ τόπον), an entity pushes or pulls another, whereupon there is locomotion (φορά). But because there is no common form and material that transcends particular entities, there is no common way in which motion manifests; motion shows up differently for each kind of entity because of the specific forms and materials. This is what Aristotle means when he claims that "there is nothing to take up as common to these which is, as we say, a this, a quantity, a quality, or any of the other categories" (200b34-201a1). It is precisely because of the categorial structure of natural entities that motion is prohibited from having its own generic classification. Were there such a thing as a genus of motion, there would have to be a *univocally* common form and material prior to the highest categories; but, again, there is *nothing* higher than these (μηθενός γε ὄντος παρὰ τὰ εἰρημένα, 201a2-3). But there is no common form and material in general that applies to all things in the same way since form and material extend only as far as the categories, and even then, they manifest as causes *distinctly* per category.[70] Thus, there is no common, univocal way by which motion manifests. Instead, motion occurs differently with respect to each category.

Before we move to the fourth and final argument in Aristotle's remarks on the categories, there are several peculiarities with the structure of Aristotle's third argument that will help us to understand better its connection to the second argument and thereby how it sets up the fourth. The third argument can be read as responding dialectically to the concerns in the second

[70] *Cf.* Phil. *In Phys.,* 349.15-26 where he reports Theophrastus' view on interwoven categories.

argument. First, the dialectical force can be seen in Aristotle's use of both φαμέν (200b35) and λέγεται (200b29). Whereas Platonists hold that ("it is said," λέγεται, 200b29) there is a general class of contrariety/relation in which *all* motion manifests, "we say" (φαμέν)—i.e. Aristotle and his followers—that motion is *always* (αεὶ, 200b33) with respect to specific categories, wherein there is nothing common to each so as to indicate a class of things beyond them. Whereas the Great and the Small is a sort of genus of motion for Plato, Aristotle distinguishes his position by assuming instead that motion manifests only within the confines of categorial being, wherein there is nothing common amongst them which could function as a prior kind of thing. Secondly, the way Aristotle formulates the inference at 200b32-33 is another indication of this view: "there is no motion apart from *particular entities*" (οὐκ ἔστι δὲ κίνησις παρὰ τὰ πράγματα). Aristotle's use of τὰ πράγματα is noteworthy; it concretizes motion as something inextricable from particular entities. The term πράγμα signifies a matter of fact, some concrete phenomenon in the physical world, which has a precise categorial determination—e.g. this human being writing a book, an instance of the color blue attributed to a chair, a particular magnitude of five feet tall, or a specific location such as the house in which a person dwells. Aristotle's use of τὰ πράγματα is to reinforce the concrete character of categorial motion. Because motion is always *in* the mover and the moved with respect to whatever categorial determination the mover and the moved might have, motion belongs to particular entities according to those very determinations.

In short, there is nothing strictly generic about motion. It is here that motion shows up as an aporia for Aristotle, although he is not terribly explicit that this is the case. The aporia may be formulated as follows: Because the causes of motion are form and material, and because there is no common form and material under which the categories may be classed, motion will manifest only with respect to the specific form and material that is found in each category. Therefore,

motion is as equivocal as its causes. Motion, then, fails to meet the standards of scientific definition because these definitions are univocally constructed (at least when strictly formulated in terms of a genus and some essential differentia). What then is motion? What 'kind' of thing is it? Just what sort of definition may even be assigned to it? *That* motion exists is clear enough from the fact that it is essential to nature (*Phy.* II.1, 192b13-14). But *what* motion is and how this 'definition' is to be articulated is certainly problematic. The typical method of looking for definitions (*APo.* II. 1-2) does not work here. But Aristotle still needs a definition of some sort. Physics is a science after all. Thankfully, Aristotle does not abandon the issue. It is in the fourth and final argument that Aristotle tacitly points to a solution, due to how contrariety manifests analogically in each category. A solid understanding of this final argument will put us on firmer footing when we look to the definition of motion momentarily.

Because every concrete entity is categorially determined with respect to some form and material, and because there is nothing in addition to (παρά) these determinations, motion *is* just in these ways. There is no genus of motion; for, there is no common form and material which could account for a generically robust meaning of motion. Argument four, though, provides a crucial caveat: each category still has a *similar* (ὁμοίως) or analogical contrarietal structure consisting of at least one pair of contraries that correspond to a form and its privation, i.e. actuality and potentiality:

> However, each of the categories manifests *in* all things in two ways: substantially as either the form or privation of something, qualitatively as either white or black, quantitatively as either complete or incomplete, and similarly for place as either up or

down, or as light or heavy. Thus, there are as many types of motion and change as there are types of being (*Phy.* III.1, 201a3-9).

Every category is in terms of actuality and potentiality, since each has its own sort of form and material. As we know, this is because every mover has some actuality which acts upon the potentiality of what is moved—i.e. a specific relation between a pair of contraries. And this is exactly what Aristotle's examples illustrate here. The two ways (διχῶς) each kind of thing manifests in all things (ὑπάρχει πᾶσιν) is with respect to that kind's actuality/form and corresponding potentiality/material, where the latter is understood as a *privation* (στέρησις).[71] The form-privation structure is present in each category admitting motion.[72] This is most obvious in the category of substance, but we find similar manifestations of a form-privation structure in quality, quantity, and place—the categories in which Aristotle locates motion in argument three. Whatever is actually white has the potential to become black because black is the privation of white, and as a privation, the substrate has the capacity to take on its respective form. A similar argument may be given for quantity and place, since what is complete and incomplete or up/down and heavy/light correspond to some specific form and its privation—a pair of contraries. Each category is vested with actuality and potentiality, some form and material, which pertains to contrariety. Each kind of thing has a similar contrarietal structure. But because

[71] John Anton, *Aristotle's Theory of Contrariety* (NY: Humanities Press, 1957), pp. 60-61 follows Aristotle's text literally by restricting the form-privation structure to the category of substance. While Aristotle does use form-privation at 201a4-5 to index substance specifically, I agree with Halper that that very same structure generally befits all contraries. See Halper, "Aristotle's Scientific Method," pp. 78-80. See also *Phy.* V.1-3 for a more thorough account of the role of contrariety in categorial motions.

[72] See James Bogen, "Change and Contrariety in Aristotle," *Phronesis* 37 (1), p. 2. Bogen pp. 8-11 also recognizes a sort of homonymy of motion because of how contraries differ metaphysically across genera. He argues that this seems more to do with "the unclarity of Aristotle's usage of the term 'genus'" (p. 8). See also Anton, *Theory of Contrariety*, pp. 77-78.

each pair is always *particular* to the kind in which it manifests, however, whatever similarity there is, is not due to a common genus under which all of them fall univocally. So, how do we define motion?

§2.1.2. First definition of motion (*Phy.* III.1, 201a9-201a19)

Aristotle's goal in *Phy.* III.1 is to set down a definition of motion as a solution to the foregoing category problem. Aristotle gives the definition at three places: (1) 201a9-11, (2) 201a27-29, (3) 201b8-11. Definitions (1) and (2) are forged to address the foregoing categorial problems, while definition (3) sets up the problem for *Phy.* III.2: the indefiniteness and incompleteness of motion. Definition (1) addresses the transcategorial function of actuality and potentiality. For each category that admits of motion, a specific formulation of the definition may be constructed with respect to how that particular actuality and potentiality appears in that category, e.g. in the category of quality, alteration is the actuality of what is alterable *qua* alterable (τοῦ μὲν ἀλλοιωτοῦ, ᾗ ἀλλοιωτόν, ἀλλοίωσις, 201a11-12). Through analogical reasoning, thereby, Aristotle generalizes from each specific formulation so as to define motion as the actuality of a potentiality *qua* potential. For the definition (2), Aristotle returns to the role of the category of relation. Because every motion requires both an active mover and something passively moveable, every motion will be both active and passive at the same time, but not in the same respect. Definition (3) shows how motion is to be defined as something necessarily incomplete—indefinite—because motion in a way retains the potential as it happens. Motion is potentiality at work.

What we ought to notice is that Aristotle sets down the first definition of motion (201a9-11) with respect to his foregoing categorial remarks: "*Since* these divisions have been made

according to each genus with respect to actuality and potentiality, motion is the actuality of a potentiality insofar as there remains a potential" (διῃρημένου δὲ καθ᾽ ἕκαστον γένος τοῦ μὲν ἐντελεχείᾳ τοῦ δὲ δυνάμει, ἡ τοῦ δυνάμει ὄντος ἐντελέχεια, ᾗ τοιοῦτον, κίνισίς ἐστιν).[73] Definitions indicate, most of all, independent things—substances—since definitions give the articulation of the essence, and essences pertain primarily to substances (*APo.* I.4). How can there be a definition of motion, then, if motion is not an independent thing? Attributes also have definitions (*APo* I.4), and motion is an attribute. Indeed, motion is *per se* of nature. But, definitions of attributes are just as univocal as those of substances. All definitions with respect to the nature/essence require a *single* genus in order for it to have an essential differentia which differentiates what is defined from other members of the same genus; the definiens of an entity is composed of a univocal genus that is specified by a *per se* attribute. How, then, is there a single articulation of motion if there are categorially/generically different motions, but no overarching genus under which all fall? To be sure, Aristotle provides a definition at 201a9-11; but *how* is this definition of motion possible, let alone one required by a science, and what might be his reasoning for drawing it up? My suggestion is that it is by the analogical reasoning sketched in *Topics* 18. Simplicius directs out attention to the account of similarity (τὸ ὁμοίος) in *Topics* I.18 and analogical demonstrations in *APo* II.17:

> Having said that one can find nothing common to the different sorts of change, [Aristotle] made clear what sort of common element he is denying by shifting the discussion to the categories in which there is change when he said 'which is neither a particular thing nor a

[73] A *prima facie* indication of this seems to be how Aristotle frames the definition as following from a genitive absolute (which can sometimes function to indicate an inference). Literally, the first clause reads "*Having made these divisions* [διῃρημένου]," where the genitive absolute διῃρημένου signals the established assumptions previously made in 200b26-201a9.

> quantity nor a quality.' […] But if change is equivocally named, how does [Aristotle] define it? For there are no definitions of the equivocally named, or else the definition of the equivocally named are also equivocal. For since 'what is primary' [ἀρχή] is equivocally named, the definition that says that 'what is primary' is the first each thing is also equivocal [*Top.* I.18, 108b27]. Moreover, the actuality of the changed *qua* changed will be equivocal. For the product of the equivocals is equivocal. Therefore, he had to add this lemma lest someone who heard the definition of change should think that the definition is given of it as a single genus, and so that it should be clear that the account is given through an equivocal expression as being of an equivocal expression [*APo.* II.17, 99a16-17] (modified Urmson translation).[74]

According to Simplicius, Aristotle denies motion the strict generic character that each category enjoys. That is to say, unlike substance, quality, quantity, and place, motion is not itself properly generic. So, Simplicius asks the pertinent question: *how* ought it be defined? That is to say, not only what does the definition itself need to look like, but also what is the *procedure* for arriving at it? To be sure, Aristotle is far from explicit. We know that insofar as the causes of motion differ categorially, motion is different. How then is Aristotle able to locate what each type of motion has in common without a prior genus of motion?

We have in the *Physics*, as Simplicius suggests, what Aristotle discusses at *Top.* I.18, 108b23-33: the rendering of things that are "fundamentally distinct" (ἐν τοῖς πολὺ διεστῶσι,

[74] Simplicius, *In Phys.* 403.36-404.15. Though Simplicius does not acknowledge it, Aristotle's discussion in *Topics* I.18, 108b7-31 comes from Archytas (DK47A22 = *Meta.* H.2, 1043a19-25). See Carl A. Huffman, *Archytas of Tarentum: Pythagorean, Philosopher and Mathematician King* (Cambridge: CUP, 2005), pp. 490-505.

108b23) into a definition by means of "the intellectual grasp of likeness" (ἡ τοῦ ὁμοίου θεωρία).[75] That is to say, there is a way to discover a definition through analogical reasoning:

> As for the rendering of definitions, because it is possible to survey [συνορᾶν] what is the same in each thing, we will not be at an impasse concerning what is necessary to set down before us as the definition(s) of a genus; for what is most common to each thing is what must be predicated of the genus. Similarly, for things which are fundamentally distinct [ἐν τοῖς πολὺ διεστῶσι], the intellectual grasp of likeness [ἡ τοῦ ὁμοίου θεωρία] is useful for finding definitions, such as when the calm in the sea and the quietness in the air are the same (for each is a sort of rest) and also between the point on a line and the unit in number (for each is a sort of principle) (*Top.* I.18, 108b24-27).

Notice how Aristotle's examples are framed as analogical arguments: "…the calm in the sea and the quietness in the air are the same (since each is *a sort* of stillness), as well as a point on a line and a unit in a number (since each is *a sort* of principle)" (ταὐτὸν γαλήνη μὲν ἐν θαλάσσῃ, νηνεμία δ᾽ ἐν ἀέρι (ἑκάτερον γὰρ ἡσυχία), καὶ στιγμὴ ἐν γραμμῇ καὶ μονὰς ἐν ἀριθμῷ (ἑκάτερον γὰρ ἀρχή)). The arguments work in the way described at *APo.* II.17, 99a15-16, such that demonstrations are analogous when their middle terms are similar. Aristotle never elaborates why the middle terms must be analogous nor does he develop an explicit protocol for how to

[75] Admittedly, Aristotle never uses κατ᾽ἀνάλογον or ἀναλογία in *Topics* I.18. However, his examples of likenesses are clearly analogies (*Top.* I.18, 108b23-33; *Top.* I.17). See Alexander, *In Arist. topicorum*, p. 118, 6-30, Muskens, *op. cit.,* p. 14, and Julius Pacius, *In Porphyrii Isagogen et Aristotelis Organum Commentarius* (Hildesheim: Georg Olms, 1966), p. 565.

draw up the analogy.[76] But I suggest that what we find in *Topics* I.18 and *APo* II.17 shows how Aristotle thinks it possible to see the unity of categorially distinct entities without impugning their distinct natures. The implicit syllogisms in the *Topics* passage above are as follows:

1^{α}	1^{β}
1. The sea is calm.	1. The air is quiet.
2. <u>Calmness is to be at rest.</u>	2. <u>Quietude is to be at rest.</u>
3. Therefore, the sea is at rest.	3. Therefore, the air is at rest.

2^{α}	2^{β}
1. Every line has a point.	1. Every number has a unit.
2. <u>A point is a principle</u>.	2. <u>A unit is a principle.</u>
3. Therefore, the line has a principle.	3. Therefore, the number has a principle.

The procedure for discovering the commonality between two (or more) fundamentally distinct entities seems to be by surveying the syllogisms for each thing in order to apprehend the similarity between their middle terms. By surveying syllogisms 1^{α} and 1^{β}, we notice that calmness:sea::quietude:air because the middle terms in each syllogism function similarly. The same procedure applies to 2^{α} and 2^{β}. The analogy between all the middle terms in the above syllogisms looks something like $1^{\alpha}2:1^{\beta}2::2^{\alpha}2:2^{\beta}2$. This is because we are able to grasp (θεωρία) that calmness and quietude are both ways in which the sea and the air are at rest by seeing that

[76] Commenting on *APo.* II.14, 98a20-23, Eustratius, *In analy. poster. lib. sec.*, 230.15 interprets Aristotle's use of analogy as useful for "picking out and inquiring into *causes*" (τοῦ ἐκφέρειν καὶ αἰτιολογεῖν). *Cf.* Pacius, *op. cit.*, p. 342.

their middle terms operate similarly in each account. The grasping comes by way of stepping back to survey (συνορᾶν) how their middle terms operate similarly in each account. 'Rest' and 'principle' are not genera in their own right because each manifests different in different things. For example, the earth is at rest at the center of the universe while fire rests at the edge. Many things have fundamentally different principles. However 'rest' (ἡσυχία) and 'principle' (ἀρχή) are terms that group together widely divergent instances without univocally saying what those particular things are in themselves because those instances pertain to different kinds of things. In other words, analogies allow Aristotle to unify very different things together when there is no genus to be found.[77] Understood in *themselves*, ἡσυχία and ἀρχή are terms that model their more exact types. What Aristotle takes as the class—ἡσυχία—does not stipulate any specific characteristics of the particular sorts of rests. The 'stillness of the winds' (νηνεμία) is specific to the air, whereas the 'stillness of the water' (γαλήνη) is specific to the sea. They are both different natures.[78] Thus, the terms are not commutable since they are not the same type of rest. One would never say νηνεμία ἐν θαλάσσῃ or γαλήνη ἐν ἀέρι. Nevertheless, each functions in a similar way for the sea and the water. The same applies to the analogy between ἀρχαί; the point pertains only to the line and the units only to number (*APo.* I.31, 88a30-35; I.32, 80a31-34), but points and units are sorts of indivisible things.

The account of motion in the *Physics* functions analogically insofar as the already available accounts of motion of the natural kinds—the categories—are structurally similar. That is to say, there is an analogy between middle terms in each particular scientific account as what is responsible for demonstrating the particular way those things come to be. The different

[77] See also Muskens, *op. cit.,* p. 29 and Alexander, *In Arist. topicorum,* 124.11-29.
[78] LSJ *s.v.* νηνεμία and γαλήνη.

motions may fall under a single definition insofar as they are one by analogy, as evident by how Aristotle frames the definition of motion by examples from the categories (*Phy.* III.1, 201a9-19):

> Since the distinction has been made for each kind of being between actuality and potentiality, motion is the actuality of a potentiality *qua* potential, such as how alteration is the actuality of the alterable *qua* alterable, increase and decrease is the actuality of the increasable, and its contrary, of the decreasable (since there is no common name for both), generation and destruction is the actuality of the generable and destructible, and the actuality of what is movable in place is locomotion. That this is motion is clear from the following: for whenever the build*able* [οἰκοδομητόν], so long as it is said as such, is in actuality something being built, and this is the activity of building. It is *similar* [ὁμοίως] for learning, healing, rolling, leaping, ripening, and aging (*Phy.* III.1, 201a9-19).

The overall analogy may be framed as follows: just as generation/destruction is the actuality of the generable/destructible *qua* generable/destructible, alteration is the actuality of the alterable *qua* alterable, etc., because each activity is *a sort* of actuality of a potentiality *qua* potential. Aristotle is able to define motion as the actuality of a potentiality *qua* potential because he has surveyed (συνορᾶν) how actuality and potentiality operate specifically in each category. Every type of motion consists of a particular ability (-τον), the material, which is activated by and worked out in an activity that is governed by the final actuality, the form. Notice also that building (οἰκοδόμησις) represents substantial change, learning (μάθησις) and healing (ἰάτρευσις) pertain to alteration, rolling (κύλισις) and leaping (ἅλσις) are instances of locomotion, and

ripening (ἄδρυνσις) and aging (γήρανσις) are processes of growth and decay, or increase and decrease (*cf. Phy.* V.2, 226a24-226b8). Again, Aristotle has the categories in clear view—substance, quality, place, and quantity respectively. In each category, there is a specific potential that is actualized because of the material and form pertinent to that kind of thing. The definition of motion is discovered by surveying the similarities between the middle terms in categorially different accounts of motion.[79] Take for example, the following reconstructed accounts of housebuilding, healing, aging, and rolling:

3^{α}: Housebuilding (substantial change)

1. The imposing of the shape of a house onto wood and brick is housebuilding.
2. Housebuilding is an actuality of the generable *qua* generable
3. Therefore, the imposition of the shape of a house on wood and brick is an actuality of the generable *qua* generable.

3^{β}: Healing (qualitative change)

1. The proportionate heating of the humors is healing.
2. Healing is an actuality of the alterable *qua* alterable.
3. Therefore, the proportionate heating of the humors is an actuality of the alterable *qua* alterable.

[79] Aristotle does not explicitly lay out the following syllogisms. I have reconstructed them to illustrate how one might give an account of each example of motion. *Cf.* Simplicius, *In Phys.* 416.9-16.

3^{γ}: Aging (quantitative change)

1. The gradual increasing of time in the human body is aging.
2. Aging is an actuality of the increasable *qua* increasable.
3. Therefore, the gradual increasing of time in the human body is an actuality of the increasable *qua* increasable.

3^{δ}: Rolling (locomotion)

1. The rotation along a magnitude is rolling
2. Rolling is an actuality of the locatable *qua* locatable.
3. Therefore, rotation along a magnitude is an actuality of the locatable *qua* locatable.

Each syllogism has a middle term that pertains to a motion that falls within a specific category. From there, we can generalize one step further by taking a closer look at each middle term: each is some sort of actuality of a potentiality *qua* potential. The analogy looks a bit like this: $3^{\alpha}2:3^{\beta}2::3^{\gamma}2:3^{\delta}2$. Each of these terms is also commutable. But unlike each specific motion, 'the actuality of a potentiality *qua* potential' does not indicate any particular form and material. Only the specific middle terms do. Instead, the definition of motion is a model for how the actuality of the form and the potentiality of the material function in causally similar ways for different types of motions. While the definition refers to no particular definiendum, as an analogical definition, it articulates more generally the similarities between the causes of each categorial motion. It consolidates into a single expression the model of specific actualities and potentialities in each category. In other words, similarity is a kind of geometry (ἀνάλογος). Each category has a contrarietal structure that specifies the form and material/privation—the causes—necessary for

its respective motion. Even though Aristotle objects to a general class of contrariety, as he thinks we find in Plato, there is at least a similarity that spans the categories with respect to it. Since contrariety pertains to a form and material, it also involves actuality and potentiality. In this way, motions have similar architectures. Motion is proportionally similar across categories insofar as each motion indicates the transformation from a categorially specific potentiality/material into its respective actuality/form. It takes a bit of imagination to see this since there is no immediate concrete individual that gets defined as κίνησις. To see this, let me jump ahead to the end of *Phy*. III.1.

At the end of III.1, we see more clearly the heuristic power behind the analogy: "This activity [i.e. housebuilding], then, is a sort of motion. Even more importantly, a *similar* account *will be fitted onto* [ἐφαρμόσει] the other motions" (*Phy.* III.1, 201b13-15). Notice Aristotle's geometric language, ἐφαρμόσις.[80] While in other places, Aristotle uses ἐφαρμόσις with respect to a universal as univocally applied (*APo* I.9 and I.31-32), this does not mean that it is used exclusively in this way because of its usage here in *Phy.* III.1 In addition to the definition of motion, the equivocal way the definition of the soul applies to the soul's particular forms (*DA* II.3, 414b20-25) is another clear example of when it cannot. I suggest that it indicates more of a geometric insight—a *model* of particulars:

[80] For a later technical geometrical use of ἐφαρμόσις, see Euclid's use of ἐφαρμόζειν, "to coincide with," in his Common Notion 4: "Things which *coincide with* [τὰ ἐφαρμόζοντα] one another are equal to one another." See Euclid, *The Thirteen Books of the Elements,* trans. Thomas Heath (New York: Dover Publications, 1956), p. 224. In his commentary on Common Notion 4, Thomas Heath remarks that "[i]t seems clear that the Common Notion, as here formulated, is intended to assert that superposition is a legitimate way of proving the equality of two figures which have the necessary parts respectively equal, or, in other words, to serve as an *axiom of congruence*," *ibid.,* p. 225. I point to Euclid's use of this term and Health's explanation only as a possible metaphor for understanding how Aristotle might be using ἐφαρμόσις here in the *Physics*. The definition of motion may be applied 'congruently' or as a sort of 'superposition' of the more specific types of categorial motions.

> It is now evident that a single definition can be given of soul only in the same sense as one can be given of 'figure' [σχήματος]. For, as in that case there is no figure apart from triangle and those that follow in order, so here there is no soul apart from the forms of soul just enumerated [sc. vegetative, appetitive, and rational]. It is true that a common account may be given for a figure which *fits onto* [ἐφαρμόσει] all figures, but will in no way *be* any specific character of a figure (*DA* II.3, 414b20-25, modified Smith translation).

Aristotle defines the soul as "the first actuality of the body with the capacity for life" (*DA* II.1, 414a27). Much debate has been had over its meaning—that's not my current interest. What is important for my analysis of the definition of motion is the way Aristotle treats the definition of the soul as like that of 'figure' (σχῆμα)—equivocally as a model.[81] While Aristotle never explicitly defines σχῆμα, here Aristotle means roughly 'geometrical shape.' There are many geometrical shapes, however, and each has *their own* univocal definition. But there is no individual thing to which 'geometrical shape' pertains univocally. 'Circle' is defined as a plane

[81] Averroes and Simplicius do not believe the definitions of the 'soul' and 'figure' are *simply* equivocal. In his *Long Commentary on the De Anima* II.30, Averroes says that Aristotle makes clear that the definition of the soul "is neither equivocal nor univocal" and thereby argues that soul and figure "are not among the definitions of equivocal names (since if it were so, then Geometry would be Sophistics), nor also from the genera [of things] which are said in a univocal way […] That is, the definition is not univocal but [still] it is possible for all figures, although they differ, that they have a broad universal definition fitting them all, although they differ a great deal in definition and in being. Likewise, it is possible for those different powers to have one universal definition fitting for them all, just as the definition of figure fits all the figures and is specifically proper to none" (Taylor translation, p. 138). *Cf.* Simp. *In Arist. de An.,* 107.7-14 and 107.32-35. To my ear, however, Averroes' account still sounds like the type of definition set down in *Topics* I.18—an equivocal definition of widely divergent things. I agree with Philoponus' more straightforward reading of equivocity. See Phil. *In Arist. de An.*, 255.26-37.

figure whose boundary is equidistant from a fixed point, whereas 'triangle' is a plane figure the sum of whose angles equal 180 degrees. As we can see, 'figure' appears in each definition, but it itself does not have a strict definition of its own.[82] But, Aristotle does imply a sort definition of figure at *DA* II.3 in order to illustrate how the definition of the soul is equivocally *fitted onto* specific forms of the soul: what has the capacity to be divided serially into posterior shapes.[83] The circle contains all geometrical shapes whereas the triangle contains only triangles. Solid figures contain planes and lines, the line contains points, while points are indivisible. Just like figures, then, the soul, as 'the first actuality of the body with the capacity for life,' equivocally fits onto to the particular forms: just as the human soul contains all three faculties (rationality, sensation, and nutrition), animals contain two (sensation and nutrition), and the vegetative soul contains only one (nutrition). This is what Aristotle means by "the instances of soul and figure are exactly parallel" (*DA* II.3, 414b28-29).

Nevertheless, what we need to heed for our present purposes is that the definitions for both the soul and figure are *equivocally applied*. Again the particular forms of the soul and the forms of figure each have their own proper definitions, but there is no common soul or figure apart from these. The "common account" (λόγος κοινός) of soul or figure is not with respect to specific characteristics (λόγος ἴδιος)—as one would find in the account of the human soul or the circle—but are, I suggest, general *mappings* of distinct natures that lack a strictly common account. As a mapping, the generalized formulation may be fitted onto (ἐφαρμόσις) particular

[82] *Cf.* John Vella, *Aristotle: A Guide for the Perplexed* (New York: Continuum, 2008), p. 97.
[83] Vella, *ibid.* suggests that "[t]here is a single definition of all these figures, i.e. a certain number of lines that enclose a space." Philoponus, *In Arist. de An.*, 255.28-29 defines 'figure' as "that which is surrounded by a certain boundary or boundaries" (σχῆμά ἐστι τὸ ὑπό τινος ἤ τινων ὅρων περιεχόμενον). But Aristotle never explicitly offers a definition in these terms. Simplicius does not suggest one either. However, here in *De Anima,* the definition of figure I believe Aristotle intimates is one that illustrates the equivocity of the soul's definition as pertaining to different faculties. The definition of figure that I suggest is functional and illustrative in this way.

accounts in order to measure the degree to which something may be appropriately called by a term, such as 'figure' or 'soul'—even 'motion.' The generalized formulation is not a universal (καθόλου), but a *model* that functions proportionally as a λόγος κοινός, a geometric account that maps the specific instances without determining the distinct natures (λόγος ἴδιος) corresponding to each type of thing.

Just as with geometric figures and souls, categorially specific instances of motion can be fitted onto its generalized formulation. We know that the very reason the highest categories are distinct is due to how form and material operate as causally different in each. In the same fashion, then, were the definition of motion rigorously universal—a strict definition—it would have to determine the character of the form and material of each instance. The particular instantiation of the universal is meant in the same way as the universal. The universal's univocity governs the character of the form and type of material of any particular falling under it. Human beings and horses are animals in the same respect—both have the capacity for sensation. The λόγος κοινός of motion must function not as a universal, but architectonically as a general pattern by which other instances of motion are modeled, in the very way that Aristotle describes at *DA* II.3, 414b20-25.[84] This might very well be the genius of Aristotle's definition of motion. It is rooted in a geometric metaphor, whose applicability extends methodologically beyond logic and mathematics. The analogical character of motion expresses how the causes of motion—form and material—operate proportionally across categories, given that we know the individual accounts of each type. By surveying how form and material in each category account for each respective type of motion, we can track the middle terms in each account, each syllogism, in order to find what is similar between them as causes. The definition of motion is the product of a

[84] *Cf.* Philoponus, *In Arist. De An.*, 256.28-257.1.

geometric metaphor that points to a *pattern* between the causes of motion in the highest categories of being.

§2.1.3. Second and third definition of motion (*Phy.* III.1, 201a19-29)

The second and third definition of motion clarify the roles of material and form in motion, respectively. The second definition, given at 201a27-29, is with respect to the role of material in motion: "the actuality of the potential, whenever the actuality is at work, not so long as it is itself but as long as it is movable." In other words, there is motion only insofar as the activity *retains* the potential since the material is still present in the activity as it is being shaped into the new form. The third definition is given specifically with respect to substantial change—housebuilding—in order to show that motion is always the actualization of a *specific* form: "the actuality of the buildable, so long as something is buildable, is the activity of building" (201b8-10). But before Aristotle draws up these definitions, Aristotle still needs to address what motion's precise role is in the category of relation:

> Now since some of these are both in potentiality and actuality, though not simultaneously and in the same respect, but such as what is actually hot and potentially cold, many things are at once acting and affecting one another, for all things will be both active and affective simultaneously. Thus, the moving thing is naturally movable; for everything of this sort moves what is moved and itself. Indeed, it seems to certain people that every moving thing is moved; but the merit of thing in view [ὅπως ἔχει] will be clarified from other considerations (for there is something that is both a mover and motionless). But the

> actuality of the potential, whenever the actuality is at work, not so long as it is itself but as long as it is movable, it is a motion (*Phy.* III.6, 201a19-29)

Because motion is defined in terms of both actuality and potentiality, it must be clarified also to what extent and how something can be *both* actual and potential in the same thing regardless of the type of motion. Both a mover acting upon what is moved and what is moved being affected by the mover *together* constitute a single motion. While something can be both actual and potential simultaneously, it cannot be so in the same respect. Aristotle will return to this in *Phy.* III.3, but for now the issue is resolving the difficulty of how the mover and what is moved can be both actual and potential simultaneously. The active mover corresponds to the actuality which imparts the form—the efficient cause—whereas the passive moveable entity has the potential to take on that form, which Aristotle makes clear in *Meta.* Δ.15:

> The active and the passive imply an active and passive capacity and the actuality of the capacities, such as that which is capable of heating is related to that which is capable of being heated because the former can heat the latter, and again, that which is heating is related to that which is being heated and that which is cutting to that which is being cut because they are actually doing these things (*Meta.* Δ.15, 1021a14-19).

Again, the causes of motion are at work here. The form of a composite entity has built within it the potential to become its contrary. A tacit but crucial point to notice is that the material also has an actuality too—the bronze is the potential for a statue, and what is cold is able to become hot; both the bronze and the cold are actualities in their own right. Therefore, both the mover and

what is moved are actualities. What is actually hot is contrary to something actually cold because the former is potentially cold inasmuch as the latter is potentially hot. Since the mover imparts the contrary form by acting upon whatever is able to take on that form and since the form is the actuality, the action corresponds to the actuality. That upon which the mover acts, then, must be a specific sort of material, which has the potential to be shaped into the new form. What follows from this is that the mover and what is moved necessarily belong to the category of relation in terms of activity and passivity, just as Aristotle infers at 200b28-31. What does this have to do with the definition of motion, especially since it is presented in the context of the categories? In the realm of nature—categorial being—everything is predicated by a relation between a mover, as something active, and what is moved, something passive. This is because of what motion is; but this does not mean that this relation is analogical as motion.

Because every motion requires an active mover and something passively movable, Aristotle captures this in his second formulation: "But the actuality of the potential, whenever the actuality is at work, not so long as it is itself but as long as it is moveable, it is a motion" (*Phy.* III.1, 201a27-29). However, to clarify this formulation, Aristotle turns our attention to the role of the material in motion by speaking to the function of the adverb ᾗ in the definition:

> Now, "insofar as" [ᾗ] means this: the bronze is potentially a statue, and yet motion is not the actuality of the bronze so long as it is [actually] bronze, for it is not the same to be bronze and to be with respect to a certain potential. Were they the same simply and in account, then the actuality of the bronze so long as it is bronze would be the motion. But they are not the same, as it has been said (and it is clear from contrariety, for the ability to be healthy and the ability to be sick are different; were they the same, then health and

> sickness would be the same also. But it is the substrate for health and sickness—either water or blood—that is one and the same). Since they are not the same, just as color and the ability to be seen are not, it is obvious that motion is the actuality of a potential insofar as there is a potential (*Phy.* III.1, 201a29-201b5).

I call this the ᾗ-qualification. My reading of it here is in terms of *material's* status in the generative process.[85] It pertains to the degree to which the material is *at work* in motion. Aristotle must ask whether (1) there is motion when there is the material, simply being what it is or (2) if there is motion when the bronze is actively being shaped into the statue? He answers 'no' to the former, and 'yes' to the latter. Since the material has its own form that accounts for the potential(s) that is actualized in motion, Aristotle needs to clarify when and to what extent the material is involved in motion.

Bronze has the potential to be fashioned into a statue because of its own form. Insofar as the bronze remains itself, there is no motion because it is not actively directed towards becoming the statue. The sculptor has yet to act on the bronze in such a way as to forge it into a statue. When acted upon by that mover, the potential in the bronze is energized or 'put to work' becoming a statue. But, how does this happen and what does this mean? Aristotle's use of ᾗ in the definition of motion is deictic. We say 'bronze insofar as it is bronze' when we *pick out* the bronze's own actuality, its form. But, 'bronze insofar as it is the actuality of the potential for a statue' pertains to its capacity's directedness toward another actuality—motion. Scholars have

[85] Lang, *Order of Nature*, p. 59 argues similarly: "[Aristotle's] point concerns the ways in which a thing may be *potential* [i.e. material]. Bronze, Aristotle's first example, may be potential either as bronze, *according to its definition*, or as a work of art" (emphasis mine).

described this in terms of orders of actuality and potentiality.[86] Motion is a second-order potentiality and, thereby, a first-order actuality. The first-order potentiality pertains to the latent character of an entity's potential to take on a new form—the material as it is with respect to itself, e.g. 'bronze *qua* bronze.' It is that level of being whose potentiality has not yet been put to work in the generative activity. Bronze *qua* (ᾗ) bronze is not merely an indeterminate amalgam of 'stuff.' As proximate material, it has a determination of its own. Its form pertains to the proper *ratio* of the elements, its λόγος. This is the bronze's essence. That very ratio is, nevertheless, *why* bronze is appropriate material for forging. Bronze may be defined as what has the capacity to become a statue. But, as it is being forged into a statue, the governing form is no longer that of the bronze, but the statue. This is because the statue, as the final cause, accounts for the proper steps through which the bronze's capacity to become a statue is to be appropriately and efficiently assimilated into the final product. Bronze has its own λόγος, a proper ratio of elements that determine its capacities, e.g. the ability to become a statue comes from a certain proportion of elements that allow it to be molded into a statue. When the bronze is itself—*insofar as* the bronze is directed toward its own form—there is no motion; its potential to become a statue is not yet at work because it still has its own form. But, when the bronze is being shaped into a statue, there is motion.

Simply because the bronze has the potential to become a statue due to *its own* form, not only does it not follow that when it remains unused, there is motion, but also that its own form is the very reason the material is able to be set into motion. Were the bronze *qua* bronze the same as the bronze *qua* being forged into a statue, both "simply" and "in account," then the bronze *itself* would need to be defined in the same way as the process of becoming a statue. Simply

[86] Myles Burnyeat, "*De Anima* II 5," *Phronesis* 47 (1), pp. 28-90; Edward Halper, "Aristotle on the Knowledge of Nature," pp. 93-116; Kosman, "Definition of Motion," pp. 40-62.

having the proper proportion of elements, which enables bronze to be appropriate material for forging, would be the very activity of forging. This is not the case because the substrate is the bronze's own form—the proportion of the elements. Having the appropriate amount of earth, water, air, and fire is what gives it the ability to take on or lose the *shape* (μόρφη) of the statue. Having and lacking the μόρφη of the statue are the contraries inherent to the bronze. The potential to be healthy or sick is *in* the substrate, in the very way that Aristotle argues at *Phys.* II.1, because the human body consists of water and blood, an excess or deficiency of which pertains to sickness and whose equilibrium with the other humors is health. Therefore, the ᾗ-qualification allows Aristotle to show how potentiality operates in motion: the material's determination, whether its own or that for which it has the potential. The bronze *qua* bronze and the bronze *qua* being forged into a statue are both potentially a statue. This is because the potential is inherent to the form of the bronze, just as the form of any proximate material determines its capacities for motion. But only the latter pertains to motion.

The upshot of Aristotle's comments on the ᾗ-qualification is once again that we see how motion is by analogy. Each kind of thing has a material substrate and a pair of contraries. Recall from *Phys.* I.6 that the substrate must persist in the change from one contrary to another, where one contrary is predicated of the substrate potentially while the other is predicated actually.[87] This is why Aristotle must articulate the ᾗ-qualification in the first place. Again, Aristotle's examples are indicative of this. For substantial change, Aristotle references water/blood whose contraries are health and sickness (201a35-201b3). And qualitatively, color has the ability to be seen given the luminosity of a translucent medium: brightness and darkness. We might say that in quantity, there is some physical magnitude, as the material substrate, which admits of increase

[87] For substantial change, the persistence of the substrate is peculiar because even the substrate seems to undergo a change. See Halper, "Aristotle's Scientific Method," p. 79.

and decrease. For place, the container is the substrate into and out of which an entity changes place. In each category, therefore, motion manifests differently with respect to how the material substrate functions with a pair of contraries (*cf. Phy*. I.7). Motion is always within the substrate because motion occurs between contraries and contraries are in the substrate. But this means that the determination of the material substrate *across categories* is by analogy. Just as the bronze either has or is deprived of the shape of the statue, color becomes either bright or dark. And just as the bronze *qua* bronze is not yet oriented towards becoming the statue, the color *qua* dark is not yet being seen, for each is actually the privation of the form for which it is potentially. In short, in each genus there is a governing form and its privation is predicated of its respective material substrate. Therefore, the second formulation of motion implicitly addresses both the third and fourth categorial remarks insofar as there is no common material substrate and common set of contraries; rather substrates and contraries are analogical. In every category admitting of motion, we see that motion is not separate from natural entities because it is always in a *specific* material substrate.

Still, the material is not what governs motion. Such priority belongs to the form into which the material is shaped (*Phy.* II.1)—the final cause. This brings us to the third formulation of the definition: "the actuality of the buildable, insofar as something is buildable, is the activity of building." Priority in generation belongs not to the material and the potential, but to the form and actuality because these govern the generative activity:

> It is clear then that this is motion and that motion happens at the very moment when [τότε κινεῖσθαι ὅταν] its actuality *is*, neither before nor after. For each thing sometimes admits of activity [ἐνεργεῖν] and at some other times not, such as what is buildable: the actuality

> of the buildable, insofar as something is buildable, is the activity of building (for the activity is either the actuality of the buildable or the house). But whenever there *is* a house, the ability to be built *goes no further* [οὐκέτ᾽οἰκοδομητὸν ἔστιν]; yet what is being built is *that* with the ability to be built. Thus, the activity of building is an actuality. This activity, then, is a sort of motion. Even more importantly, a *similar* [ἐφαρμόσει] account will be suitable for the other types of motions (*Phy.* III.1, 201b5-15).

The third definition of motion is to address the status of actuality in the definition as a corollary to the second definition. And notice how Aristotle defines motion with respect to a specific type. What applies to this will apply analogously to other motions. The concern in the second formulation pertained to the status of the material as 'that from which' motion begins; the concern in the third is the status of the final form '*for* which' the material is the potential, the τέλος, since this is the actuality. Motion is neither the material out of which the new form is actualized nor is it the completed actuality. Aristotle is framing motion between two stable actualities, where motion is the incomplete actuality in between. As it turns out, however, this points to another aspect of the ᾗ-qualification in the second formulation: the preservation of the material in the act of motion. When the material is actively directed towards the form that is being actualized, the material remains more or less throughout the process. The bronze does not entirely cease being bronze either as it is being forged into the statue or when the statue is completed. The stone out of which a house is built remains even when the house is fully built. This means that in whatever activity that brings about the final actuality, the potentiality persists throughout. As the material is being fashioned, the activity itself is inherently indefinite not simply because the new form has yet to manifest fully, but more so because of how the activity

of working out the new form *preserves* the potential. Themistius describes this as "saving" (σώζουσα) the potential.[88] This is why a first-order actuality *is* a second-order potentiality. The first-order actuality is the gradual manifestation of the final actuality precisely because it is the activity that retains the potential. Motion occurs only *insofar as* the bronze's potentiality to be fashioned into a statue is *put to work* (*tätigen*) in the forging process. I call this motion's *active potentiality* (*tätig Möglichkeit*).[89]

But, what does it mean to say that motion is an active potentiality as opposed to an inactive potentiality? The inactive potentiality is a first order potentiality, or simply the potential to move as inherent to the substrate. This potential is not yet motion in any way, e.g. the bricks lying dormant in a heap. But, when this potentiality is actualized—brought to completion *as* motion—what is actualized is a second order potentiality, building *qua* the bricks being actively used.[90] An active potentiality is of a higher order than a first order potentiality because its actuality is of the sort that maintains the potentiality to be that very activity. What this means, however, is that the actualization of the potential to move is not a motion.[91] If it were, there

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[88] Themistius, *In Phys.* 213.1ff. Philoponus, *In Phys.* 351.9-12, remarking on *Phy.* III.1, 201a9, speaks of the actuality of motion in terms of *saving* the potential: "So, Aristotle says that motion is the actuality of what is potential, insofar as it is so disposed to be potentially, so that motion has both potential and actual, while still saving the potential [ἔτι σῳζομένου τοῦ δυνάμει]." The best way, I think, to read this is that *so long as* the material substrate is present, potentialities are retained by the substrate. Simplicius, *In Phys.* 414.3-6 says "retaining its potentiality" (μένοντος ἐν αὐτῷ τοῦ δυνάμει).

[89] I borrow this helpful term from Martin Heidegger, *Grundbegriffe der aristotelische Philosophie* (Frankfurt am Main: Klostermann, 2002), p. 378. For my extended analysis on Heidegger's concept of *tätig Möglichkeit* as it applies to Aristotle's definition of motion, see Joseph P. Carter, "Heidegger's *Sein zum Tode* as Radicalization of Aristotle's Definition of *kinesis*," *Epoché: A Journal for the History of Philosophy* 18 (2), pp. 473-502.

[90] Kosman, *op. cit*., pp. 51-53 offers a similar, but more detailed interpretation.

[91] Notice that I am not saying that motion itself is the actualization, but rather the actualization that pertains to the change (μεταβολή) *into* a motion (κίνησις). This is to differentiate motion's own generation from what it is, when it is. Kosman, for instance, would mostly likely call what I am saying to be motion's actualization an "energization" since "energization takes place not

would be an infinite regress of motions, for the actualization of a motion would be a motion, which would have been actualized by another motion *ad infinitum*. Rather, the actualization of the potential to move is simply the change into the very activity of moving. This change is instantaneous (*Phy.* VIII.6, 258b16-20; *Meta.* Z.15, 1039b26-29 and H.3, 1043b15-16).[92] This is because instantaneous change is what *simply* comes to be without com*ing*-to-be, (γένεσις), since this is not a contrarietal change but a change in terms of contradiction (*Phy.* V.1, 225a25-29). That is, there is no *passage* between first and second order potentialities. Thus, motion—opposed to its actualization—is not instantaneous, because it happens along a continuum (225a12-20).

Because of the preservation of the material in the generative process, it does not follow that the potentiality in the bronze is 'exhausted.' This is a hallmark of active potentialities. We ought not read οὐκέτ᾽οἰκοδομητὸν ἔστιν as *completely* destroying or eliminating the ability to be built. Here, οὐκέτι ἔστιν is ambiguous. It is usually read as 'completely destroyed' or 'eliminated.' Instead, I suggest we read it as 'going no further' or 'no longer manifest.' For, when the house is completed, the stone remains, even though its ability to be the house goes no further than the full manifestation of the shape (μορφή) that governs the motion. The potential for housebuilding is no longer active, even though the stone remains. It is, after all, a *stone* house. The material substrate is never eliminated. The final actuality is of such a character that it preserves the potential because, as a composite, it cannot exist without the substrate. Because the potentiality for the house is predicated of the stone as part of the stone's own form, the stone

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between second and third terms [i.e. motion and the end of motion], but between the first and second" (Kosman, "Definition of Motion," p. 55, emphasis mine). What Kosman is rightly trying to avoid is saying that the there is a passage between first and second order potentialities.

[92] Aristotle's typical way of articulating instantaneous change is that it is possible for something "to be at one time and not to be [at another] without *coming*-to-be or *being*-destroyed [εἶναί ποτε καὶ μὴ εἶναι ἄνευ γενέσεως καὶ φθορᾶς]" (*Phy.* VIII.6, 258b17-18). For example, at *Phy.* VIII.3, 253b25-26, Aristotle speaks of freezing (πῆξις) as happening suddenly (ἀθρόα γίγνεται). See also Bowin, *op. cit*., pp. 14-15.

does not cease to be stone when the house is built, at least not entirely. The bronze from which a statue is forged is still retains that very potential to be a bronze especially when the statue is fully actualized. Otherwise, the statue would cease to be a *bronze* statue. Evidence for this is the fact that both the statue and the house are able to be *disassembled* into bronze and stone respectively because in the destructive process, the governing form is that of the material, for it is the *privation.* Statues and houses are quite often destroyed, whose ruins are the very materials out of which they were made.[93]

My emphasis on the preservative aspect of the definition of motion points to an important dispute in the contemporary scholarship. The lynchpin of the dispute has long been what to make of the conjunction of ἐντελέχεια in the first formulation of the definition.[94] As a human being, I am capable of walking. My ambulatory capacity exists as a capacity as long as I'm a functioning human being. But, merely having this capacity, especially while sitting or standing around, does not mean that I am actively walking, i.e. *moving*. Furthermore, the completion of my walk is not the motion. What if we say then, in similar terms as Ross, that my walking is the *"realization* of the potential to walk"?[95] This is wrongheaded on at least two counts. Linguistically, because the English term "realization" means "motion," Ross makes the definition "vacuous" or circular.[96] More seriously, it does not make sense of the definition ontologically. If what defines motion is a realization, which itself is a motion, a motion will precede the motion, *ad infinitum*. That is to say, the motion will always pertain to my *going* to walk, or to *becoming* a walker, and not to the

[93] N.B. While this is typical for artifacts, natural substances such as living beings do not decompose into their material substrate.
[94] For example, see A. Anagnostopoulos, "Change in Aristotle's *Physics* 3," pp. 45-49; Kosman, "Definition of Motion," pp. 56-60 and "The Activity of Being in Aristotle's *Metaphysics*," pp. 202-203; Gill, "Aristotle's Distinction," p. 10; Heinaman, "Circular Change?" pp. 27-30; Hussey, *op. cit.*, pp. 58-60.
[95] Ross, *Aristotle's* Physics, pp. 536-537.
[96] Kosman, " Definition of Motion," p. 41.

actual activity of walk*ing*.[97] Reading ἐντελέχεια as "realization" makes it so that there is an unnecessary process of *becoming able* to move prior to the motion. This is unnecessary, since a capacity for something like walking is *built into* a nature. My capacity to walk need not be realized as a capacity before I walk because it is part of my nature. Thus, there is no need for it to be actualized *as* this capacity.

To emphasize another of Kosman's points, and to set the context for the discussion in *Phy.* III.2 concerning the indefiniteness of motion, I suggest that we read the definition as "the *full visibility/manifestation* of the potential *so long as it retains* the potential."[98] What this tells us is how motion is an inherently incomplete, indefinite activity. For, the *full* manifestation of motion is its very incompleteness. In this way, the activity is the *actuality* of something indefinite.[99] For, whatever is incomplete is indefinite, because definiteness pertains to what is complete. What is complete is in a determinate state. Neither the pile of bricks nor the completed house is the motion of building precisely because neither has the *phenomenality* of incompleteness. The phenomenality of the pile of bricks and the completed house pertains to something *finished*. As discussed above, the material by itself has its own actuality and the new form is a complete actuality. Insofar as these are finished, therefore, they have a definite form. But, the activity *between* them is fully unfinished—i.e. indefinite. For, the activity of building pertains *both* to the form of the bricks *and* the form of the house albeit *incompletely*. As the building is occurring, there is always *another* brick to be laid on the way to finishing the house. As unfinished, the activity is indefinite. Neither the pile nor the house itself makes building

[97] *Ibid*., p. 45.
[98] "Motion, in other words, is not the actuality of a potentiality in the sense of the actuality which results from a potentiality, but rather in the sense of an actuality which is a potentiality *in its full manifestation*," *Ibid*, p. 50, emphasis mine.
[99] Joseph Owens, "Aristotle—Motion as Actuality of the Imperfect," *Paideia: Special Aristotle Issue* (1978), pp. 120-132 calls this an 'imperfect actuality.'

visible as an unfinished, indefinite actuality because these are finished activities. When the builder puts the bricks to work, however, the bricks are acted upon in such a way that their buildability is now the visible work performed *toward* the end. The building, then, is the *visibly* incomplete, indefinite activity of *becoming* a house. Motion's incompleteness points to the full manifestation of something indefinite.

We see something *analogous* in the activity of heating. Something cold becomes hot by a hot thing acting upon what is cold, where what is cold *remains able* to be hot throughout the activity of heating until the appropriate temperature is reached. Just like the bricks in building, there is always *another* degree that is increased (or decreased, if the activity is cooling) until the appropriate temperature is reached. But until then, the activity of heating remains indefinite, since the material that is being worked on is both hot and cold in such a way that it never remains in the same determinate state. By not remaining the same, it is indefinite. In becoming hotter, the thing's potential for hotness is put to work, or is manifested in the activity of heating, but in an incomplete way. The activity is still unfinished because the potentiality remains. The character by which something is definite and finite is being in a finished state. Until something is at rest in the new form, the motion is indefinite, since the form is what establishes a thing's definiteness.

However, if motion is inherently indefinite, and yet motions are analogical, how then are we to understand indefiniteness with respect to the categories? Is it possible to use indefiniteness as what is common to all motions in order to class motion univocally on its own? This is, in fact, what some of Aristotle's predecessors attempted. However, as Aristotle shows in *Phy.* III.2, it is still not possible to class motion on its own with respect to indefiniteness precisely because this is not a *cause* of motion. To see this, let us turn to Aristotle's dialectical account of Plato and the Pythagorean's view of motion's indefiniteness.

§2.2. *Phy.* III.2-3: Motion's incompleteness and the turn toward the infinite

No matter how one parses the definition of motion—whether speaking to its analogical character or to its inherent indefiniteness—in the end, we are left with a deeply perplexing, but no less appropriate ontology of motion. We know that because motion is *between* contraries, both of which are stable, *determinate* forms, motion is therefore an indeterminate, incomplete activity. Motion is therefore indefinite (ἀόριστον).[100] We have also learned that motion is analogical. Now, in *Phy.* III.2, motion's analogical status and its indefiniteness converge with respect to Plato and the Pythagoreans. They argued that because of its indefiniteness, motion should be classed with other indefinite things like 'difference' and 'plurality.' Aristotle, I will argue, interprets this as treating motion *univocally* because it assumes that *whenever* difference and plurality obtain, so does motion, because their indefiniteness is assumed to be the cause of motion. Aristotle places motion's indefiniteness in the context of how his predecessors used it as a *reason* to mistakenly class motion by itself apart from the things to which motion is attributed.

On one hand, this is not a surprising strategy, since Aristotle spent most of *Phy.* III.1 establishing how motion cannot be classed on its own because it is *in* many different things. Any position which tries to class motion by itself needs to be corrected. Furthermore, as I discussed above, it also makes sense for Aristotle to have a dialectical treatment of his predecessor's views and the common beliefs about motion, although it is peculiar that this treatment now comes after Aristotle's own account. Nevertheless, since it is directly relevant to the subject matter, Aristotle discusses and corrects his predecessors' views on the indefiniteness of motion because they use it as a way to class motion by itself, when this in fact is not possible, as I will show.

[100] *Cf. Top.* VI.4, 142a17-20.

On the other hand, however, there is another aspect to Aristotle's discussion of motion's indefiniteness in *Phy.* III.2 that we need to notice. One might expect Aristotle to have examined more closely the character of motion's indefiniteness itself. After all, as indefinite, motion is a strange entity—it is the actuality of something incomplete, even though by definition, actualities are fundamentally complete. As discussed, motion is the full manifestation of a *potentiality at work* towards a new form, where the presence of potentiality is why motion remains incomplete. Motion, in this way, is something 'completely incomplete.' The indefiniteness of motion has an actuality, and as such, seems to pertain to something whose complete manifestation is its very incompleteness. But this is the extent of the account. Aristotle does not elaborate further here in *Phy.* III.2. Why not? Let me suggest that even though Aristotle's aim is to correct his predecessors' classification of motion with respect to indefiniteness, the discussion is also a signpost for the account of the *infinite* (ἄπειρον) in *Phy.* III.4-8. To be sure, Aristotle does not explicitly distinguish between ἄπειρον and ἀόριστον in his writings; but, at least with respect to motion, they are practically interchangeable for Aristotle.[101] Since motion is an incomplete

[101] In the case of motion, indefiniteness points to the infinite. For, as an incomplete activity, motion still maintains an *excess* of potentiality while the new form is coming into being. This coincides with Aristotle's claim at *Meta.* Δ.15, 1021a3ff that the indefinite is a type of relation *to that in which there is an excess*. I take this as another description of incompleteness. This in turn correlates with the definition of the infinite in *Phy.* III.6 as 'that beyond which there is always something more.' But, it is not clear if this correlation applies to all of Aristotle's uses of the indefinite. While Aristotle certainly defines ἄπειρον and extensively accounts for it (*Phy.* III.4-8), he only briefly defines ἀόριστον at *APr.* I.13, 32b11 as that which is both what is and is not the case. The indefinite is, in a way, the existence of a contradiction; *cf. Meta*. Γ.4, 1007b19-28 where, in his discussion of the principle of non-contradiction, Aristotle identifies the indefinite with non-being. See also *Meta*. Z.11, 1037a27; Θ.6, 1049b2; N.10, 1087a17 and *Phy*. IV.2, 209b9 for the indefiniteness of matter and potentiality; *DA* III.11, 434a4 for how the indefiniteness of motion is a model for the way animals have imagination; and *Meter*. IV.4, 382a16 for how hardness and softness are indefinite with respect only to themselves and not to the faculty of touch. For instances where Aristotle uses ἀόριστον explicitly in conjunction with ἄπειρον, see *EE* VII.14, 1247b12; *Meta.* K.8, 1065a26; *Phy.* II.5, 196b28-29; *Phy.* III.6, 207a26-27.

actuality, it is something *unbounded*—infinite. As such, the infinite is something indeterminate. We do know that Aristotle will define the infinite as 'that outside of which there is always something' (*Phy.* III.6, 207a1). Motion's indefiniteness is an indication of the existence of the infinite in nature because if motion is by definition something incomplete and outstanding, and indefiniteness pertains to an excess, which is essentially the definition of the infinite, it makes sense to take motion's indefiniteness as another way of talking about the infinite. As such, I take Aristotle's acknowledgement that motion's indefiniteness is "difficult to grasp precisely [ἰδεῖν], but one that *is* nevertheless" (202a2-3) as an indication for further inquiry into the infinite, which Aristotle addresses *immediately* after the account of motion. In *Phy.* III.4-8, he looks to motion as a way to explain the infinite. At this point, however, he postpones this account until *Phy.* III.4-8. For now, it is important for Aristotle to correct previous views that use motion's indefiniteness to class motion univocally.

Aristotle begins *Phy.* III.2 showing that his predecessors class motion univocally with the indefinite:

> That motion has been spoken about correctly [καλῶς] is clear also from those things others say concerning motion and from the fact that motion is not easy to define in another way. For one could not place motion and chance into another genus, even though it is clear to some who consider where it is placed, supposing that motion is difference, inequality, and non-being. It is not necessary, however, that something is moved by these, whether it were something different, unequal, or some non-being. Nor even is change to or from these, instead of from *those things* which are opposed [ἐκ τῶν ἀντικειμένων]. A reason to place motion into these genera is that it seems to be something indefinite

> [ἀόριστον] and that the principles in the other column are *indefinite*, since they are privative; for none of these or anything like them is the same thing [of a sort] or belongs to any of the other categories (*Phy.* III.2, 201b16-27).

The Platonists identify motion with difference (ἑτερόντητα), inequality (ἀνισότητα), and non-being (τὸ μὴ ὄν) (*Soph.* 256d-e; *Tim.* 57e-58c) and the Pythagoreans class motion in the column pertaining to the principles of plurality, since motion, as indefinite, pertains to none of the principles of unity in the first column of the table of opposites (*Meta.* A.5, 986a21-28). The principles in the second column are privative (τὸ στερητικαί); as privative, they pertain to nothing complete because privativeness pertains to a *lack* or *what is not yet*. Aristotle argues that both the Platonists and the Pythagoreans class motion in a single respect because of its inherent indefiniteness. Aristotle's objection to this must be understood in the context of motion's transcategorial status. Does motion deserve its own classification because motion's indefiniteness seems to pertain to difference (Plato)? Should it be classed in the column of plurality with other indefinite, privative entities (Pythagoreans)? Aristotle's dialectical account stays on course with why and how motion is a category problem, but for another reason: its inherent indefiniteness. Aristotle's response to the Platonists and Pythagoreans builds off the insights of III.1. On one hand, the ἔνδοξα are correct (καλῶς) since motion is in fact an incomplete activity. Because it exists, it needs to be classed somewhere. They class motion along with other indefinite things since motion too is indefinite. This is because Plato and the Pythagoreans *identify* motion with things like difference and inequality. Because the Platonists *define* motion by difference, inequality, and non-being, then whenever these obtain, so does motion. That is to say, difference and inequality ought to function as *causes* of motion because

they a supposed to say why motion is indefinite. After all, motion is a sort of becoming-different, and as incomplete it seems to be a kind of non-being. Such reasoning requires motion to be meant in one way. In this way, motion is defined univocally. The Pythagoreans commit the same error. As indefinite, they class motion with plurality. Nevertheless, there may be multiple things, either potentially or actually without admitting motion:

> But a reason that motion seems to be indefinite is that it corresponds neither to potentiality [only] nor actuality [only]; for what is potentially a certain size is neither necessarily moved nor what is actually that size, but motion seems to be the actuality of a certain sort—an incomplete one. This is because the potential predicated of the actuality is incomplete. Because of this, then, it is difficult to grasp what motion is. For it must correspond either to the privation, the potential, or the complete actuality [ἐνέργεια ἁπλῆν], but none of these appears to admit of motion. This leaves, therefore, the way that has been suggested—that there is a certain actuality, the sort already discussed, one that is difficult to grasp precisely [ἰδεῖν], but one that *is* nevertheless (*Phy.* III.2, 201b27-202a3).

Simply because of its inherent indefiniteness does not mean that motion must be classed on its own apart from the other categories in which it manifests. But this is, in essence, a *category* mistake. Motion's indefiniteness is an insufficient criterion to class it with other indefinite entities. For, many differences and pluralities exist without admitting motion. They are not causes of motion. Simply because my father is taller than my mother does not *entail* that a change has occurred. Another example is that there are many different kinds of beings. The

essential difference between an animal and a house does not mean that there is motion. The mistake the Platonists and Pythagoreans commit, therefore, is to class motion as the *same kind of thing* as other indefinite things because things like difference and plurality are not causes of motion. Motion cannot be classed univocally even though all motions appear to be indefinite. The difficulty (χαλεπόν) in grasping motion is, therefore, twofold: not only is its actuality incomplete, hence indefinite, but it is also not restricted to a single category. The ἔνδοξα mistakenly take indefiniteness as a reason or cause to class motion in one way.

The thrust of Aristotle's objections to both Plato and the Pythagoreans is that none of the so-called classes of indefinite things are *causes* of motion. It appears to harken back to the dialectical treatment of the natural scientists (οἱ φυσικοί) at *Phy.* I.4, 187a16-20, wherein Aristotle likens their understanding of contrariety (τὸ ἐναντία) to Plato's understanding of excess and deficiency (ὑπεροχὴ καὶ ἔλλειψις):

> The physicists, however, give two causes [for motion]. There are some who make the underlying body one thing, e.g. one of the three elements [i.e. fire, water, or air] or another thing denser than fire and rarer than air, and then *generate* everything from it by making multiple things through condensation and rarefaction. (These things [i.e. condensation and rarefaction] are contraries, which generally pertain to excess and defect, just as Plato's the Great and the Small, despite the fact that he makes it material and makes unity to be form, whereas others say that unity is the underlying material, and the contraries are the differentia—i.e. the forms (*Phy.* I.4, 187a16-20.)

What is interesting here is how Aristotle treats excess and deficiency as a sort of Platonic 'category' (καθόλου) of contrariety in order to describe how the φυσικοί understood the elements' relationship to each other. Contrariety pertains to the more and the less, just as we see at *Cat.* 9, 11b1-8. Thus, it is a relation. But it is not only relation that admits of contrariety. For Aristotle, contrariety is structurally necessary in *every* category that admits of motion, for they are causes of motion—form and material. But, for Plato, excess/deficiency and great/small *are* general classes of contrariety. In fact, for Plato, contrariety points to a single class of change, or at least a single, univocal *principle* of change. Evidence for this is found in the so-called 'digressive' sections of the *Statesman* 283c-285b. At *Statesman* 283b-c, the Stranger raises a problem for the younger Socrates: have they been too verbose on the subject of weaving? Has their discussion been in *excess*? To answer this, the Stranger, in the famous digression, thinks it necessary to address excess and deficiency *in general* (πέρι...πάσης ὑπεροχῆς τε καὶ ἐλλειψεως, *Statesman* 283c11-d1). The appropriate expertise to which excess and deficiency correspond is the art of measurement (ἡ που μετρητική), since knowledge of excess and deficiency *in general* requires that one knows how to determine what is too much or too little for any craft—knowledge of the *common contrariety* of the Great and the Small in conjunction with the knowledge of what is necessarily *generated* (τὸ μὲν κατὰ τὴν πρὸς ἄλληλα μεγέθους καὶ σμικρότητος κοινωνίαν, τὸ δὲ [τὸ] κατὰ τὴν τῆς γενέσεως ἀναγκαίαν οὐσίαν, 283d7-9). In what sense is the contrariety (τὴν πρὸς ἄλληλα) of the Great and the Small common (κοινωνίαν) and why is the other part of measurement associated with generation? Plato's answer arrives later in the digression, where we see that every expertise aims not at what is too much nor too little, but at what is generated according to an appropriate measure (μετρίον)—the form. This means that the Great and the Small is *common* to every expertise with respect to that *out of which* the

product is generated. But as such, the Great and the Small is itself *indefinite*. The determining factor is the measure. The appropriate measure is the form, which acts upon the Great and the Small, since the latter is indefinite material, just as Aristotle's claims at *Phy.* I.4, 187a19. As the measure, the form is that in respect of which there is excess and deficiency, since excess is what is greater than the form and deficiency is what is lesser than it. Because the Great and the Small pervade everything as a sort of common indefinite material, whatever is generated is necessarily according to some form acting upon it, since the form is the appropriate measure. Plato's understanding of contrariety, taken in terms of the Great and the Small, is something like a thoroughgoing 'category' of motion. This is even implied in part of the argument from contraries at *Phaedo* 71a-b. Whether this is called "excess and deficiency," "the Great and the Small," or "the More and the Less," for Plato (according to Aristotle) motion seems to be some sort of category of contrariety in which all generated things fall.

Before Aristotle turns to the account of the infinite in *Phy.* III.4-8, there is another issue that Aristotle addresses in the last part of III.2 and in III.3, which will be important for how the infinite is actual by way of the activity of division.[102] For any motion, there is a mover and something moved (202a3-4). How many motions are there, then? There is only one, since motion is in what is moved, not the mover:

> Now, every moving thing is something moved, just as it has been said, where what is movable is potentially something else and whose motionlessness is rest (for that to which motion is predicated is that whose motionlessness is rest [ᾧ γὰρ ἡ κίνησις ὑπάρχει, τούτου ἡ ἀκινησία ἠρεμία]). For the moving *thing* [τοῦτο] is the same as the activity

[102] See §4.1.2 below.

> towards this [i.e. the thing at rest], so long as it is this sort of activity [τὸ γὰρ πρὸς τοῦτο ἐνεργεῖν, ᾗ τοιοῦτον, αὐτὸ τὸ κινεῖν ἐστι]. But the mover acts by contact, thus at the same time also affects [what is movable]. Because of this, motion is the actuality of the movable, so long as it is something movable, and the moving happens [συμβαίνει δὲ τοῦτο] by contact with the mover, and hence is affected simultaneously. And the moving thing will always bear/produce a certain form—either substantially, qualitatively, or quantitatively—which will be the principle and cause of the motion, whenever there is motion, e.g. the actual human being brings forth another human from what is potentially human (*Phy.* III.2, 202a3-12).

Aristotle's argument is that every motion requires a mover, since the mover is necessary for producing (οἴσεται) the form in what is moved. But, how is this relevant to Aristotle's categorial concerns? Admittedly, Aristotle sets up a latter discussion of the unity of motion in *Phy.* V.1-2 by appealing to contact (θίξις). However, one of Aristotle's points here is that regardless of what type of motion, the *relationship* between every mover and moved will be the same in kind. Every motion is predicated by contact—an active mover transmitting a form to what is passively moved, which is a relation. Contact is the relationship between mover and moved and it is univocal for every type of motion (*Meta.* Δ.15, 1021a14-19). For, in each motion, the mover and the moved relate in the same way in every motion, such that some agent acts upon a patient. Water is passively heated by contact with an active heat source in the same way that a car is propelled by contact with the road surface. Even in substantial generation and destruction, contact is necessary respectively for sexual activity and any sources of harm or sickness. However, while the relation between mover and moved is predicated in the same way for every

motion, the equivocity still holds with respect to the causes of motion. Aristotle raises this issue in his categorial remarks at the beginning of III.1 and returns to it here.

The Platonists especially class motion with relation because the relation between mover and moved seems to require difference and inequality since these things are indefinite. As Aristotle sees it, however, motion cannot be classed in this way because it is *in* the moved entity. On one hand, all motions are univocally the same because of the kind of relationship between mover and moved. In this way, Aristotle agrees with the ἔνδοξα. Yes, every motion admits of the same type of relation. The mover transmits the form to something moved by contact. On the other hand, though, because motion happens in what is able to be moved (τὸ κινήτον), and different kinds of things are moved, every motion is equivocally the same. Just because the contact between every mover and moved entity is univocal does not entail that motion is the same in each case. Because motion is predicated of different kinds of things, it must be equivocal.

An aporia results, however: if the actuality of the mover actualizes the potential in the moved, is there one or are there two actualities? Even though Aristotle is not explicit, I suggest that this the impasse with which he begins at III.3, 202a13:

> Now, the problem is solved since motion is in the movable. Motion is the actuality of the movable brought about by a mover. And the actuality of the mover is not different, for it is necessary that actuality be in both. While the movable is according to its potential, the mover is by its activity [τῷ ἐνεργεῖν], and the mover is able to act upon the movable. Thus, in like manner, there is one actuality in both, just as one to two and two to one is the same interval, as well as ascending and descending scales. For while these are one,

the account is not. This is similar to the mover and that which is moved (*Phy*. III.3, 202a13-21).[103]

The actuality of the mover and that of what is moved are one, even though the motion is only in what is moved, because they have the same form. As I write outside, for example, I am heated by the sun. The change from cool to warm, light to dark, occurs in my body—the sun remains unaffected. Why aren't these two different actualities? It is because The sun and my body are predicated by the same kind of qualities. My body's capacity to become hot and golden is the same type of quality as the actual heat and gold color of the sun. Therefore, my body has in it the same pair of contraries as the sun; my body becomes hotter and darker insofar as the change follows the same path as that which is in the sun (were the sun to undergo such a change). Just as the interval between one and two in whatever subject of which this quantity is predicated, the range between hot and cold or white and black is the same in whatever subject being moved by the prior subject. For a subject to undergo change by another subject imposing a form on it, the form must be the same in kind as the latter. In this way, the actuality between two subjects is the same. The heat actualized in my body is the same quality as that of the sun.

In another sense, however, the actuality of the mover is different than that of the moved by virtue of the fact that the motion is *only in what is moved*.[104] The actuality of the mover is

[103] Sachs, *Aristotle's Physics*, p. 80, suggests this impasse refers to Zeno, but he does not provide much evidence to this effect. I find it better to read the beginning of *Phy.* III.3 by problematizing the end of III.2.

[104] In a way, movers are also in motion, but only as the efficient cause. While sculpting a statue, the sculptor's body is in motion, but the form being imposed on the statue remains unchanged in the sculptor's mind. This is why the motion is technically not in the mover. See *Phy.* V.6 for how locomotion seems to take priority over the other motions. Ultimately, Aristotle will need to argue for an ultimate unmoved mover to avoid an infinite regress of the causes of motions. See *Phy.* VIII.5-6 for this account.

complete, whereas the actuality of what is moved is incomplete—indefinite. The form of the agent is the same in the patient, albeit in different degrees; one is completely actual, the other incompletely. The active potentiality of the motion in the moved means it has the same actuality as the mover while still being distinct from each other in terms of degree. As Aristotle says, "generally it is correct to say" (ὅλως δ'εἰπεῖν...κυρίως) that the agent and patient are different degrees of the same activity:

> However, it is not absurd that another actuality be in something else (for teaching is the actuality of the teacher in something that is still undivided, and is something that is a part of it [ἀλλὰ τοῦδε ἐν τῷδε],[105] nor is there nothing that prevents one and the same thing from belonging to two things (not as in *being* the same thing, but in the sense of a potential that is predicated of an actual thing). Nor must a teacher be a learner—neither if the action and affection is the same nor even if the sense of there being an account for the essence of each (e.g. clothing and a piece of cloth), but as the road from Thebes to Athens, and vice versa, just as it was said before. For not everything which is the same belongs to the same things in one way or another, but only to that whose being is the same [οὐ γὰρ ταὐτὰ πάντα ὑπάρχει τοῖς ὁπωσοῦν τοῖς αὐτοῖς, ἀλλὰ μόνον οἷς τὸ εἶναι τὸ αὐτό]. Nor is it even the case if teaching is the same as learning, or learning as teaching, just as it is not the case if division is one of the things divided, and if what is divided from here to here and from there back are one and the same (*Phy.* III.3, 202b5-16).

[105] See Aristotle's *Categories* 2.

Just as Aristotle argued earlier at 202a13-21, the relation between contraries of the same kind is univocal for both mover and moved—it pertains to the same form and privation. The difference pertains to the degrees of actuality of the relata. While the knowledge in the teacher is completely actual, whereas that in the learner is potential or even actively potential (if the learner is actively learning), the relationship between actual and potential knowledge is meant in the same way. Even though agent and patient often manifest in two different subjects, a mover and moved, the *path* of the motion between them is one because it involves the same steps in either direction. Teaching and learning follow the same path, since the steps by which the teacher educates the student are the same as those the student follows in the process of learning. The steps by which my skin becomes warmer and darker are the same as those the sun takes in actively heating and darkening my skin. Though the sun and I are different substances, the sun heating my skin and my skin being heated are the same actuality—the same motion—because the quality acting upon my skin is one in kind with the quality being actualized in it. Heat in the sun is the same species of quality as that in my skin, differing only in degrees. This is what Aristotle means by "the actuality of what is part of something is different in account [τῷ λόγῳ] from the actuality of what is acted upon by something else" (202b21-22). However, the difference in account, Aristotle argues, is the *degree* to which the mover is completely actual and the latter is incompletely so—an active potentiality. The sun and the teacher are not changed as my body is heated/changes color and the student learns, respectively; the former are full actualities, whereas the latter are incomplete actualities. This is because motion is necessarily in what is moved.

§2.3. Motion and its implications for the account of the infinite

In *Phy.* III.1, Aristotle shows that motion must be defined analogically and that it exists incompletely as an active potentiality. If the infinite is in any way connected to motion, will it be just as analogical as motion? Furthermore, how does the active potentiality of motion as an incomplete actuality help Aristotle understand the actuality of the infinite? In *Phy.* III.6, Aristotle answers this by showing that the infinite is actual specifically with respect to a quantitative change. While it is true that all motions are indefinite as they are occurring, I will argue that this is by analogy to quantitative change, specifically the act of division. Secondly, just as each type of motion is incomplete and indefinite insofar as the potentiality has yet to be exhausted, we will see in *Phy.* III.6 that the activity of division has the same character because there is always more of the magnitude able to be divided. In *Phy.* III.2, we learned Plato and the Pythagoreans *identified* motion with the infinite. They believed this because motion has an indefinite character. However, in *Phy.* III.6, Aristotle will show instead that the infinite is an *attribute* of the moving body. The activity of division is a change that occurs *within* a substance, since bodies and other magnitudes are things that undergo division. The fact that that motion is *in* the moved and not in the mover means that if the infinite is connected to motion in anyway, the infinite will be in what is moved as well, and not something separate. But, before we see how Aristotle argues for this in *Phy.* III.6, let us look at Aristotle's general account of the existence of the infinite in *Phy.* III.4 and then at Aristotle's arguments in *Phy.* III.5 for the ways in which the infinite cannot exist separately as an actual physical body.

CHAPTER 3

THE EXISTENCE OF THE INFINITE (*PHY.* III.4-5)

§3.1. Motion, bodies, and the existence of the infinite (*Phy.* III.4)

Like motion, the infinite is a strange entity. As we've seen from *Phy.* III.2, because motion is neither the latent potentiality associated with the material nor the completed actuality of the new form, it is the incomplete actuality occurring in between. Motion is an *unfinished* activity. It is, therefore, indefinite—in other words, it is infinite. The unfinished or indefinite character of motion parallels Aristotle's definition of the infinite at the end of *Phy.* III.4 as 'that outside of which there is always something.' Therefore, Aristotle does not deny the infinite's existence. Somehow it exists with respect to motion, at least. In fact, Aristotle is in agreement with his predecessors that the infinite exists because it seems to be generally accepted as a sort of source of being (ἀρχήν τινα τιθέασι τῶν ὄντων, 203a3-4), specifically as a source of motion as I will argue. In the first part of this chapter, I will argue that *Phy.* III.4 set downs evidence for the *existence* of the infinite with respect to motion. While the scholarship speaks little to Aristotle's dialectical inquiry in *Phy.* III.4, I argue that Aristotle appeals to the 'the common opinions' and the beliefs of his predecessors—the ἐνδοξά—as reliable proof for *at least* the existence of the infinite because most believe in its connection to motion as a principle of generation and we know that motion exists.[106] What *Phy.* III.4 does not tell us, though, is the *essence* of the infinite.

[106] Apart from the ancient and medieval commentators, few contemporary readers provide a systematic treatment of *Physics* III.4-5. Of those that do, even fewer assess how Aristotle's analysis of his predecessors contributes to his positive views of the infinite in III.6-8; see

In *Phy.* III.4, Aristotle only uncovers enough evidence for its existence to point the inquiry in the right direction: if motion exists, so does infinity. And motion most certainly exists.

While Aristotle is not challenging its existence, he does object to what his predecessors believed to be the nature of the infinite—an actual physical body. In the second part of this chapter, I will show that in *Phy.* III.5, Aristotle assesses the ἐνδοξά to show that, contrary to accepted beliefs, an actual infinite body does not exist either substantially or attributively. Even though Aristotle agrees with his predecessors *that* the infinite exists because there is motion, he objects to the view that it is actual like other physical entities: there is no actual infinite substance nor actual infinite attribute of a physical body precisely because either actuality *destroys* motion. The reason for this is that Aristotle's litmus test for the infinite's actual existence is categorial being. For if the infinite exists in the physical world, then it must belong to at least one of the categories as an actual body. As Aristotle sees it, though, the ἐνδοξά about an actual infinite body fail since it is only with respect to the categories that actual physical bodies exist and the categories are always finite. This means that the infinite cannot be either an actual substance or an actual attribute. By the end of the assessment in *Phy.* III.5, however, we find out that Aristotle now has a problem of his own: the infinite exists, but it cannot be either a

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Apostle, *op. cit.,* pp. 226-233, Hussey, *op. cit.*, pp. 72-82, Ross, *Aristotle's* Physics, pp. 541-554, Wagner, *op. cit*., pp. 65-75. Others mostly critically assess—and often dismiss—the merits of Aristotle's testimony of the Presocratics; see Harold Cherniss, *Aristotle's Criticism of Presocratic Philosophy* (New York: Octagon Books, 1976); Walter Burkert, *Lore and Science in Ancient Pythagoreanism,* trans. Edwin L. Minar, Jr. (Cambridge, MA: Harvard University Press, 1972), pp. 28-52, and Malcolm Schofield, *An Essay on Anaxagoras* (Cambridge, UK: CUP, 1980), pp. 43-64. Cherniss' account is highly critical of Aristotle's treatments, even "excessively severe" (Hussey, p. 73). On the other hand, Walter Burkert, *op. cit.,* p. 46, reminds us that, even though Aristotle reports his predecessors in his own terms, Aristotle is still the richest and most reliable resource for Pythagorean doctrines as "unaffected by the achievements of Socratic-Platonic dialectic." Whatever the legitimacy of Aristotle's testimonies is, *Physics* III.4-5 is much more than a doxography for indexing and collating Presocratic beliefs. While it is important to be circumspect regarding Aristotle's abilities as a doxographer, we should investigate how the dialectical treatment motivates Aristotle's own arguments for the existence of the infinite.

substance or an attribute. But nothing else exists for Aristotle besides substances and their attributes. So, how does infinity exist? This sets up the problem with which *Phy.* III.6 begins.

§3.1.1. The dialectical account of the infinite (*Phy.* III.4, 203a5-203b2)

Aristotle turns to Plato and the Pythagoreans first (203a5-16) as evidence for the existence of the infinite. Plato argues that the infinite is the Great and Small (τὸ μέγα καὶ τὸ μικρόν), which is the so-called 'Indefinite Dyad' or 'two infinites' (δύο τὰ ἄπειρα), whereas the Pythagoreans identify it with evenness (τὸ ἄρτιον). What is interesting is that according to Aristotle, both the Platonists and the Pythagoreans claim that the infinite is a substance (οὐσία):

> Those like the Pythagoreans and Plato treat the infinite as something in itself, not as accidental to something else but as *itself* a *substance* [οὐσίαν αὐτὸ ὂν]. On one hand, the Pythagoreans put the infinite in physical things (for they do not separate number from these things), and treat the infinite as 'what is beyond the heavens' [τὸ ἔξω τοῦ οὐρανοῦ ἄπειρον]. On the other hand, for Plato, nothing exists beyond the body [of the universe], including the forms on account of the fact they do not exist anywhere, and yet the infinite is present is both physical things and the forms. There are those [i.e. the Pythagoreans] who say that the infinite is the Even. For this is that which is cut off and limited by the Odd and admits of [παρέχειν] the infinite in beings. An indication of this is what happens to numbers: for having set down the gnomon around the unit and [being] different [than the unit] [τὸ ἕν καὶ χωρὶς], on one hand the form *always becomes different* [ἀλλὸ ἀεὶ γίγνεσθαι], but on the other hand, the form is one. But Plato has two infinites [i.e. the Indefinite Dyad], which is the Great and the Small (*Phy.* III.4, 203a4-16).

Both Plato and the Pythagoreans treat the infinite as a substance but in different respects.[107] For Plato, the infinite exist by itself as the Great and the Small, which Aristotle interprets as an independent thing, which is a substance. The Great and the Small must exist independently because it functions as a material source of motion separate from physical things and upon which the Forms act, for it is that out of which physical things are generated by the Forms (*Meta.* A.9. 992b7-9).[108] On the other hand, as we know from *Meta.* N.5, everything is constituted by a number for the Pythagoreans, since the world is physical and the physical is quantifiable. But unlike the Platonists, who treat mathematical entities as separate, immaterial entities, the physical world *itself* is nothing but numbers for the Pythagoreans (*Meta.* A.6, 987b27f).[109] Evenness, as opposed to oddness, is the source of motion precisely because evenness pertains to plurality and plurality admits of change.[110] The substantiality of the infinite is the very plurality of the physical world. Nevertheless, Aristotle's aim here is to point out that for both the

[107] Themistius, *In libros Arist. de Anima,* 79.28-80 and 80.25-17 shows that for both Plato and the Pythagoreans, the Great and the Small is the infinite as a material principle of generation/motion. See also Burkert, *op. cit.*, pp. 15-28, Halper, *One and Many: A-Δ*, pp. 170-174; Kahn, *Pythagoras and the Pythagoreans* (Indianapolis: Hackett Publishing Co., 2001), pp. 63-72; and Leonid Zhmud, *Pythagoras and the Early Pythagoreans* (Cambridge, UK: CUP, 2012), pp. 412-432.

[108] Apostle, *Aristotle's* Metaphysics, p. 267, commenting on *Meta.* A.9, 992b7-9, confirms this: "The material principle of a Form is the Great and the Small, *which is motion*; so the Forms, having this principle shall be moving or capable of it." Sometimes this is associated with the *receptacle* at *Timaeus* 48c-53b. See also *Phy.* IV.2, 209b33-210a2.

[109] Kahn, *Pythagoras,* p. 63. *Cf.* Burkert, *op. cit.,* pp. 40-41 and Francis Cornford, "Mysticism and Science in the Pythagorean Tradition," *Classical Quarterly* 16 (1922), pp. 137-150.

[110] Concerning how the Pythagoreans make the Even itself materiality *as identical with* plurality, in contrast to Plato who makes the Dyad the principle of plurality, see Pseudo-Alexander, *In Arist. Meta.* 796.21-24 and 30-33. See also Christian August Brandis, *A Study of the Lost Books of Aristotle:* On Ideas *and* On the Good *or* On Philosophy, ed. Orrin F. Summerell, Studia Philosophica et Historica 27 (Frankfurt am Main: Peter Lang, 2005), pp. 67-69. Syranius, *In Arist. Meta.* 166.17-19 however conflates Plato and the Pythagoreans: "Nor did Plato, who demonstrates the indefinite dyad by means of inequality, depart from Pythagoras, who proves it by means of plurality (Brandis translation); see also Brandis, *op. cit.*, p. 68.

Platonists and the Pythagoreans, the infinite exists substantially as an independent material source of motion.

Plato himself argues for connection between materiality and infinity at *Philebus* 24a-25a where he speaks to the More and the Less (τὸ μᾶλλον καὶ ἧττον), which is a variation on the Great and the Small.[111] There, Plato portrays the infinite in connection to becoming (γίγνεσθαι) and progression (προχωρείν, πρόειμι), as opposed to finality (τέλος, τελευτή) and stillness (παύω, ἡχυσία).

> **Socrates**
>
> Consider then. What I ask you to consider is difficult and debatable; but consider it all the same. In the first place, take hotter and colder and see whether you can conceive any limit of them, or whether the More and Less which dwell in their very nature do not, so long as they continue to dwell therein, rule out the *generation* of an end [τέλος...γίγνεσθαι]; for were that which is generated to be finished [γενομένης γὰρ τελευτῆς], the More and Less would most certainly be finished. **[**…]
>
> **Socrates**
>
> Always, then, the argument shows that these two have no end; being endless, they indeed *become* infinite [ἀτελῆ δ᾽ ὄντε δήπου παντάπασιν ἀπείρω γίγνεσθον].

[111] See Kenneth Sayer, *Metaphysics and Method in Plato's* Statesman (Cambridge: CUP, 2006), pp. 158-161. It is generally accepted that Plato's 'the More and the Less,' 'the Great and the Small,' 'excess and defect,' are all synonyms for the Indefinite Dyad. See James G. Lennox, "Aristotle on Genera, Species, and 'The More and the Less," *Journal of the History of Biology* 13 (2), pp. 321-346 for Aristotle's use of 'the More and the Less' in his biological accounts.

Protarchus

Most emphatically, Socrates.

Socrates

I am glad you responded, my dear Protarchus, [24c] and reminded me that the word "emphatically "which you have just used, and the word "gently" have the same force as "more" and "less." For wherever they are present, they do not allow any definite quantity to exist; they always introduce in every instance a comparison—more emphatic than that which is quieter, or vice versa—and thus they create the relation of More and Less, thereby doing away with fixed quantity. For, as I said just now, if they did not abolish quantity, but allowed both it and the measure to make their appearance in the abode of the More and Less, the emphatically and gently, those latter would be banished from their own proper place. When once they had accepted definite quantity, they would no longer be hotter or colder; for hotter and colder are always *progressing* and *never stationary*; but quantity is at rest and does not progress [προχωρεῖ γὰρ καὶ οὐ μένει τό τε θερμότερον ἀεὶ καὶ τὸ ψυχρότερον ὡσαύτως, τὸ δὲ ποσὸν ἔστη καὶ προιόν ἐπαύσατο]. By this reasoning hotter and its opposite come to be infinite [ἄπειρον γίγνοιτ'] (*Philebus* 24a-25a, modified Fowler translation).

Even *prima facie*, we can see that motion is integral to Plato's account of the infinite. But, what is not so clear is how the infinite functions as a material source of motion in this account. To see this, let us look more closely at the argument. The argument consists of two parts: (a) the More and Less are infinite because the More and Less are unfinished (ἀτελή) and as such admit of motion (b) the More and Less are ἀτελή because the More and Less pertain to material which is

limited by form. Part (a) reveals that the More and Less admit of motion since things like the hot and the cold themselves are indeterminate and indeterminacy indicates a lack of stability or as Plato says, a state of being—finished (τελευτή). Only things at rest are 'finished.' Thus, the hot and the cold—the More and Less by extension—admit of motion. But, notice the way Plato describes the infinitude of the More and Less: it is always in terms of becoming (γίγνεσθαι). The first argument is as follows: For if the More and Less had a limit, it would (Plato claims) "be finished." But the More and Less itself is unfinished (ἀτελή) because it lacks a specific quantity. Therefore, they are infinite. Now, insofar as they are ἀτελή, they are not at rest. Thus, the hot and cold are in motion insofar as they are More and Less. The More and Less necessarily admit of becoming (γίγνεσθαι) and progression (προχωρείν). That is to say, the More and Less are endless, and as such they are the sources of motion. For, if they are unfinished, then they are not at rest.

Even in the first argument, Plato hints at motion as an indication that the More and the Less must be infinite. But in the second argument, he turns to quantity. Quantity pertains to the *measure* of the More and the Less, the μετρίον. The measure is what limits the More and Less, since it is the form.[112] Quantity, then, is a sort of form imposed on the More and the Less, for the More and the Less are determined by whatever measure they obtain, and the form pertains to the measure. This implies that the More and the Less, for Plato, is the material. Now, because material is inherently indefinite, it is ἀτελή. Therefore, the More and the Less are ἀτελή because they are material. But, as ἀτελή, the More and the Less are *always* progressing and never at rest, whereas a specific quantity, because it is the measure—the form—is at rest. The infinite, for Plato, is an independent source of motion. As independent, it is a substance. Furthermore, even

[112] *cf.* Sayer, *op. cit.*, pp. 157. See also *Statesman* 283c-285d and §2.2 above.

though Aristotle does not say it in so many words, Plato's account of the infinite is really an account of one of the sources of generation.

We find a similar motivation in the Pythagorean account at *Phy.* III.4, 203a10-16. A careful consideration of this passage reveals that Aristotle's interest the Pythagoreans' notion of the 'Even' (τὸ ἄρτιον) is in how it functions as a sort of source of motion:

> There are those [i.e. the Pythagoreans] who say that the infinite is the Even. For this is that which is cut off and limited by the Odd admits of [παρέχειν] the infinite in beings. An indication of this is what happens to numbers: for having set down the gnomon around the unit and [being] different [than the unit] [τὸ ἕν καὶ χωρὶς], on one hand the form *always becomes different* [ἀλλὸ ἀεὶ γίγνεσθαι], but on the other hand, the form is one (*Phy.* III.4, 203a10-16).[113]

Part of the difficulty with this passage is the dearth of direct Pythagorean source material regarding the function of the Even, the Odd, and the gnomon.[114] But this should not deter us

[113] The ambiguity of the syntagm καὶ χωρὶς is well known. See Ross, *Aristotle's* Physics, pp. 543-544. I suggest that we read it more straightforwardly: insofar as it is placed around the unit, the gnomon is *different* (χωρίς) in *kind* from the unit. Any gnomon is that which surrounds, not that which is surrounded. That is to say, the gnomon is *separable*. It is not the squared number but that by which something is squared, which means it is different, for the square does not square itself according to the Pythagorean account.

[114] Unraveling Aristotle's terse report of the Pythagorean gnomon has a long history in the scholarship. See Ross, *Aristotle's* Physics, pp. 543-544 for the ancient commentators' interpretations and Burkert, *op. cit.*, p. 33 fn. 27 for a thorough treatment of the modern reception. Recently, Monica Ugaglia and Fabio Acerbi, "Aristotle on placing gnomons round (*Ph.* 3.4, 203a10-15)," *Classical Quarterly*, 65 (2), pp. 587-608 revisited this debate by challenging both the ancient and modern interpretations, specifically Simplicius and the traditional Milhaud-Burnet reading; see Gaston Milhaud, *Les philosophes-géomètres de la Grèce: Platon et ses prédécesseurs* (Paris: Felix Alxan, 1900), pp. 115-117 and John Burnet, *Early Greek Philosophy* (London: Adam & Charles Black, 1908), pp. 110-117. The Milhaud-

from understanding *Aristotle's* assessment of the Pythagorean gnomon. I claim that it pertains to how Aristotle thinks they used the Even and Odd gnomon to demonstrate motion and rest.[115] We know that the Even and the Odd fall within the Pythagorean columns (συστοιχία) of unlimited and limit respectively (*Meta.* A.5, 986a24-26; *Phy.* III.2, 201b24-27).[116] But, this does not tell us much as to *how* the Even functions as not only the infinite, but also, as I have suggested, as a source of motion. We first need to understand what Aristotle means by τὸ ἕν at 203a14. Aristotle claims that the gnomon surrounds *the unit* (τὸ ἕν), which may be odd or even. It has a concrete meaning for the Pythagoreans—a pebble (ψῆφος). We know that the gnomon is the geometric figure formed by the L-shaped arrangement of pebbles. The gnomon originates in isopsephy, the arithmetic procedure of counting pebbles—dot-arithmetic. The first pebble is the μονάς—the unit. So, τὸ ἕν pertains to the pebble(s) which the L-shaped gnomon surrounds. If the magnitude of the gnomon is odd, it will retain the same form as the unit—squaredness. The counter places three pebbles around the first, resulting in the first squared number, which is four. Although the counter cannot place the previous gnomon around the four, since it requires 5 pebbles to square four, the fact that the subsequent gnomon is different does not change the form of original unit.

Burnet interpretation, however, is still the most elegant interpretation as it shows how the geometric and algebraic reading of the Odd and the Even gnomons are commensurate, whereas the ancient readings only allow for a geometric reading. For the strictly geometric reading, see Simplicius, *In Phy.* 457.25-458.7.

[115] To be sure, the Pythagoreans themselves seemed not to have clarified how the infinite as the Even accounts for motion; see *Meta.* A.8, 990a8-11. However, here Aristotle seems to provide a way that the Pythagoreans could have used their account of the Even and Odd to address motion and rest.

[116] Aristotle also refers to the columns as the 'table of opposites' (*Meta.* A.5, 986a21-28) See also Burkert, *op. cit.*, p. 51-52 and Zhmud, *op. cit.*, pp. 449-452.

$$\frac{n}{n}$$

Each consecutive unit remains a squared number regardless of how many times this procedure is applied. The sides remain proportional $\left(\frac{n}{n}\right)$; therefore, the unit square is always *one* with the odd gnomon. In this way, oddness pertains to limit and sameness for the Pythagoreans because regardless how many times the odd gnomon is applied to the unit square, whatever that unit square might be, the *form* remains the same. For Aristotle, this has greater significance. Aristotle's point seems to be that oddness is the form as it is *at rest*. That is to say, oddness is a sort of principle of rest. There is no *motion* with respect to oddness because it *is* limitedness and sameness.

What then is the function of the Even? Suppose that the gnomon has a magnitude of four, an even number. Following the same procedure as with the odd gnomon, when the even gnomon is applied to the first even unit, two, we see a change in form, for the resulting figure is not square. In fact, regardless of the arrangement, the resulting figure will be always oblong—it will always have *disproportionate sides*:

$$\frac{n}{n+1} < \frac{n}{n} < \frac{n+1}{n}$$

Each application of an even gnomon produces an iteratively distinct form, which is 'always becoming different' (ἀλλὸ ἀεὶ γίγνεσθαι) insofar as the ratio of the sides is always less than or greater than one $\left(\frac{n}{n+1} < \frac{n}{n} < \frac{n+1}{n}\right)$. This continues *ad infinitum.* The Even is always unfinished. It is never *quite* one. Every application of an even gnomon changes the proportionality of the unit. This change continues infinitely. When the odd gnomon is applied to the unit, the form of the unit remains one—that is to say, it stays the same squared figure. But, when the gnomon is even, the form 'always becomes different,' changing from one oblong shape to another. The significance of ἀλλὸ ἀεὶ γίγνεσθαι pertains to how Pythagorean evenness, as infinite, is a material principle of motion.[117] When limited by the Odd, the proportion stays the same. It is always *one*. Thus, the unit is 'at rest,' so to speak. Taken with an even number, however, the proportion changes form indefinitely; the proportion is never *one*.

The takeaway from Aristotle's comments on Plato and the Pythagoreans is that where there is infinity, there must also be motion. While this is not motion as Aristotle understands it—the incomplete actuality of a moving thing—it is not entirely unlike Aristotle's. Aristotle is noticing how the infinite is connected to a sort of being-unfinished or being-different that Plato and the Pythagoreans associate with change. Aristotle will not entirely reject this at *Phy.* III.6, 206b24-26 and 206b32-33 where he references the incompleteness and constant difference of the

[117] Milhaud, *op. cit.*, pp. 118-121; *cf.* Apostle, *Aristotle's* Physics., p. 227.

day and the games as indicating the character of the infinite. However, here in *Phy*. III.4, infinity is an *independent thing* for Plato and the Pythagoreans, which for Aristotle is a substance. Both the Indefinite Dyad and the Even pertain to materiality/plurality *itself*. As such, they are independent things that account for all motion. We will see in III.5, however, that such substantiality is absurd. For now, though, it is enough to recognize that the import of Aristotle's dialectical interests in Plato and the Pythagoreans is that (1) there is precedent for believing in the existence of the infinite as substance and more importantly (2) that infinity is a source of motion because of its association with materiality itself.

Now, the dialectical account of the natural philosophers shows how motion is also central to their accounts:

> But all the physicists [οἱ δὲ περὶ φύσεως πάντες] always set down a specific nature as underlying the infinite that is different than the so-called elements, such as water, air, or 'the in-between' [τὸ μεταξὺ]. Those who make the elements finite [in number], none make them infinite [in extent]. Yet, those who make the elements infinite in number [ὅσοι ἄπειρα] say the infinite is continuous by contact, just as Anaxagoras and Democritus do, where Anaxagoras treats them as homogeneous constituents, but the Democritus makes them from shapes composed of all sorts of seeds [ἐκ τῆς πανσπερμίας τῶν σχημάτων] (*Phy*. III.4, 203a16-23).

All of the natural philosophers believe in an infinite, even Empedocles; however the infinite manifests in Empedocles' cosmos only as consequence of the everlasting nature of the cosmic

cycle.[118] Aristotle's concern here has more to do with how Anaxagoras and Democritus' is infinite is an *attribute* of a body. In so doing, Aristotle tacitly juxtaposes the account of the natural philosophers with that of Plato and the Pythagoreans. For the latter, as we recall, the infinite is a substance; but for the natural philosophers who make the elements quantitatively infinite, infinity is attributed to a physical body. It is important to see that Aristotle's juxtaposition of his predecessors is between the substantial and attributive status of the infinite. It lays the foundation for the problem he addresses in *Phy.* III.5: as a source of motion, is the infinite a substance or an attribute? We have already seen how some believe that the infinite is a substance. Let us look briefly at how Aristotle reads Anaxagoras and Democritus to understand an infinite attribute.

The key to Anaxagoras' account is the assumption that nothing comes from nothing (*Phy.* I, 187a32-34).[119] The corollary is that like comes from like. For it is not the case that something comes from nothing, since there must be something out of which generation occurs. This means that what exists is generated from something like it—another *existing* thing. Were something to come from nothing, like would come from unlike. But, everything comes from everything else (*Phy.* III.4, 203a23-25), since anything that comes to be does so from something of *that same sort* (τὸ ὁρᾶν ὁτιοῦν ἐξ ὁτουοῦν γίγνόμενον, 203a24).[120] That is to say, something comes from something. Thus, like comes form like. This applies to the parts themselves as much as to the

[118] Although Aristotle implicitly sets him aside (since he does not make the elements infinite), Empedocles still has an infinite—the cosmic cycle (DK31B17 = Simplicius, *In Phy.* 158.1-159.4; *Phy.* I.4, 187a23-24). But, Aristotle's concern here in *Phy* III.4 with the role of the infinite in the natural philosophers is how some supposed it to be a source of motion. The infinite is still a material source of motion for Empedocles because his four elements are the simplest bodies out of and into which all things are generated and destroyed in an endless cycle (*cf. DC* III.3, 302a28-30).
[119] *Cf.* Schofield, *op. cit.*, pp. 50; 53-59.
[120] *Ibid.*

whole—such as how human bodies come from flesh and bone, and flesh and bone from earth and water, and so on.

For Anaxagoras, there is no smallest element—no atom—into which the whole may be separated. Since like comes from like, the elements come from elements. Therefore, elements are *composed* of elements. So even the parts of every whole are themselves wholes composed from further elements *ad infinitum.* Every element is a body composed of additional, smaller elements. In other words, whatever is generated is a blend of homogeneous parts, i.e. elements. This is why the elements are called the *homoiomeries* from which everything was separated.[121] In fragments B1 and B6, Anaxagoras argues for this as follows:

> [B1] Together all things were ἄπειρα both in quantity and in smallness; for the small was boundless too. And as all things were together nothing was manifest by reason of its smallness. For air and aether dominated all things, both being ἄπειρα. For these things are the greatest in the totality [ἐν τοῖς σύμπασι] both in quantity and in size. [B6] And since there are portions equal in number of the large and the small, so too would everything be in everything. And it is not possible for things to be isolated, but everything has a portion of everything. Since it is not possible for there to be a least, it would not be possible for things to be isolated nor would anything be able to come to be by itself, but as it was in the beginning, so at present all things are still together. And in all things many things are present, and of things being separated there is an equal number in both the larger and the smaller (DK54B1 and DK54B6, Graham translation).

[121] For a fuller account of the *homoiomeries* in Aristotle's own account of generation and destruction, see *GC* II.7 and Timothy J. Crowley, *Aristotle on the Matter of Elements*, DPhil Diss., (University of Oxford, 2009), pp. 184-206.

There must have been an originating whole (τὸ σύμπας) out of which everything was separated because, as Anaxagoras assumes, everything is composed of the same material.[122] Because there is no smallest part (nor largest), the *homoiomeries* are infinite. Thus, the originating whole is quantitatively infinite insofar as everything has an indefinite number of parts. What this means is that the infinite is an attribute of every body. [123] For, if the parts of the whole universe are numerically infinite, then the parts of any generated, composite body must be just as infinite. For the parts of the latter are also wholes constituted *ad infinitum* from the same kind of elements.

When it comes to Democritus, the way Aristotle describes his infinite is peculiar. At 203a34-203b1, Aristotle attributes Democritus' infinite to what Aristotle calls 'the common body' (τὸ κοινὸν σῶμα). But, what is this common body and how is it infinite? Nowhere else in Aristotle's writings do we find this description of Democritus' atomism, let alone in Democritus' extant writings. By τὸ κοινὸν σῶμα, Aristotle likely means how the atoms are 'generically one,'

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[122] What is interesting about Anaxagoras' account, however, is how there appears to be a second infinite, or at least another thing to which infinity is ascribed—*nous*:

> Everything has a portion of everything, but *nous* is ἄπειρον, self-ruling, mixed with nothing else, and is alone all by itself […] And nothing is completely separated nor segregated the one from the other except *nous*. *Nous* is all alike, both the larger and the smaller. No other thing is like anything else, but each one is and was most manifestly those things of which it has the most (DK54B12, modified Graham translation).

Nous is clearly a moving cause for Anaxagoras, as reported correctly by Aristotle (DK54B13 = Simplicius, *In Phy.* 300.27-301.1; *cf. DA* I.2, 404a25-404b6). What Aristotle does not disclose is that *nous,* for Anaxagoras, is also infinite. While Aristotle mentions *nous* as a principle of generation (*Phy.* III.4, 203a28-32), he does not mention that *nous* is infinite. Because *nous* is *like* everything, and everything is infinite, *nous* must be also infinite. Even though Aristotle does not mention Anaxagoras' infinite *nous*, it is still safe to say, I believe, that this 'other' infinite is still an attribute. See Schofield, *op. cit.,* pp. 14-22 for an interpretation of *nous*'s infinity as omniscience.

[123] Still, there is also a qualitative infinite for Anaxagoras because of his nonatomistic ontology. Aristotle elsewhere describes the *homoiomeries* as 'invisible' (ἀοράτων ὁμοιομερῶν, *DC* 302b3). Aristotle could be reading fragment B4 where Anaxagoras argues that "before these things were separated, when they were all together, not so much as a color was manifest…" (DK54B4 = Simplicius, *In Phy.* 34.21-26; 156.4-9).

since each atom functions as the same kind of indivisible material substrate underlying the different arrangements, positions, and shapes. This helps us see why Democritus' infinite is *quantitative* by Aristotle's understanding. There are an infinite number of indivisible bodies—atoms—differing in arrangement (ταξίς), position (θέσις), and shape (σχήμα).[124] Furthermore, they are held together by contact.[125] Each is generically one by being the same kind of indivisible body and held together in the same way. From Aristotle's perspective, then, Democritus' atoms are the generic substrate for the many ways they manifest. What Aristotle does not mention here, and what might help us understand his description of Democritus' infinite is that *void* is 'where' all the atoms exist and, more importantly, *move.* The common body must exist within a void, for bodies exist somewhere and that in which bodies exist must be empty. Without the void, there would be no motion, since that into which an entity moves must be empty. The void allows for the infinite number of atoms. For, being completely empty, void is non-being. Thus, it is limitless, since limit is ascribed only to existing things. Because the void is limitless, and only a being can be the limit of another, the atoms within the void must therefore be infinite.

It is important to point out that even amongst the natural philosophers, the infinite is considered by the likes of Anaximander to be a *first* unchanging source of motion, or a first principle. In this way, Anaximander's infinite is more of a substance than an attribute. Aristotle's second argument at *Phy.* III.4, 203b7-15 concerning generation and destruction indicates this:

> Still, [the infinite] is both ungenerated and indestructible, since it is a certain sort of source [ἀρχή]. For it is necessary that what is generated admit of an end [λαβεῖν τέλος] and that everything being destroyed be finished [τελευτὴ]. Because of this, as we have

[124] See DK67A6 = *Meta.* A, 985b4-20 and DK68A45 = *Phy.* I, 188a22-26.
[125] Furley, "Aristotle and the Atomists on Infinity," p. 88.

> already said, while there is no source of the infinite, the infinite seems to be a source of other things, both surrounding and governing everything, just as those who make no other causes than the infinite, such as intelligence or friendship. Furthermore, it is treated as divine, since it is seen as deathless and indestructible, just as Anaximander and most of the physicists say (*Phy.* III.4, 203b7-15).

The infinite is ungenerated and indestructible—unchanging—since it is a *certain sort* of source (ὡς ἀρχὴ τις οὖσα). By 'certain sort' (τις), Aristotle likely means that for Anaximander the infinite is a first source because its unchanging character is the ground for itself while being the ground of everything else. But, how so? For the natural philosophers, the τέλος of generation is destruction. We need to pay close attention to Aristotle's use of τέλος here. Primarily, for Aristotle, the τέλος of generation is not destruction, but the form that completes the generative process. And yet, there is another sense, one that Aristotle uses here in this passage: τελευτή, as that which is finished or comes last (*Meta.* Δ.16, 1021b25-30). The τέλος of generation, as Aristotle portrays the naturalists, is what comes last, namely death (τελευτή) and destruction (φθορᾶς). There is no destruction, however, of the infinite, since as a source it is ungenerated. Nothing ungenerated is destructible, particularly as a first source, since there would have to be something into which it is destroyed, which would have to be some other prior material source. Thus, it would not be first. But assuming that the infinite is a source and not grounded by one, it is necessarily ungenerated and indestructible. Thus, Anaximander's infinite is a first source. While this makes clear that Aristotle thinks that the naturalists hold that the infinite is a source, it remains to be seen how the infinite itself is not without a reason (μάτην), as Aristotle claims (*Phy.* III.4, 203b5). Because it is ungenerated and imperishable, and because nothing is

groundless (μάτην = ἄναρχος), even the infinite must have a ground. But since it comes from nothing else, it must be grounded in itself. Of course, what follows from this is that the infinite is divine, such as Anaxagoras' *nous* or Empedocles' *philia* (203b10-15). Anything unchanging and imperishable is necessarily divine. As such, Anaximander's ἄπειρον, by virtue of underlying all things as a fundamental substrate, is divine.[126]

In summary, the dialectical survey aims, at least in part, to uncover how Aristotle's predecessors view the infinite as a source of motion. Most if not all of Aristotle's predecessors believed that their infinite is a source of motion. However, in addition to what Aristotle's predecessors argued, the ἔνδοξα include five generally accepted beliefs (πίστις) to support the existence of the infinite in addition to being a source of motion: the belief (1) that time is infinite (*Phy.* III.4, 203b16), (2) that every magnitude is infinitely divisible (*Phy.* III.4, 203b16-18), (3) that there is a material plenum (*Phy.* III.4, 203b18-20; *cf.* πανπλῆρες, *GC* I.8, 325a29), (4) that there is an infinite extension (*Phy.* III.4, 203b20-22), and (5) the belief in the infinitude of numbers and mathematical objects (*Phy.* III.4, 203b22-25). Each belief seems to be supported by corresponding phenomena. Time is infinite for Plato, for example, as the sempiternality of the heavenly motions (*Tim.* 38b-c). Because of the regularity and enduring character of the circular motion of the heavenly bodies, their positions by which we count years, seasons, months, days, and hours have no beginning or end—i.e. infinite. Their unending circular motion imitates the eternality of the divine mind. Secondly, that there is a continuum is evident enough by its use in mathematics, for magnitudes are continuous by nature and infinite divisibility pertains to the continuum. Correlatively and thirdly, the existence of the plenum is evident by the endurance of the continuum in generation and destruction. The plenum is supposed to be something like a

[126] See Kahn, *Anaximander*, pp. 42-46; 237-238 for a nice summary of Aristotle's general treatment of the attribution of divinity to ἄπειρον.

single actual infinite body into and out of which there is endless generation and destruction. Were there no continuum, or were the continuum to bottom out into indivisibles, generation and destruction would cease, since an essential attribute of the material plenum is its infinite divisibility, as Leucippus believed (*GC* I.8). Aristotle later rejects this at *Phy.* III.8, 208a9-12. Now, fourthly Aristotle's apparent acceptance of infinite by extent should also be met with caution. Here, in III.4, he accepts it dialectically, for it is conceivable that because the finite is always defined by a boundary, that boundary must be defined by contact with another, assuming that the finite is always bounded by something else *ad infinitum*. However, at *Phy.* III.8, 208a12-15, he rejects this assumption.[127]

The fifth phenomenon—that numbers and mathematical objects are infinite—is the most trustworthy (κυριώτατον). The fact that numbers and mathematical magnitudes are infinite because in thought (τῇ νοήσει) we are able to think of a greater number or to construct the increasingly larger (or smaller) magnitudes. All the ancients believed this, even the atomists. Still, even though Aristotle accepts the claim that numbers and mathematical objects are infinite, he nevertheless rejects the noetic justification also at *Phy.* III.8, 208a15-19, for an actual infinite in thought is accidental because it does not necessarily correspond to any physical phenomenon. Nevertheless, it is the corresponding belief that what lies beyond the heavens (τὸ ἔξω τοῦ οὐρανοῦ), or what transcends the cosmos, that interests Aristotle here because it reveals an important problem, mostly directed at the Pythagoreans. Numbers and mathematical magnitudes *are* physical things for them. As such, they are *somewhere,* for all physical things are either inside or outside something. But if numbers, magnitudes, and thereby everything they constitute are infinite, so too must the entire cosmos be infinite. Insofar as the cosmos contains everything,

[127] See §4.3 below.

and everything that exists is, for the Pythagoreans, amounts to an infinite, the cosmos must be infinitely large.

§3.1.2. The definitions of the infinite (*Phy.* III.4, 203b30-204a7)

Before we see how Aristotle argues against the existence of an actual infinity body in III.5, we should explain how Aristotle addresses the four definitions of the infinite given at 204a3-7 with respect to motion.[128] Aristotle first defines the infinite as (1) "what is impossible to go through in such a way as not to have the natural capacity to be gone through/broken apart, just as the invisibility of the voice" (τὸ ἀδύνατον διελθεῖν τῷ μὴ πεφυκέναι διιέναι, ὥσπερ ἡ φωνὴ ἀόρατος, 204a3-4). What is peculiar about this definition is that (a) the qualification "in such a way as not to have the capacity to be gone through" (τῷ μὴ πεφυκέναι διιέναι) is apparently redundant to "what is impossible to go through" (τὸ ἀδύνατον διελθεῖν) and (b) Aristotle never explains what it means for the voice to be invisible (ἡ φωνὴ ἀόρατος) and how it illustrates this sort of infinite. The apparent redundancy is that, given the usual sense of διελθεῖν and διιέναι, it is borderline tautological to say that the infinite is impossible "to go through" (διελθεῖν) because it lacks the natural capacity "to be gone through" (διιέναι). Lacking the natural capacity is analytic to the claim that the infinite is τὸ ἀδύνατον διελθεῖν. Why then the qualification? I suggest the problem lies in the ambiguity of διιέναι. Read as the present infinitive of δίειμι, "to traverse, go through" the redundancy exists. But, read as διίημι, "to go through by breaking apart, to divide" the definition makes more sense. In the latter sense, there is a possible pun here

[128] Simplicius, *In Phys.*, 410.20-27 and Philoponus, *In Phys.* 470.36-471.14 treat infinite by addition and division (204a6-7) as a fifth definition. However, Hussey, *op. cit.*, p. 77-78 argues that there are only four definitions. He treats 204a6-7 as a corollary (ἔτι) to the fourth definition at 204a4-6: "the successive steps of the 'journey' [*sc*. διέξοδον] involve adding something or dividing something." I agree with Hussey since infinite by division and addition assumes the fourth definition; see §4.1.2 below.

(διιέναι ≈ διίημι): the infinite is what is impossible to traverse because lacking the natural capacity to be traversed is also to lack the natural capacity to be divided, i.e. it is indivisible. There is no way to *begin* to traverse this sort of infinite because no magnitude exists to be divided in such a way as ever to traverse it. In order words, this sort of infinite is like a point, for it would be like the invisibility of the voice.

But what does Aristotle mean by the analogy to the invisibility of the voice (ἡ φωνὴ ἀόρατος)? Admittedly, the locution ἡ φωνὴ ἀόρατος occurs only here in the *Physics* and the *Metaphysics* K.10, 1066b9-11. In both places, Aristotle never explains the implications for the invisibility of the voice. What is the connection, then, between invisibility and infinity? It is rather concrete once we look carefully. In *Sense and Sensibilia*, Aristotle argues that every physical body must be visible because to some degree bodies have color (*SS* 3, 439a25-30). For, without color, it is impossible to discern where a body's magnitude begins and ends, for color is the limit of the transparency of the surface of a body (*SS* 3, 439a30-33; 439b10-14); color *is* the visibility of a surface. The opaquer the surface, the more defined its limits appear. If, then, something is colorless, it is invisible. Thus, invisibility implies a limitless entity, since it would be impossible to discern the limits of the entity's surface. In turn, this entity cannot be extended, since every magnitude belongs to a body and all bodies are limited by virtue of their *visible* surfaces. Therefore, whatever is invisible is unextended like a *point*. The voice is invisible like a point. This means that the voice is also *indivisible* because points are indivisible. Thus, invisibility pertains or at least indicates indivisibility. So, when Aristotle compares this sort of infinity to the invisibility of a voice, he means to say that there is an infinite like a point, which lacks the natural capacity to be divided because it is not a magnitude; it is the very division of a magnitude. This reinforces my earlier suggestion for how to read διιέναι. According to the first

definition, it is impossible to 'break [the infinite] apart' (διιέναι ≈ διίημι) insofar as it lacks a magnitude which one may even begin to traverse, since traversal requires dividing a magnitude. In short, this sort of infinite is intraversable insofar as it lacks any *entry*; there is an infinite that lacks the natural capacity even to begin to be traversed.

But, where the first definition pertains to an infinite without entry, we can see how the second definition (*Phy.* III.4, 204a5) is the opposite: an infinite that one may begin to traverse but is impossible to complete (ἀτελεύτητον)—one with no escape. This sort of infinite likely pertains to an actual infinite physical magnitude. It has to pertain to an extended magnitude of some sort since the definition implies an entry point where one may begin any traversal. But, as ἀτελεύτητον, this infinite is inescapable insofar as the magnitude being traversed lacks an end—a limit—at which one may exit or complete the traversal. So, on one hand, we have the infinite without entry, as if it were a point, and on the other the infinite whose entry point lacks a corresponding point of escape.

Aristotle also names a third infinite that is difficult (μόγις) and rare (μόλις) to come to an end or escape (*Phy.* III.4, 204a6). Aristotle, as usual, does not elaborate.[129] Simplicius and Philoponus, however, give an interesting and plausible suggestion: a labyrinth.[130] The labyrinth is seemingly infinite because it is difficult and rarely complete, full of passages that lead us to nowhere. What is important to notice is that this infinite implies an *actual* point of entry and *actual* escape, but whose traversal is likely burdensome and disorienting (περικάμψαντες πάλιν), in a similar way to how Socrates speaks about the labyrinth at *Euthyphro* 291b-c. Whatever

[129]LSJ, *s.v.* ἄπειρος (Tragic) as "entangled without escape"; *cf.* Hussey, *op. cit.*, pp. 77. See also Arist. *Ag.* 1382 ἄπειρον ἀμφιβλήστον, Soph. fr. 526 (Nauck) ἄπειρον χιτών, and Eur. *Or.* 25 ἄπειρον ὕφασμα.
[130] Simplicius, *In Phys.* 470.23-25 and Philoponus, *In Phys.* 410.1-8.

traversal may be made in this sort of infinite, one is always twisted about, never reaching the end, in spite of the fact that there is an *actual* end.

The fourth definition (*Phy.* III.4, 204a5-7) is a qualified version of the second, I suggest. Just like the infinite without escape, this infinite is never completed. However, the difference lies in Aristotle's use of πεφυκὸς ἔχειν. This infinite is 'what *has the natural capacity* for traversal' (τὸ διέξοδον…ὅ πεφυκὸς ἔχειν) but does not *have* a traversal or limit (μὴ ἔχει διέξοδον ἢ πέρας). That is to say, there is something that is *able* to be traversed indefinitely *by nature*, but whose traversal is never completed. At first glance, this sounds like the second definition. What is the difference? Here, it is possible to read πεφυκὸς ἔχειν against μὴ ἔχει διέξοδον ἢ πέρας as a contrast between potentiality and actuality. For, all magnitudes—i.e. any physical entity—have the natural capacity to be divided without reaching an indivisible quantum, or a limit, since magnitudes are continuous by nature. But, this implies neither that there is no actual (second definition) end nor that there is an actual point of completion (third definition). There *might* be. But, simply because a magnitude is able to be divided infinitely does not mean that this must ever be actualized. Aristotle then briefly claims at *Phy.* III.4, 204a6-7 as a corollary to the fourth definition that the infinite may be defined by addition (κατὰ πρόσθεσιν) and by division (κατὰ διαίρεσιν), since the magnitude may be divided endlessly. Aristotle will argue for this in greater detail in *Phy.* III.6-7.[131]

With definition four and its corollary, we find an infinite into which there is an actual beginning and a *possible* end. Aristotle casts this infinite, at least tacitly, as what is often referred to as the 'potential infinite,' and is what he will adopt in the positive account in *Phy.* III.6 after ruling out definitions 1-3 in *Phy.* III.5. Looking over every definition, especially the order in

[131] See §4.1.2 and 4.2 below.

which they are presented, Aristotle already charts a course for how the physicist inquires into the nature of the infinite: First, does the infinite exist like a point, as that whose limitlessness precludes any traversal, any entry or exit, whatsoever? Or is it something that one may begin traversing, but come to no actual end precisely because no end is even possible? If neither of these, is the infinite, then, like a labyrinth, with both an actual beginning and actual end, but whose completion is inhibited by a burdensome traversal? Or if none of these is a viable definition, is the infinite that which has the capacity to be traversed—what has an actual entry and a possible escape—but is never actually traversed? *Phy.* III.5 answers the first three questions with a resounding no, since there is no actual infinite body whatsoever. However, *Phy.* III.6 answers the fourth question with an appeal to the incompleteness of motion.

For Aristotle, it is clear from both the ἔνδοξα and the phenomena that there is an infinite because of a connection to motion. Nevertheless, the ἔνδοξα compete with each other since some believe that the infinite is a substance (like Plato, the Pythagoreans, and Anaximander) whereas others think it is an attribute of a body (such as Democritus and Anaxagoras). Additionally, the phenomena, including the definitions, are ambiguous concerning the same issue: what kind of thing is the infinite and why? Is it a substance or an attribute? There are only two options, as far as Aristotle is concerned:

> The inquiry into the infinite leads to an *aporia*: for many impossibilities result both if the infinite exists or not. If it exists, is it a *substance or essentially attributed* to something by nature [ὡς οὐσία ἢ ὡς συμβεβηκὸς καθ᾽αὐτὸ φύσει τινί]? Or in neither respect, but still something infinite or infinitely many? The most significant *aporia* for the physicists is whether there is an infinite physical magnitude (*Phy.* III.5, 203b30-204a3).

An infinite physical magnitude is either a substance or an attribute. For Aristotle, those are the only options for its existence. The physicist must investigate which it is. We have good reasons dialectically to believe that the infinite exists. But, Aristotle's predecessors are divided on the issue; some like Plato and the Pythagoreans argue that the infinite is a substance since it is an immaterial thing which serves as a source of motion, while others like the natural philosophers claim that the infinite is an attribute because physical bodies are composed of infinitely many parts. Therefore, the dialectical account reveals that it is both. This, however, cannot be the case, for nothing is both a substance and an attribute. Which is it? What we can now see from Aristotle's dialectical inquiry how he sets up the guiding problem for III.5: the infinite exists, but it seems to be both a substance and an attribute, at least according to Aristotle's predecessors. Since this cannot be the case, Aristotle needs to determine which it is. Is it a substance or an attribute? In *Phy.* III.5, Aristotle will show that the infinite is neither an actual substance nor an actual attribute. This is the problem in which Aristotle finds himself by the end of III.5 and with which he begins *Phy.* III.6

§3.2. The refutation of an actual infinite body (*Phy.* III.5)

The claim that Aristotle advances in *Phy.* III.5 is that there is no actual infinite substance or actual infinite attribute. For, if an actual infinite substance exists, then it is either immaterial or physical. If it is neither an immaterial nor physical substance, then it must be an attribute of a physical body. Aristotle will first show that the infinite is not an actual immaterial substance since immaterial things are indivisible but the infinite seems to be divisible as a quantity. However, Aristotle will show in turn that it is neither an actual physical substance nor actual

attribute—not even an actual quantity. The reason is that categorial being is the context of the problem. Categorial being only admits finite bodies, since physical substances pertain to a bodies which are bound by surfaces, and attributes, including quantity, are *definite* features of the bodies.

Additionally, according to Aristotle, if there were an actual infinite thing, either substantially or attributively, that would *destroy* motion, since an actual infinite body undermines contrariety and place, both of which we know are required for motion. Even though motion itself is incomplete and indefinite, the *goals* to which motion aims are finite. At some point, motion must stop. This is because motion is always between contraries, which are determinate differences (e.g. hot and cold, educated and uneducated, etc.). Furthermore, locomotion is between determinate places. But, as we will see, if there were an infinite body, both its contraries and its place would be infinite. Thus, there would be no motion. We will see that in order to save the infinite, Aristotle will need to show how it exists in such a way that does not destroy motion, which will be the goal of the positive account of the infinite in *Phy.* III.6-8.

§3.2.1. Objections to an actual infinite substance (*Phy.* III.5, 204a8-205a7)

Aristotle addresses the immaterial substantiality of the infinite in terms of the first definition of the infinite (204a4-5) and the Platonists' account. As we have seen, the Platonists claimed that the infinite exists as a separate, immaterial substance. Aristotle suggests that the only way a separate, immaterial being might be plausible as infinite is if it were like the invisibility of the human voice—i.e. something like a point (204a13-17).[132] This recalls the first

[132] Platonists like Speusippus argued that points were the sources of the generation of magnitudes. For Speusippus' account of the definition of the point, see Leonardo Tarán, *Speusippus of Athens: A Critical Study with a Collection of the Related Texts and Commentary*

definition of the infinite: invisibility of the voice is like the indivisibility of a point. The infinite would be extensionless, for points are extensionless. As such, the infinite would be indivisible, since only what is unextended is indivisible. But, it is impossible for the infinite to be indivisible since the infinite necessarily pertains to magnitudes and magnitudes are divisible. The infinite must pertain to a magnitude because it is understood in terms of what cannot be traversed (ὡς ἀδιεξίτητον, 204a14; *cf.* 204a4-9) and traversability is always with respect to a magnitude.[133] And magnitudes belong to physical bodies. Therefore, the infinite cannot exist separately (χωριστόν) from physical things.

Notice that in order to object to an immaterial infinite substance, Aristotle invokes the general sense we get from the definitions of the infinite that it somehow pertains to an extended thing. For, if the infinite is defined as something that cannot be entirely traversed, and traversal always requires a magnitude, then the infinite must somehow be a sort of quantity of a physical body. Now, here is where Aristotle's argument gets interesting. As a quantity, is the infinite then a sort of *attribute*? Aristotle suggests this at *Phy* III.5, 204a15-204b10. As a quantity, the infinite must be divisible into parts, for magnitudes are divisible into parts. But as a quantity, the infinite cannot be a substance, let alone an immaterial thing. (Keep this in mind as we turn to the positive account of the infinite in *Phy.* III.6.). For, no immaterial thing has parts. Since divisibility is part

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(Leiden: Brill, 1981), pp. 457-459. See also Apostle, *Aristotle's* Metaphysics, p. 431 n. 26, which is in reference to *Meta.* M.9, 1085b29 where Aristotle claims that "some thinkers" generate magnitudes from points. However, at *Meta.* M.9, 1085b30-35 objects to this, arguing similarly as he does here in *Phy.* III.5: "But the parts of an interval cannot even be indivisible like those of plurality whose parts are units; for a number is composed of indivisibles, but a magnitude is not" (Apostle translation).

[133] Aristotle reiterates this at *Meta.* K.10, 1066b2-7: "The infinite cannot be something which is separate *and* sensible. For if it is neither a magnitude nor a plurality, but its substance is to be infinity itself, and not an attribute, it will be indivisible; for that which is divisible is either a magnitude or a plurality. But if it is indivisible, it is not infinite, except in the sense in which the voice is invisible. But this is not the sense in which people use the term [infinite] nor that which we are seeking, but that which is untraversable" (Apostle translation).

and parcel of the physical world, the infinite, therefore, cannot be an actual immaterial substance. So, Aristotle undermines the possibility of an immaterial substance by appealing to how the infinite by definition is a sort of quantity. As such, the infinite appears to be an attribute. But, Aristotle does not draw this conclusion so immediately.

Interestingly enough, at this point in the account, Aristotle does not yet explore how the infinite might or might not be an actual attribute of a body. Instead, we see that at *Phy.* III.5, 204a35-204b11 there is a curious interlude, which at first glance seems like an interruption to the argument:

> Perhaps, however, a more general inquiry can be made, whether there is an infinite in mathematical objects, in thinking [ἐν τοῖς νοητοῖς], and in things with no magnitude. We are currently examining physical things and things we have made a part of our method, and now whether or not among these things there is a *body which is infinite in the direction of increase* [σῶμα ἄπειρον ἐπὶ τὴν αὔξησιν]. Examined logically, it would appear to follow that there is no such sort of body; for if the account of a body [σώματος λόγος] is "that which is limited by a surface" [τὸ ἐπιπέδῳ ὡρισμένον], then there is no infinite body, neither intelligibly nor physically. Additionally, it is not possible for a number to exist as something separate and infinite, since a number or that which has a number is numerable. Therefore, if what is numerable may be numbered, then it might be possible to traverse an infinite. But to be sure [δὲ μᾶλλον], these *are* considered from the following physical problems: the infinite is neither a composite nor a simple body. (*Phy.* III.5, 204a34-204b11).

The function of this passage is to set up the remainder of *Phy.* III.5 as pertaining only to the physical world by distinguishing between the physical problems of an infinite from more metaphysical and formal issues pertaining to the existence of an immaterial infinite.[134] Aristotle's concern here is that the infinite exists only with respect to the physical world (περὶ τῶν αἰσθητῶν καὶ περὶ ὧν ποιούμεθα τὴν μέθοδον), not as it might be separate from the physical world. The infinite is a physical problem because it pertains to the possibility or impossibility of an infinite body *in the direction of increase*. That is to say, physics examines whether there is a *quantitatively large* infinite. As we have already shown, this is because the infinite pertains to magnitudes which are capable of being traversed, and magnitudes are quantities which belong to physical bodies with respect to increase and decrease. This includes numbers too, because magnitudes are able to be numbered insofar as magnitudes have sizes, weights, and plurality. Number and magnitude pertain to quantity, and as we know, quantity falls within categorial being, which in turn pertains to physical things. Furthermore, number is an attribute of wholes and parts, which strictly belong to physical bodies. In this way, Aristotle agrees with the Pythagoreans insofar as number strictly belongs to physical things. Any actual existence of the infinite apart from magnitudes is immaterial—even in thought (ἐν τοῖς νοητοῖς). And Aristotle has already shown that this is impossible since an immaterial actual infinite would be separable and therefore indivisible. Therefore, with respect to an infinite substance, or even as an attribute,

[134] *Cf.* Hussey, *op. cit.,* p. 79 and Ross, *Aristotle's* Physics, 548-549. Ross suggests that Aristotle is tacitly addressing the relevance of the infinite with respect to the Platonic divisions between αἰσθητά, μαθηματικά, and νοητά. Hussey claims that this section pertains to "formal" arguments against an infinite: "The question of whether there is a self-subsistent Infinite is not germane to physics and has to be treated with 'formal' arguments, i.e. ones not drawing on any particular science." While I agree that Aristotle is distinguishing issues not relevant to physics, I suggest that these issues are actually germane to a particular science—metaphysics. Metaphysics must ask whether things like mathematical objects may admit an infinite insofar as they are immaterial. In other words, these issues fall outside of the scope of physics.

the most relevant question that the physicist asks is does the infinite exist as an actual *physical body*? Based on the analysis of an infinite immaterial substance, the infinite seems instead to be an attribute, namely a quantity. And, to be sure, attribution to a body is a physical problem for the infinite. For the time being, however, Aristotle postpones investigating whether the infinite is an actual attribute in order to assess first whether or not the infinite is a physical substance at *Phy.* III.5, 204b1-205b1. He will return to examine the possibility of an infinite attribute shortly at *Phy.* III.5, 205a7-205b1.

Physical bodies exist substantially in two ways: compositely (σύνθετα) or simply (ἁπλά) (*Phy.* III.204b11). Aristotle shows here that the infinite is neither a composite (*Phy.* III.5, 204b10-22) nor a simple body like a physical element (204b22-205a7). First, there is no infinite composite body as a substance, since the elements that constitute the composite are finite in number and in power (204b10-22). For any composite entity, its constituent parts—its elements (τὰ στοιχεῖα)—are necessary finite in number (πλείω). Since the whole body is finite, so too are its parts, since the number of actual parts cannot exceed the actual whole body; the whole is greater than the parts (at least for Aristotle). If an infinite whole existed with infinite parts, then the infinite whole would be greater than that of the parts. But this is absurd since no infinite is greater than another. Nevertheless, the fact that a composite body is necessarily composed of a finite number of elements is not enough to show that there is no infinite composite. What is also required is that the *powers* (δυνάμεις), or essential qualities (παθή), of the elements balance (ἰσάζειν), such that they are proportional to each other.[135] The elements composing a body must be proportional to each other, since composites have *ratios* (λόγοι) of elements, such as how

[135] The essential powers or qualities of the elements are: fire is hot and dry, air is hot and wet, water is cold and wet, and earth is cold and dry. For a recent thorough treatment of the elements and their mutual interaction, see Crowley, *op. cit.*, pp. 91-95.

different colors are determined by the specific ratios of black and white.[136] For, the proportion or ratio of elements is the *form* of the composite. Because the elements' powers balance each other, their qualities must be *limited*. Otherwise, one element would destroy the others insofar as the element with infinite power would overtake those with finite power. This is the heart of Aristotle's objection to an infinite composite substance: were there an element, say fire, whose qualities were infinitely hot and dry, it would always overpower (ὑπερβαλείν) the other elements *within* the composite; thus the ratio would be destroyed, which destroys the composite. For, as Aristotle argues, the element infinite in power would *exceed* the total amount of the finite element's power. The coolness of the water, were it infinite, would squelch the heat of fire entirely and the powers of the other elements, were they only finite in power. Thus, this would destroy all other bodies. One element infinite in power destroys composite being. However, this is not actually the case; clearly composites exist, for the ratios constituting composites are in fact limited and determinate ('having single specific amount,' μόνον ἀριθμόν τινα ἔχον, 204b17-18); thus none of the elements overpower each other. There is, in fact, only so much fire an entity has and can have relative to another without destroying the other finite elements altogether. Whatever power a composite body has as a whole is, therefore, determined by the *finite* ratio between the powers belonging to its elements. A body's qualities or powers are necessarily finite, and moreover, finite *relative* to that of another body.

The next possibility that Aristotle raises is if each element were extendedly infinite (204b19-22), not just one, could there be an infinite composite body as a substance? This, however, would destroy directionality. A body is extended in every direction—up, down, left, right, front, and back. But this is only finitely the case. Solids have determinate dimensions in all

[136] Crowley, *op. cit.*, p. 110. See also *Phy.* I.5, 188b23-26 and *cf. GC* II.7 on 'composition' and 'mixtures' of elements. See also *SS* 4, 442a13-17.

directions, for not all solids have the same dimensions and not all bodies are in the same location. For, many bodies limit and are limited by each other. Different bodies exist in different places relative to other bodies. The desk on which I use my computer to type this dissertation has a specific height, length, and breadth, and my computer with its own dimensions sits on top (i.e. above) of my desk, whereas my chair sits in front of the desk. This is true even for the elements that constitute these things. Fire exists above the air since fire moves upward, whereas earth exists below it at the center of the universe. Sometimes, certain amounts of fire are below the air, such as a campfire, or earth above water such as a rock thrown into the air. But if each element were infinite in extent, there would be no directionality nor even displacement, for each element would be extended in all directions to the point of no differentiation between the elements; in other words, the infinite would be a single actual intraversible body (ἀπεράντως διεστηκός, 204b21).[137] There would be only one element since no distinction could be made between one infinite element and another. The elements are in fact distinguished by their natural places, which corresponds to different directions. But an infinite element destroys directionality. But, because there is a plurality of finitely extended bodies, no infinite element exists. Thus, no infinite composite body exists.

Directions are *relational* attributes with *definite* points of reference such that something is 'up' insofar as there is something else 'below' relative to at least two finite bodies. What this implies is that relation cannot admit of an infinite. It is clear from the phenomena that directions exist because there are multiple finitely extended bodies in different places. So, in summary, if there were an infinite composite body, its elements would be necessarily infinite, either one or all of them. But, the elements are finite in number, extent, and in power. As we can see, regardless

[137] *Cf. Phy.* III.4, 204a5-6 for the first definition of the infinite.

of whether one or all bodies are infinite, an infinite body would destroy the very plurality of constitutive parts required by composite being. Therefore, there is no infinite composite substance.

Nevertheless, what if the infinite were a substance, if it *itself* were a simple body apart from the other elements instead of being composed by elements (*Phy.* III.5, 204b22-205a7) The first way Aristotle objects to this is by showing that there is no infinite physical body that is one and simple (ἕν καὶ ἁπλοῦν, 204b22), in terms of Anaximander's substratum ('that which is beyond the elements, from which the elements are generated, τὸ παρὰ τὰ στοιχεῖα, ἐξ οὗ ταῦτα γεννῶσιν, 204b23-24). Although Anaximander is unnamed, Aristotle's description is indeed a reference to him.[138] Anaximander calls this 'other thing' (ἕτερον) the ἄπειρον, since it is that unbounded, indefinite body which is in-between (τὸ μέταξυ) the determinate elements which serves as a principle of generation. Unlike Heraclitus and Thales, who make the elements of fire and water respectively the principles of generation, Anaximander makes the infinite a simple physical body *separate* from the elements as the source of their generation (ἐξ οὗ ταῦτα γεννῶσιν) in order to prevent the elements from destroying each other (204b13-22).[139] Nevertheless, according to Aristotle, the problem with Anaximander's ἄπειρον is that it is a perceptible body (σῶμα αἰσθητόν) *apart* from the elements since it is a physical ἀρχή.[140] But this is absurd for Aristotle. Aristotle rightly treats Anaximander's ἄπειρον as itself a physical thing,

[138] Ross, *Aristotle's* Physics, p. 549 and Simplicius, *In Phy.* 479.32-480.8

[139] Kahn, *Anaximander*, p. 233 points out that Anaximander's ἄπειρον is in fact a physical body, "primarily a huge, inexhaustible *mass*, stretching endlessly in every direction. […] The Boundless is in fact what we call infinite space, the antecedent for the atomistic void as well as for the Receptacle or Nurse of generation in Plato's *Timaeus*. But *this space is not yet thought of in abstraction from the material which fills it*," emphasis mine.

[140] See *ibid.,* pp. 30-34 on the doxographical difficulties verifying that Anaximander himself identified his ἄπειρον as a fundamental ἀρχή. See also *ibid.*, pp. 235-237 on the cosmological role ἄπειρον serves as an ἀρχή for Anaximander.

since it would have to be physical in order for the elements to be generated from and resolved back into it. But, this is impossible since one of the requirements for perceptibility is *physical contact* with the elements. This means that the ἄπειρον must be equally *here* (ἐνταῦθα) in the perceptible world just like the elements. In other words, it would be a perceivable thing. However, no such body even *appears* to us (φαίνεται). Aristotle's objection to Anaximander is that even if there were an infinite physical body existing apart from the elements as their source of generation, it would necessarily be as perceptible as the elements since it would be another *phenomenon*—it would have to *appear* to us *here* as another perceptible body (*cf. DC* I.10, 279b18-19). No such body appears to us, however. Again, the problem with Anaximander's ἄπειρον is not the infinitude of such a body; it is that no such body even *appears to us*. The simplest bodies that appear to us are the elements—fire, air, water, and earth—and it is only out of these that generation occurs.

What if, however, the infinite were one of the elements serving as a source of generation for the others (204b35-205a7) instead of being separate from the elements? Could the infinite exist simply (ἁπλῶς) as a substance in this way? Just as we see with Anaximander's ἄπειρον, the issue with Heraclitus' everliving fire, Thales' water, or Anaximenes' air is not whether the simple body is itself infinite. The issue is whether elements are themselves capable of mutual transformation. Even as *finite* entities (κἂν ᾖ πεπερασμένον), neither fire, water, air nor earth can be the single generative source of the other elements. For, no element can change into another. Were all things generated from and resolved back into a single fundamental element such as fire, coldness would come from heat and wet from dry. In other words, water would come from fire. But, as Aristotle argues at *DC* I.10, 279b21-30, there is no generation whatsoever from a single element because elements are incapable of being otherwise:

> Suppose that the world was formed out of elements which were formerly otherwise. Then if their condition was always so and could not have been otherwise, the world could never have come into being. And if the world did come into being, then clearly, their condition must have been capable of change and not eternal: after combination therefore they will be dispersed, just as in the past after dispersion they come into combination, and this process either has been or could have been, indefinitely repeated. But if this is so, then the world cannot be indestructible, and it does not matter whether change of condition has actually occurred or remains a possibility (*DC* I. 10, 279b21-30, Stocks translation).

In short, for everything to come from and be resolved into a single element, that element *itself* must be able to change. Its infinitude is ancillary to this issue. The reason Aristotle gives at *Phy.* III.5, 205a6-7 is that every change is from contrariety. Fire, were it that fundamental element, would have the *capacity* to change from hot to cold and dry to wet. Except that, fire is hot and dry *by nature*, and for this very reason opposed to water. Fire itself lacks the capacity to be otherwise. This is why the infinite cosmic cycle is impossible if everything had to come from one simple element. Aristotle sidesteps the problem of an infinite simple body by showing that even as finite bodies, the elements cannot be generative principles. The very simplicity of the element bars its being otherwise (*cf. Meta.* A.3). Heraclitus' everlasting fire, or Thales' water into which everything is resolved and comes to be again cannot be an infinite source of generation because even as finite, the elements are fundamentally opposed. Thus, none of them could be the source of another element's generation, even if they were infinite. Again, were air

(which is cold) to come from fire (which is hot), fire would be subsequently destroyed because its heat would be extinguished.

For Aristotle, therefore, the infinite exists neither compositely nor simply. What follows, it seems, is that the infinite does not exist in any way as a physical substance, for both composites and simples are the only types of physical substances for Aristotle. The next important takeaway is that the arguments against a composite and simple infinite undermine the second definition of the infinite because it undermines motion. Were the infinite *either* a composite or a simple element, there would be an actual intraversible extended entity (ἀπεράτως διεστηκός). There would be an infinite body in the direction of increase—i.e. an intraversible extended entity. As we have seen, however, an intraversible extended entity destroys contrariety. Thereby, there would be no motion. For, as we learned from *Phy.* III.1, motion is always between contraries. So, the second definition of the infinite makes motion impossible. We need to keep in mind how much motion remains a central theme. While the supposition of a simple infinite body seems to be an attempt to save contrariety and motion, it inevitably accomplishes the opposite by reducing the element to a single material source that is ultimately incapable of any change because of its incapacity for contrariety. We can see, then, that as a composite or simple body, the infinite would be a substance that destroys motion.

Before we turn to Aristotle's arguments against an actual infinite attribute, let us take stock of his objections thus far. In this section, Aristotle has examined whether the infinite is an immaterial or material substance. Aristotle first denies the existence an infinite immaterial substance. If the infinite *itself* were a substance, it might as well exist as an independent being as Plato claims. That is, it would be an immaterial thing separate from categorial being. As such, it would be indivisible. However, Aristotle argues that as what is traversable without end (204a4-

5), the infinite must be something extended, for extended objects like magnitudes are traversed. So, if the infinite is not immaterial, is it a material or physical substance? After refuting the existing of an infinite immaterial substance, Aristotle examines the beliefs of the natural philosophers in the existence of an infinite physical substance. This too is absurd. For, physical substances have parts, and parts are *distinct* from the whole by being *less* than it. That is to say, the infinite would have to be a whole physical body with parts. But, a part of infinity is no less than the whole infinite (204a26-27). Infinitely many parts would necessarily exist; but parts are as finite as the whole, either in number, power, or extent. Thus, the infinite is not a substance, either immaterially or materially.

What if the infinite is an attribute of a body? As we have already seen, Aristotle seems to advance this view, especially since the infinite seems to be an attribute of number and magnitudes (204a17-19). In fact, this is how Aristotle objected to the Platonists regarding the infinite's separability—the infinite cannot exist separately from physical things because it *seems to be* a sort of quantity. Now that Aristotle has ruled out an actual infinite substance, he returns to an infinite attribute as a possibility. But, as I will show, the way Aristotle undermines the existence of an actual infinite attribute is by demonstrating that *none* of the accidental categories actually admit an infinite body, including quantity. If there is an infinite physical body, its attributes must also be infinite, since attributes derive their being from that to which they are attributed. But, all the ways a physical thing may be attributed are categorial, which are necessarily finite. To see this, let us turn to the next section of the text.

§3.2.2. Objections to an actual infinite attribute (*Phy.* III.5, 205a7-205b1)

Aristotle's objections to an actual infinite attribute are specifically in terms of the homogeneity or heterogeneity of the parts of a whole physical body. Assume that there is an infinitely extended physical body (as Anaxagoras and Democritus do; *cf. Phy.* III.4, 203a19).[141] If the whole body were infinite, so too would be its parts, for the parts are on account of the whole. The question then is, would those parts be infinitely homogeneous or heterogeneous with respect to each other and the whole? Aristotle first addresses infinite homogeneity (ὁμοειδές) at *Phy.* III.5, 205a10-19:

> It is clear from the following that an infinite physical body is impossible. For, it is the nature of every physical body to be somewhere [που], such that [καὶ] a specific place exists for each, which is the same for both the part and its whole, such as the whole earth and a clod of earth, and all of fire and a spark. So, if [the infinite body] is homogeneous [ὁμοειδές], then it will be either motionless or always carried about. But these are impossible. For, to be sure, why is there a 'down' or an 'up' or any other direction [ἢ ὁπουοῦν]? I say, for example, if there were a clod of earth, where would it move to or stay still? For the infinite place of the body would be the same in kind [as that of the part]. Then will the body occupy the whole place? How so? What then would it be or where would it rest or move? In this way [ἄρα], it will not be moved. Would it be moved altogether? Then it would never standstill. (*Phy.* III.4, 205a10-19).

[141] Furley, "Aristotle and the Atomists on Infinity," p. 88.

Infinite homogeneity pertains to the identity between the places and bodies. If the parts are entirely the same as the whole *with respect to place*, or even the same place as each part, motion and place are destroyed because there would be only one body. For motion and place to exist, there needs to be multiple bodies. For, every body is contained somewhere (ποῦ) by something else. What this means is that even the parts of each body have places, for parts often contain other parts, e.g. the head contains the brain and the seats in a car contain padding. Now, assume that each part were infinitely homogeneous with the whole in which it exists. This would include their places. Were there a clod of earth, and were it infinitely homogeneous with the whole earth, it would not move. For, the infinite clod would be extended in the same way as the occupying place as the whole—they would take up the same amount of space. This is because the place of the infinite homogeneous part—the clod—would be the *same in kind* (συγγενοῦς) and *in the same respect* (αὐτῇ) as that of the whole earth in which it is located. Essentially, the part and whole would be indistinguishable; there would be no difference between the clod and the whole earth. Thus, there would be no place, since only one body would exist and nothing else to contain it. But clearly, there is a difference between clods and the whole earth. The phenomena, again, are enough to show that this is false. Parts clearly occupy different finite magnitudes and not everything takes up the same amount of volume. Were a part to occupy the same place as an infinite homogeneous body it would take up the same volume as the whole body; the size of the part would be no different than the whole. But, this is clearly not the case. Clods of earth are smaller than the whole; in fact, some are smaller/larger than other clods. The evidence is rather simple: there are *other* clods of earth. Thus, parts do not occupy the whole place. Furthermore, some things are at rest while others are moving. This is because the parts of the whole occupy different places, even though they have the same natural place. The clod of earth falls to the

center because that is the nature of the earth; but this does not mean that the clod is in the same place *in the same respect.* Some clods even displace other clods *away* from the center. The same can be said for fire. A spark or flame moves upward because fire's natural place is at the edge of the cosmos. But clearly not all fire is in the same place, even though all fire *naturally* moves in the same direction. This is why it is possible to distinguish between parts of the whole and the whole—the existence of different, heterogeneous places in the universe. Thus, there is no infinite homogeneous body because the attributes of a body are finite and heterogeneous.

Now, what if the infinite body were infinitely heterogeneous? Aristotle addresses this after homogeneity (*Phy.* III.5, 205a19-25). However, the argument against infinite heterogeneity, unlike that against homogeneity, cannot proceed by undermining the heterogeneity of place. At least it cannot do so entirely. Aristotle accepts heterogeneous places fundamentally. This is precisely why infinite homogeneity is false for Aristotle. Instead, Aristotle observes what happens to place and relation when, in an infinitely heterogeneous universe, the parts *themselves* are either finite or infinite. For the sake of the argument, Aristotle tacitly assumes the Democritean belief that the body of the universe is one only by contact (*cf. GC* I.8):

> But if the whole is heterogeneous, the places will also be heterogeneous. With respect to the former, the whole body will not be one except by contact; with respect to the latter, the parts will be either finite or infinite in kind [τῷ εἴδει]. The parts could not be finite in this way (for if the whole is infinite, some parts such as fire or water would be infinite in extent, while others not; but what is of this sort [τὸ τοιοῦτον] would destroy contrariety (*Phy.* III.5, 205a19-25).

Assume there are infinitely many places. This is to say that the universe is infinitely filled with innumerable indivisible corpuscles—atoms—whose unity is a *contiguous* whole. What follows (ἔπειτα) is that these atoms are parts of the contiguous body, which are either finite or infinite in kind (τῷ εἴδει). The argument here divides into two parts: (1) If some of the parts of the infinitely heterogeneous body are finite, then contrariety is destroyed because the infinite parts overpower them (*cf.* 204b22-25). (2) But, if all the parts were infinite, neither the parts nor their respective places would fit together exactly (ἀπαρτίζειν)—i.e. either there would exist a void or a body would be without a place. Both inferences are absurd. Let us work out the arguments to understand why.

The first part of Aristotle's argument reiterates a previous objection (καθαπὲρ εἴρηται πρότερον) that contrariety is destroyed when an element finite in power combines with another infinite in power (204b14-19). Were the universe infinitely heterogeneous in such a way that some of its parts were finite in power while others were infinite—e.g. water and fire respectively—then the finite part would be destroyed.[142] Similar to the argument at 204b14-19 and b24-29, the infinite element cannot be limited by the others that constitute the whole body. Remember also that the whole body is constituted by its ratio of elements, which is determined by how much the power of each element limits and is limited by the contrary in another element. A body has a certain temperature only because the parts of the body are balanced between a limited amount of hot and cold. But, if some of its parts are finite in power and others infinite, such that the heteronomy pertains to how at least one part has limited power and another unlimited, then this results in the destruction of contrariety. Thus, there would be no such thing

[142] Thales and Anaximenes made water and air respectively the *simple* infinite material source since their natural places are *ambiguously* (ἐπαμφατερίζει) up and down instead of fire and earth (though no one really seems to favor earth as a candidate) since these define the outer and inner limits of the universe.

as temperature because the parts would not be able to balance between hot and cold. Furthermore, there would be neither hot nor cold at all, since these have determinate degrees. But, the infinite heat of fire would overtake the limited coolness of water. The result of this is the destruction of motion, such that no change in temperature or any qualitative change would exist. For, motion is always between contraries. Additionally, there is no *place* for an infinite element because place is the limit of the containing body, but the infinite element is not bounded or contained since it would be indeterminately extended. So, if even one of the parts were different insofar as it were infinitely extended, then no other part would be contained nor would the infinite part be contained. Therefore, were the infinitely heterogeneous body composed of both finite and infinite elements, place would be undermined.

What if, however, *every* part were infinite and simple (*Phy.* III.5, 205a29-205b1)? Since each part is an element, and elements are defined by their natural places, the number of natural places would be infinite as well. Since each element, as simple, is different because of its powers and natural place, not only would each be infinite in power but also in number. But that too undermines place:

> If the parts are infinite and simple, then both the elements and their places will be infinite [reading with *Meta.* 1067a21, no comma after ἄπειροι]. But this is impossible, given that their places are finite, then the whole also is necessarily finite. For it is impossible for a body and its place not to fit together exactly since neither is any place quantitatively larger than the *admitted* [ἐνδέχεται] body (and *a fortiori* there will be no infinite body) nor is the body quantitatively larger than its place. The reason for this is that either there will be a void or a natural body will exist without a place (*Phy.* III.5, 205a29-205b1).

Aristotle's objection turns on how both elements and their places would be numerically infinite in magnitude. Here is where an infinite seems to fail even in the category of quantity. For, an infinite number of elements necessitates an infinitely extended universe. The universe would need to be infinitely large to accommodate bodies differing infinitely in size and number. For, they would exist either with gaps in between each other (if they are contiguous), or nowhere at all (οὐδαμοῦ). It is clear that simples at least exist somewhere; fire, air, water, and earth all have natural places. It follows, then, that there will be a void between simples since no body would fit together exactly. Place has the power to admit (ἐνδέχεται) only as much of a body as it is able in addition to the body's own power to be contained by that place. For there to be infinitely many bodies held only contiguously, their respective places must be infinitely *larger* to admit the bodies. The reason is that if the containing body only touches the contained body, there would be gaps in between—a void. So, the boundary of the container would need to be larger than what is contained. The volume of the contained body will be smaller than the capacity of the container, if the gaps exist between container and the contained. Therefore were there infinitely many contiguous bodies, their places would be infinitely more. However, this cannot be the case. There are only as many places as the bodies they contain. This is because, for Aristotle, a place and its body by definition do, in fact, share a boundary—they are continuous. In this argument, Aristotle seems to assume the definition of place that he makes explicit in *Phy.* IV.4, 212a20-21, which is the *innermost limit of the surrounding body*. The corollary is that space is the outermost limit of the surrounded body. Because the limits of the container and contained are continuous, both fit together exactly. This means that the size of a body correlates to the size of its place. Therefore, there are no gaps—the boundaries of the body and its container are *continuous*. As such, there is

no need for the place to be larger than what it contains since both would share a boundary. So, no infinitely heterogeneous body exists with respect to size and place, since the number of places are as finite as the number of occupying bodies since place is continuous with what it contains. Given Aristotle's analysis of both homogeneous and heterogeneous infinite bodies, neither is possible because place is *finitely heterogeneous*.

§3.2.3. Final refutation of an actual infinite body (*Phy.* III.5, 205b1-206a8)

Aristotle's final refutation of an actual infinite body turns on the impossibility of Anaxagoras' view that the infinite is motionless (Ἀναξαγόρας δ'ἀτόπως λέγει περὶ τῆς τοῦ ἀπείρου μονῆς, 205b1-2).[143] What is Aristotle's motive here, though? Well, if the infinite body is located neither in a single homogeneous place (because place is necessarily heterogeneous) nor in infinitely heterogeneous places (since the number of places are finite), where would it be located, if an infinite body exists? Anaxagoras answers this by treating the infinite as in itself a motionless whole:

> Anaxagoras speaks absurdly concerning how the infinite is at rest with respect to place. For he says that the infinite fixes itself. This is because it is in itself (since nothing else surrounds it), as if wherever something is, it is there by nature. But this is not true, since something could be located somewhere by force and not naturally. So, however much it is the case that the whole [universe] does not move (for that which is fixed in itself and something in itself is necessarily motionless), it is still necessary to show *why* it is

[143] Schofield, *op. cit.*, p. 158 fn. 27.

> motionless by nature, one might say. For it is insufficient to make an unsubstantiated claim like this (*Phy.* III.5, 205b1-8).

Aristotle's objection to Anaxagoras is that the infinitude of a body is irrelevant to its natural place of rest. Anaxagoras assumes that the whole universe is infinite in extent.[144] As such, it is 'in itself fixed' (τὸ στηριζόμενον), for it is contained by nothing else (οὐδὲν περιέχειν). So, the whole must be completely at rest with respect to place. In other words, there is nothing else into which the infinite may move, if it is fixed in itself. But, this assumes that being infinite accounts for being completely at rest. This is Anaxagoras' mistake. Being at rest pertains to a body's *natural place*, a body's finitude. Take the whole earth for example. It is at rest (μονή) at the center of the universe. But this is not because it is infinite or because it is incapable of changing place. It remains at the center of the cosmos because earth is heavy by nature, and whatever is heavy remains at the center (*Phy.* III.5, 205b14-18). In other words, the whole earth remains where it is because of the *nature* (φύσις) of the earth. Even if the earth were infinite, such would be ancillary to being at rest at the center of the universe because the *cause* of the earth's location is its nature (*Phy.* III.5, 205b11-14). Whether infinite or finite, the earth would move to and rest at the center because of its nature. Fire supports itself at the edge of the universe and earth remains at the center because these are their natural places. The same applies to water and air. Furthermore, this does not entail that the elements are motionless; in fact, natural place is part what accounts for a body's capacity to change locations. Since a part occupies the same kind of natural place as the whole, wherever the whole remains at rest, so too will its parts. Were the whole earth or all of fire incapable of motion, all its parts would be just as motionless. But, a

[144] *Ibid*.

spark and a clod of earth can be moved to other parts of the universe *by force* (βίᾳ), even without all of fire or earth moving along with it. Natural place determines how and where an entity's locomotion occurs, not whether something is infinite or finite.

Three more absurdities seem to follow from Anaxagoras' position, all reiterating earlier problems: (1) No weight will exist, for heaviness and lightness is determined by how much a body moves toward or away from the center of the universe; but nothing will be able to move toward or away from the center if the whole is infinite and fixed in itself. The infinite body would have to be divided in half at the center and have corresponding extremities for there to be any movement and from either location. But the extended *fixed* infinite has neither extremes nor a center. And according to Aristotle, there is certainly a center from which and toward which bodies move. Therefore, weight exists but the infinite body does not (*Phy.* III.205b25-31). (2) If the infinite body were fixed in itself, the corollary would be the existence of only one place—the in-itself of the infinite body—due to there being only one body. There are, however, many different regions occupied by different things. For Aristotle, both (1) and (2) are absurd because they do not accord with the *phenomena*. I suggest that this is the tenor of Aristotle's final remarks at 205b24-206a9. (3) As *perceptible* bodies (σῶμα αἰσθητόν), their phenomenality attests to the fact that bodies move to and from *definite* locations determined by their natures, even if some bodies are hindered from their natural place. It is impossible for there to be an infinite body, if every body *qua* perceptible has a specific natural place. The *phenomenality* of place and motion is incompatible with an infinite body. From our experience, physical bodies exist with places that determine how and where they move (205a24-31).

The finitude of the cosmos is because things have *natures*. Natures pertain to determinate individuals with specific characteristics. One of the defining characteristics of the cosmos is

having a center and a circumference. This is part of the essential finitude of the cosmos. An indication of this is quantity itself (τὸ πόσον), as Aristotle argues in the closing remarks at *Phy.* III.5, 206a1-7. *Any* quantitative determination pertains to a concrete, physical magnitude that itself has limits. The category of quantity indicates the essential finitude of the physical universe. Quantity always refers to a finite amount, such as two or three cubits (δίπηχυ ἢ τρίπηχυ, 206a4). Our ability to discover standards of measurement requires that the physical world be *inherently* divisible, which already assumes *determinate* magnitudes. And so it seems that the infinite physical magnitude is not even capable of being a quantity. For Aristotle, to be extended (διαστάσις) is to have a definite magnitude in multiple directions and dimensions, and this is part of what it means to be somewhere. Extendedness requires physical limits, since not everything is in the same location. Since there is no void, everything then has a determinate magnitude because it is limited by where it is, either naturally or by force.

§3.3. Guiding problem for *Phy.* III.6-8: What is the infinite if not a substance or an attribute?

Before turning to the positive account of the infinite in *Phy.* III.6-8, let us take stock of Aristotle's objections to an actual infinite. The general objection Aristotle has leveled throughout III.5 is that there is no actual infinite body either substantially or attributively. The reason is that none of the categories admit of an actual infinite body. The infinite is neither an actual substance, quality, quantity, or relation. When addressing the Platonists and the Pythagoreans at the outset of III.5, Aristotle rules out the first definition of the infinite immediately because no motion or place is possible if there is an infinite immaterial substance in the way that a point exists—*indivisibly*. But for there to be motion, there must be multiple places, which requires a magnitude to be divisible into different places. As a result, Aristotle shows that the infinite cannot be an

immaterial substance (*cf. Meta.* K.10, 1066b2-7). But that's not even the worst of it! No infinite physical substance exists because neither the whole body nor its parts are infinite in power or extent. Furthermore, were there an infinite perceptible body, at least one of the other accidental categories would admit an actual infinite. In other words, the infinite would be an actual attribute of a body. But, inasmuch as no physical substance is actuality infinite, none of the remaining categories of being may admit an actual infinity either. And if none of the categories admit an actual infinite, then it seems that the infinite cannot be an attribute, since the accidental categories are attributed of the substance. For the moment, it is worth quickly reiterating how none of the accidental categories admit of an infinite.

Not only is it impossible for an actual infinite to manifest quantitatively (204b4-10; 206a2-9), but also qualitatively with respect to powers and affectivity (204b12-22). If there were an actual infinite body, then the elements out of which it would be generated must also be infinite in power (δύναμις), e.g. infinitely warm, cold, large, small, etc. But, if any one element were infinite in power, it would destroy the others. An infinite power does away with contrariety (τἀναντία, 204b13). The power of fire is to heat and that of air is to cool. But, if the air itself were infinite whereas the fire were finite, then nothing would ever be warmed since the power of the air is proportional to the amount of air itself; thus, the air would always overtake all the other powers in other things (204b14-19). However, the air is indeed warmed by fire—the sun. So, an infinite physical body is not possible seeing that it undermines qualitative change by doing away with contrariety insofar as the power of the quality must be proportional to the body. That infinity does away with contrariety in qualities, in turn, leads to the impossibility of an infinite relation. In the *Categories*, Aristotle is clear that for any relation, the relation is known in virtue of the relata. Since the relata are finite, so too is the relation (*Cat*. 7, 8a37-8b3). If there were an

infinite physical body, however, such as fire or air, then since these are related by their contrary attributes, their relation would also be infinite. This, however, undermines the possibility of anything being cooled or warmed, as there would be no distinction between them *qua* infinite. If both cold and warmth are infinite, and if *many* infinites are absurd (204a26), then there is really no cold or warmth into which something changes because there is no *difference* (204b27-30). There is no distinct relata to be related, if the relation is infinite precisely because there is no relation at all! The relata must be *definite* for there to be a relation. So, there is no actual infinite physical body because its infiniteness would destroy the relationality that is requisite for contrariety.

Aristotle shows that there is no actually infinite physical body most interestingly in the way he undercuts the possibility of an actually infinite quantity. In general, for any quantity, there is a specific amount. Aristotle appeals to *place* (τόπος) as a way to discount an infinite quantity. As a *continuous* thing, place is a quantity. For any natural thing, it has a proper place (205a10-12). Earth belongs at the center and fire in the heavens. Thus, there is a specific place for each natural thing. If their places are infinite, however, then wherever they are located will be the same; all places will be entirely homogeneous, ultimately indistinguishable from one another because there will be no *definite* place in contrast to another. But, this means there will be no distinction between up/down, left/right, front/back—no contrariety (205a14-16; 205b32-35). Thus, if place is infinitely homogeneous, there is no place. Thus, there is no motion. But, it is clearly possible to throw a rock upwards and have it fall down. It moves upward because it is forced out of its natural place, and falls downward because the rock seeks to be at the center. That is, there *is* motion. Thus, place cannot be infinite homogeneously. An infinite place homogenizes all places.

What if, however, place were infinitely heterogeneous? This would require that the universe (τὸ πᾶν, 205a20) be infinitely divided *actually* into infinitely many parts, since each of the parts would be a specific body that individually marks off a particular place. But, if the universe were entirely heterogeneous, some bodies would be infinite and *others* finite; fire would be infinite and air finite, or vice versa. Once again, this only destroys the contrariety entailed by fire and air by reintroducing a qualitative and relational infinite, which has been shown to be absurd (204b14-19); the infinity of the fire would overtake the finitude of the air. But the qualities of the elements, their powers, are finite, and the relationship between them is also finite precisely because relation requires *determinate* relata. On top of this, given that a physical body is a determinate magnitude, the place it occupies must be just as determinate as the body, since place is denoted by the boundaries of a thing. Thus, there is no infinite body precisely because an *actual* place, as a quantity, is not infinite.

For all its diversity, Aristotelian ontology is rather simple. There are only two ways to exist—substantially and attributively. *Phy.* III.4 implied dialectically that the infinite seems to be both. But *Phy.* III.5 has shown us that it is neither. There is no infinite physical body because none the categories admit an actual infinite. By showing that there is no actual instance of the infinite in any category, and because an actual thing exists categorially, there is no actual infinite, even *attributively*. What this means is that the infinite seems to be neither an actual substance nor an actual attribute. However, Aristotle now has a serious problem, one that he recognizes at the outset of *Phy.* III.6: if there is no infinite *at all*, then time will have a beginning and end, magnitudes will be divisible into a smallest point, and number will be exhausted. But none of these things are true for Aristotle. Furthermore, we have good reasons from *Phy.* III.4 to believe that it in fact exists: time (203b16-17), divisibility (203b18-20), limit (203b20-22), and physical

magnitudes (203b22-20). But, if to *be* is to be actual, and yet none of the categories admit of an actual infinite even attributively, how can Aristotle preserve time, magnitude, and number by appealing to the infinite? How can the infinite be attributed to any of these things, if its attribution is called into question the finitude of categorial being? How does the infinite exist at all, if neither as an actual substance nor an actual attribute of a body? To solve this problem, in *Phy.* III. 6-8, Aristotle appeals to *his* understanding of motion as an active potentiality.

CHAPTER 4

THE POSITIVE ACCOUNT OF THE INFINITE (*PHY.* III.6-8)

§4.1. Aristotelian actual infinity (*Phy.* III.6)

As we have seen from *Phy.* III.5, the infinite is neither an actual substance nor an actual attribute. This leaves Aristotle with a significant problem at the outset of *Phy.* III.6. The infinite needs to exist since it is a necessary condition of time, magnitudes, and number:

> Now, it is clear that if the infinite simply does not exist [μὴ ἔστιν ἄπειρον ἁπλῶς], then multiple impossibilities follow. For time will have a specific beginning and point of cessation [τις ἀρχὴ καὶ τελεθτή], magnitudes will be not divisible into further magnitudes, and number will not be infinite. But, whenever this distinction has been made and neither appears possible [φαίνηται ἐνδέχεσθαι], it is necessary to make further clarification [διαιτητοῦ δεῖ] such that it is clear that there is a way in which the infinite exists and another in which it does not (*Phy.* III.6, 206a9-14).

The 'distinction' (διωρισμένων) Aristotle references here pertains to the infinite simply not existing at all (μὴ…ἁπλῶς, 206a9) and the infinite existing in a qualified sense. It is not the case that there is no infinite whatsoever. For, were there no infinite, time would have a beginning and end, magnitudes would be indivisible, and number would be finite. But, time is infinite insofar as time has neither a beginning nor an end, and is thus without a limit; magnitudes are continuous

quantities (which by definition are infinity divisible); and numbers may be added *ad infinitum*. So, the infinite must exist in some qualified sense. But in what qualified way does the infinite exist, if neither as an actual substance nor as an actual attribute, especially if *only* substances and their attributes populate the entire landscape of Aristotelian ontology? This is why 'further clarification' is needed (διαιτητοῦ δεῖ).

One of the major components of the standard reading of *Phy.* III.6-8 is that because Aristotle refutes the existence of an actual infinite body in *Phy.* III.5, this rules out any actuality of the infinite whatsoever.[145] Accordingly, the infinite must exist only potentially. Now, the infinite cannot even be potentially a substance, because were this the case, at some point it would be actualized into physical, extended body, given an infinite amount of time. Furthermore, were it potentially a substance, there would already have been an actual substance from which it was generated, for the actual is prior to the potential, just as an actual human being precedes and is that from which a potential human is generated. So, according to the standard reading, it must be some sort of potential attribute. The evidence for this is that physical magnitude has the capacity to be divided indefinitely. The infinite is an attribute of the physical magnitude *only* potentially; it can never be fully actualized even as an attribute. It is impossible to make all infinite divisions.

The standard reading is correct that the infinite is indeed a potential. But infinity cannot be *simply* potential—it must also be actual in some measure. For, the scope of actuality and potentially includes things that are both actual and potential at the same time, such as incomplete activities like motion, as we have learned from *Phy.* III.1-2. The problem with the reading that the infinite exists only potentially is that it assumes that actuality pertains only to completely formed finite physical bodies. It assumes that for the infinite to be actual, the magnitude would

[145] See §1.2.2 above for a full discussion of this literature.

have to be completely divided infinitely. It solves the problem of how the infinite exists as an attribute by assuming that the infinite can be either only actually or only potentially. While Aristotle has indeed shown that there is no actual infinite substance or actual attribute, Aristotle has *only* undermined the possibility of a *completely infinite body,* either as an infinitely extended substance or something with infinitely many actual parts.[146] But, he has not ruled out any actuality of the infinite whatsoever. This is because incomplete actualities exist! So, I suggest we think of the problem differently.

Here, at the intersection of Aristotle's refutation of an actual infinite body in *Phy.* III.5 and the demand for the existence of the infinite in *Phy.* III.6, Aristotle is left with the key problem which the account of the infinite has been anticipating: The infinite must be actual somehow because it exists. But, it seems to be neither an actual substance nor an actual attribute. What then is its actuality, if there seem not to be any other options besides a substance or attribute? This is the problem with which Aristotle wrestles in *Phy.* III.6-8. In this chapter I will argue that on one hand, the infinite needs to exist in such a way that its potentiality is never completely actualized. The impossibility of a fully actual infinite was, after all, the whole point of the argument of *Phy.* III.5. On the other hand, though, the infinite cannot exist *solely* as a potential, since this would entail a pure potentiality—the existence of which is unlikely in any respect, which I will address below. Since the infinite can be neither completely actual nor purely potential, I will argue that infinity is both actual and potential much like motion—an

[146] Massie, *op. cit.* p. 579 argues similarly that "[w]hat Aristotle rejects here [at *Phy.* III.6, 206a21-25] is not at all the concept of actual infinity *simpliciter*, but the idea that infinity could be actual in the sense of something simultaneously given as a whole: that is, that infinity could be something *complete, achieved and separate*. The so-called Aristotelian rejection of actual infinity in Book 3 of the *Physics* concerns, in fact, the *impossibility of an actually infinite body (a body that would infinitely add something to itself)*. This, Aristotle argues, is never what a body is. But it does not follow that infinity can only be in potency and never in act," emphasis mine.

active potentiality. When Aristotle argues that the infinite is both actual and potential like the day and Olympic games (*Phy.* III.6, 206a18-25), I will also argue that Aristotle is proximately attributing infinity to certain types of motion that are actual *insofar as* they are always incomplete. Like with motion, Aristotle is pointing to what I call the infinite's complete incompleteness. But since not all motions are the same as we learned from *Phy.* III.1, which type of motion will be the most demonstrative of the infinite in this way? Since the infinite is a sort of quantity, I will argue that the infinite is most manifest as a proximate attribute of a particular sort of quantitative motion—the act of dividing and adding. I will lastly show that this is possible only because of what physical substance is—a being capable of motion. The infinite exists as a proximate attribute of the activity of division because it exists as a *per se* attribute of the moving substance insofar as the substance is the subject of the division and addition.

§4.1.1. The active potentiality of the infinite (*Phy.* III.6, 206a14-206b3)

Aristotle begins his account with a typical distinction: being is meant in two ways (206b14-15), actually and potentially. Aristotle then claims that the infinite exists in two additional ways, by division (διαιρέσει) and by addition (προσθέσει). The two types of infinity, because they are ways of being, are therefore either potential or actual:

> Now, being is said with respect to what is potentially and to what is completely actual; additionally, on one hand the infinite exists by addition and on the other by division. It has been said that there is no actually infinite magnitude, but there is infinite by division. For it is not difficult to refute the existence of indivisible lines. What remains, then, is the infinite as a potentiality (*Phy.* III.6, 206a14-18).

Here, Aristotle briefly outlines the positive account of the infinite in *Phy.* III.6. The infinite exists potentially with respect to division. But 'potentially' in what way? Aristotle first addresses how the infinite exists potentially as something like motion, which is an active potentiality or an incomplete actuality (206a14-206b3). Following this, Aristotle gives an account of how this infinite exists by division and addition (206b3-206b33). In this section, I will analyze Aristotle's first set of arguments. Then in the next section I will analyze the arguments to show how Aristotle accounts for this sort of actuality of the infinite as it pertains to division and addition.

Because there is no actually infinite physical body, no extended object (i.e. body, plane, or line) is actually and completely infinite. The corollary to this is that there is no actually infinite number of existing things; for if there were an actual infinite physical body, then it would have an actual infinite number of parts, just as the atomists argued. As Democritus and Anaxagoras argued, the parts are prior to the whole because actual bodies are composed of indivisible atoms. Indivisible atoms *compose* divisible bodies. So the infinite body of the universe would be composed of infinitely many parts. But, as we know from *Phy.* III.5, nothing has an infinite number of actually existing parts. As a result, it *seems* then that infinite by addition is impossible *a fortiori* because the sum total of things in the universe including their parts will always be finite since the whole universe is finite. (Granted that Aristotle does not explicitly draw this conclusion here, but it seems implied at least in respect to the sum of actually existing things. For if there is only a finite number of exiting things in the universe, then their parts will be finite as well, since parts will never exceed the whole. Nevertheless, he will return to infinite addition later in *Phy.* III.6 to show that it is possible *only* as the inverse of infinite by division.) But, we do know that magnitudes are infinitely divisible by nature because they are continuous quantities (*Cat.* 6, 4b4-6). As continuous, the magnitude has the *capacity* for an

infinite number of points which are generated by division. So, Aristotle argues, the infinite must exist potentially by division.

However, Aristotle does not end the argument here. In fact, there is a problem: in what way is the infinite potential by division? In one respect, potentiality pertains to the proximate material that takes on its corresponding form, just as how bronze is able to become a statue:

> But it is not necessary to take potentiality as if something were potentially a statue, since this *will* [ἔσται] be a statue, and in this way an infinite *will* be something which is an actuality [οὕτω καὶ ἄπειρον ὃ ἔσται ἐνεργείᾳ] (*Phy.* III.6, 206a18-21).

Call this a *telic potentiality*. A telic potentiality is understood with respect to the *actualizable telos* of the motion. This type of potential is *defined* with respect to a specific goal that can be fully actualized. For it will be actualized after a finite number of steps in a finite amount of time in the process of producing a new form. Aristotle's use of the future active indicative ἔσται is also important. The bronze *will be* a statue at the end of the forging activity. Bronze eventually becomes a statue because bronze statues do in fact exist. The latter is ontologically prior to the former, even though the former is temporally prior. The reason for this is that it is the nature of this kind of potential to be fully actualized since it is defined by an actually existing goal. It is the one of the natures of bronze to be proximate material for *actual* statues. Proximate material *will* become its corresponding form once it is set in motion, unless the process is impeded. The same can be said for locomotion or alteration. My capacity to walk from my desk to the kitchen is a potential that can and will be realized once the motion begins just as much as a student who has the potential to learn calculus will eventually learn it. The reason is that a telic potentiality is

defined with respect to an actual form that can be fully actualized. Having the capacity to walk and the capacity for mathematical knowledge are proximate materials which have specific goals that can and will be realized in a finite amount of time (assuming no impediments).

Infinite by division is not a potential in this sense. Infinite division will never be completed insofar as 'completion' means the exhaustion of the potential. For one reason, unlike the bronze statue, no fully actual infinite body exists as prior. Now, because of this, we could simply infer that since the infinite does not have the potential to be fully actualized, it is *only* a potential and leave it at that. That is to say, the infinite is always and only a potential. Nevertheless, there is a significant problem with this. As only a potentiality, the infinite would be, in effect, a *pure* potentiality.[147] However, were the infinite a pure potentiality, ironically enough, it would commit Aristotle to the existence of a formless entity. For, suppose that the infinite were a pure potential and assume that it exists. The infinite would then be pure material, for pure potentiality is without any actuality and that which lacks actuality is formless.[148] Thus, a formless entity would exist. But all existing things, even *qua* material, have forms, e.g. the form of bronze is its ratio of elements. So, the infinite cannot be only potentially. But for the sake of

[147] The existence of Aristotelian pure potentialities is widely debated. My view is that Aristotle entertains it only counterfactually and that all potentialities in Aristotle have some measure of actual determination one way or another because actuality is always ontologically prior to potentiality. Frans A.J. de Haas, *John Philoponus' New Definition of Prime Matter: Aspects of its Background in Neoplatonism and the Ancient Commetary Tradition* (Leiden: Brill, 1997), p. 72 points out that for Aristotle, material (and by association potentiality) is always with respect to something 'out of which' something is generated. That is to say, material/potentiality is always connected to a specific, actual thing, which exists prior to the potential. *Cf.* Heinz Happ, *Hyle: Studien zum aristotelischen Materie-Begriff* (Berlin: Walter de Gruyter, 1971), pp. 302-305 on Aristolte's discussion of *prima materia* in *GC* II.2 as "nicht…als einen Substanzteil isolieren" whose ontological status is "den des ungetrennten Möglichseins." See also Heinz, pp. 569-581. *Cf.* Christopher Byrne, "Prime Matter and Actuality," *Journal of the History of Philosophy* 33 (2), pp. 197-224 for a nice corrective to the traditional reading of prime matter as a pure undetermined potential.

[148] On the identity of prime material and pure potentiality, see Byrne, *op. cit.,* pp. 198-203.

the argument, suppose that the form of the infinite were materiality itself such that the form of pure potentiality were *ipso facto* its formlessness. Then it would be either immaterial or material. An example of this is Plato's Indefinite Dyad or Anaximander's ἄπειρον, respectively. For Aristotle, were it immaterial, it would not be a potential at all since only physical things have potentiality. What if a pure potential infinite were instead a material entity, such as Anaximander's ἄπειρον? Pure potentiality, as a material thing, is essentially an unbounded physical body. But, the infinite would again be an actual physical body, which we know is absurd. So, were the infinite *only* a potentiality, Aristotle would be committed to the same positions as some of his predecessors, the very ones he goes to great lengths to reject in *Phy.* III.5, such as Plato's Indefinite Dyad or Anaximander's ἄπειρον, each of which is a separate, limitless actual body. They are, in effect, pure potentialities by virtue of the fact that they are sorts of undifferentiated 'reservoirs' into which and out of which finite bodies are generated and destroyed.

How, then, does the infinite exist as a potential? Remember that potentiality is meant in as many ways as being (206a21).[149] One sense is the kind of potential we find in motion—an active potentiality. This is the very sense we see in Aristotle's analogy to the day and the Olympic games:

> What remains, then, is the infinite as a potentiality. But, it is not necessary to take 'potentiality' just as if something were potentially a statue, since what is potentially a statue *will be* a statue, and in this way the infinite *will be* in actuality. But since being is said in many ways, just as the day and the Olympic games come to be as always

[149] *Cf. Phy*. VIII.4, 255a30-255b13 and *Meta*. Δ.12.

different, in this way the infinite exists [potentially]. (For with respect to these things, there is something that is *both* potentially *and* actually, for the Olympic games exist in the sense that the games are *both* capable of occurring *and* that they *are* occurring) (*Phy.* 206a18-25).

The infinite exists with respect to something that is always changing (τῷ ἀεὶ ἄλλο καὶ ἄλλο γίγνεσθαι) such as the day and the games. But what does this mean? I suggest that the infinite is an attribute of motion insofar as motion is an incomplete actuality. Since the infinite is neither *completely* actual nor *completely* potential, and since the infinite does in fact exist, it must be something that is both *incompletely* actual and *incompletely* potential. What kind of thing is this? It is something like an active potentiality—motion.[150] However, it cannot be a motion that actually comes to an end such as the generation of the statue. It must be a motion that itself always remains incomplete. This is the significance of the day and the games. For, unlike the statue, the day and the games are *atelic potentialities*, which are actual *by virtue* of their incompleteness; they are *goalless* activities.[151] In other words, as opposed to the telic potentiality of the bronze or almost any other material, an atelic potentiality has the nature to be actualized *always and only* incompletely. In a way, this dissertation is a sort of atelic potentiality since there will always be something more to say about its subject matter. The dissertation is not fully actualized at its submission or even after its defense. For its actuality is precisely of the nature

[150] Simplicius, *In Phy.* 497.13-19 also explicitly connects the concomitance of actuality and potentiality in the infinite to the definition of motion. *Cf.* Bowin, *op. cit.* p. 247. Hintikka, "Aristotelian Infinity," pp. 199-202 appeals to both time and what he thinks is Aristotle's 'principle of plenitude' to make sense of the analogy to the day and the games. See also Oskar Becker, *Grösse und Grenze der Mathematischen Denkweise* (Freiburg: Verlag Karl Alber, 1959), pp. 83 for the connection between this analogy and time.

[151] *Cf.* Bowin, *op. cit.,* p. 241 for a discussion of *goalless* activities, which I call *atelic potentialities.*

that more writing and information can be added to it or refined. This is like the day and the games. They are actual and potential *at the same time* insofar as they pertain to something that is "always taken one after the other" (τῷ ἀεὶ ἄλλο καὶ ἄλλο λαμβάνεσθαι, 206a27-28).[152] For, the day is fully what it is *insofar as* there is always more of the day left to happen. The Olympic games, as they are happening, are fully manifest to the extent that there are still more events left to complete. While it is true that the day eventually comes to an end when the sun passes below the horizon or when it has run its twenty-four hour circuit (depending on how one interprets the extent of 'the day'), and that the Olympic games culminate in the closing ceremonies, the being-at-the-end of either activity is not their actualities. In other words, they are *completely incomplete* as activities.[153] For as Massie argues,

> [a]t any moment, the day has already started and yet, it still remains ahead of us. Similarly the ἀγών (the season of the Olympic gatherings) goes on for days and days. Yet, this going on has nothing to do with a succession of identical moments or a numerical series; the games always bring something new; as long as they go on, the outcome remains undecided. The day and the games are in potency inasmuch as they do not form a completed whole, but they are also in as much [sic] as they are ongoing processes. Of these phenomena we must say that as long as they are *actual,* they are

[152] John J, Cleary, *Aristotle and Mathematics: Aporetic Method in Cosmology and Metaphysics*, (Leiden: E.J. Brill, 1995), p. 83 nicely points out that, from what Aristotle says at *Phy.* III.5, 206a27ff., this is "a general account of the mode of being of the infinite in terms of the *continual taking* (τῷ λαμβάνεσθαι) of one thing after another, with each thing that is taken being finite but *always different*," (emphasis mine).

[153] I thank Elizabeth Brient for her assistance in working out this interpretation.

> precisely not concluded. The day that embraces the now of our existence and the season of the games always contain some unfinished business.[154]

The day is not fully actual when the clock strikes midnight, nor are the games complete when the last race is run. Rather the day is fully actual at every moment there is more left to the day and the games are at their fullest when there are more games left to be played. This is because when the day turns to night or when the games cease, there is nothing left to be taken of them. But the actuality of the day and the games is precisely such that *more* is always possible at every actual moment! The essential nature of the infinite is that there is always something more to be taken, which coincides with Aristotle's 4th and 5th definitions of the infinite (*Phy.* III.4, 204a5-7). In short, the infinite is attributed to a certain motion that never exhausts the potential, or an activity that is fully manifest in its incompleteness. For, the infinite requires the constant *preservation* of the potential. But, if the potential within the motion is constantly preserved in order for the infinite to exist, then the motion to which the infinite belongs also can never come to an end. Aristotle's point, here, is that the infinite is both actual and potential because its nature is to belong to an activity which is completely incomplete, which seems to be motion, or at least a particular kind of motion.

To support this further, we can look to *Phy.* V.4, where Aristotle argues for this 'complete incompleteness' of motion with respect to the continuity of the thing in motion:

> Additionally, completeness is said to be one, either generically, in species, or according to its substance, and just as with other things completeness and wholeness pertain to

[154] Massie, *op. cit.,* p. 579.

> unity. But, there is another sense in which what is incomplete is said to be one *insofar as it is continuous* [ἔστι δ' ὅτε κἂν ἀτελὴς ᾖ μία λέγεται, ἐὰν μόνον ᾖ συνεχής] (*Phy.* V, 228b11-15).

I contend that this is a reference to what I have called the 'complete incompleteness' of motion. There is something curious about what Aristotle says briefly after this discussion of differences in continuity. Motion is continuous because motion is in or at least follows along a magnitude, and magnitudes are necessarily continuous. Motion's continuity ensures its completeness, and yet it is also incomplete by nature. How can motion be both complete and incomplete? Even more problematically, according to Aristotle, continuity is to show how motion is one in its very incompleteness. In other words, the completeness of continuity is to *account for* motion's incompleteness; motion is 'completely incomplete' due to continuity.[155] Now, what sense does this make? Let me suggest that Aristotle is serious about assigning a kind of unity to motion that actually gives motion's incompleteness intelligibility, which is the very intelligibility which Aristotle admitted was a difficulty in III.2. Motion is always the one motion, since it is the same throughout the activity. Walking does not change when I am walking because the walking is continuous in the activity, holding itself together along the way to some end. In this way, the walking is complete. It is *one* activity. But, walking is actual only insofar as I am not yet at the new place. In this way, walking is incomplete. As such, precisely by holding itself together in one activity towards some end not yet attained, motion is complete in its incompleteness. Were it not for continuity, there would be no motion, for there would be no way for motion to be *one* activity while also being incomplete.

[155] *Cf.* Bowin, *op. cit.* p. 247.

At this point, it is crucial to consider how the day and the games *qua* motions help us understand the ontological status of the infinite as an attribute. Again, the infinite cannot be a substance; were it a substance, it would come to be as a finite physical body just like the statue. The infinite would have a determinate goal that *will be* actualized at the end of a process. But this is absurd, because the infinite is not a substance like a human, horse, or a statue precisely because it is not the sort of entity that is determined by a form that *will be* actualized at some point in the generative process. Thus, the infinite cannot be something that becomes a substance. So, if not a substance, then it must be an attribute. But, it cannot be like *any* attribute, for attributes *also* reach a point of completion. The reason for this, as we have seen in *Phy.* III.1 and again in III.5, is because motion is always between *determinate* contraries. More importantly, in *Phy.* III.5, Aristotle has shown that the infinite cannot be an attribute in the sense of the actual sum of existing parts, contra Democritus and Anaxagoras. But, since infinity does have an actuality, and since it is not a substance in *any case*, it must be some sort of actual attribute. How is this possible? So far we know that the infinite has a similar character of motion as an active potentiality; motion, with respect to itself, is indefinite. But the infinite is certainly not the same thing as motion because motion is not understood primarily with respect to itself. For the most part, motions are determined primarily by their *goals* at which they terminate. Even though motion is different in each category, each comes to an end eventually, even the heavenly motions. The heavenly motions are finite in magnitude, even though their circuits may be repeated indefinitely, since every beginning point is also the end. Therefore, while the infinite is like motion, it is not the same as motion. So, on one hand, motions are finite because they are defined with respect to their respective goals. On the other hand, motion is indefinite with respect to itself because as motion is happening there is more left to the activity. This, I believe,

is where the infinite is actualized as an *attribute* of motion. For, the infinite is actual only with respect to the motion as the motion is occurring. The actual infinite is an attribute of *something incompletely actual*, and it is from this that the infinite derives its actuality.

But, how is this possible when motions do in fact come to an end? So far, it sounds as if the infinite applies to all motions in the same way. But, this is not the case. Even though all motions seem to be indefinite *insofar as* the motion is occurring, they eventually come to an end. Aristotle needs a specific motion that *never* ceases, or a motion that *always* maintains its indefinite character. There is one type of motion that does not come to an end because its *nature* does not have a goal at which the motion terminates—the quantitative acts of division (δίαιρεσις). Division is a quantitative change within the magnitude because it pertains to *decrease* or *reduction* of the magnitude, and 'decrease' belongs to the category of quantity. While most motions come to an end by nature, division is one motion that *by nature* does not come to an end because indivisible quanta do not exist. Remember that Aristotle claimed earlier that the infinite is potential by division. While Aristotle had to clarify how the infinite is potentially, he did not elaborate on this explicitly with respect to division. At *Phy.* III.6, 206a25-206b3, he returns to this issue. Surprisingly, the infinite falls under the category of quantity, since it is defined as an amount 'outside of which there is always something more.' How is this possible, when in *Phy.* III.5, Aristotle has shown that quantity does not admit of an actual infinite? Even though Aristotle has shown that quantity does not admit of an actual infinite, remember that it was with respect to the *size* and *number* of bodies. Aristotle's objection to a quantitative infinite pertains to a body being extended infinitely beyond the limits of the universe, as well as an infinite number of parts and wholes. However, this is only in the sense of *increase*; Aristotle's objection to a quantitative infinite pertained to the character of a physical

body as a fundamentally bounded entity. But, this leaves room for the infinite to exist quantitatively with respect to the *act* of division *in the direction of decreasing* the magnitude, since physical bodies are fundamentally divisible.

However, before Aristotle argues for infinity by division in the direction of decrease (206b3-33), he makes an important distinction between the infinite in time and the species in contrast to infinite divisions in the magnitude because he also needs to account for infinite by addition (προσθέσις) by way of division:

> Clearly, the infinite in time and humans is different from the infinite in the divisions of magnitudes. For generally the infinite is in the way: by always taking one thing after another, where what is taken is always finite, but always different. [Yet, being is said in many ways. So it is not necessary to grasp the infinite as a *this*, such as a human or a house, but as the day and the games are said, whose being is not as a specific substance that has already been generated, but as always coming to be and being destroyed, each being finite, but always different.][156] But [when this happens] in magnitudes, what has been taken remains, but for time and humans, that which is destroyed does not (*Phy.* III.6, 206a25-206b3).

Aristotle is distinguishing between motions that do and do not exemplify the active potentiality of the infinite. To be sure, this is not to say that the infinite does not pertain to time or to generation and destruction. These are infinite insofar as there is always something that comes

[156] See Ross, *Aristotle's* Physics, pp. 555-556 for the MSS history for why 206a29-206b3 are likely an alternate version of the argument that was later incorporated into the MSS. Ross suggests that it is likely a scholion, but Simplicius, *In Phy.* 495.6-17 argues that Aristotle might have added it later.

into being after another has been destroyed. Both time and generation are indeed infinite, but only by *similarity* to the division of a magnitude. Aristotle seems to think they are not properly infinite. Why is this the case? Time and human beings are infinite insofar as both instants (τὸ νύν) and individual humans are generated without beginning and end. Time eternally follows a specific substance's locomotion—the motion of the sun and heavenly spheres—and human beings are generated substantially one after another. The problem is that each instant is destroyed the moment another follows after it and human beings eventually perish after others are generated.[157] The reason that this is important is that infinite by addition is impossible with time and human beings precisely because that which is taken—i.e. counted—does not persist.[158] It is impossible to add instants or things *ad infinitum* because of their impermanence; there is always a finite amount of human beings at any given moment and instants exist only one at a time. However, divisions within a magnitude do in fact persist (at least theoretically). The magnitude

[157] Bowin, *op. cit.,* p. 239 argues that the problem Aristotle is addressing here is really a red-herring to Aristotle's attempt to connect infinity with motion because generation and time concern "a problem of infinite *pre*cession rather than infinite *suc*cession." In other words, generation and time fail to be actually infinite because the things and instants which precede those that come into being do not persist. And this is indeed part of Aristotle's point here. But this problem of persistence that Aristotle seems to be wrestling with is, Bowin thinks, largely irrelevant to the issue of *future* succession of things that could be counted or divided. But, what Bowin does not consider is how Aristotle is simply using the issue of persistence to rule out certain types of motions which do not adequately express the infinite as that in which something more can be taken. Only existing things can be counted or divided. Neither human beings nor instants persist long enough to be counted *along with* future beings to be counted. However, divisions in the magnitude actually persist along with possible future divisions, which is consistent with the infinite as both actual and potential. *Cf.* Bowin, *op. cit.,* pp. 239-241 and Hussey, *Aristotle's* Physics, pp. 81-82.

[158] Admittedly, Aristotle's following locution is ambiguous: 'by always taking one thing after another, where what is taken is always finite, but always different' (τῷ ἀεὶ ἄλλο καὶ ἄλλο λαμβάνεςθαι, καὶ τὸ λαμβανόμενον, ἀεὶ εἶναι πεπερασμένον, ἀλλ᾽ ἀεί γε ἕτερον καὶ ἕτερον, 206a27-29). Aristotle could be referring either to division or addition here. Aristotle does not exactly clarify here. I suggest that it pertains to both because it sets up the next arguments about the convertibility of division and addition, which requires divisions to persist for the conversion to work.

may be divided *ad infinitum* because of continuity. As a quantitative motion, the act of division does not destroy the previous divisions, for the magnitude *persists* as it undergoes division. We are able to count divisions (both actual and imagined) in a finite magnitude *ad infinitum* since divisions *persist* in a magnitude without the magnitude becoming greater than the actual universe. So, not only does Aristotle need to address infinite by division and its actuality, but also infinite by addition, since it seems to be possible always to count another number *ad infinitum.* To see this, let's turn to *Phy.* III.6, 206b3-206b33 where Aristotle shows how infinite by division and by addition are the same because they are convertible (ἀντεστραμμένως) in the same activity.

§4.1.2. Division, addition, and the finite cosmos (*Phy.* III.6, 206b3-33)

By this point, Aristotle has faced two hurdles: (1) to show that there is a sort of actuality that admits an infinite and (2) that this kind of an actuality does not require an infinitely extended body. The argument from the day and the games (206a17-25) begins to address the first problem. The infinite is an active potentiality because it is an attribute of motion. But not just any motion; the infinite needs to be an attribute of a quantitative change—division and addition. As we will see in this section, the reason for this is that the infinite is properly defined as a quantity, 'that outside of which there is always something *more*." To see this, Aristotle first needs to show how the act of division necessarily and actually happens in physical magnitudes. From there, he shows that infinite by division is convertible (ἀντεστραμμένως) into infinite by addition, since the divisions are prior to being counted. But even though the divisions are prior, this does not mean that the activity is complete, for as soon as the very first division begins, another division is *always* possible. The reason for this is that for every division, another line segment is made,

which in turn can be divided. In short, for every set of two points, another point can and *will* be found between them (given an infinite time). At *Phy.* III.6, 206b3-33, Aristotle will argue more specifically why the infinite properly belongs to the category of quantity with respect to division and addition.

As to the second problem, if there is an actual infinite in any way, Aristotle needs to preserve his finitism. Division and addition are infinite not by being or even becoming an extended body, but by being inherently incomplete activities *within* a finite magnitude by *reducing* (ἐπὶ τὴν καθαίρεσιν) a body into its parts and in turn counting those parts in the direction of increase (ἐπὶ τὴν αὔξην). In other words, in the same act of breaking down a finite body, one may add the parts of the magnitude by counting them.[159] An actual infinitely extended magnitude is not necessary. Through the convertibility of division and addition, therefore, Aristotle preserves the finitude of the cosmos while also allowing for an actual infinite *activity*. We must also keep in mind that even though the infinite will belong properly to division and addition as quantitative changes, because motions are analogous, changes in the other categories will be similar to the infinite in quantity since in each other type it is possible to find a sense in which there is always something more to be taken. For example, substantially, another individual can always be generated, qualitatively, the color spectrum is a sort of continuum and as such it can be 'divided' like a magnitude. For locomotion, the similarity will be with respect to the heavenly motions, since it is the only kind of change of place that is unending.

[159] For more in-depth analyses on Aristotle's discussion of the convertibility of division and addition as well as geometric and arithmetic sequences in the division of lines, see Oskar Becker, *Grösse und Grenze,* pp. 83-85; Thomas Health, *Mathematics in Aristotle* (Bristol, UK: Thoemmes Press, 1998), pp. 107-110; Philoponus, *In Phy.* 468.22-470.25; Ross, *Aristotle's* Physics, p. 557; Simplicius, *In Phy.* 497.11-498.31 and 504.1-509.21; and Wagner, *Aristoteles: Physikvorlesung,* pp. 524-525.

The infinite is, by definition, intraversable; it is that outside of which there is always something more. So if it is ever to manifest in a magnitude, the magnitude must never be traversed. Yet, every magnitude is traversable because magnitudes pertain to physical bodies, and all physical bodies are necessarily finite. So, how is the infinite by division possible in magnitudes when all magnitudes are traversable? It is by dividing magnitude into smaller and smaller segments according to the same proportion by which the original segment is first divided:

> The infinite by addition is in some way the same as that by division, for the latter is converted into addition. For that which is seen to be divided [ὁρᾶται] goes to infinity, in the same way as what is added appears [φανεῖται] with respect to what is limited. For if in the finite magnitude something limited is taken by the same proportion *in* what is already taken hold of [προσλαμβάνῃ τῷ αὐτῷ λογῷ], not completing the magnitude by the same proportion with respect to the whole, the finite magnitude is not traversed (*Phy.* III.6, 206b3-9).

This method of division is called 'division in the direction of reduction.' By taking the same proportion of each subsequently smaller segment, the divisions never exhaust magnitude. For, no magnitude may be traversed when subsequent parts of the whole magnitude are divided by the same proportion by which the whole was originally divided—i.e. a geometric sequence. This applies to both physical and theoretical magnitudes. Assume that the length of magnitude AB equal to 1. Bisect AB at C. The proportion is 1/2. Now, bisect CB at D. While CB stands to AB by ½, DB stands to AB by ¼. In order to reduce AB further to 1/8, we take not ¼ of DB, but ½

again, since each division is "taken by the same proportion in what is already taken hold of" (προσλαμβάνῃ τῷ αὐτῷ λογῷ). The reason for this is that it is not with respect to the whole magnitude AB the divisions are made, but to each subsequently smaller parts of the magnitude. In doing so, there is always a remainder of the whole left to be taken after every division, e.g. 1, 1/2, 1/4, 1/8, 1/16, 1/32, …, ∞.[160] Therefore, when division occurs geometrically with respect to a *segment* of the magnitude, but not the whole, the whole magnitude may be divided *ad infinitum* and never traversed.

What is important to notice here is that division is immediately convertible to addition insofar as the divisions may be counted at the same time divisions are made. For, as the number of countable units—the divisions—increases, the magnitude is reduced at the same time. The number of divisions increases only as the lengths of the segments within the finite magnitude decreases. The convertibility of infinite division to infinite addition is therefore inversely proportional. This makes sense only if the magnitude remains untraversed (οὐ διέξεισι τὸ πεπερασμένον) in the direction of reduction. To be sure, what is counted is always a finite amount, an ἀριθμός, whereas the counting as an activity is infinite because it is always incomplete. Every division is finite, but because each is also different, one may literally 'set down something with respect to another,' or *to add* (προστιθέναι). Nevertheless, the *activity* is infinite since it is always possible to make another division and thereby add or count another unit. Therefore, counting goes hand-in-hand with division. Division and addition are the same

[160] Ross, *Aristotle's* Physics, pp. 50-51; 556 suggests it is by adding the fractions of segments. While this is certainly possible, it does not capture how division and addition can be accomplished in the same activity as simply counting the points that are marked off by the divisions at the same time as making the divisions. Ross's suggestion requires two different procedures, whereas counting divisions as divisions are made are two aspects on the same activity.

(τὸ αὐτό) insofar as they are convertible in the *same activity*.[161] Infinite by addition may be converted from division immediately because the divisions actively being made in the magnitude may be counted at the same time the divisions are made.

What happens, however, if the proportion *increases* with respect to the whole? The magnitude is traversed: "But, if the proportion is increased in such a way as always to *constrain* an amount equal to the magnitude, the magnitude is traversed, since the whole magnitude is taken up by whatever part that has been limited" (206b9-12). In other words, the whole magnitude is traversed when the proportion of the divisions increase *arithmetically* with respect to the whole magnitude. This method of division is 'division in the direction of increase.' That is to say, the proportion increases by addition. The result is that the whole magnitude is used up. Let EF be a magnitude equal to 1. Quarter EF at G. EG is ¼ of EF. Now, instead of quartering EG, bisect EF at H. By adding EG and GH, this increases the total proportion to ½ of EF. Now bisect HF at I. EH plus HI increases the proportion to ¾ of EF. It is possible to take only the remaining quarter IF since this will increase the proportion to 1, which equals or 'constrains' (περιλαμβάνων) the whole magnitude EF. The proportion increases *with respect to the whole* because each division is added to the previous. In a way, Aristotle is talking about something *analogous* to locomotion. For, assume that the distance from my desk to my kitchen is 24ft. Assume also that my stride is about 3ft. I will traverse the distance in roughly 8 steps, since with each stride, the total distance traveled increases until I reach the kitchen. Aristotle's point is that this method of division does not admit an infinite, since eventually the whole universe would be traversed. Not only, then, is infinite by division in the direction of increase impossible, but also infinite by addition, since there will be only a finite number of divisions. Division in the

[161] See Massie, *op. cit.,* p. 578.

direction of increase will always run up against the limit of every magnitude. Without more of the magnitude to divide, there is nothing else left to add.

The significance of Aristotle's discussion of division, addition, and their convertibility is how they are *actual changes* that a magnitude undergoes. Division and addition are incomplete activities belonging to a magnitude in which the infinite manifests because they pertain to the *actualization* of the magnitude's potentiality to be divided without reaching a limit:

> In no other way, then, does the infinite exist except *both* potentially *and* by reducing [the magnitude] (and also in actuality [καὶ ἐντελεχείᾳ δὲ] just as we speak of the day and the games to be).[162] In this way, the infinite exists potentially like material, and not on account of itself in the way the finite exists. To be sure, infinite by addition also exists potentially in this way, which is in a certain way the same as by division. For something will always exist beyond what is taken, which will not exceed every magnitude, just like how in the direction of division something goes beyond what has been marked-off, which will always be a smaller part (*Phy.* III.6, 206b12-20).

At first glance, it would be easy to interpret Aristotle as arguing that the infinite exists only as a potential since the magnitude is infinite only by reduction and because the magnitude potentially contains an infinite number of points. But remember, the infinite is *both* actual *and* potential. On

[162] Ross, *Aristotle's* Physics*,* p. 556 provides a helpful way to read Aristotle's terse claim here: "The punctuation I have adopted seems on the whole most likely to represent the course of Aristotle's thought. He first makes a statement which reproduces what he has said in [206a16-18] that the infinite exists potentially, and by way of division, (i.e. as the infinitely divisible, not as the infinitely extended). He then remarks that parenthetically that (while it does not exist at any time as a given entity), *it does exist actually in the special sense that, when the division of a line is going on, a process which is in principle endless is being progressively actualized, as a day or a contest is progressively actualized. (cf. a21-5)*" emphasis mine.

one hand, the infinite is a potentiality like material, but on the other it is also actual or 'at-work' like the day and the games. *As soon as* the very *first* division is made, the infinite is fully actualized precisely because the activity will *always* remain incomplete, regardless of how long or how short the interval of time between further divisions. This is because the actuality of the infinite has everything to do with *lacking* an end to the activity. It is a *goalless* or *atelic* activity. Remember, the infinite is both actual and potential in a very special sense. The actuality of the infinite is the inherent incompleteness of the activity of division *in the direction of reduction.* For, just like the day and the games, the infinite is actual in the act of division because when the magnitude actively undergoes division and is being reduced, there is always something able to be divided (*cf. Meta.* Θ.6, 1048b14-17).[163] Because the act of division happens in physical magnitudes, which are continuous, the activity is inexhaustible in the direction of reduction.[164]

For a better understanding of division as a motion that continues indefinitely *within the magnitude,* it is crucial to recall from *Phy.* III.3, 202a13-14 that motion occurs only in what is moved and not in the mover. This is because the actuality of what is moved is incomplete, whereas the actuality of the mover is complete. Teaching is the complete actuality whereas learning is the incomplete actuality, or the motion, for teaching has knowledge but learning is the gaining of knowledge. The reason for this is that the form in the mover imparts the motion to what is moved as the efficient cause, and this form is completely actual. Therefore, as a complete actuality, the mover is not moving. However, the motion is in that to which the form is imparted—what is moved. The learner does not yet have complete knowledge. The act of division, as an actuality, must be understood in the same two senses: the complete actuality of

[163] At *Meta.* Θ.6, 1048b14-17, Aristotle makes clear that division is a 'never-ending activity' (τὸ μὴ ὑπολείπειν) that itself 'exhibits' (ἀποδίδωσι) an actuality which exists potentially—an active potentiality. *Cf.* Bowin, *op. cit.,* p. 241.
[164] *Cf.* Hussey, *Aristotle's* Physics, p. 84.

the divider imposing a division in the magnitude and the incomplete actuality of what is being divided, i.e. the magnitude undergoing division. Just like teaching, the divider is not changing; it is not being divided. Like the learner, the magnitude changes, since the act of division is the magnitude *qua* actively being reduced. In this way, the infinite is actualized in the act of dividing the magnitude, since *this* is the motion.[165] Of course, the key difference between learning and division is that the former has a definite *telos*—knowledge—whereas division does not. For, the very nature of division *in the direction of reduction* is not to have a *telos.*[166] The act of division, specifically to reduce the magnitude, is an *atelic potentiality.* The actuality of the infinite is the *manifestation* of the *magnitude's* inexhaustible capacity to be divided insofar as the *magnitude* is actively undergoing division.

At this point, it is crucial for Aristotle to show that the infinite's attribution to the acts of division does not conflict with the finitism of the universe. To do this, Aristotle draws from the account of convertibility. Unlike his predecessors, to account for the possibility of numerical infinity, one actually needs a thoroughly divisible but also finite universe. Unlike the actual infinite physical body (τὸ ἄπειρον ἐντελεχείᾳ σῶμα αἰσθητόν), infinite by division/addition does not exceed every possible physical magnitude because they are activities belonging to finite things. Divisions occur within determinate magnitudes, which are counted:

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[165] *Cf.* Clearly, *Aristotle and Mathematics,* p. 83 argues this by emphasizing the agency of the divider: "What is significant about the account [at 207a27ff] is its emphasis on the *activity of some agent* by means of which the potential infinite is *realized*. Since Aristotle holds (*Met.* 1049a11 ff.) that every genuine potentiality in nature must be realized at some time or another if nothing prevents it." While Cleary is indeed right to emphasize the role of the agent-divider, it is crucial to distinguish it from the line undergoing the division: the divider is not in motion while the line being divided is in motion. The infinite is being realized (though never to completion) in the latter.

[166] Bowin, *op. cit.,* pp. 240-241.

> To be sure, infinite by addition also exists potentially in this way, which is in a certain way the same as by division. For something will always exist beyond what is taken, which will not exceed every magnitude, just like how in the direction of division something goes beyond what has been marked-off, which will always be a smaller part.
>
> As such, it is not even possible in potentiality to exceed all things by addition, unless accidentally there happens to be an actual infinite [body], just as the physicists argue that the infinite is what is beyond the body of the cosmos, such as air or some other certain thing of that sort [ἄλλο τι τοιοῦτον] (*Phy.* III.6, 206b16-24).

Here, Aristotle builds off the convertibility of division and addition to show that contrary to the physicists (φυσιολόγοι), infinite by addition is possible not on account of a body existing beyond the cosmos (τὸ ἔξω σῶμα τοῦ κοσμοῦ, 206b23), but precisely because finite magnitudes are able to be divided endlessly because of their continuity. For, within a finite body 'there will always be something beyond which to take' (ἀεὶ...τι ἔξω ἔσται λαμβανείν, 206b17-18). By way of reduction, there is always more magnitude beyond each division. We would be remiss not to notice how Aristotle retains and modifies the notion of 'the beyond' (τὸ ἔξω). Infinite by addition is τὸ ἔξω because one must always be able to add one more number and this is possible only insofar as another division is able to be made *in the body*. For, there is always another possible division *beyond* the previous divisions *within* the finite body. In this way, Aristotle does not object to the physicists' assumption of τὸ ἔξω *in principle*. He agrees that for the infinite to exist, there must always be something beyond which to take, since the infinite pertains to intraversibility. The issue, however, is *where* τὸ ἔξω exists. Is 'the beyond' outside of the finite body or in the body? As Aristotle sees it, it is in the body. The physicists' notion of τὸ ἔξω is

misplaced. An actually infinite physical body existing beyond the finite body of the cosmos is unnecessary at best. Instead, in any finite magnitude, there is always something extra to take in the way of reduction. Aristotle preserves his finitism by placing 'the beyond' in the finite physical substance.

Interestingly enough Aristotle's infinite by division and addition also retains features of Plato's Indefinite Dyad. At 206b24-33, Aristotle's polemic against the Indefinite Dyad reveals its redeeming qualities. Because infinite division and addition convert inversely, proceeding in contrary directions, Aristotle believes he makes better use of Plato's "two infinites" (δύο τὰ ἄπειρια), the Great and the Small (τὸ μέγα καὶ τὸ μικρόν):[167]

> If there is no actual infinite physical body [that overtakes the limits of the cosmos], it is said that there is no potential infinite by addition, except as we said inversely with respect to division, since for Plato, it is because of *this* [i.e. the convertibility of division and addition] that he makes two infinites [δύο τὰ ἄπειρια]. For, in the direction of increase, it overtakes [every magnitude] and goes to infinity in the direction of reduction. Although he makes two infinites, he does not use them. For the infinite in the direction of reduction does not exist in number (for the monad is the smallest number) nor in the direction of increase (for he makes number only as big as the decad) (*Phy*. III.6, 206b24-33).

Infinity in the direction of increase is like Plato's Greatness (τὸ μέγα); in the direction of reduction, the infinite is like Smallness (τὸ μίκρον). For Aristotle, τὸ μέγα pertains to how there is no upper limit to number, since addition is always possible in the direction of increase. It is

[167] δύο τὰ ἄπειρια is Aristotle's reference to Plato's τὸ μέγα καὶ τὸ μικρόν. See *Phy.* III.4, 203a15-16. See also Ross, *Aristotle's* Physics, p. 557. *Cf.* Simplicius, *In Phy.* 499.1-500.2.

always possible to add one more number. However, this is possible only because of infinite division *in the direction of decrease,* which seems to be Aristotle's modification of τὸ μίκρον.[168] So, together, the convertibility of infinite division and addition is a sort of revision of the Great and the Small, but only insofar as division and addition occur within the finite body. This points to Plato's other mistake, which was to treat the Great and the Small as a separate substance; just like the physicists' τὸ ἔξω, the Great and the Small is really *in* the finite magnitude since the magnitude is continuous. Aristotle's additional complaint is that Plato does not use the dyad appropriately, since number for Plato (according to Aristotle), is actually *limited* both in the direction of increase and reduction, since for Plato number has a lower limit of one and an upper limit of ten.

Because infinite by division and addition are convertible, Aristotle does not jettison the ἔνδοξα entirely. Infinite division preserves the physicists' notion of τὸ ἔξω and the dyadic structure of Plato's τὸ μέγα καὶ τὸ μικρόν, since in the direction of decrease there is always *something beyond* which to divide and count. Because of the convertibility of division and addition, the existence of Aristotle's infinite avoids the pitfalls admitted by an infinite physical body or the Indefinite Dyad. Aristotle's infinite is an attribute of the *acts* of division and addition within a finite physical magnitude. But if the infinite is an attribute of a motion, it must also be an attribute of the substance. This is now what Aristotle has to show, since motion itself is not a sufficient substrate for an *essential* attribute because motion itself is an attribute of the substance.

[168] See fn. 175 below.

§4.1.3. *Per se* attribution of the actual infinite (*Phy.* III.6, 206b33-207a32)

The peculiarity of an Aristotelian infinite is that it must exist actually in a finite universe but not *as* a physical body. So far it looks like infinity exists as an attribute of the acts of division and addition—motions. But, this is only its *proximate* attribution. As motions, division and addition are only the proximate subjects of the infinite, for division and addition are primarily attributes of the substance. Division and addition as activities are properly attributes of the substance because they occur primarily in whole finite bodies. The whole is prior to any division. Does this mean that the infinite is also an attribute of the body? Was it not shown in *Phy.* III.5 that infinity cannot be the case? This is a key question for Aristotle at this point. For, the infinite cannot exist simply as an attribute of motion because motion itself is not what is really undergoing the division. Substantial bodies undergo division. Furthermore, we need to ask *why* division and addition admit any infinite at all. What is the cause of this? How do we *know* that division and addition admit the infinite necessarily and not accidentally?

Aristotle begins the argument by defining infinity in response to the physicists and the Platonists as 'not that outside of which there is nothing, but that outside of which there is always something' (οὗ ἀεὶ τι ἔξω ἐστί, τοῦτο ἄπειρόν ἐστιν, 207a1-2).[169]

[169] Simplicius, *In Phy.* 717.23 – 718.1 takes the ἀριθμός of circular motion, which is time, to be Aristotle's modification to Plato's view that time is the moving image of eternity (*Timaeus* 37d5-7). Indeed, what this shows is that while Aristotle criticizes Plato and the Platonists for equating time with the circuit itself, Aristotle still does not dismiss entirely the Academic doctrines: "Now observe that Aristotle, best of all men, well understood Plato's conception of time. For what is the 'eternal image proceeding according to number' [κατ' ἀριθμόν ἰοῦσαν αἰώνιον εἰκόνα] other than time existing according to the number that is neither the paradigmatic number [τὸν παραδειγματικὸν, i.e. *eidetic number*] nor monadic number [τὸν μοναδικόν, i.e. an ideal unit, Plato's 'intermediary number'], but that which is viewed as an image according to order of motion [τάξιν], that is to say, the before and after. For the number of motion *qua* motion has, at different times [ἄλλοτε], a different before and after, and both introduces the number and determines it according to order" (modified Urmson translation).

> It follows then that the infinite is contrary to what others say: the infinite is not that outside of which there is nothing, but that outside of which there is always something. An indication of this is that they say that a ring without a bezel is infinite, since there is always something to take. However, even though they speak on account of some sort of similarity, this is not the infinite *in the fullest sense* [οὐ μέντοι κυριώς]. For this must be the case *and* the same thing cannot be twice. This does not happen in the circle, but always one division following after another. So, the infinite is what one takes *quantitatively* outside of which there is always something to take (*Phy.* III. 206b33-207a8).

Aristotle clearly ascribes the infinite to division as a quantitative activity, since it is only in quantity that we find something *capable* of unending division—the magnitude. Only in the magnitude is there always something beyond which to take. But, here, Aristotle's argument changes. Here, he speaks directly to the nature of circular motion instead of directly addressing the division of the magnitude. He distinguishes his infinite from those who say that the infinite resembles (καθ'ὁμοιότητα) circular motion. The difficulty with Aristotle's argument is ascertaining not only whom he targets, but more so why it is not possible for the infinite to appear in the act of dividing a circle. Even though in a certain sense the circle may be divided infinitely, since it is indeed a magnitude, it does not capture the infinite in the fullest sense (οὐ μέντοι κυριώς) as in the way it appears in dividing a linear magnitude. The reason is this: when dividing a circle, one *always* returns to the beginning. In fact, *every* point is both the beginning and end of the circle. Thus, the circle is *finite.* The circle does not capture infinity in the fullest sense because when dividing the circle, one returns to the beginning in the same way as when

increasing the proportion so as to complete the magnitude (206b7-8). Even though infinite divisions are possible in the circle since the circuit is also a magnitude, the circle is primarily *traversable*. Circular motion is properly finite.

For instance, in *De Caelo* I.5, 272b28-273a5, Aristotle makes it clear that neither the heaven, the heavenly bodies, nor the revolution of the bodies is infinite because they are necessarily traversable insofar as the amount of time that it takes to complete the circuit is always finite:

> If the heaven is really infinite, but moves in a circle, then it will have to be the case that in a finite amount of time, it *has traversed* the infinite [ἄπειρον…διεληλυθώς]. For assume that the heaven, which is fixed, is infinite, and that which moves in it is equal to it. As such, when the infinite body [which moves in the infinite heaven] *has completed* its revolution [ὥστ᾽ εἴπερ περιελήλυθε κύκλῳ ἄπειρος ὤν], then it *has traversed* [διελήλυθεν] an infinite equal to it in a finite amount of time. But this is impossible. Correlatively, if the time in this revolution is finite, then it is necessary that the magnitude which the body *has traversed* [διελήλυθεν] must also be finite. For [δὲ] the magnitude *traversed* [διελήλυθεν] is equal to the time. Because of this, the time [taken by the body to traverse the magnitude] is finite (*DC* I.5, 272b28-273a5).[170]

Let us first be clear on Aristotle's *reductio*. The heaven is either finite or infinite. Assume that the heaven is infinite. This entails that its circuit is infinite, for the heaven is defined by the circuit of the heavenly body, which as heavenly, is also infinite. Now, the time it takes for the

[170] Translating δὲ as 'For.'

body to finish its revolution is finite, since the circuit *qua* 'circuit' is itself already complete. To be infinite, however, is *always* to be incomplete. Therefore, the time it takes for the heavenly body to complete its circuit is both finite and infinite with respect to the same thing—the circuit. But, this is absurd, for the circuit and the time cannot be both infinite and finite. Therefore, the heaven *and* the time it takes to complete the revolution are finite. The same argument applies to the magnitude and the body moving in the circuit. What is peculiar about this argument is that the *reductio* is accomplished by appealing to time as a way to bring out the contradiction that an infinite heaven completes its revolution in a finite amount of time. Notice how Aristotle pairs διελήλυθεν (the perfect of διέρχομαι, 'to traverse; pass through') with περιελήλυθε (the perfect of περιέρχομαι, "to come around, to revolve").[171] To *have* completed a revolution (περιεληλυθέναι) is to *have* traversed it (διεληλυθέναι). So, the time of celestial revolution must be finite, since the revolution is *traversable*. Nevertheless the infinite is intraversable in the direction of reduction. To be sure, the circle may be traversed over and over again endlessly—time, in fact, follows this path. Again, there is certainly a *similarity*. However, as Simplicius argues, the similarity is only with respect to how the number of cycles are countably indefinite. Simplicius shows us how that the proper sense of the infinite is division in the *direction of reduction*, not the repetition of a circuit, precisely in spite of the supposed unending character of circular motion:

> It is as if such limitlessness [for Aristotle] were in a straight line and not cyclical, as in the case of circles, whose rotations can counted since they repeatedly start at the same point and are not strictly limitlessness, but only through some similarity, since there is

[171] Both terms share the root ἔρχομαι, "to come; set out."

always something beyond *in each case*. But the infinite must also never reach the same point. So, *as a good definition*, we say that beyond which, taken quantitatively, it is *always possible to find an extra quantum* (modified Urmson translation).[172]

The circle may be traversed repeatedly one circuit after another and thereby counted *analogously* as if there is always another division to be made and counted. But the most important point is that that the circuit is traversed each cycle. By contrast, the infinite is by definition intraversable *with respect to the magnitude,* not a number of cycles. While it is possible to divide the circle in the direction of reduction, this is only if one treats the circle *as if* it were a linear magnitude, i.e. by marking off at least two divisions to create a line segment, and then divide geometrically as described at *Phy.* III.6, 206b3-9. The infinite in the fullest sense, therefore, appears in the magnitude *in the direction of reduction*, not by indefinite repetition of the circuit.

The next question we must ask is *why*? Why is division in the direction of reduction that in which the infinite appears and not in something like circular motion? In short, why is the act of division necessarily and essentially infinite, whereas circular motion is not? We know that the infinite must exist somehow as an attribute and that the activity of division actualizes the magnitude's capacity to be divided *ad infinitum*. What ultimately explains this phenomenon? What is its cause? As it turns out, it is the physical *substance*, which Aristotle assumes to be a

[172] Simplicius, *In Phy.* 500.20-501.7. Aquinas, in his *Commentary on Aristotle's Physics,* 11.384 argues similarly to Simplicius adding that each part that is taken must be unique: "For some say that rings are infinite because of the fact that they are circular and because it is always possible to take a part in addition to a part already taken. But *this is not said properly but according to a certain similitude.* For in order for a thing to be infinite, it is required that beyond which any part taken there be some other part, in such a way that the part which was previously taken *was never taken again. But this is not so in the circle.* For the part which is taken after another part is different only from the part which has just been taken, but not from all the parts previously taken. For one part can be taken many times, as it is clear in circular motion" (emphasis mine)

whole finite body; physical substances can be essentially defined as finite *continuous* bodies. The trick to see this is by *first* encountering the infinite as it manifests in division. The act of division as a quantitative change is the *threshold phenomenon*—the actuality—in which we first encounter the infinite.[173] While the infinite is proximately attributed to quantitative change, it is *per se* primarily with respect to the finite continuous body: Because the infinite manifests proximately in the activity of division in the direction of reduction, and because it is only in the magnitude that there is always something more to be divided, the infinite *as an activity* must also belong *essentially* to the magnitude. But the magnitude, as a continuous quantity, ultimately belongs to the *whole* physical substance because physical substances are continuous by nature—they are whole finite bodies. Just as color belongs to the body by way of the surface, so the infinite belongs to the physical substance by way of the act of dividing the magnitude of the substance. Therefore, the actual infinite is a *per se* attribute of the physical thing *qua* extended but only insofar as it is actively being divided.

As Aristotle defines it, the infinite is a quantity (κατὰ τὸ ποσόν); just as Simplicius claims above, "as a good definition [of the infinite], we say that beyond which, *taken quantitatively*, it is always possible to find an extra quantum."[174] But quantities belong to physical substances. Magnitudes, as continuous quantities, necessarily belong to finite physical substances. Since division happens in magnitudes, the actual infinite too is *per se* of physical substances, but *only insofar as* the substance is *actually* undergoing division.[175] For, just as motion is actual because of the moving substance, the infinite is actual in the act of division. That is to say, the infinite is actual if and only if a magnitude is actively being divided because

[173] On the role of 'threshold phenomena' especially with respect to motion, see §1.1 above.
[174] Simplicius, *In Phy.* 500.6-7.
[175] Bowin, *op. cit.,* p. 243.

the activity must always remain incomplete.[176] The actuality of the infinite, however peculiar it is, is explained by the nature of the actual physical substance, since the substance has within it the actualizable capacity to be divided infinitely as an extended body, albeit *incompletely*. The way Aristotle discovers the infinite's essential attribution to the physical substance is through the active division of the physical substance *qua* magnitude. While infinity is proximate to quantity, specifically a quantitative change, it is *per se* primarily of the physical substance because the latter is the implicit subject to which quantity is predicated.

In the final arguments in III.6, Aristotle's concern returns to how the infinite can exist as 'that outside of which there is always something' when no material thing exists beyond the whole universe. In other words, *where* is the infinite? To round out how the infinite is a *per se* attribute of the substance, Aristotle returns to the final problems raised at the end of III.5 regarding the *place* of the infinite:

> For it is true that the infinite is the material of the totality of the magnitude, and is what is potentially whole, but not actually; and it is divisible not only in respect to reduction but also in addition, which are convertible; but [δὲ] it is even whole and limited though not by itself but by something else; and it does not contain but is contained insofar as it is infinite. Hence it is also unknowable insofar as it is infinite, for the material does not

[176] *Ibid.,* pp. 247-249. However, Bowin argues that it is only the *potential* infinite that is *per se* of quantity, the kind of actual infinite I argue here. Furthermore, he does not extend the infinite's *per se* attribution to the substance. It is possible that were Bowin to have seen the activity of division as ultimately an activity belonging to the physical body, he would have concluded similarly that the infinite belongs essentially to the substance. Bowin does not explicitly deny that such attribution cannot be traced back to the substance, but he does not go beyond the category of quantity in his analysis.

have form. So it is clear that the infinite is on account of the part rather than that of the whole (*Phy.* III.6, 207a21-27).

The infinite does not exist in itself *beyond* the finite as the sum total of all the homogeneous parts, and as such containing everything. Instead, the infinite is as Parmenides argued: it is *contained* by the finite, just as form contains material (*Phy.* III.6, 207a15-25). Just like materiality itself, the infinite cannot be known apart from the finite thing in which it exists. However, the infinite can be *accounted* for by dividing any whole, any physical body, in the direction of reduction because it is only in this sense that the infinite exist 'not on account of itself, but on account of something else' (οὐ καθ'αὑτὸ, ἀλλὰ καθ'ἄλλο, 207a24). In this way, it may be known because the act of dividing a body is the actuality that makes the infinite intelligible. Infinity is actual, and thereby intelligible, with respect to physical substance when and only when the substance is actively undergoing division.

Infinity ultimately belongs to the physical substance but only in a specific manner. However, this is not immediately clear to us. The *cause* of the infinite is not the act of division itself, but the *finite individual* that is being divided. This is because part of the nature of a whole finite body is to be divided. To see this, division and addition must be treated as threshold phenomena that provide an indication that something else is the cause of infinity. We begin with division and addition, and work backwards to their cause—the physical substance as a whole finite body. What looks like a surprising twist, the infinite turns out to be an actual, albeit *incomplete* quantity of a whole finite body because all attributes belong to the substance. But, has Aristotle not already show in *Phy.* III.5 that the infinite cannot be actually attributed to a finite body as a quantity? While it is true that no finite body can be actually infinite, neither in

extent nor in number of parts, this does not preclude the infinite from belonging to it *insofar as it is moving.* That is to say, Aristotle can show that the actuality of the infinite is derived from its *per se* attribution to the finite body whose *natural* capacity to be divided is actualized in the act of division. The infinite ultimately and essentially belongs to the whole finite body as the physical substance because substances *by nature* can be broken down into parts without reaching an indivisible atom. The activity is always incomplete. Infinity is something like material whose potentiality is actualized incompletely by an activity that ultimately belongs to a finite being; the infinite is *in* the finite, just as material is in the form.

§4.2. *Per se* attribution, convertibility, and the physical substance (*Phy.* III.7)

Now, in *Phy.* III. 7, Aristotle returns to the argument for convertibility in order to reinforce the infinite's *per se* attribution to the physical substance: infinite addition is on account of the physical substance as much as infinite division is because the form of the substance is ontologically prior to their convertibility. Before division or addition may happen, the whole finite body must exist. Infinite by division and by addition convert because of how bodies are defined as continuous wholes *containing* material parts:

> In no way does the account admit of an infinite by addition that exceeds every magnitude but by division there is. For material and the infinite are contained within [bodies], while the form contains. (*Phy.* III.7, 207a33-207b1).

The individual whole governs how the magnitude even admits of division. Ultimately, the universe is the first *physical* whole prior to any other. Such cosmological priority, nevertheless,

is not the governing factor. Ontological priority belongs to whole finite bodies, for the form of any body is necessarily finite.

The reason that a finite universe prohibits an infinitely large magnitude is because of the ontological priority of form in nature (*Phy.* II.2). Form is the *physical* limit of a thing, since it is the body's boundary which contains the material. But, not only does form limit the physical size of magnitudes, it also places limits on how small numbers can be. Interestingly enough, the way that Aristotle argues for this is by tacitly revising Plato's Indefinite Dyad:

> It is also reasonable [εὐλόγως] that the limit in number is *in the direction of what is lesser* [τὸ ἐν μὲν τῷ ἀριθμῷ εἶναι ἐπὶ μὲν τὸ ἐλάχιστον πέρας], whereas *in the direction of what is more* [ἐπὶ δὲ τὸ πλεῖον] number always exceeds every finite amount [πλήθους]. It is the *opposite* with respect to magnitudes [ἐπὶ δὲ τῶν μεγεθῶν τοὐναντίον], such that they are exceeded *in the direction of the smaller* [ἐπὶ μὲν τὸ ἔλαττον], whereas there is no infinite magnitude *in the direction of the greater* [ἐπὶ δὲ τὸ μεῖζον]. The cause [αἴτον] is that the one is indivisible, which could be whatever one in particular (such that a human being is one human instead of many), whereas number is *many ones* and *a specific amount* [ἕνα πλείω καὶ πός' ἄττα]. Therefore, it is necessary to come to a stop at the indivisible (for 'three' and 'two' are names [of more than *one* thing] just as with each of the other numbers), but it is always possible to imagine [νοῆσαι] what is more. For the bisections of magnitudes are infinite. Therefore, infinite is potentially, but not actually; and yet, what is taken always exceeds every finite amount (*Phy.* III.7, 207b1-13).

First, let us look at the text, then at the argument. Textually, this is a crucial passage for understanding how Aristotle wrestles with Plato's Indefinite Dyad. It is important to notice how, at 207b1-5, Aristotle revises Platonic terminology related to the Indefinite Dyad in order to speak about the directionality of infinite by division and infinite by addition within a whole finite body. As we know, infinite by addition is with respect to 'the direction of increase' (ἐπὶ τὴν αὔξην) whereas infinite by division is with respect to 'the direction of reduction' (ἐπὶ τὴν καθαίρεσιν). The Platonic terms are 'the more (than)' (τὸ πλεῖον) and 'the lesser' (τὸ ἐλάχιστον), and 'the greater' (τὸ μεῖζον) and 'the smaller' (τὸ ἔλαττον).[177] Their grouping in the text is a plausible indication that Aristotle is targeting the Indefinite Dyad. However, by adding the adverb ἐπί, 'towards, in the direction of,' to the Platonic terms, Aristotle gives them a concrete orientation, whereby he can show how they actually pertain to division and addition within finite magnitudes. In this way, Aristotle *indexes* the Indefinite Dyad *within* the whole finite body. Notice the following parallels: 'the direction of what is more' (ἐπὶ τὸ πλεῖον) is similar to 'the direction of increase' (ἐπὶ τὴν αὔξην) just as 'the direction of the lesser (ἐπὶ τὸ ἐλάχιστον) is similar to 'the direction of reduction' (ἐπὶ τὴν καθαίρεσιν). Then, 'the direction of increase' (ἐπὶ τὴν αὔξην) is similar to 'the direction of the greater' (ἐπὶ τὸ μεῖζον) just as 'the direction of the smaller' (ἐπὶ τὸ ἔλαττον) is similar to 'the direction of reduction' (ἐπὶ τὴν καθαίρεσιν). In other words, both 'the More and the Less' and 'the Great and the Small' pertain to the whole finite body's ability to be added and divided. In this way, Aristotle *internalizes* and *concretizes* Plato's infinite by placing it *within* physical bodies.

[177] The terms 'the more (than)' (τὸ πλεῖον) and 'the lesser' (τὸ ἐλάχιστον) are variants of the More and the Less (τὸ μᾶλλον καὶ ἧττον), whereas 'the greater' (τὸ μεῖζον) and 'the smaller' (τὸ ἔλαττον) are variants of the Great and the Small (τὸ μέγα καὶ τὸ μικρόν). For Plato's use of the More and the Less see *Philebus* 24a-25a. For the Great and the Small, see *Statesman* 283c-285d. Sayer, *op. cit.,* pp. 154-168 also provides a thorough treatment of these terms in Plato's writings.

With this in mind, let us look at the argument. We know that infinite by division converts to infinite by addition insofar as the divisions may be counted. As the magnitude is reduced in size by division, number increases by counting the divisions. What this means is that number is infinite in the direction of increase since there is there is no greatest number of parts into which a physical body may be divided. The lack of an upper summative limit is due to the indefinitely possible divisions in the physical substance *qua* magnitude. For the physical magnitude is by nature continuous. In turn, this means that the smallest number is 'two' (δύο), for 'one' (ἕν) is actually the *whole* body since it is the lower limit with respect to addition. 'Two' is the result of the first division, given that the whole body is prior division. This means that the whole body is prior to infinite by addition insofar as it determines the lowest boundary condition for counting.[178] So, number is surpassed in the direction of increase, but not in the direction of reduction; infinite by addition has no upper limit, but it does have a lower limit precisely because the form of the whole body *contains* the countable parts prior to any division. The form of the body is the limit. But what sense should we make of Aristotle's conclusion that "there is an infinite potentially, but not actually; but what is taken always goes beyond every determinate amount"? While there is no actual infinite that is a whole physical substance, what is divided or 'taken' from the finite substance always leaves a remainder. The infinite goes in the direction of the *parts* as the body is broken down.

Number and the magnitude exist *in* the physical substance (*cf. Meta.* N.4-5). So, whatever potential infinite they enjoy *qua* quantities is on account of the whole physical substance. Again, the *per se* attribution of the infinite to the physical substance, in the end, explains the infinite's

[178] For the priority of the individual substance in the generation of numbers and counting, and how numbers are attributes of substances, see Halper, "Some Problems in Aristotle's Mathematical Ontology," pp. 140-143.

per se attribution to quantitative change. We can see this especially with time, motion and the magnitude. Each is infinite.[179] However, they ultimately depend on the substance:

> The infinite is not the same in the magnitude, in motion, and in time as if it had a single nature [ὡς μία τις φύσις], but what is posterior is said with respect to what is prior, such that on one hand motion [is said with respect to the magnitude] since the magnitude is that over which something moves, alters or increases, while time is on account of motion. We use these things now, but later we will also speak to what each is and the reason why every magnitude is divisible into magnitudes (*Phy.* III.7, 207b21-27).

The nascent point at 207b21-27, which echoes Aristotle's previous argument at 206b25-207a3, is that the physical substance is divided with respect to different things so as to account for the infinite in time, motion, and the magnitude. The infinite in time is derived from the infinite in motion, which in turn is derived from the substance as it is being divided. The priority belongs first to the substance, then to motion, and lastly to time. For, temporal instances follow the before and after of locomotion, locomotion follows places along a magnitude, and the magnitude is divided with respect to the whole physical substance. Time follows motion insofar as time is the number of the before and after of the *moving substance* (*Phy.* III.7, 207b13-15; *Cf. Phy.* IV.11, 218b30-219b2).[180] When attributed to time, for example, infinity *qua* intraversibility pertains neither to the body in time nor to the body's revolution—a span of time—for not only are both of

[179] See also Simplicius, *In Phy.* 509.22-510.14 on the difference between the attribution of the infinite to motion and to the magnitude.
[180] Ursula Coope, *Time for Aristotle:* Physics *IV. 10-14* (Oxford: Oxford University Press, 2005), 85-109 and Julia Annas, "Aristotle, Number and Time," *Philosophical Quarterly* 25 (99), 97-113 are some of the few interpreters who stress counting as an activity defining time, although neither recognizes the peculiar character of ἀριθμούμενον the counting activity.

these finite and traversable, as argued in *Phy.* IV.12, they are that *by* which the count is made (ᾧ ἀριθμοῦμεν), not the *activity* of counting itself (τὸ ἀριθμούμενον). Rather, the intraversibility of time pertains to the endlessness of its *number*, the count. But this number ultimately belongs to the *substance* in motion.

Now, because all of this happens within *quantity*, how does the account of the infinite save mathematics, as Aristotle claims at the opening of III.6? In other words, how is the mathematician able to rely on infinity *as a principle* (ἀρχή) of mathematics without contradicting cosmic finitism? This was, after all, part of the aim of the inquiry. The mathematician may generate a finite quantity—be it numerical or extended—as large as one prefers for the sake of mathematical proof (τὸ δεῖξαι) without undermining the finite limits of the universe. Again, the nature of the physical substance as an embodied form plays a pivotal role. The requisite material needed for mathematical procedures used either for geometric or arithmetic purposes is the continuous magnitude. Aristotle argues for this at the outset of his refutation of an infinite cosmos in *De Caelo* I.5:

> First, is there an infinite body, as the majority of the ancient philosophers thought, or is this an impossibility? The answer to this question, either way, is not unimportant, but rather all important, to our search for truth. It is this problem which practically always has been and may be expected to be the source of the differences of those who have written about nature as a whole, since the least initial deviation from the truth is multiplied later a thousandfold. Admit, for instance, the existence of a minimum magnitude, and you will find that the minimum which you have introduced causes the greatest truths of mathematics to totter. The reason is that the principle in question is

> great in power rather than in extent [τούτου δ᾽αἴτιον ὅτι ἡ ἀρχὴ δυνάμει μείζων ἢ μεγέθει]; hence that which was small at the start turns out a giant at the end. The infinite possesses this power of principles, and indeed *in the sphere of quantity possesses it in the highest degree* (*DC* I.5, 271a1-14, modified Hardie & Gaye translation).

Notice that between the accounts in *Phy.* III.6-7 and *DC* I.5, Aristotle does not jettison the ἔνδοξα concerning the infinite's status as an ἀρχή of mathematical proof. The work it performs not only for mathematical proof, but also for physics and its relevant spheres of inquiry, is that of the necessary—material—cause for time, motion, magnitude, and ultimately the finite physical substance. Without the infinite, there is no quantity, for number and magnitude are able to be as large as possible. I suggest this is what Aristotle means by the infinite as a material cause (φανερὸν ὅτι ὡς ὕλη τὸ ἄπειρον αἴτιόν ἐστι, *Phy.* III.7, 207b35). Infinity, it might be said, is *per se* of the physical substance because without it, nature is powerless to persist. The whole physical substance as form containing material, the infinite in the finite, secures the infinite's place in nature. While there is no lower limit for magnitudes, the whole physical substance is certainly the upper limit. Ultimately, the cosmos as a whole is that very physical body which limits all other magnitudes. But, *every* whole—regardless if it is an individual human, insect, or the smallest wholes in Aristotle's universe (the elements)—is prior to and contains the infinite. The form of every body contains its material in such a way that while the material grants the physical substance the power to be infinite in two different directions, the form restricts that power as a whole.

§4.3. Remaining arguments against a separable infinite (*Phys.* III.8)

At III.8, 208a8-22, Aristotle gives three final arguments against the separability (ἀφωρισμένον) of an actual infinity: (1) An actual infinite physical body is unecessary (οὔτε...ἀναγκαῖον ἐνεργείᾳ ἄπειρον εἶναι σῶμα αἰσθητόν, 208a8-9) to preserve continual generation and destruction; (2) simply because something is limited does not entail reciprocal limitation by another body *ad infinitum*, as if limitation required contact (τὸ ἅπτεσθαι); and (3) infinite numerical and extended magnitudes are *accidentally* actual in thought (τῇ νοήσει) because the excess and defect upon which such abstractions rely are not actual in the concrete thing (οὗ ἐπὶ τοῦ πράγματος). Arguments 1-3 refute three of the five purported phenomena supporting the infinite given at *Phy.* III.4, 203b16-25: (1) the inexhaustibility of generation and destruction (b18-20); (2) limits require (ἀνάγκη) a limiter (b20-22); and (3) that number and the mathematical magnitude are inexhaustible in thought (b22-25).[181] Still, why address these previous problems here at the end of the account? It is, I suggest, to settle the issue that the existence of any sort of infinite must be *predicated essentially* of the physical, finite substance, and not be itself a substance, either materially or immaterially.

First, no actual, physical infinite is necessary to account for the inexhaustibility of generation and destruction because entities come to be and resolve into other *finite* entities. An Anaxagorean plenum or Democritean atomism is gratuitous, for the material from which and into which entities transform is already in the finite universe potentially insofar as all material belongs to already existing entities (*GC* I.3, 318a13-23; 319a10-28).[182] Furthermore, the finite substrate remains throughout any motion. No generation or destruction is possible without it. Therefore, an actual infinite physical body serving as a plenum is unnecessary. The finite,

[181] *Cf.* Ross, *Aristotle's* Physics, p. 561.
[182] See Furley, "Aristotle and the Atomists on Infinity," pp. 85-91.

physical universe supplies all the material necessary at any given moment because even if the amount is always finite, material in general is inexhaustible because of the continuity of physical bodies.[183]

Concerning the phenomenon of limits, Aristotle thinks that the view in which limits always require an external limiter *ad infinitum* conflates limitation, or finitude (τὸ πεπερασμένον), with contact (τὸ ἅπτεσθαι). While contiguity necessarily entails an external entity, since touch requires a relation between two different physical entities, having a limit does not entail such a relation (208a13). A limit pertains to a whole, where wholeness is that outside of which nothing extra exists (*Phy.* III.6, 207a9-10; *Meta*. Δ.17, 1022a4-5). The finite entity is defined καθ'αὑτό. Any contact it admits, then, is accidental to what it is itself. So, there is no actually infinite physical substance necessarily with respect to a succession of limits. The implicit corollary is that since the universe is complete and whole, its finitude is not determined by any external body *ad infinitum*. The same applies to any physical body.

Now, the charge against an actual infinite in thought (τὸ τῇ νοήσει, 208a14-22) redresses the ἔνδοξον that number and *mathematical* magnitudes are noetically inexhaustible (203b22-25). Recall from the earlier ἔνδοξον that an actual infinite in thought admits something beyond the heavens (τὸ ἔξω τοῦ οὐρανοῦ)—an immaterial infinite. Such would exist καθ'αὑτό, thus necessarily. Aristotle's point here, however, is that thought, at least human cognition, is counterfactually accidental. For, one may be able to imagine an object to be larger or small than it is in fact (ὅτι ἐστί), even so much as to be infinite, but that thought is accidental to the actual size of the object in the physical world. The excessive size in thought is not actually predicated of the corresponding physical phenomenon, nor is such excess even necessary to its existence.

[183] *Cf.* Tamer Nawar, "Aristotelian Finitism," *Synthese* 192 (2015), pp. 2350-2352.

Certainly, the mathematician may do well to imagine infinitely extended lines in order to prove the parallel postulate, but this does not entail that this geometric object exists necessarily as a matter of fact.[184]

Aristotle's point, again, is that the finite physical substance, as a matter of fact, governs however big or small a number or magnitude can be imagined because it is in respect to the concrete substance that such excess or defect is able to be abstracted. Even though it is possible to imagine myself as a homunculus, it is only counterfactually possible considering my actual physical height. That is to say, I can imagine myself to be a small human being only comparatively with respect to my actual physical height. With respect to the infinite, any number imaginable is possible because we may extrapolate from the fact that any act of division may proceed *ad infinitum*. But this is possible only because that imagination is still with respect to an actual physical body from which more or less may be abstracted. That very possibility is because infinity is *per se* primarily of the finite physical substance, not of the thinking. The necessity pertains not to whatever may be counterfactually actualized in thought, but to the actual concrete physical substance.

Aristotle's positive account of the infinite defends two things: that the potential infinite exists also as a special kind of actuality and that very actuality does not violate the finitism of physical substances. Both are because of the way the infinite manifests in the act of division within the whole finite bodies. Proximately, the infinite appears as an attribute of division. But, division itself is essentially attributed to the physical substance. So, the infinite is not proper to division itself, but to the *body* as it is being divided. Throughout III.6, the answer has been staring us in the face: the infinite is *per se* of the physical substance primarily by virtue of the

[184] *Cf.* Hintikka, "Aristotelian Infinity," pp. 202-203.

fact that natural substances are extended bodies and every extended body is continuous. The degree to which infinity exists depends on how much actuality one is willing to grant potentialities themselves. Whatever actuality it has must exist derivatively as an attribute. The infinite, regardless of any actuality it might have on the part of another thing, cannot itself exist separately as a finite entity (ἀφωρισμένον, 208a6). That is to say, it cannot be a substance *in any way*. Infinity, instead, is *in* the finite as an attribute. In *Phy.* III.6-8, Aristotle confirms the last two suggested definitions posed at the end of III.4. The attempt to reduce the magnitude by division and thereby to add those divisions reveals the way it is intraversible. This is how the infinite exists in a finite universe. Aristotle attempts to solve the problem by attending to the phenomena and by revising the ἔνδοξα. *Phy.* III.6, 206b33-207a8 provides an implicit demonstration of a phenomenon in need of explanation—the act of division as indefinite—and 207a8-30 brings the ἔνδοξα into agreement with it. The *per se* attribution of the infinite to the physical substance entails its actuality in some measure as an attribute of a quantitative change. The infinite is actual as a special kind of potentiality, one that is not, indeed cannot be, fully actualized, so as to preserve time, number, and the magnitude.

CONCLUSION

The aim of this dissertation has been to show that despite Aristotle's rigorous arguments against an actual infinite body, and despite the standard interpretation that what follows from these arguments is the existence of a purely potential infinite, there is still a context in nature where the infinite is and must exist as an actuality: the activities of division and addition. Aristotle's infinite is not itself a body; the actual infinite is in the finite insofar as finite bodies actively undergo division without arriving at an indivisible quantum. There is always more of a magnitude to divide. The ingenuity of Aristotle's account turns on both the inherent incompleteness of motion and the continuity of finite physical magnitudes. The intersection of motion's incompleteness and the continuity of physical magnitudes allow the infinite to exist as an attribute of the moving body in terms of an active potentiality. The infinite is *in* the finite *actively*. For sure, there is no actual infinite *body* because none of the categories admit it. But, the categories admit *other* things than physical bodies; they admit motion! Motion is Aristotle's way of saving the infinite and he does so while also preserving his strict finitism.

Nevertheless, there remain several problems requiring further investigation. First, as we have seen in *Phy*. III.5, Aristotle's universe is absolutely finite. Aristotle's universe is an absolute physical whole, for there is no other physical magnitude beyond the body of the universe.[185] This is the very reason why an actual infinite magnitude is impossible. Therefore, it

[185] While it is true that the unmoved mover exists beyond the cosmos, since it is *the* final cause of the motions of the heavenly spheres and thereby all other terrestrial motions, the significance of the absolutism of Aristotle's universe pertains to the limits of its *physicality*; there is no

would be nonsense to talk about 'something' outside of the universe for Aristotle. Nevertheless, for this very reason, Aristotle's finite universe is still phenomenologically peculiar—*where* is Aristotle's universe, *even if* it is an absolute whole? Is it not possible to imagine an *empty space* existing beyond and containing the body of the cosmos? Phenomenologically, it *appears to us* as if there must be 'something' beyond the cosmos, if only empty space, since we seem to experience the universe still to exist *somewhere*. For Aristotle, it is a *body* after all, and all finite bodies are contained by something else. In the antithesis of Kant's first antinomy (*KrV* A427/B455-A429/B457), Kant raises this concern with respect to empty space:

> As to the second point [of the antithesis], first assume the opposite, namely that the world is finite and bounded in space; then it exists in an empty space, which is not bounded. There would thus be encountered not only a relation between things in space, but also a relation of things to space. *Now, since the world is an absolute whole, besides which there is encountered no object of intuition, and hence no correlate of the world to which the world could stand in relation, the relation to empty space would be a relation of the world to no object.* Such a relation, however, and hence also the boundlessness of the

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physical body outside of and relative to Aristotle's cosmos. But the unmoved mover can exist beyond and relative to the universe while not violating the absolutism of the universe's body. This is because the unmoved mover it not itself a body. It exists beyond the universe in terms of its *separateness* from the physical world entirely, and not by *extending* beyond the limits of the universe. The unmoved mover lacks material, for if it were a physical thing, it would have the capacity to move and thus not be unmoved. But because it does not have material, it cannot stand in relation to the body of the universe as limiting the universe as if by contact. For this account, see *Phy.* VIII.5-6 and *Meta.* Λ.7; 9.

> world by empty space is nothing; *therefore, the world is not bounded at all in space, i.e. in its extension it is infinite.*[186]

Kant's antithesis challenges Aristotle's absolute physical universe. Aristotle would indeed agree that there is "no correlate of the world to which the world could stand in relation," if by "correlate" Kant means another physical body. This is because, for Aristotle, the universe does not need to be limited by an external limiter. Remember that, at the end of *Phy.* III.8, Aristotle rejects the view that limits require a limiter *ad infinitum* because it would entail an infinite magnitude. So, nothing outside of the universe stands in relation to it as another physical body. However, based on Kant's *reductio*, even if it is true that being limited does not require a limiter, the *body* of Aristotle's universe would still stand in relation to an empty space because it needs to be *somewhere*. But, in empty space, there is nothing in relation to which the universe could be *contained.* And yet, all finite bodies are contained by places. Since the body of the universe would not be contained in empty space, therefore, it must be *unbounded*—infinite. For Aristotle, this 'empty space' is essentially a void. But, however much Aristotle insists on the non-existence of a void (*Phy.* III.6-9), can he avoid what seems like the necessary existence of empty space beyond the universe, since, as a body, the universe still needs to exist in a place? In short, if there

[186] Immanuel Kant, *Critique of Pure Reason*, trans. Paul Guyer and Allen Wood (Cambridge, UK: CUP, 1998), p. 471, emphasis mine. It is important also to note that the thesis of Kant's first antinomy reiterates general spirit of Aristotle's objection to an actual infinite magnitude. According to Kant, for there to be an infinite spatial magnitude, the totality of its parts would have to exist *simultaneously*; that is to say, all the parts of infinite body would necessarily exist in actuality and completely at the same time. For Kant, this is impossible to comprehend, since it would take an infinite amount of time for us to *add the parts* (i.e. "successive synthesis"). For Aristotle, however, it is not so much that it is impossible for us to go through all infinitely many parts (even though it is true Aristotle that we are in fact unable to do so), but that the simultaneity of an actual infinite number of parts undermines the possibility for motion.

is nothing outside of the *body* of the universe, can the universe truly be finite? In relation to what is the universe *in a place*, if it is finite?

The antithesis of Kant's first antinomy raises a phenomenological concern about the *relative* locality of Aristotle's cosmos. Can we experience the universe as truly absolute and finite, if there is nothing outside of it? Kant's forms of intuition, specifically space, speak to how Aristotle's arguments for an absolutely finite universe seem opposed to the very *possibility* of our experience of such a thing since we experience bodies as contained and bounded by places. That is to say, if Aristotle's universe is in fact a *body,* then we expect it to be located in space. And this is important, especially if we aim to give a *physical* account. Physics requires some degree of observation of bodies in places, even for Aristotle. So, even if there were an absolute bounded universe, *we* would still expect to *experience* it as located in a *boundless* empty space. Kant makes this clear when he argues that:

> But here we are talking only about the *mundus phaenomenon* and its magnitude, where one can in no way abstract from the intended conditions of sensibility without removing the being itself. The world of sense, if it is bounded, necessarily lies in infinite emptiness. If one wants to leave this out, and hence leave out space in general as the *a priori* condition of the possibility of experiences, then the whole world of sense is left out (*KrV* A433/B461).[187]

That is to say, we experience finite bodies, including the body of the universe, with respect to the world of appearance. The consequence of this, however, is the body of the universe would have

[187] *Ibid.,* p. 473.

to exist in an infinite empty space, which would necessarily be just as much a part of the universe. But this means that the universe would in fact be infinitely extended. How might Aristotle address this problem? Possible answers to this problem require further research into Aristotle's view of *place* in *Phy.* IV.1-5 and the character of the cosmos in *Phy.* VIII and *De Caelo.* Aristotle would have to show that the body of the universe does not need to be contained by another other than it in order to avoid Kant's objection.

The second problem is that in what respect(s) does infinite division and addition occur in nature? As activities, division and addition seem to require *human cognition*. Division and addition, as Aristotle presents them in *Phy.* III.6, seem to require human thinking because they are at least *mathematical calculations*. A divider, as an agent, makes mathematical judgments regarding the proportion(s) according to which the magnitude is reduced. In turn, addition requires a *counter* in order to add one unit/division to the previous. But, is it sufficient for Aristotle's account for division and addition to be included as a part of physics, if so much of these activities seem to require a human agent? To what extent does division and addition occur in nature apart from human thinking? In one respect, we could argue that division occurs in nature with respect to the way things are broken down when decomposing or being destroying. Division might even be a kind of destruction, if division by reduction is understood in the sense of *breaking down* the magnitude. For example, a table or living organism may undergo division infinitely insofar as the process of decay will not reach an indivisible atom. But, what then happens to that decaying material? Is decay even a type of division? It is unclear, therefore, whether division occurs in nature only with respect to human thinking or also in apart from it. As for the natural occurrence of addition, in *Phy.* IV.14, Aristotle raises this issue with respect to time, since time seems to require a human being to count the positions of the sun and the

heavens—instants—in order to calculate the before and after of their motions. And since we know that, for Aristotle, time is infinite by virtue of not having a beginning nor end, does this require that there has been and always will be human beings to account for time? If so, addition does seem to require a human being. But, does this call into question time's relevance to the study of physics, if a human subject seems to be required? Even in the account of time in *Phy.* IV.14, this is unclear. Answers to these questions require further investigation into the nature of Aristotelian mathematics, human cognition, the nature of material, the continuum, time, and generation and destruction in *De Generatione et Destruction, Metaphysics* M-N, as well as the account of time in *Phy.* IV.10-14.

In addition to the foregoing problems, this dissertation leaves open possibilities for future research in Aristotle's broader use of the account of motion in the *Physics*. The current dissertation is in the context of what I think is a wider application of motion as a threshold phenomenon. The *Physics* presents an elegant picture of *per se* attribution insofar as the essential attributes of nature—infinity, place, time, and continuity—can be traced back to the physical substance by beginning with how they first manifest in motion. Its elegance is enhanced even more once we realize that this reading is consistent with Aristotle's method of research outlined in *Phys.* I.1, 184a16-184b14, that of following a path (ὁδός) from what is more familiar and clearer to us to what is known better and simply by nature. In one sense, with respect to us, infinity, place, time, and continuity are intrinsic to motion because it is always with respect to motion that they *first* manifest, and motion appears clearly for us. For us, motion is the proximate subject to which these four entities are most often visible. There is always an indefinite, incomplete aspect to motion (*Phys.* III.2; III.6); motion is always in some place or another (IV.1-5); time follows along with and is the measure of motion (IV.10-14); and it is by

looking to how motion is always from one thing to another (ἔκ τινος εἴς τι)—i.e. contrariety—that we first gain insight into continuity and by extension the unity of motion (*Phy.* V-VI). But, just as we have seen with the infinite, all these attributes, including motion, are properly *per se* of substances. While proximately and for us motion may be taken as the substrate, all five attributes (motion included) are properly *per se* of the substance. Even though motion is, in a way, what underlies the other attributes, it is not the substance, and the substance is *per se* most of all. Only the substance, as an independent thing (οὐσία), is what it is through itself (αὐτὰ καθ'αὑτά). So, there is more to be done to work carefully through each inquiry in *Physics* III-VI to illuminate the *per se* status of each attribute, taking the account of motion (III.1-3) as key to each inquiry. Not only is motion the *terminus a quo* for the discussion in books III-VI, it is also a critical element *within* each inquiry. Aristotle investigates how the infinite, place, time, and continuity are intricately connected to motion, and proceeds in light of this to resolve *aporiai* related to each attribute in order to demonstrate scientifically how each is a *per se* attribute of the physical beings. In short, a larger project is necessary to show how Aristotle can find the *per se* attributes of natural substances by investigating how and why they are so intimately tied to motion.

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