PROFILES AND PREDICTORS OF MATH ACHIEVEMENT IN EARLY ELEMENTARY SCHOOL

by

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(Under the Direction of Martha Carr)

ABSTRACT

The longitudinal study examined fourth grade children's profiles of strategy-use and the contributions of children's second grade behaviors to their fourth grade profiles. The longitudinal study followed 206 children in the second-, third-, and fourth-grades to assess changes in strategy-use and achievement as children solved multi-digit computation and word problems. Each year children were assessed on types of strategies selected, fluency, accuracy, confidence in math abilities, spatial abilities, and math competency.

Clustering analyses were run on fourth grade strategy-use to determine whether there were distinct groups of children as a function of strategy-use. Next, discriminant analyses were performed to determine whether fourth grade cluster group membership could be predicted by second grade strategy-use, fluency, accuracy, confidence, spatial ability, performance on a math competency test. Following this, a MANOVA was performed on fourth grade math competency subtest scores using cluster group membership as the independent variable to determine whether group membership affect all, or only some, areas of mathematics competency.

Results indicate that fourth grade children separate into groups of cognitive strategy-users, transition strategy-users, and manipulative strategy-users. The groups were differentiated by competency, fluency, accuracy, and spatial abilities. Twenty-five percent of the 4th-grade children were classified as manipulative strategy-users with limited proficiency in addition and subtraction problem-solving skills. Second grade cognitive strategy-use, performance, and fluency predicted fourth grade group membership in the cognitive and transition strategy-use groups. The study also determined that fourth grade strategy-use contributed differentially to the various domains of mathematics.

INDEX WORDS: elementary school mathematics; mathematics achievement; strategy use; spatial abilities; fluency; accuracy

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CHAPTER 1

REVIEW OF LITERATURE

Theories of Math Development

Introduction

Researchers index the development of children's mathematical knowledge about number, counting, and numerical relationships by the changes in children's strategy-use during addition and subtraction computations (Geary, 1994). The construction of mathematical knowledge is a process of continuous, progressive change, and it emerges from the interactions between children and their environments. Each math experience builds upon the former, and a conceptual framework of information is created that, over time enables the emergence of mathematical reasoning and accurate computations and problem solving (Beilin, 1994). Developmental theories and models aid in the identification and explanation of the cognitive components and processes associated with the changes in children's mathematical strategy-use, such as how counting-based strategies evolve, how children select strategies, and the typical pattern of strategy development. Longitudinal studies examine the pathways of how children's strategies change over time and provide markers identifying significant contributors to strategy development.

Schema-Models of Strategy Development

Children's mathematical knowledge is conceptualized in terms of schema (Baroody, 1992; Steffe, Cobb, & von Glasersfeld, 1988). Schema are action sequences of procedures that are implemented when solving a problem, and the term implies that conditions inherent to the problem are recognized as requisite conditions for the execution of specific actions (von Glasersfeld, 1995). The notion of schema indicates a well-integrated or well-structured knowledge base of conceptual, procedural, and utilization competence that is most likely represented in multiple forms (Baroody, 1992; Greeno, Riley, & Gelman, 1984). Math strategies are embedded within the developing schemes and operations, and they change as a function of the children's emerging knowledge (Carr & Hettinger, 2002).

The developmental progression of children's math knowledge stem from children's early counting schemes (Steffe, 1992). Through counting schemes, children develop the foundational mathematical structure referred to as the number sequence, which in turn supports the development of part-to-whole mathematical reasoning and children's concept of number (Baroody, 1987; Fuson, 1992; Greeno et al., 1984). Each stage progressively engenders more abstract knowledge about number that allows the student to shift from concrete representations, or manipulative strategies to mental operations, or cognitive strategies. Initially, children must physically represent both addends and physically touch or move each object as it is counted when solving addition problems (Carpenter, Fennema, & Franke, 1996; Carpenter & Moser, 1984). For example, an early learner would physically represent the counts of three and four with counting chips in the problem 4 + 3. To solve the problem, the child would then recount all the chips in a number sequence. The advent of the initial number sequence allows the child to mentally hold the count of four in her head and physically represent the count of three with chips. To answer the problem, the child would begin the number sequence count with the cardinal value of four and would utilize the chips to count-on three more.

The schema based model of development developed by Baroody (Baroody & Ginsburg, 1986; Baroody & Tiilikainen, 2003) highlights the acquisition of basic number combinations as fundamental to math knowledge. It asserts that the number combinations of 0 through 9 contain the structure, patterns, and relationships that create the foundational principles of children's knowledge of number and general mathematical skills (Kilpatrick, Swafford, & Findell, 2001). The model assumes that conceptual, procedural, and factual knowledge form an integrated unit in children's math knowledge, and the schema structures are arranged in an hierarchical network of semantic memory (Anderson & Pirolli, 1984; Bjorklund, Muir-Broaddus, & Schneider, 1990). As children's mathematical knowledge develops, the number of and associations of the semantic nodes change to allow for modified and new knowledge. The network engenders children's selection of more fluent and more accurate strategies that produce increasingly more efficient answers during calculations (Baroody, 1987, 1995; Baroody, Ginsburg, & Waxman, 1983). For example, as children develop the conceptual understanding of how numbers relate to each other and recognize number combination patterns, decomposition and retrieval strategies emerge (Baroody, 1989; Baroody et al., 1983).

The schema models recognize the significance of young children's experiences with physical objects in the development of counting schemes and early procedures. In general, most children gradually transition from using concrete manipulative strategies to cognitive strategies over the first three years of school. Children who are not capable of abstract representations of number benefit from the use of concrete representations for computations (Boulton-Lewis, 1998); however, concrete representations are cognitively demanding in that children must transform the elements of the problem onto their concrete representations, a process that is effortful and time consuming (Boulton-Lewis, 1993a, 1993b, 1998). If the representations are not transparent and easily identified by children, the use of concrete representations can be laborious (DeLoache, Miller, & Pierroutsakos, 1998) constraining other cognitive activities, such as problem identification and monitoring of procedures.

To briefly summarize, schema models assert that mathematical schemes and strategies emerge in a specific order, and that higher level strategies emerge from lower level strategies. Young children construct their initial math schemes and strategies through the active manipulation of physical objects. With experience children internalize their initial concepts of number (i.e., mathematical counting schemes), and they rely less on physical representations. The developing abilities to mentally represent numbers provokes children to progressively create more sophisticated strategies, such as cognitive strategies that result in more abstract understandings of the principles of number relationships. With practice, children develop complex cognitive knowledge structures regarding the interrelationships of number combinations that result in the selection of more efficient, cognitive-based strategies in numerical operations.

Information Processing Approaches to Strategy Development

The initial studies regarding the acquisition of mathematical skills in young children assumed that procedural skill and strategy knowledge were age dependent and stage-like in manifestation (Ilg & Ames, 1951; Siegler & Shipley, 1995). For any given type of problem, an appropriate strategy would emerge that would function as the dominant strategy for computations with less sophisticated strategies being discarded in favor of more efficient ones. For example, the work of Groen and Parkman (1972) asserted that first-grade children consistently solve addition problems using the *min strategy*. Children using the *min strategy* would solve the problem 6 + 3 by starting at 6 (the larger addend) and counting upward 3 counts (i.e., counting "7, 8, 9"). The regression analyses used in the study (Groen & Parkman, 1972) indicated that the smaller addend in an addition problem was the best predictor of solution times. Using similar methods, Ashcraft (Ashcraft, 1982) concluded from studies of elementary children that first-graders regularly implemented the *min strategy*, fourth-graders and older children consistently use *retrieval*, and third-graders sometimes employ both strategies.

Children acquire and employ several different kinds of math strategies across problem sets and within any specific type of problem (Fuson, 1988; Geary, 1994; Ginsburg, Klein, & Starkey, 1998; Siegler, 1996). Siegler's model of Strategy Choices and Strategy Discovery Simulation (SCADS) (Crowley, Shrager, & Siegler, 1997; Shrager & Siegler, 1998; Siegler, 1996), posits that multiple strategies exist within a child's cognitive repertoire at any given time with each of the strategies competing for selection. With practice and maturation, children select more effortful but more efficient strategies to solve problems. Initial use of a new strategy may not reveal the efficiency, but the variance in children's selections enables the strategy to be practiced. The change in strategy selection is seen as a series of overlapping waves with different strategies implemented more often at different ages or in different contexts. Overtime, new strategies are constructed and implemented as older less efficient strategies gradually disappear from the repertoire (Chen & Siegler, 2000). In general, the change in strategy generation and selection is considered to be a gradual transition (Alibali, 1999) especially among children with several strategies in their repertoire. The driving force in the development and application of higher order strategies is the child's continuous process of self-modification to confidently select the most fluent and most accurate procedures during calculations.

The strategy selection process is explained by the model of distribution of associations (DOAM) (Siegler & Shrager, 1984). The underlying assumption of the model is that children's favored strategy is retrieval; however, when children are unsure of the answer or faced with a difficult problem, they will select a back up strategy, such as counting-on for accuracy reasons. The distributions of associations model is composed of two interacting components that account for strategy selection: the associated probabilities of numerical facts and an individual's confidence criterion. The ability to retrieve an arithmetic fact is represented by a distribution of associative strengths between a problem (e.g., 3 + 5) and all the possible answers to that problem (e.g., 7, 8, 9). If most of the strength associated with the problem is concentrated on one answer (e.g., 8), then the distribution is described as peaked, and if the strength associated with the problem is spread among several possible answers (e.g., 7,8, 9), the distribution is described as flat. According to the model, problems with relatively peaked distributions are more likely to be solved by retrieval, and problems with relatively flat distributions are more likely to be solved with backup strategies, such as counting-on. The strength of association is also a measurement of fluency, in that retrieval strategies are fast.

Each child also generates a confidence in ability criterion, the threshold that must be exceeded by the associative strength of a retrieved answer for that answer to be expressed. If the associative strength does not exceed the confidence criterion, the child may retrieve another answer or solve the problem with a backup strategy. The model signifies the importance of confidence in children's selection of mathematical strategies with more confident children using retrieval (and perhaps cognitive strategies) over manipulative-based strategies.

In sum, Siegler's strategy choice models address strategic variability in children's mathematical procedural behaviors, and the models describe the mechanisms and processes for changes in children strategy selection. The acquisition of new strategies is a process that is best understood in the context of children's prior knowledge. With practice and maturation, children become more accurate, more fluent, and more confident in the utilization of any particular strategy; and they begin the process of selecting or discovering a more efficient strategy (Siegler & Jenkins, 1989)

The next section will focus on several variables thought to affect the emergence of mathematical knowledge and strategies. Fluency and accuracy have been proposed to support the emergence of new conceptual knowledge by freeing cognitive resources (Royer, Tronsky, Chan, Jackson, & Marchant, 1999). Increased fluency and accuracy on basic math facts should allow students the resources to develop higher level strategies. Self-confidence has been found to predict mathematics achievement and is thought to support children's acquisition of new strategies that might require effort and for which they may not necessarily have high accuracy (Casey, 1996; Eccles, Wigfield, Harold, & Blumenfeld, 1993) As students practice strategies and number combinations they should become more confident in their use and be more likely to use them. Spatial ability is linked to mathematics achievement and is thought to support computations (Casey, Pezaris, & Nuttall, 1992; Hegarty & Kozhevnikov, 1999). It may be that spatial ability supports the emergence of more complex cognitive strategies by allowing children to more efficiently represent number. Each variable will be discussed as predictors of mathematics knowledge and strategies.

Correlates of Math Achievement

Fluency and Accuracy

Fluency is the speed of processing in computations, and it is a critical component to the process of strategy development (Baroody & Ginsburg, 1986; Siegler & Shrager, 1984; Steffe, 1992). An underlying assumption in developmental theories is that over time, children select more efficient strategies. For example, Steffe's model posits that with experience, children's developing fluency with counting schemes leads to the emergence of number sequences, and to increasingly more abstract strategy-use. Baroody posits that with computational experience, children develop accuracy and fluency in basic operations; and, therefore, select more efficient strategies in the effort to reduce cognitive demands in children. Siegler's DOAM model directly addresses the importance of fluency in that the strength of association between a problem and its answer is calculated as a fluency measurement.

Other research indicates that fluency is important to mathematical achievement. First, fluency predicts mathematical performance, and numerous studies have provided empirical evidence of the robustness of the relationship between fluency and mathematical achievement (e.g., Canobi, 2005; Geary et al., 1991; Royer et al., 1999). In addition, fluency in the retrieval of math facts (i.e., the arithmetic values of all combinations of 1 to 9) functions as an antecedent to the development of higher order mathematical calculations and computational abilities (Royer et al., 1999; Tronsky & Royer, 2002).

Fluency operates in the working memory system (Adams & Hitch, 1998), and working memory has limited capacity that constrains the processing of information (Bjorklund, 2000; Case & Okamoto, 1996; Swanson, 2006). Fluency increases as a function of practice (Goldman, Pellegrino, & Mertz, 1988; Hiebert & Wearne, 1996; Royer et al., 1999), and as children become more fluent, less space is needed in the working memory for the execution of operations. Working memory is also limited in duration (Schneider & Pressley, 1989), and requires rapid and efficient computations in order to be effective. Fluent processing decreases the possibility that information will decay in memory, especially when children employ complex strategies that demand more than one operation (Kintsch, 1988).

Retrieval is associated with increased fluency in mathematics computations because it requires minimal operating space and minimal time for execution (Hecht, 2002; Siegler, 1987; Tronsky & Royer, 2002). Retrieval of math facts is a significant predictor of children's achievement. For example, in comparisons of gifted, normal, and mathematically disabled children (Geary & Brown, 1991), gifted children scored the highest rates of correct retrieval trials and disabled children scored the lowest. According to Royer et al., (1999), it is the early development of fluent retrieval of correct answers to addition and subtraction problems that provides the fundamental foundations for later development of various math competencies. Fluent cognitive processing and retrieval frees memory space for other higher order cognitive processes, such as constructing a representation of a problem, recognizing similarities to problems in long-term memory, activating appropriate computational procedures, and monitoring the results.

Accuracy is the number of problems that study participants correctly solve (Carr & Davis, 2001; Hecht, 2002). As children become more accurate in their procedural skills, children also improve in achievement performance (Geary, Brown, & Samaranayake, 1991; Siegler, 1996; Siegler & Jenkins, 1989). For example, Siegler's (1988) study of good students, not-so-good students and perfectionist students examined children's strategy use, accuracy in computations, and speed of processing. The good students and perfectionists were characterized by accuracy in strategy-use and high retrieval rates (i.e., an index of speed), and they scored significantly better on the computation and word problem subtests of standardized achievement measures than the not-so-good students.

Confidence in Math Abilities

A positive relationship between confidence in learning math and math achievement was reported in an early study, (Crosswhite, 1972), and several studies have validated the relationship (e.g.,Reyes, 1984). Although the focus of most research on confidence as it relates to mathematics achievement has been on gender differences (Carr & Davis, 2001; Carr, Steiner, Kyser, & Biddlecomb, in press; Eccles et al., 1993; Ewers & Wood, 1993), a few studies provide insights as to how confidence relates to the development of mathematical knowledge. Other research examines the interplay of confidence and achievement.

Research in the development of confidence in math abilities (Eccles et al., 1993) indicates that children's beliefs about their competence in doing mathematics differs from their beliefs regarding the importance of the mathematics task. The factors are referred to as competency beliefs and subjective task values. Siegler's (Siegler & Shrager, 1984) model of the distribution of associations (Siegler & Shrager, 1984) speaks directly to issues of competency beliefs. The model is composed of two components, the associated probabilities of numerical facts and an individual's confidence criterion. The confidence criterion is a measurement of an individual's style (Kerkman & Siegler, 1997), in that some children require higher levels of confidence than others to answer problems. Studies of individual differences in strategy choice (Kerkman & Siegler, 1993, 1997; Siegler, 1988) demonstrate the significant impact of children's self-confidence in strategy selection and accurate computations. The research identified three groups of children based on their patterns of strategy use and errors in addition and subtraction. One group was identified as the "perfectionists," and the children were characterized by little evidence of retrieval strategies and by high accuracy in computations. According to Siegler (1988), "perfectionists" are children with peaked distributions of associations and high thresholds of confidence. These children test high in performance, and they often use backup strategies to ensure their accuracy. Children classified as "good students" employed the retrieval strategy on most problems and were relatively accurate in retrieval and backup strategy use. These children display relatively peaked distributions of associations and relatively relaxed thresholds of confidence in comparison to "perfectionists." Children in the third group were identified as "not-so-good students," in that they were less accurate in retrieval and backup strategies. "Not-so-good students" had relatively flat distributions of associations and fairly low levels of confidence criteria, and they made more mistakes in any strategy that they select.

It is widely accepted that math achievement contributes to increased confidence in one's math abilities. The Ewers and Woods (1993) study, compared groups of children on assessments of confidence and math performance. As predicted, the high achieving children scored significantly higher on the assessments of confidence and made significantly fewer over-estimations in the prediction accuracy assessments. The study implies that high achieving children possess a realistic assessment of one's mathematical abilities. Confidence and achievement share an iterative relationship. For example, Casey (1996) reports that individual's skill in using spatial manipulations contributes to children's feelings of self-confidence in mathematical computations.

Spatial Abilities

Although none of the developmental theories address the contribution of spatial abilities in the development of math strategies, the extant literature indicates a consistent correlation between spatial ability and mathematical performance (Casey, Nuttall, Pezaris, & Benbow, 1995; Hegarty & Kozhevnikov, 1999). Currently there is a renewed interest in the academic community regarding the importance of spatial abilities as cognitive skills that are necessary for the educational challenges in the modern technological world (Levine, Huttenlocher, Taylor, & Langrock, 1999; Liben, 2006). It has been suggested the ability to mentally rotate items implies a predisposition to process information using spatial, or abstract strategies (Casey, 1996), and empirical research indicates that spatial abilities are evident in children as young as four-years of age (Levine et al., 1999). Given the relationship of spatial abilities to math achievement and the contemporary interest in types of cognitive processes, spatial abilities was included as a correlate of math achievement in the study.

Spatial ability refers to one's skill in representing, transforming, generating, and retrieving symbolic, nonlinguistic information (Linn & Petersen, 1985). Mental rotation and spatial visualization are types of spatial abilities and have been correlated to mathematical performance (Casey et al., 1995; Hegarty & Kozhevnikov, 1999). Spatial abilities predict performance on word problems, by allowing individuals to create schematic representations of the word problems (Hegarty & Kozhevnikov, 1999; van Garderen, 2006).

Halpern (Halpern & Wright, 1996) asserts that to understand the interplay of spatial abilities and mathematical performance, it is necessary to provide a model that analyzes the cognitive processes that are activated when solving problems (Just & Carpenter, 1985). Rather than relying on the type of information that is used in problem solving, such as verbal or quantitative skills, the process-oriented model examines the underlying cognitive processes employed in the execution of the task. For example, some mathematics problems can be solved through retrieval of facts and procedures from memory whereas other problems require visual representation and manipulation of information in working memory.

Research Questions

The literature provides theories and models of how elementary school-aged children develop mathematical knowledge. As children develop skills in solving addition and subtraction problems, children's performance behaviors develop characteristic achievement profiles defined by their repertoire of strategies correctly implemented.

Children's strategy-use evolves over time and the developmental trajectory posits that strategies change from primarily manipulative strategy-use to an increasing reliance on cognitive strategy-use. The strategy change demonstrates a move to more efficient strategy-use (Baroody & Ginsburg, 1986; Siegler & Shrager, 1984; Steffe, 1992) and an increasing proficiency in using abstract representations of number in solving computation and word problems. Whereas, children's profiles of math achievement can be indexed by children's strategy-use, the literature also posits that changes in achievement is supported by measurements of fluency, accuracy, confidence in math abilities, and spatial abilities.

The current study examines fourth grade children's profiles of math achievement and the contributions of children's second grade behaviors to their fourth grade profiles. The primary purpose of the study was to determine group differences in strategy-use and other correlates of math achievement that would identify and predict group achievement. It was believed that the early identification of group differences would allow teachers to plan effective classroom curriculum that would support the early development of strategies that promote math achievement. Three research questions are identified in this study. The first question concerns descriptive profiles of achievement in fourth grade children. The second question examines from a longitudinal perspective the predictors of children's math performance and is divided into two parts: identification and prediction. The last question explores how children's strategy use is related to competency in different mathematical domains.

Q.1: *Are there distinct groups of children identifiable by the patterns of mathematical strategies in the fourth grade? Is one of these groups characterized by high levels of manipulative strategy use?*

Theoretical and empirical research assert that children's cognitive strategies are derived from manipulative strategies during the early elementary school years (Baroody, 1987; Fuson, 1988; Gelman & Gallistel, 1986; Siegler & Jenkins, 1989): however, some children do not develop cognitive math strategies within the normal developmental time frame (Geary & Brown, 1991; Geary et al., 1991). It was hypothesized that children in the fourth grade will cluster into multiple groups on the basis of the strategies they use. It was further hypothesized that the group strategy-use clusters would be identifiable by other correlates of math achievement assessments, such as assessments of competency, total correct, fluency, accuracy, confidence, and spatial ability. It was expected that children in the manipulative strategy-use cluster would score significantly lower in achievement assessments (i.e., total correct in computations) than children in the cognitive strategy-use cluster (Biddlecomb & Carr, 2006).

Q.2A: *Given that there are multiple group clusters, what are the differences in the second grade assessments of strategy-use that describes the differences among the fourth grade multiple group clusters? What are the differences in the second grade assessments of competency, total correct, fluency, accuracy, confidence, and spatial ability that describe the differences among the fourth grade multiple group clusters?*

Longitudinal data provides information regarding developmental changes in children's knowledge and performance. Identifying the patterns of strategy-use that were employed by second graders allows for the identification of group differences among the multiple group clusters as second graders (Huberty & Olejnik, 2006). It was believed that group differences could be determined on the basis of second grade strategy-use and correlates of mathematics achievement. The ability to do so would allow teachers to better assess students with the goal of better instruction.

Q.2B: *Do children's second grade assessments of strategy use predict fourth grade multiple group cluster membership? Do children's second grade assessments of CRCT, total correct, fluency, accuracy, confidence, and spatial ability predict fourth grade multiple group cluster membership?*

It was hypothesized that second grade children can be reliably classified into fourth grade group clusters as a function of second grade strategy-use and second grade correlates of math achievement. The early identification of children in clusters allows for the opportunity for early mathematical intervention for low achieving children (Geary, 1990; Gersten, Jordan, & Flojo, 2005; Griffin, Case, & Siegler, 1994). Mathematical achievement is assumed to be a composite of the interactions of children's strategy use, fluency, accuracy, confidence, and spatial ability (Carr et al., in press).

Q.3: *Is there a relationship between cluster membership and Criterion Referenced Competency Test (CRCT) math achievement tests' scores in the fourth grade? Do children identified in the manipulative strategy-use cluster score significantly lower on all math sub tests than children in the cognitive strategy-use cluster?*

Some literature suggested that children with manipulative strategies would differ significantly in achievement scores from children with cognitive strategies (Canobi, Reeve, & Pattison, 1998; Fuson, 1992; Geary et al., 1991; Goldman et al., 1988), with manipulative strategy-users scoring significantly lower in all subtests of the mathematics CRCT. Other researchers believe that different areas of mathematics require different skills and this would suggest that performance on CRCT subtests will not be uniformly high for cognitive users and uniformly poor for manipulative strategy users.

CHAPTER 2

METHODS

The Data Set

The data used in the current research is part of a larger study that was designed to examine gender differences in mathematical achievement over a three year period (Carr et al., in press). The longitudinal data set includes measures of strategy-use, achievement, fluency, accuracy, confidence in math abilities, spatial abilities, and math competency (i.e., criterion referenced achievement assessments) on 206 children. The independent variables were selected because they predict mathematical performance (Canobi, 2005; Casey, 1996; Casey et al., 1995; Geary et al., 1991; Siegler, 1988) and because gender differences exist in these variables.

Two hundred forty-one children, 118 boys and 123 girls, from 35 elementary classrooms in seven schools in northeastern Georgia participated in the longitudinal study of second-, third-, and fourth-graders. In the first year of the study, the mean age of the students was 7.5 years (SD = .62). Seventy-one percent of the sample were Caucasian, 24% were African American, 3% were Asian, and 2% were Latino. Participating children were recruited with the written permission of their parents and the school. In the third year of the study (i.e., the fourth grade), 206 children, 100 boys and 106 girls participated in the project. Families moving out of the school district impacted the study as evidenced by the 14.5% attrition rate over the 3-year longitudinal study; however, the demographic parameters of the sample did not vary significantly. Seventy-two percent of the returning children in the fourth grade were Caucasian, 23% African American, 3% Asian, and 2% Hispanic. Only children who returned signed permission forms and individually agreed to take part in the assessments participated in the study. At the conclusion of the testing session, children were given a pencil for their participation in the study.

Procedures and Materials

During the fall of the second grade and the fourth grade school years, the participating children were assessed for strategy-use, achievement (i.e., total correct), fluency (i.e., speed of processing), accuracy, confidence in math abilities, and spatial abilities. The following spring children completed statewide CRCT testing (i.e., math competency), and the mathematical overall and subtest scores were obtained from school records.

The children were interviewed individually outside the classroom for about 50 minutes by two graduate student researchers. Prior to the testing session, the children were told that the researchers were interested in finding out how kids do math and that they would be given some math problems to complete. After the children were assured that the procedures were solely designed to understand how kids do math, they were individually asked to participate. Half of the children were assessed by one investigator for fluency, accuracy, confidence in math abilities, and spatial abilities prior to the assessment of strategy-use and total correct conducted by the other investigator. The testing sequence was reversed for the other children.

During the second grade assessments, children were video-taped while completing the measurements of strategy use, fluency, and accuracy; and the tapes coded at a later time. The spatial ability measure was completed using pencil and paper forms.

In the fourth grade, only the strategy-use assessment was video-taped and coded later. Fourth grade assessments of fluency, accuracy, and spatial abilities were conducted on a laptop computer using Cognitive Aptitude Assessment Software (CAAS) that collects button press and vocalization latency data. The confidence in math abilities measure at both time points was completed in pencil and paper forms, each question being read aloud by the research investigator.

Strategy-Use

Prior to the assessment, the children were given the following instructions. "I have some math problems for you to solve, and you can solve them any way that you want to. After you give me the answer, I am going to ask you about how you got your answer." Children were given counters and paper and pencils to use during the strategy-use evaluation. Each math problem was read aloud from a printed card before being placed in front of the child. Following the instructions, the investigator presented two sets of randomly selected problems, and the children worked at their own pace to solve each problem. If a child was unable to solve a problem within two minutes, the investigator gave the child the option of moving on to the next problem. After each problem was solved and an answer was given by the children, the investigator asked the children to tell how they solved it.

In the second grade two sets of ten problems each were presented to each child. The first set of computation problems consisted of ten (5 addition, 5 subtraction) double- or triple-digit problems with solutions ranging from 3 to 595. The second set consisted of ten (5 addition, 5 subtraction) word problems with solutions ranging from 9 to 101. Half of the computation and word problems required children to borrow or carry. (See

Appendix A for the Second Grade Math Problems.) In the fourth grade one set of ten computation problems and one set of twelve word problems were presented to each child. The first set consisted of ten (5 addition, 5 subtraction) double- or triple-digit computation problems with solutions ranging from 59 to 1075, and the second set consisted of twelve (6 addition, 6 subtraction) word problems. Eighteen of the problems required children to borrow or carry. (See Appendix B for the Fourth Grade Math Problems.)

The strategies were coded for accuracy (i.e., correct answer to the problem) and classified as manipulative or cognitive strategies. Determination of strategy-use was based on the children's reports of strategy-use and the observations of the children's behaviors from the video-tapes. The method of observing strategy-use and children's retrospective report of strategy used has been found to be a valid indicator of the strategies children use (Siegler, 1987, 1989).

Strategies were coded as manipulative, or concrete representational, strategies when there was evidence of children using counters, counting on fingers, counting dots, or using hatch marks on paper (Carr & Jessup, 1997; Siegler, 1989). The counting-all-manipulative was identified when each number value in the problem was physically represented. Other coded manipulative strategies were counting-on-manipulative, counting-up-manipulative, and counting-back-manipulative, and were identified when children mentally used a place-holder to represent one value in the problem and concretely represented the other value with counters, fingers, dots, or hatchmarks. Children's overt behaviors, such as any movement of the fingers which suggested the fingers were acting as concrete representations were considered to be manipulative.

The cognitive strategies were defined as strategies in which the children reported using thinking to solve the problems, and there was no evidence of the child using manipulatives (Carr & Jessup, 1997; Siegler, 1989). Cognitive counting strategies were observable when children moved their lips during the solution. The cognitive strategies coded were counting-on-head, counting-up-head, decomposition, and retrieval. Decomposition strategies were classified when children reported mentally breaking the number values into groups of 10's and 1's and calculating the answers as sets of 10's and 1's, or when children separated the values into smaller subsets of problems (i.e., 5 + 4 decomposes into 4 + 4 + 1). Retrieval strategies were identified from children's self-reports and by their quick responses. Retrieval indicates automatic knowledge of the problem's answer.

Standard algorithm strategies were classified when children computed the multi-digit numbers in the proper column order, from right to left (i.e., ones-place, tens-place, hundreds-place, etc.). The standard-algorithm-manipulative strategy was identified when the children physically counted the numerical values in the standard column order from the right column (the one's-place) to the left columns (the ten's- and hundred's-place values). The standard-algorithm-head strategy was coded when the children mentally counted or retrieved the numerical values in the standard order. Modified algorithm strategies were identified when the children computed the multi-digit numbers working from the left column (i.e., the ten's-place) to the right column (i.e., the one's-place). The modified-algorithm-manipulative strategy was identified when the children physically counted the numerical values beginning in the left column and working to the right columns. The modified-algorithm-head strategy was coded when the children mentally counted or retrieved the numerical values in the left column working to the right columns.

In the second grade assessments, an additional strategy-use category was identified, rule-10. The rule-10 strategy is a teacher taught strategy to use with double-digit computations when the numbers are designated sets of tens, such as 20 or 30. Children are taught to increment the ten's place only. For example, to solve 20 + 33, children are to add 2 sets of 10's to the 33 to compute the answer of 53. The rule-10 strategy is specific only to problems with designated sets of tens and does not generalize to problems of decomposition. In the fourth grade, children were also coded for standard-algorithm-mix and modified-algorithm-mix, when children used a combination of cognitive and manipulative strategies across place values during computations. For example, standard-algorithm-mix was coded when the child used manipulative counting on fingers for the one's place value computation but used the cognitive retrieval strategy for the ten's place value computation. Refer to Table 1 for detailed descriptions of each type of strategy that was coded.

Scores of strategy-use were calculated by summing across correct responses within each strategy-use category. Across the 20 second grade problems and the 22 fourth grade problems, the number of correct solutions within each strategy-use category was tallied to create categorical scores for each child. In the second-grade, inter-rater reliabilities of strategy assessments were calculated as the ratio of observational agreements to total number of observations. Inter-rater reliabilities were .97 for manipulative strategy use and .98 for cognitive strategies. In the fourth grade, inter-rater reliability was calculated using Cohen's Kappa (Huck, 2000). Fifteen percent of the cases were randomly selected, yielding the inter-rater reliability kappa = .897.

Total correct

Achievement was assessed as the total number of correctly answered computation and word problems that were administered during the strategy-use assessment. Possible scores for second graders range from zero to twenty, and fourth graders scores range from zero to twenty-two.

Fluency

The counters, paper, and pencils were removed from the testing area following the strategy-use evaluation. Each student was then presented with 5 addition and 5 subtraction single-digit combinations to assess the student's speed and accuracy in basic mathematical facts. The children were told to solve the problems as quickly as they could without counting in their heads or using any manipulatives (i.e., fingers). Fluency was calculated by averaging the reaction times of the correctly answered problems. Second-grade children were presented the problems in paper form and scores were assessed from the video-tapes. Scoring was calculated as a function of the reaction time and accuracy of all the problems. The time of each frame of video-taped data was measured as 1/3 second, and the number of frames between the investigator's presentation of the problem and the child's correct answer were counted. Fourth-grade children were presented the same problems on the computer, and scores were automatically calculated from the CAAS software. Scores were recorded as the number of seconds per problem. (See Appendix C for Fluency Combinations.)

Accuracy

Accuracy was calculated as the percentage of number combinations the children answered correctly in the 10 single-digit combinations presented in the *Fluency* assessment. Scores range from zero to 100. The correlation between fluency and accuracy was not significant in the second grade (r = .10) but the scores were significantly correlated in the fourth grade (r = -.34, p <.01).

Confidence in Math Abilities

The mathematics competence scale (Eccles et al., 1993) was administered to assess the children's confidence in their mathematical abilities. The measurement is comprised of six questions and ratings on a 7-point Likert type scale. Children responded to questions about how good they are in mathematics, how difficult they think mathematics is, their expectations about future performance in mathematics, and how good they would be at learning something new in mathematics. Possible scores summed across the six questions range from six to forty-two. (See Appendix D for Mathematics Competence Scale.)

Each question was read aloud by the investigator, and bar-values of the 7-point Likert type scale were explained for each question. Children were given a pencil and told to circle the bar that best represented the answer to each question. Reliability data for this scale have been gathered for second and fourth grade students and are .78 and .83, respectively (Eccles et al., 1993). For the current study, the internal consistency was .71 for the second-graders and .75 for fourth graders

Spatial Ability

The Vandenberg Test of Mental Rotation (Vandenberg & Kuse, 1978) was administered to assess children's three-dimensional spatial ability. The test consists of three sample problems and twenty test items, each with 4 answer choices. The task required the children to select two images from the possible four answers that matched a target form. Two of the images were rotated versions of the three-dimensional target form, and two images were distracter forms that differed structurally from the target. The test assessment controlled for chance by allocating two points when the children selected both correct forms and zero points for all other answers. Possible scores range from zero to forty. (See Appendix E for Vandenberg's Mental Rotation Test.)

The children were presented the task as a game in which the goal was to "find shapes that match other shapes." The investigator discussed the idea of 3-dimensional rotation by showing the children a pencil box, and talking about how the pencil box looked different as the investigator rotated it. Following the demonstration, the children completed three sample problems. To ensure the children's understanding of the task, corrective feedback was given when the child selected the wrong form, and formative feedback was given for correct answers. After completing the sample items, the investigator presented each test item to the child. Children were allowed to work at their own pace until completion. The second-grade children were presented the task in pencil and paper form; whereas, the fourth-grade children were presented the task on a computer screen utilizing the CAAS software.

Reports of internal consistency of the measure on 439 elementary school-aged children ($\underline{M}$ = 9.91 years) yielded $\alpha = .62$ (Quaiser-Pohl, Lehmann, & Eid, 2004).

Internal consistency for the second-grade and fourth-grade children tested in this study was .56 and .78, respectively. The low reliability of the measure for the second-grade children is indicative of the task difficulty for young children.

Mathematics Competency

Children's scores on the mathematics portion of the CRCT assessed mathematics competency. The second grade test is comprised of five subtests, Number Sense and Numeration, Geometry and Measurement, Patterns and Relationships/Algebra, Computation and Estimation, and Problem Solving; and an additional subtest, Statistics and Probability was administered in the fourth grade. The CRCT is administered annually to all grade levels in Georgia public schools. Scores for the total test were used to assess the impact of predictor variables on a broad range of mathematics skills and knowledge.

The mathematics test is comprised of two sections, each with 35 problems. The CRCT is designed to measure student acquisition and understanding of the knowledge, concepts, and skills as set forth in the revised Georgia Quality Core Curriculum and Georgia's Performance Standards (Georgia Department of Education, 2007). Total test time is 120 minutes with additional time for a break between the sections. The test format is multiple-choice in which children select from one of three possible answers in the second grade or four possible answers in the fourth grade. Performance scores range from 150-450 and are ranked in three levels: Exceeds the Standards includes scores at or above 350; Meets the Standards are scores between 300-349; and Does Not Meet the Standards are scores below 300.

As part of the assessment of Number Sense and Numeration, children might be asked another way to write seven hundred and eighty-four, to determine whether numbers are smaller or larger than each other, or to determine place-value knowledge of how many 100s, 10s, and 1s are represented in a number. Fourth graders might also be asked to select a number that is a multiple of 7. For the Geometry and Measurement subtest children might be shown a shape and asked to name it or to match the shape as seen from another angle. Fourth graders are also expected to use appropriate units and measurements in measuring tasks. For the problems on the Patterns and Relationships/Algebra subtests children might be given a series of prices paid for increasingly larger groups of pencils, and children would be asked to give the price and number of pencils that would be next in the pattern. Fourth graders might be asked to complete charts that represent a trend, such as the number of books that would be checked out in December. The Computation and Estimation problems require children to add or subtract numbers and/or fractions. Fourth grade children might be asked to coordinate two pieces of information such as number of tickets sold in one year and estimate the number of tickets that would be sold in two years. In the fourth grade, the problems on the Statistics and Probability subtest ask children to collect, organize, and interpret data from charts and graphs. Children are asked to determine probability of a given event in terms of equally likely, most likely, or least likely. For example, the problem might read, "A box contains 5 blue crayons, 11 purple crayons, 6 green crayons, and 4 red crayons. If a crayon is chosen from the box at random, which color is LEAST likely to be chosen?" For the Problem Solving subtest, second graders are given word problems such as "John as 25 T-shirts in his drawer. His mother took out 12 that did not fit him anymore. How many T-shirts does he have now?" An example of a problem solving task on the fourth grade subtest would be, " Paul bought a toy for $0.45. He gave the clerk $1.00. The clerk gave him two quarters and one other coin. What was the other coin? A quarter, a dime, a penny, or a nickel."

Table 1

Types and Definitions of Addition and Subtraction Strategies

Strategy	Definition
Counting-all-manipulative	Addition or subtraction. Represents each number with counters and counts each counter to find sum or difference.
Counting-on-manipulative	Addition, mentally represents one addend and uses manipulatives to count on the second addend.
Counting-up-manipulative	Subtraction, mentally represents the subtrahend and counts up to the minuend using manipulatives.
Counting-back- manipulative	Subtraction, mentally represents the minuend and counts back the subtrahend value using manipulatives.
Counting-on-head	Addition, mentally represents one addend and mentally counts on the second addend.
Counting-up-head	Subtraction, mentally represents both numbers with the child mentally counting up from the subtrahend to the minuend.
Decomposition	Addition or subtraction. Child breaks numbers into smaller numbers and recomposes the values, frequently by using tens or sets of tens.
Retrieval	Fast solutions for which the child reports just knowing the answer or the answer popping into their

	heads.
Standard-algorithm-manipulative	Uses manipulatives to solve starting in right column and solves each column sequentially.
Standard-algorithm-head	Counts in head, decomposes, or retrieves to solve starting in right column and solves each column sequentially.
Standard-algorithm-mixed	Uses combination of manipulative and cognitive strategies across columns starting from the right column and solves each column sequentially.
Modified-algorithm-manipulative	Uses manipulatives to solve starting in the left column and solves each column sequentially
Modified-algorithm-head	Uses combination of manipulative and cognitive strategies across columns starting from the left column and solves each column sequentially.
Modified-algorithm-mixed	Uses combination of manipulative and cognitive strategies across columns starting from the left column and solves each column sequentially.
Rule-10	Teacher taught strategy for problems with numbers that are sets of tens (e.g., 33 + 20), for which the child mentally increments the 30 by 2 tens.

CHAPTER 3

RESULTS

Statistical analyses were carried out in multiple steps, each addressing one of the research questions of the present research. All analyses were conducted using the software package SPSS 14.0.

Q1: *Are there distinct groups of children identifiable by the patterns of mathematical strategies in the fourth grade? Is one of these groups characterized by high levels of manipulative strategy use?*

Cluster analysis was performed to separate children into groups based on their fourth grade profiles of correct strategy-use on 22 addition and subtraction problems comprised of 10 computation problems and 12 word problems (Aldenderfer & Blashfield, 1984; Hair & Black, 2000). Each math problem that was correctly executed was categorized as to the type of strategy the child used to solve the problem. No differences in strategy-use or total correct were found between the computation problems and the word problems. Across the 22 problems, each strategy-use category was tallied, and the sums of each strategy-use category were the individual's raw scores. The input strategy-use variables entered in the cluster analysis were the manipulative strategies of counting-all-manipulative, counting-on-manipulative, counting-up-manipulative, counting-back-manipulative, and standard-algorithm-manipulative; the cognitive strategies of counting-on-head, counting-up-head, decomposition, retrieval, modified-algorithm head, and standard-algorithm-head; and the mixed strategy-use of modified-algorithm-mixed, and standard-algorithm-mixed.

Hierarchical clustering and the agglomerative method were selected for the clustering procedure and are often used by social science researchers (Canobi, 2004; Canobi et al., 1998). The analysis partitions children's strategy selections by calculating a distance measure between each child's strategy-use pattern and every other child's strategy-use pattern, and it groups the two cases that have the least distance, or the greatest similarity, into a cluster of two. The measure of similarity in the program is squared Euclidean distance, the sum of squared differences between matching variables for each case. The analysis continues as it re-computes the distance measures all over again, and combines either the next two cases that are the closest or combines the next case with the cluster of two already formed. The process is iterative until all children's strategy patterns are grouped.

Once the clustering algorithm divided children into groups, ANOVAs were used to determine the strategy-use measures on which significant differences among the groups were present. Next, the sources of differences among the groups on each strategy-use measure were probed further through the use of Bonferroni post-hoc comparisons. Lastly, in order to provide convergent validation for the identified cluster groups, ANOVA tests and follow-up Bonferroni post-hoc comparisons were performed to compare the groups on six measures of correlates of math achievement that were not used to generate the cluster solution (Aldenderfer & Blashfield, 1984). The six additional measures were CRCT, total correct, fluency, accuracy, confidence, and spatial ability.

Table 2 presents the means and standard deviations of the input strategy-use variables utilized by the fourth grade children. The data was inspected for irregularities prior to clustering, and potential outliers were identified as unusually high or low scores on any particular strategy. Three children were eliminated because of unusually high scores. For example, one child used decomposition correctly to solve 14 problems. Table 3 presents the correlations of the input variables used in the determination of cluster membership.

Initially the results of the cluster algorithm's three- and four-group solutions were compared. In the three-cluster and four-cluster groupings, groups 1 and 2 did not differ in the solutions as to group size or central tendencies. The difference between the clustering solutions was found in the further delineation of group 3 of the three-cluster solution. Group 3 of the three-cluster solution separated into groups 3 and 4 in the four-cluster solution. The original hypothesis predicted that clusters would form according to types of strategy use: cognitive strategy-users, mixed or transitional strategy-users, and manipulative strategy-users. The significant central tendencies of group 3 of the three-cluster solution and groups 3 and 4 of the four-cluster solution indicated that children in any of these groups would be classified as manipulative strategy-users. The further differentiation of the 4-cluster solution did not provide helpful information regarding the discrimination of the hypothesized groups. In the interest of parsimonious solutions, the three-cluster solution was selected.

Examination of the ANOVA tests conducted to determine significant differences among the groups according to strategy use indicated that each group's performance was readily interpretable. Significant differences among the clustering groups were found in the manipulative strategies of standard-algorithm-manipulative, $F(2, 200) = 59.65$, $p < .001$; count-on-manipulative, $F(2, 200) = 3.60$, $p = .03$; count-all-manipulative, $F = 3.75$, $p = .03$. Significant differences among the groups were also found in the cognitive strategies standard-algorithm-head, $F(2, 200) = 445.21$; $p < .001$; and decomposition, $F(2, 200) = 3.34$, $p = .04$; and the mixed strategy standard-algorithm-mix, $F(2, 200) = 68.50$, $p < .001$. Table 4 presents the means and standard deviations of the cluster centroids (i.e., between group differences) for each clustering group and the results of the post-hoc comparisons. The three groups were labeled cognitive strategy-users, transitional strategy-users and manipulative strategy-users.

The cognitive group (n = 70) was characterized by the high usage of the cognitive strategies standard-algorithm-head and decomposition, the low usage of the manipulative strategies standard-algorithm-manipulative and count-on-manipulative, and the low usage of standard-algorithm-mix. No child in the cognitive cluster group selected the manipulative strategy count-all-manipulative. The manipulative group (n = 52) was characterized by the low selection of the cognitive strategies standard-algorithm-head and decomposition; the high usage of the manipulative strategies standard-algorithm-manipulative, count-on-manipulative, and count-all-manipulative; and the high usage of standard-algorithm-mix. Lastly, the transitional strategy use group (n = 81) was characterized by mean values lower than the cognitive group and higher than the manipulative group. The means of the strategies of the transitional group were between the cognitive group and the manipulative group.

The Bonferroni post-hoc comparisons indicated that the cognitive group was significantly different than the manipulative group in standard-algorithm-manipulative, standard-algorithm-head, and standard-algorithm-mix. The cognitive group was significantly different than the transition group in the usage of standard-algorithm-head and standard-algorithm-mix. The transition group was significantly different than the manipulative group in the strategies standard-algorithm-manipulative, standard-algorithm-head, and standard-algorithm-mix.

Convergent validation for the cluster groups was conducted by performing ANOVAS and Bonferroni post-hoc comparisons on the measurements of CRCT, total correct, fluency, accuracy, confidence, and spatial ability. Significant differences among the groups were found in all six variables. Significant differences were found in CRCT, $F(2, 187) = 13.56$, $p < .001$; total correct, $F(2, 200) = 32.04$, $p < .001$; fluency, $F(2, 198) = 19.27$, $p < .001$, accuracy, $F(2, 198) = 4.38$, $p = .01$; confidence, $F(2, 200) = 3.12$, $p = .046$; spatial ability $F(2, 199) = 9.60$, $p < .001$. Table 5 reports the between group differences of the variables and the results of the post-hoc comparisons.

To summarize, the fourth grade children clustered into 3 groups according to the types of strategies they correctly implemented while solving calculation and word problems. The groups were identified as cognitive strategy-users, transition strategy-users, and manipulative strategy-users. The groups differed significantly in strategy-use and on all measures of the correlates of math achievement.

Q2A: *Given that there are multiple group clusters, what are the differences in the second grade assessments of strategy-use that describes the differences among the fourth grade multi-group clusters? What are the differences in the second grade assessments of competency (i.e., CRCT), total correct, fluency, accuracy, confidence, and spatial ability that describe the differences among the fourth-grade multi-group clusters?*

Descriptive discriminant analysis (DDA) is the statistical procedure that describes group differences from response variable scores (Huberty, 2005). The procedure examines linear composites of outcome variables to identify constructs that underlie the group differences and the structural dimensions of the constructs. The linear composites are referred to as linear discriminant functions (LDFs), and the correlations between LDFs and individual outcome variables are designated as "structure r's". Structure r's recognize group difference by mean vectors. The variables that share the most variance with a given LDF define what attribute the LDF represents; however, the labeling of functions is primarily a substantive decision (Huberty & Olejnik, 2006).

Strategy-Use Variables

A descriptive discriminant analysis was conducted to describe the differences in second grade strategy-use among the fourth grade 3-group clusters. From the second grade strategy-use assessments, the variables entered were the manipulative strategies modified-algorithm-manipulative, standard-algorithm-manipulative, count-all-manipulative, count-on-manipulative, count-up-manipulative, count-back-manipulative; and the cognitive strategies modified-algorithm-head, standard-algorithm-head, count-on-head, decomposition, and rule-10. Table 6 presents the means and standard deviations of the strategy-use variables as a function of cluster-group membership. The overall Wilks' lamba was significant, $\Lambda = .70$, $\chi^2 (22, \underline{N} = 202) = 69.83$, $p < .001$, indicating that overall the strategy-use variables differentiated among the three groups of cognitive strategy-users, transition strategy-users, and manipulative strategy-users. Twenty-five percent of the variability of the scores for the first linear discriminant function (LDF) is accounted for by differences among the three cluster groups. The second discriminant function was not significant.

Table 7 presents the within-groups correlations between the strategy-use variables and the respective LDF scores. Based on these coefficients, it is apparent that the cognitive strategies modified-algorithm-head, standard-algorithm-head, decomposition, and rule-10 primarily define the first LDF; therefore, LDF_1 is labeled cognitive strategies. The cluster-group means are generally consistent with this interpretation. The means of the cognitive strategy-users on modified-algorithm-head (M = 2.16) were higher than the transition strategy-users (M = 1.96), which in turn were higher than the manipulative strategy-users (M = .65). Likewise, the means of the cognitive strategy-users on rule-10 (M = .59) were higher than the transition strategy-users (M = .34), which were higher than the manipulative strategy-users (M = .21). The means of the cognitive strategy-users on decomposition (M = .47) were higher than the transition strategy-users (M = .06) and the manipulative-users (M = .06). The means of the cognitive strategy-users on standard-algorithm-head (M = 2.90) were higher than the transition strategy-users (M = .91) and the manipulative users (M = 1.48).

Correlates of Math Achievement Variables

A descriptive discriminant analysis was conducted to describe the differences in second grade assessments of CRCT, total correct, fluency, accuracy, confidence, and spatial ability among the fourth grade 3-group clusters. Table 8 presents the means and standard deviations of the second grade correlates of math achievement variables as a function of cluster-group membership. The overall Wilks' lamba was significant, $\Lambda = .70$, χ^2 (12, N = 201) = 69.82, $p < .001$, indicating that overall the variables differentiated among the three groups of cognitive strategy-users, transition strategy-users, and manipulative strategy-users. In addition, the residual Wilks' lamba was significant, $\Lambda = .94$, χ^2 (5, N = 201) = 12.03, $p = .03$. This test indicated that the variables differentiated significantly among the three cluster groups after partialling out the effects of the first LDF. Because these tests were significant, both functions were interpreted. Twenty-six percent of the variability of the LDF_1 scores was accounted for by differences among the three cluster groups, and 6% of the variability of LDF_2 scores was accounted for by the differences among the three cluster groups.

Table 9 presents the within-groups correlations between the correlates of math achievement predictors and the respective LDF scores. From these results, the first construct is defined primarily by fluency, CRCT scores, and total correct. As these variables are components of high performance, the first function is labeled performance achievement. The cluster-group means of LDF_1 are consistent with this interpretation. The means of the cognitive strategy-users on fluency are (M = 1.96) are lower than the transition strategy-users (M = 2.54) which were lower than the manipulative strategy-users (M = 3.16). A low fluency score indicates fewer seconds in the speed of processing. The means of cognitive strategy-users on the CRCT (M = 342.27) were higher than the transition strategy-users (M = 327.08), which in turn were higher than the manipulative strategy-users (M = 319.39). Likewise, the means of the cognitive strategy-users on total correct (M = 9.96) were higher than the transition strategy-users (M = 8.35), which were higher than the manipulative strategy-users (M = 7.25).

The second construct is defined by the relationships of confidence and accuracy. The cluster-group means of LDF_2 are somewhat inconsistent. The means of the transition strategy-users (M = 35.09) on confidence were higher than the cognitive strategy-users (M = 33.91), which are higher than the manipulative strategy-users (M = 32.88). The means of the cognitive users on accuracy (M = 8.11) were higher than the transition strategy-users (M = 7.29) and the manipulative strategy-users (M = 7.65).

To summarize, significant group differences in the second grade were found among the groups according to the cognitive strategies implemented while solving second grade calculation and word problems. Significant group differences in the second grade were also found among the groups according to the second grade performance achievement measurements of CRCT, total correct, and fluency.

Q2B: *Do children's second grade assessments of strategy use predict fourth grade multiple group cluster membership? Do children's second grade assessments of CRCT, total correct, fluency, accuracy, confidence, and spatial ability predict fourth grade 3-group cluster membership?*

Predictive discriminant analysis (PDA) is a procedure to determine the likelihood that a rule based on a given sample will be expected to classify groups in future samples (Huberty & Olejnik, 2006). The expectation of accuracy of classification is referred to as actual hit rates and the analysis is conducted by an external classification method. The PDA classification analysis was done by cross-validation, a method in which each case is classified by the functions derived from all cases other than the case selected to determine the accuracy of the linear functions.

Strategy-Use Variables

Results of the PDA indicate that overall, 48% of second graders will be correctly classified by strategy use. Examination of Table 10 indicates hit rates of 47.1% for children in the cognitive strategy-use cluster, 62.5% for the children in the transition strategy-use cluster and 25% for children in the manipulative strategy-use cluster. Fifty-three percent of the original sample was correctly classified.

Correlates of Math Achievement

Results of the PDA regarding the prediction of cluster membership by second grade correlates of math achievement indicate that overall, 54.7% of the children will be classified correctly. Examination of Table 11 indicates hit rates of 62.9% for children in the cognitive strategy-use cluster, 58.8% for the children in the transition strategy-use cluster, and 37.3% for the manipulative strategy-use cluster. Fifty-seven percent of the original sample was correctly classified. In summary, measures of second grade cognitive strategy-use and performance achievement predict fourth grade cluster membership in the cognitive strategy-use group and the transition strategy-use group.

Q3: *Is there a relationship between cluster membership and CRCT math achievement tests' scores in the fourth grade? Do children identified in the manipulative strategy-use cluster score significantly lower on all math sub tests than children in the cognitive strategy-use cluster?*

A one-way multivariate analysis of variance (MANOVA) was conducted to determine the relationship between the three groups of fourth grade strategy users (cognitive strategy-users, transition strategy-users, and manipulative strategy-users) and the six subtests of the CRCT mathematics achievement assessment (Number Sense and Numeration, Geometry and Measurement, Patterns and Relationships/Algebra, Statistics and Probability, Computation and Estimation, and Problem Solving). The statistical procedure tests equality of means among the three groups on the six subtests as well as equality of means among the three groups on linear combinations of the six subtests. Significant differences were found among the three groups on the dependent measures, Wilks' Λ = .83, F (12, 362) = 2.88, $p = .001$. The multivariate η^2 based on Wilks' Λ was .09, indicating a medium to large effect size of the multivariate variance of the CRCT subtests was associated with the grouping factor. Table 12 contains the means and standard deviations on the dependent variables for the three groups.

Analyses of variances (ANOVA) on each dependent variable were conducted as follow-up test to the MANOVA. Using the Bonferroni correction, each ANOVA was tested at the .008 level, and significant differences were found for four of the subtests. The ANOVA on the Number Sense and Numeration subtest was significant, F (2,186) = 6.54, p =.002, $\eta^2 = .07$, as was the ANOVA tests on the Statistics and Probability subtest, F (2,186) = 6.14, $p = .003$, $\eta^2 = .06$; the Computation and Estimation subtest, F (2,186) = 14.80, $p < .001$, $\eta^2 = .14$; and the Problem Solving subtest, F (2,186) = 8.62, $p < .001$, η^2 = .09.

Post hoc analyses to the univariate ANOVA for the subtests consisted of conducting pairwise comparisons to find which groups were differentiated by the mathematical subtest scores. Each pairwise comparison was tested at the .008 divided by 3 or .003 level. The manipulative group scored significantly lower than the cognitive group on the Number Sense and Numeration subtest, the Statistics and Probability subtest, the Computation and Estimation subtest, and the Problem Solving subtest. No differences were found among the groups on the Geometry and Measurement subtest and Patterns and Relationships/Algebra subtest. As a point of interest, the cognitive users were differentiated from the transition group on the Computation and Estimation subtest.

The cognitive group scored significantly higher (see Table 12). In summary, manipulative strategy-users scored significantly lower than the cognitive strategy-users in four of the six CRCT math achievement subtests.

Table 2

Descriptive Statistics for Strategies Correctly Used in the Fourth Grade (N = 203)

Strategy	M	SD
Count-All-Manipulative	.05	.28
Count-On-Manipulative	.10	.38
Count-Up-Manipulative	.02	.14
Count-Back-Manipulative	.01	.10
Standard-Algorithm-Manipulative	.66	1.29
Count-On-Head	.01	.07
Count-Up-Head	.02	.14
Decomposition	.17	.69
Retrieval	.06	.35
Modified-Algorithm-Head	.12	.50
Standard-Algorithm-Head	10.44	5.03
Modified-Algorithm-Mix	.04	.25
Standard-Algorithm-Mix	3.11	3.06

Table 3

Correlations of Strategy-Use Variables for Cluster Membership Fourth Grade (N = 203)

	1	2	3	4	5	6	7	8	9	10	11	12	13
1 Mod. Algorithm Head	---												
2 Mod. Algorithm Mix	.04	---											
3 Std. Algorithm Manipulative	-.07	.12	---										
4 Std. Algorithm Head	-.09	-.13	-.56**	---									
5 Std. Algorithm Mix	-.14*	.01	.28**	-.51**	---								
6 Count On Manipulative	.01	.11	.10	-.20**	.07	---							
7 Count On Head	-.02	-.01	-.04	.12	-.07	-.02	---						
8 Count All Manipulative	.06	.40**	.19**	-.18*	.00	.09	-.01	---					
9 Count Up Manipulative	-.04	.12	-.07	-.03	.12	-.04	-.01	-.03	---				
10 Count Up Head	-.04	-.03	-.07	.01	-.04	-.04	-.01	-.03	.24**	---			
11 Count Back Manipulative	-.03	.18**	-.01	-.05	.01	-.03	-.01	-.02	.35**	-.01	---		
12 Decomposition	.19**	-.02	-.11	.07	-.15**	.03	-.02	-.04	.12	.07	.05	---	
13 Retrieval	.30**	-.03	-.09	.02	-.14**	-.05	-.01	-.03	-.02	-.02	-.02	.39**	---

Note. * Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).

Table 4

Cluster Centroids: Between Group Differences for Strategy Use Fourth Grade (N = 203)

Strategy	Clusters			F statistic (2,200)
	Group 1 (n = 70) Cognitive *M (SD)*	Group 2 (n = 81) Transition *M (SD)*	Group 3 (n = 52) Manipulative *M (M/SD)*	
Std.-Algorithm-Manipulative	.04 (.20)[b]	.35 (.55)[c]	1.98 (1.90)[bc]	59.65***
Std.-Algorithm-Head	15.86 (2.57)[ab]	9.84 (1.82)[ac]	4.10 (2.07)[bc]	445.21***
Std.-Algorithm-Mix	.67 (1.18)[ab]	3.59 (2.45)[ac]	5.64 (3.26)[bc]	68.50***
Count-On-Manipulative	.03 (.17)	.10 (.41)	.21 (.50)	3.60*
Count-All-Manipulative	.00 (.00)	.04 (.19)	.14 (.49)	3.75*
Decomposition	.33 (1.05)	.12 (.46)	.02 (.14)	3.34*

Note. * Significant at the 0.05 level (2-tailed).
*** Significant at the 0.001 level (2-tailed).
[a] Post Hoc tests reveal mean differences between Group 1 and Group 2
[b] Post Hoc tests reveal mean differences between Group 1 and Group 3
[c] Post Hoc tests reveal mean differences between Group 2 and Group 3

Table 5

Convergent Evidence of Between Group Differences in Fourth Grade Cluster Membership

Measure	Clusters: Group 1 Cognitive *M (SD)*	Group 2 Transition *M (SD)*	Group 3 Manipulative *M (SD)*	F statistic
Total Correct N = 203	17.11 (2.48) [ab] n = 70	14.46 (3.31) [ac] n = 81	12.27 (4.29) [bc] n = 52	32.04*** df (2,200)
Fluency N = 201	1.85 (.77) [b] n = 70	2.08 (.68) [c] n = 80	2.80 (1.17) [bc] n = 51	19.27*** df (2,198)
Accuracy N = 201	9.71 (.59) [b] n = 70	9.63 (.64) [c] n = 80	9.31 (1.07) [bc] n = 51	4.38* df (2,198)
CRCT N = 190	330.18 (25.89) [ab] n = 61	316.58 (21.22) [a] n = 81	307.60 (21.88) [b] n = 48	13.56*** df (2,187)
Confidence N = 203	30.86 (3.85) n = 70	29.65 (3.77) n = 81	29.01 (4.70) n = 52	3.12* df (2,200)
Spatial Ability N = 202	15.09 (8.56) [ab] n = 70	10.37 (6.47) [a] n = 81	10.71 (5.65) [b] n = 51	9.60*** df (2,199)

Note. * Significant at the 0.05 level (2-tailed).
*** Significant at the 0.001 level (2-tailed).
[a] Post Hoc tests reveal mean differences between Group 1 and Group 2
[b] Post Hoc tests reveal mean differences between Group 1 and Group 3
[c] Post Hoc tests reveal mean differences between Group 2 and Group 3

Table 6

Means and Standard Deviations of Second Grade Strategy-Use Variables as a Function of Fourth Grade Cluster Membership (N = 202)

Variables	Clusters		
	Group 1 Cognitive (n = 70) *M (SD)*	Group 2 Transition (n = 80) *M (SD)*	Group 3 Manipulative (n = 52) *M (SD)*
Modified-Algorithm-Manipulative	.16 (.44)	.51 (1.11)	.40 (.80)
Modified-Algorithm-Head	2.16 (3.09)	1.96 (2.67)	.65 (1.44)
Standard-Algorithm-Manipulative	.21 (.70)	.26 (.57)	.46 (1.15)
Standard-Algorithm-Head	2.90 (4.36)	.91 (2.03)	1.48 (2.77)
Count-On-Manipulative	1.40 (1.80)	1.83 (2.00)	1.70 (1.82)
Count-On-Head	.27 (.54)	.30 (.56)	.35 (.79)
Count-All-Manipulative	.29 (.70)	.41 (.91)	.60 (1.23)
Count-Up-Manipulative	.26 (1.00)	.09 (.48)	.06 (.24)
Count-Back-Manipulative	1.16 (1.60)	1.65 (1.71)	1.19 (1.52)
Decomposition	.47 (.96)	.06 (.29)	.06 (.31)
Rule-10(Taught Strategy)	.59 (.91)	.34 (.71)	.21 (.54)

Table 7

Structure r's for Strategy-Use Variables

Variable	LDF_1	LDF_2
Modified-Algorithm-Head	**.305**	.574
Standard-Algorithm-Head	**.430**	-.431
Decomposition	**.555**	-.226
Rule 10	**.355**	.082
Count-On-Manipulative	-.153	.163
Count-On-Head	-.074	-.072
Count-All-Manipulative	-.205	-.184
Count-Up-Manipulative	.229	-.037
Count-Back-Manipulative	-.116	.441
Mod.-Alg.-Manipulative[1]	-.285	.299
Std.-Alg.-Manipulative[2]	-.170	-.275

Note. [1] Modified Algorithm Manipulative
[2] Standard Algorithm Manipulative

Table 8

Means and Standard Deviations of Second Grade Correlates of Math Achievement Variables as a Function of Fourth Grade Cluster Membership (N = 201)

	Clusters		
Variables	Group 1 Cognitive (n = 70) *M (SD)*	Group 2 Transition (n = 80) *M (SD)*	Group 3 Manipulative (n = 51) *M (SD)*
CRCT	342.27 (25.80)	327.08 (22.70)	319.39 (22.58)
Total Correct	9.96 (3.98)	8.35 (3.49)	7.25 (3.53)
Fluency	1.96 (.75)	2.54 (1.18)	3.16 (1.29)
Accuracy	8.11 (1.43)	7.29 (1.87)	7.65 (1.80)
Confidence	33.91 (5.68)	35.09 (5.12)	32.88 (7.18)
Spatial Ability	8.11 (5.21)	5.79 (4.20)	6.20 (4.40)

Table 9

Structure r's for Correlates of Math Achievement Variables

Variable	LDF_1	LDF_2
CRCT	**.662**	-.054
Total Correct	**.492**	-.136
Fluency	**-.711**	.434
Accuracy	.272	**.543**
Confidence	.033	**-.593**
Spatial Ability	.349	.397

Table 10

External Linear Classification Analysis for Fourth Grade Cluster Membership by Second Grade Strategy-Use

	Predicted Group Membership					
	Cognitive		Transition		Manipulative	
Group Membership	n	%	n	%	n	%
Cognitive	33	**47.1**	28	40.0	9	12.9
Transition	13	16.3	50	**62.5**	17	17.5
Manipulative	8	15.4	31	59.6	13	**25.0**

Note. Overall percent of correctly classified cases = 52.5%
Cross-validated grouped cases correctly classified = 47.5%

Table 11

External Linear Classification Analysis for Fourth Grade Cluster Membership by Second Grade Correlates of Math Achievement Variables

	Predicted Group Membership					
	Cognitive		Transition		Manipulative	
Group Membership	n	%	n	%	n	%
Cognitive	44	**62.9**	23	32.9	3	4.3
Transition	21	26.3	47	**58.8**	12	15.0
Manipulative	9	17.6	23	45.1	19	**37.3**

Note. Overall percent of correctly classified cases = 56.7%
Cross-validated grouped cases correctly classified = 54.7%

Table 12

Means and Standard Deviations on CRCT Subtest Variables of Cluster Membership Fourth Grade

Subtest	Group 1 Cognitive (n = 61) *M*	*(SD)*	Group 2 Transition (n = 81) *M*	*(SD)*	Group 3 Manipulative (n = 47) *M*	*(SD)*
Number Sense and Numeration	331.40[b]	(32.19)	320.25	(34.11)	308.51[b]	(30.55)
Geometry and Measurement	322.97	(29.02)	316.22	(25.58)	310.87	(24.56)
Patterns and Relationships/Alg[1]	327.46	(31.15)	320.88	(29.68)	310.02	(28.01)
Statistics and Probability	330.00[b]	(20.61)	321.23	(27.40)	313.28[b]	(24.86)
Computation and Estimation	333.85[ab]	(28.09)	317.90[a]	(25.03)	305.68[b]	(28.66)
Problem Solving	337.28[b]	(34.41)	318.84	(33.06)	311.79[b]	(34.46)

Note. [1]Patterns and Relationships/Algebra

[a] Post Hoc tests reveal mean differences between Group 1 and Group 2

[b] Post Hoc tests reveal mean differences between Group 1 and Group 3

[c] Post Hoc tests reveal mean differences between Group 2 and Group 3

CHAPTER 4

DISCUSSION

The current research was designed to explore the math achievement profiles in a 3-year longitudinal study of 206 children. The results of the current study show that not all fourth grade children move from manipulative based strategies to cognitive based strategies when solving addition and subtraction problems. Approximately twenty-five percent of the children in the sample continued to use primarily manipulative based strategies in solving two- and three-digit arithmetic problems in the fourth grade. This profile of strategy use was accompanied by low fluency, poor spatial skills and poor achievement. For some children in the fourth grade, success in math computations remains illusive.

Three research questions were addressed that concern children’s arithmetic strategy use while solving addition and subtraction problems. First, I examined whether fourth grade children would separate into identifiable groups according to the types of strategies they correctly used to solve computation and word problems. The second research question examined the children's second grade assessments for descriptive differences among the fourth grade 3-groups, and the ability to predict fourth grade 3-group membership from the second grade assessments. The second grade assessments were comprised of measurements of strategy-use and the correlates of math achievement of mathematics competency, total correct, fluency, accuracy, spatial ability, and confidence. The third question dealt with the relationship between the identified groups of fourth grade strategy-users and their achievement in various domains of mathematics, as defined and measured in the fourth grade.

Are there distinct groups of children identifiable by the patterns of mathematical strategies in the fourth grade?

Yes, the results clearly indicate that fourth grade children can be clustered into three identifiable groups according to the strategies they successfully use when solving addition and subtraction problems. The three groups were labeled as cognitive strategy-users, transition strategy-users, and manipulative strategy-users. In support of the finding, convergent evidence of between cluster differences was found in measurements of competency (i.e., CRCT), total correct, fluency, accuracy, and spatial ability. Thirty-four percent (n = 70) of the fourth graders were classified as cognitive strategy-users and were characterized by frequent use of the cognitive strategies including standard-algorithm-head and decomposition, the low usage of the manipulative strategies including standard-algorithm-manipulative and count-on-manipulative, and the low usage of standard-algorithm-mix. Cognitive users were also differentiated by high CRCT scores, high ratings of spatial abilities, and correctly solving the most multi-digit math problems.

Forty percent of the children (n = 81) were classified as transition strategy-users, and the group was characterized by mean values located between the cognitive group and the manipulative group on standard-algorithm-head and standard-algorithm-mix strategy frequencies, and total correct. The group also used standard-algorithm-manipulative strategy significantly less often than the manipulative group. The transition group was significantly lower than the cognitive group on CRCT scores and spatial abilities. In measurements of fluency and accuracy, the transition group did not differ from the cognitive group, as both groups were more fluent and more accurate than the manipulative group.

Twenty-six percent of the children (n = 52) were classified as manipulative strategy-users, and were characterized by the high use of the manipulative strategies standard-algorithm-manipulative, count-on-manipulative, and count-all-manipulative, the high usage of standard-algorithm-mix, and the low selection of the cognitive strategies standard-algorithm-head and decomposition. The manipulative strategy-use group scored the lowest on the CRCT, were the least fluent and least accurate, scored the lowest ratings of spatial abilities, and correctly solved the fewest multi-digit math problems.

Children in the cognitive strategy-use group and the transition strategy-use group were successful in mathematical performance assessments (i.e., CRCT and total correct); whereas, children in the manipulative group were not. The manipulative strategy-use children were the low achievers in the fourth grade, and they comprised 26% of the sample. On average, these children correctly answered 55% of the double-digit computation and word problems, and the mean CRCT mathematics composite score (i.e., 308) for the group average fell just at acceptable levels for state standards (i.e., 300).

The manipulative group in this study is similar to children classified as having mathematical learning difficulties and disabilities (Mazzocco, 2007). Research by Geary and colleagues (Geary, 1990, 1993; Geary, Hamson, & Hoard, 2000) found that the patterns of strategy selection, response time, and accuracy in calculations are distinctive indicators of developmental changes in children's mathematical knowledge and discriminate children who have difficulties in mathematics. According to the research (Geary, 1990, 2004; Ostad, 1997), children with math learning difficulties (MLD) select more immature manipulative counting strategies during calculations, are more inaccurate when selecting retrieval strategies, and are slower in the recall of basic number combinations than typically achieving children.

Whereas some of these learning disabilities are likely biologically-based, others may occur as a result of a poor environment (Gersten et al., 2005). In a series of studies examining the developmental progression of strategy development and computation skills in children, Geary and colleagues (Geary, 1990; Geary et al., 1991) delineated group differences between children who were mathematically developmentally-delayed from children with significant mathematical learning disabilities. Developmentally-delayed children benefited from remedial education, in that these children exhibited mature changes in the selection of strategies moving from manipulative counting strategies to cognitive strategies. According to the research, roughly 6-8% of children have serious cognitive deficits in mathematics (Geary, 2005), while many of the low-achieving children exhibit development delays. The research suggests that teaching practices (e.g.,Fuson & Briars, 1990; Fuson & Secada, 1986; Griffin, 2004; Griffin et al., 1994) can improve learning for many of the low performing children in the manipulative strategy-use group in this study.

The qualitative and quantitative similarities between the manipulative strategy-use group identified in the present study and children who have been classified as MLD children by research is not meant to imply the generalization of classification between groups. Although the current study was based upon the same strategy-selection developmental model of mathematical knowledge as Geary, the children were not sorted into low-achievers according to percentile rankings (i.e.,25^{th}). Rather, the children were grouped according to their correct strategy use during computations. However, it is of interest to note that strategy use by the manipulative group in the fourth grade was typical of the strategy use of Geary's (1990) children with MLD.

Two points should be made. First, the current study provides evidence that a significant group of children do not develop proficiency in the addition and subtraction skills that are necessary for success in computations by the fourth grade. Given the importance of math in a technological society (Rivera-Batiz, 1992) and the integral relationship of addition and subtraction skills to other domains of mathematics (Baroody & Tiilikainen, 2003; Kilpatrick et al., 2001), the failure of 25% of children to develop proficient addition and subtraction skills is unacceptable. The persistent and inefficient reliance on manipulative strategies by children at this time period in their mathematical career is troublesome. Second, it is unclear how changes in education practices will impact the development of math knowledge for the children in the manipulative strategy group. Let us examine the results of Question 2 for further discussion on the educational implications of the study.

What are the differences in the second grade assessments of strategy-use that describes the differences among the fourth grade 3-group clusters?

In the second grade strategy-use analysis, LDF_1indicated that a certain subset of cognitive strategies (i.e., modified-algorithm-head, standard-algorithm-head, decomposition, and rule-10) discriminated between children who were later classified as being in either the cognitive or transition group. Second graders who correctly use the subset of cognitive strategies in computations of two- and three-digit problems most likely have developed, or are in the process of developing, a good conceptual knowledge of place value. Other cognitive strategies, such as counting-on-head did not discriminate groups, likely because these counting strategies do not require such conceptual knowledge. The implication that children possess knowledge of place value, as evidenced by children's correct cognitive strategy-use, is in line with other research.

The longitudinal research in children's developing mathematical thinking by Fennema, Carpenter, Jacobs, Franke, and Levi (1998), for example, suggests that children's correct usage of modified algorithms during early skill acquisition in the computation of multi-digit arithmetic problems indicates conceptual understanding of number and place value. In modified algorithm use, children are combining units specifically because the units are common to each number of the addition or subtraction problem. As children develop proficiency in the computations of multi-digit calculations, the modified algorithms are replaced by the more efficient use of standard algorithms. The current research found a similar pattern of development in the cognitive cluster group and the transition cluster group.

Conceptual knowledge of place value is evident in the other discriminating cognitive strategies of second graders. The use of these strategies suggests good conceptual understanding of counting (Fuson, 1988; Steffe et al., 1988; Steffe, Thompson, & Richards, 1982) and number relationships (Baroody & Ginsburg, 1986; Baroody & Tiilikainen, 2003). The rule-10 strategy is a teacher taught strategy to use with double-digit computations when the numbers are designated sets of tens, such as 20 or 30. Children are taught to increment the ten's place only. Decomposition requires the ability to break numbers into hundreds, tens, and ones; and, therefore, reflect conceptual knowledge about place value (Canobi et al., 1998; Canobi, Reeve, & Pattison, 2003; Fuson, 1990). Although the standard-algorithm-head strategy can be taught and used as a rote procedure, the data in this study suggests that when children correctly use it at the beginning of the second grade, they likely had some knowledge of place value. As a general rule, the standard algorithm is introduced in the second grade curriculum (Fuson, 2003). Likely, the children who had a poor understanding of place value at the beginning of the second grade continued to use manipulatives to solve standard algorithm problems.

What are the differences in the second grade assessments of CRCT, total correct, fluency, accuracy, confidence, and spatial ability that describe the differences among the fourth grade 3-group clusters?

In the second grade correlates of math achievement analysis, two LDFs were identified that discriminated among the groups. The first construct, labeled performance achievement, was comprised of fluency, CRCT, and total correct, and the construct supports the research that links fluency with achievement (Canobi, 2005; Geary et al., 1991; Royer et al., 1999). The finding that achievement predicts group membership in the cognitive and transition strategy-use groups is not surprising, given that the best predictor of future performance is past performance. Clearly children who start the second grade with a high mastery of the skills necessary to solve math problems correctly will continue to achieve success in numerical computations.

Identifying the construct of LDF_2 is difficult substantively and statistically, as the interpretation of the negative relationship of confidence to accuracy is troublesome. Statistically, confidence does not correlate with accuracy in the second grade, nor does confidence differentiate among the groups in the second grade. However, when combined with the accuracy in the linear composite, confidence does appear to contribute to the function. Further research using other measures of confidence in math abilities might provide a better understanding of the proposed relationship.

Do children's second grade assessments of strategy use predict fourth grade 3-group cluster membership? Do children's second grade assessments of CRCT, total correct, fluency, accuracy, confidence, and spatial ability predict fourth grade 3-group cluster membership?

Yes, the predictive discriminant analysis indicated that the cognitive strategies correctly implemented by second graders reliably predict fourth grade cluster membership in the cognitive strategy-use group and the transition strategy-use group. Cognitive strategy-use in the second grade did not predict the fourth grade children in the manipulative strategy-use group. Second grade performance achievement scores also reliably predict cluster membership in the cognitive strategy-use and the transition strategy-use groups, but does not predict membership in the manipulative strategy-use group.

Given the importance of high stakes performance testing in contemporary classrooms (United States Department of Education, 2007), it is imperative to identify the teaching practices that readily produce solid conceptual and procedural knowledge of number relationships and numerical computations for elementary school children. The current research indicates that promoting the development of cognitive strategies is an effective pathway to children's success on performance achievement assessments. The research supports the emergence of the mental construction of number as a symbolic representation. The study also draws attention to the pitfalls of over-using manipulatives as teaching aids in the classroom (Boulton-Lewis, 1993a, 1998; Boulton-Lewis & Tait, 1994). Clearly, manipulatives are useful to young children who are mastering counting sequences and beginning procedural skills (Steffe, 1994). However, the educational value of manipulatives diminishes when children's strategy use does not develop appropriately. The use of manipulatives as counting aids should be closely monitored in the classroom.

Is there a relationship between cluster membership and CRCT math achievement tests' scores in the fourth grade? Do children identified in the manipulative strategy-use cluster score significantly lower on all math sub tests than children in the cognitive strategy-use cluster?

The results of the current study indicate that the manipulative strategy-use group performs significantly lower on tests measuring Number Sense and Numeration, Computation and Estimation, Statistics and Probability, and Problem Solving than the cognitive strategy-use group. There were no differences between the strategy-use groups when scores on the subtests of Geometry and Measurement, and Patterns and Relationships/Algebra were used as dependent measures.

An underlying assumption prevalent in the mathematics education (Kilpatrick et al., 2001; NCTM, 2000) and the cognitive psychology (Geary, 1994; Siegler & Shrager, 1984) literatures is that proficiency in basic math operations is fundamental to the development of mathematical knowledge. Our results are inconsistent with this assumption about mathematics. Although subtest performance intercorrelated, the results indicate that the skills developed by the high performing cognitive group did not generalize across the six subtests. Specifically, the cognitive group did not perform significantly better than the manipulative group in Geometry and Measurement or in Patterns and Relations/Algebra.

Intuitively one can grasp the connected relationships among the subtests (i.e., mathematical domains) of number sense, computation, statistics, and problem solving. At the fourth grade level, these subtests' questions all required a numerical manipulation or operation to answer the problems correctly. For example, the subtest of Problem Solving required fourth graders to organize and interpret data in order to solve word problems using basic arithmetic operations. The findings indicate that cognitive strategy users extend their proficiency in numerical computations within these mathematical domains. It also suggests that manipulative strategy-users are constrained by their low numerical skills to achieve success in these domains.

The disconnect between cognitive strategy-users and achievement in the subtests representing early knowledge of the domains of geometry and algebra requires further investigation. It is possible that there is little or no connection between proficiency in numerical computations and early knowledge of geometry and algebra. Another possible explanation for the disconnect between groups' achievement scores and tests of geometry and algebra is the tenuous relationship between children's early domain specific knowledge and the instruments used to assess that knowledge. According to the CRCT guidelines (Georgia Department of Education, 2007), the subtest of Geometry and Measurement was designed for two purposes: to identify and compare the fundamental characteristics of shapes, and to select and use appropriate units and instruments in measurement. A close examination of the test questions indicates that the correct answers assess knowledge of specific facts. For example, a geometry question requires the child to select the geometric figure in which a dotted line is a line of symmetry. An example of a measurement test question asks, "What unit would be used to measure the length of a car? Centimeter, gram, meter, ton." The questions of the subtest appear to assess basic factual knowledge that is necessary to support geometric operations, but do not require procedural operations or skills.

The subtest of Patterns and Relationships/Algebra (Georgia Department of Education, 2007) was designed to measure children's ability to identify number relations and to recognize, describe, generalize, and predict patterns, rules of patterns, and sequences. For example, one test question required fourth graders to complete a table that indicated a consistent and progressive change over three time points of 15 incremental units. In another question, children were presented with a bar chart indicating progressive incremental changes of 50 units over four months. The question asked, "If the trend continues, about how many books will be checked out in January?" The ability to interpret a chart as well as the ability to recognize the characteristics of a pattern were necessary skills for achievement on the subtest.

The close examination of the test questions on the subtests of Geometry and Measurement, and Patterns and Relationships/Algebra suggests that the two subtests measure factual knowledge that is representative of beginning domain specific knowledge. The test questions do not demonstrate integrated connections to other domains of mathematical knowledge or mathematical operations. The groups in the study were created based on their selection of strategies to solve computation and word problems, not retrieve factual information about geometry or visual patterns.

Limitations of the Study

The research conducted in the current study assessed children who reside in the northwestern section of rural Georgia. Whereas the children were functioning at grade level for the standards of Georgia, generalizing the results to children in urban settings or in more affluent school districts might prove to be troublesome. No assessments of socio-economic status were collected in this sample, nor were measurements of children's general intellectual abilities. Likewise, no assessments concerning the type or quality of mathematical instruction were collected.

The data collected in the study was part of a larger research project examining gender differences in mathematical achievement. Other variables that are linked to the development of mathematical knowledge and strategy selection (e.g., working memory or reading ability) were not assessed in the larger project. Regarding the variable assessments, the reliability of the spatial ability measurement for the second graders is also problematic. The low reliability of the measure is indicative of the task difficulty for young children. Given that spatial skills were influential in the strategy selection of the children, and that spatial skills consistently indicate mathematical success, further research in the development of age-appropriate mental rotation assessments would be most beneficial.

Summary

Children who correctly use cognitive strategies to solve multi-digit computation and word problems are the children who achieve success in elementary school mathematics. Unfortunately, the results of the current study indicate that about 25% of fourth grade children persist in the use of inefficient manipulative strategies to solve addition and subtraction problems. Fourth grade manipulative strategy-users do not demonstrate proficiency in arithmetic calculations and minimal competency in other domains of mathematics. The study highlights the integral relationship of early cognitive strategy use in young learners with subsequent proficiency in the fourth grade. It suggests that second grade cognitive strategy-use develops in conjunction with increases in conceptual knowledge, specifically knowledge of place value. Results of the study indicate that fourth grade math competencies require different kinds of math knowledge. The classroom implications of the study indicate that children's strategy-use profiles are appropriate indicators of children's mathematical knowledge. Although the results highlight the importance of cognitive strategy-use and fluency early in children's math experiences, the study is correlational. Recommendations for classroom instruction will require further experimental research regarding the impact of specific instruction on the variables identified in the study.

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APPENDICES

APPENDIX A

Second Grade Math Problems

Computation problems

41 + 13 = 54	82 + 7 = 89	89 + 99 = 188
306 + 188 = 494	97 + 3 = 100	85 – 14 = 71
34 – 21 = 13	703 – 108 = 595	20 – 17 = 3
36 – 5 = 31		

Word problems

1. Farmer Jones has 38 chickens and 10 ducks. How many birds does he have altogether?

2. Miss Smith's second grade class has 18 students. Miss Sawyer's second grade class has 19 students. How many students are there in the second grade?

3. John ate 87 French fries from his plate and 14 from his brother's plate. How many French fries did John eat?

4. Chris has 16 kids on his soccer team. Cheryl has 20 kids on her soccer team. If they played a game together, how many kids would be playing soccer?

5. The computer lab at school has 23 computers. If 13 more computers are added, how many computers will be in the lab?

6. Brandon has 96 marbles. He plays with them outside and loses 15. How many does he have left?

7. Will's father buys a package of 25 carrot sticks at the store. Will eats 6. How many carrot sticks are left over for Will's sister?

8. Martha and Brian are making necklaces in art class. Brian has 45 beads and gives Martha 27. How many beads does Brian have left to make his necklace?

9. The school lunchroom buys 72 apples for lunch. They serve 63 of them at lunch. How many apples are left over for tomorrow's lunch?

10. The pet store has 55 parakeets in a cage. Twelve of them fly away. How many are left?

APPENDIX B

Fourth Grade Math Problems

Computation problems

89 + 99 = 188	59 + 29 = 88	111 + 79 = 190
450 + 625 = 1075	306 + 188 = 494	81 – 22 = 59
174 – 69 = 105	384 – 286 = 98	85 – 14 = 71
703 – 108 = 595		

Word problems

1. John ate 87 French fries from his plate and 14 from his brother's plate. How many French fries did John eat?

2. The computer lab at school has 23 computers. If 13 more computers are added, how many computers will be in the lab?

3. On Monday Ben made 43 sandwiches. Ray made 12 more sandwiches than Ben. Rachel made 24 more than Ray. How many sandwiches did Rachel make?

4. John scores 651 points on a video game. Lori scores 418 points more than John. Alex scores 163 more points than Lori. How many points does Alex score?

5. The distance between Georgetown and Lincoln Park is 37 miles. The distance between Lincoln Park and Athens is 79 miles. What is the distance between Georgetown and Athens?

6. Sandy had 59 stickers. Her father gave her 28 more stickers for her birthday. How many stickers did Sandy have then?

7. Jesse writes two pages on a computer. He writes 234 words on page 1 and 188 words on page 2. How many more words does he need for a 500-word story?

8. Martha and Brian are making necklaces in art class. Brian has 45 beads and gives Martha 27. How many beads does Brian have left to make his necklace?

9. The school lunchroom buys 72 apples for lunch. They serve 63 of them at lunch. How many apples are left over for tomorrow's lunch?

10. The Art Club raised $437 by selling cookies. The Music Club raised $673 by washing cars. How much more did the Music Club raise?

11. The school has 436 pencils to give away and they give away 194 pencils. How many pencils do they have left?

12. Raymond’s lunch break at school is 55 minutes long. He spent 17 minutes in the hot lunch line and 19 minutes eating lunch. How much time did he have left?

APPENDIX C

Fluency Combinations

Fluency problems

5 + 1 = 6	7 – 0 = 7
2 + 3 = 5	6 – 2 = 4
8 + 2 = 10	9 – 8 = 1
7 + 4 = 11	8 – 5 = 3
0 + 9 = 9	4 – 3 = 1

APPENDIX D

Mathematics Competence Scale

ID #____________

Grade_______

1. How good in math are you?

Not at all good | Very good
|__________|__________|__________|__________|__________|__________|

2. If you were to list all the students in your class from the worst to the best in math, where would you put yourself?

One of the worst | One of the best
|__________|__________|__________|__________|__________|__________|

3. Some kids are better in one subject than in another. For example, you might be better in math than in reading. Compared to most of your school subjects, how good are you at math?

A lot worse in math than in other subjects | A lot better in math than in other subjects
|__________|__________|__________|__________|__________|__________|

4. In general, how hard is math for you?

Not at all hard | Very hard
|__________|__________|__________|__________|__________|__________|

5. How well do you expect to do in math this year?

Not at all well | Very well
|__________|__________|__________|__________|__________|__________|

6. How good would you be at learning something new in math?

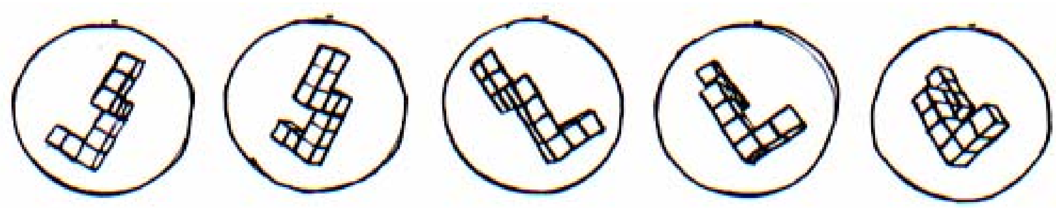
Not at all good | Very good
|__________|__________|__________|__________|__________|__________|

APPENDIX E

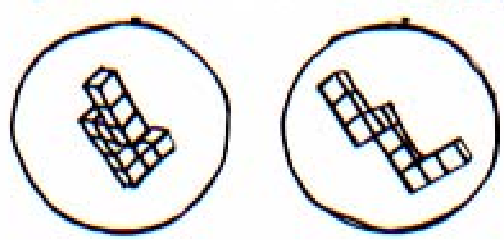
Vandenberg's Mental Rotation Test Sample Problems

ID ________________________
Grade______________________

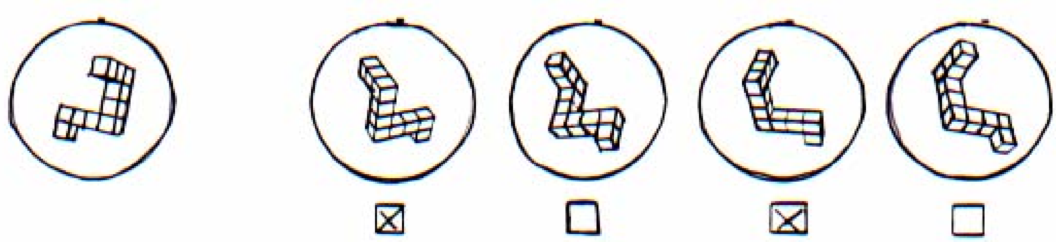
This is a test of your ability to look at a drawing of a given object and find the same object within a set of dissimilar objects. The only difference between the original object and the chosen object will be that they are presented at different angles. An illustration of this principle is given below, where the same single object is given in five different positions. Look at each of them to satisfy yourself that they are only presented at different angles from one another.



Below are two drawings of new objects. They cannot be made to match the above five drawings. Please note that you may not turn over the objects. Satisfy yourself that they are different from the above.



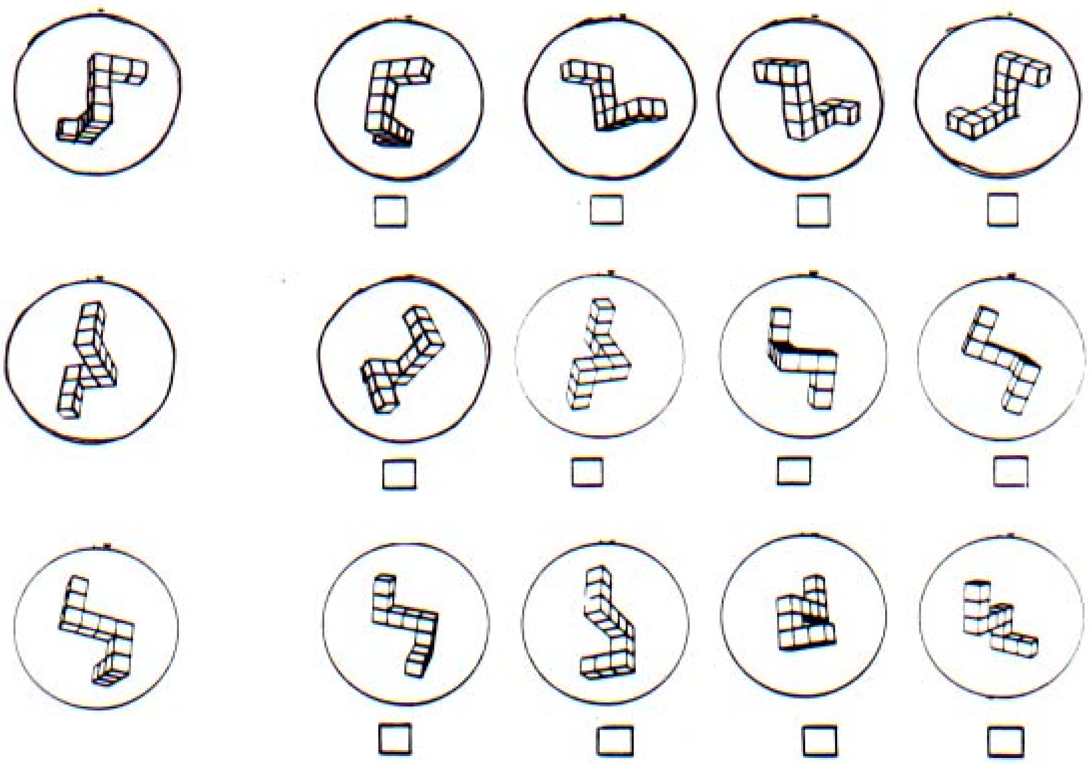
Now let's do some sample problems. For each problem there is a primary object on the far left. You are to determine which two of four objects to the right are the same object given on the far left. In each problem always <u>two</u> of the four drawings are the same object as the one on the left. You are to put Xs in the boxes below the correct ones, and leave the incorrect ones blank. The first sample problem is done for you.



Go to the next page.

Adapted by S.G. Vandenberg, University of Colorado, July 15, 1971. Revised instructions by H. Crawford, U. of Wyoming, September, 1979.

Do the rest of the sample problems yourself. Which two drawings of the four on the right show the same object as the one on the left? There are always two and only two correct answers for each problem. Put an X under the two correct drawings.



Answers: (1) first and second drawings are correct
(2) first and third drawings are correct
(3) second and third drawings are correct