TRANSFERRING DEMAND: SECONDARY TEACHERS' SELECTION AND ENACTMENT OF MATHEMATICS TASKS FOR ENGLISH LANGUAGE LEARNERS

by

ZANDRA URSULA DE ARAUJO

(Under the Direction of Denise A. Spangler)

ABSTRACT

In this study I examined 3 secondary mathematics teachers' selections and enactments of mathematical tasks for their English language learner students. More specifically, I attended to the cognitive demand of the tasks as they moved through the three successive phases of the Mathematical Tasks Framework (Stein & Smith, 1998). I examined the tasks the teachers selected, the modifications the teachers made to the tasks to accommodate their ELL students, and the aspects of the classrooms that contributed to the maintenance or decline of cognitive demand during implementation.

The participants were secondary mathematics teachers who taught a ninth grade, mathematics class comprised entirely of ELLs. I employed a qualitative, multiple case study design. The primary data sources included a survey, interviews, observations, and classroom artifacts. I administered a survey to each teacher prior to conducting interviews or observations and observed each teacher's classroom daily for two weeks. I conducted daily interviews with each of the teachers prior to each observation and conducted two extended interviews after the two weeks of observation. The classroom artifacts included the tasks presented to the students. I analyzed the data using the constant comparison method decoupled from grounded theory. This involved many rounds of inductive coding where I identified themes and collapsed them into broader categories.

The teachers routinely selected highly repetitive, low cognitive demand tasks focused on increasing procedural fluency. These tasks were selected in part because of the teachers' perceptions of students, lack of resources and training, and focus on standardized testing. The teachers often modified tasks during set up to lessen the number of words and lower the mathematical rigor as they attempted to accommodate their ELL students. During implementation, student-centered communication tended to maintain the cognitive demand of tasks. Understanding the ways in which teachers select, modify, and enact curriculum materials to accommodate their ELL students is an important step in understanding effective teaching strategies for ELLs. The findings of this study suggest that teachers require additional training and resources to select and modify curriculum materials for ELL students that are both mathematically rigorous and promote communication.

INDEX WORDS: Mathematics Education, English Language Learners, Curriculum

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DEDICATION

To my mom and dad who worked very hard and sacrificed a lot so that I could have a fantastic education and a wonderful life. Muito obrigada!

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CHAPTER 1

CHANGE CREATES NEW CHALLENGES: THE PROBLEM STATEMENT

Throughout my life I have heard people refer to mathematics as the universal language. It is unclear how this long held notion came about. It stands to reason that the symbolic nature of mathematics may have played a large part in this common saying. For example, I may very well enter a mathematics classroom in another country and see the teacher write something such as $2x + 3 = 5$ on the board. Though I may not speak the language, I would be able to solve this equation for x, a common goal for problems of this nature. This scenario, however, does not take into account a common trend in American mathematics classrooms.

The United States' mathematics curriculum has undergone a multitude of shifts in the past century (Schoenfeld, 2004). The Common Core State Standards, arguably one of the most substantive changes to the United States' mathematics curriculum, are to be implemented by the vast majority of states in the near future. Increasingly, there is a call for school mathematics to focus on problem solving and connect to real life (e.g., Common Core State Standards Initiative, 2010a; National Council for Teachers of Mathematics, 2009). This shift in emphasis from computation toward a curriculum centered on reasoning and sense making has led to higher language demands for all students (Khisty & Chval, 2002).

As the curriculum continues to shift, the population of U.S. students is also undergoing changes. One such change is the dramatic increase in students whose native language is not English. The intersection of the changes in school mathematics and the U.S. population are presenting new challenges for teachers who may not be adequately prepared for either the change in curriculum or the student population. Remillard and Cahnmann (2005) asserted “As mathematics education moves away from rote procedures and rules and toward conceptual thinking, teachers in bilingual and bicultural settings find themselves negotiating unfamiliar mathematical terrain across languages and cultures” (p.172).

To illustrate the changes to the school mathematics curriculum and its potential impact on non native speaking students, consider the following scenario. Ella is an American middle school student whose mathematics skills are on grade level as measured by her state’s standardized test. She is a monolingual English speaker who happens to visit a Portuguese middle school mathematics classroom quite similar to her own. As Ella takes a seat in the back of the room, the teacher begins the class.

> Bom dia classe! Hoje eu tenho um desafio para voces. Meu amigo Diego mora 2mu ma fazenda, e em sua fazenda existem algumas cabras e umas galinhas. Diego olhando em torno da fazenda contou um total de 52 pernas entre as cabras e as galinhas. Ele esta tentando descobrir exatamente quantas cabras vivem em sua fazenda. A mae de Diego disse que possuem um total de 16 animais. Voces podem ajudar Diego descobrir quantos dos animais sao cabras?

Ella tries to pay close attention as the teacher talks, but as a non-Portuguese speaking student, she finds herself a bit perplexed. Ella watches as the teacher writes the following on the board.

> 52 pernas entre as cabras e as galinhas
>
> Total de 16 anamias
>
> Quantos dos animais sao cabras?

Ella recognizes some of these words from the teacher's initial discussion. She immediately recognizes the numerals, as they are the only terms on the board with which she is familiar. She also sees the word "total," which she takes to mean the same as the English word. Ella has some prior exposure to the Spanish language in the US, so she incorrectly identifies the word "dos" as "two." After a mere 10 minutes, each of the Portuguese students has handed in a paper while Ella continues to ponder the situation, unsure of what the teacher wants of her.

Ella was unable to answer the Portuguese teacher's problem. This does not necessarily mean Ella does not understand the mathematics of the problem. In fact, Ella could easily have solved the problem if the teacher had presented it in English as follows:

> Good morning class! I have a challenge for you today. My friend Diego lives on a farm and on his farm there are some goats and some chickens. Diego was looking out at his farm and counted a total of 52 animal legs. Diego is trying to figure out exactly how many goats there are on his farm. Diego's mother said that they have 16 animals in all. Can you help Diego figure out how many of the animals are goats?

If mathematics was truly a universal language, one might expect Ella to be able to communicate her solution to others regardless of her or their native language. Conversely, the universality of mathematics would allow Ella to understand the solution regardless of the country in which the problems were presented. Though this may be true for mathematical problems focused on procedures and devoid of context, these types of problems no longer constitute the entirety of the school mathematics curriculum for many students.

This task is a high cognitive demand task (Stein, Smith, Henningsen, & Silver, 2009), meaning it requires mental effort to understand and solve the problem. Many mathematics tasks focused on building mathematical understanding and problem solving skills are high in cognitive demand. These tasks often require students to interpret a given scenario or problem and explain their solutions. The linguistic demands of such tasks are often greater than low cognitive demand tasks. As the school mathematics curriculum moves toward more rich, high cognitive demand tasks one must consider what the impact of such tasks will be on those students who are not native English speakers.

Background

The achievement gap in mathematics between English language learners (ELLs) and their native English speaking counterparts is well documented (Ballantyne, Sanderman, & Levy, 2008). As mathematics teachers and educators continue to try to close this achievement gap, the demographics of U.S. students are rapidly changing. Currently, ELLs comprise approximately 11% of the students in U.S. public schools. This percentage represents a 51% increase in the decade since the 1997-1998 school year (National Center for English Language Acquisition, 2011). ELLs are the fastest growing segment of the U.S. public schools. One study predicted that ELLs will comprise 40% of U.S. students by 2030 (Thomas & Collier, 2002). Many states that have had very small ELL populations have experienced large increases in ELLs in recent years. From the 1997-98 to the 2007-08 school years the U.S. student population grew by 7.22%, while over half the states had greater than 100% increases in ELL students, and 11 states experienced growth exceeding 200% (National Center for English Language Acquisition, 2011).

With these dramatic increases comes a new set of challenges for many school districts. Many teachers who have no experience or training related to teaching ELL students now have several ELL students in their classrooms. One report found that only 29.5% of teachers who teach ELLs have had preparation to teach these students effectively (Ballantyne et al., 2008). Additionally, recent results from standardized tests reveal that this quickly growing segment of students continues to reside on the lower end of the achievement gap in mathematics (Fry, 2007). These situations highlight the growing need to train both in-service and preservice teachers to teach ELL students effectively.

In addition to the change in the student population, curriculum materials for school mathematics have undergone or are undergoing substantial changes. These changes not only affect what mathematical content teachers must teach but also how teachers should teach the content. The impetus for many of these changes was National Council for Teachers of Mathematics' (NCTM) 1989 publication of the *Curriculum and Evaluation Standards for School Mathematics*. Mathematics educators often refer to the curriculum materials built upon the foundation laid by NCTM as "standards-based." Many of these standards-based curriculum materials contain tasks emphasizing student-centered classrooms, conceptual learning, problem solving, mathematical reasoning, and communication.

NCTM recently released *Focus in High School Mathematics: Reasoning and Sense Making* (2009), a document that is supposed to serve as a framework for the development of future secondary school mathematics curricula and resources. A companion document to the *Focus, A Teacher's Guide to Reasoning and Sense Making* (2010a) outlines instructional strategies that teachers should use to help students make sense of mathematics. These strategies include the selection of worthwhile tasks that develop students' mathematical understanding, the creation of a classroom environment where engagement in mathematical thinking is the norm, and the effective orchestration of purposeful discourse to encourage student reasoning and sense making.

In 2005, the state of Georgia began implementation of the Georgia Performance Standards (GPS). The GPS is a reform-based mathematics curriculum created in part to change the way teachers teach mathematics in Georgia. The Georgia Department of Education (DoE) describes the GPS for mathematics as a curriculum that encourages students' mathematical reasoning, evaluation of mathematical arguments, and use of mathematical language to communicate ideas precisely (Georgia DoE, 2009). The GPS also includes a framework for each mathematics course. The framework includes a collection of tasks that "represents the depth, rigor and complexity expected of all students… [but] does not represent a complete curriculum" (Georgia DoE, 2010, p. 3).

Due to the increased emphasis on the use of tasks in Georgia and in other curricula, I focused my study on mathematical tasks. I chose Stein and Smith's (1998) definition of mathematical task as a portion of the classroom centered on the development of a mathematical concept. Under this definition, a mathematical task could entail a single problem or an entire class period.

NCTM has discussed the importance of choosing "worthwhile tasks" for many years. I have chosen to define worthwhile tasks as mathematical tasks that contain a high level of cognitive demand (Stein et al., 2009). Although I acknowledge there are times when tasks with lower cognitive demand are necessary, I think there are far more opportunities for students' mathematical knowledge to grow with the use of high cognitive demand tasks, and research supports this position (e.g., Stein, Smith, Henningsen, & Silver, 2009). In *the Principles and Standards for School Mathematics* (2000), NCTM stated that worthwhile tasks offer an opportunity to present mathematical concepts to students. Worthwhile tasks often present opportunities for students to engage in mathematics connected to real-world experiences. Furthermore, worthwhile mathematical tasks should allow students to explore multiple solution paths.

As a high school teacher, I often found it quite difficult to maintain the cognitive demand when using high cognitive demand tasks with students. It was particularly challenging when implementing these types of tasks with ELL students. Thus, as both a mathematics educator and teacher, I think this area of study is important to mathematics education.

Research Questions

Though NCTM, the state of Georgia, and the Common Core State Standards Initiative have issued statements regarding the importance of using appropriate strategies when teaching ELL students, none has provided resources describing specific strategies. Teachers looking for help must rely on what little training their districts may offer or on resources they can find on their own. As a high school teacher in Florida, a state with a very large ELL population, I received training in general strategies (speak slowly, repeat important information, use gestures, avoid idioms, etc.) but never received specialized training on strategies for the mathematics classroom, therefore, I designed this study with the purpose of exploring how teachers and students may impact the cognitive demand of mathematical tasks during the various phases of implementation. This study's results could guide future research on implementing high cognitive demand tasks not just with ELL students but all students.

Stein and Smith (1998) proposed three phases of task implementation. The first of these phases is the tasks as they appear in the curriculum materials. The task then moves into the set up phase as the teacher indicates her expectations for the students' work on the task. Finally, the students implement the task in the classroom with or without the teacher's intervention. The following research questions relate to each of these phases, respectively:

1. How do teachers choose mathematical tasks for use with their ELL students?
    a. What are the characteristics of the tasks they select?

    b. What factors influence the teachers' selection of tasks?

2. What modifications, if any, do teachers make to mathematical tasks prior to their implementation with ELL students?
    a. What factors influence the teachers' decisions to modify or not modify the tasks?
    b. In what ways, if any, do these modifications affect the cognitive demand of the tasks?
3. What aspects of the classroom appear to contribute to the maintenance or decline of high cognitive demand in mathematical tasks?

Significance

The selection and implementation of mathematical tasks is an important part of a teacher's practice and students' learning. Kloosterman and Walcott's (2010) examination of NAEP results concluded that there exists a "positive relationship between what is taught and what is learned" (p. 101). This implies that the types of problems used in the classroom impact the type of learning that occurs. NCTM stated that the tasks used in classrooms create the opportunities for students' mathematical learning (1991). Similarly, Stein et al. (2009) wrote that the choice of mathematical tasks impacts the level and type of student thinking in the mathematics classroom. Due to the impact task selection has on students' opportunities to learn, several studies have examined teachers' uses of mathematical tasks (e.g. Boston & Smith, 2009; Stein et al.). However, in a review of the literature I found no studies that specifically examined teachers' selection and use of tasks with secondary ELL students. This gap in the literature and the need to create and enhance learning opportunities for ELL students in particular has led to my interest in this area of research.

This study fills a gap in the literature regarding the implementation of mathematics tasks with ELLs. As the linguistic demands of mathematics classrooms increase, teachers of ELLs are responsible for finding and executing instructional strategies that aid their students. Understanding the modifications teachers make to curriculum materials to accommodate their ELL students and the resulting impact these modifications have on the mathematical rigor or cognitive demand of the curriculum materials is an important first step to creating effective teaching strategies for ELLs.

Definitions

Throughout the literature, researchers have used particular terms in a number of ways. For example, there exist well over 100 different meanings for the term curriculum (Portelli, 1987). In this section I have provided the definitions for key terms used throughout this study. In providing the reader with the meaning I ascribed to these terms, I hope to alleviate potential misinterpretations.

Cognitive Demand:	The cognitive demand of a task refers to the type and amount of thinking needed to successfully complete a task (Stein et al., 2009).
Curriculum:	“[Curriculum refers to] the complete set of learning experiences and activities that the student undergoes” (Burkhardt, Fraser, & Ridgeway, 1990, p. 6).
English Language Learner:	“[English language learners are] those students who are not yet proficient in English and who require instructional support in order to fully access academic content in their classes” (Ballantyne et al., 1008, p. 2).

Sheltered Mathematics Course:	A sheltered mathematics class is a class comprised entirely of English language learners.
Standards-Based Curriculum:	A standards-based curriculum is a curriculum built upon the foundation laid by NCTM’s (1989) *Standards for School Mathematics.*
Task:	“[A task is] a segment of classroom activity that is devoted to the development of a particular mathematical idea. A task can involve several related problems or extended work, up to an entire class period, on a single complex problem” (Stein & Smith, 1998, p. 269).

CHAPTER 2

REVIEW OF THE LITERATURE

Few researchers have examined mathematics curriculum use with English language learners, the focus of the present study. Therefore, in this chapter I review studies from two separate bodies of literature related to the focus of my study. First, I examine literature related to mathematics curricula, particularly teachers' interactions with curriculum materials. I then consider literature related to the mathematics education of English language learners. I seek to provide an overview of what researchers have learned in reference to teaching mathematics to ELL students. I close the chapter with the theoretical framework I have woven together from several sources in order to bridge these bodies of literature and answer this study's research questions.

Teachers and Curriculum Materials

A large portion of teachers' jobs entails selecting and enacting curriculum materials for their students. Ben-Peretz (1990) is one of several researchers who have asserted, "The ways in which teachers handle the curriculum determine, to a large extent, the learning processes in their classrooms" (p. 23). Because of the important role teachers play in selecting and enacting curriculum materials, I focused my study on examining this relationship as it pertains to teaching ELLs. In this section I discuss the literature related to teachers' curriculum use. I begin with a look at how I have chosen to define curriculum. I then move to a discussion of mathematical tasks and their place in the curriculum. I close this section with an examination of studies related to how teachers use curriculum materials.

Defining Curriculum

The term curriculum can refer to a number of different notions (Ben-Peretz, 1990; Howson, Keitel, & Kilpatrick, 1981; Marsh, 2009). Over 20 years ago, Portelli (1987) wrote that there existed more than 100 definitions of curriculum in the literature and stated that these definitions of curriculum fall into three categories. First, there are those that classify curriculum by the content. Second, one may define curriculum in terms of classroom activities or experiences. Finally, one may define curriculum as a plan. Marsh also classified the various definitions of curriculum. Marsh's categorizations included the "purposes of the goals of the curriculum," the "contexts within which the curriculum is found," and the "strategies used throughout the curriculum" (p. 4). Though both Marsh and Portelli examined a number of definitions of curriculum, both noted the inevitable incompleteness of such a definition, no matter how well thought out. Portelli likened the efforts of those attempting to define curriculum to a well-prepared centaur hunter; that is to say, no matter how well prepared, the effort will be fruitless.

With the multitude of definitions for curriculum, I thought it important to discuss which of these I ascribe to curriculum for this study, albeit perhaps setting myself up as yet another centaur huntress. Though the definition may fall incomplete, I hope to provide readers with a clear understanding of my use of the term. Howson et al. (1981) discussed the importance of defining curriculum and called for a more broad definition of curriculum than was commonly described at the time they wrote their publication stating, "Curriculum therefore, must mean more than syllabus – it must encompass aims, content, methods and assessment procedures" (p. 2). Similarly, Marsh (2009) cautioned readers to avoid too narrow a definition.

I have adopted one such broad view of curriculum for this study. This particular definition comes from Burkhardt, Fraser, and Ridgeway (1990) and defines curriculum as “the complete set of learning experiences and activities that the student undergoes” (p. 6). This definition would fall under Portelli’s (1987) second classification of curriculum definitions, those that define curriculum through the activities. Using Marsh’s (2009) classifications, this definition refers to the “contexts within which the curriculum is found” (p. 5). Encompassed in this definition are, among other things, textbooks, standards, tasks, and other supplemental resources.

In addition to defining curriculum in general, there are different types of curriculum discussed in the literature. Of the multitude of classifications of curriculum in the literature, I found Burkhardt et al.’s (1990) most helpful. Burkhardt et al. wrote about six types of curriculum. First, there is the ideal curriculum set forth by experts. The available curriculum consists of the teaching materials to which teachers have access. The adopted curriculum is what districts or states mandate teachers to teach. For example, using this classification one would refer to state standards as the adopted curriculum. The implemented curriculum is what the teachers teach in the classroom, while the achieved curriculum is what students learn. Finally, the tested curriculum is defined by the tests that students are to take. I return to these notions of curriculum as I later discuss the phases of task implementation used in my theoretical framework.

Classifying Tasks

I have chosen to focus my study on one particular aspect of the school mathematics curriculum, mathematics tasks. I have adopted Stein and Smith’s (1998) definition of a task as “a segment of classroom activity that is devoted to the development of a particular mathematical idea. A task can involve several related problems or extended work, up to an entire class period, on a single complex problem" (p. 269). My focus on tasks rather than curriculum in general stems from my desire to study the teachers' use of curriculum materials. Tasks are the activities teachers select for students to enact. Doyle and Carter (1984) wrote, "The study of tasks, then, provides a way to examine how students' thinking about subject matter is ordered by classroom events" (p. 130). Teachers are largely responsible for selecting particular tasks from their available curriculum but do so with the understanding that the tasks should meet the requirements of the adopted curriculum. Some researchers have chosen to define curriculum in a general sense by the tasks teachers use. For example, Doyle and Carter (1984) stated, "The curriculum consists of a set of academic tasks that students encounter in classrooms" (p. 130). I find this view of curriculum to be overly narrow as it does not take into account other factors, such as standards. I do, however, assert that the implemented curriculum by and large consists of the tasks teachers choose to use with students.

As teachers, my colleagues and I used a number of descriptions of problems such as real world, drill, and word problems. In mathematics education research there also exist different means of classifying mathematics tasks. Swan (2008) classified tasks by the processes with which they help the learner develop mathematical understandings. Becker and Shimada (1997) classified problems as either closed or open-ended, depending upon the number of correct solutions. Another classification found in the literature relates to a problem's worth in helping students learn mathematics.

The notion of classifying tasks according to their worth is presented by several sources. One such classification of this type is NCTM's notion of worthwhile tasks (NCTM, 2007). NCTM described worthwhile mathematical tasks as follows: "Worthwhile mathematical tasks are those that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite students to speculate and to pursue their hunches" (p. 33). Hiebert et al. (1997) discussed a similar notion of worth, which they referred to as appropriate tasks. Those tasks deemed appropriate problematized mathematics for students and engaged them in mathematical thinking but were commensurate with students' abilities. Though discussing the worthwhileness of tasks may be useful in some situations, I prefer to focus on the mathematical thinking the learner must use to complete tasks.

Somewhat related to classifications of a task's worth are classifications of the type of mathematical thinking required of students to complete tasks. Schoenfeld (1994) espoused his support for a "problem-based curriculum" focused on rich mathematics tasks that required high levels of student engagement and thinking stating, "The historical curriculum focus has been on such lists of topics; it has ignored issues of problem solving strategies, metacognition, belief, mathematical culture" (p. 72). The types of tasks to which Schoenfeld referred are those that go beyond procedures and algorithms. Though Schoenfeld provided characteristics of the rich tasks he described, he did not provide categorizations of different types of tasks.

Ten years prior to Shoenfeld's 1994 essay, Doyle and Carter (1984) examined tasks in an English class. Their work resulted in a discussion of two types of tasks. The first type was those referred to as major assignments. Doyle and Carter described major assignments as "higher-level tasks" (p. 145) requiring more explanation and thinking. In contrast, they categorized minor tasks as "usually accomplished with recurring and routinized procedures or…in close association with familiar assignments" (p. 145). Over a decade later, Stein and Smith (1998) built on Doyle and Carter's work as they developed a classification describing tasks' levels of cognitive demand. I discuss this notion in detail in a section to follow as it is a component of my theoretical framework. Though other, similar classifications exist to classify the mathematical thinking required to complete tasks (e.g., Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956), I have chosen to use Stein and Smith's due to its focus on mathematics tasks in particular.

Teachers' Interactions with Curriculum Materials

Teachers play a pivotal role in selecting and enacting curriculum materials for students. Bruner (1977) wrote, "A curriculum is more for teacher than it is for pupils…If it [curriculum] has any effect on pupils, it will have it by virtue of having had an effect on teachers" (p. xv). This statement mirrors others made by researchers such as Ben-Peretz (1990) and highlights the importance of the teacher-curriculum relationship on student learning. Several researchers have examined this relationship between teachers and curriculum materials. Sherin and Drake (2009) reviewed the literature and discussed three key processes in which teacher curriculum use falls—studies on adapting materials, reading materials, and evaluating materials. For the purposes of this study, I have focused my review of teacher curriculum use studies on those that discus the adapting of materials. I have chosen this focus because of my interest in how teachers select and use materials for ELL students.

Earlier studies examining teachers' curriculum interactions often centered on teachers' use of textbooks, with researchers viewing the textbook as the sole determinant of curriculum (e.g., Chambliss & Calfee, 1998; Love & Pimm, 1996; Walker, 1976). Many of these studies examined how closely teachers followed the textbook or which aspects of the textbook teachers relied upon most heavily (Remillard, 2005). More recently, studies have arisen in response to standards-based curriculum materials (e.g., Cohen, 1990; Llloyd, 1999; Remillard & Bryans, 2004). The authors of these studies tend to hold a broader definition of curriculum and take into account resources other than textbooks.

Remillard and Bryans (2004) studied middle school teachers' uses of standards-based mathematics curricula. This study resulted in a framework for describing the ways in which teachers use curriculum materials. Intermittent and narrow refers to a minimal use of curriculum materials. Teachers who use curriculum materials in this way tend to rely upon teaching routines and other resources. Adopting and adapting refers to the use of curriculum materials to provide the "general structure and content" (Remillard & Bryans, p. 374). Finally, piloting refers to use of all curriculum materials as intended by the developers. Other researchers have proposed similar ideas; in particular Brown and Edelson (2003) discussed the adapting, offloading, and improvising of curricular resources, ideas roughly correlated to those proposed by Remillard and Bryans. Though other frameworks exist, I found these two the most helpful when describing teachers' use of tasks.

The Mathematics Education of English Language Learners

In the time since my parents immigrated to the United States in the 1960s, researchers have made great strides in mathematics education research regarding ELLs. The literature related to this area is relatively new, with much of the research occurring in the past two decades due to projects such as the Center for the Mathematics Education of Latino/as (CEMELA). In this section I discuss the literature pertaining to the mathematics education of English language learners. In particular, I examine how researchers, including myself, define this segment of the student population. I then discuss findings related to the teaching of ELLs and the few mathematics curriculum studies focused on ELLs.

Defining ELLs

Different states use different phrases to describe the subset of students I refer to as English language learners. Some of the more common phrases describing students whose first language is not English are English as a Second Language (ESL), English Speakers of Other Languages (ESOL), Limited in English Proficiency (LEP), and English Learner (EL). In addition to the variety of terms for English language learners, researchers often carry out research on this group of students in conjunction with other foci. These foci might be bilingual education studies or studies focused on a particular ethnicity such as Latino/as.

For this study I have chosen to use the term English language learners (ELLs) because this classification avoids the deficient classification connoted by the LEP label. Furthermore, researchers in mathematics education more commonly use the descriptor ELL. The U.S. Department of Education refers to “national-origin-minority students who are limited-English-proficient” (U.S. Department of Education, 1999, p. 3) as ELLs. I have adopted Ballantyne et al.’s (2008) definition of ELLs as “those students who are not yet proficient in English and who require instructional support in order to fully access academic content in their classes” (p. 2).

Kersaint, Thompson, and Petkova (2009) discussed 10 different formats of ELL instruction in the United States. In some states, such as Georgia, some schools create sections of content courses comprised entirely of ELLs, so called sheltered courses. The students in these courses typically have yet to pass an English proficiency test. Schools that do not have sheltered courses due to low numbers of ELL students often place ELL students in mainstream mathematics courses with native English speakers. Similarly, schools will typically mainstream ELL students who have passed an English proficiency test and require teachers to accommodate these students until they no longer require services. It is important to note that different states use different instruments to assess students’ language abilities; therefore, classifications may vary state to state.

Teaching ELLs Mathematics

Studies related to English language learners are a relatively small, but growing, subset of mathematics education literature. Early research on ELL students focused on the challenges students encountered completing or interpreting language-rich problems (Téllez, Moschkovich, & Civil, 2011). These early studies framed the teaching of ELL students as a problem and failed to value students' abilities to speak multiple languages as an asset. Gradually, studies have shifted from a focus on computation to a more broad focus on ELL students' reasoning and problem solving in mathematics (Téllez et al.). These more recent studies also aim to understand the ELL students' multilingual assets as a means of leveraging mathematical understandings (e.g., Moschkovich, 2002).

In 2008 CEMELA was established with funding by the National Science Foundation. CEMELA was a joint partnership among the University of Arizona, the University of California Santa Cruz, the University of Illinois Chicago, and the University of University New Mexico. CEMELA is described as "an interdisciplinary, multi-university consortium focused on the research and practice of the teaching and learning of mathematics with Latino students in the United States" (CEMELA, 2012, para. 1). Because a high proportion of Latinos/as is labeled ELL, much of CEMELA's work related to the mathematics education of ELLs. The focus of many of the CEMELA studies was the examination of the mathematics education of Latino/as and their culture (e.g., Díez-Palomar, Simic, & Carley, 2007; López Leiva, 2010; Turner, Varley Gutiérrez, Simic-Muller, & Díez-Palomar, 2009) and on family involvement in students' mathematics learning (e.g. Civil, 2009; Acosta-Iriqui, Civil, Díez-Palomar, Marshall, & Quintos, 2011). Another area in which literature on ELLs is more abundant, both within and outside of CEMELA, is assessment (e.g., Abedi & Herman, 2010; Fernandes, Anhalt, & Civil, 2010;

Solano-Flores, 2003). Studies directly examining the teaching of mathematics to ELLs are less common, especially at the secondary level.

Existing studies examining teaching mathematics to ELLs tend to provide a number of recommendations for ELLs guided by either empirical research or reviews of existing literature. Though these recommendations may seem to be attributes of good teaching in general, Coggins, Kravin, Coates, and Carroll (2007) explained,

> If we teach mathematics by following commonly accepted ‘best practices,’ we may actually overlook English learners, because they have very specific needs. On the other hand, if we teach mathematics in ways that benefit English learners, then all students will benefit from the rich repertoire of strategies designed to create access to mathematics content. (p. ix)

Other researchers, including myself, support this view of accommodating those students historically underserved by schools and the resulting positive impact on all students (e.g., Ladson-Billings, 1995).

There exist a number of books written for teachers of ELL students. These books provide descriptions and examples of instructional strategies for ELLs based upon the extant literature in this area (e.g., Coggins, Kravin, Coates, & Carroll, 2007; Echevarría, Vogt, & Short, 2010; Kersaint, Thompson, & Petkova, 2009). A major theme of many of these books is the development of ELL students’ mathematical vocabularies, also referred to as academic language. The strategies suggested to aid in this endeavor include use of multiple representations, especially visual, increased classroom communication, and building on students’ culture and prior knowledge (Coggins et al.; Echevarría et al.; Kersaint et al.).

Building students’ academic language is an especially important and challenging aspect of teaching ELLs mathematics. I use the term academic language to mean “the specialized words and phrases related to content, procedures, the activity of learning, and expression of complex thinking process" (Coggins et al., 2007, p. 15). Academic language means more than learning words; it suggests the meaningful use of words in mathematical contexts. Barnett-Clarke and Ramirez (2004) further explicated, "not only do students need explicit instruction to read and write mathematical symbols and words, they also need to learn how to express mathematical ideas orally and with written symbols" (p. 57). Several researchers have discussed the importance of going beyond the learning of words in order to build ELL students' mathematical understandings (e.g., Brenner, 1994; Garrison & Mora, 2005; Moschkovich, 2002). Khisty and Chval (2002) wrote the following regarding ELLs' learning of vocabulary.

> The words represent meanings that are waiting to be developed and eventually internalised. Therefore, which words are presented to the students and how they are developed are vitally important. Just as important is that students have opportunities to use these words in their talk and as they work. (p. 155)

Because of the potential difficulties ELL students encounter as they try to develop an academic language alongside a second language, this area is, understandably, of great importance to researchers who examine the teaching of mathematics to ELLs.

The use of multiple representations has been advocated by NCTM for all students for a number of years (NCTM, 2000) and is an important accommodation to support ELL students' development of academic language. Common in the ELL literature is the suggestion that teachers include visual representations such as graphic organizers or diagrams when introducing new vocabulary to students (e.g., Brenner, 1994; Coggins et al., 2007; Echevarría et al. 2010; Kersaint et al., 2009). Multiple representations might also include manipulatives or technology (Coggins et al., 2007). Coggins et al. explained the importance of multiple representations for ELLs by noting, "When English learners are exposed to multiple representations of a concept, including various concrete representations, they have increased access to verbal information and more opportunities to develop mental models and solid understanding" (p. 42). Garrison and

Mora (2005) echoed this sentiment saying that when teachers relate new vocabulary to concrete materials for ELLs, the terms tend to be easier to recall and understand.

Many researchers have asserted that classroom communication is also crucial to ELLs' development of academic language (Brenner, 1994; Coggins et al., 2007; Khisty & Morales, 2004; Moschkovich, 1999). Moschkovich (2010) explained,

> Researchers in vocabulary acquisition agree that the best way for students to develop mathematical vocabulary is have opportunities provided for them to actively use mathematical language to communicate about and negotiate meaning for mathematical situations…One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical ideas, rather than on pronunciation, vocabulary, or low-level linguistic skills. (p. 21)

Because of ELL students' limited proficiency in English, this focus on communication may seem difficult to achieve, especially in classrooms with monolingual teachers; however, many experts agree that it is through communication that students will improve English proficiency as well as mathematical understandings.

One particular aspect of classroom communication that is stressed throughout the literature is questioning. Coggins et al. (2007) explained the importance of questioning,

> Particularly for English learners, the questions that a teacher asks affect students' access to a lesson, the level of engagement, and the degree of mathematical learning that takes place. Questions can act as a catalyst for the use of spoken language. (p. 73)

While the use of questioning is important, studies have reported that teachers tend to use a higher proportion of low level questions when teaching ELL students (Gall, 1984; Hill & Flynn, 2008). In addition to questioning, researchers have discussed the importance of allowing students to use oral and written communication in the mathematics classroom as a means of building academic vocabulary (Barnett-Clarke & Ramirez, 2004). These communications might occur in only

English or, as in the case of bilingual classrooms, occur in both the students' native languages and English.

The incorporation of students' cultures and prior knowledge into mathematics instruction has also been the focus of several studies in the mathematics ELL literature. Many ELL researchers whose work focuses on mathematics education have suggested scaffolding tasks to build on students' prior knowledge and cultures (e.g., Coggins et al., 2007; Echevarría et al. 2010; Kersaint et al., 2009). Experts who suggest this approach to mathematics teaching tend to perceive students' knowledge of multiple languages and cultural experiences as levers to learning mathematics rather than barriers. In addition to teaching strategies that bring in students' cultures, there have been curriculum studies that have similar foci, which I discuss in the following section.

ELLs and Mathematics Curricula

The purpose of my study was to understand teachers' selections and enactments of mathematics tasks for ELL students. This study bridges the areas of mathematics curriculum research and the teaching of ELLs. Studies examining mathematics curricula and ELLs are less common that those on teaching ELL students mathematics. I have already discussed studies related to teachers' uses of curriculum materials in general. In this section I discuss studies that examined curriculum and curriculum use with ELLs.

Many projects that involve both ELL students and curriculum have focused on culturally relevant curricula. A rather recent subset of mathematics education called ethnomathematics focuses on this area (Barton, 1996; D'Ambrosio, 2006). Other studies tend to focus on the development of curriculum materials for ELLs (e.g., Freeman & Crawford, 2008) or the evaluation of a curriculum's appropriateness for ELLs (e.g., Khisty & Radosavljevic, 2010;

Lipka et al., 2005). Far fewer researchers have examined how teachers select or use curriculum materials for ELL students (Chval, 2010).

A recent study by Pitvorec, Willey, and Khisty (2011) is one of the few existing studies examining teachers' curriculum use with ELLs. In this study, Pitvorec et al. examined teachers' use of *Finding Out/Descubrimiento*, a curriculum created over 40 years ago by bilingual education experts to accommodate ELLs. Examining the use of this curriculum, the authors developed a framework intended to guide curriculum development and implementation for ELLs. The framework suggested that one must first adopt an ideology that embraces students. Those responsible for ELLs' mathematical learning must then attend to three "interactional spaces" (p. 419)—language and communication, curriculum materials, and learning communities. Though I found this framework helpful to consider, I found it lacked the specificity necessary to be of use in my analysis. Furthermore, the framework seemed to be aimed specifically at bilingual settings, limiting its application in the vast majority of classrooms.

Remillard and Cahnmann (2005) also examined teachers' use of curriculum materials with ELLs in 2 third grade classrooms. They proposed the model for mathematics teaching in culturally diverse mathematics classrooms shown in Figure 1. The authors claimed that "accomplishing genuine change in urban classrooms requires teachers to push their practices along both continua, which involves integrating sound mathematics teaching with practices that are culturally contextualized" (p. 175). In examining this model, conceptually oriented mathematics corresponds to what Stein and Smith (1998) refer to as high cognitive demand tasks, and procedurally oriented tasks correspond to low cognitive demand tasks. Culturally contextualized refers to the need for mathematical learning "to be embedded in classroom contexts that are accessible to students" (Remillard & Cahnmann, p. 175). Remillard and

Cahnmann made it clear that this culture does not have to be the students' home culture but could be shared experiences from the classroom. They also stated the importance of teachers selecting, creating, or modifying tasks that are both rigorous and appropriate for their students. Though I had initially included the cultural contextualization of tasks as part of my theoretical framework I have since removed it because, after analyzing my data, I found no instances cultural contextualization of tasks.

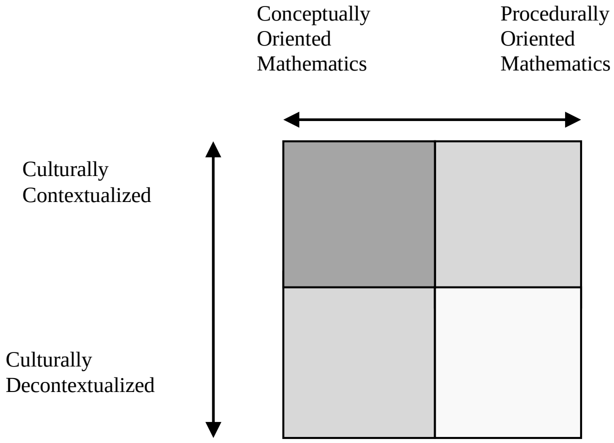

Figure 1. Relationship between cultural contextualization and conceptual tasks. Adapted from "Researching Mathematics Teaching in Bilingual-Bicultural Classrooms," by J. T. Remillard, and M. Cahnmann, 2005, *Language, literacy, power, and schooling* in T. McCarty (Ed.), (pp.169-188) Hillsdale, NJ: Erlbaum. p. 175. Copyright 2005 by Lawrence Erlbaum Associates.

The dearth of studies related to teachers' curriculum use with ELL students prompted my interest in this area. In the following section I provide a detailed description of the theoretical framework I have adopted for my study. This framework brings together aspects of curriculum studies and the ELL literature to allow me to examine how teachers select and enact mathematics tasks with ELLs.

Theoretical Framework

> Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students. (NCTM, 2000, p. 11)

Teachers must make appropriate accommodations to teach all students. No Child Left Behind (2001) mandated that all students, including ELL students, meet state-defined standards by the 2013-2014 school year. Therefore, teachers, school districts, and universities are under pressure to find effective strategies to teach ELL students.

I developed my research questions to understand teachers' selections and enactments of mathematical tasks with ELL students. In particular, my research questions correlate to the factors influencing the task set up and implementation phases developed by Stein, Grover, and Henningsen (1996). The rectangles in Figure 2 illustrate these phases, while the circles in Figure 2 represent the factors that may influence the cognitive demand between the phases. Examining the types of tasks teachers select from their curriculum materials is the focus of the first question. Determining what factors affect the set up phase is the focus of the second research question. With the third question, I sought to find factors influencing the final phase, the mathematical task as implemented by the students. I present my theoretical framework in accordance with these phases and include a discussion of cognitive demand in the following sections.

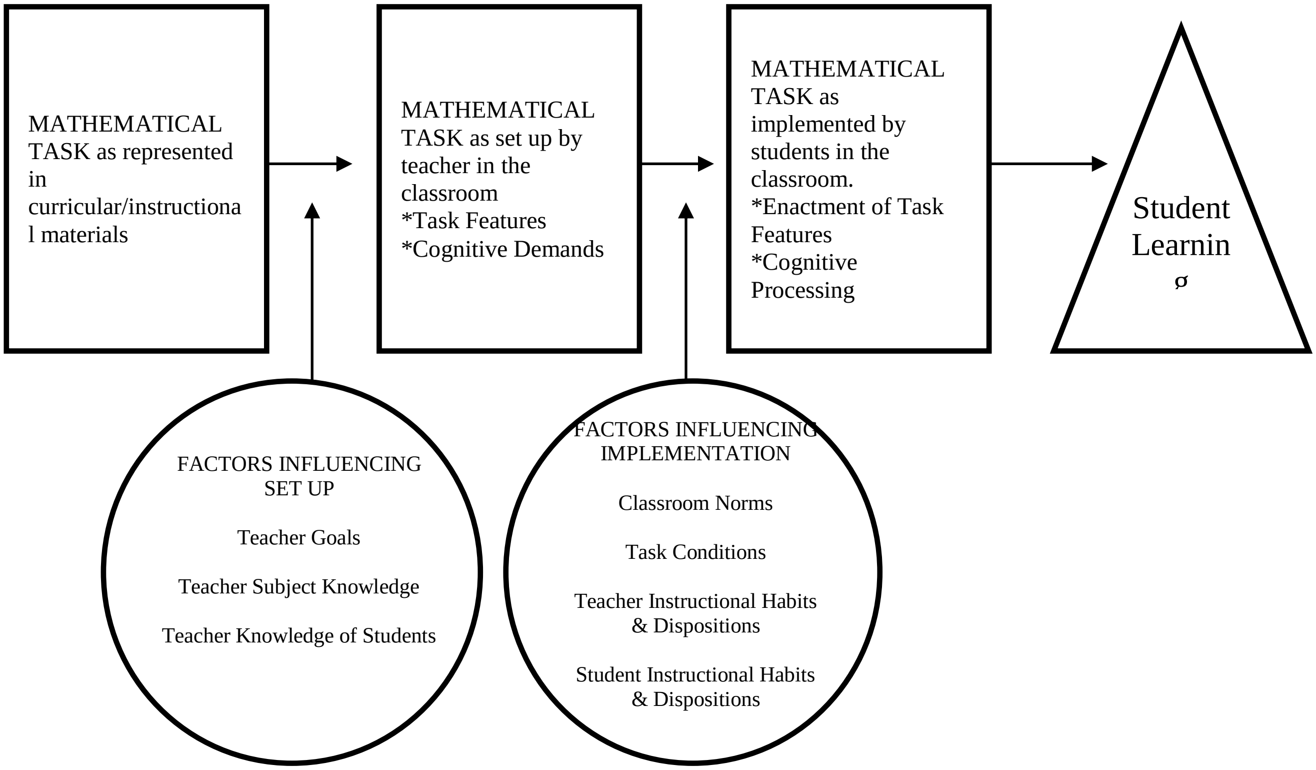

Figure 2. Relationship among task-related variables and student learning. Adapted from "Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms," by M. K. Stein, B. W. Grover, and M. Henningsen, 1996, *American Educational Research Journal, 33,* p. 459. Copyright 1996 by the American Educational Research Association.

Cognitive demand. In order to respond to my first research question I examined the tasks as they appeared in the curriculum materials, the first phase of Stein et al.'s (1996) framework. This phase is akin to what Burkhardt et al. (1990) referred to as the available curriculum. I began by categorizing the cognitive demand of the mathematical tasks using the Task Analysis Guide developed by Stein et al. (2009). Cognitive demand refers to the level of thinking required to complete a task. The Task Analysis Guide defines four levels of cognitive demand, two related to high cognitive demand and two related to low cognitive demand.

The two levels related to low cognitive demand are *Memorization Tasks* and *Procedures without Connections Tasks*. Tasks categorized as Memorization Tasks focus on the reproduction of mathematical facts or formulas previously learned by students (Stein et al.). Asking a student to determine what fraction is equivalent to 0.20 is an example of this type of task. Procedures without Connections Tasks are characterized as being algorithmic in nature. For example, asking students to determine the price of an item that costs $480 and is on sale for 20% off is a Procedures without Connections task because the use of the algorithm is evident due to the nature of the problem (Stein et al.). It is important to note that each of these categorizations is dependent upon students' knowledge. For example a task that may be a memorization task for one grade level may not be classified as such in prior grades where the necessary facts have yet to be learned.

The first of the levels related to high cognitive demand is termed *Procedures with Connections Tasks*. As the name implies, these types of tasks utilize procedures; however, the purpose of the procedures is for students to further develop mathematical understanding (Stein et al., 2009). An example of this type of task would be to ask students to identify decimal and percent equivalents for a proper fraction utilizing a 10 x 10 grid (Stein et al., p. 3). Procedures with connections tasks differ from procedures without connections tasks in that they require students to use the procedures so as to "build connections to underlying concepts and meaning" (Stein et al., p. 2). Procedures without connections tasks fail to connect to the underlying mathematical concepts, allowing students to complete a procedure and move on without the mathematical thinking required to delve deeper into the reasons for the use of the procedure or how the procedure may connect to other mathematical concepts or representations. The final level is *Doing Mathematics Tasks.* In these tasks students must discover mathematical concepts and their connections to one another (Stein et al.). The following is an example of a Doing Mathematics Task,

> Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of area that is shaded, b) the decimal part of area that is shaded, and c) the fractional part of area that is shaded. (Stein et al., p. 3)

The cognitive demand of a task may change during each of the three successive phases of task implementation. Therefore, when discussing the cognitive demand of tasks used in a classroom, it is important to note the phase in which the task resides. In the following sections I examine the set up and implementation of tasks and the factors that may impact the cognitive demand at each of these phases.

Mathematical tasks as set up by the teacher. Smith and Stein (1998) identified factors associated with both the maintenance and decline of cognitive demand in tasks. In Figure 2, the circle on the left is the focus of my second research question. This circle focuses on what happens between the selection and setup phases of the task. The set up phase is the teacher's explanation to students about the expectations for the work they are to complete and the resources they are to use (Stein et al., 2009). This phase may be construed as the beginning of the curriculum as implemented (Burkhardt et al., 1990). The factors that may impact this phase include the teacher's subject knowledge, goals for the lesson, and teacher's knowledge of his or her students (Stein at al., 1996).

Teacher subject knowledge could manifest in several areas. First, if a teacher does not understand the mathematical goals of the task as presented in the curricular materials, he or she may alter it and lose some of the original mathematical goals. Similarly, the teacher may miss some of the mathematical connections inherent in the task’s original presentation. Furthermore, I include pedagogical knowledge within the area of teacher subject knowledge. This implies that teacher subject matter knowledge can impact pedagogical decisions such as the amount of time provided for the task and the required student products. This could also affect whether the teacher remains focused on students understanding mathematical ideas or simply providing correct responses. Though I acknowledge the potential impact teacher subject and pedagogical knowledge may have on tasks, I did not measure these attributes for this study.

In terms of goals for the lesson, teachers ultimately decide the goals for the lesson (Brown, 2009). Thus, if the teacher’s goals for the lesson differ from the goals of the task as defined in the curricular materials, he or she may alter the task to reflect his or her goals. For example, a teacher may choose a mathematical task from the curricular materials in which the stated objective is for students to understand ellipses. If the teacher’s goal is for students to graph an ellipse from the standard equation, he or she may alter the task to omit any questions or references in the task that go beyond the graphing of an ellipse.

The teacher’s knowledge of her students influences several factors. For instance, knowing students’ abilities and prior mathematical knowledge could influence whether the task appropriately builds on student thinking and reasoning and thus is appropriate for the students’ abilities. Knowing one’s students could also influence whether the teacher provides sufficient time for exploration. Additionally, knowledge of one’s students could lead to the inclusion of cultural contextualization in the tasks as defined by Remillard and Cahnmann (2005). Cultural contextualization is the use of contexts in mathematical tasks that are relevant to students’ lives, either outside of or within the mathematics classroom.

Mathematical tasks as implemented by the students. The framework for my third question builds on the work of three separate studies, relying most heavily on the Stein et al. (1996) framework. No one framework seemed to suit the purposes of my study, so I utilized aspects of frameworks developed from non-ELL classrooms and ELL classrooms. Including aspects from these different settings allowed me to analyze classroom observational data to fully explore my third research question.

In Figure 2, the circle on the right is the focus of my third research question. This circle refers to the factors influencing the implementation phase of the task. The implementation phase begins as the students start work on the task and ends when the class moves on to a new task. The factors in this circle could impact the cognitive demand between the setup and implementation phases of the task. I discuss each of these factors and the guiding frameworks for each below.

Mathematics Tasks Framework. Within the circle on the right are four categories. The first, *classroom norms*, refers to the teacher's expectation of what work the students will do and what the expectation is for quality and accountability. *Task conditions* refer to the extent to which the tasks build on prior knowledge and the matching of tasks to students' abilities. The *teacher instruction habits and dispositions* refer to what type of help the teacher provides to students when they are having trouble with a problem and how long a teacher will allow the students to struggle prior to providing that help. Finally, *student learning habits and dispositions* refers to the amount of time students are willing to struggle when working on difficult mathematical problems and the amount of self-monitoring in which students engage (Stein et al., 1996).

In addition to the aforementioned factors, Stein and Smith (1998) identified the routinization of aspects of the task and the students' lack of accountability for the process as contributing to the decline of cognitive demand of tasks. I think these factors affect the implementation phase of the task and thus have placed them into the category of teacher instruction habits and dispositions described previously. In terms of the maintenance of high-level cognitive demand, Stein and Smith suggested students' engagement in self-monitoring and teachers' uses of scaffolding, the modeling of high-level performance, and pressing for meaning as crucial. I have placed student self-monitoring in the category of student learning habits and dispositions and each of the others into teacher instruction habits and dispositions.

Although I agree that these factors seem to affect the cognitive demand of mathematical tasks, I think that including factors that address the classroom communication in more depth are necessary. Furthermore, the ELL literature asserts that communication is a key factor in helping ELLs develop academic language (e.g., Coggins et al., 2007; Moschkovich, 2002). One of the main challenges in teaching ELL students is communication due to the difference in the student's native language and the language in which he or she receives instruction. Therefore, I have drawn on Hufferd-Ackles, Fuson and Sherin's (2004) work to further explore classroom communication factors potentially influencing the cognitive demand of mathematical tasks.

Math-Talk Learning Community. Hufferd-Ackles et al. (2004) developed a framework for examining mathematical discourse in the classroom. This framework was initially developed in a study conducted in a predominately Latino elementary classroom. Their framework described what they have termed a Math-Talk Learning Community and consists of action trajectories for both the students and the teacher. Their model has four components: questioning, explaining mathematical thinking, sources of mathematical ideas, and responsibility for learning.

Each of these components can fall in one of four levels. The levels (0-3) correspond to a change from a teacher-centered classroom to a classroom where the teacher acts as a co-teacher and co-learner (Hufferd-Ackles et al.).

Questioning refers to whether the questions are teacher or student generated and the types of responses these questions generate. Therefore, the shift in this component would be from a classroom where the teacher asks short response questions to one in which the students are posing more why questions to one another. The second component focuses on the extent to which the teacher asks students to explain their mathematical thinking. The sources of mathematical thinking could shift from the teacher (level 0) to the students (level 3). Finally, the responsibility for learning refers to the shift in students from passive recipients to active participants in mathematical understanding. For example, in a level 0 classroom, the students may sit and listen as the teacher states the correct answers and tells them what they are required to know. In a level 3 classroom, on the other hand, the students would help one another to understand the mathematical ideas and correct each other's misunderstandings (Hufferd-Ackles et al., 2004).

I substituted this framework for what Stein and Smith referred to as *Teacher Instructional Habits and Dispositions* and *Student Learning Habits and Dispositions,* collapsing these categories into what I call *Classroom Habits and Dispositions*. As previously discussed, teachers' and students' habits and dispositions suggest features of pedagogical and learning characteristics that can impact how both the teacher and students engage in classroom activities. Included in these characteristics are things such as the degree to which a teacher will allow his or her students to work on a challenging problem, the type of help that the teacher will provide to struggling students, and the degree to which students will continue to work on challenging problems (Stein et al., 1996).

The use of the Math Talk Learning Community framework allowed me to investigate these areas in a more detailed way. Although the Math Talk framework is sufficient for the categorization of the classroom habits and dispositions, it did not address the areas of classroom norms and task conditions. I think these areas have a great impact on the cognitive demand of tasks during the implementation phase; thus, I found that using the Math Talk framework alone was inadequate for my study.

Building on each of these frameworks and collapsing overlapping categories, I have arrived at the following model (Figure 3) that encompasses the classroom aspects I sought to explore in my third question. This model is a replacement for the right circle in Figure 2. Classroom norms refer to the negotiation between the teacher and students as to what work is to be completed, what the quality of that work will be, and what the students are accountable for producing. Task conditions are the conditions that the teacher imposes on the task. These include using tasks that build on student knowledge and that are appropriate to their ability level. I also include the time allotted for the completion of the task. I explored the classroom habits and dispositions via the Math-Talk community framework. The use of this framework in the analysis of my data as well as the methodology I employed in collecting my data is discussed in the following chapter.

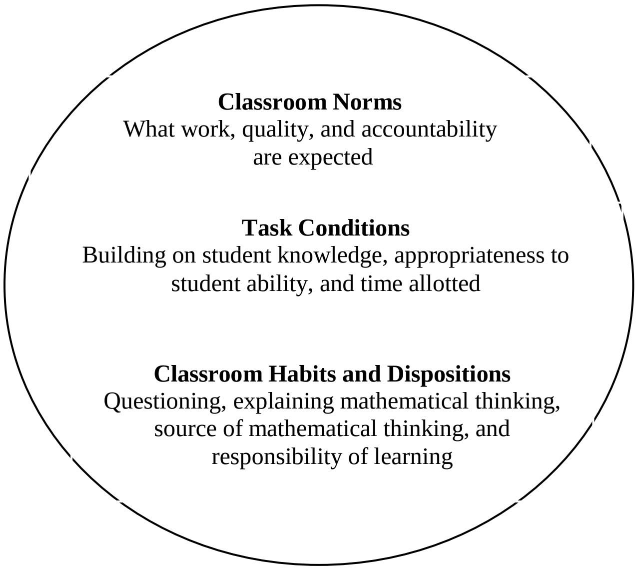

Figure 3. Factors influencing the cognitive demand of tasks as implemented by the students. Adapted from “Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms,” by M. K. Stein, B. W. Grover, and M. Henningsen, 1996, *American Educational Research Journal, 33,* p. 459. Copyright 1996 by the American Educational Research Association.

CHAPTER 3

METHODS

In this chapter I discuss the methodology used for this study. In particular, I discuss the choice of a case study design, the setting and participants, my subjectivities, and the specifics regarding the collection and analysis of the data. I collected all the data for this study during the spring of 2011. In thoroughly describing the methods used, I hope to allow the reader access to the purposes behind my decisions. I begin with a discussion of the decision to use a case study design.

Case Study Design

The purpose of this study was to explore how teachers select and enact mathematics tasks with ELLs. In order to accomplish this purpose, I employed a qualitative research methodology. Using qualitative research methods allowed for a deep exploration of the relationships and events that occurred between the teachers and their ELL students in the mathematics classroom. Patton (2002) referred to qualitative methods as facilitating "study of issues in depth and detail" (p. 14). Maxwell (2005) discussed several goals well served by qualitative methodology including understanding the meaning of events, experiences, or actions for participants in the study, understanding the participants' contexts and that context's possible influence on the participants' actions, understanding unanticipated events or influences, and understanding the processes that lead to events or actions. Qualitative research methods allowed for a detailed account of the classroom environment in context through the participants' words and actions.

In choosing a particular tradition of inquiry with which to conduct my study, I first re-examined the study's goals. I decided case study would be the most appropriate methodology because the purpose of a case study is to get an in depth examination of a case in a particular context. Stake (1995) explained "case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances" (p. xi). deMarrais and Lapan (2004) described that case studies "seek to answer focused question by producing in-depth descriptions and interpretations over a relatively short period of time" (p. 218). Due to my desire to obtain an in depth analysis of secondary ELL classrooms in context, the case study method was the best fit.

Three secondary ELL mathematics teachers served as the cases for this study. These cases were the units of analysis for a multiple-case study design. Multiple-case study allowed for a more thorough understanding about a particular topic and for theorizing in a broader context (Berg, 2007). Yin (2009) suggested that multiple-case studies can be "considered more compelling, and the overall study is therefore regarded as more robust" (p. 53). The multiple-case study design afforded me the opportunity for both a better understanding and results that are more robust.

Participants and Setting

I employed a purposeful, criterion based sampling for this study (Patton, 2002) in which I examined the selection and implementation of mathematical tasks for ELLs by secondary mathematics teachers. Thus, I selected mathematics teachers who taught classes entirely comprised of ELL students, so called sheltered classes. Guidance counselors placed students in the sheltered mathematics classes due to recommendations from previous teachers and test scores on district English proficiency exams.

In the following sections I introduce the reader to each of the three participants. One of the strengths of a qualitative methodology is the ability to highlight the participants' voices (Patton, 2002). In order to understand the decisions these teachers made in the classroom, it is important to first understand the teachers and their backgrounds. In addition to providing a description of each of the participants I describe the schools at which they taught and the students in the classrooms I observed.

Three teachers participated in this study. I chose three because I wanted to observe classrooms with differing dynamics. If I had chosen a single classroom, I would have no comparison to how other classrooms operate. If I had chosen two classrooms, I think it would have been possible that I would have gotten two similar classrooms or two very different classrooms. As it turned out, each of the cases provided a slightly different perspective on the research questions but had enough similarities to allow me to draw some general conclusions related to the teachers' practices.

I found the process of selecting the teachers somewhat challenging. Initially, I wanted to focus my study on teachers who regularly enacted high cognitive demand tasks. Due to time and cost, I limited my search to a two-hour radius of my home. I first identified school districts within my search radius that contained sizeable ELL populations. I did this by looking on the Georgia School Report Card website (Georgia Department of Education, 2011) and researching school districts with the largest ELL populations. I then contacted the ELL coordinators at these school districts to get the names of high school mathematics teachers who taught sheltered mathematics courses. This narrowed my list considerably as not all districts provided sheltered mathematics courses. I then contacted the teachers to ask if they would be willing to participate in my study. Initially, 6 teachers expressed a willingness to participate; however after explaining what participation would entail, this list narrowed to one teacher. The teacher, Natalie Hunter, was a teacher with whom I had worked with in the past. Natalie Hunter put me in contact with her colleague, Guy Dubois, who also agreed to participate. A friend suggested one additional teacher who agreed to participate. These three teachers comprised my final participants. The difficulty of finding teachers willing to participate in studies, especially with traditionally underserved students, seems to be a common occurrence as I have discussed this issue with a number of colleagues who have faced similar challenges.

Meg Thomas[1] and Turner High School. Old farmhouses and manufactured homes sitting on large tracts of land dotted with cows and horses lined the two lane, bucolic road leading to the town of Turner. The number of churches along this rural drive seemed too great for the seemingly sparse number of inhabitants of the area. A few taxidermy shops, a bait shop, and a gas station seemed to be the only commerce on the way to Turner. Boarded up gas stations with analog pumps and abandoned corner stores served as evidence the area used to be more bustling. As one gets closer to the town of Turner, several planned communities pop up and the density of houses increases. The city's downtown seemed to have come across hard times since its heyday. There were several abandoned storefronts in the downtown area but also some signs of renewal with new shops donning signs announcing "now open." Abandoned mills and industrial spaces now stood empty on the edges of downtown, remnants of the once booming industries that have since gone defunct. Turner High School was just outside of downtown in a residential neighborhood.

Turner High School was a midsized school of about 1300 students in grades 9-12. Classified as a Title 1 school, over half of the students at Turner were eligible for free or reduced price lunch. The student body of Turner High School was approximately 6% Asian, 13% Black,

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[1] All names are pseudonyms

11% Hispanic, 65% White and 5% Multi-Racial. As of 2009, the school had classified about 4% of Turner's students as English language learners (Georgia Department of Education, 2011).

The school district appeared to have kept Turner High School in good condition as the paint, flooring, and furniture appeared in good repair. School spirit was evident at Turner High School. Visitors to Turner saw the mascot prominently featured throughout the school and on many items of the students' clothing. Ms. Thomas' room was located two halls down from the entrance. The Turner High School spirit was again evident in Ms. Turner's classroom.

When I entered the classroom, my gaze traveled to the numerous decorations around the classroom. Ms. Thomas had decorated nearly every inch of the classroom. She had painted the school's colors everywhere from the plaid painted rocking chair to the polka dotted bookshelves. Ms. Thomas' personal touches extended to the bulletin boards and various other spaces filled with pictures of her with students. Two of the bulletin boards were specifically devoted to graduation pictures and contained motivational messages such as "The tassel is worth the hassle."

Ms. Thomas had arranged her desks in two mirrored halves. Each half consisted of rows of desks running from the wall to the middle of the classroom. The middle of the classroom was left open for people to walk from the door to Ms. Thomas' desk. The desks were quite close to one another, allowing students to easily interact during class.

For this study, I specifically targeted experienced teachers who taught sheltered, high school mathematics courses and were ESL certified. A friend who was aware of my search for such a teacher put me in touch with Ms. Thomas, one of his colleagues.

Meg Thomas was in her sixth year teaching high school mathematics. She began teaching immediately after earning her Bachelor's degree in mathematics education at a local university and was in her late twenties. Meg was a monolingual white woman. She was certified in both ESL and gifted education. She earned a Master's degree in instructional technology from an online university.

Ms. Thomas was a bubbly woman who smiled easily. Her southern upbringing was evident in her southern drawl as she greeted her students at the door and called them "baby." Ms. Thomas kept up-to-date on her students' interests and extracurricular activities, often congratulating students on homeruns or other outstanding performances. She mentioned she regularly attended student activities because she enjoyed supporting her students and the school.

Turner High School was on a block schedule, meaning students have four classes each semester that meet for approximately 90 minutes daily. The focus of this study was Ms. Thomas' third block, ninth grade sheltered Mathematics 1 course coupled with mathematical support. Mathematics 1 was a ninth grade mathematics course in Georgia and was considered an integrated mathematics course because it contained units on algebra, geometry, and statistics. The support was provided because of the students' levels of English proficiency. Due to the extra support needed for the course, the class met for the entire academic year instead of a single semester. The school had classified each of the 14 students in the class as an English language learner who required additional support due to language limitations, earning the course the "sheltered" moniker. Ms. Thomas' mathematics class allowed for additional support due to its small size and extended length. The two weeks I spent in Meg Thomas' classroom spanned two units. Meg spent the first eight observation days on a quadrilaterals unit and the last two days focused on the beginnings of a probability unit.

There were 14 students in Ms. Thomas' third period classroom. Four of the students were Hmong and the other 10 were Latino/a. The school ESOL coordinator placed the students into this class due to their scores on an English proficiency test. The students varied greatly in their spoken English abilities, with some of the students able to hold conversations in a seemingly effortless manner and others struggling to find the English words to convey their thoughts. The students were mostly ages 14 and 15 and were in the ninth grade for the first time.

Natalie Hunter, Guy Dubois, and Easton High School. The two other participants in this study taught at Easton High School, located in the city of Easton. The drive to Easton was reminiscent of my drive to Turner. The scenery along the highway leading to town alternated between rural areas and small towns. The rural areas included acres and acres of farmland filled with various animals and animal housing. The small towns varied with some consisting of little more than a single newly developed strip mall and others having a traditional downtown area.

Easton was a small city of less than 40,000 residents. The city’s main industry was poultry processing plants, which drew a substantial number of immigrant workers, both documented and undocumented, primarily from Mexico and Latin America. Easton High School’s student demographics mirrored that of the residents. Easton High School was a Title I school, and its student body was 4% Asian, 22% Black, 49% Hispanic, and 23% White, and 2% other (Georgia Department of Education, 2011). Approximately 20% of the students at Easton High School were receiving services due to their classification as an English language learners. During the 2009-2010 school year, 69% of the students at Easton High School were eligible for free or reduced price lunch.

Easton High School exhibited as much school spirit as Turner High School. Their mascot was prominently featured throughout the school buildings as were student-made signs cheering on the various sports teams. The school was in good repair and each of the students wore an identification card hanging from a lanyard around his or her neck. Both Natalie Hunter and Guy Dubois' classrooms were housed in the ninth grade area of the building.

There were many similarities among Guy Dubois and Natalie Hunter's classrooms. Both classrooms were relatively unadorned save a few student-made art pieces and mass-produced mathematics posters. Both teachers had configured the desks into rows facing the front of the room. Each teacher had a desk area filled with stacks of papers and a laptop provided by the school. The rooms' stark nature lacked the personality of Meg Thomas' room but somewhat mirrored the distance these teachers maintained with their students during class.

Natalie Hunter had been teaching for six years, four of which had been at Easton High School. Initially receiving degrees in special education and then law, Ms. Hunter transitioned to teaching mathematics after passing the state assessment for mathematics certification. In addition to certifications in mathematics and special education, Ms. Hunter also passed the state assessment for ESL certification, which led to her assignment teaching the ninth grade sheltered mathematics course.

Natalie Hunter was a witty woman who was quick to trade barbs with students before class. She was a monolingual white female in her early thirties. She was honest and open with students but maintained a professional demeanor throughout class. Ms. Hunter came to class early each day and allowed students to hang out and talk in her room before the start of class.

Easton High School, like Turner, was on a 4 by 4 block schedule. The class period that I observed was a sheltered Mathematics 1 classroom similar to Meg Thomas' class with one structural difference. At Turner High School, the sheltered Mathematics 1 course was taught as a single course over an entire school year, at Easton High School sheltered Mathematics 1 course was taught in two parts over two semesters. This allowed students who failed the first part to retake it prior to moving on to part 2. Natalie Hunter's class was the first part of this course, Mathematics 1 part 1. The students were in this course for the first time, having taken a basic skills class the prior semester. All 21 students in Natalie Hunter's class were Latino/a and had varying degrees of English proficiency.

Across the hall from Natalie Hunter was Guy Dubois. Guy Dubois was a white, French native in his mid-thirties who was fluent in French, English, and Spanish. He had been teaching mathematics for six years. Mr. Dubois' undergraduate majors were computer science and mathematics. Mr. Dubois also earned a Master's degree in mathematics education. Mr. Dubois coached the soccer team at Easton High School, and many of his players came in before class to talk or hang out before the first bell. Mr. Dubois had a great rapport with his students; he was quick to smile and was able to joke with his students. On his lanyard he wore a pin that read "I'm too pretty to do math."

I observed Guy Dubois's sheltered Mathematics 1 part 2 class. The students in his class completed sheltered Mathematics 1 part 1 with Natalie Hunter the previous semester. Of the 15 students in Mr. Dubois' class, 12 were Latinos/as primarily from Mexico 1 was from the Congo, 1 was from India, and 1 was from Vietnam. The students varied greatly in English proficiency from speaking no English to speaking English in a seemingly fluent manner. During my first seven days in Mr. Dubois' classroom I observed a unit on triangles. After the first seven days Mr. Dubois was out for several days due to a family emergency. When he returned I conducted the last three days of observations, during which time he focused his instruction on quadrilaterals and the distance and midpoint formulas.

Researcher's Subjectivities

“Qualitative inquiry, because the human being is the instrument of data collection, requires that the investigator carefully reflect on, deal with, and report potential sources of bias and error” (Patton, 2002, p. 51). In this study, as in any other, it is important for the reader to understand the inevitable impact my experiences and perspectives have had in shaping the ways in which I collected and analyzed the data. Therefore, I find it appropriate to discuss my subjectivities so that the reader may understand the lens through which the study unfolds.

I am a first generation American. My mother and father immigrated to this country in the mid 1960s from Portugal and Brazil, respectively. Due in part to the difficulties my parents faced attending public schools speaking little or no English, they chose to raise my sister and me solely as English speakers. Therefore, though many of my family members were ELLs, I consider myself a monolingual English speaker, although I can communicate in Portuguese at a basic level.

My family holds education as the key to the American dream. They encouraged me to study hard and earn good grades so that I would have an easier life than they did. Perhaps due in part to this encouragement, I developed a true love of school and learning. I entered school at the age of 6 and have yet to leave.

After earning a degree in mathematics from the University of Florida, I received a position as a high school mathematics teacher in Orlando, Florida. During my teaching career I taught in a large urban school district with a diverse student body. The schools I taught at had large numbers of ELL students, the majority of whom were from Spanish speaking countries. Though I tried my best to teach all of my students and engage them in problem solving activities, I found my interactions with ELL students frequently devolved into rote memorization and skills practice. My experiences and frustrations teaching ELLs fueled my desire to learn new strategies for enacting challenging tasks with ELL students.

Data Collection

When utilizing qualitative research methodology, it is important to use multiple sources of data (Patton, 2002; Yin, 2009). Patton touted the benefits of triangulation, saying that through the triangulation of data sources researchers can overcome much of the skepticism that arises from the use of a single method. I chose to gather data from a variety of sources to address each of my research questions. In using multiple data sources, I hoped to better understand and interpret the results of my data analysis.

Data Sources

For my first research question regarding teachers' selections of tasks, I asked each of the participants to complete a survey. I then conducted a variety of interviews and classroom observations and collected curricular materials. In order to address the second research question, I used data from the survey, classroom observations, interviews, and samples of student work. I based the results of the third research question on data collected from classroom observations, interviews, and samples of student work. I provide a summary of these data source in relation to the questions they address in Appendix A. In the following sections, I expand on the collection and analysis of each of these data sources.

Survey. Prior to meeting with the teachers in person, I provided each participant with a survey that consisted of three parts. The purpose of the first part of the survey was to collect background data on each of the teachers. The second part focused on teachers' practice in general and the third part focused on the teachers' experiences teaching ELLs. I adapted the survey from The Mathematics Georgia Performance Standards Knowledge Survey used by

Edenfield in her 2010 dissertation. Edenfield adapted several of the items in the second part of the survey from a survey created by Ross, McDougall, Hogaboam-Gray, and LeSage (2003) to measure elementary teachers' implementations of standards-based mathematics instruction. The survey included both open ended and Likert scale items. I have provided a copy of the survey in Appendix B.

Classroom observations. In order to examine the teachers' enactment of mathematics tasks, I thought it important to observe and record their practice. Therefore, I conducted classroom observations in each of the teachers' classrooms. I observed each teacher for a period of two weeks or ten consecutive class periods. I conducted the observation during the same class period each day and observed a sheltered ninth grade Mathematics 1 class for each teacher.

I chose to conduct each cycle for two weeks of typical instruction (e.g., not during standardized testing) to ensure that I was able to capture classroom interactions on multiple tasks. I video recorded each observation with one or two video cameras. I initially planned to use two cameras for each observation; however, during many of the observations the second camera was not needed because of the teachers' use of direct instruction for a large part of the lesson. The teachers' use of direct instruction allowed me to capture the teachers' actions and interactions with a single camera and allowed me more opportunity to focus on taking field notes. On the occasions I used a single video camera, I followed the teacher as he or she provided instruction or worked with students. On those days I used two cameras, one camera focused on the teacher while the other focused on a particular group of students with which the teacher regularly interacted. In addition to video recordings of each class period, I also recorded field notes.

I partially transcribed each of the observations using the lesson graph (as described in Izsák, 2008) format presented in Appendix C. During the set up of the task, I focused the observation on the teacher. This focus on the teacher entailed focusing my primary camera’s video recordings on the teacher as he or she provided directions, passed out papers, or answered student questions. During the implementation phase, I followed the teacher as he or she visited various groups around the classroom. I focused the primary camera’s video recordings on the teacher as he or she moved around the classroom to various students or worked at the front of the classroom. During each observation I attempted to focus on the protocol provided in Appendix D. I chose to follow the teacher because the focus of my study during the implementation phase is the interaction between the teacher and the students. After collecting the data, I partially transcribed instances where the teacher spoke either to the class as a whole or small groups of students.

Daily planning interviews. In order to learn more about the teachers’ selection of mathematics tasks, I conducted a daily planning interview with each teacher prior to that day’s classroom observation. These interviews allowed me to ask about the tasks the teachers selected, the reasons for choosing the tasks, and other instructional decisions (Appendix E). These interviews also allowed me to follow up on events that occurred in the previous day’s class. I audio recorded each of the pre-observation interviews and took field notes. I fully transcribed the audio recordings verbatim. I conducted 9 planning interviews with Meg Thomas, 8 with Natalie Hunter, and 7 with Guy Dubois. . There were several occasions where parent meetings or other events conflicted with the interview time, resulting in a missed interview.

Curriculum materials. During the daily planning interviews, I asked each teacher to provide me with copies of the curriculum materials he or she had chosen for the lesson. These materials came from a variety of sources including textbooks, web searchers, software programs, and the teachers' own creation. Because of my interest in the types of tasks selected, having access to these tasks for later analysis was immensely important for this study. Having copies of these tasks also allowed me to determine the cognitive demand and other characteristics of the tasks as presented in the curriculum materials. Having a copy of the task as presented in the curriculum materials from which it was derived allowed me to assess the cognitive demand of the task as written and then to determine whether the cognitive demand of the task changed during the task set up and implementation.

Student work. Because a task's written instructions or set up do not guarantee an accurate prediction of how a student may interpret or enact it, I collected samples of student work. During several tasks I collected student responses and made copies of these responses for later analysis. These tasks were chosen because the students appeared to provide varied responses. For many of the other tasks it was unnecessary to collect samples of the students' work due to the students' answers being identical to what the teacher presented on the board. I only collected student work samples from those students whose parents had signed consent forms. The student work I did collect allowed me to see how the students enacted the task and whether or not the students carried out the task the way the teacher had envisioned.

Post-observation interview. I conducted an extended, semi-structured interview with each teacher after the two-week observation cycle. The interviews lasted between one to two hours. The purpose of this interview was to follow up on questions that came up during the observation cycle. It also allowed me to elicit the teacher's thoughts on the lessons I observed and their selection of tasks. I have provided the interview protocol in Appendix F.

Curriculum interview. After collecting all of the aforementioned data, it seemed that the data related to how teachers modified tasks, the second research question regarding the teachers' modifications to tasks, was rather thin. The teachers did not modify many tasks; instead they mainly chose tasks already devoid of context or text. In order to gather more data regarding this question I designed another interview. I refer to this final interview as a curriculum interview.

The curriculum interview lasted between one and two hours for each of the teachers. For this interview I brought in two sets of curriculum materials, one set related to triangle congruence and the other related to the Pythagorean Theorem. I chose these topics because both topics appear in the ninth grade mathematics courses I observed. Each set of materials included samples from several different textbooks. I purposefully selected materials from traditional textbooks, standards-based textbooks, and some that I considered somewhere in between the two extremes (Appendix G). During the interview I asked teachers to critique the units in terms of their appropriateness for ELL students. I have provided the interview protocol in Appendix H.

Data Management

Qualitative research produces a vast amount of data. I converted all of the data into electronic files and keep them on a password-protected computer. I also have the hard copies of data including audio tapes, video tapes, field notes, and student work, which I will keep in a locked cabinet for five years, after which I will destroy the data. I created an audit trail in order to record the major events and decisions that occurred during the study. I used Express Scribe to transcribe the audio and video files. I analyzed all of the transcriptions and other data (field notes and student work) using Microsoft Word and Excel. In the following section I describe in detail the procedures I used to analyze the data for this study.

Data Analysis

Qualitative data analysis requires the researcher to manage and organize vast amounts of data into manageable chunks. To begin this process, I first analyzed each individual case and then conducted a cross-case analysis. To analyze the data I used the constant comparative method decoupled from grounded theory (Glaser & Strauss, 1967). The analysis of each case required several rounds of coding. First, I identified initial themes in each of the data sources based upon my own observations and field notes. I then employed open coding (Strauss & Corbin, 1990) as I read the data to verify the initial themes and identify any additional themes. The open coding led to the creation of an initial codebook. I used Excel to create the codebook, and I continually updated it throughout the data analysis process.

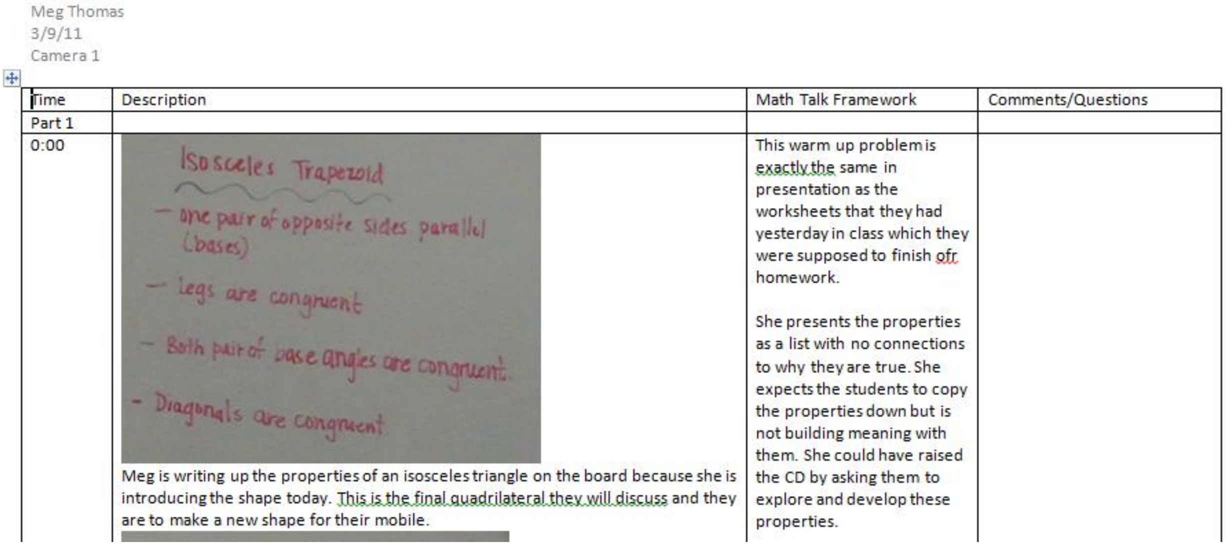
After the open coding, I employed axial coding (Strauss & Corbin, 1990) in order to code each data source based upon the initial themes. This required several more passes through the data, during which I attended to the broader categories for each phase of the task as presented in the theoretical framework. For example, in those instances related to the implementation phase, I attended to passages related to classroom norms, task conditions, and classroom habits and dispositions. Next, I further refined these themes by collapsing codes and creating sub-themes. I then analyzed across the data sources to look for differences and similarities. Finally, after analyzing each of the cases individually, I employed a cross-case analysis. This entailed looking for commonalities and differences from case to case. I have provided a more in depth explanation of this process below.

Survey. After receiving all of the surveys, I compiled each of the teacher's responses into a single document. I then looked for themes related to their teaching practice. After analyzing the other data sources I returned to the survey to examine the teachers' responses in terms of the themes that emerged.

Curricular materials. I scanned each of the curriculum materials I collected during my observations in order to create electronic copies. I then created an Excel file to analyze each of the tasks. In the Excel sheet I noted the date the task was used, the time spent on the task, a summary of the task, the class format in which the task was implemented, the cognitive demand of the task during the various phases, questions, and comments related to the task, and other characteristics of the tasks. In order to determine the cognitive demand of the tasks I classified them into the categories in the *Task Analysis Guide*. After I completed this round of analysis I looked for commonalities among the tasks for each teacher and then across the teachers. This led to the creation of common themes of the tasks used by all three teachers.

Interview data. Following each interview, I recorded my immediate reactions and preliminary thoughts on emerging themes. I fully transcribed each interview verbatim using Express Scribe. I then employed open coding (Strauss & Corbin, 1990), to generate codes based on my initial pass through the data. I repeated the open coding process three times, once for each research question. As I coded, I pulled out the relevant passages from the transcripts into a separate summary document to reduce the amount of data. I used my field notes to compare the observational data to the transcriptions. Then, using axial coding (Strauss & Corbin), I went through the transcriptions several more times attending to the themes from my theoretical framework. Finally, I analyzed across interviews of a single participant and then across interviews of all participants to determine similarities and differences. This resulted in the final themes that comprise my findings.

Classroom observations. I partially transcribed each classroom observation into a lesson graph (as described in Izsák, 2008) using Microsoft Word. I used these lesson graphs to organize the data into episodes determined by the mathematical task on which the class is working. The graph includes a partial transcription of what events occurred, screen shots of the video from the observation, and my notes regarding preliminary analysis of these episodes (Figure 4). After completing all of the lesson graphs I reviewed the preliminary analysis and created codes while attending to the aspects included in my theoretical framework and any unexplained phenomena. Throughout this process, I consulted with my major professor in order to establish reliability of the chosen codes.



Meg Thomas
3/9/11
Camera 1

Time	Description	Math Talk Framework	Comments/Questions
Part 1			
0:00	Isosceles Trapezoid – one pair of opposite sides parallel (bases) – legs are congruent – Both pair of base angles are congruent. - Diagonals are congruent Meg is writing up the properties of an isosceles triangle on the board because she is introducing the shape today. This is the final quadrilateral they will discuss and they are to make a new shape for their mobile.	This warm up problem is exactly the same in presentation as the worksheets that they had yesterday in class which they were supposed to finish ofr homework. She presents the properties as a list with no connections to why they are true. She expects the students to copy the properties down but is not building meaning with them. She could have raised the CD by asking them to explore and develop these properties.	

Figure 4. Lesson graph example.

Student work. I analyzed student work to determine if the students carried out the task as the teacher intended. For example, if the teacher asked for multiple solutions, I examined the student work in order to verify whether students found more than one solution. Similarly, I looked for instances of students providing only numerical responses when the teacher asked for written responses. I classified the student work into one of the four categories of cognitive demand found in the *Task Analysis Guide* (Stein et al., 2009) described in the theoretical framework. This classification allowed me to determine whether the cognitive demand of the task as completed by the student changed from the task as set up by the teacher.

Validity and Reliability

The validity of qualitative data stems from the richness of the information presented, the cases that are chosen, and the skills of the researcher. In employing both methods, triangulation and the triangulation of sources, I strengthened the internal validity of the study (Patton, 2002). I have addressed my own biases in hopes that the reader will gain insight into the lens through which I collected and analyzed the data. I also consulted with my major professor in developing and verifying the codes. It is through these methods I maintained a high degree of validity for this study.

The reliability of a study refers to the ability of the reader to recreate the study as presented. I have provided in depth discussion of the methodology and presented my own subjectivities to aid the reader in this process. I also maintained an audit trail in order to record a timetable of events related to this study. In providing an in depth analysis and discussion of the results, the reliability of my study was enhanced.

Limitations

In this study, as with any other, there are certain limitation of the design and findings. Though my choice of a case study design afforded me the opportunity to study in depth the teachers and their practice as related to their selection and enactment of mathematics task for ELLs, it also presents constraints. Stake (1995) clearly stated that a researcher would not choose a case study for the purpose of generalizing his or her results. Rather, Stake explained, “The real business of case study is particularization, not generalization. We take a particular case and come to know it well, not primarily as to how it is different from others but what it is, what it does" (p. 8). Indeed, I make no claims that what I have found in studying these three teachers is indicative of teachers as whole. I do claim, however, that studying these three teachers has allowed me to examine, in detail, a case of how these particular teachers' instructional practices and decisions informed the answers to my research questions.

In addition to the limits of the generative power of this study, it is important to note that I entered the teachers' classrooms for two weeks. While I believe the conclusions I have come to are indicative of typical instruction by these teachers, it is possible that observations during different points in time would have yielded slightly different outcomes. For example, my time in Guy Dubois' class was quite near the yearly end of course test, which may have made his emphasis of the standards more explicit than at other points in the year. By triangulating my data sources, I believe I have minimized this limitation to the best of my ability.

CHAPTER 4

FINDINGS

In this chapter I present the findings related to the research questions I first posed in Chapter 1 and that I have restated here to aid the reader.

1. How do teachers choose mathematical tasks for use with their ELL students?
    a. What are the characteristics of the tasks they select?
    b. What factors influence the teachers' selection of tasks?
2. What modifications, if any, do teachers make to mathematical tasks prior to their implementation with ELL students?
    a. What factors influence the teachers' decisions to modify or not modify the tasks?
    b. In what ways, if any, do these modifications affect the cognitive demand of the tasks?
3. What aspects of the classroom appear to contribute to the maintenance or decline of high cognitive demand in mathematical tasks?

For each of these questions I present the relevant findings and evidence for the conclusions I have drawn. I have organized this chapter by research question in order to present the reader with a thorough, coherent response to each question. I begin with the findings related to the first question.

Mathematics Tasks as Represented in the Curriculum Materials

In this section I discuss the tasks the teachers selected while in the first phase of the Mathematics Task Framework (Stein & Smith, 1998), the tasks as presented in the curriculum materials. The evidence related to this first question comes from the tasks I collected from each of the teachers, the surveys the teachers completed, and the various interviews I conducted with each teacher. It is important to recall I have defined tasks as a portion of class time aimed at developing a certain mathematical concept (Stein & Smith, 1998). Under this definition, I have excluded summative assessments such as chapter tests because their purpose is to assess mathematical ideas rather than develop them. Furthermore, those activities where teachers were discussing concepts and did not expect the students to produce a product were not included. The teachers provided me with several tasks they had planned to use but ultimately did not, mostly due to time constraints. I also chose not to include these tasks in this particular analysis because I was unable to follow the tasks through the phases of implementation. I first discuss the characteristics of the tasks the teachers selected in terms of their cognitive demand and other characteristics. I then explore the reasons for the teachers' task selections.

Characteristics of Tasks

In order to understand what ties the tasks together I examined the curriculum materials the teachers selected, the surveys the teachers completed, and the interviews that I conducted. I found there are four common attributes of the tasks the teachers selected. First, I discuss the sources from which the teachers selected the tasks. I then discuss the cognitive demand and repetitive procedural aspects of the tasks. I conclude with an examination of the teachers' choices of tasks to build academic vocabulary.

Sources. On the survey, all three teachers marked that they agreed that one of the primary responsibilities of a teacher is to select and develop mathematics tasks. Each of the teachers in this study had access to mathematics textbooks adopted by their respective schools, although each of the teachers frequently abandoned these textbooks in favor of other curriculum materials for their ELL students. Of the 42 tasks I observed, just 1 of those tasks came directly from a textbook.

All three teachers had access to a Mathematics 1 textbook published by Carnegie Learning. Carnegie Learning advertises this textbook series as standards based (Carnegie Learning, 2011). The majority of problems in the Carnegie Learning textbook presented a real-world context and aimed to facilitate students' learning as they uncovered mathematical concepts. In addition to the Carnegie Learning textbook, Guy Dubois and Natalie Hunter each had a classroom set of Mathematics 1 textbooks by McDougall Littell. These textbooks featured chapters that began with an overview of the content and relevant definitions followed by two sets of problems designed to foster proficiency through repetition and procedures.

In lieu of the textbooks, the teachers favored either creating or finding tasks by searching the internet for specific key words, borrowing resources from other teachers, or using problem-generating software. Meg Thomas stated numerous times when asked where she found a particular task, "I made it." On the survey, Ms. Thomas echoed this sentiment stating, "I use the textbook very little, maybe some practice problems is all I get from that (the textbook)." She went on to elaborate her selection of tasks, noting, "I search the internet a lot for simplified versions of activities, but mostly start from scratch making my own materials." Ms. Thomas was proud of the fact she created and found her own asks and viewed herself as resourceful in this regard.

Guy Dubois and Natalie Hunter favored problem-generating software as the source of their tasks. Guy used a test generator that came with the McDougall Littell textbooks, while Natalie typically used a software program called Infinite Algebra. Both teachers expressed that they did not know of many suitable curriculum resources for ELL students. Typically, when I arrived before school I found Guy and Natalie staring at the computer screen, intently focused on choosing problems for the day. I have summarized the sources from which the teachers selected tasks in Table 1.

When asked about the source of their tasks on the survey, the answers did not necessarily align to what I observed in their classrooms. In reference to the statement, "The district-provided textbook and materials are the main sources for mathematics in my classroom," Meg and Natalie strongly disagreed while Guy agreed. At first glance, Natalie's response may not have seemed consistent with my findings; however, it seemed from our discussions she was not including Infinite Algebra in the district-provided textbooks and materials. All 7 of the tasks she chose from software programs were from the Infinite Algebra program.

Table 1

Sources of Observed Tasks

Teacher	Total Tasks Observed	Tasks from Textbooks	Tasks from Software Programs	Tasks Created or Found from Other Sources
Meg Thomas	20	0	0	20
Natalie Hunter	11	1	7	3
Guy Dubois	11	0	7	4

The remainder of this section describes the findings related to the characteristics of the tasks the teachers selected. I first examined the cognitive demand of the tasks they used in their classrooms and then looked for themes that tied the tasks together.

Cognitive Demand. I focused my initial analysis on the cognitive demand of the tasks. The tasks teachers chose often included a number of problems that, taken together, comprised the task. I arrived at the classifications of the tasks by assigning each of the problems in a particular task a numerical value from 1-4 using the Stein and Smith classification (1998). The numbers correlated to the level of cognitive demand of the problem; therefore a problem assigned a 1 is of the lowest level of cognitive demand, memorization, while a task assigned a 4 was a doing mathematics task. I then averaged the numerical values of all the problems in the task to arrive at the final numerical value describing the cognitive demand of the task. For example, one of Meg's chapter review tasks contained 30 problems of which 25 were procedures without connections, 2 were procedures with connections, and the remaining 3 were memorization. After assigning values to each of the problems and averaging those values, this review task yielded a cognitive demand of 1.97.

As I undertook the cataloging of the tasks, I encountered a dilemma. I identified several tasks that fell outside of the cognitive demand levels proposed by Stein and Smith (1998). These tasks seemed to be lower in cognitive demand than memorization tasks because students were not required to recall facts. Meg Thomas used one such task on the first observation day. The purpose of the task was for students to rewrite the properties of a kite found on the board onto a piece for the quadrilateral mobiles they had been building. I determined that though this task was situated in a mathematics classroom, the type of thinking required to complete this task was not mathematical in nature. Thus, I labeled this task as not mathematical and assigned it a value of 0. The results of my analysis of the tasks' cognitive demand are presented in Table 2.

Table 2

The Cognitive Demand of the Tasks Selected

Level of Cognitive Demand		# of Tasks		
		MT	NH	GD
Not Mathematical (0-.49)		4	1	2
Low Cognitive Demand	Memorization (.5-1.49)	7	0	6
	Procedures without Connections (1.5-2.49)	4	10	3
High Cognitive Demand	Procedures with Connections (2.5-3.49)	5	0	0
	Doing Mathematics (3.5-4)	0	0	0
TOTAL		20	11	11

As is evident from the table, the teachers primarily selected low cognitive demand tasks for their students. During one of the pre-observation interviews, Ms. Thomas stated that for her this was a purposeful decision. Although she did not use the term cognitive demand, Ms. Thomas stated that she considered the type of thinking required when selecting tasks for her ELL students. She said she selected easier tasks requiring less sophisticated mathematical reasoning for her sheltered course than she did for her other classes. This purposeful choice of low level tasks evidenced her decision to lower the cognitive demand in order to accommodate her ELL students.

Guy Dubois and Natalie Hunter also purposefully chose lower demand tasks for their ELL students. Ms. Hunter often stated that her students needed a lot of practice. In an interview she stated the importance of "just going over it and over it and over it and over it and over it," a sentiment that led her to give students this practice by providing a number of similar problems with different numbers. These practice problems were predominately procedures without connections tasks. Mr. Dubois also expressed a similar position, stating that the students needed to practice these types of problems in order to build procedural fluency.

Ms. Thomas' statements regarding her desire to provide the class with low level tasks is consistent with the low cognitive demand classification of the majority of tasks she used during my stay in her classroom. Mr. Dubois and Ms. Hunter's decision to provide students with a lot of practice is also evident in the large number of repetitive tasks provided to their students. There were several high cognitive demand problems within the tasks the teachers chose; however, the scant number prevented the overall classification of tasks containing these problems as high cognitive demand.

During the curriculum interviews, the teachers frequently eschewed higher cognitive demand tasks in favor of those having a lower cognitive demand. Many of the characteristics leading to a task's classification as high cognitive demand are characteristics the teachers found undesirable or unnecessary for their ELLs. For example, Stein and Smith (1998) listed "complex and nonalgorithmic thinking" as a characteristic of a doing mathematics task. Natalie and Guy often referred to this characteristic as discovery learning, a type of learning described by Guy as "hard to implement." Natalie stated that students "regard the time (spent on discovery tasks) as free time." When discussing her thoughts on a *Core-Plus Mathematics* investigation I presented during the curriculum interview, Natalie Hunter described her frustrations,

> It's almost like they're (the textbook company) trying too hard. Does that make sense? Like they're "Oh we're going to be great and wonderful and we're going to do all of this discovery, so we're going to do all of this," and it's like you're still overwhelming kids with so much stuff. They're totally lost and very frustrated, and so is their teacher.

Meg, on the other hand, stated that she thought discovery tasks were good for ELL students. However, it was evident from our conversations that Meg equated group work with discovery learning. Indeed, as stated earlier, the tasks Meg selected were primarily low in cognitive demand and classified as procedures without connections. She described these tasks as “discovery” due to her students working cooperatively on them. I explore this notion in greater depth when discussing the implementation of tasks. From the tasks I collected, it is clear the vast majority of the problems was algorithmic in nature or could be solved through students recalling of basic facts.

Nonalgorithmic thinking is not the only characteristic of high cognitive demand tasks the teachers avoided when selecting tasks for their ELL students. The teachers frequently referred to tasks requiring students to prove concepts, another characteristic of high cognitive demand tasks, as too difficult for ELL students. When discussing a proof of the Pythagorean theorem in the Jacobs (2003) geometry unit, Guy stated “I think I will skip the proof part,” a statement he and the other teachers made several times during the curriculum interviews. Meg stated that she endeavored to find problems with just one answer, a characteristic of many low cognitive demand tasks. Natalie also discussed her reluctance to use questions with multiple answers due to the confusion it caused her students. Upon completing the analysis related to the cognitive demand of the tasks, I determined it necessary to examine the tasks further. This further analysis uncovered other qualities of the tasks the teachers chose for their students, which I discuss in the following sections.

Repetition and procedures. A close examination of the tasks the teachers chose revealed highly repetitive problems focused on procedural learning. The problems centered on students learning certain procedures and did not contain any real world or contextual applications of such procedures. During the pre-observation interviews, Ms. Thomas discussed the focus of problems during the quadrilateral unit saying, “I wanted something with them learning the properties (of quadrilaterals).” The problems she chose to aid students in learning these properties had students write in properties to justify calculations for finding missing quantities in quadrilaterals. These problems involved Ms. Thomas creating a number of problems identical to one another except for the numbers involved. Ms. Thomas thought this repetition was important saying, “I’m trying to emphasize those properties every day.” Natalie and Guy did not explicitly state that they chose problems for repetition but did say they chose problems for students to practice and master skills. An examination of their tasks revealed an assortment of tasks quite similar to those Meg Thomas chose. The software programs both Guy and Natalie utilized allowed the teachers to create easily a large number of similar problems.

Overall, there was an emphasis on students’ memorization of properties. For example, Meg Thomas’ gave students a problem stating that the length of the top and bottom sides of a parallelogram were $x + 2$ and $2x - 8$, respectively. She wanted the students to understand that the expressions could be equated because opposite sides of a parallelogram are congruent. There was no attempt to explain why this property holds true or to connect it to real world applications. This type of expectation was common among each of the teachers in this study. The tasks chosen by Natalie and Guy also reflected their desire to get students to learn mathematical properties as evidenced by the large number of problems devoted to a particular skill that comprised the majority of their tasks.

The vast majority of tasks asked for students to “solve” for or “find” a value. The teachers seemed to think students’ proficiency completing mathematical procedures was equivalent to a mastery of concepts. The teachers’ discussions of their successful lessons during the final interview reflected this view. For example, Guy stated he thought the midpoint and distance formula task was the most successful because his students did well on the task and the corresponding quiz. Both the task and the quiz were entirely comprised of procedures without connections problems. Therefore, the type of learning Guy found satisfactory reflected students' abilities to complete algorithms rather than understanding how the mathematical concepts related to the algorithms or why the algorithms worked. Natalie also discussed her students' learning in terms of their abilities to accurately carry out mathematical procedures. The teachers' assessments of the textbook units during the curriculum interviews also evidenced their fondness of these types of procedures based problems.

The high number of low cognitive demand tasks chosen by the teachers correlated with this focus on procedures over process. The number of tasks requiring students to produce a response beyond a numerical value was quite small. Meg Thomas included a "because" blank on her quadrilateral worksheets. These "because" blanks led me to classify those tasks as procedures with connections instead of procedures without connections; however as discussed in a later section, the expectations for the responses did not maintain the cognitive demand. Aside from Meg's "because" blanks, I was challenged to find a task requiring an explanation of the students' solution processes. Several problems on two of Natalie Hunter's tasks asked students to "explain what it (a functional value) means in terms of the problem." None of the problems on the tasks Guy Dubois selected asked for an explanation.

When asked about some of the more standards-based curriculum materials the teachers had access to, such as the state's Frameworks and the Carnegie Learning textbooks, Guy and Natalie questioned their ELL students' abilities to learn from this approach. Guy stated, "They [the district personnel] say that they [the students] should learn with the book and we can give them the book, but no, no, no, I don't think it works like that." Natalie Hunter shared this sentiment, laughing at the idea of her ELL students learning from what she considered “discovery learning” tasks. Meg Thomas liked the idea of these materials but thought they required significant modifications prior to implementation with ELL students. The modifications she proposed involved transforming more cognitively demanding tasks into procedural tasks. The aspect of discovery learning Ms. Thomas seemed to value was the cooperative nature of the instructional format many of the tasks encouraged.

Though each of the teachers discussed the value of word problems relevant to students, very few of the tasks the teachers selected during my observations included a context at all. The only tasks situated in a context were several problems from worksheets Natalie Hunter and Meg Thomas created. Natalie Hunter’s contextualized problems came from the aforementioned set on function notation. These problems were similar to the following,

> The function $f(c) = 5 + .25c$ represents the amount of money Felicity receives in allowance each month for having completed *c* chores. What is the value of *c* such that $f(c) = 25$?

Meg Thomas tasked students with coming up with real world scenarios that were independent, dependent, mutually exclusive, and overlapping when discussing probability. Though the teachers discussed the value of bringing students’ cultures into the mathematics classroom, the tasks they chose did not reflect this notion. Despite the lack of contextualized problems, the teachers did focus many of their tasks on another aspect they greatly valued, vocabulary.

Vocabulary. A common goal among the teachers was to build ELL students’ mathematical vocabularies. In talking with the teachers, it seemed their use of the term vocabulary referred to learning words, the meaning of those words, and the relevant mathematical contexts related to the words; a notion akin to academic language (Coggins et al., 2007). In each of the final interviews, the teachers stated that their focus on vocabulary was a major difference between their mainstream and sheltered mathematics courses. Thus, the teachers selected mathematics tasks they thought aided students in achieving this goal. Natalie Hunter stated that she "pushed the vocabulary" in her sheltered course. When I asked Guy Dubois whether he emphasized vocabulary more in his sheltered course, he responded, "Oh yeah, yeah, yeah!" Similarly, Meg Thomas discussed the importance of vocabulary leading her to use graphic organizers with her ELL students, a tool she did not use with her mainstream students. The teachers' selection of tasks for their ELL students clearly reflected the importance they placed on vocabulary.

In order to help his students learn the vocabulary, Guy Dubois wrote a long list of words on the whiteboard for the students to define. In order to define the words, Guy expected the students to look up each term in the textbook's glossary. Similarly, Natalie Hunter presented her students with a list of words on the projector. She also expected her students to use a glossary or their notes to define the list of words. Though not included in my analysis of tasks' cognitive demands, both Natalie and Guy gave students vocabulary quizzes. These quizzes were comprised of fill in the blank questions. Natalie Hunter included a word bank on her quiz from which students were to choose the correct answer. Guy Dubois did not include a word bank on his quiz, instead expecting students to recall the terms from memory.

Meg Thomas had students learn vocabulary but took a more visual approach. For important new terms Meg Thomas introduced during my observations, she had students fill out a Frayer or modified Frayer model (Frayer, Fredrick, & Klausmeier, 1969). These models were similar to the model she used for the term "kite" as shown in Figure 5. In addition to these graphic organizers, Ms. Thomas also had her students create flashcards and mobiles to illustrate and display important terms. These too were projects she stated she would not have used in non-sheltered mathematics classrooms. As with most tasks the teachers selected during my observations, these vocabulary centered tasks tended to be highly repetitive and low in cognitive demand. In fact, the majority of the *not mathematical* tasks that I coded as level 0 were tasks focused on building the students' mathematical vocabularies.

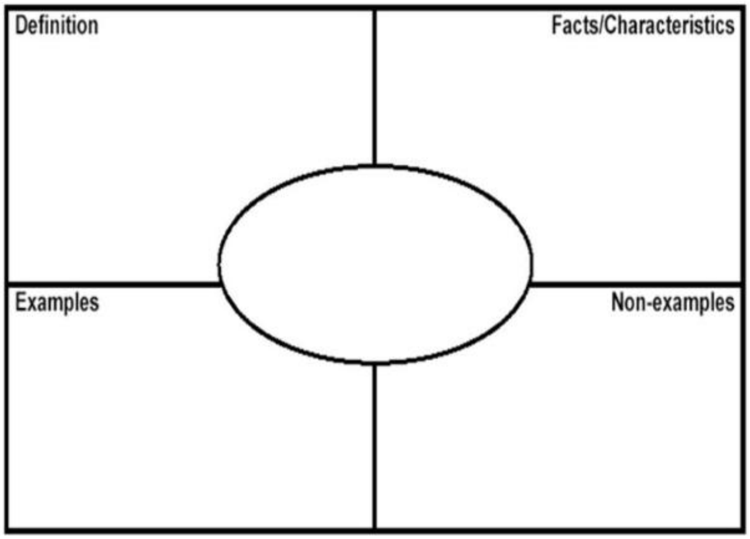


Figure 5. Meg Thomas' Frayer model template.

Several researchers have discussed the importance of going beyond the learning of words in order to build ELL students' mathematical understandings (e.g., Brenner, 1994; Moschkovich, 2002). Khisty and Chval (2002) wrote the following regarding ELLs' learning of academic language.

> The words represent meanings that are waiting to be developed and eventually internalised. Therefore, which words are presented to the students and how they are developed are vitally important. Just as important is that students have opportunities to use these words in their talk and as they work (p. 155).

All teachers in my study stated the importance of vocabulary for all students and stated that their focus on vocabulary was especially important for their sheltered mathematics students. Although the teachers' stated goal for many tasks was to build academic vocabulary, the tasks they selected seldom achieved this goal. The mathematical meaning of terms was a missing part of the activities, leaving teachers focusing on helping students to learn words without their associated meaning in a mathematical context. Moschkovich addressed the narrow view of mathematics for ELLs focused on learning vocabulary devoid of communication and context, saying,

> Although an emphasis on vocabulary may have been sufficient in the past, this perspective does not include current views of what it means to learn mathematics…In general, learning to communicate mathematically is now seen as a central aspect of what it means to learn mathematics (p. 192).

Moshckovich also described the difference between a register, knowing meanings for words in a particular context, and lexicon, knowing the words in terms of "phonology, morphology, syntax, and semantics" (p. 194). The teachers in my study typically helped students build their mathematical lexicons, falling short of aiding students in building mathematical registers.

The activities chosen by the teachers were frequently solitary exercises requiring no mathematical understanding. The teachers' failure to use the terms in context and allowance for resources that reduced many of the tasks to transcription activities led to my classification of these tasks as non-mathematical activities. I have termed tasks of this type *word activities*. This terminology highlights the difference between a focus on words without meaning versus activities that help build academic language (Coggins et al., 2007).

A common example of a word activity is a word find with math words. Completing the word find does not help the student to attach meaning to the mathematical terms. Indeed, someone could replace the mathematics terms with any word and the goal of the activity would remain the same–to match letters in a word on a list to letters in a word on the word find. Therefore, this constitutes a word activity rather than a vocabulary activity.

Each of the teachers in this study presented his or her students with word activities. Guy Dubois' lengthy list of terms required students only to copy the definitions out of the textbook's glossary. Similarly, both Natalie Hunter and Meg Thomas asked their students to create flashcards for important mathematical terms. In order to create the flashcards, the teachers expected the students to find definitions for the terms in their notes or textbook and copy them onto note cards. This transfer of words from one source to another is something I term a transcription activity, an idea discussed in a later section.

Motivation activities. Meg Thomas frequently selected tasks I refer to as motivation activities. These activities were those in which students were engaged in crafting or other hands on activities. These activities typically lost the original mathematical goal. I chose the term motivation activities because Ms. Thomas selected these activities in order to engage and motivate students to learn mathematics. In many cases these motivation activities, on the surface, appeared to be mathematical in nature due to their enactment in a mathematics classroom and the inclusion of mathematical words. Upon closer examination, however, it was clear that these activities did not contain a mathematical goal, leading to their categorization as non-mathematical activities.

Ms. Thomas selected these activities as a means of accommodating her ELL students. She discussed these activities as a fun way to get students involved in mathematics. One such activity was the creation of the quadrilateral mobiles. Though the students were required to copy the properties of quadrilaterals from the board on their mobile pieces, this was simply a transcription activity. The students spent the vast majority of this time cutting the pieces into the appropriate shape and decorating the mobile with markers. The entire activity spanned approximately two weeks with each mobile piece taking approximately 30-40 minutes to create, half a class period.

Classroom games can also fall into the category of motivation activities. Ms. Thomas played “trashketball” with her ELL students, something she said she would not do in a regular class. This game, meant to serve as a review, required students to answer questions in small groups. If a group’s answer was correct, they received the opportunity to throw the trashketball from one of three lines into a waste bin. The further the line was from the bin, the more points the shot earned the group if it made it into the bin. Clearly, throwing a trashketball into a waste bin does not engage students in mathematical activity, though it did motivate the students to participate. This was not the only portion of the game that lost a mathematical focus. The questions Ms. Thomas had students answer were from the prior night’s homework assignment. Therefore, students had only to write down a response from a paper. In theory, the students could argue between answers if they differed; in practice however, they deferred to the student perceived as the smartest.

In looking across all of the teachers, I can best describe the tasks they selected as low cognitive demand tasks aimed at building procedural fluency and mathematical vocabulary through repetition. The teachers chose these tasks because they thought they would help their sheltered ELL students be successful learners of mathematics. In the following section, I explore the specific factors influencing the selection of these types of tasks.

Factors Influencing Task Selection

Once I established an understanding of the types of tasks the teachers selected for their students, I explored the factors influencing their choice of tasks. During the interviews and on the survey the teachers discussed reasons for choosing tasks for their sheltered classes. In this section I discuss the three main findings related to the second sub question, What are the characteristics of the tasks the teachers choose?

Teachers' perceptions of students. In analyzing the interviews and observations, I found evidence suggesting the teachers' perceptions of students influenced their choice of tasks. The teachers developed these perceptions of their students both because of their personal experiences with ELL students and because of stereotypes they held about ELL students. In this section I discuss these perceptions and the resulting influence they exerted on the teachers' choices of tasks.

The teachers explicitly cited students' difficulties with English as a factor when selecting tasks. The teachers acknowledged that the students in their sheltered course varied greatly in terms of English proficiency; however, they often referred to the students as a homogenous group of students who greatly struggled with English. Though several of the students spoke little to no English, others had tested out of sheltered courses for the upcoming school year, suggesting their English capabilities were at or near proficient. Because the teachers perceived the students as quite lacking in overall English proficiency, the teachers avoided tasks with challenging language and those tasks requiring students to provide written explanations. Ms. Hunter stated that she was unable to use problems centering on jokes and riddles due to her students' limited proficiency in English. "My ELL kids don't get the joke, because like 'What happened at the flea circus? The dog ran away with the show'...they are like 'What, Miss?' They're like 'I don't get it'." She cited a similar response from her ELL students when trying to implement a state task based upon a riddle from *Alice in Wonderland*. These experiences with students seemed to influence Ms. Hunter's decision to provide her students with problems that were largely decontextualized and free of difficult language.

In the curriculum interview, Mr. Dubois examined a *Core-Plus Mathematics* unit and quickly dismissed it stating, "too much word…when they see too much word, too much writing and all of that they think it's going to be hard." Ms. Thomas also discussed her reluctance to provide "wordy" tasks. When examining a unit from a geometry book she stated, "This is very wordy…I would think they would lose this; they wouldn't understand what was going on." These types of statements were common throughout the curriculum interviews and when discussing the state developed tasks with the teachers. The teachers discounted the ELL students' abilities to complete tasks that contained what they deemed too many words, leading to their selection of decontextualized, procedures based tasks.

In addition to avoiding "wordy" problems, the teachers shunned formal proofs with their ELL students. This seemed to be largely due to the teachers' doubts regarding the students' abilities to comprehend mathematical proofs. The teachers seemed to base these doubts upon their perceptions of the students' lack of both language and mathematical abilities. During the curriculum interviews, the teachers were apprehensive about students' abilities to complete rigorous proofs. Mr. Dubois stated, "I think I will skip the proof part" while Ms. Hunter said, "looking at those proofs [in a textbook] seems, that makes me want to turn to the next page." During my time in the teachers' classrooms, I did not observe many instances of the teachers developing or demonstrating formal proofs of mathematical concepts. At several points the teachers stated that something was true "because…" but would quickly follow it with statements such as "but you just need to know" how to complete some procedure. For example, Mr. Dubois had his students learn how to construct the incenter of a triangle with a straight edge and compass; however, after they completed the task he told the students,

> Ok, so what you need to know about that. You need to be able to recognize that this is to build the angle bisector and incenter. You need to be able to tell that the incenter is where my three angle bisectors meet. For example, if I tell you what do we build with the angle bisector you need to be able to tell me that it's the incenter.

After constructing the incenter that class period, he never again referenced the construction. He only expected students to know the definition and recognize a drawing representing the incenter. The notion that students need only to recognize or know a concept rather than understand why was common among all the teachers.

Throughout the interviews and survey, the teachers expressed the sentiment, both directly and indirectly, that ELL students lacked the mathematical abilities of their native speaking peers and thus required simplified tasks. In further discussing the types of tasks best suited for their students, the teachers often correlated the students' limited proficiency in English with limited mathematical proficiency. This correlation seemed to have led to the teachers' selection of the highly repetitive, low cognitive demand tasks described in the prior section.

Each of the sheltered classes I observed contained students with varying mathematical abilities. The school did not place students in the classes due to mathematical difficulties but due to difficulties with the English language. Each school had mathematics courses for students struggling with mathematics. The sheltered courses I visited contained some students who would have been in these courses but also contained students whose mathematical abilities were at grade level. Therefore, the teachers' blanket statements regarding their ELL students' mathematical abilities seemed to be conflating the students' English and mathematical capabilities.

Meg Thomas explained that she wanted to be sure her students were not "bogged down in the math" as she explained in the following excerpt.

> I try to make sure that whenever they solve equations that they need for quadrilaterals, they don't get decimals or fractions. I try to make them like, just so that they learn the process for what they are doing and don't get bogged down in the math. Because even you've seen probably, when they are solving equations they're not good at solving equations sometimes…so I try to not make them get bogged down in that math so I try to make them easy ones to solve.

Ms. Thomas repeatedly stated her preference for ELL students not to get “bogged down in the mathematics” during several of the planning interviews and on the survey. Ms. Thomas stated to me that she did include fractions and decimals in her other, non-sheltered courses. The decision to include non-integer values in her other courses suggests that Ms. Thomas’ perception of her ELL students’ mathematical abilities differed from those of her native English speaking students. Ms. Thomas further evidenced this sentiment in her response to my question regarding the advice she would give to other ELL teachers, “I guess what I would advise them to do for any kind of concept that they had to teach is just to try to get down on a lower level.”

When asked about his choice of tasks for his sheltered class, Mr. Dubois stated that he had to go slower with his sheltered class than in his non-ELL class where he said he didn’t “feel like I have to do everything with them.” He also discussed his reluctance to provide the students with too many mathematical symbols because they would find it confusing, a view the other teachers shared. This statement along with other discussions with Mr. Dubois seemed to suggest he perceived differences in the mathematical abilities between his sheltered and non-sheltered students.

Mr. Dubois often stated his ELL students lacked the prerequisite skills that his non-ELL students had acquired, a sentiment shared by the other teachers in this study. Ms. Hunter repeatedly referenced her students’ lack of basic skills and also their lack of number sense with statements such as “They don’t have number sense” and “Most of them [ELL students] don’t understand the concepts well enough for that kind of stuff [graduation test problems].” Similarly, Ms. Thomas remarked that tasks for ELL students would ideally begin with a focus on the basic skills they were missing. Though the ELL classes contained students of a wide range of mathematical abilities, the teachers frequently spoke of their ELL students as a homogenous group. These blanket statements about sheltered mathematics students extended to areas other than mathematical abilities.

The teachers frequently made statements reflecting a deficit view of their ELL students. The teachers often used this deficit view to justify the selection of less challenging tasks. For example, when I presented Ms. Hunter with a task from a Spring Board textbook and explained that it was developed by the College Board in an effort to better prepare students for Advanced Placement classes, she stated, “[That] is awesome, but my kids aren’t going there [to AP courses]; they’re going to work at Fieldo or Tyson [the local chicken processing plants].” Each of the teachers tended to stereotype the ELL students as young adults who were likely not to finish high school and instead head straight to a life of manual labor. Though there is evidence that ELLs have a higher dropout rate than non-ELLs (NCELA, 2008), the teachers often used this point to defend the lowered expectations they held for their ELL students.

Mr. Dubois and Ms. Hunter also thought their students had difficulty “discovering” mathematical concepts. Instead of allowing students to persevere and discover topics, Mr. Dubois and Ms. Hunter thought that direct instruction was a more effective instructional strategy. Ms. Hunter stated, “In my experience they [ELL students] have a hard time with non-traditional teaching; it’s almost like they don’t respect it.” Many of the high cognitive demand tasks in published curricula require students to “discover” ideas as teachers provide guidance in the form of questioning rather than direct instruction. Both Mr. Dubois and Ms. Hunter stated that this type of instruction was ineffective for ELL students and said they were reluctant to try such tasks with their sheltered classes.

On the other hand, Ms. Thomas stated that she preferred discovery type tasks for her sheltered classes and seldom used them for her non-sheltered students. During an interview she stated, “I think it’s important to do group work and let them discover things themselves as much as possible.” She stated several times that her sheltered students responded better to this type of learning. However, in examining the types of tasks she deemed to be discovery tasks, it seemed that Ms. Thomas often equated group work with discovery learning. Many of the tasks Ms. Thomas referred to as discovery tasks were low cognitive demand tasks that students worked in groups to complete. For example, Ms. Thomas considered the station activity a discovery task. In this task students rotated to various stations around the classroom. At each station, the students were to read clues related to a particular quadrilateral and decide which quadrilateral was represented by the clues. Though the students were working in small groups they were not discovering any new concepts. Instead, the students were simply reviewing what they had learned in prior class periods.

The teachers’ perceptions of their students’ status as English language learners also impacted the task selection in other ways. On several occasions Ms. Hunter stated that her ELL students came from a culture that did not value cooperative learning. This led her to avoid choosing tasks requiring group work. Conversely, Ms. Thomas’ perceptions of ELL students led to her choice of tasks suitable for group work. As she explained, “The ELL kids respond better when they work in a small group with each other.” She made this comment as she contrasted sheltered students from mainstream students, leading me to infer that this accommodation was made due to the students’ status as ELLs. The notion that certain groups of students respond better to cooperative learning or traditional teaching is not one backed by the literature. There are, however, several studies that discuss the value of standards-based practice for all students (e.g., Riordan & Noyce, 2001).

Mr. Dubois did not specifically reference students' culture influencing the task format but did state that his students would benefit from tasks that were more culturally relevant, an idea backed by findings from Cahnmann and Remillard (2002). During the curriculum interview, Mr. Dubois explained this view,

> I would like to have a different test for my ELLs on the same standard, but I am tired of the question about the baseball field. I would like a question about the soccer field …I just would like better questions sometimes, not easier or what, but better, more related to what they [ELLs] do.

Natalie Hunter also expressed her approval of tasks that tied into what she considered more culturally relevant contexts in her classroom. When discussing a problem during the curriculum interview, Ms. Hunter stated, "I like the fabric (referring to a picture in the problem) just because of the pattern but that's, I'd do more pattern than the, because a lot of Hispanics still do house-wifey [*sic*] art stuff." Ms. Thomas said that while she liked the idea of bringing in her students' culture she thought it difficult to do in a mathematics classroom.

> In that culture class [she took for her ELL endorsement] they always talked about how to bring in the culture into your activities and I thought, "Well that'd be so easy to do if you were teaching social studies or even English, but in math, that is really hard to bring in their culture, other than to put them in a word problem.

Though each of the teachers discussed their students' culture ideally influencing their task selection, this influence was not evident in the tasks they selected during my time in their classrooms.

The teachers' perceptions of students led to their selection of tasks they thought would best help their ELL students succeed in mathematics. It is important to note that the success to which I refer is teacher perceived. The teachers implied that they thought if the students passed their mathematics class this would be a successful mathematical experience. The teachers did not seem to expect their students to succeed in upper level mathematics courses or to continue on to college mathematics for the most part. Though some of the perceptions led to the teachers' selection of repetitious, procedures based tasks I described in a previous section, these perceptions also guided the teachers to set up tasks in a different manner than they would have in a non-sheltered class. I discuss this notion in detail in relation to the response to the second research question.

Focus on the standards and testing. In both Easton and Turner High School the importance administrators and teachers placed on making Adequate Yearly Progress was evident. The morning announcements at both schools repeatedly emphasized the importance of benchmark tests to students. The teachers often discussed the pressures of getting their students ready for the end of course test during interviews. They also discussed the importance of the end of course test with their students throughout my observations.

Both Turner and Easton High School participated in standards-based grading. This practice required teachers to grade students based upon their mastery of the state mathematics standards for a particular unit. Ms. Thomas embraced this practice and often referenced it when discussing her plans for the day. When she chose problems, the first criterion she referenced was the state standard the tasks covered. "I always base on the standards, making sure they [her students] do exactly what the standard says; I interpret what the standard [says]." Her focus on the standards was evident throughout my time in her classroom. On several of her assignments she typed the standards in bold as a heading for the problems related to a particular standard.

Mr. Dubois and Ms. Hunter also guided their task selection by the standards but primarily referenced the standards in terms of the state's end of course test. They frequently stated the need for students to master the standards in order to pass the end of course test. For example, Mr. Dubois frequently told his students they needed to be able to use the distance formula if they saw coordinates or points on a graph because that is how it might appear on the end of course test. Ms. Hunter expressed the importance of her students passing the test on a number of occasions both to me during interviews and to students during class. In an interview, she stated that although she thought the test was a “reading comprehension test” more than a mathematics test, the students had to perform well to go on to the next course.

Perhaps in part because I conducted the observations close to testing time, the teachers seemed to follow a “teach to the test” mentality. The problems I examined during my time in the teachers’ classrooms seemed to focus on the standards for the day and avoided problems that addressed other standards. As Ms. Thomas stated in the survey, “I try to cut out any part that isn’t directly related to the standard.” This practice prevented Meg and the others from choosing tasks connecting to other ideas or concepts. For example, when working on arithmetic sequences, Ms. Hunter did not tie the sequences to graphical representations and real world applications because they were not a part of the standard. When discussing points of concurrency, Mr. Dubois stated on numerous occasions to his students, “You need to make sure you learn how to recognize them (points of concurrency). You won’t have to build them, but you will have to recognize them.” The teachers’ focus on the standards and the corresponding tests often resulted in the teachers selecting procedures-focused problems in order to help students master the standards.

Lack of Resources. Though each of the teachers had received an ESL endorsement, each noted a lack of available resources for teaching ELL students. These resources included not only curriculum materials but also professional development and collaboration. In this section I discuss these perceived deficits as described by the teachers and the influence the lack of resources had on their selection of tasks.

Each of the teachers stated that the curriculum materials and resources he or she had were insufficient for teaching ELL students. On the survey, in response to which curriculum materials she thought were best suited for teaching ELL students, Ms. Thomas wrote, “I use the textbook very little, maybe some practice problems is all I get from that.” Ms. Thomas frequently noted that she had to get creative and find new sources of curriculum materials for her ELL students whereas she mostly used the textbook for her non-ELL classes.

Natalie Hunter and Guy Dubois also stated they abandoned their textbooks in favor of the problems they generated from their software programs. Ms. Hunter noted that she liked the software program because she could make a large number of problems for practice. The software programs she and Mr. Dubois used did not include many high cognitive demand tasks. The lack of readily available high cognitive demand tasks appropriate for ELLs seems to have led to Mr. Dubois and Ms. Hunter’s exclusion of these types of tasks. The teachers thought the tasks the state provided in their frameworks, many of which were more standards-based, were not well suited for ELL students. In her final interview Ms. Thomas explained her feelings toward the state’s tasks,

> I think you know, we talk about, you talk about the tasks all the time, and I’m not good at using them in my classroom, but if someone had a sheltered like, version of the task I think that would be very helpful.

Guy Dubois stated that the state’s tasks were just too time consuming to implement with his sheltered students, a sentiment shared by Natalie Hunter.

The teachers also expressed the notion that there is simply a shortage of curricular resources for ELLs in general. During the final interview Meg explained her own lack of experience stating, “I guess I’m not that experienced when it comes to sheltered, to have a lot in my bank to pull from.” This lack of experience led Ms. Thomas to search the Internet for resources. The teachers all noted that they would like to have access to more resources for their sheltered courses and that they were open to trying new things should they come across them.

The lack of curricular resources was not the only area the teachers found deficient. When asked in the final interview about their training to teach ELL students, all of the teachers noted they did not think it was sufficient. Meg Thomas stated that the other teachers in the courses for her ESL certification were primarily elementary teachers or teachers in areas other than mathematics. Because she was the only secondary mathematics teacher in her ESL courses, Ms. Thomas said that the facilitators did not provide her with specific strategies for her content area. Ms. Thomas thought many of the materials the school and district trainings provided her with were not particularly useful for her sheltered mathematics classroom. “That was what I always struggled with in those [ESL certification] classes was how to apply it to mine.” Guy Dubois also took courses to receive his ESL certification. Although he found one of the courses helpful, the other two provided no support directly related to mathematics teaching. Natalie Hunter did not take any courses for ESL certification; instead she passed a test that provided her with ESL certification. When asked about ESL training, Ms. Hunter stated that there were no math specific ESL trainings, although she thought they would be useful. Ms. Hunter went on to explain that she drew on her special education resources and training when teaching sheltered students. It seemed as though she thought the two groups of students, special education and ELL, shared many learning characteristics, though only one of her ELL students was classified as in need of special education services. Ms Hunter’s propensity to draw connections between special education and ELL strategies is consistent with her tendency to conflate her ELL students’ difficulties in English and mathematics.

The teachers' lack of training for teaching ELL students, specifically in mathematics, seemed to impact their task selection. The teachers stated that they were unsure of appropriate resources. This left the teachers feeling as though they were on their own to find appropriate resources, most of which were as described in the prior section. Each of the teachers expressed interest in learning of new resources but also stated a lack of time to commit to finding and learning how to use them.

Collaboration is another resource the teachers found lacking. Though Mr. Dubois and Ms. Hunter had the possibility of collaborating with one another as the only sheltered mathematics teachers at Easton High School, they did not do so. Both stated a lack of time as a big aspect, as well as the fact they were teaching two different parts of the course. Ms. Thomas commented on the lack of collaborative opportunities in her school and district. She noted that because she was the only sheltered mathematics teacher in the school she had no one with whom to plan. She suggested,

> It would be helpful, like if, counties within, like schools within the same RESA (regional educational service agency) would kind of plan their sheltered classes together…if we did that, you know, I wouldn't feel like I was always alone in my planning because sometimes I really question what I'm doing in the classroom and is it best.

Ms. Thomas attempted to overcome the perceived lack of resources by modifying tasks she found through internet searches to accommodate her ELL students. Though it is not clear what role additional resources would have played in these teachers' instruction had they been present, it is reasonable to suggest the additional resources would carry some influence in their task selection.

Mathematics Tasks as Set Up by the Teachers

In this section I discuss the tasks as they appeared in the second phase of the Mathematics Task Framework (Stein, Grover, & Henningsen, 1996), the tasks as set up by the teacher. In order to focus my analysis, I attended to the observational data where the teachers discussed the task with their students prior to implementation. I also used interview data that involved the teachers discussing how they modified specific tasks or approached modifications for their ELL students in general. Finally, I provide evidence from the curriculum interviews where the teachers suggested how they would modify and present the tasks to their students. I begin with a discussion of the modifications the teachers made or stated they would make to tasks. I then discuss the impact these modifications had on the cognitive demand of the tasks. I close with a discussion of the factors impacting the teachers' decisions to modify the tasks.

Modifications to the Tasks

All of the teachers discussed modifications they made or would make to tasks prior to implementing the tasks with their sheltered mathematics classes. I have classified the teachers' task modifications into two categories—modifications related to the task's content and modifications related to the instructional format. Although Stein et al. (1996) discussed these two categories under a singular category they termed *Task Features,* I purposefully distinguished between features related to the tasks' content and instructional format. This distinction arose from my discussions with the teachers, particularly from the ways in which they discussed the tasks and the modifications they made to the tasks for their ELL students. I discuss each of these in the following section.

Task content. The teachers discussed the need to modify the content of tasks for their ELL students. I use the term content to refer to the features of the task including the written presentation, the numerical values included in the task, and the task's visual presentation. I begin with a discussion regarding the teachers' modifications to the written presentation of the tasks.

Throughout my discussions with the teachers, each stated the need to modify the language of tasks for his/her sheltered courses. The teachers said they had to “cut out a lot of words” and “simplify” the tasks for ELLs. When asked if they modified tasks for all of their classes, all of the teachers stated that they did on occasion but in general did not have as great a need for these modifications in their non-sheltered classes. Each of the teachers had access to Carnegie Learning’s *Mathematics 1* textbooks. This textbook typically presented students with word problems to build a particular mathematical concept. The teachers discussed their thoughts regarding the language in the textbook on several occasions. Ms. Thomas stated, “I really do scale down [problems for her sheltered class]. They are not as difficult as what is in the textbook; but my regular class, they do problems out of the book.” Ms. Hunter and Mr. Dubois shared this sentiment, often stating they thought the textbooks did not meet their ELL students’ needs. Ms. Hunter wrote, “I rarely use the book” in response to a survey item asking about her thoughts of the district-provided materials, and she frequently referenced the book as being ill suited to her ELL students. During my observations, I only witnessed her using one task from the textbook. The lack of textbook use provides further evidence of the teachers’ stated views regarding the ineffectiveness of the textbook for sheltered mathematics classes.

The idea of simplifying language also extended into the state’s tasks, some of which were greater than 10 pages in length. Meg Thomas discussed her frustration with the tasks by noting, “They are [long] and they are really, really, wordy and when I used them with sheltered I have to really simplify them; like I cut out a lot.” During the curriculum interview, Ms. Hunter discussed her view on the state’s task with which I had presented her. “For my ELL kids, that’s a lot of pages and they go ‘Nah, I’m good.’ It’s a lot of words, a lot of vocabulary that just, and even two different methods [of solution], they look at me like ‘Which one is best’?” Ms. Hunter repeated this sentiment throughout our interviews whenever I asked her about state tasks. Guy Dubois shared a similar view on the state's tasks, stating that they took too much time to implement and were much too wordy for his ELL students, although he did like some of the real world contexts they provided.

The notion of simplifying or cutting down the language also came up during the curriculum interviews. When presented with tasks from standards-based textbooks such as *Interactive Mathematics Program* (*IMP)* and *Core-Plus Mathematics*, Guy Dubois repeatedly described the amount of words being problematic for his students, a sentiment shared by the other teachers. When reading over a unit from the Spring Board series, Ms. Thomas stated,

> This is very wordy. I would think, I would think they would lose this; they wouldn't understand what was going on …. So like this kind of language would really confuse them [her ELL students], but I am sure it is a good activity, I would have to do a lot of clipping and re-wording.

The other teachers shared similar thoughts throughout the curriculum interview when presented with what they considered "wordy" tasks.

In addition to simplifying the language, the teachers discussed the need to simplify the mathematical content for their ELL students. Ms. Thomas discussed this idea during an interview,

> I try to make sure that whenever they solve equations that they need for quadrilaterals, they don't get decimals or fractions. I try to make them like, just so that they learn the process for what they are doing and don't get bogged down in the math … I try to make them easy ones to solve. They, I give them a lot, like on parallelograms. A lot of times I don't give them variables in the angles; I'll just say an angle is 122 degrees instead of saying that it is $3x + 100$ degrees or whatever, because, I just, to make sure they understand what that property means I guess. But in a regular class I do make them figure out, like they'll have a lot of variables in the angles instead of having the exact angle measure. I guess that's really, pretty much how I scale them down; I just try to simplify them a little bit.

Ms. Thomas discussed the need to lower the mathematics difficulty of tasks on several occasions during my time in her classroom. She noted that she thought the added difficulty prevented students from mastering the standards.

Though Ms. Hunter and Mr. Dubois did not explicitly state the need to simplify the content of tasks for their ELL students, I did witness several instances of them describing modifications that would simplify the mathematics. One such instance occurred during the curriculum interview with each of the teachers when Mr. Dubois stated, “I think I would skip the proof part.” His reluctance to bring in formal proofs to his class was evident during my observations. During an interview he stated that it would be nice for his students to know why things work but that it was not necessary for the end of course test. Ms. Hunter and Ms. Thomas also stated their reluctance to complete formal mathematical proofs with their sheltered courses. The simplification of content also extended to the way the teachers presented the mathematical content. Mr. Dubois repeatedly stated he would not use particular problems or units with students because they would get lost in all of the mathematical symbols. Similarly, Ms. Hunter stated that her students would get confused when looking at mathematical representations with too many symbols, a notion shared by Ms. Thomas.

Related to simplifying the mathematical content, the teachers discussed their desire to modify tasks so that they had only one solution or one solution path. When discussing modifications to a particular task, Meg Thomas stated, “I wanted really to just have one answer.” This desire for a single answer led Ms. Thomas to modify a task she found on the Internet so that there was only one correct answer. Natalie Hunter and Guy Dubois noted they thought presenting students with a multitude of ways to solve a particular problem created unnecessary confusion. Therefore, they preferred to set up tasks with a particular solution method in order to preempt their students' confusion. On the survey, Natalie Hunter remarked, "If the kids find an alternate solution I'm delighted; however, when teaching a new topic I've found that group instruction on more than one method is problematic." I witnessed further evidence of the teachers' decisions to set up tasks with a single solution throughout my time in their classrooms. I also saw further evidence of their reluctance to pursue alternate solution paths during the task implementation as noted in another section of this study.

Beyond discussion related to simplifying the mathematics and the presentation of the task, Ms. Thomas also stated that her task modifications for sheltered students often included visual representations. In response to the question, "Do you modify materials for your ELL students? If so, how?", Meg Thomas wrote "Yes, I try to do multiple representations of materials – pictures, words, definitions, alternative prompts… I do graphic organizers for vocabulary and skills." Ms. Thomas included one such modification on a task related to the properties of quadrilaterals. The task was comprised of a chart that listed the properties of quadrilaterals in the first column and the names of the quadrilaterals along the second row. For each quadrilateral, students were to place a check mark if a particular property applied to the quadrilateral. In the first row, Ms. Thomas included a heading "Picture." Students were to draw the picture of the quadrilateral in this column. Ms. Thomas explained,

> I want them to draw their picture of the shape up here (referring to the first row), and I've used this chart before in other classes but I don't have that row on there, so that's kind of how I've changed it for them. It helps them to see the picture of the shape as opposed to the name.

Ms. Thomas also described other visual modifications to tasks for her ELL students throughout my time in her classroom.

Ms. Hunter and Mr. Dubois did not directly discuss making modifications of this type with their sheltered students; however, during the curriculum interviews they did express approval of tasks that included visual representations. Ms. Dubois stated that he liked several tasks because they included “graphs (pictures).” Similarly, Ms. Hunter explained how she would change a particular task, “It’s like, I would take the information from it (the task) and produce it in a different way, a lot less words, a lot more spoken, maybe talk about, you know but, have a diagram.” The idea of using visual representations to connect concepts to language was a recurrent theme throughout my discussions with the teachers.

Instructional format. In addition to modifications to the tasks’ content, the teachers modified the instructional format they used for the tasks they selected for their sheltered courses. I use the term instructional format to refer to the arrangement of students, time allowed for a task, and the resources the teachers provided students during the teachers’ explanation of the task set up. I discuss the instructional format as a modification because the tasks the teachers selected did not specifically mention an instructional format the teachers should use when implementing the tasks. Therefore, the teachers’ decisions regarding the instructional format arose during the set up phase as they discussed the set up with students. Furthermore, the teachers often stated that the instructional formats they chose for their sheltered students served as a modification to their typical routine used with non-sheltered students.

Each of the teachers discussed the arrangement of students as a modification to the tasks they used, though the arrangements differed among the teachers. Meg Thomas discussed her use of small groups within her sheltered course, a practice she avoided with her non-sheltered students. Indeed, during my observations, Ms. Thomas set up the vast majority of tasks as small group activities. She stated that this was important for her ELLs because they could help one another as they completed the assignments. When discussing a particular lesson, Ms. Thomas explained, “I don’t do as much small group [for non-sheltered classes] as what I do in the sheltered, so, that is kind of my way of modifying this lesson." Similarly, Guy Dubois often assigned problems and then encouraged students to work with and help one another. Encouraging students to communicate and work with one another in the mathematics classroom is a strategy supported by literature on effective teaching practices for ELLs (e.g., Coggins et al., 2007).

Natalie Hunter differed from the other teachers in this regard. When setting up a task, Ms. Hunter often stated to students that they were to complete the assignment individually or to do their own work. My observations confirmed this expectation as the students sat in rows and seldom conferred with one another about mathematics. Throughout my time in her classroom, Ms. Hunter did not assign particular groups of students to work with one another, a set up frequently utilized by Ms. Thomas. Ms. Hunter stated her preference for direct teaching on a number of occasions, often stating that her sheltered students did not value cooperative learning and got off task too easily.

The teachers often provided students with time limitations as they set up the tasks. For example, before a task that required students to rotate between stations, Ms. Thomas told students they would have 5 minutes at each station. Ms. Hunter stated that she purposefully did not spend an extended amount of time on any one task, "As a management technique and as a boredness [*sic*] issue, I am trying not to do the same thing for like more than 20 minutes." Mr. Dubois typically provided students with as much time as they needed, frequently checking in with students to gauge how much more time they would need to complete the task. The exception to this was his time restraints on the daily quizzes. Guy Dubois used these quizzes as a means of assessing the prior day's learning. He typically provided the students with 10 minutes to complete these quizzes before collecting them. The time restraints set up by the teachers seemed intended to focus student activity on mathematics and eliminate off task behavior. The teachers frequently relaxed these restraints during the actual implementation, a point I discuss later in response to the third research question.

In terms of resources provided during task set up, the teachers encouraged their students to draw on graphics, vocabulary aids, and manipulatives as they worked on tasks. Because the scope of this study did not include an in-depth examination of the teachers' non-sheltered courses, I cannot claim the teachers used these resources exclusively when setting up tasks for their ELLs, although, in some instances, as noted below, the teachers did explicitly state this was the case.

Ms. Thomas frequently told students that they could "use their mobiles" as part of her task set up. The mobiles she referred to consisted of cutouts of the quadrilaterals attached by string to a coat hanger (Figure 6). On each cutout the students had written the properties related to that particular quadrilateral. Ms. Hunter and Mr. Dubois encouraged the students to refer to their notes or the textbook as they completed tasks. Ms. Hunter also brought in algebra tiles as an aid for students as they completed a task involving sums and differences of polynomials. Calculators were present in each of the classrooms, and the teachers frequently encouraged calculator use for basic computations.

Figure 6. A mobile made by one of Meg Thomas' students.

During the curriculum interviews the teachers stated that they appreciated tasks that included hands on resources. One such task came from the *Georgia Frameworks* (Georgia Department of Education, 2008). In this task, students examined different college pennants to determine whether they were similar triangles. When discussing this task, Guy Dubois stated that he liked it "because I could probably find those (pennants)…I could get them interested in creating their own school." Natalie Hunter appreciated that a task included a moveable triangle created of plastic that had vertices that allowed students to turn the sides of the triangle to create different angles and sides that were able to extend to different lengths. This triangle could allow students to explore the triangle congruence theorems. Meg Thomas also expressed her approval of tasks that allowed students to engage with hands on activities, such as an *IMP* task requiring students to make a scale model of a television. The teachers stated that the inclusion of these manipulatives aided their ELL students in forming connections to the mathematics and better understanding concept, a position supported by researchers whose work centers on ELL students (e.g., Coggins et al., 2007; Echevarría, Vogt, & Short, 2010; Kersaint, Thompson, & Petkova, 2009).

Modifications' Impact on Cognitive Demand

In this section I discuss what impact, if any, the teachers' modifications had on the tasks' cognitive demand. It is important to note that the intent of this section is to respond to my second research question, which addresses the tasks as set up by the teacher. Several of the modifications previously described had the potential to change the cognitive demand; however, it was not clear in which direction this change went until I examined the task in the implementation phase. Therefore, in this section I attend only to the modifications that changed the tasks in a noticeable direction *prior* to implementation. I do address the other modifications and the resulting changes to cognitive demand in response to my third question in a section to follow.

None of the modifications set up by the teachers resulted in an increase in cognitive demand prior to implementation. Therefore, the modifications led to one of two outcomes: the maintenance or decline of the cognitive demand. I first discuss those modifications that maintained the cognitive demand and then describe the modifications that resulted in a lowering of the cognitive demand.

Of the modifications I have described, several of them did not result in a change in the cognitive demand. These modifications instead contributed to the maintenance of cognitive demand. The modifications that maintained cognitive demand included the use of visual representations, the time constraints, and the inclusion of resources. The teachers often included visual representations such as a picture on their worksheets but did not explicitly connect the visual representations to the intended task outcomes, except perhaps in the case of Meg Thomas' chart. More typically, the representations were included to help students visualize concepts. The lack of explicit connections of the visual representations to the task or inclusion of reasoning about the representations as part of the outcome prevented the representations from increasing the demand. Conversely, the mere presence of these representations did not lower the demand as they were not a vital part of the task beyond an illustrative reference.

The time constraints the teachers placed on the tasks helped to prevent the tasks from devolving into non-mathematical activity. Though time in itself cannot raise the cognitive demand, Stein et al. (2009) cited time as a task feature that can aid in the maintenance of cognitive demand. Though the teachers sometimes avoided placing time constraints on tasks, I could not gage the impact of the lack of time constraints until I examined the tasks during the implementation phase. This is mainly due to the related factor of the teachers' management of the facilitation of the task during the allotted time for the task. For example, sometimes the lack of time constraints allowed the teacher to interact and ask questions of students as they worked on the task, while other times the students used the extra time to engage in non-mathematical activities.

The provision of resources during the task set up did not impact the cognitive demand prior to implementation. For the most part, the teachers suggested to students that they could use calculators, visual aids, textbooks, etc. but did not explicitly discuss how they should use them in conjunction with the task. Therefore, the inclusion of these resources did not work to raise or lower the cognitive demand. In the case of the calculators, the outcome of the tasks was not to perform calculations but rather to use calculations to arrive at some other answer. For example, Meg Thomas had students solve for a missing variable in order to determine angle measures. The calculator may have aided the students in performing basic computations, but it did not achieve the ultimate outcome of determining the measure. It is, however, important to note that during task implementation several resources resulted in a drastic reduction of cognitive demand, a phenomenon I describe in detail in response to the third research question.

As discussed earlier, the majority of tasks selected by the teachers were already low in cognitive demand. Therefore, the modifications that led to a decline in cognitive demand often resulted in memorization level or non-mathematical tasks. The teachers' decisions to simplify the language of tasks often resulted in the set up of tasks lowering the cognitive demand. Many of the tasks I witnessed were selected because of their lack of difficult language; therefore the curriculum interviews provided the best evidence of how "getting at the mathematics" resulted in a lowered cognitive demand.

The curriculum interviews provided further insight into the modifications the teachers thought appropriate for high cognitive demand, standards-based tasks and the resulting impact on cognitive demand. In general, the teachers thought a text heavy problem obfuscated the mathematics for their ELL students. For example, when Ms. Thomas was discussing a task during the curriculum interview she stated,

> This is a little wordy too…I think they might could do this after they had a grasp of what the concepts were, I know it's reviewing the same thing, but I think it would be good for them to do it over and over and over again, and see it different ways, like this was very simply worded (referring to the beginning of the task) and then this is a little more advanced wording to me…I would think they would need something before it.

Ms. Thomas thought that the wording made the problem more difficult mathematically for the students. She said they needed to practice the concepts before they could do the "wordy" task. However, it is important to point out that this particular task was meant to be an introduction to the concept of similar triangles and might be considered more of an investigation type task. On a number of occasions, the teachers referenced the need to simplify the written context around a problem to get at the mathematics. For these teachers, the elimination of the context resulted in procedures without connections tasks. These tasks consisted of problems devoid of context that required little or no justification from students. On a number of occasions, the context described in a task would require students to interpret the situation and tie their numerical responses to the situation. The elimination of this connection lowered the cognitive demand, a phenomena of which the teachers were not aware. For example, Natalie Hunter explained her frustration with a *Core-Plus Mathematics* unit,

> That [the unit] really feels like a task; it really does, but I still have the same problem with it–the vocabulary is so high. You're asking all of these crazy questions; you're doing all of this stuff, and it's almost like you don't, at the end they're [the students] like "I don't know what just happened."

She went on to say that the reading and context really complicated what students were supposed to do mathematically. This idea was also evident in the other teachers' evaluations of the curriculum materials.

The teachers' modifications to the mathematical content also led to a lowering of the cognitive demand. This was the only modification that had the intentional outcome of a lowered cognitive demand. The teachers' avoidance of proof in their sheltered course led to lowered expectations in terms of students' justification of answers. In Meg Thomas' class, when students were asked to provide a reason for a solution, "plugging it in" was an acceptable response. Guy Dubois informed students that it was sufficient to associate a point of concurrency with a term without knowing the why, leading to students' memorization of terms. Similarly, Natalie Hunter seldom asked for justifications as students worked on procedures-based tasks. Indeed, she often glossed over those parts of tasks that asked students to explain why. The teachers' decisions to avoid difficult mathematics so as not to confuse students resulted in the students experiencing mathematics through low cognitive demand tasks. The persistent use of low cognitive demand tasks is in opposition to the task literature, which suggests that a variety of tasks are important for student learning (Stein et al., 2009).

In addition to avoiding proof, the teachers' reluctance to embrace multiple solution paths or tasks with multiple solutions lowered the cognitive demand. Stein et al. (2009) discussed the inclusion of multiple solutions and solution paths as a feature of high cognitive demand tasks. By actively modifying and setting up tasks to contain one path to a single solution, the teachers lowered the cognitive demand of the tasks.

The other modifications to tasks, the grouping of students in particular, had the potential to impact the cognitive demand; however it was not until I observed the task implementation that I could describe the ways in which these impacts occurred. I revisit this and the other modifications and the resulting impact on cognitive demand in response to the third research question. In the following section, I provide a discussion of the factors leading to the teachers' decisions to make the aforementioned modifications to the tasks they used in their sheltered mathematics courses.

Factors Influencing the Teachers' Decisions to Modify

The reasons for the teachers' decisions to modify tasks for their ELLs were the same as those I discussed in response to the first question about the factors that influenced the teachers' selections of tasks. The teachers' perceptions of their ELL students, focus on the standards and testing, and lack of resources contributed to their decisions to modify the tasks. In this section I revisit each of these areas and relate the findings to those described in the literature on which I based my theoretical framework.

The teachers' perceptions of their students in terms of the students' English language capabilities, mathematical capabilities, and cultural values influenced the teachers' decisions of which tasks to present to their ELL students as well as whether and how to modify the tasks. These findings align to similar findings by Stein et al. (1996) regarding the factors influencing the maintenance or decline of cognitive demand of tasks. In their work, Stein et al. discussed *Teacher Knowledge of Students* as one such factor potentially impacting the cognitive demand of tasks as they move from the task as written to the task as set up by the teacher. I have interpreted *Teacher Knowledge of Students* as Teachers' Perceptions of Students in order to avoid the quantifiable connotation the term knowledge tends to carry in educational literature. The deficit view the teachers held regarding their ELL students' abilities led to lowered expectations, which, in turn, led to modifications to the format and content of the tasks that lowered the cognitive demand, a finding consistent with those noted by Stein et al. (2009).

I have collapsed the teachers' adherence to state standards and testing expectations, as well as their differing goals for ELL students, into the category *Teachers' Goals* developed by Stein et al. (1996). If one wanted to refine further the *Teachers' Goals,* one could do so by categorizing the goals as internal or external to the teachers. Internal goals would be those based upon the teachers' experiences and thoughts of what their students should accomplish. The external goals would include those generated by administration or government, such as the state standards and pressure to make Adequate Yearly Progress. In either case, the goals teachers held for their students influenced their decisions to select and modify tasks for their sheltered classes.

The final factor influencing set up identified by Stein et al. (1996) was *Teachers' Knowledge of Subject Matter*. Though it seems as though each of the teacher's subject matter knowledge would have an impact on their interpretation of a particular task, the scope of this study did not address this issue directly. In order to make claims about the teachers' subject matter knowledge I would have to administer an assessment to gauge their knowledge or make inferences based on other factors such as educational background or observational data. In terms of the number of prior mathematics courses, Guy Dubois certainly had completed the greatest number of mathematics courses, followed by Natalie Hunter, and then Meg Thomas; however I was not privy to their performance in these courses or the particular content the courses covered. Therefore, I cannot make claims to the influence this factor had on the teachers' decisions to set up tasks in a particular way.

Mathematics Tasks as Implemented by the Students

In this section I examine the tasks in the final phase of the Mathematical Tasks Framework (Stein, Henningsen, & Grover, 1996), the *tasks as implemented by students*. This phase consists of the activities that occurred after the teachers set up the tasks, as the students began work on the tasks. The primary data sources for this response were the classroom observations, interviews, student work, and the tasks selected by teachers. Due to my focus on the teacher, I centered my analysis of the classroom observations on the teachers' interactions with students during this phase.

I first discuss the classroom norms and the ways in which these norms impacted the cognitive demand. I then discuss the task conditions and revisit several of the modifications the teachers made to the tasks during the set up phase and discuss how those modifications impacted the cognitive demand during the implementation phase. I then examine the common occurrences and patterns of interactions during this phase, specifically those pertaining to the classroom habits and dispositions I discussed in the theoretical framework and explain the impact that these habits and dispositions had on the cognitive demand of the tasks.

Classroom norms. The classroom norms refer to what work, quality, and accountability the teachers expected of students as they implemented the tasks. The classroom norms are the general expectations the teachers hold for their classes rather than the expectations for a particular task. For example, Meg Thomas typically provided students with clear expectations as she set up her activities. The following is one such instance where as she introduced a quadrilateral activity where students would move between various stations, she clearly stated her expectations.

> We are going to practice today with more quadrilaterals, but I do not want you using your mobiles today; we've got to learn our properties without our mobiles. So we've planned an activity for you today and what you're going to do, you're going to work with one other person, but I am going to decide who you are working with. You have a recording sheet, okay, and there's ten different stations. There's going to be ten bags standing up and at each station there's a bag that says clues and then there's a bag that has the number 1 on it. Now, this is where you are going to put your answer and this is where you get your question. So when you reach into your bag, you pull this out and it has three or four characteristics on it, and this one says, "My two diagonals are congruent to each other, all of my sides are congruent to each other, all of my angles are congruent to each other," and what I want you to do with you and your partner is to figure out what shape I am describing for you on this sheet of paper, ok? And when you find that shape, whatever you think that your answer is, then you're going to have a stack of shapes that Miss D has cut out for you, so you can thank her. So with your shapes if you think that shape that we just described was a trapezoid, then you are going to take the trapezoid and you're going to put it into the other bag for me, so that's your way of putting your answer into your bag.

Ms. Thomas also stated a time limit of 5 minutes for each station. Mr. Dubois and Ms. Hunter seldom provided such detailed instructions for their tasks. Instead, the norm was that once the papers were passed out students, were to begin work by reading directions on their own.

Many of the tasks I witnessed began with the teacher passing out the task. The teacher would then read the directions written on the task and proceed to do one or more of the problems for the class. Natalie Hunter started nearly all of her tasks in this way. In the following excerpt, Ms. Hunter had passed out the task without setting it up. She began implementation as follows.

> In your hands you have the chapter 2 test review, the chapter 2 test review. This covers, wait, function notation, sequences, and rate of change, function notation, sequences, (addresses a student) We're talking about sequences, function notation, and rate of

> change. Alright so what's the first one say? Provide an example (addresses a student's behavior). Alright, function notation. Alright, what's function notation?

As the implementation continued, Ms. Hunter went through each problem with students. The expectation was that Ms. Hunter would ask the questions and write the answers on the board while students copied down what she did. Each of the students' responses matched Ms. Hunter's work upon the conclusion of the task. Ms. Hunter seemed to have a greater share of accountability for learning than her students did. There was no formally stated expectation for cooperative learning as was typically present in Meg Thomas' classroom.

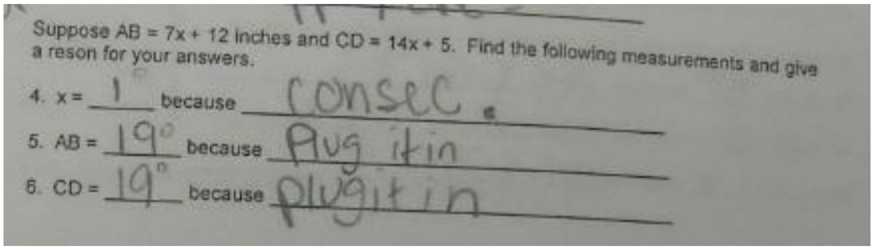
Activities such as this in which the teacher provided the mathematical thinking while students copied down responses are what I refer to as transcription activities. Perhaps the best description of this type of activity is through the example of a court stenographer. Stenographers are paid to create written records of court proceedings. Stenographers sit in on a multitude of different cases and listen to testimony that witnesses provide in technical language. Though a stenographer witnesses and records the courtroom proceedings, it is unlikely he or she understands the details of all of the recordings. This is understandable as the goal of the stenographer is to create a written record in real time, not to understand what he or she writes. Each of the teachers had instances of tasks that devolved into transcription type activities as they were implemented. The classroom norm in Mr. Dubois and Ms. Hunter's classrooms seemed to be that when students struggled, the teacher would take over the mathematical thinking, thus transforming the tasks into transcription activities.

Guy Dubois' classroom norms were similar to those in Natalie Hunter's classroom. Typically, Mr. Dubois would give a brief lecture during which time students were expected to copy down the notes and ask and answer questions. After the lecture, he typically provided students with practice problems. A typical implementation began with Guy Dubois working out the first problem as an example of the work he expected and then asking students to continue to work. He did not set up formal groups; however, the classroom norm was that students could work together on tasks.

As previously discussed, the teachers varied in their allowances for students to interact as they worked on tasks that were not teacher guided. Ms. Thomas typically assigned students to groups and discussed her expectations for their participation within the groups. Ms. Hunter and Mr. Dubois did not typically assign students to groups. In Ms. Hunter and Mr. Dubois' classrooms, the lack of formally stated norms for cooperative learning seemed to create disparate task experiences for the students in terms of cognitive demand. Often, the students who better understood a particular concept would take on the responsibility for learning, leading to the maintenance of cognitive demand for them. The other students who worked with these students would generally follow their lead and copy down what the lead student wrote. As a result, the second student would experience a decline in the cognitive demand because the first student assumed it.

In some instances, a student would take time to explain an answer and the process that led him or her to that solution to another student. This type of exchange typically occurred between students who spoke the same native language, and the exchange was typically transacted in that language. In this case, the first student experienced a higher level of cognitive demand. Though not always explicitly stated to students, the teachers seemed supportive of these types of interactions as a means of aiding their ELL students. On several occasions, the teachers mentioned that they encouraged students to explain a particular solution to another student in their home language; however, I only encountered a teacher-initiated situation of this type on one occasion. During a class discussion on the Triangle Inequality Theorem, Guy Dubois asked a student to explain a concept to her classmate who was absent the previous day. The student explained in Spanish to her classmate, also a native Spanish speaker. Mr. Dubois was able to listen and chime in as she explained the concept because he was also fluent in Spanish. In this case, the teacher facilitated the increase in demand for the student explaining the concept. Though not typically teacher initiated, I did frequently observe students less comfortable with English asking their classmates for explanations in their home language in this way.

Because the majority of the tasks implemented were low in cognitive demand, the teachers typically expected students to produce only a numerical response for problems. I witnessed evidence of this throughout my observations as teachers worked out example problems for students and in the samples of student work. In several instances the tasks asked students to provide an explanation. The classroom norms related to what comprised an acceptable mathematical explanation were quite lax in each of the teacher's classrooms. For example, on the tasks where Meg Thomas asked students to fill in the "because," students filled in things such as those in Figure 7, and these were deemed acceptable by the teacher. As previously stated, the teachers discussed their lowered expectations regarding proof for their sheltered students. Meg Thomas stated that she allowed ELL students to have their own way of doing proofs, which was a simplified version of proofs. The teachers' satisfaction with imprecise mathematical explanations contributed to a decline in cognitive demand.



Suppose AB = 7x + 12 inches and CD = 14x + 5. Find the following measurements and give a reson for your answers.

4. x = 1 because consec.

5. AB = 19° because Plug it in

6. CD = 19° because plug it in

Figure 7. Sample student response deemed acceptable.

In terms of classroom norms, I define accountability in terms of the teachers' expectation that students would complete and submit work. Student accountability for completion of work varied among teachers and among tasks. The teachers frequently collected both homework and class work assignments from students; however, there were times that a significant portion of the class failed to turn in an assignment. The teachers perceived the failure to turn in assignments as a normal part of their jobs and noted that some students had just quit trying. It is unclear if the teachers experienced similar rates of homework returns in their other courses as I did not inquire about their non-sheltered courses. The teachers also stated their frustrations with having limited recourse for students who no longer tried. Though each of the teachers made efforts to get students to complete work, the students who failed to complete work seemed to have little accountability for their lack of effort. For example, Natalie Hunter discussed her students who had given up by saying, "I went back and filled in all of the stuff where they had to turn stuff in and I put the zeros in, and the only people who didn't get a zero were (names three students)…they're just not turning it in or I don't have it." Though this low level of accountability for completing work did not impact the cognitive demand of the tasks, it did result in an increase in non-mathematical activity for those students who chose not to complete tasks.

Task conditions. An examination of the task conditions proved more difficult than the other dimensions of implementation. The task conditions refers to the extent to which tasks build on the students' prior knowledge, the appropriateness of a task in terms of students' abilities, and the time allotted for task completion. In this section I examine each of these areas and discuss the resulting influence these facets had on the tasks' cognitive demand.

Stein et al. (2009) found the extent to which tasks build on students' prior knowledge and the appropriate match of tasks to students may impact the cognitive demand of the tasks. I did not measure student knowledge for this study. Therefore, I am unable to make claims as to the extent to which tasks built on prior knowledge or were appropriate for students. I can however, discuss the teachers' perspectives on these areas.

As previously discussed, the teachers frequently stated that the students lacked the prerequisite skills necessary to complete particular tasks. The teachers frequently stated the mismatch as something they had no control over rather than a point they needed to address with modifications to the task. The mismatch seemed to lead the teachers to work out more of the problems with and for students in order to try to counteract the lack of prerequisite skills. This outcome served to lower the cognitive demand because the teacher took on the mathematical thinking for the students.

The teachers' perceptions of students' lack of prerequisite skills also led teachers to encourage calculator use during tasks. A large portion of Mr. Dubois' lesson on distance formula involved him showing students how to enter numbers in the calculator. The other teachers also discussed using the calculators for basic calculations because their ELL students lacked these skills. As discussed earlier, the purpose of the tasks was seldom to perform basic arithmetic; therefore, the calculators were an aid but did not lower the cognitive demand of the tasks.

When the teachers witnessed students struggling on a task, they often suggested resources other than calculators to aid the students during implementation. The teachers' recommendations of these resources related to the task conditions, particularly the appropriateness of the task to the students' abilities. Rather than providing tasks well matched to the students, the teachers brought in resources that alleviated the thinking required by students, instead transforming the task into one of lower cognitive demand. It is important to note that the introduction of the resources alone did not lower the cognitive demand; rather it was the teachers' suggested uses of the resources as a means to alleviate productive struggle that lowered the cognitive demand. The productive struggle is a necessary part of learning as discussed by NCTM (2010b), “Productive struggle with complex mathematical ideas is crucial to learning during problem solving” (p. 4). The teachers gave students resources that allowed students to look to the resources for solutions rather than reason out solutions for themselves.

Meg Thomas frequently told students to use their quadrilateral mobiles or flashcards to help them as they worked on tasks. This type of intervention led to a decrease in cognitive demand as students were no longer required to persevere and reason mathematically; instead they had only to read properties off of their mobile or flashcard. Similar situations occurred in Natalie Hunter and Guy Dubois’ classrooms. The teachers would tell students to use their books or vocabulary sheets to find an answer rather than reason through the task. The resources went beyond mobiles, flashcards, and books. Many times the teacher became the source of mathematical ideas, as previously discussed. Regardless of the resource, these types of occurrences lowered the cognitive demand of the tasks for the students implementing them.

As discussed earlier as a factor of set up, the time teachers allotted for tasks played a role in the cognitive demand. Earlier, I discussed the time constraints set up by teachers as contributing to the maintenance of cognitive demand. As I followed the tasks through to the implementation phase, it was clear that the teachers seldom upheld the time they presented students with during the set up. Several scenarios were common during my observations. Sometimes the teachers would tell students that their time to work on the task was over and that they were ready to collect the assignment. At that point, several students might complain and say they had yet to finish the task. These complaints led the teachers to provide additional time for the students. Typically, the original time set up for the task seemed appropriate for the task.

Therefore, when the teacher provided added time, a substantial group of students would already have completed the task, leaving the students with no mathematical activities in which to engage. The students who requested additional time sometimes used the time to complete the task and sometimes used the additional time to talk about non-mathematical topics.

Classroom habitats and dispositions. In discussing the classroom habits and dispositions in this study, I draw upon the frameworks presented in the second chapter, in particular the Math Talk Community (Hufferd-Ackles, et al., 2004). The four components of this framework—questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning—helped me to focus my analysis of the classroom activities. I discuss each of the four components separately, though I would be remiss to say these separations are clear-cut. Indeed, as acknowledged by the framework's authors, there exists much overlap among the categories. Therefore, though discussed separately, there are similarities and overlap in my discussion of these aspects. Furthermore, it is impossible for me to link causation from one category to the next. For example, my analysis of the data suggests a link between the questioning and responsibility for learning, but I cannot state that one causes the other. In addition to discussing the teachers' classrooms in terms of these components, I relate the teachers' shifts between the various levels of the frameworks with the resulting impact these shift had on the cognitive demand of the tasks.

Questioning. Questioning encompasses the types of questions asked, responses given, and source of the questions (Hufferd-Ackles et al., 2004). I classified each of the teachers' classrooms as level 1 in the area of questioning (Hufferd-Ackles et al., 2004). This classification means that the teachers had some focus on student thinking but still focused mainly on answers. The teachers also served as the predominant source of questioning in the classroom. Ms.

Thomas' classroom seemed to be on the threshold of level 2 because her students would ask questions of one another during tasks; however, I categorized her classroom as level 1 because the majority of her questions remained focused on answers and she was often satisfied with short answers.

Questioning proved to be an important factor to consider when examining the cognitive demands of tasks in the classrooms. Though each of the classrooms was level 1, there were exchanges that included more in depth, student led questioning. On several occasions, the teachers' use of questioning helped to maintain and, although quite rare, elevate the level of cognitive demand. In the following excerpt Ms. Hunter was leading the class through a task focused on sequences. The task consisted of the chart pictured in Figure 8. I classified this task as a procedures without connections task, low in cognitive demand. The students were working a row that contained the sequence $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}$.

Ms. Hunter: Alright, I'm dealing with two things, the top number and the bottom number right? Two things are happening. What am I doing to the numerator or the top number?

Mario: Multiply by two.

Ms. Hunter: I multiply each one by two? So 8 times two is?

Mario: 16.

Marisol: 16 over 243.

Ms. Hunter: How did you get 243?

Marisol: Because I multiplied it by 3.

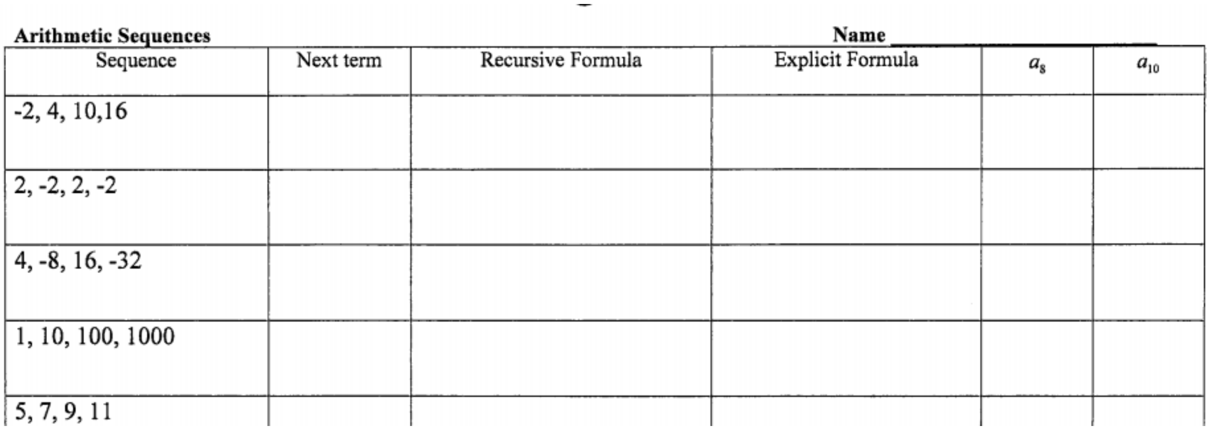
Ms. Hunter: Why did you multiply by 3?

Marisol: Because three times three equals nine, nine times three equals 27, 27 times three equals 81, 81 times three equals 243.

Mike: What? I don’t understand (to Student 2)

Marisol: You multiply.

This excerpt shows the teacher moving from short, numerical answers to asking why students performed a particular calculation. It also includes a student asking another student to clarify, though the second student’s response is not detailed. The chart did not ask students to explain how they found the next term in the sequence or justify their answers, characteristics of higher cognitive demand tasks. In this case, questioning helped to raise the demand, though perhaps not entirely to procedures with connections, but certainly somewhat higher than the task presented. The use of the question “why” proved to be one of the most frequently observed and effective ways to raise or maintain the cognitive demand.



Arithmetic Sequences **Name** ____________

Sequence	Next term	Recursive Formula	Explicit Formula	a_8	a_{10}
-2, 4, 10,16					
2, -2, 2, -2					
4, -8, 16, -32					
1, 10, 100, 1000					
5, 7, 9, 11					

Figure 8 Natalie Hunter’s Sequences Chart

Another such example of questioning raising demand came from Meg Thomas' classroom. In this excerpt Ms. Thomas was going over a probability task with students (Figure 9).

Ms. Thomas: Number five, anybody? (Juan volunteers)

Juan: It's like the same thing. Six; the dice got six sides so one out of six.

Ms. Thomas: Very good; the dice has six sides, but there is only one six on that dice so one out of six again, very good. Number six. (Alejando volunteers)

Alejandro: I got one-third.

Ms. Thomas: Tell me how you got one-third. That's correct.

Alejandro: When you add, like 6 white marbles, and 7 green marbles, and 8 marbles, you get 21.

Ms. Thomas: So that means there are 21 what?

Alejandro: Marbles.

Ms. Thomas: in the…

Alejandro: Bag.

Ms. Thomas: Very good.

Alejandro: And you have to see how many like, possibilities of getting one green, and it's seven.

Ms. Thomas: Because there are seven green ones right?

Alejandro: Yeah.

Ms. Thomas: So seven out of 21, and that reduces to give me?

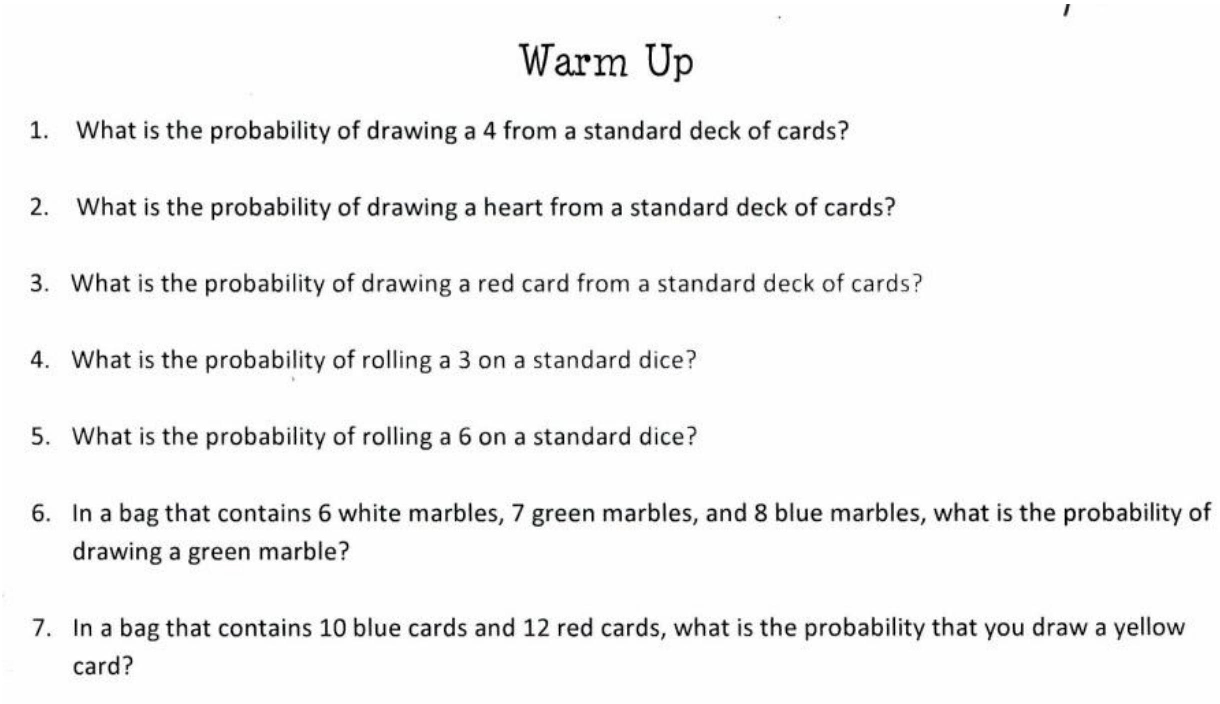
Alejandro: One-third

Ms. Thomas: One out of three. Very good. Number seven. Alright Ileana again

Ileana: It says in a bag that contains 10 blue cards and 12 red cards, what is the probability that you will draw a yellow card? And the answer is zero out of 22 because there are no yellow cards.

Ms. Thomas: Good, that’s not going to happen; you are not going to reach your hand into this bag and pull out a yellow card because there are no yellow cards in there. So 0 out of 22, and if you reduce that further then your probability is just zero; that’s not going to happen ever.

Ms. Thomas continued to go over the remainder of the problems in this way. The task only required students to provide the probability. Ms. Thomas’ questioning required students to explain their reasoning, which increased the cognitive demand of the task. It is important to note that Ms. Thomas did not encourage students to interact with one another during this time. The exchanges were all between Ms. Thomas and one student at a time. However, it was clear the students were listening to one another as they often referenced one another’s answers. This excerpt also evidences the short, fill in the blank type answers frequently used by each of the teachers, a type of questioning I address in the following passage.



Warm Up

1. What is the probability of drawing a 4 from a standard deck of cards?
2. What is the probability of drawing a heart from a standard deck of cards?
3. What is the probability of drawing a red card from a standard deck of cards?
4. What is the probability of rolling a 3 on a standard dice?
5. What is the probability of rolling a 6 on a standard dice?
6. In a bag that contains 6 white marbles, 7 green marbles, and 8 blue marbles, what is the probability of drawing a green marble?
7. In a bag that contains 10 blue cards and 12 red cards, what is the probability that you draw a yellow card?

Figure 9. Meg Thomas' probability task.

Though questioning proved to be an effective means of maintaining cognitive demand, I also witnessed many instances of questioning that led to non-mathematical thinking. Frequent question formats used in each classroom included oral fill in the blank type statements and questions asking students to give the term described by the teacher. Consider for example the following excerpt from Meg Thomas' classroom in which she was discussing how to find the sum of the interior angles of a polygon.

> So what's worth 180? What shape always has a 180 in it? (student says triangle) Triangle, so what we would do is divide any shape that you were given into triangles, but you had to do it a certain way. What kind of segments did you have to draw to get the triangles? Don't say those little lines. Do not say those little lines; they have a name. What's the name of those little lines? Look through notes look for anything to find out the name of those little lines.

The questions Ms. Thomas asked required students to recall facts, which could be viewed as a memorization task. However, she encouraged students to look it up in the book, which turned it into non-mathematical activity. A similar example comes from Natalie Hunter's classroom.

> Ms. Hunter: What do we call a sub n?
>
> Veronica: Cool
>
> Ms. Hunter: We call it cool; what else do we call it? I wrote it up there in purple (referring to the board)
>
> America: Detention!

Again, the teacher, Ms. Hunter, asked students for a term and suggested they look at the board to identify the term. On one side of the board was "the *n*th term," which was the response Ms. Hunter was hoping students would say. On the other side of the board, also in purple, was the detention list with the word "detention" in large letters. Each of the teachers frequently asked questions of these types in the classroom. Though visual representations are proposed as a means of aiding ELL students (e.g., Coggins et al., 2007), it is not simply the inclusion of the visual representation but the way in which it is utilized that will work to enhance mathematics instruction for ELL students, a point also evident in the above excerpt.

In general, the teachers supplemented their lessons and tasks with low level questions. As stated in an earlier section, Meg Thomas consciously avoided high-level questions with her sheltered students. The other teachers did not explicitly state they avoided challenging questions, but the observational data revealed the vast majority of questions were memorization level questions. The use of questions focused primarily on answers rather than process seemed to set in place the expectation that correct answers were the ultimate mathematical goal in the classrooms, a characteristic of low cognitive demand tasks.

Explanation of mathematical thinking. Explanation of mathematical thinking refers to who is explaining the mathematical ideas and the depth with which one explains the mathematical ideas (Hufferd-Ackles, et al., 2004). In this category, I again classified each of the classrooms as level 1. This classification acknowledges that the teachers would sometimes probe student thinking, but the explanations the students provided tended to be brief. Furthermore, the teachers were often satisfied with a single student strategy.

During my observations, the explanation of mathematical thinking was seldom student generated. More frequently, as evidenced in the aforementioned excerpts from Meg Thomas and Natalie Hunter's classrooms, students explained their thinking only when prompted by the teacher. This was also the case in Guy Dubois' classroom where he frequently used the question "why" during whole class discussions after students gave single word responses. It is important to note, however, that the students' responses to his question of why something was so were often simplistic. For example, the following exchange occurred when the class was discussing the Triangle Inequality Theorem,

Mr. Dubois: Gabriella, next one (referring to a triangle with sides 8,3, and 1)

Gabriella: No.

Mr. Dubois: Because it is smaller.

Mr. Dubois: Yes, what is smaller?

Gabriella: When you add.

Mr. Dubois: Yes (moves on to the next student)

Mr. Dubois had asked the previous students to explain his or her thinking, but the students still did not provide explanations until prompted by Mr. Dubois. Exchanges such as this were common throughout the time I spent in the teachers' classrooms. Students rarely explained why or how they arrived at a solution without the teacher prompting this response. Furthermore, many of their responses lacked depth. This may result from the lack of such expectations in the tasks presented to students and the classroom norms developed over time by the teachers. In either case, the lack of student explanation typically aligned with the maintenance of the tasks' already low cognitive demand.

Several opportunities arose in which the teachers' prodding for student explanations of mathematical thinking could have raised cognitive demand; however, the teachers' acceptance of brief, often superficial responses, prevented this from occurring. One such example comes from Meg Thomas' classroom. In this task Meg Thomas asked students to solve for missing quantities such as angle or side measures in a given quadrilateral. Next to each answer blank Ms. Thomas also included the word "because" and another blank in which students were to write a property that justified their solution. Meg Thomas presented one of the only tasks that included problems asking students to justify their responses, though the justifications were not mathematically rigorous as previously discussed.

The lack of in depth explanations seemed to correlate with the types of task the teachers selected. Most of the tasks did not ask for explanations, and none of the tasks asked students to present more than one solution strategy. Ms. Thomas did implement one task that allowed students to come up with unique solutions. This particular task was a type of graphic organizer in which students were expected to come up with scenarios that represented various types of events (Figure 10). The discussions around this task seemed to lead to more student involvement in explaining mathematical activities as Ms. Thomas walked around asking students to explain why they chose a particular answer. In this case, the explanations she expected required more thinking from students, thus increasing the cognitive demand of the task.

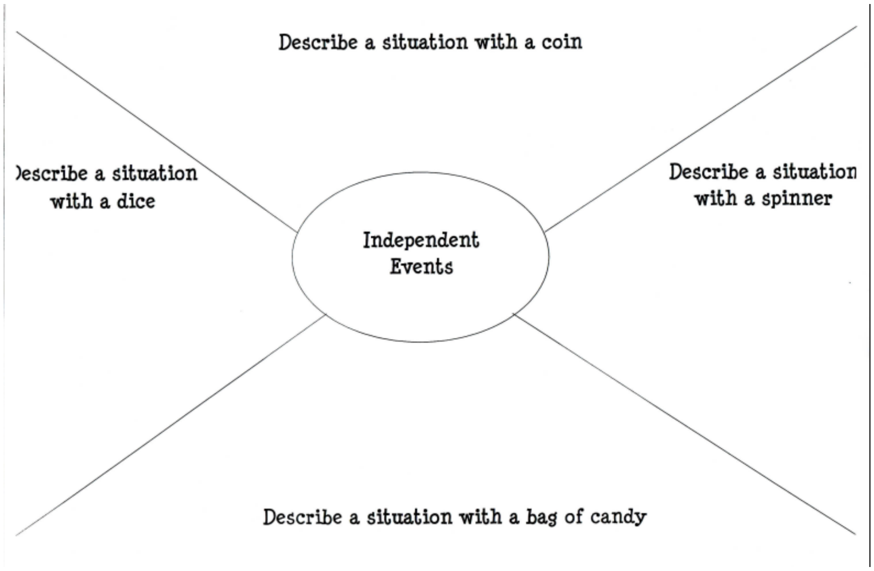


Figure 10. Meg Thomas' graphic organizer task for probability.

Hufferd-Ackles et al. (2004) discussed the strong relationships among the various aspects of their Math Talk Learning Community. In my analysis, the relationship between the explaining of mathematical thinking and the questioning aspects of the classrooms seemed especially strong. Because the teachers asked the majority of the questions in the classroom, the students seldom volunteered their own thoughts. They instead opted to wait for the teacher to ask a question before providing input. The teachers tended to focus on one student response at a time, seldom asking other students to comment on or evaluate their classmates' responses. This seemed to set in place a classroom expectation that one solution is the norm. Furthermore, because the questions tended to focus on answers and acceptable answers were often quite brief, the teachers did not expect students to provide in depth explanations. Conversely, in those instances when questioning helped to raise the cognitive demand, the students played a greater role in the explaining of their mathematical ideas, leading to for a higher responsibility for mathematical thinking.

Source of mathematical thinking. Hufferd-Ackles et al. (2004) described the source of mathematical thinking in terms of the party responsible for introducing mathematical ideas and the extent to which students and teachers explore the mathematical ideas raised. Throughout my time in the classrooms, the teachers served as the primary source of mathematical ideas. I classified both Ms. Hunter and Mr. Dubois' classrooms as level 0 in this aspect of classroom habits and dispositions, whereas I labeled Ms. Thomas' classroom as level 1. Ms. Hunter and Mr. Dubois tended to be the preeminent source of mathematical ideas in their classrooms. A typical class period would find Mr. Dubois and Ms. Hunter in front of the class at the white board leading the students through the lesson. In contrast, Ms. Thomas preferred to walk around as students worked cooperatively on lessons. As she walked around, Ms. Thomas would offer help and remained the primary source of mathematical ideas; however, she did make efforts to elicit some student ideas.

During many of the tasks, the teachers were quick to intervene when students seemed to have difficulties. The interventions often provided students with explicit directions on what steps to take next to reach a solution. Many times the intervention included the teacher taking over the task and answering the problems for the students. In each of these cases, the teacher shifted the balance toward him or herself as the source of mathematical ideas. The teachers acknowledged that they intervened more in their sheltered classes than in the non-sheltered classes. Ms. Hunter typically worked out the entire task with students if they seemed unsure of a problem. Ms. Hunter explained her decision to do more direct instruction for ELL students.

> They're (her ELL students) really funny about; they want direct instruction just about. Or that group, when I had them in the fall and I tried to do groups and discovery type stuff

> they just were like, they acted like I wasn't teaching or like it had no value. And I don't know if that's a cultural thing or if it's just that group of kids.

Mr. Dubois also frequently stepped in to go over tasks with students if they seemed unsure of how to proceed. Ms. Thomas intervened but often did so as she moved between small groups of students. Her interventions typically consisted of telling students how to proceed and ensuring they arrived at a solution. In each of the classrooms, students were not expected to persevere if tasks seemed too difficult; rather they relied on the teacher to take over such tasks. Understandably, when the teachers served as the source of mathematical ideas, the students experienced a lowered cognitive demand.

In addition to the teacher taking over as the source of mathematical ideas, other resources seemed to replace students as the source of mathematical ideas. For example, Ms. Thomas frequently told students to "use your mobiles" to identify a particular property for a given quadrilateral. Similarly, Guy Dubois and Natalie Hunter referred students to notes and the textbook to find needed information rather than asking questions to trigger student thinking. The reliance on resources such as the mobiles and textbooks for mathematical ideas lowered the cognitive demand for the students.

Another occurrence related to the explanation of mathematical thinking was the teachers' failure to pick up and extend students' thinking. For example, Mr. Dubois limited a student's explanation, saying he only wanted one property from each student.

> Mr. Dubois: So, you have to say something or draw something. You have to say something smart that has to do with one of these seven things (quadrilaterals).
>
> Jack: Alright, a square is like four, 90 degree angles.
>
> Mr. Dubois: Ok, so a square has four, 90 degree angles.

Jack: And uh, four equal sides too.

Mr. Dubois: Wait, I said you say one thing. I know you like to talk.

Jack: I did, I said *and.*

Mr. Dubois: I know, that was a second one.

There were many instances similar to this in each of the other teachers' classrooms. In Ms. Hunter's classroom, students would frequently call out answers. She often ignored incorrect answers, instead focusing on correct answers. The failure to follow up on student reasoning resulted in missed opportunities to raise the cognitive demand. For example, the following excerpt is from a lesson on adding and subtracting polynomials in Ms. Hunter's classroom. Ms. Hunter presented students with algebra tiles, and she was trying to get students to understand how to use the algebra tiles.

Ms. Hunter: Do they fit on there perfect? (asking if the unit tiles fit along the long side of the rectangular tile perfectly)

Maria: No.

Ms. Hunter: No? They don't fit perfect do they?

Marisol: So they are supposed to fit, like perfectly?

Ms. Hunter: Is it exactly 5? (Addressing Maria, ignoring Marisol's question)

Maria: No.

Ms. Hunter: You are measuring still is what I think you are doing, ok.

Maria: Can I use a ruler?

Ms. Hunter: No, you can't use a ruler, just the one and the x is all you've got. So what's the side? X, right? So how do I find the area?

Javier: You multiply by one.

Ms. Hunter: Multiply what by what to find area?

Javier: By one. x by

Ms. Hunter: (cutting off student) We just did this! Come on, how do you find area? Guys?

Marisol: By five.

Ms. Hunter: What?

In this exchange, Natalie Hunter missed several opportunities to explore student ideas and questions. Had she listened to Javier and built on his reasoning, she could have had the opportunity to help the class understand his error by evaluating his response. This type of error analysis is characteristic of more cognitively demanding task implementations These types of exchanges were common in each of the classrooms as the teachers seldom picked up on incorrect student answers. The teachers often led students to solve problems in a particular way rather than build on novel solution paths. The teachers stated that this was a conscious decision enacted to avoid the confusion generated by multiple solutions. The teachers' lack of demand for in depth student explanations and multiple solution paths contributed to the maintenance of low cognitive demand during implementation.

The responsibility for learning. The responsibility for learning refers to the extent to which students are active learners in the classroom (Hufferd-Ackles et al., 2004). I classified Ms. Hunter and Mr. Dubois' classrooms as level 0, reflecting a teacher-centric responsibility for learning. In each of their classrooms, the teachers responded to student answers with a statement of verification or, in the case of incorrect answers, a correct solution strategy. This practice led students to be passive mathematics learners. I classified Ms. Thomas' classroom as level 1 because she had begun to set up her classroom to encourage and facilitate her students' growth as active mathematics learners through her selection of tasks and instructional format.

Throughout my observations, it was evident that when the students served as passive learners, the cognitive demand remained low. In those instances, although far less frequent, where students were more actively involved in learning, the cognitive demand was maintained or even increased. In order to illustrate, I present two scenarios. The first vignette is from Natalie Hunter's classroom. In this excerpt, Ms. Hunter was beginning a task on rate of change, a new concept for her students.

Ms. Hunter: How do you find the average? Average, how do you find the average? You add them all up and divide by how many there are, right? And your average, what's something, what's an example of an average? What's something that you got, oh, I don't know, Friday that has your average on it? Anybody know?

Marisol: Your report card.

Ms. Hunter: Your progress report has your average on it right? And that's your grade. You add up all your grades and divide by how many there are. Ours are a little bit more complicated than just that, but that's the idea, right? So, average rate of change would be another name for?

Belinda: Slope.

Ms. Hunter: Slope. Average rate of change would be another name for slope. So on the first one on the worksheet that you have in your hand, what would be another name for average rate of change? Or a way to define it? I could call it what? I could call it… You don't know? I just told you like three times. What could you call it?

Belinda: The slope.

Ms. Hunter: Slope. Está bien. You could call it the slope.

Belinda: That's number one right?

Ms. Hunter: Yeah, isn't that what is says? It says average rate of change? (Natalie turns on the projector). So another word for average rate of change would be? I said it like eight times, I'll say it one more, the slope.

In this excerpt, Ms. Hunter carried the responsibility for learning. She asked a number of questions to which students did not respond. She continued in this manner for quite some time before a student finally stated the slope. The student then asked about the answer to number one, further evidence of the normal classroom behavior where Ms. Hunter provided the answers as students recorded them. In this instance, Ms. Hunter lowered the cognitive demand of the task as she took over the responsibility of learning. The class carried out the remainder of the task in a similar fashion.

This second vignette is from Meg Thomas' classroom and illustrates the students taking on a more active role in their mathematical learning due to the structures Ms. Thomas set in place. In this excerpt, Ms. Thomas was discussing the students' solutions to a task that involved students moving to various stations. At each station there were clues describing a particular quadrilateral. Students were to choose the quadrilateral they thought the clues described, drop it in a bag, and then move on to the next station.

Ms. Thomas: Alright, then y'all are going to help me with three. I am not going over it; we're going to discuss and argue about what answers you put in the bag.

Somebody clear off the board and somebody bring them (the answers) to me. (student tapes the answers from the bag onto the white board) Alright, now, on this one, let's write up our properties. We're split, we've got two people voted parallelograms, one trapezoid, one isosceles trapezoid, and two rhombus. So, let's argue our answers. (Writes the clues on the board as she talks) Both pairs of opposite sides congruent, both pair consecutive angles supplementary, and my two diagonals bisect each other. K? You all argue.

Michael: It's the first two, the first two!

Ms. Thomas: The parallelogram?

Michael: Yup

Ms. Thomas: Why? (Several students start explaining at the same time) Everybody's talking to me, and I don't know what you're saying. So Michael you seem to know what you're talking about; I need you to come teach me why you think it's a parallelogram.

At this point, Michael came to the board to explain his reasoning to the class. Meg Thomas continued to facilitate a discussion until the class came to an agreement. In this case, Ms. Thomas' classroom exhibited many of the features of a level 3 classroom. She encouraged the students to consider one another's mathematical contributions. She also asked students to state whether they agreed or disagreed with one another's answers and to tell why. During this activity, the entire class actively engaged in learning. The task itself required students only to drop the names of the quadrilaterals in the bags based on clues, leading to its initial categorization as a memorization level task. Meg Thomas' facilitation shifted the responsibility of learning toward the students and resulted in an increase in cognitive demand for this task.

In addition to the classroom environments fostered by the teachers, the tasks the teachers selected also affected the responsibility for learning. Natalie Hunter and Guy Dubois both expressed their disinclination to use what they referred to as "discovery learning" tasks. These tasks involved students problem solving and working toward the development of mathematical ideas. These types of tasks, when well designed, allow students to explore and construct mathematical ideas for themselves, leading to a higher responsibility for learning. This is mainly because the teacher must step back from the role of the mathematical authority and become a facilitator who aids students. Because Guy Dubois and Natalie Hunter avoided these tasks, they presented information to students and remained the source of mathematical ideas. Meg Thomas frequently stated that she enjoyed discovery learning tasks; however as stated previously, she tended to classify any cooperative learning activity in this way. However, because Ms. Thomas encouraged small group work and often avoided direct instruction, her students often took a more active role in learning as compared to the other two classrooms.

The link between the source of mathematical ideas and responsibility for learning also seems quite strong. The more students were encouraged to take on an active role in the classroom, the more mathematical ideas they raised. In those instances when teachers shifted control to students, the cognitive demand of tasks rose and students appeared to participate more actively in the lesson. Conversely, when teachers refused to relinquish control of the lesson, students passively experienced the mathematics as they ceded the presentation of mathematical ideas to the teacher. This resulted in a lowered cognitive demand.

In general, as the classroom habits and dispositions shifted toward the student, the cognitive demand of a particular task increased. As the classroom activity shifted toward the teacher, the cognitive demand generally remained at a low level, sometimes devolving into non-mathematical activities.

A Discussion of Implementing Tasks with ELLs

During my time in the teachers' classrooms, those tasks that did not decline in cognitive demand or devolve into non-mathematical shared two key characteristics. First, each of these tasks retained focus on a mathematical goal. Second, the implementation of these tasks involved classroom communication that was not shifted entirely toward the teacher. In this section I discuss these two aspects in detail.

Maintaining focus on a mathematical learning goal is vital to maintaining tasks' cognitive demand when working with ELLs. Hiebert, Morris, Berk, and Jansen (2007) discussed the importance of explicitly stated learning goals in the mathematics classroom, noting that "Formulating clear, explicit learning goals sets the stage for everything else" (p. x). I include the term mathematical when discussing learning goals because the teachers in this study frequently stated learning goals for their sheltered classes that were not mathematical in nature.

The teacher determines the mathematical learning goals for a particular lesson, but outside influences likely affect the teacher's development of the goals. For example, a teacher might state that he or she is teaching a particular standard for the day's lesson. Though the standard influences the teacher's learning goals, the interpretation of the standard by the teacher is what constitutes the learning goal. Brown (2009) compared teachers' interpretations of curriculum materials to a musician's interpretation of sheet music, highlighting the room for interpretation teachers have when enacting curriculum materials. An example comes from

Natalie Hunter’s classroom. Ms. Hunter described the lesson as covering the following state standard, “MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques” and corresponding substandard “Recognize sequences as functions with domains that are whole numbers.” Ms. Hunter’s interpretation of the standard was that students were to understand arithmetic sequences: “We do geometric [sequences] in Math 2. They’re [geometric sequences] in the Carnegie textbook, but they are not part of Math 1 standards.” Ms. Hunter focused her lesson only on the algebraic representation of the sequences as evident from the chart task she used during this lesson (see Figure 8). Therefore, the learning goal Ms. Hunter focused on for this particular lesson was the algebraic manipulation of arithmetic sequences.

In addition to maintaining focus on mathematical learning goals, the tasks that maintained cognitive demand featured classroom communication during implementation. The findings of this and other studies suggest that a focus on communication is an important aspect of teaching mathematics to ELLs (Brenner, 1994; Coggins, Kravin, Coates, & Carroll, 2007; Khisty & Morales, 2004; Moschkovich, 1999). The communication about mathematics to which I refer includes both written and oral communication. In looking across my data, I found it especially helpful to consider classroom communication in terms of the four aspects of Hufferd-Ackles et al.’s (2004) Math-Talk Learning Community: explaining of one’s reasoning, questioning, sources of mathematical ideas, and responsibility for learning. These aspects, when shifted toward the student, supported students as they engaged in high cognitive demand mathematics tasks.

Communication serves to support the mathematical goal of a meaningful task. One could imagine the mathematical practices as bricks and communication serving as the mortar connecting the bricks to one another. An example from Ms. Thomas' classroom further exemplifies this relationship. When Ms. Thomas led the students in a discussion about kites, she was able to use oral and written communication to support her students as they developed an understanding of what constitutes a kite in the mathematical register. The students answered questions and presented examples and non-examples of kites, each of which the class discussed. This allowed the class to come to a consensus regarding the key characteristics of a kite and the development of a graphic organizer to help understand kites. The students served as the source of many of the mathematical ideas during this lesson and did much of the explaining. In this instance and others where the mathematical goal was maintained, students were encouraged to communicate, and the teacher served to facilitate student generated communication, the cognitive demand was maintained and the students remained engaged in the lesson.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Finding strategies to improve the educational outcomes of ELLs is imperative as they are the fastest growing segment of U.S. students but continue to reside on the lower end of the achievement gap (Genessee, Lindholm-Leary, Saunders, & Christian, 2005; Thomas & Collier, 1997). Though the majority of teachers now have at least one ELL student in their classrooms, only about one-third of teachers have received training in effective teaching strategies for ELL students (Ballantyne, Sanderman, & Levy, 2008). The mismatch between training and the realities of teaching leaves many teachers to their own devices as they seek out, create, or modify curriculum materials for their ELL students.

In this study I examined 3 high school mathematics teachers' selections and enactments of mathematical tasks for their ELL students. More specifically, I attended to the cognitive demand of the tasks as they moved through the three successive phases of the Mathematical Tasks Framework (Stein & Smith, 1998) in order to answer the following research questions.

1. How do teachers choose mathematical tasks for use with their ELL students?
    a. What are the characteristics of the tasks the teachers select?
    b. What factors influence the teachers' selections of tasks?
2. What modifications, if any, do teachers make to mathematical tasks prior to their implementation with ELL students?
    a. What factors influence the teachers' decisions to modify or not modify the tasks?

    b. In what ways, if any, do these modifications affect the cognitive demand of the tasks?

3. What aspects of the classroom appear to contribute to the maintenance or decline of high cognitive demand in mathematical tasks?

I employed a qualitative, multiple case study design (Yin, 2009). Because of my focus on ELLs, I selected teachers who taught a significant number of ELL students. The three teachers in this study taught different variations of a ninth grade mathematics course comprised entirely of English Language Learners. Because the classes were purposely comprised entirely of ELL students they were termed “sheltered” mathematics courses. Each teacher completed a background survey (Appendix B). The survey questions focused on the teachers’ experiences teaching ELL students, specifically with regard to the instructional strategies and curriculum materials they used when teaching ELLs. I observed each teacher’s sheltered mathematics class for two weeks. In addition to video recording each observation to later create partial transcripts, I wrote field notes. Prior to each observation, I conducted a short interview during which I asked about the lesson for the day. Upon the conclusion of the two-week observation cycle, I conducted two extended interviews with each teacher. During the first of these interviews, I asked teachers about the lessons I had observed. The second interview was what I refer to as a curriculum interview. In this interview I brought in units from various textbook series and asked teachers to discuss each of the units in terms of their usefulness for ELL students. All interviews were audio recorded and fully transcribed. The final source of data came from classroom artifacts, including copies of the tasks the teachers used and samples of student work.

I analyzed the data using the constant comparison method decoupled from grounded theory. This involved many rounds of inductive coding where I identified themes and collapsed them into broader categories. I first analyzed the data for each teacher and then conducted a cross-case analysis. In the following section, I discuss the main findings of this study.

Conclusions

Mathematics tasks evolve as they progress through the various phases of implementation. The task as written in the curriculum materials may be all but unrecognizable when observed during enactment. I presented the findings in relation to each of the phases of the Mathematical Tasks Framework (Stein & Smith, 1998) and have done the same with the study's conclusions. Therefore, in this section I discuss the major findings as they relate to each of the successive phases of task implementation.

Tasks as Written

The teachers abandoned their textbooks in favor of other resources because they thought the textbooks provided by their schools were overly wordy, and therefore too difficult, for ELL students. The teachers typically focused on the quantity of words rather than on the content of the words as they selected tasks for their ELL students. In lieu of the textbooks, the teachers often generated problems using a software program or found problems by searching the internet. The resulting problems were typically low in cognitive demand and contained very few words other than the directions.

The vast majority of tasks selected by the teachers were highly repetitive and low in cognitive demand. The teachers selected these problems to help the students build procedural fluency through repetition. The tasks were typically devoid of a context, a decision the teachers made to avoid the word count required to present a context. The tasks typically consisted of a worksheet with a number of similar problems. Most of the problems asked for students to solve for or find a value. The number of tasks requiring students to produce a response beyond a numerical value was quite small. The teachers seemed to place a great importance on their ELL students' proficiency in completing mathematical procedures.

A common purpose of the low cognitive demand tasks selected by the teachers was to build students' mathematical vocabularies. The teachers said that they focused on vocabulary to a greater extent with their ELL students than with their native speaking students. Many of these tasks were memorization level tasks. Others fell into the category of non-mathematical activity due to the disconnect between the terms and their mathematical meaning. These tasks typically required students only to copy definitions from a source such as a glossary.

The teachers' reasons for selecting tasks fell into three categories. First, the teachers' perceptions of their students played a significant part in their task selections. The teachers seemed to correlate their ELL students' limited proficiency in English with limited mathematical abilities. The perceived lack of mathematical abilities led the teachers to choose low cognitive demand tasks commensurate with their students' abilities. In addition to perceiving ELL students as having a lower mathematical capacity, the teachers tended to hold lower expectations for their ELL students due to stereotypes. These stereotypes ranged from ELLs not going on to college to Latinos not valuing cooperative learning tasks. In each case, the stereotypes led the teachers to select less demanding tasks for their sheltered classes. The choice of relying predominately on low cognitive demand tasks runs contrary to studies that assert a variety of tasks is important (e.g., Stein et al., 2009).

The second factor impacting task selection was the teachers' focus on standardized testing and the standards. The teachers frequently discussed the importance of end of course tests and making Adequate Yearly Progress to their students and to me. When discussing how they planned lessons, the teachers said they first examined the standard on which the lesson was to focus and selected tasks that supported their interpretation of the standard. The focus on standardized testing was evident as the teachers frequently discussed the students' need to know a particular way of doing something because it would be on the end of course test. Similarly, the teachers frequently stated that students did not need to prove or know why some concepts worked because they would not be on the end of course test. The procedural focus of the standardized assessments contributed to the teachers' selections of procedural tasks.

The final factor impacting task selection was the teachers' perceived lack of resources. The teachers frequently lamented the lack of adequate curriculum materials for their sheltered students. The teachers thought the recent shift to what the state referred to as a standards-based curriculum created new challenges for their ELL students because the new curriculum brought about increased language demands. The deficient curriculum materials led the teachers to seek out and create resources they perceived to be better suited for their students. In addition to a lack of materials, the teacher thought they lacked the support necessary to be effective teachers. Though each of the teachers had received an ESL certification, they added a lack of professional development to their list of deficient resources. The teachers stated that additional training or support from ESL faculty would be an appreciated supplement.

Tasks as Set Up

Lacking what they deemed adequate resources for their ELL students, the teachers modified existing curriculum materials. These modifications were akin to the notion of adaptations described by Brown and Edelson (2003) in which the teachers adopt some aspects of the curriculum but alter others. The teachers modified both the task content and the instructional format set up for the tasks. With regard to the former, the teachers' modifications included simplifying both the tasks' language and mathematics. Typically, modifications to the language consisted of the teachers deleting words or eliminating a back story. The modifications to the mathematics included lowering expectations of proof, eliminating potentially confusing mathematical symbols, rewriting to prevent multiple solutions, and circumventing non-integer solutions. The teachers undertook these modifications specifically to make the tasks more suitable for their ELL students.

In addition to modifying task content, the teachers modified the instructional format for the tasks. The modifications teachers made to the instructional format included the arrangement of students, time allowed, and resources provided. The teachers routinely modified the format typically used for their mainstream classes when teaching ELL students. These modifications to format consisted of the teacher using or avoiding group work. The teachers also stated that they allowed ELL students more time to complete tasks. The extended time seemed to relate to the teachers' perceptions of their students as less mathematically able than their native speaking peers. Frequently, the teachers presented their ELL students with resources such as calculators and visual aids as a means of modifying their instruction for ELL students.

None of the modifications the teachers made increased the cognitive demand of the tasks; the modifications either maintained or lowered the cognitive demand, a result consistent with Stein et al.'s (2009) findings. Though the modifications to the instructional format did not impact the cognitive demand during set up, the modifications to the task content did. The teachers' simplification of the mathematical content was the only modification the teachers made to intentionally lower the cognitive demand. The teachers simplified the mathematics in an attempt to make problems less confusing for their ELL students. The teachers seemed to base this modification upon a false correlation between the students' English proficiency and mathematical abilities. Though the teachers discussed simplifying the words in tasks so as not to impact the mathematics, my analysis revealed this was not the case. Indeed, the removal of a context and requirements for student explanations routinely resulted in lowered cognitive demand.

Tasks as Implemented

As students and teachers implemented tasks, there were several aspects that impacted the cognitive demand. The classroom norms governing acceptable explanations and solutions seemed to lower or maintain already low cognitive demand. The teachers' classroom norms regarding satisfactory responses were consistent with the level of tasks selected. The tasks routinely asked only for a solution and rarely required students to provide an explanation. Similarly, the teachers were satisfied with students providing mathematically imprecise responses and seldom probed students to expound upon answers.

During implementation, the use of some resources lowered the cognitive demand to non-mathematical activity. The teachers encouraged students to rely upon resources such as flashcards and textbooks to complete tasks already quite low in cognitive demand. The provision of these resources transformed the tasks into transcription activities, effectively eliminating the need for mathematical thinking.

In general, the classroom communication served to maintain and on rare occasions, elevate the cognitive demand of tasks. Using Hufferd-Ackles et al.'s (2004) Math-Talk Community Framework, I examined the use of questioning, responsibility for learning, explanations of mathematical thinking, and sources of mathematical ideas in the classrooms. When these aspects shifted away from the teachers to the students, the cognitive demand was often maintained or even elevated. Conversely, when these aspects shifted toward the teacher, the cognitive demand lessened, sometimes devolving into non-mathematical activity. These findings agree with researchers who suggested communication is vital to ELLs' development of academic language and mathematical understandings (Brenner, 1994; Coggins, Kravin, Coates, & Carroll, 2007; Khisty & Morales, 2004; Moschkovich, 1999).

Implications

This study fills an important gap in the literature regarding the implementation of curricula with ELLs. This research can impact teachers' practice in order for them develop strategies that improve the mathematics education of ELL students. Knowing how to select or modify curriculum materials in ways that maintain the mathematical rigor is important for students to build mathematical understanding.

Teachers should focus their selection and enactment of mathematics tasks on a mathematical learning goal. Teachers should maintain focus on the mathematical learning goal throughout the phases of task implementation to ensure the task does not devolve into non-mathematical activity. This is especially important as teachers develop goals for their ELL students. These goals, such as the development of academic language, should maintain mathematical rigor and purpose. The development of the mathematical learning goals should take into account the mathematical standards for which teachers are responsible but avoid an overly narrow interpretation that limits connections.

In addition to maintaining focus on mathematics, teachers should work to facilitate student-centered communication in the classroom. Teachers might examine their practice in terms of the aspects set forth by Hufferd-Ackles et al. (2004)—questioning, responsibility for learning, source of mathematical ideas, and explaining of mathematical thinking. The examination should include a critical evaluation of where on the teacher to student scale the communication lies. As I found in this study, when communication in these areas shifts toward the student, the cognitive demand is maintained or elevated. Furthermore, many leading researchers in the area of the mathematics education of ELLs assert the importance of rich student discussions in the mathematics classroom as a means of building students' mathematical understandings (e.g., Coggins et al., 2007)

The teachers in this study tended to discuss mathematical ability and language ability interchangeably. Teachers should avoid conflating student proficiency in the English language with student ability in mathematics. In this study, this unfounded link resulted in lowered mathematical expectations and a lowering of cognitive demand. Teachers should instead focus on getting to know students as individual mathematical learners. English language learners are a heterogeneous group, and mathematics instruction should reflect the diversity of experiences, thoughts, and abilities these students bring to the classroom. In doing so, teachers may avoid the sweeping generalization of ELL students as a homogeneous group in terms of both language and mathematical abilities. Teachers should maintain high expectations for all students and bring students up to meet these expectations, rather than holding low expectations that do not help students grow as mathematical learners.

Selecting and implementing high cognitive demand mathematics tasks is an area in which many teachers struggle (Stein et al., 2009). The teacher in this study struggled to use these types of tasks with ELL students; however, on the occasions I observed these teachers' non-sheltered classes there too was a noticeable lack in high cognitive demand tasks employed. Teacher educators must continue to work with preservice and inservice teachers to provide ample opportunities to observe and enact high cognitive demand tasks with students. These models will help teachers to better understand what these tasks look like when enacted and will provide opportunities for teachers to observe the learning outcomes that result from the use of these types of tasks.

Teacher educators should work to meet the growing need for professional development for teachers in the area of mathematics education of ELL students. U.S. teachers are currently underprepared for teaching ELL students. As the number of ELL students continues to rise, the mismatch between well-prepared teachers and ELL students will continue to grow unless teacher educators and other stakeholders prioritize teacher preparation that includes strategies for effectively teaching ELLs. Teacher educators and professional developers can build on the findings of this study to develop strategies to better prepare teachers for this rapidly increasing population of students. These strategies should include fostering student-centered, mathematical communication, selecting appropriate mathematical tasks, modifying curriculum materials, and providing appropriate resources.

Teacher preparation programs should broach the subject of teaching ELL students. Mathematics teacher educators should explicitly address the teaching of ELL students and provide strategies and resources to increase ELL students' opportunities for learning. The majority of U.S. teacher candidates are monolingual white women (Villegas & Lucas, 2007). Making preservice teachers aware of ELL students' needs and strategies that meet these needs is important. Teacher educators could include research findings and reports that discuss effective strategies in their courses. Discussions of curriculum materials appropriate for ELL students as well as the modifications one can make to curriculum materials should be included in teacher preparation.

Less than half the states require training on ESL strategies for teacher licensure (NCELA, 2008). These courses tend to focus on general strategies for all content areas instead of strategies focused specifically on mathematics education. Policy makers may want to consider including mathematics specific strategies in the licensure requirements for mathematics teachers. The teachers in this study reflected upon the difficulty of finding strategies aimed directly at mathematics. Furthermore, they were unsure how to apply some of the general strategies they learned to their mathematics classrooms. Including mathematics specific strategies and ideas could aid in better preparing mathematics teachers to teach ELL students.

Finally, this research could also allow curriculum developers to understand the challenges teachers encounter when selecting, modifying, and enacting curriculum materials for ELLs. Curriculum developers should understand that many teachers did not receive training to teach ELL students. This lack of training leaves teachers to their own devices as they select curriculum materials. Curriculum developers should include written recommendations for using their materials with ELL students. These recommendations should be specific to a particular task rather than a general statement in the teacher's guide. The training provided by curriculum developers to acclimate teachers to use their curriculum should include tips for ELL students. These considerations can lead to improvement in curriculum materials to support teachers of ELLs.

Future Research

This study may lead to research along a number of avenues. One such avenue is examining teachers who routinely select high cognitive demand tasks in ELL classrooms. In general, the teachers in my study selected tasks already low in cognitive demand. As noted by Stein et al. (2009), it is uncommon for tasks to increase in cognitive demand. Therefore, when the tasks selected by these teachers lowered in cognitive demand, they tended to fall outside of the cognitive demand classifications provided by Stein et al. into non-mathematical activity.

Future research could investigate the prevalence of this phenomenon in classrooms in which teachers begin with high cognitive demand tasks.

The classes examined in this study were sheltered mathematics classrooms comprised entirely of ELL students. Future research might explore teacher task selection and enactment in mainstream classrooms with ELL students. Researchers might exam teachers’ task selections in these classrooms to understand if the presence of a smaller proportion of ELL students yields similar results.

Researchers might examine whether strategies aimed at improving the educational outcomes of ELLs would positively impact other students as well. This might be particularly insightful with students who struggle to read at grade level because their language difficulties may carry similar implications to the difficulties faced by ELL students. This line of research may be particularly meaningful since as of 2009, one-quarter of all U.S. eighth graders read below the basic level (National Center for Education Statistics, 2010).

As I noted in a prior chapter, I did not measure student or teacher knowledge. Therefore, these aspects and their potential relation to the maintenance or decline in cognitive demand remain unexplored. Future research might include these measures in order to better understand factors that may impact teachers’ task selection and enactment. Measuring student knowledge may allow researchers to determine whether the inclusion of a greater number of high cognitive demand tasks yields greater learning gains or allows students to think more flexibly about mathematical concepts.

Building professional development to counter those factors that contributed to a lowering of cognitive demand also presents interesting research opportunities. Researchers could study professional development focused on effective teaching strategies for ELL students. The researchers could observe practice before and after the professional development to better understand what impact, if any, the professional development had on the teachers' practice and on student learning.

Finally, the development of high school level tasks that are high in cognitive demand and linguistically accessible could provide interesting insights into the teaching and learning of ELL students. Though textbooks such as *Pitfalls and Pathways* (Barnett-Clarke, Ramirez, & Coggins, 2009) target ELL students, these textbooks are unlikely to be adopted by districts with low numbers of ELL students. Furthermore, few, if any, of these types of textbooks target high school students. The development of rich mathematical tasks that encourage communication and maintain mathematical rigor would aid teachers in search of such tasks. Researchers could examine the implementation of such tasks and measure student learning in such tasks as opposed to the modification of existing tasks. It might also be interesting to examine the learning outcomes of native English speakers on such tasks, especially those that do not read on level.

Closing Remarks

All of the teachers in this study cared greatly about the success of their students. They selected tasks they thought were well suited for their ELL students. The teachers intended for the modifications to their teaching and instruction to improve the educational outcomes of their students.

With this study I sought to better understand the process of task selection and enactment. Knowing the strategies teachers employ when teaching ELL students is quite relevant to US education. English language learners have historically been at the lower end of the achievement gap (Fry, 2008) and tend to have lower graduation rates (NCES, 2009). Finding strategies to improve the educational outcomes of these students is imperative as ELLs are the fastest growing segment of U.S. students (Thomas & Collier, 1997). Furthermore, understanding how to best teach ELL students will likely result in better strategies for all students.

As the communication demands of mathematics classrooms continue to increase, teachers of ELLs are responsible for finding and executing instructional strategies to aid their students. Understanding the ways in which teachers select and enact tasks to accommodate their ELL students is an important first step to creating effective teaching strategies for ELLs. I encourage future researchers to continue to find ways to improve the educational outcomes of not just ELL students, but also all students as we move into the era of the Common Core State Standards. I challenge teachers both old and new to continue to find ways to engage all students in mathematics through meaningful discussions. Finally, I hope that we as a country avoid viewing English language learners as deficient and instead embrace their knowledge and culture as assets in the classroom and community.

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APPENDIX A

DATA IN RELATION TO RESEARCH QUESTIONS

<table>
<tr><td colspan="3">1. How do teachers choose mathematical tasks for use with their ELL students?
a. What are the characteristics of the tasks they select?
b. What factors influence the teachers’ selection of tasks?</td></tr>
<tr><td>Survey</td><td>Daily Planning Interviews, Final Interview, and Curriculum Interview</td><td>Curricular Materials</td></tr>
<tr><td>Determine the sources of tasks and where they find the tasks</td><td>Determine how teachers choose the tasks</td><td>Establish the cognitive demand of the chosen tasks prior to set up</td></tr>
<tr><td colspan="3">2. What modifications, if any, do teachers make to mathematical tasks prior to their implementation with ELL students?
a. What factors influence the teachers’ decisions to modify or not modify the tasks?
b. In what ways, if any, do these modifications affect the cognitive demand of the tasks?</td></tr>
<tr><td>Classroom Observation</td><td>Daily planning Interviews and Final Interview</td><td>Curriculum Interview</td></tr>
<tr><td>Verification how the task is presented during set up phase and it’s resulting cognitive demand as set up</td><td>Determine teachers’ perspective on what modifications were made, if any, and rationale for modifying (or not) and to determine the cognitive demand of the task the teacher plans on implementing</td><td>Determine how teachers might modify high cognitive demand tasks</td></tr>
<tr><td colspan="3">3. What aspects of the classroom appear to contribute to the maintenance or decline of high cognitive demand in mathematical tasks?</td></tr>
<tr><td>Classroom Observations</td><td>Post-Observation Interviews</td><td>Student Work</td></tr>
<tr><td>Observe classroom aspects and the cognitive demand of the task as implemented</td><td>Determine teachers’ perspective of the task’s cognitive demand during implementation</td><td>Verifying that the task was carried out as set up by the teacher</td></tr>
</table>

APPENDIX B

SURVEY

Directions: Please complete this on your own. Please do not discuss with others or refer to outside sources in completing this survey. When you have completed the survey, please email it back to me at dearaujo@uga.edu.

Part 1: Background Information

1. Which mathematics courses have you taught?

2. Which mathematics courses are you currently teaching?

3. How long have you been teaching?

4. How long have you taught in this school district? How long have you taught in this school?

5. What degrees have you received? What institution(s) granted the degrees?

6. What type of teacher preparation did you complete? (Circle those that apply)
    a. Undergraduate mathematics education preparation
    b. Undergraduate degree outside education; Masters' mathematics education
    c. Undergraduate degree outside education; Alternative preparation program (please describe)
    d. Other (please describe)

7. In what areas are you certified to teach?

8. Do you hold an ELL endorsement? What was required for this endorsement? How long ago did you receive the endorsement?

9. Do you speak any languages other than English? If so please state the language(s) and your fluency level.

Part 2: Teaching Practice

Indicate your agreement with each of the following statements by circling the appropriate column word/phrase. If you would like to expand on an answer, please include those comments beneath your response.

1. I believe that one of my primary responsibilities as a teacher is to select and develop mathematical tasks.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

2. I like to use problems with multiple solutions / paths often in my classes.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

3. I like my students to master basic procedural skills before they tackle complex problems.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

4. I encourage students to use manipulatives and other representations to explain their mathematical ideas to each other.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

5. Creativity, reasoning, and problem solving are fostered in my classes.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

6. I regularly engage students in real-life math problems that are of interest to them.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

7. When students are working on math problems, I put more emphasis on getting the correct answer than on the process.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

8. I don’t necessarily answer students’ math questions but rather let them puzzle things out for themselves.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

9. In my math classes, students learn best when they can work together to discover mathematical ideas.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

10. The district-provided textbook and supporting materials are the main sources for mathematics in my classroom.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

11. I received adequate preparation to teach English language learners.

N/A	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Comments: ____

Part 3: Understanding your Classroom

ELL students

1. What preparation have you had to teach ELL students?

2. What challenges, if any, does the new curriculum present to ELL students?

3. If applicable, how do you differentiate your teaching for ELL students?

Materials for Teaching Mathematics

1. What curriculum materials have you used / will you use to teach the mathematics? Are there other materials you would like to have to better teach the mathematics?

2. If you have used / plan to use the state frameworks tasks, how do you decide which ones and how much of each task to use?

3. Do you modify materials for your ELL students? If so how?

4. What curriculum materials do you find are best suited for teaching ELL students?

THANK YOU FOR YOUR PARTICIPATION!

APPENDIX C

LESSON GRAPH FORMAT

Time:	Description & Phrases	Themes	Comments
Insert Time Stamp Here	• Partial Transcriptions and screen shots go here • Each row will include an episode of related events	Address instances related to theoretical framework	Discuss task phase and questions or general comments
	•		
	•		

APPENDIX D

OBSERVATION PROTOCOL

1. How does the teacher present the task to students?

2. How do the students implement the task?

3. How does the teacher interact with students during the task?

4. How do the students interact with one another during the task?

5. How does the teacher wrap up the task?

APPENDIX E

DAILY PLANNING INTERVIEW PROTOCOL

1. What lesson are you planning to teach today?
2. What mathematical task(s) will you be using?
3. Where did you find the task?
4. What resources will you use with the task?
5. What are your mathematical goals for the task(s)?
6. What modifications have you made to the task(s), if any?
7. Why have you chosen (or not) to modify this task?
8. How will you introduce the task(s)?
9. How will students work on the task(s)?
10. How will you assess the students?
11. What products will the students produce, if any?

APPENDIX F

FINAL INTERVIEW PROTOCOL

OBSERVATIONS

1. What was the most successful lesson?
2. What would you do differently next time?
3. What would you do the same?
4. What was your least successful lesson?
5. What would you do the same? Different?
6. Were any of the materials specifically chosen with ELLs in mind?
7. Ideal curriculum for ELLs? Class set up?

BACKGROUND

1. Tell me more about your ELL certification.
    a. Courses
    b. Experiences
    c. What you remember
2. How did these students get placed in your class?
3. How can they test out?
4. Will they continue to be sheltered next year?
5. Are there any contextualized tasks you used this year?

VIEWS

1. Does good teaching look different in a sheltered classroom?
2. What is most important thing to consider when teacher ELLs? Advice to new ELL teachers?
3. Do you modify your materials for ELLs in different classes?
4. As a sheltered math teacher, what do you see your responsibilities and goals are?
    a. Are these different from your non-sheltered classes?
5. What type of resources do you think would benefit you as a sheltered mathematics teacher?

APPENDIX F

TEXTBOOK UNITS USED IN CURRICULUM INTERVIEW

Triangle Congruence Units			
Unit Label	**Curriculum Unit**	**Unit Details**	**Classification**
A1	Georgia Department of Education. (2008). *Mathematics 1 frameworks student edition: Unit 3 geometry gallery*. Retrieved from https://www.georgiastandards.org/Frameworks/Pages/BrowseFrameworks/math9-12.aspx	Triangles Learning Task (Unit 3, p. 14)	Standards Based Unit
A2	Jacobs, H. R. (2003). *Geometry: Seeing, doing, understanding*. New York, NY: W. H. Freeman and Company.	Lesson 6 SSS Congruence (p.163)	Blend, mostly traditional
A3	College Entrance Examination Board. (2004). *SpringBoard mathematics with meaning: Geometry*. New York, NY: College Entrance Examination Board.	Truss your Judgment (p.69)	Standards Based Unit
A4	Jurgensen, J. W., Brown, R. G., & Jurgensen, J. W. (1993). *Geometry*. Evanston, IL: McDougal Littell.	4-2 Some Ways to Prove Triangles Congruent (p.122)	Traditional
A5	Larson, R., Boswell, L., & Stiff, L. (2003). *Geometry*. Evanston, IL: McDougall Littell.	4.3 Proving Triangles are Congruent: SSS and SAS (p.212)	Traditional
A6	Long, A. (2007). *Georgia high school mathematics 1*. Evanston, IL. McDougall Littell.	4.8 Prove Triangles Congruent by SSS (p.236)	Traditional
Triangle Congruence Units			
B1	*Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E. (2008). Core plus mathematics course 1*. Columbia, OH: McGraw-Hill.	Investigation 4 Getting the Right Angle (Unit6, p.378)	Standards Based (NSF)
B2	Larson, R., Boswell, L., & Stiff, L. (2003). *Geometry*. Evanston, IL: McDougall Littell.	9.2 The Pythagorean Theorem (p. 535)	Traditional
B3	Jurgensen, J. W., Brown, R. G., & Jurgensen, J. W. (1993). *Geometry*. Evanston, IL: McDougal Littell.	8-2 The Pythagorean Theorem (p.290)	Traditional
B4	Jacobs, H. R. (2003). *Geometry: Seeing, doing, understanding*. New York, NY: W. H. Freeman and Company.	Lesson 5 – The Pythagorean Theorem (p.365)	Blend, Mostly Traditional
B5	Fendel, D., Resek, D., & Alper, L., (1996). *Interactive mathematics program year 1*. Emeryville, CA: Key Curriculum Press.	Investigation 2 – Television Screens and Pythagoras (Unit 5, p.362)	Standards Based (NSF)